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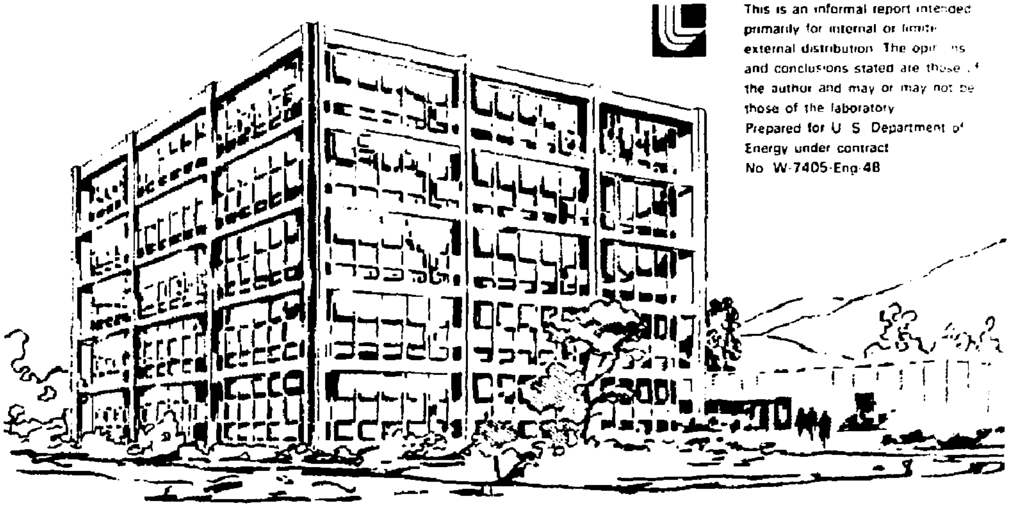

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ORBIT-AVERAGED KINETIC EQUATIONS FOR AN AXISYMMETRIC MIRROR MACHINE

ABSTRACT

The orbit-averaged kinetic equation suitable for studying collisional transport in an axisymmetric mirror machine is derived and reduced to a three-dimensional equation in phase space in the case of $\nu_c \ll \omega_b \ll \Omega$ where ν_c is the collision frequency, ω_b is the bounce frequency, and Ω is the gyrofrequency. We found that the gyrophase dependence of the distribution function, ignored in previous Fokker-Planck calculations, is responsible for the diffusion of guiding centers and should be retained when the Larmor radius is comparable to the plasma scale length. The complete Fokker-Planck equation for a square-well magnetic mirror is given as a special case to clarify the approximations and assumptions involved in the radial Fokker-Planck code of Futch. The quasilinear transport code of Matsuda and Berk is also discussed relative to the orbit-averaged Fokker-Planck equation.

1. INTRODUCTION

The numerical solution of kinetic equations with Fokker-Planck collision terms has been used extensively to study mirror reactors with neutral beam injection, neutral-beam heating of tokamak plasmas, and two component tokamaks.[1,2] Since it is not practical to solve the kinetic equation in six-dimensional phase space on a present-day computer, some simplifying assumptions are usually made to solve the problem in one- or two-dimensional phase space.

One of the important assumptions in the Fokker-Planck codes to date is that the distribution functions are independent of gyrophase angle.[3] This assumption is valid when the Larmor radius of the particles is much smaller than the scale length of the system, as in tokamaks. In mirror machines, however, the Larmor radius of high-energy particles can be comparable to the plasma radius. In such cases the gyrophase dependence of the distribution function cannot be ignored; in fact it is essential for the description of diffusion of particle-guiding centers as shown in Section 2.2. The other assumption widely used is to neglect spatial dependence of the distribution functions. Some Fokker-Planck codes take into account the effects of spatial inhomogeneity by averaging the collision terms over a particle orbit. A code that solves the Fokker-Planck equation averaged over bounce motion between mirror throats is such an example.[4]

In this report we shall first derive the kinetic equation averaged over a periodic particle orbit to include the effects of both radial and axial inhomogeneity in an axisymmetric mirror machine and point out the importance of the gyrophase dependence of the distribution function. We shall then examine the radial Fokker-Planck code of Futch[1] in the light of our averaged Fokker-Planck equation. We shall also discuss inclusion of the quasilinear diffusion due to rf turbulence in connection with the quasilinear radial-transport code of Matsuda and Berk.[5]

