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CALCULATION OF BWR LIMIT CYCLE AMPLITUDE USING GALERKIN'S METHOD

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This paper describes the application of Galerkin's method[1] to estimate the amplitude of boiling water reactor (BWR) limit cycle oscillations. It will be shown that Galerkin's method can be applied to a model of BWR dynamics consisting of the point kinetics equations and the LAPUR[2] generated feedback transfer function to calculate the time history of small amplitude limit cycles. This allows results from the linear frequency domain code LAPUR to be used to calculate nonlinear time domain information.

The BWR dynamics model consists of the point kinetics equations as the forward path, and the LAPUR-calculated feedback transfer function, H(s), which accounts for the channel thermal hydraulics, fuel heat transfer, and Doppler reactivity coefficient, as the feedback loop (Fig. 1). The reactivity may be written as the product of the linear feedback transfer function and the neutron density:

$$\rho(t) = \mathcal{L}^{-1}[H(s)n(s)] \qquad (1)$$

For small amplitude limit cycles, the main nonlinearity is the product of neutron density and reactivity in the point kinetics equations. Substituting the reactivity expression into the point kinetics equations results in a single, nonlinear differential equation which is solved using Galerkin's method.

In Galerkin's method, a solution comprised of products of trial functions and unknown constants is substituted into the differential equation to form a residual. Equations for the unknown constants are obtained by setting the product of the residual and the trial functions, integrated over one fundamental period, to zero.

Since the limit cycle is periodic, the assumed solution for the neutron density can be expanded in Fourier series.

$$n(t) = R + \sum_{k=1}^{j} \left[P_k Sin(k\omega t) + Q_k Cos(k\omega t) \right] \qquad (2)$$

Applying Galerkin's method results in 2j + 1 nonlinear algebraic equations for the Fourier coefficients. An additional equation for the fundamental frequency, ω, is obtained by setting the product of the assumed neutron density time derivative and the residual, integrated over one period, to zero. The resulting 2j + 2 nonlinear algebraic equations are solved using a numerical algorithm.

The LAPUR transfer function for a case with a decay ratio of 1.02 was used as input to the Galerkin solution. The results for different numbers of harmonics in the assumed solution are shown in Table 1. Note how the solution converges as the number of harmonics is increased. A time history of the 8 harmonic solution is shown in Fig. 2.

It can be concluded that Galerkin's method can be used in conjunction with the LAPUR feedback transfer function and the point kinetics equations to obtain limit cycle information in the time domain. It has also been shown that the Galerkin method solution converges as the number of terms in the solution increases. The technique described here is being incorporated into the LAPUR code, allowing the amplitude of limit cycle oscillations to be estimated.

REFERENCES

1. J. N. Reddy, *Energy and Variational Methods in Applied Mechanics*, John Wiley & Sons, New York, 1984.

2. P. J. Otaduy, "Modeling the Dynamic Behavior of Large Boiling Water Reactor Cores," Ph.D. dissertation, University of Florida, 1979.

Table 1

Results from the Galerkin Solution

Parameter	Number of Harmonics							
	1	2	3	4	5	6	7	8
ω	0.4289	0.4280	0.4271	0.4266	0.4265	0.4265	0.4265	0.4265
R	0.0045	0.0230	0.0340	0.0380	0.0400	0.0400	0.0400	0.0400
P_1	0.2400	0.5400	0.6500	0.7000	0.7100	0.7200	0.7200	0.7200
Q_1	******	0.0220	-.1270	-.0860	-.0830	-.0780	-.0800	-.0800
P_2	******	0.0100	-.0960	-.0740	-.0740	-.0720	-.0720	-.0720
Q_2	******	-.1500	-.2200	-.2600	-.2700	-.2800	-.2800	-.2800
P_3	******	******	-.0650	-.0980	-.1050	-.1080	-.1070	-.1070
Q_3	******	******	0.0480	0.0460	0.0490	0.0470	0.0480	0.0480
P_4	******	******	******	0.0220	0.0270	0.0270	0.0280	0.0280
Q_4	******	******	******	0.0320	0.0390	0.0410	0.0400	0.0400
P_5	******	******	******	******	0.0120	0.0140	0.0140	0.0140
Q_5	******	******	******	******	-.0120	-.0140	-.0140	-.0140
P_6	******	******	******	******	******	-.0060	-.0070	-.0070
Q_6	******	******	******	******	******	-.0044	-.0046	-.0046
P_7	******	******	******	******	******	******	-.0028	-.0013
Q_7	******	******	******	******	******	******	******	0.0033
P_8	******	******	******	******	******	******	******	0.0013
Q_8	******	******	******	******	******	******	******	0.0002
Max. Amp.	0.2400	0.6950	0.9820	1.1250	1.1800	1.1980	1.2040	1.2060

