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Reliability Indexes for Power Systems

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ABSTRACT

Three main objectives were accomplished during this project. The first objective was to review the traditional treatment of power system reliability indexes. A qualitative analysis of both generation and transmission indexes is presented along with a detailed examination of the analytic and computational algorithms which have been developed. Several algorithms are given that are computationally more efficient than those commonly given in the literature. The second objective was to advance the state of the art by developing a unified probability framework for the definition of power system reliability indexes. The key idea in this approach is the notion of a reliability indicator as an observable characteristic central to describing the reliability of a system. Using this concept, reliability indexes are defined as parameters of probability laws of reliability indicators, thereby providing new insights for the understanding and computation of reliability indexes. The final objective was to evaluate all of the reliability indexes in terms of their usefulness to utility planners and consumers and to determine the characteristics required for quantification of the worth of reliability. Based on interviews with both utility planners and regulators, a ranking of reliability indexes was developed.

EPRI PERSPECTIVE

PROJECT DESCRIPTION

The evolution of criteria for planning generation, transmission, and distribution systems has led to the development of reliability indexes that are responsive to the basic planning parameters. The primary purpose of reliability indexes is to serve as a basis for power system planning. If an index can be shown to aid in the planning of the decision-making process, and if computation of the index is economically practical, then system planners can be expected to use them. Today's increased cost of new facilities, scarcity of capital, and concerns with environmental issues have led to an intense interest in the development of reliability indexes that measure the impact on the ultimate customer. At least one potential advantage of reliability indexes would be to serve as a basis for an objective comparison of the reliability of alternative system configurations.

This project (RP1353-1) develops analytic expressions and computational requirements for bulk power system reliability indexes and applies the indexes to a simplified utility test system. During the development, primary consideration was given as to whether the indexes were accurately reflecting the kind of service the customer was getting, were valuable in estimating the worth of reliable service, were helpful in communicating with nontechnical people or were easy to compute. As a result, four primary indexes were selected: (1) expected hourly loss of load, (2) expected energy not supplied, (3) frequency of loss of load, and (4) expected number of customers cut off. All of these are computed on an annual basis.

PROJECT OBJECTIVES

The objectives of this project are (1) to review the power system reliability indexes currently in use, (2) to advance the state of the art by developing a unified probability framework for the definition of power system reliability indexes, (3) to evaluate the reliability indexes in terms of their usefulness to utility planners and consumers, and (4) to determine the characteristics required for quantifying reliability.

PROJECT RESULTS

The project's accomplishments are the following:

1. Reviewed data requirements, assumptions, and calculation methods for existing reliability indexes

2. Demonstrated the computational feasibility of generation and transmission indexes using simplified utility test systems

3. Developed the mathematical basis and relationship between indexes and developed analytic expressions for the indexes

4. Ranked the reliability indexes according to criteria for measuring service quality

5. Identified the characteristics for quantifying reliability

This report can serve as a handbook for power system planners since the reliability indexes most basic to the electric utility industry have been identified and given firm mathematical definitions.

Recommendations for future research include (1) the development of computationally efficient models for generating unit and transmission line outage rates and (2) the development of calculation methods for reliability indexes when applied to the generation and transmission components.

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SUMMARY

There are three main objectives which this report accomplishes. The first is to review the traditional treatment of power system reliability indices. A qualitative analysis of both generation and transmission indices is presented along with a detailed examination of the analytical and computational algorithms which have been developed. Several algorithms are given which are more computationally efficient than those commonly given in the literature. The second objective is to advance the state-of-the-art by developing a unified probability framework for the definition of power system reliability indices. The key idea in this approach is the notion of a reliability indicator as an observable characteristic which is central to describing the reliability of a system. Using this concept, reliability indices are defined as parameters of probability laws of reliability indicators thereby providing new insights for the understanding and computation of reliability indices. The final objective is to evaluate all of the reliability indices in terms of their usefulness to utility planners and consumers and to determine the characteristics required for quantification of the worth of reliability.

OVERVIEW OF RELIABILITY INDICES

During the development and evaluation of the reliability indices, primary consideration was given to the following attributes:

1. Responsiveness to basic questions considered during the planning of the generation, bulk-power, area-supply and distribution components and the overall power system.

2. Reflection of service quality from the customer viewpoint, i.e., events which have a direct and physical impact on the ultimate customer.

3. Suitability for quantifying reliability worth.

4. Helpfulness in communicating with non-technical people.

5. Feasibility for computation.

These five attributes led to the consideration of four primary reliability indices listed below in order of increasing difficulty to calculate.

1. Hourly loss of load expectation during a year.

2. Expected energy not supplied during a year.

3. Frequency of loss of load during a year, also termed the expected number of occurrences during a year.

4. Expected number of customers not supplied during a year.

Table S.1 shows the units for the above indices as applied to generation, bulk power, area supply and distribution as well as the total system.

TABLE S.1
ILLUSTRATION OF BASIC RELIABILITY INDICES
FOR THE FOUR SYSTEM COMPONENTS

	HLOLE	EENS	FLOL	ECNS
GENERATION BULK POWER AREA SUPPLY DISTRIBUTION	HOURS	MEGAWATT-HOURS	OCCURRENCES	CUSTOMERS
SYSTEM				

Hourly loss of load expectation has the dimension of hours during the study period, expected energy not supplied has the dimensions of megawatt-hours, frequency of loss of load has the dimension of number of occurrences and expected customers not supplied has the dimensions of number of customers interrupted one or more times during a study period.

