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PROCEEDINGS

OF

SUMMER INSTITUTE ON PARTICLE PHYSICS

July 28 - August 8, 1986

PROBING THE STANDARD MODEL

Program Directors

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PREFACE

The fourteenth SLAC Summer Institute on Particle Physics was held from July 28 to August 8, 1986. Three hundred and forty-three physicists from 12 countries gathered to study the subject of "Probing the Standard Model." The format of past Institutes was followed in which the first seven days was organized as a "Summer School" and the last three as a "Topical Conference." The school focussed on tests of the "Standard Model" with excellent lectures by Haim Harari, Bruce Winstein, Rick Field, Jonathan Dorfan, Fred Gilman and Rafe Schindler. We also had a very topical set of lectures on Computing, Data Acquisition and Custom Design of Electronics by Paul Kunz, Martin Breidenbach and Ray Larsen. The latest results from theory and experiment were shared in the Topical Conference. The last afternoon of the Topical Conference was devoted to a Drell-a-bration, a tribute to Sid Drell, SLAC's Deputy Director, in celebration of his sixtieth birthday. His colleagues and former students gave talks on topics of particular interest to him.

We continued last year's experiment of having "provocateurs" for the afternoon discussion sessions, and would like to thank the following people for enlivening this year's afternoons: Jon Bagger, Ikaros Bigi, Greg Dubois, Paula Franzini, Howard Haber, Gail Hanson, Tom Himel, Marek Karliner, Michael Levi, Bryan Lynn, Usha Mallik, Sherwood Parker, Alfred Petersen, Blair Ratcliff, Jack Ritchie, Steve Shapiro, and Dave Sherden.

Finally, we would like to thank Eileen Brennan for organizing and running the meeting and for editing these proceedings. Her planning, organization and the hard work and good humor of she and her staff were central to the success of the Institute.

Gary Feldman
Frederick J. Gilman
David W. G. S. Leith
Program Directors

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SUMMER SCHOOL LECTURES



Electroweak Interactions - Standard and Beyond*

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ABSTRACT

Several important topics *within* the standard model raise questions which are likely to be answered only by further theoretical understanding which goes *beyond* the standard model. In these lectures we present a discussion of some of these problems, including the quark masses and angles, the Higgs sector, neutrino masses, W and Z properties and possible deviations from a pointlike structure.

Lectures delivered at the 1986 SLAC Summer Institute

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I. OVERTURE

I.1 Plan of Lectures

It is often claimed that the *standard* model is fully understood and that the physics *beyond* it is essentially unknown. However, somewhere between "*standard*" and "*beyond*" there is a border area - a gray fringe containing topics which are part of the standard model but which are far from being well understood. These lectures are devoted to several such topics, including the systematics of fermion masses and angles, the Higgs sector, neutrino physics, W and Z properties and possible deviations from a pointlike behavior.

We do not attempt to discuss in detail any specific theory which goes beyond the standard model. However, such theories often have important implications for the topics listed above. Consequently, we will comment on some of these implications whenever necessary, referring to general classes of theories including left-right symmetric models, grand-unified theories, Supersymmetry, horizontal symmetries, composite models and the so-called "String Inspired Phenomenology" (SIPH).

The present chapter deals with preliminaries, introducing the standard model, its parameters, its theoretical "loose ends" and the classes of theories which go beyond it. We also introduce the general ground rules of SIPH. We then move on to five specific topics.

The first topic deals with fermion masses and mixing angles. In the most minimal version of the standard model there are 18 arbitrary parameters. Of these, 13 arise from the quark and lepton mass matrices, corresponding to 9 masses, 3 angles and one phase. In chapter II we present a brief discussion of several issues related to these poorly understood parameters.

Chapter III deals with the Higgs particles, their properties, their accompanying Higgsinos in Supersymmetric theories and the possible existence of super-

multiplets including quarks, leptons and Higgsinos.

Chapter IV is devoted to neutrinos, their masses, neutrino oscillations in vacuum and in matter, the recent proposal concerning the solar neutrino puzzle and the neutrino spectrum in SIPH.

Chapter V deals with future probes of the W and Z bosons, including the possibility of additional Z 's in SIPH and in Left-Right-Symmetric theories. We also discuss the possibility of a substructure for W and Z and deviations from the normal W and Z gauge couplings.

We conclude with a final chapter discussing the future of multi-TeV accelerator physics in view of the decreasing cross sections for all processes among pointlike particles. We consider the possibility of future deviations from a pointlike behavior.

I.2 Counting the Parameters of the Standard Model

The *minimal* version of the standard model is based on the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ with three generations of quarks and leptons and one physical Higgs particle. If we assume that there are no right-handed neutrinos and that there is no strong CP violation, the minimal model contains 18 arbitrary parameters:

- (i) Three gauge couplings for the three gauge groups. These can be chosen e.g. as g_1 , g_2 , g_3 or α_s , $\sin^2 \theta_W$. At present energies g_1 and g_2 are of the same order of magnitude but g_3 is substantially larger (or, equivalently, $\alpha_s > \alpha$, $\sin^2 \theta_W \sim O(1)$).
- (ii) Two parameters representing the Higgs sector, even in the absence of fermions. These can be chosen, e.g. as M_W and M_Δ or, alternatively, as $\langle \phi \rangle$ and M_Δ . We have no detailed information about M_Δ but expect it to be within one order of magnitude from M_W .

- (iii) Nine masses for the six quarks and three charged leptons of the three generations. The nine mass values are spread over at least five orders of magnitude.
- (iv) Three generalized Cabibbo angles for the quark sector.
- (v) One Kobayashi-Maskawa (KM) phase for the quark sector.

The possible existence of "strong" CP-violation would add a 19th arbitrary parameter (whose value must be tiny).

Of these parameters, three are fundamental gauge couplings representing the strength of the three fundamental interactions at present energies. We may hope to relate them to each other only if we succeed in unifying two or more of these interactions. The other 15 parameters are related, in one way or another, to the Higgs sector of the theory. Their origin is obscure. Any attempt to reduce the number of these parameters must involve a deeper understanding of the symmetry breaking mechanism of the model.

The standard model may have several different extensions which will add no fundamental new physics but will increase the number of arbitrary parameters. The three most direct extensions are the following:

- (i) *Neutrino masses.* If neutrinos are not exactly massless, we start by adding three additional neutrino mass parameters. However, the existence of these masses opens the door to generation mixing among the leptons, allowing for three leptonic Cabibbo angles and one leptonic KM phase. If the neutrinos have both Dirac and Majorana masses, the number of parameters is even larger, but, in that case, additional Higgs (and possibly Goldstone) particles must exist. The existence of non-vanishing neutrino masses therefore adds at least seven new parameters, possibly many more.
- (ii) *Additional Higgs particles.* The standard model may include any number of Higgs doublets without changing its main features. However, the introduction of such additional doublets opens the way to a variety of additional

terms in the Higgs potential. The couplings of the new Higgs fields as well as their masses and vacuum expectation values are additional free parameters.

- (iii) *Additional generations.* It is entirely possible that additional generations of quarks and leptons, following the pattern of the first three generations, will be discovered. Since we have no reason to expect precisely three generations, we should not consider the possible existence of additional generations as a major extension of the model. However, a fourth generation will add nine additional arbitrary parameters (if all neutrinos are massless) and at least fourteen parameters if neutrinos have masses.

We therefore conclude that even the least controversial extensions of the standard model are likely to increase its number of arbitrary parameters to anywhere between 25 and 40.

It would have been bad enough if the standard model included 18 or 25 or 40 arbitrary parameters whose observed experimental values obeyed some simple patterns. For instance, a reasonable unbiased guess in the minimal standard model would suggest that M_W , M_Z and all quark and lepton masses are roughly of the same order of magnitude. If that were the case, we might still wonder about the origin of so many independent parameters but their general behavior would have posed no striking puzzles. Instead, we have $\frac{m_t}{M_W} \approx 6 \times 10^{-6}$, quark masses ranging from 4 MeV to at least 30 GeV, etc. Not only we cannot calculate the various parameters, we have no understanding of their general orders of magnitude and no explanation for the observed hierarchy of masses.

1.3 "Loose Ends" of the Standard Model

There are no experimental facts which force us to go beyond the standard model (with the possible exception of the non-vanishing baryon number of the universe) and there are no theoretical internal inconsistencies within the frame-

work of the model. Consequently, all possible motivations for expecting physics beyond the standard model are, to a certain extent, a matter of taste. Nevertheless, there is a wide-ranging consensus among high energy physicists that there must be some new physics beyond the standard model. Every physicist may have his or her own list of motivations. I present here my own list:

- (i) *The Fine Tuning Problem.* Why is M_t of order M_W and not of order M_{Planck} ?
- (ii) *The Generation Puzzle.* Why do we have several generations? What distinguishes among them? Why do w have different orders of magnitudes for fermion masses in different generations?
- (iii) *The Quark-Lepton Connection.* How are the quarks and the leptons related to each other? Why do they have simple charge ratios? What is the origin of the miraculous anomaly cancellation between quarks and leptons in one generation?
- (iv) *The Origin of P and CP Violation.* Is parity conserved at short distances? Is it broken explicitly or spontaneously at present energies? Why are neutrinos massless or very light? What is the origin of "weak" and "strong" CP-violation?
- (v) *The Unification Problem.* Can we unify the three basic fundamental interactions of the standard model, represented by the three commuting gauge groups?
- (vi) *The Gravity Connection.* Can we construct a quantum field theory of gravitational interactions and relate it to the three interactions of the standard model? If we can do it, can we relate the physics of the Planck scale to the physics of present energies or to anything which may become accessible experimentally within the next few decades?
- (vii) *"The 4-3-2-1 Puzzle".* Why 4 dimensions of space-time? Why $SU(3)$? Why $SU(2)$? Why $U(1)$?

None of these questions can be answered by the standard model. All of them require explanation. All such explanations can only come from some new physics beyond the standard model. The new physics is likely to emerge from a combination of new theoretical ideas and, most important, new experimental results which will indicate deviations from the predictions of the standard model. It is disappointing that no such experimental results exist at the moment. We must hope that they will appear soon. We now turn to a brief discussion of the general classes of experiments which may give us the required hints for the new physics.

1.4 Common Features of "Beyond Standard" Theories

Several theoretical approaches and numerous explicit models based on these approaches have been proposed for describing the physics beyond the standard model. All of them assume that the standard model will remain an excellent approximation at low energies (say, below $E \sim O(M_W)$). Among the various approaches we might mention Technicolor, Horizontal Symmetries, Left-Right Symmetry, Supersymmetry, Grand Unification, Compositeness of Quarks, Leptons and possibly W and Z, and – last but not least – Superstring Theory. Many models are based on combinations of the above ideas (e.g. Supersymmetric Grand Unified Theories with Horizontal Symmetries, etc.). What is common to all of these approaches is the existence of a new fundamental underlying theory, valid at energies well above present energies, leading to an effective low energy approximation which is consistent with the standard model.

In all "beyond standard" theories there is always a new high energy scale $\Lambda \gg M_W$ which characterizes the new Lagrangian. In some cases we may have several such scales, all larger than M_W . At energies around or above Λ many new phenomena are always expected. In particular, every theory predicts a large number of new particles with $M \sim O(\Lambda)$. We do not know the value (or values) of Λ . It can be anywhere between $O(\text{TeV})$ and M_{Planck} . It is not likely to be below

1 TeV or else some indirect effects would have probably been already observed.

If and when we can do experiments at $E \sim O(\Lambda)$, there would be no great difficulty in discovering evidence for the new physics. We would be able to produce particles with $M \sim O(\Lambda)$ and will directly observe effects due to any possible new fundamental interactions. However, within the next 20 years, experiments at $E \sim O(\Lambda)$ will be possible only if $\Lambda \sim 1 \text{ TeV}$ and a multi-TeV collider is constructed. If Λ is larger and/or if such a collider is delayed, we will be reduced to experimentation at energies well below Λ .

At such energies we can still learn about the new "beyond standard" physics by using indirect experimental methods. The crucial ingredient here is the existence, in all "beyond standard" theories, of particles which are much lighter than Λ . Such particles are *approximately massless* on the Λ scale. We have mentioned earlier that the typical expected mass scale for all particles is actually $O(\Lambda)$. How do we then obtain approximately massless particles in all "beyond standard" models?

The answer is simple and well-known. A theory with a typical scale Λ allows massless particles if and only if there is some symmetry principle which protects these particles from acquiring a mass. We know at least four such principles:

- (i) *Gauge Symmetry.* An unbroken gauge symmetry provides us with a mechanism for obtaining massless vector particles (e.g. photons, gluons and possible technigluons or hypergluons).
- (ii) *Chiral Symmetry.* A chiral symmetry may prevent spin $\frac{1}{2}$ particles from acquiring a mass (e.g. left-handed neutrinos in a minimal standard model without right-handed neutrinos).
- (iii) *Goldstone mechanism.* A spontaneously broken global symmetry will produce massless spin 0 Goldstone bosons (such as pions, axions, majorons etc.).
- (iv) *Supersymmetry.* The above three mechanisms can produce massless parti-

cles with spins $1, \frac{1}{2}, 0$ respectively. When combined with supersymmetry, any one of them will lead to additional massless particles with "neighboring" spin values. Thus a massless spin- $\frac{1}{2}$ fermion can occur as a result of chiral symmetry or it can be the supersymmetric partner of a massless gauge boson or a massless Goldstone boson.

It is therefore not too difficult to produce particles with masses well below Λ . In fact, it is fairly easy to account for *exactly* massless particles. It is much more difficult to allow for particles which have small, finite masses.

The existence of particles with masses which are much smaller than Λ enables us to describe physics phenomena at energies well below Λ in terms of a "low-energy" effective Lagrangian. Such a Lagrangian will, however, contain traces of the original theory at the Λ scale which led to it. All experiments at energies below Λ relate to this phenomenological effective Lagrangian.

1.5 Classes of Experiments which Probe "Beyond Standard" Theories

How can we probe a new theory which corresponds to a characteristic energy scale Λ by performing experiments at energies well below Λ ?

We assume that some new theory, beyond the standard model, is described at $E \sim O(\Lambda)$ by a new fundamental Lagrangian $\mathcal{L}_{NEW}$. At energies $E \ll O(\Lambda)$, $\mathcal{L}_{NEW}$ leads to a low-energy effective Lagrangian $\mathcal{L}_{EFF}$ which describes low energy processes involving particles whose masses obey $M \ll O(\Lambda)$. The effective Lagrangian may be schematically written as:

$$\mathcal{L}_{EFF} = \mathcal{L}_{SM} + \mathcal{L}_{LP} + \mathcal{L}_{HD} + \mathcal{L}_G$$

Each of the components of $\mathcal{L}_{EFF}$ represents a large class of low energy phenomena which can be tested experimentally. We now consider these terms:

- (i) $\mathcal{L}_{SM}$. This is the Standard Model Lagrangian. The experimental success of the standard model tells us that $\mathcal{L}_{EFF}$ cannot be very different from $\mathcal{L}_{SM}$ or else we would easily detect experimental phenomena which go beyond the standard model. We therefore conclude that the most obvious and severe constraint on $\mathcal{L}_{NEW}$ is that it must lead to a low-energy Lagrangian which approximately reproduces the standard model.
- (ii) $\mathcal{L}_{LP}$. These are terms in the Lagrangian representing interactions of additional Light Particles with couplings of "normal" strength (i.e. coupling constants comparable to ordinary standard model couplings). Such particles have masses $M \ll O(\Lambda)$ but do not appear in the standard model. They may correspond to simple extensions of the standard model, such as an additional generation of quarks and leptons or additional Higgs doublets. They might also correspond to entirely new features which are required in some "beyond standard" models. The most important example of such particles are the supersymmetric partners of all standard model particles which appear in supersymmetric theories. If a given $\mathcal{L}_{NEW}$ leads to terms of the type $\mathcal{L}_{LP}$, there is no *a priori* reasons for $\mathcal{L}_{LP}$ to be less significant than $\mathcal{L}_{SM}$. However, since experimentally there is no evidence for $\mathcal{L}_{LP}$, we must conclude that its additional light particles must be somewhat heavier than the corresponding particles in $\mathcal{L}_{SM}$. The possible discovery of a $\mathcal{L}_{LP}$ term will not enable us to determine the value of Λ but will certainly provide us with hints concerning possible extensions of the standard model or some of the features of the new Lagrangian $\mathcal{L}_{NEW}$.
- (iii) $\mathcal{L}_{HD}$. These are High-Dimension terms ($d > 4$), reflecting the new physics at $E \sim O(\Lambda)$. A typical term in $\mathcal{L}_{HD}$ would be an effective four-fermion term of dimension six, preceded by a coefficient of order $\frac{1}{\Lambda^2}$:

$$\mathcal{L}_{HD} \sim \frac{g^2}{\Lambda^2} \bar{f}_i f_j \bar{f}_k f_l.$$

There could also be $\mathcal{L}_{HD}$ terms of dimension 5, 7 or more, always accom-

panied by coefficients which are inversely proportional to positive powers of Λ . Such terms may lead to additional, nonstandard, contributions to existing processes (such as $e^+ + e^- \rightarrow e^+ + e^-$) or to new processes which cannot occur in the standard model (such as $p \rightarrow e^+ + \pi^0$ or $\mu \rightarrow e + \gamma$). In the first case we expect to observe experimental deviations from quantitative predictions of the standard model. In the second case we should begin to observe processes for which we presently have only experimental upper limits. In both cases the amplitudes are substantially smaller than typical standard model amplitudes because of the $\frac{1}{\Lambda^2}$ factor in $\mathcal{L}_{HD}$. Any observation of an experimental effect due to a $\mathcal{L}_{HD}$ term will provide us with some information on Λ . A precise determination of Λ requires knowledge of the effective coupling g .

- (iv) $\mathcal{L}_G$. These are terms of dimension four, involving Goldstone or pseudo-Goldstone particles which are generated by symmetry breaking at a scale Λ . Their (Yukawa) couplings are inversely proportional to Λ . Typical examples involve particles such as axions, majorons, familons etc. Their couplings to fermions are typically of the form:

$$\mathcal{L}_G = \frac{m_f}{\Lambda} \chi \bar{f} f$$

where χ is the Goldstone (or pseudogoldstone) particle. Such couplings are clearly weaker than ordinary standard model couplings. For sufficiently large values of Λ they cannot be detected in terrestrial experiments and the only available information may come from astrophysical and cosmological arguments.

From the above analysis it is quite clear that we have several classes of experiments which may probe the new physics of the Λ scale at energies well below Λ . Some of these experiments can actually be performed at fairly low energies (e.g. measuring $(g-2)_{e,\mu}$ probes terms of the $\mathcal{L}_{HD}$ type). Others may not even require accelerators (e.g. searches for proton decay). All present experiments and most

experiments in the next few decades will belong to the classes of experiments described here. We will somehow have to learn about $\mathcal{L}_{NEW}$ by probing $\mathcal{L}_{EFF}$.

1.6 String Inspired Phenomenology (SIPH)

String theory is certainly the first serious candidate for a quantum theory of gravity with the added attraction of an intimate connection between gravity and other gauge interactions. The theory itself is remarkable. However, it relates to a ten-dimensional world with 496 gauge-group generators and it makes predictions for physics at the Planck scale. The standard model describes physics at an energy scale which is 17 orders of magnitude lower in a world with four dimensions and with twelve gauge bosons. Somehow we must learn to make the transition between these two situations.

Eventually one would hope to derive the number of space-time dimensions at our present energies as well as all the parameters of the standard model (including the choice of its gauge group) from the fundamental ten-dimensional theory at the Planck scale. At the present time we do not know how to do this. We are therefore reduced to *assuming* that somehow six of the ten dimensions "compactify" and that, after the compactification, we remain with a gauged subgroup of the original large gauge group which was, presumably, $E_8 \times E_8$. The leading candidate for the subsidiary gauge group is E_6 whose 27-dimensional multiplets allegedly contain quarks, leptons, Higgsinos and their supersymmetric partners. The E_6 symmetry is broken to a subgroup containing $SU(3)_c \times SU(2)_L \times U(1)$. That subgroup may include at least one (possibly two) extra neutral Z' , and perhaps an extra $SU(2)$.

There is no proof that the above scenario is a necessary consequence of the $E_8 \times E_8$ heterotic string theory. On the contrary, it is perfectly possible that the remaining gauge group after compactification is $SU(5)$ or $SO(10)$ or some other group. It is also possible that the extra Z boson (or bosons) will appear at intermediate energies between the Planck scale and M_W , leaving no observable effects at present energies. On the other hand, there is no clear experimental

evidence against the E_6 scenario and it may be useful to pursue it both as an example of a String Inspired Phenomenology (SIPH) or merely as a candidate Grand Unified Theory with no Strings attached.

We will therefore discuss here and in the next chapters several implications of this possible phenomenology.

The ground rules are the following¹: We assume that the *gauge couplings* obey the relations of an E_6 symmetry. There are 78 gauge bosons, corresponding to the adjoint representation of E_6 . The corresponding gauginos are also in a 78 of E_6 . All other fermions (quarks, leptons and Higgsinos) are in 27 or $\overline{27}$ representations of E_6 . We will usually discuss only the three "normal" 27 representations and ignore the possible $\overline{27}'$ s. There are also corresponding 27's containing the supersymmetric partners: squarks, sleptons and Higgses.

The masses of all gauge bosons, matter fermions and their corresponding supersymmetric partners may be anywhere between the present low-energy scale and the Planck scale. The 27 of E_6 has the following $SO(10)$ decomposition:

$$27 = 16 + 10 + 1$$

while the $SO(10)$ representations contain the following $SU(5)$ multiplets:

$$16 = 10 + \overline{5} + 1; \quad 10 = 5 + \overline{5}$$

It is clear that only 15 of the 27 states correspond to the usual quarks and leptons. They, together with an additional neutrino N , form the 16 dimensional multiplet of $SO(10)$. The remaining 11 fermions are:

- (i) A charge $-\frac{1}{3}$ quark, usually denoted by g , belonging to the 5 of $SU(5)$ and to the 10 of $SO(10)$.
- (ii) The antiquark $\bar{g}$.
- (iii) A doublet of Higgsinos H^+, H^0 in the 5 of $SU(5)$ and 10 of $SO(10)$.

(iv) The antiparticles of the above Higgsinos.

(v) An extra neutrino-like particle S , which is an $SU(5)$ and $SO(10)$ singlet.

The g -quark is "dangerous" because it may lead to proton decay as well as to flavor-changing neutral currents. Most models would suggest that its mass is well above M_W , probably around the GUT scale. We will return to the neutrino spectrum in section IV.4. The remaining 19 states may be light and they may form three generations of quarks, leptons, Higgsinos and their associated supersymmetric partners.

Strictly speaking, in String models the Higgs particles need not belong to the same 27 multiplets as the matter fermions. They may be part of "incomplete" multiplets and their Yukawa couplings need not obey the usual E_6 relations.

However, the Higgs states must belong to 27 or $\bar{27}$ representations, in contrast with the usual situation in Grand Unified Theories ($SU(5)$, $SO(10)$ or E_6) in which the Higgs particles belong to representations which are different from those of the matter fermions.

It is important to understand that what we call SIPH is not a well-defined framework and that any confirmation or failure of its predictions will neither confirm nor destroy the fundamental concepts of String Theory. Unfortunately, the predictive power of String Theory is still unsatisfactory. On the other hand, SIPH leads to a variety of interesting and useful phenomenological observations which may be helpful, independently of the validity of String Theory. We return to some of them in every one of the following chapters.

II. MASSES, ANGLES AND PHASES – STANDARD AND BEYOND

II.1 Experimental Values

In the minimal standard model we have 13 parameters representing the fermion masses (nine parameters), mixing angles (three) and KM-phase (one). If

we assume that the top quark will be found somewhere in the 30-60 GeV range and that the observed value of ϵ in the $K^0 - \bar{K}^0$ system is fully accounted for by the KM-phase, we can quote either precise values or reasonable estimates for all of these parameters. The values are (all masses in GeV units):

First generation masses: $m_u = 0.004$; $m_d = 0.007$; $m_e = 0.0005$.

Second generation masses: $m_c = 1.3$; $m_s = 0.15$; $m_\mu = 0.1$.

Third generation masses: $m_t = 45 \pm 15$; $m_b = 5$; $m_\tau = 1.8$.

Mixing between adjacent generations: $\theta_{12} = 0.22$; $\theta_{23} = 0.05$.

Mixing between "distant" generations: $\theta_{13} \sim 0.01$.

KM-phase: $\delta \sim 90^\circ \pm 30^\circ$.

A few comments are in order:

- (i) All masses of the five known quarks are approximate, but their order of magnitude is correct and we will not need here more than that.
- (ii) The top-quark mass may still turn out to be above the range listed here. In fact, the only upper limit² we have is $m_t \leq 180 \text{ GeV}$, obtained from the maximum value of $m_t - m_b$ consistent with the present experimental value of the parameter $\rho = \frac{M_W^2}{M_Z^2 \cos^2 \theta_W}$.
- (iii) We are using a choice of mixing angles which is the most convenient for most considerations. We will comment on this issue in detail in section II.3.
- (iv) Direct searches for $b \rightarrow u$ transitions give only an upper limit for θ_{13} . No direct determination of δ is available. However, if we want to explain the observed value of ϵ and the upper limit on $\frac{\Gamma(b \rightarrow u)}{\Gamma(b \rightarrow c)}$ in terms of a minimal three-generation standard model without additional Higgs particles or "beyond standard" physics, we must assume θ_{13} and δ values in the general range quoted above.

Consequently, all the above numbers should be considered as an approximate representation of the emerging pattern of mass, angle and phase values. Some

modifications are still possible, especially as a result of "beyond standard" ideas.

In searching for regularities among the above 13 parameters, it is instructive to consider the *dimensionless ratios of fermion masses in different generations*. In fact, we may wish to consider the following:

Quantities relating generations 1 and 2:

$$\sqrt{\frac{m_u}{m_c}} = 0.06; \sqrt{\frac{m_d}{m_s}} = 0.22; \sqrt{\frac{m_e}{m_\mu}} = 0.07; \theta_{12} = 0.22.$$

Quantities relating generations 2 and 3:

$$\sqrt{\frac{m_c}{m_t}} \sim 0.18; \sqrt{\frac{m_s}{m_b}} = 0.18; \sqrt{\frac{m_\mu}{m_\tau}} = 0.24; \theta_{23} = 0.05.$$

Quantities relating generations 1 and 3:

$$\sqrt{\frac{m_u}{m_t}} \sim 0.01; \sqrt{\frac{m_d}{m_b}} = 0.04; \sqrt{\frac{m_e}{m_\tau}} = 0.017; \theta_{13} \sim 0.01.$$

II.2 Numerology

So far, no one has offered a satisfactory explanation for the observed pattern of masses and angles. Eventually, we might hope that some new physics will enable us to calculate the exact values of some or all of these parameters. But before we attempt to do that, we should have at least some qualitative understanding of the general orders of magnitude and the observed hierarchy of mass values. Here we are essentially reduced to naive "numerological" attempts and to possible relations between mass ratios and mixing angles.

In order to pursue some of these attempts, we should first inspect the observed pattern of the parameter values and try to identify simple regularities.

A brief inspection indicates that all mixing angles and square roots of mass ratios connecting *adjacent* generations are of order $\frac{1}{10}$. In fact, if we arbitrarily define a parameter $\alpha = 0.1$, the following empirical relations are not too wrong:

$$\theta_{ij} \sim O(\alpha^{|i-j|}); \quad \sqrt{\frac{m_i}{m_j}} \sim O(\alpha^{|i-j|}).$$

We will refer to this empirical pattern as "Numerology I".

A somewhat more detailed numerical observation is the fact that all the above mass ratios and angles actually cluster around three values: 0.2; 0.05; 0.01. Consequently, one may introduce a parameter λ such that $\lambda \sim 0.22$ and:

$$\sqrt{\frac{m_d}{m_s}} \sim \sqrt{\frac{m_c}{m_t}} \sim \sqrt{\frac{m_s}{m_b}} \sim \sqrt{\frac{m_\mu}{m_\tau}} \sim \theta_{12} \sim \lambda;$$

$$\sqrt{\frac{m_u}{m_c}} \sim \sqrt{\frac{m_e}{m_\mu}} \sim \sqrt{\frac{m_d}{m_b}} \sim \theta_{23} \sim \lambda^2;$$

$$\sqrt{\frac{m_u}{m_t}} \sim \sqrt{\frac{m_e}{m_\tau}} \sim \theta_{13} \sim \lambda^3.$$

We refer to this empirical pattern as "Numerology II".

"Numerology I" is a simple, easy to remember pattern. However, it is correct only within factors of two. "Numerology II" is much more accurate, but seems to follow an irregular pattern.

At present, the above numerical observations are useful either as a simple method of remembering the orders of magnitude of the parameters or as an approximation procedure for certain calculations, keeping terms up to a certain order of α or λ . There is no convincing explanation or theoretical foundation for the observed pattern.

As we will see in the next sections, these naive numerical observations may provide us with some guidance in attempting to obtain relations between mass ratios and mixing angles.

11.3 A Recommended Choice of Mixing Angles and Phases

The mixing among the three generations of quarks is defined by a unitary 3×3 matrix V whose matrix elements can be parametrized in terms of the three generalized Cabibbo angles and a single KM-phase. In the general case of N generations, we have an $N \times N$ matrix, described in terms of $\frac{1}{2}N(N-1)$ angles and $\frac{1}{2}(N-1)(N-2)$ phases. Clearly, there are many ways of choosing the angles and phases. Among the well-known choices: the original KM-choice³ (probably the least convenient for any purpose), the Maiani choice⁴ (convenient for angles but less so for phases), the Wolfenstein choice⁵ (convenient for phases but based on "Numerology I" for angles) and others. We strongly recommend that the standard choice of angles and phases become the choice first introduced by Chau and Keung⁶ for three generations (incorporating the main ideas of both Maiani and Wolfenstein) and later generalized⁷ to the case of N generations.

In this choice every angle has a clear and direct relation to one matrix element of the matrix V . All angles are denoted by θ_{ij} (for any $j \cdots i > 0$), representing the mixing among generations i and j . Each phase is denoted by δ_{ij} (for any $j \cdots i > 1$), and the related $e^{i\delta_{ij}}$ factor always multiplies the corresponding $\sin \theta_{ij}$. Assuming that the pattern of "Numerology I" persists in the general case of N generations, the above choice of parameters obeys, for any N and for all $j \cdots i > 0$:

$$V_{ij} = s_{ij}(1 + O(\alpha^4))$$

where $s_{ij} = \sin \theta_{ij}$ for $j \cdots i = 1$ and $s_{ij} = \sin \theta_{ij} e^{i\delta_{ij}}$ for $j \cdots i > 1$. In practice, this means that all V_{ij} values above the main diagonal are given, to an accuracy of three or more significant figures, by the corresponding values of s_{ij} .

The explicit form of the matrix V in the case of three generations is:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}^* & c_{12}c_{23} - s_{12}s_{23}s_{13}^* & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}^* & -c_{12}s_{23} - s_{12}c_{23}s_{13}^* & c_{23}c_{13} \end{pmatrix}$$

where $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij} e^{i\delta_{ij}}$.

A detailed discussion of our recommended choice of parameters can be found in Reference 7.

11.4 A Fourth Generation?

There is no known fundamental reason for the existence of three generations of quarks and leptons. There is no good argument for or against the existence of additional generations. Accepting the pattern of "Numerology I", we would guess that the mixing angles of a possible fourth generation with the first two are probably very small: $\theta_{14} \sim 10^{-3}$, $\theta_{13} \sim 10^{-2}$. It is unlikely that the fourth generation will have a substantial influence on low energy quantities involving the first two generations, with the possible exception of C^{12} -violating amplitudes in the $K^0 - \bar{K}^0$ system. Even in this latter case, we do not expect fourth generation effects to dominate, but they may lead to terms which are comparable to those induced by the third generation particles.

There are several interesting experimental and theoretical constraints concerning a possible fourth generation:

- (i) The UA1 collaboration⁸ obtained a lower limit of 41 GeV for the mass of a possible fourth generation charged lepton σ . Note that this limit already indicates that

$$\frac{m(\sigma)}{m(\tau)} > \frac{m(\tau)}{m(\mu)}$$

while the $\frac{\tau}{\mu}$ mass ratio is smaller than the $\frac{e}{\mu}$ mass ratio.

- (ii) Cosmological and astrophysical considerations seem to limit the number of light neutrino generations to at most four.
- (iii) Measurements of the Z width should soon provide us with strong limits on the number of light neutrinos. At present, however, they cannot compete with the astrophysical bound.

- (iv) The present value of the ρ -parameter in the standard model is within 2% from its expected value of 1. This leads to an upper limit⁹ on the mass difference between the two quarks of a hypothetical fourth generation. We obtain⁷

$$m(t') - m(b') < 180 \text{ GeV}.$$

If we assume (for no good reason) that the approximate relation $\frac{m(c)}{m(s)} \sim \frac{m(b)}{m(t)}$ carries over to a fourth generation, we conclude that $m(t')$ must be below 200 GeV. However, we cannot exclude heavier $t' - b'$ pairs which are almost degenerate.

- (v) If the mass difference within a hypothetical fourth generation of quarks allows the decays $t' \rightarrow b' + W^+$, $t' \rightarrow b' + \phi^+$ where ϕ^+ is a charged Higgs particle, such decays should dominate over the usual weak decays $t' \rightarrow b' + c^+ + \nu_c$, $t' \rightarrow b' + u + \bar{d}$.
- (vi) If quark masses in the fourth generation exceed a few hundred GeV's, the Yukawa couplings of these quarks may become strong, leading to a variety of unpleasant effects of the so-called "strong weak interactions". However, such a situation cannot be excluded and it may very well happen.

Our overall conclusion from the above assortment of comments is the following: There is no need for additional generations. If they exist, they are not likely to solve or to illuminate any presently existing problem in the standard model. Extrapolating present mass patterns and using various bounds it is reasonable to guess that at most one additional generation exists. If it does, the t' -mass should not be too far from 200 GeV and the b' -quark and the σ -lepton would be lighter than the Z .

II.5 Why Do We Expect Relations Between Masses and Angles?

Within the standard model, all masses and angles are free parameters. There

are no relations among them. It appears that the standard model would remain self-consistent for any set of mass and angle values.

Within some new "beyond standard" theory which describes physics at a high energy scale Λ , we may be able to calculate *all* the masses, angles and phases, starting from some new set of (hopefully few) fundamental parameters.

Until such a time comes, it may be interesting to try to find some relations among the observed mixing angles and the pattern of masses. We may not be able to derive the masses and the angles from first principles, but we may be able to relate quantities which we do not yet know to compute.

Why do we believe that such relations must exist? Within the standard model, there are several "low-energy" quantities which we can calculate both in the tree approximation and in higher orders. We often discover that some low-energy quantity depends on the masses of intermediate particles which can be exchanged in a one-loop diagram. That, by itself, is no surprise. However, our physics intuition tells us that it is unlikely that a low-energy quantity will become indefinitely larger if the mass of such an intermediate particle increases. Such is the case at least in three simple examples which we now list:

- (i) $\Delta M(K_S^0 - K_L^0)$. In this case the contribution of the top quark is such (because of the GIM mechanism) that for $m_t \rightarrow \infty$ we find $\Delta M \rightarrow \infty$.
- (ii) $\mu \rightarrow e + \gamma$. Here, again, a GIM mechanism operates. The rate of the process depends on the masses of intermediate neutrinos in a way which does not disappear for $m_\nu \rightarrow \infty$.
- (iii) The ρ -parameter of the standard model gets a contribution⁹ from any pair of quarks with charges $\frac{2}{3}$, $-\frac{1}{3}$ which have a non-vanishing coupling to W^+ (in other words: when the relevant mixing angle does not vanish). Here, again, we may consider e.g. the contribution of a loop with a t-quark and a d-quark. If we hold everything else fixed and send the t-quark mass to infinity, we obtain a divergent contribution to M_W (and to ρ).

In all of these cases, there is a very simple way out of the paradox. The contribution of the intermediate quark or lepton is always multiplied by a mixing angle. If we assume that the mixing angle *must* decrease when the fermion mass increases to infinity, we will encounter no difficulty whatsoever. Thus, for instance, if the angle θ_{23} is proportional to $\sqrt{\frac{m_t}{m_t}}$, the contribution of the t-quark to $\Delta M(K_S^0 - K_L^0)$ will not "explode" when $m_t \rightarrow \infty$. Similarly, if $\theta_{13} \rightarrow 0$ fast enough for $\frac{m_d}{m_t} \rightarrow 0$, the t-quark contribution to the ρ -parameter will not "explode".

We have therefore reached a remarkable conclusion: We have supplemented the standard model by a simple physical assumption stating that low-energy quantities must remain stable when masses of intermediate particles in higher order corrections increase indefinitely. We then find that this simple assumption forces us to have relations between masses and angles. More specifically: It tells us that mixing angles between a given pair of generations must decrease when the mass ratios of the fermions in the same generations decrease. We cannot derive a precise relation but the necessity of having some such relation is a significant result.

Since both the masses and the angles are obtained in the standard model from the mass matrices (which, in turn, are based on the Yukawa couplings of the Higgs fields), we must therefore conclude that *within the mass matrices*, some new symmetries or relations must exist. It is possible that some elements of the mass matrices vanish because of some new symmetry or that some otherwise unrelated matrix elements become related as a result of some new principle. Only such relations can yield the necessary connections between masses and angles.

We can now formulate two approaches to the problem of understanding the observed values of the masses, angles and phases:

- (i) *The theoretical approach.* We search for the new theory, discover the new Lagrangian $\mathcal{L}_{NEW}$, derive the new symmetries which appear in the mass matrices and find the resulting relations among masses, angles and phases.

- (ii) *The phenomenological approach.* We start from the observed pattern of masses and angles. Assuming what we earlier called "Numerology I" or "Numerology II" and imposing mass-angle relations of the type suggested above (i.e. $\theta_{ij} \sim \sqrt{\frac{m_i}{m_j}}$) we search for simple patterns in the mass matrices. On the basis of these, we guess the new symmetry or principle and then, hopefully, try to start building a convincing new model for the new physics at the high-energy scale.

Clearly, the first method is superior, if we can pursue it. No one has succeeded in doing so. The second method is less ambitious and much less profound. Several interesting attempts have been made along its lines but no great success can be reported. In the following section we briefly review some such attempts, mainly in order to show the type of work that can be done, at present.

II.6 Playing with Mass Matrices

Consider the quark mass matrix for the case of three generation. For simplicity, we assume that all mass matrices are Hermitian (in general they are not, but we are *only illustrating the methods here*). The simplest game one can play is to assume that certain matrix elements vanish (presumably as a result of a new symmetry of the Higgs Yukawa couplings). With a sufficient number of vanishing matrix elements, one can derive new relations between masses and angles.

The best known ansatz is the one proposed by Fritzsch¹⁹ several years ago. According to his hypothesis, the 3×3 mass matrices for the up and down sectors have the form:

$$M_u = \begin{pmatrix} 0 & X_u & 0 \\ X_u^* & 0 & Y_u \\ 0 & Y_u^* & Z_u \end{pmatrix}; \quad M_d = \begin{pmatrix} 0 & X_d & 0 \\ X_d^* & 0 & Y_d \\ 0 & Y_d^* & Z_d \end{pmatrix}.$$

In this case we can express all masses, angles and phases in terms of eight real parameters. Since we have ten measurable quantities (six masses, three angles

and one phase) we may obtain two relations. These relations are, at present, consistent¹¹ with the available experimental information.

Another ansatz, based on a different theoretical motivation has been proposed by Stech.¹² He postulates different forms for the mass matrices in the up and down sectors. According to Stech:

$$M_u = S; \quad M_d = \beta S + A$$

where S and A are, respectively, a symmetric and an antisymmetric 3×3 matrix. Here, again, we are able to describe the ten measurable quantities in terms of a smaller number of parameters, obtaining relations which are, at the present time, consistent with the data.

Using the empirical fact that:

$$\frac{m_u}{m_c} \ll \frac{m_d}{m_s}$$

we obtain from both the Fritzsch ansatz and the Stech ansatz:

$$\theta_{12} \sim \sqrt{\frac{m_d}{m_s}}$$

We also obtain, for the Fritzsch case:

$$\theta_{23} \sim \sqrt{\frac{m_s}{m_b}} - \sqrt{\frac{m_c}{m_t}}$$

and for the Stech case:

$$\theta_{23} \sim \sqrt{\frac{m_s}{m_b}} - \frac{m_c}{m_t}$$

All of these results are consistent with the data. Moreover, both schemes provide us with an explanation to one interesting feature of the pattern of masses and angles. We have noticed in section II.2 that θ_{23} was significantly smaller than

θ_{12} . In fact, in what we called "Numerology II" we gave them the values $\theta_{23} \sim \lambda^2$, $\theta_{12} \sim \lambda$. Now we learn that the smallness of θ_{23} is related to the similar values of $\frac{m_s}{m_b}$ and $\frac{m_c}{m_t}$ while the difference between $\frac{m_s}{m_b}$ and $\frac{m_c}{m_t}$ is related to the fact that θ_{12} is larger. Thus, both the Fritzsch and the Stech guesses account for an important regularity in "Numerology II".

This is precisely the type of qualitative features which we may be able to understand by "playing" with mass matrices.

Another interesting exercise was recently proposed by Gronau et al.¹³ They subscribe to *both* the Fritzsch and the Stech hypotheses and combine them to suggest the following mass matrices:

$$M_u = \begin{pmatrix} 0 & A & 0 \\ A & 0 & B \\ 0 & B & C \end{pmatrix}; \quad M_d = \theta \begin{pmatrix} 0 & A & 0 \\ A & 0 & B \\ 0 & B & C \end{pmatrix} + \begin{pmatrix} 0 & ia & 0 \\ -ia & 0 & ib \\ 0 & -ib & 0 \end{pmatrix}.$$

Here all masses, angles and phases are expressed in terms of only six real parameters (A, B, C, θ, a, b) and the predicted relations are still in reasonable agreement with the existing data.

The above "games" can teach us something about the physics beyond the standard model only if they can be based on some reasonable theoretical foundations. Typically, one would have to introduce some kind of a "horizontal symmetry" according to which different generations are labeled by different values of a new (spontaneously broken) quantum number. By applying such a symmetry to the fermion sector and to the Higgs sector, one immediately obtains selection rules preventing certain Higgs particles from coupling to certain fermions, depending on their generation. In this way we obtain vanishing matrix elements in the mass matrices, leading to one pattern or another.

Unfortunately, all the "horizontal symmetries" which were suggested so far, appear to be fairly artificial in the sense that they are designed to produce a specific ansatz for the mass matrices without explaining or solving other important issues of the standard model. Nevertheless, we believe that the problem of

masses and angles is so important that we should continue to pursue it even at the simple-minded level described here with the hope of obtaining some clues to the real mystery behind the experimentally observed pattern. In section III.3 we will briefly return to this problem, suggesting yet another simple form for the mass matrices.

II.7 Masses, Angles and SIPH

String theories lead to a new interesting explanation for the existence of generations. The observed particles, which are approximately massless on the Planck scale, are assumed to be the so-called "zero-modes" of the string, obtained after the compactification of the ten-dimensional space-time into the usual four dimensional space-time. The most widely discussed version of String Theory is the one based on an $E_8 \times E_8$ gauge group, where the remaining four-dimensional symmetry is a broken E_6 . In that case, all matter fermions (as well as their supersymmetric partners) belong to 27 and 27 representations of E_6 . The number of 27-dimensional multiplets and the number of the 27 multiplets are determined by the topological properties of the relevant compactified manifold. We are then naturally led to the existence of several massless multiplets of fermions. This is considered a great triumph for string theories, since it provides us for the first time with a theoretical reason for the replication of fermions which have a small mass on the Planck scale.

Models can be constructed with almost any number of generations. In particular there are several variants which lead to exactly three generations of quarks and leptons. In principle, the theory should dictate the number of generations. However, at the present time we can only *choose* a solution with the correct number of generations and study it.

The theory does not provide us with a clear indication of the features that distinguish between particles belonging to different generations. There is no explicit quantum number which labels the generations in such models.

We also have no idea on how to proceed with an explicit calculation of the quark and lepton masses. In principle, the Yukawa couplings of all Higgs fields are computable from the fundamental string interactions. The mass values should then follow. In practice, not only we do not know how to derive the relevant numbers but we also do not yet have a qualitative argument either for the observed scale of the fermion masses ($10^{-18} - 10^{-22}$ in Planck units) or for the mass ratios among different generations (i.e. our "Numerology I and II").

One is reduced again, at least for the time being, to "playing games" with symmetries of the mass matrices. The first interesting attempt in that direction, within the framework of SIPH, was recently made by Greene et al.¹⁴ They considered a three generation model with a Z_3 discrete symmetry, in which E_6 is broken into a Grand Unified $SU(3) \times SU(3) \times SU(3)$ gauge group, which is then spontaneously broken into a subgroup containing the standard model. They obtain quark mass matrices which are consistent with the observed masses, but the only explicit prediction is $m_u = 0$, which can be considered satisfactory or not, depending on one's point of view. We hope that future analysis of mass matrices along similar lines will enable us to derive additional relations among masses, angles and phases and eventually lead even to a complete calculation of the parameters of the standard model. This seems, however, a distant goal.

III. HIGGS - STANDARD AND BEYOND

III.1 Introduction

Twenty years ago, in 1966, some of the most interesting puzzles in physics were the following:

- The $\mu - e$ puzzle.
- Calculating the proton-neutron mass difference.
- Calculating the Cabibbo angle.

- The origin of CP violation.
- Why are the neutrinos massless?

Today, twenty years later, we still face the same five questions. The first three have now been generalized, respectively, to “the generation puzzle”, understanding the quark masses, and calculating the generation mixing angles. The last two problems remain unchanged (except that we now have three neutrinos).

The common feature of all of these old unsolved problems is their direct dependence, in the standard model, on the Higgs parameters of the theory. The Higgs sector remains the most mysterious sector of the standard model.

In the minimal version of the standard model we have only one physical neutral Higgs particle. In simple extensions of the standard model we may have one or several Higgs particles; they may be neutral or charged; they may be light or heavy; their interactions may be weak or strong. There are no experimental hints concerning any of these properties. Some of these questions are extremely important and their answer may have a deep influence on the future of theoretical and experimental particle physics.

A trivial example can illustrate this: Imagine a situation in which we have at least two Higgs doublets. This is the case in many “beyond standard” models. It follows that we must have at least one charged positive Higgs particle ϕ^+ . Assume further that the top quark is found below the W -mass and that the mass of the physical ϕ^+ obeys:

$$m_t - m_b > M_{\phi^+}.$$

For m_t anywhere between 30 GeV and M_W , there is a fair chance that the above inequality is obeyed. In such a case, the dominant decay of the t -quark is likely to be $t \rightarrow b + \phi^+$. Most produced t -quarks will yield charged Higgs particles. This would certainly change both our plans for studying t -decays and our hopes for performing Higgs-physics experiments.

More significant consequences occur if Higgs particles are very heavy and if they have strong interactions.

The present chapter deals with an assortment of topics related to the Higgs sector of the standard model, using hints which may be derived from “beyond standard” ideas.

III.2 One or Many? Charged or Neutral? Light or Heavy?

The first question we wish to address is : Do we have one physical Higgs particle or several?

Allowing ourselves complete freedom with the choice of Yukawa couplings, the standard model is perfectly self-consistent with having one physical Higgs particle whose Yukawa couplings differ from each other by at least five orders of magnitudes (if neutrinos are massless) and possibly by nine or more orders of magnitudes (if neutrinos have small Dirac masses). This is extremely unattractive but not necessarily incorrect.

The success of the Weinberg mass relation $M_W = M_Z \cos \theta_W$ requires that the only Higgs bosons contributing to the W and Z masses are $SU(2)$ doublets. We may have as many doublets as we wish without influencing this relation. Higgs triplets or higher $SU(2)$ representations would have to have extremely small vacuum expectation values or else their contributions to the W and Z mass will destroy the mass relation. Singlet Higgs fields are harmless but also useless, with one exception: We may have an $SU(2)$ -singlet which transforms nontrivially under some other symmetry (gauged, global or discrete), serving as a symmetry breaking mechanism for that other symmetry without influencing the Weinberg mass relation or other features of the standard model. Such is the case in Left-Right Symmetric theories, in some Grand Unified Theories, in majoron schemes and in Horizontal symmetry schemes.

If we have more than one doublet we immediately face several consequences:

- (i) We must have charged Higgs bosons. Those are easier to detect experimentally.
- (ii) We may have several scales for the vacuum expectation values of the different Higgs fields. This may allow us to have Yukawa couplings which cover a much smaller range of values, the hierarchy of quark and lepton masses being attributed to the different vev's rather than to different Yukawa couplings.
- (iii) The GIM mechanism is not guaranteed in a model with many Higgs multiplets. The additional Higgs doublets must obey certain constraints¹⁵ or be sufficiently heavy to avoid flavor changing neutral transitions.
- (iv) We have an additional source of CP violation, beyond the KM-phase. It is logically possible that CP-violation is due *only* to the Higgs sector.¹⁶ However, since we know that we have at least three generations and there is no reason to assume that the KM-phase is small, we assume that part of the CP-violating effects is definitely due to the KM-phase while another part may be due to multi-Higgs effects. The interplay between these two sources of CP-violation has not been sufficiently studied¹⁷ and we suspect that a complete understanding of CP-violation in light hadron processes may not be possible without it.
- (v) There are several quantities related to "beyond standard" physics which depend on the number of Higgs fields. A well-known example is the SU(5) prediction for the proton lifetime which decreases by almost a full order of magnitude with every additional Higgs doublet,¹⁸ as a result of the Higgs influence on the rate in which the coupling constants "run". There are several other examples of such a dependence.

We should note at this point that practically *all* "beyond standard" models predict the existence of several Higgs multiplets. Since we are fairly confident that there is physics beyond the standard model and that some of the present ideas are likely to be among the correct ingredient of a future "beyond standard"

theory, we argue that the existence of several Higgs particles (including charged ones) is almost certain. We believe that it does not make too much sense to rely on predictions which are based on the single Higgs hypothesis.

The unknown mass of the Higgs particle(s) leads to another well-known ambiguity which we will not discuss in these notes. It is well known that as the Higgs mass approaches the *TeV* range, Higgs couplings must become strong,¹⁹ leading to copious production of Higgs particles and longitudinal *W*'s,²⁰ to possible bound states of Higgs particles and to an entirely new range of hadron-like physics at the *TeV* scale. This possibility may become even more complicated if additional generations of quarks and leptons exist with masses larger than 0.5 *TeV* or so. In that case, the Yukawa couplings may become large, preventing us from using perturbative methods, and adding the heavy quarks and leptons⁹ to the list of particles "enjoying" strong weak interactions. Note, however, that such heavy quarks and leptons must come in approximately degenerate pairs, implying the existence of heavy left-handed neutrinos.

III.3 Generations of Higgs Particles?

We can think of at least two motivations for considering the possibility that Higgs particles, like quarks and leptons, come in generations. The first motivation is based on supersymmetry. In supersymmetric models we must have Higgses and Higgsinos, leptons and sleptons. All of them may be SU(2) doublets. There is no fundamental difference between the properties of Higgsinos and leptons, or between Higgses and sleptons. While we do not know the reason for the existence of repetitive generations, these reasons may apply equally to leptons and to Higgsinos, leading to generations of quarks, leptons and Higgsinos, accompanied by the corresponding squarks, sleptons and Higgses.

In such a case, each generation would have to include at least the usual quarks and leptons and four Higgsinos arranged in two doublets $(\hat{H}^+, \hat{H}^0), (\hat{H}^0, \hat{H}^-)$.

Counting left-handed states only (particles and antiparticles) we obtain a minimum of 19 states per generation (instead of the usual 15).

A second, independent motivation for suggesting generations of Higgs fields follows from the observed mass hierarchy of the fermions in the three known generations. The different energy scales for the fermion masses in different generations may be due to:

- (i) Yukawa couplings of different orders of magnitude (as in the minimal standard model).
- (ii) Several Higgs fields possessing vev's of different orders of magnitude.
- (iii) Masses in different generations being due to different powers of the same vev.

The last two possibilities seem to be more natural than the first one. The second possibility requires that the masses in a given generation are actually dominated by the vev of a corresponding Higgs field, with all Yukawa couplings being of the same order of magnitude.

It is interesting to ask whether it is possible to construct a phenomenological model in which Higgs fields appear in generations, possessing the same generation labels as ordinary quarks and leptons.

We have studied this possibility and found some interesting consequences. We assume that all quarks and leptons possess some generation label X such that the three generations are labeled by $X = X_1, X_2, X_3$. We then assume that we also have three Higgs doublets with the same X -values, respectively, belonging to the same generations. The label X may correspond to a gauged quantum number, a global symmetry or a discrete symmetry. The X -symmetry must, of course, be spontaneously broken by the three Higgs doublets, leading to non-diagonal mass matrices, generation mixing, etc. However, the Lagrangian, including all Yukawa couplings, is assumed to conserve X -symmetry.

We then find that there is only one set of X -values (up to a multiplicative

factor) which does not lead to contradictions or to trivial solutions. It is:

$X_1 = 2; X_2 = 1; X_3 = 0$. The resulting quark or lepton mass matrix has the form:

$$M_{u,d} = \begin{pmatrix} 0 & 0 & \lambda_1 \langle \phi_1 \rangle \\ 0 & \lambda_1' \langle \phi_1 \rangle & \lambda_2 \langle \phi_2 \rangle \\ \lambda_1 \langle \phi_1 \rangle & \lambda_2 \langle \phi_2 \rangle & \lambda_3 \langle \phi_3 \rangle \end{pmatrix}.$$

If we now assume:

$$\frac{\langle \phi_1 \rangle}{\langle \phi_2 \rangle} \sim \frac{\langle \phi_2 \rangle}{\langle \phi_3 \rangle} \sim \alpha \sim 0.1,$$

we obtain mass values and mixing angles which obey the general pattern suggested by "Numerology I" in section II.2, i.e.

$$\theta_{ij} \sim O(\alpha^{|i-j|}); \quad \sqrt{\frac{m_i}{m_j}} \sim O(\alpha^{i-j}).$$

Furthermore, we find that the predictions of such a scheme are consistent with the known values of the masses and angles. It should be interesting to pursue this form of the mass matrices and to see whether it leads to useful constraints among masses and angles.

The above pattern of Higgs fields and quarks and leptons in each generation should appear in some Grand Unified Supersymmetric models. We have already indicated that the smallest number of left-handed fermions in each generation in such a model must be 19. We do not know of any reasonable gauge group which has a 19 or a 20-dimensional representation, consistent with the above set of quantum numbers. The smallest schemes which can accommodate two Higgs doublets and a full generation of quarks and leptons in one large multiplet are E_6 and its subgroup $SU(3) \times SU(3) \times SU(3) \times Z_3$. Both of these groups appear in SIPH models in which we encounter 27-dimensional multiplets containing the above 19 or 20 states. The remaining states in these multiplets are g , $\bar{g}$ and S (see section I.6).

IV. NEUTRINOS – STANDARD AND BEYOND

IV.1 Massless or Light?

Neutrinos are either exactly massless or extremely light. If they are exactly massless, there must be a symmetry principle which prevents them from acquiring a mass to all orders in the standard model and in any possible “beyond standard” physics. We do not know any such symmetry and no one has made a proposal for it. If neutrinos are very light, compared with all other quarks and leptons, there still must be a convincing reason which explains why it is the neutrino and no other particle, which happens to be so light. Fortunately, there is such a class of mechanisms and they apply only to neutrinos. Only the neutrinos can have both a Dirac and a Majorana mass, leading to a 2×2 mass matrix even in the case of one generation. If, for some reason, the right-handed neutrino has a large Majorana mass (corresponding to the scale Λ of some new physics) and the left-handed neutrino have no Majorana mass (or a tiny Majorana mass), we obtain a mass matrix of the form:

$$\begin{pmatrix} 0 & m \\ m & M \end{pmatrix}$$

with eigenvalues:

$$m(\nu_1) \sim \frac{m^2}{M}, \quad m(\nu_2) \sim M.$$

The eigenstates ν_1, ν_2 are approximately equivalent to ν_L, ν_R , respectively. The Majorana mass M originates from the new physics at the scale Λ , and the Dirac mass m is, presumably, of the same order of magnitude as the mass of the corresponding charged lepton.

The above mechanism²¹ (often referred to as the “see-saw” mechanism) appears in Left-right Symmetric models, in some Grand Unified Theories (especially various versions of $SO(10)$) and in some Horizontal Symmetry schemes. In all

of these cases the standard model provides the Dirac masses for the neutrinos and no Majorana mass. The Majorana mass is due to some “beyond standard” effects at the Λ scale and it applies to the right-handed neutrino and not to the left-handed one because of their different transformation properties under the gauge group in question. The scale Λ must be at least $O(2 \text{ TeV})$ in LRS models, approximately $O(10^{15} \text{ GeV})$ in GUTs and could be somewhere in between in Horizontal Symmetry schemes. The present experimental upper limits on the masses of the three neutrinos imply only $M \geq 50 \text{ GeV}$. Note that M need not be equal to Λ . In fact, if M is due to some Higgs field with a vev of order Λ , we expect $M = \bar{A}\Lambda$ where $\bar{A}$ is a Yukawa coupling of that Higgs field. Since Yukawa couplings are likely to be smaller than one, we expect $M \leq \Lambda$.

The above argument, which is quite general, leads us to suspect that neutrinos are actually light but not massless and that they enjoy both Majorana and Dirac masses, leading to one extremely light and one heavy particle for each neutrino generation. The existence of neutrino masses immediately implies a large number of additional effects including neutrino oscillations, leptonic Cabibbo-like angles, leptonic KM phase, possible neutrino decays, etc.

IV.2 Neutrino Oscillations in Vacuum and in Matter

Assuming that neutrinos have small masses, we may express the mass eigenstates ν_i for $i = 1, 2, 3$ in terms of the “weak” eigenstates ν_e, ν_μ, ν_τ . If we neglect the small mixing of antineutrinos into the ν_i eigenstates (as a result of the “see-saw” mechanism), we obtain a unitary 3×3 matrix similar to the matrix V discussed for the quark sector in section II.3. There is no reason to assume that the leptonic generation mixing angles vanish. If they do not, we expect neutrino oscillations. A beam of $\bar{\nu}_e$, say, ν_e may eventually contain ν_μ s or ν_τ s as a result of such oscillations. The effect may be amplified when the neutrinos go through dense matter,^{22,23} in analogy with the familiar situation in the $K^0 - \bar{K}^0$ system.

In order to understand the main physics features of this effect, let us consider

the case of two generations of neutrinos. All the qualitative results remain valid in the more realistic case of three generations.

In the case of two generations there is only one mixing angle θ defined by:

$$\nu_e = \nu_1 \cos \theta + \nu_2 \sin \theta$$

$$\nu_\mu = -\nu_1 \sin \theta + \nu_2 \cos \theta.$$

The squared mass matrix which is relevant for the description of oscillations can be written as:

$$\begin{pmatrix} m_1^2 \cos^2 \theta + m_2^2 \sin^2 \theta & \frac{1}{2} \Delta \sin 2\theta \\ \frac{1}{2} \Delta \sin 2\theta & m_1^2 \sin^2 \theta + m_2^2 \cos^2 \theta \end{pmatrix}.$$

This can be rewritten as:

$$\frac{1}{2} \Sigma \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} -\Delta \cos 2\theta & \Delta \sin 2\theta \\ \Delta \sin 2\theta & \Delta \cos 2\theta \end{pmatrix}$$

where:

$$\Sigma = m_2^2 + m_1^2; \quad \Delta = m_2^2 - m_1^2.$$

It is clear that if $\Delta = 0$ or $\theta = 0$ there are no oscillations. If θ is small, the oscillations must be small. The mixing and the oscillations are controlled, as always, by the ratio between the off-diagonal matrix element ($\Delta \sin 2\theta$) and the difference between the two diagonal matrix elements ($2\Delta \cos 2\theta$). The unit matrix ($\frac{1}{2}\Sigma I$) contributes to neither of these quantities and remains outside the "game". Experiments can search for oscillations (i.e. starting with neutrinos of one type and looking for neutrinos of the other type in the beam) or for depletion (i.e. starting with a known flux of neutrinos of a given type and measuring that same flux as a function of distance). A large number of neutrino oscillation experiments of both types have been performed and no convincing effects have been observed, leading to upper bounds on θ for a given Δ . All the experiments searched for oscillations (or depletion) in vacuum.

The formalism for oscillations in matter is similar but differs in one extremely important feature. The mean free path of ν_e in matter is different from that of ν_μ because of the reaction:

$$\nu_e + e \rightarrow e + \nu_e$$

which can proceed via W exchange, while ν_μ has no analogous reaction at low energies. The evolution of a neutrino beam, when it goes through matter, will be influenced by this reaction which will add an effective squared mass term to the above matrix. The revised squared mass matrix will now be:

$$\begin{pmatrix} m_1^2 \cos^2 \theta + m_2^2 \sin^2 \theta + A & \frac{1}{2} \Delta \sin 2\theta \\ \frac{1}{2} \Delta \sin 2\theta & m_1^2 \sin^2 \theta + m_2^2 \cos^2 \theta \end{pmatrix}$$

where:

$$A = 2\sqrt{2}GN_e E$$

and G is the Fermi constant, N_e is the density of electrons in the matter traversed by the neutrinos and E is the neutrino energy. This can, again, be rewritten as:

$$\frac{1}{2}(\Sigma + A) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} A - \Delta \cos 2\theta & \Delta \sin 2\theta \\ \Delta \sin 2\theta & -A + \Delta \cos 2\theta \end{pmatrix}.$$

Here, again, the unit matrix has no effect on the mixing. However, the important ratio is now: $\frac{\Delta \sin 2\theta}{2(\Delta \cos 2\theta - A)}$. It is clear that we now have a new situation in which the parameter A plays a crucial role. We can easily see that the effective mixing angle in matter θ_m obeys:

$$\sin^2 2\theta_m = \frac{\Delta^2 \sin^2 2\theta}{(A - \Delta \cos 2\theta)^2 + \Delta^2 \sin^2 2\theta}.$$

For $A = 0$ we clearly obtain $\theta_m = \theta$. However, in the very special situation in which

$$A = \Delta \cos 2\theta$$

we find $\sin^2 2\theta_m = 1$, yielding maximal mixing, regardless of the value of θ , as long as $\theta \neq 0$. These are the "resonant" neutrino oscillations in matter²³ which

can actually convert a pure neutrino beam of one type into a beam dominated by the other type of neutrinos even for an extremely small vacuum mixing angle, as long as the process is adiabatic.

The immediate potential practical application of this idea is the case of the missing solar neutrinos. It is well known that only a small fraction of the expected flux of solar neutrinos is detected on earth. There are several possible explanations for this, including modifications of the model of the solar interior, neutrino decays and several other exotic proposals. The experiment itself must still be verified by an independent apparatus. However, all of the above explanations are not very likely to solve the puzzle. The resonant neutrino oscillations in matter provide us with yet another potential explanation of the solar neutrino puzzle. It turns out²⁴ that, for a small vacuum mixing angle θ , the solar mass density and the neutrino energies are such that neutrinos with energies above several MeV 's, originating in the solar interior, may well undergo resonant neutrino oscillations, converting from ν_e to ν_μ or ν_τ . In that case, the number of ν_e 's observed on earth will be smaller than expected, in detectors which are sensitive to these higher energy neutrinos. On the other hand, other planned detectors such as the Gallium detector, are sensitive to lower energy neutrinos, enabling them to observe the full neutrino flux without suffering resonant oscillations.

In the Δ - θ plane, three general solutions were found, all leading to observable effects in the flux of solar neutrinos. The first solution²⁴ assumes a small θ in vacuum, yielding $\Delta \sim 10^{-4} \text{ eV}^2$ and, for, $m_2 \gg m_1$:

$$m(\nu_2) \sim 0.01 \text{ eV}.$$

The second solution²⁵ allows arbitrary values of θ and leads to Δ values ranging from 10^{-4} eV^2 (for small θ) to 10^{-7} eV^2 (for large θ). The resulting allowed range of $m(\nu_2)$ is between 10^{-2} and 10^{-4} eV . The third solution²⁶ corresponds to large vacuum mixing angles and is physically much less surprising and less appealing than the other two solutions.

In the above results, ν_2 could be either ν_μ or ν_τ , depending on the neutrino type whose mass obeys the condition for resonant oscillations at the relevant matter densities and neutrino energies. Future solar neutrino experiments will be able to test this hypothesis and to distinguish among the various solutions.

IV.3 Neutrino Masses

We have no experimental evidence for non-vanishing neutrino masses. As outlined above, we do have several theoretical arguments for expecting such masses. We now proceed to review the various sources of information concerning the question of the masses of the three known generations of neutrinos.

Experimentally, we know that:

$$m(\nu_e) < 40 \text{ eV}; \quad m(\nu_\mu) < 250 \text{ keV}; \quad m(\nu_\tau) < 70 \text{ MeV}.$$

In the case of the ν_e mass there is one claim²⁷ of an observed nonvanishing mass of the order of 30 eV . Several recent experiments²⁸ are beginning to contradict this claim and the issue should be resolved within the next couple of years.

If all three neutrinos are stable, we have a cosmological limit²⁹ on their masses. Assuming that the neutrinos cannot contribute to the universe *more* than its *total* energy density, one can show that all masses of stable neutrinos should be below $O(100 \text{ eV})$. This is not far from the present direct experimental limit on ν_e but it is approximately six orders of magnitude below the present limit for ν_τ .

The above limit does not apply to unstable neutrinos. In such a case, cosmological considerations give only a constraint relating the lifetime and the mass of an unstable neutrino.³⁰ By combining this constraint with theoretical estimates of the decay amplitudes of the relevant neutrino, it is possible, under fairly general assumptions,³¹ to exclude neutrino masses above 100 eV .

On the other hand, if the "see-saw" mechanism²¹ is in effect and if its origin is a Grand Unified Theory with a typical energy scale of $\Lambda \sim 10^{15}$ GeV, we expect the neutrino masses to be much smaller. For instance, assuming the above value of Λ and the relations:

$$m(\nu_i) = \frac{[m(\ell_i)]^2}{\Lambda}$$

we expect:

$$m(\nu_e) \sim 10^{-13} \text{ eV}; m(\nu_\mu) \sim 10^{-8} \text{ eV}; m(\nu_\tau) \sim 10^{-6} \text{ eV}.$$

In this case it is unlikely that we can ever see effects of nonvanishing neutrino masses. It is also unlikely that the solar neutrino puzzle has any relation to the resonant matter oscillations. We can reverse the argument and ask what should be the value of Λ which, through the see-saw mechanism, could yield the necessary mass range for resonant neutrino oscillations which solve the solar neutrino puzzle. The answer depends, of course, on whether the ν_e converts into ν_μ or into ν_τ as well as on the vacuum value of θ . The obtained range of Λ -values for all of these cases is between 10^9 and 10^{13} GeV. This is a "no-man's-land" for most "beyond standard" theories with the possible exception of some String models which would like an intermediate energy scale half way (on a logarithmic scale) between the Planck scale and the weak interaction scale.

Another interesting question is how the ratios among the neutrino masses relate to the mass ratios of charged leptons. The simplest see-saw models seem to predict:

$$\frac{m(\nu_i)}{m(\nu_j)} = \left[\frac{m(\ell_i)}{m(\ell_j)} \right]^p$$

where $p = 2$ for a model in which all generations have the same Majorana masses and $p = 1$ for models in which Majorana masses of different generations are proportional to the Dirac masses of the same generations.

The summary of this somewhat confused discussion is that neutrino masses are likely to be well below their present experimental limits and they probably obey a generation hierarchy which is not very different from that of the corresponding charged leptons. We suspect that the mass of ν_τ will be found somewhere between 100 eV and 10^{-6} eV with the ν_μ and ν_e masses scaled down by the corresponding ratios of the squared masses of the charged leptons. Our "guess" is admittedly extremely poor, as it leaves no less than eight available orders of magnitude for each neutrino mass. However, it excludes six orders of magnitude between the present experimental limit and the top of our predicted range. Unfortunately, we cannot say more, at present, and some of our colleagues would not subscribe even to the above guess.

IV.4 Neutrino Proliferation in SIPH

This is neither the place nor the time to present a detailed discussion of neutrinos and neutrino-like states in SIPH. All we want to state are a few simple observations which may give us a preliminary impression of the complexity of the neutrino spectrum in such a theory. Each generation (or each 27-dimensional representation of E_6) contains five neutrinos. We may characterize each of these five states by their $SO(10)$ and $SU(5)$ representations. We will also mention the $SU(2)_L$ classification, although it is completely determined by the two other parameters. For each neutrino we then list three numbers: (d_1, d_2, d_3) for the dimensionality of the representations of the three groups. The five neutrino states in each generation are:

$$\nu_L(16, 5, 2), N(16, 1, 1), \tilde{H}^0(10, 5, 2), \tilde{H}^0(10, \bar{5}, 2), S(1, 1, 1).$$

In a somewhat more familiar terminology these are a left-handed neutrino, a right-handed neutrino, a Higgsino and its anti-particle and a singlet neutrino. Since we must have at least three generations, we must have at least 15 neutrino states, five per generation. To these we must add several neutral colorless

Gauginos ($\tilde{\gamma}, \tilde{Z}, \tilde{Z}', \tilde{Z}''$) which may mix with the above states. A complete understanding of the neutrino spectrum would then involve the diagonalization of a mass matrix involving at least 19 states, four of which belong to the adjoint representation of E_6 and 15 describing the three generations of matter particles. To these one may wish to add E_6 singlets and possible members of incomplete $27 + \bar{27}$ multiplets. The complete neutrino spectrum is incredibly complicated.

It would appear that such a proliferation would allow unlimited freedom in choosing the mass parameters and in obtaining an appropriate "generalized see-saw" mechanism for the masses of the three observed light neutrinos. However, it appears that this is not the case. Neither the tree approximation nor the one-loop approximation seem to lead to the existence of three very light $SU(2)$ -doublet neutrinos. A detailed discussion of this problem³² is beyond the scope of these lectures.

V. PROBING W AND Z – STANDARD AND BEYOND

V.1 Introduction

The discovery of the W and Z bosons is clearly a great triumph of the standard model. Given the experimental value of $\sin^2 \theta_W$ obtained from neutral current experiments, one can predict the masses of W and Z , including one-loop radiative corrections², to be:

$$M_W = \frac{38.7 \text{ GeV}}{\sin \theta_W}; \quad M_Z = \frac{77.3 \text{ GeV}}{\sin 2\theta_W}.$$

This is consistent with present experiments. We expect a much better measurement of M_Z as soon as SLC is ready. The two pp colliders at CERN and Fermilab should soon be able to improve the accuracy of the W -mass determination.

There is no direct evidence yet for the WWW and $WWWW$ couplings required by the nonabelian nature of the $SU(2)$ gauge group (except for the obvious

existence of an electric charge for W^+ and W^-). In particular, we have no measurement of the magnetic moment of the W . Such measurements are likely to come only from difficult processes such as $\bar{p} + p \rightarrow W^+ + \gamma + \text{anything}$ at $\bar{p}p$ colliders or from $e^+ + e^- \rightarrow W^+ + W^-$ at the second stage of LEP in the 1990's.

"Beyond standard" models may require additional W and Z bosons. Interest in such a possibility has recently increased as a result of the possibility of a single additional Z' in SIPH. It is amusing that such a Z' can still be as light as 130 GeV and not be detected by present experiments. We discuss this in section V.2.

On the other hand, the limits on right-handed W 's are much stronger. We have a lower bound³³ of approximately 2 TeV for $M(W_R)$. We discuss this issue in section V.3.

Another interesting possibility which was already briefly mentioned above in section III.2 is that of a strongly interacting Higgs particle. In such a case the longitudinal component of the W will also participate in such strong interactions. At energies of many TeV's the longitudinal W may then become the "pion of the weak interaction", being copiously produced²⁰ in any collision of quarks or leptons.

The present experimental limit on possible substructure of W and Z are still only around 1 TeV, possibly even less. This together with the $O(\text{TeV})$ limits on quark and lepton substructure, still allow for the possibility of detecting such effects within the next decade. We discuss this in section V.4.

V.2 Low Lying Z -Bosons and SIPH

The possibility of adding an additional $U(1)$ gauge group to the standard model can, of course, be studied on its own, without reference to any specific theory at higher energy scales. All we have to do is assume that the complete gauge group is $SU(3)_c \times SU(2) \times U(1) \times U(1)$ and that the extra Z -boson is sufficiently heavy so as not to modify the low-energy predictions of the standard

model for all neutral current phenomena which have already been experimentally tested.

In the above scenario, we would have no information on the detailed couplings of the extra Z to quarks and leptons. In other words, the new $U(1)$ charges of all standard model particles cannot be determined without additional information.

Only if we have a “beyond standard” theory at higher energies, which at low-energies yields the new extended gauge group, can we hope to determine the properties of the extra Z -boson. An example of such a theory is the simple version of the heterotic string theory based on $E_8 \times E_8$, with a leftover broken E_6 operating in the four-dimensional world below the Planck scale.

In such a theory, the E_6 gauge symmetry is broken by the so-called “Wilson loop” operators which transform like the adjoint 78-dimensional representation of E_6 . In order to see how this leads to additional Z 's, we may consider the decomposition of the 78 multiplet under $SU(3)_c \times SU(3)_L \times SU(3)_R$. We obtain:

$$78 \rightarrow (3, 3, 3) + (\bar{3}, \bar{3}, \bar{3}) + (8, 1, 1) + (1, 8, 1) + (1, 1, 8).$$

Among these, the only ones which can break E_6 and leave $SU(3)_c \times SU(2) \times U(1)$ intact are the $(1, 8, 1)$ and $(1, 1, 8)$ operators. However, the particular combination of these operators which breaks the symmetry must conserve the usual $U(1)$ of the standard model. It is easy to see that it must therefore conserve at least one additional $U(1)$ within $SU(3) \times SU(3) \times SU(3)$. If it conserves exactly one such $U(1)$, it can be chosen as $Y_L + Y_R$ (where $Y_{L,R}$ is the $U(1)$ charge which commutes with $SU(2)$ within $SU(3)_{L,R}$).

The simplest way to achieve this is to suggest that E_6 is actually broken at the Planck scale into $SU(3) \times SU(2) \times U(1) \times U(1)$ and that the latter symmetry persists down to energies well below 1 TeV. In this case we would have an extra low mass Z' boson with well defined couplings to all particles.

Before we briefly address the relevant phenomenological issues, we hasten to add that the above scenario is *definitely not* a necessary consequence of the

heterotic string theory. It may be the simplest scenario, but other variations are possible. We have already mentioned that, to begin with, E_6 is not the only possible leftover gauge group. Even if it were, it might be broken to a larger subgroup. In fact, the following possibilities exist for the surviving gauge group after E_6 has been broken by the Wilson loop operators:

$$SU(3) \times SU(3) \times SU(3)$$

$$SU(3) \times SU(2) \times SU(2) \times U(1) \times U(1)$$

$$SU(3) \times SU(2) \times U(1) \times U(1) \times U(1)$$

$$SU(3) \times SU(2) \times SU(2) \times U(1)$$

$$SU(3) \times SU(2) \times U(1) \times U(1)$$

All of these groups are subgroups of E_6 , and they all contain the standard model group as well as at least one additional Z' . The first case corresponds to a GUT and must be further broken by a “normal” Higgs mechanism at a relatively high energy scale. However, in the last four cases the remaining gauge groups may survive down to energies well below the typical GUT scale. In each of these cases we have either one or two extra Z -bosons. Both of them, one of them or none of them may be below 1 TeV. If both are, they may mix and we cannot determine the precise couplings of the lowest lying extra Z . If both of them are at masses well above 1 TeV (say, around the so-called “intermediate mass scale” of 10^{11} GeV) we expect no detectable effects at low energies. Only if the remaining gauge group contains *exactly one* extra Z and the symmetry breaking pattern is such that the extra Z is below 1 TeV, we can expect clear experimental signatures of a new neutral Z -boson with well-known couplings to all other particles.

Several authors³⁴ have analysed the experimental constraints on additional Z -bosons which are presently available or which are expected at the pp colliders, SLC, LEP, HERA and the SSC. One can distinguish among three kinds of experimental effects:

- (i) Direct observation of a Z' . At sufficiently high energies such a particle can be produced both at hadron colliders and at lepton colliders. The present mass limits allow the observation of a Z' even at the Fermilab collider, LEP II and HERA.
- (ii) Direct contributions of heavier Z' -bosons to specific amplitudes. This will happen in almost any process to which the ordinary Z contributes and the resulting amplitudes and cross sections will be modified.
- (iii) Indirect effects due to the mixing of a new Z' with the ordinary Z . These could modify the properties of the lowest Z -boson, including its mass, width and other features. These last effects may be the first ones to be observed, once we have an e^+e^- collider at the Z mass.

The search for a possible extra Z is interesting on its own merit, regardless of its possible relation to SIPM.

V.3 Right-Handed Weak Bosons

One of the most straightforward extensions of the standard model is the Left-Right symmetric theory (LRS) in which the gauge group is $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. In this type of theory all left-handed quarks and leptons are in $(\frac{1}{2}, 0)$ representations of the LRS group while their right-handed counterparts are in $(0, \frac{1}{2})$ representations. The usual Higgs field ϕ is in a $(\frac{1}{2}, \frac{1}{2})_0$ multiplet. In the minimal version of the theory³⁵ we also have an extra $SU(2)$ -triplet Higgs field Δ . Because of the left-right symmetry we must then have a Δ_L in a $(1, 0)_2$ and a Δ_R in a $(0, 1)_2$. Parity is spontaneously broken by giving Δ_L^0 and Δ_R^0

different vev's. We denote:

$$\langle \Delta_L^0 \rangle = v_L; \quad \langle \Delta_R^0 \rangle = v_R; \quad \langle \phi \rangle = k.$$

We learn that in order to preserve the standard model predictions at low energies we must have:

$$v_R \gg k \gg v_L.$$

We immediately obtain the following results:

- (i) The masses of the additional W and Z are related to v_R .
- (ii) The right-handed neutrino gets a Majorana mass related to v_R , leading to a "see-saw" mechanism and to a light left-handed neutrino.
- (iii) All Higgs fields other than the usual standard model Higgs have masses of order v_R . Some of the additional neutral Higgs particles induce flavor-changing neutral currents.
- (iv) The absolute values of right- and left-handed generalized Cabibbo angles are identical, but extra phases appear, leading to CP-violating effects which are smaller than ordinary weak interactions amplitudes by a factor $\left[\frac{M(W_L)}{M(W_R)} \right]^2$.

All of the above features are quite attractive, but the main question remains unanswered: What is the value of v_R and the mass of W_R ?

Direct searches for right-handed currents give lower limits of a few hundred GeV's for $M(W_R)$. However, a much stronger limit can be obtained from the $K_S^0 - K_L^0$ mass difference. That limit has an amusing history which is worth reviewing:

- (i) The standard model contribution to $\Delta M(K_S^0 - K_L^0)$ is dominated by the "box diagram" involving the exchange of two ordinary W 's. That diagram gives approximately the correct value for ΔM . The leading additional

contribution in the LRS model is an identical diagram, with one of the ordinary “left-handed” W ’s replaced by a W_R . The success of the standard model calculation means that the new diagram should contribute less than the usual diagram. When the new diagram is inspected superficially, it appears to be suppressed (relative to the usual diagram) only by a factor of $\left[\frac{M(W_L)}{M(W_R)}\right]^2$. There must also be a numerical factor corresponding to the different chiralities involved, but it appears to be of order one. If that were the case, all we would be able to deduce would be:

$$\left[\frac{M(W_L)}{M(W_R)}\right]^2 < 1$$

yielding a useless bound $M(W_R) > M(W_L)$.

- (ii) Enter Beall, Bander and Soni.³³ They were the first to calculate the numerical factor preceding $\left[\frac{M(W_L)}{M(W_R)}\right]^2$ in the relative strength of the LRS box diagram and the standard model box diagram. They found that the numerical factor (which was allegedly of order one) was actually 430. The new result, *assuming equal left-handed and right-handed Cabibbo angles*, was:

$$430 \left[\frac{M(W_L)}{M(W_R)}\right]^2 < 1.$$

This leads to a much stronger bound for $M(W_R)$:

$$M(W_R) \geq 1.7 \text{ TeV}.$$

- (iii) However, while there are good reasons to expect an equal *magnitude* for the left- and right-handed Cabibbo angles, there is no reason to assume that they are equal in *phase*. Actually, the contribution of the LRS diagram should contain an additional arbitrary relative phase ϕ and the correct

result becomes:

$$430 \left[\frac{M(W_L)}{M(W_R)}\right]^2 \cos \phi < 1.$$

Since there is no apriori reason for ϕ to have any specific value, no bound can be derived and the 1.7 TeV result appears to be lost.

- (iv) The 1.7 TeV bound can be rescued by observing that the CP-violating parameter ϵ gets a contribution from the *same* box diagram involving one W_L and one W_R . That contribution depends on the *same* arbitrary phase ϕ . The LRS contribution to ϵ is:

$$\epsilon_{LRS} = \frac{1}{2\sqrt{2}} 430 \left[\frac{M(W_L)}{M(W_R)}\right]^2 \sin \phi.$$

We can now combine the contributions to ΔM and to ϵ and derive a new bound³⁰:

$$430 \left[\frac{M(W_L)}{M(W_R)}\right]^2 < \sqrt{1 + 8\epsilon_{LRS}^2}.$$

Assuming that ϵ_{LRS} is not much larger than $\epsilon \sim O(10^{-3})$, we may safely neglect ϵ_{LRS}^2 and recover:

$$M(W_R) \geq 1.7 \text{ TeV}.$$

- (v) The above limit is a strict limit valid for any value of ϕ . However, in order to get $M(W_R) = 1.7 \text{ TeV}$ we must actually have $\phi \leq 0.5^\circ$. Such a small value of ϕ is allowed but appears to be unnatural. There is no reason for ϕ to be small. If we arbitrarily assume that ϕ is not very small (say, $\phi \geq 5^\circ$), we immediately obtain³⁶ a much stronger bound on the mass of W_R :

$$M(W_R) > 5 \text{ TeV}.$$

This bound is based on reasonable “hand-waving” but is not as solid as the 1.7 TeV bound.

(vi) The contribution of the neutral Higgs particle to ΔM can also be computed.

Assuming that it is not larger than the standard model box diagram, we obtain a lower bound on the Higgs mass. It should be³⁷ somewhere above 5 – 10 TeV. We expect the Higgs mass to be of the same general order of magnitude as $M(W_R)$. Consequently, we conclude again that $M(W_R)$ is likely to be at least around 5 – 10 TeV. The Higgs diagram also contributes to ϵ .

The summary of all of these steps is the following: We definitely know that $M(W_R)$ is above 1.7 TeV. We strongly suspect that it is actually *at least* around 10 TeV. If the LRS diagrams (both the Higgs contribution and the $W_L - W_R$ box) contribute a *substantial* part of ϵ we also know that $M(W_R)$ cannot be much larger than,³⁸ say, 100 TeV. However, if it provides a negligible contribution to ϵ , W_R could be substantially heavier.

The LRS theory by itself does not provide answers to any of the problems of the standard model, except for the parity problem and possibly the CP and the neutrino mass problems. It certainly does not shed any light on Unification, Substructure, the Fine Tuning problem or the Generation Puzzle. However, LRS is built into some of the more detailed models such as $SO(10)$, E_6 and into some composite models. In some of these models the scale of $M(W_R)$ is determined uniquely.

V.4 Possible Substructure of W and Z

One of the two candidate solutions to the “fine tuning” problem is the suggestion that Higgs particles are composite objects (the other solution being supersymmetry). If Higgses are composite, so are the longitudinal W and Z that are “born” from the Higgs field. It is entirely consistent to assume that these are the *only* composite objects among the particles of the standard model. However, one may also entertain the hypothesis that other particles possess substructure. The prime candidates for such a substructure are the quarks and leptons, and

the next candidates may be the W and Z bosons.

The scenario for composite W and Z goes as follows: The $SU(2)$ gauge symmetry is replaced by a global $SU(2)$, guaranteeing that all W and Z couplings obey the usual ratios and that the Weinberg mass relation is preserved. The W and Z consist of some subparticles bound together by a new fundamental interaction (possibly, but not necessarily, a color-like interaction). The usual weak interactions are then induced³⁹ as residual interactions among the composite quarks, leptons, W 's and Z 's, in analogy to the usual hadronic forces being residual color interactions. Properties such as universality of W and Z couplings can be recovered by assuming Z -dominance of the electromagnetic currents. We may, in fact, use the present known accuracy of $\epsilon - \mu$ universality and quark-lepton universality in order to estimate the lowest bound on the mass of a possible excited W or Z . We obtain values around 600 GeV.⁴⁰

An important possible experimental test of a W substructure (or, for that matter, of any deviation from a minimal nonabelian gauge coupling for the W) is the measurement of the anomalous magnetic moment of the W . Experimentally, we have no direct information on κ_W . The strongest *indirect* limit is obtained⁴¹ from the contribution of a $W - \gamma$ loop to the W -mass. Such a contribution would lead to a deviation from the observed accuracy of the Weinberg mass relation (the ρ -parameter) unless we have $\kappa < 0.01$. Direct measurements of κ are unlikely to achieve such a level of accuracy within the next decade.

We have no clear idea about the compositeness scale of the W (if it is indeed composite). However, if that scale is too high above $M(W)$, we will face a new difficult problem. We will not be able to explain why $M(W)$ should be very small compared to its own compositeness scale. The only reasonable solution seems to be that, if W and Z are composite, the relevant energy scale should not be above 1 TeV or so. This is perfectly consistent with the compositeness scale of the Higgs particle according to Technicolor models. It is also consistent with all available experimental information on W and Z and with the experimental

limits on quark and lepton compositeness.

If there is a substructure at the 1 TeV scale, a machine like the SSC cannot fail to uncover it.

VI. ON THE FUTURE OF ACCELERATOR HIGH-ENERGY PHYSICS

VI.1 Limits on Pointlike Behavior

All present experimental data are consistent with a "pointlike" behavior of all standard model particles. The phrase "pointlike" refers to the minimal couplings of the standard model Lagrangian. The only corrections to these couplings, allowed by the standard model, are the usual radiative corrections which are calculable and small.

At the same time, one cannot exclude the possibility that some or all of the standard model particles do have some internal structure at distance scales below the ones presently probed by experiments. Technicolor models and composite models for quarks, leptons and W and Z bosons necessarily lead to such a substructure. On the other hand, Grand Unified theories do not allow for any substructure at least up to an energy scale of the order of 10^{15} GeV. String theory insists on a substructure at the Planck scale, but (at least in its present version) does not allow for a deviation from pointlike behavior at lower energies.

In discussing models which suggest substructure,⁴² we should distinguish among four possible versions:

- (i) The most conservative departure from an overall pointlike behavior is the suggestion that Higgs particles are the *only* composite objects among the standard model particles. This is the approach advocated by the original Technicolor models and it is designed to solve the fine-tuning problem by avoiding fundamental scalars. Since we have no experimental evidence for Higgs particles, we clearly have no direct tests of their possible substructure.

However, theoretical arguments suggest that Higgs compositeness will solve the fine-tuning problem only if the compositeness scale is not too far from 1 TeV. Here one may wish to distinguish between Technicolor schemes in which the new scale must be of the order of 1 TeV and composite Higgs models in which one has more flexibility. In all cases we end up with a scale between 1 and 10 TeV.

- (ii) The next logical step may be to assume that not only Higgs particles, but also quarks and leptons are composite. Here the main motivation is to explain the proliferation of parameters and the replication of generations in terms of some common substructure and a smaller number of building blocks. Experimentally, we have several *direct* tests of quark and lepton compositeness. All *model-independent* tests lead to lower bounds of the order of 1 TeV for the compositeness scale. These tests include the measurements of $(g-2)_{e,\mu}$, observed cross sections for $e^+ + e^- \rightarrow e^+ + e^-$ and $e^+ + e^- \rightarrow \mu^+ + \mu^-$ and upper limits on deviations from quark pointlike behavior in deep inelastic scattering experiments and in e^+e^- collisions. The summary of all of these limits is that quarks and leptons could still have a typical "radius" of the order of an inverse TeV, without contradicting any known experimental data. There are additional important bounds which are *model-dependent*. The absence of processes such as $\mu \rightarrow e + \gamma$, $\mu \rightarrow 3e, K^0 \rightarrow e^+\mu^-$, $K^+ \rightarrow \pi^+e^-\mu^+$, yields bounds of order 100 TeV. Stronger bounds above 1 PeV can be obtained from $\Delta M(K_S^0 - K_L^0)$. However, these bounds are valid only if the relevant processes are not suppressed or forbidden by the quantum numbers of the model. Since we do not have an accepted convincing description of the generation pattern, we cannot use these limits as a valid estimate on the possible "size" of quarks and leptons. We can only state that if quarks and leptons do have a "size" of order 1 TeV⁻¹, the resolution of the generation puzzle will occur on a different energy scale which is unlikely to be below 1 PeV. A similar conclusion follows from the present bounds on proton

decay. If quarks and leptons are composite, either proton decay is exactly forbidden, or it is strongly suppressed by some dynamical mechanism (for instance – it may occur only in third order in the new fundamental interaction) or the compositeness scale is of the order of the usual GUT scale. We may not like this as an attractive theoretical option, but it is perfectly possible⁴³ that the “size” of quarks and leptons is given by one energy scale, the generation puzzle is solved at a different scale and baryon number violation occurs at yet another scale.

- (iii) A more radical possibility is to suggest³⁹ that, not only Higgs, quarks and leptons are composite, but also W and Z . We have briefly discussed this possibility in section V.4 and indicated that, again, the compositeness scale is unlikely to be too different from 1 TeV . Experimentally, this is perfectly possible.
- (iv) Finally, it is conceivable that *all* particles of the standard model (including the gluon and the photon) have some substructure. In that case it is likely that the relevant scale is not too far from the Planck scale. This is the case in String Theory.

The overall situation is therefore the following: The actual “size” of all Higgs particles, quarks, leptons and W and Z can still be as large as 1 TeV^{-1} . The scale for Higgs, W and Z substructure must be around that number while the “size” of quarks and leptons could assume any value above 1 TeV . It is entirely possible that there are several compositeness scales.

VI.2 Cross Sections and Pointlike Behavior

Any two-body reaction $a + b \rightarrow c + d$ among four pointlike particles, which is dominated by a direct-channel exchange and takes place at an energy scale well above the masses of the four particles, will follow a high-energy behavior of the form $\frac{1}{s}$. Examples include $e^+e^- \rightarrow \tau\tau$ and $q\bar{q}$ collisions leading to lepton pairs, quark

pairs, W -pairs etc. The exceptions to this rule are resonances (such as ψ , T or Z in e^+e^- scattering), possible new thresholds and other effects signalling the onset of some new physics.

If all standard model particles are pointlike up to the GUT scale or the Planck scale (as is the case e.g. in SIPH), we expect that most of the interesting amplitudes above 1 TeV or so will follow the $\frac{1}{s}$ behavior. On the other hand, if some particles have a substructure, we expect departures from the $\frac{1}{s}$ behavior at energies around the compositeness scale or somewhat below it. Thus, if quarks and leptons have a size of the order of TeV^{-1} , we expect the amplitudes for e^+e^- and $q + \bar{q}$ scattering to follow a pattern which is different from the $\frac{1}{s}$ pointlike behavior. At energies well above the compositeness scale, we actually expect a more or less constant cross section, in accordance with a simple diffractive picture (assuming that the binding force of the constituents inside quarks and leptons is a sufficiently strong force). For instance, at $E \sim 10\text{ TeV}$ we expect:

$$\sigma(e^+ + e^- \rightarrow \mu^+ + \mu^-) = 10^{-39}\text{cm}^2 \text{ (for pointlike behavior);}$$

$$\sigma(e^+ + e^- \rightarrow \mu^+ + \mu^-) = 10^{-33}\text{cm}^2 \text{ (based on } \sigma \sim 2\pi R^2 \text{ with } R \sim \text{TeV}^{-1}\text{).}$$

The difference between these two possibilities is remarkable, especially if you are planning an experiment at a high energy collider.

It is perhaps too optimistic to expect substructure at energies which are as low as 1 TeV . However, in the absence of such a substructure, we may run out of interesting measurable cross sections in the next few generations of high-energy accelerators.

VI.3 A Wild Extrapolation: Accelerator Experiments in 2030

Historically, the center-of-mass energy of the highest energy accelerators have advanced by one order of magnitude approximately every twelve years. We are

presently reaching energies of order 1 TeV (more precisely: We should soon have almost 2 TeV at the Fermilab collider but the "useful" energy of quark and gluon subprocesses is only of the order of, say, 0.5 TeV). The graduate students attending this summer school are now in their mid-twenties. They will therefore retire approximately around the year 2030. Can we try to predict how accelerator experiments will look at that time?

Extrapolating the progress of accelerators over the last fifty years, we conclude that the highest energy available in the year 2030 should be above 1 PeV ($\approx 10^5$ TeV). This would presumably require a linear collider with an entirely new technology which will allow for a much higher acceleration per unit length than anything imaginable now. We all know that to do physics at such energies will require innovations in accelerator physics, new detector technology, new computing capabilities and, of course, a lot of money. However, we wish to address here an entirely different question. It is not enough to have the money and the technology, the accelerator, the detectors and the computers. We need to be able to study interesting physics and that requires a sufficient number of events of the interesting types.

If string theory is right (or if Grand Unified Theories are right), we expect cross sections to continue to reflect a pointlike behavior at energies between 1 TeV and 1 PeV. The cross section for $e^+ + e^- \rightarrow \mu^+ + \mu^-$ at $E = 1$ PeV would then be approximately 10^{-42}cm^2 . Cross sections for other processes such as $e^+ + e^- \rightarrow W^+ + W^-$ will be somewhat larger but of the same general order of magnitude. In order to accumulate, say, only 1000 events per year in any given such process, one will then need a luminosity of $10^{39} \text{cm}^{-2} \text{sec}^{-1}$ (!). One should never underestimate future technology, but such a luminosity really appears to be beyond any reasonable extrapolation. In particular, if we simply scale the preliminary parameters recently discussed as a possibility for an electron collider at the 1 TeV range, "we will find that a beam size of the order of 10^{-6}cm (!) is required for such a luminosity in a linear collider, assuming everything else remains the same as in the 1 TeV parameters.

The situation will, of course, be very different if some substructure is uncovered anywhere between 1 TeV and 1 PeV. All cross sections involving the particles whose substructure is unraveled will become much larger than the above estimates. A substructure at 1, 10, 100 TeV, respectively, would yield $e^+ + e^- \rightarrow \mu^+ + \mu^-$ cross sections of the order of 10^{-33} , 10^{-35} , 10^{-37}cm^2 , respectively, allowing experimentation at much lower luminosities.

Do we have the right to expect that "physics will be good to us"? We do not know. At least in two cases in the past, cross sections ended being much larger than the then current standard predictions. Deep inelastic electron scattering was expected by many to show no events. Hadronic final states in e^+e^- collisions were expected to be few and far between. In both cases, experiments showed that the existence of pointlike constituents led to cross sections which were much larger than otherwise predicted. It is ironic that we now have to base our hopes for a large cross section on the option of "losing" that same pointlike behavior. If that does not happen, the best detectors may not be able to detect anything by the time our present graduate students reach retirement age.

Our agenda for the next few years is then to continue benefiting from existing machines, start exploiting the Fermilab collider, SLC and TRISTAN, continue building LEP and HERA, continue planning the SSC and possible Large Hadron Colliders and Large Linear Colliders, continue to search for the new theory of the physics beyond the standard model, develop new accelerator physics ideas, and, last but not least, pray for a decent cross section at high energies!

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CP AND OTHER EXPERIMENTAL PROBES OF ELECTROWEAK PHENOMENA

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1. INTRODUCTION

In these lectures we will treat some topics where probes of electroweak phenomena are likely either to confirm the "Standard Model" or point to phenomena which lie beyond it. We will discuss briefly the current status on the measurement of important parameters of the model, limits on the existence of other phenomena beyond it, and the important measurements of the charged current couplings. Then we will give a more extensive treatment of CP nonconservation in the K^0 system and the prospects for improvement there. The signatures for CP nonconservation in the heavy quark systems will be discussed. Finally, we will briefly discuss the various searches for phenomena beyond the Standard Model in the study of rare or "forbidden" decays, particularly of the K meson.

The current status of the measurements of the parameters of the Standard Electroweak model are given in reference 1. The intermediate vector boson masses and widths are well measured with a total of about 700 identified decays. Major improvement is expected when the two new

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colliders become operational in this country: the Tevatron at Fermilab and the SLC at SLAC. The extracted values of $\sin^2 \theta_W$ as determined by many different techniques are in impressive agreement. The techniques, including the ratio of the W and Z masses, parity violation in atoms, polarized electron Deuteron scattering, elastic muon-neutrino electron scattering, and inelastic muon-neutrino nucleon scattering, have different sensitivities to radiative corrections and to the mass of the as yet unobserved top quark.

2. NEW LEPTONS

Limits on the existence of new heavy charged leptons can be obtained from data at the SppS collider.⁽²⁾ The technique is sensitive to those leptons from W decay and the signature is one or two jets plus missing energy. Topological cuts are imposed to bias against W decays to the tau-neutrino and the method is thought to be sensitive to new sequential leptons up to a mass of about 70 GeV. At present, the limit is around 42 GeV. While there is some debate as to whether this technique is viable or not, it is definitely worth pursuing in that the hadron colliders provide an enormous window for such new phenomena.

The hadron collider experiments can also provide important limits⁽¹⁾ on the number of light neutrinos. The most sensitive method uses the ratio of cross section times branching ratio for W's to electron neutrino to that for Z's to electron-positron. By taking the production cross section ratio, the leptonic branching ratios, and the W width from theory, one can in principle deduce the Z total width. When one then assumes a top quark mass, the number of neutrinos coupling to the Z can be deduced. The UA1

and UA2 data, when treated as described, look to be consistent with at most one additional neutrino beyond those associated with the three charged leptons. Again there is much uncertainty and model dependence in this result, but it is impressive that the hadron colliders show the promise of being able to address this important area of physics.

The best limit on the number of new generations comes from the combination⁽³⁾ of data from a number of experiments at electron-positron facilities designed to look for the reaction $e^+e^- \rightarrow Z^0 \rightarrow \gamma \nu \bar{\nu}$. The best result comes from the ASP group where a limit of 7.5 neutrinos can be set. The limit from the combination is $N_\nu \leq 4.8$ (90% confidence) which is a very impressive result. The technique will be important for both SLC and LEP where real Z's will be produced. By sitting about 3 GeV above the Z peak, at a luminosity of $10^{30} \text{ cm}^{-2} \text{ sec}^{-1}$, one would expect one photon (above 1 GeV) per 8 hour shift per generation!

3. CHARGED CURRENT COUPLINGS

The matrix of these nine numbers (we assume three generations and for now no CP violation) can be written as follows:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

There are determinations⁽⁴⁾ only of the upper six elements as the top quark has not as yet been observed.

The element V_{ud} is measured using the eight super allowed $0^+ \rightarrow 0^+$ $J=1$ transitions in nuclear beta decay where, at the current level of

accuracy, radiative corrections are important. The result is $V_{ud} = 0.9747(10)$ where the quoted error is solely a result of theoretical uncertainties.

The element V_{us} has been determined in two ways: from charged and neutral Kaon beta decays,⁽⁵⁾ and from Hyperon beta decays.^(5,6) From the Kaon decays, the result is $V_{us} = 0.2196(20)$ while from the Hyperon decays it is $V_{us} = 0.2210(20)(30)$ where the second error is theoretical.

The elements V_{cd} and V_{cs} are measured using data⁽⁷⁾ on neutrino and antineutrino induced charm production from an isoscalar target. With one combination of the particle and antiparticle cross sections, one can eliminate the contribution from the strange sea and thereby isolate the reaction $\nu d \rightarrow \mu c$ with the subsequent decay of the c quark providing an opposite sign muon pair. The result is $V_{cd} = 0.207(24)$. The same data can be combined to isolate the strange sea contribution but uncertainties there result in only that $V_{cs} \geq 0.59$ (90% confidence).

The B meson lifetime is governed primarily by the size of the element V_{bc} so that its measurement, so far by means of non-Gaussian impact parameter distributions, can yield a determination of the matrix element. The latest result⁽⁸⁾ for the B meson lifetime is given by $\tau_B = 1.13(10)(7)(17) \times 10^{-12}$ sec. The different measurements from MAC, Mark II, DELCD, JADE, and TASSO have been combined with common systematic errors factored out. The first error is statistical, the second results from a "pedestal" uncertainty in the extrapolated impact parameter, and the third is a systematic error due to uncertainties in the b quark hadronization. The experiments are also insensitive to any possible differences in lifetime between the B^0 and the B^+ mesons. From this number, the resultant

matrix element is then $V_{cd} = 0.046(10)$ where the error also includes the uncertainty in the b quark mass itself.

The last element for which there is any direct information is V_{ub} . The best information comes from a careful examination of the momentum spectrum of electrons emitted from B meson decays. While there has recently been considerable uncertainty, both experimental and theoretical, the lack of a signal above background above about 2.4 GeV provides an upper limit for the matrix element. Both the CLEO and ARGUS data contribute to the result that $V_{ub} \leq 0.012$ (90% confidence).

One can use this data to look for a new generation which is something outside of the Standard Model. Since this matrix of charged current coupling constants must be unitary, any apparent lack of unitarity with three generations would signal additional ones. The only significant test at present results from looking at the "length" of the upper row of elements:

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.9982(22).$$

Thus a limit can be placed on the coupling of a possible fourth generation but in practice it is not terribly stringent.

We have so far treated only the real parts of the matrix of charged current coupling constants the KM matrix. The possibility that some of these elements are imaginary and that therefore CP violation can be viewed as a phenomenon directly related to the mixing of the quark generations is discussed later together with the experimental status of measurements of CP violation both in the mixing and in the two pion decays of the neutral Kaon.

4. CP VIOLATION IN NON 2 π KAON DECAYS

The K_S decay to three pions is being studied⁽⁹⁾ at Fermilab. There the (predominantly) CP violating transition is $K_S \rightarrow \pi^+ \pi^- \pi^0$ and this amplitude is measured in interference with the dominant one from K_L in an initially (nearly) pure K^0 beam. No interference is seen and the current result for the magnitude of the parameter η_{+-0} (the ratio of the above two amplitudes) is $-0.023(29)$. This represents a major improvement over previous limits but is still rather far from the expected level of about 2.3×10^{-1} . The group has much more data in hand, however, and expects to eventually have a result with a sensitivity in the expected range. Of course there could always be a big surprise.

The Yale-BNL group⁽¹⁰⁾ has performed a beautiful set of measurements searching for a T-violating polarization of the muon perpendicular to the pion-muon plane in the decays of the charged and neutral Kaon to $\pi\mu\nu$. Their result, $P_\mu = -0.0019(36)$ can have implications on non-Standard Model theories for CP nonconservation but negligible polarization is expected with the KM mechanism. The experiment had about 20×10^6 events and was still statistics limited so that it would be possible to consider improving this measurement.

5. CP STUDIES AT LEAR

A group⁽¹¹⁾ operating at the LEAR facility at CERN is planning on extensive measurements of the CP violating parameters in the Kaon system. The facility makes use of the ACOL ring to provide an intense source of anti-protons which annihilate at rest in a gaseous target producing $K^+ \pi^- K^0$ (and the charge-conjugate state) at a rate of about 10^8 per day providing

therefore a means of tagging initially produced Kaons or anti-Kaons. The technique allows direct comparisons between particle and anti-particle decay rates and thus, in principle, is a very clean way of addressing direct CP non-conservation. First running is expected in 1988 and the ultimate accuracy on ϵ'/ϵ is claimed to be in the range of 1 to 2×10^{-3} .

6. CP VIOLATION IN HEAVY FLAVOR SYSTEMS

The KM model predicts that CP violating effects will show up in heavy flavor systems. There are three such systems that can now be considered: the D^0 anti- D^0 system, the B_d^0 anti- B_d^0 system and the B_s^0 anti- B_s^0 system. It is easy to show that for these heavy systems, the lifetime difference is expected to be small with respect to the lifetime itself, in contrast to the $K_L K_S$ system so that it will not be possible to produce a beam of one of the weak eigenstates as one can with the K_L . However, when one considers the mass differences relative to the lifetime, the B_s^0 system is expected to show a sizable effect and thus there may be substantial mixing. This is of course not CP violating, but such an effect could be valuable in searches for CP nonconservation. Indeed, the UA1 group⁽¹²⁾ at the CERN collider has recently reported an observation of an excess of same sign muons which, after various topological cuts, is given by

$$R = (N_{++} + N_{--})/N_{+-} = 0.46(7)(5)$$

where the first error is statistical and the second systematic. Their Monte-Carlos predict something in the range of 0.25 in the absence of mixing. They conclude that the "observed R is consistent with full B_s^0

mixing and no B_d^0 mixing.* It would obviously be highly desirable to have a vertex detector to better distinguish against backgrounds in this type of measurement.

(Note added: The ARGUS group has reported a sizable mixing in $B_d^0 \bar{B}_d^0$ of the order of 25%. If confirmed, this will alter the conclusion of the UA1 group; it also indicates a rather heavy top quark.)

Some of the signatures for CP nonconservation in the heavy quark systems (particularly the $B^0 \bar{B}^0$ system) are as follows:

1. Non-exponential decay to a CP eigenstate, for example $D^+ D^-$. This was in fact the way that CP non-conservation was established first in the K^0 system with the two charged pion mode. Such an observation would be definitive and completely independent of production dynamics. However, with just one mode observed, one can not know whether CP is violated in the mixing or in the decay.

2. The observation of two decays of the same CP from a $B^0 \bar{B}^0$ pair produced in a $J^P = 1^-$ state. Again, this process cannot distinguish between CP non-conservation in mixing or in decay.

3. The observation of a charge asymmetry in double semi-leptonic decays, again from a 1^- state of $B^0 \bar{B}^0$. This is definitively CP violation in the mixing.

4. A rate difference between particle and anti-particle decays (charged or neutral) to CP conjugate states resulting from final state interactions.

5. Interference between particle and anti-particle to the same final state by means of mixing. This requires no final state interactions!

The last two of these techniques are discussed in great detail in a recent article⁽¹³⁾ by Bigi and Sanda. The latter is particularly attractive in that a CP violating effect can be observed in an asymmetry in the time evolution of particle and anti-particle decays to the same final state even in the absence of CP violation in the mixing and of more than one final state accessible in the decay. This may be at first paradoxical in that for the K^0 system it is well known that CP violation in the decay to two pions can only show up by means of the interference between the $I=0$ and $I=2$ final states. Were there a method whereby one could "beat the $\Delta I=1/2$ rule" it would be greatly welcomed. It is instructive to convince oneself that it cannot be done in the Kaon system and to understand why.

The magnitudes of many of the various effects in the B^0 system have been calculated in the beautiful paper⁽¹⁴⁾ by Buchala, Slominski, and Steger. Certain final states have very large asymmetries but with correspondingly small branching fractions so that samples exceeding 10^7 B decays are needed making it unlikely that CP violation, at the level predicted by the Standard Model, will be seen at any of the machines of this decade. Nevertheless it is important to search because seeing the phenomena outside the K^0 system would add greatly to our understanding and because the Standard Model may not have anything to do with the observed CP violation.

7. RARE OR FORBIDDEN PROCESSES

Finally, we briefly consider those rare or highly suppressed processes that, if seen, would constitute a violation of the Standard Model.

There are presently four experiments on the floor at BNL which are searching for such processes in Kaon decays. This entire class of experiments is beautifully described in the recent review⁽¹⁵⁾ by Littenberg.

The forbidden K_L decay with the most attention (two dedicated experiments) is that to muon-electron. The goal is a measurement at the 10^{-12} level and at that level of sensitivity, one can limit the mass of new "horizontal" bosons to greater than about 200 TeV! The process can go in various technicolor, supersymmetric, or composite models. Experimentally the detectors have to operate under extreme conditions where multiple Kaon decays within the resolving time of the apparatus are a serious problem. Also, the triggering schemes have become quite sophisticated with several layers of on-line processors in order to keep the dead-time to a minimum. In these respects, such efforts directly relate to some of the experimental problems that will be encountered in experimentation at the SSC.

We now turn our attention to the phenomenology of CP violation in the neutral Kaon system.

8. $K^0 - \bar{K}^0$ MIXING

As a result of the violation of strangeness conservation in the weak interaction, the phenomenon of neutral Kaon mixing occurs. Then, if CP is a good symmetry, we would expect two orthogonal eigenstates of the weak Hamiltonian which are themselves CP eigenstates.

The mixing phenomenon, even in the absence of CP conservation, can be described by the time-dependent Schroedinger equation

$$H\psi = (id/dt)\psi \quad (1)$$

where H can be written as:

$$H = M - i\Gamma/2. \quad (2)$$

While H itself is non-Hermitian, both M (the mass matrix) and Γ (the decay matrix) are in general separately Hermitian so that the Hamiltonian can then be written as:

$$H = \begin{pmatrix} M - i\Gamma/2 & M_{12} - i\Gamma_{12}/2 \\ M_{12}^* - i\Gamma_{12}^*/2 & M - i\Gamma/2 \end{pmatrix}. \quad (3)$$

The two component wave function ψ is the amplitude to be a K^0 , $\bar{K}^0$; $M = \langle K^0 | M | K^0 \rangle = \langle \bar{K}^0 | M | \bar{K}^0 \rangle$ where the last equality follows from CPT invariance. Similarly,

$$\Gamma = \langle K^0 | \Gamma | K^0 \rangle = \langle \bar{K}^0 | \Gamma | \bar{K}^0 \rangle; \quad M_{12} = \langle K^0 | M | \bar{K}^0 \rangle; \quad \Gamma_{12} = \langle K^0 | \Gamma | \bar{K}^0 \rangle.$$

The weak eigenstates are found by diagonalizing H :

$$K_{L,S} = \frac{K^0 + a\bar{K}^0}{\sqrt{1 + |a|^2}} \quad (4)$$

where

$$a = \frac{M_{12} - i\Gamma_{12}/2}{M_{12}^* - i\Gamma_{12}^*/2}. \quad (5)$$

The $K_L - K_S$ mass difference is given by

$$\Delta m = 2|M_{12}| \quad (6)$$

while the width difference is

$$\Delta\Gamma = -2|\Gamma_{12}|; \quad (7)$$

the choice of the minus sign results from the empirical observation that the heavier of the two states is the longer lived.

It is clear that the above measurable quantities, Δm and $\Delta\Gamma$, are expressed independent of whatever phase convention is adopted. This is not the case for the mixing parameter α . It is easily seen that the phase of α can be chosen at will by the appropriate adjustment of the relative K^0 and $\bar{K}^0$ phase (which is not in itself measurable) so that in fact only its modulus is physically relevant.

The modulus of α is equal to one only if M_{12} and Γ_{12} are relatively real. In this case, the weak eigenstates are also CP eigenstates so that the measure of CP violation in $K^0 - \bar{K}^0$ mixing is the phase difference between the off-diagonal elements of the mass and decay matrices.

Simplifying the above expression for α , we have

$$\begin{aligned} |\alpha| &= 1 + \frac{|\Gamma_{12}|^2/2}{|M_{12}|^2 + \frac{1}{4}|\Gamma_{12}|^2} \operatorname{Im}\left(\frac{M_{12}}{\Gamma_{12}}\right) \\ &= 1 + \frac{x}{2(x^2 + \frac{1}{4})} \phi_{\Delta S=2} \\ &= 1 + \frac{\phi_{\Delta S=2}}{2} \end{aligned} \quad (8)$$

where we have used $(11) \quad x \equiv \Delta m/\Delta\Gamma = 0.477$, and

$$\phi_{\Delta S=2} \equiv \arg \left(\frac{M_{12}}{\Gamma_{12}} \right).$$

The parameter $\phi_{\Delta S=2}$ (its departure from 0 or π) is a measure of the degree of CP nonconservation in the mixing. It is most easily determined from a measurement (12) of the semileptonic charge asymmetry in K_L decays:

$$\delta_L = \frac{\text{Rate}(\pi e \bar{\nu}) - \text{Rate}(\pi \bar{e} \nu)}{\text{Rate}(\pi e \bar{\nu}) + \text{Rate}(\pi \bar{e} \nu)}$$

$$= 3.30(12) \times 10^{-3}$$

$$= \frac{1 - |\alpha|^2}{1 + |\alpha|^2} \sim -\frac{\phi_{\Delta S=2}}{2}. \quad (9)$$

Thus, from the experimental result, we have

$$|\alpha| = 0.99671(13); \text{ and} \quad (10)$$

$$\phi_{\Delta S=2} = -6.58(26) \times 10^{-3}. \quad (11)$$

The experimental knowledge of the off-diagonal matrix elements can then be expressed as:

$$\frac{M_{12}}{\Gamma_{12}} = -0.4773(23)(1 - i6.58(26) \times 10^{-3}). \quad (12)$$

9. DYNAMICAL ORIGIN OF CP VIOLATION IN $K^0\bar{K}^0$ MIXING

What is the origin of the phase difference between the off-diagonal elements of the mass and decay matrices? To get some insight into the possibilities, we express both to second order in perturbation theory:

$$M_{12} = \langle K^0 | H_W | K^0 \rangle + \sum_n \frac{\langle K^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{E_n - M_{K^0}}, \quad (13)$$

where the first term represents a possible $\Delta S=2$ term in the weak Hamiltonian and the second term is the sum over all virtual intermediate states, and,

$$\Gamma_{12} = 2\pi \sum_F \rho_F \langle K^0 | H_W | F \rangle \langle F | H_W | K^0 \rangle \quad (14)$$

where ρ_F is the density of final states for the state F (2π , 3π , 4π , ...).

From the above expression, we can delineate some possibilities for the origin of the ~ 6.5 $\pi\pi$ phase difference:

- A. All real and virtual K meson transition elements are relatively real. In this case, the phase difference arises from the first term in expression (13). This hypothesis is known as the Superweak Model;⁽¹⁷⁾ if true, it has little consequence outside the neutral K system. While not in favor, this hypothesis is nevertheless consistent with all experimental information in that CP violating effects have yet to be seen in the K meson transitions. Discarding this term, we have
- B. All virtual matrix elements are relatively real. In this case, the 6.5 $\pi\pi$ phase could arise from a difference in phase between, for example, the $K \rightarrow 2\pi$ matrix elements with $I=0$ and $I=2$ final states. This possibility can be ruled out as a result of the small amount of $\Delta I=1/2$ violation.
- C. All real K meson decay matrix elements are relatively real. In this case, the phase difference could arise from some heavy state(s) with orthogonal contributions, an example being the state of top quark-anti top quark. This hypothesis would have no consequence within the Kaon system; however the heavy states could be produced in higher energy collisions, or they might have significance in the mixing and decays in other systems.
- D. The phase difference arises from heavy virtual states (again, for example, $t\bar{t}$) with small contributions to K meson decays. With this possibility, one has the hope of studying CP violation in the K meson decays themselves (e.g. the 2π and even the 3π

and $\gamma\gamma$ modes). The Kobayashi-Maskawa model⁽¹⁸⁾ is one with the promise of such studies.

10. CP VIOLATION IN $K \rightarrow 2\pi$ TRANSITIONS

We now consider the transitions $K_{L,S} \rightarrow \pi^+\pi^-$ and $\pi^0\pi^0$. There are two decay amplitudes, A_0 and A_2 , depending upon the isotopic spin of the final state pions.

- A. Suppose A_0 and A_2 are relatively real (no direct CP violation). In this case, we have, after some algebra,⁽¹⁹⁾

$$\eta_{+-} = \frac{\text{amp}(K_L \rightarrow \pi^+\pi^-)}{\text{amp}(K_S \rightarrow \pi^+\pi^-)} = \frac{-ix}{2x + i} \phi_{\Delta S=2} \quad (15)$$

From the known values for x and $\phi_{\Delta S=2}$, we get the following prediction:

$$\eta_{+-} = 2.27(8) \times 10^{-3} e^{i43.7^\circ(?)}, \quad (16)$$

which is in excellent agreement with the experimental value⁽¹⁶⁾

$$\eta_{+-} = 2.279(26) \times 10^{-3} e^{i44.6^\circ(1.2^\circ)}. \quad (17)$$

- B. Consider now the case of a small phase difference between A_1 and A_2 . We let $\phi_{\Delta S=1} \equiv \arg(A_2/A_0)$; then,⁽²⁰⁾

$$\eta_{+-} = \eta_{+-}(1 - 0.1 \frac{\phi_{\Delta S=1}}{\phi_{\Delta S=2}}) \quad (18)$$

$$\frac{\eta_{+-}}{\eta_{00}} = 1 - 0.3 \frac{\phi_{\Delta S=1}}{\phi_{\Delta S=2}} \quad (19)$$

$$\left| \frac{\eta_{+-}}{\eta_{00}} \right|^2 \sim 1 - 0.6 \frac{\phi_{\Delta S=1}}{\phi_{\Delta S=2}} \quad (20)$$

The current experimental information comes from two experiments

$$\begin{aligned} \left| \frac{\eta_{+-}}{\eta_{00}} \right|^2 &= 0.972(35) \text{ Chicago Saclay}^{(21)} \\ &= 1.010(49) \text{ BNL-Yale}^{(22)} \end{aligned} \quad (21)$$

Averaging these results gives

$$\left| \frac{\eta_{+-}}{\eta_{00}} \right|^2 = 0.925(28) \quad (23)$$

with the following value for the ratio of the two angles measuring the degree of CP violation in the $\Delta S=1$ and $\Delta S=2$ interactions:

$$\frac{\phi_{\Delta S=1}}{\phi_{\Delta S=2}} = 0.025(47). \quad (24)$$

Thus there is as yet no evidence for a "direct" CP non-conservation, the sensitivity to a non-zero phase angle between A_0 and A_2 being now at the level of a few hundred micro-radians.

11. RELATION TO THE "STANDARD" PHASE CONVENTION

In the phase convention of Wu and Yang,⁽²³⁾ one chooses A_0 , the $K \rightarrow 2\pi$, $I=0$ transition amplitude, to be real. Then the long lived state is written

$$K_L \sim (1 - \epsilon)|K^0\rangle - (1 + \epsilon)|\bar{K}^0\rangle \quad (25)$$

where ϵ is a small, complex mixing parameter. In this convention, one has

$$\text{Arg}(\epsilon) \sim \pi/4 \quad \text{and} \quad (26)$$

$$\text{Re} \epsilon \sim -\frac{\phi_{\Delta S=2}}{4}. \quad (27)$$

The parameter ϵ' is defined by

$$\epsilon' \equiv \frac{i}{\sqrt{2}} \epsilon (\delta_2 - \delta_0) \frac{\text{Im}(A_2)}{A_0} \quad (28)$$

and its relation to $\phi_{\Delta S=1}$ is approximately given by (see ref. 28)

$$\epsilon'/\epsilon \sim \frac{1}{10} \frac{\phi_{\Delta S=1}}{\phi_{\Delta S=2}}. \quad (29)$$

12. STANDARD MODEL EXPECTATIONS FOR CP VIOLATION IN THE KAON SYSTEM

In the standard model of the weak interactions, CP nonconservation is accounted for by the Kobayashi-Maskawa (KM) model.⁽¹⁸⁾ The authors pointed out that with six quark flavors, an irreducible complexity to the charged current coupling matrix among the quarks would provide for CP non-conservation. Not only is the phenomenon of CP violation then unified with that of quark mixing but as well CP violation is predicted among the heavier quark systems, particularly in B^0 decays as was discussed in section 6. While such effects may be observable in the future, the experimental effort has so far been concentrated upon the observation of the very small direct CP violating Kaon decays which also arise in the KM model. The 2π decays can sense the heavy quark intermediate states through the $\Delta I=1/2$ "penguin" amplitude which then interferes with a $\Delta I=3/2$ amplitude involving only light quarks. These amplitudes are diagrammed in Figure 1.

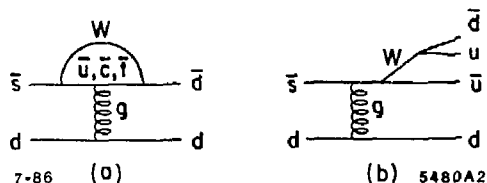


Figure 1. Diagrams contributing to $K^0 \rightarrow 2\pi$. (a) is the "penguin" which is pure $\Delta I=1/2$; in the KM model, it has a phase difference with respect to the $\Delta I=3/2$ part of (b).

The calculations for $\phi_{\Delta S=1}$ involve considerable uncertainty; they have been refined⁽¹⁴⁾ since the publication of the results above and it now appears that, with three generations,

$$|\phi_{\Delta S=1}| \sim (0.02 \text{ to } 0.08) |\phi_{\Delta S=2}| \sim 100 \text{ to } 400 \mu\text{r}. \quad (30)$$

The sign is uncertain, and, with more than three generations, lower values are possible.

13. CURRENT EXPERIMENTS ON CP VIOLATION IN $K^0 \rightarrow 2\pi$

At present there are two experiments nearly ready to take data with sensitivities, to $\phi_{\Delta S=1}$ in the range of 50 μr . These are, first, the

Chicago-Fermilab-Princeton-Saclay experiment at FNAL (E731) which is an outgrowth of the Chicago-Saclay experiment cited above, and, second, the CERN-Dortmund-Edinburgh-Maintz-Ursay-Pisa-Siegen experiment at CERN (NA31). These efforts will be briefly treated here; more details about the CERN experiment will be found in the contribution to the topical conference by D. Cundy.

Both experiments need to measure the four rates $K_{L,S} \rightarrow \pi^+\pi^-$, $\pi^0\pi^0$ with high statistics, low backgrounds, and minimal systematic errors, the latter representing the greatest challenge.

The detector for the Fermilab experiment is shown in Figure 2. For that experiment, a new beam-line was constructed making full use of the 800 GeV primary protons, the 20 sec beam pulse, large targeting angles, very clean collimation, and low Z absorbers to provide two closely spaced, clean, high intensity K_L beams. One of the beams passes through a thick regenerator so that both K_L and K_S decays to a given channel are studied simultaneously thereby significantly reducing one important source of systematic error. A new high rate drift-chamber spectrometer was constructed and many electromagnetic veto planes were installed to aid in the rejection of the copious $K_L \rightarrow 3\pi^0$ mode. The regenerator itself was "active" in order to reject inelastically regenerated events with particle production. Photons are detected in a large finely-segmented lead-glass array.

The experiment alternates between two phases - the charged mode and the neutral mode. For the charged mode running, only a simple two-track trigger is needed and statistics are readily obtained. For the neutral mode, a thin lead counter is inserted at the end of the decay region and

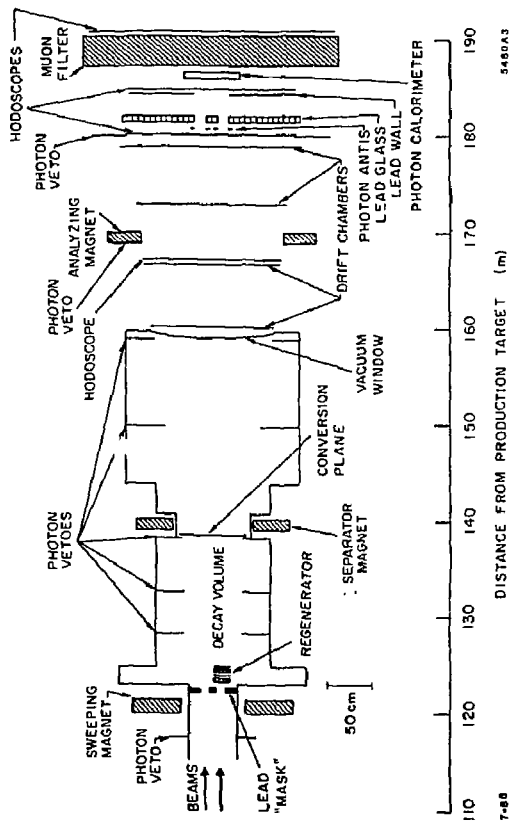


Figure 2. Elevation View of the FNAL (E731) Experiment

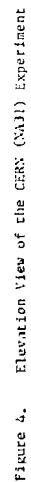
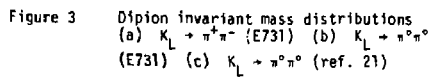
one converted photon is required in the trigger together with more than 25 GeV deposit in the lead glass array.

This experiment will run in 1987 and will collect in excess of 10^5 $K_L \rightarrow 2\pi^0$ events. The experiment had a test run in 1985; the quality of the data can be seen in Figure 3 where, for a sample from this test run, is plotted the $\pi^+\pi^-$ and $\pi^0\pi^0$ invariant mass distributions. The latter distribution is also shown for the earlier Chicago-Saclay experiment,⁽²⁾ the background has been reduced by about a factor of five.

The detector for the CERN experiment is shown in Figure 4. Again the detector and beam-line are new, and the technique is different from the Ferrelab experiment. Both K_S decays modes are simultaneously recorded with a close target which is moved throughout the decay region. A far target is used for recording the K_L modes. The electromagnetic detector is a large liquid argon calorimeter and the charged pions are detected with MWPC's and a hadron calorimeter; no magnet is employed. No conversion is required and the acceptance is very high. Four planes of anti-counters are employed to help reject $K_L \rightarrow 3\pi^0$ decays.

This experiment will run in 1986 and should collect about 2.5×10^5 $K_L \rightarrow 2\pi^0$ events.

As mentioned above, it is likely that systematic uncertainties will ultimately determine the sensitivities of the two experiments. The FNAL experiment is relatively immune to changes in detector response as a function of time or intensity; the CERN experiment is not very dependent upon an accurate Monte-Carlo simulation of the detector response. Both are critically dependent upon knowledge of the absolute energy scale for each decay mode.



14. TEST OF CPT INVARIANCE⁽²⁶⁾ IN K^0 DECAY

Both groups mentioned in the previous section have proposed to test⁽²⁶⁾ CPT invariance in the decay of the neutral Kaon by measuring the phase difference $(\phi_{+-} - \phi_{00})$ between η_{+-} and η_{00} to a precision of 1° . We note that in the absence of any direct CP violation, η_{+-} and η_{00} should have identical phases (see (15) above):

$$\eta_{+-} = \eta_{00} = \frac{-ix}{2x + i} \phi_{\Delta S=2} \quad (31)$$

$$\phi_{+-} = \phi_{00} = \arctan 2x = 43.7^\circ (2). \quad (32)$$

As noted earlier the measured value for ϕ_{+-} is in good agreement with the prediction; however the best measurement⁽²⁷⁾ of ϕ_{00}

$$\phi_{00} = 54.5^\circ (5.3) \quad (33)$$

differs by about two standard deviations. Possible explanations for this deviation are

1. Actually $\phi_{+-} \sim \phi_{00} \sim 43.7^\circ$ and the value for ϕ_{00} represents a fluctuation.
2. CPT is violated in $K^0 \rightarrow 2\pi$. This will account for $\phi_{+-} \neq \phi_{00}$. However, CPT must as well be violated in the $K^0 - \bar{K}^0$ mixing since otherwise one would expect that $\phi_{+-} \neq \arctan 2x$.
3. CPT is a good symmetry; however the assumption that direct CP violation is small - which leads to the equality of ϕ_{+-} and ϕ_{00} - is incorrect. The experiments described above which look for a direct CP violation are in fact only sensitive to the component of the amplitude which is in the direction of $\arctan 2x$. The phase of ϵ' is in the direction $\delta_2 - \delta_0 + \pi/2$ which,

according to the results of several consistent but somewhat indirect determinations,⁽²⁸⁾ is near 45° . However, if we ignore these determinations and if the phase of ϵ' were -45° , we would then expect a splitting between ϕ_{+-} and ϕ_{00} .

A 10° difference would then result from $|\epsilon'/\epsilon| \sim 0.06$.

However, one would then expect that ϕ_{+-} would differ from $\arctan 2x$ by about 3° .

Thus the first scenario is the most likely; nevertheless the 1° measurements of the phase difference will provide the most accurate test of CPT symmetry in a decay amplitude.

15. CONCLUSION

It is interesting to note that there are many reasons for doing physics at an energy scale above 1 TeV which come from studies of the Kaon, a particle with a center of mass energy of only 0.5 GeV. These come from studies of rare decays, of CP violation, and from the very small size of the $K_L K_S$ mass difference. There is no sign that these and other precision studies will not continue to be fruitful in the future.

16. ACKNOWLEDGEMENTS

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**The Bound on the Number of Neutrinos
from $e^+e^- \rightarrow \gamma\nu\bar{\nu}$ at PEP and PETRA**

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ABSTRACT

Searches for single unaccompanied photons in e^+e^- collisions at PEP and PETRA are combined. The upper limit on the single-photon process $e^+e^- \rightarrow \gamma\nu\bar{\nu}$ is used to obtain a bound on the number of light-neutrinos.

Provocateur paper presented by T. Lavine supplementing B. Winstein's lecture.

Radiative neutrino pair production at PEP and PETRA energies receives contributions from t -channel $W^\pm$ exchange and from s -channel virtual Z^0 annihilation. Neutrino pair production is sensitive to the total number of neutrinos through the Z^0 to which all neutrinos couple. Additional neutrinos besides ν_e , ν_μ , and ν_τ may indicate the presence of additional fermion generations.

In the local (point-coupling) limit, the lowest order $e^+e^- \rightarrow \gamma\nu\bar{\nu}$ cross section for arbitrary total number N_ν of massless neutrinos is¹

$$\frac{d^2\sigma(e^+e^- \rightarrow \gamma\nu\bar{\nu})}{dx_\gamma d\cos\theta_\gamma} = \frac{G_F^2 \alpha}{6\pi^2} \left\{ 2 + \frac{N_\nu(V_e^2 + A_e^2) + 2(V_e + A_e) \left[1 - s(1-x_\gamma)/M_Z^2 \right]}{\left[1 - s(1-x_\gamma)/M_Z^2 \right]^2 + \Gamma_Z^2/M_Z^2} \right\} \quad (1)$$

$$\times \frac{s(1-x_\gamma)}{x_\gamma \sin^2\theta_\gamma} \left[\left(1 - \frac{1}{2}x_\gamma \right)^2 + \frac{1}{4}x_\gamma^2 \cos^2\theta_\gamma \right]$$

where V_e and A_e are the vector and axial vector couplings, respectively, of the Z^0 to electrons, Γ_Z is the total width of the Z^0 , and $x_\gamma = 2E_\gamma/\sqrt{s}$. The three

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additive terms in the large curly brackets in equation (1) are due, respectively, to production of ν_e pairs through the virtual $W^\pm$ exchange diagrams, to production of all N_ν neutrinos through the virtual Z^0 diagram, and to interference between these processes for ν_e pair production. The sensitivity to N_ν of the cross section (1) for $e^+e^- \rightarrow \gamma\nu\bar{\nu}$ depends on s and on the minimum x_γ . In the limit of small s and small minimum x_γ , the cross section would be proportional to $1 + \frac{1}{2}N_\nu$ if $\sin^2\theta_W$ were $\frac{1}{4}$.

The total width Γ_Z is sensitive² to the unknown mass of the top quark and to the possible presence of a new heavy charged lepton. However, the cross section (1) for $e^+e^- \rightarrow \gamma\nu\bar{\nu}$ is practically insensitive to the properties of these unknown particles at PEP and PETRA energies.

Cross section (1) is for massless neutrinos and is additionally suppressed by the factor $\frac{1}{4}\beta_\nu(3 + \beta_\nu^2)$ for heavy Dirac neutrinos or β_ν^3 for heavy Majorana neutrinos, where $\beta_\nu = (1 - 4m_\nu^2/s)^{1/2}$. Henceforth, all neutrinos will be assumed to be light Dirac fermions with $\beta_\nu \approx 1$.

Single-photon searches have been reported by the MAC³ and ASP⁴ Collaborations at PEP and by the CELLO Collaboration⁵ at PETRA. The MAC, ASP, and CELLO single-photon searches are summarized in Table 1. The single-photon yield calculated from $e^+e^- \rightarrow \gamma\nu\bar{\nu}$ in each search is

$$n = L\epsilon\sigma \quad (2)$$

where L is the time-integrated luminosity, ϵ is the search efficiency averaged over the acceptance, weighted by the differential cross section, and σ is the cross section (1) integrated over the acceptance.

Search	$\sqrt{s}$ GeV	Acceptance Cuts	$L\epsilon$ pb^{-1}	Expected Yield ($N_\nu = 3$)	Observed Yield
MAC ³	29.0	$E_{\perp\gamma} > 4.5, 2.0, 2.6 \text{ GeV}$ $\theta_\gamma > 40^\circ$	27, 50, 42	1.1	1
ASP ⁴	29.0	$E_{\perp\gamma} > 1 \text{ GeV}$ $\theta_\gamma > 20^\circ$	66	2.2	1
CELLO ⁵	42.6	$E_\gamma > 2.1 \text{ GeV}$ $\theta_\gamma > 34^\circ$	13	0.7	0
Combined				4.0	2

TABLE 1. Single-photon searches. The expected yields quoted are calculated for $e^+e^- \rightarrow \gamma\nu\bar{\nu}$ with the three known neutrinos. The $\sqrt{s}$ for CELLO is an average appropriate for the quoted integrated luminosity.

The yield from $e^+e^- \rightarrow \gamma\nu\bar{\nu}$ in each search can be calculated from equation (2) using the appropriate acceptances and lowest order cross sections. The acceptance cuts, integrated luminosity, and average efficiency for detecting radiative pair production of neutrino-like particles in each search are specified in Table 1. The three running periods in the MAC search have,³ respectively, $L = 36, 80$, and 61 pb^{-1} , and $\epsilon = 74, 62$, and 69% . For the ASP search,⁴ $L = 115 \text{ pb}^{-1}$ and $\epsilon = 57\%$. For the CELLO search,⁵ $L = 37.6 \text{ pb}^{-1}$ and $\epsilon = 35\%$. The CELLO data come from PETRA running at several different values of $\sqrt{s}$; the value used, $\sqrt{s} = 42.6 \text{ GeV}$, is an average appropriate for the quoted integrated luminosity.⁵

The combined single-photon yield expected from $e^+e^- \rightarrow \gamma\nu\bar{\nu}$ for the three known neutrinos is 4.0 events, divided among the searches as indicated in Table 1. The two events seen by the MAC and ASP searches are consistent with this

	MAC ³	ASP ⁴	CELLO ⁵	Combined
N_ν	< 17	< 7.5	< 15	< 4.8

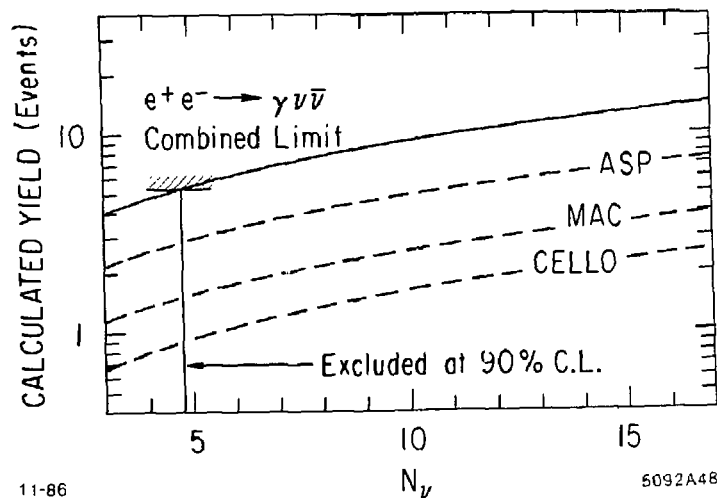
TABLE 2. Limits at 90% CL from single-photon searches.

combined expected yield; the probability of the combined searches seeing two events or less is 24%.

An upper limit on the total number N_ν of light-neutrinos can be calculated assuming that $e^+e^- \rightarrow \gamma\nu\bar{\nu}$ is the only reaction that produces single-photons. The 90% CL upper limit on the $e^+e^- \rightarrow \gamma\nu\bar{\nu}$ yield in the combined searches then is 5.3 events, based on the two single-photons observed. The 95% CL upper limit is 6.3 events.

The single-photon yield from $e^+e^- \rightarrow \gamma\nu\bar{\nu}$ in each search and the combined yield, calculated using the lowest order cross section (1) assuming $M_Z = 92.7 \text{ GeV}/c^2$ and $\sin^2\theta_W = 0.22$, are plotted in Figure 1 as a function of the unknown total number N_ν of light-neutrinos. The 5.3 event 90% CL upper limit on the combined single-photon yield corresponds to the limit $N_\nu < 4.8$, as indicated in Figure 1. The 6.3 event 95% CL upper limit on the combined yield corresponds to $N_\nu < 6.1$. As discussed above, this limit on N_ν is independent of the mass of the top quark or a new heavy lepton. Limits on N_ν from each search, and the limit obtained by combining the searches, are summarized in Table 2.

The single-photon searches at PEP and PETRA have provided restrictive limits on the number of neutrinos. These limits are expected to improve signifi-



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1. Calculated $e^+e^- \rightarrow \gamma\nu\bar{\nu}$ combined yield as a function of the unknown total number N_ν of light-neutrinos. Dashed curves indicate the calculated yield in each search. The 5.3 event 90% CL upper limit on single-photons in the combined searches corresponds to the limit $N_\nu < 4.8$.

cantly as similar searches, with estimated sensitivities of 0.2 light-neutrinos, are performed at the SLC and LEP.

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PERTURBATIVE QCD: "K-FACTORS"¹

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ABSTRACT

A pedagogical series of lectures on the perturbative applications of Quantum Chromodynamics (QCD) is presented. Detailed calculations are made for e^+e^- annihilations, deep inelastic scattering, and the Drell-Yan process. Emphasis is placed on the way one handles the infrared and mass singularities that arise in QCD. Two regularization schemes, massive glue and dimensional regularization, are studied. The origin and significance of "K-factors" is explained.

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1. INTRODUCTION

During the last ten years, a new framework to describe strong interaction physics has emerged: quantum chromodynamics (QCD). The forces among the colored quarks are generated by the exchange of colored vector gluons which are coupled to the quarks in a gauge-invariant manner. Although the theory is well defined, precisely what it predicts for experiment is not always clearly known. At high momentum transfer, Q , the effective coupling between quarks and gluons becomes small which permits the use of perturbation theory. Yet every real process involves both high and low Q together and precisely how to separate these parts and how to gain knowledge of the low Q uncalculable pieces is not always completely clear.

In these lectures I will discuss in detail how mass singularities arise in perturbative QCD and how they are handled. In addition, we will examine carefully the beyond leading order corrections to deep inelastic electron proton scattering and the Drell-Yan process that lead to the so called "K-factor". I have decided to discuss the origin of "K-factors" in detail because I feel that many people associate the word "fudge-factor" with "K-factor" and this is simply not true. QCD differs from the naive parton model and some of these differences manifest themselves in the form of "K-factors". The fact that experiment differs from the naive parton model in precisely the way predicted by perturbative QCD is a great success for the theory.

It is instructive to examine two schemes for regularizing the infrared and mass singularities. This will illustrate clearly those quantities that depend on the regularization scheme and those that do not. Of course, it must be true that no experimental observable depend on the manner in which the regularization is done. One scheme that we will use is intuitively easy to understand and involves giving the gluon a fictitious mass, m_g . [1,2] I call this scheme the "massive gluon" (MG) scheme. The other regularization scheme we will use is more elegant and involves performing the calculations in N rather than 4 dimensions. [3] This scheme is called "dimensional regularization" (DR).

We will begin with the Feynman rules for QCD in Fig. 1 [4] and perform many detailed calculations. I feel that the best way to learn about perturbative QCD is by doing perturbative QCD. I hope that experimenters as well as theorists will be able to follow the lectures and that both will find them useful. I will begin in Section II with an examination of the perturbative QCD corrections in e^+e^- annihilations. Pedagogically this is the best place to start. We will develop the tools necessary in analyzing deep inelastic electron proton scattering (Section III) and the Drell-Yan process (Section IV).

I do not have time in these lectures to calculate in detail the ultraviolet divergences whose renormalization results in an effective QCD coupling that varies with momentum transfer, Q . For completeness, I will simply outline the situation.

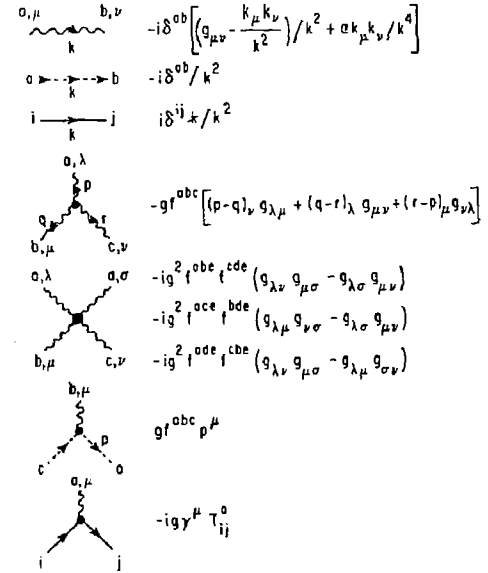


Fig. 1. Feynman rules for QCD. The solid and wavy lines are quarks and vector gluons, respectively. The dotted lines are (non-existent) "ghost" particles that must be included when one calculates in non-axial gauges.

In Quantum Electrodynamics (QED) there is one parameter, the coupling, that must be determined experimentally. Suppose that we decide to experimentally measure the strength of electromagnetic interactions by scattering an electron off an electron and comparing the result with a theoretical calculation. The rate for electron-electron scattering is affected by the presence of vacuum polarization diagrams like that shown in Fig. 2. One immediately runs into trouble, however, since the one electron loop correction to the photon propagator diverges like $\log(\lambda)$, where λ is some ultraviolet cutoff that can be arbitrarily large. In particular, the leading order bubble contribution is

$$\alpha_0 B(Q^2) = -\frac{\alpha_0}{3\pi} [\log(\lambda^2/Q^2) + \frac{5}{3}]$$

for $Q^2/m_e^2 \gg 1$. (1.1a)

and

$$\alpha_0 B(Q^2) = -\frac{\alpha_0}{3\pi} [\log(\lambda^2/m_e^2) - \frac{1}{5} \frac{Q^2}{m_e^2}]$$

for $Q^2/m_e^2 \ll 1$. (1.1b)

where $q^2 = -Q^2$ is the q -momentum squared of the virtual (spacelike) photon and m_e is the electron mass. The coupling α_0 is the "bare" electric charge ($\alpha_0 = e_0^2/4\pi$). It is convenient to define an effective coupling, $\alpha_{\text{eff}}(Q^2)$, that incorporates all the vacuum polarization bubbles. Namely,

$$\alpha_{\text{eff}}(Q^2) = \alpha_0 (1 + \alpha_0 B(Q^2) + \alpha_0 B(Q^2) \alpha_0 B(Q^2) + \dots), \quad (1.2a)$$

yielding

$$\alpha_{\text{eff}}(Q^2) = \alpha_0 / (1 - \alpha_0 B(Q^2)). \quad (1.2b)$$

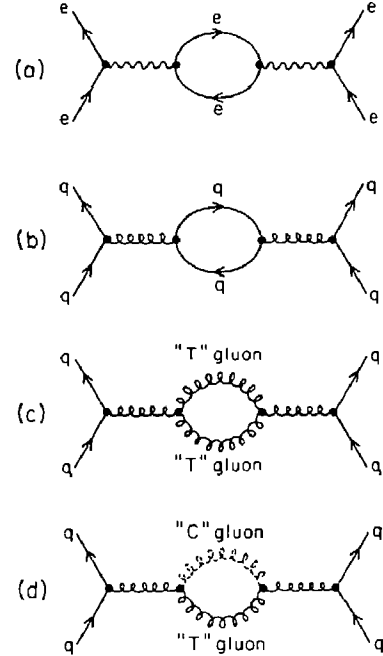


Fig. 2. (a) Lowest order vacuum polarization correction to the electric charge. (b) Lowest order correction to the quark-gluon coupling due to a virtual quark-antiquark pair. (c) Lowest order correction to the quark-gluon coupling due to a virtual pair of transverse ("T") gluons in the coulomb gauge. (d) Lowest order correction to the quark-gluon coupling due to a virtual pair of gluons, one transverse ("T") and one "Coulomb" ("C") in the coulomb gauge.

or

$$1/\alpha_{\text{eff}}(Q^2) = 1/\alpha_0 - B(q^2). \quad (1.2c)$$

The procedure for handling the ultraviolet divergences like those appearing in $B(q^2)$ in (1.1) is called renormalization. One defines an experimental electric charge, α , by the large distance behavior of the electric potential (Thompson limit)

$$\alpha = \alpha_{\text{eff}}(Q^2 \rightarrow 0), \quad (1.3)$$

which experimentally is about $1/137$. All results of calculations are now reexpressed in terms of the experimental coupling, α , rather than the unobservable bare coupling, α_0 . In terms of α , the effective coupling is given by

$$1/\alpha_{\text{eff}}(Q^2) = 1/\alpha - \{B(q^2) - B(0)\}, \quad (1.4)$$

where the quantity $\{B(q^2) - B(0)\}$ is now independent of the artificial ultraviolet cutoff λ . The cutoff λ is now sent to infinity while holding α constant. From (1.1a) and (1.2b) we see that the large q^2 behavior of the effective coupling is given by

$$\alpha_{\text{QED}}(Q^2) = \alpha/[1 - (\alpha/3\pi)\log(Q^2/m_e^2)]. \quad (1.5)$$

In QED as Q^2 increases, $\alpha_{\text{eff}}(Q^2)$ increases. No matter how small an α one has, one can always increase Q^2 to a point where $\alpha_{\text{QED}}(Q^2)$ becomes infinite. This means that perturbation theory breaks down at high Q^2 in QED. One needs to include higher and higher orders in α_{QED} as Q^2 increases. At low Q^2 , on the other hand, $\alpha_{\text{QED}}(Q^2)$ is small ($\alpha \approx 1/137$) and perturbation theory works well.

The physical reason for the rising effective charge with the increased Q^2 of the probing photon is illustrated in Fig. 3a. If Q^2 is small then the photon cannot resolve small distances and "sees" a

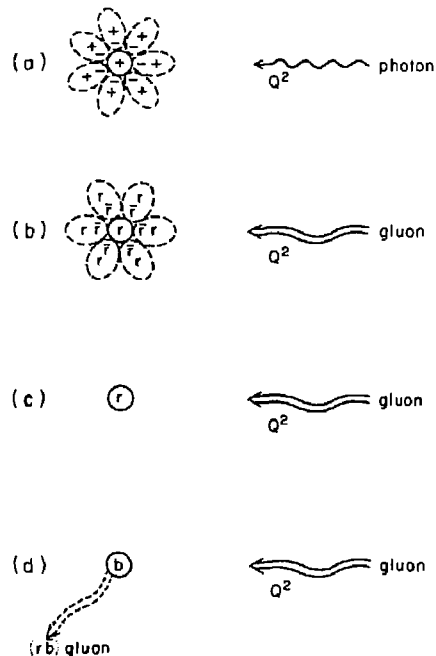


Fig. 3. (a) Illustration of how vacuum polarization in QED will "shield" a bare positive charge when placed in a vacuum. (b) The same shielding as in (a) but for a "red" charge in QCD. (c) and (d) Show how a "red" charge can, in QCD, radiate away its red charge, r , and become a blue charge, b , via the emission of a virtual $r\bar{b}$ gluon.

"point" charge shielded by the vacuum polarization of the infinite sea of electron-positron pairs. As Q^2 increases, the photon "sees" a smaller and smaller spatial area and the shielding effect is less.

In QCD the behavior of the effective coupling constant is strikingly different. The reason for this difference is the feature of QCD, that the gluons carry charge (color) and interact with each other. The amount of the contributions of the various diagrams in Fig. 2b and Fig. 2c is gauge dependent. However, the situation is most clear in the Coulomb gauge. In this gauge, the lowest order bubble contribution to the gluon propagator is given for large Q^2 by

$$\alpha_0 \beta_{\text{QCD}}(Q^2) = -\alpha_0 \tilde{a} \log(\lambda^2/Q^2). \quad (1.6)$$

where λ is the ultraviolet cutoff and α_0 is the bare quark-gluon coupling and where

$$\tilde{a} = -\beta_0/4\pi, \quad (1.7a)$$

with

$$\beta_0 = -(\frac{2}{3} n_f + 5 - \epsilon), \quad (1.7b)$$

where n_f is the number of quark flavors. The $+\frac{2}{3} n_f$ and the $+5$ come from the quark loop and the transverse gluon loop in Fig. 2, respectively, and are of the same sign as the QED case. These contributions must be positive since the diagrams can be cut across the bubble and represent contributions to the physical rate for producing quark pairs or transverse gluon pairs which must be positive. The -16 in (1.7b) comes from the diagram with one transverse and one "Coulomb" gluon in the bubble (Fig. 2d). This contribution need not be positive since the instantaneous "Coulomb" gluon is not physical. If $\frac{2}{3} n_f < 11$ then β_0 is positive and $\tilde{a}$ is negative in contrast to the QED case (1.1a). As for the QED case, the ultraviolet divergences in (1.6) are handled by renormalization. Here, however, we cannot define the "experimental charge" by the $Q^2 \rightarrow 0$ limit of $\alpha_{\text{eff}}(Q^2)$ as we did in (1.3). We instead choose some Q^2 say $Q^2 = \mu^2$, to define the coupling and express all predictions in terms of the coupling at this point (called the renormalization point or subtraction point). The effective coupling is then given by

$$1/\alpha_{\text{eff}}(Q^2) = 1/\alpha(\mu^2) - (\beta(Q^2) - \beta(\mu^2)). \quad (1.8)$$

As before, the quantity $(\beta(Q^2) - \beta(\mu^2))$ is independent of the arbitrary cutoff λ . By the use of (1.6), we see that the leading order behavior of the coupling in QCD is

$$\alpha_{\text{eff}}(Q^2) = \alpha_s(Q^2) = \alpha(\mu^2)/[1 + (\alpha(\mu^2)\beta_0/4\pi)\log(Q^2/\mu^2)], \quad (1.9)$$

which approaches zero as $Q^2 \rightarrow \infty$ (asymptotic freedom) as illustrated in Fig. 4. This means that for QCD, perturbation theory should work well at high Q^2 (short distances) but break down at small Q^2 (large distances) where $\alpha_s(Q^2)$ becomes large and hopefully (?) confines quarks within hadrons.

The nature of the QCD coupling constant $\alpha_s(Q^2)$ takes a bit of getting used to. In QED it is easy to define what one means by the charge of an electron e . It is related to the large distance behavior of the electric potential (Thomson limit). One cannot do this for QCD since the $Q^2 \rightarrow 0$ limit of $\alpha_s(Q^2)$ cannot be calculated by perturbation theory. Instead we had to define some arbitrary point μ^2 at which the coupling is $\alpha(\mu^2)$. It, however, does not matter which point μ^2 one chooses. If one chooses instead the point M^2 then the two couplings are related (to lowest order) by

$$1/\alpha_s(\mu^2) - \tilde{a} \log(Q^2/\mu^2) = 1/\alpha_s(M^2) - \tilde{a} \log(Q^2/M^2), \quad (1.10a)$$

or

$$1/\alpha_s(\mu^2) + \tilde{a} \log(\mu^2) = 1/\alpha_s(M^2) + \tilde{a} \log(M^2). \quad (1.10b)$$

Thus, in some sense, the "real" parameter in the theory is not $\alpha(\mu^2)$ or μ^2 but rather a mass scale, A , that is independent of μ^2 and is given to this order by

$$\log(A^2) = 1/(\tilde{a}\alpha(\mu^2)) + \log(\mu^2). \quad (1.11)$$

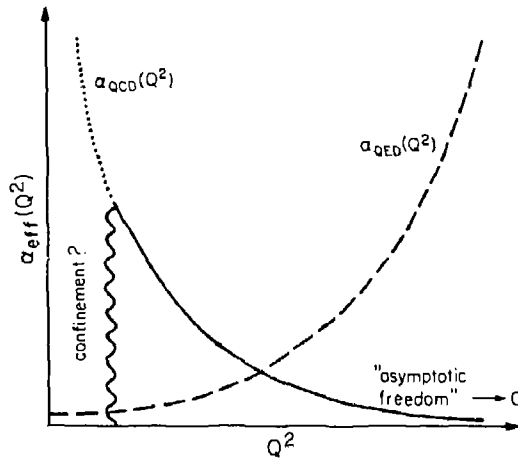


Fig. 4. Illustration of the behavior of the effective coupling calculated in perturbation theory for QED and QCD. In QED the effective coupling, $\alpha_{\text{QED}}(Q^2)$, is small at small Q^2 , but becomes large at large Q^2 (small distances). In QCD, on the other hand, the effective coupling is large at small Q^2 (large distances) where confinement may occur, but decreases toward zero as Q^2 increases ("asymptotic freedom"). Perturbation theory should work well for QCD (QED) at large Q^2 (small Q^2).

In terms of the mass scale Λ , the effective coupling is given by

$$\alpha_s(Q^2) \equiv \kappa_s^2(Q^2)/4\pi = \frac{4\pi}{\beta_0 \log(Q^2/\Lambda^2)} \quad (1.12a)$$

with

$$\beta_0 = 11 - \frac{2}{3} n_f. \quad (1.12b)$$

It is interesting to notice that (1.12) becomes infinite at $Q^2 = \Lambda^2$ which corresponds to a distance of about 0.5 Fermi for $\Lambda \approx 500$ MeV. (This is extremely crude, however, since (1.12) is not applicable when $\alpha_s(Q^2)$ becomes large.)

A physical reason for the behavior of $\alpha_s(Q^2)$ is illustrated in Fig. 3. Quark-antiquark vacuum polarization shields the color charge as was the case in QED. However, now since the source can radiate charge (i.e., change from red to blue by emitting a red-blue-bar gluon), the charge is no longer located at a definite place in space. It is diffusely spread out due to gluon emission and absorption. As one increases the Q^2 of the incoming gluon probe, thereby looking at smaller and smaller spatial distances, it becomes less likely to find the "charge" (red in Fig. 3). This latter effect is stronger than the former and the effective charge thus appears weaker and weaker as the Q^2 of the probe increases.

II. ELECTRON-POSITRON ANNIHILATIONS

A. Order α_s Corrections to the Total Rate for e^+e^- Hadrons

1. Naive Parton Model Result

The differential cross section for the electromagnetic process $e^+e^- \rightarrow \mu^+\mu^-$ shown in Fig. 5a is

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{cm}} = \frac{1}{64\pi^2} \frac{2}{E_{\text{cm}}} |M(e^+e^- \rightarrow \mu^+\mu^-)|^2$$

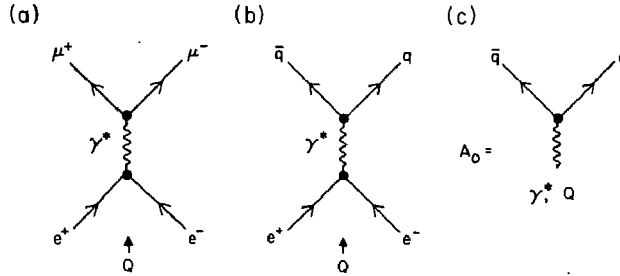


Fig. 5. (a) Lowest order Feynman diagram for the process $e^+e^- \rightarrow \mu^+\mu^-$. (b) Lowest order diagram for the process $e^+e^- \rightarrow \bar{q}q$. (c) Lowest order amplitude, A_0 , for the "decay" of a virtual photon, γ^* , of invariant mass Q into a quark-antiquark pair.

$$= \frac{1}{4} \frac{\alpha^2}{Q^2} (1 + \cos^2 \theta_{cm}), \quad (2.1)$$

where masses have been neglected and where the center of mass energy $E_{cm} = Q$ and the electromagnetic coupling $\alpha = e^2/4\pi$. The solid angle is given by, $d\Omega_{cm} = d(\cos \theta_{cm}) d\phi_{cm}$, where θ_{cm} is the scattering angle between the outgoing μ^+ and the incoming e^+ . Integrating over the angles θ_{cm} and ϕ_{cm} gives the total cross section

$$\sigma(e^+e^- \rightarrow \mu^+\mu^-) = \frac{4\pi}{3} \frac{\alpha^2}{Q^2}. \quad (2.2)$$

Replacing the $\mu^+\mu^-$ pair with a quark-antiquark pair (Fig. 5b) gives

$$\sigma(e^+e^- \rightarrow q\bar{q}) = (3) \frac{4\pi}{3} \frac{\alpha^2}{Q^2} e_q^2, \quad (2.3)$$

which is identical to $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ except for the factor of 3 which comes from summing over the three quark colors and the factor e_q^2 , where e_q is the charge of the quark q in units of the electric charge e . Assuming that quarks turn into hadrons with unit probability one arrives at the famous parton model prediction for the ratio of the total cross section $e^+e^- \rightarrow \text{hadrons}$ to the total cross section for $e^+e^- \rightarrow \mu^+\mu^-$. Namely,

$$\begin{aligned} R(e^+e^-) &\equiv \sigma(e^+e^- \rightarrow \text{hadrons}) / \sigma(e^+e^- \rightarrow \mu^+\mu^-) \\ &= 3 \sum_{i=1}^{n_f} e_{q_i}^2, \end{aligned} \quad (2.4)$$

where the sum is over all quark pairs that can be produced at the given CM energy Q .

It is convenient to visualize this process as an e^+e^- pair which annihilates producing a virtual photon, γ^* , which then "decays" into either a muon pair or a quark-antiquark pair (Fig. 5c). From this

point of view we arrive at (2.4) by considering the two-body differential decay rate

$$dW = \frac{1}{2E_{cm}} |M|^2 d^6R_2, \quad (2.5)$$

where d^6R_2 is the two-body phase-space factor

$$d^6R_2 = \frac{d^3p_1}{(2\pi)^3(2E_1)} \frac{d^3p_2}{(2\pi)^3(2E_2)} (2\pi)^4 \delta^4(q-p_1-p_2). \quad (2.6)$$

The total decay is deduced by integrating dW in (2.5) over the 4-momentums of the decay products p_1 and p_2 subject to the constraint

that $p_1 + p_2 = q$, where q is the 4-momentum of the virtual photon, γ^* . Integrating over the 3-momentum of particle 2 yields

$$\int d^3p_2 \delta^4(q-p_1-p_2) = \delta(q-E_1-E_2). \quad (2.7)$$

Next we integrate over the direction of particle 1 using

$$\frac{d^3p_1}{2E_1} = \frac{1}{2} E_1 \sin\theta_1 d\theta_1 d\phi_1 dE_1. \quad (2.8)$$

where I have taken particles 1 and 2 to be massless and θ_1 and ϕ_1 are angles with respect to an arbitrary choice of axis. Integrating over θ_1 and ϕ_1 yields

$$\int \frac{d^3p_1}{2E_1} = \frac{4\pi}{2} E_1 dE_1. \quad (2.9)$$

The final integration over E_1 is accomplished using the conservation of energy δ -function

$$\frac{\delta(Q-E_1-E_2)}{2E_2} = \delta[(q-p_1-p_2)^2] = \delta(Q^2-2E_1Q), \quad (2.10)$$

giving

$$\int \frac{E_1 dE_1}{2E_2} \delta(Q-E_1-E_2) = \frac{E_1}{2Q} = \frac{1}{4}, \quad (2.11)$$

since $E_1 = Q/2$. Hence,

$$R_2 = \int d^6R_2 = \frac{1}{8\pi}. \quad (2.12)$$

and since the matrix element does not depend on any of the variables of integration the total decay rate is

$$W = \frac{1}{2Q} |M|^2 R_2 = \frac{1}{16\pi Q} |M|^2. \quad (2.13)$$

The matrix element for a virtual photon to decay into a muon pair is

$$|M(\gamma^* \rightarrow \mu^+ \mu^-)|^2 = 4e^2 Q^2 = 16\pi\alpha Q^2, \quad (2.14)$$

where the polarization states of the virtual photon have been summed over by using Feynman's trick of setting

$$\sum_{pol} \epsilon_\mu \epsilon_\mu^* = -g_{\mu\nu}, \quad (2.15)$$

where ϵ is the polarization 4-vector of the virtual photon and $g_{\mu\nu}$ is the matrix $g_{00} = 1$, $g_{ij} = -\delta_{ij}$. Combining (2.13) and (2.14) gives

$$W(\gamma^* \rightarrow \mu^+ \mu^-) = \alpha Q. \quad (2.16)$$

Similarly,

$$W(\tau^* \rightarrow q\bar{q}) = \sigma_0 = 3\alpha \frac{e_q^2}{Q}. \quad (2.17)$$

and the ratio $R = W(\tau^* \rightarrow q\bar{q})/W(\tau^* \rightarrow \mu^+\mu^-)$ is the same as that arrived at in (2.4). I will call the rate $\tau^* \rightarrow q\bar{q}$ the Born term, σ_0 , and the amplitude in Fig. 5c the Born amplitude, A_0 .

2. Real Gluon Emission

We now consider the three-body "decay" of a virtual photon, τ^* , into a quark-antiquark pair and a real gluon as shown in Fig. 6. The three-body differential decay rate is

$$dW = \frac{1}{2E_{\text{cm}}} |M|^2 d^3R_3, \quad (2.18)$$

where the three-body phase space factor is given by

$$d^3R_3 = \frac{d^3p_1}{(2\pi)^3(2E_1)} \frac{d^3p_2}{(2\pi)^3(2E_2)} \frac{d^3p_3}{(2\pi)^3(2E_3)} (2\pi)^4 \delta^4(q - p_1 - p_2 - p_3). \quad (2.19)$$

Equations (2.18) and (2.19) replace the two-body formulas (2.5) and (2.6) and to arrive at a total rate we must integrate over the 3-momentums of three decay particles (1 = antiquark, 2 = quark, 3 = gluon).

It is convenient to define the dimensionless, energy variables

$$x_i = 2E_i/Q \quad (i = 1, 2, 3). \quad (2.20)$$

If we neglect the masses of the decay products then

$$p_i \cdot p_j = \frac{1}{2} Q^2(1 - x_k) \quad (2.21)$$

and the Lorentz invariants become

$$s \equiv (p_1 + p_3)^2 = Q^2(1 - x_2) \quad (2.22a)$$

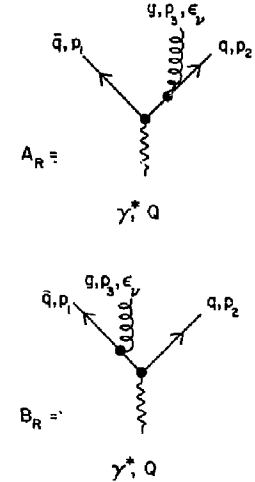


Fig. 6. Lowest order diagrams for the production of a real gluon, g , with momentum and polarization given by p_3 and ϵ_ν , respectively, in the "decay" of a virtual photon, $\tau^* \rightarrow q\bar{q}g$.

$$t \equiv (p_2 + p_3)^2 = Q^2(1-x_1) \quad (2.22b)$$

$$u \equiv (p_1 + p_2)^2 = Q^2(1-x_3). \quad (2.22c)$$

As in (2.7) integrating over the 3-momentum of particle 3 gives

$$\int d^3 p_3 \delta^4(q-p_1-p_2-p_3) = \delta(Q-E_1-E_2-E_3). \quad (2.23)$$

The remaining energy conservation δ -function implies that $Q = E_1 + E_2 + E_3$ or from (2.20)

$$x_1 + x_2 + x_3 = 2. \quad (2.24)$$

We must now integrate (2.19) over the directions of particles 1 and 2 similar to (2.9). The result is

$$\int \int \frac{d^3 p_1}{2E_1} \frac{d^3 p_2}{2E_2} = \frac{(4\pi)(2\pi)}{4} E_1 dE_1 E_2 dE_2 \int_{-1}^1 dz, \quad (2.25)$$

where $z = \cos\theta_{12}$ is the relative angle between particle 1 and particle

2 (we can pick the $\hat{z}$ axis in the direction of particle 1). The integration over z can be carried out using

$$\begin{aligned} \frac{\delta(Q-E_1-E_2-E_3)}{2E_3} &= \delta[q-p_1-p_2-p_3]^2] \\ &= \delta[Q^2\{1-x_1-x_2 + \frac{1}{2}x_1x_2(1-z)\}]. \end{aligned} \quad (2.26)$$

resulting in

$$\int_{-1}^1 dz \frac{\delta(Q-E_1-E_2-E_3)}{2E_3} = \frac{2}{x_1x_2Q^2} \quad (2.27)$$

so that

$$d^2 R_3 = \frac{Q^2}{16(2\pi)^3} dx_1 dx_2. \quad (2.28)$$

At this point we cannot integrate the phase-space factor $d^2 R_3$ any further since the matrix element in (2.18) will depend in general on x_1 and x_2 . Combining (2.28) and (2.18) gives a differential decay rate

$$dW = \frac{Q}{32(2\pi)^3} |M|^2 dx_1 dx_2 \quad (2.29)$$

which when written as a differential cross section becomes

$$\frac{d\sigma}{dx_1 dx_2} = \frac{Q}{32(2\pi)^3} |M|^2. \quad (2.30)$$

The matrix element squared for the decay of a virtual photon into a quark, antiquark, and a gluon is given by

$$|M(\gamma^* \rightarrow q\bar{q}g)|^2 = 4e^2 e_q^2 g_s^2 (S_{11} + S_{22} + S_{12}). \quad (2.31)$$

where the factor of 4 comes from a sum over the final colors (there are four distinct color combinations) and where

$$S_{11} = |B_R|^2 = \frac{8t}{s} = \frac{8(1-x_1)}{(1-x_2)}, \quad (2.32a)$$

$$S_{22} = |A_R|^2 = \frac{8s}{t} = \frac{8(1-x_2)}{(1-x_1)}, \quad (2.32b)$$

$$S_{12} = 2A_R B_R^* = 8(2Q^4 - 2Q^2 t - 2Q^2 s)/(st)$$

$$= 8 \left(\frac{2}{(1-x_1)(1-x_2)} - \frac{2}{(1-x_1)} - \frac{2}{(1-x_2)} \right). \quad (2.32c)$$

where the amplitudes A_R and B_R are shown in Fig. 6. The three pieces S_{11} , S_{22} and S_{12} are gauge dependent with the results in (2.32) holding only for the Feynman gauge (which is implemented by using (2.15) to sum the gluon polarization states). The sum

$$\begin{aligned} S_{11} + S_{22} + S_{12} &= |A_R + B_R|^2 \\ &= 8 \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}. \end{aligned} \quad (2.33)$$

is, of course, gauge invariant. Combining (2.30) with (2.31) and (2.33) yields

$$\frac{1}{\nu_0} \frac{d\sigma}{dx_1 dx_2} = \left(\frac{2\alpha_s}{3\pi} \right) \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}. \quad (2.34)$$

where the Born term σ_0 is given in (2.17) and the strong coupling is $\alpha_s = g_s^2/4\pi$.

The order α_s correction to the Born term total cross section, σ_0 , in (2.17) from the emission of a real gluon, $\sigma(\text{real})$, is arrived at by integrating (2.34) over the kinematically allowed range of x_1 and x_2 . The boundary of the allowed phase-space region for massless quarks and gluons is given by the implicit solution of

$$stu = 0. \quad (2.35a)$$

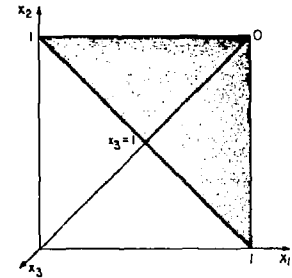
or in terms of x_1

$$(1-x_1)(1-x_2)(1-x_3) = 0. \quad (2.35b)$$

Namely, the triangular region shown in Fig. 7a with

DALITZ PLOT FOR 3 PARTICLE FINAL STATE

(a) Massless Particles



(b) Massive Gluon: $\beta = m_g^2/Q^2$

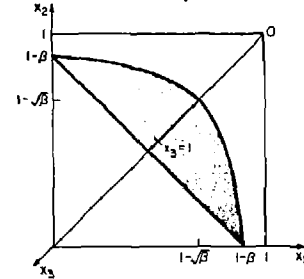


Fig. 7. (a) Dalitz plot for the "decay" of a virtual photon with invariant mass Q into a massless quark, antiquark and gluon with fractional energies $x_i = 2E_i/Q$. The shaded region is the allowed kinematic range. (b) Same as (a) except now the gluon is given a fictitious mass, m_g , and $\beta = m_g^2/Q^2$.

$$0 \leq x_1 \leq 1, \quad (2.36a)$$

$$1 - x_1 \leq x_2 \leq 1. \quad (2.36b)$$

Hence,

$$\sigma(\text{real}) = \left(\frac{2\alpha_s}{3\pi}\right) \sigma_0 \int_0^1 dx_1 \int_{1-x_1}^1 dx_2 \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}. \quad (2.37)$$

and we are left with a problem that will occur over and over as we perform perturbative QCD calculations. The differential cross section in (2.34) diverges as x_1 or x_2 goes to 1 and $\sigma(\text{real})$ in (2.37) is infinite. The origin of this divergence is clear. Consider the invariant t in (2.22b). In the massless limit we have

$$t = 2p_2 \cdot p_3 = 2E_2 \omega (1 - \cos \theta_{23}), \quad (2.38)$$

where θ_{23} is the angle between $\vec{p}_2$ and $\vec{p}_3$ and where E_2 and ω are the energy of the outgoing quark and gluon, respectively. The differential cross section diverges as $t \rightarrow 0$ which occurs when the energy of the gluon goes to zero ($\omega \rightarrow 0$, "soft divergence") or when the outgoing quark and gluon become parallel ($\cos \theta_{23} \rightarrow 1$, "parallel divergence"). The first type of divergence is usually referred to as an infrared divergence (it occurs in the limit $m_g \rightarrow 0$ and $m_q \neq 0$, where m_g and m_q are the gluon and quark masses, respectively), while the second is referred to as a mass singularity (it occurs as $m_q \rightarrow 0$ with $m_g \neq 0$).

In order to proceed we must decide on some way of regularizing the infrared and mass singularities. In addition, it better be true that no experimental observables depend on the manner in which we perform the regularizing. Before we proceed to examine regularization schemes it is instructive to integrate (2.34) over the region

$$s \geq m^2, \quad (2.39a)$$

$$t \geq m^2, \quad (2.39b)$$

where m is an arbitrary mass (for example a pion mass). The result is the total cross section for producing a real gluon whose invariant mass with both the quark and antiquark is always greater than m^2 .

$$\sigma_m(\gamma^* \rightarrow q\bar{q}g) = \int_\beta^{1-\beta} dx_1 \int_{1-x_1}^{1-\beta} dx_2 \frac{d\sigma}{dx_1 dx_2}, \quad (2.40)$$

where $\beta = m^2/Q^2$. The integrals in (2.40) are straightforward but not easy! This illustrates how quickly one runs into complicated math even though (2.34) appears to be a simple looking formula. It helps to rewrite

$$\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} = \frac{2}{(1-x_1)(1-x_2)} - \frac{(1+x_1)}{(1-x_2)} - \frac{(1+x_2)}{(1-x_1)}. \quad (2.41)$$

The second and third terms are easy to integrate.

$$\begin{aligned} \int_\beta^{1-\beta} dx_1 \int_{1-x_1}^{1-\beta} dx_2 \frac{(1+x_1)}{(1-x_2)} &= \int_\beta^{1-\beta} dx_1 \int_{1-x_1}^{1-\beta} dx_2 \frac{(1+x_2)}{(1-x_1)} \\ &= -\frac{3}{2} \log(\beta) - \frac{5}{4}. \end{aligned} \quad (2.42)$$

where terms which vanish in the limit $\beta \rightarrow 0$ have been dropped. The $\log(\beta)$ term arises from

$$\int_{1-x_1}^{1-\beta} \frac{1}{1-x_2} dx_2 = \log\left(\frac{x_1}{\beta}\right). \quad (2.43)$$

Integrating the first term in (2.41) is tough. The result is

$$\int_\beta^{1-\beta} dx_1 \int_{1-x_1}^{1-\beta} dx_2 \frac{1}{(1-x_1)(1-x_2)} = \log^2(\beta) - \log(\beta) \log(1-\beta)$$

$$+ \text{Li}_2(1-\beta) - \text{Li}_2(\beta), \quad (2.44)$$

where again I have dropped terms that vanish in the limit $\beta \rightarrow 0$ and I have used

$$\begin{aligned} \int_{\beta}^{1-\beta} \frac{\log(x_1)}{1-x_1} dx_1 &= \int_{\beta}^1 \frac{\log(x_1)}{1-x_1} dx_1 - \int_{1-\beta}^1 \frac{\log(x_1)}{1-x_1} dx_1 \\ &= \text{Li}_2(1-\beta) - \text{Li}_2(\beta), \end{aligned} \quad (2.45)$$

where the dilogarithm function, $\text{Li}_2(x)$, is defined by

$$\text{Li}_2(x) = - \int_0^x \frac{\log(1-p)}{p} dp. \quad (2.46)$$

Combining terms and using

$$\text{Li}_2(\beta) + \text{Li}_2(1-\beta) = \frac{\pi^2}{6} - \log(\beta) \log(1-\beta), \quad (2.47a)$$

and the fact that

$$\text{Li}_2(1) = \frac{\pi^2}{6}, \quad (2.47b)$$

we arrive at

$$\sigma_m(\tau^* \rightarrow q\bar{q}g) = \left(\frac{2\alpha_s}{3\pi}\right) \sigma_0 (2\log^2(\beta) + 3\log(\beta) + \frac{\pi^2}{3} + \frac{5}{2}), \quad (2.48)$$

where terms that vanish in the limit $\beta \rightarrow 0$ have been dropped.

The cross section in (2.48) has a term that diverges like $\log^2(\beta)$ as $\beta \rightarrow 0$ which comes from the region in which both x_1 and x_2 approach 1. In addition, there is a term that diverges like $\log(\beta)$ and there are terms that are finite as $\beta \rightarrow 0$. As Q increases the cross section

in (2.48) increases like $\log(Q)$. The $\log^2(\beta)$ term behaves like $\log^2(Q)$ at large Q , but the coupling constant in (1.12) behaves like $1/\log(Q)$ resulting in a net $\log(Q)$ dependence of (2.48) as Q becomes large.

3. Virtual Gluon Corrections

The vertex correction amplitude, A_v , in Fig. 8 interferes with the Born amplitude A_0 in Fig. 5c to produce a correction that is also of order α_s . Integration over the momentum, k , of the virtual gluon gives

$$\begin{aligned} \sigma_v(\text{virtual}) &= \int \frac{d^4k}{(2\pi)^4} (2A_0 A_v^*) \\ &= \left(\frac{4}{3}\right) \sigma_0 2g_s^2(-i) \int \frac{d^4k}{(2\pi)^4} \frac{N(p_1, p_2, k, q)}{(p_1-k)^2 (p_2+k)^2 k^2}, \end{aligned} \quad (2.49)$$

where the numerator $N(p_1, p_2, k, q)$ is given by

$$\begin{aligned} N(p_1, p_2, k, q) &= -2q^2 + \frac{8(p_1 \cdot k)(p_2 \cdot k)}{q^2} + (1 + 2\eta)(p_2 \cdot k - p_1 \cdot k) \\ &\quad + \eta k^2 - \frac{4\eta(p_1 \cdot k)(p_2 \cdot k)}{k^2}, \end{aligned} \quad (2.50)$$

and where $\sigma_v(\text{virtual})$ refers to the order α_s contribution arising from the vertex correction. The η in (2.50) is a gauge parameter and comes from using

$$\frac{-i}{k^2} [g_{\mu\nu} + \eta \frac{1}{k^2} k_\mu k_\nu] \quad (2.51)$$

for the gluon propagator ($\eta = 0$ is the Feynman gauge, $\eta = -1$ is the Landau gauge).

To evaluate $\sigma_v(\text{virtual})$ we make use of a technique developed by Feynman called "Feynman parameterization". We use the fact that

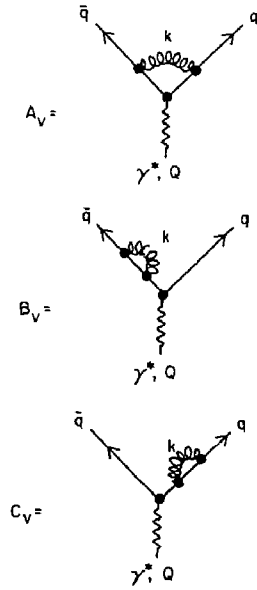


Fig. 8. Virtual gluon corrections to the process $\gamma^* \rightarrow q\bar{q}$.

$$\frac{1}{ab} = \int_0^1 dy \frac{1}{[ay+b(1-y)]^2}. \quad (2.52)$$

and we set

$$a = (p_1 - k)^2 = k^2 - 2p_1 \cdot k, \quad (2.53a)$$

$$b = (p_2 + k)^2 = k^2 + 2p_2 \cdot k, \quad (2.53b)$$

so that

$$ay + b(1-y) = k^2 - 2k \cdot p_y \quad (2.53c)$$

with

$$p_y = yp_1 - (1-y)p_2, \quad (2.53d)$$

which yields

$$\sigma_V(\text{virtual}) = \left(\frac{4}{3}\right) \alpha_0^2 g_s^2 (-1) \int \frac{d^4 k}{(2\pi)^4} \int_0^1 dy \frac{N(p_1, p_2, k, q)}{(k^2 - 2p_y \cdot k)^2 k^2}. \quad (2.54)$$

The integration over k involves a $\frac{1}{k^2}$ and a $\frac{1}{k^4}$ piece. Namely,

$$\frac{1}{k^2} N(p_1, p_2, k, q) = \frac{q^2}{k^2} N_1(p_1, p_2, k, q) + \frac{q^2}{k^4} N_2(p_1, p_2, k, q), \quad (2.55a)$$

with

$$N_1(p_1, p_2, k, q) = -2 + \frac{8(p_1 \cdot k)(p_2 \cdot k)}{q^4} + (4+2\eta) \frac{(p_2 \cdot k - p_1 \cdot k)}{q^2} + \eta \frac{k^2}{q^2}, \quad (2.55b)$$

and

$$N_2(p_1, p_2, k, q) = -4\eta \frac{(p_1 \cdot k)(p_2 \cdot k)}{q^2} \quad (2.55c)$$

We now use

$$\frac{1}{c^2 d} = \int_0^1 dx \frac{2x}{[cx+d(1-x)]^3} \quad (2.56)$$

and

$$\frac{1}{c^2 d^2} = \int_0^1 dx \frac{6x(1-x)}{[cx+d(1-x)]^4} \quad (2.57)$$

with

$$c = k^2 - 2p_y \cdot k \quad (2.58a)$$

$$d = k^2 \quad (2.58b)$$

so that

$$\begin{aligned} cx+d(1-x) &= k^2 - 2xp_y \cdot k = (k - xp_y)^2 - x^2 p_y^2 \\ &= k^2 - C \end{aligned} \quad (2.58c)$$

where

$$K = k - xp_y \quad (2.58d)$$

and

$$C = x^2 p_y^2 \quad (2.58e)$$

Shifting from an integral over k to one over K using $k = K + xp_y$ yields

$$\sigma_V(\text{virtual}) = \left(\frac{4}{3}\right) \sigma_0 2e_s^2 (-1) \int \frac{d^4 k}{(2\pi)^4} \int_0^1 dy \int_0^1 dx \left\{ \frac{2xq^2 N_1(k \rightarrow K + xp_y)}{[K^2 - C]^3} + \frac{6x(1-x)q^2 N_2(k \rightarrow K + xp_y)}{[K^2 - C]^4} \right\} \quad (2.59)$$

The shifted numerator N_1 is evaluated using the fact that

$$\frac{(p_1 \cdot k)(p_2 \cdot k)}{q^4} \rightarrow \frac{1}{8} \frac{K^2}{q^2} - \frac{1}{4} x^2 y(1-y) \quad (2.60a)$$

$$\frac{(p_2 \cdot k - p_1 \cdot k)}{q^2} \rightarrow \frac{1}{2} x \quad (2.60b)$$

$$\frac{K^2}{q^2} \rightarrow \frac{K^2}{q^2} - x^2 y(1-y) \quad (2.60c)$$

where terms odd in K are dropped since they contribute nothing to the integral over K and

$$K_\mu K_\nu \rightarrow \frac{1}{4} K^2 g_{\mu\nu} \quad (2.61)$$

In addition,

$$p_y^2 = -y(1-y)q^2 \quad (2.62a)$$

and

$$C = -y(1-y)x^2 q^2 \quad (2.62b)$$

so that the shifted numerators become

$$N_1(k \rightarrow K + x p_y) \rightarrow -2 - (2+\eta)x^2 y(1-y) + (2+\eta)x + (1+\eta) \frac{k^2}{q^2} \quad (2.63a)$$

and

$$N_2(k \rightarrow K + x p_y) \rightarrow -\eta \left(\frac{k^2}{2} - x^2 y(1-y) q^2 \right), \quad (2.63b)$$

where I have rearranged several of the terms in N_1 .

The integral over the momentum K is performed using

$$\int \frac{d^4 K}{(2\pi)^4} \frac{(K^2)^R}{[K^2 - C]^M} = \frac{i(-1)^{R+M}}{16\pi^2} C^{R-M+2} \frac{\Gamma(R+2)\Gamma(M-R-2)}{\Gamma(M)}, \quad (2.64)$$

yielding

$$\begin{aligned} \sigma_v(\text{virtual}) = & \left(\frac{2\alpha_s}{3\pi} \right) \sigma_0 \left[-\frac{1}{2} (2+\eta) + \int_0^1 dx \int_0^1 dy \frac{(-2+(2+\eta)x)}{y(1-y)x} \right] \\ & + \left(\frac{4}{3} \right) \sigma_0 \frac{2g_s^2(-i)}{2\pi^4} \int_0^1 dx \int_0^1 dy \int \frac{d^4 K}{(2\pi)^4} \frac{2x(1+\eta)k^2}{[K^2 - C]^3}. \quad (2.65) \end{aligned}$$

The second term comes from last term of N_1 in (2.63a) and is ultra-violet divergent like $\log(K)$ as $K \rightarrow \infty$. In the Landau gauge (i.e. $\eta=1$) this term is absent but one is still left with the infrared divergences that occur in the x and y interactions of the first term of (2.65).

The vertex correction in (2.65) is not only infinite, it is gauge dependent. To get a gauge independent result one must add the order α_s self-energy corrections. The self energy correction shown in Fig. 9 is given by

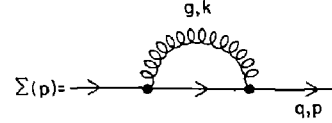


Fig. 9. Lowest order self energy correction for a quark q with momentum p .

$$\Sigma(p) = (-g_s^2) \int \frac{d^4 k}{(2\pi)^4} \gamma_\alpha \frac{\not{p}-\not{k}}{(p-k)^2} \gamma_\beta \left(\frac{g_{\alpha\beta}}{k^2} + \eta \frac{k_\alpha k_\beta}{k^4} \right), \quad (2.66)$$

where again I have used the gluon propagator in (2.51). It will turn out that $\Sigma(p)$ is proportional to $\not{p}$ so that I can write

$$\Sigma(p) = -i\not{p}\bar{\Sigma}. \quad (2.67)$$

The contribution to the total $\gamma^* \rightarrow q\bar{q}$ rate from the virtual self-energy amplitudes in Fig. 8 is then

$$\begin{aligned} \sigma_s(\text{virtual}) &= \int \frac{d^4 k}{(2\pi)^4} \frac{1}{2} (2A_0 B_V^* + 2A_0 C_V^*) \\ &= \left(\frac{4}{3}\right) \sigma_0 \frac{1}{2} (2\bar{\Sigma} + 2\bar{\Sigma}) = \left(\frac{4}{3}\right) \sigma_0 2\bar{\Sigma}. \end{aligned} \quad (2.68)$$

The factor of 1/2 comes from the usual convention of putting one-half of this correction into self-energy and one-half in the wave function renormalization. To evaluate $\Sigma(p)$ in (2.66) we use both

$$\frac{1}{ab} = \int dx \frac{1}{[ax+b(1-x)]^2} \quad (2.69a)$$

and

$$\frac{1}{ab^2} = \int dx \frac{2(1-x)}{[ax+b(1-x)]^3} \quad (2.69b)$$

with

$$a = (p-k)^2 = k^2 - 2p \cdot k + p^2 \quad (2.70a)$$

$$b = k^2 \quad (2.70b)$$

$$ax + b(1-x) = k^2 - C \quad (2.70c)$$

with

$$K = k - px \quad (2.70d)$$

and

$$C = -p^2 x(1-x). \quad (2.70e)$$

The factors in (2.66) become

$$\gamma_\alpha (\not{p}-\not{k}) \gamma_\alpha = -2\not{p}-\not{k} \rightarrow -2\not{p}(1-x) \quad (2.71a)$$

$$\begin{aligned} \frac{K(\not{p}-\not{k})K}{k^4} &= -\frac{1}{k^2} (\not{p}-\not{k}) + \frac{\eta}{k^4} 2p \cdot k K \\ &\rightarrow -\frac{1}{k^2} \not{p}(1+x) + \frac{2\not{p}}{k^4} \left(\frac{k^2}{4} + x^2 p^2 \right). \end{aligned} \quad (2.71b)$$

After shifting k to $K + px$ and dropping terms odd in K we have

$$\begin{aligned} \bar{\Sigma} &= (-g_s^2) i \int_0^1 dx \int \frac{d^4 K}{(2\pi)^4} \frac{[-2(1-x)\not{p} - \eta(1+x)]}{[K^2 - C]^2} \\ &+ \int_0^1 dx \int \frac{d^4 K}{(2\pi)^4} \frac{\eta(1-x)[K^2 + 4x^2 p^2]}{[K^2 - C]^3}. \end{aligned} \quad (2.72)$$

As was the case with the vertex corrections $\sigma_V(\text{virtual})$ in (2.65), the self-energy corrections are gauge dependent and in general, contain both infrared and ultraviolet divergences.

At this point we cannot proceed without a procedure for regularizing the ultraviolet divergences in $\sigma_s(\text{virtual})$ and $\sigma_s(\text{virtual})$. If we handle these divergences correctly we will find that the ultraviolet divergences cancel and that the sum

$$\sigma(\text{virtual}) = \sigma_v(\text{virtual}) + \sigma_s(\text{virtual}). \quad (2.73)$$

is gauge invariant and contains only infrared divergences. If we then regularize these infrared divergences together with the infrared divergences in $\sigma(\text{real})$ in (2.37) we will find that the sum $\sigma(\text{real}) + \sigma(\text{virtual})$ is finite and contains no divergences of any kind.

4. Real Gluon Emissions - Massive Gluon Scheme

In these lectures we will consider two regularization schemes. In the "massive gluon scheme" (MG) one regularizes the real and virtual corrections by giving the gluon a fictitious mass, m_g . This scheme breaks gauge invariance and is applicable only if the triple gluon vertex is not involved. At order α_s the triple gluon vertex does not play a role so everything is okay.

For the case of a massive gluon, the differential cross section in (2.34) becomes

$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 \left(\frac{2\alpha_s}{3\pi} \right) \frac{1}{(1-x_1)(1-x_2)} (x_1^2 + x_2^2 + \beta[2(x_1 + x_2) - \frac{(1-x_1)^2 + (1-x_2)^2}{(1-x_1)(1-x_2)}] + 2\beta^2), \quad (2.74)$$

where $\beta = m_g^2/Q^2$. In addition, the region of integration now becomes

$$0 \leq x_1 \leq 1-\beta,$$

$$1 - \beta - x_1 \leq x_2 \leq \frac{1-x_1-\beta}{1-x_1}, \quad (2.75)$$

as shown in Fig. 7b. Integrating over x_2 results in

$$\frac{d\sigma}{dx_1} = \left(\frac{2\alpha_s}{3\pi} \right) \sigma_0 \frac{1+x_1^2}{1-x_1} \log\left(\frac{x_1(1-x_1)}{\beta}\right) - \frac{3}{2} \frac{1}{1-x_1}$$

$$+ \frac{1}{2} x_1 + \frac{1}{2} + \frac{\beta(2-x_1)}{(1-x_1)^2} + \frac{1}{2} \frac{\beta^2}{(1-x_1)^3}, \quad (2.76)$$

where I have dropped some terms that vanish in the limit $\beta \rightarrow 0$. The first term comes from

$$\int_{x_2^{\min}}^{x_2^{\max}} \frac{1}{1-x_2} dx_2 = \log\left(\frac{x_1+\beta}{\beta}\right) \rightarrow \log\left(\frac{x_1(1-x_1)}{\beta}\right). \quad (2.77)$$

Care must be taken in not dropping the β and β^2 terms in (2.76) for these terms give finite contributions when integrating over x_1 . For example,

$$\int_0^{1-\beta} \frac{\beta}{(1-x_1)^2} dx_1 = 1, \quad \int_0^{1-\beta} \frac{\beta^2}{(1-x_1)^3} dx_1 = \frac{1}{2}. \quad (2.78)$$

We can thus replace the β and β^2 terms in (2.76) by a δ -function contribution as follows

$$\begin{aligned} \frac{d\sigma}{dx_1} = & \left(\frac{2\alpha_s}{3\pi} \right) \sigma_0 \frac{1+x_1^2}{1-x_1} \log\left(\frac{x_1(1-x_1)}{\beta}\right) - \frac{3}{2} \frac{1}{1-x_1} \\ & + \frac{1}{2} x_1 + \frac{1}{2} + \frac{5}{4} \delta(1-x_1). \end{aligned} \quad (2.79)$$

Integrating over x_1 and using

$$\int_0^{1-\beta} \frac{1}{1-x_1} dx_1 = -\log(\beta), \quad (2.80a)$$

$$\int_0^{1-\beta} \frac{1}{1-x_1} \log(1-x_1) dx_1 = -\frac{1}{2} \log^2(\beta). \quad (2.80b)$$

$$\int_0^{1-\beta} \frac{1+x_1^2}{1-x_1} \log(x_1) dx_1 = -\frac{\pi^2}{3} + \frac{5}{4}. \quad (2.80c)$$

yields

$$\sigma_{MC}(\text{real}) = \left(\frac{2\pi}{3\pi}\right) \sigma_0 \left\{ \log^2(\beta) + 3 \log(\beta) - \frac{\pi^2}{3} + 5 \right\}. \quad (2.81)$$

which, as expected, contains a $\log^2(\beta)$ and a $\log(\beta)$ divergence as $\beta \rightarrow 0$.

5. Virtual Gluon Corrections - Massive Gluon Scheme

The ultraviolet divergences in $\sigma_v(\text{virtual})$ and $\sigma_s(\text{virtual})$ in (2.65) and (2.72) can be regularized by multiplying the gluon propagator by a convergence factor so that

$$\frac{1}{k^2} \rightarrow \frac{1}{k^2} C(k), \quad (2.82a)$$

where

$$C(k) = \frac{L}{L-k^2}. \quad (2.82b)$$

and where L is large. For small k , $C(k)$ is approximately equal to one, but at large k it behaves like $1/k^2$ causing the integrals to converge.

In the massive gluon scheme the infrared divergences are handled by replacing $1/k^2$ by $1/(k^2 - m_g^2)$ in the gluon propagator. Both the infrared and ultraviolet divergences can be treated simultaneously by the replacement

$$\frac{1}{k^2} \rightarrow - \int_{\frac{m_g^2}{2}}^L \frac{d\ell}{(k^2 - \ell)^2} \quad (2.83)$$

in the gluon propagator. With this replacement the self-energy correction in (2.72) becomes

$$\bar{\Sigma}(p) = (-g_s^2)(-i) \int_{\frac{m_g^2}{2}}^L d\ell \int_0^1 dx \int \frac{d^4 K}{(2\pi)^4} \frac{2(1-x)[-2(1-x)]}{[K^2 - C]^3}. \quad (2.84)$$

where C in (2.70e) is now given by

$$C = -p^2 x(1-x) + \ell(1-x). \quad (2.85)$$

and where for simplicity I have set $\eta=0$ (Feynman Gauge). Integrating over ℓ and K gives

$$\begin{aligned} \bar{\Sigma}_{MC}(p) &= (-g_s^2) \frac{1}{16\pi^2} \int_0^1 dx \, 2(1-x) \log \left[\frac{-p^2 x(1-x)}{-p^2 x + m_g^2} \right] \\ &= (-g_s^2) \frac{1}{16\pi^2} \log(L/m_g^2), \end{aligned} \quad (2.86)$$

where terms that vanish in the limit $m_g^2/L \rightarrow 0$ have been dropped. Hence from (2.68),

$$\sigma_s(\text{virtual}) = -\left(\frac{2\pi}{3\pi}\right) \sigma_0 \log(L/m_g^2). \quad (2.87)$$

for the Feynman Gauge ($\eta=0$). Similarly the replacement (2.83) and $\eta=0$ gives

$$\sigma_0(\text{virtual}) = \left(\frac{4}{3}\right) \sigma_0 \frac{2g_s^2(-i)}{\int_{\frac{2}{m_g}}^L d\ell \int \frac{d^4 K}{(2\pi)^4} \int_0^1 dy \int_0^1 dx}$$

$$\frac{6x(1-x)q^2[2+2x^2y(1-y)-2x-\frac{k^2}{2}]}{[k^2-C]^4} \quad (2.88)$$

for (2.65), where C has changed from (2.62b) to

$$C = -y(1-y)x^2q^2 + \ell(1-x). \quad (2.89)$$

Integrating over K yields

$$\sigma_V(\text{virtual}) = \left(\frac{2\alpha_s}{3\pi}\right) \sigma_0 \int_{\frac{2}{m_g}}^L d\ell \int_0^1 dy \int_0^1 dx$$

$$\left\{ \frac{x(1-x)q^2[2+2x^2y(1-y)-2x]}{[-y(1-y)x^2q^2+\ell(1-x)]^2} + \frac{2x(1-x)}{[-y(1-y)x^2q^2+\ell(1-x)]} \right\}. \quad (2.90)$$

and integrating over ℓ gives

$$\sigma_V(\text{virtual}) = \left(\frac{2\alpha_s}{3\pi}\right) \sigma_0 \int_0^1 dy \int_0^1 dx$$

$$\left\{ \frac{xq^2[2+2x^2y(1-y)-2x]}{[-y(1-y)x^2q^2+m_g^2(1-x)]} - \frac{x^2q^2[2+2x^2y(1-y)-2x]}{[-y(1-y)x^2q^2+L(1-x)]} \right.$$

$$\left. + 2x \log\left(\frac{-y(1-y)x^2q^2+L(1-x)}{-y(1-y)x^2q^2+m_g^2(1-x)}\right) \right\}. \quad (2.91)$$

With $q^2 > 0$ the denominators in (2.91) can vanish over the range of the x and y integrations. We can avoid problems by requiring that

$$\bar{Q}^2 = -q^2 > 0, \quad (2.92)$$

and after we have performed the integrals we can analytically continue to the $q^2 > 0$ (timelike) region. The integrals in (2.91) are tedious but straightforward. The result is

$$\sigma_V(\text{virtual}) = \left(\frac{2\alpha_s}{3\pi}\right) \sigma_0 \{-\log^2(\bar{\beta}) - 3\log(\bar{\beta}) - \frac{7}{2} - \frac{2\pi^2}{3} + \log(L/m_g^2)\}, \quad (2.93a)$$

where

$$\bar{\beta} = m_g^2/\bar{Q}^2 \quad (2.93b)$$

with $\bar{Q}$ given by (2.92). Combining this with $\sigma_S(\text{virtual})$ in (2.87) gives

$$\sigma_{\text{HC}}(\text{virtual}) = \sigma_V(\text{virtual}) + \sigma_S(\text{virtual})$$

$$= \left(\frac{2\alpha_s}{3\pi}\right) \sigma_0 \{-\log^2(\bar{\beta}) - 3\log(\bar{\beta}) - \frac{7}{2} - \frac{2\pi^2}{3}\}. \quad (2.94)$$

which contains only infrared divergences. The dependence on the ultraviolet cutoff L has dropped out.

6. Complete Order α_s Correction - Massive Gluon Scheme

In the massive gluon scheme the order α_s virtual corrections are given by (2.94). Namely,

$$\sigma_{\text{HC}}(\text{virtual}) = \left(\frac{2\alpha_s}{3\pi}\right) \sigma_0 \{-\log^2(\bar{\beta}) - 3\log(\bar{\beta}) - \frac{7}{2} - \frac{2\pi^2}{3}\}. \quad (2.95)$$

with $\bar{q}$ given in (2.93b) and $\bar{q}^2 = -q^2 > 0$ (i.e. q^2 is spacelike). For q^2 timelike we analytically continue using

$$\log(-q^2) = \log(q^2) - i\pi, \quad (2.96a)$$

$$\log^2(-q^2) = \log^2(q^2) - 2i\pi \log(q^2) - \pi^2. \quad (2.96b)$$

and arrive at

$$\sigma_{MC}(\text{virtual}) = \left(\frac{2\alpha_s}{3\pi}\right) \sigma_0 \{-\log^2(\beta) - 3\log(\beta) - \frac{7}{2} - \frac{2\pi^2}{3} + \pi^2\}. \quad (2.97a)$$

with

$$\beta = \pi^2/Q^2 \quad (2.97b)$$

and $Q^2 = q^2 > 0$ (i.e. q^2 timelike), and where I have kept only the real part. The π^2 in (2.97a) comes from the analytic continuation from the spacelike to the timelike region of q^2 . Combining this with the result in (2.81) for the real gluon corrections, we arrive at

$$\begin{aligned} \sigma_{MC}(\text{real}) + \sigma_{MC}(\text{virtual}) &= \left(\frac{2\alpha_s}{3\pi}\right) \sigma_0 \left\{-\frac{\pi^2}{3} + 5 - \frac{7}{2} - \frac{2\pi^2}{3} + \pi^2\right\} \\ &= \left(\frac{2\alpha_s}{3\pi}\right) \sigma_0 \left(\frac{3}{2}\right) = \frac{\alpha_s}{\pi} \sigma_0. \end{aligned} \quad (2.98)$$

Both the $\log^2(\beta)$ and $\log(\beta)$ terms cancel out in the sum leaving a finite result in the limit $m_{\bar{q}} \rightarrow 0$. Thus the total rate for a virtual photon to decay into partons has the following perturbation series

$$\sigma_{\text{tot}}^{e^+e^-} = \sigma_0 \left(1 + \frac{\alpha_s}{\pi} + \dots\right) \quad (2.99a)$$

As discussed in the introduction, higher order ultraviolet divergences can be absorbed into the definition of the coupling constant giving

$$\sigma_{\text{tot}}^{e^+e^-} = \sigma_0 \left(1 + \frac{1}{\pi} \alpha_s(Q^2) + \dots\right), \quad (2.99b)$$

where $\alpha_s(Q^2) = 4\pi/(\beta_0 \log(Q^2/\Lambda^2))$ is the familiar running coupling constant in (1.12) and Λ is the QCD perturbative parameter that sets the scale.

7. Real Gluon Emission - Dimensional Regularization Scheme

Before proceeding with the three-body decay $\gamma^* \rightarrow q\bar{q}g$ in N dimensions, we must recalculate the Born term $\gamma^* \rightarrow q\bar{q}$ in N dimensions. The two-body differential decay rate in N dimensions is

$$dW = \frac{1}{2E_{\text{cm}}} |M|^2 d^{2N-2}R_2, \quad (2.100)$$

where the two-body phase-space factor

$$d^{2N-2}R_2 = \frac{d^{N-1}P_2}{(2\pi)^{N-1}(2E_1)} \frac{d^{N-1}P_2}{(2\pi)^{N-1}(2E_2)} (2\pi)^N \delta^N(q - p_1 - p_2). \quad (2.101)$$

replaces (2.6). As in (2.7) integrating over p_2 yields

$$\int d^{N-1}P_2 \delta^N(q - p_1 - p_2) = \delta(Q - E_1 - E_2), \quad (2.102)$$

however,

$$\begin{aligned} \frac{d^{N-1}P_1}{2E_1} &= \frac{1}{2} E_1^{N-3} \sin^{N-3} \theta_1 \sin^{N-4} \theta_2 \dots \\ &\dots \sin \theta_{N-3} d\theta_1 d\theta_2 \dots d\theta_{N-2} d\phi_1 \end{aligned} \quad (2.103)$$

replaces (2.8). Here the angles $\theta_1, \dots, \theta_{N-2}$ are angles with respect to the axes in $N-1$ dimensions. If $N=4$ (2.103) reduces to (2.8) with θ_2 in (2.103) being the ϕ angle, ϕ_1 . The matrix element in (2.100)

does not depend on these angles and they can be integrated out by repeated use of the formula

$$\int_0^\pi \sin^n \theta = \sqrt{\pi} \frac{\Gamma(\frac{n}{2} + \frac{1}{2})}{\Gamma(\frac{n}{2} + 1)}, \quad (2.104)$$

yielding

$$\int \frac{d^{N-1} p_1}{2E_1} = 2^{N-3} \pi^{(N-2)/2} \frac{\Gamma(\frac{1}{2}(N-2))}{\Gamma(N-2)} E_1^{N-3} dE_1, \quad (2.105)$$

which replaces (2.9). The final integration over E_1 is again accomplished using (2.10) which gives

$$\int \frac{E_1^{N-3} dE_1}{2E_2} \delta(Q-E_1-E_2) = \frac{E_1^{N-3}}{2Q} = \frac{Q^{N-4}}{2^{N-2}}. \quad (2.106)$$

Combining (2.105), (2.106) and (2.101) yields

$$R_2 = \int d^{2N-2} R_2 = 2^{1-N} \pi^{1-N/2} \frac{\Gamma(\frac{N}{2}-1)}{\Gamma(N-2)} Q^{N-4}. \quad (2.107)$$

which reduces to (2.12) when $N=4$. The problem now is that the phase-space factor R_2 has dimensions that depend on N . If $N=4$ then R_2 is dimensionless, but for other values of N it is not. To keep R_2 dimensionless for any N one introduces a "dimensional regularization mass", m_D , and writes (2.107) as

$$R_2 = \frac{1}{2^{N-1} \pi^{N/2-1}} \frac{\Gamma(\frac{N}{2}-1)}{\Gamma(N-2)} \left(\frac{Q}{2}\right)^{2N/2-2}, \quad (2.108)$$

where m_D replaces the gluon mass in the massive gluon regularization scheme.

In N dimensions the matrix element squared for a virtual photon to decay into a quark-antiquark pair is given by

$$|A(\gamma^* \rightarrow q\bar{q})|^2 = 6(N-2)e_q^2 e^2 Q^2 = 24\pi(N-2)\alpha e_q^2 Q^2. \quad (2.109)$$

and combining this with (2.107) gives

$$\sigma(\gamma^* \rightarrow q\bar{q}) = \sigma_0 = 3\alpha e_q^2 \frac{Q}{2^{N-1} \pi^{N/2-2}} \frac{\Gamma(\frac{N}{2})}{\Gamma(N-2)} \left(\frac{Q}{2}\right)^{2N/2-2} m_D^{-2}. \quad (2.110a)$$

or

$$\sigma_0 = 3\alpha e_q^2 Q \frac{1}{2^{\epsilon} \pi^{\epsilon/2}} \frac{\Gamma(2+\frac{\epsilon}{2})}{\Gamma(2+\epsilon)} \left(\frac{Q}{2}\right)^{2\epsilon/2} m_D^{-2}. \quad (2.110b)$$

where $N=4+\epsilon$. Of course, (2.110b) reduces to (2.17) when $\epsilon = 0$.

To evaluate the three-body decay of a virtual photon into a quark, antiquark, and a gluon we proceed in an analogous manner. The three-body differential decay rate in N dimensions is

$$d\sigma = \frac{1}{2E_{cm}} |A|^2 d^{3N-3} R_3. \quad (2.111)$$

where

$$d^{3N-3} R_3 = \frac{d^{N-1} p_1}{(2\pi)^{N-1} (2E_1)} \frac{d^{N-1} p_2}{(2\pi)^{N-1} (2E_2)} \frac{d^{N-1} p_3}{(2\pi)^{N-1} (2E_3)} (2\pi)^N \delta^N(q-p_1-p_2-p_3). \quad (2.112)$$

As in (2.23) integrating over p_3 gives

$$\int d^{N-1} p_3 \delta^N(q-p_1-p_2-p_3) = \delta(Q-E_1-E_2-E_3). \quad (2.113)$$

To integrate over the directions of p_1 and p_2 one makes use of (2.103)

and (2.105) which results in

$$\iint \frac{d^{N-1}p_1}{2E_1} \frac{d^{N-1}p_2}{2E_2} = \frac{2^{N-3} \pi^{N-2}}{\Gamma(N-2)} E_1^{N-3} dE_1 E_2^{N-3} dE_2 \int_{-1}^1 dz (1-z^2)^{N/2-2}, \quad (2.114)$$

where as in (2.25) I have chosen the axis to be in the direction of particle 1 and again $z = \cos\theta_{12}$, where θ_{12} is the relative angle between particle 1 and particle 2, and where all decay particles have been taken to be massless. Changing to the x_i variables in (2.20) and using (2.26) gives

$$\int_{-1}^1 dz (1-z^2)^{N/2-2} \frac{\delta(Q-E_1-E_2-E_3)}{2E_3} = \frac{2(1-z^2)^{N/2-2}}{x_1 x_2 Q^2}, \quad (2.115)$$

and

$$d^2R_3 = \frac{Q^2}{2^{2N-4} (2\pi)^{N-1} \Gamma(N-2)} \left(\frac{Q^2}{2}\right)^{N-4} (1-z^2)^{N/2-2} x_1^{N-4} dx_1 x_2^{N-4} dx_2, \quad (2.116)$$

where

$$z = 1 - 2(1-x_1-x_2)/(x_1 x_2), \quad (2.117)$$

and where again a "dimensional regularization mass", m_d , has been introduced to give d^2R_3 the same dimensions for any N . Equation (2.116) reduces to (2.28) when $N=4$ and as before we cannot integrate d^2R_3 any further without knowing the matrix element since, in general, it will depend on x_1 and x_2 . In anticipation of this let me write

$$|M|^2 = 32 \pi^2 e_s^2 g_s^2 F(x_1, x_2), \quad (2.118)$$

so that

$$\frac{1}{\sigma_0} \frac{d\sigma}{dx_1 dx_2} = \left(\frac{2\alpha_s}{3\pi}\right) \frac{\left(\frac{Q^2}{2}\right)^{N/2-2} F(x_1, x_2)}{2^{N-4} \pi^{N/2-2} \Gamma(\frac{N}{2})} x_1^{N-4} x_2^{N-4} (1-z^2)^{N/2-2}. \quad (2.119)$$

where σ_0 is given in (2.110). The total rate is arrived at by integrating over the allowed region of x_1 and x_2 . Namely

$$\sigma_{DR}(\text{real}) = \sigma_0 \left(\frac{2\alpha_s}{3\pi}\right) \frac{\left(\frac{Q^2}{2}\right)^{N/2-2}}{2^{N-4} \pi^{N/2-2} \Gamma(\frac{N}{2})} \int_0^1 dx_1 x_1^{N-4} \int_{1-x_1}^1 dx_2 x_2^{N-4} (1-z^2)^{N/2-2} F(x_1, x_2), \quad (2.120)$$

where "DR" denotes the dimensional regularization scheme and "real" implies the production of a real gluon by the process $\gamma^* \rightarrow q\bar{q}g$.

The nested integrals in (2.120) are painful to evaluate and it is somewhat easier to "decouple" the integrals by defining a variable v such that

$$x_2 = 1 - vx_1, \quad (2.121a)$$

and to express everything in terms of x_1 and v which both run from 0 to 1. Using

$$\frac{(1-z^2)^{N/2-2}}{x_1 x_2} = \frac{2^{N-4} v^{N/2-2} (1-v)^{N/2-2} (1-x_1)^{N/2-2}}{x_1 (1-vx_1)^{N-3}}. \quad (2.121b)$$

we arrive at

$$\sigma_{\text{DR}}(\text{real}) = \sigma_0 \left(\frac{2\alpha_s}{3\pi} \right) \left(\frac{Q^2}{4\pi m_D^2} \right)^{N/2-2} \frac{1}{2^{N-4} \pi^{N/2} \Gamma(\frac{N}{2})}$$

$$\int_0^1 dx_1 x_1^{N-3} (1-x_1)^{N/2-2} \int_0^1 dv v^{N/2-2} (1-v)^{N/2-2} F(x_1, v). \quad (2.122)$$

The evaluation of the matrix element squared for the decay of a virtual photon into a quark, antiquark, and gluon in N dimensions is straightforward. The result for $F(x_1, v)$ in (2.116) and (2.122) is

$$F(x_1, v) = \frac{[(v^2+1)x_1^2 - 2vx_1 + 1]}{x_1 v(1-x_1)} + \frac{[(v^2-v+1)x_1^2 - x_1 + 1]}{vx_1(1-x_1)} \epsilon$$

$$+ \frac{1}{4} \frac{[(v^2-2v+1)x_1^2 + 2(v-1)x_1 + 1]}{vx_1(1-x_1)} \epsilon^2, \quad (2.123)$$

where $N=4+\epsilon$ and where terms of order ϵ^3 and higher have been dropped. It is necessary to keep both the ϵ and ϵ^2 term in (2.123) since the integrations in (2.122) will produce $1/\epsilon$ and $1/\epsilon^2$ factors. Using

$$\int_0^1 dx x^a (1-x)^b = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}, \quad (2.124)$$

it is straightforward but tedious to insert (2.123) into (2.122) and perform the integrations. The result is

$$\sigma_{\text{DR}}(\text{real}) = \sigma_0 \left(\frac{2\alpha_s}{3\pi} \right) \left(\frac{Q^2}{4\pi m_D^2} \right)^{\epsilon/2} \frac{\Gamma^2(1 + \frac{\epsilon}{2})}{\Gamma(1 + \frac{3}{2}\epsilon)} \left\{ \frac{8}{\epsilon^2} - \frac{6}{\epsilon} + \frac{19}{2} \right\}, \quad (2.125)$$

where some terms that vanish in the limit $\epsilon \rightarrow 0$ have been dropped. Equation (2.125) has not yet been expanded in a power series in ϵ . To

do so one uses

$$\frac{\Gamma^2(1 + \frac{\epsilon}{2})}{\Gamma(1 + \frac{3}{2}\epsilon)} = 1 + \frac{1}{2} \gamma_E \epsilon + \frac{1}{48} (6\gamma_E^2 - 7\pi^2) \epsilon^2 + \dots, \quad (2.126)$$

where γ_E is Euler's constant ($\gamma_E = 0.5772157$) and

$$\left(\frac{Q^2}{4\pi m_D^2} \right)^{\epsilon/2} = \exp \left[\frac{\epsilon}{2} \log \left(\frac{Q^2}{4\pi m_D^2} \right) \right] = 1 + \frac{1}{2} \log \left(\frac{Q^2}{4\pi m_D^2} \right) \epsilon$$

$$+ \frac{1}{4} \log^2 \left(\frac{Q^2}{4\pi m_D^2} \right) \epsilon^2 + \dots, \quad (2.127)$$

Inserting (2.126) and (2.127) into (2.125) and dropping terms that vanish when $\epsilon \rightarrow 0$ gives

$$\sigma_{\text{DR}}(\text{real}) = \sigma_0 \left(\frac{2\alpha_s}{3\pi} \right) \left\{ \frac{8}{\epsilon^2} + \left(4 \log \left(\frac{Q^2}{4\pi m_D^2} \right) + 4\gamma_E - 6 \right) \frac{1}{\epsilon} \right.$$

$$\left. + \log^2 \left(\frac{Q^2}{4\pi m_D^2} \right) + (2\gamma_E - 3) \log \left(\frac{Q^2}{4\pi m_D^2} \right) + \gamma_E^2 - 3\gamma_E - \frac{7\pi^2}{6} + \frac{57}{6} \right\}. \quad (2.128)$$

As in the massive gluon scheme $\sigma_{\text{DR}}(\text{real})$ contains both a log squared and a log divergence as $m_D \rightarrow 0$.

5. Virtual Gluon Corrections - Dimensional Regularization Scheme

In N dimensions the self energy corrections in (2.72) becomes

$$\tilde{\Sigma} = (-i\pi^{\frac{N}{2}}) i \int_0^1 dx \int \frac{d^N K}{(2\pi)^N} \frac{[(2-N)(1-x)-\eta(1+x)]}{[K^2 - c]^2}$$

$$+ \int_0^1 dx \int \frac{d^N K}{(2\pi)^N} \frac{\eta(1-x) \left[\frac{4K^2}{N} + 4x^2 p^2 \right]}{[K^2 - C]^3}, \quad (2.129)$$

where C is given by (2.70e) and where (2.71) are replaced by

$$\gamma_\alpha(p-K)\gamma_\alpha \rightarrow (2-N)p(1-x), \quad (2.130a)$$

$$\frac{K(p-K)K}{k^4} \rightarrow -\frac{1}{k^2} p(1-x) + \frac{2x}{k^4} \left(\frac{K^2}{N} + x^2 p^2 \right), \quad (2.130b)$$

where, as before, terms odd in $K = k - px$ give no contribution and (2.61) becomes

$$K_\mu K_\nu \rightarrow \frac{1}{N} K^2 g_{\mu\nu}, \quad (2.131)$$

The integration over K is performed using

$$\int \frac{d^N K}{(2\pi)^N} \frac{(K^2)^R}{[K^2 - C]^N} = \frac{i(-1)^{R-N}}{(16\pi^2)^{N/4}} C^{R-N+N/2} \frac{\Gamma(R + \frac{N}{2})\Gamma(N-R - \frac{N}{2})}{\Gamma(\frac{N}{2})\Gamma(N)}, \quad (2.132)$$

which replaces (2.64). The integrals over x are done with the aid of

$$\int_0^1 dx x^{R-1} (1-x)^{M-1} = \frac{\Gamma(R)\Gamma(M)}{\Gamma(R+M)}, \quad (2.133)$$

The result is

$$\bar{\Sigma}_{DR} = \frac{g_s^2}{(16\pi^2)^{N/4}} \left(\frac{-p^2}{2} \right)^{2N/2-2} \frac{1}{m_D} \left(1 - \frac{N}{2} \right) (1+\eta) \frac{\Gamma(2 - \frac{N}{2})\Gamma(\frac{N}{2} - 1)}{\Gamma(N-2)} \quad (2.134a)$$

or

$$\bar{\Sigma}_{DR} = \frac{g_s^2}{16\pi^2} \left(\frac{-p^2}{2} \right)^{\epsilon/2} \frac{\Gamma(1 - \frac{\epsilon}{2})\Gamma(1 + \frac{\epsilon}{2})}{\Gamma(1+\epsilon)} (1+\eta) \frac{(\epsilon+2)}{\epsilon(\epsilon+1)}, \quad (2.134b)$$

where $N=4+\epsilon$. In the dimensional regularization scheme the self energy corrections vanish when $\eta = -1$ (Landau gauge). Therefore when working in this scheme one usually chooses the Landau gauge and ignores the self energy corrections. Namely,

$$\bar{\Sigma}_{DR} = 0 \quad (2.135)$$

$\eta = -1$ (Landau gauge).

Furthermore $\bar{\Sigma}_{DR}$ in (2.134b) is zero for any value of η provided $p^2 = 0$ (on shell) and $\epsilon > 0$. This is a subtle point, but one can show that it is consistent to set $\bar{\Sigma}_{DR} = 0$ for any η and any ϵ provided $p^2 = 0$ (on shell).

In N dimensions the vertex correction in (2.49) becomes

$$\sigma_V(\text{virtual}) = \left(\frac{4}{3} \right) \sigma_0 2k_s^2 (-1) \int \frac{d^N k}{(2\pi)^N} \frac{N(p_1 \cdot p_2 \cdot k \cdot q)}{(p_1 - k)^2 (p_2 + k)^2 k^2}, \quad (2.136)$$

with

$$\begin{aligned} N(p_1 \cdot p_2 \cdot k \cdot q) &= -2q^2 + \frac{S(p_1 \cdot k)(p_2 \cdot k)}{q^2} + (4 + 2N)(p_2 \cdot k - p_1 \cdot k) \\ &+ \eta k^2 - \frac{4\eta(p_1 \cdot k)(p_2 \cdot k)}{k^2} + (N-4)k^2. \end{aligned} \quad (2.137)$$

Equation (2.50) becomes

$$\sigma_V(\text{virtual}) = \left(\frac{4}{3} \right) \sigma_0 2k_s^2 (-1) \int \frac{d^N k}{(2\pi)^N} \int_0^1 dy \int_0^1 dx$$

$$\left(\frac{2\pi q^2 N_1(k \rightarrow K+xp_y)}{[k^2-C]^{-1}} + \frac{Gx(1-x)q^2 N_2(k \rightarrow K+xp_y)}{[k^2-C]^{-1}} \right), \quad (2.138)$$

with the shifted numerators $(k \rightarrow K+xp_y)$ in (2.63) becoming

$$N_1(k \rightarrow K+xp_y) \rightarrow -2-(2+\eta)x^2y(1-y)+(2+\eta)x + (\eta+1+\epsilon)\left(\frac{1}{N}\right)\frac{k^2}{q}, \quad (2.139a)$$

$$N_2(k \rightarrow K+xp_y) \rightarrow -\eta\left(\frac{2}{N}\right)k^2-x^2y(1-y)q^2, \quad (2.139b)$$

where $N=4+\epsilon$ and C and p_y are given by (2.62). As before terms odd in K are dropped since they do not contribute to the integral over K and (2.131) replaces (2.61). The integrals over K , x , and y are performed using (2.132) and (2.133) with the result

$$\sigma_{DR}(\text{virtual}) = \left(\frac{2\pi}{3\pi} \right) \sigma_0 \left(\frac{\bar{Q}^2}{4\pi m_D^2} \right)^{\epsilon/2} \frac{\Gamma(1-\frac{\epsilon}{2})\Gamma^2(1+\frac{\epsilon}{2})}{\Gamma(1+\epsilon)} \left(-\frac{8}{\epsilon} + \frac{6}{\epsilon} - \frac{(8+4\epsilon)}{(1+\epsilon)} \right), \quad (2.140)$$

where σ_0 is given by (2.110) and where m_D is the dimensional regularization mass and (as was the case in the massive gluon scheme), I have taken

$$\bar{Q}^2 = -q^2 > 0 \quad (2.141)$$

(i.e. q^2 spacelike) in order to avoid singularities within the regions of integration. Expanding in powers of ϵ gives

$$\frac{\Gamma(1-\frac{\epsilon}{2})\Gamma^2(1+\frac{\epsilon}{2})}{\Gamma(1+\epsilon)} = 1 + \frac{1}{2}\gamma_E\epsilon + \frac{1}{48}(6\gamma_E^2 - \pi^2)\epsilon^2 + \dots \quad (2.142)$$

and using (2.127) equation (2.140) becomes

$$\sigma_{DR}(\text{virtual}) = \left(\frac{2\pi}{3\pi} \right) \sigma_0 \left(-\frac{8}{\epsilon} + \frac{1}{\epsilon} (-4 \log \left(\frac{\bar{Q}^2}{4\pi m_D^2} \right) - 4\gamma_E + 6) \right.$$

$$\left. -\log^2 \left(\frac{\bar{Q}^2}{4\pi m_D^2} \right) - (2\gamma_E - 3) \log \left(\frac{\bar{Q}^2}{4\pi m_D^2} \right) - \gamma_E^2 + 3\gamma_E + \frac{\pi^2}{6} - 8 \right), \quad (2.143)$$

which is valid for q^2 spacelike as defined in (2.141).

9. Complete Order α_s Corrections - Dimensional Regularization Scheme

In the dimensional regularization scheme the order α_s virtual corrections are from (2.143)

$$\sigma_{DR}(\text{virtual}) = \left(\frac{2\pi}{3\pi} \right) \sigma_0 \left(-\frac{8}{\epsilon} + \frac{1}{\epsilon} (-4 \log \left(\frac{\bar{Q}^2}{4\pi m_D^2} \right) - 4\gamma_E + 6) \right. \\ \left. -\log^2 \left(\frac{\bar{Q}^2}{4\pi m_D^2} \right) - (2\gamma_E - 3) \log \left(\frac{\bar{Q}^2}{4\pi m_D^2} \right) - \gamma_E^2 + 3\gamma_E + \frac{\pi^2}{6} - 8 \right) \quad (2.144)$$

where $\bar{Q}^2 = -q^2 > 0$ (i.e. q^2 spacelike). For q^2 timelike we analytically continue using (2.96) to arrive at

$$\sigma_{DR}(\text{virtual}) = \left(\frac{2\pi}{3\pi} \right) \sigma_0 \left(-\frac{8}{\epsilon} + \frac{1}{\epsilon} (-4 \log \left(\frac{Q^2}{4\pi m_D^2} \right) - 4\gamma_E + 6) \right. \\ \left. -\log^2 \left(\frac{Q^2}{4\pi m_D^2} \right) - (2\gamma_E - 3) \log \left(\frac{Q^2}{4\pi m_D^2} \right) - \gamma_E^2 + 3\gamma_E + \frac{\pi^2}{6} - 8 + \pi^2 \right), \quad (2.145)$$

where now $Q^2 = q^2 > 0$ (i.e. q^2 timelike) and where I have only kept the terms that are real.

Combining this with the result in (2.128) for the real gluon corrections

$$\sigma_{\text{DR}}(\text{real}) = \left(\frac{2\alpha_s}{3\pi}\right) \sigma_0 \left\{ \frac{8}{\epsilon^2} + \frac{1}{\epsilon} \left(4 \log\left(\frac{Q^2}{4\pi m_1^2}\right) + 4\gamma_E - 6 \right) \right. \\ \left. + \log^2\left(\frac{Q^2}{4\pi m_1^2}\right) + (2\gamma_E - 3) \log\left(\frac{Q^2}{4\pi m_1^2}\right) + \gamma_E^2 - 3\gamma_E - \frac{7\pi^2}{6} + \frac{57}{6} \right\}. \quad (2.146)$$

gives

$$\sigma_{\text{DR}}(\text{real}) + \sigma_{\text{DR}}(\text{virtual}) = \left(\frac{2\alpha_s}{3\pi}\right) \sigma_0 \left(-\frac{7\pi^2}{6} + \frac{57}{6} + \frac{\pi^2}{6} - 6 + \pi^2 \right) \\ = \left(\frac{2\alpha_s}{3\pi}\right) \sigma_0 \left(\frac{3}{2}\right) = \frac{\alpha_s}{\pi} \sigma_0, \quad (2.147)$$

which is finite in the limit $\epsilon \rightarrow 0$ and is precisely the same as the result obtained in (2.98) using the massive gluon scheme!

B. Quark and Gluon Fragmentation Functions

1. Quark and Gluon Distributions

Equation (2.79) gives the cross section for producing a quark in the process $\gamma^* \rightarrow q\bar{q}g$ carrying fractional energy x (in the massive gluon scheme),

$$\frac{d\sigma_{\text{MG}}}{dx} = \left(\frac{2\alpha_s}{3\pi}\right) \sigma_0 \left\{ \frac{1+x^2}{1-x} \log\left(\frac{x(1-x)Q^2}{m_g^2}\right) - \frac{3}{2} \frac{1}{1-x} + \frac{1}{2} x + \frac{1}{2} + \frac{5}{2} \delta(1-x) \right\}. \quad (2.148)$$

This differential cross section has a single logarithmic divergence as $m_g \rightarrow 0$, but we know that if we integrate over x and add the virtual corrections that the result is finite. Namely,

$$\int_0^1 \frac{d\sigma_{\text{MG}}}{dx} dx + \sigma_{\text{MG}}(\text{virtual}) = \frac{\alpha_s}{\pi} \sigma_0 \quad (2.149a)$$

or

$$\int_0^1 \left\{ \frac{d\sigma_{\text{MG}}}{dx} + (\sigma_{\text{MG}}(\text{virtual}) - \frac{\alpha_s}{\pi} \sigma_0) \delta(1-x) \right\} dx = 0. \quad (2.149b)$$

Here it is very useful to define "plus functions" in the following way:

$$\int_0^1 \left(\frac{d\sigma_{\text{MG}}}{dx} \right)_+ dx = 0 \quad (2.150)$$

with

$$(F(x))_+ \equiv \lim_{\beta \rightarrow 0} \{ F(x) \theta(1-x-\beta) - \delta(1-x-\beta) \int_0^{1-\beta} F(y) dy \}. \quad (2.151)$$

For $x < 1-\beta$ $(F(x))_+ = F(x)$, but

$$\int_0^1 (F(x))_+ dx = 0. \quad (2.152)$$

From (2.148) and (2.149) we see that

$$\frac{1}{\sigma_0} \left(\frac{d\sigma_{\text{MG}}}{dx} \right)_+ = \left(\frac{\alpha_s}{2\pi} \right) P_{q \leftarrow q}(x) \log(Q^2/m_g^2) + \alpha_s f_{\text{MG},q}^{e^+e^-}(x), \quad (2.153)$$

where

$$P_{q \leftarrow q}(x) = \frac{4}{3} \left(\frac{1+x^2}{1-x} \right)_+ = \frac{4}{3} \left(\frac{1+x^2}{1-x} \right)_+ + \frac{3}{2} \delta(1-x), \quad (2.154)$$

and

$$\alpha_s f_{\text{MG},q}^{e^+e^-}(x) = \left(\frac{2\alpha_s}{3\pi} \right) \left((1+x^2) \left(\frac{\log(1-x)}{1-x} \right)_+ + \frac{1+x^2}{1-x} \log(x) \right)$$

$$-\frac{3}{2} \frac{1}{(1-x)_+} + \frac{1}{2} (1+x) + \left(\frac{\pi^2}{3} - \frac{15}{4}\right) \delta(1-x). \quad (2.155)$$

Furthermore we have

$$\int_0^1 P_{q \rightarrow q}(x) dx = 0, \quad (2.156a)$$

and

$$\int_0^1 \alpha_s f_{\text{RG},q}^{e^+e^-}(x) dx = 0. \quad (2.156b)$$

since

$$\int_0^1 (1+x^2) \left(\frac{\log(1-x)}{1-x} \right)_+ dx = \frac{7}{4}, \quad (2.157a)$$

$$\int_0^1 \frac{1+x^2}{1-x} \log(x) dx = -\frac{\pi^2}{3} + \frac{5}{4}. \quad (2.157b)$$

In addition from the definition (2.151) we see that

$$\frac{1}{(1-x)_+} = \frac{1}{1-x} \theta(1-x-\beta) + \log(\beta) \delta(1-x-\beta) \quad (2.158a)$$

and

$$\left(\frac{\log(1-x)}{1-x} \right)_+ = \frac{\log(1-x)}{1-x} \theta(1-x-\beta) + \frac{1}{2} \log^2(\beta) \delta(1-x-\beta). \quad (2.158b)$$

The quark distribution $(da/dx)_+$ in (2.153) contains both real and virtual corrections.

To arrive at the quark distribution in the dimensional regularization scheme we integrate (2.119) over x_2 . The result is

$$\frac{d\sigma_{\text{DR}}}{dx} = \left(\frac{2\alpha_s}{3\pi} \right) \sigma_0 \left(\frac{x^2(1-x)Q^2}{4\pi m_D^2} \right)^{\epsilon/2} \int_0^1 dv v^{N/2-2} (1-v)^{N/2-2} F(x,v), \quad (2.159)$$

where $F(x,v)$ is given by (2.123) and $N = 4+\epsilon$. Integrating over v yields

$$\frac{d\sigma_{\text{DR}}}{dx} = \left(\frac{2\alpha_s}{3\pi} \right) \sigma_0 \left(\frac{x^2(1-x)Q^2}{4\pi m_D^2} \right)^{\epsilon/2} \frac{\Gamma(1+\frac{\epsilon}{2})}{\Gamma(1+\epsilon)} \left\{ \frac{1+x}{1-x} \frac{2}{\epsilon} - \frac{3}{2} \frac{1}{1-x} - \frac{3}{2} x + \frac{5}{2} \right\}, \quad (2.160)$$

where I have dropped some of the terms that vanish as $\epsilon \rightarrow 0$. Using the expansion

$$\frac{\Gamma(1+\frac{\epsilon}{2})}{\Gamma(1+\epsilon)} = 1 + \frac{1}{2} \gamma_E \epsilon + \frac{1}{16} (2\gamma_E^2 - \pi^2) \epsilon^2 + \dots, \quad (2.161)$$

we arrive at

$$\begin{aligned} \frac{d\sigma_{\text{DR}}}{dx} &= \left(\frac{2\alpha_s}{3\pi} \right) \sigma_0 \left\{ \frac{1+x}{1-x} \frac{2}{\epsilon} + \frac{1+x}{1-x} \left[\log \left(\frac{x^2(1-x)Q^2}{4\pi m_D^2} \right) + \gamma_E \right] \right. \\ &\quad \left. - \frac{3}{2} \frac{1}{1-x} - \frac{3}{2} x + \frac{5}{2} \right\}. \end{aligned} \quad (2.162)$$

and

$$\frac{1}{\sigma_0} \left(\frac{d\sigma_{\text{DR}}}{dx} \right)_+ = \left(\frac{\alpha_s}{2\pi} \right) P_{q \rightarrow q}(x) \log(Q^2/m_D^2) + \alpha_s f_{\text{DR},q}^{e^+e^-}(x), \quad (2.163)$$

where $P_{q \rightarrow q}(x)$ is the same function that appears in (2.154) and

$$\alpha_s f_{DR,q}^{e^+e^-}(x) = \left(\frac{2\alpha_s}{3\pi}\right) \left\{ (1+x^2) \left(\frac{\log(1-x)}{1-x} \right)_+ + \frac{2(1+x^2)}{1-x} \log(x) \right. \\ \left. - \frac{3}{2} \frac{1}{(1-x)_+} - \frac{3}{2} x + \frac{5}{2} + \left(\frac{2\pi^2}{3} - 6 \right) \delta(1-x) \right\} \\ + \frac{\alpha_s}{2\pi} P_{q-q}(x) \left\{ \frac{2}{\epsilon} + \gamma_E - \log(4\pi) \right\}. \quad (2.164)$$

Equations (2.153) and (2.163) illustrate a general feature that is common with many of the calculations we will be performing in these lectures. The quark distributions are regularization scheme dependent. The coefficient of the $\log(Q^2)$ term is universal (scheme independent) but the "little-f" functions in (2.155) and (2.164) are different and depend on the scheme. It is true, however, that the "little f" functions have a unique integral. Namely

$$\int_0^1 f_{DR,q}^{e^+e^-}(x) dx = \int_0^1 f_{MC,q}^{e^+e^-}(x) dx = 0, \quad (2.165)$$

which is necessary to insure that (2.149) is true regardless of the regularization scheme chosen. Furthermore equations (2.153) and (2.163) explicitly contain the gluon mass and the dimensional regularization mass, respectively. We believe that QCD gives finite results for experimental observables in the limit that m_g or m_D goes to zero. We also believe that experimental observables cannot depend on the regularization scheme chosen. This means that do/dx in (2.153) or (2.163) better not be an experimental observable or we are in serious trouble. Finally, from the point of view of a perturbation series the first term in (2.153) or (2.163) does not become small as Q increases. Since $\alpha_s(Q^2)\log(Q^2)$ is of order one, as is any term of the form $[\alpha_s(Q^2)\log(Q^2)]^n$, terms of this form must be summed in order to arrive at a valid perturbation expansion.

In a similar manner one can construct the cross section for producing a gluon in the process $\gamma^* \rightarrow q\bar{q}g$ carrying fractional energy x . In the massive gluon scheme one gets

$$\left(\frac{d\sigma_g}{dx} \right)_+ = \left(\frac{\alpha_s}{2\pi} \right) P_{q-g}(x) \log(Q^2/m_g^2) + \alpha_s f_{MC,g}^{e^+e^-}(x). \quad (2.166)$$

with

$$P_{q-g}(x) = \frac{4}{3} (1+(1-x)^2)/x \quad (2.167)$$

and

$$\alpha_s f_{MC,g}^{e^+e^-}(x) = \left(\frac{2\alpha_s}{3\pi} \right) \left(2 \frac{1}{x} (1+(1-x)^2) \log(x) - 2x \right). \quad (2.168)$$

In general the "little-f" functions must be labeled by the process and the scheme since they depend on both.

2. Single Hadron Inclusive Cross Section

The next step is to embed the parton cross sections within the desired hadronic process. What is measured experimentally is the probability of observing an outgoing hadron in the process $\gamma^* \rightarrow$ hadrons carrying a certain fraction z of the available energy.

$$z = 2E_h/Q. \quad (2.169)$$

This inclusive single hadron cross sections is given by

$$d\sigma^h(z, Q^2) = \left(\frac{d\sigma^q}{dy} \right) dy D_{o,q}^h(x) dx \\ + \left(\frac{d\sigma^{\bar{q}}}{dy} \right) dy D_{o,\bar{q}}^h(x) dx \\ + \left(\frac{d\sigma^g}{dy} \right) dy D_{o,g}^h(x) dx. \quad (2.170)$$

where $(d\sigma^q/dy)dy$ is the probability of finding a quark with energy

$$E_q = \frac{1}{2} y Q \quad (2.171)$$

and $D_{a,q}^h(x)dx$ is the probability that a quark of energy E_q fragments into a hadron carrying fractional energy

$$x = E_h/E_q. \quad (2.172)$$

Similarly $(d\sigma^g/dy)dy$ is the probability of finding a gluon with energy $E_g = \frac{1}{2}yQ$ and $D_{o,g}^h(x)dx$ is the gluon fragmentation function. The "outside" experimental variable z is related to the two "inside" parton variables x and y as follows.

$$x = z/y \quad (2.173)$$

and $0 \leq x \leq 1$ implies that $z \leq y \leq 1$. Thus,

$$\begin{aligned} \frac{d\sigma^h}{dz}(Q^2) &= 3\alpha_0 \int_0^1 \frac{dy}{y} \left\{ \sum_{i=1}^{n_f} e_{q_i}^2 [D_{o,q_i}^h\left(\frac{z}{y}\right) + D_{o,q_i}^h\left(\frac{z}{y}\right)] \right. \\ &\quad \left[\left(1 + \frac{\alpha_s}{\pi}\right)\delta(1-y) + \frac{\alpha_s}{2\pi} P_{q-q}(y) \log(Q^2/m^2) + \alpha_s f_q^{e^+e^-}(y) \right] \\ &\quad \left. + 2 \sum_{i=1}^{n_f} e_{q_i}^2 D_{o,g}^h\left(\frac{z}{y}\right) \left[\frac{\alpha_s}{2\pi} P_{q-g}(y) \log(Q^2/m^2) + \alpha_s f_g^{e^+e^-}(y) \right] \right\}. \quad (2.174) \end{aligned}$$

where I have summed over n_f quark flavors and three quark colors and where $P_{q-q}(y)$ and $P_{q-g}(y)$ are given by (2.154) and (2.167), respectively. The "little- f " functions are the scheme dependent functions in (2.155), (2.164) and (2.168) and $m = m_g$ or m_o depending on the scheme. The fragmentation functions $D_{o,q}^h$ and $D_{o,g}^h$ are not experimental observables. They contain the non-perturbative information on how the quarks turn into outgoing hadrons and all we know is that energy is conserved so that

$$\sum_{\text{All } h} \int_0^1 x D_{o,q_i}^h(x) dx = 1. \quad (2.175a)$$

$$\sum_{\text{All } h} \int_0^1 x D_{o,q_i}^h(x) dx = 1. \quad (2.175b)$$

This is sufficient to insure the normalization condition

$$\sum_{\text{All } h} \int_0^1 \frac{1}{2} z \frac{d\sigma^h}{dz}(Q^2) dz = \sigma_{\text{tot}}^{e^+e^-} \quad (2.176)$$

where, $\sigma_{\text{tot}}^{e^+e^-}$ is given by (2.99b).

If we now define experimentally observable fragmentation functions according to

$$\frac{d\sigma^h}{dz}(Q^2) = 3\sigma_{\text{tot}}^{e^+e^-} \sum_{i=1}^{n_f} \frac{e_{q_i}^2}{z} (D_{q_i}^h(z, Q^2) + D_{q_i}^h(z, Q^2)). \quad (2.177)$$

then to order α_s (in the massive gluon scheme)

$$\begin{aligned} D_q^h(z, Q^2) &= \int_0^1 \frac{dy}{y} \left\{ D_{o,q}^h\left(\frac{z}{y}\right) \left[\delta(1-y) + \frac{\alpha_s}{\pi} P_{q-q}(y) \log(Q^2/m_g^2) + \alpha_s f_{MG,q}^{e^+e^-}(y) \right] \right. \\ &\quad \left. + D_{o,g}^h\left(\frac{z}{y}\right) \left[\frac{\alpha_s}{2\pi} P_{q-g}(y) \log(Q^2/m_g^2) + \alpha_s f_{MG,g}^{e^+e^-}(y) \right] \right\}. \quad (2.178) \end{aligned}$$

We cannot calculate $D_q^h(z, Q^2)$ at a given Q^2 since the "bare" fragmentation functions $D_{o,q}^h(x)$ and $D_{o,g}^h(x)$ are unknown. Because of this the "little- f " functions $f_q^{e^+e^-}$ and $f_g^{e^+e^-}$ are not directly experimentally observable. This is fortunate because they are regularization scheme dependent. However it is true that

$$\int_0^1 y [f_q^{e^+e^-}(y) + f_g^{e^+e^-}(y)] dy = 0 \quad (2.179)$$

in any scheme, which together with

$$\int_0^1 y [P_{q-q}(y) + P_{q-g}(y)] dy = 0, \quad (2.180)$$

insures that if energy is conserved by $D_{0,q}^h$ and $D_{0,g}^h$ (i.e. (2.175) are satisfied) then it is also conserved by $D_q^h(z, Q^2)$.

At this point let me define the convolution

$$C(z) = A * B \equiv \int_z^1 \frac{dy}{y} A\left(\frac{z}{y}\right) B(y), \quad (2.181)$$

whereupon (2.178) becomes

$$\begin{aligned} D_q^h(z, Q^2) &= D_{0,q}^h * \left(1 + \frac{\alpha_s}{2\pi} P_{q-q} \log(Q^2/m_g^2) + \alpha_s f_{HG,q}^{e^+e^-}\right) \\ &+ D_{0,q}^h * \left(\frac{\alpha_s}{2\pi} P_{q-q} \log(Q^2/m_g^2) + \alpha_s f_{HG,g}^{e^+e^-}\right) \end{aligned} \quad (2.182)$$

and

$$D_{NS}^h(z, Q^2) = D_{0,NS}^h * \left(1 + \frac{\alpha_s}{2\pi} P_{q-q} \log(Q^2/m_g^2) + \alpha_s f_{HG,q}^{e^+e^-}\right). \quad (2.183)$$

where the non-singlet fragmentation function is defined by

$$D_{NS}^h(z, Q^2) \equiv D_{q_1}^h(z, Q^2) - D_{\bar{q}_1}^h(z, Q^2). \quad (2.184)$$

which is easier to discuss since the gluon term $D_{0,g}^h$ drops out.

The fragmentation functions in (2.182) and (2.183) still appear to diverge like $\log(m_g)$ in the limit $m_g \rightarrow 0$. Since we believe that all observable quantities should be well behaved in the limit of zero gluon mass (or in the limit in which the dimensional regularization mass goes to zero), this divergence must be an artifact of the way we have done the calculation. For example, we have divided the observable $D_{NS}^h(z, Q^2)$ into two terms, $D_{0,NS}^h(x)$ and $P_{q-q}(y) \log(Q^2/m_g^2)$. This latter term diverges as $m_g \rightarrow 0$ but $D_{NS}^h(z, Q^2)$ must remain finite. This means that $D_{0,NS}^h(x)$ must also diverge as $m_g \rightarrow 0$ in such a way that the product in (2.183) is finite. The function $D_{0,NS}^h(z)$ must, therefore, have the form

$$D_{0,NS}^h(z) = \tilde{D}_{0,NS}^h * \left(1 + \frac{\alpha_s}{2\pi} P_{q-q} \log(m_g^2/\Lambda^2) + \dots\right). \quad (2.185)$$

where Λ is a mass scale that is related to the size of hadrons and where $\tilde{D}_{0,NS}^h$ is finite in the limit $m_g \rightarrow 0$. The $m_g \rightarrow 0$ mass singularities are "factored" off into $D_{0,NS}^h$ as follows

$$\begin{aligned} D_{NS}^h(z, Q^2) &= D_{0,NS}^h * \left(1 + \frac{\alpha_s}{2\pi} P_{q-q} [\log(Q^2/\Lambda^2) + \log(\Lambda^2/m_g^2)]\right) \\ &= D_{0,NS}^h * \left(1 + \frac{\alpha_s}{2\pi} P_{q-q} \log(\Lambda^2/m_g^2)\right) \\ &* \left(1 + \frac{\alpha_s}{2\pi} P_{q-q} \log(Q^2/\Lambda^2)\right) + \text{order}(\alpha_s^2) \\ &= \tilde{D}_{0,NS}^h * \left(1 + \frac{\alpha_s}{2\pi} P_{q-q} \log(Q^2/\Lambda^2)\right) + \text{order}(\alpha_s^2). \end{aligned} \quad (2.186)$$

The mass singularity $\log(m_g^2)$ has been absorbed into the unknown function $\tilde{D}_{0,NS}^h$ which is well behaved in the limit $m_g \rightarrow 0$. The scale Λ can be taken to be the perturbative parameter in (1.12) which sets the

value of $\alpha_s(Q^2)$ at a given scale Q and is also related to the size of hadrons. At this order the "factorization of the mass singularities" is a trivial property of logarithms. It has been shown, however, that, to all orders of perturbation theory, one can factor out and absorb these singularities into the functions $\tilde{D}_{0,q}$ and $\tilde{D}_{0,g}$.

