CONF-810214--5

LA-UR-81-682

TITLE: ICRF OSCILLATIONS OF AN INHOMOGENEOUS PLASMA CYLINDER

MASTER

AUTHOR(S): T. E. Cayton, Los Alamos, CTR-6
H. R. Lewis, Los Alamos, CTR-6

SUBMITTED TO: Fourth Topical Conference on Radio Frequency Plasma Heating

DISCLAIMER

University of California

LOS ALAMOS SCIENTIFIC LABORATORY
Post Office Box 1663 Los Alamos, New Mexico 87545
An Affirmative Action/Equal Opportunity Employer

Form No. 836 R3
St. No. 2629
12/78

UNITED STATES
DEPARTMENT OF ENERGY
CONTRACT W-7405-ENG. 36

ICRF Oscillations of an Inhomogeneous Plasma Cylinder*

T. E. Cayton and H. R. Lewis
Los Alamos National Laboratory
University of California
Los Alamos, New Mexico 87545

ABSTRACT

We have derived a dispersion differential equation suitable for studying free and forced oscillations of a radially inhomogeneous plasma cylinder in the ion cyclotron range of frequencies (ICRF). Solving the differential equation, subject to appropriate boundary conditions, yields global eigenmodes of the cylindrical configuration; thus, our description embraces both the geometry and the physics relevant in the ICRF. The derivation begins with the equations of the Vlasov(ion)-fluid(electron) model. An approximate solution of the ion Vlasov equation is obtained analytically for a general screw pinch equilibrium by restricting the ion gyroradius to small, but finite values. However, the frequency of oscillation is not assumed to be small compared with the ion cyclotron frequency, and the pitch of the magnetic field lines is not restricted to large values. Thus, we have generalized and extended the theory of finite gyroradius effects by including ion cyclotron damping, arbitrary diffuse cylindrically symmetric equilibrium profiles, and other effects. The approximate solution of the ion Vlasov equation together with the Ampere equation yield the dispersion differential equation. The solutions of the equation explicitly exhibit two different length scales: a MHD length scale, the pinch radius; and a microscopic scale, the ion gyroradius. In the limit of vanishingly small ion gyroradii, our model reduces to the guiding center plasma model.

INTRODUCTION

Experiments have demonstrated that the ions of a high-β plasma column are efficiently heated by resonant magnetoacoustic oscillations.[1] In these experiments the magnetoacoustic resonance frequency turned out to be of the order of the ion cyclotron frequency; the resonant oscillations themselves corresponded to high-Q eigenmodes of the particular geometric configuration. This suggests that both the geometry and the detailed ion response to the excitation are important aspects of resonant magnetoacoustic oscillations. Previous descriptions of these oscillations in hot-ion pinches involve very restrictive assumptions about both the geometry and the ion response.[1] In particular, the geometry has been restricted in previous theories to the near θ pinch where the axial component of magnetic field is much larger than any of the other components; likewise, ion dynamics have been restricted to low frequencies, $\omega \ll \Omega_{ci}$, an assumption that is almost always violated in actual experiments. In the present theory we relax both restrictions. The present theory should be applicable to both the experiments mentioned and ones of current interest: the Reversed Field Z-Pinch.

THEORY

We consider small amplitude oscillations of a straight, cylindrically symmetric, hot-ion plasma column. This plasma cylinder is surrounded by a vacuum region, and finally by a coaxial, rigid, perfectly conducting shell. Oscillations are excited by surface currents that flow on another coaxial cylindrical surface located between the plasma column and the conducting shell. The idealized excitation coil divides the vacuum region into two annular volumes.

The unexcited plasma is inhomogeneous. Plasma density, ρ, pressure, p, and the magnetic field components, B_θ and B_z, are functions only of the radial coordinate, r, in cylindrical polar coordinates. The equilibrium quantities satisfy the equations of ideal magnetohydrostatics, a property of the Vlasov-fluid model[2] which we use to describe the plasma. Current in the unexcited inhomogeneous plasma is carried solely by electrons. The macroscopic ion current density and the flow velocity vanish. But because of this, the ion guiding centers must be drifting, and there is a corresponding electric field in the stationary reference frame. The electron fluid transfers momentum to the ion fluid via the electrostatic field. In this respect, these equilibria differ from the usual description found in ideal MHD or guiding center plasma theory where the guiding centers are a rest, and there is no equilibrium electric field.