Calculating reliability for the generation system only is easier than for the other system components. The third section of this report describes the mathematical definition of three deterministic indices and nine probabilitistic indices as shown in Table S.2.

TABLE S.2

GENERATION RELIABILITY INDICES

DETERMINISTIC INDICES

1. Percent reserve based on peak load
2. Percent reserve based on installed capacity
3. Reserve equal to several large units

PROBABILISTIC INDICES

1.	HLOLE	Hourly loss of load expectation.
2.	LOLE	Loss of load expectation, also referred to as loss of load probability, LOLP.
3.	POPM	Probability of positive margin.
4.	Q	Quality index.
5.	PLOL	Probability of loss of load.
6.	EENS	Expected energy not supplied.
7.	XLNS	Conditional expectation of load not supplied, also referred to as XLOL (Expected Loss of Load).
8.	FLOL	Frequency of loss of load.
9.	DLOL	Duration of loss of load.

To aid the reader in understanding these 12 indices for generation, a small FORTRAN program has been written and supplied in the report. By changing various input data, the reader is encouraged to verify for himself the strength and weaknesses of each index on his own system. Through a detailed comparison of the computational algorithms commonly used for the calculation of the indices this section shows that once the work has been done to calculate <u>any</u> of the probabilistic indices the remaining indices listed above can be determined with little additional cost for any desired margin state.

One of the most important areas for further work on reliability indices is the development of computationally efficient models for bulk power transmission, as discussed in the fourth section. Bulk power refers to both inclusion of generating unit outage rates and transmission line outage rates, especially those lines that interconnect companies, power pools, and regions of the utility system.

Table S.3 displays the five deterministic indices and six probabilistic indices considered for bulk power systems.

TABLE S.3

BULK POWER SYSTEM RELIABILITY INDICES

<u>DETERMINISTIC INDICES</u>

1. Maximum load not supplied.
2. Maximum energy not supplied.
3. Minimum load supplying capability.
4. Minimum simultaneous interchange capability.
5. Maximum line flow.

<u>PROBABILISTIC INDICES</u>

1. HLOLE Hourly loss of load expectation.
2. LOLE Loss of load expectation.
3. EENS Expected energy not supplied.
4. FLOL Frequency of loss of load.
5. BPII Bulk power interruption index.
6. BPECI Bulk power energy curtailment index.

Although many more bulk power reliability indices were found in the literature, they all shared a common difficulty; completely calculating the value of such indices fully recognizing all contingencies exceeds computer time and cost limits on most systems. The bulk power indices are discussed using failure effects analysis (FEA) as a framework. The advantage of the FEA approach is that it is flexible enough to accommodate a variety of assumptions and varying degrees of computational accuracy. The computation of the transmission indices is illustrated using a small sample system. The computations and sample results of combined generation-transmission are also demonstrated using the sample network.

PROBABILITY MODELS FOR POWER SYSTEM RELIABILITY INDICES

Section 5 shows how the most commonly used reliability indices can be defined in terms of probability laws of observable characteristics of a power system. The approach is to start with the single numerical-valued characteristic, load (demand) not supplied. Observation over time of load not supplied is then used as a basis for defining several additional observable characteristics which describe reliability. For example, the cumulative amount (integral) of load not supplied is energy not supplied over the observation period.

The name "reliability indicator" is used in Section 5 as a general name for empirical observations like load not supplied and energy not supplied. Thus, a reliability indicator is an observable characteristic which is important for purposes of describing "reliability". The concept of a reliability indicator is a central idea upon which the entire discussion in Section 5 is based. The viewpoint is that reliability indices are parameters of probability laws of reliability indicators. One important advantage of this approach is that it clearly shows how each reliability index is related to the observed performance of the system.

Reliability indicators are classified into three fundamental categories - point reliability indicators, interval reliability indicators, and duration reliability indicators. The categories are distinguished by the manner in which the reliability indicator is observed. Point reliability indicators consider only the systems condition at the observation time, without regard to previous times. An example is load not supplied. Interval reliability indicators consider the system condition over a period of time. An example is energy not supplied. The third category, duration reliability indicators, are observations of the duration of time between specified occurrences. An example is loss of load duration.

Each of the interval and duration reliability indicators can be derived from observation over time of a point reliability indicator. Therefore, the names for the point reliability indicators can be used as "roots" for naming all of the other reliability indicators. The probability law of each reliability indicator forms a basis for at least one reliability index. For numerical-valued reliability indicators the basic reliability index is the expectation (expected value) of the reliability indicator. For the loss of load event, which is not numerical-valued, the probability of the event is the basic reliability indicator. The uniform terminology used to name the reliability indicators can easily be extended to the basic reliability indices. It is only necessary to add the word expectation, or probability, to the reliability indicator name to form the index name. Furthermore, the reliability indices can be classified according to the same three categories as the reliability indicators.

One advantage of classifying the reliability indicators and indices into three basic categories is that the capabilities of current analytical methods can conveniently be summarized in terms of each category. For each of the point reliability indices, the entire probability law can be computed in terms of basic input data on equipment capacity and load demand. This means that such indices as expected load not supplied can be calculated directly from the probability law of load not supplied. Furthermore, additional parameters of the probability law can also be computed if desired. Examples of such parameters are percentiles of the distribution for higher moments, such as the variance (or standard deviation).