Since $\tilde{D}_{0,NS}(x)$ in (2.186) is unknown we still cannot calculate the observable $D_{NS}^h(z, Q^2)$. We might try to calculate the change in $D_{NS}^h(z, Q^2)$ as we change Q^2 . However, since $\alpha_s(Q^2) \log(Q^2)$ is of order one, we cannot do this unless we sum all terms of the form $[\alpha_s(Q^2) \log(Q^2)]^n$.

3. Summing Leading Logarithms - Q^2 Dependent Fragmentation Functions

Let us examine the double differential cross section

$$\frac{1}{\sigma_0} \frac{d\sigma}{dx_1 dx_2} = \left(\frac{2\pi}{3s}\right) \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} \quad (2.187)$$

from yet another point of view. This time we introduce Sudakov or light cone variables

$$E_1 + (p_z)_1 = z_1 Q \quad (2.188a)$$

$$E_1 - (p_z)_1 = y_1 Q \quad (2.188b)$$

where as shown in Fig. 10, the z -axis is defined in the direction $\vec{p}_2 + \vec{p}_3$. The Sudakov variables for parton 2 (quark) and parton 3 (gluon) satisfy

$$z_2 + z_3 = 1, \quad (2.189a)$$

$$y_2 + y_3 = \hat{t} = (1-x_1), \quad (2.189b)$$

$$z_2 y_2 = z_3 y_3 = z_2 (1-z_2) \hat{t} = k_T^2 / Q^2, \quad (2.189c)$$

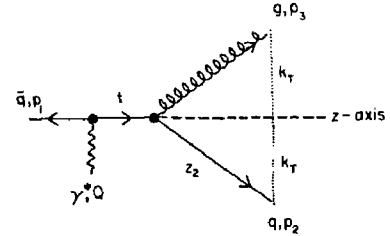


Fig. 10. Illustration of the "decay" of a quark, q , with invariant mass $\sqrt{t}$ into a gluon, g , and a quark carrying a fraction z_2 of its $E+p_z$. The outgoing gluon has a transverse momentum, k_T , relative to a z -axis defined along the direction of the original quark.

$$z_2 = x_2 - \hat{t}(1-x_2)/(1-\hat{t}), \quad (2.189d)$$

where

$$\hat{t} = t/Q^2, \quad (2.190)$$

and where t is defined in (2.22b) and where k_T is the transverse momentum relative to the z -axis (i.e. relative to $\vec{p}_2 + \vec{p}_3$). The differential cross section (2.187) when written in terms of $z = z_2$ and the invariant mass, t , of parton 2 and 3 becomes

$$\frac{1}{\sigma_0} \frac{d\sigma}{dzdt} = \left(\frac{2\alpha_s}{3\pi}\right) \left\{ \frac{1+z^2}{t(1-z)} - \frac{2(1-z)(1-z)}{(1-z)} + \frac{\hat{t}(1+(1-z)^2)}{(1-z)t} \right\}. \quad (2.191)$$

In the region where $k_T^2/Q^2 = z(1-z)\hat{t}$ is small this cross section can be approximated by the first term. The approximation

$$\left(\frac{1}{\sigma_0} \frac{d\sigma}{dzdt}\right)_{LPA} = \left(\frac{2\alpha_s}{3\pi}\right) \frac{1+z^2}{(1-z)t}, \quad (2.192)$$

is known as the "leading pole" approximation (LPA) and when integrated over $\hat{t}$ produces the leading log singularity. The virtual corrections can be included by writing (2.192) in terms of a "+" function" as follows:

$$\left(\frac{1}{\sigma_0} \frac{d\sigma}{dzdt}\right)_{LPA} = \frac{\alpha_s(t)}{2\pi t} P_{q-q}(z), \quad (2.193)$$

where $P_{q-q}(z)$ is the same as (2.154). Namely,

$$P_{q-q}(z) = \frac{4}{3} \left(\frac{1+z^2}{1-z}\right)_+. \quad (2.194)$$

Integrating the leading pole (2.193) over $\hat{t}$ gives the leading log (LL) piece of (2.153). For example,

$$\left(\frac{1}{\sigma_0} \frac{d\sigma}{dz}\right)_{LL} = \int_{\beta}^1 \left(\frac{1}{\sigma_0} \frac{d\sigma}{dzdt}\right) d\hat{t} = \frac{\alpha_s}{2\pi} P_{q-q}(z) \log(Q^2/m_g^2), \quad (2.195)$$

where I have taken $\beta = m_g^2/Q^2$ and where, for the moment, I have ignored the t dependence of α_s .

We will interpret (2.192) as the probability that a quark of invariant mass $\sqrt{t}$ propagates and "decays" into a quark and gluon carrying z and $(1-z)$, respectively, of its $E + p_z$. We would like to construct final states with, for example, n gluons by multiplying the LPA probability by itself n times (i.e. independent emission) as illustrated in Fig. 11. Unfortunately, if we examine the contributions to the LPA probability from the three terms in (2.32) we find

$$S_{11} = |B_R|^2 = 8(1-x_1)/(1-x_2) \xrightarrow{LPA} 8(1-z)/\hat{t} \quad (2.196a)$$

$$S_{22} = |A_R|^2 = 8(1-x_2)/(1-x_1) \xrightarrow{LPA} 0 \quad (2.196b)$$

$$S_{12} = 2A_R B_R^* = 8\left(\frac{2}{(1-x_1)(1-x_2)} - \frac{2}{(1-x_1)} - \frac{2}{(1-x_2)}\right) \xrightarrow{LPA} 8 \frac{2z}{(1-z)t}, \quad (2.196c)$$

where the terms were calculated using the Feynman gauge. The sum of the three terms, of course, reproduces the LPA result in (2.192). However, since the interference term (2.196c) contributes to the result, we cannot simply multiply the probabilities of successive emissions. If interference terms are present then we must add the amplitudes and then square. It does not appear as if we have "independent" emission. These interference terms would correspond to diagrams in which the gluons in the ladder in Fig. 11 are crossed.

Fortunately each of the three terms in (2.196) is gauge dependent, only the sum is gauge independent. We can move the contributions around so that, for example, S_{11} alone contains the complete LPA approximation and the interference term goes to zero. In an axial gauge the sum over gluon polarization states is replaced by

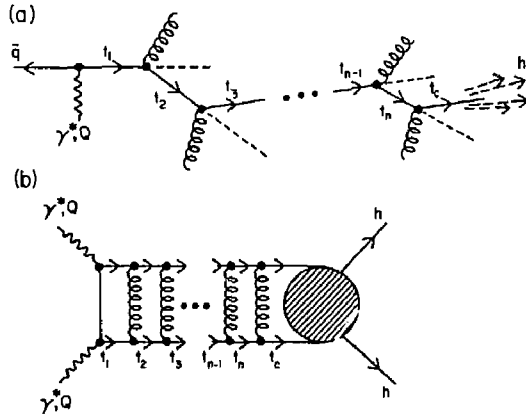


Fig. 11. (a) Illustration of the case where an initial quark produced by the "decay" of a virtual photon of invariant mass Q emits n gluons and has its invariant mass degraded from t_1 to t_c ($Q^2 \geq t_1 \geq t_2 \geq \dots \geq t_{n-1} \geq t_c$) whereby it subsequently fragments into a hadron, h . (b) Square of the amplitude for the process in (a) in the "leading pole" approximation. In an axial gauge interference terms do not contribute to leading order and the cross section takes on a simple ladder form.

$$\sum_{\text{poL}} \epsilon_{\mu} \epsilon_{\nu}^* = -g_{\mu\nu} + \frac{n_{\mu} k_{\nu} + n_{\nu} k_{\mu}}{(n \cdot k)} - \frac{n^2 k_{\mu} k_{\nu}}{(n \cdot k)^2}. \quad (2.197)$$

where k is the gluon momentum and n is an arbitrary 4-vector satisfying $e \cdot n = 0$ (usually one also takes $n^2 = 0$ as well). With the choice, for example, of

$$n = Q - p_2/x_2, \quad (2.198)$$

we have

$$S_{11} = 8 \frac{1+x_2^2}{(1-x_1)(1-x_2)} \xrightarrow{\text{LPA}} 8 \frac{1+z^2}{(1-z)t} \quad (2.199a)$$

$$S_{22} = 8 \frac{1-x_1}{1-x_2} \xrightarrow{\text{LPA}} 0 \quad (2.199b)$$

$$S_{12} = 8 \frac{-2}{(1-x_2)} \xrightarrow{\text{LPA}} 0. \quad (2.199c)$$

In this gauge (and in any axial gauge) the cross section for the emissions of n gluons in the LPA approximation has the simple ladder structure shown in Fig. 11 and the probability of emitting n gluons becomes the product of each emission,

$$\left(\frac{d\sigma/\sigma_0}{dz_1 \dots dz_n dt_1 \dots dt_n} \right)_{\text{LPA}} = \left[\frac{\alpha_s(t_1)}{2\pi t_1} P_{q-q}(z_1) \right] \left[\frac{\alpha_s(t_2)}{2\pi t_2} P_{q-q}(z_2) \right] \dots \left[\frac{\alpha_s(t_n)}{2\pi t_n} P_{q-q}(z_n) \right], \quad (2.200)$$

where $P_{q-q}(z)$ is given by (2.194) and where the invariant masses are ordered according to

$$Q^2 \geq t_1 \geq t_2 \geq \dots \geq t_{n-1} \geq t_n \quad (2.201a)$$

or

$$1 \geq \hat{t}_1 \geq \hat{t}_2 \geq \dots \geq \hat{t}_{n-1} \geq \hat{t}_n. \quad (2.201b)$$

It is now easy to obtain the precise form of the leading log terms in the perturbation series and to sum these terms to all orders. For simplicity I will take the non-singlet fragmentation function in (2.184) and I will include in the sum all subprocesses in which the final quark in Fig. 11 has an invariant mass greater than some cut-off, t_c . The first term in the series is simply

$$D_{NS}^h(z, Q^2) = D_{NS}^h(z, t_c) \quad (2.202)$$

and corresponds to no emitted gluons. The next term involves one gluon emission

$$\frac{1}{\sigma} \frac{d\sigma}{dz} = P_{q \rightarrow q}(z) \int_{t_c}^1 \left(\frac{\alpha_s(t)}{2\pi t} \right) d\hat{t}, \quad (2.203)$$

where

$$\alpha_s(t) = 4\pi/(\beta_0 \log(t/\Lambda^2)) \quad (2.204)$$

with β_0 given by (1.7b). To integrate (2.203) we change variables to

$$\kappa = \frac{2}{\beta_0} \log(\alpha_s(t_c)/\alpha_s(t)), \quad (2.205)$$

so that

$$\frac{d\kappa}{dt} = \frac{\alpha_s(t)}{2\pi t} \quad (2.206)$$

and the integral becomes

$$\int_0^\kappa d\kappa = \kappa. \quad (2.207)$$

The series now becomes

$$D_{NS}^h(z, Q^2) = D_{O,NS}^h + \kappa P_{q \rightarrow q} * D_{O,NS}^h, \quad (2.208)$$

where

$$D_{O,NS}^h = D_{NS}^h(z, t_c). \quad (2.209)$$

The next term arises from the emission of two gluons

$$\frac{1}{\sigma} \frac{d\sigma}{dz_1 dz_2} = P_{q \rightarrow q}(z_1) P_{q \rightarrow q}(z_2) \int_{t_c}^1 \frac{\alpha(t_1)}{2\pi t_1} d\hat{t}_1 \int_{t_c}^{\hat{t}_1} \frac{\alpha(t_2)}{2\pi t_2} d\hat{t}_2. \quad (2.210)$$

After changing variables to κ we are left with the "nested" integral

$$\int_0^\kappa d\kappa_1 \int_0^{\kappa_1} d\kappa_2 = \frac{1}{2} \kappa^2. \quad (2.211)$$

The factor of $\frac{1}{2}$ is crucial and arises because of the way the integrals are "nested" (the area of the triangle formed by dividing a square along its diagonal is $\frac{1}{2}$ the area of the square). The series now becomes

$$D_{NS}^h(z, Q^2) = D_{O,NS}^h + \kappa P_{q \rightarrow q} * D_{O,NS}^h + \frac{1}{2} \kappa^2 P_{q \rightarrow q} * D_{O,NS}^h. \quad (2.212)$$

It is easy to see that the series is that of an exponential. The n -th term involves the nested integral

$$\int_0^{\kappa} dk_1 \int_0^{\kappa_1} dk_2 \dots \int_0^{\kappa_{n-1}} dk_n = \frac{\kappa^n}{n!}, \quad (2.213)$$

and the infinite sum becomes

$$D_{NS}^h(z, Q^2) = \exp(\kappa P_{q \rightarrow q}) D_{NS}^h(\tau_c). \quad (2.214)$$

This equation allows us to calculate $D_{NS}^h(z, Q^2)$ in terms of its value at some reference point $\tau_c = Q_0^2$ with $Q > Q_0$ and is accurate provided τ_c is large enough so that $\alpha_s(\tau_c)$ is small and hence perturbation theory is valid. Equation (2.214) is the integral form of the Altarelli-Parisi Equation.[5] If we differentiate it with respect to κ we get

$$dD_{NS}^h(z, Q^2)/d\kappa = P_{q \rightarrow q} * D_{NS}^h(Q^2) \quad (2.215)$$

or

$$\begin{aligned} \frac{dD_{NS}^h(z, Q^2)}{d\tau} &= \frac{\alpha_s(Q^2)}{2\pi} P_{q \rightarrow q} * D_{NS}^h(Q^2) \\ &= \frac{\alpha_s(Q^2)}{2\pi} \int_z^1 \frac{dy}{y} P_{q \rightarrow q}\left(\frac{z}{y}\right) D_{NS}^h(y, Q^2), \end{aligned} \quad (2.216)$$

where

$$\tau = \log(Q^2/\Lambda^2) \quad (2.217)$$

and

$$\frac{d\kappa}{d\tau} = \frac{\alpha_s}{2\pi}. \quad (2.218)$$

Equation (2.216) is the usual form of the equation that govern the Q^2 evolution of the non-singlet fragmentation function $D_{NS}^h(z, Q^2)$. The solution of (2.216) is (2.214). At leading log order the "little f" functions do not contribute.

III. DEEP INELASTIC ELECTRON PROTON SCATTERING

1. The Naive Parton Model

In the naive parton model, one defines parton distributions, $G_{h \rightarrow q}(x)$, as the number of quarks q with fractional of momentum between x and $x + dx$ within a hadron of type h of high momentum. In particular, some of the functions necessary to describe the quark distributions in a proton are:

$$\begin{aligned} u(x) &\equiv G_{p \rightarrow u}(x), & \bar{u}(x) &\equiv G_{p \rightarrow \bar{u}}(x), \\ d(x) &\equiv G_{p \rightarrow d}(x), & \bar{d}(x) &\equiv G_{p \rightarrow \bar{d}}(x), \\ s(x) &\equiv G_{p \rightarrow s}(x), & \bar{s}(x) &\equiv G_{p \rightarrow \bar{s}}(x), \end{aligned} \quad (3.1a)$$

where u , d and s refer to up, down and strange quarks, respectively, and $\bar{u}$, $\bar{d}$ and $\bar{s}$ to their antiquarks. The distribution of gluons within a proton is defined by

$$g(x) \equiv G_{p \rightarrow g}(x), \quad (3.1b)$$

where g stands for gluon. These distributions satisfy the following sum rules:

$$\int_0^1 [u(x) - \bar{u}(x)] dx = 2 \quad (3.2a)$$

$$\int_0^1 [d(x) - \bar{d}(x)] dx = 1 \quad (3.2b)$$

$$\int_0^1 [s(x) - \bar{s}(x)] dx = 0. \quad (3.2c)$$

That is the net number of each kind of quark is just the number one arrives at in the simple non-relativistic quark model. In addition, momentum conservation implies

$$\int_0^1 \sum_{i=1}^{n_f} x(G_{p=q_i}(x) + G_{p=q_i^-}(x) + xg(x)) dx = 1, \quad (3.3)$$

where n_f is the number of quark flavors (i.e., u, d, s, ..., etc.). The distributions in a neutron are arrived at using isospin symmetry, which implies that $G_{n=u}(x) = G_{p=d}(x) = d(x)$, $G_{n=d}(x) = u(x)$, $G_{n=s}(x) = s(x)$, etc.

Deep inelastic scattering structure functions $w_1(x, Q^2)$ and $w_2(x, Q^2)$ are measured by experimentally examining the energy, E' , and angle θ_{LAB} of the scattered electron in the process $e+N \rightarrow e'+X$.

$$\frac{d\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{Q^4} \{2w_1 \sin^2(\frac{\theta_{LAB}}{2}) + w_2 \cos^2(\frac{\theta_{LAB}}{2})\}. \quad (3.4)$$

where

$$\nu = E' - E \quad (3.5a)$$

is the energy loss of the electron and

$$q = p_e' - p_e \quad (3.5b)$$

is the four-momentum transfer of the electron. In addition,

$$(P \cdot q)_{LAB} = M\nu, \quad (3.5c)$$

where M is the proton mass and

$$Q^2 = -q^2 > 0 \quad (3.5d)$$

since q^2 is spacelike. The structure functions F_1 and F_2 are defined by

$$F_1(x, Q^2) = M w_1(x, Q^2) \quad (3.6a)$$

$$F_2(x, Q^2) = \nu w_2(x, Q^2) \quad (3.6b)$$

and, in addition, I will define structure functions $\mathcal{F}_1$ and $\mathcal{F}_2$ by

$$\mathcal{F}_1(x, Q^2) = 2F_1(x, Q^2) \quad (3.7a)$$

$$\mathcal{F}_2(x, Q^2) = F_2(x, Q^2)/x. \quad (3.7b)$$

Complete knowledge of the deep inelastic structure functions for electron, neutrino, and antineutrino scattering off protons and neutrons is sufficient to obtain $u(x)$, $d(x)$, $\bar{u}(x)$, $\bar{d}(x)$, $g(x)$ and $s(x) + \bar{s}(x)$. For example,

$$F_2^{ep}(x) = \frac{4}{9} x[u(x) + \bar{u}(x)] + \frac{1}{9} x[d(x) + \bar{d}(x)] + \frac{1}{9} x[s(x) + \bar{s}(x)]. \quad (3.8a)$$

and

$$F_2^{en}(x) = \frac{4}{9} x[d(x) + \bar{d}(x)] + \frac{1}{9} x[u(x) + \bar{u}(x)] + \frac{1}{9} x[s(x) + \bar{s}(x)]. \quad (3.8b)$$

In the naive parton model the structure function $\mathcal{F}_1$ are only functions of x and do not depend separately on the energy loss of the leptons, ν , or on the four-momentum transfer, Q^2 . This is because the basic interaction is assumed to be a photon of momentum q^2 interacting with a parton of momentum p ($p = \hat{p}P$) with limited transverse momentum producing a parton of momentum $p' = p + q$. The condition that $(p')^2 = m_q^2$ implies

$$2\xi(P \cdot q) + q^2 + \xi^2 M^2 = m_q^2, \quad (3.9a)$$

which as $-q^2 \rightarrow \infty$ and $P \cdot q = M\nu \rightarrow \infty$ yields

$$\xi \approx \frac{-q^2}{2P \cdot q} = \frac{Q^2}{2M\nu} \equiv x. \quad (3.9b)$$

Furthermore in the naive parton model the two structure functions in (3.7) are equal. Namely,

$$\mathcal{F}_1(x) = \mathcal{F}_2(x). \quad (3.10)$$

2. Real Gluon Emission

We now consider, as we did for e^+e^- annihilations in Section 11.A.2, the possibility that a quark can radiate a gluon before or after its interaction with the virtual photon, γ^* , as in Fig. 12. The differential cross section for the subprocess $\gamma^* + q \rightarrow q + g$ is given by

$$\begin{aligned} \frac{d\hat{\sigma}_q}{dt}(s, t) &= \frac{1}{64\pi s p_{cm}^2} |M(\gamma^* + q \rightarrow q + g)|^2 \\ &= \frac{\pi \alpha_s^2}{(s+Q^2)^2} \left(\frac{8}{3} \right) \left(-\frac{\hat{t}}{s} - \frac{\hat{s}}{t} + \frac{2Q^2(\hat{s} + \hat{t} + Q^2)}{st} \right), \end{aligned} \quad (3.11)$$

where the subscript Σ is to signify that I have used equation (2.15) to sum over the incoming virtual photon polarization states, and the superscript q labels the subprocess $\gamma^* + q \rightarrow q + g$. The invariants s, t ,

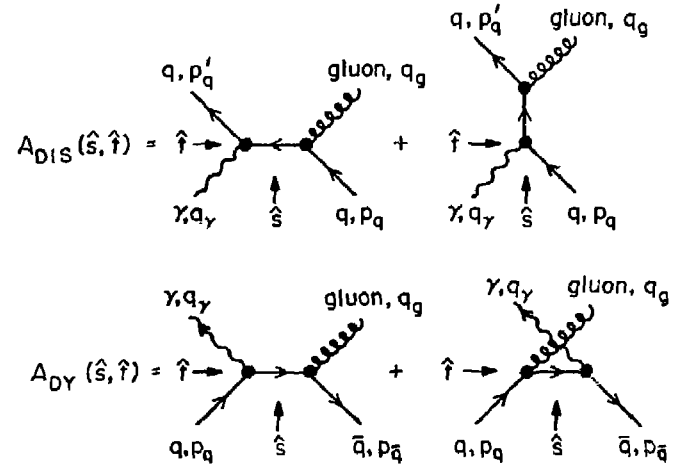


Fig. 12. Real gluon emission diagrams of order $\alpha_s(Q)$ that produce corrections to deep inelastic scattering (DIS) and to the "Drell-Yan" production of large mass muon pairs (DY).

and $\hat{u}$ are defined by

$$\hat{s} = (q_\gamma + p_q)^2, \quad (3.12a)$$

$$\hat{t} = (q_\gamma - p_q')^2, \quad (3.12b)$$

$$\hat{u} = (q_\gamma - q_g)^2, \quad (3.12c)$$

with

$$\hat{s} + \hat{t} + \hat{u} + Q^2 = 0, \quad (3.13)$$

and

$$Q^2 = -q_\gamma^2, \quad (3.14)$$

where the 4-momentums of the initial quark, p_q , final quark, p_q' , gluon, q_g , and virtual photon, q_γ are shown in Fig. 12.

Integrating (3.11) over $\hat{t}$ gives

$$\hat{\sigma}_2^g(\hat{s}) = \int_{\hat{t}_{\min}}^{\hat{t}_{\max}} \frac{d\hat{\sigma}_2^g}{d\hat{t}}(\hat{s}, \hat{t}) d\hat{t}, \quad (3.15)$$

where

$$\hat{t}_{\min} = 0, \quad (3.16a)$$

$$\hat{t}_{\max} = -(\hat{s} + Q^2). \quad (3.16b)$$

As was the e^+e^- case in (2.37) this integral is divergent and we cannot proceed without choosing a regularization scheme. The

differential cross section in (3.11) becomes infinite as $\hat{t} \rightarrow 0$ and this vanishing of $\hat{t}$ occurs for the same reason as shown in (2.38) (i.e. $\omega \rightarrow 0$ or $\theta_{23} \rightarrow 0$).

3. Initial State Gluon Corrections

One must also correct the naive parton model by including the possibility that a gluon in the initial proton can produce a quark-antiquark pair which the virtual photon then couples to as shown in Fig. 13. The differential cross section for the subprocess $\gamma^* + g \rightarrow q + \bar{q}$ shown in Fig. 13 is

$$\frac{d\hat{\sigma}_2^g}{d\hat{t}}(\hat{s}, \hat{t}) = \frac{4\pi\alpha_s e_q^2}{(s+Q^2)^2} \left(\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} + \frac{2Q^2}{tu} (\hat{t} + \hat{u} + Q^2) \right), \quad (3.17)$$

where in this case

$$\hat{s} = (q_\gamma + q_g)^2, \quad (3.18a)$$

$$\hat{t} = (p_q - q_\gamma)^2, \quad (3.18b)$$

$$\hat{u} = (p_q - q_g)^2, \quad (3.18c)$$

with (3.13) and (3.14) also holding. As in (3.11) the subscript Σ is to signify that the polarization states of the incoming virtual photon have been summed using (2.15). The superscript g labels the subprocess $\gamma^* + g \rightarrow q + \bar{q}$. This cross section diverges as $\hat{t} \rightarrow 0$ or $\hat{u} \rightarrow 0$ and as in (3.15) the integral

$$\hat{\sigma}_2^g(\hat{s}) = \int_{\hat{t}_{\min}}^{\hat{t}_{\max}} \frac{d\hat{\sigma}_2^g}{d\hat{t}}(\hat{s}, \hat{t}) d\hat{t} \quad (3.19)$$

is infinite since $\hat{t}_{\min}, \hat{t}_{\max}$ are given by (3.16). Again the origin of these divergences are the same as in (2.37) and we cannot proceed without choosing some scheme for regularizing the infrared singularities.

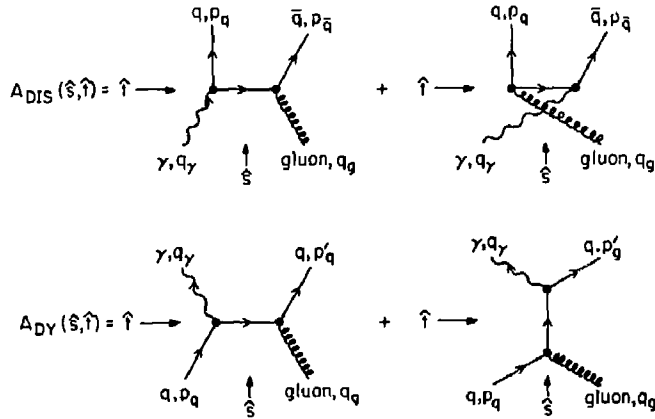


Fig. 13. Diagrams of order $\alpha_s(Q)$ that produce corrections to deep inelastic scattering (DIS) and to the "Drell-Yan" production of large mass muon pairs (DY). These diagrams produce corrections that are proportional to the probability of finding gluons with the initial hadron.

4. Order α_s Corrections - Massive Gluon Scheme

We can regulate the divergences in the subprocess $\gamma^* q \rightarrow q g$ by giving the gluon a fictitious mass $m_g^2 = m_K^2$ as we did in the e^+e^- case in Section II.A.4. The differential cross section in (3.11) becomes

$$\frac{d\hat{\sigma}_\Sigma^q}{dt}(\hat{s}, \hat{t}) = \frac{\pi\alpha_s^2}{(s+Q^2)^2} \left(\frac{8}{3}\right) \left(-\frac{1}{s} - \frac{1}{t} + \frac{2Q^2(s+t+Q^2)}{st} - \frac{m_K^2 Q^2}{t^2} - \frac{m_K^2 Q^2}{s^2}\right), \quad (3.20)$$

where terms that contribute nothing to the integral, $\hat{\sigma}_\Sigma^q$, in the limit $m_K \rightarrow 0$ have been dropped. Integrating over $\hat{t}$ with $\hat{t}_{\min}$, $\hat{t}_{\max}$ now given by

$$\hat{t}_{\min} = -\beta z Q^2 / (1-z), \quad (3.21a)$$

$$\hat{t}_{\max} = -Q^2/z + \beta Q^2, \quad (3.21b)$$

gives

$$\begin{aligned} \hat{\sigma}_{\text{MC}, \Sigma}^q(z, Q^2) &= \frac{\pi\alpha_s^2}{Q^2} \left(\frac{8}{3}\right) z \left\{ \frac{(1+z^2)}{1-z} \log\left[\frac{Q^2(1-z)}{m_K^2 z^2}\right] \right. \\ &\quad \left. - \frac{3}{2} \frac{1}{1-z} + z + 1 + \beta \frac{2z^3 - z^2}{(1-z)^2} + \frac{1}{2} \beta^2 \frac{2z^3 - z^4}{(1-z)^3} \right\}, \quad (3.22) \end{aligned}$$

where the label "MC" refers to the massive gluon scheme and where

$$\beta = m_K^2/Q^2, \quad (3.23)$$

and

$$z = \frac{Q^2}{2p_q \cdot q_\gamma} = \frac{Q^2}{(s+Q^2)}. \quad (3.24)$$

This parton subprocess must be "embedded" in the experimentally observed reaction $\gamma^* + p \rightarrow X$ as shown in Fig. 14a. Namely,

$$d\sigma(x) = G_{p-q}^{(0)}(y) dy \left(\frac{d\sigma_q}{dz} \right) dz, \quad (3.25)$$

where $G_{p-q}^{(0)}(y)dy$ is the probability of finding a quark with momentum

$$p_q = yP, \quad (3.26)$$

and $(d\sigma_q/dz)dz$ is the cross section for scattering with a value of z given by (3.24). Defining x as in (3.9b),

$$x = \frac{Q^2}{2p \cdot q_\gamma}, \quad (3.27)$$

we see that

$$z = \kappa/y, \quad (3.28)$$

and

$$\frac{1}{\sigma_0} \frac{d\sigma}{dx} = \int_x^1 \frac{dy}{y} G_{p-q}^{(0)}(y) \left(\frac{1}{\sigma_0} \frac{d\sigma_q}{dz} \right), \quad (3.29)$$

with the limits of integration coming from (3.28) with $z \leq 1$.

The structure function $\mathcal{F}_2(x, Q^2)$ is related to the total $\gamma^* q$ cross section in (3.22) as follows:

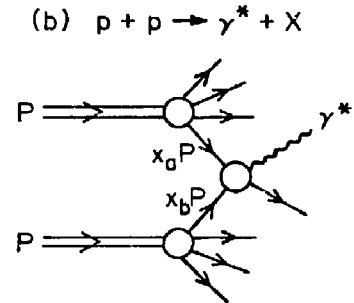
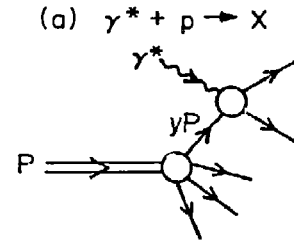


Fig. 14. (a) Illustrates how the subprocess $\gamma^* + a \rightarrow b + c$ where a , b and c are constituents (quarks or gluons) is "embedded" within the experimentally measured process $\gamma^* + p \rightarrow X$, where p is a proton and y is the fraction of momentum of the proton carried by the constituent a . (b) Illustrates how the constituent subprocess $a + b \rightarrow \gamma^* + c$ is "embedded" with the observed process $pp \rightarrow \gamma^* + X$. The virtual photon γ^* then "decays" into a muon pair. The quantities x_a and x_b are the fractional momenta carried by constituents (quarks or gluons) a and b , respectively.

$$\mathcal{F}_2(x, Q^2) = \int_x^1 \frac{dy}{y} G_{p \rightarrow q}(y) \left(\frac{Q^2}{8\pi^2 \alpha z} \right) \hat{\sigma}_2^q(z, Q^2), \quad (3.30)$$

where

$$\mathcal{F}_2(x, Q^2) = F_2(x, Q^2)/x. \quad (3.31)$$

This structure function is related to the two structure functions in (3.7) by

$$\mathcal{F}_2(x, Q^2) = \frac{3}{2} \mathcal{F}_1(x, Q^2) - \frac{1}{2} \mathcal{F}_2(x, Q^2). \quad (3.32)$$

From (3.29) and (3.30) we see that

$$\frac{1}{\sigma_0} \frac{d\hat{\sigma}_2^q}{dz} = \left(\frac{Q^2}{8\pi^2 \alpha z} \right) \hat{\sigma}_2^q(z, Q^2), \quad (3.33)$$

where σ_0 is given by (2.17). This gives

$$\left(\frac{1}{\sigma_0} \frac{d\hat{\sigma}_{\text{MC}}^q}{dz} \right)_{\text{DIS}} = \left(\frac{2\alpha_s}{3\pi} \right) \left\{ \frac{1+z^2}{1-z} \log\left(\frac{(1+z)Q^2}{z^2 m_g^2} \right) - \frac{3}{2} \frac{1}{1-z} + z + 1 + \frac{5}{4} \delta(1-z) \right\}, \quad (3.34)$$

where DIS refers to deep inelastic scattering and where the β and β^2 terms in (3.22) have been replaced by a δ -function contribution similar to what we did in (2.79).

It is interesting to compare this with the e^+e^- result in (2.148) which is

$$\left(\frac{1}{\sigma_0} \frac{d\hat{\sigma}_{\text{MC}}^q}{dx} \right)_{e^+e^-} = \left(\frac{2\alpha_s}{3\pi} \right) \left\{ \frac{1+x^2}{1-x} \log\left(\frac{x(1-x)Q^2}{m_g^2} \right) - \frac{3}{2} \frac{1}{1-x} \right.$$

$$\left. + \frac{1}{2} x + \frac{1}{2} + \frac{5}{4} \delta(1-x) \right\}. \quad (3.35)$$

Equations (3.34) and (3.35) are very similar but not quite the same. The integral over x of (3.35) was accomplished using (2.80) with the result in (2.81) that

$$(\hat{\sigma}_{\text{MC}}^q(\text{real}))_{e^+e^-} = \left(\frac{2\alpha_s}{3\pi} \right) \sigma_0 \{ \log^2(\beta) + 3 \log(\beta) - \frac{\pi^2}{3} + 5 \}. \quad (3.36)$$

The integration of (3.34) can be performed using (2.80) and yields a slightly different result

$$(\hat{\sigma}_{\text{MC}}^q(\text{real}))_{\text{DIS}} = \left(\frac{2\alpha_s}{3\pi} \right) \sigma_0 \{ \log^2(\beta) + 3 \log(\beta) + \frac{2\pi^2}{3} + 2 \}. \quad (3.37)$$

with β given by (3.23) and $Q^2 = -q^2 > 0$ (spacelike). The virtual gluon contributions in Fig. 15 are given by (2.95)

$$(\hat{\sigma}_{\text{MC}}^q(\text{virtual}))_{\text{DIS}} = \left(\frac{2\alpha_s}{3\pi} \right) \sigma_0 \{ -\log^2(\beta) - 3 \log(\beta) - \frac{7}{2} - \frac{2\pi^2}{3} \}, \quad (3.38)$$

and, as was the e^+e^- case, the total

$$\begin{aligned} (\hat{\sigma}_{\text{MC}}^q(\text{real}) + \hat{\sigma}_{\text{MC}}^q(\text{virtual}))_{\text{DIS}} &= \left(\frac{2\alpha_s}{3\pi} \right) \sigma_0 \left\{ \frac{2\pi^2}{3} + 2 - \frac{7}{2} - \frac{2\pi^2}{3} \right\} \\ &= \left(\frac{2\alpha_s}{3\pi} \right) \sigma_0 \left(-\frac{3}{2} \right) = -\frac{\alpha_s}{\pi} \sigma_0. \end{aligned} \quad (3.39)$$

is finite and independent of β as $\beta \rightarrow 0$. Equation (3.39), however, has the opposite sign from the e^+e^- case in (2.98). In this case the perturbation series is of the form $\sigma_0 \{ 1 - \alpha_s/\pi + \dots \}$ rather than

$\sigma_0 \{ 1 + \alpha_s/\pi + \dots \}$ as in e^+e^- annihilations. In e^+e^- annihilations of the final state gluon intersections are attractive (color singlet) causing the total cross section to increase. In deep inelastic scattering this is not the case and the order α_s corrections reduce the total cross section.

Now that we know that

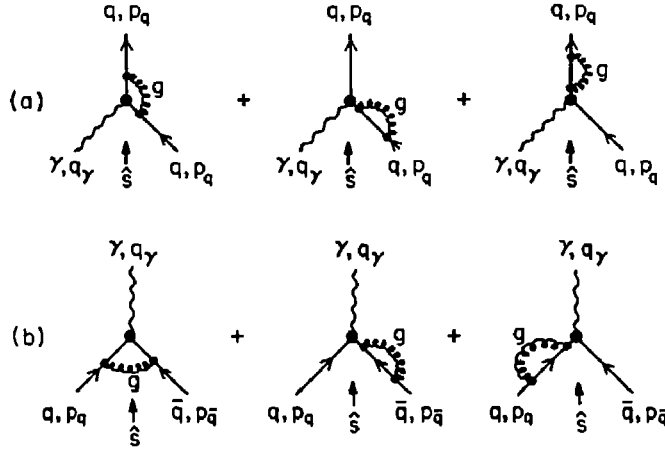


Fig. 15. Virtual gluon corrections to the deep inelastic scattering subprocess $\gamma^* + q \rightarrow q$ (a) and to the Drell-Yan subprocess $q + \bar{q} \rightarrow \gamma^*$ (b). These diagrams interfere with the order zero "Born diagram" producing effects of order $\alpha_s(Q)$.

$$\int_0^1 \frac{d\hat{\sigma}_{\text{MG},\Sigma}^q}{dz} dz + \hat{\sigma}_{\text{MG}}^q(\text{virtual}) = -\frac{\alpha_s}{\pi} \sigma_0, \quad (3.40)$$

we write

$$\int_0^1 \left(\frac{d\hat{\sigma}_{\text{MG},\Sigma}^q}{dz} + \hat{\sigma}_{\text{MG}}^q(\text{virtual}) + \frac{\alpha_s}{\pi} \sigma_0 \delta(1-z) \right) dz = 0 \quad (3.41)$$

and define "+-functions" just as we did in (2.149) and (2.150). Namely,

$$\frac{1}{\sigma_0} \left(\frac{d\hat{\sigma}_{\text{MG},\Sigma}^q}{dz} \right)_+ = \frac{\alpha_s}{2\pi} P_{q-q}(z) \log(Q^2/m_g^2) + \alpha_s f_{\text{MG},\Sigma}^{q,\text{DIS}}(z), \quad (3.42)$$

where

$$P_{q-q}(z) = \frac{4}{3} \left(\frac{1+z^2}{1-z} \right)_+, \quad (3.43)$$

is the same function as (2.154) and

$$\alpha_s f_{\text{MG},\Sigma}^{q,\text{DIS}}(z) = \left(\frac{2\alpha_s}{3\pi} \right) \left\{ (1+z^2) \left(\frac{\log(1-z)}{1-z} \right)_+ + \frac{1+z^2}{1-z} (-2\log(z)) \right.$$

$$\left. - \frac{3}{2} \frac{1}{(1-z)_+} + z + 1 - \left(\frac{\gamma_E^2}{3} + \frac{3}{4} \right) \delta(1-z) \right\}. \quad (3.44)$$

As before the "little-f" function in (3.42) depends on the process and the regularization scheme chosen. However since (3.41) must hold in any regularization scheme, the integral

$$\int_0^1 \alpha_s f_{\Sigma}^{q,\text{DIS}}(z) dz = 0 \quad (3.45)$$

is valid regardless of scheme. From (3.30) the $\mathcal{F}_2$ structure function becomes

$$\mathcal{F}_2^q(x, Q^2) = e_q^2 \int_x^1 \frac{dy}{y} G_{p-q}^{(0)}(y) \left\{ \left(1 - \frac{\alpha_s}{\pi}\right) \delta(1-z) + \frac{\alpha_s}{2\pi} P_{q-q}(z) \log(Q^2/m_g^2) + \alpha_s f_{HC, \Sigma}^{q, DIS}(z) \right\}, \quad (3.46)$$

where the superscript q signifies that this is the contribution to $\mathcal{F}_2$ from the subprocess $\gamma^* + q \rightarrow q + \gamma$. Equation (3.46) is analogous to (2.174) for the fragmentation functions.

The differential cross section for scattering longitudinal photons via the subprocess $\gamma_L^* + q \rightarrow q + \gamma$ is

$$\frac{d\sigma_L^q}{dt} = \frac{w\alpha}{(s+Q^2)^2} \frac{\alpha_s}{2} \left(\frac{4}{3}\right) (Q^2 + \hat{t} + \hat{s}). \quad (3.47)$$

This cross section is finite when integrated over $\hat{t}$ giving

$$\hat{\sigma}_L^q(z, Q^2) = w\alpha \frac{\alpha_s}{2} \left(\frac{4}{3}\right) e_q^2. \quad (3.48)$$

Here there is no need to label by a regularization scheme since no divergences were encountered.

The longitudinal structure function $\mathcal{F}_L(x, Q^2)$ is related to the total $\gamma_L^* q$ cross section in (3.48) according to

$$\mathcal{F}_L(x, Q^2) = \int_x^1 \frac{dy}{y} G_{p-q}(y) \left(\frac{-z}{y\alpha}\right) \hat{\sigma}_L^q(z, Q^2). \quad (3.49)$$

where

$$\mathcal{F}_L(x, Q^2) = F_L(x, Q^2)/x. \quad (3.50)$$

This structure function is related to the two structure functions in (3.7) by

$$\mathcal{F}_2(x, Q^2) = \mathcal{F}_2(x, Q^2) - \mathcal{F}_1(x, Q^2). \quad (3.51a)$$

and

$$\mathcal{F}_L(x, Q^2) = \mathcal{F}_2(x, Q^2) - \frac{3}{2} \mathcal{F}_1(x, Q^2). \quad (3.51b)$$

From (3.49) and (3.48) we see that

$$\mathcal{F}_L^q(x, Q^2) = e_q^2 \int_x^1 \frac{dy}{y} G_{p-q}^{(0)}(y) [\alpha_s f_L^{q, DIS}(z)]. \quad (3.52)$$

with

$$\alpha_s f_L^{q, DIS}(z) = \left(\frac{2\alpha_s}{3\pi}\right) 2z. \quad (3.53)$$

where the superscript q on $\mathcal{F}_L$ indicates that this is the contribution to the longitudinal structure function from the subprocess $\gamma^* + q \rightarrow q + \gamma$. Unlike (3.44) this "little f" function here does not depend on the regularization scheme. Integrating (3.53) over z gives

$$\int_0^1 \alpha_s f_L^{q, DIS}(z) dz = \frac{2\alpha_s}{3\pi}. \quad (3.54)$$

From (3.41b) we have

$$\mathcal{F}_2(x, Q^2) = \mathcal{F}_2(x, Q^2) + \frac{3}{2} \mathcal{F}_L(x, Q^2). \quad (3.55)$$

so that the contribution to the structure function $\mathcal{F}_2$ from the subprocesses $\gamma^* + q \rightarrow q$ and $\gamma^* + q \rightarrow q + q$ is

$$\mathcal{F}_2^q(x, Q^2) = e_q^2 \int_x^1 \frac{dy}{y} C_{p \rightarrow q}^{(0)}(y) \{ \delta(1-z) + \frac{\alpha_s}{2\pi} P_{q \rightarrow q}(z) \log(Q^2/m_g^2) + \alpha_s f_{\text{MG},2}^{\text{q,DIS}}(z) \}, \quad (3.56)$$

where

$$\alpha_s f_{\text{MG},2}^{\text{q,DIS}}(z) = \left(\frac{2\alpha_s}{3\pi}\right) \left\{ (1+z^2) \left(\frac{\log(1-z)}{1-z} \right)_+ + \frac{1+z^2}{1-z} (-2\log(z)) - \frac{3}{2} \frac{1}{(1-z)_+} + 4z+1 - \left(\frac{2z^2}{3} + \frac{9}{4} \right) \delta(1-z) \right\}. \quad (3.57)$$

This "little-f" function is scheme dependent but the integral

$$\int_0^1 \alpha_s f_{\text{MG},2}^{\text{q,DIS}}(z) dz = 0 \quad (3.58)$$

is not.

For the initial state gluon subprocess $\gamma^* + g \rightarrow q + \bar{q}$ in Fig. 13 we cannot regularize by taking $q_g^2 = m_g^2$ because then the incoming gluon could actually decay into a massless quark-antiquark pair. To regulate this process we take the incoming gluon slightly off-shell and spacelike, $q_g^2 = -m_g^2$. In this case the differential cross section in (3.17) becomes

$$\frac{d\hat{\sigma}_2^g}{dz} = \frac{\pi \alpha_s \alpha_e^2}{(s+Q^2)^2} \left\{ \frac{\hat{u}}{t} + \frac{\hat{t}}{u} + \frac{2Q^2}{tu} (t+u+Q^2) \right\}$$

$$+ \frac{2m_g^2}{tu} \left(\frac{\hat{t}+u+m_g^2}{t} - Q^2 m_g^2 \left(\frac{1}{u} + \frac{1}{t} - \frac{4}{tu} \right) \right). \quad (3.59)$$

and $\hat{t}_{\min}, \hat{t}_{\max}$ in (3.16) become

$$\hat{t}_{\min} = \hat{u}_{\min} = -m_g^2 z, \quad (3.60a)$$

$$\hat{t}_{\max} = \hat{u}_{\max} = -Q^2/z, \quad (3.60b)$$

where z is defined in (3.24) and where some terms that vanish as $m_g^2 \rightarrow 0$ have been dropped. Integrating (2.59) over $\hat{t}$ gives

$$\hat{\sigma}_{\text{MG},2}^g(z, Q^2) = \frac{2\pi \alpha_s \alpha_e^2}{Q^2} z \left\{ (z^2 + (1-z)^2) \log\left(\frac{Q^2}{z^2 m_g^2}\right) - 2 \right\}. \quad (3.61)$$

or from (3.33)

$$\left(\frac{1}{\sigma_0} \frac{d\hat{\sigma}_{\text{MG}}^g}{dz} \right)_{\text{DIS}} = 2 \left\{ \frac{\alpha_s}{2\pi} P_{g \rightarrow q}(z) \log(Q^2/m_g^2) + \alpha_s f_{\text{MG},2}^{\text{g,DIS}}(z) \right\}, \quad (3.62)$$

with

$$P_{g \rightarrow q}(z) = \frac{1}{2} (z^2 + (1-z)^2), \quad (3.63)$$

and

$$\alpha_s f_{\text{MG},2}^{\text{g,DIS}}(z) \approx - \left(\frac{\alpha_s}{2\pi} \right) [(z^2 + (1-z)^2) \log(z) + 1]. \quad (3.64)$$

The contribution to the structure function $\mathcal{F}_2$ from the subprocess

$\gamma^{*+g-q+\bar{q}}$ is thus

$$\mathcal{F}_2^g(x, Q^2) = 2e_q^2 \int_x^1 \frac{dy}{y} G_{p-g}^{(0)}(y) \left\{ \frac{a_s}{2\pi} P_{g-q}(z) \log(Q^2/m_g^2) + a_s f_{MC,2}^{g,DIS}(z) \right\}. \quad (3.65)$$

The differential cross section for the scattering of longitudinal photons via the subprocess $\gamma_L^{*+g-q+\bar{q}}$ is

$$\frac{d\hat{\sigma}_L^g}{dt} = \frac{\pi \alpha a_s e_q^2}{(s+Q^2)^2} \hat{s}. \quad (3.66)$$

and as in (3.47) one can integrate over $\hat{t}$ without any divergent terms. The result is

$$\hat{\sigma}_L^g(z, Q^2) = \pi a_s \alpha e_q^2 (1-z), \quad (3.67)$$

which from (3.49) gives a contribution to the longitudinal structure function of

$$\mathcal{F}_L^g(x, Q^2) = 2e_q^2 \int_x^1 \frac{dy}{y} G_{p-g}^{(0)}(y) [a_s f_{L,g}^{DIS}(z)] \quad (3.68)$$

with

$$a_s f_{L,g}^{DIS}(z) = \left(\frac{a_s}{2\pi}\right) 2z(1-z), \quad (3.69)$$

and a contribution to the structure function $\mathcal{F}_2$ in (3.55) of the form

$$\mathcal{F}_2^g(x, Q^2) = 2e_q^2 \int_x^1 \frac{dy}{y} G_{p-g}^{(0)}(y)$$

$$\left\{ \left(\frac{a_s}{2\pi}\right) P_{g-q}(z) \log(Q^2/m_g^2) + a_s f_{MC,2}^{g,DIS}(z) \right\}, \quad (3.70)$$

where $P_{g-q}(z)$ is given by (2.63) and

$$a_s f_{MC,2}^{g,DIS}(z) = \left(\frac{a_s}{2\pi}\right) [-z^2 + (1-z)^2] \log(z) - 1 + 3z - 3z^2. \quad (3.71)$$

If we combine (3.70) with (3.56) we get the contribution to the $\mathcal{F}_2$ structure function from the subprocesses γ^{*+q-q} , $\gamma^{*+q-g+q}$, $\gamma^{*+q-g+\bar{q}}$, and $\gamma^{*+g-q+\bar{q}}$.

$$\mathcal{F}_2(x, Q^2) = e_q^2 \int_x^1 \frac{dy}{y} (G_{p-q}^{(0)}(y) + G_{p-q}^{(0)}(y))$$

$$\{b(1-z) + \left(\frac{a_s}{2\pi}\right) P_{q-q}(z) \log(Q^2/m_g^2) + a_s f_{MC,2}^{q,DIS}(z)\}$$

$$+ 2 e_q^2 \int_0^1 \frac{dy}{y} G_{p-g}^{(0)}(y)$$

$$\left\{ \left(\frac{a_s}{2\pi}\right) P_{g-q}(z) \log(Q^2/m_g^2) + a_s f_{MC,2}^{g,DIS}(z) \right\}. \quad (3.72)$$

5. Order α_s Corrections - Dimensional Regularization Scheme

We can regulate the divergences in the two-to-two scattering subprocess $\gamma^{*+q-g+q}$ and $\gamma^{*+g-q+\bar{q}}$ by considering the scatterings to occur in N rather than 4 spacetime dimensions. In N dimensions the two-to-two cross section in Fig. 16 has the form

$$\hat{\sigma} = \frac{1}{4(p_1 \cdot p_2)} |A|^2 d^{2N-2} R_2. \quad (3.73)$$

where the two-body phase space factor $d^{2N-2} R_2$ is similar to (2.101).

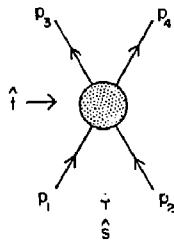


Fig. 16. Two-to-two scattering subprocess.

Namely

$$d^{2N-2}K_2 = \frac{d^{N-1}p_3}{(2\pi)^{N-1}(2E_3)} \frac{d^{N-1}p_4}{(2\pi)^{N-1}(2E_4)} (2\pi)^N \delta^N(p_3+p_4-p_1-p_2). \quad (3.74)$$

Integrating over p_4 yields

$$\int d^{N-1}p_4 \delta^N(p_3+p_4-p_1-p_2) = \delta(E_3+E_4-E_1-E_2). \quad (3.75)$$

Now if we let $y = \cos\theta_{13}$, where θ_{13} is the scattering angle between particles 1 and 3 then

$$d^{N-1}p_3 = \frac{2\pi^{(N-2)/2}}{\Gamma(N/2-1)} dp_3 p_3^{N-2} \{1-y\}^{N/2-2} dy, \quad (3.76)$$

Integrating over the magnitude of p_3 gives

$$\int \frac{1}{4E_3E_4} p_3^{N-2} \delta(E_3+E_4-E_{cm}) dp_3 = \frac{(\hat{p}_{cm}^*)^{N-3}}{4(\hat{s})^{1/2}}. \quad (3.77)$$

where

$$(\hat{p}_{cm}^*)^2 = [s - (m_3+m_4)^2][s - (m_3-m_4)^2]/(4s) \quad (3.78)$$

and

$$p_1 \cdot p_2 = (s)^{1/2} \hat{p}_{cm}^*, \quad (3.79)$$

with

$$\hat{p}_{cm}^2 = [\hat{s} - (m_1 + m_2)^2][\hat{s} - (m_1 - m_2)^2]/(4\hat{s}). \quad (3.80)$$

Combining (3.76) and (3.73) yields

$$\frac{d\hat{\sigma}}{dy}(s, t) = \left(\frac{1}{32\pi s}\right) \frac{(\hat{p}_{cm}')^{N-3}}{\hat{p}_{cm}} |\mathcal{M}|^2 \frac{(1-y)^{N-1}}{2(N-1)!} \dots \quad (3.81)$$

For the subprocess $\gamma^* + q \rightarrow g + q$ we have the following:

$$\hat{p}_{cm} = (\hat{s} + Q^2)/(2(\hat{s})^{1/2}), \quad (3.82a)$$

$$\hat{p}_{cm}' = \frac{1}{2} (\hat{s})^{1/2}, \quad (3.82b)$$

$$\hat{t} = -\frac{1}{2} (\hat{s} + Q^2)(1-y), \quad (3.82c)$$

$$z = Q^2/(\hat{s} + Q^2), \quad (3.82d)$$

$$\hat{s} = (1-z)/(zQ^2). \quad (3.82e)$$

where $Q^2 = -q_\gamma^2$. The integral of (3.81) over y is given by

$$\hat{\sigma}_{DR}^q(z, Q^2) = \left(\frac{z}{32\pi Q^2}\right) \left(\frac{(1-z)Q^2}{z^4\pi m_D^2}\right)^{\epsilon/2} \frac{1}{2^\epsilon \Gamma(1+\epsilon/2)}. \quad (3.83a)$$

where

$$I = \int_{-1}^1 dy (1-y^2)^{\epsilon/2} |\mathcal{M}(\gamma^* + q \rightarrow g + q)|^2. \quad (3.83b)$$

with $N=4+\epsilon$. In $N=4+\epsilon$ dimensions the matrix element squared is given

by

$$|\mathcal{M}_S(\gamma^* + q \rightarrow g + q)|^2 = (16\pi^2 \alpha_s^2 \epsilon_q^2) \frac{1}{6} \left\{ \frac{1}{(1-y)(1-z)} \{ 1(1-z^2 + 4yz - 4z + y^2 - 2y + 5) \right. \\ \left. + \frac{1(1z^2 - 4z + y^2 + 3)}{(1-y)(1-z)} \epsilon + \frac{4z^2 + (-4y-4)z + y^2 + 2y + 1}{(1-y)(1-z)} \epsilon^2 \right\}. \quad (3.84)$$

To evaluate the integral I in (3.83) we use

$$\int_{-1}^1 (1-y^2)^A \frac{y^B}{1-y} dy = \frac{\Gamma(A)\Gamma(\frac{B}{2} + 1)}{\Gamma(A + \frac{1}{2} B + 1)} \quad (B\text{-odd}) \\ = \frac{\Gamma(A)\Gamma(\frac{1}{2} B + \frac{1}{2})}{\Gamma(A + \frac{1}{2} B + \frac{1}{2})} \quad (B\text{-even}). \quad (3.85)$$

The integrated cross section (3.83) becomes

$$\hat{\sigma}_{DR, S}^q(z, Q^2) = \left(\frac{16\pi\alpha_s^2 \epsilon_q^2}{3Q^2}\right) \left(\frac{Q^2(1-z)}{z^4\pi m_D^2}\right)^{\epsilon/2} \frac{\Gamma(1 + \frac{\epsilon}{2})}{\Gamma(1+\epsilon)} \\ \left\{ \frac{1+z^2}{1-z} \frac{2}{\epsilon} + \frac{4z^2 - 6z + 5}{2(1-z)} + \frac{(z^2 + 2z - 1)}{2(1-z)} \epsilon + O(\epsilon^2) \right\}, \quad (3.86)$$

and using (3.33) we get

$$\left(\frac{1}{\sigma_0} \frac{d\hat{\sigma}_{DR, S}^q}{dz}\right)_{DIS} = \left(\frac{2\alpha_s}{3\pi}\right) \left(\frac{Q^2(1-z)}{z^4\pi m_D^2}\right)^{\epsilon/2} \frac{\Gamma(1 + \frac{\epsilon}{2})}{\Gamma(1+\epsilon)} \\ \left\{ \frac{1+z^2}{1-z} \frac{2}{\epsilon} - \frac{3}{2} \frac{1}{1-z} - z + 3 + O(\epsilon) \right\}. \quad (3.87)$$

where σ_0 is given by (2.110). For the e^+e^- case in (2.160) we have

$$\left(\frac{1}{\sigma_0} \frac{d\sigma_{DR}}{dx}\right)_{e^+e^-} = \left(\frac{2\alpha_s}{3\pi}\right) \left(\frac{Q^2}{4\pi m_D^2}\right)^{\epsilon/2} \frac{\Gamma(1 + \frac{\epsilon}{2})}{\Gamma(1+\epsilon)}$$

$$\left\{\frac{1+x^2}{1-x} \frac{2}{\epsilon} - \frac{3}{2} \frac{1}{1-x} - \frac{3}{2} x + \frac{5}{2} + O(\epsilon)\right\}, \quad (3.88)$$

which is similar but not the same. If we expand the Γ functions in (3.87) according to

$$\frac{\Gamma(1 + \frac{1}{2}\epsilon)}{\Gamma(1+\epsilon)} = 1 + \frac{1}{2} \gamma_E \epsilon + \frac{1}{16} (2\gamma_E^2 - \pi^2) \epsilon^2 + \dots, \quad (3.89)$$

and use (2.127) we arrive at

$$\begin{aligned} \left(\frac{1}{\sigma_0} \frac{d\sigma_{DR,\Sigma}^q}{dz}\right)_{DIS} &= \left(\frac{2\alpha_s}{3\pi}\right) \left\{\frac{1+z^2}{1-z} \left[\log\left(\frac{Q^2(1-z)}{z^2 4\pi m_D^2}\right) + \gamma_E\right] \right. \\ &\quad \left. + \frac{1+z^2}{1-z} \frac{2}{\epsilon} - \frac{3}{2} \frac{1}{1-z} - z+3\right\}, \end{aligned} \quad (3.90)$$

where terms that vanish as $\epsilon \rightarrow 0$ have been dropped.

Integrating (3.86) over z yields

$$\begin{aligned} (\hat{\sigma}_{DR}(\text{real}))_{DIS} &= \left(\frac{2\alpha_s}{3\pi}\right) \sigma_0 \left(\frac{Q^2}{4\pi m_D^2}\right)^{\epsilon/2} \frac{\Gamma(1 - \frac{\epsilon}{2}) \Gamma^2(1 + \frac{\epsilon}{2})}{\Gamma(1+\epsilon)} \\ &\quad \left(\frac{8}{\epsilon^2} - \frac{6}{\epsilon} + \frac{13}{2}\right). \end{aligned} \quad (3.91)$$

The virtual gluon contributions in Fig. 15a are given by (2.140). Namely,

$$(\hat{\sigma}_{DR}(\text{virtual}))_{DIS} = \left(\frac{2\alpha_s}{3\pi}\right) \sigma_0 \left(\frac{Q^2}{4\pi m_D^2}\right)^{\epsilon/2} \frac{\Gamma(1 - \frac{\epsilon}{2}) \Gamma^2(1 + \frac{\epsilon}{2})}{\Gamma(1+\epsilon)} \left(-\frac{8}{\epsilon^2} + \frac{6}{\epsilon} - 8\right), \quad (3.92)$$

so that

$$\begin{aligned} (\hat{\sigma}_{DR}(\text{real}) + \hat{\sigma}_{DR}(\text{virtual}))_{DIS} &= \left(\frac{2\alpha_s}{3\pi}\right) \sigma_0 \left\{\frac{13}{2} - 8\right\} \\ &= \left(\frac{2\alpha_s}{3\pi}\right) \sigma_0 \left(-\frac{3}{2}\right) = -\frac{a_s}{\pi} \sigma_0. \end{aligned} \quad (3.93)$$

This is the same result arrived at for the massive gluon scheme in (3.39)! If we define "+-functions" as in (3.41) we get

$$\frac{1}{\sigma_0} \left(\frac{d\sigma_{DR,\Sigma}^q}{dz}\right)_+ = \frac{\alpha_s}{2\pi} P_{q-q}(z) \log(Q^2/m_D^2) + \alpha_s f_{DR,\Sigma}^{q,DIS}(z), \quad (3.94)$$

where $P_{q-q}(z)$ is given by (3.43) and

$$\begin{aligned} \alpha_s f_{DR,\Sigma}^{q,DIS}(z) &= \left(\frac{2\alpha_s}{3\pi}\right) \left\{((1+z^2) \left(\frac{\log(1-z)}{1-z}\right)_+ - \frac{1+z^2}{1-z} \log(z) \right. \\ &\quad \left. - \frac{3}{2} \frac{1}{(1-z)_+} - z + 3 - \left(\frac{\pi^2}{3} + 3\right) b(1-z)\right\} \\ &\quad + \frac{a_s}{2\pi} \Gamma_{q-q}(z) \left\{\frac{2}{\epsilon} + \gamma_E - \log(4\pi)\right\}, \end{aligned} \quad (3.95)$$

with the integral of (3.95) giving zero as in (3.45). The regularization scheme dependence of this "little- f " function can again be clearly seen by comparing (3.95) with (3.44). In the dimensional regularization scheme the $\mathcal{F}_X$ structure function in (3.46) becomes

$$\begin{aligned} \mathcal{F}_{\Sigma}^q(x, Q^2) &= e_q^2 \int_x^1 \frac{dy}{y} G_{p-q}^{(0)}(y) \left\{ \left(1 - \frac{\alpha_s}{\pi}\right) \delta(1-z) \right. \\ &\quad \left. + \frac{\alpha_s}{2\pi} P_{q-q}(z) \log(Q^2/m_D^2) + \alpha_s f_{DR, \Sigma}^{q, DIS}(z) \right\}. \end{aligned} \quad (3.96)$$

with $f_{DR, \Sigma}^{q, DIS}(z)$ given by (3.95). The longitudinal structure function in (3.52) is scheme independent so

$$\begin{aligned} \mathcal{F}_2^q(x, Q^2) &= e_q^2 \int_x^1 \frac{dy}{y} G_{p-q}^{(0)}(y) \left\{ \delta(1-z) \right. \\ &\quad \left. + \frac{\alpha_s}{2\pi} P_{q-q}(z) \log(Q^2/m_D^2) + \alpha_s f_{DR, 2}^{q, DIS}(z) \right\}. \end{aligned} \quad (3.97)$$

where

$$\begin{aligned} \alpha_s f_{DR, 2}^{q, DIS}(z) &= \frac{2\alpha_s}{(3\pi)} \left\{ (1+z^2) \left(\frac{\log(1-z)}{1-z} \right)_+ - \frac{1+z^2}{1-z} \log(z) \right. \\ &\quad \left. - \frac{3}{2} \frac{1}{(1-z)_+} + 2z + 3 - \left(\frac{\pi^2}{3} + \frac{9}{2} \right) \delta(1-z) \right\} \\ &\quad + \frac{\alpha_s}{2\pi} P_{q-q}(z) \left\{ \frac{2}{\epsilon} + \gamma_E - \log(4\pi) \right\}. \end{aligned} \quad (3.98)$$

The subprocess $\gamma^* + g + q + \bar{q}$ is handled in a similar manner and the contribution to the $\mathcal{F}_2$ structure function in (3.65) is

$$\begin{aligned} \mathcal{F}_{\Sigma}^g(x, Q^2) &= 2e_q^2 \int_x^1 \frac{dy}{y} G_{p-q}^{(0)}(y) \left\{ \frac{\alpha_s}{2\pi} P_{g-q}(z) \log(Q^2/m_D^2) + \alpha_s f_{DR, \Sigma}^{g, DIS}(z) \right\}. \end{aligned} \quad (3.99)$$

where

$$\begin{aligned} f_{DR, \Sigma}^{g, DIS}(z) &= \left(\frac{\alpha_s}{2\pi} \right) \left\{ \frac{1}{2} (z^2 + (1-z)^2) \log\left(\frac{1-z}{z}\right) \right. \\ &\quad \left. + \left(\frac{\alpha_s}{2\pi} \right) P_{g-q}(z) \left\{ \frac{2}{\epsilon} + \gamma_E - \log(4\pi) \right\} \right\}. \end{aligned} \quad (3.100)$$

The contribution to the longitudinal structure function in (3.68) and (3.69) is the same since no divergences are encountered, therefore (3.70) becomes

$$\begin{aligned} \mathcal{F}_2^g(x, Q^2) &= 2e_q^2 \int_x^1 \frac{dy}{y} G_{p-g}^{(0)}(y) \\ &\quad \left\{ \frac{\alpha_s}{2\pi} P_{g-q}(z) \log(Q^2/m_D^2) + \alpha_s f_{DR, 2}^{g, DIS}(z) \right\}, \end{aligned} \quad (3.101)$$

with

$$\begin{aligned} \alpha_s f_{DR, 2}^{g, DIS}(z) &= \left(\frac{\alpha_s}{2\pi} \right) \left\{ \frac{1}{2} (z^2 + (1-z)^2) \log\left(\frac{1-z}{z}\right) + 3z - 3z^2 \right\} \\ &\quad + \left(\frac{\alpha_s}{2\pi} \right) P_{g-q}(z) \left\{ \frac{2}{\epsilon} + \gamma_E - \log(4\pi) \right\}, \end{aligned} \quad (3.102)$$

and (3.72) becomes

$$\begin{aligned} \mathcal{F}_2(x, Q^2) &= e_q^2 \int_x^1 \frac{dy}{y} \left\{ G_{p-q}^{(0)}(y) + G_{p-q}^{(0)}(y) \right\} \\ &\quad \left\{ \delta(1-z) + \frac{\alpha_s}{2\pi} P_{q-q}(z) \log(Q^2/m_D^2) + \alpha_s f_{DR, \Sigma}^{q, DIS}(z) \right\} \end{aligned}$$

$$+ 2e_q^2 \int_0^1 \frac{dy}{y} G_{p-q}^{(0)}(y) \\ \left(\frac{\alpha_s}{2\pi} P_{g-q}(z) \log(Q^2/m_0^2) + \alpha_s f_{DR,2}^{g,DIS}(z) \right). \quad (3.103)$$

6. Q^2 Evolution of the Quark and Gluon Distributions - Leading Log Sum

The quark distribution are defined from the $\mathcal{F}_2$ structure function as in (3.8). Namely,

$$\mathcal{F}_2(x, Q^2) \equiv \sum_{f=1}^{n_f} e_{q_f}^2 (G_{p-q_f}^{(2)}(x, Q^2) + G_{p-q_f}^{(2)}(x, Q^2)), \quad (3.104)$$

where the superscript (2) refers to the $\mathcal{F}_2$ structure function and where n_f is the number of quark flavors. From (3.72) or (3.103) we see that

$$G_{p-q}^{(2)}(x, Q^2) = \int_x^1 \frac{dy}{y} G_{p-q}^{(0)}(y) \{ \delta(1-z) \\ + \frac{\alpha_s}{2\pi} P_{q-q}(z) \log(Q^2/m^2) + \alpha_s f_2^{q,DIS}(z) \} \\ + \int_x^1 \frac{dy}{y} G_{p-g}^{(0)}(y) \left\{ \frac{\alpha_s}{2\pi} P_{g-q}(z) \log(Q^2/m^2) + \alpha_s f_2^{g,DIS}(z) \right\}, \quad (3.105)$$

where $z = x/y$ and where the mass m is the dimensional regularization mass m_0 or the gluon mass m_g depending on the scheme. The "little- t " functions are regularization scheme dependent and are given by (3.57) and (3.71) in the massive glue scheme and by (3.98) and (3.102) in the dimensional regularization scheme. If we define the convolution as in (2.181) then (3.105) becomes

$$G_{p-q}^{(2)}(x, Q^2) = G_{p-q}^{(0)} * \left(1 + \frac{\alpha_s}{2\pi} P_{q-q} \log(Q^2/m^2) + \alpha_s f_2^{q,DIS} \right) \\ + G_{p-g}^{(0)} * \left(\frac{\alpha_s}{2\pi} P_{g-q} \log(Q^2/m^2) + \alpha_s f_2^{g,DIS} \right), \quad (3.106)$$

which is analogous to (2.182) for the fragmentation functions. As illustrated in (2.186) the $\log(m^2)$ divergences are absorbed into the unknown distributions $G_{p-i}^{(0)}$ leaving

$$G_{p-q}^{(2)}(x, Q^2) = \tilde{G}_{p-q}^{(0)} * \left(1 + \frac{\alpha_s}{2\pi} P_{q-q} \log(Q^2/\Lambda^2) + \alpha_s f_2^{q,DIS} \right) \\ + \tilde{G}_{p-g}^{(0)} * \left(\frac{\alpha_s}{2\pi} P_{g-q} \log(Q^2/\Lambda^2) + \alpha_s f_2^{g,DIS} \right). \quad (3.107)$$

All leading log terms of the form $[\alpha_s(Q^2) \log(Q^2)]^n$ are then summed in precisely the same way we did in Section II.B.3. The result is the same as (2.214). Namely

$$G_{NS}(x, Q^2) = \exp(\kappa P_{q-q}) * G_{NS}(Q_0^2) \quad (3.108a)$$

where κ is given by (2.205) (with $C_c^2 = Q_0^2$) and where the "non-singlet" distribution is given by

$$G_{NS}(x, Q^2) = G_{p-q}(x, Q^2) - G_{p-\bar{q}}(x, Q^2). \quad (3.108b)$$

Equation (3.108) allows one to calculate the structure function at Q^2 from knowledge of the structure function at Q_0^2 ($Q^2 > Q_0^2$). Differentiating (3.108) with respect to κ yields

$$dG_{NS}(x, Q^2)/d\kappa = P_{q-q} * G_{NS}(Q^2) \quad (3.109)$$

or from (2.218)

$$\frac{dG_{NS}(x, Q^2)}{d\tau} = \frac{\alpha_s(Q^2)}{2\pi} P_{q \rightarrow q} * G_{NS}(Q^2) \quad (3.110)$$

where

$$\tau = \log(Q^2/\Lambda^2). \quad (3.111)$$

Equation (3.110) is the usual form for the Altarelli-Parisi [5] evolution equation for the non-singlet structure function.

The total number of quarks of flavor q_1 is from (3.2)

$$N_{q_1} = \int_0^1 (G_{p \rightarrow q_1}(x, Q^2) - G_{p \rightarrow \bar{q}_1}(x, Q^2)) dx, \quad (3.112)$$

and since

$$\int_0^1 P_{q \rightarrow q}(z) dz = 0 \quad (3.113)$$

(3.110) implies that

$$dN_{q_1}/d\tau = 0, \quad (3.114)$$

so that N_{q_1} does not depend on Q^2 . If we set N_{q_1} at Q_0^2 to the values

in (3.2) they will remain fixed at higher Q^2 . The net number of quarks within the proton remains constant.

The Q^2 evolution of the $G_{p \rightarrow q}(x, Q^2)$ distribution is governed by

$$dG_{p \rightarrow q}(x, Q^2)/d\tau = \frac{\alpha_s(Q^2)}{2\pi} [G_{p \rightarrow q}(Q^2) * P_{q \rightarrow q} + G_{p \rightarrow g}(Q^2) * P_{g \rightarrow q}], \quad (3.115)$$

which is arrived at by taking the derivative of (3.107) with respect to τ and dropping terms of order α_s^2 (note that $d\alpha_s/d\tau$ is of order α_s^2). The "little- f " functions do not contribute at this order of perturbation theory to the Q^2 evolution equations (3.110) and (3.115).

The gluon distribution evolves according to

$$dG_{p \rightarrow g}(x, Q^2)/d\tau = \frac{\alpha_s(Q^2)}{2\pi} \left[\sum_{j=1}^{2n_f} G_{p \rightarrow q_j}(Q^2) * P_{q \rightarrow g} + G_{p \rightarrow g}(Q^2) * P_{g \rightarrow g} \right]. \quad (3.116)$$

The functions $P_{q \rightarrow g}$ and $P_{g \rightarrow g}$ are calculated in a way similar to $P_{q \rightarrow q}$ and $P_{g \rightarrow q}$ with the result

$$P_{q \rightarrow g}(z) = \left(\frac{4}{3}\right) [1 + (1-z)^2]/z \quad (3.117a)$$

as in (2.167)

$$P_{g \rightarrow g}(z) = 6 \left[\frac{z}{(1-z)_+} + \frac{1-z}{2} + z(1-z) + \frac{(11 - \frac{2}{3}n_f)}{12} \delta(1-z) \right]. \quad (3.117b)$$

After summing the leading log terms equation (3.107) becomes

$$G_{p \rightarrow q}^{(2)}(x, Q^2) = \tilde{G}_{p \rightarrow q}(Q^2) * (1 + \alpha_s f_2^{\text{DIS}}) + \tilde{G}_{p \rightarrow g}(Q^2) * (\alpha_s f_2^{\text{DIS}}), \quad (3.118)$$

where to leading order both $\tilde{G}_{p \rightarrow q}^{(2)}$ and $\tilde{G}_{p \rightarrow g}$ are solutions of the Q^2 evolution equation (3.115) and $\tilde{G}_{p \rightarrow g}$ satisfies (3.116). The "little- f " functions in (3.118) are regularization scheme dependent which means that, at this stage, so is $G_{p \rightarrow q}^{(2)}(x, Q^2)$. However since

$$\int_0^1 \alpha_s f_2^{q,DIS}(z) dz = 0, \quad (3.119)$$

in any scheme, we have

$$\int_0^1 [C_{p-q}^{(2)}(x, Q^2) - \tilde{C}_{p-q}^{(2)}(x, Q^2)] dx = \int_0^1 [\tilde{G}_{p-q}(x, Q^2) - \tilde{G}_{p-q}^-(x, Q^2)] dx. \quad (3.120)$$

This means that the net number of quarks in the proton (3.112) is not affected by the higher order corrections that produced the "little-f" functions. This is a nice feature and it is the reason that we use the $\mathcal{F}_2(x, Q^2)$ observable to define the quark distributions.

We could define "quark distributions" from the $\mathcal{F}_1(x, Q^2)$ structure. Namely,

$$\mathcal{F}_1(x, Q^2) \equiv \sum_{i=1}^{n_f} e_{q_i}^2 (G_{p-q_i}^{(1)}(x, Q^2) + G_{p-q_i}^{(1)}(x, Q^2)). \quad (3.121)$$

where the superscript (1) refers to the $\mathcal{F}_1$ structure function. In this case (3.116) becomes

$$G_{p-q}^{(1)}(x, Q^2) = \tilde{G}_{p-q}(Q^2) * (1 + \alpha_s f_1^{q,DIS}) + \tilde{G}_{p-q}^-(Q^2) * (\alpha_s f_1^{g,DIS}). \quad (3.122)$$

where, to leading order, $G_{p-q}^{(1)}(x, Q^2)$ also satisfies (3.115). The "little-f" functions are again scheme dependent and are given by

$$\alpha_s f_1^{DIS}(z) = \alpha_s f_2^{DIS}(z) - \alpha_s f_L^{DIS}(z), \quad (3.123)$$

which means that $G_{p-q}^{(1)}(x, Q^2)$ is also scheme dependent. However, we

cannot actually calculate $G_{p-q}^{(1)}$ or $G_{p-q}^{(2)}$. If we are given $G_{p-q}^{(2)}$ or $G_{p-q}^{(1)}$ at one value of Q^2 , say Q_0^2 , we can calculate it at higher values of Q^2 from the evolution equations which, at leading order, do not involve the "little-f" functions. We now define the observable $\mathcal{F}_2(x, Q^2)$ in (3.104) to be our "reference distributions". That is we define quark distributions $G_{p-q}(x, Q^2)$ according to

$$G_{p-q}(x, Q^2) \equiv G_{p-q}^{(2)}(x, Q^2). \quad (3.124)$$

Given $G_{p-q}(x, Q_0^2)$ we can calculate $G_{p-q}(x, Q^2)$ from the Q^2 evolution equations ($Q^2 > Q_0^2$). With this definition equation (3.122) becomes

$$G_{p-q}^{(1)}(x, Q^2) = G_{p-q}(Q^2) * (1 + \alpha_s \Delta f_1^{q,DIS}) + G_{p-q}^-(Q^2) * (\alpha_s \Delta f_1^{g,DIS}). \quad (3.125)$$

or

$$G_{p-q}^{(1)}(x, Q^2) = \int_x^1 \frac{dy}{y} [G_{p-q}(y, Q^2) [\delta(1-z) + \alpha_s \Delta f_1^{q,DIS}(z)] + G_{p-q}^-(y, Q^2) \alpha_s \Delta f_1^{g,DIS}(z)], \quad (3.126)$$

where $z = x/y$ and where

$$\begin{aligned} \alpha_s \Delta f_1^{q,DIS}(z) &= \alpha_s [f_1^{q,DIS}(z) - f_2^{q,DIS}(z)] \\ &= -\alpha_s f_L^{q,DIS}(z) \end{aligned} \quad (3.127a)$$

and

$$\alpha_s \Delta f_1^{q, DIS}(z) = \alpha_s [f_1^{q, DIS}(z) - f_2^{q, DIS}(z)]$$

$$= -\alpha_s f_L^{q, DIS}(z). \quad (3.127b)$$

The Δf_1 functions are related to the longitudinal function f_L given by (3.53) and (3.69). The Δf_1 functions do not depend on the regularization scheme and tell us how much the $G_{p-q}^{(1)}(x, Q^2)$ distributions differ from the "reference distributions" $G_{p-q}(x, Q^2)$. In the naive parton model equation (3.10) holds and there is no difference between $G_{p-q}^{(1)}$ and G_{p-q} . In QCD they differ at order α_s and (3.126) tells us how to calculate $G_{p-q}^{(1)}(x, Q^2)$ in terms of $G_{p-q}(x, Q^2)$.

From (3.51a) we see that the longitudinal structure function $F_L(x, Q^2)$ is given by

$$F_L(x, Q^2) = -\alpha_s(Q^2) \int_x^1 \frac{dy}{y} \left(\sum_{i=1}^{n_f} e_{q_i}^2 (G_{p-q_i}(x, Q^2) + G_{p-q_i}(x, Q^2)) \right)$$

$$\Delta f_1^{q, DIS}(z) + \left(\sum_{i=1}^{2n_f} e_{q_i}^2 \right) G_{p-q}(x, Q^2) \Delta f_1^{q, DIS}(z). \quad (3.128)$$

which implies

$$F_L(x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} x^2 \int_x^1 \frac{dy}{y^3} \left(\frac{8}{3} F_2(y, Q^2) + \right.$$

$$\left. 2\alpha_e y G_{p-q}(y, Q^2) \left(1 - \frac{x}{y}\right) \right). \quad (3.129)$$

where

$$\alpha_e = \left(\sum_{i=1}^{2n_f} e_{q_i}^2 \right) = \frac{20}{9} \quad (\text{for } n_f=4). \quad (3.130)$$

Equation (3.129) allows us to calculate the longitudinal structure function $F_L(x, Q^2)$ in terms of the structure function $F_2(x, Q^2)$ and the gluon distribution $G_{p-q}(x, Q^2)$.

The observable $R^{DIS}(x, Q^2)$ is defined by

$$\begin{aligned} R^{DIS}(x, Q^2) &= F_L(x, Q^2) / (F_L(x, Q^2) + F_2(x, Q^2)) \\ &= R^{DIS}(x, Q^2) / (1 + R_2^{DIS}(x, Q^2)). \end{aligned} \quad (3.131a)$$

where

$$R_2^{DIS}(x, Q^2) = F_L(x, Q^2) / F_2(x, Q^2). \quad (3.131b)$$

An easy quantity to estimate in QCD is the integral of (3.131b) over x . Namely,

$$\bar{R}_2^{DIS}(Q^2) = \bar{F}_L(Q^2) / \bar{F}_2(Q^2). \quad (3.132a)$$

where

$$\bar{F}_i(Q^2) = \int_0^1 F_i(x, Q^2) dx. \quad (3.132b)$$

for $i = 2, L$. Using (3.53) and (3.69) we see that

$$\alpha_s \int_0^1 z \Delta f_1^{q, DIS}(z) dz = -\frac{\alpha_s}{2\pi} \left(\frac{8}{9} \right) \approx -0.141 \alpha_s. \quad (3.133a)$$

$$\alpha_s \int_0^1 z \Delta f_1^{g,DIS}(z) dz = -\frac{\alpha_s}{2\pi} \left\langle \frac{1}{6} \right\rangle \approx -0.027 \alpha_s \quad (3.133b)$$

so that (3.132a) becomes

$$\bar{F}_L(Q^2) = \frac{\alpha_s}{2\pi} \left\{ \left(\frac{8}{9} \right) \bar{F}_2(Q^2) + \left(\frac{1}{6} \right) a_6 G(Q^2) \right\}. \quad (3.134)$$

where $G(Q^2)$ is the total momentum carried by gluons:

$$G(Q^2) = \int_0^1 x G_{p-g}(x, Q^2) dx. \quad (3.135)$$

At $Q^2 = 16 \text{ GeV}^2$ the following is approximately true:

$$\bar{F}_2(Q^2=16 \text{ GeV}) \approx 0.164, \quad (3.136a)$$

$$G(Q^2=16 \text{ GeV}) \approx 0.514, \quad (3.136b)$$

so that

$$\bar{F}_2(Q^2=16 \text{ GeV}) \approx 0.107 \quad (3.137a)$$

$$\bar{R}(Q^2=16 \text{ GeV}) \approx 0.12. \quad (3.137b)$$

At this Q^2 , about half of $\bar{F}_L(Q^2)$ is due to the gluon term $G(Q^2)$ and about half is due to the quark term $\bar{F}_2(Q^2)$.

In all of this discussion, I have neglected corrections of order M^2/Q^2 . Such contributions cannot be calculated by perturbation theory. An estimate of the $1/Q^2$ contribution to R is

$$R(\text{primordial}) = 4 \langle k_{\perp}^2 \rangle_{\text{primordial}} / Q^2, \quad (3.138)$$

where $k_{\perp}$ is the non-perturbative "primordial" component to the transverse momentum of quarks within hadrons. The perturbative contribution to R behaves roughly as $\alpha_s(Q^2) \sim 1/\log(Q^2/\Lambda^2)$, so that at sufficiently large Q^2 this contribution dominates. However, at $Q^2 = 16 \text{ GeV}^2$, $R(\text{primordial}) \approx 0.06$ (using $R(\text{primordial}) \approx 1/Q^2$) which is certainly not negligible compared to the value of 0.12 arrived at in (3.137b) for $R(\text{perturbation})$.

IV. LARGE-MASS MUON PAIR PRODUCTION

1. The Naive Parton Model Result

In the naive parton model large mass muon pairs are produced in proton-proton collisions via the subprocess $q + \bar{q} \rightarrow \tau^+ \tau^- \rightarrow \mu^+ \mu^-$. The experimental cross section is expressed in terms of the parton subprocess as follows

$$d\sigma = G_{p-q}(x_a) dx_a G_{p-\bar{q}}(x_b) dx_b \hat{\sigma}(q + \bar{q} \rightarrow \tau^+ \tau^- \rightarrow \mu^+ \mu^-), \quad (4.1a)$$

where $G_{p-q}(x_a) dx_a$ is the probability of finding a quark with momentum

$$p_q = x_a P, \quad (4.1b)$$

and $G_{p-\bar{q}}(x_b) dx_b$ is the probability of finding an antiquark with momentum

$$p_{\bar{q}} = x_b P, \quad (4.1c)$$

where P is the momentum of the initial two protons as shown in Fig. 14b. Furthermore, if we define the dimensionless variables

$$\tau = M^2/s \quad (4.2a)$$

and

$$\hat{\tau} = M^2/s, \quad (4.2b)$$

with

$$s = (P + P)^2, \quad (4.2c)$$

and

$$\hat{s} = (p_q + p_{\bar{q}})^2, \quad (4.2d)$$

then

$$\tau = x_a x_b \hat{\tau}, \quad (4.2e)$$

since

$$\hat{s} = x_a x_b s, \quad (4.2f)$$

and the longitudinal momentum of the muon pair is given by $P_L = p_q - p_{\bar{q}}$ or

$$x_L = x_a - x_b \quad (4.3)$$

with $x_L = 2P_L/\sqrt{s}$. The total cross section in (4.1a) is the same as that in (2.2) except for an extra factor of 1/3 due to color. Namely,

$$\hat{\sigma}(q + \bar{q} \rightarrow \tau^+ \tau^-) = \left(\frac{1}{3}\right) \left(\frac{4\pi}{3}\right) \frac{\alpha^2}{M^2} = \sigma_0. \quad (4.4)$$

For this subprocess

$$\hat{s} = M^2, \quad (4.5a)$$

so that

$$\hat{\tau} = 1. \quad (4.5b)$$

From (4.2c) and (4.3) we see that x_a and x_b are completely specified in terms of τ and x_L according to

$$x_a x_b = \tau \quad (4.6a)$$

$$x_a - x_b = x_L \quad (4.6b)$$

and (4.1a) becomes

$$\sigma = \frac{d\sigma_{DY}}{dM^2 dx_L} (s, M^2) = \frac{4\pi\alpha^2}{9M^2} \frac{1}{(x_a + x_b)} \sum_{i=1}^{n_f} e_{q_i}^2 (G_{p-q_i}(x_a) G_{p-q_i}(x_b) + G_{p-q_i}(x_a) G_{p-q_i}(x_b)). \quad (4.7)$$

where I have included n_f quark flavors and included the contribution from $\bar{q} + q \rightarrow \mu^+ \mu^-$. The DY refers to the "Drell-Yan" process $p + p \rightarrow \mu^+ \mu^- + X$. Integrating over x_L gives

$$\sigma = \frac{d\sigma_{DY}}{dM^2} (s, M^2) = \left(\frac{4\pi}{9}\right) \frac{\alpha^2}{M^2} \int_{\tau}^1 \frac{dx_a}{x_a} \sum_{i=1}^{n_f} e_{q_i}^2 (G_{p-q_i}(x_a) G_{p-q_i}(x_b) + G_{p-q_i}(x_a) G_{p-q_i}(x_b)). \quad (4.8)$$

where $x_b = \tau/x_a$.

2. Real Gluon Emissions

We now consider the possibility that the initial quark or antiquark can radiate a gluon before annihilating into a virtual photon as shown in Fig. 12. The differential cross section for the subprocess $q + \bar{q} \rightarrow \gamma^* + g$ is

$$\begin{aligned} \frac{d\hat{\sigma}_q}{dt}(\hat{s}, \hat{t}) &= \frac{1}{64\pi s} \frac{\hat{\sigma}_q^2}{P_{cm}^2} |M(q + \bar{q} \rightarrow \gamma^* + g)|^2 \\ &= \frac{\pi\alpha_s^2}{s^2} \left(\frac{8}{9} \right) \left(\frac{\hat{u}}{t} + \frac{\hat{t}}{u} + \frac{2M^4}{tu} - \frac{2M^2}{t} - \frac{2M^2}{u} \right), \end{aligned} \quad (4.9)$$

where

$$\hat{s} = (p_q + p_{\bar{q}})^2, \quad (4.10a)$$

$$\hat{t} = (q_\gamma - p_q)^2, \quad (4.10b)$$

$$\hat{u} = (p_g - p_q)^2. \quad (4.10c)$$

as shown in Fig. 12. The invariant mass of the virtual photon, M^2 , is

$$q_\gamma^2 = M^2 \quad (4.11)$$

with

$$\hat{s} + \hat{t} + \hat{u} = M^2. \quad (4.12)$$

The superscript q refers to the annihilation proton $q + \bar{q} \rightarrow \gamma^* + g$ and the DY refers to the Drell-Yan production of muon pairs.

The integral of (4.9) over $\hat{t}$ is given by

$$\hat{\sigma}_{DY}^q(\hat{s}) = \int_{\hat{t}_{\min}}^{\hat{t}_{\max}} \frac{d\hat{\sigma}_q}{dt}(\hat{s}, \hat{t}) d\hat{t}, \quad (4.13)$$

where

$$\hat{t}_{\min} = 0 \quad (4.14a)$$

$$\hat{t}_{\max} = M^2 - \hat{s}. \quad (4.14b)$$

and diverges for precisely the same reason as the deep inelastic scattering integral (3.15). Again we cannot proceed without a regularization scheme.

3. Initial State Gluon Corrections

One must also correct the naive parton model by including the "Compton" subprocess $g + q \rightarrow \gamma^* + q$ shown in Fig. 13. The differential cross section for this subprocess is

$$\frac{d\hat{\sigma}_g}{dt}(\hat{s}, \hat{t}) = \frac{\pi\alpha_s^2}{s^2} \left(\frac{1}{3} \right) \left(-\frac{\hat{t}}{\hat{s}} - \frac{\hat{s}}{\hat{t}} + \frac{2M^2(\hat{s} + \hat{t} - M^2)}{\hat{s}\hat{t}} \right) \quad (4.15)$$

where in this case

$$\hat{s} = (p_q + q_g)^2 \quad (4.16a)$$

$$\hat{t} = (q_\gamma - p_q)^2 \quad (4.16b)$$

$$\hat{u} = (q_\gamma - q_g)^2 \quad (4.16c)$$

and as before $q_\gamma^2 = M^2$ is the virtual photon mass squared. The integral

$$\hat{\sigma}_{DY}^g(\hat{s}) = \int_{\hat{t}_{\min}}^{\hat{t}_{\max}} \frac{d\hat{\sigma}_{DY}^g}{d\hat{t}}(\hat{s}, \hat{t}) d\hat{t}, \quad (4.17)$$

is again infinite since $\hat{t}_{\min}$ and $\hat{t}_{\max}$ are given by (4.14) and we cannot proceed without a regularization scheme.

4. Order α_s Corrections - Massive Gluon Scheme

We can regulate the divergences in the subprocess $q + \bar{q} \rightarrow \tau^+ + g$ by giving the gluon a fictitious mass $q_g^2 = m_g^2$ as we did for deep inelastic scattering in Section III.4. The differential cross section in (4.9) becomes

$$\begin{aligned} \frac{d\hat{\sigma}_{DY}^q}{d\hat{t}}(\hat{s}, \hat{t}) &= \frac{\pi\alpha_s e_q^2}{\hat{s}^2} \left(\frac{8}{3} \right) \left(\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} + \frac{2M^4}{\hat{u}\hat{t}} \right. \\ &\quad \left. - \frac{2M^2}{\hat{t}} - \frac{2M^2}{\hat{u}} - \frac{m_g^2 M^2}{\hat{u}^2} - \frac{m_g^2 M^2}{\hat{t}^2} \right), \end{aligned} \quad (4.18)$$

where terms that do not contribute to the integral, $\hat{\sigma}_{DY}^q$, in the limit $m_g \rightarrow 0$ have been dropped.

The integration of (4.18) over $\hat{t}$ as defined in (4.13) is a bit more difficult than the other cases we have considered so far. One must be careful to keep the exact form for $\hat{t}_{\min}$ and $\hat{t}_{\max}$. Namely,

$$\hat{t}_{\min, \max} = -M^2((1-\hat{\tau}-\beta\hat{\tau}) \pm [(1-\hat{\tau})^2 + \beta\hat{\tau}(\beta\hat{\tau}-2\hat{\tau}-2)]^{1/2})/2\hat{\tau} \quad (4.19)$$

with

$$\beta = m_g^2/M^2, \quad (4.20)$$

where $\hat{\tau}$ is defined by (4.2b). The result is

$$\hat{\sigma}_{DY}^q(\hat{\tau}) = \frac{\pi\alpha_s e_q^2}{\hat{s}} \left(\frac{8}{3} \right) \left\{ 2 \frac{(1+\hat{\tau}^2)}{1-\hat{\tau}} \log(\hat{t}_{\max}/\hat{t}_{\min}) - 4(1-\hat{\tau}) \right\}, \quad (4.21)$$

where $\hat{t}_{\max}$ and $\hat{t}_{\min}$ are given by (4.19).

This parton subprocess must be "embedded" in the experimentally observed process $p + p \rightarrow \mu^+ \mu^- + X$, where M is the mass of the muon pair and s is the proton-proton CM energy squared as shown in Fig. 14b. Namely,

$$\sigma(s, M^2) = G_{p \rightarrow q}^{(0)}(x_a) dx_a G_{p \rightarrow \bar{q}}^{(0)}(x_b) dx_b \left(\frac{d\hat{\sigma}_{DY}^q}{d\hat{t}} \right) d\hat{t}, \quad (4.22)$$

where $G_{p \rightarrow q}^{(0)}(x_a) dx_a$ is the probability of finding a quark with momentum

$$p_q = x_a P \quad (4.23a)$$

and $G_{p \rightarrow \bar{q}}^{(0)}(x_b) dx_b$ is the probability of finding an antiquark with momentum

$$p_{\bar{q}} = x_b P \quad (4.23b)$$

where P is the momentum of the initial two protons, respectively. Furthermore

$$\hat{s} = x_a x_b s \quad (4.23c)$$

with s and $\hat{s}$ defined by (4.2c) and (4.2d), respectively, and

$$\hat{\tau} = M^2/\hat{s} = x_a x_b \hat{\tau} \quad (4.23d)$$

The quantity $(d\hat{\sigma}_{DY}^q/d\hat{\tau})d\hat{\tau}$ is the probability that the two quarks will annihilate and produce a $\mu^+\mu^-$ pair with mass M . From (4.22) we have

$$\frac{d\hat{\sigma}_{DY}^q}{d\hat{\tau}}(s, M^2) = \left\{ \int_{\hat{\tau}}^1 \frac{dx_a}{x_a} \right\} \left\{ \int_{\hat{\tau}/x_a}^1 \frac{dx_b}{x_b} C_{p-q}^{(0)}(x_a) C_{p-q}^{(0)}(x_b) \left(\frac{d\hat{\sigma}_{DY}^q}{d\hat{\tau}} \right) \right\}, \quad (4.24)$$

where the q corresponds to the contribution from the annihilation subprocess and the limits of integration are determined from (4.23d) using the fact that $0 \leq \hat{\tau} \leq 1$. The double differential cross section is related to $d\sigma/dt$ by

$$\frac{d\hat{\sigma}_{DY}^q}{dM^2 dt} (q + \bar{q} \rightarrow \mu^+ \mu^- + g) = \left(\frac{\alpha}{3\pi M^2} \right) \frac{d\hat{\sigma}_{DY}^q}{d\hat{\tau}} (q + \bar{q} \rightarrow \gamma^* \rightarrow \mu^+ \mu^- + g), \quad (4.25)$$

where the factor of $(\alpha/3\pi M^2)$ comes from integrating over the muon pair angular distributions. Integrating over t yields

$$\frac{1}{\sigma_0} \frac{d\hat{\sigma}_{DY}^q}{d\hat{\tau}} = \left(\frac{3}{4\pi^2} \right) \hat{\sigma}_{DY}^q, \quad (4.26)$$

where σ_0 is the Born cross section given in (4.4). From (4.26) and (4.21) we see that

$$\begin{aligned} \left(\frac{1}{\sigma_0} \frac{d\hat{\sigma}_{HC}^q}{d\hat{\tau}} \right)_{DY} &= 2 \left(\frac{2\alpha_s}{3\pi} \right) \left\{ \frac{1+\hat{\tau}^2}{1-\hat{\tau}} \log \left(\frac{(1-\hat{\tau})^2 M^2}{\hat{\tau}^2 m_g^2} \right) - 2(1-\hat{\tau}) \right. \\ &\quad \left. + [2\log^2(2) - \frac{\pi^2}{6}] \delta(1-\hat{\tau}) \right\}, \end{aligned} \quad (4.27)$$

where the δ -function term comes from the fact that

$$\int_{2\sqrt{\beta}}^1 \frac{\log(\hat{t}_{\min}/\hat{t}_{\max})}{1-\hat{\tau}} d\hat{\tau} = \frac{\pi^2}{12} - \log^2(2) + \int_{2\sqrt{\beta}}^1 \frac{\log(\hat{\beta}\hat{\tau}^2/(1-\hat{\tau})^2)}{1-\hat{\tau}} d\hat{\tau}, \quad (4.28)$$

where $\hat{t}_{\min}, \hat{t}_{\max}$ are given by (4.19). Integrating (4.27) over $\hat{\tau}$ from $\hat{\tau}_{\min} = 2\sqrt{\beta}$ to one as in (4.28) gives

$$(\hat{\sigma}_{HC}^{(real)})_{DY} = \left(\frac{2\alpha_s}{3\pi} \right) \sigma_0 (\log^2(\beta) + 3\log(\beta) + \pi^2). \quad (4.29)$$

The virtual corrections in Fig. 15b are the same as the e^+e^- case in (2.98) but with $Q^2 = M^2$ (q^2 timelike). Namely,

$$(\hat{\sigma}_{HC}^{(virtual)})_{DY} = \left(\frac{2\alpha_s}{3\pi} \right) \sigma_0 \left\{ -\log^2(\beta) - 3\log(\beta) - \frac{7}{2} - \frac{2\pi^2}{3} + \pi^2 \right\} \quad (4.30)$$

and as has been true in every case we have considered, the total

$$\begin{aligned} (\hat{\sigma}_{HC}^{(real)} + \hat{\sigma}_{HC}^{(virtual)})_{DY} &= \left(\frac{2\alpha_s}{3\pi} \right) \sigma_0 \left\{ \pi^2 - \frac{7}{2} - \frac{2\pi^2}{3} + \pi^2 \right\} \\ &= \left(\frac{2\alpha_s}{3\pi} \right) \sigma_0 \left\{ \frac{4\pi^2}{3} - \frac{7}{2} \right\} = \alpha_s \sigma_0 \left\{ \frac{8\pi}{9} - \frac{7}{3\pi} \right\}, \end{aligned} \quad (4.31)$$

is finite and independent of β as $\beta \rightarrow 0$. This can be compared to the e^+e^- case in (2.98)

$$(\hat{\sigma}_{HC}^{(real)} + \hat{\sigma}_{HC}^{(virtual)})_{e^+e^-} = \frac{\alpha_s}{\pi} \sigma_0 \quad (4.32)$$

and the deep inelastic scattering case in (3.39)

$$(\hat{\sigma}_{HC}^{(real)} + \hat{\sigma}_{HC}^{(virtual)})_{DIS} = -\frac{\alpha_s}{\pi} \sigma_0. \quad (4.33)$$

In this case the perturbation series behaves like

$$\sigma_{\text{tot}}^{\text{DY}} = \sigma_0 \left(1 + \left(\frac{8\pi}{9} - \frac{7}{3\pi} \right) \alpha_s + \dots \right) \quad (4.34a)$$

or

$$\sigma_{\text{tot}}^{\text{DY}} = \sigma_0 (1 + 2.05 \alpha_s + \dots). \quad (4.34b)$$

The order α_s corrections are very large and positive. The coefficient of the α_s term is so large that one must worry about the contribution from the α_s^2 and higher order terms. Can we believe the first two terms in (4.34) adequately reproduces the sum for $\alpha_s \approx \frac{1}{3}$ or $\frac{1}{4}$? For $\alpha_s = \frac{1}{3}$ the second term represents a 66% increase over the Born term!

Actually the situation is better than it looks. Keeping track of the π^2 that came from the analytic continuation from the spacelike (deep inelastic scattering) to the timelike (Drell-Yan) region of q^2 we have

$$\begin{aligned} (\hat{\sigma}_{\text{MC}}(\text{real}) + \hat{\sigma}_{\text{MC}}(\text{virtual}))_{\text{DY}} &= \left(\frac{2\alpha_s}{3\pi} \right) \sigma_0 \left(\pi^2 - \frac{7}{2} + \pi^2 \right) \\ &= \alpha_s \sigma_0 \left(\frac{2\pi}{9} - \frac{7}{3\pi} + \frac{2\pi}{3} \right), \end{aligned} \quad (4.35a)$$

where the third term comes from the analytic continuation of $\log^2(q^2)$ in (2.97). The series is now

$$\sigma_{\text{tot}}^{\text{DY}} = \sigma_0 (1 - 0.045 \alpha_s + \frac{2\pi}{3} \alpha_s + \dots), \quad (4.35b)$$

but the $\log^2(q^2)$ term in (2.96) or (2.98) can be shown to exponentiate. I will not have time to discuss this in these lectures, but it is well known that one can sum the leading double logs and that they form an exponential series. This means that the part of the series in (4.35) coming from the analytic continuation of the $\log^2(q^2)$

term also exponentiates. [6] Namely

$$\sigma_0 \left(1 + \frac{2\pi}{3} \alpha_s + \dots \right) = \sigma_0 \exp\left(\frac{2\pi}{3} \alpha_s\right) \quad (4.36)$$

and the series in (4.35) becomes

$$\begin{aligned} \sigma_{\text{tot}}^{\text{DY}} &= \sigma_0 \left(1 + \left(\frac{2\pi}{9} - \frac{7}{3\pi} \right) \alpha_s + \frac{2\pi}{3} \alpha_s + \dots \right) \\ &= \sigma_0 \exp\left(\frac{2\pi}{3} \alpha_s\right) \left(1 + \left(\frac{2\pi}{9} - \frac{7}{3\pi} \right) \alpha_s + \dots \right) \\ &= \sigma_0 \exp\left(\frac{2\pi}{3} \alpha_s\right) (1 - 0.045 \alpha_s + \dots), \end{aligned} \quad (4.37)$$

which is a well behaved perturbation series. For $\alpha_s = \frac{1}{3}$ (4.37) becomes

$$\begin{aligned} \sigma_{\text{tot}}^{\text{DY}} &= \sigma_0 \exp(0.698) (1 - 0.015 + \dots) \\ &= \sigma_0 (1.96) \end{aligned} \quad (4.38)$$

which is about a factor of 2 greater than the Born term!

Now that we have determined the integral over $\hat{\tau}$ we know that

$$\int_0^1 d\hat{\tau} \left\{ \frac{d\hat{\sigma}_{\text{MC}}^{\text{q}}}{d\hat{\tau}} + (\hat{\sigma}_{\text{MC}}(\text{virtual}) - (\sigma_{\text{tot}}^{\text{DY}} - \sigma_0)) \delta(1-\hat{\tau}) \right\} = 0. \quad (4.39)$$

where $\sigma_{\text{tot}}^{\text{DY}}$ is given by (4.37) and we define "+-functions" just as in (2.150). Namely,

$$\frac{1}{\sigma_0} \left(\frac{d\hat{\sigma}_{\text{MC}}^{\text{q}}}{d\hat{\tau}} \right)_+ = \left(\frac{\alpha_s}{2\pi} \right) 2P_{q-q}(\hat{\tau}) \log(M^2/m_e^2) + 2\pi_s \hat{\sigma}_{\text{MC}}^{\text{q,DY}}(\hat{\tau}), \quad (4.40)$$

where

$$P_{q-q}(\hat{\tau}) = \frac{4}{3} \left(\frac{1+\hat{\tau}^2}{1-\hat{\tau}} \right)_+ \quad (4.41)$$

is the same function as (2.154) and

$$\alpha_s f_{MC}^{q,DY}(\hat{\tau}) = \left(\frac{2\alpha_s}{3\pi} \right) \{ 2(1+\hat{\tau}^2) \left(\frac{\log(1-\hat{\tau})}{1-\hat{\tau}} \right)_+ - 2 \left(\frac{1+\hat{\tau}^2}{1-\hat{\tau}} \right) \log(\hat{\tau}) - 2(1-\hat{\tau}) - \frac{2\pi^2}{3} \delta(1-\hat{\tau}) \}, \quad (4.42)$$

and where

$$\int_0^1 \alpha_s f_{MC}^{q,DY}(\hat{\tau}) d\hat{\tau} = 0, \quad (4.43)$$

is valid regardless of the scheme. Inserting (4.40) into (4.24) yields

$$s \frac{d\sigma_{DY}^q}{dM^2}(s, M^2) = \left(\frac{4\pi}{9} \right) \frac{\alpha_s^2 e^2}{M^2} \int_{\tau}^1 \frac{dx_a}{x_a} \int_{\tau/x_a}^1 \frac{dx_b}{x_b} \cdot \frac{\sigma_{DY}^{(0)}(x_a) \sigma_{DY}^{(0)}(x_b)}{\left(\left(\frac{1-\hat{\tau}}{\hat{\tau}} \right) \delta(1-\hat{\tau}) \right)} + \left(\frac{\alpha_s}{2\pi} \right) P_{q-q}(\hat{\tau}) \log(M^4/m_g^4) + 2\alpha_s f_{MC}^{q,DY}(\hat{\tau}), \quad (4.44)$$

where the superscript q refers to the contribution from the annihilation subprocess $q + \bar{q} \rightarrow \gamma^* + g$. Equation (4.44) is analogous to (2.174) for the fragmentation functions and to (3.46) for the

structure functions.

For the initial state gluon "Compton" subprocess, $g + q \rightarrow \gamma^* + q$, shown in Fig. 13 we regularize by taking the gluon off mass-shell $q_g^2 = -m_g^2$. The differential cross section in (4.15) becomes

$$\frac{d\sigma_{DY}^g}{dt}(s, \hat{t}) = \frac{\pi\alpha_s e^2}{s^2} \left(\frac{1}{3} \right) \left\{ -\frac{\hat{t}}{s} - \frac{\hat{s}}{\hat{t}} + \frac{2M^2}{s\hat{t}} (\hat{s} + \hat{t} - M^2) - \frac{M_m^2}{\hat{t}^2} \right\}, \quad (4.45)$$

where the invariants $\hat{s}$, $\hat{t}$, $\hat{u}$ are given by (4.16) and $q_\gamma^2 = M^2$ and where terms that give no contribution to the integral in (4.17) in the limit $\mu \rightarrow 0$ have been dropped. In this case the limits of integration (4.14) become

$$\hat{t}_{\min} = -\frac{M_m^2}{s} \quad (4.46a)$$

$$\hat{t}_{\max} = M^2 - s \quad (4.46b)$$

and integration over $\hat{t}$ yields,

$$\sigma_{DY}^g(s) = \frac{\pi\alpha_s e^2}{s^2} \left(\frac{1}{3} \right) \left\{ \left(s + \frac{2M^4}{s} - 2M^2 \right) \log \left(\frac{\hat{s}(\hat{s}-m^2)}{M_m^2} \right) + \frac{1}{2} \left(\frac{\hat{s}-m^2}{s} \right)^2 + \frac{2M^2}{s} (\hat{s}-m^2) - 1 \right\}, \quad (4.47)$$

where I have used

$$\int_{\hat{t}_{\max}}^{\hat{t}_{\min}} d\hat{t} = \hat{s} - M^2, \quad \int_{\hat{t}_{\max}}^{\hat{t}_{\min}} \hat{t} d\hat{t} \approx -\frac{1}{2} (\hat{s}-m^2)^2$$

$$\int_{\hat{t}_{\max}}^{\hat{t}_{\min}} \frac{d\hat{t}}{\hat{t}} = \log \left(\frac{\hat{t}_{\min}}{\hat{t}_{\max}} \right) = -\log \left(\frac{(\hat{s}-\hat{M}_g^2)\hat{s}}{\hat{M}_g^2\hat{m}_g^2} \right)$$

$$\hat{m}_g^2 \int_{\hat{t}_{\max}}^{\hat{t}_{\min}} \frac{d\hat{t}}{\hat{t}^2} = -\frac{\hat{s}}{\hat{M}_g^2} \quad (4.48)$$

Changing variables to $\hat{\tau} = \hat{M}^2/\hat{s}$ and using the relationship in (4.26) gives

$$\left(\frac{1}{s_0} \frac{d\sigma_{\text{MG}}^g}{d\hat{\tau}} \right)_{\text{DY}} = \frac{\alpha_s}{2\pi} p_{g-q}(\hat{\tau}) \log(\hat{M}^2/\hat{m}_g^2) + \alpha_s f_{\text{MG}}^{g,\text{DY}}(\hat{\tau}), \quad (4.49)$$

where

$$p'_{g-q}(\hat{\tau}) = \frac{1}{2} (\hat{\tau} + (1-\hat{\tau})^2) \quad (4.50)$$

is the same function as in (3.63) and

$$\alpha_s f_{\text{MG}}^{g,\text{DY}}(\hat{\tau}) = \left(\frac{\alpha_s}{2\pi} \right) \frac{1}{2} \{ \hat{\tau}^2 + (1-\hat{\tau})^2 \} [-2\log(\hat{\tau}) + \log(1-\hat{\tau})] - \frac{1}{2} + \hat{\tau} - \frac{3}{2} \hat{\tau}^2. \quad (4.51)$$

Inserting (4.49) into (4.24) gives the gluon contribution to $d\sigma/d\tau$. Namely,

$$s \frac{d\sigma_{\text{DY}}^g}{d\hat{M}^2}(s, \hat{M}^2) = \left(\frac{4\pi}{9} \right) \frac{\alpha_s^2 e_a^2}{\hat{M}^2} \int_{\hat{\tau}}^1 \frac{dx_a}{x_a} \int_{\hat{\tau}/x_a}^1 \frac{dx_b}{x_b}$$

$$G_{p-q}^{(0)}(x_a) G_{p-q}^{(0)}(x_b) \left(\frac{\alpha_s}{2\pi} p_{g-q}(\hat{\tau}) \log(\hat{M}^2/\hat{m}_g^2) + \alpha_s f_{\text{MG}}^{g,\text{DY}}(\hat{\tau}) \right). \quad (4.52)$$

5. Order α_s Corrections - Dimensional Regularization Scheme

We can regulate the divergences in the subprocesses $q + \bar{q} \rightarrow \tau^+ + g$ and $q + g \rightarrow \tau^+ + q$ using the dimensional regularization techniques of Section III.5. Starting with (3.81) and using the fact that, in this case,

$$\hat{p}_{\text{cm}} = (\hat{s})^{1/2}/2 \quad (4.53a)$$

$$\hat{p}'_{\text{cm}} = (\hat{s}-\hat{M}^2)/(2(\hat{s}))^{1/2} = \frac{1}{2} (\hat{s})^{1/2} (1-\hat{\tau}), \quad (4.53b)$$

where $\hat{\tau}$ is defined in (4.2b), we arrive at

$$\hat{\sigma}_{\text{DY}}(\hat{\tau}) = \left(\frac{(1-\hat{\tau})}{32\pi s} \right) \left(\frac{\hat{M}^2 (1-\hat{\tau})^2}{\hat{\tau}^2 \hat{m}_D^2} \right)^{\epsilon/2} \frac{1}{2^{\epsilon} \Gamma(1 + \frac{\epsilon}{2})}. \quad (4.54)$$

where

$$1 = \int_{-1}^1 dy (1-y^2)^{\epsilon/2} |k|^2 \quad (4.55)$$

with $N=4+\epsilon$ and

$$\hat{t} = -\frac{1}{2} (\hat{s}-\hat{M}^2)(1-y) = -\frac{\hat{s}}{2} (1-\hat{\tau})(1-y) \quad (4.56a)$$

$$\hat{u} = -\frac{1}{2} (\hat{s}-\hat{M}^2)(1+y) = -\frac{\hat{s}}{2} (1-\hat{\tau})(1+y). \quad (4.56b)$$

In $N=4+\epsilon$ dimensions the matrix element squared for the annihilation subprocess is given by

$$|M_2(q+\bar{q}+\tau^*+g)|^2 = (16\pi^2 \alpha_s^2 c_q^2) \left(\frac{8}{9}\right)$$

$$\left\{ \frac{(\hat{\tau}^2 y^2 - 2\hat{\tau}^2 y^2 + y^2 + \hat{\tau}^2 + 2\hat{\tau}^2 + 1)}{(1-\hat{\tau})^2 (1-y^2)} + \frac{1-2\hat{\tau}+\hat{\tau}^2}{(1-\hat{\tau})^2 (1-y^2)} \epsilon + \dots \right\}, \quad (4.57)$$

The integral I in (4.55) is evaluated using (3.85) with the result

$$c_{DI}^q(\hat{\tau}) = \frac{\pi \alpha_s c_q^2}{s} \left(\frac{8}{9}\right) \left(\frac{M^2(1-\hat{\tau})^2}{4\pi m_D^2}\right)^{\epsilon/2} \left(\frac{2}{\epsilon}\right) \left(\frac{1+\hat{\tau}}{1-\hat{\tau}}\right), \quad (4.58)$$

or from (4.26)

$$\left(\frac{1}{\sigma_0} \frac{d\sigma_{DR}^q}{d\tau}\right)_{DY} = 2 \left(\frac{2\alpha_s}{3\pi}\right) \left\{ \frac{1+\hat{\tau}^2}{1-\hat{\tau}} \log\left(\frac{1-\hat{\tau}}{\hat{\tau}} \frac{M^2}{2}\right) + \frac{1+\hat{\tau}^2}{1-\hat{\tau}} \frac{2}{\epsilon} \right\}. \quad (4.59)$$

Integrating over $\hat{\tau}$ gives

$$(\hat{\sigma}_{DR}(\text{real}))_{DY} = \left(\frac{2\alpha_s}{3\pi}\right) \sigma_0 \left(\frac{M^2}{4\pi m_D^2}\right)^{\epsilon/2} \frac{\Gamma(1-\frac{\epsilon}{2})}{\Gamma(1-\epsilon)} \left\{ \frac{8}{\epsilon^2} - \frac{6}{\epsilon} + \frac{5}{2} \right\}. \quad (4.60)$$

The virtual corrections in Fig. 15b are given in (2.145) by

$$\begin{aligned} (\hat{\sigma}_{DR}(\text{virtual}))_{DY} &= \left(\frac{2\alpha_s}{3\pi}\right) \sigma_0 \left(\frac{M^2}{4\pi m_D^2}\right)^{\epsilon/2} \frac{\Gamma(1-\frac{\epsilon}{2}) \Gamma^2(1+\frac{\epsilon}{2})}{\Gamma(1-\epsilon)} \\ &\quad \left\{ -\frac{8}{\epsilon^2} + \frac{6}{\epsilon} - 8 + \pi^2 \right\}, \end{aligned} \quad (4.61)$$

where $q^2 = M^2$ is timelike. From the expansion in (2.142) and

$$\Gamma(1-\frac{\epsilon}{2}) = 1 + \frac{1}{2} \gamma_E \epsilon + \frac{1}{8} \left(\frac{\pi^2}{6} + \gamma_E^2 \right) \epsilon^2 + \dots \quad (4.62)$$

we see that the sum of (4.60) and (4.61) is given by

$$(\hat{\sigma}_{DR}(\text{real}) + \hat{\sigma}_{DR}(\text{virtual}))_{DY} = \left(\frac{2\alpha_s}{3\pi}\right) \sigma_0 \left(\frac{4\pi^2}{3} - \frac{7}{2}\right), \quad (4.63)$$

in the limit $\epsilon \rightarrow 0$, which is the same result as in (4.31). We now define "+-functions" as in (4.40) we arrive at

$$\frac{1}{\sigma_0} \left(\frac{d\sigma_{DR,DY}^q}{d\tau} \right)_+ = \left(\frac{\alpha_s}{2\pi}\right) 2P_{q-q}(\hat{\tau}) \log(M^2/m_D^2) + 2\alpha_s f_{DR}^{q,DY}(\hat{\tau}), \quad (4.64)$$

where $P_{q-q}(\hat{\tau})$ is the same as (4.41) and

$$\begin{aligned} \alpha_s f_{DR}^{q,DY}(\hat{\tau}) &= \left(\frac{2\alpha_s}{3\pi}\right) \{ 2(1+\hat{\tau}^2) \left(\frac{\log(1-\hat{\tau})}{1-\hat{\tau}} \right)_+ \\ &\quad - \frac{1+\hat{\tau}^2}{1-\hat{\tau}} \log(\hat{\tau}) - \left(\frac{\pi^2}{3} + \frac{9}{4} \right) \delta(1-\hat{\tau}) \} \\ &\quad + \frac{\alpha_s}{2\pi} P_{q-q}(\hat{\tau}) \left(\frac{2}{\epsilon} + \gamma_E - \log(4\pi) \right). \end{aligned} \quad (4.65)$$

The integral of $f_{DR}^{q,DY}$ over $\hat{\tau}$ vanishes as in (4.43).

The "Compton" subprocess $q + g \rightarrow \tau^* + q$ is treated in a similar fashion with the result

$$\left(\frac{1}{\sigma_0} \frac{d\sigma_{DR}^g}{d\tau}\right)_{DY} = \frac{\alpha_s}{2\pi} P_{g-q}(\hat{\tau}) \log(M^2/m_D^2) + \alpha_s f_{DR}^{g,DY}(\hat{\tau}), \quad (4.66)$$

where $P_{g-q}(\hat{\tau})$ is given by (4.50) and

$$\alpha_s f_{DR}^{g,DY}(\hat{\tau}) = \left(\frac{\alpha_s}{2\pi}\right) \frac{1}{2} \{(\tau^2 + (1-\tau)^2) \log\left(\frac{(1-\hat{\tau})^2}{\tau}\right) - \frac{3}{2} \tau^2 + \hat{\tau} + \frac{3}{2}\} \\ + \left(\frac{\alpha_s}{2\pi}\right) P_{g-q}(\hat{\tau}) \left(\frac{2}{\epsilon} + \tau_\epsilon - \log(4\pi)\right). \quad (4.67)$$

6. Drell-Yan "K-Factor"

If we combine the annihilation term in (4.44) with the "Compton" term in (4.52) and also include terms with the initial two partons interchanged, then the muon pair (or "Drell-Yan") cross section becomes (for one quark flavor)

$$s \frac{d\sigma_{DY}}{dM^2}(s, M^2) = \left(\frac{4\pi}{9}\right) \frac{\alpha_s^2 e_q^2}{M^2} \int_{\tau}^1 \frac{dx_a}{x_a} \int_{\tau/x_a}^1 \frac{dx_b}{x_b} \\ \left\{ (\tilde{G}_{p-q}^{(0)}(x_a) \tilde{G}_{p-q}^{(0)}(x_b) + \tilde{G}_{p-q}^{(0)}(x_a) \tilde{G}_{p-q}^{(0)}(x_b)) \right. \\ \left[\left(\frac{\sigma_{DY}}{\sigma_0} \right) \delta(1-\hat{\tau}) + \left(\frac{\alpha_s}{2\pi} \right) P_{q-q}(\hat{\tau}) (\log(M^2/\Lambda^2) + \log(M^2/\Lambda^2)) + 2\alpha_s f_{q,DY}(\hat{\tau}) \right] \\ + (\tilde{G}_{p-q}^{(0)}(x_a) \tilde{G}_{p-g}^{(0)}(x_b) + \tilde{G}_{p-g}^{(0)}(x_a) \tilde{G}_{p-q}^{(0)}(x_b)) \\ \left. \left[\left(\frac{\alpha_s}{2\pi} \right) P_{g-q}(\hat{\tau}) \log(M^2/\Lambda^2) + \alpha_s f_{g,DY}(\hat{\tau}) \right] \right\}. \quad (4.68)$$

where σ_{tot}^{DY} is given by (4.37). The "little-f" functions are scheme dependent and are given by (4.42) and (4.51) in the massive gluon scheme and by (4.65) and (4.67) in dimensional regularization. The $\log(m_g^2)$ or $\log(m_D^2)$ divergences have been absorbed into the $\tilde{G}_{p-q}^{(0)}$ and $\tilde{G}_{p-g}^{(0)}$ structure functions as we did in (2.186) for the fragmentation functions.

If we now define "Drell-Yan" quark distributions by the naive parton model equation in (4.8) we have

$$s \frac{d\sigma_{DY}}{dM^2}(s, M^2) = \left(\frac{4\pi}{9}\right) \frac{\alpha_s}{M^2} \int_{\tau}^1 \frac{dx_a}{x_a} \\ \sum_{i=1}^n e_i^2 \left[G_{p-q_1}^{DY}(x_a, M^2) G_{p-q_1}^{DY}(x_b, M^2) + G_{p-q_1}^{DY}(x_a, M^2) G_{p-q_1}^{DY}(x_b, M^2) \right]. \quad (4.69)$$

where $x_b = \tau/x_a$, then from (4.68)

$$G_{p-q}^{DY}(x, M^2) = \tilde{G}_{p-q}^{(0)} * \left(1 + \frac{\alpha_s}{2\pi} P_{q-q} \log(Q^2/\Lambda^2) + \alpha_s f_{q,DY} + \frac{1}{2} \alpha_s \left(\frac{8\pi}{9} - \frac{7}{3\pi} \right) \right) \\ + \tilde{G}_{p-g}^{(0)} * \left(\frac{\alpha_s}{2\pi} P_{g-q} \log(Q^2/\Lambda^2) + \alpha_s f_{g,DY} \right) \quad (4.70a)$$

and

$$G_{p-q}^{DY}(x, M^2) = \tilde{G}_{p-q}^{(0)} * \left(1 + \frac{\alpha_s}{2\pi} P_{q-q} \log(Q^2/\Lambda^2) + \alpha_s f_{q,DY} + \frac{1}{2} \alpha_s \left(\frac{8\pi}{9} - \frac{7}{3\pi} \right) \right) \\ + \tilde{G}_{p-g}^{(0)} * \left(\frac{\alpha_s}{2\pi} P_{g-q} \log(Q^2/\Lambda^2) + \alpha_s f_{g,DY} \right). \quad (4.70b)$$

where the convolution is defined in (2.16) and where I have used the unexponentiated form of σ_{tot}^{DY} in (4.34a)

$$\sigma_{tot}^{DY}/\sigma_0 \approx \left(1 + \frac{1}{2} \left(\frac{8\pi}{9} - \frac{7}{3\pi} \right) \alpha_s \right) \left(1 + \frac{1}{2} \left(\frac{8\pi}{9} - \frac{7}{3\pi} \right) \alpha_s \right). \quad (4.71)$$

These "Drell-Yan" distributions are scheme dependent since the "little-f" functions are scheme dependent. We now express them in terms of our "reference" distributions from deep inelastic scattering

$$G_{p-q}(x, Q^2) \equiv G_{p-q}^{(2)}(x, Q^2). \quad (4.72)$$

From (3.107) we see that

$$\begin{aligned} G_{p-q}^{DY}(x, M^2) &= G_{p-q}(x, M^2) * (1 + \alpha_s \Delta f_q^{DY} + \frac{1}{2} \alpha_s (\frac{8\pi}{9} - \frac{7}{3\pi})) \\ &+ G_{p-g}(x, M^2) * (\alpha_s \Delta f_g^{DY}) \end{aligned} \quad (4.73a)$$

and

$$\begin{aligned} G_{p-q}^{DY}(x, M^2) &= G_{p-q}(x, M^2) * (1 + \alpha_s \Delta f_q^{DY} + \frac{1}{2} \alpha_s (\frac{8\pi}{9} - \frac{7}{3\pi})) \\ &+ G_{p-g}(x, M^2) * (\alpha_s \Delta f_g^{DY}). \end{aligned} \quad (4.73b)$$

where

$$\begin{aligned} \alpha_s \Delta f_q^{DY}(z) &= \alpha_s (f_{qG}^{q, DY}(z) - f_{qG}^{q, DIS}(z)) \\ &= \alpha_s (f_{DR}^{q, DY}(z) - f_{DR}^{q, DIS}(z)) \\ &= (\frac{2\alpha_s}{3\pi}) \{ ((1+z)^2) (\frac{\log(1-z)}{1-z})_+ \\ &+ \frac{3}{2} \frac{1}{(1-z)_+} - 3 - 2z + (\frac{9}{4}) \delta(1-z) \}. \end{aligned} \quad (4.74)$$

and

$$\begin{aligned} \alpha_s \Delta f_g^{DY}(z) &= \alpha_s (f_{qG}^{g, DY}(z) - f_{qG}^{g, DIS}(z)) \\ &= \alpha_s (f_{DR}^{g, DY}(z) - f_{DR}^{g, DIS}(z)) \end{aligned}$$

$$= (\frac{\alpha_s}{2\pi}) \frac{1}{2} \{ (z^2 + (1-z)^2) \log(1-z) + \frac{1}{2} (9z^2 - 10z + 3) \}. \quad (4.75)$$

Equations (4.73) are not scheme dependent since the Δf^{DY} functions in (4.74) and (4.75) are the same regardless of the regularization scheme chosen! This means that we can predict the difference between quark distributions defined according to (4.69) and those defined in deep inelastic scattering by (3.104).

In leading order we neglect terms proportional to $\alpha_s(Q^2)$ and from (4.73) we see that the "Drell-Yan" distributions are the same as the deep inelastic quark distributions but with Q^2 replaced by M^2 . Namely

$$G_{p-q}^{DY}(x, M^2) = G_{p-q}(x, M^2) \quad (\text{leading order}) \quad (4.76)$$

and in leading order the Drell-Yan cross section in (4.69) becomes

$$\begin{aligned} (s \frac{d\sigma_{DY}}{dM^2}(s, M^2))_{\text{leading order}} &= (\frac{4\pi}{9}) \frac{\alpha^2}{M^2} \int_{\tau}^1 \frac{dx_a}{x_a} \\ &\sum_{i=1}^{n_f} e_{q_i}^2 [G_{p-q_i}(x_a, M^2) G_{p-q_i}(x_b, M^2) + G_{p-q_i}(x_a, M^2) G_{p-q_i}(x_b, M^2)], \end{aligned} \quad (4.77)$$

with $x_b = \tau/x_a$. However, at order α_s the Drell-Yan cross section becomes

$$\begin{aligned} (s \frac{d\sigma_{DY}}{dM^2}(s, M^2))_{\text{order } \alpha_s} &= (\frac{\sigma_{DY}^{\text{tot}}}{\sigma_0}) (s \frac{d\sigma_{DY}}{dM^2}(s, M^2))_{\text{leading order}} \\ &+ (\frac{4\pi}{9}) \frac{\alpha^2}{M^2} \int_{\tau}^1 \frac{dx_a}{x_a} \int_{\tau/x_a}^1 \frac{dx_b}{x_b} \sum_{i=1}^{n_f} e_{q_i}^2 \end{aligned}$$

$$[G_{p-q_1}(x_a, M^2)G_{p-q_1}(x_b, M^2) + (x_a \leftrightarrow x_b)]2\alpha_s \Delta f_q^{DY}(\hat{\tau})$$

$$K_{DY}^{\delta}(M^2) = \left(\frac{\sigma_{tot}^{DY}}{\sigma_0}\right) + \left(\frac{9}{3\pi}\right)\alpha_s(M^2). \quad (4.81)$$

$$+ \left(\frac{4\pi}{9}\right) \frac{\alpha_s^2}{M^2} \int_{\hat{\tau}}^1 \frac{dx_a}{x_a} \int_{\hat{\tau}/x_a}^1 \frac{dx_b}{x_b} \sum_{i=1}^{n_f}$$

$$e_{q_1}^2 [G_{p-q_1}(x_a, M^2)G_{p-q_1}(x_b, M^2) + (x_a \leftrightarrow x_b)]\alpha_s \Delta f_g^{DY}(\hat{\tau})$$

$$+ \left(\frac{4\pi}{9}\right) \frac{\alpha_s^2}{M^2} \int_{\hat{\tau}}^1 \frac{dx_a}{x_a} \int_{\hat{\tau}/x_a}^1 \frac{dx_b}{x_b} \sum_{i=1}^{n_f}$$

$$e_{q_1}^2 [G_{p-q_1}(x_a, M^2)G_{p-q_1}(x_b, M^2) + (x_a \leftrightarrow x_b)]\alpha_s \Delta f_g^{DY}(\hat{\tau}). \quad (4.78)$$

where $\hat{\tau} = \tau/(x_a x_b)$, and where Δf_q^{DY} is given by (4.74) and Δf_g^{DY} by (4.75).

The Drell-Yan "K-factor" is a complicated function and is defined to be the ratio of the order α_s result to the leading order result. Namely,

$$K_{DY}(s, M^2) = \left(s \frac{d\sigma_{DY}}{dM^2}(s, M^2)\right)_{\text{order } \alpha_s} / \left(s \frac{d\sigma_{DY}}{dM^2}(s, M^2)\right)_{\text{leading order}}. \quad (4.79)$$

Equation (4.78) shows that the "K-factor" consists of three terms,

$$K_{DY}(s, M^2) = K_{DY}^{\delta}(M^2) + K_{DY}^q(s, M^2) + K_{DY}^g(s, M^2). \quad (4.80)$$

The first term is a δ -function contribution arising from the first term in (4.78) and the δ -function piece of Δf_q^{DY} in (4.74).

From (4.34a) we have

$$K_{DY}^{\delta}(M^2) = 1 + \left(\frac{8\pi}{9} - \frac{7}{3\pi} + \frac{9}{3\pi}\right)\alpha_s(M^2) + \dots$$

$$\approx 1 + 3.0 \alpha_s(M^2) + \dots \quad (4.82)$$

which with $\alpha_s(M^2) = \frac{1}{3}$ looks like

$$K_{DY}^{\delta}(M^2) \approx 1 + 1 + \dots \quad (4.83)$$

The second term in the perturbative expansion of K_{DY}^{δ} is as big as the first and one has to doubt whether the first two terms give a good approximation to the complete series. On the other hand, part of the series exponentiates and it is (4.37) that we should use for σ_{tot}^{DY} . In this case

$$K_{DY}^{\delta}(M^2) = \exp\left(\frac{2\pi}{3}\alpha_s(M^2)\right)\left[1 + \left(\frac{2\pi}{9} - \frac{7}{3\pi} + \frac{9}{3\pi}\right)\alpha_s(M^2) + \dots\right]$$

$$\approx \exp\left(\frac{2\pi}{3}\alpha_s(M^2)\right)[1 + 0.94 \alpha_s(M^2) + \dots], \quad (4.84)$$

which with $\alpha_s(M^2) = \frac{1}{3}$ looks like

$$K_{DY}^{\delta}(M^2) \approx \exp(0.698) (1 + 0.31 + \dots) \approx 2.63. \quad (4.85)$$

Here the second term is a 30% correction to the first so it appears to be a well behaved perturbation series. For $\alpha_s = \frac{1}{3}$ K_{DY}^{δ} is about 30% larger than the naive factor of 2 arising from the unexponentiated

series in (4.82). The $k_{DY}^{\delta}(M^2)$ factor is shown in Fig. 17 for the unexponentiated series and $\Lambda = 0.4$ GeV.

The third term in (4.80) is the "gluon" contribution. It is given by

$$\begin{aligned}
 k_{DY}^g(s, M^2) = & \left\{ \int_{\tau}^1 \frac{dx_a}{x_a} \int_{\tau/x_a}^1 \frac{dx_b}{x_b} \sum_{i=1}^{n_f} \right. \\
 & e_{q_i}^2 \left[G_{p-q_i}^-(x_a, M^2) G_{p-q_i}^-(x_b, M^2) + (x_a \leftrightarrow x_b) \right] \alpha_s \Delta f_g^{\text{DY}}(\hat{\tau}) \\
 & + \int_{\tau}^1 \frac{dx_a}{x_a} \sum_{i=1}^{n_f} \int_{\tau/x_a}^1 \frac{dx_b}{x_b} \\
 & e_{q_i}^2 \left[G_{p-q_i}^-(x_a, M^2) G_{p-q_i}^-(x_b, M^2) + (x_a \leftrightarrow x_b) \right] \alpha_s \Delta f_g^{\text{DY}}(\hat{\tau}) / \\
 & \left. \left(\int_{\tau}^1 \frac{dx_a}{x_a} \sum_{i=1}^{n_f} e_{q_i}^2 \left[G_{p-q_i}^-(x_a, M^2) G_{p-q_i}^-(x_b, M^2) + (x_a \leftrightarrow x_b) \right] \right) \right\}. \quad (4.86)
 \end{aligned}$$

and is also shown in Fig. 17 at the CM energy of $W = 27.4$ GeV. The term $k_{DY}^g(s, M^2)$ is less than one as the order α_s gluon corrections slightly reduce the overall muon pair rate.

The quark term in (4.80) is given by

$$\begin{aligned}
 k_{DY}^q(s, M^2) = & \left\{ \int_{\tau}^1 \frac{dx_a}{x_a} \int_{\tau/x_a}^1 \frac{dx_b}{x_b} \sum_{i=1}^{n_f} \right. \\
 & e_{q_i}^2 \left[G_{p-q_i}^-(x_a, M^2) G_{p-q_i}^-(x_b, M^2) + (x_a \leftrightarrow x_b) \right]
 \end{aligned}$$

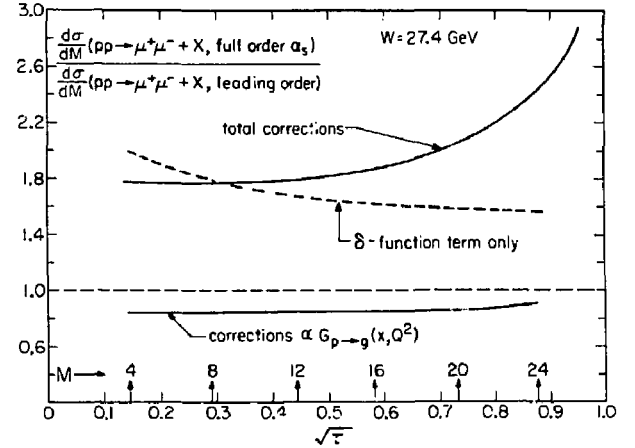


Fig. 17. Ratio of the complete order α_s results to the leading order QCD predictions for the total muon pair rate, $d\sigma/dM$, in $pp \rightarrow \mu^+\mu^- + X$ at $W = 27.4$ GeV with $\Lambda = 0.4$ GeV. The contributions from the δ -function term (unexponentiated) and the contributions from the gluon term are shown separately. The results are plotted versus both the muon pair mass, M , and $\sqrt{\tau}$, where $\tau = M^2/s$.

$$2\alpha_s \Delta f_q^{DY}(\hat{\tau}) / \left(\int_{\tau}^1 \frac{dx_a}{x_a} \sum_{i=1}^{n_f} e_{q_i}^2 \left[C_{p \leftarrow q_i}(x_a, M^2) C_{p \leftarrow q_i}^{-}(x_b, M^2) + (x_a \leftrightarrow x_b) \right] \right), \quad (4.87)$$

where Δf_q^{DY} is the same as (4.74) but with the δ -function contribution removed. Namely,

$$\alpha_s \Delta f_q^{DY}(z) = \left(\frac{2\alpha_s}{3\pi} \right) \left((1+z)^2 \left(\frac{\log(1-z)}{1-z} \right)_+ + \frac{3}{2} \frac{1}{(1-z)_+} - 3 - 2z \right). \quad (4.88)$$

As can be seen in Fig. 17 the term $K_{DY}^q(s, M^2)$ is small except when τ approaches one. The "+"-function" terms in (4.88) cause this corrections to become arbitrarily large near the kinematic boundary. This can be seen clearly in the "Drell-Yan" structure functions in (4.73). The $(\log(1-z)/(1+z))_+$ term gives

$$\frac{G^{DY}(x, Q^2)}{G(x, Q^2)} \approx 1 + \left(\frac{2\alpha_s(Q^2)}{3\pi} \right) \log^2(1-x), \quad (4.89)$$

which shows that the order $\alpha_s(Q^2)$ correction becomes arbitrarily large as $x \rightarrow 1$. Figure 18 shows the ratio $G^{DY}(x, Q^2)/G(x, Q^2)$ at $Q^2 = 50 \text{ GeV}^2$ for u and $\bar{u}$ quarks.

Actually I should not have defined the "Drell-Yan" structure functions according to (4.69). Instead I should have defined them in an analogous fashion to the fragmentation functions in (2.177) with σ_{tot} removed. If we define the "Drell-Yan" structure functions according to

$$s \frac{d\sigma_{DY}}{dM^2}(s, k^2) = \sigma_{\text{tot}}^{DY} \int_{\tau}^1 \frac{dx_a}{x_a}$$

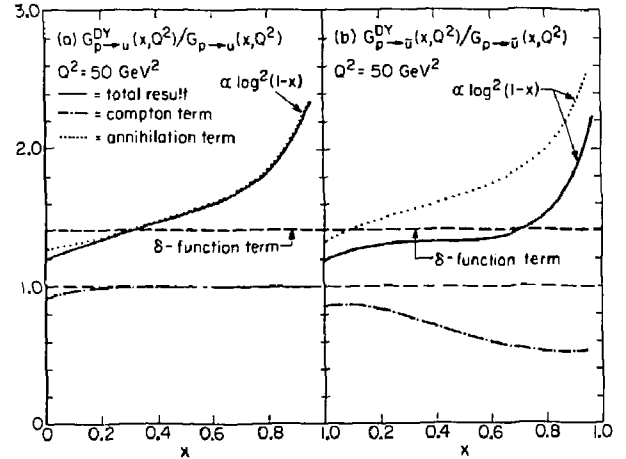


Fig. 18. Ratio of $G_{p \leftarrow u}^{DY}(x, Q^2)/G_{p \leftarrow u}(x, Q^2)$ and $G_{p \leftarrow \bar{u}}^{DY}(x, Q^2)/G_{p \leftarrow \bar{u}}(x, Q^2)$ versus x at $Q^2 = 50 \text{ GeV}^2$ with $\Lambda = 0.4 \text{ GeV}$. The functions $G_{p \leftarrow q}^{DY}(x, Q^2)$ are the full order α_s distributions defined by (4.73) for the total

muon pair rate and $G(x, Q^2)$ are the reference parton distributions defined in deep inelastic scattering. The contributions from the annihilation term and the "Compton" term are shown separately. Also shown is the contribution from the δ -function term. The functions $G_{p \leftarrow u}^{DY}$ and $G_{p \leftarrow \bar{u}}^{DY}$ are not separate observables and only have real significance when convoluted together to produce the total rate, $d\sigma_{DY}/dM^2$, in (4.69).

$$\sum_{i=1}^n e^2 \frac{1}{q_i^2} \left[\tilde{G}_{p-q_i}^{DY}(x_a, M^2) \tilde{G}_{p-q_i}^{DY}(x_b, M^2) + \tilde{G}_{p-q_i}^{DY}(x_a, M^2) \tilde{G}_{p-q_i}^{DY}(x_b, M^2) \right]. \quad (4.90)$$

where σ_{tot}^{DY} is given in (4.37) by

$$\sigma_{tot}^{DY} = \sigma_0 \exp\left(\frac{2\pi}{3} \alpha_s(M^2)\right) (1 - 0.045 \alpha_s(M^2) + \dots), \quad (4.91)$$

with $\sigma_0 = 4\pi\alpha/(9M^2)$ as in (4.4). Now in terms of the deep inelastic reference structure functions we have

$$\tilde{G}_{p-q}^{DY}(x, M^2) = G_{p-q}(x, M^2) * (1 + \alpha_s \Delta f_q^{DY}) + G_{p-g}(x, M^2) * (\alpha_s \Delta f_g^{DY}) \quad (4.92a)$$

and

$$\tilde{G}_{p-q}^{DY}(x, M^2) = G_{p-q}(x, M^2) * (1 + \alpha_s \Delta f_q^{DY}) + G_{p-g}(x, M^2) * (\alpha_s \Delta f_g^{DY}). \quad (4.92b)$$

With this definition

$$\int_0^1 (\tilde{G}_{p-q_i}^{DY}(x, M^2) - \tilde{G}_{p-q_i}^{DY}(x, M^2)) dx = \int_0^1 (G_{p-q_i}(x, M^2) - G_{p-q_i}(x, M^2)) dx, \quad (4.93)$$

since

$$\int_0^1 \alpha_s \Delta f_q^{DY}(z) dz = 0, \quad (4.94)$$

so that (at least at this order of perturbation theory) the number of quarks in the proton is the same for the Drell-Yan as well as the reference distributions.

The extraction of σ_{tot}^{DY} in (4.91) removes most of the δ -function contribution (Δf_q^{DY} still has a small δ -function term) and as long as one stays away from the $x \rightarrow 1$ region the structure functions, $\tilde{G}^{DY}(x, Q^2)$ in (4.92) do not differ greatly from the deep inelastic scattering reference distributions $G(x, Q^2)$. The major difference between the leading order and the order α_s Drell-Yan muon pair cross section lies in the multiplicative factor

$$\sigma_{tot}^{DY}/\sigma_0 = \exp\left(\frac{2\pi}{3} \alpha_s(M^2)\right) (1 - 0.045 \alpha_s(M^2) + \dots), \quad (4.95)$$

which for $\alpha_s(M^2) = \frac{1}{3}$ is about a factor of 2 as shown in (4.38). This can be compared to the e^+e^- result in (2.99) of

$$\sigma_{tot}^{e^+e^-}/\sigma_0 = (1 + 0.318 \alpha_s(Q^2) + \dots), \quad (4.96)$$

which for $\alpha_s(Q^2) = \frac{1}{3}$ is only a factor of 1.11 (i.e. about a 10% effect).

Experimentally the Drell-Yan "K-factor" is about a factor of 2 and agrees well with the order α_s prediction in (4.90) together with the exponentiated series in (4.95). [7] This is a great triumph for perturbative QCD! The experimental Drell-Yan measurements can be interpreted as a measurement of σ_{tot}^{DY} in (4.95). This measurement is as significant as the experimental measurement of the order α_s

correction to $\sigma_{tot}^{e^+e^-}$ in (2.99). The Drell-Yan experimental results deviate from the naive parton models in the precise manner predicted by perturbative QCD.

As we have seen, the "little-f" functions are process dependent. This means that the "K-factor" differs from process to process. Here we have only considered the Drell-Yan K-factor. It must be recalculated for every process of interest. For example, the "K-factor" for quark-quark scattering (i.e. the subprocess $q + q \rightarrow q + q$) has not yet been calculated. However, the exponential, $\exp(\frac{2\pi}{3} \alpha_s)$,

in (4.95) arose from the π^2 that came from the analytic continuation from the spacelike (deep inelastic scattering reference distributions)

to the timelike (Drell-Yan) region of q^2 . Hence, we might expect that whenever we try to describe a timelike (TL) process in terms of the deep inelastic (spacelike) structure functions we will have a correction term of the form

$$\sigma_{\text{tot}}^{\text{TL}}/\sigma_0 = \exp\left(\frac{2\gamma}{3} \alpha_s(Q^2)\right) (1 + B \alpha_s(Q^2) + \dots), \quad (4.97)$$

where the coefficient B must be calculated for every process under consideration and will depend on the choice of Q^2 .

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EXPERIMENTAL TESTS OF
QUANTUM CHROMODYNAMICS*

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1. INTRODUCTION

This is a write-up of two lectures given at the 1986 SLAC Summer Institute. The topic of the lectures was "Experimental Tests of Quantum Chromodynamics" and the level was supposed to be aimed at particle physicists who had recently received their doctorates. "Experimental Tests of QCD" is a vast topic and it is impossible to cover all the relevant measurements in two lectures. So I chose a series of topics which could be covered in this time period. My choice of topics presumably will not correspond to everybody else's favorite list, my emphasis of the relative importance of one area will differ from others. But those were my choices and I have in no way tried to make this write-up more complete than were the lectures. I have tried hard to emphasize the measurements and, in particular, how they relate to extracting reliable tests of QCD. I have not stressed the theoretical formalism – much of that area is covered in Rick Field's companion lectures.

QCD provides an $SU(3)$ gauge invariant, field theoretical formalism for the interactions of quarks and gluons. QCD forms the basis of our understanding of what we used to call the "Strong Interactions" – it is in fact a cornerstone of our Standard Model of $SU(3) \times SU(2) \times U(1)$. It is therefore crucial that we make objective and meaningful tests of QCD. However, this is not a simple matter. Perturbative QCD describes the interactions of partons (quarks and gluons) which at short distances ($\leq$ confinement radius) act as quasi-free constituents. However, we observe hadrons which are the long-distance manifestation of the confined partons. The bridge between the short distance (perturbative) and long distance (non-perturbative) behavior is not well understood theoretically and we are forced to use models – fragmentation models – for this bridge. The parton dynamics which we are trying to test must be inferred indirectly from the distribution of the hadronic fragments. There are some additional practical complications. Since we are trying to study behavior at the confinement scale, we must have probes which have large magnification or large momentum trans-

fer (Q^2). The perturbative behavior is characterized by a coupling constant α , which is both large (~ 0.2) and depends on Q^2 . This makes the calculations of the theoretical predictions difficult and, in particular, one is always confronted with the question as to the level of convergence associated with the finite order of the calculations. So there are significant complications in the experimental measurements and the theoretical predictions. The level and character of these complications is different for different tests – each must be examined separately to establish the validity or limitations provided by the particular comparison of an experimental measurement with a theoretical prediction. The upshot of this is that, in this reviewers mind, no single experimental measurement provides a conclusive test of the validity of QCD; there are no "smoking guns." Rather we are forced to consider measurements from a wide range of processes spanning a broad range of Q^2 . In this way one hopes to build confidence in the validity of QCD based on a wide range of positive indicators.

When we think about testing QCD, we need to remember the tremendous success of its "predecessor," the quark parton model (QPM) exemplified best by its ability to describe the scaling behavior of deep-inelastic lepton-nucleon scattering. It is clear that this model is inadequate to explain all the data we have to date. But much of the low Q^2 data can be interpreted in terms of this simple picture and, as a device, we will often pose the question of when the data begins to depart from this picture and whether QCD, in its role of replacing the QPM, accounts for the disagreements. We recall that the essence of the QPM is that partons in the nucleus are treated as free, point-like constituents which interact with a high Q^2 probe via their electric charge. It is clear that QCD must approximate this behavior at short distances while at the same time adding sufficient complexity to remedy the inadequacies of the QPM (like non-scaling). Part of the "magic" of QCD is the notion of confinement which provides just such a picture. The color force is such that at distances $\leq$ the confinement radius (≈ 1 Fermi), quarks act as free constituents. However, at distances larger than the confinement radius the color restoring force becomes increasingly strong and the

quarks cannot be plucked from the color field. De-excitation of the energy stored in the color field occurs via the emission of hadrons, the primary quark combining with an antiquark liberated from the vacuum, etc etc. Clearly then, the QCD coupling constant is not a constant but is a function of distance or in experimental terms a function of the magnification of the probe, Q^2 . So QCD, via this notion of confinement, provides a natural mechanism for mimicking the QPM at short distances while adding sufficient complexity to make it distinguishable from the QPM. The running of the QCD coupling constant is given in second order by

$$\alpha_s(Q^2) = \alpha_s^0(Q^2)[1 - b' \alpha_s^0(Q^2)(\ell n \ell n Q^2/\Lambda^2)]$$

where

$$\alpha_s^0 = (b \ell n Q^2/\Lambda^2)^{-1}, b = \frac{33 - 2N_f}{12\pi}$$

$$b' = \frac{(153 - 19N_f)}{2\pi(33 - 2N_f)}.$$

Here N_f is the number of quark flavors and Λ is the QCD scale parameter.

It is worth remembering that there exists a major ambiguity in the interpretation of perturbative expansions in QCD arising from the choice of the expansion parameter. Any observable ρ can take on the general form

$$\rho = C_0 \alpha_s(Q^2) \{1 + C_1(Q^2) \frac{\alpha_s(Q^2)}{\pi} + C_2(Q^2) \frac{\alpha_s^2(Q^2)}{\pi^2} + \dots\}.$$

The ambiguity arises in that the expansion coefficients $C_i(Q^2)$ depend both on the definition of $\alpha_s(Q^2)$, that is on the "scheme," and on the choice of scale, Q^2 . Of course, when working to all orders, ρ is always the same independent of the choice of scheme and scale. However at finite orders, which is what we contend with in real QCD tests, we must keep this ambiguity foremost in our minds. In first order, for example, a change in C_1 can be compensated for either by a change in α_s or Q^2 .

With this superficial introduction we now turn our attention to a discussion of the experimental tests.

2. TESTS IN THE e^+e^- CONTINUUM

The e^+e^- colliding beam facilities are excellent for testing QCD because the parton topologies are relatively simple. The basic process is the production of a quark (q) antiquark ($\bar{q}$) pair via a high Q^2 photon (see Fig. 1(a)). It is conventional to take $Q^2 = E_{cm}^2$, where E_{cm} is the total collision energy. This basic process provides direct production of the QCD constituents and in this sense the environment is very "clean." Measurements made at Q^2 in which Fig. 1(a) dominates cannot distinguish between QPM and QCD. To do that requires increasing the Q^2 until one sees the process in Fig. 1(b) which corresponds to the first order QCD process in which a gluon is radiated by the quark or antiquark. Higher level gluon radiation processes (both of the tree and virtual correction type) should become evident at even larger Q^2 (Fig. 1(c)). If this pattern of processes can be isolated in the data one would have a useful laboratory for testing QCD.

What would we expect to observe experimentally? At low $Q^2 \leq 200 \text{ GeV}^2$ Fig. 1(a) will dominate. Below $Q^2 \leq 25 \text{ GeV}^2$ one will see roughly spherical events with no obvious jet structure. This is because the kinetic energy of the q(or $\bar{q}$) is insufficient to provide a collimated set of hadrons. However as Q^2 is raised, the hadrons which result from the q(or $\bar{q}$) fragmentation process will follow the parton production direction and clear back-to-back, 2-jet structures should be seen. The production angular distribution of the jet-jet axis should be characteristic of underlying spin $\frac{1}{2}$ constituents. As one raises $Q^2 \geq 600 \text{ GeV}^2$ one moves into the regime of Fig. 1(b) and clear coplanar, 3-jet structures should be seen. The 3-jet kinematics should be consistent with the QCD matrix elements and should reflect the vector nature of the gluon. At even higher $Q^2 \geq 1000 \text{ GeV}^2$ one should begin to see evidence for the processes in Fig. 1(c) namely 4-jet topologies. As we will discuss below, this pattern of observations is quite clearly seen and QCD accounts remarkably well for the qualitative features of the data. Quantitative measurements of α_s are, however, difficult to achieve.

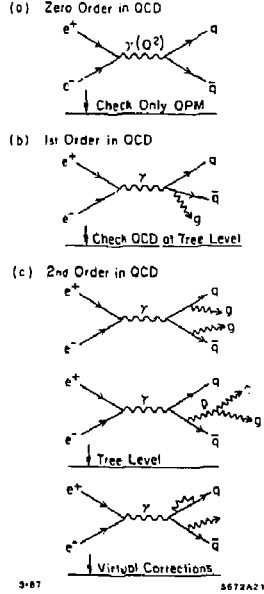


Fig. 1. Feynman diagram for the process $e^+e^- \rightarrow \text{hadrons}$ to zero order QCD(a), first order QCD(b) and second order QCD(c).

2.1 MEASUREMENT OF R

Before we proceed to the study of the $e^+e^- \rightarrow \text{hadrons}$ cross section in processes corresponding to increasing orders in perturbative QCD, we should ask why we cannot sum these pieces up and obtain a significant test of QCD from the total cross section. In the QPM, the calculation of the total cross section is straight-forward (see Fig. 1(a)). For any final state quark flavor f the differential cross section for $e^+e^- \rightarrow f\bar{f}$ is given by

$$\frac{d\sigma_{f\bar{f}}}{d\cos\theta} = \frac{\pi\alpha^2 Q_f^2}{2s} (1 + \cos^2\theta)$$

where Q_f = charge of the quark of flavor f , $s = E_{cm}^2$, θ is the polar angle of the quark relative to the incoming e^- direction, and α is the fine structure constant.

For N_f flavors

$$\sigma_{HAD} = \sum_{N_f} \int_{-1}^{+1} d\cos\theta \frac{d\sigma_{f\bar{f}}}{d\cos\theta} = \frac{4\pi\alpha^2}{3s} \sum_{N_f} Q_f^2.$$

If in fact quarks come in three colors, then

$$\sigma_{HAD} = 3 \sum_{N_f} \frac{4\pi\alpha^2}{3s} Q_f^2.$$

It is more convenient to remove the energy dependence and consider the quantity R :

$$R = \frac{\sigma_{HAD}}{\sigma_{\mu^+\mu^-}} = \sigma_{HAD} / \frac{4\pi\alpha^2}{3s} = 3 \sum_{N_f} Q_f^2.$$

This then is the QPM, or zero order QCD, result. To test QCD we need a calculation which takes into account higher orders – namely all the gluon radiation corrections. It turns out that these corrections were calculated many years ago^[1]

and are considered to be one of the least controversial and most trustworthy QCD calculations available to date. The usual term accorded the calculation of R is "gold plated." The result is

$$R(Q^2 = E_{cm}^2) = 3 \sum Q_f^2 (1 + \alpha_s/\pi + C_2(\alpha_s/\pi)^2 \dots) \quad (1)$$

where C_2 depends on the renormalization scheme. However, independent of the choice of renormalization scheme, $C_2(\alpha_s/\pi) \ll 1$ and the expansion converges. For $N_f = 5$ and $\alpha_s = 0.13$ one obtains the values for C_2 for three different renormalization schemes as shown in Table I.

Table I

SCHEME	C_2	$C_2(\frac{\alpha_s}{\pi})^2$	$C_2(\frac{\alpha_s}{\pi})^2 / \langle \frac{\alpha_s}{\pi} \rangle$
$\overline{MS}$	$1.986 - 0.115 N_f$	0.002	5%
MS	$7.359 - 0.441 N_f$	0.009	22%
MOM	$-2.193 + 0.162 N_f$	-0.002	-5%

So we have a reliable calculation of R_{QCD} . However, it differs from R_{QPM} by only $\frac{\alpha_s}{\pi} \approx 4\%$ and individual experiments have great difficulty achieving a level of precision sufficient to see such a small effect. The best experimental measurements of R come from PEP and PETRA and have measurement errors of $\gtrsim 2\%$. Where theory is in outstanding shape, the experiments are not.

Experimentally, R is an attractive quantity to study because it should be measured independently of the non-perturbative effects of hadronization. The measurement is in principle simple; one counts hadronic events ($N_{HADRONS}$, which are rather easy to isolate at high energies) and normalize to the measured luminosity ($\mathcal{L}$):

$$R = \frac{N_{Hadrons} - N_{Backg}}{A(1 + \delta)\mathcal{L}\sigma_{\mu^+\mu^-}}$$

where N_{Backg} accounts for events arising from background processes, A is the detector acceptance and δ takes into account QED radiative corrections.

The main systematic errors arise in the measurement of the luminosity (normalization of R) and the detector acceptance corrections. To illustrate these problems the systematic errors in the measurement of R by the MAC group at PEP^[2] are summarized in Table II. MAC measures $R(Q^2 = 841 \text{ GeV}^2) = 3.96 \pm 0.03 \pm 0.09$. This is typical of the precision and systematic limitations encountered by the PEP and PETRA groups. For comparison $R_{QPM} = 3.67$.

Table II

Contribution	Error(%)
Event Selection	0.36
Acceptance:	
Detector Model	1.0
QCD+Frag.	0.5
Radiative Corr(α^3)	1.0
Backgrounds	0.76
Luminosity	1.6
TOTAL	2.3

All the individual groups have labored long and hard to reduce the systematic errors, but the limitations now seem intrinsic and no significant improvement seems possible in the near future. The QED radiative corrections are included up to third order in α . The corrections to the Bhabha normalization events and the multi-hadron events cancel to some extent in the ratio R. However, it has been

estimated that a residual higher order correction (α^4) could be at the 1% level. We will discuss the resulting effect on α_s below. As a contrast to the fixed energy ($E_{cm} = 29$ GeV) measurements of MAC, Fig. 2 shows the measurements of two of the PETRA groups in the energy range from 22-36 GeV. The errors include the statistics plus the point-to-point systematic uncertainty. The TASSO data also indicate the overall normalization uncertainty. The lower curve is for the QPM model, the upper curves the prediction of QCD. The upturn of the QCD curves arises from the fact that besides the $e^+e^- \rightarrow \gamma \rightarrow$ hadrons production process one must also include the interference from the weak production $e^+e^- \rightarrow Z^0 \rightarrow$ hadrons. As with the MAC data, the conclusions are clear. The data favor the QCD prediction over the QPM but the errors make this statistically relatively weak and preclude an accurate determination of α_s via formula 1.

Recently the CELLO group at PETRA^[5] have done a slightly more systematic extraction of R using an error analysis technique which attempts to handle the point-to-point errors and the overall errors in a rigorous way. Their data is shown in Fig. 3, where as outlined before, the QCD fit contains the effect of the interference term arising from Z^0 exchange. From their data CELLO obtains:

$$(a) \alpha_s(Q^2 = 1156 \text{ GeV}^2) = 0.19 \pm 0.05$$

$$\sin^2\theta_W = 0.20 \pm 0.03$$

and fixing $\sin^2\theta_W = 0.23$,

$$(b) \alpha_s(Q^2 = 1156 \text{ GeV}^2) = 0.16 \pm 0.05.$$

The χ^2 for the fit to (a) is 3 for 7 degrees of freedom. In order to improve the error on α_s , CELLO has used the same procedure to fit the data^[4] from the MAC, PLUTO, CELLO, JADE, MARK J and TASSO experiments (Fig. 4) to obtain R and find

$$(a) \alpha_s(Q^2 = 1156 \text{ GeV}^2) = 0.165 \pm 0.04$$

$$\sin^2\theta_W = 0.236 \pm 0.020$$

and fixing $\sin^2\theta_W = 0.23$,

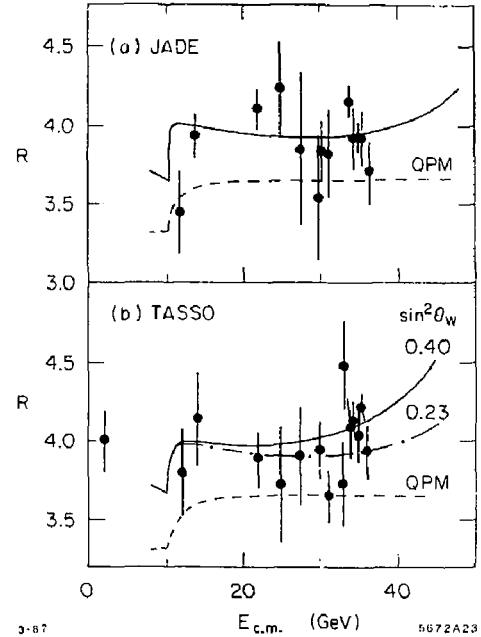


Fig. 2. Measurements of $R = \sigma_{\text{hadrons}}/\sigma_{\mu^+\mu^-}$ as a function of E_{cm} from JADE(a) and TASSO(b). The expectation of the QPM is shown as well as for QCD including electroweak effects.

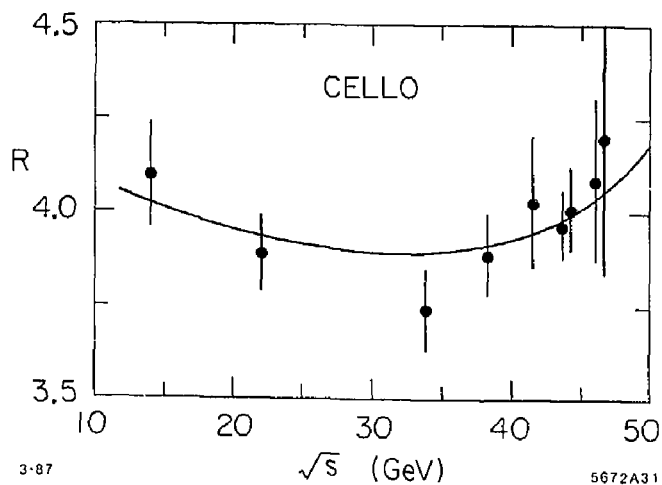


Fig. 3. Data on R from the CELLO group. Also indicated are the expectations from QPM, QCD and the electroweak interference.

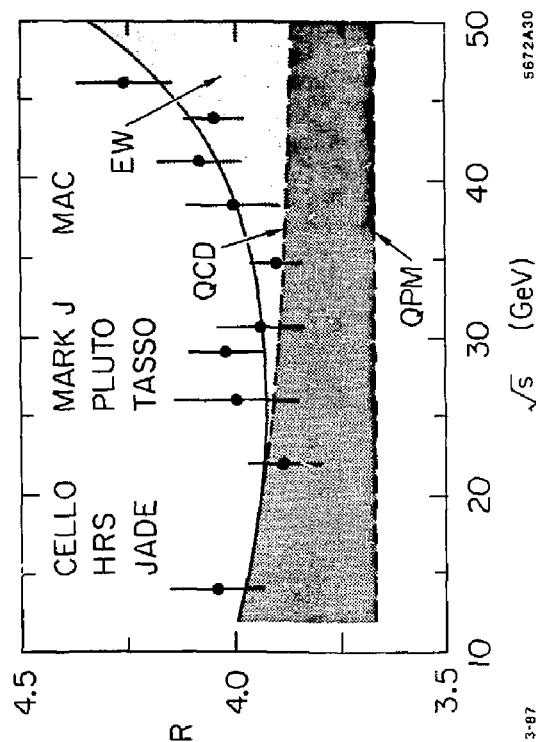


Fig. 4. Same as for Figure 3 except all the data from the experiments indicated are combined together (see Ref. 3).

$$(b) \alpha_s(Q^2 = 1156 \text{ GeV}^2) = 0.169 \pm 0.025.$$

As noted by CELLO, a 1% change in the QED radiative corrections due to higher orders would yield $\alpha_s = 0.145 \pm 0.024$ which implies a potential systematic error from this source comparable to the experimental errors.

I would conclude the R measurement discussion as follows:

1. The data do consistently support a value of R in good agreement with QCD and always larger than QPM. Individual experiments have a difficult time making a meaningful measurement of α_s .
2. If one accepts the inherent jeopardy of combining experiments (their acceptance and radiation correction software are not entirely independent) then a meaningful measurement of α_s results. With the possible exception of having $O(\alpha^4)$ QED correction calculations, we have probably reached the systematic limit of the measurement in this Q^2 range.
3. Although trivial by now, it is worth remembering that from R we clearly see that quarks come in three colors. The QPM and QCD curves shown here all assume three colors. For this non-trivial test of QCD, R is a powerful measurement.

2.2 QUALITATIVE TESTS USING SHAPE PARAMETERS

We now return to the Q^2 dependance of the event shapes to get some more detailed qualitative and quantitative tests of QCD. We will discover that a very consistent picture evolves with QCD unfolded order by order with rather impressive qualitative agreement.

There are many shape parameters and each of the groups uses its favorite one. I will not attempt to illustrate all effects with each variable. Suffice it to say that the conclusions are independent of the shape parameter/procedure used. In addition, I will not explain each shape parameter formalism but rather will refer the interested reader to the excellent review of Sau Lan Wu.^[3] In fact, all

the data in this section can be found in Ref. 5 with a more complete discussion and complete experimental references. The most familiar shape parameter used is sphericity. The sphericity axis is the symmetry axis which is chosen such that the sum of the transverse momentum squared of all tracks measured with respect to that axis is minimized. For i detected particles each with momentum $\mathbf{p}^i$ and momentum transverse to the event axis $\mathbf{P}_T^i$ the quantity S is defined as

$$S = 3/2 \frac{\sum_i (\mathbf{P}_T^i)^2}{\sum_i (\mathbf{P}^i)^2}. \quad (2)$$

The event axis becomes the sphericity axis and S the sphericity when (2) is minimized. So, spherical events have $S \rightarrow 1$, while 2-jet, cigar shaped events have $S \rightarrow 0$. In the sphericity analysis one can define an event plane which is the plane which contains the sphericity axis and has a normal for which the sum of the particle momenta, projected along this normal, are a minimum. As mentioned above the rigorous, mathematical derivation of the momentum tensor/sphericity technique is covered in Ref. 5.

All the analyses discussed in this section involve selecting events of the form $e^+e^- \rightarrow \text{hadrons}$. I will not discuss in detail how this is done for each experiment. It is sufficient to point out that these events can be isolated with high efficiency and low backgrounds. The systematics of this selection process will have no significant effect on any of the conclusions drawn from the analyses. The features most commonly used are large multiplicity (≥ 5 is typical, thus removing backgrounds from leptonic pair production) and relatively large detected energy ($\geq 0.20 E_{cm}$ to remove backgrounds from two photon processes). Typical efficiencies for these cuts are $\geq 70\%$ with background contamination of $\leq 2\%$.

The phenomena of jet production in e^+e^- annihilation was first discovered at SPEAR by the MARK I collaboration^[4] by observing the decrease in the sphericity with increasing E_{cm} in the range 3-7.4 GeV. In addition, the underlying jet axis was seen to follow a $(1 + \alpha \cos^2 \theta)$ distribution with $\alpha = 0.78 \pm 0.12$. The 2-jet structure becomes much more apparent at higher energies; at 14 GeV the

PETRA hadronic events are dominated by clearly visible back-to-back jets. The sphericity axis shows a $1 + \cos^2\theta$ distribution characteristic of spin- $\frac{1}{2}$ constituents. (Spin 0 constituents would give a $\sin^2\theta$ angular distribution.) The decrease in sphericity with increasing E_{cm} is shown for the SPEAR and PETRA (TASSO) data in Fig. 5 while the angular distribution of the sphericity axis at $E_{cm} = 34$ GeV is shown in Fig. 6.

At this level of magnification ($Q^2 \leq 200 \text{ GeV}^2$) we see that spin- $\frac{1}{2}$ constituents are clearly being produced consistent with the QPM and QCD. To differentiate between these two options we need to raise Q^2 and look for gluon radiation as shown in Fig. 1(b). For the topology of a hard gluon bremsstrahlung off the $q(\bar{q})$, coplanar 3-jet events should be seen. This is exactly what is seen both at PETRA and PEP. Each group uses its own analysis technique to demonstrate the presence of 3-jet events and quantify the agreement with QCD (see Ref. 5). Irrespective of the method, QCD does an excellent job of accounting for the observed distributions via the inclusion of the tree level branching diagram (Fig. 1(b)). Examples of clear 3-jet topologies are shown in Fig. 7.

To see the onset of the gluon radiation I show data from TASSO and MARK J. (The other PETRA experiments come to identical conclusions.) TASSO uses the sphericity analysis. Shown in Fig. 8 are the average $\langle P_t^2 \rangle_{out}$, $\langle P_t^2 \rangle_{in}$ for particles in hadronic events where *in* and *out* refer to the momentum component directions with respect to the sphericity event plane. What one sees is that in going from low center-of-mass energies (2-jet dominated) to higher center-of-mass energies, the $\langle P_t^2 \rangle_{out}$ does not change but the $\langle P_t^2 \rangle_{in}$ changes significantly. The distribution of particle momenta in the event plane relative to the sphericity axis have developed a "long tail." This tail cannot be accounted for by assuming a 2-jet model in which the typical transverse momentum in the jet fragmentation (σ_0) grows with E_{cm} . Such a model does not fit the data. One reproduces the distributions in Fig. 8 very well with first order QCD which produces planar 3-jet events which do not add to the $\langle P_t^2 \rangle_{out}$ but will broaden the P_t distribution in the event plane. The exact same conclusions can be drawn from the MARK J data

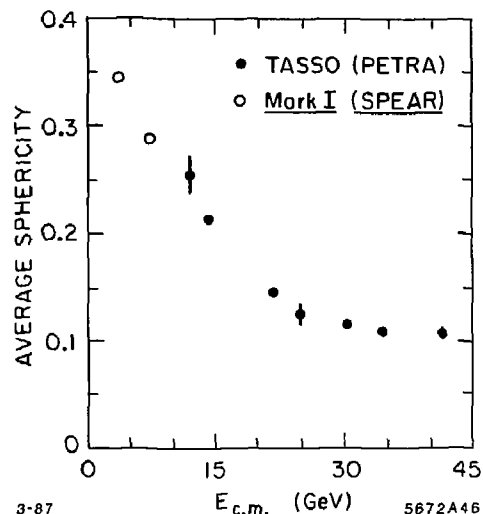


Fig. 5. The average sphericity in hadronic events for data from SPEAR (Mark I) and PETRA (TASSO). The reduction of $\langle S \rangle$ with increasing energy clearly indicates the increasing domination of 2-jet events.

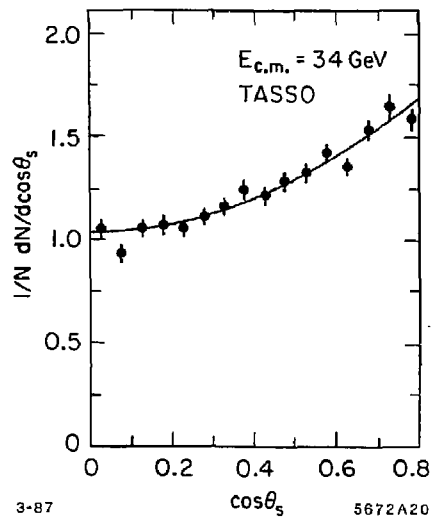


Fig. 6. The production angular distribution for the jet axes as reconstructed using the sphericity axis. The curve shows the expectation for spin- $\frac{1}{2}$ constituents, namely a $(1 + \cos^2\theta_s)$ distribution.

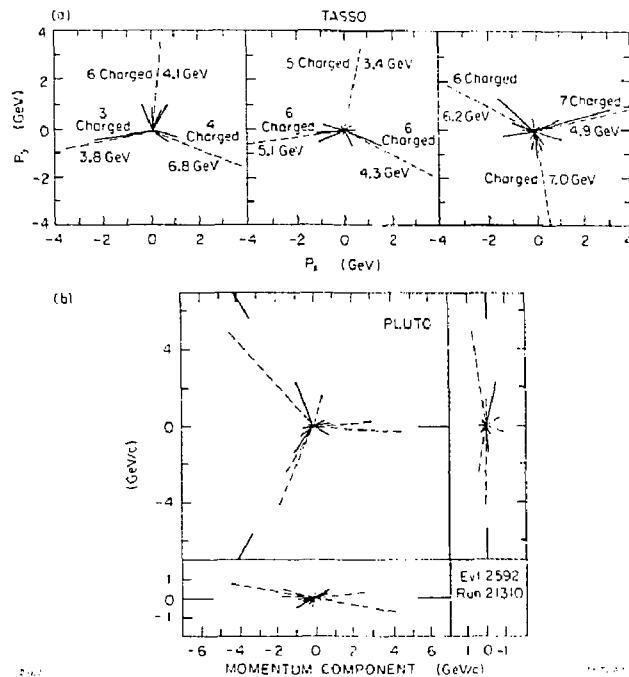


Fig. 7. Examples of 3-jet events at PETRA.

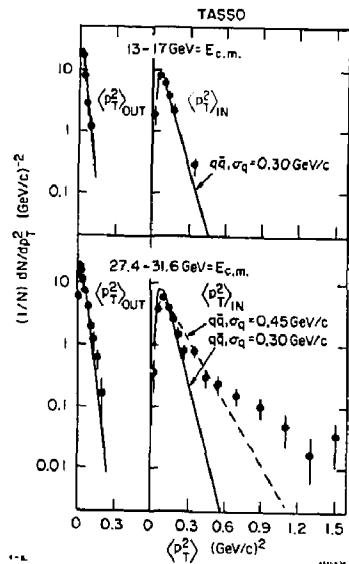


Fig. 8. The growth of the momentum in the event plane as a function of increasing center-of-mass energy is shown. The corresponding momentum component normal to the event plane shows very little growth. The curves are the predictions of 2-jet models in which σ_q represents the mean transverse momentum which characterizes the quark fragmentation. The growth in $\langle P_T^2 \rangle$ cannot be explained by such a model. This growth arises because of the presence of 3-parton events of the type $q\bar{q}g$.

(Fig. 9) where the shape variable oblateness is used. Again, simple extensions of 2-jet models do not account for the data, while first order QCD does an excellent job. So the conclusion is clear: the change in event shapes is going from $E_{cm} \approx 17$ GeV to $E_{cm} \approx 35$ GeV cannot be fit by a 2-jet hypothesis. These changes look qualitatively like those which are generated by a bremsstrahlung type process ($1/k^2$). First order QCD does a good job of reproducing the observed effects.

Further qualitative examples of QCD tests in this energy range involve testing the QCD predictions for the Dalitz plot distributions for 3 parton (jet) final states. First order QCD predicts

$$\frac{1}{\sigma_0 dx_1 dx_2} = \frac{2\alpha_s}{\pi} \left\{ \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} \right\} + 1, 2, 3 \text{ cyclic permutations} \quad (3)$$

where $x_i = 2E_i/E_{cm}$ and E_i = parton energies. To check these predictions requires reconstructing 3-jet events. Again there are several methods used - triplicity, trijettiness, cluster finding (see Ref. 5). The details of these methods are not important here, the conclusions are independent of the method. Each method chooses a sample of 3-jet events in which each particle is assigned to one of the 3 jets (see Fig.10). The jet directions are calculated from the momentum vectors of the particles assigned to the jets. The jet energies are then calculated from the jet directions assuming the partons are massless. The jet energies are ordered such that $x_1 > x_2 > x_3$. With a sample of 3-jets so selected, one can test QCD via the Dalitz plot distributions characterized by (3). The differential distribution dn/dx_1 is shown in Figs. 11 and 12 for CELLO and JADE data along with the predictions of QCD. CELLO shows curves for first order QCD using a vector gluon and first order QCD using a scalar gluon. Clearly the vector gluon is preferred and QCD does an excellent job of reproducing the data. JADE shows the prediction of both first and second order QCD. Again, QCD reproduces the data well. The TASSO group uses a slightly different variable to test the QCD matrix element. They transform the 3-jet events (along the axis x_1) into the rest frame of x_2 and x_3 . They then look at the decay angular distribution of the x_2^*

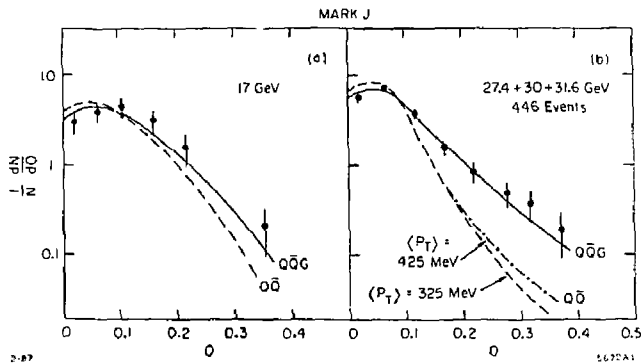
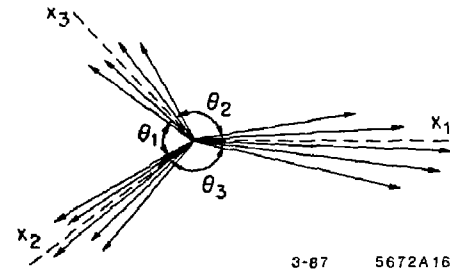


Fig. 9. The onset of 3-jet events seen in Fig. 8 is illustrated here using the variable oblateness. Again 2-jet models cannot account for the data. The predictions of first order QCD ($Q\bar{Q}G$) are explicitly shown and account well for the data.



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Fig. 10. Parameters of an idealized 3-jet event are shown where the arrows represent the particle momenta. The fractional energy of each jet is $x_i = 2E_i/E_{cm}$ and θ_i is the angle opposite the jet x_i .

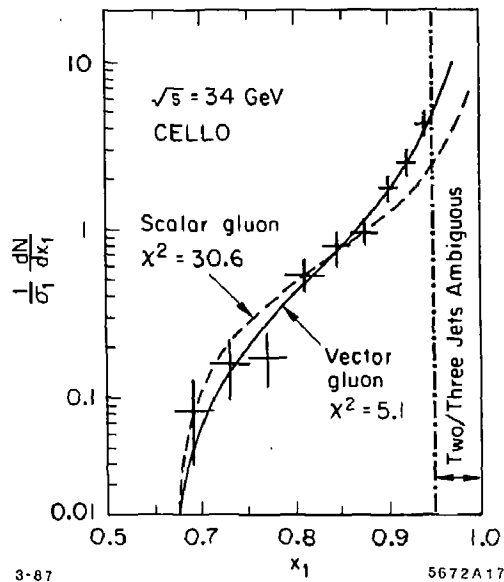


Fig. 11. The distribution of the fractional momentum of the highest energy jet in 3-jet events is shown at $E_{cm} = 34$ GeV (CELLO data). A cut at $x_1 < 0.9$ is made to select clear 3-jet topologies. QCD (solid line) gives a good representation of the data while QCD with a scalar gluon does not.

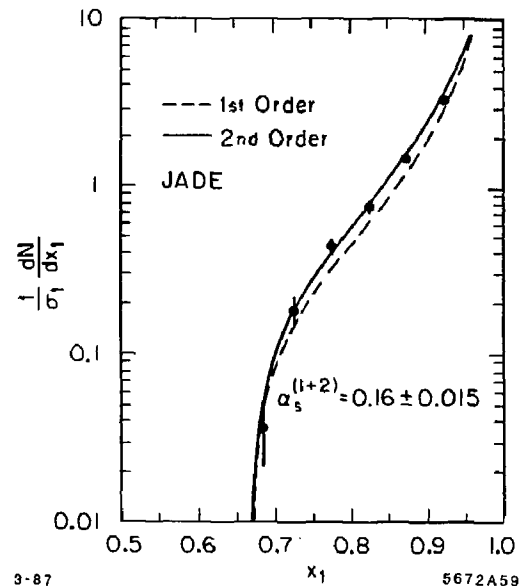


Fig. 12. The distribution of the fractional momentum of the highest energy jet in 3-jet events is shown (JADE data). The predictions of first and second order QCD are indicated. The second order prediction is a better representation of the data.

(x_3) parton (see Fig. 13). One finds that this angle, $\tilde{\theta}$, is related to the x_i 's via

$$\cos \tilde{\theta} = \frac{x_2 - x_3}{x_1}.$$

The distribution of $\cos \tilde{\theta}$ is shown in Fig. 14 with the QCD predictions for scalar and vector gluons. Again the scalar hypothesis is ruled out; the vector hypothesis gives a good description of the data.

At energies above 30 GeV at PETRA one sees direct evidence for 4-jet events – hence unfurling yet another order in QCD. Data from the JADE group (Fig. 15) show that a better accounting of the multijet topologies is obtained (L234 rather than L23) when a statistically significant 4-jet piece is added to the QCD cross section. From a fit to the 33 GeV data, JADE finds the percentage of 2, 3 and 4 cluster events to be $56.1 \pm 0.4\%$, $40.2 \pm 0.4\%$ and $3.75 \pm 0.16\%$, respectively.

We can close this section by noting that the agreement between event distributions in high energy e^+e^- collisions and the predictions of QCD is *non-trivial*. There exists no other model which can reproduce all aspects of the data. Many other models have been tried – QPM (2-jet models) with a wide variety of quark fragmentation scenarios, fire string models, etc.^[7] These other models can often be tuned to fit some aspects of the data, but never all aspects simultaneously. So if one asks, “Does the e^+e^- data require something beyond the QPM+ fragmentation,” the answer is a definitive “yes.” In addition, the qualitative features are well accounted for by QCD+fragmentation models, and there exists no other model which can claim such qualitative agreement with the data.

2.3 QUANTITATIVE TESTS FROM EVENTS SHAPES

We turn our attention now to the question of how well one can extract α_s (or Λ) from the e^+e^- event shapes? It is well known that this has been an uphill struggle and perhaps, after many years of theoretical and experimental study and toil, a clearer picture has emerged. About two years ago most of

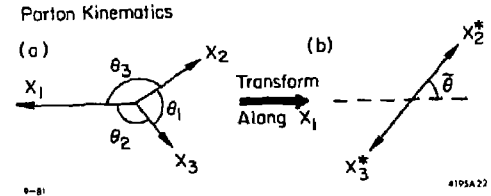


Fig. 13. (a) The kinematics for the process $e^+e^- \rightarrow q\bar{q}g$ where $x_i = E_i/E_{cm}$ and E_i are the parton energies. (b) The definition of the Ellis-Karliner angle, $\tilde{\theta}$.

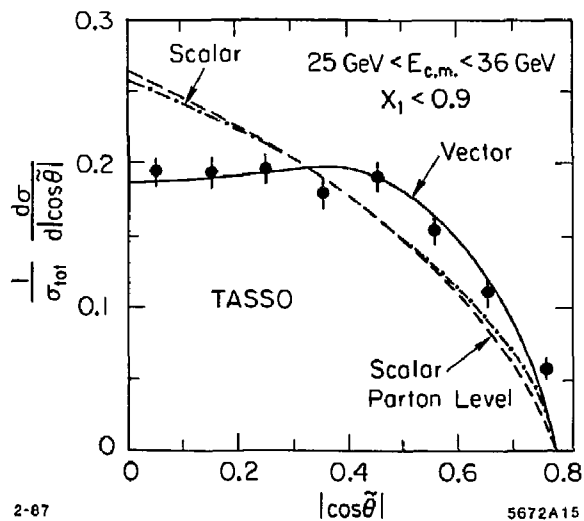


Fig. 14. The distribution of the Ellis-Karliner angle $\tilde{\theta}$, for the 3-jet events with $x_1 < 0.9$ obtained by the TASSO group. The solid curve is QCD with $\alpha_s = 0.17$, the dashed curve is the prediction for a scalar gluon.

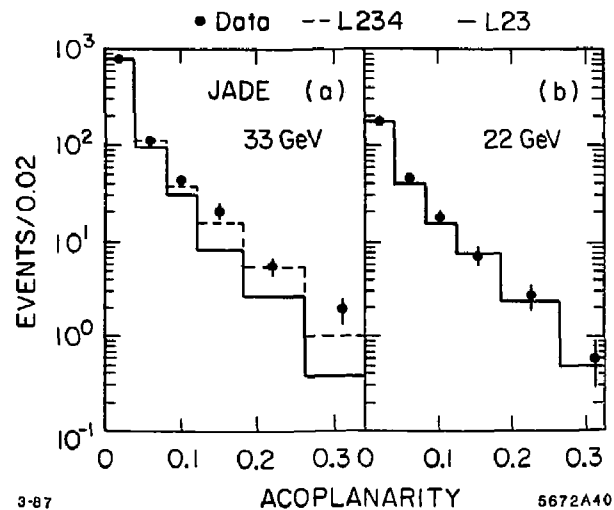


Fig. 15. Data from JADE which show that at lower energies ($E_{cm} \approx 17$ GeV) the acoplanarity distribution is well fit by a QCD model with 2 and 3 parton states only (L23). At higher energies this is no longer true and 4 parton states (L234) need to be added to obtain a good fit to the data.

the experimental groups and reviewers characterized the α_s determinations from PETRA/PEP as dependent on fragmentation effects and the schemes used for implementing $O(\alpha_s^2)$ QCD.^[1] But since then there have been some theoretical and experimental insights and more recent reviewers now feel that there is a reliable procedure which gives consistent results with significant precision. The favored procedure is the Energy-Energy Correlation Asymmetry with higher order QCD effects incorporated using the so-called ERT scheme. Where do the problems in measuring α_s arise?

The first problem is with the cutoff procedure required to distinguish different parton multiplicities. Roughly speaking, a measurement of α_s is a measurement of the ratio of the number of 3-jet events to 2-jet events. To do this one needs to specify both theoretically and experimentally what belongs to each class. When a gluon, in a $q\bar{q}g$ topology, is collinear with the quark (or antiquark) or has a vanishingly small momentum, this 3 parton topology becomes indistinguishable from the 2 parton topology. This creates an ambiguity about which parton topology the event belongs to. To avoid this problem a cutoff procedure is established which defines clearly how to resolve these ambiguous topologies. Problems arise when the α_s extracted from the data is sensitive to the cutoff procedure.

Two cutoff methods are used: a) Stermen-Weinberg (ϵ, δ) and b) parton invariant mass (y_{min}). Two partons are considered resolvable if:

- (a) Stermen-Weinberg: The energies of both partons are $> \epsilon E_{cm}/2$ and the partons are separated by an angle greater than δ .
- (b) Parton invariant mass: For parton momenta P_i and P_j $(P_i + P_j)^2 > y_{min} E_{cm}^2$.

For $O(\alpha_s^2)$ tests we have to include the graphs shown in Fig. 16 and hence we must use the cutoff procedure to partition the events into 2, 3 and 4 parton states. These $O(\alpha_s^2)$ diagrams were originally calculated by three groups:

1. Ellis, Ross and Terano (ERT).^[6]

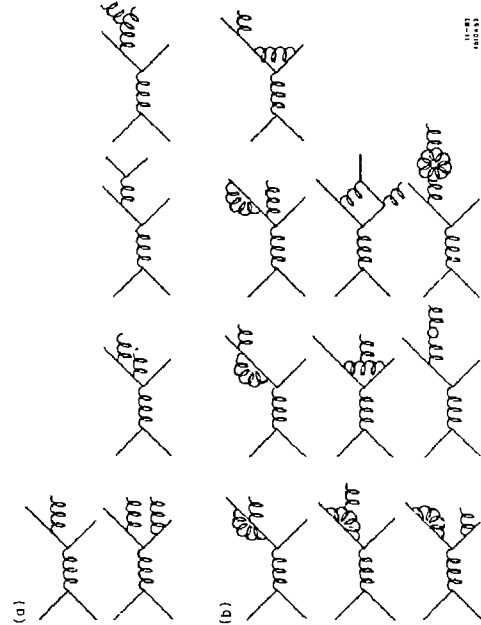


Fig. 16. (a) Tree level diagrams for 3 and 4 parton final states and (b) second order virtual parton contributions.

2. Vermaseren, Gaemers and Oldham (VGO).^[10]

3. Fabricius, Kramer, Schierholz and Schmitt (FKSS).^[11]

Methods 1) and 2) are exact and are calculated at the bare parton level while 3) is approximate incorporating jet resolution implicitly. Methods 1) and 3) have been widely used at PETRA. As discussed later, an improved version of FKSS was implemented.^[12] We refer to it here as the extended FKSS or FKSS'.

A second problem with α_s determinations arises from the fact that QCD makes calculations at the parton level and experimentalists make measurements at the hadron level. To relate experiment to theory inevitably involves a model of perturbative QCD+fragmentation. This leads to the important question about biases which result from the use of these models. For some choices of variables the fragmentation effects mask the perturbative effect which one is trying to measure. With each procedure care must be taken to investigate the effect of hadronization on the ability to extract α_s .

There have been many methods used at PETRA and PEP to measure α_s . Physicists have argued (still do) at great length about which methods are the most reliable. Most methods have significant problems - the principle one being sensitivity to fragmentation. The sceptic should look at all these methods; however, many now seem to agree that the analysis method of choice is the Energy-Energy Correlation Asymmetry (EECA). The main reasons are:

1. The EECA behaves better in second order perturbative theory than any other variable studied (i.e. $O(\alpha_s^2)$ corrections are small).
2. It is infrared stable and, hence, relatively insensitive to the choice of cutoff parameters.
3. For $E_{cm} \geq 30$ GeV fragmentation effects from $q\bar{q}$ are small. (However fragmentation effects from $q\bar{q}g$ are still significant.)

Table III summarizes the sensitivity of some commonly used variables to the inclusion of $O(\alpha_s^2)$ terms.^[13] (Taken from Ref. 13 using ERT matrix elements.) The smaller the K factor the smaller the $O(\alpha_s^2)$ corrections.

Table III

VARIABLE	INTEGRATION RANGE	$K = O(\alpha_s^2)/O(\alpha_s)$
Thrust	$T < 0.85$	18.9
Oblateness	$0 > 0.30$	3.5
EEC	$ \cos\chi < 0.75$	12.3
EECA	$ \cos\chi < 0.75$	3.1

The Energy-Energy Correlation^[14] involves using hadronic events to study the energy weighted cross section

$$\frac{1}{\sigma} \frac{d\sigma}{d\cos\chi} = \frac{1}{N} \frac{1}{\Delta(\cos\chi)} \sum_{i,j} \frac{E_i E_j}{E_{VIS}^2} \delta(\cos\chi_{ij} - \cos\chi)$$

where the sum ranges over all N events including all particles pairs i and j with energies E_i and E_j , and E_{VIS} is the detected energy in the event. The particles i and j are separated by the angle χ_{ij} . In the absence of any transverse momentum in the fragmentation, 2-jet events will give rise to peaks at $\cos\chi \approx \pm 1.0$. The presence of this hadron P_1 will provide correlations at other values of $\cos\chi$. Gluon emission will contribute an asymmetry to the energy-energy correlation. To isolate such an emission one studies the asymmetry (EECA)

$$A(\cos\chi) = \frac{1}{\sigma} \left\{ \frac{d\sigma}{d\cos\chi}(\pi - \chi) - \frac{d\sigma}{d\cos\chi}(\chi) \right\}.$$

When using $A(\cos\chi)$ to extract α_s it is customary to exclude the forward and backward regions which tend to be strongly influenced by quark fragmentation effects. Typically the region for $|\cos\chi| > 0.75$ is excluded. Table IV shows a

compilation of α_s measurements made using EECA indicating the method used for implementing the $0(\alpha_s^2)$ QCD and the sensitivity of the measurement to the non-perturbative models, SF and IF. The SF and IF models stand for string fragmentation (LUND-type model)^[15] and independent fragmentation (Ali et al.^[16] or Hoyer et al.^[17] type models), respectively. The data came from TASSO,^[18] CELLO,^[19] JADE,^[20] PLUTO,^[21] MARK J,^[22] and MAC.^[23] Looking at the table one sees several clear trends. Here α_s extracted using string fragmentation models is always larger than from independent fragmentation models. The same is true of FKSS (or extended FKSS) relative to ERT. As discussed in the TASSO paper by Gottschalk and Shatz^[24] (in collaboration with Guthrod and Shierholz) the FKSS scheme leads to α_s values which are 15-20% too large. The main contributions to this overestimate arise from the cutoff procedure used ($\approx 10\%$) and, neglecting terms of order ϵ and δ^2 (or y) ($\approx 10\%$). The extended FKSS scheme (FKSS') inserts missing four parton states which reduces this latter problem by about a factor of 2. It appears as if the ERT scheme does not suffer the problems of the FKSS scheme and we should focus our attention then on the measurements obtained with this scheme.

Table IV

Group	Ref	Year	QCD	α_s (SF)	α_s (IF)
MARK J	22	83	ERT(ϵ, δ)	0.14	0.12
CELLO	19	83	FKSS(y)	0.19	0.15-0.13
JADE	20	84	FKSS(y)	0.165	0.14-0.11
TASSO	18	84	ERT(ϵ, δ)	0.159	0.127-0.117
TASSO	18	84	FKSS'(ϵ, δ)	0.19	0.157-0.139
PLUTO	21	85	ERT(ϵ, δ)	0.145	0.136
MAC	23	85	FKSS(y)	0.185	0.14-0.11

Figures 17 and 18 show that the second order QCD corrections are under control for EECA measurements, namely that there is not a strong E_{cm} dependence. Also shown in Fig. 17 is the contribution to the EECA arising from $q\bar{q}$ fragmentation. Above $E_{cm} = 30$ GeV $q\bar{q}$ fragmentation has a negligible effect when a sensible $|\cos\chi|$ cut is used. Figure 19 demonstrates the sensitivity of the EECA measurement with respect to variations in the cutoff parameter ϵ . Similar stability for the ERT scheme is obtained for reasonable variations of δ at fixed ϵ .

The data used for the α_s determinations using the EECA and ERT scheme are shown in Figs. 20 (PLUTO), 21 (TASSO), 22 and 23 (MARK J).^[22] In all cases the limiting systematic error in α_s comes from the fragmentation models and is about 0.02. In all cases the Monte Carlo predictions for SF and IF agree well with the data. In fact for the MARK J and PLUTO data, the predictions of the models are indistinguishable in the plots. As discussed above, they agree, but with systematically different values of α_s . The results of these measurements are quoted as:

$$\begin{aligned}
 \text{PLUTO}(35 \text{ GeV})^{21}: \quad \Lambda_{\overline{MS}} &= 112 \pm 23 + 25 \text{ MeV} \quad (\text{Fit to QCD}) \\
 &= 183 \pm 31 \text{ MeV} \quad (\text{IF}) \quad (\alpha_s \approx 0.136) \\
 &= 259 \pm 40 \text{ MeV} \quad (\text{SF}) \quad (\alpha_s \approx 0.145) \\
 \text{TASSO}(35 \text{ GeV})^{18}: \quad \alpha_s &= (0.127-0.117) \pm 0.01 \quad (\text{IF}) \\
 &= 0.159 \pm 0.012 \quad (\text{SF}) \\
 \text{MARK J}(44 \text{ GeV})^{22}: \quad \Lambda_{\overline{MS}} &= 60 \pm 12^{+25}_{-26} \text{ MeV} \quad (\text{IF}) \\
 &= 150 \pm 30^{+50}_{-40} \text{ MeV} \quad (\text{SF})
 \end{aligned}$$

or in summary:

$$\begin{aligned}
 \Lambda_{\overline{MS}} &= 100 \pm 30^{+60}_{-45} \\
 \alpha_s &= 0.12 \pm 0.02 \\
 \text{MARK J}(35 \text{ GeV})^{22}: \quad \alpha_s &= 0.13 \pm 0.02
 \end{aligned}$$

One may conclude from these measurements that in the 35-44 GeV region, α_s lies between 0.12-0.16 giving $\Lambda_{\overline{MS}}$ in the range of 100-300 MeV.

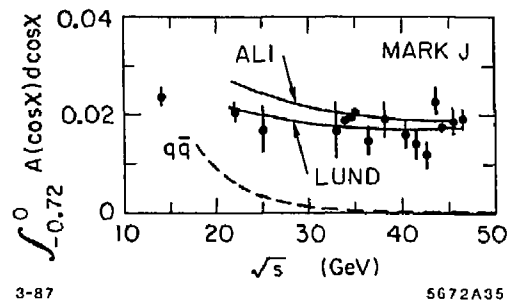


Fig. 17. The energy dependence of the integrated FECA is shown using MARK J data, along with the contribution from $q\bar{q}$ (dashed curve) and the prediction of the LUND and ALI Monte Carlo simulations.

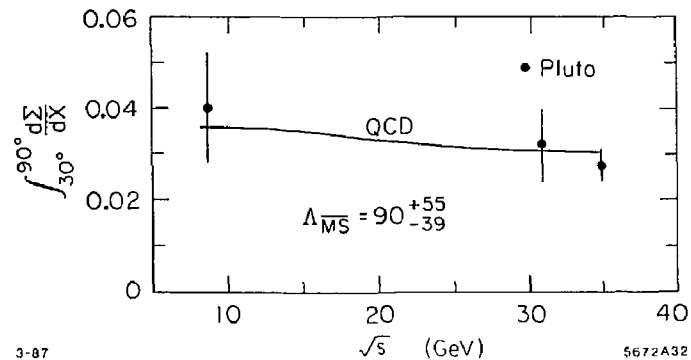


Fig. 18. The energy dependence of the integrated EECA is shown using the PLUTO data. Also shown is the prediction of QCD.

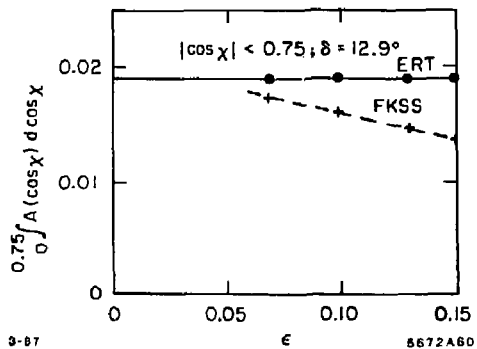


Fig. 19. Sensitivity of the integrated EECA to the cutoff parameter ϵ . Solid line is in the ERT scheme, dashed line the FKSS scheme.

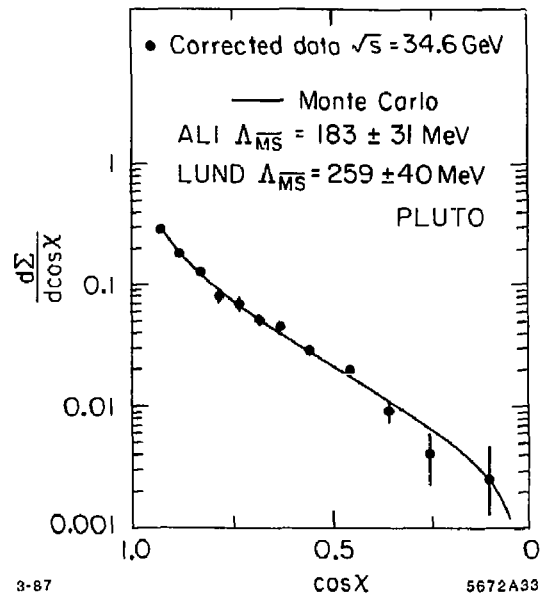


Fig. 20. The EECA as a function of $\cos \chi$ as measured by PLUTO at $E_{cm} = 34.6$ GeV. The solid line is the fit to the LUND and ALI models.

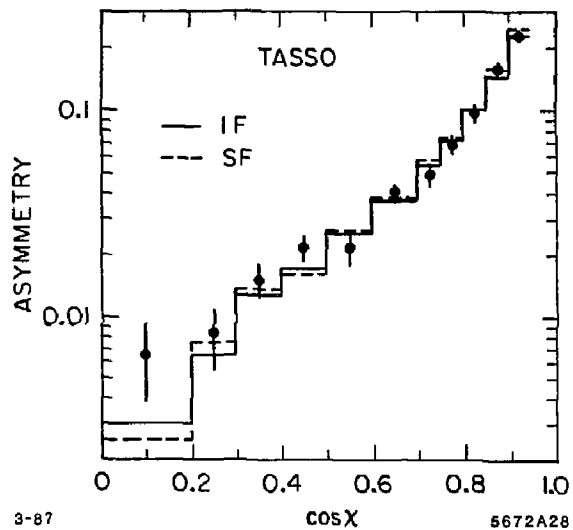


Fig. 21. The EECA as a function of $\cos\chi$ as measured by TASSO at $E_{cm} = 34.6$ GeV. The solid (dashed) line is the prediction of the ALI (LUND) model fit to the data for $|\cos\chi| < 0.7$.

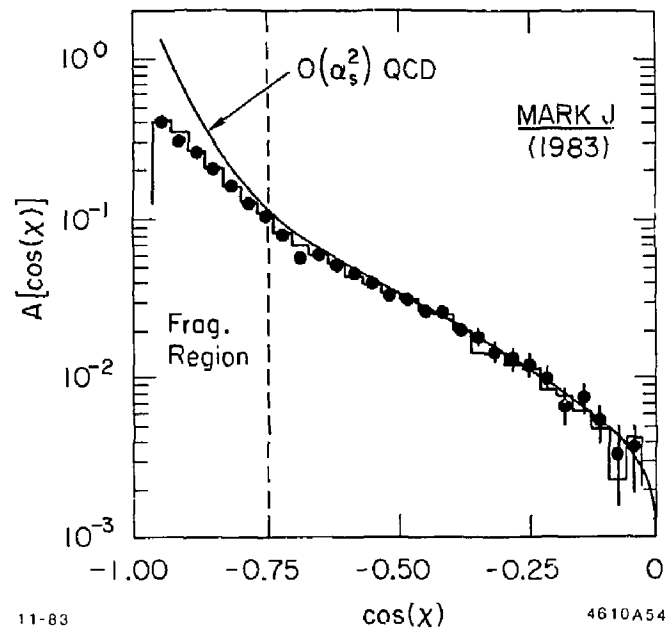


Fig. 22. The EECA as a function of $\cos\chi$ as measured by MARK J at $E_{cm} = 34.6$ GeV. The solid line is the fit to the LUND and ALI model.

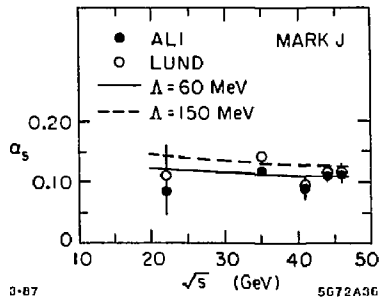


Fig. 23. The coupling constant α_s as measured by MARK J using the EECA. The solid points (open circles) are obtained using the ALI (LUND) model. Curves for Λ of 60 and 150 MeV are also shown.

2.4 DO GLUONS FRAGMENT DIFFERENTLY THAN QUARKS?

QCD predicts that high energy gluon jets should have a "softer" fragmentation function than quarks of the same energy. This prediction goes to the very heart of QCD, namely it arises from the fact that QCD is non-Abelian and, hence, the gluon has a self coupling. Referring to Fig. 24, color factors conspire to make the ggg vertex $9/4$ times "stronger" than the qqg vertex. It has been shown^[26] that for a highly perturbative topology (namely when the gluon/quark are sufficiently high energy) the jet opening angle δ (ala the Sterman-Weinberg definition) follows the relation

$$\delta_{\text{gluon}} = \delta_{\text{quark}}^{4/9}$$

where δ is measured in radians. So for example at $E_{\text{cm}} \approx 30$ GeV, $\delta_{\text{quark}} \approx 30^\circ$ would imply $\delta_{\text{gluon}} \approx 40^\circ$ while at SLC/LEP energies $\delta_{\text{quark}} \approx 10^\circ$ and $\delta_{\text{gluon}} \approx 25^\circ$. What evidence do we have for such effects?

The first evidence for softer gluon fragmentation was presented by JADE^[27] in 1983. They used 3-jet events to show that the average hadron transverse momentum, $\langle P_t \rangle$, was larger for hadrons in gluon jets than in quark jets of the same energy. A sphericity analysis was performed on the data and cuts ($Q_1 < 0.06$, $Q_2 - Q_1 > 0.07$) were made to select planar, 3-jet events. Particles in these planar events were assigned to one of three jets using the method of triplicity. The jet directions were then calculated from the vector sum of the particles which constitute each jet. Because jet directions are better measured than jet energies, the jet energies were calculated from the jet directions on the assumption that the partons are massless. These energies, E_j^4 , were then ordered such that $E_1^4 > E_2^4 > E_3^4$. Events having a jet with less than four particles or an observed energy of less than 2 GeV were removed from the sample. Both charged and neutral particles were used in the analysis. At 33 GeV the Monte Carlo models for QCD indicate that the probabilities that jets #1, #2 and #3 are the gluon are 12%, 22% and 51%, respectively and 9%, 20% and 34% at 22

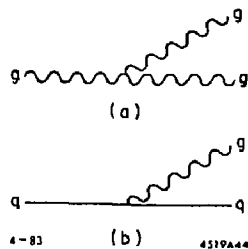


Fig. 24. The contrasting "strengths" of the triple gluon vertex and the quark-gluon vertex.

GeV. The sum of these three probabilities is not 100% because the 3-jet sample is contaminated by $q\bar{q}$ events. The thrust of the analysis is to compare jets of the same energy but with different gluon content.

Figure 25 shows the $\langle P_T \rangle$ measured relative to the jet axes for the three jets. Data are from the 22 and 33 GeV energy regions. The data are not corrected for detector biases, and both neutral and charged tracks enter into the plot. Jet #2 has a smaller $\langle P_T \rangle$ than jet #3. The Monte Carlo models predict that the gluon content of jet #2 is ~25% and that of jet #3 is ~50% for $6 < E_{2,3}^4 < 10$ GeV. The data in this jet energy region are plotted in Fig. 26 in terms of P_T . In the region of $0.2 \text{ GeV}/c < P_T < 1.5 \text{ GeV}/c$ the data were fit with $d\sigma/dP_T \propto \exp(-A_j P_T)$ and the ratio of $A_2/A_3 = 1.13 \pm 0.04$ was found indicating the jet with higher gluon content has a larger $\langle P_T \rangle$. Using charged particles only, $A_2/A_3 = 1.10 \pm 0.05$.

We return now to Figs. 25(b)-25(d). Two models were used for comparison with the data. The result of the LUND model is shown in Fig. 25(d). The model used in Figs. 25(b) and 25(c) is the independent fragmentation model of Hoyer et al.¹⁷ In Fig. 25(b) the quarks and gluons fragment identically, namely $\sigma_q = \sigma_g = 330 \text{ MeV}/c$, whereas in Fig. 25(c) $\sigma_q = 330 \text{ MeV}/c$ but $\sigma_g = 500 \text{ MeV}/c$, namely the gluon is assigned a larger primordial P_T . From Figs. 25 and 26 one can conclude that events which are gluon enriched have a larger hadron $\langle P_T \rangle$ indicating a softer fragmentation.

Recently this result has been confirmed by the Mark II group using a rather different approach. Mark II^{18a} capitalized on their large hadronic sample at $E_{cm} \approx 29 \text{ GeV}$ to select 3-jet events which were 3-fold symmetric – namely which were close to the orientation in which each jet was separated from its neighbors by 120° . Under these conditions one would expect each jet (parton) to carry an equal energy of $1/3 E_{cm}$. In addition, one would expect one-third of the jets to be gluons, two-thirds quarks.

The 3-jet events were found using a cluster algorithm. The requirement of 3-fold symmetry was that all inter-jet angles δ_j satisfy $100^\circ < \delta_j < 140^\circ$. A total

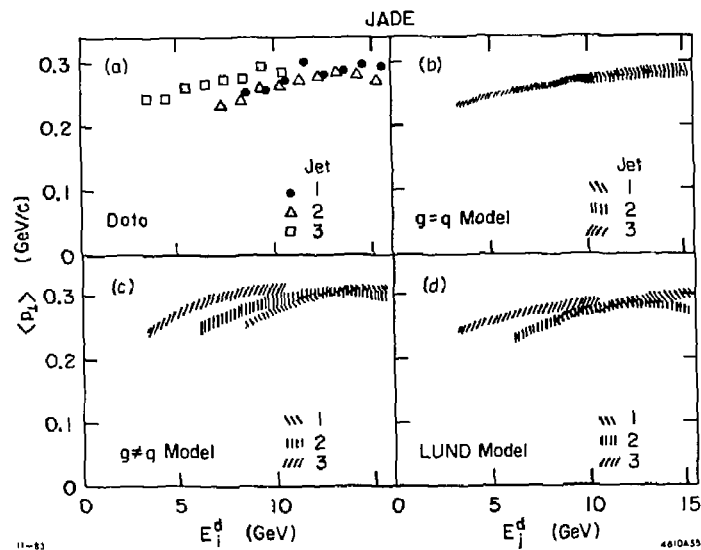


Fig. 25. The JADE measurement of $\langle p_T \rangle$ as a function of jet energy for particles in 3-jet events. (a) Data from $\sqrt{s} = 22$ and 33 GeV, (b) prediction of the Hoyer model with identical quark and gluon fragmentation, (c) prediction of the Hoyer model with broader p_T for gluon fragmentation relative to quark and (d) prediction of the LUND model.

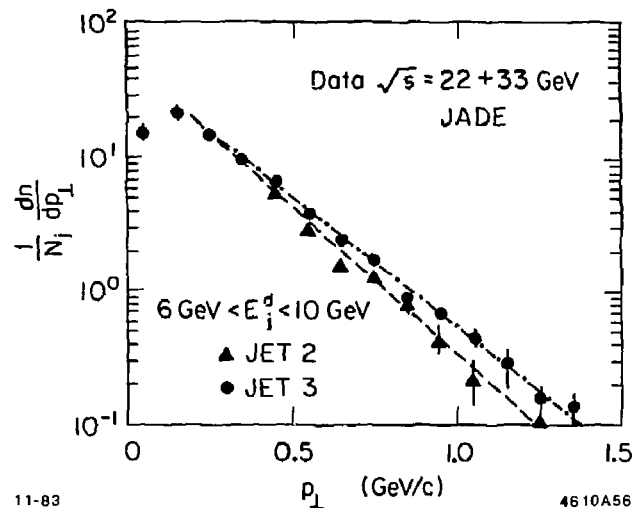


Fig. 26. The normalized differential cross section as a function of p_T for JADE data at $\sqrt{s} = 22$ and 33 GeV. Data are shown for the low and intermediate energy jet.

of 560 such events were found. The variable used to examine the fragmentation was $z_i = P_i/E_{jet}$ where P_i is the i th particle momentum and E_{jet} is the energy of the jet to which the particle was assigned. The particle z distribution was obtained and corrections were made for detector inefficiencies. This distribution is assumed to arise from events which are one-third gluon jets, two-thirds quark jets with each jet energy = 9.67 GeV. The data can be compared (see Fig. 27) with the corrected particle z distributions obtained from Mark II, TASSO, HRS and JADE in the jet energy range from 2.5 GeV to 17.5 GeV, where these data have been plotted assuming that they all come from a 2-jet topology. The 3-fold symmetric events are shown in this figure at $E_{cm}/N_{jet} = 29 \text{ GeV}/3 = 9.67 \text{ GeV}$. The different curves represent fits to the 2-jet data in the z regions indicated in the figure caption. One sees from Fig. 27 that the 3-fold symmetric events, which are gluon enriched relative to the 2-jet events, show a softer fragmentation function. This is shown more clearly in Fig. 28 where the hadron z distribution for the 3-fold symmetric events (solid points) are compared with the z distribution at $E_{cm} = 19.3 \text{ GeV}$ (as interpolated from the fits in Fig. 27) which are shown as a dashed line. In order to extract the fragmentation function of a 9.67 GeV gluon jet, the 19.3 GeV 2-jet event z distribution is used to subtract out the quark/antiquark z contribution from the 3-fold symmetric events. The resulting gluon z distribution is shown as the open points in Fig. 28. The conclusion drawn by the Mark II group is that gluon jets (open points) have a significantly softer fragmentation than quarks (dashed line) of the same energy.

The UA1 group has also measured the gluon fragmentation function.^[21] While this discussion properly belongs in Section 3, it is included here because the result is strikingly similar to the Mark II. Details of how the pp collider experiments isolate jets are discussed in Section 3. For this analysis UA1 selects events containing two jets, each with $P_T > 25 \text{ GeV}/c$, which are collinear within 30° . Charged tracks are associated with the jets if they lie within the jet cone as specified by the azimuthal angle ϕ and the pseudo-rapidity η . UA1 uses the variable $z = P_L/E_{jet}$ where P_L is the longitudinal momentum of the charged particle mea-

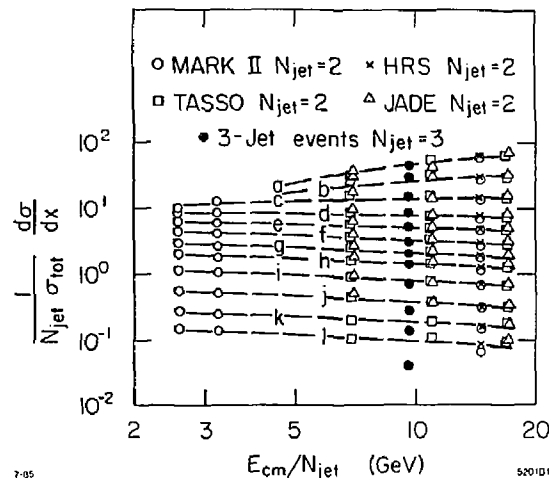


Fig. 27. The inclusive charged-particle cross section for a jet as a function of the jet energy, as measured by various experiments. The curves represent fits to the different data points for twelve z intervals which are defined by a = $0.03 < z < 0.05$, b = $0.05 < z < 0.10$, c = $0.10 < z < 0.15$, d = $0.15 < z < 0.20$, e = $0.20 < z < 0.25$, f = $0.25 < z < 0.30$, g = $0.30 < z < 0.35$, h = $0.35 < z < 0.40$, i = $0.40 < z < 0.50$, j = $0.50 < z < 0.60$, k = $0.60 < z < 0.70$, l = $0.70 < z < 0.80$. The detector corrected inclusive charged-particle distribution for 3-fold symmetric, 3-jet events at $E_{cm} = 29 \text{ GeV}$ is also shown.

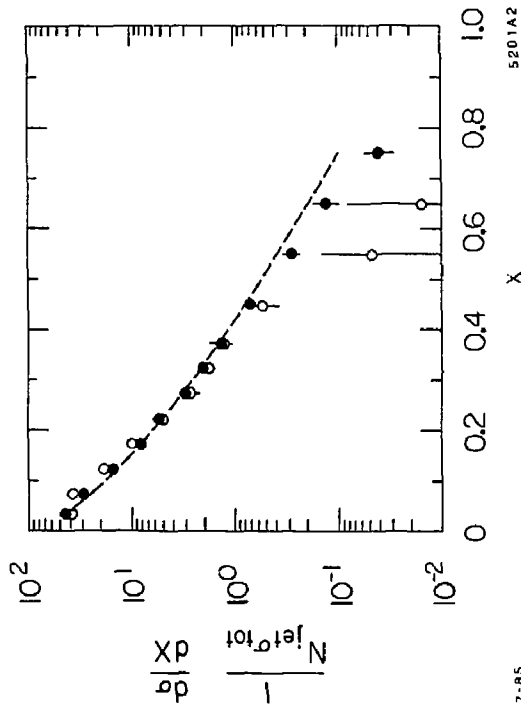


Fig. 28. The detector corrected inclusive charged-particle distribution for 3-fold symmetric 3-jet events at $E_{cm} = 29$ GeV (full symbols) in comparison with the inclusive charged-particle cross section of hadronic events at $E_{cm} = 19.3$ GeV, extrapolated from the fitted curves in Fig. 2 (dashed curve). The inclusive charged-particle distribution of a gluon jet of $E_j = 9$ GeV, assuming the subtraction discussed in the text, is shown by the open symbols.

sured relative to the jet axis. UA1 now needs a prescription for defining a gluon enriched and quark enriched region. Given the measured kinematics of the 2-jet system, one can extract the relevant 2-parton scattering variables $\hat{s}, \hat{t}, \hat{u}, x_1$ and x_2 where x_i are the fractional momentum carried by the 2-partons and $\hat{s}, \hat{t}$ and $\hat{u}$, are the Mandelstam variables of the 2-parton scattering process ($ab \rightarrow cd$). Assuming the proton structure functions as input, one can calculate, jet-by-jet, the probability that the jet is a gluon or a quark. The probability for the subprocess $ab \rightarrow cd$ is given by

$$P(ab \rightarrow cd) = F_a(x_1, Q^2) \times F_b(x_2, Q^2) \times M^2(\hat{s}, \hat{t}, \hat{u})(ab \rightarrow cd) / \sum \text{all subprocesses}$$

where $F(x_i, Q^2)$ is the structure function for the appropriate subprocess, M is the QCD matrix element for that subprocess and $Q^2 = 2\hat{s}\hat{t}\hat{u}/(\hat{s}^2 + \hat{t}^2 + \hat{u}^2)$. For each jet the probability that it is a gluon is given by

$$P(\text{jet} = \text{gluon}) = \sum_{ab} P(ab \rightarrow \text{gluon} + \text{anything}).$$

Care is taken to compare quark and gluon jets at the same Q^2 , thus avoiding quark/gluon fragmentation differences arising from non-scaling behavior. The resulting $\Gamma(g)$ (or $P(q) = 1 - P(g)$) distribution is shown in Fig. 29. The shaded regions are used as quark and gluon enriched samples.

Figure 30 shows the extracted quark and gluon fragmentation functions. The gluon fragmentation function appears softer, in agreement with the Mark II result. Figure 31 shows a comparison between the ratio of gluon and quark fragmentation functions as measured by Mark II in e^+e^- interactions at a $Q^2 = 841$ GeV^2 and UA1 in pp at $\langle Q^2 \rangle = 2000$ GeV^2 . (The two groups use slightly different variables, namely P/E_{jet} vs. P_t/E_{jet} , but this is a small effect.) The comparison is quite favorable given the experimental difficulties in obtaining these two fragmentation functions.

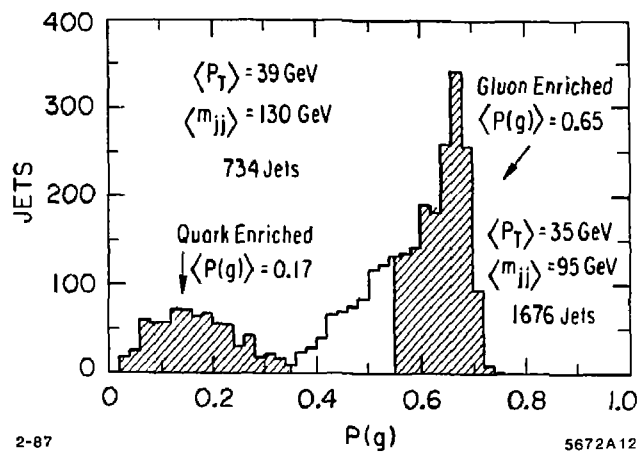


Fig. 29. The probability distribution for individual jets to be gluons. The shaded regions indicate the quark and gluon enriched samples.

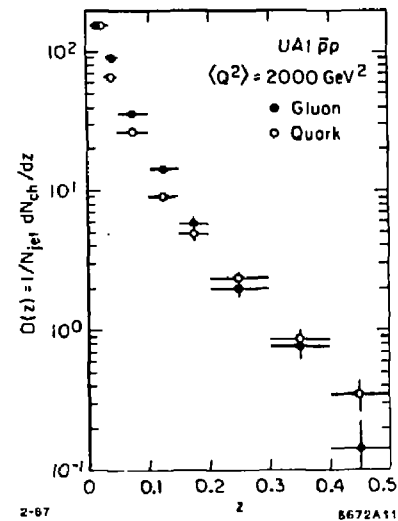


Fig. 30. Fragmentation functions for the quark-jets and the gluon jets as measured by UA1.

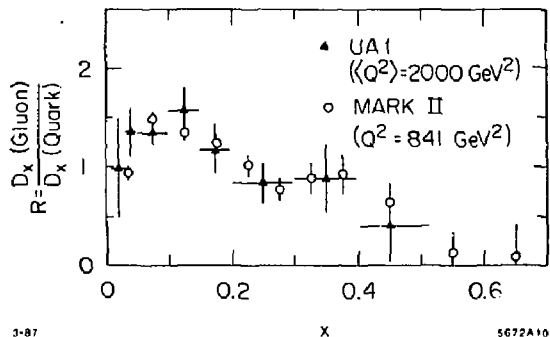


Fig. 31. The ratio of the gluon and quark fragmentation functions as a function of x , the fraction of momentum carried by the parton, are shown as measured by the Mark II and UA1.

2.5 STUDIES OF PARTICLE FLOW IN 3-JET EVENTS

Particle flow in 3-jet events has been studied by several groups.^[20] These studies give support to the string type models (LUND) and exemplify the clearest evidence that independent fragmentation models do not fully represent the e^+e^- hadronic events at high ($\gtrsim 29$ GeV) energies. Three-jet events are isolated as discussed in Section 2.2, each group using their own prescription. The conclusions of all three groups is the same and there is little reason to suspect that the 3-jet selection criteria produce any bias. Having formed the 3-jet axes, the particle or energy flow is plotted as a function of azimuthal angle with the $\phi=0$ axis aligned along the fastest jet (jet 1). The data from TPC and JADE are shown in Figs. 32 and 33. The curves shown superimposed on the data are the predictions of the various Monte Carlo models as indicated on the figures. The feature of interest in these particle flow plots is the inability of the independent fragmentation (IF) model to account for the particle density in the region between jet 1 and jet 2 which are most frequently the quark and anti-quark. There is a depletion in this region relative to the expectations of the IF model. This depletion can be enhanced by selecting heavy particles (Fig. 32(b)) or equivalently particles with significantly large momentum normal to the event plane (Fig. 33(b)). The LUND model accounts for this depletion (in fact predicted it would be there) via the "Lorentz boost" effect. In the LUND picture, the quark and antiquark can be considered to be joined by a string. As the q and $\bar{q}$ move apart, the string stretches. The emission of a gluon can be visualized as a transverse "plucking" of the string, the direction of "pluck" being the direction of the gluon. The string breaks forming two substrings. The hadronization of these $q\bar{q}$ substrings occurs in their rest frame. The Lorentz boost required to bring the hadrons into the laboratory frame results in a depletion of hadrons in the region between the q and $\bar{q}$. This is shown graphically in Fig. 34(a) where the dashed line represents the string. It is clear that if the q , $\bar{q}$ and g fragment independently (and symmetrically as they do), so asymmetry relative to the parton directions will occur (Fig. 34(b)). These data are considered important verification of the

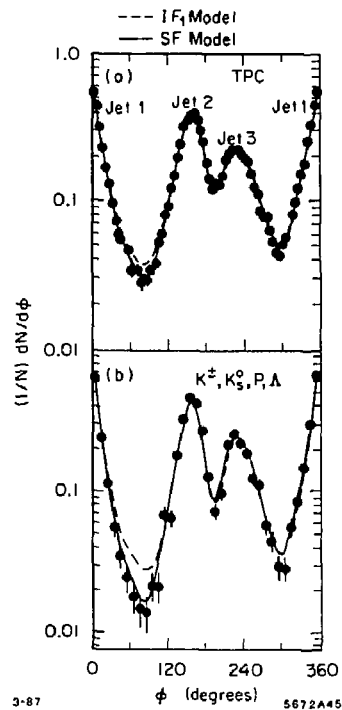


Fig. 32. The density of particles in 3-jet events as a function of the angle ϕ , where ϕ is measured in the event plane and $\phi=0$ corresponds to the direction of the fastest jet. The TPC data are shown in (a) for all particles and in (b) for heavy particles. The predictions of the string and independent fragmentation models are shown.

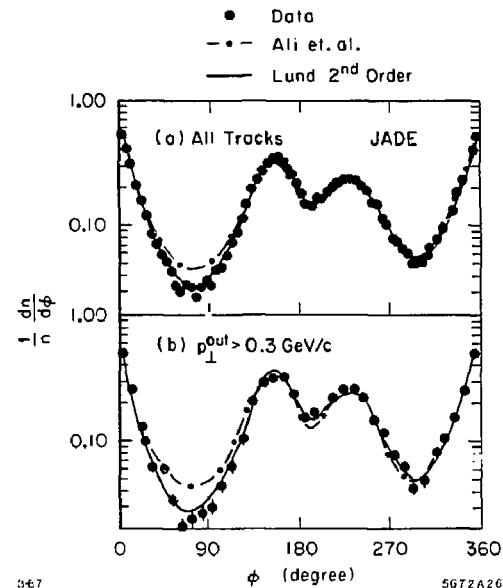


Fig. 33. The same plot as 32 except for the JADE data where (a) is for all particles and (b) is for particles with large momentum components out of the event plane.

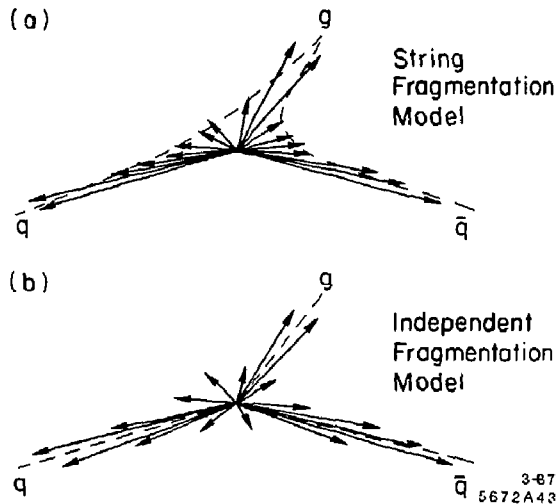


Fig. 34. Cartoon depicting the difference between the particle densities in 3-jet events as expected in the string picture (a) and the independent fragmentation picture (b).

LUND string-like description.

How does this relate to testing QCD? In a recent paper^[31] Azimov et al., calculate the radiative pattern of soft gluons in $q\bar{q}g$ events arising from the three color sources q , $\bar{q}$ and g . QCD predicts that in 3-jet events of the kind we have been considering, the production of soft gluons in the region between the quark and antiquark is reduced because of negative interference between the radiation emitted by the $q\bar{q}$ system and the gluon. The authors further postulate a duality between the soft gluon flow and hadron flow, implying that the destructive interference will be seen in the hadrons.

As outlined in Ref. 31, this QCD calculation explains the features of the data shown in Figs. 32 and 33; namely QCD can explain the string effect. These theoretical ideas have been tested further by comparing the hadron "radiation patterns" in $q\bar{q}g$ and $q\bar{q}\gamma$ events. Since the photon carries no color, there will be no interference with the $q\bar{q}$ dipole radiation pattern and no depletion should be seen in $q\bar{q}\gamma$ events. The directivity diagram for soft gluon radiation in $q\bar{q}g$ (solid lines) and $q\bar{q}\gamma$ (dashed lines) events is shown in Fig. 35. The distance from the origin represents the soft gluon density at an angle ϕ measured relative to the quark jet. (The radial scale is logarithmic in this figure.) To reiterate, we see from this figure that $q\bar{q}\gamma$ events will have more hadrons in the region opposite the photon than the $q\bar{q}g$ events will have opposite the gluon jet. Again, we have assumed a duality between soft gluon flow and hadron flow.

The comparison of $q\bar{q}\gamma$ and $q\bar{q}g$ events has been studied by the TPC^[32] and the Mark II.^[33] The analyses strongly support the QCD calculation of Azimov et al. Both groups chose 3-jet events ($q\bar{q}g$) using a cluster algorithm and select $q\bar{q}\gamma$ events by requiring two hadronic jets plus an isolated high energy ($< E_\gamma > \sim 6$ GeV) photon. The jets and photon are required to be coplanar. Because TPC was using a smaller data set than Mark II, they increased their $q\bar{q}\gamma$ sample with events containing two non-collinear hadron jets and a missing photon. Both groups make additional cuts to purify the event samples. The energies of the three

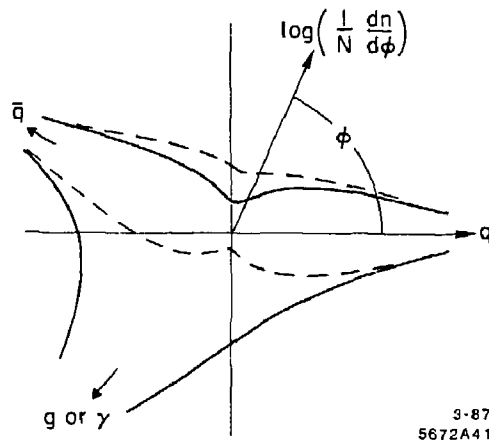


Fig. 35. Directivity diagram of soft gluon radiation in $q\bar{q}g$ (solid) and $q\bar{q}\gamma$ (dashed) events. The radial distance from the origin measures the density of soft gluons emitted at the azimuthal angle ϕ with respect to the quark jet. The radial scale is logarithmic.

jets (or γ) are calculated from the jet (γ) angles. What makes the comparison of the $q\bar{q}\gamma$ and $q\bar{q}g$ events meaningful is the similarity of the kinematics of the two event types. Table V demonstrates this is the case of the TPC analysis, where $\langle E_i \rangle$, $\langle \phi_i \rangle$ refer to the average jet energy and angle. Similar agreement is obtained by the Mark II. Table VI summarizes the number of candidate events and the probability that the lowest energy jet is a photon or a gluon.

Table V
(TPC Data)

	$q\bar{q}g$	$q\bar{q}\gamma$	$q\bar{q}(\gamma)$
$\langle E_1 \rangle$ GeV	12.7	12.7	12.2
$\langle E_2 \rangle$ GeV	10.2	10.0	9.9
$\langle E_3 \rangle$ GeV	6.1	6.3	6.9
$\langle \phi_1 \rangle$	≈ 0	≈ 0	≈ 0
$\langle \phi_2 \rangle$	153°	152°	145°
$\langle \phi_3 \rangle$	231°	229°	235°

Table VI

	TPC	Mark II	Prob. JET 3=g, γ
$q\bar{q}g$	2537	6585	60-65%
$q\bar{q}\gamma$	117	320	75-85%
$q\bar{q}(\gamma)$	1564		70%

The data is shown in Fig. 36 (TPC) and Fig. 37 (Mark II). In both analyses one sees clearly the effect predicted by QCD - namely the region opposite the gluon has a lower particle density than that opposite the photon ($\phi \approx 80^\circ$ in Figs. 36 and 37). (The difference in hadron flow at $\phi \approx 230^\circ$ is, of course, trivial

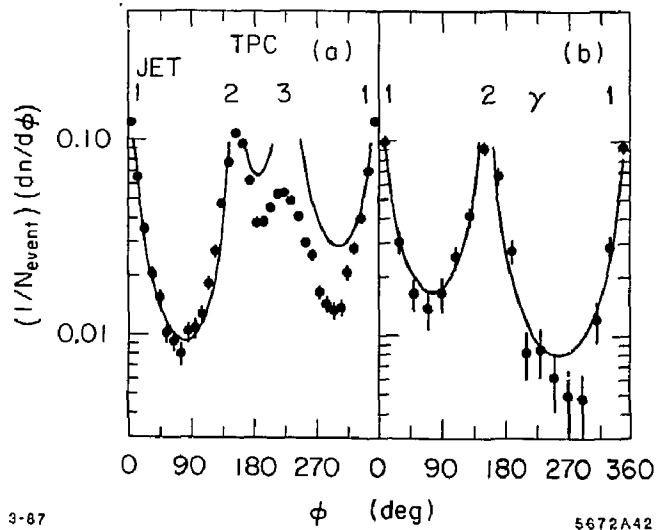


Fig. 36. TPC data showing the density of hadrons in the $q\bar{q}g$ events (a) and the $q\bar{q}\gamma$ events (b). Jet 1 and jet 2 are typically the q and $\bar{q}$. The lines are the predictions of asymptotic QCD for the flow of soft gluons.

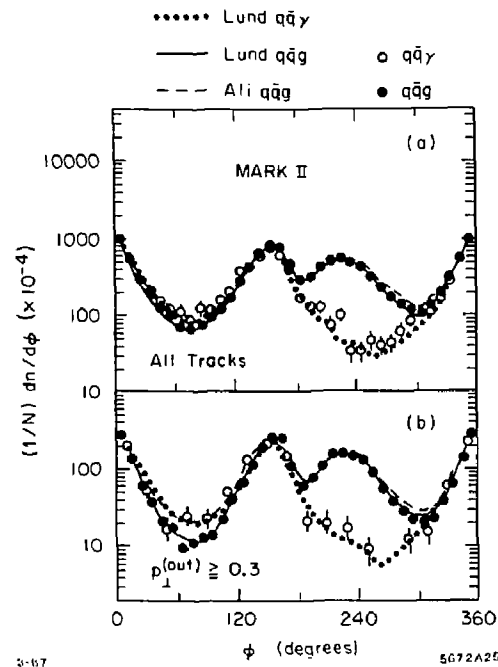


Fig. 37. Mark II data showing the density of hadrons in the $q\bar{q}g$ events (solid points) and the $q\bar{q}\gamma$ events (open points) for all tracks (a) and for tracks with large momentum out of the event plane (b). $\phi = 0$ corresponds to the direction of the fastest jet increasing to jet two and jet three, respectively. The predictions of the LUND and ALI models are indicated on the figure.

since this is the g, γ direction). The two groups display different curves. The TPC show the QCD prediction with $\phi_g = 0, \phi_g = 153^\circ$ and $\phi_{g,\gamma} = 231^\circ$. In the region of expected validity – namely not too close to the parton directions – the QCD predictions agree quite well with the hadron distributions, except in the region close to the gluon jet. This disagreement in the area of the gluon jet is not a new revelation, but presumably relates to the fact that at PEP energies we have not yet approached the asymptotic region assumed in the QCD calculations. Asymptotic QCD would predict a ratio of 9/4 for the soft gluon multiplicity in gluon versus quark jets. What is observed at these pre-asymptotic energies is more like 1.3. The Mark II shows the predictions of the LUND model and the Ali et al. independent fragmentation model. One draws the same conclusions discussed above namely the “string effect” accounts well for the data while the independent fragmentation model does not. In addition we see excellent agreement between the LUND model and the data for the $q\bar{q}\gamma$ events. To display the interference effect more quantitatively, the data can be plotted (Fig. 38) as a ratio of the particle yield for $q\bar{q}g$ versus $q\bar{q}\gamma$ using a normalized value of ϕ , namely $x = \phi/\phi_{12}$ where ϕ is the measured azimuthal angle of the particle and ϕ_{12} is the angle between jets 1 and 2. The coherent interference effect in the $q\bar{q}g$ parton geometry is clearly demonstrated. The solid line in the TPC plot is the QCD prediction assuming that jet 3 is always the gluon or photon. The dashed line allows for the fact that, in reality, the third jet is not always a gluon or photon. Hence, the solid and dashed line gives the range of the QCD predictions which is clearly in good agreement with the data. The dashed line in the Mark II data is the expected result for a model with independent fragmentation.

It is worth noting that the depletion effects seen using the hadrons in 3-jet events as discussed earlier in this section (Figs. 32 and 33) were seen in the context of comparisons with models, i.e., they were model dependent results. Comparison of $q\bar{q}g$ and $q\bar{q}\gamma$ events provides the same conclusion in a model independent manner. In addition one obtains a powerful test of the predictions of QCD for soft gluons and verification of the soft gluon/hadron duality. It

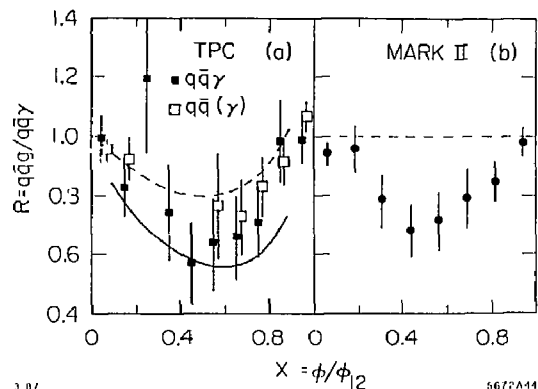


Fig. 38. The ratio of the hadron production in $q\bar{q}g$ and $q\bar{q}\gamma$ events as a function of the normalized angle $x = \phi/\phi_{12}$ where ϕ_{12} is the angle between jets 1 and 2. Both TPC and Mark II data are shown. The expected range of the QCD prediction for soft gluon flow is shown in the TPC data as solid and dashed lines; the dashed line accompanying the Mark II data is the prediction of the independent fragmentation model.

appears then that the LUND string model provides an excellent mechanism for mimicking the effects predicted by QCD.

3. TESTING QCD USING pp and $p\bar{p}$ COLLISIONS

Studying QCD in pp and $p\bar{p}$ collisions is considerably more difficult than in e^+e^- collisions. In the latter we produce the basic constituent quarks and gluons in a relatively "clean" environment. In pp (I will often use pp to imply both pp and $p\bar{p}$) collisions one is interested in the hard scattering process between either two constituent quarks (see Fig. 39) or a radiated gluon and a constituent quark or two radiated gluons. These processes are masked by the debris associated with the constituents which do not participate in the hard collision which makes the environment a lot less "clean" than in e^+e^- collisions. However, one is able to reach significantly higher Q^2 scales with pp collisions at the CERN SppS collider than at PETRA and QCD studies at $p\bar{p}$ and pp machines (ISR) are now making significant input to the information we have to test QCD. As we will see there are still experimental and theoretical problems. But these studies are still in their infancy and, as with e^+e^- tests, they will improve as these problems are confronted.

What do we need to know to untangle the QCD physics in a pp collision? We need to describe the subprocesses of parton-parton scattering ($ab \rightarrow cd$), as depicted in Fig. 40. Hence we need to know the nucleon structure functions, the matrix element describing the scattering process $ab \rightarrow cd$ and the fragmentation functions of the final state particles. Typical lowest order QCD subprocesses are shown in Fig. 41(a), where the λ_{ij} represent the relative probabilities of each subprocess. Life is not as simple as shown in Figs. 39 and 41(a); and we know it is going to be important to add the higher order (second and above) QCD effects. Some typical second order diagrams are shown in Fig. 41(b).

We would like to set up a program similar to that in e^+e^- collisions where one sees first the hard, lowest order scattering process (2-jet events), higher order

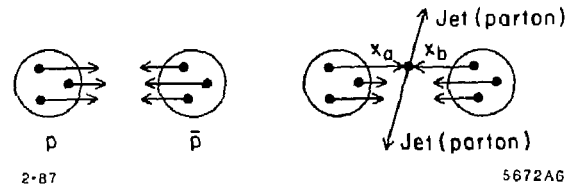


Fig. 39. Diagram of a hard scattering process in pp or $p\bar{p}$ collisions.

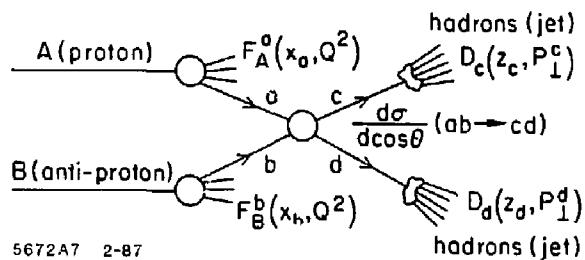


Fig. 40. Diagrams of a typical 2→2 parton scattering process. F is the structure function, $\cos\theta$ is the center-of-mass scattering angle, x is the fractional momentum carried by the parton and D is the parton fragmentation function defined by the parallel and perpendicular parton momenta z and P_T .

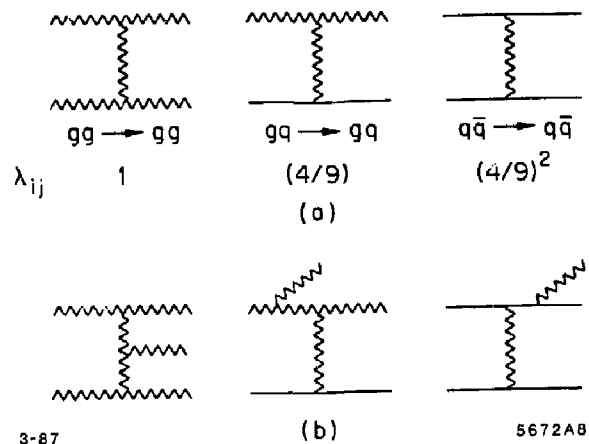


Fig. 41. (a) Typical 2→2 parton processes which lead to 2-jet events and (b) some of the higher order corrections to (a). The diagrams in (b) lead to 3-jet events when the radiated gluons are sufficiently energetic.

processes (3-, 4-jet events), measure α_s , etc. As noted before, the problems are numerous and we list below the most pressing. Firstly there are the theoretical issues:

1. How does one evaluate the cross sections, for example, for observing 2-jet events? To lowest order and neglecting non-scaling effects, the cross section for each subprocess ij factorizes:

$$\frac{d^2\sigma}{dx_1 dx_2 \cos\theta} = \frac{F_A(x_1)}{x_1} \frac{d\sigma_{ij}}{d\cos\theta} \frac{F_B(x_2)}{x_2} \quad (4)$$

where $\cos\theta$ is the center-of-mass scattering angle. As discussed by Cambridge et al.^[34] one obtains to good approximation a subprocess independent kinematic factor

$$\frac{d\sigma_{ij}}{d\cos\theta} \approx \lambda_{ij} \frac{\alpha_s(Q^2)}{\hat{s}} (1 - \cos)^{-2} \quad (5)$$

where $\hat{s}$ is the effective center-of-mass energy squared.

2. Since we are calculating the process to finite order, we are left with the dilemma about what scale (Q^2) the subprocess is occurring at.
3. One needs the quark and gluon structure functions for the proton. Combinations of theoretical and experimental input are used with the assumption

$$F(x) = g(x) + 4/9(g(x) + \bar{g}(x)).$$

4. Higher orders and the non-scaling effects in the behavior of α_s and the structure functions must also be included to extract meaningful conclusions.

Some of the experimental issues are:

1. The jets must be observed in a large background of hadrons coming from the partons which do not participate in the subprocess of interest. The experimental approach has been to concentrate on high P_T jets. However there remain problems of the assignment of particles to the jets, the jet energy scale....

2. How does one measure Q^2 ?

3. There are the usual problems of modelling the non-perturbative physics. The simulations are in general less advanced than those used in e^+e^- collisions and one is more dependent on the unknown gluon fragmentation function.

How do the experiments isolate jets? I will present data from the AFS collaboration at ISR, UA1 and UA2 at the CERN $p\bar{p}$ collider. Each group uses a slightly different technique, the details of which can be found in the experimental references given below. Typically, large energy deposits are sought in a grid defined by ϕ and θ (or η , the pseudo-rapidity). A window centered on these large deposits is used to associate smaller deposits with the primary initiator. Refinements are made to these cluster energies so that between the algorithm and the Monte Carlo simulation of the data, a robust procedure for extracting and correcting the jet energies is obtained. Striking multijet events are seen by all three groups, an example of which is shown in Fig. 42. By looking at this picture one can imagine how the jet finding algorithm works.

3.1 INCLUSIVE JET AND DIRECT PHOTON CROSS SECTIONS

We now turn to the experimental data. Figure 43(a) shows the inclusive jet cross section as a function of jet P_T from the UA1 group.^[35] What is plotted is $< d^2\sigma/dP_T d\eta >$ averaged over the rapidity region $|\eta| < 0.7$. Both data from $\sqrt{s} = 546$ GeV and 630 GeV are shown. The QCD lowest order predictions shown in the figure agree quite well with the data. Higher order QCD will affect the normalization but probably to a lesser extent the P_T dependence. The major limitation in the inclusive jet cross section as a test of QCD is the $\approx 10\%$ uncertainty in the jet energy scale which will almost certainly wipe out any sensitivity to higher order effects. Figure 43(b) shows the dimensionless (scaling) cross section $E_T^2 d^3\sigma/dp^3$ as a function of $x_T = 2P_T/\sqrt{s}$. The data at the two energies overlap; however the lever arm in $\sqrt{s}$ is much too small to expect to

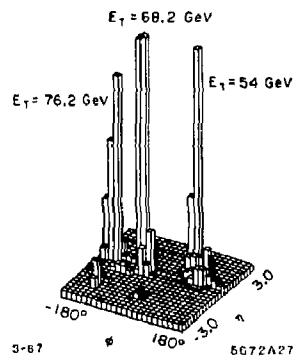


Fig. 42. A 3-jet event taken from the UA1 data. The event is shown in ϕ, η space with the vertical axis representing the energy contained in the particular ϕ, η cell. The jet transverse energies are indicated in the figure.

