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Optimization of Laser Wakefield Accelerator Parameters

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OPTIMIZATION OF LASER WAKEFIELD ACCELERATOR PARAMETERS

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Abstract

We reveal the dependencies of the LWFA performance upon such basic parameters as laser wavelength, power, and pulse duration and apply them for optimization of the plasma-channeled standard LWFA operating in a linear regime. The maximum energy gain over the dephasing distance scales proportionally to the laser peak power, while the allowed minimum laser pulse duration is proportional to the square root of the energy gain. Electron beam energy spread, emittance and luminosity tend to improve with the laser wavelength increase. These considerations, supported by quantitative examples for the 5 GeV LWFA stage, favor picosecond CO_2 laser as the optimum choice for future advanced accelerator projects.

1. Introduction

A search for the optimum laser driver for the ~5 GeV high-gradient compact laser wakefield accelerator (LWFA) is one of the DOE priorities since the Advanced Accelerator Concepts 1996 workshop[1]. The interest in such an accelerator is primarily due to its conceivable utilization in the prospective 5 TeV c.m. electron-positron collider. The result of the first exercise on this topic has been presented at the Kyoto ICFA workshop in July 1997 [2]. The 50 TW lasers of the 1-μm (solid state laser) and 10-μm (CO_2 laser) wavelength have been compared as the LWFA drivers. Both schemes demonstrate a possibility of 5 GeV acceleration per stage, while the CO_2 laser permits to attain a much higher luminosity than the solid state laser. However, the opponents argued that parameters for comparison are "chosen to the author's preference" and questioned the final conclusions.

Thus, it would be instructive to reveal the dependencies of the LWFA performance upon such basic parameters as laser wavelength and pulse duration in a generic analytical form easy for interpretation. It is the goal of this writing to uncover such relations and to apply them for optimization of the single-stage 5 GeV LWFA.

In Section 2, we identify the primary constraints that impact optimization of a laser driver for the plasma-channeled standard LWFA operating in the linear regime.

In Section 3, we show that the maximum net acceleration per stage in the plasma channeled LWFA is proportional to the laser peak power. Through the power threshold condition for the linear LWFA regime we arrive at the expression for the minimum allowed laser pulse duration.

In Section 4, we review the requirements of the electron beam quality and luminosity for the prospective accelerator, and show in Section 5 how these requirements affect the choice of the optimum laser driver parameters.

Results and conclusions of this study are summarized in Section 6.

2. Initial Constraints and Assumptions

The standard LWFA[3] is generally considered as the preferable scheme for the future high-gradient electron laser accelerators of several GeV energy gain per stage. The interest in this scheme used in the linear regime is driven by the expected relatively high stability and regularity of the plasma wake. This may offer an opportunity to achieve a reasonably good quality (emittance, monochromaticity) of the accelerated beam while maintaining the acceleration gradient ~100 times higher than with conventional RF accelerators.

In the standard LWFA scheme, plasma wave is initiated by a short laser pulse equal in duration to the half-period of the plasma wave,

$$\tau_L \approx \lambda_p / 2c, \tag{1}$$

that depends upon the plasma density, n_e,

$$\lambda_p = \frac{c}{e}\sqrt{\frac{\pi m}{n_e}}, \quad (2)$$

or

$$n_e\left[cm^{-3}\right] = 10^{21} / \lambda_p^2[\mu m]. \quad (2a)$$

The important parameter that characterizes how the laser field affects the electron is the dimensionless laser vector-potential,

$$a = eE_L / mc\omega = 0.3\, E_L\, [TV/m]\lambda[\mu m]. \quad (3)$$

Here E_L is the laser electric field amplitude related to the laser intensity, I_L, by the equation

$$E_L = \sqrt{2I_L / \varepsilon_0 c} \quad (4)$$

or

$$E_L[TV/m] = 15P_L^{1/2}[TW] / r_L[\mu m], \quad (5)$$

where P_L is the laser peak power and r_L is the radius of the laser focus spot.

There is a restriction on a imposed for the standard LWFA operating in a linear non-relativistic regime: $a \leq 1$. Thus, E_L can not exceed the value $E_L^{\max}\, [TV/m] \approx 3/\lambda[\mu m]$ defined by Eq.(3).

