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VARIATION OF THE MATTER DENSITIES OF NUCLEI FROM ^{40}Ca TO ^{68}Zn

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ABSTRACT

Analysis is made of elastic scattering of 42 MeV α-particles from 19 nuclei from ^{40}Ca to ^{68}Zn in terms of a folding model, allowing determination of matter densities in the nuclear surface region relative to that of ^{40}Ca. Specifically, values are deduced for the radii at which the matter density equals 0.016 nucleon/fm^3 as well as matter RMS radii. The relative sizes vary more slowly than $A^{1/3}$ between ^{40}Ca and ^{54}Fe and more rapidly than $A^{1/3}$ between ^{54}Fe and ^{68}Zn. The overall trend gives an increase with $A^{1/3}$ but with a bowing effect. The near equality of proton and matter RMS radii indicates that the neutron-proton radii are approximately equal to within the errors of the present analysis. The results are in reasonable agreement with those obtained with other hadronic probes and with calculated Hartree-Fock Bogoliubov densities.

KEYWORD ABSTRACT

NUCLEAR REACTIONS ^{40,42,44,48}Ca, ^{46,48,50}Ti, ^{52}Cr, ^{54,56}Fe, ^{58,60,62,64}Ni, ^{63,65}Cu, ^{64,66,68}Zn; $(\ ,\alpha)$ 42 MeV; Microscopic Optical Model Analysis; deduced matter rms radii; comparison with other results

I. INTRODUCTION

The diffractional elastic alpha-nucleus *scattering* is known to be sensitive to the surface regions of the optical potential and matter density[1-5] (∼ 10% of central value). Nuclear density parameters can be obtained through the use of a folding model; but, while those *parameters* are quite sensitive to small variations in the scattering data, their accuracy is limited by the assumptions of the model.[6,7] The accuracy is improved if one studies relative quantities since then the model errors tend to cancel. To a considerable extent all hadronic probes are limited by model errors[8,9] and the final justification of their use in extracting matter density related information lies in the consistency of their results.

In the folding model[5,10] the ground state matter density $\rho(r)$ of the target nucleus is related to the experimental data by the postulate that the optical potential, $U(r_\alpha)$, is given by a linear folding of $\rho(r)$ with a local effective α-nucleon interaction, $V_{eff}(\vec{r}-\vec{r}_\alpha)$:

$$U(r_\alpha) = (\lambda_r + \lambda_i) \int \rho(r)\, V_{eff}\, (\vec{r}-\vec{r}_\alpha)\, d\vec{r}$$

where λ_r and λ_i are empirical functions of the α-particle energy[11] which correct the lowest order expression for the optical potential.[12] The folding ansatz is of course limited in scope and therefore of restricted applicability. It is a simplification to replace a non-local, energy and density dependent operator[2,12,13] by the local interaction $U(r_\alpha)$ given by the simple folding integral. Its applicability in the diffraction region of elastic alpha scattering and at the energy range of 35 – 160 MeV has been tested by successfully predicting the observed cross sections for N=Z nuclei[5] using densities determined from electron scattering.

Recently elastic alpha scattering at backward angles was included in the analysis[1,13] in the hope of gaining more information about the nuclear interior. While scattering in this angular region is more sensitive to the nuclear interior it is at the same time susceptible to features of the reaction mechanism not adequately understood like antisymmetrization, non locality and density dependence which obviously cannot be handled by the folding model. Moreover the contribution of multistep processes is expected to be important at backward angles and so far their importance has not been adequately assessed in our view. In the present study we have therefore limited our analysis to the diffractional angular region ($\theta_{cm} \lesssim 30°$ for 62 MeV α-particles) hoping to achieve small model error but at the same time limiting the information we hope to extract to the nuclear surface region (not the central density).

The elastic scattering data of 62 MeV α-particles[5,14] from a number of intermediate mass nuclei ranging from ^{58}Ni through ^{68}Zn are here analyzed using the folding model. The model is calibrated by the requirement that the effective α-nucleon interaction when folded with the reasonably well known matter density of the N=Z nucleus, ^{40}Ca, yields an optical potential which fits experiment; therefore these results are relative to the density adopted for ^{40}Ca.

We have explored the limits of applicability of our model, its degree of sensitivity and reliability,[7] and estimates have been made of the magnitude of the model error. We stress the following characteristics of our analysis:

1) All nuclear structure information we present is relative to ^{40}Ca (our normalization reference). This is an inherent limitation to our procedure but on the other hand, provides the additional benefit of cancelling a good part of the systematic model error.

2) Since the diffractional scattering of alpha particles of a given energy primarily takes place at approximately the same density for all nuclei, model dependent error due to density dependence cancels out. Further, in the region emphasized here, 10% of the central density, the corrections due to density dependence are expected to be small.

