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INTRODUCTION

Monte Carlo radiation transport codes use a sequence of pseudorandom numbers to sample from probability distributions. A common practice is to start each source particle a predetermined number of random numbers up the pseudorandom number sequence. This number of random numbers skipped between each source particle is the random number stride, S. Consequently, the jth source particle always starts with the $j \cdot Sth$ random number providing "correlated sampling" between similar calculations.

A new machine-portable random number generator has been written for the Monte Carlo radiation transport code MCNP[1] providing user's control of the random number stride. First the new MCNP random number generator algorithm will be described and then the effects of varying the stride will be presented.

The New Random Number Generator Algorithm

MCNP has always used the congruential scheme of Lehmer[2]. A pseudorandom sequence of integers, I_n, is generated by

$$I_{n+1} = \mathrm{mod}(MI_n, 2^{48}) \ ,$$

where M is the random number multiplier, and here 48-bit integers and 48-bit floating point mantissas are assumed. The new MCNP algorithm provides user control of M with a default value of

$$M = 5^{19} = 19073486328125.$$

The random number is

$$R_n = 2^{-48} I_n \ .$$

The starting random integer of each history is

$$I_{n+S} = \mathrm{mod}(M^S I_n, 2^{48}) \ ,$$

where S is the stride. Because each random number is the least-significant (lower) 48-bits of M times the previous random number, the lower 48 bits of I_{n+S} is the same as the lower 48 bits of $M^S I_n$.

To achieve 96-bit accuracy in a machine-portable way for the above integer products, MI_n and $M^S I_n$, the MOD function is used to split 48-bit quantities into 24-bit halves, and then multiplications are done on these halves. The products of the 24-bit quantities will never exceed 48-bits. The 48-bit products can then be further manipulated to reconstruct the required 48-bit mantissa. To achieve acceptable computational speed, the faster INT function replaces the MOD function:

$$\mathrm{mod}(X,2^k) = X - 2^k \mathrm{int}(2^{-k}X) \ .$$

Historically, MCNP has used a random number stride of $S = 4297$ that is too small for many modern problems. Therefore a better default stride of

$$S = 152917_{10} = 452525_8 = 100101010101010101_2$$

has been implemented.

The Effects of Various Random Number Strides

A random number stride less than the number of random numbers required for any history causes a correlation of results because the same random number sequence is reused in adjacent histories. Fortunately, the correlation is small because the random numbers are used for different sampling purposes in most realistic problems.

Monte Carlo results are less sensitive to the choice of random number stride than might be expected. A series of twenty-six radiation transport problems were run using random number strides of 4297 and 152917. These problems included a wide variety of applications including oil well logging, criticality safety, radiation shielding, deep penetration, and electron detector modeling. No discernible effect was observed, even though in most problems some histories required more than 4297 random numbers, and sometimes more than 100000 random numbers. The oil well logging calculation was run with strides of 1, 2, 10, 100, 1000, 4217, 152917, and 5000000; and no effect was apparent.

The worst possible case occurs when the problem is so simple that the random numbers are used for only a few basic functions, and the stride is so short that the same random numbers are used for these functions in different histories. To examine this situation, a simple test problem was devised consisting of a monoenergetic, isotropic source in a 10% absorbing, 15-mean free path sphere. Random number strides of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 100, 1000, 4217, 10000, 100000, 152917 and 1000000 were tested. Fluxes were tallied at fourteen concentric surfaces, one mean free path apart, throughout the sphere. Figure 1 shows the calculated fluxes divided by the converged solution plotted against the fourteen tally surfaces for four cases: $S = 152917$ and the three worst cases of $S = 1, 2$, and 9. The fluxes converged to within one standard deviation of the mean two-thirds of the time for a stride of 152917 as expected. In the worst case where $S = 2$, the variances were underestimated. Strides of 3, 4, 5, 6, 7, and 8 were all better than strides of 4297 and 152917 whereas strides of 1,2 and 9 were worse.