To determine the eigenmodes, we solve the equations of the Vlasov-fluid model which have been linearized about the (inhomogeneous) equilibrium. The model equations are

$$\frac{\partial f}{\partial t} + \underset{\sim}{v}\cdot\frac{\partial f}{\partial \underset{\sim}{x}} + \frac{e}{m_i}\left(\underset{\sim}{E}+\frac{\underset{\sim}{v}}{c}\times\underset{\sim}{B}\right)\cdot\frac{\partial f}{\partial \underset{\sim}{v}} = 0 \ , \quad \underset{\sim}{E} + \frac{\underset{\sim}{u}_e}{c}\times\underset{\sim}{B} = 0 \ , \quad \frac{\partial}{\partial \underset{\sim}{x}}\times\underset{\sim}{E} = -\frac{1}{c}\frac{\partial \underset{\sim}{B}}{\partial t} \ , \qquad 1\text{-}3$$

$$\left(\frac{\partial}{\partial \underset{\sim}{x}}\times\underset{\sim}{B}\right)\times\underset{\sim}{B} = \frac{4\pi}{c}\underset{\sim}{J}\times\underset{\sim}{B} = 4\pi e\int d^3v \ \left(\underset{\sim}{E}+\frac{\underset{\sim}{v}}{c}\times\underset{\sim}{B}\right)f \ , \quad n_e = n_i \ . \qquad 4,5$$

Ion macroscopic quantities are defined by appropriate moments of the ion distribution function, f. [Equation (2) is the momentum equation for a cold, massless fluid, but it is possible to include more sophisticated electron models, e.g., finite electron pressure, guiding center electrons, etc.]

Because of the symmetries of the equilibrium, perturbation quantities are assumed to be of the form $g(\underset{\sim}{x},t) = g(r)\ \exp[i(m\theta+kz-\omega t)]$, where m is the azimuthal mode number, k the axial wavevector, and ω the frequency. Two other quantities are defined:

$$k_\parallel = (kB_z^{(0)}+mB_\theta^{(0)}/r)/|\underset{\sim}{B}^{(0)}| \ ; \ k_\perp = (mB_z^{(0)}/r-kB_\theta^{(0)})/|\underset{\sim}{B}^{(0)}| \ .$$

To obtain an analytic solution of Eqs. (1)-(5), we restrict our attention to cases with small, yet finite, ion gyroradii. We assume $0 < (\rho_i/a) \ll 1$, where ρ_i is the ion gyroradius ($\rho_i = v_{th}/\Omega_{ci}$), and a is the radial scale length. But unlike other small ion gyroradius theories, we do not restrict ω/Ω_{ci} , $k_\parallel v_{th}/\Omega_{ci}$ or $k_\perp v_E/\Omega_{ci}$ to small values (v_E is the electric drift speed). The linearized ion Vlasov equation reads $[(1 + L_1L_0^{-1})L_0] f_1 = \Gamma$, where the linear operators L_0 and L_1 are components of the total time derivative operator. In our description,

$$L_0 = -\Omega_{ci} \frac{\partial}{\partial\phi} - i(\omega - k_\parallel v_\parallel - k_\perp v_E) ,$$

where ϕ is the gyro-phase angle. The inverse operator L_0^{-1} is known explicitly.[3] We assume that the operator inequality $L_1L_0^{-1} \ll 1$, is valid with respect to operations on the ϕ-dependent part of the solution. This fundamental assumption allows calculation of the inverse of the total time derivative operator by the formal expansion $L_0^{-1}(1 + L_1L_0^{-1})^{-1} = L_0^{-1}(1 - L_1L_0^{-1} + \cdots)$. Application of this inverse operator yields an analytic approximation for the perturbed ion distribution function, f_1, from which is obtained the perturbation current density, $\tilde{J}_i^{(1)}$. Substituting this current density into Eqs. (3)-(5) yields a dispersion differential equation which the cylindrical modes must satisfy. For diffuse screw pinch equilibria, this prescription has been accomplished with the MACSYMA symbolic manipulation computing system (Mathlab Group, M.I.T. Laboratory for Computer Science).[4]