In contrast with point reliability indicators, it is not generally practical to compute the probability laws of the duration reliability indicators. Rather, available methods are directed at computation of only the expected value of the duration reliability indicators. Furthermore, the expected values are computed only under "steady state" conditions. The available models actually compute the steady state expected loss of load duration and the reciprocal of the steady state expected loss of load cycle duration (duration between successive loss of load occurrences). The latter quantity is called loss of load frequency, and models for computation of steady state expected loss of load duration and loss of load frequency are called "frequency and duration" models. When used in this context, the term frequency is defined as the reciprocal of the steady state expected loss of load cycle duration. Since it is simply another way of expressing a reliability index, frequency is also a reliability index, and since it is derived from a duration reliability index, frequency is a duration reliability index. Under this definition, it is not necessary (and is in fact incorrect) to use the name "expected frequency" to describe the reliability index. Furthermore, frequency is only defined under steady state conditions.

The last category of reliability index, interval reliability indices, are of the most practical interest, because these are the indices which have been used in planning models. However, methods to compute the probability laws of the interval reliability indicators are not available. Fortunately the expected value of the interval reliability indicators can be computed from the expected value of the point and duration reliability indicators. The interval reliability indices and the related point or duration index from which the interval index can be computed are shown below:

Interval Index	Related Index
Expected Loss of load points	Loss of load probability
Expected loss of load hours	Loss of load probability
Expected unserved energy	Expected load not supplied
Steady state expected loss of load occurrences	Loss of load frequency

The actual mathematical relationships between the desired indices and each related index are derived in Section 5. A summary of the equations is given in Table 5.5 of Section 5.

The first two interval indices, expected loss of load points and expected loss of load hours can be computed from the point reliability index, loss of load probability. These two interval reliability indices both measure the "amount of time" that loss of load exists. Loss of load points counts the number of specific time points (such as daily peaks) while loss of load hours counts continuous time. Expected energy not supplied is computed from the point reliability index load not supplied. The final interval reliability index, expected loss of load occurrences, is computed from loss of load frequency. The relationship holds only under steady state conditions.

The foregoing discussion has used the terminology defined in Section 5. This terminology assigns names in a consistent way using the two point reliability indicators, load not supplied and loss of load, as a base for all names. The resulting terminology is "uniform" and descriptive of the empirical quantity or parameter named, but it is not the same as the terminology which is in current industry use. A comparison of the terminology in Section 5 with common industry terminology is given in Table 5.6 of Section 5. In all sections of this report except Section 5 the industry terminology is used. It is not the intent of the terminology in Section 5 to propose new names for reliability indices in current use. Such industry conventions are established by continued usage. However, if studies such as this project suggest more logical terminology than that in current use, the new terminology may gradually come into use. In addition to the comparison of terminology in Table 5.6, comments on industry terminology are included at several points in Section 5 as an integral part of the discussion of each reliability index.

The structured approach to reliability indices in Section 5 provides a framework for considering possible additional reliability indices. Thus, all the reliability indices in Section 5 are based on the probability law of a reliability indicator, and all the reliability indicators are derived from observation over time of the fundamental reliability indicator load not supplied. Therefore, possible new indices can be considered with respect to the following three questions:

1) Are there reliability indicators which are important for describing power systems reliability but which cannot be derived from load not supplied?
2) Are there additional reliability indicators which can be described in terms of load not supplied, beyond those discussed in Section 5?
3) What additional parameters of the reliability indicators defined in Section 5 would be useful?

A short discussion of examples of possible indices in each category is given in Section 5.10. However, the effort in this project was focused on obtaining a clear understanding of the models upon which existing indices can be defined rather than on development of new indices.

In summary, the key idea in Section 5 is that reliability indices are parameters of probability laws of reliability indicators, which are empirical (observable) charateristics of a power system. Load not supplied is a fundamental reliability indicator, from which other reliability indicators can be defined. Reliability indicators, and associated reliability indices, can be classified as point, interval, and duration reliability indicators. This classification provides important insights for computation of reliability indices. Interval indices are of most importance in applications, but computation methods are based on point and duration indices. From these, the interval indices can be computed. Thus, the point and duration indices are useful primarily as a means for computing the interval indices.

EVALUATION OF INDICES

The economic evaluation of reliability performance require a variety of reliability indices. These indices, in rough order of their frequency of use in the methods surveyed, are as follows:

1) Expected unserved energy,
2) Expected duration of an interruption,
3) Average frequency of interruption,
4) Expected magnitude of load lost in an interruption,
5) Probability distribution of interruption duration,
6) Probability distribution of number of interruptions per year or other time period, and
7) Probability distribution of amount of load lost in an interruption.

Many methods, of course, make use of several of the above indices.

It is also evident that accurate economic evaluation of reliability requires that indices be calculated separately for a range of consumer classes or types of end use consumption and that indices may be required for different times of day, day of week, season of year, and geographical areas.

One general requirement implicit in all methods for the economic quantification of reliability performance is that the reliability indices utilized be computed accurately and bear close relationship to actual historical reliability performance. This is necessary because the computed cost of interruptions is usually treated in an absolute way and traded off against other system costs such as owning and operating costs to determine an "optimum" system or level of reliability performance.