2. DERIVATION OF ORBIT-AVERGED KINETIC EQUATIONS

2.1 General Formulation in an Axisymmetric Mirror

We wish to derive a kinetic equation for an inhomogeneous plasma in a mirror magnetic field, taking account of both finite gyroradius and axial bounce motion. Our starting point is the Fokker-Planck equation for the distribution function $f(\underset{\sim}{x},\underset{\sim}{v},t)$:

$$\frac{Df}{Dt} = \left(\frac{\partial f}{\partial t}\right)_c + S \quad , \tag{1}$$

where $\left(\frac{\partial f}{\partial t}\right)_c$ is the Fokker-Planck collision term, S is a source (or loss) term, and the total time-derivative operator is

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \underset{\sim}{v} \cdot \frac{\partial f}{\partial \underset{\sim}{x}} + \frac{q}{M}(\underset{\sim}{E} + \frac{1}{c}\underset{\sim}{v} \times \underset{\sim}{B}) \cdot \frac{\partial}{\partial \underset{\sim}{v}} \quad . \tag{2}$$

A quasilinear diffusion term due to rf turbulence will be included later.

We assume axisymmetry and choose cylindrical spatial coordinates (r,., z) where ψ is the ignorable coordinate. For the velocity, one usually chooses a spherical coordinate system (v,θ,ϕ) where v is the speed, θ is the pitch angle, and ϕ is the phase angle of the gyromotion. For a spatially inhomogeneous system, however, it is more convenient to define the velocity in terms of constants of the motion (ε,μ,P_ψ), where ε is energy, μ is the magnetic moment, and P_ψ is the canonical angular momentum.* (The gyrophase information is contained in P_ψ.) With this choice of variables the distribution function is of the form

$$f = f(r,z,\varepsilon,\mu,P_\psi,t) \quad , \tag{3}$$

*If the magnetic field varies appreciably over a distance of one gyroradius, then μ is not a good constant of the motion; in this case one must use a more general form of the adiabatic invariant.

and the Fokker-Planck equation is

$$\frac{\partial f}{\partial t} + \dot{r}\,\frac{\partial f}{\partial r} + \dot{z}\,\frac{\partial f}{\partial z} = \left(\frac{\partial f}{\partial t}\right)_c + S \quad . \tag{4}$$

As it stands, this equation must be solved in a five-dimensional phase space $(r,z,\varepsilon,\mu,P_\psi)$ plus time (t).

To reduce Eq. (4) to something tractable, we note that the order of magnitude of the various terms in the kinetic equation can be identified with the characteristic frequencies of the particle motion in the magnetic field:

$$\frac{\partial f}{\partial t} \sim O(\nu_c f),\ \dot{r}\,\frac{\partial f}{\partial r} \sim O(\Omega f),\ \dot{z}\,\frac{\partial f}{\partial z} \sim O(\omega_b f),\ \left(\frac{\partial f}{\partial t}\right)_c \sim O(\nu_c f) \quad , \tag{5}$$

where ν_c is the collision frequency, ω_b is the axial bounce frequency, and Ω is the gyrofrequency. We seek solutions that vary on a collisional time scale; so $\left(\frac{\partial f}{\partial t}\right)_c$ has been assigned the same magnitude as the collision term. Now, assuming that the characteristic time scales are well-separated, we introduce a smallness parameter, δ, such that

$$\frac{\nu_c}{\omega_b} \sim \frac{\omega_b}{\Omega} \sim O(\delta) \quad . \tag{6}$$

Then we solve the kinetic equation by expanding f in powers of δ:

$$f = f_0 + f_1 + \ldots\ldots \tag{7}$$

where

$$f_n \sim O(\delta^n) \quad .$$

Substituting the expansion of Eq. (7) into the kinetic Eq. (4), we obtain the following equations up to the second order:

$$\dot{r}\,\frac{\partial f_0}{\partial r} = 0 \quad , \tag{8}$$

$$\dot{r}\,\frac{\partial f_1}{\partial r} + \dot{z}\,\frac{\partial f_0}{\partial z} = 0 \quad , \tag{9}$$

$$\frac{\partial f_0}{\partial t} + \dot{r}\,\frac{\partial f_2}{\partial r} + \dot{z}\,\frac{\partial f_1}{\partial z} = \left(\frac{\partial f_0}{\partial t}\right)_c + S \quad . \tag{10}$$

From the zeroth order Eq. (8) one sees that the zeroth-order distribution must be independent of r, i.e.,

$$f_0 = f_0(z,\varepsilon,\mu,P_\psi,t) \quad . \tag{11}$$

The first-order equation, when multiplied by $1/\dot{r}$ and integrated over a complete oscillation (gyroperiod) in r, yields

$$\oint dr\,\frac{\partial f_1}{\partial r} + \frac{2\pi}{\Omega}\langle\dot{z}\rangle\,\frac{\partial f_0}{\partial z} = 0 \quad , \tag{12}$$

where

$$\frac{2\pi}{\Omega}\langle\dot{z}\rangle = \oint \frac{dr}{\dot{r}}\,\dot{z} \quad . \tag{13}$$