For the same reason of staying in a linear non-relativistic LWFA regime, the laser peak power needs to be kept below the relativistic self-focusing condition[4]

$$P_L \leq P_{cr} = 17(\lambda_p/\lambda)^2\,[GW]. \quad (6)$$

We will see in Section 3 that for the 5 GeV LWFA some form of laser channeling shall be used. As long as the relativistic self-focusing regime is not allowed for the standard LWFA scheme, we assume that a plasma channel is pre-formed and has a density step at the "wall" to satisfy the waveguiding condition (see Appendix),

$$\Delta n_e\left[cm^{-3}\right] = 1.13\times 10^{20} / r_L^2[\mu m]. \quad (7)$$

Comparing Eq.(7) with Eq.(2a) and assuming a quasi-evacuated channel "drilled" in the uniform plasma that satisfies Eq.(1) we come to the conclusion that the radius of the vacuum channel is equal to $\lambda_p/3$. If we assume just a partially rarefied plasma channel with the 10% stepped-down density, $\Delta n_e = 0.1 n_e$, then

$$r_L \approx \lambda_p. \quad (8)$$

Note, that combining Eqs.(3) and (5) under the condition of $a \leq 1$ we still arrive to Eq.(6) if $r_L = 0.6\,\lambda_p$. Thus, the conditions of $a \leq 1$ and $P_L \leq P_{cr}$ appear to be about equivalent for the plasma-channeled LWFA.

3. Acceleration gradient and net energy gain per stage

The amplitude of the accelerating field, E_a, due to the charge separation in a plasma wave is[3]

$$E_a[V/cm] = \left(a^2/\sqrt{1+a^2}\right)\sqrt{n_e\left[cm^{-3}\right]}. \quad (9)$$

In the uniform plasma, the acceleration distance within the laser focus waist is defined by the Rayleigh length

$$z_0 = \pi r_L^2 / \lambda. \quad (10)$$

We assume that the plasma channel extends the acceleration over a certain number of Rayleigh lengths. For quantitative examples in Table 1 we assume the plasma channel, $L_{ch} = 40 z_0$, which is close to experimentally demonstrated numbers[5].

Even if we succeed in creating a very long plasma channel we can not maintain the acceleration longer than the wave dephasing distance,

$$L_{ph} = \lambda_p\left(\frac{\lambda_p}{\lambda}\right)^2, \quad (11)$$

or the laser energy depletion distance,

$$L_D = c\,\tau_L/\xi^2, \quad (12)$$

where $\xi \equiv \dfrac{E_a}{E_L}$. Assuming $a \approx 1$, by Eqs. (1), (2), (3), and (9) we obtain

$$L_{ph} \approx L_D \propto \frac{\tau_L^3}{\lambda^2}. \quad (13)$$

Then, the net energy gain, ΔW, for dephasing-limited or depletion-limited interaction, scales as

$$\Delta W_{ph} \equiv E_a \times L_{ph} \propto \left(\frac{\tau_L}{\lambda}\right)^2. \quad (14)$$

The "exact" expression for the net gain may be obtained starting with Eqs. (9), (11), and (2a):

$$\Delta W_{ph}[MeV] = \frac{3a^2}{\sqrt{1+a^2}}\left(\frac{\lambda_p}{\lambda}\right)^2. \quad (15)$$

To make a simple estimate for the net gain we assume $\left(\dfrac{\lambda_p}{r_L}\right) \approx 2$, which is valid for a partially evacuated plasma channel of the length $L_{ch} \approx 0.5\ L_{ph}$. Then, using Eqs.(3), (5), and (15) we obtain

$$\Delta W_{ch}[GeV] \approx 0.1 P_L[TW]. \quad (16)$$

Keep in mind that we are not free in choosing the laser peak power in the linear LWFA regime. As we have seen previously, the restriction to the P_L scaling is adequately expressed by Eq.(6).

Another meaningful scaling law follows from Eqs.(15) and (1):

$$\tau_L[\lambda] \approx \frac{9\sqrt[4]{1+a^2}}{a}\sqrt{\Delta W_{ph}}[GeV]. \quad (17)$$

Thus, for any desirable energy gain per stage we can prescribe the minimum admissible laser pulse length, $\tau_L^{\min}$. The estimate for $\tau_L^{\min}$ obtained at a=1 and at the previous assumption of $L_{ch} \approx 0.5\ L_{ph}$ is:

$$\tau_L^{\min}[\lambda] \approx 15\sqrt{\Delta W_{ch}}[GeV]. \quad (18)$$

For example, the LWFA accelerator with the 5 GeV energy gain per stage requires $P \approx 50$ TW, by Eq.(16), and $\tau_L \geq 100$ fs, by Eq.(18) at λ=1 μm.

