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Approximation for Nonresonant Beam Target Fusion Reactivities

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Abstract

The beam target fusion reactivity for a monoenergetic beam in a Maxwellian target is approximately evaluated for nonresonant reactions. The approximation is accurate for the DD and TT fusion reactions to better than 4% for all beam energies up to 300 keV and all ion temperatures up to 2/3 of the beam energy.

1. Introduction

A series of recent experiments has led to improved determinations of the cross sections for the DT[1,2], DD[3–5], DHe^3 [4,6] (see discussion in [7]), and TT [3] fusion reactions. The present work exploits the simple energy dependence of nonresonant reaction cross sections to calculate the beam target reactivity. The method is parallel to that used by Bahcall[8] to calculate the thermonuclear reactivity.

A similar method has been used by Core[9] to develop formulae for the DT reaction. Unfortunately, the resonance in that reaction (at a center of mass energy of 50 keV) complicates the energy dependence of the cross section. As a result, the Bahcall and Core expressions for the thermonuclear reactivity are inaccurate for ion temperatures above $\sim$ 15 keV (at 30 keV the approximations are 50% too large). Core's beam target reactivity is also accurate only for relatively low values of the ion temperature. While simple fits to the numerically calculated thermonuclear reactivity of the resonant DT and DHe^3 reactions exist[10], there are apparently no similar fits to the beam target reactivity which are valid for the full range of beam voltages and target temperatures of interest.

The absence of a resonance in the DD and TT reactions permits an accurate, simple evaluation of the thermonuclear[5] and beam target reactivities (below).

2. Approximate beam target reactivity

The beam target reactivity for a monoenergetic beam at a speed $v_b = \sqrt{2E_b/m_b}$ reacting with a Maxwellian target with a thermal speed $\sigma_{th} = \sqrt{2T_i/m_t}$ is[11]

$$(\sigma v)_{bt} = \frac{1}{v_b \sigma_{th} \sqrt{\pi}} \int_0^\infty \sigma v^2 \left[e^{-\left(\frac{v-v_b}{\sigma_{th}}\right)^2} - e^{-\left(\frac{v+v_b}{\sigma_{th}}\right)^2} \right] dv. \qquad (1)$$

The complete beam target reactivity is obtained by averaging $(\sigma v)_{bt}$ over the beam energy distribution.

The evaluation of the integral can be simplified by defining the 'astrophysical S function' as the slowly varying part of the cross section:

$$\sigma(E_{cm}) \equiv S(E_{cm})/\left[E_{cm}\exp\left(-b/\sqrt{E_{cm}}\right)\right], \tag{2}$$

where $b = \pi\alpha Z_b Z_t\sqrt{2\mu c^2} = 31.4\sqrt{2\mu/m_D}\sqrt{\text{keV}}$, $E_{cm} = \mu v^2/2$, and $\mu = m_b m_t/(m_b + m_t)$ is the reduced mass.

The second exponential in Eq. (1) is ignored below because direct calculation shows that it makes a negligible contribution in all cases of interest, i.e., $E_b \leq 1$ MeV and $T_i \leq 2E\ /3$. This can be easily understood by noting that since $\sigma(E_{cm})$ is a strongly rising function of energy, the complete integrand is maximized at a speed of $v > v_b$, so $((v+v_b)/\sigma_{th})^2 > 4(v_b/\sigma_{th})^2$, and if $E_b \geq 3T_i/2$ the second exponential is of order e^{-6} in the region where the first exponential is of order 1.

Using Eq.(2) we rewrite the dominant term of Eq.(1) as

$$\langle\sigma v\rangle_{bt} = \frac{2}{\mu v_b\sigma_{th}\sqrt{\pi}}\int_0^\infty S(E_{cm})\exp\left[-\left(\frac{B}{v}+\left(\frac{v-v_b}{\sigma_{th}}\right)^2\right)\right]dv, \tag{3}$$

where $B = 2\pi\alpha Z_b Z_t c$. The rapid energy dependence of the integrand is isolated in the exponential, the argument of which

$$F(v) = B/v + \left(\frac{v-v_b}{\sigma_{th}}\right)^2$$

is maximized at a speed v_*, where

$$v_*^2(v_* - v_b) = \sigma_{th}^2 B/2.$$

The solution is

$$v_* = v_b[1/3 + \sqrt[3]{R+D} + \sqrt[3]{R-D}], \tag{4}$$

where $R = \beta/2 + 1/27$, $D = \sqrt{\beta(\beta/4 + 1/27)}$, and $\beta = (\sigma_{th}/v_b)^2 B/2v_b$. Letting

