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SURVEY, APPLICATIONS, AND PROSPECTS OF JOHNSON NOISE THERMOMETRY*

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INTRODUCTION

Significant progress in the field of Johnson noise thermometry has occurred since the 1971 survey of Kamper [1].

This paper will review the foundation work of Johnson noise thermometry, survey the basic methods which do not utilize quantum devices for noise thermometry for industrial temperatures, and present some applications of noise thermometry in temperature scale metrology and process temperature instrumentation.

THEORETICAL FOUNDATION FOR JOHNSON NOISE THERMOMETRY

Johnson noise thermometry is based on the early work of Johnson [2] and Nyquist [3]. The noise-voltage power density spectrum (S_v) in positive frequency space appearing across an unloaded resistor of value R ohms was first measured by Johnson [2] and a few months later it was shown by Nyquist [3] to be given by

$$S_v = 4hfR/[\exp(hf/kT) - 1] \quad , \tag{1}$$

where S_v has units of V^2/Hz, h is Planck's constant, k is Boltzmann's constant, f is the frequency in Hz, and T is the absolute temperature in K. If the frequency is sufficiently low that $f \ll kT/h$ ($kT/h = 6.25$ GHz at $T = 300$ K), then to a very good approximation

$$S_v = 4kTR \quad . \tag{2}$$

One circuit model for Eq. (2) is a noise voltage generator in series with a resistor of value R ohms. An alternative circuit model is a resistor R in parallel with a noise current

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generator of power density (A^2/Hz) expressed by

$$S_I = \frac{4kT}{R} \quad . \tag{3}$$

Equations (2) and (3) suggest three methods of measuring absolute temperature. In the first method, the resistor's noise voltage over a bandwidth Δf_v is amplified by a signal processor using a high input-impedance, voltage-sensitive preamplifier to obtain an output rms voltage given by

$$\left[\overline{e_{nv}^2}\right]^{(1/2)} = K_v \, (4kT\Delta f_v R)^{1/2} \quad , \tag{4}$$

where K_v is the gain constant of the signal processor which is assumed to be ideal (noiseless). Calibration of the signal processor and an auxiliary measurement of R would allow a determination of the absolute temperature. The second method uses a low input-impedance, current-sensitive preamplifier with a gain K_i and bandwidth Δf_i; to obtain T from

$$\left[\overline{e_{ni}^2}\right]^{(1/2)} = K_i \, (4kT\Delta f_i/R)^{1/2} \quad . \tag{5}$$

The third method requires the measurement of Johnson noise power by multiplying Eqs. (4) and (5) to obtain the result

$$P_n = 4kK_v K_i (\Delta f_v \Delta f_i)^{1/2} \, T \quad . \tag{6}$$

which is independent of R. By dividing Eq. (4) by (5), the value of R can be found independently of T.

The output noise voltages of Eqs. (4) and (5) are fluctuating quantities that are subject to a fundamental statistical uncertainty. This integration uncertainty, as found by Rice [4] for noise described by a Gaussian probability density function appears as an averaged signal at the output of a detector followed by an integrator or low-pass network, and is given by

$$\sigma = 100 \, (c\Delta f_n t)^{-1/2} \quad , \tag{7}$$

where t is the integration time or a characteristic time-constant of the low-pass network, Δf_n is the noise bandwidth of the noise at the input to the detector, c is a constant that depends on the noise detector and the type of averaging, and σ is the percent standard deviation of the output signal from the integrator or low-pass network. A value of $c = 1$ is appropriate for narrow-band white noise and pure integration of the output signal from a quadratic detector.

Equations (4)-(7) provide the theoretical basis for Johnson noise thermometry.

BASIC METHODS OF PRACTICAL JOHNSON NOISE THERMOMETRY

The first reported practical noise thermometer, developed by Garrison and Lawson [5] for high temperature measurement (990-1340 K), employed a technique (Fig. 1) of comparing the noise voltage of a reference resistor R_1 at temperature T_1 to the noise voltage of a sensing resistor R_s at unknown temperature T_s. The two noise voltages are made equal in the low-frequency channel (Fig. 1) by adjusting R_1. The effect of capacitance across the resistors on noise bandwidths is compensated by adjusting capacitors C_s and C_1 to obtain equal noise voltages in the high-frequency channel so that $R_1C_1 = R_sC_s$. This technique eliminates errors due to signal processor nonlinearities and long-term transfer function drifts. The error caused by preamplifier noise is also reduced by the averaging provided by the low-pass filter. The unknown temperature is found from