This breaks a common misconception that the increase of the laser peak power via pulse shortening, that is the mainstream of the femtosecond solid state laser development, leads to a more efficient driver for the future LWFA. In reality, we see that the laser pulse shortening is efficient until the relativistic self-focusing threshold (or condition a=1) is reached. Starting from this climax, further increase of the laser peak power (together with the net acceleration) is permitted only via simultaneous the laser pulse increase according to Eq. (17) and the laser energy increase in proportion $\Sigma_L \sim \Delta W^{3/2}$.

From the obtained results, we can conclude that by reducing λ we can maintain the desired energy gain in the proportionally shorter accelerator stage and at a smaller laser pulse energy (see also quantitative examples in Table 1). Is not this an advantage of the short-wavelength lasers? It would possibly be the case if the net acceleration and compactness are the only parameters for the accelerator optimization. Other important characteristics for a practical accelerator are considered in the next two paragraphs.

4. Electron beam quality and luminosity

The strategic interest in developing a compact several GeV accelerator is to use it as a construction block for a staged electron-positron or electron-gamma collider of the TeV energy range. The important cumulative characteristic which defines the collider performance is luminosity,

$$\Lambda = \frac{N_e^2 \zeta f}{4\pi\sigma_\perp^2}, \tag{19}$$

where N_e is the number of particles per bunch, ζ is a number of bunches per train, f is the repetition rate of laser pulses, and $\sigma_\perp$ is the e-beam radius at the interaction point. The required luminosity for the future collider is extremely high, $\Lambda \approx 10^{35}$ [$cm^{-2}s^{-1}$]. Increase of N_e is a way to a high luminosity. However, there is a limit to N_e defined by the condition that the self-field of the bunch does not affect appreciably the plasma wakefield structure,

$$N_e \le n_e \left(c/\omega_p\right)^3. \tag{20}$$

That means that the $N_e^{\max}$ scales proportional to λ_p. The same conclusion may be driven from the energy conservation condition. Indeed, as has been shown above, with the increase of λ_p a higher laser pulse energy is required to drive the LWFA in the linear regime. A higher laser energy permits to accelerate the proportionally bigger electron bunch charge. Thus, due to $N_e \sim \lambda_p = 2c\,\tau_L$, we gain $\Lambda \sim \tau_L^2$.

Another important parameter which enters Eq.(20) is the e-beam cross-section which depends upon the e-beam emittance. Some authors choose to rely upon the e-beam focusing in the plasma wake field to achieve a low emittance. This approach favors high transverse gradients in the plasma wake field achieved at a high plasma density. Note, however, that the maximum focusing force in a plasma wake is normally phased with the zero accelerating field. The increase in plasma density results also in noticeable collisional scattering of the electrons in the gas that competes with the focusing.

Another important characteristic for an accelerator performance is a small energy spread, preferably close to 0.1% that is customary for conventional linacs. The requirement for the small energy spread presumes that the electron bunch should be much shorter than the plasma wake period

$$\tau_b << \lambda_p / c. \tag{21}$$

This again favors a bigger λ_p. The suggestion to rely on electron micro-bunching in a plasma wake needs further verification.

Adopted in this paper, the approach to achieve the high-quality accelerated e-beams looks the most straightforward and secure. We start with the seed electron bunches small to compare with the λ_p and try to preserve their quality (emittance, energy spread) in the course of the acceleration in the plasma wave. The initial bunches with the required quality parameters and charge may be produced with the conventional technique using the photo-cathode RF gun. This approach favors a longer λ_p. Such a conservative position is also well coordinated with the primary approach of using the least risky and the most predictable linear LWFA regime. Indeed, the self-modulated and/or nonlinear LWFA (above the wave-breaking limit) promises much higher acceleration gradients than in the standard LWFA regime considered here. There are a number of suggestions of how to avoid wave-breaking instabilities by special tailoring of the laser pulse envelope or the longitudinal plasma density distribution. However, we abandon this approach at this point. We feel that it is similarly reasonable to choose the least cumbersome, classical approach to the good quality electron beam. This approach favors a relatively low plasma density and correspondingly long laser pulse duration (within the limits mentioned above).