Since the distribution function must be a single-valued function of r, the first term in Eq. (12) integrates to zero, leaving

$$\frac{2\pi}{\Omega}\langle\dot{z}\rangle\,\frac{\partial f_0}{\partial z} = 0 \quad . \tag{14}$$

From this we see that the zeroth order distribution must also be independent of z, i.e.,

$$f_0 = f_0(\varepsilon,\mu,P_\psi,t) \quad . \tag{15}$$

Putting this result back into Eq. (9) we see that the first-order distribution must not depend on r, i.e.,

$$f_1 = f_1(z, \varepsilon, \mu, P_\psi, t) \quad . \tag{16}$$

Turning to the second-order equation we can eliminate the f_2 term as in Eq. (12); the result is

$$\frac{2\pi}{\Omega}\frac{\partial f_0}{\partial t} + \frac{2\pi}{\Omega}\langle\dot{z}\rangle\frac{\partial f_1}{\partial z} = \frac{2\pi}{\Omega}\left\langle\left(\frac{\partial f_0}{\partial t}\right)_c\right\rangle + \frac{2\pi}{\Omega}\langle S\rangle \quad . \tag{17}$$

We now eliminate the f_1 term in a similar manner, multiplying by $\Omega/2\pi\langle\dot{z}\rangle$ and integrating over a complete oscillation (axial bounce period) in z. This yields the result:

$$\frac{\partial f_0}{\partial t} = \left\langle\left\langle\left(\frac{\partial f_0}{\partial t}\right)_c\right\rangle\right\rangle + \langle\langle S\rangle\rangle \ , \tag{18}$$

where the gyro- and bounce-averaged collision term is defined by

$$\left\langle\left\langle\left(\frac{\partial f_0}{\partial t}\right)_c\right\rangle\right\rangle \equiv \left(\int\frac{dz}{\langle\dot{z}\rangle}\right)^{-1}\int\frac{dz}{\langle\dot{z}\rangle}\left\langle\left(\frac{\partial f_0}{\partial t}\right)_c\right\rangle \ , \tag{19}$$

and where

$$\left\langle\left(\frac{\partial f_0}{\partial t}\right)_c\right\rangle \equiv \left(\int\frac{dr}{\dot{r}}\right)^{-1}\int\frac{dr}{\dot{r}}\left(\frac{\partial f_0}{\partial t}\right)_c \quad . \tag{20}$$

We can add the gyro- and bounce-averaged quasilinear diffusion term given by Berk[6] to the right side of Eq. (18) to obtain the complete form of the orbit-averaged kinetic equation:

$$\frac{\partial f_0}{\partial t} = \left\langle\left\langle\left(\frac{\partial f_0}{\partial t}\right)_c\right\rangle\right\rangle + \left\langle\left\langle\left(\frac{\partial f_0}{\partial t}\right)_{QL}\right\rangle\right\rangle + \langle\langle S\rangle\rangle \ . \tag{21}$$

2.2 Model Kinetic Equation in a Square-Well Magnetic Mirror

In this section we study a model problem using a simple collision operator and a magnetic square-well along the z-axis. The model collision term we use is the following:

$$\left(\frac{\partial f}{\partial t}\right)_c = C \frac{\partial}{\partial \underset{\sim}{v}} \cdot \left[\underset{\sim}{v} f + \frac{T}{M} \frac{\partial f}{\partial \underset{\sim}{v}}\right] \quad , \tag{22}$$

where C, T, and M are constants. This collision term describes drag and diffusion in velocity space. We choose a set of independent variables (r, $v_\perp$ $v_\parallel$, P_ψ) to describe the distribution function. The z-dependence has dropped out because of a magnetic square-well assumption. Perpendicular velocity $v_\perp$, parallel velocity $v_\parallel$, and canonical angular momentum P_ψ are all constants of motion under the present assumptions.