$$v = v_* + \sigma_{th}u$$

and expanding $F(v)$ we have

$$F(v) = F(v_*) + u^2[1 + 2(v_* - v_b)/(v_* + \sigma_{th}u)]. \tag{5}$$

Expanding $F(v)$ and $S(E_{cm})$ in a Taylor series in u about v_* (and neglecting higher order derivatives of S) we are led to

$$(\sigma v)_{bt} \sim \frac{2}{\mu v_b \sqrt{\pi}} \int_{-v_*/\sigma_{th}}^{\infty} [S(E_*) + S'(E_*)\mu(v_*\sigma_{th}u + \sigma_{th}^2 u^2/2)] \exp[-F(v_*)] \times \exp(-\gamma u^2)[1 + 2(v_* - v_b)(\sigma_{th}u^3/v_*^2)(1 - \sigma_{th}u/v_*)]\, du. \qquad (6)$$

where $E_* = \mu v_*^2/2$, $\gamma = [1 + 2(v_* - v_b)/v_*]$, and we have assumed $2(v_* - v_b)\sigma_{th}u^3/v_*^2 \ll 1$.

Very small additional errors are incurred by extending the lower integration limit to $-\infty$ and keeping only the terms of order u^0, u^2, and u^4 to give the final result

$$(\sigma v)_{bt} \simeq \sigma(E_*)v_* \frac{\exp\left[-(\frac{v_*-v_b}{\sigma_{th}})^2\right](v_*/v_b)}{\sqrt{\gamma}} \left\{1 + \frac{\mu\sigma_{th}^2 S'(E_*)}{4\gamma S(E_*)} + \frac{3(v_* - v_b)}{2v_*\gamma^2}\left[\frac{\mu\sigma_{th}^2 S'(E_*)}{S(E_*)} - \left(\frac{\sigma_{th}}{v_*}\right)^2\right]\right\}. \qquad (7)$$

Although Core's[9] beam target reactivity included the smaller exponential in Eq. (1) and did not extend the integral in Eq. (6) to $-\infty$, his result differs from the lowest order term in Eq. (7) by much less than 1%.

3. Application to the DD and TT reactions

The DD and TT fusion reaction cross sections are well approximated by

$$S(E_{cm}) = S_o + S_1 E_{cm},$$

for $E_b \leq E_{max}$ where (S_o, S_1, E_{max}) are (53.55 keV barn, 0.3282 barn, 700 keV) for the $D(d,n)He^3$ reaction[5], (55.72 keV barn, 0.1868 barn, 3 MeV) for the D(d,p)T reaction[5], and (190 keV barn, -0.2 barn, 700 keV) for the TT reaction[3].

The new cross section measurements for the $D(d,n)He^3$ reaction are 5% lower than Duane's fit[12] for 100 keV $< E_b <$ 150 keV, and 10–15% higher than Peres' fit[10].

A simple estimate of the importance of the target temperature is given by the lowest order correction to the cold target reactivity: $\sigma(E_{cm} = E_b/2)v_b$.

For the $D(d,n)He^3$ reaction at E_b=120 keV, $\sigma v \sim E^{1.75}$, $E_* = E_b(1-\beta)^2$, $\beta = 2T_i/E_b$, and

$$\frac{\exp\left[-\left(\frac{v_* - v_b}{\sigma_{th}}\right)^2\right](v_*/v_b)}{\sqrt{\gamma}} \simeq (1-2\beta),$$

so

$$(\sigma v)_{bt} \simeq \sigma(E_b/2)v_b(1+3T_i/E_b).$$

This shows that even when $T_i = 0.1E_b$ the finite temperature enhancement of $(\sigma v)_{bt}$ is significant. The enhancement [calculated from Eq. (1)] for a range of beam energies and temperatures is given in Table 1.

For both DD reactions the first two terms of Eq. (7) approximate $(\sigma v)_{bt}$ as calculated numerically from Eq. (1)] to 4% for E_b < 300 keV and $T_i < 2E_b/3$. The negative slope of $S(E_{cm})$ for the TT reaction leads to less cancellation in the third term of Eq. (7) and all terms are required to achieve a 4% level of accuracy over this range of beam energies and temperatures.

