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Quantum Beamstrahlung: Pulse Shaping Prospects for a Photon-Photon Collider*

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Beamstrahlung is the fractional energy loss to radiation as one beam pulse is accelerated by the electromagnetic field of the other pulse through which it passes in a collider. It is very important for high energy electron-positron linear colliders ($\sqrt{s} \sim \frac{1}{2} - 1\ TeV$) that are being studied.[1] In linear colliders the individual pulses must be formed with much larger charge densities to compensate for their much lower rate of interactions ($10^2 - 10^3$ per second) in comparison to storage rings (typically 10^5 per second). Therefore the electromagnetic fields generated by the charges in each pulse will be correspondingly larger, as will the acceleration they induce in the charges of the intersecting pulses; this in turn leads to a larger loss of energy to radiation. In the classical approximation the beamstrahlung is given by

$$\delta_{\text{classical}} = \frac{8\pi}{3} N^2 \gamma \, \frac{r_0^3}{\ell\{\pi B^2\}} \tag{1}$$

where N is the number of electrons (positrons) per pulse, for a uniformly charged cylindrical pulse of length ℓ and cross section πB^2; $\gamma = E/M$ is the beam energy and $r_0 \equiv 2 \cdot 8 \times 10^{-13}\ cm$ is the classical electron radius.

In this report we consider three questions:

1. What is the quantum result for beamstrahlung and when is the classical result valid?
2. How does the transverse beam geometry—*i.e.*, ribbon versus cylindrical pulse—affect the beamstrahlung?
3. Extending the quantum calculation to multiple photon emission, what is the photon flux and what are the prospects for a photon-photon collider?

1) Quantum beamstrahlung

Synchrotron radiation is a well-studied problem. Its application to beamstrahlung in the very high energy quantum limit was made in a seminal paper by Himel and Siegrist. The calculation can be done analytically[2] for the physical case of small disruptions—*i.e.*, for parameters such that the impact parameter of a beam particle changes fractionally only by a small amount as it traverses an intersecting beam pulse. Making this approximation which is an appropriate one for envisioned TeV colliders the quantum result for beamstrahlung for uniformly charged colliding pulses with cylindrical geometry is

$$\delta = \delta_{\text{classical}}\, F(C) \tag{2}$$

where the form factor $F(C) \to 1$ in the classical limit ($\hbar \to 0$, $C \to \infty$) and $F(C) \to C^{4/3}(0.83)$ for $C \to 0$ in the extreme quantum limit. The physical interpretation of C is the ratio of a transverse coherence length, $\ell_\perp$, to the radiation length, ℓ_{rad}:

$$C \propto \frac{\ell_\perp}{\ell_{\text{rad}}} \,. \tag{3}$$

$\ell_\perp$ is defined as the length along the trajectory of a beam particle as it traverses a pulse for it to acquire a transverse momentum $p_\perp \sim m$ from the field produced by the charges of the pulse; after travelling a distance $\ell_\perp$ the trajectory turns through an angle m/E. This angle is the same as the width of the radiation pattern from a relativistic charged particle; $\theta_{\text{tr}} \sim 1/\gamma = m/E$. Thus the radiation ceases to be coherent after a particle moves a length $\ell_\perp$ along the curving path. The second length, ℓ_{rad}, is defined as the length of the target that the electron coherently scatters from during the radiation process. According to the uncertainty principle, it is the reciprocal of the minimum longitudinal momentum transfer to the target. As shown in ref. 2

$$C = \frac{1}{4\gamma y} \left\{ \frac{\ell}{\hbar/mc} \right\} \tag{4}$$

where

$$y \equiv \frac{N r_0}{B} = r_0 \sqrt{\pi \mathcal{L}} \tag{5}$$

is a dimensionless classical variable proportional to the square root of the luminosity per pulse, $\mathcal{L}$, and we inserted $\hbar$ and c as appropriate to show that $C \propto 1/\hbar$. In the classical limit the radiation occurs locally and by (3) and (4), $C \propto 1/\hbar \to \infty$ as claimed above. In all cases currently under study as practical in the TeV energy range both lengths $\ell_{rad}, \ell_\perp \ll \ell$ and end effects of the pulse are small.[2] Using (5) we can rewrite (1) and (2) in terms of scaling variable C which, for fixed luminosity per pulse, $\mathcal{L}$, and for pulses of circular cross-section is the only parameter appearing:

$$\delta = \frac{2}{3}\alpha y \frac{F(C)}{C} ; \quad \alpha \equiv \frac{1}{137} . \tag{6}$$

The classical approximation is valid for $C \gg 1$, $F(C) \to 1$, in which case the beamstrahlung is small since for practical values of $\mathcal{L} \lesssim 10^{30} cm^{-2}$, $y \lesssim 400$.

