Idaho
National
Engineering
Laboratory

INEEL/EXT-97-01401

December 1997

Extrapolated Experimental Critical Parameters of Unreflected and Steel-Reflected Massive Enriched Uranium Metal Spherical and Hemispherical Assemblies

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INEEL/EXT-97-01401

EXTRAPOLATED EXPERIMENTAL CRITICAL PARAMETERS of UNREFLECTED and STEEL-REFLECTED MASSIVE ENRICHED URANIUM METAL SPHERICAL and HEMISPHERICAL ASSEMBLIES

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MASTER

December 10, 1997

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This paper is written under contract C96-175866 with the United States Department of Energy, administered by the Idaho National Engineering and Environmental Laboratory, although the work was performed at Rocky Flats, Colorado

Dr. Aleksandr Nikolayevich Zakharov

This paper is humbly dedicated to Dr. Zakharov, a respected Russian scientist, who died in a nuclear criticality accident in June of 1997. He and this author exhibit many professional parallels illustrated by this present paper, in preparation at the time of his death. Both have performed many hundreds of critical approach experiments over their respective careers. The particular assembly responsible for Dr. Zakharov's death bears a remarkable similarity to those described in this document. Both involved enriched uranium metal. Both were in the form of nesting hemispherical shells. Both were being manually assembled. Both involved a metallic reflector - his was copper instead of mild steel. Both experimental laboratories were large-sized rooms with thick walls.

ABSTRACT

Sixty-nine critical configurations of up to 186 kg of uranium are reported from very early experiments (1960s) performed at the Rocky Flats Critical Mass Laboratory near Denver, Colorado. Enriched (93%) uranium metal spherical and hemispherical configurations were studied. All were thick-walled shells except for two solid hemispheres. Experiments were essentially unreflected; or they included central and/or external regions of mild steel. No liquids were involved. Critical parameters are derived from extrapolations beyond subcritical data. Extrapolations, rather than more-precise interpolations between slightly supercritical and slightly subcritical configurations, were necessary because experiments involved manually assembled configurations. Many extrapolations were quite long; but the general lack of curvature in the subcritical region lends credibility to their validity. In addition to delayed critical parameters, a procedure is offered which *might* permit the determination of prompt critical parameters as well for the same cases. This conjectured procedure is not based on any strong physical arguments.

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INTRODUCTION

This document is the fifth in a series of six peer-reviewed papers written under the International Criticality Safety Benchmark Evaluation Project (ICSBEP). All six place into the public domain previously unpublished or inadequately documented experimental data generated at the Rocky Flats, Colorado, Critical Mass Laboratory (CML). The ICSBEP is administered for the Department of Energy by J. Blair Briggs of the Lockheed Martin Idaho Technologies Company, Idaho National Engineering and Environmental Laboratory (INEEL). The previous four[1,2,3,4] papers were published between 1994 and 1997.

In this fifth study, a series of sixty-nine critical parameter determinations for spherical and hemispherical thick-walled shell assemblies of 93% enriched uranium metal were performed at the Rocky Flats CML in the mid-1960s. All critical parameters were obtained from extrapolation of well-subcritical data. This was done for safety reasons. Configurations were manually assembled; and safety precluded manual assembly all the way to criticality.

In many cases, the fissile metal also had regions of mild steel in intimate contact inside and/or outside it. Other than steel, all cases were essentially unreflected except for experimental apparatus. In particular, no hydrogenous or other liquids were nearby. These results were published[5] in 1967, but that internal company document contained inadequate detail for computer codes of the 1990s. The purpose of this paper is to supply missing detail.

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[1] Robert E. Rothe, "Experimental Critical Parameters of Plutonium Metal Cylinders Flooded with Water." **INEL-96/0250.** September, 1994.

[2] Robert E. Rothe, "Experimental Critical Parameters of Enriched Uranium Solution in Annular Tank Geometries." **INEL-96-0386.** April, 1996.

[3] Robert E. Rothe, "Critical Experiments on an Enriched Uranium Solution System Containing Periodically Distributed Strong Thermal Neutron Absorbers." **INEL/EXT-97-00293.** September, 1996.

[4] Robert E. Rothe, "Critical Experiments on Single-Unit Spherical Plutonium Geometries Reflected and Moderated by Oil." **INEL/EXT-97-00665**. May, 1997.

[5] Robert E. Rothe, "Critical Masses for Partially Steel Reflected Enriched Uranium Metal Assemblies." **RFP-1021.** September, 1967.

Experiments included spherical and hemispherical geometries of enriched uranium metal, all built of nominally 3 1/3-mm-thick nesting hemispherical shells. Two assemblies were solid hemispheres; but most were hollow centered, forming thick-walled spherical or hemispherical shells. The largest spherical configuration had a nominal inside radius of 110 mm; the largest hemispherical case, 90 mm. This central cavity contained either air or mild steel. In other cases, mild steel was also placed outside the fissile metal.

Whenever steel was used, either inside and/or outside the uranium assembly, these regions, too, were built up of nesting hemispherical shells of the same 3 1/3-mm thickness. Both uranium and steel components had the same nominal radial dimensions; so either could be substituted for the other whenever desired. No case, however, found steel sandwiched between regions of uranium in this program.

The smallest component (a 20-mm-radius solid hemisphere) was omitted from every hemispherical assembly except two. This was done to form a cavity to receive an external neutron source, needed for safety. Both matching 20-mm-radius components were omitted from all spherical assemblies. This omission existed whether the innermost region contained mild steel or not. Steel on the outside was up to 80-mm thick.

This study, like all experiments performed at Rocky Flats, was used to provide nuclear criticality safety data to ensure continued safety of plant operations. Data from every program were used two ways in those early days of nuclear criticality safety. First, they were applied directly to plant operations if those operations were deemed suitably similar to experiments. The degree of similarity required was often left to the discretion of the Criticality Safety Engineer. Secondly, they were used to compare experimental results with calculations from then state-of-the-art computational methods in vogue at the time. Again, details of this comparison were often left to the discretion of the Safety Engineer.

Computational methods before the advent of Monte Carlo codes required certain conditions of geometrical symmetry. Only spherical or geometries well-approximated by infinite cylinders or slabs could be calculated. Because of that limitation, only solid and hollow *spherical* experiments - but not hemispherical ones -could have been calculated in those days.

Usually, the geometry of experimental programs at Rocky Flats bore a strong physical resemblance to some plant operation. This became almost a requirement at Rocky Flats but was possibly less true at other laboratories. This Rocky Flats practice facilitated direct comparison but limited the scope of computer validations. The present program modeled pressing and metal-forming operations at Rocky Flats. Here, thick, flat, slab-like ingots were placed in a press and formed into hemispherical shapes to be machined to final dimensions in a later operation. Steel simulated the inner punch and outer die of this operation. Results in air from this program simulated the newly-formed hemisphere removed from the press and otherwise unreflected. The criticality safety of these same components reflected and moderated by hydrogenous liquids and other materials was gleaned from other studies published as internal Rocky Flats publications[6,7,8,9] and as Journal articles[10,11,12,13]. Two still-classified reports[14,15] were also published.

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[6] Grover Tuck, "Enriched Uranium Metal Measurements, No. 1." **RFP-907**. July, 1967.

[7] B. B. Ernst, C. L. Schuske, and H. W. King, Empirical Analysis of Spherical and Hemispherical Assemblies of Enriched Uranium Metal." **RFP-937**. June, 1967.

[8] B. B. Ernst, "Critical Masses of Oil Reflected, Enriched Uranium Metal Assemblies with Polyurethane Centers." **RFP-1017.** September, 1967.

[9] Bruce B. Ernst and Grover Tuck, "Critical Masses of Spherical and Hemispherical Steel-Moderated, Oil-Reflected Enriched Uranium Metal Assemblies." RFP-1025. November, 1967.

[10] Grover Tuck, "Critical Masses of Spherical and Hemispherical Enriched Uranium Assemblies." JOURNAL OF NUCLEAR ENERGY, **23**, pp 663-672. 1969.

[11] Donald C. Coonfield, *et. al.*, "Critical Mass Irregularity of Steel-Moderated Enriched Uranium Assemblies with Composite Steel-Oil Reflectors." NUCLEAR SCIENCE AND ENGINEERING, **39**, pp 320-328. 1970.

[12] D. C. Hunt and Robert E. Rothe, "Criticality Measurements on Uranium Metal Spheres Immersed in Uranium Solution." NUCLEAR SCIENCE AND ENGINEERING, **46**, pp 76-87. 1971.

[13] D. C. Hunt and Robert E. Rothe, "A Criticality Study of Fissile-Metal and Fissile-Solution Combinations." NUCLEAR SCIENCE AND ENGINEERING, **53**, pp 79-92. 1974.

[14] E. C. Crume and G. Tuck, "Criticality Measurements and Calculations of an Enriched Uranium Sphere Reflected by Oil." **RFP-786**(classified). Date unknown.

[15] R. E. Rothe and N. L. Pruvost, "Enriched Uranium Metal Criticality Measurements and Calculations." **RFP-1027**(classified). September, 1967.

These experiments, even though quite old, should prove especially useful to the ICSBEP because of the very large masses considered. Experimental data are readily available for computer validation for masses often encountered in a production plant - up to several kilograms. Even larger masses up to about 50 kg (Godiva) are quite well understood. Little experimental data exists, however, for significantly larger masses. Confidence in any computational model used over any given range of masses would certainly be improved if that same model were found to predict criticality for significantly larger masses equally well. These results provide that missing experimental data. In summary, then, the special value of these results lie in their very large mass, far outside the normal range encountered in a production plant.

This paper requires more recollection of important information than any of the first four written under the ICSBEP. This is so because adequate documentation of some information is missing for the following reasons: (1) This author was just out of graduate school and lacked experience in data collection. (2) Documentation of more remote materials was not needed in the 1960s for computer methods then available; and the wealth of detail required for Monte Carlo codes of the 1990s was not envisioned. (3) The entire program, itself, was somewhat arbitrarily chosen by this author simply because the resulting data seemed interesting and useful. It was never an official program of the laboratory; and it never fell under the auspices of a formal, written, plan. This last reason is totally inconsistent with policies and procedures in place throughout the industry after about 1970.

This introduction ends on a somber note with a brief reference to a Russian experimental program that *appears* to have been similar to this one. Sadly, an accident during the Russian study caused the death of Dr. Aleksandr Nikolayevich Zakharov. The accident took place June 17, 1997; and Dr. Zakharov died three days later. A preliminary description of the Russian experiment reveals several commonalities between it and these aged studies at Rocky Flats. Although details have not been published in the literature, both are believed to have involved the manual assembly of nesting hemispherical shells of enriched uranium in an approach toward criticality. Both are known to have used highly enriched uranium metal. Both are known to have employed a non-fissile metal on the outside of the uranium as a neutron reflector. Copper was used in Russia, mild steel at Rocky Flats. Both were done by experienced persons in the field of nuclear criticality in what appears to be quite similar facilities. This paper is respectfully dedicated to Dr. Zakharov.

THEORY

Approaches to criticality were monitored by a method called the *Reciprocal Multiplication* technique. This was done to ensure safety. Here, the fact is used that the instantaneous neutron flux at any and every point within a system increases as criticality is approached. This flux is proportional to a neutron count rate measured at some fixed distance from the assembly being built using one or more neutron detectors. At any radius, R, of a growing assembly, this count rate, C(R), will be some factor greater than at the start of an experiment, C_o, for which R = 0. This ratio, C(R)/C_o is called the *Multiplication* of the system[16]. The inverse of this is the reciprocal multiplication already mentioned, mathematically, C_o/C(R).

At the critical radius R_c, C(R_c) would be essentially infinite relative to that initial count rate; and C_o/C(R) would, obviously, approach zero. This produces an attractive feature for graphing safe critical approaches. Extrapolating graphs to infinity (multiplication) is nebulous whereas extrapolations to zero (reciprocal multiplication) are clearly defined.

For safety, especially when humans are present, all parameters of critical approach experiments should be fixed save one. Otherwise, changes in the reciprocal multiplication curve could be attributed to either true increased multiplication or to the manner in which neutrons respond to changes in other parameters. In this program, the one variable changed was the radial thickness of the fissile metal region of the assembly. All other features remained fixed.

Another important definition is the *neutron reproduction factor, k*. It and the true multiplication, M, are related by the following equation:

$$k = M / [M + 1] = [1 + (1/M)]^{-1} \sim 1 - (1/M)$$

[16] Actually, this empirical ratio is only an approximation to the true multiplication because of other complications to the theory which are explained later.

At criticality, k equals unity; and the expression on the right is only a good approximation near there. [It is the first two terms of the series expansion of $1/(1+\varepsilon)$ when ε is small compared to unity.] The term 1/M is very close to (or at least proportional to) the reciprocal multiplication already defined. That is,

$$C_o/C(R) \sim 1/M.$$

Procedural limitations did not permit manual construction all the way to criticality; the possibility of a criticality accident was too great. The allowed limit was a multiplication of **ten**[17]. That is, assemblies could be built until the observed count rate was ten times greater than that with no fuel present. These limiting assemblies remained still well subcritical. Therefore, critical values need to be extrapolated from actually accrued subcritical data.

Any extrapolation of essentially linear data is much more reliable than that which derives from some strongly non-linear function. Figure 1 shows a comparison between the reciprocal multiplication for a selected case plotted against both its radial thickness and its mass[18]. The radial thickness plot is very close to linear between $C_o/C \sim 0.65$ and the limiting value (0.1); so the extrapolation to criticality, $C_o/C = 0$, is considered much more reliable than that for the much-more-curved mass-based function. Shown in Figure 2 is the relationship between reciprocal multiplication and the hemispherical radius of the enriched uranium shells for this same experiment. This figure is discussed more extensively in the Appendix.

In the example of Figure 1, the extrapolated critical radius is 101.7 + 20.126 = 121.8 mm and critical mass is 66.7 kg. For this example *only*, the two curves were <u>not</u> extrapolated independently[19]. The critical mass of the bottom curve was calculated from the 121.8-mm radius and the density and the mass

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[17] An inspection of tables in the Appendix reveals a few examples where this limit was exceeded. Evidently, limits were regarded more as a safety objective than as a true limit in those early days. Adherence to limits was much more respected in later years at Rocky Flats.

[18] Obviously, the two parameters are related; mass is proportional to the cube of the radius.

[19] On the contrary, all critical masses quoted in this paper *were* the result of independent extrapolations. This is the same case as example E-2 in the appendix, but the results in the appendix are from a more careful analysis.

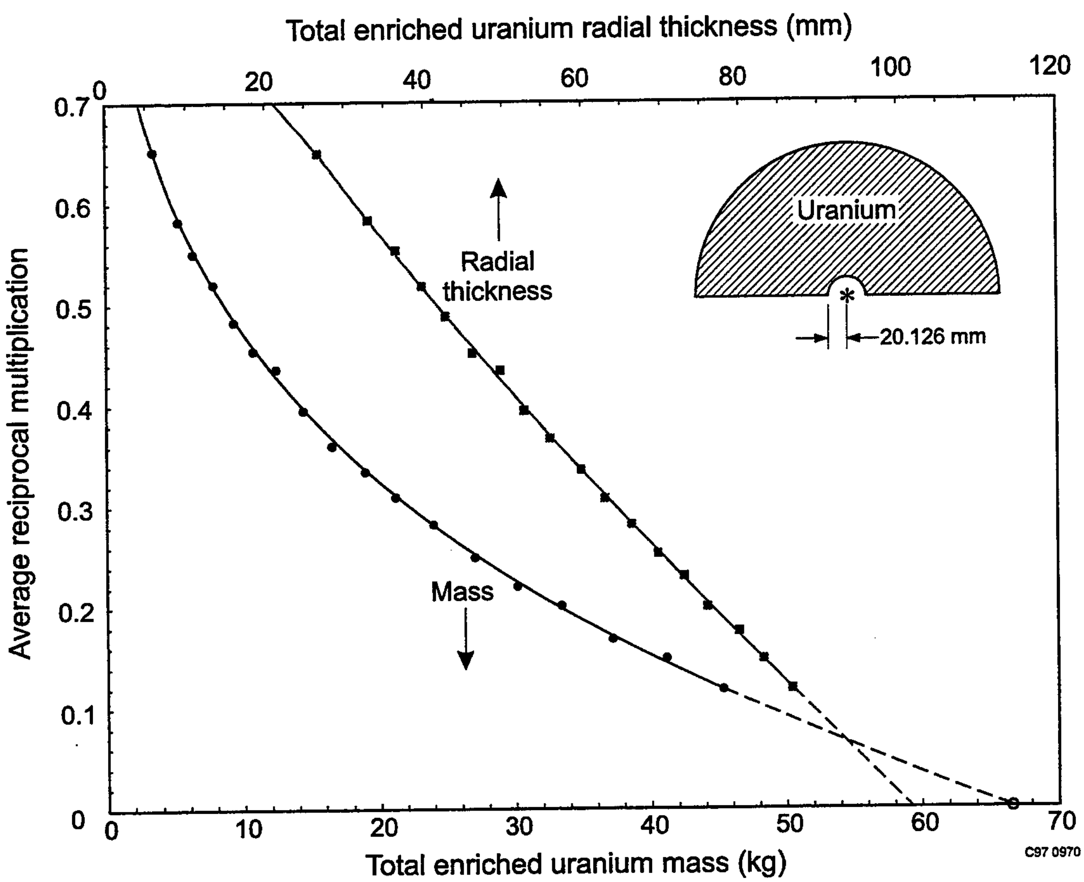

Figure 1. Reciprocal multiplication curves were much more linear when graphed against a radial parameter instead of a mass-based one. This hemispherical assembly, free of any mild steel, is one example. The open circle emphasizes that this critical mass was derived from the critical radius.

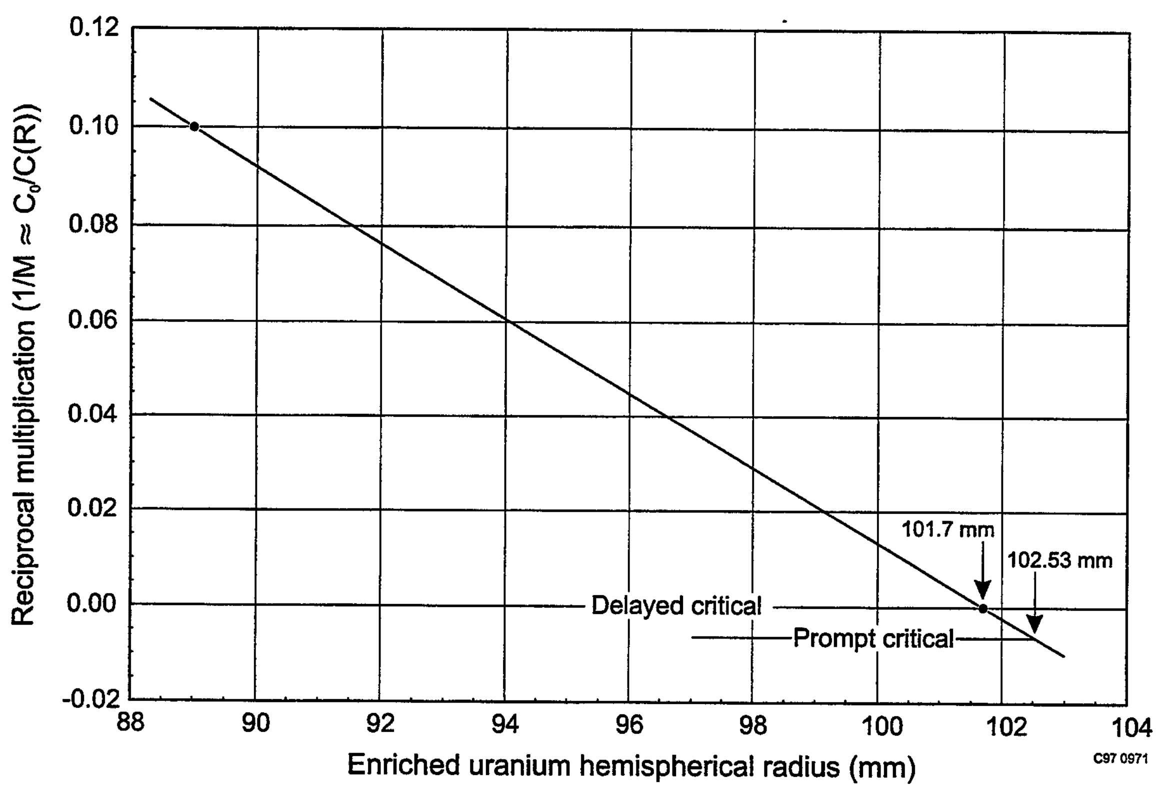

Figure 2. Possibly, prompt critical parameters can be obtained from reciprocal multiplication curves by extending the extrapolation into the physically unreal region of negative C_0/C. This illustrative example is the same experiment represented by Figure 1.

extrapolation forced to that point. Therefore, in this example only, the two curves are constrained to produce self-consistent parameters.

Assuming that this linearity of the radial function implies a closer correlation between actual multiplication and the count rate ratio, then the neutron reproduction factor, k, for any radial thickness slightly smaller than 101.7 mm and, therefore, close to criticality may be calculated by the above equation. That value of k may be compared with a computer calculation for the smaller and subcritical geometry. In summary, once validated for the critical case, the computer code may also accurately predict k for slightly subcritical systems from the data published here.

PROCEDURE

Experiments began by retrieving enriched uranium and mild steel shells from storage. Uranium was stored in ordinary commercial pressure cookers identical to those used in cooking. These provided mechanical protection, served as contamination control, and helped ensure security. Many were nested within cookers to reduce the number of containers. The total storage limit was 10.5 kg per cooker. During storage, nesting the parts never resulted in placing adjacent parts in contact with one another for fear they would stick together. Instead, one cooker contained every *other* even numbered part (02, 06, 10, 14, 18, 22, and 26), while another contained 04, 08, 12, 16, 20, 24, and 28. Two other cookers contained a similar distribution of seven odd numbered parts each. Larger components approached the safety limit with fewer shells per container; and the largest ones were stored individually. The author does not recall how or where the mild steel parts were stored. They were, of course, uranium contaminated; so that condition had to have been taken into consideration.

Retrieved parts were spread about a few meters from the assembly table. They rested on rolling carts, other tables, or any flat surface other than the experimental table itself. They were widely spaced from one another and introduced no significant reactivity. The number of parts taken from storage was only a few greater than those actually used in the subsequent experiment. The number needed could actually be quite well anticipated beforehand. These parts were spread around the room in a one-layer planar array. The minimum distance of these parts from the assembly area is recalled to be at least 2 m. The maximum may have been 5 m. The number of excess parts is recalled to have ranged from zero to, perhaps, five.

Parts were assembled in incremental stages into a growing hemispherical or spherical geometry. The detailed sequence of this construction is described later. The process has safety implications because criticality is being approached. For that reason, neutron count rates were obtained at each stage. This neutron flux was converted into a reciprocal multiplication as described previously. At each increment, this reciprocal multiplication was graphed against that radial thickness. Again, Figure 1 illustrates this procedure for one example.

Any given experiment could be characterized by four independent parameters:

(1) The geometry, either spherical or hemispherical.

(2) The inner radius of the enriched uranium region.

(3) The thickness, if any, of mild steel outside the uranium.

(4) The presence or absence of mild steel inside the uranium.

The one dependent parameter varied in every approach toward criticality was the outside radius of the enriched uranium region. The thickness was, of course the difference between this and the second parameter, above. Equally obvious, the mass of the assembly was simply the sum of the masses of the individual parts.

The reciprocal multiplication technique began by placing a small Po-Be neutron source[20] about a meter from two neutron-sensitive radiation detectors. The exact location of this source was chosen to be easily reproducible; so it could be returned at each stage. Generally speaking, this was at the bottom of the central cavity of the sphere. Counting neutrons for a fixed interval of time yielded the initial count rate, C_o. When mild steel was present outside the uranium (the 3rd parameter), it was included in this C_o count; but the interior steel, if any, (4th parameter) was not. Of course, no uranium was ever present for this initial count.

The technique continued by placing an agreed-upon number of enriched uranium parts in the proper location. Assemblies were supported on some kind of mount to keep them from falling off the experimental table. The number of shells to be assembled was clearly known by past experience or other data to be far subcritical. After returning the Po-Be source to its standard location, a new and greater neutron count rate was obtained. This yielded the first reciprocal multiplication data point. Administrative procedures called for this first increment to less than double C_o; many times, the addition of the first shells did not result in a doubling of the initial count rate.

The assembly was then dismantled and the next increment agreed upon. The size of this next addition was determined by extrapolating the reciprocal multiplication curve developed so far to the number of additional shells which would, again, less than double the neutron flux. Again, smaller increments were often the case.

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[20] The currently popular ^{252}Cf neutron source had not yet become available for common use.

These additional shells would, of course, be added to the outside of the previous uranium region; and this iterative procedure was continued until the multiplication safety limit was approached. The experiment was required to end there; so the critical parameter had to be obtained from an extrapolation of these data beyond that limit.

If the series under study had a steel-reflecting layer of thickness T outside the uranium, new uranium parts would replace some or all of the steel components. In this case, replaced steel was put away and larger shells added to the outside to retain the thickness T before the uranium was returned. Finally, the neutron source was returned to its standard position and a new neutron flux counted, converted to a reciprocal multiplication, and graphed. If the series also had interior steel, that metal was returned along with the uranium. Thus, each step along the way always had the same amount of mild steel inside and the same radial thickness of mild steel outside; and all parameters were fixed except for the radial thickness of the uranium region.

This procedure was very labor intensive, especially for spherical geometries. It required nearly complete disassembly for each increment and a lot of handling of both steel and uranium hemishells. Nonetheless, any other sequence of assembly would have called into question the meaning of an extrapolation to criticality. For example, if an arbitrary bottom hemisphere had been built too close to the expected final radial thickness and the top hemisphere built following procedures outlined in the text, the extrapolation of the reciprocal multiplication curve to criticality would *not* predict a critical sphere. Instead, it would predict the critical thickness of a top hemisphere above a fixed hemisphere of the starting size.

The extrapolation of actual subcritical data to criticality is not as obvious as it might first appear. Four related quantities can be measured for each critical configuration:

- the number of uranium metal parts,
- the outside radius of the uranium region,
- the radial thickness of the uranium region, and
- the total mass of uranium metal.

Clearly, the first three are related to radius. The outer radius equals the inner radius of the series plus the radial thickness; and the radial thickness equals the number of shells times their nominal thickness. The mass is more complicated. It is related to the cube of the radius and the density, reduced as it is due to tolerance gaps.

The first of these four was the most straightforward and, for that reason, the actual property used to generate safe critical approaches. The *number* of shells is an integer and, therefore, easy to graph quite unambiguously. That is, the actual reciprocal multiplication curves generated during the experiment consisted of graphs relating $C_o/C(\#)$ to the number, #, of uranium shells present.

For the earliest report, then, of these data (RFP-1021), *two* critical parameters were determined: the critical number of shells and the critical mass. The former was extrapolated; the latter was derived from that extrapolated number. The conversion of the first to the critical uranium mass was done according to the following procedure. The mass of the largest assembly built was obtained by summing the actual masses of its parts. This often was two dozen or more hemishells. Next, if the extrapolated number of shells predicting criticality were, for example, say, an additional 3.6 hemishells, then the mass of the *next three* actual components which could have been added if administrative procedures would have allowed it **plus** 60% of the mass of the fourth hemishell would be added to that mass. This procedure is illustrated by Table I, wherein the critical mass of one case (an unreflected, air-centered spherical assembly having an 80-mm inner radius) is calculated as done for that early report.

This procedure introduces a small arbitrariness for spherical assemblies. Roles of the largest two hemishells forming the largest spherical shell could have been reversed; or the average of these two parts could have been used for the appropriate fraction of last full spherical shell. In the example of Table I, these different approaches would reduce the 124,985 g total mass by only 1 g and 0.4 g, respectively - truly negligible compared to the precision of the graphical extrapolation of the number of parts itself.

Critical parameters determined anew in 1997 for this paper were independently extrapolated from the subcritical data. These extrapolations included the outer radius and the mass of the enriched uranium. The extrapolation methods will be discussed later in this paper.

Table I. A Sample Calculation of the Critical Mass of Enriched Uranium from the Extrapolated Critical Number of Hemishells as Done for a 1967 Rocky Flats Report.

Uranium Components	Number of Uranium parts	Mass (kg)
Largest assembly built	26	103.591[a]
Next full shell[b]	2	11.004
Next hemishell[c]	1	6.495
60% of mate to above	0.6	3.895
Total at critical	29.6	124.985 (say, 125.0)

[a] reciprocal multiplication was 0.086 for the 26 parts # 39 to # 64.
[b] mass of parts # 65 and # 66
[c] mass of part # 67. Part # 68 (6.492 kg) could have been used instead by reversing the roles of these two hemishells in the last two lines.

ENRICHED URANIUM

Rocky Flats was a major producer of both enriched uranium and plutonium components for the United States' nuclear weapons stream at the time these experimental parts were fabricated (1965). This changed a year later when the uranium work was shifted to Oak Ridge, Tennessee. After that, Rocky Flats focused on processing, manufacture, and recovery of plutonium.

Before that separation, Rocky Flats assumed it would require criticality safety data for both fissile materials for many years to come; and these data would come from the CML on site. However, uranium experiments continued even after separation because they were much less expensive. The intention was to apply uranium data to plutonium applications. The ***shape*** of a critical curve would be determined using uranium experiments and then ***normalized*** to plutonium applications with a few (more expensive) plutonium experiments. Sometimes, this normalization would be computational. The intended procedure never came into use with the advent of Monte Carlo codes.

Fabrication of these experimental parts followed standard production procedures then in use. Large slab-like ingots of high-purity enriched uranium metal were rolled into thick, flat plates of suitable thickness. These were annealed to relieve stresses introduced by rolling. The next step was to draw these into hemispheres by pressing into an outer die using an inner punch. Rough hemispheres were again annealed to relieve stresses introduced by forming. Finally, these oversized "bowls" were machined to final radial dimensions.