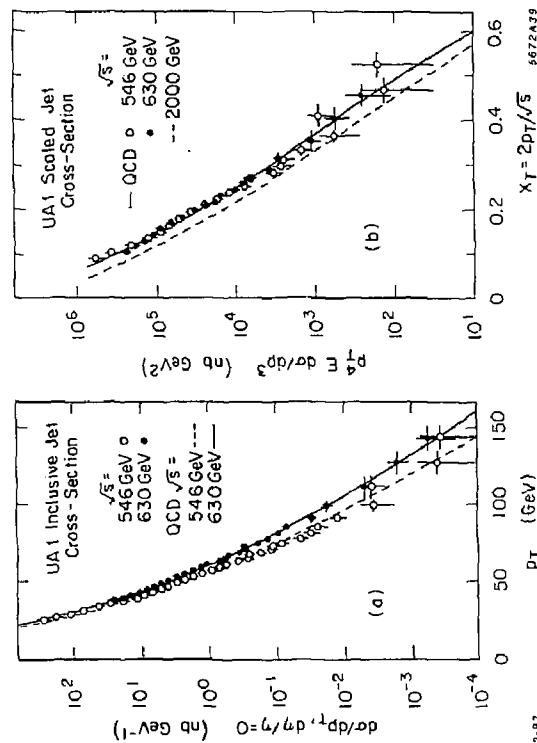


Fig. 43. (a) The inclusive jet cross section as a function of jet p_T and (b) the scaled jet cross section as measured by UA1. The data come from $\sqrt{s} = 546$ GeV and 630 GeV. Also shown are the predictions of QCD.

see the effects of non-scaling. Similar data have been published by the UA2 collaboration^[34] and the AFS collaboration.^[37]

One possible way to overcome the problems associated with measuring the inclusive jet cross section is to measure the inclusive direct photon cross section. The advantages are the absence of fragmentation effects, the good energy determination for high energy photons ($\approx 1\%$ for UA2) and the presence of $O(\alpha_s^2)$ predictions from theory. The clear disadvantage is that one has $\approx 10^4$ less cross section than for inclusive jet production and hence, the range of P_T with reasonable statistical weight is small. There is data available from the ISR experiments and UA2. I will show the UA2 data because it has an $\langle P_T \rangle \approx 4$ times larger than the ISR experiments.^[38]

The extraction of the direct photons from those coming from π^0 's and η 's is difficult. UA2^[39] requires firstly that the photon be well isolated assuming that photons from π^0 and η decays would be accompanied by other nearby hadrons. Further cuts are made using the pre-shower detection to distinguish single isolated photons from π^0 and η decays where the two decay photons have coalesced in the calorimeters. Residual background contributions are estimated and removed for each P_T bin. The corrected cross section for $\bar{p}p \rightarrow \gamma + \text{anything}$ is shown in Fig. 44 along with the $O(\alpha_s^2)$ QCD prediction. The errors in the figure are dominantly statistical. In addition there is also a 20% normalization systematic error. The QCD curves are from the calculation of Aurenche et al.^[40] When comparing the data with QCD, care must be taken to include the effects of bremsstrahlung from the final state quarks. These bremsstrahlung processes are sensitive to the isolation cuts. The two lower QCD curves shown in Fig. 44 exclude bremsstrahlung photons with the angle of the photon relative to the quark of less than 20° and 45° , respectively. The difference between these two curves represents the level of uncertainty caused by the isolation cuts. The overall agreement between QCD and the data is good and with more data in the future, these tests will become more compelling. But already they provide significant qualitative tests of $O(\alpha_s^2)$ QCD.

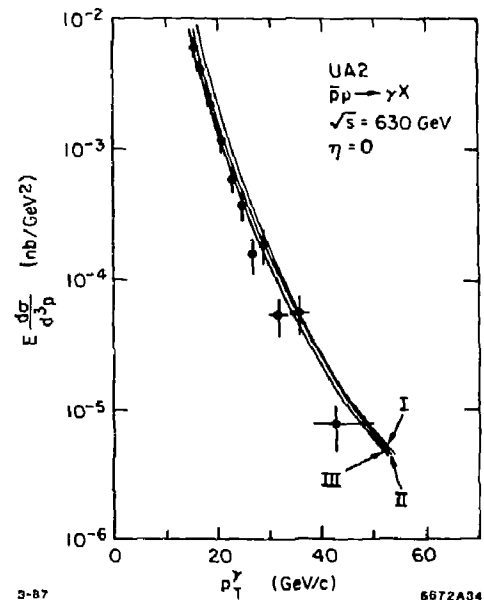


Fig. 44. The direct photon inclusive cross section as a function of P_T^γ from UA2. The curves correspond to the predictions of $O(\alpha_s^2)$ QCD. In II(III) QCD bremsstrahlung from the quarks is omitted if the angle between the quark and photon is less than 20° (45°).

The NA14 group at CERN have examined direct photon production in γN scattering.^[41] One expects a contribution from QED Compton scattering but in addition a substantial contribution from QCD processes. As with the direct photons in hadro-production, an $O(\alpha_s^2)$ calculation is available for comparison. The NA14 direct photon spectrum is shown in Fig. 45. Both statistical and systematic errors (arising mainly from the background subtraction) are shown. The dashed-dot line is QED Compton scattering while the solid line adds the $O(\alpha_s^2)$ QCD prediction.^[42] The data are clearly not in agreement with the pure QED production and the addition of the QCD contributions gives good agreement.

3.2 QUALITATIVE TESTS USING 2- AND 3-JET EVENTS

There is copious 2-jet production at the CERN $\bar{p}p$ collider. What do studies of these events tell us? From these events effective proton structure functions have been extracted by both UA1^[43] and UA2.^[44] These data are largely consistent with each other and with QCD. Intrinsically the structure function is of fundamental importance. However with respect to providing a precise quantitative test of QCD the $\bar{p}p$ structure function measurements are limited by the large systematic error on the jet energy scale. This problem can be eliminated by studying the jet production angular distribution discussed earlier in this section. We show here data from UA1.^[45] UA1 isolates events which have $\geq$ two jets, and select the two highest P_T jets ignoring the others. The subprocess center-of-mass energy is assumed to be the di-jet mass ($\sqrt{s} = M_{2\text{jet}}$) computed using corrected jet four vectors. The center-of-mass $2 \rightarrow 2$ scattering angle θ is defined relative to the average beam direction evaluated in the di-jet rest frame. The distribution in $\cos\theta$ is shown for the selected 2-jet events in Fig. 46(a). In Fig 46(b) the same data are plotted as a function of $x = (1 + \cos\theta)/(1 - \cos\theta)$. Also shown in Fig. 46 are the expectations of leading order QCD (solid lines) and leading order QCD modified to account for the dependance of α_s and the structure functions on Q^2 , i.e., non-scaling effects. To obtain the theoretical predictions required assigning relative weights to the competing subprocesses. However, since all the subpro-

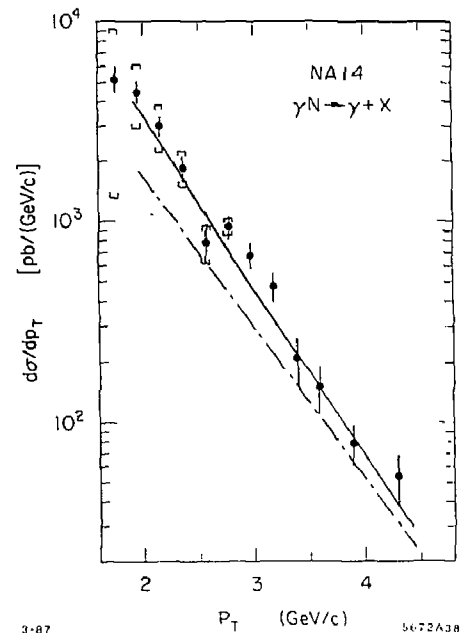


Fig. 45. The direct photon inclusive cross section as a function of p_T^2 as measured by NA14 at CERN in a photoproduction experiment. The dashed curve is the prediction for QED Compton scattering. The solid curve adds to this the effects of $O(\alpha_s^2)$ QCD corrections.

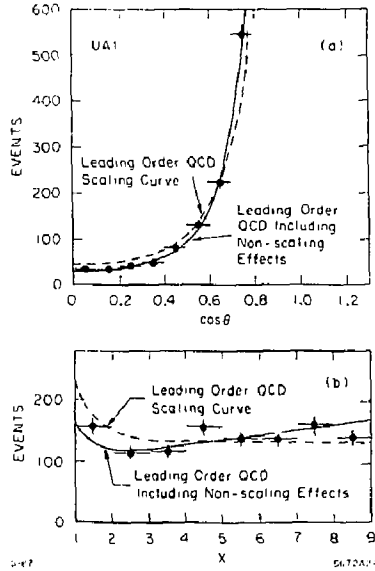


Fig. 46. (a) The 2-jet angular distribution as a function of $\cos\theta$ and (b) as a function of $x = (1 + \cos\theta)/(1 - \cos\theta)$. The dashed curves show the leading order QCD prediction, the solid line the leading order QCD prediction including the non-scaling effects.

cesses have a very similar angular distribution (see Equation 5), the result is not very sensitive to the relative subprocess weights. The data are clearly incompatible with the scaling curve, whereas the leading order QCD, including non-scaling effects, gives excellent qualitative agreement with the data. One can imagine that with time this could be fashioned into a quantitative test by measuring α_s as it enters through (5).

Data on 3-jet events is available from AFS,^[46] UA1^[47] and UA2.^[48] For three final state, massless partons at fixed subprocess $\sqrt{s}$, the parton configuration is specified by four independent variables. The leading order QCD prediction for a subprocess cross section is given by

$$\frac{d^4\sigma}{dx_1 dx_2 d\cos\theta_1 d\psi} = \left(\frac{1}{32\pi^2}\right) |M|^2$$

with

$$|M|^2 \approx (\alpha_s^2/\hat{s}) \{x_1^2 x_2^2 x_3^2 (1-x_1)(1-x_2)(1-x_3)\}^{-1}. \quad (6)$$

The kinematics are defined in Fig. 47 where x_i are the energies of the three outgoing partons (with $x_1 > x_2 > x_3$) scaled to $\sqrt{s}$, $x_T = x_i \sin\theta_i$, and the angle θ_1 and ψ are most easily understood by looking at Fig. 47. Because of the singular nature of $|M|^2$ as $x_i \rightarrow 1$ or $x_T \rightarrow 0$, experimental cuts must be made to ensure well separated 3-jet events. We will omit here a discussion of the cuts made by the three groups to isolate the 3-jet events. The interested reader is encouraged to consult Ref. 46. As is typical, each group uses somewhat different cuts and variables to display their data. For our purpose, which is to establish that leading order QCD can account for the qualitative features of the data, this is not significant.

The UA1 data are shown in Figs. 48 and 49. Figure 48 shows the 3-jet Dalitz plot variables x_1 and x_2 which are sensitive to the final state gluon radiation (see Fig. 41(b)). The solid lines are the prediction of leading order QCD (suitably weighted for each subprocess), while the dashed lines are for a phase space model.

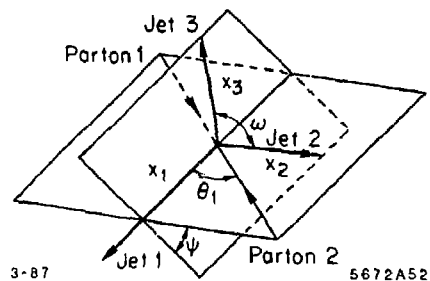


Fig. 47. The 3-jet variables as defined in the 2→3 subprocess rest frame.

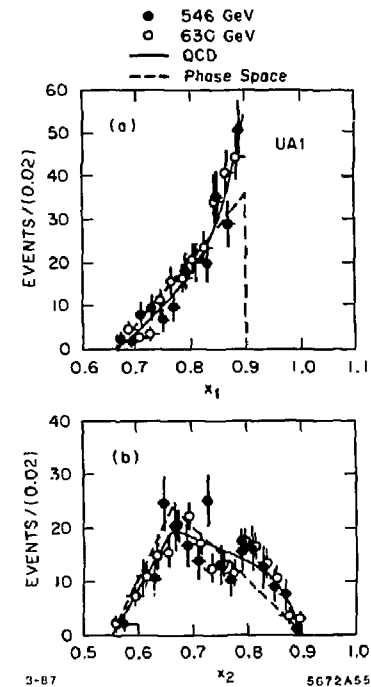


Fig. 48. UA1 3-jet Dalitz plot distributions. The solid lines are the prediction of leading order QCD, the dashed lines a phase space model.

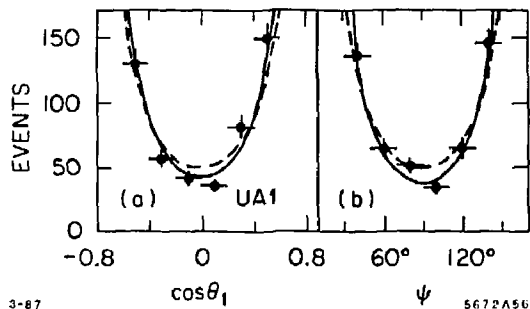


Fig. 49. UA1 3-jet angular distributions. The solid (dashed) line is the QCD prediction including (not including) scale breaking effects.

Figure 49 shows the 3-jet angular distributions which are especially sensitive to initial state gluon radiation. The curves are the prediction of leading order QCD with and without the non-scaling effects. The qualitative agreement between the predictions of leading order QCD and the data are good.

The UA2 data are shown in Figs. 50 and 51. Figure 50 shows the distribution of the angle w which is the angle between jets 2 and 3 as defined in Fig. 47. The predictions of leading order QCD and a phase-space model are shown. Figure 51 shows the Dalitz plot variables x_{23} and x_{12} where x_{ij} is the square of the invariant mass of jets i and j normalized to $\sqrt{s}$. The solid (dashed) curve is the prediction of leading order QCD (phase-space model). As with the UA1 data leading order QCD does a good job of describing the qualitative features of the 3-jet events; a phase-space model fails to reproduce the data.

Three-jet events are also observed at the ISR as discussed in Ref. 46 by the AFS group. Figure 52 shows the Dalitz plot variables for the 3-jet events, normalized to the yield of 2-jet events. Appropriate w and x_3 cuts are made to ensure well separated 3-jet events. The data are corrected for the finite size of the rapidity interval for the jets, namely the ratio of cross sections is evaluated at $\eta = 0$. The actual quantity plotted in Fig. 52(a) is

$$\frac{dN^{3jets}}{dw d\eta_1 d\eta_2 d\eta_3 / \eta_{i=0}} / \frac{dN^{2jets}}{d\eta_1 d\eta_2 / \eta_{i=0}}$$

and in Fig. 52(b) the variable w is replaced by x_3 . Also shown on the figure are the parton level lowest order QCD predictions (solid lines), the Monte Carlo predictions (shaded bands) and the Monte Carlo with the multi-jet events removed. One sees good qualitative agreement with QCD and a clear need for multi-jet production. The AFS group uses the events with $x_3 > 0.4$ and $w > 60^\circ$ for quantitative studies and, as discussed in the next section, to extract α_s . For these events the energy flow is shown in Fig. 53 where the data is compared with the Monte Carlo (modified ISAJET). The agreement is rather impressive indicating that the QCD lowest order simulation accounts well for the AFS data.

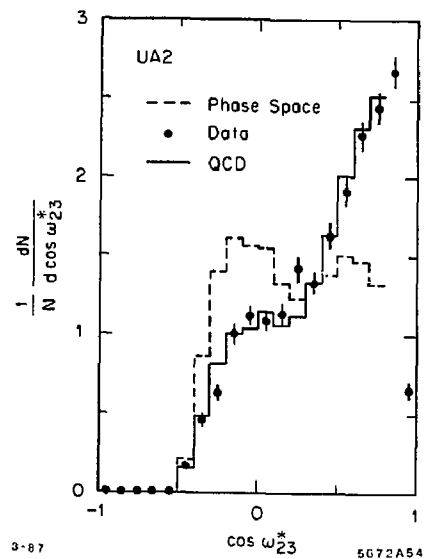


Fig. 50. UA2 data on the angle between the two least energetic jets in 3-jet events. The predictions of leading order QCD and a phase space model are shown.

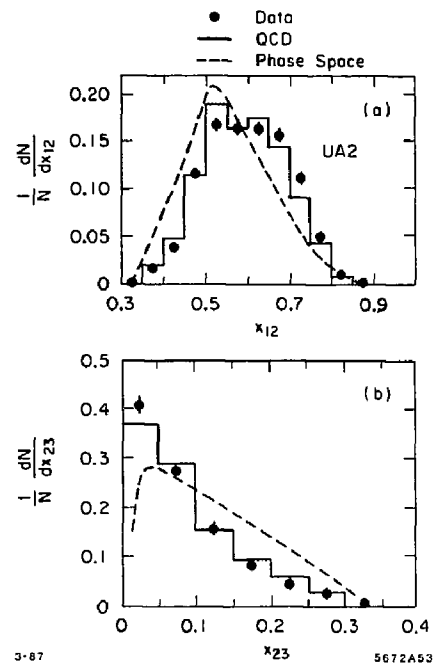


Fig. 51. UA2 3-jet Dalitz plot distributions. The curves are for leading order QCD (solid) and a phase space model (dashed).

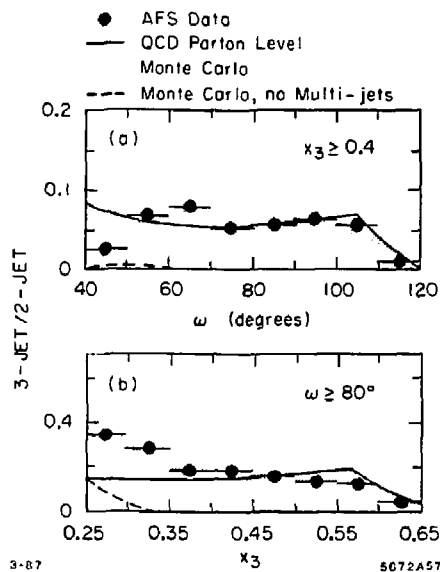


Fig. 52. The distribution of Dalitz plot variables ω and x_3 in 3-jet events. The data are corrected for trigger and filter inefficiencies and for the finite size of the rapidity gap. The yield of 3-jet events is shown normalized to 2-jet yield as described in the text. The solid line gives the lowest order QCD prediction at the parton level, the shaded bands are the Monte Carlo predictions and the dashed line the Monte Carlo with the multi-jets removed.

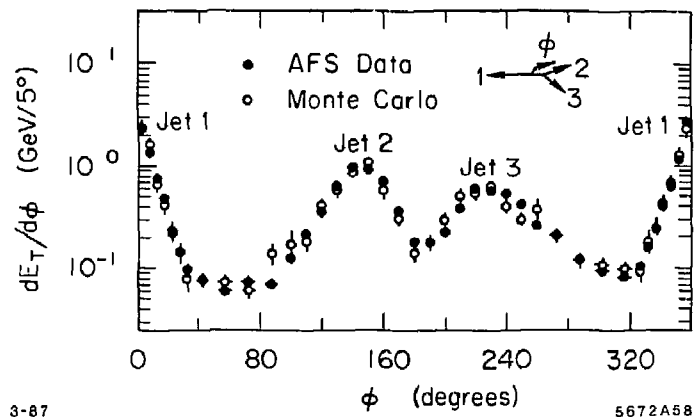


Fig. 53. The energy flow in 3-jet events as a function of the angle relative to the fastest jet. Only jets with $|\eta| < 0.7$ are included. The predictions of the Monte Carlo simulation are shown, where the data and simulation are subject to the same detector biases.

3.3 QUANTITATIVE TESTS OF QCD; MEASUREMENT OF α_s

From Equations (5) and (6) we see that if scaling holds, the 2- and 3-jet cross sections at fixed subprocess energy are given by energy-independent, dimensionless variables. In this limit, the 2- and 3-jet cross sections have the same energy dependence and the ratio will yield a number proportional to α_s :

$$\sigma_{2J} = C_{2J} \alpha_s^2 \quad \sigma_{3J} = C_{3J} \alpha_s^2$$

where C_{2J} , C_{3J} are calculable numbers which depend on the detector cuts and the relative subprocess abundances. Just as for 2-jet subprocesses, the 3-jet subprocess C_{3J} values for gg, qg and qq vary approximately as 1:4/9:(4/9)². This means that to reasonably good approximation C_{3J}/C_{2J} is a constant roughly (within 20%) independent of the combination of interacting partons. Of course, life is not so simple (as we have already seen, scaling does not hold) and in reality to lowest order in QCD

$$\sigma_{3J}/\sigma_{2J} \approx \{\alpha_s^2(Q_{3J}^2)/\alpha_s^2(Q_{2J}^2)\}$$

$$\{F(x_1, Q_{3J}^2)F(x_2, Q_{3J}^2)/F(x_1, Q_{2J}^2)F(x_2, Q_{2J}^2)\} \{C_{3J}/C_{2J}\}. \quad (7)$$

A priori, the Q^2 scale for the 3-jet and 2-jet events is unknown which makes the α_s determination from (7) yet more difficult. Figure 54(a) shows the measured ratio of 3-jet and 2-jet cross sections as a function of $\sqrt{s}$ (from UA1). The solid line is the lowest order QCD prediction for $Q_{3J}^2 = Q_{2J}^2$, the dashed line the (intuitive ?) choice of $Q_{3J}^2 = \frac{2}{3}Q_{2J}^2$. The latter choice appears to fit the data better. Figure 54(b) shows the variation of the 3- to 2-jet ratio as a function of the multijet mass (UA2). The solid curve is the prediction of QCD and the dashed lines indicate the systematic plus statistical errors. Notice in both Figs. 54(a) and 54(b) the data are consistent with the "running" of α_s predicted by the lowest order QCD. However, the data are also consistent with no "running."

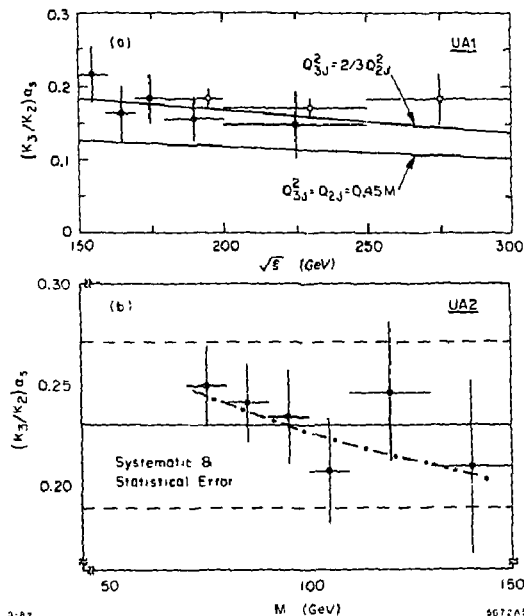


Fig. 54. a) Ratio of the 3-jet to 2-jet cross sections as a function of subprocess center-of-mass energy (UA1). The lines correspond to the QCD prediction for different choices of Q^2 . b) $\alpha_s(K_3/K_2)$ as a function of multi-jet mass from UA2. The prediction of QCD is shown along with the range of the errors.

So we see that the measurement of α_s in pp collisions is plagued by problems, mostly theoretical in nature. We bury these theoretical uncertainties in K factors and quote results for $\alpha_s(K_3/K_2)$ where K_2/K_2 accounts for the differences in the Q^2 scales for the 2- and 3-jet events and the non-scaling effects. The results from UA1 and UA2 are (see Refs. 47 and 48)

$$UA1 : \alpha_s(K_3/K_2) = 0.23 \pm 0.01 \pm 0.04 \quad < Q^2 > \approx 1000 \text{ GeV}^2$$

$$UA2 : \alpha_s(K_3/K_2) = 0.23 \pm 0.01 \pm 0.04 \quad < Q^2 > \approx 1700 \text{ GeV}^2$$

That the numbers are identical is not a careless error, but indeed true. However this is an accident and in fact the two numbers do not measure the same quantity. The main difference arises from the fact that the Q^2 definitions differ. UA1 uses an average Q^2 identical for the 2-jet and 3-jet samples, namely $< Q_{2J}/M_{2J} > = < Q_{3J}/M_{3J} > = 0.45$, whereas UA2 uses a definition which leads to a softer 3-jet Q^2 namely $< Q_{2J}/M_{2J} > = 0.48$ and $< Q_{3J}/M_{3J} > = 0.41$. In an attempt to measure the effect of the ambiguity of the Q^2 scale in the measurement of $\{K_3/K_2\}\alpha_s$, UA1 has made a jet selection which would satisfy the criteria $< Q_{3J} > \approx \frac{2}{3} < Q_{2J} >$ and find

$$\alpha_s(K_3/K_2) = 0.16 \pm 0.02 \pm 0.03.$$

The implication here is that for such a choice, (K_3/K_2) better approximates unity. This reminds us then that the ambiguity in the choice of Q^2 scale cannot be resolved until the higher order QCD corrections are calculated.

The AFS collaboration (see Ref. 46) have also extracted a value for α_s and find

$$AFS : \alpha_s(K_3/K_2) = 0.18 \pm 0.03 \pm 0.04 \quad < Q^2 > \approx 300 \text{ GeV}^2$$

in relatively good agreement with the $\bar{p}p$ results. Intrinsic differences in the three measurements are discussed at length in Ref. 46.

It is clear that the quantitative measurement of α_s in pp and pp collisions is in its infancy. We should remember the lessons of e^+e^- experiments, where both the experimental and theoretical environments are simpler, and where it has taken about six years of work to extract reliable measurements of α_s . I find the progress in the hadron collider measurements impressive and look forward to improvements in the future.

4. Measurements of α_s from Υ Decays

The decays of heavy quarkonium states provide tests of QCD. The dominant decay mode is into three gluons as shown in Fig. 55. In principle one could measure α_s from this decay, for example from $\Upsilon \rightarrow \text{Hadrons}$. However in practice such a measurement has limited precision because of an incomplete understanding of the non-perturbative contribution involving the quark wave functions. To remove this uncertainty one evaluates rather the ratio of the two similar processes $\Upsilon \rightarrow ggg$ and $\Upsilon \rightarrow \gamma\gamma\gamma$. This ratio is calculable in first order QCD, and as is pointed out by Brodsky, LePage and MacKenzie,^[49] a natural choice of $Q^2 = 0.157M_\Upsilon$ leads to small higher order corrections and the prediction

$$B_\gamma = \frac{\Gamma(\Upsilon \rightarrow \gamma\gamma\gamma)}{\Gamma(\Upsilon \rightarrow ggg)} = \frac{36q_b^2}{5} \frac{\alpha}{\alpha_s(\bar{M}\bar{S})} \left\{ 1 + (2.2 \pm 0.6) \frac{\alpha_s(\bar{M}\bar{S})}{\pi} \right\}$$

where α is the fine structure constant and q_b is the charge of the b quark ($-\frac{1}{3}$). This prediction for the measurement of α_s at $Q^2 = 0.157M_\Upsilon$ is considered to be on a sound theoretical footing. The measurement has been done by two groups, CLEO^[50] and CUSB,^[51] both running at CESR. The measurement is by no means simple since there is a large background from non-direct photons coming from decays of π^0 's and η 's. At small photon energies, $x \lesssim 0.5$ (where $x = 2E_\gamma/M_\Upsilon$) the photon spectrum is dominated by this source. For $x \gtrsim 0.5$ the dominant background is from continuum processes (i.e., non-resonant production) and the decay $\Upsilon \rightarrow q\bar{q}$. The QCD spectrum in contrast is expected to

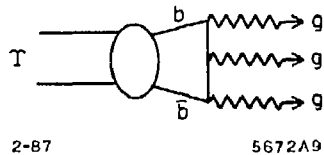


Fig. 55. The decay $\Upsilon \rightarrow ggg$.

rise linearly from $x = 0$ to $x = 1$, although radiative corrections will provide a smooth turnover as $x \rightarrow 1$.

The two groups use very different techniques for obtaining the direct photon spectrum. In addition to the Υ decay, CUSB also measures the rate for $\Upsilon' \rightarrow \gamma gg$. The CLEO group makes a measurement of the photon spectrum using their shower counters and makes the subtraction for π^0 's using the measured $\pi^\pm (= 2\pi^0)$ and π^0 x distributions. CUSB on the other hand obtains the direct photon yield from a statistical separation relying on the difference of the conversion probability of π^0 's (i.e., two photons) and direct photons. The ingredients for the algorithm are checked directly using annihilation events of the type $e^+e^- \rightarrow \gamma\gamma$. Both groups remove contributions from the continuum using data taken below the Υ and also a contribution from $\Upsilon \rightarrow q\bar{q}$. Figure 56(a) shows the CLEO observed photon spectrum and the background contributions discussed above. Figure 56(b) shows the background subtracted direct photon signal. Here the errors are statistical only. Below x of 0.5, large backgrounds preclude a meaningful measurement. Also shown in Fig. 56(b) are the fits to three theoretical spectra; the solid line for lowest order QCD, the dotted line for a calculation of Photiadis^[52] accounting for higher order QCD effects, and the dashed line a cluster model Monte Carlo simulation due to Field^[53] which accounts for hadronization effects. The χ^2 for the three fits are 14.2, 10.8 and 8.1, respectively, for 11 degrees of freedom. Comparison of the three model predictions points out a major systematic problem with this measurement, namely estimating the magnitude of the unseen portion of the photon spectrum. This problem shows up quite dramatically in the extracted branching fractions as seen in Table VII.

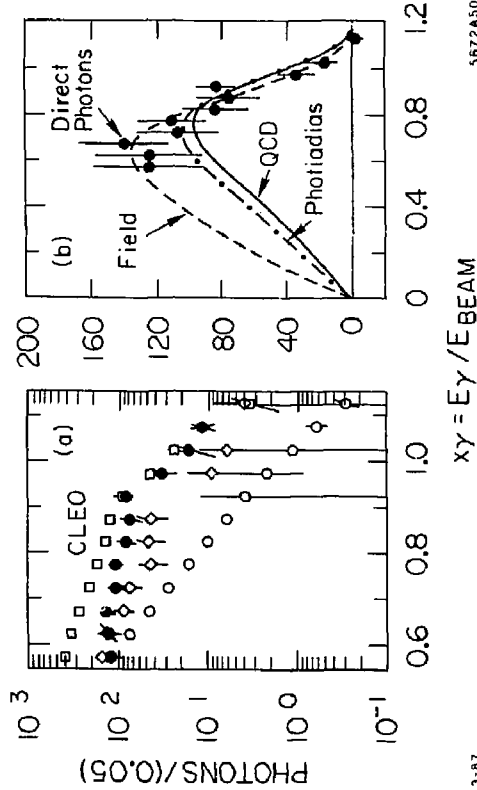


Fig. 56. a) The CLEO measured photon spectrum (open squares) and the contributions from the various background sources: continuum and $\gamma \rightarrow q\bar{q}$ (open diamonds) and π^0 and η' (open circles). The solid points are the background subtracted direct photon spectrum also shown in b) along with fits to the indicated models. The errors are statistical only.

EXPT	Model	$B_\gamma(\%)$	$\alpha_s(Q = .157M_\Upsilon)$	$\Lambda_{\overline{MS}}(\text{GeV})$
CLEO (Υ)	QCD	$1.88 \pm 0.14 \pm 0.17$	$0.40^{+0.04+0.06}_{-0.03-0.05}$	$0.37^{+0.05+0.07}_{-0.05-0.06}$
CLEO (Υ)	Phototadials	$2.03 \pm 0.15 \pm 0.16$	$0.36^{+0.04+0.04}_{-0.03-0.04}$	$0.32^{+0.05+0.06}_{-0.04-0.05}$
CLEO (Υ)	Field	$2.54 \pm 0.18 \pm 0.14$	$0.27^{+0.03+0.03}_{-0.02-0.02}$	$0.19^{+0.04+0.04}_{-0.03-0.03}$
CUSB(Υ)	QCD	2.99 ± 0.59	$0.226^{+0.067}_{-0.042}$	$0.116^{+0.105}_{-0.057}$
CUSB(Υ')	QCD	3.37 ± 1.14	$0.197^{+0.123}_{-0.055}$	$0.080^{+0.195}_{-0.059}$

The CUSB direct photon spectra for the Υ and Υ' are shown in Fig. 57 along with a fit to the spectrum $x(1-x)^a$, where a is found to be 0.17 ± 0.05 . Normalizing to the number of resonant hadronic events yields the numbers given in Table VII.

We began this chapter by noting that we had a solid QCD prediction for $B_\gamma = \Gamma(\Upsilon \rightarrow \gamma g g) / \Gamma(\Upsilon \rightarrow g g g)$. Unfortunately the results coming from the two experiments are in poor agreement indicating that there are still some unresolved experimental problems. Possibly the Crystal Ball and/or ARGUS groups at DESY could shed some light on this situation in the future.

5. Testing QCD in Deep Inelastic Lepton Scattering

Leptons being pointlike, structureless particles make excellent probes of nucleon structure (see Fig. 58). We have gained enormous insight into nucleon structure from the wealth of beautiful data from e , μ , and νN scattering experiments. I will do this magnificent experimental effort poor justice by the shortness of my remarks, the simplification of the problems and their solutions and my selective choice of data. However, this format does not permit much more than is presented with the main emphasis being placed on demonstrating the ability of QCD to explain the non-scaling behavior (i.e., departure for QPM) and the

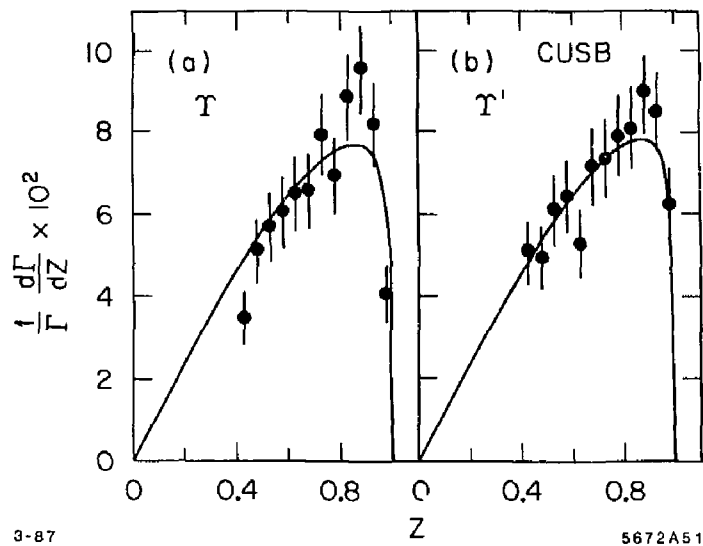


Fig. 57. Acceptance corrected direct photon spectrum from CUSB for T (a) and T' (b). The errors are statistical only. The curve is a fit to the function $x(1-x)^2$.

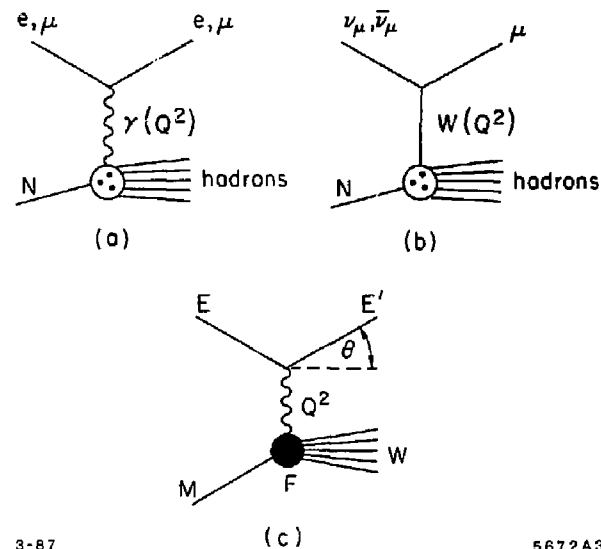


Fig. 58. Probing the nucleon structure using pointlike leptons via the electromagnetic and weak interactions.

extraction of A . For completeness a few (among many) references,^[10] far more complete than this discussion, are given.

One is able to achieve an impressive Q^2 range with lepton probes. Figure 59 illustrates the various regimes in which increasingly more sensitive tests become possible. At the lowest $Q^2 \approx 0.1 \text{ GeV}^2$ one studies elastic eN scattering and the gross nucleon structure is sensed. Raising Q^2 to $\approx 1 \text{ GeV}^2$ permits a "deep" enough probe that partons are discerned within the nucleon and scaling behavior of the structure functions is observed. At this level the data can be accommodated by a QPM picture. However, with increasing magnification the gluonic component of the nucleon is more readily seen and departures from the QPM picture are measurable. Scaling breaks down and QCD does a very good job of qualitatively explaining the data. As with all the previous discussions, a quantitative extraction of A turns out to be difficult, but a relatively narrow range of $A = (200 \pm 100) \text{ MeV}$ encompasses the most reliable measurements.

The kinematics of deep inelastic lepton scattering are shown in Fig. 58(c) where E, E' are the energy of the incoming and outgoing lepton, θ is the lepton scattering angle, Q^2 the four momentum transfer, M the mass of the nucleon fragments and the structure function F contains our a priori ignorance about the structure of the nucleon. For the case of eN or μ N scattering there are two structure functions F_1, F_2 and the differential cross section is given by

$$\frac{d^2\sigma}{dx dy} = \frac{4\pi\alpha^2}{Q^4} [F_2(Q^2, \nu)(1-y) + F_1(Q^2, \nu)xy^2]$$

where $Q^2 = 2EE'(1 - \cos\theta)$, $\nu = E - E'$, $y = \nu/E$ and $x = Q^2/2m\nu$.

The simplest example of evaluating the structure functions is in the QPM where we envisage the nucleon to contain electrically charged, spin $\frac{1}{2}$ objects and we ignore the parton/parton interactions in the scattering process (i.e., impulse approximation, quasi-free constituents). We also assume that the partons have no intrinsic transverse momentum. One obtains then the result that, if $f_i(x_i)$ is

Magnification

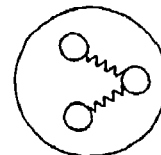
$Q^2 \approx 0.1 \text{ GeV}^2$
↓ Probe Gross Nucleon



$Q^2 \geq 1 \text{ GeV}^2$
↓ Test QPM



$Q^2 \geq 10 \text{ GeV}^2$
↓ Test QCD



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Fig. 59. Cartoon depicting the increased sensitivity to nucleon structure as the magnification (Q^2) of the probe is increased.

the probability for finding parton i with fractional momentum x_i the structure functions are given by

$$F_2(x) = \sum_i e_i^2 f_i(x)$$

$$2xF_1(x) = F_2(x).$$

In the parton model then as $Q^2, \nu \rightarrow \infty$, "magically" the structure functions become independent of Q^2 at fixed x . This is the mathematical statement of scaling.

The notion of constituent scattering is demonstrated in simplified terms (see Atwood, Ref. 53) in Fig. 60. Low Q^2 elastic ep scattering shows a normalized structure function which peaks at $\langle x \rangle = 1$ corresponding to one constituent, the gross nucleon. Quasi elastic ed (d=deuterium) scattering (two constituents) has $\langle x \rangle \approx \frac{1}{2}$ and shows some broadening due to Fermi motion. Finally with large Q^2 one achieves large magnification and the $\langle x \rangle \approx \frac{1}{3}$ indicative of three charged constituents. The scaling behavior at lowish Q^2 is seen in Fig. 61 which shows the F_2 structure function from SLAC-MIT group ep scattering for data from Q^2 of 2 GeV² to 18 GeV². In this range the scaling predicted by the OPM holds rather well.

When probing nucleon structure with $\nu(\bar{\nu})$ one gets a third structure function due to the parity violating nature of the $W^\pm$ and the cross section can be written as

$$\frac{d^2\sigma^{\nu,p}}{dx dy} = \frac{G_F^2 M E}{\pi} [F_1(x)xy^2 + F_2(x)(1-y) \pm F_3(x)xy(1-y/2)]$$

where the $+$ ($-$) is for the $\nu(\bar{\nu})$ probe, and W propagator effects are ignored as they are small ($\lesssim 10\%$ at highest Q^2). For the QPM:

$$2xF_1 = F_2, \quad xF_3 = \pm F_2$$

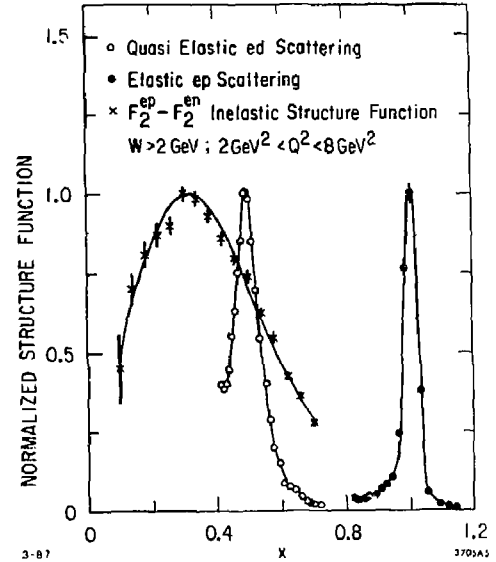


Fig. 60. Examples of the structure function as it relates to the constituent nature of the target being probed.

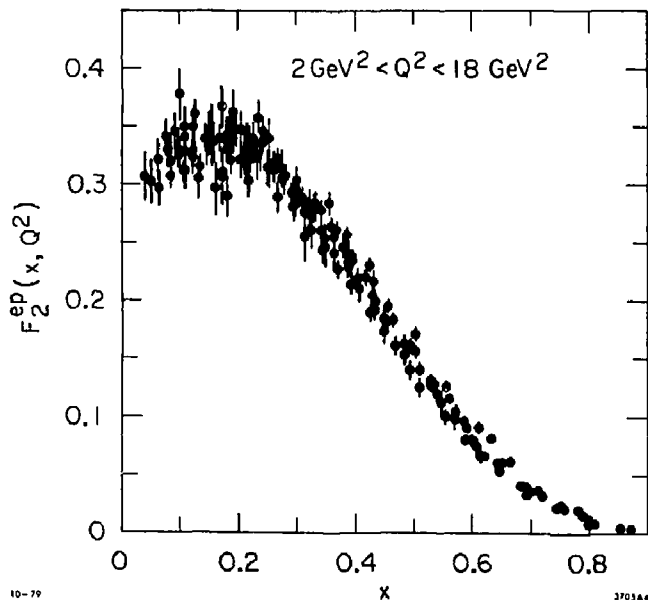


Fig. 61. F_2 structure function as a function of x as measured in ep scattering by the SLAC-MIT group. Over the range of Q^2 indicated in the figure, scaling holds rather well.

and hence,

$$\frac{d^2\sigma}{dx dy}(\nu q) = \frac{G^2 M E}{\pi} F_2(x) \{1\}$$

$$\frac{d^2\sigma}{dx dy}(\bar{\nu} q) = \frac{G^2 M E}{\pi} F_2(x) \{(1-y)^2\}.$$

We see this qualitative QPM behavior of the scattering from the valence quarks in CDHS νFe and $\bar{\nu} Fe$ data in Fig. 62. One also sees a clear indication of the presence of $q - \bar{q}$ sea quarks. To proceed further and extract meaningful qualitative and quantitative tests of QCD requires loosening the constraints of the QPM model and pushing towards higher Q^2 experiments. It is presumably well known to the reader that simple scaling does not hold and that for sufficiently high Q^2 , the structure functions are seen to depend on Q^2 for fixed x . This is illustrated in Fig. 63 for ep , μp , ed and νN scattering. Also if one measures the total momentum carried by the partons, it comprises only half of the nucleon momentum. Hence there must be partons in the nucleon which the electroweak current does not probe. In QCD the presence of gluons can qualitatively account for these effects. They are the unseen "stuff" and the emission of gluons generates transverse momentum for the quarks. This is in direct contrast to the assumption of zero transverse momentum made in the QPM. The presence of gluon emission, visible at sufficiently high Q^2 , results in a Q^2 evolution of the structure functions which shows up as scaling violations. In the spirit of Fig. 59, when we raise Q^2 we have an ever-increasing chance of "catching" a quark emitting a gluon and we go from a situation where only the quarks are visible to where the gluons emitted by the quarks are resolvable.

When a gluon is emitted from a quark inside a nucleon the fractional energy, x , of the quark is lowered and it therefore changes the distribution of quark probabilities $f(x)$. As Q^2 increases the $\langle f(x) \rangle$ of quarks will shift to lower values of x and F_2 will no longer exhibit scaling. This effect is clearly seen in Fig. 63. The evolution of the structure functions with Q^2 is predicted by QCD, and hence QCD can predict the pattern of scale breaking. This is discussed in

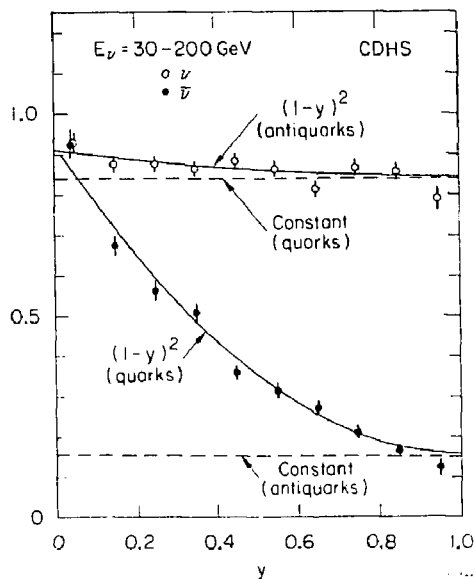


Fig. 62. The cross section for νFe and $\bar{\nu}\text{Fe}$ collisions as a function of y as measured by the CDHS group. The expected QPM functional form is seen for the scattering from the valence quarks; one sees also contributions from the sea.

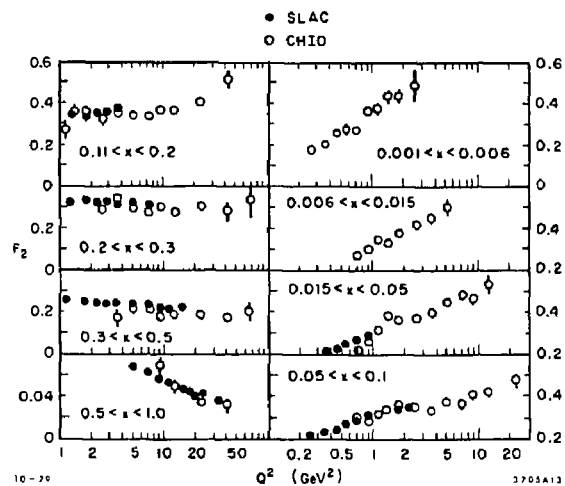


Fig. 63. The change of F_2 with Q^2 in fixed x bins. Clear scaling violations are seen.

detail in the companion lectures of Field and briefly outlined here with reference to Fig. 64. Figure 64(a) depicts the simplest situation involving the evolution of the valence quarks where the only contribution comes from gluon emission from the valence quark. The process whereby the quark with fractional momentum y evolves via the emission of a gluon to a value x is specified by the splitting function P_{qq} . QCD predicts the evolution of q :

$$Q^2 \frac{d}{dQ^2} q^v(x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} q^v(y, Q^2) P_{qq}(x/y)$$

where the splitting function is given by QCD (see Field). The situation with the sea and gluon is somewhat more complicated because there are more than one contributing process (see Fig. 64). But similar expressions can be obtained for the evolution of q^s and g . More germane to our discussion is how this translates into the structure function evolution which is given in terms of the splitting functions P_{qq} , P_{qg} and P_{gg} (all given by QCD) as follows:

$$\begin{aligned} \frac{dx F_3(x, Q^2)}{d \ln Q^2} &= \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 [P_{qq}(x/z) z F_3(z, Q^2)] \frac{x dz}{z^2} \\ \frac{dF_2(x, Q^2)}{d \ln Q^2} &= \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 [P_{qq}(x/z) F_2(z, Q^2) + 2N_f P_{qg}(x/z) G(z, Q^2)] \frac{x dz}{z^2} \\ \frac{d\bar{q}^p(x, Q^2)}{d \ln Q^2} &= \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 [P_{qg}(x/z) \bar{q}^p(z, Q^2) + N_f P_{gq}(x/z) G(z, Q^2)] \frac{x dz}{z^2} \end{aligned}$$

where G is the gluon structure function and N_f is the number of active flavors which will depend on the Q^2 range. The lower Q^2 experiments can safely ignore charm, but this is not true at higher Q^2 and the inclusion of the charm threshold is yet another detail which must be included in the analyses.

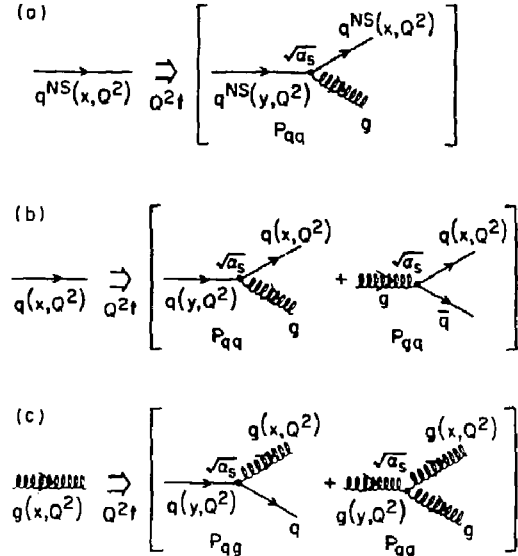


Fig. 64. The evolution of parton structure functions as described in the text for the valence quarks, the sea and the gluons.

Hence from the evolution of the structure functions one is able to measure α_s (A). In principle, the most straight-forward method is to use $x F_3$ in ν interactions since it involves the least input. It is independent of the gluon distribution, uncertainties in R, the amount of the strange sea or charm threshold effects. However it has the largest statistical error. Measurements of F_2 are the most commonly used and have the best statistical accuracy, the major problem being the unknown gluon structure function. One can go to large x ($\gtrsim 0.25$) to minimize the contribution from the gluon structure function. This also reduces the sensitivity to the sea and charm threshold effects. Or, one can use F_2 and q^0 to simultaneously extract A and G. These methods and variations thereof are used by the experimental groups. The different procedures leads to a wide range of A values each subject to its own particular problems. I have chosen values indicative of the work of these groups to indicate the range and accuracy obtained for A.

So we see that we obtain measurements of A from the evolution of the structure functions. How are the structure functions measured? They are obtained from appropriate combinations of the measured differential cross sections taking into account the complications arising from the sea, charm threshold, quark transverse momentum and mass effects and the W propagator effect.

From ν and $\bar{\nu}N$ scattering one obtains

$$F_2 = \left\{ \frac{\pi}{G_F^2 M E} \left[\frac{d^2 \sigma^{\nu}}{dx dy} + \frac{d^2 \sigma^{\bar{\nu}}}{dx dy} \right] - 2x(s-c)[1 - (1-y)^2] \right\}$$

$$1/(1 + (1-y)^2 - y^2 R/(1+R))$$

$$2xF_1 = F_2/(1+R)$$

$$xF_3 = \frac{\pi}{G_F^2 M E} \left[\frac{d^2 \sigma^{\nu}}{dx dy} - \frac{d^2 \sigma^{\bar{\nu}}}{dx dy} \right] / \{1 - (1-y)^2\}$$

where $R(x, Q^2) = \frac{y_L}{2x} = \frac{F_L}{2xF_1}$ (F_L accounts for the effects of transverse momentum of the quarks) and (s-c) accounts for the contributions from the strange and

charm quarks inside the nucleon. We expect R to be small for $x \gtrsim 0.3$. Typically experiments use fixed values of R(0-0.2) or QCD predictions. Measurements of R are given at the end of this section.

From eN and μN scattering one obtains

$$F_2 = \frac{Q^4}{8\alpha^2 \pi M E} \left(\frac{d^2 \sigma}{dx dy} \right) / \left(1 - y + \frac{y^2}{2(1+R)} \right).$$

Now to the data. Figure 65 shows the evolution of the F_2 structure function measured by CDHS.^[121] The solid line is the leading order QCD prediction, the dashed line the prediction of an Abelian vector gluon model and the dashed-dot the prediction of a scalar gluon model. The data are in good agreement with the leading order QCD but in poor agreement with the non-QCD gauge structures. Figure 66 shows the same measurement from the EMC group using a muon probe (see Ref. 60). Again leading order QCD accounts well for the scaling violations. Table VIII summarizes the A measurements from CDHS,^[121] CCFRR,^[122] CHARM,^[123] BPF,^[124] BCDMS,^[125] and EMC^[126] collaborations. The typical range of Q^2 covered by these experiments is 5-200 GeV², and A is measured using lowest order QCD for all experiments except CDHS where the $\overline{MS}$ scheme is used. These results are shown graphically in Figure 67 where the systematic and statistical errors have been added in quadrature. We may conclude that A = 200 - 100 MeV² suitably covers the range of the experimental measurements.

Measurements of R from νN scattering are shown in Fig. 68 where particular attention should be paid to the CDHS data which span a meaningful range of x. One sees that the data are in good agreement with the prediction of QCD and with the assumption of $R \approx 0$ for $x \gtrsim 0.3$ used to extract A from F_2 , as discussed above.

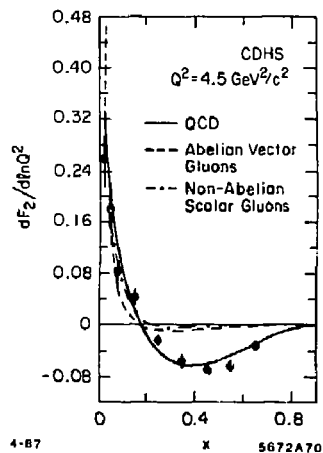


Fig. 55. The slopes of the F_2 structure function $dF_2/d\ln Q^2$ as a function of x as obtained by CDHS in μN scattering. The solid line is from a leading order QCD fit to F_2 and $\bar{q}$. The dashed lines correspond to non-asymptotically free theories of the strong interactions with scalar and vector gluons.

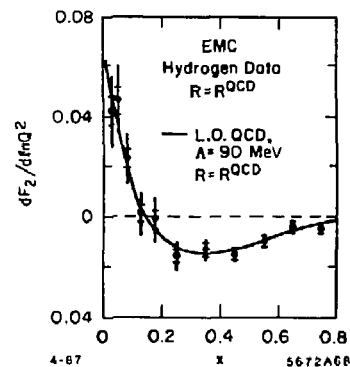


Fig. 56. The slopes of the F_2 structure function as a function of x as obtained by EMC in μN scattering. The inner error bars are statistical the outer ones systematic; a) $R = 0.0$ and b) $R = R_{QCD}$. The solid line is the leading order QCD prediction with $\Lambda = 90$ MeV.

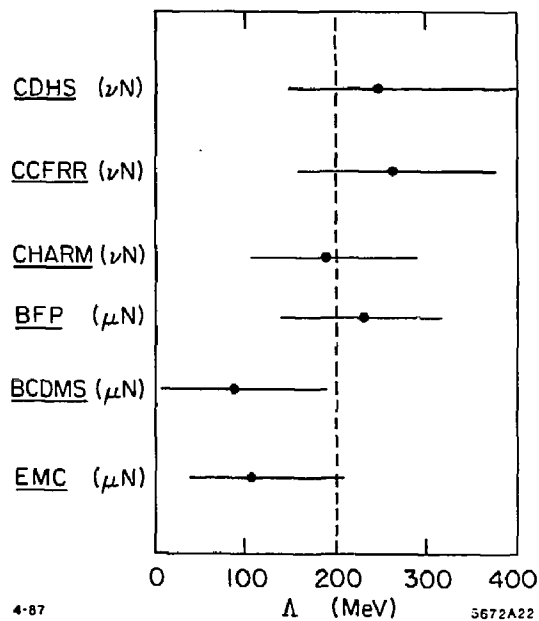


Fig. 67. Summary of the Λ_0 measurements of the experiments indicated. $\Lambda = 200 \pm 100$ MeV provides a reasonable summary of these data.

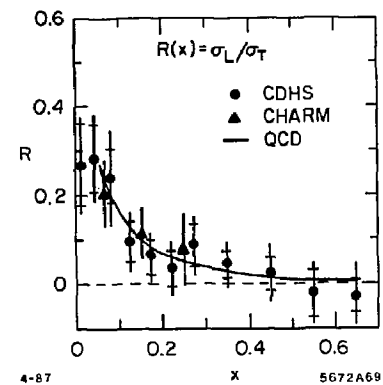


Fig. 68. Measurements of $R(x)$ from the CDHS and CHARM groups.

Table VIII

Group	REF	PROBE	Λ (MeV)
CDHS	55	ν	250^{+150}_{-100}
CCFR	56	ν	266^{+114}_{-104}
CHARM	57	ν	$190^{+70}_{-60} \pm 70$
BPF	58	μ	$230 \pm 40 \pm 80$
BCDMS	59	μ	85^{+60+90}_{-40-70}
EMC	60	μ	105^{+55+85}_{-45-45}

6. Measurement of the Photon Structure Function

The two-photon process in high energy e^+e^- collisions has been used to measure the photon structure function, F_2 , and extract from it Λ . The process is shown in Fig. 69 where hadrons are produced by the two-photon mechanism with one almost real photon ($P^2 \approx 0$) and one virtual photon ($Q^2 \neq 0$). One can envisage three rather distinct processes as shown in Fig. 70. In Fig. 70(a) at low Q^2 , both photons can turn into vector mesons (ρ 's) in which case one imagines $\rho\rho$ scattering which has a cross section which falls off like $\frac{1}{s}$ for $Q^2 > M_\rho^2$. As the probing photon becomes more virtual (Fig. 70(b)), it will begin to couple directly to the partons in the target photon thereby sensing the structure of the photon. If scaling holds, the cross section would follow a $1/Q^2$ behavior in this regime. Finally the hadronic part of the photon (Fig. 70(c)) has a pointlike component which is predicted to dominate at large Q^2 . In this region the structure function F_2 is expected to rise with Q^2 and show large scaling violations:

$$F_2 \sim \ln Q^2 \quad \text{and} \quad \sigma \sim \frac{1}{Q^2} \ln Q^2.$$

Using the variables outlined in the previous chapter we can write the differential

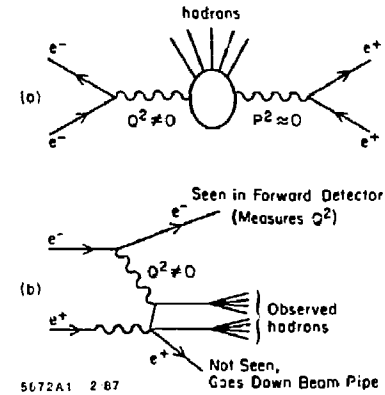


Fig. 69. a) The production of hadrons via the two-photon process in e^+e^- interactions. b) Definition of the kinematics and observables.

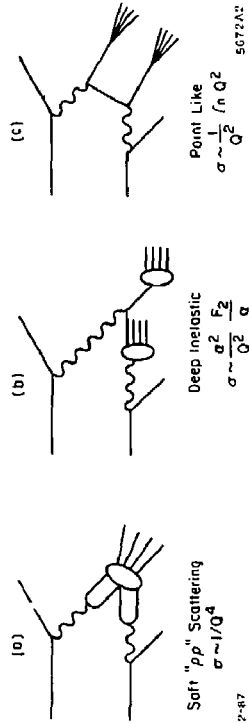


Fig. 70. Three levels of inelasticity in two-photon hadron production.

cross section for $e^+e^- \rightarrow e^+e^- + \text{hadrons}$ as

$$\frac{d^2\sigma}{dx dy} \propto \frac{1}{Q^4} \{ (1-y)F_2 + xy^2F_1 \}.$$

Since for the physics to be discussed here xy^2 is small (≈ 0.01), we can safely ignore the F_1 term. As discussed above, F_2 contains a pointlike piece which has the properties of being absolutely calculable in QCD, $F_2 \propto \frac{1}{\alpha_s(Q^2)} \sim \ln Q^2/\Lambda^2$ and unlike other F_2 structure functions, $F_2^{\text{pointlike}}$ rises with x . To counterbalance this rosy picture, the hadronic part of F_2 is not absolutely calculable in QCD and so in reality to extract Λ we need a model for F_2^{HAD} .

The experimental setup is to use a high energy, forward going, $e^\pm$ to tag the two-photon event as indicated in Fig. 69. This tagged $e^\pm$ provides the Q^2 (the $e^\pm$ having proceeded undeviated down the beam line) from $Q^2 = 2EE'$ ($1 - \cos\theta$) where $\cos\theta$ is the $e^\pm$ scattering angle, $E = E_{\text{beam}}$ and E' is the measured $e^\pm$ energy. The hadronic energy, W , is obtained from the detected hadrons: $W_{\text{vis}}^2 = \sum p_i^2 < W_{\text{true}}^2$. Since $x_{\text{vis}} = \frac{Q^2}{Q^2 + W_{\text{vis}}^2} > x_{\text{true}}$. One needs a model to correct x for the unseen hadrons. This introduces a model dependence into the determination of F_2 .

How do we confront the measured data? We can calculate the QPM prediction for F_2 :

$$F_2^{\text{QPM}} = 3(\alpha/\pi) \sum_{q,d} e_q^2 (x(1-x)^2) \ln \frac{W^2}{M_q^2} + 8x^2(1-x) - x$$

where M_q , the effective quark mass, is a parameter. One can add a VDM part to this to account for the hadronic piece via

$$F_2^{\text{VDM}} \approx 0.2\alpha(1-x).$$

The leading order QCD calculation gives for the pointlike part

$$F_2^{\text{LO}} = 3\left(\frac{\alpha}{\pi}\right) \sum e_q^2 f(x) \ln \frac{Q^2}{\Lambda^2}.$$

Higher order, regularized QCD calculations lead to

$$F_2^{H0} = F_2^R(\Lambda_{\overline{MS}}) + \Delta(x, t) + h F^{VDM}, \quad (8)$$

where $\Delta(x, t)$ is the regularization term which has been calculated by Antoniadis and Grunberg^[91] in terms of the parameter t . One must be careful to avoid double counting between the Δ and F_2^{VDM} terms, which is typically handled by introducing another parameter, h .

Does the data from tagged two-photon events support the qualitative picture outlined above? (For a comprehensive review see Ch. Berger and W. Wagner.)^[92] Figure 71 shows the cross section as a function of Q^2 for $Q^2 < 10 \text{ GeV}^2/c^2$. One sees very clearly the transition from pp scattering to deep inelastic scattering as depicted in Fig. 70. The cross section flattens out markedly for $Q^2 > M_p^2$. Can we measure mass scales via the $\ln W^2/M^2$ term which occurs in the formulae for the structure function? To study this the TPC/2 γ group have made a measurement of $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$ which can be envisaged as Fig. 70(c) with the quark lines replaced by muons. According to our QED calculation, F_2 for this process should be proportional to $\ln W^2/M_\mu^2$. The data are shown in Fig. 72 with predictions of the QED assuming $M_\mu = 50, 105$ and $200 \text{ MeV}/c^2$. With this method, M_μ is measured to $\pm 5\%$ which clearly indicates that a measurement of F_2 has considerable sensitivity to a mass scale.

Does one see the point-like structure of the photon, i.e., does F_2 increase with increasing Q^2 ? Again the answer is yes as seen in Figs. 73 and 74. Figure 73 shows the PLUTO data for F_2 as a function of x for three Q^2 bins. Figure 74 shows a compilation of data for $\langle F_2/\alpha \rangle$ as a function of Q^2 . One sees quite clearly that the structure function grows with increasing Q^2 . The growth is consistent with the QCD prediction of $a+b \ln Q^2/\Lambda^2$ but would equally well be fit with the QPM form $a' + b' \ln Q^2/M_{eff}^2$. We can see this same trend in Fig. 75 which shows F_2/α as a function of x from the PLUTO collaboration. The predictions of QCD (both lowest order and higher order, $\Lambda = 200 \text{ MeV}$) and QPM

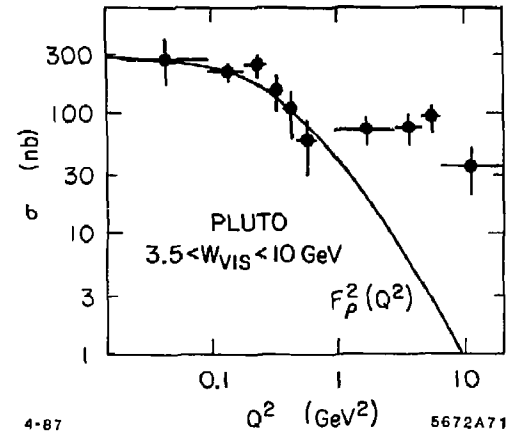


Fig. 71. The two-photon cross section as a function of Q^2 from the PLUTO group. The curve is the contribution for pp scattering.

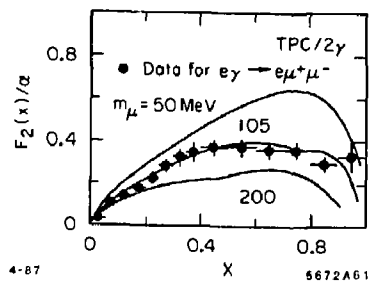


Fig. 72. The structure function for $e\gamma \rightarrow e\mu^+\mu^-$ from the TPC/2 γ group. The curves are predictions of the QPM with the muon mass as indicated.

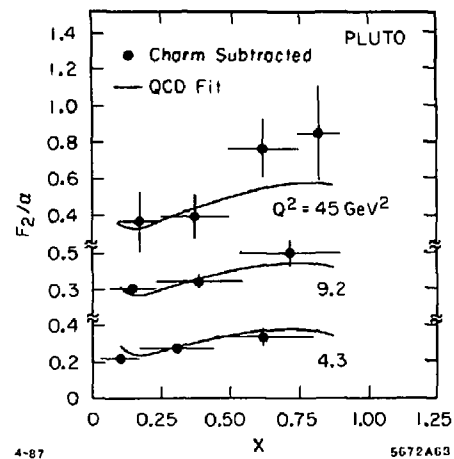


Fig. 73. The structure function for $e\gamma \rightarrow e + \text{hadrons}$ as measured by PLUTO for three Q^2 bins. The solid lines are the fits to QCD with $\Lambda = 183$ MeV.

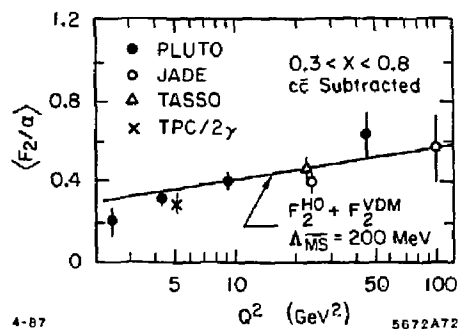


Fig. 74. Compilation of data for $\langle F_2/\alpha \rangle$ as a function of Q^2 . The clear growth of F_2 with Q^2 is seen.

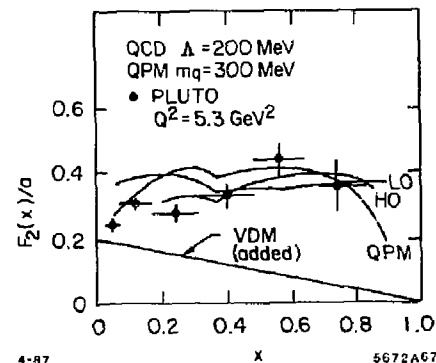


Fig. 75. The structure function for $e\gamma \rightarrow e + \text{hadrons}$ as a function of x as measured by PLUTO. The curves show the contribution from VDM which is added to the predictions of QPM and QCD with $\Lambda = 200$ MeV. Both lowest order and higher order QCD curves are shown.

are shown where the VDM piece has been added in to account for the hadronic contribution. One sees that QCD and QPM do equally well at approximating the data; it would seem that the data alone do not require anything beyond QPM. This same result is achieved by many groups (see Ref. 62). However one should note that to get the QPM to fit requires assuming quark masses of 300 MeV/c². However under the more realistic assumption of current masses, the QPM prediction is about 2-3 times too large. One may interpret this result then by saying that one is seeing clear gluon effects which are "dressing" up bare quark masses to effective masses of ~ 300 MeV/c². In the spirit then, that QCD can account for the data, how well can we extract Λ ?

All the data presented here are analyzed using the regularization scheme of Antoniadis and Grunberg. It should be pointed out that this procedure is controversial and subject to a fair amount of criticism. (For a flavor of this see the discussion of Field et al.^[63]) The data of PLUTO,^[64] TASSO^[65] and JADE^[66] are shown in Figs. 76, 77 and 78. The PLUTO data (see also Fig. 73) are fit using the form of Equation 8 with Λ , t and h as parameters. While h and t are strongly correlated, and hence are not well determined, Λ is relatively insensitive to this correlation. The value extracted is

$$\Lambda_{\overline{MS}} = 183_{-46}^{+65+46} \text{ MeV} \quad (\text{PLUTO}; Q^2 : 3 - 100 \text{ GeV}^2/c^2).$$

Both JADE and TASSO assume the hadronic piece of F_2 goes like $F_2^{\text{HAD}} = 0.2(1-x)$ and fix the value of t ($= 1, 6$, respectively) to obtain

$$\begin{aligned} \Lambda_{\overline{MS}} &= 140_{-65}^{+190} \text{ MeV} \quad (\text{TASSO}; Q^2 : 7 - 70 \text{ GeV}^2/c^2) \\ &= 250 \pm 90 \text{ MeV} \quad (\text{JADE}; Q^2 : 10 - 220 \text{ GeV}^2/c^2) \end{aligned}$$

The fits to the data are shown in the figures. All the experiments correct for the effects of charm quark production. Ignoring the fact that the three experiments

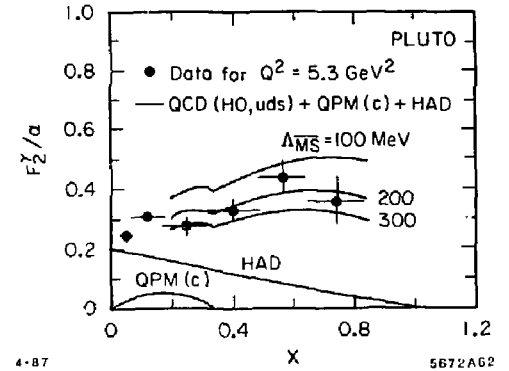


Fig. 76. PLUTO data used to extract $\Lambda_{\overline{MS}}$.

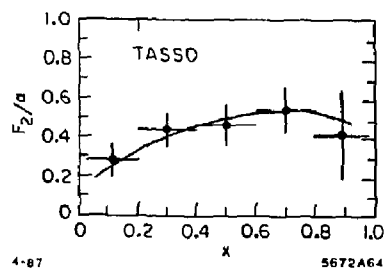


Fig. 77. TASSO data used to extract $\Lambda_{\overline{MS}}$.

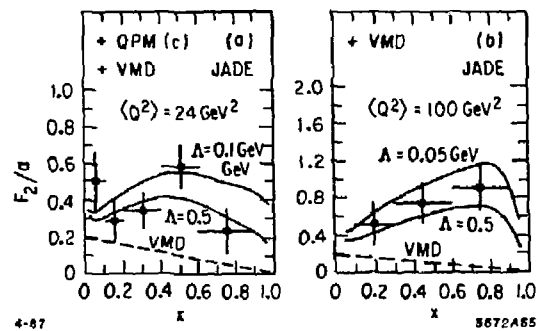


Fig. 78. JADE data used to extract $\Lambda_{\overline{MS}}$.

have different $\langle Q^2 \rangle$ and correlated errors (since their procedures are very similar), one can blindly average the three measurements to obtain

$$\Lambda_{\overline{MS}} = 195^{+60}_{-30}, \quad \langle Q^2 \rangle^2 \approx 50 \text{ GeV}^2.$$

This is an impressively precise measurement, although there still exist issues associated with the appropriateness of the regularization scheme and the use of a pion-like form factor for the hadronic component of F_2 . Some light has been shed on the latter question by the TPC/2 γ group who have used low Q^2 data ($Q^2 < 1.6 \text{ GeV}^2$) to measure F_2^{HAD} . Their data, shown in Fig. 79, agree with the form $F_2^{\text{HAD}} = 0.2\alpha(1-x)$ except at small x . So it seems that this may not be a large uncertainty since the sensitivity to Λ comes mainly from the higher x data.

7. Conclusions

We have reviewed a large body of data which relates to testing QCD. We see that the qualitative agreement between the data from a very wide range of processes and QCD is very impressive. This agreement is non-trivial in the sense that there exist no other theories or models which fit the data nearly as well.

Quantitatively we see that quarks have spin $\frac{1}{2}$ and come in three colors and that gluons are vector-like particles. We do not see direct experimental evidence for the triple gluon vertex; however most data are poorly fit ($\bar{p}p$ at CERN in particular) without such a contribution. Further indirect evidence comes from the fact that gluons appear to have a softer fragmentation function than quarks.

Extracting the QCD scale Λ is problematical and there exists no processes where a precise, non-controversial measurement can be made. All measurements seem entangled with either experimental or theoretical problems or both. However if we assume that all the measurements taken as a whole "average out" these problems, one is left reasonably satisfied with $\Lambda_{QCD} = 150 \pm 100 \text{ MeV}$.

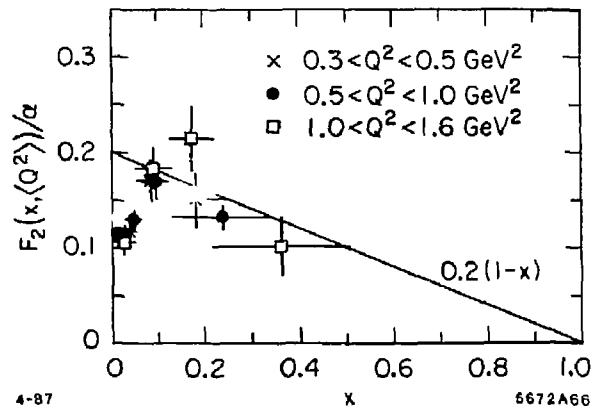


Fig. 79. The structure function as a function of x at low $\langle Q^2 \rangle$ as measured by the TPC/2 γ group. The curve is $0.2(1-x)$.

My final conclusion would be that QCD is a magnificent theoretical edifice and success. There exists no evidence from experiment to doubt its validity and, with time, I would expect the quantitative tests will be improved.

8. Acknowledgements

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PHENOMENOLOGY OF HEAVY QUARK SYSTEMS*

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1. Heavy Quark Spectroscopy

Introduction

The spectroscopy of heavy quark systems is the showcase of our understanding of hadron physics. It is sometimes even advertised as the "hydrogen atom of strong interactions".

We do indeed have a fundamental gauge theory of the strong interactions in Quantum Chromodynamics (QCD). This theory in principle explains the vast body of data that has been accumulated over the past dozen years.¹ However, as we will soon see, the connection between the fundamental theory and experimental observables is not (yet) as it is for the electroweak gauge theory, $SU(2) \times U(1)$. The situation we confront is essentially non-perturbative, and the underlying gauge theory is one-step-removed from detailed numerical confrontation with experiment.

What we have at present to accompany the data is more like a phenomenology, inspired or backed-up by QCD. At times it gives us an asymptotic form. At other times it gives an expression for the general structure of some quantity, with free parameters or hadronic matrix elements contained within it. While these latter are determined by QCD in principle, for the moment they are often only approximately calculable (at best). So we take a peek at the data and 'adjust' the parameters, thereby learning something about the nature of the solution of QCD. Then we predict additional quantities and iterate the whole process again.

This is then a place where theory and experiment intertwine; basic theory, models inspired by theory, and experiment meet and influence one another. It is

quite different from the situation in the electroweak theory where there is a well-defined and clean set of perturbative predictions to compare with experiment. In one sense this is frustrating, as one would like clean and decisive tests of the underlying theory. In another sense, this is what makes it exciting and makes the subject still worth pursuing: the interplay between theory and experiment is interesting in itself, and we often learn things which are applicable either as techniques or as results in other areas as well.

In fact, progress has been made and continues to be made.² Eventually, one has every reason to believe that we will be able to calculate the "potential" from first principles, presumably using lattice techniques. Everything then will be predicted starting from the QCD Lagrangian. We have come a long way in this direction already,³ and perhaps in the Summer Institute of a few(?) years hence we may well no longer need a talk on this subject.

The Spin Independent Potential

Let's start with the nonrelativistic, spin independent potential. Even the use of the word potential is a bit loose for we are starting with a strong interaction bound state problem and extracting from it an effective two-body, non-relativistic potential. The problem at hand is intrinsically a relativistic field-theoretic one in which the $q\bar{q}$ sector, for example, is coupled to what happens in the $q\bar{q}q\bar{q}$, $q\bar{q} + \text{gluon}$, etc. sectors as well. Some justification for the success of the "naive," non-relativistic approach have recently been given,⁴ but simultaneously questions have been raised as to the effect of what is being neglected, and how it changes the relationship between parameters in the underlying theory and the effective potential.⁵ There is even a whole, well-developed approach to understanding

some of the same body of data through QCD sum rules.⁶

With these questions in mind, we shall proceed to think in terms of a two-body potential obtained by expanding in powers of v^2/c^2 . Indeed, such an expansion does make some sense, for the smallness of v^2/c^2 in a system composed of a heavy quark and heavy antiquark encourages us to think in terms of a non-relativistic potential with spin-dependent terms which arise first in order v^2/c^2 and give rise to splittings which are smaller than those between levels of the spin independent potential. In the cases at hand v^2/c^2 is ~ 0.4 for the low-lying charmonium states and $\lesssim 0.1$ for the low-lying bottomonium states. Still, when we go to compare with the calculated energy levels with experiment we need to bear in mind that predictions from alternate potentials that differ by 10 or 20 MeV are not necessarily significant in favoring one potential over another. We may in any case be making (especially for charmonium) approximations to the exact theory in "deriving" a non-relativistic potential which render the resulting model incapable of discriminating differences at this level.

We have good theoretical guidance in two opposite regimes. At short distances, or equivalently large momentum scales, there is the property of asymptotic freedom. The running coupling becomes smaller as we decrease the distance scale at which we work, and the effective potential approaches the lowest order one gluon exchange result,

$$V(r) \rightarrow -\frac{4}{3} \frac{\alpha_s}{r} \quad (1)$$

as $r \rightarrow 0$.

Note the additional factor of $\frac{4}{3}$ compared to the usual Coulomb interaction; this arises from color. Its derivation was discussed in the Summer Institute of

10 years ago.⁷ It is so pervasive that it is worth a short derivation, so here it is again. We want the additional factor due to color. It arises from a normalized color singlet quark-antiquark wave function, $\delta_{ij}/\sqrt{3}$ in the initial and final state, a color SU(3) matrix $\lambda_{ij}^a/2$ at each quark-gluon vertex, and a color sum over the (eight) gluons, δ^{ab} , in the gluon propagator. The sum over indices gives a trace:

$$\sum_{a,b=1}^8 \sum_{i,j,k,l=1}^3 \frac{\delta_{il}}{\sqrt{3}} \frac{\lambda_{ij}^a}{2} \frac{\delta_{jk}}{\sqrt{3}} \frac{\lambda_{kl}^b}{2} \delta^{ab} = \frac{1}{3} \sum_{a=1}^8 \text{Tr} \left[\frac{\lambda^a}{2} \frac{\lambda^a}{2} \right] = \frac{1}{3} (8) \frac{1}{2} = \frac{4}{3}. \quad (2)$$

The trace of $\frac{\lambda^a}{2} \frac{\lambda^a}{2}$ is just $\frac{1}{2}$, as befits the generators of a Lie algebra (or, as may be checked for the case of the λ matrices of SU(3) directly); thus the ubiquitous factor of 4/3.

That's one regime. The second regime where we have very solid theoretical input is at the other extreme, as $r \rightarrow \infty$, and we have confinement of the quarks. From relatively general theoretical arguments we know that the potential behaves as a linear function of the distance:

$$V(r) \rightarrow kr. \quad (3)$$

There is a corresponding physical picture of a "color-electric flux tube" joining the quarks. As you pull the quarks apart, the flux tube is increased in length, at the cost of an increase in energy per unit length given by the constant k . The value of k is about 0.2 GeV².

Given those two regimes we might hope to construct the full potential. The simplest possibility is to simply add together terms with the correct functional de-

pence in the two asymptotic regimes. This is basically the Cornell potential,⁸

$$V(r) = \frac{-0.48}{r} + \frac{r}{(2.34 \text{ GeV}^{-1})^2}, \quad (4)$$

with the two coefficients having been adjusted to fit the charmonium spectrum, although the model does a quite adequate job in describing bottomonium as well.

In the late 70's Richardson combined the two behaviors in one form.⁹ Here is his potential in configuration space,

$$V(r) = \frac{8\pi}{33 - 2n_f} \Lambda \left(\Lambda r - \frac{f(\Lambda r)}{\Lambda r} \right), \quad (5)$$

with

$$f(t) = 1 - 4 \int_1^\infty \frac{dq}{q} \frac{e^{-qt}}{\ell n^2(q^2 - 1) + \pi^2}, \quad (6)$$

where it looks like two terms. What's going on is more transparent in momentum space where it can be written as one term:

$$\tilde{V}(q^2) = -\frac{4}{3} \frac{12\pi}{33 - 2n_f} \frac{1}{q^2 \ell n(1 + q^2/\Lambda^2)}. \quad (7)$$

As q^2 goes to infinity, this expression becomes precisely $\frac{4}{3} \alpha_s/q^2$, as required from one gluon exchange. In the other limit of $q^2 \rightarrow 0$, one obtains something proportional to $1/q^4$. This may be an unfamiliar behavior in momentum space, but if you Fourier transform back to configuration space, this is just a potential which is linear in r . It is by no means guaranteed that you will get the "right" coefficient to fit the data. Richardson, along with others¹⁰ who proposed modified versions of this potential, showed that you do in fact get a very reasonable, even excellent, description of the data, especially for bottomonium.

Finally, Martin has proposed the potential¹¹

$$V(r) = (5.82 \text{ GeV}) \left(\frac{r}{1 \text{ GeV}^{-1}} \right)^{0.104}. \quad (8)$$

This potential, with the absurd power of 0.104, lacks fundamental motivation (as Martin knew very well). We will use it as a kind of straw man, for it also does quite a credible job of fitting the charmonium and bottomonium data. But why?

The reason can be seen in Figure 1. Here you can see the various potentials for comparison purposes. In particular, aside from being displaced vertically from one another a little bit (which you are free to remove by adjusting the quark mass), they all have about the same behavior between 0.1 and 1 fermi. This can be seen even better looking at the inset, where r is given on a logarithmic scale and the potentials have been shifted slightly relative to one another vertically, as discussed above. Also shown are the mean radii of the psi, upsilon, etc. These are all between 0.1 and 1 fermi, and that's why the different potentials all can fit the data; the wave functions for these states mostly (but not entirely) live in this region where the potentials coincide.

Thus, where our theoretical insight is best and tells us something very well-defined for the behavior of the spin independent potential, it is mostly irrelevant to the present data. Conversely, the experiments up to now mostly tell us about a region where theory does not have much to say about the spin independent potential. In fact, one can invert the data to obtain a potential¹² which describes what happens from 0.1 to 1 fermi. Within errors, it coincides with what we have just seen in Figure 1.

Even without a particular potential and detailed calculation, we can get a good qualitative idea of what the spectrum of states will look like. In Figure 2a

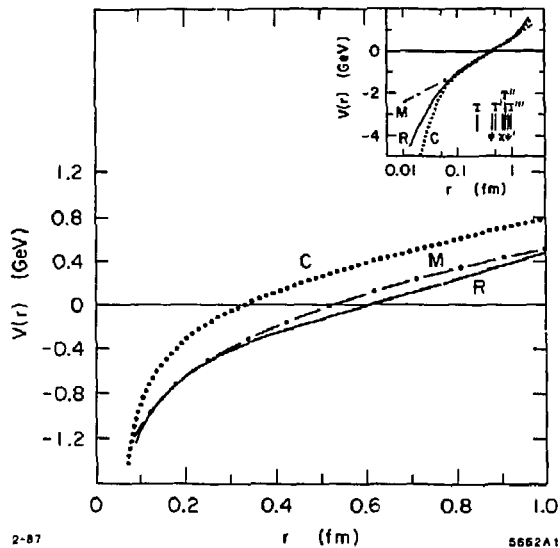


Fig. 1. Comparison of the shape of the Cornell⁸ (dotted curve), Richardson^{9,10} (solid curve), and Martin¹¹ (dash-dot curve) potentials. The inset shows the same comparison with the potentials displaced slightly on the vertical scale and a logarithmic horizontal scale, along with the mean radii of some charmonium and bottomonium states.

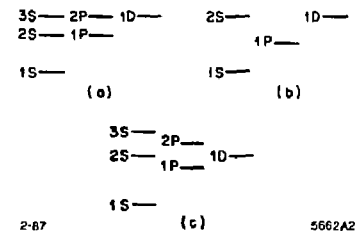


Fig. 2. The spectrum of energy levels in the case of the Coulomb potential (a), the three dimensional harmonic oscillator (2b), and a hybrid of the two (c).

is the familiar spectrum due to a Coulomb potential, which is what we have at short distances. The ground state with $\ell = 0$ is labelled 1S; its radial excitation (labelled 2S) is degenerate in the case of a Coulomb potential with the first set of $\ell = 1$ states (labelled 1P), and so on. As an example of a confining potential which we want at large distance, Figure 2b shows the levels of a three dimensional harmonic oscillator, which is more familiar than a linear potential and turns out also to be a special "boundary" case from the point of view of the ordering of levels. What will happen when we combine the two? For the energy levels we will naturally get something in between Figures 2a and 2b. This is shown in Figure 2c. The ordering, starting at the bottom, is 1S, 1P (between the 1S and 2S as for the harmonic oscillator, but closer to the 2S, as it would be degenerate with it for the Coulomb potential), 2S, 1D (above 2S as for Coulomb, but close to it, as it would be degenerate for the harmonic oscillator), etc. You can therefore get a qualitative understanding of the spectrum from quite general considerations.

There is a theorem¹³ which is quite useful in this regard and puts the qualitative ordering discussed above on a rigorous footing. It states that if $\nabla^2 V(r) > 0$ for all r , something which is true for all suggested potentials, then $E_{nS} > E_{(n-1)P}$. Related theorems are provable for the ordering of levels with other angular momenta.¹⁴

Each of the potentials discussed above can give a quantitative understanding of the levels of charmonium and bottomonium to 30 MeV or better. Even the statement that one flavor independent potential can fit both systems is nontrivial. The agreement between theory and experiment, which is shown in Schindler's lectures,¹ I regard as quite spectacular. It includes not just energy levels, but wave functions at the origin for the nS states as well. Where there is a disagree-

ment, it is difficult to know whether to blame it on the potential or on corrections due to relativistic or other effects which have been left out.

When and how will we be able to distinguish between potentials? The answer appears to be that toponium will provide the crucial system. In Figure 3 is shown the spectrum of toponium¹⁵ corresponding to m_t in the range of 40 to 50 GeV. There are 10 or more nS states below open top threshold; near that threshold there is one state per 100 MeV.

More important for the physics at hand, aspects of the spectrum of states and of the wave functions at the origin are now sensitive to the behavior of the potential at short distances. The values of the wave function at the origin are shown in Figure 4, with that for the ground state corresponding to a width into electron-positron pairs, which is proportional to the square of the wave function at the origin, of about 9 keV (from the one photon intermediate state alone). This is larger than one would expect from a naive extrapolation from the psi and the upsilon by about a factor of two. We are beginning to see the effect of the $1/r$ term in the potential pulling in the wave function. Higher levels are affected less, as seen in Figure 4, for on average they live at larger distances.

The same physical effect is shown in Table I, with the t quark mass assumed to be 50 GeV. Notice in particular how much the energy of the 1S level is pulled down by the Cornell potential (3 GeV below $2 m_t$). This is to be compared with 1.7 GeV for the Richardson and 1.4 GeV for the Martin potentials. Correspondingly the radius of the 1S state is much smaller for the Cornell potential and the 2S to 1S difference much bigger. Even more dramatic is the comparison of the wave function at the origin for the 1S state, where the Cornell result is about 3 times that for Richardson and 9 times that for Martin. Remember, the predicted

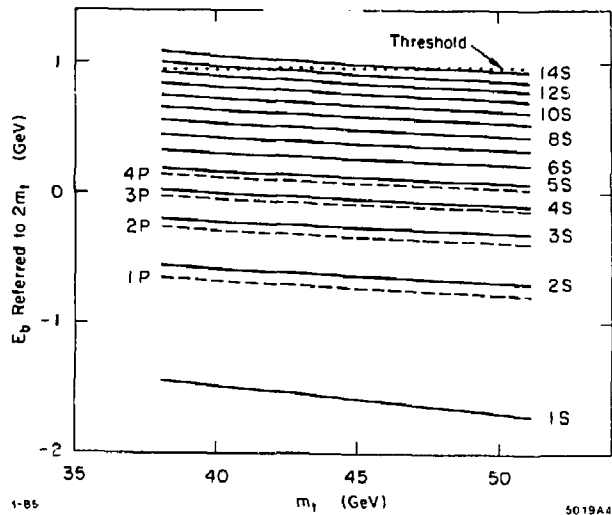


Fig. 3. The spectrum of nS states of toponium obtained¹⁵ from the Richardson potential with m_t in the range of 40 to 50 GeV.

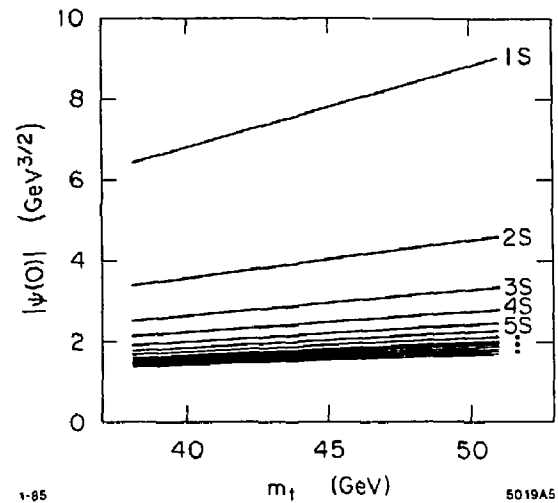


Fig. 4. The value of the wave function at the origin for toponium nS states obtained¹⁵ with the Richardson potential and m_t in the range of 40 to 50 GeV.

electron-positron width goes like these numbers squared!

Potential	E_{1S} (GeV)	$\langle r_{1S} \rangle$ (fermi)	$E_{2S} - E_{1S}$ (GeV)	$\Psi(0)_{1S}$ (GeV ^{3/2})
"Cornell"	97.1	0.028	2.2	23.3
"Richardson"	98.3	0.048	1.0	8.5
"Martin"	98.6	0.084	0.5	2.7

Table 1. Characteristics of Toponium States for Various Potentials

Before leaving this subject, we should note that this same property makes toponium a fairly sensitive place to look for extra short range forces. A good example is the presence of an extra term in the potential due to neutral Higgs exchange with enhanced couplings.¹⁶ This changes both the wave functions and the ordering of the energy levels in a characteristic fashion, and allows it to be distinguished from a simple change in the strength of the $1/r$ piece of the strong interaction potential.

The Spin Dependent Potential

Now we turn to the spin dependent potential. In its full glory it has the form:

$$\begin{aligned}
 V_{SD}(r) = & \left(\frac{\vec{S}_1 \cdot \vec{L}}{2m_1^2} + \frac{\vec{S}_2 \cdot \vec{L}}{2m_2^2} \right) \left(\frac{dV(r)}{dr} + 2 \frac{dV_1(r)}{dr} \right) \\
 & + \frac{(\vec{S}_1 + \vec{S}_2) \cdot \vec{L}}{m_1 m_2} \frac{dV_2(r)}{dr} \\
 & + \frac{1}{6m_1 m_2} \left(6\vec{S}_1 \cdot \vec{r} \vec{S}_2 \cdot \vec{r} - 2\vec{S}_1 \cdot \vec{S}_2 \right) V_3(r) \\
 & + \frac{2}{3m_1 m_2} \vec{S}_1 \cdot \vec{S}_2 V_4(r)
 \end{aligned} \tag{9}$$

as given by Eichten and Feinberg¹⁷ and discussed at previous Summer Institutes by Eichten¹⁸ and by Peskin.¹⁹ The term $V(r)$ is the spin independent potential we discussed previously. The other terms involving V_1 , V_2 , V_3 , and V_4 are not necessarily simply related to $V(r)$. As can be seen particularly clearly in Michael Peskin's lectures,¹⁹ these extra terms originate in expectation values of color electric and magnet fields which are different than those that enter in the spin independent potential; they are new objects.

Although the situation is more complicated than one might have hoped, at least initially it was possible to entertain the idea that all the new spin dependent terms are of short range. This hope was dashed when it was shown that²⁰

$$V(r) + V_1(r) = V_2(r). \tag{10}$$

Since V has a long range confining part, so must either V_1 or V_2 .

Let us use Eq. (10) to eliminate V_1 from the spin dependent potential. It now reads:

$$\begin{aligned}
 V_{SD}(r) = & \left(\frac{\vec{S}_1 \cdot \vec{L}}{2m_1^2} + \frac{\vec{S}_2 \cdot \vec{L}}{2m_2^2} \right) \left(\frac{-dV(r)}{rdr} + 2 \frac{dV_2(r)}{rdr} \right) \\
 & + \frac{(\vec{S}_1 + \vec{S}_2) \cdot \vec{L}}{m_1 m_2} \frac{dV_2(r)}{rdr} \\
 & + \frac{1}{6m_1 m_2} (6\vec{S}_1 \cdot \vec{r} \vec{S}_2 \cdot \vec{r} - 2\vec{S}_1 \cdot \vec{S}_2) V_3(r) \\
 & + \frac{2}{3m_1 m_2} \vec{S}_1 \cdot \vec{S}_2 V_4(r).
 \end{aligned} \quad (11)$$

Could it now be that the remaining new potentials V_2 , V_3 , and V_4 are short range?

Not only is there no information to contradict this possibility, but it is supported by the results of recent lattice gauge theory calculations,^{21,22} the results of some of which²² are shown in Figures 5, 6, 7, and 8. We see that V_1 (which we have eliminated from Eq. (11)) is not short range, but V_2 looks completely different; it is very short range, and similarly for V_3 and V_4 . All of this is done on a $16^3 \times 32$ lattice. It should be regarded as a qualitative result, but an important step toward the more quantitative results we can expect in the future.

Let us now go back to the spin dependent potential in the equal mass case relevant to quarkonium. We rewrite it a little bit, combining the first two terms:

$$\begin{aligned}
 V_{SD}(r) = & \frac{\vec{S} \cdot \vec{L}}{2m^2} \left(\frac{-dV(r)}{rdr} + 4 \frac{dV_2(r)}{rdr} \right) \\
 & + \frac{1}{12m^2} (6\vec{S} \cdot \vec{r} \vec{S} \cdot \vec{r} - 2\vec{S} \cdot \vec{S}) V_3(r) \\
 & + \frac{1}{6m^2} (2\vec{S} \cdot \vec{S} - 3) V_4(r).
 \end{aligned} \quad (12)$$

Now, to get a simple physical picture of what is happening, let us forget for a

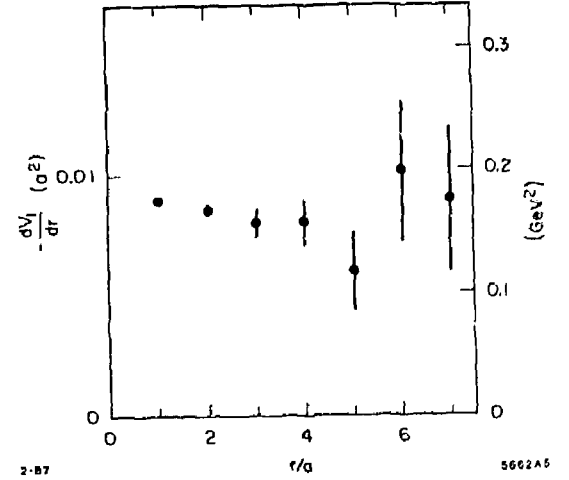


Fig. 5. Results of a lattice Monte Carlo calculation²² of the spin-dependent potential $-dV_1/dr$ as a function of radial distance in units of the lattice spacing, a .

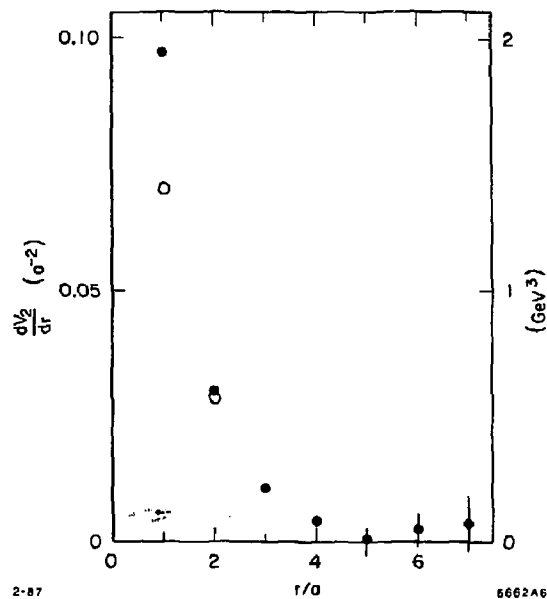


Fig. 6. Results of a lattice Monte Carlo calculation²² of the spin-dependent potential dV_2/dr as a function of radial distance in units of the lattice spacing, a . The solid points are before, and the open points after a correction for lattice artifacts described in Ref. 22.

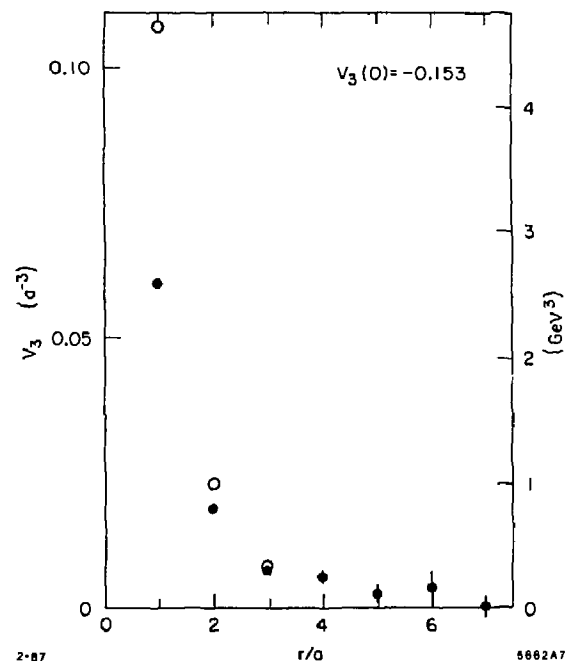


Fig. 7. Results of a lattice Monte Carlo calculation²² of the spin-dependent potential V_3 as a function of radial distance in units of the lattice spacing, a . The solid points are before, and the open points after a correction for lattice artifacts described in Ref. 22.

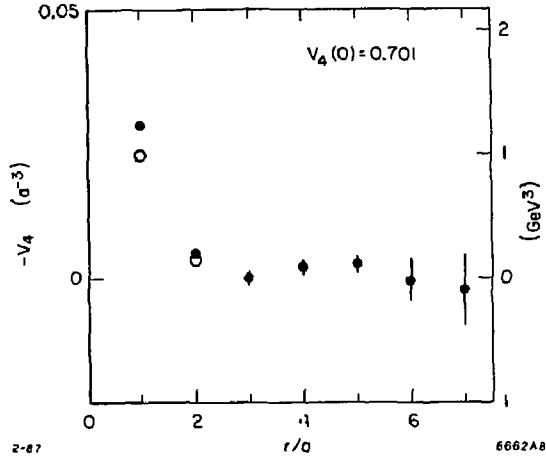


Fig. 8. Results of a lattice Monte Carlo calculation²² of the spin-dependent potential $-V_4$ as a function of radial distance in units of the lattice spacing, a . The solid points are before, and the open points after a correction for lattice artifacts described in Ref. 22.

moment the previous discussion about the spin dependent and spin independent potentials being independent entities. Let us consider what we would obtain from a (relativistic) four-fermion interaction arising from the exchange of a vector and a scalar between a quark and the antiquark of equal mass. In momentum space this is represented by an interaction:

$$L_{int} = \tilde{s}(q^2) \bar{u} u \bar{v} v + \tilde{v}(q^2) \bar{u} \gamma_\mu u \bar{v} \gamma^\mu v. \quad (13)$$

If we do an expansion in powers of v^2/c^2 , the static limit is the spin independent potential $v + s$, and the spin dependent terms give the Breit-Fermi potential, which in configuration space is:

$$\begin{aligned} V_{SD}(r) = & \frac{\vec{S} \cdot \vec{L}}{2m^2} \left(-\frac{dv(r) + ds(r)}{rdr} + 4\frac{dv(r)}{rdr} \right) \\ & + \frac{1}{12m^2} \left(6\vec{S} \cdot \vec{r} \vec{S} \cdot \vec{r} - 2\vec{S} \cdot \vec{S} \right) \left(\frac{dv(r)}{rdr} - \frac{d^2v(r)}{d^2r} \right) \\ & + \frac{1}{6m^2} \left(2\vec{S} \cdot \vec{S} - 3 \right) \nabla^2 v(r). \end{aligned} \quad (14)$$

The term $-(dv(r) + ds(r))/rdr$ in the first line is due to the familiar Thomas precession, and it is followed by usual spin-orbit, tensor (on the second line), and spin-spin (on the third line) interactions, each with a coefficient related to $v(r)$ or $s(r)$.

Now we are in a position to compare what is in Eq. (14) to the generic decomposition in Eq. (12) involving V_1 , V_2 , and V_3 . First, the spin independent potential V is here given by the sum of the vector and scalar potentials, $v + s$. Second, the spin dependent potentials V_2 , V_3 , and V_4 are all expressible in terms of derivatives of only the vector part of the potential, v . Hence, if v is related to

gluon exchange and its associated $1/r$ behavior, then the potentials V_2 , V_3 , and V_4 are all short range in character.

This encourages us to make the following division: the scalar term is long range and associated with quark confinement, while the vector term is short range (we include $1/r$ behavior as short range) and associated with gluon exchange. From the short range Coulomb-like piece one obtains the spin dependent terms we are long accustomed to in atomic physics: a spin-orbit interaction (minus the piece due to Thomas precession), a tensor interaction, and a spin-spin interaction. As you go to long range, the confining interaction, which is Lorentz scalar in character, becomes dominant. The associated physical picture²³ has a color flux tube that connects the quark and antiquark, and as they rotate around each other the flux tube rotates along with them. Consequently there are no spin dependent forces generated from this part of the potential, aside from the Thomas term which comes in with a minus sign and is generated from the spin rotation associated with Lorentz transforming from the center-of-mass to the quark or antiquark rest frame. So we get a simple way of understanding all the terms in Eq. (14). From now on we will take this identification of v and s seriously. Occasionally we will slip over to the stronger assumption that $s(r) \propto r$ and $v(r) \propto 1/r$, even to the point of thinking that we know the respective constants of proportionality.

• The Spin - Spin Interaction

The spin-spin interaction, which in the equal mass case takes the form

$$V_{SS} = \frac{1}{6m^2} (2 \vec{S} \cdot \vec{S} - 3) \nabla^2 v(r), \quad (15)$$

is the analogue for the color forces of QCD of the interaction which gives rise to

the hyperfine splittings between atomic levels. If we are brave enough to follow this analogy further and insert a $1/r$ behavior for $v(r)$, then since $\nabla^2(1/r) = -4\pi\delta^3(\vec{r})$, the spin-spin interaction is of very short range!

This delta function at the origin can be tested by noting that for quarkonium p-wave states, whose wave function at the origin vanishes, the expectation value of the spin-spin interaction should be zero. Therefore the center-of-gravity of the three states with total quark spin one and $J = 0, 1$, and 2 should be the same as the mass of the $J = 1$ state with quark spin zero:

$$\frac{5M_2 + 3M_1 + M_0}{9} = M_{\text{spin singlet}}. \quad (16)$$

(The p-wave states with total quark spin one are split in mass by the spin-orbit and tensor interactions, and the weighted average is just such as to cancel out these contributions).

For charmonium, the left-hand side of Eq. (16) is 3525.38 MeV, and an experiment in the last days of the ISR found a few candidate events with an average mass of 3525.4 ± 0.8 MeV.²⁴ For the bottomonium system, the corresponding values for the center-of-gravity are 9900.2 MeV for the 1P states and 10,261.6 MeV for the 2P states.¹ It would be very interesting to measure the mass of the corresponding singlet p wave states for bottomonium. There is a little bit of evidence from the CLEO experiment, studying $\pi\pi$ transitions from the $3S$ resonance, for a state a little below the 1P center-of-gravity.²⁵ As the $b\bar{b}$ system is more non-relativistic than $c\bar{c}$, the agreement with Eq. (16) should be excellent. Otherwise, the agreement in the charm case was an accident, and we had better take a close look at our assumptions on the short range nature of the spin-spin interaction.

Let us specialize to a system that consists of one heavy and one light quark. The assumption that $v(r)$ behaves as $1/r$ still gives a delta function at the origin in the part of the potential that gives the spin-spin interaction. Furthermore, the physical origin of this term in a quark color magnetic moment interacting with an antiquark color magnetic moment is still correct, and so it still depends inversely on the product of the quark mass and the antiquark mass (see the coefficient of V_4 in Eq. (9)). For example, the mass difference of the ground state vector and pseudoscalar states should behave as

$$M(^3S_1) - M(^1S_0) \propto \frac{|\Psi(0)|^2}{m_i m_j}. \quad (17)$$

If we use the fact that the spin-spin splitting is small and that in terms of constituent masses,

$$M(^3S_1) \sim M(^1S_0) \sim m_i + m_j, \quad (18)$$

then we can rewrite Eq. (17) in terms of mass squared,

$$M^2(^3S_1) - M^2(^1S_0) \propto \frac{m_i + m_j}{m_i m_j} |\Psi(0)|^2 \propto |\Psi(0)|^2 / \mu_{ij} \quad (19)$$

and get a result that depends on the reduced mass of the quark-antiquark system.

One the other hand, in a system composed of a heavy and a light quark we have a atomic hydrogen-like situation with the heavy quark playing the role of the nucleus and the light quark primarily living at "large" distances. The corresponding wave function is determined by the long distance part of the potential which behaves as kr . However, for a potential which behaves as r^β , the

Schrodinger equation yields a scaling law that makes²⁶

$$|\Psi(0)|^2 \propto \mu_{ij}^{\frac{3}{2+\beta}}.$$

Therefore, corresponding to the case at hand with $\beta = 1$,

$$|\Psi(0)|^2 \propto \mu_{ij},$$

and substituting this into Eq. (19), one finds^{27,28}

$$M^2(^3S_1) - M^2(^1S_0) \sim \text{const.} \quad (20)$$

This relation is compared to experiment in Table 2. The input masses come from the Particle Data Tables²⁹ except for the $F - F^*$ mass splitting where the new result from the Mark III experiment reported to this meeting is used.³⁰

Mass ² Difference	Experimental value ^{27,28} in GeV ²
$M_\rho^2 - M_\pi^2$	0.57
$M_{K^*}^2 - M_K^2$	0.56
$M_{D^*}^2 - M_D^2$	0.55
$M_{F^*}^2 - M_F^2$	0.55
$M_{B^*}^2 - M_B^2$	0.55

Table 2. Ground State Vector - Pseudoscalar Mass² Differences

The $\rho - \pi$ difference is thrown in for good measure, even though it involves only light quarks. Even the $K^* - K$ case should not be in Table 2, for the strange quark is not all that heavy. Of course, they are in Table 2 because they all agree

magnificently with each other, so much the more so now that we have the new data on the $F^* - F$ mass difference. Equation (20) works far better than it should, as not only are the "heavy" quarks involved not all that heavy, but even the statement that the wave function at the origin squared is proportional to the reduced mass is only approximate. Such superb agreement must be an accident.

Now let us return to systems with two heavy quarks. There the wave functions are not determined by the linear part of the potential and Eq. (20) should not hold. (It doesn't!) But here we can be braver yet and insert $v(r) = -4\pi\alpha_s/r$ into Eq. (15) and sandwich it between ground state vector and pseudoscalar meson wave functions to obtain³¹

$$M(^3S_1) - M(^1S_0) = \frac{32\pi\alpha_s}{9} \frac{|\psi(0)|^2}{m^2} \left(1 + O\left(\frac{\alpha_s}{\pi}\right) \right), \quad (21)$$

where even the next order QCD corrections have been calculated. If we take the measured splitting between the ψ and η_c and invert Eq. (21) to find α_s , the result³¹ is 0.3 to 0.4. This is perhaps a little bit too big, not to be regarded as very significant at this time.

• The Spin-Orbit and Tensor Interactions

Spin-orbit terms give rise to the fine structure in the old atomic physics terminology. In the case of equal constituent masses they take the form

$$V_{S.O.} = \frac{\vec{S} \cdot \vec{L}}{2m^2} \left(-\frac{ds(r)}{rdr} + 3\frac{dv(r)}{rdr} \right), \quad (22a)$$

and

$$V_{Tensor} = \frac{1}{12m^2} \left(6\vec{S} \cdot \vec{r} \vec{S} \cdot \vec{r} - 2\vec{S} \cdot \vec{S} \right) \left(\frac{dv(r)}{rdr} - \frac{d^2v(r)}{d^2r} \right). \quad (22b)$$

If we take the spin-orbit and tensor interactions and calculate their contri-

butions to the ^3P_J state masses, we get³²

$$M(^3P_2) = \bar{M} + a - 2b/5 \quad (23a)$$

$$M(^3P_1) = \bar{M} - a + 2b \quad (23b)$$

$$M(^3P_0) = \bar{M} - 2a - 4b, \quad (23c)$$

where the matrix elements a and b are defined as

$$a = \frac{1}{2m^2} \left\langle -\frac{ds}{rdr} + 3\frac{dv}{rdr} \right\rangle, \quad (24a)$$

$$b = \frac{1}{12m^2} \left\langle \frac{dv}{rdr} - \frac{d^2v}{d^2r} \right\rangle. \quad (24b)$$

We can summarize the relative values of the matrix elements in terms of one number by forming the ratio

$$r = \frac{M(^3P_2) - M(^3P_1)}{M(^3P_1) - M(^3P_0)} = \frac{2a - \frac{12}{5}b}{a + 6b}. \quad (25)$$

If only the spin-orbit term contributes, $r = 2$, while if the Coulomb-like vector part of the potential $v(r)$ is present, $r \approx 0.8$. As one turns on the scalar term, $s(r)$, the matrix element a decreases, as does r .

If you look at the experimental numbers,¹ updated with recent data, particularly from CUSB,³³ one finds¹ for charmonium $r_{\chi_c} = 0.50 \pm 0.02$, and for bottomonium $r_{\chi_b} = 0.67 \pm 0.05$ and $r_{\chi'_b} = 0.70 \pm 0.20$, for the $1P$ and $2P$ states, respectively. All these values are smaller than would result from solely a Coulomb-like vector term, and point toward a non-negligible scalar term. Moreover, the

detailed predictions from taking the vector term as $-\frac{4}{3}\alpha_s/r$ and the scalar term as kr give quite good agreement³⁴ with the data, particularly for bottomonium (recall that one expects some corrections, particularly for charmonium). The case is getting fairly good for a substantial part of the long range, confining part of the potential being scalar rather than vector.

For mesons composed of a heavy quark and a light antiquark or *vice versa* the physical situation is different, as we discussed previously in considering the spin-spin interaction. The light quark lives at larger distances, so that the Thomas term, $-ds(r)/rdr$, can "beat" the net vector term, $3dv(r)/rdr$, and the effect of the spin orbit interaction, $\langle V_{S.O.} \rangle$, can be reversed in sign. This would result in an inversion of spin multiplets³⁵ compared to atomic physics, charmonium, and bottomonium where the higher spin state lies higher: the ordering would now be $M(^3P_0) > M(^3P_1) > M(^3P_2)$. This idea might be testable in the ^3P charm meson states, labelled here D^{**} 's. A candidate state, the $D^{**}(2420)$, already has been found and must be $J = 1$ or 2 , as it decays to $D^*\pi$. If this multiplet is inverted, the $J = 0$ state, which decays to $D\pi$, will lie at a higher mass than the $D^{**}(2420)$.

Conclusion

In this brief and incomplete review of the spectroscopy of heavy quark systems, we have seen that we have a good qualitative picture of the nature of the spin-independent forces. That, plus some phenomenological potentials inspired by fundamental theory, carry us a long way. For the spin-dependent effects we even have a semi-quantitative understanding, as they are more sensitive to the short distance part of the potential and we have more insight and more tools to help us sort things out.

Eventually, we want a quantitative calculation of both the spin-independent potential $V(r)$ and the spin-dependent potentials $V_2(r)$, $V_3(r)$, and $V_4(r)$, from QCD. This will likely come in due time from improvements in computer power and in technique over the present lattice calculations.

In the meantime, to clarify the emerging picture, we need more data. We need to find or confirm the $^1P_1$ states of charmonium and bottomonium. We need to find the η_b . We need to find the other D^{**} 's. And maybe best of all, we need to see the spectrum of toponium.

2. Heavy Quark Decays

Introduction

In this second lecture we turn from the spectroscopy of hadrons involving heavy quarks to their decays. This is a comparatively new subject, but one which is already in a fairly mature state experimentally:¹ we have measurements of many D meson branching ratios, including Cabibbo suppressed nonleptonic modes and the decomposition of the semileptonic decays into exclusive channels; excellent lifetime measurements exist for both charm and bottom hadrons; a good beginning has been made on the study of exclusive decays of the $\bar{F}$, A_c and $B^{0,+}$.

On the theoretical side, we have a solid general framework within which to calculate these weak decays. In particular, this means starting with the electroweak interactions and their gauge group, $SU(2) \times U(1)$, and adding the corrections due to the strong interactions through the use of the renormalization group equation (or an equivalent formulation of the same physics), with anomalous dimensions computed from QCD.

These calculations are carried out at the quark level. A first stage in their application to actual hadrons is simply to neglect any other constituent of the decaying hadron aside from the heavy quark. In such a spectator model, as it is called, one directly carries over the quark level calculation to be the hadron level result, with the spectator quarks and gluons assumed to arrange themselves into the final state particles together with the quarks (or leptons) coming from the heavy quark at no cost or benefit to the overall rate.

From the present data on charmed particle lifetimes, it is clear that there are differences of a factor of two or so between different species.¹ To do better

than that, it is necessary to go beyond the spectator model and to consider not just what happens at the quark level but at the hadron level. In so doing, ideas such as annihilation diagrams, interference, color (mis)matching, and final state interactions have entered the discussion.²⁶

This lecture will be a brief review of the subject of the decays of hadrons containing heavy quarks. We will start at the quark level where we will spend a large proportion of the review, since we know quite precisely how to proceed theoretically and the results give a semi-quantitative description of the experimental situation as we know it today. Then we will describe the various ideas enumerated above as corrections to the spectator model, leaving a more detailed analysis of the merits of these approaches in light of the present experimental situation to others.³⁷

Weak Decays at the Quark Level

• Semileptonic Decays

It is theoretically simplest to start with semileptonic decays of the form

$$Q \rightarrow q + e\bar{\nu}_e$$

(such as $b \rightarrow ce\bar{\nu}_e$) or

$$Q \rightarrow q + e\nu_e$$

(such as $c \rightarrow se\nu_e$) which correspond to a Hamiltonian density of the form

$$\mathcal{H} = -V_{Qq} \frac{G_F}{\sqrt{2}} \bar{q}\gamma_\mu(1 - \gamma_5)Q \bar{e}\gamma^\mu(1 - \gamma_5)\nu_e. \quad (26)$$

In Eq. (26) we have used particle names in place of the corresponding spinor

operators and introduced the one factor that does not enter into the analogous expression for muon decay, the Kobayashi - Maskawa factor V_{Qq} .

The corresponding decay rate can then be easily related to that for muon decay, $G_F^2 m_\mu^5 / 192\pi^3$:

$$\Gamma(Q \rightarrow q e \bar{\nu}_e) = |V_{Qq}|^2 \frac{G_F^2 M_Q^5}{192\pi^3} F\left(\frac{m_q}{M_Q}\right) \quad (27)$$

where

$$F(\Delta) = 1 - 8\Delta^2 + \Delta^6 - \Delta^8 - 24\Delta^4 \ln \Delta. \quad (28)$$

Note again the extra Kobayashi - Maskawa factor in front and the phase space factor, F , which is unity for a massless final quark. This factor drops off rather quickly, so that $F(0.3) = 0.52$, a value relevant approximately for the $c \rightarrow s$ and the $b \rightarrow c$ transitions.

The electron (positron) energy spectrum is different in the two cases. For $b \rightarrow c e \bar{\nu}_e$ it is like that in muon decay and gives rise to a "hard" spectrum that does not vanish at the high energy end:

$$\frac{1}{\Gamma} \frac{d\Gamma}{dx} \approx \frac{12}{5} x^2 (2-x), \quad (29)$$

while for $c \rightarrow s e \bar{\nu}_e$ (and for $t \rightarrow b e \bar{\nu}_e$) it vanishes at the two ends

$$\frac{1}{\Gamma} \frac{d\Gamma}{dx} \propto 12x^2(1-x) \quad (30)$$

and where the scaled energy variable is

$$x = \frac{2E_e}{M_Q} \leq 1 - \frac{m_q^2}{M_Q^2}. \quad (31)$$

Similar results of course hold for decays involving muons or taus.

• Nonleptonic Decays

The Hamiltonian density for nonleptonic decays such as $Q \rightarrow q + u \bar{d}$ has the same basic form,

$$\mathcal{H} = -V_{Qq} \frac{G_F}{\sqrt{2}} \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) Q_\alpha \bar{u}_\beta \gamma^\mu (1 - \gamma_5) d_\beta \quad (32)$$

as for semileptonic decays, aside from the addition of the color indices α and β which are summed over the three colors to form color singlet currents. The decay rate

$$\Gamma(Q \rightarrow qu \bar{d}) = 3 |V_{Qq}|^2 \frac{G_F^2 M_Q^5}{192\pi^3} F\left(\frac{m_q}{M_Q}\right) \quad (33)$$

is also identical to the semileptonic case except for the factor of three on the right hand side due to color (we are neglecting the masses of the u and d quarks, just as we neglected those of the e and ν_e previously).

Now let us rewrite the Hamiltonian in a slightly different form:

$$\begin{aligned} \mathcal{H} = & -V_{Qq} \frac{G_F}{2\sqrt{2}} \left[c_+ \left\{ \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) Q_\alpha \bar{u}_\beta \gamma^\mu (1 - \gamma_5) d_\beta + \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) d_\alpha \bar{u}_\beta \gamma^\mu (1 - \gamma_5) Q_\beta \right\} \right. \\ & \left. + c_- \left\{ \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) Q_\alpha \bar{u}_\beta \gamma^\mu (1 - \gamma_5) d_\beta - \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) d_\alpha \bar{u}_\beta \gamma^\mu (1 - \gamma_5) Q_\beta \right\} \right] \end{aligned} \quad (34)$$

with $c_+ = c_- = 1$ initially. All we have done is to add and subtract a term which is nothing but the original expression with $Q \leftrightarrow q$. Moreover, this term would be identical to the original one if it were not for the presence of the color indices; without them the interchange $Q \leftrightarrow q$ is a Fierz transformation under which $V - A$ interactions go into themselves. In the decay rate, the three on the

right-hand side of Eq. (33) is replaced by $2c_+^2 + c_-^2$, which again is no change at all when $c_+ = c_- = 1$.

So why make a more complicated expression out of something simple? The answer lies in what happens when we turn on the strong interactions and add the effects of QCD to the purely weak interactions that we have considered up to this point. The weak interaction will be modified by the presence of strong interaction effects and c_+ and c_- will be renormalized. But they have been carefully chosen in this regard, for they only go into themselves under this renormalization, i. e. the corresponding operators, which are even or odd under interchange of color indices, do not mix through QCD corrections. Not only do the strong interactions modify c_+ and c_- from their initial value of unity, but they introduce new operators into the effective Hamiltonian. These so-called "penguin" operators, which come in beginning at the one loop level, have a different space-time structure than the $V - A \times V - A$ structure we have had up to now. We proceed to consider each of these effects and their magnitude in turn.

• The Calculation of c_+ and c_-

To calculate what happens to c_+ and c_- under renormalization due to QCD is equivalent to being able to study their behavior as one moves from one momentum scale to another. More specifically, at the momentum scale corresponding to M_W the weak interactions are characterized by the "bare" Hamiltonian density of Eq. (31) and $c_+ = c_- = 1$. We are interested in what happens when we move down to a momentum scale characteristic of hadrons, i.e. roughly the mass of the decaying heavy quark.

The study of what happens when one moves from one momentum scale to another is directly formulated through a renormalization group equation. In the

case of c_+ and c_- , they satisfy such an equation of the form:

$$\left[\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} - \gamma_{\pm}(g) \right] c_{\pm}(q/\mu, g) = 0, \quad (35)$$

where μ is some reference scale of momentum (the renormalization point) and q is a second scale at which we wish to calculate the effective weak Hamiltonian. In this equation, $\beta(g)$ is the standard beta function of the theory,

$$\beta = \mu \frac{\partial g}{\partial \mu},$$

which characterizes how the coupling changes with a change of scale. For QCD, it has the perturbation theory expansion,

$$\beta(g) = -\frac{g^3}{48\pi^2} (2n_f - 33) + \dots \quad (36)$$

where n_f is the number of quark flavors. Notice that the coefficient of g^3 is negative as long as $33 > 2n_f$. In other words, the coupling decreases as we increase the scale of momentum at which we are looking. This is just the property of asymptotic freedom; the theory of QCD becomes more and more like a free field theory as we increase the momentum scale. The quantities $\gamma_{\pm}$ are the anomalous dimensions associated with the operators $c_{\pm}$, respectively. They also can be calculated in a perturbative expansion, starting in order g^2 , where they originate in graphs where a single gluon is exchanged between fermion lines in the basic four-fermion weak interaction:³⁸

$$\gamma_+ = +\frac{g^2}{4\pi} + \dots \quad (37a)$$

$$\gamma_- = -\frac{g^2}{2\pi} + \dots \quad (37b)$$

Note that if $\gamma_{\pm} = 0$, then the combination of derivatives on the left-hand side of

the renormalization group equation can be rewritten as a total derivative:

$$\left[\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} \right] c_{\pm}(q/\mu, g) = \left[\mu \frac{d}{d\mu} \right] c_{\pm} = 0, \quad (38)$$

simply expressing the fact that $c_{\pm}$ does not change under a change of momentum scale if the anomalous dimensions are zero. In the case at hand, as we have just seen, the anomalous dimensions are non-zero and the operator coefficients $c_{\pm}$ change with scale.

We now proceed to solve this renormalization group equation.³⁹ The method of solution that follows at first looks like it is pulled out of the hat, but we will see the rationale for it, so bear with me for a moment.

We begin by defining the quantity $\bar{g}$ through an integral:

$$\ln(q/\mu) \equiv \int_{\bar{g}}^{\bar{g}(q/\mu, g)} \frac{dx}{\beta(x)} \quad (39)$$

with $\bar{g}(1, g)$. The quantity $\bar{g}$, which is dimensionless, can only be a function of the ratio of the momentum scales q and μ and the coupling g at the reference scale μ ; it is just the "running coupling" that is familiar to all of us. To see this, let us put it in a more familiar form by looking at the situation when $\bar{g}$ is small, so that we can use the perturbative result for $\beta(x) = \frac{x^4}{48\pi^2}(2n_f - 33) + \dots$ under the integral in Eq. (39). If we take the first term in this expansion we obtain on performing the integral,

$$\ln q/\mu = \frac{48\pi^2}{2n_f - 33} \frac{1}{2} \left(\frac{1}{g^2} - \frac{1}{\bar{g}^2} \right) \quad (40)$$

or

$$\alpha_s(q^2) \equiv \frac{\bar{g}^2}{4\pi} = \frac{\alpha_s(\mu^2)}{1 + \frac{33-2n_f}{12\pi} \alpha_s(\mu^2) \ln q^2/\mu^2}. \quad (41)$$

This is the standard expression for the running of α_s when it is small.

The definition of $\bar{g}$ in Eq. (39) is perfectly general though; it simply reduces to the standard form in the small coupling region upon using lowest order perturbation theory for $\beta(x)$. Moreover, it is relevant to solving our equation since it exactly satisfies the renormalization group equation with zero anomalous dimensions:

$$\left(\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} \right) \bar{g}(q/\mu, g) = 0, \quad (42)$$

as may be seen by applying the differential operator on the left-hand side of this equation to both sides of Eq. (39).

Now we are ready to solve the full equations for $c_{\pm}$. The solution of Eq. (35) is:

$$c_{\pm}(q/\mu, g) = c_{\pm}(1, \bar{g}) \exp \left(\int_{\bar{g}}^{\bar{g}} -\frac{\gamma_{\pm}(x) dx}{\beta(x)} \right), \quad (43)$$

as can be seen directly by substitution and employing Eq. (42) together with the fact that the derivative of an integral with respect to its upper limit of integration is just the integrand evaluated at that point.

We go again to perturbation theory to evaluate the integral in the exponent of Eq. (43):

$$\int_{\bar{g}}^{\bar{g}} -\frac{\gamma_{\pm}(x) dx}{\beta(x)} = \frac{6}{33 - 2n_f} \ln \frac{\bar{g}^2}{g^2}. \quad (44)$$

Therefore,

$$c_{\pm}(q/\mu, g) = c_{\pm}(1, \bar{g}) \left(\frac{\bar{g}^2}{g^2} \right)^{\frac{6}{33 - 2n_f}}, \quad (45)$$

and similarly,

$$c_-(q/\mu, g) = c_-(1, g) \left(\frac{g^2}{g^2} \right)^{\frac{-12}{33-2n_f}}. \quad (46)$$

We are interested in what happens between a momentum scale which is characteristic of the decaying hadron (which we take to be μ) and the weak scale, M_W (which we take to be q). Moreover, if we had also taken our reference momentum scale μ to be the weak scale, our coefficients should make the Hamiltonian density correspond to the "bare" density in Eq. (34), i. e., $c_{\pm}(1, g) = 1$. Therefore,

$$c_+(M_W/\mu, g) = \left(\frac{\alpha_s(M_W^2)}{\alpha_s(\mu^2)} \right)^{\frac{6}{33-2n_f}} \quad (47)$$

and

$$c_-(M_W/\mu, g) = \left(\frac{\alpha_s(M_W^2)}{\alpha_s(\mu^2)} \right)^{\frac{-12}{33-2n_f}}. \quad (48)$$

This is our sought after result for the QCD renormalization of the coefficients $c_{\pm}$.

When we recall that $\alpha_s(q^2)$ runs down as the momentum scale goes up, we see that $c_+(M_W/\mu, g) < 1$ and $c_-(M_W/\mu, g) > 1$. In fact, there is the simple relation

$$c_+^2 c_- = 1 \quad (49)$$

(which is traceable to the factor of -2 between the anomalous dimensions of the corresponding operators), so that one of the corresponding terms in the weak Hamiltonian is necessarily suppressed if the other is enhanced by the effects of QCD.

• Penguins

Before using these results to look at the overall picture of decays in the spectator model, let us take a brief look at the additional operators introduced by QCD, the "penguins". A set of lowest order graphs which contribute to the existence of "penguin" operators relevant to various quark decays is shown in Figure 9.

On the upper left is a one loop, "penguin" graph relevant to strange quark decay (and in particular, to neutral K decay). Once the loop integral is performed this diagram contributes to an effective operator whose space-time structure is $(V-A) \times V$, or equivalently a mixture of $(V-A) \times (V-A)$ and $(V-A) \times (V+A)$. The latter operator has a structure that is not in the original weak Hamiltonian density. Arguments have been made that although its relative coefficient is small, the corresponding operator has a big matrix element in K decays and that it contributes a large part of the experimentally observed amplitude.⁴⁰ This is a subject still very much under debate.⁴¹

The diagram on the upper right shows a potential "penguin" in Cabibbo suppressed charm decays. Estimates generally put its strength well below that from ordinary graphs which contribute to the same process.

In bottom decay, however, it may be possible to have processes (Cabibbo suppressed to be sure) where "penguin" diagrams give rise to contributions comparable to, or maybe even larger than, those of ordinary tree level graphs.⁴² The bottom portion of Figure 9 shows a possible example. The "penguin" diagram on the lower left contributes an effective Hamiltonian density:

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} \frac{\alpha_s}{3\pi} V_{ts}^* V_{tb} \ln(m_t^2/m_c^2) \bar{s} \gamma_{\mu} (1 - \gamma_5) b \bar{u} \gamma^{\mu} u, \quad (50)$$

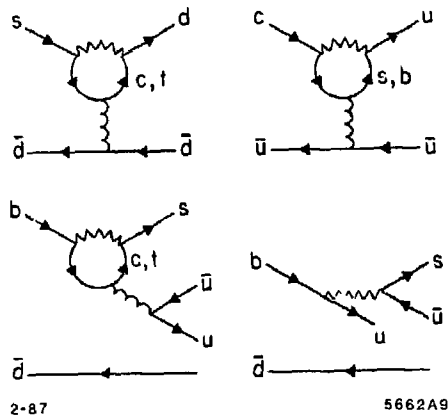


Fig. 9. Set of lowest order "penguin" graphs contributing to strange quark decay (upper left), Cabibbo suppressed charm quark decay (upper right), and Cabibbo suppressed bottom quark decay (lower left). Also shown is a spectator graph which also contributes to Cabibbo suppressed bottom quark decays (lower right).

whereas the usual spectator diagram corresponds to

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{ub} V_{us} s \gamma_\mu (1 - \gamma_5) b \bar{s} \gamma^\mu (1 - \gamma_5) u. \quad (51)$$

The "penguin" loses to the spectator graph because of the $\frac{g_s^2}{16\pi} \ln(m_t^2/m_c^2)$ that arises from having one loop and the presence of the gluon, but it wins because of the Cabibbo (or more exactly, Kobayashi-Maskawa) factor $V_{ub}V_{us}^*$, which involves zero and one generation jumps, as compared to $V_{ub}V_{ub}^*$, which involves two and one generation jumps, respectively. Depending in part on how small V_{ub} is (something still not known), it could well be that the spectator is the lesser of the two contributions. Then, for example, in the decays $B_d \rightarrow K^+ \pi^-$ or $B_s \rightarrow \phi K^0$ the "penguin" contribution may be dominant.⁴³

• Decays in the Spectator Model

What does all this mean numerically for the decay of the various quark flavors? First consider the strange quark. The statement that $c_- > 1$ corresponds to the enhancement of the $\Delta I = 1/2$ amplitude in strange particle decay, which is what one desires in order to be in accord with experiment. However, it already requires some stretching to get a factor of 3 or 4 in the amplitude, while what is needed is something like a factor of 20. Another piece of physics, perhaps "penguins" (see the discussion above), is required in addition to the QCD enhancement of c_- .

For the charm quark, if we set $\mu = m_c$, we find $c_- \sim 2$ and $c_+ \sim 1/\sqrt{2}$. At the quark level the Cabibbo allowed decay channels are $c \rightarrow s \bar{e} \nu_e$, $c \rightarrow s \bar{\mu} \nu_\mu$, and $c \rightarrow s \bar{d} u$. In the spectator model, all charmed hadrons would have the same lifetime and the same semileptonic branching fraction, which would be identified

with that for the charm quark as if it decayed in isolation from other hadronic constituents:

$$BR_{\text{semileptonic}} \sim \frac{1}{2 + 2c_+^2 + c_-^2} \sim 14\%. \quad (52)$$

For the bottom quark, with $\mu = m_b$, the QCD enhancement (suppression) of c_- (c_+) is less than that for charm: $c_- \sim 1.5$ and $c_+ \sim 0.8$. In this case we have an expanded list of decay channels at the quark level: $b \rightarrow ce\bar{\nu}_e$, $b \rightarrow e\mu\bar{\nu}_\mu$, $b \rightarrow c\tau\bar{\nu}_\tau$, $b \rightarrow cd\bar{u}$, and $b \rightarrow cs\bar{s}$. We have neglected decays where the final c quark is replaced by a u quark (using the experimental result¹ that $b \rightarrow u/b \rightarrow c$ is small). The corresponding semileptonic branching ratio is

$$BR_{\text{semileptonic}} \sim \frac{1}{2.2 + 1.2(2c_+^2 + c_-^2)} \sim 15\%, \quad (53)$$

where the semileptonic decays involving $c\tau\bar{\nu}_\tau$ and $c\bar{s}s$ have been given an approximate phase space weight which is 0.2 times that for $c\bar{e}\nu_e$.

These days, everyone is quick to point out that these results do not agree with experiment, e.g., the D^0 and the D^+ lifetimes differ by a factor of two or so, the average B semileptonic branching ratio is about 12%, etc.¹ Before we go on to investigating the shortcomings of the spectator model, let me emphasize that this is not so bad – I only wish that I was able to calculate so simply everything else involving strong interactions to a factor of two or better in the rate! The spectator model does provide a very useful qualitative and even semi-quantitative basis for calculating the weak decays of heavy quarks.

With that stressed, let it also be said that we should and can do better theoretically. We shall then look beyond the spectator model at what are the effects of the other quarks and gluons present in the initial or final state.

Beyond the Spectator Model

• Leptonic Decays

The simplest decay processes that involve the erstwhile spectator quarks are those where the initial quark and antiquark in a meson annihilate to yield a purely leptonic final state; this must be a non-spectator process. The prototype for all such decays is $\pi^- \rightarrow \mu^- \bar{\nu}_\mu$, and as for the pion, the decay rate for any pseudoscalar meson P has the form:

$$\Gamma(P \rightarrow \ell \bar{\nu}_\ell) = \frac{G_F^2 f_P^2 M_P m_\ell^2}{8\pi} \left(1 - \frac{m_\ell^2}{M_P^2}\right)^2, \quad (54)$$

where f_P is the pseudoscalar decay constant, the analogue of f_π .

For $D^+ \rightarrow \bar{\mu}\nu_\mu$, this yields a branching ratio

$$BR(D^+ \rightarrow \bar{\mu}\nu_\mu) = 4.3 \times 10^{-3} \sin^2 \theta_c \left(\frac{f_D}{f_K}\right)^2 \frac{\tau_{D^+}}{10^{-12} \text{ sec}}. \quad (55)$$

As we expect⁴⁴

$$f_D \sim f_F \sim f_B \sim f_K,$$

the branching ratio for this mode is to be found down at the level of 2×10^{-4} .

The one case of a heavy meson with an appreciable leptonic branching ratio is the F (now renamed the D_s , but we retain the old name here). The particular decay of relevance is $F^+ \rightarrow \tau^+ \nu_\tau$, where a calculation as above gives:

$$BR(F^+ \rightarrow \tau^+ \nu_\tau) = 4.3 \times 10^{-2} \left(\frac{f_F}{f_K}\right)^2 \frac{\tau_{F^+}}{10^{-12} \text{ sec}}. \quad (56)$$

Here we have neither a Cabibbo nor a helicity suppression (because of the high tau mass), and given the present F lifetime¹ we can expect a branching ratio of nearly 2%.

In the case of a vector meson decaying weakly to leptons, we no longer have the helicity suppression and the rate does not involve a factor of m_l^2 :

$$\Gamma(V \rightarrow l \bar{\nu}_l) = \frac{G_F^2 f_V^2 M_D^2}{24\pi} \left(1 - \frac{m_l^2}{M_D^2}\right)^2 \left(2 + \frac{m_l^2}{M_D^2}\right). \quad (57)$$

However, for all the known vector mesons, such a mode is swamped by strong or electromagnetic decays; even for the vector meson containing a t quark it is overwhelmed by other weak decays.

• Semileptonic Decays

We have considered these decays at the quark level previously, which presumably provides an approximate, smoothed inclusive sum of the actual exclusive modes such as $D \rightarrow K e \bar{\nu}_e$, $D \rightarrow K^* e \bar{\nu}_e$, etc. If the number of available exclusive channels is small, as in D semileptonic decay, then they can be untangled experimentally¹ and calculated individually theoretically.

Consider, for example, $D \rightarrow \bar{K} e \nu_e$. This decay has a standard matrix element

$$\mathcal{M} = -V_{cs} \frac{G_F}{\sqrt{2}} f_+(q^2) (p_K^\mu + p_D^\mu) \bar{\nu}_e \gamma_\mu (1 - \gamma_5) e, \quad (58)$$

where $q = p_D - p_K$ is the four-momentum carried by the lepton pair. The corresponding rate is

$$\Gamma(D \rightarrow \bar{K} e \nu_e) = |V_{cs}|^2 \frac{G_F^2 M_D^2}{192\pi^3} 8 \int_0^{(M_D - m_K)^2} dq^2 |f_+(q^2)|^2 |\bar{q}|^3, \quad (59)$$

where $|\bar{q}|$ is taken in the D rest frame. If the form factor f_+ had no q^2 dependence,

then the last integral can be done explicitly

$$8 \int_0^{(M_D - m_K)^2} dq^2 |f_+(q^2)|^2 |\bar{q}|^3 = |f_+(0)|^2 \cdot \frac{1}{4} F(m_K/M_D). \quad (60)$$

The function $F(\Delta = m_K/M_D)$ in Eq. (60) is just that introduced in Eq. (28) from the decay rate calculation at the quark level. In fact, the whole expression for the decay rate for the exclusive mode $D \rightarrow \bar{K} e \nu_e$ is just $\frac{1}{4} |f_+(0)|^2$ times that for the inclusive rate at the quark level if we replace M_D and m_K by the corresponding masses of the "heavy" quarks they contain, m_c and m_s . Numerically, $F(m_K/M_D) = 0.60$, and inserting the measured behavior¹ of the form factor f_+ increases the integral by a factor of 1.3. The measured rate for this decay can be used to obtain an expression for $|V_{cs}|$ in terms of $|f_+(0)|$, since all the other quantities in Eq. (59) are known.

This calculation of exclusive channels one by one can be carried a step further by using a model of the possible final states and their matrix elements to systematically calculate semileptonic decays as a sum of exclusive channels. Such a calculation has recently been carried out using the quark model for the final state resonances and their matrix elements,⁴⁵ with results for the electron energy spectrum in D meson decay as shown in Figure 10. A comparison is also made there of the sum of the exclusive channels and the inclusive decay calculated at the quark level. The overall rates (integrated area under the curves) in the two cases are quite close in value, while the exclusive calculation gives a somewhat softer electron spectrum. A similar comment holds for B decay into charm, as shown in Figure 11. When it comes to B decay into non-charmed final states however, the sum of the exclusive channels involving comparatively low-lying final states which is shown in Figure 12 falls well short of giving the same rate as

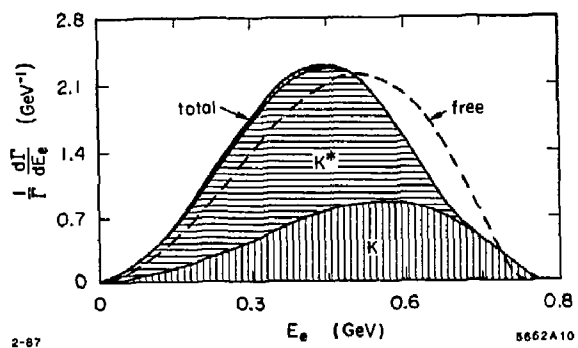


Fig. 10. The components of the electron spectrum from different hadronic channels in semileptonic D decays involving strange meson final states according to Ref. 45.

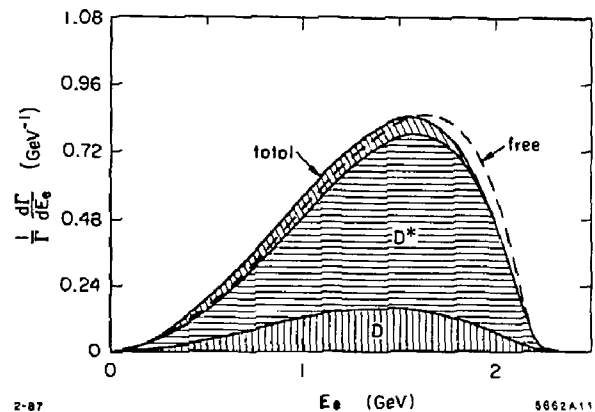


Fig. 11. The components of the electron spectrum from different hadronic channels in semileptonic B decays involving charmed meson final states according to Ref. 45.

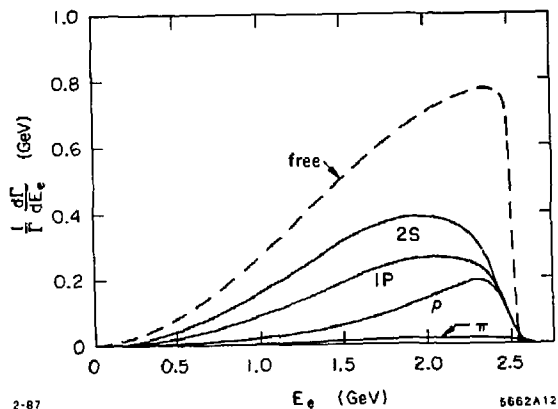


Fig. 12. The components of the electron spectrum from different hadronic channels in semileptonic B decays involving non-charmed meson final states according to Ref. 45.

the inclusive decay calculated at the quark level. It yields a much softer electron energy spectrum as well. Usually, it is precisely near the endpoint of the spectrum where one would tend to trust the calculation involving a discrete sum of exclusive states, as opposed to the quark level calculation, which at best can be a smooth averaging of the discrete sum. It is also the shape of the spectrum near the endpoint which is critical in sorting out $b \rightarrow u$ from $b \rightarrow c$. The decidedly softer spectrum in Figure 12 for the former process makes experiment less sensitive to $b \rightarrow u$; there are less restrictive limits on $b \rightarrow u$ when the data are analyzed in terms of it.

• Nonleptonic Decays

There are a number of ways that have been proposed to account for the deviations from the spectator model in nonleptonic decays. We examine them briefly in this section.^{36,37}

Final State Interactions

Once created by the weak interaction, the final hadrons undergo strong interaction scattering effects. If the pair of hadrons has an energy which is below inelastic threshold for that channel, it can be proven rigorously that the amplitude must have the phase of the elastic scattering process characterized by the appropriate quantum numbers at that energy.⁴⁸ For a process in which the final state is composed, for example, of two possible isospins, the corresponding portions of the weak amplitude each pick up a final state interaction factor with a phase that is appropriate to scattering in that particular isospin state. Such phases can completely change predictions made for the weak amplitudes alone by destroying the phase relationship between different amplitudes. For example, consider the decay $D^0 \rightarrow K^0 \pi^0$ in which the final $K\pi$ system has isospin $1/2$ or

3/2:

$$A(D^0 \rightarrow K^0 \pi^0) = \sqrt{\frac{1}{3}} A_{1/2} + \sqrt{\frac{2}{3}} A_{3/2}. \quad (61)$$

As we will see shortly, this amplitude is predicted to be very small due to color mismatches in one picture of such decays. If this is recast in terms of the isospin decomposition of Eq. (61), the small net amplitude must come about when

$$A_{1/2} \sim -\sqrt{2} A_{3/2}. \quad (62)$$

Once we add final state interactions, the same decay amplitude becomes

$$A(D^0 \rightarrow K^0 \pi^0) = \sqrt{\frac{1}{3}} A_{1/2} e^{i\delta_{1/2}} + \sqrt{\frac{2}{3}} A_{3/2} e^{i\delta_{3/2}}, \quad (63)$$

where we have used a simple phase factor to represent the final state interactions, even though π^0 is well above the inelastic threshold for $K\pi$ scattering at the energy of the D . Even with this somewhat symbolic notation, the point is well made⁴⁷ that with sizable strong interaction phases of opposite sign, the cancellation between the two terms on the right-hand side of Eq. (61) can be turned around into a positive enhancement, totally obscuring the underlying relation obtained from the weak interactions alone. Even more, strong interaction rescattering can produce a final state that is not allowed in a particular model from weak processes alone. Such is the case for the process $D^0 \rightarrow \bar{K}^0 \phi$, which is not permitted to occur through the weak interactions without the presence of annihilation graphs (see below), but could occur through the chain⁴⁸

$$D^0 \rightarrow K^{*0} \eta \rightarrow \bar{K}^0 \phi, \quad (64)$$

where the first step is already allowed in the spectator model and the second step is a purely strong interaction rescattering process.

Color Factors and (mis)Matching

Consider a quark level process like $c_\alpha \rightarrow s_\alpha + d_\beta u_\beta$, where color indices have been reinstated (and where repeated, they are summed over the three colors), that takes place inside a D^0 meson with quark content $c_\alpha \bar{u}_\alpha$. If we proceed naively to form final hadrons out of the resultant quarks and antiquarks, then the color indices for the combination $(u\bar{d})(s\bar{u})$, e.g., $\pi^+ K^-$, automatically "match" to form color singlet hadrons. The color indices for $(u\bar{u})(s\bar{d})$, e.g., $\pi^0 \bar{K}^0$, are mismatched. Only one time in three are the indices appropriate for forming a color singlet. Thus we expect that the second process will be down a factor of 9 from the first. Actually the prediction is⁴⁹

$$\frac{\Gamma(D^0 \rightarrow \bar{K}^0 \pi^0)}{\Gamma(D^0 \rightarrow K^- \pi^+)} = \frac{1}{18}, \quad (65)$$

because there is an additional factor of one-half coming from the square of the π^0 wave function, $\frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d)$. When the usual QCD corrections are put in, the prediction is reduced further to $\sim 1/40$. This (and other cases of such color suppression) is in gross disagreement with experiment,¹ where these two widths are within a factor of two of each other. The color matching or mismatching can be destroyed if we allow soft gluons to transfer color from one final quark to another at no cost in rate. However, there may be a modified version of such color factors that is of relevance, and indeed they play a role in some of the recent attempts to understand D decays systematically.^{50,51}

Interference

Unlike the example above where the two different ways of combining final quarks and antiquarks led to two distinct hadronic final states, there are cases

where the final state hadrons are the same. An example of this is given in Figure 13, where the two ways of combining the final quarks in D^+ decay, say into two pseudoscalar mesons, leads to the same final state, i. e., $\bar{K}^0\pi^+$. By the laws of quantum mechanics the amplitudes are coherent, and in this particular case are found to interfere destructively. This reduces (at least this) particular partial width and it can be argued that this is a mechanism for reducing the total width of the D^+ and hence increasing its lifetime and semileptonic branching fraction as compared to the D^0 and F^+ , for which this interference does not occur (in the Cabibbo allowed decays).^{52,53} This mechanism seems to be present at some level, although it has been argued that it is probably not enough of an effect by itself to give a factor of two difference in the D^+ and D^0 lifetimes.⁵⁴

Annihilation

Graphs like those in Figure 14 are suppressed for the decays of a pseudoscalar meson into light final fermions because of the $V \sim A$ character of the weak interaction. This is precisely the same physics that gives the factor of m_l^2 in Eq. (54) and causes the amplitudes for $\pi^- \rightarrow e\bar{\nu}_e$ and $K^- \rightarrow e\bar{\nu}_e$ to be suppressed by a factor of m_e/m_μ compared to $\pi^- \rightarrow \mu\bar{\nu}_\mu$ and $K^- \rightarrow \mu\bar{\nu}_\mu$; we sometimes say the former processes are "helicity suppressed" compared to the latter.

In the case of hadronic decays as in Figure 14, this suppression may be removed by emitting gluons. Consider, for example, the annihilation diagram for F decay in Figure 14, where a gluon is radiated by the c or $\bar{s}$ quark in the initial state, or where the gluon is present as a constituent to from the beginning.⁶⁵⁻⁶⁷ At the vertex where the $c\bar{s}$ pair annihilate into a W they are no longer in a spin zero state (in fact, they are necessarily have spin one); the helicity suppression is gone. The width of hadrons for which such a process can contribute to the

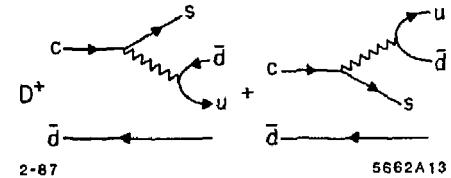


Fig. 13. Diagrams contributing to $D^+ \rightarrow \bar{K}^0\pi^+$ decay, illustrating potential interference between amplitudes arising from two different ways of combining final quarks into the same final state hadrons.

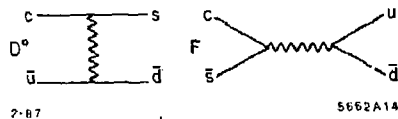


Fig. 14. Annihilation graphs contributing to D^0 and F^+ decay.

weak decays will increase; the only question is the magnitude of the effect. In particular, the width of the D^0 and F^+ should increase, while that for the D^+ should not in the dominant Cabibbo-allowed modes (but the annihilation graph does contribute to Cabibbo-suppressed modes of the D^+). In addition, specific exclusive hadronic modes, like $D^0 \rightarrow \bar{K}^0 \phi$ and F decay to hadrons not containing strange quarks, are permitted through the annihilation graph and not through the spectator graph by itself (without final state interactions, see above).

There is increasing evidence that annihilation graphs do contribute to a measurable fraction of D decays. We very much need further quantitative calculations and experimental measurements of D and F decay with which to compare them. Even if such effects are important in D and F decay, they should be much less so in B decay, and this needs experimental checking as well. So while the situation with weak decays of hadrons containing heavy quarks is getting clearer, and we do have a semi-quantitative theory/model of these processes, much remains to be done both theoretically and experimentally.

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On Our Theoretical Understanding of Charm Decays*

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ABSTRACT

A detailed description of charm decays has emerged. I sketch the various concepts involved. Although this description is quite successful in reproducing the data the chapter on heavy flavour decays is far from closed. Relevant questions like on the real strength of weak annihilation, Penguin operators, etc. are still unanswered. I try to identify important directions in future work, both on the experimental and theoretical side.

Provocateur paper presented by I. Bigi supplementing F. Gilman's lecture.

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1. Enlightened History

To recount history the way it actually happened is at times more confusing than illuminating. Charm decays seem to be such a case. Therefore, I will describe history the way it should have happened.

Treating charm decays in complete analogy to muon decays – which is the so-called spectator picture – one finds for the charm lifetime (ignoring all QCD corrections)

$$\tau(\text{charm}) \simeq \frac{1}{5} \left(\frac{m_\mu}{m_c} \right)^5 \tau(\mu) \sim 7 \times 10^{-13} \text{ sec} \quad (1)$$

for $m_c = 1.5 \text{ GeV}$. Comparing that with the experimental findings¹ $\tau(D^0) \sim 4.4 \times 10^{-13} \text{ sec}$, $\tau(D^+) \sim 10^{-12} \text{ sec}$ one should first note how remarkably close the naive prediction (1) is to these data; for one has to keep in mind that Eq. (1) represents an extrapolation over more than six orders of magnitude! Having said that it is then fair to state that the agreement between the prediction (1) and the data is not perfect since $\tau(D^+)/\tau(D^0) \sim 2.2$ experimentally while naively one expects $\tau(D^+) = \tau(D^0)$. This shows that hadronic effects are important – yet they are less rampant than for kaons where one has $\tau(K^\pm)/\tau(K_s) \sim 135$.

The MARK III analysis¹ reveals that most non-leptonic D decays lead to two-body final states of the type $D \rightarrow PP$, PV where $P[V]$ denotes a pseudoscalar [vector] meson. The remainder could largely be made up by VV final states. (It is true that this dominance of two-body final states had not been anticipated; yet in the noble tradition of “Monday morning quarterbacking” one can argue that one should have guessed it.)

At this point one concludes that a detailed understanding of charm decays requires inclusion of ordinary hadronic effects like form-factors, final state inter-

actions etc.; gross features on the other hand can be reproduced from fairly simple quark level calculations up to a factor of two or so as another manifestation of the idea of duality.

2. The Art of Theoretical Engineering

Stech and co-workers² had developed a comprehensive framework for describing all two-body decay modes of charm (and bottom) states before detailed data existed. Their prescription is based on five ingredients:

- (i) The usual effective quark operators for charm decay are employed:

$$\mathcal{L}(\Delta C = 1) \propto \frac{c_+ + c_-}{2} \bar{s}_L \gamma_\mu c_L \bar{u}_L \gamma_\mu d_L + \frac{c_+ - c_-}{2} \bar{u}_L \gamma_\mu c_L \bar{s}_L \gamma_\mu d_L \quad (2)$$

where the coefficients $c_\pm$ contain the QCD radiative corrections.¹

- (ii) Only quark decay diagrams are retained while W exchange diagrams are ignored.
- (iii) The quark currents in (2) are replaced by the corresponding hadronic currents J_H when computing transition amplitudes. This is treated as a trivial procedure for diagrams like Fig. 1(a) where the colors are already properly aligned; for diagrams with the topology of Fig. 1(b) one introduces an a-priori undetermined new parameter ξ since the colors are not automatically matched: (Naively, by just counting degrees of freedom one would have $\xi \sim 1/N_C = 1/3$.) Thus one writes for the transition amplitude

$$T(D \rightarrow f) \propto a_1 \langle f | (\bar{s}_L \gamma_\mu c_L)_H (\bar{u}_L \gamma_\mu d_L)_H | D \rangle + a_2 \langle f | (\bar{u}_L \gamma_\mu c_L)_H (\bar{s}_L \gamma_\mu d_L)_H | D \rangle \quad (3)$$

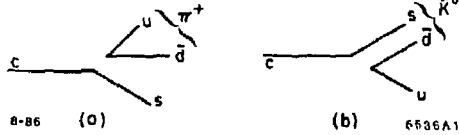


Fig. 1: (a) Quark decay diagram with color alignment. (b) Quark decay diagram without automatic color alignment.

with

$$\begin{aligned} a_1 &= \frac{1}{2} (c_+ + c_-) + \frac{\xi}{2} (c_+ - c_-) \\ a_2 &= \frac{1}{2} (c_+ - c_-) + \frac{\xi}{2} (c_+ + c_-) \end{aligned} \quad (4)$$

(iv) For two-body final states, i.e. $f = PP, PV, VV$, one employs a factorization ansatz

$$\langle f | J_H \cdot J_H | D \rangle \simeq \langle P \text{ or } V | J_H | 0 \rangle \langle P \text{ or } V | J_H | D \rangle \quad (5)$$

These simple matrix elements are given in terms of decay constants $f_\pi, f_K, f_\rho, \dots$ and nearest neighbour pole terms.

(v) Final state interactions (phase shifts, absorption etc.) are included as best as possible.

There are basically just two free parameters to be fitted, namely a_1, a_2 (although in practice one has some freedom in parameterizing the relevant final state interactions). Keeping this in mind I find the success of the Bauer-Stech fit

to some twenty D decay modes¹ very remarkable. They obtain²

$$a_1 \simeq 1.3 \pm 0.1; \quad a_2 \simeq -0.5 \pm 0.1 \quad (6)$$

Using values for $c_\pm$ (see Eq. (2)) as obtained from a perturbative QCD calculation³ – a procedure that seems reasonable although its correctness has not been proven beyond a reasonable doubt – one translates Eq. (6) into

$$\xi \sim 0 \quad (7)$$

Adding up all the two-body modes one finds in this description

$$\frac{\tau(D^+ \rightarrow PP, PV, VV)}{\tau(D^0 \rightarrow PP, PV, VV)} \sim 2 - 3 \quad (8)$$

which is easily understood. The D^0 (and F^+) decays receive incoherent contributions from the $a_1 \langle f | J_H \cdot J_H | D^0 \rangle$ and $a_2 \langle f | J_H \cdot J_H | D^0 \rangle$ terms, whereas for D^+ decays they contribute coherently to the same channel; the resulting interference has to be destructive since $a_1 \cdot a_2 < 0$ – as expected from QCD. This slows D^+ decays down while not affecting D^0 and F^+ decays. That something like this affects charm decays in a significant way was first suggested in Ref. 3 where a simple quark model illustration was given. For me it was hard to see how the necessary coherence could be maintained in genuine multi-body decays like $D^+ \rightarrow \bar{K}^0 \pi^+ \pi^0$, where $\pi^+ \pi^0$ do not form a ρ meson. The MARK III findings on the relative insignificance of such modes have erased this criticism.

3. Evidence for W Exchange?

There are other decay mechanisms that can produce a lifetime difference $\tau(D^+) > \tau(D^0)$, namely W exchange and weak annihilation diagrams (both hereafter referred to as WA) which are depicted in Fig. 2. On the Cabibbo allowed level they can contribute to D^0 , F^+ and A_1^+ but not to D^+ decays.

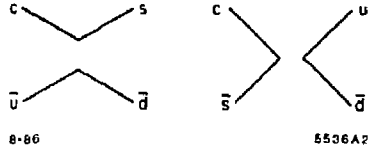


Fig. 2 : W exchange and weak annihilation diagram.

Originally they had been discarded as insignificantly small; for in analogy to $\pi \rightarrow \ell \nu$ decays one finds these contributions to be helicity suppressed by $(m_s/m_c)^2$ where m_s [m_c] denotes the strange [charm] quark mass. Prodded by early experimental data on charm lifetimes theorists found, however, ways of avoiding this helicity suppression; in the presence of gluons the $(c\bar{q})$ pair can find itself in a spin one configuration; its annihilation into a pair of light fermions will then not be impeded by small mass ratios.

There exists another effect limiting the strength of WA : weak interactions are local on the distance scales that are relevant for charm (and bottom) decays. Thus WA requires the c and $\bar{q}$ pair to come together in space for which the probability is given by the $c\bar{q}$ wavefunction at zero separation. Unfortunately we do not know at present how to calculate the wavefunction directly from the

theory in a reliable fashion. Thus no firm prediction can be made, only educated guesses.

In one of these guesses it was suggested⁴ to treat gluon bremsstrahlung off the initial quark lines by a perturbative approach which leads to

$$\Gamma_{WA, \text{perturb.}}(D^0) \propto G_F^2 \left(\frac{f_D}{m_u} \right)^2 m_D^5 \quad (9)$$

To obtain $\Gamma_{WA, \text{perturb.}} \simeq \Gamma_{\text{spect.}}$ - i.e. to blame $\tau_{D^+} \sim 2 \tau_{D^0}$ solely on WA - one has to require⁵

$$\frac{f_D}{m_u} \sim 2.2 \quad (10)$$

Using constituent masses, i.e. $m_u \sim 330$ MeV, the requirement of Eq. (10) translates into $f_D \sim 720$ MeV. This is much larger than most guesstimates: using QCD sum rules, potential models or bag models one finds f_D ranging between 100 and 230 MeV. Alternatively one can equate the measured $D^+ - D$ mass difference with the quantity that is obtained in an one-gluon-exchange ansatz

$$m(D^+) - m(D) \simeq \frac{8\pi}{27} \frac{M_D}{m_c m_u} \alpha_s f_D^2 \quad (11)$$

which leads to $\sqrt{\alpha_s} f_D \sim 200$ MeV.

The MARK III group has just now derived an upper bound on f_D by searching for $D^+ \rightarrow \mu^+ \nu$:

$$f_D \leq 340 \text{ MeV} \quad (12)$$

Hence one concludes that WA implemented via perturbative gluon bremsstrahlung can at best introduce a 25% difference in $\tau(D^+)$ and $\tau(D^0)$.

There appears an easy way to avoid such a conclusion: if one uses smaller mass values for the up quark, say $m_u \sim 100$ MeV, one can increase the strength of WA at will. However, I harbor grave doubts that Eq. (9) represents a bona fide perturbative calculation: the problem is not that one integrates down to m_u but that almost the whole contribution originates from the regime m_u to $\sim 2 m_u$. Therefore it does not make much sense to me to lower m_u even further. The emergence of gluons should then be treated as a non-perturbative phenomenon by putting them into the hadronic wavefunction.⁶ Such an ansatz leads to

$$\Gamma_{WA, \text{ non-pert.}}(D^0) \propto G_F^2 \bar{f}_D^2 m_D^3 \quad (13)$$

where $\bar{f}_D$ contains the wavefunction describing the $(c\bar{q} + \text{gluon})$ configuration. The quantity $\bar{f}_D$ is often identified with f_D measured in $D \rightarrow \ell\nu$ decays, yet such a procedure is in general not correct. For the c and $\bar{u}$ quarks annihilate in the presence of gluon fields; therefore it will not lead to a purely leptonic final state and bounds on $D \rightarrow \ell\nu$ per se and do not teach us something on the $c\bar{q} + \text{glue}$ wavefunction.

The preceding discussion shows that WA could produce a 25% lifetime difference "perturbatively" or considerably more non-perturbatively. Considering this vagueness in the prediction it is then suggestive to analyze whether special decay modes could clearly reveal the significance of WA . Two decay modes have been enlisted to testify in favor of WA : semi-leptonic D^0 decays and the $D^+ \rightarrow \bar{K}^0 \phi$ mode.

(i) Semi-leptonic D^0 decays

The measured semi-leptonic branching ratio of $D^0 - b_{SL}(D^0) \sim 7.5\%$ - is compared with the alleged prediction in the spectator ansatz - $b_{SL}(D^0) \sim 14\%$. If the

latter were a firm prediction it would clearly establish that WA is essential for describing D^0 decays. Unfortunately this is not the case. For the prediction quoted above is based on a very simple quark level calculation: $b_{SL}(D) \sim \frac{1}{2 + 2c_+^2 + c_-^2}$, where the naive assignment $\xi = \frac{1}{3}$ was used, which - as discussed above - does not describe the non-leptonic D decays.

Instead a more reasonable ansatz is given by⁷

$$b_{SL}(D^0) \sim \frac{1}{2 + \frac{3}{2}(c_+^2 + c_-^2) + \frac{3\xi}{2}(c_+^2 - c_-^2)} \quad (14)$$

which gives $b_{SL}(D^0) \sim 10 - 14\%$ for $\xi = 0$. But the main point is - as stated in the beginning - that these simple quark level computations cannot be trusted to better than a factor of two in D decays. Indeed, if one adds up all the two-body modes in the Bauer-Stech description one finds

$$\frac{BR(D^0 \rightarrow \ell\nu P \text{ or } V)}{BR(D^0 \rightarrow PP, PV, VV)} \sim 8\% \quad (15)$$

Therefore, one cannot cite the measured semi-leptonic branching ratio of D^0 as firm evidence for WA .

(ii) $D^0 \rightarrow \bar{K}^0 \phi$

It had been suggested some time ago⁸ that observing $D^0 \rightarrow \bar{K}^0 \phi$ with a branching of not much less than 1% would establish WA . Experimentally, a branching ratio of slightly more than 1% was indeed found.¹ However, it has been pointed out recently by Donoghue⁹ and by Stech² that this evidence is not compelling: it is quite conceivable that in particular the very prominent mode $D^0 \rightarrow K^- \rho^+$ could - via rescattering - generate some rate for $D^0 \rightarrow \bar{K}^0 \phi$ even in the absence of WA . In passing it should be noted that these rescattering diagrams

do involve $q\bar{q}$ annihilation; however this occurs due to the strong interactions with a typical range of 1 fermi and not the pointlike weak interactions; thus it is clearly distinct from WA .

These issues will be clarified considerably once detailed data on F decays are available, in particular $BR(F \rightarrow \ell + X)$ and $BR(F^+ \rightarrow \pi \rho, \omega \pi)$. The latter branching ratio must amount to several percent if WA is significant.

To summarize the status of WA in charm decays:

- $A \sim 20\%$ contribution to $\tau(D^0, F^+)$, and possibly more to $\tau(A_1^+)$, is a reasonable though not firm ball park estimate.
- Its strength could actually be considerably larger; however a phenomenological need for its contributions has at present not been established in an unequivocal fashion.

At this point the natural question arises why worry about WA since there is no really compelling evidence for it at present and since it is not based on highly lucid concepts. There are several reasons why one has to be concerned about the strength of WA :

- (a) Establishing the presence of gluons in the hadronic wavefunction would represent a very nice (though not totally) surprising result.
- (b) A comparison of $D^0 \rightarrow K^+ K^-$ with $D^0 \rightarrow \pi^+ \pi^-$ can give us some information on the strength of penguin transitions; yet the presence of WA can affect the conclusion.
- (c) The MARK III analysis of $D^0 \bar{D}^0 \rightarrow (K^\pm + \pi'^\pm)(K^\mp + \pi'^\mp)$ transitions has yielded some marginal evidence for $D^0 - \bar{D}^0$ mixing¹ (with a strength well beyond Standard Model expectations). Unfortunately the same final state

can be produced by doubly Cabibbo suppressed D decays; WA would affect them.

- (d) A difference in the lifetimes of bottom mesons - $\tau(B^\pm) \neq \tau(B^0)$ - would severely affect any conclusion that one draws on the size of the KM parameter $V(bu)$ and on $B^0 - \bar{B}^0$ mixing when studying semi-leptonic B decays. Scenarios involving WA can be scaled up to make predictions on $\tau(B^\pm)/\tau(B^0)$ by using $\tau(D^+)/\tau(D^0)$ as input. Treating WA perturbatively one finds

$$\frac{\tau(B^\pm)}{\tau(B^0)} \simeq 1 + \frac{\tau(D^+)}{\tau(D^0)} \times \frac{m_c}{m_b} \quad (16a)$$

whereas a different scaling law applies for the non-perturbative treatment

$$\frac{\tau(B^\pm)}{\tau(B^0)} \simeq 1 + \frac{\tau(D^+)}{\tau(D^0)} \times \left(\frac{m_c}{m_b}\right)^2 \quad (16b)$$

In both cases one estimates

$$\frac{\tau(B^\pm)}{\tau(B^0)} \lesssim 1.2 \quad (17)$$

Although I consider this a fairly safe prediction, one would prefer to have a measurement of it.

- (e) If WA were a significant contributor to D decays it would still have some impact on B decays. Interference between WA and quark decay would then allow certain CP asymmetries to show up that otherwise were absent.¹⁰

4. "Computational Power to the Masses": The $1/N$ Approach

The prescription of Stech and co-workers works quite well for D decays. Nevertheless it is fair to say that

- (a) Its description of the data is not perfect; very recently a new reason for concern has appeared: MARK III has found that only slightly more than 50% of the $D \rightarrow \ell \nu K \pi$ transitions come from $D \rightarrow \ell \nu K^*$. Not only is this result quite different from theoretical expectations,² but it raises – by extrapolation – serious worries about our ability to understand semi-leptonic B decays and extract $|V(bu)/V(bc)|$ from there.
- (b) It is not very elegant.

The fit to the data that a priori could have yielded any value for ξ seems to favor $\xi \simeq 0$; first indications suggest that such a value allows to describe two-body B decays as well. It was noted by Buras and coworkers¹¹ that a consistent application of the $1/N$ approach – N stands for the number of colors – could bring simplicity back to the theoretical description. Its basic rules are indeed simple:

- (i) Use the usual transition operators $O_{\pm}$ with coefficients $c_{\pm}$ as obtained from perturbative QCD.
- (ii) Expand the appropriate matrix element into powers of $1/N$:

$$\langle M_1 M_2 | \mathcal{L}_{eff} | D \rangle = \sqrt{N} \left\{ b_0 + \frac{b_1}{N} + O\left(\frac{1}{N^2}\right) \right\} .$$

In practice only the leading term with coefficient b_0 is retained.

- (iii) To compute the coefficient b_0 , one draws all the quark diagrams; the hadrons are described by their valence quarks only.

- (iv) Every closed quark loop yields a factor N ; every hadron introduces a normalization factor $1/\sqrt{N}$; a factor $1/\sqrt{N}$ enters also through quark-gluon couplings.

These rules are easy to apply: only valence quarks have to be considered and final state interactions are ignored to leading order in $1/N$ (see rule (iv)). Although this approach certainly increases the transparency and simplicity of the theoretical description and is very user-friendly, it does not provide a fully satisfactory framework:

- (a) The non-leading terms in $1/N$ are dropped by fiat; our theoretical understanding is therefore not advanced – unless at least the first non-leading corrections are computed.
- (b) Although the overall fit to the data is not bad there are obvious discrepancies, for example $BR(D^0 \rightarrow \bar{K}^0 \phi)$ and $BR(D^0 \rightarrow K^+ K^-)/BR(D^0 \rightarrow \pi^+ \pi^-)$ typically come out too small.

This presumably means that final state interactions etc. cannot be ignored, i.e. that non-leading terms are significant for D decays.

5. Summary

We do have now a very decent approximate description of charm decays. There is no clearly established need to have WA as the major source of the lifetime differences. The maturity level we have reached is such that we can address fairly subtle issues:

- (i) does WA really contribute $\sim 20\%$ to $\tau(D^0)$, $\tau(F^+)$? Is it more or is it less?¹²

(ii) Are Penguin operators relevant for Cabibbo suppressed charm decays?¹²

(iii) Do we understand doubly Cabibbo suppressed decays?

Continuing analysis of even more decay modes of $D^\pm$, D^0 , $F^\pm$ and Λ_c and of exclusive B decays will help to clarify these issues. Just one simple example: does $\frac{BR(D^+ \rightarrow \pi^0 \pi^+)}{BR(D^+ \rightarrow \bar{K}^0 \pi^+)} = \frac{1}{2} \tan^2 \theta_c$ really hold or not? A violation of this relation would yield very useful information on $SU(3)_{\text{Flavour}}$ violations like f_π/f_K etc. On the theoretical side it would represent a major advance if the contributions that are non-leading in $1/N$ could be computed consistently.

A close feed-back between theory and experiment has clearly improved our understanding of heavy flavour decays. There is every reason to believe that this story will repeat itself in the future: final success – after some ups and downs.

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Toponium- Z^0 Mixing*

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ABSTRACT

Provocateur paper presented by P. Franzini supplementing F. Gilman's lecture.

The subject of Z^0 -toponium interference is briefly reviewed. The qualitative features of the Z^0 mixing with one $t\bar{t}$ state are discussed. Effects of mixing with the full $t\bar{t}$ spectrum, of the smearing due to beam spread, and of different potentials, are then shown.

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1. Introduction

In this talk, I would like to discuss a particular aspect of heavy quark phenomenology, of relevance when $2m_t \approx m_{Z^0}$. Why do we expect toponium- Z^0 mixing to be of interest? From the absence of flavor-changing neutral currents in B decay, we are confident that the bottom quark must have an as-yet-unobserved partner. Experimentally, $m_t < 23$ GeV is excluded, while UA1 data suggests a top quark of mass between 30 and 50 GeV. It appears quite possible that $t\bar{t}$ bound states will have masses near that of the Z^0 (93 GeV), and thus vector ($J^{PC} = 1^{--}$) $t\bar{t}$ states (henceforth V) could be nearly degenerate with the Z^0 . We expect the effects of $V - Z$ mixing to be seen soon, at both SLC and LEP.

I first present a qualitative way of understanding the nearly complete destructive interference of the Z boson with one V state. I then present the results of the Z mixing with the full spectrum of toponium states (when the Z and V are nearly degenerate); I show the effects of finite beam width on the cross sections and asymmetries. I then display the striking effects that remain if the Z is relatively far away from the V (10 - 20 GeV), and conclude by contrasting the effects of the Richardson^[4] potential, the Cornell^[11] potential, and a non-standard Higgs sector.

This talk is based on work done in collaboration with Fred Gilman and Gregory Athanasiu.^[11-13]

2. Mixing of the Z^0 with a Single $t\bar{t}$ State

I shall discuss the simplified case of only two states, the Z and one vector ($J^{PC} = 1^{--}$) toponium resonance, V . I begin with a qualitative argument to show that the interference is indeed destructive. Let us consider the process $e^+e^- \rightarrow f\bar{f}$, where f denotes an arbitrary final fermion state. This occurs predominantly as $e^+e^- \rightarrow Z_0 \rightarrow f\bar{f}$, while another contribution is $e^+e^- \rightarrow Z_0 \rightarrow V_0 \rightarrow Z_0 \rightarrow f\bar{f}$ (for now, we neglect the small contributions due to γ couplings). The first term has an amplitude proportional to the propagator $1/(s - M_{Z_0}^2 + i\Gamma_{Z_0}M_{Z_0})$, and therefore to $1/\Gamma_{Z_0}$ on the peak of the Z_0 resonance. If, for simplicity, we choose the Z_0 and V_0 resonances to be degenerate, the amplitude from the second contribution is similarly proportional to $1/(i\Gamma_{Z_0}i\Gamma_{V_0}i\Gamma_{Z_0})$. Thus we have a relative minus sign between these two amplitudes, i.e., destructive interference.

We can extend this argument by replacing the Z_0 propagator by the iterated series

$$\text{---} + \text{---} + \text{---} + \text{---} + \dots$$

where the solid line denotes the Z_0 and the double line the V_0 . Using a phenomenological $Z_0 - V_0$ coupling a , we get the amplitude to be proportional to

$$\begin{aligned} & \frac{1}{s - M_{Z_0}^2} + \frac{1}{s - M_{Z_0}^2} \cdot \left(a \cdot \frac{1}{s - M_{V_0}^2} \cdot a \cdot \frac{1}{s - M_{Z_0}^2} \right) \\ & + \frac{1}{s - M_{Z_0}^2} \cdot \left(a \cdot \frac{1}{s - M_{V_0}^2} \cdot a \cdot \frac{1}{s - M_{Z_0}^2} \right)^2 + \dots = \frac{s - M_{V_0}^2}{(s - M_{Z_0}^2)(s - M_{V_0}^2) - a^2} \end{aligned} \quad (2.1)$$

(Here, and often in what follows, we will use $M_{Z_0}^2$ to represent the full expression $M_{Z_0}^2 - i\Gamma_{Z_0}M_{Z_0}$.) For energies a few GeV away from a V_0 resonance, $(s - M_{Z_0}^2)(s - M_{V_0}^2)$ is large compared to a^2 ; as expected, we recover the Z_0 propagator. On the V_0 resonance we get zero for the amplitude—thus we have complete destructive interference.

The amplitude exactly vanishes only if we make some simplifying assumptions:

(1) I have ignored the virtual photon contribution to the process $e^+e^- \rightarrow \mu^+\mu^-$. This is a good approximation, since the photon, by definition, contributes an R-value of about* one, while the R-value on the Z_0 peak is 200. (Note that on the Z_0 peak, the Z amplitude is imaginary while that of the photon is real, so that there is no $\gamma - Z$ interference. However, in general we must compute $Z\gamma V$ mixing. The effect of the photon is small enough to be negligible, except in the determination of the asymmetry parameters.)

(2) I have implicitly assumed that the width of the V_0 is zero. The expression $s - M_{V_0}^2$ really represents $s - M_{V_0}^2 + iM_{V_0}^2\Gamma_{V_0}$ which can only be zero (for a physically allowed value of s) if $\Gamma_{V_0} = 0$. This is also a good approximation, since the expected width of a tf $1S$ state (using the Richardson potential) is about 100 keV, compared to $\Gamma_Z = 2.7$ GeV.

(3) Finally, I have ignored the "direct" couplings of the V_0 , that is, the V_0 coupling to fermions through the photon instead of through the Z_0 . This approximation is analogous to, and comparable in magnitude with, the second one.

We can also determine the mass and width of the physical eigenstates by diagonalizing the mass matrix (see Ref. 1 for details). The shifts in M_Z and in Γ_V (those in M_V and Γ_Z , respectively, are equal and opposite to the shifts shown) are shown in Fig.1 for the Z mixing with a $1S$ toponium state described by the Richardson potential. While the mass shifts, and shifts in Γ_Z as well, are insignificant, the shift in Γ_V is very impressive. The dashed line indicates the width of the toponium state without mixing.

* The contribution is not exactly one, because R-value is given by the actual photon cross section divided by the QED point cross section with α defined at the electron mass scale.

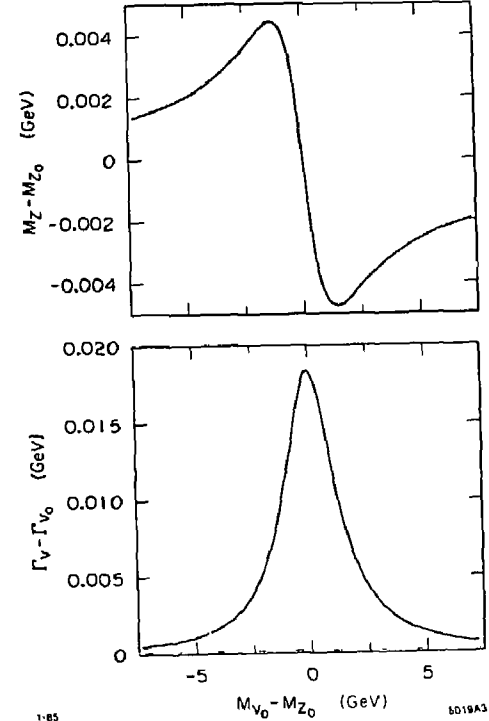


Figure 1. Changes in M_Z and Γ_V due to mixing of the Z state with the ground state of toponium as a function of the mass difference of the bare states (M_{Z_0} is held fixed at 93 GeV, while M_{V_0} is varied; the subscript 0 denotes unmixed states).

3. What We Will See: Many States, Smearing, and All That

Considering the Z interfering with the full set of toponium states below threshold we obtain cross sections such as that shown in the first part of Fig. 2. Of course, real machines, such as SLC and LEP, will not resolve these very narrow spikes; we must convolute the curves with a Gaussian (with width related to the beam spread) in order to approximate what will be measured. We have

$$\sigma_{\text{SMEARED}} = \int_{-\infty}^{\infty} dw' \sigma(w') \frac{1}{\sqrt{2\sigma}} e^{-w'^2/2\sigma^2} \quad (3.1)$$

where $\sigma = \sqrt{2}\sigma_{\text{BEAM}}$. In the second part of Fig. 2, I show R for the Z alone, and for the Z interfering with toponium states, convoluted with Gaussians appropriate to $\sigma_{\text{beam}} = 40$ MeV and 100 MeV. LEP is expected to run (without wigglers) at the former beam width; SLC is expected to achieve the latter, and perhaps with special effort, the former.

I next remark that even for a V relatively far away from the Z , the enhancement due to mixing should be quite noticeable (see Fig. 3). The height of the peak does not decrease, though its width does. The smeared height is therefore greatly reduced, but should be compared to the also much reduced background due to the Z . Note that to get comparable statistics to those obtained on the Z , one must run for far longer.

I now present smeared polarization and forward-backward asymmetries for various values of M_{to} . These are found by calculating the cross sections (for each individual helicity configuration), smearing them, and then taking the appropriate differences and ratios. Since the asymmetries also crucially depend on the $ZV\gamma$ interference, the results do not seem to have a simple qualitative explanation. In Fig. 4 I show the asymmetries; the effects are in fact more striking for V moderately far away from Z .

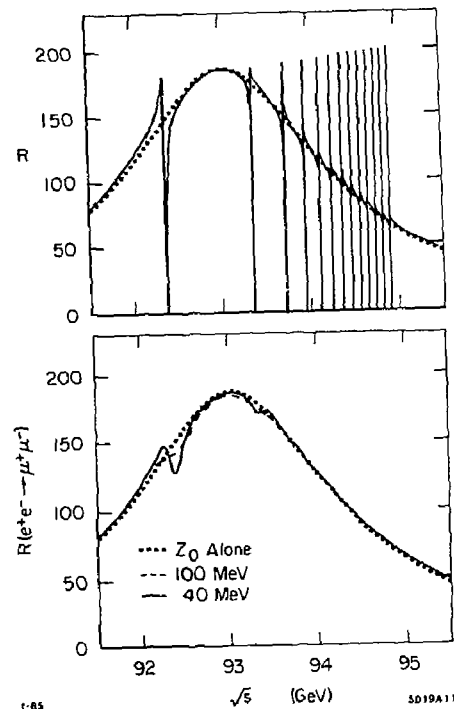


Figure 2. (top) $R(e^+e^- \rightarrow \mu^+\mu^-)$ for several toponium states mixing with the Z (Richardson potential, $m_t = 47$ GeV). The dotted line is the Z_0 alone.

(bottom) $R(e^+e^- \rightarrow \mu^+\mu^-)$, smeared, for various expected beam widths.

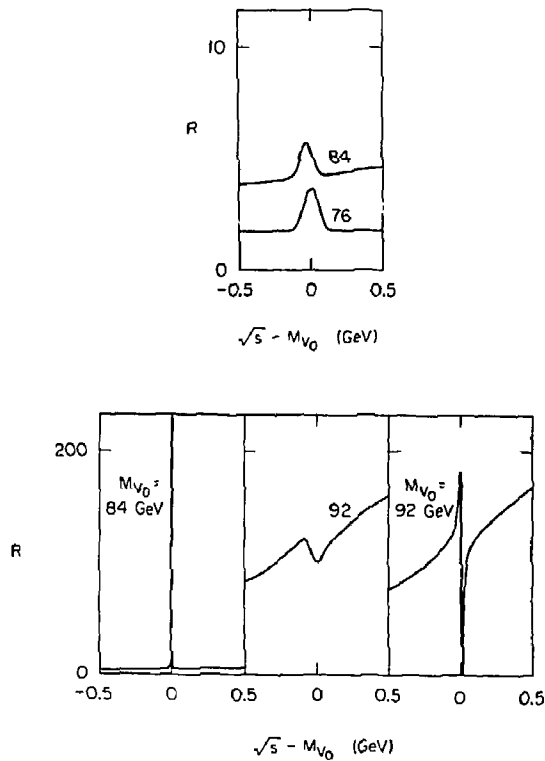


Figure 3. $R(e^+e^- \rightarrow \mu^+\mu^-)$ smeared and not, for $M_{V_0} = 76, 84$ and 92 GeV.

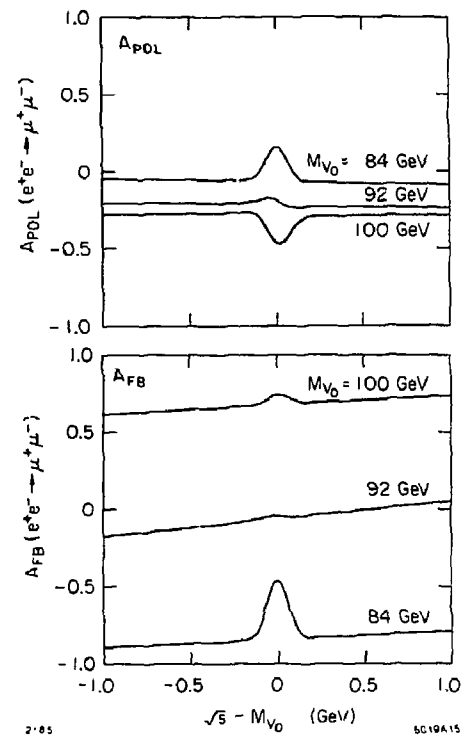


Figure 4. A_{POL} and A_{FB} for three different values of M_{V_0} .

All the results I have shown so far used the Richardson potential. I shall briefly show the effects of using the Cornell potential, and the Richardson potential combined with a non-standard Higgs sector. Consider the two-Higgs doublet model of Glashow, Weinberg and Paschos,^[9] where one Higgs couples to up-type quarks, and one to down-type. There is a neutral-Higgs (H_0) exchange contribution to the toponium potential, where the H_0 coupling is enhanced by the vacuum-expectation-value ratio ξ/η (ξ being the VEV of the Higgs coupling to down-type quarks and η to up-type). The extra contribution is an attractive Yukawa, in momentum space

$$-\left(\frac{\xi}{\eta} \frac{gm_t}{2M_W}\right)^2 \frac{1}{m_H^2 + q^2} \quad \text{or} \quad -\left(\frac{\xi}{\eta} \frac{gm_t}{2M_W}\right)^2 \frac{e^{-r m_H}}{4\pi r} \quad (3.2)$$

in coordinate space. This addition has the effects of increasing the wavefunctions at the origin, since it pulls in the wavefunctions, and of lowering states (increasing binding energies); it changes the level spacings, since it affects the lowest lying states the most. Finally, if the Higgs term is strong enough* it has a very curious effect—it causes the 2S state to lie below the 1P. This effect does not happen for any standard quarkonium potential, and is related^[17] to the fact that $\Delta V(r) < 0$ for the Higgs potential and not so for any standard quarkonium potential. In Fig. 5, I show $R(e^+e^- \rightarrow \mu^+\mu^-)$, smeared ($\sigma_{\text{beam}} = 40$ MeV), for Richardson alone, Cornell alone, and Richardson with Higgs[†]. Note the qualitative similarity between the second and third figure.

In summary, we have seen that toponium and the Z_0 almost completely destructively interfere. Toponium states pick up a large width from mixing—the 1S state, with a bare width of 100 keV, can acquire a width of as much as 20 MeV (using the Richardson potential). While the beam widths of machines such as SLC and LEP will greatly blur the sharp spikes that we find, effects will

* that is, ξ/η equals about 5, if we are using the Cornell potential, or 10, for Richardson.

† the parameters have been chosen to be dramatic; they are all but excluded by BB mixing.^[11]

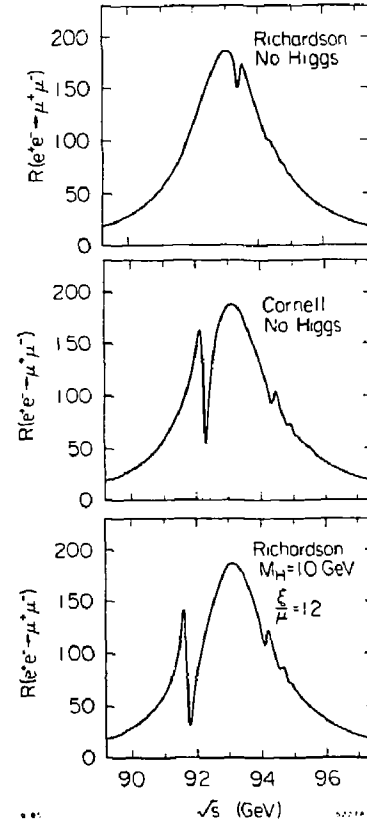


Figure 5. Effects of varying quarkonium potential.

be visible as wiggles in cross sections and asymmetry parameters. The exact potential for toponium (and thus exactly what we will see) is not very well known. The Higgs (in a two-Higgs model) can have noticeable effects, but it may be hard to distinguish these effects from those of different potentials; the 2S-1P level inversion is a possible qualitative difference, if the Higgs couplings are rather large.

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Heavy Quark Spectroscopy and Decay[†]

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1. INTRODUCTION

The understanding of $q\bar{q}$ systems containing heavy, charmed, and bottom quarks has progressed rapidly in recent years, through steady improvements in experimental techniques for production and detection of their decays. These lectures are meant to be an experimentalist's review of the subject. In the first of two lectures, the existing data on the spectroscopy of the bound $c\bar{c}$ and $b\bar{b}$ systems will be discussed. Emphasis is placed on comparisons with the theoretical models described in greater detail in the lectures of F. Gilman, published in these proceedings. The second lecture covers the rapidly changing subject of the decays of heavy mesons ($c\bar{q}$ and $b\bar{q}$), and their excited states. Additional theoretical material is available in the article of I.I.Y. Bigi, published in these proceedings. The topics of CP violation and mixing are covered in the lectures of B. Winstein. In combination, the spectroscopy and decays of heavy quarks are shown to provide interesting insights into both the strong and electroweak interactions of the heavy quarks.

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2. THE SPECTROSCOPY OF CHARM AND BEAUTY^[1]

The bound systems of two heavy charmed or bottom quarks below the threshold for production of the corresponding mesons provide a unique laboratory from which to test the flavor independence of the strong interactions, and our ability to work from the relativistic $c\bar{c}$ system into the nonrelativistic $b\bar{b}$ and $t\bar{t}$ systems. The general properties of these system, and the hitherto unobserved third generation toponium system are summarized in Table I. The properties of these systems will be shown to be determinable through detailed measurements of the level splittings, the fine and hyperfine structure, the transition rates, and the decays. In the first sections, the general structure of the two systems is reviewed.

Table I. Bound Systems of Heavy Quarks

	CHARMONIUM	BOTTOMONIUM	TOPONIUM
q mass	1.5 GeV	4.5 GeV	?
q charge	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$
$\langle \beta^2 \rangle$	.15-.25	< .1	?
$\sqrt{\langle r^2 \rangle}$	.4 f_m	.2 f_m	$M_q > 80 \text{ GeV} < .05$
# Bound States	8	22	10 s-wave, 200 total

2.1 Charmonium

In Fig. 1 a typical level scheme for a $c\bar{c}$ bound system is shown. The details of the level structure are determined by the assumed form of the interquark potential. Figure 1 is based for example on the QCD inspired Cornell Model:^[2]

$$V(r) = -\frac{4\alpha_s}{3r} + \lambda r. \quad (1)$$

Here the $\frac{1}{r}$ term represents the strong Coulombic-like potential that binds the system. Unlike the electromagnetic interactions in the hydrogen atom, the coupling α_s should diminish with the effective q^2 or quark masses, thus exhibiting the strong interaction property of asymptotic freedom. The second term represents an ansatz for the long range part of the potential, exhibiting the second property of the strong interaction, that of confinement. No explicit flavor dependence is

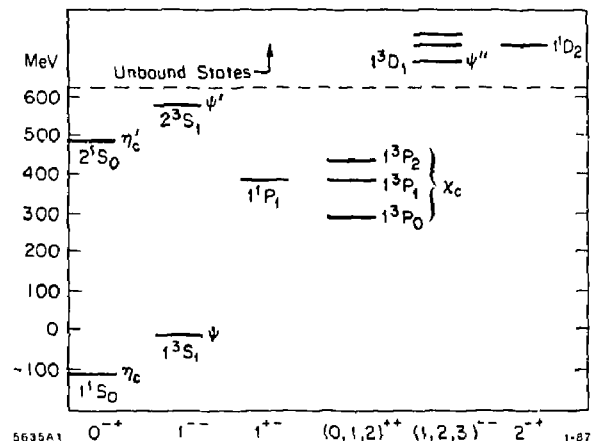


FIG. 1. The lowest lying states of charmonium.

exhibited in this potential. Under these assumptions, two narrow S-wave states are expected below the open charm threshold. The splitting of these ^3S states is determined by the strength and form of the spin independent $\frac{1}{2}$ part of the potential. Nonrelativistic models should reliably predict these splittings. The addition of the confinement term to the potential shifts the center of gravity (COG) of the otherwise degenerate P-wave states below that of the 2^3S state.

The detailed features of the level structure are analogous to the hydrogen atom. There is both fine and hyperfine structure which must be introduced by relativistic terms accounting for spin and orbital angular momentum of the quarks. The theoretical constraints on these parts of the potential are described in detail elsewhere in these proceedings.^[4] The three ^3P_J states are themselves degenerate in the absence of a spin-orbit ($L \cdot S$) interaction. The spin-spin ($S \cdot S$) interaction breaks the degeneracy of the $^1S_0$ and the $^3S_1$ states. The $^1P_1$ remains degenerate at the COG of the ^3P_J states if the form the potential is short range. Tensor forces can contribute to the fine structure, and also mix states of the same J but different L, such as the 2^3S_1 and the 1^3D_1 .

The possible transitions and decays of the charmonium states are shown in Fig. 2. The M1 (magnetic) dipole transitions follow the selection rules: $L_i = L_f$ and $P_i = P_f$, while the E1 (electric) dipole transitions require $L_i = L_f \pm 1$, $P_i = -P_f$, and $|J_i - 1| \leq J_f \leq J_i + 1$, where P is the parity of the state. While the $^3S_1$ and $^3D_1$ states are directly produced in e^+e^- collisions and are connected by $\pi\pi$ and η strong interaction transitions, the $^1S_0$ and $^3F_2$ states can only be produced through the electromagnetic cascades from higher mass charmonium states. The $^1P_0$ state cannot be excited in e^+e^- , nor be reached through hadronic decays of the 2^3S_1 because of the limited phase space. It can be produced however in $p\bar{p}$ formation experiments. All the states have significant hadronic widths, which in principle can be used to determine their quantum numbers and spectroscopic assignment.

2.2 Bottomonium

The level structure of the bound $b\bar{b}$ system can be extrapolated reliably from Eq. (1), under the assumption of flavor independence, once the parameters of the potential are determined from the charmonium system. Such an extrapolation is shown in Fig. 3. The general features are determined by the addition of a third narrow $^3S_1$ bound state below threshold for open B meson production.

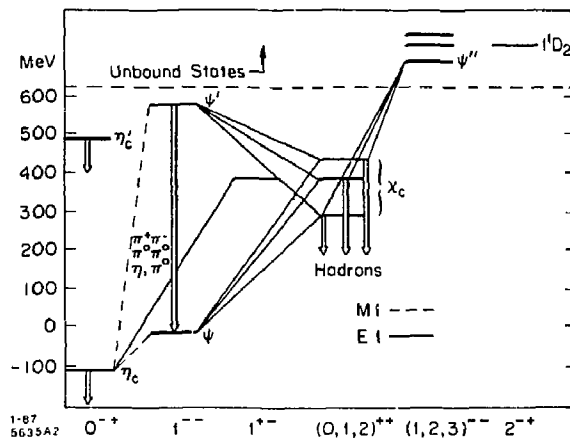


FIG. 2. The transitions and decays of charmonium states.

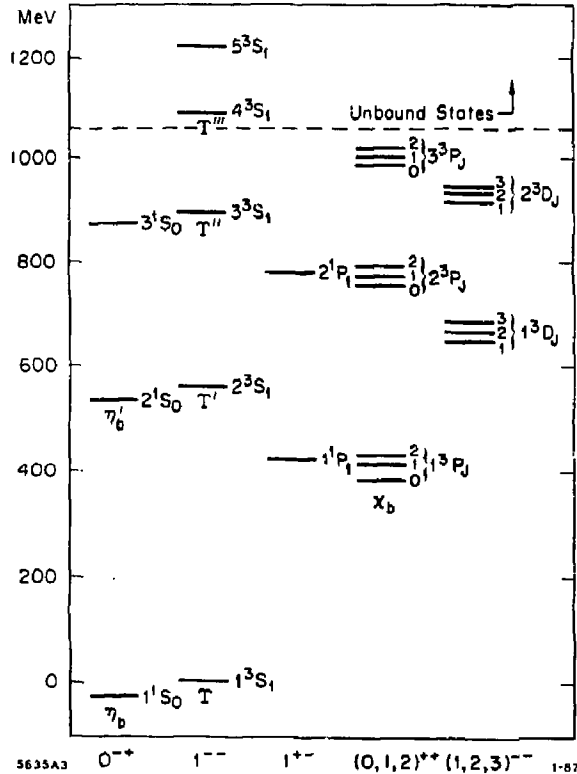


FIG. 3. The bound $b\bar{b}$ system.

This in turn implies a richer and more complicated spectroscopy, owing to the presence of additional P-wave multiplets, a third $1S_0$ state, and the D-wave states below open beauty threshold. The fine and hyperfine splittings are smaller than the corresponding ones in the charmonium system, posing a more difficult experimental challenge.

The radiative transitions are shown in Fig. 4. There are now several levels of cascades through the $3P_J$ states, and weaker ones through the $3D_J$ states. The hadronic transitions are considerably more complicated, as can be seen in Fig. 5. Both two- π and three- π transitions are now energetically allowed. One interesting difference from charmonium is the accessibility of the 1^1P_1 state through a hadronic $\pi\pi$ transition from the 3^3S_1 state. The subsequent decay should have a large radiative width into the 1^1S_0 state.

2.3 Experimental Determination of Resonance Parameters

The most precise determinations of the masses and widths for $c\bar{c}$ and $b\bar{b}$ states comes from e^+e^- production. In e^+e^- storage rings the energy spread is small and the center of mass energy can typically be moved in few tenths of an MeV steps. Thus the possibility of measuring an excitation curve exists:

$$\sigma(e^+e^- \rightarrow \psi \rightarrow f) = \frac{2\pi^2(2J+1)}{M_\psi^2} \frac{\Gamma_{ee}\Gamma_f}{(E - M_\psi)^2 + \Gamma_{tot}^2/4}. \quad (2)$$

Here Γ_f can be the partial width to hadrons (Γ_{had}), electron pairs (Γ_{ee}) or muon pairs ($\Gamma_{\mu\mu}$). The observed data is corrected by convoluting (2) with the machine energy spread, and the effects of radiative corrections.

One way of extracting Γ_{ee} , Γ_{had} , and $\Gamma_{\mu\mu}$ is simply to fit to the excitation curve into each of the three final states. The more common technique uses the fact that the area (A) under (2) for hadronic final states is given by:

$$A = \frac{6\pi^2}{M_\psi^2} \frac{\Gamma_{ee}\Gamma_{had}}{\Gamma_{tot}} C_{rad}.$$

The radiative corrections (C_{rad}) are incorporated into the lineshape. Under the

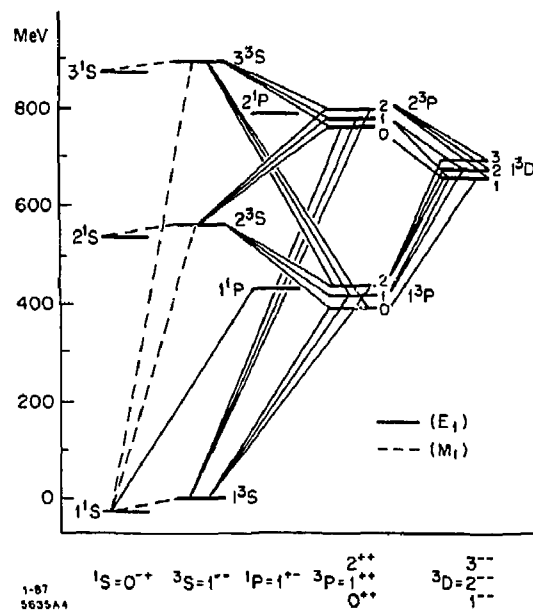
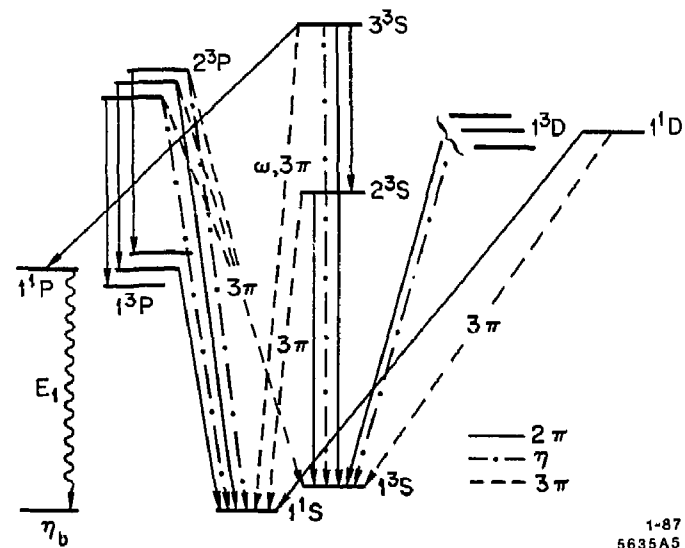
FIG. 4. Radiative transitions in the $b\bar{b}$ system.

FIG. 5. Hadronic transitions in the $b\bar{b}$ system.

assumption of lepton universality, ($\Gamma_{\mu\mu} = \Gamma_{ee} = \Gamma_{\tau\tau}$) and assuming:

$$\frac{\Gamma_{had}}{\Gamma_{tot}} = 1 - NB_{\mu\mu},$$

then

$$\Gamma_{ee} = \frac{A M_\psi^2}{6\pi^2} \frac{1}{(1 - NB_{\mu\mu})}.$$

Here, $N=2$ for charmonium and $N=3$ for bottomonium. Hence measuring $B_{\mu\mu}$ and the integral of the hadronic cross section, gives Γ_{ee} . The total width (Γ_{tot}) is $\Gamma_{ee}/B_{\mu\mu}$, and $\Gamma_{had} = \Gamma_{tot} - N\Gamma_{ee}$.

To find $B_{\mu\mu}$, it is only necessary to measure the ratio of μ -pairs to hadrons on the resonance. Then:

$$B_{\mu\mu} = \frac{1}{\frac{\Gamma_{had}}{\Gamma_{\mu\mu}} + N}$$

since $\Gamma_{tot} = \Gamma_{had} + N\Gamma_{\mu\mu}$.

2.3.1 S-wave and D-wave bound states of $c\bar{c}$ and $b\bar{b}$.

Below open charm threshold, the S-wave states (the ψ and ψ') appear as narrow states ($\Gamma \sim 100$ KeV) in the hadronic cross section. Once above threshold for charmed meson production, the structure of the cross section becomes considerably more complex. The $^3D_1$ state (the ψ'') lies just above D-meson threshold and has a width of ~ 25 MeV, typical of a strongly decaying meson of that mass. It appears to decay largely to charmed meson pairs.^[4] Above the ψ'' however, the cross section becomes complex in structure,^[4] (see for example Fig. 6), owing to the thresholds that open for the production of new charmed meson final states ($DD, DD^*, D^* \bar{D}^*, D, \bar{D}, D, \bar{D}^*, \dots$ etc.). Figure 6 also shows the prediction of the position from two potential models,^{[4][11]} for the 3^3S_1 and 4^3S_1 states. The structures in the cross section in the 4 GeV region are roughly reproduced by the coupled-channel model shown in Fig. 7.

The cross section data near 10 GeV show a similar structure^[11] (see Fig. 8). The three lowest lying $b\bar{b}$ bound states (the $^3S_1$ states) are narrow, and are followed by a broader state just above threshold for open B production. This fourth state (the T'') decays predominantly to B_s and B_d mesons. A little higher in energy,

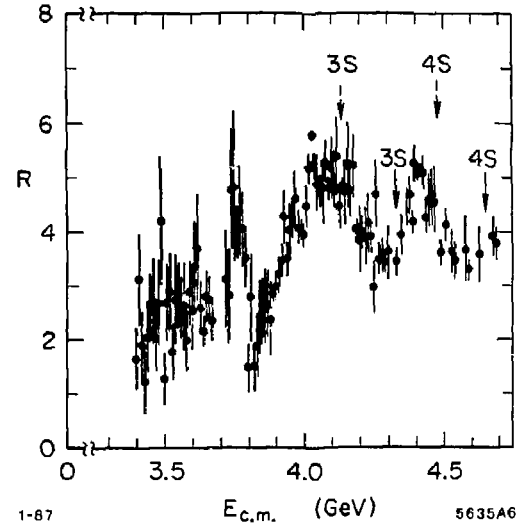


FIG. 6. The e^+e^- cross section divided by $\sigma_{\mu\mu}$, the point cross section, in the regime above charm threshold.^[5] The solid arrows indicate the positions of the 3S and 4S states from Eichen *et al.*, while the dashed arrows are the same from Buchmuller and Tye.

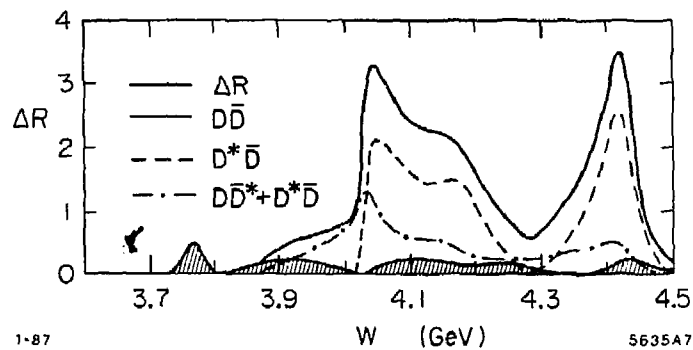


FIG. 7. Coupled channel model of Ref. [6].

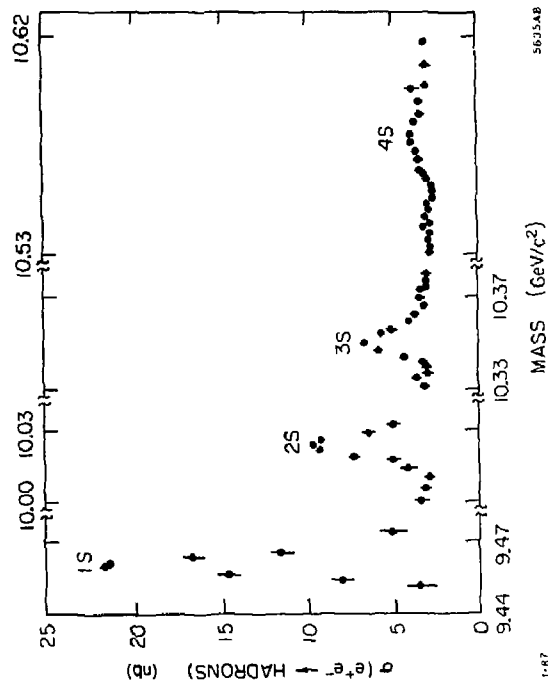


FIG. 8. Data from Ref. 8 of the Υ region.

the structure of the cross section becomes complex again.^[9] This region is shown in detail in Fig. 9. There is evidence for at least two additional resonances, which correspond well in position with the potential model calculations for the next two $^3S_1$ states, the $T(5S)$ and $T(6S)$. Structure above the 4^3S_1 state is also complicated by the opening of $B\bar{B}^*$, $B^*\bar{B}^*$, and B_s thresholds. The $T(7S)$ is expected to have a mass of ~ 11.2 GeV.

Table II summarizes the current data on the $c\bar{c}$ S-wave states, while Table III summarizes the $b\bar{b}$ data. In charmonium, even the most naive calculation, namely the relative leptonic widths of the ψ and ψ' is difficult, owing to mixing of the ψ' with the nearby ψ'' . In the coupled channel model of Eichten *et al.*, the mass and Γ_{ee} for the ψ are fixed and $\Gamma_{ee}(\psi')$ calculated to be 2.3 KeV, compared with the observed value of 2.1 KeV. This is basically a measure of how well the model computes the square of the radial wave function (R_0) at the origin. It is seen to be good to about 10%.

Table II. Comparison of $^3S_1$ State Charmonium Parameters^[10]

	$\Gamma_{ee}(\text{KeV})$	$B_{ee}(\%)$	$\Gamma_{\mu\mu}(\text{KeV})$	$B_{\mu\mu}(\%)$	$\Gamma_{had}(\text{KeV})$	$\Gamma_{tot}(\text{KeV})$	Mass (MeV)
1S	$4.75 \pm .51$	6.9 ± 0.9	$4.85 \pm .51$	$6.9 \pm .9$	57.3 ± 10.9	63 ± 6.3	3096.93 ± 0.09
2S	$2.05 \pm .21$	0.88 ± 0.13		0.77 ± 0.17	224 ± 56	228 ± 56	3686.00 ± 0.10
3S	0.75 ± 0.15 0.77 ± 0.23				dominant dominant	$52 \pm 10 \text{ MeV}$ $78 \pm 20 \text{ MeV}$	4028 ± 2 4160 ± 20
4S	0.5 ± 0.1				dominant	$43 \pm 15 \text{ MeV}$	4415 ± 6

Table III. $^3S_1$ Bottomonium States^{[10] [11][12]}

	$\Gamma_{ee}(\text{KeV})$	$B_{ee}(\%)$	$B_{\mu\mu}(\%)$	$\Gamma_{tot}(\text{KeV})$	Mass
1S	1.224 ± 0.050	2.82 ± 0.31	2.78 ± 0.22 $2.4 \pm 0.2^{[11]}$	43.1 ± 3.1	9460.0 ± 0.19
2S	0.537 ± 0.033		1.8 ± 0.44	30 ± 7.3	10233.7 ± 0.34
3S	0.402 ± 0.031		0.3 ± 1.5 $1.5 \pm 0.4^{[12]}$	12.0 $26 \pm 7^{[12]}$	10355.5 ± 0.5
4S	0.19 ± 0.04 CLEO 0.28 ± 0.04 CUSB			$20.0 \pm 4.5 \text{ MeV}$ 25.0 ± 2.5	10577.5 ± 4.0
5S	0.22 ± 0.09 CLEO 0.37 ± 0.07 CUSB			$110.4 \pm 13 \text{ MeV}$	10685 ± 8
6S	0.095 ± 0.05 CLEO 0.156 ± 0.04 CUSB			$79 \pm 16 \text{ MeV}$	11010 ± 9

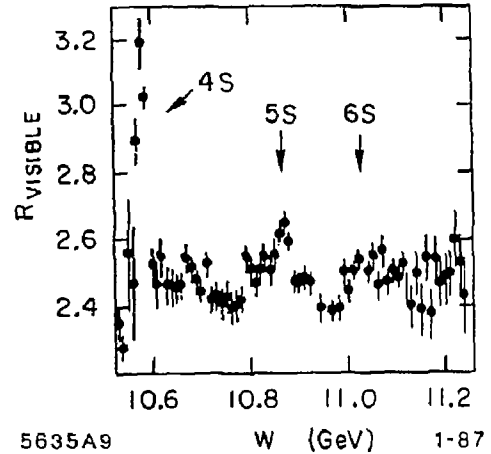


FIG. 9. The region above open beauty threshold from D. Lovelock *et al.*, Ref. 9.

In the $b\bar{b}$ system, the parameters of the potential derived from the $c\bar{c}$ system should provide a good description of the level splittings if the potential is truly flavor independent. From the model of Eichten *et al.*, the 2S-1S splitting is predicted to about 5%. If the model is adjusted to make it correct, then the 3S-1S is good to about 1% and the 4S-1S is good to about 4%. Recall that these splittings depend principally on the short range part of the potential. The leptonic widths of the 2S and 3S are calculated to within 10%, if the parameters for the 1S are used. Published measurements of the 4S are poorer, with large discrepancies between experiments.

2.4 The P-Wave States

2.4.1 $^1P_1$ and ^3P_J states of $c\bar{c}$.

These states have quantum numbers $J^{PC} = 1^{+-}$ and 1^{++} , and hence are not directly excitable in e^+e^- collisions. They must be studied through the electromagnetic transitions from higher S-wave states, as was shown in Figures 2 and 4. Three experimental approaches are employed for studying these states. They are the inclusive photon spectrum, reconstruction of the full cascade, or reconstruction of the first radiative transition with the subsequent hadronic decay of the P-wave state. These are pictured in Fig. 10. The inclusive spectrum, combined with the exclusive full cascade measurement, determines the branching ratio for the P-wave states into $\gamma\psi$. In conjunction with the hadronic P-wave decays, the hadronic branching fractions are determinable. In the $c\bar{c}$ system, the primary photon has an energy of 130 to 270 MeV, while the level splittings are typically 50 MeV. In the $b\bar{b}$ system, the primary photon is 100 to 160 MeV, and the splittings only about 25 MeV. Typical glass detectors (NaI, BGO, Pb-glass) have resolutions of about 5 MeV at 100 MeV, while the use of e^+e^- pair gamma conversions improves the resolution to 1 to 2 MeV but at a cost of about one order of magnitude in detection efficiency.

The inclusive photon spectrum from the Crystal Ball detector^[13] is shown in Fig. 11. The fourth line is the merged Doppler shifted lines from the recoiling P-wave state. The typical resolution obtainable from hadronic decays of the P-wave states is shown in Fig. 12.

The measurement of the natural widths of the P-wave mesons in the $c\bar{c}$ system can in principle be extracted from the gamma ray lineshape in the $\psi' \rightarrow \gamma\psi$

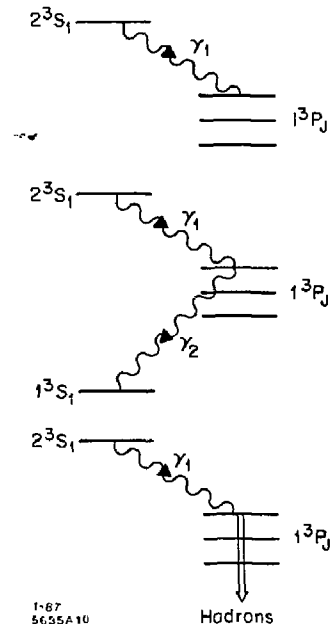


FIG. 10. Three approaches to observing P-wave states.

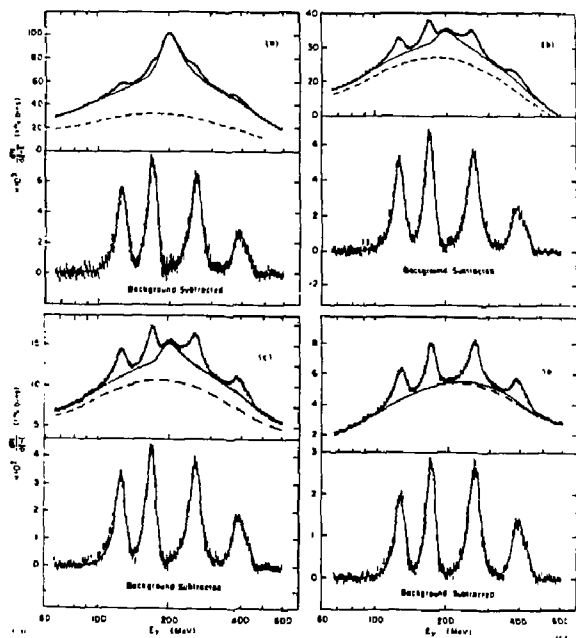


FIG. 11. Crystal Ball inclusive photon spectrum.^[13]

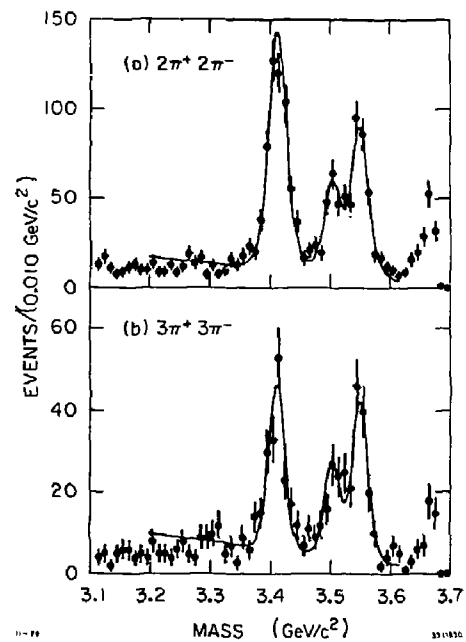


FIG. 12. Example of hadronic decays of P-wave states, from Ref. 106.

transition, if the detector resolution were adequate. The Crystal Ball has recently determined values for these widths, given in Table IV.^[14]

Table IV. Widths of P-Wave $c\bar{c}$ States

State	Width (γ) (MeV)	Width $p\bar{p}$ (MeV)
$^3P_2$	≤ 3.8	$2.6^{+1.4}_{-1.0}$
$^3P_1$	$0.8 - 4.9$	≤ 1.3
$^3P_0$	$13.5 \pm 3.3 \pm 4.2$	—

A more precise measurement was recently obtained^[14] by performing a formation experiment using p in the ISR storage ring, and a hydrogen gas jet target. The center of mass energy spread is small ($\delta E_{cm} \approx \pm 0.4$ MeV). The p momentum is stepped to produce a scan over the P-wave mass (see Fig. 13):

$$p\bar{p} \rightarrow \chi_J \rightarrow \psi + \gamma$$

where χ_J are the common names corresponding to the ^3P_J states. The excitation curve is measured by counting events from $\psi \rightarrow e^+e^-$. The results are shown in Table IV for the narrower χ_J states. In addition, the masses $M_{\chi_1} = 3511.3 \pm 0.4 \pm 0.4$ MeV and $M_{\chi_2} = 3556.9 \pm 0.4 \pm 0.5$ MeV were measured.

The 1^1P_1 (η_c) and 2^1P_1 (η'_c) states of the $c\bar{c}$ system have been measured by the inclusive photon technique. These splittings, being less than 100 MeV are a particular experimental challenge. The η'_c has only been observed in one experiment,^[15] while the existence of the η_c has been confirmed through its hadronic decays^[16] and extensively measured.^[17] The η_c and η'_c signal in inclusive photons are shown in Fig. 14, while Fig. 15 shows typical hadronic decays of the η_c .

2.4.2 1^1P_1 and 3^3P_J states of $b\bar{b}$.

As indicated in Figure 4, the radiative transitions in the $b\bar{b}$ system are more complex because of the presence of two sets of P-wave states (usually denoted χ_b and χ'_b). A summary of the measurements of the χ_b states from the observation of the photons in the transitions $T(2S) \rightarrow \gamma + \chi_b^{J=0,1,2} \rightarrow \text{anything}$ are shown

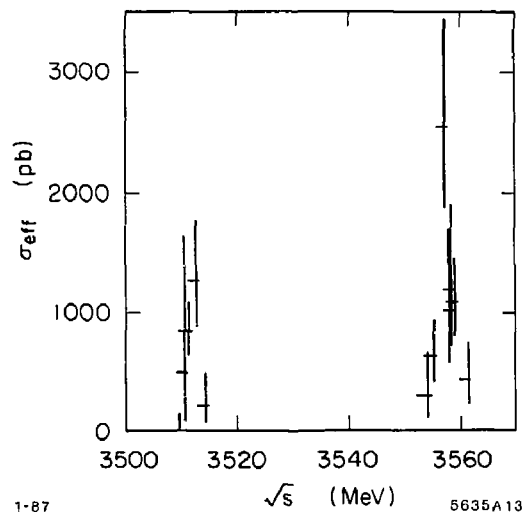


FIG. 13. Formation experiment^[14] to measure masses and widths of the χ_c states.

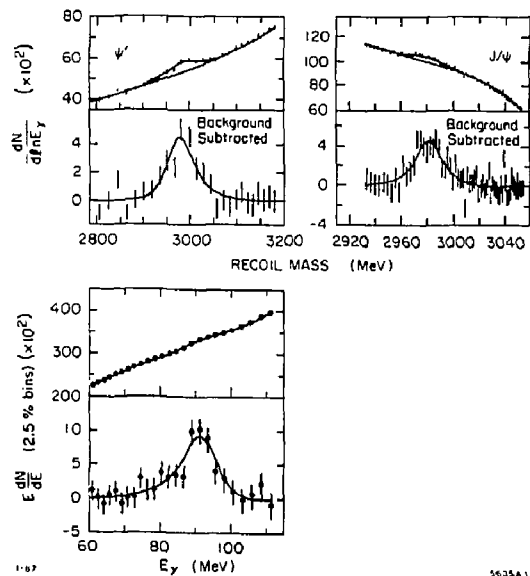


FIG. 14. Inclusive photons from Crystal Ball show signals for both the η_c and the η_c' states.

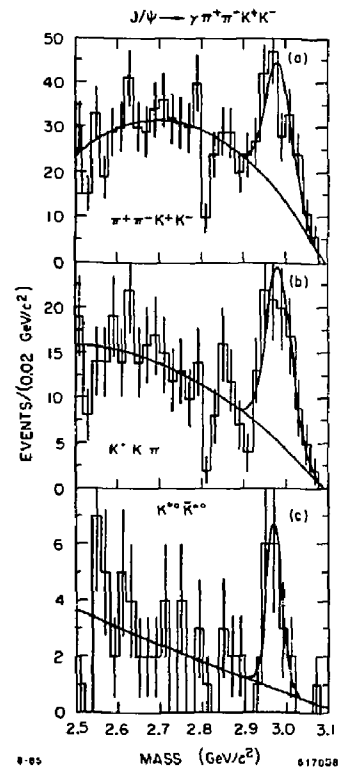


FIG. 15. Hadronic decays of the η_c from Mark III.^[107]

in Fig. 16. In these measurements, the 1 MeV resolution of the converted pairs in the ARGUS detector separates the states unequivocally. The transitions to the 2^3P_J multiplet from the reaction $T(3S) \rightarrow \gamma + \chi_b^{J=0,1,2} \rightarrow \text{anything}$ are less well established. The first inclusive measurements are shown in Fig. 17, indicating the presence of the multiplet, but not clearly separating the states. The $J=2, 1$ and 0 states of the χ_b^J have fitted lines of 122 ± 5 , 100 ± 2 and 84 ± 2 MeV respectively. New data ^[101] using CUSB-II (a BGO augmented device) has improved the resolution of these states.

2.5 Experimental and Theoretical Comparisons

This section summarizes the data on E1, M1 and hadronic transitions in the charmonium and bottomonium systems, comparing the data with theoretical expectations. More details of the models under discussion appear in the parallel lectures of F. Gilman.

2.5.1 Charmonium splittings.

Table V summarizes the average energies and branching fractions of the χ_c states. As was noted, in the absence of a confining term the COG of ^3P_J states would remain degenerate with the 2^3S_1 . The spin dependent part of the potential would break the degeneracy of the multiplet, but leave the singlet ($^1P_1$) state unchanged at the COG. The splitting of the ^3P_J provides information on the Lorentz structure of the spin dependent part of the potential. The parameter R , defined :

$$R = \frac{{}^3P_2 - {}^3P_1}{{}^3P_1 - {}^3P_0} \quad (3)$$

is used as a measure of the splitting. In the charmonium system $R = 0.50 \pm 0.02$. A purely Coulombic spin dependent potential would give $R \approx 0.8$. If however there is a scalar contribution, then R is decreased, while a vector contribution would increase it further. The value of 0.5 suggests that these added terms are scalar in their Lorentz structure.

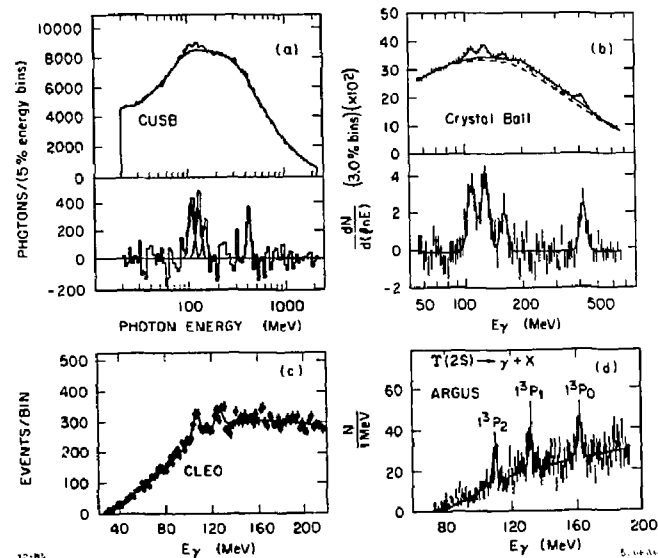


FIG. 16. Recent summary of inclusive photons from the $T(2S)$.^[108]

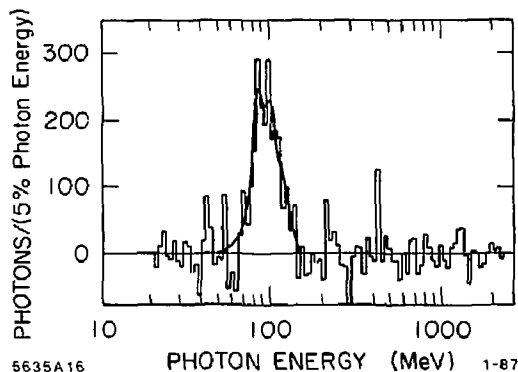


FIG. 17. Early CUSB inclusive photon data on the $T(3S)$ showing the narrowly resolved lines of the $2P$ multiplet.^[8]

Table V. Summary of Data on the χ_c Transitions^[10]

χ_c	Energy ($2^3S_1 \rightarrow 3P_1$)	$Br(2^3S_1 \rightarrow ^3P_1)$	$Br(3P_1 \rightarrow ^3P_1)$	Γ_{had} (MeV)
$J = 2$	129.7 ± 0.4	$7.83 \pm 0.82\%$	$14.8 \pm 1.7\%$	$2.6^{+1.4}_{-1.0}$
$J = 1$	175.3 ± 0.5	8.65 ± 0.81	26 ± 3	< 1.3
$J = 0$	271.1 ± 1.1	9.35 ± 0.80	0.7 ± 0.2	$13.5 \pm 3.3 \pm 4.2$

2.5.2 Charmonium E1 and M1 transitions.

The E1 rates are summarized in Table VI, along with the nonrelativistic coupled channel model of Eichten *et al.* The E1 rates for such transitions are given:

$$\Gamma(2^3S_1 \rightarrow 1^3P_J) \propto q^2 (2J+1) k^3 \langle r \rangle^2$$

where k is the photon energy, and $\langle r \rangle$ is the overlap integral of the dipole operator (r) between the initial and final states. While the transitions $\chi_c \rightarrow \gamma\psi$ are in fairly good agreement, the primary transitions $\psi' \rightarrow \gamma\chi_c$ have predicted E1 rates about a factor of 2 off from the data. McClary and Byers^[19] recently looked at relativistic corrections in the charm system. Their explanation is shown schematically in Fig. 18 they point out that the $2S \rightarrow 1P$ transition is sensitive to corrections of $O(\frac{v^2}{c^2})$ which may shift the node of the radial wavefunction of the $2S$ state, right at the peak of the $1P$ wavefunction, thus reducing the transition rate. This is not the case for the $1P \rightarrow 1S$ transition. The corrected numbers for the E1 transitions are shown in Table VI, and are in good agreement with the data.

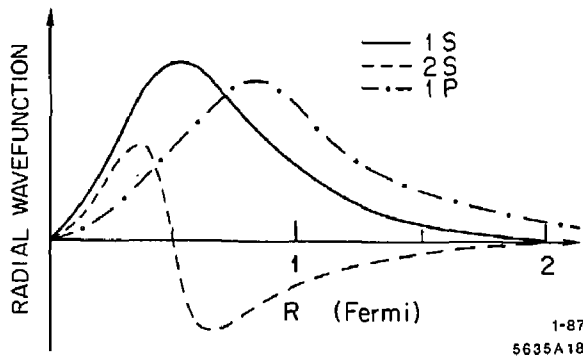


FIG. 18. Schematic showing the relative position of nodes in the S and P wave states, and their effect on transition rates.

Table VI. E1 Transitions in Charmonium^{[10][11]}

Charmonium Transition	Angular Momentum (J)	Nonrelativistic Width (KeV)	Relativised Width (KeV)	Coupled Channel (KeV)	Data (KeV)
$\psi' \rightarrow \gamma \chi_c$	0	45	19	16	20.1 ± 4
	1	40	31	23	18.6 ± 4
	2	27	27	22	16.8 ± 4
$\chi_c \rightarrow \gamma \psi$	0	121	128	117	95 ± 37
	1	250	270	240	≤ 252
	2	362	347	305	429^{+270}_{-169}

Measurements of M1 transitions are considerably poorer as is seen in Table VII where data and theory are compared:

Table VII. M1 Transitions in Charmonium^{[9][11]}

Charmonium Transition	Branching Ratio (%)	Width (KeV)	Model (KeV)
$\psi \rightarrow \gamma \eta_c$	1.27 ± 0.36	0.8 ± 0.24	2
$\psi' \rightarrow \gamma \eta'_c$	0.28 ± 0.06	0.6 ± 0.18	1

There is no evidence of any discrepancy owing to the large errors that remain in the data.

2.5.3 The $^1P_1$ charmonium state.

The $p\bar{p}$ gas jet formation experiment mentioned in Section 2.4.1 has also searched for the singlet-P state. Five candidate events have been found with a mass of $3525.4 \pm 0.8 \pm 0.5$ MeV. This state is seen to be close to the COG of the three ^3P_J states, 3525.38 MeV, confirming the expectation that the hyperfine spin-spin splitting of the P-wave states should effect both the singlet and the COG of the triplet equivalently, creating no net displacement.

2.5.4 Bottomonium COG and splittings.

Table VIII summarizes the average energies and branching fractions of the χ_b and χ'_b states. New data from CUSB-II^[12,14] has been included in the table. The COG of the χ_b and χ'_b are 9900.2 MeV and 10261.6 MeV, respectively. The COG as noted is a sensitive measure of the long range confining term in the potential. Table IX is a comparison between various models^{[7] [14] [24] [25] [26] [27] [28] [29] [30]} which predict the COG of the χ_b states. These models generally use the $\Upsilon(1S)$ mass as input to set the scale. As can be seen, the relativistic potential models do very well in predicting the COG.

Table VIIIa. Bottomonium 1^3P_J Masses and Widths^[14]

χ_b	Energy	Br(%)	Br(%)
	$\langle 2^3S_1 \rightarrow 1^3P_J \rangle$	$\langle 2^3S_1 \rightarrow 1^3P_J \rangle$	$\langle 1^3P_J \rightarrow 1^3S_1 \rangle$
$J = 2$	109.5 ± 0.6	6.6 ± 0.9	22 ± 4
$J = 1$	130.7 ± 0.7	6.7 ± 0.9	35 ± 8
$J = 0$	162.3 ± 1.3	4.3 ± 1.0	< 6

Table VIIIb. Bottomonium 2^3P_J Masses and Widths^[14]

χ'_b	Energy	Br(%)	Br(%)
	$\langle 3^3S_1 \rightarrow 2^3P_J \rangle$	$\langle 3^3S_1 \rightarrow 2^3P_J \rangle$	$\langle 3^3P_J \rightarrow 2^3S_1 \rangle$
$J = 2$	84.1 ± 1.7	12.7 ± 4.1	-
$J = 1$	99.7 ± 1.7	15.6 ± 4.2	-
$J = 0$	122.1 ± 5.0	7.6 ± 3.5	-

Table IX. Theoretical Predictions for the COG^{[7] [14] [24] [25] [26] [27] [28] [29]}

Models	$M_{\text{COG}}(\chi_b) = 9900.2 \text{ MeV}$	$M_{\text{COG}}(\chi'_b) = 10261.6 \text{ MeV}$
PM	Khare 9871	-
NP	Eichten, Feinberg 9925	10275
B	Buchmuller 9888	10250
RP	Moxhay, Rosner 9906	10262
R ^{1/2}	Gupta <i>et al</i> 9900	10258
RP	McClary, Byers 9923	10267
NP	Richardson 9896	10250
P	Martin 9861	10242

PM = PHEN. POT. NP = NON-REL. POT. B = BAG RP = REL. POT.

The parameter R (see Eq. (3)) measuring the multiplet splitting is calculated from Table IX to be:

$$R_{\chi_b} = 0.67 \pm 0.05 \quad R_{\chi'_b} = 0.70 \pm 0.20.$$

These values are higher than in the charmonium system. Table X summarizes many of the theoretical models which predict R for the $b\bar{b}$ system. Again the relativistic potential models appear to track the data quite well. The large values of R in the $b\bar{b}$ system compared with charmonium's 0.5, are still below the 0.8 value predicted for a purely Coulombic potential, going in the right direction for an additional scalar confinement term. The larger values suggest however that potential is more strongly vector-Coulombic on average over the wavefunction at the shorter distances probed in the heavier $b\bar{b}$ system.

Table X. Summary of ^3P_J (χ_b , χ'_b)^{[24][21][7][27][10][28][22][23][27][29]}

Models	Data	$R\chi_b = 0.67 \pm 0.05$ PDG	$R\chi'_b = 0.70 \pm .20$ PDG
PM	Khare	0.50	-
NP	Eichten,Feinberg	0.80	0.80
B	Buchmuller	0.73	0.78
RP	Moxhay,Rosner	0.42	0.42
RP	Gupta <i>et al</i>	0.68	0.70
RP	McClary,Byers	0.45	0.48
BR	Baacke	0.73	0.77
RP	Kang	0.52	0.57
RP	Beavis	0.96	1.0

The absolute size of the fine structure splitting is defined:

$$\Delta_{fs} = {}^3P_2 - {}^3P_0.$$

This splitting is sensitive directly to the strong coupling constant (α_s), and is found from Table IX to be:

$$\Delta_{fs}^* = 52.8 \pm 1.4 \text{ MeV} \quad \Delta_{fs}' = 38.0 \pm 5.3 \text{ MeV}.$$

Table XI compares these values with those of numerous models of the $b\bar{b}$ system. The general trend of a reduction in the absolute splitting is seen to arise in all models, while the absolute magnitude is only close for some of the fully relativised models, and those that treat relativistic effects perturbatively.

Table XI. Theoretical Predictions for the Fine Structure Splittings^{[7][21][22][26]}

Models	Data	$\Delta_{fs}\chi_b \approx 52.8 \pm 1.4$	$\Delta_{fs}\chi'_b = 38.0 \pm 5.3$
NP	Eichten Feinberg	51.0	36.0
B	Buchmuller	38.0	32.0
RP	Moxhay, Rosner	37.0	28.0
RP	Gupta <i>et al</i>	42.0	34.0
BR	Baacke	52.0	42.0

2.5.5 Bottomonium E1 transitions.

The E1 rates for the $2S \rightarrow 1P$ transitions are summarized in Table XII, along with the predictions of several models.^[6] Because the E1 rates scale as k^3 , the predictions have been scaled to the correct photon energies, when calculating the total E1 widths in Table XII. New CUSB-II data for the $3S \rightarrow 2P$ transitions are also included in the table. The agreement is generally good, and suggests that the relativistic corrections are less important in this case, than the $c\bar{c}$.

Table XII. E1 Transitions of Bottomonium^{[7][24][25][22][6]}

Models	2^3P_0	2^3P_1	2^3P_2	$\Sigma_{\text{total}}^{\text{corrected}\dagger}$	$\Sigma\chi'_b$
Khare	0.56	1.06	1.6	3.0	
Buchmuller	1.3	2.4	2.8	5.6	6.4
Kang	1.4	2.8	3.6	4.7	5.3
Moxhay,Rosner	1.0	2.1	2.2	5.4	6.9
EXPT (KeV)	$1.3 \pm .4$	2.0 ± 0.5	2.0 ± 0.5	5.3 ± 0.8	$4.3^{+3.6}_{-1.7}$

† From reference 8. ‡ CUSB-II data.^{[14][12]}

There is no data on M1 transitions in the bottomonium system.

2.5.6 Hadronic widths of the χ_b states.

The total width of each χ_b^f state is the sum of its E1 width, $\Gamma_{E1}(2^3P_J \rightarrow 1^3S_1)$, plus its hadronic width, $\Gamma(3P_J)$. Inclusive photon measurements give the $Br(2^3S_1 \rightarrow 1^3P_J + \gamma)$. Exclusive measurements of the cascades give the product:

$$Br(2^3S_1 \rightarrow 1^3P_J + \gamma) \cdot Br(3P_J \rightarrow \gamma 1^3S_1) \cdot Br(1^3S_1 \rightarrow l^+l^-).$$

Hence, dividing by the leptonic branching ratio of the 1^3S_1 and by $Br(2^3S_1 \rightarrow 1^3P_J + \gamma)$ gives $B_J = \frac{\Gamma_{E1}}{\Gamma_{tot}}$. Combining this with the definition of Γ_{tot} gives:

$$\Gamma_{had} = \Gamma_{E1} \left(\frac{1}{B_J} - 1 \right). \quad (4)$$

Using theoretical E1 widths for the $1P \rightarrow 1S$ transitions is reasonable in light of the good agreement for the $2S \rightarrow 1P$ rates.^[10] The results are shown in Table XIII, in comparison with the theoretical values from Kuang and Yan^[11] using the QCD multipole expansion. The agreement is rather good. The ggg and $q\bar{q}g$ decays are seen to be smaller than the gg decays of the $J=0,2$ states.

Table XIII. Hadronic Widths of the χ_b States^{[10],[11]}

2^3P_J	$J=0$	$J=1$	$J=2$
B_J	< 6	35 ± 8	22 ± 4 (%)
$< \Gamma_{E1} >^\dagger$	27 ± 5	33 ± 5	27 ± 5 KeV
Γ_{HAD}	> 423	61 ± 29	96 ± 33 KeV
Typ. Theor. Est.	380	30-80	100-200
Had. Decays	gg	$(ggg, q\bar{q}g)$	gg

[†] from Ref. 30.

2.5.7 Hadronic transitions in $c\bar{c}$ and $b\bar{b}$.

The QCD multipole expansion has been used^[12] to scale the hadronic transitions of $\psi' \rightarrow \psi$, to those of the $b\bar{b}$ system. The ratios between $c\bar{c}$ and $b\bar{b}$ are largely determined by the quark masses, the size of the initial states, and the available phase space (PS):

$$\frac{\Gamma(\Upsilon' \rightarrow \pi\pi\Upsilon)}{\Gamma(\psi' \rightarrow \pi\pi\psi)} = \frac{< r_{\Upsilon'}^2 >}{< r_{\psi'}^2 >} \approx \frac{1}{16}, \text{ and}$$

$$\frac{\Gamma(\Upsilon' \rightarrow \eta\Upsilon)}{\Gamma(\psi' \rightarrow \eta\psi)} = \left(\frac{M_c}{M_b} \right)^4 \left(\frac{PS(\Upsilon)}{PS(\psi)} \right)^3 \approx \frac{1}{300}.$$

Results for ψ transitions are given in Table XIV, along with the expectations and measurements in the Υ system in Table XV. The agreement between data and theory is seen to be quite good.

Table XIV. Hadronic Transitions of Charmonium^[10]

Transition	Br(%)
$\psi' \rightarrow (\pi\pi)^n \psi$	50.3 ± 4.2
$\psi' \rightarrow (\pi^0\pi^0)\psi$	28.7 ± 3.0
$\psi' \rightarrow \eta\psi$	2.66 ± 0.44
$\psi' \rightarrow \pi^0\psi$	0.1 ± 0.03

Table XV. Hadronic Transitions of Bottomonium^[10]

Transition	Theory(%)	Experiment (%)
$\left\{ \begin{array}{l} \Upsilon \rightarrow \pi^+\pi^-\Upsilon \\ \Upsilon' \rightarrow \pi^0\pi^0\Upsilon \end{array} \right\}$	25-29	$\left\{ \begin{array}{l} 18.8 \pm 1.0 \\ 8.6 \pm 1.1 \end{array} \right\} 27.4 \pm 1.5$
$\Upsilon' \rightarrow \eta\Upsilon$	0.04%	$< 0.2 @ 90\% \text{ C.L.}$
$\Upsilon'' \rightarrow \pi^+\pi^-\Upsilon$	2-5	4.5 ± 0.8
$\Upsilon'' \rightarrow \pi^+\pi^-\Upsilon'$	2-3	3.1 ± 2.0

2.5.8 The $1P_1$ bottomonium state.

As was indicated in section 2.2, the additional phase space in the $b\bar{b}$ system may allow a sizeable rate (about 1%) for $\Upsilon(3S) \rightarrow \pi^+\pi^-\Upsilon(1P_1)$. The $1P_1$ state should lie at the COG of the χ_b states, at a mass of 9900.5 ± 1.3 .^[12] The recoil mass from opposite charge pion pairs at the $\Upsilon(3S)$ is shown in Fig. 19. Evidence^[12] for a narrow bump with 335 ± 135 events is seen, at a mass of 9894.8 ± 1.5 MeV. While only a 2.5σ effect, the peak lies close the COG of the χ_b states, and has a

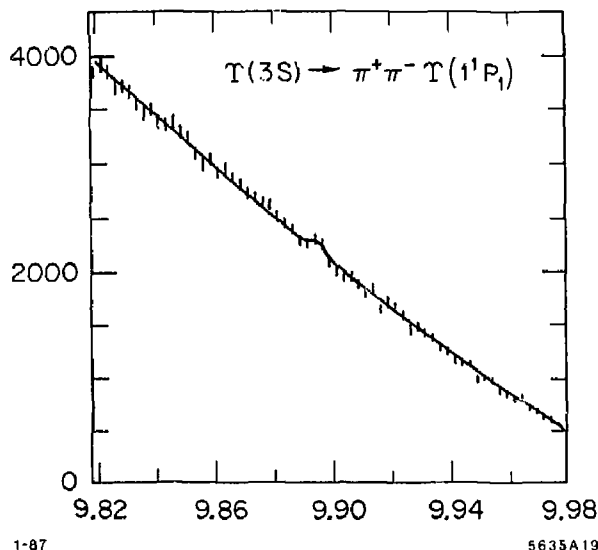


FIG. 19. Recoil mass from $\pi^+\pi^-$ pairs at the $\Upsilon(3S)$ from CLEO.

branching ratio of $0.37 \pm 0.15\%$, thus making it an excellent candidate for the 1^1P_1 state.

3. STATES OF EXCITED CHARM AND BEAUTY

I deal here with the data on the vector and orbitally excited states of charm and beauty mesons. This section lies naturally midway between the spectroscopy of the bound $c\bar{c}$ and $b\bar{b}$ states and the weak decays of the ground state charmed (cq) and b -mesons (bq), discussed in the subsequent section.

In the standard parton model, the light u , d , and s quarks are expected to combine with the heavier charmed (c) quark to form the three lowest lying pseudoscalar states: D^0 ($c\bar{u}$), D^+ ($c\bar{d}$) and D_s ($c\bar{s}$).^[141] Analogous B meson states also exist, denoted B_u , B_d and B_s . In addition the heavy state B_c should also exist. Spectroscopically, these correspond to the 1^1S_0 states. The D^0 , D^+ and B_d , B_u form isotopic doublets; the D_s , B_s and B_c are isosinglets. With the exception of the B_c , these states have been isolated in either e^+e^- annihilation, hadroproduction, photoproduction, or ν -scattering experiments. The masses^[110] ^[141] and lifetimes^[141] of the groundstates are summarized in Table XVI.

Table XVI. Ground and Excited States of Charmed Mesons^[110] ^[141] ^[141]

Charmed Meson	Quark Content	Mass GeV/c^2	Width (MeV) (Lifetime $\times 10^{-13}$)	J^P Assignment
D^0	$c\bar{u}$	1864.6 ± 0.6	$(4.43^{+2.19}_{-0.17})$	0^-
D^+	$c\bar{d}$	1869.3 ± 0.6	$(10.29^{+0.54}_{-0.43})$	0^-
D_s^+	$c\bar{s}$	1970.5 ± 2.5	$(3.85^{+0.65}_{-0.48})$	0^-
B_u	$b\bar{u}$	5277.6 ± 0.6	-	0^-
B_d	$b\bar{d}$	5280.4 ± 0.8	-	0^-
D^{*0}	$c\bar{u}$	2007.2 ± 2.1	≤ 5.0	1^-
D^{*+}	$c\bar{d}$	2010.1 ± 0.7	≤ 2.2	1^-
D_s^{*+}	$c\bar{s}$	2110 ± 6	≤ 22	1^-
B^*	$b\bar{q}$	$\Delta M \approx 0.052 \pm 0.005$	-	1^-
D^{**0}	$c\bar{u}$	2420 ± 6	70 ± 21	$1^+, 2^+$

Each ground state meson is expected to have a vector state ($^3S_1$) corresponding to the parallel alignment of its constituent quark spins. The D^{*0} and D^{*+} are now well established.^[39] The excited state of the D_s has only recently been established in e^+e^- annihilation,^{[40],[41]} through both its direct decay, and its associated production ($e^+e^- \rightarrow D_s \bar{D}_s^*$) near threshold. The B^* has only been seen indirectly through the gamma ray transition to the B_u or B_d meson.^[35]

As in the spectroscopy of light quark mesons, a set of orbitally excited charmed mesons is also expected with the lowest lying states having spectroscopic and quantum number assignments: $^1P_1$ (1^+) or ^3P_J ($0^+, 1^+$ and 2^+), and masses typically 500 MeV/c² higher than the ground states.^[41]

A typical set of mass splittings expected for bound $c\bar{q}$ states in both non-relativistic and relativistic potential models are indicated in Fig. 20. The first candidate for an orbitally excited state (D^{*10}) has only recently been observed.^[42] The experimental evidence for the state, is shown in Fig. 21.

