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CP VIOLATION: THE STANDARD MODEL AND LEFT-RIGHT SYMMETRIC ELECTROWEAK MODELS

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ABSTRACT

We review and discuss CP violation in the minimal standard model of the electroweak interactions, and in general left-right symmetric electroweak models based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)$. We point out that in left-right symmetric models the contribution to ϵ' due to W_L-W_R mixing could be as large as the present experimental limit. Some other effects of W_L-W_R mixing are also considered.

INTRODUCTION

This year is the 20th anniversary of the discovery of CP violation. CP violation has been found so far only in the neutral kaon system, and there the only observed effect is a nonzero value[1]

$$|\epsilon_{expt}| = (2.28 \pm 0.05) \times 10^{-3}$$
$$Re\epsilon_{expt} = (1.64 \pm 0.06) \times 10^{-3} \tag{1}$$

of the parameter[2]

$$\epsilon \simeq \frac{e^{i\pi/4}}{\sqrt{2}} \left(\frac{m'}{\Delta m} + \frac{ImA_0}{ReA_0}\right) \tag{2}$$

$[A_\ell = A(K^0 \to 2\pi(I = \ell)),\ \Delta m \equiv m_{K_L} - m_{K_S} = 2ReA(K^0 \to \bar{K}^0)$, $m' \equiv ImA(K^0 \to \bar{K}^0)]$ involved in $K_L \to 2\pi$ decays. New measurements of the second parameter

$$\epsilon' \simeq \frac{1}{\sqrt{2}} e^{i(\delta_2-\delta_0+\pi/2)} \frac{ReA_2}{ReA_0} \left(\frac{ImA_2}{ReA_2} - \frac{ImA_0}{ReA_0}\right) \tag{3}$$

describing $K_L \to 2\pi$ decays have been reported at this conference. The present value is[3]

$$(\epsilon'/\epsilon)_{expt} = -0.0046 \pm 0.0053 \pm 0.0023 \tag{4}$$

(the first error is statistical, the second one is systematic), from which one can deduce[4]

$$-0.0163 < \epsilon'/\epsilon < 0.0078 \text{ (90\% C.L.)} \ . \tag{5}$$

The origin of CP violation is still unknown. But CP violation has become less a mystery, since in current theories there are many possible sources of CP violation. An important question is whether the observed CP violation is a manifestation of the usual weak interactions. In the first part of this talk I shall review briefly what one can say at present regarding this question. In the rest of the talk I shall discuss CP violation in general left-right symmetric electroweak models based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)$.

CP VIOLATION IN THE MINIMAL STANDARD MODEL

In the minimal standard model[5] (the standard model with one Higgs doublet) there is no CP violation in the Higgs sector, but for three generations the W-quark couplings involve a CP-violating phase.[6] The $\Delta S = -1$ Hamiltonian in the local limit is of the form

$$H^{\Delta S=-1} = f\left[(\hat{S} - \hat{P}) - iK'(\hat{S}_{c,t} - \hat{P}_{c,t})\right], \tag{6}$$

where

$$\hat{S}_{n,\ell} = \left(\{\bar{n}\gamma_\mu d, \bar{s}\gamma_\mu n\}_+ + \{\bar{n}\gamma_\mu\gamma_5 d, \bar{s}\gamma_\mu\gamma_5 n\}_+\right) - (n \to \ell)$$

$$\hat{P}_{n,\ell} = \left(\{\bar{n}\gamma_\mu d, \bar{s}\gamma_\mu\gamma_5 n\}_+ + \{\bar{n}\gamma_\mu\gamma_5 d, \bar{s}\gamma_\mu n\}_+ - (n \to \ell)\right) ,$$

$\hat{S} = \hat{S}_{u,c} + K\hat{S}_{c,t}$, $\hat{P} = \hat{P}_{u,c} + K\hat{P}_{c,t}$, $f = -2^{-3/2}Gs_1c_1c_3$, $K = s_2^2 + s_2c_2t_3c_\delta/c_1$, $K' = s_2c_2t_3s_\delta/c_1$ ($s_1 \equiv \sin\theta_1, t_3 \equiv \tan\theta_3$, etc.; δ is the CP-violating phase; the quark fields are interacting fields).[7] The contributions ϵ_{KM} and ϵ'_{KM} of the standard model to ϵ and ϵ' are proportional to the CP-violating parameter $K' \approx s_2s_3s_\delta$ and are given by[8]

$$\epsilon'_{KM}/\epsilon_{expt} = \left(-1/20\sqrt{2}\right) |\epsilon_{expt}|^{-1} (\mathrm{Im}A_0/\mathrm{Re}A_0) \approx 8.4\, s_2s_3s_\delta \quad , \tag{7}$$

$$\epsilon_{KM} = e^{i\pi/4}(s_2s_3s_\delta)\, BF\left(m_t,\, s_2(s_2 + s_3c_\delta)\right) \quad , \tag{8}$$

where B parameterizes the matrix element of the $\Delta S = 2$ operator and F is a function of the indicated quantities. The value of B has been extracted from $K \to \pi\pi$ data using PCAC and SU(3) symmetry, with the result $B \approx +0.33$ (Ref. 9). Equation (8) involves the assumption that the long-distance contribution to $\mathrm{Im}A(K^0 \to \bar{K}^0)$ can be ignored in the Wu-Yang phase convention.[10] The experimental bound on b-quark branching ratios $B(b \to u)/B(b \to c) < 0.05$ fixes F to be positive.[11]

The measured b-lifetime and the previous bound on the b-quark branching ratios set an upper bound on $s_2(s_2 + s_3c_\delta)$ and this implies an upper bound F_{max} on F (Ref. 12).