Each part was machined to a nominally 3 1/3-mm-thick hemispherical shell less tolerances required for a slip-fit. The only exceptions to this were the two smallest components: nominally 20-mm-radius hemispheres weighing 296 g each. Figure 3 shows eleven parts viewed from above. The smallest (lower right) was the part not used in this program. The shell to its left fit inside the larger shell to the left of the bottom row. The two slipped, in turn, inside the shell to the left of the middle row. Then, all three fit inside the shell to the right of that one and so on. All eleven parts shown here formed a hemisphere about 53.3 mm in radius. It would have weighed 5719 g if the parts shown were odd; 5725 g, if even.

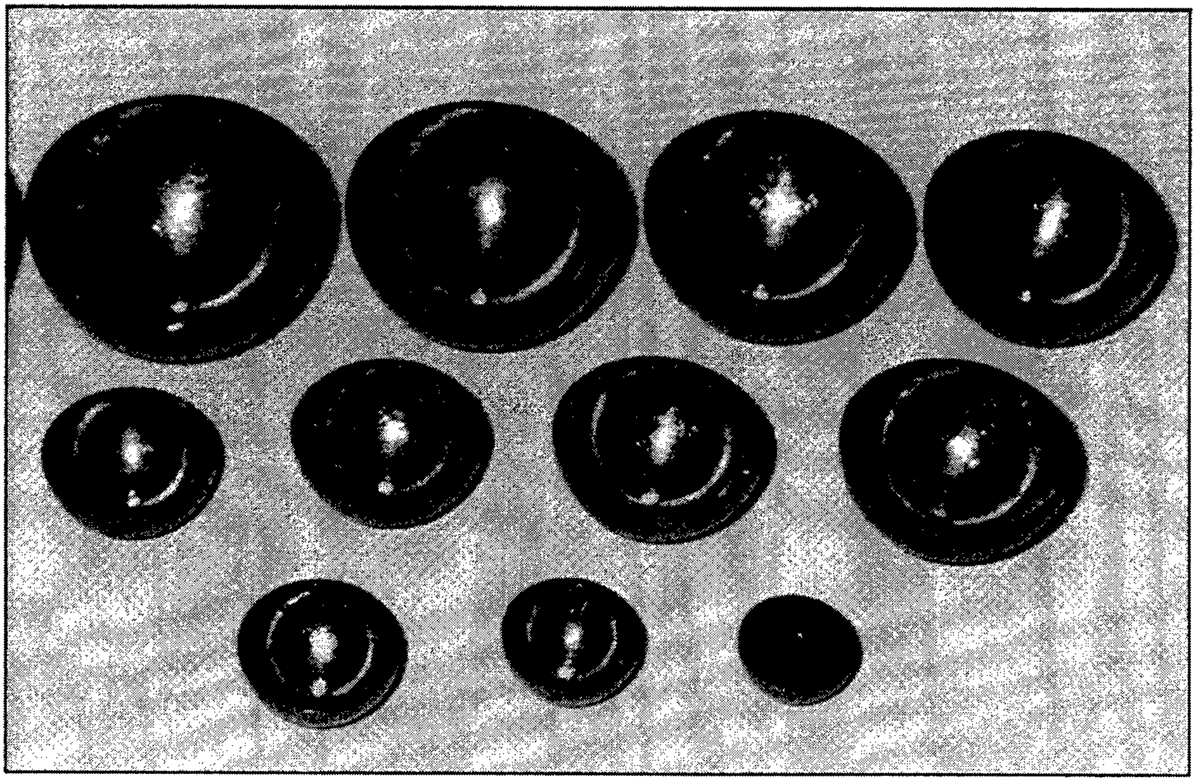

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Figure 3. Eleven of eighty nesting enriched uranium hemispherical shells are shown. Even-numbered shells could form one hemisphere; odd-numbered, the other. Shells were black in color because of an oxide coating; and they were shiny because of a thin coat of grease used to control contamination.

Isotopic composition was determined during forming and machining operations according to routine Rocky Flats production procedures. These results were then merely reported along with the delivered product. Subsequent analyses could not be made of the metal itself without damaging the machined finish. The uranium was analyzed, however, in later years using the oxide rubbed off a surface during cleaning. Table II reports two measurements of isotopic content. The "1965" column is associated with initial fabrication; the "1971" column, from oxide. Both columns sum to 100%; and this causes one to wonder if the ^{235}U content was measured directly. It might have been assumed to be the difference between 100% and the sum of the other three measured isotopes. This is not recalled for certain. Values in the table are here offered as the best available some 30 years later. The ^{233}U content is not reported, although this author does recall occasional measurements of this isotope by the Analytical Laboratory. Whenever measured, this isotope was always found to be "less than the detectable limit". This somewhat hazy recollection may pertain to the metal, although most are recalled to have pertained to the enriched uranium solution, also used in a great many experiments at the laboratory. The best guess, three decades later, is that the enriched uranium metal contained negligible amounts of ^{233}U.

Table II. Isotopic Content of Enriched Uranium Metal

Uranium Isotope	Weight-Percent	
	1965	1971
233	not recorded	
234	1.00	1.02
235	93.19	93.16
236	0.40	0.47
238	5.41	5.35

Fortunately, all records from the Rocky Flats CML are being retained at the Los Alamos National Laboratory Archives[21]. This collection does contain additional Analytical Laboratory measurements pertaining to isotopic composition. Possibly, a refined average for the isotopic distribution could be obtained from a survey of those data; but that effort was not deemed justified at this writing.

[21] Roger A. Meade, Archivist. Archive number: **A-96-051**.

No measurement of metallic impurities within the fissile metal could be found during an exhaustive search of available records. Because parts were fabricated at Rocky Flats following normal procedures, however, a good estimate could be recovered from the Rocky Flats archives. This would yield at least nominal values for this missing information.

The bulk density of the uranium metal was probably the nominal 18.664 mg/mm^3, often quoted in textbooks. A survey of documents pertaining to these specific parts specifies 18.675 $\pm$ 0.05 mg/mm^3, very close to the nominal value. One early publication lists this density as 18.76 $\pm$ 0.06 mg/mm^3; but this is thought to be a typographic error.

The effective density of an assembled configuration was reduced due to the necessary machining tolerances on each shell. The inside radius of any given component had to be sufficiently larger than the outside radius of the next smaller component to permit the two to slip-fit together. A typical such gap was about 0.1 mm. Those gaps plus five small holes drilled through each component reduced the overall effective density of an assembly to about 18.13 $\pm$ 0.07 mg/mm^3. Specific densities within this range are presented for each assembly in the 69 tables in the Appendix.

Table III describes each hemishell precisely giving its inside radius, outside radius, and mass. These are the values measured at manufacture (1965) and the ones employed throughout this paper. All represent finished parts after machining and with all holes drilled. Odd numbered parts are given to the left, even, to the right. Subsequent weighings - even 5 and 32 years later - revealed only small differences of seldom more than one gram[22]; one gram is about the readability of a certified precision balance. Such weighings always followed a careful cleaning which included a solvent removal of residual grease and oils and a gentle soft-paper wiping of loose oxide.

[22] More often than not, these changes were *increases* in mass rather than loss due to abrasion or the rigors of chemical cleaning. A possible explanation of increased weight might be the additional absorbed oxygen in the surface coat as the black oxide of uranium builds up.

Table III. Physical Parameters of the Enriched Uranium Hemishells Manufactured in the 1960s for the Rocky Flats Critical Mass Laboratory

Odd-Numbered Hemishells					Even-Numbered Hemishells				
Part #	Inside Radius (mm)	Outside Radius (mm)	Mass[a] (g)	Total Mass[b] (g)	Part #	Inside Radius (mm)	Outside Radius (mm)	Mass[a] (g)	Total Mass[b] (g)
01	c	20.015	296	0	02	c	20.009	296	0
03	20.126	23.371	176	176	04	20.126	23.377	176	176
05	23.475	26.696	233	409	06	23.473	26.698	234	410
07	26.800	30.035	302	711	08	26.791	30.027	302	712
09	30.127	33.352	377	1088	10	30.123	33.351	376	1088
11	33.437	36.697	465	1553	12	33.44	36.698	466	1554
13	36.798	40.025	555	2108	14	36.801	40.024	554	2108
15	40.168	43.381	653	2761	16	40.162	43.376	652	2760
17	43.457	46.697	767	3528	18	43.463	46.698	766	3526
19	46.786	50.045	890	4418	20	46.783	50.039	890	4416
21	50.173	53.372	1005	5423	22	50.128	53.358	1013	5429
23	53.464	56.692	1147	6570	24	53.458	56.693	1150	6579
25	56.794	60.027	1288	7858	26	56.790	60.015	1286	7865
27	60.113	63.346	1445	9303	28	60.121	63.344	1440	9305
29	63.451	66.707	1612	10915	30	63.441	66.696	1612	10917
31	66.784	70.025	1779	12694	32	66.792	70.030	1777	12694
33	70.060	73.296	1949	14643	34	70.098	73.338	1951	14645
35	73.417	76.658	2134	16777	36	73.428	76.665	2130	16775
37	76.824	80.027	2349	19126	38	76.711	80.027	2342	19117
39	80.128	83.364	2527	21653	40	80.075	83.292	2511	21628
41	83.462	86.683	2722	24375	42	83.443	86.680	2741	24369
43	86.782	89.996	2945	27320	44	86.764	89.995	2953	27322
45	90.095	93.328	3188	30508	46	90.104	93.329	3179	30501
47	93.418	96.667	3442	33950	48	93.432	96.683	3450	33951
49	96.771	99.999	3656	37606	50	96.775	100.001	3658	37609
51	100.119	103.340	3912	41518	52	100.104	103.336	3918	41527
53	103.445	106.696	4207	45725	54	103.427	106.685	4208	45735
55	106.743	110.009	4464	50189	56	106.773	110.013	4461	50196
57	110.113	113.348	4733	54922	58	110.112	113.315	4729	54925
59	113.439	116.660	5003	59925	60	113.444	116.670	5025	59950
61	116.765	119.987	5323	65248	62	116.785	120.015	5326	65276
63	120.108	123.358	5660	70908	64	120.111	123.363	5650	70926
65	123.486	126.492	5509	76417	66	123.507	126.505	5495	76421
67	126.671	130.030	6495	82912	68	126.639	129.996	6492	82913
69	130.085	133.321	6599	89511	70	130.087	133.313	6598	89511
71	133.432	136.690	6982	96493	72	133.34	136.707	6973	96484
73	136.789	140.014	7262	103755	74	136.787	140.010	7244	103728
75	140.085	143.317	7619	111374	76	140.096	143.322	7613	111341
77	143.420	146.656	7984	119358	78	143.530	146.657	7952	119293
79	146.801	150.043	8415	127773	80	146.798	150.062	8413	127706

a) These are the same masses as listed in the "1965 Manufactured" columns of Table IV.
b) Sum of the mass of all smaller hemishells and the one associated with each entry except that the 296 g center core hemisphere is excluded.
c) This inside radius would be zero except that the cylindrical pole hole drilled through each component renders this parameter ill-defined.

The right-hand-most column of each half of the table gives the total mass of all components in an assembled hemispherical geometry up to that hemishell; but it does not include the 296 g smallest part (nominally 20-mm-radius). Taking the appropriate two masses from this column and subtracting them yields the mass of the corresponding thick-walled hemispherical shell. Then, using this mass, the outside radius of the largest shell, and the inside radius of the smallest, the density of the hemispherical assembly may also be calculated.

Individual densities of hemispherical shells calculated from masses and radii of Table III range from 18.10 to 19.25 mg/mm^3 with an average of 18.73 mg/mm^3. These densities compare well with the nominal 18.664 mg/mm^3 textbook bulk density mentioned earlier. The greatest densities are for parts 37 and 78. The masses of parts 37 and 78 are reasonably close to those of their counterparts; but the radial dimensions seem to be significantly different. It should be noted that for the larger hemishells, a small error in radius will result in a relatively large discrepancy in density.

The variation in density of individual components manufactured at the same time and with the same bulk material, as these parts were, is almost certainly not a function of material variations. Rather these variations demonstrate the human errors in measuring radial dimensions.

Masses (only) have been remeasured on other occasions. A comparison is presented in Table IV. The first column repeats the 1965 manufactured values from Table III. The next represents a reweighing of the cleaned first 36 parts in April of 1970. These parts were among the most often used at the CML. All 80 parts were again weighed in 1997 - 32 years after manufacture - just prior to being given to the Los Alamos National Laboratory (LANL) for use in their critical experiments facility. This is the third column. Other than being weighed at Rocky Flats, not much is known about these 1997 masses; but some care in measurement is assumed for two reasons: (1) results agree so well with the 1965 masses and (2) this was an interagency transfer of accountable material.

One additional column of masses is presented in Table IV; but the date and origin of this set is not known. A typed list containing dimensions and masses of all 80 hemishells was discovered during a records search in preparation for the present paper. For each part number, both inside and outside radii are *identical*

Table IV. A Comparison of Masses (in grams) of the Enriched Uranium Hemishells Over More Than Three Decades and From a Variety of Sources

Odd Numbered Hemishells					Even Numbered Hemishells				
Part #	1965 Manufactured	1970 Reweighing	1997 Transfer	Unknown Origin	Part #	1965 Manufactured	1970 Reweighing	1997 Transfer	Unknown Origin
01	296	296	296	296	02	296	296	296	296
03	176	176	175	170	04	176	176	176	171
05	233	233	233	225	06	234	234	234	225
07	302	302	302	290	08	302	302	302	290
09	377	377	377	360	10	376	376	375	361
11	465	465	465	444	12	466	465	466	444
13	555	555	554	528	14	554	554	554	527
15	653	653	652	621	16	652	652	651	621
17	767	767	767	729	18	766	766	766	728
19	890	890	889	846	20	890	890	890	845
21	1005	1005	1005	950	22	1013	1012	1011	958
23	1147	1147	1146	1085	24	1150	1150	1149	1087
25	1288	1288	1288	1222	26	1286	1286	1286	1218
27	1445	1445	1445	1365	28	1440	1441	1441	1361
29	1612	1612	1612	1528	30	1612	1612	1611	1527
31	1779	1779	1779	1680	32	1777	1778	1777	1679
33	1949	1949	1950	1842	34	1951	1951	1952	1846
35	2134	2134	2135	2021	36	2130	2131	2130	2019
37	2349		2349	2182	38	2342		2342	2256
39	2527		2527	2395	40	2511		2512	2378
41	2722		2722	2582	42	2741		2742	2594
43	2945		2946	2781	44	2953		2954	2795
45	3188		3188	3012	46	3179		3179	2178
47	3442		3443	3251	48	3450		3451	3254
49	3656		3657	3461	50	3658		3658	3459
51	3912		3912	3692	52	3918		3918	3704
53	4207		4206	3975	54	4208		4208	3983
55	4464		4464	4249	56	4461		4462	4216
57	4733		4733	4473	58	4729		4730	4470
59	5003		5003	4722	60	5025		5025	4730
61	5323		5323	5001	62	5326		5327	5015
63	5660		5661	5334	64	5650		5650	5338
65	5509		5509	5201	66	5495		5496	5189
67	6495		6496	6129	68	6492		6491	6122
69	6599		6600	6217	70	6598		6598	6197
71	6982		6983	6582	72	6973		6974	6811
73	7262		7262	6842	74	7244		7246	6838
75	7619		7618	7188	76	7613		7612	7175
77	7984		7984	7540	78	7952		7951	7291
79	8415		8415	7910	80	8413		8413	7965

to the 1965 dimensions to *six significant figures*. That high precision suggests that the origin of both was the same; the probability of independent measurements being so identical with that precision is nil. These two pages, however, also contain mass data - expressed to an unbelievable *nine* significant figures. These masses differ significantly from all the other mass data. They are assumed to be in error but are recorded here out of a sense of complete and accurate reporting.

The 1970 reweighing data combined with the 1997 transfer to LANL substantiates the 1965 masses because of the very small differences observed. The data of unknown origin are not considered credible because of the discrepancies and because these data calculate to unreasonably low densities - about 17.63 mg/mm^3.

Masses in Table III and repeated in the first column of Table IV were obtained at Rocky Flats as a routine step during manufacture. Although the specific balance used is not known, procedures at Rocky Flats would have required a certified precision balance. The second column masses were obtained at the Rocky Flats CML using their 5 kg balance[23] certified by the Rocky Flats Standards Group. Transfer masses (1997) were also obtained at Rocky Flats and probably employed another certified balance.

Even-numbered parts were always assembled to form one hemispherical shell; odd-numbered parts, the other. No assembly mixed odd and even parts in the same hemisphere. No effort was made to keep either set on the top or bottom of a spherical assembly. Hemispherical geometries always used even-numbered parts.

Figure 4 shows two nested hemispheres flanked by the next larger shells. One hemisphere is composed of even-numbered parts; the other, odd. Parts were actually so well machined that the plane surface of an assembled hemisphere <u>never</u> revealed individual shells except upon very close inspection. Parts were intentionally nested poorly for this photograph to illustrate nesting.

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[23] Masses in excess of 5 kg were obtained by offsetting one side of the balance with certified known masses of a few kg.

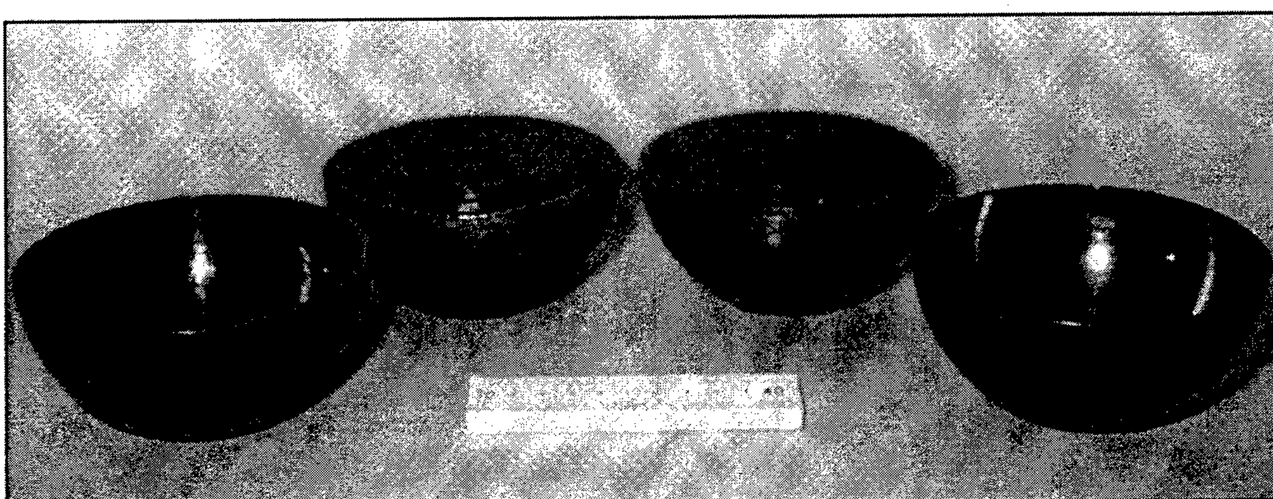

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Figure 4. Enriched uranium hemispheres actually nested so well that individual shells could not be seen in the surface of the equatorial plane except upon very close inspection. These hemispheres were intentionally misaligned to highlight nesting.

Each hemishell had five holes drilled in it, one at the pole and four smaller ones equally spaced in a plane parallel to the equatorial plane and a little below it. The pole hole was 7.14 mm in diameter with a tolerance of +0.13 mm and -0.05 mm. Both faces were counterbored to relieve sharp edges. The mass of enriched uranium removed amounted to about 3 g, including counterbores. The purpose of the pole hole was to receive a 6.35-mm-diameter rod to align nested shells. No alignment rod was needed in this program although other programs used them.

Five such rods had been machined of the same enriched uranium metal; but these were never used because they never were found to be a useful length. Instead, steel or stainless steel rods, commercial threaded screw stock, or hollow tubes were used in these other programs.

The four smaller holes (pry holes) were intended to be used only if two or more nested shells became stuck together through oxidation, vacuum, congealed grease, or any other physical mechanism. Happily, this worry proved unfounded; shells never stuck together. Each hole was 3.18 mm in diameter. They were drilled parallel to the equatorial plane (not radially) and one-third of the outside radius down from this plane toward the pole. Each hole reduced the weight of a shell by about 0.5 g, 2 g for all four holes.

Figure 5 shows a cross section of these components. Although drawn from a construction drawing last revised in February of 1965, the figure represents actual shells well.

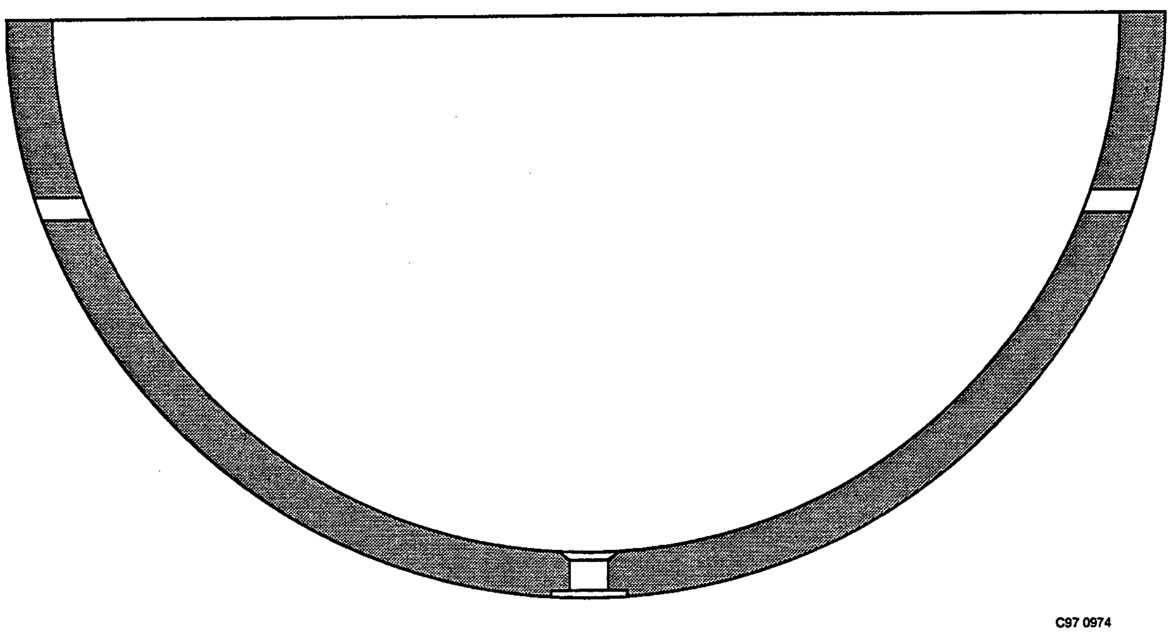

Figure 5. Both enriched uranium and mild steel hemishells were machined from this construction drawing. Tolerances were kept extremely small on these high-quality components.

OTHER MATERIALS

Hemispherical mild steel shells were sometimes nested to build up a steel interior inside hollow spherical or hemispherical enriched uranium assemblies. Sometimes, similar but larger shells were added to the outside of the uranium region to form a steel reflecting layer. Of course, sometimes no steel was used at all.

Steel parts were machined to the same nominal dimensions as the uranium; so each shell was nominally 3 1/3 mm thick less machining tolerances required for a slip-fit. The bulk density of the steel was 7.86 mg/mm^3; and values around that appear on many Rocky Flats documents associated with machining. This density was reduced to about 7.65 mg/mm^3 because of tolerance gaps and assorted holes drilled into the hemishells[24]. More accurate densities for specific combinations of shells in any application can be calculated from available data. This reduction in density is about the same as found for uranium components.

Precise outer radii and masses for steel hemishells are listed in Table V. Apparently, no inner radii were measured. The machining requirement was simply that adjacent parts must slip-fit within one another. Most thicknesses and some density data were recorded. Thicknesses were found for most parts after #20. These data are culled from a number of sources and are believed consistent with one another. Thicknesses are discussed further later.

Parts #01 through #20 were delivered in June of 1965, #21 through #50, the next month, and #51 through #70 in October of that year. The outside radius of each was recorded on the Rocky Flats "Internal Transfer" document for the shipment. Masses of the first 80 parts were measured in May of 1966, well after receipt and well after some use. In all cases where the mass was also reported on the plant's "In Process Inspection Form", generated during machining, no discrepancies were noted between the two. Parts numbered #81 through #108 and #115 and #116 contained both the outside radius and mass on these same inspection forms; and their transfer document supports radial dimensions. The inspection form for shells #109 through #114, unfortunately, did not record a mass.

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[24] One reference to this effective density quotes 7.62 mg/mm^3. The difference is well within measurement uncertainty.

Table V. Physical Parameters of the Mild Steel Hemishells as Manufactured in the 1960s for the Rocky Flats Critical Mass Laboratory

Odd-Numbered Hemishells								Even-Numbered Hemishells							
Part #	Outside Radius (mm)	ΔR[a] (mm)	Mass (g)	Part #	Outside Radius (mm)	ΔR[a] (mm)	Mass (g)	Part #	Outside Radius (mm)	ΔR[a] (mm)	Mass (g)	Part #	Outside Radius (mm)	ΔR[a] (mm)	Mass (g)
01[c]	19.992		125	59	116.65	3.23	2112	02[d]	20.036		125	60	116.62	3.24	2112
03	23.340		74	61	120.01	3.21	2235	04	23.348		74	62	120.00	3.23	2228
05	26.665		98	63	123.38	3.28	2394	06	26.708		98	64	123.34	3.28	2397
07	30.002		128	65	126.56	3.00	2323	08	30.013		128	66	126.52	2.99	2308
09	33.350		159	67	129.93	3.30	2706	10	33.327		158	68	130.05	3.24	2688
11	36.716		195	69	133.28	3.15	2723	12	36.721		196	70	133.28	3.20	2741
13	40.025		233	71	136.67	2.97	2806	14	40.015		233	72	136.63	3.15	2857
15	43.366		276	73	140.03	2.69	3072	16	43.358		275	74	140.00	2.58	3046
17	46.703		322	75	143.33	2.75	3194	18	46.670		321	76	143.37	2.72	3189
19	50.051		372	77	146.65	3.20	3308	20	50.046		371	78	146.65	3.18	3325
21	53.373	3.23	425	79	150.03	2.86	3538	22	53.370	3.23	426	80	150.05	2.82	3535
23	56.667	3.24	453	81	153.20	3.25	3713	24	56.708	3.21	481	82	153.24	3.25	3710
25	60.015	3.23	540	83	156.69	3.26	3860	26	60.018	3.23	542	84	156.69	3.25	3849
27	63.350	3.23	606	85	159.99	3.23	3998	28	63.350	3.23	607	86	160.05	3.19	3976
29	66.693	3.23	674	87	163.32	3.25	4199	30	66.698	3.23	672	88	163.32	3.25	4184
31	70.023	3.24	745	89	166.65	3.26	4371	32	70.002	3.23	743	90	166.67	3.25	4397
33	73.353	3.24	827	91	170.00	3.25	4522	34	73.363	3.24	820	92	170.02	3.23	4513
35	76.665	3.23	891	93	173.21	3.00	4420	36	76.665	3.23	895	94	173.19	3.00	4387
37	80.035	3.25	983	95	176.66	3.25	4920	38	80.046	3.25	989	96	176.64	3.25	4921
39	83.383	3.23	1060	97	179.91	3.18	5056	40	83.360	3.23	1066	98	179.85	3.18	5018
41	86.693	3.23	1152	99	183.34	3.20	5272	42	86.685	3.23	1155	100	183.32	3.20	5255
43	90.007	3.24	1247	101	186.54		5418	44	89.995	3.23	1243	102	186.64	3.23	5463
45	93.332	3.23	1336	103	189.94	3.18	5632	46	93.340	3.23	1338	104	190.09	3.20	5665
47	96.693	3.25	1449	105[c]	193.45		5877	48	96.680	3.25	1451	106	189.94[b]		5702
49	100.005	3.21	1538	107[c]	196.90		5642	50	100.005	3.21	1540	108[c]	196.70		5676
51	103.34	3.23	1645	109[c]	200.43			52	103.35	3.23	1648	110[c]	200.15		
53	106.69	3.25	1774	111[c]	201.04			54	106.70	3.25	1768	112[c]	201.04		
55	109.90	3.18	1846	113[c]	206.73	3.24		56	110.02	3.24	1874	114[c]	206.55		
57	113.35	3.23	1977	115[c]	210.19	3.24	6899	58	113.33	3.21	1971	116[c]	210.19	3.21	6899

[a] An estimate of the radial thickness of the shell only obtained from an average of the thickest and thinnest measurement.
[b] Probably an error, should be about 193 mm.
[c] Not used during this program.
[d] Used only for too hemispherical experiments.

The outside radius for shell #106 is entered in the table as recorded even though it is believed in error. That shell should have had a radius of about 193.3 mm, not 189.94 mm. The smaller radius is associated with part #103; a simple copying error is suspected. Shells #111 and #112, also, appear to have a wrong radius; a value close to 203 mm was expected, not 201.04 mm. This also may have been a copying error. Since no masses were found, however, the possibility exists that both shells (larger than any uranium part) may have been machined very thin: 0.61 and 0.89 mm, respectively. If so, shells #113 and #114 would have been abnormally thick. A copying error appears more likely because one radial thickness (#113) was measured and would be inconsistent with the thin/thick shell hypothesis.

A graph of the first 60 shells reveals a fairly uniform, but, of course, non-linear, increase in mass. Above that, however, masses seem to scatter about expected values. This is probably due to slight variations in thickness for these larger shells. The functional relationship between mass and outside radius is expected to approach a more linear one for large shells. In the limit of infinite radius, the two would be truly linearly related.

The table also presents the thickness of most shells. These are a little less certain because they were obtained from maximum/minimum thicknesses obtained during machining of each shell. The average of those limits is presented in the table; and that is not necessarily a true average because the distribution between maximum and minimum thickness is probably not uniform. The range between limits was also rather large, averaging about 0.12 mm with a few as large as 0.28 mm. Tabled thicknesses should be considered to have an uncertainty of $\pm$ 0.06 mm. Still another anomalous observation is that those few thickness measurements recorded directly in metric units tended to be a little smaller than those measured with a commercial American micrometer.

An average thickness for these steel shells, quoted in one early publication, was 3.28 mm, quite similar to the uranium. This is close to the radial thickness estimated from density information assuming each shell is simply thinner than its nominal 3 1/3-mm-thick goal. If an equal amount had been machined away from each surface to produce the desired tolerance gap, the effective density would be that reduced mass divided by the unreduced volume. The density ratio equals the mass ratio since volumes are the same; and masses are proportional to the thickness of a shell. The ratio of these steel densities[25] is 0.9733; so the thickness calculated this way would be 3.24 mm, in fair agreement with a rough average of the thicknesses given in the table.