5. Requirements of the laser driver for LWFA

As has been shown, the accelerator luminosity scales according to $\Lambda \sim \tau_L^2$. This dependence may be even enhanced if to take into account less e-beam emittance degradation at the reduced plasma density. However, even the quadratic dependence significantly relaxes the requirements of the laser driver repetition rate and the average power. Indeed, we can still maintain the required luminosity at the elevated laser pulse duration if we reduce quadratically the required laser repetition rate, $f^{\min} \sim \tau_L^{-2}$. Then, the average laser power calculated from

$$P_{av} = P_L \tau_L f \tag{22}$$

scales as $P_{av} \sim \frac{1}{\tau_L}$. As long as we keep the ratio $\frac{\tau_L}{\lambda}$ constant in order to maintain the desired net acceleration per stage (see Section 3), we finally get $P_{av} \sim \frac{1}{\lambda}$. This conclusion is illustrated by the examples in columns 1 and 2 of Table 1 which summarizes the laser driver parameters for the 5 GeV LWFA.

TABLE 1. Prospective comparative characteristics for multi-stage LWFA driven with 1-μm and 10-μm lasers

	Column number			
Laser parameters	1	2	3	4
Laser wavelength, λ [μm]	10	1	10	1
Energy [J]	50	5	1.5	15
Pulse length, τ_L [ps]	1	0.1	0.3	0.3
Peak power, P_L [TW]	50	50	5	50
Focal spot radius, r_L [μm]	360	36	120	120
Peak intensity, I [W/cm^2]	10^{16}	10^{18}	10^{16}	10^{17}
Laser field, E_L [TV/m]	0.33	3.3	0.33	1
Dimensionless laser strength, a	1	1	1	0.3
Repetition rate, f [kHz]	0.2	20	2	2
Average power [kW]	10	100	3	30
Wakefield parameters				
Plasma density, n_e [cm^{-3}]	3×10^{15}	3×10^{17}	3×10^{16}	3×10^{16}
Plasma wavelength, λ_p [μm]	600	60	200	200
Critical laser power, P_{cr} [TW]	60	60	6	600
Efficiency, $\xi = E_a/E_L$ [%]	1.2	1.2	3.6	0.17
Acceleration gradient, E_a [GeV/m]	4	40	12	1.7
Free space interaction length, πz_0 [cm]	10	1	1	10
Pump depletion length, L_D [cm]	280	28	90	900
Phase detuning length, L_{ph} [cm]	220	22	7	700
Assumed channel length, L_{ch} [cm]	125	12.5	5.5	125
Energy gain (with channeling), ΔW_{ch} [GeV]	5	5	0.7	2.1
Accelerator parameters				
Electrons/bunch, N_e	3×10^9	3×10^8	10^9	10^9
Number of bunches per pulse, ζ	3	3	3	3
Normalized emittance ε_n [m]	4×10^{-7}	4×10^{-7}	4×10^{-7}	4×10^{-7}
Spot size at interaction point [Å]	7	7	7	7
Luminosity, Λ [cm^{-2}s^{-1}]	10^{35}	10^{35}	10^{35}	10^{35}
Bunch repetition rate, $f\zeta$ [kHz]	0.6	60	6	6
Number of stages	500	500	3.570	1190
Total length* [m]	725	162	890	1725

* Total length of the 2.5 TeV multistage accelerator is estimated under assumption of the 20 cm long dead space between the plasma channels, e.g., for the laser beam focusing.

These examples, together with the generic analytical approach elaborated in this paper, illustrate that the ponderomotively strong long-wavelength laser in combination with a relatively low plasma density tends to improve the LWFA performance (within the selected spectral range). To make the author's point even more clear and to abate any residual concerns regarding the "arbitrary" choice of the input laser parameters, we evaluate two extra cases (columns 3 and 4). These examples illustrate that deviations from the optimum laser driver parameters result in deterioration of the accelerator performance.

For example, in column 3 we estimate the impact of the laser pulse reduction below the minimum calculated by Eq.(18). We see that in order to stay in the linear regime we need to reduce the laser peak power in proportion $P_L \sim \tau_L^2$. Because of the higher plasma density, the acceleration gradient improves inversely proportional to the τ_L. However, due to the significant reduction in L_{th}, the net gain per stage drops dramatically. Together with the reduction in N_e, this leads to less attractive accelerator performance (consider the seven times higher number of stages and two times higher total plug-in power consumption by the laser system).