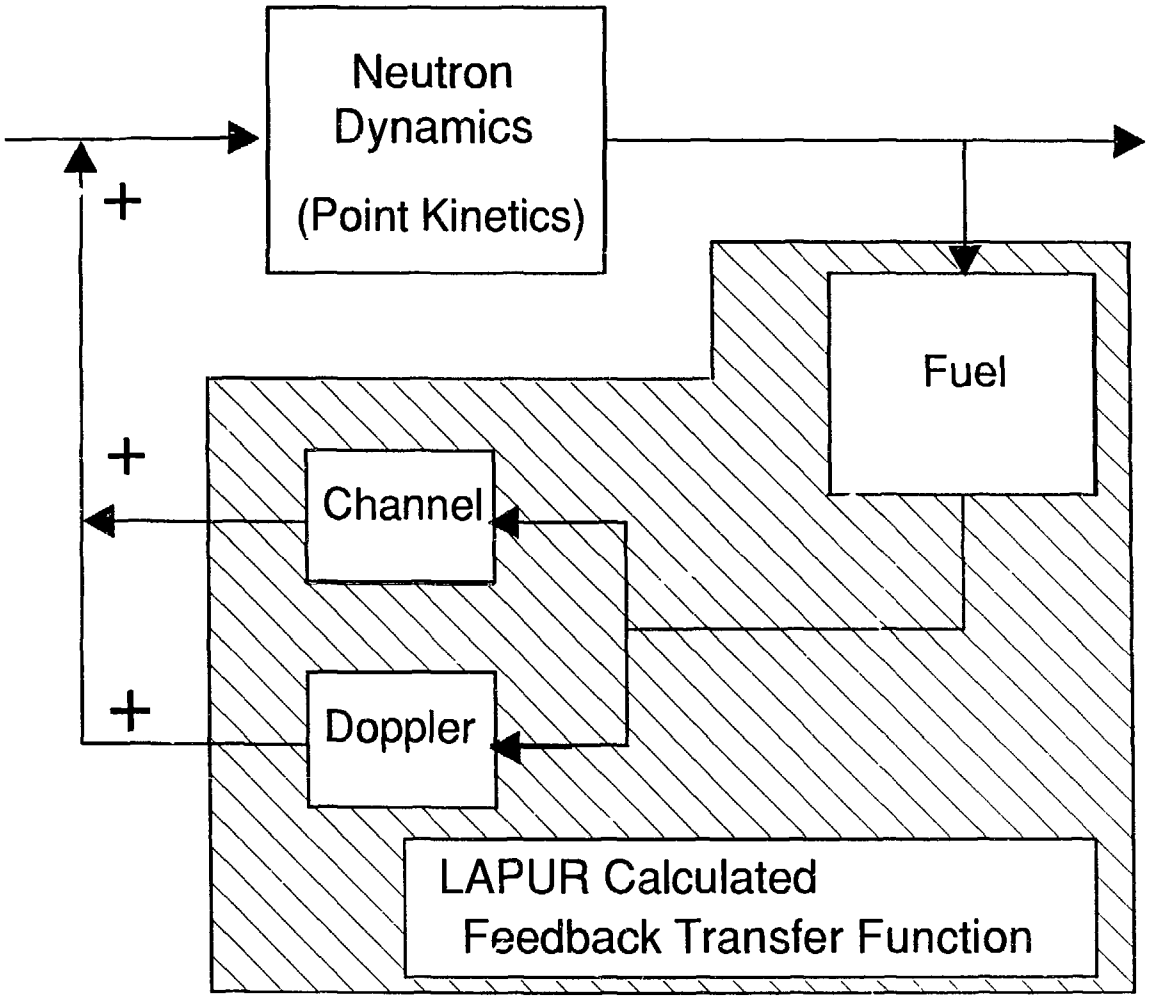

Figure 1. Block Diagram of BWR Dynamics

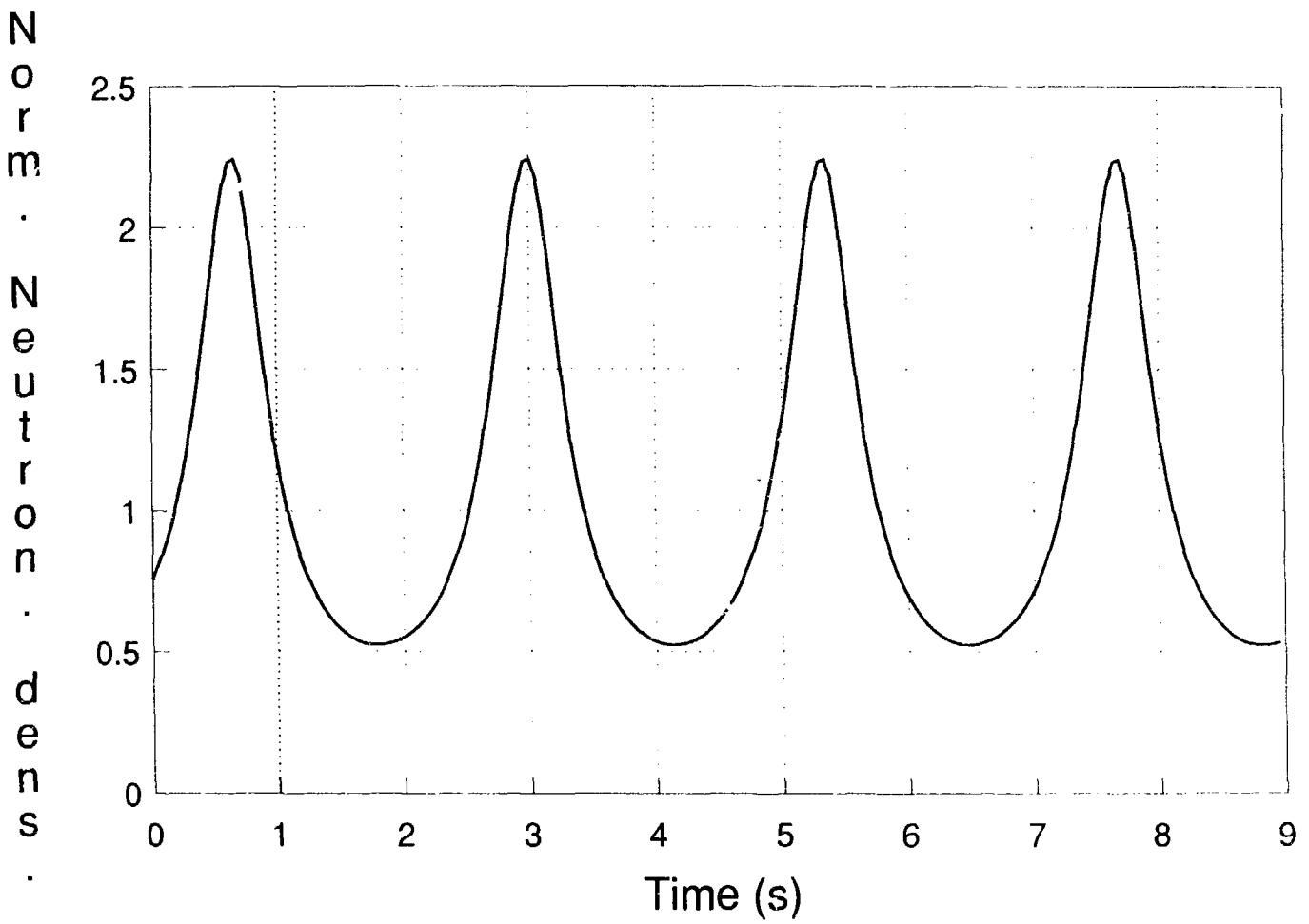

Figure 2
8 Harmonic Galerkin Solution

Calculation of BWR Limit Cycle Amplitude using Galerkin's Method

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Presentation Outline

- Background information on BWR stability.
- Description of the calculations.
- Presentation of the calculation results.