Each steel shell had a 7.14-mm-diameter pole hole intended for use in tying an assembly together; but no such tie bolt was used in this manual assembly study. Four 3.17-mm-diameter pry holes were drilled below the equator as in the uranium; and these were intended for use only if two shells should ever stick together. They never did; so these holes were never used. All five holes are identical in size, location, and orientation to those drilled into the uranium.

The mild steel was type SAE 1018. Other than the principal element, iron, metallic impurities included between 0.15 and 0.20 wt-% carbon, between 0.60 and 0.91 wt-% manganese, and a maximum of 0.04 wt-% phosphorus and 0.05 wt-% sulphur. The source of these impurities' ranges is not certain. Because steel shells were not all machined at one time, the material probably was not all from the same batch. Still, all steel was nominally the same SAE 1018.

Most other experimental programs using these steel and uranium parts involved eventual immersion into some kind of fluid. A coating of petroleum jelly was used in these other programs both to protect both metals against corrosion and to exclude seepage of moderating liquids. That jelly was not needed in this study; so all metal parts were wiped reasonably dry for these experiments. Complete removal, however, was difficult. The four small pry holes may have contained some residual jelly. A very thin coat may have remained on component surfaces; but this thickness (if any) is difficult to estimate. An evaluator may choose to ignore this material or perform a sensitivity study to determine its effect. The petroleum jelly was 85% carbon and 14.8% hydrogen. It had a density of 0.816 mg/mm^3. Impurities, in parts per million, included: aluminum, 20; calcium, 7; copper, 23; and iron, 5. All others added to less than 5 ppm. The source of these impurity values is not certain.

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[25] The similar ratio for uranium, including tolerance gaps and holes, was 0.97246 suggesting that both materials had comparable tolerance gaps.

TEMPERATURE

These experiments were performed at room temperature. This probably varied between about 18 and 21 °C, depending upon the season. This guess is based on past temperature recordings of other experimental programs in the same facility. No logged entries identify temperatures in this room during these early years. Experiments appear to have been done between August, 1966, and the following February. This suggests fall and winter conditions; so, probably, the lower end of the above range is more likely correct.

Uranium metal is not a strong alpha particle emitter. Unlike plutonium, which is quite warm to the touch due to energy dissipated within the metal through radioactive decay, uranium remains cool to the touch. Both uranium and steel shells probably remained very close to room temperature during the entire program.

APPARATUS

Many details of this experimental program are difficult to document three decades later. This is especially true of the equipment and hardware used. Fortunately, the most important information, critical-approach data leading to critical parameters, was very well documented. Several reasons explain this dearth of equipment details. (1) The whole study was a somewhat informal program initiated by this author alone; and that did not seem to warrant a logbook for detailed record keeping at the time. Thus, any written information now available had been recorded on loose pages at best. (2) No detailed Experimental Plan, which would have contained at least intentions about the experiment, was written. Such documents were not required then. (3) The current (1990s) need for a wealth of descriptive detail to large distances from the fissile material, itself, was not foreseen. Finally, (4) the ultimate usefulness of this data was not immediately appreciated.

In spite of these shortcomings, safety considerations which became common in the 1970s and beyond were still employed. For example, the entire program was carefully thought out and discussed with others before any experiments were performed. An external neutron source was employed to manifest any actual reactivity changes into changes in neutron count rate. Two independent neutron-sensitive detectors were used to generate reciprocal multiplication approaches. Redundancy minimized the possibility of a single instrument's failure causing an accident. Finally, two knowledgeable persons participated in almost every critical approach.

A consequence of this minimal documentation is that some information now considered important must be recalled from memory 30 years and 1600 experiments later. Fortunately, this author recalls this study quite well. Still, the possibility exists that some details, especially concerning apparatus, may be less certain. Whenever that is the case, the fact will be duly noted.

Tables

Two different tables were used to support experiments at a comfortable working height. Both were made of steel; but no analysis of either steel's composition is known. One is referred to as the "heavyweight" table because it had been manufactured at Rocky Flats out of steel angle stock. It was stout. Most spherical experiments and those steel-centered hemispherical ones which were not reflected by steel on the outside were performed on this table. The other is the "lightweight" table; it had been purchased commercially. Most hemispherical experiments were performed on this except for those steel-centered ones which had no external steel reflector.

The **heavyweight table** measured 2.13 m long by 0.61 m wide. The working surface stood 0.96 m above the concrete floor. It is recalled to have been constructed of horizontal and vertical lengths of steel angle stock welded together. A reasonable geometrical description of the table's frame would be to consider it an open rectangular parallelepiped with each of 12 edges composed of angle stock. This author recalls that the angle stock was about 75 mm wide; but it could have been as large as 100 mm. It was perhaps 6 mm thick. It would not have been much thicker but could have been a little thinner. These estimates are based on standard sizes available in the industry.

The table's top surface was solid and probably steel, although it could have been aluminum. If aluminum, it would have been 13 mm thick. If steel, it could have been between 6 and 13 mm thick. This detail is, unfortunately, not at all recalled. Again, estimates are based on materials commonly found at Rocky Flats. A small number of experiments were recorded as being performed with a "50-mm-thick" plastic slab covering the table. These experiments are identified by reference to this tabletop in the table's title in the Appendix. Although this top is not recalled by this author, written documents report its use; and these are believed. This plastic would almost certainly have been one of the commercial forms of polymethylmethracrylate. The thickness of 50 mm is probably quite accurate. It should be assumed to cover the entire tabletop.

The table extended orthogonally out from a thick concrete wall. The distance between the closest edge of the table and the wall probably was about 100 mm. The table is recalled to have been a considerable distance from any other concrete wall or other neutron-reflecting surface. A more specific location along the east wall of the room in which experiments were performed is not recalled. Experimenters worked from both sides of this table; but they always stepped away from the area while neutrons were being counted.

Both radiation detectors stood vertically at the wall end of this table. They were taped to lengths of 76 mm x 76 mm steel angle stock, 6.4 mm thick, attached to the table's vertical members. Counters were backed by a 0.3-m square sheet of 1.6-mm-thick cadmium bent to a right angle and taped to the angle stock. This cadmium reduced wall-returned neutrons seen by the detectors. The full length (0.35 m) of each detector was covered by a 353-mm by 113-mm plastic block to moderate neutrons from the fissile assembly into the thermal energy range more easily counted by the detectors. Each block was 50 mm thick. Figure 6, which is derived from a similar figure in Reference 5 (RFP-1021), shows a plan view of this table with the approximate location of the center of fissile assemblies marked by a small rosette. Proportional counters (~ 50 mm diameter) are shown at the corners of the left end of the table.

The **lightweight table** was a little different. A small portion of it is shown in the photograph of Figure 7 which is included only because it illustrates construction. This table measured 1.53 m long by 0.71 m wide. The working surface stood 0.87 m above the same concrete floor. It was constructed of lightweight sheet metal, probably 1.5 to 2.5 mm thick. The top surface was the same metal; and legs were formed of the same stock in a "T" cross section. Legs were inset a little from each corner. Figure 8 shows an elevation of this table with the approximate location of the center of fissile assemblies marked by a rosette. This table was, in fact, a commercial steel table often found in industrial settings. It was probably bolted together.

The location of this table within the room is not accurately recalled, although this author does recall that it was not very close to any concrete wall. The table was probably about centered in an open area in the southeast corner of the room and reasonably well-removed from significant neutron reflectors. Experimenters could walk all around this table; but, as before, they stepped away during counting.

Both radiation detectors rested side by side on the floor almost directly below the fissile assembly. They were backed by a large sheet of 1.6-mm-thick cadmium to reduce floor-returned neutrons seen by the detectors. The full length (0.35 m) of each detector was covered by a 480 mm by 690 mm plastic block to moderate neutrons from the fissile assembly into the thermal energy range more easily counted by the detectors. This block was 50 mm thick.

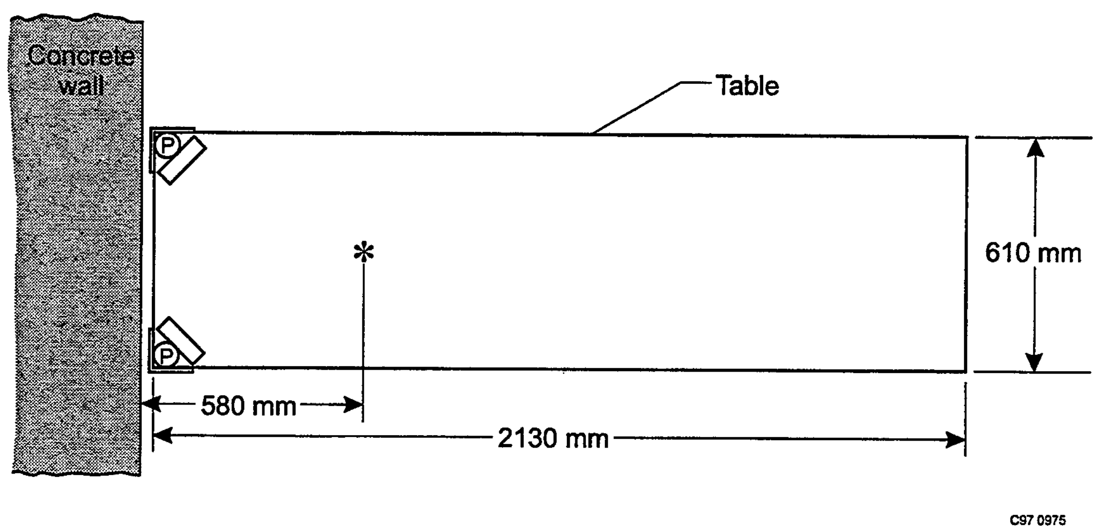

Figure 6. The heavy-weight table was used on many experiments. It was fabricated by welding together steel structural shapes. The material of its top surface is not known, an unfortunate omission. One end of the table was close to a thick concrete wall.

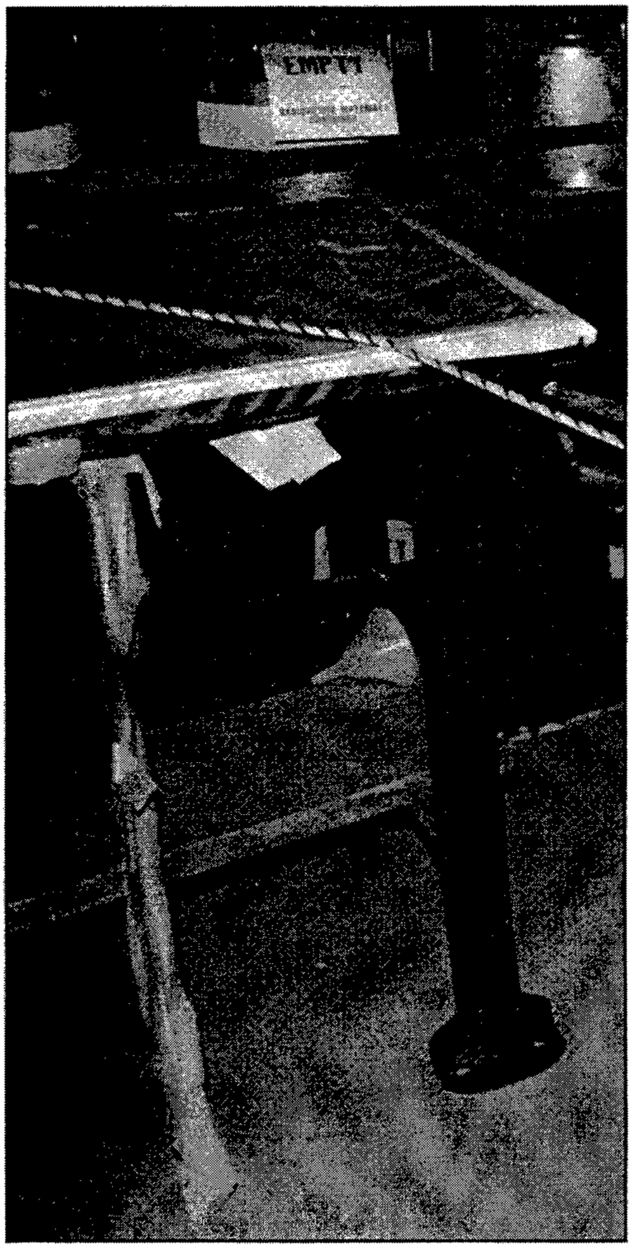

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Figure 7. The light-weight table was used on remaining experiments. It was a commercial table purchased for general laboratory use. It was not close to any significant environmental reflector. The sheet plastic covering shown was not present during experiments.

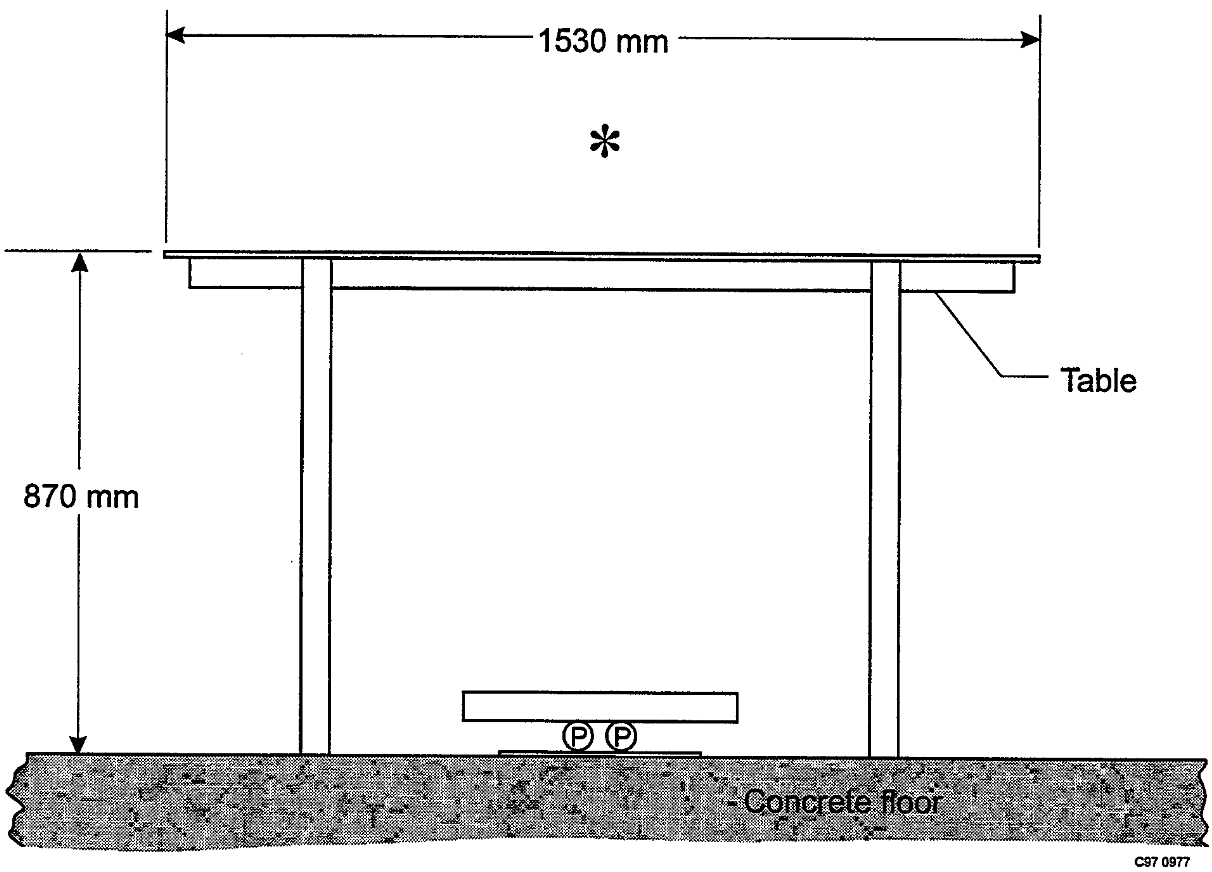

Figure 8. Some assemblies were built directly upon the thin surface of the light-weight table. An external neutron source was placed in a 20-mm-radius cavity in the center; and neutron detectors were placed on the floor.

Assembly Mounts

All spherical assemblies were built on an aluminum support consisting of a short length of a vertical, right circular, cylindrical shell welded to a thin, round, bottom disk. Actually, three such mounts were made; they differed in the diameter of the vertical cylinder. Their outer diameters, designed to accommodate spheres of various size, were 81, 136.5, and 208 mm with thicknesses of 3.2, 4.8, and 6.4 mm, respectively. All three cylinders were 203 mm tall. The top edge was chamfered to fit spherical loads better. The circular aluminum disk was 0.41 m in diameter by 12.7 mm thick. A cross section of one of these mounts is shown in Figure 9. The specific aluminum alloy is not known. The disk had a short length of aluminum bar stock welded vertically at its center. The bar stock was either 12.7 or 19 mm in diameter and 90 ± 13 mm tall. This was threaded to receive a mounting bolt for all other programs; but no such bolt was used in these studies. Stability relied on the weight of the assembly.

Unfortunately, the specific mount used on each experiment was only specified on one experiment. There, the "smaller cylinder mount" had been used. This suggests to this author that the other two were more common; but even that is conjecture.

Mounts used with hemispherical assemblies are less well known. At lease three are known to have been used. Some experiments were done directly on the table's flat surface with the pole down. Assemblies rested on their 7.14-mm-diameter pole hole, relying on this small flat to keep them from rocking. Such a procedure would not have been allowed in subsequent decades but was deemed acceptable at the time; workers paid attention not to bump the table holding a delicately balanced hemisphere. Still other experiments recognized this potential; and the hemisphere was constructed on a cork ring somewhat shaped like a large donut with an almost square cross section. Use of this ring was probably not recorded consistently and may have occurred more often than noted. Although a mental picture of these cork rings is quite clear, no example could be found to permit accurate dimensions. The outside diameter of a cork ring was about 150 ± 20 mm, the radial thickness about 50 ± 20 mm, and the vertical thickness about 40 ± 10 mm. With such a ring, the pole hole of any assembly would have been only a few millimeters above the table's surface.

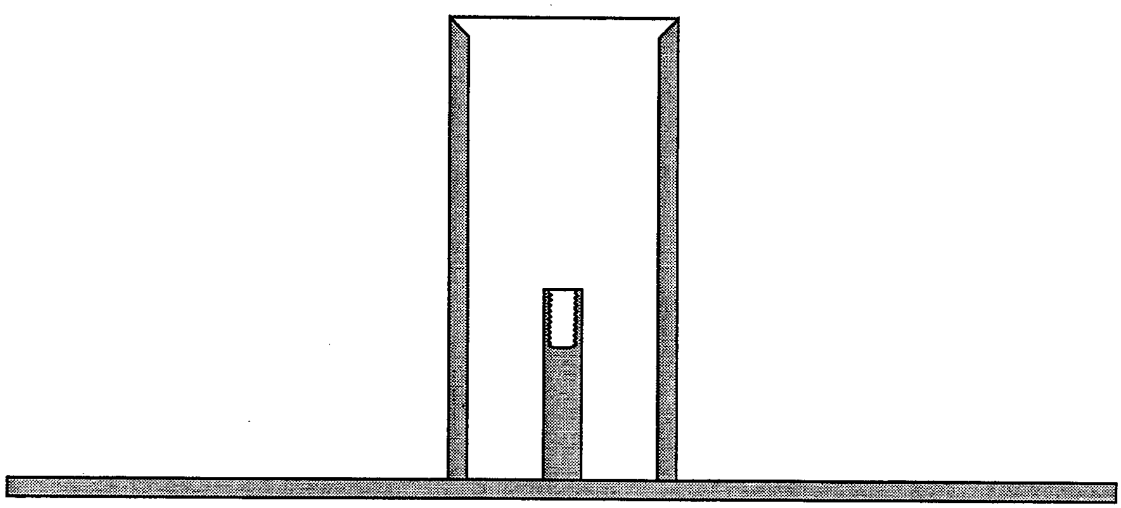

Figure 9. The mount for spherical assemblies (shown in cross section) was constructed of an aluminum cylinder welded to a flat disk. Cylinders of three diameters were fabricated; but which mount was used on which experiment was seldom recorded.

A third support was used for hemispherical experiments having no steel outside the uranium but with steel in the central cavity. It was a welded grid of heavy aluminum bars attached to a pair of aluminum end plates. A drawing is shown in Fig 10. This mount was designed for water-reflected experiments in another program. There, it would be stood vertically as shown in the figure. In this application, however, the support was rotated 90° counter clockwise to lay horizontally. Hemispheres were assembled with their pole hole up, not down. The equatorial plane of the hemisphere was 80 mm above the table's surface.

The overall length between end plates of this massive mount was 0.71 m maintained by three aluminum bars. Although their size is not specified, these bars probably were the same as the three orthogonal, horizontal, aluminum bars upon which the assembly, itself, rested directly: 51 mm wide by 9.5 mm thick. The latter three were about 0.41 m long. End plates were probably oval rather than round; but this is inferred from the figure and not recalled specifically. The thickness of the end plates, also, is not given but was probably 13 mm.

No other materials stood close enough to the fissile assembly under construction during neutron counting intervals to be considered any significant neutron reflection. Even the experimenters moved a considerable distance away from the table at these times to avoid any influence from human beings.

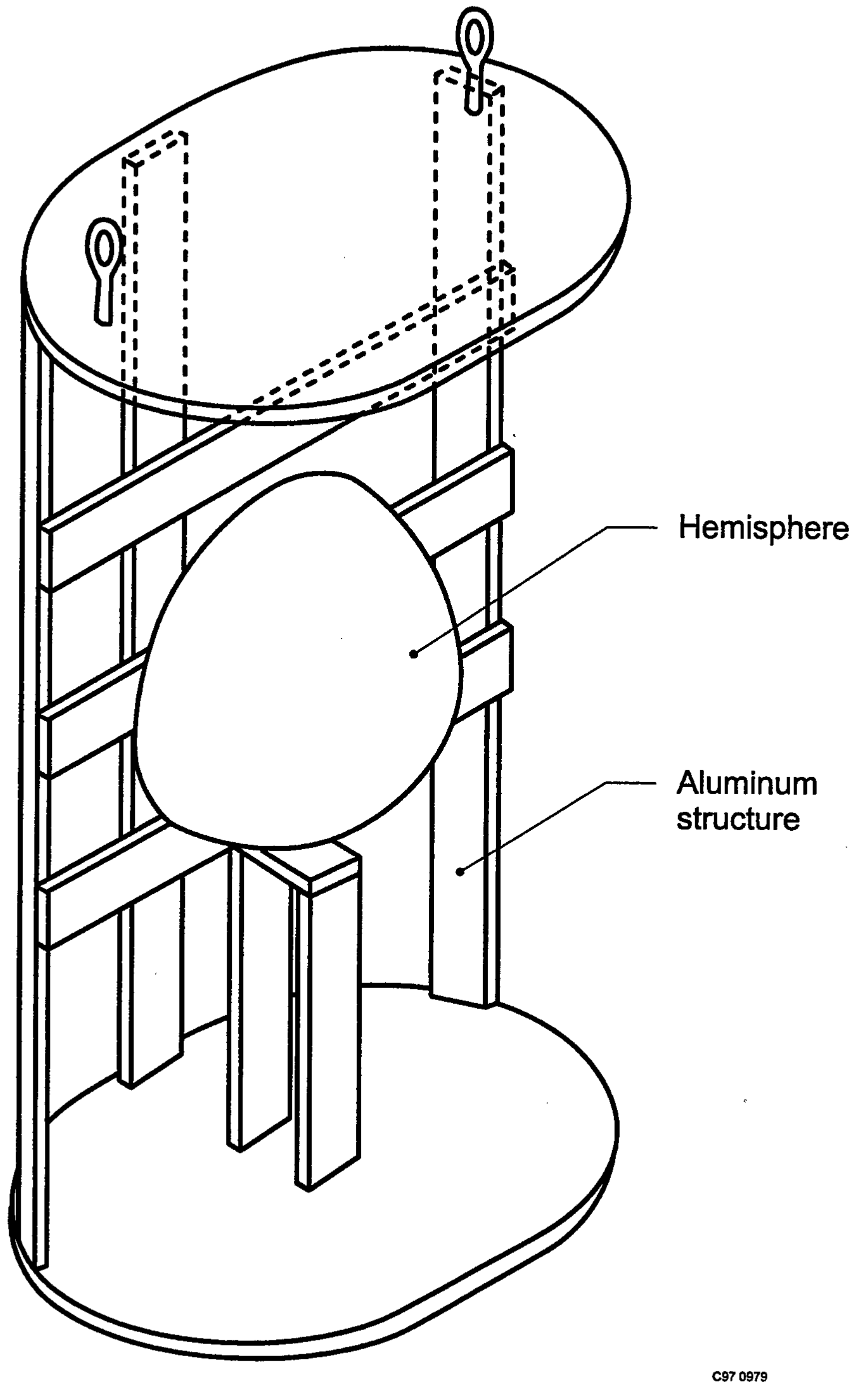

Figure 10. Hemispherical assemblies were supported in a number of ways. One was this aluminum mount made for another program. In that program, this mount kept the equatorial plane of hemispheres vertical as shown here; but, for these manual-assembly experiments, the mount was rotated 90° to place the pole up.

ENVIRONMENT

Experiments were performed in Room 101, called the Assembly Room, of the Rocky Flats Plant's Critical Mass Laboratory (CML). Most of the 1700 critical and critical-approach experiments performed at Rocky Flats between 1964 and its closure in 1989 were performed in that room. It is a large concrete room containing only a few items large enough and/or close enough to the fissile assemblies to provide any significant neutron reflection.

Assembly Room

The interior of this room measured 11.28 m in the east/west direction by 10.67 m in the other. It was 9.75 m high. Concrete walls and ceiling were formed in one, continuous, monolithic (seamless) pour in 1964. The north wall was 1.52-m thick; but the other three were only 1.22 m. The north wall was thicker because people occupied rooms to the north; and the small additional shielding would further protect them from radiation during experiments. The ceiling varied between 0.61-m and 0.71-m thick. The concrete floor was poured separately and was 0.15-m thick but rested directly upon compacted earth.

A sketch of this room is contained in Figure 11. Two rosettes mark the general location of the two tables used in this program. The rosette for assemblies on the heavyweight table close to the east wall is fairly well-defined in the east/west direction but may have been closer to the south wall than shown. The location of the lightweight table is not at all well known except that it was somewhere in the southeast portion of the room and far from any wall. An overhead photograph of the room, taken many years after these experiments, is shown in Figure 12 with the north wall at the top (same orientation as the previous figure). Much of the equipment seen in this figure was <u>not</u> present during this program: sets of trays at the bottom, the cabinet and two carts to the lower right, the cylindrical tank and its hardware near right center, and the house and other equipment seen on both halves of the Horizontal Split Table. This table which <u>was</u> present during this program is in the center of the photograph. These experiments were performed, then, somewhere in the lower third of this photograph. Again, rosettes mark approximate table locations.

The Assembly Room's concrete walls contained two layers of crossed steel rebar to strengthen the concrete. One layer was about 80 mm in from the outer surface; the other, the same distance out from the

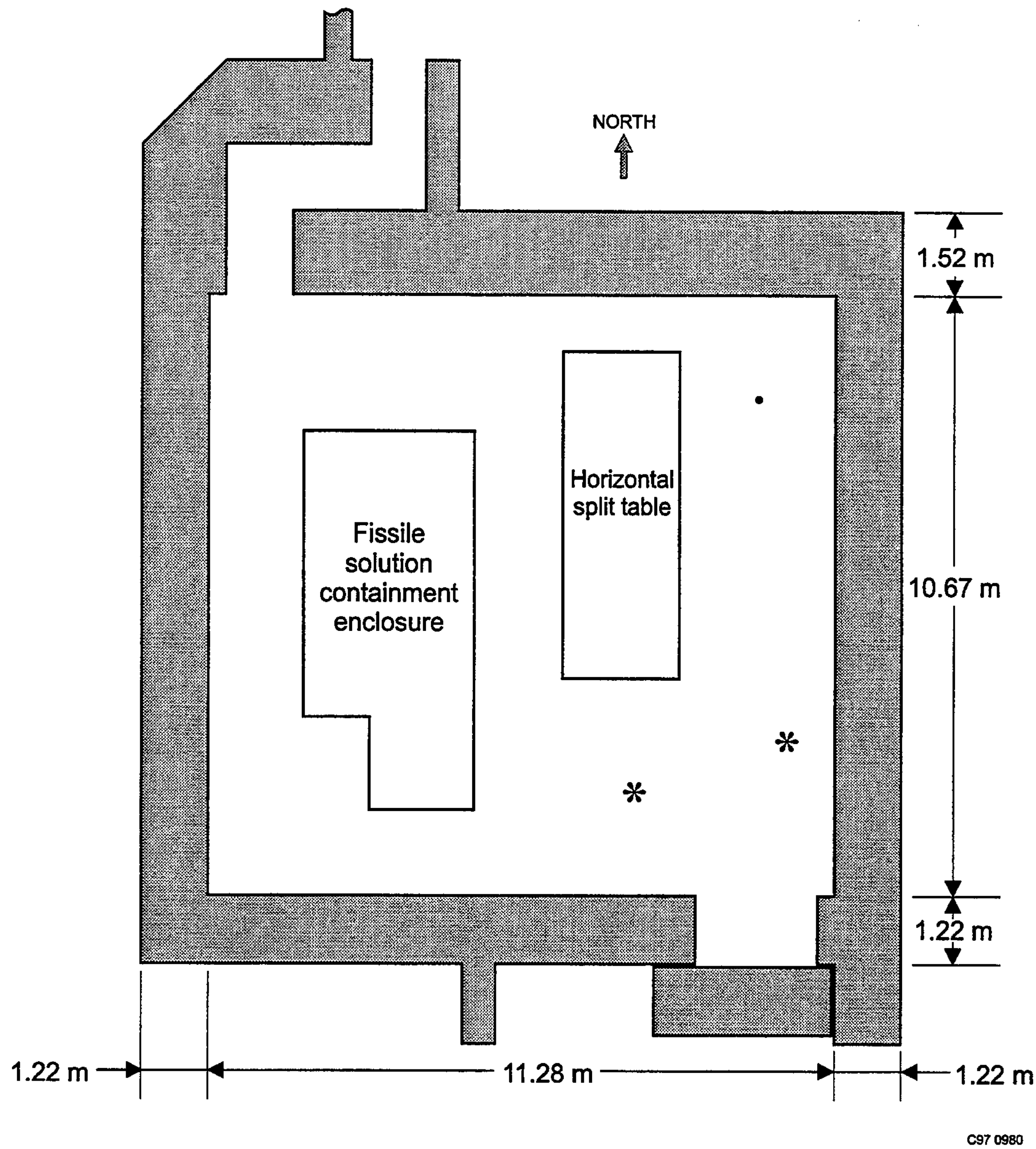

Figure 11. The thick-walled concrete room in which experiments were performed was called the "Assembly Room". The rosette close to the east wall approximately locates the centers of assemblies built on the heavyweight table. The other rosette provides a similar location for the lightweight table. The dot in the northeast corner pertains to Figure 13.

