15

ISPO-140
PNL-3712
UC-15

ISPO TASK C.35

CALCULATION OF PARAMETERS FOR INSPECTION PLANNING AND EVALUATION—LOW ENRICHED URANIUM CONVERSION AND FUEL FABRICATION FACILITIES

FEBRUARY 1981

USA — PROGRAM FOR TECHNICAL ASSISTANCE TO IAEA SAFEGUARDS

IAEA

Department of Energy

Safeguards & Security

P. T. Reardon
M. F. Mullen
N. L. Harms

PACIFIC NORTHWEST LABORATORY
OPERATED BY
BATTELLE MEMORIAL INSTITUTE

PACIFIC NORTHWEST LABORATORY
operated by
BATTELLE
for the
UNITED STATES DEPARTMENT OF ENERGY
Under Contract DE-AC06-76RLO 1830

Printed in the United States of America
Available from
National Technical Information Service
United States Department of Commerce
5285 Port Royal Road
Springfield, Virginia 22151

Price: Printed Copy $_______*; Microfiche $3.00

*Pages	NTIS Selling Price
001-025	$4.00
026-050	$4.50
051-075	$5.25
076-100	$6.00
101-125	$6.50
126-150	$7.25
151-175	$8.00
176-200	$9.00
201-225	$9.25
226-250	$9.50
251-275	$10.75
276-300	$11.00

ISPO-140
PNL-3712
UC-15

CALCULATION OF PARAMETERS FOR INSPECTION PLANNING AND EVALUATION - LOW ENRICHED URANIUM CONVERSION AND FUEL FABRICATION FACILITIES

P. T. Reardon
M. F. Mullen
N. L. Harms

February 1981

Prepared for
International Safeguards Project Office
Program for Technical Assistance
to the International Atomic Energy Agency

Pacific Northwest Laboratory
Richland, WA 99352

SUMMARY

As part of Task C.35 (Calculation of Parameters for Inspection Planning and Evaluation) of the U.S. Program of Technical Assistance to IAEA Safeguards, Pacific Northwest Laboratory has performed some quantitative analyses of IAEA inspection activities at low-enriched uranium (LEU) conversion and fuel fabrication facilities. This report presents the results and conclusions of those analyses.

Implementation of IAEA safeguards at LEU conversion and fuel fabrication facilities must take into account a variety of practical problems and constraints. One of the key concerns is the problem of flow verification, especially product verification. There are basically two kinds of difficulties:

- The amount of inspection effort required for full flow verification can be quite large.
- Given current measurement technology, no fully satisfactory measurement strategy for product verification is available.

The objective of this report is to help put the problem of flow verification in perspective by presenting the results of some specific calculations of inspection effort and probability of detection for various product measurement strategies.

Three basic product verification strategies can be defined:

- verification of finished fuel assemblies
- verification of fuel rods, supplemented by other inspection activities to cover fuel assemblies
- verification of pellet stacks prior to their insertion into fuel rods, supplemented by other inspection activities to cover fuel rods and fuel assemblies.

A fourth possibility is an appropriate combination of the three basic strategies; this combined approach has some advantages from the practical point of view.

In order to provide quantitative information about the advantages and disadvantages of the various strategies, eight specific cases were examined.

For each case, the technical approach was to define model facilities, outline the inspection strategy, and calculate the probability of detection and required inspection effort. When all cases were completed, the results were analyzed. On the basis of the conclusions, a number of supplemental inspection activities were identified and quantitative assessments were made of their impact on the overall inspection approach.

For each of the eight case studies, plant throughputs of 200, 400 and 600 tons of uranium per year and plant inventories equal to 34% and 81% of annual throughput were considered. Operator measurement errors consistent with international standards of measurement accuracy were assumed. The following IAEA detection goals were postulated:

- significant quantity = 75 kg ^{235}U contained in low-enriched uranium
- detection probability = 95%
- false alarm probability = 5%
- detection time on the order of one year

The key conclusion of the study concerns the effectiveness of the verification strategy that includes a) a limited variables sampling plan for verification measurements at the pellet loading station, b) a sampling plan for quantitative NDA verification measurements of fuel rods, and c) an attributes sampling plan for fuel assembly verification. The probability of detection provided by this approach depends on the diversion scenario, but the following generalizations are possible:

- For diversion through the MUF-$\hat{D}$ path, the probability of detection tends to be smaller than the detection probability used for planning, except when the facility is small or the measurements are very good.
- For diversion by gross defects, the detection goal can be achieved provided that sufficient inspection resources are available to perform the required number of attributes tests. This conclusion applies for all forms of material in the facilities.
- For diversion via partial defects and biases, the probability of detection depends on where in the facility the diversion occurs. If it occurs before the pellet loading station, then it can be detected (if sampling is random

or nearly so) although the probability of detection may be smaller than the detection goal. If it occurs after the pellet loading operation (or if sampling is not random), then diversion via partial defects or biases probably cannot be detected without supplemental inspection activities of some kind (such as surveillance).

- Flow verification of the product stratum dominated the error variances, with significant contributions from the waste and scrap strata. Improved methods of dealing with waste and scrap will aid the probability of detection.
- The effect of more than doubling the inventory size generally was small on the probability of detection. This was because error variances were dominated by flow strata.

CONTENTS

TABLES

FIGURES

1.0 INTRODUCTION

The United States Program of Technical Assistance to IAEA Safeguards, managed by the International Safeguards Project Office (ISPO), includes several tasks in the general area of inspection effort modeling. The first of these was Task C.5, Estimation of Inspection Effort for Chosen Inspection Procedures. The objective of Task C.5 was to develop and demonstrate a method which provides a systematic way of determining inspection effort and safeguards effectiveness as a function of safeguards goals, facility models, and safeguards approach. This task produced two documents which describe the method used, and a computer program package, INSPECT, which is useful in determining safeguards effort and effectiveness.[1,2]

The next task was C.19, Model for Analysis of the Impact of Safeguards Criteria. The objective of Task C.19 was to apply the method developed in C.5 to analyze the effect on inspection effort and safeguards effectiveness of varying safeguards goals and procedures. A set of input parameters to INSPECT was systematically varied using an experimental design (Box-Behnken design). Results were analyzed using graphs, response surfaces and regression analysis.

The IAEA found the INSPECT programs to be very useful for studies of material accountancy, and is in the process of applying them to comprehensive calculations related to mixed-oxide and high-enriched uranium fuel element fabrication, low-enriched uranium conversion and fuel element fabrication, and reprocessing facilities. A part of this activity is to be accomplished under ISPO Task C.35, Calculation of Parameters for Inspection Planning and Evaluation. This paper deals with LEU conversion and fuel fabrication facilities.

The two primary objectives of this study are as follows: 1) to quantify certain aspects of the effectiveness of possible IAEA inspection approaches at LEU conversion and fuel fabrication facilities; and 2) investigate the effect of large plant inventories (up to 80% of throughput) on the detection probability and the corresponding inspection effort. The inspection approaches include verification of fuel assemblies, verification of fuel rods, verification of pellets at the rod loading station, or some combination of the three approaches.

The technical approach taken in this study is as follows:

- Define the reference facility;
- Outline the inspection approaches to be analyzed;
- Use INSPECT to analyze the various inspection approaches, and note the pertinent system responses;
- Consider further measures which will increase inspection effectiveness.

2.0 THE REFERENCE FACILITY

The reference facility is defined by the collection of many parameters. Some of the important physical parameters are the amount of throughput, material types (strata), batch and item sizes and weights, and inventory sizes. The reference facility also defines operator and inspector measurement capabilities necessary to calculate the operator's variance of MUF (material unaccounted for) and the inspector's difference ($\hat{D}$) statistic. These statistics play a key role in determining the probability of detection of diversion of a significant quantity by material accounting techniques.

2.1 STRATA DESCRIPTIONS

In the model plant, various sized throughputs and inventories were developed for this study. The throughput sizes were 200, 400, and 600 tons of low enriched uranium, and each throughput size had two inventory sizes associated with it to make up the six different cases. The size of the inventory was based on a percentage of the throughput, and for this study these percentages were chosen to be 33.7% and 80.6%. The beginning and ending inventories contain the same number of items and amount of material at the beginning of the period, and at the end of the period. Tables 2.1 and 2.2 summarize the amount of material in inventories per stratum for the 200 ton throughput facility with 33.7% and 80.6% of throughput on hand, respectively. Stratum data for the 400 and 600 ton facilities were scaled up by increasing the number of batches in a stratum by two and three times. This will keep the relative amount of material in inventory at 33.7% and 80.6% of throughput. Appendix A contains the strata data for all six facilities used in this study.

TABLE 2.1. Inventory for 200 Ton Facility with 33.7% Throughput on Hand

Strata	Avg. Item Weight (kg U)	No. of Batches	No. Items Per Batch	Total U in Strata (kg)	Percent of Throughput
UF_6 Cylinders	1,450	20	1	29,000	14.5
UO_2 Powder	13	33	30	12,870	6.4
UO_2 Pellets	13	17	30	6,630	3.3
U_3O_8 Powder	20	5	20	2,000	1.0
Scrap	10	20	50	10,000	5.0
Waste	0.5	500	1	250	0.1
Miscellaneous	4	250	1	1,000	0.5
Fuel Rods	2.9	32	63	5,846	2.9
				67,596	33.7

TABLE 2.2. Inventory for 200 Ton Facility with 80.6% Throughput on Hand

Strata	Avg. Item Weight (kg U)	No. of Batches	No. Items Per Batch	Total U in Strata (kg)	Percent of Throughput
UF_6 Cylinders	1,450	48	1	69,600	34.8
UO_2 Powder	13	79	30	30,810	15.4
UO_2 Pellets	13	40	30	15,600	7.8
U_3O_8 Powder	20	12	20	4,800	2.4
Scrap	10	47	50	23,500	11.8
Waste	0.5	1,187	1	594	0.3
Miscellaneous	4	593	1	2,372	1.2
Fuel Rods	2.9	76	63	13,885	6.9
				161,161	80.6

2.2 MEASUREMENT CAPABILITY

It was assumed that the measurement capability of the operator was consistent with international standards of accountability. For LEU conversion and fuel fabrication facilities, the expected standard deviation of MUF (σ_{MUF})

is 0.3% of throughput.(7) Table 2.3 summarizes operator and inspector measurement error data for the material types involved. Appendix B contains all the bulk measurement, sampling, and analytical error standard deviations used in this study. It is assumed that all between-laboratory components are zero. It is also assumed that the inspector has the same measurement capability as the operator for all strata except the product stratum of fuel rods, which reflects the assumption that the operator bases his estimate of fuel rod contents on measurements at the rod loading station, and uses NDA techniques (rod scanner, for example) for quality assurance and inventory, not material accounting. The inspector currently must rely on NDA measurements of the fuel rods. Table 2.4 summarizes the standard deviation of MUF as a percentage of throughput for the six cases of this study. The measurement error standard deviations assumed in the study give a MUF that comes close to meeting the international standard. The large inventory cases of 80.6% exceed the standard somewhat. The reason this occurs is given the measurement capability, increasing the amount of material will increase MUF. To observe the effects of large inventories, one must hold the measurement standard deviations constant. The effect on MUF was not large as these are flow dominated facilities.

The various material accounting approaches are studied by changing the proper measurement error to reflect changing verification methods. This is further explained in the Section 3.

2.3 INSPECT PARAMETERS

Calculation of the inspection effort and the material accounting effectiveness depend a great deal on the INSPECT parameters used.(2) These parameters include the goal quantity, the false alarm probability, the variance inflation factor, and the crossover point.

The significant quantity of ^{235}U in low-enriched uranium has been set at 75 kg of contained ^{235}U. It is assumed that this plant produces 3% enriched fuel, hence the significant quantity is 2500 kg. This figure is used as the goal quantity in all calculations.

TABLE 2.3. Operator and Inspector Measurement Error Data by Material Type (Relative Standard Deviations)

Strata	Operator		Inspector	
	Random	Systematic	Random	Systematic
UF_6 Cylinders				
Weighing	0.00133	0.00163	0.00133	0.00163
Sampling	0.0017	0.00144	0.0017	0.00144
Analysis	0.00148	0.00140	0.00148	0.00140
UO_2 Powder				
Weighing	0.00113	0.00113	0.00113	0.00113
Sampling & Analysis	0.00128	0.0012	0.00128	0.0012
UO_2 Pellets				
Weighing	0.00113	0.0009	0.00113	0.0009
Sampling & Analysis	0.0012	0.00098	0.0012	0.00098
U_3O_8 Powder				
Weighing	0.00113	0.00113	0.00113	0.00113
Sampling & Analysis	0.0021	0.0021	0.0021	0.0021
Scrap				
Weighing	0.002	0.00163	0.002	0.00163
Sampling & Analysis	0.02	0.0163	0.02	0.0163
Waste				
Weighing	0.0	0.0	0.0	0.0
Sampling & Analysis	0.0875	0.05625	0.0875	0.05625
Miscellaneous				
Weighing	0.00113	0.0009	0.00113	0.0009
Sampling & Analysis	0.00182	0.00163	0.00182	0.00163
Fuel Rods				
Weighing	0.00083	0.00083	0.0	0.0
Sampling & Analysis	0.0012	0.0012	0.01	0.01

TABLE 2.4. Standard Deviation of MUF for the Model Facilities

Plant Size (kg)	Inventory Size (% of Throughput)	σMUF (kg)	σMUF (% of Throughput)
200,000	33.7	595.8	0.298
400,000	33.7	1188.6	0.297
600,000	33.7	1781.3	0.297
200,000	80.6	743.8	0.372
400,000	80.6	1483.3	0.371
600,000	80.6	2222.8	0.370

The false alarm probability was set at 0.05 per year. It was assumed that there was one inventory per year at the 200 ton facilities, and two inventories at the 400 and 600 ton facilities. For the 400 and 600 ton facilities it was necessary to adjust the false alarm probability for each material balance period such that on a yearly basis it was kept at the desired level. Using the rough approximation that the accounting statistics for successive material balance periods are uncorrelated, then the simple model

$$\alpha = 1 - (1 - \alpha')^n$$

where:

α = yearly goal false alarm probability

α'= single material balance period false alarm probability

n = number of material balance periods per year

can be used to determine what the false alarm probability should be for any given inventory period.

For two material balance periods a year (n= 2) and a goal false alarm probability of 0.05, it is found that the single inventory false alarm probability is α' = 0.0253. While only an approximation because successive material balance statistics are correlated, it is a good approximation in this instance.

The other parameter that requires some explanation is the crossover point, γ. The crossover point is a constant that describes the ability of the attributes tester to detect a defect of a given size in a particular stratum. For example, if γ = 0.10 for a stratum, then this means that a defect equal in size to 10% of the item value would not be detected by attributes inspection, although a larger defect would be.[4] In other words, the smaller γ is set, the more sensitive the inspector's attribute methods. This has an important effect in determining the variables sample size for the inspector.

The variables sampling plan has two objectives: 1) detection of partial defects; and 2) detection of bias defects. In order to fulfill these objectives two variables sampling sizes are calculated. These are denoted as follows:

- NV_1 = variables sample size to detect partial defects
- NV_2 = variables sample size to detect bias defects

The actual variables sampling plan is chosen to protect against both the partial defects and bias defects diversion paths. To do this, the larger of NV_1 and NV_2 is chosen. Past experience has shown that NV_1 generally dominates NV_2 for LEU conversion and fuel fabrication facilities. The crossover point enters the calculation in the following manner:

$$NV_{1i} = N_i \left[1 - (\beta)^{1/r_i}\right]$$

where N_i = number of items in the stratum

β = nondetection probability

r_i = number of defects that would be needed to acquire material amount M

and $r_i = \dfrac{M}{\gamma \bar{X}_i}$.

For example, if M = 2500 kg U

N_i = 4000

β = 0.05

$\bar{X}i$ = 20 kg = average item weight in the stratum

and the inspector's attribute sensitivity is poor (γ = 0.4), then

NV_{1i} = 39.

If the inspector's attribute sensitivity is good (γ = 0.05), then

NV_{1i} = 5.

For this study, the value of γ was taken to be 0.05.

In this manner, the dominance of NV_1 over NV_2 is reduced. By making improvements in the attributes testers, both diversion paths are still covered, but the number of samples taken (hence the manpower requirements) are greatly reduced. It should also be noted that if a rod scanner is one of the attribute testers, and has a measurement standard deviation, σ, of about 1%, then a crossover point, γ, of 5% is about to four to six times σ. If the lower range of the instrument is taken conservatively as six times the relative standard deviation, then a rod scanner essentially detects with 100% certainty all removals over 5% of the nominal amount of uranium in an item.

A final INSPECT parameter to be set is the variance inflation factor. This parameter arises because the variance of $\hat{D}$ under the hypothesis that there is no diversion through small data falsifications may be smaller than that under the alternative hypothesis that some may be thus diverted. This is because under the alternative hypothesis, the diverter will likely choose to not falsify all items by the same amount. Thus a statistical sampling error will be introduced because the variance of $\hat{D}$ will depend on which items were selected to be inspected. Hence, the variance inflation factor is used to relate the variance of $\hat{D}$ under the null hypothesis (no diversion) to its variance under the alternative hypothesis (diversion) during the planning stage. This is shown as follows:

$$\sigma_{\hat{D}_r|H_1}^{\;2} = F\sigma_{\hat{D}_r|H_0}^{\;2}$$

where F denotes the inflation factor. Part F of the Safeguards Technical Manual recommends a value of 4.

Table 2.5 summarizes the key INSPECT parameters used in this study. Appendix F contains variables sample size results for NV_1 and NV_2 for the model facilities.

TABLE 2.5. Summary of INSPECT Parameters

Parameter	Symbol	Value
Goal Quantity	M	2500.0 kg
False Alarm Probability	α	0.05
Nondetection Probability	β	0.05
Variance Inflation Factor	F	4.0
Crossover Point	γ	0.05

3.0 MATERIAL ACCOUNTING INSPECTION APPROACHES

The objective of IAEA safeguards is stated in INFCIRC/153 as follows:

- The Agreement should provide that the objective of safeguards is the timely detection of diversion of significant quantities of nuclear material from peaceful nuclear activities to the manufacture of nuclear weapons or of other nuclear explosive devices or for purposes unknown, and the deterence of such diversion by the risk of early detection.
- To this end the Agreement should provide for the use of material accountancy as a safeguards measure of fundamental importance, with containment and surveillance as important complementary measures.
- The Agreement should provide that the technical conclusion of the Agency's verification activities shall be a statement, in respect of each material balance area, of the amount of material unaccounted for over a specific period, giving the limits of accuracy of the amounts stated.

To fulfill this objective, and according to safeguards agreements, the Agency performs different types of inspections. This study concerns itself only with the routine inspections.

The Agency must perform product verification at some stage of the material balance. There are several options open to the Agency. These are illustrated in Figure 3.1. The Agency could close the material balance with the verification of UO_2 pellets at the rod loading station (option 1), the verification of welded fuel rods awaiting assembly (option 2), and the verification of finished fuel assemblies (option 3). Each option has its advantages and disadvantages as will be discussed on a case by case basis.

