

## Measurements of heavy quark fragmentation at TRISTAN

(The AMY Collaboration)

### ABSTRACT

The momentum spectrum of muons contained in multihadron  $e^+e^-$  annihilation events at an average center-of-mass energy  $\sqrt{s}=57.3$  GeV is used to study the fragmentation functions for heavy quarks. We obtain average values for the fragmentation variable,  $x_E = E_{hadron}/E_{beam}$  for the  $b$ - and  $c$ -quark of  $\langle x_E \rangle_b = 0.77^{+0.12}_{-0.11}$  and  $\langle x_E \rangle_c = 0.57^{+0.08}_{-0.10}$ , respectively.

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## Introduction

An understanding of the fragmentation process is a necessary component of a complete theoretical description of the process  $e^+e^-$  annihilation into hadrons. While perturbative QCD can exactly calculate  $e^+e^- \rightarrow q\bar{q}(g)$  cross sections to second order in the QCD coupling strength  $\alpha_s$ , the hadronization process of quarks and gluons into observable particles is described by non-perturbative phenomenological models such as the independent-jet and color-string models. For experimental studies of fragmentation, the reactions  $e^+e^- \rightarrow c\bar{c}$  ( $b\bar{b}$ ) into hadrons are particularly useful since in these cases the original quarks are distinguishable from the secondary ones.

In the fragmentation models, at the end of hadronization process the energy and momentum of the original quark is transferred to the primary hadrons according to a phenomenological function  $f(z)$ , where  $z$  is defined as  $(E+p)$  of the hadron divided by  $(E+p)$  of the original quark. This function is expected to scale with energy and depend only on the quark flavor. For the most models for heavy quark fragmentation, the average energy fraction taken by the primary heavy hadron is given by

$$\langle z \rangle \sim 1 - \frac{1\text{GeV}}{m_Q}, \quad (1)$$

where  $m_Q$  is the mass of the heavy quark [1] From this, we can expect that  $\langle z \rangle$  for the  $b$ -quark is larger than that for the  $c$ -quark [2],[3]. To date, insufficient statistics have limited experimental results to measurements of  $\langle z \rangle$ ; the functional form of  $f(z)$  has not been determined experimentally. Among the many forms that have been proposed for  $f(z)$ , [1]-[4]. that of

Peterson et al.[3], [3], which has only one parameter to be determined from experimental data, is the most widely used. It is given by

$$f(z) \propto \frac{1}{z[1 - \frac{1}{z} - \frac{\epsilon_Q}{(1-z)}]^2}, \quad (2)$$

where  $\epsilon_Q$  is a free parameter that has to be determined experimentally for each heavy quark  $Q$ . It is expected to be approximately equal to

$$\epsilon_Q \sim \frac{M_q^2}{M_Q^2}, \quad Q = c\text{- or } b\text{-quark, } q = u, d, \text{ or } s\text{-quark,} \quad (3)$$

i.e., the ratio of the squares of the masses of the light and heavy quarks forming the primary hadron. Even though  $z \equiv \frac{(E+p_{\parallel})_{hadron}}{(E+p)_{quark}}$  is the theoretically preferred definition of scaling variable, we chose  $x_E \equiv \frac{E_{hadron}}{E_{beam}}$  in this analysis due to ambiguity in the evaluation of  $(E + p)_{quark}$ , which is not directly measurable [5]. Since  $E_{quark}$  is the quark energy after accounting for initial-state radiation and final-state photon and gluon radiation,  $E_{quark}$  is always smaller than  $E_{beam}$  and, therefore,  $x_E$  is always larger than  $z$ .

Experimentally, two types of analyses have been used to determine the mean value of  $\langle x_E \rangle$ , the ‘‘hardness’’ of heavy quark fragmentation function: one is the reconstruction of heavy hadrons from their hadronic decay products, thereby obtaining  $x_E$  directly; the other uses hadronic events containing leptons that come from the semi-leptonic decay of heavy hadrons and inferring an average value of  $x_E$  from the lepton momentum spectrum. While the former is preferred, it has limited statistical significance because of the low efficiency for reconstructing heavy hadrons, especially hadrons containing  $b$ -quarks. The results reported here are based on the latter approach.

Since the secondary production of  $c$ - and  $b$ -quarks in the fragmentation process is expected to be suppressed due to their heavy masses [6]. [6], virtually all heavy mesons and baryons contain either the primary quark or its weak decay product. The longitudinal-momentum spectrum of leptons from primary hadrons provides information concerning the fragmentation of heavy quarks; the lepton's momentum component transverse to the event axis,  $p_t$ , provides information on quark identification. In our analysis we used the thrust axis as the event axis.