Based upon the discussions held with members of Public Service Commissions and utility personnel the general basis for the ranking of the indices, in relative order of importance, are:

1) significance and meaningfulness to consumers,
2) measurability from historical records of system reliability performance,

3) usefulness to planners,
4) acceptability to regulatory agencies, and
5) usefulness in economic evaluation of reliability.

A basic conclusion is that the trend in system reliability indices is, and should be, toward indices which have the closest possible relationship to attributes of service reliability which are significant and meaningful to the ultimate consumer. It is believed that this emphasis on consumer-related reliability indices will tend to focus attention on the ultimate goal of system reliability; will tend to unify indices which are used in generation, transmission and distribution systems; and will tend to produce indices which could be used in economic evaluation of reliability.

The second basis for ranking indices, measurability from historical records, is also felt to be very important. Indices which can be measured from historical records will be, by definition, physically significant - an important attribute and closely related to the first ranking basis. Measurability from historical records also implies that computed reliability indices can be verified against historical performance thereby providing a means of checking the accuracy of calculation models. Thus, the use of indices which are physically measurable should foster the use of more accurate modeling methods and should tend to produce index values having greater absolute significance.

The usefulness to planners of an index is very largely the extent to which the index is sensitive to and reflects planning parameters of interest. Planners are often satisfied with indices having only relative significance for choosing between alternatives. However, planners seem to be favoring more physically-based, absolute significant, indices for the future primarily because such indices will be better received outside the planning environment.

The factors which make an index acceptable to regulatory agencies are fundamentally similar to those factors of concern to planners and utility management. However, an overriding concern of regulatory agencies is that reliability indices be simple, unambiguous, and intuitively appealing. In the future, regulatory agencies would appear to favor consumer-related, physically-measurable, indices which can or have been varified against historical experience.

The usefulness of reliability indices in making economic evaluations of system reliability does not seem to be an important concern at this time or for the near future. Both planners and regulatory agencies presently view the accuracy of economic evalutions of reliability with great skepticism. However, most seem to agree that economic evaluation of reliability would be a good approach if practical and accurate methods for such evaluations can be found. Further, indices which are selected on the basis of significance to consumers and physical measurability will also satisfy most if not all, of the requirements of economic evaluation methods.

Using the above bases the following ranking of generic reliability indices was developed. Note that, in many cases, a single index is meaningful only if used in conjunction with other indices was developed.

1) Frequency of interruption, load loss, capacity shortage, or other capacity margin event. Frequency is interpreted to include all indices which measure the average or expected number of interruptions or outages in a given time period. This includes the "expected number of days of capacity deficiency" as used in traditional generation reliability studies as well as the modern and precisely defined "frequency" indices used in both generation and T&D studies.
2) Expected duration of interruption, load loss, capacity shortage, or other capacity margin event.
3) Expected magnitude of interruption or capacity shortage given such an event.
4) Expected unserved energy in a specified time period. Note this index can be computed if the first three above are known. Note further that this index is, perhaps, the most descriptive <u>single number</u> index which exists.
5) Probability distribution of interruption duration. This index, if it can be called that, is needed where the expected value of interruption duration does not adequately describe the range of possible durations or where consumers are significantly nonlinearly sensitive to interruption duration.
6) Probability distribution of the number of interruptions per time interval. This index is needed where consumers are significantly nonlinearly sensitive to the time between interruptions.

7) Probability distribution of the magnitude of interruption. This index is needed when interruption impact is significantly nonlinear with respect to interruption magnitude.

8) All other indices including, in particular, those indices which express reliability in terms of probability of "successful" or "unsuccessful" operation. These indices are judged to have much less physical significance than those listed above and to be generally less desirable.

There are, of course, many methods for computing the above indices. Some methods for computing given indices contain more detailed modeling than others and thereby give greater accuracy and permit the impact of more planning variables to be evaluated. The methods to be used in given studies obviously should be those which give the required accuracy at least computing cost for the specific application.

FUTURE WORK NEEDS

The reliability indices most basic to the electric utility industry have been identified and given firm mathematical definitions. The industry is now in a position to research the calculation methods for these reliability indices when applied to the generation, bulk power, area supply, and distribution components, as well as the combined system. Specifically, the bulk power and area supply reliability indices are hampered by the complicated network calculations that must be repeated thousands of times before accurate reliability expectations result. Partial results based on the most significant calculations must be accepted at present. Defining acceptable approximations is an area where much work is needed.

In Section 5 the formulation of power system reliability indices in terms of fundamental empirical characteristics provides a solid foundation. The application of these probability concepts to existing computation methods is a logical next step. The objective would be to identify and validate the essential assumptions and limitations of the methods, show the relationship between the various methods, and relate industry terminology and symbols in a common framework.

SECTION 1

PROJECT INTRODUCTION

1.1 BACKGROUND

Historically, the evolution of planning criteria in each of the three power system sectors (generation, transmission, and distribution) has led to the development of reliability indices which are responsive to the basic planning parameters of that sector. Although the underlying objective of reliability criteria is to provide a basis for balancing cost and reliability, the primary application of reliability indices has been as a consistent basis for planning. The design level of the indices has not been selected by use of precise analytical methods. Rather, design criteria have evolved gradually, over a long period of time, based on subjective and intuitive judgements coupled with observation of actual experience.