One has therefore $|s_2s_3s_\delta| > |\epsilon_{KM}|/BF_{max}$, so that

$$|\epsilon'_{KM}/\epsilon_{expt}| > 8.4\, |\epsilon_{KM}|/BF_{max} \quad . \tag{9}$$

Let us now assume that the standard model accounts for the observed CP violation, i.e., that $\epsilon_{KM} = \epsilon_{expt}$. Then (since $\mathrm{Re}\,\epsilon_{expt} > 0$) it follows from Eq. (8) that $s_2s_3s_\delta > 0$, implying that ϵ'/ϵ is positive.[13] Equation (9) now gives a lower bound

$$\epsilon'/\epsilon > 8.4\, |\epsilon|/BF_{max} \equiv (\epsilon'/\epsilon)_{min} \quad . \tag{10}$$

on ϵ'/ϵ.[11,12,8] $(\epsilon'/\epsilon)_{min}$ is a decreasing function of m_t. For m_t = 150 GeV one obtains $(\epsilon'/\epsilon)_{min} = 10^{-3}$, whereas m_t = 40 GeV corresponds to $(\epsilon'/\epsilon)_{min} = 3 \times 10^{-2}$ (see Ref. 8). The experimental limit (4) requires $m_t \gtrsim 60$ GeV ($m_t \gtrsim 35$ GeV if B = 0.66).

What would one conclude if a future measurement results in a value of ϵ'/ϵ that significantly violates the bound (10) for a known value of m_t? The possibilities include: our methods of calculating nonleptonic amplitudes are much less reliable than we presently believe; a fourth quark family may be present; a new interaction must exist to account for the observed CP violation. Note that even if the standard model is not fully responsible for ϵ_{expt}, it could still give a sizeable contribution (of arbitrary sign) to ϵ'.

Some further consequences of the standard model for CP violation:

- time-reversal-violating effects in leptonic and semileptonic processes are absent in first order in the weak interactions;
- the electric dipole moment of the neutron D_n is predicted to be $D_n \approx (10^{-29}-10^{-32})$ecm (assuming $\epsilon_{KM} = \epsilon_{expt}$).[14] It should be noted that if the P,T-violating term in the QCD Lagrangian is present, D_n would receive an additional contribution, which can be as large as the present experimental limit $[(D_n)_{expt} < 4 \times 10^{-25}$ (90% C.L.) (Ref. 15)];
- the CP-violating part of the quantity $\eta_{+-0} (\equiv \langle\pi^+\pi^-\pi^0|H_W|K_S\rangle / \langle\pi^+\pi^-\pi^0|H_W|K_L\rangle$ can be shown[16] (using PCAC and current algebra) to satisfy the relation $|\eta_{+-0}(0) - \epsilon| = (2/3)\, |\eta_{+-0} - \eta_{00}|$ $[\eta_{+-0}(0)$ is the value of $\eta_{+-0}(s_1, s_2 = s_1, s_3)$ at $s_3 = s_0]$.

Further CP-violating quantities of interest include the rate asymmetry $\Delta(\Lambda) \equiv (\Gamma - \bar{\Gamma})/(\Gamma + \bar{\Gamma})$ and the quantity $\Delta_\alpha \equiv (\alpha + \bar{\alpha})/(\alpha - \bar{\alpha})$ for $\Lambda \to p\pi^-$ and $\bar{\Lambda} \to \bar{p}\pi^-$ decays ($\alpha \equiv$ parity-violating asymmetry parameter), and the slope asymmetry $\Delta(\tau) \equiv [a(\tau^+) - a(\tau^-)]/[a(\tau^+) + a(\tau^-)]$ for $K^\pm \to \pi^\pm\pi^\mp\pi^\pm$ decays.[17]

The rate asymmetry $\Delta(\Lambda)$ can be written generally as[18-20]

$$\Delta(\Lambda \to p\pi^-) = \sqrt{2}\, |A_3^S/A_1^S|\, \sin(\delta_1 - \delta_3)\sin(\phi_1 - \phi_3)$$
$$\approx (5 \times 10^{-3})\sin(\phi_1 - \phi_3) \quad , \tag{11}$$

where A_1^S, A_3^S are the S-wave amplitudes for the I = 1/2 and I = 3/2 final states, δ_1, δ_3 are the corresponding final-state scattering phase shifts, ϕ_1, ϕ_3 are the phases of A_1^S and A_3^S. In the absence of CP violation A_1^S and A_3^S are real, and thus $\Delta(\Lambda) = 0$. Assuming the dominance of Penguin-type diagrams, $\Delta(\Lambda)$ is predicted[19-21] in the standard model to be $\Delta(\Lambda) \approx 10^{-2}\, s_2 s_3 s_\delta$. From Eqs. (5) and (7) one has $|s_2 s_3 s_\delta| \lesssim 2 \times 10^{-3}$, yielding $\Delta(\Lambda) \lesssim 2 \times 10^{-5}$.