Because we do not measure  $E_{hadron}$  directly, we extract a relation between  $x_E$  and  $p$  muons from the semileptonic decays of the heavy quarks, as well as the relation between  $p_t$  of muons and their parent quark-type, from a Monte Carlo simulation. We then fit this to the corresponding distribution observed in the data using the fragmentation function as a weighting factor.

### Detector Description

The AMY detector is a compact detector with cylindrical geometry based on a 3-Tesla solenoid magnet and optimized for lepton identification. The detector is described elsewhere [7]. Charged particles are detected by an inner tube-type tracking chamber and a cylindrical drift chamber (CDC) over the polar angle region of  $|\cos\theta| < 0.87$  with a momentum resolution of  $\Delta p_t/p_t \sim 0.7\% \times p_t(\text{GeV}/c)$ . Neutral particles are detected by a cylindrical shower counter (SHC) over the angular range of  $|\cos\theta| < 0.73$  with an energy resolution of  $\sigma_E/E = 23\%/\sqrt{E(\text{GeV})} + 6\%$ . Calorimeters mounted on the magnet pole tips in both end-cap regions measure forward Bhabha events that are used to determine the luminosity.

The muon detector (MUO) is located outside of the magnet and consists of six sextants. Each sextant of the MUO consists of four layers of planar aluminium tube drift chambers for position measurements and one layer of scintillation counters for the measurements of the time-of-flight of particles that penetrate the material of the SHC, the superconducting coil of the magnet, and the iron return yoke. This material corresponds to about 9 nuclear absorption length; the minimum muon momentum for penetrating the absorber and reaching the MUO is 1.9 GeV/c. The angular coverage of the muon drift chambers is  $|\cos\theta| < 0.74$  and the tracking efficiency is better than 98%.

#### Analysis of Inclusive Muon Events

Inclusive muon events are selected from the sample of events that pass our standard hadronic event selection criteria [8] and have at least one muon candidate. Charged tracks are identified as muon candidates if they register hits in at least 3 out of 4 layers of the muon tracking chamber after penetrating through the absorber.

To reject backgrounds due to hadron punchthrough of the absorber and the decay-in-flight of charged  $\pi$  and K mesons, we require the distance between the MUO hit position and extrapolated position of a candidate muon's CDC track to be within  $2\sigma$  of the deviation expected from multiple coulomb scattering. This distance, which is a smoothly varying function of the extrapolated track momentum at the muon chamber, is 14 cm at 5 GeV/c [9]. The overall detection efficiency for muons in the angular region of  $|\cos\theta| < 0.73$  is 80%.

We classified inclusive muons as follows.

- Sources of primary (prompt) muons:

- 1)  $b \rightarrow c \mu^- \bar{\nu}_\mu$ ;
- 2)  $c \rightarrow s \mu^+ \nu_\mu$ ;
- 3) cascade decay;  $b \rightarrow c X$ ;  $c \rightarrow s \mu^+ \nu_\mu$ .

- Sources of background:

- 4a) decay-in-flight of  $\pi^\pm$  and  $K^\pm$  mesons;
- 4b) hadron punchthrough;
- 4c) mis-identification.

The estimated fraction of primary muons among the muon track candidates is 72%. The fraction of background tracks due to punchthrough and decay-in-flight of  $\pi$  and K mesons are 9% and 12%, respectively. We estimate that 7% of the muon sample are mis-identified tracks, which are mainly caused by mis-matches between CDC and MUO tracks.

The data sample used for this analysis was collected at center-of-mass energies from 50 to 61.4 GeV. The total integrated luminosity is  $40.5 \text{ pb}^{-1}$  and the total number of inclusive muons in the sample is 231. Figure 1 shows the total- and transverse-momentum distributions of the muon candidates in the data sample together with the expectations from the Monte Carlo simulation.