$$T_s = \frac{T_1R_1}{R_s} \quad . \tag{8}$$

Commutation switches allow the same measurement system to interrogate both the reference and sensing resistors. To obtain the value of the temperature by this comparison technique, three quantities are measured: the reference temperature T_1, the value of the reference resistor R_1, and the value of the sensing resistor R_s. Garrison and Lawson claimed uncertainties as low as ±0.1%, but Hogue [6] estimated that their results were subject to errors up to ±0.5%. A major contribution to the error is the dependence of noise generated in the preamplifier on source impedance. Pursey and Pyatt [7] developed a new thermometer that uses reference and sensor resistors of equal impedance. In their design, balance is achieved by attenuating the noise voltage from the resistor which is at the higher temperature. The authors [7] concluded that, with further research, errors due to the preamplifier noise could be reduced to ±0.1% and, perhaps, lower.

Several other noise thermometry systems have been published [7-11], all based on the voltage comparison method. All are equipped with commutation switches, except the system developed by Fink [9]. His system (suggested by Garrison) uses a cross-correlation scheme which incorporates a sensing resistor and its associated capacitance in an R-C, π-section, two-port network. The noise voltage at each port is measured, and, after proper adjustment of the π-section parameters, the unknown temperature is determined from the values of two reference resistors and the sensing resistor and the (known) temperatures of the reference resistors. Fink reported an uncertainty of ±1% between 1.3 K and 4.2 K, and ±0.2% in the range from 77 to 90 K.

A simpler correlation scheme for noise thermometry was suggested by Shore and Williamson [12]; it uses the correlator-amplifier system (Fig. 2) of Brophy, Epstein, and Webb [13] to reduce the noise contribution of the preamplifiers, which is a major source of error in noise thermometry--especially at low temperatures. The outputs from two voltage amplification channels, driven by the noise voltage from a single sensor are multiplied and averaged (Fig. 2) to reduce the uncorrelated noise from the two channels while emphasizing the correlated noise generated by the sensing resistor. The scheme is still subject to the statistical integration uncertainty expressed by Eq. (7) and it requires careful matching of the transfer functions of the two channels. Practical realizations of the scheme were published by several authors [14-16].

The correlator-amplifier system (Fig. 2) is most useful at low temperatures and with noise voltage preamplifiers having high equivalent noise resistance, R_n. Over the years, values of R_n have been decreased from a value of 1000 Ω in 1959 for a vacuum-tube preamplifier [8] to 44 Ω in 1974 for a junction field effect transistor (JFET) input stage [17], and to 24 Ω in 1979 for a differential preamplifier with parallel JFETs in the input [18]. The lowest published value of R_n is 10.6 Ω for a wideband, feedback, single-ended, voltage preamplifier designed by Blalock [19]. The recent availability of low-noise preamplifiers may eliminate the more complex correlation techniques in future noise thermometers, except perhaps for high-accuracy thermometers used for temperature scale metrology at low temperatures.

A different approach to noise thermometry signal processing was developed by Brodskii and Savateev [20]. The temperature of a resistor is measured by counting the number of times N that the noise voltage levels exceed a reference level V in a unit time. This level-crossing process is described by the equation

$$N = A \exp [- V^2/(B + CT)] \quad , \tag{9}$$

where the constants A, B, and C depend on the value of the sensing resistor, the transfer function of the signal processor, and the noise of the signal processor. The sensitivity of this method varies with the temperature T being measured, since the output N is a very nonlinear function of T [Eq. (9)]; consequently, a rather complex calibration procedure is required. Although this method does not need an rms-to-dc converter with its inherent nonlinearities, it requires a special-purpose pulse height discriminator, which is no easier to implement than a high-linearity rms-to-dc converter [21]. Work at the Oak Ridge National Laboratory (ORNL) on noise pulse counting techniques shows no improvement in accuracy over the rms-to-dc conversion technique for the same measurement time, and no significant simplification or improvement in the signal processor. However, the counting technique allows greater discrimination against some types of nonthermal noise [21].