3) We have verified that as long as one proceeds consistently, using effective interactions which are independent of target, the results for the densities are altered by less than the quoted errors. The assumption that the effective interaction is independent of the target nucleus has been tested for N = Z nuclei.[5] It is reassuring that the ranges of our effective α-nucleon interaction are in reasonable accord with those resulting from calculations[5,15] and are very close to those used in phenomenological models of free α-nucleon scattering.

II. EXPERIMENT AND PROCEDURE

The data for the present analysis are those of an earlier study[4] of scattering from intermediate mass nuclei ranging from ^{40}Ca through ^{62}Ni, supplemented by measurements[14] for ^{64}Ni, ^{63,65}Cu, and ^{64,66,68}Zn targets. The experimental procedures followed in the latter measurements were nearly identical to those of Ref. 4. The principal improvement was the use of a position sensitive detector, 2.3" wide; this permitted the more rapid acquisition of data, which was especially important for mapping out the diffraction minima. In the latter study, cross sections were also measured for three of the nuclei studied previously, ^{40}Ca, ^{48}Ca, and ^{62}Ni; the radii extracted (see below) agree with those we obtain from the earlier measurements to within ± 0.02 fm.

For the folding model analysis, the matter density for ^{40}Ca was assumed to be twice the proton density obtained from an analysis of electron scattering data[16] in terms of a 3-parameter Fermi charge density (c = 3.7984 fm, a = 0.583 fm, and w = -.18). The resulting density is in agreement with a more recent result[17] and with Hartree-Fock densities,[18,19] especially in the important surface region.

Analyses of the data were carried out independently at MIT and at the University of Washington (UW). The two different procedures, both based on similar principles are:

(MIT) The optical potential was obtained from Eq. (1) using a Gaussian α-nucleon interaction $\exp(-\alpha|\vec{r}-\vec{r}_i|^2)$. The strength parameters V_R and V_I were determined by requiring a fit to the data for ^{40}Ca. A value of 0.[illegible] fm^{-2} for α, the square of the inverse range of the α-nucleon interaction, led to the best fit but deviations of ±0.05 fm^{-2} gave values of χ^2 which exceeded the minimum by only 50% or less (see discussion of model error). The matter densities of other nuclei were taken to have the Fermi form, properly normalized, whose radius and diffuseness parameters, were varied to give a best fit to the measured cross sections. We have explored the error introduced by the use of an analytical parameterization of ρ(r) and found that this is not the dominating contribution to the model error.[7]

(UW) As an intermediate step, the data were analyzed in terms of a standard four-parameter Woods-Saxon potential. It is familiar, however, that analyses of low energy, forward angle data pin down only the real potential in the tail region; specifically, we found here that the radius at which the real potential equals: -2.4 MeV, $R_{2.4}$, was determined to within ±0.03 fm (relative to ^{40}Ca) although the diffuseness parameter was determined only to within ±0.0[illegible] fm. The parameters of the effective α-nucleon interaction were fixed by the requirement that when this interaction is folded with the matter density of ^{40}Ca it yields the value and slope of the real part of the optical potential at $R_{2.4}$. When the effective interaction was taken to have the Gaussian form, this resulted in the value 0.36 fm^{-2} for α. The matter densities of the other nuclei were then assumed to have the Fermi form with diffuseness fixed at 0.5 fm; the radius parameter c was varied so that after folding one obtains the correct value of the real part of the optical potential at $R_{2.4}$. Typical fits to data with the MIT and UW procedures are shown in Figures 1 and 2. The results of the two procedures agree well within the errors and the average values are presented here.

III. RESULTS AND ERRORS

The biggest source of error in our results is not due to experimental limitations but rather due to the inadequacy of the phenomenology (model error). The model assumes constant values for λ_r, λ_I, and α, independent of A as well as a Fermi form for $\rho(r)$. We have explored the model errors by relaxing these constraints for several targets and alpha particle energies and found in general that χ^2 did not decrease by more than 50%. The details of these considerations will be presented in a future publication.[7] For the present purposes we can summarize these findings by noting that a reasonable estimate of the model error for any deduced quantity ξ can then be given by the deviation $\Delta\xi$ that induces an increase in χ^2 by 50% above its minimum.

We present in the following figure and tables the quantities: $r_{0.016}$ the radius at which the density is 0.016 nucleons/fm^3 (approximately 10% of central density); $R_{2.4}$, the radius at which the real part of the optical potential $U(r_\alpha)$ takes the value of -2.4 MeV, and R, the matter RMS radius.

