SAN098-1873C
SAND--98-1873C
CONF-980913--

Faster shot-record depth migrations using phase encoding

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Summary

Phase encoding of shot records provides a means of imaging a number of shots within a single migration. This results in a reduction in the required computation for a complete image, a reduction by the number of shots used in each individual migration, trading this increase in speed for additional noise in the resulting image. Some methods for phase encoding have been shown to limit this noise to a tolerable range when combining several shots, enabling speed ups of a factor of a few (Ober et al., 1997). In this paper, we present a use of phase encoding which allows faster imaging by an order of magnitude or more, with the additional benefit that the individual migrations can be stopped whenever the answer is "good enough." This approach may ultimately render 3-D frequency-domain prestack depth migration cost effective.

Introduction

Most companies today use Kirchhoff methods to perform their production 3-D prestack depth migrations, largely because they are relatively inexpensive and are flexible with respect to image size and data geometry. However, they are usually limited to single-valued, high-frequency operators. Recursive frequency-domain shot-record migration naturally accounts for multi-path arrivals and finite frequency effects, phenomena which frequently occur in the regions of complex geology being explored today. Unfortunately the cost of these methods, especially for marine surveys with hundreds of thousands of shots, is currently prohibitive for their straightforward application.

There are a couple of traditional approaches to reducing the cost of recursive frequency-domain migration. To make each migration less expensive, much effort has been made to develop more efficient yet still accurate operators. Another approach is to use a subset of the available shots to reduce the number of migrations, while attempting to maintain sufficient signal-to-noise ratio and subsurface coverage.

In this paper, we focus on a technique called phase encoding which uses a frequency-domain prestack migration method and all the available shot records to calculate the correct image, within the limitations of your velocity model and migration method. The computation is reduced by combining a number of shots within a single migration. However, there is no free lunch — phase encoding trades an increase in speed for additional noise in the image.

The question is, how much noise can be tolerated for a given decrease in cost? Figure 1 contains two images of the Marmousi data set, with the second image requiring only $\frac{1}{10}$ the amount of computation. Is this additional noise tolerable?

Theory

Phase encoding is applicable in the context of frequency-domain recursive migration methods in which each shot record and its associated source wavelet are Fourier transformed and downward propagated into the earth's interior using a one-way wave equation. The resulting source and receiver wavefields for shot j, $S_j(\underline{x},\omega_n)$ and $R_j(\underline{x},\omega_n)$, respectively, can then be crosscorrelated to produce an image for the shot record (Claerbout, 1971):

$$I_j(\underline{x}) = \sum_{n=1}^{N_\omega} S_j^*(\underline{x},\omega_n) R_j(\underline{x},\omega_n), \qquad (1)$$

The final image is then the sum of the images produced by the migration of the individual shot records. There is clearly tremendous potential for speed up if we could perform the summation over shots (at least partially) before migration.

However, the straightforward approach results in unwanted cross terms. Linearity of one-way wave equations allows us to create any linear combination of the surface wavefields before downward propagation, resulting in combined wavefields for the sources and receivers:

$$S(\underline{x},\omega_n) = \sum_{j=1}^{N_{shot}} a_{j,n} S_j(\underline{x},\omega_n) \qquad (2)$$

and

$$R(\underline{x},\omega_n) = \sum_{j=1}^{N_{shot}} a_{j,n} R_j(\underline{x},\omega_n). \qquad (3)$$

However, this approach breaks down when we apply the (nonlinear) imaging condition to the combined wavefields:

$$I_a(\underline{x}) = \sum_{j,k=1}^{N_{shot}} \sum_{n=1}^{N_\omega} a_{j,n}^* a_{k,n} S_j^*(\underline{x},\omega_n) R_k(\underline{x},\omega_n). \qquad (4)$$

Now if the coefficients have magnitude one, i.e. can be written in the form $a_{j,n} = e^{i\phi_{j,n}}$, then each $j = k$ summation reduces to one of the desired shot-record images and the correct (stacked) image,

$$I(\underline{x}) = \sum_{j=1}^{N_{shot}} \sum_{n=1}^{N_\omega} S_j^*(\underline{x},\omega_n) R_j(\underline{x},\omega_n), \qquad (5)$$