2) Ribbon Beams

The field strengths generated by the pulses are reduced when the circular cross section of a cylindrical pulse is distorted to an ellipse of the same area. Therefore the resulting acceleration of a beam particle is also reduced, as is the energy loss to radiation. As defined in ref. 2 this reduction can be summarized in the aspect ratio $1/G$ defined as the ratio of the mean value of the semi-major and minor axes to their average:

$$\frac{1}{G} = \frac{2\sqrt{a_x a_y}}{a_x + a_y} \leq 1 \ . \tag{7}$$

The result is[2]

$$\delta^{\text{ellipse}} = \frac{2}{3}\,\frac{\alpha y}{G}\,\frac{F(CG)}{(CG)} \ . \tag{8}$$

The fact that $\delta^{\text{ellipse}}_{\text{classical}} \propto 1/G^2$ follows from the fact that $(\text{acceleration})^2 \propto (\text{field strength})^2 \propto (1/G)^2$. Since $G \geq 1$, the argument CG of the form factor increases with increasing values of the aspect ratio and we move toward the classical region for fixed C.

3) Multiple photon emission and prospects for a $\gamma - \gamma$ collider

Interest in this possibility is kindled by the fact that the expected gamma flux due to beamstrahlung for TeV linear colliders now under study is one to two orders of magnitude larger than that calculated in the Weiszacker-Williams equivalent-photon approximation, and the resultant photon spectrum is hard.[3]

This result introduces a new dimension into the design and application of $e^{\pm}$ linear colliders. For constant luminosity, the beam pulses can be shaped as desired in order to either maximize or minimize the fields and the resulting beamstrahlung. The beamstrahlung can be reduced by forming beams with very thin ribbon-like cross sections, *i.e.*, $G \sim 5$, permitting study of e^+e^- interactions at maximum energy and with narrow energy spread. Oppositely, in order to study high energy photon-photon processes, the collider can be tuned by shaping the beam pulses with smaller aspect ratio, *i.e.*, $G \sim 2$, in order to increase the flux of hard photons. We note that the multiple photon contributions substantially alter the spectral distribution of radiated photons but have little effect on the fractional energy loss δ.

The basic calculation proceeds from the rate equation for $P_e(x,t)$, defined as the probability of finding an electron with a fraction x of its incident momentum at a fractional depth t within the pulse:

$$\frac{dP_e(x,t)}{dt} = \int_x^1 dz\, T(x,z)\, P_e(z,t) - p(x)\, P_e(x,t) \ , \tag{9}$$

where the two terms on the right hand side are the standard "source" and "sink" contributions that represent falling into and out of the momentum fraction x as a result of radiating a photon. $T(x,z)\, dt\, dx$ is the probability that an electron with momentum fraction z will radiate a photon with momentum fraction $(z - x)$ and end up within dx of x after traversing a fraction dt of the pulse length ($0 < t < 1$).

$$p(z)\, dt \equiv \int_0^z dx\, T(x,z)\, dt \tag{10}$$

is the probability that the electron with z radiates one photon while traversing dt.

Analogously, the rate equation for the probability of finding a photon with momentum fraction v at depth t is

$$\frac{dP_{ph}(v,t)}{dt} = \int_x^1 dx\, T(x-v,x) P_e(x,t) \ . \tag{11}$$

Equations (9) and (11) include only the first generation of shower development; neglected is subsequent pair conversion of the high energy radiation formed in the pulses. We have also proceeded classically above in calculating the probabilities for multiple photon emission since for $y \gg 1$ (see Eq. 5 and the subsequent discussion) we can chop the pulse into thin slices of thickness ℓ/y such that radiation from successive slices is incoherent.