If the target temperature is restricted to be no higher than 30 keV, however, the first term of Eq. (7) is accurate to 3% for all three of the reactions considered here over the entire range of E_b for which the cross sections fits are valid (see Fig. 1 for E_b <200 keV). In most circumstances, therefore, the lowest order approximation provides a convenient, computationally inexpensive evaluation of Eq. (1) for the DD and TT reactions.

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Table 1: Beam target reactivities for the $D(d,n)He^3$ reaction

E_b (keV)	σv_b ($10^{-24}m^3$ sec)	Enhancement: $(\sigma v)_{bt}/\sigma v_b$		
		T_i = 5 keV	T_i = 15 keV	T_i = 30 keV
50	1.01	1.8	3.3	5.6
100	5.11	1.2	1.6	2.1
150	10.5	1.1	1.3	1.5

Figure caption

Figure 1. The relative error for the first term of Eq. (7) compared to numerical integration of Eq. (1). The relative error of the D(d,p)T reaction is less than 1% for all T_i less than 30 keV.

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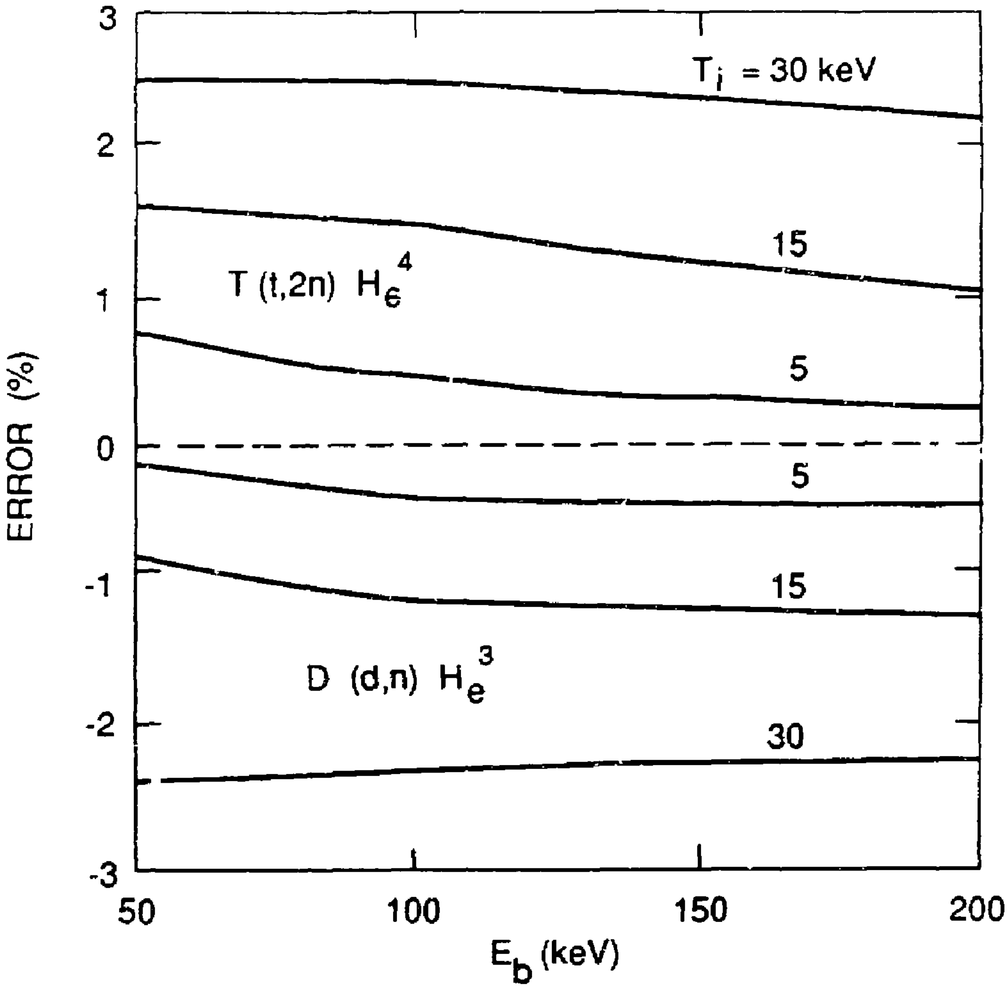

Fig. 1