In recent years, increased cost of new facilities, scarcity and cost of capital, and concerns with environmental and other social issues have led to an intense interest in development of reliability indices which are based on events which have a direct and physical impact on the ultimate customer. One advantage of such measures would be their potential ability to serve as an "absolute" basis for determining an "adequate" level of service reliability.

In summary, the primary purpose of reliability indices is to serve as a basis for system planning. The correlation on a particular utility system between the reliability indices and the important planning parameters is critical to the acceptance of the index. If an index can be shown to aid in the planning decision-making process, and if computation of the index is economically practical, then application by planners can be expected.

1.2 OVERVIEW OF REPORT

In order to facilitate the presentation of the results in a logical order, this report is divided into six sections. A brief review of these sections follows.

Section 1 - Project Introduction

This section outlines the objectives of the study and presents a brief summary of the results. Also included is an outline of the remainder of the report.

Section 2 - Overview of Reliability Indices

Section 2 contains a qualitative overview of power system reliability in general, as well as a more specific look at the generation and transmission indices which have been developed.

This section begins with a discussion of where power system reliability research is leading, and the "ideal" reliability indices of the future. A hierarchy of the indices and a qualitative discussion of the computational processes involved is then presented. In addition to examining some of the specific indices used in generation and transmission, this section also takes a look at some of the special problems associated with power system reliability evaluation.

Section 3 - Calculation of Generation Reliability Indices

This section presents an examination of the analytical expressions for each generation reliability index and details the various algorithms which have been developed for their calculation. Some new and powerful computationally efficient algorithms are developed for the determination of the frequency, duration, and magnitude of outages. These algorithms are responsive to the hourly variation of the loads and the uncertainty inherent in the demand forecast. Included in this section, and related appendices, are a "hand example" showing the detailed calculation of each index, as well as a computer program demonstrating the calculation of the indices for a large power system.

Section 4 - Calculation of Transmission Reliability Indices

This section is similar to Section 3 except that here the transmission indices are examined. A logical structure for the determination of reliability indices is presented along with the necessary algorithms for their calculation. Also examined are some of the problems involved in representing the transmission system in reliability calculations. Various indices are calculated for a small sample system showing the difficulties involved in determining a combined generation and transmission reliability.

Section 5 - Probability Models for Power System Reliability Indices

Section 5 expresses each important reliability index in terms of a general probability model. The key idea in this approach is the notion of a reliability indicator as an observable characteristic of a power system which is important for describing the reliability of the system. Using this concept, reliability indices can be defined as parameters of probability laws of reliability indicators. Load not supplied is a fundamental reliability indicator, from which other reliability indiators can be derived.

Reliability indicators, and associated reliability indices, can be classified as point, interval, and duration reliability indicators. This classification provides important insights for computation of reliability indices. Interval indices are of most important in applications, but computation methods are based on point and duration indices. Thus, the point and duration indices are useful primarily as a means for computing the interval indices. Another result of the classification of reliability indicators is that the two point reliability indicators - load not supplied and loss of load - provide a basis for a uniform and consistent nomenclature for both reliability indicators and reliability indices.

Section 6 - Evaluation of Indices

The first part of this section develops "yardsticks" to evaluate the various reliability indices in terms of their usefulness to utility planners, their physical measurability, and their meaningfulness to utility customers. The remainder of this section then ranks the indices according to these criteria, and defines the characteristics necessary for the quantification of the worth of reliability.

1.3 RESULTS

The results of this project are:

- A review of calculation methods, data requirements, and assumptions involved in existing reliability indices.
- A demonstration of selected indices on a synthetic system.
- An analytical statement of indices and the mathematical relationships between indices.

- A ranking of existing indices according to criteria for measuring service quality.

- A survey of methods for reliability worth quantification.

Section 2

OVERVIEW OF RELIABILITY INDICES

2.1 INTRODUCTION

Section 2 focuses on the reliability indices useful in answering utility system planning questions. The many indices used in generation and bulk power planning are mentioned with particular emphasis on four primary reliability indices. These four reliability indices apply to all four components of the system: generation, bulk power (which includes generation), area supply and distribution.

Section 2.2 presents a visualization of system status indicators used in conceptualizing an imaginary reliability center. Reliability indices can be related to the probability laws of these status indicators.

Section 2.3 presents a look-ahead to the desired results of applying the four primary reliability indices to the four basic components of the utility system.

Section 2.4 discusses the computational structure necessary to compute the four primary reliability indices. Section 2.5 focuses on a special case, the generation system without considering operating requirements and interconnections. For this special case, many reliability indices can be calculated as a computer program in Appendix C confirms. Section 2.6 extends the generation discussion to include operating and interconnection considerations. When these considerations are added it is not possible, at present to compute the desired indices.

Section 2.7 presents the bulk power reliability indices. Again, the discussion focuses on the challenges to calculating probabilistic reliability indices. Section 2.8 concludes by summarizing of the needs for future work based on this section. Section 2.9 contains the references for this section.

2.2 VISUALIZING STATUS INDICATORS

What is a way of visualizing the indicators referred to by reliability indices?

Imagine a room called the "Electric Utility Reliability Index Center". This room has lights and dials, called reliability status indicators, on the walls. The lights, when they're on, indicate certain conditions exist on the system. The dials, reliability meters, show magnitudes of random variables on the system. The definitions in the glossary below will be used in the following paragraphs.