CONCLUSION

A new MCNP random number generator has been written providing a larger default stride and user control of both stride and multiplier in a machine-portable way. The effects of different values of the stride have been examined. In simplistic problems a small stride can cause an underestimation of the variance. But for realistic problems, Monte Carlo convergence is surprisingly insensitive to the stride, even when extreme values such as

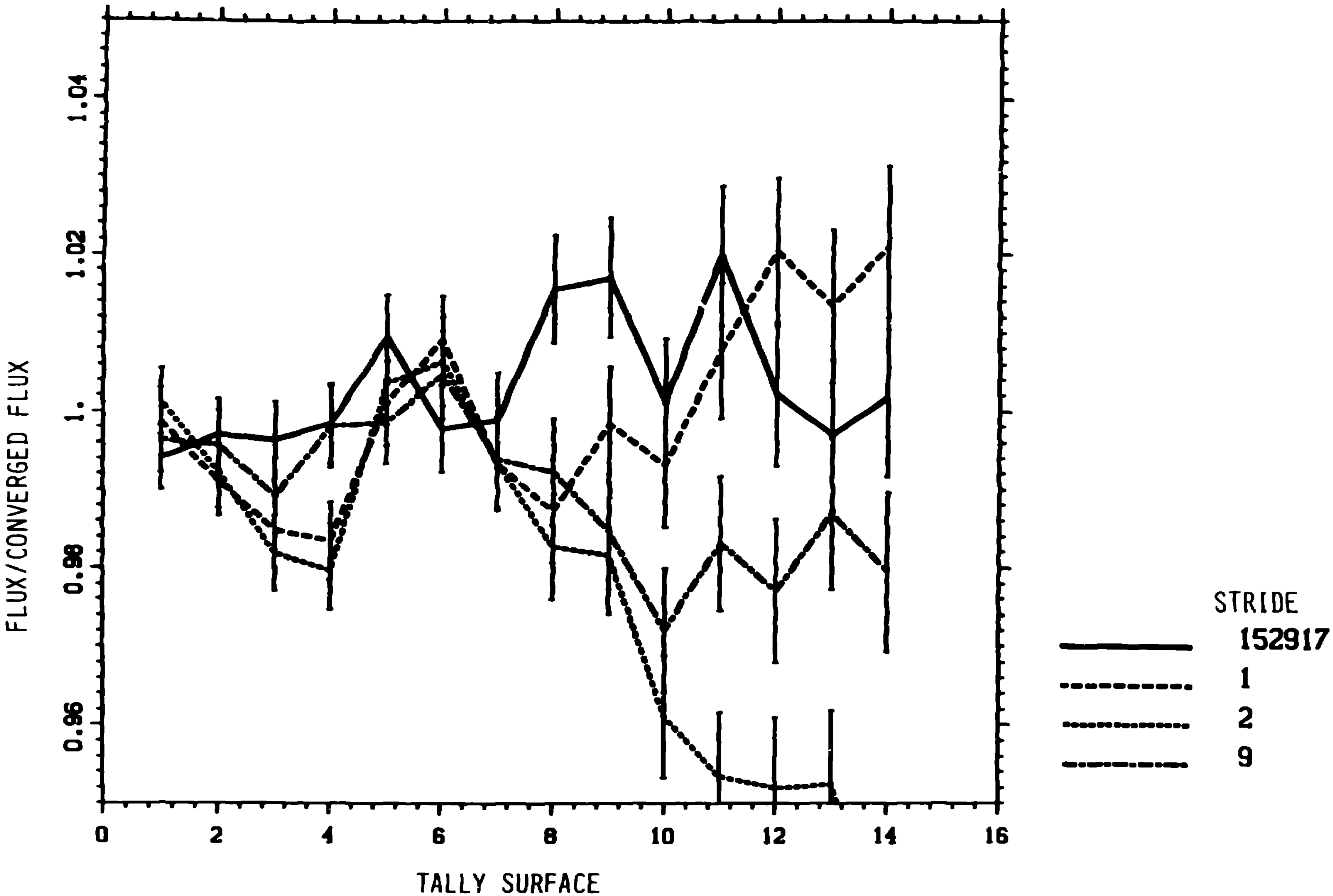
$S = 1$ are used, because the random numbers are used to sample different processes.

REFERENCES

1. J. F. Briesmeister, Editor, "MCNP - A General Monte Carlo Code for Neutron and Photon Transport, Version 3A," Los Alamos National Laboratory report, LA-7396-m, Rev. 2, 1986.

2. D. H. Lehmer "Mathematical Methods in Large-Scale Computing Units," *Am. Comput. Lab. Harvard Univ.*, **26**, 141-146, 1951.

FIGURE 1


1.04
1.02
1.
0.98
0.96
FLUX/CONVERGED FLUX
0
2
4
6
8
10
12
14
16
TALLY SURFACE
STRIDE
152917
1
2
9