    1) Comparison with a numerical solution.

    2) Comparison with measured data.

Three types of instabilities have been observed in BWRs

- Control system instabilities - Due to out of tune controllers.

- Channel thermal hydraulic instabilities - Related to the momentum dynamics in two-phase flow.

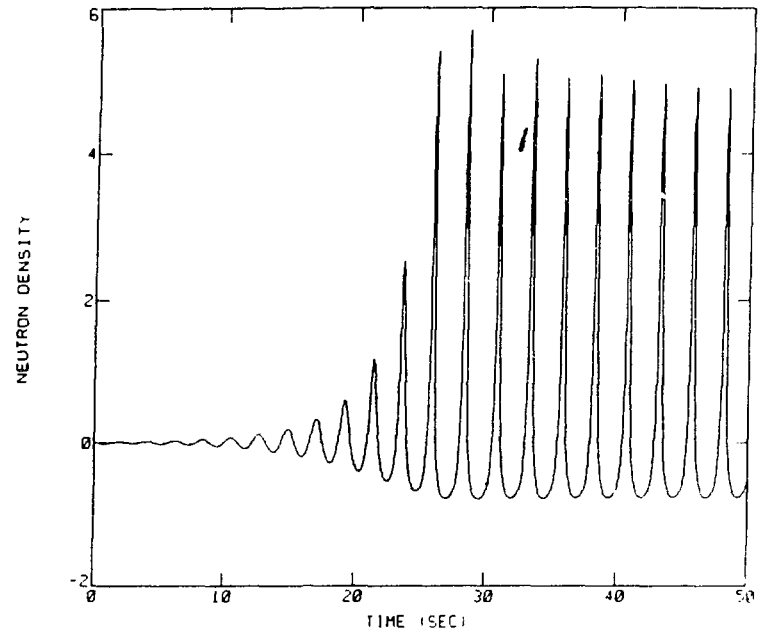
- Reactivity instabilities - Coupling of the reactor neutronics with the channel thermal hydraulics.