We divided the data into 15  $(p, p_t)$  bins: three total-momentum bins ( $p < 4 \text{ GeV}$ ,  $4 \leq p < 6 \text{ GeV}$ ,  $p \geq 6 \text{ GeV}$ ) and five transverse-momentum

bins ( $p_t < 0.5$  GeV,  $0.5 \leq p_t < 1.0$  GeV,  $1.0 \leq p_t < 1.5$  GeV,  $1.5 \leq p_t < 2.0$  GeV, and  $p_t \geq 2.0$  GeV). The number of expected inclusive muons in each  $p$  and  $p_t$  bin is given by

$$\begin{aligned}
 N_{exp}(p, p_t) = & N_{bkg}(p, p_t) + \\
 & \epsilon(p, p_t)[2N_{c\bar{c}} \cdot BR_c \sum_j W(j)P_c(j, p, p_t) + \\
 & 2N_{b\bar{b}}\{BR_b \sum_j W(j)P_b(j, p, p_t) + \\
 & BR_c \sum_j W(j)P_{bc}(j, p, p_t)\}]. \tag{4}
 \end{aligned}$$

Here,  $N_{bkg}$  is the background which includes both hadron fake and mis-identification,  $\epsilon(p, p_t)$  is efficiency for detecting prompt muons in the AMY detector,  $N_{c\bar{c}}$  and  $N_{b\bar{b}}$  are the expected number of  $c\bar{c}$  and  $b\bar{b}$  events in the total multihadronic events, and  $BR_c$  and  $BR_b$  are the average semi-leptonic branching ratios.  $W(j)$  is the weighting factor in the  $j$ -th bin of  $x$ ; this contains the fragmentation function information. The  $W(j)$  values are obtained by integrating  $f_Q(x)$  in each  $x$  bin,

$$W(j) = \int_j dx \beta f_Q(x), \quad j = 1, 5 \tag{5}$$

where  $\beta$  is a velocity of heavy hadron (we set  $\beta = 1$  in our fit) and  $f_Q$  is given by

$$f_Q(x) = N/[x(x - 1/x - \epsilon_Q/(1 - x))]^2. \tag{6}$$

For the last term in eq.(4), which accounts for cascade decays, we used the fragmentation function of  $b$ -quark.  $P_c$ ,  $P_b$  and  $P_{bc}$  are the probabilities that the semi-leptonic decay of a heavy hadron with  $x$  in  $j$ -th bin will produce

a muon in  $(p, p_t)$  bin. The subscripts  $b$  and  $c$  denote the parent quark of the muon,  $bc$  denotes muons from the semileptonic decay of  $c$ -quarks from  $b$  decays. In order to obtain these probabilities, we generated hadronic events with a flat fragmentation function using the Lund 6.2 event generator and subjected them to the detector simulation and the event selection procedures for inclusive muon events.

In our fit, we fixed the average semi-muonic branching ratio for bottom hadrons,  $BR(b \rightarrow \mu X) = 11.8 \pm 0.5\%$ , which is a combined result from continuum and  $\Upsilon(4S)$  measurements, and for charmed hadrons we used  $BR(c \rightarrow \mu X) = 8.6 \pm 0.9\%$ , which is an average from PEP and PETRA experiments [12]. [12].

From a minimum  $\chi^2$  fit with  $\epsilon_b$  and  $\epsilon_c$  treated as free parameters we obtain

$$\epsilon_b = 0.017^{+0.019}_{-0.013}$$

$$\epsilon_c = 0.197^{+0.185}_{-0.095},$$

with  $\chi^2/d.o.f. = 1.44$ . The quoted errors include statistical errors and the correlations between fit parameters. From this we obtain  $\epsilon_c/\epsilon_b=11.4$ , which agrees well with the value  $\epsilon_c/\epsilon_b \sim M_b^2/M_c^2 = 5^2/1.5^2 = 11.1$  that is suggested by Eq. (3). These values of  $\epsilon_b$  and  $\epsilon_c$  correspond to the average values for  $x_E$  of

$$\langle x_E \rangle_b = 0.77^{+0.018}_{-0.10}$$

$$\langle x_E \rangle_c = 0.57 \pm 0.06.$$

Figure 2 shows the  $p$  and  $p_t$  distributions of the inclusive muons together

with the results from the fit. Figure 3 displays  $\langle x_E \rangle_b$  and  $\langle x_E \rangle_c$  together with the results from other experiments [12],[13],[14],[15].