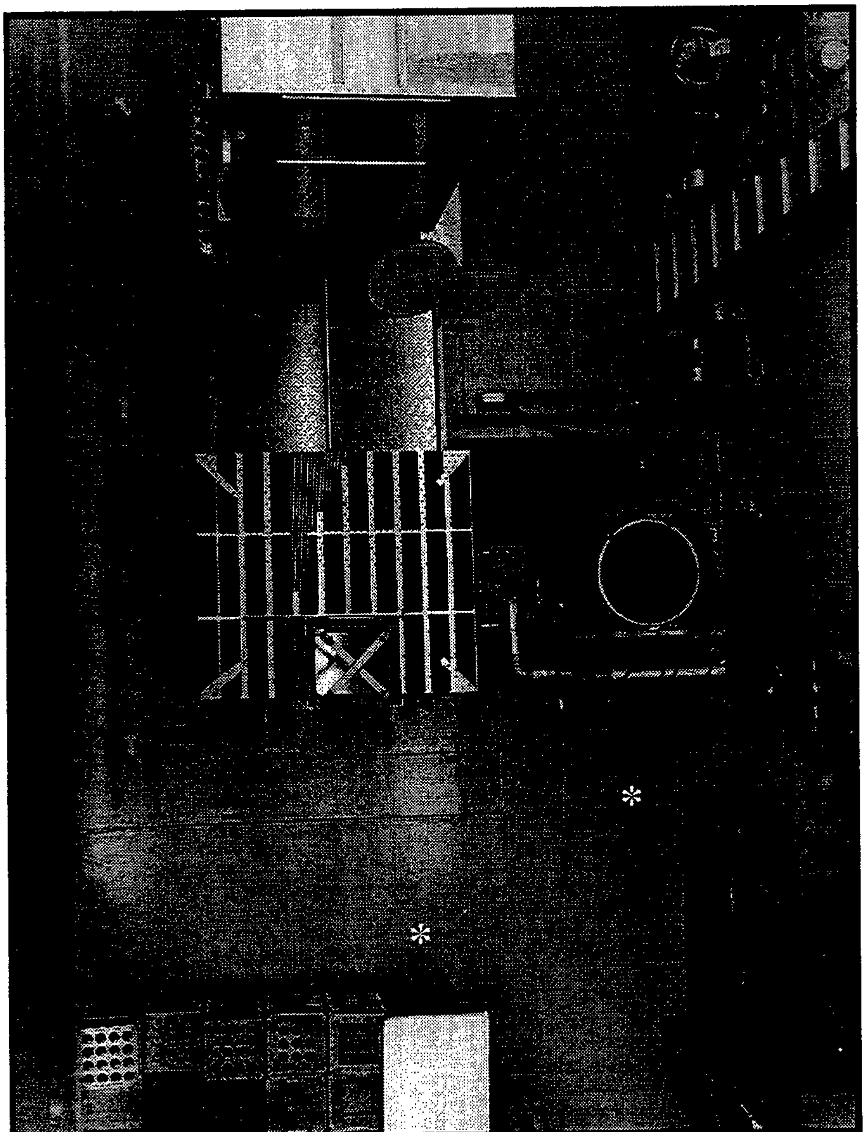

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Figure 12. This overhead photograph is oriented the same as Figure 11 and shows both areas where experiments were performed by rosettes. It shows the room in the late 1970s, long after this study was finished; much of the equipment shown was not present then.

inner. Horizontal rebars were #8 on 0.3-m centers. Vertical bars were #6 on the same centers. Approximately 7,000 kg of steel strengthens the concrete.

Specifications for the concrete poured in 1964 were quite common for industrial applications. Type I Portland cement was used at 307 kg/m^3. The maximum water content in the fresh mix was 30 kg/m^3; and the water had to be pure. Allowed aggregate sizes ranged from 6 to 18 mm; and this rock had to be low in amorphous siliceous materials.

In later years, the room would be painted a number of times. These were not just cosmetic; instead, paintings were aimed at improving the leak-tight integrity of the room. The first painting had probably not yet been done at the time of these studies, although this detail is not recalled for certain.

Two doorways penetrated this room. One in the north wall at the west end was a 1.07-m-wide by 2.44-m-tall passageway used for personnel access. Solid fissile materials and small experimental components were introduced here too. The passageway extended the full thickness of the north wall plus 1.07 m (2.59 m total) before making a 90° turn east. The wall backing the first passageway was also very thick. A similar turn back north after a 1.52-m-long hallway completed a Z-shaped labyrinth. The purpose of this labyrinth was to prevent radiation streaming out of the room in the event of a nuclear criticality accident. It might pass through the closed steel door; but it would not make the two right-angle turns to propagate down the hallway.

The second opening was diagonally across the room. It was in the south wall but at the east side. This was an equipment doorway connecting directly to the out-of-doors. The opening was twice as large (2.44-m square) to accommodate movement of larger and heavier components. This equipment opening was backed by a sliding concrete shield door. This massive shield was 1.07-m thick. Its 3.05-m-wide by 2.78-m-high size effectively would stop any radiation streaming south out of the room.

Both openings could be closed off and sealed by closing strong, steel, "blast doors" with a rubber seal between them and doorjamb. These doors are omitted from the sketch for clarity. One such door existed at the north passageway; and two existed at the south opening. Each door was 1.22-m wide by 2.59-m high and 0.10-m thick, although the fairly thick door was constructed as a honeycomb to reduce its weight. All three blast doors were made of steel; and each can be modeled as two 6.4-mm-thick plates on either face separated by 25 linear meters of honeycomb material (steel) 6.4-mm thick by 90-mm wide. Each door weighed about 425 kg.

Both blast doors in the south wall were closed during these experiments because no equipment would be moved in or out then. The blast door at the north access way, however, was intentionally left open for safety reasons. This provided an egress route for personnel in some unforeseen event.

Equipment

The closest large item to either experimental table was the Horizontal Split Table. It was situated within the east portion of the Assembly Room. Its overall dimensions were about 5.4 m long by 2.2 m wide; and it rose about 0.7 m above the floor. Although the table had a complicated geometry of honeycombed steel webbing and structural steel channel, a conservative approximation to its steel content would be a 25-mm-thick vertical rectangular shell measuring 5.4 m by 2.2 m on the outside supporting two horizontal and co-planar table tops. Each top would be 1.9-m-long by 2.2-m-wide and 50 mm thick. This is the most steel that the table would have contained. One tabletop would be located at each end of the rectangle. The long dimension of this table was parallel to the east wall. The southeast corner of the Split Table was 2.44 m west of the east wall and 4.18 m north of the south wall. This very heavy table was never moved during the lifetime of the laboratory.

The fissile solution containment enclosure was in the west half of the room. A very early photograph of it, even before the first experiment of any kind at this laboratory, is Figure 13. The picture also shows a small portion of the Horizontal Split Table in the foreground. The containment enclosure was about centered north/south within the Assembly Room; and it was also about centered in the west half of the room. The room was constructed mostly of stainless steel (1.6 mm thick); but it had an estimated 30% of its surface covered with 13-mm-thick plastic windows. The room was 5 m north/south by 3 m and stood 6 m tall.

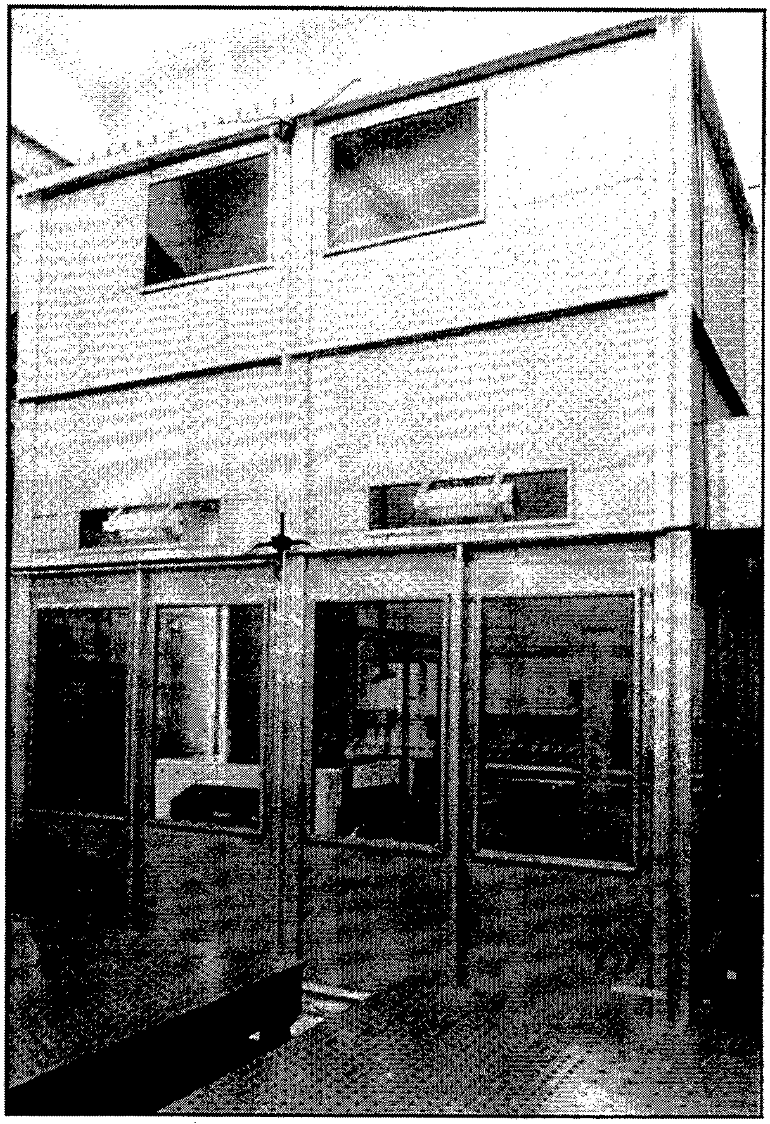

Figure 13. This 1964 photograph of the Assembly Room predates any experimentation at the Rocky Flats laboratory. The view is from the east wall at about the middle of the north half of the Horizontal Split Table, seen in the foreground. The east wall of the Fissile Solution Containment Enclosure is seen behind the table. A small dot on Figure 11 locates the cameraman.

An air-handling deck existed in the southwest corner of the room but well above the floor. This deck can not be seen in any figure. It was an all-steel structure 4.8 m east/west by 2.4 m and stood 4.5 m above the floor. It was constructed of about 30 m of nominally 0.2-m channel iron.

The room's travelling crane (also not seen) was a heavy-duty industrial "5-Ton" crane built into the room a short distance below its ceiling. Its massive steel I-beams ran east/west with a bridge for orthogonal movement.

EXPERIMENTAL RESULTS

This paper, unlike others in the International Criticality Safety Benchmark Evaluation Project series, does *not* report parameters for *actual critical* systems. This is so because of safety considerations. Critical approaches were constructed manually; and the risk of an accident was too great to permit criticality while personnel were present. Dozens of other programs at Rocky Flats did, in fact, intentionally achieve criticality; but, for these, reactivity was added remotely so the danger to personnel was eliminated.

Even these other experiments never were *precisely* critical. That is almost impossible to attain because a very long time is required to establish a truly infinite reactor period. Instead, critical parameters were always interpolated between two very closely similar states. One was slightly above and one slightly below criticality. Thus, precise criticality is never actually achieved in any experimental study; it is always either interpolated between bracketed data or extrapolated from subcritical data.

Critical parameters from this study are the result of a rather lengthy extrapolation of subcritical data. Long extrapolations are considered valid in this study because of the nearly linear relationship between parameters. Any extrapolation of almost linear data is always better than an extrapolation of a function displaying a great deal of curvature. The outside radius of assemblies appears to be very nearly linearly related to reciprocal multiplication. When graphed against mass, on the other hand, much more curvature is noted.

In spite of arguments claiming linearity, even the radial dependence is not perfectly linear. Some small curvature exists. For that reason, every critical parameter presented in this paper also contains the original subcritical data. This permits each reader to draw personal conclusions as to the critical value without being forced to accept this author's personal bias.

Both subcritical data and extrapolated critical parameters are contained in 69 tables located in the Appendix to this paper. They are placed there merely to avoid a major interruption to the written text. These tables are both complex and need to be free of ambiguity. Therefore, each portion of them is discussed separately.

Format

Tables A through D pertain to spherical geometries; Tables E through H, *hemi*spherical cases. Many tables are found within each letter designation; and each corresponds to a single experiment.

Tables A-1 to A-5 refer to spheres with no steel anywhere; and four Tables B pertain to those few steel-free experiments performed with a thick plastic surface covering the table. Eleven Tables C describe cases with no interior steel but some steel outside; and nine Tables D, to experiments having steel inside the uranium. Similarly, Tables E-1 to E-6 refer to hemispheres with no steel at all, four Tables F to those few steel-free experiments performed with slight changes in apparatus[26], fifteen Tables G to cases with no interior steel but some steel outside, and, finally, fifteen Tables H to experiments having steel inside the uranium. Within each set, tables are arranged in ascending order of inside radius. Additionally, tables pertaining to steel-reflected cases are further arranged in ascending order of this thickness.

The 29 spherical cases represent 26 different combinations of uranium and steel thicknesses. The other three are steel-free cases performed close to two different table surfaces which could reflect neutrons differently. The 40 hemispherical cases represent 32 different combinations of uranium and steel; and the others are experiments performed with small changes in apparatus.

All tables present both actual subcritical data and extrapolated critical parameters for a wide variety of independent variables. Spherical cases span six different inside radii, ranging from IR = 20.126 mm (neutronically, quite similar to a solid sphere) to thick-walled shells of very large inside radius (110.113 mm). For very large IR, neutron physics approaches that for a solid slab of the same material; but this is strictly true only in the limit of infinite radius. Hemispherical cases span seven inner radii, not all the same as the spherical cases. Results for two solid hemispheres are reported.

Tables representing steel describe systems having steel inside and/or outside the uranium. Steel outside was present in five different thicknesses up to nominally 80 mm. When inside, the steel almost always filled the central cavity except for a small (20 mm radius) spherical pocket which held the neutron source.

[26] Always to be compared with some case from Tables E.

Two solid hemispheres were exceptions; there, the neutron source merely rested on the plane surface at its center.

All 69 tables are divided into three sections. The upper presents the actual subcritical reciprocal multiplication data actually obtained during an experiment. This untreated data may be graphed however one chooses to obtain independent evaluations of criticality, free of this author's personal bias. On the other hand, his personal interpretation *is* contained in the middle section. There, extrapolated critical values are tabled for three parameters: the *number* of uranium hemishells, the *outside radius* of the largest, and the total uranium *mass*. The last two are shown in bold-faced font because of their greater importance. They are determined by up to four independent means. The range between the four suggest the uncertainty in the extrapolation. The bottom section contains a conjecture as to the incremental increases in outside radius and mass which might yield prompt criticality. Smaller-sized font signifies decreased certainty in the validity of the procedure.

Definitions

A precise definition of each column heading and its content for the common format of all eight sets of tables is given below. This is provided to preclude any ambiguity.

The table's title identifies the geometry as spherical or hemispherical and also specifies the inside radius of the uranium. This radius was simply taken from Table III for the smallest uranium hemishell (hemispherical assembly) or pair of hemishells (spherical) used. The two smallest shells of a spherical load differed a very little; so the *average* inside radius, rounded off to 5 significant figures, is quoted. The title also identifies the use of steel.

Some tables have an extra row across the top; these pertain to experiments which included steel in either location. The row specifies the location and *nominal* thickness, but not the precise thickness, of the material. Steel inside uranium appears to the left; that outside, to the right. These are nominal thicknesses and assume the region is composed of nesting shells a little less than 3 1/3 mm thick each. Precise dimensions and masses of steel parts used may be obtained from Table V[27].

The following paragraphs describe each column in all three sections of all 69 Tables A through H. These are discussed separately for each section.

Subcritical Data:

Number of Hemishells

The actual number of uranium hemishells built into an assembly. This *number of parts* is never the same as *part numbers* from Table III because smaller shells were omitted to form the thick-walled shell. For hemispherical assemblies, even-numbered parts were arbitrarily used. For spherical loads, the number of parts was usually even; but a slightly asymmetric sphere was occasionally constructed with one more shell on one half than the other. This was needed to fill in the reciprocal multiplication graph. These configurations are enclosed in parentheses.

Average Outside Radius

The outside radius of the largest shell (hemisphere) or pair of matching shells (sphere) used. These radii also come from Table III. When the two differed a little as in spherical cases, the average outside radius is given, rounded off to three decimal places. Radii enclosed in parentheses again represent the occasional asymmetric spheres mentioned above; so they are the average of two adjacent components from Table III.

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[27] Consider Table D-9 as an illustrative example. The uranium spherical shell had a nominal inside radius of 80 mm and steel both inside and out. Since the smallest steel core was omitted to make room for the neutron source, the inside steel extended from a radius of about 20 mm to 80 mm. From Table V, steel parts #03 and #04 through #37 and #38 would have been used. These 18 pair form a spherical shell 60 mm thick. The external steel reflector was 80 mm thick. The largest uranium assembly actually built (18 shells) had an outside radius of 110.011 mm according to Table D-9; so the steel reflector would have been constructed using mild steel parts #57 and #58 through #103 and #104 (24 pair of shells). Each of the four assemblies in the top section of Table D-9 would have used a different set of 24 pair of shells for the outside steel.

Total Mass (g)

The total uranium mass of shells actually used. This is obtained from Table III by either of two calculations: sum entries from the Mass column or take the appropriate two entries from the Total Mass column and subtract them. Again, asymmetric spheres are noted in parentheses.

Density (mg/mm^3)

The mass from the preceding column divided by the volume of the thick-walled shell. That volume is calculated from the outside radius in the second preceding column and the inside radius in the table's title. Ideally, these densities should be almost constant. They only varied slightly because tolerance gaps varied and holes had differing influence. No densities are shown for the occasional asymmetric spheres, although they could be calculated if desired.

Reciprocal Multiplication, C_o/C

The ratio of the neutron flux observed at the start of an experiment, C_o, to that obtained for the assembly being built. This was discussed in detail in an earlier section of this paper. This same parameter was graphed independently against each of the first three parameters[28]: number, outside radius, and mass. This parameter is also enclosed in parentheses for asymmetric spheres.

Data for air-centered and steel-centered cases of otherwise similar assemblies were obtained concurrently. This was done to minimize handling components. A consequence of this detail is that C_o for steel-centered cases was actually that for air-centered configurations; so the reciprocal multiplication measured is really only *proportional* to that which would have been obtained otherwise. The reader will note that some reciprocal multiplications obtained for larger steel-centered hemispheres are larger than unity. This safety concession will not affect extrapolations to delayed criticality but may call into question estimates of prompt criticality in these few cases.

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[28] A reciprocal multiplication may be considered equally to be $C_o/C(\#)$, $C_o/C(OR)$, or $C_o/C(M)$ where # = the number of shells, OR = the outer radius of the largest, or M = the total mass, respectively.

Delayed Critical Extrapolation:

The first column lists the four methods that were employed to obtain critical parameters. Not all four were used in all cases. For example, the critical number of uranium shells was extrapolated in the 1960s and was not repeated in 1997.

RFP-1021

This reference tabled only critical masses determined by this author in the 1960s extrapolating only the number of uranium shells to criticality. Critical masses, then, were determined from this as described earlier and illustrated in Table I.

A few cases reported here were not included in the earlier publication and therefore, show no entry in this area of the 69 tables.

Smooth Curve

A set of commercial, plastic "ship's curves" were available as pencil-guiding templates to extrapolate actual subcritical data. This author's philosophy was to select the best one to fit data as well as possible by eye. Then, that template was used to predict criticality. The template[29] always had the same or greater radius of curvature (closer to linear) in the extrapolation region than in the data region. Use of templates of smaller radii possibly would introduce unjustified physics.

Not only was the same template used for all smooth curve fits; but the same general *region* of that template nicely fit all data. Thus, this author recognizes a strong systematic nature to his results for the entire set of 69 experiments. On the other hand, any bias introduced by the template will, at least, be systematic. This is discussed again in the Uncertainties Section. Another factor intentionally considered when making these fits granted greater importance to smaller reciprocal multiplications; little concern was paid to data above $C_o/C \sim 0.4$.

[29] One particular template seemed to work remarkably well in all cases. It had been marketed by Keuffel and Esser Company as their curve #1864-48. Only the outside edge was used. This information is offered to help readers reproduce this author's extrapolations.

The greater linearity of radial curves over mass-based ones is especially obvious in Figure 14. This and Figure 15 are examples of extrapolations for all three parameters: number of uranium hemishells, outside radius of the largest, and total uranium mass. The first figure pertains to one spherical case; the other, to a hemispherical one. Many hemispherical cases actually appeared to fit straight lines better than any curved template. In fact, the hemispherical data of Figure 15 are *intentionally* connected only with a straight line to illustrate the enhanced linearity observed. These two figures are derived from Tables C-8 and H-12, respectively.

Critical values for the two radial parameters, the **Number of Shells** and the **Average Outside Radius**, were independently extrapolated to criticality. The former, in the 1960s; the latter, 1997. This independence minimized pre-conceived biases. The two radial parameters are not absolutely proportional to one another because individual shells did vary slightly in thickness, although the *number* is insensitive to this detail. The difference is small.

The critical **Mass** is obtained from yet another independent extrapolation of subcritical data using the ship's curve. Typically, greater curvature was found in mass-based curves. Both are lengthy extrapolations; and neither is expected to be more accurate than, say, 1 or 2%. Still, radial extrapolations should be better than mass because of smaller curvature.

The **density** enclosed in braces in the next column is a visual average of densities over the largest few subcritical assemblies. Its smaller precision is reflected through fewer significant figures.

All three independently extrapolated critical parameters and the presumed density of the critical assembly may be combined with other, better-known parameters in a test of self consistency. The volume of the uranium region may be calculated from the inside radius in the table's title and the extrapolated critical outside radius. This volume times the density is another measure of the critical mass and may be compared with the independently extrapolated value. This exercise has not been done in every case; but three examples, selected at random, are offered: (1) Table A-1 predicts a mass of 62 kg calculated by this procedure, a little more than 3% larger than that obtained by the mass extrapolation. (2) Table A-5 predicts a mass of 187 kg

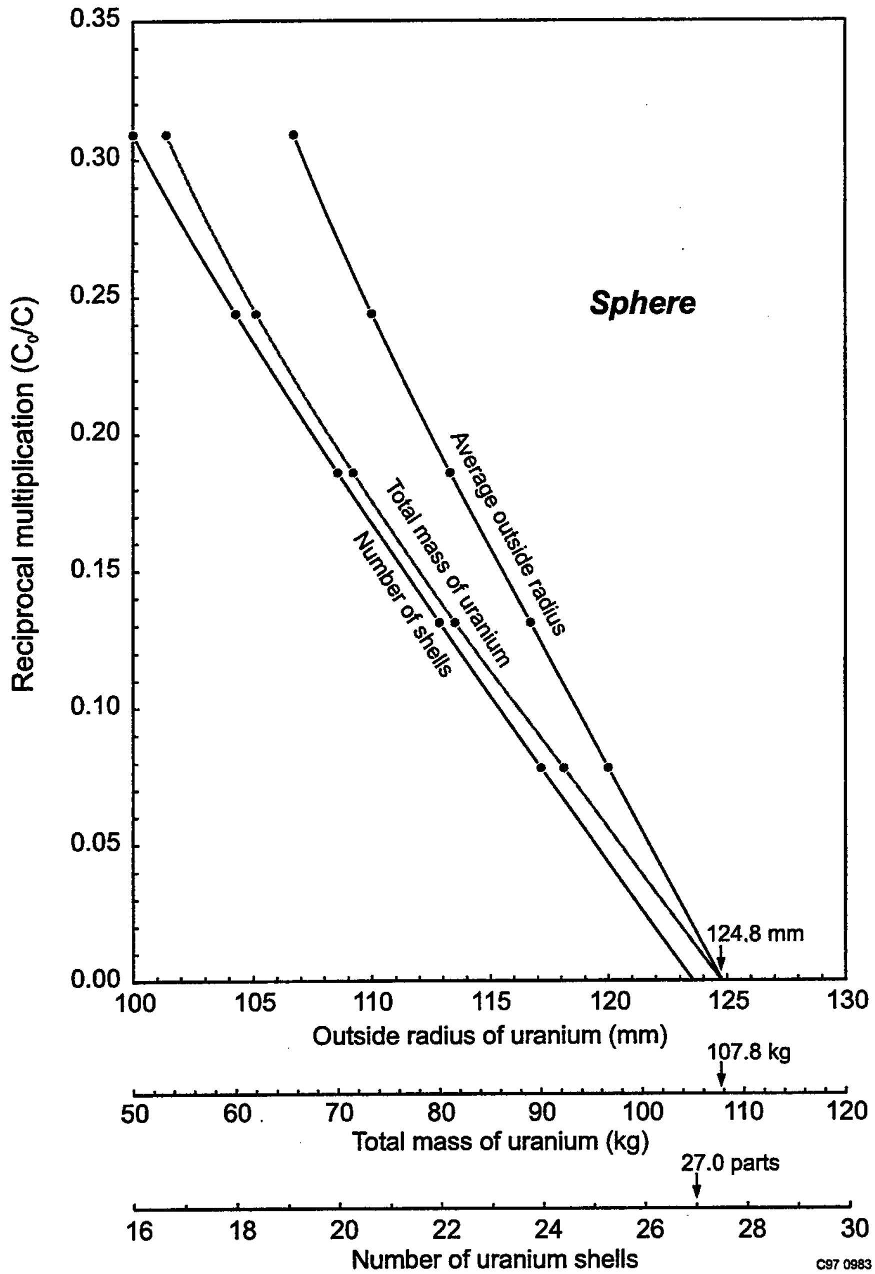

Figure 14. Extrapolations of all three parameters to criticality were almost linear, but not precisely so, for spherical assemblies. Radial parameters tended to be closer to linear than mass-based ones.

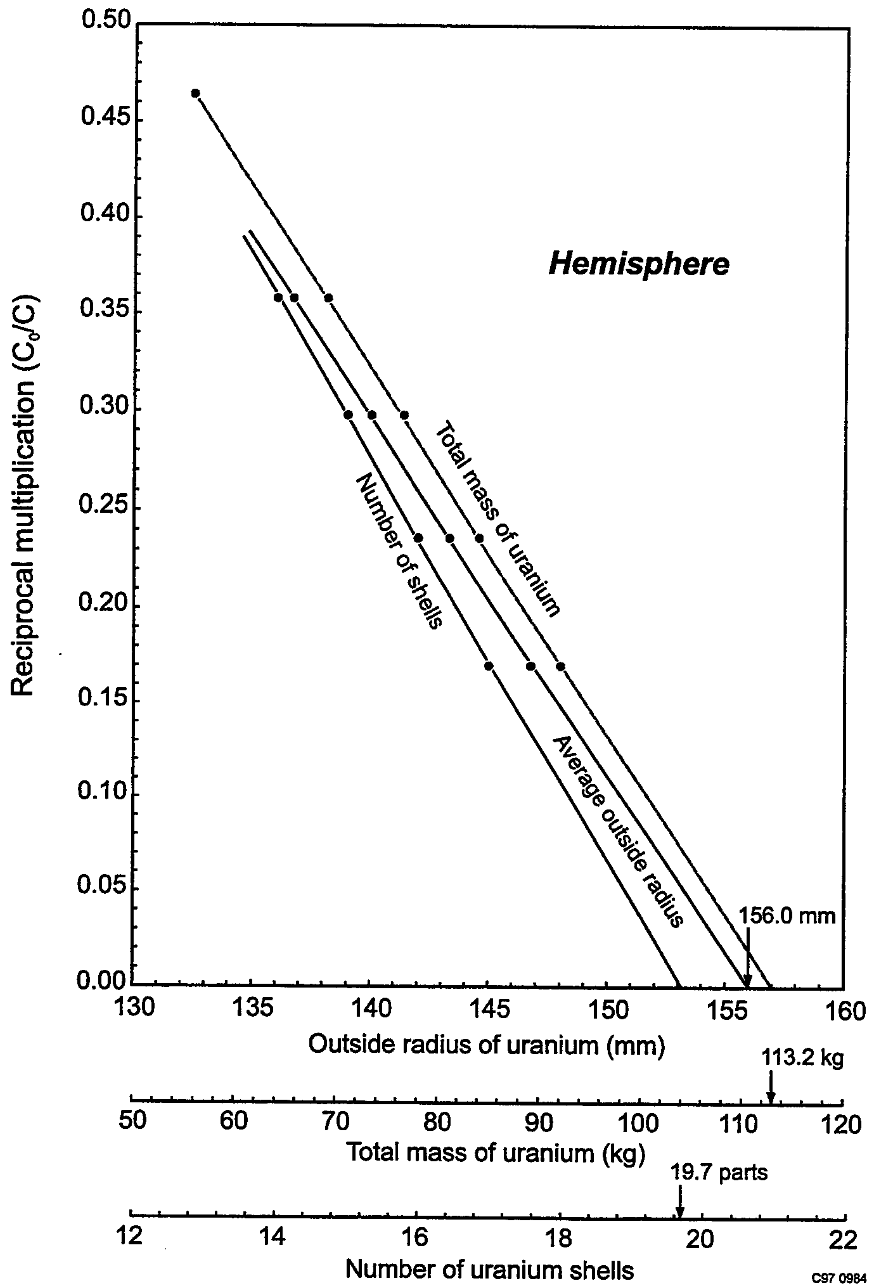

Figure 15. Extrapolations to criticality for hemispherical cases were often very nearly linear. All three curves in this example are straight lines.

from the radius extrapolation, in moderate agreement with the mass extrapolation. (3) Table C-10 predicts a mass of 85.8 kg, fair agreement.