Δ_α can be written as[18,19]

$$\Delta_\alpha \approx -\tan(\phi_S - \phi_P)\tan(\delta_1 - \delta_{11}) \approx -0.1\tan(\phi_S - \phi_P) \quad , \tag{12}$$

where ϕ_S and ϕ_P are the phases of A_1^S and of the P-wave amplitude A_1^P, respectively. $\Delta_\alpha = 0$ in the absence of CP violation. Although Δ_α would vanish in the approximation of Penguin dominance for all the amplitudes involved (since one would have $\phi_S = \phi_P$), one may have in general $\phi_S - \phi_P \approx \phi_S, \phi_P$.[21] Δ_α is then expected to be[21] (using for

ϕ_S the phase of the coefficient of the Penguin operator[8]) $\Delta_\alpha = -0.2\, s_2 s_3 s_\delta$, implying $|\Delta_\alpha| \lesssim 4 \times 10^{-4}$.

The slope asymmetry is given by[22]

$$\Delta(\tau) = \tan\delta \left[\frac{\mathrm{Im}(\beta_{1c} - \frac{1}{2}\beta_{3c})(2\alpha_{1c} - \alpha_{3c})^* + \sqrt{3}\,\mathrm{Im}\gamma_{3c}(2\alpha_{1c} - \alpha_{3c})^*}{\mathrm{Re}(\beta_{1c} - \frac{1}{2}\beta_{3c})(2\alpha_{1c} - \alpha_{3c})^* + \sqrt{3}\,\mathrm{Re}\gamma_{3c}(2\alpha_{1c} - \alpha_{3c})^*}\right], \tag{13}$$

where $\alpha_{\ell c}$, $\beta_{\ell c}$ ($\ell = 1,3$), and γ_{3c} are defined by $\langle\pi^+\pi^+\pi^-|H|K^+\rangle = i[(2\alpha_{1c} - \alpha_{3c}) + (\beta_{3c} - \frac{1}{2}\beta_{3c} + \sqrt{3}\,\gamma_{3c})y]$; $\alpha_{\ell c} = \alpha_\ell + i\alpha'_\ell$, $\beta_{\ell c} = \beta_\ell + i\beta'_\ell$ ($\ell = 1,3$), $\gamma_{3c} = \gamma_3 + i\gamma'_3$. The parameters α_ℓ, β_ℓ ($\ell = 1,3$), γ_3, and α'_ℓ, β'_ℓ ($\ell = 1,3$), γ'_3 describe the matrix elements of the CP-conserving and of the CP-violating Hamiltonian, respectively. They are real if final-state interactions are neglected. In Eq. (13) δ describes the final-state interactions in the I = 1 symmetric three-pion state. The final-state interaction in other states have been neglected for simplicity.

The slope asymmetry, like the quantity Δ_α, is not suppressed in general by the ratio of $\Delta I = 3/2$ and $\Delta I = 1/2$ matrix elements. In the standard model, however, $\Delta(\tau)$ vanishes approximately in the absence of $\Delta I = 3/2$ transitions. The commutator relation $[Q_5^k, H_{p.c.}] = -[Q^k, H_{p.v.}]$ [Q_k(Q_5^k) are the vector (axial-vector) charges], which holds in the standard model, makes it possible to express the $K^\pm \to 3\pi$ amplitudes in terms of the physical $K^\pm \to 2\pi$ matrix elements using soft-pion techniques. The soft-pion treatment leads to the relation[16] $\alpha'_1/\alpha_1 = \beta'_1/\beta_1$, which eliminates the contribution of the $\beta^*_{1c}\alpha_{1c}$ interference term in (13) (Ref. 23). The slope asymmetry is found to be[24] $\Delta(\tau) \approx 10\, \epsilon'_{KM} \tan\delta$, so that $|\Delta(\tau)| \lesssim (4 \times 10^{-4})\tan\delta$. The value of δ is not known. Using the estimate[25] $\tan\delta \approx a_0 m_\pi$, where a_0 is the I = 0 π-π scattering length, and the current algebra prediction[26] $a_0 \approx 0.20\, m_\pi^{-1}$, one obtains $|\Delta(\tau)| \lesssim 8 \times 10^{-5}$. The present experimental value is[27] $\Delta(\tau)_{expt} = -0.0070 \pm 0.0053$. A new experiment using present technology could improve the sensitivity by a factor of 10 to 100 (Ref. 28).

CP VIOLATION IN LEFT-RIGHT SYMMETRIC ELECTROWEAK MODELS

Although the standard model is consistent at present with all data including the observed CP violation, the electroweak interaction may yet turn out to be associated with a gauge group larger than $SU(2)_L \times U(1)$.