The D^0 , D^{*+} , and D_s , being the lightest charmed mesons, and the B_u , B_d , B_s , and B_c being the lightest bottom mesons, must decay weakly through a charm-changing charged current. The details of these decays will encompass the greater part of the next lecture. The vector states D^{*0} , D^{*+} , and B^* decay strongly and electromagnetically to the ground states through $\pi^\pm$, π^0 , and γ emission. Some of these transitions (such as $D^{*0} \rightarrow \pi^- D^+$) are energetically forbidden (see Fig. 22). While all the decays have been measured, there are still discrepancies in the branching fractions, owing to the difficulty of the measurements. The charm-strange D_s^{*+} , being an isosinglet, cannot decay strongly to the D_s^+ via π^0 emission. The γ transition is uninhibited, and is expected to dominate the D_s^{*+} decay. The world average for the mass difference between the D_s^+ and D_s^{*+} is now measured to be $132 \pm 5 \pm 4$ MeV/c², forbidding an isospin-violating decay through π emission. The difference in squared masses between vector and pseudoscalar states for both the D , D_s , and B_q lie close to the constant found for all lighter mesons (see Table XVII) to be expected if the meson wavefunction at the origin is determined by the long range confining part of the potential.^[41]

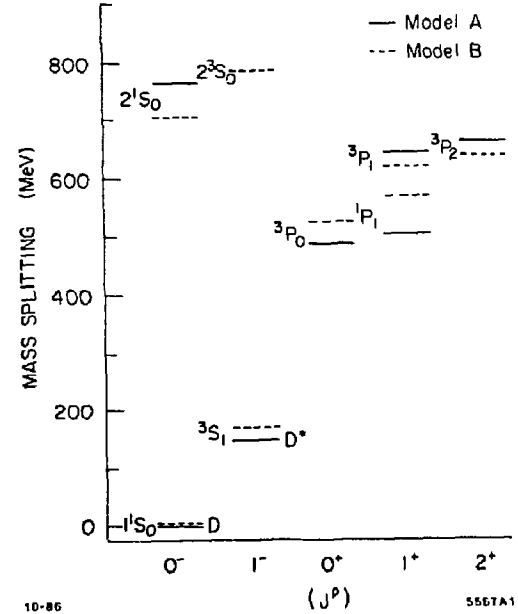


FIG. 20. Expected states for D and D_s mesons. Model A from Eichten *et al.*, Model B from Godfrey and Isgur, Ref [41].

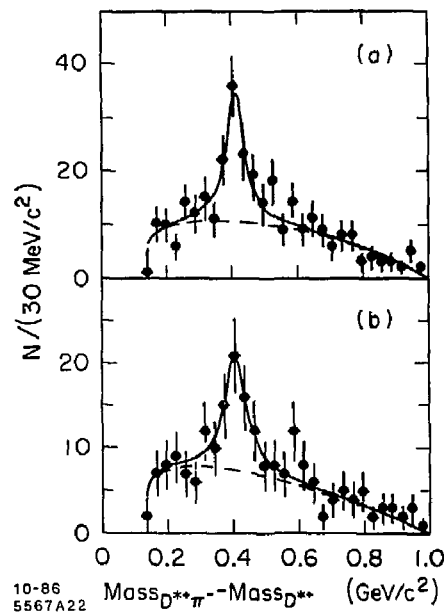


FIG. 21. D^{*0} candidate from Albrecht et al., Ref. [42].

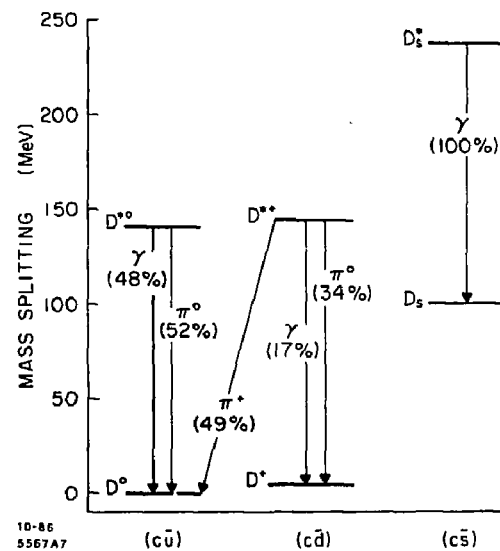


FIG. 22. Transitions of the lowest lying charmed vector mesons.

Table XVII. Difference in $(\text{Mass})^2$ for Pseudoscalar and Vector Mesons.⁽⁴⁴⁾

Mesons	$(\text{Mass})^2$ Difference
$\rho - \pi$	0.574
$K^* - K$	0.556
$D^* - D$	0.546
$D_s^* - D_s$	0.55 ± 0.01
$B^* - B$	0.55 ± 0.05

The lowest-lying charmed orbitally excited states are at sufficiently high masses to allow the possibility of strong π decays to both the ground states and the vector states from the $^1P_1$, $^3P_1$, and $^3P_2$ states. Parity conservation in the strong decay allows the $^3P_0$ to decay only to the ground state, through single π emission. Widths of 50 to 100 MeV/c² are expected for all these decays, making it difficult to distinguish the multiplet of states whose mass splitting should be comparable. Mixing between the singlet and triplet J=1 states may further complicate the picture. Multipion and other strong decays are also likely to occur for these states when energetically allowed. At present, the only candidate for one or more of these 1^+ or 2^+ states is the 70 MeV/c² wide resonance $D^{*0}(2420)$, decaying to $D^{*+}\pi^-$.⁽⁴⁵⁾ This state appears to play a significant role in charm fragmentation at high energies. One might also expect it to be present in B meson decays.

4. WEAK DECAYS OF CHARM AND BEAUTY MESONS

The following sections encompass the second lecture on heavy quark decays. I concentrate here on the issues surrounding weak hadronic and semileptonic decays of charm and beauty mesons. Much of the associated theory and phenomenology is available in these proceedings, in the lectures of F. Gilman, and the article of I.I.Y. Bigi.

4.1 Lifetimes of Charm and Beauty

In the most naive picture of heavy meson decay, all species of charm and beauty mesons have characteristic lifetimes associated with the weak decay of the

heavy quark within the meson:

$$\Gamma_i = \frac{G_F^2 M_Q^5}{192\pi^3} \times \frac{\text{Weak Mixing}}{\text{Angles}} \times \frac{QCD}{\text{Factors}} \quad (5)$$

$$\Gamma_{\text{tot}} = \frac{1}{\tau} = \sum \Gamma_i.$$

Here, the index i refers to any of the final states of the $W^\pm$ decay (see Fig. 23), and light quark masses are ignored.

The first evidence for differing lifetimes between charm meson species came from the semileptonic decays. The most recent and precise values available are shown in Table XVIII:

Table XVIII. Semileptonic Branching Ratios⁽⁴⁶⁾

Meson	$B_l(\%)$
D^0	$7.5 \pm 1.1 \pm 0.4$
D^+	$17.0 \pm 1.9 \pm 0.7$
D_s	unmeasured

If one assumes isospin symmetry for the semileptonic decays and that the Cabibbo suppressed decay widths are small, then a relation between the semileptonic branching ratios and the lifetimes of the charged and neutral mesons exist:

$$\frac{\tau^+}{\tau^0} = \frac{\Gamma_0}{\Gamma_+} = \frac{\Gamma_0^{SL}}{\Gamma_+^{SL}} / \frac{\Gamma_0^{SL}}{\Gamma_+^{SL}} = \frac{Br(D^+ \rightarrow l^+ + X)}{Br(D^0 \rightarrow l^+ + X)}. \quad (6)$$

The experimental results are summarized in Table XIX:

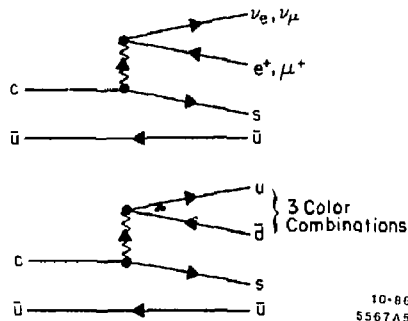


FIG. 23. Spectator model for heavy quark decays.

Table XIX. Lifetime Ratios Through Semileptonic Decays⁽⁴⁴⁵⁾⁽⁴⁴⁷⁾⁽⁴⁴⁸⁾

Quark System	Experiment	Lifetime Ratio (τ_c^l/τ_s^l)
Charm	DELCO	≥ 4.3 at 90% C.L.
Charm	MARK II	$3.1^{+4.2}_{-1.4}$
Charm	MARK III	$2.3^{+0.5}_{-0.4} \pm 0.1$
Beauty	CLEO	$2.3 \geq \tau_c^l/\tau_s^l \geq 0.49$

The measurements of MARK II and MARK III are *direct*, using events where one charmed particle is *tagged* in a hadronic decay thus separating charged and neutral mesons while the other decays semileptonically. The DELCO and CLEO results are not direct, as they count single and double leptons inclusively, and rely on knowing the fraction of charged and neutral mesons initially produced. Furthermore, as the mass of the quark increases, corrections for complex final states and cascades through lighter flavors and baryons make the semileptonic branching ratio determination more difficult and model dependent, even near the threshold for meson production.

Charmed meson and beauty-hadron lifetimes have also been measured more directly by geometrical techniques relying on the displacement of their decay vertex from the production vertex. In going from momenta of 1 to 10 GeV/c the decay length (λ) for a D meson changes from ~ 170 to $750 \mu\text{m}$ while for a B meson it goes from ~ 150 to $360 \mu\text{m}$. Two general strategies are in use. The *visual* techniques of bubble chambers, silicon strip detectors, and emulsions, and the primarily *collider*-based precision drift chamber technique. The visual techniques measure the decay length directly, having measurement errors $\delta\lambda \ll \lambda$. These devices are employable in fixed target experiments, where momenta in the lab are high, making multiple scattering effects less important.

The collider techniques rely on multiple measurements, each with $\delta\lambda \gg \lambda$, to achieve a statistical estimate of the decay length. A hybridization of these techniques is now emerging with the proposed use of silicon strip and CCD devices at higher energy colliders where the multiple scattering in these thicker devices poses less of a problem. The collider techniques break down further into two types. The full vertex reconstruction of final state (for example, a D^0 tagged through its

cascade $D^{*+} \rightarrow D^0 \pi^+$ with $D^0 \rightarrow K^- \pi^+$), or the statistical measurement of the impact parameter of one of the tagging particles in the decay (for example, the lepton in a semileptonic heavy quark decay). These techniques are described in more detail in other references.^[44] An example of the impact parameter technique from CLEO is shown in Fig. 24. Note the importance of good signal to background. One of the revolutions in such measurements for the charmed mesons has recently occurred with the Fermilab experiment E691 (TPS). This experiment used silicon microstrip detectors just beyond a Be target, and followed by an elaborate spectrometer with good particle identification. The apparatus resided in a 260 GeV/c tagged photon beam, wherein charm production is thought to proceed largely through gamma-gluon fusion. Typical decay lengths are mm to cm, making lifetime measurements rather simple and bias free. The typical mass spectra for charmed meson decays, with their decay length curves, are shown in Fig. 25. Table XX summarizes the most recent measurements of charm lifetimes.

The averages from this table are largely dominated by E691:

$$\begin{aligned}
 \tau_{D^0} &= 4.34_{-0.22}^{+0.24} \times 10^{-13} \\
 \tau_{D^{*+}} &= 10.1_{-0.6}^{+0.7} \times 10^{-13} \\
 \tau_{D_s^+} &= 3.5_{-0.6}^{+0.8} \times 10^{-13} \\
 \frac{\tau_{D^{*+}}}{\tau_{D^0}} &= 2.33 \pm 0.21 \\
 \frac{\tau_{D_s^+}}{\tau_{D^0}} &= 0.81 \pm 0.19 .
 \end{aligned} \tag{7}$$

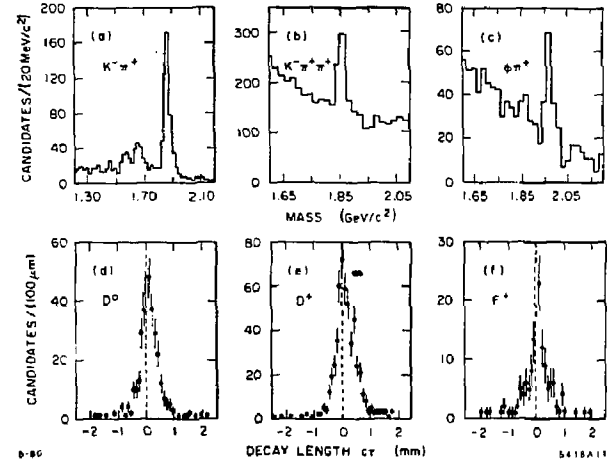


FIG. 24. Impact technique used to measure lifetimes of charmed particles from CLEO.

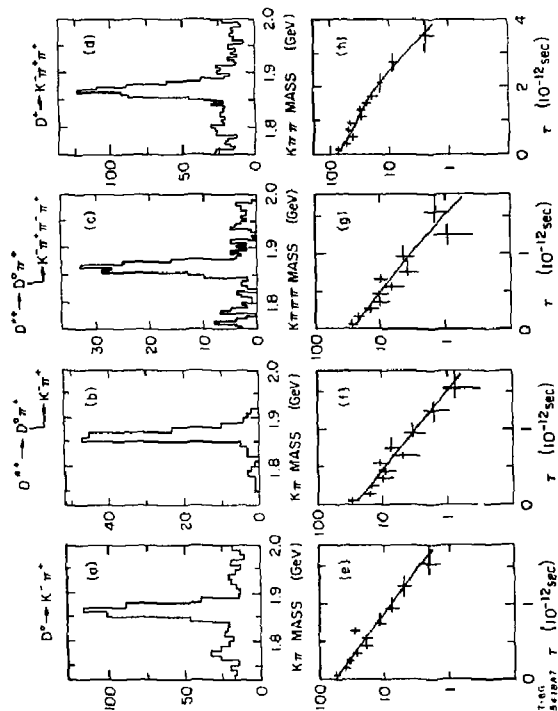


FIG. 25. E691 results for D decays.

Table XX. Summary of Lifetime Measurements[†]

Expt.		D^0		D^{*+}		D_s^{*+}
E691	672	$4.4 \pm 0.2 \pm 0.2$	480	$10.9 \pm 0.8 \pm 0.6$	35	$4.2 \pm 0.9 \pm 0.6$
CLEO	317	$5.0 \pm 0.7 \pm 0.4$	247	$11.4 \pm 1.6 \pm 1.0$		
DELCO	269	$4.6 \pm 1.5 \pm 0.7$ 0.6				
MKII	66	$4.7 \pm 0.9 \pm 0.5$	16	$8.9 \pm 3.8 \pm 1.3$ 2.7		
E531	58	$4.3 \pm 0.7 \pm 0.1$ 0.2	23	$11.1^{+4.4}_{-2.9}$	6	2.6 ± 1.0 1.1
SHF	50	$6.1 \pm 0.9 \pm 0.3$	48	$8.6 \pm 1.3 \pm 0.7$ 0.3		
WA58	44	$3.4 \pm 1.2 \pm 0.7$ 0.8	27	$5.0 \pm 1.5 \pm 1.9$ 1.0		
NA16	16	$4.1 \pm 1.3 \pm 0.9$	15	$8.4 \pm 3.5 \pm 2.2$		
NA27	129	$4.2 \pm 0.5 \pm 0.4$	147	$10.6 \pm 1.3 \pm 0.9$		
NA11	26	$3.7 \pm 1.0 \pm 0.5$ 0.7	28	$10.6 \pm 3.6 \pm 1.6$ 2.4	12	$3.1 \pm 1.2 \pm 0.8$
TASSO	13	$4.3 \pm 2.0 \pm 0.8$			7	$3.4 \pm 2.0 \pm 1.6 \pm 0.7$
HRS	53	$4.2 \pm 0.9 \pm 0.6$	114	$8.1 \pm 1.2 \pm 1.6$	13	$3.5 \pm 2.4 \pm 1.8 \pm 0.9$
NA1	51	$4.3 \pm 1.4 \pm 0.9$	98	$9.5 \pm 3.1 \pm 1.9$		

[†] Compiled by V. Lüth, reference 38.

The lifetime ratio evaluated by measuring the individual species is notably close to that obtained by the semileptonic decay ratios of charm in Table XIX, suggesting that the assumptions of Eq. (6) are valid for charmed mesons.

Unlike the D mesons, no individual measurements of B_u or B_d have yet been performed. It is likely⁽¹⁰⁾ that the difference in lifetimes will be less pronounced, owing to the smaller size of the QCD corrections in B -decay. Average measurements of B hadron lifetimes have been made by use of the impact parameter technique on leptons in events that have been topologically selected to be enriched in $b\bar{b}$ quarks. These techniques have been extended to use hadrons from the b -quark fragmentation as well. Table XXI summarizes recent results.⁽¹¹⁾

Table XXI. B-Hadron Mean Lifetimes

Experiment	B-hadron Lifetime
JADE	$1.8^{+0.1}_{-0.3} \pm 0.4$
MAC	$1.16^{+0.17}_{-0.17} \pm 0.07$
MKIIa	$0.85^{+0.17}_{-0.17} \pm .21$
MKIIb	$1.25^{+0.28}_{-0.19} \pm .50$
DELCO	$1.17^{+0.27+0.17}_{-0.22-0.16}$
TASSO	$1.83^{+0.38+0.17}_{-0.37-0.16}$

The average value is thus ~ 1.2 ps for the B-hadron, a remarkably long lifetime when compared to the naive expectation of Eq. (5), suggesting the severe reduction of the width due to the size of the Kobayashi-Maskawa(KM) parameter. This value along with the average semileptonic branching ratio for B-hadrons can be used to extract limits on the KM matrix elements V_{bu} and V_{bc} .^[42] These however rely on the assumptions that B-hadrons all have similar lifetimes and semileptonic branching fractions. The measurements of Table XXI are all at similar energies and thus have similar admixtures of B-hadrons. Measurements of average semileptonic branching ratios have been made both from mixtures of B_u and B_d at the $T(4S)$ (determined to $\sim 20\%$) and from higher energy data containing unknown B-hadron admixtures (where the lifetime measurements were obtained). The results are in rough agreement,^[43] and give average semileptonic and semimuonic branching ratios of about $11 \pm 1\%$, thus suggesting that the average lifetime of all species are consistent with one another at this level.

4.2 Patterns of Hadronic and Semileptonic Weak Decays

The remainder of the lecture will concern the hadronic and semileptonic decay measurements of heavy quark systems. Emphasis will be placed on how they can be used to understand the difference in charm lifetimes as well as predict the size of effects in the B meson system.

4.2.1 Contributions to charm and beauty meson widths.

The tree level diagrams thought to contribute to heavy meson decays are summarized in Fig. 26. Both charm and beauty have principle decays through the

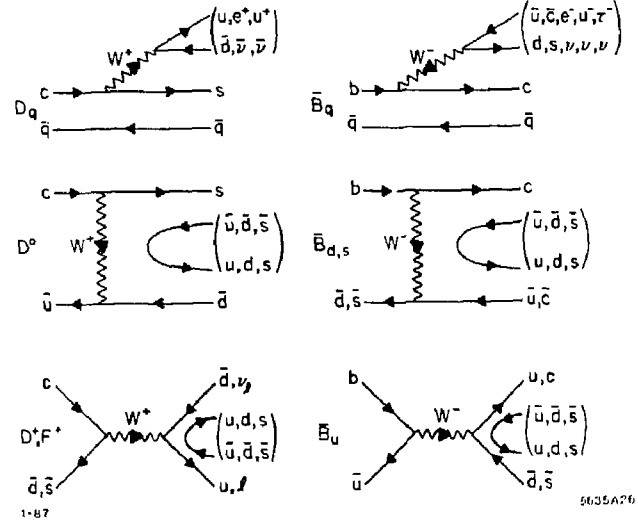


FIG. 26. Principle diagrams leading to charm and beauty meson decays.

so-called spectator graphs. In the spectator diagram, the light quark of the meson is passive. The D meson and B meson decay is similar, except that additional channels for $\bar{B} \rightarrow \tau^- X$ and $\bar{B} \rightarrow c\bar{q}(\bar{s})$ are possible. This naive model would predict semileptonic branching fractions of about 20% for charm and 17% for beauty.

The QCD corrections to the non-leptonic decays modify Eq. (5), and account for hard gluon exchanges among the quarks:

$$\Gamma_i = \frac{G_F^2 M_Q^6}{192\pi^3} \times \left\{ \frac{\text{Weak Mixing}}{\text{Angles}} \right\} \times \frac{2c_+^2 + c_-^2}{3}. \quad (8)$$

Here, c_+ and c_- are the so called Wilson Coefficients. They are manifestly a function of the strong coupling constant (α_s) and are hence a function of the mass scale of the interaction. The two coefficients are not independent, being related through the expression $c_- \approx \sqrt{1/c_+}$. The dependence of the coefficients on the mass scale are indicated in Fig. 27. At infinite masses, or equivalently in the regime of free-quarks, the coefficients go towards unity, recovering Eq. (5). At lower masses, the effect is to enhance the nonleptonic contribution, and thus diminish the semileptonic width. It has been shown that the coefficients, when calculated to next order, continue to move in this direction, although the enhancement appears to diminish (see Fig. 27). For the charm system $c_- \approx 2.3$ while for the beauty system $c_- \approx 1.6$.

The effect of this nonleptonic enhancement on the semileptonic branching fraction (B_l) is shown schematically in Fig. 28. It is calculated from the simple expression:

$$Br(c \rightarrow e + X) = \frac{1}{2 + 2c_+^2 + c_-^2} \quad (9)$$

Here, the 2 in the denominator comes from equal contributions from semileptonic muon and electron contributions. At the tree level in B-decay the $b \rightarrow c\ell\bar{\nu}_\ell$ also contributes in the denominator, as do $b \rightarrow c\bar{s}$ transitions, modifying this slightly. The effect is small because of limited phase space. For charm, the nominal ratio of c_-/c_+ is about 3, while for beauty it is somewhat lower (~ 2); for top mesons, it would approach unity. These values lead to semileptonic branching fractions being reduced to about 14% for charm, and about 15% for beauty, *without regard to the flavor of the spectator quark*.

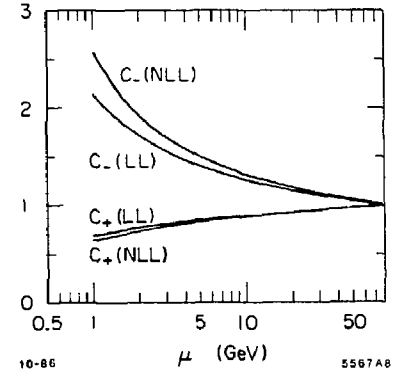


FIG. 27. The Q^2 dependence of the Wilson coefficients in leading and next-to-leading log approximation. The calculation is only up through the charm-flavor.

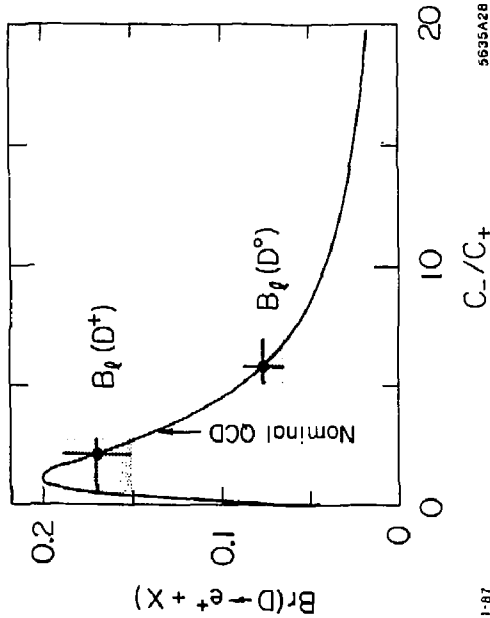


FIG. 28. The semileptonic branching fraction for charm versus the ratio of coefficients c_-/c_+ .

These crude estimates when view along with the data are however very enlightening and point to the need for a more sophisticated treatment of the problem. First, they predict average values of B_l which disagree with average values from experiments (about 11% for charm and beauty both). Second, they do not account for the difference between D^0 and D^+ semileptonic branching fractions, nor will they do so when the difference between B_u and B_d is actually measured. To understand the difference, it is necessary to look at differences which might arise in the weak hadronic sector. Two principle mechanisms have been proposed to understand the observed differences in hadronic widths between the D^0 and the D^+ . These are W-exchange or W-annihilation, and Pauli interference, both operative in the charm and beauty system. These mechanisms are depicted in Fig. 29 and Fig. 30.

4.2.2 Weak flavor annihilation.

The most direct way to enhance the D^0 , the D_s , or the B^0 is to add additional diagrams denoted as W-exchange and W-annihilation, respectively.^[16] The W-annihilation graph is also present for Cabibbo-suppressed D^+ and B^+ decays. These graphs historically have been ignored because at the quark level they are helicity suppressed ($\propto \frac{M_q^2}{M_D^2}$) and require a large wavefunction overlap of initial state quarks ($\propto \frac{f_B}{M_D^2}$, $\propto \frac{f_B}{M_D^2}$ or $\propto \frac{f_B}{M_D^2}$):

$$\Gamma_{EXCH}^D = \frac{G_F^2}{8\pi} (m_{q_1}^2 + m_{q_2}^2) M_D f_D^2 \frac{(2c_+ - c_-)^2}{3},$$

$$\Gamma_{ANN}^D = \frac{G_F^2}{8\pi} (m_{q_1}^2 + m_{q_2}^2) M_D f_D^2 \frac{(2c_+ + c_-)^2}{3} \sin^2(\theta_2).$$

It has been argued that the helicity suppression may be removed by the presence of gluons in the meson wavefunction,^[15] or by the radiation of gluons from the light quark vertex.^[16] The former is largely a non-perturbative effect, the latter, perturbative. This leaves the wavefunction overlap factor which is expected to be small owing to the small values (~ 150 MeV/c) of f_D , f_{D_s} and f_B .^[17]

Recent work^[18] suggests that a dynamical mechanism such as the presence of a resonance with quantum numbers equal to that of a $\bar{K}$ and mass close to the D^0 , could also enhance the annihilation contribution to charm decays. Such a

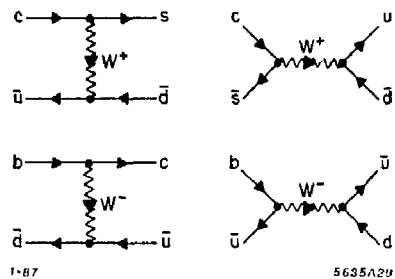


FIG. 29. W-exchange and W-annihilation in D and B decays.

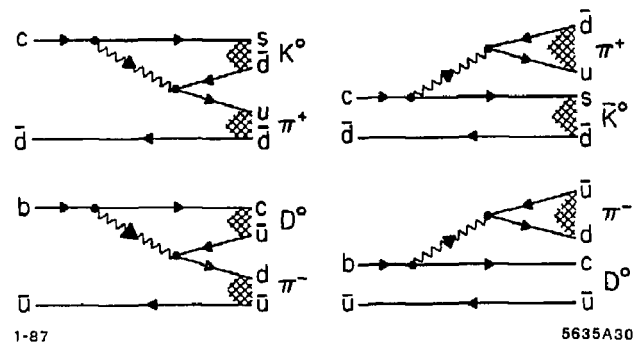


FIG. 30. Pauli interference in B and D decay.

mechanism is unlikely to be present for B mesons, as their higher masses place them out of the light quark resonance region.

Experimentally, certain decays of the D^0 , such as $D^0 \rightarrow \bar{K}^0 \phi$, $\bar{K}^0 K^0$, and $\bar{K}^0 K^{*0}$, should be clear signatures for W-exchange.^[131] Here, the $\bar{u}$ quark of the initial state is absent in the final state meson. For the D_s meson, final states with no net strangeness and no $s\bar{s}$ content (such as $\rho\pi$), would be characteristic of W-annihilation. Recent work^{[132][133]} however has suggested that rescattering effects, or non-planar diagrams (see Fig. 31) may lead to final states that mimic the non-spectator decays. Flavor annihilation $u \rightarrow \bar{d}$ occurs through the strong interaction, rather than the weak one. The possibility of rescattering being significant is increased when the channels through which rescattering is to occur, are themselves many times larger than the final states in question. The situation will remain unresolved until there is a substantial increase in the world data.^{[133][134]}

4.2.3 Pauli interference.

The D^+ and B_c can receive enhancement in its Cabibbo-suppressed decays through W-annihilation diagrams with the caveats of the previous sections. More importantly, the leading D^+ and B_c Cabibbo-allowed decays may be suppressed by cancellation of final state amplitudes in the presence of strong color clustering and QCD sextet enhancement.^[135] Figure 30 indicates how color clustering leads to identical final state amplitudes which interfere in the D^+ due to the relative minus sign. To the extent that the coefficient $c_- \gg c_+$, a cancellation can occur for pseudoscalar-pseudoscalar decays, while pseudoscalar-vector decays may be enhanced.^[136]

The interference can also arise in charm (beauty) decays at the quark level, before hadronisation, from the presence of two identical $d(u)$ quarks in the final state. In the case of charm, for example, the D^+ width then receives an extra term:¹³

$$\Gamma^{\text{int}}(D^+) = -(c_-^2 - 2c_+^2) \frac{12\pi^2}{M_D^2} f_D^2 F_0. \quad (10)$$

This term is negative for $c_- \gg c_+$. More detailed calculations (e.g. potential and bag models) show that the effect of interference in charm decays ranges from a few percent to as large as $\approx 50\%$ and may thus account for much of the D^0 and

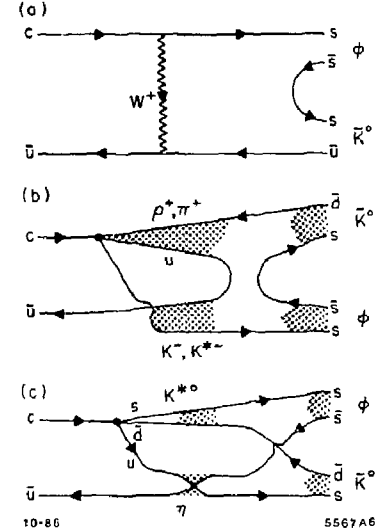


FIG. 31. (a) W-exchange leading to $\bar{K}^0 \phi$, and (b)-(c) Non-planar diagrams simulating the same D^0 W-exchange final state.

D^+ lifetime difference.^[9] The effect in the B system should be present but smaller owing to the larger mass and slightly smaller value of c_- .

4.2.4 Color suppression and the role of gluons

One final effect that is of theoretical interest, is the role that soft, non-perturbatively treated gluons may play in heavy meson decay. The next-to-leading-log calculation^[10] of additional gluons leads to corrections which are small (see Fig. 27). Soft, non-perturbative gluons may however play an important role as pointed out in Section 4.2.2 controlling the degree of W-exchange and W-annihilation as well as the overall level of nonleptonic enhancement. Early attempts to calculate hadronic matrix elements^[11] led to predictions which were very sensitive to the QCD corrections. An example of the calculation of the ratio of $\Gamma(\bar{D}^0 \rightarrow \bar{K}^0 \pi^0) / \Gamma(D^0 \rightarrow K^- \pi^+)$ is shown in Fig. 32, where a very sharp minimum is seen close to the nominal QCD values for c_-/c_+ . This has been frequently referred to as *color suppression*, and would occur in a similar fashion for decays like $D^0 \rightarrow \bar{K}^{*0} \pi^0$, $D^+ \rightarrow \phi \pi^+$ and $B^0 \rightarrow \bar{K}^0 \psi$. The origin of the effect is seen in Fig. 33 where the color matching naively reduces amplitude (a) by 3 relative to amplitude (b). Isospin accounts for a factor of $1/\sqrt{2}$ and QCD further reduces the relative rate to as little as $\sim 1/40$.

A naive way to reduce color suppression, is to evaluate the Wilson coefficients at a smaller mass scale, such that c_-/c_+ is considerably greater. In essence, this approach can be interpreted as an attempt to increase the non-perturbative contributions beyond the QCD expectation. While this is an ad-hoc approach, it simultaneously reduces the theoretical estimate of the semileptonic branching ratios for D^0 and D^+ , leaving at least the D^0 closer to experiment.

Recent work in calculating hadronic matrix elements has removed the singularities associated with color matching, through the introduction of an additional parameter (ξ) the so-called color screening parameter.^[12] This parameter has some phenomenological basis, being related to the QCD $1/N$ expansion^[13] (N is the number of colors). Fitting of data (described in the following sections) yields a value of $\xi \approx 0$ instead of the naive $1/3$. This relaxes color suppression in all channels and provides a reasonably good description of the data.^[12]

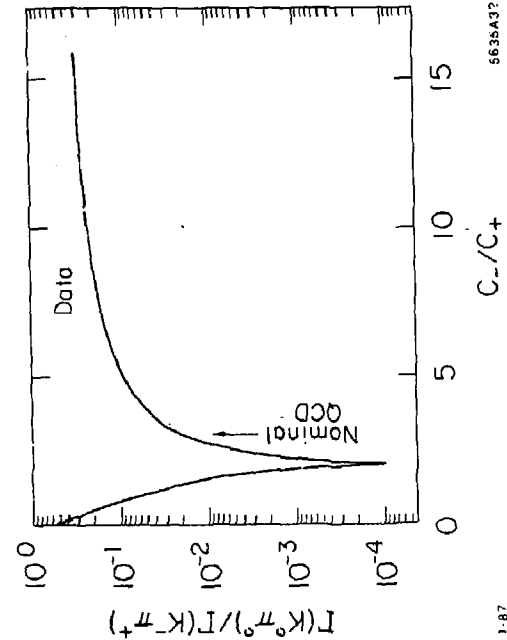


FIG. 32. Prediction for the ratio of $D^0 \rightarrow \bar{K}^0 \pi^0$ to $D^0 \rightarrow K^- \pi^+$.

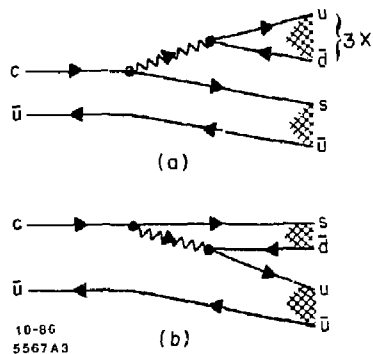


FIG. 33. Origin of color suppression in the spectator model.

4.3 Data on Charmed Meson Decays

A significant fraction of the Cabibbo-allowed and Cabibbo-suppressed decays of D^0 and D^+ have now been measured. The bulk of the information comes from e^+e^- storage ring experiments at the $\psi(3770)$ resonance. Working at slightly higher energies, information on D_s decays has been obtained. Some more recent measurements are coming out high energy machines like CESR, PEP and PETRA, where the extra Lorentz boost improves detection efficiencies.

This was instrumental in the discovery of the D_s and certain rare D^0 decays, and the measurements of the D_s lifetime. Finally, new data is expected in the near future from the photoproduction experiment E691, which may serve to "close the book" on many (but probably not all) the issues of charmed D and D_s decay.

4.3.1 Data on hadronic decays of charmed D^0 , D^+ and D_s

Let us first summarize the experimental data available now on charmed meson rates. ^{[193] [174] [175] [176] [177]} Table XXII summarizes the Cabibbo-allowed decays of the D^0 and D^+ with their production cross section times branching ratio ($\sigma \cdot B$) at the $\psi(3770)$ from the major experiments.

Table XXII. Cabibbo-Allowed Decays of D Mesons
 $\sigma \cdot Br(nb)$ at $\sqrt{s} = 3.77$ GeV

Decay Channel	MARK III ^[174,175]	MARK II ^[176]	LGW ^[177]
$D^0 \rightarrow K^- \pi^+$	$0.25 \pm 0.01 \pm 0.01$	0.24 ± 0.02	0.25 ± 0.03
$\bar{K}^0 \pi^0$	$0.11 \pm 0.02 \pm 0.01$	0.16 ± 0.08	-
$K^0 \eta$	$0.09 \pm 0.04 \pm 0.01$	-	-
$\bar{K}^0 \omega$	$0.19 \pm 0.07 \pm 0.05$	-	-
$\bar{K}^0 \phi$	$0.05^{+0.03+0.02}_{-0.02-0.01}$	-	-
$K^- \pi^+ \pi^0$	$0.76 \pm 0.04 \pm 0.08$	0.68 ± 0.23	1.4 ± 0.6
$\bar{K}^0 \pi^+ \pi^-$	$0.37 \pm 0.03 \pm 0.03$	0.30 ± 0.08	0.46 ± 0.12
$\bar{K}^0 K^+ K^-$	$0.05^{+0.02+0.01}_{-0.01-0.01}$	-	-
$K^- \pi^+ \pi^+ \pi^-$	$0.53 \pm 0.03 \pm 0.05$	0.69 ± 0.11	0.36 ± 0.11
$\bar{K}^0 \pi^+ \pi^- \pi^0$	$0.67 \pm 0.11 \pm 0.15$	-	-
$D^+ \rightarrow \bar{K}^0 \pi^+$	$0.14 \pm 0.01 \pm 0.01$	0.14 ± 0.03	0.14 ± 0.05
$K^- \pi^+ \pi^+$	$0.39 \pm 0.01 \pm 0.03$	0.38 ± 0.05	0.36 ± 0.06
$\bar{K}^0 \pi^+ \pi^0$	$0.42 \pm 0.08 \pm 0.08$	0.78 ± 0.48	-
$\bar{K}^0 \pi^+ \pi^+ \pi^-$	$0.31 \pm 0.03 \pm 0.03$	0.51 ± 0.18	-
$K^- \pi^+ \pi^+ \pi^0$	$0.18 \pm 0.04 \pm 0.04$	-	-
$K^- \pi^+ \pi^+ \pi^- \pi^+$	-	≤ 0.23 at 90% CL	-

Detailed measurements of the pseudoscalar vector decays of the D mesons are summarized in Table XXIII. ^{[174] [176]} It is seen from the table the predominance of quasi two-body decays of the D^0 and D^+ .

Table XXIII. Pseudoscalar-Vector Content of the
Three-body Cabibbo-Allowed Modes[†]
 $\sigma \cdot Br(nb)$ at $\sqrt{s} = 3.77$ GeV^{[174] [176]}

Channel	Fraction(%)	$\sigma \cdot Br(nb)$
$D^0 \rightarrow K^- \pi^+ \pi^0$		
$K^- \rho^+$	$74.0 \pm 4.6 \pm 5.0$	$0.56 \pm 0.05 \pm 0.07$
$K^+ \pi^+$	$12.9 \pm 2.7 \pm 2.0$	$0.30 \pm 0.06 \pm 0.06$
$\bar{K}^0 \pi^0$	$7.6 \pm 3.3 \pm 2.0$	$0.09 \pm 0.04 \pm 0.03$
nonresonant	$5.5 \pm 4.4 \pm 3.0$	$0.04 \pm 0.03 \pm 0.02$
$D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$		
$\bar{K}^0 \rho^0$	$16.8 \pm 5.3 \pm 2.5$	$0.06 \pm 0.02 \pm 0.01$
$K^+ \pi^+$	$63.9 \pm 7.6 \pm 4.5$	$0.36 \pm 0.05 \pm 0.04$
nonresonant	$19.3 \pm 8.6 \pm 3.5$	$0.07 \pm 0.03 \pm 0.01$
$D^+ \rightarrow \bar{K}^0 \pi^+ \pi^0$		
$\bar{K}^0 \rho^+$	$86.5 \pm 9.1 \pm 5.0$	$0.37 \pm 0.08 \pm 0.07$
$K^0 \pi^+$	$7.0 \pm 4.3 \pm 4.0$	$0.09 \pm 0.06 \pm 0.06$
nonresonant	$6.5 \pm 5.5 \pm 4.0$	$0.03 \pm 0.02 \pm 0.02$

[†] These results are preliminary.

Finally, Table XXIV contains information of the magnitude of many Cabibbo-forbidden decays. The ratios are quoted here to reduce systematic errors and thus allow more precise comparison with theoretical models.

Table XXIV. Cabibbo-Suppressed Decays of D Mesons
Relative Rates and Br(%)^{[17][18][19]}

Decay Channel	Ratio
D⁰ Decays	
$\frac{\Gamma(\pi^+\pi^+)}{\Gamma(K^-\pi^+)}$	$0.033 \pm 0.010 \pm 0.006$
$\frac{\Gamma(K^-K^+)}{\Gamma(K^-\pi^+)}$	$0.122 \pm 0.018 \pm 0.012$
$\frac{\Gamma(K^0K^0)}{\Gamma(K^-\pi^+)}$	≤ 0.11 at 90% C.L.
$\frac{\Gamma(K^{*0}K^0+cc)}{\Gamma(K^{*0}\pi^+)+\Gamma(K^-\pi^+)}$	≤ 0.034 at 90% C.L.
$\frac{\Gamma(K^{*0}K^+\pi^+cc)}{\Gamma(K^{*0}\pi^+)+\Gamma(K^-\pi^+)}$	0.05 ± 0.03
$\frac{\Gamma(\pi^+\pi^+\pi^0)}{\Gamma(D^0\pi^+\pi^0)}$	$0.011 \pm 0.004 \pm 0.002$
$\frac{\Gamma(\pi^+\pi^+\pi^+\pi^-)}{\Gamma(D^0\pi^+\pi^0)}$	$0.015 \pm 0.006 \pm 0.002$
D⁺ Decays	
$\frac{\Gamma(K^0K^+)}{\Gamma(K^+\pi^+)}$	$0.317 \pm 0.086 \pm 0.048$
$\frac{\Gamma(\pi^+\pi^0)}{\Gamma(K^+\pi^+)}$	≤ 0.15 at 90% C.L.
$\frac{\Gamma(\pi^+\pi^+\pi^+)}{\Gamma(K^-\pi^+\pi^0)}$	$0.042 \pm 0.016 \pm 0.010$
$\frac{\Gamma(K^-K^+\pi^+)}{\Gamma(K^-\pi^+\pi^+)}$	$0.059 \pm 0.026 \pm 0.009$
$\frac{\Gamma(\phi\pi^+)}{\Gamma(K^-\pi^+\pi^+)}$	$0.084 \pm 0.021 \pm 0.011$
$\frac{\Gamma(K^{*0}K^+)}{\Gamma(K^-\pi^+\pi^+)}$	$0.048 \pm 0.021 \pm 0.011$

Very little data on D_s decays is available; In Table XXV are listed the observed decays of the D_s from both hadroproduction and e^+e^- experiments.^{[14][15][16][17]}

Table XXV. Decays of the D_s Mesons^{[14][15][16][17]}
(†) indicates a preliminary result

Experiment	$\sqrt{s}$ (GeV)	Beam	Channel	Mass (MeV/c ²)
CLEO	10	e^+e^-	$\phi\pi^+$	$1970 \pm 5 \pm 5$
TASSO	14-25	e^+e^-	$\phi\pi^+$	$1975 \pm 9 \pm 10$
ARGUS	10	e^+e^-	$\phi\pi^+$	$1973 \pm 3 \pm 3$
ARGUS	10	e^+e^-	$\phi\pi^+\pi^-\pi^+$	$1976 \pm 5 \pm 3$
TPC	29	e^+e^-	$K^-K^+\pi^+$	$1948 \pm 28 \pm 10$
HRS	29	e^+e^-	$\phi\pi^+$	$1963 \pm 3 \pm 3$
ACCMOR	200	$(\pi, N), (K, N)$	$K^-K^+\pi^+$	1975 ± 4
ARGUS	10	e^+e^-	$\bar{K}^{*0}K^+$	<i>seen</i>
MARK III	4.14	e^+e^-	$\phi\pi^+$	$1973 \pm 4 \pm 4^\dagger$
MARK III	4.14	e^+e^-	$\bar{K}^{*0}K^+$	<i>seen</i>
MARK III	4.14	e^+e^-	$\bar{K}^0K^+$	<i>seen</i>
TPS	260	γN	$\phi\pi^+$	<i>seen</i>
TPS	260	γN	$\bar{K}^{*0}K^+$	<i>seen</i>

4.3.2 Data on semileptonic and pure leptonic D^0 and D^+ decays

In a previous section we have discussed the inclusive semileptonic decays of charm. Recently, a number of new measurements have been made on the pure leptonic and exclusive semileptonic decays of D mesons. First, by use of tagging, it is possible to look at the exclusive decay modes of semileptonic D-decay. These are expected to be dominated by $D \rightarrow KI^+\nu_l$ and $D \rightarrow K^*I^+\nu_l$.^[14] Table XXVI summarizes these measurements:

Table XXVI. Exclusive Semileptonic Decays

Decay Mode	Events Sig. (Bkd.)	Branching Ratio (%)
$D^0 \rightarrow K^- e^+ \nu_e$	47 (2.1)	$3.9^{+0.6}_{-0.6} \pm 0.6$
$D^0 \rightarrow K^- \pi^0 e^+ \nu_e$	7 (1.1)	$1.7^{+0.9}_{-0.7} \pm 0.6$
$D^0 \rightarrow \bar{K}^0 \pi^- e^+ \nu_e$	9 (0.7)	$2.2^{+0.9}_{-0.7} \pm 0.4$
$D^0 \rightarrow \pi^- e^+ \nu_e$	3 (0.9)	$0.4^{+0.4}_{-0.3} \pm 0.1$
$D^0 \rightarrow K^- \mu^+ \nu_\mu$	56 (9.4)	$4.1^{+0.7}_{-0.7} \pm 1.2$
$D^0 \rightarrow \bar{K}^0 \pi^- \mu^+ \nu_\mu$	20 (8.5)	$2.7^{+1.1}_{-1.0} \pm 1.6$
$D^+ \rightarrow \bar{K}^0 e^+ \nu_e$	15 (1.1)	$6.3^{+2.0}_{-1.6} \pm 1.1$
$D^+ \rightarrow K^- \pi^+ e^+ \nu_e$	24 (1.2)	$3.9^{+0.9}_{-0.8} \pm 0.7$
$D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$	37 (8.9)	$10.2^{+2.2}_{-2.1} \pm 3.6$

One interesting feature of this data is that the D_{14} decays appear only partially consistent with pure K^* ; there being some room in the fit for a non-resonant component. This has been estimated to be about 45% of all $K\pi\nu_l$,^[15] although it is based on a small number of events beyond the K^* , as shown in Fig. 34.

Another recent (preliminary) measurement is the $Br(D^+ \rightarrow \mu^+ \nu_\mu)$. This pure leptonic decay is expected at a rate governed by the decay constant f_D . The decay constant may thus be unambiguously measured by observing this decay:

$$\Gamma_{D^+ \rightarrow \mu^+ \nu} = \frac{G_F^2}{8\pi} f_D^2 m_D m_\mu^2 |V_{cd}|^2 (1 - (m_\mu/m_D)^2)^2.$$

The data^[16] gives a 90% C.L. upper limit on the branching ratio of 8.4×10^{-4} . Using a D^+ lifetime of $(10.1^{+0.7}_{-0.6}) \times 10^{-13}$ s,^[17] and $|V_{cd}|^2 = 0.0506 \pm .0065$,^[18]

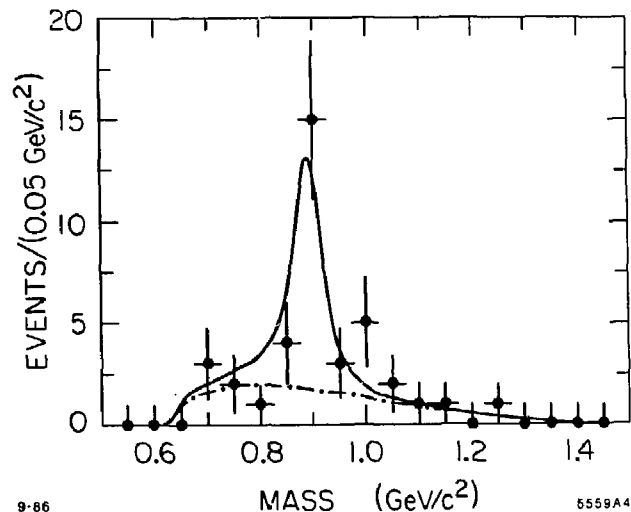


FIG. 34. Fit to $K\pi$ system in D_{e4} decays,^[15] indicating the possibility of both K^* and non-resonant contribution ns.

then the 90 % C.L. branching ratio limit corresponds to $f_D = 310 \text{ MeV}/c^2$. When the errors on τ_{D^+} and $|V_{cd}|^2$ are included, a 90 % C.L. upper limit on f_D of $340 \text{ MeV}/c^2$ is obtained (see Fig. 35).

4.4 Interpretation of the Charm Meson Data

The data on exclusive charm decays is seen to be rather rich, allowing us to address many of the theoretical questions posed in Section 4.2.

4.4.1 Color-suppression.

First we see that color-suppression or color-matching expected from the simplest QCD calculation of hadronic matrix elements appears to be largely absent in both D and D_s decay. From the previous tables we can extract:

$$\frac{\Gamma(D^0 \rightarrow K^0 \pi^0)}{\Gamma(D^0 \rightarrow K^- \pi^+)} = 0.45 \pm 0.08 \pm 0.05$$

$$\frac{\Gamma(D^0 \rightarrow K^{*0} \pi^0)}{\Gamma(D^0 \rightarrow K^{*-} \pi^+)} = 0.29 \pm 0.14 \pm 0.09$$

$$\frac{\Gamma(D^0 \rightarrow K^0 \rho^0)}{\Gamma(D^0 \rightarrow K^- \rho^+)} = 0.11 \pm 0.04 \pm 0.02$$

$$\frac{\Gamma(D^+ \rightarrow \phi \pi^+)}{\Gamma(D^+ \rightarrow K^- \pi^+ \pi^+)} = 0.08 \pm 0.02 \pm 0.01$$

$$\frac{\Gamma(D_s^+ \rightarrow K_s^0 K^+)}{\Gamma(D_s^+ \rightarrow \phi \pi^+)} = 0.44 \pm 0.12 \pm 0.21$$

In no instance is a significant suppression observed for color mismatched decays. It may be argued that final state interactions may play a significant role in D decays.^[10] However, in *all* cases, the suppression expected from the naive spectator model is not present. In particular, since this appears to be true for both D^0 , D^+ , and D_s decays one must seek a common explanation for the effect. It seems unlikely that a conspiracy of final state interactions produces the effect. As noted earlier, the ad-hoc approach of increasing c_-/c_+ would also largely remove

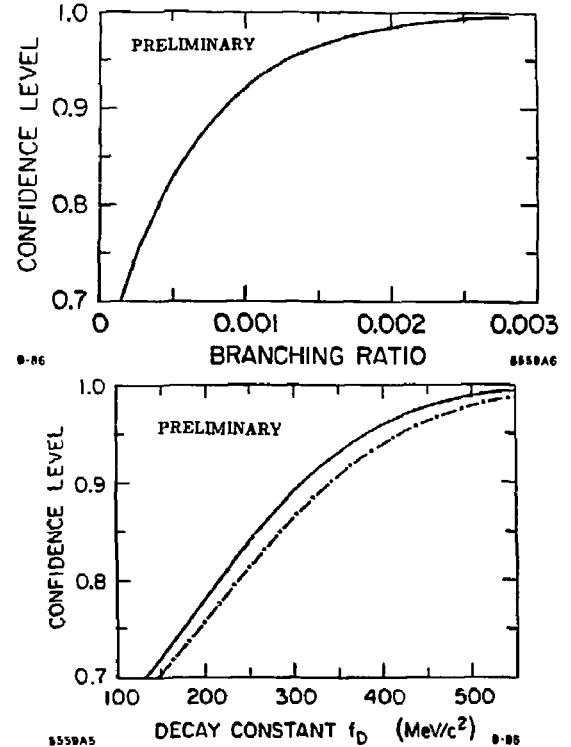


FIG. 35. Shown is the confidence level (C.L.) for the result as a function of (a) $B(D^+ \rightarrow \mu^+ \nu_\mu)$, and (b) f_D . The limit calculation is described in Ref. [40]. The dashed curve in (b) includes the effects of lowering the values of τ_{D^+} and $|V_{cd}|^2$ by their errors.

cancellations, however it is clear from the data which have reasonable statistics, that this alone cannot - even asymptotically- reproduce the measurements. The most naive interpretation would attribute the lifting of the precise color-matching required by the perturbative calculations to the presence of soft (non-perturbative) gluons in the meson wavefunction. As noted earlier, one attempt to quantitatively introduce the effect is by the *screening factor*(ξ) discussed in Section 4.2.4; taking $\xi \approx 0$ largely removes these cancellations. It should be noted once again that the need to introduce the parameter $\xi \approx 0$ to get the weak hadronic decays correct also reduces B_I for the D^0 .^[17]

$$B_I(D^0) = \frac{1}{2 + \frac{3}{2}(c_+^2 + c_-^2) + \frac{3\xi}{2}(c_+^2 - c_-^2)}. \quad (11)$$

Using the nominal values of $c_{\pm}(q^2)$ and $\xi = 0$ a value of $\sim 11.5\%$ is obtained for B_I , in better agreement with the data than what is obtained using $\xi = \frac{1}{3}$ (for $N=3$ colors) and the expression in Eq. (8) for B_I . The value for $B_I(D^+)$ requires an additional term discussed in Section 4.4.4.

4.4.2 Non-spectator processes.

The search for direct evidence for W-exchange graphs in D^0 decays can be summarized by the following results:

$$Br(D^0 \rightarrow \bar{K}^0 \phi) \approx 1.5\%$$

$$\frac{\Gamma(D^0 \rightarrow \bar{K}^0 K^0)}{\Gamma(D^0 \rightarrow K^- \pi^+)} \leq 0.11 \text{ at } 90\% C.L.$$

$$\frac{\Gamma(\bar{K}^{*0} K^0 + c.c.)}{\Gamma(K^{*-} \pi^+) + \Gamma(K^- \rho^+)} \leq 0.034 \text{ at } 90\% C.L.$$

The first channel, $D^0 \rightarrow \bar{K}^0 \phi$, is clearly seen (see Fig. 36) by three experiments,^{[16][17]} between experiments due to their assumptions of the backgrounds. The channel is Cabibbo-allowed and occurs at a rate which is surprisingly large, in that it is consistent with that for *ordinary* pseudoscalar-vector decays after a reduction for the limited phase-space and a factor for the removal of an $s\bar{s}$

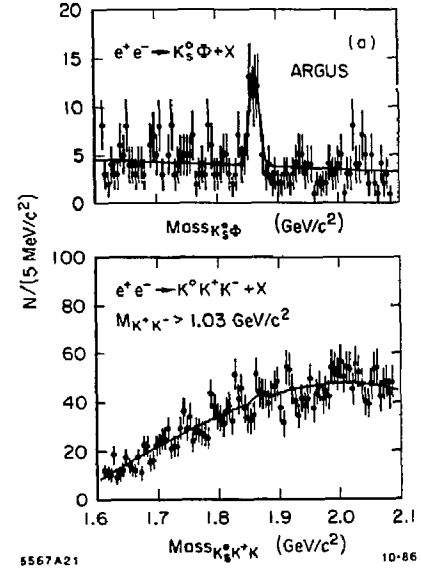


FIG. 36. $D^0 \rightarrow \bar{K}^0 \phi$ in e^+e^- annihilation.

pair from the vacuum is taken into account.^[60] This would suggest that if W-exchange is present, it proceeds at a rate which is largely uninhibited. The same non-perturbative gluon effects suspected for the absence of color cancellations, may also lift the helicity suppression of these channels. Because of the surprisingly large value for the branching ratio, alternate explanations have been proposed, as discussed in Section 4.2.2. This decay may arise for example from *small* rescattering out of the very *large* $K^- \rho^+$ channel, as opposed to the W-exchange mechanism itself. The second decay ($D^0 \rightarrow \bar{K}^0 K^0$) is Cabibbo-suppressed and is suppressed in exact SU(3). The limit is already below the value measured for the $K^+ K^-$ decay (see Table XXIV), but it is not stringent enough to give additional information. The third channel ($D^0 \rightarrow \bar{K}^{*0} K^0$) is Cabibbo-suppressed but not SU(3)-suppressed. While the value of the limit is preliminary,^[60] it is intriguingly small considering the size of $\text{Br}(D^0 \rightarrow \bar{K}^0 \phi)$.

The current D_s measurements given in Table XXV do not provide unique information on the presence of W-annihilation graphs; they all may arise from spectator amplitudes as well. Only measurements such as $D_s \rightarrow \rho \pi, \omega \pi, \dots$, will answer the question of W-exchange and W-annihilation, as would inclusive measurements of D_s decays opposite tagged D_s .

4.4.3 Interference effects in D^+ decays.

Evidence for interference as discussed in section 4.2.3, in exclusive decays is derived from the following ratios:

$$\frac{\Gamma(D^+ \rightarrow \bar{K}^0 K^+)}{\Gamma(D^0 \rightarrow \bar{K}^0 \pi^+)} = 0.32 \pm 0.09 \pm 0.05 \quad (12)$$

$$\frac{\Gamma(D^+ \rightarrow \pi^+ \pi^0)}{\Gamma(D^+ \rightarrow \bar{K}^{*0} \pi^+)} \leq 0.15 \text{ at } 90\% \text{ C.L.} \quad (13)$$

$$\frac{\Gamma(D^+ \rightarrow \bar{K}^{*0} K^+)}{\Gamma(D^0 \rightarrow \bar{K}^{*0} \pi^+)} = 0.21 \pm 0.17 \pm 0.15 \quad (14)$$

As can be seen from Figure 30, interference effects are expected for both $\pi^+ \pi^0$ and $\bar{K}^{*0} \pi^+$, but not for $\bar{K}^0 K^+$ or $\bar{K}^{*0} K^+$. Thus, since each of the numerators in (12) to (14) are Cabibbo-suppressed, one expects values close to $\tan^2(\theta_c) \approx 0.055$

for the ratios. Expression (13) however, is expected^[61] to be given by $\frac{1}{2}\tan^2(\theta_c)$ although as pointed out earlier, SU(3)-breaking and final-state interactions may alter the value.^[62] The deviation from equality in partial widths expected under exact SU(3) for the well measured Cabibbo-suppressed decays $D^0 \rightarrow \pi^+ \pi^-$ and $D^+ \rightarrow K^+ K^-$ (see Table XXIV) sets the scale for the size of these effects in charm decay.^[63] While (13) is clearly consistent with expectations, (12) and (14) are considerably larger, even including the possibility of large SU(3)-violations or final-state interactions. This is then entirely consistent with the pattern expected for interference among D^+ final state amplitudes, which may lead to a longer D^+ lifetime.

If interference is prevalent exclusively, then it leads to a decrease in the width and lengthening of the lifetimes of the D^+ and D_s states. One would introduce into Eq. (11), a term as in Eq.(10), to estimate the effect inclusively for the D^+ . That most charm decays appear to be quasi-two-body, with only a small nonresonant component, strengthens the argument that interference is a major effect in determining the total widths.

4.4.4 The pure leptonic decays and the total widths.

The decay constant f_D is a direct measure of the overlap of the wavefunctions of the heavy and light quarks in the D meson.^[64] It thus plays a fundamental role in setting the scale for processes such as weak flavor-annihilation and Pauli interference invoked to account for the differences in D^+ and D^0 lifetimes.^[46] A measurement of f_D also provides a stringent test of potential model^[65] and QCD sum rule^[66] calculations. In addition, it allows reliable estimates of other heavy meson decay constants (f_F , f_B , etc.), which are difficult to obtain due to the large theoretical uncertainties in extrapolating from f_s and f_K to the nonrelativistic heavy quark mesons. The decay constant also is essential in evaluating the magnitude of operators leading to $D^0 \bar{D}^0$ and $B^0 \bar{B}^0$ mixing.^[64] Calculations of the pseudoscalar decay constants obtain values which either increase (QCD sum rule method^[66]) or decrease (both relativistic and non-relativistic potential^[67] and bag model methods^[68]) with the meson mass. While our result does not probe the small values of f_D suggested by the bag model or QCD sum rule calculations (160 $\rightarrow$ 280 MeV/c), it restricts the range of values predicted by recent potential model calculations (208 $\rightarrow$ 450 MeV/c). One important point to make is that the

limit obtained excludes the very high values of f_D which have been suggested^[60] as an explanation for the large observed ratio of $\tau(D^+)/\tau(D^0)$. The latter estimate is of a perturbative nature, and is used to break the helicity suppression that would otherwise reduce the contribution of the non-spectator processes to charm decay. The value of f_D cannot be used to eliminate the non-perturbative techniques of reducing helicity suppression, such as the addition of gluons to the meson wavefunction.^[64]

4.5 Data on Hadronic Weak Decays of Beauty Mesons

As indicated in Section 4.2.1, the decays of B mesons arise largely from the $b \rightarrow c$ transition yielding final states containing D^0 and D^+ and D_s mesons, and their vector partners. The D_s fraction is expected to be small, requiring either a Cabibbo-suppressed decay, or the fluctuation of the vacuum to an $s\bar{s}$ state. There is also the possibility of decays through the heavier D^{**} orbitally excited mesons. The additional features of B decay allow however for the $b \rightarrow c\bar{s}$ transitions providing the possibility of $c\bar{s}$ final states (ψ, ψ', χ_c , etc...). As indicated in Fig. 37, B decays to baryons are also expected to be present at a small level. One final interesting decay would be that of the heaviest B_c meson where the single quark decay of the b and the c would compete favorably owing to the relative sizes of the KM matrix elements.

Table XXVII summarizes the measurements of hadronic B decays. These results all come from data taken at the T(4S), and thus correspond exclusively to the decays of B_u and B_d mesons. No data on the heavier states exists, except in the inclusive analyses at PEP and PETRA energies, where such states presumably contaminate the B_u and B_d sample.

Table XXVII also contains the published values^[67] for the ratio of $(b \rightarrow u)/(b \rightarrow c)$. The theoretical uncertainty in evaluating this quantity now sheds doubt on the validity of the measurement. These questions are addressed more thoroughly in the lectures of B. Winstein in these proceedings.

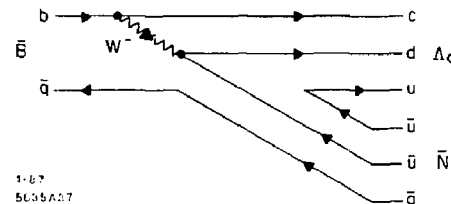


FIG. 37. Diagrams leading to B meson decays containing baryons.

Table XXVII. B Meson Branching Ratio(%)^{[100] [101] [102] [103]}

Decay Modes	CLEO	ARGUS	CUSB
$\bar{B}^0 \rightarrow D^+ \pi^-$	$0.14 \pm 0.19 \pm 0.05$		
$\bar{B}^0 \rightarrow D^0 \pi^+ \pi^-$	$1.6 \pm 0.9 \pm 0.6$ (7 ± 5) [†]		
$\bar{B}^0 \rightarrow D^{*+} \pi^-$	$0.35 \pm 0.14 \pm 0.11$ (1.7 ± 0.5 ± 0.5) [†]	$0.25 \pm 0.15 \pm 0.15$ $0.40 \pm 0.20 \pm 0.20$	
$\bar{B}^0 \rightarrow D^{*+} \pi^- \pi^0$		$1.1 \pm 0.6 \pm 0.6$	
$\bar{B}^0 \rightarrow D^{*+} \pi^- \pi^- \pi^+$		$2.4 \pm 0.7 \pm 1.1$	
$B^- \rightarrow D^0 \pi^-$	$0.4 \pm 0.1 \pm 0.1$ (1.1 ± 0.6) [†]		
$B^- \rightarrow D^+ \pi^- \pi^-$	$0.9 \pm 0.5 \pm 0.3$		
$B^- \rightarrow D^{*+} \pi^- \pi^-$	$0.3 \pm 0.2 \pm 0.1$ (2.7 ± 1.7) [†]	$0.4 \pm 0.2 \pm 0.2$	
$B^- \rightarrow D^{*+} \pi^- \pi^- \pi^0$		$3.5 \pm 1.1 \pm 2.1$	
$B^- \rightarrow D^{*+} \rho^-$	$8.1 \pm 2.9^{+5.9}_{-2.4}$		
$B^0 \rightarrow \psi X$	$1.1 \pm .2 \pm .2$	$1.2 \pm .2 \pm .2$	
$B \rightarrow \psi' X$		0.50 ± 0.23	
$B \rightarrow \psi_{\text{direct}} X$		0.90 ± 0.30	
$B \rightarrow \psi K^-$	$0.09 \pm 0.06 \pm 0.02$	< 0.20	
$B \rightarrow \psi K^{*0}$	$0.41 \pm 0.19 \pm 0.03$	0.44 ± 0.27	
$B \rightarrow \psi X$	$2.3 \pm 0.6 \pm 0.6$		
$B \rightarrow p X X$	≥ 3.6 at 90% C.L.		
$B \rightarrow \Lambda X$	≥ 2.2 at 90% C.L.		
$B \rightarrow u X$	10.6 ± 1.2		11.2 ± 1.3
$B \rightarrow e X$	12.0 ± 0.9		13.2 ± 1.6
$B \rightarrow D^0 X$	$39 \pm 5 \pm 4$	$50 \pm 7 \pm 8$	
$B \rightarrow D^+ X$	$17 \pm 4 \pm 4$	$23 \pm 6 \pm 5$	
$B \rightarrow D^* X$	$23 \pm 3^{+7}_{-4}$		
$(B \rightarrow D_s X)(D_s \rightarrow \phi \pi)$	0.004 ± 0.001	$0.002 \pm 0.001 \pm 0.001$	
$b \rightarrow u/b \rightarrow c$	≤ 4.0 at 90% C.L. [‡]		≤ 5.5 at 90% C.L. [‡]

† Previously published values. ‡ See Ref. 97.

4.6 Interpretation of the B Meson Data

While the B system is expected to behave closer to the naive spectator picture, the detailed measurements of specific, exclusive decay modes do not yet exist to prove this assertion. Under these circumstances, and almost without exception, it has been experimentally necessary to resort to inclusive measurements at the T(4S) to establish the pattern of decays. The results and conclusions are discussed in the following sections.

Because these measurements are inclusive, one should keep in mind that they are normalized to the height of the T(4S) resonance, which is assumed to decay entirely to $B\bar{B}$ pairs. This assertion has been tested by CLEO,^[101] and the limit on non- $B\bar{B}$ decays of the T(4S) is found to be $\leq 3.8\%$ at 90% C.L. This is analogous to the case at the $\psi(3770)$. A major uncertainty remains however in the assignment of the relative production $B_u \bar{B}_u : B_d \bar{B}_d$, because of the proximity to threshold. For example, a mass difference of 2 MeV produces a ratio $\frac{N_u^+}{N_d^+} = 1.2$ while a mass difference of 4 MeV produces a ratio $\frac{N_u^+}{N_d^+} = 1.5$. Using the same mass scale for the T(4S) of 10579.8 MeV, the mass difference of B_u and B_d are given:^[101]

$$M(B_d) - M(B_u) = 3.1 \pm 1.4 (\text{CLEO})$$

$$M(B_d) - M(B_u) = 2.4 \pm 1.6 (\text{ARGUS}).$$

This implies that an uncertainty in the relative fractions of B_u and B_d produced at the T(4S) remains.

4.6.1 Color suppression in B decays.

As was seen in charm decays, no evidence for a suppression resulting from color mismatch appears to be evident in the data. In B decay, the analogous effects would be present in comparisons of $\bar{B}_d \rightarrow D^0 \pi^0$ to $\bar{B}_d \rightarrow D^0 \pi^+$, (and analogous vector-pseudoscalar channels) although the size of the suppression should be smaller.^[102] Neither of these channels are well measured.

A cleaner and better measured system is that of the B decays containing bound $c\bar{c}$. The total B meson branching fraction to a final state with a charmed and anti-charmed quark is expected to be 10% to 15% based on the spectator model. In some fraction of these, the final state will contain a $c\bar{c}$ meson. These are seen in

Fig. 38 to have the same *color matching* topology as in the D meson case. The theoretical estimates^[101] of the size of this component range from ~ 0.4 to $\sim 3.0\%$ for the inclusive ratio $\Gamma(b \rightarrow \psi)/\Gamma(b \rightarrow \text{all})$. The higher values have ignored color matching, the middle values of $\sim 2\%$ assume some color-suppression, and the smallest values $\sim 0.5\%$ include the full QCD correction. Theoretical uncertainties arise from phase space effects and hadronization of the $c\bar{c}$ system into detectable ψ mesons. The next section discusses this question. As can be seen in Table XXVII, the experimental values are about $1.1 \pm 0.3\%$, consistent with the lack of a full suppression just as was seen in charm decays.

4.6.2 Charm in beauty decay.

Since the branching ratios of the charmonium states are well known, the absolute branching ratios for $B \rightarrow \psi$ are determinable and are seen from Table XXVII to be within range of expectations.

The rate for D_s production in B decay can only be estimated, since the absolute branching ratio for D_s into $\phi\pi^+$ (the only channel observed) is not known. From Table XXVII, one sees that the ARGUS and CLEO results are in poor agreement. Taking the CLEO value and assuming the branching ratio of say 3% for $D_s \rightarrow \phi\pi^+$, one obtains a rate for $B \rightarrow D_s + X \approx 13\%$ which is consistent with expectations^[104] of about 9% for D_s production from both $b \rightarrow c(\bar{u}d)$ and $b \rightarrow c(\bar{s}s)$ combined.

The previous section discussed the issue of color-suppression signatures in B meson decay. One sidelight is the hadronization process in beauty decays. In charm decays, it was seen that a large fraction of the decays appear to be quasi-two body. It is of course interesting to see if this is the case in B decay, as it results in a calculational simplification. To this end, in the absence of good exclusive reconstruction efficiency, it is necessary to look inclusively at the *first* daughters of B meson decay, namely D_s and ψ mesons.

The Z distributions from CLEO and ARGUS for the D_s and ψ mesons in B decay are shown in Fig. 39. If the CLEO data are correct, 65% of the D_s arise from quasi two-body production, presumably, in conjunction with a similarly hard D^0 or D^+ . The production through W-exchange or internal (D_s -K) decay appear to be largely absent.

The data from ARGUS indicate that ψ production is qualitatively softer, with a significant fraction of ψ coming from ψ' . A large fraction of the spectrum does

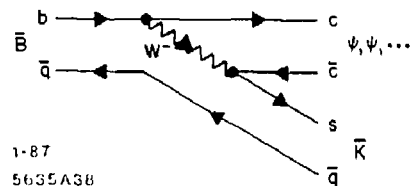


FIG. 38. Diagrams leading to bound charm in B meson decay.

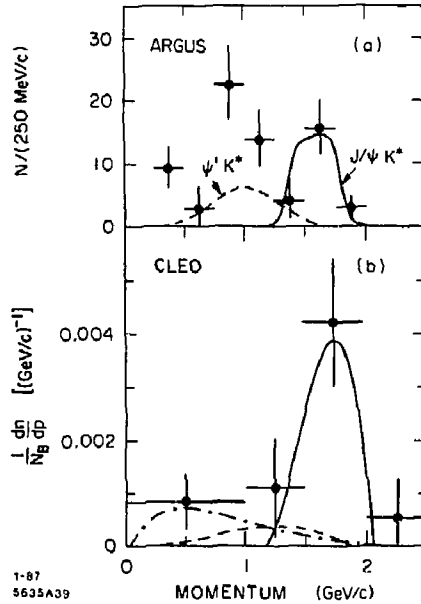


FIG. 39. (a) Z distribution for D_s in B decays. (b) Z distribution for ψ in B decays.

appear to be quasi two-body, with channels such as ψK , ψK^* and $\psi' K^*$ being large.

4.6.3 Baryons in B decay.

While D^0 and D^+ mesons are too light to decay through baryonic channels, the B mesons may have a sizeable rate. Estimates have been made based on the assumption that the quarks in the B decay can pair into diquarks, and pick up a $q\bar{q}$ pair from vacuum fluctuations to make pairs of baryons. These were pictured in Figure 37. The rates estimated in Ref. 105 are given:

$$\frac{Br(\bar{B} \rightarrow \Lambda_c \bar{N})}{\bar{B} \rightarrow (c\bar{q})(d\bar{u})} = 2 \rightarrow 15\%$$

$$\frac{Br(\bar{B} \rightarrow N \bar{N})}{\bar{B} \rightarrow (u\bar{q})(d\bar{u})} = 1 \rightarrow 2\%$$

$$\frac{Br(\bar{B} \rightarrow \Lambda_c \bar{\Lambda}_c)}{\bar{B} \rightarrow (c\bar{q})(s\bar{e})} = 5 \rightarrow 24\%$$

$$\frac{Br(\bar{B} \rightarrow \Lambda \bar{\Lambda}_c)}{\bar{B} \rightarrow (u\bar{q})(s\bar{e})} = 1 \rightarrow 7\%.$$

The rate for baryon-antibaryon production from B mesons is 4 to 26% overall.^[105] The data, from CLEO^[94] is consistent with this range, being posed in lower limits for p and Λ inclusive production in Table XXVII.

4.6 Conclusions On the Decays of Open Charm and Beauty

The extensive exclusive measurements of D^0 and D^+ mesons provide a reasonably firm basis for understanding the charm meson width. The decays of D^0 and D^+ appear to have a large quasi two-body component. There is evidence for color-suppressed decays, decays in which interference is occurring, and decays where flavor-annihilation appears to be present. Studies of the semileptonic decays point to hadronic matrix elements that are relatively simple, being dominated by form factors that are well modelled by simple poles. Detailed information on the D_s meson is still lacking. The latter would provide the most direct means of

checking the current ideas on interference, flavor-annihilation and the apparent dominance of quasi two-body decays of charm. An alternative is to measure the factors (such as the KM angles and the weak decay constants) that make up the theoretical estimate of the rate for each of these processes. While $D_s \rightarrow \tau \nu$ should provide an experimentally accessible measurement of the weak decay constant f_{D_s} , stringent limits on f_D from $D \rightarrow \mu \nu$ already provide information on the size of perturbative effects that may allow flavor-annihilation to proceed at a measurable rate. Current models favor a small amount of weak flavor-annihilation using decays (such as $D^0 \rightarrow K^0 \phi$) as evidence. Such decays are however under suspicion as possible arising from flavor mixing or rescattering at the strong interaction level.

The decays of light B mesons (B_u and B_d) appears to be dominated as expected by the $b \rightarrow c$ transition. The determination of the $b \rightarrow u$ fraction has been clouded both by theoretical uncertainties and by experimental difficulty. The lack of a global picture for B hadronic decay has also been hampered by the lack of data on exclusive channels. Inclusive studies of $B \rightarrow D_s, \psi$, and D project a spectator-like picture. The smallness of the semileptonic branching ratio however leads one to believe that the nonleptonic sector of B decay may yet hold some surprises. The key to understanding B decay will be in the ability to separate the species B_u and B_d and to systematically study the exclusive decays. The heavy B mesons B_s and B_c will also be interesting, the latter providing the possibility of a significant fraction of single quark decays, which otherwise won't be seen until the t-quark is discovered.

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SOME ASPECTS OF COMPUTING
IN HIGH ENERGY PHYSICS*

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These lectures have two distinct parts. The first is on computer networking and the status of network installation for HEP. The second part is on vector and parallel processing and its success, or lack thereof for HEP.

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I. COMPUTER NETWORKING

I.1 INTRODUCTION

A variety of network services are being provided for High Energy Physics with a variety of network protocols. This diversity reflects the diversity of opinion of what are the most important perceived needs. This diversity is also influenced by the style of computing for a particular collaboration. For example, a collaboration based on centralized computing, typically the mainframe computer at the national laboratory, puts emphasis on remote logon and remote printing, while one that is based on distributed computing puts emphasis on file transfer and process to process communications. In both styles and for communications outside the collaboration, electronic mail and file transfer are important.

The major networks in use by HEP are BITNET, PHYSNET, X.25 based networks, Data switches/terminal multiplexors, and MFENET. Note that there is very little development of networking software or hardware being done by HEP. All of the above are implemented with off-the-shelf items that can be bought from various vendors or available from other sources. They may only require minor maintenance or modification to be used. This, however, is only true for wide-area networking as within the local area of one site, there are development projects which these lectures will not cover.

I.2 UNDERSTANDING NETWORKING TERMINOLOGY

Like many specialized subjects, computer networking is not easy to understand until its basic structure and terminology is understood. It is therefore useful to take a look at the International Standards Organization's (ISO) model

of networking called the Open Systems Interconnect (OSI) model as shown in Fig. 1.

7	Application
6	Presentation
5	Session
4	Transport
3	Network
2	Link
1	Physical

Fig. 1. ISO OSI model of networking layers.

In this model the complete networking package is broken down to seven layers of functionality, each communicating with the next layer up and down with well defined procedures. This description of the layers, although not exactly precise, should be sufficient for a basic understanding. At the highest level, the application layer, we have the user commands and application programs. After processing user input or a call from a program, it communicates with the presentation layer which handles data storage or I/O via the session layer which interfaces to the operating system of the computer. The transport layer prepares packets of data to be sent over the network which will include the addresses of the destination and the sender. The routing of these packets is handled by the network layer. The next layer down, the link layer, is where the protocol for bits and bytes, along with checksums is handled. And finally, at the lowest layer, we have the physical definition of standards for wires, how many volts to signal a bit, etc.

It should be noted that this model is very idealized. In practice it isn't always so obvious for a given task, to which layer it should be assigned. This model was defined after some networks already existed and its purpose is for setting up new networking standards that will be international in scope and independent of any particular computer operating system.

To understand networks currently used by HEP, a simpler model of networking layers is very useful, as shown in Fig. 2. In this model, the highest layer is a name of an organization which sponsors or organizes the interconnection of computers, or is just a name for a set of interconnected computers. The next layer down is a package of software commands or programs, which the users sees when he uses the network. The transport layer is a package of software which implement a set of protocols to send files or messages on the network. Finally, at the lowest layer, we have the protocols for the wires and voltages on them to define what is a bit, a byte, a packet, etc.

4	Organization
3	Command Package
2	Transport Package
1	Line Protocol

Fig. 2. Non-ISO model of networking layers.

To see how this model works, let's take a network that most HEP people are familiar with: BITNET. The term BITNET is the name of the network, thus corresponds to the organization layer of the model. To use BITNET, one needs some commands. To send mail, for example, on a computer running VM, one

would use the NOTE (or at SLAC the MAIL) command, or on a computer running VMS, one would use the VMS MAIL command. These commands, along with those to send a file or a message, belong to the command package layer of the network. On VM, they are part of the operating system commands; on VMS, they are also part of the operating systems but augmented with commands that come with the software product called Jnet. The transport layer of BITNET is implemented by a software product called the Remote Spooling and Communications Subsystem (RSCS) which works with the VM operating system. For a VMS computer, the Jnet product contains a RSCS emulation package. The line protocol used by BITNET is IBM's BiSync protocol, which both the RSCS and Jnet products understand. Thus we see how the four layers of this model form a coherent whole.

There are some wrinkles, however, even in this simple model. For example, Jnet can drive lines in two different ways. The first is the BiSync lines as used by RSCS. But the other is to use lines which are under control of DECnet. DECnet is also a software product for the transport package layer, so in effect we have this layer subdivided with Jnet on top of DECnet. DECnet itself can use one of three different line protocols: DDCMP, X.25, or Ethernet. So we have the situation where some BITNET connections are made with Jnet over DECnet via an Ethernet cable. It should now be clear that not every word that ends with the letters *net* describe objects which are alternatives to the same thing.

In the following sections, the most widely used networks used by HEP will be described. The reader should keep the simple model in mind, when trying to understand the components of these networks.

1.3 BITNET

BITNET is an international network of more than 1500 computers at over 500 institutions in 23 countries. In Europe, the same network is called EARN and in Canada it is called NETNORTH, but it is really one physical network managed by several organizations. Although the network protocol is IBM's VM-based RSCS, the current composition of operating systems on the network is 36% VMS, 35% VM, 12% UNIX, 7% MVS, and 10% all others. The computers on BITNET are interconnected by leased land lines at 9600 baud (land lines are required by RSCS). Figure 3 shows the current topology of the network in the States. The costs of leased lines vary from \$400 per month for 20 miles in states with high tariff rates to \$2,700 per month for coast to coast lines. Each site pays the cost for the line it uses to join the network and may help another site defray the costs of a long line. There are no other charges related to the use of the network, although a fixed yearly charge to finance the network information center will probably be instituted soon. In Europe, the international lines, and some of the domestic lines are paid for by a grant from IBM until the end of 1987. Leased line charges are considerably higher in Europe, especially for international lines, than in the States.

About 160 computers used by HEP at nearly 80 institutions are on BITNET. BITNET is used extensively for its fast reliable electronic mail and small file transfer. As an example, SLAC sends out on BITNET over 10,000 files per month totaling over 140 MBytes, with about an equal amount of traffic being received. To Europe alone, SLAC sends over 3000 files totaling 30 MBytes per month. In a typical month, over 40% of SLAC's 1300 users use BITNET at least once. Mail delivery times are typically measured in minutes during times of peak network

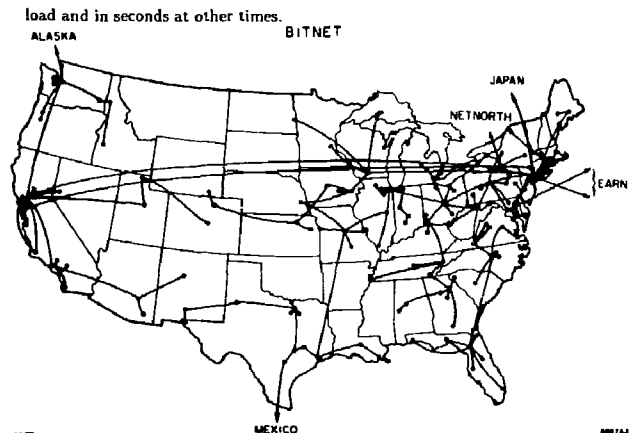


Figure 3. BITNET Topology, December 1986.

BITNET also has the feature of interactive messages between users on different computers, which may seem frivolous (some people say annoying), but is actually a very useful feature. It permits the implementation of interactive conferences and interactive network file servers besides allowing one to send short messages interactively instead of sending mail. The interactive feature also gives one the impression that a remote user is as close to you as a user on your own machine, which has the effect of encouraging information exchange.

One difficulty with BITNET is that large file transfer is done by store-and-forward at each node, thus needlessly extending the elapsed time to transfer a large file and tying up a link while it is being transferred (the latter is supposed to be fixed with the next release of RSCS). Also BITNET only allows a user to send

a file but not to 'fetch' a file unless additional remote software is provided (such as an interactive file server). However, on such a heterogeneous network, the send-only nature becomes a feature because of the potential of *hackers* stealing software.

1.4 PHYSNET

PHYSNET is a term used for a collection of over 600 computers, connected using DECnet protocols, at laboratory and many university sites. About half of the computers are used by HEP at about 60 sites. The network started by extending the PEP ring DECnet to LBL, U.C.Riverside, and UCLA to support the TCP collaboration. The Two-Gamma group added U.C.Santa Barbara and U.C.San Diego and the DELCO group added Caltech. Later, the HRS collaboration leased a satellite link to Argonne, and land lines to Michigan, Purdue, and Indiana. The network remained at that size for a number of years until the CDF and D0 collaborations started expanding it coast to coast. Its growth was thus ad hoc; one could even say accidental. Throughout the network 9600 baud leased land and satellite lines are used. It also has been extended to Europe via CERN to the existing DECnet there and INFN DECnet in Italy. At over a dozen sites, there are VAXes on this network that are on BITNET as well. The current topology of the DECnet in the States is shown in Figure 4.

DECnet is clearly the choice of collaborations based on distributed computing with DEC computers. It allows all the services expected from a network and it integrates very well with the VMS operating system. In particular, file transfer in either direction is as easy from disk to disk on one machine as from disk to disk on two machines 6000 miles apart. Also process to process communication is natural

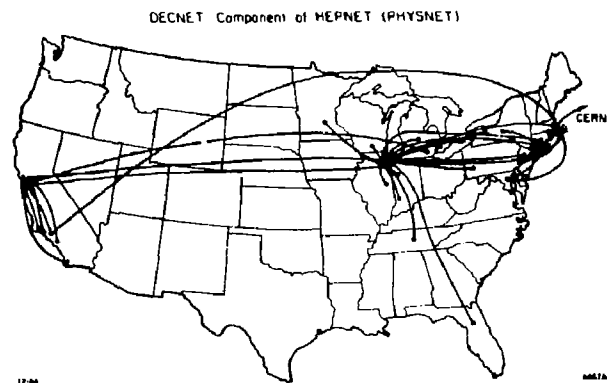


Figure 4. PHYSNET Topology, December 1986.

on DECnet, which allows centralized databases for a collaboration as well as the automatic updating of programs throughout the network, features that are used heavily by the collaborations based on DECnet. It is harder to get usage statistics from DECnet so exactly how much PHYSNET is used is unknown, however, it is known that the 9600 baud line between SLAC and Argonne saturates during peak periods.

One difficulty with DECnet is that needed network links must be working to proceed with communications. For this reason using DECnet for electronic mail does not work too well over a wide area where it is possible for a needed link to be down. DECnet allows login to remote computers on the network, but this service is not completely satisfactory when using the editor and does not

provide a connection to non-DEC computers such as the mainframes located at the laboratories.

1.5 DATA SWITCHES/TERMINAL MULTIPLEXORS

Remote logon via statistical multiplexors and data switches are extensively used in the States. These are mainly lines from university sites to the HEP laboratories. There are 14 lines to Fermilab and 13 to SLAC, for example. The current topology of leased lines with multiplexors is shown in Figure 5.

HEP LEASED LINES FOR TERMINALS

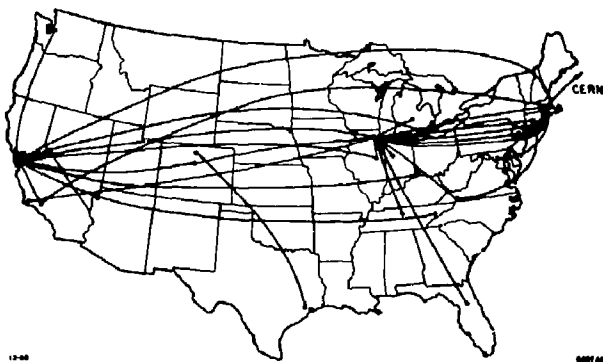


Figure 5. HEP network for terminal multiplexors, December 1986.

To analyze this network in terms of even the simple model is difficult. At the top level, the network has no name. User commands are implemented mostly in hardware, such as SLAC's Micom data switch, to choose which computer to

connect a terminal. The transport layer is handled by the multiplexors, which allow many terminals to share one physical line. The line protocol is defined by the combination of multiplexors and modems.

One can understand why this extensive network came into being if one considers the alternatives which are still being used by some:

1. Public phone lines. Some sites use public phone lines to connect to a remote site with 1200 baud modems. For low usage, such as accessing SPIRES at SLAC, this service is adequate. Long distance telephone costs are only about \$30 per hour during the weekday and up to 60% discount at other hours. However, for large usage this method becomes very expensive and 1200 baud is too slow for serious use of a remote computer.
2. Dial-in to Public Packet Switching Networks (PPSN). The PPSNs in the States have dial-in ports available in over 500 cities so they are almost always only a local phone call away. These networks use the X.25 standard protocol for layers 1-3 of the ISO OSI model and with a PAD (Packet Assembler/Dis-assembler) effectively provide layer 4, thus allowing one to connect a terminal to any computer with the same facilities. Most of the major laboratories are connected to PPSNs to this degree. At a certain level of usage, this method is less expensive than public phone lines and, of course, the only way to Europe (due to modem incompatibilities). The costs are about \$5.50 per hour plus the packet or byte charges in the States. Domestic rates in Europe are much lower, but international rates much higher, especially trans-Atlantic connections. Because of the dial-in, one is limited to 1200 baud.
3. Direct connection to PPSN. For higher baud rates, one needs a leased line

connection to PPSN. This costs about \$1,200 per month in the States (a factor 10 less in Europe) plus connect time and packet or byte charges. However, the quality of the service is not too good, since there are large echo delays which make use of full screen editors frustrating and the real throughput seems to be about 2000 baud instead of the 9600 baud one is paying for.

On the other hand, the quality of remote logon service via leased lines with statistical multiplexors is very high and the costs are quite reasonable in the States. For example, the land line from the University of Tennessee to SLAC is about \$1,300 per month from MCI (one of the three telephone companies that could supply the line). A 9600 baud satellite circuit costs about \$600/month (independent of distance) plus the land line cost to the nearest earth station at each end. Fortunately, there are earth stations near SLAC and Fermilab, but some university sites need such a long line to an earth station that it is less expensive to lease a land line all the way to the laboratory.

Most sites run a number of terminals at 9600 baud in full duplex mode, so that full screen editors can be used as easily as at the laboratory site. Some of these leased lines are via satellite links which does cause an echo delay of 1/2 second. Users of these links say that they get used to this delay and don't notice it after a while except when positioning the cursor with a full screen editor.

Many of these leased lines also run some channels in the opposite direction for printers. From SLAC the printers are driven by the same RSCS that drives BITNET and from Fermilab an equivalent system on the Cybers drives the printers. Also DECnet shares the bandwidth on some of the leased lines. A few sites runs all three protocols on the same 9600 baud line.

I.6 COLOURED BOOKS NETWORKS

Coloured Books (with English spelling) refers to a suite of protocols that closely follow the ISO OSI model. Each layer is described in a book of a different color, thus the name. The implementation was designed for and used by research organizations sponsored by the Science and Engineering Research Council in the UK, and the network was originally called SERCNET. Now it has been opened to the general academic community and called JANET (Joint Academic Network). JANET is a leased line network with 9600 and 48000 baud lines, using X.25 protocols at the lower three layers and the corresponding Coloured Book protocols for the upper layers.

There is another Coloured Book based network used by HEP called LEP3NET. It is a collection of 9600 or 16800 baud leased land lines radiating from MIT and used by the LEP3 collaboration. LEP3NET differs from JANET in that it also runs the DECnet protocols on the upper layers in parallel with the Coloured Book ones. In fact, it is LEP3NET's use of DECnet that extends PHYSNET to Europe because their Caltech computer was already on PHYSNET.

The Coloured Books suite, allows mail, file-transfer, and remote logon. Since the lower layer protocols are based on X.25 standards which is available from the PPSNs on an international scale, easy connections can be made between computers on the leased line and PPSN part of networks. The performance of Coloured Books networks has mixed reviews, but one needs to be careful whether the reported performance was based on using PPSNs or leased lines.

1.7 MFENET

MFENET was setup by the plasma physics community so remote users could gain access to the supercomputers at the Magnetic Fusion Energy center at Livermore, California. As the DOE's Energy Research computer is also there, MFENET is starting to be used by HEP to gain access to the supercomputer time allocated to HEP. Use is mostly by theorists for lattice gauge calculations and accelerator physicists for modeling.

MFENET supports terminal login to the MFE computers as well as remote job entry/retrieval from a VAX. It is a nonstandard protocol and there have been mixed reviews about its performance. MFENET has been approved for an upgrade called MFENET-II, which should be complete by 1989.

1.8 OTHER NETWORKS

Other large networks are hardly used by HEP. Mostly, they are used via gateways from BITNET to reach a few sites not otherwise reachable. ARPANET contains mostly the computer science and Department of Defense research community. Their lower layer protocols is based on TCP/IP. USENET is a network for the UNIX community and is thus not used much by HEP because most HEP VAXes run VMS. However, two UNIX machines used by theorists are reachable via USENET. A few HEP computers are only reachable via national networks that use the X.400 protocols. These protocols are the ISO OSI application layer for mail and the most used implementation comes from University of British Columbia with a software package called EAM. These national networks follow the policy of most European telephone authorities in using PPSNs for the transport and lower layers. Thus they currently suffer from additional costs for each mail item

sent, but the X.400 protocols themselves are probably the wave of the future as networks converge on international standards.

1.9 FUTURE U.S. HEPNET

As a result of the recent HEPAP Subpanel on Computing, it has been realized that a coordinated approach to networking within HEP might be better than the ad hoc growth being experienced so far. It would allow sharing and central funding of much higher speed lines and avoid the duplication of effort and apparent waste in having so many transcontinental leased lines. It would also provide the needed coordination for leased lines to Europe and Japan. An example of the poor coordination that has existed so far is the fact that the DECnet connection between Harvard and MIT is broken if SLAC's SLD VAX goes down.

An example of what a U.S. HEPNET might look like is the following. Trunk lines of 56K baud would run between SLAC, Fermilab, and Brookhaven. LBL and Argonne would use their existing microwave links to SLAC and Fermilab, respectively. University sites would have 9600 baud feeders to one of the laboratories, but not necessarily the nearest one. This is because the costs don't vary with distance that much, so university groups will want to connect to the laboratory of their prime interest.

There are a number of issues that need to be resolved before a U.S. HEPNET comes into existence. The choice of protocol or protocols needs to be made. A management structure needs to be setup with appropriate funding mechanisms. Equipment choices may need to be made at a national level instead of locally. One will also need to make choices on an international scale for the connection to Europe and Japan. There are also some who question if it is technically feasible

to share such lines without degradation of current service and if HEPNET would be cheaper than the current situation.

In another development, DOE has declared that there must be coordination of networking activities across all the Energy Sciences divisions, which include the HEP and MFE communities. A new network, ESNET, would be designed and operated by the MFENET staff. Whether this initiative is good for HEP remains to be seen.

I.10 CONCLUSION

Networks are playing an important role in the HEP program. They have become the foundation for many collaborations and will be even more so in the future. They are also rapidly reducing the number of telephone calls and displacing telex and postal mail as the means of communication within the HEP community.

Like the computer industry as a whole, networking technology is rapidly changing, costs are coming down, and speeds are going up. It will be interesting to compare the status of U.S. HEP network today with what the community will have five years from now.

II. VECTOR and PARALLEL PROCESSING

II.1 A BIT OF HISTORY

Use of computers in HEP has come a long way as this short historical summary shows. In the 1960's, HEP detectors were counters, spark chambers, and bubble chambers. The data acquisition systems, by today's standards were slow and produced a small amount of data. The fastest computers of that era were sequential machines. HEP was using these supercomputers of the day and they were quite adequate for our needs.

In the 1970's, detectors moved to entirely electronic readout with first proportional chambers, then drift chambers. Data acquisition systems became modest in size as well as the amount of data produced. The fastest computers split into two paths, bigger sequential machines and vector processors, i.e. the supercomputers. HEP found that *productivity tools* were more important than raw CPU speed and our codes didn't vectorize very well, thus followed the route of the sequential machines and losing our expertise on the supercomputers.

In the 1980's, detectors became massive data generators with data acquisition systems that can easily saturate tape writing speeds. The large computers are similar, but faster, than the ones in the 1970's and a new generation of supercomputers matured. New options for processing, namely *parallel processing*, was introduced.

Experimental HEP now has more needs for computing than we can afford to acquire by traditional means. To these needs, we also have to add the new needs of the accelerator physicist and those of the theorist. Thus, at schools like this one, we look at a wider range of solutions than those that have been traditional

considered *best* for HEP. Namely, we must look at vector and parallel processors as well as the standard sequential machines.

11.2 HEP COMPUTER NEEDS

HEP computing needs in the past have been dominated by experimentalist's event processing, Monte Carlo generation, and analysis requirements which have become very large with the advent of large 4π detectors. For the HEP experimentalist, about 1/3 of the CPU cycles are needed for event processing. The event can be 50K to 200K bytes of data and the processing may take 10 to 100 seconds of CPU time on a mainframe.

HEP's event processing needs are rather unique in the field of large scale processing and frequently misunderstood by industry, computer scientists, and others. The problem for the experimentalist is not finding a computer with sufficient power to process one event. Rather the problem is that 1-2 events per second are collected while the detector is running, thus millions of events are collected per year. Event processing must keep up with event collection or a backlog develops, there would not be feed back to the running detector, and physics results will not be presented in a timely fashion. To analyze the events processed, comparison is made with a set of events which have been generated by Monte Carlo simulation techniques. This set of events should be at least several times the number of real events and will also take a considerable amount of CPU cycles to generate and process. The processing of real and Monte Carlo events typically use up about 2/3 of the CPU cycles available at an HEP laboratory.

The event processing is not the whole problem. The development of the programs, which can easily reach 500K lines of FORTRAN, is also a problem of

considerable magnitude. This development is done by 10-50 physicists working over a number of years without real events to judge their progress. When real data is available from an imperfect detector they must work rapidly to correct, tune, and otherwise make the programs work properly on a short time scale so that physics results can be extracted. To this, there is of course, many other development jobs such as calibration and alignment work, and analysis of the event samples. This development and analysis work dictates a computer with good *productivity tools* which may not be the one with the fastest CPU per unit cost.

The needs of the HEP theorist are of the more traditional type known well to industry and computer scientist as *scientific and engineering* computing. That is, they consist of a model of a physical system on a lattice of points where one needs to do a large number of floating point calculations on each point and many iterations must be done for the system to stabilize into some state. The needs of the HEP accelerator designers is one of tracking a large number of particles through a grid of dipole, quadrupole, and sextupole magnets to understand the stability of a particular design. Both of these kinds of problems are similar in nature to problems in other fields of science. They both have a relatively small amount of code and they vectorize well, unlike the event processing problem of the experimentalist.

11.3 VECTOR PROCESSING

The term *vector processor* and the allied term *array processor* are somewhat misnomers for a style of computer architecture which is based on a simple fact: there is no way with a given technology that floating point arithmetic is going to

be as fast as a binary add. Also there is no practical way that random memory access time is going to be as fast as a binary add. Thus in any computer, a single floating point operation is going to take multiple CPU cycles. For example, a floating point add may be divided into a number of cycles show in Fig. 6. The operations performed in each cycle are as follows:

1. Fetch operands from memory or register files.
2. Prenormalize the mantissa with the smallest exponent.
3. Add the mantissa.
4. Postnormalize the resulting mantissa and correct the exponent if necessary.
5. Store the results in memory or register file.

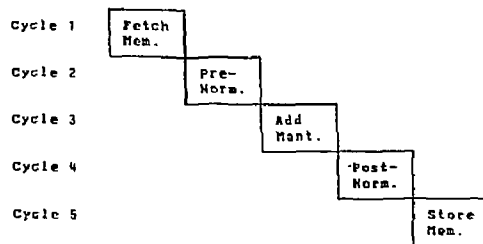


Fig. 6. Example of multiple cycles of floating point add.

With an appropriate computer architecture, this operation over a number of operand pairs can be made faster by overlapping the steps or pipelining them. Thus as shown in Fig. 7, one can do 3 floating point adds in only 7 cycles instead of the 15 it would take if done sequentially.

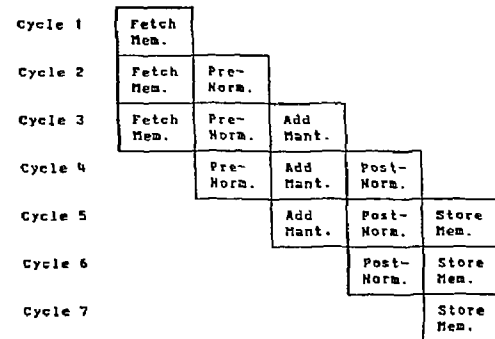


Fig. 7. Example of pipelined cycles of floating point add.

This pipelining only works well if the data operands are in an orderly pattern, which the FORTRAN programmer knows as a vector or an array, thus the terms vector or array processor is used for one that can perform in this manner.

Since the introduction of the Cray-1 supercomputer in the mid-1970's, there has been renewed interest in this type of architecture. Figure 8 is a block diagram of a typical supercomputer, in this case the Amdahl 1200 (known as the Fujitsu VP-200 in Japan). This computer consists of the scalar unit, the vector unit, the main storage unit and the channel processors.

The scalar unit fetches and decodes all instructions. When an instruction is of scalar type, it is executed in the scalar unit; otherwise it is issued immediately to the vector unit. The scalar unit of this machine is essentially an IBM S/370 compatible processor with a speed about twice that of the processors in

the IBM 3081K.

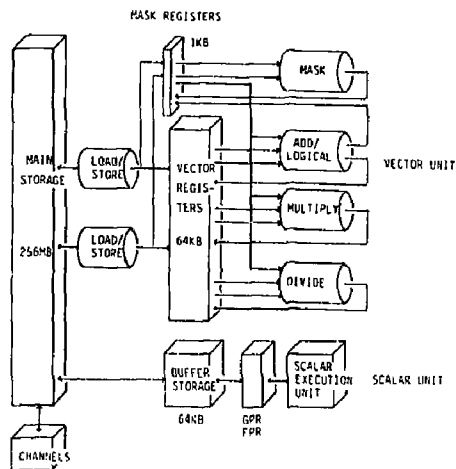


Fig. 8. Block diagram of typical supercomputer, the Amdahl 1200.

The vector unit consists of vector registers, mask registers, an add/logical pipeline, a multiply pipeline, a divide pipeline, a mask operation pipeline and load/store pipelines. The arithmetic pipelines fetch operands from the vector registers and stores results back into them and they can operate concurrently. The mask registers control conditional vector operations, that is, each bit in the mask controls whether the arithmetic operation should occur on the corresponding element of the vector, without interrupting the flow through the pipeline. The use of such a feature will be described later.

Each of the pipelines operate with a 7 ns machine cycle (except the divide pipeline), thus giving a peak performance of 570 MFLOPS (mega floating point operations per second). The basic bottleneck of such machines is not having the data you want to feed the pipes in the vector registers. For this reason, this machine has load/store pipelines which can move data between the 256 MByte main memory and the vector registers at 2.1 GByte/sec each.

Other supercomputers share most of the characteristics of the Amdahl 1200, differing, of course, in the size and speed of the various subsystems. Each is trying to optimize the performance for their customer base.

11.4 HEP USE OF VECTOR PROCESSING COMPUTERS

To the HEP experimentalist, these new machines look impressive but even more inappropriate than previous supercomputers. That is, it is difficult for the experimentalist to understand how to use these machines effectively. To the theorist and accelerator physicist, however, these new machines look good as their codes do vectorize. The trend towards very large memory sizes in supercomputers is very much needed and the CPU speed must also be very fast to be able handle the very large lattices one can now put into memory.

The supercomputer to a theorist is what an accelerator is to an experimentalist. That is, once a model of the physical world is running on the supercomputer the theorist can perform his *experiments* to study the validity of the model or study the states the model can produce. The HEP community is used to the idea of spending money to build accelerators for the experimentalist. However, they are not used to the idea of spending money on supercomputers for the theorist, nor are the theorist used to getting together to make proposals to obtain the

money they need to buy equipment. Good theory results can be obtained from supercomputers, but the HEP community as a whole needs to decide how much money they are worth and make the appropriate investments.

However, the QCD lattice gauge calculations the theorists would like to do on a supercomputer is enormously computationally intensive. To obtain a reasonable result, one would put a proton on a 32^4 lattice which is 1 million sites. At each site there are four gauge boson links, each represented by 3×3 complex matrix and each of the four flavors of quarks requires three complex numbers. Thus we have a total of about 100 words per site and a requirement of 100 Mword of memory for the problem; a staggering number to the experimentalist.

The theorist measures a computer's performance by the "event" rate it can deliver. An event is typically 100 iterations over the lattice before taking a sample of the state of the system. The total calculations needed is about 5K FLOPS per site times the 10^6 sites times the 100 iterations which adds up to 500 GFLOPS per "event". One event thus takes 500 seconds on a 1 GFLOP supercomputer. A reasonable number of events are needed, say 1000, which would take a solid week of dedicated time on a 1 GFLOP machine! But this is not all, many runs are needed as one varies the input parameters such as the value of g^2 , mass of the quarks, etc.; then multiply by the theorists who want to this kind of work. It is indeed an enormous computational load.

11.5 RECODING EXPERIMENTAL CODE TO ACHIEVE VECTORIZATION

In spite of the experimentalist's pessimism on supercomputers, one paper at the Amsterdam conference on Computing in High Energy Physics showed some interesting results with HEP code. It was given by Kenichi Miura of Fujitsu

Limited and entitled "Vectorization of Monte Carlo Codes on the FACOM VP-200 for High Energy Physics Applications."

Miura worked with the FOWL Monte Carlo code, which is a tracking code, he obtained from CERN. He first compiled the code, with all vectorization in the compiler turned off, and ran it on the FACOM VP-200. With vectorization turned off, the compiler does not generate any vector machine instructions, thus this measures the speed of the scalar processor. A run lasted 105.9 seconds instead of 753.6 seconds on an IBM 370/168; a speed up of about a factor of 7. Then, without changing the code, he turned on the vectorization in the compiler and found that the code ran slower (111.9 seconds)!

The fact that there was no speed up is not surprising since this type of code does not deal very much with vectors. The fact that the code was slower with vectorization turned on is a bit strange until one thinks about it. The only vectors in the code are the 3-vectors of the particle momentum. But the vector pipelines of the machine, although fast, are relatively long, thus there is a start-up time penalty. Thus operating of vectors of length 3 takes longer using the vector pipeline instructions, then to do 3 sequential scalar operations. One needs vector lengths in the hundreds to realize the full potential of a vector machine. This result is typical of all supercomputers, not just the FACOM VP-200.

We now describe several techniques for hand vectorization of the FOWL code that was done by Miura in order to achieve efficient use of the vector processor. Note that the hand vectorization here refers to the process of restructuring the original scalar codes using FORTRAN language, in order to expose more parallelism to the compiler.

The basic strategy for code restructuring is to process many events in one

pass, that is, to bring the event loop to the innermost DO-loop instead of the outermost one. This includes modifications to the data structures. Single variables in the original scalar code are now changed to arrays. Temporary variables in a loop may be left scalar but they are actually defined in vector registers. The change of the code structure also necessitates the conversion of all the user-defined subroutines and functions from scalar form to vector form. The compiler automatically performs this conversion where ever it is syntactically possible to do so, for example the trigonometric functions. As for the random number generations, a new pseudo-random number routines had to be developed for this code. This routine generates random numbers in an array at a rate of 80 million real random numbers per second.

One of the most important issues in vectorizing Monte Carlo codes is how to vectorize IF tests. It is well known to the experimentalist, that IF statements riddle both our Monte Carlo and reconstruction codes. The IF statements can be categorized into two types as discussed below.

The first type is the 'case'-like IF where subsequent computations differ in the TRUE and FALSE branches. It is generated by the IF-THEN-ELSE structure in FORTRAN. The FACOM VP-200 system has three methods vectorizing this type of IF statement in its architecture, with the compiler picking the optimal one. One of the easiest to understand is the use of the masked arithmetic instructions as shown in Fig 9. In scalar mode, either the calculations in P2 or P3 are done following the IF, then the DO-loop iterates for the next set. In vector mode, the IF statement is done as a vector instruction with the only result being a string of 1's or 0's depending on whether the IF results was TRUE or FALSE. This string is called a 'mask' and is stored in the mask registers of the machine. Then the

calculations in P2 are done 'under the mask' which means they are only done if the corresponding mask bit is '1', and there is machine instructions to allow for this 'do under mask'. The calculations in P3 are done under the corresponding anti-mask with all the 0's and 1's inverted. On sufficiently long vector lengths, this procedure is faster than operating in scalar mode.

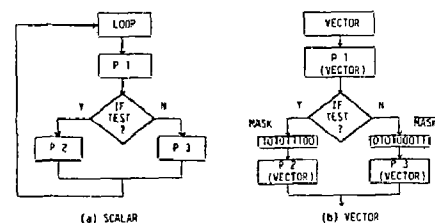


Fig. 9. Feed-Forward type IF.

The second kind of IF structure is the DO-WHILE IF which is typically used for rejection technique. Trials have to be iterated for each event until a given condition is satisfied. The number of trials is probabilistic and varies from one event or one track to another. It takes some semantic modifications to the code in order to vectorize this type of IF statement. The approach taken by Miura was to define two temporary buffer arrays called "stacks", one for successful events and the other for failing events. By using these stacks DO-WHILE IF statements

can be reduced to IF-THEN-ELSE type statements since immediate feedback paths can be eliminated. The vector data editing instructions of the machine, such as compress instruction and the indirect addressing capability on array data are used by the compiler to fill the stacks. Figure 10 illustrates this.

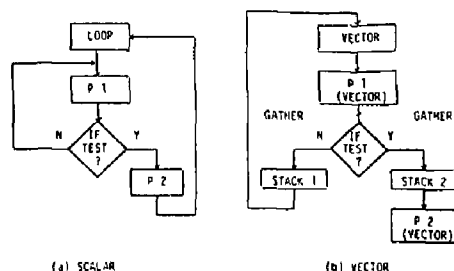


Fig. 10. Feed-Backward type IF.

There is yet one more effect that required additional recoding. The Monte Carlo code is a series of rejection tests. Thus the output stack of a rejection loop is smaller than the input stack, thus the next rejection loop works on a smaller vector. To prevent this loss of machine efficiency, each stack was refilled to some maximum before starting the rejection procedure by overlapping events.

The results of this work brought the processing time down for one run to 32.7 seconds in vector mode, which is a factor of 3 improvement over scalar mode. Curiously enough, if the modified code was compiled in scalar mode, it also ran

faster, being only 81.6 seconds for a run which is a 30% improvement. This result is also typical and shows that the original disorderly IF statements degrade the performance of the code on pure scalar machines as well.

One could ask the following questions about this work.

- Miura worked alone in improving the code for his vector processor, understanding the workings of the code only from comment cards. Could the author of the code have done an even better job in speeding up the code?
- After the conversion of the code to a vector processor, is it still maintainable and/or can be further developed in the sense we have come accustomed to or has it become very difficult to understand or modify?
- Are the techniques applied to obtain the speed up general, or was Miura lucky with this one code? Can the techniques be applied to real event processing with all the exceptions the code has for a real detector (as opposed to an idealized detector)?
- Although the code runs faster than on any scalar machine, the vector machine costs more than the scalar one. Has there been an improvement in cost effectiveness?

Nevertheless, this paper shows some validity to what many are beginning to suspect, that vector processors should be looked at more carefully by the HEP community. It would be a shame if a vendor of one of these machines had to show us the techniques to make them effective in HEP, if they are there.

11.6 RECODING RECONSTRUCTION CODE AT THE LOWEST LEVEL

Another example of recoding HEP code comes from the track reconstruction code of the Mark III detector. This time the code was recoded at the lowest level: the basic pattern matching for finding tracks. The technique was also applied to track finding for a Fermilab fixed target experiment by the Florida State Group.

The first step is to generate a track dictionary by a "geometry" program which draws circles from the beam line through the detector in the $r - \phi$ plane and notes which sets of drift chamber cells lie on each. This method is illustrated in Fig. 11. Each dictionary entry is one distinct set of these cells. To keep the dictionary small only circles which correspond to transverse momenta of greater than 50 MeV are drawn. Because the data from the detector is unpacked cell by cell, it is natural to structure the dictionary not only as a list of cells on each track, but also inversely as a list of tracks that pass through each cell.

Having set up these tables once, the pattern recognition is ready to begin on events. During this phase, as each cell is unpacked and identified, the program sets bits in a two-dimensional bit array called PATARY with one row for each layer in the drift chamber and one column for each track in the dictionary. For each hit cell, one bit is set for each track that might have caused the hit. These bits then indicate which of the drift chamber layers on any given track are actually hit as shown in Fig. 12.

It should be noted that at this lowest level of reconstruction the code is already amenable to exploiting the vector instructions of some supercomputers. This is so because one can take as one long vector the list of all hit cells, and operate on that vector to fill the PATARY array.

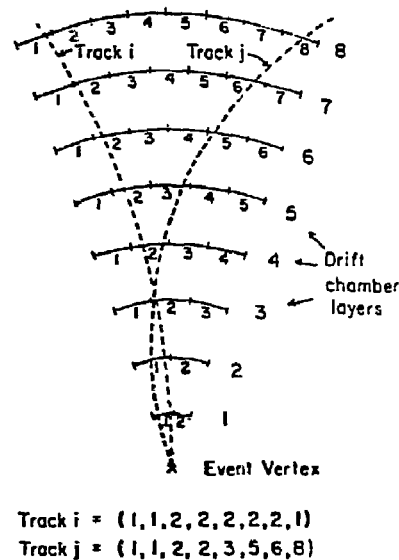


Fig. 11. Schematic representation of dictionary generation.

These ideas may seem trivial, but they are critical to exploiting vectorization. Rather than doing pattern recognition serially (track-to-track), information is developed and stored from primitive operations on all cells (as described above), then all clusters of cells in layers (named objects), then all clusters of objects over layers (named bundles of track candidates), and finally the isolated tracks themselves. At each step, long vectors can be made up of objects, bundles, or tracks.

x denotes a bit that is set.

Of course, the real life situation isn't that simple. For example, the actual code is more complex in order to allow for cell inefficiencies which leads to tracks without the full complement of hit cells. Also, unlike the Florida State Group, the Mark III group never had a chance to actually run this vectorizable code on a computer with vector capabilities. Nevertheless they predicted that such methods could save up to a factor of 5 in the track finding and fitting time. And as Miura found, the code ran faster on scalar processors as well, with a measured three fold increase in speed over a conventional approach.

The other method of getting more performance is to try to exploit the inherent parallelism of a program and have these parts run in parallel on separate processors. These processors can be either tightly coupled or loosely coupled; in

For the JEP event processing, it is clear that events can be processed in parallel. That is, the method of having one event processed by one processor *works* as was clearly demonstrated as far back as 1979 by users of the 168/E. It has also been shown that the method is not sensitive to the method of coupling. It is working equally well with tightly coupled processors such as the Elxsi computers or loosely coupled processors such as the FPS-164. Trying to exploit parallelism within one event, however, has so far been less effective because the overall execution time can easily be slowed down by the non-parallel part of the program, even with tightly coupled processors.

The parallel processing efficiency can be understood by a few trivial calculations. Let us first define an efficiency, ξ . Let T_{req} be the time it takes to do a set of computations by on a single processor, and T_{par} be the time to do the same set of computations on a system of n parallel processors. The efficiency ξ is then defined as

where a perfectly efficient system of n parallel processors, the parallel processing time would be exactly $1/n$ th of the sequential processing time.

-301-

$$T_{seq} = t_s + t_p$$

where t_s is the part that can only be run in serial and t_p is the part that can be run in parallel. The total parallel processing time is thus written

$$T_{par} = t_s + t_p/n + t_{oh}$$

where t_{oh} is the overhead time in communicating between the two parts. Working through the equations with the assumption that $nt_s \ll t_p$ we find

$$\xi \cong 1 - (n-1) \frac{t_p}{t_s + t_p} - n \frac{t_{oh}}{t_s + t_p} .$$

One can immediately draw two conclusions from the above equation. First, to be efficient, the time spent in parallel processing must be much larger than the time that must be spent handling the sequential step. A given problem must meet this criteria to be efficient on a parallel processing system. Note also the ratio t_s/t_p must be smaller as the number of processors increase, or, to look at it in an easier way, for a given ratio of t_s/t_p , one can only use so many processors before the efficiency goes down. When the efficiency goes down, one is spending money for more processors without realizing their full potential.

The second conclusion one can draw is that the overhead must be small compared to the processing time. If we ignore the sequential processing time, the above equation could be roughly expressed as

$$\xi \cong 1 - \frac{t_c}{t_p}$$

where t_c is the communications time. Expressed more simply, the parallel processing time must be much larger than the time spent communicating results between processors or between the processors and a host. This can be achieved in two ways. One should make sure each processor has a sufficient fraction of the problem to do before it needs to communicate and that this amount is minimal. In the case where the problem is expressed on a lattice of sites, each processor should have a large enough number of sites in its own memory to increase its processing time relative to the communication time spent when it reads data from sites in neighboring processors. Lacking the ability to structure the problem in this way, the other way to reduce communication time is to have shorter and/or faster communications paths, which could be a large investment in additional hardware.

These criteria for parallel processing efficiency are easy to satisfy with a large range of scientific and engineering calculations. This fact and the fact that it is relatively easy to design a multi-processor system with today's technology has led to large numbers of commercially available parallel processing systems. To illustrate this, Table I is taken from a review of this market by one of the popular trade journals, *Electronics*. Note their heading of the table, implying that the marketplace is in fact crowded with product offerings.

Table I. Tabulation of Parallel Processing Systems in the Marketplace.

Company	Product	Price	Parallelism	Connectivity	Memory	Processor
Accelerated Processors	Model 10	\$85,000	4 to 12 groves of 8 ALU's	reconfigurable	N.A.	N.A.
Alliant Computer System	FX18	\$270,000	up to 20	bus	global, up to 64 megabytes	64-bit CMOS gate array
Ametek	System 14	\$75,000 up	16 to 256	Hypercube	local, up to 256 megabytes	16-bit 90186/80287
Bull, Beranek & Newman	Butterfly	\$40,000 up	1 to 256	switching system	shared and local	16-bit MC 68000
Denelcor	HEP1	\$1 million to \$2 million/execution module	1 to 16 execution modules	shuttle network	global	64-bit ECL
ELSYS	6400	\$600,000	up to 12	bus	global, with local cache, up to 800 megabytes	64-bit ECL gate arrays
Enric Computer	Multimax	\$114,000	up to 20	bus	global, local, up to 32 megabytes	32-bit NS 32032
Flexible Computers	Flex-32	\$150,000 up	up to 20/bus up to 2,400 total	bus	global, local, 96 megabytes/cache	32-bit NS 32032
Gemini Computers	Trusted Multiple Microcomputer	\$45,000 up	1 to 8	bus	shared, local, up to 128 megabytes	16-bit 80286
Intel Scientific Computers	386C	\$150,000 up \$520,000	32 to 128	Hypercube	local, 248 megabytes	16-bit 80286, 80287
International Parallel Machines	IP-1	\$50,000	1 master processor, up to 8 processors	crossbar-like switch	global, up to 40 megabytes	32-bit
Lorel	Itatain	\$65,000 up	5 to 256 data; 6 processor	bus	shared, local, up to 14.5 megabytes	16-bit NS 32016
Merko	Computing Surface	\$220,000 to \$300,000	up to 128	four nearest neighbors	local, 4-K bytes/4 processors	32-bit Inmos T414 transputer
Multidown Systems	N.A.	VAX range	multiple registers	N.A.	global, more than 1 megabyte	gate arrays
Seuze	Seuze Ten	\$100,000 up	16 to 1,024	Hypercube	local, up to 160 megabytes	32-bit custom VLSI
Saxpy Computer	Saxpy-1M	\$2 million	32 processors	parallel systolic	shared, up to 512-K bytes	32-bit custom
Seymour Computer Systems	Balance 8000	\$60,000	up to 12	bus	global, up to 28 megabytes	32-bit NS 32032
Seymour Systems	Seymour System	\$200,000	up to 64	bus	global, up to 252 megabytes	36-bit MC 68010
Thinking Machines	Connection Machine	N.A.	64,000 to 1,000,000	Hypercube	global, 500 megabytes	1-bit custom

II.9 CURRENT STATE OF PARALLEL PROCESSING IN HEP

The parallel processing projects used in HEP come in two basic forms: those used for theoretical calculations and those used for experimental event processing. They have quite different systems, each designed for their own goals.

A representative sample of those projects used by theorists is given in Table II. Each takes a problem expressed as a lattice of points, with nearest neighbor interactions, than assign a large number of lattice sites to each processor. This is done to minimize the communication time between processors compared to the calculation time; one of the necessary criteria discussed above. Perhaps the best description on how this is actually done, can be found in the papers written about the CosmicCube.

Table II. Parallel Processor Projects from HEP Theory

Name	Group	Processors
APE	INFN	3081/E + custom
STAC	Boston U	custom + Weitek
	Southampton	Transputer
ANL/STAR	Argonne	ST-100
	Columbia	80286 + custom
MACE	C.M.U.	8086/8087
CosmicCube	Caltech	8086/8087

Some of these systems have the processors tightly coupled, that is one processor can read and/or write to the memory of another. This is needed to communicate the state of nearest neighbors on a lattice, something that is unavoidable in

this type of calculation. Because of the enormity of the QCD calculations many of these systems have quite specialized processors, requiring either hand coding in the low level micro-code of the processor or requiring a new compiler to be written. Fortunately, the amount of code involved is quite small, so the programming is not a large burden.

The experimentalists have been using parallel processing systems for quite some time. The earliest was the 168/E system, shown in Fig. 13, which was installed at SLAC in 1979. In this system nine processors were interfaced to a parallel bus with one end of the bus interfaced to a host mainframe computer. This is shown in Fig. 14 and is an example of a loosely coupled system which is quite adequate for event processing.

Other systems have been used at laboratories and university sites. The 3081/E processor was designed as a SLAC/CERN collaboration as a replacement for the 168/E and systems using them are in operation at SLAC, CERN, UCSC, MIT, and other places. The Fermilab ACP (Advanced Computer Project) group has built such systems using commercially available 32 bit microprocessors. The Weizmann Institute-designed 370E has been used at many sites in Europe.

These systems work well because it is relatively easy to satisfy the criteria for high parallel processing efficiency. All that is needed is to run the system so that each processor handles one event completely and thus events are run in parallel. In so doing, the parallel processing part is easily much longer than the sequential processing part. The communication time between the processors is reduced to zero, as the processing of each event is independent of the others. The only sequential processing part and overhead is the input/output transfer times for the raw data buffer and the data summary results. Because of this, the

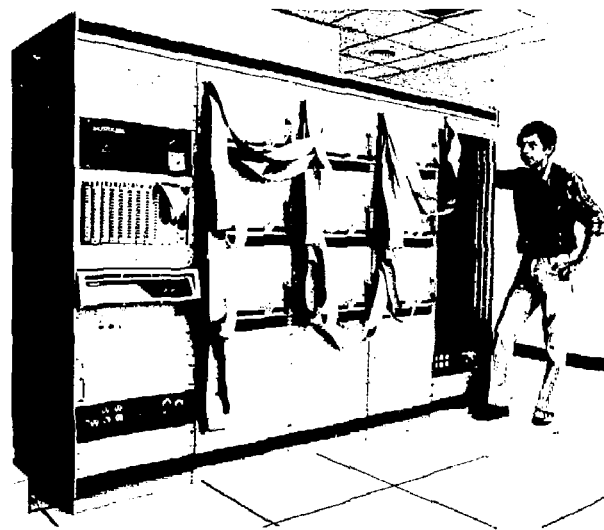


Fig. 13. The 168/E processor system at SLAC.

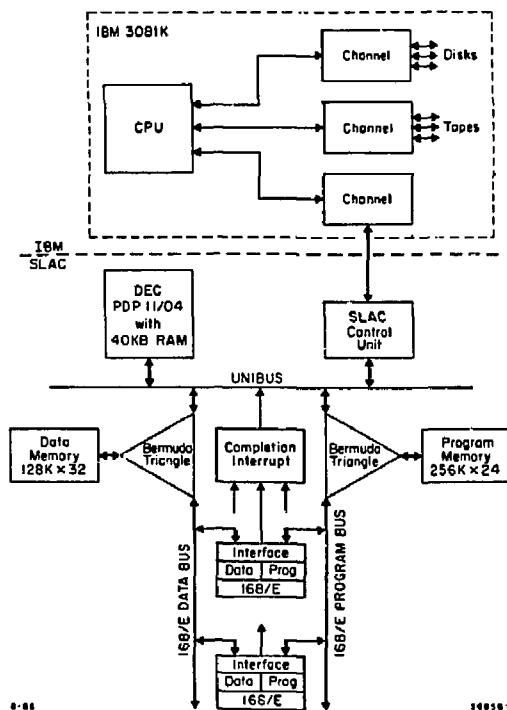


Fig. 14. Block diagram of 168/E processor system at SLAC.

systems are not very sensitive to how the processors are coupled to a host, and in many cases one does not even need a high transfer rate of data to or from the processors.

Because of the very large size of event reconstruction or Monte Carlo codes, one of the emphasis on each of these projects is the ability to handle code written in FORTRAN which can be ported to the processors with little or no modifications. In most cases, ability to run FORTRAN is far more important than speed or costs of the processors.

The question has been frequently raised of whether parallel processing technique will continue to be valid in the SSG era where a single detector will require 1000-2000 VAX 11/780 equivalent processing power. The answer seems to be affirmative and can be understood from the following simple arguments:

- A data acquisition computer with a certain I/O bandwidth recorded the data at the detector.
- What ever the power of the parallel processors (as long as they can run the complete program), one will add enough of them to obtain the required total CPU power.
- As long as rate of event processing is not greatly different from the original data acquisition rate, then the host computer with I/O capacity at least equal to the data acquisition computer will be sufficient to run a processor farm.

It should be emphasized the processors used in these parallel processing projects, which are designed and built within the HEP community, are not real computers. By stripping the hardware down to the bare processing essentials,

they are a very cost effective means to obtain the large scale computing that is needed.

Alternatives to the one-event/one-processor scheme has been frequently considered. In these alternate schemes, one has tried to specialize the processors for certain phases of the event reconstruction, such as pattern finding, or track fitting. These schemes run into a number of problems...

- It takes a considerable amount of development effort to design, debug, and install a specialized processor. As there are so many stages in event processing, there would be a need for many specialized processors.
- In such a scheme, one event would flow through many processors, which increases the total communication time. As intermediate results, are generally larger than the original raw data, the communication time is even worse.
- The throughput of such a system is only as fast as the slowest event in the system at any one time. Since event processing times differ greatly, this can lead to long times when some processors are idle, waiting to send their data to the next stage, which could lead to huge processor inefficiencies.

With these potential drawbacks in all schemes where an event is processed by many processors and with the very high efficiency that can easily be achieved in the one-event/one-processor scheme, it is not clear one will be ever be able to invent a scheme more cost effective than the one-event/one-processor one. In fact it might be *hard enough* to just come close to the same efficiency.

In considering using one of these processor systems, there are a number of issues that should be examined by potential users. Some of which were addressed by users and implementers:

- The user friendliness of the interface between the processors and the host.
- The difficulty of porting the user code to the processors, or writing special code for it.
- The importance of uniformity between the processors, the host computer, and the computer where the code was originally developed.
- The speed versus cost of processor.
- The procedures for checking that the processing system is working correctly.
- The real costs and time scale of manufacturing and maintenance of the system.

An ideal system would address all these issues well, but as things go some systems excel on some issues but not on all of them.

Since on-line triggering and filtering systems have become more sophisticated, the distinction between them and off-line processing systems has become blurred. There was a number of systems with an orientation towards on-line systems that could be considered as candidates for off-line systems as well and vice-versa. The same set of issues are discussed but with different emphasis for the on-line environment.

11.10 CONCLUSION

HEP physicists are quite active in the field of vector and parallel processors, and this activity extends over off-line event and Monte Carlo processing, on-line trigger/filtering, theoretical calculations, and accelerator design. Although only HEP's theoretical work seems appropriate for vector processors, there are some signs they could be used for experimental work as well and their use is worthy

of further study. Use of parallel processors in HEP is well established in all areas with many systems in use and still more being planned.

The next question to the experimentalist, is how are they going to analyze all the events that get processed? That is, it is less clear how vector or parallel processing is going to help with the histogramming, cutting, fitting, etc. These processes one would like to be interactive procedures and yet can consume a lot of I/O resources, not just CPU cycles.

DATA ACQUISITION FOR HIGH
ENERGY PHYSICS EXPERIMENTS*

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Introduction

The general scope of Data Acquisition Systems range from the feedthroughs on the actual detectors to the storage racks for the data recording media as sketched in Fig. 1. The systems are concerned with the conversion of interesting, volatile signals usually to a stable, digitized format. The problems involved are interesting because of the boundary conditions: engineering, economics, required resolution and range, rates, radiation damage, and physical size available for the components. Systems that will be considered here are limited to those for e^+e^- and hadronic colliding beam problems.

The basic components of a Data Acquisition System are indicated in Fig. 2. Component tasks of a Data Acquisition System are preamplification and signal conditioning, digitization, sparsification, data correction, data reduction, and data recording. Implementation of these tasks require considerations for buffering of the data, multiplexing of the data paths, and triggering systems to control the Data Acquisition System.

In these lectures I will attempt to develop a conceptual understanding of the building blocks required for these systems and their relationships to each other in real systems. I will attempt to be pedagogic and introduce a variety of data acquisition issues, rather than attempt to review the field for professionals. The hard problems of actually designing circuits are not touched! I will illustrate with an example of SLD and I will then speculate on an approach for an SSC detector.



Figure 1 The range of a HEP Data Acquisition System.

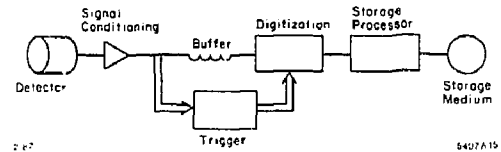


Figure 2 The basic components of a Data Acquisition System.

Design Requirements & Goals

The design requirements for a system begin with the ability to "measure" signals adequately to permit the reconstruction of events. Obviously this maximizes the information gained from the detector, and implies that the processing for each element must adequately consider the resolution and dynamic range of the signals involved. It is necessary to maximize the good data which is recorded, involving a trigger to separate events of interest from background; to minimize deadtime both from the trigger system itself and the read-in process for selected events; to assure adequate calibration of the system; to insure adequate reliability and component lifetime of the system; and to provide adequate diagnostics for timely repair of the system when a component fails. It is necessary to minimize the impact on other detector components by considering the dead space caused by the Data Acquisition System components and the multiple scattering and showering produced by those components. Finally, and obviously, it is necessary to minimize costs. It is a challenge to meet all of these goals for systems with large numbers of channels. Figure 3¹⁻⁶ indicates the gain in channel counts for selected detectors as a function of time from 1960 through that expected for the SSC. It is particularly hard to meet these requirements in hadronic collider environments.

Different machine environments present quite distinct design parameters for a Data Acquisition System. The e^+e^- storage rings are characterized by small cross sections -- corresponding to a $\mathcal{L} \sigma_i$ of approximately 1 Hz and a medium bunch separation in the range of 1 to 10 μ sec. An e^+e^- collider has the same low $\mathcal{L} \sigma_i$ but has an immense bunch separation of order msec. The pp or $p\bar{p}$ rings are characterized by high cross sections with $\mathcal{L} \sigma_i \geq 10$ MHz and medium bunch

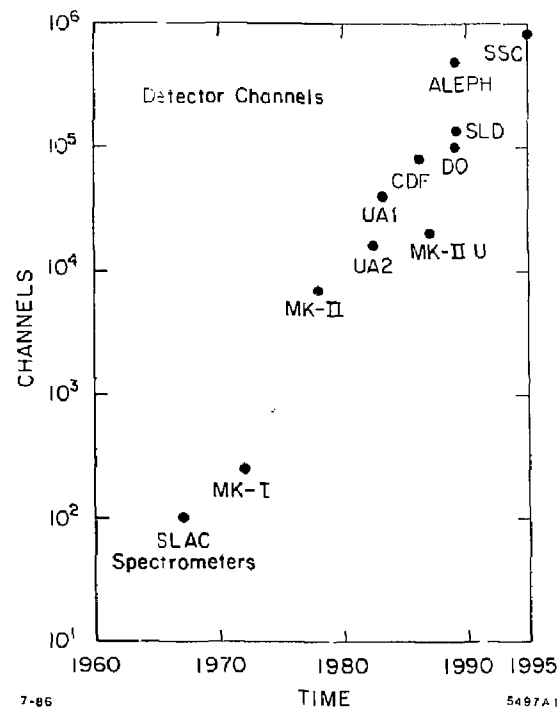


Figure 3 Some examples of the increase in detector channels versus time. The instrumentation for the SLAC spectrometers utilized discrete transistors; MKI utilized Small Scale Integration (SSI) and some Medium Scale Integration (MSI), while in the 1980 and beyond detectors are making extensive use of LSI, VLSI, custom VLSI, and hybrids.

separations of μsecs ranging to small bunch separations of 15 nsecs in the case of the SSC. SSC bunch spacing, cross sections, and luminosity correspond to 1.4 interactions per crossing.

Leading issues for future development include costs, rates, thermal management, radiation hardening, and size. Present cost estimates for SSC detectors indicate that the Data Acquisition Systems amount to roughly one-third of the total cost. Rate considerations range from the multiple hits on a drift chamber wire due to the density of tracks in a high energy jet, to the multiple hits in calorimeter cells from multiple interactions within the resolving time of the device. Current electronic technology tends towards the use of high power in producing high speed circuits. Low rate systems can keep the average power down, but this is not an option for a SSC detector. Radiation estimates for the end walls of a SSC drift chamber range from 10^0 to 10^3 rads per year, due to particles from the interaction region.⁷ This is greater than or equal to limits for conventional CMOS VLSI. As to size, if some proposed SSC drift chambers were outfitted with conventional cable plants consisting of a twisted pair per channel, the cable plant would have a cross sectional area of 1 M².

Digitization

The basic process of digitization includes the measurement of amplitudes for calorimeters, dE/dx , charge division, and signals from silicon vertex detectors; measurement of time differences for drift chambers and time-of-flight systems; the process of waveform recording which is basically the synchronous measurement of amplitudes for time projection chambers, drift chambers, and high rate calorimeters; and the measurement of "hits" for hodoscopes and muon detector

systems. Basic architectures for amplitude measurement are indicated in Fig. 4. In the liquid argon calorimeter the signal from the detector is AC coupled to a charge-sensitive preamplifier, goes through a shaping stage, and is then fed to a gated Analog-to-Digital Converter (ADC). If the liquid argon is replaced by a scintillator viewed by a photomultiplier, the photomultiplier serves as a preamplifier and shaping stage and the signal can be fed directly to the gated ADC. Fast signals may be measured by integration over a gating period, or they may be sampled at their peak using a sufficiently fast ADC, or a "sample-and-hold" circuit may be employed to present an amplitude to a slower ADC. The basic scheme of a sample-and-hold circuit is indicated in Fig. 5(a). The input signal is fed through a switch to a capacitor which follows the voltage waveform. When the switch is opened, the buffer presents the voltage on the capacitor to following circuits. A dual slope ADC is sketched in Fig. 5(b). During the gating period, an operational amplifier is used to integrate the input signal. After the gating period, the operational amplifier is switched to a reference signal of opposite sign and a clock is used to measure the time required for the signal on the integrating capacitor to integrate to zero, thus producing a digital signal proportional to the initial charge. Dual slope ADCs have good integral and differential linearity and have wide dynamic ranges up to 16 bits. Dual slope ADCs tend to be slower than successive approximation and flash ADCs. The structure of a successive approximation ADC is sketched in Fig. 5(c). It uses a Digital-to-Analog Converter (DAC) and a comparator to compare the generated reference voltage to the input voltage. In each of n clock cycles, the "next" most significant DAC bit is turned on. If the DAC current is less than the input current, then the data bit is left on. The cycle is repeated n times. Successive approximation ADCs are

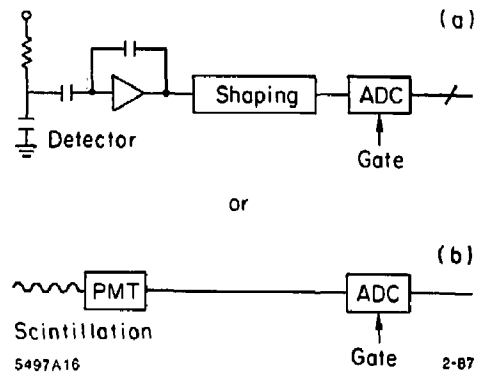


Figure 4 Basic architecture for measurement of a signal's amplitude.

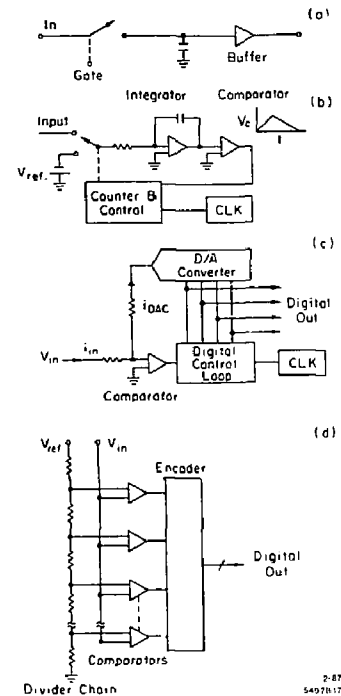


Figure 5 (a) Conceptual design of a sample-and-hold circuit, (b) Conceptual design of a Dual Slope ADC, (c) Conceptual design of a Successive Approximation ADC, (d) Conceptual design of a Flash ADC.

available with resolutions up to 12 bits, moderate speed (12 bits in approximately 1 μ sec), moderate cost, and with modest differential linearity. A flash ADC is shown in Fig. 5(d). An n bit flash ADC utilizes $2^n - 1$ comparators to digitize the input voltage in essentially a single cycle. As shown in the figure, all of the comparators whose input voltage is below their reference voltage are on and all of the others are off. An encoder is used to convert the information from the full set of comparators to a standard digital code. Flash ADCs have speeds in the range of 10 to 20 nsecs but because of the $2^n - 1$ comparators, are typically limited to less than 10 bits. They tend to be expensive, but modern production techniques are driving the costs within range. The divider chain in the flash ADC need not be linear, thus permitting a wider dynamic range of signals to be measured.

A block diagram of a time difference measurement system suitable for drift chamber or time-of-flight applications is indicated in Fig. 6(a). The input signal is amplified and standardized with a threshold discriminator and is then used to start a Time-to-Digital Converter. A standard stop signal stops the conversion and a digital output is presented. Of course, the stop and start signals may be interchangeable. The Time-to-Amplitude-Conversion (TAC) technique is shown in Fig. 6(b). The start and stop signals are used to set and reset a flip-flop which controls a current source charging a capacitor. A capacitor output is buffered to an Analog-to-Digital Converter and has a voltage proportional to Δt . After the Analog-to-Digital Conversion cycle, the capacitor is reset by a switch and the cycle may be repeated. Figure 6(c) shows a dual slope TDC, which is essentially a hybrid of the TAC and the dual slope ADC. Again the start and stop signals control a flip-flop controlling a current source which is used to charge a capacitor to a voltage level proportional to the time difference. After the stop signal, the

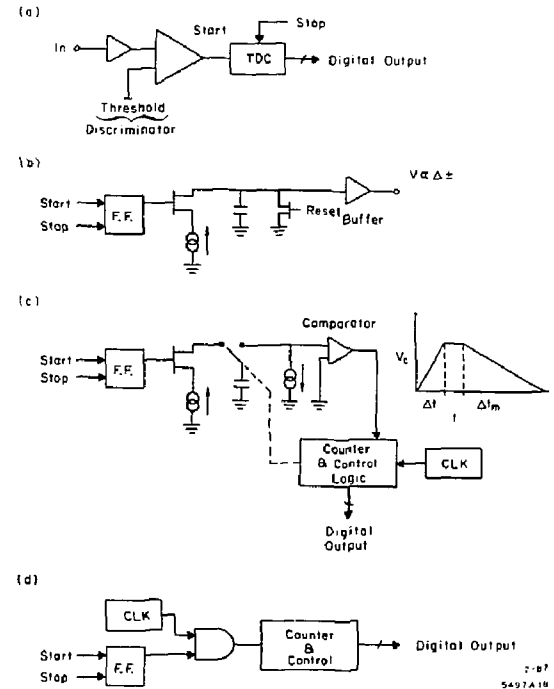


Figure 6 (a) Block diagram for a time difference measurement, (b) Conceptual design of a Time to Amplitude Converter, (c) Conceptual design of a Dual Slope Time to Digital Converter, (d) Conceptual design of a Gated Clock Time to Digital Converter.

capacitor is switched to an opposite sign standard current source and the time required to discharge the capacitor to zero measured as the number of clock ticks into the control logic. Finally, the gated clock scheme is sketched in Fig. 6(d) in which the flip-flop is used to control a clock which is directly counted to produce the digital output. This scheme is appropriate to both coarse resolution time measurements using relatively slow clocks and counters, and recently to higher performance systems using faster clocks and counters.

Waveform recorders synchronously measure the amplitude of the inputs signal at a high enough frequency to reproduce the waveform shape for time projection chambers and drift chambers. Normally, adequate timing information is easily extracted from the set of amplitude measurements thus eliminating hardware discriminators and TDCs. By recording the actual waveform, such systems have essentially the ultimate in ultimate resolution, and produce correlated amplitude measurements of the signals for charge division and dE/dx systems. A basic drawback is the production of perhaps a factor of 100 more data per channel than an ordinary TDC/ADC system. Two basic waveform recording techniques are used. In the unbuffered scheme shown in Fig. 7(a) a flash ADC runs at full rate into a fast digital memory. Advantages in comparison to the buffered scheme are that the system can operate at the full rate of the digital memory and that no calibration of the signal from the flash ADC is required. Possible limitations are the resolution and dynamic range of the flash ADC, and again compared to some analog buffered schemes, the relatively modest speed of the flash ADC. Negative considerations include the power consumption and cost of the flash ADC and memory for such a system.

The buffered analog storage scheme indicated in Fig. 7(b) uses an analog

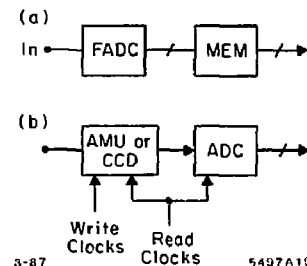


Figure 7 (a) Conceptual designs of Waveform Recording systems, Unbuffered, (b) Conceptual designs of Waveform Recording systems, Buffered analog storage.

memory operating in a fast in-slow out mode to buffer the input data to an ADC. The analog storage might be an Analog Memory Unit (AMU) which is a series of sample-and-hold circuits, or it might be a Charge Coupled Device (CCD). Disadvantages of this scheme are that it is dead during the time of readout at least in simple schemes, and that cell-to-cell corrections might be needed. Its advantages include wide dynamic range, speeds of order 200 MHz, modest power, and that multiplexed sharing of the output ADCs is both possible and natural in such systems.

The Analog Memory Unit has been developed by a Stanford/SLAC Collaboration for SLD.⁸ It is a VLSI array of 16×16 sample-and-hold circuits as indicated in Fig. 8.

Multiplexing

Multiplexing in Data Acquisition Systems generally means sharing of expensive components in the time rather than the frequency domain. Multiplexing lowers the ultimate system speed and may cause undesirable deadtime. A simple example of multiplexing is indicated in Fig. 9(a) where a set of digitizers for some set of detector channels share a common digital data bus, a computer interface, and a common computer and data storage system. Perhaps the "ultimate" multiplexing system is indicated in Fig. 9(b) - a magnetostrictive spark chamber readout scheme. Here a magnetostrictive wand is placed at the end of a large set of spark chamber wires. A sense coil and preamp drive a multithit time measurement system. The wand, preamp, cables, time digitizer, etc. are all shared in this scheme.

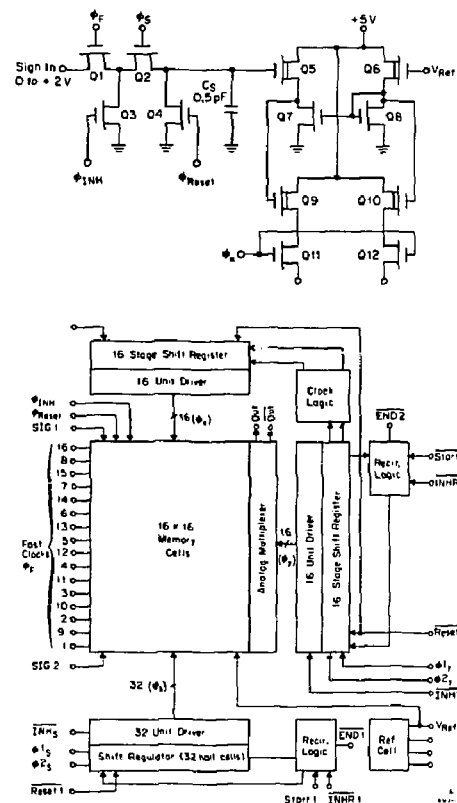


Figure 8 Block diagram of the Analog Memory Unit.

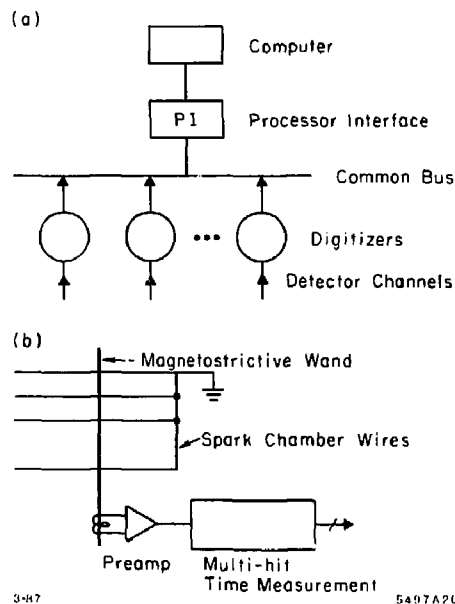
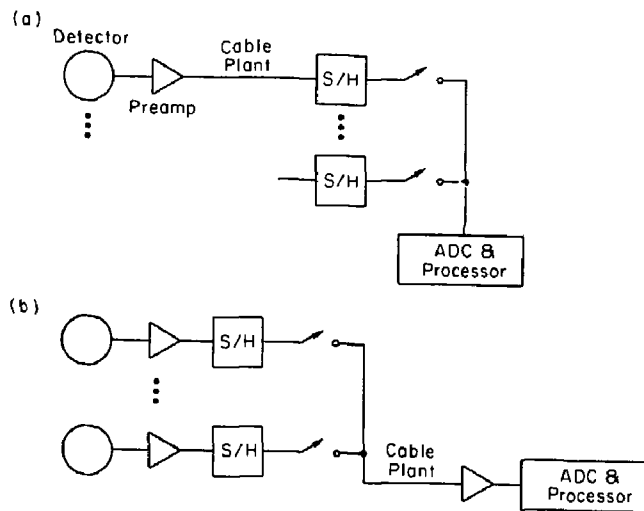


Figure 9 (a) A simple example of multiplexing showing the sharing of a digital bus, processor interface, and computer, (b) The "ultimate" multiplexing situation using a magnetostrictive wand for spark chamber readout.

Several recent Data Acquisition Systems share ADCs, thus implying that analog storage of the shaped input signals is needed. These schemes divide into two classes. The first, illustrated in Fig. 10(a) has the preamplifier located on the detector. The preamps are connected via a cable plant to a set of sample-and-holds and a shared ADC and processor. Technical considerations limited the multiplexing levels to 32 sample-and-hold or Time-to-Amplitude Conversion channels per CAMAC module, and about 600 such channels per ADC. These systems take about 30 msec to process a crate worth of channels. This system has been used by Mark II, Mark III, ASP, MAC, LASS, and is intended for use by BEPC and others.

An interesting topological distortion of the previous scheme is shown in Fig. 10(b). Here the sample-and-holds are adjacent to the preamplifiers on the detector, their outputs are multiplexed, and a much reduced cable plant is used to transmit analog signals to the ADC and processor. The cable plant is shared and the signal storage is on or in the detector. Current examples of such systems share a 125-400 channels per cable and take 2-50 msec of processing time. Such a system is in fabrication for SLD, and a system with the ADC adjacent to the sample-and-hold and transmitting digital data on the cable plant is in use by CDF.¹ The photograph of the hybrid AMU (HAMU) developed for SLD is shown in Fig. 11. It contains 16 of the 256 sample-and-hold per chip AMU circuits.



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Figure 10 (a) A shared ADC system with preamps separated from the sample-and-holds, (b) A shared cable plant and ADC system.

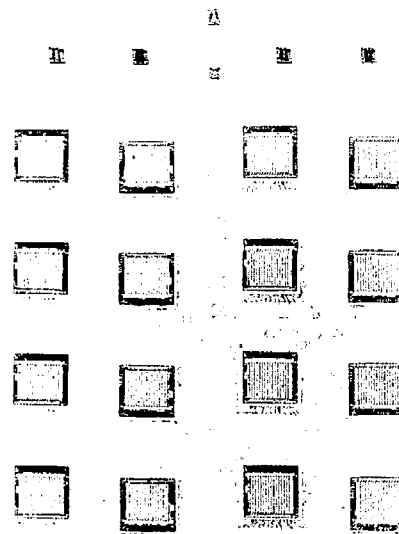


Figure 11 Photograph of the hybrid AMU.

Data Sparsification, Correction and Reduction

Sparsification or data compression is utilized to reduce the burden on the back end of Data Acquisition Systems. Early systems recorded the data from every channel in the system and a particular channel was identified by its position in the data block. Subsequently, channels with only pedestal information or zeroes were suppressed and address information was added to the data stream. The current strategy is to output coordinates which are more physical, in the sense of processing calorimeter data to remove the baseline and pedestals or to process waveforms to extract the arrival times and charge integrals.

Zero suppression techniques utilize hardware in the digitizer modules to detect the zeroes and suppress the readout cycle, typically using the CAMAC Q-scan mechanism. Subsequent developments used firmware processor logic typified by the SLAC BADC and similar commercial products.⁹ The CDF RABBIT system utilizes analog thresholds and baseline subtractions prior to the digitization circuit to accomplish sparsification. A digital custom VLSI circuit is being developed for SLD to accomplish data suppression and compression as well as data correction for waveform recording and calorimetry.

Data correction involves the application of calibration corrections which may apply:

- globally: for example, the drift chamber common start time or a calorimeter scale factor;
- to data acquisition channels: for example, a preamp gain correction;
- to a data acquisition cell: for example, an AMU cell offset;
- to a detector channel itself: for example, a drift chamber wire position.

The corrections may be applied by on-line or off-line computers, by programmable crate controllers such as the SLAC BADC, by a programmable FASTBUS processor such as the SSP, MX, or 168/E, by programmable VME or Multibus processors, or by fixed program custom chips such as the Data Correction Unit (DCU).^{1, 10-14}

A block diagram of the Data Correction Unit is indicated in Fig. 12. The device performs a piece-wise linear correction to the input data, operating as a digital pipeline at 750 nsecs per step. The correction constants and the data are loaded synchronously. The corrected data are passed to one of two compaction sections. In calorimetry mode, the hardware performs the range determination and the baseline subtraction. The logic of the wire chamber mode is sketched in Fig. 13. The DCU snips the indicated waveform from the longer data stream of the AMU. The FIFO allows the leading edge of the waveform to be included in the snipped section even though a high trigger threshold is employed. Counting circuitry keeps the trailing edge of the waveform included with the snipped record. A photograph of the mask for the DCU is shown in Fig. 14.

The data reduction process covers the range of algorithms from sophisticated, complex, and often changed analysis implemented on general purpose computers to digital filters used to deconvolve drift chamber or calorimeter data. Rather sophisticated level 3 trigger computers may be used to implement algorithms similar to those of the general purpose computers. Devices proposed or used for level 3 trigger computers include the 168/E, the 3081/E, the DEC MicroVAX, the FASTBUS VAX, and the ACP.^{2, 15-17} Other possibilities include the semiprogrammable data acquisition modules such as the Waveform Sampling Module

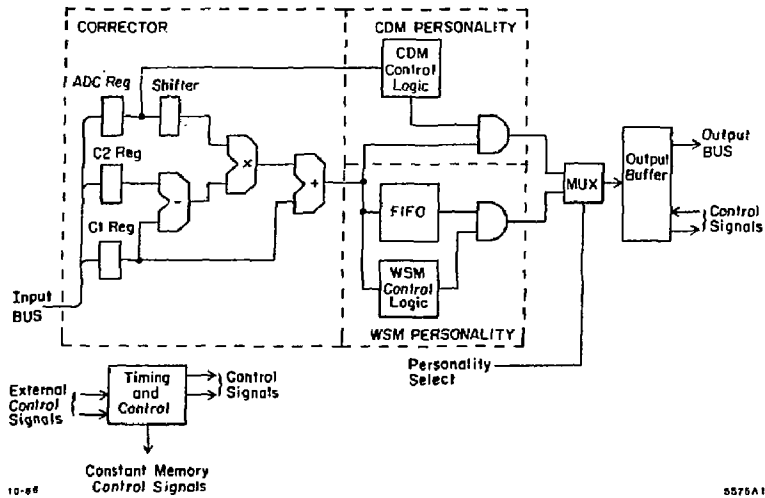
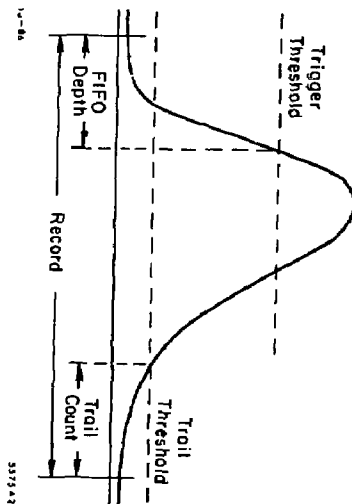


Figure 12 Block diagram of the Data Correction Unit.

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Figure 13 The operation performed by the data compression circuit of the DCU using the WSM Personality section.



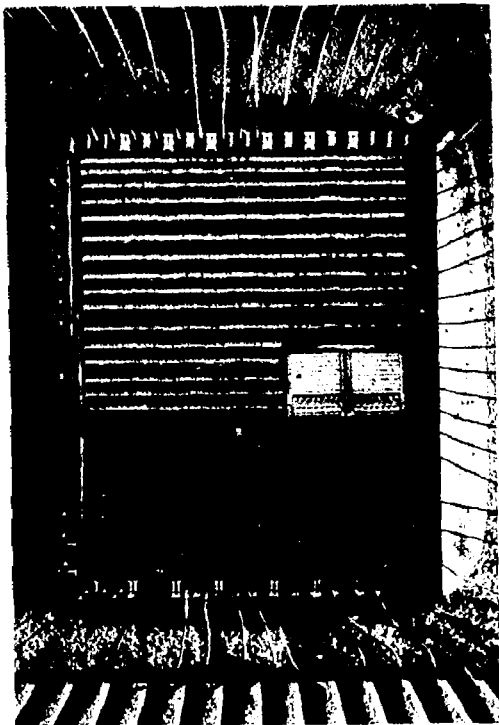


Figure 14 A photograph of the DCU.

(WSM) being developed for SLD which I will describe later. There is a clear trend to large increases in the "equivalent computing capacity" located towards the front end of the Data Acquisition System.

Data Acquisition Buses & Standards

Two significant data acquisition buses have been developed by and for high energy physics. These bus standards include specifications for mechanical crates; for modules with standard connections to a standardized back plane; and for communication protocols which include the electrical standards in the crates, the cables to the computer interface, and standardized software access to and from the modules. The first, and still widely used system, is CAMAC. CAMAC is a 24-bit, single master system, running at a rate of about 1 MHz. Various needs have forced development of exceptions to the single master in CAMAC, but they do not fit naturally within the architecture.

Because of the natural multiplexing and concentration towards the back of a Data Acquisition System, there is a need for a higher performance bus than CAMAC, in fact with greater or equal performance to that of a computer bus. Such a bus needs features for arbitrary data transfers between master and slave modules. The FASTBUS system is a 32-bit bus with multiple masters, very high speed, (≥ 2 MHz) and mechanics designed for high power, large boards, and multiple input cables.

Extensive experience with these standards indicate that they significantly ease engineering development tasks because of their well developed protocols and hardware and software interfaces; encourage laboratory interchanges; and encourage commercialization. However, on any very large system they are probably an

imperfect match to any given problem. Commercial standards that have been used in HEP include the VME based on the Motorola 68000 series of microprocessors and Multibus based on the Intel 8086 bus series of microprocessors.^{12,13}

Many hybrid schemes have been developed. There is a pressing need for standards and protocols for high speed serial digital and analog transmission, which might utilize either electrical cables or fiber optic transmission lines.

Data Recording

The amount of data per event in various detectors has been growing prodigiously over the years and is crudely indicated in Fig. 15. In e^+e^- machines the data recording limitation is usually the luminosity of the machine itself. For example, SLC and LEP produce of order 1 million events per year. For hadronic colliders the usual limitation is the ability to analyze the data that can be recorded. Estimates of the computing power required to analyze CDF or SLD data are about 100 mip secs per event. Note that the VAX 11/780 is roughly a 1 mip machine. A crude rule of thumb is that event processing is about one-third of the experiment computing load, so that about 3×10^8 CPU secs are needed per million events. Since this processing power is required in less than one year, a facility with ≥ 30 mip is needed. This is a reasonable match to laboratory computing center facilities today. I note, however, that the SSC Off-line Computing and Networking Group has estimated that 10,000 mip will be required after the first year of SSC operation, consisting of ≥ 200 mip in a central analysis machine with the remainder in processor farms in order to process $10^8 - 10^9$ events per year.¹⁸

The planned event sizes of the current generation of detectors range from

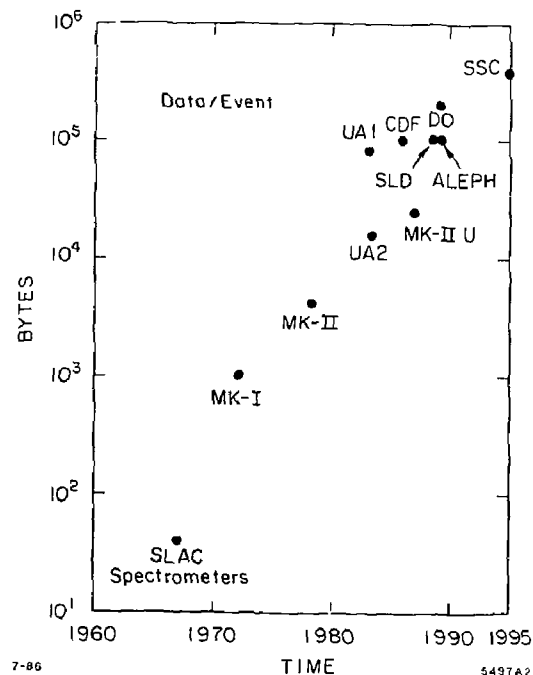


Figure 15 Some examples of the increase in data per event versus time.

100 to 400 Kbytes, and begin to put strains on available storage media. A 6250 BPI tape corresponds to about 150 Mbytes, or 400 - 1500 events, which implies several tapes per hour. Present optical disk WORM (Write Once-Read Multiple) technology has ≥ 1000 Mbytes per side on two-sided 12-inch disks. However, the current writing speeds of order 200 Kbytes/sec are marginal.

Triggers

Because of the data recording and processing limitations, triggering systems are needed to reject machine backgrounds and enhance the concentration of events of physics interest. The usual model is that of a multistage trigger: level 1 is a fast, simple, possibly analog trigger operating in about 1 μ sec to reduce the rate to level 2. Level 2 is a more sophisticated, usually digital, track and calorimeter cluster finding trigger operating in about 10 μ secs which reduces the rate to level 3. Level 3 is a more sophisticated system of highly programmable processors, usually many identical machines operating in parallel, which can take 10 to 1000 msec each to present data to the on-line computer and a recording system. The indicated times for each trigger are typical for hadron colliders, while e^+e^- machines usually have more relaxed requirements. A helpful distinction to these three levels is the state of organization of the data.

In level 3 the data is through the processing of digitization, sparsification, correction, and perhaps some reduction. The events have been assembled into buffers by "Event Builders." The full events are passed to individual processors to be strategically analyzed. Strategic analysis implies working on the features of the event most likely to permit that event to be rejected quickly. An impression of the scale may be gained from some present and planned detectors. CDF is

using an Advanced Computing Project (ACP) cluster corresponding to about 20 11/78 zeroes equivalent. D0 is planning to use about 50 MicroVAX IIs with dual ported memory. SLD is intending to use the FASTBUS VAX system of about 15 MicroVAX IIs interfaced to FASTBUS, although the motivation is primarily the optimization and balancing of on-line and off-line computing loads rather than a necessary reduction in data storage requirements. It is estimated that an SSC 4 π detector will require the equivalent of about 500 11/78 zeroes. There is general agreement that the output rate from the level 3 trigger should be between 1 - 10 Hz.

The two basic schemes for level 1 triggers are indicated in Figs. 16(a) and 16(b). If the level 1 trigger time is greater than the machine interpulse period, the scheme equivalent to 16(a) can be utilized. The delay might be cable, an AMU or CCD operating in a fast in - fast out mode, multiple AMUs or CCDs operating in the fast in - slow out mode, or flash ADCs driving a digital memory. The essential feature is that an analog or digital delay of some form is required while the first level trigger performs. For moderate to slow machines the explicit delay may be finessed as indicated in scheme B. Here the data is buffered in either analog or digital form. If the trigger decision is negative, then the data of the next event simply replaces the old data in the buffer.

Deadtime is encountered when an element being entered in the system is busy. The system designer typically chooses a level 2 scheme with no parallel buffers, modest efficiency of the level 2 trigger, and low cost as shown in Fig. 17(b); or parallel buffers with higher efficiency and the higher cost of the buffers, as in Fig. 17(a). The tradeoffs between cost and deadtime are usually made at this stage.

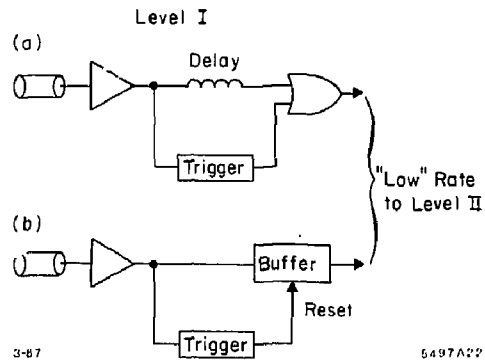


Figure 16 Basic schemes for level 1 triggers: (a) Delay signal during trigger decision, (b) Store signal and re-store if trigger decision negative.

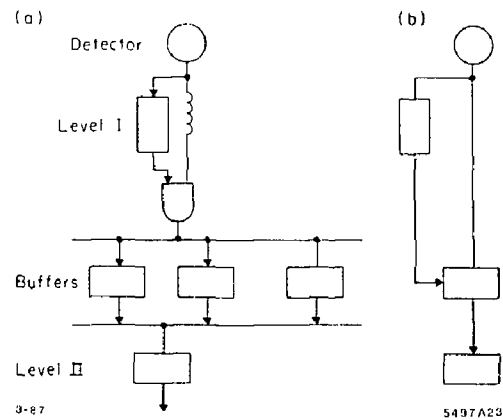


Figure 17 Level 2 trigger schemes with (a) multiple buffers, (b) a single buffer.

Trigger strategies for level 1 at e^+e^- machines are almost trivial. A threshold on the number of hit cells in the tracking system, and simple energy thresholds in calorimetry implemented in modest digital logic or analog sums, is usually adequate. The situation is much more difficult in hadronic colliders since $\cos\theta$ weighted calorimetric sums in the high rate environment are required.¹⁹ Mechanisms to enforce cuts on P_t , total E_t , and missing E_t may be used along with schemes to identify isolated leptons or jets. Level 2 schemes on e^+e^- machines usually employ a track finding trigger. This can be implemented as a coarse resolution dictionary look-up procedure using memories to store pre-established patterns.^{20, 22} Level 2 triggers on hadronic colliders may include track finding triggers, tracking in transition radiation detectors, *muon finding and tracking*, and isolation of electrons.^{22, 23}

System Optimization

Most of the work on Data Acquisition System optimization that I am aware of is extremely informal, and that which is presented here is primarily due to Siskind.²⁴ The components of a system are digitization, sparsification, data correction, and data reduction. The order of these components in the data acquisition chain is semi-arbitrary. Stages of sparsification and reduction may lower the performance requirements of subsequent steps by reducing their bandwidth requirements. Storage or buffering may be inserted between any steps, including stages upstream of the digitization. Such storage may be necessary for the trigger. However, if a stage is unbuffered, then the subsequent stages must run at the speed and duty factor of the predecessor. The cost of the system is the sum of the costs of the processing, the buffering, and the interconnections. A concept

of processor efficiency (ϵ_p) can be defined as the fraction of time a processor operates on data which will ultimately be retained or, at least, sent to level 3. We claim that efficient processing requires buffering.

Consider the case of no buffering and no multiplexing as shown in Fig. 18(a). For an e^+e^- machine,

$$\epsilon_p(ADC) = R \cdot \tau_{ADC} \approx 2 \cdot 10\mu s \approx 10^{-3}\% \quad ,$$

where R is the trigger rate and τ_{ADC} is the ADC processing time. A scheme with multiplexing is sketched in Fig. 18(b). Let N_m equal the number of multiplexed channels per ADC. A deadtime of about 10% with a trigger rate of 2 Hz implies that $N_m \cdot \tau_{ADC} < 50$ msec which implies $N_m \leq 5000$ for $\tau_{ADC} \approx 10$ μ secs. Considerations of mechanics, cabling, and reliability probably limit N_m to approximately 1000 channels. Therefore,

$$\epsilon_p(ADC) \approx 2 \cdot 10^{-5} \cdot 10^3 = 2\% \quad .$$

A scheme with multiplexing and buffering is indicated in Fig. 18(c). Figure 18(a) and 18(b) are alternative buffers for the ADC. Using the same deadtime assumptions as before, $N_m \cdot \tau_{ADC} \leq 300$ msec. For 1000 channels and $\tau_{ADC} \approx 10$ μ secs, double buffering is worthless, but, consider the digitization and processing of 128 channels of waveform recorder cells with a modestly faster ADC:

$$\epsilon_p = R \cdot N_m \cdot \tau_{ADC} = 2 \cdot 6.5 \times 10^4 \cdot 1.5 \times 10^{-6} \approx 20\% \quad .$$

Note that the multiplexing and buffering can be digital or analog and can be anywhere in the chain. Figure 19 shows the deadtime versus event processing

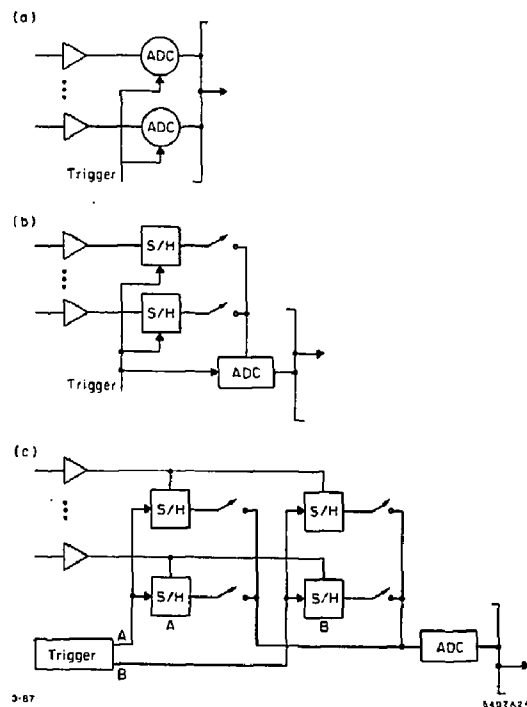


Figure 18 Block diagrams of ADC systems to illustrate the idea of processor efficiency: (a) Without buffering and multiplexing. (b) With multiplexing of the ADC. (c) With multiple buffers for each channel and multiplexing of the ADC.

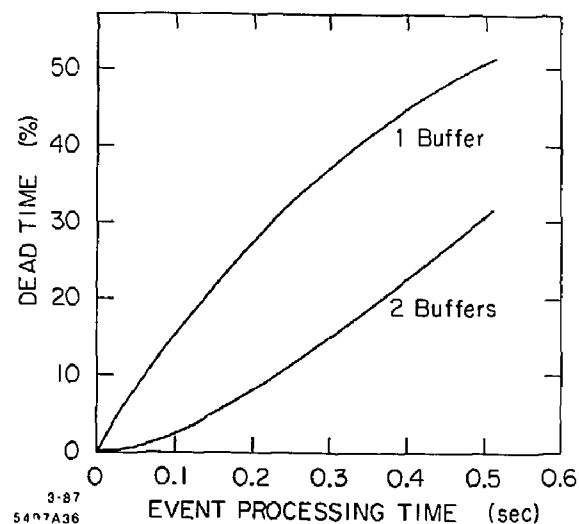


Figure 19 Deadtime versus event processing time for the scheme of Fig. 18(c). The input rate is 2 Hz.

times for a system modelled after the scheme of Fig. 18(c). For a rate of 2 Hz and with deadtime fixed at 10%, Fig. 20 indicates the maximum processor efficiency as a function of the number of buffers.

The global characteristics of the system are determined by the order of digitization, sparsification, data correction, and data reduction; by ϵ_p ; by the amount of storage; and by the amount of multiplexing. The "ideal" system has maximal ϵ_p , maximal multiplexing, and minimal buffering. The relative weights in this mix depend on the current state of the art in processing signal transmission, data storage, etc.

The SLD Data Acquisition System

The SLD is a large solenoidal detector being designed and built to study e^+e^- collisions at 100 GeV at SLC. A quadrant view is shown in Fig. 21.²⁵ Its major systems are:

- A CCD vertex detector having approximately 250 CCD chips arranged in two concentric cylinders about the beampipe. Each CCD has 223,000 pixels, and each pixel is 22 microns square.
- The tracking system consisting of the central drift chamber and two sets of planar endcaps. The central drift chamber utilizes charge division readout to enhance the track finding. The system has a total of approximately 12,000 waveforms sampling channels to provide timing, optimal two-track resolution, charge division, and dE/dx measurement.
- Particle identification is provided by a system of Čerenkov Ring Imaging Detectors (CRID)s arranged in barrel and endcap geometry. Liquid and

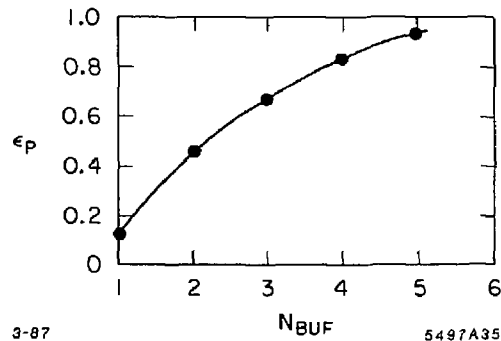


Figure 20 Processor efficiency versus number of buffers at fixed input event rate and deadtime.

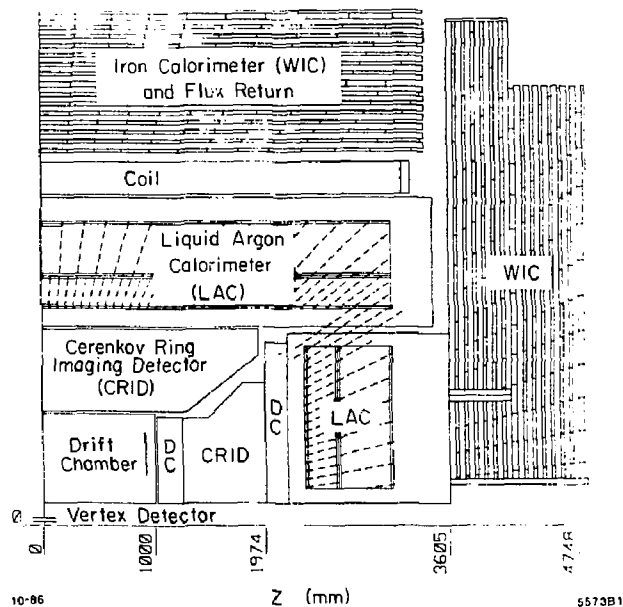


Figure 21 A quadrant view of SLD.

gas radiators produce rings of photons imaged onto photon detectors having three-dimensional readout of the photo-electron position utilizing wire position, drift time, and charge division. The system has approximately 22,000 channels similar to those of the tracking system.

- A lead liquid argon calorimeter (LAC) is used to provide electromagnetic and hadronic calorimetric measurements. The system has two electromagnetic and two hadronic readout segments and is arranged in projective tower geometry. The system has approximately 44,000 channels.
- Tails of showers leaking from the LAC are measured in the Warm Iron Calorimeter (WIC) consisting of layers of iron alternated with Frascati Plastic Tube (limited streamer mode) readout. The WIC is equipped with pads matching the towers of the LAC and has about 10,000 channels. Alternate sides of the Frascati Plastic Tubes are equipped with strip readout to form a muon identification system. There are about 110K channels of muon strips.
- A luminosity monitor utilizing a silicon/tungsten sandwich with about 2,000 channels similar to the LAC.

The design of the SLD Data Acquisition System is strongly guided by the characteristics of the SLC environment, in particular the low repetition rate of 180 Hz and the small e^+e^- cross section. The structural guidelines include:

- situating the preamplifiers with integral calibration and test circuits adjacent to or on relevant detectors;
- analog buffering of signals on the detectors;
- multiplexing of these analog buffers to the outside world utilizing analog fiber optics cables to minimize the cable plant;²⁶

- digitization of these signals by standard fiber optic receiver/ADC modules mounted on the FASTBUS auxiliary backplane;
- sparsification and correction of the data by the digital correction unit;
- reduction by microprocessors at the module data acquisition level and by further crate level programmable processors. Very similar FASTBUS modules are being designed for the waveform systems (Tracking and CRID) and for calorimetry (LAC and WIC).

Because of the SLC rate of 180 Hz, no formal level 1 trigger is required, and data is passed directly to a level 2 trigger. This trigger has track finding and calorimetric energy cuts computed by trigger processors which are programmable FASTBUS modules, and makes a decision to accept or reject the event in the 5.6 ms interpulse period. The expected trigger rate of 1 - 2 Hz permits 50 ms for event readout with a deadtime of less than 10%. The long readout time allows extensive multiplexing of the analog storage elements in the detector, minimizing cable plant and readout hardware. The calorimetry trigger data is extracted from the actual data by the Calorimetry Data Module and thus requires collection of all the data in about 2 msec. The drift chamber trigger data flow is independent of the waveform data, permitting use of the 50 ms for the waveform readout. The level 3 trigger is expected to be implemented as a set of 15 FASTBUS MicroVAX II processors.

The overall organization of the LAC electronics is indicated in Fig. 22. There are 192 channels per optical fiber and 600 channels associated with each flange on the calorimeter bulkhead. Each FASTBUS board services eight optical fibers and two crates are sufficient for the entire system. A block diagram of the electronics is shown in Fig. 23. Signals are amplified in 8-channel hybrid preamplifiers which

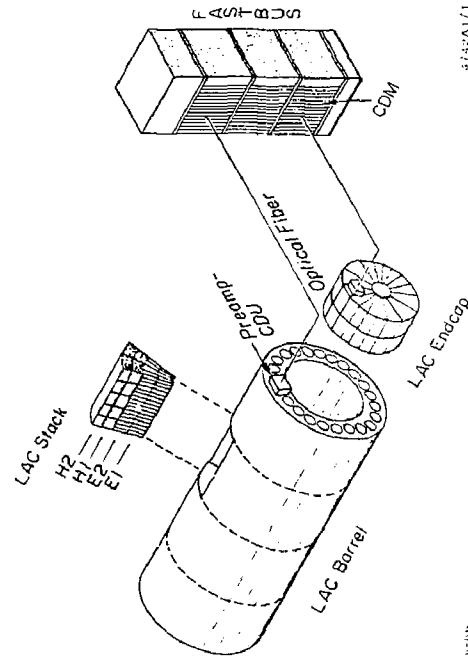


Figure 22 Organization of the Liquid Argon Calorimeter electronics.

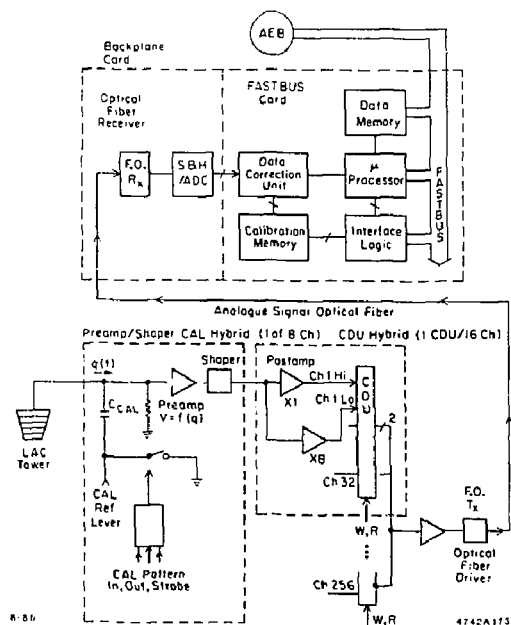


Figure 23 Block diagram of the LAC electronics.

include calibration circuitry with a routing section to determine which channels receive calibration pulses. A photograph of the preamplifier is shown in Fig. 24. Signals from the preamps are routed to the Calorimetry Data Unit (CDU) hybrid which is based on another VLSI chip analogous to the AMU. The CDU chip has 32 channels of sample-and-hold and each channel has four sampling cells. The CDU hybrid has gain of one and gain of eight buffers for each channel, thus implementing a dual range system to achieve the required dynamic range. Each CDU services 16 LAC channels and 12 CDU hybrids are ganged on an optical fiber driver to the FASTBUS system. The structure of the CDU chip is indicated in Fig. 25. The optical fibers are received by an 8-channel card located on the auxiliary FASTBUS back plane. The Calorimetry Data Module contains a Data Correction Unit, calibration memory, a microprocessor, and associated interface logic. The Data Correction Unit does a piecewise linear correction to the incoming data, determines the appropriate gain range to use, and subtracts the baseline before passing the data to the microprocessor to prepare data for the final data stream and perform energy sums for the calorimetric trigger. Six preamplifier hybrids and three CDU hybrids comprise one daughter-board as shown in Fig. 26(a) and sets of daughter-boards are grouped on a mother-board as shown in Fig. 26(b).

The higher level FASTBUS organization of the system is shown in Fig. 27. The data flow for the calorimetric trigger begins with the assembly of cluster sums in each Calorimetry Data Module. Data are read from each module through the pair of SIs by the trigger processor at the second level. The trigger processor performs the global calorimetric sums and cuts, evaluates data from the Drift Chamber Trigger Module, and makes the accept/reject decision. After the

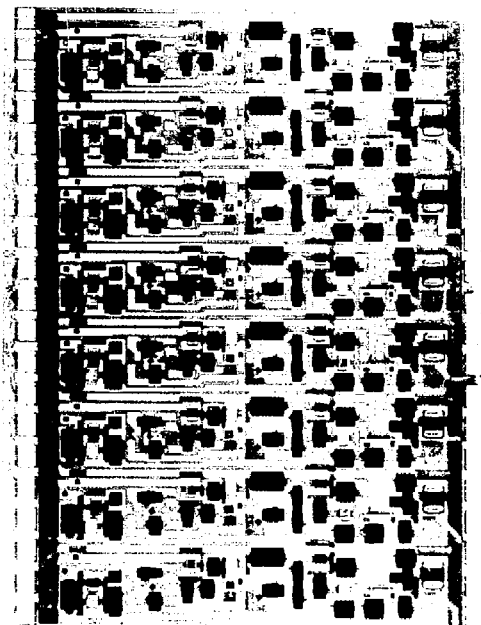


Figure 24 Photograph of the LAC preamplifier.

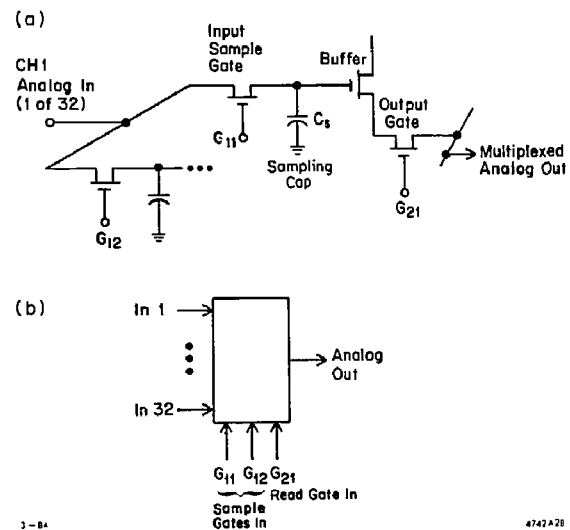


Figure 25 Block diagram of the Calorimetry Data Unit chip.

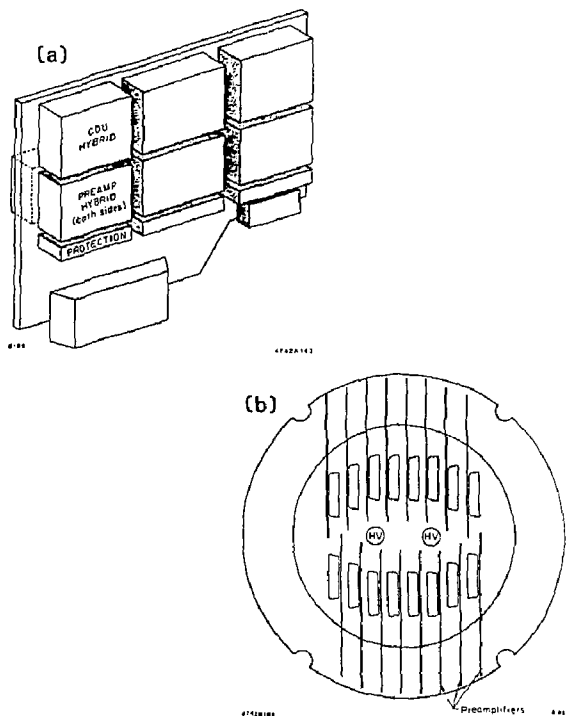


Figure 26 Physical organization of the LAC front end: (a) Layout of a 48-channel daughter board, (b) Indication of the grouping of daughter boards to service 600 channels.

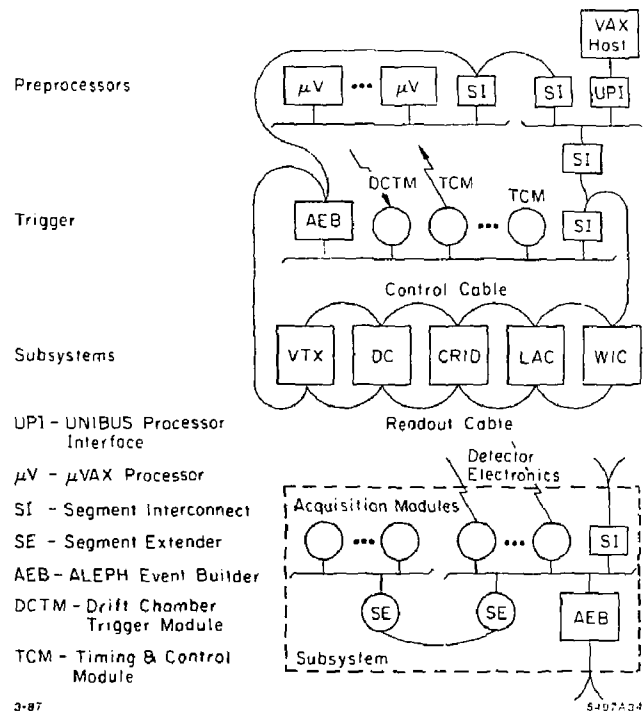


Figure 27 FASTBUS organization of the SLD Data Acquisition System.

calorimetry data module has produced its trigger sums, and if the event is accepted, it prepares a bitmap of hit towers. Bitmaps are passed this time through the data level segment extenders to the calorimeter event builder. The event builder uses the bitmaps to make clusters of hits and their associated neighbors. This is inherently a two-dimensional problem which cannot be solved at the time of data collection by the individual Calorimetry Data Modules. The data then flow from the event builder, flow out of AEB cable slave, only 1 SI between μ VAX and AEB to the SI and FASTBUS VAX and eventually through the processor interface to the host.

The WIC readout electronics are shown in Fig. 28. Since the WIC has a smaller dynamic range than the LAC, a single range system is used and a hybrid having 32 preamplifiers and 1 CDU chip is employed. The WIC utilizes the same fiber optic receivers and calorimetry data modules as the LAC.

The organization of the waveform sampling electronics for the tracking system and for the CRID are indicated in Fig. 29. Signals from each end of the wire are amplified and fed to AMUs. Sixty-four channels of AMU are ganged together over a single optical fiber and transmitted to a Waveform Sampling Module (WSM). Corresponding sets of AMUs are interlaced to feed each WSM channel. The block diagram of the system including the internals of the Waveform Sampling Module is shown in Fig. 30. For the tracking system, eight channels of preamplifier with associated calibration and calibration routing circuitry are organized as a single hybrid. Figure 31 is a photograph of the hybrid preamplifier. The WSM utilizes the previously described DCU to make piecewise linear corrections to the AMU cells and compact the data. The DCU handles the data from both ends of each wire to ensure that the snipped section corresponds from one end

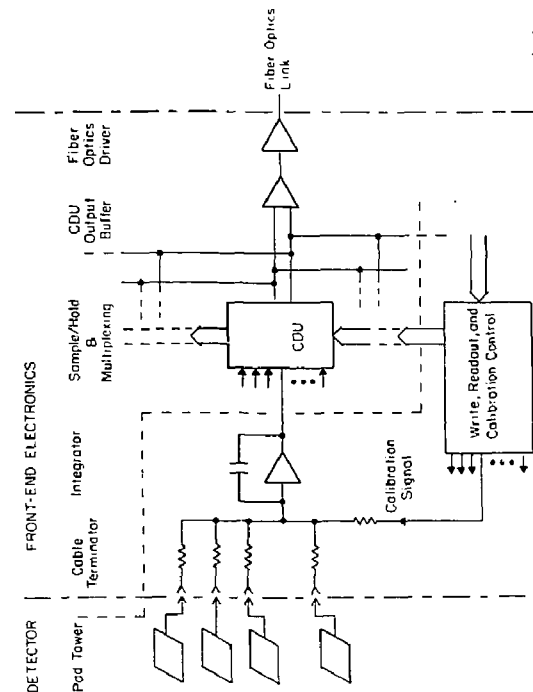


Figure 28 Block diagram of the WIC calorimeter readout electronics.

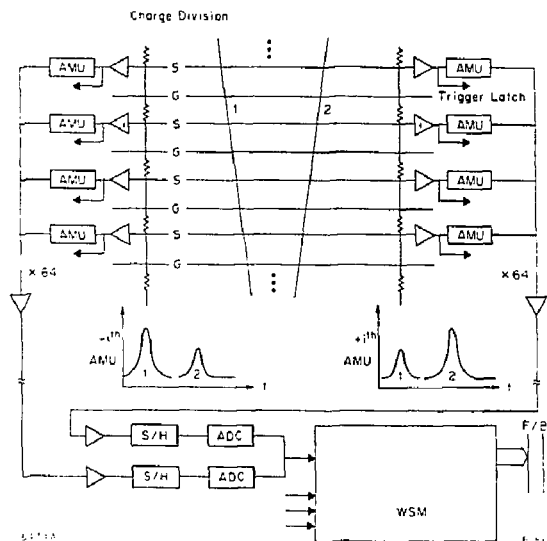


Figure 29 Organization of the Waveform Sampling electronics for the Tracking Systems and the CRID.

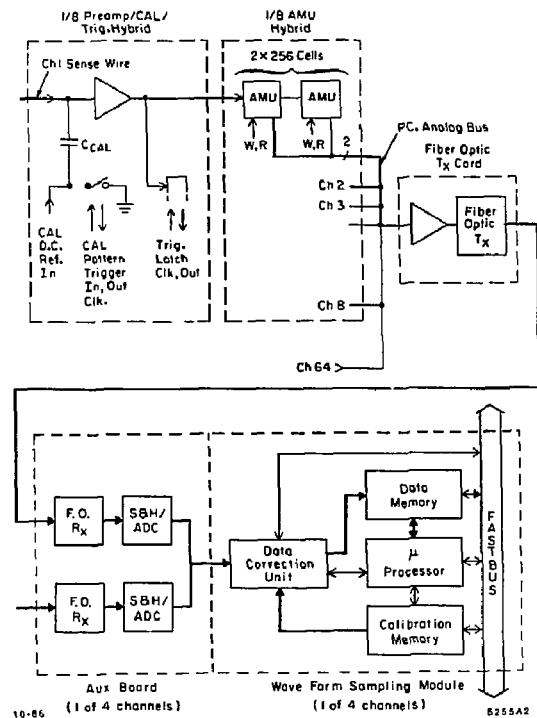


Figure 30 Block diagram of the Wave Form Sampling system.

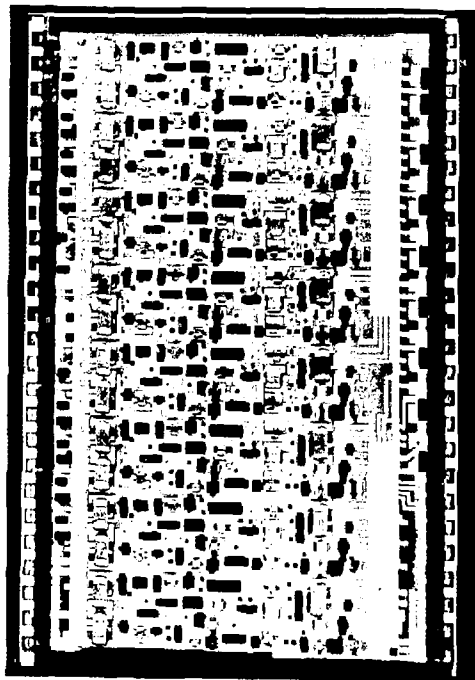


Figure 31 Photograph of the hybrid drift chamber preamplifier. A second layer contains the trigger, calibration, and power pulsing circuitry.

of the wire to the other. Figure 32 shows AMU output from a drift chamber prototype. Figure 32(a) is the output corresponding to a single track while Fig. 32(b) shows the output from seven closely spaced tracks. Note that a track separation corresponding to 1 millimeter is readily resolved.

Since the processing of the tracking system AMU data requires about 50 msec, a completely separate data path is utilized for the trigger data. The scheme is indicated in Fig. 33. Each preamplifier output channel is viewed by a threshold comparator whose output is fed through a parallel in, serial in-out shift register, which is implemented as a semi-custom gate array and is installed as a component of the preamplifier hybrid. The gate arrays are electrically chained together in sets corresponding to about 1500 sense wires, and read out at 10 MHz over optical fibers to Drift Chamber Trigger Module. The Trigger Module reduces the data to a bit map of "hit" drift chamber cells for dictionary look up and track finding by the Trigger Processor.

In both the CDM and WSM, the DCU and the calibration memory operates synchronously with the acquisition of data from the analog storage sections. However, the data memory associated with the microprocessor is sufficiently large to allow it to effectively serve as the buffer for multiple events, thereby significantly increasing the processing efficiency of the microprocessor. The processor will be a Motorola 68020, in contrast to the digital signal processor of earlier designs.

The CCD signal processing system is shown in Fig. 34. Hybrid preamplifiers are located adjacent to the vertex detector itself and signals are fed over strip lines to amplifier modules located in FASTBUS. A subsampler produces four signals with 5 ns relative delays, so that effectively 16 samples at 5 ns intervals are fed to a flash ADC. The correlated sampling module sums 4 actual samples

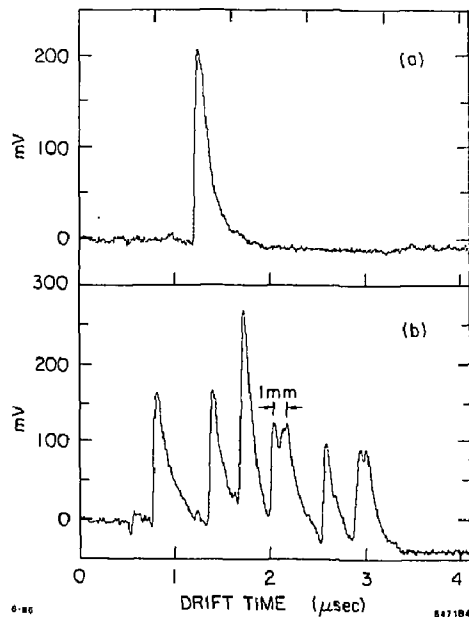


Figure 32 (a) Waveform output for a single track, (b) Waveform output for seven closely spaced tracks.

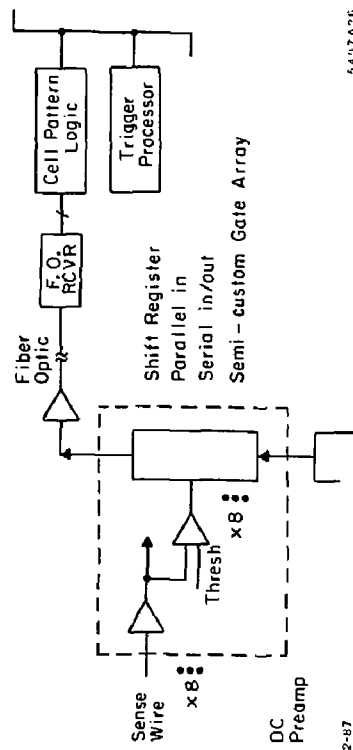


Figure 33 Block diagram of drift chamber trigger data path.

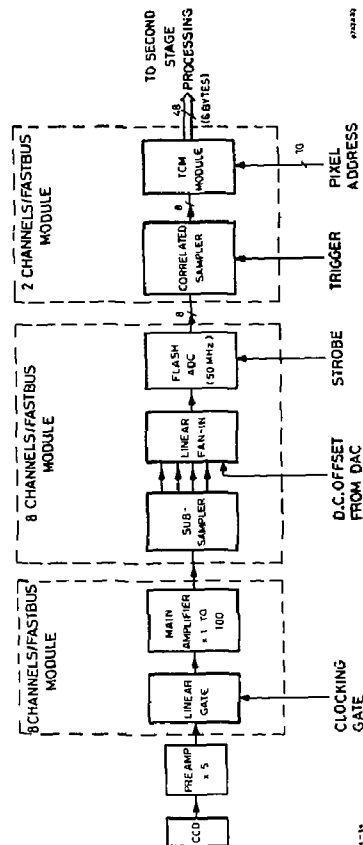


Figure 34 Block diagram of CCD signal processing system.

and subtracts them from those of the previous pixel. The TCM is a Thresholding, Clustering, and Memory module. It produces clusters found by amplitude cuts and adds address information to the data stream. The block diagram corresponds to the logic required for each CCD chip, and it processes approximately 223,000 pixels in less than 50 ms. The requirements of the second stage processing are set by the need to reduce the background level from synchrotron radiation photons. The details of this system are still being designed.

The WIC muon strip readout scheme is indicated in Fig. 35. The logic is implemented as a monolithic CMOS circuit except for the preamplifier section employing discrete components on a 8 channel hybrid. Small boards of 32 channels are physically located in the laminations of the calorimeter, and groups of about 3500-channels are read by a Motorola 68020 controlled module over fiber optic lines. The system is not used in the trigger during normal SLAC operation, but fast "or" outputs are used for a cosmic ray trigger.

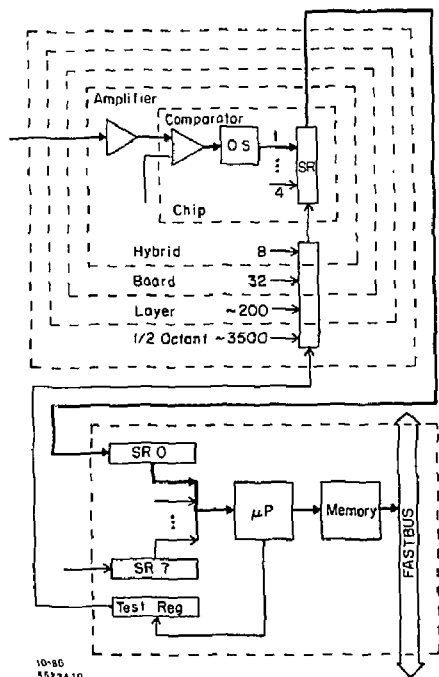


Figure 35 Block diagram of the WIC muon tracking system.

A summary of the data flow and event processing power in the SLD system is indicated in Table I. Section A shows for each subsystem the amount of digitized data, the size of the calibration memory used in data correction by the DCUs, and the amount of data from the DCU after data compression. Section B of Table I shows the processing requirements for one event at a rate of two events per sec. The data stream at both the digitization level and the sparsification level is completely dominated by the vertex detector. Rather conservative assumptions were used for the background level in the CCDs, and the actual situation may be substantially easier. The calibration data required is proportional to the number of analog storage cells in the system, and is dominated by the tracking and CRID systems. The number of constants per cell, of course, is set by the desired resolution and the inherent nonlinearity of the system. The total calibration data requirement is in the range of 200 megabytes, which is substantial by current standards. The expected processing power is associated with the FASTBUS modules for each system, although the estimates for the CCD is still rough. The 2400 mips is large compared to the processing power available anywhere else in the system, including the off-line. The second and third columns indicate the amount of processing power, in thousands of instructions per word available to reduce the data associated with a single event; and the expected size of the output data stream from the FASTBUS module level.²⁷

Table I(a)

Subsystem	Digitized Data (Mbytes)	Calibration Data (Mbytes)	DCU Output (Mbytes)
Vertex	58	0	2
Tracking	14	126	0.5
CRID	16	134	0.05
Calorimeters	1	8	0.13
Total	89	278	2.7

Table I(b)

Subsystem	Processing Power (MIPS)	Instructions/Word Available	Data Out (Kbytes)
Vertex	1000	1000	40
Tracking	400	800	50
CRID	500	10000	25
Calorimeters	500	4000	20
Total	2400		135

SSC Speculations

I will now offer some speculations on Data Acquisition Systems for a central tracking system for an SSC detector. Consider the SSC detector sketched in Fig. 36 taken from the Detector Cost Model Advisory Panel Report.³ The number of channels in each of its major systems is shown in Table II.

Table II

Subsystem	Channel Count
Si VXD	-
Wire VXD	16K
Central Tracking	132 - 232K
End Cap Tracking	20K
Forward Tracking	24K
EM Calorimetry	340 - 510K
Hadron Calorimetry	43K
Catcher Calorimetry	43K
Muon	164K
Total	782 - 1052K

Note that the central tracking system and the electromagnetic calorimeter have roughly a factor of 20 more channels than SLD, and the entire system has an order of magnitude more channels. The Detector Cost Evaluation Panel estimated a Data Acquisition System cost of \$106M out of a total detector cost of \$297M. The basic assumptions for this exercise are taken from the SSC Conceptual Design Report, the Report of the Triggering, Data Acquisition, and Computing Workshop, and the SSC Detector R & D Report.^{7, 28, 29} The machine is assumed to have a luminosity of $10^{33} \text{cm}^{-2} \text{s}^{-1}$ a bunch separation equivalent to

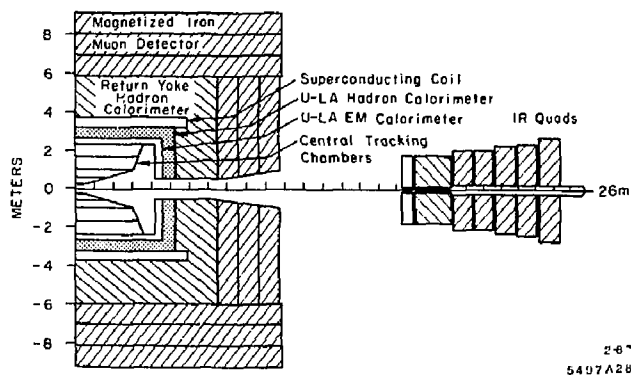


Figure 30 Quadrant view of SSC Model Detector A.

15 nsec, and an average of 1.4 interactions per crossing, implying 10^8 pp interactions per sec. We will assume a "standard" architecture of three trigger levels, a system with 10% deadtime, and will attempt to maximize ϵ_p , maximize the multiplexing, and minimize the buffering. The general model of the data flow is taken from Lankford and Dubois and is shown in Fig. 37.²⁷ The 6×10^7 crossing per sec are reduced to $10^4 - 10^5$ triggers per sec by the level 1 trigger operating in a decision time of 1 μ sec. This rate is reduced another two orders of magnitude by the level 2 trigger operating in about 10 μ secs. I will sketch the drift chamber system through the level 2 processing.

We assume the central drift chamber has approximately 175,000 sense wires per end, arranged in 100 layers with radii between 25 and 235 cm. A cell width of ± 0.5 cm is consistent with the Detector R & D taskforce radiation damage limit. The drift velocity is assumed to be 5 cm/ μ sec. We will take a slightly different approach from Cooper et al., and require some multiplexing on the detector bulkhead to alleviate the cable hole problem.³⁰ Note that the model detector has an area of 2 cm² per sense wire available, and that SLD, which is densely tiled with two layers of electronics, has 6 cm² per sense wire. The Data Acquisition System will be assumed to contribute less than 100 μ to the drift chamber resolution. This approach is somewhat less conservative than that presented in the Triggering Workshop, and is not intended to be an existence proof.

The level 1 scheme is shown in Fig. 38(a). An AMU associated with each sense wire samples the data at 250 MHz or every 4 nsecs. The full event drift time is 100 nsecs corresponding to 25 AMU cells. The resolution σ is approximately 60 microns. The required 1 μ sec delay for the level 1 trigger corresponds to 256

GENERAL MODEL OF DATA FLOW

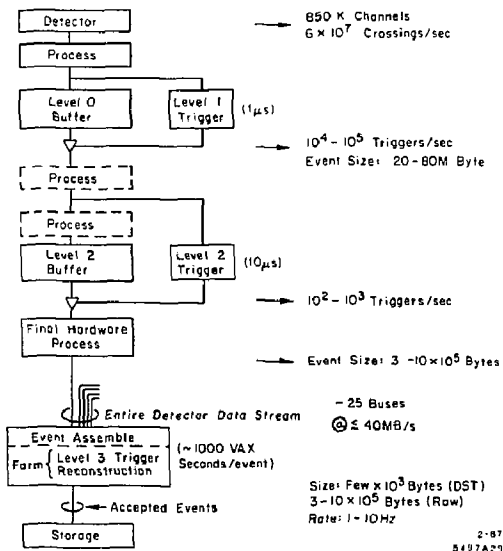


Figure 37 Model of detector data flow.²⁷

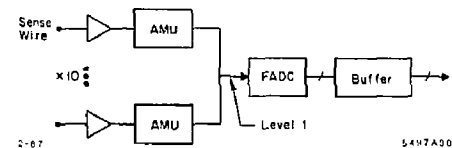


Figure 38(a) Scheme for level 1 buffering digitization.

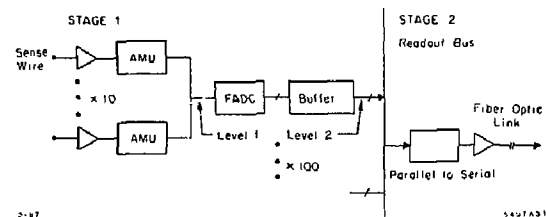


Figure 38(b) Level 2 scheme indicating level 1 multiplexing.

AMU cells, which is ideal.

A single 250 MHz 8-bit flash ADC is assumed to service ten AMU channels. Since the flash ADC runs after the level 1 trigger, the digitization time for 10 channels is 1 μ sec. With a level 1 trigger rate of $10^4 - 10^5$ per sec, the deadtime goal of 1 - 10% with no alternate buffers can be achieved.

The buffer must accept byte wide parallel data at a 250 MHz rate. It must hold several events during the level 2 decision time. For average events: 10^5 events/sec $\cdot$ 10 wires/event $\cdot$ 25 cells/wire $\cdot$ 10 μ s/level 2 trigger = 250. Consequently, a buffer of 1K bytes should be adequate.

The organization at stage 2 is indicated in Fig. 38(b). One-hundred of the previously described systems are gathered on a common readout bus, which is active upon receipt of a level 2 trigger. The output of the bus goes to a digital serial fiber optic link, operating at 250 M bits/sec. The link data rate of 250 M bits/sec and the level 2 trigger rate of 10^3 events per sec imply that the multiplexing of 25 bytes per wire and 1000 wires per fiber is satisfactory. The readout bus is a byte wide bus operating at 25 MHz. This should be compared with FASTBUS operating at about 5 MHz. With this multiplexing, a few hundred fibers will be satisfactory for the central tracking chamber. The space requirement is still a delicate issue. A monolithic preamp and an associated AMU in 2 cm^2 is plausible. A flash ADC and buffer would be required to fit in 20 cm^2 . The readout bus and its associated control logic could use 2000 cm^2 . The thermal management and radiation damage issues are still to be addressed.

I assume that the level 1 trigger is calorimetric, but that the level 2 trigger will require tracking information. A scheme for moving data from the flash ADC to the level 2 trigger is sketched in Figure 39. Data corresponding to 10 sense

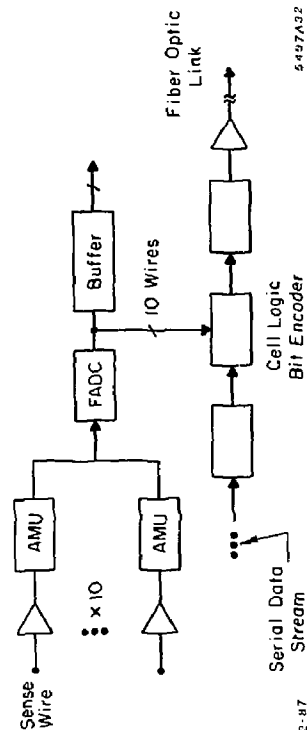


Figure 39 Level 2 tracking trigger data path.

wires is moved into the cell logic bit encoder in the $1 \mu\text{sec}$ interval after the level 1 trigger in which the FADC is active. The function of the cell logic bit encoder is to reduce these 10 bytes to 2 bits representing the activity on the 10 sense wires. If we assume 100 cell encoders per serial link at 2 bits per 10 wires, then a data link with the same performance as that used for the output buffers will deliver the data to the level 2 trigger in about $1 \mu\text{sec}$. This changes the required delay time for the level 2 trigger by about $2 \mu\text{secs}$. Consequently, the previously assumed 1 Kbyte buffer is satisfactory. Presumably, one would attempt to integrate the cell encoder, FADC, buffer, and control logic on a common substrate!

A high-band width data processor is being designed by Siskind, and is sketched in Fig. 40.¹⁶ We would need two such data processors per optical fiber. The event corresponds to $1000 \text{ wires} \times 25 \text{ cells}$, and the rate would be $\frac{1}{2} \times (100 - 1000) \text{ Hz}$. The DCU is a sparsifier and data corrector operating as a 10 MHz pipeline, which is about 7 times faster than the SLD rate. Occupancy of any AMU cell is expected to be about 8% per machine crossing. The data correction is performed by a local farm of 16 digital signal processors (DSPs). Each DSP is expected to have twice the performance of the SLD microprocessors located in the CDMs and WSMs. It is believed (perhaps naively) that about 100 instructions per data word will be adequate for the data correction. This device needs extensive R & D, but a prototype processor is being designed and built to test the feasibility of this architecture.

This approach probably depends on the elimination of cell-by-cell corrections in the next generation of analog memories and on the cost relative to an implementation with a flash ADC per wire. The system is multiplexed at the bandwidth limit of most of the post level 1 components. The sparsification

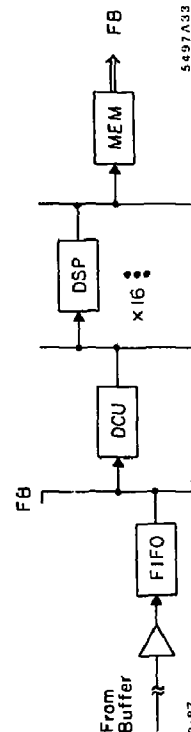


Figure 40 High bandwidth data processor.

and data reduction are occurring at the far end of the optical fiber links, which may be more pessimistic than is necessary. The calorimeter could use a similar system. It will need multiple samplings per event to deconvolve the pileup. It will also need a wider dynamic range than the drift chamber system. This will presumably be done with a multiple gain range system. There are possible and interesting similarities to modern developments in radar processing for the high bandwidth deconvolution and reduction problem. Also, there are rather good opportunities for R & D for the SSC detectors!

Acknowledgments

Much of the work that was described here is general knowledge in the field, and most of the rest is due to many friends and colleagues too numerous to name. However, particular mention must go to Ray Larsen, Dave Sherden, Eric Siskind, Dave Nelson, and the SLD Engineering and Data Acquisition Groups.

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TOPICAL CONFERENCE TALKS



Resonant Neutrino Oscillations[†]

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Abstract

Analytic results are derived for the electron neutrino survival probability after passage through a resonant oscillation region. This survival probability together with a sophisticated model of the production distribution of the solar neutrino sources and the solar electron number density are used to study the effects of resonant neutrino oscillation in the solar interior on the current and proposed solar electron neutrino experiments.

[†] Invited talk at the topical conference of the 14th SLAC Summer Institute on Particle Physics, August 5-8, 1986.

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Recently, Mikheyev and Smirnov¹ have shown that the matter neutrino oscillations of Wolfenstein² can undergo resonant amplification in the solar interior thereby reducing the flux of electron neutrinos emerging from the Sun. This mechanism may be the solution to the solar neutrino puzzle.^{3,4} Subsequently, Bethe⁵ and others⁶ have refined and restated the Mikheyev and Smirnov idea, pointing out that there are three general regions of parameter space in which the solar electron neutrino flux is sufficiently reduced. In this seminar, I review an analytic calculation⁷ for the electron neutrino survival probability after passage through a resonant oscillation region. Then, I outline a calculation⁸ which uses this result, together with a relatively sophisticated solar model for the production distribution of solar neutrino sources and the solar electron number density, to generate contour plots of electron neutrino capture rates in the mass difference squared - vacuum mixing angle plane, for both chlorine (³⁷Cl) experiment and the proposed gallium (⁷¹Ga) detector.

If neutrinos are massive then the flavor and mass eigenstates are not necessarily identical, however a general neutrino state can always be written in the flavor basis,⁹

$$|\nu(t)\rangle = c_e(t) |\nu_e\rangle + c_\mu(t) |\nu_\mu\rangle. \quad (1)$$

For an ultra-relativistic plane wave state propagating in the vacuum, the Dirac equation for this state reduces to the following Schrodinger-like equation,

$$i \frac{d}{dt} \begin{pmatrix} c_e \\ c_\mu \end{pmatrix} = \frac{\Delta_0}{2} \begin{pmatrix} -\cos 2\theta_0 & \sin 2\theta_0 \\ \sin 2\theta_0 & \cos 2\theta_0 \end{pmatrix} \begin{pmatrix} c_e \\ c_\mu \end{pmatrix}, \quad (2)$$

after an overall change of phase. The $\Delta_0 \equiv \delta m^2/2k$, where $\delta m^2 = (m_\mu^2 - m_e^2)$, the neutrino squared mass difference, and k is the neutrino energy. The θ_0 is the vacuum mixing angle. This evolution equation is trivially solved in terms of the mass eigenstates (the eigenvectors of the two by two matrix). Thus, if a neutrino is produced as an electron neutrino, then the probability of detecting an electron neutrino at some later time, t' , is

$$P_{ee}(t') = \frac{1}{2} + \frac{1}{2} \cos^2 2\theta_0 + \frac{1}{2} \sin^2 2\theta_0 \cos\left(\int_{t'}^{t'} \frac{\delta m^2}{2k} dt\right). \quad (3)$$

The last term describes the phenomena of vacuum neutrino oscillation, where the oscillation length, L_0 , is given by

$$L_0 = \frac{4\pi k}{\delta m^2} \sim 200 km \frac{k}{10 MeV} \frac{10^{-4} eV^2}{\delta m^2}. \quad (4)$$

The value of the neutrino energy, 10 MeV, is typical of the neutrinos from the Sun and δm^2 of $10^{-4} eV^2$ will be determined by the electron number density at the solar center.

In matter, the Dirac equation is modified by the weak interactions of the neutrino. Coherent forward scattering contributes to the evolution of this plane wave state. The neutral current interactions of the neutrino with the electrons, protons and neutrons in matter make a contribution proportional to the identity matrix, and therefore only change the overall phase of the neutrino state. The charge current interactions only effect the electron neutrino component because of the absence of the charge partner to the ν_e in matter and the low energy of the neutrinos. For cold matter (temperature $\ll 1 MeV$) this contribution is proportional to the number density of electrons, N_e . Again, after factoring out a piece proportional to the identity matrix, the neutrino evolution equation becomes²

$$i \frac{d}{dt} \begin{pmatrix} c_e \\ c_\mu \end{pmatrix} = \frac{\Delta_N}{2} \begin{pmatrix} -\cos 2\theta_N & \sin 2\theta_N \\ \sin 2\theta_N & \cos 2\theta_N \end{pmatrix} \begin{pmatrix} c_e \\ c_\mu \end{pmatrix}, \quad (5)$$

with Δ_N and θ_N determined by

$$\Delta_N \cos 2\theta_N = \Delta_0 \cos 2\theta_0 - \sqrt{2} G_F N_e,$$

$$\Delta_N \sin 2\theta_N = \Delta_0 \sin 2\theta_0,$$

and where G_F is the Fermi constant. Of course the sign and the coefficient of the N_e term require careful calculation.

At an electron density, N_e , the matter mass eigenstates are

$$\begin{aligned} |\nu_1, N\rangle &= \cos \theta_N |\nu_e\rangle - \sin \theta_N |\nu_\mu\rangle \\ |\nu_2, N\rangle &= \sin \theta_N |\nu_e\rangle + \cos \theta_N |\nu_\mu\rangle \end{aligned} \quad (6)$$

with eigenvalues $E_1 = -\Delta_N/2$ and $E_2 = \Delta_N/2$. Resonance occurs when the difference in these eigenvalues is minimum, that is, when

$$N_e^{res} = \delta m^2 \cos 2\theta_0 / 2\sqrt{2} G_F. \quad (7)$$

In Figure (1) the eigenvalues are plotted as functions of the electron number density. At the resonance density the matter mixing angle $\theta_N^{res} = \pi/4$, which

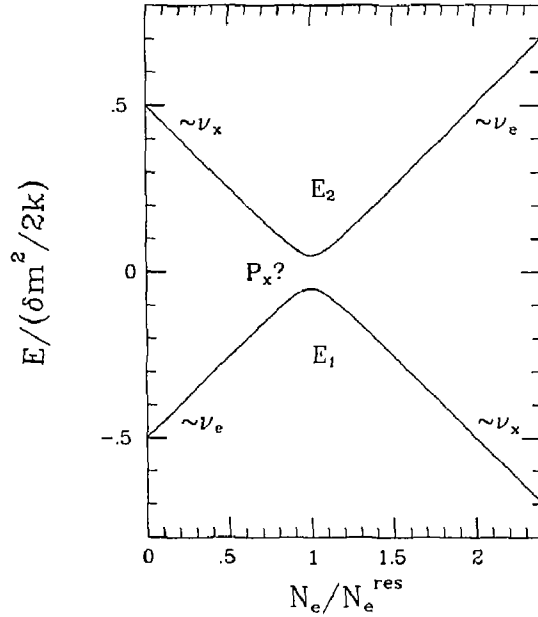


Figure (1): Energy eigenvalues for the matter eigenstates as a function of electron number density.

corresponds to maximal mixing between the two flavor states. To convert N_e into a mass density one uses

$$m_N N_e^{res} = (\rho Y_e)^{res} \sim 10^2 \text{ g cm}^{-3} \cos 2\theta_0 \frac{10 \text{ MeV}}{k} \frac{\delta m^2}{10^{-4} \text{ eV}^2}, \quad (8)$$

where m_N is the nucleon mass, ρ the mass density and Y_e is the ratio of electrons to nucleons. Now we can understand why $\delta m^2 \approx 10^{-4} \text{ eV}^2$ is interesting because the mass density of the solar core is $\sim 10^2 \text{ g cm}^{-3}$. In Table I the important parameters for this process are given for a variety of values of the electron number density.

Table I
Parameters for Resonant Neutrino Oscillations

N_e	0	N_e^{res}	$2N_e^{res}$	$\rightarrow \infty$
$E_2 - E_1$	$\frac{\delta m^2}{2k}$	$\frac{\delta m^2}{2k} \sin 2\theta_0$	$\frac{\delta m^2}{2k}$	$\rightarrow \infty$
θ_N	θ_0	$\frac{\pi}{4}$	$\frac{\pi}{2} - \theta_0$	$\rightarrow \frac{\pi}{2}$
L_N	$L_0 = \frac{4\pi k}{\delta m^2}$	$L_0 / \sin 2\theta_0$	L_0	$\rightarrow 0$

The idea of resonant neutrino oscillations as a way of reducing the flux of electron neutrinos and thereby solving the solar neutrino puzzle is to produce at least some of the neutrinos above the resonant density. This means that these neutrinos are produced mainly in the matter mass eigenstate $|\nu_2, N\rangle$. Then, if the electron number density changes sufficiently slowly as neutrino exits the Sun these neutrinos remain in the $|\nu_2, N\rangle$ state which at zero density is mainly the ν_e neutrino flavor state. That is, there is little crossing between the two matter mass eigenstates.

In Figures (2a)-(2d), I have solved the neutrino evolution equation, numerically, for an electron neutrino produced at the center of an exponential electron density distribution which approximates the solar electron density profile. The adiabatic crossing cases are Figures (2a) and (2b) whereas in Figures (2c) and (2d) there is significant crossing between the two adiabatic states. The transition from adiabatic to non-adiabatic passage occurs when the neutrino state does not have time to change its flavor character as the neutrino passes through resonance. At resonance, the neutrino can change its character in a resonant

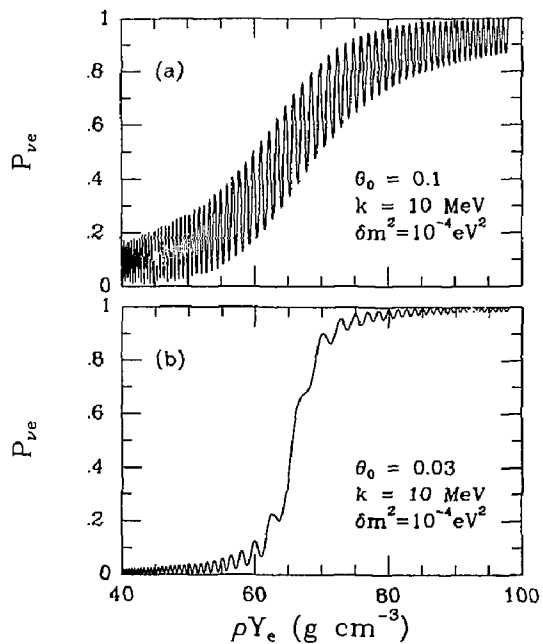


Figure (2a) and (2b): Electron neutrino probabilities as a function of electron density for an electron neutrino produced at the center of the Sun. For these values of θ_0 resonance crossing is adiabatic.

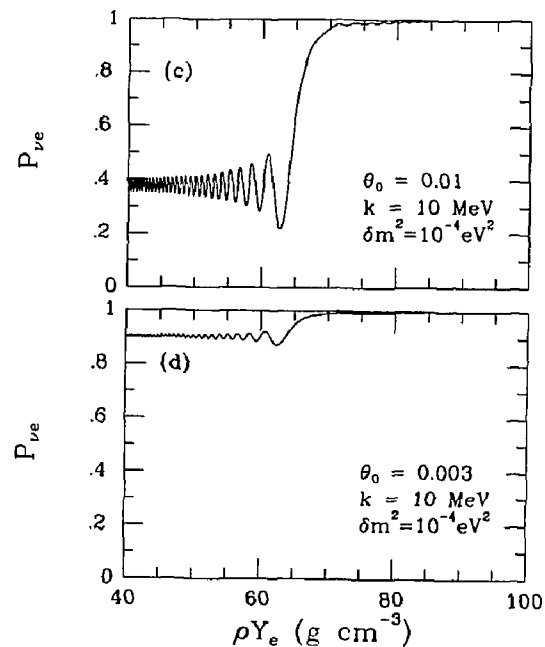


Figure (2c) and (2d): Electron neutrino probabilities as a function of electron density for an electron neutrino produced at the center of the Sun. For these values of θ_0 resonance crossing is non-adiabatic.

oscillation length, $L_0/\sin 2\theta_0$. Here the width of the resonance region is $R, \tan 2\theta_0$, for an exponential profile of scale height, R_s . These two lengths are of the same order, for a scale height equal to 0.092 times the radius of the Sun, when $\theta_0 \approx 0.01$ for a neutrino energy of 10 MeV and a $\delta m^2 = 10^{-4} \text{eV}^2$.

For a slowly varying electron density, the matter mass eigenstates evolve independently in time; that is $e^{-i \int_{t_0}^t E_1 dt} |\nu_1, N(t)\rangle$ and $e^{-i \int_{t_0}^t E_2 dt} |\nu_2, N(t)\rangle$ are the adiabatic states. Therefore, it is convenient to use these states, as the basis states, in the region for which there are no transitions (away from the resonance region). As a neutrino goes through resonance these adiabatic states may be mixed, but on the other side of resonance, the neutrino state can still be written as a linear combination of these states. That is, a basis state produced at time t , going through resonance at time t_r , and detected at time t' is described by

$$e^{-i \int_t^{t'} E_1 dt} |\nu_1, N(t)\rangle \rightarrow a_1 e^{-i \int_{t_r}^{t'} E_1 dt} |\nu_1, N(t')\rangle + a_2 e^{-i \int_{t_r}^{t'} E_2 dt} |\nu_2, N(t')\rangle$$

or

$$e^{-i \int_t^{t'} E_2 dt} |\nu_2, N(t)\rangle \rightarrow -a_2^* e^{-i \int_{t_r}^{t'} E_1 dt} |\nu_1, N(t')\rangle + a_1^* e^{-i \int_{t_r}^{t'} E_2 dt} |\nu_2, N(t')\rangle$$

where a_1 and a_2 are complex numbers such that $|a_1|^2 + |a_2|^2 = 1$. The relationship between the coefficients, for these two basis states, is due to the special nature of the wave equation, Eqn.(5). The phase factors have been chosen so that coefficients a_1 and a_2 are characteristics of the transitions at resonance and are not related to the production and detection of the neutrino state.

Hence, the amplitude for producing, at time t , and detecting, at time t' , an electron neutrino after passage through resonance, is

$$A_1(t) e^{-i \int_t^{t'} E_1 dt} + A_2(t) e^{-i \int_t^{t'} E_2 dt}$$

where

$$A_1(t) = \cos \theta_0 (a_1 \cos \theta_N e^{i \int_{t_r}^t E_1 dt} - a_2^* \sin \theta_N e^{i \int_{t_r}^t E_2 dt})$$

$$A_2(t) = \sin \theta_0 (a_2 \cos \theta_N e^{i \int_{t_r}^t E_1 dt} + a_1^* \sin \theta_N e^{i \int_{t_r}^t E_2 dt}).$$

Thus the probability of detecting this neutrino as an electron neutrino is given by

$$P_{\nu_e}(t, t') = |A_1(t)|^2 + |A_2(t)|^2 + 2|A_1(t)A_2(t)| \cos(\int_{t_r}^{t'} \Delta_N dt + \Omega)$$

with $\Omega = \arg(A_1^* A_2)$.

The detection averaged electron neutrino survival probability is easily calculated as

$$P_{\nu_e}(t) = \frac{1}{2} + \frac{1}{2}(|a_1|^2 - |a_2|^2) \cos 2\theta_N \cos 2\theta_0 - |a_1 a_2| \sin 2\theta_N \cos 2\theta_0 \cos(\int_{t_r}^t \Delta_N dt + \omega)$$

with $\omega = \arg(a_1 a_2)$. The last term demonstrates that the phase of the neutrino oscillation at the point the neutrino enters resonance can substantially effect this probability, see Figure (3). Therefore, we must also average over the production position, to obtain the fully averaged electron neutrino survival probability^{7,10} as

$$\overline{P_{\nu_e}} = \frac{1}{2} + \left(\frac{1}{2} - P_x\right) \cos 2\theta_N \cos 2\theta_0 \quad (9)$$

where $P_x = |a_2|^2$, the probability of transition from $|\nu_2, N\rangle$ to $|\nu_1, N\rangle$ (or vice versa) during resonance crossing. The non-resonance crossing case is trivially obtained by setting $P_x = 0$.

Also, if the electron neutrinos are produced at a density much greater than the resonance density, so that $\cos 2\theta_N \sim -1$, then

$$\overline{P_{\nu_e}} \approx \sin^2 \theta_0 + P_x \cos 2\theta_0. \quad (10)$$

Thus for small θ_0 , in this limit, the survival probability is just equal to the probability of level crossing during resonance passage.

Similar calculations can also be performed for the case of double resonance crossing (neutrinos from the far side of the Sun). Here we must average not only over the production and detection positions of the neutrino but also over the separation between resonances. This sensitivity to the separation of the resonance can be understood as the effect of the phase of the oscillation as the neutrino enters the second resonance region. The fully average probability of detecting an electron neutrino is the same as Eqn.(9) with P_x replaced by

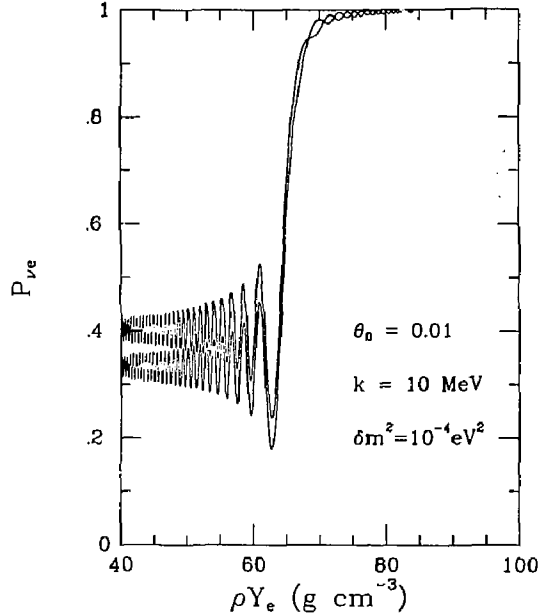


Figure (3): Similar to Figure (2) but these two cases were started at $\rho Y_e = 85.0 \pm 0.3 \text{ g cm}^{-3}$. Note the difference in the phase of the oscillation as the neutrino enters the resonance region.

$P_{1z}(1 - P_{2z}) + (1 - P_{1z})P_{2z}$ (the classical probability result). Therefore, the generalization to any number of resonance regions, suitable averaged, is obvious.

To calculate the probability, P_z , I make the approximation that the density of electrons varies linearly in the transition region. That is, a Taylor series expansion is made about the resonance position and the second and higher derivative terms are discarded;

$$N(t) \approx N(t_r) + (t - t_r) \frac{dN}{dt} \Big|_{t_r}. \quad (11)$$

In this approximation the probability of transition between adiabatic states was calculated by Landau and Zener.¹¹ This is achieved by solving the Schrödinger equation, Eqn.(5), exactly in this limit. Applying their result to the current situation^{7,12} gives

$$P_e = \exp \left[-\frac{\pi \sin^2 2\theta_0}{2 \cos 2\theta_0} \left| \vec{n} \cdot \nabla \ln N_e \right|_{t_r} \frac{\delta m^2 / 2k}{\delta m^2 / 2k} \right] \quad (12)$$

where the unit vector, $\vec{n}$, is in the direction of propagation of the neutrino. Equations (9) and (12) demonstrate that only the electron number density, at production, and the logarithmic derivative of this density, at resonance, determine the electron neutrino survival probability. It should be emphasized here, that this result assumes that the neutrino state is produced before significant transitions take place and thus Eqn.(12) is not valid for neutrinos produced in the transition region.

From Eqn.(12) the size of the transition region can be determined. There are significant transitions ($P_z > 0.01$) if $\theta_0 < \theta_{crit}$ where θ_{crit} satisfies

$$\frac{\sin^2 2\theta_{crit}}{\cos 2\theta_{crit}} = 3 \frac{1}{\Delta_0} \left| \frac{1}{N} \frac{dN}{dt} \right|_{t_r}. \quad (13)$$

Hence, the maximum separation between the eigenstates for which transitions take place is $\Delta_0 \sin 2\theta_{crit}$. Therefore, the transition region is defined by

$$\Delta_N < \Delta_0 \sin 2\theta_{crit}. \quad (14)$$

This can only happen if $\theta_0 < \theta_{crit}$. In this transition region, the maximum variation of the electron number density from the resonant value is $\pm \delta N$, where

$$\frac{\delta N}{N(t_r)} = \sin 2\theta_{crit}.$$

Thus, the size of the transition region is

$$|t - t_r| = \sin 2\theta_{crit} / \left| \frac{1}{N} \frac{dN}{dt} \right|_{t_r}.$$

This is the maximum $|t - t_r|$ for which the linear approximation must be good, so that Eqn.(12) gives a reasonable estimate of the probability of crossing. For an exponential density profile, the Taylor series expansion is an expansion in $\sin 2\theta_{crit}$, so that small θ_{crit} is an excellent approximation.

Before applying these results to the solar model in detail, let us first consider an exponential electron number density profile which is a good approximation for the solar interior except near the center. In Figure (4), I have plotted the electron neutrino survival probability contours at the earth in the $\delta m^2/2\sqrt{2}kG_F N_c$ versus $\sin^2 2\theta_0/\cos 2\theta_0$ plane for such an exponential density profile. Here, the solar central electron number density, N_c , is also the number density at the point where the neutrinos are produced. This plot depends only on the properties of the Sun and this dependency is through the combination $R_s N_c$, where R_s is the scale height. For this figure, I have used an N_c corresponding to a density of 140 g cm^{-3} and $Y_e = 0.7$ and a scale height R_s of 0.092 times the radius of the Sun.

Above the line $\delta m^2/2\sqrt{2}kG_F N_c = 1/\cos 2\theta_0$ in this plot, the neutrinos never cross the resonance density on their way out of the Sun. Here, the probability of detecting an electron neutrino is close to the standard neutrino oscillation result. Below this line, the effects of passing through resonance comes into play. Inside the 0.1 contour "triangle", there is only a small probability of transitions between the adiabatic states as the neutrino passes through resonance. To the right of this contour triangle, the probability of detecting a neutrino grows, not because of transitions, but because both adiabatic states have a substantial mixture of electron neutrino at zero density. To the left and below the 0.1 contour triangle, the probability grows because here there are significant transitions between the adiabatic states as the neutrino crosses resonance.

More precisely, the solar electron neutrino capture rate for a detector characterized by an electron neutrino capture cross section, $\sigma(E)$, and energy threshold E_0 , is

$$\sum_{\text{processes}} \int_{E_0}^{\infty} \frac{d\Phi_\nu}{dE} \sigma(E) dE. \quad (15)$$

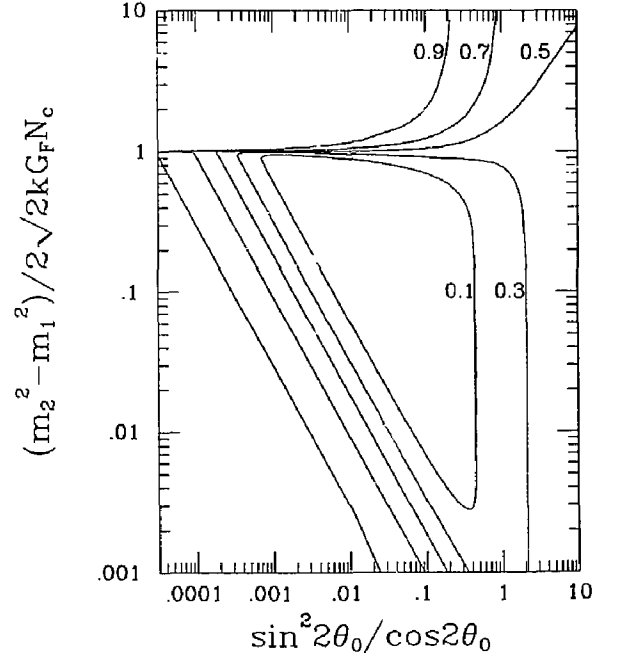


Figure (4): Electron neutrino survival probability contours for an exponential solar electron density profile and an electron neutrino produced at center of the Sun.

The sum is taken over all neutrino sources in the Sun and $d\Phi_\nu/dE$ is the differential electron neutrino flux of a given source at the earth's surface. To include the reduction in the electron neutrino flux from the Sun due to resonant neutrino oscillations, the differential electron neutrino flux for each process ν as calculated as

$$\frac{d\Phi_\nu}{dE} \propto W(E) \int_{\text{sun}} dV \overline{F_\nu} \frac{df}{dV} \quad (16)$$

where $W(E)$ is the standard weak interaction energy distribution for the neutrinos of a given process and df/dV is the fraction of the standard solar model flux coming from a given solar volume element for this process. The solar electron number density profile, $\rho Y_e/m_N$, and the values of df/dV for the various processes were taken from Bahcall's solar model,¹³ see Figure (5). The $d\Phi_\nu/dE$ was normalized for each process by demanding that the energy and solar volume integrations of Eqn.(15) yield the capture rates given in the Table II,⁴ when $\overline{F_\nu} \equiv 1$. The cross sections used, for both ^{37}Cl and ^{71}Ga detectors, are given in Figure (6).

Table II
Neutrino Sources and Capture Rates (SNU)

Process	$E_\nu^{\text{max}}(\text{MeV})$	^{37}Cl	^{71}Ga
^8B	14.06	4.3	16
^7Be	0.861(90%) +0.383(10%)	1.0	27
p-p	0.420	0	70
pep	1.44	0.23	2.5
^{13}N	1.199	0.08	2.6
^{15}O	1.732	0.26	3.5
Total		5.9	122

In Figures (7) and (8), we present electron neutrino capture rate contours (iso-SNU contours) for the ^{37}Cl and ^{71}Ga experiments as a function of δm^2 and $\sin^2 2\theta_0/\cos 2\theta_0$ for this solar model. The 1σ deviations from the Davis et al.³ result of 2.1 SNU are the 2.4 and 1.8 iso-SNU contour lines in Figure (8). The generic structure of these total SNU plots is due to the superposition

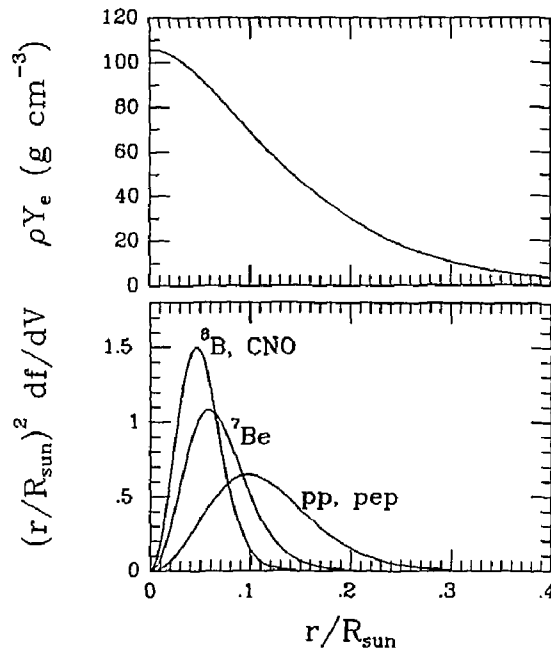


Figure (5): Electron number density and the fractional neutrino emissivity per shell versus the radial distance from the center of the Sun.

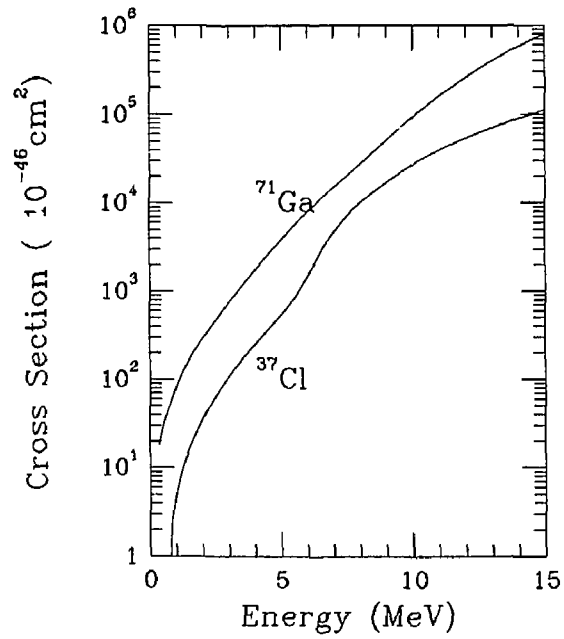


Figure (6): The cross sections for the Chlorine and Gallium detectors as a function of energy.

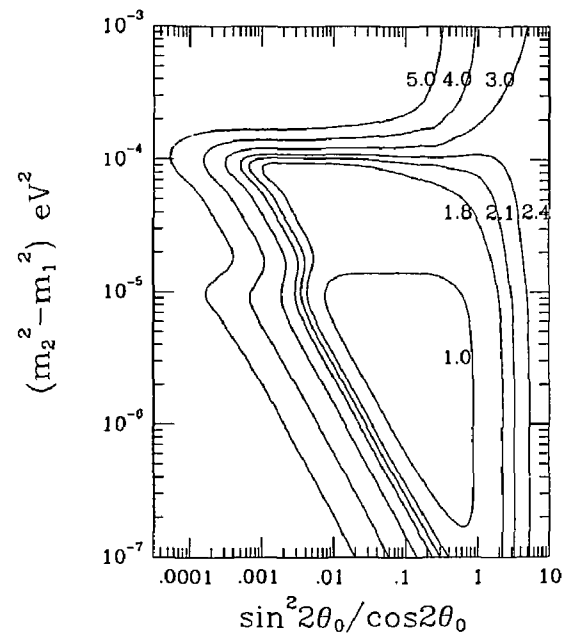


Figure (7): Iso-SNU contours for the ^{37}Cl experiment using Bahcall's solar model. The contours are labeled with their the corresponding SNU values.

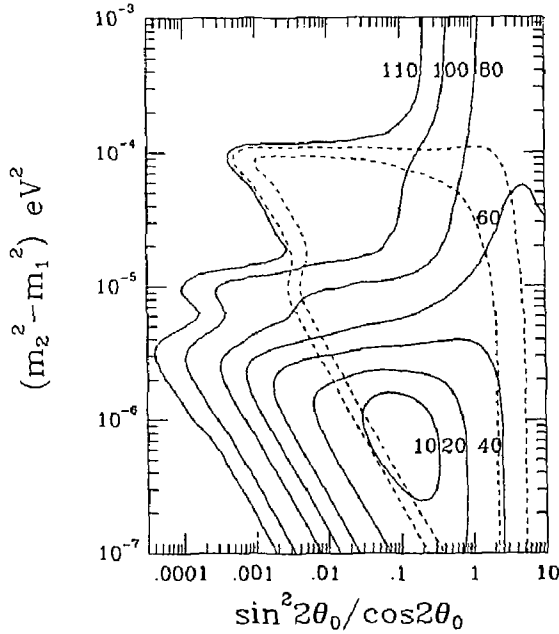


Figure (8): Iso-SNU contours for a ^{71}Ga detector using Bahcall's solar model. The solid contours are labeled with their appropriate ^{71}Ga SNU values and the dashed contours are the 1σ deviations from the Davis ^{37}Cl experimental result.

of triangular iso-SNU contours associated with each individual neutrino source contributing to a given total SNU value. These individual contours owe their shape to the appropriate iso-probability contour, Figure (4), and their position is determined by the typical energy scale and production electron density of the individual neutrino source. For each neutrino source the resonance mechanism becomes important, provided $\theta_0 > 0.01$, as soon as δm^2 becomes small enough so that the average resonant electron density for that source is less than the solar electron density at the production site. This occurs when δm^2 is approximately equal to 1.5×10^{-4} , 1.2×10^{-5} , and 3.7×10^{-6} eV² for the ^8B , ^7Be and pp neutrinos respectively. Below these values the individual neutrino sources have contours which are diagonals of slope minus one coming from the form of the transition probability between adiabatic states, Eqn.(12). The intersection of these diagonal lines with the turning on of resonance for ^8B , ^7Be and pp is responsible for the shoulders at small $\sin^2 2\theta_0 / \cos 2\theta_0$ in the contour plots. The vertical sections of the contours, at large θ_0 , occur because for large θ_0 both adiabatic states have a large component of electron neutrino.

From Figure (8), we see that the results of the ^{71}Ga experiment can range from 10 to 110 SNU and still be compatible with the ^{37}Cl experiment. In general, a given gallium contour crosses the 2.1 ± 0.3 chlorine contour at least twice and therefore the results of the ^{71}Ga experiment will leave a two-fold degeneracy in $(\delta m^2, \theta_0)$ -space. If one accepts the theoretical prejudice against large vacuum angles provided by see-saw models,¹⁴ this degeneracy is removed. Unfortunately, the degeneracy is continuous for that region of parameter space corresponding to a ^{37}Cl rate of 2.1 ± 0.3 SNU and a ^{71}Ga rate greater than 100 SNU. In this region *only* the ^8B neutrinos are effected by the resonance phenomena. Also, in this region of parameter space the two experiments will not be able to distinguish between a small temperature change at the solar core and the resonant neutrino oscillation mechanism. This is due to the relatively strong temperature dependence of the ^8B neutrino flux.¹⁵ It is only when the ^{71}Ga SNU rate is depleted below that of merely removing the ^8B component (i.e., appreciably less than 110 SNU), so that reduction of the less temperature sensitive neutrinos (^7Be and pp) becomes necessary, that the resonant oscillation mechanism becomes a likely solution to the solar neutrino problem.

I wish to thank Rocky Kolb and Terry Walker for discussions.

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8

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15. In the case where a ${}^{71}\text{Ga}$ rate of $\gtrsim 100$ SNU is measured, a measurement of the ${}^8\text{B}$ solar neutrino spectrum (see Rosen and Gelb Ref. 6) or the flavor independent solar neutrino flux (see S. Weinberg, contribution to the XXIII International Conference on High Energy Physics, Berkeley, CA., July, 1986) would allow us to distinguish between changes in the solar model and resonant neutrino oscillations.

RECENT RESULTS FROM THE UA1 EXPERIMENT AT CERN

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Helsinki-Kiei-Imperial College, London and Queen Mary College, London-MIT-
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ABSTRACT

Results based on a sample of events with large missing transverse energy corresponding to 715 nb^{-1} of data from the UA1 experiment at the CERN pp Collider are presented. High transverse momentum tau-leptons from W decays are observed for the first time through their semi-hadronic decay modes. The first direct tests of the $e - \mu - \tau$ universality of the weak charged couplings at $Q^2 = m_W^2$ is provided.