Equations (9) and (11) can be integrated[3] numerically in terms of known single photon radiation probabilities, $T(x,z)$. We define the photon flux through a pulse by

$$F(v) \equiv \int_0^1 dt\, P_{\rm ph}(v,t) \,.$$

The equivalent luminosity for beamstrahlung photon-photon collisions at total mass W is given by

$$\frac{d\mathcal{L}_{\gamma\gamma}}{dW^2} = \int_0^1 dv_1 \int_0^1 dv_2\, F(v_1)F(v_2)\,\delta(sv_1v_2 - W^2) \,,$$

or

$$\frac{d\mathcal{L}_{\gamma\gamma}}{dz} = 2z \int_0^1 \frac{dv}{v}\, F(v)\, F\left(\frac{z^2}{v}\right) \,, \qquad (12)$$

where $z^2 = W^2/s = W^2/4E^2$.

Using this definition of $\mathcal{L}_{\gamma\gamma}$, the cross section to form a final state X at mass W is given by

$$\begin{aligned} &\frac{d}{dz}\,\sigma_{\rm beamstrahlung}(e^+e^- \to e^+e^-X) \\ &\quad = \left[\frac{d}{dz}\,\mathcal{L}_{\gamma\gamma}\right] \sigma_{\gamma\gamma\to X}(W) \,, \end{aligned} \qquad (13)$$

In the narrow resonance approximation for a "particle" X of mass m_X (13) reduces to

$$\begin{aligned} &\sigma_{\rm beamstrahlung}(X) \\ &\quad = \left[z\,\frac{d}{dz}\,\mathcal{L}_{\gamma\gamma}\right]_{z=m_X/\sqrt{s}} \left\{\frac{4\pi^2(m_X \to 2\gamma)}{m_X^3}\right\} \,. \end{aligned} \qquad (14)$$

The equivalent luminosity is shown in Figs. 1 and 2 for two choices of luminosity

$$C = 1.5 \;\&\; y = 400, \text{ or } \mathcal{L} \sim 0.7\ 10^{30} cm^{-2} \;\&\; \ell \sim 0.9mm$$
$$C = 0.5 \;\&\; y = 800, \text{ or } \mathcal{L} \sim 2.8\ 10^{30} cm^{-2} \;\&\; \ell \sim 0.3mm\,. \tag{15}$$

The sensitivity to the beam geometry is illustrated by the three curves plotted in each figure corresponding to different values of the pulse aspect ratio, $G = 1$, 2 and 5 respectively. The latter values correspond to a flat beam with aspect ratios $a_x/a_y = 15$ and and 100, respectively. The virtual photon curves are drawn as dotted lines for a 1 TeV collider. For a 600 GeV collider with the same values of C and y, the results differ only slightly since the beamstrahlung photon-photon flux does not depend explicitly on the beam energy, and the virtual flux decreases only slightly due to its logarithmic energy dependence.

This calculation shows that two photon reactions via beamstrahlung can be significantly larger than previously appreciated on the basis of virtual photon processes. Note also that the beamstrahlung source works with electron-electron as well as positron-electron colliders. These hard photons may lead to troublesome backgrounds or, conversely, raise the attractive possibility of doing interesting new physics with linear colliders in the 500 $GeV - 1\ TeV$ energy range that is now being studied. The colliders can be tuned by beam shaping to minimize or to maximize the beamstrahlung contribution as demanded by experiment.

FIGURE CAPTIONS

1) The photon-photon luminosity relative to the incident electron-positron fluxis plotted for the case $C = 1.5$ and $y = 400$. The effect of pulse shaping is illustrated by the three values of G. The virtual two photon flux is also plotted as the dotted line for a collider energy of 1 TeV.

2) The same as Fig. 1 but for machine parameters of $C = 0.5$ and $y = 800$. The increase in luminosity is evident.