Least Squares Fit

Reciprocal multiplication curves plotted against either radial function exhibit only a slight tendency to curve outward toward greater radius than that predicted by linearity. Although no physics suggests a quadratic relationship, that would be the next higher order to assume in attempting to fit data. The method of Least Squares was selected to obtain this measure of the critical **outer radius**. These calculations were performed using a commercial computer application[30]. A discussion of the mathematics associated with this approach is presented later. The statistically derived equation for the best fit is given in each table.

The critical **Mass** associated with this method was calculated from this outer radius determined by the method of Least Squares and other known parameters. As an example, the critical Total Mass for Table A-1 (58.7 kg) was calculated from:

$$M_c = [4\pi/3]\{(OR)^3-(IR)^3\}\rho,$$

$$58.6 \times 10^6 = [4\pi/3]\{(92.1)^3-(20.126)^3\}\{18.10\}.$$

where (OR) is from the Least Squares fit, (IR) comes from the table title, and the density, ρ, is that in braces. Values in the tables used many more significant figures than shown here; so small round off discrepancies, such as found in this example (58.6 kg vs 58.7 kg), may be expected.

Last Two Points

A linear extrapolation of just the last two data points often yields a lower bound to the critical **outer radius** because all cases appear to curve outward at least a little. No reciprocal multiplication curves exhibited opposite curvature. This is not universally the case because statistical fluctuations and other experimental uncertainties sometimes distort the last two data points just a little. This estimate of the critical outside radius and the critical mass ignores a lot of data; only two points are involved. Still, results are surprisingly good.

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[30] Microsoft Excel, method function TREND.

The critical **mass** in the fourth column for this method was calculated the same way as for the Least Squares method described two paragraphs earlier and assumed the same density.

That density, enclosed in braces in the 5th column, is somewhat arbitrary. It is a subjective average of densities over the larger few actually built assemblies. Its greater uncertainty is reflected in fewer significant figures.

Prompt Critical Increment (One Dollar):

The Least Squares fit, quadratic or linear, as well as the last two points' linear extrapolation to the critical outside radius were used to estimate **prompt** criticality by merely extending the reciprocal multiplication curve to -0.00665. This concept was discussed elsewhere. The extended extrapolation by both these methods estimates the *increment* in the **Average Outside Radius** above delayed criticality which corresponds to prompt criticality.

The associated prompt critical **Mass** *increment* was calculated for both methods using the outside surface area of the delayed critical assembly, the estimated incremental increase in radius to prompt criticality, δ(OR), and the same average density as before. Again, Table A-1 provides an example; the example (1.53 kg) refers to the method using just the last two data points:

$$\Delta M(1\ \$) = (4\pi)(OR)^2\delta(OR)\rho,$$

$$1.54 \times 10^6\ \text{mg} = (4\pi)(90.9)^2\delta(0.82)\{18.10\}.$$

As before, round-off error introduces a small difference. The same average density was used as in the middle section of the table because increments are small and the density would not change much.

Auxiliary Information

Finally, one additional row spans the table just below the prompt critical section. This contains assembly details related to which table was used and which mount supported the fissile material. Much of this information was either recorded or is well recalled. Whenever neither is the case, a question mark acknowledges uncertainty in the entry. All spherical assemblies were built on one of the three cylindrical mounts shown in Figure 9. The one used was almost never specified; only one case (Table A-3) identified the smallest one. Tables associated with hemispherical assemblies contain one additional detail - the location of the pole hole (up or down).

A Discussion of Least Squares

The mathematics of the Least Squares fit method of analysis, described above, is both interesting and important to understanding this approach. Two quadratic functions could have been fit to the data. The first thought, based on one's common bias regarding the xy-plane in Cartesian coordinates, would be the parabola:

$$C_o/C = \mathbf{A}R^2 + \mathbf{B}R + \mathbf{C};$$

but this choice would be ill-advised. The region of this parabola fit by the data would be the left-hand arm of an "upward open" parabola (positive coefficient of the quadratic term). Therefore the region of the extrapolation has *more curvature* than the region containing data points. This would appear to introduce unjustified physics.

A better choice - and the one adopted in all but 13 cases - was to fit the data to the parabola:

$$R = \mathbf{A}(C_o/C)^2 + \mathbf{B}(C_o/C) + \mathbf{C}$$

This parabola opens to the right in the C_o/C vs R plane; and the extrapolated region has a little *less* curvature than that fit to experimental data, consistent with this author's intuition. This is also consistent with the author's use of the ship's curve template. Even though smaller, curvature continues in the same direction along the entire reciprocal multiplication curve. No physical principals would suggest that the radius-of-curvature ought to change direction in these simple and straightforward systems.

Limitations of this Least Squares curve fitting routine as a statistical procedure are clearly recognized. In addition to no physical basis for a quadratic functional dependence, many cases had only four subcritical data pair. Sometimes, the best fit appeared to be the linear regression,

$$R = A(C_o/C) + B,$$

instead of quadratic. This was true in 12 hemispherical cases and one spherical one; and this difference in numbers supports the oft-noted observation that all hemispherical curves were more nearly linear than spherical ones. Two reasons prompted this subjective decision to prefer linear regression: (1) The data, regardless of the number of data pair sometimes just naturally fell along a straight line. (2) Sometimes four data points exhibited so much scatter that either could have been used. Fitting the higher-powered quadratic with so few points seemed unjustified.

Using this Least Squares fit procedure, the constant, **C**, is the estimate of the delayed critical radius. The radius for the prompt critical conjecture would be simply R(-0.00665); and the prompt critical incremental radial increase would be: R(-β)-R(0).

<u>Critical Mass Data Summary</u>

The middle sections of all 69 Tables A through Tables H present critical parameters determined in four different and independent ways. The parameters include the mass of enriched uranium in spherical or hemispherical assemblies, the outer radius of a thick-walled shell whose inner radius is given in the table's title, and the number of these shells. The remainder of this Experimental Results Section will focus specifically on the critical mass.

Four methods of determining this parameter included: (1) The critical mass originally published in 1967. (2) The critical mass determined by drawing a smooth curve through subcritical data points on the mass-based reciprocal multiplication curve. This estimate was always completed <u>before</u> the remaining two to eliminate subconscious bias. (3) The critical mass calculated from the critical radius obtained from the Least Squares fit method and the density. Finally, (4) The critical mass calculated from a straight line extrapolation of the outside radius curve for just the last two data points and, again, the density. This mass often tended to be a lower bound to the "best" critical mass. This was usually true because most curves were observed to curve outward just a little; but the small amount of scatter in the actual experimental data occasionally predicted otherwise. The last three critical masses were all determined in 1997 as part of the preparation of this paper.

Except that the fourth method ignores some data, none of the four are claimed superior to another. Consequently, an *average* of the four is adopted in this paper as the published critical mass. Still, some subjectivity entered into this average. In four instances, one of the three 1997 results seemed an outlier compared to other methods. There, the offending mass was dropped from the average. This subjective decision is signified in the tables by enclosing the ignored mass in asterisks. For example, the "smooth curve" mass in Table F-1, *61.7* , was later excluded from the average.

These average critical masses are presented in Tables VI and VII. The first presents average masses for spherical data; the latter, hemispherical. Both illustrate the dependence on reflector steel thickness from left to right; and the upper and lower portions tabulate masses with and without mild steel inside the cavity of the thick uranium shell. Uncertainties are non-standard; they represent one-half the range between the four (sometimes three) masses averaged. The four entries shown in italics in Table VII reflect the subjective rejection of one method in determining the average as discussed above.

Figures 16, 17, and 18 display these same data graphically. Curves represent no physical model nor mathematical function; they are simply guides to aid the reader. Error bars on Figure 16 are the same non-standard uncertainties listed in Tables VI and VII. Data points and error bars at 60.117 mm (spherical) and 60.121 mm (hemispherical) would intersect another; so only half of each symmetrical error bar is shown in a concession to clarity. Error bars have been omitted from Figures 17 and 18 (also for clarity); but they still exist and can be read from Tables VI and VII.

Figure 16 shows the increase in critical mass as the inside radius of the thick-walled shell increases. Both spherical (•) and hemispherical data (x) are presented. The open circle for a solid sphere (IR = 0) corresponds, of course, to the well-known Godiva[31] assembly. It agrees with the present data very nicely,

[31] The Godiva assembly is an unreflected sphere of 93.15% enriched ^{235}U. Rick Patternoster of Los Alamos National Laboratory quoted a mass of 53.6 kg for Godiva in a private communication.

Table VI. Critical Masses (kg) for Spherical Geometries Averaged Over the Four Methods of Determining Them.

Uranium Inside Radius (mm)	Nominal[a] Thickness of Mild Steel Reflection Outside Uranium (mm)					
	0[c]	10	20	40	50	80
No Mild Steel Inside Uranium Shell						
20.126	57.6 ± 1.2	48.2 ± 2.3	43.8 ± 1.2	38.7 ± 1.2		34.0 ± 0.5
20.126	(51.3 ± 4.4)					
40.165	71.0 ± 1.7[d]	61.5 ± 1.7		48.8 ± 1.3		42.6 ± 0.4
60.117	(94.4 ± 4.8)					
80.102	125.8 ± 1.0	109.5 ± 1.5	100.7 ± 1.9	85.4 ± 1.5		75.1 ± 0.8
80.102	(126.4 ± 1.3)					
100.112	(164.6 ± 0.7)					
110.113	186.0 ± 1.4					
Mild Steel Inside Uranium Shell [a,b]						
40.165	72.3 ± 0.9	61.1 ± 2.0		49.6 ± 1.0		42.5 ± 0.6
80.102	126.3 ± 1.0		100.4 ± 1.0	87.3 ± 0.8	82.5 ± 0.5	76.8 ± 0.7

a Refer to Table V for precise dimensions

b Except for the center 20-mm-radius cavity to accommodate an external neutron source

c Experiments (4 masses) enclosed in parentheses were performed with a 50-mm-thick plastic table top

d A repeated measurement yielded 71.2 ± 1.7 kg

Table VII. Critical Masses (kg) for Hemispherical Geometries Averaged Over the Three or Four Methods of Determining Them.

Uranium Inside Radius (mm)	Nominal[a] Thickness of Mild Steel Reflection Outside Uranium (mm)					
	0	10	20	40	60	80
			No Mild Steel Inside Uranium Shell			
0	*68.8 ± 0.7*[c,f]					
20.126	68.0 ± 3.1[d]	62.2 ± 1.2	59.3 ± 0.7	*53.5 ± 0.5*[f]		49.1 ± 0.3
20.126	(66.5 ± 1.1)[e]					
40.162	*76.7 ± 0.5*[f]	70.1 ± 1.5	63.7 ± 1.0	59.6 ± 0.5		54.8 ± 0.8
60.121	94.1 ± 1.9					
80.075	115.6 ± 1.5	103.4 ± 2.1	98.5 ± 0.9	89.7 ± 1.1	84.7 ± 0.2	
90.104	128.3 ± 3.1	117.3 ± 1.7	109.5 ± 1.8	99.0 ± 1.2		
			Mild Steel Inside Uranium Shell [a,b]			
40.162	74.4 ± 2.6	67.2 ± 0.4	61.8 ± 0.1	56.9 ± 0.2		52.8 ± 0.4
66.792	*92.3 ± 1.3*[f]					
80.075	103.1 ± 1.8	87.8 ± 2.9	85.1 ± 0.9	76.9 ± 0.2	72.5 ± 0.6	
90.104	114.8 ± 2.6	100.4 ± 1.2	92.5 ± 0.9	83.4 ± 1.1		

a Refer to Table V for precise dimensions
b Except for the center 20-mm radius cavity to accommodate an external neutron source
c Repeated measurement on heavyweight table yielded 68.3 ± 1.6 kg
d Repeated measurement on heavyweight table yielded 68.7 ± 1.8 kg
e Experiment mass enclosed in parentheses was performed with a 50-mm-thick plastic table top on top of the heavyweight table; a repeated measurement on the same configuration yielded 69.6 ± 3.5 kg.
f Four masses shown in italics are the average of one fewer methods than available because that one simply appeared to be a significant outlier relative to the others

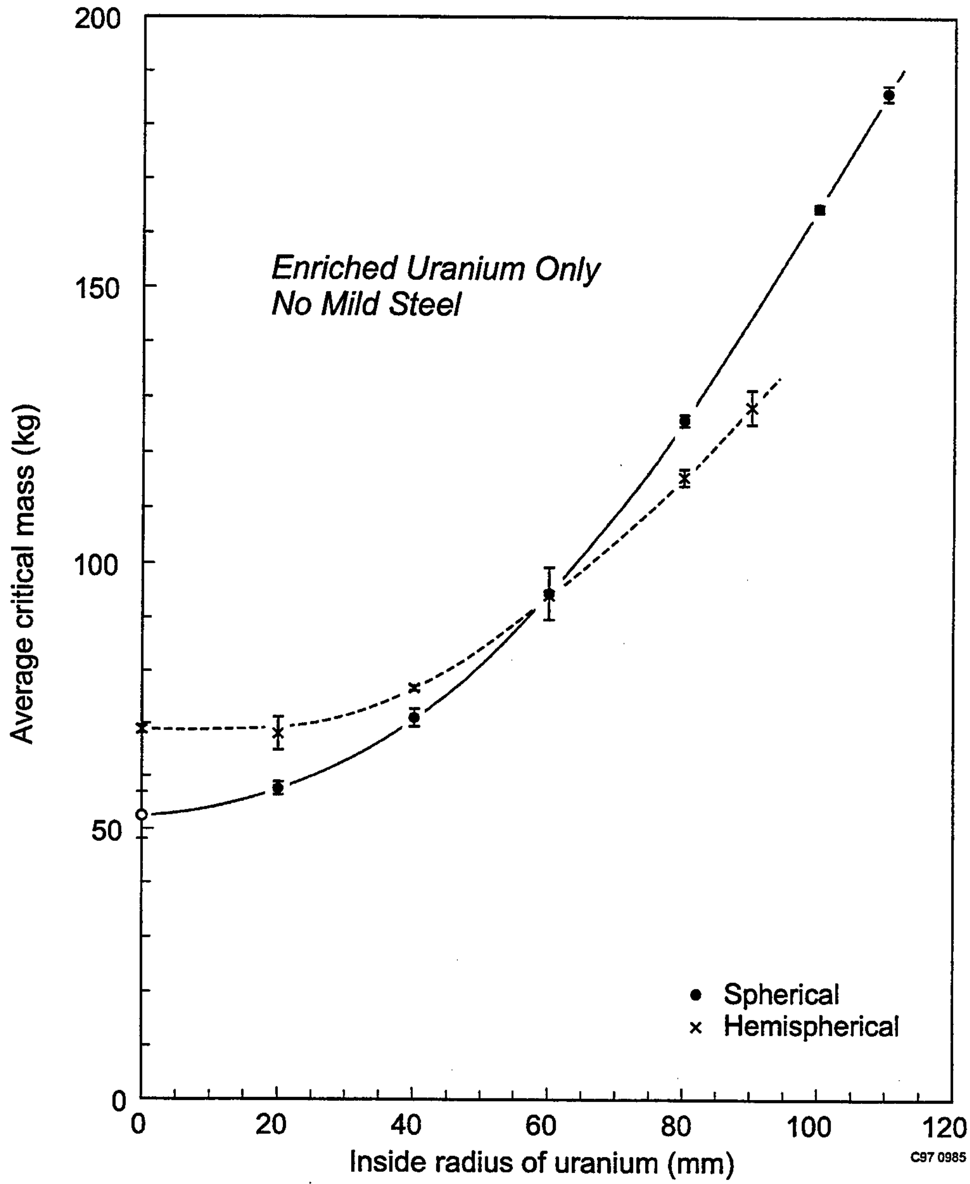

Figure 16. The critical mass increases with any departure from a solid spherical geometry. Thick-walled spherical shells are represented by dots; hemispherical ones, by x's. Error bars are not statistical and are explained in the text. The Godiva mass appears as an open circle.

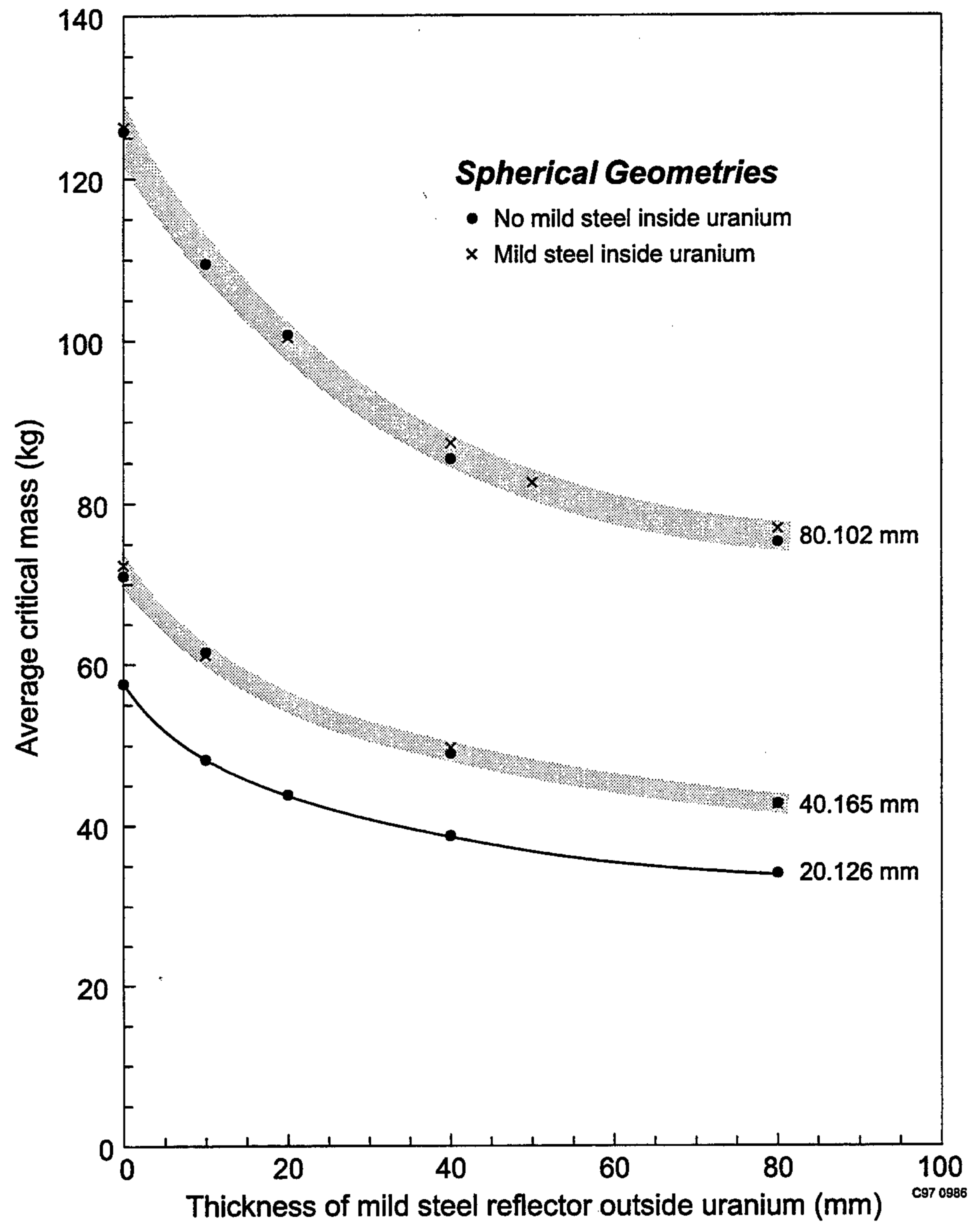

Figure 17. Steel outside uranium spherical assemblies decreased the critical mass because the metal acted simply as a neutron reflector. The influence of steel inside spherical shells (x's) was small. Inside radii of the uranium are shown to the right of each shaded set.

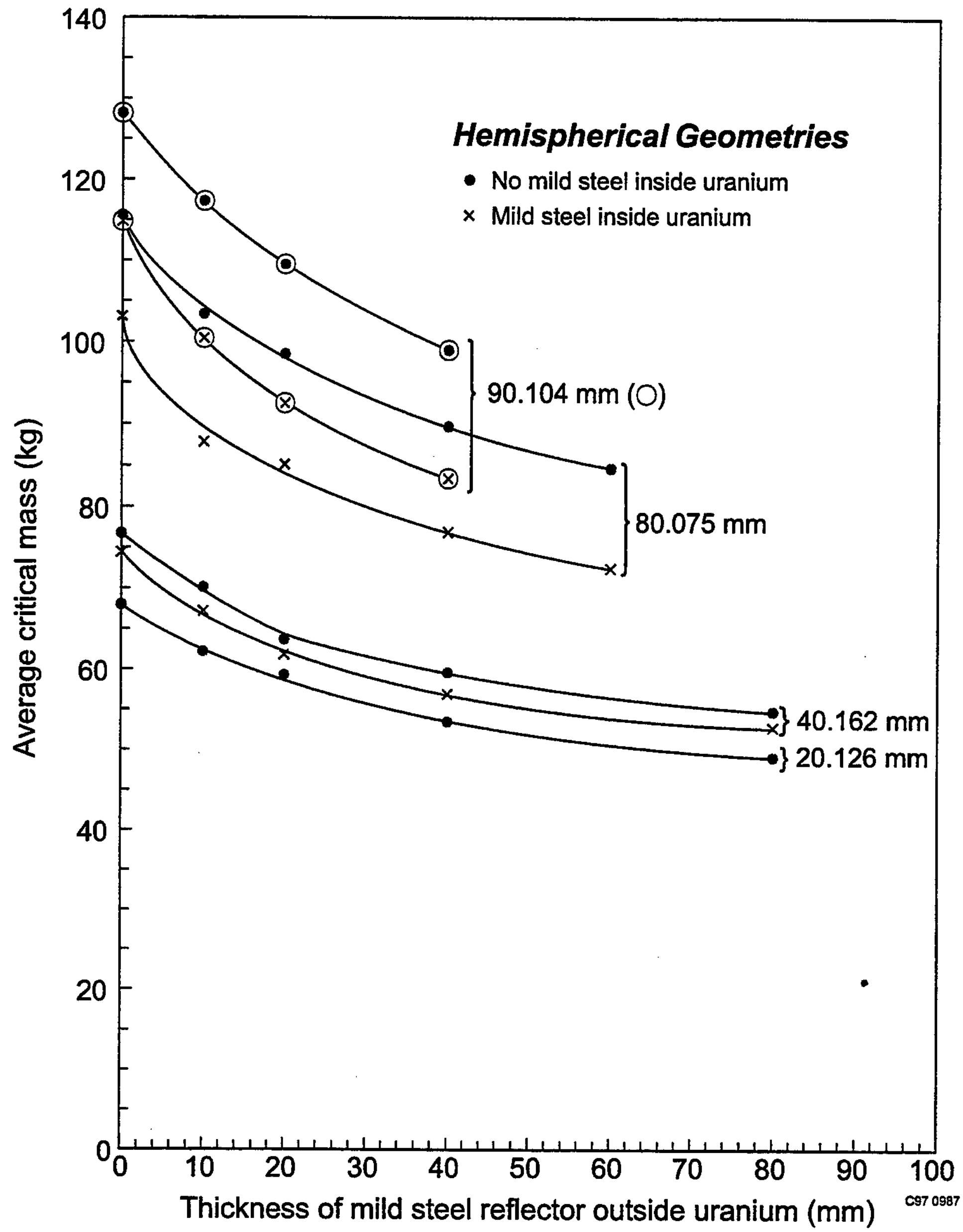

Figure 18. Steel outside the uranium again decreased critical mass for hemispherical assemblies; but the influence of steel inside the central cavity was quite pronounced. Inside radii of the uranium are shown to the right of each bracketed set.

although no adjustment has been made for small differences in density[32]. Any departure from both solid and spherical geometry is expected, based on simple physical arguments, to increase critical mass; and both curves exhibit that feature. *Hollow* spheres have a greater critical mass than a solid sphere; and a solid *hemi*sphere also has a greater critical mass than a solid sphere. The hemispherical point at 20.126 mm does not fit the free-hand curve well; but it also has a large uncertainty.

Right hand limits to both curves, where the inside radius of shell assemblies become infinite, warrant discussion. The hemispherical case approaches the mass of an infinitely large slab of enriched uranium metal of some thickness. That thickness would be simply the critical thickness of a slab of unreflected enriched uranium metal; and its mass would, of course, be infinite. The spherical case, on the other hand, approaches **twice** that mass because it represents the critical condition for **two** large slabs of the same uranium separated infinitely far apart; and both slabs would have the *same thickness*. Another representation of this same concept would have been to graph the radial thickness of the thick-walled shell as a function of the inner radius. Then, both curves would have approached the same thickness; but the spherical case would simply have had another similar region infinitely far away.

The figure shows that the critical mass of both geometries is the same at about IR = 60 mm. To the left, spherical geometries are more reactive because all portions of the spherical shell interact with one another. To the right, both hemishells of a full spherical shell are becoming increasingly de-coupled from one another. In the limit of a very large inside radius sphere, both hemishells essentially do not interact with one another at all.

The effect of laminating mild steel to the outside of a uranium sphere is presented in Figure 17. This steel clearly serves as a neutron reflector because critical masses decrease as this region increases in thickness. Curiously, the presence or absence of mild steel inside the uranium makes little difference. Both kinds of data are shown by the symbols (•) and (x) in the figure for two inner radii. In most cases, the two fell well within one another's error bars. If any trend could be noted, steel-centered cases appeared to exhibit an increased

[32] The popular course "Nuclear Criticality Safety: 5-Day Training Course", **LA-12388-M**, John Schlesser (Ed), offered by the Los Alamos National Laboratory teaches that an increase in critical mass is inversely proportional to the square of the density ratio.

critical mass over its steel-free counterpart. Apparently, inner steel serves *less* as a reflector or a moderator and more as an absorber of neutrons. Only the curve without interior steel is presented for IR = 20.126 mm.

Figure 18 presents data similar to the previous one but for hemispherical geometries. The influence of steel outside hemispheres is somewhat smaller than for comparable spheres. In one comparison, the mass of the hemispherical case decreased about 30% as the 80 mm of steel was added while an equal-sized sphere decreased about 40%. This is expected because spheres were reflected with much more steel. (The plane surface of hemispheres was not reflected at all.) The influence of steel inside the hemispherical cavity, however, was much greater than for spherical cases. Critical masses were significantly smaller with the center cavity filled. This also agrees with intuition because an entire new surface (the hemispherical depression in the equatorial plane) is now being reflected. For larger hemispherical shells, this about doubles the amount of reflected surface area. As expected, the magnitude of the difference between mild steel centers or none became smaller at smaller inner radii simply because less steel was involved. In the limiting case, on the other hand, of hemispherical shells of very large inner radii, both steel regions would serve as reflectors to an almost slab-like region of uranium.

UNCERTAINTIES

The most important uncertainty associated with this entire study is, without question, the extrapolation of subcritical data to criticality. Extrapolations were long - very long. A random survey revealed that the extrapolated critical *mass* was often 30% or more greater than the largest assembly built. Extrapolated *outer radii* were a little better, usually between 10% and 15%. The *number* of hemishells was even better yet; extrapolations were only about 10% or less greater in most cases.

Two kinds of extrapolation uncertainties must be recognized; and the attempt to evaluate each is discussed separately. First, a *systematic* uncertainty relates to the general *shape* of reciprocal multiplication curves in the extrapolation region. Second, a *random* uncertainty is superimposed on top of this and relates to statistical and other experimental parameters associated with measurement error.

This author made a consistent assumption as to the general shape of reciprocal multiplication curves. That assumption could be completely wrong in which case all critical values would be systematically wrong by about the same amount and in the same direction; but his assumption was at least systematic. In the extrapolation region, assumptions were: (1) curves should not increase curvature beyond that exhibited in the data region, and (2) curves should not reverse curvature. These assumptions were <u>not</u> based on any clear physical principals; instead, they were based on a fear of an artificial introduction of physics that did not actually exist.

Any other systematic assumption could have produced a little more or somewhat less curvature. A closer experimental approach to criticality would have resolved the question; but further experiments are no longer possible. The evaluator is limited to existing data. No physical argument can be conceived that would ever suggest reciprocal multiplication curves ought to reverse curvature in the final approach toward criticality. No other experimental program at Rocky Flats, spanning more than three decades, ever featured reciprocal multiplication curves exhibiting a tendency[33] to reverse curvature.

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[33] The sole exception to this generalization would be the plutonium array studies where water was added past layers of the fissile metal. There, such a complicated sequence of physical factors influenced neutron movement that wild and unruly reciprocal multiplication curves were common. Even then, these factors were understood and anticipated; so safety was never compromised. Even in those plutonium studies, a point was reached - still well away from criticality - where curvature no longer changed in nature; and extrapolations based on still-well-subcritical data would have predicted the same critical parameter as was actually achieved later in the real experiment.

Whatever error may have been introduced through any false assumption about the shape of the extrapolation does not destroy the merit of the program. This study featured a family of closely-allied experiments with only slowly-changing systematic variations in parameters. The same general shape for all curves can be expected for such a family. Spherical and hemispherical geometries should feature similar families of curves; and even these two families ought to display remarkable similarity. This fact can be used to good advantage.

Even if the *wrong* general shape had been assumed, that same systematic error ought to exist in all parameters. Thus, if an evaluator found, say, **all** cases systematically low or high in predicting criticality by about the same amount, then all 69 cases probably contained similar extrapolation error. Whatever adjustments might improve agreement in one case should foster the same improvement agreement in all.

One simplistic attempt was made to estimate the systematic error introduced by this author's personal bias in data analysis. Four cases were selected at random out of the entire field of 69. For each, the last four or five reciprocal multiplication values were plotted as a function of outside radius. The size of the dots was about the size of the uncertainty in C_o/C. No curves were drawn connecting the plotted points. Admittedly, these four graphs still contained this author's bias in plotting points; but this was presumed small. About two dozen copies of this set of four pages of graph paper were distributed to a wide variety of people. They were asked to independently extrapolate all four sets of data and record *their* estimate of the critical outside radius.