3.1 CASE 1: ASSEMBLY COLLAR

For this case (option 3) it was assumed that the product verification was done using an NDA technique on the fuel assemblies. To investigate this method using INSPECT, the random and systematic analytical measurement error standard deviations were set at 0.01 (1%) for all product material strata. Product material occurs in beginning and ending inventories, as well as in shipments. It

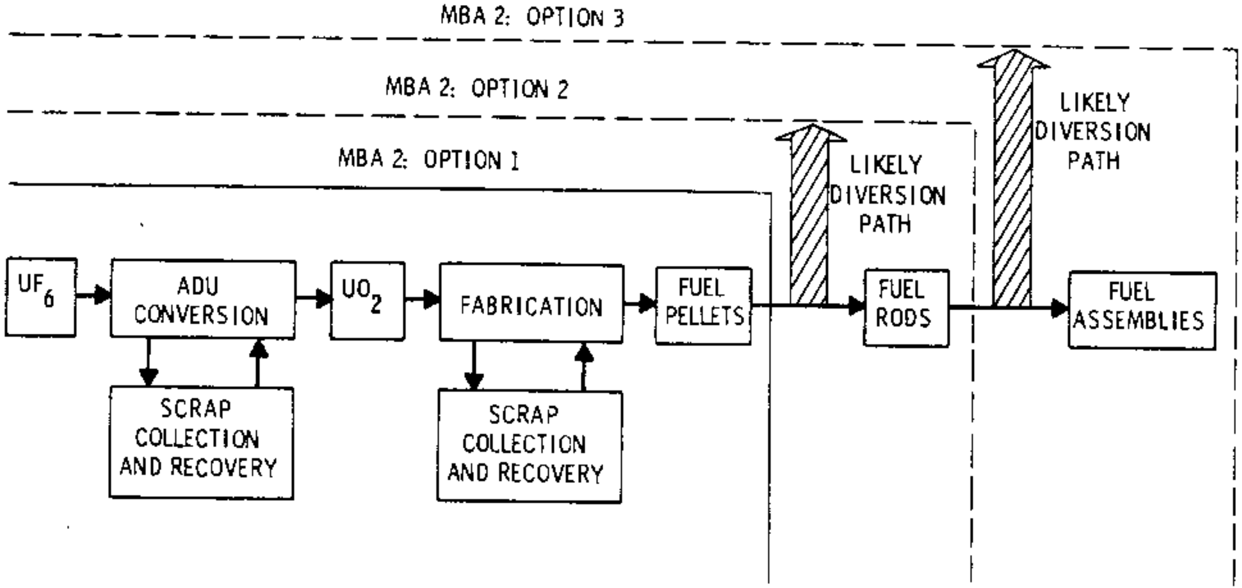

FIGURE 3.1. Material Balance Verification Options

is felt this value of measurement error (0.01) is optimistic for the yet uncompleted assembly collar being developed.[8] INSPECT runs using measurement error data of 5% and 10% were also examined, but the probability of detection achieved for these cases was very small.

There are many potential advantages to verifying fuel assemblies. Data on individual fuel assemblies is reported to the Agency. It would be difficult for the facility operator to alter the assemblies after verification as the inspection could be timed to correspond with shipment of the assemblies. Time between inspections could also be arranged such that the inspector could randomly sample from the entire material throughput. Also, all diversion paths throughout MBA are covered by a verification which spans the entire material balance area.

The biggest disadvantage to verifying fuel assemblies is that a suitable measurement capability does not exist at this time. Weighing the assembly is of little value as a possible substitute for enriched uranium is depleted uranium which would not alter the assembly weight. SAM-II use can verify enrichment of the outer rods, but verification of the interior rods is not possible. Film insertion between fuel rods to attempt to verify interior rods is being developed, but appears to require strict operating conditions and long exposure times.[8] Los Alamos Scientific Laboratory is developing a portable collar that would fit around the assembly. At present a suitable measurement technique just does not exist.

3.2 CASE 2: FUEL ROD SCANNER

Since assembly scanning may not be feasible, the next choice would be to scan the fuel rods (option 2). This involves active neutron interrogation, and determines the integrated total of ^{235}U in the fuel rod. These devices, while reasonably quick, are not very portable and are fairly expensive. The inspector could possibly use the facility's rod scanner with independent calibration standards. For this study, the random and systematic analytical measurement error standard deviations were set at 0.005 (0.5%).

Verifying fuel rods instead of fuel assemblies has opened up a new diversion path, alteration of the rod after inspection. In order to divert 75 kg of ^{235}U by rod falsification nearly 1000 dummy fuel rods would have to be manufactured.

Random sampling is also possible but requires substantial inspector effort. Rod storage areas in most facilities are large enough to hold about a one-month supply, hence inspections would have to be more frequent than monthly. Records traceability of fuel rod data to a reported quantity is possible, but the number of records to be audited is huge as a typical plant may produce 100,000 fuel rods per year. A sampling scheme may be worked out such that all records need not be audited.

3.3 CASE 3: PELLET VERIFICATION - CURRENT MEASUREMENT CAPABILITY

The next inspection approach to be examined was verifying the pellets at the rod loading station (option 1). This approach allowed for much better measurement of the product material strata. This is important because as measurement capability improves, detection probability improves. The inspector's random and systematic analytical measurement standard deviations for the product strata were 0.001. These are approximately the same in magnitude as those of the operator. The biggest advantage to verifying at this stage is the good measurement capability.

The biggest problem with verifying at this stage of the process is the lack of random sampling from the entire material throughput. Inspection activities to cover fuel rods and fuel assemblies must be added. The key diversion paths that open up concern items that have been verified, but falsified at a later date. These include material substitution in a finished fuel rod, or the substitution of a complete new rod to replace the one containing verified pellets. Another problem is the deliberate falsification of enrichment (overstating the enrichment for the purpose of diverting ^{235}U), as well as the pellet weights. Measurement capabilities are normally good enough to detect this type of diversion, but lack of random sampling may hamper effective verification.

Pellet verification will give the best material accountancy verification of the three approaches. If coupled with rod scanning as a check for dummy rods, diversion detection is probably maximized.

3.4 CASE 4: PELLET VERIFICATION - FUTURE HIGH QUALITY MEASUREMENT CAPABILITY

For this case it was assumed that very good chemical techniques were used to verify the pellet contents. The inspector's random and systematic measurement error standard deviations were set at 0.0005. This value may not be achievable at this time, but this case may serve as a lower bound on inspector measurement capability and the ability to verify material balance data.

3.5 CASE 5: FUEL ROD SCANNING WITH SCRAP MEASUREMENT AT 1%

One of the largest measurement uncertainties is associated with the scrap strata in beginning and ending inventory. As the inventory grows larger, these strata make a significant contribution to the standard deviations of MUF and $\hat{D}$. As expected, the larger these standard deviations get, the smaller the detection probability for the material balance verification.

For this case the inspector's random and systematic analytical measurement error standard deviation for scrap is 0.01, and for the product strata 0.005.

3.6 CASE 6: PELLET VERIFICATION WITH SCRAP MEASUREMENT AT 1%

This case analyzes the pellet verification safeguards approach with improved scrap measurement capability. The inspector's random and systematic analytical measurement error standard deviations for the product strata are 0.001, and for the scrap strata 0.01.

3.7 CASE 7: PELLET VERIFICATION WITH SCRAP MEASUREMENT AT 0.05%

This case analyzes the pellet verification safeguards approach with still further improved scrap measurement capability. Better techniques will be required in dealing with scrap for this capability to be reached. Work will have to be done in homogenizing the scrap and reducing it to a form that is measurable with high accuracy.

3.8 CASE 8: HIGH QUALITY PELLET VERIFICATION WITH SCRAP MEASUREMENT AT 0.05%

In terms of measurement capability, this case is the most optimistic of those examined, and also the least likely with today's techniques. It is being examined to represent a best case for the inspector in material accounting.

The eight cases are summarized in Table 3.1.

TABLE 3.1. Summary of Inspection Approach Cases

Case	Product Verification Strata	Inspector Product Measurement Errors		Inspector Scrap Measurement Errors	
		Random	Systematic	Random	Systematic
1	Fuel Assemblies	0.01	0.01	0.02	0.0163
2	Fuel Rods	0.005	0.005	0.02	0.0163
3	Fuel Pellets	0.001	0.001	0.02	0.0163
4	Fuel Pellets	0.0005	0.0005	0.02	0.0163
5	Fuel Rods	0.005	0.005	0.01	0.01
6	Fuel Pellets	0.001	0.001	0.01	0.01
7	Fuel Pellets	0.001	0.001	0.005	0.005
8	Fuel Pellets	0.0005	0.0005	0.005	0.005

4.0 INSPECTION EFFECTIVENESS AND EFFORT RESULTS

The diversion of material can occur in many ways. Figure 4.1 shows the diversion analysis in a fault tree format. Given the diversion (top event), there are two possibilities: either the diverter attempts to cover up the diversion by data falsification, or else the diverter relies on the inherent noise level of the system to hide the diversion. Most likely, a diverter would pursue a mixed strategy to avoid detection.[1]

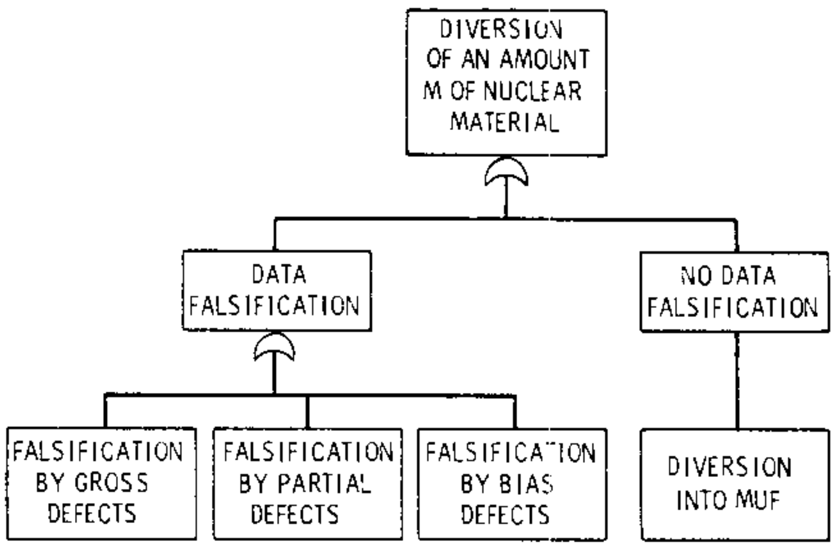

FIGURE 4.1. Basic Diversion Tree

4.1 EFFECTIVENESS

The four basic diversion paths of Figure 4.1 are reduced in INSPECT to two in order to make the mathematics more tractable. This is shown in Figure 4.2. Gross defects and partial defects are lumped together into a diversion path called "large defects." Large defects are those which can be detected by a single attributes or variables type measurement. The path involving bias defects and diversion into MUF are combined into a path called "diversion into MUF-$\hat{D}$." The mathematics underlying the reduced diversion tree is explained in Part F of the Safeguards Technical Manual.

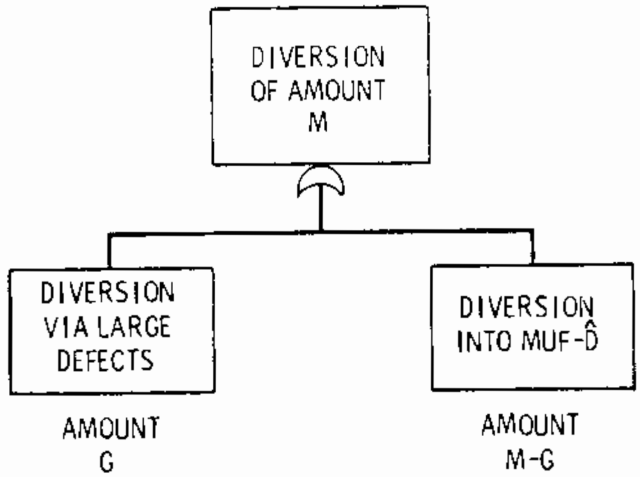

FIGURE 4.2. Reduced Diversion Tree

Appendix E contains the effectiveness data for the eight inspection cases outlined in Section 3. This data includes the goal quantity, the optimal amount to divert via large defects, the probability of failing to detect a diversion by the large defects paths, the probability of failing to detect a diversion by the MUF-$\hat{D}$ path, the overall probability of failing to detect a diversion, the induced nondetection probability based on NV_2 (bias detection), and the percent probability of detecting a diversion.

Figure 4.3 illustrates the behavior of the probability of detection versus plant size for the first four inspection approaches. Similar figures for all the inspection approaches are located in Appendix E. The first thing that becomes apparent is that detection probability drops off with increasing plant size. This was true for all inspection approaches. With the measurement errors for a case fixed, it becomes harder and harder to detect the diversion of a goal quantity as that goal quantity becomes a smaller and smaller fraction of the throughput.

In this study the goal quantity, expressed as a fraction of throughput, ranged from 1.25% for the 200-ton plant to 0.42% for the 600-ton plant. As could be expected, when the measurement errors for the product strata became smaller, the detection probability for each plant size increased. It should be noted from Figure 4.3 that Case 3 (pellet measurement = 0.001) and Case 4 (pellet measurement = 0.0005) give nearly identical results for each inventory

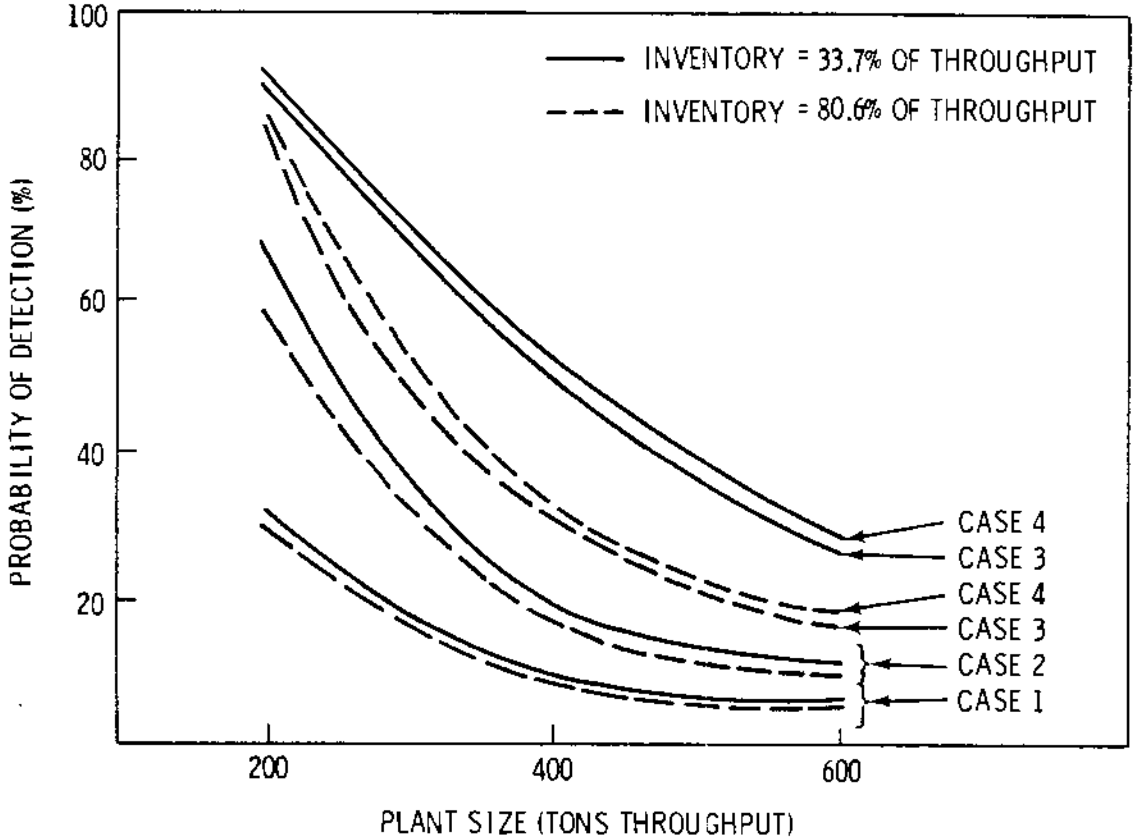

FIGURE 4.3. Probability of Detection vs. Plant Size for Various Inspector Product Measurement Capabilities

size, respectively. This result occurs because the product strata contribution to the variance of $\hat{D}$ becomes dwarfed by the other strata contributions, such as UF_6 cylinders and scrap. This is clearly shown in the tables of Appendix D where the major contributors to the variance of $\hat{C}$ are listed.

Figure 4.3 also shows the effect of increasing the inventory from 33.7% of throughput to 80.6%. In all four cases the increase in inventory size decreased the detection probability. Table 4.1 summarizes this effect for all eight cases. There are two related forces at work. As the amount of material in inventory gets larger, the inspector's variance of $\hat{D}$ grows larger more quickly than the variance of MUF. This is because the inspector has larger measurement errors, and has a limited sample size. As the difference of the variances grows, the probability of detection of a significant quantity decreases. This causes the decrease in detection probability for all cases. The other force involved is

TABLE 4.1. Inventory Size Effect on Detection Probability

Case	Inventory Size (%)	Detection Probability (%) for Plant Size (Tons Throughput) 200	400	600
1	33.7	31.8	9.3	6.4
	80.6	30.2	8.9	6.2
2	33.7	67.2	19.4	11.3
	80.6	59.3	16.6	10.0
3	33.7	90.5	50.1	26.5
	80.6	84.5	31.7	17.1
4	33.7	90.8	53.6	28.1
	80.6	86.1	32.8	17.6
5	33.7	68.1	19.7	11.5
	80.6	63.4	18.0	10.0
6	33.7	90.8	53.6	28.5
	80.6	89.0	40.8	21.5
7	33.7	91.1	55.6	29.7
	80.6	90.2	47.0	24.7
8	33.7	91.5	59.1	31.8
	80.6	90.5	49.7	26.1

the relative importance of the flow or inventory strata in their contribution to the variance. Cases 1 and 2 were dominated by the flow strata variances, hence a change in inventory did not affect detection probability much. Cases 3 and 4 had much improved measurement capability in the flow strata, hence inventory variances became relatively more important. The increased inventory size led to a substantial decrease in the detection probability. Cases 5, 6, 7, and 8 had fairly small decreases in detection probability as they included improved measurement capability in the scrap stratum, as well as the improved flow strata measurements. Cases 3 and 6 yielded similar results, and required no new developments in measurement technology. For Case 6, reducing the variance contribution of scrap can be accomplished by reducing the total amount of material on hand by timely recovery, or by improving the techniques of measuring. This can be accomplished by changing the scrap into a more measureable form (by dissolving, for example), or improving the actual measurement method. If improvements in scrap handling and measurement are not made, then Case 3, pellet verification at the fuel rod loading station, would appear to give the most desirable material verification results.

Another important effectiveness parameter is the nondetection probability based on NV_2, the variables sample size to detect bias defects. This nondetection probability refers to diversion in the $MUF-\hat{C}$ path which is bias defects and diversion into MUF. $MUF-\hat{D}$ is an important statistic as it is independent of the operator's systematic errors which may or may not be truly known to the inspector.[10] Examining the data in Appendix E shows that when plant size was 400 or 600 tons throughput, there was no material taken as large defects. Hence, the total detection probability was almost identical to the bias detection probability already discussed. The slight difference is due to the fact that the total detection probability is based on NV, while the bias detection probability is based on NV_2, and they are not identical numbers. This implies for the large, throughput-dominated plants the measurement capability as modeled is not good enough to detect a bias loss of a goal quantity of 75 kg ^{235}U at the 95% confidence level. As before, the probability of detection drops off substantially as the plant size increases, but as the inspector's measurement capability for the product stratum improves, the probability of detection for bias losses increases.

4.2 INSPECTION EFFORT

The Agency performs different types of inspections. These inspections include routine inspections, ad hoc inspections, and special inspections. This study concerns itself only with the routine inspections.

For a typical flow verification, inspection activities could be as follows:[1]

- Examination of records, verification of self-consistency and consistency with reports, updating book inventory, filling in all documents for samples, preparing samples for shipment;

- Application, examination, removal and replacement of seals;
- Servicing of surveillance equipment and review of films or tapes;
- Selection of items to be sampled, and observation of sampling;
- Item identification, counting and measurements of fuel rods and assemblies (NDA);
- Observation of the calibration and carrying out of calibration for necessary measurement equipment;
- Activities at the rod loading station (sampling, NDA);
- Verification of the quality of operator's measurement system including analytical and NDA equipment using independent standards.

Some of the major activities for inventory verification include:

- The above flow verification activities;
- Verification of the operator's physical inventory taking for completeness and accuracy;
- Weighing of containers with nuclear material on the basis of a random sampling plan;
- Taking accountability samples;
- Identification and counting of fuel assemblies and rods, and the use of NDA techniques for their verification.

These many activities can be grouped into five major categories as was done in ISPO Task C.5.(1) These are as follows:

- Planning and preparation
- Audit
- Check and service containment and surveillance devices
- Post-inspection and evaluation
- Verification of measurement data

Appendix F contains all the data and assumptions used for calculating man-days of effort for the five categories for each facility size.

The maximum routine inspection effort (MRIE) is defined as the maximum number of man-days or man-years of inspection per annum allowable for a facility as provided for in paragraph 80 of INFCIRC/153. For low enriched uranium fuel fabrication and conversion facilities, the value of MRIE in man-days is given as follows:

$$MRIE = 100 + 0.4\ E$$

where E = throughput in effective kilograms.