A. $r_{0.016}$ and $R_{2.4}$

The matter densities are most reliably specified in the surface region. More precisely, studies using the MIT procedure show the uncertainty in the matter densities is least in the vicinity of 10% of the central density. For the best range, $\alpha = 0.30\ \mathrm{fm}^{-2}$, the value of $r_{0.016}$ is insensitive to the exact range of the effective interaction; a change of more than 20% in α was necessary to produce a change in $r_{0.016}$ of $\pm$ .03 fm. Therefore, as a measure of the matter density in the surface region we have evaluated $r_{0.016}$ using both the UW and MIT procedure; with but one exception the difference in extracted values with these two procedures has not exceeded 0.02 fm. The same statement applies to the value of $R_{2,4}$.

We estimate that the uncertainty in the listed values of $r_{0.016}$ relative to that of ^{40}Ca, is within the range $\pm$ 0.05 fm. This is an estimate of the combined model errors (which is larger than the $\pm$ .03 fm caused by the uncertainty in the range of the interaction). Considerable care was taken in both sets of experiments[4,14] to measure accurately the scattering angles and relative bombarding energies; as discussed in Reference 4, the uncertainties in these quantities lead to an uncertainty in differences of strong absorption radii which is within $\pm$ 0.03 fm. When this is compounded in quadrature with the model errors one obtains $\pm$ 0.05 fm as the total error.

The averages of the values of $r_{0.016}$ and $R_{2,4}$ obtained are listed in Table I for all 19 nuclei of this survey. It is interesting to note that the difference, $R_{2,4} - r_{0.016}$, is essentially constant (2.60 $\pm$ 0.03 fm). A constant difference is expected on the basis of the folding model and reflects the size of the α-particle folded with a finite range nucleon nucleon interaction.

B. RMS Radii

It is possible to obtain R ($\equiv \langle r^2 \rangle^{1/2}$ the matter RMS radius) from our knowledge of $\rho(r)$ in the surface region although such a determination is model dependent. The additional error in this extrapolation, however, is less than one might at first think since the integrand in the formula for the r.m.s. radius, $r^4 \rho(r)$, is strongly peaked in the region where the density has been best determined. Thus one finds that the value of $r_{0.016}$ effectively determines R for the nuclei here considered. For example, when the density is taken to have the two parameter Fermi form, the variation in R is found to be only ± 0.02 fm for the reasonable range in values of the diffuseness parameter, a = 0.50 ± 0.05 fm. So, under the rather safe assumption that the neutron density does not exhibit wild oscillations an overall uncertainty of no more than ± .02 for the RMS radius relative to that of ^{40}Ca can be assumed.

The question naturally arises, "How dependent are the radii listed in Table I on the density adopted for ^{40}Ca?" We have found that the values of $r_{0.016}$ and $R_{2.4}$ relative to those for ^{40}Ca, are essentially independent of the fine details of the ^{40}Ca density as long as the RMS radius is not changed. If the parameters of the ^{40}Ca density are altered so that RMS radius is changed by 0.02 fm the value of $r_{0.016}$ and $R_{2.4}$ for any other nucleus is found to change by essentially the same amount. (The strength of the effective interaction is recalibrated to the new density by the requirement that the ^{40}Ca angular distribution again be fit.)

A plot of $r_{0.016}$, R, and R_p the proton RMS radius, versus $A^{1/3}$ is presented in Figure 3 and the numerical values are also listed in Table I. The results of accurate muonic x-ray[20] and electron scattering data have provided values for the charge RMS radii, R_c. We have then corrected these for the finite proton size whose RMS charge radius is 0.836 fm[22], the neutron charge form factor and other relativistic corrections[21] to obtain the values of R_p presented in Table I and Figure 3.

IV. DISCUSSION OF RESULTS

Concerning the results shown in Figure 3 and Table I we note the following general trends:

1) Except for the shift between ^{48}Ca and the Ti isotopes, the values of $r_{0.016}$ and R increase smoothly and monotonically.

2) Superimposed on this general trend is a "bowing effect" wherein the radii of nuclei from ^{40}Ca to ^{58}Ni increase more slowly than in the case of nuclei from ^{58}Ni to ^{68}Zn. This bowing effect for matter radii is similar to what was concluded previously[4] concerning strong absorption radii. The bowing effect is seen in the plot of $r_{0.016}$ and more strongly for R and R_p.

3) To within our stated uncertainties there is a general equality between R and R_p so that the difference, $R_m - R_p$, is typically close to zero. It is quite remarkable how well the matter radii track the proton radii. This trend persists despite the fact that the degree of neutron excess varies widely as we move from ^{40}Ca to ^{68}Zn. Although the mean values of R for a few of the middle nuclei lie below R_p, we cannot claim, bearing in mind the stated uncertainty in our analysis, that these differences are significant. It follows as a corollary that R_n (the neutron RMS radii) $\simeq R_p$ to within the uncertainties of the analysis.