These people were not all scientists; the only requirement was that they be intelligent and responsible persons who understood the notion of extrapolation. They included the author's family, friends, and colleagues from Rocky Flats. Results were surprising and revealed much about how people interpret such data. Some tended to force fit a linear curve or one closer to linear than the data seemed to indicate, recalling that "half the points should be above my line, half below, with the sum of distances to the line about equal." This category of grapher missed the true curvature that was quite obvious to others. A second group of people did not seem to recall the author's advice to "put a little more confidence into higher multiplications" and to "try not to introduce increased curvature in the extrapolation range". A third observation is that drawing only four graphs is different than all 69; and the difference is important. The similarity in curvature for all 69 curves has been mentioned often and should influence the drawing of each one. Even though specific data for other cases do not enter into a particular extrapolation, the *tendency* should contribute to confidence in extrapolating each case. Unfortunately, these several other graphers did not have that background "feel"; and their extrapolations are different because of it.

The four cases selected for independent extrapolation came from Tables A-3, D-5, G-13, and H-1. Results of this simple study of the critical outer radius are listed below based on 11 different evaluations:

case	average	standard deviation	appendix
A-3	102.5	± 1.6	99.8
D-5	129.4	± 0.5	129.4
G-13	157.0	± 1.1	156.6
H-1	126.3	± 1.2	126.9

The second case (D-5) agrees quite well with this author's prior and independent "smooth curve" determination. The other two methods of estimating this parameter, the quadratic equation from the method of least squares and the use of only the last two data points, also fall well within the small standard deviation of other people's results. The last case (H-1) also shows good agreement, although the standard deviation was quite large. The other two methods also fall within a standard deviation. Case G-13 shows fair agreement with the author's effort and the other two methods; but, once again, the standard deviation is large. The author's estimate for case A-3 fell below anyone else's, suggesting his error. The method of least squares and its fit to a quadratic equation fell well within the standard deviation of other people's estimates.

In summary, then, this ancillary study involving a somewhat random selection of people to evaluate only a few critical outer radii is probably open to considerable question. If any conclusion were to be drawn, the accuracy in critical radii contained in the Appendix might be considered to be ± 0.5 to 1.5 mm.

All this discussion about possible uncertainties in the techniques of extrapolation to critical parameters presented in this paper prompts this author to offer the following specific recommendation. It is suggested to whichever code validator is assigned this document for inclusion in the notebooks of the *International Criticality Safety Benchmark Evaluation Project*.

1) Calculate all cases exactly as quoted here in Tables VI and VII.

2a) If criticality is generally predicted ($k_{eff} = 1.0$), stop.

2b) If criticality is generally either high or low by about the same amount, select a few cases (say, 4 to 6) for the following steps.

3) Adjust the published "critical" parameters until $k_{eff} = 1.0$ is achieved on the average.

4) Re-draw those 4 to 6 reciprocal multiplication curves including the newly-adjusted critical point.

5) Compare these reciprocal multiplication curves with the author's to determine a sense of his personal bias which may have yielded erroneous values.

6) Apply this new knowledge of how these curves *should have been drawn* to all 69 cases. This will yield a new set of critical parameters.

7) Compare this new set with new calculations as in step #1.

This iterative approach is altogether legitimate. The author's personal bias was completely subjective and arbitrary. Any analyst might well have obtained a different set (either better or worse) of critical parameters from the subcritical data provided. This recognition, in the first place, is what prompted publication of the subcritical data in all 69 tables in the Appendix. Stated differently, this paper **confidently** presents a very large set of closely allied subcritical data. It then **tenuously** offers one person's interpretation of these data to criticality by his preferred extrapolation procedure.

Figures 16, 17, and 18 support arguments of slow and systematic variation of parameters related to critical masses. The overall self-consistency of the entire field of data is remarkable, even though slightly incorrect assumptions may have been used in extrapolating. All expected variations, based on intuitive physics principals, of one parameter relative to another are, indeed, found in these three sets of curves.

Random uncertainties associated with this study are probably mostly statistical in nature. For any critical approach, the external neutron source, neutron detectors, and environmental reflectors were maintained reliably constant. Only fissile material was interposed between source and counters. Typical neutron fluxes at the start of an approach (C_o) were between 10,000 and 30,000 counts. The statistical uncertainty in 20,000 counts is about 140 neutrons - about 0.7%. For an assembly with a multiplication of ten, the statistical uncertainty for 200,000 neutrons would be about 0.2%. Combining the two, the statistical uncertainty in reciprocal multiplications is about 0.75%. Including other unspecified sources of statistical error, reciprocal multiplications are probably correct to about $\pm$ 1%.

Masses and dimensions of fissile metal and mild steel components are so well known that they should contribute no significant uncertainty. The amount of residual grease or oil on these surfaces is expected to contribute more; but even this would be very small. The presence of unrecalled environmental reflectors is another possible source of error. Documentation was weaker in the 1960s than later decades; still, mental images of the experimental apparatus do not suggest any important omissions. Even human beings involved in experiments made a conscious effort to step a distance away from the assembly while neutrons were being counted. This avoided neutron reflection and/or moderation by human bodies.

In summary, critical masses quoted in this paper are probably accurate to about $\pm$ 1 or 2%. Within that, systematic uncertainty (accuracy) is probably larger than random uncertainty (precision). Uncertainties of both kinds associated with radial parameters are smaller than those related to mass.

Lack of laboratory analyses for non-fissile materials is expected to be relatively unimportant. Evaluators may assume nominal compositions for the various metals used without much error. Amounts of these materials were small; and they were usually quite far away from the construction.

Where measured parameters are reported, an uncertainty in the last significant figure given may be assumed. This applies to room dimensions, table and mount dimensions, and similar features. Even the reciprocal multiplication values at each stage of construction is expressed to only three significant figures, reflecting a one-percent uncertainty in the parameter.

This paper is written using metric dimensions throughout, consistent with modern technical reporting. All apparatus, however, was built in the United States using materials commonplace there. These are manufactured in dimensions such as inches. For example, metal sheets and plates, rods, and structural shapes used to form tables, mounts, screws, and a host of other experimental components would be built using American dimensions. For example, nominal dimensions of one-quarter inch, 0.03125 inch, and so on would be common. The conversion of these units into metric units can increase uncertainty.

For example, a metal plate used to fabricate some component may have had a nominal thickness of, say, one-quarter inch. Considering tolerances normally allowed in such US manufacture, this thickness should be given as 0.25**0** inches. Neither 0.25 inches nor 0.2500 inches properly describes the material because some uncertainty is understood to exist in the last significant figure. The importance of this discussion lies in the number of significant figures used in the conversion to metric units. This stock **should** be reported as 6.35 mm thick - neither 6.4 mm nor 6.350 mm thick. Sometimes, the thickness 6.4 mm is incorrectly reported. Unfortunately, the correct precision was not always reported in this paper or the preceding ones in the benchmark study. This is the author's oversight.

Consequences of this conversion error apply to this paper as well as all previous four papers written under the International Criticality Safety Benchmark Evaluation Project. For any of the five papers, whenever written text states or common sense suggests that a component was made using materials manufactured in the United States, these materials may be assumed to have dimensions appropriate to such American manufacture. Thus, an evaluator is at liberty to determine the closest fractional inch dimension associated with a published metric dimension, assume that was the actual thickness used, and convert this American dimension back to metric units to any number of significant figures desired.

This error is heightened whenever multiple layers of the same material might be encountered. For example, if a number of one-quarter-inch-thick plates were stacked in an experiment, the accumulated error of using 6.4 mm instead of 6.35 mm would most certainly be significant. This was the case in one previous report within this benchmark series; but this compounding of errors does not appear to affect the present document.

Obviously, some common sense is called for. A thin sheet metal shim may have been composed of stock sold as "1/32 inch" thick. Conversion of this dimension to 0.79375 mm would be wrong. That is far too precise. A rounded off 0.7938 (or even 0.794) would better represent the true expected thickness.

Plastic stock such as poly methylmethracrylate also warrants separate consideration. These sheets are usually cast by the manufacturer and rarely exhibit the nominal dimension. Most such stock is up to 1% too thin. In this paper, the 50-mm-thick plastic plate that covered one of the tables in some cases is probably better represented by the stated thickness, not 50.8 mm as suggested by the above discussion.

ANOMALOUS NEUTRON FLUX

A situation occurred during this program that has never been satisfactorily understood to this day. That missing explanation has concerned this author ever since. Its possible consequences precipitated his fervent dedication to greatly enhanced safety in subsequent experiments. Was the phenomenon simply a manifestation of well-understood physics principals (neutron moderation); or was it some other phenomenon?

Details of the incident are vague. A hemisphere was being assembled to best recollection; and that construction followed procedures outlined elsewhere. The pole hole is recalled to have been down placing the equatorial plane and the neutron source cavity at the top. The inside radius of this cavity was small: possibly 20, 40, or 60 mm. No steel is recalled to have been involved, at least inside the assembly. The last reciprocal multiplication data point had been counted; and the multiplication is recalled to have been near ten.

Based on these recollections, the assembly in question may have been the heaviest one built from Tables E-2, E-3, or E-4.

The experiment finished, the configuration was ready for disassembly. The external neutron source was removed simply by picking it up with two or three fingers and the thumb of a gloved hand to be stored to prevent unnecessary radiation exposure. During this source movement, however, the neutron flux suddenly increased about a factor of three the instant fingers and thumb entered the source-holding cavity.

Experiments were equipped with an *audible* indication of the instantaneous neutron flux in addition to visual readout. When fingers touched the source at the bottom of the hemispherical depression, the frequency of the audible clicks indicating the instantaneous multiplication about tripled. This increase in flux was confirmed visually, noting the two reciprocal multiplication meters decrease to about 0.03 at that instant.

The neutron flux immediately returned to normal levels as the source was withdrawn and left the vicinity. Whether or not this situation was recorded on a strip-chart recorder is not recalled. Even the presence of such a device for these early experiments is not at all recalled, although later experiments (1970s and later) would certainly have had this capability.

One possible explanation of this phenomenon is that the soft tissue of the fingers and thumb only moderated neutrons into an energy range more suitable to the boron trifluoride neutron detectors. The increased count rate would be due to the greater efficiency in detecting slower neutrons rather than an actual increase in the number of neutrons. Not much else was available to slow neutrons down; so these experiments lived on quite high-energy neutrons. This process would only correspond to an increase in detector efficiency.

Interested readers are encouraged to calculate the above situation for candidate configurations to determine the neutronics actually taking place. The hydrogenous composition of fingers and thumb would have to be approximated.

ACKNOWLEDGMENTS

This paper is written under the auspices of the *International Criticality Safety Benchmark Evaluation Project*, funded by the United States Department of Energy. The project is administered by J. Blair Briggs of Lockheed Martin Idaho Technologies Company. This author is grateful to Mr. Briggs for the opportunity to publish these data in the peer-reviewed literature before the author's full retirement. This is the fifth of six papers presenting previously unpublished or inadequately documented results from criticality experiments performed at the Rocky Flats Critical Mass Laboratory. These six documents will mark a satisfying closure to this author's long career in the arena of critical experiments.

Special recognition is due Mr. Grover Tuck (deceased) for his design of the enriched uranium metal hemishells. His careful attention to sizes and machining tolerances has resulted in a nearly ideal set of experimental fissile components. As proof of this, the entire set of 80 shells has been send to Los Alamos National Laboratory for their continued use in nuclear criticality safety experiments.

Several colleagues participated in one or more experiment. Not all were "Certified Experimenters"; that designation had not yet been introduced by the mid-1960s. Any individual who understood the physics involved was both welcome and invited to assist. Records indicate that persons involved in at least one experiment included: Harold E. Clark, Bruce B. Ernst, Lynn A. FitzRandolph, Douglas C. Hunt, Howard W. King, and Grover Tuck. In a few cases, they performed experiments under the author's direction but without his presence.

All were involved in the field of nuclear safety at Rocky Flats. Hunt and Tuck later went on to become "Certified Senior Experimenters" (equivalent to Reactor *Supervisors*). Clark and Ernst became "Certified Experimenters" (equivalent to Reactor *Operators*). The other two did not continue in experimental aspects of the nuclear industry. Sadly, both Douglas Hunt and Grover Tuck, friends as well as colleagues, have died - Hunt in a mountain climbing accident and Tuck of natural causes.

The patient and capable efforts of Christine White and Peggy Shiffer are, as in past papers under this program, gratefully recognized. They prepared figures and tables, respectively. This was rendered more difficult because of separation; they work in Idaho, the author, Colorado.

White and Shiffer deserve special recognition for efforts on the many tables in the Appendix. Chris White performed the many tedious Least Squares fits and even exercised sound judgement in cases for which the quadratic form assumed was a poor choice. Peggy Shiffer showed remarkable patience with this author as she prepared many dozens of tables containing entries with as many as six significant figures.

A very careful and detailed review of this paper was performed by Virginia Dean; and many hours of tedious checking the text, tables, and figures are greatly appreciated. She was able to correct many of the author's errors and oversights before the final printing.

APPENDIX

Sixty-nine tables of experimental results are presented in an Appendix rather than the body of the paper to avoid a complete interruption of the flow of written text. The first 29 tables pertain to spherical geometries; the remaining 40, hemispherical. The tables were discussed thoroughly in the body of this paper.

Each table contains either four or five segments. The top section contains nominal information as to the thickness of mild steel regions inside and outside the uranium. When no steel was used, this section is absent. One section found in all tables contains the raw and untreated subcritical data accrued during the 1960s. The next section always contains extrapolated delayed critical information. Its importance is emphasized through bold-faced font. Critical masses are expressed in kilograms; and this is shown by italics. The next-to-last section contains conjectured prompt critical information, expressed as an incremental addition to delayed criticality. Reduced certainty in this information is represented through use of a smaller font. The final section of each table contains three details related to apparatus described in detail in the text: (1) whether the lightweight or heavyweight table was used, (2) the orientation of the pole hole of a hemispherical assembly (not relevant in spherical cases), and (3) the nature of the mount used to support the fissile load.

Most entries in these 69 tables are derived from calculated parameters containing more significant figures that those shown. These were "rounded off" to 3 or 4 significant figures as presented in the delayed and prompt critical sections of the tables; and that number of figures is reasonable for the assumed uncertainty in these critical parameters. Whenever that rounding procedure yielded a "5" but the un-rounded parameter fell *below* 5, that rounded off "5" is underlined: $\underline{5}$. The consequence of this common approach is that any further rounding should merely drop a $\underline{5}$.

Tables A through D, 29 in all, refer to spherical geometries. Five Tables A involved no mild steel. Four Tables B had a thick plastic slab covering the experimental table. Eleven Tables C had no steel inside the spherical shell but did have steel outside. The last nine Tables D did have steel inside the spherical shell whether or not steel was found outside the uranium.

Tables E through H, 40 in all, refer to hemispherical cases. Six Tables E involve no mild steel. Four Tables F represent a small variety in apparatus details. Fifteen Tables G had no steel inside the hemispherical shell but did have steel outside. Finally, the last fifteen Tables H did have steel inside the hemispherical shell whether or not steel was found outside the uranium.

These 69 tables contain an enormous amount of information which had to be entered into computer format from handwritten drafts of the tables. The task was rendered even more difficult because many entries contained 5 or 6 significant figures. Use of normal, bold, italic, and smaller-sized fonts was employed to communicate important aspects of the data. Some entries are enclosed in parentheses, others in braces, and still others in asterisks; and all this nomenclature has meaning. Proofreading these 69 tables proved difficult; and errors may have escaped detection. To maximize accuracy in this transcription of this most important data of the entire paper, the hand-done drafts of these 69 tables will be stored to reduce information from this document into the desired format at the Los Alamos National Laboratory Archives so that they are available for evaluators.

A Conjecture

Theoretical aspects of the mathematical relationship between parameters may be pushed even further thanks to the almost linear relationship between radial thickness and reciprocal multiplication (or count rate ratio). The radial thickness of a **prompt critical** configuration may, possibly, also be calculated! This notion is not rigorous; at best, it is only conjecture. The merit to this paper lies in its tabulation of delayed critical parameters. If the following discussion proves fruitful, so much the better. If not, it should be dismissed with no decrease in the paper's merit.

For enriched uranium, the difference in reactivity between delayed and prompt criticality, often referred to as "one dollar", is 0.00665[34]. That is, prompt criticality corresponds to:

$$k = 1 + \beta = 1 + 0.00665 = 1.00665.$$

[34] This constant, also denoted β, does vary a little with the specific system; but 0.00665 will be used in this discussion.

The neutron reproduction factor is greater than unity for systems in excess of delayed criticality. Ignoring the restriction to subcritical systems for the earlier relationship between k and the multiplication, M:

$$k = 1 - (1/M),$$

the two relations may be combined at prompt criticality:

$$k = 1 + \beta = 1 - (1/M) = 1 - C_o/C(R).$$

Stated differently, prompt criticality would occur when

$$C_o/C(R) = -\beta = -0.00665.$$

A linear continuation of these very-close-to-linear reciprocal multiplication curves into the *physically unreal* domain of *negative count rates* **may** predict prompt criticality. Figure 2 illustrates this conjecture for the linear radial curve of the first figure. The extrapolated data point at criticality (101.7 mm) and the actual data point at a multiplication of ten (89.0 mm) are extended linearly into the realm of negative count rates. The conjecture is that this extension *may* predict that prompt criticality. If so, it would occur at:

$$R_c(\text{prompt}) = 102.53 \text{ mm!}$$

This conjecture may be described as being "just a little wrong" or "just a little outside the realm of physically possible"! This concept shares some similarity with the mathematical notion of *Analytic Continuation* in complex variables.

The hope is that some Monte Carlo code user will calculate neutron reproduction factors for the conjectured prompt critical configurations presented in the Appendix. If these compare well with k = 1.00665, the conjecture deserves further consideration. If not, this conjecture should be discarded.

SPHERICAL GEOMETRIES

Table A-1. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Spherical Shell of Inside Radius 20.126 mm and No Mild Steel in Contact with the Uranium.

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Average Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm³)	C_o/C
	28	66.702	21832	18.059	.201
actual	30	70.028	25388	18.078	.172
subcritical	32	73.317	29288	18.116	.142
reciprocal	(33)	(74.998)	(31422)		(.129)
multiplication	34	76.662	33552	18.106	.114
data	(35)	(78.346)	(35899)		(.102)
	36	80.027	38243	18.102	.0883
Delayed Critical Extrapolation					
RFP 1021			*57.5*		
smooth curve	42.9	**93.9**	***58.0***	{18.10}	0.0000
least squares		**92.1**	***58.7***		
$R = (96.35)(C_o/C)^2 - (145.67)(C_o/C) + (92.13)$					
last two points		**90.9**	***56.3***		
Prompt Critical Increment (one dollar)					
least squares		0.97	*1.88*	{18.10}	-0.00665
last two points		0.82	*1.53*		
Apparatus Details					
table: heavy				mount:	Figure 9

Table A-2. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Spherical Shell of Inside Radius 40.165 mm and No Mild Steel in Contact with the Uranium.

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Average Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	22	76.662	29336	18.155	.249
subcritical	24	80.027	34027	18.144	.206
reciprocal	26	83.328	39065	18.151	.171
multiplication	28	86.682	44528	18.125	.134
data	30	89.996	50426	18.127	.100
Delayed Critical Extrapolation					
RFP-1021			*71.0*	{18.13}	0.0000
smooth curve	36.0	**99.8**	***69.6***		
least squares		**100.8**	***72.9***		
$R = (73.06)\,(C_o/C)^2 - (115.41)\,(C_o/C) + (100.83)$					
last two points		**99.7**	***70.4***		
Prompt Critical Increment (one dollar)					
least squares		0.77	*1.78*	{18.13}	-0.00665
last two points		0.65	*1.47*		
Apparatus Details					
table: heavy				mount: Figure 9	

Table A-3. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Spherical Shell of Inside Radius 40.165 mm and No Mild Steel in Contact with the Uranium.

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Average Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	22	76.662	29336	18.155	.291
subcritical	24	80.027	34027	18.144	.207
reciprocal	26	83.328	39065	18.151	.170
multiplication	28	86.682	44528	18.125	.133
data	30	89.996	50426	18.127	.100
Delayed Critical Extrapolation					
RFP-1021			*71.0*	{18.13}	0.0000
smooth curve	36.0	**99.8**	***69.6***		
least squares		**100.8**	***72.9***		
$R = (75.31)\ (C_o/C)^2 - (115.99)\ (C_o/C) + (100.82)$					
last two points		**100.0**	***71.1***		
Prompt Critical Increment (one dollar)					
least squares		0.77	*1.79*	{18.13}	-0.00665
last two points		0.67	*1.52*		
Apparatus Details					
table: ?				mount: Figure 9 (smallest)	

Table A-4. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Spherical Shell of Inside Radius 80.102 mm and No Mild Steel in Contact with the Uranium.

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Average Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_0/C
actual	18	110.011	62142	18.149	.279
subcritical	20	113.332	71604	18.153	.226
reciprocal	22	116.665	81632	18.147	.176
multiplication	24	120.001	92281	18.146	.127
data	26	123.361	103591	18.140	.081
Delayed Critical Extrapolation					
RFP-1021			***125.0***	{18.14}	0.0000
smooth curve	29.6	**129.1**	***127.0***		
least squares		**129.5**	***126.1***		
$R = (31.78)\ (C_0/C)^2 - (78.807)\ (C_0/C) + (129.53)$					
last two points		**129.3**	***125.1***		
Prompt Critical Increment (one dollar)					
least squares		0.53	*2.01*	{18.14}	-0.00665
last two points		0.49	*1.85*		
Apparatus Details					
table: heavy ?				mount: Figure 9	

Table A-5. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Spherical Shell of Inside Radius 110.113 mm and No Mild Steel in Contact with the Uranium.

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Average Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	14	133.317	78637	18.149	.371
subcritical	18	140.012	107098	18.138	.246
reciprocal	20	143.320	122330	18.153	.190
multiplication	22	146.657	138266	18.144	.137
data	(23)	(148.355)	(146681)		(.111)
Delayed Critical Extrapolation					
RFP-1021			***185.6***	{18.14}	0.0000
smooth curve	27.4	**156.0**	***186.0***		
least squares		**156.1**	***187.5***		
$R = (32.03)\ (C_o/C)^2 - (73.268)\ (C_o/C) + (156.09)$					
last two points		**155.6**	***184.8***		
Prompt Critical Increment (one dollar)					
least squares		0.49	*2.71*	{18.14}	-0.00665
last two points		0.43	*2.40*		
Apparatus Details					
table: light ?				mount: Figure 9	

Table B-1. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Spherical Shell of Inside Radius 20.126 mm and No Mild Steel in Contact with the Uranium but Experiments Had A 50-mm-Thick Plastic Tabletop.

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Average Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	18	50.042	8834	18.000	.344
subcritical	22	56.693	13149	18.034	.272
reciprocal	26	63.345	18608	18.056	.208
multiplication	28	66.702	21832	18.059	.179
data	30	70.028	25388	18.079	.151
Delayed Critical Extrapolation					
RFP-1021					
smooth curve	40.8	**88.9**	***47.0***	{18.09}	0.0000
least squares		**90.6**	***55.8***		
$R = (95.66)\ (C_o/C)^2 - (150.85)\ (C_o/C) + (90.62)$					
last two points		**88.0**	***51.0***		
Prompt Critical Increment (one dollar)					
least squares		1.01	*1.88*	{18.09}	-0.00665
last two points		0.79	*1.39*		
Apparatus Details					
table: heavy				mount: Figure 9	

Table B-2. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Spherical Shell of Inside Radius 60.117 mm and No Mild Steel in Contact with the Uranium but Experiments Had A 50-mm-Thick Plastic Tabletop.

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Average Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	18	89.996	38919	18.160	.308
subcritical	22	96.675	52178*	18.151	.210
reciprocal	24	100.000	59492	18.145	.166
multiplication	26	103.338	67322	18.135	.122
data	(27)	(105.014)	(71530)		(.104)
Delayed Critical Extrapolation					
RFP-1021					
smooth curve	40.7	**112.7**	***88.6***	{18.14}	0.0000
least squares		**114.1**	***96.4***		
$R = (48.97)\ (C_o/C)^2 - (93.389)\ (C_o/C) + (114.12)$					
last two points		**114.7**	***98.1***		
Prompt Critical Increment (one dollar)					
least squares		0.62	*1.85*	{18.14}	-0.00665
last two points		0.62	*1.86*		
Apparatus Details					
table: heavy				mount: Figure 9	

* The author's initial version of this table specified 51818 grams, but this may have been in error. The density was originally listed as 18.026.

Table B-3. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Spherical Shell of Inside Radius 80.102 mm and No Mild Steel in Contact with the Uranium but Experiments Had A 50-mm-Thick Plastic Tabletop.

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Average Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	16	106.691	53217	18.137	.400
subcritical	20	113.332	71604	18.153	.277
reciprocal	22	116.665	81632	18.147	.218
multiplication	24	120.001	92281	18.146	.162
data	(25)	(121.681)	(97941)		(.133)
	26	123.361	103591	18.140	.105
Delayed Critical Extrapolation					
RFP-1021					
smooth curve	29.8	**129.5**	***125.1***	{18.14}	0.0000
least squares		**129.9**	***127.6***		
$R = (14.52)\,(C_o/C)^2 - (63.910)\,(C_o/C) + (129.93)$					
last two points		**129.7**	***126.6***		
Prompt Critical Increment (one dollar)					
least squares		0.43	*1.64*	{18.14}	-0.00665
last two points		0.40	*1.53*		
Apparatus Details					
table: heavy ?				mount: Figure 9	

Table B-4. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Spherical Shell of Inside Radius 100.112 mm and No Mild Steel in Contact with the Uranium but Experiments Had A 50-mm-Thick Plastic Tabletop.