Following the procedure used in Section 2.1 we obtain for $\nu_c/\Omega \ll 1$:

$$\frac{\partial f_0}{\partial t} = \frac{1}{\tau} \oint \frac{dr}{\dot{r}} \left(\frac{\partial f_0}{\partial t}\right)_c \quad , \tag{23}$$

where f_0 is a function of $v_\perp$, $v_\parallel$, and P_ψ, and

$$\tau = \oint \frac{dr}{\dot{r}} = \frac{2\pi}{\Omega}$$

To calculate the right-hand side of Eq. (23) we first express Eq. (22) in terms of $v_\perp$, $v_\parallel$, and ϕ:

$$\left(\frac{\partial f}{\partial t}\right)_c = C \left\{ \frac{1}{v_\perp} \frac{\partial}{\partial v_\perp} \left(v_\perp^2 f + \frac{T}{M} v_\perp \frac{\partial f}{\partial v_\perp}\right) + \frac{\partial}{\partial v_\parallel} \left(v_\parallel f + \frac{T}{M} \frac{\partial f}{\partial v_\parallel}\right) \right.$$
$$\left. + \frac{1}{v_\perp} \frac{\partial}{\partial \phi} \left(\frac{T}{M} \frac{1}{v_\perp} \frac{\partial f}{\partial \phi}\right) \right\} \quad . \tag{24}$$

Noting that $P_\psi = Mrv_\perp \sin\phi + qrA_\psi/c$ (Fig. 1); thus $\partial/\partial\phi = Mr\dot{r}\partial/\partial P_\psi$, and substituting Eq. (24) in Eq. (23), we obtain

$$\frac{\partial f_0}{\partial t} = C \left\{ \frac{1}{v_\perp} \frac{\partial}{\partial v_\perp} \left(v_\perp^2 f_0 + \frac{T}{M} v_\perp \frac{\partial f_0}{\partial v_\perp}\right) + \frac{\partial}{\partial v_\parallel} \left(v_\parallel f_0 + \frac{T}{M} \frac{\partial f_0}{\partial v_\parallel}\right) \right.$$
$$\left. + \frac{\partial}{\partial P_\psi} \left(\frac{MT}{v_\perp^2} \langle r^2 \dot{r}^2 \rangle \frac{\partial f_0}{\partial P_\psi}\right) \right\} \quad , \tag{25}$$

where

$$\langle r\,\dot{r}^2\rangle = \frac{1}{\tau}\int \frac{dr}{\dot{r}}\, r^2\dot{r}^2 \quad .$$

Note that the first and second terms on the right-hand side of Eq. (25) were not affected by the orbit integral because they are independent of r.

The first two terms in Eq. (25) represent drag and diffusion in $v_\perp$ and $v_\parallel$. The physical meaning of the third term of Eq. (25) can be made more explicit by considering a case with a uniform constant magnetic field. Noting that the guiding center position R is related to the canonical angular momentum $P_\psi = M\Omega(R^2 - \rho^2)/2$ where ρ is the Larmor radius, as seen from Fig. 1, we can write Eq. (25) as

$$\frac{\partial f_0}{\partial t} = C\left\{\frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\left(v_\perp^2 f_0 + \frac{T}{M}\, v_\perp \frac{\partial f_0}{\partial v_\perp}\right) + \frac{\partial}{\partial v_\parallel}\left(v_\parallel f_0 + \frac{T}{M}\frac{\partial f_0}{\partial v_\parallel}\right)\right.$$

$$\left. + \frac{T}{2M\Omega^2}\frac{1}{R}\frac{\partial}{\partial R}\left(R\,\frac{\partial f_0}{\partial R}\right)\right\} \quad , \qquad (26)$$

where $\langle r^2\dot{r}^2\rangle$ has been evaluated to be equal to $1/2(R^2v_\perp^2)$. The meaning of the third term in Eq. (26) is now clear; it represents diffusion of the guiding-center positions. This shows that the gyrophase dependence of the distribution function has to be retained when one studies radial-transport problems with arbitrary large Larmor radius. It can be easily shown on the other hand that when the Larmor radius is small compared with the radial scale length of a plasma, the gyrophase dependence can be ignored, which is the case for the Fokker-Planck equation applied to tokamak problems.

2.3 Fokker-Planck Equation in a Square-Well Magnetic Mirror

We now derive the orbit-averaged kinetic equation with the Fokker-Planck collision term under the same assumption as in the previous section, namely, a magnetic square-well in z and uniform magnetic field in r with $\nu_c \ll \Omega$. We choose the speed v, the pitch angle θ, and the radial postion of guiding center R as three coordinates representing velocity space. Note that v, θ, and R are constants of motion.