The basic perturbation quantity is $\xi_\perp$, the displacement of the electron fluid perpendicular to $\underline{B}^{(0)}$; it is related to the perturbed electric field. We use the representation $\tilde{\xi}_\perp = \xi_r\hat{r} + \xi_y\hat{y}$, where $\hat{y}=\underline{B}\times\hat{r}$, and $\underline{B}=\underline{B}^{(0)}/|\underline{B}^{(0)}|$; ξ_r and ξ_y satisfy a pair of coupled, second-order ordinary differential equations. The coefficient in these equations are extremely complicated in the general case. Numerical methods are being developed to solve the system. A FORTRAN code produced by MACSYMA has been compiled. Some features of the solution are:

1) The basic radial wavelength is of the order of the cylinder radius, a.
2) The modes exhibit features that vary rapidly with r -- a skin depth of the order of the ion gyroradius appears.
3) Ion cyclotron resonance(fundamental and harmonics) and Cerenkov resonance (n=0 subharmonic) are included in the ion response.
4) The ion dynamics couple the shear Alfvén and fast magnetoacoustic waves. There is no Alfvén continuum -- we have an ion kinetic Alfvén wave.

In the special case of a cylinder partially filled with homogeneous plasma, the equations simplify and we can obtain analytic solutions. For m=0 (axisymmetric) modes, the differential equations read

$$(A+A_1)\nabla_\perp^2 u - (k^2-K_1^2)u - A_2\nabla_\perp^2(iv) - K_2^2(iv) = 0 ,$$

$$A_6\nabla_\perp^2(iv) + (k^2-K_1^2)(iv) + A_5\nabla_\perp^2 u \quad K_2^2 u = 0 ,$$

where $u=\xi_r'(r)$, $v=\xi_y'(r)$, and the coefficients are defined as follows

$$A = 1 - \beta\zeta_0 Z(\zeta_0) ,$$

$$A_1 = (\beta/4)\zeta_0[Z(\zeta_{-2}) + Z(\zeta_{-1}) + Z(\zeta_{+1}) + Z(\zeta_{+2})] ,$$

$$A_2 = -(\beta/4)\zeta_0[Z(\zeta_{-2}) - Z(\zeta_{-1}) + Z(\zeta_{+1}) - Z(\zeta_{+2})] ,$$

$$A_5 = -(\beta/4)\zeta_0[Z(\zeta_{-2}) + Z(\zeta_{-1}) - Z(\zeta_{+1}) - Z(\zeta_{+2})] ,$$

$$A_6 = -(\beta/4)\zeta_0[Z(\zeta_{-2}) - Z(\zeta_{-1}) - Z(\zeta_{+1}) + Z(\zeta_{+2})] ,$$

$$(\kappa^2 - K_1^2) = k^2 - (\Omega_{ci}^2/c_A^2)\zeta_0[Z(\zeta_{-1}) + Z(\zeta_{+1})]/2 ,$$

$$K_2^2 = (\Omega_{ci}^2/c_A^2)[\omega/\Omega_{ci} - \zeta_0[Z(\zeta_{-1}) - Z(\zeta_{+1})]/2] .$$

Here, $Z(x)$ is the plasma dispersion function and $\zeta_{\pm n} = (\frac{\omega \pm n\Omega_{ci}}{kv_{th}})$.

Note that the ion skin depth, (c_A/Ω_{ci}), the Cerenkov resonance, ζ_0, and the ion cyclotron resonances, ζ_{-2}, ζ_{-1}, ζ_{+1}, and ζ_{+2} are included in this description. By taking proper limits these equations reduce to those given in Ref. 1, and finally, to the guiding center plasma result. At the perturbed plasma-vacuum interface we must match this plasma solution with the TM and TE components of the vacuum electromagnetic field. Jump conditions are being evaluated.

ACKNOWLEDGMENT

*This work was performed under the auspices of the U. S. Department of Energy.

REFERENCES

1 T. E. Cayton and H. R. Lewis, Phys. Fluids 23, 109(1980); also see references cited therein.
2 J. P. Freidberg, Phys. Fluids 15, 1102(1972).
3 H. R. Lewis and C. E. Seyler, to be published.
4 H. R. Lewis, to be published.