An attractive extension of the standard model is a description of the electroweak interactions in terms of a left-right-symmetric theory based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)$ (Ref. 29). The fermions in these models are assigned to representations of $SU(2)_L \times SU(2)_R \times U(1)$ in a left-right-symmetric manner: the left- (right-) handed fermions are doublets of $SU(2)_L$ [$SU(2)_R$] and singlets of $SU(2)_R$ [$SU(2)_L$] (Ref. 30). The presence of right-handed couplings allows for new sources of CP violation to be present, which might be responsible for the observed CP violation.[31,32] We note that studies of CP violation in these models are of interest regardless of whether or not they account for the observed CP violation, in particular since they give rise to new types of CP-violating effects.

The couplings of the charged gauge bosons W_L and W_R to the fermions are given by

$$\mathcal{L} = \frac{g_L}{2\sqrt{2}} W_L \bar{P} \Gamma_L U_L N + \frac{g_R}{2\sqrt{2}} W_R \bar{P} \Gamma_R U_R N + \text{leptonic terms} + \text{H.c.} \quad , \tag{14}$$

where $\Gamma_{L,R} = \gamma_\lambda(1 \mp \gamma_5)$ (the Lorentz indices have been suppressed), $\bar{P} = (\bar{u},\bar{c},\ldots)$, $\bar{N} = (\bar{d},\bar{s},\ldots)$; $u,d,\ldots$ are the quark mass eigenstates; U_L and U_R are the quark mixing matrices. They are unrelated, unless some ad hoc symmetries are introduced in the Higgs sector. We shall leave them to be independent. W_L and W_R are linear combinations of the mass eigenstates W_1 and W_2

$$\begin{aligned} W_L &= \cos\zeta\, W_1 + \sin\zeta\, W_2 \quad , \\ W_R &= e^{i\omega}(-\sin\zeta\, W_1 + \cos\zeta\, W_2) \quad . \end{aligned} \tag{15}$$

It is instructive to discuss first CP violation in the four-quark model and without W_L-W_R mixing.

A. Model for Two Generations with $\zeta = 0$

This is the original model, proposed by Mohapatra and Pati.[31] The most general form of U_L and U_R is given by [33,34]

$$U_L = \begin{pmatrix} \cos\theta_L, & \sin\theta_L \\ -\sin\theta_L, & \cos\theta_L \end{pmatrix}, \quad U_R = \begin{pmatrix} e^{i\alpha}\cos\theta_R, & e^{i\beta}\sin\theta_R \\ -e^{-i\gamma}\sin\theta_R, & e^{i(\beta-\alpha-\gamma)}\cos\theta_R \end{pmatrix}. \tag{16}$$

The $\Delta S = -1$ Hamiltonian is of the form

$$H^{\Delta S=-1} = [f_L + f_R\, e^{i(\alpha-\beta)}]\hat{S}_{u,c} - [f_L - f_R\, e^{i(\alpha-\beta)}]\hat{P}_{u,c} \quad , \tag{17}$$

where $f_{L,R} = (g_L^2/16m_{L,R}^2)\sin\theta_{L,R}\cos\theta_{L,R}$. The operators $\hat{S}_{u,c}$ and $\hat{P}_{u,c}$ are defined as in Eq. (6).

An immediate consequence of the Hamiltonian (17) is that $\varepsilon' = 0$ (Ref. 31), since the resulting A_0 and A_2 have the same phase[35]:

$$\text{Im}A_0/\text{Re}A_0 = \text{Im}A_2/\text{Re}A_2 = -r\sin(\alpha-\beta) \quad , \tag{18}$$

where

$$r = (g_R^2 m_L^2/g_L^2 m_R^2)\,(\cos\theta_R/\cos\theta_L)\,(\sin\theta_R/\sin\theta_L) \quad . \tag{19}$$

Stringent constraints on the parameters of the model come from the $K^0 \to \bar{K}^0$ amplitude.[36] With the matrices (16), the dominant contribution from right-handed currents (box diagrams involving W_L and W_R) has the form

$$A_{LR}(K^0 \to \bar{K}^0) = (-1/2)\Lambda r\, e^{i(\alpha-\beta)} A_{LL}(K^0 \to \bar{K}^0) \quad , \tag{20}$$

where A_{LL} is the usual contribution involving two W_L's and Λ is a factor found to be ~430 (Ref. 36).[37] From Eq. (20) one has

$$(\Delta m)_{LR} = [-Ar\cos(\alpha-\beta)](\Delta m)_{LL} \quad , \tag{21}$$

$$\epsilon_{LR} = -2^{-3/2}e^{i\pi/4}r\sin(\alpha-\beta)\,[2 + A/(1 - Ar\cos(\alpha-\beta))] \quad . \tag{22}$$

Requiring $(\Delta m)_{LR} < (\Delta m)_{expt}$ and $|\epsilon_{LR}| < |\epsilon_{expt}|$ yields[33]

$$|r|\,|\cos(\alpha-\beta)| \lesssim A^{-1} = 3\times 10^{-3} \quad , \tag{23}$$

$$|r|\,|\sin(\alpha-\beta)| \lesssim (3\times 10^{-3})(1 + A/2)^{-1} = 1.5\times 10^{-5} \tag{24}$$

and therefore also $$|r| \lesssim 3\times 10^{-3} \quad . \tag{25}$$

Note that for smaller A (Ref. 37) the above constraints would be weaker.