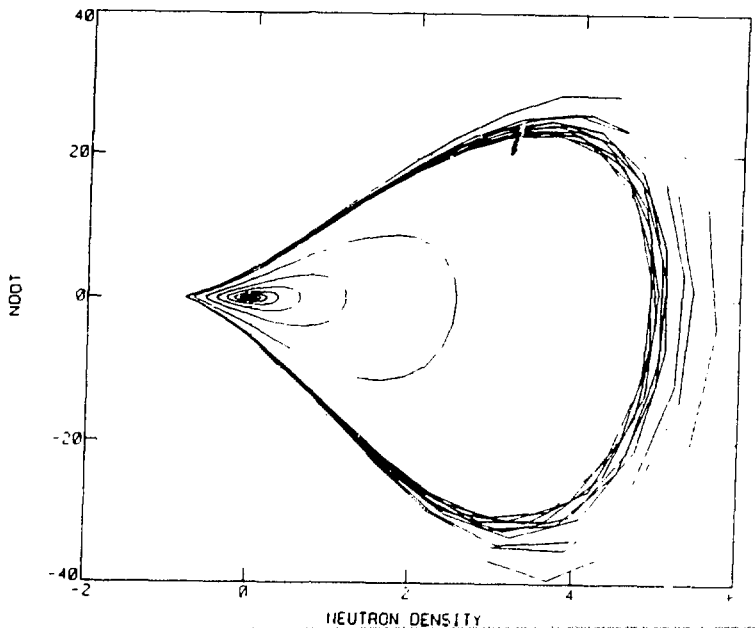
    1) Core wide instability.

    2) Regional instability.

Reactivity instabilities result in power oscillations.

- Oscillation amplitude grows initially with time.
- Because of the systems nonlinearity, the oscillations are attracted to a steady-state limit cycle.





Prediction of instability and its effects

- Linear frequency domain codes - LAPUR, FABLE - Predict the degree of stability and the stability boundary. Relatively small codes, can be run quickly and inexpensively.

- Nonlinear time domain codes - TRAC, RETRAN - Predict BWR behavior in both the stable and the unstable regions. Can perform steady-state or transient calculations. Large codes, detailed reactor models, computationally intensive, relatively expensive.

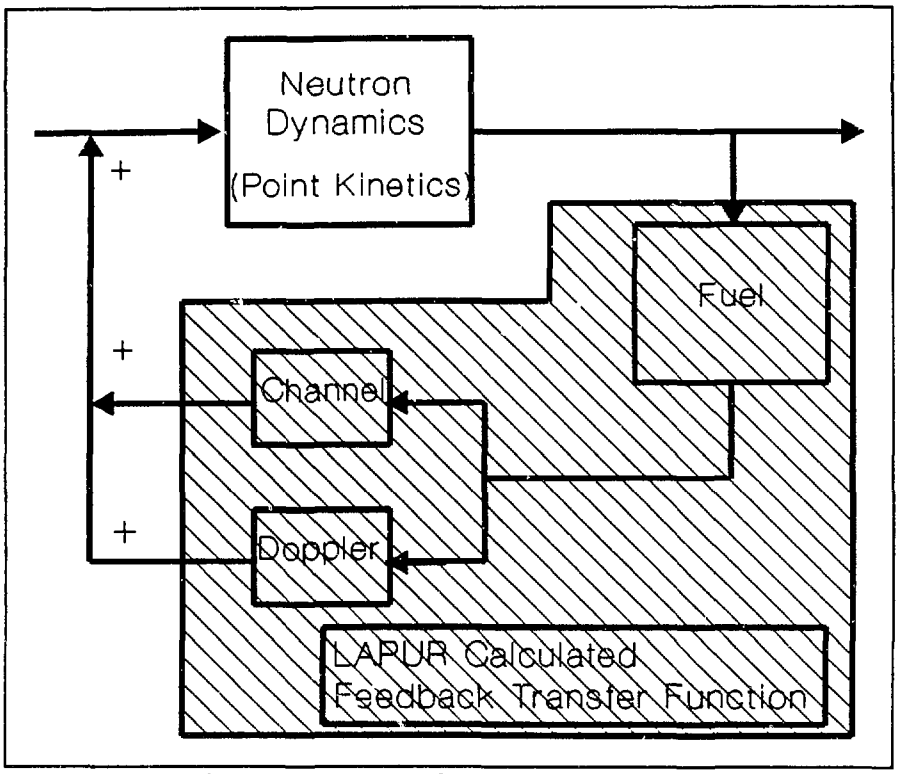
The calculation technique described in this paper uses results from linear frequency domain codes to predict steady-state BWR limit cycle behavior.