In other example, illustrated by column 4, we increase the laser pulse duration above the calculated optimum (minimum) value. However, due to the recognized solid state laser technology limitations, we can not keep up with τ_L by upscaling $P_L \sim \tau_L^2$ and $\Sigma_L \sim \tau_L^3$. In addition, the rarefied plasma can not provide tight laser beam channeling (note, that in all examples we assume r_L=0.6 λ_p). As a result, a drops from 1 to 0.3 and we lose a lot in the acceleration gradient. In spite of the longer interaction length, the net acceleration per stage is also down. The number of the acceleration stages and the total accelerator length is up. The requirements to the average laser power are relaxed, but still beyond any realistic expectations for the solid state laser technology.

Anyone who prefers to select laser parameters according to personal preference can build new columns to Table 1 following the procedure outlined in this paper.

6. Conclusions

Based on generic constraints applicable to the plasma channeled standard LWFA operating in a linear regime, we obtained a simple linear relation between the laser peak power and the maximum net energy gain per acceleration stage. For example, for the target 5 GeV energy gain per the LWFA stage, we need the laser driver of the ~50 TW peak power.

The maximum acceptable laser peak power for the linear LWFA regime scales with the laser wavelength and the pulse duration as $P_L^{\max} \sim \left(\frac{\tau_L}{\lambda}\right)^2$. As a result, we arrive to the square root dependence of the minimum laser pulse duration upon the required net gain. For example, the 5 GeV gain calls for $\tau_L \geq 100$ fs at λ=1 μm or $\tau_L \geq 1$ ps at λ=10 μm. This flashes the light to a common misunderstanding that the increase of the laser peak power to the petawatt level via laser pulse shortening to a few cycles ensures a more efficient driver for the future LWFA.

In addition to the acceleration per stage, the important parameters of the practical accelerator design are: the electron beam monochromaticity, emittance, and luminosity. All these parameters are improved with the increase of the plasma wavelength that scales, at the fixed net gain, proportionally to the laser wavelength. Such scaling results from the quadratic dependence of the electron ponderomotive potential (linear dependence of the vector-potential a upon the laser wavelength). As a result, the long-wavelength radiation permits maintaining a high acceleration at the reduced plasma density.

This strongly favors using CO_2 lasers in the next advanced accelerator projects aimed towards future high-gradient e^--e^+ colliders. The additional important argument in favor of this choice is a potential of the picosecond terawatt CO_2 laser technology towards developing high repetition rate lasers of the multi-kilowatt average power[6].

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APPENDIX (Optical guiding in plasma channels)

In a plasma, the refractive index, η, depends upon n_e and λ according to the equation

$$\eta = \eta_0 \sqrt{1 - \frac{e^2 \lambda^2 n_e}{\pi c^2 m}} = \eta_0 \sqrt{1 - \frac{n_e}{n_{cr}}}, \qquad \text{(A1)}$$

where $n_{cr} = \pi mc^2 / e^2 \lambda^2$, known as the critical electron density. For $n_e << n_{cr}$, Eq.(A1) takes the form

$$\eta \approx \eta_0 \left(1 - \frac{n_e}{2n_{cr}}\right). \qquad \text{(A2)}$$

The idea of a preformed plasma waveguide is based on changing the refractive index in a medium due to a developed plasma density gradient according to the equation

$$\frac{\Delta\eta}{\eta_0} = -\frac{\Delta n_e}{2n_{cr}}. \qquad \text{(A3)}$$

For an EM wave incident onto the plasma layer at the angle β_0 the condition for reflection is according to Snell's law

$$\eta(r) \le \eta_0 \sin\beta_0, \qquad \text{(A4)}$$

or for oblique incidence $\eta(r) - \eta_0 \equiv \Delta\eta(r) \le -\eta_0 \theta_0^2 / 2$, where $\theta_0 \equiv 90^0 - \beta_0$, and by Eq.(A3)

$$\Delta n_e \ge n_{cr} \theta_0^2. \qquad \text{(A5)}$$

If we consider a cylindrical plasma layer with the "wall height" to satisfy Eq.(A5), then for a focused Gaussian beam having a diffraction divergence

$$\theta_0 = \frac{\lambda}{r_L \pi} \qquad \text{(A6)}$$

the condition of optical guiding inside a plasma cylinder is

$$\Delta n_e \ge \frac{mc^2}{e^2 r_L \pi} \equiv (r_e r_L^2 \pi)^{-1}, \qquad \text{(A7)}$$

where r_e=2.82×10^{-13} cm is the classic electron radius, or

$$\Delta n_e \left[cm^{-3}\right] = 1.13 \times 10^{20} / r_L^2 [\mu m]. \qquad \text{(A7')}$$

Note, that the waveguiding condition, Eq.(A7), does not depend upon the laser wavelength.

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