In order to investigate scaling, we extracted a relation between  $\langle x_E \rangle$  and  $\langle z(rec) \rangle$ , where  $z = E_{hadron}/E_{quark}$  and  $z(rec)$  denotes the fact that it is reconstructed from the generated Monte Carlo four-vectors [10],[11]. By calculating  $\langle x_E \rangle$  and  $\langle z(rec) \rangle$  for a given  $\epsilon$  using the Monte Carlo generator, we obtain

$$\begin{aligned}\langle z(rec) \rangle_b &= 0.87^{+0.07}_{-0.09} \\ \langle z(rec) \rangle_c &= 0.65^{+0.05}_{-0.06}.\end{aligned}$$

Since  $\langle z(rec) \rangle$  depends on  $E_{quark}$ , it is sensitive to the choice of values for the parameters  $\alpha_s$  and  $y_{min}$  used in the QCD event generator. From the changes in the extracted value for  $\langle z(rec) \rangle$  when we vary  $\alpha_s$  from 0.12 to 0.2 and  $y_{min}$  from 0.01 to 0.04, we obtained systematic errors due to a choice of these parameters of 0.03 and 0.02 for  $\langle z(rec) \rangle_b$  and  $\langle z(rec) \rangle_c$ , respectively. Our extracted value of  $\langle z(rec) \rangle$  is shown in Fig. 4. The results from DELCO and TPC are extracted from  $\langle x_E \rangle$  [11]; the others are obtained by using  $z(rec)$  instead of  $x$  in the fits.

We considered the following sources of systematic errors:

1. uncertainty in the level of hadron background in the muon sample;  
(We allowed a  $\pm 30\%$  uncertainty in punchthrough background [9],[16].)
2. uncertainty of the efficiency for finding prompt muons;
3. uncertainty of the number of  $b\bar{b}$  and  $c\bar{c}$  events;
4. uncertainty in the average semi-leptonic branching ratio;

5. uncertainty in the direction of the parent quark; (In multijet events the direction of the fragmentation and event axis can differ. We estimate the magnitude of this effect by performing the same analysis using only two-jet events.)
6. the effect of the first bin; (Since the first bin of the fit [ $p < 4$  GeV and  $p_t < 0.5$  GeV] contains the largest fraction of background, we reperformed the analysis with the first bin eliminated.)

Estimates of the overall systematic error are listed in Tables 1 and 2. In these tables the numbers in parentheses is the relative error (in percent). As can be seen from the table, the major systematic error comes from the uncertainty in the direction of the fragmentation axis and the uncertainty in the fraction of hadron backgrounds in the muon sample. In general, the results for the  $c$ -quark is more sensitive to the parameters of fit than those for the  $b$ -quark.

### Result and Discussion

From the analysis of inclusive muon events we obtained averages for the fragmentation variables for the  $b$ - and  $c$ -quarks of  $\langle x_E \rangle_b = 0.77^{+0.12}_{-0.11}$ , and  $\langle x_E \rangle_c = 0.57^{+0.08}_{-0.10}$ , respectively. The value of  $\langle x_E \rangle_b$  can be compared with the recent measurement from the L3 [13] and ALEPH[14] experiments, which are  $\langle x_E \rangle_b = 0.69 \pm 0.04$  and  $\langle x_E \rangle_b = 0.67^{+0.04}_{-0.03}$ , respectively. A comparison of the PEP/PETRA with LEP results, shown in Fig. 3, hints at an  $E_{beam}$  dependence of  $\langle x_E \rangle_b$ , as expected from the effects of gluon emission. Our results do not have enough precision to confirm this  $E_{beam}$

dependence. The results for  $\langle z(rec) \rangle$  are consistent with the scaling of  $\langle z(rec) \rangle$ , as can be seen in Fig. 4. Our results of  $\langle z(rec) \rangle_b = 0.87^{+0.12}_{-0.11}$  and  $\langle z(rec) \rangle_c = 0.65^{+0.07}_{-0.13}$  agree well with the average values of all the data shown in Fig. 4,  $0.85 \pm 0.02$  and  $0.68 \pm 0.02$ , respectively.