- Only valid where the linear thermal hydraulics model from the frequency domain codes is accurate.

The technique for estimating steady-state BWR limit cycle behavior entails;

- Selecting a BWR dynamic model.
- Obtaining expressions for all state variables in terms of the neutron population, n(t).
- Using these expressions to form a non-linear differential equation for n(t).
- Applying Galerkin's method to obtain an approximate solution for n(t).

The dynamic model consists of the point kinetics equations as the forward path and the LAPUR-calculated power-to reactivity transfer function as the feedback loop.



Block diagram of BWR dynamic model.

An expression for the precursor populations in terms of the neutron population can be obtained from the point kinetics equations.

- Since n(t) is periodic during steady-state limit cycle oscillations, it may be written as

$$n(t) = R + \sum_{n=1}^{\infty} \left\{ P_n Sin(n\omega t) + Q_n Cos(n\omega t) \right\} .$$

- This expression can be used with the precursor equations to form expressions for the steady-state component of the precursors.

- The unknowns in these expressions are the undetermined coefficients in the expression for n(t).

An expression for the reactivity in terms of the neutron density can be obtained from the power-to-reactivity transfer function.

- Let the transfer function (frequency response) be given by $H(\omega) = A(\omega) + iB(\omega)$.
- Then, $\rho(t) = H(\omega)\, n(t)$, since $n(t)$ and $\rho(t)$ are purely harmonic functions.
- Again, the unknowns in this expression are the undetermined coefficients in the expression for $n(t)$.

A single equation equivalent to a non-linear differential equation for n(t) is obtained by substituting the expressions for $\rho(t)$ and $c_m(t)$ into the remaining point kinetics equation.

- The solution of the original system of non-linear differential equations has been reduced to determining the undetermined constants in the expression for n(t).

If a finite number of harmonics is used in the expressions for n(t), the equation cannot be satisfied exactly;

- The resulting error is called the residual, $\mathbb{R}$

$$
\begin{aligned}
\mathbf{R} = \sum_{n=1}^{j}\sum_{l=1}^{j} \{ & P_l(A_nP_n - B_nQ_n)S_lS_n + (B_nP_lP_n + (A_l + A_n)P_lQ_n - B_lQ_lQ_n)S_lC_n \\
& + Q_l(B_nP_n + A_nQ_n)C_lC_n\}\sum_{n=1}^{\infty}\{[(n\omega[\Lambda + \sum_{m=1}^{M}\left(\frac{\lambda_m\beta_m}{\lambda_m^2 + n^2\omega^2}\right)] - B_n(R+1))Q_n \\
& + (R(A_0 + A_n) - n^2\omega^2\sum_{m=1}^{M}\left(\frac{\beta_m}{\lambda_m^2 + n^2\omega^2}\right) + A_n)P_n]S_n \\
& + [(R(A_0 + A_n) - n^2\omega^2\sum_{m=1}^{M}\left(\frac{\beta_m}{\lambda_m^2 + n^2\omega^2}\right) + A_n)Q_n \\
& - (n\omega[\Lambda + \sum_{m=1}^{M}\left(\frac{\lambda_m\beta_m}{\lambda_m^2 + n^2\omega^2}\right)] - B_n(R+1))P_n]C_n\} + A_0R(R+1)
\end{aligned}
$$

- Galerkin's method, a weighted residual technique, is used to determine the values of the constants.

Weighted residual techniques may be used to form approximate solutions to linear and non-linear differential equations.

- Assume the form of the solution; The solution is comprised of a sum of products of trial functions and undetermined constants.
- Trial functions must satisfy the boundary conditions and should be members of a complete set.
- Since the assumed solution is not exact, there is some error, or residual $\mathbb{R}$.
- The "best" solution minimizes some measure of $\mathbb{R}$ summed over the region in which the solution applies. This "best" solution is obtained by adjusting the values of the undetermined constants in the assumed solution.
- The assumed solution converges to the exact solution as more terms are included.

The error minimization is performed by setting an integral of $\mathbb{R}$ and a weighting function, integrated over the region in which a solution is sought, to zero;

$$\int_{t=t_0}^{t_1} \mathbb{R}(t)\,\omega(t)\,dt = 0.$$

- The choice of weighting functions determines the particular weighted residual method employed.
- In Galerkin's method, the sets of trial functions and weighting functions are the same
- Different weighting functions result in different rates of convergence and in various degrees of computational complexity.