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Average Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	16	126.499	77623	18.152	.365
subcritical	18	130.013	90610	18.112	.302
reciprocal	20	133.317	103807	18.140	.238
multiplication	22	136.699	117762	18.125	.179
data	24	140.012	132268	18.134	.121
	(25)	(141.666)	(139887)		(.092)
Delayed Critical Extrapolation					
RFP-1021				{18.13}	0.0000
smooth curve	28.4	**146.8**	***164.0***		
least squares		**147.1**	***165.4***		
$R = (9.364)\ (C_o/C)^2 - (59.701)\ (C_o/C) + (147.08)$					
last two points		**146.9**	***164.5***		
Prompt Critical Increment (one dollar)					
least squares		0.40	*1.95*	{18.13}	-0.00665
last two points		0.38	*1.86*		
Apparatus Details					
table: heavy ?				mount: Figure 9	

Table C-1. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Spherical Shell of Inside Radius 20.126 mm but With Mild Steel Regions in Contact with the Uranium.

inside - none -	Nominal Mild Steel Thickness Around Uranium (mm)	outside 10.0

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Average Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	20	53.365	10852	18.013	.344
subcritical	24	60.021	15723	18.040	.259
reciprocal	28	66.702	21832	18.059	.189
multiplication	30	70.028	25388	18.078	.156
data	32	73.317	29288	18.116	.123
	(33)	(74.989)	(31418)		(.107)
Delayed Critical Extrapolation					
RFP-1021			***48.7***	{18.11}	0.0000
smooth curve	40.0	**86.0**	***45.7***		
least squares		**87.5**	***50.3***		
$R = (72.00)\,(C_o/C)^2 - (124.24)\,(C_o/C) + (87.53)$					
last two points		**86.2**	***47.9***		
Prompt Critical Increment (one dollar)					
least squares		0.83	*1.45*	{18.11}	-0.00665
last two points		0.69	*1.17*		
Apparatus Details					
table: light ?				mount: Figure 9	

Table C-2. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Spherical Shell of Inside Radius 20.126 mm but With Mild Steel Regions in Contact with the Uranium.

inside	Nominal Mild Steel Thickness Around Uranium (mm)	outside
- none -		20.0

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Average Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	20	53.365	10852	18.013	.337
subcritical	24	60.021	15723	18.040	.256
reciprocal	28	66.702	21832	18.059	.176
multiplication	30	70.028	25388	18.078	.140
data	32	73.317	29288	18.116	.106
Delayed Critical Extrapolation					
RFP-1021			***44.9***	{18.10}	0.0000
smooth curve	38.6	**83.7**	***42.6***		
least squares		**83.7**	***43.9***		
$R = (37.94)\ (C_o/C)^2 - (102.73)\ (C_o/C) + (83.71)$					
last two points		**83.6**	***43.6***		
Prompt Critical Increment (one dollar)					
least squares		0.68	*1.09*	{18.10}	-0.00665
last two points		0.64	*1.02*		
Apparatus Details					
table: light ?				mount: Figure 9	

Table C-3. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Spherical Shell of Inside Radius 20.126 mm but With Mild Steel Regions in Contact with the Uranium.

inside - none -	Nominal Mild Steel Thickness Around Uranium (mm)	outside 40.0

Actual System or Extrapolated Parameters	Enriched Uranium Region: Number of Hemishells	Enriched Uranium Region: Average Outside Radius (mm)	Enriched Uranium Region: Total Mass (g) or *(kg)*	Enriched Uranium Region: Density (mg/mm^3)	Reciprocal Multiplication C_o/C
actual	20	53.365	10852	18.013	.339
subcritical	24	60.021	15723	18.040	.245
reciprocal	26	63.345	18608	18.056	.201
multiplication	28	66.702	21832	18.059	.162
data	30	70.028	25388	18.078	.123
	(31)	(71.672)	(27339)		(.102)
Delayed Critical Extrapolation					
RFP-1021			*39.3*	{18.08}	0.0000
smooth curve	36.4	**80.5**	*37.8*		
least squares		**81.3**	*40.1*		
$R = (46.53)\,(C_o/C)^2 - (98.262)\,(C_o/C) + (81.31)$					
last two points		**79.7**	*37.7*		
Prompt Critical Increment (one dollar)					
least squares		0.66	*0.98*	{18.08}	-0.00665
last two points		0.52	*0.75*		
Apparatus Details					
table: light ?				mount: Figure 9	

Table C-4. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Spherical Shell of Inside Radius 20.126 mm but With Mild Steel Regions in Contact with the Uranium.

inside	Nominal Mild Steel Thickness Around Uranium (mm)	outside
- none -		80.0

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Average Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	20	53.360	10852	18.013	.358
subcritical	24	60.021	15723	18.040	.249
reciprocal	26	63.345	18608	18.056	.197
multiplication	28	66.702	21832	18.059	.146
data	30	70.028	25388	18.078	.099
Delayed Critical Extrapolation					
RFP-1021			*34.3*		
smooth curve	34.3	77.3	*33.3*	{18.08}	0.0000
least squares		77.2	*34.2*		
$R = (23.02)\ (C_o/C)^2 - (74.733)\ (C_o/C) + (77.17)$					
last two points		77.0	*34.0*		
Prompt Critical Increment (one dollar)					
least squares		0.50	*0.67*	{18.08}	-0.00665
last two points		0.47	*0.63*		
Apparatus Details					
table: light ?				mount: Figure 9	

Table C-5. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Spherical Shell of Inside Radius 40.165 mm but With Mild Steel Regions in Contact with the Uranium.

inside - none -		Nominal Mild Steel Thickness Around Uranium (mm)		outside 10.0	
Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Average Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	20	73.317	25072	18.176	.261
subcritical	22	76.662	29336	18.155	.215
reciprocal	24	80.027	34027	18.144	.173
multiplication	26	83.328	39065	18.151	.134
data	28	86.682	44528	18.125	.0972
Delayed Critical Extrapolation					
RFP-1021			***61.3***	{18.14}	0.0000
smooth curve	33.3	**97.1**	***60.0***		
least squares		**96.5**	***63.3***		
$R = (72.60)\,(C_o/C)^2 - (107.61)\,(C_o/C) + (96.45)$					
last two points		**95.5**	***61.3***		
Prompt Critical Increment (one dollar)					
least squares		0.72	*1.52*	{18.14}	-0.00665
last two points		0.61	*1.26*		
Apparatus Details					
table: light ?				mount: Figure 9	

Table C-6. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Spherical Shell of Inside Radius 40.165 mm but With Mild Steel Regions in Contact with the Uranium.

inside - none -	Nominal Mild Steel Thickness Around Uranium (mm)	outside 40.0

Actual System or Extrapolated Parameters	Enriched Uranium Region: Number of Hemishells	Enriched Uranium Region: Average Outside Radius (mm)	Enriched Uranium Region: Total Mass (g) or *(kg)*	Enriched Uranium Region: Density (mg/mm^3)	Reciprocal Multiplication C_o/C
actual	16	66.702	17616	18.129	.344
subcritical	20	73.317	25072	18.176	.227
reciprocal	22	76.662	29336	18.155	.178
multiplication	24	80.027	34027	18.144	.128
data	(25)	(81.678)	(36538)		(.105)
Delayed Critical Extrapolation					
RFP-1021			***48.1***	{18.15}	0.0000
smooth curve	29.2	**88.8**	***47.8***		
least squares		**89.9**	***50.3***		
$R = (45.13)\,(C_o/C)^2 - (83.065)\,(C_o/C) + (89.92)$					
last two points		**89.2**	***49.1***		
Prompt Critical Increment (one dollar)					
least squares		0.55	*1.02*	{18.15}	-0.00665
last two points		0.48	*0.87*		
Apparatus Details					
table: light ?				mount: Figure 9	

Table C-7. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Spherical Shell of Inside Radius 40.165 mm but With Mild Steel Regions in Contact with the Uranium.

inside - none -	Nominal Mild Steel Thickness Around Uranium (mm)	outside 80.0

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Average Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	16	66.702	17616	18.129	.351
subcritical	20	73.317	25072	18.176	.217
reciprocal	22	76.662	29336	18.155	.156
multiplication	24	80.027	34027	18.144	.096
data					
Delayed Critical Extrapolation					
RFP-1021			*42.3*	{18.15}	0.0000
smooth curve	27.1	**85.8**	*42.6*		
least squares		**85.8**	*43.1*		
$R = (22.84)\ (C_o/C)^2 - (62.521)\ (C_o/C) + (85.83)$					
last two points		**85.4**	*42.4*		
Prompt Critical Increment (one dollar)					
least squares		0.42	*0.70*	{18.15}	-0.00665
last two points		0.37	*0.62*		
Apparatus Details					
table: light ?				mount: Figure 9	

Table C-8. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Spherical Shell of Inside Radius 80.102 mm but With Mild Steel Regions in Contact with the Uranium.

inside - none -	Nominal Mild Steel Thickness Around Uranium (mm)	outside 10.0

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Average Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	16	106.691	53217	18.137	.309
subcritical	18	110.011	62142	18.149	.244
reciprocal	20	113.332	71604	18.153	.186
multiplication	22	116.665	81632	18.147	.131
data	24	120.001	92281	18.146	.078
Delayed Critical Extrapolation					
RFP-1021			*110.2*	{18.15}	0.0000
smooth curve	27.0	124.8	*107.8*		
least squares		125.4	*110.7*		
$R = (34.63)\ (C_o/C)^2 - (71.186)\ (C_o/C) + (125.36)$					
last two points		124.9	*109.1*		
Prompt Critical Increment (one dollar)					
least squares		0.47	*1.70*	{18.15}	-0.00665
last two points		0.42	*1.49*		
Apparatus Details					
table: heavy ?				mount: Fig 9	

Table C-9. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Spherical Shell of Inside Radius 80.102 mm but With Mild Steel Regions in Contact with the Uranium.

inside	Nominal Mild Steel Thickness Around Uranium (mm)	outside
- none -		20.0

Actual System or Extrapolated Parameters	Enriched Uranium Region: Number of Hemishells	Enriched Uranium Region: Average Outside Radius (mm)	Enriched Uranium Region: Total Mass (g) or *(kg)*	Enriched Uranium Region: Density (mg/mm^3)	Reciprocal Multiplication C_0/C
actual	12	100.000	36972	18.160	.417
subcritical	18	110.011	62142	18.149	.207
reciprocal	20	113.332	71604	18.153	.148
multiplication	22	116.665	81632	18.147	.091
data					
Delayed Critical Extrapolation					
RFP-1021			***100.8***	{18.15}	0.0000
smooth curve	25.5	**123.6**	***102.6***		
least squares		**122.4**	***100.5***		
$R = (29.37)\,(C_0/C)^2 - (66.071)\,(C_0/C) + (122.44)$					
last two points		**122.0**	***98.9***		
Prompt Critical Increment (one dollar)					
least squares		0.44	*1.51*	{18.15}	-0.00665
last two points		0.39	*1.32*		
Apparatus Details					
table: heavy ?				mount: Figure 9	

Table C-10. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Spherical Shell of Inside Radius 80.102 mm but With Mild Steel Regions in Contact with the Uranium.

inside - none -	Nominal Mild Steel Thickness Around Uranium (mm)	outside 40.0

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Average Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm³)	C_o/C
actual	12	100.000	36972	18.160	.393
subcritical	14	103.338	44802	18.142	.315
reciprocal	16	106.691	53217	18.137	.239
multiplication	18	110.011	62142	18.149	.169
data	20	113.332	71604	18.153	.098
Delayed Critical Extrapolation					
RFP-1021			***85.9***	{18.15}	0.0000
smooth curve	22.8	**118.0**	***83.5***		
least squares		**118.2**	***86.5***		
$R = (11.04)\ (C_o/C)^2 - (50.700)\ (C_o/C) + (118.21)$					
last two points		**117.9**	***85.6***		
Prompt Critical Increment (one dollar)					
least squares		0.34	*1.08*	{18.15}	-0.00665
last two points		0.31	*0.99*		
Apparatus Details					
table: heavy ?			mount: Figure 9		

Table C-11. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Spherical Shell of Inside Radius 80.102 mm but With Mild Steel Regions in Contact with the Uranium.

inside	Nominal Mild Steel Thickness Around Uranium (mm)	outside
- none -		80.0

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Average Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	12	100.000	36972	18.160	.373
subcritical	14	103.338	44802	18.142	.278
reciprocal	16	106.691	53217	18.137	.193
multiplication	18	110.011	62142	18.149	.110
data					
Delayed Critical Extrapolation					
RFP-1021			*75.1*	{18.14}	0.0000
smooth curve	20.7	**114.8**	*74.5*		
least squares		**114.8**	*76.0*		
$R = (14.44)\ (C_o/C)^2 - (45.156)\ (C_o/C) + (114.82)$					
last two points		**114.4**	*74.7*		
Prompt Critical Increment (one dollar)					
least squares		0.30	*0.90*	{18.14}	-0.00665
last two points		0.23	*0.79*		
Apparatus Details					
table: heavy ?				mount: Figure 9	

Table D-1. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Spherical Shell of Inside Radius 40.165 mm With Mild Steel Regions in Contact with the Uranium.

inside 20.0	Nominal Mild Steel Thickness Around Uranium (mm)	outside - none -

Actual System or Extrapolated Parameters	Enriched Uranium Region: Number of Hemishells	Enriched Uranium Region: Average Outside Radius (mm)	Enriched Uranium Region: Total Mass (g) or *(kg)*	Enriched Uranium Region: Density (mg/mm^3)	Reciprocal Multiplication C_o/C
actual	16	66.702	17616	18.129	.432
subcritical	22	76.662	29336	18.155	.283
reciprocal	26	83.328	39065	18.151	.194
multiplication	30	89.996	50426	18.127	.115
data	32	93.329	56793	18.123	.078
Delayed Critical Extrapolation					
RFP-1021			*73.3*	{18.13}	0.0000
smooth curve	36.6	**100.3**	*71.6*		
least squares		**100.6**	*72.4*		
$R = (43.00)\ (C_o/C)^2 - (97.010)\ (C_o/C) + (100.60)$					
last two points		**100.4**	*71.8*		
Prompt Critical Increment (one dollar)					
least squares		0.65	*1.49*	{18.13}	-0.00665
last two points		0.60	*1.37*		
Apparatus Details					
table: heavy ?				mount: Figure 9	

Table D-2. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Spherical Shell of Inside Radius 40.165 mm but With Mild Steel Regions in Contact with the Uranium.

inside 20.0		Nominal Mild Steel Thickness Around Uranium (mm)		outside 10.0	
Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Average Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	20	73.317	25072	18.176	.268
subcritical	22	76.662	29336	18.155	.221
reciprocal	24	80.027	34027	18.144	.180
multiplication	26	83.328	39065	18.151	.140
data	28	86.682	44528	18.125	.101
Delayed Critical Extrapolation					
RFP-1021			***61.3***	{18.13}	0.0000
smooth curve	33.3	**95.3**	***59.0***		
least squares		**96.3**	***62.9***		
$R = (54.87)\ (C_o/C)^2 - (100.62)\ (C_o/C) + (96.31)$					
last two points		**95.4**	***61.0***		
Prompt Critical Increment (one dollar)					
least squares		0.67	*1.42*	{18.13}	-0.00665
last two points		0.57	*1.19*		
Apparatus Details					
table: light ?				mount: Figure 9	

Table D-3. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Spherical Shell of Inside Radius 40.165 mm but With Mild Steel Regions in Contact with the Uranium.

inside 20.0	Nominal Mild Steel Thickness Around Uranium (mm)	outside 40.0

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Average Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	16	66.702	17616	18.129	.359
subcritical	20	73.317	25072	18.176	.241
reciprocal	22	76.662	29336	18.155	.186
multiplication	24	80.027	34027	18.144	.137
data	(25)	(81.678)	(36538)		(.112)
Delayed Critical Extrapolation					
RFP-1021			***50.4***	{18.15}	0.0000
smooth curve	30.0	**89.5**	***48.6***		
least squares		**90.0**	***50.6***		
$R = (38.14)\ (C_o/C)^2 - (78.677)\ (C_o/C) + (90.03)$					
last two points		**89.1**	***48.8***		
Prompt Critical Increment (one dollar)					
least squares		0.52	*0.97*	{18.15}	-0.00665
last two points		0.44	*0.79*		
Apparatus Details					
table: light ?				mount: Figure 9	

Table D-4. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Spherical Shell of Inside Radius 40.165 mm but With Mild Steel Regions in Contact with the Uranium.

inside 20.0		Nominal Mild Steel Thickness Around Uranium (mm)		outside 80.0	
Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Average Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	16	66.702	17616	18.129	.369
subcritical	20	73.317	25072	18.176	.232
reciprocal	22	76.662	29336	18.155	.167
multiplication	24	80.027	34027	18.144	.103
data					
Delayed Critical Extrapolation					
RFP-1021			*42.6*	{18.14}	0.0000
smooth curve	27.3	**85.5**	*41.8*		
least squares		**85.7**	*42.9*		
$R = (13.59)\,(C_o/C)^2 - (56.531)\,(C_o/C) + (85.71)$					
last two points		**85.4**	*42.5*		
Prompt Critical Increment (one dollar)					
least squares		0.38	*0.63*	{18.14}	-0.00665
last two points		0.35	*0.58*		
Apparatus Details					
table: heavy ?				mount: Figure 9	

Table D-5. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Spherical Shell of Inside Radius 80.102 mm but With Mild Steel Regions in Contact with the Uranium.

inside	Nominal Mild Steel Thickness Around Uranium (mm)	outside
60.0		- none -

Actual System or Extrapolated Parameters	Enriched Uranium Region: Number of Hemishells	Enriched Uranium Region: Average Outside Radius (mm)	Enriched Uranium Region: Total Mass (g) or *(kg)*	Enriched Uranium Region: Density (mg/mm^3)	Reciprocal Multiplication C_o/C
actual	16	106.691	53217	18.137	.324
subcritical	22	116.665	81632	18.147	.179
reciprocal	24	120.001	92281	18.146	.131
multiplication	26	123.361	103591	18.140	.086
data					
Delayed Critical Extrapolation					
RFP-1021			*126.3*	{18.14}	0.0000
smooth curve	29.8	129.4	*125.0*		
least squares		129.8	*127.0*		
$R = (15.25)\ (C_o/C)^2 - (76.180)\ (C_o/C) + (129.78)$					
last two points		129.8	*127.0*		
Prompt Critical Increment (one dollar)					
least squares		0.51	*1.95*	{18.14}	-0.00665
last two points		0.50	*1.91*		
Apparatus Details					
table: heavy ?				mount: Figure 9	

Table D-6. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Spherical Shell of Inside Radius 80.102 mm but With Mild Steel Regions in Contact with the Uranium.

inside 60.0	Nominal Mild Steel Thickness Around Uranium (mm)	outside 20.0

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Average Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	12	100.000	36972	18.160	.370
subcritical	18	110.011	62142	18.149	.197
reciprocal	20	113.332	71604	18.153	.144
multiplication	22	116.665	81632	18.147	.091
data					
Delayed Critical Extrapolation					
RFP-1021			***100.5***	{18.15}	0.0000
smooth curve	25.4	**122.3**	***99.4***		
least squares		**122.7**	***101.3***		
$R = (16.96)\ (C_o/C)^2 - (67.593)\ (C_o/C) + (122.69)$					
last two points		**122.4**	***100.3***		
Prompt Critical Increment (one dollar)					
least squares		0.45	*1.54*	{18.15}	-0.00665
last two points		0.42	*1.43*		
Apparatus Details					
table: heavy ?				mount: Figure 9	

Table D-7. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Spherical Shell of Inside Radius 80.102 mm but With Mild Steel Regions in Contact with the Uranium.

inside 60.0	Nominal Mild Steel Thickness Around Uranium (mm)	outside 40.0

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Average Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	12	100.000	36972	18.160	.374
subcritical	16	106.691	53217	18.137	.235
reciprocal	18	110.011	62142	18.149	.169
multiplication	20	113.332	71604	18.153	.102
data					
Delayed Critical Extrapolation					
RFP-1021			***88.0***	{18.15}	0.0000
smooth curve	23.1	**118.5**	***86.5***		
least squares		**118.6**	***87.6***		
$R = (5.565)\ (C_o/C)^2 - (51.721)\ (C_o/C) + (118.56)$					
last two points		**118.4**	***87.1***		
Prompt Critical Increment (one dollar)					
least squares		0.34	*1.10*	{18. }	-0.00665
last two points		0.33	*1.05*		
Apparatus Details					
table: heavy ?				mount: Figure 9	

Table D-8. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Spherical Shell of Inside Radius 80.102 mm but With Mild Steel Regions in Contact with the Uranium.

inside 60.0	Nominal Mild Steel Thickness Around Uranium (mm)	outside 50.0

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Average Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm³)	C_o/C
actual	12	100.000	36972	18.160	.517
subcritical	16	106.691	53217	18.137	.328
reciprocal	18	110.011	62142	18.149	.218
multiplication	(19)	(111.672)	(66875)		(.165)
data	20	113.332	71604	18.153	.112
Delayed Critical Extrapolation					
RFP-1021			*82.4*	{18.15}	0.0000
smooth curve	22.1	**116.9**	*82.2*		
least squares		**117.1**	*83.1*		
$R = -(32.812)(C_o/C) + (117.13)$					
last two points		**116.8**	*82.2*		
Prompt Critical Increment (one dollar)					
least squares		0.22	*0.68*	{18.15}	-0.00665
last two points		0.21	*0.65*		
Apparatus Details					
table: heavy				mount: Figure 9	

Table D-9. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Spherical Shell of Inside Radius 80.102 mm but With Mild Steel Regions in Contact with the Uranium.

inside 60.0	Nominal Mild Steel Thickness Around Uranium (mm)	outside 80.0

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Average Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	12	100.000	36972	18.160	.362
subcritical	14	103.338	44802	18.142	.279
reciprocal	16	106.691	53217	18.137	.196
multiplication	18	110.011	62142	18.149	.117
data					
Delayed Critical Extrapolation					
RFP-1021			*77.6*		
smooth curve	21.2	115.2	*76.6*	{18.14}	0.0000
least squares		115.0	*76.5*		
$R = (5.435)\ (C_o/C)^2 - (43.414)\ (C_o/C) + (115.01)$					
last two points		114.9	*76.3*		
Prompt Critical Increment (one dollar)					
least squares		0.29	*0.87*	{18.14}	-0.00665
last two points		0.28	*0.84*		
Apparatus Details					
table: heavy ?				mount: Figure 9	

HEMISPHERICAL GEOMETRIES

Table E-1. Experimental Subcritical and Extrapolated Critical Parameters for a Solid Enriched Uranium Hemisphere and No Mild Steel in Contact with the Uranium.

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	20	83.292	21924	18.116	.555
subcritical	24	96.683	34247	18.093	.361
reciprocal	25	100.001	37905	18.098	.315
multiplication	26	103.336	41823	18.097	.262
data	27	106.685	46031	18.100	.216
Delayed Critical Extrapolation					
RFP-1021					
smooth curve	31.5	**121.6**	***61.7***	{18.10}	0.0000
least squares		**121.5**	***68.1***		
$R = -(68.860)(C_o/C) + (121.54)$					
last two points		**122.**	***69.5***		
Prompt Critical Increment (one dollar)					
least squares		0.46	*0.77*	{18.10}	-0.00665
last two points		0.48	*0.83*		
Apparatus Details					
table: light		pole hole: down		mount: none	

Table E-2. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemisphere Shell of Inside Radius 20.126 mm and No Mild Steel in Contact with the Uranium.

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	19	83.292	21628	18.127	.372
subcritical	23	96.683	33951	18.100	.246
reciprocal	24	100.001	37609	18.104	.216
multiplication	25	103.336	41527	18.102	.185
data	26	106.685	45735	18.105	.153
Delayed Critical Extrapolation					
RFP-1021			*67.0*	{18.10}	0.0000
smooth curve	30.9	**122.6**	***64.4***		
least squares		**123.2**	***70.6***		
$R = (2.218)(C_o/C)^2 - (108.19)(C_o/C) + (123.22)$					
last two points		**122.7**	***69.8***		
Prompt Critical Increment (one dollar)					
least squares		0.72	*1.24*	{18.10}	-0.00665
last two points		0.70	*1.19*		
Apparatus Details					
table: light		pole hole: up		mount: not specified	

Table E-3. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemisphere Shell of Inside Radius 40.162 mm and No Mild Steel in Contact with the Uranium.

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	16	93.329	28393	18.120	.338
subcritical	18	100.001	35501	18.124	.271
reciprocal	20	106.685	43627	18.122	.203
multiplication	21	110.013	48088	18.126	.172
data	22	113.315	52817	18.140	.140
	23	116.670	57842	18.130	.107
Delayed Critical Extrapolation					
RFP-1021			*76.6*	{18.13}	0.0000
smooth curve	26.4	**127.2**	***71.6***		
least squares		**128.1**	*77.3*		
$R = (15.15)\,(C_o/C)^2 - (107.86)\,(C_o/C) + (128.07)$					
last two points		**127.5**	*76.3*		
Prompt Critical Increment (one dollar)					
least squares		0.72	*1.34*	{18.13}	-0.00665
last two points		0.68	*1.25*		
Apparatus Details					
table: light		pole hole: down		mount: cork ring	

Table E-4. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemisphere Shell of Inside Radius 60.121 mm and No Mild Steel in Contact with the Uranium.

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	16	113.315	47060	18.154	.292
subcritical	17	116.670	52085	18.142	.249
reciprocal	18	120.015	57411	18.137	.212
multiplication	19	123.363	63061	18.137	.172
data	20	126.505	68556	18.112	.138
	21	129.996	75048	18.102	.100
Delayed Critical Extrapolation					
RFP-1021			*95.1*	{18.11}	0.0000
smooth curve	23.9	**139.8**	*91.7*		
least squares		**139.8**	*95.5*		
$R = (37.87)\,(C_o/C)^2 - (101.99)\,(C_o/C) + (139.83)$					
last two points		**139.2**	*94.0*		
Prompt Critical Increment (one dollar)					
least squares		0.68	*1.51*	{18.11 }	-0.00665
last two points		0.61	*1.35*		
Apparatus Details					
table: light		pole hole: down		mount: not specified	

Table E-5. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemisphere Shell of Inside Radius 80.075 mm and No Mild Steel in Contact with the Uranium.

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_0/C
actual	14	126.505	57304	18.107	.352
subcritical	16	133.313	70394	18.111	.255
reciprocal	17	136.707	77367	18.095	.209
multiplication	18	140.010	84611	18.107	.165
data	19	143.322	92224	18.117	.122
Delayed Critical Extrapolation					
RFP-1021			*115.7*	{18.12}	0.0000
smooth curve	21.8	**152.9**	***114.0***		
least squares		**153.2**	***116.9***		
$R = (21.26)\,(C_0/C)^2 - (83.215)\,(C_0/C) + (153.16)$					
last two points		**152.7**	***115.7***		
Prompt Critical Increment (one dollar)					
least squares		0.55	*1.48*	{18.12}	-0.00665
last two points		0.51	*1.36*		
Apparatus Details					
table: light		pole hole: down		mount: not specified	

Table E-6. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemisphere Shell of Inside Radius 90.104 mm and No Mild Steel in Contact with the Uranium.

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	14	136.707	69162	18.111	.329
subcritical	15	140.010	76406	18.122	.279
reciprocal	16	143.322	84019	18.132	.234
multiplication	17	146.657	91971	18.125	.190
data	18	150.062	100384	18.103	.142
Delayed Critical Extrapolation					
RFP-1021			***128.6***	{18.11}	0.0000
smooth curve	21.1	**160.4**	***125.1***		
least squares		**161.2**	***131.3***		
$R = (19.22)\ (C_o/C)^2 - (18.064)\ (C_o/C) + (161.24)$					
last two points		**160.1**	***128.0***		
Prompt Critical Increment (one dollar)					
least squares		0.54	*1.60*	{18.11}	-0.00665
last two points		0.47	*1.38*		
Apparatus Details					
table: light		pole hole: down		mount: not specified	

Table F-1. Experimental Subcritical and Extrapolated Critical Parameters for a Solid Enriched Uranium Hemisphere and No Mild Steel in Contact with the Uranium.

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	20	83.292	21924	18.116	.572
subcritical	24	96.683	34247	18.093	.379
reciprocal	25	100.001	37905	18.098	.325
multiplication	26	103.336	41823	18.097	.280
data	27	106.685	46031	18.100	.230
Delayed Critical Extrapolation					
RFP-1021					
smooth curve	32.0	**122.0**	***66.3***	{18.10}	0.0000
least squares		**122.4**	***69.5***		
$R = -(68.334)\ (C_o/C) + (122.41)$					
last two points		**122.1**	***69.0***		
Prompt Critical Increment (one dollar)					
least squares		0.45	*0.77*	{18.10}	-0.00665
last two points		0.45	*0.76*		
Apparatus Details					
table: heavy		pole hole: down		mount: not specified	

Table F-2. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 20.126 mm and No Mild Steel in Contact with the Uranium.

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	20	86.680	24369	18.092	.308
subcritical	22	93.329	30501	18.096	.252
reciprocal	24	100.001	37609	18.104	.195
multiplication	25	103.336	41527	18.102	.166
data	26	106.685	45735	18.105	.138
Delayed Critical Extrapolation					
RFP-1021			*67.0*	{18.10}	0.0000
smooth curve	31.0	**120.7**	*67.0*		
least squares		**122.8**	*70.0*		
$R - (117.44)\ (C_o/C) + (122.88)$					
last two points		**123.2**	*70.6*		
Prompt Critical Increment (one dollar)					
least squares		0.78	*1.34*	{18.10}	-0.00665
last two points		0.80	*1.37*		
Apparatus Details					
table: heavy		pole hole: up		mount: not specified	

Table F-3. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 20.126 mm and No Mild Steel in Contact with the Uranium. Experiments had a 50-mm-Thick Plastic Tabletop.

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
	19	83.292	21628	18.127	.310
actual	20	86.680	24369	18.092	.284
subcritical	21	89.995	27322	18.100	.256
reciprocal	22	93.329	30501	18.096	.228
multiplication	23	96.683	33951	18.100	.199
data	24	100.001	37609	18.104	.172
	25	103.336	41527	18.102	.145
	26	106.685	45735	18.105	.118
Delayed Critical Extrapolation					
RFP-1021				{18.10}	0.0000
smooth curve	30.3	**121.2**	***65.2***		
least squares		**121.1**	***66.9***		
$R = (4.473)\ (C_o/C)^2 - (122.76)\ (C_o/C) + (121.05)$					
last two points		**121.3**	***67.4***		
Prompt Critical Increment (one dollar)					
least squares		0.82	*1.36*	{18.10}	-0.00665
last two points		0.82	*1.37*		
Apparatus Details					
table: heavy		pole hole: up		mount: Figure 10	

Table F-4. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 20.126 mm and No Mild Steel in Contact with the Uranium. Experiments had a 50-mm-Thick Plastic Tabletop[a].

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	16	73.338	14645	18.101	.479
subcritical	18	80.027	19117	18.097	.415
reciprocal	20	86.680	24369	18.092	.355
multiplication	21	89.995	27322	18.100	.322
data					
Delayed Critical Extrapolation					
RFP-1021					
smooth curve	30.3	**125.0**	***66.4***	{18.10}	0.0000
least squares		**124.8**	***73.4***		
$R = (2.521)\ (C_o/C)^2 - (108.76)\ (C_o/C) + (124.83)$					
last two points		**122.3**	***69.1***		
Prompt Critical Increment (one dollar)					
least squares		0.72	*1.28*	{18.10}	-0.00665
last two points		0.67	*1.14*		
Apparatus Details					
table: heavy		pole hole: down		mount: Figure 9	

a Tables F-3 and F-4 differ only in the polar orientation of the hemispheres and the mount used.

Table G-1. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 20.126 mm but With Mild Steel Regions in Contact with the Uranium.

inside - none -	Nominal Mild Steel Thickness Around Uranium (mm)	outside 10.0

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	20	86.680	24369	18.092	.352
subcritical	22	93.329	30501	18.096	.280
reciprocal	24	100.001	37609	18.104	.207
multiplication	25	103.336	41527	18.102	.165
data	26	106.685	45735	18.105	.129
Delayed Critical Extrapolation					
RFP-1021			***62.6***	{18.10}	0.0000
smooth curve	29.5	**120.1**	***60.7***		
least squares		**118.3**	***62.4***		
$R = -(89.356)(C_o/C) + (118.25)$					
last two points		**118.7**	***63.1***		
Prompt Critical Increment (one dollar)					
least squares		0.59	*0.95*	{18.10}	-0.00665
last two points		0.62	*0.99*		
Apparatus Details					
table: light		pole hole: down		mount: none	

Table G-2. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 20.126 mm but With Mild Steel Regions in Contact with the Uranium.

inside - none -		Nominal Mild Steel Thickness Around Uranium (mm)		outside 20.0	
Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	18	80.027	19117	18.097	.421
subcritical	22	93.329	30501	18.096	.258
reciprocal	24	100.001	37609	18.104	.181
multiplication	25	103.336	41527	18.102	.144
data	26	106.685	45735	18.105	.107
Delayed Critical Extrapolation					
RFP-1021			*59.2*	{18.10}	0.0000
smooth curve	28.9	117.2	*58.7*		
least squares		116.8	*60.0*		
$R = (22.04)\ (C_o/C)^2 - (96.549)\ (C_o/C) + (116.77)$					
last two points		116.4	*59.4*		
Prompt Critical Increment (one dollar)					
least squares		0.64	*1.00*	{18.10}	-0.00665
last two points		0.60	*0.93*		
Apparatus Details					
table: light		pole hole: down		mount: none	

Table G-3. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 20.126 mm but With Mild Steel Regions in Contact with the Uranium.