All of the preceeding methods of noise thermometry involve a measurement of open-circuit noise voltage and an auxiliary measurement of resistance. The quantity defined by Eq. (6)--termed "virtual power" by Johnson in his classic paper [2]--is the product of the open-circuit noise voltage and the short-circuit noise current of a resistor and is independent of the value of the resistor. A Johnson noise power thermometer (JNPT) based on Eq. (6) was developed by Borkowski and Blalock [19] in 1974. The system (Fig. 3) uses a single resistor, a voltage preamplifier with a high input-impedance and a current preamplifier with a low input-impedance to alternately sample the resistor noise through commutating reed relays at the preamplifier inputs. A stable noise source with a level large compared to the preamplifier noise was developed [22] to facilitate calibration of the JNPT amplifiers. For field use, the JNPT (Fig. 3) is equipped to perform the sample-and-hold, multiplication, and data correction functions with a minicomputer which also controls both the switching and the automatic calibration functions.

The JNPT was also implemented [19, 23] in pure analog form, using a probe containing two sensing resistors that maintain a constant resistance ratio as their temperature varies. The noise voltage of one resistor and the noise current of the other resistor are processed simultaneously. These signals are multiplied by an analog multiplier, and the product signal, after resistance-capacitance low-pass filtering, is displayed by a digital panel meter for direct indication of absolute temperature.

APPLICATIONS OF NOISE THERMOMETRY IN TEMPERATURE SCALE METROLOGY

Several high-accuracy noise thermometers have been reported for applications in the determination of the temperature scale.

Grovini and Actis, following extensive contributions [11, 24, 25] to noise thermometry in the temperature range above 670 K, have recently reported [26] the development and use of a noise thermometer to measure differences between the thermodynamic scale and the IPTS-68 (International Practical Temperature Scale of 1968). They carefully measured absolute temperature referenced to the ice point (273.15 K) with two different realizations of a noise voltage thermometer using a signal processor similar to the low-frequency channel of the Garrison-Lawson (GL) system (Fig. 1). In the first realization, the rms voltages of two equal resistors at different temperatures were compared (method of [7]). Balance of the two output voltages was achieved with a precision attenuator switched into the signal processing chain when the higher noise voltage was measured. In the second realization, a modified version of the classical GL system, the balance was achieved in two steps: first, the resistance ratio [Eq. (8)] was preadjusted to match the IPTS-68 kelvin temperature ratio; after which an attenuator in the higher level noise measurement was adjusted to achieve an accurate balance which is described by

$$T_s = a^2 \frac{R_1}{R_s} T_1 + (a^2 - 1) \frac{R_n}{R_1} T_1 \quad . \qquad (10)$$

Since a^2, the power attenuator ratio, is in practice so near unity, a measurement of R_n to within a few percent is adequate. Resistors R_1 and R_s were alumina insulated, platinum resistance thermometers with resistance values near 600 Ω at 1230 K. In both realizations, the noise voltage measurements were made in the frequency band from 20 to 120 kHz and a low-noise ($R_n \approx 120$ Ω) preamplifier with selected JFETs was used. The average uncertainty of the experimental results of Crovini and Actis was ±0.034% (99% confidence limits). Their results showed that the IPTS-68 is lower than the thermodynamic scale between 900 and 1230 K; the maximum difference is 0.56 ± 0.20 K (99% confidence limits) near 1090 K.

A high-resolution noise thermometer for measurements in the temperature range from 90 to 100 K was reported by Pickup [27]. The measurement system employed was a modified GL system; the preamplifier input circuit consisted of a JFET cascoded with a bipolar transistor second stage, yielding an R_n of ~300 Ω. The signal processor bandpass extends from ~10 kHz to 200

KHz for the low-frequency channel and the sensing resistor is ~10 kΩ. The temperature of a 3-kΩ reference resistor was (~25°C) as was indicated by a platinum resistance thermometer calibrated against the triple point of water. Pickup estimates that the overall uncertainty of his results is ± 0.0078% at 90 K and ± 0.0096% at 97 K (99% confidence limits) for 7-h integration times. His results show that the IPTS-68 is lower than the thermodynamic scale by 3 ± 6.99 mK at 90.17 K and 8 ± 9.32 mK at 97 K (99% confidence).

Another high-resolution noise thermometer for measurement of temperatures near 4 K was developed by Klein, Klempt, and Storm [18]. Their thermometer system measures noise voltage in a frequency band from 2 to 15 kHz and employs a correlator-amplifier system with a low noise (R_n = 24 Ω) differential preamplifier achieved by a parallel JFET input stage. The temperature sensing resistor is about 10 kΩ. A thorough analysis of error sources led the authors to conclude that the total uncertainty in their measurements was 0.30 mK.