Applying Galerkin's method results in the following non-linear algebraic equations which can be solved simultaneously for the undetermined constants in the solution for n(t).

$$(1) \quad \int_{t=0}^{\frac{2\pi}{\omega}} \mathbb{R}\,SIN(k\omega t)\,dt = 0\,,\ (k = 1, 2, 3, \ldots j)\,,$$

$$(2) \quad \int_{t=0}^{\frac{2\pi}{\omega}} \mathbb{R}\,COS(k\omega t)\,dt = 0,\ (k = 1, 2, 3, \ldots j)\,,$$

$$(3) \quad \int_{t=0}^{\frac{2\pi}{\omega}} \mathbb{R}R\,dt = 0$$

- Get "j" equations of type (1), "j" equations of type (2), and one equation from (3)

One final equation is needed to solve for ω; This is obtained by setting the integral of $\mathbb{R}$, integrated over one cycle with respect to n, to zero.

$$\int_{one\ cycle} \mathbb{R}\,dn = \int_{t=0}^{\frac{2\pi}{\omega}} \mathbb{R}\dot{n}\,dt = 0$$

- An analogy in mechanical systems - The sum of work performed on a mechanical system over one cycle during steady-state oscillations must be zero.
- Geometrical interpretation - This condition, along with the condition that $\int_{t=0}^{\frac{2\pi}{\omega}} \mathbb{R}\,dt = 0$, forces the average value of the line representing the relationship between $\mathbb{R}$, t, and n to lie in the $\mathbb{R} = 0$ plane.

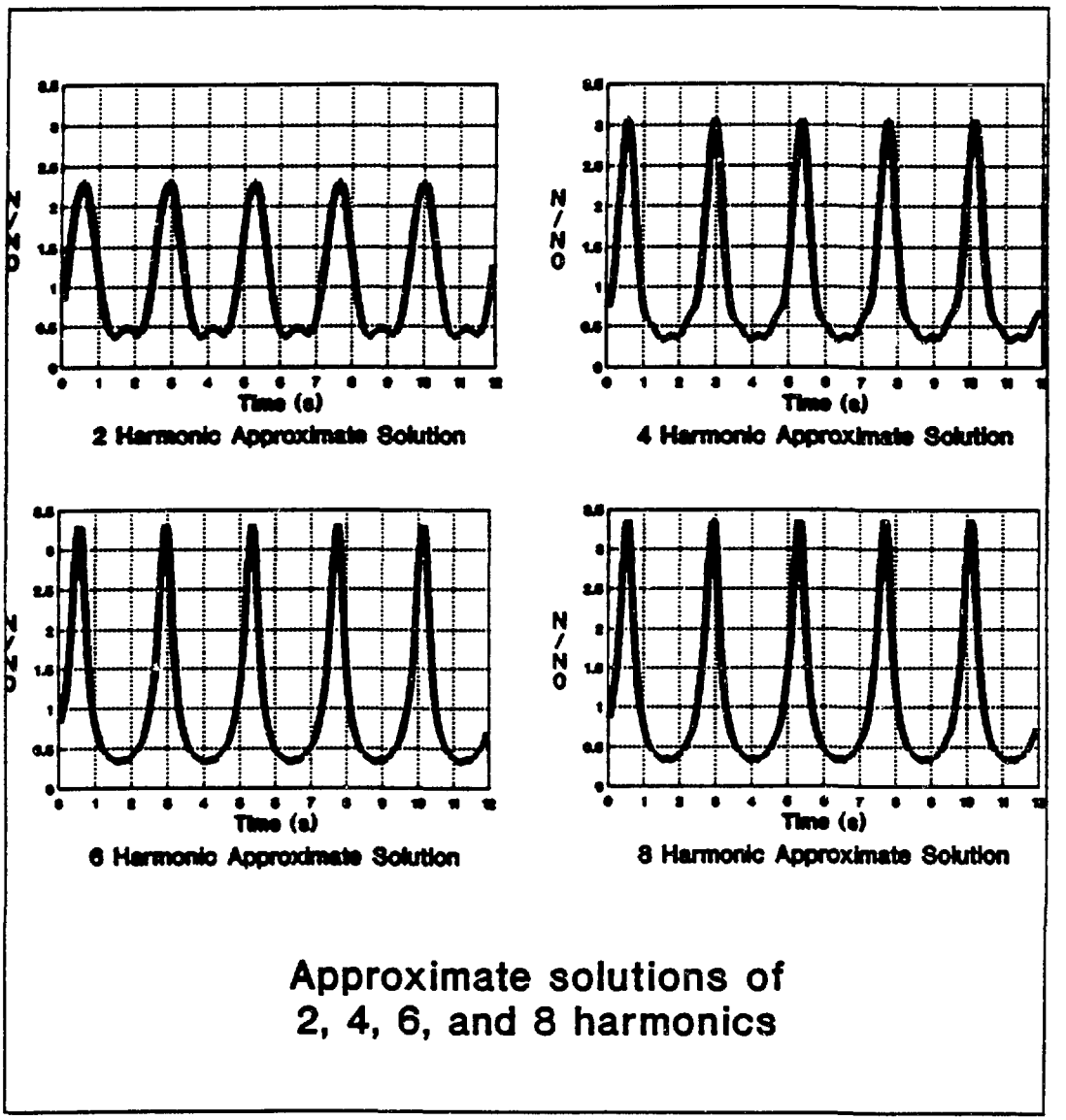
A computer code was written to generate the equations and solve for the undetermined constants in approximate solutions containing up to 20 harmonics.

