MASTER

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ANALYSIS OF THE LOFT MODULAR DRAG DISC TURBINE TRANSDUCER (MDTT) SPRING FOR COMPRESSIVE BUCKLING

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IDAHO NATIONAL ENGINEERING LABORATORY

DEPARTMENT OF ENERGY

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ANALYSIS OF THE LOFT MODULAR DRAG DISC TURBINE TRANSDUCER (MDTT) SPRING FOR COMPRESSIVE BUCKLING

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EG&G Idaho, Inc.

LOFT TECHNICAL REPORT
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ABSTRACT

The LOFT Modular Drag Disc Turbine Transducer (MDTT) springs (for range 2, ρV^2 = 4900 lbm/ft-sec^2) were analyzed to determine the static ρV^2 load needed to cause a buckling failure.

The static load needed to cause elastic buckling was found to be equivalent to a ρV^2 value of 431,000 lbm/ft-sec^2 according to classical buckling theory, but could be as low as ρV^2 = 100,000 lbm/ft-sec^2 due to uncertain end fixity and other spring imperfections.

Note by R. E. Ford: REF 8/9/78

Because the load required to cause elastic buckling is so much larger than the loads the spring is expected to be exposed to, the probability of a buckling failure is very small. In the event a spring buckles, the only consequence will be the loss of that measurement. No parts would be detached from the DTT in the event of a spring buckling.

CONTENTS

ANALYSIS OF THE LOFT MODULAR DRAG DISC TURBINE TRANSDUCER (MDTT) SPRING FOR COMPRESSIVE BUCKLING

I. SCOPE OF INVESTIGATION

The LOFT MDTT springs (for range 2, ρV^2 = 4900 lbm/ft-sec^2) were analyzed to determine the static ρV^2 load needed to cause a buckling failure.

II. METHOD OF ANALYSIS

Static equilibrium considerations along with spring force, moment, and deflection formulas[2] were used to calculate spring forces and maximum bending moments due to the fluid dynamic pressure (ρV^2) acting on the MDTT drag disc. Dimensional data, material properties, and drag disc forces from References 1 and 4 were used. The critical load for elastic buckling of the springs was calculated using methods in Reference 3.

III. RESULTS

Theoretical compressive elastic buckling of the 4900 ρV^2 range MDTT spring was found to occur at a ρV^2 value of 431,000 lbm/ft-sec^2. Plastic hinge formation in the spring was calculated to occur at ρV^2 = 738,500 lbm/ft-sec^2 and, thus, does not occur before elastic buckling.

The observed spring buckling failure occurred at a ρV^2 of less than 350,000 lbm/ft-sec^2. Dimensional inaccuracies and reduced spring end fixity may cause the actual spring compressive buckling load to decrease by a factor of 4 or more, causing the spring to buckle at a ρV^2 value around 100,000 lbm/ft-sec^2. Dynamic effects may also reduce the critical buckling ρV^2 value.

IV. REFERENCES

1. W. R. Mosby, "Stress Analysis of the LOFT Modular DTT Flowmeter for LOCE Transients (L1-5 and L2-4)", LOFT Technical Report LTR 141-77, March 3, 1978.

2. R. J. Roark and W. C. Young, Formulas for Stress and Strain, 5th Edition, McGraw-Hill, 1975.

3. S. P. Timoshenko and J. M. Gere, Theory of Elastic Stability, McGraw-Hill, 1961.

4. Drawing 036-0806,25-220-208189C, "LOFT Drag Disc Module Assembly", September 1, 1977.

APPENDIX

MDTT SPRING BUCKLING

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LTR141 83

FORM EG&G-1592
(Rev. 5-77)

CALCULATION WORK SHEET

Page A-ii of ____ Pages

Subject MDTT Spring Buckling — Date ____

Prepared By W.R. Mosby — Checked ____ Work Request ____

Contents

EG&G Idaho, Inc.

LTR141 83

FORM EG&G-1592
(Rev. 5-77)

CALCULATION WORK SHEET

Subject MDTT Spring Buckling Date

Prepared By W.R. Mosby Checked Work Request

1. General

a. Geometry

Figure 1 shows the MDTT transducer spring and drag disc carrier assembly (references 1 and 4).