<table>
<tr><td colspan="2">inside
- none -</td><td colspan="2">Nominal Mild Steel Thickness Around Uranium (mm)</td><td colspan="2">outside
40.0</td></tr>
<tr><td rowspan="2">Actual System or Extrapolated Parameters</td><td colspan="4">Enriched Uranium Region</td><td rowspan="2">Reciprocal Multiplication
C_o/C</td></tr>
<tr><td>Number of Hemishells</td><td>Outside Radius (mm)</td><td>Total Mass (g) or (kg)</td><td>Density (mg/mm^3)</td></tr>
<tr><td>actual</td><td>18</td><td>80.027</td><td>19117</td><td>18.097</td><td>.427</td></tr>
<tr><td>subcritical</td><td>20</td><td>86.680</td><td>24369</td><td>18.092</td><td>.339</td></tr>
<tr><td>reciprocal</td><td>22</td><td>93.329</td><td>30501</td><td>18.096</td><td>.248</td></tr>
<tr><td>multiplication</td><td>24</td><td>100.001</td><td>37609</td><td>18.104</td><td>.162</td></tr>
<tr><td>data</td><td>25</td><td>103.336</td><td>41527</td><td>18.102</td><td>.119</td></tr>
<tr><td colspan="6">Delayed Critical Extrapolation</td></tr>
<tr><td>RFP-1021</td><td></td><td></td><td>*60.0*</td><td rowspan="5">{18.10}</td><td rowspan="5">0.0000</td></tr>
<tr><td>smooth curve</td><td>27.9</td><td>110.9</td><td>52.9</td></tr>
<tr><td>least squares</td><td></td><td>112.7</td><td>53.9</td></tr>
<tr><td colspan="4">$R = (6.887)\ (C_o/C)^2 - (76.262)\ (C_o/C) + (112.65)$</td></tr>
<tr><td>last two points</td><td></td><td>112.6</td><td>53.8</td></tr>
<tr><td colspan="6">Prompt Critical Increment (one dollar)</td></tr>
<tr><td>least squares</td><td></td><td>0.53</td><td>0.76</td><td rowspan="2">{18.10}</td><td rowspan="2">-0.00665</td></tr>
<tr><td>last two points</td><td></td><td>0.52</td><td>0.74</td></tr>
<tr><td colspan="6">Apparatus Details</td></tr>
<tr><td colspan="2">table: light</td><td colspan="2">pole hole: down</td><td colspan="2">mount: none</td></tr>
</table>

Table G-4. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 20.126 mm but With Mild Steel Regions in Contact with the Uranium.

inside - none -	Nominal Mild Steel Thickness Around Uranium (mm)	outside 80.0

Actual System or Extrapolated Parameters	Enriched Uranium Region: Number of Hemishells	Enriched Uranium Region: Outside Radius (mm)	Enriched Uranium Region: Total Mass (g) or *(kg)*	Enriched Uranium Region: Density (mg/mm^3)	Reciprocal Multiplication C_o/C
actual	18	80.027	19117	18.097	.415
subcritical	20	86.680	24369	18.092	.320
reciprocal	22	93.329	30501	18.096	.224
multiplication	24	100.001	37609	18.104	.129
data					
Delayed Critical Extrapolation					
RFP-1021			***49.3***	{18.10}	0.0000
smooth curve	26.8	**109.6**	***49.2***		
least squares		**109.0**	***48.8***		
$R = (0.5236)\,(C_o/C)^2 - (70.065)\,(C_o/C) + (109.02)$					
last two points		**109.1**	***48.9***		
Prompt Critical Increment (one dollar)					
least squares		0.47	*0.63*	{18.10}	-0.00665
last two points		0.47	*0.63*		
Apparatus Details					
table: light		pole hole: down		mount: none	

Table G-5. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 40.162 mm but With Mild Steel Regions in Contact with the Uranium.

inside	Nominal Mild Steel Thickness Around Uranium (mm)	outside
- none -		10.0

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	14	86.680	22261	18.123	.369
subcritical	16	93.329	28393	18.120	.293
reciprocal	18	100.001	35501	18.124	.229
multiplication	20	106.685	43627	18.122	.164
data	21	110.013	48088	18.126	.132
	22	113.315	52817	18.140	.101
Delayed Critical Extrapolation					
RFP-1021			*69.9*		
smooth curve	25.2	**123.5**	*68.7*	{18.13}	0.0000
least squares		**125.0**	*71.7*		
$R = (40.43)\ (C_o/C)^2 - (119.09)\ (C_o/C) + (125.02)$					
last two points		**124.1**	***70.1***		
Prompt Critical Increment (one dollar)					
least squares		0.79	*1.41*	{18.13}	-0.00665
last two points		0.71	*1.24*		
Apparatus Details					
table: light		pole hole: down		mount: cork ring	

Table G-6. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 40.162 mm but With Mild Steel Regions in Contact with the Uranium.

inside - none -	Nominal Mild Steel Thickness Around Uranium (mm)	outside 20.0

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	14	86.680	22261	18.123	.371
subcritical	18	100.001	35501	18.124	.226
reciprocal	20	106.685	43627	18.122	.153
multiplication	21	110.013	48088	18.126	.117
data					
Delayed Critical Extrapolation					
RFP-1021			*64.3*	{18.12}	0.0000
smooth curve	24.2	**120.4**	*62.7*		
least squares		**120.9**	*64.6*		
$R = -(92.279)(C_o/C) + (120.92)$					
last two points		**120.0**	*63.1*		
Prompt Critical Increment (one dollar)					
least squares		0.61	*1.02*	{18.12}	-0.00665
last two points		0.57	*0.93*		
Apparatus Details					
table: light		pole hole: down		mount: cork ring ?	

Table G-7. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 40.162 mm but With Mild Steel Regions in Contact with the Uranium.

inside - none -	Nominal Mild Steel Thickness Around Uranium (mm)	outside 40.0

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	14	86.680	22261	18.123	.374
subcritical	18	100.001	35501	18.124	.208
reciprocal	20	106.685	43627	18.122	.130
multiplication	21	110.013	48088	18.126	.090
data					
Delayed Critical Extrapolation					
RFP-1021			***60.0***	{18.12}	0.0000
smooth curve	23.4	**118.1**	***59.4***		
least squares		**118.0**	***59.9***		
$R = (15.47)\ (C_o/C)^2 - (89.500)\ (C_o/C) + (117.98)$					
last two points		**117.5**	***59.1***		
Prompt Critical Increment (one dollar)					
least squares		0.60	*0.94*	{18.12}	-0.00665
last two points		0.55	*0.87*		
Apparatus Details					
table: light		pole hole: down		mount: none	

Table G-8. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 40.162 mm but With Mild Steel Regions in Contact with the Uranium.

inside - none -	Nominal Mild Steel Thickness Around Uranium (mm)	outside 80.0

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	14	86.680	22261	18.123	.381
subcritical	16	93.329	28393	18.120	.286
reciprocal	18	100.001	35501	18.124	.198
multiplication	19	103.336	39419	18.120	.152
data	20	106.685	43627	18.122	.108
Delayed Critical Extrapolation					
RFP-1021			*54.6*	{18.12}	0.0000
smooth curve	22.4	**115.4**	***53.9***		
least squares		**115.2**	***55.5***		
$R = (12.32)\,(C_o/C)^2 - (79.498)\,(C_o/C) + (115.15)$					
last two points		**114.9**	***55.1***		
Prompt Critical Increment (one dollar)					
least squares		0.53	*0.80*	{18.12}	-0.00665
last two points		0.51	*0.76*		
Apparatus Details					
table: light		pole hole: down		mount: none	

Table G-9. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 80.075 mm but With Mild Steel Regions in Contact with the Uranium.

inside	Nominal Mild Steel Thickness Around Uranium (mm)	outside
- none -		10.0

Actual System or Extrapolated Parameters	Enriched Uranium Region: Number of Hemishells	Enriched Uranium Region: Outside Radius (mm)	Enriched Uranium Region: Total Mass (g) or *(kg)*	Enriched Uranium Region: Density (mg/mm^3)	Reciprocal Multiplication C_o/C
actual	10	113.315	35808	18.158	.367
subcritical	12	120.015	46159	18.136	.293
reciprocal	14	126.505	57304	18.107	.224
multiplication	16	133.313	70394	18.111	.153
data	17	136.707	77367	18.095	.118
Delayed Critical Extrapolation					
RFP-1021			***104.4***	{18.10}	0.0000
smooth curve	20.5	**148.0**	***100.6***		
least squares		**148.6**	***104.8***		
$R = (17.28)\ (C_o/C)^2 - (102.42)\ (C_o/C) + (148.56)$					
last two points		**148.1**	***103.8***		
Prompt Critical Increment (one dollar)					
least squares		0.68	*1.71*	{18.10}	-0.00665
last two points		0.64	*1.61*		
Apparatus Details					
table: light		pole hole: down		mount: none	

Table G-10. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 80.075 mm but With Mild Steel Regions in Contact with the Uranium.

inside	Nominal Mild Steel Thickness Around Uranium (mm)	outside
- none -		20.0

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_0/C
actual	12	120.015	46159	18.136	.311
subcritical	14	126.505	57304	18.107	.232
reciprocal	15	129.996	63796	18.095	.188
multiplication	16	133.313	70394	18.111	.150
data	17	136.707	77367	18.095	.110
Delayed Critical Extrapolation					
RFP-1021			***98.2***	{18.10}	0.0000
smooth curve	19.8	**145.8**	***97.7***		
least squares		**146.3**	***99.4***		
$R = (15.28)\ (C_0/C)^2 - (89.387)\ (C_0/C) + (146.35)$					
last two points		**146.0**	***98.6***		
Prompt Critical Increment (one dollar)					
least squares		0.60	*1.45*	{18.10}	-0.00665
last two points		0.56	*1.37*		

Apparatus Details		
table: light	pole hole: down	mount: none

Table G-11. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 80.075 mm but With Mild Steel Regions in Contact with the Uranium.

inside - none -	Nominal Mild Steel Thickness Around Uranium (mm)	outside 40.0

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	12	120.015	46159	18.136	.304
subcritical	14	126.505	57304	18.107	.211
reciprocal	15	129.996	63796	18.095	.163
multiplication	16	133.313	70394	18.111	.119
data					
Delayed Critical Extrapolation					
RFP-1021			***89.9***	{18.10}	0.0000
smooth curve	18.9	**142.5**	***88.6***		
least squares		**142.7**	***90.7***		
$R = (23.30)\ (C_o/C)^2 - (81.717)\ (C_o/C) + (142.70)$					
last two points		**142.3**	***89.7***		
Prompt Critical Increment (one dollar)					
least squares		0.54	*1.26*	{18.10}	-0.00665
last two points		0.50	*1.15*		
Apparatus Details					
table: light		pole hole: down		mount: none	

Table G-12. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 80.075 mm but With Mild Steel Regions in Contact with the Uranium.

inside - none -	Nominal Mild Steel Thickness Around Uranium (mm)	outside 60.0

Actual System or Extrapolated Parameters	Enriched Uranium Region: Number of Hemishells	Enriched Uranium Region: Outside Radius (mm)	Enriched Uranium Region: Total Mass (g) or *(kg)*	Enriched Uranium Region: Density (mg/mm^3)	Reciprocal Multiplication C_o/C
actual	12	120.015	46159	18.136	.296
subcritical	14	126.505	57304	18.107	.199
reciprocal	15	129.996	63796	18.095	.148
multiplication	16	133.313	70394	18.111	.099
data					
Delayed Critical Extrapolation					
RFP-1021			***84.6***	{18.10}	0.0000
smooth curve	18.0	**140.1**	***84.5***		
least squares		**140.1**	***84.9***		
$R = (4.689)\,(C_o/C)^2 - (69.406)\,(C_o/C) + (140.1\underline{5})$					
last two points		**140.0**	***84.6***		
Prompt Critical Increment (one dollar)					
least squares		0.46	*1.03*	{18.10}	-0.00665
last two points		0.45	*1.00*		
Apparatus Details					
table: light		pole hole: down		mount: none	

Table G-13. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 90.104 mm but With Mild Steel Regions in Contact with the Uranium.

inside - none -	Nominal Mild Steel Thickness Around Uranium (mm)	outside 10.0

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual subcritical reciprocal multiplication data	11	126.505	49099	18.131	.317
	12	129.996	55591	18.115	.280
	13	133.313	62189	18.130	.240
	14	136.707	69162	18.111	.200
	15	140.010	76406	18.122	.168
	16	143.322	84019	18.132	.132
	17	146.657	91971	18.125	.099
Delayed Critical Extrapolation					
RFP-1021			*116.6*	{18.13}	0.0000
smooth curve	19.8	**156.5**	***115.4***		
least squares		**156.9**	***118.8***		
$R = (37.22)\ (C_o/C)^2 - (107.17)\ (C_o/C) + (156.86)$					
last two points		**156.7**	***118.2***		
Prompt Critical Increment (one dollar)					
least squares		0.71	*2.00*	{18.13}	-0.00665
last two points		0.67	*1.88*		
Apparatus Details					
table: light		pole hole: down		mount: none	

Table G-14. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 90.104 mm but With Mild Steel Regions in Contact with the Uranium.

inside	Nominal Mild Steel Thickness Around Uranium (mm)	outside
- none -		20.0

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	12	129.996	55591	18.115	.288
subcritical	14	136.707	69162	18.111	.201
reciprocal	15	140.010	76406	18.122	.163
multiplication	16	143.322	84019	18.132	.122
data					
Delayed Critical Extrapolation					
RFP-1021			***109.6***		
smooth curve	19.0	**153.8**	***108.1***	{18.13}	0.0000
least squares		**154.3**	***111.6***		
$R = (30.22)\ (C_o/C)^2 - (92.972)\ (C_o/C) + (154.25)$					
last two points		**153.2**	***108.7***		
Prompt Critical Increment (one dollar)					
least squares		0.62	*1.68*	{18.13}	-0.00665
last two points		0.54	*1.44*		
Apparatus Details					
table: light		pole hole: down		mount: none	

Table G-15. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 90.104 mm but With Mild Steel Regions in Contact with the Uranium.

inside - none -	Nominal Mild Steel Thickness Around Uranium (mm)	outside 40.0

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm³)	C_o/C
actual	8	116.670	32628	18.187	.472
subcritical	12	129.996	55591	18.115	.278
reciprocal	14	136.707	69162	18.111	.180
multiplication	15	140.010	76406	18.122	.134
data					
Delayed Critical Extrapolation					
RFP-1021			*100.0*	{18.12}	0.0000
smooth curve	18.0	149.5	*97.6*		
least squares		149.4	*98.8*		
$R = (2.880)\ (C_o/C)^2 - (70.684)\ (C_o/C) + (149.40)$					
last two points		149.6	*99.4*		
Prompt Critical Increment (one dollar)					
least squares		0.47	*1.19*	{18.12}	-0.00665
last two points		0.48	*1.22*		

Apparatus Details		
table: light	pole hole: down	mount: none

Table H-1. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 40.162 mm but With Mild Steel Regions in Contact with the Uranium.

inside	Nominal Mild Steel Thickness Around Uranium (mm)	outside
20.0		- none -

Actual System or Extrapolated Parameters	Enriched Uranium Region: Number of Hemishells	Enriched Uranium Region: Outside Radius (mm)	Enriched Uranium Region: Total Mass (g) or *(kg)*	Enriched Uranium Region: Density (mg/mm^3)	Reciprocal Multiplication C_o/C
	16	93.329	28393	18.120	.375
actual	18	100.001	35501	18.124	.302
subcritical	19	103.336	39419	18.120	.267
reciprocal	20	106.685	43627	18.122	.231
multiplication	21	110.013	48088	18.126	.193
data	22	113.315	52817	18.140	.154
	23	116.670	57842	18.130	.115
Delayed Critical Extrapolation					
RFP-1021			*73.2*		
smooth curve	25.8	**126.9**	*71.8*	{18.13}	0.0000
least squares		**127.2**	*75.7*		
$R = -(89.95)(C_o/C) + (127.23)$					
last two points		**126.6**	*77.0*		
Prompt Critical Increment (one dollar)					
least squares		0.60	*1.10*	{18.13}	-0.00665
last two points		0.57	*1.04*		
Apparatus Details					
table: heavy ?		pole hole: up		mount: Figure 10	

Table H-2. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 40.162 mm but With Mild Steel Regions in Contact with the Uranium.

inside	Nominal Mild Steel Thickness Around Uranium (mm)	outside
20.0		10.0

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_0/C
actual	16	93.329	28393	18.120	.408
subcritical	18	100.001	35501	18.124	.311
reciprocal	20	106.685	43627	18.122	.216
multiplication	21	110.013	48088	18.126	.170
data	22	113.315	52817	18.140	.125
Delayed Critical Extrapolation					
RFP-1021			*67.1*	{18.13}	0.0000
smooth curve	24.7	122.6	*66.8*		
least squares		122.7	*67.6*		
$R = (10.74)\ (C_0/C)^2 - (76.305)\ (C_0/C) + (122.68)$					
last two points		122.5	*67.3*		
Prompt Critical Increment (one dollar)					
least squares		0.51	*0.87*	{18.13}	-0.00665
last two points		0.49	*0.83*		
Apparatus Details					
table: light		pole hole: down		mount: none	

Table H-3. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 40.162 mm but With Mild Steel Regions in Contact with the Uranium.

inside 20.0		Nominal Mild Steel Thickness Around Uranium (mm)		outside 20.0	
Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	14	86.680	22261	18.123	.516
subcritical	18	100.001	35501	18.124	.306
reciprocal	20	106.685	43627	18.122	.199
multiplication	21	110.013	48088	18.126	.146
data					
Delayed Critical Extrapolation					
RFP-1021			***61.8***	{18.12}	0.0000
smooth curve	23.8	**119.3**	***61.8***		
least squares		**119.2**	***61.9***		
$R = -(63.065)(C_o/C) + (119.24)$					
last two points		**119.2**	***61.8***		
Prompt Critical Increment (one dollar)					
least squares		0.42	*0.68*	{18.12}	-0.00665
last two points		0.42	*0.68*		
Apparatus Details					
table: light		pole hole: down		mount: none	

Table H-4. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 40.162 mm but With Mild Steel Regions in Contact with the Uranium.

inside	Nominal Mild Steel Thickness Around Uranium (mm)	outside
20.0		40.0

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_0/C
actual	14	86.680	22261	18.123	.519
subcritical	18	100.001	35501	18.124	.277
reciprocal	20	106.685	43627	18.122	.161
multiplication	21	110.013	48088	18.126	.104
data					
Delayed Critical Extrapolation					
RFP-1021			*56.8*	{18.12}	0.0000
smooth curve	22.8	**116.3**	*56.9*		
least squares		**116.2**	*57.1*		
$R = (6.770)\ (C_0/C)^2 - (60.458)\ (C_0/C) + (116.23)$					
last two points		**116.1**	*56.9*		
Prompt Critical Increment (one dollar)					
least squares		0.40	*0.62*	{18.12}	-0.00665
last two points		0.39	*0.60*		
Apparatus Details					
table: light		pole hole: down		mount: none	

Table H-5. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 40.162 mm but With Mild Steel Regions in Contact with the Uranium.

inside 20.0		Nominal Mild Steel Thickness Around Uranium (mm)		outside 80.0	
Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	16	93.329	28393	18.120	.395
subcritical	18	100.001	35501	18.124	.254
reciprocal	19	103.336	39419	18.120	.190
multiplication	20	106.685	43627	18.122	.124
data					
Delayed Critical Extrapolation					
RFP-1021					
smooth curve	21.9	**113.4**	***53.0***	{18.12}	0.0000
least squares		**113.5**	***53.0***		
$R = (13.03)\ (C_o/C)^2 - (56.163)\ (C_o/C) + (113.47)$					
last two points		**113.0**	***52.3***		
Prompt Critical Increment (one dollar)					
least squares		0.37	*0.55*	{18.12}	-0.00665
last two points		0.34	*0.49*		
Apparatus Details					
table: light		pole hole: down		mount: none	

Table H-6. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 66.792 mm but With Mild Steel Regions in Contact with the Uranium.

inside 46.7	Nominal Mild Steel Thickness Around Uranium (mm)	outside - none -

Actual System or Extrapolated Parameters	Enriched Uranium Region: Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	Reciprocal Multiplication C_o/C
actual	16	120.015	54359	18.141	.310
subcritical	17	123.363	60009	18.141	.255
reciprocal	18	126.505	65504	18.115	.207
multiplication	19	129.996	71996	18.104	.152
data					
Delayed Critical Extrapolation					
RFP-1021			***92.8***	{18.11}	0.0000
smooth curve	22.0	**140.3**	***90.7***		
least squares		**140.2**	***93.3***		
$R = (12.62)\ (C_o/C)^2 - (69.195)\ (C_o/C) + (140.24)$					
last two points		**147.8**	****103.0****		
Prompt Critical Increment (one dollar)					
least squares		0.39	***0.97***	{18.11}	-0.00665
last two points		0.35	***0.88***		
Apparatus Details					
table: heavy		pole hole: up		mount: Figure 10	

Table H-7. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 80.075 mm but With Mild Steel Regions in Contact with the Uranium.

inside 60.0	Nominal Mild Steel Thickness Around Uranium (mm)	outside - none -

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	14	126.505	57304	18.107	.379
subcritical	15	129.996	63796	18.095	.322
reciprocal	16	133.313	70394	18.111	.262
multiplication	17	136.707	77367	18.095	.208
data	18	140.010	84611	18.107	.146
Delayed Critical Extrapolation					
RFP-1021			***102.7***	{18.11}	0.0000
smooth curve	20.3	**148.3**	***101.6***		
least squares		**148.6**	***105.1***		
$R = -(58.116)(C_o/C) + (148.6\underline{5})$					
last two points		**147.8**	***103.0***		
Prompt Critical Increment (one dollar)					
least squares		0.39	*0.97*	{18.11}	-0.00665
last two points		0.35	*0.88*		
Apparatus Details					
table: heavy		pole hole: up		mount: Figure 10	

Table H-8. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 80.075 mm but With Mild Steel Regions in Contact with the Uranium.

inside 60.0	Nominal Mild Steel Thickness Around Uranium (mm)	outside 10.0

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	10	113.315	35808	18.158	.697
subcritical	12	120.015	46159	18.136	.537
reciprocal	14	126.505	57304	18.107	.387
multiplication	16	133.313	70394	18.111	.225
data	17	136.707	77367	18.095	.114
Delayed Critical Extrapolation					
RFP-1021			***90.7***		
smooth curve	18.8	**141.7**	***86.4***	{18.10}	0.0000
least squares		**141.9**	***89.0***		
$R = -(40.762)(C_o/C) + (141.95)$					
last two points		**140.2**	***85.0***		
Prompt Critical Increment (one dollar)					
least squares		0.27	*0.62*	{18.10}	-0.00665
last two points		0.20	*0.45*		
Apparatus Details					
table: light		pole hole: down		mount: none	

Table H-9. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 80.075 mm but With Mild Steel Regions in Contact with the Uranium.

inside 60.0		Nominal Mild Steel Thickness Around Uranium (mm)		outside 20.0	
Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_0/C
actual	12	120.015	46159	18.136	.563
subcritical	14	126.505	57304	18.107	.374
reciprocal	15	129.996	63796	18.095	.284
multiplication	16	133.313	70394	18.111	.191
data					
Delayed Critical Extrapolation					
RFP-1021			***84.6***	{18.10}	0.0000
smooth curve	18.0	**140.4**	***84.6***		
least squares		**140.8**	***86.4***		
$R = (5.524)\,(C_0/C)^2 - (40.071)\,(C_0/C) + (140.81)$					
last two points		**140.1**	***84.8***		
Prompt Critical Increment (one dollar)					
least squares		0.27	*0.60*	{18.10}	-0.00665
last two points		0.24	*0.53*		
Apparatus Details					
table: light		pole hole: down		mount: none	

Table H-10. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 80.075 mm but With Mild Steel Regions in Contact with the Uranium.

inside 60.0	Nominal Mild Steel Thickness Around Uranium (mm)	outside 40.0

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	8	106.685	26618	18.135	.968
subcritical	12	120.015	46159	18.136	.531
reciprocal	14	126.505	57304	18.107	.317
multiplication	15	129.996	63796	18.095	.207
data					
Delayed Critical Extrapolation					
RFP-1021			*76.8*	{18.10}	0.0000
smooth curve	16.9	136.5	*77.0*		
least squares		136.4	*76.7*		
$R = (0.4849)(C_o/C)^2 - (31.153)(C_o/C) + (136.39)$					
last two points		136.6	*77.1*		
Prompt Critical Increment (one dollar)					
least squares		0.21	*0.44*	{18.10}	-0.00665
last two points		0.21	*0.45*		

Apparatus Details		
table: light	pole hole: down	mount: none

Table H-11. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 80.075 mm but With Mild Steel Regions in Contact with the Uranium.

inside 60.0		Nominal Mild Steel Thickness Around Uranium (mm)		outside 60.0	
Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or (*kg*)	Density (mg/mm^3)	C_o/C
actual subcritical reciprocal multiplication data	8	106.685	26618	18.135	.967
	12	120.015	46159	18.136	.504
	14	126.505	57304	18.107	.273
	15	129.996	63796	18.095	.155
Delayed Critical Extrapolation					
RFP-1021			*72.5*	{18.10}	0.0000
smooth curve	16.3	**134.5**	*71.8*		
least squares		**134.4**	*72.6*		
$R = -(28.651)(C_o/C) + (134.40)$					
last two points		**134.6**	*72.9*		
Prompt Critical Increment (one dollar)					
least squares		0.19	*0.39*	{18.10}	-0.00665
last two points		0.20	*0.41*		
Apparatus Details					
table: light		pole hole: down		mount: none	

Table H-12. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 90.104 mm but With Mild Steel Regions in Contact with the Uranium.

inside	Nominal Mild Steel Thickness Around Uranium (mm)	outside
70.0		- none -

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	12	129.996	55591	18.115	.465
subcritical	14	136.707	69162	18.111	.358
reciprocal	15	140.010	76406	18.122	.298
multiplication	16	143.322	84019	18.132	.236
data	17	146.657	91971	18.125	.171
Delayed Critical Extrapolation					
RFP-1021			***112.9***	{18.13}	0.0000
smooth curve	19.7	**156.0**	***113.2***		
least squares		**156.6**	***118.1***		
$R = -(56.563)(C_o/C) + (156.62)$					
last two points		**155.4**	***114.8***		
Prompt Critical Increment (one dollar)					
least squares		0.38	*1.05*	{18.13}	-0.00665
last two points		0.34	*0.94*		
Apparatus Details					
table: heavy		pole hole: up		mount: Figure 10	

Table H-13. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 90.104 mm but With Mild Steel Regions in Contact with the Uranium.

inside 70.0		Nominal Mild Steel Thickness Around Uranium (mm)		outside 10.0	
Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_0/C
actual	13	133.313	62189	18.130	.436
subcritical	14	136.707	69162	18.111	.342
reciprocal	15	140.010	76406	18.122	.261
multiplication	16	143.322	84019	18.132	.173
data					
Delayed Critical Extrapolation					
RFP-1021			***99.3***		
smooth curve	17.9	**150.0**	***100.2***	{18.12}	0.0000
least squares		**150.5**	***101.7***		
$R = (6.537)\,(C_0/C)^2 - (42.444)\,(C_0/C) + (150.5\underline{5})$					
last two points		**150.0**	***100.3***		
Prompt Critical Increment (one dollar)					
least squares		0.28	*0.73*	{18.12}	-0.00665
last two points		0.25	*0.65*		
Apparatus Details					
table: light		pole hole: down		mount: none	

Table H-14. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 90.104 mm but With Mild Steel Regions in Contact with the Uranium.

inside 70.0	Nominal Mild Steel Thickness Around Uranium (mm)	outside 20.0

Actual System or Extrapolated Parameters	Enriched Uranium Region				Reciprocal Multiplication
	Number of Hemishells	Outside Radius (mm)	Total Mass (g) or *(kg)*	Density (mg/mm^3)	C_o/C
actual	8	116.670	32628	18.187	.934
subcritical	12	129.996	55591	18.115	.529
reciprocal	14	136.707	69162	18.111	.319
multiplication	15	140.010	76406	18.122	.215
data					
Delayed Critical Extrapolation					
RFP-1021			***93.2***	{18.12}	0.0000
smooth curve	17.2	**146.7**	***91.4***		
least squares		**147.1**	***93.0***		
$R = -(32.488)(C_o/C) + (147.07)$					
last two points		**146.8**	***92.4***		
Prompt Critical Increment (one dollar)					
least squares		0.22	*0.53*	{18.12}	-0.00665
last two points		0.21	*0.52*		

Apparatus Details		
table: light	pole hole: down	mount: none

Table H-15. Experimental Subcritical and Extrapolated Critical Parameters for an Enriched Uranium Hemispherical Shell of Inside Radius 90.104 mm but With Mild Steel Regions in Contact with the Uranium.

<table>
<tr><td colspan="2">inside
70.0</td><td colspan="2">Nominal Mild Steel Thickness Around Uranium (mm)</td><td colspan="2">outside
40.0</td></tr>
<tr><td rowspan="2">Actual System or Extrapolated Parameters</td><td colspan="4">Enriched Uranium Region</td><td rowspan="2">Reciprocal Multiplication
C_o/C</td></tr>
<tr><td>Number of Hemishells</td><td>Outside Radius (mm)</td><td>Total Mass (g) or (kg)</td><td>Density (mg/mm^3)</td></tr>
<tr><td>actual</td><td>4</td><td>103.336</td><td>14205</td><td>18.236</td><td>1.414</td></tr>
<tr><td>subcritical</td><td>8</td><td>116.670</td><td>32628</td><td>18.187</td><td>.951</td></tr>
<tr><td>reciprocal</td><td>12</td><td>129.996</td><td>55591</td><td>18.115</td><td>.483</td></tr>
<tr><td>multiplication</td><td>14</td><td>136.707</td><td>69162</td><td>18.111</td><td>.238</td></tr>
<tr><td>data</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="6">Delayed Critical Extrapolation</td></tr>
<tr><td>RFP-1021</td><td></td><td></td><td>82.9</td><td rowspan="5">{18.12}</td><td rowspan="5">0.0000</td></tr>
<tr><td>smooth curve</td><td>15.9</td><td>143.5</td><td>82.5</td></tr>
<tr><td>least squares</td><td></td><td>143.6</td><td>84.6</td></tr>
<tr><td colspan="4">$R = -(28.419)(C_o/C) + (143.60)$</td></tr>
<tr><td>last two points</td><td></td><td>143.2</td><td>83.7</td></tr>
<tr><td colspan="6">Prompt Critical Increment (one dollar)</td></tr>
<tr><td>least squares</td><td></td><td>0.19</td><td>0.44</td><td rowspan="2">{18.12}</td><td rowspan="2">-0.00665</td></tr>
<tr><td>last two points</td><td></td><td>0.18</td><td>0.43</td></tr>
<tr><td colspan="6">Apparatus Details</td></tr>
<tr><td colspan="2">table: light</td><td colspan="2">pole hole: down</td><td colspan="2">mount: none</td></tr>
</table>

M98052564

Report Number (14) INEEL/EXT-97-01401

Publ. Date (11) 199712

Sponsor Code (18) DOE/DP, XF

UC Category (19) UC-700, DOE/ER

DOE