In Table I we also present theoretical values for $r_{0.016}$, R, and R_p calculated within the framework of Hartree-Fock-Bogoliubov (H.F.B.) theory using the D1 force of Gogny.[19] These calculations are representative of "the state of the art" in mean field approximation theories. The proton distributions have been repeatedly tested in various cases by electron scattering and found to be in good agreement with empirical data.[19]

The overall agreement with our results is good, in most cases being excellent. Historically, ^{48}Ca is a case that has presented problems, especially in view of the earlier DDHF result[18] $R(^{48}Ca)-R(^{40}Ca)=0.19$ fm. We are in disagreement with this result and the DME result[18] of 0.16 fm, but consistent with the new H.F.B. calculations[23] of Decharge and Gogny.[19]

V. COMPARISON WITH OTHER PROJECTILES

Recent analyses of the elastic scattering of 0.8 GeV[24-26] and 1 GeV[27-30] protons, alpha particles[13,31,32] pion scattering,[33-35] and pion total cross sections[36] also yield nuclear matter and/or neutron distributions. We conclude by briefly relating these results to our own. We believe that the quantity most relevant for comparison is the RMS matter radius, R, relative to that for ^{40}Ca, even though for our case the quantity $r_{0.016}$, characteristic of the surface region alone, is determined more accurately.

The chain of calcium isotopes has been explored most extensively. In Table II we present the RMS matter radii differences for these isotopes $\Delta R(A) = R(A) - R(40)$ obtained using a variety of hadronic probes as well as those calculated using the density-matrix-expansion[18] and H.F.B. theory.[19]

The $\Delta R(A)$ values extracted from 0.8 - 1.0 GeV proton scattering are now in good agreement with one another; they indicate larger values for the growth of Ca isotopes than does our analysis although the discrepancies are well within our quoted respective uncertainties. We are consistent with the results of the 1.37 GeV[31] and 79 MeV[32] alpha scattering experiments and similarly are consistent with the analyses[13] of 104 MeV alpha particle scattering from ^{42}Ca and ^{44}Ca. However, the value obtained in this latter analysis[13] for the difference of the ^{48}Ca and ^{40}Ca radii is the largest entry in Table II and appears to disagree with our own result as well as most of the other values, to within the quoted uncertainties. One cannot fail to notice that α-particle experiments give the widest scatter. It should perhaps be pointed out that both in the present work as well as that of ref. 32 only the diffraction region was taken into consideration while the analysis of the 104 MeV data includes the rainbow region as well.

The agreement with the pion total cross section measurement[36] is good. We also agree with the analysis of pion scattering data[33] performed by Johnson and Bethe[34] which indicates that the measured value of $R(^{48}Ca) - R(^{40}Ca)$ is approximately 0.1 fm smaller than that predicted by Negele[illegible] and therefore in agreement with the HFB predictions.[19]

In Table III we present $R(A) - R(^{40}Ca)$, the matter RMS radii, for the Ni isotopes. One notices excellent agreement between our results and those obtained from 1 GeV proton scattering[27,30] for $^{58,60}Ni$ but possible deviations for $^{62,64}Ni$. Analysis of 0.8 GeV proton scattering from $^{58,64}Ni$ leads to $R(^{64}Ni) - R(^{58}Ni) = 0.15 \pm 0.02$ fm[25] which is between our value of 0.19 ± 0.07 fm, and the 1 GeV proton values of 0.12 ± 0.03 fm and 0.10 ± 0.02 fm. Considering the size of the errors there does not seem to be a significant problem. A possible disagreement arises from an optical model analysis of 162 MeV pion scattering[35] which gives $R(^{58}Ni) = 3.52$ fm which is somewhat smaller than the value of 3.64 ± 0.07 found here.

For ^{48}Ti Alkhazov et al.[27] find $R(^{48}Ti) - R(^{40}Ca) = 0.15 \pm 0.02$ fm which agrees with our value of 0.12 ± 0.07 fm. Using 0.8 GeV protons Paulette et al.[26] find $R(^{48}Ti) - R(^{46}Ti) = 0.06 \pm 0.02$ fm while we find 0.00 ± 0.07 fm. For ^{54}Fe Bertman et al.[26] using 0.8 GeV protons find $R = 3.57 \pm .06$ fm while we obtain $3.53 \pm .07$ fm.

VI. SUMMARY AND CONCLUSIONS

In this work we have presented the analysis of elastic alpha scattering data at 42 MeV from 19 nuclei. The analysis in terms of a microscopic optical model has yielded information about the matter density distributions of these nuclei, particularly in the surface region. The extracted matter quantities ($R, r_{0.016}$) are relative to ^{40}Ca and are given with uncertainties that primarily reflect the model errors.