- Input - point kinetics parameters, transfer function values, initial guesses of constants in the solution.
- A numerical routine solves the set of simultaneous algebraic equations generated by the code using the initial guesses.
- Output - the calculated values of the constants in the approximate solution.

The approximate solution was compared to a numerical solution to check the validity of the approximate solution technique.

- A 5-equation non-linear model of which qualitatively predicts BWR dynamics was used in this comparison. The model parameters were selected to produce limit cycle oscillations.
- A transfer function equivalent to the LAPUR feedback transfer function was calculated.
- An approximate solution was found using the computer code.
- A numerical solution was obtained using ACSL.

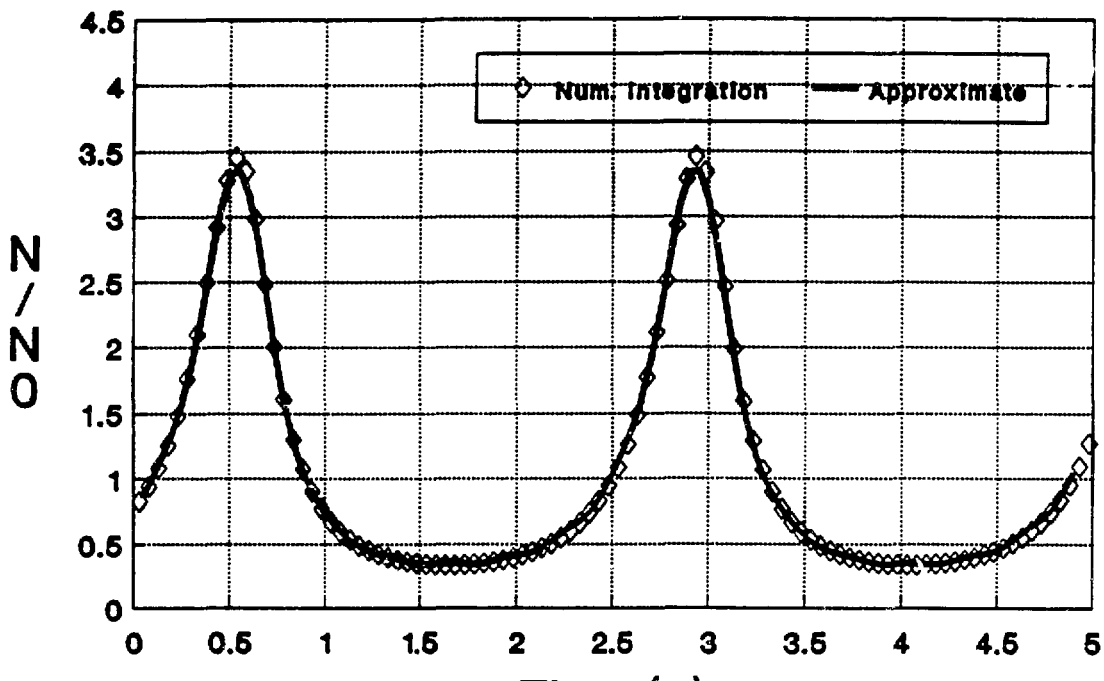
The approximate solution converges to the exact solution as more harmonics are included in the approximation.



Approximate solutions of
2, 4, 6, and 8 harmonics

The numerical an 8-harmonic approximate solution show excellent agreement.

- Implies the approximate solution technique will accurately predict steady-state limit cycle behavior if the point kinetics equations and the LAPUR feedback transfer function accurately model the BWR during limit cycle oscillations.



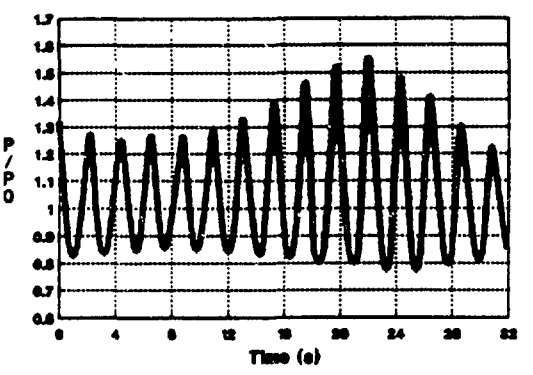
Comparison of numerical and 8 harmonic approximate solutions

An approximate solution has been calculated for the LaSalle event.