GLOSSARY FOR SECTION 2

Reliability Light - an On-Off indicator.

LOL - Loss of Load, a fundamental On-Off indicator in electric power systems.

Reliability Meter - A magnitude indicator.

LNS - Load Not Supplied (megawatts), a fundamental magnitude indicator in electric power systems.

CNS - Customers Not Supplied, a second fundamental magnitude indicator in electric power systems.

2.2.1 System Reliability Status Indicators

Exhibit 2.1 is a list of reliability status information and associated lights and dials. A lamp brings to mind the questions "How often does it light?"; "How much time is it on in a year?"; "When on, how long does it remain on?"

Exhibit 2.1

CONCEPTUAL PICTURE OF THE SYSTEM RELIABILITY CENTER

1. Loss of Load, LOL
2. Load Not Supplied, LNS
3. Customers Not Supplied, CNS — EXTERNAL

---

4. Dispatch Center in a Non-Normal State — INTERNAL
5. Neighboring Utility Asking for Help
6. Voltage Reduction
7. Number of Buses with Voltage Reductions
8. Appeals to Large Customers
9. Appeals to Public
10. Economic Dispatch Restricted
11. Transmission Line Limiting
12. Line Tripped to Remove Bottleneck
13. Cascading Resulting in Several Facilities Out
14. Voltage Out of Limits
15. Islanding Is Occurring

Dials represent numerical quantities and bring to mind questions such as "How many megawatts or customers are affected at each instant?" Questions about the accumulation of these magnitudes can be asked such as the number of megawatt-hours with load not supplied and the number of customer hours with customers not supplied in a year. These lamps and dials give a physical picture of the observations which are important for describing reliability.

The first lamp in Exhibit 2.1 is marked "Loss of Load" and is "on" whenever the utility system is experiencing a loss of load of any magnitude. The second indicator is a dial marked "Load Not Served". It normally reads "0" except when power is not being supplied, in which case the dial indicates the amount of power not being supplied. The third indicator is a dial showing the number of customers not being supplied at any moment. The loss of load lamp will be on whenever the dials show non-zero readings.

The first three status indicators are called external status conditions because they describe conditions seen by those outside the utility company. A financial or regulatory person walking into the Reliability Index Center may feel that these "External" indicators are the only reliability indices needed to judge whether the system is reliable or not. But planners and operators would like some additional information, the "Internal" indicators in Exhibit 2.1.

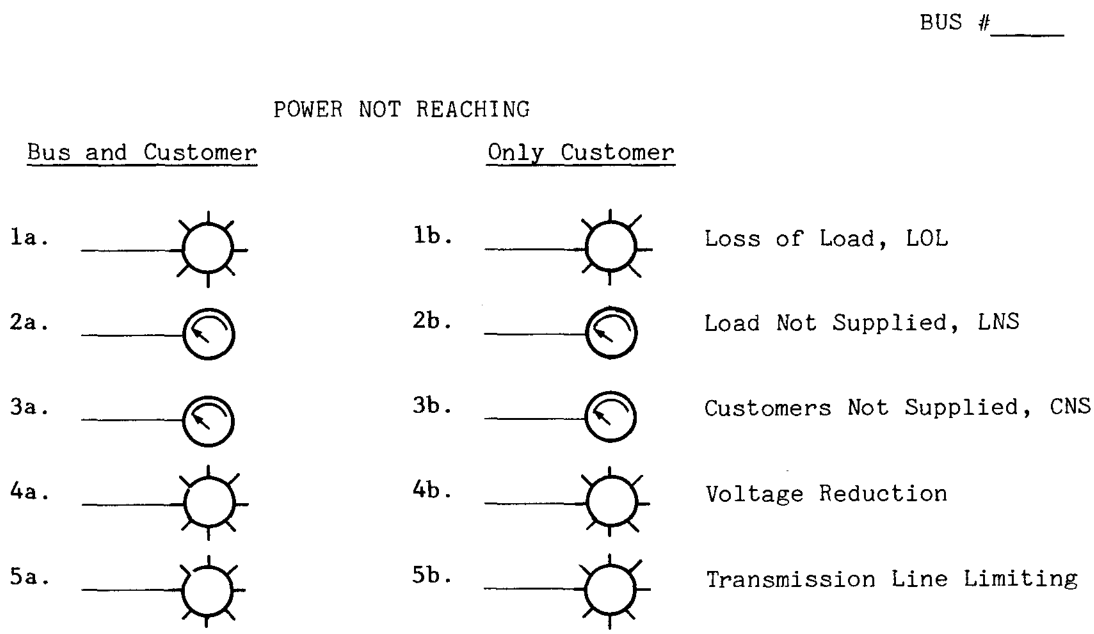
Indicators 4 through 15 are associated with the operation of the system. Whenever one or more of the lights are on the dispatch center is in some non-normal state. Indicators are included for neighboring utilities asking for help, voltage reduction, appeals to large customers, appeals to the public, a restricted economic dispatch, some transmission lines limiting the dispatch, a line tripped to remove a bottleneck, cascading is occurring, voltages are out of limits, and islanding is occurring.