Measured W and Z⁰ rates and heavy flavour cross sections are used to predict rates of missing transverse energy events from all known Standard Model processes, and thereby to place limits on possible new sources. After taking into account all known sources of missing energy events, we find a mass limit on a fourth generation charged lepton of $m_L > 41 \text{ GeV}/c^2$ (90 % C.L.) and a limit on the number of additional neutrinos of $N_\nu < 7$ (90 % C.L.). Lower limits for supersymmetric particles, namely $60 \text{ GeV}/c^2$ for the gluino and $70 \text{ GeV}/c^2$ for the squark (90 % C.L.) are also given.

Finally, two events are presented in which two energetic hadronic jets and a high energy lepton are balanced by missing energy. These events are discussed and compared to all observed W + 2-jet events.

Presented at
SLAC SUMMER INSTITUTE TOPICAL CONFERENCE
Stanford Linear Accelerator Center, Stanford, California
August 6-8, 1986

1 Introduction

The physics topics being studied at the CERN $\bar{p}p$ Collider cover a domain which extends well beyond the original motivations for this machine. [1] To deal with them all in a single summary talk is an impossible task for me. For this reason I shall only discuss two subjects:

- the physics which comes out of the events associated with high transverse missing energy,[2]
- the analysis of $W + 2$ -jets events observed in the UA1 detector.[3]

Other topics which are also part of the **mainstream** of research in UA1 have been presented this summer at the Berkeley Conference, namely:

- the study of jets and QCD tests,[4]
- jet fragmentation studies,[5]
- physical properties of the W and Z bosons,[6]
- evidence for b quark production as the source of high p_T dimuon events,[7]
- the search for $B^0 - \bar{B}^0$ oscillation in dimuon events.[8]

The analysis of events with large missing transverse energy in antiproton-proton collisions [9] is a powerful method to test the Standard Model and to search for new processes that produce one or more energetic neutrinos or new neutral weakly interacting particles. A summary of the missing energy event topologies expected from Standard and non-Standard sources is shown in Figure 1.

This method was first used in association with a requirement of an electron or a muon signal and resulted in the conclusive observation of the leptonic decays of the charged intermediate vector boson. [10] So a search for high missing energy events obviously selects the decays $W \rightarrow e\nu$ and $W \rightarrow \mu\nu$. But in the analysis presented here, these types of events are considered to be background rather than a signal.

Another strict requirement in the selection of events with high transverse missing energy is to find events where a large missing energy is produced in association with one (monojet) or more (multijet) hadronic jets. This type of events has to be extracted from a large background due to jet events. This means, that we have to reject the events in which the finite detector resolution leads to apparent missing energy aligned with the jet direction.

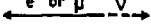
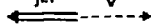
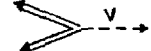
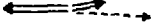
Topology		Process
	Isolated missing energy	$W \rightarrow e\nu, W \rightarrow \tau\nu \rightarrow e\nu\nu$ $W \rightarrow \mu\nu, W \rightarrow \tau\nu \rightarrow \mu\nu\nu$
monojet  multijet 		$W \rightarrow \tau\nu \rightarrow \text{hadrons} + \nu$ $p\bar{p} \rightarrow Z^0 + \text{jet(s)}$ $\quad \quad \quad \downarrow$ $\quad \quad \quad \nu\nu$ $\quad \quad \quad \downarrow$ $\quad \quad \quad N\bar{N}$ new neutrino $W \rightarrow L\nu \rightarrow q\bar{q}\nu$ } Heavy Lepton $p\bar{p} \rightarrow q\bar{q} \text{ (or } \bar{g})$ } Susy $\quad \quad \quad \downarrow$ $\quad \quad \quad q\bar{q} + \bar{\gamma}$
	non-isolated missing energy	$W \rightarrow c\bar{s}$ $Z^0 \rightarrow c\bar{c}, b\bar{b}$ $p\bar{p} \rightarrow c\bar{c}, b\bar{b}$ $p\bar{p} \rightarrow \tau\bar{\tau}$ $p\bar{p} \rightarrow q\bar{q} \text{ (or } \bar{g})$
		Jet fluctuation background

Figure 1: Schematic representation of topologies of large missing energy events expected from Standard and non-Standard Model processes, as well as from jet fluctuation.

Dominant sources of validated missing energy plus hadronic jet events (Figure 1) are expected to be produced by:

- the decay of a W into a tau lepton, the tau decaying semi-hadronically,
- the production of a high transverse momentum Z^0 recoiling against a quark or a gluon jet, the Z^0 decaying into two neutrinos ($Z^0 \rightarrow \nu\bar{\nu}$),
- heavy flavour production with subsequent semi-leptonic decay of the heavy quark ($W \rightarrow c\bar{s}, W \rightarrow t\bar{b}, Z \rightarrow c\bar{c}, Z \rightarrow b\bar{b}, Z^0 \rightarrow t\bar{t}$, or direct production of $c\bar{c}, b\bar{b}, t\bar{t}$),
- $Z^0 \rightarrow \tau^+\tau^-$ where the tau decays semi-leptonically.

In addition to these expected Standard Model sources, there are a number of new physics sources that would be expected to produce events with significant missing E_t . The potential new physics sources, which contribute to events with missing energy (Figure 1) are:

- $W \rightarrow L\nu$ decay; $L \rightarrow \bar{u}d\nu$ or $L \rightarrow \bar{c}s\nu$, where L is the sequential heavy charged lepton from the fourth generation of quarks and leptons.[11]
- any additional low mass neutrino species with the standard coupling to the Z^0 would produce missing E_t events. Missing energy events can therefore be used to constrain the total number of generations.
- production of supersymmetric particles. Supersymmetric models [12] contain a rich spectrum of new particles. These particles will eventually decay into known particles plus the lightest supersymmetric particle, which should be stable and noninteracting, so producing missing energy.

The analysis presented here tries to search for these new physics sources in the data and in the absence of a signal, to give significant limits on such new processes.

2 The Missing Energy Technique

The UA1 experiment [1], was originally designed to observe intermediate vector bosons in the decay channels $W \rightarrow e\nu$ and $W \rightarrow \mu\nu$. The emission of the neutrino is signalled by the apparent momentum unbalance in the transverse direction. The missing energy vector is defined as:

$$\Delta E^{miss} = -\sum_i E_i u_i$$

where u_i is the unit vector between the collision point and the centre of the i^{th} calorimeter cell in which one has observed the energy deposition E_i . In order to measure reliably the missing energy in an event, one requires a detector covering the whole solid angle, i.e. a hermetic detector. The UA1 detector [1] covers the full angular range down to a polar angle of 0.2° with a minimal fraction of insensitive areas.

If particle masses are neglected for a perfect 4π calorimeter, one expects for minimum bias events:

$$\Delta E^{miss} = 0.$$

If one or more neutral, noninteracting particles are emitted with a total momentum p_ν , then:

$$\Delta E^{miss} = p_\nu.$$

At the collider, a significant fraction of the energy escapes detection along the beam pipe, making a missing energy measurement not possible. So in practice, only the components ΔE_y and ΔE_z transverse to the beam direction can be accessed.

For the standard events, where the energy balance is expected to be determined by overall calorimeter resolution, the components of the transverse missing energy are neatly centered around zero and have an approximately Gaussian shape with a RMS width:

$$\sigma_{p_\perp} = 0.5 \sqrt{E_t}$$

where E_t is the scalar sum (in GeV) of observed transverse energy from all calorimeter cells. The modulus of the resultant transverse energy vector:

$$\Delta E_t^{miss} = \sqrt{\Delta E_y^2 + \Delta E_z^2}$$

is distributed exponentially in the variable $(\Delta E_t^{miss})^2$. The resulting transverse missing energy is estimated with an attenuation factor:

$$\sigma = 0.7 \sqrt{E_t}$$

and we define the significance N_σ of the observed missing transverse energy as:

$$N_\sigma = \Delta E_t^{miss} / \sigma.$$

3 Observation of High E_T^{miss} Events

The objective of the missing transverse energy event selection is to isolate events containing high p_T neutral weakly interacting particles. The first application of this technique led to the discovery of the W particles.[10] But an inclusive selection of missing transverse energy events provides both the sample of $W \rightarrow e\nu$ decays, and a significant number of additional events in which the missing energy recoils against one (the so-called monojet) or more high energy jets. Eight monojets have been reported by us in a paper [9] based on 118 nb^{-1} integrated luminosity.

The selection of missing transverse energy events is in fact made more difficult by the presence of backgrounds that can fake missing E_T . These backgrounds are of four types:

- *external backgrounds:* The E_T^{miss} is generated by an energy deposition in the calorimeter that does not come from pp collision (cosmic rays, beam halo interactions,...),
- *instrumental effects:* The E_T^{miss} is due to electronic noise, accidental loss of detection capability, problem in event reconstruction,
- *jet fluctuation:* The E_T^{miss} is caused by the finite detector resolution. The shower (electromagnetic and/or hadronic) produced in the detector can have measured energies that can fluctuate substantially from their true energy. The E_T^{miss} direction in these events is usually aligned with the jet direction (Figure 1),
- *fake E_T^{miss} ,* due to jets pointing towards regions in the calorimeter with reduced sensitivity.

To minimise all these backgrounds, we have required that the E_T^{miss} is significantly different from zero ($N_\phi \geq 4$) and that the missing energy is isolated, i.e. not accompanied by significant jet activity.

3.1 Selection of the 4σ Event Sample

The analysis presented in this paper is based on data recorded during run period 1983-85 and corresponding to an integrated luminosity of 715 nb^{-1} at pp centre of mass energies of 546 GeV (118 nb^{-1}) and 630 GeV (597 nb^{-1}).

The event selection has been designed to obtain a background-free sample of events with missing E_T . Events were recorded that satisfied one or more of the following hardware triggers[13]:

- *Electromagnetic transverse energy trigger,* requiring E_T greater than 10 GeV in two adjacent electromagnetic elements,
- *Jet E_T trigger,* requiring a jet with $E_T > 25 \text{ GeV}$,
- *Transverse energy imbalance trigger,* requiring a jet with $E_T > 15 \text{ GeV}$ together with missing transverse energy greater than 17 GeV. The energy imbalance trigger was not used for the 546 GeV data.

To define the experimental missing transverse energy data sample, the following selection criteria were used off line:

- *missing transverse energy $E_T^{\text{miss}} > 15 \text{ GeV}$ and $N_\phi \geq 8$.* This rejects the majority of events with small apparent but real E_T^{miss} ,
- *one or more jets* [14] observed in the calorimeter with $E_T > 12 \text{ GeV}$,
- *one or more tracks observed in the drift chambers with $p_T > 1 \text{ GeV}/c$ within a cone of size $\Delta r = 0.4$ centered on a jet axis ($\Delta r^2 = \Delta\phi^2 + \Delta\eta^2$).* This removes external and instrumental backgrounds not correlated with charged tracks coming from the main vertex.

Moreover we have rejected events which did not satisfy fiducial cuts to avoid fake missing E_T caused by jets going into regions of detector with reduced sensitivity.

After these loose cuts, we have obtained about 6000 events out of $1.3 \cdot 10^7$ triggers written on tape. Figure 2(a) shows the distribution of number of signals (N_s) for these events.

The background from jet fluctuation is expected to have an exponential behavior, which clearly appears as a linear fall-off on the log scale plot (Figure 2(a)). At larger N_s , a break is apparent in the distribution, which is due to $W \rightarrow e\nu$ events and other real missing E_T events and some remaining backgrounds.

To reduce the jet fluctuation background additional cuts were used:

- *isolation of the missing transverse energy,* by rejecting events with a jet observed in the calorimeter (and/or in the central detector), with respectively $E_T > 8 \text{ GeV}$ (and/or $p_T > 5 \text{ GeV}/c$) within ± 30 degrees in azimuth of the missing transverse energy direction,
- *veto of coplanar jet activity,* by rejecting events with a calorimeter jet of $E_T > 8 \text{ GeV}$ (and/or a central detector jet with $p_T > 5 \text{ GeV}/c$) within 150 - 210 degrees in ϕ of the direction of the highest E_T jet (trigger jet),
- *veto of events with an electron or muon candidate in the $9 \leq N_\phi \leq 4$ data sample in order to remove contributions from $W \rightarrow e\nu$ and $W \rightarrow \mu\nu$ decays.*

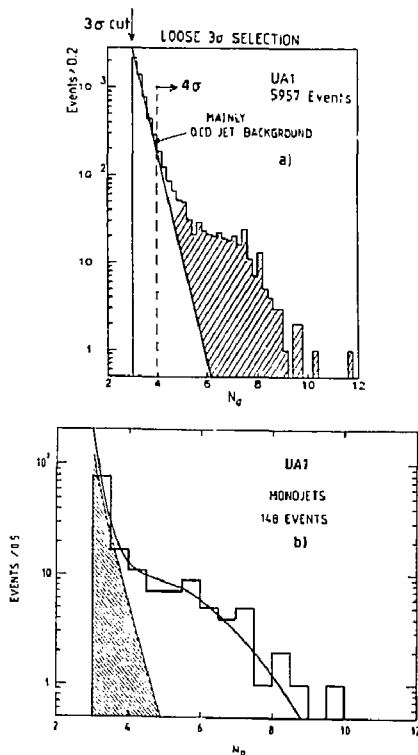


Figure 2: Distribution of $N_0 = E_t^{miss} / 0.7 \sqrt{E_t}$ for the missing transverse energy events (a) fulfilling the loose selection criteria. The solid line correspond to the contribution of the jet fluctuation background. (b) fulfilling the tight selection cuts. The shaded region correspond to jet fluctuation background. The solid line is the sum of all estimated Monte Carlo contribution (see section 3).

At the end of this tight selection, all events were checked visually on a graphic display where 30 events have been rejected as being background or as having uncertain missing energy due to event reconstruction problems.

A total of 384 events have been obtained, which consist of 222 electron candidates and 11 muon candidates coming from W decays, 53 monojets and 3 two-jets events (with for both jet $E_t > 12$ GeV) with $N_0 > 4$, and 95 monojets with $3 \leq N_0 < 4$. This last data sample will be used to check the background estimations. No events with three or more jets have been observed with $N_0 \geq 4$.

In Figure 2(b), we present the plot of N_0 for the 148 monojet events. One clearly sees that there is still a large number of events clustering near the $N_0 = 3$ cut, which could indicate that in this data sample we are still not free from jet fluctuation background.

3.2 Contribution of the Jet Fluctuation

This background was evaluated by a Monte Carlo technique using ordinary jet events with no missing E_t from fully reconstructed UA1 jet data. The energy of these measured jets was randomly fluctuated about their measured values, according to the known calorimeter resolution as obtained from test beam results and after Monte Carlo simulation studies. The number of events generated in this way corresponds to an integrated luminosity of 3180 nb^{-1} .

The distribution of N_0^2 presented in Figure 3 for the jet data is well described by the jet fluctuation background contribution.

To further study the effect of the jet fluctuation background on the 4σ sample, we have relaxed the criteria on coplanar jet activity. Figure 4 presents the distribution of the largest jet transverse energy (E_t^{back}) within ± 30 degrees in azimuth of the direction opposite to the trigger jet or within ± 30 degrees in azimuth of the missing transverse energy direction.

It can be seen from Figure 4 that the Monte Carlo distribution provided a good description of the data.

Other checks of the reliability of this technique have been made by confirming that the effect of loosening various E_t^{miss} selection cuts are correctly predicted by the jet fluctuation Monte Carlo.

It is also clear from Figure 4 that when we apply a cut of $E_t^{back} = 8 \text{ GeV}$ on the data as in our selection:

- the jet fluctuation background is highly reduced. It is estimated that only 3.8 events remain in our 4σ sample.
- the main contribution to the 4σ sample comes from the $W \rightarrow \tau\nu$.

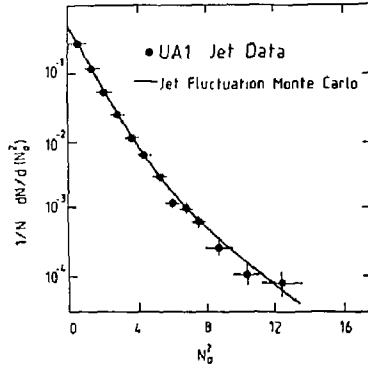


Figure 3: Distribution of N_0^2 from a sample of UA1 jet data compared with the prediction from the jet fluctuation Monte Carlo.

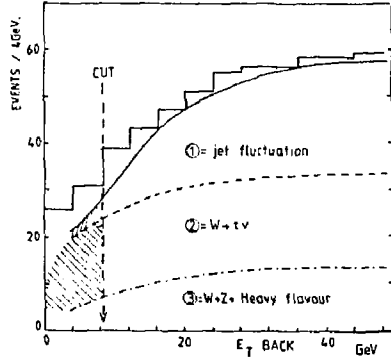


Figure 4: Distribution of the largest jet transverse energy (E_T^{back}) within ± 30 degrees in azimuth of the direction opposite to the trigger jet or within ± 30 degrees in azimuth of the missing transverse energy direction. The solid curve represent the sum of the events which come from jet fluctuation background (region 1), the $W \rightarrow \nu\bar{\nu}$ events (region 2), and the other contributions from the Standard Model (region 3).

3.3 Contributions from Standard Model Sources

These contributions were evaluated using a modified version of the ISAJET Monte Carlo '15' in which the spectator particle parameters have been adjusted to be consistent with UA1 data and which was extended to provide a full simulation of the UA1 detector including hardware trigger. Events were generated for all Standard Model processes expected to give missing E_T events. These processes are defined in Table 1. The statistics of the generated Monte Carlo samples correspond to approximately ten times the actual recorded luminosity. Processes involving a W or a Z^0 particle were also simulated using a second method taking events from our $W \rightarrow e\nu$ sample. The electron was removed and a random τ decay of W was superimposed on top of the spectator particle. The results from this second method are in complete agreement with the ISAJET calculations within the statistics available from our W and Z^0 data sample.

MONTE CARLO SAMPLES

Process	Monte Carlo	Normalisation	Comments
QCD Jet	Jet Fluctuation	UA1 Jet Data Two jets with $E_T^{\text{jet}} \geq 15$ GeV	
$pp \rightarrow e\bar{e}^{(*)}$ $pp \rightarrow b\bar{b}^{(*)}$	ISAJET '15	UA1 inclusive μ data $10 \leq p_T^e \leq 15 \text{ GeV}/c$	no top quark
$W \rightarrow e\nu$ $W \rightarrow \mu\nu$ $W \rightarrow \tau\nu$ $Z^0 \rightarrow \nu\bar{\nu}$ $Z^0 \rightarrow \tau^+\tau^-$	ISAJET Mixed events	$\sigma B(W \rightarrow e\nu)_{\text{UA1}}$ $\sigma B(Z^0 \rightarrow e^+e^-)_{\text{UA1}}$	$\bullet p_T^{\nu}$ spectrum from UA1 data $\bullet \tau$ polarisation included
$W \rightarrow e\bar{e}^{(*)}$ $Z \rightarrow e\bar{e}^{(*)}$ $Z \rightarrow b\bar{b}^{(*)}$	ISAJET	$\sigma B(W \rightarrow e\nu)_{\text{UA1}}$ $\sigma B(Z^0 \rightarrow e^+e^-)_{\text{UA1}}$	top quark not included

Table 1: Physics and background Monte Carlo processes used for the study of the E_T^{miss} data sample. The $(*)$ is attached to semi-leptonic decays.

3.4 Comparison with the Data

This comparison is shown in Figure 2(b), where we present the histogram of

N_e for the 148 selected monojet events (3σ sample) and the prediction for the expected jet fluctuation background (shaded region) and the contributions from the expected Standard Model sources (dashed curve).

It appears clearly that at low N_e , the background is still dominated by the jet fluctuation.

We give in Table 2, the contributions from the different channels. A total contribution of 127.4 ± 3.8 events (where the error is statistical error from Monte Carlo generation) has to be compared to the 148 selected events.

The experimental jet transverse energy (E_T^{jet}) and the missing transverse energy (E_T^{miss}) distributions (Fig. 5 (a) and (b)) are well reproduced by the Monte Carlo predictions.

In order to minimise the background, we decide to consider only the events with $N_e \geq 4$ in the following sections.

CONTRIBUTIONS TO THE 3σ SAMPLE

PROCESS	MONTÉ CARLO CONTRIBUTION
Heavy Flavour	11.9 ± 2.0
all W and Z decays	13.5 ± 1.7
$W \rightarrow \tau\nu; \tau \rightarrow \text{hadrons} + \nu$	39.7 ± 1.8
Jet fluctuation	62.3 ± 2.0
TOTAL	127.4 ± 3.8

Table 2: Predicted event rates for processes giving large missing transverse energy passing all experimental event selection cuts with $N_e \geq 3$.

3.5 The 4σ Missing Transverse Energy Data Sample

From now on, we work with the 4σ sample defined with the following selection criteria:

- $E_T^{miss} \geq 15$ GeV and $N_e \geq 4$,
- at least one jet with $E_i \geq 12$ GeV,

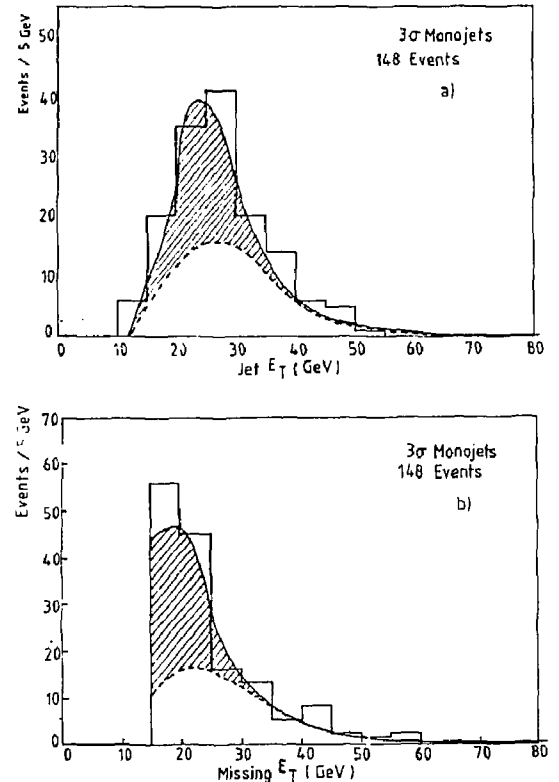


Figure 5: (a) Jet E_T distribution, (b) Missing E_T distribution, for the 148 3σ monojet events. The solid curves represent the contributions from the jet fluctuation background (shaded region) plus the Standard Model contributions (dashed line).

- isolated E_T^{miss} : No jet with $E_T \geq 8$ GeV in the calorimeter (or $p_T \geq 5$ GeV/c in the central detector) found within ± 30 degrees in azimuth along the E_T^{miss} direction or back to back to the trigger jet direction.

This selection produces 56 events. After subtracting the expected background from jet fluctuation (3.8 events), one is left with a signal of 52.2 events. The expected contribution from known Standard Model processes is 48.2 events.

A detailed breakdown of the processes is given in Table 3. One sees that the dominant contribution comes from $W \rightarrow \tau\nu$; $\tau \rightarrow \text{hadrons} + \nu$. All other contributions are smaller and come from a large variety of sources although most are due to the decays of the W or Z⁰ bosons.

CONTRIBUTIONS TO THE $L_s < 0$ DATA SAMPLE

Process	Events (total)	Events with $L_s < 0$	Events with $L_s < 0$ and $E_T^{\text{miss}} < 40$ GeV
$W \rightarrow e\nu$ $W \rightarrow \mu\nu$ $W \rightarrow \tau\nu$; $\tau \rightarrow \text{leptons}$	3.6	2.0	1.4
$W \rightarrow \tau\nu$; $\tau \rightarrow \text{hadrons} + \nu$	36.7	8.0	7.1
$W \rightarrow c\bar{s}$	< 0.1	< 0.1	< 0.1
$Z^0 \rightarrow \tau^+\tau^-$	0.5	0.1	0.1
$Z^0 \rightarrow \nu\nu$ (3 neutrino species)	7.4	7.1	5.6
$Z \rightarrow c\bar{c}$ and $Z \rightarrow b\bar{b}$	< 0.1	< 0.1	< 0.1
$c\bar{c}$ and $b\bar{b}$ (direct production)	0.2	0.2	0.2
Jet Fluctuation	3.8	3.4	3.4
TOTAL	52.2	20.8 ± 6.1	17.8 ± 4.7

Table 3: Predicted event rates for processes giving large missing transverse energy passing all experimental event selection cuts ($N_s \geq 4$ and $L_s < 0$).

In terms of number of jets with $E_T \geq 12$ GeV, the 4 σ sample contains 53 monojet events, three di-jet events and no higher multiplicity. The distributions of the trigger jet E_T and of the E_T^{miss} for the 56 events are well described by the Monte Carlo (Figure 6) curves corresponding to the total contributions from the expected sources.

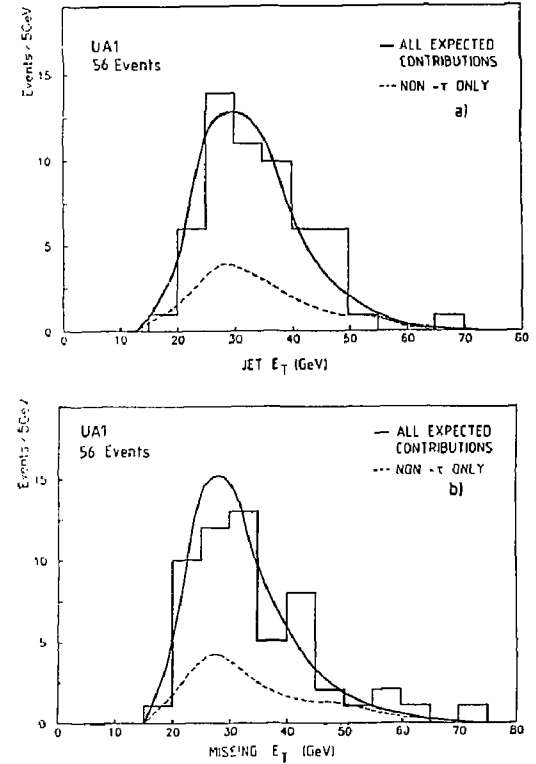


Figure 6: Distributions (a) of the trigger jet transverse energy (E_T^{jet}) and (b) of the transverse missing energy (E_T^{miss}) for the 56 events from the 4 σ data sample. The curve represents the total contribution from expected sources.

4 Study of the $W \rightarrow \tau\nu$ Decay

4.1 Event Isolation

The $W \rightarrow \tau\nu$ decay is the dominant contribution to our selected sample. So to study the decay properties of the $W \rightarrow \tau\nu$ events, we need to isolate these events. For that we shall develop selection criteria by which hadronic jets from tau decays can be distinguished from quark or gluon jets. We shall make use of previously measured properties of semi-hadronic $\tau \rightarrow \text{hadrons} + \nu$ decays, namely the low multiplicity, low mass jet.

The signature of the tau is a high transverse momentum narrow hadronic jet with associated missing transverse energy. For this purpose, we define three variables particularly suitable for selecting taus:

- *fraction of jet energy (F)* contained in a small cone of size $\Delta r = 0.4$ relative to the energy contained in the jet[14] definition cone of $\Delta r = 1.0$ ($\Delta r^2 = \Delta\phi^2 + \Delta\eta^2$).
- *angular separation (R)* between the leading track of the jet, as measured in the central detector drift chambers, and the jet axis as measured in the calorimeter.
- *charged particle multiplicity (n)* of tracks with $p_t > 1 \text{ GeV}/c$ within a cone of $\Delta r = 0.4$ about the calorimeter jet axis.

The distributions of F, R, and n from the tau Monte Carlo and from a sample of inclusive jet data are shown in Figure 7. As expected, the jets from tau decay, typically have larger values of F, smaller values of R and lower multiplicity n than for jets from an inclusive jet data sample. For each event, we assign a tau likelihood:

$$\mathcal{L}_\tau = \ln(P_F P_R P_n)$$

where P_F , P_R , and P_n are the values of the predicted tau distributions for the measured variables F, R, and n for the trigger jet (highest E_T^{jet}). The distribution of $\mathcal{L}_\tau$ for the data, tau Monte Carlo and jet data are shown in Figure 7 (d). The cut $\mathcal{L}_\tau > 0$ keeps 78% of the $\tau \rightarrow \text{hadrons} + \nu$ decays and rejects 89% of the ordinary jet events.

Therefore we define our final τ sample as those events with $\mathcal{L}_\tau > 0$.

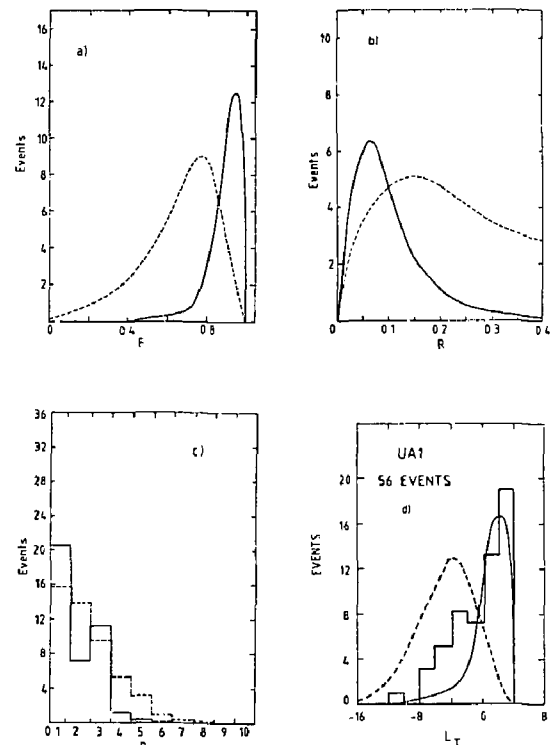


Figure 7: Distribution of (a) F, (b) R, (c) n and $\mathcal{L}_\tau$ from the tau Monte Carlo (solid curve and solid line histogram) and from jet data (dashed curve and dashed line histogram). In (d), the histogram is the $\mathcal{L}_\tau$ distribution for the 56 events in the 4 σ sample.

4.2 Properties of the $W \rightarrow \tau\nu$ Events

The total number of events predicted by the Monte Carlo from other Standard Model sources than $W \rightarrow \tau\nu$; $\tau \rightarrow \text{hadrons} + \nu$ with $\mathcal{L}_\tau > 0$ is 2.7 ± 0.6 events. This yields a net signal from $\tau \rightarrow \text{hadrons} + \nu$ of 29.3 ± 5.7 events, which agrees with the Monte Carlo prediction of 28.7 ± 1.5 events. Two examples of events with $\mathcal{L}_\tau > 0$ (a 1-prong and a 3-prongs τ candidate) are shown in Figure 8.

In both cases one observes an isolated narrow hadronic jet with large associated missing transverse energy.

The $W \rightarrow \tau\nu$ events have two neutrinos present (tau neutrino and tau-antineutrino): a hard one from the primary decay of the W and a softer from the decay of the τ . Thus, we do not expect a jacobian peak in the jet E_t distribution as observed in the electron E_t spectrum for the $W \rightarrow e\nu$ decay. The jet E_t and missing E_t are shown in Figure 9(a) and 9(b) for the 32 tau 4e events (histogram) together with the non-tau backgrounds (shaded). The calculated curves reproduce well the data.

All observed properties of our 32 events are consistent with those expected for τ particles produced in W decays. Most of the events have only one narrow hadronic jet (one event has a significant second jet with $E_t > 12$ GeV), and no other tracks or calorimeter hits with significant transverse momentum or energy. The distribution of n (22 1-prong, three 2-prongs and seven 3-prongs) is in agreement with Monte Carlo expectations where the effect of detector acceptance, photon conversion and spectator tracks have been taken into account. The jets in the 2- and 3-prong events have low invariant mass (about 1 GeV/ c^2) (Figure 10).

The momentum distribution of the tracks and the energies deposited in the electromagnetic and hadronic calorimeters are consistent with our calculations for τ decays. The angular distribution of the τ jet according to the direction of the colliding particles is also in agreement with the τ Monte Carlo calculations.

The transverse mass is a sensitive variable for deriving the mass of the parent W particle. As previously used in the analysis of the W decays, we define a transverse mass m_t as:

$$m_t^2 = 2E_t^{\text{jet}} E_t^{\text{miss}} (1 - \cos \phi)$$

where ϕ is the azimuth difference between the jet and the missing energy direction. The measured transverse mass is shown in Figure 11. We have performed a maximum likelihood fit to determine the W mass and we find the value:

$$m_t = 89 \pm 3 \pm 6 \text{ GeV}/c^2$$

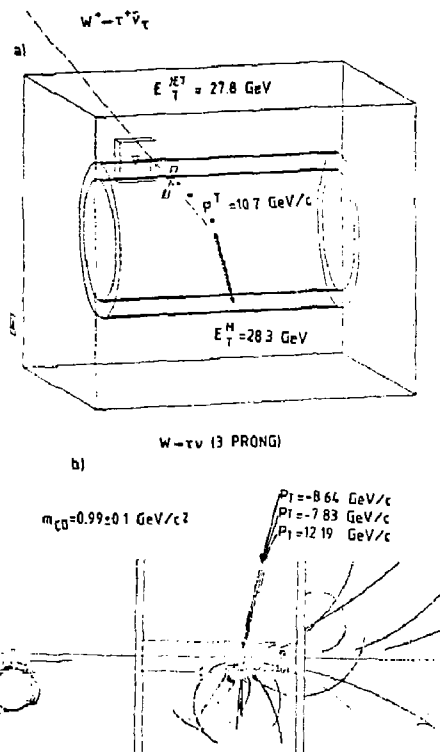


Figure 8: Graphic display of two events from the τ sample ($\mathcal{L}_\tau > 0$) (a) a 1-prong and (b) a 3-prongs. In (a) only charged tracks with $p_t > 1$ GeV/ c and calorimeter cells with $E_t > 1$ GeV are displayed. In (b) all raw digitisations recorded by the central detector drift chamber are shown.

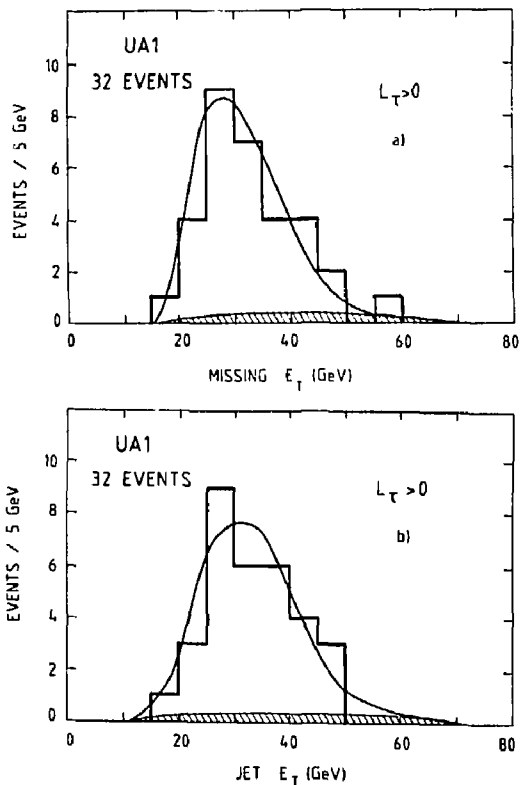


Figure 9: Distribution of (a) jet E_T and (b) missing E_T for the sample of 32 4e events with $L_\tau > 0$ (histogram). The curve is the expectation from Monte Carlo with the shaded region being the non-tau background.

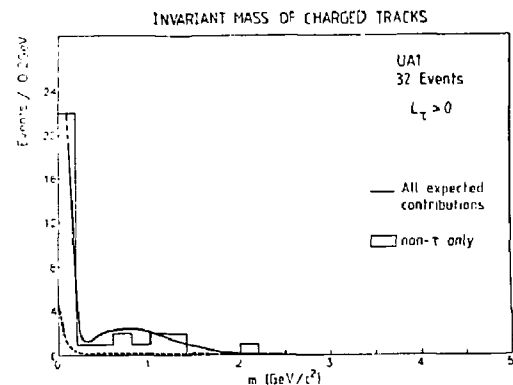


Figure 10: Effective mass of charged tracks for 1-, 2- and 3-prongs jets from the 32 $L_\tau > 0$ events. The solid curve is the expectation from Monte Carlo with the shaded region being the non-tau background

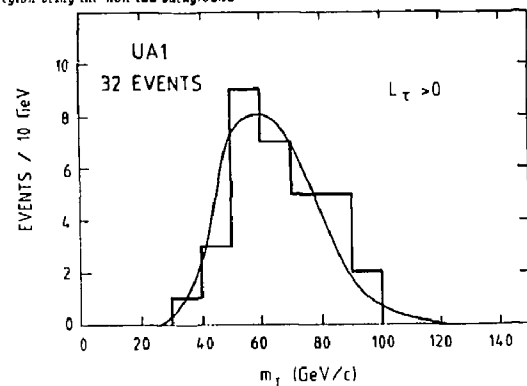


Figure 11: Transverse mass distribution for the tau sample (histogram). The curve shows the expectation from the Monte Carlo for a W of $83.5 \text{ GeV}/c^2$.

where the first error is statistical and the second comes from systematic errors of the absolute energy scale determination.

The expected transverse mass distribution for $m_W = 83.5 \text{ GeV}/c^2$ is shown in Figure 11 for comparison with the data.

4.3 $W \rightarrow \tau\nu$ Cross Section, Universality Test

From the rate of observed $W \rightarrow \tau\nu$ events, we can calculate the cross section for our process. The calculated acceptance for $W \rightarrow \tau\nu$ is $6.8 \pm 0.4\%$, which is made up of the following components: $\tau \rightarrow \text{hadrons} + \nu$ branching ratio (64%), trigger efficiency (45%), E_T^{miss} event selection (30%), and tau selection cut $\mathcal{L}_\tau > 0$ (78%).

We deduce the W production cross section times branching ratio into tau plus neutrino:

$$\sigma B(W \rightarrow \tau\nu) = 600 \pm 120 \pm 110 \text{ pb}$$

where the first error is statistical and the second is systematic. The systematic error is mainly due to the uncertainty of the luminosity and the prediction of the Monte Carlo tau rates.

The cross section times branching ratio for the decay $W \rightarrow e\nu$ [10] and $W \rightarrow \mu\nu$ have also been measured UA1 [6]:

$$\sigma B(W \rightarrow e\nu) = 590 \pm 50 \pm 90 \text{ pb}$$

$$\sigma B(W \rightarrow \mu\nu) = 650 \pm 70 \pm 140 \text{ pb}$$

providing direct confirmation of the electron-muon universality of the weak coupling of the W. Since the discovery of the third lepton family (τ) more than ten years ago, [16] the τ has been observed only through its coupling into virtual photon (with a small contribution from virtual Z^0 particle) in e^+e^- collisions [17, and in upilon decay, [18].

Electron-muon universality of weak charged current is verified precisely at low energy by comparison of the branching ratios $\pi \rightarrow \mu\nu$ and $\pi \rightarrow e\nu$. This measurement [19] gives the relative strengths of the W couplings to $e\nu$ (g_e) and $\mu\nu$ (g_μ) to be $g_e/g_\mu = 0.9939 \pm 0.0057$. The universality of the strength of the W coupling to $\tau\nu$ (g_τ) has been checked at low energy by analysis of τ decay rates, made possible by recent measurements of the τ lifetime, [20]. From a world average of decay rate measurement, g_τ/g_μ is unity to a precision of about 5%.

Our data allow the first check of the universality of the weak charged couplings at high energy. We may express our measured ratio in terms of the weak charged coupling constants g_e and g_τ :

$$\Gamma(W \rightarrow \tau\nu) / \Gamma(W \rightarrow e\nu) = g_\tau^2 / g_e^2$$

(neglecting very small phase space differences for the electron and τ) from which we deduce that:

$$g_\tau / g_e = 1.01 \pm 0.09 \pm 0.03.$$

From comparison of the $W \rightarrow \mu\nu$ and $W \rightarrow e\nu$ rates, we derive the analogous result that:

$$g_\mu / g_e = 1.05 \pm 0.07 \pm 0.08.$$

These data provide direct experimental verification of $e - \mu - \tau$ universality for weak charged couplings at $Q^2 = m_W^2$.

5 The Non-tau Sample

Figure 12 gives the scatter plot of jet transverse energy (E_T^{jet}) versus tau likelihood ($\mathcal{L}_\tau$). Figure 13 displays a typical monojet event.

It was shown in the last section, that the 32 events above the line $\mathcal{L}_\tau = 0$ mainly come from $W \rightarrow \tau\nu$ decay. By requiring now $\mathcal{L}_\tau < 0$, most of the $W \rightarrow \tau\nu$ events are eliminated. The remaining sample of 22 relatively broad monojets and two di-jets have relatively large contributions from:

- known decays of W and Z^0 ,
- decays of heavy flavours,
- fluctuations of detector response to ordinary jets.

The total expected contribution from known processes for $\mathcal{L}_\tau < 0$ is 20.8 events. The breakdown of these contributions in the different Standard Model sources is given in Table 3.

The absolute normalisation of the cross sections is obtained from experimental data, using our full W and Z^0 data sample. The W transverse momentum spectrum generated by ISAJET was modified to agree with the measured UA1 spectra up to $p_T^W = 40 \text{ GeV}/c$. Above $40 \text{ GeV}/c$, where only two events are observed, the spectrum was extrapolated using the ISAJET slope. The resulting p_T^W spectrum is consistent with perturbative QCD calculations [21] over a wide range of p_T^W (Figure 14).

Because of limited Z^0 statistics (53 events), the p_T spectrum for the Z^0 could not be determined with sufficient accuracy, and was deduced by scaling the W spectrum according to theory.

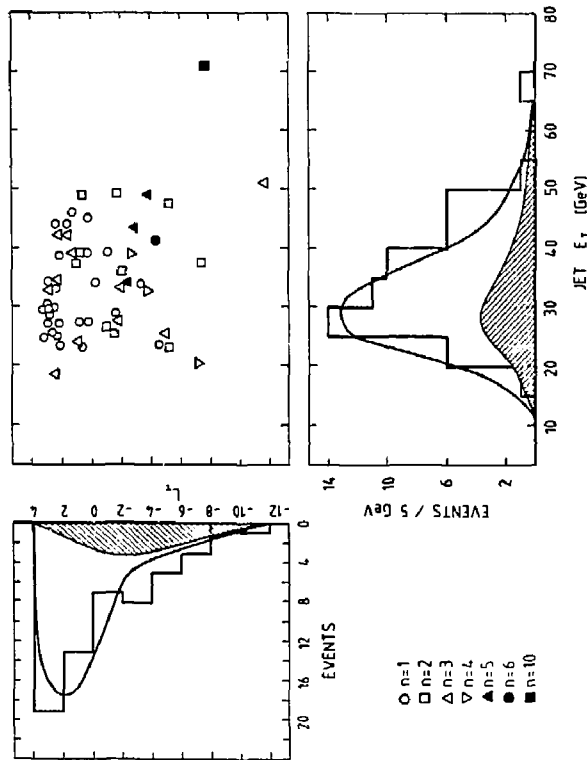


Figure 12: Scatter plot of transverse energy of the highest E_T jet in the event versus the likelihood L_T for 56 events. The different symbols indicate different charged multiplicities of the jet ($\circ$: 1-prong, $\square$: 2-prong, $\triangle$: 3-prong, ∇ : 4-prong, $\blacktriangle$: 5-prong, $\bullet$: 6-prong, $\blacksquare$: 10-prong). The projections for E_T and L_T are plotted for the data (histograms), all predicted contributions (solid line) and non-lau contributions (shaded area).

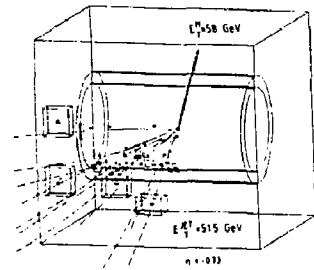


Figure 13: Event display of a monojet event in the UA1 detector. Only tracks with $p_T > 1 \text{ GeV/c}$ and calorimeter cells with $E_T > 1 \text{ GeV}$ are displayed.

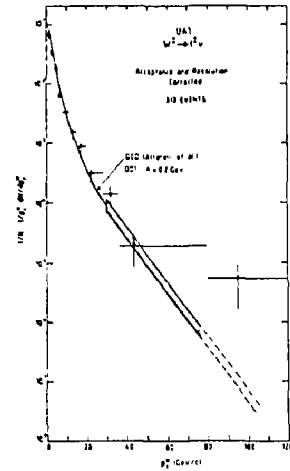


Figure 14: W transverse momentum distribution for the UA1 $W \rightarrow e\nu$ and $W \rightarrow \mu\nu$ data samples, corrected for detector acceptance and resolution. The full curve is the modified ISAJET spectrum used for the missing energy analysis. The shaded region is the result of the perturbative QCD calculation.[21]

In addition, we have calculated the expected contributions of missing energy events from the top quark [22] due to processes $W \rightarrow tb$, $Z^0 \rightarrow tt$, and direct tt . For a top quark of mass $40 \text{ GeV}/c^2$, the number of expected events is (4.3 ± 2.2) . The data clearly allow room for these small contributions. [23]

The total error on the Monte Carlo prediction of 20.8 events with $L_T < 0$ is ± 6.1 events. This error includes:

- the statistical errors on the measurement of the high transverse momentum W and Z^0 cross section (4.6 events),
- the statistical errors of the Monte Carlo generation (2.0 events),
- the uncertainties in the jet energy scale (0.7 events),
- the QCD backgrounds (0.7 event),
- the τ acceptance (0.5 event),
- the heavy flavour cross section uncertainty (0.2 event).

5.1 Properties of the $L_T < 0$ Events

We observe 24 events with $L_T < 0$. As we have seen, the Monte Carlo predictions of the known processes (see Table 3), give a total of 20.8 ± 6.1 events. The jet E_T and missing E_T distribution are shown in Figure 15(a) and 15(b) (histogram) together with the expected contributions (solid curve). The overall agreement between the data and the Monte Carlo is rather good, although we observe a slight excess of events at large values of jet E_T . The majority of these events are expected to come from high p_T Z^0 production ($Z^0 \rightarrow \nu\nu$). This could be consistent with the possible excess of high p_T W events seen in the $W \rightarrow e\nu$ and $W \rightarrow \mu\nu$ channels [3], and will be discussed in the next section of this paper. The distribution of the transverse energy for the second highest E_T jet in the events is shown in Figure 16.

The data sample contains 12 events with $E_T^{\text{jet2}} > 5 \text{ GeV}$ and two events with two jets with $E_T > 12 \text{ GeV}$. These events will be more deeply discussed in the next section.

We can conclude that, within the presently available statistics, the large missing E_T sample can be understood in term of known Standard Model processes. But we can use the non-tau sample to place limits on a variety of possible new physics processes.

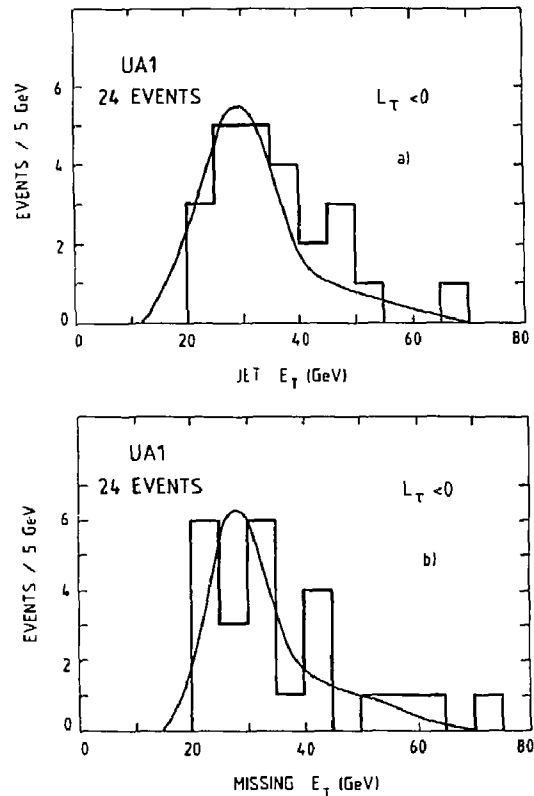


Figure 15: (a) Jet transverse energy distribution (b) Missing transverse energy distribution, for events passing the cut $L_T < 0$ (24 events, histogram) compared with all expected contributions (solid line).

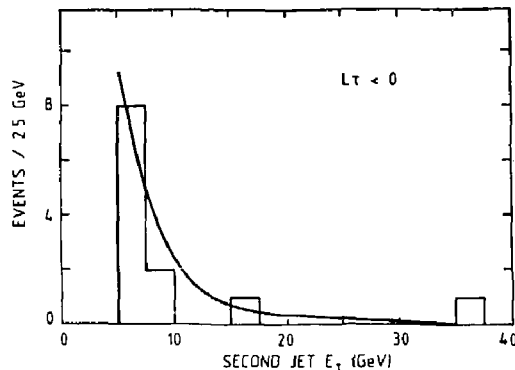


Figure 16: Distribution of transverse energy of second highest E_T jet ($E_T^{jet 2} > 5 \text{ GeV}$) for events from the non-tau sample (histogram). The curve shows the expectations from the Monte Carlo.

5.2 General Limit on New Physics Processes

We first derive a general limit on the total cross section for any new process which gives isolated transverse energy events with $E_T^{miss} > 4 \sigma$ and $L_T < 0$.

The number of observed events is 24 with an expected contribution of $20.8 \pm 5.0 \pm 1.1$. The first error includes the statistical errors. The second one is the systematic error due to uncertainty in:

- the absolute energy scale (0.7 event),
- normalisation of the jet fluctuation background (0.7 event),
- the tau acceptance (0.5 event),
- the heavy flavour cross section (0.2 event).

Taking into account both statistical and systematic errors, one obtains the following general limit on the total cross section for any new physics process :

$$\sigma \epsilon < 20 \text{ pb} (90\% \text{ C.L.})$$

where ϵ is the efficiency for events from a given process to pass all the event selection cuts and the requirement of $L_T < 0$.

5.3 Heavy Lepton

The existence of three generations of leptons and quarks is well established, while the possibility of a fourth generation is allowed by the Standard Model. Direct searches in e^+e^- collisions for a fourth generation charged lepton have placed a lower mass limit of $22.7 \text{ GeV}/c^2$, at 90 % confidence level, [24].

The weak coupling of a new generation of particles to the W and Z⁰ would be expected to be of universal strength. This universality has been explicitly tested (section 4.3) to about 10 % for the leptonic decay modes of the W into electron, muon and tau families.

We have analysed in detail the effects of the W decays into a new sequential heavy charged lepton, where the heavy lepton decays semi-hadronically:

$$W^+ \rightarrow L\nu; L \rightarrow u\bar{d}\nu \text{ or } L \rightarrow c\bar{s}\nu \text{ (or charged conjugate states)}.$$

The signature is one or two high transverse momentum jets with an associated missing transverse energy. The contributions from this process was evaluated with ISAJET Monte Carlo with full detector simulation. The matrix element in ISAJET was modified to account for spin effects in the production and decay of the heavy lepton, [25]. Events were generated for ten different values of m_L ranging between 20 and 75 GeV/c^2 . The event rates are predicted from

the measured $W \rightarrow e\nu$ rates including the effects of phase space,[26] trigger acceptance and event selection cuts.

The highest E_T jet in the heavy lepton Monte Carlo events usually has $L_T < 0$ and $E_T^{jet} < 40$ GeV,[25] so we restrict ourselves to this region to set limits on the allowed range of heavy lepton mass.

The net acceptance for a heavy lepton of mass 40 GeV/c² decaying semi-hadronically, is 6.2 %, compared to 10.6 % for the tau.

The expected contribution of mono- and multijets events to a $L_T < 0$ and $E_T^{jet} < 40$ GeV sample (where 17 events are observed, while the predicted contributions total 17.8 events with a total error of ± 4.7 events) is shown as function of m_L in Figure 17.

With the jet definition[14] used, most of the heavy lepton decays are predicted to give monojet event when the data selection is applied. The number of two-jet events ranges from 2.5 events for $m_L = 25$ GeV/c² to 0.3 event for $m_L = 65$ GeV/c². For the monojet events the prediction is large at low mass (16.3 events for $m_L = 20$ GeV/c²) and falls to zero as m_L approaches the W mass.

Taking into account the existence of the additional neutrino coupling to the Z^0 , we derive the limit:

$$m_L > 41 \text{ GeV}/c^2 \text{ (90\% C.L.)}$$

5.4 Additional Neutrino Families

Another possible source of events would be the existence of additional neutrinos coupling to the Z^0 . A major component of the expected physics contributions to the selected data (7.1 out of 20.8 events) comes from the process $Z^0 \rightarrow \nu\bar{\nu}$, where we have assumed three neutrino families (Table 3).

The existence of additional light neutrinos or of any other noninteracting neutral particle coupling to the Z^0 would be expected to produce missing E_T events in the same way.

As discussed earlier, such events would be preferentially selected at high Z^0 transverse momentum. Because of uncertainties in the predicted rates of high p_T W's and Z^0 's, we restrict ourselves to the region $E_T^{jet} < 40$ GeV. For $L_T < 0$, we calculate a contribution of 1.8 events from $Z^0 \rightarrow \nu\bar{\nu}$, for each neutrino species.[31] We derive the limit on the number of light extra-neutrino species:

$$N_\nu < 7(90\% \text{C.L.})$$

All the statistical and systematic errors on the data and on the Monte Carlo calculations have been taken into account. Inclusion of the top quark contribution

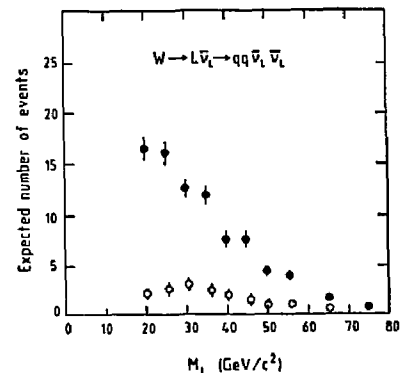


Figure 17: Expected contribution to the region $L_T < 0$, $E_T^{jet} < 40$ GeV from heavy lepton production as a function of the heavy lepton mass (full data points). The open data points show the contribution for events containing two or more jets with $E_T > 18$ GeV.

in the Monte Carlo calculations would lower the limit on the number of new neutrino species.[6]

We present in Figure 18 more restrictive limits on N_ν obtained from:

- $\bar{p}p$ Collider measurements,[6]
- e^+e^- experiments,[28],[29],[30],[31]
- cosmological arguments based on big bang nucleosynthesis calculations,[33] and [34]

Although our limit is not the lowest one, it should be stressed that this is an independent measurement and is the first use of the gluon tagging method of neutrino counting. This method is somewhat equivalent to the one proposed for e^+e^- collisions [32] where one plane to perform ν counting by setting the machine energy above the Z^0 resonance and detecting a radiated photon (rather than a gluon).

We may also place an upper limit on the partial width Γ_P of the Z^0 into any light neutral noninteracting particles (assuming a partial width of 180 MeV for each decay, appropriate for $Z^0 \rightarrow \nu\bar{\nu}$ decays) of:

$$\Gamma_P < 1.8 \text{ GeV} (90\% \text{ C.L.}).$$

The net acceptance for detecting a missing energy event from $Z^0 \rightarrow \nu\bar{\nu}$ decay is 1.8 %.

5.5 Limit on Masses of Super-Symmetric Particles

We have investigated the acceptance of the present missing transverse energy event selection for decays of massive supersymmetric particles which have stable and noninteracting photinos as decay products. There are a large number of possible scenarios to consider, depending on particle masses and other free parameters. We have chosen the most simple model with[35]:

- The photino is the lightest supersymmetric particle with $m_{\tilde{\gamma}} = 0$,
- the first five squark masses are degenerate $m_{\tilde{u}} = m_{\tilde{d}} = m_{\tilde{s}} = m_{\tilde{b}} = m_{\tilde{t}}$,
- the Higgsino, Wino, Zino, sleptons and top squark are ignored.

This leaves only the mass of the squark and the mass of the gluino as free parameters.

LIMITS ON NEUTRINO FAMILIES

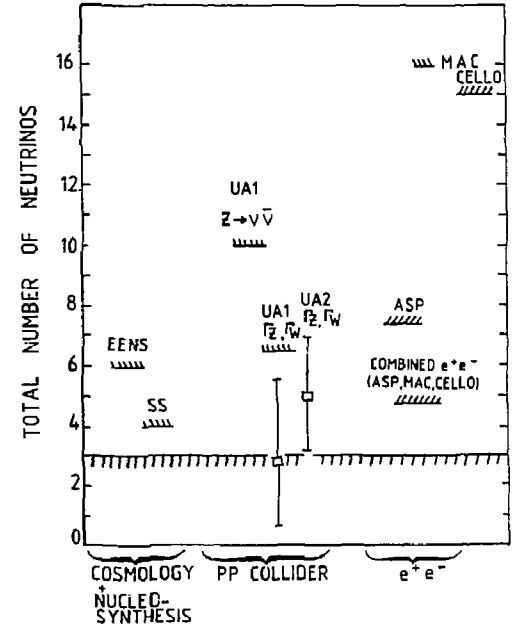


Figure 18: Present limit of the number of light neutrino species.

There are two main regimes for decay modes of the squarks and gluino:

$$\text{if } m_{\tilde{q}} > m_{\tilde{g}} \quad \tilde{q} \rightarrow q \tilde{g} \quad \text{and} \quad \tilde{g} \rightarrow q \tilde{q} \quad \bar{q},$$

$$\text{if } m_{\tilde{q}} < m_{\tilde{g}} \quad \text{then } \tilde{g} \rightarrow \tilde{q} \tilde{q}^* \quad \text{and} \quad \tilde{q} \rightarrow q \tilde{g}.$$

Since supersymmetric particles would be pair-produced, this could result in as many as six quarks in the final state with two photinos to carry the missing E_T .

The contributions to our 4σ sample were evaluated with ISAJET with full detector simulation. Figure 19 (a) shows the distributions of the number of jets per event with $E_T^{\text{jet}} > 12$ GeV for events which pass our selection criteria and for $m_{\tilde{q}} = 60$ GeV/c² and $m_{\tilde{g}} = 70$ GeV/c². The missing E_T for these events is shown in Figure 19(b).

One sees that supersymmetric events are produced with large value of E_T^{miss} and in association with typically two or three high E_T jets.

The overall acceptance for supersymmetric events is relatively low for our selection. It varies between 1 to 5 % for squark and gluino masses between 60 and 90 GeV/c² for example. This is due to the fact that supersymmetry events are not usually characterised by an isolated E_T^{miss} .

For $m_{\tilde{q}} = 60$ GeV/c² and $m_{\tilde{g}} = 70$ GeV/c² for example, we expect 4.6 monojet and 15.1 multijet events in our non-tau sample.

To set limits on the allowed range of squark and gluino masses, we consider the 4σ multijet sample (two events) with $L_T < 0$. The expected contribution to this sample from known Standard Model processes is $2.8 \pm 1.7 \pm 0.3$ events.

The limits at 90 % C.L. on supersymmetric particle masses derived from this sample is shown in Figure 20, where limits derived from e^+e^- annihilations [31] and beam dump experiments are also plotted. [30]

From this plot one can extract the following limits:

$$m_{\tilde{q}} > 70 \text{ GeV}/c^2 \text{ and } m_{\tilde{g}} > 60 \text{ GeV}/c^2 \text{ (90\% C.L.)}.$$

Note that we cannot yet exclude the region $3 < m_{\tilde{g}} < 5$ GeV/c² and $m_{\tilde{q}} > 100$ GeV/c² (the light gluino window).

We continue to investigate the optimisation of the E_T^{miss} selection cuts to try to further improve the squark and gluino mass limits.

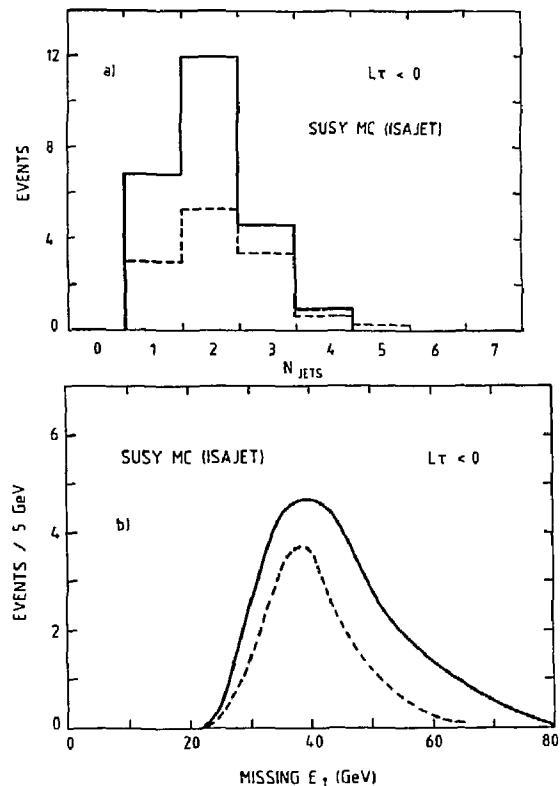


Figure 19: Distribution of (a) the number of reconstructed jets with $E_T^{\text{jet}} > 12$ GeV and (b) the missing E_T for supersymmetric events passing the isolated 4σ cuts. The events were generated with $m_{\tilde{q}} = 60$ GeV/c² and $m_{\tilde{g}} = 90$ GeV/c² (full histogram and full curve) or with $m_{\tilde{q}} = 70$ GeV/c² and $m_{\tilde{g}} = 60$ GeV/c² (broken histogram and broken curve).

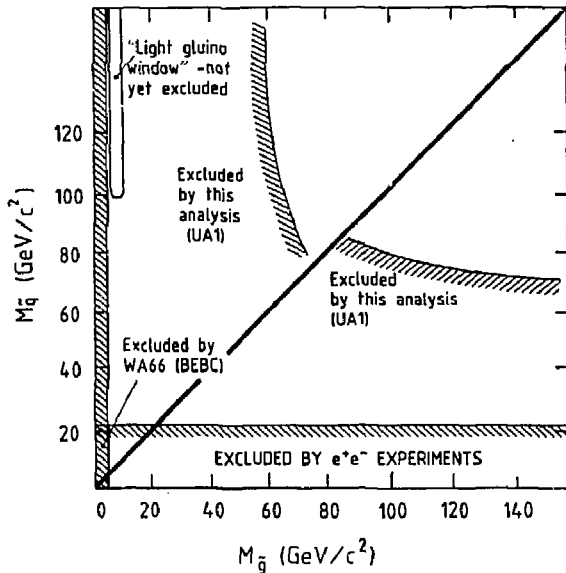


Figure 20: The 90% confidence level limits on squark and gluino masses derived from non-tau multijet sample (2 events). Also shown are limits from e^+e^- annihilations [84] and beam dump [86] experiments.

6 High Transverse Momentum Intermediate Vector Bosons

6.1 Event Characteristics

Using the 1982-1985 data, UA1 has collected a sample of 255 $W \rightarrow e\nu$ decays and 57 $W \rightarrow \mu\nu$ decays [6]. We have seen (Section 5) that the W transverse momentum distribution is well described by QCD predictions [21] in the region $p_t^W < 40 \text{ GeV}/c$ (Figure 14). We have also mentioned the presence of two events one from the electron and one from the muon channels), which deviate from the theoretical prediction at high reconstructed p_t^W .

In both cases two energetic jets are observed in addition to the charged lepton and the missing transverse momentum.

The parameters of these events, labelled A and B are listed in Table 4.

EVENT A: A graphical display of this event is shown in Figure 21(a). The W^- candidate decays in $(\mu + \nu\mu)$ channel. The reconstructed transverse momentum of the $(\mu\nu)$ system is $p_t^W = 83 \pm 9 \text{ GeV}/c$. The muon has transverse momentum $p_t^\mu = 21.4 \pm 3 \text{ GeV}/c$. Although the muon passes all the W isolation criteria, we note the presence of a nearby oppositely charged track with $p_t = 1.9 \pm 0.1 \text{ GeV}/c$. The two-track invariant mass is $2.6 \pm 0.2 \text{ GeV}/c^2$. Two energetic hadronic jets are observed, with transverse momenta of $(73 \pm 8) \text{ GeV}/c^2$ and $(22 \pm 3) \text{ GeV}/c^2$ respectively. The two-jet invariant mass is $(71 \pm 7) \text{ GeV}/c^2$ and the three body mass (W -jet+jet) is $(304 \pm 24) \text{ GeV}/c^2$. This later mass has been obtained from the muon, neutrino and the two-jet four vectors. Since we do not measure the longitudinal component of the neutrino momentum (in the direction of the incoming beams), it has been calculated by imposing the W

HIGHEST p_t W AND E_t^{miss} TWO JETS EVENTS

Event	p_t^W (GeV/c)	$E_t^{\mu 1}$ (GeV)	$E_t^{\mu 2}$ (GeV)	m_{Wj1} (GeV/c ²)	m_{j2} (GeV/c ²)
A $W \rightarrow e\nu$	83 ± 9	73 ± 8	22 ± 3	304 ± 24	71 ± 7
B $W \rightarrow \mu\nu$	110 ± 11	94 ± 10	19 ± 3	287 ± 21	96 ± 9
C E_t^{miss}	83 ± 9	71 ± 7	40 ± 4	268^{+28}_{-21}	121 ± 10
D E_t^{miss}	73 ± 8	52 ± 5	21 ± 3	204^{+20}_{-14}	47 ± 6

Table 4: Event parameters for events containing two hadron jets, large missing energy and a lepton (A and B) or only missing energy (C and D).

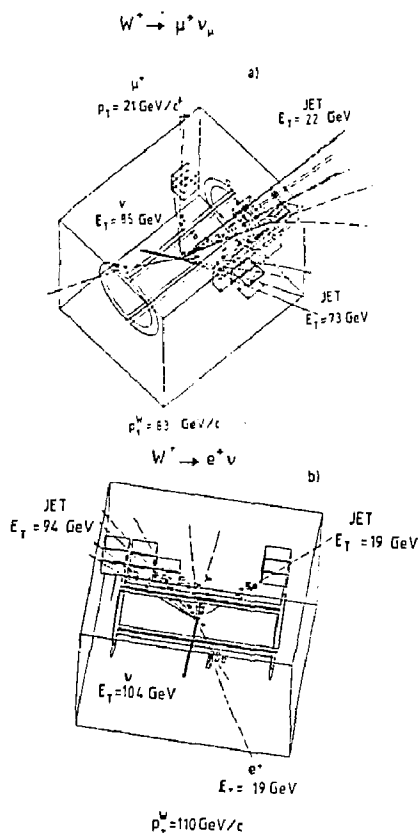


Figure 21: Graphic display of calorimeter cells ($E_T > 1 \text{ GeV}$) and charged tracks ($p_T > 1 \text{ GeV}/c$) observed in the UA1 detector for: (a) event A and (b) event B.

mass ($m_{\mu\nu} = 83.5 \text{ GeV}/c^2$) on the $(\mu\nu)$ -system. The two-fold ambiguity arising from this procedure [10], is resolved by choosing the solution which gives the lowest W momentum. The alternative solution would give a $(W+\text{jet}+\text{jet})$ mass of $(255 \pm 17 \text{ GeV}/c^2)$.

EVENT B: A graphical display of this event is shown in Figure 21(b). The W^+ candidate decays in the $(e^+\nu_e)$ channel. The positron has transverse energy $E_T^e = (19.1 \pm 1.2) \text{ GeV}$, and the reconstructed transverse energy of the $(e\nu)$ -system is $(110 \pm 11) \text{ GeV}/c$. Once again we note that, although the positron satisfies all the W isolation criteria, there are two nearby charged tracks, both with negative charge. Their transverse momenta are $p_T = (0.7 \pm 0.1) \text{ GeV}/c$ and $(3.2 \pm 0.1) \text{ GeV}/c$. The three-track invariant mass is $(1.7 \pm 0.3) \text{ GeV}/c^2$. The missing transverse energy in the event is $(104 \pm 11) \text{ GeV}$. As for event A, two energetic hadronic jets are observed, with momenta of $(94 \pm 10) \text{ GeV}/c$ and $(19 \pm 3) \text{ GeV}/c$. The two-jet invariant mass is $(96 \pm 9) \text{ GeV}/c^2$, and the three-body mass $(W+\text{jet}+\text{jet})$ is $(287 \pm 21) \text{ GeV}/c^2$. This latter mass has been obtained by the same method applied to event A. If the alternative solution for the longitudinal component of the neutrino momentum is chosen, we obtain a $(W+\text{jet}+\text{jet})$ -mass of $(453 \pm 29) \text{ GeV}/c^2$.

There is a striking similarity between the topologies of events A and B. In particular, both events contain two energetic hadronic jets recoiling against the W candidate.

As also mentioned in Section 4, that in the missing transverse energy event sample with $Z_s < 0$ and $E_T^{\text{miss}} > 40 \text{ GeV}$, there also exist two striking two-jets events with high E_T^{miss} , which we label here C and D. They are displayed in Figure 22.

If the activity at large p_T^W is statistically and significantly higher than expected, this would lead to a larger number of events in the E_T^{miss} sample with high E_T . It is then possible to interpret these two events as Z^0 decays in the $(\nu\bar{\nu})$ channel. The parameters for C and D are summarised in Table 4.

In event C the $(\text{jet}-\text{jet})$ -mass is $(121 \pm 20) \text{ GeV}/c^2$. Furthermore the minimum mass of the $(Z+\text{jet}-\text{jet})$ -system is $(268^{+21}_{-21}) \text{ GeV}/c^2$, close to the mass scale associated with events A and B. The minimum mass has been obtained by identifying the missing transverse momentum in the event with the Z transverse momentum, and choosing the longitudinal momentum component of the Z that minimises the $(Z+\text{jet}-\text{jet})$ -mass. The transverse momentum of the invisible Z is $83 \pm 9 \text{ GeV}/c$. There is a clear topological and kinematical similarity between event C and events A and B.

In event D, the $(\text{jet}-\text{jet})$ -mass is $(47 \pm 6) \text{ GeV}/c^2$. The minimum mass of the $(Z+\text{jet}-\text{jet})$ -system is $(204^{+24}_{-24}) \text{ GeV}/c^2$. Although similar in topology, to events A, B and C, this event has a significantly smaller $(\text{jet}-\text{jet})$ -mass.

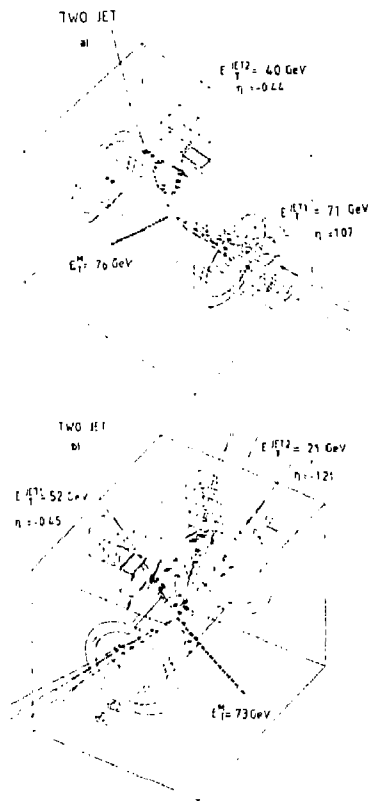


Figure 22: Graphic display of calorimeter cells ($E_T > 1 \text{ GeV}$) and charged tracks ($p_T > 1 \text{ GeV}/c$) observed in the UA1 detector for: (a) event C and (b) event D.

6.2 Other Properties

In Figure 23, we show the distribution of the fraction of events with $p_T > p_T^0$ for $W \rightarrow e\nu$ and $W \rightarrow \mu\nu$ events, and the distribution of the fraction of events with $E_T^{\text{miss}} > E_T^0$ for the 24 events from the $L_T < 0$ sample. The solid curves are:

- the QCD prediction of Reference [21], (Figure 23(a)),
- Monte Carlo prediction including all expected contributions from the Standard Model (Figure 23(b)).

One sees that up to $p_T^0 = 40 \text{ GeV}/c$ and $E_T^0 = 40 \text{ GeV}$, the curves reproduce well the data. For $p_T^0 > 40 \text{ GeV}/c$ and $E_T^0 > 40 \text{ GeV}$, the calculated contributions start to deviate from the data.

In order to have a better understanding of these high p_T^W and E_T^{miss} events, we have selected the sub-sample of leptonic W decays in which two hadronic jets are reconstructed, each with $E_T^{\text{jet}} > 10 \text{ GeV}$. There are eight such (W+jet+jet) events, including events A and B. The (W+jet+jet)-mass is shown as a function of p_T^W in Figure 24.

There are six events which cluster in region $p_T^W < 40 \text{ GeV}/c$ and $m_{W,jj} < 160 \text{ GeV}/c^2$, then a large gap with no event and finally events A, B and C with $p_T^W > 80 \text{ GeV}/c$ and $m_{W,jj} > 280 \text{ GeV}/c^2$. The six lower p_T^W events have p_T^W and $m_{W,jj}$ distributions which are well described by the QCD expectation for (W+jet+jet) events. The curves shown in Figure 24 have been obtained using the lowest order (W+jet+jet) Monte Carlo program of reference [37].

Figure 25 shows the correlation between the jet-jet mass m_{jj} and the three-body mass $m_{W,jj}$ for the eight (W+jet+jet) events and the two di-jets from the E_T^{miss} selection. Figure 26 shows the correlation between m_{jj} and p_T^W .

The resulting QCD calculation for (W+jet+jet) predicts $(3.3 \pm 0.9 \pm 0.6)$ events for the selected sample, where the first error reflects the uncertainty of the detector simulation procedure and the precision of the knowledge of the integrated luminosity, and the second error reflects the theoretical uncertainty associated with the choice of Q^2 scale and the underlying structure functions. If the jet transverse-momentum cut is lowered from $10 \text{ GeV}/c$ to $9 \text{ GeV}/c$, corresponding to a 10% uncertainty in the jet energy scale, the predicted number of (W+jet+jet) events increases to $(4.2 \pm 1.1 \pm 0.8)$.

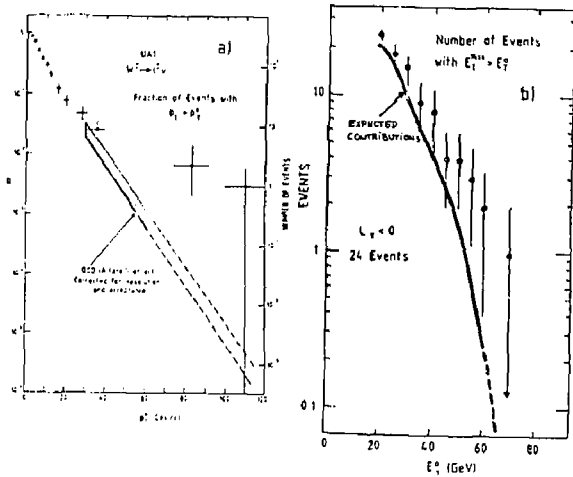


Figure 23: Distribution of the fraction of events with (a) $p_T > p_T^0$ for the W events, the curve shows the QCD prediction of Reference [97] and (b) $E_T^{miss} > E_T^0$ for the events from the E_T^{miss} data sample, the curve is the calculation for all expected contributions from the Standard Model. The data have been corrected for detection and selection inefficiencies and for the effects of detector resolution.[6]

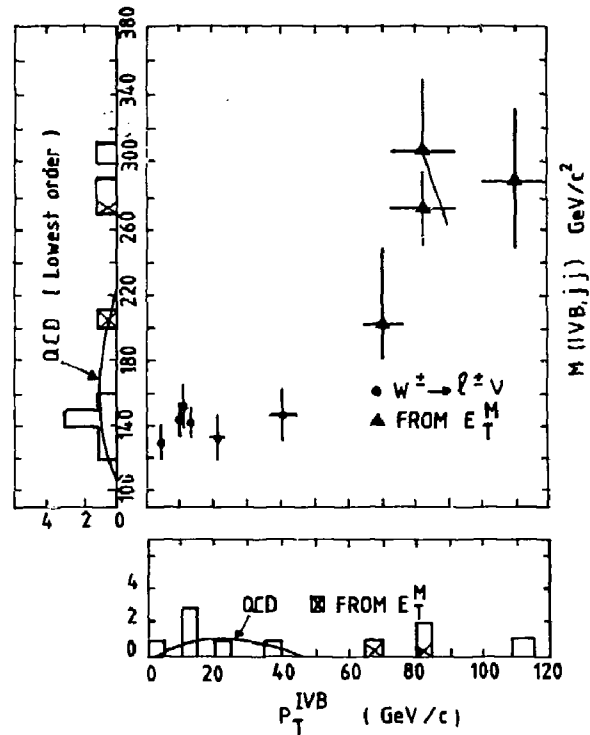


Figure 24: Correlation between the W transverse momentum and the $(W+jet+jet)$ -mass for the eight W events in which two hadronic jets with $E_T^{jet} > 10$ GeV are observed. The curves on the projections of the distributions are the predictions from the explicit second order (in α_s) QCD calculations [97] with the normalisation from theory. The two di-jets events from the E_T^{miss} selection have been also plotted.

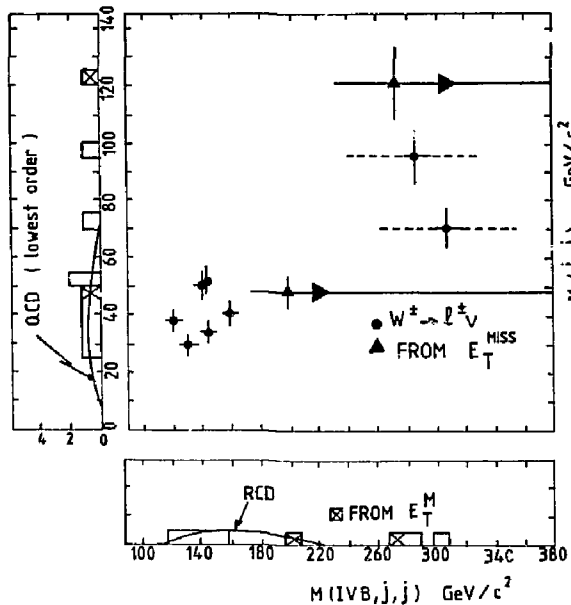


Figure 25: Correlation between the $(W+jet+jet)$ -mass and the $(jet+jet)$ -mass. The event sample and curves are as for Figure 24.

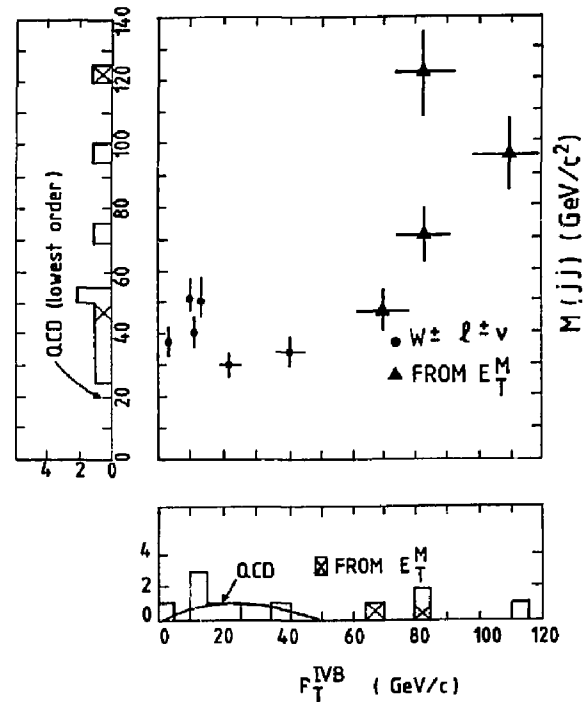


Figure 26: Correlation between the W transverse momentum and the $(jet+jet)$ -mass. The event sample and curves are as for Figure 24.

In all cases the QCD calculation gives a reasonable description of the six lower p_t^W events, but a very low probability to find an event with p_t^W greater than or equal to the measured value for event A or B. The combined probability to find an event in a region of the p_t^W vs. m_{jj} plane which is equally or less likely than the measured position of event A or B is 0.01 or 0.003 respectively. In computing these probabilities we have taken into account the uncertainties in the measurement of p_t^W and m_{jj} for events A and B.

We conclude that the rate and kinematical distributions for the sample of six (W+jet+jet) events with $p_t^W < 40$ GeV/c are well understood in the framework of QCD higher order corrections to the bare Drell-Yan production of charged vector bosons at the collider. However, the presence of events A, B and C is not easily accommodated by QCD calculation.

7 Conclusion

We have shown that it is possible to select an inclusive sample of missing transverse energy events in the UA1 detector exposed to $\bar{p}p$ collisions at the CERN SPS-Collider.

A sample of 56 isolated large E_T^{miss} events has been selected for which the residual background from jet fluctuation is small.

We have used these events for checking the Standard Model sources of large p_t neutrinos. It was shown that the experimental data sample is dominated by processes involving W and Z⁰ productions.

The largest contribution to the data is from $W \rightarrow \tau\nu$ decays; $\tau \rightarrow \text{hadrons} + \nu$. We succeed to isolate a nearly pure $W \rightarrow \tau\nu$ sample and measure its properties. Consequently universality of the weak charged current coupling to the $e \rightarrow \mu \rightarrow \tau$ leptons was tested at 10% level from measurements of $W \rightarrow e\nu$, $W \rightarrow \mu\nu$ and $W \rightarrow \tau\nu$ branching ratios at $Q^2 = m_W^2$.

Our selection method is also used to make sensitive searches for new physics sources of missing E_T . We used here a sample of 24 events ($L_T < 0$), and it was shown that these events were consistent with the expectations of the Standard Model. This sample is dominated by residual $W \rightarrow \tau\nu$ and $Z^0 \rightarrow \nu\bar{\nu}$ decays.

But as significant event rates are expected from a variety of non-standard physics processes, we used our data to place constraints:

- on the number of additional neutrinos species:

$$N_\nu < 7 \text{ (at 90\% C.L.)}$$

- on the mass of a new sequential heavy lepton:

$$m_L > 41 \text{ GeV}/c^2 \text{ at 90\% C.L.}$$

- on the mass of supersymmetric particles.

The predicted event rates from processes involving $W^\pm$ or Z^0 are very sensitive to the W, Z transverse momentum distribution at high p_t^W or p_t^Z . In the region $p_t^W < 40$ GeV/c the measured W transverse momentum distribution is in agreement with the expectations for weak bosons production at the collider by the QCD improved Drell-Yan mechanism. However, there are two events in our W samples in which the reconstructed p_t^W is abnormally large. Both events have two energetic hadronic jets produced in association with the W-candidate. We have compared these events with other (W+jet+jet) events in our event samples, and find:

- In addition to the two high- p_t^W events, there are six other W events in which two hadronic jets with $E_T > 10$ GeV are produced. The rate and kinematical properties of these six events are consistent with the expectations of an explicit second order QCD calculation.
- The QCD calculation gives a very low probability for observing the two highest- p_t^W events.

Finally, although it is premature to draw definite conclusions with current statistics, we have observed a third event in the E_T^{miss} analysis with two hadronic jets and similar topology and kinematics to the high- p_t^W events.

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LATEST RESULTS FROM THE UA2 EXPERIMENT

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Pavia - Perugia - Pisa - Saclay (CEN) Collaboration

Abstract

A study has been made of the decays $W \rightarrow e\nu$ and $Z \rightarrow e^+e^-$, using the UA2 detector at the CERN pp Collider. The data correspond to an integrated luminosity of 142 nb^{-1} at a centre of mass collision energy $\sqrt{s} = 546 \text{ GeV}$, and 768 nb^{-1} at $\sqrt{s} = 630 \text{ GeV}$. Measurements of the Standard Model parameters from samples of 251 W decay and 39 Z decay candidates are compared with expectations of the Standard Electroweak Model. Jet cross sections are presented and compared with the predictions of QCD and with data at lower energies. An exclusive sample of three-jet events is analysed and compared with a phase-space model and with QCD. The strong coupling constant α_s is measured. An exclusive four-jet data sample is used to test perturbative QCD to order α_s^2 and to search for the existence of collisions in which more than one hard scattering among partons takes place. A status report is given on the search for the decay of the Intermediate Vector Bosons W and Z into a quark-antiquark pair.

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Part I

$W^\pm$ AND Z^0 PRODUCTION IN UA2

1. INTRODUCTION

In previous publications [1-4] UA2 has reported data on the production and decay of the W and Z bosons, via the processes

$$\bar{p} + p \rightarrow W^\pm + \text{anything} \rightarrow e^\pm + \nu_e(\bar{\nu}_e) + \text{anything}, \quad (1)$$

$$\text{and } \bar{p} + p \rightarrow Z^0 + \text{anything} \rightarrow e^+ + e^- + \text{anything}. \quad (2)$$

The data published correspond to a total integrated luminosity of 142 nb^{-1} at a centre of mass energy $\sqrt{s} = 546 \text{ GeV}$ and 316 nb^{-1} at $\sqrt{s} = 630 \text{ GeV}$. In a subsequent run during 1985, the integrated luminosity at $\sqrt{s} = 630 \text{ GeV}$ was increased to 768 nb^{-1} . This represents the totality of data collected by the UA2 detector. We report final results on those aspects of W and Z production and decay which are relevant to comparisons with the Standard Electroweak Model.[5]

2. APPARATUS AND DATA ANALYSIS

The UA2 detector [3-4,7-9] consists of a vertex detector surrounded in the central region by a highly segmented, tower structured calorimeter with complete cylindrical symmetry in azimuth and full coverage in the polar range ($20^\circ < \theta < 160^\circ$). Charge particle momentum measurement is provided in the regions ($20^\circ < \theta < 40^\circ$; $140^\circ < \theta < 160^\circ$), where only electromagnetic calorimetry is present. Figure 1 shows a schematic view of the longitudinal cross section of the detector in a plane containing the beam axis.

The UA2 calorimeter, which is used to measure both, the electron energy and the jet energies, is divided into two regions.

In the central region ($40^\circ < \theta < 140^\circ$) the calorimeter is divided into 240 cells, each covering 15° in ϕ and 10° in θ . [7] Each cell is segmented

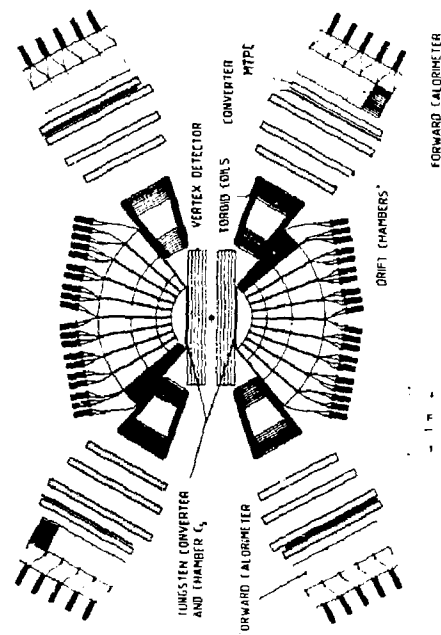


Figure 1: Longitudinal cross section of the UA2 detector

longitudinally into a 17 radiation lengths thick electromagnetic compartment (lead-scintillator) followed by two hadronic compartments (iron-scintillator) of two absorption lengths each.

In the forward region [8] ($20^\circ < \theta < 37.5^\circ$ and $142.5^\circ < \theta < 160^\circ$) the calorimeter is divided into 120 cells upstream and downstream the interaction point. Each cell is segmented longitudinally into a 24 radiation lengths thick electromagnetic compartment (lead-scintillator) followed by a six radiation lengths thick lead-scintillator sandwich used as a hadron veto.

The response of the calorimeters to electromagnetic and hadronic showers has been studied at the CERN PS and SPS machines using electron, muon and hadron beams from 1 to 70 GeV. [8-9] Electromagnetic showers are measured with an energy resolution of $\sigma_E/E = 0.14/\sqrt{E}$ (E in GeV), whereas the energy resolution for single pions varies from 32% at 1 GeV to 11% at 70 GeV, approximately proportional to $E^{-1/4}$.

2.1 Calorimeter energy measurements

Of importance for the accurate measurement of the Standard Model parameters using the electron decay modes of the W and Z bosons are :

- i. the accurate measurement of the energies of identified electrons, and
- ii. the selection with a well-measured efficiency of event samples that are minimally contaminated by backgrounds, generally resulting from hadronic jets that satisfy the criteria used to identify electrons in the detector.

Electrons are identified in the UA2 detector over the full azimuthal range, $0^\circ < \phi < 360^\circ$, and in polar angles $20^\circ < \theta < 160^\circ$ with respect to the beam line. In the central region, clusters of energy deposition are obtained by joining cells which share a common side and contain at least 0.4 GeV. The cluster energy E_{cl} is defined as $E_{cl} = E_{em} + E_{had}$, where E_{em} is the sum of the energies deposited in cells of the electromagnetic compartment of the calorimeter and E_{had} is the corresponding sum for the hadronic compartments. Since the response of the calorimeter to electrons and hadrons is different, two values of the energy are retained, corresponding to whether the calorimeter cluster is considered to result from an electromagnetic or from a hadronic shower.

In the forward region, clusters are reconstructed as for the central calorimeter. Since the 240 forward calorimeter cells are far from the interaction point, and their size is large compared with that of an electromagnetic shower, any cluster of electromagnetic origin should consist of at most two adjacent cells. For showers of hadronic origin, the absence of hadronic calorimetry prevents an energy measurement from the calorimeters alone, and information from the momenta of reconstructed charged tracks in the preceding magnetic spectrometer is included.

Although each calorimeter cell was initially calibrated in an electron beam, and the energy resolution for isolated electrons was measured to be, on average, $\sigma_E = 0.14/\sqrt{E}$ (E in GeV), the most important energy measurement error results from systematic uncertainties of the calorimeter calibration. The uncertainty on the absolute scale of energy measurement in the central calorimeter, after an operating period exceeding 5 years, is $\pm 1.5\%$, with an additional cell-to-cell calibration uncertainty of rms 2.5%. A further uncertainty results from the time variation of the light attenuation properties of the calorimeter scintillator. We estimate a total systematic uncertainty on the measured electron energy of $\pm 1.6\%$. These estimates have been confirmed by a recent recalibration of 40 cells of the central calorimeter.

In the case of the forward calorimeters, 50 cells were recalibrated in July 1986. As a result of this recalibration, the energy value assigned to forward electrons of the W and Z samples has been changed on average by -4.4%. In particular, we note that the energy has been re-evaluated for the sample of forward electrons collected during the 1985 run, and also for previously published data samples. [1-4] We estimate an uncertainty on the absolute energy scale of $\pm 2.5\%$, with an additional cell-to-cell calibration uncertainty of 2.5%.

2.2 Identification of events satisfying the $W \rightarrow e\nu$ hypothesis

Events are selected from a hardware trigger which requires a transverse energy deposition $E_T > 10$ GeV in any 2×2 cell matrix of the electromagnetic calorimeter, in coincidence with a minimum bias signal from small angle hodoscopes [10] which are used to suppress backgrounds not resulting from pp collisions.

An electron candidate is defined to be a reconstructed calorimeter cluster of transverse energy $E_{T,C} > 10$ GeV which satisfies a set of criteria characteristic of isolated high- p_T electrons:

- i. the cluster of energy deposition in the calorimeter must have small lateral dimensions and a small energy leakage in the hadronic compartment as expected for an isolated electron,
- ii. a charged particle track which points to the cluster must be reconstructed and the pattern of energy deposition in the calorimeter must be consistent with that expected from an isolated electron incident along the track direction,
- iii. in the forward directions, the reconstructed track momentum must be consistent with the associated calorimeter energy deposition,
- iv. a hit must be recorded in preshower counters located behind ≈ 1.5 radiation length thick converters that precede the calorimeters, and this hit must be aligned with the reconstructed track and have a pulse height characteristic of an electron shower.

Details of the analysis criteria used for electron identification are noted in Refs. [3] and [4]. The efficiency, η , of identifying high- p_T electrons in the region of the central calorimeter is measured from the data themselves to be $\eta = 0.71 \pm 0.07$. In the forward regions, the efficiency is estimated to be $\eta = 0.79 \pm 0.03$.

For each event the neutrino transverse momentum, $\vec{p}_T^\nu$, is defined to be equal to the missing transverse momentum, $\vec{p}_T^{\text{miss}}$, which is obtained from the expression

$$\vec{p}_T^{\text{miss}} = -\vec{p}_T^e - \sum \vec{p}_T^{\text{cl}} - \vec{p}_T^{\text{SP}} \quad (3)$$

where the sum extends over all observed clusters (excluding the electron itself) of transverse energy $E_T^{\text{cl}} > 3$ GeV. The vector $\vec{p}_T^{\text{SP}}$ is the total transverse momentum carried by the system of all other particles not associated with clusters exceeding 3 GeV transverse energy. From measurements of $\vec{p}_T^e$ and $\vec{p}_T^\nu$, the transverse mass m_T^{ev} is evaluated to be

$$m_T^{\text{ev}} = \sqrt{[2 p_T^e p_T^\nu (1 - \cos \Delta\phi)]} \quad (4)$$

where $\Delta\phi$ is the angle between $\vec{p}_T^e$ and $\vec{p}_T^\nu$.

A total of 5340 events contain at least one electron candidate satisfying $p_T^e > 11$ GeV/c. Figure 2 shows the p_T distribution of the electron candidates in the (p_T^e, p_T^ν) plane, and separately for the p_T^e

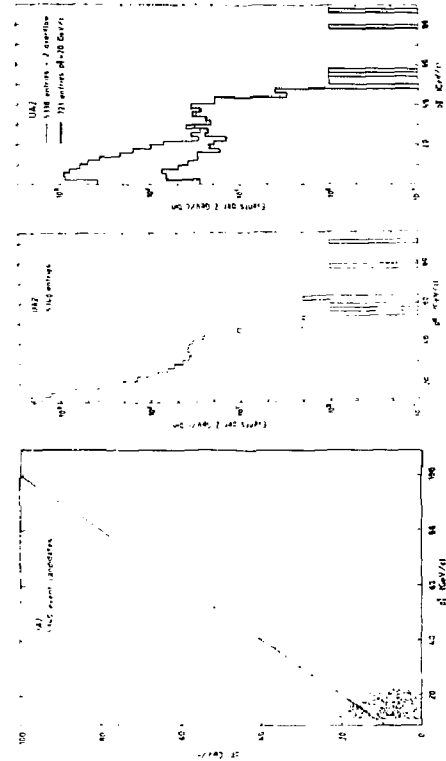


Figure 2: a) Electron candidates in the (p_T^e, p_T^ν) plane, b) p_T^e projection, c) p_T^ν projection; shaded area with $p_T^e > 20$ GeV/c.

and p_T^ν projections. If more than one electron candidate is selected, that of largest p_T is retained. In Figure 2 the $W \rightarrow e\nu$ signal is clearly visible as a clustering of events with $p_T^e \approx p_T^\nu$. The superimposed line represents $p_T^\nu = p_T^e$. Also included in the sample are $Z \rightarrow e^+e^-$ events, with large p_T^e and small p_T^ν .

The region of low p_T^e is dominated by hadronic background and for this reason we restrict the W sample to electron candidates of $p_T^e > 20$ GeV/c. The resulting $m_{e\nu}$ spectrum of 722 events is shown in Fig. 3a. The expected signal from $W \rightarrow e\nu$ decay using $m_W = 80.2$ GeV is superimposed as dashed line on this figure. Also shown, as a dashed-dotted line, is the summed contribution from $W \rightarrow \tau\nu$ decays (5.2 ± 0.5 events) and from Z decays for which one electron escapes the detector acceptance (9.6 ± 1.6 events). The solid line shows the total of all expected contributions to the $m_{e\nu}$ spectrum, including the hadronic background contribution (11.6 ± 2.1 events for $m_{e\nu} > 50$ GeV) from misidentified hadrons or hadronic jets.

Of all the $W \rightarrow e\nu$ decays within the acceptance of the apparatus, 80% are expected to satisfy the kinematic requirements $p_T^e > 20$ GeV/c and $m_{e\nu} > 50$ GeV. Therefore these selections have been applied to the final $W \rightarrow e\nu$ sample of 251 events used in the studies of Section 3. The p_T^e distribution of this sample is shown in Fig. 3b, again with superimposed background estimates.

In the distributions of Fig. 2 and Fig. 3b, we note the existence of one event containing an electron candidate of $p_T^e = 77.4$ GeV/c. The event has no associated jet activity, and a neutrino with $p_T^\nu \approx 80$ GeV/c is reconstructed as in Eqn. (3), opposite in azimuth to the electron. The transverse mass is evaluated using Eqn. (4) to be $m_{e\nu} \approx 156$ GeV. Background contributions to this event from two-jet events in which one jet fakes the electron and the second jet is outside the detector acceptance, or from a beam halo particle hitting the calorimeter in coincidence with a genuine pp collision, are negligible. However, events of large $m_{e\nu}$ are expected via the processes $u\bar{d} \rightarrow e^+\nu_e$ or $\bar{u}d \rightarrow e^-\nu_e$ mediated by W exchange in the s-channel, and we estimate that 0.07 such events should be observed with $p_T^e > 70$ GeV/c in the W sample. This estimate is insensitive, to within $\pm 10\%$, to the structure function parameterisation used. This event has been excluded from the W event sample for the measurements described in Section 3.

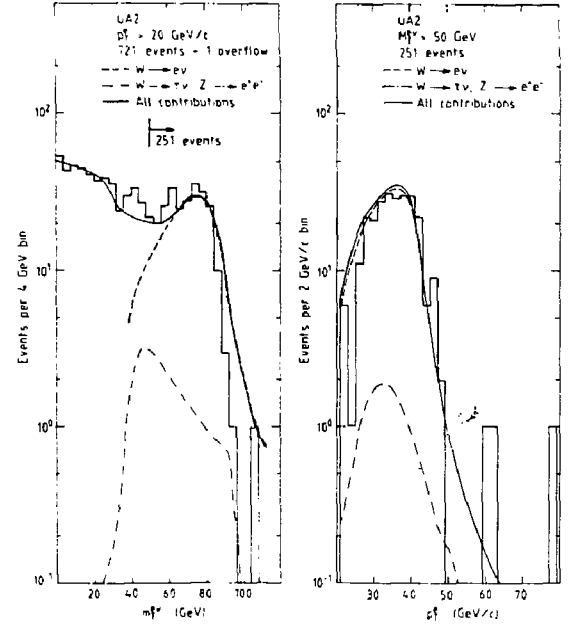


Figure 3: a) Transverse mass spectrum with $p_T^e > 20$ GeV/c.
b) Transverse momentum spectrum with $m_T^{e\nu} > 50$ GeV/c².

2.3 Identification of $Z \rightarrow e^+e^-$ decays

The Z trigger requires the simultaneous presence of two depositions of electromagnetic transverse energy, each exceeding ≈ 5 GeV, in 2×2 -cell matrices separated in azimuth by at least 60° . As for the W trigger, a minimum bias signal is required in coincidence. In the following analysis, clusters are formed as for the W search, and a selection $E_T > 10$ GeV on the cluster transverse energy is made. If the selection criteria used for the W - analysis are used on both clusters of the $Z \rightarrow e^+e^-$ candidate, the detection efficiency is low ($\approx 50\%$). However, because of the increased rejection resulting from the requirement of two clusters of energy with lateral and longitudinal profiles consistent with those of isolated electrons, less stringent selection criteria can be applied while maintaining good Z identification efficiency and good rejection of hadronic background. Using the W sample, we estimate that the efficiency in the central region for the identification of isolated electrons, from the selections noted below, is 14% higher than the efficiency of electron identification quoted in Section 2.2.

An initial selection is made on the lateral and longitudinal shower profile of each cluster, as in item (i) of Section 2.2. To improve the electron efficiency, the limit on energy leakage into the hadronic compartments of the calorimeter is increased by a factor 1.5. Additional rejection against hadronic background is obtainable from the requirement that the energy deposition due to each electron candidate is well isolated from other calorimetric energy in the event. We require less than 7 GeV within a cone of 30° about the electron candidate. The resulting distribution of mass, m_{ee} , is shown in Fig. 4a. The Z peak is clearly visible; the sample includes 54 events satisfying $m_{ee} \geq 76$ GeV, with an estimated background of 14 events.

The final sample is obtained by requiring that at least one electron candidate satisfies the criteria (ii) and (iii) of Section 2.2, except that (again to improve the electron efficiency) the requirement of spatial matching between the track and preshower signal is relaxed from $d = 10$ mm to $d = 14$ mm and the charge Q of the preshower signal is required to satisfy $Q > 2$ mip (minimum ionising equivalents). A total of 39 events satisfy $m_{ee} \geq 76$ GeV and are attributed to the Z, with an estimated background of 1.3 events. The distribution of m_{ee} for the final sample is shown in Fig. 4b. The contribution of QCD background processes is superimposed as a dashed line.

The process of internal bremsstrahlung can produce events of the type $Z \rightarrow e^+e^-\gamma$; one such event is included in the final event sample.[2-4,12] This event consists of a 24 GeV photon separated in

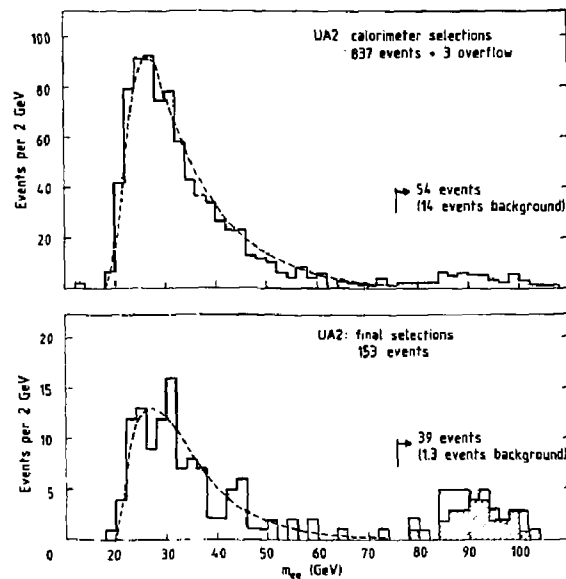


Figure 4: a) Effective mass, m_{ee}^{eff} , with calorimetric selections only, b) Effective mass, m_{ee}^{eff} , for one electron satisfying also track criteria.

space by 31° from an electron of 11 GeV. The probability of seeing at least one (ee) event having an (ee) configuration less likely than that observed in the sample of 39 Z decays is estimated to be ≈ 0.4 .

Figure 2a includes an event containing an isolated electron candidate of $p_T^e = 90.1$ GeV/c. The electron is balanced by a hadronic jet of $E_T = 77 \pm 8$ GeV. A careful examination of the jet topology indicates the overlap of an electron having $p_T^e \approx 22$ GeV/c with a hadronic jet having $E_T \approx 52$ GeV. The electron pair mass is then estimated to be $m_{ee} \approx 91$ GeV. The natural interpretation of this event is therefore the production of a Z having transverse momentum $p_T^Z \approx 70$ GeV/c, with an associated jet that overlaps one of the electrons from $Z \rightarrow e^+e^-$ decay. This event does not pass the calorimetric selections of the Z analysis, and is excluded from the Z sample.

3. PHYSICS RESULTS RELATED TO THE STANDARD MODEL

Using the data samples described in the previous section, we now discuss properties of the data relevant to tests of the Standard Model.

3.1 The W and Z masses and widths. Limits on the number of neutrino types.

An estimate of the W mass, m_W , is obtained from a comparison of the $m_{T^{e\nu}}$ distribution of Fig. 3a in the range $m_{T^{e\nu}} > 50$ GeV, with that expected from W decay. Results of the best-fit comparison are superimposed on Fig. 3. A Monte Carlo program is used, which generates the $dN/dm_{T^{e\nu}}$ distribution for different values of m_W . The distribution of $m_{T^{e\nu}}$ depends only weakly on the W production mechanism. Nevertheless, the Monte Carlo program takes into account our understanding of W production and decay, and a full simulation of the detector response to $W \rightarrow e\nu$ events.

The W longitudinal momentum distribution is obtained from the quark (antiquark) structure functions of the proton (antiproton) as parameterised in Ref. [13]. The W transverse momentum, p_T^W , is generated from the distribution of Ref. [14], which agrees well with the data. [6] The decay is described by standard (V-A) coupling with cecay parameters given by the Standard Model.

The best fit to the experimental distribution is

$$m_W = 80.2 \pm 0.6(\text{stat}) \pm 0.5(\text{sys}_1) \pm 1.3(\text{sys}_2) \text{ GeV}, \quad (5)$$

and if the W width, Γ_W , is fitted as an additional free parameter, $\Gamma_W < 7$ GeV (90% confidence level). The statistical uncertainty in (5) takes into account the resolution of the energy measurement, and also cell-to-cell uncertainties of the energy calibration. Systematic uncertainties of the mass measurement have two major contributions, which are quoted separately. The uncertainty (sys_1) of Eqn. (5) is mainly due to possible systematic biases in the evaluation of p_T^{ν} , and consequently $m_{T^{e\nu}}$. The second systematic uncertainty (sys_2) reflects the measurement uncertainty on the global energy scale of the calorimeter calibration. The influence on the mass fit of the (dominantly low- p_T) background contribution in Fig. 3a is small.

The W mass can alternatively be estimated from a fit to the observed p_T^e distribution of Fig. 3b. In this case, the evaluation is less sensitive to the p_T^{ν} evaluation, but is more sensitive to the detailed shape of the p_T^W distribution. Selecting a sub-sample of events for which $p_T^W < 15$ GeV/c, the m_W measurement is consistent with the result of (5).

Using a relativistic Breit-Wigner shape modified by the mass resolution, the Z mass is evaluated from a fit to the mass values of a sub-sample of 26 events for which both electron energies are accurately measured and for which $m_{ee} > 76$ GeV. The result is

$$m_Z = 91.5 \pm 1.2(\text{stat}) \pm 1.7(\text{syst}) \text{ GeV}. \quad (6)$$

where the systematic uncertainty results mainly from the calorimeter energy scale uncertainty of the central and forward calorimeters. From (5) and (6), we measure

$$m_Z - m_W = 11.3 \pm 1.3(\text{stat}) \pm 0.5(\text{sys}_1) \pm 0.8(\text{sys}_2) \text{ GeV}, \quad (7)$$

where (sys_1) again results from systematic uncertainties of the p_T^{ν} evaluation in (5), and (sys_2) reflects the differing global energy scale uncertainties of the forward and central calorimeters.

A direct measurement of the width, Γ_Z , of the Z is difficult since a precise knowledge of the shape of the mass resolution is required. The average measurement error is estimated to be 3.1 GeV, which is of the same order as the expected Z width. Following [4] we obtain

$$\Gamma_Z = 2.7 \pm 2.0(\text{stat}) \pm 1.0(\text{syst}) \text{ GeV} \\ < 5.6 \text{ GeV (90\% confidence level),}$$

where the quoted systematic error reflects the uncertainty of the average measurement error. These measurements are in good agreement with previously published results,[4] and with measurements from the UA1 Collaboration.[15]

As described in Ref.[4], we can obtain an independent but model-dependent [16] estimate of the ratio of total widths, Γ_Z/Γ_W , from

$$\Gamma_Z/\Gamma_W = R^{\text{exp}} \cdot R^{\text{th}} \cdot R^{\text{lept}}$$

where $R^{\text{exp}} = \sigma_W^{\text{ev}}/\sigma_Z^{\text{ec}}$ is the experimentally determined cross section ratio from this experiment, where R^{th} is the ratio σ_Z/σ_W obtained theoretically, and $R^{\text{lept}} = \Gamma_Z^{\text{ec}}/\Gamma_W^{\text{ev}}$ is the ratio of leptonic partial widths as evaluated from the Standard Model. We measure [6]

$$\begin{aligned} R^{\text{exp}} &= 7.2 \quad (\text{stat}), \\ &< 9.52 \text{ (90\% confidence level),} \\ &< 10.42 \text{ (95\% confidence level).} \end{aligned} \quad (8)$$

Both R^{th} and R^{lept} depend on $\sin^2\theta_W$, explicitly through the neutral current couplings and implicitly via the W and Z masses. However the product $R^{\text{th}}R^{\text{lept}}$ is constant to within 1% over the range $\sin^2\theta_W = 0.232 \pm 0.009$ (see Eqn. 14). A more important uncertainty results from the use of different sets of structure functions in the theoretical calculation of R^{th} [14,17]. Recent estimates [18] give values of R^{th} ranging between 0.285 and 0.325. Using R^{exp} as in (8), and the value $R^{\text{th}} = 0.305 \pm 0.020$, we evaluate

$$\begin{aligned} \Gamma_Z/\Gamma_W &= 0.82 \quad (\text{stat}) \pm 0.06 \text{ (theor),} \\ &< 1.09 \pm 0.07 \text{ (theor) (90\% confidence level),} \\ &< 1.19 \pm 0.08 \text{ (theor) (95\% confidence level),} \end{aligned} \quad (9)$$

where the theoretical error reflects the uncertainty on R^{th} as quoted above.

The ratio of total widths is sensitive to both the number of neutrino types and the mass of the top quark, m_t , [19]. Assuming that the charged members of any new family and also any other unknown particle are too massive to contribute significantly to W or Z decays, we can make an estimate of the number of light neutrino types including the expected variation of Γ_Z/Γ_W with m_t and by varying $\sin^2\theta_W$ in the range

0.232 ± 0.009 . From the 95% confidence limit using the conservative upper estimate of $R^{\text{th}} = 0.325$, the data exclude more than seven neutrino types when no requirement is made on m_t . This limit decreases to four neutrino types in the case $m_t > 71$ GeV.

3.2 Measurement of the Standard Model parameters

The masses of the weak bosons, m_W and m_Z , are two essential parameters of the Standard Model. In its minimal expression, it relates them to the fine structure constant α , the Fermi constant G_F and the weak mixing angle θ_W via the following relations [21]:

$$m_W^2 = A^2 / [(1 - \Delta r) \sin^2\theta_W] \quad (10)$$

$$m_Z^2 = A^2 / [(1 - \Delta r) \sin^2\theta_W \cos^2\theta_W] \quad (11)$$

where [22] :

$$A = (\pi\alpha/\sqrt{2}G_F)^{1/2} = 37.2810 \pm 0.0003 \text{ GeV.}$$

In relations (10) and (11), the quantity Δr accounts for the effects of one-loop radiative corrections on the W and Z masses and has been computed to be [21,23,24]

$$\Delta r = 0.0711 \pm 0.0013 \quad (12)$$

assuming that $m_t = 35$ GeV and that the mass of the Higgs boson is $m_H = 100$ GeV. This quantity is insensitive to m_H but would deviate from (12) if the top quark were very massive ($\Delta r = 0$ for $m_t \approx 270$ GeV with $\sin^2\theta_W = 0.232$). [24-26]

From a measurement of the ratio m_W/m_Z , which is free from the common systematic uncertainty of calorimeter energy calibration on the W and Z mass scale, a direct measurement of $\sin^2\theta_W$ is provided via the relation

$$\sin^2\theta_W = 1 - (m_W/m_Z)^2. \quad (13)$$

From Eqn. (13), we evaluate

$$\sin^2\theta_W = 0.232 \pm 0.025(\text{stat}) \pm 0.010(\text{syst})$$

where the quoted uncertainty includes the contribution of a ± 0.5 GeV systematic error on the value of m_W which is not related to the energy

calibration of the calorimeter (see Eqn. 5). This result is independent of other experiments, and of theoretical uncertainties.

By using accurate existing measurements of A , [22] and the value of Δr in (12), a more precise measurement of $\sin^2\theta_W$ is obtainable from a best fit to Eqns. (10) and (11). We obtain

$$\sin^2\theta_W \approx 0.232 \pm 0.003(\text{stat}) \pm 0.008(\text{syst}). \quad (14)$$

These results are in excellent agreement with previously published UA2 and UA1 results [4,15] and with those obtained in low energy neutrino experiments,[27-30] which average to :

$$\sin^2\theta_W \approx 0.232 \pm 0.004(\text{exp}) \pm 0.003(\text{theor}), \quad (15)$$

where the weighted mean is evaluated on the basis of quoted experimental errors, assuming a charmed quark mass $m_c = 1.5$ GeV, and ignoring the uncertainty on the theoretical error due to $\bar{m}_c$.

The results are summarised in Fig. 5 where confidence contours (68% level) in the $(m_Z - m_W, m_Z)$ plane are shown, taking into account the statistical error only (i), and with statistical and systematic errors combined in quadrature (ii). The region bounded by curves (a) and (b) is allowed by the average of recent low-energy measurements.[27-30] Curve (c) is the Standard Model prediction for $\rho = 1$ with known radiative corrections, and curve (d) is the expectation without radiative corrections.

Any departure from the minimal Standard Model will induce modifications to the above formalism. In particular values of $\sin^2\theta_W$ deduced from Eqn. (10), from Eqn. (13), or from low energy neutrino experiments, will generally differ. All existing measurements are in excellent agreement with the predictions of the minimal Standard Model; nevertheless, they can be used to place limits on possible deviations from the minimal Standard Model. In particular the quantity [31]

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W} \quad (16)$$

is sensitive to the Higgs sector (more precisely it depends on the isospin structure of the Higgs fields, but only weakly on their masses). Assuming the value of Δr in (12), we measure from (16)

$$\rho = 1.001 \pm 0.028(\text{stat}) \pm 0.006(\text{syst}),$$

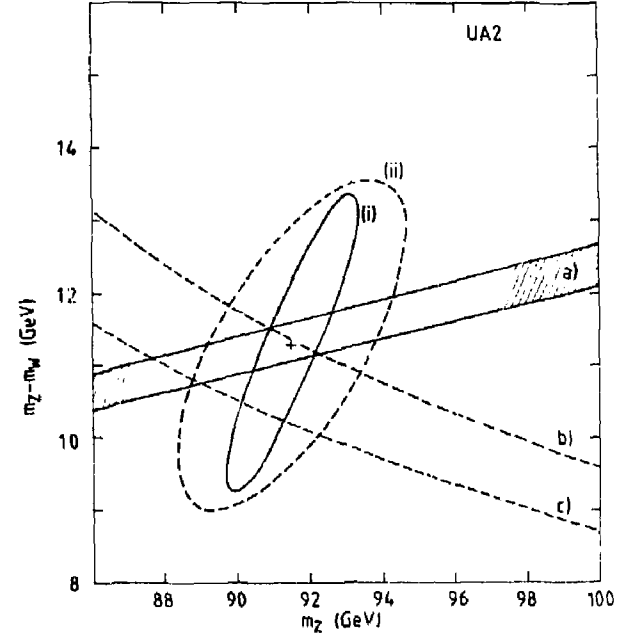


Figure 5: Confidence contours in the $(m_Z - m_W, m_Z)$ plane.

in good agreement with the minimal Standard Model prediction of $\rho = 1$.

The relations (10) and (11) may also be used to measure the radiative correction parameter Δr , which may deviate [24-26] from its calculated value in (12) if, for example, a new fermion family existed with a large mass splitting within isospin doublets, or if there existed additional gauge bosons, or if as noted above the top quark were very massive. Eliminating $\sin^2\theta_W$ from Eqns. (10) and (11) we obtain

$$1 - \Delta r = (A^2/m_W^2)/(1 - [m_W^2/m_Z^2]),$$

from which we deduce

$$\Delta r = 0.068 \pm 0.087(\text{stat}) \pm 0.030(\text{syst}), \quad (17)$$

in agreement with the minimal Standard Model prediction of (12). The value of $\sin^2\theta_W$ in (15), from low energy experiments, can be used in equations (10) and (11) to provide a more accurate measurement of Δr . We measure

$$\Delta r = 0.068 \pm 0.022(\text{stat}) \pm 0.032(\text{syst}). \quad (18)$$

We therefore conclude that the existing data are consistent with, but barely sensitive to, the existence of radiative corrections from known processes (see Fig. 5).

3.3 Charge asymmetry of the decay $W \rightarrow e\nu$

The magnetic spectrometers of the UA2 detector allow a measurement of the electric charge in the forward regions ($20^\circ < \theta < 37.5^\circ$ and $142.5^\circ < \theta < 160^\circ$) where a distinctive charge asymmetry is expected in $W \rightarrow e\nu$ decay. [4] Assuming a universal (V-A) coupling of the W to fermions, the electron (positron) angular distribution takes the form

$$d/d(\cos\theta^*) \propto (1 - q\cos\theta^*)^2 + 2q\alpha\cos\theta^* \quad (19)$$

where the angle θ^* is measured with respect to the incident proton in the W rest frame, where q is the sign of the electron or positron charge, and

$$\alpha = [(1 - x^2)/(1 + x^2)]^2. \quad (20)$$

The sensitivity of the asymmetry measurement to the exact form of the angular distribution is largest for values of $\cos\theta^*$ close to ± 1 , corresponding to small values of p_T^e . Therefore, we relax the $m_{T^{\text{ev}}}$ cut of Section 2.2 and consider all electron candidates with $p_T^e > 20$ GeV/c and $m_{T^{\text{ev}}} > 40$ GeV that are detected in the forward regions. This leaves a total of 47 events with an estimated background of 4.2 ± 0.3 events from misidentified hadrons. The $W \rightarrow \tau\nu$ and Z decay contaminations of this sample are estimated to be respectively 0.6 events and 1.2 events.

A unique value of $\cos\theta^*$ is not calculable for W decays because the longitudinal neutrino momentum, p_L^ν , is not measured. The requirement $m_{T^{\text{ev}}} = m_W$ results in two solutions for p_L^ν . For events in which both solutions are allowed, the solution corresponding to the smaller absolute value of the W longitudinal momentum is chosen. Seven events, which because of the limited measurement accuracy satisfy $m_{T^{\text{ev}}} > m_W$, are excluded from the analysis. Furthermore, transformations of the electron four-momentum to the W rest frame are unique only if $p_T^W = 0$, and the quarks have no transverse momentum. For $p_T^W \neq 0$ the initial parton directions are not known and the convention of Ref. [32] is used.

A Monte Carlo program is used to correct the $\cos\theta^*$ distribution for the effects of the detector acceptance and resolution. The background-subtracted and acceptance-corrected $\cos\theta^*$ distribution is shown in Fig. 6 and is consistent with the expected form $(1 - q\cos\theta^*)^2$ of Eqn. (19) with $\alpha=0$, modified to take into account higher-order QCD contributions to W production. [11]

To extract a value of α from these data we use a Monte Carlo program to compare the expected two dimensional distributions $f^\pm(p_T^e, \theta_e)$, for positrons and electrons separately, with those observed. Following the analysis of [4] we measure α to be consistent with zero, as expected for V-A coupling. We determine $\alpha < 0.35$ (90% confidence level), corresponding in Eqn. (20) to $0.51 < |x| < 1.97$.

3.4 Cross section for inclusive $W^\pm \rightarrow e^\pm\nu$ and $Z^0 \rightarrow e^+e^-$ production

The cross section, σ_W^e , for inclusive W production, followed by the decay $W \rightarrow e\nu$, is determined from the number of $W \rightarrow e\nu$ decays, after subtracting the various background contributions. From the event samples quoted in Section 2.2, after having taken into account the integrated luminosity $\mathcal{L}$, the detector acceptance and the overall efficiency of the electron identification criteria averaged over the central and forward detectors, we get

$$\sigma_W^e = 570 \pm 100 (\text{stat}) \pm 70 (\text{syst}) \text{ for } \sqrt{s} = 546 \text{ GeV and}$$

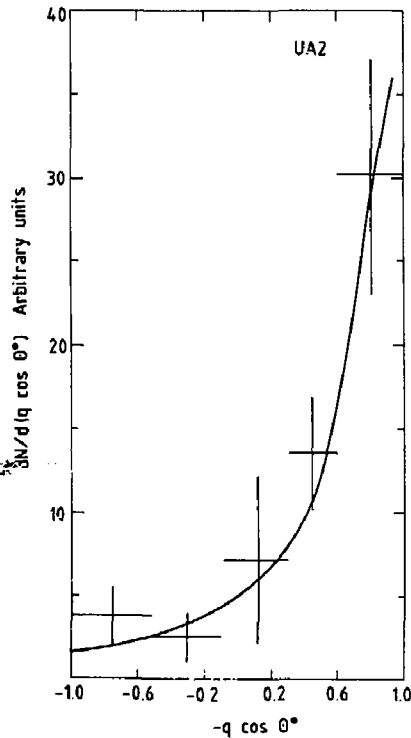


Figure 6: Charge asymmetry distribution $dN/d\cos\theta^*$.

$$\sigma_W^e = 610 \pm 50 \text{ (stat)} \pm 70 \text{ (sys)} \text{ for } \sqrt{s} = 630 \text{ GeV.}$$

We have also measured the cross section, σ_Z^e , for inclusive Z^0 production, followed by the decay $Z^0 \rightarrow e^+e^-$:

$$\sigma_Z^e = 110 \pm 40 \text{ (stat)} \pm 10 \text{ (sys)} \text{ for } \sqrt{s} = 546 \text{ GeV and}$$

$$\sigma_W^e = 70 \pm 10 \text{ (stat)} \pm 10 \text{ (sys)} \text{ for } \sqrt{s} = 630 \text{ GeV.}$$

These values are in good agreement with UA1 measurements [15] and with theoretical expectations.[14]

4. CONCLUSIONS

We have described measurements of the Standard Model parameters, using the final UA2 data samples for the processes

$$\begin{aligned} \bar{p} + p &\rightarrow W^\pm + \text{anything} \\ &\rightarrow e^\pm + \nu_e(\bar{\nu}_e) + \text{anything, and} \end{aligned}$$

$$\begin{aligned} \bar{p} + p &\rightarrow Z^0 + \text{anything} \\ &\rightarrow e^+ + e^- + \text{anything.} \end{aligned}$$

The measured boson masses are

$$m_W = 80.2 \pm 0.6 \text{ (stat)} \pm 0.5 \text{ (sys}_1) \pm 1.3 \text{ (sys}_2) \text{ GeV,}$$

where (sys₁) is a systematic uncertainty which results mainly from possible systematic biases in the evaluation of p_T^{ν} , and (sys₂) is from the $\pm 1.6\%$ energy scale uncertainty, and

$$m_Z = 91.5 \pm 1.2 \text{ (stat)} \pm 1.7 \text{ (syst)} \text{ GeV,}$$

with the systematic uncertainty resulting in this case from the energy scale uncertainty. The values are, within errors, consistent with previously published values [4] and with published data of the UA1 experiment.[15] From the above measurements of m_W and m_Z we evaluate

$$\sin^2\theta_W = 0.232 \pm 0.025 \text{ (stat.)} \pm 0.010 \text{ (syst.).}$$

If in addition accurate low-energy measurements of α and G_F are used, together with recent calculations of the radiative correction Δr , we obtain

$$\sin^2\theta_W = 0.232 \pm 0.003(\text{stat}) \pm 0.008(\text{syst}).$$

There is no evidence of deviations from the predictions of the minimal Standard Model. If deviations are expressed in terms of the parameter ρ ($= 1$ in the minimal Standard Model), we obtain

$$\rho = 1.001 \pm 0.028(\text{stat}) \pm 0.006(\text{syst}).$$

Furthermore, the existing data are consistent with (but barely sensitive to) the expected modification of the Standard Model parameters due to known radiative corrections, even if recent low-energy neutrino measurements are taken into account.

From measurements of the ratio of the boson widths, Γ_Z/Γ_W , we describe model-dependent limits on the number of neutrino types allowed by the existing data. Within 95% confidence limits, the data allow up to seven neutrino types if no restriction is placed on the top mass m_t , and up to four neutrino types in the case if a top quark mass $m_t > 71$ GeV.

The measured W and Z production cross sections are consistent with previously published data and, within known theoretical uncertainties, are well described by QCD-modified predictions of the Standard Model.

Part 2

RECENT RESULTS ON HIGH p_T JETS IN UA2

1. INTRODUCTION

The successful operation of the CERN $\bar{p}p$ Collider has provided the opportunity to examine high-energy collisions between quarks and gluons with increasingly large statistics. This part describes the final states containing two or more high- p_T jets, produced by the partons that emerge from a hard scattering.

Hadronic jets were first observed by UA2 during the Collider run in 1981. [33] The inclusive cross sections and the dynamical properties of the production and fragmentation process have been extensively studied since then by the UA2 [34-36] and UA1 [37] Collaborations. The data have been compared with the predictions of the standard theory of the strong interactions, quantum chromodynamics (QCD).

The inclusive cross sections are measured from data taken at different values of the centre of mass energy $\sqrt{s}$. The distributions show an increase with increasing $\sqrt{s}$ in good agreement with the QCD calculations. The analysis of the inclusive cross sections in terms of a model, [38] in which the quarks are composite, is used to establish an upper limit on the scale of a hypothetical quark substructure.

An exclusive sample of events containing three hard jets in the final state is analysed and compared with the QCD predictions. The angular distributions of the three jets show evidence for both initial and final state bremsstrahlung, in good agreement with a QCD model to leading order in the strong coupling constant α_s . The yield of three-jet events relative to that of two-jet events provides a measure of α_s .

The exclusive four-jet data sample allows a test of perturbative QCD to order α_s^4 . A preliminary study of such events finds a good agreement with the theory and a significant deviation from a phase-space model. Another possible mechanism is also considered, in which more than one hard scattering among partons takes place. [39] The preliminary conclusion is that the data sample is inconsistent with the presence of a large contribution from multi-parton scattering.

Finally, a status report is given on the search for the decay of the Intermediate Vector Bosons W and Z (IVB) into a quark-antiquark pair. The observation at the SPS Collider of the production and the leptonic decay of the IVB's provides evidence for the validity of the Standard Model expressions of the couplings between quarks and IVB's. As a direct consequence, the IVB's are expected to decay into quark-antiquark pairs with large branching fractions. However, important experimental difficulties have to be faced: the signal is expected to appear as an excess of only a few percent over the QCD background of two-jet events. This demands both a very good mass resolution and a large integrated luminosity. For this reason, special selection criteria have been adopted, which are expected to give a mass resolution of the order of $\sim 10\%$ in the IVB mass region. Preliminary evidence for a signal is found, in good agreement with the Standard Model expectations.

2. EVENT SAMPLES

The UA2 detector [7-9] has already been described in Section 2. The results, presented in this part, are based on jet identification and energy measurement in the central calorimeter, [9] which detects electromagnetic and hadronic showers over the full azimuth (ϕ) range and in the pseudo-rapidity region $|\eta| \leq 1$.

As already pointed out in Section 2, electromagnetic showers are measured with an energy resolution of $\sigma_E/E = 0.14/\sqrt{E}$ (E in GeV), whereas the energy resolution for single pions varies from 32% at 1 GeV to 11% at 70 GeV, approximately proportional to $E^{-1/4}$. The systematic uncertainty on the energy calibration is estimated to be less than $\pm 1.5\%$ for the electromagnetic calorimeter and less than $\pm 6\%$ for the hadronic one. They combine in a global systematic error of less than $\pm 4\%$ on the jet energy scale.

The data presented here were collected at $\sqrt{s} = 630$ GeV and correspond to an integrated luminosity $\mathcal{L} = 0.74 \text{ pb}^{-1}$. They were recorded by selecting $p\bar{p}$ collisions which deposited large transverse energy into the central calorimeter. Three different triggers were used:

- i. "one-jet trigger": the transverse energy deposited in any azimuthal wedge ($\Delta\phi = \pm 60^\circ$) had to be larger than a given threshold, set at ~ 30 GeV;
- ii. "two-jet trigger": the transverse energies deposited in any two opposite azimuthal wedges (ϕ_1 and $\phi_2 = \phi_1 + \pi$, $\Delta\phi_1 = \pm 30^\circ$,

$\Delta\phi_2 = \pm 60^\circ$) are both required to exceed a given threshold, set at ~ 20 GeV during the 1984 data-taking period (0.31 pb^{-1}), at ~ 15 GeV for part of the 1985 (0.27 pb^{-1}) and at ~ 12.5 GeV for the rest of the 1985 data (0.15 pb^{-1});

- iii. total transverse energy trigger (used in multi-jet studies): the total transverse energy in the central calorimeter (ΣE_T) was required to exceed a given threshold, set at ~ 60 GeV.

Background from sources other than $p\bar{p}$ collisions was suppressed at the trigger level by requiring a coincidence with two signals ("minimum bias" trigger) obtained from scintillator arrays covering the angular range $0.44^\circ < \theta < 2.84^\circ$ on both sides of the interaction region. [10] As in previous work [34-36] a small background contamination, resulting from the interaction of beam halo particles in the UA2 calorimeter, is reduced off-line to a negligible level ($< 2\%$ independently of ΣE_T) after the application of sharp timing cuts and rejection of events having characteristic background configurations in their pattern of energy deposition.

A simple clustering algorithm has been adopted, [34] taking advantage of the fine granularity of the calorimeter. All cells which share a common side and have a cell energy $E_{\text{cell}} > 400$ MeV are joined into a cluster. The direction of the cluster is measured from its centroid to the center of the interaction region. In each event the clusters are ordered in decreasing transverse energies denoted by $E_{T1} > E_{T2} > E_{T3} > \dots$. The individual clusters are taken defined to be massless by setting $p_T = E_T$.

3. INCLUSIVE JET CROSS SECTIONS

The cross sections for the reactions

$$p\bar{p} \rightarrow \text{jet} + \text{anything} \quad (21)$$

$$p\bar{p} \rightarrow \text{jet} + \text{jet} + \text{anything} \quad (22)$$

have been measured from the observed number of jets using the integrated luminosity $\mathcal{L}$ for normalisation and a Monte Carlo simulation to account for detector acceptance. In order to partly account for final state gluon radiation, the cluster momentum is modified by adding to it the vectors of all the clusters having $E_T > 3$ GeV and separated by an angle $\cos \omega > 0.2$. In the presence of such secondary clusters the jet acquires a mass and its p_T differs from its E_T . The results are

summarised for $\sqrt{s} = 546$ GeV and $\sqrt{s} = 630$ GeV in Fig. 7a for the inclusive jet production $d^2\sigma/dp_T d\eta$ at $\eta=0$ and in Fig. 7b for two-jet production $d\sigma/dm$.

The errors include the statistical error and an energy dependent systematic uncertainty coming from the acceptance functions. An additional systematic error of 45%, arising from an energy independent uncertainty of the acceptance functions (35%), calorimeter calibrations (30%) and luminosity (8%), is not included in the plots.

As can be seen in Fig. 7a, the analysis of the 1985 data agrees well with previously published results at $\sqrt{s} = 630$ GeV (the errors on the 1985 data are almost identical to those of the 1984 data). A clear increase of the jet production cross sections with $\sqrt{s}$ is visible in Fig. 7. In order to examine this increase in more detail, the ratio of cross sections at the two $\sqrt{s}$ values has been evaluated and is shown in Fig. 8 after suitable rebinning. Only statistical errors are given since the systematic errors approximately cancel in the ratios.

The inclusive jet cross sections have also been measured in pp collisions at the ISR.[40] These results can be compared to those as shown in Fig. 8 by using the scaled invariant cross section $p_T^{-4} E d\sigma/dp^2$ as a function of $x_T = 2 p_T/\sqrt{s}$. Scale breaking effects can be parametrised in the form

$$E d\sigma/dp^2 = p_T^{-n} \cdot f(x_T) \quad \text{with} \quad (23)$$

$$f(x_T) = A \cdot (1-x_T)^m \cdot x_T^{-2}.$$

A global fit give $n = 4.74 \pm 0.06$, $m = 6.54 \pm 0.15$.

All these measurements can be compared with QCD calculations: the curves shown in Fig. 7 and 8 have been obtained from a leading order (in α_s) QCD calculation using $q^2 = p_T^2$ as a scale ($\Lambda = 0.2$ GeV) and the structure functions of Ref. [41]. This prediction describes the cross sections at both Collider energies and their increase with $\sqrt{s}$. However, the $\pm 45\%$ systematic error on the measurements precludes an accurate evaluation of the effects coming from higher order α_s contributions (K-factors).

The inclusive jet cross section data at $\sqrt{s} = 630$ GeV have been analysed [34] in terms of deviations from QCD, including the effects of a hypothetical super-strong interaction binding preons inside quarks, following the parametrisation of Ref. [38], with the choice $g^2/4\pi = 1$ for the coupling constant. In this model a substructure of partons would

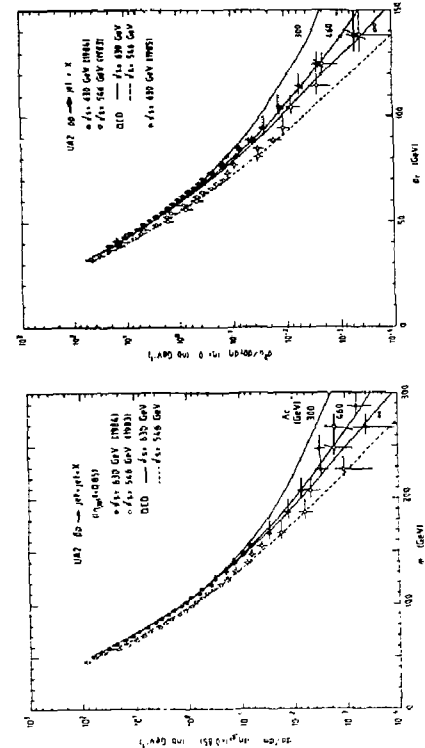


Figure 7: Inclusive single-jet and two-jet cross sections.

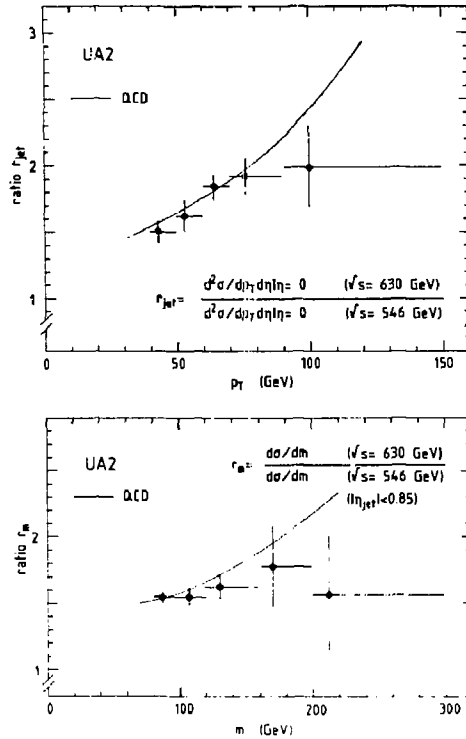


Figure 8: Ratio of the inclusive cross sections at different values of $\sqrt{s}$.

manifest itself as a contact interaction visible at momentum transfers well below the characteristic energy scale Λ_c . Finite values of Λ_c would produce an excess of events compared to ordinary QCD predictions ($\Lambda_c = \infty$) at large p_T values. Hence the analysis is possible to the extent that the main uncertainties (systematic errors and ignorance of the K-factor) are approximately constant over the p_T range, so that the data can be normalised in the low p_T region and deviations can be observed in the high p_T tail.

In Fig. 7 the results are plotted for the pure QCD calculation ($\Lambda_c = \infty$) and for the best fit ($\Lambda_c = 460$ GeV), together with the expected behaviour for $\Lambda_c = 300$ GeV as an illustration of the sensitivity to Λ_c . When both theoretical and experimental uncertainties are taken into account, a lower limit is obtained of $\Lambda_c = 370$ GeV at 95% C.L., [34] which agrees with recently reported results from the UA1 Collaboration. [37]

4. THREE-JET EVENTS

4.1 Data reduction

In the study of multi-jet events the standard cluster algorithm has been modified in order to achieve a better angular resolving power for pairs of jets. First, primary clusters are obtained as described in Section 2. In a second phase, the clusters are reprocessed using the same algorithm and applying a higher threshold, set at 5% of the total energy of the primary cluster. Any primary cluster cell with an energy below threshold is redistributed among secondary clusters according to their relative location and energy content. The fraction of events for which the clustering algorithm gives two clusters is a measure of the angular resolving power. It is found [35] to have a sharp distribution with an approximately energy-independent cut-off at $30^\circ \pm 10^\circ$.

In each event, the secondary clusters are sorted in order of decreasing transverse energy and labeled 1,2,3, etc... The following conditions are applied to all the events containing three such clusters :

$$E_{T^1} + E_{T^2} + E_{T^3} > 70 \text{ GeV}, \quad (24a)$$

$$E_{T^3} > 10 \text{ GeV}, \quad E_{T^4} < 10 \text{ GeV}, \quad (24b)$$

$$|\eta_i| < 0.8, \quad i = 1,2,3. \quad (24c)$$

$$|\vec{p}_{T^1} + \vec{p}_{T^2} + \vec{p}_{T^3}| < 20 \text{ GeV}. \quad (24d)$$

Condition (24a) ensures that the configuration of the three leading jets is unbiased by the trigger threshold. Condition (24b) retains events in which the third cluster is likely to be associated with the hard collision rather than with spectator fragments (the condition on E_T^{cl} is an exclusive topology selection). Condition (24c) defines a fiducial volume in which the jet energy measurements are reliable. Condition (24d) excludes events with a large imbalance in transverse momentum, mostly due to the presence of hard jet(s) outside the acceptance. There are $\sim 12,300$ events which satisfy the above conditions. Similarly a sample of two-jet events is defined in which Relations (24) are replaced by similar conditions applied to the first two jets only. This sample contains $\sim 25,500$ events.

Finally, a Lorentz transformation to the three-jet centre of mass system (CMS) is applied to each event of the sample. The three leading jets are then re-ordered according to their CMS momenta $\vec{p}_i^*$.

4.2 The QCD and phase-space models

The experimental distributions associated with the three-jet sample are compared to the predictions of a QCD model, implemented in a Monte Carlo simulation of the UA2 detector.[35]

To leading order in α_s , the cross section for producing a system of n final state partons with mass $\sqrt{s}$ is expressed in terms of elementary subprocesses, in which two incident partons, i and j , carrying fractions x_i and x_j of their nucleon parent momentum, interact :

$$\sigma_n^{\text{LO}} = (|\alpha_s|^n / s) \int \sum_{ij} F_i(x_i) F_j(x_j) Q_{ij}^n \Phi_n(dx_i/x_i) (dx_j/x_j) \quad (25)$$

where Φ_n is the n -body phase-space factor and $x_i, x_j \leq \hat{s}$. Explicit expressions for the contribution Q_{ij}^n of each elementary subprocess are available in the literature for $n = 3$,[42] $n = 4$,[43] and $n = 4$,[44] In order to emphasize those features peculiar to QCD, the distributions of several variables are also compared with the predictions derived from a phase-space model, obtained by setting $Q_{ij}^n \equiv 1$ in Equation (25). In the quark and antiquark case, the structure functions $F_i(x)$ are taken from low- q^2 neutrino data,[45] evolved to the q^2 -range exploited by the Collider. The gluon structure function is evaluated from the two-jet data of this experiment,[36] assuming that the inclusive K-factor takes the value $K = 2$. In Equation (25) both the structure functions and the strong coupling constant α_s are functions of q^2 , which is defined as the square of the largest transverse momentum among the final state partons.

As a result of the bremsstrahlung nature of the gluon radiation spectrum, Q_{ij}^n diverges when the mass of one of the parton pairs in the initial or final states approaches zero. These divergences are cancelled by non-leading contributions to the topological cross sections associated with lower parton multiplicities in the final state. In order to ensure that the parton configuration in the final state is free of the above divergences, two cut-offs are defined : only partons having a transverse momentum in excess of 8 GeV and parton pairs having an opening angle in excess of 40° are considered. Within these cut-offs, the ratio K_n^{LO} between the topological cross section σ_n and its leading order approximation σ_n^{LO} is a quantity which can, in principle, be calculated, although this has not yet been done. Lacking a calculation of K_n^{LO} , the two-jet and three-jet data are compared to expressions proportional to $K_2 \alpha_s^2$ and $K_3 \alpha_s^3$ respectively, with the result that the ratio is sensitive to the quantity $\alpha_s K_3/K_2$ instead of simply α_s .

Final state partons are made to fragment into hadrons following a method based on the Field-Feynman algorithm [46] and modified to reproduce the cluster radius distribution observed in the present experiment.[34]

The acceptance of the UA2 detector and the details of the energy response of the central calorimeter are simulated in a Monte Carlo programme, which reproduces the experimental details of relevance.[34] The underlying event, associated with spectator interactions, is simulated by superimposing actual minimum bias events onto the jets produced by the hard collision.

4.3 Description of the sample

In the CMS the jet momenta are completely defined by six independent variables. In order to compare the experimental distribution with QCD and with the phase-space model, variables are chosen which are directly related to dynamical properties.

Three such variables are the angles which describe the orientation of the plane defined by the three momenta $\vec{p}_i^*$. Figure 9a shows the distribution of $\cos \theta^*$, the angle between $\vec{p}_1^*$ and the beam direction. This distribution has been normalised to one at $\cos \theta^* = 0$ and corrected for the limited acceptance of the calorimeter. Also the corrected distribution for two-jet events [36] is shown. The shapes are very similar and in agreement with the expectations for vector gluon exchange. The curve represents the parton level calculation of the cross section for the $gg \rightarrow ggg$ sub-process, computed using Equation (25) and the transverse

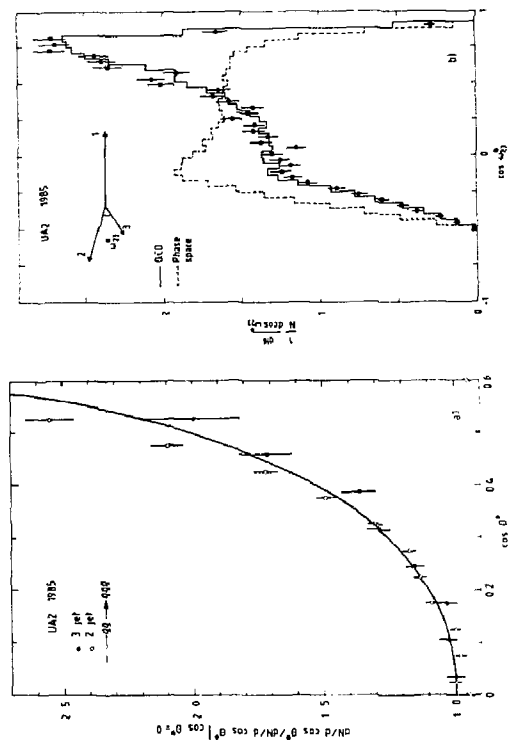


Figure 9: a) Angle between the first jet and the beam direction;
b) Angle between the second and the third jet.

momentum and angular separation cut-offs previously described. The curves for the other sub-processes are not significantly different, and the overall agreement with QCD is very good.

The remaining three variables depend on the internal configuration of the three-jet system, independent of its orientation. Their features can be shown in the form of a Dalitz plot, by defining scaled variables which are functions of the two-body masses m_{ij} :

$$x_{ij} = m_{ij}^2 / s. \quad (26)$$

The distribution of x_{12} versus x_{23} is shown in Fig. 10. If the events were distributed according to a phase-space density, the population would be uniform across the plot. Instead, as predicted by QCD, there is an increased density in the region of large x_{12} and small x_{23} (corresponding to bremsstrahlung of a soft third jet), compared to that of small x_{12} (corresponding to equal sharing of energy among the three jets). The absence of events near $x_{12} = 1$ and $x_{23} = 0$ is a result of the event selection of Relations (24) and of the two-jet angular resolving power.

The dynamical features of the scattering can also be studied by examining different variables. The angle ω_{23} between $\vec{p}_2^*$ and $\vec{p}_3^*$ is expected to peak at small values (compatible with the angular resolving power of the detector) where the contribution of final state bremsstrahlung is largest. In Fig. 9b, the distribution of $\cos \omega_{23}^*$ is compared to QCD and phase-space, both corrected for detector effects. The expected enhancement at small values of $\cos \omega_{23}^*$ can be clearly seen, in significant disagreement with the phase-space model.

4.4 The strong coupling constant α_s

In the present section, the quantity $\alpha_s K_3/K_2$ is evaluated by adjusting α_s in the QCD model until the theoretical value of the ratio between the three-jet cross section and the two-jet cross section (R_{QCD}) is equal to its experimental value, R_{exp} .

To this end, the two-jet and three-jet data samples are redefined using stricter selection criteria which are better suited to a quantitative comparison with the QCD model:

- i. secondary cluster pairs, as defined in Section 4.1, are merged into a single cluster, if they have an opening angle smaller than 50° ;

- ii. the 10 GeV threshold used in relation (24b) is replaced by a 15 GeV threshold to further reduce the contamination from spectator interaction products. The 8 GeV transverse momentum cut-off used in the QCD model generation is accordingly replaced by a 12 GeV cut-off.

The experimental value R_{exp} is simply calculated as the ratio between the number of three- and two-jet events, $R_{\text{exp}} = 0.186 \pm 0.003$.

The selection criteria are also applied to the Monte Carlo event samples. The value of R_{QCD} is the ratio between the two cross sections, σ_3^{QCD} and σ_2^{QCD} . Events generated from σ_3^{LO} (σ_4^{LO}) and having only two (three) jets obeying the selection criteria are included in σ_2^{QCD} (σ_3^{QCD}).

By varying α_s in the QCD model, the value of the strong coupling constant that makes R_{QCD} equal to R_{exp} is

$$(\alpha_s K_3/K_2) = 0.236 \pm 0.004 (\text{stat.}) \pm 0.040 (\text{syst.}) \quad (27)$$

The effects of the dependence of α_s on q^2 are shown in Fig. 11, in which the value of $\alpha_s K_3/K_2$ is plotted in different bins of the multi-jet mass M . The expected effect, due to the q^2 -dependence of α_s is less than $\pm 7\%$ over the full mass range. The data are in agreement with the expected variation (dashed line), but the deviation from the mean value (dotted line) obtained over the full range is not significant. More data are necessary to observe the variation of α_s with q^2 .

The systematic error quoted in Equation (27) takes into account effects due to the uncertainty in the fragmentation process ($\Delta\alpha_s/\alpha_s = \pm 10\%$), in the contributions from the spectator scattering ($\pm 10\%$), in the structure functions ($\pm 7\%$), in the energy response of the calorimeter ($\pm 3\%$), in the contribution from the four-jet sample ($\pm 8\%$) and in the dependence of α_s on the p_T of the multi-jet system ($\pm 4\%$).

The choice of q^2 in the scaling violation factors is somewhat arbitrary. Another choice would not affect α_s , but would only modify the QCD parameter Λ trivially, as long as the two-jet and three-jet samples are treated equally. However, it is possible to choose q^2 definitions which alter the average q^2 of the two-jet sample with respect to that of the three-jet sample. As an example, by replacing the choice $\sqrt{q^2} = \text{Max}(p_T^i)$ by $\sqrt{q^2} = \langle p_T^i \rangle$, reduces the three-jet $\langle q^2 \rangle$ while leaving the mean value of q^2 for the two-jet sample unchanged. The result is an increase in the value of R_{QCD} and hence a decrease in the measured value of α_s by 25%.

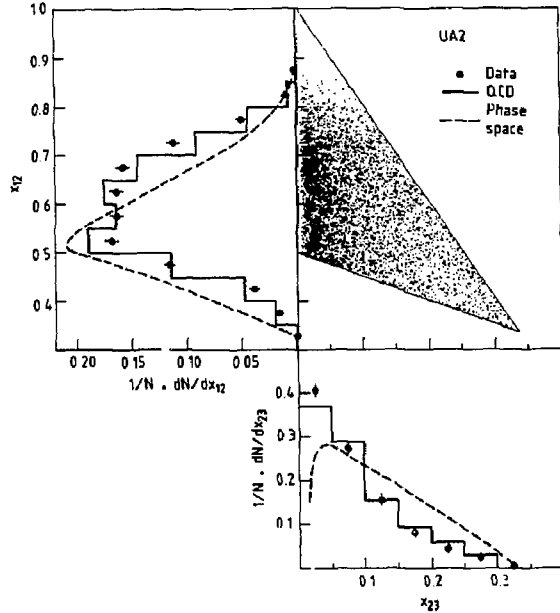


Figure 10: Three-jet Dalitz plot.

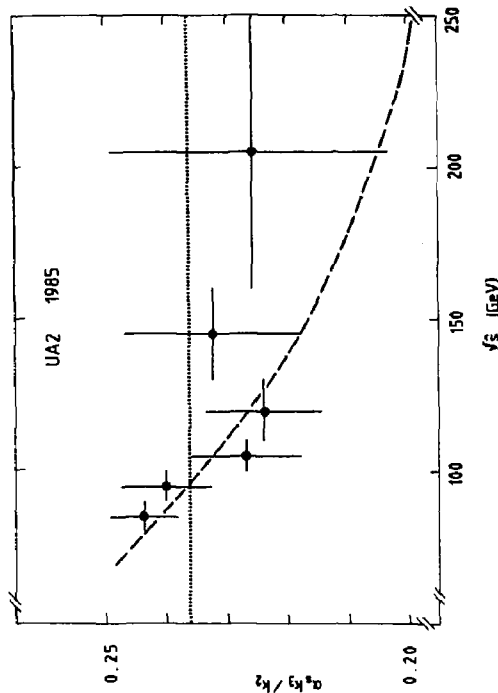


Figure 11: Dependence of α_s on the multi-jet mass.

The result (27) is very similar to that of the UA1 Collaboration,[37] obtained when using the same q^2 -scale for two- and three-jet events.

In conclusion, it is noted that conceptual rather than instrumental limitations preclude a more accurate and more reliable measurement of α_s using the present method: a first-order calculation of K_3/K_2 and a deeper understanding of the relevant q^2 -scales are the next questions to be addressed.

5. FOUR-JET EVENTS

5.1 Description of the sample

The four-jet sample is selected according to the criteria described in Section 4.1, with the Conditions (24) applied to the highest four jets of the event. Secondary clusters with an opening angle smaller than 30° are then merged into a single cluster. This reduces the sensitivity to fragmentation effects while still retaining good angular resolution, and results in a sample of ~ 2200 events.

The four-jet data can be described in terms of nine variables: three describing the orientation of the jet system and six describing the internal configuration of the jets. This large number of variables makes the description of this final state rather complex. The jets do not lie in a plane, nor can they be described by simple Dalitz plots. As for the three-jet sample, those variables are chosen, which are more directly correlated with the underlying dynamics.

As for the three-jet analysis, comparisons will be made between the data, a parton level QCD model based on recent calculations by Kunszt and Stirling,[44] and a phase-space model obtained by setting $Q_{ij}^{(1)}$ in Equation (25) equal to one. At this stage of the analysis, only the shapes of the distributions have been studied, by normalising the area of the distributions to one.

The sphericity of the event, calculated from the jet momenta in the CMS is shown in Fig. 12a. It is apparent that the observed event shape agrees well with the QCD model, and is significantly less spherical than phase-space. A more detailed understanding of the features of four-jet events can be obtained by examining the space angles among the final state jets. Figure 12b shows the distribution of $\cos \omega_{23}$. The data shows a significant enhancement above the phase-space model in the region of small angular separation, indicating the presence of bremsstrahlung.

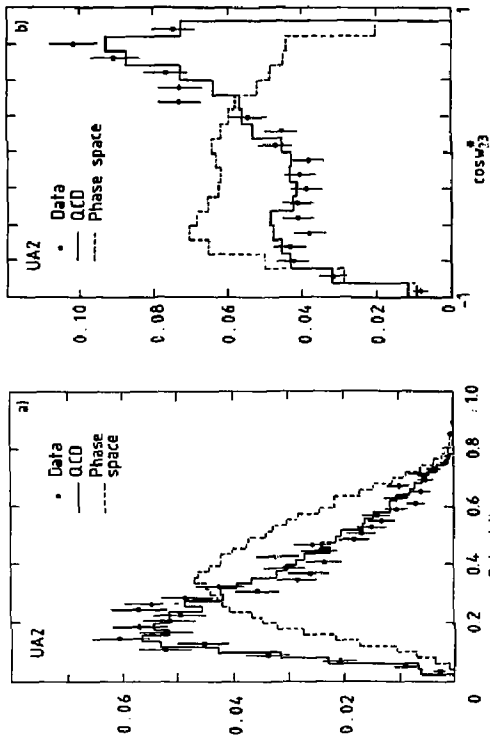


Figure 12: a) Sphericity of the four-jet system;
b) Angle between the second and the third jet.

5.2 Comparison between QCD and multi-parton interactions

Several authors have predicted an additional source of four-jet events,[39] the multi-parton mechanism, in which multiple hard parton collisions take place in a single hadron collision. This mechanism becomes important for small values of the parton momentum fraction, where the density of partons in the nucleon becomes very large. The simplest form of this process, in which two independent pairs of partons interact, can be described as follows :

$$\sigma_{mp} \sim \int G(x_1, x_2) G(x_3, x_4) d\hat{\sigma}_{12} d\hat{\sigma}_{34} \quad (28)$$

where the G are double structure functions and the $d\hat{\sigma}$ are the cross sections for the two parton sub-processes. A quantitative prediction of the yield of the process (28) requires the knowledge of the double parton wave function and, therefore, can only be attempted within the context of a model. Following Ref. [39], a simple model has been constructed, in which the double structure functions factorise in terms of single parton densities $\{G(x_1, x_2) \sim F(x_1) F(x_2)\}$. The effects of soft gluon radiation from the initial state partons are simulated by giving each jet-jet system a transverse momentum, whose distribution has been adjusted to agree with the measured two-jet data.[36] An estimate of the number of multi-parton collisions which are present in the data sample requires a detailed simulation of the fragmentation and detector effects on the parton energy scale and has not yet been performed. Instead, the normalised distributions of the double collision model will be compared with the single scattering process (Equation 25) and with the data.

The most characteristic feature of multi-parton events should be the appearance of pair-wise correlations among the jets. To search for these correlations, transverse variables in the lab frame are chosen, since they are relatively insensitive to centre of mass motion. A simple variable of this kind is the difference between the azimuthal angle of the leading jet, and the azimuthal angles of the other jets in the event. This variable, called ϕ_{lead} , is sensitive to the presence of a second jet opposite to the leading jet. The distribution of this variable, with three entries per event, is shown in Fig. 13a. The data agree well with the QCD model, and show no sign of the narrow peak that is expected for the multi-parton process. An alternative variable is the p_T unbalance in the event, defined by the expression :

$$p_T^2(\text{unb}) = 2 \min (\vec{p}_T^i + \vec{p}_T^j)^2 \quad (29)$$

where i is chosen to minimise the p_T unbalance. This variable should take only small values for the multi-parton process since there will be a

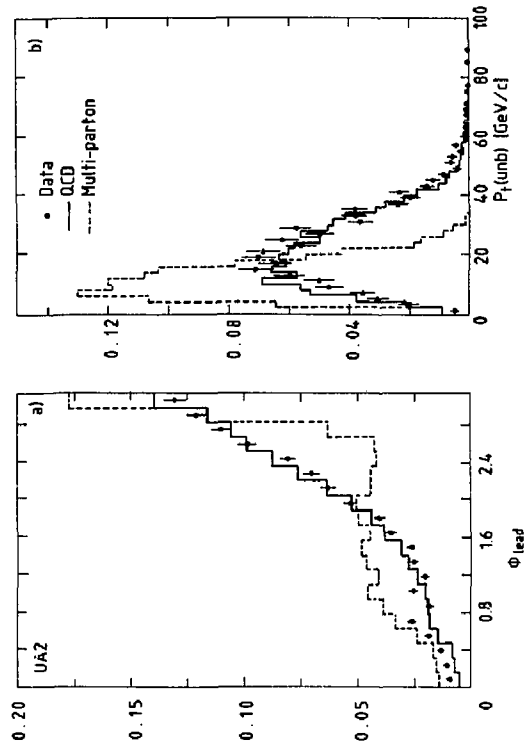


Figure 13: a) Azimuthal difference between the leading and the other jets
b) p_T (mb) - See Equation (29) in the text.

second jet balancing the p_T of the leading jet. The observed distribution is shown in Fig. 13b, and agrees well with the QCD model. Again, there is no sign of the enhancement expected at small p_T unbalance from the multi-parton mechanism.

Two preliminary conclusions emerge from this discussion. The first is that the observed four-jet distributions agree well with a leading order QCD calculation, and differ significantly from four-body phase space. The second is that in the present data there is no evidence for the additional contribution from the multi-parton processes.

6. SEARCH FOR THE HADRONIC DECAYS OF W AND Z BOSONS

A study of the invariant mass distribution of jet pairs observed in the central calorimeter has been made, in order to search for an excess of events in the region of the W and Z bosons arising from the hadronic decays of the IVB's.

6.1 Selection criteria

The selection criteria were chosen to provide the best mass resolution compatible with retaining a large data sample. Special cuts and a different jet definition algorithm have been applied. Events which show a large departure from the average behaviour are excluded, since their mass resolution will be worse. The high mass tail is particularly dangerous in the present case, since the expected signal is superimposed on a steeply falling distribution.

The jet definition algorithm starts from the clusters defined in Section 2. Then, jet directions are defined using the cluster centroids and the centre of the detector. Finally, the energy of a jet is taken as the sum of the energies of all calorimeter cells having their centre within a cone of angle ω around the jet axis. The value of ω is adjusted in order to minimise energy measurement errors. This procedure requires a variable which is sensitive to the jet energy resolution. One such variable is the difference between the average values of p_T^{ξ} and p_T^{η} , the components of the transverse momentum of the jet pair projected on the bisectors of the jet transverse momenta. While the component p_T^{ξ} , parallel to the jet axis, is mostly affected by energy measurement errors, p_T^{η} is only influenced by angular measurement errors, which produce a much smaller effect.[36] If the cone is chosen too large, more energy from the

underlying event is incorrectly associated with the jet. If the cone is chosen too small, the energy of the original parton is not well-contained. Thus, there is an optimum value for the cone size, which is found to be $\cos \omega = 0.6$.

In order to ensure sufficient containment, only jets having their axes within a fiducial volume, $|\cos \theta| < 0.6$, are considered. Corrections to the energy and polar angle of each jet, accounting for the lack of calorimeter coverage outside the interval $40^\circ < \theta < 140^\circ$, are applied. They are evaluated from a study performed on a sample of well contained jets and amount to

$$\Delta\theta \text{ (radians)} = -0.05 \cos \theta, \quad (30)$$

$$\Delta E = 0.06 E (\cos^2 \theta + |\cos \theta|). \quad (31)$$

The following selection criteria are adopted :

i. criteria applied to each jet :

- the energy ΔE measured between the two cones having $\cos \omega = 0.5$ and $\cos \omega = 0.7$ must not exceed 4 GeV;
- the fraction f_{em} of jet energy measured in the calorimeter electromagnetic compartment must exceed $0.235 - 0.094 \ln(E/40\text{GeV})$;
- the fraction f_{H2} of jet energy measured in the second hadronic compartment must not exceed $0.410 + 0.087 \ln(E/40\text{GeV})$;

ii. criteria applied to each jet pair :

- the mass m_{c1} of the pair, calculated using cluster energies, must not differ from the two-jet invariant mass m by more than 17 GeV;
- the transverse momentum p_{T1} , expressed in GeV, must not exceed $24 + 11.5 \ln(m/80\text{GeV})$.

The cuts were tuned to reject a fixed fraction — typically 5 to 10% each — of the event sample, independent of the jet-pair mass. They reduce the event sample to a fraction $r_{cut} = 0.66$ of its original population. Their efficiency to retain IVB decays may be slightly larger, to the extent that quark jets may be expected to pass slightly tighter cuts than the gluon jets which dominate the event sample from which r_{cut} has been evaluated.

Two quantities related to the amount of transverse energy not used in the jet definition are also introduced :

- $\bar{E}_{T,C}$, in the central calorimeter outside the jet definition cones;
- $\bar{E}_{T,F}$, in the electromagnetic compartments of the forward calorimeters.

These quantities monitor a possible contamination from soft collisions. Such a contamination was studied using the cluster algorithm to define jets [48] and is expected to be negligible even though the present algorithm produces wider jets. The $\bar{E}_{T,F,C}$ distributions of events passing the selection criteria and having a jet-pair mass in the IVB region ($70 < m < 100$ GeV) are shown in Fig. 14. Such distributions are expected to be similar for $W, Z \rightarrow q\bar{q}$ and for $W \rightarrow e\nu$ events, since jet fragments outside the jet-definition cones carry approximately the same transverse energy as spectator fragments inside the jet-definition cones. A cut $\bar{E}_{T,C} < 10$ GeV, $\bar{E}_{T,F} < 6$ GeV is applied to the sample. It retains $\approx 60\%$ of the jet-pairs in the IVB region.

6.2 Estimate of the mass resolution

The p_{T1} and p_{T2} distributions (shown in Fig. 15) of events passing the cuts and having a jet-pair mass in the IVB region, $70 < m < 100$ GeV, have average values $\langle p_{T1} \rangle = 6.25$ GeV and $\langle p_{T2} \rangle = 4.33$ GeV. Accounting for the effect of angular measurement errors brings this latter value down to 4.25 GeV. The quantity $Q = \{ \langle p_{T1}^2 \rangle - \langle p_{T2}^2 \rangle \}^{1/2} \approx 4.6$ GeV is related to the mass resolution Δm . To evaluate Δm from Q , a set of events are generated from the observed two-jet events by altering the jet energies by random amounts chosen with a Gaussian distribution of zero mean and given variance. The dummy events obtained in this way are filtered through the selection criteria defined above. An energy resolution of $\approx 12.6\%$ is found to be necessary to increase Q by another 4.6 GeV, corresponding to a mass resolution $\Delta m/m = 9\%$.

An independent evaluation of Δm is obtained from a Monte Carlo simulation of the experiment, which includes all the effects relevant to the measurement of jet-pair masses. Another purpose of this simulation is to evaluate the average mass values at which the W and Z signals are expected and to predict their magnitudes. The structure of the simulation program describing the IVB production mechanism and the UA2 detector is the same as in previous work [34-36]. A number of parameters have been adjusted with particular care in view of their relevance to the present study :

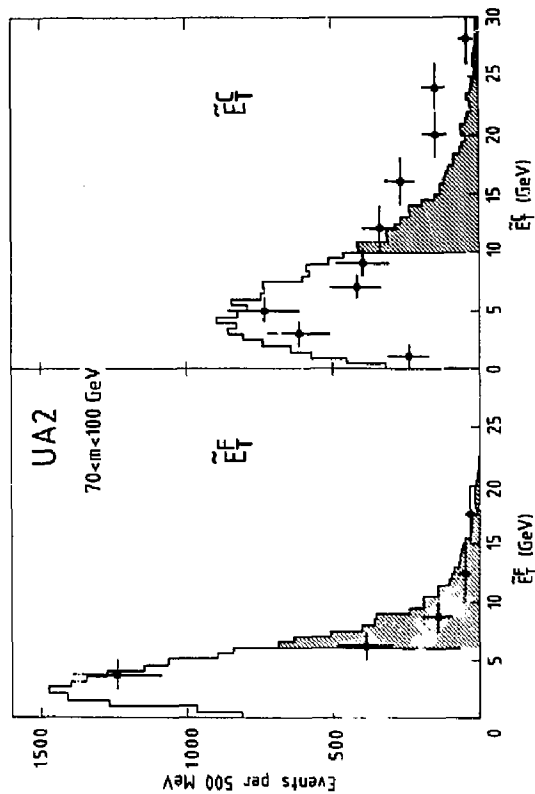


Figure 14: Distributions of the transverse energy not used in the jet definition.

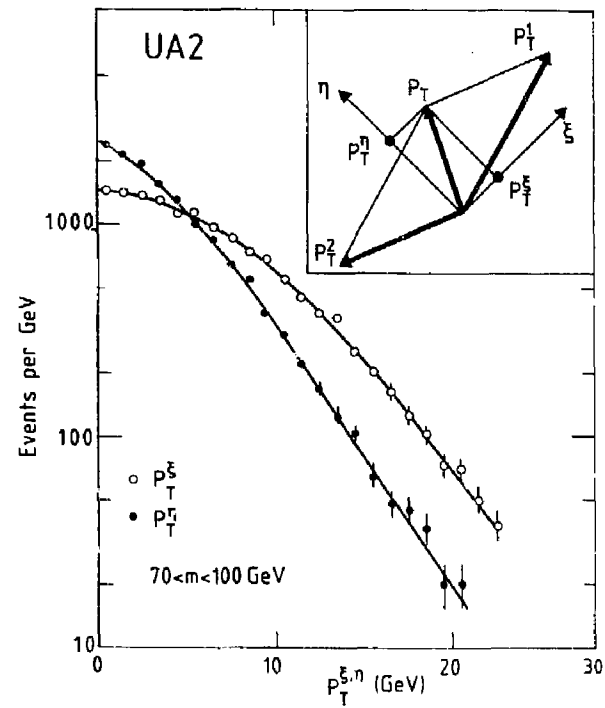


Figure 15: Distributions of the components of the jet transverse momentum.

- i. The calorimeter response to low energy particles, ≤ 2 GeV, (jet fragments include such particles) was not extensively studied in the test beam measurements [9] and is extrapolated from higher energy data. As a check on this procedure, the density of transverse energy per unit of rapidity in the central region has been studied for a sample of "minimum bias" events. The measurement is found to be in good agreement with a calculation based on Collider data on the low transverse momentum particle spectrum and on its composition.[49]
- ii. The fraction of jet energy outside the definition cone has been measured for quark jets at electron Colliders up to $\sqrt{s} \approx 46$ GeV.[50] In the Monte Carlo simulation it is extrapolated to higher energies using the fragmentation algorithm,[51] which reproduces the lower energy data.
- iii. Particles produced in the collective interaction of the spectator partons and gluons from initial state bremsstrahlung uniformly populate the central rapidity region. Some of these particles happen to fall inside the jet-definition cones, thus causing an increase of the measured jet energies. This effect is independent of the IVB decay mode and can be studied from a sample of identified $W \rightarrow e\nu$ events observed in the UA2 detector.
- iv. The jet-pair mass m is calculated under the assumption that jets are massless, defining the jet momentum p to be equal to its energy E . The error on m resulting from this approximation is very small, $< 1\%$ on the average.

A summary of various factors affecting the mass scale and the mass resolution in the IVB region is presented in Table 1.

6.3 Preliminary results

The $Z - W$ mass difference is too small, compared with the expected experimental resolution, to allow the two peaks to be resolved. Therefore, an excess of events should be observed in a wide mass region, typically $65 < m < 105$ GeV.

A quantitative measurement of the signal yield above the level of the strong interaction background requires an interpolation from the two control regions on either side of the signal. Therefore, the mass distribution can be used only in the range not affected by distortions, such as threshold biases due to the trigger or contamination by soft

Table 1: Estimate of the mass scale and resolution.

Effect	Mass shift (GeV)	Mass resolution (GeV)
1. Energy leakage outside definition cone	-3.4 ± 1.5	3.8
2. Spectator contribution inside the cone	$+4.2 \pm 0.7$	3.0
3. Calorimeter response	-3.1 ± 3.0	6.4
4. Energy calibration	0 ± 3.4	-
Ideal detector (1.-2.)	$+0.8 \pm 1.7$	4.8
Combination of all effects (1.-4.)	-2.3 ± 4.8	8.0
Direct estimate from $p_T^{\xi,\eta}$	-	7.4

collisions.[48] As discussed in Section 2, the two-jet data have been taken with different thresholds. The mass range which is free from trigger biases is found by studying the mass dependence of the ratio between high- and low-threshold data. The observed values are $m = 45, 49$, and 57 GeV for the events taken with threshold set at $E_T = 12.5, 15$ and 20 GeV respectively. The three data samples are then combined with relative normalisations evaluated using the unbiased region $m > 57$ GeV.

In Fig. 16 the mass distribution of the events which pass all the cuts is shown. Each event is given a weight $(m/100 \text{ GeV})^3$ to reduce the effects due to the use of wide mass bins in a steeply falling distribution. A structure is visible, in qualitative agreement with the signal expected from W and Z decays.

The control regions are used to parametrise the shape and size of the strong interaction background. Parametrisations of the form $m^{-\alpha} \exp(-\beta m)$, $m^{-\alpha} \ln(\beta/m)$, and $m^{\alpha} \ln m$ are in good agreement with the data. As an example, the function $dN/dm \sim m^{-\alpha} \exp(-\beta m)$ fits the control regions, with a $\chi^2 = 4.1$ for 7 degrees of freedom. When extended to the whole mass range, including the signal region, the same function gives a poor fit, with a $\chi^2 = 21.1$ for 12 degrees of freedom.

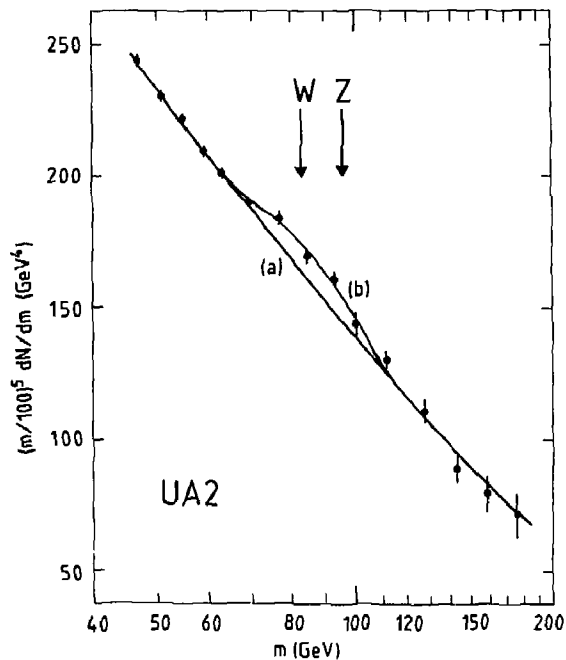


Figure 16: Two-jet mass distribution.

A quantitative evaluation of the number of events in the signal and a measurement of its significance is obtained by fitting the data, distributed in 1 GeV wide mass bins, to a form

$$dN/dm = A \{ m^{-\alpha} \exp(-\beta m) + \xi S(m, m_0) \} \quad (32)$$

where $S(m, m_0)$ is the sum of two Gaussian distributions, representing W and Z signals respectively. The W (Z) distribution is taken to have a mean value m_0 ($1.14 m_0$) and R.M.S. 8 GeV (9 GeV). The relative normalisation between the two Gaussians is assumed equal to the expected ratio (≈ 3) between the numbers of observed W and Z decays. The parameters α , β and ξ are adjusted to maximise the likelihood function, the constant A being calculated each time to provide the appropriate normalisation. The mass parameter m_0 is either set to the expected value, $m_0 = 78.5$ GeV, or treated as an additional free parameter. In the latter case m_0 is found to be 82 ± 3 GeV, the significance of the signal is 3.3 standard deviations, and its magnitude corresponds to 632 ± 190 events.

The number of events found using the fit can be compared with the results of the simulation described in Section 6.1, which takes into account the Standard Model predictions and the efficiency of the selection criteria. First, the number of signal events in the absence of $\bar{E}_T^{F,C}$ cuts is evaluated, using a cut efficiency of $\epsilon_{\text{cut}} = 0.66$, as discussed in Section 6.1. The effect of the $\bar{E}_T^{F,C}$ cuts is estimated either from the background events observed in the signal region, or from the $W \rightarrow e\nu$ events, modified to account for the presence of jet fragments outside the definition cones. This calculation results in an expected number of events which is approximately one standard deviation below the observed signal. This result does not vary if the $\bar{E}_T^{F,C}$ cut is relaxed in both the data and in the Monte Carlo.

Other fits, using either a different parametrisation of the strong interaction background or different data subsamples obtained by altering the selection criteria, have been performed. Fits to data samples which do not satisfy two or more selection criteria, give no evidence for a signal. Subsamples of data which pass the selection criteria always present a signal with a statistical significance between 2 and 3.5 standard deviations. No known systematic effect seems capable of producing such a signal.

In conclusion it may be noted that the hadronic decay of the IVB's is one of the simplest test-cases of the ability of future Collider experiments to analyse multiparticle final states in terms of hadron jets identified with their parent partons. The observed signal, at the level of ~ 3 standard

deviations above the copious and steeply falling strong interaction background, is in good agreement with Standard Model expectations. However, more data are needed to place it on firmer ground. It will be a challenge for the upgraded UA2 detector [52] at ACOL [53] to show stronger evidence for the signal and to measure the $W, Z \rightarrow q\bar{q}$ branching fractions.

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RESULTS ON θ , FROM THE CERN HIGH-ENERGY NEUTRINO EXPERIMENTS OR: WHY θ ? HOW θ ?

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1. INTRODUCTION

The hard core of the Standard Model is the electroweak gauge theory determined by the two independent local symmetries $SU(2)_L$ and $U(1)$. From the very illuminating lectures of Bruce Winstein, we have learned that there are no published experimental data that contradict the Standard Model, and we can safely state that the amount of data—from the parity-violating emission of light to the existence, at the expected mass, of the massive vector bosons—that constitute the experimental construction of the Standard Model is really very impressive. In fact, it is so impressive that probably the statistically expected violations of the Standard Model are kept in the drawer of some poor experimentalist who is waiting for the chance to repeat the experiment.

For the physics of 'fundamental particles', we are living in the Standard Era (1970-?). Is this simple formulation of gauge theory the forerunner of the string tyranny? Let us still hope that the experimental method will prove to be the strongest.

What shall we do in the Standard Era? The lesson has already been taught us by the ancestor of the Standard Model: the full beauty of the predictive power of QED has been shown in many elegant experiments, such as those on the Lamb shift, the $g-2$, the Josephson effect, and others.

In the Standard Model the strength of the forces is described by two independent coupling constants, g and g' , attached to $SU(2)_L$ and $U(1)$, respectively:

$$\begin{matrix} SU(2)_L & \times & U(1) \\ g & & g' \end{matrix}$$

The four gauge bosons—three ($W_{1,2,3}$) from $SU(2)_L$ and one (B) from $U(1)$ —give rise to four physical particles: two charged, the $W^\pm$, and two neutral, the γ and the Z^0 ; γ is the familiar photon, the fastest boson in the Universe ($m_\gamma = 0$); Z^0 is the recently discovered, heaviest elementary particle ($m_Z = 91.5 \pm 1.2 \pm 1.7$ GeV, the latest UA2 value).

The fundamental quantities of the Standard Model— g , g' , m_W , m_Z —are related to the angle θ_w that mixes the two symmetry-related bosons W_3 and B , with zero electric charge, to produce the physical γ and Z through the following very simple relations:

$$\tan \theta_w = g'/g \quad (1)$$

$$\cos \theta_w = m_W/m_Z \quad (2)$$

Equations (1) and (2) are real fundamental predictions in the sense that the measurement of the coupling constants g and g' dictates the relation between the masses of the physical

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bosons. The requirement that the massless boson coming from the W_3 -B mixing be the well-known photon gives the electromagnetic connection (this is not a prediction)

$$g \sin \theta_w = e$$

identifying the electric charge e as the projection of the g coupling constant.

The above relations are well represented geometrically by the Gatto triangle (Fig. 1).

Now that we know *why*, in the Standard Era, we have to measure θ_w , let us learn *how* to do so.

2. METHODS OF MEASURING θ_w

If the relation

$$\cos \theta_w = m_W / m_Z$$

is taken as the definition of θ_w , we have the following recipe:

Measure the W and Z masses and take the ratio.

If m_W or m_Z are not measured very accurately, the muon decay can help out. The μ decay is described by the charged-current (CC) graph of Fig. 2. In the Fermi approximation (four-fermion contact interaction),

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2} = \frac{e^2}{8m_W^2 \sin^2 \theta_w}$$

$$m_W^2 = \frac{2^{-1/2} e^2}{G_F \sin^2 \theta_w} \quad \sin \theta_w = \frac{2^{-1/2} e}{G_F^{1/2} m_W}$$

or

$$\frac{1}{2} \sin 2\theta_w = \frac{2^{-1/2} e}{m_Z G_F^{1/2}}$$

These relations are valid at the tree level. The weak angle θ_w can be derived from the measurement of m_W or m_Z and the values of e (accurately measured, for instance, in the Josephson effect: $1/\alpha = 137.035963 \pm 0.000015$) and of G_F , also known very precisely from the μ decay: $G_F = 1.16637(2) \times 10^{-5} \text{ GeV}^{-2}$.

3. θ_w FROM νe AND $\bar{\nu} N$ SCATTERING

The Z coupling to the fermion in the Standard Model is given by

$$\epsilon(f_L) = I_3 - q \sin^2 \theta_w$$

$$\epsilon(f_R) = -q \sin^2 \theta_w$$

The scattering of ν_e and $\bar{\nu}_e$ on free electrons is mediated only by Z for flavour conservation (Fig. 3).

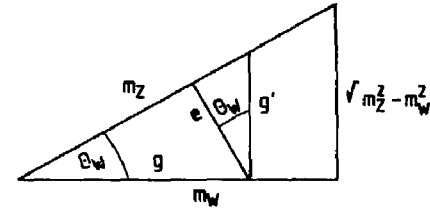


Fig. 1 The Gatto triangle

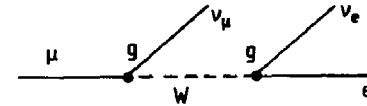


Fig. 2 Muon decay

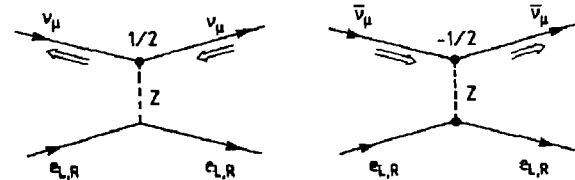


Fig. 3 Neutrino and antineutrino scattering on free electrons

Owing to the different helicity of ν_e and $\bar{\nu}_e$ in the differential cross section, the angular dependence of σ and $\bar{\sigma}$ is different, so the integrated cross sections σ and $\bar{\sigma}$ contain a different combination of $\sin^2 \theta_e$:

$$\sigma \propto \epsilon(c_1)^2 + \epsilon(c_2)^2/3,$$

$$\bar{\sigma} \propto \epsilon(c_2)^2 + \epsilon(c_1)^2/3,$$

and

$$R = \sigma/\bar{\sigma} = 9 \left[1 + \frac{1}{3} F(\theta_e)/(1 + F(\theta_e)) \right]$$

where $F(\theta_e) = 16 \sin^2 \theta_e / (1 - 4 \sin^2 \theta_e)$. The measurement of R is particularly appealing, both for experimental and for theoretical reasons. The values of $R(\theta_e)$ and $F(\theta_e)$ versus θ_e are given in Fig. 4. The experimental arguments in favour of the R measurement are as follows:

- i) The efficiency ϵ for finding isolated electrons in the detector, as required in the measurement of σ and $\bar{\sigma}$, is the same provided the energy spectra of the incident ν_e and $\bar{\nu}_e$ beams are equal. So the value of the ratio R of $\sigma/\bar{\sigma}$ becomes independent of the knowledge of ϵ . The value of ϵ can hardly ever be computed exactly because the electron signals have to be extracted from the large background coming from the reaction of $\bar{\nu}_p$ on nucleons with $\approx 10^4$ times better probability than from the reaction of ν_e .
- ii) In order to derive σ and $\bar{\sigma}$ from the isolated-electron events (Fig. 5), the monitoring of the ν_e and $\bar{\nu}_e$ fluxes is done using the relatively high probability reactions such as elastic $\nu_e p$ scattering, or the inclusive νN reactions whose cross sections are, in principle, better known than those of the $\bar{\nu}_e p$ reactions.

From the theoretical point of view, the elastic scattering of two point-like leptons, ν_e and e , can be computed exactly to all orders of perturbation, and the ratio of σ and $\bar{\sigma}$ is not affected by possible errors in the computation of the higher-order corrections that are derived by means of lengthy and difficult computer programs. Table 1 and Fig. 6 give the values of $\sin^2 \theta_e$ measured in different experiments, and the same values when radiative corrections are applied. The $\nu_e e$ experimental values change only by 1%.

In spite of the simplicity of the θ_e derivation from points (i) or (ii), at present the most precise value of $\sin^2 \theta_e$ comes from the high-statistics experiments of ν_e scattering on nucleons recently completed at the CERN Super Proton Synchrotron (SPS). The results were presented by the CDHS and CHARM Collaborations [1, 2] at the 1986 International Conference on High-Energy Physics held at Berkeley, California. Llewellyn Smith has recently shown [3] that if $\sin^2 \theta_e$ is measured in semileptonic neutrino interactions, then, contrary to previous claims in the literature, the error due to unknown dynamical (higher-twist) correction to the QCD parton model is small provided that target contains equal amounts of d-quarks and u-quarks (isoscalar target).

The neutral-current (NC) νN interactions are due to the reaction shown in Fig. 7. The charged-current (CC) $\nu_e N$ interactions (Fig. 8) involve only left-handed d-quarks in the initial state (here the contributions from the $q\bar{q}$ sea are neglected). For an isoscalar target and in the Standard Model, the NC to CC ratio is given by

$$R = \sigma^{NC}(\nu_e N \rightarrow \nu_e X) / \sigma^{CC}(\nu_e N \rightarrow \mu^- Y)$$

$$= \left(\frac{1}{2} - \sin^2 \theta_e \right) + \frac{5}{9} \sin^2 \theta_e (1 + r);$$

$$r = \sigma(\bar{\nu}_e N \rightarrow \mu^+ X) / \sigma(\nu_e N \rightarrow \mu^- Y).$$

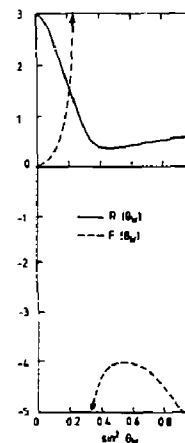


Fig. 4 Values of the functions $R(\theta_e)$ and $F(\theta_e)$

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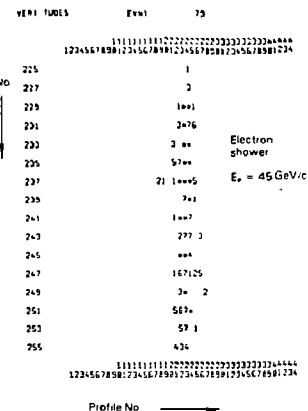


Fig. 5 Example of an isolated electromagnetic shower in the CHARM detector

Table 1: Values of $\sin^2 \theta_W$ before and after electroweak radiative corrections are included for a variety of experiments. The values $m_t \approx 36$ GeV and $m_H \approx m_Z$ were employed in the radiative correction.[†]

Experiment	$\sin^2 \theta_W^{\text{unc.}}$	Radiative Correction	$\sin^2 \theta_W \equiv 1 - m_W^2/m_Z^2$
Cs Atomic P.V. (Paris - Boulder)	0.221 ± 0.027	+0.007	0.228 ± 0.027
eD Asymmetry (Yale - SLAC)	0.224 ± 0.020	-0.006	0.218 ± 0.020
$(-)$ $\nu \mu^+$ (BNL - CHARM)	0.212 ± 0.023	≤ 0.001	0.212 ± 0.023
$(-)$ $\nu \mu^+$ (BNL)	$0.220 \pm 0.016^{+0.025}_{-0.031}$	small	$0.220 \pm 0.016^{+0.023}_{-0.031}$
ν_μ^N deep-inel. (CDHS, CHARM) (CCFR, FMM)	0.242 ± 0.006	-0.011	0.231 ± 0.006
$m_W = 80.0 \pm 1.4$ GeV (UA1 - UA2)	0.213 ± 0.008	+0.016	0.229 ± 0.008
$m_Z = 92.3 \pm 1.7$ GeV (UA1 - UA2)	0.205 ± 0.011	+0.022	0.227 ± 0.011
World Average			0.229 ± 0.004

[†] Taken from Morcone's talk given at the 23rd Int. Conf. on High Energy Physics, Berkeley, California, 1986.

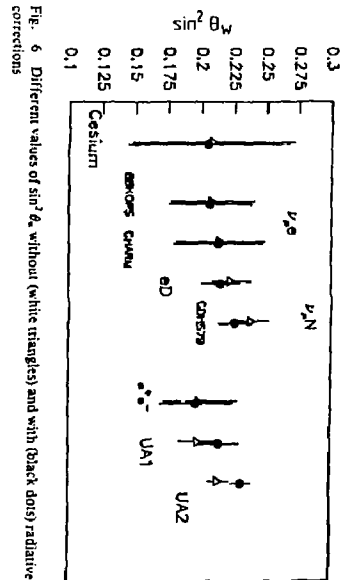


Fig. 6 Different values of $\sin^2 \theta_W$ without (white triangles) and with (black dots) radiative corrections

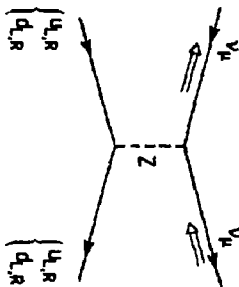


Fig. 7 Neutral-current νN interactions

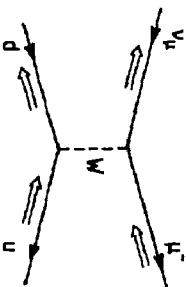


Fig. 8 Charged-current νN interactions

The recipe for precise θ_n ($\delta \sin^2 \theta_n = 0.005$) is:

$$\text{Measure: } \Delta R/R = \pm 1\%$$

and

$$\Delta r/r = \pm 3\%$$

The measurement of R to 1% requires high-statistics ($\approx 10^5$ CC) interactions and a good knowledge of the NC and CC detection efficiency. The ν_μ flux-measurement drops in the ratio R . The measurement of r can be done with an error of $\approx 3\%$: $\sin^2 \theta_n$ is about $1/4$ and therefore $\sin^2 \theta_n \approx 1/16$, so that the term containing r is reduced.

Two recent experiments (CDHS and CHARM [1, 2]) have reported on $\sin^2 \theta_n$ from semileptonic neutrino interactions from high-statistics data collected with an improved narrow-band (neutrino) beam NBB. The high-flux neutrino beam is obtained by 6×10^{18} 400 GeV protons impinging on a Be target of $\approx 1\lambda$.

Before describing the two detectors and their experimental results, I will briefly recall the main features of the NBB. A narrow-band beam of neutrinos is produced by the decay of a monochromatic, positively charged, unstable hadron crossing a vacuum tunnel, called the decay tunnel (Fig. 9). The following two processes dominate the two-body decay source of the NBB:

$$\pi^+ \rightarrow \mu^+ \nu_\mu (100\%) \pi_{\pi 2},$$

$$K^+ \rightarrow \mu^+ \nu_\mu (63.5\%) K_{\pi 2}.$$

The two-body kinematics of the decay $h^+ \rightarrow \mu^+ \nu_\mu$ impose a relation between the hadron momentum p^+ , the neutrino angle, and the neutrino energy:

$$E_\nu^{(h)}(\theta_n) = p^+ [(m_h^2 - m_\mu^2)/(m_h^2 + p^{+2} \sin^2 \theta_n)].$$

The neutrino angle θ_n in the laboratory system varies from 0 to $\theta_{\max}$, defined by the detector acceptance,

$$0 < \theta_n < R/l_{\text{detec.}},$$

where $l_{\text{detec.}}$ is the distance between the middle of the decay tunnel and the detector and R is the detector radius. For a value of $R/l_{\text{detec.}} = 2 \text{ mrad}$, the energy ranges between:

$$p^+ [(m_h^2 - m_\mu^2)/(m_h^2 + 4 \times 10^{-6} p^{+2})] < E_\nu^{(h)} < p^+ [(1 - m_\mu^2)/m_h^2].$$

The two values of m_h (m_π , m_K) give rise to the bichromatic beams shown in Fig. 10. For hadrons of 160 GeV/c momentum, as chosen in the recent high-statistics experiments at CERN [1, 2] the energy range of the neutrino beam was

$$\begin{aligned} 10 < E_\nu < 70 \text{ GeV} & \quad \text{for } \nu \text{ from } \pi \text{ decay,} \\ 100 < E_\nu < 153 \text{ GeV} & \quad \text{for } \nu \text{ from } K \text{ decay.} \end{aligned}$$

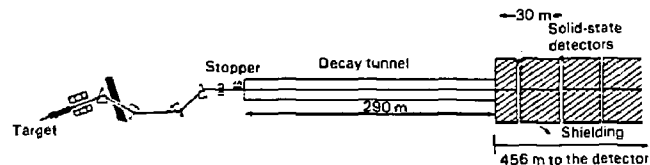


Fig. 9 Schematic layout of the neutrino beam line at CERN (1984)

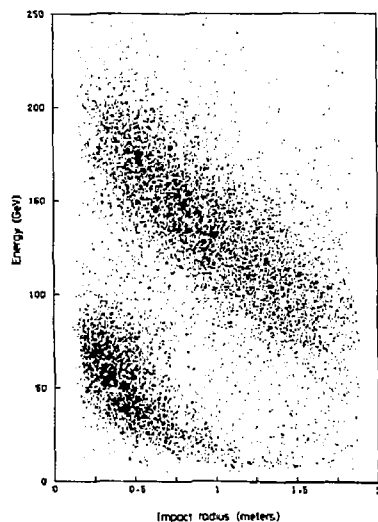


Fig. 10 Relation between the energy and the radial coordinate of the interaction point of CC events as measured by the CDHS Collaboration. The lower-energy concentration of dots corresponds to pion decay, the higher energy one to kaon decay.

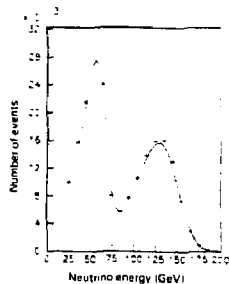


Fig. 11 Projection of Fig. 10 on the energy axis.

The energy spectrum of the neutrino beam is illustrated in Fig. 11. The energy dispersion is due to the decay along the 300 m decay tunnel, to the divergence of the hadron beam, and to the contamination of the three-body decay of the kaon:

$$\begin{aligned} K_{s2} &\rightarrow \pi^0 \mu \nu_\mu \\ K_{s2} &\rightarrow \pi^0 e \nu_e \end{aligned}$$

The elements of the CERN NBB are shown in Fig. 12. The relative weight of the π and K components is measured with the aid of a gas Cherenkov counter. The pressure curve is shown in Fig. 13.

The neutrino spectrum as measured by the CDHS Collaboration, reconstructing the total energy of the CC events, is shown in Fig. 14 for different values of R.

4. THE DETECTORS

The CDHS and CHARM calorimeters for neutrino physics, like other calorimeters in this field, have a large target mass in order to compensate for the low νN interaction probability [$\sigma(\nu N) \approx E_\nu \times 10^{-38} \text{ cm}^2 \text{ GeV}^{-1}$]. The transverse dimension ($\phi \approx 1-2 \text{ m}$) of the calorimeter is such that most of the neutrino beam angular opening can be covered. In fact, neutrino beams with a typical angular dispersion of a few milliradians produce a spot, at a distance of 1-2 m, to which the transverse dimension of the calorimeter has to be matched. The longitudinal dimension is a reasonable compromise between the infinite length required by statistics and the finite financial resources of all existing laboratories. The CDHS and CHARM detectors contain these general neutrino calorimeter features, and differ only in the emphasis placed on certain aspects of neutrino physics. The CDHS give priority to high statistics and detector homogeneity, choosing Fe as the target and integrating the muon spectrometer function with the calorimetry. The CHARM detector emphasizes the physics of the neutral-current interaction, and its light fine-grain calorimeter is followed by the muon spectrometer at the end of the calorimeter.

Figures 15a,b show schematic layouts of the CDHS and CHARM calorimeters. In Figs. 16a,b,c we see an example of a CC ν_μ event. The main features of the two calorimeters are listed in Table 2.

5. EXPERIMENTAL RESULTS AND DATA ANALYSES

5.1 Results from the CDHS Collaboration

The CDHS raw data collected under different experimental conditions (^3He beam dumping, cosmic subtraction) are presented in Table 3a. The separation between CC and NC was made on the basis of the penetration of the event (Fig. 17). The muon of the CC events traverses all the apparatus at high energy, and labels the event according to the length of the penetration, as shown in Fig. 17.

The experimental acceptance and the beam distributions are introduced by a Monte Carlo simulation, and the experimental data compare well with the expected result, as shown in Figs. 18a,b. The CC and NC events for ν_μ and $\bar{\nu}_\mu$ exposures are shown in Table 3b after all corrections have been made. The following results are derived from the corrected data:

$$\begin{aligned} R_1 &= 0.3072 \pm 0.0025 \pm 0.0021 \\ R_2 &= 0.382 \pm 0.015 \pm 0.006 \end{aligned}$$

$$t = 0.393 \pm 0.005 \pm 0.013$$

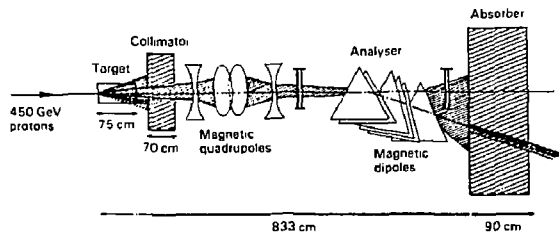


Fig. 12 Detail of the hadron beam at CERN (1984).

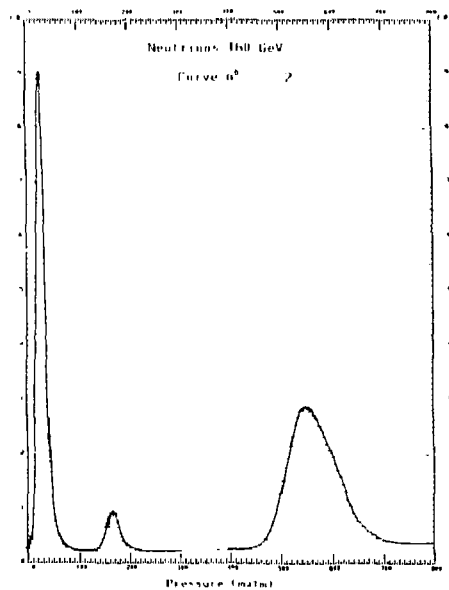


Fig. 13 π/K ratio in the primary hadron beam producing the neutrinos at CERN.

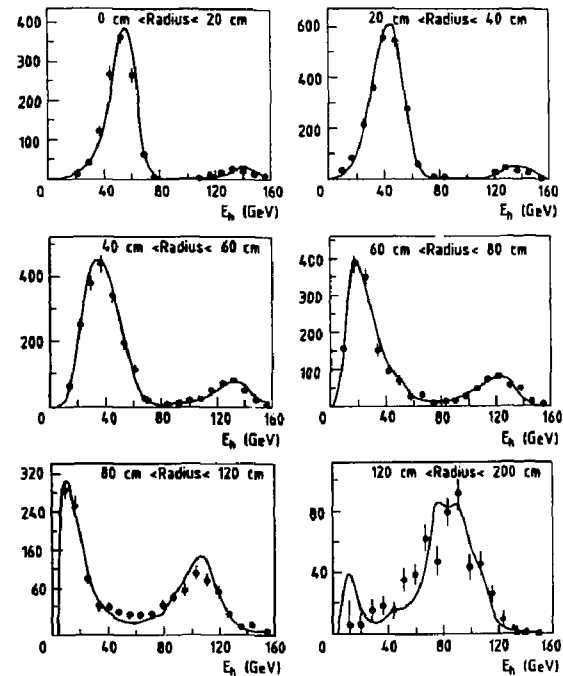


Fig. 14 The neutrino spectrum measured for different radii of the CERN detector.

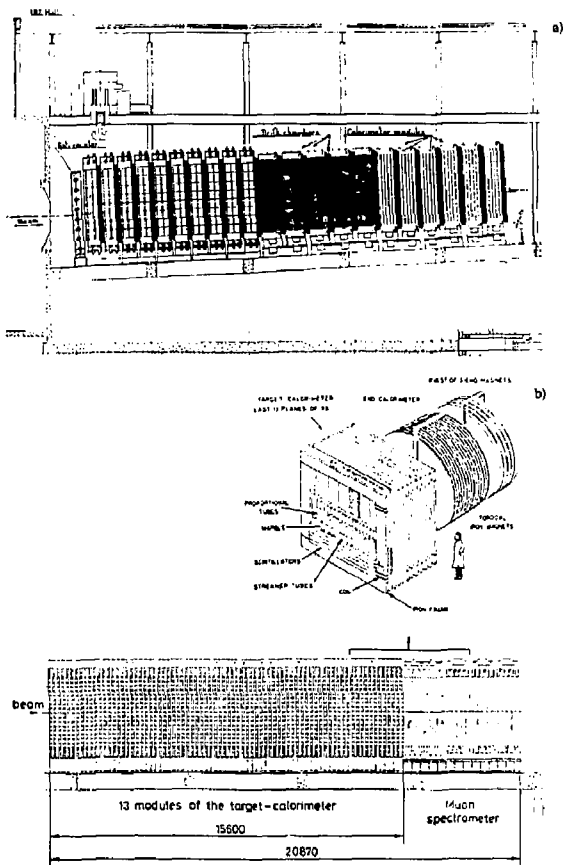


Fig. 15 Schematic layout of the CDHS (a) and CHARM (b) calorimeters.

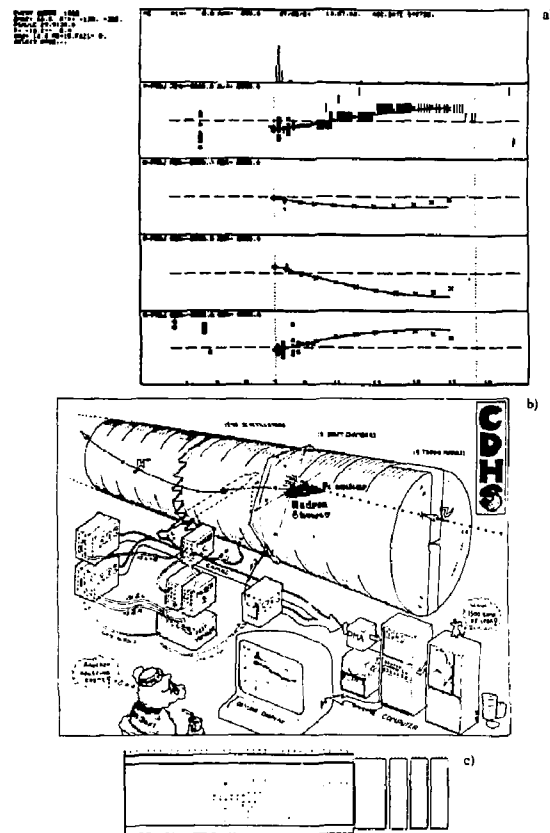


Fig. 16 An event display of a CC ν_e interaction as seen by the CDHS Collaboration: a) a computer display view; b) an artist's view; c) the same type of event, seen by the CHARM detector.

Table 2
Properties and performance of the CDHS and CHARM calorimeters

	CDHS	CHARM
Major physics goals	» physics; emphasis on structure functions	» physics; emphasis on neutral-current events
Approx. size (t)	1400	150
Technique	2.5 cm or 5 cm Fe plates sampled with scintillators and drift chambers	8 cm marble plates sampled with scintillators and proportional tubes
No. of read-out channels	3600 PMs 4000 drift wires	≈ 1600 scintillators ≈ 10 ⁴ proportional tubes
α_1 (for 1 GeV) hadrons photons	58 (70%), 2.5(5) cm 23%	53% 20%
Comments	Optimized for high density and $p_{\parallel}$ mass	Optimized for angular resolution for missing-momentum determination
Muon spectrometer (σ_{μ}/p)	$1/(0.1/N)$ + $[(2.7 \times 10^{-2} p_{\parallel})/(N^2)]^{1/2}$	

Table 3
Results from the CDHS Collaboration

a) Raw data	NC cand	Norm.	CC cand	Norm.	R	NC cand	Norm.	CC cand	Norm.	R
Normal beam	60936 ± 252	10567 ± 103	137863 ± 371	10219 ± 101	0.4420 ± 0.0021	312 ± 14	2967 ± 55	5573 ± 75	261 ± 16	0.532 ± 0.012
Beam Cosmic- corrected	1120 ± 25	5 ± 2	9 ± 2	4 ± 2				5 ± 2	1 ± 1	
Normal data (cosmic-corrected)	59816 ± 253	10562 ± 103	137844 ± 371	10215 ± 101	0.4319 ± 0.0021	2655 ± 57	2967 ± 55	5568 ± 75	260 ± 16	0.477 ± 0.012
With beam Cosmic- corrected	1272 ± 36	212 ± 15	1645 ± 41	181 ± 14		234 ± 15	144 ± 5	144 ± 12	24 ± 5	
Beam stopper Cosmic- corrected	352 ± 15	1 ± 1	2 ± 1	1 ± 1		112 ± 8	1 ± 1	0	0	
Data with beam stopper	925 ± 39	211 ± 15	1643 ± 41	182 ± 14		122 ± 17	29 ± 5	144 ± 12	24 ± 5	
Complete data sample	56896 ± 277	9896 ± 113	132657 ± 393	9640 ± 110	0.4289 ± 0.0024	2367 ± 70	228 ± 21	5228 ± 80	203 ± 20	0.473 ± 0.015
b) Corrected sample										
NC long candidates	+159 Correction	-57 Correction	-158 Correction	-57 Correction		+7 Correction	-2 Correction	-6 Correction	-2 Correction	
CC short candidates	57055 ± 282	9830 ± 113	132899 ± 393	9583 ± 110	0.4306 ± 0.0025	2374 ± 70	226 ± 21	5222 ± 80	201 ± 20	0.455 ± 0.015
	-9642 Correction		+9526 Correction			-183 Correction		+168 Correction		
	47413 ± 303		142035 ± 437		0.3318 ± 0.0025	2191 ± 72		5390 ± 85		0.406 ± 0.015
	-3016 Correction		+2498 Correction			-103 Correction		+73 Correction		
R_{CC}	44704 ± 303		144513 ± 411		0.3072 ± 0.0025	2088 ± 73		5463 ± 81		0.382 ± 0.015

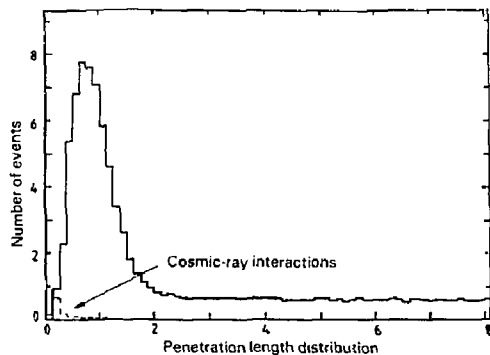


Fig. 17 Distribution of the penetration length of the events used for the separation of CC and NC events.

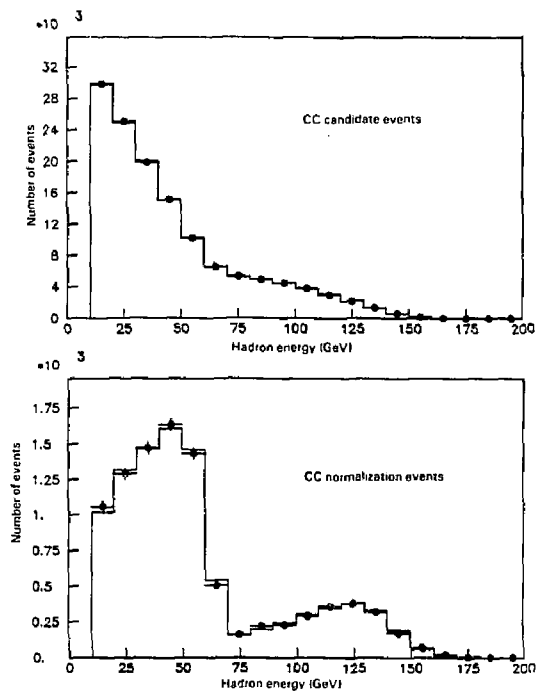


Fig. 18 Distribution of the hadronic energy for candidate CC and normalization CC events. The line shows the same distributions simulated by Monte Carlo.

The value of $\sin^2 \theta_w$ determined by R_e and r according to the Llewellyn Smith procedure is

$$\sin^2 \theta_w = 0.225 \pm 0.005 \pm 0.003 (\text{th.}) + 0.013 (m_c - 1.5) .$$

The different corrections to $\sin^2 \theta_w$ from the CDHS data are shown in Table 4 and schematically in Fig. 19.

A previous value of $\sin^2 \theta_w$ given in a 'mural publication'*) is shown in Fig. 20; it agrees quite well within experimental errors.

5.2. Results from the CHARM Collaboration

The CHARM detector, in the same CERN NBB line, has collected the sample of neutrino interactions shown in Table 5.

The separation of the CC and NC events is done on the basis of the single event, as shown in Fig. 21. The CHARM calorimeter, optimized for NC events, is lighter ($\rho_{\text{target}} = 2.7 \text{ g cm}^{-3}$, $Q_{\text{eff}} = 1 \text{ g cm}^{-3}$) and more finely grained, so the muon can be traced down to a low momentum ($p_\mu \approx 1 \text{ GeV/c}$) and the energy threshold of the hadronic shower is as low as 4 GeV. The performance of the beam and of the detector is well known, as is shown by the agreement between the simulated data and the experimental results (Fig. 22).

The NC candidate events (defined by the absence of muons (zero muon events)) contain the CC events produced by the small contamination from ν_e particles (Fig. 23, shaded region). The good beam monitoring (K/π ratio) allows the determination of the CC cross section from ν_e . From the data of Table 5, the following results are derived:

$$R_e = 0.3093 \pm 0.0031 , \\ r = 0.456 \pm 0.012 ,$$

and $\sin^2 \theta_{w, \text{exp}} = 0.2356 \pm 0.0050$. The corrections to be applied to $\sin^2 \theta_{w, \text{exp}}$ are illustrated in Table 6, and the final value is

$$\sin^2 \theta_w = 0.236 \pm 0.005 \pm 0.003 (\text{th.}) + 0.012 (m_c - 1.5) .$$

As the results from the two detectors are obtained by independent methods, they can be combined by adding the errors quadratically, giving

$$\sin^2 \theta_{w, N} = 0.230 \pm 0.003 .$$

This value of $\sin^2 \theta_w$ can be compared with the most precise value of $\sin^2 \theta_w$ derived from the measurement of m_W and m_Z at the CERN $p\bar{p}$ Collider:

$$\sin^2 \theta_w = 0.232 \pm 0.003 \pm 0.008 .$$

The global fit of all $\sin^2 \theta_w$ measurements [4] is

$$\sin^2 \theta_w = 0.231 \pm 0.003 .$$

*) Those interested may 'read' it on the wall of CERN Building 180.

Table 4
Corrections and errors on $\sin^2 \theta_w$

Parameter	$\Delta \sin^2 \theta_w$ correction and error
Mixing of quark families	$+0.0031 \pm 0.0003$
Non-isoscalarity of iron $d_V/u_V = 0.39 \pm 0.04$	-0.0090 ± 0.0009
Longitudinal structure function QCD prediction $\pm 100\%$	$+0.0006 \pm 0.0006$
Fraction of antiquarks $(\bar{u} + \bar{d})/(u + d) = 0.13 \pm 0.02$	$+0.0021 \pm 0.0003$
Fraction of strange antiquarks $\bar{s}/\bar{d} = 0.45 \pm 0.10$ for $E_h = 30 \text{ GeV}$	$+0.0046 \pm 0.0010$
Fraction of charmed sea $c/s = 0.3 \pm 0.3$	$+0.0005 \pm 0.0005$
Charm quark mass $m_c = 1.5 \pm 0.3 \text{ GeV}$	$+0.011 \pm 0.004$
Radiative corrections $m_l = 45 \text{ GeV}, m_h = 100 \text{ GeV}$	-0.011 ± 0.002
Total uncertainty for a fixed charm quark mass	± 0.003
Total correction and total systematic uncertainty	$+0.0019 \pm 0.005$

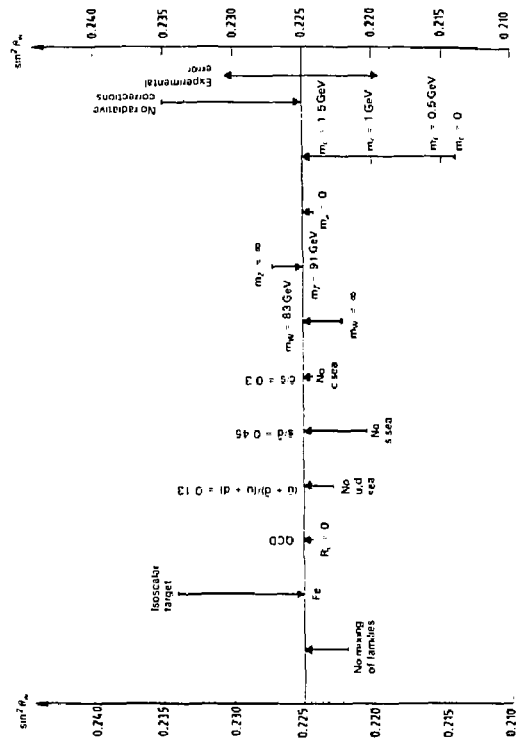


Fig. 19 Corrections to $\sin^2 \theta_w$ done on their data by the CDHS Collaboration.

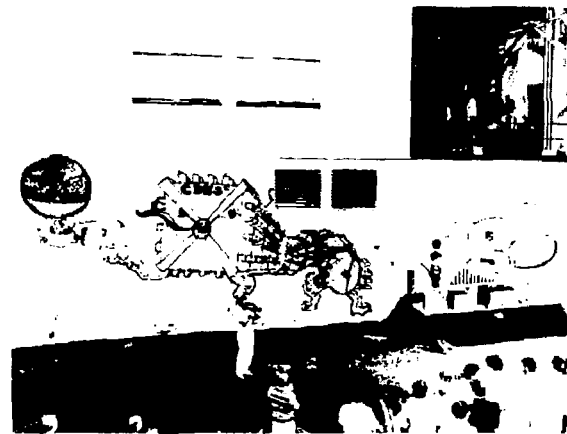


Fig. 20 Mural display of a $\sin^2 \theta_{\text{e}}$ value given by the CDHS Collaboration in September 1984

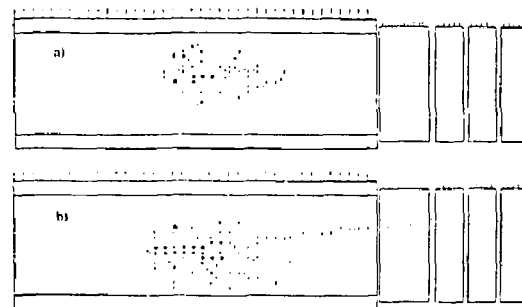


Fig. 2) Neutral-current (a) and charged-current (b) events recorded in the CHARM detector.

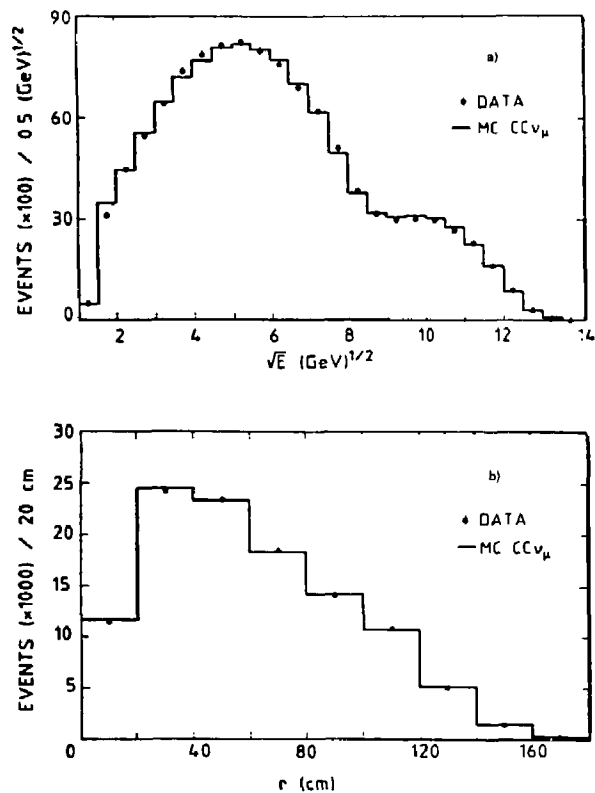


Fig. 22 a) Energy distribution of CC neutrino-induced data as measured in the CHARM detector.
b) Radial distribution of the same events.

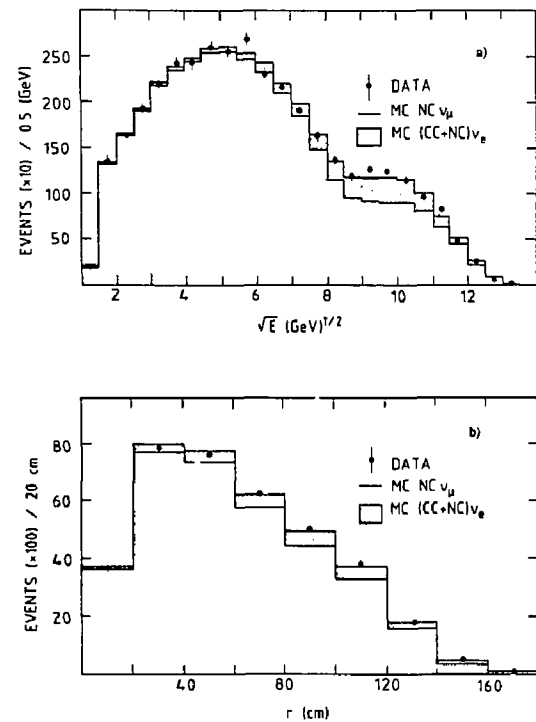


Fig. 23 Same distributions as in Fig. 22a and 22b for 0μ events. In the Monte Carlo simulation the CC and NC electron-neutrino-induced events have been separated and are shown as dashed regions.

Table 5

Neutrino event numbers ($E_\nu \geq 4$ GeV)

	Neutral current	Charged current
Uncorrected data sample	39239 $\pm$ 198	108472 $\pm$ 329
Trigger + filter efficiency	7 $\pm$ 4	0 $\pm$ 0
Scan correction	40 $\pm$ 40	60 $\pm$ 44
Cosmic and WB correction	-2310 $\pm$ 87	-4311 $\pm$ 119
Difference in energy cut	-	0 $\pm$ 129
Lost muons correction	-3737 $\pm$ 105	3735 $\pm$ 105
π and K decay	1892 $\pm$ 50	-1835 $\pm$ 50
K_{e3} charged current	-1768 $\pm$ 68	-106 $\pm$ 6
K_{e3} neutral current	-532 $\pm$ 20	-33 $\pm$ 2
Corrected event numbers	32831 $\pm$ 283	105982 $\pm$ 408

Table 6

Corrections to $\sin^2 \theta_w$ ($E_\nu \geq 4$ GeV)

Source	$\Delta \sin^2 \theta_w$	Theor. uncertainty
Muon mass	+0.0011	$\pm$ 0.0001
$W^\pm$ thresholds, F_1	+0.0005	$\pm$ 0.0005
KM mixing matrix		$\pm$ 0.0010
Strange sea at $m_c = 0$	-0.0074	$\pm$ 0.0010
Charm sea at $m_c = 0$	+0.0015	$\pm$ 0.0010
Radiative corrections	-0.0092	$\pm$ 0.0020
Total uncertainty (fixed m_c)		$\pm$ 0.0030
Charm mass ($m_c = 1.5$ GeV/ c^2)	+0.0140	
Total ($m_c = 1.5$ GeV/ c^2)	+0.0005	$\pm$ 0.0030

6. CONCLUSIONS

What have we learned from the previous precision experiments? The results of the experiments illustrated in the foregoing are from cross sections (^3He scattering), from the masses of both the $W^\pm$ and the Z^0 , or from one of these two masses and the α (Josephson effect) and G_F (muon lifetime). The value of $\sin^2 \theta_w$ is obtained in the Standard Model by computing the cross section and the masses at some perturbative level. In the Standard Model the three quantities $\sin^2 \theta_w$, m_Z , and m_W are not independent, but for the test in which we are interested it is worth treating them as if they were. The value of $\sin^2 \theta_w$ derived from the comparison between a measured cross section and one computed at a given order of perturbation expansion, can be different from that computed from α , G_F , and m_W at the same order. Only a full calculation would give equal results.

Since all calculations in the Standard Model are done by perturbation expansion, the comparison of $\sin^2 \theta_w$ extracted from different experiments has to be done in an equivalent way. This is shown in the graph (Fig. 24) from the talk given by G. Altarelli at the Berkeley Conference [5]; this illustrates that it is only after radiative correction that the $\sin^2 \theta_w$ from different processes can be brought into agreement. The $\sin^2 \theta_w$ value from the measurements quoted in Fig. 24— νN , m_W , and m_Z —has tested the Standard Model at the quantum level (higher loop), much as in the past the Lamb shift or the $g - 2$ tested QED. What is still missing is the same accuracy, but we hope to reach this level with the advent of the SLC and LEP, and the results from CHARM 2.

WHAT IS TESTED?

- 1) The tree level
- 2) The radiative corrections now important for comparing different experiments.

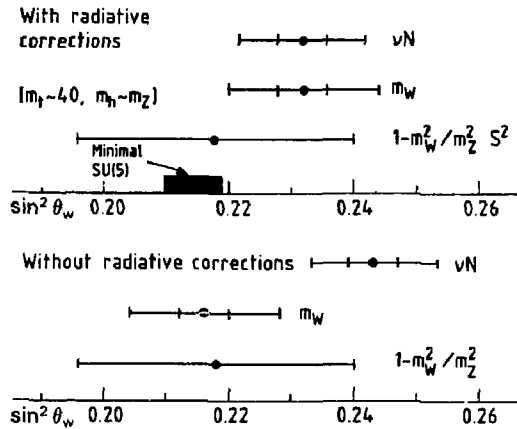


Fig. 24 Concluding remarks on $\sin^2 \theta_w$ precision measurements taken from the talk of Altarelli [5].

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Physics from PEP*

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ABSTRACT

Recent results from PEP experiments are reviewed. Included are searches for SUSY particles and measurement of the number of neutrino flavors, two-photon production of resonances pertinent to glueball spectroscopy, τ lepton decays, QCD and electroweak tests, and heavy quark lifetimes.

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A large part of the PEP physics program was completed earlier this year as the storage ring was shut down to make preparations for a major luminosity upgrade. When PEP resumes operation in 1987 in a new "microbeta" configuration the TPC/Two-Gamma collaboration will pursue a broad range of investigations exploiting large event samples and the added information from a new precision vertex chamber. The Mark II detector has been moved to the SLC ready to take beam in early 1987. The MAC, HRS, DELCO and ASI operations are complete according to present plans. The data analysis efforts of all these groups continue to be very productive.

From among the many physics contributions of the past year or so from PEP experiments I've selected the following topics for this review:

1. Searches for annihilation to weakly-interacting neutral particles (neutrino counting and SUSY search)
2. Two-photon annihilation into resonant $KK\pi$ states in the $\sqrt{s}$ region (glueball candidate)
3. τ lepton decays
4. QCD, fragmentation and hadron production
5. Electroweak tests
6. Heavy flavor lifetimes

1. Single Photon Production

The ASP and MAC groups have measured the cross section for electron-positron annihilation into final states containing no detectable particles except for a single photon that is presumed to come from bremsstrahlung off one of the initial-state electrons. The photon serves to tag otherwise invisible reactions such as production of $\nu\bar{\nu}$ pairs via a Z^0 in the s -channel or of $\tilde{\gamma}\tilde{\gamma}$ (photino) pairs via a $\tilde{e}$ (selectron) in the t -channel. The first reaction is interesting because in a picture with n generations each of which has a massless neutrino, the cross

section gives a measure of the number of generations. (Here "massless" means light compared with the overall center of mass energy ($\sqrt{s}$) so that one is well above threshold for the neutrinos of all generations.)

Evidently the optimal energy for the neutrino counting experiment would be one near the Z^0 mass, where the annihilation (after radiation of the photon) is resonant. This is clear from Fig. 1, in which the cross section for neutrino pair production is plotted as a function of $\sqrt{s}$.^[1] This experiment is prominent on the agenda of the SLC and LEP experiments. Looking at it the other way around, one needs to be well below M_Z to do a search for SUSY particle production, to avoid the neutrino pair "background". A plausible assumption is that the photinos are among the lightest SUSY particles; if their masses are comfortably below $\sqrt{s}$ the production rate will be limited by the mass of the virtual scalar electron which appears in the propagator. In Fig. 1 the photino pair cross sections are illustrated for selectron masses within the sensitivity of the PEP experiments.

To make a sensitive measurement of either process one seeks to detect the lowest energy photons possible, maximizing the phase space available for the pair production. The limitation comes principally from the background represented by radiative Bhabha events in which the electrons have such small polar angles as to go undetected. The detector must be free of dead zones where electrons might hide. The beam pipe itself is a dead zone for all detectors below some polar angle. If one computes the largest transverse energy with respect to the beam ($E_\perp$) that an electron could have and still go undetected, then by transverse momentum conservation one gets a minimum photon $E_\perp$ for which the Bhabha hypothesis can be ruled out. Other backgrounds include beam-gas production of π^0 's and rare configurations of cosmic rays. Shower tracking and shape requirements help insure that only photons produced at the interaction point (IP) are accepted; timing cuts help reject cosmic rays. The relevant detector parameters are summarized in Table 1^[2] for the PEP and PETRA detectors that have reported results on these reactions.

For MAC^[3] the last step in the event selection process is to display the

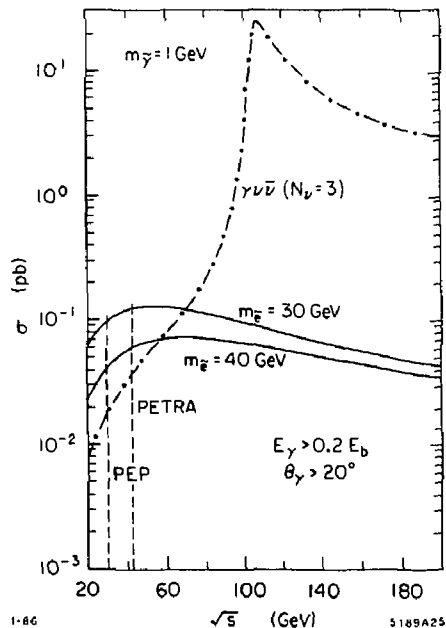


FIGURE 1. Cross sections for $e^+e^- \rightarrow \gamma\nu\bar{\nu}$ and $e^+e^- \rightarrow \gamma\gamma\gamma$ as functions of center of mass energy.

TABLE 1. Summary of single-photon search experiment parameters.

	ASP	MAC	CELLO	MARK J
$\int L dt$ (pb $^{-1}$)	115	36,80,61	37.6	27.8
$\sqrt{s}$ (GeV)	29	29	42.6	42
$E_{\perp min}$ (GeV)	1.0	4.5,2.0,2.6	$E_{\gamma} > 2.1$	6
θ_{min} (degrees)	20	40	34	37
θ_{veto} (mrad)	21	175,66,84	50	87
Δt (ns)	1.5 $\rightarrow$ 1.1	75	45 $\rightarrow$ 25	0.5
efficiency (%)	57	53-69	35	67
$N_{\gamma\nu\nu}$ expected	2.2	1.1	0.7	
$N_{observed}$	1	1	0	0

events surviving all cuts on shower quality, timing, geometric acceptance, etc., as a distribution in photon $E_{\perp}$. The results are shown in Fig. 2. One sees the residual radiative Bhabha background below the final minimum $E_{\perp}$ cuts, and the one event (in Fig. 2b) that is retained in the final sample.

The ASP experiment,^[2,4] with its very small veto angle, rejects Bhabhas for $E_{\perp}$ values as low as 600 MeV. The remaining backgrounds at this level are ones for which the final state particles do not have all of the original e^+e^- energy, such as beam-gas collisions and cosmic rays. A scatterplot is given in Fig. 3 of the photon $E_{\perp}$ vs. the impact parameter, R , of the photon track with respect to the IP in the xz or yz projection. One sees the background that is easily eliminated by a combination of timing, R and $E_{\perp}$ cuts. One event remains in the final sample.

The event yields from these experiments are summarized in the last two lines of Table 1, along with the expectations for neutrino pair production according to the standard model with three generations. The 90% confidence upper limits for N_{ν} from ASP, MAC and CELLO are 7.5, 16, and 15, respectively.

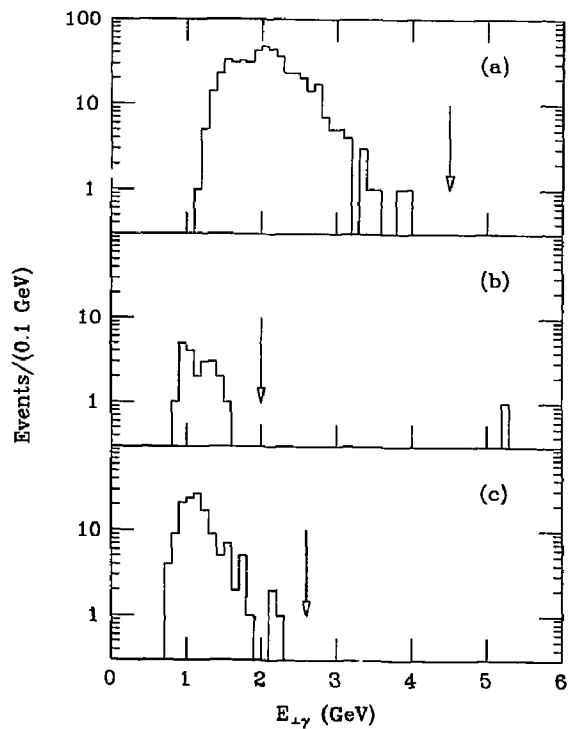


FIGURE 2. MAC single photon candidates vs. $E_{\perp}$. The three distributions represent distinct configurations of the detector, for which the minimum accepted $E_{\perp}$ values were 4.5, 2.0, and 2.6 GeV, respectively, as indicated by the arrows.

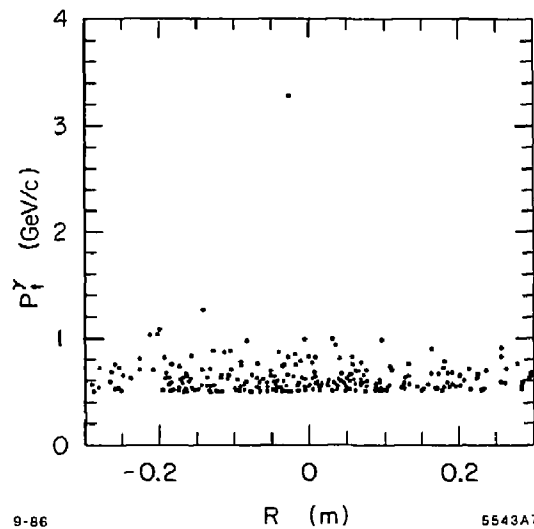


FIGURE 3. Single photon candidates from ASP plotted vs. $E_{\perp}$ (p_T^{γ}) and photon track impact parameter (R).

Taking the experiments in combination^[6] we see that altogether two events are observed when 4.0 are expected. From the plot (Fig. 4) of expected yield vs. N_ν we read the N_ν value corresponding to the 90% confidence limit of 5.3 events implied by the observed two from all experiments:

$$N_\nu < 4.9 \text{ (all } e^+e^- \text{ experiments),}$$

a rather impressive result.

Going beyond the standard model, it's clear that none of the experiments sees evidence for additional sources of single photon events. Limits are deduced for the cross sections of both photino and sneutrino pair production as functions of the propagator mass. With the assumption that the produced particle mass is small compared with $\sqrt{s}$, the 90% confidence lower limits for masses of the selectron and wino are given in Table 2. (There are differences among the groups' methods of assigning the limits in the presence of an estimated background, which slightly affect the numbers quoted.)

TABLE 2. Neutrino number upper limits and SUSY mass lower limits (90% CL) from e^+e^- experiments. The produced particles are assumed massless, and $\tilde{e}_{L,R}$ are assumed degenerate.

	ASP	MAC	CELLO	MARK J
N_ν	7.5	16.	15.	
$M_{\tilde{e}} \text{ (GeV}/c^2\text{)}$	65.7	50.	37.7	33.
$M_{\tilde{W}} \text{ (GeV}/c^2\text{)}$	60.	51.	40.	37.

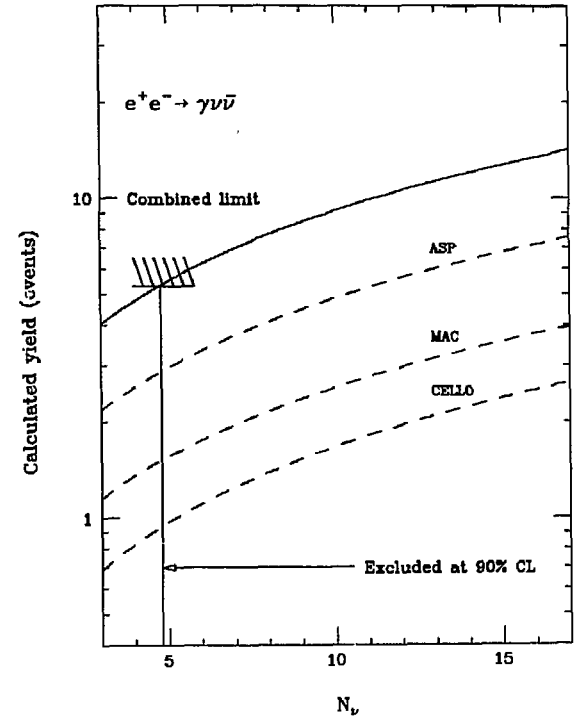


FIGURE 4. Expected event yields from the e^+e^- experiments as a function of the number of neutrino flavors.

2. Glueball Spectroscopy

The Mark II and TPC/Two-Gamma collaborations have presented contributions to the spectroscopy of states coupled to two photons with substantial decays to the $K_S^0 K^+ \pi^-$ final state, in the mass region of the ι and E mesons. The ι is an $I = 0$, $J^{PC} = 0^{-+}$ state with quantum numbers well established from J/ψ decay, and in the new naming scheme^[4] is called the $\eta(1440)$. The other state in this mass region, the E , alias $f_1(1420)$,^[4] has been seen in several hadronic experiments and has $I = 0$, but some experiments favor $J^P = 1^+$, others 0^- . By producing these states with electron-photon scattering one can hope to untangle the angular momentum, through the Q^2 -dependence, and the quark/gluon composition, through the coupling to two photons.

The two-photon production reaction is

$$e^+e^- \rightarrow e^+e^-X,$$

where X is required in these analyses to be $K_S^0 K^+ \pi^-$ (exclusively). There are two subsamples: those in which one of the very forward-going electrons in the final state is detected ("tagged" events, for which $Q^2 > 0$), and those in which both electrons are within the vacuum pipe, for which $Q^2 \approx 0$ ("untagged"). We may also view this as electron-photon scattering, since one of the photons is nearly real (both, in the untagged case). Now a spin-1 resonance has no coupling to two real photons, so the observation of a state in the tagged sample only suggests $J = 1$. Conversely, with the untagged sample one can search for production of spinless states.

The $K_S^0 K^+ \pi^-$ mass distribution for the Mark II^[1] untagged sample is shown in Fig. 5a. The only structure evident is the $\eta_c(2980)$, for which a width times branching ratio of $0.15^{+0.11}_{-0.06} \pm 0.05$ is inferred. The spinless $\eta(1440)$ would be allowed; from these data we conclude

$$\Gamma(\eta(1440) \rightarrow \gamma\gamma) < 2 \text{ keV}.$$

For the $Q^2 > 0$ case (Fig. 5b) a clear peak is seen in the $K_S^0 K^+ \pi^-$ spectrum

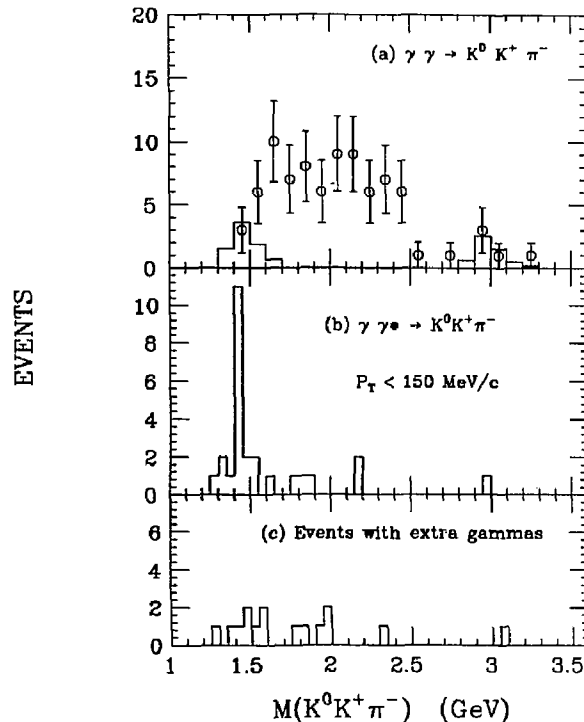


FIGURE 5. Mark II invariant mass distributions for (a) $\gamma\gamma \rightarrow K_S^0 K^+ \pi^-$ (untagged, $Q^2 = 0$), (b) $\gamma^* \gamma \rightarrow K_S^0 K^+ \pi^-$ (tagged, $Q^2 > 0$), and (c) $\gamma^* \gamma \rightarrow K_S^0 K^+ \pi^- (\pi\pi^0)$. In (a), the histograms are Monte Carlo predictions for $\eta(1440)$ and $\eta_c(2980)$.

(but not in the background spectrum of events with extra gammas, Fig. 5c). The Q^2 -dependence implied by Figs. 5a and 5b together suggests spin one for this state, perhaps the $f_1(1420)$. The Dalitz plot for the thirteen signal events in terms of $K\pi$ subenergies is consistent with the predominance of $K^*\bar{K}$, in contrast with the sideband and extra-gamma background events. A search in the $\eta\pi^+\pi^-$ mass spectrum shows no peak, further evidence that the resonance is not $\eta(1440)$, since the latter has a substantial branching ratio to this decay through the $\omega_0(980)$ intermediate state.

The Mark II observations confirm those made earlier by the TPC/Two-Gamma collaboration. The TPC/Two-Gamma untagged^[8] $K_S^0 K^+ \pi^-$ data are shown in Fig. 6a, superimposed upon the prediction for $\eta(1440)$ formation. There is no evidence of the $\eta(1440)$; the upper limits deduced are

$$\Gamma(\eta(1440) \rightarrow \gamma\gamma) \cdot B(\eta(1440) \rightarrow K\bar{K}\pi) < 1.6 \text{ keV},$$

or with the assumption of a 70% branching ratio,

$$\Gamma(\eta(1440) \rightarrow \gamma\gamma) < 2.2 \text{ keV}.$$

The tagged data,^[9] Fig. 6b reveal a clear peak which is nicely fit by the predicted shape of the $f_1(1420)$.

The TPC/Two-Gamma untagged and tagged data are combined in a plot of $B(K\bar{K}\pi)\Gamma_{\gamma\gamma}$ vs. Q^2 ^[10] under the separate assumptions of spin zero and spin one for the resonance. As expected, the upper limit quoted above for the untagged data rules out spin zero, while the data are consistent with spin one at all Q^2 .

Questions remain about the $^*_{\pi}/E^*$ region, as the hadronic experiments give so far conflicting results. Apparently there are at least two $I = 0$, $C = +1$ resonances, the $\eta(1440)$ decaying to $K\bar{K}\pi$ and $\eta\pi\pi$ (including $\omega_0\pi$ and $K^*\bar{K}$), and the $f_1(1420)$, decaying at least in part to $K\bar{K}\pi$ (including $K^*\bar{K}$).

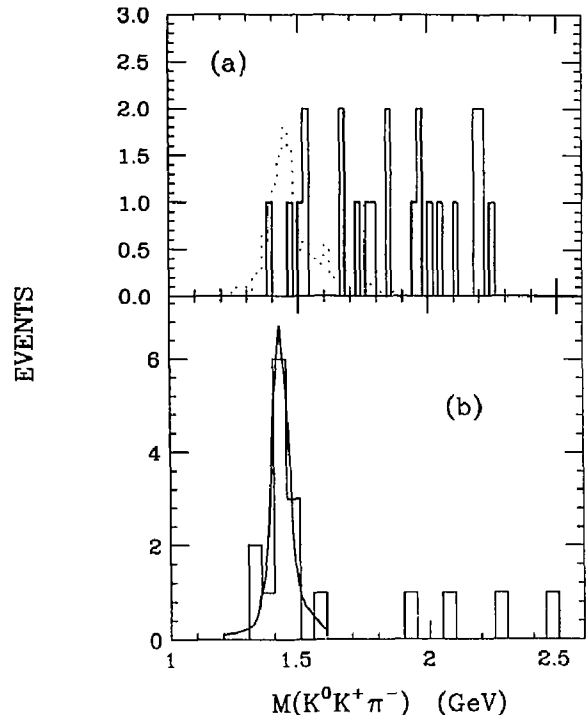


FIGURE 6. TPC/Two-Gamma $\gamma\gamma \rightarrow K_S^0 K^+ \pi^-$. (a) Untagged mass spectrum (solid histogram) compared with expectation for formation of $\eta(1440)$ (dotted histogram); (b) Tagged mass spectrum (histogram) with the prediction (smooth curve) for $f_1(1420)$.

The limits from both experiments on the $\eta(1440)$ coupling to $\gamma\gamma$ are significant when compared with what one would expect given the rate for $J/\psi \rightarrow \gamma\eta(1440)$. They constitute evidence that the $\eta(1440)$ is a glueball with little admixture of $q\bar{q}$ components. That is, glue has no direct coupling to photons. The $J/\psi \rightarrow \gamma\eta(1440)$ decay provides a normalization for the $\eta(1440)V$ coupling, where V means any vector, including the photon. The $f_1(1420)$, on the other hand, is consistent with being mainly $u\bar{u}$ and $d\bar{d}$.^[9]

More on this subject will be found in the talk by Usha Mallik in these proceedings.

3. Tau Decay Modes and Branching Fractions

The production and decay of the τ lepton represent a laboratory rich in opportunities for testing the standard model. In the last couple of years we've seen considerable refinement of the measured branching fractions, with many of the contributions coming from PEP experiments. The phenomenology has been worked out in considerable detail in several papers including the early work of Tsai^[10] and a recent survey by Gilman and Rhie.^[11]

With respect to hadronic decays, the experiments have mostly been limited to statements about the number of charged prongs and whether or not these are accompanied by any neutrals. The theorist, on the other hand, deals with specific states (π, K, ρ, a_1 , etc.), or n -pion states with specific isospin quantum numbers, which can be related through CVC, PCAC, etc., to known couplings. From the data and interpolations based upon such considerations, Gilman and Rhie^[11] found a deficit in the sum of decay rates to exclusive one-prong modes, relative to the observed inclusive one-prong rate. Mainly this was a problem of the isospin-constrained allocation of multipion states between the one- and three-prong modes. Ways out included possibly stretching the error limits a bit on some of the measurements, or invoking decays with isosinglet neutrals (e.g., η).

At last summer's conferences several new measurements confirmed a number of the branching fractions, especially for leptonic, ρ and inclusive three-prong modes, which only increased the significance of the discrepancy. The known exclusive one-prong modes added up to $78.0 \pm 2.0\%$ while the inclusive one-prong fraction was measured to be $86.8 \pm 0.3\%$. We'll see that new data show indications that one-prong multineutrals have higher than expected rates, so the picture may be coming together.

3.1 ONE-PRONG MULTINEUTRAL DECAYS

Inclusive multineutrals

From the TPC/Two-Gamma collaboration^[12] comes a new measurement of the decay mode

$$\tau^- \rightarrow \nu_\tau \pi^- \pi^0 (n h^0), \quad n \geq 1,$$

where h^0 is a π^0 or an η . Essentially the same final state, namely

$$\tau^- \rightarrow \nu_\tau \pi^- (\geq 2 h^0),$$

is extracted by the Mark II group^[13] from a global fit of events classified according to charged and neutral multiplicity and particle identification. These results and those of a constrained fit from TPC/Two-Gamma^[14] are combined in Table 3.^[15] (Decays with five or more charged tracks contribute less than 0.15%, see below.)

The results are generally consistent with the more precise previous world average values. We notice, however, that the new measurements for the modes $\pi^- (\geq 2 h^0)$ are about five percent higher than the (partly theoretical) previous number. That is, the branching fraction for this set of modes exceeds the expectation for three and four pion one-prong decays. The latter are assumed to be mainly $a_1(1270)$ (see below) and $\rho(1600)$, respectively. The question then is, what are the h^0 's? Perhaps η 's?

TABLE 3. Tau branching fractions from the Mark II and TPC constrained fits, together with the TPC π^+ multineutral measurement. All fractions are in percent.

Decay mode $\tau^- \rightarrow \nu_\tau +$	Mark II	TPC/Two-Gamma	Previous world average
all 1-prong	$86.7 \pm 1.0 \pm 0.7$	$84.5 \pm 0.8 \pm 1.0$	86.8 ± 0.3
$e^- \bar{\nu}_e$	$19.1 \pm 0.8 \pm 1.1$	$18.0 \pm 1.2^{+1.9}_{-1.6}$	17.9 ± 0.4
$\mu^- \bar{\nu}_\mu$	$18.3 \pm 0.9 \pm 0.8$	$18.1 \pm 1.2^{+0.5}_{-0.4}$	17.2 ± 0.4
$\pi^- (\geq 0h^0)$	$47.8 \pm 1.6 \pm 1.3$	$46.8 \pm 1.2^{+0.5}_{-1.0}$	$41.1 \pm 1.9^*$
π^-	$10.0 \pm 1.1 \pm 1.4$		10.9 ± 1.4
ρ^-	$25.8 \pm 1.7 \pm 2.5$		22.1 ± 1.2
$\pi^- (\geq 2h^0)$	$12.0 \pm 1.4 \pm 2.5$	$13.9 \pm 2.0^{+1.9}_{-2.1}$	$8.1 \pm 0.4^*$
all 3-prong	$13.3 \pm 1.0 \pm 0.7$	$15.3 \pm 0.8 \pm 1.0$	13.1 ± 0.3

*Theoretical (isospin and CVC).