The IAEA safeguards glossary defines an effective kilogram as a special unit used in safeguarding nuclear material which reflects its strategic value. For uranium with an enrichment of 1% and above, an effective kilogram is the weight in kilograms multiplied by the square of the enrichment. Table 4.2 summarizes the MRIE for the facilities.

TABLE 4.2. MRIE for LEU Conversion and Fuel Fabrication Facilities

Plant Size (kg Throughput)	$(\text{Enrichment})^2$	Throughput in Effective kg (E)	MRIE (Man-Days)
2×10^5	0.0009	180	172
4×10^5	0.0009	360	244
6×10^5	0.0009	540	316

Of the five categories of effort, three involve time spent at facilities. These are audit, check and service containment and surveillance equipment, and verification of measurement data (sampling plan effort). Table 4.3 summarizes this effort for the facilities. Complete data is found in Appendix F. In no plant was the MRIE exceeded, and in the worst case barely half the maximum effort allowable was used. It does not appear that the MRIE is a constraint for LEU fabrication and conversion plants.

What could be a constraint in some cases is the actual routine inspection effort (ARIE). The ARIE is defined in the IAEA glossary as the inspection effort in man-days per annum agreed for a facility between the IAEA and the State. The ARIE is equal to or less than the MRIE, and is included in the

TABLE 4.3. Effort at Facilities for Routine Inspections

Plant Size (Tons Throughput)	Inventory Size (%)	Man-Days of Effort Audit	C/S Devices	Average Sampling	Average Total	Percent of MRIE
200	33.7	21	1	25	47	27.3
200	80.6	26	1	37	64	37.2
400	33.7	43	1	52	96	39.3
400	80.6	59	1	78	138	56.6
600	33.7	58	1	75	134	42.4
600	80.6	80	1	112	193	61.1

facility attachment of the subsidiary arrangements. The value of ARIE could vary from facility to facility. The final column of Table 4.3 shows the expended effort at the facility as a percentage of MRIE which may or may not exceed the ARIE for a particulary facility.

The sampling effort is made up of these components:[1]

- attributes sampling effort, NA
- variables sampling effort, NV
- item check effort.

For this study the attributes sampling effort typically made up about 50% of the total sampling plan effort, with variables sampling and item counts each accounting for about 25%. The attributes sampling plan was constant for each plant size for a given inventory, hence variations in the variables sample size due to changing inspection approach had little effect on the total sampling effort.

The change in inventory size had more of an effect on sampling effort than the changing inspection approach. Increasing the inventory size from 33.7% of throughput to 80.6% increased the sampling effort about 50% for all facilities. This effect is shown in Figure 4.4. However, much of the total effort involved in inspections of LEU facilities is not the sampling effort, and the increased inventory size made for a 25% increase in the total effort.

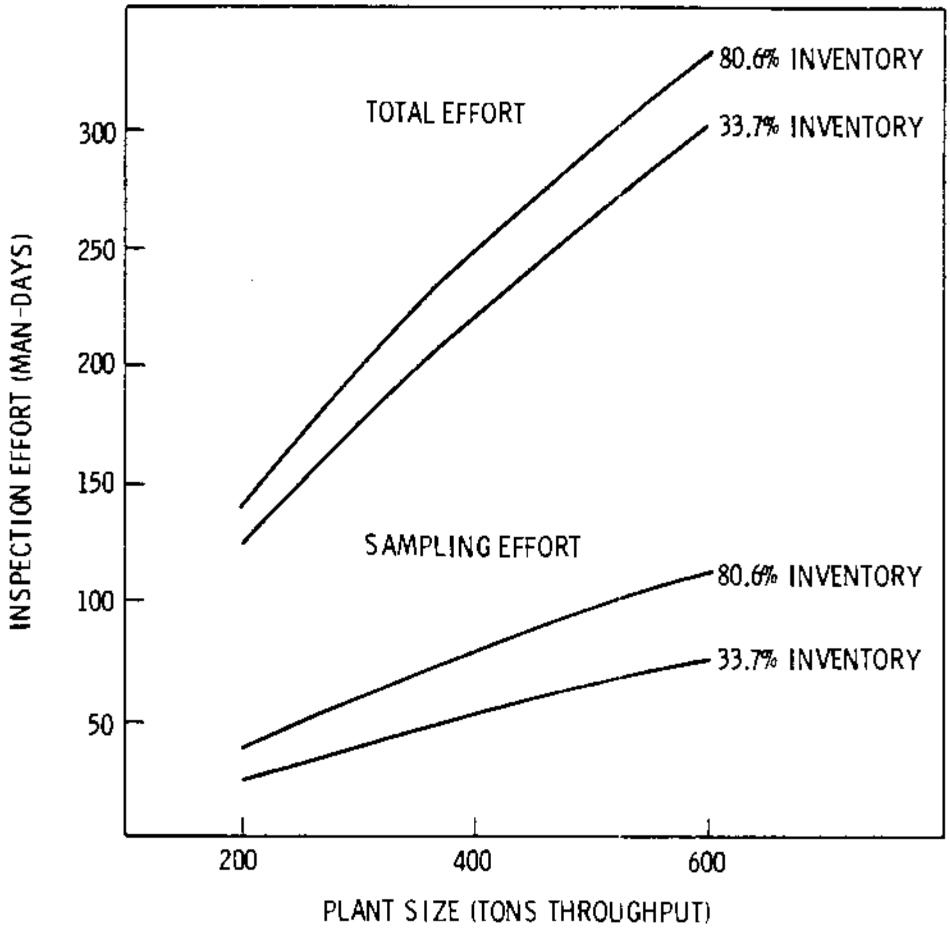

FIGURE 4.4. Average Total Effort and Sampling Effort in Man-Days for Various Plant Sizes and Inventories

4.3 EFFECTIVENESS FOR RESTRICTED VARIABLES SAMPLE SIZE

Completion of the full variable and attribute sampling plan calculated with INSPECT may not be achievable. Reasons for this include calculated sample sizes may overwhelm the analytical laboratory capacity of the Agency, lack of inspection manpower may not permit taking of all samples, and a complete sampling plan may cost too much to fully execute. For these reasons, a restricted sampling plan may have to be used. The restricted plan was studied by limiting the number of variables samples to a range of 40 to 50, and then calculating the material accounting effectiveness. The complete attributes plan is still in effect. This was done for the Case 6 measurement error data, fuel pellet verification with 1% scrap measurement.

To distribute the samples over the strata, the total number of variables samples, NV, was normalized to forty, and each stratum's sampling total multiplied by this normalizing factor. This gave a reduced sampling total for each stratum. For example, the 400-ton, 33.7% inventory plant required 93 variables samples to be taken. The UF_6 cylinder stratum in receipts made up 27 of these samples. For the restricted sample, this stratum would have

$$\frac{40}{93} \times 27 \sim 12 \text{ samples.}$$

Fractions of samples are rounded to the next highest whole number, and each stratum had to have at least one sample. For this reason, each plant did not have exactly 40 samples. The number of samples to detect gross defects, NA, was not altered from the full sampling plan.

As could be expected, taking fewer samples decreased the material accounting effectiveness. The only facility unaffected is the 200-ton, 33.7% inventory facility which required only 38 variables samples in the complete sampling plan. Table 4.4 summarizes the sample sizes and effectiveness for each plant size. It is interesting to note that for the larger facilities, cutting the variables sample size by a large amount did not affect the detection probability too much. One rapidly approaches the point of diminishing returns for variables samples in these facilities.[1]

TABLE 4.4. Detection Probability for Restricted Variables Sample Size

Plant Size (Tons Throughput)	Inventory Size (% of Throughput)	Restricted Variables Sample Size	Full Variables Sample Size	Detection Probability (Restricted Plan)	Detection Probability (Full Plan)
200	33.7	38	38	90.8	90.8
200	80.6	43	93	61.9	89.0
400	33.7	46	93	41.5	53.6
400	80.6	45	144	32.7	40.8
600	33.7	49	144	25.1	28.5
600	80.6	44	167	16.6	21.5

5.0 SUPPLEMENTAL INSPECTION ACTIVITIES FOR VARIOUS DIVERSION PATHS

Examination of the inspection approaches pointed out clearly that effective material accounting depends a great deal on inspector measurement capability, and this capability is most important in the flow strata. The best measurement capability for the inspector in the fuel product stratum is achieved at the fuel rod loading station. This in turn produced the best detection probabilities for all facilities examined. Fuel rod and fuel assembly verification do not appear to yield satisfactory detection probabilities.

There are several difficulties associated with verifying at the pellet loading station. The most important is that the Agency does not have the resources to do full flow verification at the fuel rod loading station. Current efforts will be limited to facility visits once or twice a month, the taking of pellet samples during the visit, measurement of some fuel rods using a rod scanner or SAM-II, and some type of fuel assembly measurement. Outside rods of the fuel assembly can be checked for content using the SAM-II, but complete assembly verification is under development at this time.

Ideally, the fuel product stratum to base verification on would be fuel assemblies. Unfortunately, the measurement capability does not yet exist to measure fuel assemblies accurately. Recent developments of a coincidence collar for the measurement of unirradiated fuel assemblies by the Los Alamos Scientific Laboratories show promise for the near future. This is also discussed in Section 3.1.

Backing up to verifying loaded fuel rods awaiting assembly also has many problems. Adequate measurement capability does not yet exist to assure the detection probability goals set forth by the Agency. A new diversion path is also opened; namely, alteration, or substitution of the fuel rod after verification, but prior to assembly. This path is shown in Figure 3.1. The alteration of the fuel rod could occur in three ways; either the substitution of material in the original rod, substitution of a completely new rod, or the removal without replacement of the rod. Removal without replacement of enough rods to acquire a goal quantity (862) does not seem likely. If no attempt to replace them is made, item checks would show the loss. Substitution of material would seem to be the most credible path.

Verifying at the pellet loading station produced the best material accounting case largely due to the improved measurement capability. This inspection approach also opens up diversion paths. As with the fuel rod verification, substitution of material could occur after verification. Also, removal of material after verification could occur. As before, removal of enough material to make a goal quantity without some sort of item substitution would lead to 862 fewer rods, a fuel rod item count would show this.

5.1 SUBSTITUTION OF MATERIAL

The ideal material for substitution of low enriched uranium is natural or depleted uranium. This would require some sort of NDA technique to discover discrepancies if one is to leave all the fuel rods intact.

Table 5.1 contains rod production data for the model facilities. This table is helpful in dealing with yearly material flows in the plant. The table also includes rod inventory and the average amount of time a rod remains in inventory before being made into fuel assemblies. The smaller inventory plants turn rods to bundles in about 11 days, while the larger inventory plants take 26 days.

TABLE 5.1. Rod Data for the Model Facilities

Plant Size (Tons Throughput)	Inventory Size (%)	Rods Produced Per Year	Rods Produced Per Month	Rods in Inventory	Rod Storage Time (Days)
200	33.7	68,275	5,690	2,016	11
400	33.7	136,550	11,380	4,032	11
600	33.7	204,825	17,070	6,048	11
200	80.6	68,275	5,690	4,788	26
400	80.6	136,550	11,380	9,576	26
600	80.6	204,825	17,070	14,364	26

The diverter could subsitute 100% of the rod, or some fraction of a rod. Obviously, the higher the percentage of the rod he substitutes, the fewer defects necessary to obtain a goal quantity. To detect this type of diversion with 95% probability, an attributes test can be performed on a population sample. The size of this sample, Na, is found as follows:

$$Na = N\,[1 - 0.05^{1/r}]$$

where $r = \dfrac{\text{Goal Quantity}}{\text{Average Item Weight}} = \dfrac{M}{X} = \dfrac{2500\ \text{kg}}{2.9\ \text{kg}}$.

N= Number of items.

For fuel rods, r= 862. Table 5.2 summarizes the number of defects for various substitution strategies, and the number of samples needed to detect such falsification.

TABLE 5.2. Defects by Substitution and Sampling Plan for 95% Detection Probability

Defect Size as Percentage of Item Weight	No. of Defects	Attribute Samples per Year for 95% Detection Plant Size (Tons) 200	400	600
100.00	862	237	474	711
50.00	1,724	119	237	356
25.00	3,448	59	119	178
10.00	8,620	24	47	71
1.25	68,960	3	6	9

For the small inventory plants, frequent flow verification inspections would be required to assure that the entire population of rods had a chance to be sampled. For the larger inventory plants, monthly inspections would be required for example. Using a SAM-II detector, and scanning the entire rod, as many as four rods an hour can be examined for substitution. The SAM-II would be effective in determining substitutions using natural or depleted uranium. If the inspector assumes there is 100% material substitution (giving the smallest number of defects) the maximum number of samples results. This assumption would then ensure all other substitution schemes in Table 5.2 were covered. Table 5.3 shows the number of samples per visit needed to be

TABLE 5.3. Number of Samples Taken per Monthly Inspector Visit

Defect Size as Percentage of Item Weight	Plant Size (Tons Throughput) 200	400	600
100.00	20	40	60
50.00	10	20	30
25.00	5	10	15
10.00	2	4	6
1.25	1	1	1

taken to fulfill the sampling plan. In the largest plants, two days would be required to sample the necessary rods if the flow verification was monthly.

Other possibilities for detecting fuel substitution that are under consideration include examination of data from the reactors where fuel is sent. Key data would include neutron fluxes at detector points, radial and axial power distributions, and core reactivity levels in the form of soluble boron concentration levels for criticality in a PWR's, critical control rod patterns for BWR's, and shutdown margins.[9] The reactor cycle and burnups could also be affected.

To summarize this diversion path, the diverter will be creating a large number of defects in order to obtain a goal quantity. The more defects that are created, the easier it will be for the inspector to detect a peculiar rod. To ensure that the inspector samples from the entire fuel rod population, frequent visits may be required. With frequent, short visits, and proper use of NOA equipment, diversion by material substitution will be difficult.

5.2 PELLET WEIGHT OR ENRICHMENT FALSIFICATION

Most of the diversion calculated by INSPECT was not by gross defects but by diversion into MUF-$\hat{D}$. This is made of bias defects and diversion into MUF.

Another possible diversion scenario could then be the purposeful overstatement of enrichment or weights of pellets to mask removal of material elsewhere in the facility. Removal after the fuel rod loading station was discussed in the previous section. It is quite common for LEU fabrication and conversion facilities to keep natural UF_6 on-site to adjust the enrichment of feed UF_6 slightly. The 200-ton facility runs 6,000 kg of ^{235}U through the facility a year. If 75 kg of ^{235}U were siphoned out at the UF_6 station and natural UF_6 substituted, the overall plant enrichment of the pellets drops from 3.00% to 2.96%. This small difference is probably not detectable by NDA methods after the fuel rod loading station. Table 5.4 summarizes the enrichments and time required to divert one goal quantity for the various plants. If it is assumed that there is a week between the time the facility knows an inspector is coming and the time he arrives, it is safe to assume the pellets at the loading station that the inspector sees will actually be 3.00%, not overstated pellets.

TABLE 5.4. Necessary Enrichment and Time Frame to Divert 75 kg ^{235}U by Enrichment Falsification

Enrichment (%)	Time Necessary to Divert a Goal Quantity for Plant Size (Tons) 200	400	600
2.96	1 yr.	6 mo.	4 mo.
2.98	--	1 yr.	8 mo.
2.99	--	--	1 yr.

One possible solution opens up because the rods containing overstated pellets must still be in inventory awaiting assembly when the inspector arrives. If the Agency were allowed to perform destructive chemical analysis of some pellets in a rod, the falsification could be shown.

For a pressurized water reactor core, approximately 70 assemblies per year are loaded into the core. For a 17 x 17 matrix with 25 nonfuel rods per assembly, this amounts to 18,480 rods per reload. For a boiling water reactor core replacing 244 assemblies per year in an 8 x 8 matrix and one nonfuel rod, this amounts to 15,372 rods per reload. Table 5.5 shows the yearly production of the various plants, and indicates the number of reloads per year the facility produces. Current practice is to reload PWR's with one uniform enrichment, and BWR's usually contain four enrichments per assembly for the reload. Excluding the United States and the U.S.S.R., the world ratio of PWR to BWR is about 3 to 1, hence we will assume the average facility produces reloads in this ratio.

TABLE 5.5. Rod Production for Different Sized Facilities

Plant Size (Tons Throughput)	Rods Produced Per Year	Rods Produced Per Month	PWR Reloads Per Year	BWR Reloads Per Year
200	68,275	5,690	3	1
400	136,550	11,380	6	2
600	204,825	17,070	9	3

The next question is how many rods per reload or batch should be sampled in this manner. To study this, power curves for a couple of situations are shown. The first case is Figure 5.1. Here the stated enrichment (null hypothesis) is 3.00. The variance of the measured enrichment, which reflects process variance and measurement variance is σ = 0.02 on the enrichment, or 0.67%. This variance is representative of quality control practices in the United States.

The power curves represent the probability of making a Type II error, β. That is the probability of accepting a false null hypothesis. The value of β is large if alternatives to the null hypothesis are close to the value of the null hypothesis, for example, an alternative enrichment value of 2.98 instead of 3.00. Conversely, the value of β is small if the alternative and null hypothese differ substantially. This reflects the fact that alternatives which are only slightly different from the null hypothesis will be difficult to discover, and large differences will be easier to recognize. The method of calculation of the power of the test is shown in Appendix G.

If only one sample is taken (N=1), Figure 5.1 shows that the probability of making a Type II error is less than 10% (i.e., the power = 1-β is greater than 90%) for enrichments 2.90 to greater than 2.94. For an enrichment alternative of 2.96, the power is still 64%. As the number of samples increases slightly, the power makes substantial improvement, particularly at the low end of alternative enrichments. One sample is likely to be adequate under these conditions.

Figure 5.2 shows power curves for a slightly different case. The null hypothesis is still 3.00, but it is assumed that the variance is much worse, σ = 0.05 of enrichment, or 1.67%. As can be expected, the larger the variance,

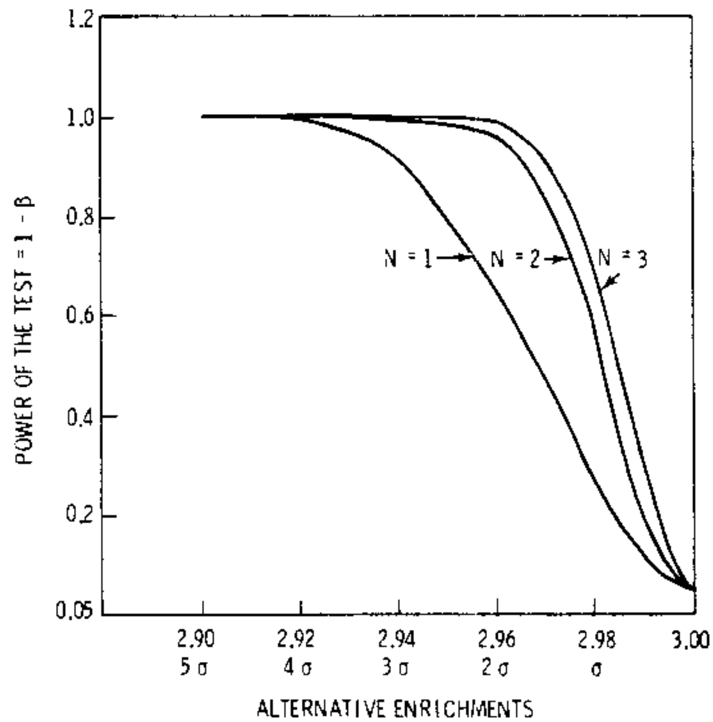

FIGURE 5.1. Power Curves for Pellet Enrichment Falsification, μ = 3.00, σ = 0.02

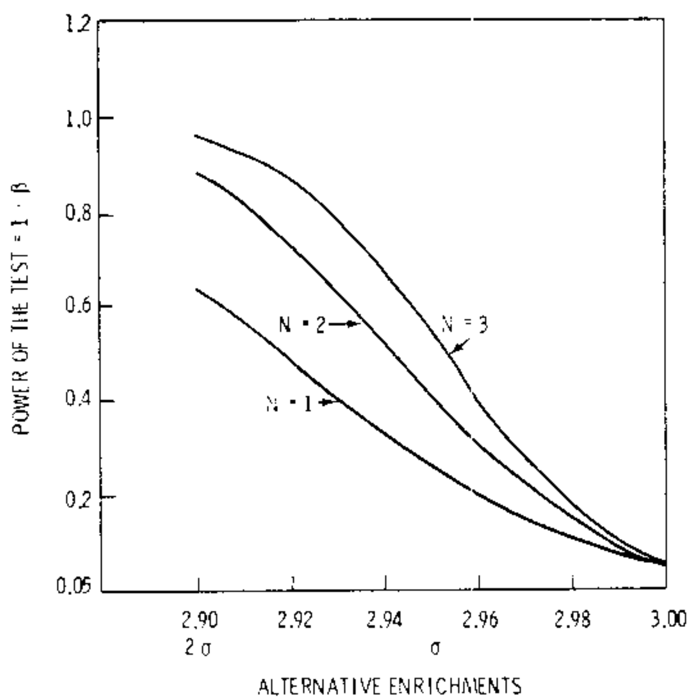

FIGURE 5.2. Power Curves for Pellet Enrichment Falsification, μ = 3.00, σ = 0.05

the more difficult to discover a significant discrepancy. Even under these poor conditions, the number of samples need not be large to give a reasonable power.