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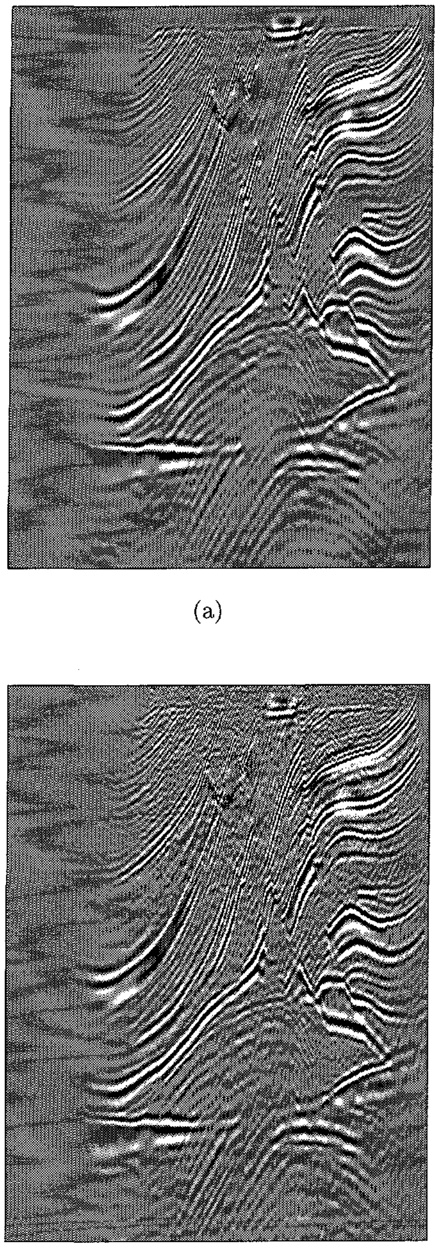

(a)

(b)

Fig. 1: Two images of the Marmousi data set, differing only in the level of random noise and the computational cost. Both images contain the same physical information, all the shot records and the same finite-difference frequency-domain migration code were used. However, the first migration used the standard approach to shot-record migration, while the second migration used random phase encoding and required only $\frac{1}{10}$ the computer time.

is contained in I_a. However, the unwanted $j \neq k$ (cross) terms,

$$\delta I_a(\underline{x}) = \sum_{j \neq k}^{N_{shot}} \sum_{n=1}^{N_\omega} e^{i(\phi_{k,n} - \phi_{j,n})} S_j^*(\underline{x}, \omega_n) R_k(\underline{x}, \omega_n), \quad (6)$$

are unphysical crosscorrelations between unrelated source and receiver wavefields. Unfortunately they are potentially as large as the correct shot-record images; clearly we should not in general choose the phases $\phi_{j,n}$ to be zero.

The research question is, how can we choose the phases so as to minimize the cross terms? A related practical issue is, how much noise can be tolerated?

Application

A number of schemes have been examined for phase encoding several shots within a single migration (Ober et al., 1997; Romero et al., 1998). The best of these schemes is a modified chirp function, which results in only 4% error when combining two adjacent shots in each individual migration in the complete imaging of the Marmousi data set. However, the error for this scheme rises rapidly when combining more than two shots per migration.

A more robust encoding scheme is also one of the simplest, that of using phases which are uniform random variables. This scheme performs relatively poorly when combining two adjacent shots, yielding a relative error of 16% for the Marmousi image. However, the error grows slowly with the number of shots used in each migration. In Figure 2, we plot the L_2 norm of the difference between the correctly migrated image for the Marmousi data set (calculated with a single shot per migration) and images where a number of randomly phase-encoded shot records were migrated together. The number of shots per migration, displayed on the bottom axis, is also the factor by which the computational cost is reduced.