$\tau^- \rightarrow \nu_\tau \pi^- \eta X$

The $\eta^0(550)$ has been searched for by Crystal Ball¹ and by HRS. The Crystal Ball group see an η peak in the $\gamma\gamma$ mass distribution, but have not extracted a branching ratio so far. The HRS data are shown in Fig. 7, the $\gamma\gamma$ mass distribution for 1-1 and 1-3 topologies and for the combined sample.^{11(c)} The η peak region is estimated to contain 43 signal and 60 background events, leading to the branching fraction

$$B(\tau^- \rightarrow \nu_\tau \pi^- \eta X^0) = (5.0 \pm 1.0 \pm 1.5)\%.$$

This would account for the observed branching fraction for $\pi^- (\geq 2h^0)$. The $\pi^- \eta (J^{PG} = 1^{--})$ mode is forbidden for a first class current, so this rather large $\pi^- \eta$ inclusive rate, if correct, must reflect higher-multiplicity modes such as $\eta 2\pi$ and $\eta \eta \pi$. This is a bit surprising in view of the limited phase space for these modes.

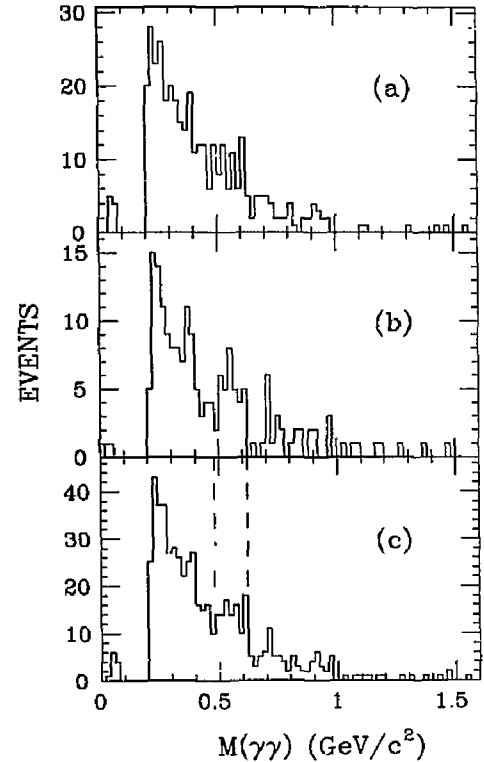


FIGURE 7. HRS inclusive $\gamma\gamma$ mass distributions for τ pairs. (a) 1-1 prong topology; (b) 1-3 topology; (c) combined. Dashed lines indicate the η region.

3.2 $\tau^- \rightarrow \nu_\tau \pi^- \omega$

The ARGUS and HRS groups have studied the decay $\tau^- \rightarrow \nu_\tau \pi^- \pi^0 \pi^+$. A clear ω signal is seen in the $\pi^- \pi^0 \pi^+$ system, while the $\omega \pi^-$ spectrum shows no evidence of structure. A spin-parity analysis (ARGUS) establishes $J^P = 1^-$ for the $\omega \pi$ system, consistent with the absence of second-class currents, which would be required for either of the other two possibilities, $J^P = 1^{++}$ or 0^{++} . The two measurements for the branching fraction are

$$B(\tau^- \rightarrow \nu_\tau \pi^- \omega) = (1.5 \pm 0.4)\% \quad (\text{ARGUS}), \text{ and} \\ \simeq 1.7\% \quad (\text{HRS}).$$

These are almost all three-prong decays.

3.3 $\tau^- \rightarrow \nu_\tau \pi^- \pi^- \pi^+$

The decay $\tau^- \rightarrow \nu_\tau \pi^- \pi^- \pi^+$ is dominated by the $a_1(1270)$, which decays in turn through $\rho \pi$. Both Mark II and ARGUS have performed spin-parity analyses of the $\rho \pi$ system and find it to be predominantly $J^P = 1^+$, as expected for a_1 . The mass and width are determined by these groups and by MAC and DELCO. The results are summarized in Table 4. The formula used by the PEP groups to fit the mass spectrum is given by

$$\frac{dN}{dm^2} = m^n (m_r^2 - m^2)^2 (m_r^2 + 2m^2) \frac{\Gamma(m)}{(m_r^2 - m^2)^2 + m_r^2 \Gamma(m)^2}.$$

The power n as chosen by the different groups differs by a range of four. This accounts for much of the lack of agreement among the measurements.

TABLE 4. Mass and width measurements for the $3\pi(a_1)$ system in $\tau^- \rightarrow \nu_\tau \pi^- \pi^- \pi^+$. The quantity n in the second column is the power of m in the mass formula.

Experiment	n	m_a (MeV/c ²)	Γ_a (MeV/c ²)	Branching fraction (%)
DELCO	+2	1056 ± 30	476 ± 140	5.0 ± 1.0
MARK II	-1	1194 ± 20	462 ± 70	7.8 ± 0.9
ARGUS		1046 ± 11	521 ± 27	5.6 ± 0.7
MAC	-2	1169 ± 19	411 ± 76	7.8 ± 0.8
PDG ⁽⁶⁾		1275 ± 28	316 ± 45	

3.4 EXCLUSIVE AND RARE MODES, LIFETIME

I mention briefly here several other recent τ decay measurements from PEP experiments. The new lifetime results from HRS and MAC, discussed in the last part of this talk, may be combined with earlier data to give the new world average

$$\tau(\tau) = 0.286 \pm 0.014 \text{ ps.}$$

This combined with the calculated electronic decay width gives an independent value for the electronic branching fraction:

$$B(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e) = (17.9 \pm 0.9)\%,$$

in excellent agreement with the value in Table 3 (see also Table 6 below). The direct branching fraction measurements are still more precise than the lifetime by about a factor of two.

The HRS group^[11] have now isolated 13 five-prong events, of which six include a π^0 , and have set a limit on the fraction of seven-prong decays. A significant improvement in precision for the $\nu_\tau \pi^-$ branching fraction has been

achieved by MAC^[18]. The TPC/Two-Gamma group^[14] have measured several modes containing kaons. These results are summarized in Table 5.

TABLE 5. Some recent τ branching fraction measurements.

Decay mode $\tau^- \rightarrow \nu_\tau +$	Experiment	Branching fraction (%)	Previous world average
π^-	MAC	$10.6 \pm 0.4 \pm 0.8$	10.9 ± 1.4
$K^- (\geq 0\pi^0)$	TPC	$1.6 \pm 0.4 \pm 0.2$	1.5 ± 0.4
$K^- (\geq 1\pi^0)$	TPC	$1.2 \pm 0.5^{+2}_{-4}$	0.8 ± 0.3
$K^{*-} (\geq 0\pi^0)$	TPC	$1.4 \pm 0.9 \pm 0.3$	1.4 ± 0.3
$3\pi^- 2\pi^+$	HRS	0.051 ± 0.020	0.07 ± 0.03
$3\pi^- 2\pi^+ \pi^0$	HRS	0.051 ± 0.022	0.07 ± 0.03
$4\pi^- 3\pi^+ n\gamma$	HRS	< 0.038 (90% CL)	

3.5 SUMMARY OF τ BRANCHING FRACTIONS

The world average values of the branching fractions including the new measurements are summarized in Table 6. The last two rows indicate that the tally of exclusive modes for each topology now differ from the corresponding inclusive topological branching fractions by only about 1.5 standard deviations.

TABLE 6. Summary of world average branching fraction values.

Decay mode $\tau^- \rightarrow \nu_\tau +$	Branching fraction	
	1-prong	3-prong
$e^- \bar{\nu}_e$	17.9 ± 0.4	
$\mu^- \bar{\nu}_\mu$	17.5 ± 0.3	
π^-	10.9 ± 0.6	
K^-	0.7 ± 0.2	
ρ^-	22.1 ± 1.1	
K^{*-}	1.1 ± 0.3	0.3 ± 0.1
$\pi^- (\geq 2\pi^0)$	13.0 ± 2.0	
$3\pi^\pm$		6.4 ± 0.4
$3\pi^\pm \pi^0$		5.2 ± 0.5
Total	83.2 ± 2.4	11.9 ± 0.7
Inclusive n-prong	86.8 ± 0.3	13.1 ± 0.3

4. Jets and Inclusive Hadron Production

A number of studies of inclusive hadron production have been made recently at PEP. I would like to discuss two of these.

4.1 COMPARISON OF $q\bar{q}g$ AND $q\bar{q}\gamma$ EVENTS

The TPC/Two-Gamma^[19] and Mark II^[20] groups have studied the angular flow of hadrons in three-jet events and in events containing two hadron jets plus one isolated gamma ray. The measurement tests a QCD prediction^[21] of destructive color field interference in the region between the quark and antiquark for events with a hard gluon. The idea is illustrated in Fig. 8, which shows a polar plot of the particle flow. Planar events are aligned according to the quark jet directions. Balancing these is either a gluon jet or a photon. The QCD interference effect depletes the region between quark and antiquark only for the $q\bar{q}g$ events, while the $q\bar{q}\gamma$ events serve as the control.

A cluster algorithm is used to sort multihadron events into jets. The label quark is assigned to the most energetic jet, antiquark to the next. The comparison is made between planar three-jet and two-jet-gamma events of similar topologies. A polar plot for the TPC/Two-Gamma data, Fig. 9, shows clearly the three lobes corresponding to the $q\bar{q}g$ hadron jets, the asymmetric two lobes for the $q\bar{q}\gamma$ events, and the difference between the two classes of events in the region between about 40° and 130° . The same quantity for the Mark II data appears in Fig. 10, with the predictions of the Lund QCD plus string fragmentation Monte Carlo curve pretty well hidden by the data points. Also shown is the Ali independent-jet prediction, which disagrees with the $q\bar{q}g$ data, as expected.

Finally we show in Fig. 11 the ratio of particle yields for $q\bar{q}g$ and $q\bar{q}\gamma$ for both experiments as functions of azimuth between the q and $\bar{q}$ jets normalized to the azimuthal separation of the two jets. This ratio directly demonstrates the destructive interference for the $q\bar{q}g$ events. Note that the same qualitative features are expected from the string fragmentation picture: in $q\bar{q}g$ events the

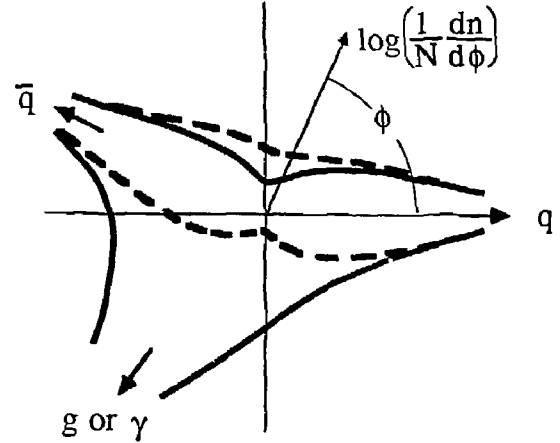


FIGURE 8. Expected particle flow for $q\bar{q}g$ (solid curves) and $q\bar{q}\gamma$ (dashed curves) with respect to the indicated quark, antiquark and gauge boson directions.

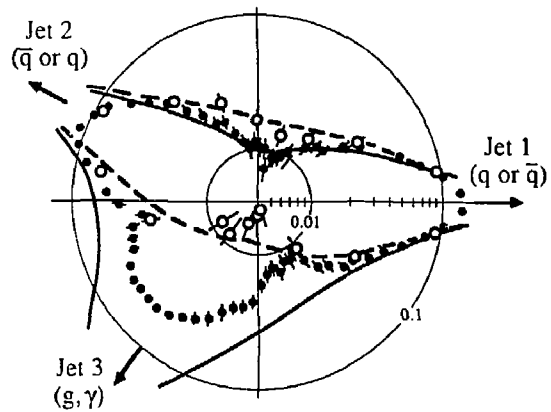


FIGURE 9. Log-polar plot of $\frac{1}{N} \frac{dN}{d\phi}$ for the TPC $q\bar{q}g$ (solid points) and $q\bar{q}\gamma$ data.

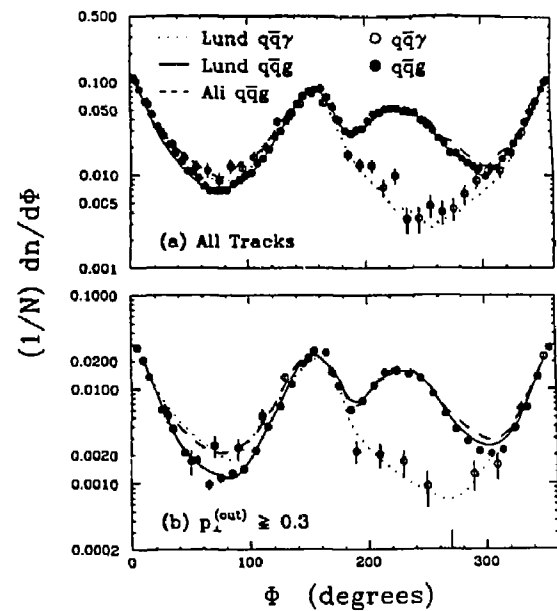


FIGURE 10. Particle flow vs. azimuth for the Mark II data. In (a), all tracks are included; in (b), only those tracks with transverse momentum with respect to the event plane exceeding 300 MeV/c are included.

color lines connect each quark with the gluon, while in $q\bar{q}\gamma$ events a single color line connects the quarks. The soft hadrons arise from breaking of the string and so populate the corresponding azimuthal regions. The two descriptions agree quantitatively to lowest order in $1/N_{\text{colors}}$.

4.2 HADRON PRODUCTION IN e^+e^- AND pp COLLISIONS

The TPC/Two-Gamma group^[29] have made a comparison of the transverse momentum ($p_\perp$) distributions of light hadrons with those found in pp collisions at the ISR,^[23] in the central region of rapidity (y) where the spectator quarks in pp should have little effect. The dE/dx measurements of the particle tracks in the TPC facilitate identification of pions, kaons and protons over most of the momentum range up to about 2 GeV/c. The results are shown in Fig. 12. We find that the two production reactions are quite similar. That the pp yields fall rather faster than those from e^+e^- is not unexpected, and may be attributed to the contribution in pp of gluon-quark and gluon-gluon scattering at high $p_\perp$, with dependence as $p_\perp^{-4}$, vs. the hard-gluon ($p_\perp^{-2}$ -dependence) in e^+e^- .

The ratios of particle flavors vs. $p_\perp$ are shown in Fig. 13. There is good agreement between the two reactions, suggesting that similar mechanisms control central-region particle production.

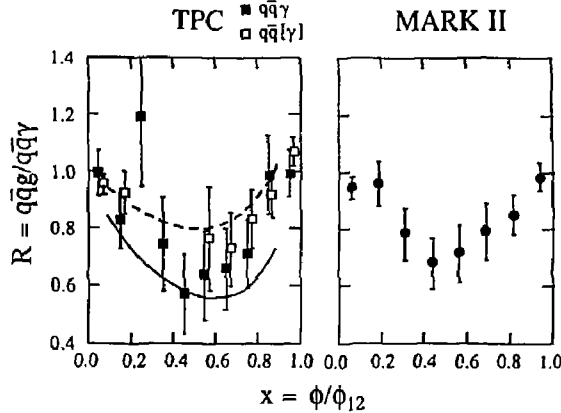


FIGURE 11. Ratio of $q\bar{q}g$ to $q\bar{q}\gamma$ particle flows for the TPC/Two-Gamma and Mark II measurements. Open symbols in the TPC data represent events in which the gamma is inferred from kinematics rather than detected directly. The solid curve is the QCD prediction if it is assumed the gluon jet is correctly identified in each event. The dashed curve gives the prediction with allowance for gluon jet misassignment.

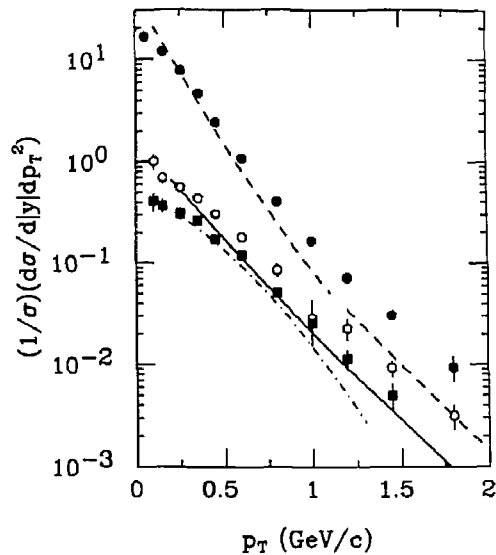


FIGURE 12. Particle yields from e^+e^- at $\sqrt{s} = 29$ GeV, $|y| < 1$ (TPC, symbols), and from pp at $\sqrt{s} = 53$ GeV, $y \approx 0$ (Ref. 23, curves), for π^- (solid circles and dashed curve), K^- (open circles and solid curve), and $\bar{p}$ (solid squares and dot-dashed curve). Typical uncertainties in the pp data are 10%.

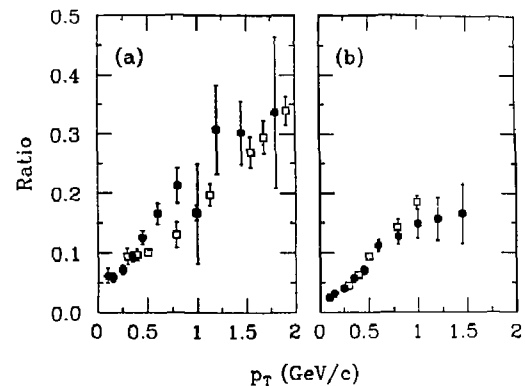


FIGURE 13. Ratio of (a) K^- to π^- and (b) $\bar{p}$ to π^- production vs $p_{\perp}$. The solid circles and open squares represent e^+e^- and pp reactions, respectively.

5. Electroweak Measurements

Some very good measurements of the forward-backward asymmetry due to the interference between photon and Z^0 diagrams in lepton pair production have been reported by PEP experiments before this year. Here I'd like to discuss some recent measurements of the effects in the light quark forward-backward asymmetry and in the τ lepton polarization asymmetry.

5.1 INCLUSIVE $A\bar{A}$ ASYMMETRY

The HRS group^[24] have reconstructed several hundred $\Lambda \rightarrow \pi p$ decays in multihadron events. The forward and backward region πp mass distributions are separately fit to a peak plus background. The background fraction decreases with $z \equiv 2E_\Lambda/\sqrt{s}$, so the net observed asymmetry increases in magnitude, as we see in Fig. 14 which shows the acceptance-corrected asymmetry vs. $z_{\min}$. The data agree within statistics with the prediction from the Lund Monte Carlo, also shown in Fig. 14. The best trade-off between statistical precision and sample purity is achieved with the requirement $z > 0.3$, which gives the value

$$A = (-23 \pm 8 \pm 2)\%.$$

By way of comparison, the asymmetry calculated with the assumption that light quarks do not contribute is $A = -3\%$, which would be inconsistent with the measurement. The conclusion is that the electroweak effects in light quark production have been observed in this experiment.

5.2 INCLUSIVE LIGHT QUARK CHARGE ASYMMETRY

The MAC group^[25] have made a measurement of the net $q\bar{q}$ forward-backward asymmetry by studying the net charge of the jets in multihadron events. For events passing cuts to minimize hard gluon radiation the jet charge

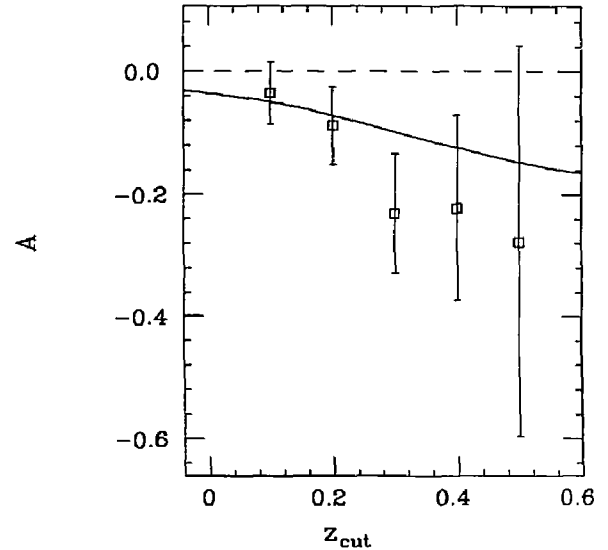


FIGURE 14. HRS $A\bar{A}$ asymmetry vs. $z_{\min}$. The curve is a prediction from the Lund Monte Carlo.

is defined as

$$Q_{jet} = \sum_i \eta_i^\kappa Q_i,$$

where η_i is the rapidity of charged particle i and the exponent κ is chosen to maximize the efficiency for seeing oppositely-charged jets. With $\kappa = 0.2$, 49,402 of 80,380 events are retained as oppositely-charged jet pairs. The expected asymmetry contributions by quark flavor are (see next section)

$$\begin{aligned} A_{uu} &= A_{cc} = +0.09, \\ A_{dd} &= A_{ss} = -0.18 \quad \text{and} \\ A_{bb} &= -0.16. \end{aligned}$$

The net asymmetry expected at the quark level is $\langle A \rangle_{parton} = +0.018$. When the dilution of parton charge information caused by fragmentation is accounted for, the prediction becomes $\langle A \rangle_{jet} = +0.022$. The measured asymmetry is extracted from the polar angular distribution, shown in Fig. 15. The fit gives

$$A = +0.028 \pm 0.005,$$

where the error quoted is statistical.

The systematic errors are established from studies of the sensitivity of the charge retention efficiency to the parameters of the Lund and Webber Monte Carlo calculations. The dominant uncertainty comes from the fraction of strange quarks in the soft quark pair sea. One check is provided by the results of a similar measurement made previously^[24] of the jet charge asymmetry for events with two hadron jets and a hard gamma ray. In that case the asymmetry expected from QED is large, and the measured value was in good agreement with that prediction.

The asymmetry is proportional to the "average" axial-vector coupling of

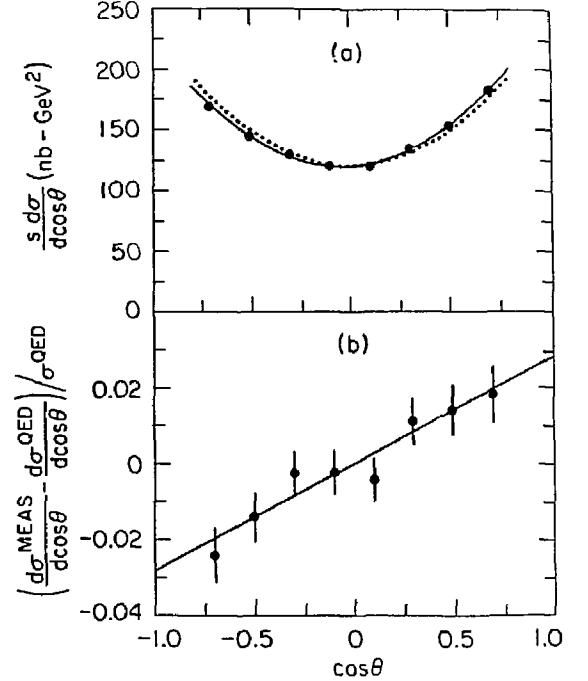


FIGURE 15. MAC signed jet pair angular distribution. (a) Observed cross section compared with pure QED (dotted curve), and best-fit (solid curve) predictions. (b) Normalized excess over the QED Born cross section, with the best-fit curve.

the quarks to the Z^0 : we assume the relation

$$g_A^d = g_B^d = g_A^b = -g_A^u = g_A^c \equiv \langle g_A^q \rangle,$$

and find

$$g_A^c \langle g_A^q \rangle = -0.34 \pm 0.06 \pm 0.05,$$

in satisfactory agreement with the expected value $-\frac{1}{2} \cdot \frac{1}{2} = -0.25$.

5.3 τ POLARIZATION

The MAC group^[27] have made a measurement of the vector neutral current coupling of the τ from an analysis of the polarization of the produced taus. To put this problem in context, let us first recall the τ pair differential cross section to order αG :

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s} [(1 - r g_V^e g_V^\tau)(1 + \cos^2\theta) - 2r g_A^e g_A^\tau \cos\theta],$$

where

$$r = \frac{G}{\pi\sqrt{2}\alpha} \frac{s}{1 - s/M_Z^2}.$$

The quantity $r \simeq 0.3$ at the PEP energy of 29 GeV. The couplings in the standard model are

$$g_A = T_3 = -\frac{1}{2}$$

for all of the charged leptons, and

$$g_V = T_3 - 2\sin^2\theta_W Q = -\frac{1}{2}(1 - 4\sin^2\theta_W) \simeq -0.04.$$

Here T and Q are the weak isospin and electric charge, respectively. Define the ratio R of total cross section to the QED value, the average τ polarization $\langle P \rangle$, the forward-backward asymmetry A_{FB} and the forward-backward asymmetry

of the polarization A_P . These quantities are given in terms of standard model parameters in Table 7. A measurement of all four observables determines the full set of couplings. The near vanishing of the vector couplings consequent to $\sin^2\theta_W$ being close to $\frac{1}{4}$ means that all the observables are small except for A_{FB} ; we may say that the scale of the weak effect in this reaction is set by $\tau g_A^2 \simeq 0.08$ at this energy.

TABLE 7. Electroweak interference observables.

Observable	Standard model prediction	
$R - 1$	$-r g_V^e g_V^\tau$	-0.001
$\langle P \rangle$	$r g_V^e g_A^\tau$	0.01
A_{FB}	$-\frac{2}{3} r g_A^e g_A^\tau$	-0.06
A_P	$\frac{3}{4} r g_A^e g_V^\tau$	0.0075

The τ decay most sensitive to the polarization, and the simplest to analyze, is the decay to a ν_τ plus a (pseudo)scalar, e.g., the pion. Because of fermion helicity conservation and the pure $V - A$ charged current, the neutrino direction of flight tends to be opposite the initial τ spin. In terms of the pion center of mass decay angle θ' , the decay rate is proportional to $1 + P \cos\theta'$. This translates into a linear dependence of the rate upon the pion laboratory momentum. Thus the mean value of p_π reflects the polarization, with analyzing power $\frac{1}{2}$. The pion branching fraction is about 0.1, so the statistical precision of the measurement is a factor $(\frac{1}{2}\sqrt{2} \times 0.1)^{-1} \simeq 7$ worse than that of a measurement of A_{FB} , ignoring efficiency corrections, for the same number of produced tau pairs. The results for the τ polarization and its asymmetry are

$$\langle P \rangle = -0.02 \pm 0.07 \pm 0.11,$$

$$A_P = 0.06 \pm 0.08.$$

The latter leads to

$$g_V^e = (-0.52 \pm 0.62) \cdot (1 \pm 0.012),$$

assuming the standard value for the electron's axial coupling. The second factor gives the small multiplicative systematic error.

6. Lifetime Measurements

There are new measurements or updates of the lifetimes of the τ lepton, charm mesons and bottom hadrons from HRS, DELCO and MAC.

6.1 HRS LIFETIME MEASUREMENTS

The HRS group added a vertex detector in 1983 which has enabled them to measure lifetimes of the τ , D^0 , D^+ and D_s^+ ($= F$). The vertex detector contains two double cylindrical layers of mylar-wall drift tubes, a new cell design that has been adopted by several other groups. Point measurement errors around $100 \mu\text{m}$ are achieved.

τ lifetime

Tau pairs of the 1-3 prong topology are selected and the vertex formed by the three-prong decay is reconstructed.^[34] The distance of this vertex from the interaction point, projected into the xy plane and onto the line of flight of the three charged particle system gives the projected decay length. The decay length itself is obtained by scaling by $1/\sin\theta$. The average decay length is about $700 \mu\text{m}$, while the typical error for one event is about $800 \mu\text{m}$. The sample contains 626 events, yielding the result

$$\tau(\tau) = (2.8 \pm 0.2 \pm 0.2) \times 10^{-13} \text{ s}.$$

D^0 lifetime

Events containing D^0 s are selected with the familiar technique that exploits the special kinematics of the decay chain

$$\begin{array}{c} D^{*+} \rightarrow D^0 \pi^+ \\ \quad \downarrow \\ \quad K^- \pi^+ \end{array}$$

The difference in mass between the $K\pi\pi$ and $K\pi$ systems is sharply peaked at the $D^* - D$ mass difference, as seen in Fig. 16a for the HRS data.^[35] The energy fraction z of the $K\pi\pi$ system is required to exceed 0.4 and the angle θ^* between the K^- and D^0 directions in the D^0 rest frame to satisfy $|\cos\theta^*| \leq 0.9$, to reduce the light-quark background. For the events in the peak the $K\pi$ mass spectrum is shown in Fig. 16b, where the clear D^0 peak stands above a small background. The peak contains 140 events. After vertex quality cuts the final sample of 55 events, including 3% background, are displayed as a distribution of proper lifetimes in Fig. 16c. The lifetime for each event is inferred from the projected distance between the IP and the $K\pi$ vertex, as in the τ lifetime analysis. The mean lifetime, from a maximum-likelihood fit, is

$$\tau(D^0) = (4.2 \pm 0.9 \pm 0.6) \times 10^{-13} \text{ s}.$$

D^+ lifetime

The $D^+\pi^-$ decay of the D^{*0} , analogous to the process discussed above, is kinematically forbidden because of slight differences in mass between the different charge states of the D^* and D . Because of the excellent mass resolution of the HRS detector, it is possible to find the D^+ peak over the combinatoric background in the $K^-\pi^+\pi^+$ mass spectrum, Fig. 17a.^[36] The signal region extends from 1.83 to 1.89 GeV/c^2 ; sidebands 1.70 to 1.80 and 1.92 to 2.02 GeV/c^2 are used to study the background. The signal region contains 114 D^+ and 390 background combinations. The lifetime distributions for the peak and sideband regions are shown in Fig. 17b and 17c, respectively. The

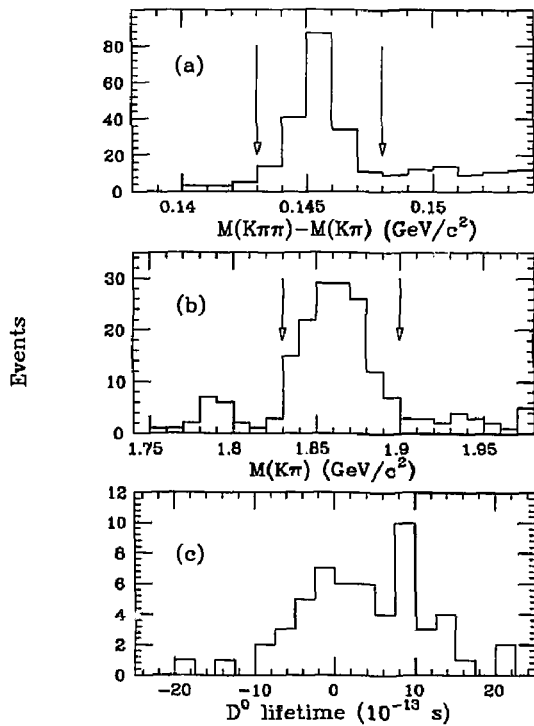


FIGURE 16. HRS D^0 lifetime sample. (a) $M(K\pi\pi) - M(K\pi)$ for $D^* \rightarrow D$ candidates. Arrows indicate the cuts defining the peak region. (b) $K\pi$ mass spectrum for events in the D^* peak of (a). (c) Proper lifetime distribution for candidates in the D peak of (b).

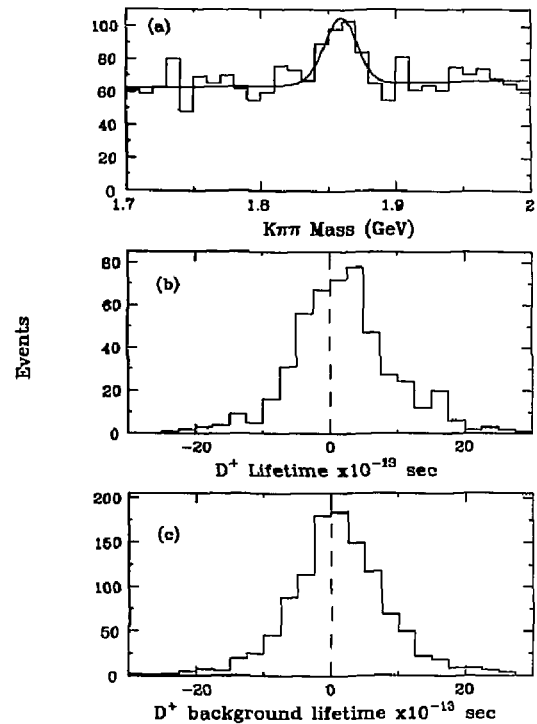


FIGURE 17. HRS D^+ lifetime sample. (a) $K^-\pi^+\pi^+$ mass spectrum. (b, c) Proper lifetime distributions for the peak and sideband regions, respectively.

width of the background distribution is dominated by the resolution; we find $\sigma \simeq 6 \times 10^{-13}$ s. The mean value of 1.25×10^{-13} s from the sideband distribution is used to estimate the background contribution to the distribution for the peak region and its systematic uncertainty. From the result of the likelihood fit and the estimated systematic error, the lifetime is found to be

$$\tau(D^+) = (8.1^{+1.2}_{-1.1} \pm 1.6) \times 10^{-13} \text{ s.}$$

D_S lifetime

The HRS group recently published^[34] a measurement of the D_S^+ lifetime based upon a sample of 17 events containing decays $D_S^+ \rightarrow \phi\pi^+$. The sample has since been enlarged to 30 events, of which an estimated six are background. The analysis is similar to that just described for the D mesons, with the result

$$\tau(D_S^+) = (3.5^{+2.4}_{-1.3} \pm 1.0) \times 10^{-13} \text{ s.}$$

6.2 DELCO b LIFETIME MEASUREMENT

The DELCO group have refined their measurement of the lifetime of bottom hadrons.^[35] The Čerenkov counters and electromagnetic calorimeters are used to identify electrons with momenta as low as 1 GeV/c. The b -enriched sample of electrons with momentum transverse to the thrust axis ($p_\perp$) greater than 1 GeV/c contains 113 measured tracks, of which 70% are direct $b \rightarrow e$ decays, 9% are $b \rightarrow c \rightarrow e$ cascades, 17% are from $c \rightarrow e$ and 4% are background. The signed impact parameter (δ) distribution for these events is shown in Fig. 18. The mean impact parameter is $259 \pm 49 \mu\text{m}$, where here the statistical error is quoted. A maximum-likelihood fit gives the result

$$\tau(b) = 1.17^{+0.27}_{-0.22} \pm 0.17 \text{ ps.}$$

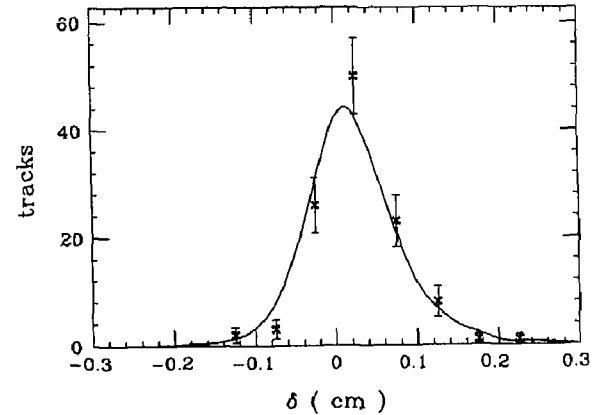


FIGURE 18. DELCO impact parameter distribution for electrons from b -hadron decays. The smooth curve is the result of a maximum-likelihood fit to the data.

6.3 MAC τ AND b LIFETIME MEASUREMENTS

The MAC group installed a precision vertex chamber (VC) in 1984, following the HRS "straw" design, but with six layers operated at four atmospheres for improved resolution. The tracking layers come unusually close to the beam line, the radius of the innermost layer being 4.6 cm. The point measurement resolution is about $50\mu\text{m}$.

τ lifetime

A new measurement of the τ lifetime^[20] based upon data recorded with the vertex chamber was made by the impact parameter method. Tau pairs of both 1-1 and 1-3 topologies yielded 8553 tracks passing the tracking quality cuts. The estimated background of multihadron and two-photon events is 4%. The signed impact parameter distribution is shown in Fig. 19. The trimmed mean of this distribution is $51.0 \pm 2.9\mu\text{m}$, from which the lifetime is found to be

$$\tau(\tau) = (2.86 \pm 0.17 \pm 0.13) \times 10^{-13} \text{ s.}$$

b lifetime

Data taken both before and after addition of the vertex chamber were analyzed with a refined impact parameter technique. As in the previous MAC measurements,^[9,11] multihadron events are selected that contain muons and electrons of momentum greater than 2 GeV/c and $p_{\perp(\text{thruster})} > 1.5 \text{ GeV}/c$. The sources of these leptons are 70% bottom decays (including cascades through charm), 16% charm decays and 14% light-quark background. There are 562 events, including 152 taken with the vertex chamber.

The track extrapolation resolution with the vertex chamber is good enough that the impact parameter measurement is limited by the beam size. To reduce this uncertainty and improve the statistical precision, all "quality" tracks in the event were used for the measurement ($p > 0.5 \text{ GeV}$ and sufficient tracking

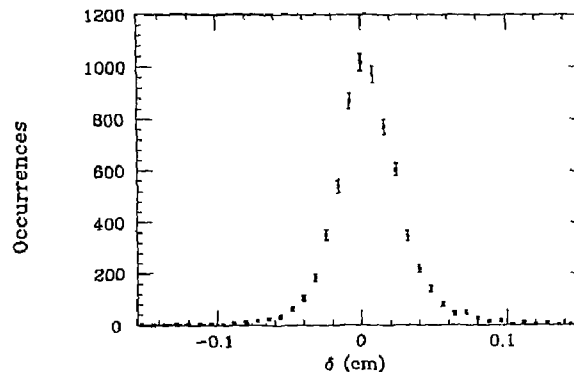


FIGURE 19. MAC impact parameter distribution for charged tracks from τ decays.

chamber measurements). About four tracks per event met the criteria. Each of these was used for a measurement of δ referenced to a point determined from fitting the remaining tracks together with the beam envelope. The results are summarized in Fig. 20. The top two pairs of distributions from a control sample illustrate the improvement in resolution that comes from the use of tracks in the event to help determine the production point. The bottom pair are for the b decay samples, which contain 1558 tracks measured before the vertex chamber was added and 441 tracks measured with the VC.

A Monte Carlo calculation provides the conversion from mean impact parameter to lifetime. The result for all the MAC data together is

$$\tau(b) = (1.16 \pm 0.16 \pm 0.07) \cdot (1.00 \pm 0.15) \text{ps.}$$

6.4 SUMMARY OF LIFETIME RESULTS

The measurements described in this chapter are collected in Table 8. The previous world averages are included for comparison.

TABLE 8. New lifetime measurements reported by PEP experiments.

Particle	1986	lifetime measurement	Previous world average*
τ	HRS	$2.8 \pm 0.2 \pm 0.2$	$2.90 \pm 0.17 \times 10^{-13} \text{ s}$
	MAC	$2.86 \pm 0.17 \pm 0.13$	$\times 10^{-13} \text{ s}$
D^0	HRS	$4.2 \pm 0.9 \pm 0.6$	$4.35 \pm 0.32 \times 10^{-13} \text{ s}$
D^+	HRS	$8.1^{+1.2}_{-1.1} \pm 1.6$	$9.01 \pm 0.75 \times 10^{-13} \text{ s}$
D_s^+	HRS	$3.5^{+2.4}_{-1.8} \pm 1.0$	$2.9^{+1.0(24)}_{-0.9} \times 10^{-13} \text{ s}$
all b	DELCO	$1.17^{+0.27}_{-0.22} \pm 0.17$	$1.26 \pm 0.16 \text{ ps}$
	MAC	$(1.16 \pm 0.16 \pm 0.07) \cdot (1.00 \pm 0.15)$	ps

*from Ref. 35, except as noted.

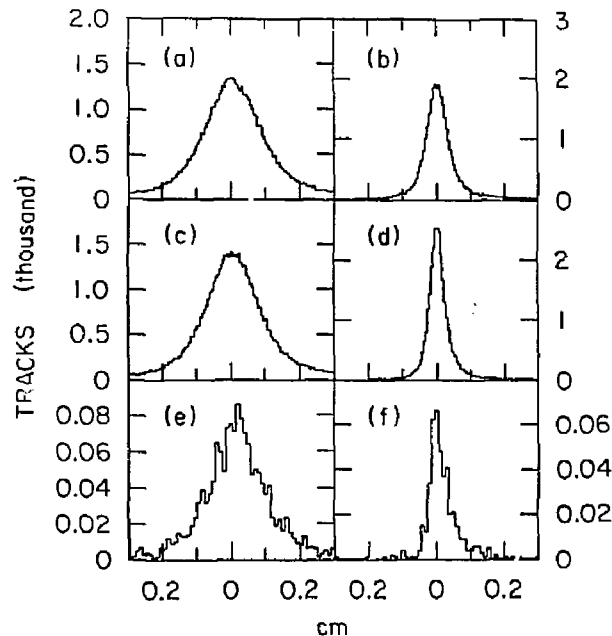


FIGURE 20. MAC impact-parameter distributions for (a) untagged multi-hadron data collected before installation of the VC with impact parameters measured relative to the beam centroid; (b) similar data after VC installation; (c) same as (a); taken relative to the average vertex; (d) same as (b); taken relative to the average vertex; (e) lepton tagged data before installation of the VC and measured relative to the average vertex; (f) similar data after VC installation.

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RESULTS FROM PETRA

A review of results from the five PETRA
experiments during the past year

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Presented at the 14th SLAC Summer Institute on Particle Physics,
Stanford, August 6th 1986

Roger Barlow 1986

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1. Introduction

During 1985 PETRA ran at 44 GeV total energy, with some running at the slightly lower energy of 38 GeV. In 1986 it is running at 33 GeV, and will continue to do so until November. Thus, in contrast to previous years, there have been no energy records broken, and no new regions to be searched for new phenomena. Furthermore the amount of data taken has been fairly modest - typically tens of inverse picobarns at each energy for each experiment - so there has not been a large increase in statistics. The new results this year have come partly from the improved performance of the upgraded detectors, and largely from the continuous analysis effort of the hard-working physicists of the five experiments, CELLO, JADE, MARK J, PLUTO, and TASSO.

A comprehensive review of the results of these hundreds of man-years of work is hardly possible within the present framework. Furthermore it would rapidly become out of date. I have therefore selected for presentation and discussion here those topics which I believe are, for one reason or another, of greatest interest and significance - other people might have chosen differently - and these are presented in Sections 2 to 7 with a brief summary of this year's other results in Section 8.

The descriptions and comments given here are those of the author, who does not claim to be omniscient or infallible in his interpretation and evaluation. Anyone requiring full and authoritative accounts should refer to those of the experiments themselves, which will be found in the references given at the end of each section, and from which the diagrams have usually been taken. References to the "Berkeley Conference" are to the 23rd International High Energy Physics Conference, Berkeley, California, in July 1986.

2. Electroweak Asymmetries

Further measurements of the Forward - Backward asymmetries for μ pairs and τ pairs have been reported by the experiments, in particular for the region around $\sqrt{s} = 38$ GeV. The latest high energy values are shown in Figures 1a and 1b, together with the Standard Model predictions (using $\sin^2\theta_W = 0.217$, $M_Z = 93$ GeV) and in Table 1.

The measured asymmetries can be interpreted in terms of standard model parameters in various ways. One is to use the form

$$A_\mu = -\frac{3}{2} \frac{a_\theta a_\mu G_F M_Z^2}{8\pi \alpha/2} \frac{s}{s - M_Z^2}$$

and similarly for τ

where assuming standard values for α , G_F , a_θ , and M_Z , the result is interpreted as a measurement of the axial lepton couplings, a_μ or a_τ , which in the Standard Model are -1. These measurements are also shown in the table. Please note that these values depend on the details of the method, such as the radiative corrections and the value used for M_Z , so that to impose consistency the couplings shown in the table have been calculated from the given asymmetries by me, using the same assumptions. The actual experiments may prefer to present slightly different values based on different assumptions. Averaging the values for a_μ and a_τ gives:

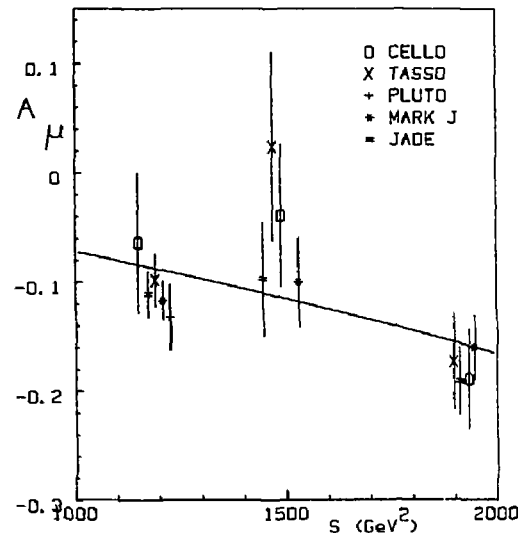
$$a_\mu = -1.09 \pm 0.06$$

$$a_\tau = -0.89 \pm 0.10$$

From which it can be noted that:

The χ^2 for μ and τ are 10.8 for 15 degrees of freedom and 13.9 for 12 degrees of freedom respectively. This shows that the two sets of data are internally consistent.

FIGURE 1A THE MUON PAIR ASYMMETRY



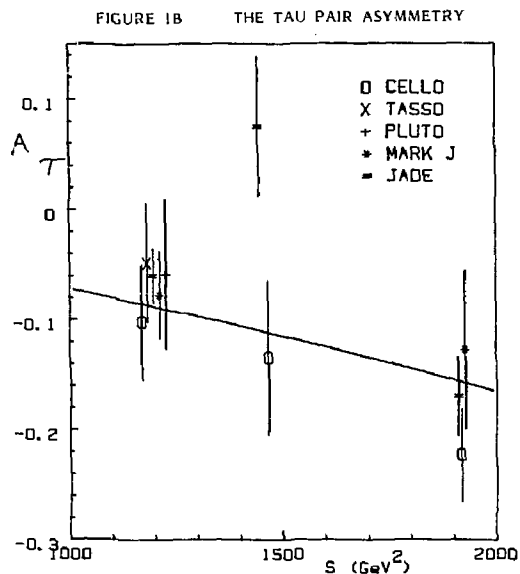


TABLE I
Asymmetry Measurements

Experiment	Energy	Asymmetry (%)	Axial Charge
For Muons			
HRS	29.0	$-4.9 \pm 1.5 \pm 0.5$	-0.82 ± 0.47
MAC	29.0	$-5.9 \pm 0.7 \pm 0.2$	-0.99 ± 0.12
Mark II	29.0	-7.1 ± 1.7	-1.19 ± 0.29
CELLO	34.2	-6.4 ± 6.4	-0.74 ± 0.74
JADE	34.4	$-11.1 \pm 1.8 \pm 1.0$	-1.27 ± 0.24
TASSO	34.5	$-9.8 \pm 2.3 \pm 0.5$	-1.11 ± 0.27
Mark J	34.6	$-11.7 \pm 1.7 \pm 0.5$	-1.32 ± 0.20
PLUTO	34.7	$-13.2 \pm 2.8 \pm 1.0$	-1.48 ± 0.33
JADE	38.0	-9.7 ± 5.2	-0.88 ± 0.47
TASSO	38.3	$+2.4 \pm 8.6 \pm 0.5$	$+0.21 \pm 0.76$
CELLO	38.3	$-3.9 \pm 6.5 \pm 0.7$	-0.35 ± 0.56
Mark J	39.1	-10.0 ± 4.1	-0.84 ± 0.35
TASSO	43.6	$-17.3 \pm 4.4 \pm 0.5$	-1.11 ± 0.28
JADE	43.7	-19.1 ± 3.1	-1.22 ± 0.20
CELLO	43.8	$-18.9 \pm 4.5 \pm 0.7$	-1.20 ± 0.29
Mark J	44.1	-16.0 ± 3.0	-1.00 ± 0.19
For Tau's			
MAC	29.0	$-5.5 \pm 1.2 \pm 0.5$	-0.93 ± 0.22
HRS	29.0	$-4.4 \pm 1.4 \pm 0.5$	-0.74 ± 0.25
Mark II	29.0	-4.2 ± 2.0	-0.71 ± 0.34
CELLO	34.2	$-10.3 \pm 5.2 \pm 1.0$	-1.19 ± 0.61
TASSO	34.4	$-4.9 \pm 5.3 \pm 1.2$	-0.56 ± 0.62
JADE	34.6	$-6.0 \pm 2.5 \pm 1.0$	-0.68 ± 0.30
Mark J	34.6	-7.8 ± 4.0	-0.88 ± 0.43
PLUTO	34.6	$-5.9 \pm 6.8 \pm 1.5$	-0.67 ± 0.79
JADE	38.0	$+7.5 \pm 6.3 \pm 1.0$	$+0.68 \pm 0.58$
CELLO	38.3	$-13.5 \pm 6.9 \pm 1.0$	-1.20 ± 0.62
JADE	43.7	$-17.0 \pm 3.6 \pm 1.0$	-1.09 ± 0.24
CELLO	43.8	$-22.3 \pm 4.1 \pm 1.0$	-1.42 ± 0.27
Mark J	43.9	$-12.8 \pm 7.0 \pm 1.5$	-0.81 ± 0.45

Both average values are compatible with the Standard Model prediction of -1 .

Although $|a_\mu|$ is larger than 1 and $|a_\tau|$ is smaller, the difference between them is 0.20 ± 0.12 , and is thus less than two standard deviations away from zero. There is no evidence for any breakdown in lepton universality.

References:

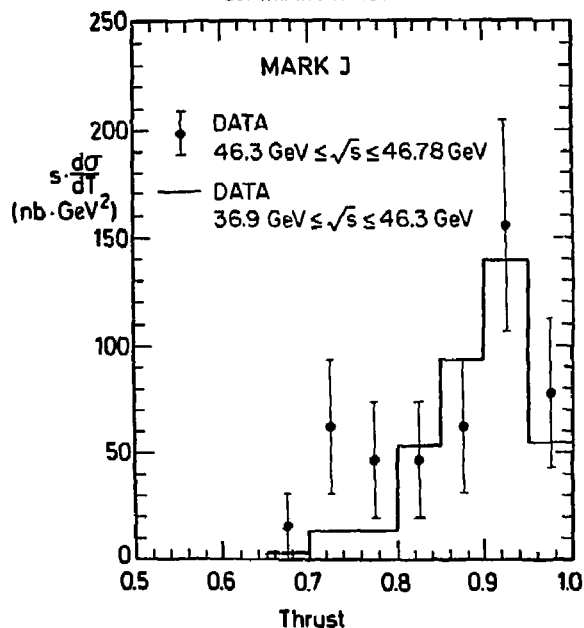
- B. Naroska. Preprint DESY 86-051, also in the Proceedings of the 21st Rencontre de Moriond.
D.H. Saxon. Preprint RAL 86-073.
W. de Boer. Talk at the 27th International Symposium on Multiparticle Dynamics, Austria, 1986.

3. Excess of low thrust hadronic events with isolated muons at the highest PETRA energies

Anomalies come and go, and over the past year or two we have learnt not to get prematurely excited over announcements of small numbers of odd events in tails of distributions. However, remembering the fable of the boy who cried "Wolf!", we should not on this account disbelieve all reports of unusual behaviour, as one day one of them is liable to turn out to be true. So, although there are certain features of the reported excess which tend to make the observer sceptical, there are also some convincing points and internal consistencies which cannot easily be explained away as background effects. Personally, I am unconvinced either way, and would be extremely loath to bet actual money on the reality or otherwise of the phenomenon. All I can do here is to present the facts, without any personal axe to grind, and let you make up your own minds.

In spring 1984, Mark J announced that they had observed 6 low thrust hadronic events containing a muon (a standard signal for the expected top quark) in the 1.4 pb^{-1} of data at the highest PETRA energies - above 46.3 GeV (the maximum ever attained was 46.78 GeV). Further high energy running took place that summer, and in another 1.4 pb^{-1} another 2 such events were found, making 8 in all. The thrust distribution is shown in Figure 2; the points are the high energy data, the solid line is the scaled expectation from lower energies. There is a plausible excess of low thrust events - in contrast to the 8 seen below a thrust of 0.8, only 1.9 would be expected from the low energy data.

FIGURE 2 THRUST OF HADRON EVENTS
CONTAINING A MUON



A further feature of these events was found in the isolation of the muon. The angle δ between the muon and the thrust axis tends to be large - 8 of the 9 muons (one of the events has two) have $|\cos \delta| < 0.7$. This is seen by comparing the T - $\cos \delta$ scatter plots for the low energy data (Fig. 3a), which do contain a few events in the low T - low $\cos \delta$ quadrant, but only as spillover from the more populated areas, with the high energy data (Fig. 3b) where the relative number in the lower quadrant is much larger, and stands out. Isolated muons are another standard signal for top quark (or similar) production. The 8 low thrust isolated muons observed contrast strongly with the 0.5 expected from the lower energy sample although, as Mark J carefully point out, the $\cos \delta$ cut was made a posteriori and the statistical significance of the difference is therefore lessened.

The JADE experiment has rather less luminosity (1.7 pb^{-1} as opposed to 2.8 pb^{-1}) and a similar analysis produces 5 low thrust isolated muons, against an expectation of 0.6 from the low energy sample. (The $\cos \delta$ cut is now a priori: The same cuts as used by MARK J were applied to the JADE data, so the comparison is meaningful). Figures 4a and 4b show the equivalent of Figures 3a and 3b for the JADE data.

The agreement in the overall numbers and shapes looks convincing particularly as presented in Fig. 5. Is this really some new physics process? There are arguments for and against.

FIGURE 3A THRUST AND ISOLATION
BELOW 46.3 GEV

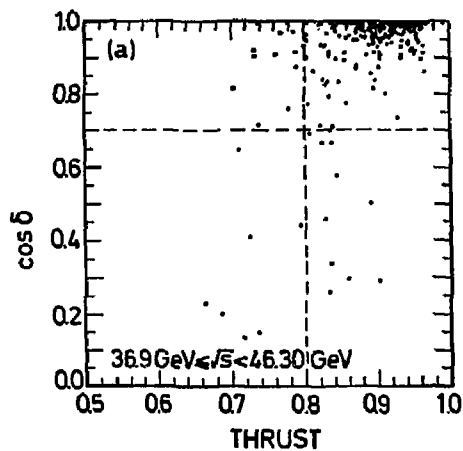
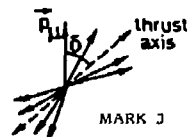


FIGURE 3B THRUST AND ISOLATION
ABOVE 46.3 GEV

MARK J

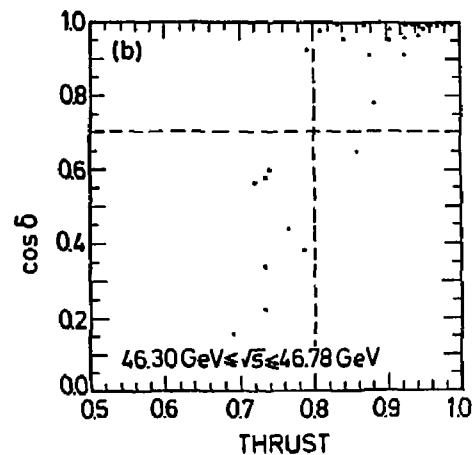


FIGURE 4A - THRUST AND ISOLATION
BELOW 46.3 GEV

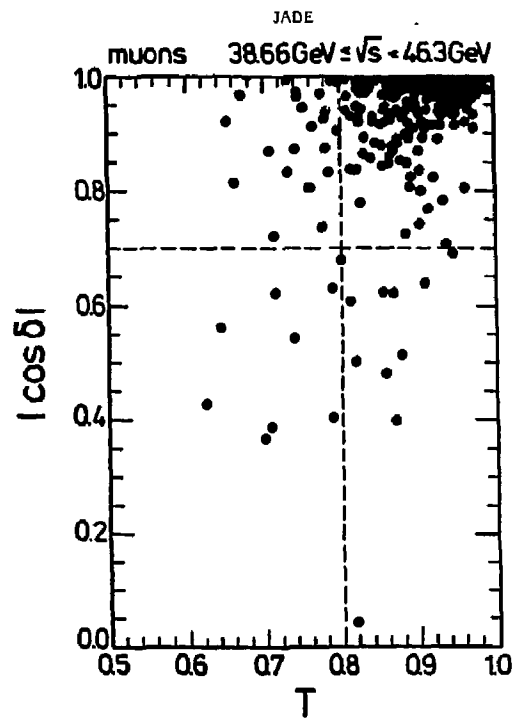


FIGURE 4B - THRUST AND ISOLATION
ABOVE 46.3 GEV

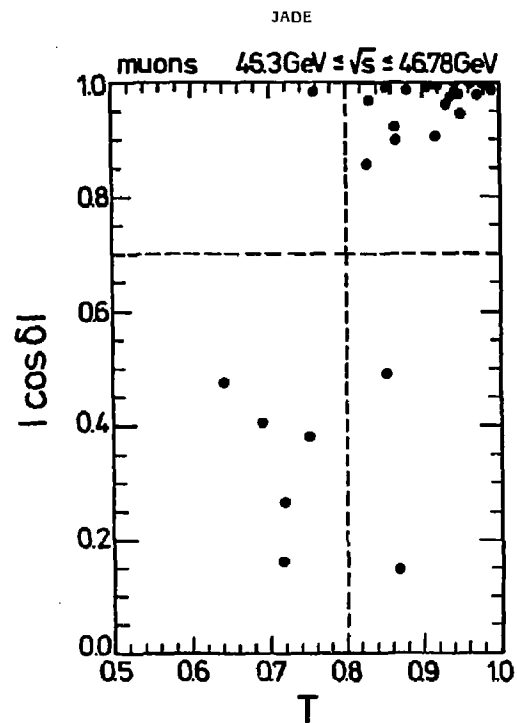
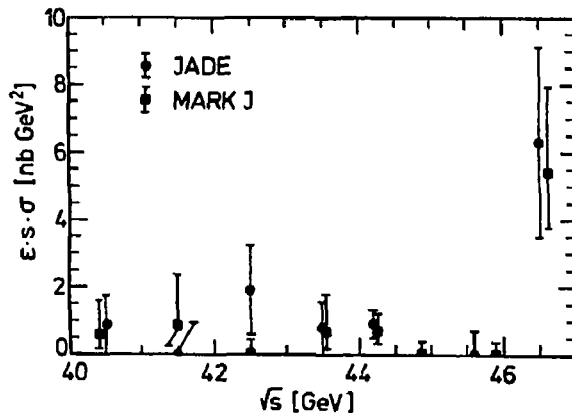


FIGURE 5 PRODUCTION RATE OF LOW THRUST
ISOLATED MUONS AS A FUNCTION OF
ENERGY.



THE ERROR BARS SHOULD NOT BE INTERPRETED
TOO LITERALLY.

Arguments against - the case for the Prosecution:

These events are either background, or a statistical fluctuation, because:

The other PETRA experiments do not see any effect. The amounts of data taken and muon detection acceptances are shown in Table 2. The TASSO result is inconclusive. The CELLO data, on the other hand, are irreconcilable.

Beam conditions were extremely bad during this running. Particles were lost from the beams at a high rate, and a lot of associated background and synchrotron radiation came down the beampipes onto the detectors. The JADE endwall muon chambers were very badly affected; many had to be switched off as they were drawing high currents. And yet four of the five JADE muons in question occur in the endwall chambers.

The JADE muons are of low quality. If tighter cuts are applied, such as the ones used for b quark studies, then 3 of the 5 "muons" are rejected.

No similar effect is seen for electrons. As shown in Table 2, no low thrust events with isolated electrons are observed by JADE, CELLO, or TASSO, which is irreconcilable with what would be expected from the MARK J/JADE muon numbers, if $e-\mu$ universality is not violated.

TABLE 2

Luminosities, Acceptances and Signals for
PETRA Experiments above $\sqrt{s} = 46.3$ GeV

Muons

	Mark J	JADE	CELLO	TASSO
Integrated Luminosity (pb ⁻¹)	2.8	1.7	2.8	1.1
Acceptance	70%	62%	72%	38%
Signal (events)	7	5	1	1
Expected Background	0.8	0.56	0.65	0.3

Electrons

	JADE	CELLO	TASSO
Integrated Luminosity (pb ⁻¹)	1.7	1.4	1.1
Acceptance	46%	55%	29%
Signal (events)	0	0	0
Expected Background	0.7	0.2	0.3

Numbers taken from J.G. Branson's talk at Berkeley.

Arguments in Favour - the Case for the Defence

These events are genuine. They are the first sign of new and exciting physics, because:

The events show no obvious signs of background: they look perfectly normal.

Noise was high at other high energies, but these events are not seen. As Figure 5 shows, there is a real threshold. Lead shielding was installed at JADE during 1984, which reduced the noise, but two of the endwall events occurred after this.

Although four of the five JADE muons are in the endwalls (three are entirely in the endwalls, one in the barrel, and one is shared between them), the solid angle covered by the endwalls is large enough for this not to be remarkable. The angular distribution of all muons does not show any excess in the endwalls, and the distributions above and below the threshold at 46.3 GeV are the same.

Although three of the five JADE muons are not of highest quality, visual inspection reveals that in all cases this is understandable due to problems with edges and corners of the detector, or noise confusing the track finder. If not "gold plated", they are at least "silver plated", and their credibility is good.

If there were an excess of apparent muons due to background, this would produce more muons at all values of thrust, and an excess of high thrust muons is not observed. While an excess of "muons" due to the high background in these two experiments at PETRA is perfectly credible (though there is no evidence or reason to suppose that it exists), it is not credible that this mechanism only produces muon candidates in low thrust events, for isolated tracks.

The Verdict

The safe conclusion can only be to wait and see what happens at TRISTAN and SLC. However if the events are a genuinely new effect, some conclusions may be drawn.

1. This is not the top quark. The numbers are too small. They are however compatible with a new $1/3(b')$ quark.
2. The muon momentum is generally low, below 5 GeV/c except for three above 10 GeV/c.
3. Two events, one from each experiment, have two muons.
4. Many of the events also contain electromagnetic clusters, from photons or electrons.
5. The missing energy is small. This means that this is not a new heavy lepton.
6. The event shapes are planar. Both experiments agree on this. This makes the b' quark explanation untenable, as the decay of two heavy quarks would produce a spherical event.

This last point is probably (though perhaps it should not be) the strongest argument against accepting this as a new phenomenon - it is very hard to devise a mechanism for producing low thrust, planar events with isolated muons. Hopefully theorists will regard this as a challenge to their ingenuity - at any rate for the next year or so.

References:

- Mark J: Physical Review D 34 681 (1986).
Michael Kuhlen: Preprint DESY 86-052, also in the proceedings of the 21st Rencontre de Moriond.
J.G. Branson: Talk at the Berkeley Conference.

4. Lifetime measurements with the TASSO vertex detector

The TASSO vertex detector contains eight layers of sense wires, with a resolution of 100 μm , some of which are angled to provide stereo information. About 50 pb^{-1} have been accumulated with this chamber operational, and to be competitive with such a comparatively small data sample, the collaboration have been inspired to produce some interesting developments in analysis techniques.

4.1 The D^0 lifetime

The D^0 mesons were found by the usual $D^* - D^0$ mass difference technique, in the D^0 decay channels

$$D^0 \rightarrow K^- \pi^+, \quad D^0 \rightarrow K^- \pi^+ \pi^0, \quad D^0 \rightarrow K^- \pi^+ \pi^0$$

(and their charge conjugates). The π^0 in the third channel is not observed, but if it is of low energy a satellite peak may be seen.

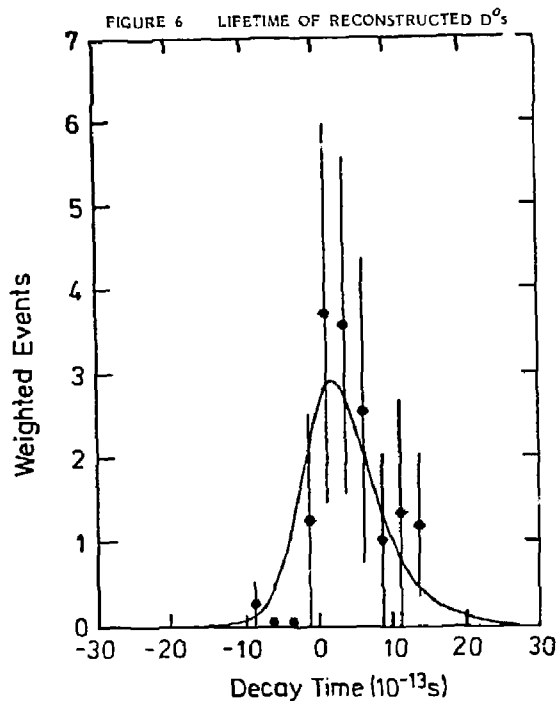
After selection cuts on the mass difference and on the track quality, the decay vertex position is then found by doing a 3-D fit, where the vertex position and track parameters are adjusted simultaneously to give the best fit to the measured points on the tracks. The tracks were then kinematically constrained to the D^0 mass, which improves the vertex resolution by 16%. Cuts on the vertex quality leave 15 events, 11 in the $K^- \pi^+$ channel, two each from the other two channels. The decay distances are converted into proper-times, as the momentum is known, and these are shown, weighted by their appropriate errors in Fig. 6. A maximum likelihood fit, including the background fraction and the effect of a finite D lifetime (some of the D^* 's come from B mesons) gives

$$\tau_{D^0} = 4.3^{+2.0}_{-1.4} \cdot 10^{-13} \text{ sec}$$

where many sources of systematic error have been considered, the largest being due to the uncertainty in the detector resolution.

References:

- TASSO Collaboration: "A Measurement of the D^* Lifetime" DESY 86-027.
D.H. Saxon. N.I.M. A234 258 (1985).
G.E. Forden and D.H. Saxon. N.I.M. A248 439 (1986).
See also N.I.M. A247 586 (1986).



4.2 The $F^{\pm}$

F mesons are found in the decay channel

$$F^{\pm} \rightarrow \phi \pi^{\pm}$$

$$\phi \rightarrow K^+ K^-$$

with cuts and fitting techniques similar to the D^0 analysis. Nine candidates are selected in the mass range $1.920 - 2.020 \text{ GeV}/c^2$, and they give a preliminary value for the lifetime of

$$\tau_F = 3.4^{+2.9}_{-1.6} \pm 0.7 \cdot 10^{-13} \text{ sec.}$$

Reference: G.E. Forden. Talk at the Berkeley Conference.

4.3 The B lifetime

Most measurements of the B lifetime have used the distribution in the impact parameter for samples of events selected to have a high B hadron content. TASSO have introduced two new methods providing a refreshing change of approach. Both use the whole data sample, with no selection or enhancement using high P_T leptons or otherwise.

In the first of these, the Vertex Method, after some fairly non-stringent cuts on the event and track quality, the best 3-track vertex is found. The distance between the beam spot and this "decay" vertex is then computed in the xy plane, according to the formula

$$L = \frac{x \sigma_{yy} \cos \varphi + y \sigma_{xx} \sin \varphi - \sigma_{xy}(x \sin \varphi + y \cos \varphi)}{\sigma_{yy} \cos^2 \varphi - 2 \sigma_{xy} \sin \varphi \cos \varphi + \sigma_{xx} \sin^2 \varphi}$$

where x and y are the differences between the "decay" vertex and the beam spot, φ is the azimuthal angle of the sphericity axis, and the σ terms are the combined variances and covariances of the beam spot and "decay" vertex. This assumes that the decay particle starts near

the beam point, travels at angle ϕ , finishing near the decay vertex, and adjusts the start and end points to minimise the joint χ^2 .

Monte Carlo studies show that for a b quark event the average decay distance produced by this algorithm is large - $756 \mu\text{m}$ if the lifetime is 1.5 ps . For light (uds) quarks it is small (though not zero) at $22 \mu\text{m}$. For charm quarks, with their short lifetime and soft fragmentation, it is fairly small, at $148 \mu\text{m}$. The value from the data is $141 \pm 16 \mu\text{m}$, which translates (knowing the fractions of quarks of different flavours in the sample) into the preliminary result

$$\tau(B) = 1.50^{+0.37}_{-0.29} \pm 0.28 \text{ ps.}$$

The second technique is called the Dipole Method. It has the advantage that only the tracks of the event concerned are involved - there is no dependence on the beam spot.

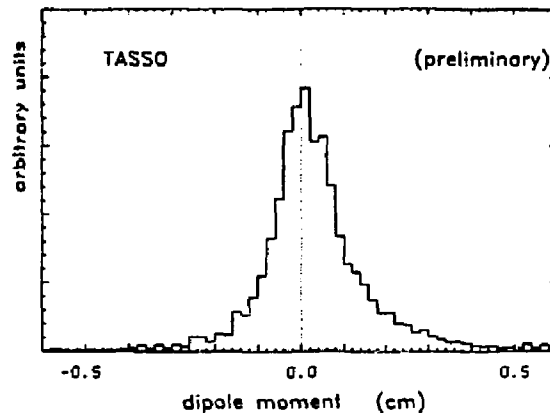
Again, the xy plane is used, and a "dipole axis" is defined as lying in the direction of the sphericity axis. Each charged track intersects this axis at some position r along the axis, measured relative to an arbitrary zero, and the position of the dipole perpendicular to its direction is found by minimising a weighted sum over all tracks of $(r_i - \bar{r})^2$. Having found the axis, tracks are classed as belonging to either the "plus" or "minus" jet, depending on whether their momentum lies along or against the dipole direction. The (weighted) mean values of r for the two jets are found, and the difference is the "dipole moment".

The data show a clearly asymmetric dipole moment distribution (Figure 7). The relation between the mean dipole moment and the B lifetime have to be established by Monte Carlo, and the preliminary result is :

$$\tau(B) = 1.62^{+0.33}_{-0.29} \pm 0.25 \text{ ps.}$$

Reference: D. Strom. Talk at the Berkeley Conference.

FIGURE 7 DIPOLE MOMENT DISTRIBUTION FROM THE DATA



5. Charm production in two photon collisions

When two high energy photons collide to produce hadrons, this is believed to occur largely through production of a quark-antiquark pair to which they both couple. The cross section is thus proportional to the fourth power of the quark charge involved, so that charmed quarks should be very significant. Even so, until recently there has been no direct evidence for charm production in two photon processes, but two different studies have now observed it.

5.1 Exclusive production of the η_c .

PLUTO was upgraded for $\gamma\gamma$ studies by the addition of taggers and forward spectrometers, and these have been used in searching for the process:

$$\gamma\gamma \rightarrow \eta_c \rightarrow K_S^0 K^\pm \pi^\mp.$$

To select a clean sample of exclusive $\gamma\gamma \rightarrow \eta_c$ events, four charged tracks of net charge zero are required, with no additional photons and a small missing transverse momentum.

The K_S^0 is identified by its separate secondary vertex. There is a potential background to this from photon conversion, and if one of the tracks in the "V" is compatible with pointing back to the origin cuts are applied to the opening angle, decay distance, and electromagnetic energies. The resulting K^0 mass peak is very clean (Figure 8). Cuts on the particle identification of the $K^\pm$ and $\pi^\mp$ are not applied - the presence of a K^0 in the event implies (by associated production) that another strange particle must also be present. Thus each event gives two entries in the $K_S^0 K^\pm \pi^\mp$ mass distribution, shown in Figure 9, for the $K^+ \pi^-$ and $K^- \pi^+$ assignments.

FIGURE 9 MASS OF PION PAIRS SATISFYING VERTEX CRITERIA

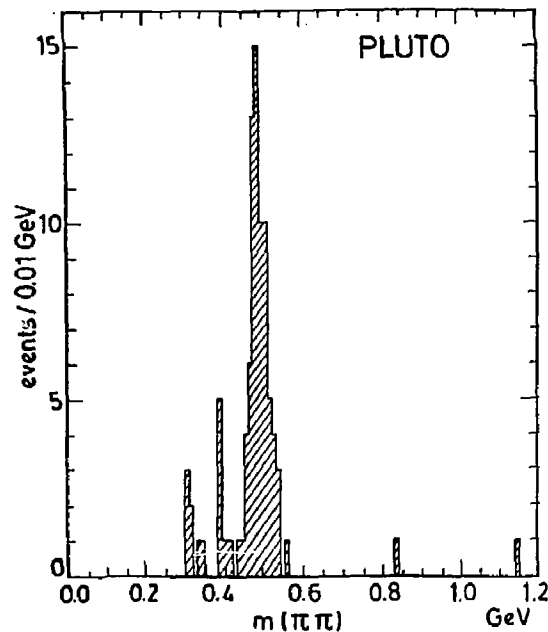
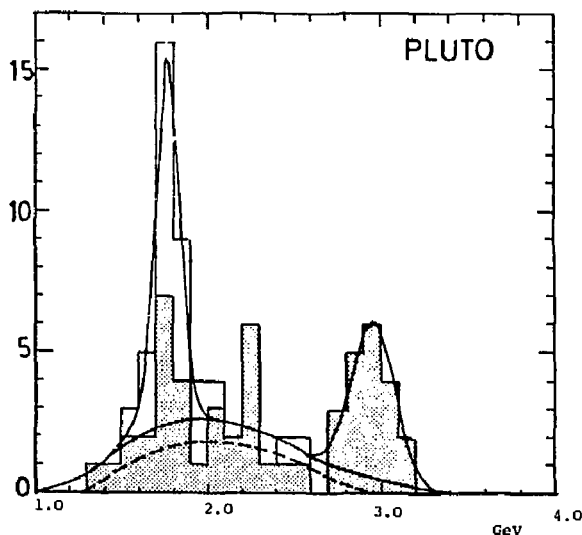


FIGURE 9 MASS DISTRIBUTION OF η_c CANDIDATES



SHADED HISTOGRAM SHOWS THE DATA REMAINING AFTER THE K^0 CUT. CURVES ARE FITS TO THE RESONANCES AND BACKGROUND.

The plot is dominated by two peaks, at 1.8 and 3.0 GeV. The lower peak is believed to be a reflection from the process

$$\gamma\gamma \rightarrow f'(1525) \rightarrow K_S^0 K_S^0$$

with the mass shifted upwards from 1.5 to 1.8 GeV as the " K_S^0 " is actually a π . If it is required that the mass of the two charged tracks be incompatible with the K^0 mass, then this removes the lower peak but, as shown by the shaded area in the figure, does not affect the upper one, which is taken as being due to the η_c . It contains 10 events, each of which gives two entries - there is little difference between the masses from the two assignments. One of the ten is tagged as a two photon event, the other nine are not.

Monte Carlo studies show that the detection efficiency varies only slowly with mass, so the peak is not an artifact produced by a rising acceptance on top of a falling $\gamma\gamma$ luminosity. Its value depends on the form factor of the η_c , which could plausibly behave like a ρ , (which gives 6.3%) or the ϕ (in which case it is 5.0%). Combined with the observed number of events this gives a result for the product of the width and the decay branching ratio

$$\Gamma_{\gamma\gamma}(\eta_c) \cdot B(\eta_c \rightarrow K_S^0 K_S^0 \pi^+ \pi^-) = 0.5^{+0.20}_{-0.15} \pm 0.1 \text{ KeV.}$$

To extract the radiative width requires the branching ratio, which is currently not well established. Using the Particle Data Group value of 3.5% gives

$$\Gamma_{\gamma\gamma}(\eta_c) = 14 \pm 12 \text{ KeV}$$

whereas the recent value of 1.5% from Mark III gives

$$\Gamma_{\gamma\gamma}(\eta_c) = 33 \pm 20 \text{ KeV}$$

where the error quoted includes statistical and all systematic errors, including the error on the branching ratio. Until the branching ratio situation clears up, the width-branching ratio product is a more meaningful number.

Reference: PLUTO Collaboration, DESY report 85-130.

3.2 Inclusive D^+ production

In this study by JADE a pure sample of high energy two photon collision events was selected by requiring events to have at least four charged hadrons, a total longitudinal momentum of at least 12 GeV/c, a mass of at least 4 GeV/c², and to be tagged as a two photon event by an electron of at least 60% of the beam energy. Within these, all combinations were tried in searching for the decay:

$$D^{*+} \rightarrow D^0 \pi^+$$

followed by

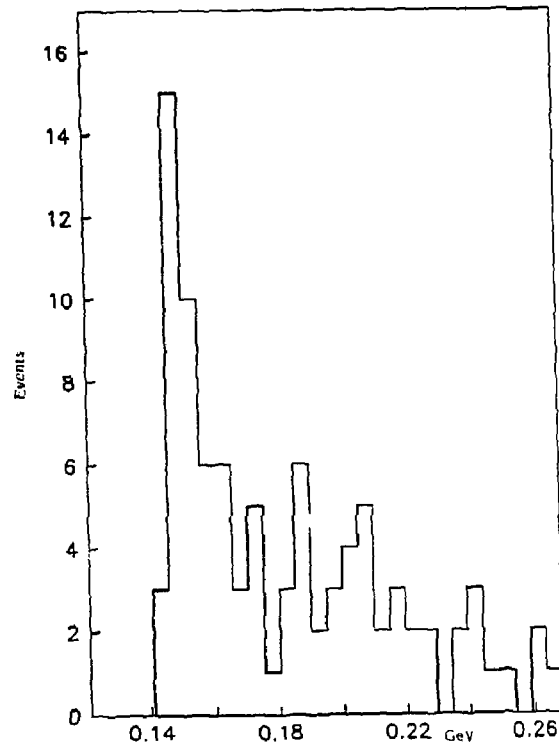
$$D^0 \rightarrow K^+ \pi^- \pi^0$$

exploiting the small reconstruction error on the D^+ - D^0 mass difference. As usual, the charge conjugate states are implied by the above equation, and will be throughout. The $K^+ \pi^- \pi^0$ mode was chosen as it has a highest branching fraction, and JADE has good π^0 identification by combining photons in the lead glass. (The channel $D^0 K^+ \pi^+$ has been investigated but the result is not statistically significant.)

The " D^0 " combinations are then formed from the π^0 and two oppositely charged tracks, where the "kaon" has to have an acceptable dE/dx measurement. These are accepted as possible D^0 mesons if their mass lies in the range 1.45 - 2.20 GeV/c² and, subject to some further cuts, combined with a further pion. The resulting distribution in mass differences, $M(D^+) - M(D^0)$, is shown in Figure 10; a sharp peak is seen containing 32 events.

The background to this figure is estimated from " D^0 " combinations in the mass range 2.3 - 3.0 GeV/c², for which the mass difference distribution has no peak, and contains 13 events in the relevant region. (A similar exercise using the same mass range but choosing combinations with all three charged particles of the same sign also show no peak.) The net number of events due to the signal thus emerges as 19±3.

FIGURE 10 D^+-D^0 MASS DIFFERENCE



As one would expect, the Vector Dominance model fails to describe either the properties of these events (such as mean p_T) or their number. The basic box diagram of the quark parton model describes the shapes well, but predicts only 3.0 ± 1.2 events, in contrast to the 19 observed. This may be due to the QCD corrections to the basic QPM box diagram, which are already known to be significant in the production of high p_T jets.

Reference: J.A. Skard. Talk at the Berkeley Conference.
 JADE Collaboration. Submitted to Physics Letters.
 J. Nye. Ph. D. Thesis, Lancaster University (1986).

5. QCD Studies

Many analyses have been reported in the study of QCD, α_s , and hadronic jets. Here are three of them.

5.1 R and α_s

The measurement of R (the ratio of the hadronic cross section to the QED muon cross section) is the only means of measuring the strong coupling constant, α_s , which is not bedevilled by theoretical uncertainties from cutoffs, fragmentation effects, etc. Unfortunately it is a hard quantity to measure precisely, as it requires accurate knowledge of the luminosity and of the acceptance (by the online trigger and software selection) of the detector. CELLO have attacked this by combining their own results with those of the other experiments at PETRA and PEP to increase the statistical accuracy.

The values of R are shown in Figure 11. The simple quark model predicts a value of $1/3$, which is seen to be too low, the excess being due to the QCD corrections, which have been calculated to second order, and a rise at higher energies as the effect of the Z^0 propagator makes itself felt.

The combination of the results from different experiments is non-trivial due to the important systematic errors in the normalisation, so the measurements at different energies are correlated if they come from the same experiment, but not from different experiments. The way such a systematic error applies to different points must be incorporated in any χ^2 which is minimised by including it in the covariance between the two errors. Using this technique, fitting α_s and $\sin^2\theta_W$ gives

$$\sin^2\theta_W = 0.24 \pm 0.02$$

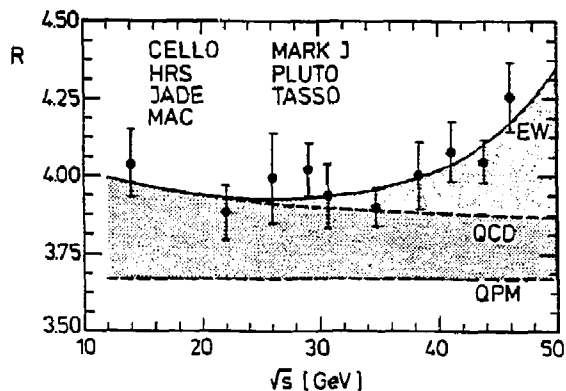
$$\alpha_s = 0.17 \pm 0.03 \quad \text{at } 34 \text{ GeV, in 2nd order QCD.}$$

This, unlike some "measurements" with much higher quoted precision, is not subject to severe hidden theoretical ambiguities.

Reference: W. de Boer. Invited talk at the XVIth International Symposium on Multiparticle Dynamics, Austria, July 1986.

CELLO Collaboration. Contribution to the Berkeley Conference.

FIGURE 11 COMBINED R VALUES



6.2 The Planar Triple Energy Correlation

The Planar Triple Energy Correlation is a new variable used by Mark J for a study of QCD and measurements of α_s . It is analogous to the usual energy-energy correlation, but instead of considering pairs of particles (or calorimeter cells, depending on the nature of the detector) separated by an angle χ , one considers triplets, between which there are three angles, χ_1 , χ_2 , and χ_3 .

Only those triplets where the three angles sum to 360° (or close to it) are considered; this selects only those particles in a planar configuration, which is natural for studying three jet events, and has the added benefit that the distribution need only be considered as a function of two of the angles, χ_1 and χ_2 . Each triplet is weighted by the product of the three energies, giving

$$\frac{1}{\sigma} \frac{d^2\sigma}{d\chi_1 d\chi_2} = \frac{1}{N \delta\chi_1 \delta\chi_2} \sum_{\text{events}} \sum_{i,j,k} \frac{E_i E_j E_k}{E_{\text{vis}}^3} \quad (\text{planar})$$

Like the energy-energy correlation, it has the nice feature that no axis finding, minimising or maximising is needed. It is claimed to be insensitive to QCD cutoff parameters, and detector effects.

Mark J have used the number of events in the χ_1, χ_2 distribution in the region where two jet effects are small, and obtain, for second order QCD using the matrix elements of Ellis, Terrano and Ross

$$\alpha_s = 0.125 \pm 0.005 \pm 0.012 \text{ at } 35 \text{ GeV};$$

$$\alpha_s = 0.121 \pm 0.007 \pm 0.010 \text{ at } 44 \text{ GeV};$$

$$\Lambda_{\overline{MS}} = 110 \pm 30 \begin{matrix} +70 \\ -55 \end{matrix} \text{ MeV.}$$

Systematic errors are dominated by uncertainties in the fragmentation scheme, as usual, and are probably best regarded as guesstimates. It is certainly unclear to me how the systematic error of 0.012 at 35 GeV, say, is obtained by comparing the two results found of $\alpha_s=0.112$ from the Ali model and $\alpha_s=0.147$ from the LUND model (indeed it is not clear how the quoted average of $\alpha_s=0.125$ is derived from these two numbers).

Reference: Mark J: MIT Technical Report 148, June 1986.

6.3 Multijet Production

Three jet events were discovered early at PETRA, and were a major contribution to the acceptance of QCD - the third jet was much more visible and convincing than relations between structure functions. To establish the non-Abelian nature of QCD in a similarly graphic way will require studies of four jet production, as these are largely (though not exclusively) produced by the gluon self-coupling. The JADE group has therefore made a detailed study of multijet events.

Although some events do have a clear four jet structure, such classifications cannot be strictly demarcated, as there is no real dividing line between hard, perturbative, bremsstrahlung gluons and soft fragmentation gluons. An algorithm is therefore used which combines particles into jets up to some invariant mass M_{cut} , normally expressed in terms of the variable $y_{cut} = M_{cut}^2/E_{vis}^2$, and the number of "jets" found in an event depends explicitly on the value of y_{cut} .

For example, for $y_{cut}=0.04$, at $\sqrt{s}=34$ GeV, the numbers of jets found are:

	<u>2 jets</u>	<u>3 jets</u>	<u>4 or more jets</u>
Data	56.1±0.4	40.2±0.4	3.75±0.16
LUND Model	53.2±0.3	44.0±0.3	2.85±0.12
Webber Model	58.5±0.4	37.8±0.4	3.68±0.15

where the numbers show the percentage of events with a given number of jets (so the rows sum to 100%) for the data, and also for two models which fit other aspects of the data well (including the "string effect"). The LUND model has the full second order QCD matrix elements and string fragmentation, whereas the Webber model uses leading log first order QCD matrix elements, and cluster fragmentation.

As can be seen, the LUND model predicts too many three jet events and too few four jet events, and the Webber model, predicts too few three jet events but gets the number of four jet events right.

The behaviour is the same for other values of y_{cut} , and at other energies, as is shown in Figures 12 and 13. It also appears in the distribution in Acoplanarity (a variable sensitive to multijet production), which shows that the discrepancy is not just a by-product of the cluster-finding method. As the numbers of partons generated by the two models exhibit the same pattern of behaviour as the numbers of jets found, there is a strong suggestion that the problem lies in the QCD matrix elements used, rather than the different fragmentation schemes, in the two models. Certainly, adjustment of the various fragmentation scheme parameters cannot cure either discrepancy without ruining the good description of the other features of the data.

It may be possible that the mechanism of the Webber model can be adjusted, perhaps even in a "natural" way, so as to produce the correct three jet fraction; this is still a very open question. It could be that third order QCD corrections would cure the LUND model, but calculations are not available. Changing α_s cannot help, as increasing α_s to increase the number of four jet events would make the excess of three jet events even worse, and vice versa. Indeed, a consequence of this problem with the LUND Monte Carlo is that if α_s is determined from the data from some global quantity such as energy-energy correlations, the result is 10% higher than when it is determined from the number of three jet events - the value of α_s needs to be higher to make up the deficiency of multi-jet events - another point to be remembered when considering "precision" measurements of this quantity.

Reference: JADE Collaboration DESY Preprint 86-086.

RECENT RESULTS FROM ARGUS

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Representing

The Argus Collaboration¹

1. Introduction

The ARGUS experiment, running at the DORIS II storage ring, has been collecting data on e^+e^- collisions near 10 GeV region since the end of 1982. The ARGUS detector, designed specifically for large acceptance and high resolution spectroscopy in the Υ resonances region, has produced a prolific amount of new results in recent months. Some of these are presented here. The subject matters include topics from charm and beauty spectroscopy and weak decays of charm mesons, beauty mesons and the tauon.

The amounts of data on which these results are based, in terms of integrated luminosities and the number of multihadron events at each energy region, are shown in Table 1.

Table 1. Summary of Data Sample to the end of 1985

Region	$\int L dt \text{ pb}^{-1}$	Number of events
$\Upsilon(1S)$	23.2	331128
$\Upsilon(2S)$	38.1	284198
$\Upsilon(4S)$	60.0	307906
Continuum	32.1	140213
Total	153.4	1063445

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The outline of the rest of the paper is as follows. In Section 2, a brief summary of the main features of the ARGUS detector is presented. In Section 3, we present an update on charm spectroscopy of $D^{*0}(2420)$ and $D_s^{*+}(2110)$ (formerly $F^{*+}(2110)$). Here and in the following, references to a specific charge state implies the corresponding charge conjugate state also. In Section 4, results on two recently discovered decay channels, $D^0 \rightarrow \bar{K}^0 \phi$ and $D_s^+ \rightarrow \bar{K}^{*0} K^+$ will be presented. Section 5 describes the determination of a new upper limit on $D^0 - \bar{D}^0$ mixing. Section 6 presents a preliminary result on the measurement of τ life-time. Section 7 describes the reconstruction of B decays into exclusive channels that contain one $D^{*+}(2010)$ and a number of pions, up to three. Section 8 gives a brief summary.

2. The ARGUS Detector

The ARGUS detector is a solenoidal magnetic spectrometer with a field of 0.8 T. The main characteristics of the detector, and the triggering scheme are described in Ref. 2. Only a brief description will be given here for completeness. The detector consists of a cylindrical vertex drift chamber (VDC) with 594 sense wires, a cylindrical main drift chamber with 5940 sense wires arranged in 36 concentric layers, a time-of-flight (TOF) system with 64 scintillators in the barrel region and with 48 scintillators in each of two endcaps, 1280 lead-scintillator-sandwich shower counters in the barrel region and 240 in each endcap, and a muon identification system with 1744 proportional counter tubes that surround the coils and iron yoke of the magnet.

Resolutions for detector components are specified below:

- momentum resolution of $\sigma_p/p = 0.012$ at 1 GeV/c with the main drift chamber
- dE/dx resolution of $\sigma(E)/E = 4.5\%$
- the Time-of-Flight counters time resolution of $\sigma(t) = 220$ ps

- the shower counter energy resolution of $\sigma(E)/E = \sqrt{0.07^2 + 0.08^2}/E$, where E is the energy in GeV.

The following two aspects of the detector are worth emphasizing here. The shower counters are located inside the coil to obtain higher acceptance and better resolution of photons and electrons. The sense wires of the main drift chamber are equipped to measure specific ionization in addition to drift-time. Section 5 (and Ref. 3) describes how charged particles are identified by combining dE/dx information and the TOF measurements. The method described there is now extended to include other detector components (shower counters, muon-chambers) for the purpose of identifying all particles.

3. New Results on Charm Spectroscopy

3.1 Updates on $D^{*0}(2420)$

Earlier this year ARGUS reported on evidence for a new charmed meson at a mass of $2.42 \text{ GeV}/c^2$ decaying into $D^{*+}(2010)\pi^-$ channel.⁴ A natural interpretation is that it is a manifestation of one or two of the long awaited P-wave mesons of $c\bar{u}$ system. Recently, Rosner gave an extensive theoretical review of mesons composed of one heavy quark (Q) and a light anti-quark ($\bar{q}$), emphasizing that this is an ideal testing ground for the one-body problem in hadron physics.

In the limit of one very heavy quark, the system approaches the one-body system as in hydrogen atom, and the light quark's dynamics tends to a universal limit.⁵ In addition, P-wave $Q\bar{q}$ system would probe relativistic effects, and forces at long distance.

Possible P states in the usual spectroscopic notation are $^3P_{2,1,0}$ and $^1P_1$. However, as in $J^P = 1^+$ kaon resonances, $^3P_1$ state and $^1P_1$ state are expected to mix with each other. In the limit of one heavy quark, the total spin is no longer a good quantum number. Instead, one couples light quark's spin to the relative angular momentum ($L=1$) to obtain a system of $j=3/2$ or $j=1/2$, where

j is the total angular momentum of the light quark. Coupling j with the spin of the heavy quark, one obtains two $j=3/2$ states ($2_{3/2}^+$ and $1_{3/2}^+$, in J_P^J notation), and two $j=1/2$ states ($1_{1/2}^+$ and $0_{1/2}^+$). The pattern of mass splitting is such that two $j=3/2$ states form one closely spaced doublet, and two $j=1/2$ states form another closely spaced doublet.⁵ Updated compilation of mass predictions by a number of authors for P-wave states of $c\bar{u}$ and $c\bar{s}$ systems are given in Rosner.⁶ For example, the mass splitting between the two $j=3/2$ states ranges from 8 to 60 MeV/ c^2 .

Since the first report, which was based on 82 pb⁻¹ of data, ARGUS collected an additional 69.9 pb⁻¹ of data, mostly on the T(4S). In order to confirm our initial analysis, the original study has been repeated without change with the new data sample. The result provides an independent confirmation of our first result.

The decay chains used are the following:

$$D^{*0}(2420) \rightarrow D^{*+}(2010)\pi^-$$

$$D^{*+}(2010) \rightarrow D^0\pi^+$$

$$D^0 \rightarrow K^-\pi^+$$

$$\rightarrow K^-\pi^+\pi^-\pi^+$$

$$\rightarrow K^-\pi^+(\pi^0), \pi^0 \text{ unobserved.}$$

The small Q value for the decay, $D^{*+}(2010) \rightarrow D^0\pi^+$, provides an excellent resolution³⁰ for the $D^{*+}(2010)$, D^0 mass difference and hence the background free sample of $D^{*+}(2010)$. A mass difference method was employed to look for bumps in $D^{*+}(2010)\pi^-$ mass distribution. All selections are the same as in our previous publication.⁴ In addition to the usual mass selections on D^{*+} and D^0 , two selections are made on the $D^{*+}(2010)\pi^-$ system. These are:

- $x_p > 0.6$, ($x_p \equiv p/p_{\max}$), where p is the momentum of the $D^{*+}(2010)\pi^-$ system.

- $\cos\theta < 0$, where θ is the angle, in the $D^{*+}(2010)\pi^-$ rest frame, between the D^{*+} direction and the $D^{*+}(2010)\pi^-$ system line of flight.

The resulting mass difference distributions from the original study based on 82.4 pb⁻¹ data, for the new sample based on 69.9 pb⁻¹ data and the combined result are shown in Fig. 1(a), (b) and (c), respectively. Fitting these distributions to a sum of a Breit-Wigner form for the signal plus threshold factor multiplied by a second order polynomial for the background, we obtain the results given in Table 2. The masses and widths are consistent between the old and the new data sets. The combined result has a significance of 5.5 σ . The event rate in the new data is somewhat lower than the earlier rate, but the two are consistent within errors.

Table 2. $D^{*0}(2420)$ parameters for the three data sets

Data set	Original	New	Combined
$M[D^{*0}(2420)]$ (MeV/ c^2)	2420 ± 6	2432 ± 8	2426 ± 6
FWHM width MeV/ c^2	70 ± 21	54 ± 16	75 ± 20
Number of events	135^{+34}_{-29}	75^{+22}_{-19}	192^{+55}_{-35}

The division of the signal among the three D^0 channels for the full data set is shown in Fig. 2(a), (b) and (c). Breit-Wigner fits with fixed mass and width yielded 37 ± 12 , 60 ± 23 and 61 ± 21 events in the three decay channels. The ratio of the observed number of events in the $K^-\pi^+$ and $K^-\pi^+\pi^-\pi^+$ channels is consistent with expectation.

Following the procedures described in our previous publication,⁴ we find $17^{+6}_{-5} \pm 6\%$ of the observed $D^{*+}(2010)$ are produced from the $D^{*0}(2420)$.

As for spin-parity, only $J^P = 0^+$ is ruled out for the observed decay mode, $D^*\pi$. Spin-parity determination of this object would require detailed study of angular distributions and investigation of other channels notably $D\pi$. Closely spaced doublet structure, if realized, would present further complications. The

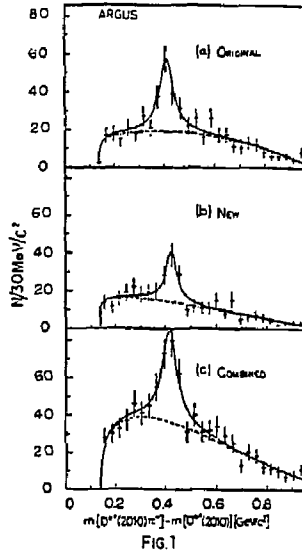


Figure 1. Mass difference distribution, $M[D^{*+}(2010)\pi^-] - M[D^{*+}(2010)]$, with selections $x_F > 0.6$ and $\cos\theta < 0.0$, for three sets of data.

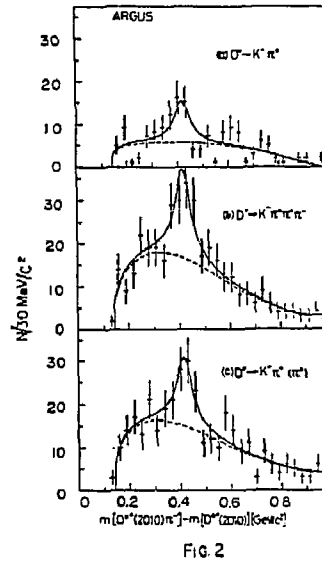


Figure 2. The same for three D decay channels.

present event sample is rather small for these purposes. The expected increase in the ARGUS data sample, by 60% by the end of 1986, will remedy the situation somewhat. Also, other experiments such as Fermilab E691 and CLEO are expected to shed new light on this subject in the near future.

3.2 New Results on $D_S^{*+}(2110)$

The D_S^{*+} (formerly F^{*+}) is a state composed of $c\bar{s}$ quarks with quantum numbers, $J^P = 1^-$ and $I = 0$. Isospin conservation forbids its decay into $D_S^+(1971)\pi^0$. This leaves the radiative decay channel $D_S^{*+}(1971)\gamma$ as the dominant mode. In 1984, ARGUS and TPC reported observation of D_S^{*+} into $D_S^+\gamma$.^{6,7} The particle data group does not consider these observations sufficiently compelling to regard this state as established.⁸ We report here much stronger evidence of D_S^{*+} based on more data, 151 pb^{-1} of luminosity as compared to 43.7 pb^{-1} earlier.

The D_S^{*+} was selected using the decay chain:

$$D_S^{*+} \rightarrow \phi\pi^+, \quad \phi \rightarrow K^+K^-.$$

Figure 3 shows the $\phi\pi^+$ mass distribution. The presence of a strong D_S^{*+} signal is evident. The weaker peak is due to the Cabibbo-suppressed decay mode of $D^+(1869)$ meson. Selections similar to those in our earlier work⁹ were applied to obtain this distribution. These are:

- $1.012 < M(K^+K^-) < 1.028 \text{ GeV}$,
- $P_{D_S} > 2.5 \text{ GeV}/c$. The fragmentation function of D_S^{*+} as observed in ARGUS is shown in Fig. 4. It indicates hard fragmentation for D_S^{*+} , as hard as in $D^{*+}(2010)$, which justifies this selection.
- $\cos\theta_\phi > 0.8$, where θ_ϕ is the angle, with respect to the D_S^{*+} boost direction, of the ϕ in the D_S^{*+} rest frame.
- $|\cos\theta_{K^+}| > 0.4$, where θ_{K^+} is the helicity angle of the K^+ in the ϕ rest frame with respect to the π direction.

j is the total angular momentum of the light quark. Coupling j with the spin of the heavy quark, one obtains two $j=3/2$ states ($2_{3/2}^+$ and $1_{3/2}^+$, in J_J^P notation), and two $j=1/2$ states ($1_{1/2}^+$ and $0_{1/2}^+$). The pattern of mass splitting is such that two $j=3/2$ states form one closely spaced doublet, and two $j=1/2$ states form another closely spaced doublet.⁵ Updated compilation of mass predictions by a number of authors for P-wave states of $c\bar{u}$ and $c\bar{s}$ systems are given in Rosner.⁵ For example, the mass splitting between the two $j=3/2$ states ranges from 8 to 60 MeV/ c^2 .

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The decay chains used are the following:

$$\begin{aligned} D^{*0}(2420) &\rightarrow D^{*+}(2010)\pi^- \\ D^{*+}(2010) &\rightarrow D^0\pi^+ \\ D^0 &\rightarrow K^-\pi^+ \\ &\rightarrow K^-\pi^+\pi^-\pi^+ \\ &\rightarrow K^-\pi^+(\pi^0), \pi^0 \text{ unobserved.} \end{aligned}$$

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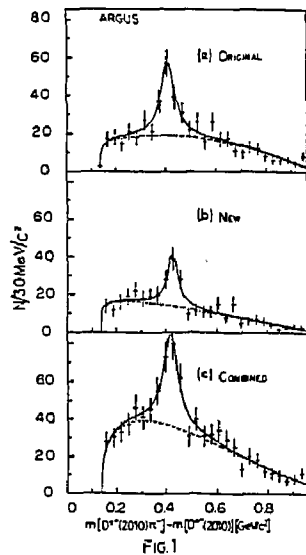


Figure 1. Mass difference distribution, $M[D^{*+}(2010)\pi^-] - M[D^{*+}(2010)]$, with selections $x_p > 0.6$ and $\cos\theta < 0.0$, for three sets of data.

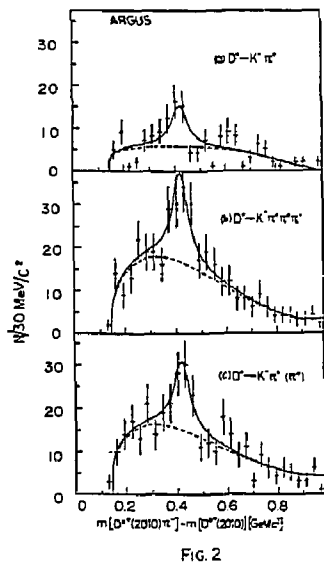


Figure 2. The same for three D decay channels.

present event sample is rather small for these purposes. The expected increase in the ARGUS data sample, by 60% by the end of 1986, will remedy the situation somewhat. Also, other experiments such as Fermilab E691 and CLEO are expected to shed new light on this subject in the near future.

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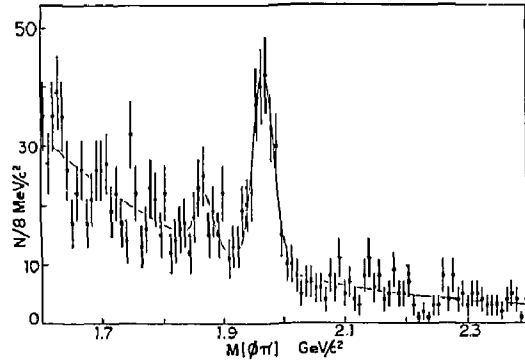


Figure 3. $M(\phi\pi^+)$ distribution showing $D_S^+(1971)$ and $D^+(1869)$ peaks. For details consult the text.

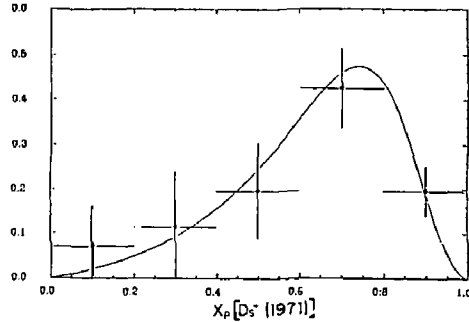


Figure 4. Fragmentation function for $D_S^+(1971)$. The solid line is a fit to the Peterson *et al.*'s formula¹⁰ with $\epsilon = 0.09^{+0.04}_{-0.03}$.

See Ref. 9 for more details.

The mass difference distribution, $\Delta M \equiv M(\phi\pi^+\gamma) - M(\phi\pi^+)$, for those $\phi\pi^+$ combinations in the $D_S^+(1971)$ region ($1.949 < M(\phi\pi^+) < 1.989$ GeV/c^2) is shown in Fig. 5. The energy distribution of photons coming from the D_S^{*+} decay ranges from 50 to 500 MeV and peaks near 150 MeV. Due to enormous background γ rays and phototube noises in the low energy region, the mass distribution is sensitive to the γ energy selection. Figure 5 was obtained with selections $E_\gamma > 0.15$ GeV and $x_p > 0.4$ ($x_p \equiv p_{D_S^+}/p_{\text{max}}$). The enhancement near 0.14 GeV is present in all selections of E_γ and x_p .

We interpret this as evidence for production of $D_S^{*+}(2110)$ and determined its mass as follows.

Similar mass difference distribution, using the $\phi\pi^+$ combinations in the "side band" of $D_S^+(1971)$ does not show such enhancement. We use the shape of this side band distribution as the background line shape and fit the data to a Gaussian distribution with the width fixed at 21.7 MeV, which is the detector mass resolution as deduced with simulated events. The results obtained are:

$$\Delta M = 141.0 \pm 5.3 \text{ MeV}/c^2 ;$$

$$N = 70^{+13}_{-12} \text{ events} .$$

This is a 5.8σ effect. Our earlier result⁶ had a significance of 4.1σ . The mass difference must be corrected for the E_γ selection. Finite resolution of the E_γ introduces systematic shift in the mass difference distribution when strong E_γ selection is applied. This shift can be determined with simulated events. Figure 6 shows this effect for three E_γ selections.

Correcting for this shift, we obtain

$$M_{D_S^{*+}} - M_{D_S^+} = 137.3 \pm 5.3 \pm 7.0 \text{ MeV}/c^2 .$$

Combining with our own $M_{D_S^+}$ determination,

$$M_{D_S^{*+}} = 1969.0 \pm 1.7 \text{ MeV}/c^2 ,$$

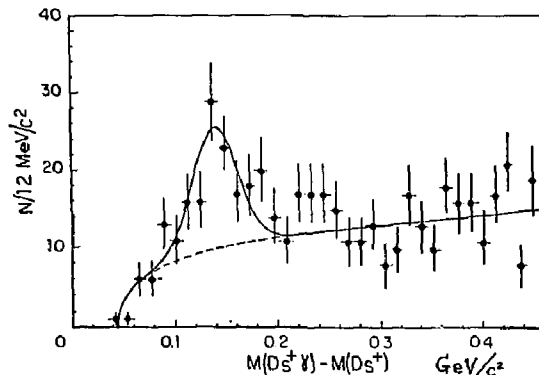


Figure 5. $M(D_S^{*+} \gamma) - M(D_S^{*+})$. Selections are $E_\gamma > 0.15$ GeV, $x_D > 0.4$. The background line shape is obtained using events from the $D_S^{*+}(1971)$ side band.

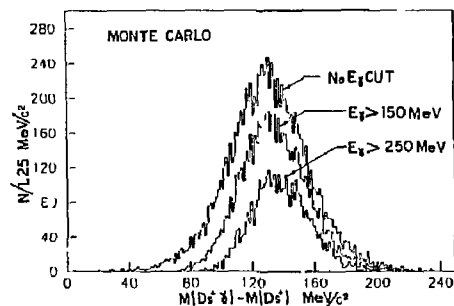


Figure 6. Monte Carlo Mass difference distributions, $\Delta M = M(D_S^{*+} \gamma) - M(D_S^{*+})$, for three E_γ selections. Peaks of distributions is seen to increase with increasing E_γ selection.

we obtain

$$M_{D_S^{*+}} = 2106.3 \pm 8.9 \text{ MeV}/c^2.$$

Although preliminary in some aspects, this new result confirms our earlier observation of D_S^{*+} but at a level much more significant than before. Recently MARK III group reported¹¹ observation of D_S^{*+} in a reaction $e^+e^- \rightarrow D_S^{*+} D_S$ at SPEAR. Their result, $M_{D_S^{*+}} = 2106.8 \pm 1.8 \pm 6.2 \text{ MeV}/c^2$ is in agreement with our result.

In conclusion, new ARGUS data provides strong confirmation of our earlier observation of $D_S^{*+}(2110)$ meson.

4. New Results on Charm Decays

In the simplest model of charm particle decay, known as the spectator model, the charm quark decays as if free; the light quark plays the role of a spectator; and all charmed particles are expected to have more or less the same lifetime. The large lifetime difference between the D^0 and D^+ mesons ($\tau(D^+)/\tau(D^0) = 2.4$) has been the clearest symptom of shortcomings of the spectator model (see, for example, reviews by Gilman and by Schindler in this SLAC Summer Institute, Refs. 15 and 16). Recently, ARGUS has discovered two decay processes, $D^0 \rightarrow \bar{K}^0 \phi$ and $D_S^{*+} \rightarrow \bar{K}^{*0} K^+$, in which decay diagrams other than the spectator diagrams are believed to play important roles. Relevant diagrams are the W-exchange diagram for the $D^0 \rightarrow \bar{K}^0 \phi$, and the flavor annihilation diagram and the color mismatched spectator diagram for $D_S^{*+} \rightarrow \bar{K}^{*0} K^+$ decay. The W-exchange diagram is absent in decay of D^+ , possibly accounting for its longer life.

Only highlights and summarizes of latest ARGUS results on these two channels are given here. For more detailed account, reader is referred to relevant Refs. 12 and 21.

4.1 Updates on $D^0 \rightarrow \bar{K}^0 \phi$

This decay mode of the D -meson was first observed by ARGUS¹² with an unexpectedly large branching ratio of $\text{Br}(D^0 \rightarrow \bar{K}^0 \phi) = (1.43 \pm 0.45)\%$. It has subsequently been confirmed by CLEO and MARK III.^{13,14} The simplest way that this process can proceed in a quark picture is via a W -exchange diagram as shown in Fig. 7, which is helicity suppressed and color mismatched. However, soft gluons are believed to reduce the helicity suppression effect. Also, it has been shown recently that color suppression is not evident in D^+ decays.²⁰

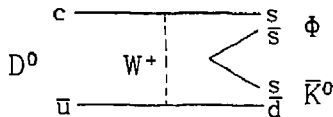


Figure 7. W -exchange diagram for the decay $D^0 \rightarrow \bar{K}^0 \phi$.

The importance of this decay channel in the context of the large lifetime difference between D^0 and D^+ was stressed previously and reviewed extensively by Gilman and by Schindler in this SLAC Summer Institute.^{15,16} Recently, Donoghue showed that under certain conditions final state rescattering effects can produce the reaction even when the W -exchange diagram is not present,¹⁷ casting some doubts on the simplest interpretation discussed above.

The result presented here is based on a larger data sample, corresponding to a total integrated luminosity of 152 pb^{-1} , compared to 82 pb^{-1} previously. The new ARGUS vertex chamber¹⁸ was operational for about 50 pb^{-1} of new running, and results in a significant improvement in the amount of data due to a 60% increase in the efficiency of K_S^0 reconstruction.

The invariant mass distribution of $K_S^0 K^+ K^-$ is shown in Fig. 8. A clean D^0 signal is evident. The signal has 205 ± 38 events at a mass of $1864.3 \pm$

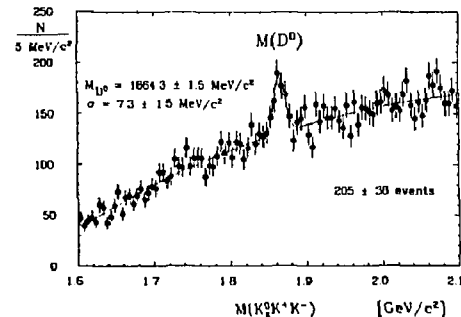


Figure 8. $K_S^0 K^+ K^-$ mass distribution for events from e^+e^- interactions at cm energies near 10 GeV.

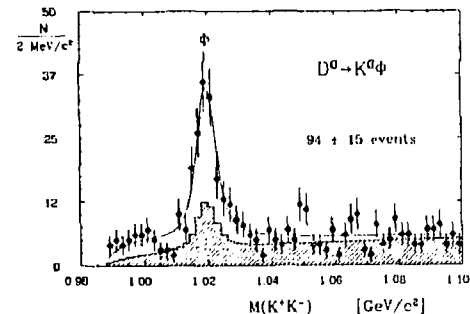


Figure 9. $K^+ K^-$ mass spectrum for events with $|M(K_S^0 K^+ K^-) - M(D^0)| < 16.2 \text{ MeV}/c^2$ (points with error bars). The hatched histogram gives the contribution which is not correlated with a D^0 meson.

1.5 MeV/c². Strong $D^0 \rightarrow K_S^0 \phi$ contribution to this three-body channel can be seen in the $M(K^+K^-)$ distribution of events from the D region, viz., by demanding $|M(K_S^0 K^+ K^-) - M(D^0)| < 16.2 \text{ MeV}/c^2$ [Fig. 9]. A prominent ϕ signal is evident. The part of the K^+K^- mass spectrum which is unrelated to the $D^0 \rightarrow K_S^0 \phi$ decay can be determined from sidebands above and below the D^0 (hatched histogram). There is a weak unrelated ϕ signal on a phase-space-like background from the sidebands. However, a clear excess of ϕ events exist on the D^0 band. As a further check the helicity angle θ , where θ is the angle between the K^+ and K_S^0 in the rest frame of the ϕ meson, was examined. As expected for the decay chain, $J^P = 0^- \rightarrow 0^- 1^-, 1^- \rightarrow 0^- 0^-$, we observe $\cos^2 \theta$ behavior (not shown).

More directly, the $M(K_S^0 \phi)$ distribution shows a very clean D^0 peak on a low background. Figure 10 shows $M(K_S^0 \phi)$ distribution with $x_p > 0.5$ selection, which reduces background since the fragmentation process of charm particle production in the continuum is found to be hard (for example, see Fig. 4 for the x_p distribution of $D_S(1971)$). Fitting to the mass spectrum, we obtain 55.9 ± 8.5 events for the process $D^0 \rightarrow K_S^0 \phi$ ($\phi \rightarrow K^+ K^-$). To obtain the branching ratio we studied another D^0 decay channel, $D^0 \rightarrow K_S^0 \pi^+ \pi^-$. With the same x_p cut, we obtain 777 ± 91 and $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ events. See Fig. 11. Use of these two numbers, after a correction for small difference in the acceptances of these two similar channels, yields:

$$\frac{\text{Br}(D^0 \rightarrow K_S^0 \phi)}{\text{Br}(D^0 \rightarrow K_S^0 \pi^+ \pi^-)} = 0.155 \pm 0.033$$

which, using $\text{Br}(D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-) = (7.6 \pm 0.7 \pm 0.8)\%$, yields

$$\text{Br}(D^0 \rightarrow \bar{K}^0 \phi) = (1.18 \pm 0.25 \pm 0.17)\%.$$

Recent results from CLEO¹³ and MARK III¹⁴ are listed below for comparison. All three experiments agree within errors.

$$\text{Br}(D^0 \rightarrow \bar{K}^0 \phi) = (1.18 \pm 0.40 \pm 0.17)\%, \quad [\text{CLEO}]$$

$$\text{Br}(D^0 \rightarrow \bar{K}^0 \phi) = (1.1^{+0.7}_{-0.5} \pm 0.4)\%, \quad [\text{MARK III}]$$

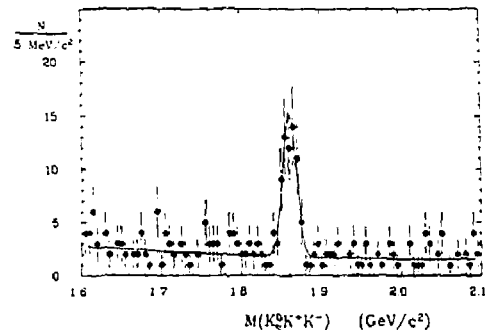


Figure 10. $K_S^0 K^+ K^-$ mass spectrum for events with $1.01 < M(K^+ K^-) < 1.03 \text{ GeV}/c^2$ and $x_p > 0.5$ ($x_p \equiv p/p_{\text{max}}$).

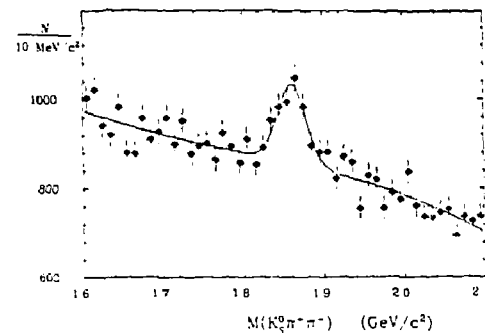


Figure 11. $K_S^0 \pi^+ \pi^-$ mass spectrum for events with $x_p > 0.5$.

It can be safely concluded that the decay $D^0 \rightarrow \bar{K}^0 \phi$ is firmly established experimentally. Its theoretical interpretation in terms of W-exchange or other models such as final state interaction appears to require more detailed theoretical and experimental studies.

4.2 Observation of $D_S^+ \rightarrow \bar{K}^{*0} K^+$ Decay

CLEO first established $D_S(1971)$ through its $\phi\pi^{19}$ decay mode. Since then many experiments confirmed the $\phi\pi$ mode. ARGUS reported earlier $\phi(3\pi)$ decay mode as well as the $\phi\pi$ mode.⁹ The spectator model can account for both of these modes. There are, however, two other diagrams which may make significant contributions as discussed above. In the context of $\bar{K}^{*0} K^+$ final state, on which we report here, these are shown in Fig. 12.

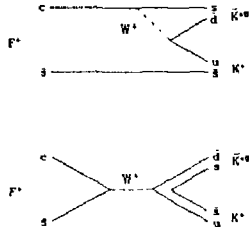


Figure 12. Color mismatched spectator and annihilation diagrams for the decay $D_S^+ \rightarrow \bar{K}^{*0} K^+$.

The data sample used for this analysis corresponds to a total integrated luminosity of 149.1 pb^{-1} (see Table 1). In forming $\bar{K}^{*0} K^+$ mass spectrum, the following selections were applied:

- all participating tracks were required to originate from the primary vertex with $\chi^2 < 36$.
- all K^- and π^+ were combined, and those with its invariant mass lying within $50 \text{ MeV}/c^2$ of $\bar{K}^{*0}$ mass were taken as $\bar{K}^{*0}$.
- a momentum selection on the $\bar{K}^{*0} K^+$ of $p(\bar{K}^{*0} K^+) > 2.5 \text{ GeV}/c$ was applied to reduce background.
- $\cos \theta_{\bar{K}^{*0}} > 0.2$, where $\theta_{\bar{K}^{*0}}$ is the angle of the $\bar{K}^{*0}$ in the D_S^+ rest frame with respect to the D_S^+ boost direction. This cut eliminates reflections from the $D \rightarrow \bar{K}^{*0} \pi^+$ process producing events in the signal region due to misidentification of π^+ as K^+ .
- $|\cos \theta_{K^+}^{K^-}| > 0.5$, where $\theta_{K^+}^{K^-}$ is the helicity angle of the K^- with respect to the K^+ in the $\bar{K}^{*0}$ rest frame. In the decay $D_S^+ \rightarrow \bar{K}^{*0} K^+$, the angular distribution should be of $\cos^2 \theta$ form. Thus, this helicity angle cut removes 50% of uniformly distributed background, while retaining 87.5% of the D_S^+ signal.

Figure 13 gives $\bar{K}^{*0} K^+ \pi^+$ mass distribution with all of the above cuts applied. An enhancement near $1970 \text{ MeV}/c^2$ is evident. A similar mass spectrum with the " $\bar{K}^{*0}$ " mass taken in the side bands outside the $\bar{K}^{*0}$ mass, which is defined to be the mass regions 0.75 to $0.80 \text{ GeV}/c^2$ and 1.0 to $1.05 \text{ GeV}/c^2$, is shown in Fig. 14. There is no similar enhancement in this side-band spectrum.

We have made careful studies of possible reflection problems and conclude that with the anti-reflection selection described above the net effect is negligible.²¹

We interpret this enhancement as evidence for $D_S(1971)$ meson and proceed to determine its branching ratio relative to $D_S^+ \rightarrow \phi\pi^+$. The fitted number of D_S^+ mesons are $87.2_{-21.0}^{+12.1}$, where the D_S^+ mass is taken to be $1970 \text{ MeV}/c^2$ from the $\phi\pi^+$ channel and the Gaussian width is fixed at $16.0 \text{ MeV}/c^2$, which is obtained from detector simulation. Comparing this to our own $D_S^+ \rightarrow \phi\pi^+$ result, we find:

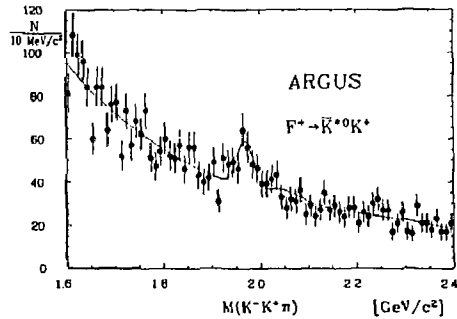


Figure 13. Mass distribution of $K^-\pi^+K^+$ with $M(K^-\pi^+)$ in the $\bar{K}^{*0}$ region. See text for other selections.

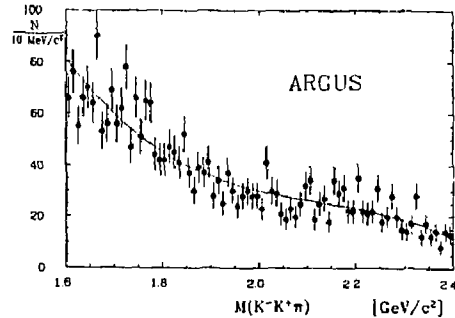


Figure 14. Mass distribution of $K^-\pi^+K^+$ with $M(K^-\pi^+)$ in the $\bar{K}^{*0}$ side band intervals, 0.75-0.80 and 1.00-1.05 GeV/c^2 . Other selections are the same as in Fig. 13.

$$\frac{\text{BR}(D_S^+ \rightarrow \bar{K}^{*0} K^+)}{\text{BR}(D_S^+ \rightarrow \phi \pi^+)} = 1.44 \pm 0.37.$$

For more details, readers are referred to Ref. 21.

To conclude, there is strong evidence for the decay mode, $D_S^+ \rightarrow \bar{K}^{*0} K^+$, in the ARGUS data, and its rate is comparable to that of the $D_S^+ \rightarrow \phi \pi^+$ mode. This result has now been confirmed by the Fermilab E691 Experiment as reported at the Berkeley Conference (1986) and this SLAC Summer Institute (1986).²² Although detailed theoretical calculations are lacking, this result on $D_S^+ \rightarrow \bar{K}^{*0} K^+$ lends further support to the importance of the non-spectator processes in the decay of charm mesons.

5. An Upper Limit on $D^0 - \bar{D}^0$ Mixing

We have looked for $D^0 - \bar{D}^0$ mixing in the cascade decay of D^{*+} , using the excellent particle identification of the ARGUS detector. No mixing was observed and we report an upper limit on $D^0 - \bar{D}^0$ mixing of 2.3% (90% confidence level).

Quark flavors are not conserved in weak interactions and the $K^0 \rightarrow K^0$ transition is allowed via the $\Delta S = 2$ box diagram.²³ The $D^0 - \bar{D}^0$ mixing is, however, highly suppressed in the six quark standard model of electroweak interaction.²⁴ Experimental upper limits are still orders of magnitude away from the values predicted in the standard model, but a better upper limit is desirable to test the high $D^0 - \bar{D}^0$ mixing rate predicted by certain theories beyond the standard model.

The two most common methods used to search for $D^0 - \bar{D}^0$ mixing are:

1. the study of the di-muon events (or tri-muon events in the case of μ -scattering experiments) arising from the semi-leptonic decay of the charm anti-charm hadron pairs²⁵ and
2. full reconstruction of the D^0 meson from its daughter particles in the cascade decay of the D^{*+} .²⁶

The best upper limits have been obtained using the di-muon events in lepto- and hadro- production of the charm. However, these results are model-dependent, since one makes assumptions on: the charm production cross section, the semi-leptonic branching fractions of all charmed particles, and the D^0 meson production ratio compared to the rest of the charmed particles produced. We follow the second method, and report an upper limit of 2.3%. The best published upper limit obtained previously in this way is 8.1% from DELCO (Yamamoto *et al.*²⁶). Recently, IIRS (Abachi *et al.*²⁶) reported an upper limit of 4.0%. On the other hand, MARK III reported observation of three wrong-sign events which could be interpreted as due to mixing at 1% level (see Ref. 16 for review).

The Q value of the decay, $D^{*+} \rightarrow D^0 \pi^+$, is only 5.8 MeV and a clean narrow signal can be observed in the $\Delta M = M(D^{*+}) - M(D^0)$ spectrum.³⁶ In the subsequent weak decay of the $D^0(c\bar{u})$, the c quark decays to an s quark, and so the sign of the charged daughter kaon is negative; thus, the sign of the charged kaon from the D^0 decay is opposite to the sign of the pion from the D^{*+} decay. If mixing were to occur, the signs would be the same. The amount of mixing can be determined by comparing the signal in the same sign combination with that in the opposite sign combination. We call the opposite sign mode a "right charge" combination, and the same sign mode a "wrong charge" combination. We use the following two decay channels of the D^0 which contain a charged kaon.

$$D^{*+} \rightarrow (D^0) \pi^+ \rightarrow (K^- \pi^+) \pi^+$$

$$D^{*+} \rightarrow (D^0) \pi^+ \rightarrow (K^- \pi^+ \pi^- \pi^+) \pi^+$$

The data sample corresponding to 153 pb^{-1} was used in the present analysis (see Table 1).

The particle identification for the charged particle is carried out in the same way as described earlier.³ Briefly, to each charged track e, μ , π , K, p mass hypotheses are assigned, and the χ^2 value for each hypothesis is obtained from

the dE/dx and TOF measurements. The normalized probability, p_i , for each mass hypothesis is calculated,

$$p_i = \frac{w_i \cdot e^{-\chi_i^2/2}}{\sum_j w_j \cdot e^{-\chi_j^2/2}} \quad i, j = e, \mu, \pi, K, p$$

where w_i are *a priori* weights. Since the pions are more abundant, the weight ratio is set to 1 : 1 : 5 : 1 : 1. If a track has a probability greater than 5% for a certain mass hypothesis, we accept that mass hypothesis. If more than one hypothesis is accepted for a track, each hypothesis is treated equally.

The general procedure we follow is to form the $K\pi$ and $K3\pi$ invariant masses, make the D^0 selection, add appropriate charged pions, plot the ΔM spectrum, and count the number of events inside the D^{*+} signal region. The contributions from the double Cabibbo-suppressed decays, e.g., $D^0 \rightarrow K^+ \pi^-$, are expected to have a rate reduced by a factor of $\sin^4 \theta_c$, and are therefore neglected.

The background contributions from particle misidentification in the Cabibbo-suppressed decays: $K^+ K^-$, $\pi^+ \pi^-$, $\pi^+ \pi^- \pi^+ \pi^-$, and the double misidentification of the charge conjugate states, can be eliminated by requiring that there be no $K-\pi$ ambiguity for the daughter particles of the D^0 , i.e., a pion must not have a kaon probability greater than 5% and *vice versa*. To further reduce the contamination, a tight cut is applied to the D^0 selection: the $K\pi$ mass must lie within 30 MeV of the D^0 mass, and the $K3\pi$ mass within 25 MeV, as Monte Carlo study shows that the particle misidentification greatly broadens the D^0 invariant mass.

A momentum cut of $P_{D^{*+}} > 2.5 \text{ GeV}/c$, along with the tight D^0 selection, virtually removes the combinatorial background.

Figure 15 shows the ΔM plot for both sign modes. We take $143 < \Delta M < 148 \text{ MeV}$ as the signal region and find 272 events in the "right sign" and 10 events in "the wrong sign". The backgrounds are estimated by fitting the histograms with a Gaussian plus third order polynomial using the MINUIT

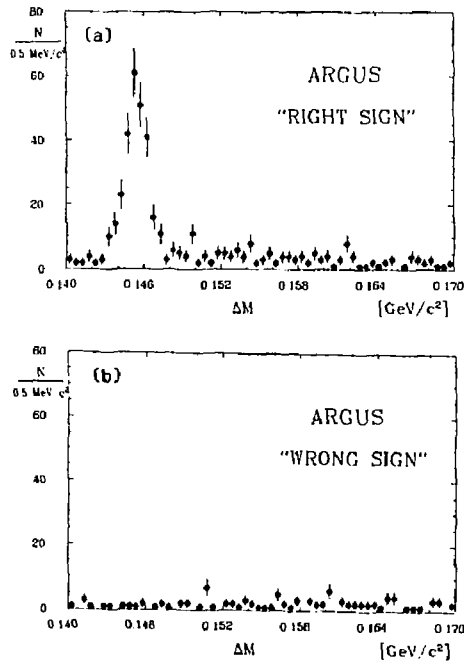


Figure 15. (a) The mass difference distribution for the "right sign" charge combinations. (b) The mass difference distribution for the "wrong sign" charge combinations.

fitting programs. The expected backgrounds are 31 for the "right sign" and 12 for the "wrong sign"; the background is higher in the "right sign" mode, because of the additional combinations coming from the events with D^{*+} . We conclude that there is no evidence for $D^0 - \bar{D}^0$ mixing and proceed to calculate the upper limit at 90% confidence level.

The determination of the confidence level of ratio of small numbers, is by no means trivial, especially when one has measurements with background. Errors on the ratios of small numbers in the classical approach is discussed by James and Roos.²⁷ Instead, we follow the likelihood density method of inference in Bayesian theory.²⁸

In our case, we are interested in the mixing rate, $p = N/(N + M)$, where M and N are the true, but unknown number of events in the "right" sign and the "wrong sign" mode. We measure instead, m and n with background B_m and B_n . The likelihood density for Poisson distributed m and n with mean values $M + B_m$ and $N + B_n$ is,

$$L(M, N) = e^{-(M+B_m)} e^{-(N+B_n)} \frac{(M+B_m)^m}{m!} \frac{(N+B_n)^n}{n!}.$$

After changing variables from (M, N) to (p, N) , we integrate over N to find the likelihood density $L(p)$, for nonvanishing B_m and B_n ,²⁹

$$L(p) = \int_0^\infty L(p, N) dN \\ = \frac{1}{Z} (1-p)^m p^n \sum_{j=0}^m \sum_{i=0}^n (i+j+1)! \binom{m}{j} \binom{n}{i} \left(\frac{B_m}{1-p} \right)^{m-j} \left(\frac{B_n}{p} \right)^{n-i},$$

where Z is a normalization factor such that $\int_0^1 L(p) dp = 1$. By inserting the values, $m = 272$, $n = 10$, $B_m = 31$, and $B_n = 12$, we obtain the 90% C.L. upper limit of 2.3%.

The mixing rate is related to the mass difference (δm) and the difference in the inverse lifetimes ($\delta \lambda$) of the two CP mass eigenstates,³⁰

$$\tau = \frac{(\delta m)^2 + \frac{1}{4}(\delta \lambda)^2}{2\lambda^2 + (\delta m)^2 - \frac{1}{4}(\delta \lambda)^2},$$

where λ is the mean lifetime. With $\lambda = 4.30 \times 10^{-13}$ s (see Ref. 35), we obtain the limit, $\delta m \leq 3.6 \times 10^{-10}$ MeV.

In conclusion, we have observed no evidence for $D^0 - \bar{D}^0$ mixing and set an upper limit of 2.3% at the 90% confidence level. The best earlier measurement reported is 0.5% (90% C.L.) from E615 at Fermilab, Louis *et al.*,²⁵ in which like-sign muon pairs produced by 255 GeV/c pions are counted. The result relies, as discussed earlier, on certain assumptions regarding production and decay of D^0 .

Our result is free of model dependencies, as the D^0 mesons are fully reconstructed.

6. τ Lifetime Measurement with ARGUS VDC

We report here very preliminary results on a measurement of the lifetime of the τ lepton. These results were obtained with a data sample corresponding to approximately 40 pb⁻¹ of data collected in 1985 in the region of the T(4S).

The installation of a high resolution vertex chamber¹⁸ has made lifetime measurements possible at ARGUS. Briefly, the vertex chamber has an inner radius of 6 cm, an outer radius of 14 cm and a length of 109 cm. There are 594 sense wires arranged in a close packed hexagonal array and each drift cell has an inscribed radius of 0.45 cm. There is approximately 3% of a radiation length between the interaction point and the first sense wire layer of the vertex chamber. By plotting the distance of closest approach between Dhabha tracks, we find that we can extrapolate tracks to the origin with an accuracy of 95 μ m in the x-y plane. For multi-hadron tracks the resolution becomes worse by a factor 1.4.

We selected τ events with a procedure similar to that described in the ARGUS ν_τ mass limit.³¹ The method of extracting the τ lifetime in an e^+e^- collision environment is by now well established, and we refer to a review article (see e.g. Ref. 32). Only a brief summary of our result is given below.

We use two different topologies, 4-prong events and 6-prong events. In the 4-prong events the decay length in the x-y plane is measured with respect to the beam crossing point. For 6-prongs, the decay length between the two τ 's is measured, again in the x-y plane. A number of cuts are made to ensure the quality of the data sample and to reduce the background. We end up with 1403 4-prong events with $(11.3 \pm 1.0)\%$ background and 100 6-prong events with $(54.6 \pm 3.0)\%$ background. The decay length distributions of 4- and 6-prong events are given in Figs. 16 and 17, respectively.

The decay length data are fit, using the maximum likelihood method, to the usual formula which has the exponential decay convoluted by the experimental resolution of decay length determination. For the 4-prong events we obtain for the mean lifetime:

$$\tau = (3.03 \pm 0.26 \pm 0.30) \times 10^{-13} \text{ s}$$

where the first error is statistical and the second systematic.

For 6-prongs we obtain:

$$\tau = (2.52 \pm 0.76 \pm 0.35) \times 10^{-13} \text{ s}.$$

These two independent results are then combined to yield:

$$\tau = (2.92 \pm 0.36) \times 10^{-13} \text{ s}.$$

This result is still very preliminary. It compares well with the world average of $(2.94 \pm 0.12) \times 10^{-13}$ s, as reported at the Berkeley conference (1986).³⁵ We

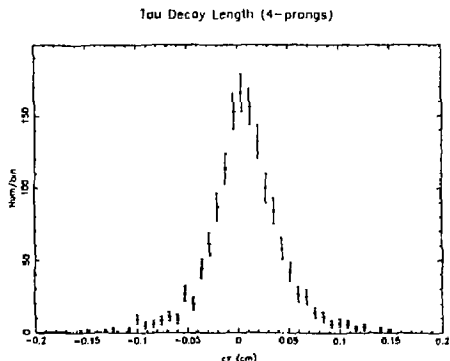


Figure 16. Proper decay length (cr) of selected 4-prong events.

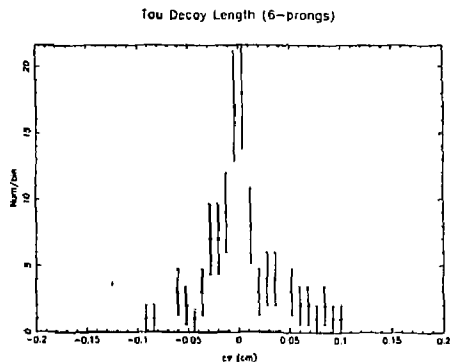


Figure 17. Proper decay length (cr) of selected 6-prong events.

anticipate to have a much larger data sample, by a factor of three, by the end of 1986, thus enabling us to make a fairly significant τ lifetime measurement in the very near future.

7. Reconstruction of B Mesons

The reconstruction of B meson decays in exclusive channels is important in understanding basic features of systems composed of a heavy quark and a light quark. First, it allows a determination of the masses of charged and neutral B mesons, and their branching ratios into various decay channels. Second, the reconstructed B mesons are essential for obtaining tagged B mesons to study Kobayashi-Maskawa matrix elements, $B^0 - \bar{B}^0$ mixing and CP-violation. Recent theoretical and experimental reviews on B physics can be found in Refs. 34 and 35.

The e^+e^- data at T(4S) is a copious source of B decays. For example, in the present ARGUS data there should be about 50,000 pairs of $B\bar{B}$ decays. However, the kinematics of their production and decay makes the task of reconstruction extremely difficult. First, B mesons are produced at very low momentum in the laboratory frame, viz. approximately 400 MeV/c. Decay products are then distributed spherically, and the jet structure observed at higher momentum, which could attribute a set of tracks to a particular B decay, is absent. For $sn\bar{s}$, the average charged multiplicity in non-leptonic B decays⁴¹ is around 6, leading to an enormous combinatorial problem. Furthermore, theoretical expectations for the branching ratios⁴² for the simplest channels such as $D\pi$, $D^*\pi$, $D^*\rho$ are below 1% level.

First evidence for fully reconstructed B mesons, with about 10 candidates, was reported in 1983 by CLEO⁴⁰ with higher branching ratios than expected.

The results presented here are based on ARGUS data corresponding to an integrated luminosity of 59 pb^{-1} collected at T(4S) energy. As it will be described below, we observe 71 ± 11 reconstructed B meson decays above back-

ground.

Exclusive B decays were reconstructed through the cascade decay chain:
 $B \rightarrow D^{*+} \rightarrow D^0$. Specifically, the following decay channels were used:

$$\begin{aligned}\bar{B}^0 &\rightarrow D^{*+}\pi^- \\ \bar{B}^0 &\rightarrow D^{*+}\pi^-\pi^0 \\ \bar{B}^0 &\rightarrow D^{*+}\pi^-\pi^+\pi^- \\ B^- &\rightarrow D^{*+}\pi^-\pi^- \\ B^- &\rightarrow D^{*+}\pi^-\pi^-\pi^0\end{aligned}$$

followed by:

$$D^{*+} \rightarrow D^0\pi^+$$

followed by:

$$\begin{aligned}D^0 &\rightarrow K^-\pi^+ \\ D^0 &\rightarrow K_S^0\pi^+\pi^- \\ D^0 &\rightarrow K^-\pi^+\pi^0 \\ D^0 &\rightarrow K^-\pi^+\pi^+\pi^-\end{aligned}$$

The $D^{*+} \rightarrow D^0\pi^+$ decay plays the pivotal role by providing background free identification of D^{*+} by means of the well known technique of exploiting the fine resolution inherent in measuring the $D^{*+} D^0$ mass difference.³⁶

In selecting participating particles in the B decay, the following selection procedures were applied:

- Charged tracks must extrapolate to a vertex of relevance with $\chi^2 < 36$.
- Charged particles are identified on the basis of specific ionization in the drift chamber and the time of flight information.³
- For π^0 reconstruction, each photon is required to have energy greater than 40 MeV.³⁷

- Mass constraint fits are performed to the intermediate states, i.e., D^{*+} , D^0 , K_S^0 and π^0 .

In the search for B candidates two principal cuts are applied:

1. Beam-energy constraint fits, i.e., $(E_B)_{fit} = E_{beam}$ are performed on those B-candidates for which $|E_B - E_{beam}| < 3\sigma(E_B)$, where E_B is the total energy of the particles participating in the B decay, E_{beam} is the beam energy of the e^+e^- collision.
2. We require the probability for the sum of all χ^2 from particle identification and from kinematic fitting to exceed 1%.

The energy constraint fit translates the good momentum resolution for the B mesons into a good mass resolution, without biasing the background distribution. The B meson mass scale after this fit will be directly correlated with the beam energy scale of DORIS.

Combinations passing these cuts, and whose mass lies in the region of interest, i.e., between $M(T(3S))/2$ and $M(T(4S))/2$, are considered to be B meson candidates. Due to the high multiplicity of reconstructed B candidates, there are events containing more than one candidate. Up to 40% of events of some decay chains have more than one B decay candidates. There are two classes of multiple candidates events:

1. Two B candidates consist of the same set of stable particle tracks that come from a true B decay. However, two of these particles may exchange their respective positions in the decay chain, giving rise to multiple candidates, having almost the same mass values.
2. The B candidate is built by tracks, some or all of which do not come from a true B decay (combinatorial background).

The first class of candidates tends to enhance a signal in the B mass spectrum artificially, whereas the second enhances the background. Thus, only one

candidate can be accepted per event and per decay chain. We do this by choosing the one with the largest probability as calculated from the sum of all χ^2 contributions from particle identification, kinematic fitting and the beam energy constraint fitting.

The mass spectrum of all B candidates thus selected is shown in Fig. 18. We observe a signal of 71 ± 11 events at a mass of the (5277.1 ± 0.8) MeV/c² with a Gaussian width of 4.5 MeV/c². This mass is obtained by using a beam energy scale which sets the mass of $\Upsilon(4S)$ to 10577 MeV/c² (see Ref. 8). If we select only those decay chains with a small combinatorial background, we find a very clean B meson sample of about 29 events (Fig. 19), which can be used for tagging purposes.

The sample is divided into neutral and charged B mesons (Figs. 20 and 21), and fits are made to obtain the masses of the B mesons. The shape of the background used in these fits are determined from event mixing, from wrong charge combinations and from the mass distributions obtained in the continuum below the $\Upsilon(4S)$. It can be described by the form:

$$\frac{dN}{dM} \sim M \cdot \sqrt{1 - \frac{M^2}{E_{\text{beam}}^2}}$$

which is derived assuming the background is uniformly distributed in phase space.

From separate fits to these distributions, we obtain:

$$M(B^0) = (5278.2 \pm 1.0 \pm 3.0) \text{ MeV}/c^2$$

$$N(B^0) = 40 \pm 8 \text{ events};$$

$$M(B^\pm) = (5275.8 \pm 1.3 \pm 3.0) \text{ MeV}/c^2$$

$$N(B^\pm) = 32 \pm 7 \text{ events};$$

and

$$M(B^0) - M(B^\pm) = 2.4 \pm 1.6 \text{ MeV}/c^2.$$

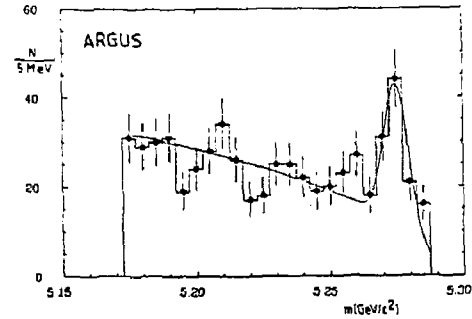


Figure 18. Mass distribution of B meson candidates.

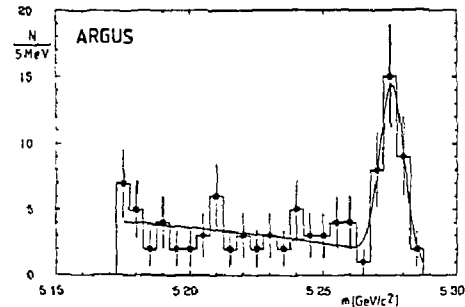


Figure 19. Mass distribution of B candidates for selected decay chains with small combinatorial background.

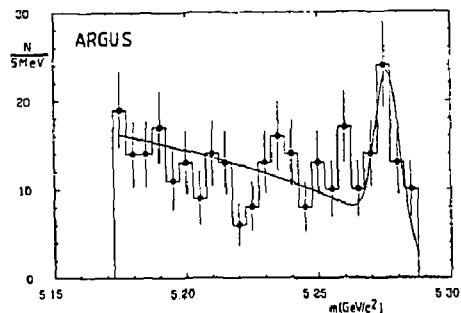


Figure 20. Mass distribution of B^0 , $\bar{B}^0$ candidates.

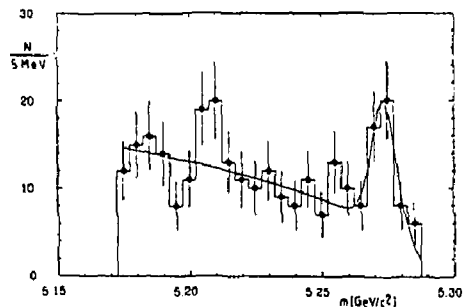


Figure 21. Mass distribution of $B^\pm$ candidates.

CLEO reported new results on the masses of the B mesons at the Berkeley Conference (1986);³⁵ these are reproduced below for comparison (systematic errors are not specified below):

$$M(B^0) = (5281.0 \pm 0.9) \text{ MeV}/c^2;$$

$$M(B^\pm) = (5277.9 \pm 1.1) \text{ MeV}/c^2;$$

and

$$M(B^0) - M(B^-) = 3.1 \pm 1.4 \text{ MeV}/c^2.$$

Agreement between these two experiments can be regarded as fair at the present level of experimental uncertainties.

Branching ratios for the five exclusive decay channels are obtained by assuming that the $T(4S)$ decays into B^+B^- pairs and $\bar{B}^0B^0$ pairs with equal probability. The branching ratios of the D^0 channels are taken from Refs. 38 and 39. The results are given in Table 3:

Table 3
Number of events and branching ratios for five exclusive B -decay channels.

	Number of Events	Branching Ratio
$\bar{B}^0 \rightarrow D^{*+}\pi^-$	5 ± 2.5	$(0.2 \pm 0.1 \pm 0.1)\%$
$\bar{B}^0 \rightarrow D^{*+}\pi^-\pi^0$	8 ± 4	$(1.0 \pm 0.5 \pm 0.6)\%$
$\bar{B}^0 \rightarrow D^{*+}\pi^-\pi^+\pi^-$	27 ± 7	$(2.2 \pm 0.8 \pm 1.0)\%$
$B^- \rightarrow D^{*+}\pi^-\pi^-$	7 ± 3	$(0.4 \pm 0.2 \pm 0.2)\%$
$B^- \rightarrow D^{*+}\pi^-\pi^-\pi^0$	24 ± 7	$(3.8 \pm 1.1 \pm 2.2)\%$

A comparison with earlier CLEO results⁴⁰ on the decays $\bar{B}^0 \rightarrow D^{*+}\pi^-$ and $\bar{B}^0 \rightarrow D^{*+}\pi^-\pi^+\pi^-$ shows strong disagreement. Recently, CLEO has reported revised branching ratios,⁴³ which are in much better agreement with those presented here.

In conclusion, we have reconstructed B mesons in the decays $B \rightarrow D^* + n\pi$ ($n = 1, 2, 3$) and determined branching ratios and masses for the $\bar{B}^0$ and B^- meson. The branching ratios, where comparable, are in better agreement with recent theoretical predictions⁴⁴ than the much larger previously reported values.⁴⁰

8. Conclusions

The ARGUS detector has continued to perform well during the past year, producing an abundance of new results. Highlights on charm spectroscopy, charm decay, $D^0\bar{D}^0$ mixing, tauon lifetime measurement, and reconstruction of B mesons have been presented here.

To summarize:

- We have confirmed the existence of a P-wave charm meson $D^{*0}(2420)$ with new data. We have seen that the $D_s^*(2110)$ is now well established with more ARGUS data and is now confirmed by MARK III results.
- We have shown that nonspectator processes manifest themselves in two decay processes with large branching ratios:

$$\begin{aligned} \text{BR}(D^0 \rightarrow \bar{K}^0 \phi) &= (1.18 \pm 0.25 \pm 0.17)\% , \quad \text{and} \\ \text{BR}(D_s^+ \rightarrow \bar{K}^{*0} K^+) &= (1.44 \pm 0.37) \times \text{BR}(D_s^+ \rightarrow \phi \pi^+) . \end{aligned}$$

- We have arrived at a new limit on $D^0\bar{D}^0$ mixing, less than 2.3% at 90% confidence level.
- We have begun to measure the lifetime of tauon with the new vertex chamber.
- We have reconstructed 71 ± 11 B mesons decaying into $D^{*+} + n\pi$, ($n = 1, 2$, or 3), enabling us to make significant measurements of the masses and a few of the branching ratios of the B mesons.

More data accumulated in the 1986 running period will provide an 80% increase in T(4S) data, and a 60% increase in data relevant to continuum physics, such as charm physics and tauon physics. Thus, we anticipate significant improvement in 1987 on most of our recent results.

Acknowledgements

It is a pleasure to thank the members of the ARGUS collaboration on whose hard work this report is based. I express my special thanks to those who had the prime responsibilities in some of the physics topics discussed here and who were so generous with their time and advice: Hartwig Albrecht, Uwe Binder, David Wilkinson, Doug Gingrich, Bob Orr, Peter Kim, Nowhan Kwak, David MacFarlane, Janis McKenna, Paul Padley, Henning Schröder and Jae-Chul Yun. I also wish to thank the organizers of the SLAC Summer Institute for pleasant and stimulating meeting.

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D_s Production Data From Mark III*

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Representing the Mark III Collaboration†

Abstract

Preliminary results on D_s decays from the Mark III at SPEAR are presented. The data were taken at a e^+e^- center of mass energy of $\sqrt{s} = 4.14$ GeV with a total integrated luminosity of 6.3 ± 0.46 pb. The decay $e^+e^- \rightarrow D_s D_s^*$, $D_s \rightarrow \phi\pi$ is observed, yielding a D_s mass of $1973 \pm 4 \pm 4$ MeV/c² and a D_s^* mass of $2110.8 \pm 1.9 \pm 3.2$ MeV/c² with a rate of $\sigma(e^+e^- \rightarrow D_s^+ D_s^{*-}, D_s^{*-} \rightarrow \gamma D_s^+ \rightarrow \phi\pi^+) = 36 \pm 7 \pm 13$ pb. There is evidence for two other modes, $D_s^+ \rightarrow K^{*0} K^+$ and $D_s^+ \rightarrow K_S^0 K^+$, which are observed with rates of $\sigma(e^+e^- \rightarrow D_s D_s^*) \cdot B(D_s^+ \rightarrow K^{*0} K^+) = 31 \pm 6 \pm 11$ pb and $\sigma(e^+e^- \rightarrow D_s D_s^*) \cdot B(D_s^+ \rightarrow K_S^0 K^+) = 16 \pm 3 \pm 5$ pb.

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This paper presents preliminary results on D_s and D_s' decays from the Mark III experiment at SPEAR. The data were taken in spring 1986 at a center mass energy of $\sqrt{s} = 4.14 \text{ GeV}/c^2$. This paper is divided into five sections. The first is an introduction that briefly reviews the expected D_s decay modes, the $D_s' - D_s$ mass difference and the expected D_s production rate. The next three sections discuss separately the analyses on the three modes, $D_s^\pm \rightarrow \phi\pi^\pm$, $D_s^\pm \rightarrow K^{*0}K^\pm$ and $D_s^\pm \rightarrow K_S^0 K^\pm$ produced from $e^+e^- \rightarrow D_s D_s'$. The last section summarizes the results.

1. Introduction

The D_s meson, a bound state of $c\bar{s}$ quarks, is expected to decay via the standard spectator diagram shown in Fig. 1a). The decay modes from this diagram include $D_s \rightarrow \phi\pi$ which has been experimentally well established.¹ Other possible diagrams are the internal W emission diagram in Fig. 1b) and the annihilation diagram in Fig. 1c). The internal W emission diagram is expected to be suppressed from color counting. Some decay modes predicted from these non-spectator diagrams are $D_s \rightarrow \bar{K}^* K$ and $D_s \rightarrow \bar{K}^0 K$.

The D_s' , the vector partner of the pseudoscalar D_s meson, will have a slightly higher mass than the D_s . The D_s' decay should be similar to that of the D_s except that the D_s' should decay only via $D_s' \rightarrow \gamma D_s$ because it has no isospin since the D_s has no u or d quarks. The simple quark model predicts a mass formula of²

$$m(q_1 q_2) = m_1 + m_2 + a \frac{\vec{s}_1 \cdot \vec{s}_2}{m_1 m_2}$$

where m_1 and m_2 are the quark masses, $\vec{s}_1$ and $\vec{s}_2$, the quark spins and a is a constant to be fitted from experimental data. This formula produces the following simple linear mass difference relation:

$$m_{D_s'} - m_{D_s} = \frac{m_c}{m_u} (m_{K^*} - m_K) = \frac{m_c}{m_u} m_{D^*} - m_{D^*} = \frac{m_c m_s}{m_u^2} (m_{D_s'} - m_{D_s}).$$

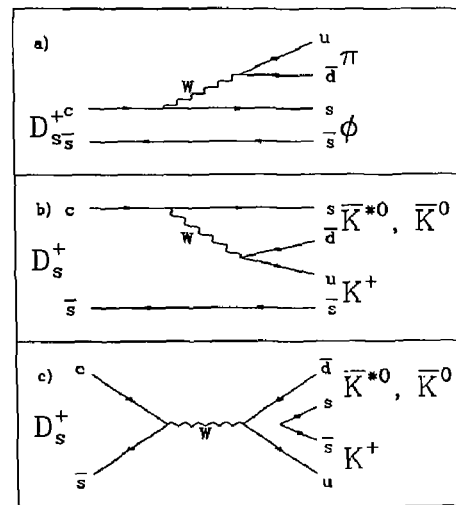


Fig. 1 D_s Weak decay diagrams

Using values for the quark masses of

$$m_u = m_d = .310 \text{ GeV}/c^2, m_s = .480 \text{ GeV}/c^2, m_c = 1.65 \text{ GeV}/c^2$$

yields the following predictions listed under model 1:

Mass Difference	Model 1	Model 2	Experiment
$K^+ - K^-$	407 MeV/c ²	470 MeV/c ²	399 MeV/c ²
$D^+ - D^-$	118 MeV/c ²	128 MeV/c ²	145 MeV/c ²
$D_s^+ - D_s^-$	94 MeV/c ²	142 MeV/c ²	140 ? MeV/c ²

Another model predicts a mass squared difference, $m^2(\text{vector}) - m^2(\text{pseudoscalar})$, that is constant.³ This difference predicts roughly equality between the difference of mass squared;

$$m_p^2 - m_\pi^2 = m_{K^+}^2 - m_{K^-}^2 = m_{D^+}^2 - m_{D^-}^2 = m_{D_s^+}^2 - m_{D_s^-}^2.$$

The results are listed under model 2. This predicts that the $D_s^+ - D_s^-$ mass difference is very close to the $D^+ - D^-$ mass difference or $\sim 140 \text{ MeV}/c^2$.

The previous experimental data on the D_s^+ comes from the TPC and ARGUS experiments.⁴ The TPC group measured a D_s mass of $m_{D_s} = 1948 \pm 28 \pm 10 \text{ MeV}/c^2$ and a mass difference of $m_{D_s^+} - m_{D_s^-} = 139.5 \pm 8.3 \pm 9.7 \text{ MeV}/c^2$. The ARGUS group obtained a mass difference of $m_{D_s^+} - m_{D_s^-} = 144 \pm 9 \pm 7 \text{ MeV}/c^2$ using an D_s mass of $m_{D_s} = 1965 \pm 3 \text{ MeV}/c^2$.

The Mark III Collaboration took data in spring 1986 in the center mass region $\sqrt{s} \approx 4 \text{ GeV}/c^2$ to confirm the D_s^+ and to search for new D_s decays. The D_s^+ 's and D_s^0 's are expected to be produced via associated production of $c\bar{s}$ and $s\bar{c}$ quarks, in the modes $e^+e^- \rightarrow D_s^\pm D_s^\mp$, $D_s^\pm D_s^0$ and $D_s^0 D_s^{\pm\mp}$.

The center mass energy was chosen in order to optimize the search for the D_s^+ . Assuming a D_s mass of $1970 \text{ MeV}/c^2$ and a D_s^0 mass of $2100 \text{ MeV}/c^2$ the following thresholds are obtained:

Mode	$\sqrt{s}$
$D_s \bar{D}_s$	3.94 GeV/c ²
$D_s \bar{D}_s^0$	4.107 GeV/c ²
$D_s^0 \bar{D}_s^0$	4.20 GeV/c ²

The easiest method to detect the D_s^+ is in the recoil against the D_s in the $e^+e^- \rightarrow D_s \bar{D}_s^0$ mode. Therefore the center mass energy was limited to $4.107 < \sqrt{s} < 4.20 \text{ GeV}/c^2$. The final choice was $\sqrt{s} = 4.14 \text{ GeV}/c^2$ which is roughly in between the limits. There could be threshold effects which could enhance or reduce the rate.

The D_s production rates can be crudely estimated. The formula for the $D_s \bar{D}_s$ cross section,

$$\sigma_{D_s \bar{D}_s} = \sigma_{\mu^+ \mu^-} \cdot 3 \left(\frac{2}{3} \right)^2 \quad (.15)$$

contains the relevant factors. The $\sigma_{\mu^+ \mu^-}$ is the point μ pair cross section. The factor 3 is for color. The $\frac{2}{3}$ factor is the charge of the charm quark and 0.15 factor is the probability of producing a strange quark pair from the sea. Inserting the numbers yields, $\sigma_{D_s \bar{D}_s} \approx 1 \text{ nanobarn}$, at $\sqrt{s} = 4.14 \text{ GeV}/c^2$. This prediction does not include any contributions from threshold effects which could be sizeable. The data run at Mark III from December 1985 to February 1986 produced a total integrated luminosity of $6.3 \pm 0.46 \text{ picobarns}$. This would produce roughly 6,000 $D_s \bar{D}_s$ pairs. Assuming an $D_s \rightarrow \phi\pi$ branching ratio of 3%, about 360 $D_s^\pm \rightarrow \phi\pi^\pm$ events should be produced in this data.

2. Analysis of the Mode $e^+e^- \rightarrow D_s D_s^*, D_s \rightarrow \phi\pi$

The analysis of the mode $e^+e^- \rightarrow D_s D_s^*, D_s \rightarrow \phi\pi$, is performed on the full data sample at a center mass energy of $\sqrt{s} = 4.1407 \text{ GeV}/c^2$ with an integrated luminosity of $6.3 \pm .46 \text{ pb}$. Events were selected with the following requirements:

- 1) Require at least three charged tracks ($\sum_i q_i = \pm 1$);
- 2) TOF identification, $t_{\text{meas}} - \frac{t_K + t_\pi}{2} > 0$;
- 3) ϕ mass requirement, $1.0095 < m(K^+ K^-) < 1.0295 \text{ GeV}/c^2$.

In the TOF requirement, t_{meas} is the measured times of the charged track and $t_K(t_\pi)$ is the predicted time of flight for a kaon (pion) hypothesis. This selects tracks that have a measured time that is closer to the predicted kaon time than to the pion time. This timing requirement is applied to both oppositely charged tracks. The $K^+ K^-$ mass distribution of pairs of tracks satisfying this selection is shown in Fig. 2. There is clear evidence for the decay mode $\phi \rightarrow K^+ K^-$. The peak has a fitted mass of $1019.6 \pm 0.4 \text{ MeV}/c^2$ which agrees with the Particle Data Group value for the ϕ mass of $1019.6 \pm .1 \text{ MeV}/c^2$.

To obtain $\phi\pi$ events, oppositely charged tracks satisfying this ϕ mass requirement are combined with each other charged track in the event which is assumed to be a pion. The reconstructed mass is plotted versus its recoil mass in Fig. 3. There is a cluster of events near a $\phi\pi$ mass of $1.97 \text{ GeV}/c^2$ and a recoil mass of $2.1 \text{ GeV}/c^2$. Projecting the recoil mass with a requirement that the reconstructed mass $1.925 < m(\phi\pi) < 2.025 \text{ GeV}/c^2$, yields clear evidence for a D_s^* near $2.1 \text{ GeV}/c^2$ as shown in Fig. 4. The projection of the converse plot of the $\phi\pi$ mass with requirements that the recoil mass $2.05 < m(\text{recoil}) < 2.025$, and $1.97 < m(\text{recoil}) < 2.05 \text{ GeV}/c^2$ are shown in Fig. 5a) and 5b). The former requirement selects the D^* in the recoil from the reaction $e^+e^- \rightarrow DD^*$ and in Fig. 5a) a clear D peak appears at $m_{D^*} = 1854 \pm 9 \pm 7 \text{ MeV}/c^2$. The latter requirement selects the D_s^* from the reaction $e^+e^- \rightarrow D_s D_s^*$ and in Fig. 5b) a clear D_s signal appears with a fitted mass of $1773 \pm 4 \pm 4 \text{ MeV}/c^2$.

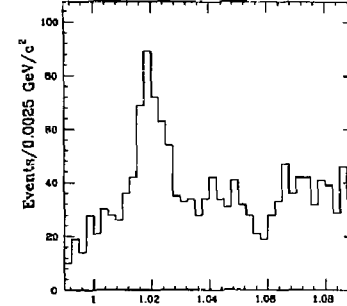


Fig. 2 $K^+ K^-$ invariant mass distribution

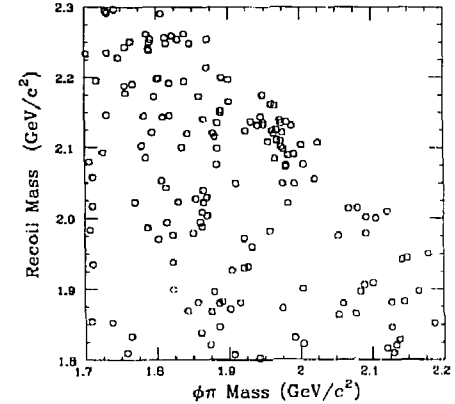


Fig. 3 Scatter Plot of $\phi\pi$ mass versus recoil mass

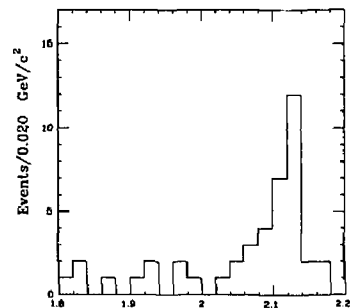


Fig. 4 Recoil mass distribution

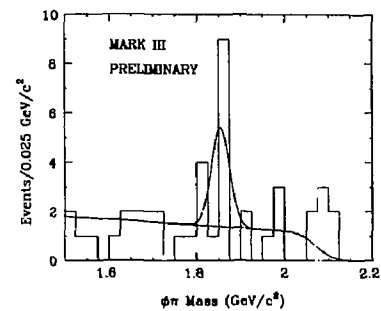


Fig. 5a $\phi\pi$ mass distribution with D requirement

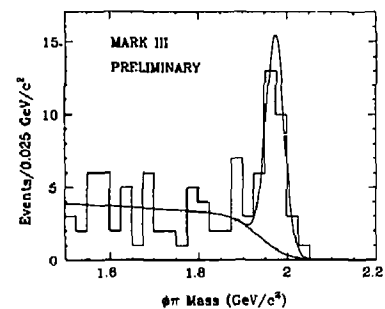


Fig. 5b $\phi\pi$ mass distribution with D_s^+ requirement

The $\cos\theta$ angle of the K^+ with respect to the direct of the ϕ is plotted in Fig. 6. The shape is consistent with $\cos^2\theta$ as expected for a pseudoscalar D_s decaying into $\phi\pi$, $\phi \rightarrow K^+K^-$.

To obtain a precise D_s^* mass, constraints are applied to the D_s^* mass calculation, assuming $e^+e^- \rightarrow D_s D_s^*$. The D_s^* mass is calculated as

$$m_{D_s^*} = \sqrt{E_{D_s^*}^2 - p_{D_s^*}^2}$$

$$m_{D_s^*} = \sqrt{(\sqrt{s} - E_{D_s})^2 - p_{D_s}^2}$$

$$m_{D_s^*} = \sqrt{(\sqrt{s} - \sqrt{m_{D_s}^2 + p_{D_s}^2})^2 - p_{D_s}^2}$$

where $E_{D_s(D_s^*)}$ and $p_{D_s(D_s^*)}$ are the energy and momentum of the $D_s(D_s^*)$. The constraint is applied by fixing the $\sqrt{s}$ and m_{D_s} to known values. The world average of 1970.5 ± 2.5 MeV/c² is used for the D_s mass in the constraint. The only measured parameter that enters in the calculation is the momentum of the D_s . The errors in the D_s^* mass and the $D_s^* - D_s$ mass difference that are produced from the uncertainties of the D_s mass are,

$$\delta m_{D_s^*} \cong -\delta m_{D_s}$$

$$\delta (m_{D_s^*} - m_{D_s}) \cong -2\delta m_{D_s}.$$

The mass difference error has a factor 2 larger error than the D_s^* mass error. The constrained mass of the D_s^* is shown in Fig. 7. The constrained recoil mass distribution is a superposition of 1) the recoil mass against the D_s produced with the D_s^* in the decay process $e^+e^- \rightarrow D_s D_s^*$, and 2) the recoil mass against the D_s which decays from the D_s^* in the decay process $e^+e^- \rightarrow D_s D_s^*$, $D_s^* \rightarrow \gamma D_s$. The Monte Carlo simulation of the former process produces a narrow peak near 2.11 GeV/c² and the latter process produces a curve with a box like shape from 2.05 to 2.15 GeV/c². The background obtained from mixing random data events

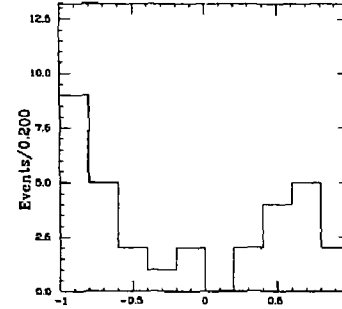


Fig. 6 $\cos\theta_K$ distribution

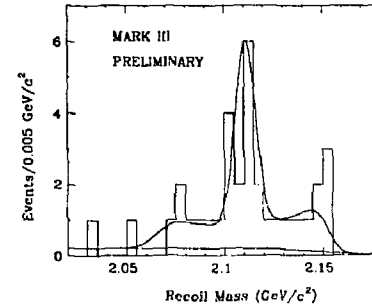


Fig. 7 Constrained Recoil Mass distribution

and this produces a flat distribution. These curves and their sum are shown in Fig. 7.

The D_s^+ mass is determined by a maximum likelihood fit. The mass of the D_s^+ is allowed to vary but the shape of the curve as determined by the Monte Carlo simulation is fixed. The fitted D_s^+ mass is

$$m_{D_s^+} = (2110.8 \pm 1.9 \pm 3.2) \text{ MeV}/c^2.$$

The first error is from the fit. The second error is the systematic error which includes the error in varying the cuts of $1.6 \text{ MeV}/c^2$, the uncertainty in the center mass energy $\sqrt{s}$ of $1.2 \text{ MeV}/c^2$ and the error of the D_s mass of $2.5 \text{ MeV}/c^2$. This result is consistent with the previous experimental measurements of the D_s and D_s^+ masses. The result supports the theoretical models which predict a constant mass squared difference.

The production rate of the D_s is obtained by fitting the number of events in Fig. 5b) with a maximum likelihood fit to a Gaussian plus a background shape. The background shape is determined from a mass distribution created by randomly combining ϕ 's of one event with π 's in another event. The total number of fitted events is 29.4 ± 5.4 . The detection efficiency is 6.3%. The resulting cross section is

$$\sigma(e^+e^- \rightarrow D_s^+ D_s^{*\mp}) \cdot B(D_s^+ \rightarrow \phi \pi^\pm) = (36 \pm 7 \pm 13) \text{ pb}.$$

If the $D_s D_s^+$ production rate is 1 nanobarn as estimated in the previous section the branching ratio for $D_s \rightarrow \phi \pi$ is roughly

$$B(D_s^+ \rightarrow \phi \pi^+) \cong 4\%.$$

This is in good agreement with previous measurements.

3. $D_s^+ \rightarrow K^{*0} K^+$ Analysis

This analysis exploits the knowledge of the D_s^+ mass by applying it as a constraint to improve the mass resolution and reject background. The following requirements are applied to the data:

- 1) Require at least three charged tracks ($\sum_i q_i = \pm 1$);
- 2) Apply 1-C fit to, $e^+e^- \rightarrow D_s^+ K^+ K^{*\mp}$, where the D_s^+ is not measured but the D_s^+ mass constraint is applied at $m=2110.8 \text{ MeV}/c^2$;
- 3) Require the TOF identification for the kaons as described in the previous section;
- 4) Apply a K^* mass requirement $.79 < m(K\pi) < .99 \text{ GeV}/c^2$;
- 5) Apply a $\cos \theta_K$ requirement of $|\cos \theta_K| > 0.5$.

After applying requirements 1-3, the resulting $K\pi$ mass distribution is shown in Fig. 8. There is a clear K^{*0} peak. The fitted values are $m = 896 \pm 4 \text{ MeV}/c^2$ and $\Gamma = 28 \pm 19 \text{ MeV}/c^2$ which agree with Particle Data Group values of, $m_{K^{*0}} = 892 \pm 0.3 \text{ MeV}/c^2$ and $\Gamma_{K^{*0}} = 51 \pm 0.8 \text{ MeV}/c^2$. To improve the signal to background ratio an additional requirement is applied on the angular distribution of the charged kaon in the rest frame of the K^{*0} . The theoretical angular distribution is $\cos^2 \theta$. Requiring $|\cos \theta_K| > 0.5$ reduces the signal by 12% and the background by 50%. The resulting $K^{*0} K^+$ mass distribution is shown in Fig. 9. There is a clear peak near $1.97 \text{ GeV}/c^2$. A background distribution of the $K^+ K^- \pi^\pm$ mass is obtained with a $K\pi$ requirement of $.75 < m(K^\pm \pi^\mp) < .80$ and $1.0 < m(K^\pm \pi^\mp) < 1.05$, as shown in Fig. 10. This is fitted with a polynomial and subtracted from the events in Fig. 9 to produce the background subtracted plot shown Fig. 11. Fitting the peak with a Gaussian results in 24 events. The Monte Carlo efficiency is 9.75%. The resulting branching ratio is

$$\sigma(e^+e^- \rightarrow D_s D_s^+) \cdot B(D_s^+ \rightarrow K^{*0} K^+) = 31 \pm 6 \pm 11 \text{ pb}.$$

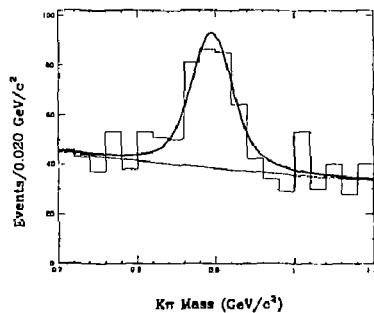


Fig. 8. $K\pi$ invariant mass distribution

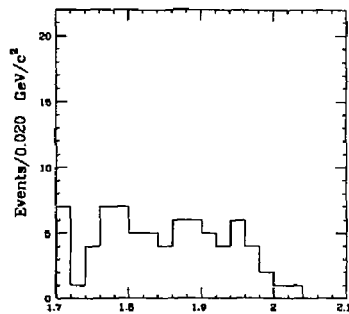


Fig. 10. K^0K background mass distribution

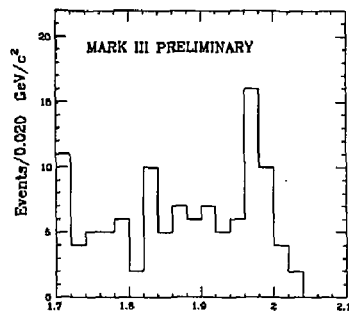


Fig. 9. K^0K invariant mass distribution

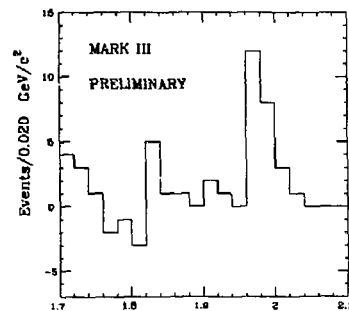


Fig. 11. K^0K background subtracted mass distribution

Several checks have been performed. The same analysis technique is applied to detect the $D_s \rightarrow \phi\pi$ events. Taking the same events produced in this analysis, the K^+K^- mass is plotted versus the $K^+K^-\pi^\pm$ masses shown in Fig. 12. There is a clear cluster of events near the ϕ mass and the D_s mass. The $K^+K^-\pi$ mass distribution with the ϕ mass requirement around the K^+K^- mass is shown in Fig. 13. There is a clear D_s peak demonstrating the effectiveness of this method.

Another check for feed down from other processes is performed by reconstructing Monte Carlo events of the processes, $e^+e^- \rightarrow D\bar{D}^*$ and $e^+e^- \rightarrow D^*\bar{D}^*$ which are expected to be produced. Generating the expected number of events, including the known D decay modes, and applying the same analysis requirements results in the $K^+K^-\pi^\pm$ mass distribution of the expected background from D decays. No spurious D_s peak is created in these processes.

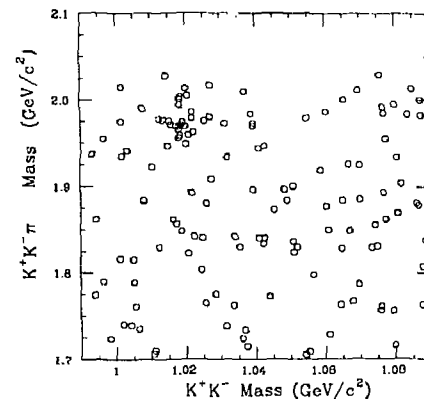


Fig. 12. K^+K^- mass versus $K^+K^-\pi$ mass

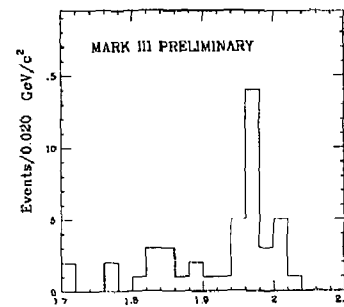


Fig. 13. $\phi\pi$ invariant mass distribution

4. $D_s^+ \rightarrow K_s^0 K^+$ Analysis

The $D_s^+ \rightarrow K_s^0 K^+$ analysis is similar to that of the previous section. The analysis steps are listed below.

- 1) Require at least three charged tracks ($\sum_i q_i = \pm 1$);
- 2) Reconstruct all oppositely charged tracks using the vector momentum at intersection of the tracks in the x-y plane and require the charged mass pair to satisfy $447 < m(\pi^+ \pi^-) < 547 \text{ MeV}/c^2$;
- 3) Require kaon TOF identification as described in the previous section for the third track;
- 4) Fit (2-C) the events to $e^+ e^- \rightarrow D_s^+ K_s^0 K^\pm$, where the D_s^+ is not measured and the mass constraints of the D_s^+ ($m=2110.8 \text{ MeV}/c^2$) and K_s^0 are imposed;
- 5) Require the decay vertex of the K_s^0 to have a positive decay length.

The reconstructed $\pi^+ \pi^-$ mass distribution is shown in Fig. 14. The mass and resolution are $476.6 \pm .12$ and $5.2 \pm .13 \text{ MeV}/c^2$. The decay length is calculated imposing the direction of the K_s^0 relative to the beam as a constraint and is required to have a positive decay length. This reduces the non- K_s^0 background. The 2-C fit to $e^+ e^- \rightarrow D_s^+ K_s^0 K^\pm$ where the D_s^+ and K_s^0 masses are constrained improves the resolution on the $K_s^0 K^+$ mass.

The $K_s^0 K^+$ mass distribution is shown in Fig. 15. There is evidence for a signal at $1.97 \text{ GeV}/c^2$. The background is smoothly varying to zero. The peak contains 32 events. The Monte Carlo efficiency is .16. The resulting cross section is

$$\sigma(e^+ e^- \rightarrow D_s^+ D_s^-) \cdot B(D_s^+ \rightarrow K_s^0 K^+) = 16 \pm 3 \pm 5 \text{ pb.}$$

Several checks were performed on the data. The data was subjected to the same analysis except fit to $e^+ e^- \rightarrow D^{*\pm} K_s^0 K^\mp$. This should detect the decay $D^+ \rightarrow K_s^0 K^+$. A small D^+ signal is observed near $1870 \text{ MeV}/c^2$ as expected.

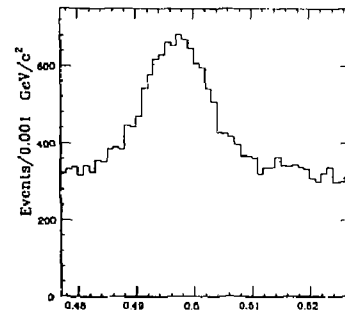


Fig. 14. $\pi^+ \pi^-$ mass distribution

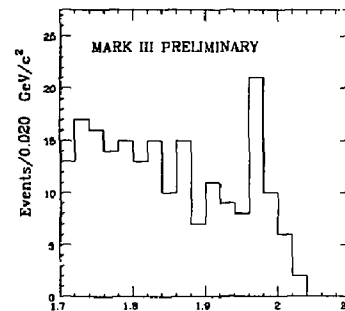


Fig. 15. $K_s^0 K^\pm$ mass distribution

To check that the signal is not due to $D_s^\pm \rightarrow \pi^+\pi^-K^\pm$, the analysis was redone with the K_s^0 mass constraint changed from $m = 497.7 \text{ MeV}/c^2$ to $397.7 \text{ MeV}/c^2$ and $597.7 \text{ MeV}/c^2$. No D_s signal is observed from this process.

To demonstrate that the D_s signal is not due to feed down from D^* decays in the data, Monte Carlo events of $e^+e^- \rightarrow D^*\bar{D}$ and $D^*\bar{D}^*$ events were generated and reconstructed. These events were analysed with the same requirements. There is little spurious D_s signals produced in these events.

5. Summary

In conclusion, associated $D_s D_s^*$ production via the $D_s \rightarrow \phi\pi$ decay mode has been observed at $\sqrt{s} = 4.14 \text{ GeV}/c^2$. The D_s^* mass was measured to be

$$m_{D_s^*} = 2110.8 \pm 1.9 \pm 3.2 \text{ MeV}/c^2.$$

There is evidence for the decay modes $D_s^{*\mp} \rightarrow K^{*0}K^\pm$ and $K_s^0 K^\pm$ modes. This is the first observation of the latter mode.

The preliminary branching ratios for these decays at $\sqrt{s} = 4.14 \text{ GeV}/c^2$ are

$$\sigma(e^+e^- \rightarrow D_s D_s^*) \cdot B(D_s^{*\pm} \rightarrow \phi\pi^\pm) = 36 \pm 7 \pm 13 \text{ pb};$$

$$\sigma(e^+e^- \rightarrow D_s D_s^*) \cdot B(D_s^{*\pm} \rightarrow K^{*0}K^\pm) = 31 \pm 6 \pm 11 \text{ pb};$$

$$\sigma(e^+e^- \rightarrow D_s D_s^*) \cdot B(D_s^{*\pm} \rightarrow K_s^0 K^\pm) = 16 \pm 3 \pm 5 \text{ pb}.$$

In the future more analysis remains to be done with his new data from Mark III. This two body decay will allow a spin-parity test of the D_s and the D_s^* . The search will be extended to more modes. This includes the $\eta\pi$ and $\rho\pi$ modes. The total number D_s events (83 at present) may be increased in order to attempt a semileptonic measurement of the D_s and to measure the absolute branching ratios of D_s decays.

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RECENT RESULTS ON UPSILON PHYSICS FROM CUSB-II

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New results on $T(3S)$ spectroscopy and new particle searches have been obtained using CUSB-II, a high resolution bismuth germanate electromagnetic spectrometer. No evidence is seen for long lived axions, light neutral Higgs's, or gluinos in radiative T decays. Confidence level upper limits on their branching ratios are presented, together with the excluded gluino mass range. The branching fraction of the $T(3S)$ into muons, $B_{\mu\mu}(3S)$, was measured to be $(1.53 \pm 0.29 \pm 0.2)\%$, the $T(3S)$ total width to be 25.5 ± 5.0 keV, $a_0 = 0.17 \pm 0.014$ and $\sqrt{s} = 148 \pm 50$ MeV. By observing the $T(3S) \rightarrow \chi_b(2P) \gamma \rightarrow T(2S) \gamma \gamma \rightarrow \ell \ell \gamma \gamma$ decay chain we resolved completely the J=2 and J=1 $\chi_b(2P)$ states. We obtain product ratios for the decay chains and derive the hadronic widths of the $\chi_b(2P)$ states. We also resolved the $\eta(0)$ state in the inclusive photon spectrum and measured the fine structure of the $\chi_b(2P)$ state.

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1. INTRODUCTION

The CUSB detector at the Cornell Electron Storage Ring (CESR) has been used to study the spectroscopy of triplet bound $b\bar{b}$ states, known as T 's ($^3S_1$) and X_b 's ($^3P_{2,1,0}$).¹ The T 's are resonantly produced in e^+e^- collisions and decay predominantly via $b\bar{b}$ quark annihilation into three gluons which subsequently hadronize mostly into pions. For a small percentage of the time ($\approx 1-3\%$) they decay into a pair of leptons. The resulting signature of two back to back leptons, when the leptons are either electrons or muons, each with $E_M/2$, allows their essentially background free identification. Although the P states of the T system (X_b 's) cannot be produced directly in e^+e^- collisions, they can be reached through electric dipole (E1) photon transitions from T 's (and can decay to the T 's via photon emission). A summary of T spectroscopy is shown in Figure 1 which shows the level diagram of the bound $b\bar{b}$ system indicating the major pion transitions (double lines), photon transitions (single lines), and final states. The ones observed with CUSB-I are shown as solid lines. The first generation experiments contributed greatly to our understanding of the spin independent part of the interquark forces. Precision measurements of the fine and hyperfine splittings of the $b\bar{b}$ states are needed for corresponding progress in the study of the spin dependent interquark forces.

Besides T spectroscopy, the T system provides us with an excellent laboratory in which to search for new particles including 1) standard model particles such as the axion, and neutral Higgs, and 2) supersymmetric particles such as gluinos. We have searched for such particles in radiative T decay; no evidence for any new particles has been found, but new upper limits on their production have been obtained.

2. THE CUSB-II DETECTOR

The CUSB-II detector has been designed to provide the necessary improvement in em energy resolution that is required to carry out the precision spectroscopy program outlined above.² The CUSB-II detector consists of a high resolution bismuth germanate (BGO) electromagnetic calorimeter inserted in the Nal array of CUSB-I. Figure 2 is a perspective drawing of the whole detector seen through partially cut away counters used for the muon trigger. The inset shows a detail of the BGO cylinder array. This cylinder consists of 36 azimuthal sectors covering 10 degrees in ϕ . Each sector is divided into two polar halves, covering the θ ranges $45^\circ-90^\circ$ and $90^\circ-135^\circ$. Each sector, 12 radiation lengths (X_0) thick at $\theta=90^\circ$, is subdivided into 5 radial layers, for a total of 360 crystals in the whole array. Between the beam pipe and the BGO cylinder are 72 $1/8''$ thick scintillators, one in front of each BGO sector. These counters were used for charged particle veto, instead of the mini jet chamber shown in Figure 2, since the latter was not installed until very recently. The BGO cylinder is surrounded by an 8 X_0 square array of 328 Nal crystals, arranged in 5 radial layers, 32 azimuthal sectors,

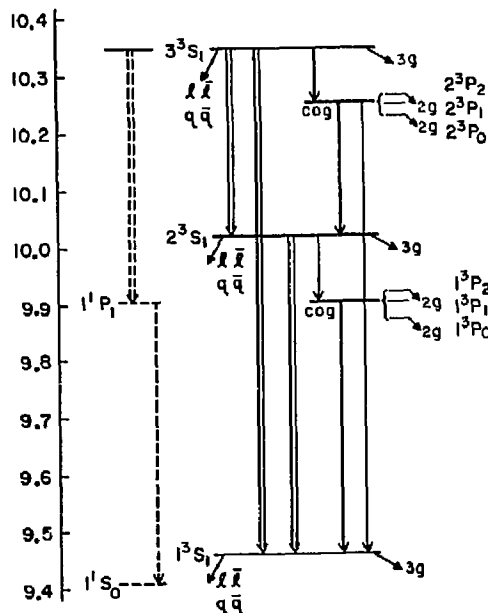


Figure 1. Level diagram of the bound $b\bar{b}$ system. The observed transitions are shown as solid lines; single for photon transitions, and double for hadronic transitions. The dashed line transitions have not yet been observed.

CUSB-II

COLUMBIA
STONY BROOK

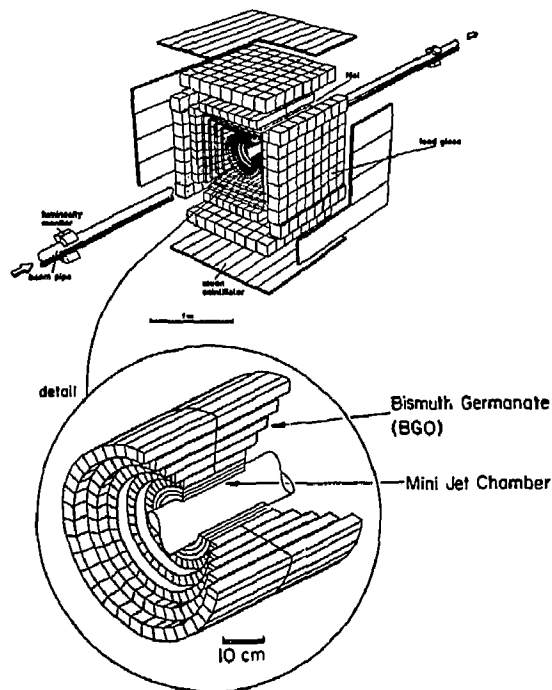


Figure 2. Schematic view of CUSB-II. Inset shows the BGO cylinder.

and 2 polar halves. Between NaI crystal layers are four proportional chambers with x and y cathode strip readout, used for tracking noninteracting charged particles. The NaI array is surrounded by four square arrays of 8x8 lead glass blocks, each 7 X_0 thick. Minimum ionizing noninteracting particles are identified, by their energy loss, 5 times in BGO, 5 times in NaI and once in lead glass, for a total of 2.5 nuclear interaction lengths. The detector covers a solid angle of 66% of 4π . Outside the lead glass, plastic scintillator counters cover the four sides of the detector. These counters cover $\approx 29\%$ of the total solid angle and provide our dimuon trigger and give time of flight information for cosmic ray rejection.

The BGO cylinder has significantly improved the energy resolution for electromagnetic showers, as is demonstrated in Figure 3, which shows a comparison of the resolution obtained for the original CUSB-I NaI detector (dashed line) vs. that obtained for the CUSB-II BGO detector (solid line). The resolution has been improved by more than a factor of two from $3.9\%/ \sqrt{E_\gamma}(\text{GeV})$ to $1.8\%/ \sqrt{E_\gamma}(\text{GeV})$.

3. SEARCH FOR NEW PARTICLES

A. AXIONS

Axions were introduced to avoid parity violation in strong interactions. The Peccei-Quinn symmetry³ leads to the existence of a pseudoscalar boson called the axion.^{4,5} In the minimal axion model in which there are two Higgs doublets, the coupling to fermions depends on the fermion mass and on a parameter x for 'up' like quarks and $1/x$ for 'down' like quarks, where $x = \langle \phi_1 \rangle / \langle \phi_2 \rangle$, whose value is unknown, is the ratio of vacuum expectation values (vev) of the two Higgs fields. Since the coupling is proportional to mass, one expects substantial branching ratios for radiative J/ψ and T decays to $a + \gamma$. The branching ratio is given by $BR(T \rightarrow a + \gamma) = G_F m_T^2 / \pi \alpha / 2 (x^2 \text{ or } 1/x^2)$, where the mass of the axion is given by $m_a = 75 \times (x+1/x)$, or $m_a \approx 150 \text{ keV}$ for $x=1$. Such an axion was excluded by combining CUSB⁶ and Crystal Ball⁷ results; the only assumption made was that the axion decayed outside our detector. With the reports of monochromatic positrons produced in heavy ion collisions,⁸ interest in nearly visible axions was renewed.⁹ If the observed signal were due to an axion, then the mass would be $\sim 1.8 \text{ MeV}$, which in turn implies a much lower symmetry breaking scale or a value of $x=1/25$. Such a small value of x leads to a large increase in the branching ratio (by about a factor of 600), but a short lifetime ($\sim 4 \times 10^{-12} \text{ s}$) thereby invalidating the previous results, which assumed that the axion decayed outside the detector. We have reanalysed a partial T(1S) data sample of 133,000 events!¹⁰ from 7 pb^{-1} of integrated luminosity, for $T \rightarrow a + \gamma$ events of the type $e e \rightarrow \gamma \gamma$ and $e e \rightarrow e \gamma \gamma$ (depending on the axion lifetime). These events have an easily identifiable topology consisting of two nearly back to back em showers; the background is due to QED events, and can be readily measured from continuum data. After background subtraction, we are left with an

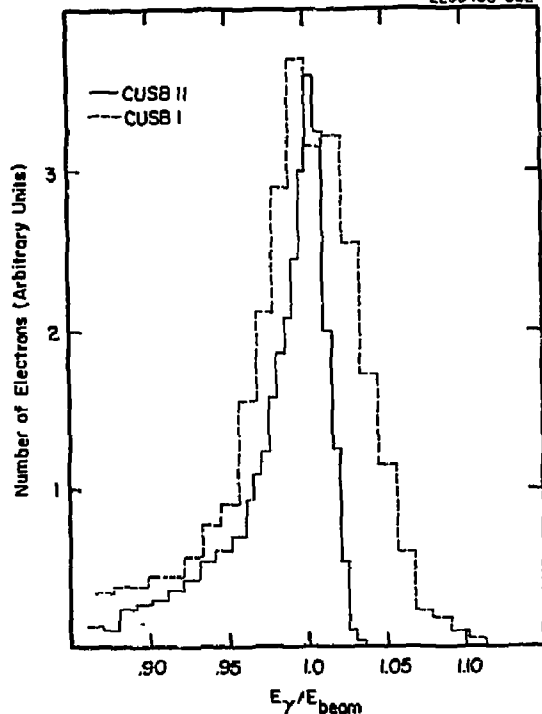


Figure 3. Electromagnetic energy resolution for Bhabha events. The dashed line is the resolution in the CUSB-I NaI detector, and the solid line is the resolution for the new CUSB-II BGO cylinder.

excess of 38 ± 30 events from which we derive upper limits on the branching ratio to $\gamma + a$ vs. the mean decay length $d = \gamma c \tau$ (curve (a) in Figure 4). The other curve (b) comes from our older result.⁴ For comparison, we note that the expected branching ratio is $\sim 25\%$, far above the upper limits of $.01\%$ to $.5\%$ seen in Figure 4.

B. LIGHT NEUTRAL HIGGS

In the minimal standard model there must be at least one neutral Higgs boson, which if light enough would be mainly produced in the decays of upsilons via the Wilczek⁵ mechanism. In that case the branching ratio is given by $BR(T \rightarrow \gamma H) = BR(T \rightarrow \mu\mu) \times (G_F m_T^2 / \pi \alpha / 2) \times (1 - (m_\mu/m_T)^2)$; last year CUSB had measured upper limits below that level,¹¹ but meanwhile radiative corrections¹² have lowered the expected branching ratios by $\sim 50\%$, thus placing them below present upper bounds. However, we can recast the experimental limits as upper bounds on the ratio $\kappa = \langle \phi_1 \rangle / \langle \phi_2 \rangle$ (same as for the axion above) due to a two Higgs doublet model, such as one would expect from supersymmetric models. The experimental upper limit on this ratio, κ , vs. Higgs mass is shown in Figure 5 for both the original Wilczek formula (lower curve) and the radiatively corrected formula (upper curve). Soon, perhaps, SUSY models will be predictive enough to confront these limits.

G. GLUINOS

In supersymmetric theories there exist supersymmetric partners for the elementary particles. The spin 1/2 partners of the gluons are known as gluinos, $\tilde{g}$, and they have been searched for by experiments at colliders and beam dumps,¹³ with the limits usually expressed in terms of the squark mass. Until now, light gluinos (mass < 5 GeV) have not been unambiguously excluded. Since gluinos carry color and are expected to interact with a uniquely determined strength, we would expect that $\tilde{g}\tilde{g}$ bound states should exist, and that they should have very similar properties to those of the T and ψ . We refer to those bound states as gluinium, $\tilde{G}$. We have searched for such gluinium states in radiative T decays, where we can use the machinery of heavy quark decays to calculate the expected rates. The rates and the total widths have been calculated by Goldman and Haber¹⁴ (extrapolating from similar decays for the η_c) and by Keung and Khare¹⁵ (in terms of the rate for $T \rightarrow \gamma g$, which is experimentally measured¹⁶). We have analyzed 400,000 T decays in a partially upgraded detector (i.e. only a portion of the BGO was in place¹⁷) for monochromatic photons in hadronic final state and isolated photon events. No structure is observed in either of the photon spectra (i.e. photons in the NaI or BGO). A maximum likelihood analysis of the combined NaI and BGO spectra yields the 90% CL upper limit shown in Figure 6 (lower curve). Note that the horizontal axis is expressed in terms of the gluino mass, which we define to be half the gluinium mass. The theoretical predictions derived from References 14 and 15 are shown as the upper curves in Figure 6, from which we conclude that gluino masses above .6 GeV and below 2.2 GeV

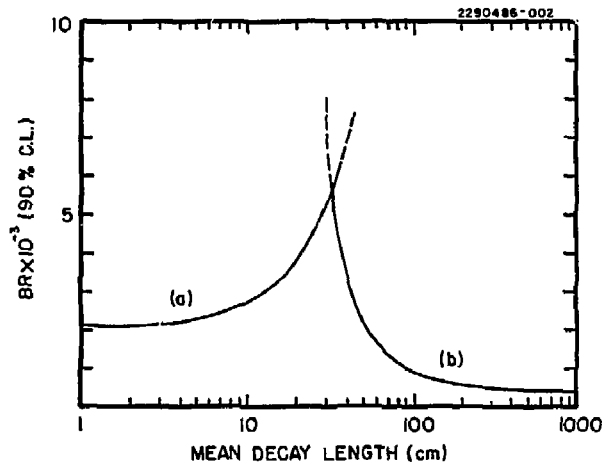


Figure 4 The 90% CL upper limit for $BR(T \rightarrow \gamma)$ vs. mean decay length of a , where (a) assumes that $a \rightarrow e$ in or before the tracking system, and (b) assumes that a decays outside the active detector volume.

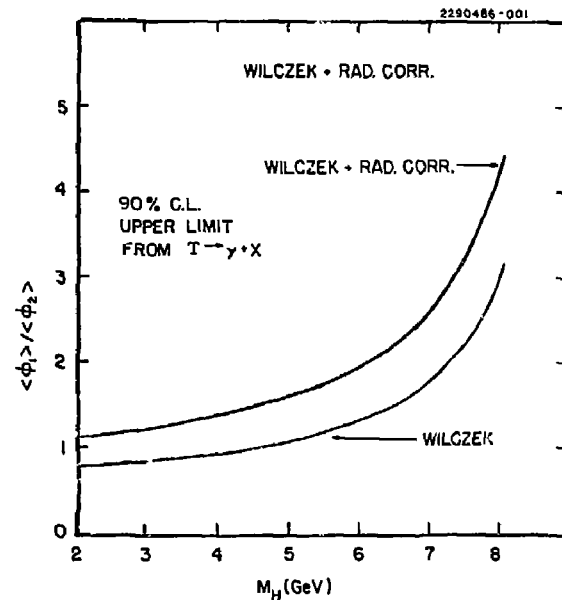


Figure 5. The 90% CL upper limit for $x = \langle \phi_1 \rangle / \langle \phi_2 \rangle$ from radiative T decay for the simple Wilczek formula (lower curve), and the radiatively corrected formula (upper curve).

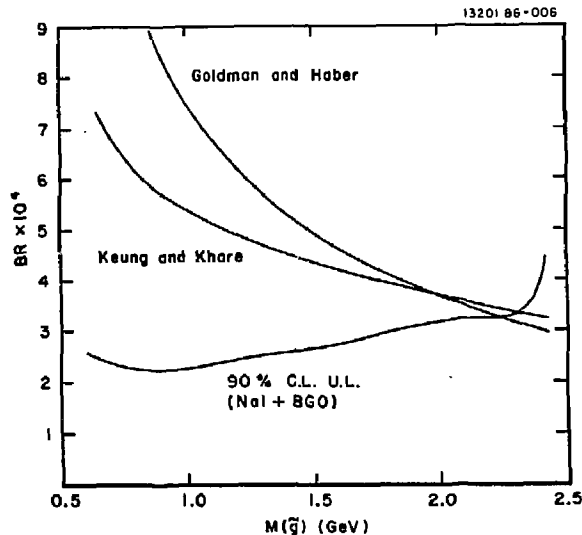


Figure 6. The upper two curves are theoretical models of references 14 and 15. The lower curve is the experimental upper limit for the $BR(T \rightarrow \gamma + \tilde{G})$ vs. gluino mass for the combined NaI and BGO data.

are excluded. Figure 7 shows the region excluded by CUSB (whose results are independent of the squark mass) compared to ee collider results and beam dump results.

4. THE BRANCHING RATIO FOR $T(3^2S_1) \rightarrow \mu\mu$

The members of the T family of mesons provide valuable testing grounds for model of the strong interaction. An important parameter to be determined experimentally is $B_{\mu\mu}$, the branching fraction into two muons. Together with the e^+e^- decay width, it allows the determination of the total decay width and of the partial widths into other channels. From $B_{\mu\mu}$, one can calculate Γ_{ggg} and therefore α_s , the coupling constant of QCD, and $\Lambda_{\overline{MS}}$, the QCD scale parameter.

We obtain $B_{\mu\mu}$ by measuring the increase of the dimuon yield at the peak of the resonant cross section relative to its value in the continuum. This increase is expected to be only of the order of 8% at the $T(3S)$. Apart from differences due to radiative corrections, muons from the QED process $e^+e^- \rightarrow \mu^+\mu^-$ and from $e^+e^- \rightarrow T(3S) \rightarrow \mu^+\mu^-$ have the same angular distribution. Most systematic uncertainties in the determination of the excess muon yield at the $T(3S)$ cancel if experimental values for resonance and continuum yields are obtained in the same detector with the same cuts applied to the data. Since the branching ratio for upsilons, above the b -flavor threshold, into muons is negligible ($B_{\mu\mu}(T(4S)) \approx 10^{-5}$), we can therefore use all the data collected on the continuum and at the $T(4S)$ resonance as "continuum" data. All data is normalized to the actual luminosity as obtained from the observed large angle Bhabha scattering yields.

Data were taken at the $T(3S)$ ($\approx 41 \text{ pb}^{-1}$) and $T(4S)$ ($\approx 19.5 \text{ pb}^{-1}$) peaks and in the continuum just below the $T(4S)$ ($\approx 9.5 \text{ pb}^{-1}$). The dimuon hardware trigger required a coincidence among muon counters on opposite sides of the detector, plus at least 100 MeV in the outer three layers of NaI and BGO (dimuons lose 180 MeV in NaI and 170 MeV in BGO). This trigger is 100% efficient for muons entering the counters and is dominated by soft particles crossing counters and the BGO and NaI arrays. Most background is trivially removed by requiring that the observed energy signals be consistent with two noninteracting particles with origin at the interaction point. Only events with two muon tracks and no other energy clusters in BGO or NaI are used for the determination of $B_{\mu\mu}$. The remaining background consists of cosmic ray muons, mostly vertical, in accidental coincidence with the beam bunch crossing time in the hardware trigger acceptance window of $\approx 30 \text{ ns}$. This background can be identified and rejected by using muon time of arrival and tracking information, as shown in the arrival time plot of Figure 8(a) for events that trigger the horizontal muon trigger counters, and (b) for 'vertical' events (notice that cosmic rays are easily recognized as a second peak). We measure the efficiency for detecting the $\mu^+\mu^-$ pairs from $T(3S)$ decay by comparing the number of dimuon events observed in the continuum to the number calculated from the luminosity and the continuum cross section. This gives the continuum

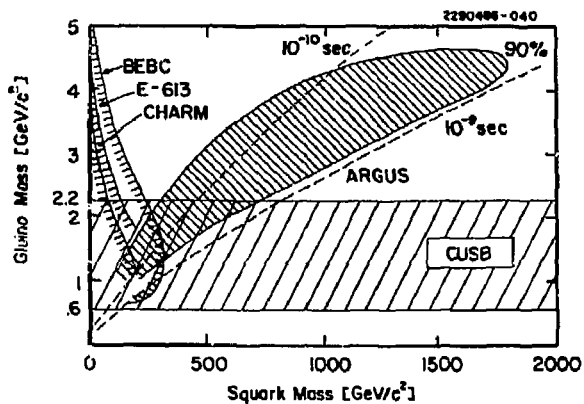


Figure 7. The gluino mass region excluded by CUSB, together with the beam dump and ARGUS limits.

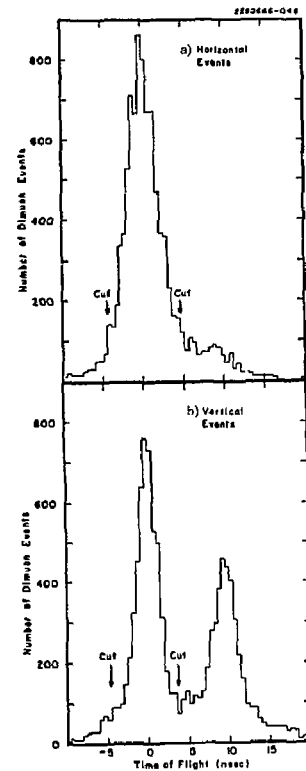


Figure 8. Muon scintillator timing for (a) horizontally incident muons, and (b) for vertically incident muons. The timing cuts are shown on the plots.

efficiency which is corrected, using the Monte Carlo of Berends and Kleiss¹⁸ for initial state radiation.

We obtain

$$B_{\mu\mu}[T(3S)] = (1.53 \pm 0.29 \pm 0.21) \%$$

where the first error is statistical and the second systematic. This value is to be compared to the previous measurement of $(3.3 \pm 2.0) \%$.¹⁹ Since all decay widths into channels involving $b\bar{b}$ annihilation scale as $|\psi(0)|^2/M^2$ one can also write²⁰

$$B_{\mu\mu}[T(3S)] = B_{\mu\mu}[T] \times \{(1 - B_{\pi\pi}[T(3S)] - B_{E1}[T(3S)])\},$$

where $B_{\pi\pi}$ and B_{E1} are the branching fractions for $T(3S) \rightarrow (T(2S) + \pi\pi)^{21}$ and $T(3S) \rightarrow \chi_b(2P) \gamma$.²² Using $B_{\mu\mu}[T] = 0.028 \pm 0.002$ one obtains $B_{\mu\mu}[T(3S)] = (1.56 \pm 0.18) \%$ in good agreement with our direct measurement.

The measured value of $B_{\mu\mu}$ together with our measurement of the $T(3S)$ leptonic width $\Gamma_{ee} = 0.39 \pm 0.02 \text{ keV}^{23}$ determines the total width of the $T(3S)$ to be $\Gamma_{\text{tot}}[T(3S)] = 25.5 \pm 5 \text{ keV}$.

Using this value for $B_{\mu\mu}$, we can calculate $\Gamma_{ggg}/\Gamma_{\mu\mu}$, where Γ_{ggg} is the 3 gluon decay width, and we obtain $\Gamma_{ggg}/\Gamma_{\mu\mu} = 28.6 \pm 7.0$. Our $B_{\mu\mu}[T(3S)]$ corresponds to

$$\alpha_s[0.48M_{T(3S)}] = 0.17 \pm 0.014, \quad \overline{M_{\overline{MS}}} = 148 \pm 50 \text{ MeV}.$$

These results compare well with our previous determinations of

$$\alpha_s[0.48M_{T(1S)}] = 0.172 \pm 0.010, \quad \overline{M_{\overline{MS}}} = 140 \pm 30 \text{ MeV}, \text{ and}$$

$$\alpha_s[0.48M_{T(2S)}] = 0.154 \pm 0.019, \quad \overline{M_{\overline{MS}}} = 92 \pm 40 \text{ MeV}.^{24}$$

5. $T(3S) \rightarrow \chi_b(2P) \gamma \rightarrow [T(2S)] \gamma \gamma \rightarrow \mu^+ \mu^- (e^+ e^-) \gamma \gamma$ AND HADRONIC WIDTHS OF THE $\chi_b(2P)$ STATES

The 2P states of the upsilon system cannot be produced directly in e^+e^- annihilations, instead they can be observed in E1 photon transitions from higher lying S states. CUSB-1 has observed both the 1P and the 2P states in this way,^{22,25} however since the fine structure splitting of the 2P states of the upsilon system is so small (15 to 20 MeV), the three spin states ($J=0,1,2$) that make up the 2P state were not explicitly resolved. The E1 photons from these transitions have been observed by two methods: 1) in the inclusive photon spectrum, and 2) in a study of the decay chain $T(3S) \rightarrow \chi_b(2P) \gamma \rightarrow [T(2S)] \gamma \gamma$ where the T's decay leptonically. The second class of events usually shows superior energy resolution because they are free from both the large photon background and from overlaps of energy depositions from neighboring particles.

In the following we report on the first $\chi_b(2P)$ results from CUSB-11, starting with an analysis of 'exclusive' events. The data reported in this section comes from an integrated luminosity of 41.4 pb^{-1} collected at the $T(3S)$ peak energy. A total of 2.89×10^5 hadronic e^+e^- annihilation events were observed, corresponding to 1.82×10^5 $T(3S)$ produced. We also collected data above the b flavor threshold and at the continuum above the $T(3S)$ (integrated luminosity $\approx 15.6 \text{ pb}^{-1}$).

For exclusive event candidates, we can classify the pair of photons according to their energies. For pairs of photons which come from transitions via an intermediate state, the sum of the two energies must equal the energy difference between the initial and final state T's (of which there are two possible final states, $T(1S)$ or $T(2S)$). Thus candidate photon pairs must lie on either of the two dashed lines shown in Figure 9, which is a scatter plot of the higher energy photon (E_{high}) versus the lower energy photon (E_{low}) for $\mu\mu\gamma\gamma$ events at the $T(3S)$ peak. Similarly, Figures 10 and 11 are the corresponding plots from $e\gamma\gamma$ events on the $T(3S)$ and $e\gamma\gamma$ events obtained from 15.6 pb^{-1} of continuum running. In all three plots the vertical dotted line is the E_{γ} lower energy cut and the diagonal dotted line is the reflection boundary. Doppler broadening of up to 35 MeV can occur for the second photon for the transition $T(3S) \rightarrow \gamma\gamma T$.

The data cluster, in Figures 9 and 10, around 80-100 MeV for the lower energy γ , and either around 230 or 760 MeV for the higher energy photon, confirming their origin as being due to the cascade chain $T(3S) \rightarrow \chi_b(2P) \gamma \rightarrow (T(2S) \text{ or } T(1S)) \gamma\gamma$ respectively. The distribution of Figure 11 has a higher density of events in the region where both γ 's have low energy. This indicates that background to the $e\gamma\gamma$ events come from multiple soft radiation of the electrons, as confirmed by our Monte Carlo calculations.

Table 1. Summary of fit results for exclusive event analysis.

Final State	E_{γ} (MeV)	σ_E (MeV)	Events	$\chi^2/\text{d.o.f.}$
(1^3S_1)	85.4 ± 0.6	2.8	34.5 ± 6.1	6.9/22
$+(2^3S_1)$	100.1 ± 0.5	3.2	48.7 ± 7.2	
background level = 0.5 event/2.5 MeV bin				
(1^3S_1)	85.2 ± 0.7	2.8	22.1 ± 6.7	8.6/22
	100.6 ± 0.9	3.2	14.5 ± 2.8	
background level = 0 event				
(2^3S_1)	85.8 ± 1.1	2.8	12.2 ± 3.8	6.8/22
	100.4 ± 1.1	3.2	29.4 ± 5.7	
background level = 0.5 event/2.5 MeV bin				

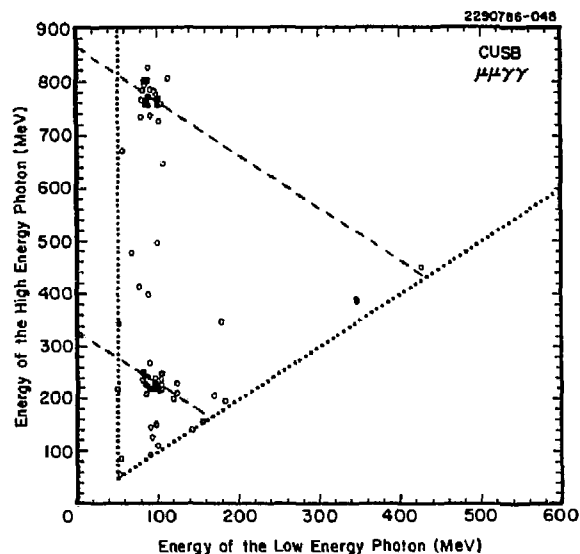


Figure 9. Scatter plot of the high energy photon vs. the low energy photon for exclusive events at the T(3S) energy with two photons and two muons in the final state. The horizontal dashed lines represent the energy difference between the initial and final state T. The vertical dotted line is the low energy photon cut.

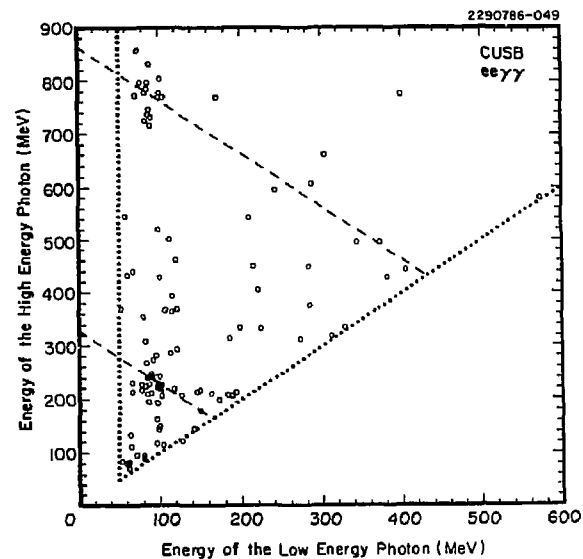


Figure 10. Scatter plot of the high energy photon vs. the low energy photon for exclusive events at the T(3S) energy with two photons and two electrons in the final state. The horizontal dashed lines represent the energy difference between the initial and final state T. The vertical dotted line is the low energy photon cut.

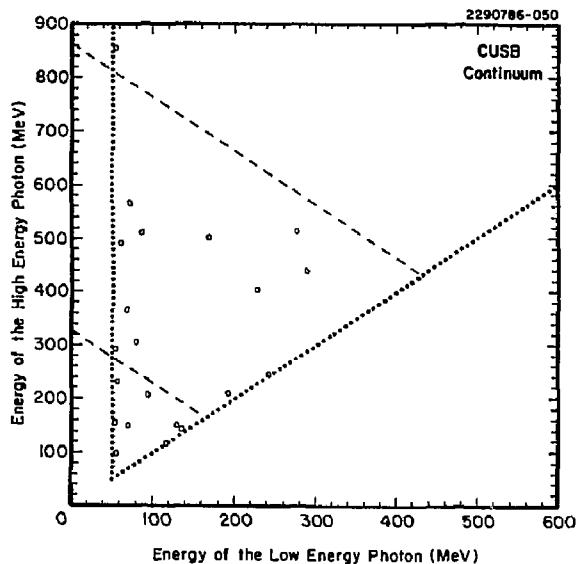


Figure 11. Scatter plot of the high energy photon vs. the low energy photon for exclusive events on the continuum. Note that no clustering is observed.

Figure 12 shows the energy spectrum of $E_{\gamma\text{low}}$ ($130 > E_{\gamma\text{low}} > 65$ MeV) for $T(3S) \rightarrow \chi_b(2P) \gamma \rightarrow [T(2S)] T(1S) \gamma \gamma \rightarrow 2\ell \gamma \gamma$ candidates, i.e. events for which the sum of the two photon energies lie between 250 and 375 MeV or between 800 and 920 MeV. Figures 13(a) and 13(b) are similar spectra for $T(3S) \rightarrow \chi_b(2P) \gamma \rightarrow T(2S) \gamma \gamma \rightarrow 2\ell \gamma \gamma$ candidates (the sum of the two photon energies lie between 250 and 375 MeV), and $T(3S) \rightarrow \chi_b(2P) \gamma \rightarrow T(1S) \gamma \gamma \rightarrow 2\ell \gamma \gamma$ candidates (the sum of the two photon energies lie between 800 and 920 MeV) respectively. The superimposed curves are fits using two Gaussians of width $\sigma_E = 1.8\% \sqrt{E_\gamma} \text{ (GeV)}$, the resolution determined from Bhabha events and Monte Carlo modellings. Two completely resolved photon lines are evident in all plots. All the fits are statistically excellent and are consistent with each other, as seen in Table 1.

By dividing the CUSB-II product branching ratios by the CUSB-I inclusive transition rates of $3^3S_1 \rightarrow 2^3P_{J,0}$ of $12.7 \pm 4.1\%$, $15.5 \pm 4.2\%$, and $7.6 \pm 3.5\%$ respectively,²² we obtain the branching ratios from the 2^3P_J states to the 2^3S_1 , 1^3S_1 states. CUSB-II efficiencies used for computing the product branching ratios are included in Table 2. In the product branching ratio calculations we use $B_{\mu\mu}(1^3S_1) = 0.028 \pm 0.002$ and $B_{\mu\mu}(2^3S_1) = 0.017 \pm 0.004$ as input.

Table 2. Summary of product branching ratios.

2^3P_J State	ϵ	$\text{BR}(3^3S_1 \rightarrow 2^3P_J \gamma) \times \text{BR}(2^3P_J \rightarrow 2^3S_1 \gamma)$	$\text{BR}(2^3P_J \rightarrow 2^3S_1 \gamma)$
J=2	0.21	$(1.9 \pm 0.7)\%$	$(15 \pm 8)\%$
J=1	0.25	$(3.8 \pm 1.2)\%$	$(24 \pm 10)\%$
J=0†	0.20	$(0.3 \pm 0.2)\%$	$(4 \pm 3)\%$
2^3P_J State	ϵ	$\text{BR}(3^3S_1 \rightarrow 2^3P_J \gamma) \times \text{BR}(2^3P_J \rightarrow 1^3S_1 \gamma)$	$\text{BR}(2^3P_J \rightarrow 1^3S_1 \gamma)$
J=2	0.22	$(2.0 \pm 0.4)\%$	$(16 \pm 6)\%$
J=1	0.26	$(1.1 \pm 0.3)\%$	$(7 \pm 3)\%$
J=0	0.21	$< 0.2\% \text{ at } 90\% \text{ C.L.}$	$< 3\% \text{ at } 90\% \text{ C.L.}$

† The 1.75 events with $E_\gamma \approx 125$ MeV are assumed to be due to this transition.

We used the measured branching ratios for $3P_J \rightarrow 3S_1 + \gamma$ transitions (BR_J) together with EL rates calculated using potential models to obtain the hadronic widths of the P-states. These are given by:

$$\Gamma_{\text{hadronic}}(^3P_J) = \Gamma_{\text{EL}}(J) [1 - \text{BR}_J] / \text{BR}_J$$

where J is the state's total angular momentum. The widths so

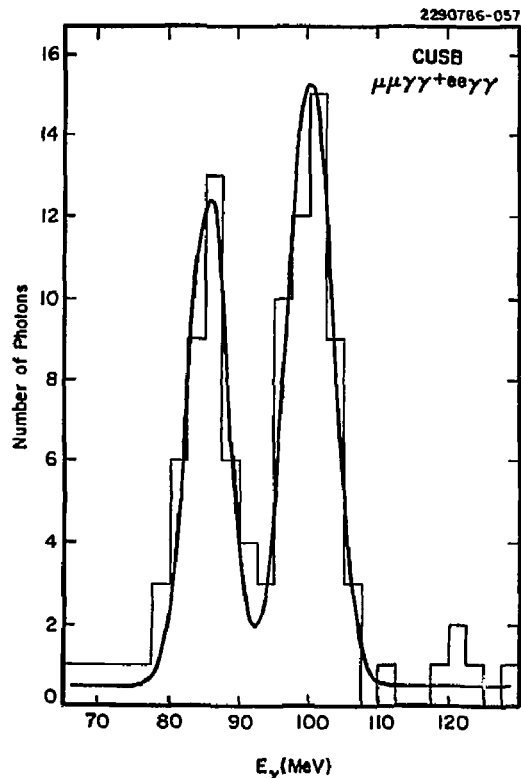


Figure 12. Energy spectrum of $E_{\gamma \text{ low}}$ for $T(3S)-\chi_b(2P)-\gamma-T(2S)+T(1S)-\gamma\gamma\gamma$ candidates, i.e. E_γ lie between 250 and 375 MeV, or between 800 and 920 MeV.

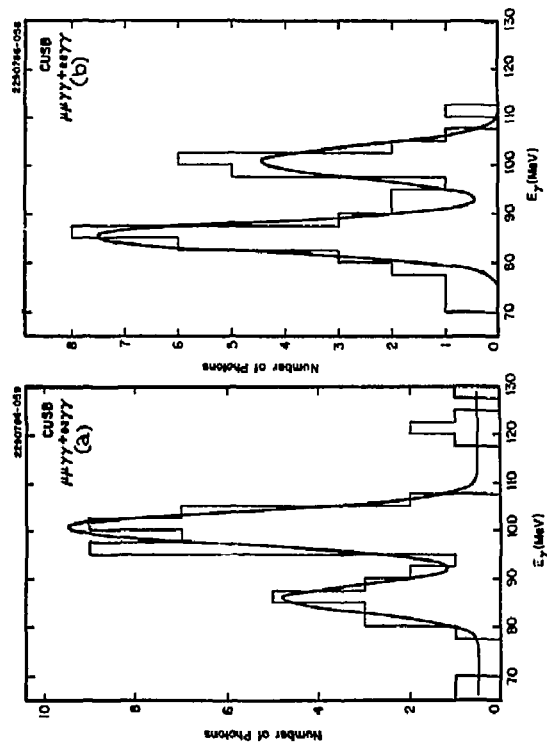


Figure 13. (a) Energy spectrum of $E_{\gamma \text{ low}}$ for $T(3S)-\chi_b(2P)-\gamma-T(2S)-\gamma-T(1S)-\gamma\gamma\gamma$ candidates, 375 < E_γ < 250 MeV. (b) Energy spectrum of $E_{\gamma \text{ low}}$ for $T(3S)-\chi_b(2P)-\gamma-T(1S)-\gamma\gamma\gamma$ candidates, 920 < E_γ < 800 MeV.

obtained are shown in Table 3 for five potential models: GRR,²⁴ MR,²⁷ MB,²⁸ EI,²⁹ BT.³⁰

QCD predictions of the hadronic widths by Olsson et al.³¹ using the lowest order QCD calculations of Barbieri et al.³² (with $\alpha_s=0.165$) are scaled by us from 1^3P_J to 2^3P_J :

$$\begin{aligned}\Gamma_{\text{had}}(2^3P_2) &= 123 \text{ keV}, \\ \Gamma_{\text{had}}(2^3P_1) &= 38 \text{ keV}, \\ \Gamma_{\text{had}}(2^3P_0) &= (15/4) \times \Gamma_{\text{had}}(2^3P_2) = 440 \text{ keV}.\end{aligned}$$

Table 3. Hadronic widths of the $2P$ states for various models.

Model	J	$\Gamma_{\text{El}}^{2P_J \rightarrow 2S}$	$\Gamma_{\text{had}}^{2P_J}$	$\Gamma_{\text{El}}^{2P_J \rightarrow 1S}$	$\Gamma_{\text{had}}^{2P_J \rightarrow 1S}$
GRR	2	23	122±78	9.7	30±24
GRR	1	19.4	50±32	9.2	102±49
GRR	0	14.9	379±370	8.5	>280 at 90% C.L.
MR	2	16	78±54	14	60±35
MR	1	14	31±23	12	145±64
MR	0	12	304±298	7.5	>248 at 90% C.L.
MB	2	13.7	68±46	10.6	44±26
MB	1	13.6	35±23	7.2	82±38
MB	0	12	311±298	1.3	>33 at 90% C.L.
EI	2	17	85±57	13	54±32
EI	1	17	40±28	13	156±69
EI	0	17	429±422	13	>434 at 90% C.L.
BT	2	16	81±54	9	33±22
BT	1	16	41±27	9	104±48
BT	0	16	407±397	9	>296 at 90% C.L.

†All widths are measured in keV's.

We note that the hadronic widths obtained from using the El rates for the $2P_J \rightarrow 2S$ transitions (left column of the above table) are in general agreement with the QCD expectations listed above. This situation is very similar to the one we studied using Γ_{El} rates for $1P_J \rightarrow 1S$ transitions and argues that with improved statistical and systematic accuracy we can hope to get a good determination of hadronic widths of the P states using this indirect method, as well as obtaining checks on QCD relations. In fact, even at present we confirm the predicted ratio of 15:4 for the width of the J=0 and J=2 states.

However, we find an inversion of the relative widths for the J=2 to J=1 states when we use the measured BR's and model calculations of El rates for the $2P_J \rightarrow 1S$ transitions (right column of the above table). This effect is partly due to the fact that we measure a larger BR for the J=2 state than for the J=1 state while the potential models do not predict a correspondingly smaller absolute El transition rate from the J=1 state relative to the J=2 state. It is known that these transition rates are very sensitive to relativistic corrections, as can be seen by examining the range of $\Gamma_{\text{El}}(2P_J \rightarrow 1S)$ listed in the table (the last two models do not exhibit J dependence because they are nonrelativistic potential models). If this effect persists as we accumulate more statistics, we may have found one of the few anomalies in the heavy quarkonium systems.

6. FINE STRUCTURE OF THE $\chi_b(2P)$ STATES

The data used in the following is from a run partially marred by excessive background. The presently recovered portion corresponds to an integrated luminosity of 23 pb^{-1} collected at the T(3S) peak energy, yielding a total of 1.46×10^5 detected hadronic events corresponding to $\approx 1.03 \times 10^5$ produced T(3S).

The photon search codes used are based on the CUSB-1 photon algorithms which use longitudinal segmentation for identifying electromagnetic showers. Because of the improved resolution of the new calorimeter fine tuning is necessary to optimize the somewhat orthogonal requirements of high efficiency and resolution. Acceptance and the resolution function are then obtained by adding photons of fixed energies generated with "EGS" to real hadronic events. Figure 14 shows the recovered peak for 100 MeV Monte Carlo (MC) photons and the resolution curve obtained. The preliminary acceptance*efficiency determined for photons of energy 100-200 MeV in decays $T(3S) \rightarrow \gamma + \chi_b(2P) \rightarrow \gamma + \text{hadrons}$ is $10 \pm 2\%$, with $\sigma_E/E \approx 1.9\%$, which is ≈ 1.2 times worse than the optimal $\sigma_E/E = 3.2\%$ obtained from low (≈ 4) multiplicity events. For photons of energy between 70 and 1000 MeV we find the rms energy spread function well described by $\sigma_E/E = (\kappa \alpha / \sqrt{E}) (\text{GeV})$ with $\kappa = 2.2$.

In Figure 15 we show the inclusive photon spectrum obtained at the T(3S) peak energy, plotted in constant $3\% E_\gamma$ energy bins. It has several definite, distinct structures, standing out on top of a large background, as shown by curves in Figure 15. The data was fitted with five resolution functions of energy dependence as specified above, but with κ , area and position as free parameters, plus a polynomial to account for the background. The fit yields $\kappa = 2.2 \pm 0.2$, in good agreement with the Monte Carlo simulation results discussed above. Figure 16(a) shows the spectrum obtained after subtracting the polynomial background curve shown in Figure 15. We note that the first two lines are partially merged but the third is well resolved. Figure 16(b) shows the two individual contributions to the first peak. In the CUSB-1, the three signals were merged into one peak. We identify these three lines with the

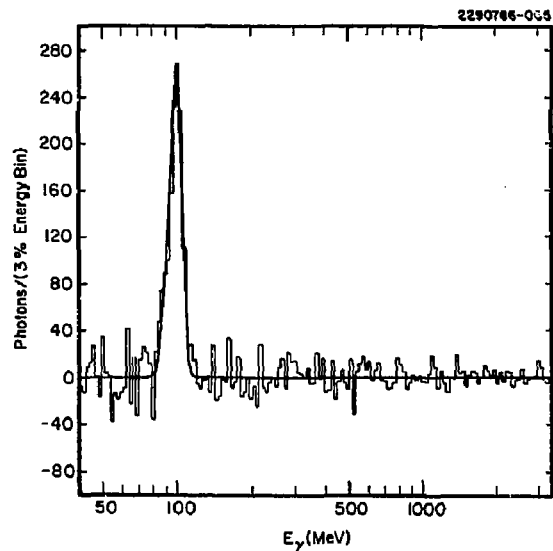


Figure 14. The energy resolution for CUSB-II using MC simulated 100 MeV photons.

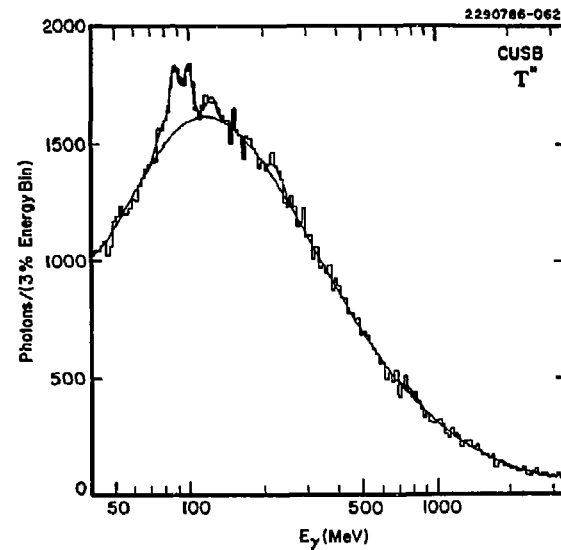


Figure 15. The inclusive photon spectrum from $e^+e^- \rightarrow \text{hadrons}$ at the $T(3S)$ energy.

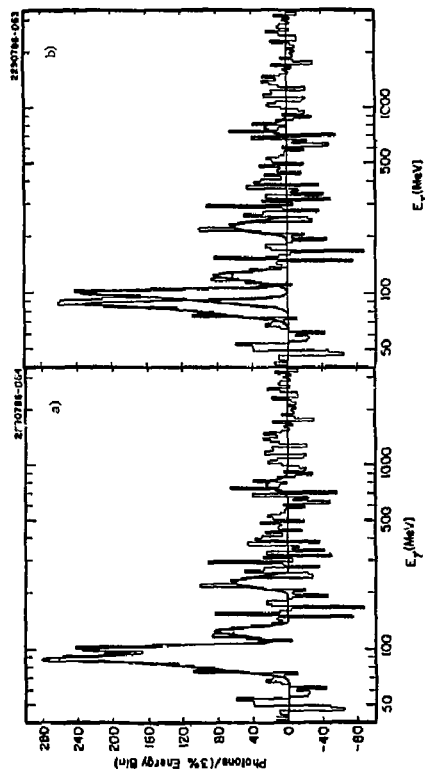


Figure 16. (a) Background subtracted inclusive photon spectrum with fit. (b) Individual line decomposition.

$T(3^3S_1) \rightarrow \chi_b(2^3P_{2,1,0}) \gamma$ transitions. The fourth peak at E_γ of ≈ 230 MeV, is actually the merged result of two signals of equal strength which are separated 15 MeV apart, and is due to the photon transitions from the $\chi_b(2^3P_{2,1})$ to the $T(2^3S_1)$. These transitions are seen for the first time in an inclusive photon spectrum. The fifth peak falls at an energy which corresponds to the $\chi_b(2^3P_{2,1})$ to the $T(1^3S_1)$ photon transitions and therefore is suggestive of this source. However, it is barely a two sigma effect, and we still rely on "exclusive" events (of the previous section) where $T(3S) \rightarrow \gamma \chi \gamma \gamma T \rightarrow \ell \ell \gamma \gamma$ to obtain the branching ratios for $2^3P_J - 1^3S_1$.

In Table 4 we list the peak position, the area, and the branching ratio for each of the five peaks. The quantities in the parentheses are the published CUSB-1 values, included for comparison. They are in good agreement with our new results.

Table 4. Fit parameters from the inclusive photon analysis.

Transition	Energy (MeV)	Events	Branching Ratio(%)
$3^3S_1 \rightarrow 2^3P_2$	86.5 ± 0.7	1060 ± 176	$12.8 \pm 2.1 \pm 2.6$
$3^3S_1 \rightarrow 2^3P_1$	99.3 ± 0.8	972 ± 158	$11.7 \pm 1.9 \pm 2.3$
$3^3S_1 \rightarrow 2^3P_0$	124.2 ± 2.3	441 ± 162	$5.3 \pm 2.0 \pm 1.1$
$3^3S_1 \rightarrow 2^3P_2$	(84.2 ± 2)		(12.7 ± 4.1)
$3^3S_1 \rightarrow 2^3P_1$	(101.4 ± 3)		(15.6 ± 4.2)
$3^3S_1 \rightarrow 2^3P_0$	(122.1 ± 5)		(7.6 ± 3.5)
$\langle E_\gamma \rangle$	$95.0 \pm 0.3 \pm 1.0$		$(94 \pm 0.5 \pm 1.6)$
$\langle M(2^3P_J) \rangle$	$10260.0 \pm 0.3 \pm 1.7$		$(10260.9 \pm 0.5 \pm 1.6)$
$2^3P_{2,1} \rightarrow 2^3S_1$	227.3 ± 5.0	386 ± 141	$4.7 \pm 1.7 \pm 0.9$
$2^3P_{2,1} \rightarrow 1^3S_1$	773.0 ± 26.5	152 ± 81	$1.8 \pm 1.0 \pm 0.4$

† These are the product branching ratios
 $BR(3^3S_1 \rightarrow 2^3P_J \gamma) \times BR(2^3P_J \rightarrow 2(1^3S_1) \gamma)$.

In the following we shall use for fine splitting the values

$$M(2^3P_2) - M(2^3P_1) = 14 \pm 0.6 \text{ MeV and} \\ M(2^3P_1) - M(2^3P_0) = 24.4 \pm 2.3 \text{ MeV,}$$

obtained from combining our exclusive results and the above values.

Eichten and Feinberg²³ (EF) first developed a generalized formulation of the spin dependence of quark antiquark interactions and expressed it in terms of spin-orbit, spin-spin, and tensor interactions derived from a vector potential, V_V , and possibly a scalar potential, V_S . Various phenomenologists postulated particular

V_V 's and V_S 's, for example. EF assumed no scalar component ($V_S=0$) and the vector part V_V to be composed of a short range piece, $V_{ge}-(4/3)a_s/R$, corrected to first order in a_s , and a long range piece due to the longitudinal color electric field which transforms as the fourth component of a Lorentz vector ($\gamma^0\gamma^0$). This scheme is known as "electric confinement". Moxhay and Rosner²⁷ (MR) also chose $V_S=0$, but their V_V is given by a Richardson potential modified such that a long range tensor force remains. McClary and Byers²⁸ (MB) chose instead a linear scalar potential ($V_S=A+BR$), and $V_V=V_{ge}$. Similarly, Gupta et al²⁶ (GRR) assume a linear scalar potential and obtained V_V from QCD including corrections to order a_s^2 after adjusting several parameters to obtain agreement with charmonium and T level spacings. For both these two latter cases the confining potential arises from an effective scalar exchange and is referred to, in the literature, as "scalar confinement".

In Table 5 we compare the measured $3^3S_1 \rightarrow 2^3P_J$ E1 transition rates, $\Gamma_{E1}(J)$ in keV and reduced widths, $\Gamma_{RW}(J) = \Gamma_{E1}(J)/(E_{3^3S_1} \times (2J+1))$ normalized to the $J=1$ case, with three most recent calculations which take into account relativistic corrections: GRR,²⁶ MR,²⁷ and MB.²⁸

Table 5. Comparison of E1 and reduced widths.

$\Gamma_{E1}(J)$	Experiment	GRR	MR	MB
$J = 2$	$3.3 \pm 0.8 \pm 0.7$	2.9	2.7	2.6
$J = 1$	$3.0 \pm 0.8 \pm 0.6$	2.7	2.8	2.4
$J = 0$	$1.4 \pm 0.6 \pm 0.3$	1.5	1.4	1.0
$\Sigma J = 2, 1, 0$	$7.7 \pm 1.7 \pm 1.5$	7.1	6.9	6.0

$\Gamma_{RW}(J)$	Experiment	GRR	MR	MB
$J = 2$	0.99 ± 0.21	0.97	1.17	1.05
$J = 1$	1	1	1	1
$J = 0$	0.70 ± 0.26	0.52	0.74	0.9

There is a very good overall agreement between data and experiment. It is also apparent that the transition rates cannot distinguish between the models. We can also compare the measured fine structure with various models. Following Rosner³⁴ we can write the 3^3P masses as:

$$M(3^3P_2) = \bar{M} + a - 2b/5, \quad M(3^3P_1) = \bar{M} - a + 2b,$$

$$M(3^3P_0) = \bar{M} - 2a - 4b;$$

where
$$a = (1/2M_Q^2) < 3V_V' / R - V_S / R >.$$

and
$$b = (1/12M_Q^2) < V_V' / R - V_V'' >.$$

and compare the experimental values of a and b with four models (see Table 6): EF,³³ MR,²⁷ MB,²⁸ and GRR.²⁶

Table 6. Comparison of fine structure.

Parameter	EXPERIMENT	EF	MR	MB	GRR
a	9.9 ± 0.5	10.7	6.5	14.6	9.2
b	2.4 ± 0.3	1.3	2.1	4.2	1.8

While no model is singled out, those using "scalar confinement" appear to be favored. The same conclusion follows from the 1^3P_J fine structure and the original argument of Büchmüller³⁵ that a long range confinement potential must transform as a Lorentz scalar. Pantaleone, Tye and Ng³⁶ recently argued that the *ad hoc* postulation of non-perturbative long range forces is most unappealing from a fundamental viewpoint. They derive in a self consistent way the $\chi_b(2P)$ fine structure (accurate to 1 MeV) in terms of the $\chi_b(1P)$ splitting for their model and for scalar confinement models. Their results for the splittings in MeV are given in Table 7, with their model in the first column and scalar confinement ones in the second column. Our measurements are in the third column:

Table 7. Comparison of fine structure with Ref. 36.

Splitting	no scalar conf. (MeV)	scalar conf. (MeV)	experiment (MeV)
$M(2^3P_2) - M(2^3P_1)$	16.2	14.9	14.0 ± 0.6
$M(2^3P_1) - M(2^3P_0)$	21.8	22.3	24.4 ± 2.3

While more data are clearly needed, the present results seem to favor the need of an *ad hoc* scalar long range interaction to describe the fine structure of the P -wave $b\bar{b}$ states.

7. CONCLUSION AND OUTLOOK

Our first run with CUSB-II has shown that it performs as expected. Aside from obtaining a new improved (5x) measurement of $B_{\text{BR}}(3S)$ and an intriguing preview of precision $\chi_b(2P)$ spectroscopy, we measured a resolution improvement of over a factor of two over CUSB-II. It augurs well for our hopes of eventually seeing the singlet bound $b\bar{b}$ states (shown as dashed lines in Figure 1).

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RECENT RESULTS FROM THE CRYSTAL BALL EXPERIMENT*

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ABSTRACT

This report reviews several recent analyses from the Crystal Ball collaboration. The major topics discussed are the search for new states in radiative $T(1S)$ decays, the search for lepton number-violating and inclusive η decay modes of the τ , and results from $\gamma\gamma$ physics.

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1. Introduction

From 1982, when the Crystal Ball detector^[2] became operational at the DORIS II storage ring, to about 1984, the experiment's primary objective was Υ spectroscopy. The χ_b states were resolved and their masses, spins and hadronic widths were measured, the $\pi\pi$ hadronic transition and upper limits for the η hadronic transition between the $\Upsilon(2S)$ and $\Upsilon(1S)$ were measured, and searches for the η_b and η'_b states were made^[3]. These results can be classified as "bread and butter" physics because they are interesting and there was confidence the measurements were possible before they were begun.

From about 1984 to the present, more speculative analyses have been completed, many of which attempt to test the Standard Model. Going beyond the Standard Model involves looking for processes with small branching ratios and/or couplings. Such searches necessitate a large sample of clean events. For the Crystal Ball experiment, this translates to a large $\Upsilon(1S)$ data set because the $\Upsilon(1S)$ is very narrow, ($\Gamma(\Upsilon) \approx \frac{2}{3}\Gamma(\psi)$), and its decays are OZI suppressed so that rare decays are easier to detect.

Several radiative decay modes of the $\Upsilon(1S)$ have been examined by the Crystal Ball group. The search for narrow resonances, in particular a Higgs boson, has been made in the inclusive photon spectrum. A non-minimal Higgs, or any particle decaying predominantly to the $\tau\tau$ final state, has also been sought in radiative $\Upsilon(1S)$ decays. The search for radiative $\Upsilon(1S)$ decays to all-neutral final states has been performed. Finally, radiative decays to unseen particles have been investigated.

The Crystal Ball has also collected a large data set on the $\Upsilon(4S)$ and nearby continuum so that the total luminosity collected at DORIS II is about 250 pb^{-1} . Many analyses become possible with such large data sets; in particular, about 500 K τ and 150 K B decays have been recorded. From the analyses using these data, this report concentrates on the search for lepton number-violating τ decays and for inclusive η decays of the τ . Results from B decays have been

presented recently^[4] and will not be included here. Finally, resonant π^0 , η and η' production and $\pi^0\pi^0$ production near threshold, from two photon interactions are discussed.

2. The Inclusive $\Upsilon(1S)$ Photon Spectrum^[4]

The goal of this analysis is to search for new states by observing the inclusive photon energy spectrum. A narrow resonance in the energy spectrum indicates the existence of a new state X produced by the process $\Upsilon(1S) \rightarrow \gamma X$. This analysis is based on an integrated luminosity of about 51 pb^{-1} corresponding to approximately 0.44×10^6 produced $\Upsilon(1S)$ events.

Figure 1 shows the final inclusive photon spectrum plotted in 2.0% bins. Note the detector resolution for the Crystal Ball is

$$\frac{\sigma_E}{E} = \frac{2.7 \pm 0.2\%}{\sqrt{E}}.$$

This spectrum is fit with a polynomial background with the expected NaI line-shape fixed at 1% intervals along the spectrum. No narrow structures are ob-

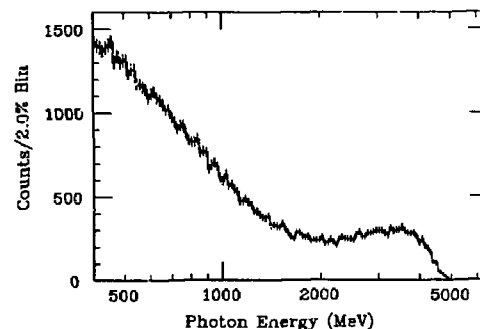


Fig. 1 The final $\Upsilon(1S)$ inclusive photon spectrum. No obvious narrow structures consistent with the decay $\Upsilon(1S) \rightarrow \gamma X$ are indicated.

served; the most significant structure lies at $E_\gamma = 4188$ MeV with a significance of less than 1.7.

Because no evidence for a new state is seen, an upper limit for the process $T(1S) \rightarrow \gamma X$ is calculated. An essential element needed to extract upper limits for this process is the photon efficiency as a function of energy. Since the efficiency for this process depends on the properties of the state X , some assumptions about its decay modes must be made. In this analysis, X is assumed to decay to all possible fermion-antifermion pairs energetically accessible, where the coupling between X and the $f\bar{f}$ pairs is assumed to be proportional to the fermion mass. These assumptions are those expected for a minimal Higgs particle.

For photon energies above 4.00 GeV, the decay $T(1S) \rightarrow \gamma X \rightarrow \gamma c\bar{c}$ is not energetically possible. In this case, the decay $X \rightarrow \tau\bar{\tau}$ is the dominant decay mode. Similarly, above photon energies of 4.06 GeV, the τ decay mode becomes inaccessible and the dominant decay mode of X is into strange quark pairs, $X \rightarrow s\bar{s}$. Finally, for photon energies above 4.68 GeV, the dominant decay modes of X are into light quark pairs and μ pairs. The photon efficiency as a function of photon energy for each of these decay modes is calculated from Monte Carlo simulations. The overall photon efficiency takes into account the $c\bar{c}$, $\tau\bar{\tau}$ and $s\bar{s}$ threshold effects and is the average of efficiencies of the energetically available decay modes, weighted by the mass squared and the color factor of the decay fermions. Figure 2 shows the final photon efficiency as a function of photon energy. The dashed vertical lines indicate the $c\bar{c}$, $\tau\bar{\tau}$ and $s\bar{s}$ thresholds. The thresholds are assumed to turn on quickly, so phase-space factors may smooth this result. The rapid drop in efficiency between 4 and 5 GeV is primarily due to Bhabha rejection cuts.

The 90% confidence level upper limit curve for the decay $T(1S) \rightarrow \gamma X$ is shown in Figure 3. This result can also be plotted as a function of recoil mass rather than the photon energy as shown in Figure 4.

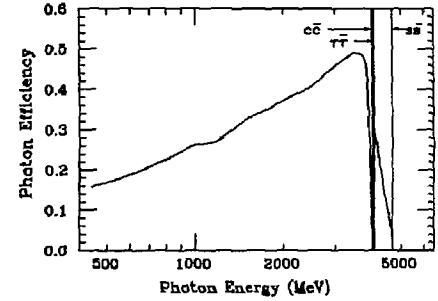


Fig. 2 The photon efficiency for the process $T(1S) \rightarrow \gamma X$ as a function of photon energy. The state X is assumed to decay into fermion pairs with a coupling proportional to the mass of the fermion. The vertical dashed lines show the kinematic thresholds for the relevant fermions.

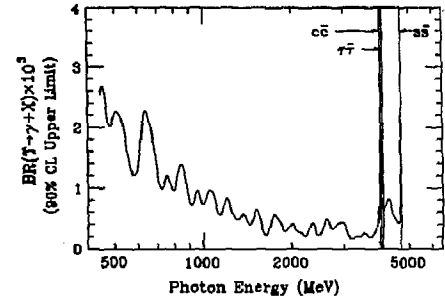


Fig. 3 The 90% confidence level upper limit for the process $T(1S) \rightarrow \gamma X$ as a function of photon energy. The assumptions on the decay of X are found in the text. The vertical dashed lines show the kinematic thresholds for the relevant fermions.

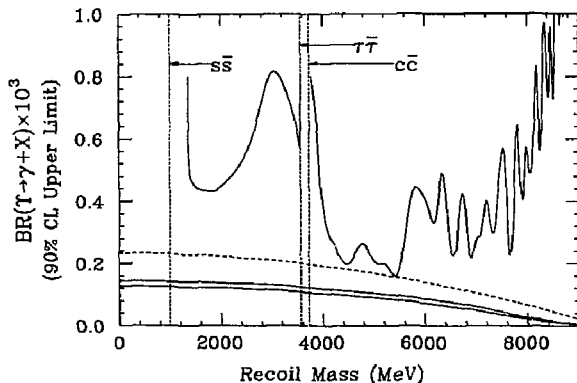


Fig. 4 The 90% confidence level upper limit for the process $\Upsilon(1S) \rightarrow \gamma X$ as a function of recoil mass. The vertical dashed lines show the kinematic thresholds for the relevant fermions. The horizontal dashed line corresponds to the lowest order calculation for the Wilczek mechanism. The two solid horizontal lines indicate the range of the theoretical estimate of the first order QCD radiative corrections to the Wilczek calculation.