An overall smooth monotonic increase between ^{40}Ca and ^{68}Zn is observed (except for a discontinuous jump between ^{48}Ca - ^{48}Ti) characterized by a "bowing effect". Nuclei seem to grow slower than the overall trend from ^{40}Ca to ^{58}Ni and faster from ^{58}Ni to ^{68}Zn. Our data indicate that neutron matter densities track the proton distributions fairly well despite the wide fluctuations of neutron excess present in this region of the periodic table.

For eleven of the nuclei we have made comparisons to theoretical calculations[19] based on the Hartree-Fock-Bogoliubov theory. An overall agreement is present although in some cases that is achieved at the extreme limit of our uncertainty. There is now generally satisfactory agreement between the radii in this mass region using various hadronic probes. In particular there is better agreement for the important case of ^{48}Ca than was apparent a few years ago, both from an experimental and theoretical point of view.

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Table 1

Nucleus	$R_{2.4}$	$r_{0.016}$ exp.	$r_{0.016}$ the.	R_{matter} exp.	R_{matter} the.	R_c	R_p exp.	R_p the.
^{40}Ca	7.37	4.78	4.79	3.38	3.366	3.483	3.393	3.389
^{42}Ca	7.41	4.81	4.856	3.40	3.404	3.513	3.430	3.39[illegible]
^{44}Ca	7.46	4.86	4.916	3.42	3.441	3.524	3.446	3.406
^{48}Ca	7.51	4.92	5.015 4.963	3.44	3.491 3.477	3.482	3.414	3.41[illegible] 3.476
^{46}Ti	7.54	4.95		3.50		3.614	3.532	
^{48}Ti	7.55	4.98	5.010	3.50	3.509	3.599	3.521	3.484
^{50}Ti	7.58	5.00	5.055	3.50	3.536	3.576	3.503	3.491
^{52}Cr	7.65	5.05	5.100	3.53	3.569	3.645	3.567	3.554
^{54}Fe	7.68	5.08		3.54		3.694	3.612	
^{56}Fe	7.75	5.15		3.60		3.739	3.660	
^{58}Ni	7.83	5.22	5.272	3.64	3.679	3.777	3.695	3.684
^{60}Ni	7.89	5.28	5.340	3.70	3.722	3.815	3.735	3.707
^{62}Ni	7.99	5.38		3.78	3.769	3.844	3.764	3.734
^{64}Ni	8.04	5.44	5.468	3.83	3.815	3.862	3.781	3.762
^{63}Cu	8.01	5.40		3.79		3.888	3.808	
^{65}Cu	8.07	5.45		3.83		3.908	3.826	
^{64}Zn	8.05	5.45		3.83		3.932	3.852	
^{66}Zn	8.10	5.50		3.86		3.955	3.874	
^{68}Zn	8.19	5.59		3.94		3.970	3.887	

$R_{2.4}$ is the radius at which the real part of the optical potential equals -2.4 MeV (errors ± .03). $r_{0.016}$ is the radius at which the matter density equals 0.016 nucleons/fm^3 (errors ± .05). R is the RMS matter radius (errors ± .07). R_c is the change RMS radius from reference 20 (errors ± .005). R_p is the proton RMS radii obtained from R_c using the corrections of reference 21 (error ± .006) all radii in fm. Theoretical results are those of reference 19 (see footnote 23).

Table II

Isotopic differences in the matter RMS radii of the Ca isotopes, ΔR

Method	Ref.	42-40	44-40	48-40
		ΔR (fm)		
present[a]		.02 ± .07	.04 ± .07	.06 ± .07
79 MeV (α,α)[b]	32			.05 ± .04
104 MeV (α,α)	13	.06 ± .03	.10 ± .03	.18 ± .04
1.37 GeV (α,α)[b]	31	.06 ± .03	.09 ± .03	.12 ± .03
1.0 GeV (p,p)	28	.04 ± .06	.08 ± .06	.12 ± .06
1.0 GeV (p,p)	29	.055 ± .02	.07 ± .02	.10 ± .02
1.0 GeV (p,p)	30	66 ± .03	.07 ± .03	.09 ± .03
1.0 GeV (p,p)	27	.05 ± .02	.07 ± .02	.10 ± .02
0.8 GeV (p,p)	24	.07 ± .02	.10 ± .02	.09 ± .02
[illegible] (σ_{tot})	36		.05 ± .06	.08 ± .06
H.F. (DME)[d]	18	.044	.086	.16
H.F. (DDHF)	18			.19
H.F.B. (D1)	19	.038	.075	.125[c]
H.F.B. + RPA[e]	19			.108

a) quoted error based on experimental, fitting and estimate of model error

b) quoted error based on experimental and fitting errors only.

c) see footnote 23

d) these results are due to the theory of Negele and Vautherin[18] as given in reference 24.

e) in this calculation the densities of both ^{40}Ca and ^{48}Ca are derived as described in reference 19 but then in addition a self consistent Random Phase Approximation (RPA) calculation is performed using the same force (D1) that generated the H.F.B. basis.