### Acknowledgements

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systematic errors	$\epsilon_b$	$\langle x_E \rangle_b$	$\epsilon_c$	$\langle x_E \rangle_c$
bkgd +30%	0.0363	0.716 (-6.7%)	0.296	0.530 (-6.7%)
fake -30%	0.0084	0.811 (+5.7%)	0.134	0.603 (+6.2%)
Br( $b \rightarrow \mu X$ ) +1 $\sigma$	0.0193	0.760 (-0.9%)	0.204	0.564 (-0.6%)
Br( $b \rightarrow \mu X$ ) -1 $\sigma$	0.0154	0.775 (+1.0%)	0.190	0.5708 (+0.6%)
Br( $c \rightarrow \mu X$ ) +1 $\sigma$	0.0214	0.754 (-1.8%)	0.243	0.549 (-3.3%)
Br( $c \rightarrow \mu X$ ) -1 $\sigma$	0.014	0.781 (+1.8%)	0.156	0.589 (+3.7%)
Efficiency +1 $\sigma$	0.0210	0.755 (-1.7%)	0.221	0.557 (-1.9%)
Efficiency -1 $\sigma$	0.0140	0.781 (+1.8%)	0.174	0.579 (+2.0%)
R( $b\bar{b}$ ) +1 $\sigma$	0.024	0.747 (-2.7%)	0.217	0.559 (-1.6%)
R( $b\bar{b}$ ) -1 $\sigma$	0.0121	0.790 (+2.9%)	0.173	0.577 (+1.6%)
R( $c\bar{c}$ ) +1 $\sigma$	0.020	0.758 (-1.2%)	0.235	0.551 (-2.9%)
R( $c\bar{c}$ ) -1 $\sigma$	0.0150	0.777 (+1.2%)	0.161	0.586 (+3.3%)
Two-jet events	0.0052	0.837 (+9.0%)	0.433	0.494 (-13.0%)
No first bin	0.0181	0.765 (-0.4%)	0.199	0.567 (-0.2%)
Total	0.768	$\begin{matrix} +0.080 & +0.089 \\ -0.098 & -0.060 \end{matrix}$	0.568	$\begin{matrix} +0.060 & +0.048 \\ -0.062 & -0.088 \end{matrix}$

Table 1: The systematic errors for  $\langle x_E \rangle$ . We indicate how the results change when we increase and decrease each item by its estimated uncertainty. R( $b\bar{b}$ ) and R( $c\bar{c}$ ) are the fractions of hadronic events that are  $b\bar{b}$  and  $c\bar{c}$ , respectively.

systematic errors	$\epsilon_b$	$\langle z(rec) \rangle_b$	$\epsilon_c$	$\langle z(rec) \rangle_c$
Bkgd +30%	0.0363	0.812 (-6.3%)	0.296	0.597 (-8.2%)
Bkgd -30%	0.0084	0.904 (+4.2%)	0.134	0.682 (+4.9%)
Br( $b \rightarrow \mu X$ ) +1 $\sigma$	0.0193	0.867 (-0.03%)	0.204	0.638 (-1.9%)
Br( $b \rightarrow \mu X$ ) -1 $\sigma$	0.0154	0.882 (+1.7%)	0.190	0.650 (-0.1%)
Br( $c \rightarrow \mu X$ ) +1 $\sigma$	0.0214	0.853 (-1.6%)	0.242	0.624 (-4.1%)
Br( $c \rightarrow \mu X$ ) -1 $\sigma$	0.0139	0.878 (+1.2%)	0.156	0.672 (+3.3%)
Efficiency +1 $\sigma$	0.0210	0.850 (-1.9%)	0.221	0.633 (-2.7%)
Efficiency -1 $\sigma$	0.0140	0.880 (+1.5%)	0.174	0.658 (+1.2%)
R( $b\bar{b}$ ) +1 $\sigma$	0.0235	0.857 (-1.2%)	0.217	0.641 (-1.5%)
R( $b\bar{b}$ ) -1 $\sigma$	0.012	0.894 (+3.2%)	0.178	0.660 (+1.5%)
R( $c\bar{c}$ ) +1 $\sigma$	0.020	0.857 (-1.1%)	0.235	0.628 (-3.5%)
R( $c\bar{c}$ ) -1 $\sigma$	0.0150	0.882 (+1.7%)	0.161	0.663 (+1.9%)
Two-jet events	0.0052	0.949 (+9.5%)	0.433	0.562 (-13.6%)
No first bin	0.018	0.863 (-0.5%)	0.199	0.645 (-0.9%)
Total		0.867 $\begin{smallmatrix} +0.068 & +0.098 \\ -0.091 & -0.061 \end{smallmatrix}$		0.650 $\begin{smallmatrix} +0.019 & +0.042 \\ -0.062 & -0.112 \end{smallmatrix}$

Table 2: The systematic errors for  $\langle z(rec) \rangle$ . The notation is the same as that used for Table 1. The overall systematic error is estimated to be  $\pm 0.03$  and  $\pm 0.02$  for  $\langle z(rec) \rangle_b$  and  $\langle z(rec) \rangle_c$ , respectively.