It follows that the model can account for the observed CP violation if $|r|$ is in the range $1.5\times 10^{-5} \lesssim |r| \lesssim 3\times 10^{-3}$ and $|\sin(\alpha-\beta)|\,(=1.5\times 10^{-5}/|r|)$ satisfies $5.7\times 10^{-3} \lesssim |\sin(\alpha-\beta)| < 1$. Such values of the parameters are not ruled out by any available data.[38] Some further consequences:

- in first order all P-conserving, T-violating observables in semileptonic processes vanish, and P,T-violating observables not involving neutrino polarization are proportional to neutrino masses;[33,38]
- D_n arises only in fourth order in the W-quark couplings.[31] The magnitude of this contribution is expected to be less than $\sim 10^{-26}$ ecm;
- $\eta_{+-0} = \eta_{000} = \epsilon_{expt} + 2ir\sin(\alpha-\beta)$ (Ref. 31), so that $\eta_{+-0} = \eta_{000} = \epsilon_{expt}$; $|\eta_{+-0} - \eta_{+-}| = 2|r\sin(\alpha-\beta)|$ (Ref. 31), implying $|\eta_{+-0} - \eta_{+-}| \lesssim 3\times 10^{-5}$ (the equality sign corresponds to $\epsilon_{LR} = \epsilon_{expt}$);
- the rate asymmetry (11) (as well as the rate asymmetries of other nonleptonic hyperon decays) vanishes, since $\phi_1 = \phi_3 = r\sin(\alpha-\beta)$;
- the phase difference $\phi_S - \phi_P$ in the expression for Δ_α [Eq. (12)] is $\phi_S - \phi_P = 2r\sin(\alpha-\beta)$. Consequently [see Eq. (24)] $|\Delta_\alpha| \lesssim 3\times 10^{-6}$ for $\Lambda \to p\pi^-$, $\bar\Lambda \to \bar p\pi^+$ decays;
- the slope asymmetry $\Delta(\tau)$ [Eq. (13)] in $K^\pm \to \pi^\pm\pi^\pm\pi^\mp$ decays vanishes, since all the independent $K^\pm \to \pi^\pm\pi^\pm\pi^\mp$ matrix elements acquire the same CP-violating phase.

B. Three-Generation Model with $\zeta = 0$

It is straightforward to extend the four-quark model to incorporate the third quark family.[33,34,39] For three generations U_L, U_R contain altogether six mixing angles and seven unremovable CP-violating phases. U_L can be written in the form of the usual Kobayashi-Maskawa matrix, containing three mixing angles θ_i^L (i = 1,2,3) and a CP-violating phase δ_L. U_R contains the mixing angles θ_i^R (i = 1,2,3) and the remaining phases.

Let us consider a scenario in which the Kobayashi-Maskawa-type CP violations are negligible so that the dominant CP violation is due to relative phases between the left- and right-handed couplings. One finds then[33]

$$\mathrm{Im}A_0/\mathrm{Re}A_0 = -r\sin(\alpha-\beta) + (K_R - K_L)hr\sin(\alpha-\beta) \qquad (26)$$

$$\mathrm{Im}A_2/\mathrm{Re}A_2 = -r\sin(\alpha-\beta) \quad , \qquad (27)$$

where $h \equiv f_L\langle\pi\pi(I=0)|\,\hat{P}_{c,t}\,|K^0\rangle/\mathrm{Re}A_0 \approx 0.5$ (from the results for $\mathrm{Im}A_0$ in the standard model) and r is given by Eq. (19) with $\sin\theta_m\cos\theta_m$ replaced by $(-\sin\theta_1^m)\cos\theta_3^m\cos\theta_1^m$ (m = L,R). K_L and K_R are defined as in Eq. (6), K_L (K_R) involving parameters from U_L (U_R). Thus, unlike in the four-quark model, ϵ' vanishes only if $\theta_1^L = \theta_1^R$ (Refs. 33 and 39). Equations (26) and (27) yield

$$|\epsilon'/\epsilon_{expt}| = 8.4\ |K_R - K_L|\ |r\sin(\alpha-\beta)| \quad . \qquad (28)$$

Assuming that there is no significant cancellation in $(\Delta m)_{LR}$ and ϵ_{LR} among terms involving different combinations of mixing angles or phases, nor with other possible contributions, the constraints (23) and (24) remain valid and in addition, the same constraints also hold for $|(K_R - K_L)r\cos(\alpha-\beta)|$ and $|(K_R - K_L)r\sin(\alpha-\beta)|$, respectively. As a consequence one finds[33]

$$|\epsilon'/\epsilon_{expt}| \lesssim \kappa h/10(\Lambda+2) \approx 1.3\times10^{-4}\,\kappa \quad , \qquad (29)$$

where $\kappa = |K_R - K_L|$ if $|K_R - K_L| < 1$ and $\kappa = 1$ for $|K_R - K_L| > 1$. The equality sign in Eq. (29) applies when the phase $(\alpha-\beta)$ accounts for ϵ_{expt}.[40]

For η_{+-0}, η_{000} one obtains[33]

$$\eta_{ijk}(0) = \epsilon_{expt} + 2ir\sin(\alpha-\beta) + (K_R - K_L)(z_{ijk} - h)ir\sin(\alpha-\beta) \quad , \qquad (30)$$

where $z_{ijk} \equiv \langle\pi_i\pi_j\pi_k|\,\hat{S}_{c,t}\,|K_1\rangle/\langle\pi_i\pi_j\pi_k|\,\hat{S}_L\,|K_2\rangle$. The results of Ref. 16 yield $z_{+-0} = z_{000} \approx 0.8\,\mathrm{Re}A_2/\mathrm{Re}A_0$, so that $\eta_{+-0} = \eta_{000} \approx \epsilon_{expt}$ and $|\eta_{+-0} - \eta_{+-}|$ is approximately of the order of 10^{-5} or less.