Figure 4 indicates that only for Higgs mass around $5.5 \text{ GeV}/c^2$ does this analysis come close to the Wilczek estimate^[6] for the branching ratio of a minimal Higgs particle. For a Higgs mass below about $4 \text{ GeV}/c^2$, the efficiency drop, due to Bhabha rejection in the hadron selection routines, causes a large increase in the corresponding upper limit. A different Bhabha rejection algorithm, tuned for this analysis, might improve the upper limit for Higgs' masses in the 1 to 4 GeV/c^2 range. The decay modes open to the Higgs for masses below $1 \text{ GeV}/c^2$ are sufficiently different from the decays into $c\bar{c}$ and $s\bar{s}$ that an entirely different analysis would be required to separate such decays from QED events. For a Higgs mass above $6 \text{ GeV}/c^2$, both a slow decrease in photon efficiency and an

increase in the number of background photons causes a rise in the upper limit. In no mass range does this analysis rule out a minimal Higgs to the 90% confidence limit for the latest estimates of the $\Upsilon(1S)$ branching ratio which includes QCD radiative corrections^[7]

The above analysis can be applied to different assumptions on the decay modes of X . The major difference is that the photon efficiency may have different kinematic thresholds depending on the couplings and the possible decay products involved. If X decays predominantly through low multiplicity exclusive channels, for example, the $\tau\bar{\tau}$ final state predicted to dominate the Higgs decay in some non-minimal models, this analysis will have a lower sensitivity for their detection. The small band between the $\tau\bar{\tau}$ and $c\bar{c}$ thresholds in Figure 3 shows this directly. On the other hand, as long as X decays into multi-hadron final states, the detection efficiency remains high, and the upper limits found here will be approximately unchanged.

In summary, searching for the Higgs boson in radiative $\Upsilon(1S)$ decays appears the most direct method available for discovering a light Higgs. Unfortunately, no Higgs' masses have been experimentally ruled out by this method. To reach the current theoretical estimates for a Higgs' mass above 8 GeV would require a sample of about 10 million $\Upsilon(1S)$ decays gathered with a detector having a sensitivity five times that of the Crystal Ball. This would be an enormous undertaking spanning many years of data-taking.

3. The Semi-Exclusive Search for Radiative Decays $\Upsilon(1S) \rightarrow \gamma X \rightarrow \gamma\tau\bar{\tau}$

This search is also motivated by the possibility of observing a new state, including a Higgs boson, in radiative $\Upsilon(1S)$ decays^[8]. In some non-minimal models^[9] the decay $H \rightarrow c\bar{c}$ is suppressed and the dominant decay is $H \rightarrow \tau\bar{\tau}$. The analysis presented above for the inclusive photon spectrum is also sensitive to this decay, but not quite to the level found here.

The data set used for this analysis corresponds to approximately 220 K produced $\Upsilon(1S)$ events. To efficiently tag these $\gamma\tau\bar{\tau}$ events, one τ is required to

decay to $e\bar{\nu}_e\nu_\tau$ and the other to $\mu\bar{\nu}_\mu\nu_\tau$. Thus, the final state observed is an electron, a muon and a photon. Figure 5 shows the photon spectrum for events passing the $\Upsilon(1S) \rightarrow \gamma\tau\bar{\tau}$ selection criteria. The entries in this spectrum are consistent with τ pair production with initial state bremsstrahlung. Figure 6 shows the corresponding 90% confidence level upper limit for the branching ratio $BR(\Upsilon(1S) \rightarrow \gamma X \rightarrow \gamma\tau\bar{\tau})$. These limits are higher than the rates expected by the Wilczek mechanism, however, they can be used to constrain the vacuum expectation values for two-Higgs doublet models.

4. The Search for the Decay $\Upsilon(1S) \rightarrow \gamma + \text{Unseen Particles}$ ^[10]

This analysis, originally motivated by O. Nachtmann^[11] looks for the $\Upsilon(1S)$ decaying to a photon and any number of undetected particles. These unseen particles are not necessarily resonant; for example Nachtmann discusses the three-body decay $\Upsilon(1S) \rightarrow \gamma\lambda\bar{\lambda}$ where λ is a supersymmetric Goldstone fermion. This would be a very difficult experiment to perform by running on the $\Upsilon(1S)$ because the final state would be a single, non-resonant photon. To lower backgrounds, 57 pb⁻¹ of $\Upsilon(2S)$ data, corresponding to approximately 185 K events, were used to tag $\Upsilon(1S)$ events by the hadronic transition $\Upsilon(2S) \rightarrow \pi^0\pi^0\Upsilon(1S)$. Because each of the two π^0 s decay to two photons, and the $\Upsilon(1S)$ decays to a photon plus unseen, the final state has five photons. Events consistent with two π^0 s and one remaining photon are selected and the mass recoiling opposite the $\pi^0\pi^0$ system is calculated. The energy of the unpaired photon is plotted against this recoil mass in Figure 7. The range indicated by the vertical dashed lines shows the $\Upsilon(1S)$ mass band. No events in this band with photon energy above 1200 MeV are found, which leads to the upper limit

$$BR(\Upsilon(1S) \rightarrow \gamma + \text{unseen}) < 2.3 \times 10^{-3}, \quad M_{\text{unseen}} < 8.1 \text{ GeV}/c^2.$$

This limit is only valid if the lifetime of the unseen particles is greater than 10⁻⁷ seconds.

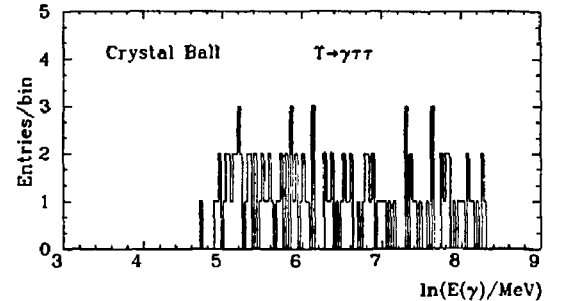


Fig. 5 The photon spectrum for events passing $\Upsilon(1S) \rightarrow \gamma\tau\bar{\tau}$ selection criteria. The events shown are consistent with QED.

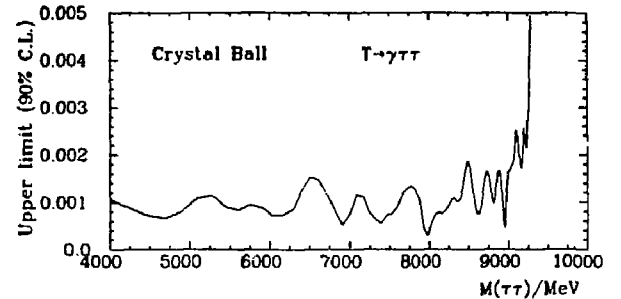


Fig. 6 The 90% confidence level upper limit for the process $\Upsilon(1S) \rightarrow \gamma\tau\bar{\tau}$ as a function of the $\tau\bar{\tau}$ invariant mass.

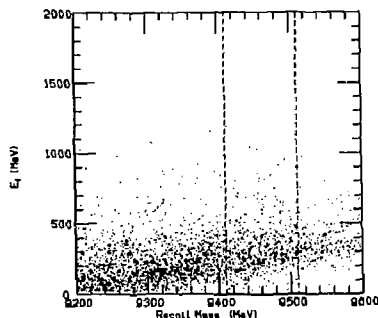


Fig. 7 E_γ vs recoil mass scatter plot. The vertical dashed lines indicate the $T(1S)$ mass window. No events in this band above 1200 MeV are detected.

5. The Search for Exclusive $T(1S)$ Decays to All-Neutral Final States^[12]

The measurement of radiative decay modes of heavy bound $q\bar{q}$ systems may provide insight into the formation mechanism and the gluonic content of the light mesons produced as well as into some features of the bound $q\bar{q}$ system. Such decays have been measured on the J/ψ with branching ratios on the order of 10^{-3} and have yielded interesting results on the low mass meson sector^[13]. These decays should also be present at the $T(1S)$, but theoretical predictions of branching ratios range over several orders of magnitude.

This analysis is based on 306 K $T(1S)$ events. The all-neutral decay modes considered are

$$\begin{aligned} T &\rightarrow \gamma\eta & \eta &\rightarrow 3\pi^0, 2\gamma \\ T &\rightarrow \gamma\eta' & \eta' &\rightarrow 2\pi^0\eta & \eta &\rightarrow 3\pi^0, 2\gamma \\ T &\rightarrow \gamma f_2 & f_2 &\rightarrow 2\pi^0. \end{aligned}$$

Because there can be up to 11 photons in the final state, many of which overlap, the individual photon energies and directions cannot be reconstructed. A

method called the "Global Shower Technique"^[14] is used to calculate the invariant mass of a cluster of photons. First the direction of the center of the energy cluster is found using $\vec{c} = \frac{1}{E} \sum_i \vec{c}_i E_i$ where $\vec{c}_i$ is the direction vector from the interaction point to the crystal center, E is the total deposited energy in the cluster, and the sum runs over all crystals in the cluster with energy E_i . The width of the cluster is then calculated using $S = \frac{1}{E} \sum_i (\vec{c} - \vec{c}_i)^2 E_i$. Finally, the invariant mass of the cluster is given by $M = \sqrt{S - S_\gamma} E$ where S_γ is the width of a single photon as determined from Monte Carlo studies.

Events consistent with a single photon recoiling against a neutral energy cluster are selected. The top histogram in Figure 8 shows the invariant mass of the energy clusters in these events. The other histograms show the mean and width expected from Monte Carlo simulations of each of the three decay modes listed above. Fitting the top plot with these Monte Carlo distributions leads to the upper limits shown in Table 1.

The $T(1S) \rightarrow \gamma\eta$ and $T(1S) \rightarrow \gamma\eta'$ results are the only experimental limits reported for these quantities, however, the theoretical estimates are anywhere from 1 to 3 orders of magnitude below these results. The $T \rightarrow \gamma f_2$ limit presented here is close to one theoretical prediction, but the CLEO collaboration has already reported more restrictive upper limits which rule out this mode^[15].

6. The Search for $r \rightarrow e\gamma$ and $r \rightarrow e\pi^0$ Decay Modes^[16]

This search is motivated by the prediction of inter-family transitions in composite model theories^[17] and by the historical precedent of searching for such transitions. The data sample used in this analysis consists of 61 pb⁻¹ corresponding to 124 K r leptons for the $r \rightarrow e\gamma$ search and 37 pb⁻¹ or 76 K r leptons for the $r \rightarrow e\pi^0$ search. A semi-inclusive approach is used to obtain a high detection efficiency for the r decays investigated here. Monte Carlo studies indicate a high spatial correlation between the electron and the gamma or π^0 for these decay modes. An electron and a gamma or π^0 are required to be in one hemisphere, defined by the gamma or π^0 direction, with no additional particles

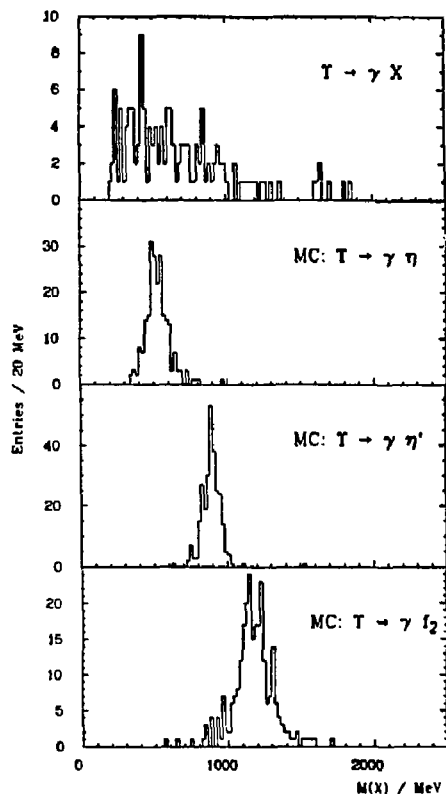


Fig. 8 The top plot shows the observed invariant mass for the state X consistent with the process $T(1S) \rightarrow \gamma X$. The other 3 plots indicate the mean and width expected for the corresponding decay mode calculated from Monte Carlo simulations.

Theoretical Predictions			
	$T \rightarrow \gamma + \eta$	$T \rightarrow \gamma + \eta'$	$T \rightarrow \gamma + f_2$
Intemann <i>Phys. Rev. D</i> 27 (1983) 275	6.3×10^{-7} 1.3×10^{-7}	2.5×10^{-6} 5.3×10^{-7}	
Deshpande, Eilam <i>Phys. Rev. D</i> 25 (1982) 270	1.5×10^{-4}	$\approx 10^{-3}$	
Körner <i>et al.</i> <i>Nucl. Phys. B</i> 229 (1983) 115	3.4×10^{-5}	1.6×10^{-4}	1.4×10^{-4}
Experimental Results			
CLEO preliminary CLNS 86/714			$< 4.8 \times 10^{-5}$
Crystal Ball preliminary 90% C.L.	$< 3 \times 10^{-4}$	$< 1 \times 10^{-3}$	$< 4 \times 10^{-4}$

Table 1 Theoretical predictions and experimental results for exclusive radiative decays of the $T(1S)$.

in this hemisphere. The decay products of the other τ lepton in the opposite hemisphere are required to have low multiplicity; 1 to 3 charged tracks and an arbitrary number of neutrals. This second τ lepton is not analysed further, thus the term "semi-inclusive."

Events consistent with radiative Bhabha events are rejected. Figures 9 and 10 show the $e\gamma$ and $e\pi^0$ invariant mass spectra; no significant signal with the expected width is seen at the τ mass. Thus, the 90% confidence level upper limits are calculated to be

$$BR(\tau \rightarrow e\gamma) < 3.4 \times 10^{-4}$$

$$BR(\tau \rightarrow e\pi^0) < 4.4 \times 10^{-4}.$$

The above numbers represent the best existing limits for these reactions.

The limit on $\tau \rightarrow e\gamma$ can be translated into a lower limit on the composite-

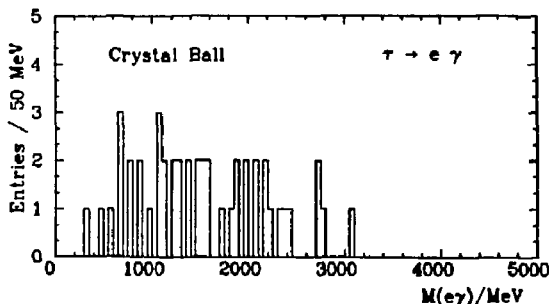


Fig. 9 The invariant $e\gamma$ mass. No enhancement at the τ mass is observed.

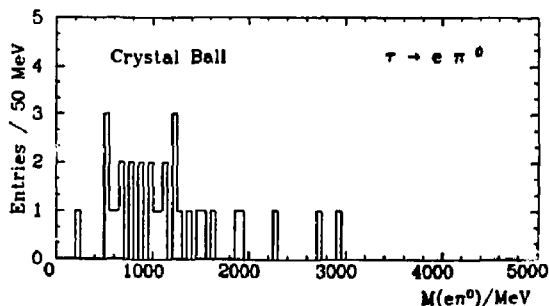


Fig. 10 The invariant $e\pi^0$ mass. No enhancement at the τ mass is observed.

ness mass scale: $\Lambda/\sqrt{\alpha} > 65 \text{ TeV}^{[17]}$ This limit is somewhat model dependent, in particular, the unknown coupling constant α can vary between 1 and the τ Yukawa coupling, ≈ 0.01 . No explicit limit calculations have been done for $\tau \rightarrow e\pi^0$, a reaction which is also of potential interest for composite models.

7. Search for $\tau \rightarrow \eta X$ Decays^[18]

This analysis is motivated by the apparent discrepancy between the published inclusive and exclusive measurements of the τ branching fractions into 1 charged prong plus neutrals. Table 2 shows the inclusive and sum of exclusive branching fractions of the τ for 1 and 3 charged prongs compiled from the most recent results^[19]. These preliminary results soften the apparent discrepancy in the one charged prong mode but still allow undiscovered τ decay modes.

Recent τ Decay Summary	1 Charged Prong	3 Charged Prongs
Sum of Exclusive Decays (%)	83.2 ± 2.4	11.9 ± 0.7
Inclusive Measurement (%)	86.8 ± 0.3	13.1 ± 0.3
Difference	3.6 ± 2.4	1.2 ± 0.8
Difference (Published)	8.8 ± 2.0	0.4 ± 0.8

Table 2 A comparison between the inclusive and the sum of exclusive τ decay modes for 1 and 3 prong decays. These values summarize recent preliminary results. The difference calculated from published numbers is also given for comparison.

The data set used for this analysis consists of 82 pb^{-1} corresponding to about 90 K $\tau^+\tau^-$ pairs; about half the available data. To select events, one τ is tagged with the electron from the $\tau \rightarrow e\bar{\nu}_e\nu_\tau$ decay. The other τ is required to decay to one charged particle plus ≥ 3 photons. The cut at three or more photons was chosen because the decay $\tau^- \rightarrow \nu_\tau\pi^-\eta$, with only two photons in the final state, is forbidden as a first class current. Other likely decays, for example $\tau^- \rightarrow \nu_\tau\pi^-\eta\pi^0$, have four or more photons. The cut was set at three

in case one photon was undetected.

The points in Figure 11 show the "electron" spectrum. "Electron" is in quotes to indicate that at the lower energies shown, π and ρ particles contribute to this plot from the $\tau \rightarrow \pi\nu$ and $\tau \rightarrow \rho\nu$ decay modes. This is perfectly acceptable because they also tag τ decays. The central question is, how does one have confidence that these decays are really $\tau^+\tau^-$ events and not low-multiplicity hadronic decays which mimic τ pairs. To estimate the background contribution to the spectrum of points in Figure 11, a Monte Carlo simulation of $\tau^+\tau^-$ events was subjected to the same selection cuts. The histogram in Figure 11 shows the absolute Monte Carlo calculation for the $\tau^+\tau^-$ contribution. The background level is given by the excess the data points have over the histogram. The data and Monte Carlo agree in shape and magnitude, indicating the data sample is very nearly pure $\tau^+\tau^-$ production, assuming the Monte Carlo efficiency is correct.

Finally, photons not consistent with π^0 decays are paired and their invariant mass calculated. Figure 12 shows the π^0 subtracted $\gamma\gamma$ invariant mass spectrum. A clear η signal appears, indicating the existence of $\tau \rightarrow \eta X$ decays.

Several checks have been made to rule out the possibility that the η signal is from hadronic decays:

- The "electron" spectrum agrees with the Monte Carlo in shape and amplitude, as discussed above.
- The η signal appears as expected if the "electron" spectrum is cut at 1500 MeV to enhance real electrons, if harder pattern cuts are used to select electrons, or if exactly four photons are required from the τ decay.
- Three-gluon and $q\bar{q}$ Monte Carlo events and separated beam data show no η signal.
- No Monte Carlo events simulating $\gamma\gamma \rightarrow \eta, \eta'$ pass the selection cuts.
- The η signal is not from B decays; the data sample of 82 pb^{-1} consists of $\Upsilon(1S)$ and $\Upsilon(2S)$ events only.

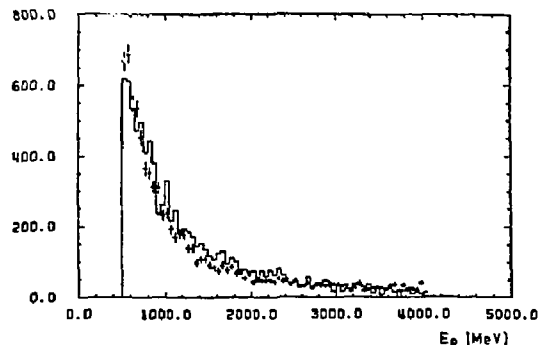


Fig. 11 The "electron" energy spectrum. At lower energies, π and ρ particles enter this plot. The points correspond to the data while the histogram is derived from a Monte Carlo simulation. The points and data are not normalized to each other.

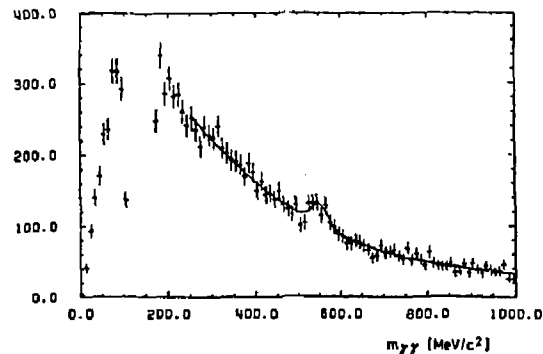


Fig. 12 The invariant $\gamma\gamma$ mass spectrum. Pairs consistent with π^0 s are subtracted.

Although there is evidence for the decay $\tau \rightarrow \eta X$, this analysis is preliminary and awaits further background studies and efficiency calculations.

8. Results on Photon-Photon Collisions

This is a brief review of recent Crystal Ball two photon results. Figure 13 shows the $\gamma\gamma$ invariant mass plot for the process $\gamma\gamma \rightarrow X \rightarrow \gamma\gamma$, for 50 pb^{-1} of data^[20]. The π^0 , η and η' are all clearly seen in this one plot. A transverse-momentum cut of $|\sum \vec{P}_T| < 0.1 M_{\gamma\gamma}$ was used here. A more detailed analysis of each of these three states and an analysis specific to the process $\gamma\gamma \rightarrow \eta' \rightarrow \eta\pi^0\pi^0$ ^[21] gives the following results

$$\Gamma(\pi^0 \rightarrow \gamma\gamma) = 7.8 \pm 0.4 \pm 0.9 \text{ eV}$$

$$\Gamma(\eta \rightarrow \gamma\gamma) = .51 \pm .02 \pm .06 \text{ KeV}$$

$$\begin{aligned} \Gamma(\eta' \rightarrow \gamma\gamma) &= 5.0 \pm 0.6 \pm 0.8 \text{ KeV} \quad (\eta' \rightarrow \gamma\gamma) \\ &= 4.1 \pm 0.3 \pm 0.8 \text{ KeV} \quad (\eta' \rightarrow \eta\pi^0\pi^0). \end{aligned}$$

The analysis of $\gamma\gamma \rightarrow \eta' \rightarrow \eta\pi^0\pi^0$ can also be used to derive the upper limit for $\Gamma(X \rightarrow \gamma\gamma)$ from the process $\gamma\gamma \rightarrow X \rightarrow \eta\pi^0\pi^0$ for an isoscalar X , assuming $\Gamma_X = 50 \text{ MeV}$. The 90% confidence level upper limit is shown in Figure 14. No radially excited 0^{-+} state is seen, in particular, the $\eta(1275)$ is not observed.

Finally, the reaction $\gamma\gamma \rightarrow \pi^0\pi^0$ is studied below the f_2 to measure the non-resonant two photon production of $\pi^0\pi^0$ and to search for scalar resonances^[22]. Figure 15 shows the invariant mass of the $\pi^0\pi^0$ system for 90 pb^{-1} of data. The cross section is plotted in arbitrary units because some efficiency factors have not yet been determined. No narrow states below the f_2 are seen, but a sizable $\pi^0\pi^0$ production is evident.

9. Summary and Conclusions

Experimentally, $\Upsilon(1S) \rightarrow \gamma + H$ searches are just beginning to reach the sensitivity needed to test theoretical predictions. Excluding Higgs masses below about $5 \text{ GeV}/c^2$ seems within reach of the next round of planned experiments

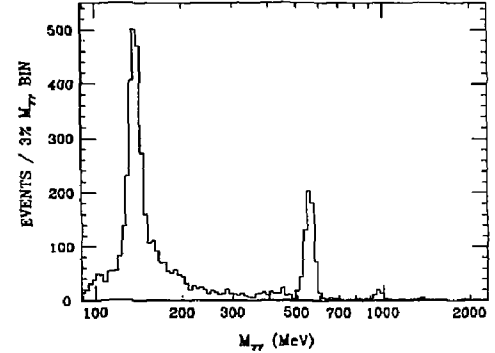


Fig. 13 The invariant $\gamma\gamma$ mass spectrum.

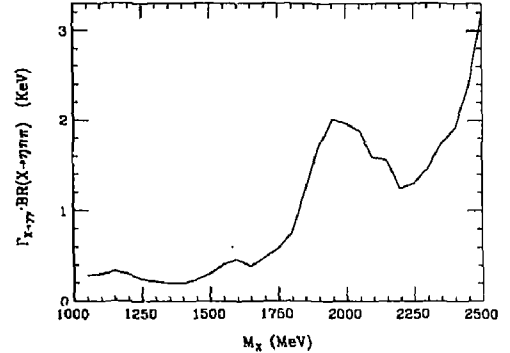


Fig. 14 The 90% confidence level upper limit for the $\gamma\gamma$ production of an isoscalar state X , assuming $\Gamma_X = 50 \text{ MeV}$. No radially excited 0^{-+} state is observed.

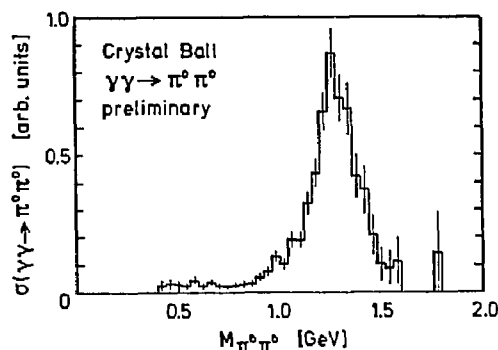


Fig. 15 The invariant $\pi^0\pi^0$ mass spectrum. No new states are observed below the f_2 .

but excluding masses above about 8 GeV will probably require new production and detection techniques for radiative $T(1S)$ decays. This assumes the theoretical prediction does not drop further; because the first order QCD radiative corrections to the Wilczek calculation reduce the rate by about a factor of two, higher-order corrections may further suppress this prediction. No exclusive decay modes of the $T(1S)$, excluding the leptonic decays, have been observed to date. Thus the T is relatively barren compared to the J/ψ .

No hint of compositeness has been observed in τ decays. The upper limits quoted for the $\tau \rightarrow e\gamma$ and $\tau \rightarrow e\pi^0$ decays are the best in existence. Evidence has been presented for an inclusive η decay mode of the τ , but further background studies and efficiency calculations are needed.

The two photon width of the π^0 , η and η' have been measured. No new states are seen in the $\eta\pi^0\pi^0$ final state, or in the $\pi^0\pi^0$ final state below the f_2 in two photon interactions.

These results show no deviation from those expected in the Standard Model.

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J/ψ SPECTROSCOPY FROM MARK III*

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Abstract

The MARK III detector at the SPEAR e^+e^- storage ring at SLAC has accumulated a data sample of 5.8×10^6 J/ψ produced. The status of the $\xi(2230)$ observed in the radiative J/ψ decay is described. The status of the glueball candidates $\eta(1440)$ $\{\iota(1440)\}$ and $f_2(1720)$ $\{\theta(1720)\}$ are probed with a systematic comparison between the radiative and the hadronic decays of J/ψ . Finally, an understanding of quark correlations is attempted from a systematic study of the J/ψ decaying into Vector-Pseudoscalar, Vector-Tensor and Vector-Scalar nonets

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1. Introduction

The discovery of charm was not only a giant step toward establishing the present standard model of electroweak interactions, but also was largely responsible for the boost in the 'religion' of QCD. A major interest and novelty in the realm of QCD has recently been to observe gluonic bound states. Such states are not accounted for in the quark model; however, according to QCD gluons are strongly interacting and they should form bound states^[1] either between themselves (pure glueballs), or with the quarks and antiquarks (hybrids). The few potential candidates, which have previously been observed in radiative decays of the J/ψ ,^{[2] [3] [4]} still need to be established firmly as glueballs. The search for new candidates within this new spectroscopy continues. In a broader sense, any new physics, coming from the J/ψ , may be interesting and informative. Studies of the $\eta_c(2980)$, the charmed pseudoscalar isosinglet state into which the J/ψ decays through a magnetic dipole transition, also yield important information on charmonium decay.

In addition to the above, J/ψ decays can be used as a laboratory to study light quark spectroscopy. The initial state of J/ψ , a very sharp and narrow resonance, has a uniquely defined spin parity $J^{PC} = 1^{--}$. This, along with the low multiplicity of its decays, renders the J/ψ far more convenient for such studies than the fixed target experiments where the initial spin-parity is uncertain and the final states often contain a lot of hadronic debris.

All J/ψ decays are Okubo-Zweig-Iizuka (OZI) suppressed.^[5] A large data sample of J/ψ provides the opportunity to study both the Single-OZI (SOZI) and the Double-OZI (DOZI) suppressing mechanisms. Comparison of the radiative and the hadronic decays provides not only insight into the decay mechanisms, but also helps in differentiating between the conventional $q\bar{q}$ and the 'new' spectroscopy.

The J/ψ decays into baryons provide a wealth of information about baryon interactions. Furthermore, the J/ψ , an $SU(3)$ singlet, should hadronically decay into a baryon-antibaryon pair where both (the baryon and the antibaryon) belong to octets or both belong to the decuplets. Mixed decays where one of the two (the baryon or the antibaryon) belongs to an octet and the other to a decuplet are $SU(3)$ forbidden. Observation of such decays, along with the $SU(3)$ allowed decays, yields information about different couplings and their possible interference effects.

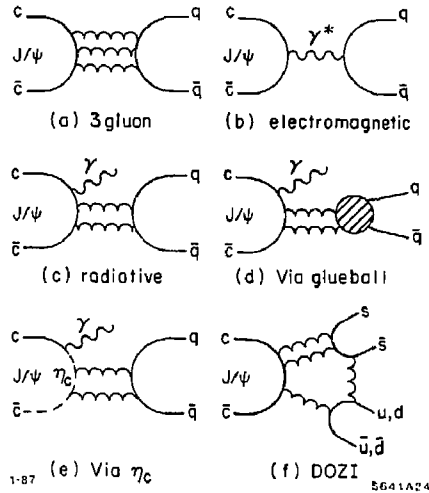
The principal decay mechanisms of the J/ψ are explained below. Figure 1 displays these in the order of strength. The mass of the J/ψ is below the charmed meson pair threshold, so direct decay into charmed mesons is forbidden. Therefore, the strong decay proceeds through (at least) three gluon exchange, as shown in Figure 1(a). (One gluon exchange is forbidden because it carries color to the final state and two gluon exchange is forbidden by charge conjugation for a color singlet.) The final state does not contain any charm quark, hence the decay is OZI suppressed. The decay strength is proportional to α_s^3 , and is given explicitly by^[6]

$$\Gamma(J/\psi \rightarrow 3g) = \frac{16}{9\pi}(\pi^2 - 9) \frac{5}{18} \alpha_s^3 \frac{|\psi(0)|^2}{M_{J/\psi}^2},$$

where $\psi(0)$ is the value of the radial wavefunction of the J/ψ at the origin. This represents $\sim 62\%$ of the total J/ψ branching ratios. Figure 1(b) is the electromagnetic decay through a virtual photon exchange. This is proportional to α and is calculated as

$$\Gamma(J/\psi \rightarrow e^+e^-) = 4(Q_C\alpha)^2 \frac{|\psi(0)|^2}{M_{J/\psi}^2},$$

where $Q_C = 2/3$ is the charge of the charmed quark. This diagram represents $\sim 29\%$ of the J/ψ branching ratios. Figure 1(c) is the radiative decay mechanism and has been the hunting ground for the glueball searches. The strength,



1. (a) Strong decay proceeding through $c\bar{c}$ annihilation into three gluons; (b) Electromagnetic decay through $c\bar{c}$ annihilation into one photon; (c) Electromagnetic decay into a final state of one photon and two gluons (radiative decay); (d) The two gluons forming a color singlet possible bound state (glueball); (e) Radiative decay through a magnetic dipole transition to η_c ; (f) Doubly disconnected diagrams (double OZI suppression).

proportional to $\alpha\alpha_s^2$ is calculated as^[1]

$$\Gamma(J/\psi \rightarrow \gamma gg) = \frac{32}{9\pi} (\pi^2 - 9) \alpha_s^2 \alpha Q_C^2 \frac{|\psi(0)|^2}{M_{J/\psi}^2}$$

and represents $\sim 7\%$ of the total J/ψ branching ratio for the radiative decay. Since the photon in the final state does not carry color, the two gluons could form a color singlet bound state. Figure 1(d) shows a possible gluonic intermediate bound state decaying into light quark mesons. It is this intermediate step that makes the radiative decays the 'hunting ground' of the 'glueball' candidates. Figure 1(e) shows a special case of a radiative decay where the J/ψ decays into an η_c through a magnetic dipole transition before the $c\bar{c}$ annihilate. This mechanism is responsible for $\sim 1\%$ of the total J/ψ branching ratio and provides a very good tool for unraveling the η_c decays as mentioned earlier. Figure 1(f) shows the DOZI decay mechanism, which is comparatively small, but may not be negligible in the interference effects with other amplitudes. This is described later.

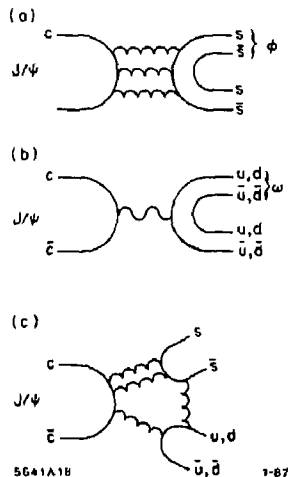
FLAVOR TAGGING:

The implications of some of these decays are explained in Fig. 2. Figure 2(a) and 2(b) show the quasi-two-body decays of the case of Fig. 1(a) and 1(b). The vector meson nonet being ideally mixed, the $\phi(1020)$ contains only strange quark (s), and the $\omega(783)$ contains only up(u) and down(d) quarks. In fact the wavefunctions of these two isoscalars are, to a very good approximation,

$$\phi = |s\bar{s}\rangle$$

$$\omega = \frac{|u\bar{u} + d\bar{d}\rangle}{\sqrt{2}}.$$

Hence, if a ϕ (Fig. 2(a)) or an ω (Fig. 2(b)) is identified in the J/ψ decay, because of the continuation of the quark lines, the recoil system will contain the $s\bar{s}$ (Fig. 2(a)) or the $u\bar{u}$, $d\bar{d}$ (Fig. 2(b)) quark content of the respective mesons. This provides a very useful technique to determine the quark contents of various



2. Quasi two-body J/ψ decay through $c\bar{c}$ annihilation into (a) a ϕ and the $s\bar{s}$ part of a meson, (b) an ω and the $(u\bar{u} + d\bar{d})$ part of a meson, (c) an $s\bar{s}$ and an $u\bar{u}, d\bar{d}$ state through DOZI mechanism.

resonances by observing their production in association with a ϕ and an ω in the J/ψ decays. This, however, is not exact because of the presence of DOZI decays, which, though small, do not obey the quark correlation as seen from Fig. 2(c).

According to the generalized G-parity^[1] conservation, the J/ψ can decay hadronically into the following meson nonets

$$\begin{aligned}
 J/\psi &\rightarrow \text{Pseudoscalar (P)} + \text{Vector (V)} \\
 &\rightarrow \text{Tensor (T)} + V \\
 &\rightarrow \text{Scalar (S)} + V \\
 &\rightarrow \text{Axial Vector (A)} + V
 \end{aligned}$$

and radiatively,

$$\begin{aligned}
 J/\psi &\rightarrow \gamma + P + P \\
 &\quad \gamma + V + V.
 \end{aligned}$$

The flavor tagging technique can be generalized to the whole vector nonet, where recoils against each member of the vector nonet can be studied. Such a systematic study would yield the strength of the different amplitudes e.g. strong (isospin conserving), electromagnetic (isospin violating) and SU(3) violating.

The MARK III detector at the SPEAR e^+e^- storage ring at SLAC has collected a data sample of 5.8×10^6 J/ψ produced, over two separate running periods of 1982-83 and 1985. The present paper deals with the radiative decays in Section 2, starting with the status of the $\xi(2230)$ and then searches in other radiative channels, along with the η_c decays. Section 3 deals with the status of the glueball candidate $\eta(1440)$, first observed^[1] in the $K\bar{K}\pi$ mode of the radiative decay of the J/ψ . The $K\bar{K}\pi$ and $\eta\pi^+\pi^-$ channels, were studied in both radiative decays and in hadronic decays against a ϕ and an ω recoil. The controversy in the 1280-1500 MeV/ c^2 mass region over the structures $f_1(1420)/\eta(1440)$ $\{E(1420)/\psi(1440)\}$ and the $\eta(1275)/f_1(1285)$ $\{\eta(1275)/D(1285)\}$ are discussed. Section 4 describes the status of the $f_2(1720)$ in $K\bar{K}$ and $\pi^+\pi^-$ and the comparison between radiative

and hadronic decays. The quark structure of the mesons and strength of the amplitudes are determined by observing reactions of the type

$$J/\psi \rightarrow 1^{--} + 0^{-+} \quad (1)$$

$$J/\psi \rightarrow 1^{--} + 2^{++} \quad (2)$$

$$J/\psi \rightarrow 1^{--} + 0^{++} \quad (3)$$

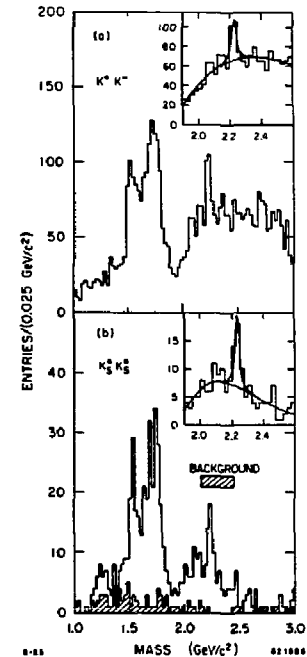
for complete nonets. Sections 5 and 6 describe (1), an ongoing analysis of (2) and the beginning of a systematic study of (3).

2. Radiative Decays

2.1 THE STATUS OF $\xi(2230)$

MARK III reported^[1] the observation of a narrow resonance $\xi(2230)$ in radiative J/ψ decay into K^+K^- in 1983, with a data sample of 2.8×10^6 produced J/ψ . Several theoretical interpretations, as to the nature of the $\xi(2220)$, were subsequently proposed e.g., a glueball or a hybrid state,^[10] an ordinary high spin mesonic state,^[11] and others.^[12] An additional statistics of 3×10^6 J/ψ from the 1985 data, confirmed^[13] the previous observation. Figures 3(a) and 3(b) show the K^+K^- and the $K_S^0 K_S^0$ mass spectra from the combined data sample in the radiative J/ψ decay.

Both spectra are similar in their primary features. Three clear peaks are seen in each, with the first one at ~ 1525 MeV/c², corresponding to the $f_2'(1525)$ $\{f'(1525)\}$, the $s\bar{s}$, $I=0$, member of the 2^{++} nonet. (The 2^{++} nonet is almost ideally mixed.) The second peak is at ~ 1720 MeV/c², the $f_2(1720)\{\theta(1720)\}$, which is a prime glueball candidate^[13] ^[14] and will be described later. The third peak (a narrow peak on top of a wide bump centered around 2100 MeV/c²) at ~ 2230 MeV/c² corresponds to the $\xi(2230)$. The $K_S^0 K_S^0$ spectrum contains



3. (a) K^+K^- effective mass spectrum in the radiative decay $J/\psi \rightarrow \gamma K^+K^-$,
(b) $K_S^0 K_S^0$ effective mass spectrum from the decay $J/\psi \rightarrow \gamma K_S^0 K_S^0$.

little background since decays like $J/\psi \rightarrow K_S^0 K_S^0 \pi^0$ and $J/\psi \rightarrow K_S^0 K_S^0$ are forbidden by C parity conservation. This is demonstrated in the Dalitz plots, Figures 4(a) and 4(b), for the charged and the neutral channels respectively. Diagonal bands from the above three resonances are observed ($f_2'(1525)$ and $f_2(1720)$ overlapping), as indicated in both plots. However, Fig. 4(a) contains a vertical and a horizontal band parallel to the two boundaries arising from $K^\pm$ production in the reactions $J/\psi \rightarrow K^{*\pm} K^\pm$, misidentified as $K\bar{K}\gamma$. The accumulation of events on the two boundaries are from the misidentified decay $J/\psi \rightarrow e^+ e^- \gamma$.

A maximum likelihood fit of a Breit-Wigner line shape in the 1900-2600 MeV/c² region yielded the parameters of the $\xi(2230)$ as

$$m(\xi) = 2230 \pm 6 \pm 14 \text{ MeV}/c^2$$

$$\Gamma(\xi) = 26_{-16}^{+30} \pm 17 \text{ MeV}/c^2$$

for the charged mode, and,

$$m(\xi) = 2232 \pm 7 \pm 7 \text{ MeV}/c^2$$

$$\Gamma(\xi) = 18_{-16}^{+23} \pm 10 \text{ MeV}/c^2$$

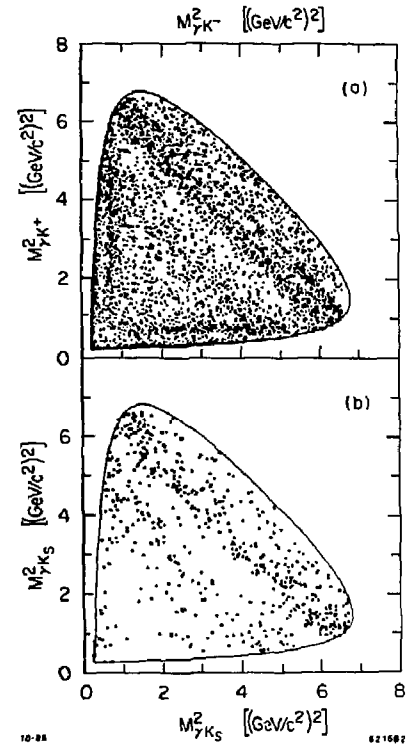
for the neutral mode. The first error corresponds to the statistical and the second error corresponds to the systematic error. The measured branching ratios of the two decay modes were

$$B(J/\psi \rightarrow \gamma \xi) \cdot B(\xi \rightarrow K^+ K^-) = (4.2_{-1.4}^{+1.7} \pm 0.8) \times 10^{-5}$$

$$\text{and } B(J/\psi \rightarrow \gamma \xi) \cdot B(\xi \rightarrow K_S^0 K_S^0) = (3.1_{-1.3}^{+1.5} \pm 0.7) \times 10^{-5}.$$

The ratio of the branching ratios is consistent with the $\xi(2230)$ being an isoscalar. Details have been presented elsewhere.⁽¹⁴⁾

A preliminary spin-parity analysis, using a maximum likelihood technique, has been performed in the relatively background free $K_S^0 K_S^0$ mode, to determine



4. Dalitz plot distributions for the decays (a) $J/\psi \rightarrow \gamma K^+ K^-$ and (b) $J/\psi \rightarrow \gamma K_S^0 K_S^0$.

the J^P of the $\xi(2230)$. This maximum likelihood technique using the helicity amplitudes was employed earlier^[137] to determine the spin-parity of the $f'_2(1525)$ and the $f_2(1720)$ with a smaller data sample. Figure 5 (a),(b),(c) show the $\cos\theta_{K_S^0}$ distributions for the $f'_2(1525)$ region [1420-1550 MeV/c²], $f_2(1720)$ region [1620-1820 MeV/c²] and the $\xi(2230)$ region [2180-2280 MeV/c²] respectively, where $\theta_{K_S^0}$ is the polar angle, with respect to the photon direction of one of the K_S^0 's in the $K_S^0 K_S^0$ center-of-mass frame. The spins determined for both the $f'_2(1525)$ and the $f_2(1720)$ were 2 (the $K_S^0 K_S^0$ system can only have even spin and positive parity, i.e., 0^+ , 2^+ , 4^+ ...). The analysis assigned a minimum spin of 2 to ξ , although a spin 4 assignment could not be excluded.^[141] (The effect of the 2100 MeV/c² region was examined separately and its contribution was estimated as well as possible.) Due to the meager statistics a full Partial Wave Analysis (PWA) was not performed.

The DM2 collaboration, with a data sample of 8.6×10^6 produced J/ψ , reported^[142] the lack of observation of a narrow resonance near 2230 MeV/c². However, the GAMS collaboration has presented evidence^[143] for a narrow structure in $\eta\eta'$ at ~ 2220 MeV/c² with $J^P \geq 2^+$, in the reaction

$$\pi^- p \rightarrow \eta\eta' + n$$

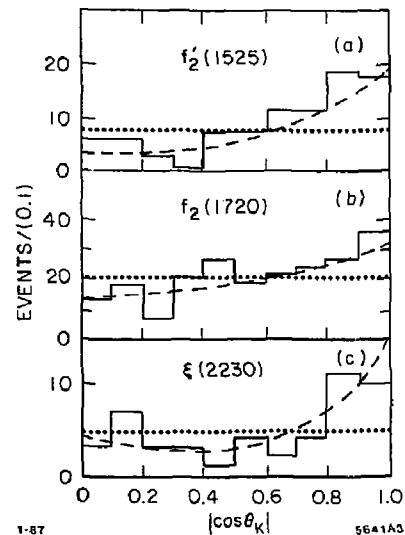
at 38 and 100 GeV/c.

The LASS collaboration also reported^[141] observation of a state at ~ 2200 MeV/c² decaying into $K_S^0 K_S^0$ and $K^+ K^-$ in the reactions

$$K^- p \rightarrow \Lambda K_S^0 K_S^0 \quad \text{and} \quad K^- p \rightarrow \Lambda K^+ K^-$$

at 11 GeV/c. A moment analysis prefers the spin-parity of this state to be 4^{++} .

The search for other decay modes of the $\xi(2230)$ by MARK III and the upper limits in various channels have been previously presented.^[144]



5. The $\cos\theta_K$ distributions for events in the region (a) 1420-1550 MeV/c², (b) 1620-1820 MeV/c², and (c) 2180-2280 MeV/c². The histograms are the data. The dotted lines are the Monte Carlo predictions for the spin 0 and the dashed lines for the spins 2 hypotheses.

2.2 $J/\psi \rightarrow \gamma\phi\phi$

Three glueball candidates were reported^[123] to decay into $\phi\phi$ in the fixed target reaction $\pi^-p \rightarrow \phi\phi + n$. Hence the $\xi(2230)$ was searched for in the radiative $\phi\phi$ decay of the J/ψ . The observation of the $\eta_c(2980)$ decay into $\phi\phi$ in the reaction

$$J/\psi \rightarrow \gamma \phi \quad \phi$$

$$\quad \quad \quad \searrow \quad \swarrow$$

$$\quad \quad \quad K^+ K^- \quad K^+ K^-$$

was published^[124] earlier by MARK III with a smaller data sample. Recently, the DM2 collaboration has published their results in this decay mode.^[124]

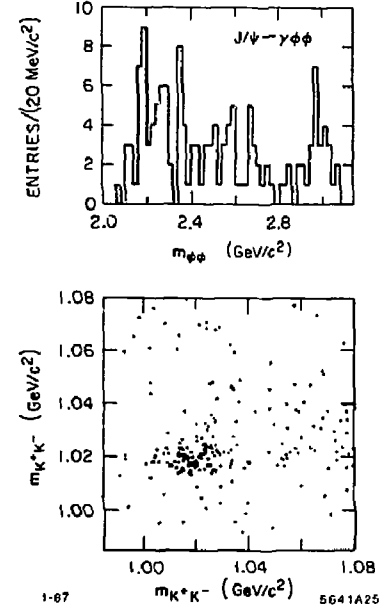
The $\phi\phi$ effective mass distribution is plotted in Fig. 6(a); the evidence for $\phi\phi$ production from the four charged kaons is shown in the scatter plot in Fig. 6(b). Figure 6(a) shows clear production of the $\eta_c(2980)$, while the lower mass region around 2200-2400 MeV/c² shows interesting structures. The charged kaon detection efficiency in the lower mass region is critical because of the kinematics. A careful evaluation of the efficiency and detection of the ϕ in decay modes e.g., $K_S^0 K_L^0$ and $\pi^+\pi^-\pi^0$, which do not suffer from similar problems, are underway. Presently, the branching ratio in the $\phi\phi$ mass region of 2100-2400 MeV/c² is measured to be

$$B(J/\psi \rightarrow \gamma\phi\phi) = (4.0 \pm 0.5 \pm 0.8) \times 10^{-4}.$$

The spin-parity of the relevant structures are also being examined.

2.3 $J/\psi \rightarrow \gamma\omega\phi$

One of the proposed decay modes of a hybrid^[10] $\xi(2230)$, is thought to be $\omega\phi$. An additional motivation for studying this decay mode was to complete a systematic study of the $\eta_c \rightarrow 1^{--}1^{--}$ decays, which was published^[124] earlier (with the smaller data sample), without the $\omega\phi$ mode. The various decay



6. (a) $\phi\phi$ mass spectrum from the reaction $J/\psi \rightarrow \gamma\phi\phi \rightarrow \gamma 4K^\pm$, (b) Scatter plot of K^+K^- vs K^+K^- .

mechanisms of the $\eta_c(2980)$ are shown in Fig. 7(a). It was suggested^[31] that the measurement of this decay mode would yield important insight into the relevant diagrams contributing to the $\eta_c(2980)$ decay. If flavour - SU(3) were exact, the reduced branching ratios $\tilde{B}(\eta_c \rightarrow \phi\phi)$, $\tilde{B}(\eta_c \rightarrow \rho^+\rho^-)$, $\tilde{B}(\eta_c \rightarrow \omega\omega)$, and $\frac{1}{2}\tilde{B}(\eta_c \rightarrow K^{*+}K^{*-})$ should all be equal. These measurements are being repeated with the present high statistics data.

Figure 7(b) shows the $\omega\phi$ effective mass plot in the decay,

$$J/\psi \rightarrow \gamma \omega \quad \phi$$

$$\quad \quad \quad \swarrow \quad \searrow$$

$$\quad \quad \quad \pi^+ \pi^- \pi^+ \quad K^+ K^-$$

A hint of the $\xi(2230)$ is visible with a few events at 2230 MeV/c². An upper limit was estimated with a conservative approach,

$$B(J/\psi \rightarrow \gamma \xi(2230)) \cdot B(\xi(2230) \rightarrow \omega\phi) < 5.9 \times 10^{-5},$$

at a 90% C.L. Note for comparison that the branching ratios of the J/ψ into K^+K^- and $K_S^0 K_S^0$ are

$$B(J/\psi \rightarrow \gamma \xi(2230)) \cdot B(\xi(2230) \rightarrow K^+ K^-) = (4.2_{-1.4}^{+1.7} \pm 0.8) \times 10^{-5},$$

and

$$B(J/\psi \rightarrow \gamma \xi(2230)) \cdot B(\xi(2230) \rightarrow K_S^0 K_S^0) = (3.1_{-1.3}^{+1.6} \pm 0.7) \times 10^{-5}.$$

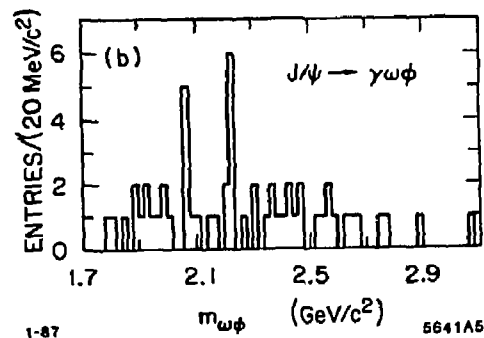
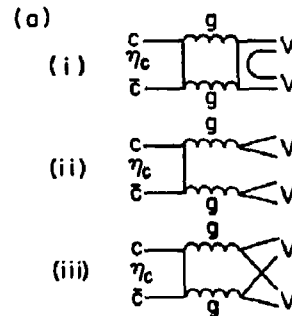
No signal for the $\eta_c(2980)$ is visible. The upper limit at 90% C.L. is

$$B(J/\psi \rightarrow \gamma \eta_c(2980)) \cdot B(\eta_c(2980) \rightarrow \omega\phi) < 1.3 \times 10^{-5}.$$

The inclusive branching ratio into $\gamma\omega\phi$ was measured as

$$B(J/\psi \rightarrow \gamma\omega\phi) = (1.40 \pm 0.25 \pm 0.28) \times 10^{-4}.$$

In summary,



7. (a) Mechanisms for $\eta_c \rightarrow VV$ decay. In (ii) the produced mesons are color-octet states, but supposedly turn into color singlets by soft gluon exchanges, i.e. final state interactions. (b) $\omega\phi$ mass spectrum from the reaction $J/\psi \rightarrow \gamma\omega\phi$.

- i) The narrow state $\xi(2230)$, first observed by MARK III has been established. The spin-parity determination by a maximum likelihood fit to the helicity amplitudes favors $J^P \geq 2^+$.
- ii) The $\eta_c(2980)$ was observed in the $\phi\phi$ decay mode with increased statistics in an ongoing analysis of $J/\psi \rightarrow \gamma\phi\phi$. The mass region between 2200-2400 MeV/c² shows interesting structures.
- iii) The radiative decay $J/\psi \rightarrow \gamma\omega\phi$ was observed with a few events at 2230 MeV/c². No significant $\eta_c(2980)$ signal was observed. A complete analysis of $\eta_c \rightarrow 1^- 1^- 1^-$ with increased statistics is underway and should yield important insight into the decay mechanism.

3. The 1400 - 1500 MeV/c² Region and the 1280 MeV/c² Region

The 'old' and the 'new' spectroscopy came to meet in a bitter feud in the 1400-1500 MeV/c² mass region. The $f_1(1420)$ $\{E_1', 420\}$ has been around for over twenty years as a member of the 1^{++} axial vector nonet, primarily containing $s\bar{s}$ quarks.^[14] However, a recent high statistics PWA^[15] claimed the spin-parity of the $f_1(1420)$ to be 0^{-+} . The $\eta(1440)$, a pseudoscalar glueball candidate, was and has been observed in radiative J/ψ decays. As mentioned earlier, while the radiative decays could be the ideal place to look for glueballs, the ordinary $q\bar{q}$ mesons are also observed here. Therefore, with the possibility of the $f_1(1420)$ being a pseudoscalar, it has been suggested^[14] that the $f_1(1420)$ is the same as, or part of, the $\eta(1440)$. To address this question, the radiative decay $J/\psi \rightarrow \gamma K\bar{K}\pi$ was compared with the hadronic decays, $J/\psi \rightarrow \omega K\bar{K}\pi$, and $J/\psi \rightarrow \phi K\bar{K}\pi$.

The $K\bar{K}\pi$ Decay Mode

3.1 $J/\psi \rightarrow \gamma K\bar{K}\pi$

The $\eta(1440)$ was observed by MARK III in the decay modes $J/\psi \rightarrow \gamma K^\pm K_S^0 \pi^\mp$, and $J/\psi \rightarrow \gamma K^+ K^- \pi^0$ and a complete Dalitz plot analysis was performed^[20] with the earlier data sample. The spin-parity of the $\eta(1440)$ was determined to be 0^- , and the mass and the width

$$\begin{aligned} m &= 1456 \pm 5 \pm 6 \quad \text{MeV/c}^2 \\ \Gamma &= 95 \pm 10 \pm 15 \quad \text{MeV/c}^2 \end{aligned}$$

in $K^\pm K_S^0 \pi^\pm$ mode, and

$$\begin{aligned} m &= 1461 \pm 5 \pm 5 \quad \text{MeV/c}^2 \\ \Gamma &= 101 \pm 10 \pm 10 \quad \text{MeV/c}^2 \end{aligned}$$

in $K^+ K^- \pi^0$ mode. The measured branching ratios were

$$B(J/\psi \rightarrow \gamma \eta(1440)) \cdot (\eta(1440) \rightarrow K^\pm K_S^0 \pi^\mp) = (16.5 \pm 1.0 \pm 2.7) \times 10^{-4}$$

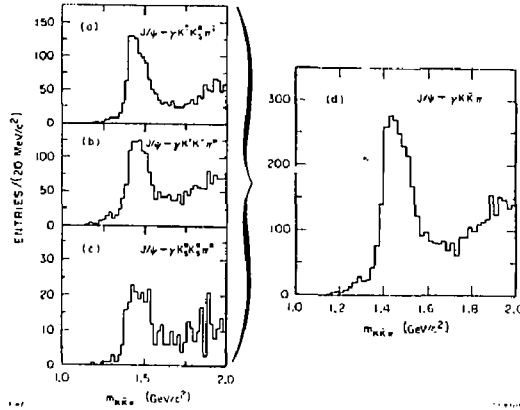
and,

$$B(J/\psi \rightarrow \gamma \eta(1440)) \cdot (\eta(1440) \rightarrow K^+ K^- \pi^0) = (8.2 \pm 0.4 \pm 1.4) \times 10^{-4}.$$

These are in agreement with the isospin zero prediction of 2:1. The branching ratio into $K\bar{K}\pi$, corrected for the unseen decay modes was

$$B(J/\psi \rightarrow \gamma \eta(1440)) \cdot (\eta(1440) \rightarrow K\bar{K}\pi) = (50 \pm 3.0 \pm 8.0) \times 10^{-4}.$$

Figure 8 (a), (b) and (c) show the $K^\pm K_S^0 \pi^\pm$, $K^+ K^- \pi^0$ and the $K_S^0 K_S^0 \pi^0$



8. The $\eta(1440)$ signal in $K\bar{K}\pi$ from the decay modes (a) $J/\psi \rightarrow \gamma K^\pm K_S^0 \pi^\mp$, (b) $\gamma K^+ K^- \pi^0$, (c) $\gamma K_S^0 K_S^0 \pi^0$, and (d) the sum of the three, $J/\psi \rightarrow \gamma K\bar{K}\pi$.

effective mass spectra respectively from the reactions

$$J/\psi \rightarrow \gamma K^\pm K_S^0 \pi^\mp$$

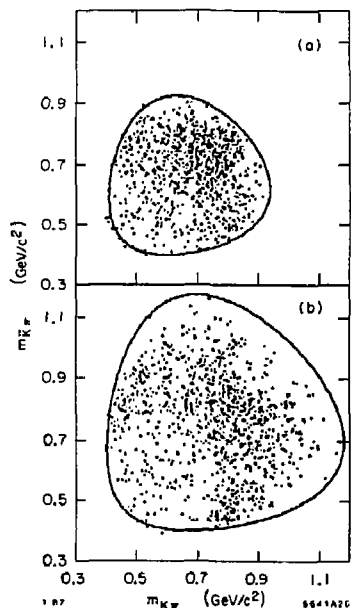
$$J/\psi \rightarrow \gamma K^+ K^- \pi^0$$

$$\text{and } J/\psi \rightarrow \gamma K_S^0 K_S^0 \pi^0$$

from the recent high statistics J/ψ data sample. Clear peaks at $\sim 1440 \text{ MeV}/c^2$ are observed in all three spectra. However, they could not be described well by a simple Breit-Wigner parametrization of the data. Figure 8(d) shows the three spectra combined. The $K\bar{K}\pi$ final states can arise from either $K^*(892)K$ or $a_0(980)\{\delta(980)\}\pi$ intermediate states. In the $K^*(892)K$ mode, the threshold effect can be important. To observe possible substructures, the $\eta(1440)$ peak was split into a lower [1360-1460 MeV/c^2] and an upper mass [1460-1580 MeV/c^2] band. Figure 9(a) and (b) show the Dalitz plots for these two mass regions. In the upper mass region, away from the $K^*(892)K$ threshold, the Dalitz plot indicates presence of the $K^*(892)$. In the lower mass region, kinematic effects make it hard to distinguish between the two isobars $K^*(892)$ and the $a_0(980)$. In addition, the $a_0(980)$ decays mostly into $\eta\pi$ and partially into $K\bar{K}$ which is suppressed by phase space. Hence, one has to keep in mind that the observed spectrum can be the result of a threshold effect, a coupled channel effect or possible interferences between different intermediate states. A complete PWA program is necessary to unfold the spectrum and such a study is underway.

3.2 $J/\psi \rightarrow \omega + K\bar{K}\pi$ and $\phi + K\bar{K}\pi$

As mentioned earlier, the $q\bar{q}$ mesons are copiously produced in the hadronic decays of the J/ψ . The $f_1(1420)$, was first seen^[120] in pp annihilations at rest in the $K\bar{K}\pi$ decay mode, and its spin-parity was assigned to be 0^- . Its subsequent observations in hadroproduction experiments showed^{[121] [122]} the state to have a $J^{PC} = 1^{++}$ and a dominant K^*K decay^[121] mode. Some recent experiments observed the $f_1(1420)$ decay into $K\bar{K}\pi$ ^[121] and $\eta\pi\pi$ ^[123] through a $a_0(980)\pi$



9. Dalitz plot distribution of the events in the $\eta(1440)$ region of (a) 1360-1460 MeV/c² and (b) 1460-1580 MeV/c².

intermediate state. So, comparing the resonance structures in the final states $KK\pi$ and $\eta\pi\pi$ produced in radiative decays and in association with a ϕ and an ω in hadronic decays of the J/ψ , address the following questions at the same time; i) do we see the $\eta(1440)$ in hadronic decays, ii) is $f_1(1420)$ distinct from the $\eta(1440)$, and iii) what is the $f_1(1420)$.

3.3 $J/\psi \rightarrow \omega KK\pi$

The following decay modes were observed;

$$J/\psi \rightarrow \omega K^+ K^- \pi^0 \rightarrow \pi^+ \pi^- \pi^0 \quad i)$$

and,

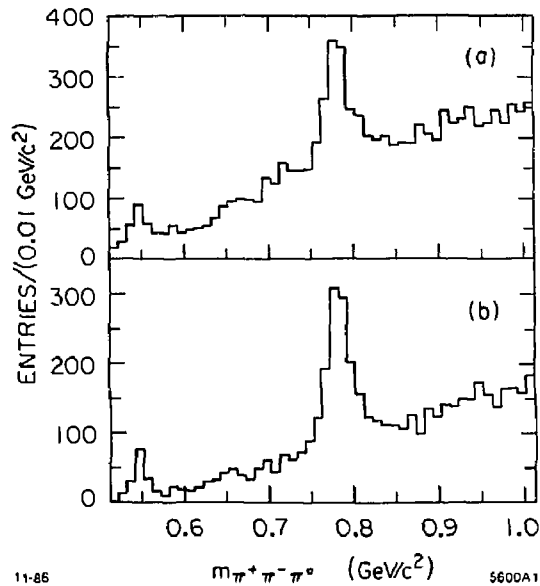
$$J/\psi \rightarrow \omega K_S^0 K_S^0 \pi^\pm \rightarrow \pi^+ \pi^- \pi^0 \quad ii)$$

Figure 10(a) and (b) show the reconstructed $\pi^+ \pi^- \pi^0$ spectra from i) and ii) respectively. Clean ω signals are visible in both cases.

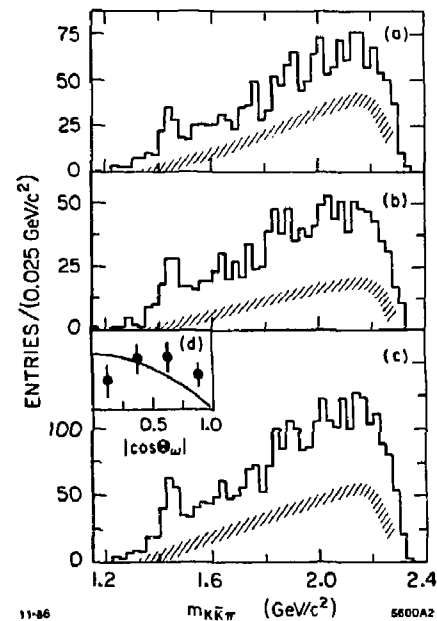
Figure 11(a) and (b) show the $K^+ K^- \pi^0$ invariant mass from i) and the $K_S^0 K_S^0 \pi^\pm$ invariant mass from ii) recoiling against the ω .

The spectra are very similar in nature. Figure 11(c) contains the sum of 11(a) and (b).^{*} A clear peak at ~ 1440 MeV/c² is observed. Analysis of i) and ii) was first performed separately, and then together; they were found to be consistent with each other. Details are given elsewhere.^[14] Fitting the combined spectrum with a Breit-Wigner (and a polynomial background), one obtains the

* The background from events not containing a ω (marked by the shaded area) is estimated from observing the $KK\pi$ events associated with the events from the sidebands of the ω .



10. The $\pi^+\pi^-\pi^0$ invariant mass distributions from the reactions (a) $J/\psi \rightarrow \omega K^+ K^- \pi^+$ with two possible entries per event and (b) $J/\psi \rightarrow \omega K^+ K_S^0 \pi^+$ with six possible entries per event.



11. (a) $K^+ K_S^0 \pi^+$ invariant mass distribution from the reaction $J/\psi \rightarrow \omega K^+ K_S^0 \pi^+$, (b) $K^+ K^- \pi^+$ invariant mass distribution from $J/\psi \rightarrow \omega K^+ K^- \pi^+$, and (c) the sum of the two. The shaded bands show the estimate of the background. (d) distribution of $|\cos \theta_\omega|$ with prediction.

mass and the width of the peak,

$$m = 1442 \pm 5^{+10}_{-17} \text{ MeV}/c^2$$

$$\Gamma = 40^{+17}_{-13} \pm 5 \text{ MeV}/c^2$$

The width, which is given as $24 < \Gamma < 84 \text{ MeV}/c^2$ at a 90% C.L., is not consistent with that of the $\eta(1440)$.^[24] Table 1 lists the relevant branching ratios, as well as the results of the Breit-Wigner fits for each decay mode individually and combined. The angular distribution of the ω , recoiling against the $1440 \text{ MeV}/c^2$ peak was studied in order to distinguish between 0^- and 1^+ of the spin-parity of the state. Figure 11(d) shows the distribution of the normal to the ω decay plane in the helicity system of the ω . The solid curve is the prediction for a $J^P = 0^-$ state. The data clearly is very different from the pseudoscalar assumption (a fit yields a 6% probability). A coupled channel analysis^[24] was performed to the $K\bar{K}\pi$ system assuming that the $K\bar{K}\pi$ consisted of three contributions, i) a K^*K intermediate state, ii) an $a_0(980)\pi$ intermediate state, and iii) an isotropic distribution. The K^*K was observed to be the dominant contribution and the resonant structure was assigned a $J^P = 1^+$ by this technique. If the DOZI contributions were neglected, the $f_1(1420)$, an axial-vector-like object, produced in association with an ω in J/ψ hadronic decay should have a substantial amount of u, d quark content.

3.4 $J/\psi \rightarrow \phi K\bar{K}\pi$

The following decay modes were examined,

$$J/\psi \rightarrow \phi \begin{cases} K^+ K^- \pi^0 \\ K^+ K^- \end{cases} \quad \text{i)}$$

and

TABLE 1. Measured Branching Ratios into $K\bar{K}\pi$ and $\eta\pi\pi$ Decay Modes

Decay mode $J/\psi \rightarrow$	Object	Mass MeV/c^2	Width MeV/c^2	Branching Ratio $\times 10^{-4}$
$\gamma + K^\pm K_S^0 \pi^\mp$ $\gamma + K^+ K^- \pi^0$	$\eta(1440)$	$1456 \pm 5 \pm 6$ $1461 \pm 5 \pm 5$	$95 \pm 10 \pm 15$ $104 \pm 10 \pm 10$	$(J/\psi \rightarrow \gamma\eta(1440)) \cdot (\eta(1440) \rightarrow K\bar{K}\pi)$ $= 50 \pm 3 \pm 8$ $= 49 \pm 2 \pm 8$
$\omega + K^\pm K_S^0 \pi^\mp$ $\omega + K^+ K^- \pi^0$	inclusive inclusive	inclusive inclusive	inclusive inclusive	$29.5 \pm 1.4 \pm 1.0$ $53 \pm 14 \pm 14$
$\omega + K^\pm K_S^0 \pi^\mp$ $\omega + K^+ K^- \pi^0$	$f_1(1420)$	$1449 \pm 7^{+10}_{-16}$ $1440 \pm 7^{+10}_{-17}$	$44^{+20}_{-16} \pm 13$ $34^{+22}_{-16} \pm 5$	$(J/\psi \rightarrow \omega f_1(1420)) \cdot (f_1(1420) \rightarrow K\bar{K}\pi)$ $(f_1(1420) \rightarrow K^\pm K_S^0 \pi^\mp) = 2.1^{+0.9}_{-0.7} \pm 0.6$ $(f_1(1420) \rightarrow K^+ K^- \pi^0) = 1.3^{+0.5}_{-0.4} \pm 0.3$
$\omega + K\bar{K}\pi$ $\phi + K^\pm K_S^0 \pi^\mp$ $\phi + K\bar{K}\pi$	inclusive inclusive $\eta(1440)$	$1442 \pm 5^{+10}_{-17}$ inclusive 1420-1440 1460	$40^{+17}_{-13} \pm 5$ inclusive 40-60 92	$(f_1(1420) \rightarrow K^+ K^- \pi^0) = 6.8^{+1.0}_{-0.8} \pm 1.7$ $= 7.0 \pm 0.6 \pm 1.0$ $(J/\psi \rightarrow \phi f_1(1420)) \cdot (f_1(1420) \rightarrow K\bar{K}\pi) < 1.2 \otimes 90\% \text{ C.L.}$ $(J/\psi \rightarrow \phi\eta(1440)) \cdot (\eta(1440) \rightarrow K\bar{K}\pi) < 2.1 \otimes 90\% \text{ C.L.}$
$\gamma + \eta\pi^+ \pi^-$ $\gamma + \eta\pi^+ \pi^-$	$\eta(1275)$ or $f_1(1285)$?	1283 fixed 1283 fixed	$14^{+20}_{-15} \pm 10$ 20 fixed 26 fixed	$(J/\psi \rightarrow \phi f_1(1285)) \cdot (f_1(1285) \rightarrow K\bar{K}\pi) = 0.6 \pm 0.2 \pm 0.1$ $(J/\psi \rightarrow \gamma\pi) \cdot (\pi \rightarrow a_0(980)\pi) \cdot (a_0(980) \rightarrow \eta\pi)$ $= 2.7 \pm 0.8 \pm 0.2$ $= 3.2 \pm 1.1 \pm 0.3$
$\gamma + \eta\pi^+ \pi^-$ $\gamma + \eta\pi^+ \pi^-$	$X?$	1362 $\pm$ 6 1400 $\pm$ 7	69 $\pm$ 23 62 $\pm$ 16	$(J/\psi \rightarrow \gamma X) \cdot (X \rightarrow a_0(980)\pi) \cdot (a_0(980) \rightarrow \eta\pi)$ $= 5.2 \pm 1.2 \pm 0.5$ $= 5.9 \pm 1.8 \pm 0.5$
$\omega + \eta\pi^+ \pi^-$ $\phi + \eta\pi^+ \pi^-$	$f_1(1420)?$ $\eta(1275)$ or $f_1(1285)?$	$1421 \pm 8 \pm 10$ $1283 \pm 6 \pm 10$	$45^{+32}_{-25} \pm 15$ $24^{+40}_{-14} \pm 10$	$(J/\psi \rightarrow \omega f_1(1420)) \cdot (f_1(1420) \rightarrow \eta\pi\pi) = 9.2 \pm 2.4 \pm 2.8$ $(J/\psi \rightarrow \phi\pi) \cdot (\pi \rightarrow \eta\pi\pi) = 1.6^{+0.6}_{-0.5} \pm 0.4$

i) Results obtained from the initial 2.7×10^6 data sample.

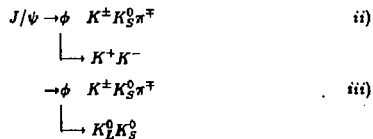
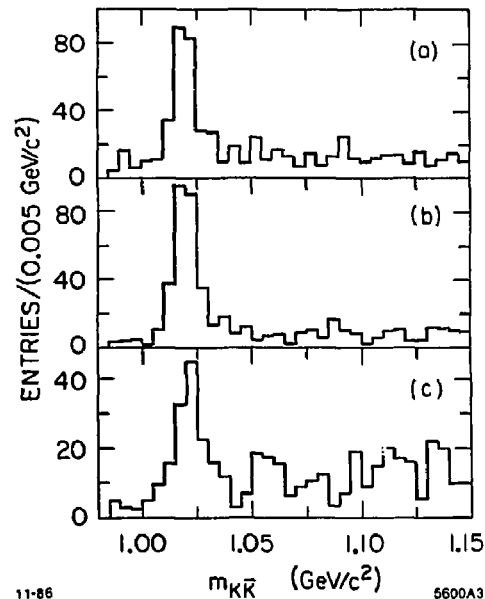


Figure 12(a), (b) show the reconstructed K^+K^- spectra from i) and ii) respectively, and Fig. 12 (c) shows the $K_L^0 K_S^0$ spectrum from iii); a clear ϕ signal is apparent in all three plots. Figure 13 presents the combined $K\bar{K}\pi$ spectrum produced in association with a ϕ from i), ii) and iii). No clear structure is visible in the 1440 MeV/c² region. Apart from a small signal at 1280 MeV/c², the rest of the spectrum is dominated by a broad phase space like distribution.^[34] The upper limits at a 90% C.L. for the $f_1(1420)$ and the $\eta(1440)$ production in association with the ϕ are quoted in Table 1. The inclusive branching ratios are also listed. The mass and the width of the structure at 1280 MeV/c² was measured to be

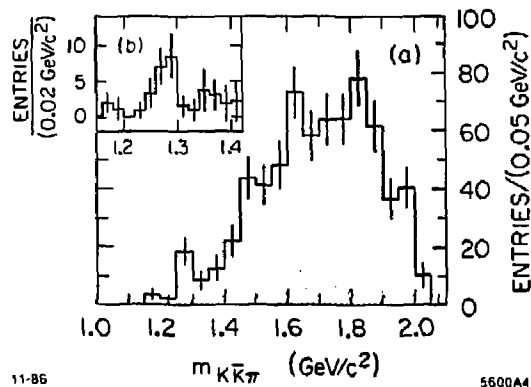
$$\begin{aligned}
m &= 1279 \pm 6 \pm 10 \text{ MeV}/c^2 \\
\Gamma &= 14_{-14}^{+20} \pm 10 \text{ MeV}/c^2,
\end{aligned}$$

consistent with those of the $f_1(1285)$ $\{D(1285)\}$, an isosinglet member of the 1^{++} nonet. Figure 14 shows the three $K\bar{K}\pi$ systems recoiling against a γ , an ω and a ϕ .

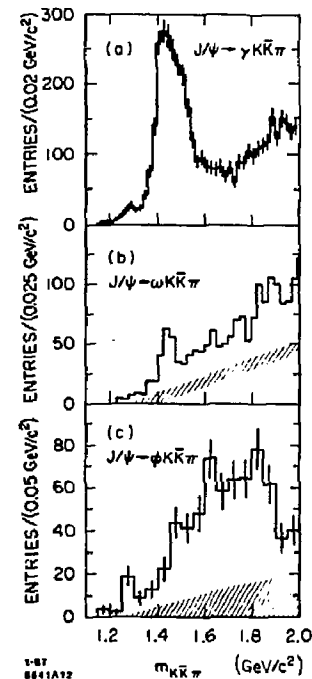
Neither the $f_1(1420)$ nor the $\eta(1440)$ is observed in the $K\bar{K}\pi$ mode recoiling against a ϕ . If the DOZI contributions could be neglected, this would strongly suggest that the $f_1(1420)$ does not have any substantial strange quark content. Furthermore, a structure consistent with the $f_1(1285)$, commonly believed to contain u, d quarks, is observed to be produced recoiling against a ϕ .



12. K^+K^- invariant mass distributions from (a) $J/\psi \rightarrow \phi K^+K^- \pi^0$ with four possible entries per event, (b) $J/\psi \rightarrow \phi K^\pm K_S^0 \pi^\mp$ with two possible entries per event, and (c) $K_S^0 K_L^0$ invariant mass distribution from $J/\psi \rightarrow \phi K^\pm K_S^0 \pi^\mp$ with up to six entries per event. Background from events not containing K_S^0 is subtracted in (b) and (c).



13. (a) Summed $K^+K^-\pi^+$ and $K^\pm K_S^0\pi^\mp$ invariant mass distributions from the reactions $J/\psi \rightarrow \phi K^+K^-\pi^+$ and $J/\psi \rightarrow \phi K^\pm K_S^0\pi^\mp$. The shaded area shows the estimate of the background. (b) Detail of 1200 MeV/c^2 mass region after selecting $m(K\bar{K}) < 1150 \text{ MeV}/c^2$.



14. $K\bar{K}\pi$ mass spectra recoiling against (a) a γ , (b) and ω and (c) a ϕ . Shaded area represents the estimated background.

The $\eta\pi^+\pi^-$ Decay Mode

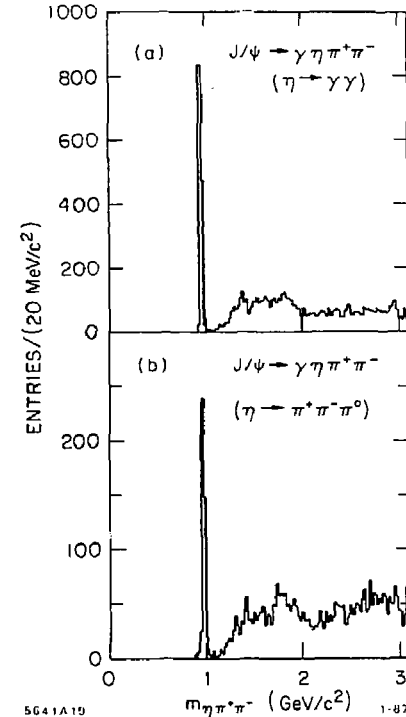
In the $K\bar{K}\pi$ system, an important question in identifying the resonance, has been isolating the isobaric intermediate state of its decay, namely, K^*K and $\omega(980)\pi$. The $\omega(980)$ has a large decay branching ratio into $\eta\pi$. Hence an ideal place to look for a solution to the $K\bar{K}\pi$ puzzle was in the $\eta\pi\pi$ system.

3.5 $J/\psi \rightarrow \gamma\eta\pi^+\pi^-$

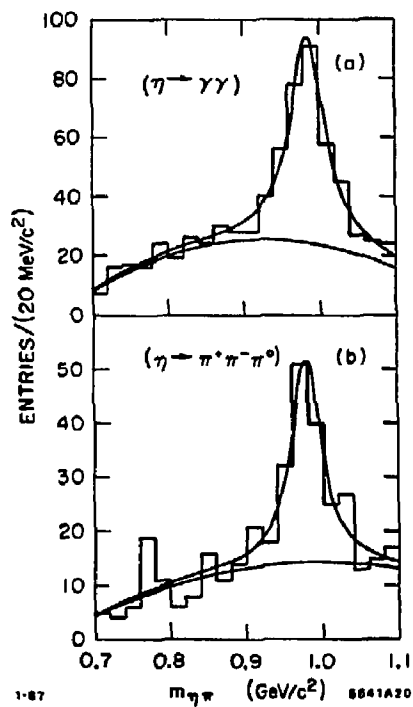
The following two decay modes were observed,

$$\begin{array}{ll} J/\psi \rightarrow \eta\pi^+\pi^- + \gamma & i) \\ \quad \downarrow \gamma\gamma & \\ J/\psi \rightarrow \eta\pi^+\pi^- + \gamma & \\ \quad \downarrow \pi^+\pi^-\pi^0 & ii) \end{array}$$

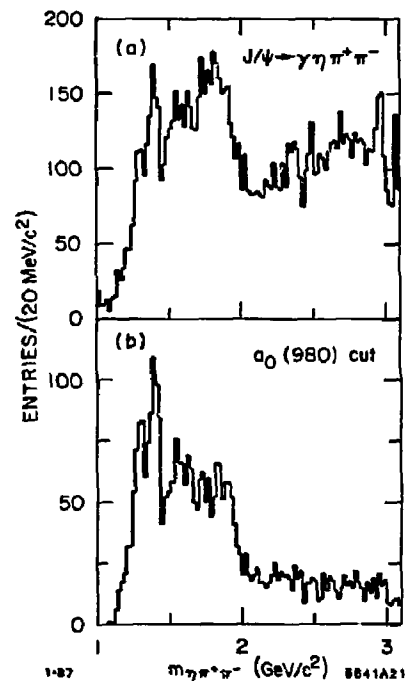
the invariant $\eta\pi^+\pi^-$ mass distributions from i) and from ii) are displayed in Fig. 15(a) and Fig. 15(b) respectively. A very sharp peak at $\sim 960 \text{ MeV}/c^2$ in both plots indicate copious $\eta'(958)$ production. Figure 16 (a),(b) show the $\eta\pi^\pm$ invariant mass spectra from i) and ii), in which copious $\omega(980)$ production is evident. The combined spectrum, with the $\eta\pi^+\pi^-$ invariant mass above $1000 \text{ MeV}/c^2$ is presented in Fig. 17(a), and shows many interesting structures. However, a sharp dip instead of a peak exists at the conventional mass of the $\eta(1440)$. Several possible explanations exist,^{[101] [102]} one of these being that the dip is caused by destructive interference^[103] between different decay modes of the $\eta(1440)$. Figure 17(b) depicts the $\eta\pi^+\pi^-$ spectrum where an intermediate $\omega(980)$ production was required. The $1200 \sim 1600 \text{ MeV}/c^2$ region is thereby emphasised, and a narrow peak at $\sim 1280 \text{ MeV}/c^2$ and another narrow peak at $\sim 1390 \text{ MeV}/c^2$ are apparent. The exact mass and the width, assigned to the $\sim 1390 \text{ MeV}/c^2$ structure by a Breit-Wigner parametrization, are listed in Table 1. The respective branching ratios are also listed in Table 1.



15. The $\eta\pi^+\pi^-$ mass spectra in the reaction $J/\psi \rightarrow \gamma\eta\pi^+\pi^-$ where the η decays into (a) $\gamma\gamma$ and (b) $\pi^+\pi^-\pi^0$.



16. Example of $a_0(980)$ formation in the decay $J/\psi \rightarrow \gamma \eta \pi^+ \pi^-$, e.g. (a) in $\eta \pi^+$ where η decays into $\gamma \gamma$ and (b) in $\eta \pi^-$ where η decays into $\pi^+ \pi^- \pi^0$.



17. (a) $\eta \pi^+ \pi^-$ effective mass spectrum where the two η decay modes are summed, (b) $\eta \pi^+ \pi^-$ effective mass spectrum where an intermediate $a_0(980)$ formation is required.

The lower peak at 1285 MeV/c² is consistent with $f_1(1285)$, however it is also consistent with the $\eta(1275)$ reported^[11] in the PWA of the $\eta\pi^+\pi^-$ system in fixed target experiments. The second peak occurs at a mass lower than the conventional $f_1(1420)$ mass. A complete PWA is required to clarify the situation. The mass region above 1500 MeV/c² is not discussed here, but shows interesting structures.

3.6 $J/\psi \rightarrow \omega\eta\pi^+\pi^-$

The following decay mode was investigated,

$$J/\psi \rightarrow \omega \eta \pi^+ \pi^-$$

$$\pi^+ \pi^- \pi^0 \leftarrow \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \gamma \gamma$$

Figure 18(a) presents the $\eta\pi^+\pi^-$ invariant mass spectrum recoiling against an ω . The $\eta'(958)$ is clearly visible along with a peak at the nominal $f_1(1285)$ and a peak at the $f_1(1420)$ mass. The masses and the widths were determined to be

$$m = 1283 \pm 6 \pm 10 \text{ MeV}/c^2$$

$$\Gamma = 14^{+10}_{-14} \pm 10 \text{ MeV}/c^2$$

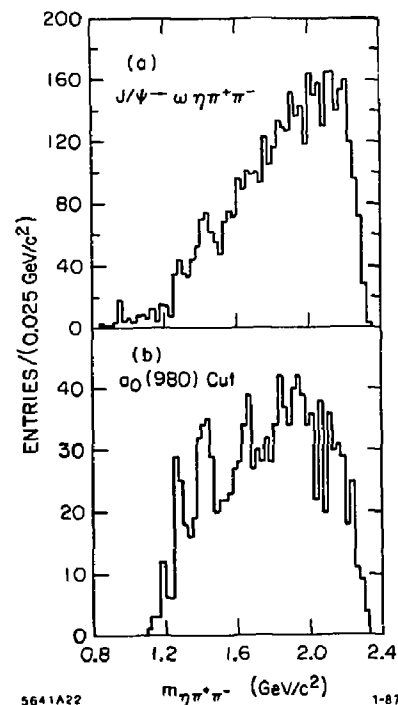
and

$$m = 1421 \pm 8 \pm 10 \text{ MeV}/c^2$$

$$\Gamma = 45^{+32}_{-23} \pm 15 \text{ MeV}/c^2$$

respectively, for the 1285 MeV/c² and the 1420 MeV/c² peaks. The corresponding branching ratios are listed in Table 1.

A study of the substructure in $\eta\pi^+\pi^-$ yielded a significant amount of the $a_0(980)$ production. Figure 18(b) represents the $\eta\pi^+\pi^-$ invariant mass spectrum requiring that $\eta\pi$ system formed an $a_0(980)$. The $f_1(1285)$ region, seems consistent with all of the resonance proceeding through an intermediate state of $a_0(980)$, while in the $f_1(1420)$ region, the resonance proceeds dominantly through the $a_0(980)$. It is worth noting that the higher mass region in this $\eta\pi^+\pi^-$ spectrum in Fig. 18(a) seems very promising in terms of structures.



18. (a) $\eta\pi^+\pi^-$ invariant mass spectrum in the decay $J/\psi \rightarrow \omega\eta\pi^+\pi^-$, (b) $\eta\pi^+\pi^-$ invariant mass spectrum requiring an intermediate $a_0(980)$ formation.

3.7 $J/\psi \rightarrow \phi \eta \pi^+ \pi^-$

The decay mode observed was,

$$\begin{array}{c} J/\psi \rightarrow \phi \eta \pi^+ \pi^- \\ K^+ K^- \leftarrow \quad \rightarrow \gamma \gamma. \end{array}$$

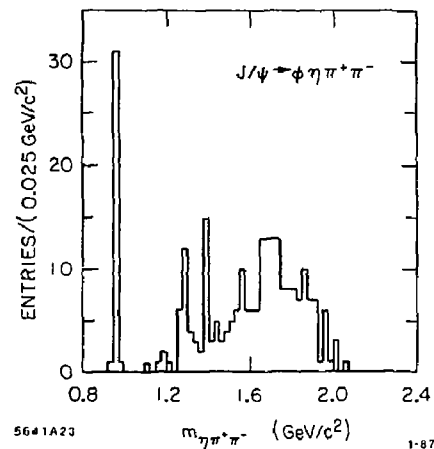
Figure 19 shows the invariant $\eta \pi^+ \pi^-$ mass recoiling against the ϕ . A clean spectrum with a large $\eta(958)$ signal is seen. At $\sim 1285 \text{ MeV}/c^2$, a peak compatible with the $f_1(1285)$ is observed. The mass and the width as determined by a Breit-Wigner parametrization were,

$$\begin{aligned} m &= 1283 \pm 6 \pm 10 \text{ MeV}/c^2 \\ \Gamma &= 24_{-14}^{+20} \pm 10 \text{ MeV}/c^2. \end{aligned}$$

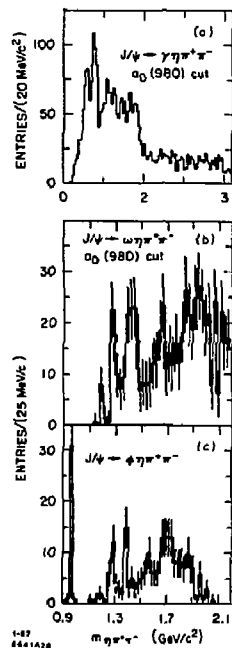
Again, similar to the spectrum of the $K\bar{K}\pi$ system recoiling against a ϕ , any production of the $\eta(1440)$ or the $f_1(1420)$ is not visible (a single high bin is observed). The relevant branching ratios are listed in Table 1. Similar to the previous decay modes, study of the substructure in the $\eta \pi^+ \pi^-$ system revealed that the $1285 \text{ MeV}/c^2$ resonance decayed through an intermediate $\omega_0(980)$ state.

Figure 20 presents the $\eta \pi^+ \pi^-$ spectrum recoiling against a γ , an ω and a ϕ . Figure 20(a) and (b) present the spectrum with the requirement of an $\omega_0(980)$ intermediate state. The conventional $\eta(1440)$ is not observed in the $\eta \pi \pi$ invariant mass spectrum in the radiative J/ψ decay. A narrow structure at $\sim 1400 \text{ MeV}/c^2$ is observed, along with an $f_1(1285)$ -like structure at the appropriate mass. In the hadronic decay of the J/ψ , an $f_1(1285)$ -like and an $f_1(1420)$ -like structure are seen to be produced in association with an ω , while only an $f_1(1285)$ like structure is seen to be produced in association with a ϕ .

To recapitulate, at the time the $\eta(1440)$ was first observed, the $f_1(1420)$ was established as the $s\bar{s}$ member and the $f_1(1285)$ as the $(u\bar{u} + d\bar{d})$ member of the 1^{++} family. These interpretations stemmed from the mass formula^[11]



19. The $\eta \pi^+ \pi^-$ invariant mass spectrum in the decay $J/\psi \rightarrow \phi \eta \pi^+ \pi^-$.



20. The $\eta\pi^+\pi^-$ (background subtracted) invariant mass spectra from the reactions (a) $J/\psi \rightarrow \gamma\eta\pi^+\pi^-$, (b) $J/\psi \rightarrow \omega\eta\pi^+\pi^-$ and (c) $J/\psi \rightarrow \phi\pi^+\pi^-$.

$$m_{a_1(1270)} = m_{f_1(1285)}$$

$$2m_{Q_A(1340)} = m_{f_1(1285)} + m_{f_1(1420)}$$

and from the fact that the $f_1(1420)$ was observed to decay into $K\bar{K}\pi$, while the $f_1(1285)$ decayed primarily into $\eta\pi\pi$. Recently, as discussed earlier, the $\eta(1440)$ is established as a 0^- , and the identity of the $f_1(1440)$ as an axial vector or a pseudoscalar state as well as an $s\bar{s}$ state has been seriously questioned. Several questions are raised by the MARK III data which need to be answered, e.g. whether the peaks seen in $K\bar{K}\pi$ and in $\eta\pi\pi$ (consistent with the $f_1(1420)$) recoiling against an ω are the same objects, and whether the peak in $\eta\pi\pi$ in the radiative decay is the same. Curiously, in the recoil against the ϕ , no peak is observed at the $f_1(1420)/\eta(1440)$ mass region. Several explanations have been attempted^{[10] [11]} to accommodate all the data in this region from the J/ψ decay, fixed target reactions and the radiative and the two photon width measurements. One of these^[10] suggests that the $K\bar{K}\pi$ peak observed in association with an ω in the J/ψ decay as well as that observed in the 2γ reaction^[12] is a hybrid $q\bar{q}\rho$ 1^-+ exotic state, and the state observed in $\eta\pi\pi$ in association with an ω is a different state.

The $1280 \text{ MeV}/c^2$ region is somewhat complicated because of the presence of the $\eta(1275)$ (the radial excitation of the η), in addition to the $f_1(1285)$. Assuming that MARK III data shows the $f_1(1285)$ in $\eta\pi\pi$ in associated production with ϕ and ω in the J/ψ decay, it casts doubt on the pure ($u\bar{u} + d\bar{d}$) interpretation of the $f_1(1285)$, if DOZI correlations were to be ignored.

4. In Search of the $f_2(1720)$

As discussed earlier, it will be a triumph for QCD if a strongly interacting gluonic bound state or a hybrid state, namely a bound state of quarks and gluons is observed. Identifying such a state will be an experimental success. The $f_2(1720)$ has been a prime candidate for such a state. It was first observed^[1] in radiative J/ψ decay, $J/\psi \rightarrow \gamma\eta\eta$. MARK III observed^[17] the $f_2(1720)$ in radiative decay in $K\bar{K}$ and $\pi^+\pi^-$, in its initial 2.7×10^6 data sample. The measured branching ratio into K^+K^- was,

$$B(J/\psi \rightarrow \gamma f_2(1720)) \cdot \{f_2(1720) \rightarrow K^+K^-\} = (4.8 \pm 0.6 \pm 0.9) \times 10^{-4}.$$

The mass and the width were

$$\begin{aligned} m &= 1720 \pm 7 \text{ MeV}/c^2 \\ \Gamma &= 132 \pm 15 \text{ MeV}/c^2. \end{aligned}$$

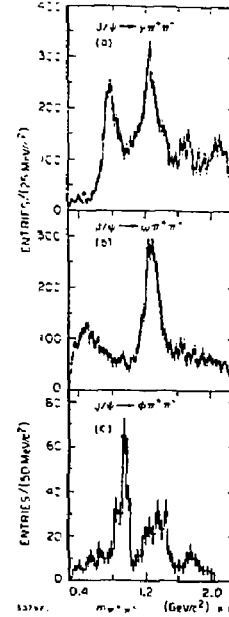
The spin parity was determined to be 2^+ . Figure 3(a) and 3(b) show the K^+K^- and $K_S^0\bar{K}_S^0$ invariant mass spectra from the updated data sample in the reactions $J/\psi \rightarrow \gamma K^+K^-$ and $J/\psi \rightarrow \gamma K_S^0\bar{K}_S^0$, respectively. A clear $f_2(1720)$ is seen in both plots, along with the $f_2'(1525)$. In the $K_S^0\bar{K}_S^0$ mode, a maximum likelihood fit to the helicity amplitudes^[18] yielded $J^P = 2^+$ for both the $f_2'(1525)$ and the $f_2(1720)$. Observation of the $f_2(1720)$ in the decay mode $J/\psi \rightarrow \gamma\pi^+\pi^-$ was also reported^[17] by MARK III. The mass and the width of the $f_2(1720)$ from this decay mode were measured to be

$$\begin{aligned} m &= 1713 \pm 15 \text{ MeV}/c^2 \\ \Gamma &= 130 \text{ MeV}/c^2 (\text{fixed}). \end{aligned}$$

The branching ratio was measured as

$$B(J/\psi \rightarrow \gamma f_2(1720)) \cdot \{f_2(1720) \rightarrow \pi^+\pi^-\} = (1.6 \pm 0.4 \pm 0.3) \times 10^{-4}.$$

Figure 21(a) shows the invariant $\pi^+\pi^-$ mass spectrum from the complete data sample. The low mass peak at $\sim 700 \text{ MeV}/c^2$ is feed-through events from the



21. The $\pi^+\pi^-$ effective mass spectra in the reactions (a) $J/\psi \rightarrow \gamma\pi^+\pi^-$, (b) $J/\psi \rightarrow \omega\pi^+\pi^-$, and (c) $J/\psi \rightarrow \phi\pi^+\pi^-$.

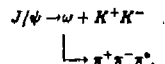
$J/\psi \rightarrow \rho^0 \pi^0$ decay mode, where one photon has not been detected. The $f_2(1270)$ ($f(1270)$) is observed along with a shoulder possibly due to the $f_2'(1525)$ production. The $f_2(1720)$ is clearly observed at the expected mass. A structure is seen at ~ 2100 MeV/c². This was previously reported^[17] as possible evidence of production of the $f_4(2030)$ ($h(2030)$). A detailed analysis is underway. An analysis of the radiative decay mode $J/\psi \rightarrow \gamma \eta \eta$ is also in progress.

Hadronic Searches

4.1 $\omega K^+ K^-$ AND $\phi K^+ K^-$

The comparison of a state produced in the radiative decay mode and a similar state produced in hadronic decay modes proved to be a useful tool to differentiate between the 'glue' and the quark content of the state, as described earlier. The same technique was employed to understand the nature of the $f_2(1720)$. Figure 22 displays the three $K^+ K^-$ spectra.

Figure 22(b) displays the $K^+ K^-$ effective mass spectrum observed in association with an ω in the reaction



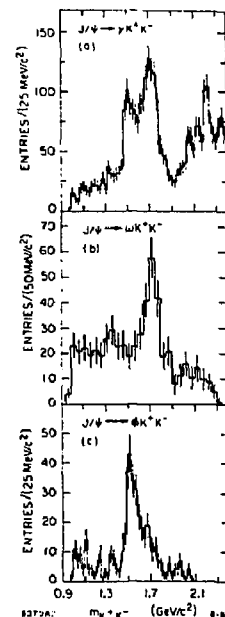
An $f_2(1720)$ -like peak is observed with a mass and a width of

$$\begin{aligned} m &= 1731 \pm 10 \pm 10 \text{ MeV}/c^2 \\ \Gamma &= 110^{+45}_{-35} \pm 15 \text{ MeV}/c^2. \end{aligned}$$

These parameters are consistent with those of the $f_2(1720)$. The measured branching ratio was

$$B(J/\psi \rightarrow \omega' f_2(1720)') \cdot (f_2(1720)' \rightarrow K\bar{K}) = (4.5^{+1.3}_{-1.1} \pm 1.0) \times 10^{-4}.$$

The $K_S^0 \bar{K}_S^0$ mode yields consistent results, although with smaller statistics.



22. The $K^+ K^-$ effective mass spectra in the reactions (a) $J/\psi \rightarrow \gamma K^+ K^-$, (b) $J/\psi \rightarrow \omega K^+ K^-$, and (c) $J/\psi \rightarrow \phi K^+ K^-$.

Figure 22(c) shows the K^+K^- spectrum recoiling against a ϕ in the decay mode,

$$\begin{array}{l} J/\psi \rightarrow \phi \quad K^+K^- \\ \quad \quad \quad \downarrow \\ \quad \quad \quad K^+K^- \end{array}$$

A prominent $f_2'(1525)$ is observed, with a clear shoulder on the high mass side. A coherent fit with standard $f_2(1720)$ parameters describe the data very well. A detailed study with the proper mix of the angular distributions is underway.

4.2 $\omega\pi^+\pi^-$ AND $\phi\pi^+\pi^-$

The radiative $\pi^+\pi^-$ spectrum was compared with the $\pi^+\pi^-$ spectra in hadronic decays, produced in association with a ϕ and an ω , as shown in Figure 21. Figure 21(b) displays the $\pi^+\pi^-$ spectrum from the decay mode,

$$\begin{array}{l} J/\psi \rightarrow \omega \quad \pi^+\pi^- \\ \quad \quad \quad \downarrow \\ \quad \quad \quad \pi^+\pi^-\pi^0 \end{array}$$

As expected from the quark correlations, a clear $f_2(1270)$ is observed. No obvious structure at the $f_2(1720)$ is seen. The $\pi^+\pi^-$ spectrum is described later in some detail. Figure 21(c) presents the $\pi^+\pi^-$ spectrum from the decay mode,

$$\begin{array}{l} J/\psi \rightarrow \phi \quad \pi^+\pi^- \\ \quad \quad \quad \downarrow \\ \quad \quad \quad K^+K^- \end{array}$$

A clear $f_0(975)\{S(975)\}$ and a broad structure in the 1400-1500 MeV/c² are observed and will be discussed later. A structure, similar in mass and width of those of the $f_2(1720)$, is observed.

In summary, the $f_2(1720)$ has been observed in the radiative decays of the J/ψ into K^+K^- , $K_S^0\bar{K}_S^0$ and $\pi^+\pi^-$, with large branching ratios. The analysis of the radiative decay mode, $J/\psi \rightarrow \gamma\eta\eta$ is in progress. Comparisons with the

hadronic channels yield interesting results. An $f_2(1720)$ -like structure is observed in $K\bar{K}$ produced in association with an ω , and in $\pi^+\pi^-$ produced in association with a ϕ , where the conventional $q\bar{q}$ mesons are not expected to be produced profusely. In $K\bar{K}$, in association with a ϕ , the $f_2(1720)$ -like structure is seen as only a shoulder to the $f_2'(1525)$, although a detailed analysis will yield more definitive results.

5. Quark Correlations

The study of quark correlations can be extended to whole meson nonets, namely the members of any nonet recoiling against members of the vector nonet. The vector nonet was chosen because it is ideally mixed. The nonet correlations investigated so far are,

$$\begin{array}{l} J/\psi \rightarrow \text{Vector} + \text{Pseudoscalar} \\ \quad \quad \quad + \text{Tensor} \\ \quad \quad \quad + \text{Scalar} \end{array}$$

All the Vector-Pseudoscalar decay branching ratios were measured^[40] from the previous data sample. By observing and measuring the decays

$$\begin{array}{l} J/\psi \rightarrow \omega\eta \\ \quad \quad \quad \rightarrow \phi\eta \end{array}$$

and

$$\begin{array}{l} J/\psi \rightarrow \omega\eta' \\ \quad \quad \quad \rightarrow \phi\eta' \end{array}$$

the quark contents of the η and the η' were determined.^[40] From measuring all the decay modes, shown in Table 2, the strong, the electromagnetic and the SU(3) violating amplitudes were determined.

A similar study is almost completed for the Vector-Tensor case, and has begun for the Vector-Scalar.

TABLE 2. $J/\psi \rightarrow V\text{-}P$ Decay Amplitudes^{[40],[47]}

Decay mode	Amplitude	Amplitude with DOZI
$\rho^+\pi^-, \rho^0\pi^0\rho^-\pi^+$	$g + e$	$g + e$
$K^{*+}K^-, K^{*-}K^+$	$g - h + e(2 - x)$	$g - h + e(2 - x)$
$K^{*0}\bar{K}^0, \bar{K}^{*0}K^0$	$g - h - 2e(\frac{1+x}{2})$	$g - h - 2e(\frac{1+x}{2})$
$\omega\eta$	$(g + e)\chi_\eta$	$(g + e)\chi_\eta + \sqrt{2}\gamma g(\sqrt{2}\chi_\eta + Y_\eta)$
$\omega\eta'$	$(g + e)\chi_{\eta'}$	$(g + e)\chi_{\eta'} + \sqrt{2}\gamma g(\sqrt{2}\chi_{\eta'} + Y_{\eta'})$
$\phi\eta$	$(g - 2h - 2ex)Y_\eta$	$(g - 2h - 2ex)Y_\eta + \gamma(g - h)(\sqrt{2}\chi_\eta + Y_\eta)$
$\phi\eta'$	$(g - 2h - 2ex)Y_{\eta'}$	$(g - 2h - 2ex)Y_{\eta'} + \gamma(g - h)(\sqrt{2}\chi_{\eta'} + Y_{\eta'})$
$\rho^0\eta$	$3e\chi_\eta$	$3e\chi_\eta$
$\rho^0\eta'$	$3e\chi_{\eta'}$	$3e\chi_{\eta'}$
$\omega\pi^0$	$3e$	$3e$
$\phi\pi^0$	0	0

$X_\eta \rightarrow \sqrt{2}(u\bar{u} + d\bar{d})$ content of η , $Y_\eta \rightarrow s\bar{s}$ content of η ,

$X_{\eta'} \rightarrow \sqrt{2}(u\bar{u} + d\bar{d})$ content of η' , $Y_{\eta'} \rightarrow s\bar{s}$ content of η' ,

$g \rightarrow SU(3)$ symmetric strong amplitude, $e \rightarrow$ electromagnetic amplitude,

$h \rightarrow SU(3)$ violating amplitude, $x \rightarrow$ ratio of s and u quark magnetic moments,

$r \rightarrow$ ratio of DOZI and strong amplitudes.

5.1 THE $I = 0$ CORRELATION

$J/\psi \rightarrow \phi\pi^+\pi^-$

Figure 21(c) shows the $\pi^+\pi^-$ spectrum recoiling against a ϕ . As mentioned earlier, the $f_0(975)$ is observed in its characteristic asymmetric way. The $f_0(975)$, in the $q\bar{q}$ scheme is an $I = 0$, $s\bar{s}$ member of the 0^{++} nonet. Several other interpretations of $f_0(975)$ exist, including a $q\bar{q}q\bar{q}^{[41]}$ or a molecule,^[42] and as two states close to each other, one an $s\bar{s}$ meson and the other, a glueball candidate^[43] (a pole in $\pi\pi$ and another in $K\bar{K}$). In the $s\bar{s}$ scheme, the $f_0(975)$ would prefer to decay into $K\bar{K}$, but, being below $K\bar{K}$ threshold, decays mostly into $\pi\pi$. However, once the invariant mass is above the $K\bar{K}$ threshold, it decays mostly to $K\bar{K}$ and hence the sharp fall-off in $\pi\pi$ on the high mass side of the $f_0(975)$. The branching ratio was measured using the standard coupled channel Flatté parametrization^[44] (which is an approximation, a simple extension of a formulation for non-relativistic systems) and also using a more exact formalism which is relativistic. The same branching ratio was obtained with both parametrizations,

$$B(J/\psi \rightarrow \phi f_0(975)) \cdot (f_0(975) \rightarrow \pi^+\pi^-) = (2.3 \pm 0.3 \pm 0.6) \times 10^{-4}.$$

In the 1300 - 1500 MeV/ c^2 mass region, one or more structures are visible. There are simply speculations regarding their identities, because of limited statistics. One might attribute these structures to the $f_2(1270)$ and the $f_0(1300)\{f(1300)\}$. The structure at ~ 1750 MeV/ c^2 has been discussed previously.

$J/\psi \rightarrow \phi K^+K^-$

Figure 22(c) shows the K^+K^- spectrum recoiling against a ϕ . A clear $f_2'(1525)$ is observed. This is expected from the quark correlations, since the 2^{++} nonet is almost ideally mixed and $f_2'(1525)$ contains $s\bar{s}$. As mentioned earlier, a shoulder possibly due to the $f_2(1720)$ production is present. The branching ratio derived for $f_2'(1525)$ production does not depend substantially on the ' $f_2(1720)$ '

parametrization. The measured branching ratio is

$$B \cdot (J/\psi \rightarrow \phi f'(1525)) \cdot (f'(1525) \rightarrow K\bar{K}) = (6.4 \pm 0.6 \pm 1.6) \times 10^{-4},$$

where the masses and the width of the $f'_2(1525)$ were fixed at

$$\begin{aligned} m &= 1520 \text{ MeV}/c^2 \\ \Gamma &= 75 \text{ MeV}/c^2. \end{aligned}$$

$$\underline{J/\psi \rightarrow \omega \pi^+ \pi^-}$$

Figure 21(b) presents the $\pi^+ \pi^-$ spectrum recoiling against an ω in the decay

$$\begin{aligned} J/\psi &\rightarrow \omega \pi^+ \pi^- \\ &\quad \searrow \pi^+ \pi^- \pi^0. \end{aligned}$$

The $f_2(1270)$ is very pronounced. The branching ratio was measured as

$$B \cdot (J/\psi \rightarrow \omega f_2(1270)) \cdot (f_2(1270) \rightarrow \pi^+ \pi^-) = (27.7 \pm 1.4 \pm 7.0) \times 10^{-4}.$$

The lower mass region features a broad enhancement at $\sim 500 \text{ MeV}/c^2$ which was seen in previous experiments. Several speculations exist as to the nature of it. The angular distributions of the ω (recoiling against this structure) in its helicity frame behaves like $\sin^2 \theta$, consistent with the ω being aligned and the low mass structure having spin 0.

There is a small structure at $\sim 970 \text{ MeV}/c^2$ consistent with some $f_0(975)$ production, with a branching ratio $\sim 10^{-4}$ from simple event counts.

$$\underline{J/\psi \rightarrow \omega K\bar{K}}$$

As discussed earlier, Fig. 22(b) displays the $K^+ K^-$ spectrum from the decay mode,

$$\begin{aligned} J/\psi &\rightarrow \omega K^+ K^- \\ &\quad \searrow \pi^+ \pi^- \pi^0. \end{aligned}$$

None of the conventional $q\bar{q}$ type mesons are seemingly observed. An upper limit for $f'_2(1525)$ production is measured as

$$B \cdot (J/\psi \rightarrow \omega f'_2(1525)) \cdot (f'_2(1525) \rightarrow K\bar{K}) < 1.2 \times 10^{-4} \quad \text{at } 90\% \text{ C.L.}$$

The $K_S^0 \bar{K}_S^0$ channel, although lower in statistics, shows similar features.

5.2 THE $I = 1/2$ CORRELATION

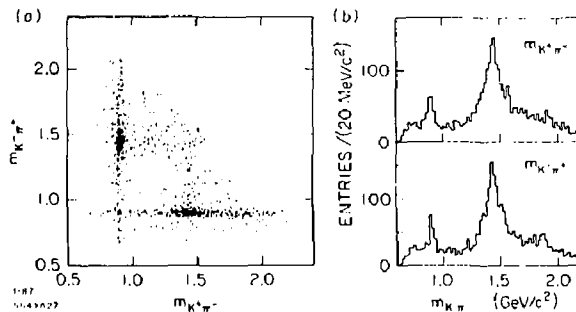
The following decay mode was observed

$$\begin{aligned} J/\psi &\rightarrow K^*(892)^0 \bar{K}^{*2}(1430)^0 + cc. \\ K^+ \pi^- &\quad \swarrow \quad \searrow K^- \pi^+ \end{aligned}$$

Figure 23(a) shows the plot of $K^- \pi^+$ vs $K^+ \pi^-$ effective mass. A band due to $K^*(892)$ (or cc) production is observed. Figure 23(b) shows the $K^- \pi^+$ spectrum recoiling against the $K^*(892)$. A very large peak at the $\bar{K}^{*2}(1430)^0$ is evident. The production branching ratio was measured as

$$B \cdot (J/\psi \rightarrow K^*(892)^0 \bar{K}^{*2}(1430)^0) + cc = (120 \pm 20 \pm 22) \times 10^{-4}.$$

There is a hint of a shoulder on the low mass side of the $\bar{K}^{*2}(1430)^0$. The $I = 1/2$ member of the 0^{++} family, the $K^{*0}(1350)$ $\{\kappa(1350)\}$ is reported to be $\sim 1350 \text{ MeV}/c^2$ with a width of $\sim 250 \text{ MeV}/c^2$. However, it has so far been observed only in PWA. Hence, further analysis is needed to clarify the $K^{*0}(1350)$.



23. (a) Scatter plot of $K^-\pi^+$ vs $K^+\pi^-$ effective mass in the reaction $J/\psi \rightarrow K^{*0}(892) \overline{K}^{*0}(1430) + \text{cc.}$ (b) The $K^-\pi^+$ and $K^+\pi^-$ projections.

5.3 THE $I = 1$ CORRELATION

The associated production with a ρ was observed in the following decay mode,

$$J/\psi \rightarrow \rho a_2(1320) \\ \pi\pi \leftarrow \rho \leftarrow \eta\pi\pi$$

in all three charged states, namely $\rho^+ a_2^-(1320)$ and $\rho^- a_2^+(1320)$. The final state observed was $\eta\pi^+\pi^-\pi^0$. (The $\rho^\pm(\rho^0)$ decays into $\pi^+\pi^-(\pi^+\pi^-)$ and the $a_2^\pm(1320)$ decays into $\eta\pi^\mp(\eta\pi^0)$.)

The same final state also yielded information about the reaction

$$J/\psi \rightarrow \rho a_0(980) \\ \pi\pi \leftarrow \rho \leftarrow \eta\pi\pi$$

in all three charged states i.e. $\rho^\pm a_0^\mp(980)$ and $\rho^0 a_0^0(980)$.

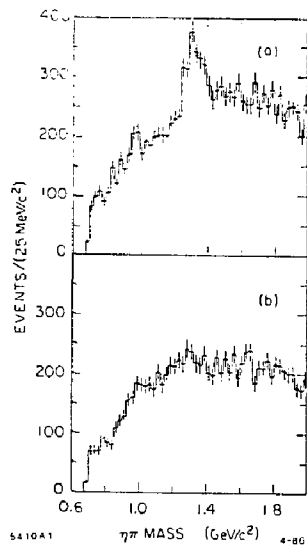
Figure 24 displays the combined spectra of $\eta\pi^+\pi^-$, $\eta\pi^0$, and $\eta\pi^+\pi^-$, recoiling against a ρ^+ , ρ^0 , and ρ^- , respectively. A peak from the $a_2(1320)$ production over a smooth background is apparent. However, no obvious signal for the $a_0(980)$, correlated with the ρ production is seen. The branching ratio for $\rho a_2(1320)$ is measured as

$$B(J/\psi \rightarrow \rho a_2(1320)) = (118 \pm 8 \pm 29) \times 10^{-4}.$$

An upper limit for the $\rho a_0(980)$ production was obtained as

$$B(J/\psi \rightarrow \rho a_0(980)) \cdot (a_0(980) \rightarrow \eta\pi) < 4.4 \times 10^{-4} \text{ at } 90\% \text{ C.L.}$$

Table 3 lists some of the relevant branching ratios. The quark correlations seem to exist, at least qualitatively. The branching ratios for the associated productions



24. (a) Sum of $\eta\pi^+$, $\eta\pi^-$, and $\eta\pi^0$ mass spectra recoiling against a ρ in the reactions $J/\psi \rightarrow \rho a_2(1320)$ and $\rho a_0(980)$ where $a_2(1320)$ and $a_0(980)$ decay into $\eta\pi$ (all three charged states). (b) Sum of $\eta\pi^+$, $\eta\pi^-$, and $\eta\pi^0$ associated with ρ sidebands.

TABLE 3. Measured Branching Ratios into K^+K^- and $\pi^+\pi^-$

Decay mode $J/\psi \rightarrow$	Object	Mass MeV/c ²	Width MeV/c ²	Branching Ratio $\times 10^{-4}$
$\gamma + K^+K^-$	$f_2'(1525)$	$1525 \pm 10 \pm 10$	85 ± 35	$[J/\psi \rightarrow \gamma f'(1525)] [f_2'(1525) \rightarrow K^+K^-] = 3.0 \pm 0.7 \pm 0.6$
$\gamma + K^+K^-$	$f_2(1720)$	$1720 \pm 10 \pm 10$	130 ± 20	$[J/\psi \rightarrow \gamma f_2(1720)] [f_2(1720) \rightarrow K^+K^-] = 4.8 \pm 0.9 \pm 0.9$
$\omega + K^+K^-$	$f_2'(1525)$	1520 fixed	75 fixed	$[J/\psi \rightarrow \omega f_2'(1525)] [f_2'(1525) \rightarrow K\bar{K}] = 1.2 \pm 0.90^{+0.9}_{-0.8}$ C.L.
$\omega + K^+K^-$	$f_2(1720)$?	$1731 \pm 10 \pm 10$	$110^{+46}_{-35} \pm 15$	$[J/\psi \rightarrow \omega f_2(1720)] [f_2(1720) \rightarrow K\bar{K}] = 4.5^{+1.3}_{-1.1} \pm 1.0$
$\phi + K^+K^-$	$f_2'(1525)$	1520 fixed	75 fixed	$[J/\psi \rightarrow \phi f_2'(1525)] [f_2'(1525) \rightarrow K\bar{K}] = 6.4 \pm 0.6 \pm 1.6$
$\gamma + \pi^+\pi^-$	$f_2(1720)$	1713 ± 5	130 fixed	$[J/\psi \rightarrow \gamma f_2(1720)] [f_2(1720) \rightarrow \pi^+\pi^-] = 1.6 \pm 0.4 \pm 0.3$
$\gamma + \pi^+\pi^-$	$f_2(1270)$	1269 ± 13	180 fixed	$[J/\psi \rightarrow \gamma f_2(1270)] [f_2(1270) \rightarrow \pi^+\pi^-] = 11.5 \pm 0.7 \pm 1.9$
$\gamma + \pi^+\pi^-$	$f_0(975)$	974 fixed	35 fixed	$[J/\psi \rightarrow \gamma f_0(975)] [f_0(975) \rightarrow \pi^+\pi^-] = 0.7 \pm 0.07^{+0.07}_{-0.06}$ C.L.
$\omega + \pi^+\pi^-$	$f_2(1270)$	1277 fixed	182 $\pm$ 10	$[J/\psi \rightarrow \omega f_2(1270)] [f_2(1270) \rightarrow \pi^+\pi^-] = 27.7 \pm 1.4 \pm 7.5$
$\phi + \pi^+\pi^-$	$f_0(975)$	coupled channel	coupled channel	$[J/\psi \rightarrow \phi f_0(975)] [f_0(975) \rightarrow \pi^+\pi^-] = 2.3 \pm 0.3 \pm 0.6$

† Results obtained from the initial 2.7×10^6 data sample.

of $\omega f_2(1270)$, $\phi f_2'(1525)$, $K^*(892) \overline{K}^*(1430) + cc$, and $\rho a_2(1320)$ in J/ψ decays are large, while $\omega f_2'(1525)$ and $\phi f_2(1720)$ are not. However, there is a striking difference between the branching ratios into the $\phi f_2'(1525)$ and $\omega f_2(1270)$ modes even after considering the effects of phase-space. The question of SU(3) violation is important. The question of how rigorous the quark correlations are, i.e. to what extent the DOZI processes are significant, is discussed later.

A systematic study of the scalars has started as is evident from Table 3. Perhaps most importantly, it could yield the answer to the question as to the nature of the scalars. The lack of observation of $\rho a_0(980)$ production raises questions about the $q\bar{q}$ interpretation of the $a_0(980)$, according to which the wavefunction of the $a_0(980)$ is

$$|a_0(980)\rangle = \frac{1}{\sqrt{2}} |u\bar{u} - d\bar{d}\rangle.$$

This is to be compared with similar associated production branching ratios of $\rho a_2(1320)$ and $\rho \pi^{(441)}$ ($\sim 100 \times 10^{-4}$), where both the $a_2(1320)$ and π are well established $q\bar{q}$ mesons ($|\rho\rangle = \frac{1}{\sqrt{2}} |u\bar{u} - d\bar{d}\rangle$, $|\pi\rangle = \frac{1}{\sqrt{2}} |u\bar{u} - d\bar{d}\rangle$).

Several explanations,^{[44][45]} including a molecule interpretation of the $a_0(980)$ relating to the 27-width^[44] exist.

6. The DOZI Interpretation

Table 2 lists the Vector-Pseudoscalar decay modes expressed in terms of the strong, the electromagnetic and the SU(3) violating amplitudes. The SU(3) violation is present in both the strong and the electromagnetic amplitudes. The mixing angle of the Pseudoscalar nonet, (alternatively, the quark contents of the η and the $\eta'(958)$) were determined^[46] by fitting these amplitudes to the data.[‡]

However, the recent measurement of the two photon width^[44] of the $\eta'(958)$

[‡] The η contained only u, d, and s quarks, while the $\eta'(958)$ had room for something extra.

disagreed with this conclusion and was consistent with it containing only u, d, and s quark as well.

Following the suggestions^{[44] [47]} that the DOZI amplitude may be small but not negligible, and assuming that DOZI amplitude = $r \times$ strong amplitude, one can replace the previous expressions with^[47] the new ones shown also in Table 2. It should be noted that the isospin conservation is still respected by the DOZI process. Currently, some of the Vector-Pseudoscalar decay modes are being updated with the present data sample and a refit of the amplitudes shown in Table 2 with the DOZI is in progress.

7. Conclusion

The baryonic decays are not discussed in the present paper, but a complete analysis is being pursued.

Interesting structures were observed^{[148] [149] [150] [151]} in $\rho\rho$,^[148] $\omega\omega$,^[149] and $\gamma\rho$ ^[150] in radiative decays of the J/ψ . These are currently being investigated with the complete data sample.

In short, the J/ψ data from MARK III has yielded some very interesting results and raised numerous important questions. A very rich analysis program is in progress and promises to address these issues.

For further datataking, the innermost drift chamber (trigger chamber) in the MARK III detector will be replaced by a high precision vertex detector. This will improve trigger efficiency, track resolution and track reconstruction efficiency, in particular for short tracks, while ensuring an efficient background rejection (e.g. cosmic rays). With the expected upgrade in SPEAR luminosity, MARK III will continue to be a source of very rich physics.

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NEW RESULTS ON MESONS CONTAINING STRANGE QUARKS*

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ABSTRACT

Recent results on strange and strangeonium mesons are presented. The data come from a high sensitivity study (4.1 ev/nb) of K^-p interactions at 11 GeV/c using the LASS spectrometer at SLAC. The complete leading orbitally-excited K^* series up through $J^P = 5^-$ and a substantial number of the expected underlying states are observed decaying into $K^- \pi^+$, $\bar{K}_s^0 \pi^+ \pi^-$, and $K \eta$ final states, and new measurements are made of their masses, widths, and branching ratios. Production of strangeonium states via hypercharge exchange is observed into $K_s^0 K_s^0$, $K^- K^+$, and $K_s^0 K^\pm \pi^\mp$ final states. The leading orbitally-excited ϕ series through $J^P = 3^-$ is clearly seen and evidence is presented for additional high spin structure in the 2.2 GeV/c² region. No $f_2(1720)$ is observed. The $K_s^0 K^\pm \pi^\mp$ spectrum is dominated by $1^+(K^* \bar{K} + \bar{K}^* K)$ production in the region below 1.6 GeV/c². These results are compared with data on the same systems produced by different production mechanisms.

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1. Introduction

The spectroscopy of light quark mesons continues to be an important area of investigation in high energy physics. There is now a substantial amount of experimental activity in the intermediate mass region between 1 and 2 GeV/c², particularly from the colliding beam machines, and there is an awareness that these data are important in the search for exotic objects and in the testing of basic features of the $q\bar{q}$ interaction. Making a significant contribution to these studies via "old-fashioned" hadroproduction requires quite substantial improvements in the sensitivity of the data compared with the many earlier experiments, as well as very high quality data with good resolution and good acceptance over a wide variety of physics channels. Today, I will describe some recent results from the LASS collaboration which address several important issues in the strange and strangeonium meson systems.

2. The Experiment

The experiment I will be discussing was performed by a collaboration of physicists from two Japanese and two U.S. institutions using the LASS spectrometer facility¹ which is shown in Fig. 1. LASS has 4π geometrical acceptance with excellent angular and momentum resolution, full azimuthal symmetry, excellent particle identification, and a high rate triggering capability. It is situated in an RF-separated beam which delivers an 11 GeV/c K beam of high purity (typically a 70/1 or better K/π ratio before tagging by beam Čerenkov counters). It contains two large magnets filled with tracking detectors. The first magnet is

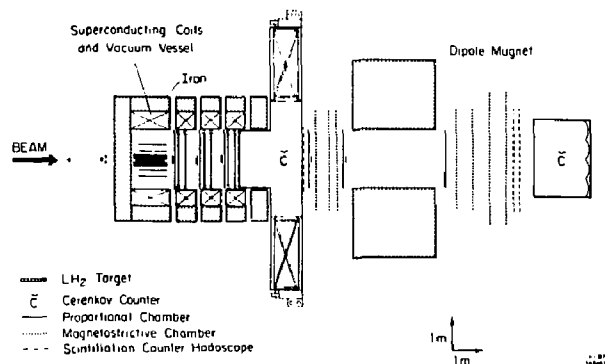


FIG. 1. The LASS spectrometer.

a superconducting solenoid with a 22.4 kG field parallel to the beam direction. This is followed by a 30 kG-m dipole magnet with a vertical field. The solenoid is effective in measuring the interaction products which have large production angles and relatively low momenta. High energy secondaries, which tend to stay close to the beam line, are not well measured in the solenoid, but pass through the dipole for measurement there. Particle identification is provided by a Čerenkov counter ($\check{C}_1$) and a time-of-flight hodoscope (TOF) which fill the exit aperture of the solenoid, and by a Čerenkov counter ($\check{C}_2$) at the exit end of the dipole spectrometer. In addition, the dE/dx ionization energy loss in the cylindrical proportional chambers which surround the target is measured to separate wide angle protons from π 's in the $1/\beta^2$ region below 800 MeV/c.

The trigger for this experiment requires two or more charged particles to exit the target. It is formed by cluster counting logic attached to a set of proportional chambers which conceptually form a box surrounding the target. The trigger is essentially σ_{tot} , except for the all neutral final states, and is quite clean. About 85% of all triggers are good physics events. For an experiment with such an open trigger, the sensitivity of about 4 ev/nb for the K^- beam incident, which will be discussed today, is very high. This leads to a processed data sample of about 115 million events, which leads in turn to one of the major difficulties in performing this experiment. The data analysis burden is very large, requiring the equivalent of about three IBM 3081/K years for completion. The task was shared between Nagoya University, utilizing a dedicated FACOM M200 at the High Energy Laboratory and the University Center's M382, and a nine processor 168/E farm at SLAC.

3. Motivation

Substantial progress in understanding the physics of meson systems has been made during the 10 year period following the "November Revolution" with the discovery of the heavy quarkonia, and their detailed study in the e^+e^- colliding machines. However, it remains clear that complementary studies of the light quark spectra are extremely important and provide access to important features of the spectroscopy which otherwise remain closed. First, as Fred Gilman discussed earlier in this school,² the light quark spectrum probes a different piece of the $q\bar{q}$ potential. In particular, the cc and $b\bar{b}$ systems probe the short range behavior of the potential while an excited light quark system, such as one that is spun up in orbital angular momentum, allows the study of the strength and Lorentz structure of the confining term.

Second, the hadroproduction mechanism is sufficiently different than production in colliding e^+e^- machines that the experimentally accessible excitations are nearly orthogonal. Figures 2 (a) and (b) are level diagrams (called Grottrian plots by nuclear physicists) which illustrate this for the cc and strange spectra respectively. The levels are arranged so that the states with quark spins antiparallel ($S = 0$) are on the left, while states with quark spins parallel ($S = 1$) are on the right. Orbital excitations appear as columns in each section which increase from $L = 0$ (S wave) on the left to $L = 3$ (F wave) on the right. Radial excitations appear as towers going up each of these columns. The positions at which the levels appear in mass should be considered as illustrative only for purposes of this discussion. The primary emphasis here is on the experimentally known excitations. In Fig. 2(a), the cc states which are included in the latest summary

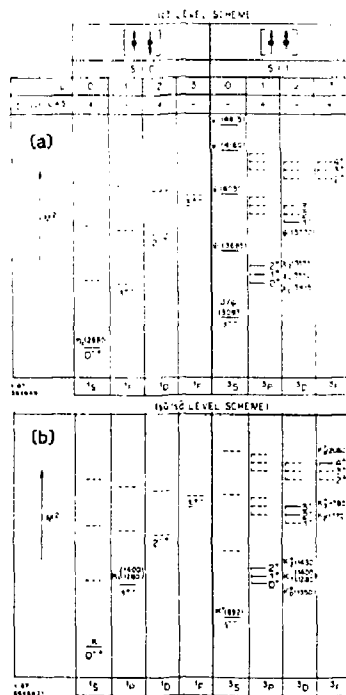


FIG. 2. Level diagrams for $q\bar{q}$ meson systems summarized in PDG tables: a) for charmonium states; b) for strange meson states. Mass levels are illustrative only and classification ambiguities, if any, have been neglected.

table of the Particle Data Group³ are shown. It should be noted that there are classification ambiguities in a few cases for the higher lying 1^- resonances, but the basic experimentally observed level structure is a very beautiful tower of 1^- radial excitations, with only the beginnings of the orbital towers building up. The reason for this is that the 1^- states are directly produced in e^+e^- collisions while the observation of higher orbital states requires decays from the produced 1^- states and therefore are much rarer and more difficult to observe. This may be contrasted with the situation in K^* spectroscopy as shown in Fig. 2(b). Once again the states shown are taken from the 1986 PDG summary tables. There are no candidate radial states. However, the first few levels of the orbital excitation ladders can be clearly seen, both in the $S = 0$ and the $S = 1$ sectors. So it is clear that the production area which is used in the study of a spectroscopy is of vital importance in determining the states, and the features of the spectrum, which can be observed. In particular, e^+e^- collisions are without peer in producing clear 1^- radial towers, but the less specific hadroproduction mechanism is essential to the study of the higher orbital excitations.

Third, the recent speculation that some of the states observed in the 1 to 2.2 GeV/ c^2 mass region in e^+e^- production may be exotic objects such as "glueballs" points up once again the importance of understanding the ordinary $q\bar{q}$ states in this same mass region in order to understand whether any particular state is "unusual", and different production mechanisms may be crucial in this process. For example, the production of light mesons decaying to $K\bar{K}$ in K^-p interactions is dominated at small values of momentum transfer by hypercharge exchange processes (K and K^* exchange) and would be expected (and is known) to be

dominated by the production of strangeonia ($s\bar{s}$ mesons), while the production of the same final states via radiative decays from the J/ψ might be expected to contain a rather large admixture of glueballs. So the combination of these different production mechanisms should be very powerful in sorting out the nature of any new or unusual states which are observed.

4. The Strange Mesons

For a variety of reasons, the strange mesons appear to be an excellent place to try to understand a pure $q\bar{q}$ spectrum. First, flavored (i.e., K) beams are available which allow the strange mesons to be produced cleanly with large cross sections. Second, the charge-exchange channels are dominated by the well understood π exchange mechanism, which allows studies of $K\pi$ scattering via extrapolation, and a clear look at both orbital excitations and the underlying states. Third, neutral K s are visible via the K_S^0 decay, so that it is rather easy to study all charges of final states with the good resolution of tracking detectors rather than using a neutral detector such as a shower counter. Finally, the $q\bar{q}$ final states have overt flavor so that there is no isoscalar-isovector mixing and no confusion with glueballs. The reaction

$$K^- p \rightarrow K^- \pi^+ n \quad (1)$$

is an ideal place to study the orbital excitation ladder. This final state is topologically simple, is restricted to only the natural spin-parity series, and has a large cross section which is dominated by π exchange at small values of momentum transfer ($t' = t - t_{min}$). Many features of this channel are illustrative of

the physics and analysis methods used throughout this talk, particularly in the strangeonium sector, so I would like to discuss some of these features in detail. The invariant mass for Reaction (1) is shown in Fig. 3 for all 730,000 events with $|t'| < 1.0$ (GeV/c)². The spin-parity $J^P = 1^- K^*(892)$ and $2^+ K_2^*(1430)$ mesons can be clearly seen as can a higher mass structure in the $3^- K_3^*(1780)$ region. However, even with the enormous statistics of this plot, there is little evidence for additional structure in the high mass region where additional higher spin resonances would be expected. There are several different reasons for this. Reaction (1) contains not only K^* resonances, but nucleon (N^* and Δ) resonances as well. The large total amount of nucleon resonances produced in Reaction (1) is clearly seen in the invariant $n\pi^+$ mass plot shown in Fig. 4. It reflects rather smoothly into the $K^-\pi^+$ invariant mass distribution, but much less smoothly into the angular structure. In this situation, the easiest thing to do is cut out this portion of the phase space. This can only be done if the statistics of the experiment are very high since such a cut puts holes into the acceptance which must be corrected. With the large statistics of this experiment, we remove the events with $n\pi^+$ masses below 1.7 GeV/c² from the subsequent analysis. The remaining sample, which is shown in the cross-hatched histogram of Fig. 3, contains 385,000 events. However, even with the elimination of the nucleon resonances, the structure observable in the plot is essentially unchanged. The fundamental reason for this is that the $K\pi$ elasticity drops as a function of mass, so that the visible cross section for a given K^* resonance to decay in this channel decreases with increasing mass; in addition, the level structure of the spectroscopy, as shown in Fig. 2, leads to a large number of overlapping resonances in the region above 2.0

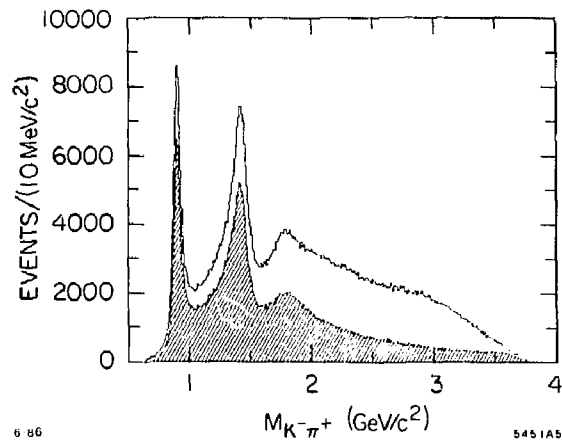


FIG. 3. The invariant $K^- \pi^+$ mass for the reaction $K^- p \rightarrow K^- \pi^+ n$; the unshaded curve contains all events while the cross-hatched curve contains events with N^* 's removed (see text).

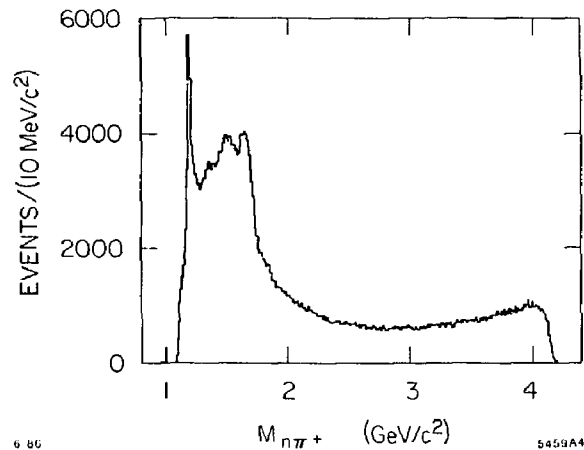


FIG. 4. The invariant $n \pi^+$ mass for the reaction $K^- p \rightarrow K^- \pi^+ n$.

GeV^2/c^2 . These effects conspire to yield the featureless distribution (observed in Fig. 3) at high mass.

Nevertheless, a great deal of interesting structure is hidden in this plot, as becomes evident when we increase its dimensionality. Figure 5 is a scatter plot of $K\pi$ invariant mass against the cosine of the t -channel decay angle of the K in the $K\pi$ center of mass (θ_{tJ}). The $K^*(892)$ stands out as a clear band, while there are big bumps, at both forward and backward θ_{tJ} , which indicate the $K_2^*(1430)$. As we continue to higher M_{K^*} 's, the structure becomes more complex. For example, the large bump around $\cos(\theta_{tJ}) = 0.5$ in the $1.8 \text{ GeV}^2/c^2$ region, and a corresponding hole in the backward region, are associated with the $K_3^*(1780)$; the prominent backward peak just above $2.0 \text{ GeV}^2/c^2$ turns out to be related to the $K_4^*(2060)$.

Many other complex features are apparent in this plot, but it is also apparent that the understanding of the nature of these structures requires a detailed angular analysis. Today, we will concentrate on the leading K^* states in this channel, and so will discuss only the simplest analysis of this type, which is a spherical harmonic moments analysis. We select the data for this analysis to emphasize the π exchange contribution by requiring $t' > 0.2 \text{ (GeV}^2/c^2)$. For pure π exchange, only moments with $M = 0$ are allowed, and a resonance of spin J can appear in moments up to $L = 2J$. In general, the leading orbitally excited resonances are expected to be the lowest lying states of high spin so that they will dominate the highest moments required at a given mass. For example, Fig. 6 shows the even $L, M = 0$ moments required to describe the data in the mass region below $1.88 \text{ GeV}^2/c^2$. Moments are not used in a particular mass region if they are consistent

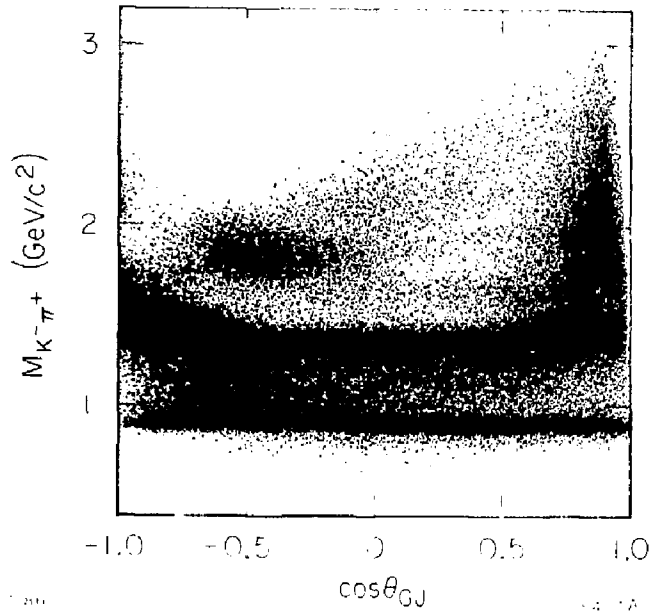


FIG. 5. The $\cos\theta_{tJ}$ vs. M_{K^*} scatter plot for events with $t' > 0.2 \text{ (GeV}^2/c^2)$; events near $\cos\theta_{tJ} = 1$ at high mass are removed by the V^0 cut and by the removal of $K^- p$ elastic scattering.

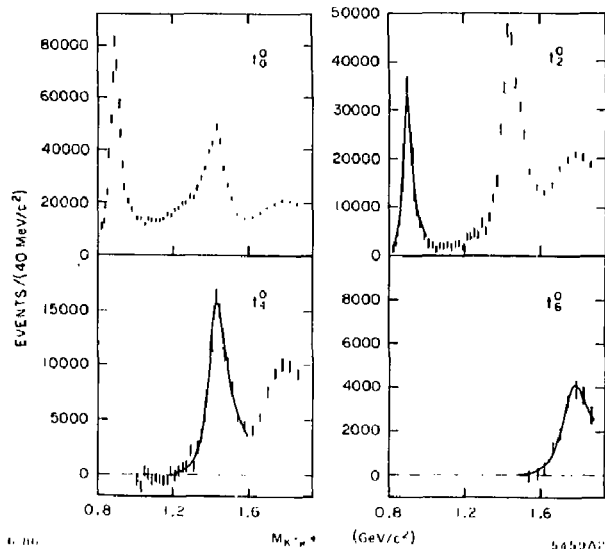


FIG. 6. The unnormalized 1-even, $M = 0$ $K^- \pi^+$ moments for the mass region below $1.88 \text{ GeV}/c^2$; bin sizes vary from 10 to $40 \text{ MeV}/c^2$, with events normalized to $40 \text{ MeV}/c^2$ bins. The curves are described in the text.

with zero there. The $1^- K^*(892)$, $2^+ K_2^*(1430)$, and $3^- K_3^*(1780)$ are clearly seen in t_2^0 , t_4^0 , and t_6^0 , respectively. Each state dominates the highest moment required in the relevant mass region, and also appears with lower prominence in moments with lower L . Breit-Wigner fits to the $L = 2J$ moment give new measurements of the masses and widths of these states, indicated in Table I.

Table I

The parameters for the 1^- , 2^+ , and 3^- states from a fit to the leading moments. The indicated errors are statistical and systematic respectively.

Resonance	J^P	Mass(MeV/c^2)	Width(MeV/c^2)
$K^*(892)$	1^-	$897.0 \pm 0.7 \pm 0.7$	$49.9 \pm 1.7 \pm 0.8$
$K_2^*(1430)$	2^+	$1433.0 \pm 1.6 \pm 0.5$	$115.8 \pm 2.7 \pm 1.6$
$K_3^*(1780)$	3^-	$1778.1 \pm 6.4 \pm 1.3$	$185.9 \pm 23.3 \pm 12.3$

Having observed the leading K^* states that are rather well understood, let us now look at candidates for higher orbital excitations. Figure 7 shows the required moments with $L > 6$, $M = 0$, in the region above $1.8 \text{ GeV}/c^2$. Moments, not shown, with $L > 10$ are consistent with zero. The peaks in the t_8^0 and t_{10}^0 moments, and the interference structures in the t_6^0 and t_8^0 moments are naturally interpreted as confirming the $4^+ K_4^*(2060)$ and demonstrating the existence of a $5^- K^*$ around $2.38 \text{ GeV}/c^2$. However, the large errors on the moments make it difficult to determine the parameters of these resonances from the leading moments alone. The curves shown in Fig. 7 result from a simple fit to all 21 moments in this mass region with $L \leq 10$, $M \leq 1$. The high spin F , G , and H waves are parameterized as relativistic Breit-Wigner forms. Background terms

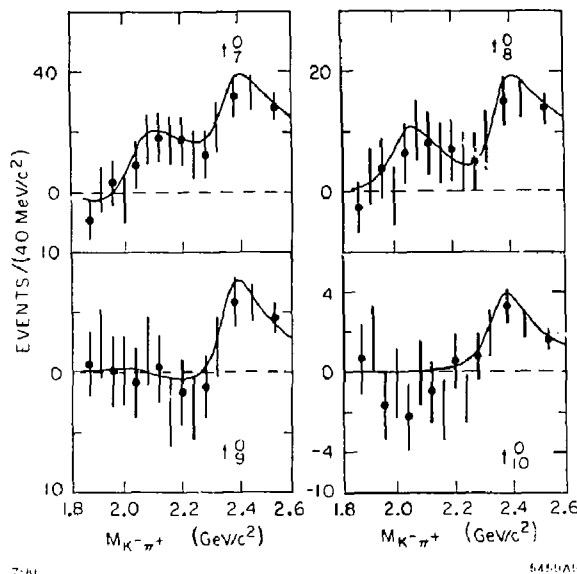


FIG. 7. The unnormalized $7 \leq L \leq 10$, $M = 0$ $K^-\pi^+$ moments for the mass region above $1.88 \text{ GeV}/c^2$. Moments with $L > 10$ are consistent with zero. The moments are plotted for overlapping bins; black dots indicate the independent set of mass bins used for the fit described in the text.

which are linear in both amplitude and phase are used for the lower spin waves and added to the F , G , and H wave terms as well. The smaller $M = 1$ moments are related to the $M = 0$ moments using a parametrization of earlier energy independent PWA results. The significance of the spin 5 state in this model is about 5σ . The masses and widths are shown in Table II.⁴

Table II

The parameters for the 4^+ and 5^- states from a fit to all moments. The indicated errors are statistical and systematic respectively.

Resonance	J^P	Mass (MeV/c^2)	Width (MeV/c^2)
$K_4^*(2060)$	4^+	$2062 \pm 14 \pm 13$	$221 \pm 48 \pm 27$
$K_5^*(2380)$	5^-	$2382 \pm 14 \pm 19$	$178 \pm 37 \pm 32$

The reaction

$$K^- p \rightarrow \bar{K}_s^0 \pi^+ \pi^- n \quad (2)$$

is an important source of information on the inelastic decay modes of the K 's, and makes possible the observation of states with unnatural spin-parity. The invariant mass distribution ($M_{K\pi\pi}$), shown in Fig. 8, appears to show the expected leading $K_2^*(1430)$ and $K_3^*(1780)$ resonances over a substantial background. However, an analysis of these data⁵ with the SLAC-LBL three-body PWA program⁶ reveals that over 2/3 of this spectrum is resonant. Moreover, the peaks around 1.45 and $1.8 \text{ GeV}/c^2$ contain not only the expected leading resonances, but other underlying states with comparable intensities.

Even though I will not discuss the model in detail, it may be useful to briefly

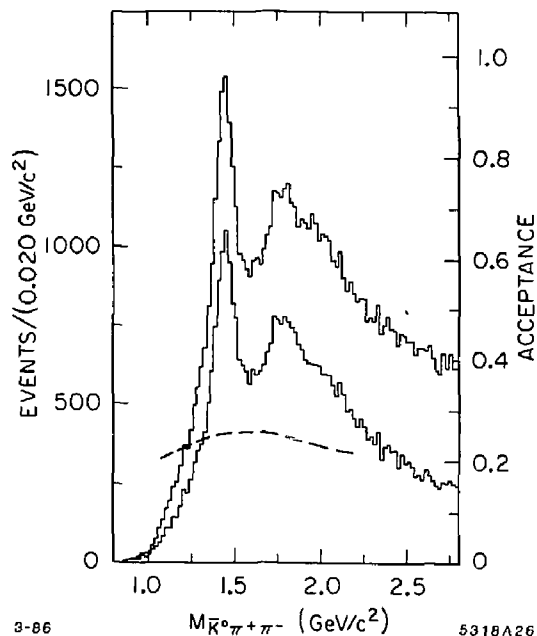


FIG. 8. The $\bar{K}^0 \pi^+ \pi^-$ invariant mass distribution. The outer histogram is for all events while the inner histogram contains only events with $|t'| \leq 0.3 \text{ (GeV/c)}^2$. The dashed line gives the final acceptance after all cuts.

review the approach taken in the analysis, and the labeling of the resulting three-body partial wave amplitudes. The model describes the $K\pi\pi$ final state as a superposition of two-body states made up of isobars, which decay into two particles, and a bachelor particle. The isobars of importance in this analysis are the $K^*(892)$, $K_2^*(1430)$, $\rho(770)$; a three-body phase space term is also required. The labeling of the amplitude components of the final state is given by the series of quantum numbers $J^P M^\eta$ (isobar) L , as illustrated in Fig. 9, where J is the total spin and P is the parity of the final state combination; M is the magnetic substate; η approximates the naturality of the exchange in the t channel; (isobar) is the isobar state; and L is the relative orbital angular momentum between the isobar and bachelor meson.

Let us begin by looking at the decomposition of the cross section into its spin-parity contributions. Figure 10 shows the natural spin-parity part. The 2^+ , 3^- , and 4^+ cross sections appear to contain the same leading K^* states we just described in the $K\pi$ channel at 1430, 1780, and 2060 MeV/c^2 respectively. There is also a substantial amount of structure in the 1^- wave around 1.4 and 1.8 GeV/c^2 , and in the 2^+ wave around 2.0 GeV/c^2 . In fact, as we will discuss below, a further decomposition of the 1^- amplitude into the different isobar partial waves shows that the structure is caused by two 1^- states, at $\sim 1420 \text{ MeV/c}^2$ and 1740 MeV/c^2 , and that essentially the entire natural spin-parity sector is resonant. On the other hand, the unnatural spin-parity waves, shown in Fig. 11, are only partially resonant. There is a large structure near 1400 MeV/c^2 corresponding to the $K_1(1400)$, but the other waves are rather smooth and featureless. The parameters of the resonant states are estimated by fitting

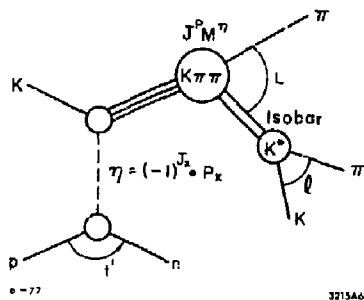


FIG. 9. Schematic diagram for $K \rightarrow K\pi\pi$ production describing the variables used by the isobar model.

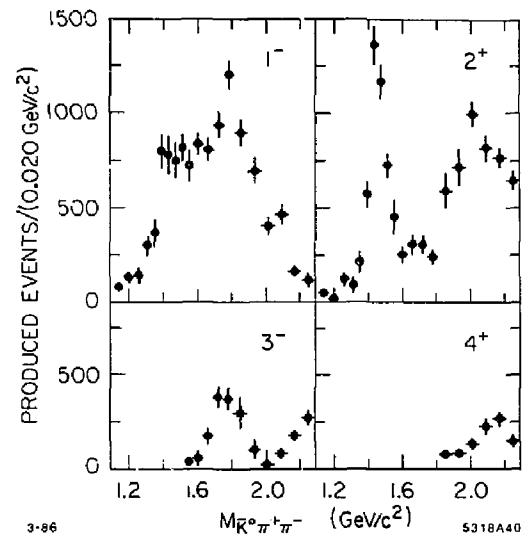


FIG. 10. The natural spin-parity wave sums for the $\bar{K}^0 \pi^+ \pi^-$ final states. All partial waves of the same J^P are summed coherently.

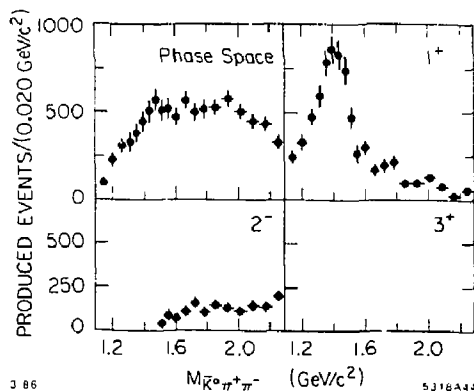


FIG. 11. The unnatural spin-parity wave sums for the $K^0 \pi^+ \pi^-$ final state. All partial waves of the same J^P are summed coherently.

the partial wave amplitudes to Breit-Wigner resonances plus simple background terms. In the natural spin-parity sector, on which we concentrate today, we do a simultaneous fit to both 1^- waves, plus the $2^+ K^* \pi$, $3^- K^* \pi$, and $3^- \rho K$, using the known behavior of the three leading resonances to constrain the relative phase behavior of the 1^- waves. The fit requires two 1^- resonances, one at approximately $1.42 \text{ GeV}/c^2$, and the other at approximately $1.74 \text{ GeV}/c^2$. In Fig. 12, the intensities and phases of the leading $2^+ K^* \pi$, $3^- K^* \pi$ and $3^- \rho K$ are shown, along with the results of the five wave model fit just discussed, while in Fig. 13, we show the model fit to the 1^- amplitudes. The masses and widths of the 1^- resonant states are indicated in Table III.

Table III

The parameters for the two underlying 1^- states from the five wave fit described in the text. The indicated errors are statistical and systematic respectively.

Coupling	Mass (MeV/ c^2)	Width (MeV/ c^2)
$K^* \pi$	$1420 \pm 7 \pm 10$	$240 \pm 18 \pm 12$
$K^* \pi, \rho K$	$1735 \pm 10 \pm 20$	$423 \pm 18 \pm 30$

In order to understand the nature of these states, it is useful to consider two more pieces of information. First of all, in the $K^+ \pi^+ \pi^-$ channel there is a large resonant state with an elasticity of around 0.35 in the $1.75 \text{ GeV}/c^2$ region. However, the elasticity in the $1.4 \text{ GeV}/c^2$ region is less than 0.1, indicating that the coupling of the lower state to the two-body channel is strongly suppressed.⁷ This suppression is corroborated by the production characteristics of the three-body amplitudes as shown in Fig. 14. The $|p'|$ dependence of the 1^- amplitudes in

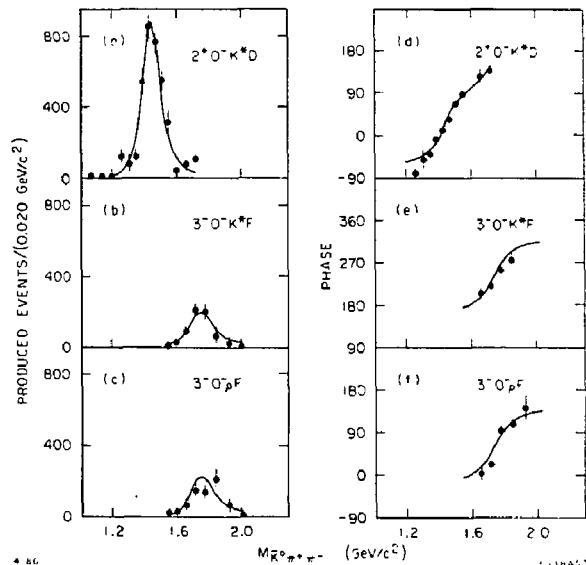


FIG. 12. The leading 2^+ and 3^- K^* resonant waves observed in $\bar{K}^0 \pi^+ \pi^-$, compared with the predictions of the five wave model discussed in the text.

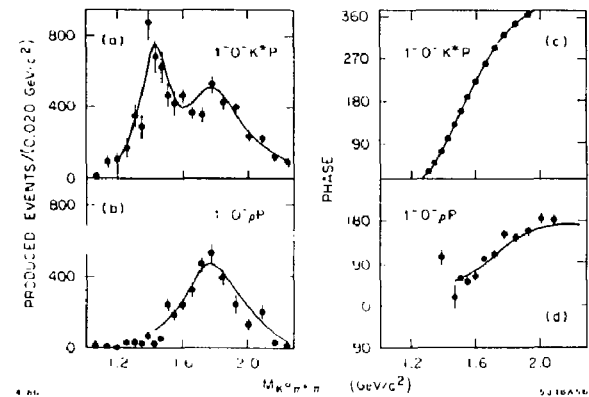


FIG. 13. The $\bar{K}^0 \pi^+ \pi^- 1^-$ waves compared with the predictions of the five wave model discussed in the text.

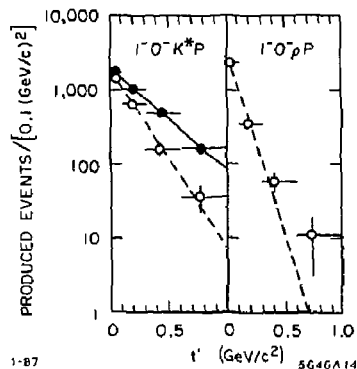


FIG. 14. The $|t'|$ dependencies of the 1^- partial waves. The solid dots and line are for the $1.42 \text{ GeV}/c^2$ mass bin. The open dots and dashed lines are for the $1.79 \text{ GeV}/c^2$ mass bin. The partial waves are modeled by a curve of the form $e^{it'}$.

the 1.73 to $1.85 \text{ GeV}/c^2$ region, shown by the open circles, is quite steep as would be expected from π exchange, while the slope of the 1^- amplitude in the 1.37 to $1.47 \text{ GeV}/c^2$ region, shown by the closed circles, is flatter as would be expected from B exchange, for example. It should be noted that the relative phase between the 1^- state at $\sim 1420 \text{ MeV}/c^2$ and the $K_2^*(1430)$ is $\sim 90^\circ$, whereas the phase between the 1^- state at $\sim 1735 \text{ MeV}/c^2$ and the $K_2^*(1780)$ is $\sim 0^\circ$. This is also indicative of a different production mechanism for the two 1^- states.

This behavior leads naturally to our preferred classification of these two 1^- states. Though mixing is not entirely excluded, it is simplest to associate the higher state with the 1^3D_1 state based on the small $L \cdot S$ splitting and the similarity of the widths and branching ratios with typical quark model calculations. The lower state then becomes mostly the first radial excitation of the $K^*(892)$. The suppression of the $K\pi$ decay mode of this lower state is understood in some models as being a dynamical effect resulting from the presence of a node in the radial wave function.

There is an additional new structure which can be seen in these three-body amplitudes. Figure 15 shows the behavior of the 2^+ amplitudes. In addition to the well-known leading $K_2^*(1430)$, a large structure is evident in the mass region around $2.0 \text{ GeV}/c^2$. We have fit these amplitudes in the region above $1.69 \text{ GeV}/c^2$ to a model which incorporates a Breit-Wigner resonance and a linear coherent background. The phase is essentially constrained by the leading $3^- K^*\pi$, as incorporated in the five wave fit described above. The fit, which is indicated by a solid line in Fig. 15, gives a mass of $\sim 1.97 \text{ GeV}/c^2$ with a width of $\sim 0.37 \text{ GeV}/c^2$. However, since a substantial background is required, the single

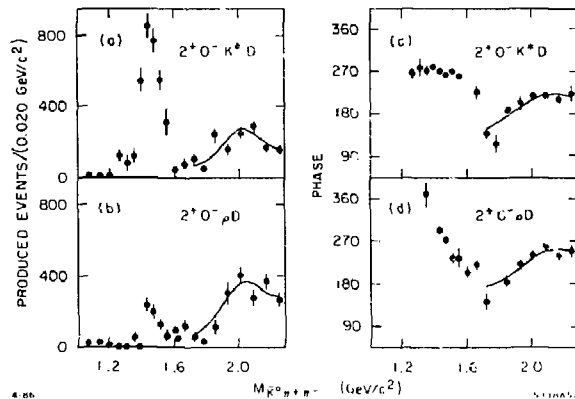


FIG. 15. The high mass $\bar{K}^0 \pi^+ \pi^- 2^+$ waves. The solid line represents the fit to the high mass region described in the text.

resonance fit is not unique. In fact, the data can be fit equally well in a two resonance model with the second resonance at a somewhat higher mass.

Figure 16 summarizes the states we have observed in the $\bar{K}_s^0 \pi^+ \pi^-$ final state along with our preferred quark model assignments. The shaded regions correspond to the experimental errors on the mass values, while the stars indicate the mass values predicted by the model of Godfrey and Isgur.⁸ The J -values of the leading states are linear in mass squared, and the $L \cdot S$ splitting appears to be quite small. In general, the model does rather well. The greatest difficulty is with the 1^- radial state which is predicted to lie significantly above the observed state.

Other good testing grounds for mesonic models are their predictions for decay rates and branching ratios. In particular, the K^* mesons should decay into the $K\eta$ final state. However, the visible cross sections are expected to be rather small and the final state is difficult to study experimentally, so there is very little data available from earlier experiments. In the present experiment, we have looked at events which satisfy a 1C kinematic fit to the channel

$$K^- p \rightarrow K^- \pi^+ \pi^- \pi^0 p \quad (3)$$

Events which satisfy a 4C fit to $K^- \pi^+ \pi^- p$ are rejected. The resulting $\pi^+ \pi^- \pi^0$ mass spectrum is shown in Fig. 17. There is a large η signal over some background. The shaded regions serve as controls for background subtraction. Figure 18 shows the invariant $K^- \eta$ mass distribution after subtraction of the control regions and a set of cuts to remove the Y^* and N^* overlap. The spectrum is dominated by a single bump with a mass and width which are consistent with

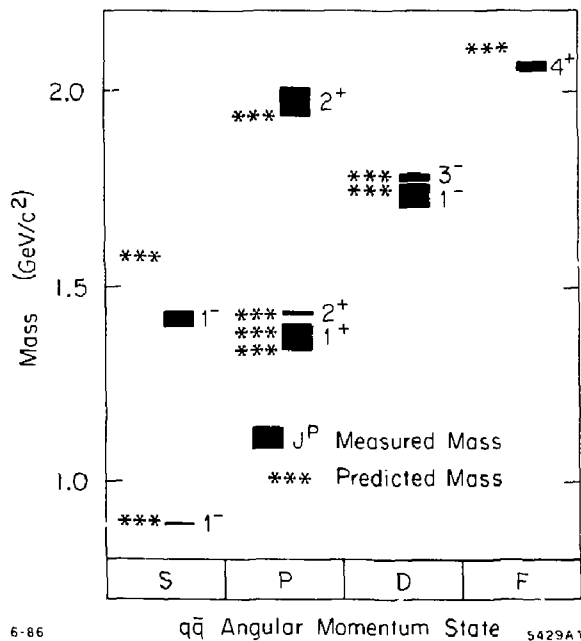


FIG. 16. Level diagram for the resonant $K^0 \pi^+ \pi^-$ amplitudes observed in this experiment. The shaded regions indicate the measured uncertainty in the mass position. The predicted masses are taken from Ref. 8.

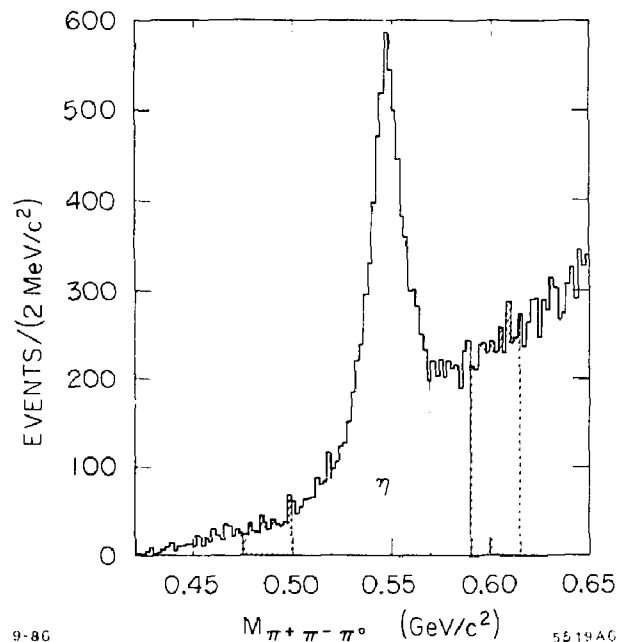


FIG. 17. The $\pi^+ \pi^- \pi^0$ invariant mass from the reaction $K^- p \rightarrow K^+ \pi^- \pi^0 p$. The vertical lines indicate the region used to define the η signal. The shaded regions are used to estimate the background under the $K\eta$ signal.

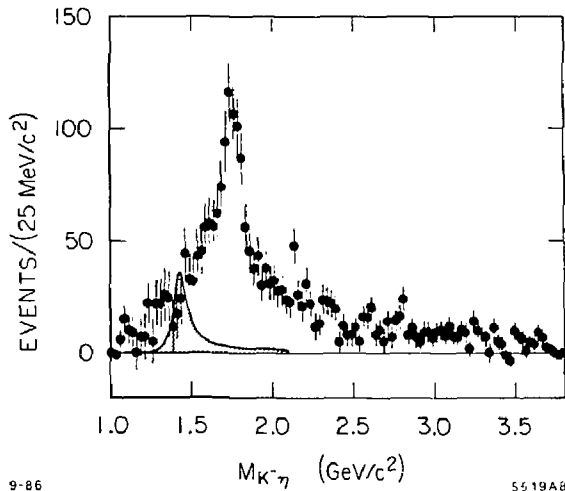


FIG. 18. The background subtracted $K\eta$ invariant mass distribution after N^* and Y^* cuts. The shaded curve shows the signal expected for a $K^*(1430) \rightarrow K\eta$ branching ratio of 0.5%.

the $3^- K_3^*(1780)$ resonance. Preliminary results of the moments analysis also require substantial production of a spin 3 resonance. Assuming that $K_3^*(1780)$ dominates the region, the observed cross section corresponds to a branching ratio $K_3^*(1780) \rightarrow K\eta$ of $\sim 2.5\%$. In contrast, there is no evidence at all for a decay of the $K_2^*(1430)$ to $K\eta$. The shaded area centered at $1.43 \text{ GeV}/c^2$ shows the signal expected for a $K_2^*(1430) \rightarrow K\eta$ branching ratio of 0.5%, which is clearly a conservative upper limit on the decay. Though the branching ratios for the 2^+ and 3^- leading K^* resonances differ by at least a factor of five, the results of standard models⁸ appear to agree at least qualitatively with this result.

Figure 19 summarizes the K^* spectrum observed to date in this experiment, most of which we have discussed today. The observed leading states in the orbital ladder now extend all the way through the $5^- K^*$. Many of the expected underlying states have now been seen and there are good candidates for several radial states. We have seen π transitions from most of these states, as well as transitions to vector, and in some cases, tensor mesons. Rare decays into final states such as $K\eta$ are also beginning to be observed. The clear experimental picture now emerging provides powerful tests of existing $q\bar{q}$ models, and imposes important constraints on the predictions which might result from future modifications to them.

5. Strangeonium Mesons

The ability of the LASS spectrometer to reconstruct V^0 decays provides a very clean way to study the production of strangeonium mesons. In all cases discussed today, we will look only at fully constrained channels with a slow Λ recoil. Thus, the backgrounds are small; resolution is very good; particle identification requirements are minimized; and the resulting geometrical acceptance is nearly flat. As discussed earlier, the dominant production process in a K induced reaction at small t with a Λ recoil is hypercharge exchange. This leads us to expect very clean production of the strangeonia into channels containing a $K\bar{K}$ in the final state.

The $K\bar{K}$ invariant mass spectrum from the reaction

$$K^- p \rightarrow K^- K^+ \Lambda \quad (4)$$

shown by the open histogram in Fig. 20 confirms these expectations. The distribution is dominated at low mass by the production of the classic strangeonia, the $1^- \phi(1020)$ and the $2^+ f_2'(1520)$, while there is another smaller bump at $1.86 \text{ GeV}/c^2$, the $\phi_s(1850)$ to be discussed below, which is expected to be the next state on the strangeonium orbital ladder. In general, the $K^- K^+$ (strangeonium) invariant mass spectrum is very reminiscent of the $K^- \pi^+$ (strange) spectrum, Fig. 3, with an appropriate shift in the mass scale to account for the additional constituent s quark. This should be contrasted with production of the same final state with a π beam, as observed by the OMEGA spectrometer experiment of Evangelista et al.,⁹ shown in the cross-hatched histogram of Fig. 20, where no

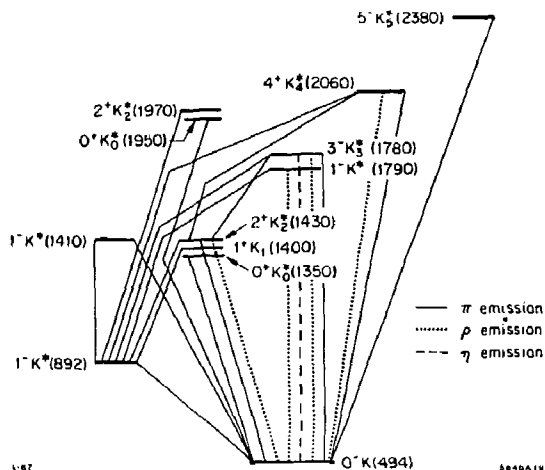


FIG. 19. Level diagram summarizing strange meson states and transitions observed in this experiment.

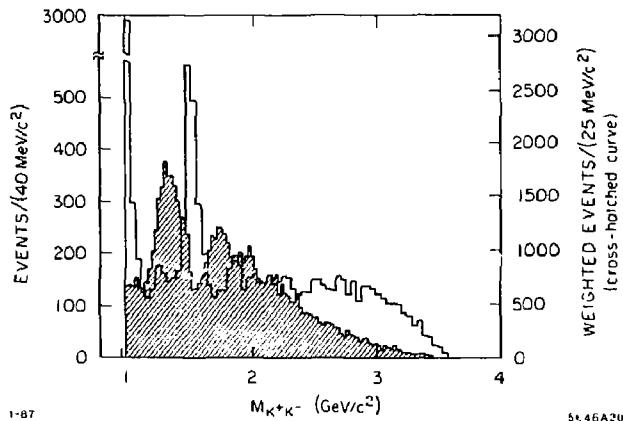


FIG. 20. Comparison of the K^-K^+ final state produced in the reaction $K^-p \rightarrow K^-K^+\Lambda$ from this experiment with the reaction $\pi^-p \rightarrow K^-K^+n$ from Ref. 9.

strangeness is exchanged. The bumps observed are associated with minority decay modes of objects without hidden strangeness, such as $f_1(1270)$, and $a_2(1320)$, interfering with strangeonium production.

Given that it is natural to expect strangeonium production in hypercharge exchange processes, the reactions

$$K^-p \rightarrow K_s^0 K^+ \pi^- \Lambda \quad (5)$$

should be the ideal place to study the 1^+ strangeonium mesons which might be expected to lie in the $1.4 \text{ GeV}/c^2$ region, and for which evidence has been claimed in earlier experiments.^{3,10} The invariant mass for the combined channels, shown in Fig. 21, has clear structure in the mass regions just above $1.5 \text{ GeV}/c^2$ and around $1.85 \text{ GeV}/c^2$, close to the positions expected for the leading 2^+ and 3^- $s\bar{s}$ states, but little activity in the region below $1.5 \text{ GeV}/c^2$ except for a sharp rise at K^*K threshold. However, the major features are so reminiscent of the strange three-body $K_s^0 \pi^+ \pi^-$ channel discussed earlier, which contained complicated structure in the peak regions, as to make us very cautious about associating these structures with any known states until the results of a full PWA are available.

In spite of a long history of confusion regarding the data in the $1.4\text{--}1.5 \text{ GeV}/c^2$ region, the classical "E" meson (now called the $f_1(1420)$) has generally been taken as the 1^{++} strangeonium state,³ so the lack of any clear structure in these data in the $1.4 \text{ GeV}/c^2$ region is somewhat disappointing. Given the rather narrow ($56 \text{ MeV}/c^2$) width of the $f_1(1420)$, it is worth looking at the low mass spectrum plotted in $20 \text{ MeV}/c^2$ bins, as given in Fig. 22, to investigate the K^*K threshold region in more detail. There appears to be a small amount of $f_1(1285)$ production

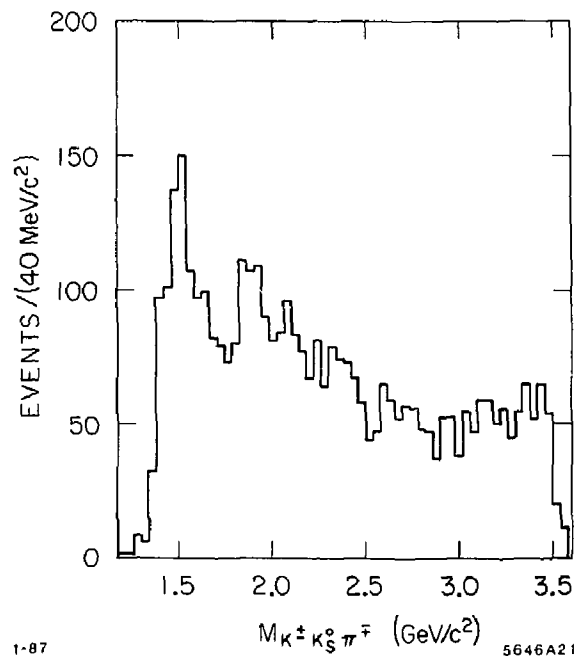


FIG. 21. The summed $K_S^0 K^\pm \pi^\mp$ invariant mass spectrum for masses between threshold and $3.6 \text{ GeV}/c^2$.

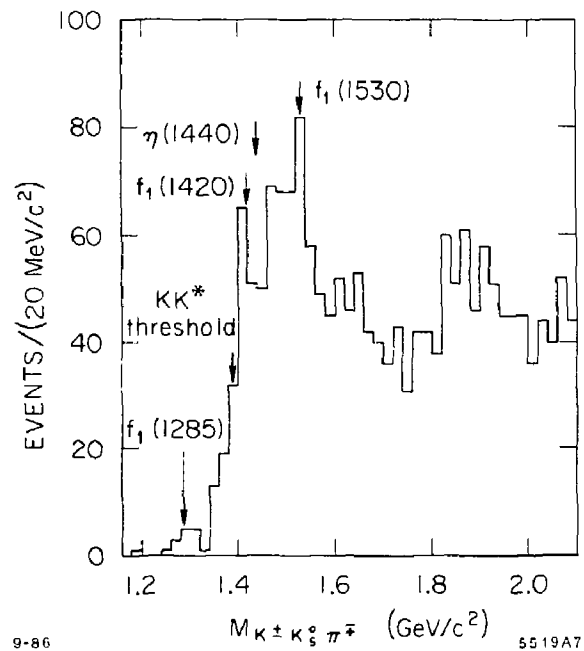


FIG. 22. The summed $K_S^0 K^\pm \pi^\mp$ invariant mass spectrum in the mass region below $2.1 \text{ GeV}/c^2$ in $20 \text{ MeV}/c^2$ bins.

followed by a sharp rise to what may be a small peak just above K^*K threshold. However, not only is this structure of limited statistical significance, but it lies some 10–20 MeV/c² below the accepted mass value for the $f_1(1420)$ meson. The clearest structure is the rise at threshold. Overall, the impression of the peaks in the low mass region is that they look rather similar to but somewhat weaker than those produced into the same final state with π beams, which is not what would be expected if they are states with dominant $s\bar{s}$ content.

These final states are dominated by the production of $(K^*\bar{K} + \bar{K}^*K)$ for all masses. In particular, for the mass region below 1.64 GeV/c², this can be seen very clearly in the Dalitz plot of Fig. 23. However, the amounts of K^* and $\bar{K}^*$ are substantially different, which implies that this region is not dominated by the production of a single resonance.

Preliminary results from the PWA analysis of these channels indicate more clearly the nature of the dominant structure around 1.52 GeV/c². The number of events required to perform a fit with the isobar model is rather large, which has forced us to use rather wider bins than we would prefer in this region. Figure 24 shows all the waves required to fit these data in the mass region below 1.76 GeV/c² summed over isobars. The total cross section is dominated by the unnatural spin-parity waves everywhere and in particular by the 1^+ wave below 1.7 GeV/c². The peak at 1.52 GeV/c² is 1^+ and so does not correspond to the $(K^*\bar{K} + \bar{K}^*K)$ decay mode of the $f_2'(1520)$. In fact, this is as expected both in SU(3) and other more modern models⁸ and results from the small amount of phase space and the large spin inhibition factor of this decay.

The 1^+ waves are all K^* isobars while the 0^- wave is a δ isobar. We find

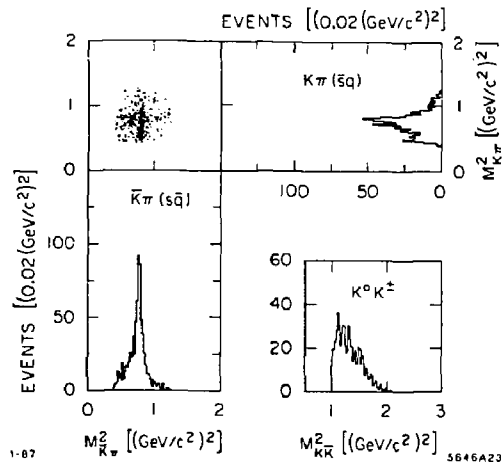


FIG. 23. Dalitz plot for the summed $K^*K^+\pi^-$ final states for the mass region $1.40 \leq M_{K^*K^+} \leq 1.64$ GeV/c². Also shown are projections for the strangeness ± 1 $K\pi$ isobars and the $\delta(K\bar{K})$ isobar.

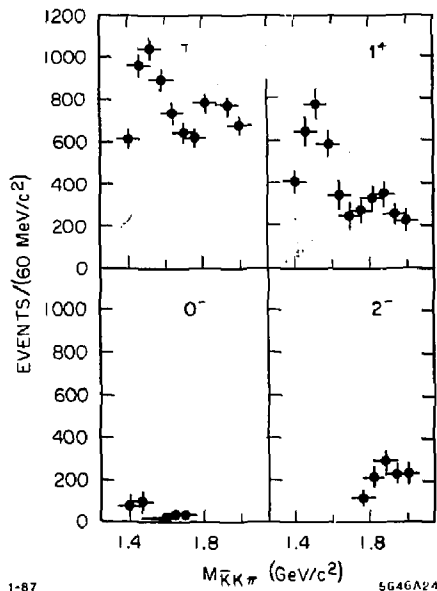


FIG. 24. The spin-parity intensity distributions required to describe the $K_s^0 K^\pm \pi^\mp$ channels below $1.76 \text{ GeV}/c^2$. All partial waves of the same J^P are summed coherently.

no evidence for $0^-\delta\pi$ anywhere, although we cannot totally exclude a small production cross section around $1.42 \text{ GeV}/c^2$. The $1^+ K^+$ waves form a large bump centered at about $1.52 \text{ GeV}/c^2$ with a width of around $100 \text{ MeV}/c^2$, which it is tempting to ascribe to a " D' (1530)" [$f_1(1530)$] resonance previously claimed by Gavillet et al.¹¹ However, since the 1^+ wave dominates this region so completely, we are unable to make a convincing case for resonant phase motion. Moreover, the unequal production of K^+ and $\bar{K}^+$ requires a more complex explanation than a simple one-resonance model. An attempt to better understand these data and their interpretation is in progress.

Not only do the $K\bar{K}\Lambda$ final states in Reaction (4) and the reaction

$$K^- p \rightarrow K_s^0 K_s^0 \Lambda \quad (6)$$

provide a look at the hadroproduction of strangeonia, but they also can provide revealing comparisons with the $K\bar{K}$ spectra produced in radiative J/ψ decay, which might be expected to be glue-enriched. Reaction (4) couples to all natural spin-parity states, while Reaction (6) is restricted to the even spin states only. After restricting the data to events with $|t'| \leq 2.0 \text{ (GeV}/c^2)^2$, both channels are very clean and the normalization agrees well between them in the $f_2'(1520)$ region. The $K\bar{K}$ invariant masses shown in Fig. 25 contain the expected leading $1^-\phi(1020)$, seen only in the $K^- K^+$ channel, as well as the $f_2'(1520)$ in both channels. There is also evidence for a third leading orbitally excited strangeonium state, the $\phi_3(1850)$, which we will discuss below. The primary difference between the spectra, apart from the restriction to only even spin in the $K_s^0 K_s^0$, is what appears to be a large continuum in the high invariant mass region of

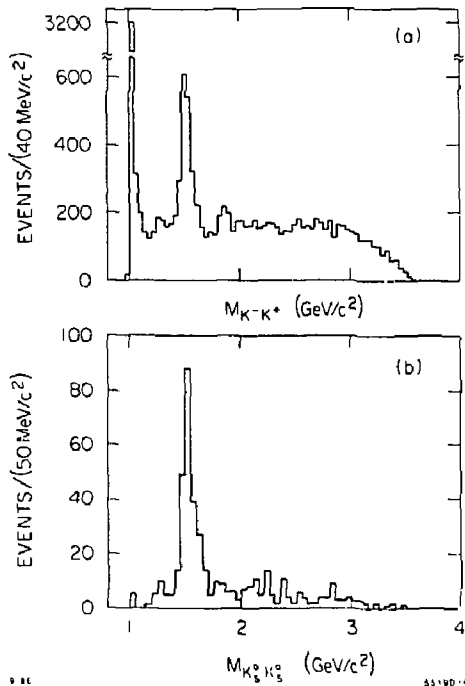


FIG. 25. The $K\bar{K}$ invariant mass spectrum for; (a) the reaction $K^- p \rightarrow K^- K^+ A$; and b) the reaction $K^- p \rightarrow K_S^0 K_S^0 A$.

the $K^- K^+$ spectrum. This background results from the diffractive production of $N^* \rightarrow K^+ A$, as is clear from the Dalitz plot of Fig. 26. On the other hand, N^* production in the $K_S^0 K_S^0$ channel is small. The N^* background in the $K^- K^+$ channel becomes dominant in the region above $2.0 \text{ GeV}/c^2$, and reduces the effective sensitivity of the $K^- K^+$ channel compared to $K_S^0 K_S^0$ even though the visible cross section of the $K^- K^+$ is much larger.

In order to understand the high mass structures in the $K^- K^+$ data, we repeat the moments analysis technique described earlier for the $K^- \pi^+$ channel. The resulting moments distributions are shown in Fig. 27. The acceptance corrected mass spectrum, t_0^0 , shows a clear peak around $1.86 \text{ GeV}/c^2$ and similar structures appear in all moments up to t_8^0 . These structures confirm the existence of a $J^P = 3^-$ ϕ -like object in this mass region. Breit-Wigner fits, shown for t_0^0 and t_8^0 , provide estimates of the parameters of this resonance. The t_0^0 moment is assumed to be dominated by the pure resonance while the t_8^0 moment is given a simple linear background. The fits to the t_0^0 give a mass of $1854 \pm 9 \text{ MeV}/c^2$ with a width of $64 \pm 21 \text{ MeV}/c^2$, while the fit to the t_8^0 moment gives consistent values of 1885 ± 26 and $86 \pm 30 \text{ MeV}/c^2$, respectively. Preliminary results from the amplitude analysis confirm this result.

An object which has been observed in the mass region around $1.7 \text{ GeV}/c^2$ in the radiative J/ψ decays is the " θ " [$f_2(1720)$]. It has a spin-parity 2^+ and is about 150 MeV wide.¹³ Since it has even spin, it should be seen most conspicuously in hadroproduction in the $K_S^0 K_S^0$ channel. There are two decay modes which have been observed with approximately equal strength (the $K\bar{K}$ and the η). There are a few (weak) claims for other decay modes but it appears to be

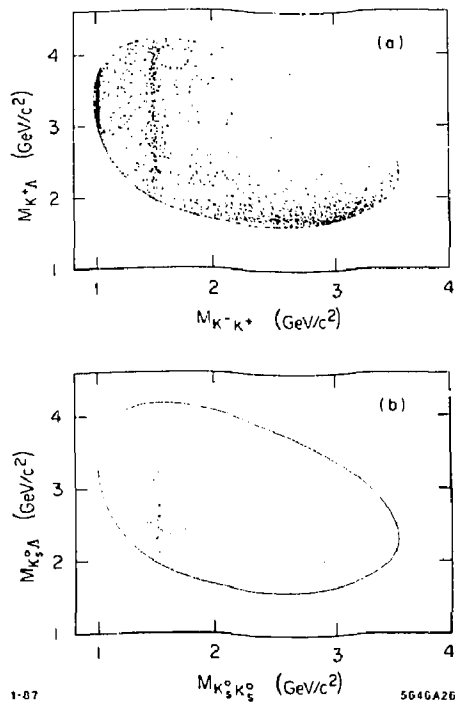


FIG. 26. Dalitz plots for a) the reaction $K^- p \rightarrow K^- K^+ A$; b) the reaction $K^- p \rightarrow K_S^0 K_S^0 A$.

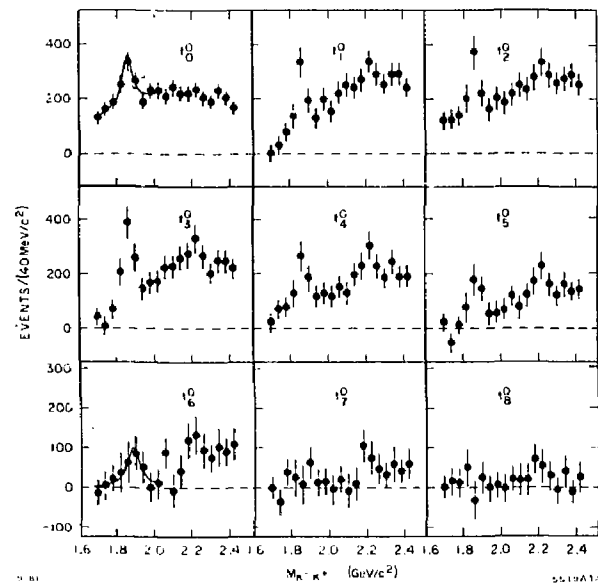


FIG. 27. The unnormalized $K^- K^+ M = 0$ moments from threshold to 2.44 GeV/c^2 .

at least reasonable from the J/ψ decay data to guess that the partial width of " θ " going to $K\bar{K}$ is around 75 MeV, substantially larger than the $f_2'(1520)$ to $K\bar{K}$ partial width. Figure 28 compares the $K_s^0 K_s^0$ hadroproduction data with the radiative J/ψ data from the MARK III experiment.¹² The data from LASS have been multiplied by 0.127 to normalize the $f_2'(1520)$ peaks in the two experiments. There is clearly no evidence at all for production of a $f_2(1720)$ in LASS. It appears to be suppressed by at least an order of magnitude compared to $f_2'(1520)$ production, unlike the production via radiative J/ψ . This would appear to require either that there are some large $f_2(1720)$ decay modes waiting to be discovered, and that there is a mechanism for suppressing simple decay modes like $\pi^-\pi^+$; or that the exchange mechanism is very different than for the nearby $f_2'(1520)$. Either way, the implication is rather strong that the $f_2(1720)$ is not a conventional strangeonium object in spite of its strong $K\bar{K}$ decay mode.

In contrast, data from this experiment and the MARK III do appear to be consistent in the high mass region around 2.2 GeV/c² where a narrow $X(2220)$ [called the $\zeta(2220)$] has been claimed. Figure 29 compares $K_s^0 K_s^0$ mass distributions for the two experiments in the mass region between 1.8 and 2.7 GeV/c². The data are normalized to have the same number of total events in this mass interval which leads to multiplying the acceptance corrected LASS data by 0.42. The data are clearly compatible. While the statistics of this channel are too limited to perform a definitive spin-parity analysis, it is clear that the events are not distributed isotropically in the t -channel helicity frame. Figure 30 shows the $K_s^0 K_s^0$ spectrum for events in the forward region where $\cos\theta_{CJ} > 0.85$. The cut enhances the 2.2 GeV/c² region. Equivalently, the inset to Fig. 30 shows

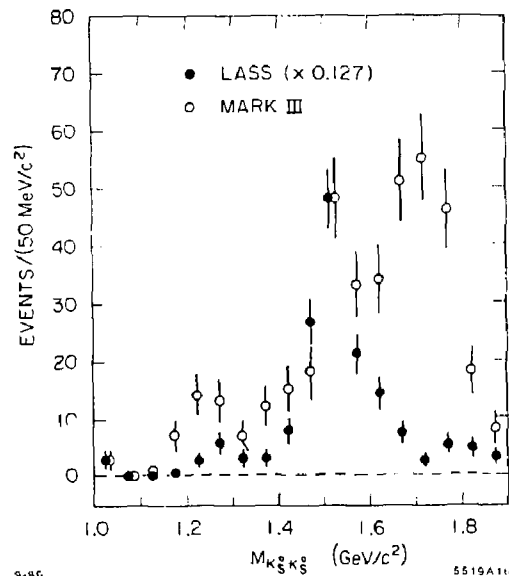


FIG. 28. The acceptance corrected $K_s^0 K_s^0$ invariant mass distribution produced in this experiment from threshold to 1.9 GeV/c² compared with the same final state produced in radiative J/ψ decay as seen in the MARK III (Ref. 12).

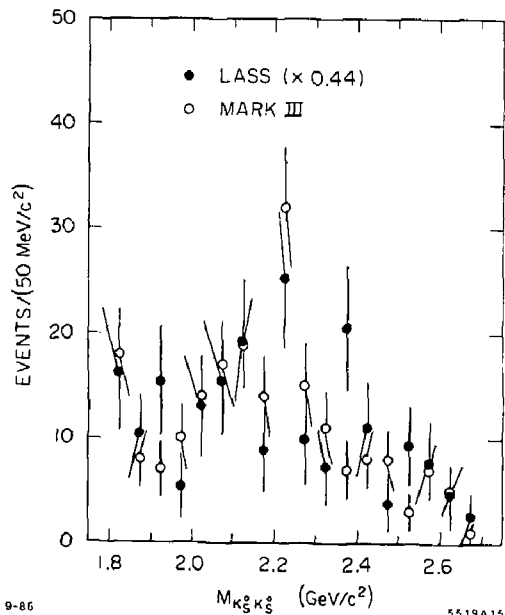


FIG. 29. The acceptance corrected $K_s^0 K_s^0$ invariant mass distribution produced in this experiment in the region $1.8 \leq M_{KR} \leq 2.7$ GeV/c^2 compared with the same final state produced in radiative J/ψ decay as seen by the MARK III [Ref. 12].

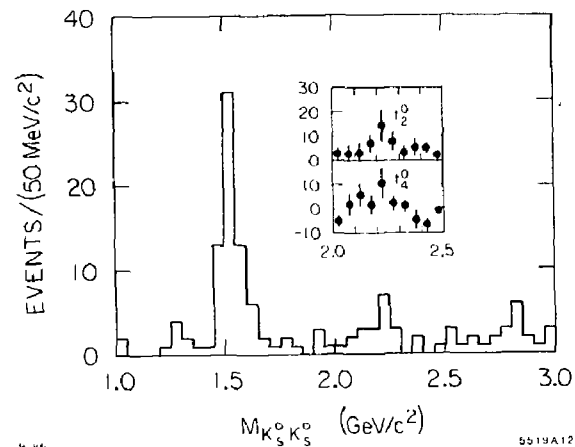


FIG. 30. The $K_s^0 K_s^0$ invariant mass spectrum for events with $\cos \theta_{GJ} > 0.85$. Inset are the $L = 2$ and $L = 4$, $M = 0$ moments in the 2.2 GeV/c^2 region. Moments with $L > 4$ are consistent with 0.

that the t_2^0 and t_4^0 moments also have structure at $2.2 \text{ GeV}/c^2$. Though higher moments are consistent with zero, this may simply result from a lack of statistics. All in all, the data from this channel appear to confirm the MARK III result that a rather narrow object whose spin is at least two exists at $2.2 \text{ GeV}/c^2$. The K^-K^+ channels are not so directly comparable because of the large N^+ diffractive background in the LASS experiment which produces a background underneath the strangeonium production. However, this background should be smooth in K^-K^+ mass, and even though it leads to substantial moments up to t_6^0 , it should not cause any structure in them. The K^-K^+ moments shown in Fig. 27, do show structure in the $2.2 \text{ GeV}/c^2$ region in all moments up to t_6^0 . Although not statistically compelling, this is most simply interpreted as evidence for a spin 4 object at the same mass. Taking these results together, the simplest interpretation appears to be that we are seeing evidence for the production of the $L = 3$ strangeonium triplet expected to lie in this mass region on the basis of quark models.⁸

6. Conclusions

The variety of topics addressed today indicates that there remains a great deal of important physics to be learned in the light quark sector, provided that the data are of sufficient quality and sensitivity. The strange mesons provide a clear avenue to a spectroscopy of pure $q\bar{q}$ states, and good progress has been made in understanding where the states lie and their decay modes. There are now good candidates for most of the underlying states expected in the region below $2.0 \text{ GeV}/c^2$, and the complete leading orbitally excited K^* series up to a

$J^P = 5^-$ at $2380 \text{ MeV}/c^2$ has been observed. Moreover, most of these states have been demonstrated to decay into more than one final state, and rare decay modes, such as the $K\eta$ decay of the $K_2^*(1780)$, have been seen. The QCD based spectroscopy models are successful in explaining the broad outlines of the strange spectrum, but they do have some difficulty in explaining the detailed behavior. For example, the $1^- K^*(1410)$ state is most naturally explained as the first radial excitation of the $K^*(892)$, but it lies too low in mass to be easily explained by most of the models.

The "strangeonium" states produced in hypercharge exchange provide an important alternative window into the search for unusual states, such as glueballs, as well as a direct approach to the $s\bar{s}$ spectrum. Detailed comparisons of the states observed here with those observed in e^+e^- collisions are of particular value in attempts to elucidate their composition. The $K_2^0 K^\pm \pi^\mp$ final state gives evidence for weak $f_1(1285)$ production and perhaps even some evidence for narrow structure in the "E" [$f_1(1420)$] region. However, the production seems very small for an $s\bar{s}$ resonance. The largest structure around is a $1^+(K^* \bar{K} + K^* \bar{K})$ bump at $1.52 \text{ GeV}/c^2$, but it is difficult to prove it is resonant since the amount of K^* and $\bar{K}^*$ production in the region is very different. The $K\bar{K}$ final states clearly show the expected leading $s\bar{s}$ series up to a $J^P = 3^-$ state at 1860 MeV . Data in the high mass " $\xi(2220)$ " [$X(2220)$] region look remarkably like those from MARK III, and provide evidence for structure whose spin is at least 2^+ (and perhaps 4^+), as would be expected in a quark model. On the other hand, the data are completely different from the MARK III data in the region of the " θ " [$f_2(1720)$], which raises interesting questions about the nature of the object which has been seen in e^+e^- collisions.

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CHARM HADROPRODUCTION IN pp INTERACTIONS AT
400 GeV/c AND 800 GeV/c

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1. INTRODUCTION

Charm hadroproduction characteristics have been determined in many experiments at different energies. However, difficulties in the comparison of these results arise from the large correction factors which have to be applied. In fact, most of the experiments use low acceptance spectrometers or have limited phase-space trigger and detection volume; those using a heavy material target suffer from the unknown of the A dependence of the charm cross section.

We present here a comparison of the results of two charm hadroproduction experiments at different energies. In these experiments the above mentioned difficulties have been greatly reduced by the use of a high resolution hydrogen bubble chamber which, at the same time, is a proton target and allows the detection, with very high efficiency, of the production and decay points of the charm particles. The background is therefore considerably reduced.

2. EXPERIMENTAL SET-UP

Both experiments are performed using the same small bubble chamber LEHC together with spectrometers:

- The experiment NA27 consists of an exposure of LEHC and the European Hybrid Spectrometer (EHS) to a 400 GeV/c proton beam from the CERN SPS. The main characteristics of this set-up could be found in Refs. [1-2]. The major modifications consist of the improvement of the charged particle identification with the addition of a Čerenkov plus a transition radiation detector and the detection of neutral hadrons by two hadronic calorimeters.

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- The second experiment E743 also used LEBE but with the Fermilab MultiParticle Spectrometer (MPS) in an 800 GeV/c proton beam. In this apparatus charged particles identification is provided by two Čerenkovs and a transition radiation detector.

LEBC is a rapid cycling (30 Hz) hydrogen bubble chamber. Its two views conventional optical system provides a two track resolution which is less than 20 μm . The bubble density is of the order of 80 bubbles per centimeter. This high resolution yields a high efficiency for the detection of charm decay vertices. The chamber has a useful visible volume of $10 \times 5 \times 0.4 \text{ cm}^3$. A typical picture is shown in Fig. 1.

In both experiments the spectrometers readout and camera flash systems are activated by an interaction trigger which use scintillators to define a good beam particle and high resolution multiwire proportional chambers to define a secondary multiplicity greater than two.

3. ANALYSIS PROCEDURE

The charmed particles are detected by observing their decay in the bubble chamber. Each picture is scanned twice. Each of the independent scans are guided by an upstream measurement of the beam track producing the trigger. After locating the predicted interaction in the hydrogen fiducial volume, a careful search is made for secondary vertex candidates.

These vertices are classified as either C_n for charged decays or V_n for neutral decays, n being the number of charged decay products. A photograph of NA27 with a V_4 and a C_3 is shown in Fig. 1.

A description of the two samples used in this analysis is given in Table 1.



Fig. 1 Bubble chamber photograph of a charm event. A four prong neutral decay (V_4) and a three prong charged decay (C_3) are shown.

TABLE 1

Characteristics of the two samples used

Experiment √s GeV	NA27 27	E743 39
Interactions	1 014 000	109 000
sensitivity (nb)	31.3 ± 0.9	3.97 ± 0.16
charm events	326	
charm decays	484	
V2	163	-
V4	52	16
V6	2	0
C1	65	-
C3	137	34
C5	7	4

The statistics presented in Table 1 for E743 corresponds to only 25% of the final sensitivity. As the analysis of these data is preliminary and since the C1 and V2 decay samples suffer from large strange particle background, we base the analysis in E743 only on C3 and V4. All events with decays found are measured on a high precision measuring machine and a full reconstruction of the event is done using the information of the spectrometer. The final step in the data processing chain (performed up to now in the experiment NA27 only) is the kinematic fitting of decay vertices, including particle identification to resolve ambiguities, where possible.

4. DIFFERENTIAL CHARM CROSS SECTIONS

This analysis, as the one on DD correlations described in the following paragraph, has been performed for the moment on the NA27 data. We use only the decays which have a well determined x_F and p_T^2 values. The selection method used is the same as the one described in Ref. [2]. Following this, we obtain a sub-sample of 72 decays with $x_F \geq 0$. To construct the differential cross sections distributions, each event is assigned a weight derived from the spectrometer acceptance and the visibility in the bubble chamber.

The essential features of the distributions are apparent in the unweighted data.

The transverse momentum p_T^2 distribution is presented in Fig. 2 and can be fitted to the form $\exp(-a p_T^2)$. We find $a = 1.21 \pm 0.14 \text{ (GeV/c)}^{-2}$ which corresponds to $a \langle p_T^2 \rangle = 0.83 \text{ (GeV/c)}^2$.

In Table 2 a summary of other measurements of this parameter is shown.

TABLE 2

Summary of $d\sigma/dp_T^2$ behaviour in some experiments

Experiment	Reaction	Energy (GeV/c)	$a \text{ (GeV/c)}^{-2}$	$\langle p_T \rangle \text{ MeV/c}$
NA16	pp	360	1.1 ± 0.3	750 ± 120
NA27	pp	400	1.21 ± 0.14	-
CCFES	pFe	350	-	920 ± 140
NA16	$\pi^- p$	360	1.1 ± 0.3	850 ± 120
NA27	$\pi^- p$	360	$1.18^{+0.18}_{-0.16}$	-
NA18	$\pi^- \text{Pneon}$	340	≈ 1.1	780 ± 140
NA11	$\pi^- \text{Be}$	175, 200	$1.0^{+0.2}_{-0.1}$	-
CCFES	$\pi^- \text{Fe}$	278	0.7 ± 0.15	-
WA42	$\pi^- \text{Be} \rightarrow A$	135	$1.1^{+0.7}_{-0.4}$	-

The agreement between all these results is good; the mean $\langle p_T^2 \rangle$ value is always near equal to 1.0 (GeV/c)^2 , independently of the incoming beam nature.

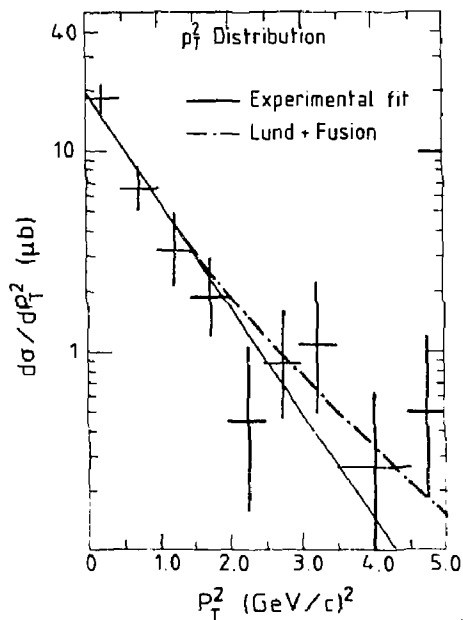


Fig. 2 $d\sigma/dp_T^2$ distribution: 72 decays of NA27.

In Fig. 3 the $d\sigma/dx_F$ distribution is presented; the solid curve shows the result of a fit of the D production spectrum to the form $(1-x_F)^n$, with $n = 4.8 \pm 0.7$. If we separate the sample into $\bar{D}$ -mesons containing a quark which could be a valence quark of the incoming proton and D-mesons which have no quark in common with the incident proton, we find no significant difference in the x_F behaviour. This has already been observed in other p-nucleon experiments. However, this type of "leading" effect does exist in π^-p interactions as it can be seen in Table 3.

TABLE 3

Summary of $d\sigma/dx_F$ behaviour in some experiments

Reaction Energy GeV		Acceptance $(1 - x_F)^n$ $n =$	Two Components Fit			
			n_1 central	n_2 leading	$\%$ central	
pp	360	$x_F > 0.0$	1.8 ± 0.8	No leading effect		
pp	400	$x_F > 0.0$	4.8 ± 0.7			
pFe	350	$x_F > 0.2$	5.0 ± 0.8			
π^-p	360	$x_F > 0.0$	2.8 ± 0.8	6 ± 3	1 ± 1	70 ± 0
π^-p	360	$x_F > 0.0$	3.8 ± 0.6	$7.5^{+2.5}_{-1.5}$	$0.7^{+1.0}_{-0.7}$	80^{+11}_{-18}
π^-Fe	278	$x_F > 0.2$	$2.1 \pm 0.5 (p^+)$ $1.6 \pm 0.3 (p^-)$	$5.9^{+3.2}_{-1.6}$	$0.9^{+0.9}_{-0.6}$	76 ± 16
π^-Be	175,200	$x_F > 0.0$ $x_F > 0.2$	2.9 ± 0.6 1.2 ± 0.5	No leading effect		
π^-Freon	340	$x_F > 0.0$	$0.7^{+0.9}_{-0.7}$			
π^-Be	135	-	1.7 ± 0.7			

The QCD fusion model can be used to calculate the D meson production spectra. This calculation is made of three parts: the structure functions of the parton inside the proton, the calculation of the

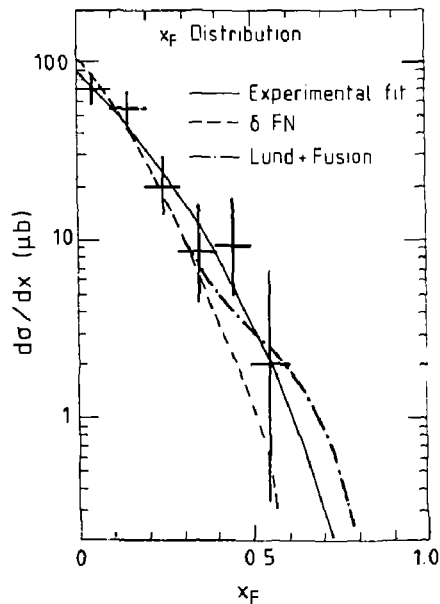


Fig. 3 $d\sigma/dx_F$ distribution: 72 decays of NA27.

cross sections for the intrinsic processes $gg \rightarrow c\bar{c}$ and $q\bar{q} \rightarrow c\bar{c}$ and some description of the $c \rightarrow D$ fragmentation process.

We use the structure functions determined by Duke and Owens in Ref. [3]. The intrinsic parton transverse momentum k_T is described by the distribution

$$dN/dk_T^2 = e^{-k_T^2/\langle k_T^2 \rangle}.$$

The subprocess cross sections are given by Combridge Ref. [4] and we used the Lund model to describe the $c \rightarrow D$ fragmentation.

In Figs. 2 and 3 the predictions of these QCD calculations are shown. The $d\sigma/dp_T^2$ distribution is sensitive to the parton intrinsic k_T value. With $\langle k_T^2 \rangle = 0$ the QCD calculation gives a $\langle p_T^2 \rangle$ value of the order of 0.6 (GeV/c)^2 which does not agree with our data. The data are well reproduced (Fig. 2) by a $\langle k_T^2 \rangle$ value equal to 0.64 (GeV/c)^2 (as suggested by the results of Ref. [5] scaled to our energy).

The $d\sigma/dx_F$ distribution shape is sensitive mainly to the fragmentation of the c quark to the D meson, as it can be seen on Fig. 3 where we can compare no fragmentation at all (δ function) to the Lund process.

5. CORRELATIONS

For 158 events we have detected the two decays. When we apply cuts to reject unclear decays we are left with a sample of 102 charm-anticharm pairs.

The distribution of the azimuthal ϕ_T angle (angle between the two decays in a plane transverse to the beam direction) is shown in Fig. 4(a). Its shape is very sensitive to the k_T value; if $k_T = 0$

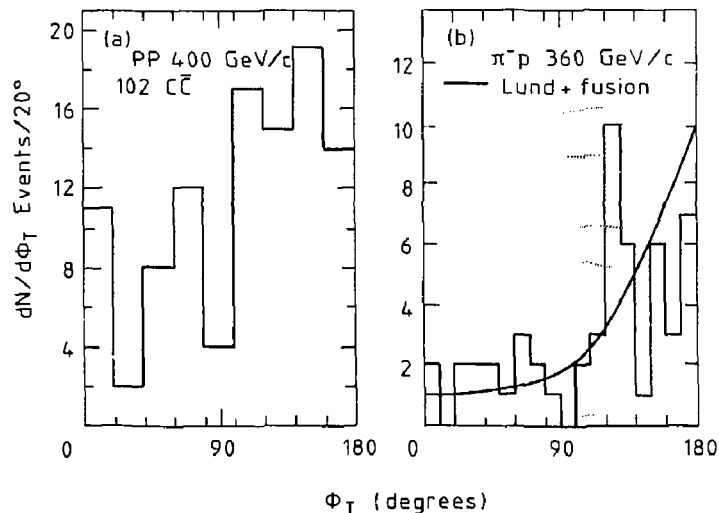


Fig. 4 Azimuthal angle Φ_T of a $D^0 \bar{D}^0$ pair in a transverse plane, to the beam direction.

(a) pp at 400 GeV/c

(b) $\pi^- p$ at 360 GeV/c.

the two charm-anticharm would be produced back-to-back in the transverse plane and the Φ_T angle would be a δ function at $\Phi_T = 180^\circ$. Here a $\langle k_T^2 \rangle$ value of 0.64 (GeV/c)^2 also agrees with the experimental result. In Fig. 4(b) the same Φ_T distribution is presented for 53 $D^0 \bar{D}^0$ pairs produced in $\pi^- p$ interactions at 360 GeV/c. The same asymmetry could be seen between the number of decays with a Φ_T value smaller or bigger than 90° .

6. INCLUSIVE CROSS SECTIONS

In order to calculate the charm cross section we use the following formula

$$\sigma(D) = \frac{N_{\text{obs}}(D) \cdot W_{\text{mc}}}{S \cdot BR \cdot \epsilon_s}$$

where $N_{\text{obs}}(D)$ is the number of observed D decays, W_{mc} is a Monte Carlo computed correction factor which takes into account the cuts described below, S is the sensitivity of the sample and BR the branching ratio for the particular D decay mode used. The scanning efficiency ϵ_s is equal to 0.90 ± 0.05 .

The decay selection criteria are designed to restrict the data to a pure sample of D events for which the overall detection probability is high. First we restrict the sample to the clear charm topologies with a very small strange particle background: then we use V_4 decays for $\text{Br}(D^0 \rightarrow D^+ D^-)$ cross section determination and $C3$ decays for the $D^{\pm}$ cross section.

We define an impact parameter y for each decay track as $L \tan \theta$, where L is the decay length and θ the decay track angle relative to the decaying particle direction. A D decay is included in $N_{\text{obs}}(D)$ if its decay length L is $\geq 1 \text{ mm}$ in NA27, $\geq 2 \text{ mm}$ in E743, its minimum impact parameter $y_{\text{min}} \geq 20 \text{ } \mu\text{m}$ and its maximum impact parameter $\geq 100 \text{ } \mu\text{m}$ ($50 \text{ } \mu\text{m}$) and $\leq 1.5 \text{ mm}$ (2 mm) for $C3$ (V_4) decays. In addition we demand that the laboratory angles of all but one decay track be

≤ 150 mrad. These cuts remove topologically ambiguous decays and suppress the contamination of F and A_C decays. They result in a sample of 75 and 21 $C3$ in NA27 and E743 and in a sample of 32 and $V4$ in NA27 and E743 respectively.

The weight w_{mc} is calculated from a Monte-Carlo generation of D^* s produced with a $(1-x_F)^n e^{-ap_T^2}$ spectrum, where n and a have the values found in the NA27 sample.

The branching ratios are: $BR(D^{*+} \rightarrow C3) = 0.43 \pm 0.10$ and $BR(D^{*0}/\bar{D}^{*0} \rightarrow V4) = 0.17 \pm 0.04$. We checked the stability of the cross sections against the cut values. In Figs.5 and 6 the D^{*+} , $D^{*0}/\bar{D}^{*0}$ cross sections results are plotted as a function of the y_{min} cut. As the x_F and p_T^2 dependence at 800 GeV/c is not known, we have tried different values for n and a parameters. In Figs.5 and 6, the cross sections obtain for $(1-x_F)^9$ assumption are shown; variations of $\sim 50\%$ in the parameters n and a cause changes in cross sections of less than 10%.

For a y_{min} cut of 20 μ m we obtained the cross sections presented in Table 4.

TABLE 4

Inclusive cross sections for $y_{min} \geq 20 \mu$ m

	NA27 $\sqrt{s} = 27$ GeV	E743 $\sqrt{s} = 39$ GeV
$\sigma(D^{*+})_{all} x_F$ (μ b)	12.5 ± 1.5	33 ± 7
$\sigma(D^{*0}/\bar{D}^{*0})_{all} x_F$ (μ b)	21.9 ± 3.8	26^{+21}_{-13}
$\sigma(D/\bar{D})_{all} x_F$ (μ b)	34.4 ± 4.2	59^{+22}_{-15}

The errors quoted above are statistical errors. Additional systematic 25% uncertainty is linked to the 25% branching ratios uncertainty. We compute the ratio R of the total inclusive $D/\bar{D}$ cross sections at 800 GeV/c and 400 GeV/c:

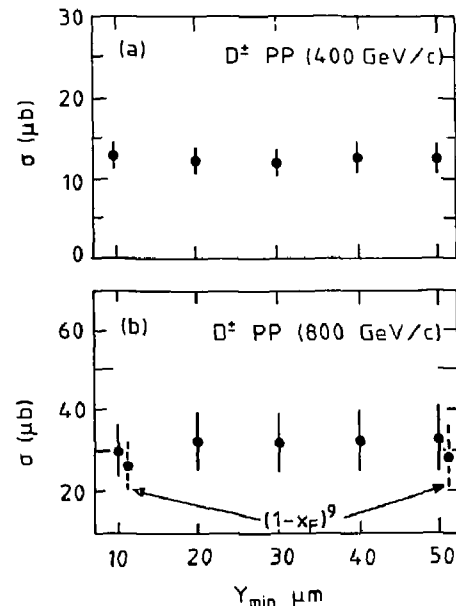


Fig. 5 D^{*+} inclusive cross section as a function of the y_{min} cut

(a) pp at 400 GeV/c

(b) pp at 800 GeV/c.

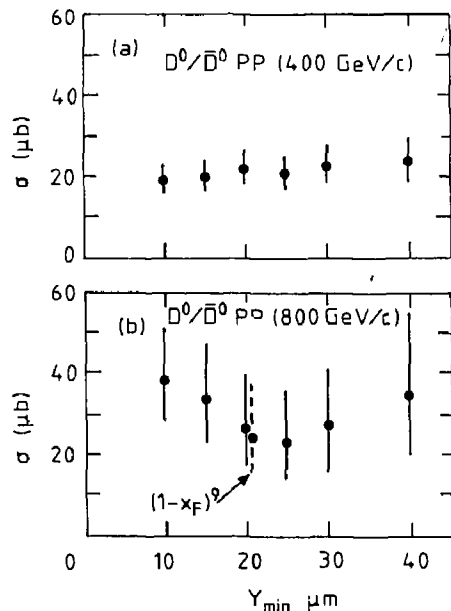


Fig. 6 $D^0/\bar{D}^0$ inclusive cross section as a function of the $y_{\min}$ cut
(a) pp at 400 GeV/c
(b) pp at 800 GeV/c.

$$R = \frac{\sigma(D/\bar{D}) \text{ at } \sqrt{s} = 39 \text{ GeV}}{\sigma(D/\bar{D}) \text{ at } \sqrt{s} = 27 \text{ GeV}} = 1.7^{+0.6}_{-0.5}$$

for $y_{\min} \geq 20 \mu\text{m}$. The sensitivity of this ratio to the $y_{\min}$ cut is presented in Fig. 7.

Experimental results on $\sigma(pp + D\bar{D} + X)$ are plotted in Fig. 8 as a function of energy.

We compare our experimental results to predictions of the QCD fusion model. The fusion model cross section estimates depend sensitively on assumed values of different parameters (charm quark mass, effective threshold and scale parameter); however, the $\sqrt{s}$ dependence is less sensitive to the values of these parameters. Therefore we choose to plot the ratio of cross section rather than the absolute cross section values. The QCD prediction for this ratio is shown in Fig. 9 as a function of $\sqrt{s}$. The range of values at each energy comes from using a variety of structure functions R_{eth} , [3] and [6] and varying the charm quark mass between 1.2 to 1.4 GeV/c².

7. CONCLUSION

At the present level of statistics the ratio of the cross sections at $\sqrt{s} = 27$ GeV and $\sqrt{s} = 39$ GeV is in good agreement with the result of a fusion model calculation.

However, the cross section values obtained at the ISR ($\sqrt{s} = 60$ GeV) are typically a factor ~ 10 higher than those at $\sqrt{s} = 27$ GeV (Fig. 8). It appears in Fig. 9 that the fusion model calculations do not support such a rapid increase in the cross section.

The fusion model calculation predicts the correct shape of both the x_F and p_T^2 spectra if we assume an intrinsic transverse momentum k_T^2 for the partons equal to 0.64 (GeV/c)² and a $c \rightarrow D$ fragmentation using the Lund scheme.

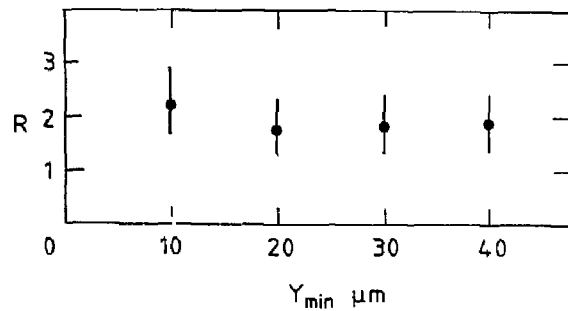


Fig. 7 Distribution of

$$R = \frac{\sigma(pp \rightarrow D/\bar{D} + X)_{800 \text{ GeV}/c}}{\sigma(pp \rightarrow D/\bar{D} + X)_{190 \text{ GeV}/c}}$$

as a function of the $y_{\min}$ cut.

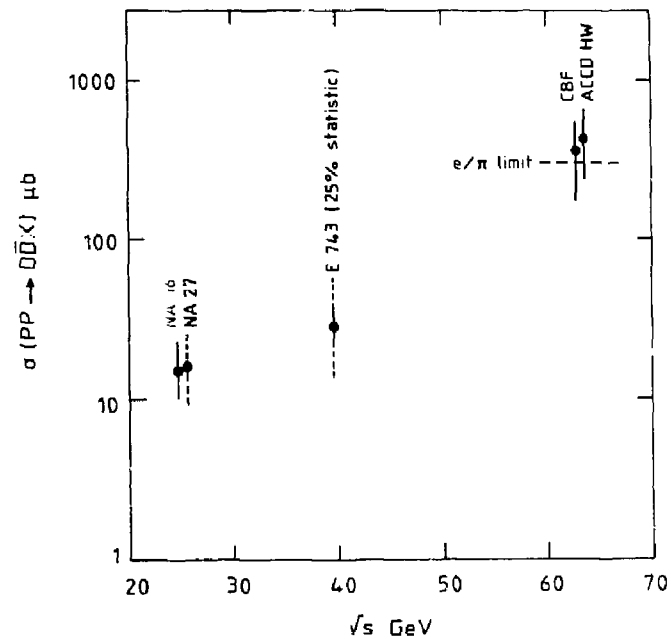


Fig. 8 Cross section of the reaction $pp \rightarrow D\bar{D}X$ as a function of $\sqrt{s}$.

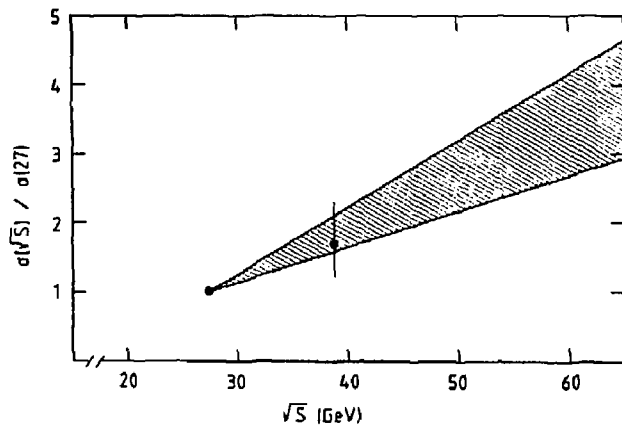


Fig. 9 Total $D/\bar{D}$ cross section normalised to the value at $\sqrt{s} = 27$ GeV versus c.m. energy.

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ON THE DISCOVERY OF VERY HIGH ENERGY POINT SOURCES

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ABSTRACT

A cosmic accelerator such as Cygnus X-3 has a luminosity of at least one million suns. This observation may resolve the long-standing question of the origin of very high energy cosmic rays. Its flux at Earth is roughly 10^7 photons km^{-2} year^{-1} with energy $1-10^5$ TeV. We discuss tagged photon experiments exploring particle structure in this energy range. We show how possible underground muon observation of point sources raises rather than answers questions. We discuss these questions in the context of some recent experimental results.

1. γ -RAYS FROM COSMIC SOURCES AND THEIR DETECTION

A crash course in the detection of cosmic particles is given in Fig. 1. Particles in the cosmic ray beam with large interaction cross section with matter (hadrons, photons, ...) interact near the top of the atmosphere. They initiate an electromagnetic shower (either directly or via $\pi^0 \rightarrow \gamma\gamma$ production). This shower can have a muon component from π production followed by the decay $\pi \rightarrow \mu\nu$. Muons of GeV energy can be detected by shallow detectors shielded by a few meters of earth or concrete. The TeV muons can reach deep underground *proton-decay* detectors. These detectors can also reveal the presence in the cosmic ray flux of weakly interacting particles such as the neutrino. They ignore the atmosphere but can produce muons inside or in the rock above or below deep underground detectors via the usual $\nu + \text{nucleon} \rightarrow \mu$ weak interaction; see Fig. 1.

As suggested in Fig. 1 the electromagnetic showers initiated by cosmic ray hadrons are not spectacularly different from those initiated by γ -rays from an astronomical source, making astronomy in the TeV-band of the electromagnetic spectrum a challenge. A primary photon converts to an e^+e^- pair after 1 radiation length λ_R in the atmosphere which is about 25 radiation lengths thick, with

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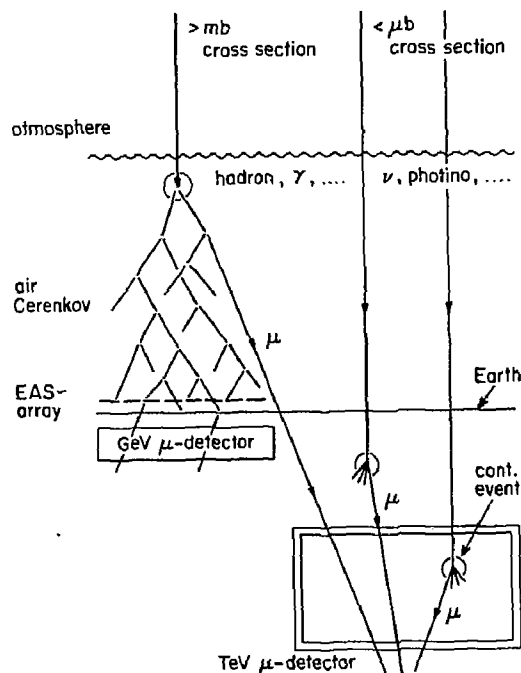


Fig. 1. Summary of observational techniques for cosmic particles with large and small interaction cross sections with matter.

$\lambda_p \approx 37 \text{ g/cm}^2$. In subsequent radiation lengths the electromagnetic particles further lose energy by bremsstrahlung $e^\pm \rightarrow \gamma e^\pm$ and pair production $\gamma \rightarrow e^+ e^-$; see Fig. 2. The prominent feature that can set γ -rays apart from background cosmic ray hadrons is their low muon content. The number of muons in a γ -shower is typically a few percent of that in a hadron shower in which muons are abundantly generated by the decay of the produced $\pi^\pm$. In γ -initiated showers processes resulting in muons are characterized by small cross sections: $\gamma + \text{nucleon} \rightarrow \pi$ photoproduction followed by the decay $\pi \rightarrow \mu \nu$, production and subsequent semi-leptonic decay of charm quarks and $\gamma \rightarrow \mu^+ \mu^-$ pair production which is suppressed by a large factor $(m_e/m_\mu)^2$ relative to $\gamma \rightarrow e^+ e^-$. The muon production through these channels has been calculated.¹ An illustrative result is shown in Fig. 3 where the number of muons in excess of 1 GeV is plotted as a function of shower size (number of electrons N_e) for γ - and nucleon-initiated showers. The result is qualitatively easy to understand: muons are the progeny of hadrons and the photon is hadronic to $O(\alpha)$, i.e., to order 10^{-2} .

To the experimentalist the shower of Fig. 2 looks like a pancake of electromagnetic energy about $10^2 \sim 10^3 \text{ m}^2$ in size and a few nanoseconds thick, moving down the atmosphere at the speed of light. Showers in excess of 10 TeV reach the ground, as shown in Fig. 1. By recording the arrival time of the pancake in several detectors of an extensive particle array, the arrival direction of the shower can be reconstructed from the timing sequence. Lower energy showers do not reach the ground. A 100 GeV photon will produce about 100 electrons at 10 km altitude. Their Čerenkov light does, however, reach earth and can be collected by mirrors viewed by phototubes. The angular spread of the Čerenkov cone is only about 1.5° around the parent photon direction.

Experiments recording the arrival direction of cosmic particles are, of course, as old as cosmic ray physics. The subject emerged on the forefront when in 1983 the Kiel group, after mapping cosmic rays in the sky for five years, reported² a 4 σ excess coming from the direction of Cygnus X-3. High energy cosmic rays are, after all, not totally isotropic! No other source was found at a significant statistical level. Roughly 20 experiments³ have by now found evidence for the emission of very high energy γ -rays from the direction and with the characteristic time structure (more about that later) of the binary star Cygnus X-3. Some made repeated observations. An incomplete compilation of results is shown in Fig. 4. This source seen in radio, infra-red, MeV- γ and X-ray experiments is observed to emit TeV photons by Čerenkov telescopes. Ground-based detectors

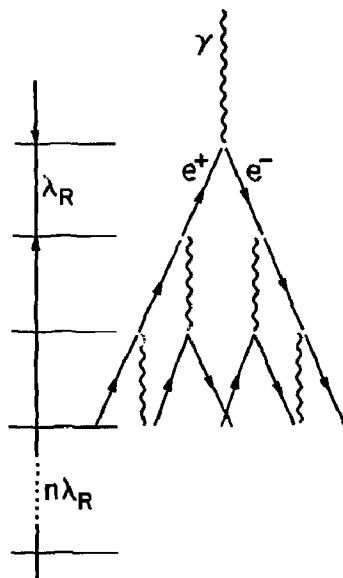


Fig. 2. The γ -ray initiated cascade in the atmosphere. Each layer represents a radiation length λ_R . The number of layers is limited by the maximum energy of the source, e.g. 10^5 TeV for Cygnus K-3.

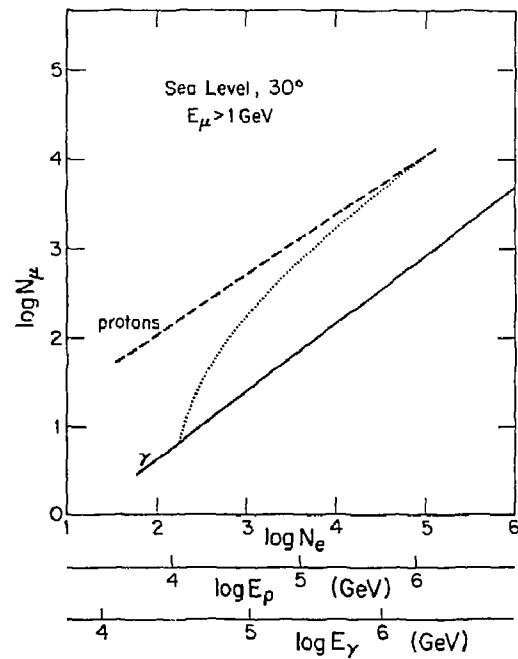


Fig. 3. Number of muons in excess of 1 GeV (N_μ) versus shower size (i.e. number of electrons N_e) for γ - (solid line) and proton- (dashed line) initiated showers. The dotted line shows the muon signature in a model where the photon becomes strongly interacting at very high energies. The horizontal energy scales show the conversion of shower size to initial proton or γ -ray energy.

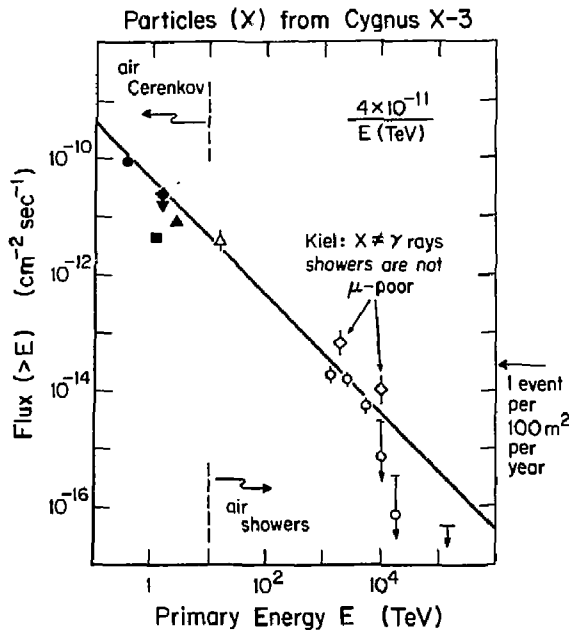


Fig. 4. Integral flux of very high energy particles from the X ray binary Cygnus X-3. We will call these particles γ -rays although the Kiel experiment challenges this identification. Note the flatter E^{-1} energy dependence compared to the $E^{-1.7}$ fall-off of the cosmic ray flux.

suggest emission all the way up to 10^5 TeV, while the Haverah park array suggest a cut-off at that energy (open circles in Fig. 4). The flux can be approximated by⁴

$$F(>E) = \frac{4 \times 10^{-11}}{E(\text{TeV})} \text{ particles cm}^{-2}\text{sec}^{-1} \quad (1.1)$$

for $E \gtrsim 0.1$ TeV. We have determined (1.1) from time-averaged data. Ground arrays, such as Kiel, yield for certain epochs fluxes higher by more than one order of magnitude. Some TeV experiments have identified periods of increased activity of a few minutes of duration.⁵ The Cygnus beam has a very rich time structure which has not been deciphered.³ Periodicities of 4.8 hours, 19 days and one year have been suggested. Only the 4.8-hour bunching of the beam is established; high energy emission occurs very sporadically in these 4.8-hour periods. Other very high energy emitters have been reported: Hercules X-1, Crab pulsar, 4U0115 + 63, PSR 1953, LMC X-4, Centaurus A, Vela pulsar, M31, PSR 1802 and the galactic plane. Some of these certainly require confirmation.

It must be emphasized that the surface experiments do not detect the primary particle, only its atmospheric cascade. The primaries are assumed to be photons because they are the only known particle that is neutral, stable (and hence capable of travelling in straight lines in the galactic magnetic field over very long distances) and initiates air-showers. At this point the only cloud in this firmament of exciting results is the observation by the Kiel group² that the on-source showers from Cygnus show some deficit in the number of muons with respect to hadron showers; the muon content exceeds however, the expected value by over one order of magnitude. This result has been debated,⁶ e.g., on the basis of punch through in the 2 m of concrete used to identify the muons in a shallow detector as in Fig 1, but never successfully challenged. This started speculations⁷ that the particles carrying the radiation from Cygnus X-3 are not photons.² We will return to this later.

2. COSMIC ACCELERATORS

As a particle physicist one wonders how an X-ray binary living at the far edge of the galactic plane (Cygnus is at least 12 kpc away) is beaming 10^5 TeV particles at us with a luminosity making a bump in the background cosmic ray spectrum. In Fig. 5 we sketch the general picture^{8,9} of a binary source of high energy photons. The system consists of a compact star in a 4.8 hour orbit with a star that has not yet collapsed. The compact partner somehow accelerates protons, perhaps by a pulsar mechanism¹⁰ or through conversion of

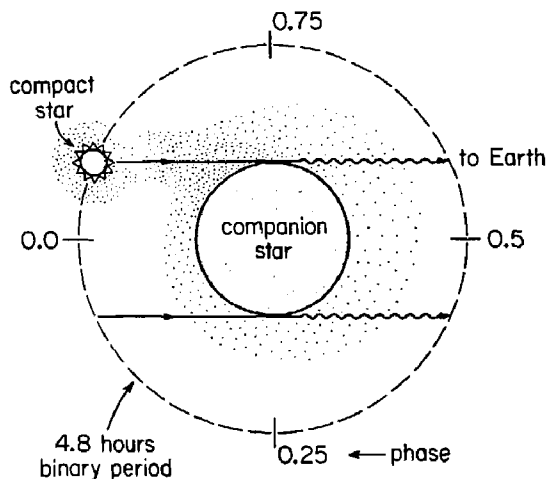


Fig. 5. The Cygnus accelerator beams particles to earth produced in the beam dump at phases 0.2 and 0.8 of the pulsar which accelerates the primary beam. The dots represent matter accreting on the pulsar and material blown off the companion.

energy from accretion of matter from the companion star.^{11,12} The accelerated particles then interact with the companion or the surrounding gas (see Fig. 5) to produce a cascade of secondaries, the stable end products of which are as in any beam dump experiment photons, neutrinos, protons, antiprotons, electrons and positrons. The charged particles are injected into the galaxy as cosmic rays, though the electrons and positrons especially will be much degraded in the source. Some fraction of photons and neutrinos from

$$\begin{aligned} p &\rightarrow \pi^0 \rightarrow \gamma \\ p &\rightarrow \pi^\pm \rightarrow \nu \end{aligned} \quad (2.1)$$

will escape the source and travel in straight paths. They may be detected on earth if their production is sufficiently prolific. As in any beam dump any new particle for which the beam is above threshold will be produced and beamed to earth if it is sufficiently stable and neutral, e.g.

$$p \rightarrow \tilde{g} \rightarrow \tilde{\gamma}. \quad (2.2)$$

In all but a few experiments (e.g., Kiel² and Haleakala⁵) a signal cannot be established without exploiting the fact that on-source showers are a 4.8 hour pulsed signal on a continuous (and typically 100 times larger) cosmic ray background. It is customary to define a phase angle from 0 to 1 which traces the binary revolution as shown in Fig. 5. The 0.0 corresponds to X-ray minimum when the compact star is eclipsed by its companion. In the model shown the beam-on positions are in the vicinity of phases 0.2 and 0.8. A compilation¹ of emission phases is shown in Fig. 6 along with the phase distribution of showers from the Cygnus direction as measured by the Haverah park array. The picture is a warning that the structure of the accelerator is complex. Its structure could be different for different energies. If the accreting matter provides the target material, emission at the other phases is possible; see Fig. 7.

The primary proton beam in Fig. 5 must of course reach energies up to 10^5 TeV to account for the secondary spectrum shown in Fig. 4. Beams of 10^5 TeV might sound out of the ordinary. The basic reason why such energies are achieved can, however, be understood on the basis of a dimensional argument. The typical pulsar is shown in Fig. 8. It is 10 km in diameter and accelerates particle beams along the dipole axis of its magnetic field which can reach values of 10^{12} gauss. Like a lighthouse the beams spin with pulsar periods of $10^{-3} \sim 1$ sec around a rotation axis not coincident with the dipole axis. The EMF of

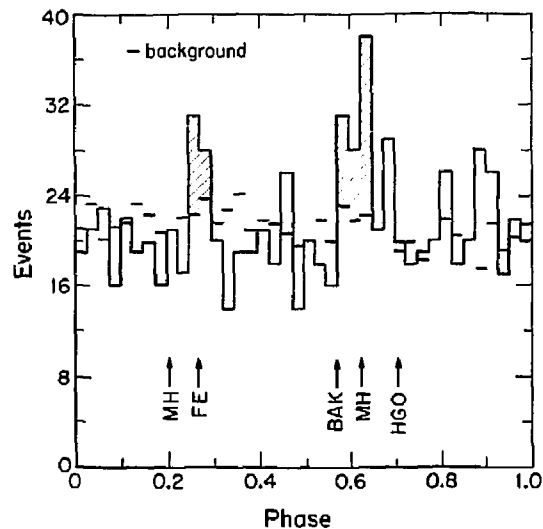


Fig. 6. Phase distribution of the radiation in the direction of Cygnus X-3 as observed by the Haverah park extensive air shower array. Emission is also observed in the general vicinity of $\phi \approx 0.2$ and $0.6 \sim 0.7$ by Baksan, Fly's Eye, Mount Hopkins and Haleakala (HGO).

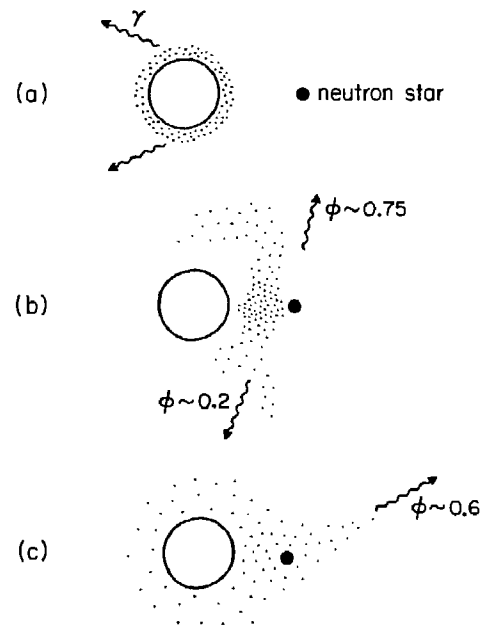


Fig. 7. Models where the accreting matter itself ((b) and (c)) rather than the companion's atmosphere ((a) and Fig. 5) provide the target material for the beam dump. Emission at other phases is possible.

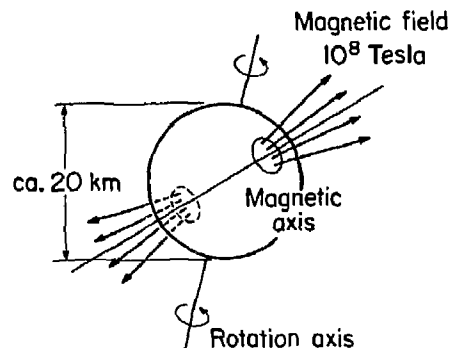


Figure 8. Pulsar

such a system is

$$\mathcal{E} = B\ell v \text{ or } B \frac{\partial \phi}{\partial t} = B \frac{\ell^2}{t} = 10^{16} \sim 10^{19} \text{ eV}. \quad (2.3)$$

It should be pointed out here that no pulsar period had been identified for Cygnus X-3 until the Durham group¹³ found evidence for a 12 msec period. This result has been recently confirmed by Haleakala Gamma Observatory (HGO).¹⁴ It is the first novel astronomical result of very high energy astronomy and should once and for all establish the existence of TeV γ -rays from point sources. So if the energy is easy to understand, what is the big deal? We discuss this next.

3. THE PUZZLE OF THE ORIGIN OF VERY HIGH ENERGY COSMIC RAYS

The intriguing property of Cygnus X-3 is its spectacular energy output which results in the flux enhancement in the Kiel detector over uniform cosmic ray background. No other source in the detector's field of view could match this power. The energy output can be inferred⁹ from the observed flux in our earth-based apparatus. The energy output at the source is obtained from the experimentally observed flux of about $10^{-10} \text{ erg cm}^{-2} \text{ sec}^{-1}$ above 1 TeV, shown in Fig. 4, after correcting for the fact that

- (i) we only catch a fraction of the $4\pi R^2$ ($> 1.5^{+0.6}_{-0.4} \text{ cm}^2$ for a distance of 10 kpc) emission,
- (ii) we have to take into account the duty cycle of the accelerator (≈ 0.02 as inferred from phase plots such as Fig. 6),
- (iii) only a fraction (0.1) of the energy goes into $p + \pi^0 \rightarrow \gamma$, and finally that
- (iv) some γ 's are absorbed on the 3° K background along the way.

These corrections are somewhat model-dependent but a conservative estimate yields

$$L > 10^{39} \text{ ergs sec}^{-1} \quad (3.1)$$

i.e., more than one million times the total energy output of the sun. This luminosity of the accelerator exceeds the Eddington limit for spherically symmetric accretion onto a one solar mass compact object.

The mean free path of cosmic γ -rays is shown as a function of their energy in Fig. 9. The primary γ -flux is depleted due to the process $\gamma\gamma \rightarrow e^+e^-$, although secondary electrons can regenerate some photons by $e\gamma \rightarrow \gamma e$. Especially for

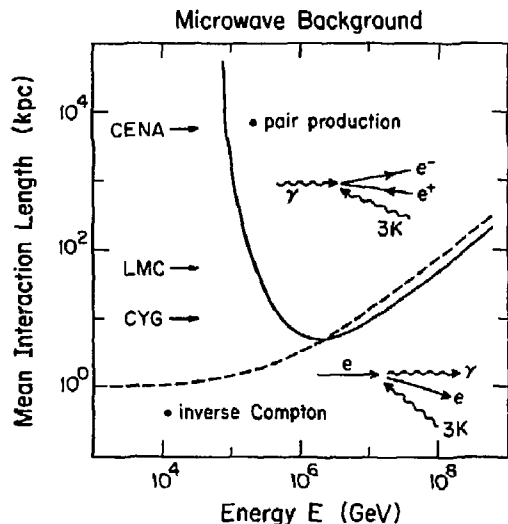


Fig. 9. Mean interaction length of a photon of energy E in the microwave background¹⁵ due to the process $\gamma\gamma(3K) \rightarrow e^+e^-$. At higher energies the absorption is reduced due to the regeneration of photons by the secondary electrons $e\gamma(3K) \rightarrow \gamma e$. The arrows indicate the distance in kpc of Cygnus X-3 and some tentatively identified extra-galactic high energy γ -emitters LMC X-4 and Centaurus A.

extra-galactic sources such as Centaurus-A or LMC X-4 absorption plays a crucial role. For a given γ -ray flux their expected neutrino flux is greatly enhanced compared to galactic sources. Neutrinos, unlike photons, stream freely to earth through the 3° K photon background. Experiments such as IMB are becoming sensitive to the expected ν -flux for LMC X-4. Also γ -rays of extra-galactic origin are so strongly absorbed in the 10^5 TeV energy region, see Fig. 9, that observation in that energy region would essentially exclude that the high energy radiation of these point sources is exclusively carried by photons. This provides us with the possibility to check the puzzling indication of the Kiel experiment.

Let us return to the result that the Cygnus X-3 accelerator has the incredible luminosity given by (3.1). This discovery might have solved^{9,16} the old problem of the origin of very high energy cosmic rays. It has been known for some time that the cosmic-ray spectrum can be understood up to perhaps 10^{16} eV in terms of shock wave acceleration in supernova remnants.¹⁷ Although the spectrum shows a kink at 10^{15} eV as shown in Fig. 10, cosmic rays with much higher energies are observed and cannot be accounted for by this mechanism. A simple estimate¹⁸ will reveal that the Cygnus accelerator's power (3.1) is more than adequate by itself to supply all the cosmic rays in the galaxy in the interval $10^{15} - 10^{17}$ eV! We first calculate the energy density in this interval from the observed cosmic ray flux $I(E)$ shown in Fig. 10. We find

$$\rho_E = \frac{4\pi}{c} \int_{10^{15}}^{10^{17}} EI(E)dE \approx 4 \times 10^{-16} \frac{\text{ergs}}{\text{cm}^3}. \quad (3.2)$$

The power of the source required to fill our galaxy continuously with an energy flux ρ_E is

$$\frac{\rho_E V_{\text{galaxy}}}{\tau} \approx 5 \times 10^{38} \frac{\text{ergs}}{\text{sec}}. \quad (3.3)$$

Here τ is the mean confinement time of cosmic rays of about 10^{16} eV inside the galactic disk. This number is known to be about 10^7 years for low energy cosmic rays, but the confinement of the very high energy ones is of course reduced; transport calculations yield $\tau \approx 2 \times 10^5$ years.⁹ This and (3.2) determine the required power (3.3). Only about 10% of this power or 5×10^{37} ergs sec^{-1} is the galaxy's need for the very high energy interval $10^{15} \sim 10^{17}$ eV. This is only 5% of the estimated power (3.1) generated by Cygnus X-3. We conclude that Cygnus X-3 need only be on 5% of the time to generate all cosmic rays not accounted for by supernovas! Although the source is indeed very variable (see

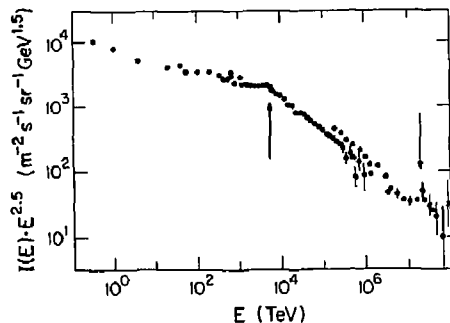


Fig. 10. Cosmic ray spectrum with hints of structure indicated by the arrows.

section 7), this is a reasonable number based on the last decade of observations. Note that the other identified TeV γ -ray emitters are likely to play a role in this problem. The very highest energy cosmic rays (beyond the kink indicated by the second arrow in Fig. 10) are likely to be of extragalactic origin, but also here point sources could play a primary role with e.g., LMC X-4 already identified in the TeV-band.

4. PARTICLE PHYSICS WITH COSMIC ACCELERATORS?¹⁹

Whereas Cygnus X-3 is possibly closing a chapter in cosmic ray physics, it might be opening a new one in particle physics. As Fig. 2 suggests, Cygnus is truly a HERA in the sky; primary photons populate the atmosphere with an electromagnetic cascade. The interactions of these particles can be studied by ground-based detectors. The potential of such experiments for particle physics should not be judged against conventional cosmic ray experiments which have a distinguished but somewhat checkered record. Here the direction and characteristic time structure of the radiation from a point source can be used to tag a beam of known composition (γ -rays) with well-understood interactions (described by QED) with the target atmosphere. An experiment using a tagged cosmic photon beam, therefore, overcomes the classic hurdles in interpreting cosmic ray experiments. New physics cannot be confused with some *mundane* (at least from the particle physics point of view) change in chemical composition of the primary particles or of their hadronic interaction with atmospheric nuclei.

The structure of the atmospheric beam is extremely simple and can be calculated using linear shower theory.²⁰ For the primary E^{-1} spectrum of Eq. (1.1) at the top of the atmosphere we obtain a flux of photons

$$N_{\gamma}(> E, z) = \frac{1}{2} A E^{-1} \quad (4.1)$$

as a function of energy and depth in the atmosphere described in terms of linear column density $z(\text{g}/\text{cm}^2)$. Here A is the constant in Eq. (1.1). Notice that the number of photons of a given energy E is independent of depth z . This somewhat surprising result can be understood as follows: while photons lose energy with depth, others of energy E are generated from showers with $E' > E$. Energy loss and feed-down are in equilibrium for a E^{-1} spectrum, hence the z -independent result of Eq. (4.1). Every radiation length of atmosphere contains the same number of photons of energy E . The number of radiation lengths is of course limited to some number n_{max} which is determined by the maximum

energy of the source (10^5 TeV for Cygnus, see Fig. 4) and by the fact that a photon will lose half of its energy per radiation length. Therefore,

$$n_{\max}(\text{radiation lengths}) \approx \frac{\ln(E_{\max}/E)}{\ln(2)}, \quad (4.2)$$

where $n_{\max} \approx 15$ for 1 TeV γ -rays. It is also important to remember that we are observing roughly equal numbers of γ 's and electrons interacting above a detector; see Fig. 2.

The beam is free. The question is what the detector requirements are to achieve statistics and signal/noise to do a quality photoproduction experiment at energies inaccessible to artificial accelerators and to possibly probe the much-anticipated new structure in particle physics at a scale of $(\sqrt{2} G_F)^{-1/2} \approx 0.25$ TeV. As we have seen the most discriminating signature of the photon-induced shower against the hadron-induced cosmic ray background showers is the scarcity of muons. The two essential components of a tagged photon experiment are therefore obvious:

- (i) an array of scintillation counters with fast timing or a lattice of simple single mirror Čerenkov telescopes giving an observation of the electromagnetic component of the shower with good angular resolution on the arrival direction,
- (ii) an array of shielded counters identifying the content of (GeV) muons in the shower.