2.2.2 Bus Reliability Status Indicators

The reliability center also brings in reliability information for five indices associated with each bulk power substation or bus. In Exhibit 2.2, a bus would have a lamp indicating loss of load due to power not reaching that bus and a second lamp for any customers not receiving power. For example, a customer with a power pole knocked down because of an automobile would indicate loss of load by lighting the "Power Not Reaching Only Customers" lamp, 1b, but lamp 1a would be "off". The other four indications shown in Exhibit 2.2 correspond to load not supplied, customers not supplied, voltage reduction and transmission line limitations in effect. These indications and more would be of interest to power system operators and planners if the information could be made available.

Exhibit 2.2

CONCEPTUAL PICTURE OF THE BUS RELIABILITY CENTER



2.2.3 External System Indicators

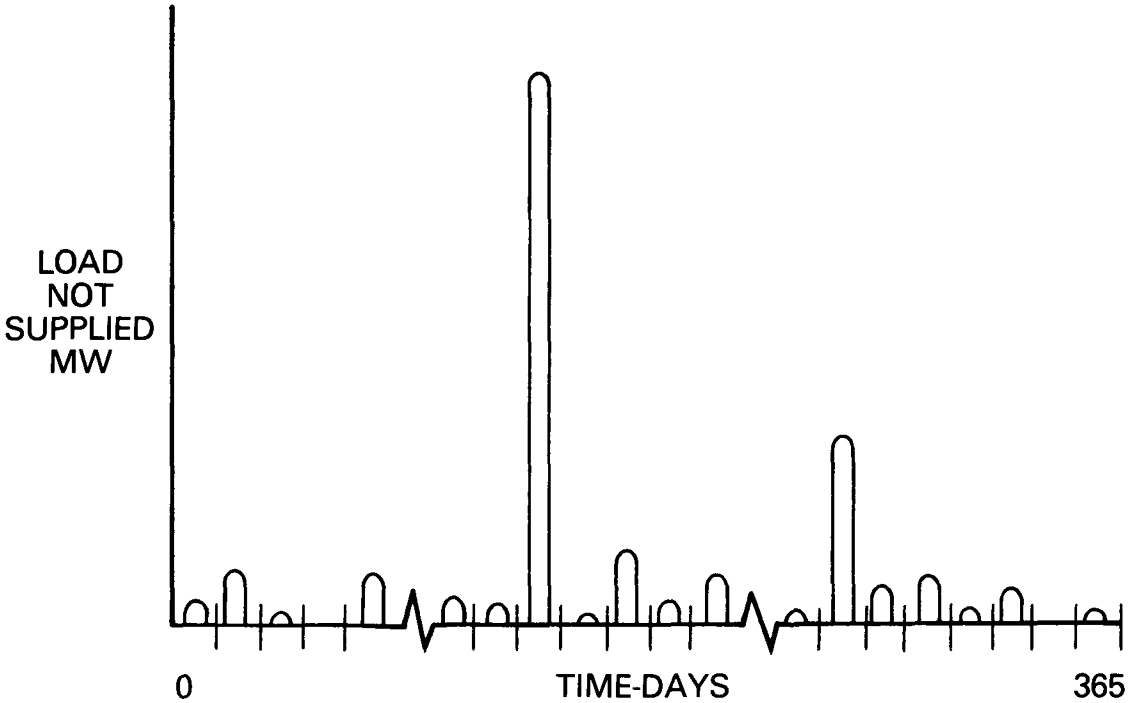
The 15 system indicators and 10 indicators per bus show the many types of information useful for described reliability status. Reliability indices based on the number of times each light is on in a month or year, the magnitude of the readings on each of the dials each time it occurs, the accumulation or integration of the meter readings to record the megawatt-hours of unserved energy or number of customers unserved, would help in the operation and planning of power systems. On the following pages we will discuss the type of indices that could be calculated based on the external indicators in Exhibit 2.1, loss of load, load not supplied, and customers not supplied. Indices for internal status indicators and bus status indicators are left for further research.

2.2.4 Observations of System Load Not Supplied

To introduce the statistics for the three external reliability status indicators, it will be helpful to refer to the observation of the Load Not Supplied, LNS, over time, Exhibit 2.3. First, note that Load Not Supplied occurs almost every day, it is a frequent occurrence. The number of occurrences in a year will be close to the number of days in a year.

Exhibit 2.3

ILLUSTRATIVE SAMPLE RECORD
LOAD NOT SUPPLIED



The second observation about Exhibit 2.3 is that the magnitude of LNS varies widely, with many small occurrences and a few large ones.

Three statistics are important in consolidating our understanding of a Load Not Supplied record such as Exhibit 2.3:

Number of occurrences during a year.
Energy accumulated during the year.
Hours accumulated during a year.

Estimates of the expected value of these statistics for future years become three primary reliability indices:

1. Expected Number of Occurrences during a year (called Frequency of Loss of Load)
2. Expected Energy Not Supplied during a year.
3. Expected hours load not supplied in a year (called Hourly Loss of Load Expectation).

A similar sample record of the Customers Not Supplied observations could be drawn. The three statistics are:

Number of occurrences during a year, identical to the LNS statistics.
Customers accumulated during a year, a new statistic different from LNS.
Hours accumulated during a year, identical to the LNS statistics.

The expected value of the new statistics is a fourth primary reliability index:

4. Expected Customers Not Supplied during a year.

These four indices will be illustrated in the following paragraphs.