The rate asymmetry $\Delta(\Lambda)$ is given by $\Delta(\Lambda) \approx (5\times10^{-3})r\sin(\alpha-\beta)\,(K_R - K_L)\big(\langle p\pi^-|\hat{S}_{c,t}|\Lambda\rangle/\langle p\pi^-|\hat{S}_L|\Lambda\rangle\big)$, so that (using the standard-model results for the ratio of the matrix elements, and the bound $|K_R - K_L||r\sin(\alpha-\beta)| \lesssim 1.5\times10^{-5}$) $|\Delta(\Lambda)| \lesssim 2\times10^{-7}$. Similarly one obtains $\Delta_\beta(\Lambda) \lesssim 10^{-5}$, and $|\Delta(\tau)| \approx 0.2\tan\delta|(K_R - K_L)r\sin(\alpha-\beta)| \lesssim 3\times10^{-6}|\tan\delta|$.

The general implications for T violation in semileptonic processes and for D_n are the same as in the four-quark model.

C. Models with W_L-W_R Mixing

In general one expects the parameter ζ to be different from zero. We shall discuss the effects of W_L-W_R mixing for simplicity in the framework of the four-quark model. The essential conclusions would be the same in the six-quark model.

For $\zeta \neq 0$ the p.c. and p.v. $\Delta S = -1$ Hamiltonians acquire a new term

$$H'_{p.c.} = -f_L(g_R/g_L)\ \zeta\big([\cos\theta_R\, e^{i(\omega+\alpha)}/\cos\theta_L + \sin\theta_R\, e^{-i(\omega+\beta)}/\sin\theta_L]\hat{S}'_u$$

$$- [\cos\theta_R\, e^{-i(\omega+\beta-\alpha-\gamma)}/\cos\theta_L + \sin\theta_R\, e^{i(\omega-\gamma)}/\sin\theta_L]\hat{S}'_c\big) \quad (31)$$

and

$$H'_{p.v.} = f_L(g_R/g_L)\ \zeta\big([\cos\theta_R\, e^{i(\omega+\alpha)}/\cos\theta_L - \sin\theta_R\, e^{-i(\omega+\beta)}/\sin\theta_L]\hat{P}'_u$$

$$+ [\cos\theta_R\, e^{-i(\omega+\beta-\alpha-\gamma)}/\cos\theta_L - \sin\theta_R\, e^{i(\omega-\gamma)}/\sin\theta_L]\hat{P}'_c\big) \quad ,(32)$$

respectively, where $\hat{S}_n = \{\bar{n}\gamma_\mu d,\ \bar{s}\gamma_\mu d\}_+ - \{\bar{n}\gamma_\mu\gamma_5 d,\ \bar{s}\gamma_\mu\gamma_5 d\}_+$, and $\hat{P}'_n = \{\bar{n}\gamma_\mu d,\ \bar{s}\gamma_\mu\gamma_5 n\}_+ - \{\bar{n}\gamma_\mu\gamma_5 d,\ \bar{s}\gamma_\mu n\}_+$ $(n = u,c)$. The contribution $A'_{0,2}$ of (32) to $\text{Im}A_{0,2}$ is expected to be dominated by the W-exchange diagram for the $\bar{s}d \to \bar{u}d$ transition.[34] For ϵ', which no longer vanishes,[31,33,34,41] one obtains then

$$|\epsilon'| = 2^{-1/2}\ |\text{Re}A_2/\text{Re}A_0|\,|\chi|\,|\zeta g_R/g_L|$$

$$\times\ |\sin(\omega + \alpha)\cos\theta_R/\cos\theta_L + \sin(\omega + \beta)\sin\theta_R/\sin\theta_L| \quad , \qquad (33)$$

where $\chi \equiv \big(\langle\hat{P}'_u\rangle_2/\langle\hat{P}_{u,c}\rangle_2 - \langle\hat{P}'_u\rangle_0/\hat{P}_{u,c}\rangle_0\big)$ $\big(\langle\hat{P}_u\rangle_2 \equiv \langle 2\pi(I = 2)|\hat{P}'_u|K^0\rangle$, etc.$\big)$. Since the W-exchange diagram for $\bar{s}d \to \bar{u}d$ contributes to both $\Delta I = 1/2$ and $\Delta I = 3/2$ transitions, one expects $\text{Im}A'_0$ and $\text{Im}A'_2$ to be of the same order of magnitude, i.e., that $(\text{Re}A_2/\text{Re}A_0)\chi$ is of the order of 1. An estimate[34] yields $\chi \approx 12$. Using this value, the experimental limit (5) implies

$$|\zeta g_R/g_L|\ \ |\sin(\omega + \alpha)\cos\theta_R/\cos\theta_L + \sin(\omega + \beta)\sin\theta_R/\sin\theta_L| \lesssim 10^{-4} \quad . \qquad (34)$$