The complete Fokker-Planck collision term in terms of v, θ, and ϕ is

$$\left(\frac{\partial f}{\partial t}\right)_c = -\left[\frac{1}{v^2}\frac{\partial}{\partial v}(j_v v^2) + \frac{1}{v\sin\theta}\frac{\partial}{\partial\theta}(j_\theta \sin\theta) + \frac{1}{v\sin\theta}\frac{\partial j_\phi}{\partial\phi}\right] \quad , (27)$$

where

$$j_v = f\frac{\partial h}{\partial v} - \frac{\partial f}{\partial v}\frac{\partial^2 g}{\partial v^2} - \frac{1}{v^2}\frac{\partial f}{\partial\theta}\left(\frac{\partial^2 g}{\partial v\partial\theta} - \frac{1}{v}\frac{\partial g}{\partial\theta}\right) - \frac{1}{v^2\sin^2\theta}\frac{\partial f}{\partial\phi}\left(\frac{\partial^2 g}{\partial v\partial\phi} - \frac{1}{v}\frac{\partial g}{\partial\phi}\right),$$

$$j_\theta = \frac{f}{v}\frac{\partial h}{\partial\theta} - \frac{\partial f}{\partial v}\frac{\partial}{\partial v}\left(\frac{1}{v}\frac{\partial g}{\partial\theta}\right) - \frac{1}{v^2}\frac{\partial f}{\partial\theta}\left(\frac{\partial g}{\partial v} + \frac{1}{v}\frac{\partial^2 g}{\partial\theta^2}\right)$$

$$- \frac{1}{v^2\sin^2\theta}\frac{\partial f}{\partial\phi}\left(\frac{1}{v}\frac{\partial^2 g}{\partial\phi\partial\theta} - \frac{\cot\theta}{v}\frac{\partial g}{\partial\phi}\right) \quad ,$$

$$j_\phi = \frac{1}{v\sin\theta}\frac{\partial h}{\partial\phi} - \frac{\partial f}{\partial v}\frac{\partial}{\partial v}\left(\frac{1}{v\sin\theta}\frac{\partial g}{\partial\phi}\right) - \frac{1}{v^2}\frac{\partial f}{\partial\theta}\frac{\partial}{\partial\theta}\left(\frac{1}{v\sin\theta}\frac{\partial g}{\partial\phi}\right)$$

$$- \frac{1}{v^2\sin^2\theta}\frac{\partial f}{\partial\phi}\left(\sin\theta\frac{\partial g}{\partial v} + \frac{\cot\theta}{v}\frac{\partial g}{\partial\theta} + \frac{1}{v\sin\theta}\frac{\partial^2 g}{\partial\phi^2}\right) \quad ;$$

and h and g are given by

$$h(\underset{\sim}{x},\underset{\sim}{v},t) = 4\pi e^4 \sum_b \frac{Z^2 Z_b^2}{M M_b} \ln \Lambda_b \int d\underset{\sim}{v}' \frac{f_b(\underset{\sim}{x},\underset{\sim}{v}',t)}{|\underset{\sim}{v} - \underset{\sim}{v}'|}$$

and

$$g(\underset{\sim}{x},\underset{\sim}{v},t) = 2\pi e^4 \sum_b \frac{Z^2 Z_b^2}{M^2} \ln \Lambda_b \int d\underset{\sim}{v}' |\underset{\sim}{v} - \underset{\sim}{v}'| f_b(\underset{\sim}{x},\underset{\sim}{v}',t) \quad .$$

For isotropic Rosenbluth potentials h and g the averaged Fokker-Planck Eq. (23) becomes

$$\frac{\partial f_0}{\partial t} = -\frac{1}{v^2}\frac{\partial}{\partial v}\left[v^2 \left\langle\frac{\partial h}{\partial v}\right\rangle f_0 - v^2 \left\langle\frac{\partial^2 g}{\partial v^2}\right\rangle \frac{\partial f_0}{\partial v}\right] + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left[\frac{\sin\theta}{v^3}\left\langle\frac{\partial g}{\partial v}\right\rangle \frac{\partial f_0}{\partial \theta}\right]$$

$$+ \frac{1}{R}\frac{\partial}{\partial R}\left[\frac{1}{\Omega^2 R v^3 \sin^2\theta} \left\langle r^2 \dot{r}^2 \frac{\partial g}{\partial v}\right\rangle \frac{\partial f_0}{\partial R}\right] \quad , \qquad (28)$$

where the relation $\partial/\partial\phi = (r\dot{r}/\Omega R)\partial/\partial R$ is used. Here, f_0 is a function of v, θ, R, and t, and $\langle\ \rangle$ represents the orbit integral:

$$\langle I\rangle = \frac{\Omega}{2\pi}\oint \frac{dr}{\dot{r}} I(r,v) \quad , \qquad (29)$$

with

$$\dot{r} = \pm \frac{\Omega}{2r}\sqrt{4R^2r^2 - (R^2 + r^2 - \rho^2)^2} \quad .$$

Note that although h and g are assumed isotropic, the orbit-averaged quantities $\langle\frac{\partial h}{\partial v}\rangle$, $\langle\frac{\partial g}{\partial v}\rangle$, $\langle\frac{\partial^2 g}{\partial v^2}\rangle$, and $\langle r^2\dot{r}^2 \frac{\partial g}{\partial v}\rangle$, are generally functions of v, θ, and R.