Some of the latest developments in precision noise thermometry for temperature scale metrology are reported in this conference by Klempt and Storm [28] and by Pickup [29].

APPLICATIONS OF NOISE THERMOMETRY IN PROCESS TEMPERATURE INSTRUMENTATION

Brixy applied the GL method, modified by using the correlator-amplifier technique [13], to measurements of temperatures in nuclear reactors [30-32]. His preamplifier has a JFET-bipolar transistor cascode input with R_n ≈ 50 Ω. Brixy suggests some techniques [32] for solving problems arising from the use of long cables to connect the sensing resistor to the preamplifier and that it is possible to make accurate temperature measurements even with cables as long as 100 m. He reported that the measurement inaccuracy was <0.1% over a range from 300 to 1200 K were obtained under laboratory conditions, and <0.5% over a range from 300 to 500 K in high radiation fields in a nuclear reactor. Brixy concluded that noise thermometry is especially suitable for the measurement of temperature in nuclear reactors, and that no additional noise would be contributed from ionization produced in the sensing resistor by gamma rays and neutrons.

Development of the JNPT [19] at ORNL has continued since the early feasibility studies of 1971 and this work has resulted in several practical applications in process temperature measurements. Temperatures and sensor resistances are routinely measured by Johnson noise with uncertainties less than ±0.5% (99% confidence) for sensing resistors from 50 to 300 Ω, and temperature range of from 273 to 1000 K, using signal cables as long as 18 m. Calibration of the signal processor independently of the sensing resistor and connecting cable is accomplished by injection of a high-level noise signal of known spectral density directly into the inputs of the voltage and current preamplifiers [22]. The Johnson noise generated in the cable itself and the effect of the cable on the system transfer function are accounted for by applying data reduction algorithms. One algorithm, developed by Blalock [34], uses a lumped-element model for the cable has been employed successfully for most cables <20 m in length. For longer cables, a different algorithm based on a distributed model was developed by Agouridis [34].

The JNPT was used to measure temperatures of uranium fuel during irradiation in a nuclear reactor.

The JNPT sensor, made of rhenium wire with a resistance value near 80 Ω at 1670 K, was installed in the centerline of the fuel irradiated in ORNL High Flux Intensity Reactor. After 4500 hours of high-temperature (1570-1770 K) high-radiation exposure, the decalibration of the JNPT was negligible even though 80% of the rhenium transmuted to osmium [35]. Occasional bursts of microphonic noise caused by vibrations of the loosely supported rhenium coil contaminated the JNPT output. This defect was corrected by swaging the sensor sheaths to form a more rigid containment.

Recently the ORNL group applied the JNPT to an in situ calibration of reactor power plant platinum resistance thermometers [34]. The JNPT can independently measure both the temperature and the resistance of the plant thermometers without removal of the sensor. JNPT measurements in two power plants [34] show great promise for in situ calibration where access to the sensor can be achieved through short cables (<18 m). Long cables, already installed in power plants for dc measurements, may receive large amounts of nonthermal noise from the wide variety of noise sources. The major challenge in this application appears to be the characterization of both the noise contribution and the signal attenuation produced by long, industrial signal cables using measurements made only at the accessible end of the cable.

CONCLUSIONS AND PROSPECTS

Much progress has been made in Johnson noise thermometry since the early pioneering work of Garrison and Lawson in 1949. Progress has accelerated in the last 10 years due to rapid improvements in signal processor components and the increasing availability and power of digital computation. The difficulties noted by Kamper [1] of eliminating noise of nonthermal origin still remain and provide challenging opportunities in the design of grounding and shielding systems for successful noise thermometry.

The successful applications of noise thermometry to temperature scale metrology by Crovini, Actis, Pickup, Klein, Klempt, and Storm show the advantages of using a fundamentally linear, absolute thermometer for interpolation and extrapolation of the defining fixed points of the temperature scale.

The process temperature measurements of Brixy and the ORNL group indicate encouraging prospects for such applications. Major problems are the effects of extraneous noise pickup and random noise generated in the cable, and the effect of the cable signal transfer characteristics on the overall transfer function of the signal processor. Solution of these problem will be provided by specification of special cables for noise thermometry, development of effective techniques for measuring relevant cable parameters, and design of appropriate digital computer algorithms for data reduction and correction.

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2350