Table III

Crowth of RMS matter radii for the Ni isotopes relative to ^{40}Ca (in fm)

Method	Ref.	^{58}Ni	^{60}Ni	^{62}Ni	^{64}Ni
Present[a]		.26 ± .07	.32 ± .07	.40 ± .07	.45 ± .07
1 GeV	30	.25 ± .03	.30 ± .03	.31 ± .03	.37 ± .03
1 GeV[b]	27	.25 ± .02	.27 ± .02	.32 ± .02	.35 ± .02
H.F.B.[c]	19	.31	.36	.40	.45

a) quoted error based on experimental, fitting and model error.

b) quoted error based on experimental and fitting error only.

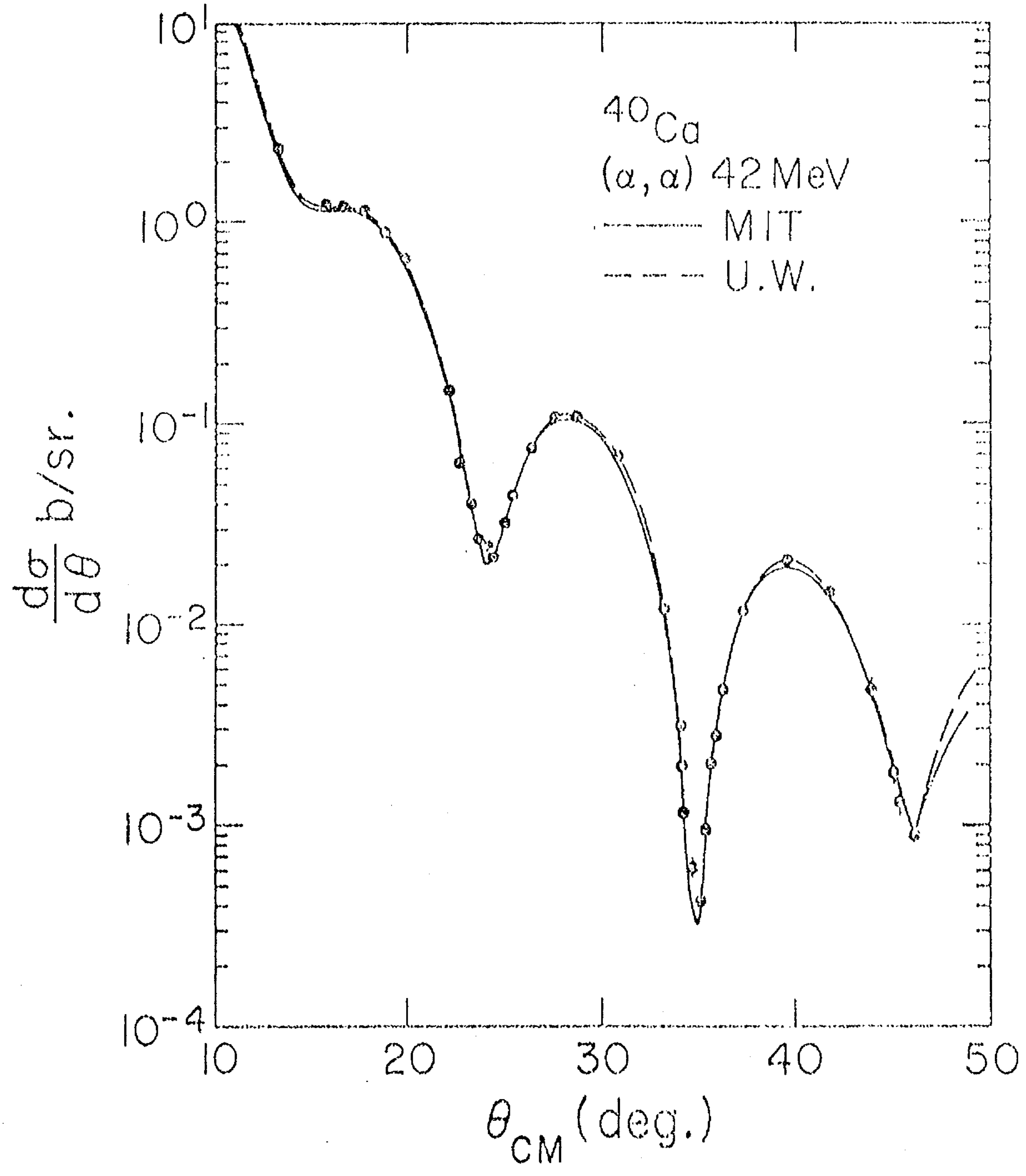
c) ref. 19., see footnote 23

FIGURE CAPTIONS

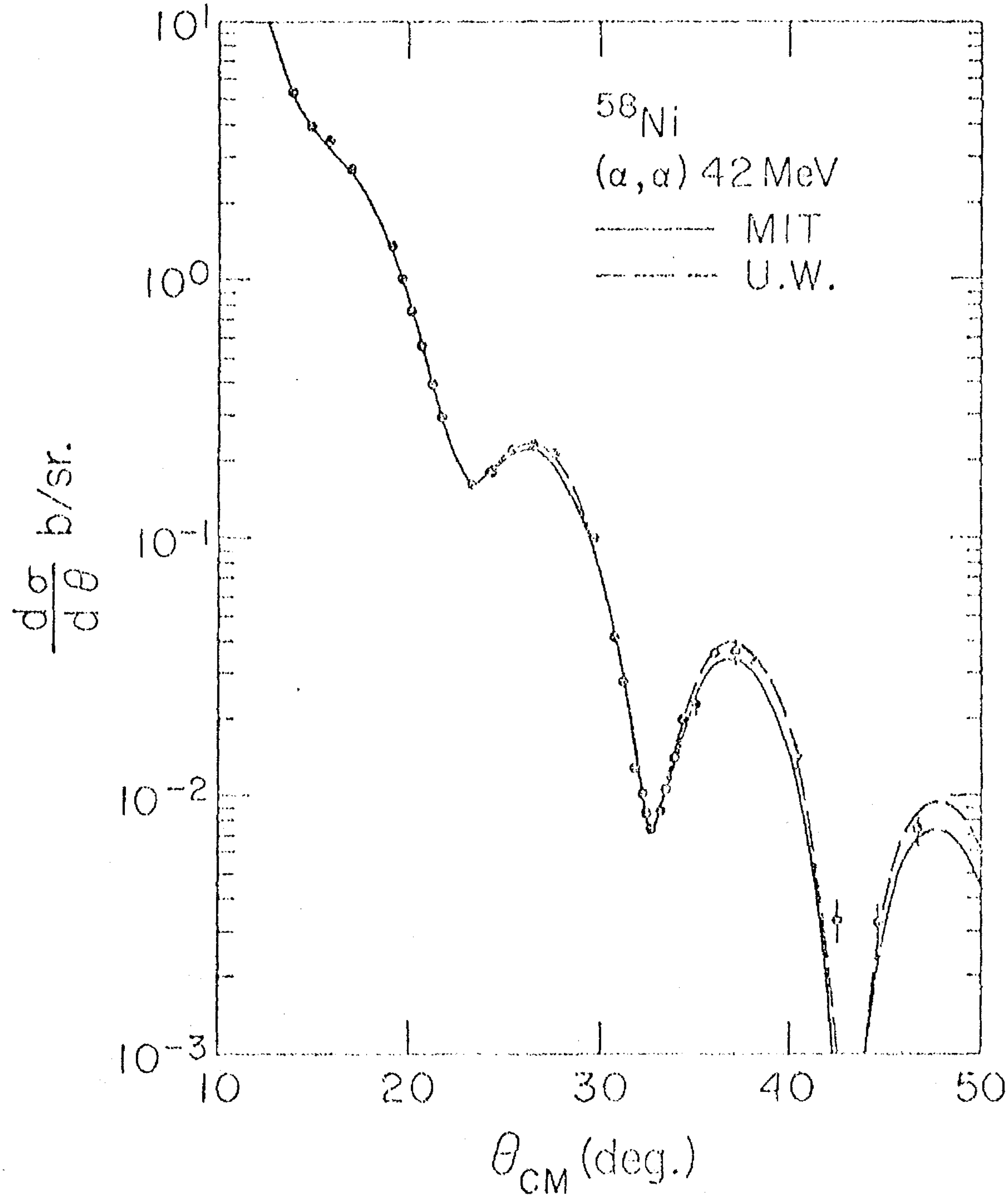
Figure 1. Fits obtained for our normalization nucleus, ^{40}Ca, by the two different procedures employed (MIT/UW - see text).

Figure 2. Typical fit for a N≠Z nucleus obtained by the two variations of the folding model employed (UW/MIT) see discussion in text.