Equation (28) is the proper kinetic equation to solve when the Larmor radius is comparable to the scale length, subject to certain boundary conditions. If we assume that there exists an absorbing radial wall at $r = a$, then we have a boundary condition $f_0 = 0$ on $R + (v \sin\theta)/\Omega = a$. The other loss boundary corresponds to end losses and is given by $\theta_L = \sin^{-1}(1/R_m)$, where R_m is the mirror ratio.

3. COMMENTS ON RADIAL FOKKER-PLANCK CODE OF FUTCH

A radial Fokker-Planck (RFP) code developed by Futch[1] has been used to study the build-up of mirror plasmas ranging from 2XIIB to a reactor. The RFP code obtains an approximate solution of Eq. (28) based on the separation of variables and assumption of locally isotropic Rosenbluth potentials. The following procedure is introduced:

(1) The θ-dependence of the orbit-averaged Fokker-Planck coefficients is removed by the replacement $\rho(v,\theta) = v \sin\theta/\Omega \approx \alpha v/\Omega$, where α is some "average" value of $\sin\theta$.

(2) This leads to solutions of the form $f_0(v,\theta,R,t) = F(v,R,t)\, M(\theta)$, and the following equations:

$$\frac{\partial F}{\partial t} = \frac{1}{v^2}\frac{\partial}{\partial v}\left[\left\langle v^2 \frac{\partial h}{\partial v}\right\rangle F - \left\langle v^2 \frac{\partial^2 g}{\partial v^2}\right\rangle \frac{\partial F}{\partial v}\right] + \frac{\lambda}{v^3}\left\langle \frac{\partial g}{\partial v}\right\rangle F + \frac{1}{R}\frac{\partial}{\partial R}\left[\frac{1}{\Omega^2 R v^3}\left\langle r^2 v^2 \alpha^2 \frac{\partial g}{\partial v}\right\rangle \frac{\partial F}{\partial R}\right] , \quad (30)$$

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial M}{\partial\theta}\right) = \lambda M , \quad (31)$$

where λ is the separation constant, i.e., the eigenvalue associated with each normal mode. Futch drops the last term in Eq. (30) and solves the truncated equation.

(3) The exact boundary condition on $f_0(v,\theta,R)$ cannot be satisfied in general unless one uses a superposition of many normal modes. Futch replaces the exact boundary condition $F(R + v \sin\theta/\Omega = a) = 0$ by $F(R + \alpha v/\Omega = a) = 0$ and uses only the lowest order normal mode for f_0, identifying F as the guiding center distribution averaged over all pitch-angles.

(4) To compute the isotropic part of the Rosenbluth potentials one needs the isotropic part of the distribution function given by

$$F_0(r,v) = \frac{1}{4\pi}\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta \, d\theta f(r,v,\theta,\phi)$$
$$= \frac{1}{\pi}\int_0^{\pi} \sin\theta \, d\theta \int_{r-\rho}^{r+\rho} \frac{R\,dR}{\sqrt{4R^2r^2 - (R^2 + r^2 - \rho^2)^2}} F(v,R) M(\theta) . \quad (32)$$

Futch now replaces $\rho(v,\theta)$ by $\bar{\rho} = \alpha v/\rho$ to obtain

$$F_0(r,v) \approx \frac{1}{\pi}\int_0^{\pi} \sin\theta d\theta M(\theta) \int_{r-\bar{\rho}}^{r+\bar{\rho}} \frac{RdR}{\sqrt{4R^2r^2 - (R^2 + r^2 - \bar{\rho}^2)^2}} F(v,R) \ . \tag{33}$$

(5) The quasilinear diffusion term [first term on the right-hand side of Eq. (36)] is averaged over θ with the lowest order normal mode assumed and is added to Eq. (30).

The terms representing atomic physics and neutral beam injection are added to Eq. (30) (without the last term on the right-hand side) and the resulting equation is solved for ions along with the rate equations for electron density and temperature.