Both goals could be achieved by having vertically stacked counters in coincidence with different shielding. The array can be triggered on specific timing sequences of the counters which correspond to showers from one or even multiple source directions. From Eq. (1.1) we can see that at 1 TeV energy even a "modest" $(100 \text{ m})^2$ array can accumulate 10^5 γ -rays per year. The number of electrons/muons at sea level is, however, insufficient to perform a realistic experiment. With a 100 TeV threshold this problem is solved but it now takes a $(1 \text{ km})^2$ array to accumulate the 10^5 /year statistics required to take a detailed look at photoproduction under difficult experimental circumstances. Some relevant information is summarized in Table 1. Such a facility²¹ would still constitute a not-too-far extension of present facilities such as the Akeno array²² in Japan. One should, however, be aware of the possibly severe punch-through problems when identifying GeV muons by their penetration through matter. Not just hadronic magnetic particles could contribute; softer ones have a low

TABLE 1. Properties of hadron- and photon-induced air showers at Akeno (depth 920 g/cm²).

Primary energy (TeV)	1	10^2
γ -flux from Cyg X-3 using (1) (km ² year ⁻¹)	10^7	10^6
Number of electrons in hadron shower	-	3800
Number of muons in hadron shower ($E > 1$ GeV)	30	1525
Number of muons in γ -shower, (see also Fig. 3)	-	40

probability to punch through but they have very large multiplicity and fake muons. For a detailed discussion, see Gaisser *et al.*, in Ref. 6. What about signal-to-background?

Present detectors (e.g. Čerenkov) telescopes achieve a nominal

$$\frac{S}{N} \approx 10^{-4}. \quad (4.3)$$

This is clearly insufficient to hope to detect traces of new particle thresholds in the tagged photon beam. This number can be greatly improved in the future when the detailed emission time structure of sources like Cygnus X-3 is well known. The binary 4.8 hour and candidate 19 day, 12 month (?) and 5 year burst (?) repetition rates of the source can be used to enhance S/N possibly by 10^2 using phase information. The μ -poor property of γ showers further enhances the signal by another factor 10^2 but not more as at that level γ showers photoproducing a π in the primary interaction are virtually indistinguishable from a hadron-induced background cosmic ray cascade. With possible improvements

in angular resolution $S/N \simeq 10^2 \sim 10^3$ can possibly be achieved. Preliminary data¹⁴ from the Haleakala Čerenkov telescope, however, suggest emission from Cygnus in bursts with a typical duration of one minute. During these bursts $S/N \simeq 1$ rather than 10^{-2} . Tagging such bursts could therefore yield data at the $10^4 \sim 10^5$ signal to noise level. It is also very easy to convince oneself that deep underground experiments observing TeV—rather than GeV—muons would not be competitive.²³

New physics can reveal itself in the experiment in two distinct ways: anomalous interactions of the photons or electrons in the atmospheric cascade (see Fig. 2) or admixture in the γ -beam of new neutral particles produced in the cosmic beam dump where the primary energies reach 10^5 TeV; see Fig. 5. We illustrate both possibilities. In Fig. 11, examples are shown of new interactions of photons and electrons with matter. In Fig. 11a, the photon couples to nucleons via new fermions f . Such interactions are certainly associated with heavy quark photoproduction with $f = c, b, t, \dots$ and $\alpha_f \simeq \alpha_e$. They have already been included in the calculations of the muon content of γ -showers shown in Fig. 3 and would be observable in the more ambitious versions of the experiment previously described. Speculation that photon interactions become strong at high energies,²⁴ though unpopular in the context of gauge theories, could easily be investigated. If at some energy $\alpha_f \simeq 1$, photon showers would eventually resemble hadron showers. The dotted line in Fig. 3 shows the result of a Monte Carlo study of the muon number dependence on energy for a new strong coupling process with 0.25 TeV threshold.

It is important to realize that electromagnetic cascades contain roughly equal numbers of electrons and photons. Composite models of quarks and leptons provide us with an example where new physics in the electron-nucleon interaction will also affect the development of the γ -initiated cascade. At very high energy, electrons have a finite size in which pointlike preons move around. They can interact with quarks (hadrons) via the exchange of preons. If these are colored, electrons can turn into quarks and begin to interact like hadrons; see the preon diagram of Fig. 11b. A drastic increase of the muon yield will result.

In both examples shown in Fig. 11, photon showers will turn mixed electromagnetic-hadronic at high energy, with enhanced muon production signaling the onset of the hadronic component. The Kiel experiment,⁶ showing an increase of the muons in the Cygnus showers by at least a factor of 10 over the QED calculation, could be interpreted along these lines.

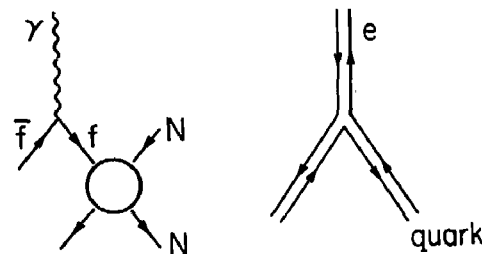


Fig. 11. Examples of new physics modifying electromagnetic cascades: (a) photons interacting with matter via a new fermion f , (b) electrons converting into quarks (or muons) via shared constituents.

The photino ($\tilde{\gamma}$) or gluino ($\tilde{g}$) supersymmetric partners of the photon and gluon provide us with postulated particles which definitely have to be produced by cosmic accelerators and could possibly be detected as admixtures in the γ -beam. The generation of particles in the Cygnus beam dump of Fig. 5 can be described by a one-dimensional evolution of the particle flux¹⁹

$$\frac{dN_j}{dE_j dz} = -\frac{1}{\lambda_j} \frac{dN_j}{dE_j} + \sum_i \int_{E_i}^{\infty} \frac{1}{\lambda_i} \frac{dN_i}{dE_i} \frac{dN_{i \rightarrow j}}{dE_j} dE_i. \quad (4.4)$$

The first term describes the depletion of the flux of particle j with interaction— or decay—length λ_j in the dump while the second term accounts for production of j by particle(s) i ; z is the column density of the dump. Assuming Feynman scaling and a E_i^{-2} spectrum of the incident proton beam, (4.4) can be written

$$\frac{dN_j}{dE_j dz} = -\frac{1}{\lambda_j} \frac{dN_j}{dE_j} + \sum_i \frac{1}{\lambda_i} \frac{dN_i}{dE_i} \langle x \rangle_{i \rightarrow j}, \quad (4.5)$$

where $\langle x \rangle_{i \rightarrow j}$ is the average momentum of j relative to its parent i

$$\langle x \rangle_{i \rightarrow j} = \frac{1}{\sigma_i} \int_0^1 dx x \frac{d\sigma_{i \rightarrow j}}{dx}. \quad (4.6)$$

Here $x = E_j/E_i$. The formalism can be easily generalized to a general power spectrum $E_i^{-\alpha}$ with $\alpha \neq 2$. If j is a decay product of a particle k as for $p \rightarrow \pi^0 \rightarrow \gamma$, the two-step generation is taken into account by a factorized generalization of (4.6)

$$\langle x \rangle_{i \rightarrow j} = \langle x \rangle_{i \rightarrow k} \langle x \rangle_{k \rightarrow j}. \quad (4.7)$$

This formalism allows for useful back-of-the-envelope calculation. From Eq. (4.5) we find the flux of secondary protons in the dump at depth z

$$\frac{dN_p}{dE} = \frac{dN_p^0}{dE} e^{-z/\lambda_p}. \quad (4.8)$$

This assumes an initial ($z = 0$) proton beam $dN_p^0/dE \sim E^{-2}$ and neglects the regeneration of secondary protons, i.e. the second term in (4.5). The γ -flux

generated by these protons via production and decay of π 's is obtained from solving (4.5) and (4.7)

$$\frac{dN_\gamma}{dE dz} = -\frac{1}{\lambda_R} \frac{dN_\gamma}{dE} + \frac{1}{\lambda_p} \frac{dN_p^0}{dE} e^{-z/\lambda_p} \langle x \rangle_{p \rightarrow \gamma} \quad (4.9)$$

with

$$\langle x \rangle_{p \rightarrow \gamma} = 2 \langle x \rangle_{p \rightarrow \pi^0} \langle x \rangle_{\pi^0 \rightarrow \gamma}. \quad (4.10)$$

The solution is

$$\frac{dN_\gamma}{dE} = \frac{dN_p^0}{dE} \frac{e^{-z/\lambda_p} - e^{-z/\lambda_R}}{\frac{\lambda_R}{\lambda_p} - 1} \langle x \rangle_{p \rightarrow \gamma}. \quad (4.11)$$

Here λ_R is of course the radiation length previously introduced to describe the depletion of photons in matter. For the nucleon target making up the dump, $\lambda_p \approx \lambda_R$ and (4.11) simplifies to

$$\frac{dN_\gamma}{dE} \approx \frac{dN_p^0}{dE} \frac{z}{\lambda_p} e^{-z/\lambda_p} \langle x \rangle_{p \rightarrow \gamma}. \quad (4.12)$$

What about new physics? We work out one example where stable photinos are produced via production and decay of gluinos $\tilde{g} \rightarrow \tilde{\gamma} q \bar{q}$. Clearly in analogy with (4.9)

$$\frac{dN_{\tilde{\gamma}}}{dE dz} = \frac{1}{\lambda_p} \frac{dN_p^0}{dE} e^{-z/\lambda_p} \langle x \rangle_{p \rightarrow \tilde{\gamma}}. \quad (4.13)$$

After integrating over z we obtain the $\tilde{\gamma}$ -flux emitted at the end of the dump

$$\frac{dN_{\tilde{\gamma}}}{dE} = \frac{dN_p^0}{dE} \left(1 - e^{-z/\lambda_p}\right) \frac{\lambda_p}{\lambda_{\tilde{g}}} \langle x \rangle_{p \rightarrow \tilde{g}} \langle x \rangle_{\tilde{g} \rightarrow \tilde{\gamma}}. \quad (4.14)$$

Comparison of (4.10), (4.11) with (4.14) gives us an idea of the expected admixture of $\tilde{\gamma}$'s in the photon flux

$$\frac{dN_{\tilde{\gamma}}/dE}{dN_\gamma/dE} = \frac{\sigma_{p \rightarrow \tilde{g}}}{\sigma_{p, \text{tot}}} \frac{\langle x \rangle_{p \rightarrow \tilde{g}} \langle x \rangle_{\tilde{g} \rightarrow \tilde{\gamma}}}{2 \langle x \rangle_{p \rightarrow \pi^0} \langle x \rangle_{\pi^0 \rightarrow \gamma}} \frac{1 - e^{-z/\lambda_p}}{z/\lambda_p} \quad (4.15)$$

$$\approx \frac{1}{3} \frac{\sigma_{p \rightarrow \tilde{g}}}{\sigma_{p, \text{tot}}}. \quad (4.16)$$

The latter relation assumes that the linear depth of the dump is small compared to λ_p (i.e. $z \ll \lambda_p$ in (4.15)) and that $\langle x \rangle_{p \rightarrow \tilde{g}} \approx \langle x \rangle_{p \rightarrow \pi^0}$. From simple decay

kinematics $\langle x \rangle_{p \rightarrow \gamma} / \langle x \rangle_{\tilde{g} \rightarrow \gamma} = 2/3$. Much more sophisticated calculations yield a similar result, which is interesting. Indeed, for GeV gluinos the production cross section is $O(mb)$. This can result into a $10^{-2} \sim 10^{-3}$ admixture of supersymmetric particles in the cosmic photon beam from (4.16). They could have characteristic signatures in underground muon detectors. (We discuss this further on.) If the gluino itself was stable, the $N_{\tilde{g}}/N_{\gamma}$ ratio would be even larger than (4.16). Gluinos would be bound in stable gluino-hadrons. They interact with the earth's atmosphere like hadrons rather than photons and would therefore be detectable through a muon excess in the muon-poor photon beam.

It is exciting to contemplate the feasibility of tagging 100 TeV photons in detectors which represent a not too far-fetched extrapolation of present facilities (e.g. Akeno). Even if no other than routine particle physics results are ever obtained, imagine their power as a telescope! However, probably the best way to illustrate how very high energy astronomy can possibly reveal particle physics is to discuss the recent experiments searching for muons from point sources. The exercise is instructive whether the present evidence is confirmed or not.

5. NEUTRINO AND MUON ASTRONOMY: IT CAN BE DONE!

The calculation (4.5)–(4.12) for $p \rightarrow \pi^0 \rightarrow \gamma$ production in the Cygnus beam dump can be copied to calculate²⁵ neutrino emission via $p \rightarrow \pi^{\pm} \rightarrow \nu$. The ν -flux can be normalized to the observed γ -flux and it would be very instructive to see these neutrinos. Since they are much more penetrating than photons their escape from the source is much less model-dependent. Estimates based on the lower limit for the source luminosity of Eq. (3.1) give an integral flux of ν -induced muons in detectors such as Soudan or Nusex of less than $10^{-14} \text{ cm}^{-2} \text{ sec}^{-1}$; recall Fig. 1. This flux is $2 \sim 3$ orders of magnitude smaller than the corresponding atmospheric γ -flux; see Fig. 12. A ν -induced upward muon signal of $\leq 1 \text{ event}/1000 \text{ m}^2/\text{year}$ is expected from the same estimate.

These estimates represent, however, lower limits. One can conceivably increase the luminosity of the accelerator (3.1) and so increase the $\pi^{\pm} \rightarrow \nu$ secondary flux without increasing the (experimentally observed!) $\pi^0 \rightarrow \gamma$ yield provided there is reabsorption of the γ 's in the dump, i.e. one increases the input value of the luminosity into the ν -flux calculation by fiddling with the correction factor (iii) leading to (3.1). Photon escape from the source is indeed very model-dependent. One has to be careful, however, as a too intense beam will critically heat the companion when it eclipses the pulsar beam at the O phase position; see Fig. 13. The heating results from the deposition of

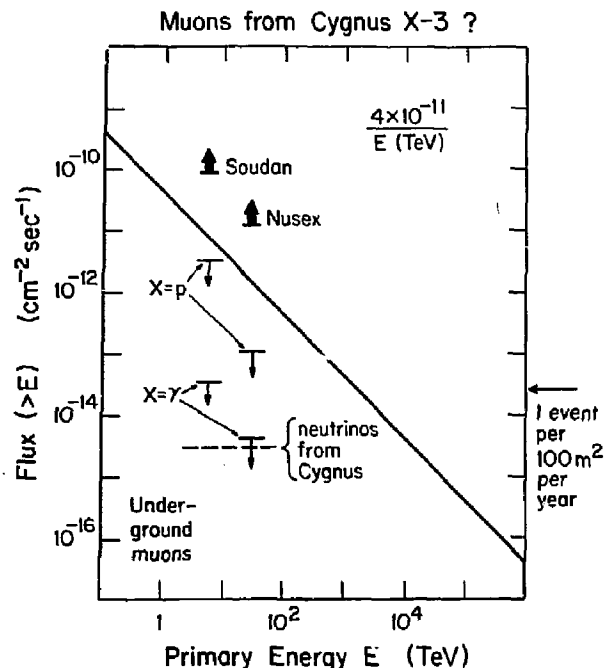


Fig. 12. Observed and calculated underground muon fluxes assuming that the source of the muons are alternatively γ -rays ($X = \gamma$), hadrons ($X = p$) interacting with the atmosphere or neutrinos produced in the Cygnus beam dump of Fig. 5 (dashed line).

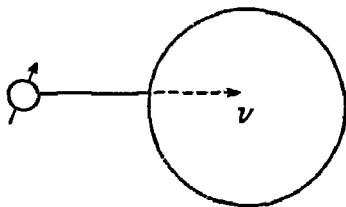


Fig. 13. At phase $\phi \approx 0$ (see Fig. 5) the pulsar's beam dumps neutrinos into the core of the companion—possibly resulting into its instability by neutrino heating.

neutrinos in the core of the companion. The companion's radius would double in a time²⁶

$$t = 10^3 \text{ years} \frac{M}{R} [f_\nu L]^{-1}. \quad (5.1)$$

Here M , R are the companion's mass and radius in solar units, L is the beam luminosity in units of 10^{38} ergs/sec and f_ν is the fraction of energy going into neutrinos. The latter number depends on the separation of the pulsar from the companion, typically²⁶ $f_\nu \approx 10^{-2}$. Equation (5.1) applied to Cygnus X-3 yields $t < 10$ years for $L \approx 10^{42}$ ergs/sec. Note that the energy $f_\nu L t$ for this situation corresponds to a temperature increase $\Delta T \approx 10^8$ K in a solar mass M ! In summary, the fact that the Cygnus binary is observed for over a decade implies an upper limit on its luminosity which must be much less than 10^{42} ergs sec⁻¹. We cannot use our incomplete knowledge of the source as an excuse to arbitrarily boost the flux prediction for not only ν 's but also for anything novel such as $\tilde{\gamma}$'s or $\tilde{g}$'s emerging from the cosmic accelerator. Conversely, if ν 's could be detected, they would give a rather direct measure of the luminosity of the source as well as more precise information on the accelerator/beam dump scenario.

Underground muon detectors can be used as an astronomical telescope even if the muon's parent particles are γ 's or neutrons rather than neutrinos. Figure 14 shows the ancestry of a 300 GeV muon observed in the Frejus detector. For present considerations the crucial fact is that the underground muon reveals the direction of the primary (on the average) 50 TeV nucleon to an angular precision

$$\theta \text{ (in mrad)} \approx \frac{\pi}{\sqrt{E_\mu \text{ (in TeV)}}}. \quad (5.2)$$

Although γ -rays are ineffective at producing muons as previously discussed, the very high energy ones in the spectrum shown in Fig. 4 will generate some TeV muons. These will produce a calculable signal in underground (proton decay) detectors which is difficult to observe as typically $N_\mu/N_e \approx 10^{-3}$, as opposed to 10^{-2} in a hadron shower where muons are abundantly generated by meson decay. We computed^{1,7} the muon signal associated with the observed atmospheric γ -ray flux given by (1.1) for the Soudan and Nusex detectors. The result is shown

12 (labeled primary $X = \gamma$) where the $\gamma \rightarrow \mu$ flux is plotted at a primary corresponding to

$$E_\mu \text{ (TeV)} = 0.5[\exp(0.4x) - 1] \quad (5.3)$$

where x is the detector depth in km of water equivalent.

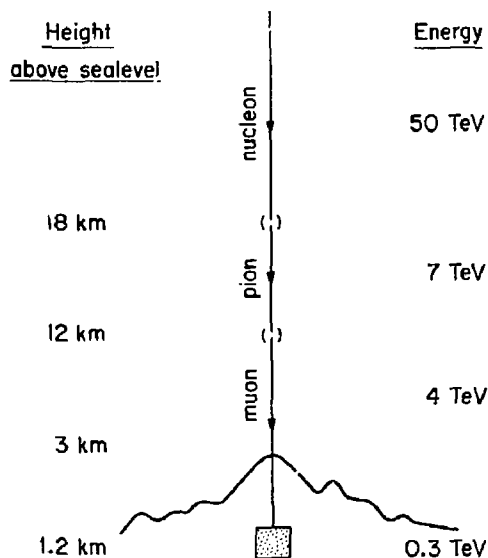


Fig. 14. A 50 TeV neutron impinging on the atmosphere is revealed as a 0.3 TeV muon in the Frejus detector.

We close with the reminder that sources such as LMC X-4 or the galactic center might be more abundant sources of neutrinos and underground muons. Muon astronomy is in our future.

6. HAVE WE ALREADY WITNESSED THE BIRTH OF MUON ASTRONOMY?

When in 1985 two experiments reported the observation of in-direction and in-phase muons²⁷ from Cygnus X-3 we literally got more than we bargained for. The observed fluxes, shown in Fig. 12 exceed the flux just calculated on the assumption that the primaries are photons by roughly three orders of magnitude for each experiment. It is possible that underground detectors rediscovered an old puzzle. The Kiel air shower array experiment,² after detecting a 4σ enhancement of the cosmic ray flu. in the direction of Cygnus X-3, performed two tests to confirm the signal. They checked that on-source showers indeed remember the 4.8 binary period but also found that the showers are not muon-poor as expected from γ -ray emission. The expected 2% muon content relative to the hadron-induced background was observed to be 70%. Is our straightforward assumption that Fig. 4 represents the high energy tail of the electromagnetic emission spectrum to be questioned? Do cosmic accelerators emit particles (referred to as X from now on) other than photons?⁷

What if X were a neutral hadron, e.g. a neutron? We can repeat the previous calculation assuming the atmospheric signal in Fig. 4 is due to hadrons (remember atmospheric experiments identify showers and not the nature of the primary particle). As expected, about 10^2 times more muons are predicted, but the assumption that X is a hadron falls short of accommodating the data by more than one order of magnitude for Soudan and two for NUSEX; see Fig. 12. To produce enough muons underground in this way would lead to more surface showers than observed.

An alternative candidate for a "conventional" explanation of the underground muon signal is that X is a neutrino. We already noted, however, that the expected signal is much too low. Moreover, it cannot be sufficiently increased by increasing the power of the source without causing instability of the companion. Given the Kiel and the underground muons results, the situation is now desperate. How desperate we will illustrate next by a series of theorems suggesting that X cannot exist!

Let us backtrack and list the properties of X implied by the observations:

- (1) The X is neutral: charged particles forget direction and time (phase) in the

3 μ gauss intergalactic field. The 10⁶ parsecs distance represents more than 10⁶ gyroradii for particles with rigidity less than 10³ TeV. This would exclude nuclei and protons anyway.

- (ii) The 4.8 hours bunching of the Cygnus beam would be lost after $L' \approx 10^4$ parsecs unless the γ factor is large enough. The time delay between the arrival of two particles with velocities v_1, v_2 which left Cygnus X-3 at the same time is

$$\delta t = \frac{L'}{v_1} - \frac{L'}{v_2} \quad (6.1)$$

or

$$\delta t \approx \frac{L'}{2c} \left[\frac{1}{\gamma_1^2} - \frac{1}{\gamma_2^2} \right] \lesssim \frac{1}{2} \text{ hour}.$$

In Eq. (6.1) we imposed the condition that the muons arrive within a rather narrow time window, approximately half an hour as observed by the experiment. From (6.1) we conclude that the muon parents must be nearly monoenergetic (which is inconceivable in the type of models sketched in Fig. 5) or that the Lorentz factors must satisfy the bound

$$\gamma = \frac{E}{M(X)} > 10^4. \quad (6.2)$$

Therefore,

$$M(X) < 10^{-4} E. \quad (6.3)$$

As for Soudan $E \approx 10^4$ GeV, we conclude conservatively that

$$M(X) \lesssim \text{a few GeV}. \quad (6.4)$$

- (iii) The lifetime of X must be sufficient to cover the 10⁴ parsecs distance, therefore

$$\tau(X) \gtrsim 10^8 \text{ sec}. \quad (6.5)$$

This is about 10⁸ neutron lifetimes.

- (iv) The observations also restrict the interaction cross section of X with matter. An upper limit is obtained from our previous observation that the muons do not originate in conventional hadronic air showers; see Fig. 12.

A way out of this is to arrange the interaction length of X to be comparable or greater than the thickness of the atmosphere so that production of the signal occurs too low for regular air shower production. This requires $\sigma(X\text{-nucleon}) \lesssim 1$ mb. However, if the cross section is made too small, the zenith angle distribution becomes very different from that of muons originating in atmospheric showers and the X particles penetrate so deep that they can interact inside the detector resulting in "contained" events. Both of these results disagree with observations. Calculations²⁸ relevant to the Soudan detector are shown in Fig. 15. A comparison with the data suggests that the cross section cannot be made smaller than 10 μ b. We therefore conclude

$$10\mu\text{b} < \sigma(XN) < 1\text{mb}. \quad (6.6)$$

Also, this discussion eliminates the possibility that neutrinos (or photinos) are the underground muon parents.

This concludes the theorem: a particle with the properties (i) - (iv) should have been discovered by accelerators. If you can imagine that such a particle has been overlooked, consider this as a challenge.

The no-go theorem does not even exploit another puzzling feature of the muon data: the angular spread of 3° around Cygnus in both experiments.^{29,30} This cannot be understood on the basis of any astronomical arguments or in terms of detector resolution which is roughly 1°. One is forced to add some large mass or transverse momentum as an ingredient of the puzzle. In summary, one can conservatively state that although scenarios can be dreamed up without contradicting accelerator information, none of these fit into the standard model or its extensions based on supersymmetry or compositeness. We illustrate this with a few random musings.

Suppose a new particle ($\tilde{q}$ or $\tilde{\eta}$) interacts with quarks in the atmosphere or rock via resonance production $\tilde{q}q \rightarrow \tilde{q}$; see Fig. 16. This is the supersymmetric analogue of the Glashow resonance $\nu_e e \rightarrow W$ suggested to detect a ν_e -beam interacting with atomic electrons in the atmosphere. The squark resonance peaks at

$$E \approx \frac{M_{\tilde{q}}^2}{\xi_q M_p}. \quad (6.7)$$

Here M_p and $M_{\tilde{q}}$ are the proton and squark masses, ξ_q is the average momentum of quarks in a proton. The peak is further broadened by the $\tilde{q}$ spectrum. Muons

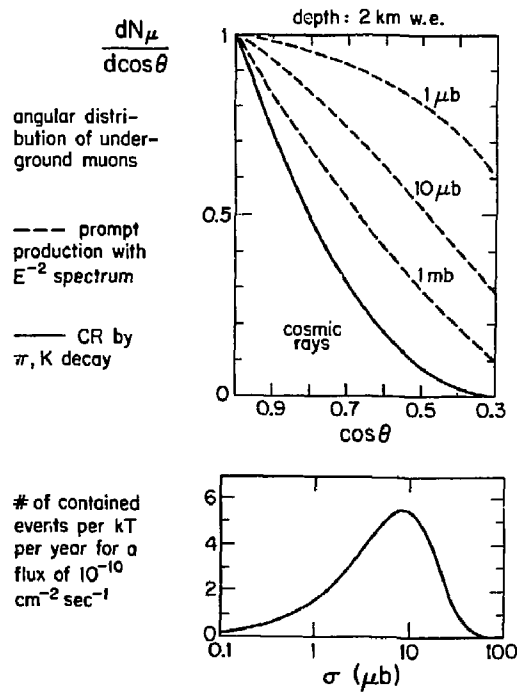


Figure 15

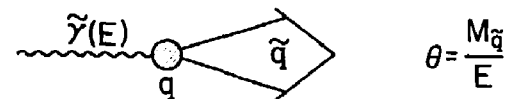


Fig. 16. Glashow resonance photino + quark $\rightarrow$ squark.

resulting from the decay of the heavy squark would have an angular spread

$$\theta \lesssim \frac{M_{\tilde{q}}}{E}. \quad (6.8)$$

For $M_{\tilde{q}} \simeq M_W$ and $E \simeq 10$ TeV (as in the experiments), Eq. (6.7) is satisfied and (6.8) predicts an angular spread of several degrees as observed. The bad news is that the cross section associated with the production of such a heavy mass

$$\sigma \simeq \frac{g^2}{M_{\tilde{q}}^2} \quad (6.9)$$

is very small and violates (6.6). In Eq. (6.9) $g^2 \simeq e^2$ is a typical standard model coupling. Reducing $M_{\tilde{q}}$ does not help as the angular spread θ is reduced accordingly; see Eq. (6.8). This is another version of the no-go theorem.

It is already clear that the calculated admixture of supersymmetric particles in the γ -beam, see Eq. (4.15), is never going to yield secondary muons at the required level. On the positive side, however, these fluxes are uncertain and can be increased like the neutrino flux in the previous section. One day the Glashow resonance might be the signature³¹ of supersymmetry in the detectors dreamed of in Section 4. Composite models³² are no better off. If one requires an $e \rightarrow \mu$ or $e \rightarrow q$ conversion through the exchange of preons as shown in Fig. 11 to populate the electromagnetic shower with muons, conversion cross sections of $O(mb)$ are required to accommodate the data. This clashes with the most conservative limits on the compositeness scale $\Lambda > 0.1$ TeV.

For those not totally discouraged, Fig. 17 shows two general roadmaps to solutions of the muon puzzle. It is, however, clear that no one has succeeded at present to explain every feature of every experiment. In Fig. 17a the particle interacts in the atmosphere with $O(mb)$ cross section. The deeper detector samples higher incident energies and therefore measures a reduced flux. This is a feature of the observations.²⁷ However, by (6.8) the angular spread should be reduced at the deeper observation level. This is not borne out by the data. In scenario (b) the incident particle interacts in the rock with $O(10\mu b)$ cross section. The energies sampled by both detectors are similar; the reduced flux seen in the deeper detector is the result of absorption in the rock. Now the angular spread could actually be larger in the deeper detector if high energy particles are preferentially absorbed (threshold behavior).³³ Only the low energy ones dribble down to the deep detector resulting in large θ by Eq. (6.8). This will leave Kiel unexplained.

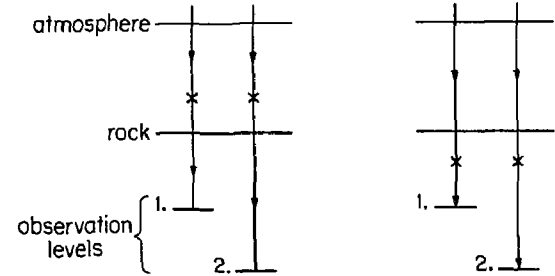


Fig. 17. Alternative scenarios where muons are produced in (a) the atmosphere or (b) the rock above two underground detectors at different depths.

One should finally also keep in mind the possibility of a "garbage" solution³⁴ of which quark nuggets is an illustration, as an alternative to new particles or interactions.

7. "DID THE MUONS GO AWAY?"

Cygnus muons are at present the only challenge to the standard model of quarks and leptons. The highest priority is to confirm the observations and as the saying goes in this business there are no experiments, only observations. One does not control the source which is known to be highly variable in radio and X-rays making controlled experiments difficult. *E.g.*, Frejus and Kamiokande have reported²⁷ upper limits that are inconsistent with signals of the strength reported by Soudan and Nusex. These measurements refer to different time intervals, however.

The long-time variability of the source is an established fact. Figure 18 shows a compilation³⁵ of epochs since September 1972 when the radio emission of Cygnus X-3 exceeded one Jansky. Notice the radioburst in 1977 which has been invoked³⁰ to explain indications of a large flux in ultra-high-energy γ -rays during that period; see Fig. 19. This was until recently the only marginal hint that activity in the radio and TeV γ bands was possibly correlated.

The TeV γ -rays by themselves are showing hints of increased activity according to a time pattern with the general structure shown in Fig. 17. Fly's Eye, Baksan, Durham and especially Haleakala have seen clear indications that the flux peaks during epochs¹ sometimes as short as one minute in duration.⁵ In fact the "art" consists of limiting the data sample to such periods to be able to successfully pull the signal out of the large background. It has been recently argued that the muon signal itself shows signs of variability.^{29,30} The Nusex phase plot is shown in Fig. 20a. The events in the signal at phase $\phi = 0.7-0.8$ seem to turn on and off when plotted as a function of time of observation; see Fig. 20b. Soudan confirms this. Its 1981-83 signal is definitely absent in the 1985-86 data; see Fig. 21. Shown is the phase plot during hot periods (i.e. two muons per 4.8 hour cycle). Most of the Frejus data are taken outside the "on" period. If one is willing to buy the Nusex/Soudan "excuse," Frejus could at best have hoped for a marginal signal in data taken at the initial stage of operation. This is consistent with their data as reported³⁷ at the Bari conference and with an analysis³⁸ comparing Frejus in summer 1985 with an expected phase plot computed from Nusex data; see Fig. 22. No evidence can be claimed, no disagreement exists. The statistics of a possible signal is so low that mixing it

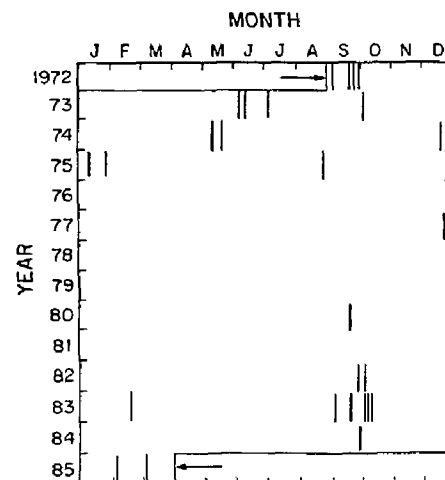


Fig. 18. Epochs between August 1972 and March 1985 when the radio emission from Cygnus X-3 exceeded one Jansky.

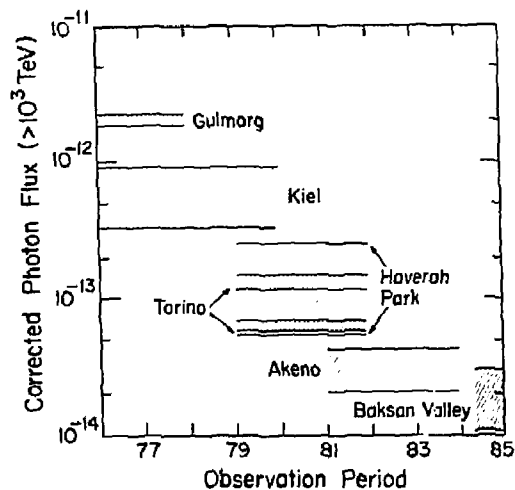


Fig. 19. Early speculation that the very high energy emission of Cygnus X-3 is decreasing since the large 1977 radio burst.

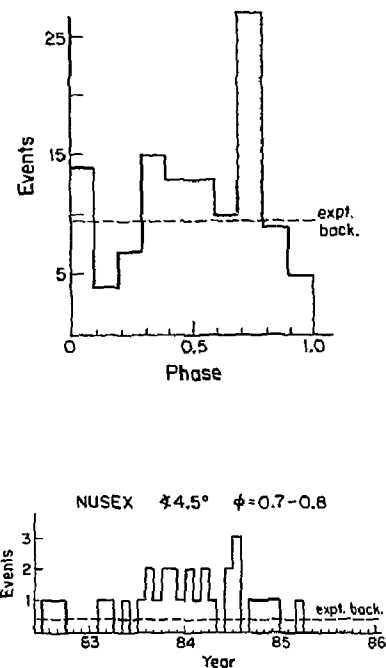


Fig. 20. (a) Muons from the direction of Cygnus X-3 as a function of their time of observation since the start of operation of the Nusex detector. (b) Phase plot of the events during the active period 1983-84, see (a). On-source data are in a cone of radius 4.5° and phase $\phi = 0.7-0.8$.

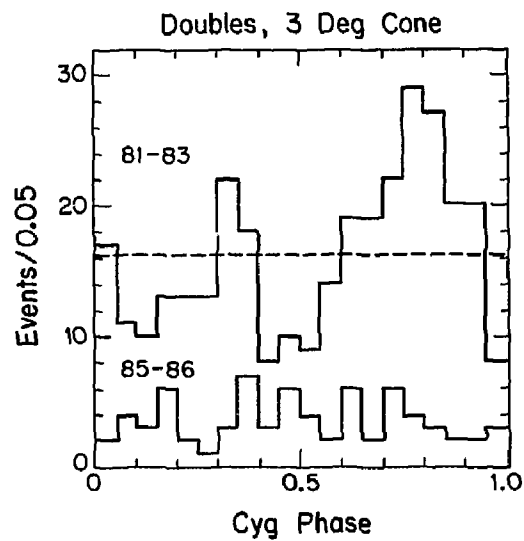


Fig. 21. Confirmation by the Soudan experiment that emission observed during 1981-83 period is not present during 1985-86. Shown is the phase of dimuons (two muons in a single 4.8 hour period) in a 3° circle surrounding the source.

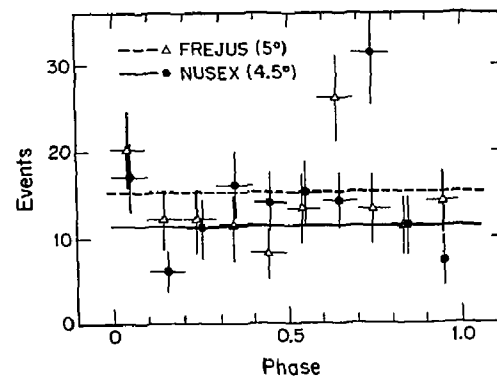


Fig. 22. Comparison of Frejus and Nusex data preceding the "quiet period" which began in 1978.

with any amount of source-off data is of course fatal.

This would all sound like a contrived excuse except for the fact that in October 1985 radio astronomers observed the largest radio burst ever.³⁵ The flux increased by up to a factor 300 in a flare lasting for a few weeks; see Fig. 23. Haleakala observed its clearest signal ever⁵ on October 12 and Baksan³⁰ witnessed a 40% increase of 100 TeV air showers on October 14. Both observations are in the vicinity of the peak of the radio flare; see Fig. 23. The case for a link between the emission in radio and TeV γ -rays has become stronger and constitutes a very interesting result. Bursts in the cosmic accelerator seem to propagate through the large cocoon of matter surrounding it to somehow trigger the radio flare. Other scenarios are conceivable. What about the muons? As shown in Fig. 24, the Soudan detector observed²⁸ $O(10)$ muons in the narrow phase bin $0.725 < \phi < 0.75$ when they expected $O(1)$. Unlike previous data the statistical significance of this is easy to evaluate as no fancy analysis over long times is involved. One simply evaluates from the data what the chance is that a "fluctuation" as shown in Fig. 24 happens in a random direction in the sky. The answer is 10^{-2} . This is rather large and the number of events small. Nevertheless, the fact that this observation actually coincides with the (early) radio flare is of course also important. In fact two of the events are in 4.8 hour cycles adjacent, but not coincident, with the October 12 Haleakala 100 sec burst. On that day the detectors were not simultaneously sensitive to the source. This discussion illustrates what we are desperately looking for: coincidence signals. *E.g.*, Utah's Fly's Eye and the Los Alamos array, which has muon capability, are only 6° apart and have therefore virtually the same field of view at the same time.

Whether other underground detectors should have observed this burst is not easily answered. However, the ten events shown in Fig. 24 correspond to one or at best a few events in a deeper detector such as Nusex for Frejus where the flux is reduced; see Fig. 12.

8. THE SOURCE 1E2259 + 586

In a systematic search for muon emission from other sources, Soudan recently reported²⁹ evidence for the X-ray binary 1E2259 + 586. Evidence was found for both the pulsar period $p_s = 6.9786271 \pm 0.0000008$ sec and the orbital period $p = 2310.40 \pm 0.09$ sec; see Fig. 25. These values are consistent with earlier X-ray observations. The experiment also finds values for the time variation of both periods $|dp_s/dt| = (-5 \pm 3)10^{-14}$ sec/sec and $dp/dt =$

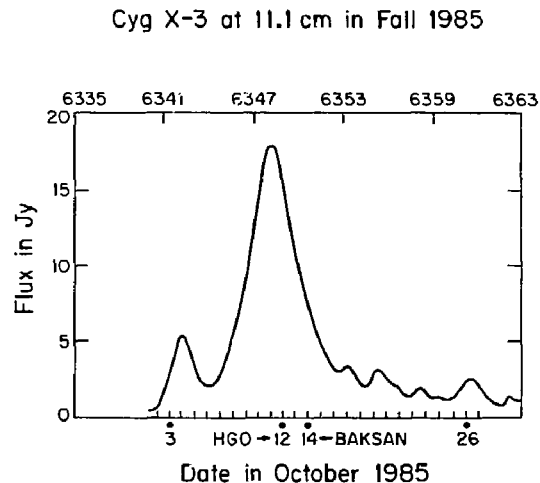


Fig. 23. Structure of the radioemission of Cygnus X-3 during the October 1985 burst. The Haleakala (HGO) and Baksan experiments observed increased emission on October 12, 14 of γ -rays with energies of approximately 1 and 100 TeV, respectively.

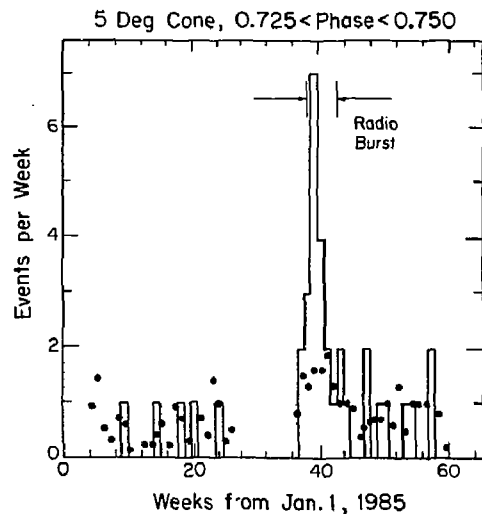


Fig. 24. Excess of muons from Cygnus X-3 in the narrow phase bin $\phi = 0.725 \sim 0.75$ during a period in October 1985 coincident with the largest radio burst observed since its discovery.

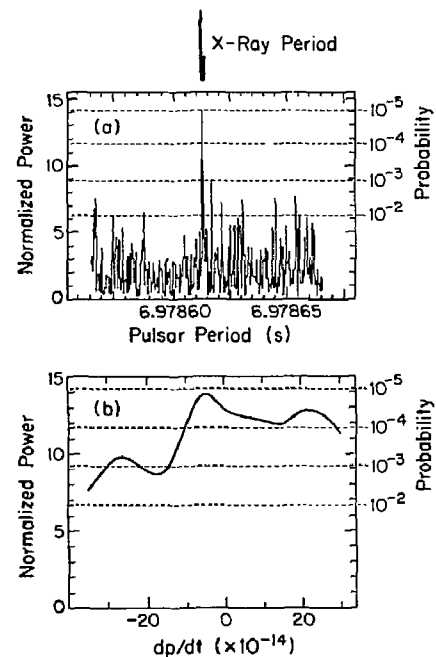


Fig. 25. (a) Power of Soudan muons from the X-ray binary 1E2259 + 586 in the fundamental and first harmonic as a function of the pulsar period p_s . (b) Same power as a function of the time derivative dp_s/dt . The arrow indicates the predicted X-ray period for $dp_s/dt \approx -5 \times 10^{-14}$ sec/sec.

$(-1.2 \pm 0.4) \cdot 10^{-8} \text{ sec/sec}$] and measures a phase distribution indicating emission at periastron, i.e. closest approach in the binary elliptic orbit of eccentricity $e = 0.55$. The most crucial implication of this observation is that all the latter facts are new and can be checked by any X-ray or γ -ray experiment, not just by another muon observation. This might not be as straightforward as it sounds as the source seems to have a reduced flux compared to Cygnus.

One can repeat the exercise of Eqs. (6.1)-(6.4) leading to a constraint on the particle mass carrying the radiation of

$$M \lesssim 2 \text{ MeV} \quad (8.1)$$

as it now keeps time to a precision of 7 sec over a distance of 3.6 kpc.

Independent of whether it will represent the smoking gun of underground muons, this source is interesting in its own right.⁴⁰ Its pulsar period is very long compared to its orbital period making observation possible without struggling with Doppler corrections. The X-ray observations constrain the masses by

$$\frac{(m \sin i)^3}{(m + m_*)^2} = 8 \cdot 10^{-3}. \quad (8.2)$$

The pulsar mass is presumably $m_* = 1.4$, therefore the companion's mass is $m = 0.28$ or more depending on the inclination $\sin i$ of the plane of motion. We are again using solar units, where m is presumably not much larger as optical observations indicate a low magnitude. The gravitational emission by these two masses m, m_* , separated by a distance $O(\text{light seconds})$, as calculated from

$$R = \left(\frac{GM}{\omega^2} \right)^{1/3} \approx 3 \times 10^8 m, \quad (8.3)$$

is impressive

$$\frac{dE}{dt} = \frac{32}{5} G^4 \frac{\mu^2 M^3}{c^5 R^5} \frac{(1 + \frac{73}{32} e^2 + \frac{37}{96} e^4)}{(1 - e^2)^{7/2}} \approx 10^{34} \frac{\text{ergs}}{\text{sec}}. \quad (8.4)$$

Here G is the Cavendish constant, $\omega = 2\pi/p$ with $p = 2300 \text{ sec}$, μ and M the reduced and total mass of m, m_* and $e = 0.55$ is the eccentricity of the binary orbit. The gravitational power could be a factor 10 larger than the estimate (8.4) if the mass m was larger than the lower limit implied by (8.2) and used

in this estimate. The binary spins up due to the emission of this radiation with $dp/dt \approx 3 \cdot 10^{-12} \text{ sec/sec}$ as estimated from

$$\frac{dp/dt}{p} \approx \frac{3}{2} \frac{dE/dt}{E} \quad (8.5)$$

with $E = GM^2/R$. The local flux from the source which is at a distance of 3.6 kpc from earth is roughly $10^{-10} \text{ ergs m}^{-2} \text{ sec}^{-1}$. The change in metric $dg_{\mu\nu}$ associated with this flux exceeds 10^{-21} !

The calculated $dp/dt \approx 10^{-12} \sim 10^{-11} \text{ sec/sec}$ is shorter than observed by Soudan reminding us that this source is likely driven by accretion as is Cygnus X-3. As the companion is less heavy the pulsar enters the Roche lobe probably only when it comes closest to its companion. This makes the observation of emission at periastron plausible.

9. CONCLUSIONS

- TeV astronomy has become reality. This field has already produced some important results:

- (i) the identification of cosmic accelerators as a (maybe "the") source of very high energy cosmic rays
- (ii) Cygnus X-3's pulsar and
- (iii) a correlation between radio and very high energy emission.

- Underground muons are a challenge to the standard model and its extensions (such as supersymmetry or compositeness). Confirmation of the data should be the highest priority. Prediction of new properties of 1E2259 + 586 could be their Achilles heel.

- The most important result: non-accelerator experiments have a 10^5 TeV accelerator.

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Recent Axion Searches⁺

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I. Motivation

The name "axion" means many things to many people, including housewives. For our purpose here we define an axion to be a "pseudo-Nambu-Goldstone boson". In particular such an axion has the following properties:

1. It is spinless, usually pseudoscalar.
2. It couples to the divergence of an almost-conserved current.
3. It is a "collective mode" associated with some form of *spontaneous symmetry break-down*.
4. It is not too massive. In the limit of exact conservation of its associated current it would be massless.

There are three kinds of axions:

1. The pion. It even exists! The almost conserved current is the axial weak $\Delta S=0$ current.
2. The original Peccei-Quinn-Weinberg-Wilczek (PQWW) axion^{1, 2} invented to evade the strong CP problem. It does not exist. It has been decisively ruled out experimentally.
3. Other axions. These will always exist, if only in the imaginations of theorists.

⁺ Talk given at the Fourteenth Stanford Linear Accelerator Center Summer Institute on Particle Physics, August 8, 1985.

The leading candidate is the "invisible" axion, inaccessible to accelerator physics but perhaps accessible by other means. But beyond this favored candidate, there are many other axions of the third kind possible. Some of these have been stimulated by experiments, such as the sighting of the $\zeta(8.3)$ or $\zeta(2.2)$. Recent axion models³ have been stimulated by the curious results⁴ from low energy heavy ion collisions at GSI (Darmstadt). Electron-positron pairs have been observed in collisions of high Z ions at an energy corresponding to the nuclei just about touching each other at closest approach. The total mass of e^+ and e^- seems to sum to 1.8 MeV, although only the laboratory energies of e^+ and e^- and not angles have been measured.

The interpretation of the GSI phenomenon as production of a pointlike particle which decays to e^+e^- is fraught with difficulty. However it has served to emphasize that in the parameter-space of lifetime and mass there existed an open window at low e^+e^- mass ($\geq$ a few MeV), provided the lifetime were short ($\sim 10^{-13}$ sec). This window has been known⁵ for some time to exist, but only if the axion were essentially decoupled from quarks.

In general, I believe it is most important to search for axions in ways which are as unbiased as possible from the fashions of the day. This is not to say that the theoretical leads (e.g. invisible axion) should not be vigorously pursued. That is necessary to do but not sufficient. And it is not easy to define the ground rules for a general search for "generic" axions. For better or worse, I have made the following try:

- i) The current J_μ associated with the axion is neutral and self conjugate. (If it is not, another can be formed from $i[Q^+, J_\mu]$, where $Q = \int J_0 dx$ is necessarily an almost-conserved charge. This channel is then a candidate axion-channel, although it does not have to be one.)
- ii) J_μ couples, if at all, universally to quarks.
- iii) J_μ couples, if at all, universally to leptons.

- iv) The axion couples to $\gamma\gamma$, if at all, via the triangle anomaly.
- v) The axion coupling to fermions is of the Goldberger-Treiman form:

$$g_{xff} = \frac{m_f}{F_x} \quad (1)$$

where F_x is the axion decay constant.

By this definition, generic axions compatible with the GSI hypothesis couple to leptons but not quarks. Their coupling to $\gamma\gamma$ is optional.

By focussing on these generic self-conjugate axions, we do not mean to imply other axions are not interesting. Examples are majorons,⁶ associated with total lepton-number nonconservation, and familons,⁷ associated with leptonic flavor nonconservation.

However, in what follows we focus on the object of search suggested by the GSI stimulus. Its properties include the following:

- i) It is electrically neutral.
- ii) It is short-lived ($\tau < 10^{-11}$ sec.).
- iii) Its mass is greater than 200 keV. Otherwise red giants would cool too rapidly via axion emission.
- iv) It decays mainly to e^+e^- , although a non-dominant decay to $\gamma\gamma$ will be considered admissible as well.
- v) It penetrates a reasonable amount of matter.
- vi) It is pointlike at least at the atomic scale.

Important constraints on its properties come from earlier beam dump experiments^{8,9} at SLAC (experiments E56 and E137, especially the former). These limit the lifetime from above. A limit on lifetime from below comes¹⁰ from the agreement of g-2 of the electron with theory; the contribution of the diagram in Fig. 1 cannot be too large. The

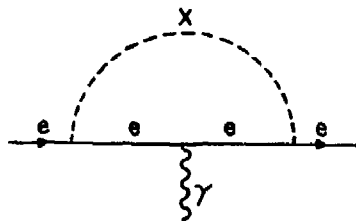


Figure 1 Axion contribution to the electron's gyromagnetic ratio.

constraints from these old measurements are shown in Fig. 2. How the beam-dump constraint comes about will be described in the next section.

II. The Basic Experimental Method

The beam-dump method is extremely simple (Fig. 3). A beam of electrons or hadrons is incident upon a dense target which absorbs the incident energy. Inside even a hadronic shower will be many pure electromagnetic cascades. We focus on them and consider axion production by electron bremsstrahlung (Fig. 4a). If there is a $\gamma\gamma$ coupling, we may also include the Primakoff process (Fig. 4b). The point is that only the mass and widths ($X \rightarrow e^+e^-$ and $X \rightarrow \gamma\gamma$) are needed to compute¹¹ these cross sections; the predicted rates are model independent. In rough order of magnitude the bremsstrahlung cross sections are roughly

$$\sigma \sim \frac{Z^2 a^2}{m_X^2} \frac{m_e^2}{F_X^2} (\log s) \quad \text{Bremsstrahlung} \quad (2)$$

$$\sigma \sim Z^2 a \frac{\Gamma(X \rightarrow \gamma\gamma)}{m_X^3} (\log s) \quad \text{Primakoff} \quad (3)$$

The $X \rightarrow e^+e^-$ width is

$$\Gamma \sim \left(\frac{m_e}{F_X} \right)^2 m_X \quad (4)$$

In both cases

$$\sigma \sim \frac{\Gamma_X}{m_X^3} \quad (5)$$

To get a rough estimate of sensitivity it is not necessary to know the above cross sections much better than presented above. Write

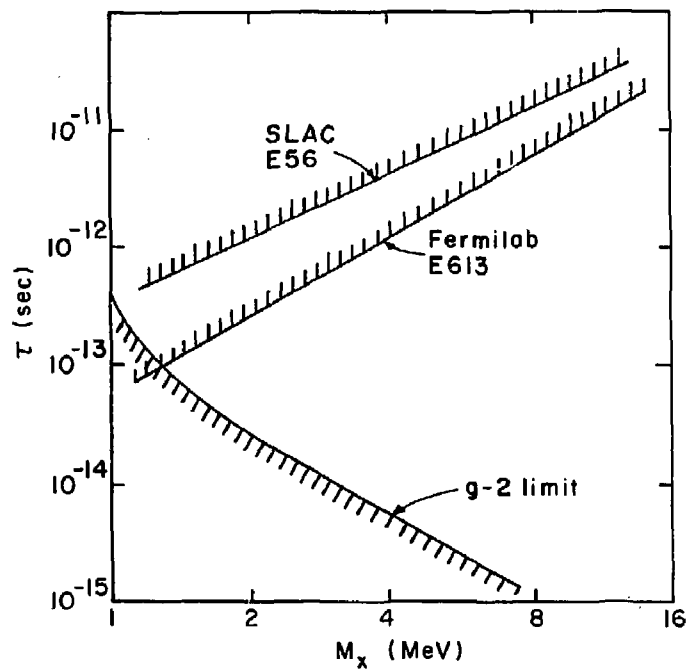


Figure 2 Constraints on m_x and τ_x from previous experiments.

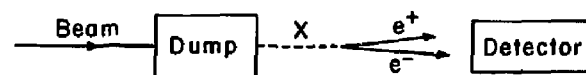


Figure 3 Schematic of a beam dump axion experiment.

For a given beam energy and geometry, the bound will, to fair approximation, be on τ/m .

III. Some Recent Experiments

Several dump experiments have been recently carried out and/or analyzed in the last few months. These have been reported¹² at the Berkeley conference and elsewhere in detail, and I will give only the briefest description here. The experiments I mention will include one using incident protons, and several using incident electrons. The limits are all displayed in the exclusion plot of Fig. 5; we now discuss them in turn. We also mention a proposal for a new experiment.

A. Fermilab Experiment E605

The primary goals of this experiment¹³ were production of high mass dihadrons and dileptons using a very large magnetic spectrometer. The (high intensity) primary beam was, however, dumped just downstream of the target within the spectrometer magnet. Beyond a 7m long decay region there existed tracking and an electron/hadron calorimeter. (Fig. 6) The copper dump was 5.5m long; the top energy of the electron/photon shower particles available for observable axion production was in excess of 400 GeV. About 4×10^{13} protons were dumped. The trigger was at least 150 GeV in the electromagnetic calorimeter. Most candidate events (~ 75) were e^+e^- pairs from muon-bremsstrahlung conversion photons and had the wrong angle. Only one candidate axion survived cuts and it was consistent with background estimates.

Within a factor 2, the exclusion limit quoted by the authors (Fig. 5) is

$$\frac{\tau}{m} = 0.3 \frac{L}{c\langle E \rangle} \sim 10^{-14} \text{ sec/MeV}. \quad (13)$$

This should be compared with Eqn. 12.

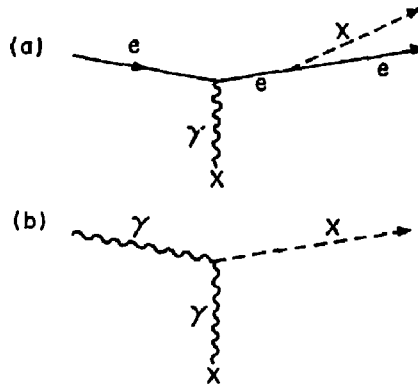


Figure 4 Axion production mechanisms:

- a) electron bremsstrahlung
- b) Primakoff production

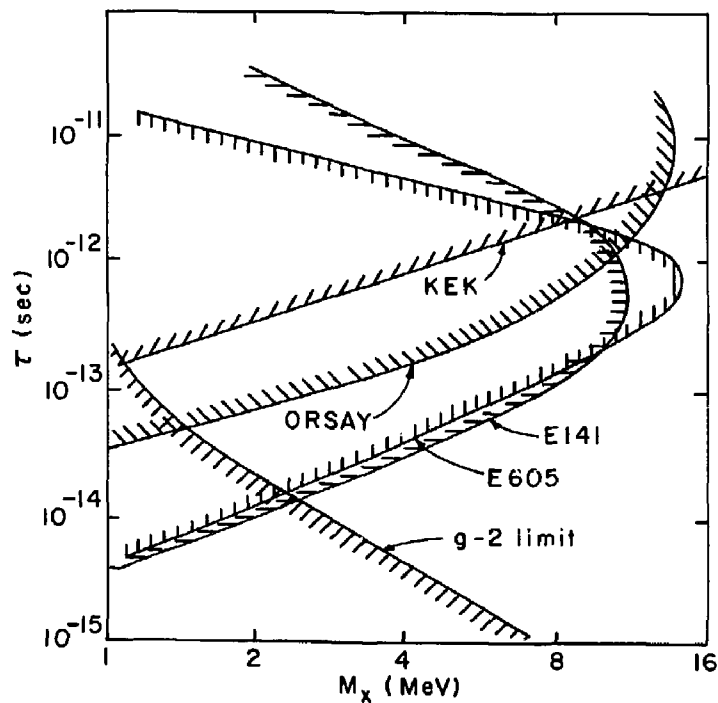


Figure 5 Constraints on m_x and τ_x from new experiments.

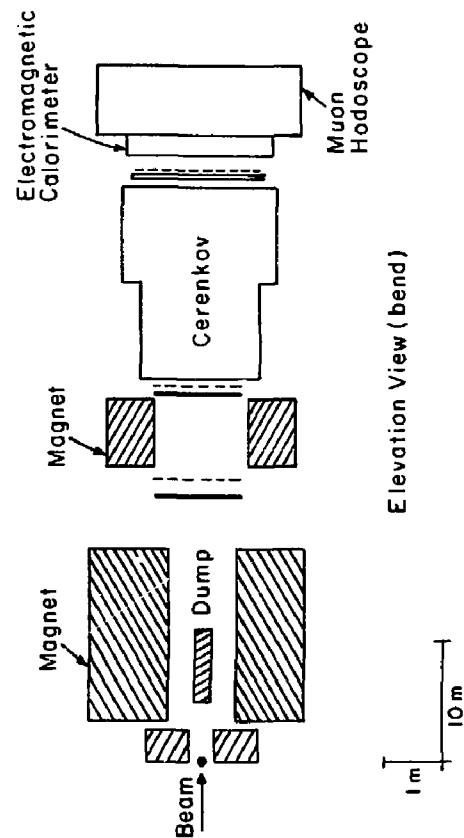


Figure 6 Layout of Fermilab E605.

B. SLAC Experiment E141

In this experiment¹⁴ the SLAC linac beam was dumped on a short tungsten target of 10 or 12 cm length. The dump was located in the beam line upstream of End Station A, wherein reside the spectrometer magnets used for electron scattering experiments. One of these, the 8 GeV spectrometer, was moved to zero degrees, where it provided measurement of the momentum spectrum of zero-degree positrons. The background appears to be predominantly from pairs produced by punch-through photons converting near the back surface of the tungsten dump. A spectrum taken with approximately 10^{15} incident electrons of 9 GeV is shown in Fig. 7, along with appropriate spectra expected from axion production. The background is not yet fully analyzed, nor are spectra taken with 18 GeV incident electrons. The exclusion limit resulting from these preliminary data is again shown on Fig. 5 and is roughly fitted by the formula

$$\frac{\tau}{m} \leq 0.25 \frac{L}{c\langle E \rangle} \approx 10^{-14} \frac{\text{sec}}{\text{MeV}} \quad (14)$$

C. Orsay Experiment

This elegant experiment,¹⁵ reported by M. Davier at the Berkeley conference, uses electrons on a split absorber of fixed number of radiation lengths (cf Fig. 8), with the gap distance variable. The total dump length varied from 11.5 to 17.5 cm. This eliminates a host of systematic errors. Again positrons were detected downstream of the dump. About 10^{15} electrons of 1.5 GeV were dumped and 1.0 and 1.15 GeV positron secondaries were examined.

The results in Fig. 5 are fit roughly by

$$\frac{\tau}{m} < 0.3 \frac{L}{c\langle E \rangle} \approx 7 \times 10^{-14} \frac{\text{sec}}{\text{MeV}} \quad (15)$$

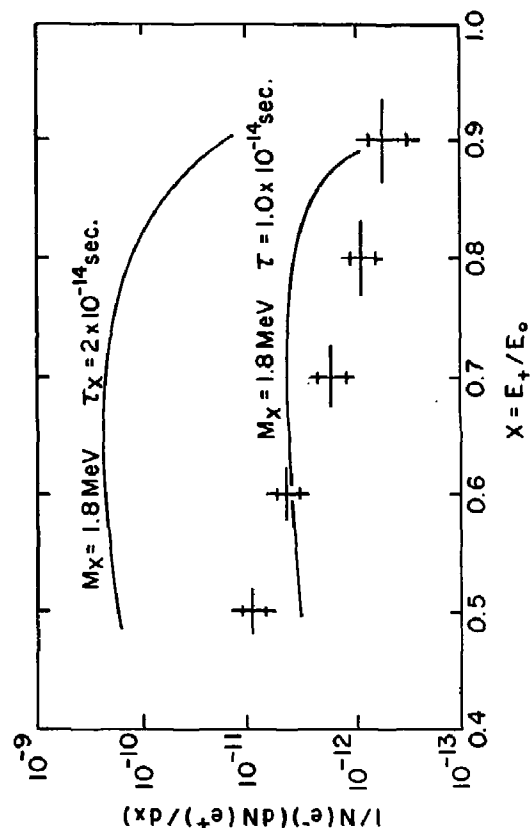


Figure 7 Spectrum of positrons emergent from the E141 beam dump, 9 GeV electrons incident.

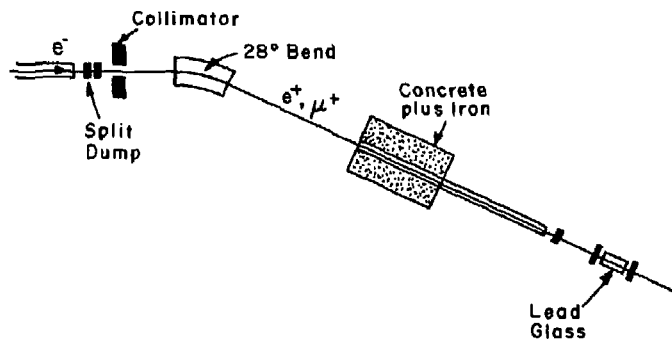


Figure 8 Layout of the Orsay experiment.

D. KEK-Kyoto Experiment

In this experiment¹⁶ 2.5 GeV electrons were incident on a thick iron dump (2.3 m thick). A pair spectrometer (MWPC's, a magnet, and a lead glass hodoscope) searched for axion decays into either e^+e^- or $\gamma\gamma$ in a 2.2m decay volume downstream of the shield. About 1.7×10^{17} electrons were dumped and no events were found. The limit exhibited in Fig. 5 can be roughly fit by the expression

$$\frac{\tau}{\hbar} \lesssim 0.1 \frac{L}{c\langle E \rangle} = 3 \times 10^{-13} \frac{\text{sec}}{\text{MeV}}. \quad (16)$$

E. Fermilab Proposal P774

This proposal exploits the high energy electron beam at Fermilab, which exists as a source of high energy γ -ray beams for photoproduction experiments. As presently envisioned, electrons would be incident on an active dump (beam contamination by pions is a problem). This is followed by a decay region, a small pair spectrometer and finally an electron/hadron calorimeter. The tungsten dump will be ~ 20 cm long. With the 500 GeV electron beam available, this should allow sensitivity to

$$\frac{\tau}{\hbar} > 0.2 \frac{L}{c\langle E \rangle} \approx 2 \times 10^{-16} \frac{\text{sec}}{\text{MeV}}. \quad (17)$$

This would represent an improvement of more than an order of magnitude over the present situation.

This proposal enjoys a first-stage approval, and probably will run in 1988.

IV. Conclusions

The beam dump experiments convincingly set an upper limit of about 10^{-14} sec for the lifetime of a pointlike axion of mass 1.8 MeV coupled predominately to e^+e^- . This, combined with the agreement of measurements of the electron g-2 with theory, excludes the interpretation of the GSI phenomenon in terms of production of a pointlike axion. We emphasize that the steep dependence of experimental yield on lifetime makes the boundary of the exclusion-domain quite sharp.

However, we must add two caveats. M. Samuel¹⁷ has recently questioned the QED calculations of g-2. If his analysis survives, there would even be an axion interpretation of the discrepancy. However, Samuel's claim is at present quite controversial.¹⁸

The second caveat is that the GSI object might still be interpreted as a particle, provided its absorption in matter were large (cross section per nucleon large compared to 10^{-28} cm²), and/or it were an extended object (at the MeV mass scale). For example positronium is not produced copiously by high energy electrons or photons, it does not penetrate a beam dump even at very high energy, and it does not contribute significantly to the electron g-2.

But irrespective of the fate of the GSI phenomenon, I believe the search for generic (and other) axions should proceed at every opportunity. The odds of discovering something are very small. But the ubiquity of spontaneous symmetry breaking in nature, together with the potential sensitivity of any such physics to phenomena beyond the standard model at very high mass scales, makes the potential rewards very great.

V. Acknowledgements

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Superstring Spectroscopy

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Introduction

For the past two years, a large part of the theoretical physics community has been locked in single-minded concentration on one highly speculative approach to the fundamental structure of matter—the theory of relativistic strings. Proponents of this theory have claimed that it provides the basic laws unifying all known interactions and that it promises the solution to some of the deepest remaining questions about Nature, including the origin of the quark and lepton generations.

Such a fundamental description of Nature seems even more wonderful because it is built up of elementary entities of a simple and concrete structure. These basic entities, the elementary strings, can in fact be readily visualized by any physicist who has developed a proper quantum-mechanical intuition. The theory of superstrings has acquired a reputation as being mathematically abstruse and formidable in the extreme, but, while it is certainly true that some unfamiliar mathematical technology is needed to perform calculations in this theory, the basic elements of the theory are remarkably accessible. The purpose of this lecture is to set out these basic elements in terms which are as pictorial as possible. (Students who wish to study this subject in a serious way should consult one of the excellent technical reviews now available.^[1-5])

Before beginning this explication, however, it is worth reviewing the main properties of string theories and, especially, of the supersymmetric version of the string theory which shows the most promise of making contact with the phenomena of elementary particle physics. This version of the theory was originally formulated in 1970 by Neveu and Schwarz,^[6] Ramond,^[7] and Thorn.^[8] It underwent a second stage of development in the early 1980's, when Green and Schwarz^[9] clarified many of its properties and pressed its interpretation as a unifying theory for all interactions.

The main properties of this superstring theory which bolster its interpretation as a fundamental theory of Nature are the following:

1. The theory requires that all particles—quarks, leptons, gauge bosons, gravitons, and their supersymmetric partners—are built of the same fundamental entities, the elementary strings. In this sense, string theories are the most elegant of all models of elementary particle substructure. One of my main tasks in this lecture will be to explain the origin of the various quantum numbers that these particles carry.
2. The theory requires that space-time be fundamentally supersymmetric. It also requires the existence of 10 space-time dimensions. This would be an excessive number if all of these dimensions were extended to the size of the 4 dimensions that are part of our everyday experience. However, the extra 6 dimensions may play a more subtle role, which the last two sections of this lecture should make clear. The most probable size for the compactified dimensions is the characteristic length of an elementary string; this is of order the Planck length, 10^{-33} cm or $(10^{19} \text{ GeV})^{-1}$.
3. The theory naturally contains as a part of its structure the gauge invariances of Yang-Mills theory and gravity. In fact, these invariances are realized as a small part of an enormous group of generalized gauge symmetries. This enormous gauge structure was first made clear in the work of Siegel.^[10]
4. Although the theory contains within it a quantum theory of gravity, it is apparently free of ultraviolet divergences. The finiteness of the theory has been shown explicitly to 1-loop order^[11,12] and a plausible intuitive argument has been given which extends this result to all orders.^[13]
5. The theory restricts the possible choices for its Yang-Mills symmetry group to only two candidates: $O(32)$ and $E_8 \times E_8$, the latter involving the largest of the exceptional groups.^[14] In fact, these two symmetry groups arise *geometrically* as solutions for the space-time structures allowed by the theory.^[15] I will explain this point in some detail at an appropriate stage of my development.

This list of properties, some of which arise rather magically from the proper-

ties of the underlying strings, should be enough to attract anyone with a speculative bent. Still, it is worth noting that string theories also occupy a privileged position within theoretical physics, as the unification point for many strands of theoretical investigation which have been actively pursued in the past decade. First of all, strings are a natural generalization of point particles, since they are objects extended in a spatial direction as well as along a world-line. From this point of view, they have long been of interest to workers in the foundations of relativistic field theories. From the list given above of the properties of string theories, it should be clear that these theories provide a natural meeting ground for workers interested in Yang-Mills fields and grand unification, supersymmetry, models of quark and lepton substructure, and gravitation. In addition, string theories have provided quite nontrivial applications for more mathematical aspects of theoretical physics—the study of 2-dimensional model field theories, and the application of higher geometry and topology to field-theoretic problems. In a certain sense, it now seems that most of the developments in theoretical physics over the past ten years were really directed toward the solution of string theory. Small wonder, then, that this theory excites so much interest in so many quarters.

I would like to conclude this brief survey of the prospects for string theory by citing the major problems which must still be solved in order to bring this theory from the level of speculation to a point where it can make concrete predictions for experiment. The most pressing problems are those which concern the conversion of the 10-dimensional space-time of string theory into a form closer to experimental reality in which 6 of these 10 dimensions are curled up to a very small size. The geometry of this compactification of dimensions determines all of the detailed properties of the system of elementary particles which would be visible at energies accessible to experiment: the number of quark and lepton generations, the gauge group which results from breaking the grand unification symmetry, the values of the strong- and weak-interaction coupling constants, and the existence and number of supersymmetric partners. The most basic aspects of how the geometry of the compact 6 dimensions determines these parameters

have been clarified by Candelas, Horowitz, Strominger, and Witten,^[16] among others. (An elementary discussion of the physics of compactification may be found in my lectures at the 1985 SLAC Summer Institute.^[17]) However, many issues, especially the mechanism of supersymmetry breaking and the relation of the weak-interaction scale to the fundamental string length scale, remain obscure. In addition, we still have no idea how Nature chooses a particular geometry for the compact 6 dimensions from among a wealth of possibilities.

In addition to these questions of quite direct physical importance, there are a number of absolutely fundamental formal questions about the superstring theory which have not yet been settled. We still do not know the complete equations of motion for the theory (though considerable progress has been made in the past year in understanding the more elementary, nonsupersymmetric case^[18,19]). We still do not have a complete set of rules for computing the perturbation theory in string interactions (though, again, there has recently been some dramatic progress in this direction^[20–22]). Finally, we have almost no idea of how to discuss string dynamics beyond perturbation theory. This last formal problem is a particularly important one, because it is known that many of the aspects of compactification that seem to us the most mysterious—which compact space is chosen, for example—are simply not determined at the level of the first perturbative loop corrections; quite plausibly, these questions can only be settled by looking beyond perturbation theory.^[23,24] It is not an uncommon occurrence in physics that the most crucial phenomenological properties of a theory arise non-perturbatively; the appearance of solids in QED and the appearance of hadrons in QCD provide two examples. In both of those cases, the connection of the phenomena to the theory was forged by quite remarkable guesses about the correct treatment of the theory in the regime of strong coupling, guesses which were motivated crucially by the findings of experiment. If string theory is to be made a predictive theory which brings new and relevant information to the study of quarks and leptons and their interactions, this will only be done through a similarly remarkable conjecture about how strings determine the form of space-time.

Let us hope that our imaginations are worthy of this challenge.

The remainder of this lecture will concern the aspects of string theory which are well-understood, and which are most easy to visualize. I will develop the theory, to the level where it produces an observable spectrum of quarks and leptons, in eight easy lessons.

Lesson 1: The Basic Quantized String

A relativistic string is an idealized 1-dimensional extended object, an object whose sole property is that it lies along some curve in space. It is, then, the natural generalization of an idealized point particle. Just as the point particle can be viewed as sweeping out a world-line as it progresses through time, the string sweeps out a 2-dimensional space-time surface, a world-sheet. A string may or may not have endpoints; one refers to a string with or without endpoints as being open or closed (see Fig. 1).



Figure 1. Typical configurations of open and closed strings.

Such an idealized object must have an extremely simple equation of motion. This equation must be relativistically invariant; it must also be local along the string, or, in a space-time view, across the world-sheet. There are two natural candidates for such an equation; these are illustrated in Fig. 2.

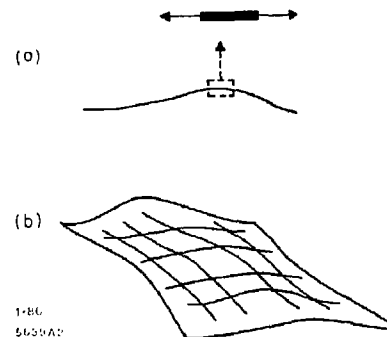


Figure 2. Rest-frame and space-time viewpoints on the equation of motion for a relativistic string.

The first candidate is best formulated in the rest frame of an infinitesimal bit of string. This string-bit has a rest energy per unit length T_0 . Stretching the string by dx would create more string, also at rest; this would cost energy $T_0 dx$. Thus, T_0 is also the (rest) tension in the string. We can then compute the net force exerted on each bit of string and, from this, deduce its motion. The second candidate is best formulated in a space-time approach. We know that the motion of a point particle in space-time is given by the geodesic principle that it should follow the shortest path between two points. The generalization of this statement to a string is that the string should sweep out a world-sheet of minimum area. Remarkably, these two formulations of the string equations of motion are consistent [25,26]. Together they give quite a clear picture of the classical mechanics of string.

To discuss a quantum-mechanical relativistic string, we should quantize the

string's vibrational modes. Let us discuss this procedure first for the open string, moving in d space-time dimensions. Let σ be a coordinate along the string, running from $\sigma = 0$ at one end to $\sigma = 1$ at the other, and let $X^i(\sigma)$ be transverse displacement of the string as a function of σ ($i = 1, \dots, d_\perp$; $d_\perp = (d - 2)$). This function obeys the boundary condition $(\partial/\partial\sigma)X^i(\sigma) = 0$ at $\sigma = 0, 1$, that is, that there should be no unbalanced transverse tension acting on the endpoint. Then we may expand $X^i(\sigma)$ in a Fourier series as follows:

$$X^i(\sigma) = x^i + \sum_{n=1}^{\infty} X_n^i \cos n\pi\sigma. \quad (1)$$

In this expression, x^i is the position of the center of mass of the string. It should be no surprise that each X_n^i turns out to be the coordinate of a harmonic oscillator. Quantizing these oscillators, one finds an expression for the energy eigenvalues of the string in terms of the harmonic oscillator ladder operators a_n^i . These correspond to the energies of relativistic particles with masses

$$m^2 = (2\pi T_0) \cdot \left[\sum_{n=1}^{\infty} n a_n^i a_n^i + (d_\perp \cdot Z) \right]. \quad (2)$$

The last term in (2) represents the zero-point energy of the oscillators. This very simple equation summarizes all of the basic properties of the string. It is relativistic, since it is an equation which gives the rest mass of the string in terms of its internal structure. It is harmonic, since the internal states which appear are those of a set of simple oscillators. And, finally, it is geometrical in assigning energy only to transverse fluctuations of the string. Displacements of the string along the string itself are not physically observable and therefore should not affect the string energy levels.

The quantization of a closed string proceeds in a similar fashion. In this case, the only boundary condition to be satisfied is that of periodicity. It is convenient to Fourier-analyze the string displacement in terms of running waves which move

to the left and to the right around the loop. Introducing a time coordinate τ on the world sheet,

$$X^i(\sigma) \approx x^i + \sum_{n=1}^{\infty} \left(X_n^i e^{2\pi i n(\sigma + \tau)} + \bar{X}_n^i e^{2\pi i n(\sigma - \tau)} + \text{c.c.} \right). \quad (3)$$

The left- and right-moving excitations form independent sets of harmonic oscillators. Denoting the ladder operators for the left- and right-moving oscillations by a_n^i , $\bar{a}_n^i$, respectively, we may write the mass formula for this case as:

$$m^2 = (4\pi T_0) \cdot \left[\sum_{n=1}^{\infty} n (a_n^i a_n^i + \bar{a}_n^i \bar{a}_n^i) + (2 \cdot d_\perp \cdot Z) \right]. \quad (4)$$

Again, this formula reflects the relativistic, quantum, harmonic, and geometrical aspects of the string.

Lesson 2: Zero-Point Energy

The last term in each of the formulae for the mass of a quantized string is the total zero-point energy of the harmonic oscillators. Normally in field theory we can simply throw away the zero-point energy, since it is not physically observable. In the analysis of the string, however, we have taken what is effectively a 2-dimensional field theory of transverse motions of the world-sheet and interpreted the eigenvalues of the Hamiltonian of this theory as the masses of particles. The zero-point energy of the field theory certainly contributes to these eigenvalues. Unfortunately, the zero-point energy of a field theory is usually infinite. We must regulate this infinity and try to make some sense of it.

Assigning a zero-point energy of $\frac{1}{2}\omega$ to each harmonic oscillator, we can write the total zero-point energy of the open string as

$$Z = \frac{1}{2} \sum_1^{\infty} n. \quad (5)$$

To define this infinite sum, add to the definition of Z a cutoff ϵ :

$$\begin{aligned} Z &= \frac{1}{2} \sum_1^{\infty} n \epsilon^{-n\epsilon} \\ &= \frac{1}{2} \left(\frac{\partial}{\partial \epsilon} \right) \sum_1^{\infty} \epsilon^{-n\epsilon} = \frac{1}{2} \left(\frac{\partial}{\partial \epsilon} \right) \left[\frac{1}{1 - e^{-\epsilon}} \right] \\ &= \frac{1}{2} \left(\frac{\partial}{\partial \epsilon} \right) \left[\frac{1}{\epsilon} - \frac{1}{2} + \frac{1}{12}\epsilon - \frac{1}{90}\epsilon^2 + \dots \right] \\ &= \frac{1}{2} \left[\frac{1}{\epsilon^2} - \frac{1}{12} + \dots \right]. \end{aligned} \quad (6)$$

The first term in this series is highly divergent as $\epsilon \rightarrow 0$. I propose that we ignore it. The next term gives a finite, cutoff-independent residual contribution:

$$Z = \frac{1}{2} \sum_1^{\infty} n \cong -\frac{1}{24}. \quad (7)$$

Brink and Nielsen^[27] have argued that the term we have omitted may be interpreted as a renormalization of the speed of light in this noncovariant calculation. In any event, there are many cross-checks which insist that the theory of strings which we are constructing can be Lorentz-invariant and self-consistent only if we define the zero-point energy by the regulated expression (7).

For our future reference, it will be useful to perform another divergent sum:

$$Z(\alpha) = \frac{1}{2} \sum_{n=0}^{\infty} (n + \alpha). \quad (8)$$

This quantity will arise as the zero-point energy of a string which runs around a region containing space-time curvature or magnetic field, as shown in Fig. 3.

In that case, the boundary condition of periodicity may become slightly more complicated: Write $X(\sigma) = X^1(\sigma) + iX^2(\sigma)$. Then the magnetic field or curvature

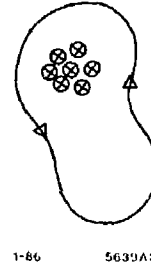


Figure 3. A closed string running around a region containing magnetic flux.

$m\pi$ be reflected in a phase factor:

$$X(\sigma = 1) = e^{2\pi i \alpha} X(\sigma = 0). \quad (9)$$

When we Fourier expand $X(\sigma)$, we must use functions which obey this new boundary condition; for example, we must replace in (3)

$$e^{2\pi i n(\sigma + \tau)} \rightarrow e^{2\pi i (n + \alpha)(\sigma + \tau)}. \quad (10)$$

Then all of the oscillator frequencies will be shifted by α , and the zero-point energy will be given by (8).

To evaluate $Z(\alpha)$, we note that it obeys the functional equation

$$Z(\alpha) = \frac{1}{2}\alpha + Z(\alpha + 1). \quad (11)$$

Substituting for $Z(\alpha)$ an arbitrary polynomial of α , we find that (11) can only

be satisfied for

$$Z(\alpha) = \frac{1}{4}\alpha(1-\alpha) + C. \quad (12)$$

We can determine the constant by noting that $Z(0) = Z(1) = Z$. Thus

$$Z(\alpha) = -\frac{1}{24} + \frac{1}{4}\alpha(1-\alpha). \quad (13)$$

The physical importance of the zero-point energy will be made clear by the central role that $Z(\alpha)$ plays in our later discussion.

Lesson 3: The Bosonic String

Now that we have clarified all of the terms in (2) and (4), we should display the spectrum of possible string states that these equations predict. Let us begin with the case of the open string:

$$m^2 = (2\pi T_0) \cdot \left[\sum_{n=1}^{\infty} n \alpha_n^\dagger \alpha_n - \frac{d-1}{24} \right]. \quad (14)$$

The ground state of the string is the state $|0\rangle$ annihilated by all of the $\alpha_n^\dagger$. Most regrettably, this state has $m^2 < 0$. The first excited state is almost as problematic:

$$\alpha_1^\dagger |0\rangle, \quad m^2 = (2\pi T_0) \cdot \left(1 - \frac{d-1}{24} \right). \quad (15)$$

The $d-1$ vector components are obviously trying to form a vector particle. However, this vector has only $d-1$ polarization states; the longitudinal polarization state is missing. This contradicts Lorentz invariance unless the vector particle is massless. Thus, we find that the open string theory we have constructed can be Lorentz-invariant only if $d-1 = 24$, that is, if $d = 26$. As an integral part of this construction, we find a massless vector field. It can be seen that, once we have

set $d = 26$, the Lorentz group automatically acts properly on all higher mass levels. For example, the next level contains states

$$\alpha_1^\dagger \alpha_1^\dagger |0\rangle, \quad \alpha_2^\dagger |0\rangle; \quad m^2 = (2\pi T_0) \quad (16)$$

of exactly the right number to form a 25×25 traceless symmetric tensor; this accounts for all of the components of a massive tensor field in 26 dimensions.

This construction works in a similar way for the closed string. In 26 dimensions, the mass formula is

$$m^2 = (4\pi T_0) \cdot \left[\sum_{n=1}^{\infty} n (\alpha_n^\dagger \alpha_n + \bar{\alpha}_n^\dagger \bar{\alpha}_n) - 2 \right]. \quad (17)$$

The ground state $|0\rangle$, and also the states $\alpha_1^\dagger |0\rangle, \bar{\alpha}_1^\dagger |0\rangle$, are tachyons, with $m^2 < 0$. But now the states

$$\alpha_1^\dagger \bar{\alpha}_1^\dagger |0\rangle \quad (18)$$

appear just at $m^2 = 0$. These states form a transverse symmetric tensor; this state would also be inconsistent with Lorentz invariance if it were not precisely massless.

It is tempting to speculate that the massless vector and tensor states that we have uncovered can be identified with the corresponding states that we see in Nature—gauge bosons and gravitons. This interpretation is surprisingly robust: when one introduces interactions into the string theory in the natural way, one finds that the low-energy scattering amplitudes for these particles agree with the predictions of Yang-Mills theory and general relativity.^[28,29] One often hears theorists mutter that these gauge-invariant equations are the only possible equations for massless vector and tensor fields; still, it is amazing that this observation constrains a system that, at first sight, has nothing to do with local field theory.

I should note that the multiplet (18) actually contains other particles in addition to gravitons, since the states shown form a transverse tensor of arbitrary symmetry. Symmetrizing and antisymmetrizing, we find:

$$\begin{aligned} \text{symmetric, transverse, traceless} &\rightarrow h^{ij} \text{ (the graviton)} \\ \text{antisymmetric, transverse} &\rightarrow b^{ij} \\ \text{trace} &\rightarrow \phi \text{ (a scalar, the dilaton)}. \end{aligned} \quad (19)$$

The antisymmetric tensor particle b^{ij} also appears in the string theory with appropriate gauge-invariant interactions.

Before we leave the subject of the closed string spectrum, I should correct one statement that I made above. In enumerating the low-mass states of the closed string, I listed the states $\alpha_1^{\dagger}|0\rangle$ and $\tilde{\alpha}_1^{\dagger}|0\rangle$ as tachyons. But, in fact, these states do not exist in the interacting string theory. I would like to explain this statement, which reveals some subtle properties of string interactions.

Generalizations of this statement will play a crucial role in later stages of our analysis. The argument for this statement proceeds in three stages, indicated diagrammatically in Fig. 4. In this first stage, we note that the states such as $\alpha_1^{\dagger}|0\rangle$ with a preponderance of left-moving excitations have net momentum P running around the closed string. The ground state, and the states (18), have $P = 0$, so the state $\alpha_1^{\dagger}|0\rangle$ is distinguished from these by a conserved quantum number. We must next ask whether interactions can couple states with $P = 0$ to states with $P \neq 0$. To answer this question, note that $P = 0$ is exactly the criterion for the string state to be invariant under rotations of σ around the loop. Now study the picture of the 3-string interaction shown in Fig. 4(b). The three strings sweep out tubes in space-time. We can make them interact by connecting the tubes in a join; in a geometrically invariant theory, we should allow the join to form in all possible ways. In particular, we must integrate over the angle θ indicated in the figure. But if the string state coming in from the

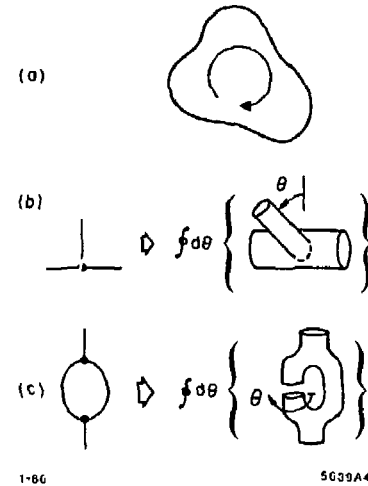


Figure 4. Three stages of the argument that states with net momentum around the string do not exist.

right is a rotationally invariant ($P = 0$) state, and the third state is joined on in a rotationally invariant way, the state going out to the left will rotationally invariant as well. This geometrical interaction, then, couples $P = 0$ states only to other $P = 0$ states. Finally, if states with $P \neq 0$ cannot be produced directly, they still might appear as intermediate states in closed-loop diagrams. This possibility is excluded by viewing string loop diagrams in the manner shown in Fig. 4(c). If we are to include all possible geometries in the join, we must sum over twists θ as indicated in the figure. The integral over θ projects out all states

but those which are rotationally invariant ($P = 0$). We find, then, that states with $P \neq 0$, although apparently present as eigenstates of the mass operator, cannot be produced by string interactions, either as real or as virtual particles. For all practical purposes, then, they do not exist.

This argument greatly simplifies the spectrum of the closed string, leaving only a single tachyon, $|0\rangle$, the massless states given in (18), and the higher-mass states with $P = 0$, such as $\alpha_1^{\dagger} \alpha_1^{\dagger} \alpha_2^{\dagger} |0\rangle$. The Lorentz group in 26 dimensions acts in a consistent way on the higher-mass states, as long as we consider only states with $P = 0$.

Lesson 4: Compactification on a Torus

We have now come roughly one-third of the way toward a theory which could be realistic. The simple, unadorned string discussed in the previous section contains vector bosons and gravitons; however, it includes no fermions. It has no well-defined gauge group. And it has a serious affliction—tachyons in both the open- and closed-string sectors. In the following two sections, I will explain how to remedy these difficulties. First, though, I would like to give a preliminary treatment of another issue which is raised by our results in the previous section: the proper interpretation of the extra spatial dimensions required for the consistency of the theory. Clearly, these extra dimensions must curl up into a compact space. Let us study, in a simple example, a possible consequence of this compactification.

Consider, then, the 26-dimensional closed string theory with one spatial dimension closed into a ring, so that space-time has the appearance of a cylinder of circumference R [Fig. 5].

To analyze the spectrum of string states in this geometry, let us Fourier-analyze $X^{\epsilon}(\sigma)$. This expansion is changed from (3) only in the compactified direction $X^{\epsilon}(\sigma)$, and there only in two simple respects: First, we must restrict



Figure 5. A space-time with one compactified dimension.

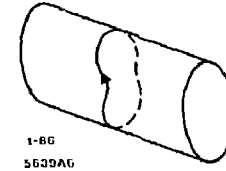


Figure 6. A closed string which winds around a compactified dimension.

the center-of-mass coordinate x^{ϵ} to $0 \leq x^{\epsilon} < R$, and, second, we must allow closed string to wind around the ϵ direction (Fig. 6).

Then

$$X^{\epsilon}(\sigma) = x^{\epsilon} + \ell \cdot R\sigma + \sum_{n=1}^{\infty} (X_n^{\epsilon} e^{2\pi i n(\sigma + \tau)} + \bar{X}_n^{\epsilon} e^{2\pi i n(\sigma - \tau)} + \text{c.c.}) ; \quad (20)$$

the integer ℓ is the number of times that the string winds around the cylinder.

To express the spectrum of string states, we should write the energy of a

string as

$$E^2 = p^2 + m^2 = \bar{p}^2 + p_c^2 + m^2, \quad (21)$$

where $\bar{p}$ is the momentum in uncompactified directions. Since the compactified dimension will be extremely small, the momentum in this direction will be physically relevant only as a part of the string energy. If we do not observe the extra dimension directly, we would say that the string states appear in $(d-1)$ dimensions with mass $\bar{m}$ given by

$$\bar{m}^2 = p_c^2 + m^2. \quad (22)$$

Taking into account the new contribution to energy cost of winding, and the fact that p_c is quantized, we find for this effective mass:

$$\bar{m}^2 = \left(\frac{2\pi k}{R}\right)^2 + T_0(\ell R)^2 + (4\pi T_0) \cdot \left[\sum (n + \bar{n}) - 2\right]. \quad (23)$$

Here k is an integer, and the last term is an abbreviation for Eq. (17).

Let us examine this formula for the particular choice $R = (2\pi/T_0)^{1/2}$. In that case, the mass formula becomes

$$\bar{m}^2 = (2\pi T_0) \cdot (k^2 + \ell^2) + (4\pi T_0) \cdot \left[\sum (n + \bar{n}) - 2\right]. \quad (24)$$

The ground state is still a tachyon: $|0\rangle$, with $k = \ell = 0$ still has $m^2 = (-2) \cdot (4\pi T_0)$. However, we now have a more interesting spectrum of zero-mass states. Let us denote the state composed of the oscillator ground state plus winding quanta k , ℓ by $|k, \ell\rangle$. This state has $P = k \cdot \ell$. We can then enumerate the states with $\bar{m}^2 = 0$ and $P = 0$. These include, of course, the states of (18) with both indices in uncompactified dimensions, plus

$$\begin{array}{ll} \bar{a}_1^\dagger |1, 1\rangle & a_1^\dagger |1, -1\rangle \\ \bar{a}_2^\dagger \bar{a}_1^\dagger |0, 0\rangle & a_2^\dagger \bar{a}_1^\dagger |0, 0\rangle \\ \bar{a}_1^\dagger |-1, -1\rangle & a_1^\dagger |-1, 1\rangle \end{array} \quad (25)$$

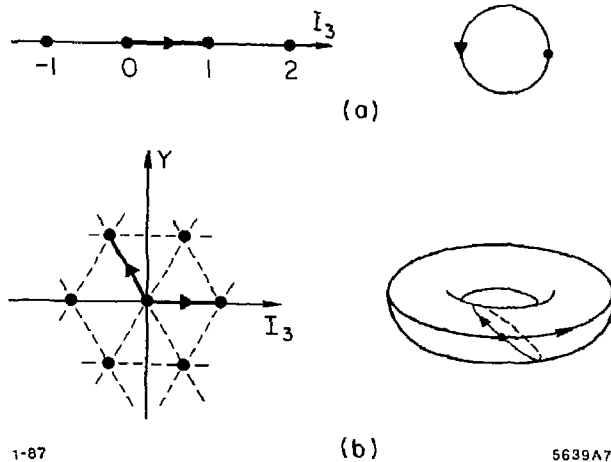
These new states form two triplets of massless vector bosons. It is extremely

tempting to conjecture that these are the gauge bosons of $SU(2) \times SU(2)$, and that the complete string theory has an $SU(2) \times SU(2)$ symmetry group. A detailed analysis shows that this is indeed the case. (For values of R other than this special choice, one finds only one zero-mass state on each side of (25) and thus a lower symmetry, $U(1) \times U(1)$.)

We can describe this phenomenon more generally as follows: Consider compactifying some number n of the extra dimensions into rings. The result is a generalized torus. A set of motions $\bar{\ell}$ carry us around the compact manifold and back to the same point. These motions form a lattice in n -dimensional space. We can, in fact, view the torus as being the full n -dimensional space, but with points related by lattice translations identified. This correspondence is illustrated in Fig. 7.

Certain lattices are closely connected to Lie groups, since the quantum numbers associated with the finite-dimensional representations of Lie groups fall at points of a lattice, called the *root lattice**. A trivial example is given by $SU(2)$: the values of I^2 for all (tensor) representations are integers; thus, the root lattice of $SU(2)$ is the 1-dimensional lattice shown in Fig. 7(a). Physicists will recognize the lattice of Fig. 7(b) as the root lattice of the group $SU(3)$. Using this language, we can state the generalization of our result above to any "simply laced" group (a class which includes $SU(N)$ and $O(2N)$ for all N and also the exceptional groups E_6, E_7, E_8): If one compactifies the closed string on a torus whose associated lattice is the root lattice of G , there is a special value of the radius of the compact space at which the compactified theory has a $G \times G$ gauge symmetry! The dimension of the compact space gives the rank of the symmetry group, the number of generators which can be simultaneously diagonalized. This number equals $(N-1)$ for $SU(N)$ (as in the two examples given), N for $O(2N)$, and k for the exceptional groups E_k .

* Properly, the root lattice includes only the quantum numbers of representations which can be built up as products of the adjoint representation. For $SU(2)$, this lattice includes the quantum numbers of the tensor, but not the spinor, representations.



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Figure 7. Examples of the correspondence between lattices in n -dimensional space and n -dimensional tori.

Apparently, closed-string theories compactified on tori can give rise to gauge symmetries in a way that is completely geometrical; this mechanism generates the gauge bosons and the gravitons in exactly the same fashion, as zero-mass, $P = 0$ closed-string eigenstates, so that these closed-string theories represent a true unification of Yang-Mills theory with gravity. It seems appropriate, then, to discard the open-string theory and pursue the theory of closed strings alone.

Lesson 5: The Superstring

Now let us begin a search for solutions to the difficulties of the bosonic string theory listed at the beginning of the previous section. Let us take up first the question of how to introduce *fermionic states of string*. In the theoretical climate of the 1980's, a natural suggestion is to replace the geometrical theory of world-sheets (2-dimensional gravity) by 2-dimensional supergravity (Fig. 8).^{*} The practical effect of this change is to replace the transverse displacement field $X^i(\sigma)$ by a supermultiplet $\{X^i(\sigma), \psi^i(\sigma)\}$. The new field $\psi^i(\sigma)$ is a fermion on the world-sheet and cannot be directly interpreted as a space-time fermion. Its influence on the theory is, as we will see, considerably more subtle.



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Figure 8. Conversion of the bosonic string world-sheet to the superstring world-sheet.

Let us, then, compute the influence of $\psi^i(\sigma)$ on the closed-string spectrum. In performing this analysis, I will treat the left-moving modes of the string in isolation from the right-moving modes. At the very end of the analysis, we can add the right-moving excitations and impose the condition $P = 0$.

^{*} Historically, though, this construction was invented first,^[a-a] and supersymmetry arose from attempts to understand its structure.^[20]

Begin by choosing periodic boundary conditions (Ramond^[7] boundary conditions) for ψ^i . Then ψ^i has a Fourier expansion analogous to (3). We can quantize the $n \neq 0$ modes of ψ^i with (anticommuting) ladder operators b_n^i to find the mass formula

$$m^2 = (4\pi T_0) \cdot \left[\sum_{n=1}^{\infty} n (a_n^{i\dagger} a_n^i + b_n^{i\dagger} b_n^i) - \frac{d_{\perp}}{24} + \frac{d_{\parallel}}{24} \right]. \quad (26)$$

The last term denotes the corresponding right-moving contributions. Note that the fermionic contribution to the zero-point energy has just the opposite sign from the bosonic contributions, so that these two terms cancel. (This is a familiar consequence of supersymmetry.) The constant terms of $X^i(\sigma)$ and its conjugate momentum form the center-of-mass position and momentum, which satisfy $[x^i, p^j] = i\delta_{ij}$. Similarly, the constant term of ψ^i plays a special role. The constant pieces of the ψ^i naturally satisfy the anticommutation relations:

$$\{\psi_0^i, \psi_0^j\} = 2\delta^{ij}. \quad (27)$$

This is exactly the defining algebra for Dirac matrices. Thus, we may represent the ψ_0^i as Dirac matrices; the string ground state must then be a zero-mass spinor. By the connection between spin and statistics, this state and all states built by applying to it the (space-time vector) operators $a_n^{i\dagger}$, $b_n^{i\dagger}$ should be fermionic particles.

It is, however, equally valid to begin with antiperiodic boundary conditions (Neveu-Schwarz^[6] boundary conditions):

$$\psi^i(\sigma = 1) = -\psi^i(\sigma = 0). \quad (28)$$

This condition also insures that bosonic quantities built out of the ψ^i are periodic around the loop. We can analyze the effect of this boundary condition by noting that it is just the condition (9) for $\alpha = \frac{1}{2}$. The quantization of the ψ^i oscillators is then shifted by $\frac{1}{2}$. The constant mode ψ_0^i no longer satisfies the boundary

conditions and so disappears. The zero-point energy of the ψ^i oscillators is given by

$$-d_{\perp} \cdot Z(\alpha = \frac{1}{2}) = d_{\perp} \cdot \left(\frac{1}{24} - \frac{1}{16} \right). \quad (29)$$

Note that we have reversed the sign, as is appropriate for fermions. Then the mass formula reads

$$m^2 = (4\pi T_0) \cdot \left[\sum_{n=1}^{\infty} (n a_n^{i\dagger} a_n^i + (n - \frac{1}{2}) b_{n-\frac{1}{2}}^{i\dagger} b_{n-\frac{1}{2}}^i) - \frac{d_{\perp}}{16} \right]. \quad (30)$$

The spectrum of this theory is as follows: The ground state $|0\rangle$ is a tachyon of mass $m^2 = (4\pi T_0) \cdot (-\frac{1}{2})$. Since there are no ψ_0^i operators, this state is a spinless boson. The first excited state is

$$b_{\frac{1}{2}}^{i\dagger} |0\rangle, \quad m^2 = (4\pi T_0) \cdot \left(\frac{1}{2} - \frac{d_{\perp}}{16} \right). \quad (31)$$

This state is a transverse vector; as we saw for the state $|15\rangle$, the present of this state is inconsistent with Lorentz invariance unless the state has precisely zero mass. This implies that the superstring with Neveu-Schwarz boundary conditions cannot be Lorentz-invariant unless $d_{\perp} = 8$, or $d = 10$. As with the bosonic string, imposition of the condition $d = 10$ makes (31) a massless gauge boson (or, after adding the right-moving excitations, a massless graviton) with gauge-invariant couplings.

We have now studied the supersymmetric string with two different boundary conditions for the fields ψ^i . Each has its advantages: Ramond boundary conditions produce massless fermions; Neveu-Schwarz boundary conditions produce the massless vector bosons. Clearly, we want to include both sets of boundary conditions in our theory. In geometrical terms, we would like to sum over world-sheets with the two sets of boundary conditions shown in Fig. 9(a) and (b).

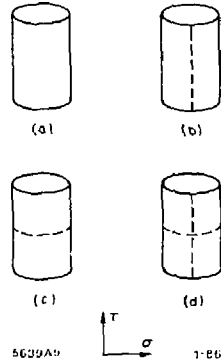


Figure 9. Possible boundary conditions which one might impose on superstring world-sheets. Each dotted line represents an antiperiodic boundary condition: Identify ψ^i on one side of the line with $-\psi^i$ on the other.

This unification of the Neveu-Schwarz and Ramond theories has a serendipitous effect, first noted by Gliozzi, Scherk, and Olive.^[31] The prescription of summing over the first two sets of boundary conditions in Fig. 9 violates geometrical invariance unless we also sum over the remaining two sets of boundary conditions shown in that figure. These latter two conditions introduce an antiperiodic boundary condition in the τ direction. Adding Fig. 9(a) and (c) or (b) and (d) is equivalent to inserting the operator

$$P_{\text{GSO}} = (1 - (-1)^F), \quad (32)$$

where F is the fermion number. This operator, called the GSO projector, removes from the spectrum states of even fermion number. Thus, the spectrum of the

Neveu-Schwarz sector

$$|0\rangle, b_{\frac{1}{2}}^{i\dagger}|0\rangle, a_1^{i\dagger}|0\rangle, b_{\frac{1}{2}}^{i\dagger}b_{\frac{1}{2}}^{j\dagger}|0\rangle, b_{\frac{3}{2}}^{i\dagger}|0\rangle, a_1^{i\dagger}b_{\frac{1}{2}}^{j\dagger}|0\rangle, \dots \quad (33)$$

is reduced to

$$b_{\frac{1}{2}}^{i\dagger}|0\rangle, b_{\frac{3}{2}}^{i\dagger}|0\rangle, a_1^{i\dagger}b_{\frac{1}{2}}^{j\dagger}|0\rangle, \dots \quad (34)$$

All states at half-integer mass levels disappear, including the tachyon. The effect of (32) on the states of the Ramond sector is to pick out one chirality for the spinors. (Recall that γ^i , which flips the chirality of a spinor, is identified with the fermion operator ψ_0^i .) If we denote the left- and right-handed massless spinors by $|La\rangle, |Ra\rangle$, the action of (32) leaves the states

$$|La\rangle, a_1^{i\dagger}|La\rangle, b_{\frac{1}{2}}^{i\dagger}|Ra\rangle, \dots \quad (35)$$

The states eliminated by this GSO projection disappear from the theory, in just the way that the $P \neq 0$ states disappeared in our argument of Lesson 3. Summing over all possible boundary conditions in the join between three strings prevents these states from being produced in scattering processes. Summing over all possible boundary conditions on the figures associated with closed-loop diagrams (as indicated in Fig. 10) keeps these states from appearing in loops. For all practical purposes, then, the states removed by the GSO projection simply do not exist in the theory.

Let us now add back the right-moving string excitations. It can be seen that it is consistent to impose Neveu-Schwarz or Ramond boundary conditions, and to perform the GSO projection, independently for the left- and right-moving parts of $\psi^i(\sigma)$. Summing over boundary conditions in this way, we find the following zero-mass states:

$$b_{\frac{1}{2}}^{i\dagger}|0\rangle \otimes \bar{b}_{\frac{1}{2}}^{j\dagger}|0\rangle, |La\rangle \otimes \bar{b}_{\frac{1}{2}}^{j\dagger}|0\rangle, b_{\frac{1}{2}}^{i\dagger}|0\rangle \otimes |La\rangle, |La\rangle \otimes |La\rangle. \quad (36)$$

The first multiplet of states contains h^{ij} , b^{ij} , and ϕ in just the manner indicated in Eq. (19). The next two multiplets provide two vector-spinors; these act as

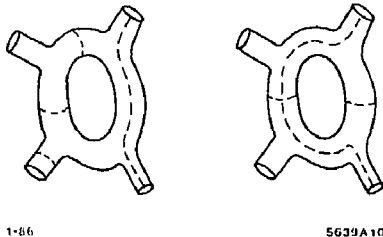


Figure 10. Some typical contributions to the sum over all possible boundary conditions for a closed-loop diagram.

gravitini, the supersymmetric partners of the graviton. The last multiplet is bosonic, and contains an array of tensor fields. All of these fields together form the content of $N = 2$ supergravity in 10 dimensions. Apparently, the theory we have constructed has not only 10-dimensional fermions but also 10-dimensional supersymmetry.

Lesson 6: The Heterotic String

The only element missing from the theory constructed in the previous section is a grand unification gauge symmetry group. In this section, we will see how to modify that theory so that a gauge group is naturally generated dynamically. The required modification is a bit bizarre: One must consider a string whose left-moving components are those of the supersymmetric string, but whose right-moving components are those of the bosonic string! This hybrid forms the *heterotic string* of Rohm, Martinec, Harvey, and Gross.^[15]

To say that this heterotic construction is problematical is something of an understatement. The supersymmetric string can be consistent only if it has 10

space-time coordinates. The bosonic string requires 26 space-time coordinates. How can these be made consistent? The solution is to find a physical interpretation for the extra 16 purely right-moving coordinate fields. To begin, write the Fourier expansion of a purely right-moving field:

$$\tilde{X}(\sigma - \tau) = \tilde{v} \cdot (\sigma - \tau) + \sum_{n=1}^{\infty} (\tilde{X}_n^i e^{2\pi i n(\sigma - \tau)} + (\text{c.c.})) . \quad (37)$$

The σ -dependence of the first term has the form of a winding; this term can be present only if we compactify these extra dimensions. For compactification to a torus, $\tilde{v}$ will be a vector of the associated lattice. The τ -dependence of this term indicates that $\tilde{v}$ is also proportional to the center-of-mass momentum $\tilde{p}$; if we compactify to a torus, this momentum will be quantized. The precise quantization rule is the following: Let $\{\tilde{e}_a\}$ denote the basic periodicities of the compact space, that is, the elementary vectors of the associated lattice. Then $\tilde{v} = \ell_a \tilde{e}_a$. The momentum $\tilde{p}$ must then satisfy $\tilde{p} \sim k_a \tilde{E}_a$, where $\tilde{e}_a \cdot \tilde{E}_b = \delta_{ab}$. If $\tilde{p}$ is also to be identified with $\tilde{v}$, then the $\tilde{e}_a$ and the $\tilde{E}_a$ must coincide, that is, the lattice must be self-dual. In that case, for an appropriate choice of the radius,

$$\tilde{X}(\sigma - \tau) = \left(\frac{2\pi}{T_0}\right)^{\frac{1}{2}} \cdot \ell_a \tilde{e}_a \cdot (\sigma - \tau) + \sum_1^{\infty} (\cdots) . \quad (38)$$

Following the steps in the derivation of Eq. (24), but inserting a factor $\frac{1}{2}$ because the winding contributions come only from the left-moving components, we find

$$\tilde{m}^2 = \frac{1}{2} (2\pi T_0) \cdot (\ell_a \tilde{e}_a)^2 + (\ell_a \tilde{e}_a)^2 + (4\pi T_0) \cdot [\sum (n + \tilde{n}) - \cdots] . \quad (39)$$

A factor $\frac{1}{2}$ also appears in the equation for P :

$$P = -\frac{1}{2} (\ell_a \tilde{e}_a) \cdot (\ell_a \tilde{e}_a) . \quad (40)$$

We can see from these formulae that the (40) will give integers and the first term of (39) will give integer multiples of $(4\pi T_0)$ only if $(\tilde{e}_a)^2 = 2$. A standard cubic

lattice is self-dual, but it does not satisfy this additional condition, so we must seek a more exotic lattice to use in our compactification. The simplest self-dual lattice in which the lattice vectors have length $\sqrt{2}$ occurs in 8 dimensions; Fig. 11 gives some idea of its structure.

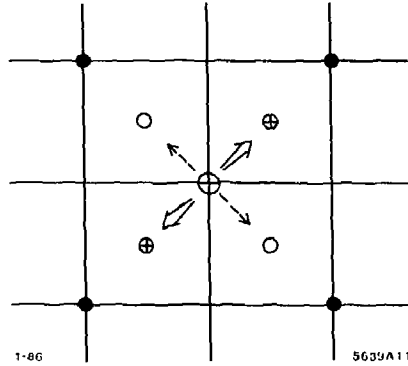


Figure 11. A representation of the 8-dimensional self-dual lattice used to compactify the heterotic string. The arrows point to the centers of hypercubes; the bold arrows point out of the paper, the dotted arrows into the paper.

The elementary vectors of this lattice consist of the points

$$\bar{e}_a = (0, 0, \pm 1, 0, \dots, 0, \pm 1, 0) \quad (41)$$

at the opposite corners of squares from the origin, plus the points

$$\bar{e}_a = (\pm \frac{1}{2}, \pm \frac{1}{2}, \dots, \pm \frac{1}{2}) \quad (42)$$

at the centers of hypercubes (such that the product of the signs is $(+1)$). The

first set of vectors generate the root lattice of $O(16)$. The second set of vectors correspond to the quantum numbers of the left-handed spinor representation of $O(16)$. Together, these multiplets comprise exactly the adjoint representation of E_8 . Compactifying the 16 right-moving dimensions using two copies of this lattice yields a gauge theory with gauge group $E_8 \times E_8$. Repeating the above construction directly in 16 dimensions yields a second self-dual lattice with $(\bar{e}_a)^2$ even; compactifying with this lattice gives an $O(32)$ gauge group.

The zero-mass, $P = 0$ states of the compactified heterotic string theory are obtained as products

$$\left(\begin{array}{c} \delta_{\frac{1}{2}}^{\mu\nu} |0\rangle \\ |La\rangle \end{array} \right) \otimes \left(\begin{array}{c} \bar{a}_1^{\dagger} |L_a = 0\rangle \\ \bar{a}_1^{\dagger} |L_a = 0\rangle \\ |L_a = \delta_{ab}\rangle \end{array} \right). \quad (43)$$

The product of the left-moving Neveu-Schwarz vector with the top state on the left gives the graviton multiplet (19). (This is also the bosonic content of $N = 1$ supergravity in 10 dimensions.) The product of the Neveu-Schwarz vector with the states of the other two forms yields a multiplet of vector bosons in the adjoint representation of $E_8 \times E_8$ or $O(32)$. The products involving the Ramond spinor give the supersymmetric partners of these states. We now have a theory with gauge bosons and fermions interacting through a large grand unification gauge group, unified in a most beautiful way with a supersymmetric theory of gravity. The theory even has a natural handedness, which will eventually be translated into the chirality of the electroweak interactions. What more can we ask of a unifying theory of Nature?

Lesson 7: Field Compactification on an Orbifold

What more, indeed! We can hardly consider this grand theory more than a philosophy unless it can give some insight into the outstanding problems of elementary particle physics. We should certainly hope that this theory will have something to say about the origin of the quark and lepton generations and the calculation of quark and lepton masses. Though it is premature to give precise predictions, I believe that the superstring theory has the power to give insight into these questions. In these last two sections, I will try to demonstrate this by discussing some physical consequences of compactification from 10 to 4 dimensions. In this section, we will warm up by compactifying ordinary field theories. In the next section, we will discuss some additional issues which arise when we compactify strings.

Onto what kind of space should we compactify the extra 6 spatial dimensions of the superstring theory? We should properly make this decision by solving the theory, but at the present level of our understanding there seem to be many possibilities. In particular, the possibility that the string prefers 10 extended dimensions has not been ruled out. Assuming, however, that the true solution to the string theory will involve compactification, several approaches have been proposed for choosing a particular form for the compact space. The first of these, due to Candelas, Horowitz, Strominger, and Witten,^[10] involves simplifying the problem by assuming that the compact space is larger than the natural length scale set by T_0 , deriving the string Einstein equations in this limit (where they reduce essentially to the equations of supergravity), and then looking for solutions. This procedure led to the Calabi-Yau spaces, 6-dimensional spaces with $R_{\mu\nu} \approx 0$, containing $SU(3)$ gauge fields which trace the curvature. These spaces gave some appealing qualitative features, including chiral fermion generations, but they are unfortunately quite complicated to deal with, since they have no symmetries and only topological properties of these spaces are known explicitly. The second method is to choose the compact space to be a torus. This approach

suffers from just the opposite difficulty—it has too much symmetry. In particular, neither the supersymmetry nor the $E_6 \times E_6$ gauge symmetry can be broken.

Fortunately, an elegant compromise between these two approaches was discovered by Dixon, Harvey, Vafa, and Witten.^[32] These authors recommend compactifying on an *orbifold*, a torus with a further identification of points related by the action of a discrete symmetry.

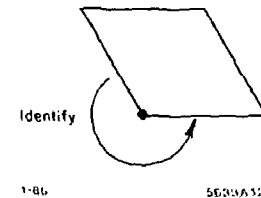


Figure 12. An example of an orbifold, obtained from the torus associated with the $SU(3)$ lattice by identifying points related by 120° rotations.

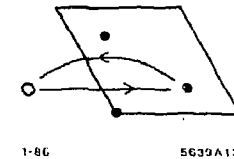


Figure 13. Fixed points of the orbifold shown in Fig. 12.

A simple example of an orbifold is shown in Fig. 12. A good way to visualize the geometry of this orbifold is to identify the points which are fixed under the

combined action of the discrete symmetry and the lattice translation. In the orbifold of Fig. 12 there are 3 such points, shown in Fig. 13. Triples of points, related by 120° rotations about the fixed point, are being identified; this process causes the neighborhood of the fixed point to be folded up into a cone, with the fixed point at its apex.

Now imagine that 2 dimensions of space are curled up into the form of the orbifold in Fig. 12. Let us study the components of fields in such a space which would be visible at low energies. We saw in our earlier study that particles which are not massless in the first approximation receive very large masses: of order $(T_0)^{1/2} \sim 10^{19}$ GeV. We will, then look for particles which are left massless after compactification; these particles will then receive GeV-scale masses from $SU(2) \times U(1)$ breaking and supersymmetry breaking effects at the weak scale.

To begin, define the effective mass after compactification in the same way that we did in the discussion of Lesson 4:

$$\tilde{m}^2 = p_{(2)}^2 + m^2, \quad (44)$$

where $p_{(2)}$ is the momentum in the compactified dimensions, and look for modes of the field for which the effective mass $\tilde{m}$ vanishes. If we start with fields which are massless in the original 10-dimensional space, we can satisfy this condition only for modes for which $p_{(2)}$ vanishes, that is, modes which are constant over the orbifold.

For a scalar field $\phi(x)$, this criterion is easily satisfied: Field configurations which are constant over the compactified dimensions will be viewed as massless scalar fields after compactification. For fields with spin, however, some subtleties arise. To explain them, I will introduce the following notation: Let indices in capital letters (M, N) run over the full 10 dimensions, indices in lower-case (m, n) run over compactified dimensions, and indices in greek letters (μ, ν) run over extended, visible dimensions. In this notation, the massless scalar field modes

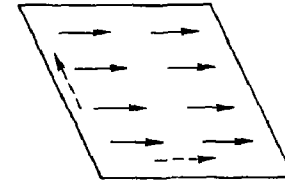
we have just described have the form:

$$\phi(x^M) = \phi(x^\mu). \quad (45)$$

Now let us try to generalize this condition for massless modes to a vector field $A^M(x^N)$. For the components of A^M which point into the uncompactified directions, the mode which is constant over the orbifold gives a massless vector field after compactification,

$$A^\mu(x) = A^\mu(x^\mu). \quad (46)$$

However, this observation fails for the components of A^M which point into the compactified dimensions.



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Figure 14. If one identifies the boundaries of this figure to form an orbifold, one must also identify the dotted tangent vectors.

The reason is shown in Fig. 14. When one identifies points related by a 120° rotation, one must also identify the directions of the vectors between these points. An A^μ configuration constant on the orbifold is represented in the figure by the solid arrows. These arrows are tangent to the lower boundary but lie at

an angle to the boundary on the left. Thus, this mode of A^m does not satisfy the boundary condition which is imposed when we identify these two boundary lines.

It is useful to formulate the boundary condition required by the orbifold a bit more abstractly. A field on the left boundary of Fig. 14 must be rotated by 240° to bring it into coincidence with a field on the lower boundary. If this rotation is implemented by an operator $R(240^\circ)$, the field will be smooth across the join if it obeys

$$\varphi = R(240^\circ) \varphi, \quad (47)$$

To find modes with $\vec{m}^2 = 0$, we must find constant fields which satisfy this criterion. The explicit form of $R(240^\circ)$ depends on the spin and spin direction

for a vector field A^μ :

$$R(240^\circ) = 1$$

for a vector field A^m :

$$R(240^\circ) = e^{\pm 4\pi i/3} \quad \text{on the combinations } A^1 \pm A^2$$

for a spinor field Ψ :

$$R(240^\circ) = e^{\pm 2\pi i/3} \quad \text{depending on the chirality}$$

Apparently, in this simple example, Eq. (47) can be solved only by scalars and vectors oriented normal to the compact dimensions.

It is possible to obtain a much more interesting result, however, by studying a slightly more complex generalization of this structure. Consider, then, the 6-dimensional torus shown in Fig. 15, consisting of three copies of the $SU(3)$ torus in three orthogonal planes.

We can turn this space into an orbifold (the Z -orbifold of Ref. 32) by identifying points related by simultaneous 120° rotations in the three planes. In addition, let us introduce $SU(3)$ gauge fields into the model, and allow a quantum of magnetic flux to point upward through one of the corners. The

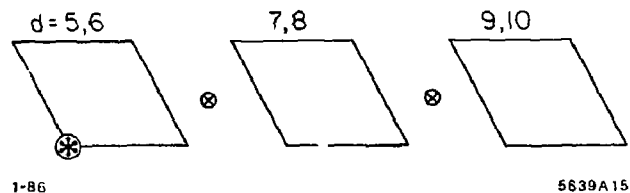


Figure 15. The 6-dimensional torus used to form the Z -orbifold.

boundary condition for a field compactified on this space reads:

$$\varphi = [R(240^\circ)]_{5,6} \cdot [R(240^\circ)]_{7,8} \cdot [R(240^\circ)]_{9,10} \cdot \mathcal{G} \cdot \varphi, \quad (48)$$

where the three rotation operators implement the rotations in the three orthogonal planes and $\mathcal{G}$ is the Bohm-Aharonov phase associated with the magnetic flux

$$\mathcal{G} = e^{i\oint dz A}. \quad (49)$$

Let us assume, further, that $\mathcal{G}$ is quantized so that, for the fundamental representations of $SU(3)$,

$$\mathcal{G} = e^{2\pi i/3} \quad \text{on the } \mathbf{3} \quad \mathcal{G} = e^{-2\pi i/3} \quad \text{on the } \bar{\mathbf{3}}. \quad (50)$$

On representations in the product of $\mathbf{3}$ and $\bar{\mathbf{3}}$, or generally on representations of zero triality, $\mathcal{G} = 1$. This association of a threefold gauge group element with a threefold rotation axis realizes the proposal of Candelas, Horowitz, Strominger, and Witten that gauge fields should trace the curvature of the compactification space.

Combining all the factors in Eq. (48), we can find a myriad of nontrivial solutions to this boundary condition, depending on the spin and $SU(3)$ representation of the field. Some examples are:

$$\begin{aligned}
 A^\mu \text{ in the } 1 \text{ or } 8: & \quad 1 \cdot 1 \cdot 1 \cdot 1 = 1 \\
 A^m \text{ in the } \mathbf{3}: & \quad e^{4\pi i/3} \cdot 1 \cdot 1 \cdot e^{2\pi i/3} = 1 \\
 \Psi \text{ in the } 1 \text{ or } 8: & \quad e^{2\pi i/3} \cdot e^{2\pi i/3} \cdot e^{2\pi i/3} \cdot 1 = 1 \\
 \Psi \text{ in the } \mathbf{3}: & \quad e^{2\pi i/3} \cdot e^{-2\pi i/3} \cdot e^{-2\pi i/3} \cdot e^{2\pi i/3} = 1
 \end{aligned} \tag{51}$$

The second and fourth lines show one of three possible solutions; the others are obtained by permuting the first three elements of the product. The solution for A^m appears as a scalar in the uncompactified dimensions. In compactification of a 10-dimensional chiral fermion, the observed 4-dimensional chirality equals the product of the chiralities evident in (51). Both of the solutions shown, then, would be observed as positive-chirality spinors in 4 dimensions. Note that, for a Ψ in the $\mathbf{3}$, there is no solution which gives a negative-chirality spinor in 4 dimensions. Thus, this orbifold is capable of producing a spectrum of light chiral fermions resulting from compactification.

Let us now apply this construction specifically to the massless sector of the $E_8 \times E_8$ heterotic string theory. The content of this sector is a multiplet of gauge bosons A^μ and gauginos Ψ in the 248-dimensional adjoint representation of each E_8 group. E_8 has a maximal subgroup $E_6 \times SU(3)$, and under this subgroup the 248 transforms as

$$248 \rightarrow (78, 1) + (27, 3) + (\overline{27}, \overline{3}) + (1, 8). \tag{52}$$

I should remind you that E_6 has often been proposed as a grand unification group, since it contains $SU(5)$ and $O(10)$ as natural subgroups, and that, with this identification, one 27 of E_6 contains 1 generation of quarks and leptons.

From (51), we can see that this set of fields $A^\mu + \Psi$ transforming according to (52) yields the following set of massless particles: a 4-dimensional vector and chiral fermion in the $(78, 1)$, which form the gauge bosons and gauginos of E_6 ; a 4-dimensional vector and chiral fermion in the $(1, 8)$, which form the gauge bosons and gauginos of an extra $SU(3)$; three 4-dimensional scalars and three 4-dimensional chiral fermions in the $(27, 3)$; and their antiparticles in the $(\overline{27}, \overline{3})$. This exercise has been a bit complex, but the result is worth it: From a simply visualized compactification geometry, we have seen quark and lepton generations—of definite chirality—emerge in a natural way as part of an effective supersymmetric grand unified theory. We have, of course, found too many generations (and more will appear in the next section), but this problem is less severe for other choices of the 6-dimensional geometry.^[33]

Lesson 8: String Compactification on a Orbifold

The generalization of the argument just given to string compactification introduces some further complications, which I would like to discuss only briefly. Closed strings can actually wrap around the compactified geometry, giving rise to new configurations which appear as light particles in 4 uncompactified dimensions.

Figure 16 illustrates the various possibilities. Figure 16(a) shows a string in a trivial configuration; all of the solutions described in the previous section correspond to this situation. Figure 16(b) shows a string which winds around the torus. These states are analogous to the winding states discussed in Lesson 4, though for the Z -orbifold one finds no new massless particles in this way. Finally, Fig. 16(c) shows a new configuration: a string runs around a fixed point from one point of the torus to a second point identified with the first under the discrete symmetry. This configuration is actually a closed loop on the orbifold. Strings in such a configuration are said to form a twisted sector.

The twisted sectors of the string theory can contain additional massless states.

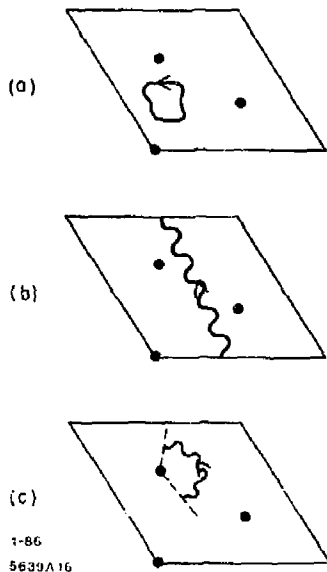


Figure 16. Three possible dispositions for a closed string on an orbifold: (a) trivial; (b) winding; (c) twisted.

To understand how this can occur, let me sketch the computation of the zero-point energy for the right-moving fields of the heterotic string in such a geometry. The nontrivial boundary conditions lead to a shift in the quantization of the oscillators in compactified directions by $\alpha = \frac{1}{3}$. The flux quantum (50) is implemented by insisting that the two identified ends of the string be shifted from one another on the E_8 lattice, so that a fixed point of the orbifold has associated with it a

dislocation in the lattice of the 16 purely right-moving dimensions. For a string in the $(27, 1)$ of $E_8 \times SU(3)$, the total zero-point energy is:

$$6 \cdot Z\left(\frac{1}{3}\right) + 18 \cdot Z(0) + \frac{2}{3}; \quad (53)$$

the last term is the energy cost of the shift on the E_8 lattice. The three terms in (53) sum to 0, so we find one 27 of fermions and one 27 of scalars in the twisted sector about each fixed point. This gives a second mechanism for producing chiral fermion generations, and their supersymmetric partners.

Some of the scalars we have uncovered will become the Higgs bosons of the electroweak theory which is derived from the string theory. The couplings of these scalars to pairs of fermions thus provide the Yukawa couplings which are responsible for the quark and lepton masses. Since both the fermions and the Higgs bosons may be visualized as string configurations on the orbifold, the couplings of these particles may be evaluated by considering transitions from one string configuration to another. In some simple models, these calculations have been carried out explicitly.^[34,35] One remarkable feature of which has been uncovered in that analysis is shown in Fig. 17. It is simplest to think of the Yukawa coupling as the amplitude for two fermion strings and one Higgs boson string to combine and disappear into the vacuum, as illustrated in Fig. 17(z). Figures 17(b) and (c) look inside the vertex for two different cases: If the three strings being coupled belong to the same twisted sector, the strings can combine and annihilate by the relatively simple process shown in Fig. 17(b). This produces a sizable Yukawa coupling: $\lambda \sim 1$. However, if the three strings belong to three different twisted sectors, their annihilation requires a nontrivial physical process on the world-surface, shown in Fig. 17(c). Essentially, one string must tunnel quantum-mechanically across the compact torus. For this case, the annihilation amplitude contains a barrier penetration factor and so is suppressed by

$$\lambda \sim e^{-\alpha T_0 R^2}, \quad (54)$$

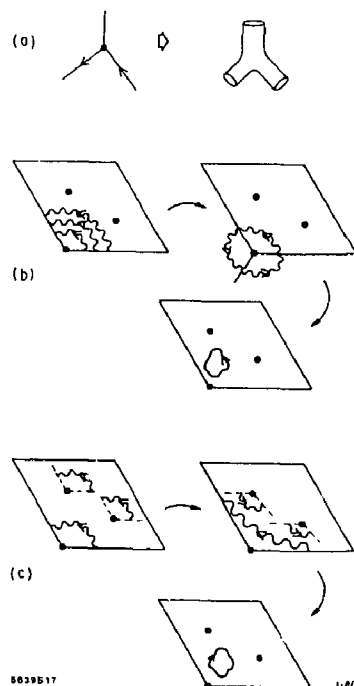


Figure 17. Illustration of the physical processes which determine the fermion Yukawa couplings.

where R is the physical size of the compactified dimensions. Thus, the hierarchical pattern of Yukawa couplings observed in Nature may have an intuitive physical origin, even if this physical picture must be visualized at distances very close to the Planck scale.

Conclusion

In this course of lessons, I have tried to explain how string theory builds up all of the types of particles which appear in physics — quarks, leptons, gauge bosons, gravitons, and others — from the same basic elements. These elements, the fundamental strings, have a dynamics which one can grasp and visualize, even if some of its mathematical features appear magical. The theory gives a geometrical unification of all known interactions. But it also gives a concrete picture of what is happening behind the unification, a picture of the internal structure of quarks and leptons and their interactions.

Critics of string theory often complain that the theory is predictive only for quantities observable at the Planck scale. I have tried to argue here that this is an overly pessimistic view. Because string theories treat quarks and leptons as dynamical entities, they allow explicit calculations of the quark and lepton Yukawa couplings. In ordinary field theory, Yukawa couplings normally cannot be computed as a matter of principle, and those models with sufficient structure to allow such computations often require a complex array of new interactions and undefined parameters. But string theory gives the promise of making definite predictions about the structure of the fermion mass spectrum, inviting a direct and nontrivial confrontation with experiment. This is, of course, an extravagant promise, but it seems to me not an unrealistic one. We will soon see whether it can be fulfilled.

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CP Violation in K-Short Decay

by

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ABSTRACT

We are performing an experiment at Fermilab (E621) to measure η_{+-0} , the parameter describing CP violation in the decay $K_S^0 \rightarrow \pi^+ \pi^- \pi^0$. By collecting $\pi^+ \pi^- \pi^0$ decays in a short neutral beam, we search for CP violation in the interference between K_S^0 and K_L^0 decays. We took data with the kaon beams produced from two targets, separated by 3 K_S^0 lifetimes for the average momentum kaon. By comparing the proper time dependence of the data from the two targets we are able to extract an upper limit on η_{+-0} that is independent of the acceptance of our detector. That upper limit is:

$$|\eta_{+-0}| = .022 \pm .026,$$

$$\arg(\eta_{+-0}) = -39^\circ \pm 80^\circ,$$

based on 86,000 events. We have collected an additional 3,200,000 events in a later run, which we are now analyzing, and we expect ultimately to achieve a sensitivity of $\delta|\eta_{+-0}| = .003$.

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This talk is actually a progress report on Fermilab experiment 621, which is designed to measure η_{+-0} , the CP violation parameter describing the decay $K_S^0 \rightarrow \pi^+ \pi^- \pi^0$. The members of our collaboration are: A. Beretvas, A. Caracciolo, T. Devlin, H. Diehl, U. Joshi, K. Krueger, P. Petersen, S. Teige, G. Thomson, Rutgers University; P. border, M. Longo, University of Michigan; N. Grossman, K. Heller, C. James, M. Shupe, K. Thorne, University of Minnesota.

After CP violation was discovered in 1964,¹ the superweak theory² was the only model to explain the data until the early 1970's when the electroweak theory of Weinberg, Salam, and Glashow (WSG) was developed. The original theory used four quarks, one doublet of Higgs bosons, W^+ , W^- , and Z^0 intermediate bosons, and the gauge group $SU(2)_L \times U(1)$, and got exactly zero CP violation. It was quickly realized that WSG had to be generalized to include CP violation: going to six quarks,³ enlarging the Higgs sector,⁴ or employing the gauge group $SU(2)_L \times SU(2)_R \times U(1)$,⁵ are the simplest ways to do this.

Table 1 shows the predictions of the superweak theory, the three generalizations of WSG, and the state of the world's data, for the two parameters of interest:

$$1 - \left| \frac{\eta_{00}}{\eta_{+-}} \right|, \text{ and } \left| \frac{\eta_{+-0}}{\eta_{+-}} \right| \approx 1.$$

Because there is some theoretical uncertainty in the predictions, I have simply written typical numbers in the table. Note that if the experiments now under way to measure η_{00} and η_{+-} find the answer 0.0, there is a two-fold ambiguity as to which model is correct. The same

Table 1. Theoretical Predictions for CP Violation Parameters

	Super Weak	Six Quarks	Three Higgs	$SU(2)_L \times$ $SU(2)_R \times$ $U(1)$	Data
$1 - \left \frac{\eta_{00}}{\eta_{+-}} \right $	0	.01	.01	0	<.015
$\left \frac{\eta_{+-0}}{\eta_{+-}} \right \approx 1$	0	.01	.5	1	<150

is true if they get a non-zero answer. In each case, if our experiment achieves the sensitivity to tell .01 from .5, our measurement clears up the ambiguity. This description of the theoretical situation may be somewhat oversimplified, because when these experimental numbers become available I am sure they will prompt a new round of theoretical activity.

The experimental number for the η_{00} and η_{+-} measurement comes from two experiments published last year.⁶ For the η_{+-0} and η_{+-} measurement I have used the Particle Data Group average: $|\eta_{+-0}| < 0.35$.⁷

We have chosen to measure η_{+-0} by looking for interference between K_L and K_S near the production target. The number of $\pi^+\pi^-\pi^0$ decays observed per unit proper time, t , is:

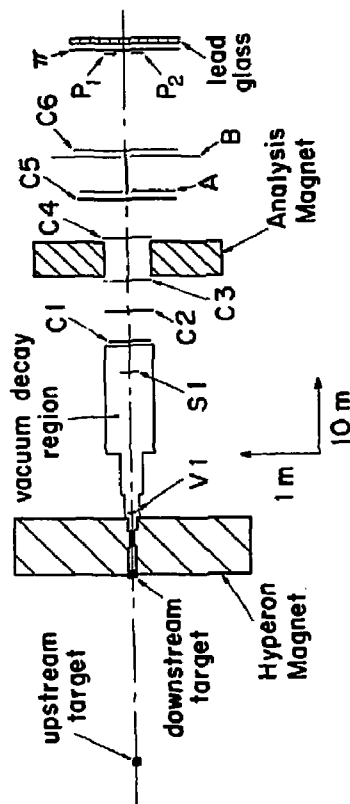
$$\frac{dN}{dt} = \frac{N_L B_{+-0}}{\tau_L} [\exp(-t/\tau_L) + |\eta_{+-0}|^2 \exp(-t/\tau_S)] \\ + 2D |\eta_{+-0}| \cos(\Delta mt + \phi_{+-0}) \exp[-\frac{t}{2}(\frac{1}{\tau_L} + \frac{1}{\tau_S})]$$

where N_L is the number of K_L in the beam, B_{+-0} is the $K_L \rightarrow \pi^+\pi^-\pi^0$ branching ratio, τ_S (τ_L) is the K_S (K_L) lifetime, D is the dilution factor (approximately constant at a value of 0.7 for our experiment), Δm is the K_L - K_S mass difference, and ϕ_{+-0} is the phase of η_{+-0} . The CP violation will show up as an oscillation of dN/dt about $\exp(-t/\tau_L)$; the total excursion of dN/dt from the exponential, in our experiment, is 0.3% if $\eta_{+-0} = \eta_{+-}$.

Figure 1 shows a plan view of our targets, hyperon magnet, and detector. The Fermilab Proton Center beam of 800 GeV/c protons was made to strike one of two targets. Consider the downstream target: it was followed immediately by a magnet, 7.3 m in length with field 35 kG, that swept away all charged particles, allowing only neutral particles to pass through the collimator in the magnet, whose defining section was 1/5 inch in diameter located 4.5 m from the target. The short neutral beam made in this way then entered our detector, a Vee spectrometer. Two scintillation counters, labeled V1 and S1 defined the decay region inside an evacuated pipe. Six MWPC's (C1 - C6) measured charged particles' trajectories, and the analysis magnet, with a p_L kick of 1.6 GeV/c, allowed us to measure their momenta. Three hodoscopes of scintillation counters (A, B, and π) were used in forming the trigger. An array of 86 lead glass counters was used to measure the vector momenta of the two gamma rays from the π^0 decay, and was used in the trigger also.

In addition, three trigger processors performed quick analyses of the patterns of hits in counters and chambers and were required in the trigger. One examined the hits in the A and B hodoscopes to pick out allowed topologies; another used the hits in the C4 and C5 proportional chambers to measure the ratio of momenta of the two charged particles: kaons decay symmetrically, while Λ hyperons, one of our biggest backgrounds, decay unsymmetrically; and the third examined the pattern of hits in the lead glass counters and π hodoscope to pick out events with two gamma ray showers.

Figure 1. Apparatus Plan View



If we had used just this much apparatus, we could have made a histogram of the number of events seen vs. proper time, dividing by the acceptance calculated by a Monte Carlo simulation of the experiment. In experiments like this the uncertainty in such a simulation is about 1%, and this would have limited our ability to measure $|\eta_{+-0}|$ to about 0.01. Not being satisfied with this limitation, we added a second target (the upstream target in Fig. 1) to the apparatus, 24.6 m from the downstream target. This distance is $3\frac{1}{3}$ for the average momentum kaon of 160 GeV/c, and changed the mixture of pure K_L^0 , pure K_S^0 , and interference for kaons produced in this target. The upstream target was placed exactly on the line defined by the downstream target and the center of the collimator in the hyperon magnet, which made the kaon beams coming from the two targets have identical acceptances in the Vee spectrometer. We then analyzed our data in terms of ratios of the number of events coming from the downstream target divided by those from the upstream target, in the same momentum and longitudinal decay position bin. Identical acceptances enter into the numerator and denominator, and cancel in the ratio. By fitting these ratios to the ratio of two dN/dt formulas, we then extract both the magnitude and phase of η_{+-0} in a way independent of the acceptance of our detector.

To minimize systematic errors, we split the Fermilab Proton Center beam in two parts, separated horizontally by about 2 inches, and struck the upstream target with one part and the downstream target with the other part. Because the beams ran in a generally northward direction, they were known as the east and west beams. The

collimator in the Hyperon magnet had two holes in it, defining two beams of neutral particles, one coming from each target, and we collected data from each simultaneously. Periodically we switched the targets to the other beam and took data that way.

We had a test run from April to July, 1984, where we studied the beam, spectrometer, and trigger, and collected 134,000 K_{s3} events. After making many improvements we continued through the 1985 Fermilab run, writing to tape 3,200,000 more events. I want to report preliminary results from the 1984 data set now. Figure 2a is a histogram of the invariant mass of $\gamma\gamma$ pairs, showing that the π^0 peak has essentially no background under it. Figure 2b shows the $\pi^+\pi^-\pi^0$ invariant mass, again with no background under the kaon peak. The x's are the data, and the o's are the prediction of our Monte Carlo simulation of the experiment. Because the data is so clean, we can make our cuts quite loose, and although the Monte Carlo simulation reproduces the data very well, we can make our analysis insensitive to any remaining shortcomings of the Monte Carlo program. To demonstrate how well the agreement is, Fig. 2c shows the proper time histogram of K_{s3} events. Here experimental resolution is good compared to the width of the distribution, and the Monte Carlo calculation simulates the data very well. With confidence in the Monte Carlo program, we now calculate the acceptances for kaons coming from each of the two targets to test whether they are actually the same. Figure 2d shows the momentum dependence of the acceptance: the histogram is for the downstream target, and the dot's are for the

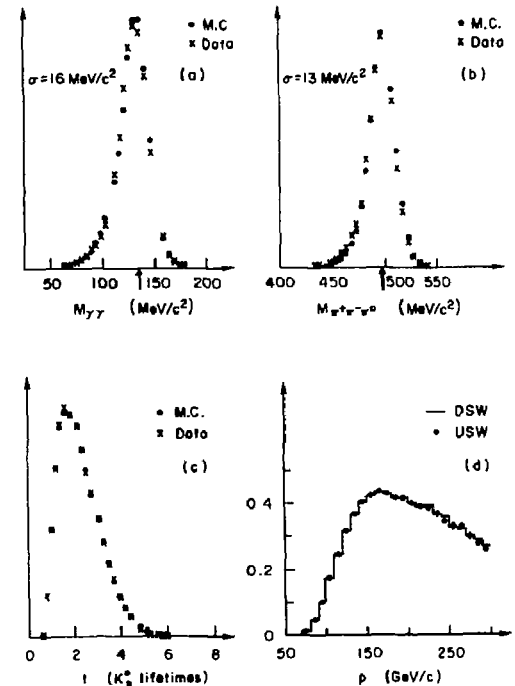


Figure 2. Data Analysis Histograms

upstream target. They are indeed the same to an accuracy much better than the statistical accuracy of the data.

From the data from the east beam striking the two targets, we form the array $N_D(p,t)$ for the downstream target, and $N_U(p,t)$ from the upstream target, and the ratio $R_e(p,t)$:

$$R_e(p,t) = \frac{N_D(p,t)}{N_U(p,t)} \frac{\int_{p,t} N_U(p,t)}{\int_{p,t} N_D(p,t)}.$$

For the west beam we also form an $R_w(p,t)$ defined the same way. We assume that the two east beam acceptances cancel and that the two west beam acceptances cancel also. Figure 3 is an average (weighted by the variances) over momentum, and over R_e and R_w . No bumps and wiggles of CP violation exhibit themselves in this graph. When we fit the R_e and R_w arrays to the appropriate ratios of the dN/dt function, we get

$$|\eta_{+-0}| = .022 \pm .026,$$

$$\text{and } \arg(\eta_{+-0}) = -39^\circ \pm 80^\circ,$$

for a data sample of 86,000 events in the momentum range 90 - 270 GeV/c. The fit has a χ^2 of 422 for 427 degrees of freedom. This represents an improvement of the world's knowledge of the upper limit on η_{+-0} of about an order of magnitude. Peter Border of the University of Michigan, the thesis student on this part of the experiment, has performed an analysis using p and z as the independent variables of the problem, and with 92,000 events has fit the data to $|\eta_{+-0}| = .041 \pm .035$, and $\arg(\eta_{+-0}) = 21^\circ \pm 89^\circ$, in good.

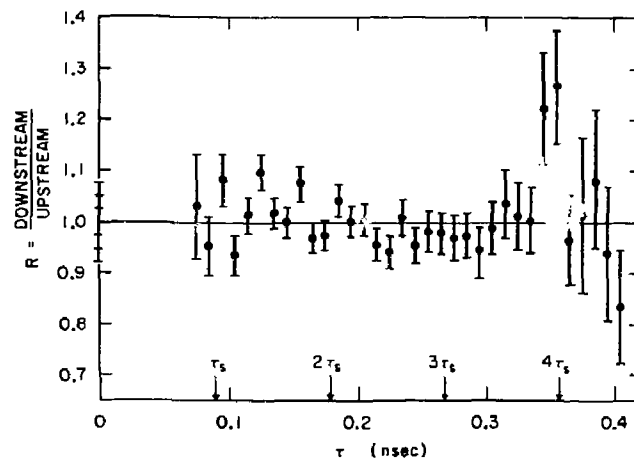


Figure 3. Proper Time Plot for 86,000 K_{π^0} Decays

agreement with my analysis. Peter's event selection and fitting processes are somewhat different from mine, and we are now working to reconcile the differences.

The expectations we have for the 1985 data are that we should have a factor of about 5 better accuracy from statistics alone, and also another factor of 2 improvement because of the better trigger. We biased the data more toward high momentum events in the 1985 run, where CP violation is a bigger effect. So we expect to achieve $\delta|\eta_{+-0}| = .003$ ultimately.

Finally, to sum up the world's knowledge, in Fig. 4 we plot the complex η_{+-0} plane, with the two previous experiments with the best statistics,⁸ and the Particle Data Group's summary also shown. Our result is labeled "1984" in Fig. 4.

This work was supported in part by the Department of Energy and the National Science Foundation.

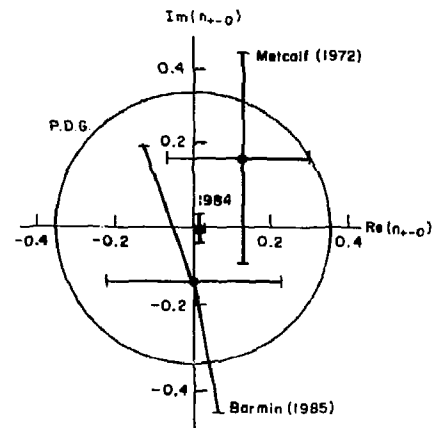


Figure 4. Complex η_{+-0} Plane

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THE CERN NEUTRAL KAON CP VIOLATION EXPERIMENT NA31

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The lectures given in this year's Summer Institute have stressed the importance of carrying out high precision experiments on CP violation in the neutral kaon system.

This paper will try and explain the basic ideas behind one of these, the CERN experiment NA31,[1] and also report some interesting results of $K_L^0 \rightarrow 2\gamma$ and $K_S^0 \rightarrow 2\gamma$.

In precision experiments the main preoccupation is to eliminate systematic errors. A classical way around this problem is to devise a way to make them cancel, and this is one of the basic ideas of NA31.

We recall that

$$\eta_{00} = \frac{A(K_L \rightarrow 2\pi^0)}{A(K_S \rightarrow 2\pi^0)} = \epsilon - 2\epsilon'$$
$$\eta_{+-} = \frac{A(K_L \rightarrow \pi^+\pi^-)}{A(K_S \rightarrow \pi^+\pi^-)} = \epsilon + \epsilon'.$$

Hence

$$\left| \frac{\eta_{00}}{\eta_{+-}} \right|^2 = 1 - 6 \frac{\epsilon'}{\epsilon}$$

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$$\frac{\epsilon'}{\epsilon} = \frac{1}{6} \left[1 - \frac{\Gamma(K_L \rightarrow 2\pi^0)}{\Gamma(K_S \rightarrow 2\pi^0)} \frac{\Gamma(K_S \rightarrow \pi^+\pi^-)}{\Gamma(K_L \rightarrow \pi^+\pi^-)} \right]$$

which can be rewritten

$$\frac{\epsilon'}{\epsilon} = \frac{1}{6} \left(1 - \frac{R_L}{R_S} \right)$$

where

$$R_L = \frac{\Gamma(K_L \rightarrow 2\pi^0)}{\Gamma(K_L \rightarrow \pi^+\pi^-)}, \quad R_S = \frac{\Gamma(K_S \rightarrow 2\pi^0)}{\Gamma(K_S \rightarrow \pi^+\pi^-)}.$$

Hence if we can measure R_L in a K_L beam, and R_S in a K_S beam, in exactly the same geometrical decay region, if possible with the same momentum spectrum, and with the same detector, then one sees that systematics from acceptance, momentum resolution, etc., all cancel, in the measurement of ϵ'/ϵ .

The experiment is aiming at a precision of $\sim 1/1000$ in ϵ'/ϵ . Statistically the accuracy is determined by the number of $K_L \rightarrow 2\pi^0$ events (the most rare, BR $\sim 10^{-3}$). As there is a gain factor of 1/6, one sees that $2 \cdot 10^8 K_L \rightarrow 2\pi^0$ is adequate. Such a statistic is relatively easy to obtain at the SPS. Using $\sim 10^{11}$ protons/pulse at 450 GeV one can get $\sim 10 K_L \rightarrow 2\pi^0$ decays/pulse in a 50 m fiducial decay region.

The decay length of K_L at 100 GeV is 3 km, so that the distribution of the decays in the 50 m fiducial region is flat. The question is how does one simulate this with K_S which have a decay length of 5 m at 100 GeV. The solution is simply to build at K_S train, with 40 stations covering exactly the same 50 m of fiducial decay region. Event rate is no problem in the K_S beam, with 10^7 protons at 450 GeV one can obtain $\sim 100 2\pi^0$ decays. The details of the K_L and K_S beams is shown schematically in Fig. 1.

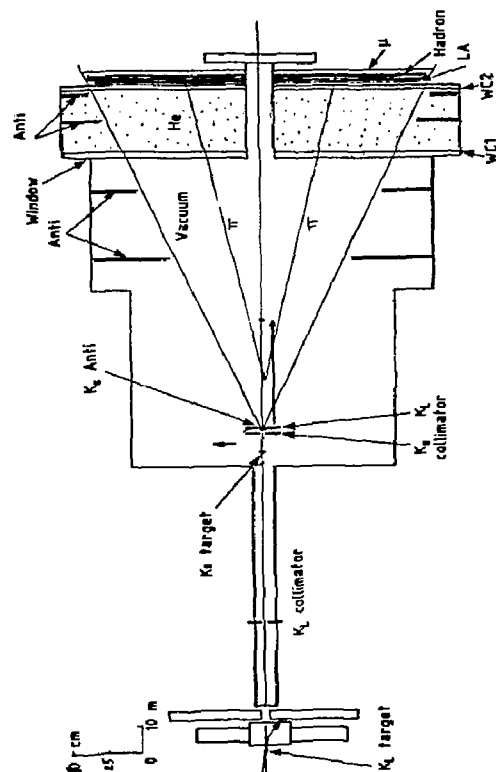


Fig. 1 Schematic layout of K_L and K_S beams and decay region.

Having got the geometrical conditions identical for the two beams, one now turns one's attention to backgrounds. The K_S is completely background free, whereas K_L has several. It is imperative in this experiment that the final backgrounds in the $K_L \rightarrow 2\pi^0$, $K_L \rightarrow \pi^+\pi^-$ samples be ~ few percent, and very well understood if the final accuracy of ~ 1/1000 in π^0/π is to be attained.

In the neutral mode the $K_L \rightarrow 3\pi^0$ branching ratio is 21.5%, compared to $\sim 10^{-3}$ for $K_L \rightarrow 2\pi^0$.

In the charged mode there are three possible sources

$K_L \rightarrow \pi^+\pi^-\pi^0$	12.5%
$\rightarrow \pi\nu\nu$	27%
$\rightarrow \pi\nu\bar{\nu}$	39%

compared to

$$K_L \rightarrow \pi^+\pi^- \quad \sim 2 \times 10^{-3}.$$

1. DETECTION OF $K^0 \rightarrow \pi^+\pi^-$

A unique feature of NA31 is that it does not have a magnet to determine the momenta of the charged pions. By using a "classical" iron scintillator calorimeter with 10 cm strips, adequate mass resolution is obtained, with the bonus that the acceptance is much larger than other experiments which use magnets.

The opening angle between the two charged pions is measured by means of wire chambers having a special precision of ~ 1 mm. The energy of each pion is measured to an accuracy of $70\%/\sqrt{E}$, giving a mass resolution of ~ 5% for a 100 GeV kaon.

The K_{S1} decays are more of a "nuisance" than a background, and they are rejected by means of muon anti-counters behind the hadron calorimeter.

For the $\pi^+\pi^-\pi^0$ decays, the $\pi^+\pi^-$ mass is < 400 MeV, however because of measurement errors they can give a background at the kaon mass. In the vast majority of these decays at least one of the γ 's from the π^0 decay hit the detector.

In the case of K_{S1} , one must be able to eliminate electrons at the 1σ level. Both the K_{S1} and K_{S2} rejection requires that a fine grain liquid argon calorimeter of 25×10^3 is placed in front of the hadron calorimeter. The energy resolution of this calorimeter is $8\%/\sqrt{E}$, and is also the key element in the detection of the neutral mode. The performance of the detector for the charged mode is shown in Fig. 2, in terms of vertex, mass and kaon energy resolution.

It is quite surprising at first sight that such a good kaon energy resolution ~ 1%, can be obtained with such a poor, ~ 10%, pion energy resolution. The kaon energy is given by the following relation:

$$E_K = \frac{1+R}{2\theta} \sqrt{4E_\pi^2 - (1+R)m_\pi^2}$$

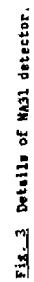
where $R = E_1/E_2$, the ratio of the pion energies and θ the angle between them. Note that the absolute hadron energy scale calibration cancels and that the kaon energy is determined purely by geometrical quantities, as in the decay vertex. Figure 3 shows the disposition of the wire chambers, liquid argon and hadronic calorimeters.

2. DETECTION OF $K^0 \rightarrow \pi^0\pi^0$

Only the energy and conversion position are measured for γ 's rays. No directions are measured.

How can one determine the vertex position from these measurements?

Consider the simple case of $K^0 \rightarrow 2\gamma$. The decay occurs at a distance z away from the detector and the γ 's have an opening angle θ_{12} and energies E_1 and E_2 , and hit the detector with a separation of R_{12} . Using the kaon mass constraint:



$$m_X^2 = E_1 E_2 \theta_{12}^2 = E_1 E_2 \frac{x^2}{z^2}.$$

For the decay $K^0 \rightarrow 2\pi^0 \rightarrow 4\gamma$

$$m_X^2 = \sum_{ij, i \neq j} E_i E_j \theta_{ij}^2 = \frac{1}{z^2} \sum_{ij, i \neq j} 2E_i E_j x_{ij}^2.$$

The decay vertex position is determined by applying the mass constraint, and has a precision of ~ 1 m for 100 GeV kaons.

The background to the $K_L \rightarrow 2\pi^0$ decay comes from $K_L \rightarrow 3\pi^0$ in which two γ rays miss the liquid argon calorimeter and the anti-counters around the decay region. On applying the mass constraint, because energy is missing, the reconstructed vertex will be smaller, i.e. nearer the detector, than the true decay vertex of the $3\pi^0$ decay. The mean displacement is about 20 m, so that the first 10 m of the 50 m decay region is almost background free, increasing rapidly beyond 25 m. Figure 4 shows this background as a function of distance. Figure 5 shows a scatter plot of the two π^0 masses, for the pairings with the smallest x^2 , for 4 γ events in the K_S beam. Figure 6 shows the same for the K_L beam. The background is $\sim 1\%$, and has the very nice feature of being flat, thus making the subtraction very reliable. The mass resolution is ~ 2.5 MeV.

Figure 7 shows the $\pi^+\pi^-$ mass plot for charged events in K_L beam. The small peak on the low mass side is due to the $\pi^+\pi^-\pi^0$, and K_S events surviving the cuts.

3. SYSTEMATIC EFFECTS

As we have discussed, in the charged mode $K^0 \rightarrow \pi^+\pi^-$ the energy scale and length scale are determined only by geometry, and therefore this scale can be considered to be the absolute and fixed scale of the experiment. In contrast, the neutral mode $K^0 \rightarrow 2\pi^0$, the energy scale also determines the length scale, so special care must be taken to

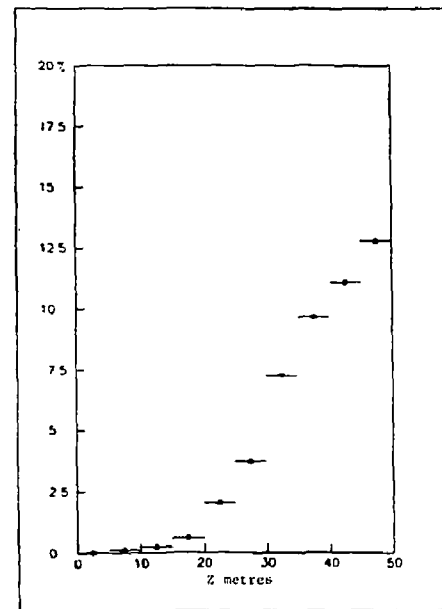


Fig. 4 Background in $K_L \rightarrow 2\pi^0$ decay as function of decay vertex position.

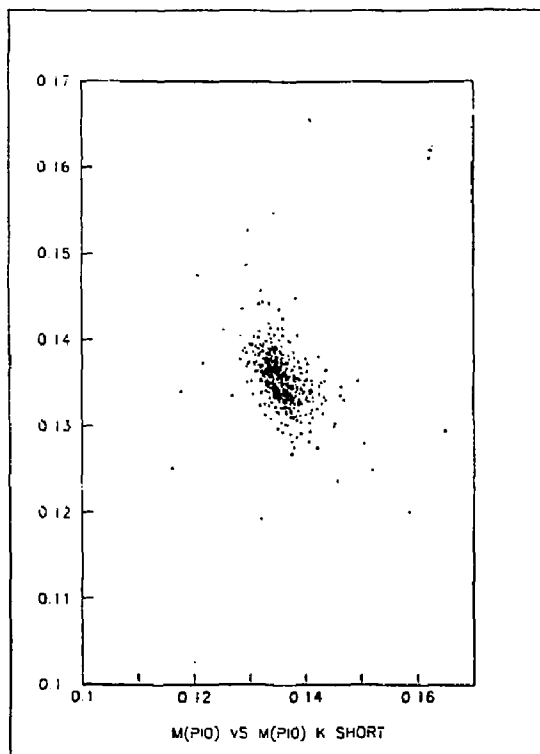


Fig. 5 Scatter plot of $\gamma\gamma$ mass for 4γ events in K_s beam.

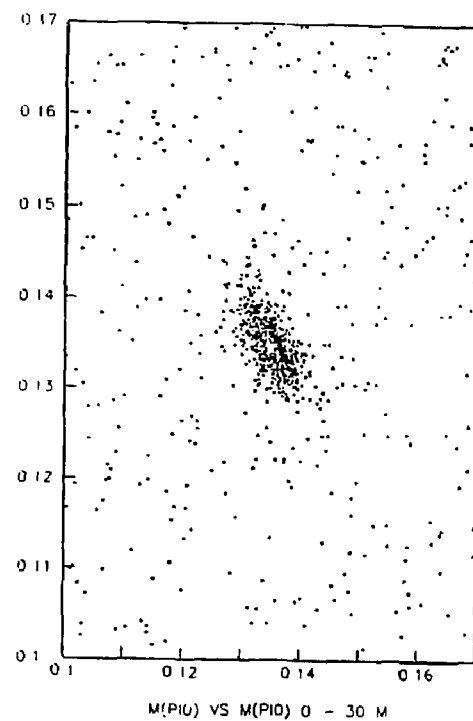


Fig. 6 Scatter plot of $\gamma\gamma$ mass for 4γ events in K_L beam.

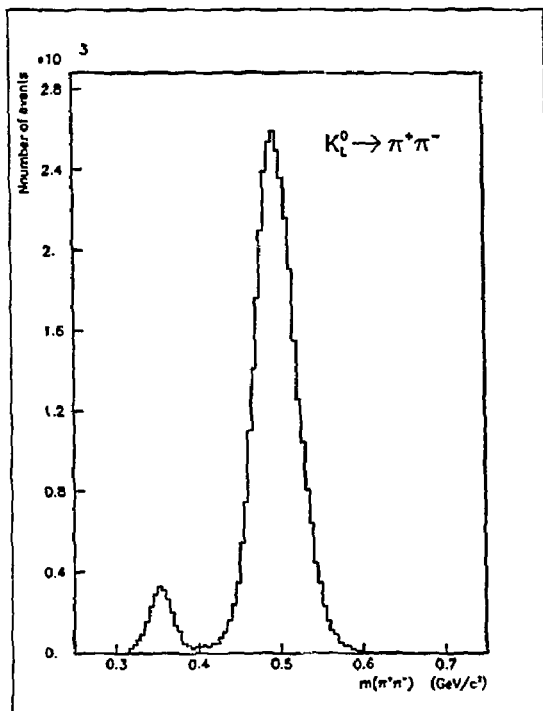


Fig. 7 $\pi^+\pi^-$ mass distribution in K_L beam.

make sure both scales are identical. This is done by placing an anti-counter in K_S beam, which anticoincides any K_S decays upstream, thus giving a sharp edge to the K_S decay distribution. This sharp edge is then fitted to be the same for the charged and neutral decays by varying the neutral energy scale.

This can be done with an accuracy of 10 cm in 100 m, i.e. $1''/\dots$. The effect of such an error on $|\eta_{00}/\eta_{+-}|^2$ is:

- (a) $\sim 1 \times 10^{-3}$ due to the decay length difference.
- (b) $\sim 1.5 \times 10^{-3}$ due to the K_S and K_L momentum spectra being of different shape.

The systematic error of ϵ'/ϵ is then well below $1''/\dots$.

At present the experiment has obtained $\sim 2 \times 10^4 K_L \rightarrow 2\gamma$, and is at present running with the aim of obtaining $\sim 2 \times 10^5 K_L \rightarrow 2\pi^0$ events. Figure 8 shows the present experimental situation [2-6] concerning ϵ'/ϵ and the expected precision from NA31.

4. RESULTS ON $K_L \rightarrow 2\gamma$ AND $K_S \rightarrow 2\gamma$

The NA31 experiment is characterized by its large acceptance, and for $K_L \rightarrow 2\gamma$ it is $\sim 50\%$. Moreover the background is very small because the BR $K_L \rightarrow 2\gamma$ is $\sim 5 \times 10^{-4}$ and the only background comes from $K_L \rightarrow 2\pi^0$ with BR $\sim 10^{-3}$. These are $K_L \rightarrow 2\pi^0$ decays, in which 2γ 's have been missed and the two detected γ rays have a first moment (i.e. energy centre of gravity) within the K_L beam profile. Figure 9 shows the first moment for the 2γ events in the K_L beam having a total energy $70 \text{ GeV} < E_{\gamma} < 150 \text{ GeV}$ and a vertex position of $5 \text{ m} < z < 47 \text{ m}$ from the K_L cleaning collimator. Events with first a moment greater than 7 cm are considered to be due to background and are used to estimate the total background in the sample. As normalisation the $K_L \rightarrow 2\pi^0$ are used, and the results are presented below.

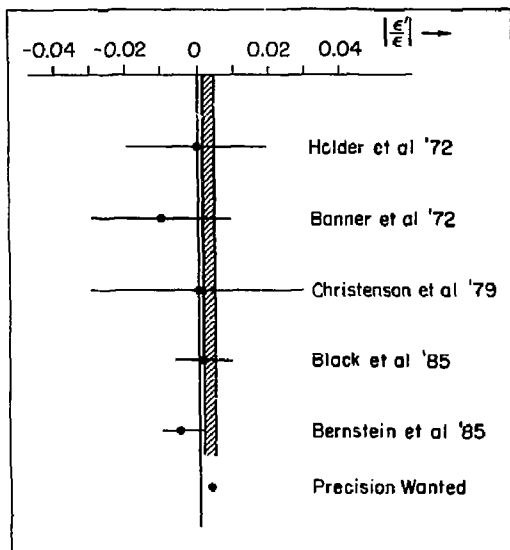


Fig. 8 World data on ϵ'/ϵ .

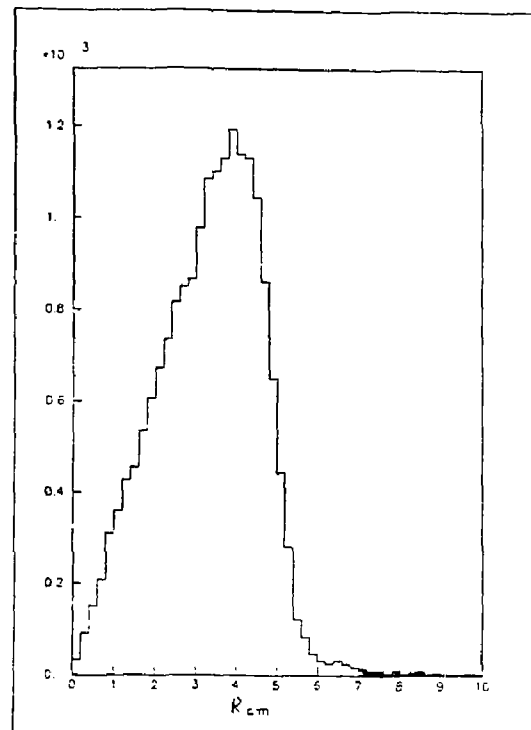


Fig. 9 First moment distribution for $\gamma\gamma$ events in K_L beam.

	2γ	$2\pi^0$
Raw events	18509	12765
Background	160 ± 60	363 ± 30
Acceptance	0.471	0.204

Corrections

$m_{\gamma\gamma}$ cut		$1.6 \pm 0.5\%$
Dalitz decays	$-0.3 \pm 0.1\%$	$2.1 \pm 0.2\%$

This gives

$$\frac{\Gamma(K_L \rightarrow 2\gamma)}{\Gamma(K_L \rightarrow 2\pi^0)} = 0.620 \pm .006 \pm 0.013 \quad (\text{the statistical and systematic errors}).$$

As the result is preliminary a systematic error of 2% is used, but eventually it will be reduced.

Using $|n_{00}|^2 = (5.2 \pm .19) \times 10^{-6}$ we obtain

$$\frac{K_L \rightarrow 2\gamma}{K_L \rightarrow \text{All}} = (5.9 \pm 0.25) \times 10^{-4}$$

which is to be compared with PDG value of $(4.9 \pm 0.4) \times 10^{-4}$. [7]

5. RESULTS ON $K_S \rightarrow 2\gamma$

At present this decay mode has not been detected because of the expected low BR $\sim 10^{-8}$ and the high background from $K_S \rightarrow 2\pi^0$. However, NA31 has the sensitivity to detect such a low branching ratio.

In the decay $K_S \rightarrow 2\pi^0$ the maximum possible transverse momentum for a γ ray is 228 MeV/c, for $K_S \rightarrow 2\gamma$ it is 249 MeV/c, i.e. 8% difference. Hence all background 2γ events coming from $K_S \rightarrow 2\pi^0$ will have a vertex reconstructed 8% smaller than the true decay vertex. It follows that the first 8% of the distance between the anti-counter in the K_S beam and the detector is background free. A preliminary analysis of the present data gives a result of

$$\frac{\Gamma(K_S \rightarrow 2\gamma)}{\Gamma(K_S \rightarrow \text{All})} < 1.4 \times 10^{-5}$$

at 90% C.L.

This result will be improved by a factor of 10, and is to be compared with the previous best limit of $< 4 \times 10^{-4}$. [8]

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DRELL-A-BRATION

(In Celebration of Sid Drell's 60th Birthday)



-685-1686

TOPICS IN ARMS CONTROL

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A few years ago on the occasion of Geoff Chew's sixtieth birthday, I commented that since having stopped being a real person, I'd gained a new profession, namely giving after dinner speeches at my friends' sixtieth birthday worship. Now, I'm aware that this is not an after dinner speech, and I'm not supposed to make cute jokes about Sid's absence of hair and things like that. I should say, however, that I've known Sid for thirty-eight years, which is a shockingly large number now that I think about it.

We first met in 1948 when I went down to Urbana to give a physics seminar at the invitation of Sid's thesis advisor, another Sid - Sidney Dancoff - one of the finest, sweetest men I've ever known. Dancoff died tragically young at the age of thirty-seven of lymphosarcoma. Probably nowadays he would have been able to survive. In fact, Dancoff had arranged for me to be offered an assistant professorship at Illinois, and then when I went down there, he proceeded to advise me to refuse it and instead to take a post-doc position at Berkeley with Bob Serber, which I did. At any rate, our Sid was the other Sid's student, and he had already begun to show talent. It was only much later that I learned of his peculiar Atlantic City background, the fact that he was a Princeton undergraduate, and you're all probably aware of the fact that what he really wanted to do at Princeton was to play football. There are many things that I could tell you about Sid the

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physicist, but I was ordered by Dick Blankenbecler to talk instead about his work on international security and arms control.

You at Stanford are well aware of his past contributions to arms control courses here, and his present half-time commitment to the Stanford program on arms control. I'm not sure what the official title is -- Center for International Security and Arms Control, or something approximately like that. What you might not know quite so well is how Sid became involved in such issues, and perhaps you don't know about some of his many important contributions to this country's national security. I take a bit of personal pride in his many accomplishments in this particular area because I played at least some role in seducing him into it. I'm not sure how Sid feels about this because with his characteristic intensity in everything that he does it certainly meant that the time and energy he had to devote to physics was diminished by these other activities, to say nothing of the impact on his family, as Sid tried to carry on two full-time professions simultaneously.

It all began in December of 1959 when Keith Brueckner, Ken Watson and I, with the strong support of Charlie Towns who was then vice-president of something called the Institute for Defense Analyses, called a meeting of -- the Young Theoretical Physicists (we were very young at the time and mostly theoretical physicists because they were all the friends of the three people that I mentioned) to form a group under the auspices of I.D.A., Institute for Defense Analyses, to work on defense issues that had a strong scientific and technical component. This group came into being, at least in part, because a number of us in various ways had begun to become involved as advisors to the government, and we were

appalled by the fact that, in the first place, we saw all the same people at all committee meetings, no matter which ones we went to, and, secondly, this was a group of people who we thought were sort of tired, World War II war horses and should be replaced by a new and vigorous breed of defense advisors.

This group was ultimately named the Jason Group. That's a name which is the source of many stories as to how it, in fact, came to pass. I will now tell you the true story. There was a computer somewhere in the bowels of the Pentagon that spit out the name "Sunrise". It was supposed to be know as "Project Sunrise", and I came home from a meeting in Washington, I told my wife that it was now going to be called "Project Sunrise." She said, "That stinks! You should call yourselves 'Jason'." And that's the way it happened. Now I could explain why she actually chose that, but it is too long a story.

Now this group, which in later years acquired a certain degree of notoriety, would meet periodically during the year and for a six or seven week period during the summers. Sid was either a charter member of this gang, or so nearly so that it makes no difference. Our mode of procedure was somewhat chaotic, viewed in retrospect. We would be presented a large menu of things that the Department of Defense was interested in, and we would choose among those, or none of the above. And we would show up in some esoteric place in the summer and sit around for a week getting briefed and talking with each other. Then in a way which was really rather marvelous to behold, people would split up into groups and start working very hard and doing interesting and important things.

Our first major summer study took place in the summer of 1961 in Brunswick, Maine on the campus of Bowdoin College. This was the time when the testing

of nuclear weapons in the atmosphere was going on and there was great interest in understanding in detail the very complicated interactions between the radiations of all kinds from the explosives and the surrounding environment. Sid and Mal Ruderman set themselves the task of understanding the infrared radiation resulting from a portion of the upper atmosphere being excited by a very high altitude (of the order of ninety kilometers) nuclear explosion. I was talking with Mal Ruderman about this work recently. He told me it was, in fact, published in some obscure journal, like the "Journal of Infrared Physics". And just to show how the grey cells deteriorate in some people, Mal said the only difference between the classified paper on the subject and what was published in this strange journal was the fact that the classified thing said that the source of the energy deposited at ninety kilometers was a nuclear explosion and that the published one did not say that. That's not true. The published one says very clearly that it's a nuclear explosion. It's others' grey cells that I'm talking about, of course. I don't know whether Sid remembered that correctly or not.

Now, I want to tell you a little bit about this problem because it's illustrative of probably a side of Sid that you never even knew about. He and Mal posed themselves the following problem: what's the infrared radiation from a layer of heated air suddenly formed at some high altitude, eighty or ninety kilometers, by an intense burst of x-rays from a nuclear explosion? Now you have to begin to sort out the various phenomena there. The O_2 and N_2 molecules don't give any dipole infrared radiation, and so they were led to consider the reactions leading to the NO molecule formation and the subsequent radiation at the fundamental mode of that which is something like 5.4μ microns.

What you're interested in is what happens after sort of the first hundred seconds when this layer is illuminated. The calculations are quite intricate, and I certainly could not reproduce them here, but I want to give you a slight feel for the problem - which would probably terrify Sid now, 25 years later, because he had to learn all kinds of things that go into such considerations. What you imagine is you have a source of kilovolt x-rays that are incident from above, you have particle densities of the order 10^{14} particles per cc. As you go down from ninety kilometers or so, the atmosphere becomes more dense, picking up of a factor of "e" about every five kilometers. The x-rays are absorbed in a layer of roughly ten or twelve kilometers in depth. You make an assumption about what the input energy is - and this is probably the place where they got into some sort of classified stuff which they fuzz over - they just happen upon a particular incident energy. Then they ask what happens.

Well, what happens is that the temperature of this layer rises from about $200^\circ C$ to about $1450^\circ C$, and you have to analyse how energy is dissipated in this layer. The x-rays produce electrons which ionize the medium and so on. Then you have to study the chemical reactions that lead to NO , determine the temperature of the layer, and finally, the infrared radiation from the vibrationally excited molecules.

Now there are many reasons to study this question for its own sake, but their involvement with I.R. got Sid and Mal involved with other technologies, such as, for example, satellites with I.R. detectors that are used to give early warning and initial trajectory analysis of strategic missiles. This is an on-going program, as you might imagine.

In the early days of satellite reconnaissance, Sid and Mal made a commitment to work almost half-time for the CIA to assist them in the design of these systems. This was in the early 1960's, and I happen to know personally that they made an enormous contribution to this effort. Their entree was through "Jason" and the fact that there was a peculiarly intelligent Director of Science and Technology in the CIA, a Stanford alumnus by the name of Bud Wheelon. Another "Jason" activity of Sid's, this time involving in addition to Ruderman, the late Henry Foley, was related to consideration of large metallic satellites moving through the ionosphere. This group pointed out that when you have a conductor moving through the earth's field, there is an induced charge separation of a magnitude just necessary to kill the " $\nabla \times B$ " electric field induced by that motion. They further noted that if there is a mechanism to have the charges conducted away, there would be an actual DC current flow, and such a mechanism for such current flow exists in the generation of things which are called Alfvén waves, which can exist in a plasma in the presence of a magnetic field. This, in turn, gave rise to a significant damping of the orbit as mechanical energy is converted into this Alfvén radiation.

There was a particular experiment, called the "Echo Satellite", that was launched some time in the remote past, and it turns out that the "drag" which these guys calculated equalled the observed "drag" on that satellite that hitherto had been attributed to collisions with the very thin atmosphere in which the thing was moving. But their most spectacular observation was that one could, by providing a source of electrical power on the satellite, convert this "drag" force into a propulsion mechanism where the satellite pushes against the earth's field.

There is no propellant involved here. After much deep thought (probably the hardest part in all of this analysis was to come up with an appropriate name), it was called an "Alfvén Propulsion Engine" in space, or the acronym AFE in Space.

I should also mention that Sid's Jason membership caused him some problems at European summer schools after the Pentagon Papers revealed the Jason involvement in the so-called electronic barrier in Vietnam. In fact only a handful of Jason's worked on that system and Sid was not one of them.

Now, Sid, while retaining his "Jason" membership, graduated, as I did and a number of others of us, to the President's Science Advisory Committee. On that committee we both served on a panel that we had been members of for a long time, called the "Strategic Military Panel". Our most important function there was trying to maintain sanity and objectivity in the analysis of a host of anti-ballistic missile systems being pushed by the Department of Defense and their various contractors. We also interacted very strongly with the Arms Control and Disarmament Agency in that particular endeavor. In this latter connection - and, if I remember correctly, a number of other "Jason" people were involved - Sid led a team that repulsed a last-ditch effort to block the ABM Treaty of 1972 by certain forces of evil who raised the specter of what was called "The SAM Breakout", where SAM stands for surface-to-air missile.

There had appeared in about 1964, or somewhere around then, a strange system in the Soviet Union. It first appeared at Leningrad, then it was seen again in Estonia. It then began to appear all over the Soviet Union. And it was clearly a missile system of some sort; it had missiles and it had radars. The immediate

assumption when this was first detected, was that this was the beginning of a massive, nation-wide ABM system. I remember very well being at a meeting - I don't know if Sid was there or not - when some young whippersnapper from TRW, I think, came to this distinguished panel of experts, all of whom were maybe two or three years older than he, and said, "That's not an ABM system; that's an air defense system!" And we laughed, and we slapped our sides and we rolled on the floor, but he was right. Nevertheless, that system was there, in a sense lurking; it was widely deployed in the Soviet Union, and still exists. The fear was that it could suddenly be upgraded and pose a threat to our offensive missile forces. Now Sid and his team showed that neither the radar nor missile performance was adequate for any kind of realistic defense. And since we already knew that the Soviet's Moscow ABM deployment wasn't really worth a damn, there were no further technical obstacles to proceed with this treaty.

It's hard to transmit the flavor of this kind of activity. The physics involved in that one and some of the other things that Sid has worked on can really be quite intricate, and it's very important that you be in a position to have these issues analyzed carefully and convincingly if you're going to have any impact on the political process.

Sid had at least two other important activities that were terribly secret, so much so that he may have had others that not even I know about. One, however, I shared in, was a panel chaired by Din Land, whom you call Edwin Land, of Polaroid Corporation. This committee had a very special and crucial role in the White House in connection with the overhead reconnaissance program. We had clearances that were so closely held that the code words describing them

themselves were classified "top secret." You weren't allowed to say the words unless you were in Washington, and even then they could be referred to only by initials. I'm exaggerating only slightly. But we were the adjudicators of many titanic turf fights involving the Air Force, the CIA, the National Security Agency, and so on. This was a very influential committee, and the country owes an enormous debt to Din Land for his leadership.

The other activity that I alluded to, that Sid was part of, was a gang of four, that met clandestinely with Henry Kissinger to advise him on strategic military issues. I'm sorry to say that, at least in my opinion, this group never won the "big ones" in their discussions with Henry, and I think that they were used by him so that if ever challenged, he could point to the high level group of advisors he had. I was not a member, and I will not reveal for reasons of national security any of the other members' names. I don't think any of them in retrospect regard the time spent as their finest hour. Sid may want to quarrel with me later about that.

Now, I've really only scratched the surface of Sid's many contributions to national security. He's an intimate of senators, he's advised the Senate Intelligence Committee. (Is that an oxymoron, senate intelligence?) He's a real "star wars" fan; he goes to Livermore constantly and he plays with Edward Teller and Lowell Wood, for example. He also publishes very penetrating analyses with his colleagues at the Stanford Arms Control Center showing the utter lunacy of "star wars" and the profound dangers that the current program poses for important U.S. security needs, to say nothing of being a negative force through international security.

Sid, i want to say I admire enormously your deep contributions to the true national security of this country. And you're also a pretty good physicist. Happy Birthday! Thank you.

COMPOSITENESS BELOW 1 TeV

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I am here to give a real physics talk, but I can't resist a few reminiscences before I start. It's hard for me to believe that Sid is sixty. I remember a long time ago when Stan Brodsky, Fred Gilman and Haim Harari turned thirty, Sid and Harriet organized a party which they called a "wake"; the last thirty years appears to be ample proof that in Sid's case there is indeed life after death.

I met Sid first when I came to Stanford as a graduate student in 1968, and as many of you remember, those were exciting times. When we weren't distracted by politics, we did a lot of interesting physics. All of us who passed through SLAC in that period of the late 1960s and early 1970s remember the openness and generosity of the Theory Group here, much the same way, perhaps, that an earlier generation cherished the spirit of Niels Bohr's Institute in Copenhagen during the 1920s and 1930s. It's not easy to create an atmosphere like that. I've learned that since leaving SLAC for the East Coast. The atmosphere in the SLAC Theory Group was largely a reflection of Sid's personality. As his students, we learned a peculiar style of physics from Sid: analytic, of course, but intensely focussed on the real world and not so concerned with elegance and form as much as with resourcefulness and ingenuity, using the tools at hand or rigging up new ones as necessary to claw our way into the belly of the problem. I'm reminded as an example of that kind of physics, of a series of papers now largely forgotten - papers by Sid and Don Levi and Tung-Mow Yan in the early 1970s - in which the parton model was set forth in tremendous detail in the context of super-renormalizable field theory. It's that kind of work which formed the physical basis for the things which our students now consider to be part of our folklore.

1. THE STRONGLY COUPLED STANDARD MODEL

To get down to physics: by good fortune my topic today, one that I've worked on recently, weaves together several threads of Sid's work throughout his career. The issue is compositeness. Not, in this case, compositeness of hadrons, but instead compositeness of quarks, leptons, and W and Z bosons. The tools we use are dispersion theory and asymptotic analysis. We make much use of the special leverage which electromagnetic interactions provide into the structure of composite particles and which lies at the heart of SLAC and of many of Sid's contributions to theoretical physics. As evidence of the vitality

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of these ideas, my talk is not nostalgic. Instead, I'm talking about a model for new physics at the frontier of our highest energy accelerators, including SLC.

I shall describe a scenario for compositeness at energies below 1 TeV in a version of the standard Glashow-Salam-Weinberg model in which the $SU(2)_L$ gauge interaction becomes strongly coupled. Mark Claudson, Eddie Farhi and I have reviewed and analyzed this model in a recent *Physical Review* article.¹ The model has a fairly long history: Larry Abbott and Eddie Farhi were the first people to suggest it as a serious alternative to the standard model of the weak interactions.² But their work relies on dynamical ideas which were developed by Bjorken³ and Hung and Sakurai⁴ and subsequently a host of other people have worked on it.⁵ It seems necessary nowadays, more than in the past, to present motivation for one's work. Most of my theoretical colleagues are working on superstring physics at Planck mass scales, and when I start talking about TeV physics, I feel obliged to present some good reasons for doing so. My first reason is to look for new physics at SLC, LEP, HERA and Tevatron energy scales. Proponents of the SSC point out that if there is new physics associated with weak symmetry breakdown it will be seen there. Often the argument is turned around to the effect that the new facilities of this decade will not see departures from the standard model. One motivation of our work is to give a counterexample to this bit of wisdom. We aim to demonstrate that there is a window for quark, lepton, W and Z compositeness at $< \text{TeV}$ energies. Windows are dangerous things. Sometimes you are presented with an open window, you stick your fingers through it looking for something and someone slams it shut quickly. It's quite possible that this window will be closed by experiments at SLC and its friends. At the moment (and I expect for the lifetime of these accelerators) it provides an interesting and illuminating alternative to the standard model.

The second motivation is to explore an important dynamical question in gauge field theories, namely: how is the Glashow-Salam-Weinberg Lagrangian in fact realized in nature? One usually assumes that the W and Z gain their mass through spontaneous breakdown of $SU(2)_L$ gauge invariance. In the simplest form: the Higgs mechanism. We are interested in an alternative mechanism for mass generation of this theory: the masses of the particles come about because of the confinement of the quanta which interact by the $SU(2)_L$ gauge interaction. We are interested in knowing which of these routes nature chooses. Of course, the question always arises "If the standard model works so well" (which it does and there are no experiments that disagree with the standard model in any significant way at the moment, and I am not going to claim that there are), "is it possible that a different and richer theory can mimic it?" The answer to this question is: "Perhaps." It depends on answering some dynamical questions about confining gauge theories that we don't yet have powerful enough computational techniques to address.

Well, that's the motivation. Let me tell you what the theory is. It is conceptually extremely simple. You simply take the Glashow-Salam-Weinberg Lagrangian, and you change the parameters a bit. You take the $SU(2)_L$ gauge

interaction, which is usually considered to be weak and treated perturbatively and you turn up its strength so that the confinement scale for $SU(2)_L$ is of order 300 GeV. Then the $SU(2)_L$ becomes confining at 300 GeV: all non-singlets are confined. The Higgs mechanism never occurs because you turn down the mass parameter of the Higgs sector so that symmetry breaking can be ignored at the scale at which the confinement takes place. That's the model.

I want to repeat that in this model the $SU(2)_L$ is confining, it's not spontaneously broken. There is no Higgs mechanism. We're dealing only with confining non-Abelian gauge symmetries. That's one of the aesthetic arguments for this theory. There aren't two kinds of gauge theories in nature: some, like QCD which are unbroken, and some like $SU(2)_L$ which are spontaneously broken. Here, all non-Abelian gauge symmetries are treated in the same way.

A remarkable feature of this theory is that in a certain technical sense, it is naturally consistent with all known weak phenomena. One of my aims is to explain how it can be that two apparently so different dynamical schemes can fit the same data. I don't want to create the impression that this subject is of only phenomenological interest. Many interesting issues of theory arise in trying to understand the strongly coupled standard model. It's a very nice example of a phenomenon known as complementarity⁶ which occurs in gauge theories with fundamental scalar particles, and allows for massless composite fermions. In a theory where the natural scale of compositeness is hundreds of GeV, it is important to have a natural explanation why quarks and leptons have masses that are so much lighter. This is an issue Sid grappled with from time to time over the past decade. In this theory there is a good possibility that it's a natural phenomenon.

Vector dominance is central to understanding the dynamics of this theory. Sid taught me to be very skeptical of vector dominance in strong interaction physics, and it is amusing to see it again in a possibly more believable context than before. The most useful way for analyzing the low energy dynamics of this theory is by dispersion theory, which I know to be dear to Sid's heart.

There are many unresolved problems in this theory, and many of the entries on this list of unresolved problems are, in fact, among the traditional motivations for considering composite models. This model does not solve the hierarchy problem - why the masses of the W and Z are so much lighter than the Planck mass. It does not solve the problem of generations. It does not solve the problem of where fermions get their masses. [You will notice that these are all problems with the Glashow-Salam-Weinberg Lagrangian. Since the Lagrangian of this theory is the same as that theory, it's not surprising it does not solve those problems.] It has another problem, namely it can't be made consistent with traditional $SU(5)$ unification because we've made the $SU(2)$ coupling constant so large that as we run it up in energy, it doesn't join the other two coupling constants at any scale. Had proton decay been detected, I would not be giving this talk. But it hasn't and unification remains a hypothetical question, and we're dealing with facts at 1 TeV.

II. QUANTA, SYMMETRIES, AND COMPOSITES

One of the pleasures of discussing this subject is to rediscover the standard model in a new light. Even if this version of the model has nothing to do with nature, it provides a wonderful insight into the standard model. It's a rare treat to take a model, turn it by hand into something apparently very different by changing its parameters, then analyze its phenomenology and find it indistinguishable from the original version of the model at low energies. To understand this physics we have to work our way through the standard model without some of the prejudices that come to us in textbooks. That's what I want to do for the next five or ten minutes. First, I'm going to list the particles that play a role in the model, then go through the symmetries, and finally talk about the composites that are formed in the strongly interacting version of the model.

A. Quanta

The quanta are all familiar. Let me define the notation. There are a large number of left-handed fermions, ψ_L^i . In fact, there are twelve doublets of left-handed fermions. They're doublets under the $SU(2)_L$. There are four generations, each of which contains a lepton doublet and a quark doublet. The quark doublet comes in three colors. The label a runs over those twelve indices. [If you ever get confused about the model I'm talking about, just look it up in a textbook. It's the same Lagrangian as the standard model.] Next are the fundamental scalar fields. They form a complex $SU(2)_L$ doublet. One convenient way to organize those four degrees of freedom is in a 2×2 matrix

$$\Omega = \begin{pmatrix} \phi_1 & -\phi_2^* \\ \phi_2 & \phi_1^* \end{pmatrix}. \quad (1)$$

Each column is an $SU(2)_L$ doublet, since ϕ_a and $\epsilon_{ab}\phi^b$ are both doublets. This is a familiar matrix representation for the scalar fields of the standard model.

Now, of course, there is a bunch of gauge bosons, ω_μ^h , associated with the $SU(2)_L$ gauge interactions. These become the W and the Z in the perturbative version of the standard model. For us, they are "gluons"; the quanta of a confining non-Abelian gauge interaction. They're not going to be observed in nature directly. There are other gauge bosons which are not affected by the confinement of these $SU(2)_L$ interactions. They include the photon, A_μ , which is the Abelian gauge boson. It does not undergo mixing with ω_μ^3 by means of spontaneous symmetry breakdown. There are also eight non-Abelian gauge particles, G_μ^a , associated with the $SU(3)$ color interaction, the QCD

gluons. In addition, there are a set of right-handed fermion fields, ψ_R^i , which play very little role in these dynamics. There is a tremendous asymmetry between left and right here. Right-handed particles do not feel the strong $SU(2)_L$ interaction. Some people find this very abnormal, but it occurs in the standard model. The left-handed electron interacts by an $SU(2)$ interaction which the right-handed electron simply doesn't feel. The only difference is that the interaction is stronger in this version.

B. Symmetries

It is essential to understand the symmetries. They are the symmetries of the standard model, but we look at them a little more carefully and perhaps with a slightly different slant. The local gauge symmetries are familiar: an $SU(3)$ of color, an $SU(2)_L$, and the $U(1)$ which will be electromagnetism. The global symmetries must be examined more carefully. For the moment, let's ignore color, electromagnetism and the Yukawa couplings that give masses to the light fermions. This is reasonable if we're interested in the dynamics at the confinement scale, which is of order 300 GeV. At that scale, QED and QCD are relatively weak. The Yukawa couplings are also small because they give rise to much smaller masses. In this limit, all twelve left-handed doublets of fundamental quarks and leptons are equivalent, and the Lagrangian is invariant under an $SU(12)$ symmetry. Furthermore, its a chiral symmetry: only the left-handed particles transform nontrivially. $SU(12)$ plays the role of the flavor symmetry of these confining interactions, just like $SU(3) \times SU(3)$ plays the role of the flavor symmetry for QCD. It is going to be an important symmetry here.

In addition, there are symmetries of the scalar sector; they are present in the standard model, and they must be looked at again here. Let me remind you what those symmetries are. The renormalizable self-interactions of the scalar particles can be written either in terms of the 2×2 matrix, Ω , or in terms of a four component real scalar field. The potential is assumed to be invariant under rotations in this four-dimensional real space. So the scalar potential has an $SO(4)$ invariance. It's well-known that $SO(4)$ is isomorphic to $SU(2) \times SU(2)$. That's particularly obvious in terms of the transformations of the 2×2 matrix, Ω . In fact, the invariance is just that we can multiply Ω on the left by an $SU(2)$ matrix U and on the right by another $SU(2)$ matrix that I'll call $V^\dagger$.

$$\Omega \rightarrow U \Omega V^\dagger \quad (2)$$

For a mnemonic, we label the symmetry $SU(2)_L \times SU(2)_R$, where L and R refer to left and right multiplication on Ω . In the standard model Lagrangian, the $SU(2)_L$ invariance is gauged, that is, this symmetry is not just global, it's been made local. The $SU(2)_R$ is just an extra global symmetry of the Weinberg-Salam model. It's responsible, in part, for the relationship $M_Z = \frac{M_W}{\cos \theta_W}$ in the standard model. The other quanta transform in a trivial way under this $SU(2)_R$. The left-handed fermions are invariant, and the right-handed fermion fields can also be chosen to be invariant.

C. Parameters

The model is going to have to fit a lot of low energy data with a very small number of parameters. Let me remind you what they are. There's the gauge coupling of the $SU(2)$ gauge theory, g_2 . Normally, g_2 is treated as a dimensionless parameter renormalized at a scale of the order G_F , and its

value experimentally is ~ 0.6 . That's not going to be the case here; g_2 is going to become large and tend to infinity as the energy drops toward the Fermi scale. Just as in QCD, it's useful to describe these interactions, not by a dimensionless constant g_2 , but instead by a dimensional parameter Λ . In order to avoid confusing it with Λ_{QCD} , I'll call it $\Lambda_{SU(2)}$. This is the mass scale characterizing the unbroken $SU(2)_L$ interaction. We trade that by dimensional transmutation for the dimensionless coupling g_2 ; g_1 is the coupling of the $U(1)$ gauge interaction. That symmetry never breaks and, therefore, this g_1 is just the electric charge e . The other parameters (also parameters of the standard model) are parameters of the scalar potentials. There's a mass scale v^2 and a quartic self-coupling, λ . My convention is that $v^2 > 0$ corresponds to spontaneous symmetry breakdown in the standard formulation.

$$\mathcal{L}_{\text{scalar}} = \frac{\lambda}{4} (\text{Tr } \Omega^\dagger \Omega - v^2)^2 \quad (3)$$

D. Composites

Now let's see what happens to this theory when the $SU(2)_L$ is confining. All physical particles have to be $SU(2)_L$ singlets, just as in QCD all particles have to be color singlets. The nicest way I know of to catalog the possible particles in such a theory is to write down local interpolating fields which are singlets under the confining gauge symmetry and see what such fields can create out of the vacuum. So, I'll look for particles that we know and love by constructing interpolating fields that will create them.

I'll start with the most complicated case, namely the light composite fermions like the electron, muon, quarks, and so on. To make an $SU(2)_L$ singlet, we combine two $SU(2)_L$ doublets. The scalars and each of the fundamental fermions are doublets under $SU(2)_L$; so, we can form a singlet simply by contracting $\Omega^\dagger$ with ψ_L^a .

$$F_L^a \equiv \Omega^\dagger \psi_L^a \quad (4)$$

which is clearly invariant under $\Omega \rightarrow U \Omega$ and $\psi_L^a \rightarrow U \psi_L^a$. ψ_L^a is not invariant under $SU(2)_R$; it is a doublet

$$F_L^a \rightarrow V F_L^a \quad (5)$$

since under $SU(2)_R$, $\Omega \rightarrow \Omega V^\dagger$ and $\psi_L^a \rightarrow \psi_L^a$.

To understand this better, let's look at the two particles that form the $SU(2)_R$ doublet. That's most easily done by going back to the complex scalar doublet notation for the scalars. There are two ways to make an $SU(2)_L$ singlet out of two $SU(2)_L$ doublets. One is by contracting $\phi^\dagger$ with ψ using the Kronecker δ ,

$$(F_L^a)_1 = \phi_i^\dagger \psi_{Lj}^a \delta_{ij} \quad (6)$$

the other is using the ϵ symbol, which for $SU(2)$ happens only to have two indices

$$(F_L^a)_2 = \phi_i \psi_{Lj}^a \epsilon_{ij} \quad (7)$$

These are in fact the two components of this $SU(2)_R$ doublet. Let's work it out for a special case. Take the fundamental electron doublet – let me call it a preon – the preonic electron doublet which contains a preonic electron neutrino and a preonic electron. Let us see what the composite states are. Equations (6) and (7) give us the following linear combinations.

$$F_L \begin{pmatrix} N_e \\ E^- \end{pmatrix}_L = \begin{pmatrix} \phi_1^\dagger e_1^- + \phi_2^\dagger \nu_{eL} \\ \phi_1 \nu_{eL} - \phi_2 e_1^- \end{pmatrix} \quad (8)$$

When we study electromagnetism in this model we will find that the top component is neutral, the lower component has large -1 , and it will become apparent eventually that these two particles have the quantum numbers and interactions of the physical electron neutrino and the physical electron. There is a certain unavoidable confusion of $SU(2)$ groups here. Each of these particles separately is a two component, left-handed Weyl spinor. Each is an $SU(2)_L$ singlet. But these two particles together form a doublet under the global $SU(2)_R$ symmetry. So "right" here refers only to which side of the Higgs field you multiply by V . It doesn't refer to chirality. For every one of the fundamental fermions, there is exactly one composite fermion doublet of this form. So we have candidates for massless composite fermions with the same quantum numbers as physical quarks and leptons.

Next, we consider composites made from the fundamental scalar fields.

First consider

$$H = \text{Tr } \Omega^\dagger \Omega. \quad (9)$$

This is clearly an $SU(2)_L$ singlet and a Lorentz scalar. It's also an $SU(2)_R$ singlet. Now, furthermore, it decouples from any massless composite fermions. The reason for that is that the massless particles that interact by the $SU(2)_L$ strong interaction are all left-handed. But a scalar particle cannot decay into $\bar{F}_L F_L$ by the same reasoning that forbids the π^0 to decay into an electron and a neutrino when the electron's mass is zero. So this scalar particle, H , has the same couplings to the physical quarks and leptons fundamental as the Higgs particle in the Weinberg-Salam model. Notice the $SU(2)_R$ quantum numbers continue to mimic the standard model's $SU(2)$ quantum numbers. There is no isovector analog of this scalar because if we try to make an interpolating field

$$\tilde{H} \equiv \text{Tr } \Omega^\dagger \tilde{\Omega} \quad (10)$$

it vanishes (Bose statistics).

Now let's look at the P waves we can make out of Ω . [I'm not going to carry this on forever; I just have to get the important objects on the table.] Take two scalars and put in a derivative

$$\tilde{W}_\mu = \text{Tr } \Omega^\dagger \tilde{\Omega} D_\mu \Omega \quad (11)$$

then this is a spin-one object. It's an $SU(2)_L$ singlet, but, because of the r -matrix, it's an $SU(2)_R$ triplet. In short, the $\tilde{W}_\mu$ has the quantum numbers of the $W^\pm$ and Z provided we identify the $SU(2)_R$ global symmetry of the fundamental Lagrangian with the $SU(2)$ of Fermi's theory, in nature. This identification continues to work because there is no isosinglet degenerate with the $\tilde{W}_\mu$. You cannot make an $SU(2)_R$ singlet vector particle analogous to $\tilde{W}'_\mu$ because if you construct that operator

$$W_\mu = \text{Tr} \Omega^\dagger D_\mu \Omega = \partial_\mu H \quad (12)$$

it is simply the derivative of the operator that creates the Higgs particle. So it does not create a new particle, it just creates the H in a P wave.

If you study this model carefully, you find that the operators of low dimensions create only the states that are known in the standard model. More complicated operators create new states. Let me list a few:

$$\begin{aligned} F_{\mu L}^a &= \text{Tr} \Omega^\dagger D_\mu \psi_L^a \\ F_{\mu R}^a &= \text{Tr} \Omega^\dagger \gamma_\mu \omega_\mu^a \psi_L^a \\ (S, T)^{ab} &= \psi_L^a \psi_L^b \\ B_{\mu b}^a &= \tilde{\psi}_L^a \gamma_\mu \psi_L^b, \text{ where} \end{aligned} \quad (13)$$

F_μ^a creates massive, composite spin-3/2 quarks and leptons. I won't analyze how it comes to be an $SU(2)_L$ singlet or how it comes to have both chiralities. We can also make $SU(2)_L$ singlets by combining two of the $SU(2)_L$ doublet fundamental fermions. That way we make difermions, states which have the quantum numbers of a lepton plus a quark, or two leptons or two quarks. They come in $(1/2, 1/2)$ -vector, $(0, 0)$ -scalar and $(0, 1) \oplus (1, 0)$ -antisymmetric tensor representations of the Lorentz group. All are candidates for excited states in this theory.

III. CONSTRAINTS ON COMPOSITENESS

Now I would like to examine whether this theory can mimic the standard model. Can it naturally satisfy the strong constraints on compositeness which are demanded by the data? I'm not going to be able to do this subject justice in so short a time, but I'd like to give a flavor of the analysis. First, I'll describe the symmetry constraints. Then, I will describe some of the general dynamical constraints, and then, finally, I will show how the study of electromagnetic interactions allows us to dissect the dynamics.

The first general symmetry constraint is to have (almost) massless fermions. Any successful composite model must give a natural explanation why the physical fermions have masses that are so small compared to the scale of compositeness. One way to do this is to introduce chiral symmetries which keep them massless until we add small Yukawa couplings that then give

them their observed masses. That's how it's done in the Weinberg-Salam Lagrangian: That's how it's done here. The $SU(12)$ chiral symmetry will keep the composite fermions massless, provided it does not break spontaneously.

In QCD, if you make the quarks massless, you don't get massless fermions but instead massless pions, because the symmetry is broken in the vacuum. 't Hooft⁶ wrote down a very beautiful series of conditions that are necessary for the chiral symmetries in a confining gauge theory to remain unbroken in the vacuum. They are not sufficient conditions, but they are an extremely restrictive set of necessary conditions. The strongly coupled standard model is perhaps the only realistic composite model which satisfies these conditions. Time does not permit me to describe them. They're simple, they're beautiful, and they're satisfied in this model. The second restriction is more obvious and also quite powerful. The confining gauge group has to be anomaly-free. We can't tolerate anomalies because they make the theory internally inconsistent or non-renormalizable or both. In general, theories with chiral fermions are in trouble with triangle anomalies. But in this theory the triangle anomalies vanish because the gauge interaction is $SU(3)$ and the coefficient of the triangle anomaly is zero in $SU(2)$. The third restriction is that an $SU(2)$ invariance must survive to low energies, because weak interaction phenomenology is dominated by an observed $SU(2)$ invariance. This theory has such a natural candidate: the $SU(2)_R$ of the Higgs sector. Finally, flavor changing neutral currents are usually a problem for any composite model because the bounds on processes that change flavor without changing charge are restrictive: compositeness limits of the order of 10^3 TeV have been placed by experiment on theories which have flavor changing neutral currents. Here, flavor changing neutral currents are suppressed because the underlying (Glashow-Salam-Weinberg) Lagrangian gives rise to the G.I.M. mechanism. I know of no other composite model with a compositeness scale under TeV which satisfies these symmetry constraints. So when people talk about "generic composite models" at TeV energies, I believe they are talking about this model.

A. Dynamical Constraints

Let me turn to dynamical constraints on the theory. Of course, the best way to understand the dynamics of a theory would be sit down and calculate. Here, it can't (yet) be done. The confining gauge theory is a very subtle one. It contains only left-handed fundamental fermion fields, and as Banks and Casher⁷ pointed out many years ago, simple models for analyzing confinement in gauge theories break chiral invariance by coupling right-handed fermions to left-handed fermions at the boundaries. The MIT bag model is an example. In the strongly coupled standard model there are no fundamental confined right-handed fermions. So the tools that we have for analyzing the dynamics of confining gauge theories don't apply to this model.

It is possible to argue that if you make the fundamental scalar field Ω heavy enough, it decouples, and when it decouples, it leaves behind a theory

which is isomorphic to QCD but with two colors and six flavors. Instead of twelve left-handed Weyl fermions you have six Dirac fermions. Since all representations of $SU(2)_L$ are self-conjugate, the particles you have called anti-particles continue to transform the same way under gauge transformations and $SU(2)_L$ is unbroken. We all would agree, I think, that two-color, six flavor QCD breaks its symmetries dynamically. So in the limit in which the scalars decouple, we know that the chiral symmetries do break spontaneously. The first dynamical constraint on the parameters of this theory is that the chiral symmetries must not break spontaneously.

The second also comes from experiment: now that the mass of the W has been measured, we know the coupling constant between left-handed fermions and the physical W particle,

$$g^2 = \frac{4G_F M_W^2}{\sqrt{2}}. \quad (14)$$

This effective coupling is measured to be approximately 0.6. In this theory, g is a strong interaction coupling among composite particles. Later, I'll come back to the question of whether 0.6 is too small to be a strong interaction coupling constant. First, let's see whether we have enough parameters in the theory – this theory doesn't have many parameters – in order to fit these two observed facts: (1) that the chiral symmetry should not break spontaneously; and, (2) that this coupling constant should be equal to 0.6. The four parameters (e , v , λ , and $\Lambda_{SU(2)}$) are restricted by the fact that the mass of the lightest vector particle is M_W . In the Glashow-Weinberg-Salam model, v is adjusted to obtain M_W . Here, loosely speaking, $\Lambda_{SU(2)}$ determines M_W . The $U(1)$ coupling is already fixed to be e . That leaves us only with two parameters. One is λ , the quartic self-coupling of the scalars, and the other is the ratio of the two mass parameters in the theory, $v^2/\Lambda_{SU(2)}^2$.

Consider, then, the dynamics as a function of $v^2/\Lambda_{SU(2)}^2$. Figure 1 shows some features of the theory at a slice of fixed value of λ . To make this graph finite, I've used $\tan^{-1} v^2/\Lambda_{SU(2)}^2$ rather than v^2/Λ^2 as the abscissa. Remember v^2 is defined so that when it is large and positive, the scalar potential is unstable at the origin and one is in the symmetry breaking mode. When v^2 is negative, the scalar potential is stable, and as v^2 becomes large in that direction, the scalar field decouples.

Now, the usual way we think about the standard model is that v is ~ 250 GeV, and $\Lambda_{SU(2)}$ is extremely small, corresponding to $\tilde{g} \sim 0.6$. In that case, $\tan^{-1} v^2/\Lambda_{SU(2)}^2$ is almost equal to $\pi/2$. At $\pi/2$, the gauge interaction is turned off and the effective coupling between the fermions and the W 's is zero. As we depart slightly from $\pi/2$, the gauge coupling rises and at some value for $v^2/\Lambda_{SU(2)}^2$, $\tilde{g}$ crosses 0.6. If we believe the standard model is implemented perturbatively, this is the point at which the world lies. Instead, we want to consider the possibility that the mass scale of the theory is set by the strong interactions of the $SU(2)_L$ sector, so $\Lambda_{SU(2)}$ is of the order of 300 GeV, v^2

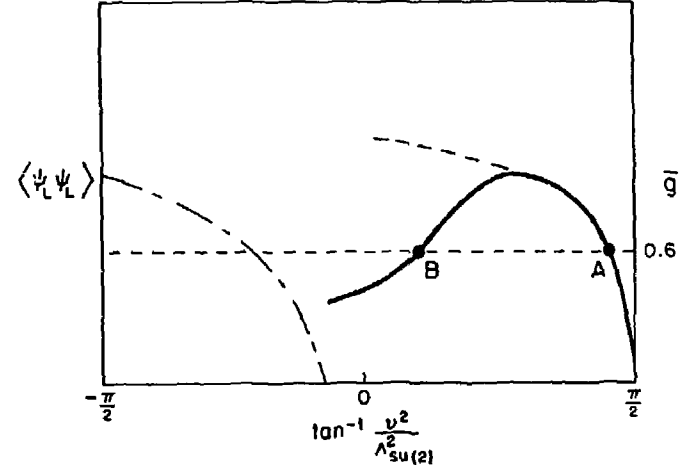


Fig. 1: Dynamical constraints on a strongly coupled standard model. $\tilde{g}$ (solid line) and $\langle\psi_L\psi_L\rangle$ are shown as functions of $\tan^{-1} v^2/\Lambda_{SU(2)}^2$ at fixed M_W and Λ , for a hypothetical case in which there is both a perturbative (A) and strongly coupled (B) version of the model.

is small, and, therefore, the inverse tangent is near zero. To describe nature, it is necessary that the curve in Fig. 1 cross 0.6 again in a region where the theory is strongly coupled. This is a crucial constraint. If it does not happen, if the curve simply continues up as shown by the dashed line, then there is no confining mode of the standard model which can be made to agree with experiment.

The problem of chiral symmetry breaking can also be described using Fig. 1. At the far left, $\tan^{-1} v^2/\Lambda_{SU(2)}^2 \approx -\pi/2$, the scalar decouples, and the chiral symmetries break. When they break we get massless bosons instead of massless fermions. For the model to be viable the point where the coupling constant passes down through ~ 0.6 must occur when the chiral symmetries remain unbroken.

These are the dynamical constraints on the model without which it simply is not a candidate for a phenomenological description of the weak interactions. It would be extremely interesting to have sophisticated enough lattice calculations for a chiral gauge theory with scalar particles or some other way to answer these equations theoretically. At the moment we have a choice: we can assume this scenario does not happen and we close up shop; or we assume it happens the way I am describing, and we go on and find out whether this theory can describe weak interaction phenomena in detail.

IV. ELECTROMAGNETISM IN THE STRONGLY COUPLED STANDARD MODEL

Describing weak phenomena in detail sound like a long haul, and I am not going to try to do that. Instead, I am going to investigate electromagnetic behavior as a way of coming to grips with general weak phenomena at low energies all at once.

First, let me point out that electric charges work out correctly. That appears confusing, but it is trivial. There is no spontaneous symmetry breakdown, so the $U(1)$ we put into the Lagrangian remains exact: the photon is massless; the coupling g_1 is just e , and, furthermore, the $U(1)$ charges of the quanta in the Weinberg-Salam model — called hypercharges there — are the electromagnetic charges. If you remember your textbook on the Weinberg-Salam model, those $U(1)$ charges are peculiar. The $U(1)$ charge for the scalar is $-1/2$. The $U(1)$ charges for the fermions are $-1/2$ for the leptons and $+1/6$ for the quarks.

In the strongly coupled standard model these are the electromagnetic charges of the fundamental, but *confined*, particles. They could be probed by deep inelastic scattering from $e.p.$, a physical electron at high enough Q^2 . Of course, in the confining version of the standard model, we don't see these fundamental particles in isolation in nature. We only see $SU(2)_L$ singlet bound states. Let's consider some of them. The particle which is going to be a candidate for the physical W comes in three charge states, schematically, $\phi^+\phi^-$, $\phi^0\phi^0$, or $\phi\phi$. If we combine the $U(1)$ hypercharge of these we find that are

$+1, 0$ and -1 , in agreement with the electromagnetic charge of the physical W . The composite fermions have $U(1)$ hypercharge which is just the sum of the hypercharge y_a of the fundamental fermion and the scalar or its conjugate with hypercharge $\pm 1/2$. If we take these peculiar hypercharges of $-1/2$ and $+1/6$ and combine them in this way, we get the usual charge assignments for quarks and leptons. If you look at the right-handed fermions, the $U(1)$ charge assignments in the Glashow-Weinberg-Salam model have been cooked up from the beginning to give them the correct electromagnetic charges. So the charges work out correctly.

Now let us use electromagnetic interactions to see how the standard model phenomena emerge at low energies. I will describe an effective Lagrangian approach which is somewhat oversimplified, but much of Ref. [1] is dedicated to removing the oversimplification.

Suppose we ignore electromagnetism, $SU(3)_{\text{color}}$ and the sources of fermion masses. Later, add fermion masses by introducing Yukawa couplings and then add color and electromagnetism in turn. An effective Lagrangian for the interactions of particles at energies below scale of compositeness has an exact $SU(12) \times SU(2)_R$ — this is the $SU(2)_R$ of the Higgs sector — global symmetry. Let us write this effective Lagrangian including all the terms of dimension four or less. This should summarize the interactions of fermions with W 's and W 's with themselves at low energies. For the moment, I will ignore exotic particles. We'll worry about putting in diquarks, dileptons, lepto-quarks and so on later.

$$\begin{aligned} \mathcal{L}_0^n = & i\bar{F}_L \not{\partial} F_L^c - \frac{1}{2} \bar{W}_{\mu\nu} \cdot \bar{W}^{\mu\nu} + \frac{1}{2} m_W^2 \bar{W}_\mu \cdot \bar{W}^\mu - g \bar{W}^\mu \cdot \bar{J}_L^\mu \\ & - \frac{1}{2} g_3 \bar{W}^{\mu\nu} \cdot (\bar{W}_\mu \times \bar{W}_\nu) + \frac{1}{4} g_4 (\bar{W}^\mu \cdot \bar{W}_\mu)^2 \\ & + \frac{1}{4} g_4' (\bar{W}^\mu \cdot \bar{W}^\nu) (\bar{W}_\mu \cdot \bar{W}_\nu) \end{aligned} \quad (15)$$

where $\bar{W}_{\mu\nu} = \partial_\mu \bar{W}_\nu - \partial_\nu \bar{W}_\mu$ and $\bar{J}_L^\mu = \frac{1}{2} F_L^2 \bar{\gamma}^\mu F_L^2$. Here, g, g_3, g_4 and g_4' are dimensionless couplings which characterize the interaction between on-shell, physical composite particles much like the ρ -nucleon, ρ^3 and ρ^4 couplings. In the SCSSM they are determined by the underlying gauge theory. However, their values are not equal to that of the gauge coupling and cannot be calculated in any known manner from the $SU(2)_L$ strong interaction dynamics.

This Lagrangian should describe the interactions of the familiar particles with one another. Note all three W 's have the same mass because of the $SU(2)_W$ symmetry. Note also that there is only one dimension four cubic coupling among three vector particles that is isospin invariant, but there are two quartic couplings. Finally, there is the single isospin invariant coupling between the W and the left-handed fermions. [I am ignoring the right-handed fermions here: they don't play any role in this discussion.] Of course, there are terms of dimension greater than four, but they are suppressed at low energies

because of derivative couplings. This Lagrangian does correctly describe weak charged current phenomena at low energies. All you need to get the charge current weak interactions is $g\bar{W}^\mu \cdot j_{L\mu}$ and the proper value for M_W . Notice, however, this is not a Yang-Mills theory: there are lots of different coupling constants — g_3, g_4, g_4', g .

Now, let's add electromagnetism. The simplest way of making a gauge invariant electromagnetic coupling would be to make the minimal substitution on what we started with. That gives a bunch of electromagnetic couplings which involve the field strength A^μ and are complicated to write down but simple to calculate. In addition, there are two other dimension four couplings which are separately gauge invariant and not given by the minimal prescription. One is an anomalous magnetic moment for the W :

$$\mathcal{L}_{\text{MAG}} = -ie\lambda \bar{F}^{\mu\nu} W_\mu^+ W_\nu^- . \quad (16)$$

Since it has a derivative of the photon field, it vanishes linearly at zero photon momentum corresponding to a magnetic moment type coupling. The second coupling is very suggestive of the old days of vector dominance. It is a gauge invariant coupling in which the W_3 , which is neutral, simply turns into a photon:

$$\mathcal{L}_{\text{VD}} = -\frac{k}{2} \bar{F}^{\mu\nu} W_{\mu\nu}^3 . \quad (17)$$

Note neither $\mathcal{L}_{\text{MAG}}$ nor $\mathcal{L}_{\text{VD}}$ is $SU(2)_R$ invariant: the $SU(2)_R$ of the scalar sector of the GWS Lagrangian is broken by the $U(1)$ interactions. In order to study weak and electromagnetic interactions together at low energies, where we can ignore higher dimension operators, we must write down all these terms, so the Lagrangians which defines our effective theory below the compositeness scale is

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \mathcal{L}_{\text{eff}}^0 + i\bar{\psi}_{Rb} \not{\partial} \psi_R^b - \frac{1}{4} \bar{F}^{\mu\nu} \bar{F}_{\mu\nu} + e a^\mu j_\mu^{\text{em}} - i e a^\mu (W^{+\nu} W_{\mu\nu}^- - W^{-\nu} W_{\mu\nu}^+) \\ & - e^2 (a^\mu a_\mu W^{+\nu} W_\nu^- - a^\mu a_\nu W_\mu^+ W_\nu^-) + e g_3 (a^\mu W_\mu^- W^{+\nu} W_\nu^3 \\ & + a^\mu W_\mu^+ W^{-\nu} W_\nu^3 - 2 a^\mu W_\mu^3 W^{+\nu} W_\nu^-) - \frac{k}{2} \bar{F}^{\mu\nu} W_{\mu\nu}^3 - i e \lambda \bar{F}^{\mu\nu} W_\mu^+ W_\nu^- \end{aligned} \quad (18)$$

where j_μ^{em} is the fermion electromagnetic current.

Now let us study, for example, the electromagnetic interaction of a composite left-handed fermion using dispersion theory. The vertex is shown in Fig. 2. The momentum transfer is q^μ , and only Dirac structure allowed is γ_μ . There is no $\sigma_{\mu\nu} q^\nu$ coupling because it would connect left to right, violating the chiral symmetry. The left-to-right coupling will be introduced when small masses are added. Since the photon breaks the global $SU(2)_R$ symmetry, we expect to find both an isoscalar coupling with some normalization y and an isovector coupling which is normalized to $\frac{\tau_3}{2}$.

$$e \langle F_L^a | J_{\text{em}}^\mu | F_L^a \rangle = e \bar{U}_L \gamma_\mu \left\{ y F_s(q^2) + \frac{\tau_3}{2} F_v(q^2) \right\} U_L \quad (19)$$

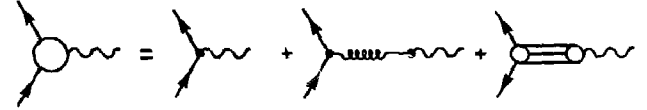


Fig. 2: Isovector electromagnetic form factor for a composite fermion. The pointlike (subtraction constant) and W -pole terms are separated out.

Let's study the isovector form factor. We know two things about it. The first is that the isovector charge of all the fermions is +1 in units where $e_3/2$ has been taken out. The second is that these are composite fermions, and therefore, the on-shell form factor should vanish as q^2 goes to infinity. Because we know $F_V(0)$, we can write a subtracted dispersion relation for the form factor.

$$F_V(q^2) = 1 + q^2 \int \frac{d\sigma^2}{\sigma^2} \frac{\rho_V(\sigma^2)}{q^2 - \sigma^2} \quad (20)$$

Let us refer back to our effective Lagrangian, and separate out the contributions to the dispersion integral which come from the terms which we already know. I'll call these the contributions from the "minimal sector" of this complicated composite theory. Other contributions come from the "exotic" sector: all the new particles or new couplings of old particles which might appear in this channel. This includes, for example, spin-3/2 fermion pairs, excited W 's Z 's and so on. The only important minimal sector contribution to ρ_V comes from the neutral W that can mix with the photon. All the needed coupling constants are written down in the Lagrangian; we can just write out this term (see Fig. 2)

$$F_V(q^2) = 1 - \frac{k g}{e} \frac{q^2}{q^2 - M_W^2} + q^2 \int \frac{d\sigma^2}{\sigma^2} \frac{\rho_E(\sigma^2)}{q^2 - \sigma^2} \quad (21)$$

where $\rho_E(\sigma^2)$ includes all exotic sector contributions to ρ_V . Now, if we use the fact that the form factor vanishes at infinity, we get a constraint, a superconvergence relation,

$$1 - \frac{k g}{e} + \int \frac{d\sigma^2}{\sigma^2} \rho_E(\sigma^2) = 0. \quad (22)$$

If the coupling of the lightest fermions to the exotic sector of the theory is weak, either because the coupling is intrinsically weak or the masses of these states are large, then perhaps it is reasonable to ignore this exotic contribution. That is going to be my approach: first ignore ρ_E , then study the impact of the exotic contributions. From this we obtain the constraint that $\frac{k g}{e} = 1$. A remarkable relationship between the W -photon mixing, the W -fermion-fermion coupling, and the electric charge. Those of you who remember the days of vector dominance, will know that is a standard relation of vector dominance.⁸

If we proceed systematically and apply this argument to all the electromagnetic form factors of all the composite particles in the theory, we come up with the following series of relationships.

$$\begin{aligned} k &= \frac{e}{g} \\ g_3 &= g \\ g_4 &= -g'_4 = g \\ \lambda &= 1 \end{aligned} \quad (23)$$

If we now take these relationships between coupling constants and put them back into the effective Lagrangian, we find that it turns into the Salam-Weinberg Lagrangian in unitary gauge (except for the Higgs sector). The transformation that does the job is:

$$\begin{aligned} Z_\mu &= \sqrt{1 - k^2} W_\mu^3 \\ A_\mu &= a_\mu + k W_\mu^1 \\ \sin \theta &= k \end{aligned} \quad (24)$$

We can eliminate all of the other coupling constants in terms of the single parameter k which now appears in our Lagrangian as the sine of the weak angle, $k = \sin \theta_W$.

So we have learned that, provided we can ignore the coupling to the exotic sector as a first approximation, then we reconstruct the standard model Lagrangian as a starting point. That means we predict the W and Z masses correctly. Also, that the theory predicts the exact form of neutral current couplings correctly, and that the bulk of radiative corrections [except for those that depend on the exotic sector] are identical to those given by the standard model — because the effective Lagrangians coincide through dimension four.

Note that all this happened without introducing any new parameters in this transformation. All we did was divide the theory up into something that reduces to the standard model plus corrections that we can now handle on a one by one basis using dispersion theory. This prepares the way for very nice program of comparing experiments at new accelerators and precision low energy tests of the standard model with the predictions of composite models, namely take the exotic particles and exotic couplings that exist in this theory, put them into the analysis that I've described and see how they feed down and affect low energy phenomena.⁹ At the moment, tests of the Glashow-Salam-Weinberg model tell us that generically the mass scale for new particles in this theory is of the order of 300 GeV. If the particles, the lepto-quarks, the dileptons, the excited W 's and Z 's were much lighter than 300 GeV this theory would be ruled out already. However, one should be cautious because some novel particles (e.g., leptoquarks, spin-2 W 's, etc.) are difficult to see at low energy and cancellation among the contributions from the exotic sector are possible, even likely.

I would like to close with another illustration of the dispersion method that I think might be of particular interest to Sid, namely the anomalous magnetic moment of the W . It is really very amusing to see how this theory works. The anomalous magnetic moment of the W in Yang-Mills theory has to be exactly one (at tree level) and it's not clear how this is constrained in the strongly coupled standard model. Sid, of course, has been deeply interested in magnetic moments throughout his career. For us, the W is a massive, composite vector particle. Its magnetic moment interaction involves the polarization of the

initial and final W , a factor of the momentum of the photon and a form factor $F_M(q^2)$.

$$\langle p' \epsilon' | J_\mu | p \epsilon \rangle = \cdots + (\epsilon'^1 \cdot q \epsilon_\mu - \epsilon \cdot q \epsilon'^1) F_M(q^2) \quad (25)$$

The contributions to the static magnetic moment $F_M(0)$, come from the terms in that effective Lagrangian. The way we have broken it up, we have a minimal electromagnetic contribution,

$$-ie A^\mu W^+ \nu W_{\mu\nu}^- + \text{h.c.} \quad (26)$$

which contributes 1 to $F_M(0)$, and we have a possible anomalous moment

$$-ie \lambda F^{\mu\nu} W_\mu^+ W_\nu^- \quad (27)$$

which contributes λ to $F_M(0)$. Since we have parameterized F_M at zero minimum transfer, we can write a subtracted dispersion relation.

$$F_M(q^2) = 1 + \lambda - \frac{2kg_3}{\epsilon} \frac{q^2}{q^2 - M_W^2} + q^2 \int \frac{d\sigma^2}{\sigma^2} \frac{\tilde{\rho}_M(\sigma^2)}{q^2 - \sigma^2} \quad (28)$$

As before, we have separated out the contribution from the tree graphs in the dimension four effective Lagrangian. The tree graph in this case is the $W^0 W^+ W^-$ coupling,

$$-\frac{1}{2} g_3 \tilde{W}^{\mu\nu} \cdot (\tilde{W}_\mu \times \tilde{W}_\nu) \quad (29)$$

combined with the W^0 photon mixing. Notice, however, that the isospin algebra gives us a factor 2 relative to the charge form factor. There are also exotic contributions, parameterized by $\tilde{\rho}_M$. These exotic contributions will give rise to deviations from the result we are about to obtain. If we demand that the magnetic moment form factor of the W vanish at large momentum transfer because the W is composite, we get a constraint,

$$1 + \lambda - \frac{2kg_3}{\epsilon} - \int \frac{d\sigma^2}{\sigma^2} \tilde{\rho}_M(\sigma^2) = 0. \quad (30)$$

Now we already know that $\frac{kg_3}{\epsilon}$ has to equal one. That constraint comes from the charge form factor of the W . If $\tilde{\rho}_M$ can be ignored, the anomalous magnetic moment of the W must be 1. So we can understand and parameterize deviations of the anomalous magnetic moment of the W from unity as limits on the exotic sector of the theory. Once again, a Yang-Mills structure emerges in this composite model at low energies, provided the exotic sector of the theory is heavy enough and/or coupled enough.

In closing, let me summarize and briefly discuss the future. To summarize the dynamical assumptions: first, it must be possible for the physical coupling constant between the W and the light composite fermions to rise through 0.6 in the perturbative sector of the theory and then fall through 0.6 in a region where

the underlying gauge interaction is strong. No one knows if this is reasonable or not. In QCD the effective coupling constant between the nucleon and the ρ meson is about eight times this large, but QCD is a very different gauge theory. It is a vector gauge theory with both left- and right-handed fermions; it breaks its chiral symmetries dynamically and it is $SU(3)$ not $SU(2)$. Second, we must assume that the $SU(2)_L$ gauge interaction, in the presence of a scalar field, does not break the chiral symmetries in the region where the $SU(2)_L$ interaction is strong. That is the only way we know to have light fermions. Finally, we already know from experiment that the mass spectrum of this composite model must have a gap between the W and Z at masses of order 100 GeV and the remainder of the spectrum at masses in excess of several hundred GeV. Is this kind of spectrum ruled out in such a gauge theory? Once again we do not know until we have solved the dynamics. So my optimistic conclusion is that it is possible that new physics will be found below one TeV and can be understood in such a theory. If so, at accelerators like SLC and others coming on in the near future, we will start to see evidence of departures from our naive implementation of the same underlying Lagrangian which gives rise to such bizarre objects as diquarks, lepto-quarks, higher spin leptons, higher spin W 's and so on. Thanks very much, and happy birthday, Sid.

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Our Changing View of Hadron Structure⁺

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We are here to celebrate Sid's birthday. Alas, I have trouble spending this time on hymns of praise to Sid. He would be squirming like mad, and I'd probably squirm too. It is the state of this talk that I don't even have anything nasty to say. But I can't not take some time talking about Sid. We all know him as a great teacher. There is plenty of evidence for that in the large collection of brilliant students in this room, not only thesis students but also others who came to Stanford as post docs and, by the time they left, knew they were Sid's students. We also know him as an author of some accomplishment. (I know better than you do!) We know Sid for his superb physics taste and what I can only call wisdom. That is something that is very hard to come by, and which all of us who come in contact with Sid learn a little, not enough, about. And, as Bob Jaffe mentioned, there is the very special atmosphere here at SLAC, which really is his doing, that I would characterize as the demand for high intellectual standards, together with a sense of humility and lack of pretense, and last but not least, having fun. In addition to all this Sid has, of course done a little research now and then. I won't go into that.

⁺In celebration of Sidney Drell's Birthday, Stanford Linear Accelerator Center, Stanford California, August 6-8, 1986.

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For me this occasion has added significance. My whole career in physics and then some has been with Sid. When I was an undergraduate at MIT I had the good fortune of taking not just one, but several courses from him. Sid always tells about the problem sessions where I'd sit in the back of the room calculating away, instead of listening to his eloquent words. He was curious about that, came back to see what I was doing and found I was doing my math homework. The reason for that was that I didn't have to do the physics homework. His plenary lectures were so good that I really didn't have to go to the problem sessions at all. But it seemed the polite thing to do.

By chance both Sid and I went from MIT to Stanford in the massive western emigration of 1956, where I stayed on as a graduate student. After finishing my thesis I stayed on as co-conspirator writing those infamous books known in some quarters as "Broken and Dull". Since then it has been a continuing close relationship as colleague and always as student. Both in terms of my own intellectual and personal growth, my relation to Sid has been very much the relation of a son to his father.

This is a celebrative occasion and I would like this to be a contribution in the traditional sense of such feasts, something like a birthday present to the honored one. Therefore, the subject should be dear to Sid's heart, and it should be the kind of talk that one loves to hear. I find that when I hear a talk I really like and then reflect on it afterwards as to why did I like it, it's often not only because the delivery was good, it was eloquent, articulate and clear, but because the subject matter was already familiar to me. So much so that I come away with the feeling that "yes, that was

a good talk, but I really understood that subject better than the speaker." I expect I can succeed in leaving Sid with that feeling. Whether I can be eloquent and articulate is another matter.

The title of this talk is not meant to be any kind of scholarly history, but something of an impressionistic retrospective of how the field has developed as I learned about it and saw it shaped, largely through the interaction with SLAC and with Sid in particular. When put all to other, it is impressive how many twists and turns there have been in trying to understand what is inside of a proton, and how hard the task has been every step of the way.

Let us start before World War II. I'm not sure how people felt about the problem at the time. Did people even ask the question, "What is the size of a proton?" and if so, on what grounds did they try to answer it? The existence of a strong force was known. There was the Yukawa meson theory and its wavelength would give a natural size for a proton. Hadrons in cosmic rays were known to attenuate in the atmosphere so there was some idea of pp cross sections at various energies, which when turned into πr^2 would give a rough estimate of size. But the prewar prehistory probably didn't lead to much that was very crisp. In the postwar period, from 1946 or so to the early fifties (1951 or 1952), things really started to happen.

The Yukawa theory was verified, and some data appeared which were more specific to nucleon size, such as the measurements of their anomalous magnetic moments. Since pions were coupled to these nucleons it would then be natural to expect that the pion cloud carry a charge around whatever kind of bare nucleon there

would be, and this would give rise to changes in the magnetic moments. In addition, there were experiments on scattering of slow neutrons from electrons. The interference between the nuclear scattering of the neutron at very low energies with the scattering of the atomic cloud gave rise to a wiggle in the angular distribution, thereby giving an estimate of the charge radius of the neutron.

And so these fragmentary pieces of evidence suggested structure in the proton and neutron. It is interesting that they already utilized an electromagnetic probe of the hadron rather than using the strong interaction. How did theorists try to interpret the data? During this time, there occurred the great revolution in quantum electrodynamics, where Schwinger's canonical transformations and Feynman's diagrams explained the electron anomalous magnetic moment. So it was natural to apply those techniques to the strong interaction problem as well. In so doing, one started with a bare nucleon plus a meson cloud, using a field theory which looked something like quantum electrodynamics. The nucleon degree of freedom replaced the electron and the pion replaced the photon, with pseudoscalar Yukawa couplings of the π 's to the nucleons. This "core + cloud" picture, from the present point of view, looks rather primitive in its approach. But that was about the only concrete technique available.

There were a host of difficulties with trying to do things this way. Unlike electrodynamics, the perturbation theory did not converge. The original calculations were done even before the Feynman-diagram perturbation theory, and must have been really arduous. There were other approaches such as strong coupling theory. One of the most interesting approximation schemes was

what is called the Tamm-Dancoff method. Murph Goldberger already mentioned that Sidney Dancoff was Sid's thesis advisor. In this method one essentially expands in terms of the number of mesons in the wave function of the nucleons. The nucleon was supposed to be a bare nucleon, with amplitude $\sqrt{Z}$ or a single meson with a certain amplitude, or a nucleon plus two mesons with a certain amplitude, etc. Then one stopped, not allowing more mesons than a given maximum in the state, and then tried to solve the field theory within that approximation. But it was found to be an inconsistent approximation, and didn't survive. The inconsistencies have to do with all the problems of relativistic quantum field theory, such as renormalization, pair creation, divergences, etc. But the main problem was the fact that one can't limit the number of degrees of freedom in the states without getting into inconsistencies.

The early fifties must have been a real low point in trying to grapple with this problem. One certainly knew that it was totally unreasonable to think of nucleons as point particles. They had to have an interior structure, but there was no reliable technique. There was little data and, as we know retrospectively, even the starting point of "core + cloud" has major deficiencies.

But during the rest of that decade, everything was uphill. The progress made during that time was very impressive and came from both experiment and theory. Many accelerators came online in the fifties. It was something of a golden age. One began to get beams of pions and photons of hundreds of MeV, and started scattering experiments. With π -nucleon scattering and pion photoproduction, the 3-3 delta resonance was discovered. Bob Hofstadter and his

collaborators scattered electrons from nuclei. In that series of beautiful measurements and, I must say, the beautiful analysis that went with it, the size and shape of nuclei were determined with great precision and elegance. It was natural that this program would, as the electron energy went up, generalize to the nucleon itself. Certainly, the focus on how to see the size of the nucleon by using electron scattering came into its own at that time. Meanwhile in the rest of the world there soon occurred a proliferation of meson resonances. At the very end of the decade, strange particle physics started flourishing and weak interaction form factors provided a new avenue into hadron structure. So the database was growing very rapidly.

The theory of nucleon structure evolved in trying to understand the 3-3 resonance and pion scattering. The big advance came in the development of the Chew-Low theory, which treated the nucleon core as a static source of infinite mass relative to the pions that were scattered from it. The theory went beyond perturbation theory and successfully accounted for the existence of the delta resonance in terms of a starting point which was still the "core + cloud" picture of bare nucleons and pions coupled with Yukawa couplings. That view evolved through the decade from its first incarnation into dispersion relations. It was found that the Chew-Low theory was essentially a simplified version of dispersion relations.

The dispersion relations themselves were formulated relativistically, generalized in a whole host of directions and caught most theorists' fancy in a way which is comparable to the way superstrings catch the fancy of theorists today. Most everyone was

working on it and for good reason. It was really a very solid development. At long last one could theoretically approach problems in the strong interactions in a way which had a certain amount of rigor. Those of the younger generation today cannot know what it was like working on shifting sands with nothing solid underneath, trying to find some firm rock on which to build a theoretical structure that would hold. Nowadays the closest analogue may be the efforts to go beyond the standard model, where there are few guide posts. But the dispersion relations related observables to each other, and allowed limited, but sound predictions to be made. But still the "core + cloud" picture was not weakened much, if at all, while this field developed.

Now as the experiments evolved (especially at the Stanford linac at HEPL), the form factors of the proton and neutron, i.e. the Fourier transforms of their charge and current distributions, were measured very beautifully. The theoretical formalism for that had to be developed and the dispersion relations were the natural tool. Bob Jaffe's talk is very reminiscent of the kinds of things being done then. These were quite successful in the sense that they anticipated the existence of the ρ and the ω mesons. They also highlighted the importance of crossing relations. One should not only look at electron + proton going to electron + proton but also should look at the crossed processes $e^+e^- \rightarrow p\bar{p}$, the latter linked to the former by analytic continuation of the scattering amplitude. When the proton form factor was viewed from that point of view, it became clear that all the states produced in e^+e^- annihilation, e.g. two pions, three pions, the ρ , and ω and so on, are very relevant in understanding the structure of the proton. Viewed from the crossing relations, the proton structure was closely related to meson

resonances, hence to pion structure. Introduction of the meson-meson interactions and what they were doing gave a very different perspective and started the liberation from the "core + cloud" picture.

Between 1960 and 1964, this dispersion theory approach to nucleon structure made much progress. The experiments were coming along and the size of protons was well-measured. And through theory there was some idea how that size was related to the parameters of the strong interactions such as the mass of the ρ and the ω . The business was quantitative and really quite encouraging. At last there was a consistent descriptive mechanism for describing the nature of proton and neutron structure.

The end of this period was really the turning point for our understanding of hadron structure as we know it now. 1964 is the year of the quark. But other conceptual revolutions were happening too. Entering the 1960's, the "core + cloud" view was I think dominant, and most people regarded the nucleon as fundamental and the delta as "derived". If the nucleon were fundamental, then an "elementary" π scatters from the "elementary" nucleon, making the not so "elementary" resonance that is called the delta. If the π were fundamental, then the π 's interacting with each other made a not-so-fundamental ρ resonance. But as the dispersion relations which evolved from Chew-Low theory and the crossing relations allowing linkage of the "scattering" channels to the "annihilation" channels entered the lore, the conceptual picture changed. Words like "Bootstrap" and "Nuclear Democracy" started entering the scene, ideas which were very revolutionary and I think very important.

The idea was that when one scatters π from the nucleon, one gets a Δ resonance because of the forces. But then somebody would look at π scattering from the Δ , and again there was a resonance and this $\pi\Delta$ resonance occurred near the mass of the nucleon. Also, the π scattered with a π via ρ exchange could make a ρ resonance. This even occurred in p - p scattering so maybe the ρ was creating a force strong enough to make itself. That was a radical notion, the "bootstrap." With this kind of thinking, which was backed up to some extent with some rough calculation, one could think of the Δ either as the derived object or as an elementary object, and the nucleon as a derived object almost as easily as the other way around. The thought of the nucleon and the π having a privileged status and everything else being composite started to weaken.

Meanwhile, other things were happening experimentally. The privileged place of the nucleon was weakened by the discovery of all its strange partners in the SU(3) octet. Along with that, the dispersion relations were generalized into what is called current algebra, that beautiful theory of flavor symmetry. It provided a solid foundation for all the ideas of weak interaction dynamics, uncluttered by the yet-to-be understood problems of how to do field theory for weak interactions as well as strong interactions. Current algebra also turned out to be a powerful tool for describing hadron structure.

With the advent of current algebra and a host of experiments, the weak decay phenomenology was enriched. Semileptonic decays of hadrons, given the lepton pair in the final state, turned out, via crossing, to be as useful in studying hadron structure as electron scattering. The species of hadrons for which one could study

structure via weak-charge form factors and the like was greatly enriched. Not only nucleons were available, but also π 's, K's, hyperons--everything that had weak decays. In addition to these experiments on weak decays, this was the era of the resonances, giving rise to the very rich strong interaction spectroscopy. It was clear that when one hit a hadron it rang in all sorts of eigenmodes; not only for non-strange but strange particles as well.

With so much raw material, the botanists got into the game and classified them into groups; SU(3) was born. As SU(3) established itself, the problem of the origin of the three in SU(3) remained obscure. There was originally the eight-fold way followed soon by 10-fold ways and 27-fold ways. Then Gell-Mann and Zweig's quarks came along in 1964. The idea then seemed rather radical. It would have been even more radical if the "bootstrap" and "nuclear democracy" had not preceded it. The idea of the nucleon and the Δ being on equal footing was in the lore, although not the idea that they are in a common hyperfine multiplet. In any case 1964 and the introduction of the quark was an obviously essential step in really finding out what was inside of the nucleon.

From 1964 to 1970 the quarks became more established, mainly through the successes of the quark model in simply interpreting the spectroscopy. The machine energies were going up. SLAC came on, AGS, and PS, the ISR, and even dreams of the Fermilab 200 BeV machine. As one went up in the energy scale one clearly started thinking of not only studying the average charge distributions of the hadrons (elastic form factors which are essentially Fourier transforms of time averaged charge or current distributions in the hadron), but also to try to find out the instantaneous distributions, as revealed

by inelastic scattering, now deep-inelastic scattering. Long before the experiments, that concept was a very familiar one, via experience with atoms, molecules and especially nuclei.

In particular, there were experiments on the inelastic scattering of electrons from nuclei done at Stanford and elsewhere. With enough excitation energy and a coarse-grained energy average the instantaneous distribution of charges and currents inside the particle being probed is seen rather than time averaged ones. The buzzwords that go with this kind of work are "sum rules", very closely related to dispersion relations, "closure approximations", i.e. the statement that the level density of the important excited states be large compared to the excitation energy, and-most of all-the "impulse approximation," meaning that the time that the probe takes to cross the object being probed to be small compared to the natural periods of motion of the object probed. All of these three concepts, of course, are intertwined.

This kind of lore was well known in terms of probing nuclei. It was natural then to generalize that lore to the problem of the nucleon. The problem was that in the case of nuclei one has a system that is non-relativistic, so that good old quantum mechanics is OK. But in the case of the nucleon clearly one was dealing with a GeV mass scale and relativistic kinematics was essential.

The answer to the problem came with the parton picture and the infinite-momentum, light cone techniques. Instead of trying to keep the nucleon at rest one goes to the opposite extreme and sends it to infinite momentum, with the lepton probe also going to infinite energy in the other direction. The nucleon thereby is

liberated from the static picture totally. The reason that this works is by now familiar. The nucleon becomes a Lorentz-contracted pancake so that the time of lepton transversal becomes very short. The motions of the ingredients inside the nucleon are slowed down by time dilatation. Thus the impulse approximation again becomes valid, so that one can do at infinite momentum what one did with nuclei at zero momentum. On the experimental side of course the deep-inelastic experiments at Stanford, neutrino interactions elsewhere, and last but not least, production of dileptons in hadron-hadron collisions everywhere has established the usefulness of this picture, which allows such a simple, powerful view of the instantaneous distribution of constituents inside the hadrons.

Here of course I have to mention the creators of the theory of dilepton production in hadron-hadron collisions, namely Naive Drell and Yan, who produced the universally known Naive Drell-Yan formula. I must say, it didn't seem naive at the time. The answer seemed too easy, and at least for me it seemed hard to believe it should work without corrections from other more important hard processes overwhelming their calculated part. But "Naive Drell" and "Naive Yan" had thought it through very well and were confident it would work. And that theoretical development has certainly enriched the field ever since in terms of giving us a way of probing the internal structure of all kinds of hadrons, beam particles as well as targets, via dilepton production by π 's, K's, antiprotons, or hyperons. In addition this process has been useful in terms of production of things like the ϕ and T , not to mention the W and the Z .

The next era is 1970 to 1974, the glory days of the gauge theories. 1970 was a banner year, with the G.I.M. mechanism for the neutral currents, and with 't Hooft establishing non-Abelian gauge theories as real theories which exist and are calculable. After a decade of mistrust and apprehension, field theory came back into fashion with a vengeance. This story is often told, and I will not go into it here. As far as hadron structure was concerned, hard collisions really came into their own in this period, when experiments here at Stanford and the neutrino experiments elsewhere probed the fractional charge of nucleon constituents by the magnitude of various structure-function ratios. The constituent spins were right because of the polarization asymmetries, R and the F_3 of neutrino processes. By 1974 the "naive" parton view had penetrated most everyone's consciousness, and more cumbersome descriptions which were based on the heritage of current algebra and dispersion relations, e.g. light cone current algebras and various moment analyses (which still have their place, I must say) largely gave way to the simple folklore of the parton model. It also became clear that 50% of the nucleon's constituents were neutral. And the quark structure was established with clarity, as opposed to "core + cloud" which had long since disappeared. The problem started focusing on the final state, and why the quarks-or at least how the quarks-did not come out after they were hit so hard in these hard collisions. Meanwhile, other things were happening to the "bootstrap" and dispersion relations, such as "duality," the Veneziano model, and string theories which led, I think, in an important way to our present picture of quark confinement and of course, now to the superstrings.

Finally, the last decade has seen a total commitment to field theory, a real theory of the strong and electroweak forces, and the convincing discoveries of the gluons, W, and Z. The fundamental theory is in place up to but not including the Higgs sector. Of course, charm and the discovery of the ψ was a big turning point with charmonium the deuteron if not the hydrogen atom of the strong interactions. Looking at charmonium in the modern sense, in terms of a quark and an antiquark bound together by a potential that has to do with gluon exchanges, is a far cry from the way one looked at the ρ as a bound state of two pions due to ρ exchange, and it certainly has shaped things ever since. Adapted to baryons, it meant the "core + cloud" picture was clearly dead. The core was replaced by three quarks. The hydrogenic picture of a "core + cloud" is replaced by a Thompson nucleon where the seeds in the plum pudding are the quarks and the pudding is the gluons. It is a long way from where one started, with a remarkable variety of viewpoints that led to it.

Now are those old viewpoints really obsolete? I think not. Bob Jaffe is using some of them very effectively in the weak interactions. Others will probably be revived in strong interaction theory as time goes on and as people grapple with nonperturbative QCD more. Duality and the Veneziano picture, which is connected to the linearly rising Regge trajectories characteristic of confining potentials, is certainly consistent with the QCD potentials and strings. The old ideas have their place and viability even today.

That finally leads to the question of whether we really understand nucleon structure now that we have QCD and all the insight of the last decade. I think the answer is both "yes" and

"no". The "yes" is that if we do not worry about first principles and strict derivations from QCD, but instead use some instinct and common sense, we understand a great deal. For example, it is incredible that the magnetic moments of not only the nucleons but the entire octet of ground state baryons have been measured quite accurately and agree with the constituent quark model at the 10% or 20% level. But whether that quark model really is derived from QCD, I do not know. The Q^2 dependence of the electromagnetic form factor is accountable by the QCD counting rules that are so much a heritage here at SLAC. And certainly the experimental life of this subject is at present very active. Things are going very well, and there appears to be a great deal of insight into the behavior of elastic form factors from QCD.

But if one now asks whether one can really derive all of these results strictly from the QCD Lagrangian, the question gets really tough. There is only one approach that I know that holds promise of derivation of some of these results from first principles, namely the lattice. Lattice QCD provides in principle a controllable nonperturbative approximation scheme, if the computer is big enough. But when we ask what that technique can really do, maybe it can get magnetic moments and some of the static properties of the nucleon. Maybe the hadron and glueball masses and other static properties can be determined. But already to calculate charge radii of nucleons is harder. To get the form factors of nucleons from lattice QCD sounds terribly hard. And can one really do the high energy and the medium distance phenomena associated with QCD dynamics (e.g. large s , small t) from these methods? Finally can one really justify, in the case of the exclusive QCD at high momentum transfers, the use of Fock-

space descriptions of the nucleon wave function? Essential in these calculations of elastic or two-body scattering processes is the notion that effectively only a finite number of gluons and quarks are in the infinite-momentum wave function. Does that really make sense, given all the history of problems with field theory and the deficiencies, if you like, of the Tamm-Dancoff method. What can one say from QCD about structure functions at very small x ? It is again a similar kind of problem. At infinite momentum, the parton model implies that there are an infinite number of constituents inside of a nucleon; it is a distribution of partons over all rapidities, with a nonvanishing mean density at any given rapidity. That adds up to infinity for the total. This is maybe another way of stating the problems that are inherent in most field theories. All of this infinite ocean of wee partons are ignored in the Fock-space approaches to exclusive QCD. It sounds a little like the early 1950's to the extent one really asks for strict construction of rigorous results from QCD.

So it may be that the next thirty years in this field will be just as active in addressing these kinds of questions, and that one of Sid's youngest student's student will eventually give another such talk as this, reviewing the progress that is yet to come. I expect that for a long time the field will continue to flourish as it is flourishing today, and it will continue to progress much as we have seen in the past.

Happy Birthday!

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