The two curves in Figure 2 represent two extremes in using combinations of shot records. The solid curve represents migrations where adjacent shots in the data set were migrated together, while the dashed line represents migrations using shots that were as widely spaced as possible, given the limits of the data.[1] Clearly, having a large physical separation between shots does help reduce the crosscorrelations between unrelated wavefields; the extreme case of only having two widely separated shots per migration, which have nearly non-intersecting regions of influence, yields only 4% error. But this strategy is of

[1]For example, when two widely separated shots were used in each migration, shots 1 and 121 (out of 240) were phase encoded and migrated together. This was stacked with the image from shots 2 and 122, and so on, up to shots 120 and 240, performing 120 migrations to produce the data point. Of course when three shots were used per migration, they started with shots 1, 81 and 161, and 80 migrations were needed for the data point.

limited usefulness; we would get about the same reduction in computational cost by migrating these shots independently and only within their aperture. While using separation to help control the error will likely contribute to any production use of phase encoding, we focus here on how best to use phase encoding for nearby shots.

As we can see from Figure 2, the total energy in the cross terms (for both degrees of separation of the shots) becomes larger than the energy in the correct image when many shots are migrated together, peaking at a noise-to-signal ratio of 1.7. In Figure 3, we see that even when all 240 shots are phase encoded and imaged in a single migration, correct structure is discernible beneath the noise. While this is clearly an unacceptable final image, it does give hope, and perhaps an indication of a fruitful approach.

Notice that the error grows much more slowly than the number of shots per migration, increasing by about one order of magnitude for two orders of magnitude reduction in the computation. But, having combined all the shots in a single phase-encoded migration, what more can be done?

In essence, this experiment samples the maximum error we can expect using random phase encoding in imaging this data set. How can we reduce the error from this point? A natural way to suppress noise is to stack redundant data. Can we calculate other phase-encoded images with different cross terms? Yes, simply by using different phases.

The dotted line in Figure 4 shows the error as a function of the number of stacked random phase-encoded migrations, using all the shots in each migration. The error decreases as the square root of the number of stacked images, as expected for random noise. For ease of comparison, we have replotted the results of Figure 2, now as a function of the number of migrations required to complete the full image. This random encoding of all shots in each migration yields lower error than the straightforward use of adjacent shots in all cases, and it is nearly as good as using widely separated shots unless the shot apertures are nearly non-intersecting.

To bring ourselves back to the initial example, Figure 1b is the result of stacking 24 migrations, each using all 240 phase-encoded shot records of the Marmousi data set, requiring only $\frac{1}{10}$ the computer time needed to migrate all 240 shots independently, shown in Figure 1a. However, we've traded speed for additional noise, as can be seen directly on Figure 1b and as quantified by the appropriate point on the dotted line in Figure 4. Is this noise acceptable?

If not, then we can use to our advantage the fact that with this approach the correct image, the imaging of all the individual shot records, is contained in the image calculated with the very first migration. All subsequent migrations can be viewed as strictly increasing the signal-to-noise ratio until it is adequate for the purposes at hand. If we aren't satisfied with the answer of a specific number of migrations, we can continue adding in migrations until we are satisfied.

The extreme is to continue to the end point of the dotted curve in Figure 4, until the image is the stack of 240 migrations each using all 240 phase-encoded shots. This image, shown in Figure 5, requires as much work as the standard approach to shot-record migration shown in Figure 1a, and the minor amount of noise is certainly tolerable. Clearly there is a point along this path where we would accept the image, and consequently require less computation than is necessary to calculate the image one shot record at a time.

Conclusions

We have demonstrated that phase encoding shot records for performing frequency-domain prestack depth migration trades speed of migration for additional noise on the image. The correct image is contained in the very first migration, masked by noise in the crosscorrelations of unrelated wave fields. However, stacking subsequent migrations will improve the signal-to-noise ratio and this process can continue until an acceptable image is reached. The question is, when is the noise tolerable? When is the image "good enough" (i.e. interpretable)? This approach allows you to decide.

Acknowledgements

This work was performed while one of the authors (SM) was employed by Cray Research, a Silicon Graphics Company, whose support has been greatly appreciated.

This work was partially supported by the United States Department of Energy under Contract DE-AC04-94AL85000, and by DOE's Office of Mathematical, Information and Computational Sciences. Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company.