Another possibility of overstatement may be the pellet weights. The pellets in the fuel rods could be made somewhat smaller than reported in design specifications, and material removed. If complete pellets are missing, or have been substituted for, the loss should be detected using NDA methods from Section 5.1. Table 5.6 shows how many rods would be effected for various amounts of weight overstatements. Weighing of pellet stacks at the loading station would be one method of checking on this type of diversion, but it will not be enough. To sample rods in inventory, an active neutron device such as a rod scanner which determines total ^{235}U in the fuel rod should be used. The sheer number of rods which would be defective at low falsification rates makes this an unattractive diversion path. Detection is almost certain by quality rod scanning.

TABLE 5.6. Pellet Weight Falsification Data to Obtain a Goal Quantity

Weight Overstatement Rate per Rod (Percent)	Pellets per Rod Equivalent	Number of Defects (Rods)	Attribute Samples per Year for 95% Detection Plant Size (Tons Throughout)		
			200	400	600
10	27.1	8,621	24	47	71
5	13.5	17,241	12	24	36
3	8.1	28,736	7	14	21
1	2.7	86,210	2	5	7

5.3 DIVERSION OF BULK MATERIAL

Another loss scenario is the diversion and coverup of bulk materials. Without an attempt to hide the diversion, it would seem that 2,500 kg of material missing would be noticed in item checks. This would be true if the material were in the form of fuel rods, or powder. The proper number of fuel rods or buckets would just not be found.

Perhaps the easiest and most attractive coverup would be overstatement of waste and scrap barrels. As these strata are the most difficult to measure, they produce large measurement variances. The requiring of timely recovery of scrap will make using that stratum as a diversion path difficult. Dealing with waste barrels is more complicated. Using a barrel scanner, random driver (active neutron system), or segmented gamma scanner will aid in the detection of falsification. Arranging to have the waste transformed into a more measurable form will also help measurement accuracies.

REFERENCES

1. Mullen, M. F., and M. A. Wincek. 1979. Estimation of Inspection Effort. ISPO-35 (PNL-2588), Pacific Northwest Laboratory, Richland, Washington.

2. Wincek, M. A., and M. F. Mullen. 1979. INSPECT: A Package of Computer Programs for Planning Safeguards Inspections. ISPO-58 (PNL-2559), Pacific Northwest Laboratory, Richland Washington.

3. International Atomic Energy Agency (IAEA). 1972. The Structure and Content of Agreements Between the Agency and States Required in Connection with the Treaty on the Non-Proliferation of Nuclear Weapons. IAEA Publication No. INFCIRC/153 (p. 9), Vienna, Austria.

4. Dixon, W. J., and F. J. Massey. 1969. Introduction to Statistical Analysis, McGraw-Hill, Inc., New York, New York.

5. Jaech, J. L. 1973. Statistical Methods in Nuclear Material Control, TID-26298, Technical Information Center, Springfield, Virgina.

6. Mullen, M. F., and P. T. Reardon. 1981. Analysis of the Input of Safeguards Criteria. ISPO-139 (PNL-3711), Pacific Northwest Laboratory, Richland, Washington.

7. Hough, G., et. al. 1979. "Technical Criteria For the Application of IAEA Safeguards." In Proceedings of a Symposium on Nuclear Material Safeguards, IAEA-SM-231/112, Vol. 1. Vienna, Austria.

8. Gardner, S. D., ed. 1980. Nuclear Safeguards Research and Development Program Status Report, July-September 1979. LA-8125-PR. Los Alamos Scientific Laboratory, Los Alamos, New Mexico.

9. Goldstein, L., Bellon, L., and Q. Dahodwala. 1979. Diversion Hazards for Light Water Reactors. ISPO-65. S. M. Stoller Corporation, New York, New York.

10. Hough, C. G., Jaech, J. L, et. al. 1974. Example of Verification and Acceptance of Operator Data-Low Enriched Uranium Fabrication. BNWL-1852, Pacific Northwest Laboratory, Richland, Washington.

APPENDIX A

PLANT MATERIAL STRATA DESCRIPTIONS FOR 200, 400 and 600 TON THROUGHPUTS WITH 33.7% AND 80.6% INVENTORIES

TABLE A-1. Notations

X(IJKQPT)	=AVERAGE ITEM WEIGHT
QOP	= OPERATOR BULK MEASUREMENT METHOD INDEX
QIN	= INSPECTOR BULK MEASUREMENT METHOD INDEX
POP	= OPERATOR MATERIAL SAMPLING METHOD INDEX
PIN	= INSPECTOR MATERIAL SAMPLING METHOD INDEX
TOP	= OPERATOR ANALYTICAL MEASUREMENT METHOD INDEX
TIN	= INSPECTOR ANALYTICAL MEASUREMENT METHOD INDEX
M(K)	= NUMBER OF BATCHES
N(K)	= NUMBER OF ITEMS PER BATCH
R(K)	= NUMBER OF SAMPLES PER BATCH (OPERATOR)
C(K)	= NUMBER OF ANALYSES PER SAMPLE (OPERATOR)
U(K)	= NUMBER OF BATCHES MEASURED BY INSPECTOR
W(K)	= NUMBER OF SAMPLES PER BATCH FOR WHICH INSPECTOR MAKES BULK MEASUREMENTS
V(K)	= NUMBER OF SAMPLES PER BATCH FOR WHICH INSPECTOR MAKES ELEMENT DETERMINATION
A(K)	= NUMBER OF ANALYSES PER SAMPLED ITEM (INSPECTOR)
B(K)	= NUMBER OF LABS USED BY INSPECTOR IN THE STRATUM

TABLE A-2. 200 Ton Plant 33.7% Inventory LEU

BEGINNING INVENTORY

STRATUM NAME AND INSPECTOR LAB INDEX	X(IJKQPT)	QOP	QIN	POP	PIN	TOP	TIN	M(K)	N(K)	R(K)	C(K)	U(K)	W(K)	V(K)	A(K)	B(K)
UF6 CYLINDERS (1 0 0 0 0 0)	1450.000	1	20	1	20	1	19	20	1	3	1	1	1	1	1	1
UO2 POWDER (1 0 0 0 0 0)	13.000	2	21	2	19	2	20	33	30	2	1	1	1	1	1	1
UO2 PELLETS (1 0 0 0 0 0)	13.000	3	22	3	21	3	21	17	30	3	1	1	1	1	1	1
U3O8 POWDER (1 0 0 0 0 0)	20.000	4	23	4	22	4	22	5	20	2	1	1	1	1	1	1
SCRAP (1 0 0 0 0 0)	10.000	5	24	5	23	5	23	20	50	2	1	1	1	1	1	1
WASTE (1 0 0 0 0 0)	0.500	6	25	6	24	6	24	500	1	1	1	1	1	1	1	1
MISCELLANEOUS (1 0 0 0 0 0)	4.000	7	26	7	25	7	25	250	1	2	1	1	1	1	1	1
FUEL RODS (1 0 0 0 0 0)	2.900	8	27	8	26	8	26	32	63	3	1	1	1	1	1	1

RECEIPTS

STRATUM NAME AND INSPECTOR LAB INDEX	X(IJKQPT)	QOP	QIN	POP	PIN	TOP	TIN	M(K)	N(K)	R(K)	C(K)	U(K)	W(K)	V(K)	A(K)	B(K)
UF6 CYLINDERS (1 0 0 0 0 0)	1450.000	9	29	9	28	9	28	138	1	3	1	1	1	1	1	1

SHIPMENTS

STRATUM NAME AND INSPECTOR LAB INDEX	X(IJKQPT)	QOP	QIN	POP	PIN	TOP	TIN	M(K)	N(K)	R(K)	C(K)	U(K)	W(K)	V(K)	A(K)	B(K)
RODS (1 0 0 0 0 0)	2.900	11	30	10	29	10	29	1085	63	3	1	1	1	1	1	1
WASTE (1 0 0 0 0 0)	0.500	12	31	11	30	11	30	3741	1	1	1	1	1	1	1	1

ENDING INVENTORY

STRATUM NAME AND INSPECTOR LAB INDEX	X(IJKQPT)	QOP	QIN	POP	PIN	TOP	TIN	M(K)	N(K)	R(K)	C(K)	U(K)	W(K)	V(K)	A(K)	B(K)
UF6 CYLINDERS (1 0 0 0 0 0)	1450.000	9	32	9	31	9	31	20	1	3	1	1	1	1	1	1
UO2 POWDER (1 0 0 0 0 0)	13.000	13	33	10	32	12	32	33	30	2	1	1	1	1	1	1
UO2 PELLETS (1 0 0 0 0 0)	13.000	14	34	13	33	13	33	17	30	3	1	1	1	1	1	1
U3O8 POWDER (1 0 0 0 0 0)	20.000	15	35	14	34	14	34	5	20	2	1	1	1	1	1	1
SCRAP (1 0 0 0 0 0)	10.000	16	36	15	35	15	35	20	50	2	1	1	1	1	1	1
WASTE (1 0 0 0 0 0)	0.500	17	37	16	36	16	36	500	1	1	1	1	1	1	1	1
MISCELLANEOUS (1 0 0 0 0 0)	4.000	18	38	17	37	17	37	250	1	2	1	1	1	1	1	1
FUEL RODS (1 0 0 0 0 0)	2.900	19	39	18	38	18	38	32	63	3	1	1	1	1	1	1

TABLE A-3. 400 Ton Plant 33.7% Inventory LEU

BEGINNING INVENTORY

STRATUM NAME AND INSPECTOR LAB INDEX	X(IJKQPT)	QOP	QIN	POP	PIN	TOP	TIN	M(K)	N(K)	R(K)	C(K)	U(K)	W(K)	V(K)	A(K)	B(K)
UF6 CYLINDERS (1 0 0 0 0 0)	1450.000	1	20	1	20	1	19	40	1	3	1	1	1	1	1	1
UO2 POWDER (1 0 0 0 0 0)	13.000	2	21	2	19	2	20	66	30	2	1	1	1	1	1	1
UO2 PELLETS (1 0 0 0 0 0)	13.000	3	22	3	21	3	21	34	30	3	1	1	1	1	1	1
U3O8 POWDER (1 0 0 0 0 0)	20.000	4	23	4	22	4	22	10	20	2	1	1	1	1	1	1
SCRAP (1 0 0 0 0 0)	10.000	5	24	5	23	5	23	40	50	2	1	1	1	1	1	1
WASTE (1 0 0 0 0 0)	0.500	6	25	6	24	6	24	1000	1	1	1	1	1	1	1	1
MISCELLANEOUS (1 0 0 0 0 0)	4.000	7	26	7	25	7	25	500	1	2	1	1	1	1	1	1
FUEL RODS (1 0 0 0 0 0)	2.900	8	27	8	26	8	26	64	63	3	1	1	1	1	1	1

RECEIPTS

STRATUM NAME AND INSPECTOR LAB INDEX	X(IJKQPT)	QOP	QIN	POP	PIN	TOP	TIN	M(K)	N(K)	R(K)	C(K)	U(K)	W(K)	V(K)	A(K)	B(K)
UF6 CYLINDERS (1 0 0 0 0 0)	1450.000	9	29	9	28	9	28	276	1	3	1	1	1	1	1	1

SHIPMENTS

STRATUM NAME AND INSPECTOR LAB INDEX	X(IJKQPT)	QOP	QIN	POP	PIN	TOP	TIN	M(K)	N(K)	R(K)	C(K)	U(K)	W(K)	V(K)	A(K)	B(K)
RODS (1 0 0 0 0 0)	2.900	11	30	10	29	10	29	2170	63	3	1	1	1	1	1	1
WASTE (1 0 0 0 0 0)	0.500	12	31	11	30	11	30	7482	1	1	1	1	1	1	1	1

ENDING INVENTORY

STRATUM NAME AND INSPECTOR LAB INDEX	X(IJKQPT)	QOP	QIN	POP	PIN	TOP	TIN	M(K)	N(K)	R(K)	C(K)	U(K)	W(K)	V(K)	A(K)	B(K)
UF6 CYLINDERS (1 0 0 0 0 0)	1450.000	9	32	9	31	9	31	40	1	3	1	1	1	1	1	1
UO2 POWDER (1 0 0 0 0 0)	13.000	13	33	10	32	12	32	66	30	2	1	1	1	1	1	1
UO2 PELLETS (1 0 0 0 0 0)	13.000	14	34	13	33	13	33	34	30	3	1	1	1	1	1	1
U3O8 POWDER (1 0 0 0 0 0)	20.000	15	35	14	34	14	34	10	20	2	1	1	1	1	1	1
SCRAP (1 0 0 0 0 0)	10.000	16	36	15	35	15	35	40	50	2	1	1	1	1	1	1
WASTE (1 0 0 0 0 0)	0.500	17	37	16	36	16	36	1000	1	1	1	1	1	1	1	1
MISCELLANEOUS (1 0 0 0 0 0)	4.000	18	38	17	37	17	37	500	1	2	1	1	1	1	1	1
FUEL RODS (1 0 0 0 0 0)	2.900	19	39	18	38	18	38	64	63	3	1	1	1	1	1	1

TABLE A-4. 600 Ton Plant 33.7% Inventory LEU

BEGINNING INVENTORY

STRATUM NAME AND INSPECTOR LAB INDEX	X(IJKQPT)	QOP	QIN	POP	PIN	TOP	TIN	M(K)	N(K)	R(K)	C(K)	U(K)	W(K)	V(K)	A(K)	B(K)
UF6 CYLINDERS (1 0 0 0 0 0)	1450.000	1	20	1	20	1	19	60	1	3	1	1	1	1	1	1
UO2 POWDER (1 0 0 0 0 0)	13.000	2	21	2	19	2	20	99	30	2	1	1	1	1	1	1
UO2 PELLETS (1 0 0 0 0 0)	13.000	3	22	3	21	3	21	51	30	3	1	1	1	1	1	1
U3O8 POWDER (1 0 0 0 0 0)	20.000	4	23	4	22	4	22	15	20	2	1	1	1	1	1	1
SCRAP (1 0 0 0 0 0)	10.000	5	24	5	23	5	23	60	50	2	1	1	1	1	1	1
WASTE (1 0 0 0 0 0)	0.500	6	25	6	24	6	24	1500	1	1	1	1	1	1	1	1
MISCELLANEOUS (1 0 0 0 0 0)	4.000	7	26	7	25	7	25	750	1	2	1	1	1	1	1	1
FUEL RODS (1 0 0 0 0 0)	2.900	8	27	8	26	8	26	96	63	3	1	1	1	1	1	1

RECEIPTS

STRATUM NAME AND INSPECTOR LAB INDEX	X(IJKQPT)	QOP	QIN	POP	PIN	TOP	TIN	M(K)	N(K)	R(K)	C(K)	U(K)	W(K)	V(K)	A(K)	B(K)
UF6 CYLINDERS (1 0 0 0 0 0)	1450.000	9	29	9	28	9	28	414	1	3	1	1	1	1	1	1

SHIPMENTS

STRATUM NAME AND INSPECTOR LAB INDEX	X(IJKQPT)	QOP	QIN	POP	PIN	TOP	TIN	M(K)	N(K)	R(K)	C(K)	U(K)	W(K)	V(K)	A(K)	B(K)
RODS (1 0 0 0 0 0)	2.900	11	30	10	29	10	29	3255	63	3	1	1	1	1	1	1
WASTE (1 0 0 0 0 0)	0.500	12	31	11	30	11	30	11225	1	1	1	1	1	1	1	1

ENDING INVENTORY

STRATUM NAME AND INSPECTOR LAB INDEX	X(IJKQPT)	QOP	QIN	POP	PIN	TOP	TIN	M(K)	N(K)	R(K)	C(K)	U(K)	W(K)	V(K)	A(K)	B(K)
UF6 CYLINDERS (1 0 0 0 0 0)	1450.000	9	32	9	31	9	31	60	1	3	1	1	1	1	1	1
UO2 POWDER (1 0 0 0 0 0)	13.000	13	33	10	32	12	32	99	30	2	1	1	1	1	1	1
UO2 PELLETS (1 0 0 0 0 0)	13.000	14	34	13	33	13	33	51	30	3	1	1	1	1	1	1
U3O8 POWDER (1 0 0 0 0 0)	20.000	15	35	14	34	14	34	15	20	2	1	1	1	1	1	1
SCRAP (1 0 0 0 0 0)	10.000	16	36	15	35	15	35	60	50	2	1	1	1	1	1	1
WASTE (1 0 0 0 0 0)	0.500	17	37	16	36	16	36	1000	1	1	1	1	1	1	1	1
MISCELLANEOUS (1 0 0 0 0 0)	4.000	18	38	17	37	17	37	750	1	2	1	1	1	1	1	1
FUEL RODS (1 0 0 0 0 0)	2.900	19	39	18	38	18	38	96	63	3	1	1	1	1	1	1

TABLE A-5. 200 Ton Plant 80.6% Inventory LEU

BEGINNING INVENTORY

STRATUM NAME AND INSPECTOR LAB INDEX	X(IJKQPT)	QOP	QIN	POP	PIN	TOP	TIN	M(K)	N(K)	R(K)	C(K)	U(K)	W(K)	V(K)	A(K)	B(K)
UF6 CYLINDERS (1 0 0 0 0 0)	1450.000	1	20	1	20	1	19	48	1	3	1	1	1	1	1	1
UO2 POWDER (1 0 0 0 0 0)	13.000	2	21	2	19	2	20	79	30	2	1	1	1	1	1	1
UO2 PELLETS (1 0 0 0 0 0)	13.000	3	22	3	21	3	21	40	30	3	1	1	1	1	1	1
U3O8 POWDER (1 0 0 0 0 0)	20.000	4	23	4	22	4	22	12	20	2	1	1	1	1	1	1
SCRAP (1 0 0 0 0 0)	10.000	5	24	5	23	5	23	47	50	2	1	1	1	1	1	1
WASTE (1 0 0 0 0 0)	0.500	6	25	6	24	6	24	1187	1	1	1	1	1	1	1	1
MISCELLANEOUS (1 0 0 0 0 0)	4.000	7	26	7	25	7	25	593	1	2	1	1	1	1	1	1
FUEL RODS (1 0 0 0 0 0)	2.900	8	27	8	26	8	26	76	63	3	1	1	1	1	1	1

RECEIPTS

STRATUM NAME AND INSPECTOR LAB INDEX	X(IJKQPT)	QOP	QIN	POP	PIN	TOP	TIN	M(K)	N(K)	R(K)	C(K)	U(K)	W(K)	V(K)	A(K)	B(K)
UF6 CYLINDERS (1 0 0 0 0 0)	1450.000	9	29	9	28	9	28	138	1	3	1	1	1	1	1	1

SHIPMENTS

STRATUM NAME AND INSPECTOR LAB INDEX	X(IJKQPT)	QOP	QIN	POP	PIN	TOP	TIN	M(K)	N(K)	R(K)	C(K)	U(K)	W(K)	V(K)	A(K)	B(K)
RODS (1 0 0 0 0 0)	2.900	11	30	10	29	10	29	1085	63	3	1	1	1	1	1	1
WASTE (1 0 0 0 0 0)	0.500	12	31	11	30	11	30	3741	1	1	1	1	1	1	1	1