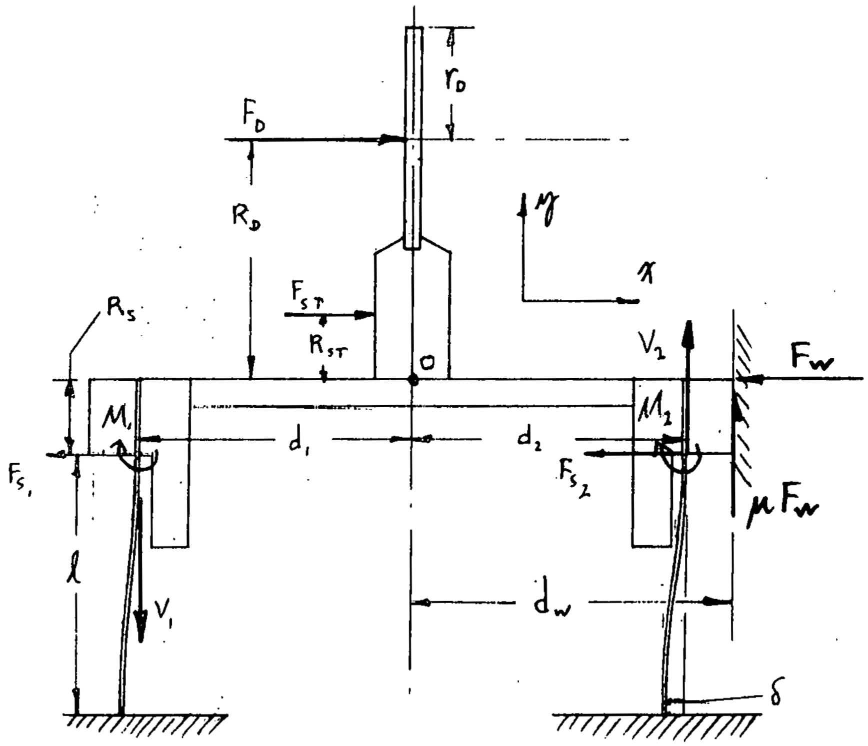

Figure 1: DTT carrier Assembly

The dimensions for the range 2 DTT ($\rho V^2 \leq 4900$) are

$r_D = .3$ in

$l = .7$ in.

$R_{ST} = \dfrac{.66 - r_D}{2} = .18$ in.

$R_D = .66$ in

$d_1 = d_2 = .73$ in.

$R_s = .2$ in.

$d_w = .857$ in.

$\delta = .05$ in.

$I = 2.9155 \times 10^{-9}$ in^4 (each spring)

CALCULATION WORK SHEET

Subject MDTT Spring Buckling Date

Prepared By W. R. Mosby Checked Work Request

b. Material (ref. 1)

Inconel X-750 AMS: 5542

$E = 31 \times 10^6$ psi, $S_y = 110{,}000$ psi = yield stress

2. Spring Buckling Loads

a. Spring effective length

Using reference 2, the stiffness of each spring is calculated to be (p. 96, case 1b)

$$\delta = \frac{W}{12EI}(\ell - a)^2(\ell + 2a) \; ; \; \text{with } a = 0,$$

$$\frac{W}{\delta} = \frac{12EI}{\ell^3}$$

The actual stiffness is measured as

$$\frac{W}{\delta} = (.842)\frac{12EI}{\ell^3}$$

An effective length, ℓ', can be used:

$$\ell\left(\frac{1}{\sqrt[3]{.842}}\right) = \underline{\ell' = 1.059\,\ell} = \underline{.7413 \text{ in}}$$

CALCULATION WORK SHEET

Subject: MDTT Spring Buckling
Date:
Prepared By: W. R. Mosby
Checked:
Work Request:

The imperfect clamping will be taken into account by using this effective spring length in the buckling analysis.

b. Critical load for buckling

The analysis of reference 3, p.51, may be used for this beam-column with initial displacement.