REFERENCES

1. R. B. Palmer, SLAC-AAS-35 (1988), Stanford University preprint SLAC-PUB-4295 (1987), SLAC-AAS-31 (1987).

2. R. Blankenbecler and S. D. Drell, Phys. Rev. D36, 277 (1987); D37, 3308 (1988).

3. R. Blankenbecler and S. D. Drell, Phys. Rev. Lett. 61, 2324 (1988).

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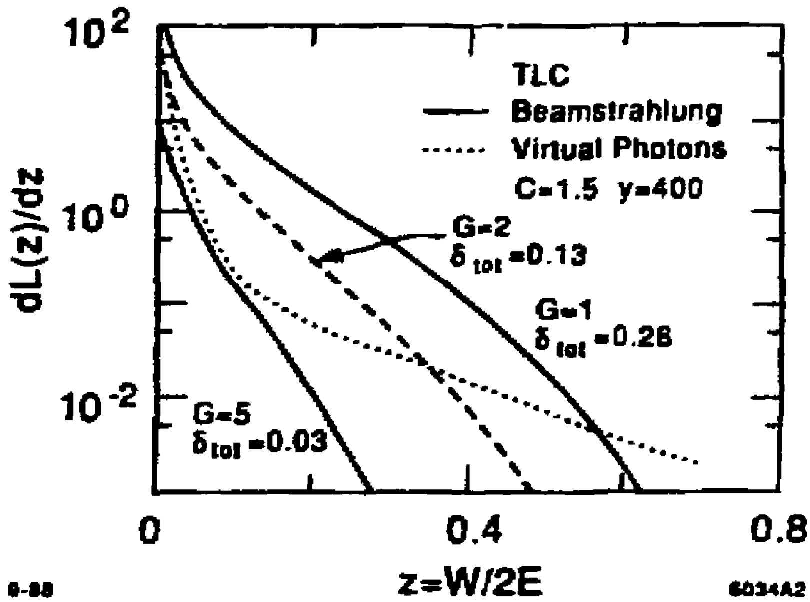

10^2
10^0
10^{-2}
dL(z)/dz
TLC
Beamstrahlung
Virtual Photons
C=1.5 y=400
G=2
δ_{tot} =0.13
G=1
δ_{tot} =0.28
G=5
δ_{tot} =0.03
0
0.4
0.8
z=W/2E
8-88
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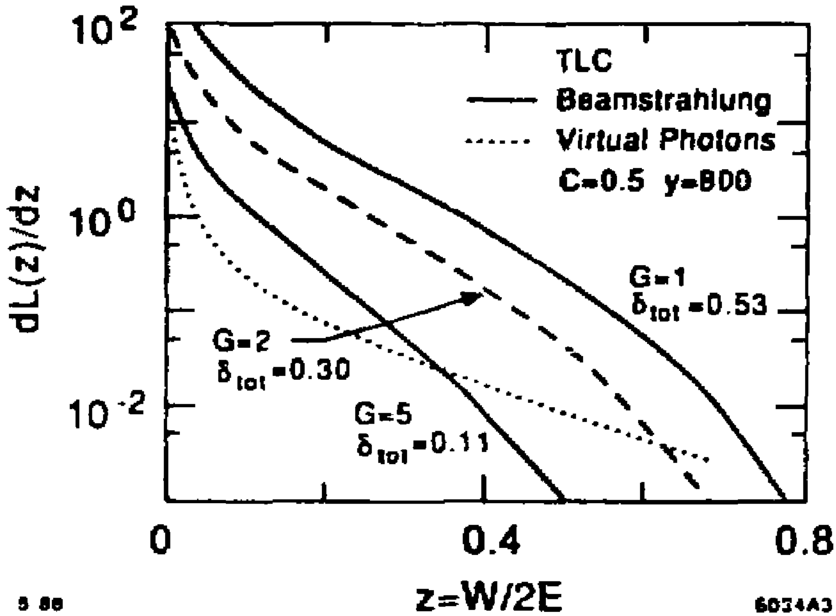
FIGURE 1



TLC
Beamstrahlung
Virtual Photons
C=0.5 y=800
G=1
δtot=0.53
G=2
δtot=0.30
G=5
δtot=0.11
dL(z)/dz
10^2
10^0
10^-2
0
0.4
0.8
z=W/2E
5 88
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FIGURE 2