ENDING INVENTORY

STRATUM NAME AND INSPECTOR LAB INDEX	X(IJKQPT)	QOP	QIN	POP	PIN	TOP	TIN	M(K)	N(K)	R(K)	C(K)	U(K)	W(K)	V(K)	A(K)	B(K)
UF6 CYLINDERS (1 0 0 0 0 0)	1450.000	9	32	9	31	9	31	48	1	3	1	1	1	1	1	1
UO2 POWDER (1 0 0 0 0 0)	13.000	13	33	10	32	12	32	79	30	2	1	1	1	1	1	1
UO2 PELLETS (1 0 0 0 0 0)	13.000	14	34	13	33	13	33	40	30	3	1	1	1	1	1	1
U3O8 POWDER (1 0 0 0 0 0)	20.000	15	35	14	34	14	34	12	20	2	1	1	1	1	1	1
SCRAP (1 0 0 0 0 0)	10.000	16	36	15	35	15	35	47	50	2	1	1	1	1	1	1
WASTE (1 0 0 0 0 0)	0.500	17	37	16	36	16	36	1187	1	1	1	1	1	1	1	1
MISCELLANEOUS (1 0 0 0 0 0)	4.000	18	38	17	37	17	37	593	1	2	1	1	1	1	1	1
FUEL RODS (1 0 0 0 0 0)	2.900	19	39	18	38	18	38	76	63	3	1	1	1	1	1	1

TABLE A-6. 400 Ton Plant 80.6% Inventory LEU

BEGINNING INVENTORY

STRATUM NAME AND INSPECTOR LAB INDEX	X(IJKQPT)	QOP	QIN	POP	PIN	TOP	TIN	M(K)	N(K)	R(K)	C(K)	U(K)	W(K)	V(K)	A(K)	B(K)
UF6 CYLINDERS (1 0 0 0 0 0)	1450.000	1	20	1	20	1	19	96	1	3	1	1	1	1	1	1
UO2 POWDER (1 0 0 0 0 0)	13.000	2	21	2	19	2	20	158	30	2	1	1	1	1	1	1
UO2 PELLETS (1 0 0 0 0 0)	13.000	3	22	3	21	3	21	80	30	3	1	1	1	1	1	1
U3O8 POWDER (1 0 0 0 0 0)	20.000	4	23	4	22	4	22	24	20	2	1	1	1	1	1	1
SCRAP (1 0 0 0 0 0)	10.000	5	24	5	23	5	23	94	50	2	1	1	1	1	1	1
WASTE (1 0 0 0 0 0)	0.500	6	25	6	24	6	24	2374	1	1	1	1	1	1	1	1
MISCELLANEOUS (1 0 0 0 0 0)	4.000	7	26	7	25	7	25	1186	1	2	1	1	1	1	1	1
FUEL RODS (1 0 0 0 0 0)	2.900	8	27	8	26	8	26	152	63	3	1	1	1	1	1	1

RECEIPTS

STRATUM NAME AND INSPECTOR LAB INDEX	X(IJKQPT)	QOP	QIN	POP	PIN	TOP	TIN	M(K)	N(K)	R(K)	C(K)	U(K)	W(K)	V(K)	A(K)	B(K)
UF6 CYLINDERS (1 0 0 0 0 0)	1450.000	9	29	9	28	9	28	276	1	3	1	1	1	1	1	1

SHIPMENTS

STRATUM NAME AND INSPECTOR LAB INDEX	X(IJKQPT)	QOP	QIN	POP	PIN	TOP	TIN	M(K)	N(K)	R(K)	C(K)	U(K)	W(K)	V(K)	A(K)	B(K)
RODS (1 0 0 0 0 0)	2.900	11	30	10	29	10	29	2170	63	3	1	1	1	1	1	1
WASTE (1 0 0 0 0 0)	0.500	12	31	11	30	11	30	7482	1	1	1	1	1	1	1	1

ENDING INVENTORY

STRATUM NAME AND INSPECTOR LAB INDEX	X(IJKQPT)	QOP	QIN	POP	PIN	TOP	TIN	M(K)	N(K)	R(K)	C(K)	U(K)	W(K)	V(K)	A(K)	B(K)
UF6 CYLINDERS (1 0 0 0 0 0)	1450.000	9	32	9	31	9	31	96	1	3	1	1	1	1	1	1
UO2 POWDER (1 0 0 0 0 0)	13.000	13	33	10	32	12	32	158	30	2	1	1	1	1	1	1
UO2 PELLETS (1 0 0 0 0 0)	13.000	14	34	13	33	13	33	80	30	3	1	1	1	1	1	1
U3O8 POWDER (1 0 0 0 0 0)	20.000	15	35	14	34	14	34	24	20	2	1	1	1	1	1	1
SCRAP (1 0 0 0 0 0)	10.000	16	36	15	35	15	35	94	50	2	1	1	1	1	1	1
WASTE (1 0 0 0 0 0)	0.500	17	37	16	36	16	36	2374	1	1	1	1	1	1	1	1
MISCELLANEOUS (1 0 0 0 0 0)	4.000	18	38	17	37	17	37	1186	1	2	1	1	1	1	1	1
FUEL RODS (1 0 0 0 0 0)	2.900	19	39	18	38	18	38	152	63	3	1	1	1	1	1	1

TABLE A-7. 600 Ton Plant 80.6% Inventory LEU

BEGINNING INVENTORY

STRATUM NAME AND INSPECTOR LAB INDEX	X(IJKQPT)	QOP	QIN	POP	PIN	TOP	TIN	M(K)	N(K)	R(K)	C(K)	U(K)	W(K)	V(K)	A(K)	B(K)
UF6 CYLINDERS (1 0 0 0 0 0)	1450.000	1	20	1	20	1	19	144	1	3	1	1	1	1	1	1
UO2 POWDER (1 0 0 0 0 0)	13.000	2	21	2	19	2	20	237	30	2	1	1	1	1	1	1
UO2 PELLETS (1 0 0 0 0 0)	13.000	3	22	3	21	3	21	120	30	3	1	1	1	1	1	1
U3O8 POWDER (1 0 0 0 0 0)	20.000	4	23	4	22	4	22	36	20	2	1	1	1	1	1	1
SCRAP (1 0 0 0 0 0)	10.000	5	24	5	23	5	23	141	50	2	1	1	1	1	1	1
WASTE (1 0 0 0 0 0)	0.500	6	25	6	24	6	24	3561	1	1	1	1	1	1	1	1
MISCELLANEOUS (1 0 0 0 0 0)	4.000	7	26	7	25	7	25	1779	1	2	1	1	1	1	1	1
FUEL RODS (1 0 0 0 0 0)	2.900	8	27	8	26	8	26	228	63	3	1	1	1	1	1	1

RECEIPTS

STRATUM NAME AND INSPECTOR LAB INDEX	X(IJKQPT)	QOP	QIN	POP	PIN	TOP	TIN	M(K)	N(K)	R(K)	C(K)	U(K)	W(K)	V(K)	A(K)	B(K)
UF6 CYLINDERS (1 0 0 0 0 0)	1450.000	9	29	9	28	9	28	414	1	3	1	1	1	1	1	1

SHIPMENTS

STRATUM NAME AND INSPECTOR LAB INDEX	X(IJKQPT)	QOP	QIN	POP	PIN	TOP	TIN	M(K)	N(K)	R(K)	C(K)	U(K)	W(K)	V(K)	A(K)	B(K)
RODS (1 0 0 0 0 0)	2.900	11	30	10	29	10	29	3255	63	3	1	1	1	1	1	1
WASTE (1 0 0 0 0 0)	0.500	12	31	11	30	11	30	11223	1	1	1	1	1	1	1	1

ENDING INVENTORY

STRATUM NAME AND INSPECTOR LAB INDEX	X(IJKQPT)	QOP	QIN	POP	PIN	TOP	TIN	M(K)	N(K)	R(K)	C(K)	U(K)	W(K)	V(K)	A(K)	B(K)
UF6 CYLINDERS (1 0 0 0 0 0)	1450.000	9	32	9	31	9	31	144	1	3	1	1	1	1	1	1
UO2 POWDER (1 0 0 0 0 0)	13.000	13	33	10	32	12	32	237	30	2	1	1	1	1	1	1
UO2 PELLETS (1 0 0 0 0 0)	13.000	14	34	13	33	13	33	120	30	3	1	1	1	1	1	1
U3O8 POWDER (1 0 0 0 0 0)	20.000	15	35	14	34	14	34	36	20	2	1	1	1	1	1	1
SCRAP (1 0 0 0 0 0)	10.000	16	36	15	35	15	35	141	50	2	1	1	1	1	1	1
WASTE (1 0 0 0 0 0)	0.500	17	37	16	36	16	36	3561	1	1	1	1	1	1	1	1
MISCELLANEOUS (1 0 0 0 0 0)	4.000	18	38	17	37	17	37	1779	1	2	1	1	1	1	1	1
FUEL RODS (1 0 0 0 0 0)	2.900	19	39	18	38	16	38	228	63	3	1	1	1	1	1	1

APPENDIX B

OPERATOR AND INSPECTOR BULK WEIGHING, SAMPLING AND ANALYTICAL MEASUREMENT ERROR DATA

TABLE B-1. Case 1 Inspector Measurement is 0.01 Bulk Measurement Error Data

BULK MEASUREMENT METHOD INDEX	RANDOM ERROR RELATIVE STANDARD DEVIATION	SYSTEMATIC ERROR RELATIVE STANDARD DEVIATION
1	0.001330	0.001630
2	0.001130	0.001130
3	0.001130	0.000900
4	0.001130	0.001130
5	0.002000	0.001630
6	0.000000	0.000000
7	0.001130	0.000900
8	0.000830	0.000830
9	0.001330	0.001630
10	0.000000	0.000000
11	0.000830	0.000830
12	0.000000	0.000000
13	0.001130	0.001130
14	0.001130	0.000900
15	0.001130	0.001130
16	0.002000	0.001630
17	0.000000	0.000000
18	0.001130	0.000900
19	0.000830	0.000830
20	0.001330	0.001630
21	0.001130	0.001130
22	0.001130	0.000900
23	0.001130	0.001130
24	0.002000	0.001630
25	0.000000	0.000000
26	0.001130	0.000900
27	0.000000	0.000000
28	0.000000	0.000000
29	0.001330	0.001630
30	0.000000	0.000000
31	0.000000	0.000000
32	0.001330	0.001630
33	0.001130	0.001130
34	0.001130	0.000900
35	0.001130	0.001130
36	0.002000	0.001630
37	0.000000	0.000000
38	0.001130	0.000900
39	0.000000	0.000000

TABLE B-2. Case 1 Inspector Measurement is 0.01
Sampling Error Data

SAMPLING METHOD INDEX	RANDOM ERROR RELATIVE STANDARD DEVIATION	SYSTEMATIC ERROR RELATIVE STANDARD DEVIATION
1	0.001700	0.001440
2	0.000000	0.000000
3	0.000000	0.000000
4	0.000000	0.000000
5	0.000000	0.000000
6	0.000000	0.000000
7	0.000000	0.000000
8	0.000000	0.000000
9	0.001700	0.001440
10	0.000000	0.000000
11	0.000000	0.000000
12	0.001700	0.001440
13	0.000000	0.000000
14	0.000000	0.000000
15	0.000000	0.000000
16	0.000000	0.000000
17	0.000000	0.000000
18	0.000000	0.000000
19	0.000000	0.000000
20	0.001700	0.001440
21	0.000000	0.000000
22	0.000000	0.000000
23	0.000000	0.000000
24	0.000000	0.000000
25	0.000000	0.000000
26	0.000000	0.000000
27	0.000000	0.000000
28	0.001700	0.001440
29	0.000000	0.000000
30	0.000000	0.000000
31	0.001700	0.001440
32	0.000000	0.000000
33	0.000000	0.000000
34	0.000000	0.000000
35	0.000000	0.000000
36	0.000000	0.000000
37	0.000000	0.000000
38	0.000000	0.000000

TABLE B-3. Case 1 Inspector Measurement is 0.01 Analytical Measurement Error Data

ANALYTICAL METHOD INDEX	RANDOM ERROR RELATIVE STANDARD DEVIATION	SYSTEMATIC ERROR RELATIVE STANDARD DEVIATION
1	0.001480	0.001400
2	0.001280	0.001200
3	0.001200	0.000980
4	0.002100	0.002100
5	0.020000	0.016300
6	0.087500	0.056250
7	0.001820	0.001630
8	0.001200	0.001200
9	0.001480	0.001400
10	0.001200	0.001200
11	0.087500	0.056250
12	0.001280	0.001200
13	0.001200	0.000980
14	0.002100	0.002100
15	0.020000	0.016300
16	0.087500	0.056250
17	0.001820	0.001630
18	0.001200	0.001200
19	0.001480	0.001400
20	0.001280	0.001200
21	0.001200	0.000980
22	0.002100	0.002100
23	0.020000	0.016300
24	0.087500	0.056250
25	0.001820	0.001630
26	0.010000	0.010000
27	0.000000	0.000000
28	0.001480	0.001400
29	0.010000	0.010000
30	0.087500	0.056250
31	0.001480	0.001400
32	0.001280	0.001200
33	0.001200	0.000980
34	0.002100	0.002100
35	0.020000	0.016300
36	0.087500	0.056250
37	0.001820	0.001630
38	0.010000	0.010000

TABLE B-4. Case 2 Inspector Measurement is 0.005 Analytical Measurement Error Data

ANALYTICAL METHOD INDEX	RANDOM ERROR RELATIVE STANDARD DEVIATION	SYSTEMATIC ERROR RELATIVE STANDARD DEVIATION
1	0.001480	0.001400
2	0.001280	0.001200
3	0.001200	0.000980
4	0.002100	0.002100
5	0.020000	0.016300
6	0.087500	0.056250
7	0.001820	0.001630
8	0.001200	0.001200
9	0.001480	0.001400
10	0.001200	0.001200
11	0.087500	0.056250
12	0.001280	0.001200
13	0.001200	0.000980
14	0.002100	0.002100
15	0.020000	0.016300
16	0.087500	0.056250
17	0.001820	0.001630
18	0.001200	0.001200
19	0.001480	0.001400
20	0.001280	0.001200
21	0.005000	0.005000
22	0.002100	0.002100
23	0.020000	0.016300
24	0.087500	0.056250
25	0.001820	0.001630
26	0.005000	0.005000
27	0.000000	0.000000
28	0.001480	0.001400
29	0.005000	0.005000
30	0.087500	0.056250
31	0.001480	0.001400
32	0.001280	0.001200
33	0.005000	0.005000
34	0.002100	0.002100
35	0.020000	0.016300
36	0.087500	0.056250
37	0.001820	0.001630
38	0.005000	0.005000

TABLE B-5. Case 3 Inspector Measurement is 0.001 Analytical Measurement Error Data

ANALYTICAL METHOD INDEX	RANDOM ERROR RELATIVE STANDARD DEVIATION	SYSTEMATIC ERROR RELATIVE STANDARD DEVIATION
1	0.001480	0.001400
2	0.001280	0.001200
3	0.001200	0.000980
4	0.002100	0.002100
5	0.020000	0.016300
6	0.087500	0.056250
7	0.001820	0.001630
8	0.001200	0.001200
9	0.001480	0.001400
10	0.001200	0.001200
11	0.087500	0.056250
12	0.001280	0.001200
13	0.001200	0.000980
14	0.002100	0.002100
15	0.020000	0.016300
16	0.087500	0.056250
17	0.001820	0.001630
18	0.001200	0.001200
19	0.001480	0.001400
20	0.001280	0.001200
21	0.001000	0.001000
22	0.002100	0.002100
23	0.020000	0.016300
24	0.087500	0.056250
25	0.001820	0.001630
26	0.001000	0.001000
27	0.000000	0.000000
28	0.001480	0.001400
29	0.001000	0.001000
30	0.087500	0.056250
31	0.001480	0.001400
32	0.001280	0.001200
33	0.001000	0.001000
34	0.002100	0.002100
35	0.020000	0.016300
36	0.087500	0.056250
37	0.001820	0.001630
38	0.001000	0.001000

TABLE B-6. Case 4 Inspector Measurement is 0.0005 Analytical Measurement Error Data

ANALYTICAL METHOD INDEX	RANDOM ERROR RELATIVE STANDARD DEVIATION	SYSTEMATIC ERROR RELATIVE STANDARD DEVIATION
1	0.001480	0.001400
2	0.001280	0.001200
3	0.001200	0.000980
4	0.002100	0.002100
5	0.020000	0.016300
6	0.087500	0.056250
7	0.001820	0.001630
8	0.001200	0.001200
9	0.001480	0.001400
10	0.001200	0.001200
11	0.087500	0.056250
12	0.001280	0.001200
13	0.001200	0.000980
14	0.002100	0.002100
15	0.020000	0.016300
16	0.087500	0.056250
17	0.001820	0.001630
18	0.001200	0.001200
19	0.001480	0.001400
20	0.001280	0.001200
21	0.000500	0.000500
22	0.002100	0.002100
23	0.020000	0.016300
24	0.087500	0.056250
25	0.001820	0.001630
26	0.000500	0.000500
27	0.000000	0.000000
28	0.001480	0.001400
29	0.000500	0.000500
30	0.087500	0.056250
31	0.001480	0.001400
32	0.001280	0.001200
33	0.000500	0.000500
34	0.002100	0.002100
35	0.020000	0.016300
36	0.087500	0.056250
37	0.001820	0.001630
38	0.000500	0.000500

TABLE B-7. Case 5 Inspector Measurement is 0.005 plus Scrap is 0.01 Analytical Measurement Error Data

ANALYTICAL METHOD INDEX	RANDOM ERROR RELATIVE STANDARD DEVIATION	SYSTEMATIC ERROR RELATIVE STANDARD DEVIATION
1	0.001480	0.001400
2	0.001280	0.001200
3	0.001200	0.000980
4	0.002100	0.002100
5	0.010000	0.010000
6	0.087500	0.056250
7	0.001820	0.001630
8	0.001200	0.001200
9	0.001480	0.001400
10	0.001200	0.001200
11	0.087500	0.056250
12	0.001280	0.001200
13	0.001200	0.000980
14	0.002100	0.002100
15	0.010000	0.010000
16	0.087500	0.056250
17	0.001820	0.001630
18	0.001200	0.001200
19	0.001480	0.001400
20	0.001280	0.001200
21	0.005000	0.005000
22	0.002100	0.002100
23	0.010000	0.010000
24	0.087500	0.056250
25	0.001820	0.001630
26	0.005000	0.005000
27	0.000000	0.000000
28	0.001480	0.001400
29	0.005000	0.005000
30	0.087500	0.056250
31	0.001480	0.001400
32	0.001280	0.001200
33	0.005000	0.005000
34	0.002100	0.002100
35	0.010000	0.010000
36	0.087500	0.056250
37	0.001820	0.001630
38	0.005000	0.005000

TABLE B-8. Case 6 Inspector Measurement is 0.001 plus Scrap is 0.01 Analytical Measurement Error Data

ANALYTICAL METHOD INDEX	RANDOM ERROR RELATIVE STANDARD DEVIATION	SYSTEMATIC ERROR RELATIVE STANDARD DEVIATION
1	0.001480	0.001400
2	0.001280	0.001200
3	0.001200	0.000980
4	0.002100	0.002100
5	0.010000	0.010000
6	0.087500	0.056250
7	0.001820	0.001630
8	0.001200	0.001200
9	0.001480	0.001400
10	0.001200	0.001200
11	0.087500	0.056250
12	0.001280	0.001200
13	0.001200	0.000980
14	0.002100	0.002100
15	0.010000	0.010000
16	0.087500	0.056250
17	0.001820	0.001630
18	0.001200	0.001200
19	0.001480	0.001400
20	0.001280	0.001200
21	0.001000	0.001000
22	0.002100	0.002100
23	0.010000	0.010000
24	0.087500	0.056250
25	0.001820	0.001630
26	0.001000	0.001000
27	0.000000	0.000000
28	0.001480	0.001400
29	0.001000	0.001000
30	0.087500	0.056250
31	0.001480	0.001400
32	0.001280	0.001200
33	0.001000	0.001000
34	0.002100	0.002100
35	0.010000	0.010000
36	0.087500	0.056250
37	0.001820	0.001630
38	0.001000	0.001000

TABLE B-9. Case 7 Inspector Measurement is 0.001 plus Scrap is 0.005 Analytical Measurement Error Data