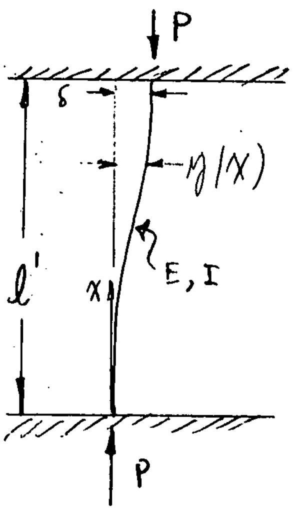

Figure 2: The DTT Spring As a Beam Column

The equation for this beam-column is

$$\frac{d^4 y}{dx^4} + k^2 \frac{d^2 y}{dx^2} = 0 \qquad (1)$$

with $k = \sqrt{\frac{P}{EI}}$

for which the general solution is $y = A \sin kx + B \cos kx + Cx + D$.

$v = y - \delta x$ is also a solution of (1), so the critical

CALCULATION WORK SHEET

Subject MDTT Spring Buckling Date

Prepared By W. R. Mosby Checked Work Request

load given in reference 3 is valid for this case, and

$$P_{cr} = \frac{4\pi^2 EI}{(l')^2} = \frac{4\pi^2 (31\times10^6)(2.9155\times10^{-9})}{(.7413)^2}$$

$$\underline{\underline{P_{cr} = 6.49 \text{ lb}}}$$

c. Load for maximum moment

(1) $M_{max} = \frac{1.5\, S_y\, I}{c} = \frac{(1.5)(110{,}000)(2.9155\times10^{-9})}{.0035} = .137$ in-lb

(2) Reference 2, p 154, case 1:

$$M_{max} = \frac{w}{k} \tan \frac{kl'}{2} \qquad (1)$$

with

$$y_A = \delta = \frac{w}{kP} \frac{C_3 C_{a3} - C_2 C_{a4}}{C_2} \qquad (2)$$

or $\quad w = \delta k P C_2 / (C_3 C_{a3} - C_2 C_{a4})$

CALCULATION WORK SHEET

Subject MDTT Spring Buckling Date

Prepared By W.R. Mosby Checked Work Request

so

$$M_{MAX} = \frac{\delta P C_2}{(C_3 C_{a3} - C_2 C_{a4})} \tan \frac{kl'}{2} \qquad (3)$$

with $a = 0$

$C_2 = \sin kl$

$C_3 = C_{a3} = 1 - \cos kl$

$C_{a4} = kl - \sin kl$

Using (3), the value of P for which $M_{MAX} = .137$ is

$$P_M = 10.95 \text{ lb}$$

The elastic buckling load is lower than this.

3. Spring force calculations

(a) Equilibrium

$\Sigma F_x = 0$

$$-F_F + F_{s_1} + F_{s_2} + F_w = 0$$

CALCULATION WORK SHEET

Subject MDTT Spring Buckling Date

Prepared By W. R. Mosby Checked Work Request

$$\Sigma F_y = 0$$

$$-V_1 + V_2 + \mu F_w = 0$$

$$\Sigma M_0 = 0$$

$$-M_F - F_{s_1} R_s - F_{s_2} R_s - M_1 - M_2 + \mu F_w d_w$$

$$+ V_1 d_1 + V_2 d_2 = 0$$

where (ref. 1)

$$F_F = F_D + F_s = 3.9894 \times 10^{-4} r_D^2 (\rho V^2) + 3.2552 \times 10^{-6} (.66 - r_D) \times \rho V^3$$

$$\underline{\underline{F_F = 3.7068 \times 10^{-5} \rho V^2}}$$

$$\underline{M_F = F_D R_D + F_{ST} R_{ST} = 2.1749 \times 10^{-5} \rho V^2}$$

(b) expressions for F_{s_1}, F_{s_2}, M_1, M_2 :

spring #1 is in tension, so ref. 2, p. 168, case (b) applies:

$$M_1 = \frac{w \, C a_3}{k \, C_2}$$

CALCULATION WORK SHEET

Subject MDTT Spring Buckling Date

Prepared By W. R. Mosby Checked Work Request

with

$$y_A = \delta = \frac{w}{kP}\left(\frac{C_3 C_{a3} - C_2 C_{a4}}{C_2}\right)$$

so

$$F_{s_1} = w = \frac{\delta k_1 V_1 C_{2_1}}{C_{3_1} C_{a3_1} - C_{2_1} C_{a4_1}}$$