We now point out some shortcomings in the RFP code when it is applied to a wide range of transport problems. First, the guiding center diffusion term is ignored. This means the RFP code may not be well-suited to study collisional transport in mirror machines such as 2XIIB and TMX (Tandem Mirror Experiment) in which the plasma scale length is only a few times or comparable to the Larmor radius. Second, even in a case where the guiding center collisional diffusion is negligible, the orbit-averaged Fokker-Planck equation is not strictly separable in v and θ as can be seen in Eq. (28). Thus the use of the lowest order normal modes for a pitch-angle distribution is generally questionable. Third, the density in the RFP code is calculated by

$$n(r) = 4\pi\int_0^{\infty} v^2 dv F_0(r,v) \approx K\int_0^{\infty} v^2 dv \int_{r-\bar{\rho}}^{r+\bar{\rho}} \frac{RdR}{\sqrt{4R^2r^2 - (R^2 + r^2 - \bar{\rho}^2)^2}} F(v,R), \tag{34}$$

where K is the normalization constant, while the proper density in the case of unseparated variables is calculated by

$$n(r) = \int_0^{\infty} v^2 dv \int_0^{\pi} \sin\theta d\theta \int_{r-\rho}^{r+\rho} \frac{RdR}{\sqrt{4R^2r^2 - (R^2 + r^2 - \rho^2)^2}} f_0(v,\theta,R) \ . \tag{35}$$

The difference between these two densities will be small when the Larmor radius is small compared with the plasma scale length or when the pitch-angle distribution is peaked near 90°. Finally, when the quasilinear diffusion due to rf turbulence is dominant, as in 2XIIB, the use of the normal mode is not consistent with the quasilinear simulation results.[7] This will make it difficult to justify Procedure (5) above. In addition, the inclusion of plasma stream for drift-cyclotron loss-cone (DCLC) stabilization in the RFP code needs to be examined carefully because the axial loss of plasma is assumed to be instantaneous in the loss cone.

The RFP code, however, applies to reactor-like plasmas in which the pitch angle scattering is important, the RF diffusion is not important, and the Larmor radius is small compared with the plasma scale length.

4. COMMENTS ON QUASILINEAR TRANSPORT CODE OF MATSUDA AND BERK

Matsuda and Berk have developed a radial transport code for mirror machines based on the quasilinear diffusion due to DCLC turbulence.[5,6] The quasilinear equation for ions reads

$$\frac{\partial F}{\partial t} = \sum_m \left(m\Omega_i \frac{\partial}{\partial v_\perp^2} + \frac{\ell}{\Omega_i} \frac{\partial}{\partial R^2}\right) \frac{4\left|\frac{e\phi_\ell}{M}\right|^2 J_m^2\left(\frac{\ell v_\perp}{\Omega_i \langle r\rangle}\right)}{\Delta\omega_\ell} \left(m\Omega_i \frac{\partial}{\partial v_\perp^2} + \frac{\ell}{\Omega_i} \frac{\partial}{\partial R^2}\right) F + \nu_d \frac{\partial}{\partial v_\perp^2}\left(v_\perp^2 F\right) + S \quad , \quad (36)$$

where m and ℓ are the ion cyclotron harmonic number and azimuthal mode number, respectively; $\Delta\omega_\ell$ is the particle correlation frequency; $\langle r\rangle$ represents the average radius; S represents such terms as neutral-beam injection, axial transit loss due to loss cone, stream input, and charge-exchange loss; and ν_d represents the energy loss due to electron drag:

$$\nu_d = \frac{8}{3}\sqrt{(2\pi)}\,\frac{nZ^2e^4m_e^{1/2}\ \ln\Lambda}{MT_e^{3/2}}\quad .$$

Equation (34) is solved simultaneously with the equations for the wave potential ϕ_k and the electron energy in the quasilinear transport code.

We point out here that (1) the quasilinear diffusion term is derived under the assumption of small Larmor radius, (2) it is orbit-averaged over both bounce and gyroperiod, and (3) the electron-drag term represents the Fokker-Planck collision term in the limit of low electron temperature, which is relevant to the present 2XIIB experiments. Since in the parameter range of the present 2XIIB experiments the pitch-angle scattering time is much longer than the quasilinear diffusion and electron drag times, Eq. (34) is adequate to simulate the 2XIIB experiments dominated by the DCLC turbulence. When the pitch-angle scattering and guiding-center diffusion are important as well as the electron drag and the quasilinear diffusion, we must add the quasilinear term [first term on right-hand side of Eq. (36)] to the orbit-averaged kinetic Eq. (28). Note that in such a case the separation of variables in v and θ is not likely to be valid.