ANALYTICAL METHOD INDEX	RANDOM ERROR RELATIVE STANDARD DEVIATION	SYSTEMATIC ERROR RELATIVE STANDARD DEVIATION
1	0.001480	0.001400
2	0.001280	0.001200
3	0.001200	0.000980
4	0.002100	0.002100
5	0.005000	0.005000
6	0.087500	0.056250
7	0.001820	0.001630
8	0.001200	0.001200
9	0.001480	0.001400
10	0.001200	0.001200
11	0.087500	0.056250
12	0.001280	0.001200
13	0.001200	0.000980
14	0.002100	0.002100
15	0.005000	0.005000
16	0.087500	0.056250
17	0.001820	0.001630
18	0.001200	0.001200
19	0.001480	0.001400
20	0.001280	0.001200
21	0.001000	0.001000
22	0.002100	0.002100
23	0.005000	0.005000
24	0.087500	0.056250
25	0.001820	0.001630
26	0.001000	0.001000
27	0.000000	0.000000
28	0.001480	0.001400
29	0.001000	0.001000
30	0.087500	0.056250
31	0.001480	0.001400
32	0.001280	0.001200
33	0.001000	0.001000
34	0.002100	0.002100
35	0.005000	0.005000
36	0.087500	0.056250
37	0.001820	0.001630
38	0.001000	0.001000

TABLE B-10. Case 8 Inspector Measurement is 0.0005 plus Scrap is 0.005 Analytical Measurement Error Data

ANALYTICAL METHOD INDEX	RANDOM ERROR RELATIVE STANDARD DEVIATION	SYSTEMATIC ERROR RELATIVE STANDARD DEVIATION
1	0.001480	0.001400
2	0.001280	0.001200
3	0.001200	0.000980
4	0.002100	0.002100
5	0.005000	0.005000
6	0.087500	0.056250
7	0.001820	0.001630
8	0.001200	0.001200
9	0.001480	0.001400
10	0.001200	0.001200
11	0.087500	0.056250
12	0.001280	0.001200
13	0.001200	0.000980
14	0.002100	0.002100
15	0.005000	0.005000
16	0.087500	0.056250
17	0.001820	0.001630
18	0.001200	0.001200
19	0.001480	0.001400
20	0.001280	0.001200
21	0.000500	0.000500
22	0.002100	0.002100
23	0.005000	0.005000
24	0.087500	0.056250
25	0.001820	0.001630
26	0.000500	0.000500
27	0.000000	0.000000
28	0.001480	0.001400
29	0.000500	0.000500
30	0.087500	0.056250
31	0.001480	0.001400
32	0.001280	0.001200
33	0.000500	0.000500
34	0.002100	0.002100
35	0.005000	0.005000
36	0.087500	0.056250
37	0.001820	0.001630
38	0.000500	0.000500

APPENDIX C

VARIANCE OF MUF RESULTS FOR ALL PLANTS

APPENDIX C

VARIANCE OF MUF RESULTS FOR ALL PLANTS

The following tables contain variance of MUF data for the operator for each plant and inventory size. The variance of MUF, standard deviation of MUF, and the percent of throughput for this standard deviation of MUF are the first three entries. Following these are the contributions to the total variance due to weighing, sampling, and analysis. The random components are not given as they make up only about 1% of the total. The final entries in the table are a few of the major strata contributing to the variances. The pairs of letters following the strata names indicate which component of the material balance it is from. The coding is as follows:

R = receipts
S = shipments
I = inventory (beginning and ending)
W = weighing variance
Sa = sampling variance
A = analysis variance

TABLE C-1. Operator Variance of MUF for 200 Ton Plant - 33.7% Inventory

	Cases 1-4	5-6	7-8
Variance of MUF (kg^2)	354,875.8	320,237.8	304,862.0
Standard Deviation of MUF (kg)	595.8	565.9	552.1
Percent of Throughput	0.298	0.283	0.276
Variance by Component			
Weighing Total (Systematic)	108,170.5	108,170.5	108,170.5
Sampling Total (Systematic)	62,448.9	62,448.9	62,448.9
Analysis Total (Systematic)	180,916.8	147,778.8	132,778.8
Major Contributions			
UF_6 Cylinders (R, W)	77,781.3	77,781.3	77,781.3
UF_6 Cylinders (R, Sa)	60,705.1	60,705.1	60,705.1
UF_6 Cylinders (R, A)	57,379.4	57,379.4	57,379.4
Fuel Rods (S, A)	56,584.7	56,584.7	56,584.7
Scrap (I, A)	53,138.0	20,000.0	5,000.0

TABLE C-2. Operator Variance of MUF for 400 Ton Plant - 33.7% Inventory

	Cases 1-4	5-6	7-8
Variance of MUF (kg^2)	1,412,824.3	1,277,272.3	1,216,522.3
Standard Deviation of MUF (kg)	1,188.6	1,130.2	1,102.0
Percent of Throughput	0.297	0.283	0.276
Variance by Component			
Weighing Total (Systematic)	432,682.2	432,682.2	432,682.2
Sampling Total (Systematic)	249,795.9	249,795.9	249,795.9
Analysis Total (Systematic)	723,667.3	591,115.3	531,115.3
Major Contributions			
UF_6 Cylinders (R, W)	311,125.2	311,125.2	311,125.2
UF_6 Cylinders (R, Sa)	242,820.3	242,820.3	242,820.3
UF_6 Cylinders (R, A)	229,517.6	229,517.6	229,517.6
Fuel Rods (S, A)	226,338.8	226,338.8	226,388.8
Scrap (I, A)	212,552.0	80,000.0	20,000.0

TABLE C-3. Operator Variance of MUF for 600 Ton Plant - 33.7% Inventory

	Cases 1-4	Cases 5-6	Cases 7-8
Variance of MUF (kg^2)	3,172,856.3	2,172,119.3	2,733,989.3
Standard Deviation of MUF (kg)	1,781.3	1,694.1	1,653.5
Percent of Throughput	0.297	0.282	0.276
Variance by Component			
Weighing Total (Systematic)	973,534.9	973,534.9	973,534.9
Sampling Total (Systematic)	562,040.8	562,040.8	562,040.8
Analysis Total (Systematic)	1,627,262.6	1,329,020.6	1,194,020.7
Major Contributions			
UF_6 Cylinders (R, W)	700,031.7	700,031.7	700,031.7
UF_6 Cylinders (R, Sa)	546,345.8	546,345.8	546,345.8
UF_6 Cylinders (R, A)	516,414.7	516,414.7	516,414.7
Fuel Rods (S, A)	509,262.4	509,262.4	509,262.4
Scrap (I, A)	478,242.0	180,000.0	45,000.0

TABLE C-4. Operator Variance of MUF for 200 Ton Plant - 80.6% Inventory

	Cases 1-4	Cases 5-6	Cases 7-8
Variance of MUF (kg^2)	533,309.1	366,779.4	283,060.7
Standard Deviation of MUF (kg)	743.8	605.6	532.0
Percent of Throughput	0.372	0.303	0.266
Variance by Component			
Weighing Total (Systematic)	91,275.0	91,275.0	91,275.0
Sampling Total (Systematic)	45,358.8	45,358.8	45,358.8
Analysis Total (Systematic)	410,202.2	227,197.6	144,360.1
Major Contributions			
UF_6 Cylinders (R, W)	45,247.7	45,247.7	45,247.7
UF_6 Cylinders (R, Sa)	35,313.9	35,313.9	35,313.9
UF_6 Cylinders (R, A)	33,379.3	33,379.3	33,379.3
Fuel Rods (S, A)	56,584.7	56,584.7	56,584.7
Scrap (I, A)	293,454.6	110,450.0	27,612.5

TABLE C-5. Operator Variance of MUF for 400 Ton Plant - 80.6% Inventory

	Cases		
	1-4	5-6	7-8
Variance of MUF (kg^2)	2,200,290.0	1,461,221.5	1,128,109.1
Standard Deviation of MUF (kg)	1,483.3	1,208.5	1,062.1
Percent of Throughput	0.371	0.302	0.266
Variance by Component			
Weighing Total (Systematic)	365,100.0	365,100.0	365,100.0
Sampling Total (Systematic)	181,435.1	181,435.1	181,435.1
Analysis Total (Systematic)	1,640,808.9	908,790.3	577,440.4
Major Contributions			
UF_6 Cylinders (R, W)	180,990.7	180,990.7	180,990.7
UF_6 Cylinders (R, Sa)	141,255.7	141,255.7	141,255.7
UF_6 Cylinders (R, A)	133,517.2	133,517.2	133,517.2
Fuel Rods (S, A)	226,338.8	226,338.8	226,338.8
Scrap (I, A)	1,173,818.5	441,800.0	110,450.0

TABLE C-6. Operator Variance of MUF for 600 Ton Plant - 80.6% Inventory

	Cases		
	1-4	5-6	7-8
Variance of MUF (kg^2)	4,940,943.0	3,283,326.5	2,535,145.3
Standard Oeviation of MUF (kg)	2,222.8	1,812.0	1,592.2
Percent of Throughput	0.370	0.302	0.265
Variance by Component			
Weighing Total (Systematic)	821,474.8	821,474.8	821,474.8
Sampling Total (Systematic)	408,229.0	408,229.0	408,229.0
Analysis Total (Systematic)	3,691,819.8	2,044,778.4	1,299,240.8
Major Contributions			
UF_6 Cylinders (R, W)	407,229.0	407,229.0	407.229.0
UF_6 Cylinders (R, Sa)	317,825.3	317,825.3	317,825.3
UF_6 Cylinders (R, A)	300,413.6	300,413.6	300,417.6
Fuel Rods (S, A)	509,262.4	509,262.4	509,262.4
Scrap (I, A)	2,641,091.5	994,050.0	248,512.5

APPENDIX D

DIFFERENCE STATISTIC ($\hat{D}$) RESULTS FOR ALL PLANTS

APPENDIX D

DIFFERENCE STATISTIC ($\hat{D}$) RESULTS FOR ALL PLANTS

The following tables contain the variance of the difference statistic ($\hat{D}$). Data is included for each plant size, and all eight cases. As with Appendix C, the following abbreviations apply:

R = receipt

S = shipments

I = inventory

W = weighing variance

Sa = sampling variance

A = analysis variance

TABLE D-1. Difference Statistic ($\hat{D}$) Data for 200 Ton Plant - 33.7% Inventory

	Case 1	2	3	4	5	6	7	8
Variance of $\hat{D}$ (Systematic)	4,633,250.5	1,681,003.1	736,284.1	706,699.1	1,616,840.5	670,011.6	640,011.6	610,423.1
Standard Deviation of $\hat{D}$	2,152.5	1,296.5	858.1	840.7	1,271.6	818.5	800.0	781.3
Weighing Total (Systematic)	111,888.6	111,888.6	111,888.6	111,888.6	111,888.6	111,888.6	111,888.6	111,888.6
Sampling Total (Systematic)	86,514.8	86,514.8	86,514.8	86,514.8	86,514.8	86,514.8	86,514.8	86,514.8
Analysis Total (Systematic)	4,083,310.3	1,131,063.4	186,344.3	156,759.4	1,100,038.0	153,209.8	138,209.8	108,621.4
Major Contributions								
UF_6 Cylinders (W, R)	106,382.3	106,382.3	106,382.3	106,382.3	106,382.3	106,382.3	106,382.3	106,382.3
UF_6 Cylinders (Sa, R)	83,027.0	83,027.0	83,027.0	83,027.0	83,027.0	83,027.0	83,027.0	83,027.0
UF_6 Cylinders (A, R)	78,478.4	78,478.4	78,478.4	78,478.4	78.478.4	78.478.4	78.478.4	78,478.4
Fuel Rods (A, S)	3,929,493.3	982,373.3	39,294.9	9,823.7	982,373.3	39,294.9	39,294.9	9,823.7
Scrap (A, I)	53,138.0	53,138.0	53,138.0	53,138.0	20,000.0	20,000.0	5,000.0	5,000.0

TABLE D-2. Difference Statistic ($\hat{D}$) Data for 400 Ton Plant - 33.7% Inventory

	Case 1	2	3	4	5	6	7	8
Variance of $\hat{D}$ (Systematic)	18,523,002.0	6,724,012.5	2,945,136.3	2,836,796.5	6,467,362.0	2,680,046.3	2,560,046.3	2,441,692.5
Standard Deviation of $\hat{D}$	4,305.0	2,593.1	1,716.1	1,681.3	2,543.1	1,637.1	1,600.0	1,562.6
Weighing Total (Systematic)	447,554.5	447,554.5	447,554.5	447,554.5	447,554.5	447,554.5	447,554.5	447,554.5
Sampling Total (Systematic)	346,059.1	346,059.1	346,059.1	346,059.1	346,059.1	346,059.1	346,059.1	346,059.1
Analysis Total (Systematic)	16,338,241.0	4,524,253.5	745,337.3	627,037.6	4,400,155.0	612,839.2	552,839.2	434,485.6
Major Contributions								
UF_6 Cylinders (W, R)	425,529.2	425,529.2	425,529.2	425,529.2	425,529.2	425,529.2	425,529.2	425,529.2
UF_6 Cylinders (Sa, R)	332,107.9	332,107.9	332,107.9	332,107.9	332,107.9	332,107.9	332,107.9	332,107.9
UF_6 Cylinders (A, R)	313,913.7	313,913.7	313,913.7	313,913.7	313.913.7	313,913.7	313,913.7	313,913.7
Fuel Rods (A, S)	15,717,973.0	3,929,493.3	157,179.3	39,294.9	3,929,493.3	157,179.8	157,179.8	39,294.9
Scrap (A, I)	212,552.0	212,552.0	212,552.0	212,552.0	80,000.0	80,000.0	20,000.0	20,000.0

TABLE D-3. Differences Statistic ($\hat{D}$) Data for 600 Ton Plant - 33.7% Inventory

	Case 1	2	3	4	5	6	7	8
Variance of $\hat{D}$ (Systematic)	41,697,280.0	15,127,052.0	6,624,580.0	6,358,315.5	14,549,589.0	6,028,127.0	5,758,127.0	5,491,831.5
Standard Deviation of $\hat{D}$	6,457.3	3,889.4	2,573.8	2,521.6	3,814.4	2,455.2	2,399.6	2,343.5
Weighing Total (Systematic)	1,006,997.6	1,006,997.6	1,006,997.6	1,006,997.6	1,006,997.6	1,006,997.6	1,006,997.6	1,006,997.6
Sampling Total (Systematic)	778,632.8	778,632.8	778,632.8	778,632.8	778,632.8	778,632.8	778,632.8	778,632.8
Analysis Total (Systematic)	36,748,808.0	10,178,583.0	1,676,110.3	1,409,845.8	9,899,361.0	1,377,899.6	1,242,899.5	976,603.8
Major Contributions								
UF_6 Cylinders (W, R)	957,440.6	957,440.6	957,440.6	957,440.6	957,440.6	957,440.6	957,440.6	957,440.6
UF_6 Cylinders (Sa, R)	747,242.7	747,242.7	747,242.7	747,242.7	747,242.7	747,242.7	747,242.7	747,242.7
UF_6 Cylinders (A, R)	706,305.8	706,305.8	706,305.8	706,305.8	706,305.8	706,305.8	706,305.8	706,305.8
Fuel Rods (A, S)	35,365,440.0	8,841,360.0	353,654.5	88,413.6	8,841,360.0	353,654.5	353,654.5	88,413.6
Scrap (A, I)	478,242.0	478,242.0	478,242.0	478,242.0	180,000.0	180,000.0	45,000.0	45,000.0

TABLE D-4. Difference Statistic ($\hat{D}$) Data for 200 Ton Plant - 80.6% Inventory

	Case 1	2	3	4	5	6	7	8
Variance of $\hat{D}$ (Systematic)	5,163,606.0	2,187,566.0	1,235,233.3	1,205,127.0	1,833,257.4	869,243.3	703,568.3	673,442.8
Standard Deviation of $\hat{D}$	2,272.4	1,479.0	1,111.4	1,097.8	1,354.0	932.3	838.8	820.6
Weighing Total (Systematic)	137,944.2	137,944.2	137,944.2	137,944.2	137,944.2	137,944.2	137,944.2	137,944.2
Sampling Total (Systematic)	103,116.7	103,116.7	103,116.7	103,116.7	103,116.7	103,116.7	103,116.7	103,116.7
Analysis Total (Systematic)	4,375,709.0	1,399,699.3	447,336.5	417,230.3	1,228,365.1	264,351.1	181,513.6	151,388.2
Major Contributions								
UF_6 Cylinders (W, R)	106,382.3	106,382.3	106,382.3	106,382.3	106,382.3	106,382.3	106,382.3	106,382.3
UF_6 Cylinders (Sa, R)	83,027.0	83,027.0	83,027.0	83,027.0	83,027.0	83,027.0	83,027.0	83,027.0
UF_6 Cylinders (A, R)	78,478.4	78,478.4	78,478.4	78,478.4	78.478.4	78,478.4	78,478.4	78,478.4
Fuel Rods (A, S)	3,929,493.3	982,373.3	39,294.9	9,823.7	982,373.3	39,294.9	39,294.9	9,823.7
Scrap (A, I)	293,454.6	293,454.6	293,454.6	293,454.6	110,450.0	110,450.0	27,612.5	27,612.5

TABLE D-5. Difference Statistic ($\hat{D}$) Data for 400 Ton Plant - 80.6% Inventory

	Case 1	2	3	4	5	6	7	8
Variance of $\hat{D}$ (Systematic)	20,654,424.0	8,750,264.0	4,940,933.0	4,820,508.0	7,333,029.5	3,476,973.0	2,814,273.3	2,693,771.3
Standard Deviation of $\hat{D}$	4,544.7	2,958.1	2,222.8	2,195.6	2,708.0	1,864.7	1,677.6	1,641.3
Weighing Total (Systematic)	551,776.6	551,776.6	551,776.6	551,776.6	551,776.6	551,776.6	551,776.6	551,776.6
Sampling Total (Systematic)	412,466.7	412,466.7	412,466.7	412,466.7	412,466.7	412,466.7	412,466.7	412,466.7
Analysis Total (Systematic)	17,502,836.0	5,598,677.0	1,789,345.9	1,668,921.1	4,913,460.5	1,057,404.5	726,054.4	605,552.7
Major Contributions								
UF_6 Cylinders (W, R)	425,529.2	425,529.2	425,529.2	425,529.2	425,529.2	425,529.2	425,529.2	425,529.2
UF_6 Cylinders (Sa, R)	332,107.9	332,107.9	332,107.9	332,107.9	332,107.9	332,107.9	332,107.9	332,107.9
UF_6 Cylinders (A, R)	313,913.7	313,913.7	313,913.7	313,913.7	313,913.7	313,913.7	313,913.7	313,913.7
Fuel Rods (A, S)	15,717,973.0	3,929,493.3	157,179.8	39,294.9	3,929,493.3	157,179.8	157,179.8	39,294.9
Scrap (A, I)	1,173,818.5	1,173,818.5	1,173,818.5	1,173,818.5	441,800.0	441,800.0	110,450.0	110,450.0

TABLE D-6. Difference Statistic ($\hat{D}$) Data for 600 Ton Plant - 80.6% Inventory

	Case 1	2	3	4	5	6	7	8
Variance of $\hat{D}$ (Systematic)	46,472,460.0	19,688,096.0	11,117,100.0	10,846,144.0	16,499,316.0	7,823,190.0	6,332,115.0	6,060,985.5
Standard Deviation of $\hat{D}$	6,817.1	4,437.1	3,334.2	3,293.3	4,061.9	2,797.0	2,516.4	2,461.9
Weighing Total (Systematic)	1,241,497.4	1,241,497.4	1,241,497.4	1,241,497.4	1,241,497.4	1,241,497.4	1,241,497.4	1,241,497.4
Sampling Total (Systematic)	928,049.9	928,049.9	928,049.9	928,049.9	928,049.9	928,049.9	928,049.9	928,049.9
Analysis Total (Systematic)	39,381,388.0	12,597,024.0	4,026,028.3	3,755,072.3	11,055,287.0	2,379,160.5	1,633,622.6	1,362,493.4
Major Contributions								
UF_6 Cylinders (W, R)	957,440.6	957,440.6	957,440.6	957,440.6	957,440.6	957,440.6	957,440.6	957,440.6
UF_6 Cylinders (Sa, R)	747,242.7	747,242.7	747,242.7	747,242.7	747,242.7	747,242.7	747,242.7	747,242.7
UF_6 Cylinders (A, R)	706,305.8	706,305.8	706,305.8	706,305.8	706,305.8	706,305.8	706,305.8	706,305.8
Fuel Rods (A, S)	35,365,440.0	8,841,360.0	353,654.5	88,413.6	8,841,360.0	353,654.5	353,654.5	88,413.6
Scrap (A, I)	2,641,091.5	2,641,091.5	2,641,091.5	2,641,091.5	994,050.0	994,050.0	248,512.5	248,512.5

APPENDIX E

MATERIAL ACCOUNTING EFFECTIVENESS RESULTS FOR ALL PLANTS AND CASES

APPENDIX E

MATERIAL ACCOUNTING EFFECTIVENESS RESULTS FOR ALL PLANTS AND CASES

This section contains the material accounting effectiveness data of the eight cases and six plants studied. The data included are the goal quantity, the optimal amount to divert via large defects, the probability of failing to detect a diversion by the large defects path, the probability of failing to detect a diversion by the MUF-$\hat{D}$ path, the overall probability of failing to detect a diversion, the probability of failing to detect a diversion by bias based on NV_2, and the percent probability of detecting a diversion.