$$M_1 = \frac{\delta V_1 \, C_{a3_1}}{C_{3_1} C_{a3_1} - C_{2_1} C_{a4_1}}$$

where I have used $P = V_1$, $k = k_1$, ...,

$$C_{2_1} = \sinh k_1 l'$$

$$C_{3_1} = C_{a3_1} = \cosh k_1 l' - 1$$

$$C_{a4_1} = \sinh k_1 l' - k_1 l'$$

$$k_1 = \sqrt{\frac{V_1}{EI}}$$

Spring #2 is in compression, so ref. 2, p. 154, case 1b. applies. The equations are of the same form as those for spring #1, except that C_2, etc, differ:

CALCULATION WORK SHEET

Subject MDTT Spring Buckling Date

Prepared By W. R. Mosby Checked Work Request

$$F_{s_2} = \frac{\delta k_2 V_2 \; C_{2_2}}{C_{3_2} Ca_{3_2} - C_{2_2} Ca_{4_2}}$$

$$M_2 = \frac{\delta V_2 Ca_{3_2}}{C_{3_2} Ca_{3_2} - C_{2_2} Ca_{4_2}}$$

$$k_2 = \sqrt{\frac{V_2}{EI}}$$

$$C_{2_2} = \sin k_{2_2} l'$$

$$C_{3_2} = Ca_{3_2} = 1 - \cos k_2 l'$$

$$Ca_{4_2} = k_2 l' - \sin k_2 l'$$

A calculator program was written which solves for V_1 and V_2 as a function of ρV^2 and calculated M_1 and M_2.

Table I gives V_1, V_2, M_1, and M_2 for the range 2 DTT spring-drag disc assembly, with $\mu = 0$. It should be noted that the foregoing

CALCULATION WORK SHEET

Subject: MDTT Spring Buckling
Date:

Prepared By: W. R. Mosby
Checked:
Work Request:

analysis Applies only if the drag disc carrier is Against the stop, for which ρV^2 is:

$$\frac{F_s}{2} = \frac{3.7068 \times 10^{-5}}{2} \rho V^2 = .05\,(.842)\,\frac{12\,(3 \times 10^{6})(2.9155 \times 10^{-9})}{(.7)^3}$$

$$\rho V^2 \geq 7183 \ \text{lbm/ft-sec}^2$$

TABLE I

Range 2 DTT Spring Forces vs. Dynamic Pressure

ρV^2 lbm/ft-sec²	V_1, lb	M_1, in-lb	V_2, lb	M_2, in-lb	Comments
10 000	.253	.0506	.253	.0481	
20 000	.402	.0513	.402	.0473	
30 000	.551	.0520	.551	.0465	
40 000	.700	.0527	.700	.0457	
[illegible]0 000	.849	.0534	.849	.0449	
431,000	6.50	.0754	6.50	-.0001	$V_2 \cong P_{cr}$; elastic buckling
738,500	10.95	.0888	10.95	-.1375	$V_2 \cong P_m$; max spring moment

CALCULATION WORK SHEET

Subject MDTT Spring Buckling Date

Prepared By W. R. Mosby Checked Work Request

4. Conclusion

This static analysis shows that a dynamic pressure of $\rho V^2 = 431{,}000$ lbm/ft-sec² is needed to cause the range 2 DTT spring to buckle elastically.

Dimensional innaccuracies, reduced end fixity, and dynamic effects may reduce the buckling ρV^2 value substantially. For example, if

$$P_{cr} = \frac{C \pi^2 E I}{(l')^2} * F$$

where

$C = 2$ instead of 4, corresponding to an end fixity between pinned and fixed ends

F = safety factor = ½

then the ρV^2 for buckling will be $\frac{431000}{4}$ or ≈ 108000 lbm/ft-sec².

The imperfect spring-end clamping may not be sufficiently taken into account by the procedure in 2. a. above.