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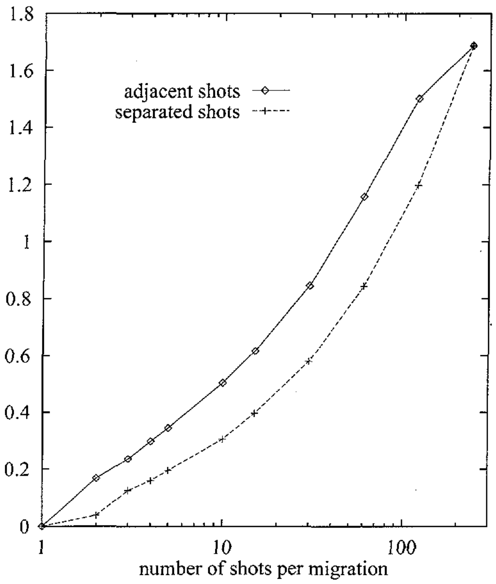

Fig. 2: The error in the Marmousi image due to phase-encoded shot-record migration, as a function of the number of shots per migration. Note that each data point represents the error in the stacked image having performed as many migrations as necessary to include all the shot records. Each square on the solid line corresponds to the error for migrating phase-encoded adjacent shot records together, while each cross on the dashed line corresponds to the error for migrating phase-encoded shots which are widely spaced.

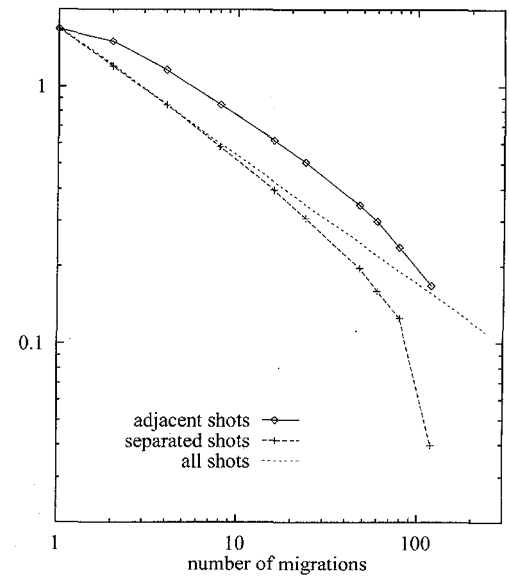

Fig. 4: The error in the Marmousi image as a function of the number of migrations. The data from Figure 2 is replotted here for ease of comparison, with the squares and crosses representing the adjacent and widely spaced phase-encoded shot records, respectively. The short-dashed curve is the error for using all 240 phase-encoded shots in each migration.

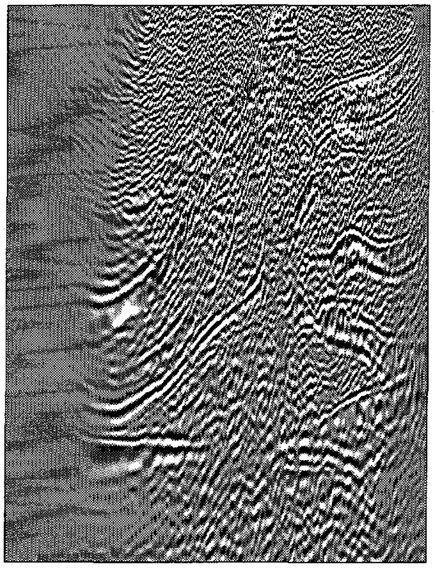

Fig. 3: An image of the Marmousi data set, where all 240 shots were phase encoded and imaged in a single migration.

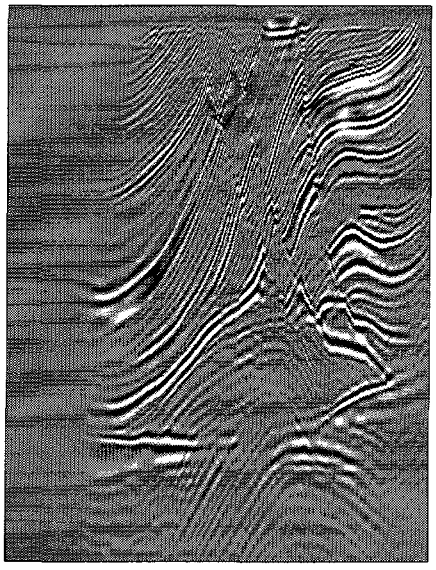

Fig. 5: An image of the Marmousi data set which is the stack of 240 migrations each using all 240 phase-encoded shots.