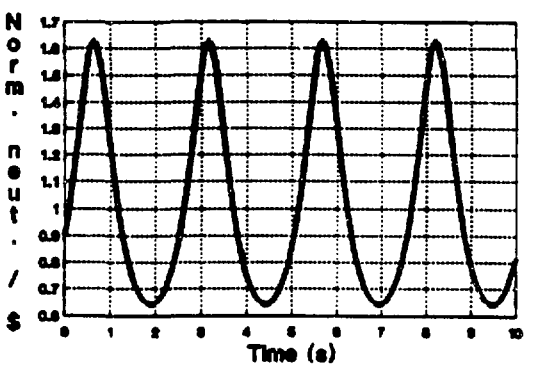
- LAPUR code used to calculate the feedback transfer function.
- LAPUR input for 43% power and 28% flow.
- LAPUR decay ratio of 1.05, ω = 0.40 Hz.
- Measured oscillation amplitudes of 20% - 100% with a period of approximately 2 seconds.

Comparing the results with measured data shows reasonably good agreement.

- Core flow and vessel water level were oscillating during the event, resulting in a fluctuating limit cycle amplitude.
- Total computer time was less than 3 minutes on a VAX 11/780.



APRM data taken during the LaSalle event

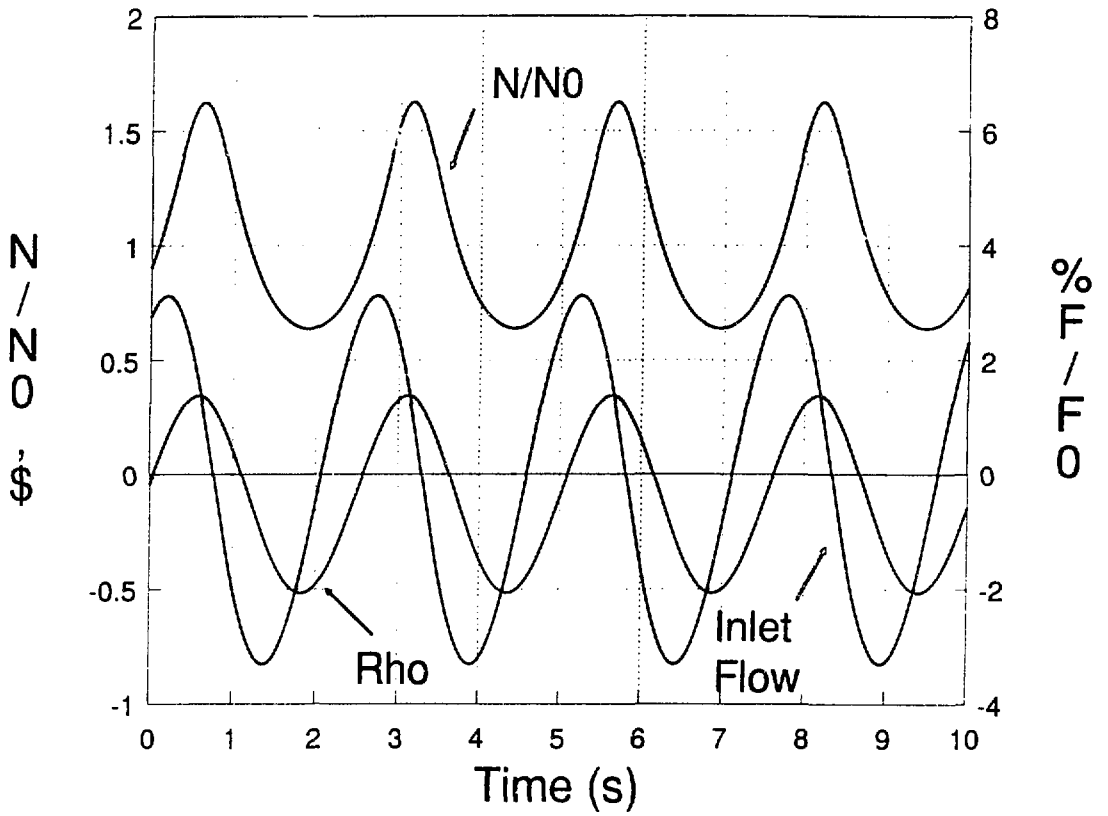


Approximate solution for LaSalle event

Additional time dependent quantities can be obtained using other transfer functions.

1) reactivity

2) flow



Conclusions:

- Galerkin's method can be used with a BWR dynamics model comprised of the point kinetics equations and the LAPUR-calculated feedback transfer function to estimate BWR behavior during stead-state limit cycle oscillations.

- The approximate solution will converge to the actual solution as additional terms are included in the approximation. Thus, the accuracy of the approximate solution is ultimately limited only by the accuracy of the dynamic model.

- The reasonably good agreement between the approximate solution and measured data shows the approximate solution technique can be used to predict BWR behavior during the limit cycle.