Among further consequences of W_L-W_R mixing is a neutron electric dipole moment generated in first order in the weak interactions.[41,42] Barring cancellations, the results of Refs. 41 and 42 and the present experimental upper limit on D_n (Ref. 15) imply[43]

$$|\zeta g_R/g_L|\ \ |\sin(\omega + \alpha)\cos\theta_R/\cos\theta_L| \lesssim 10^{-4} \qquad (35)$$

$$|\zeta g_R/g_L|\ \ |\sin(\omega + \beta)\sin\theta_R/\sin\theta_L| \lesssim 4 \times 10^{-3} \qquad (36)$$

$$|\zeta g_R/g_R|\ \ |\sin(\omega - \gamma)\sin\theta_R/\sin\theta_L| \lesssim 3 \times 10^{-4} \quad . \qquad (37)$$

A nonzero ζ gives rise to a time-reversal-violating triple correlation $\langle\vec{J}\rangle \cdot \vec{p}_e \times \vec{p}_\nu/E_eE_\nu$ in nuclear β decay with a coefficient proportional to $(\zeta g_R/g_L)\sin(\omega + \alpha)\cos\theta_R/\cos\theta_L$.[33,38] The present experimental limits[44] imply

$$|\zeta g_R/g_L|\,|\sin(\omega + \alpha)\cos\theta_R/\cos\theta_L| \lesssim 2 \times 10^{-3} \quad . \qquad (38)$$

The analogous correlation in semileptonic hyperon decays is proportional to $(\zeta g_R/g_L)\sin(\omega + \beta)\sin\theta_R/\sin\theta_L$.[33,38] An experimental study of $\Lambda \to pe^-\bar{\nu}$ decays[45] sets a weak upper bound, of the order of one, on this quantity.

The upper limits (35), (36), and (38) are the most stringent bounds at present on the quantities appearing in Eq. (33).[46] We conclude, therefore, that $\epsilon'/\epsilon_{expt}$ due to W_L-W_R mixing could be as large as the present experimental limit.

If a value of $\epsilon'/\epsilon_{expt}$ is found near the present limit and is due to the effects of right-handed currents, a T-violating correlation $\langle \vec{J} \rangle \cdot \vec{p}_e \times \vec{p}_\nu / E_e E_\nu$ with a coefficient of the order of 10^{-4} is expected either in nuclear β decay or in semileptonic hyperon decays, or both.[47] At the same time, a neutron dipole moment should be found near the present experimental limit.

Let us consider the slope asymmetry (13). Neglecting the contributions of the $\Delta I = 3/2$ component of the Hamiltonian, $\Delta(\tau)$ is given by

$$\Delta(\tau) \simeq \tan\delta\left(\frac{\beta_1'}{\beta_1} - \frac{\alpha_1'}{\alpha_1}\right) \quad . \tag{39}$$

Because of the terms (31) and (32), which are antisymmetric under the interchange of V and A, the commutator property $[Q_5^K, H_{p.c.}] = [Q^K, H_{p.v.}]$ does not hold. As a consequence, the relation $\beta_1'/\beta_1 = \alpha_1'/\alpha_1$, which holds in the standard model in the soft-pion limit, is not valid here. From Eqs. (31) and (39) we have

$$\Delta(\tau) = -(\tan\delta)(\zeta g_R/g_L)[\cos\theta_R \sin(\omega+\alpha)/\cos\theta_L - \sin\theta_R \sin(\omega+\beta)/\sin\theta_L]\Omega \quad , \tag{40}$$

where

$$\Omega = [(\langle M|\hat{S}_u'|K^+\rangle/\langle M|\hat{S}_{u,c}|K^+\rangle) - (\langle S|\hat{S}_u'|K^+\rangle/\langle S|\hat{S}_{u,c}'|K^+\rangle)] \quad . \tag{41}$$

($|S\rangle$ and $|M\rangle$ are the symmetric three-pion state and the three-pion state of mixed symmetry, respectively.)

The experimental limit $|\Delta(\tau)_{expt}| < 1.5 \times 10^{-2}$ (Ref. 27) implies (using $\tan\delta = 0.2$)

$$|\zeta g_R/g_L||\cos\theta_R \sin(\omega+\alpha)/\cos\theta_L - \sin\theta_R \sin(\omega+\beta)/\sin\theta_L| \lesssim (8\times 10^{-2})|\Omega|^{-1} \quad . \tag{42}$$

The operators $\hat{S}_u'$, $\hat{P}_u'$ have been encountered some time ago in the context of a pre-gauge-theory model of CP violation, due to Glashow.[48] A soft-pion calculation of $K \to 3\pi$ decays in this model[49] yielded large values, of the order of 20, for quantities like Ω. With $\Omega = 20$ one obtains

$$|\zeta g_R/g_L||\cos\theta_R \sin(\omega+\alpha)/\cos\theta_L - \sin\theta_R \sin(\omega+\beta)/\sin\theta_L| \lesssim 4\times 10^{-3} \quad , \tag{43}$$

which is a bound already comparable with some of the previous bounds we discussed. A calculation of Ω would clearly be of interest.