TABLE E-1. Material Accounting Effectiveness for 200 Ton Plant - 33.7% Inventory

Case	Goal Quantity	Amount Taken as Large Defects (kg)	Nondetection Probability: For Large Defects	For MUF-D Strategy	Total	Induced Based on NV_2	Probability Of Detection (%)
1	2,500.0	0.0	1.000	0.682	0.682	0.669	31.8
2	2,500.0	0.0	1.000	0.328	0.328	0.344	67.2
3	2,500.0	1,376.3	0.192	0.493	0.095	0.050	90.5
4	2,500.0	1,420.6	0.182	0.504	0.092	0.050	90.8
5	2,500.0	0.0	1.000	0.319	0.319	0.335	68.1
6	2,500.0	1,426.4	0.181	0.506	0.092	0.050	90.8
7	2,500.0	1,466.4	0.173	0.516	0.089	0.050	91.1
8	2,500.0	1,527.6	0.160	0.533	0.085	0.050	91.5

TABLE E-2. Material Accounting Effectiveness for 400 Ton Plant - 33.7% Inventory

Case	Goal Quantity	Amount Taken as Large Defects (kg)	Nondetection Probability: For Large Defects	For MUF-D Strategy	Total	Induced Based on NV_2	Probability Of Detection (%)
1	2,500.0	0.0	1.000	0.907	0.907	0.888	9.3
2	2,500.0	0.0	1.000	0.806	0.806	0.784	19.4
3	2,500.0	0.0	1.000	0.499	0.499	0.502	50.1
4	2,500.0	0.0	1.000	0.469	0.469	0.476	53.1
5	2,500.0	0.0	1.000	0.803	0.803	0.781	19.7
6	2,500.0	0.0	1.000	0.464	0.464	0.472	53.6
7	2,500.0	0.0	1.000	0.444	0.444	0.456	55.6
8	2,500.0	0.0	1.000	0.409	0.409	0.425	59.1

TABLE E-3. Material Accounting Effectiveness for 600 Ton Plant - 33.7% Inventory

Case	Goal Quantity	Amount Taken as Large Defects (kg)	Nondetection Probability: For Large Defects	Nondetection Probability: For MUF-D Strategy	Nondetection Probability: Total	Nondetection Probability: Induced Based on NV_2	Probability Of Detection (%)
1	2,500.0	0.0	1.000	0.936	0.936	0.919	6.4
2	2,500.0	0.0	1.000	0.887	0.887	0.865	11.3
3	2,500.0	0.0	1.000	0.735	0.735	0.712	26.5
4	2,500.0	0.0	1.000	0.719	0.719	0.696	28.1
5	2,500.0	0.0	1.000	0.885	0.885	0.863	11.5
6	2,500.0	0.0	1.000	0.715	0.715	0.694	28.5
7	2,500.0	0.0	1.000	0.703	0.703	0.685	29.7
8	2,500.0	0.0	1.000	0.682	0.682	0.665	31.8

TABLE E-4. Material Accounting Effectiveness for 200 Ton Plant - 80.6% Inventory

Case	Goal Quantity	Amount Taken as Large Defects (kg)	Nondetection Probability: For Large Defects	Nondetection Probability: For MUF-D Strategy	Nondetection Probability: Total	Nondetection Probability: Induced Based on NV_2	Probability Of Detection (%)
1	2,500.0	0.0	1.000	0.698	0.698	0.683	30.2
2	2,500.0	0.0	1.000	0.407	0.407	0.417	59.3
3	2,500.0	748.4	0.408	0.357	0.146	0.142	85.4
4	2,500.0	815.4	0.376	0.370	0.139	0.131	86.1
5	2,500.0	0.0	1.000	0.366	0.366	0.378	63.4
6	2,500.0	1,162.9	0.248	0.441	0.110	0.067	89.0
7	2,500.0	1,320.0	0.206	0.479	0.098	0.050	90.2
8	2,500.0	1,365.1	0.195	0.490	0.095	0.050	90.5

TABLE E-5. Material Accounting Effectiveness for 400 Ton Plant - 80.6% Inventory

Case	Goal Quantity	Amount Taken as Large Defects (kg)	Nondetection Probability: For Large Defects	Nondetection Probability: For MUF-D Strategy	Nondetection Probability: Total	Nondetection Probability: Induced Based on NV_2	Probability Of Detection (%)
1	2,500.0	0.0	1.000	0.911	0.911	0.892	8.9
2	2,500.0	0.0	1.000	0.834	0.834	0.810	16.6
3	2,500.0	0.0	1.000	0.683	0.683	0.663	31.7
4	2,500.0	0.0	1.000	0.672	0.672	0.653	32.8
5	2,500.0	0.0	1.000	0.820	0.820	0.798	18.0
6	2,500.0	0.0	1.000	0.592	0.592	0.584	40.8
7	2,500.0	0.0	1.000	0.530	0.530	0.528	47.0
8	2,500.0	0.0	1.000	0.503	0.503	0.505	49.7

TABLE E-6. Material Accounting Effectiveness for 600 Ton Plant - 80.6% Inventory

Case	Goal Quantity	Amount Taken as Large Defects (kg)	Nondetection Probability: For Large Defects	Nondetection Probability: For MUF-D Strategy	Nondetection Probability: Total	Nondetection Probability: Induced Based on NV_2	Probability Of Detection (%)
1	2,500.0	0.0	1.000	0.938	0.938	0.920	6.2
2	2,500.0	0.0	1.000	0.900	0.900	0.877	10.0
3	2,500.0	0.0	1.000	0.829	0.829	0.800	17.1
4	2,500.0	0.0	1.000	0.824	0.824	0.794	17.6
5	2,500.0	0.0	1.000	0.894	0.894	0.872	10.6
6	2,500.0	0.0	1.000	0.785	0.785	0.759	21.5
7	2,500.D	0.0	1.000	0.753	0.753	0.729	24.7
8	2,500.0	0.0	1.000	0.739	0.739	0.716	26.1

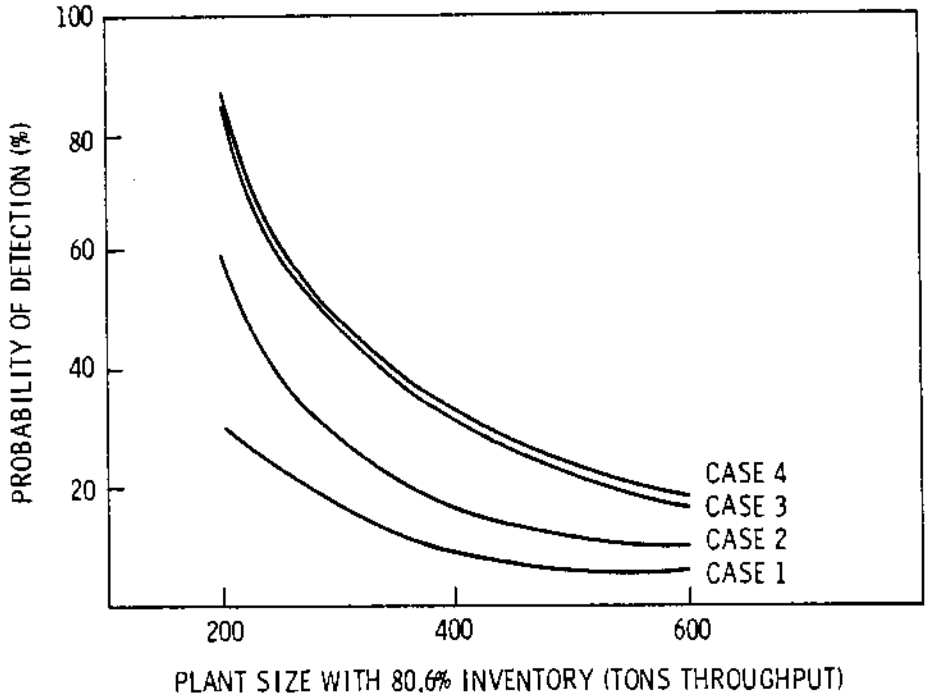

FIGURE E.1. Probability of Detection vs. Plant Size for Various Inspector Product Measurement Capabilities (80.6% Inventory)

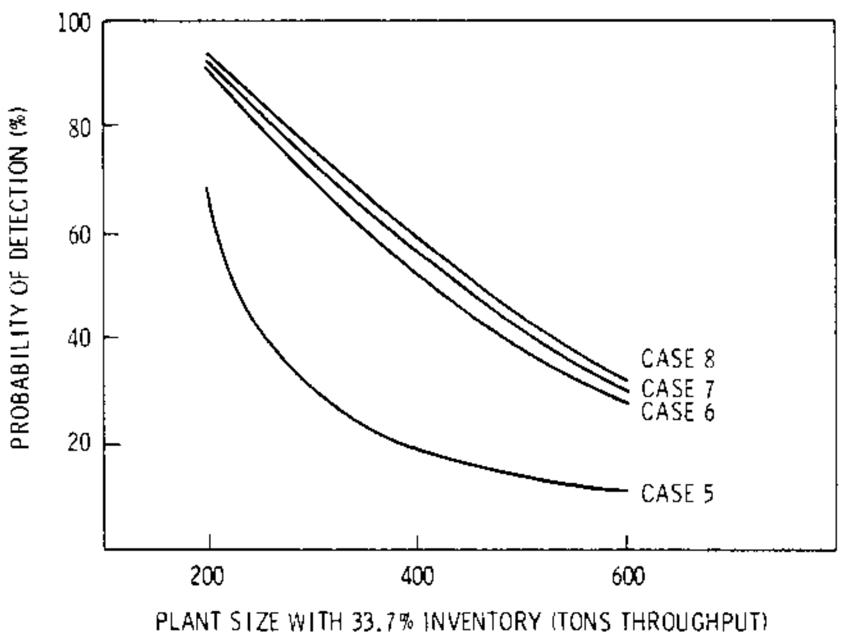

FIGURE E.2. Probability of Detection vs. Plant Size for Changing Product and Scrap Measurement Capability (33.7% Inventory)

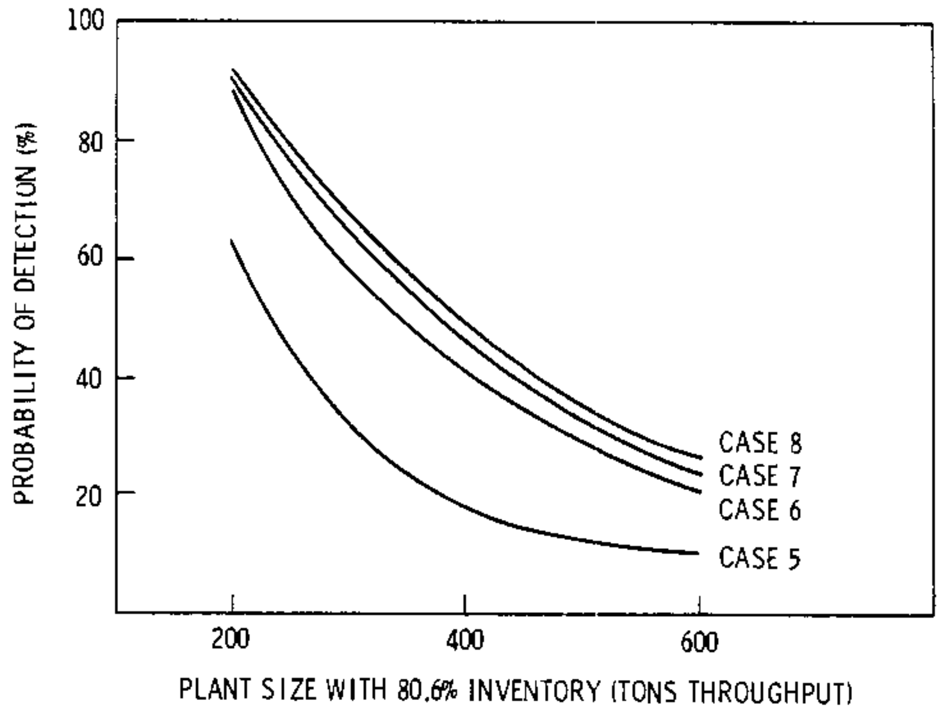

FIGURE E.3. Probability of Detection vs. Plant Size for Changing Product and Scrap Measurement Capability (80.6% Inventory)

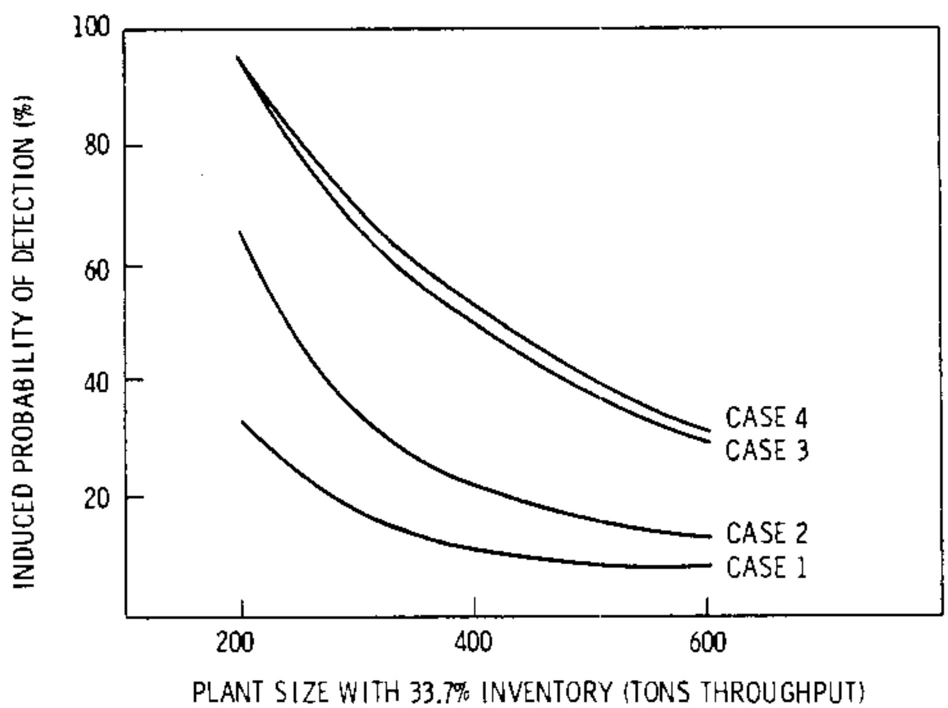

FIGURE E.4. Induced Probability of Detection Based on NV_2 with Changing Product Measurement Capability (33.7% Inventory)

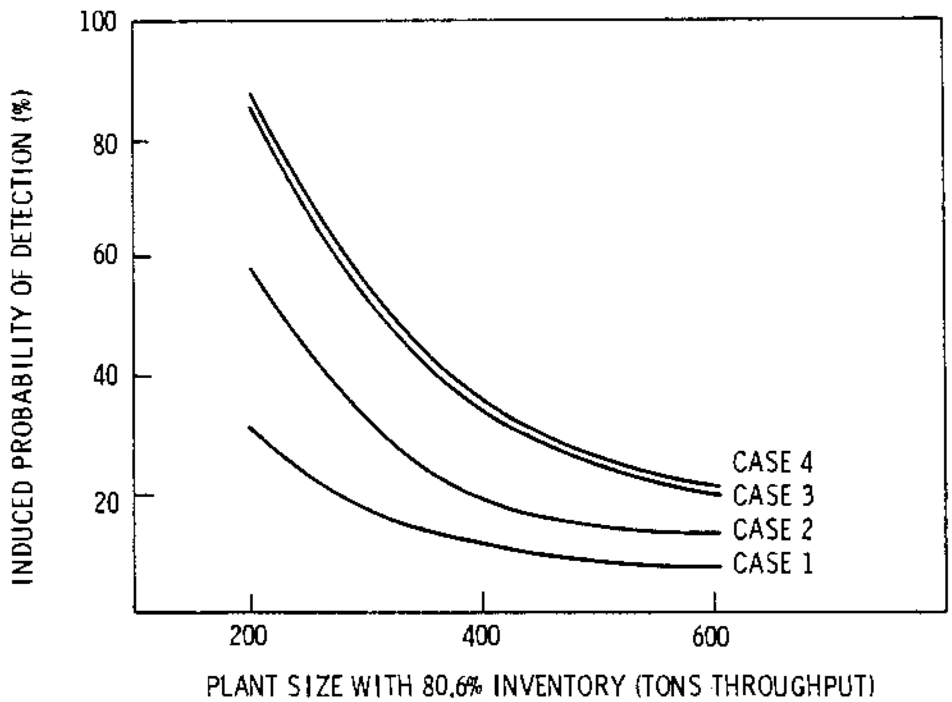

FIGURE E.5. Induced Probability of Detection Based on NV_2 with Changing Product Measurement Capability (80.6% Inventory)

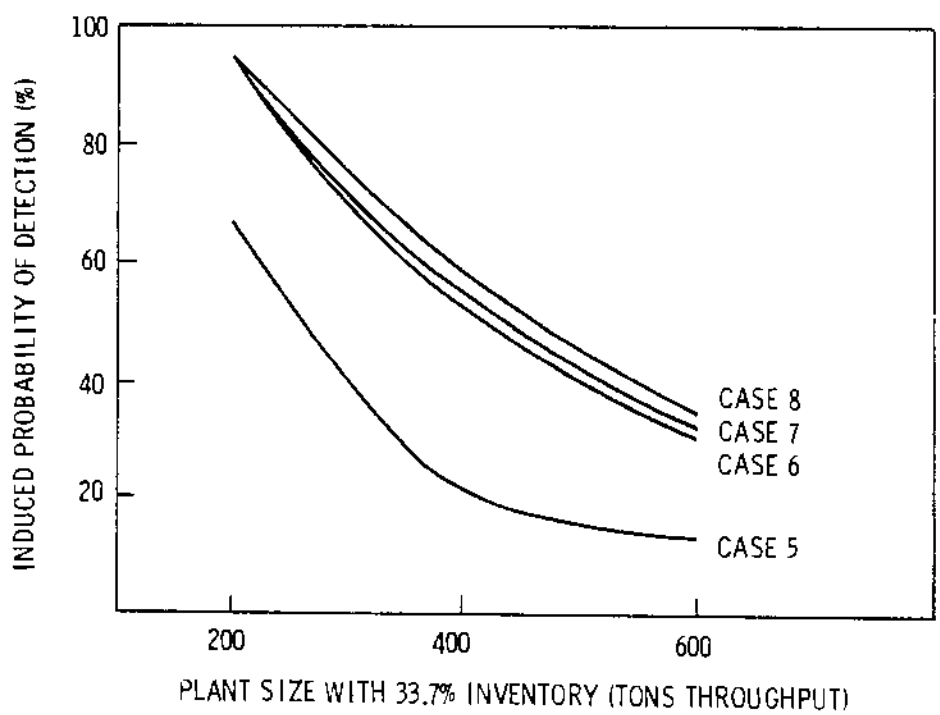

FIGURE E.6. Induced Probability of Detection Based on NV_2 with Changing Product and Scrap Measurement Capability (33.7% Inventory)