### Figure Captions

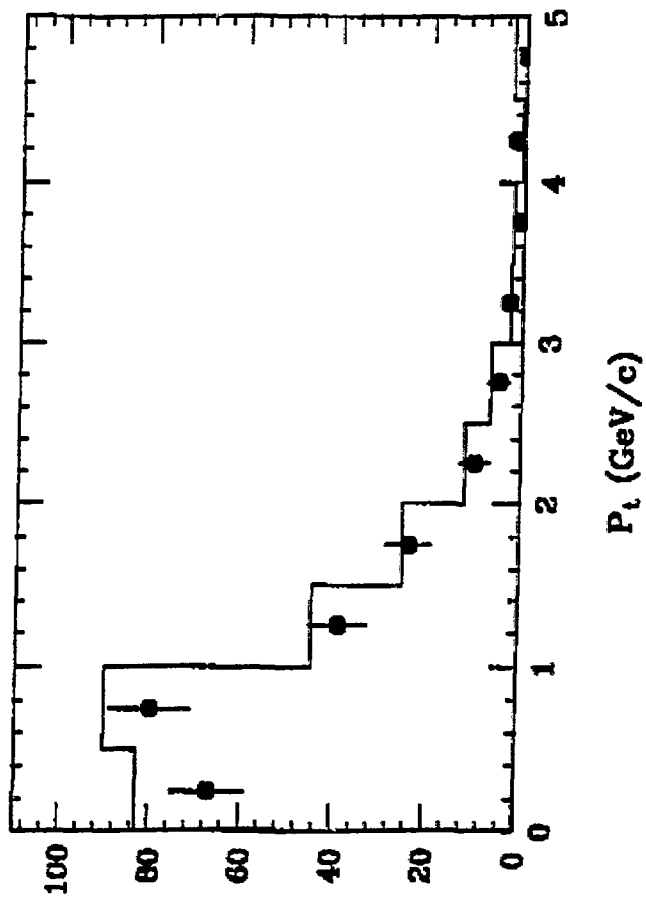
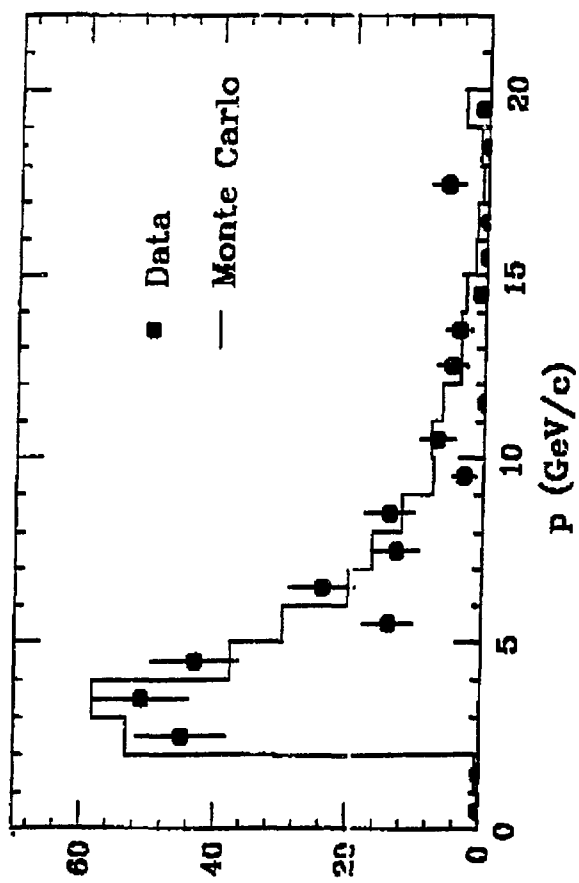
**Figure 1** The transverse (a) and total (b) momentum distribution of muons from hadronic events. The histograms indicate the results from the Monte Carlo.

**Figure 2** The data and the results of the fit for the 15 ( $p, p_t$ ) bins.

**Figure 3** A comparison of our measurements of  $\langle x_E \rangle$  with results from other inclusive lepton analyses (refs. [12],[13],[14]). Recent measurements of  $\langle x_E \rangle_c$  from TASSO and HRS, based on an analysis of  $D^*$  production, are also included [15].