5. CONCLUDING REMARKS

We have derived the kinetic equations suitable for investigating mirror plasma transport, taking into account the radial and axial dependence of the plasmas and magnetic fields. In the limit of large gyrofrequency and bounce frequency compared with collision frequency, the orbit-averaged kinetic equation is reduced to a three-dimensional equation in phase space, tractable on a high-speed computer. For a magnetic square-well case we have expressed the complete Fokker-Planck equation in terms of the three coordinates v, θ, and R [Eq. (28)] under simplifying assumptions. In a general axisymmetric case with noncircular orbits, however, the energy, the first adiabatic invariant, and the canonical angular momentum are the proper coordinates.

Our emphasis in this report is that (1) the gyrophase dependence of the distribution function needs to be retained when the Larmor radius is comparable to the plasma radial scale length and (2) the separation of variables is not strictly valid even for the isotropic Rosenbluth potentials. We also wish to note that the assumption of isotropic Rosenbluth potentials has to be examined more carefully, especially when the distribution function is peaked in pitch angle.

We have examined the RFP code of Futch when it is applied to plasma transport problems in mirror machines and identified the approximations and assumptions involved. In addition, we have briefly described the restrictions and a possible extension of the quasilinear transport code of Matsuda and Berk. While the validity of the RFP code is questionable when it is used over a wide range of parameters, e.g., when it is applied to 2XIIB experiments, a quantitative assessment of errors in the simulation results requires a code based on Eq. (28).

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FIGURE CAPTION

Fig. 1. Particle gyromotion and relationship between variables. (R: guiding center position, r: particle position, ρ: Larmor radius, $v_\perp$: perpendicular velocity, ϕ: gyrophase.) *The canonical angular momentum is given by* $P_\psi = Mrv_\perp \sin\phi + q\,r\,A_\psi/c = M\Omega(R^2 - \rho^2)/2$ for a uniform magnetic field $\underset{\sim}{B}$.

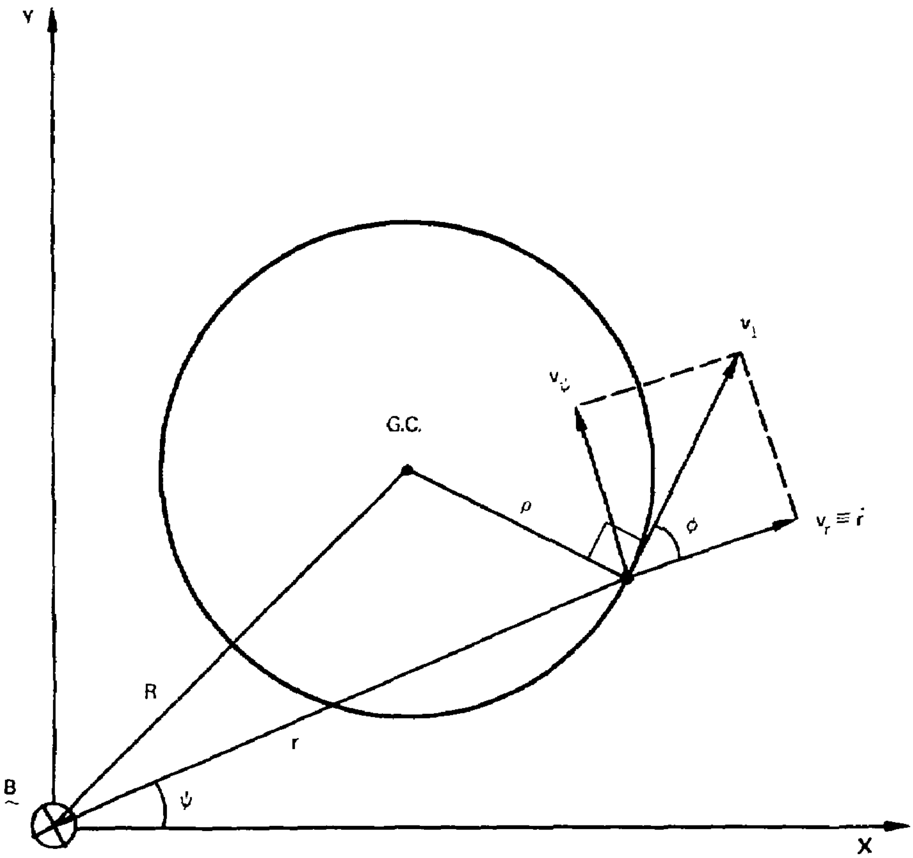

Figure 1.