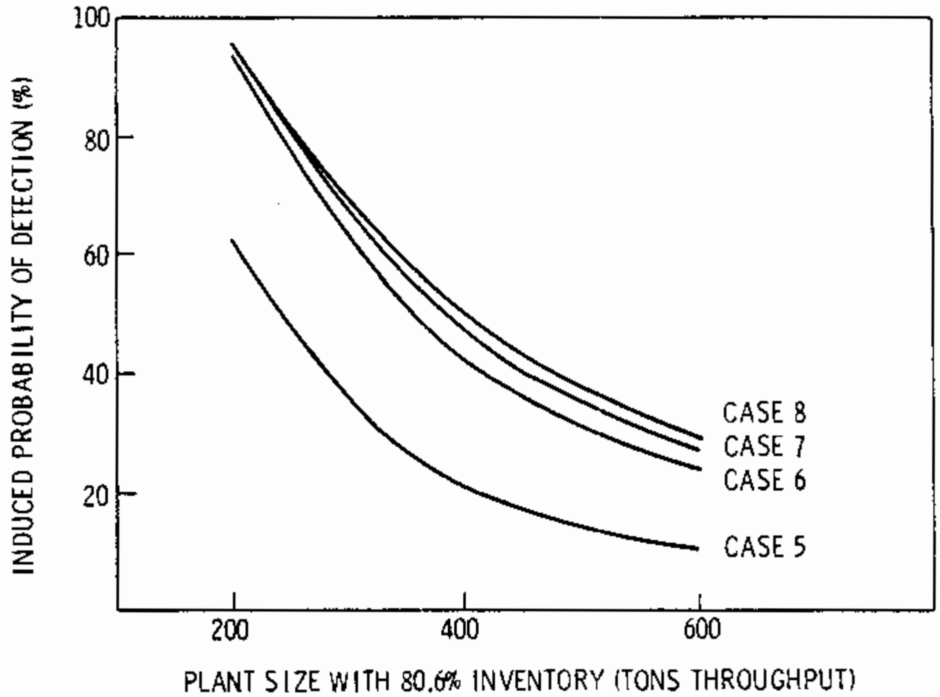

FIGURE E.7. Induced Probability of Detection Based on NV_2 with Changing Product and Scrap Measurement Capability (80.6% Inventory)

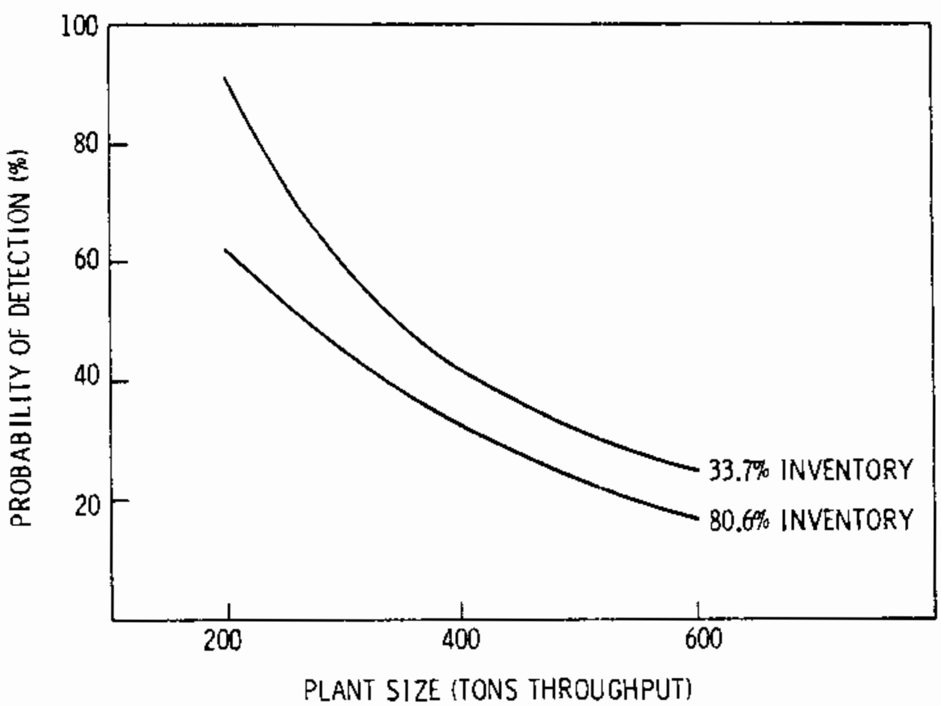

FIGURE E.8. Detection Probability for Limited Variable Sampling Plan

APPENDIX F

MATERIAL ACCOUNTING SAMPLING PLANS AND ESTIMATED EFFORT FOR ALL PLANTS

APPENDIX F

MATERIAL ACCOUNTING SAMPLING PLANS AND ESTIMATED EFFORT FOR ALL PLANTS

This section contains the effort calculations for the various plants used in this study. The following notation is used:

NA = the number of attributes samples

NV1 = sample size for partial defects not detectable by the attributes test

NV2 = sample size for bias defects

NV = maximum of NV1 and NV2 on a stratum-by-stratum basis

The sampling plan effort was calculated assuming 15 minutes of inspection effort is required for each attribute sample and one hour is required for each variables sample. It is also assumed that an item check takes five man-days for the 200-plant, 10 man-days for the 400-ton plant and 15 man-days for the 600-ton plant for those plants with 33.7% inventory. The figures for item checks are doubled for the plants with 80.6% inventory.

Each of the other categories, planning and preparation, audit, containment and surveillance device examination, and post-inspection activities, is broken into three parts. These parts are man-days spent at the facility, man-days of travel, and man-days at headquarters. Man-days at the facility and in travel can be combined to give man-days in the field. The figures used in this study are shown in Table F.1.

Table F.2 through F.7 summarize the sampling plan data, and the total effort involved.

TABLE F-1. Routine Inspection Effort for One-Year Period

Plant Size (Tons Throughput)	Number of Man-Days: At Facility	In Travel	In Field	At HQ	Total
Planning and Preparation					
200	0	19[a]	19	21[d]	40
400	0	28[b]	28	32[e]	60
600	0	41[c]	41	43[f]	84
Audit					
200	21[g]	0	21	0	21
400	43[h]	0	43	0	43
600	58[i]	0	58	0	58
C/S Devices					
200	1	0	1	0	1
400	1	0	1	0	1
600	1	0	1	0	1
Post-Inspection Activities					
200	0	19	19	21[j]	40
400	0	28	28	37[k]	65
600	0	41	41	43[l]	84

(a) Two inspectors, each one day of travel, for each of eight flow verifications, plus three inspectors, each one day of travel, for the one inventory verification.

$(2 \times 8) + (3 \times 1) = 19$ man-days

(b) Two inspectors, each one day of travel, for each of nine flow verifications, plus five inspectors, each one day of travel, for two inventory verifications.

$(2 \times 9) + (5 \times 2) = 28$ man-days

(c) Three inspectors, each one day of travel, for each of nine flow verifications plus seven inspectors, each one day of travel, for two inventory verifications.

$(3 \times 9) + (7 \times 2) = 41$ man-days

(d) Two man-days per flow verification, eight verifications per year, plus five man-days for each inventory verification.

$(2 \times 8) + (5 \times 1) = 21$ man-days

(e) Two man-days per flow verification, nine verifications per year, plus seven man-days per inventory verification.

$(2 \times 9) + (7 \times 2) = 32$ man-days

(f) Three man-days per flow verification, nine verifications per year, plus eight man-days per inventory verification.

$(3 \times 9) + (8 \times 2) = 43$ man-days

(g) Two man-days per flow verification, eight verifications per year, plus five man-days per inventory verification.

$(2 \times 8) + (5 \times 1) = 21$ man-days

(h) Three man-days per flow verification, nine verifications per year, plus eight man-days per inventory verification.

$(3 \times 9) + (8 \times 2) = 43$ man-days

(i) Four man-days per flow verification, nine verifications per year, plus 11 man-days per inventory verification.

$(4 \times 9) + (11 \times 2) = 58$ man-days

(j) Two man-days per flow verification, eight verifications per year, plus five man-days per inventory verification.

$(2 \times 8) + (5 \times 1) = 21$ man-days

(k) Two and a half man-days per flow verification, nine verifications per year, plus seven man-days per inventory verification.

$(2.5 \times 9) + (7 \times 2) = 37$ man-days

(l) Three man-days per flow verification, nine verifications per year, plus eight man-days per inventory verification.

$(3 \times 9) + (8 \times 2) = 43$ man-days

TABLE F-2. Material Accounting Sampling Plans and Estimated Effort for 200 Ton Plant - 33.7% Inventory

Case	NA	NV1	NV2	NV	Man-days of Effort: Sampling Plan	Planning & Preparation	Audit	C/S Devices	Post Inspection Activities	Total Effort
1	414	27	42	51	25	40	21	1	40	127
2	414	27	62	65	26	40	21	1	40	128
3	414	27	39	48	24	40	21	1	40	126
4	414	27	32	43	24	40	21	1	40	126
5	414	27	57	59	26	40	21	1	40	128
6	414	27	28	38	23	40	21	1	40	125
7	414	27	24	36	23	40	21	1	40	125
8	414	27	17	35	23	40	21	1	40	125

TABLE F-3. Material Accounting Sampling Plans and Estimated Effort for 400 Ton Plant - 33.7% Inventory

Case	NA	NV1	NV2	NV	Man-days of Effort: Sampling Plan	Planning & Preparation	Audit	C/S Devices	Post Inspection Activities	Total Effort
1	957	63	47	78	50	60	43	1	65	219
2	957	63	74	90	52	60	43	1	65	221
3	957	63	87	106	54	60	43	1	65	223
4	957	63	86	107	54	60	43	1	65	223
5	957	63	66	81	50	60	43	1	65	219
6	957	63	73	93	52	60	43	1	65	221
7	957	63	69	89	51	60	43	1	65	220
8	957	63	67	89	51	60	43	1	65	220

TABLE F-4. Material Accounting Sampling Plans and Estimated Effort for 600 Ton Plant - 33.7% Inventory

					Man-days of Effort					
Case	NA	NV1	NV2	NV	Sampling Plan	Planning & Preparation	Audit	C/S Devices	Post Inspection Activities	Total Effort
1	1,435	92	47	103	73	84	58	1	84	300
2	1,435	92	74	114	75	84	58	1	84	302
3	1,435	92	87	126	76	84	58	1	84	303
4	1,435	92	84	126	76	84	58	1	84	303
5	1,435	92	66	106	73	84	58	1	84	300
6	1,435	92	73	114	75	84	58	1	84	302
7	1,435	92	69	110	74	84	58	1	84	301
8	1,435	92	67	110	74	84	58	1	84	301

TABLE F-5. Material Accounting Sampling Plans and Estimated Effort for 200 Ton Plant - 80.6% Inventory

					Man-days of Effort					
Case	NA	NV1	NV2	NV	Sampling Plan	Planning & Preparation	Audit	C/S Devices	Post Inspection Activities	Total Effort
1	504	32	64	68	35	40	21	1	40	137
2	504	32	87	89	37	40	21	1	40	139
3	504	32	91	92	38	40	21	1	40	140
4	504	32	89	90	37	40	21	1	40	139
5	504	32	84	86	37	40	21	1	40	139
6	504	32	92	93	38	40	21	1	40	140
7	504	32	66	67	35	40	21	1	40	137
8	504	32	50	58	33	40	21	1	40	135

TABLE F-6. Material Accounting Sampling Plans and Estimated Effort for 400 Ton Plant - 80.6% Inventory

Case	NA	NV1	NV2	NV	Man-days of Effort: Sampling Plan	Planning & Preparation	Audit	C/S Devices	Post Inspection Activities	Total Effort
1	1,297	87	82	115	75	60	43	1	65	244
2	1,297	87	118	134	78	60	43	1	65	247
3	1,297	87	133	153	80	60	43	1	65	249
4	1,297	87	131	153	80	60	43	1	65	249
5	1,297	87	109	122	76	60	43	1	65	245
6	1,297	87	128	144	79	60	43	1	65	248
7	1,297	87	128	140	78	60	43	1	65	247
8	1,297	87	127	142	79	60	43	1	65	248

TABLE F-7. Material Accounting Sampling Plans and Estimated Effort for 600 Ton Plant - 80.6% Inventory

Case	NA	NV1	NV2	NV	Man-Days of Effort: Sampling Plan	Planning and Preparation	Audit	C/S Devices	Post Inspection Activities	Total Effort
1	1,953	126	82	144	109	84	58	1	84	336
2	1,953	126	118	165	112	84	58	1	84	339
3	1,953	126	133	184	114	84	58	1	84	341
4	1,953	126	131	184	114	84	58	1	84	341
5	1,953	126	109	149	110	84	58	1	84	337
6	1,953	126	128	167	112	84	58	1	84	339
7	1,953	126	128	158	111	84	58	1	84	338
8	1,953	126	127	160	111	84	58	1	84	338

APPENDIX G

CALCULATION OF POWER OF THE TEST FOR FUEL PELLET ENRICHMENT FALSIFICATION

APPENDIX G

CALCULATION OF POWER OF THE TEST FOR FUEL PELLET ENRICHMENT FALSIFICATION

A test of a statistical hypothesis consists of choosing a test statistic and selecting a critical region. When the hypothesis is true, the test specifies that the chance of rejecting the hypothesis is some predesigned level of significance, α. It would be an error to reject the hypothesis when it is true, and this is a type I, or α error. It is also an error to accept the hypothesis when it is in fact not true. This is a type II, or β error.

Declaring a hypothesis false should imply some knowledge of alternative situations. For example, if the hypothesis being tested is that the enrichment of the fuel pellets is $\mu = 3.00$, the alternatives may be the enrichment is really $\mu = 2.90$, or $\mu = 2.92$. If in fact, $\mu = 2.90$, there is a certain probability of rejecting the hypothesis $\mu = 3.00$. This probability of rejecting a hypothesis is called the power of the test.

The critical region for 95% confidence level for this normal one-sided test is given by

$$\frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{N}}} < -1.645$$

where μ = the test hypothesis enrichment
σ = the variance of $\overline{X}$
N = number of fuel rods tested
$\overline{X}$ = critical value of μ for 95% confidence level.

Rewriting the equation to solve for $\overline{X}$ gives

$$\overline{X} < \mu - 1.645\left(\frac{\sigma}{\sqrt{N}}\right)$$

Table G.1 summarizes these values for the cases shown in Figures 5.1 and 5.2.

TABLE G.1. Critical Values of Enrichment for 95% Confidence Level, $\mu = 3.00$

Number of Fuel Rods Sampled, N	$\overline{X}$ When $\sigma = 0.02$	$\overline{X}$ When $\sigma = 0.05$
1	2.9671	2.9178
2	2.9767	2.9418
3	2.9810	2.9525
4	2.9836	2.9589
5	2.9853	2.9632
10	2.9896	2.9740

The next step in the process is to determine the power of these critical regions against certain alternatives, μ^*. This is done by finding the critical region boundary of the normal distribution curve. The boundary is determined by

$$Z = \frac{\overline{X} - \mu^*}{\frac{\sigma}{\sqrt{N}}}$$

Tables G.2 and G.3 summarize this calculation for the cases shown in Figures 5.1 and 5.2, respectively.

TABLE G.2. Critical Region Boundary Calculation for the Hypothesis $\mu = 3.00$, $\sigma = 0.02$

Alternative Enrichment, μ^*	Number of Fuel Rods Sampled N=1	N=2	N=3	N=4	N=5	N=10
2.90	3.355	5.424	7.015	8.360	9.537	14.167
2.92	2.355	4.009	5.283	6.360	7.301	11.005
2.94	1.355	2.595	3.551	4.360	5.065	7.842
2.96	0.355	1.181	1.819	2.360	2.829	4.680
2.98	-0.645	-0.233	0.087	0.360	0.593	1.518
3.00	-1.645	-1.645	-1.645	-1.645	-1.645	-1.645

TABLE G.3. Critical Region Boundary Calculation for the Hypothesis μ = 3.00, σ = 0.05

Alternative Enrichment, μ^*	Number of Fuel Rods Sampled					
	N=1	N=2	N=3	N=4	N=5	N=10
2.90	0.355	1.182	1.819	2.356	2.826	4.680
2.92	-0.045	0.617	1.126	1.556	1.932	3.415
2.94	-0.445	0.051	0.433	0.756	1.038	2.150
2.96	-0.845	-0.515	-0.260	-0.044	0.143	0.885
2.98	-1.245	-1.081	-0.953	-0.844	-0.751	-0.379
3.00	-1.645	-1.645	-1.645	-1.645	-1.645	-1.645

The next step is to determine the areas under the normal curve corresponding to the values shown in Tables G.2 and G.3. This area represents the probability of making a type II, or β error. The power of the test is given by

$$\text{Power of test} = 1 - \beta$$

These values are shown in Tables G.4 and G.5, and portions of the data are plotted in Figures 5.1 and 5.2.

TABLE G.4. Power of the Test for the Hypothesis μ = 3.00, σ = 0.02

Alternative Enrichment, μ^*	Number of Fuel Rods Sampled					
	N=1	N=2	N=3	N=4	N=5	N=6
2.90	0.999	1.000	1.000	1.000	1.000	1.000
2.92	0.991	1.000	1.000	1.000	1.000	1.000
2.94	0.912	0.995	0.999	1.000	1.000	1.000
2.96	0.637	0.881	0.966	0.991	0.998	1.000
2.98	0.259	0.409	0.536	0.641	0.723	0.936
3.00	0.050	0.050	0.050	0.050	0.050	0.050

TABLE G.5. Power of the Test for the Hypothesis $\mu = 3.00$, $\sigma = 0.05$

Alternative Enrichment, μ^*	Number of Fuel Rods Sampled					
	N=1	N=2	N=3	N=4	N=5	N=6
2.90	0.637	0.881	0.965	0.991	0.997	1.000
2.92	0.482	0.732	0.868	0.939	0.973	0.999
2.94	0.328	0.524	0.666	0.774	0.851	0.984
2.96	0.199	0.302	0.397	0.484	0.555	0.811
2.98	0.108	0.140	0.170	0.201	0.226	0.352
3.00	0.050	0.050	0.050	0.050	0.050	0.050

DISTRIBUTION

No. of Copies		No. of Copies	
	OFFSITE		D. C. Camp ISPO Laboratory Coordinator Lawrence Livermore Laboratory P.O. Box 808 Livermore, CA 94550
	A. A. Churm DOE Chicago Patent Group 9800 South Cass Avenue Argonne, IL 60439		Jack Cusack ISPO Laboratory Coordinator Brookhaven National Laboratory Technical Support Organization Building 197 Upton, NY 11973
	G. Weisz/W. C. Bartels Safeguards and Security Department of Energy Washington, DC 20545		R. B. Perry ISPO Laboratory Coordinator Argonne National Laboratory 9700 South Cass Avenue Argonne, IL 60439
	S. C. T. McDowell Safeguards and Security Century XXI Department of Energy Washington, DC 20545		H. Menlove ISPO Laboratory Coordinator Los Alamos Scientific Laboratory P.O. Box 1663 Los Alamos, NM 87545
27	DOE Technical Information Center		R. A. Tannert ISPO Laboratory Coordinator Union Carbide Corporation P.O. Box P Mail Stop 69 Oak Ridge, TN 37830
15	Leon Green International Safeguards Project Office Building 197 Brookhaven National Laboratory Upton, NY 11973		I. G. Waddoups ISPO Laboratory Coordinator Sandia Laboratories P.O. Box 5800 Albuquerque, NM 87115
	C. D. Bingham DOE New Brunswick Laboratory 9800 South Cass Avenue Building 350 Argonne, IL 60439	5	W. Bahm International Atomic Energy Agency Wagramerstrasse 5 P.O. Box 100 A-1400 Vienna, Austria
	R. L. Heath ISPO Laboratory Coordinator Idaho National Engineering Laboratory EG & G P.O. Box 1625 Idaho Falls, ID 83401		

No. of
Copies

S. Ermakov
International Atomic Energy Agency
Wagramerstrasse 5
P.O. Box 100
A-1400 Vienna, Austria

D. Ditmars
National Bureau of Standards
Bldg. 221, Room A324
Washington, DC 20234

J. Lemming
Mound Facility
Monsanto Research Corp.
Miamisburg, OH 45342

C. E. Johnson
Exxon Nuclear Idaho, Co. Inc.
P.O. Box 2800
Idaho Falls, ID 83401

J. Carter
Oak Ridge National Laboratory
Union Carbide Corp.
P.O. Box Y-12
Bldg. 9735
Oak Ridge, TN 37830

ONSITE

2 DOE Richland Operations Office

H. E. Ransom
A. C. Walker

31 Pacific Northwest Laboratory

C. A. Bennett
R. J. Brouns
R. G. Clark
R. F. Eggers
N. L. Harms
J. A. Mahaffey
M. F. Mullen
P. T. Reardon (15)
B. W. Smith
R. J. Sorenson
Publishing Coordination (2)
Technical Information Ma (5)