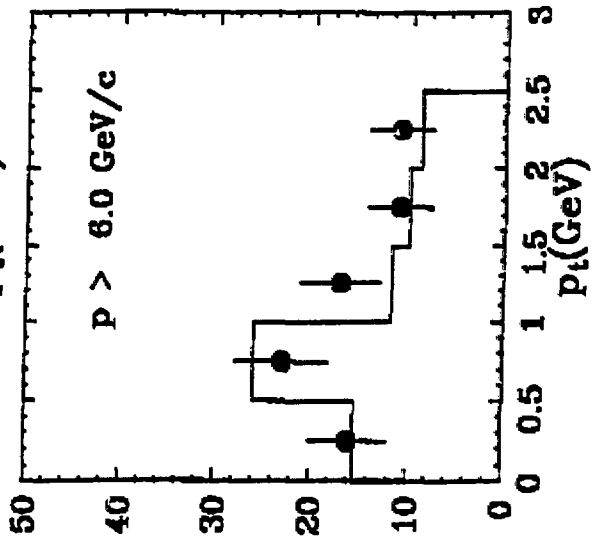
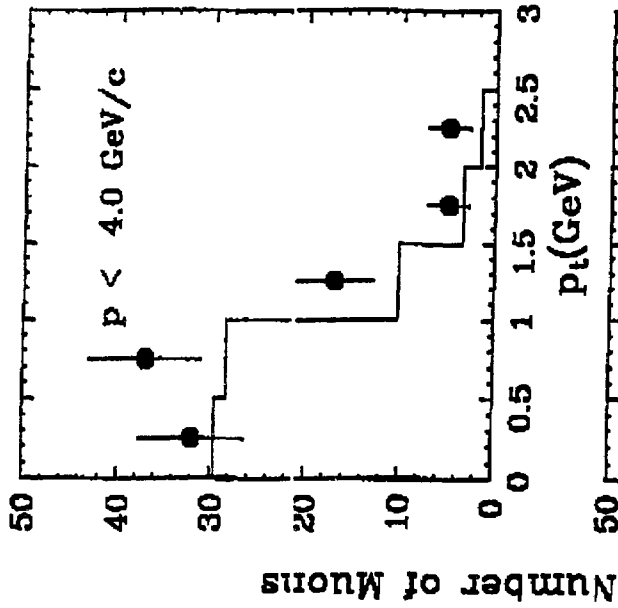
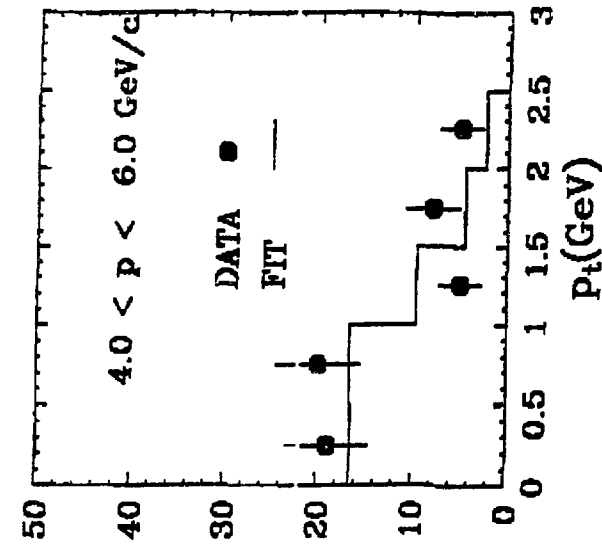
**Figure 4** A comparison of our results for  $\langle z(rec) \rangle$  with those from other experiments (taken from ref. [11]). The solid and dotted lines indicate the average value derived from the inclusive lepton data and its error. TASSO\* is a recent result from TASSO using the rapidity and charged particle multiplicities of  $b$ -quark enriched jets. [15]