Barring cancellation in Eq. (34), the experimental limit on $|\epsilon'/\epsilon_{expt}|$ implies (with $\tan\delta = 0.2$ and $\Omega = 20$)

$$|\Delta(\tau)| \lesssim 4 \times 10^{-4} \quad . \tag{44}$$

Allowing for the possibility of cancellation in ϵ', we have to turn to the bounds (35), (36), and (38), which imply

$$|\Delta(\tau)| \lesssim (8 \times 10^{-4})|\Omega| \quad . \tag{45}$$

This upper bound is even for $|\Omega| \simeq 1$ just an order of magnitude below the present limit.

For the rate asymmetry and the quantity $\Delta_\alpha(\Lambda)$, inspection yields $\Delta(\Lambda) \lesssim 10^{-5}$–$10^{-6}$ and $\Delta_\alpha(\Lambda) \lesssim 10^{-4}$–$10^{-5}$, assuming that the ratios of the matrix elements of $\hat{S}'_u$ and $\hat{S}_{u,c}$, $\hat{P}'_u$ and $\hat{P}_{u,c}$ are of the order of one. The quantity $|\eta_{+-0} - \epsilon_{expt}|$ could be of the order of ϵ_{expt}.

In the above discussion we have treated the various parameters of left-right symmetric models as independent, invoking implicitly a Higgs sector as rich as necessary to guarantee this. In models with restricted Higgs sectors some of the parameters may be related. An example is the model of Chang,[34] which has a minimal Higgs sector and P and CP broken spontaneously. As a consequence, $\theta_L = \theta_R$ and $\alpha, \beta, \gamma \simeq (m_c/m_s)(\kappa'/\kappa)\sin\tilde{\alpha}$, and also $\zeta = (m_1^2/m_2^2)\kappa'/\kappa$ and $\omega = \tilde{\alpha}$ (κ, κ', and $\tilde{\alpha}$ parameterize the Higgs vacuum expectation value; $\tilde{\alpha}$ is a CP-violating phase, providing the only source of CP violation in the model; m_1, m_2 are the masses of W_1 and W_2). It follows that $\zeta(\omega + \alpha), \zeta(\omega + \beta), \zeta(\omega - \gamma) \simeq (m_s/m_c)(m_1^2/m_2^2)\sin(\alpha - \beta)$, and since $(m_1^2/m_2^2)\sin(\alpha - \beta)$ is constrained by Eq. (24), the effects of W_L-W_R mixing are correspondingly suppressed.[18]

CONCLUSIONS

Present experiments studying $K_L \to 2\pi$ decays have already reached the level where they can sensitively probe the question whether the standard electroweak model can be considered as a potential source of the observed CP violation. Not considering the possibility that the calculations involved may be less reliable than we can presently assess, the present limit on $\epsilon'/\epsilon_{expt}$ demands that the mass of the top quark be larger than about 40-60 GeV if the three-generation minimal model of the electroweak interaction is to remain a candidate for explaining the observed CP violation.[8]

Even if the standard model accounts for the observed CP violation, additional CP violation may be present from various possible extensions of the standard model. We discussed CP violation in left-right symmetric extensions of the standard model based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)$. These models can account for the observed CP violation even if W_L-W_R mixing and the effects of the third generation are negligible.

We have shown that in models that include W_L-W_R mixing and where no other constraints are imposed on the parameters than phenomenological, $\epsilon'/\epsilon_{expt}$ could be as large as the present experimental limit. A feature that distinguishes an $\epsilon'/\epsilon_{expt}$ originating from W_L-W_R mixing from an $\epsilon'/\epsilon_{expt}$ generated by the Kobayashi-Maskawa mechanism or by some other mechanisms is the presence of a time-reversal-violating triple correlation $\langle\vec{J}\rangle \cdot \vec{p}_e \times \vec{p}_\nu/E_eE_\nu$ in nuclear β decay or semileptonic hyperon decays, or both. If a value of $\epsilon'/\epsilon_{expt}$ is found near the experimental limit and is due to W_L-W_R mixing, this correlation in neutron decay, ^{19}Ne decay and/or Λ decay is expected at the level of $\sim 10^{-4}$. A nonzero $\epsilon'/\epsilon_{expt}$ near the present limit would also imply, if it is due to W_L-W_R mixing, a neutron electric dipole moment near the present limit, and a slope asymmetry in $K^\pm \to \pi^\pm\pi^\pm\pi^\mp$ decays that could be in the observable range.

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A further source of constraints on various CP-violating combinations of the parameters involving ζ is ε_{expt}. These constraints are, however, less stringent than the constraints (25) on r because ε (and Δm) is a function of ζ^2 rather than ζ (cf. Ref. 36). Limits on various CP-conserving combinations of the parameters are not better than a few times 10^{-3} [see J. F. Donoghue and B. R. Holstein, Phys. Lett. 113B, 382 (1982)].
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