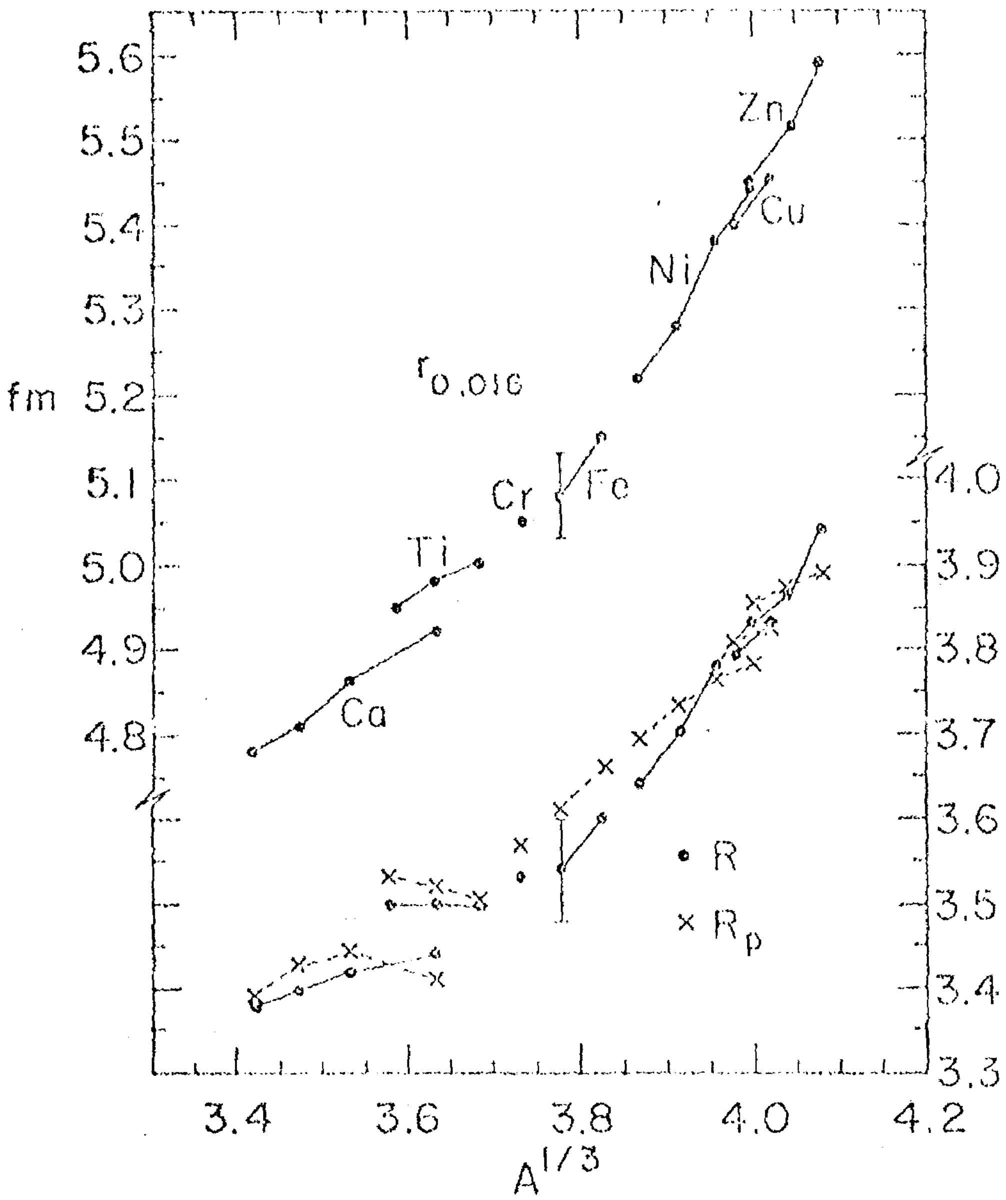
Figure 3. Relative growth for the extracted density quantities. R and Rp are the matter and proton RMS radii and $r_{0.016}$ is the radius at which the density assumes the value of 0.016 nucleons/fm^3 (see Table I and discussion in text). Easily noticeable is the "bowing effect": nuclei seem to grow slower from ^{40}Ca to ^{58}Ni and faster from ^{58}Ni to ^{68}Zn.



^{40}Ca
(α,α) 42 MeV
MIT
U.W.
$\frac{d\sigma}{d\theta}$ b/sr.
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10
20
30
40
50
θ_{CM} (deg.)



^{58}Ni
(α,α) 42 MeV
MIT
U.W.
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10
20
30
40
50
$\frac{d\sigma}{d\theta}$ b/sr.
θ_{CM} (deg.)



fm
5.6
5.5
5.4
5.3
5.2
5.1
5.0
4.9
4.8
4.0
3.9
3.8
3.7
3.6
3.5
3.4
3.3
fm
Zn
Cu
Ni
$r_{0,016}$
Cr
Fe
Ti
Ca
• R
× R_p
3.4
3.6
3.8
4.0
4.2
$A^{1/3}$