AMY  
FIGURE 1



AMY

FIGURE 2



$\chi^2/\text{dof}=1.44$

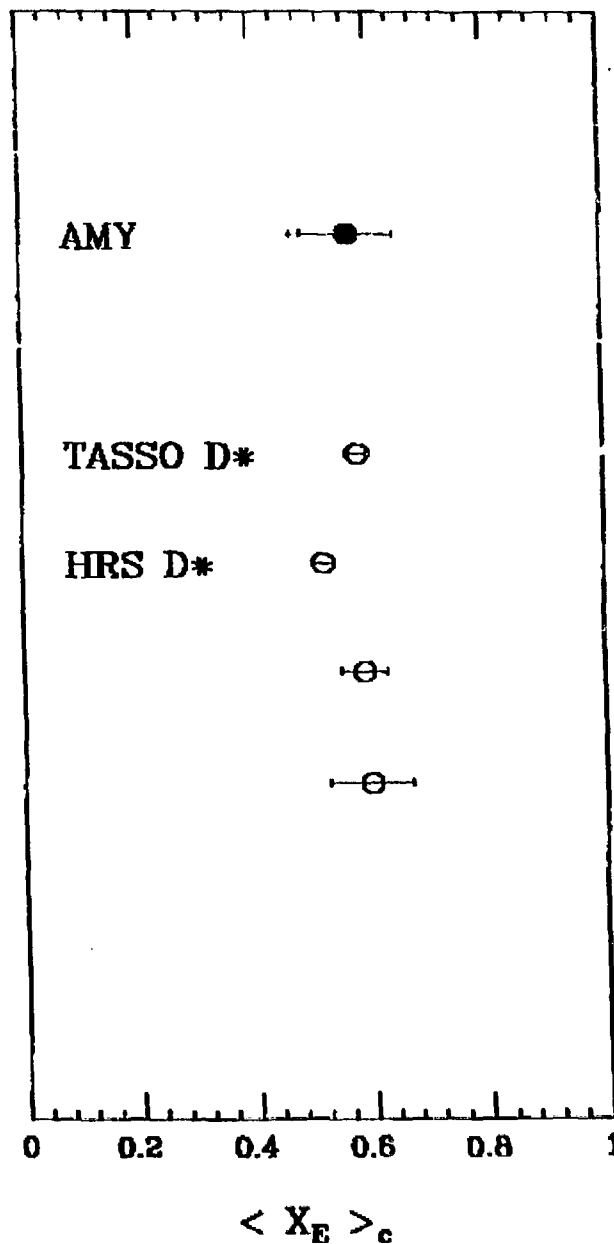
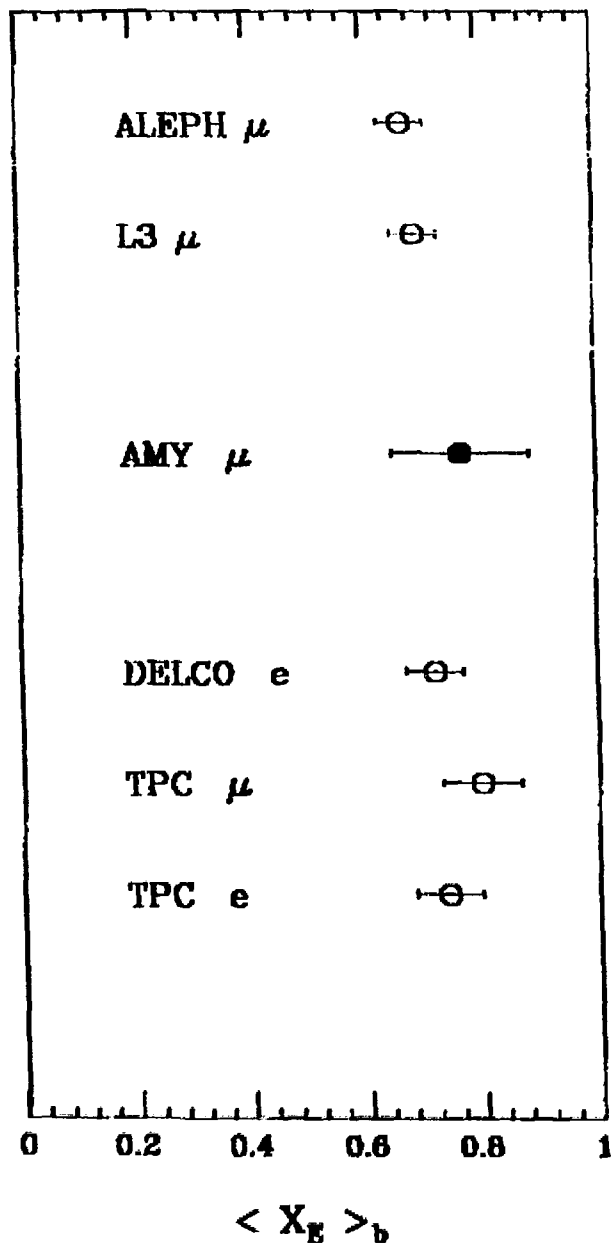
$\epsilon_B = 0.017$

$\epsilon_C = 0.196$

$BR_b = 11.8\%$  fixed

$BR_c = 8.6\%$  fixed

# Measurements of Heavy Quark Fragmentation



AMY  
FIGURE 3