2.3 LOOK AHEAD TO DESIRED RESULTS

Up to this point three external reliability status indicators have been identified. Sample records were discussed along with six statistics that are easily identifiable. Section 5 of this report develops the probability models needed to define three primary reliability indices in terms of the reliability indicators. The results in Section 5 will be used in the following paragraphs as an example is developed. This example begins with the four reliability indices that could be developed for an electric utility system.

The IEEE Reliability Test System (IEEE-PES-APMS, 1979) [2-24] provides the system data used in this illustration. The system to be studied in a member of a power pool. The pool is composed of four identical IEEE-PES-APMS systems: A, B, C, and D. Each system has a peak load of 2850 MW and an installed generation capacity at time of peak of 3375 MW. The only new data is the presence of six interconnections.

INTERCONNECTIONS

	kV	Terminals	Electrical Parameters
1	230	A 15 - B 13	same as line 16-19
2	230	A 22 - C 22	same as line 17-18
3	230	A 13 - D 15	same as line 16-19
4	230	B 22 - D 22	same as line 17-18
5	230	B 15 - C 13	same as line 16-19
6	230	C 15 - D 13	same as line 16-19

All other data are are shown in the IEEE publication. These data for system A were used to guide the selection of illustrative results. The only calculations made were for the generation indices. Therefore, the numerical quantities illustrate the general magnitudes of indices and not relationships.

2.3.1 Summary of Reliability Study

It is not unreasonable to illustrate the four primary reliability indices using values shown in Exhibit 2.4.

Exhibit 2.4

SUMMARY OF RELIABILITY STUDY
IEEE-PES Reliability Test System

Notice -- Illustrative Example Only

HIERARCHY OF RELIABILITY INDICES	YEARLY SYSTEM VIEWPOINT	YEARLY CUSTOMER VIEWPOINT
1. Frequency of Loss of Load (more accurately the Expected Number of Occurrences)	350 occurrences	0.26 occurrences
2. Expected Energy Not Supplied	1,677 MWh	3.13 kWh
3. Hourly Loss of Load Expectation	1,068 Hours	0.92 Hours
4. Expected Customers Not Supplied	66,600 customers	0.26 occurrences

Caution -- the numerical quantities in Exhibit 2.4 are illustrative only. The values are typical but were not calculated and therefore are not necessary consistent with each other.

The order of index presentation in Exhibit 2.4 is:

1. Frequency, but more accurately termed Expected Number of Loss of Load Occurrences, is important to customers as discussed in Section 6.5.

2. Expected Energy Not Supplied, which is important in economic evaluation, see Section 6.4.2.

3. Hourly Loss of Load Expectation which is necessary in computing the duration of interruptions, important to customers as discussed in Section 6.4.2. This is also the easiest to compute of the four basic reliability indices.

4. Expected Customers Not Supplied, which is important to utility planners and public service commissions as noted in Section 6.5, load supplied indices.

Frequency. The Frequency of 350 expresses the number of days on which load was not supplied during some portion of the day. An average customer may expect 0.26 loss of load occurrences during a year or nearly four years between occurrences. Frequency of loss of load is an important reliability index but should not be used alone in economic decisions.

Expected Energy Not Supplied. The second significant reliability index shown in Exhibit 2.4 is the expected energy not supplied, shown as 1677 MWh for the system and 3.13 kWh for a typical customer.

Hours Loss of Load Expectation. The third reliability index, in Exhibit 2.4, containing information not available from the first two indices, is the hours loss of load expectation. This index shows 1068 hours of loss of load during a year somewhere on the system while the average customer can expect less than one hour per year.

Expected Customers Not Supplied. The fourth significant reliability index in the heirarchy is the expected customers not supplied. During a year the system can expect 66,600 customers with one or more occurrences of load not supplied while one fourth of the customer can expect to be not supplied one or more times.

The four reliability indices introduced in Exhibit 2.4 will be detailed in the following exhibits.

2.3.2 Frequency of Loss of Load

Exhibit 2.5 amplifies the frequency numbers shown in Exhibit 2.4. The system viewpoint of all causes is 350 occurrences shown in both exhibits. It is not unreasonable that this amount could be made up of 345 occurrences in a distribution system, 5 occurrences in the area supply system, 0.3 in the bulk power system, and 0.1 occurrences in the generation and interconnection portion of the system. The "generation only" (not including ties) calculation results in 3.6 occurrences per year. The 3.6 is referred to as a dependence on supplemental capacity resources (DSCR), or the single area Loss of Load Expectation (LOLE), and is the only quantity that is easy and straightforward to calculate. All the previous quantities involve significant amounts of knowledge, experience, and judgement, in addition to computer time and man-hours.

At this point some readers may wish to debate that the expected numbers can be added up directly without subtracting the overlapping outages. Combining results is a further research effort. A second concern is that the component system results should not be added together at all. The number of occurrences of outages on the distribution system are a different kind of occurrences then outages in the bulk power. Exhibit 2.3 illustrates the problem. Frequency alone does not capture all the visual information from the sample observation. A large outage counts one occurrence just as a very small outage. That is why we do not stop with just frequency but move on to the expected energy not supplied.

Exhibit 2.5

FREQUENCY OF LOSS-OF-LOAD

Notice -- Illustrative Example Only

	YEARLY SYSTEM VIEWPOINT		YEARLY CUSTOMER VIEWPOINT	
All Causes	350	Occurrences	0.26	Occurrences
Distribution	345	"	0.20	"
Area Supply	5.	"	0.02	"
Bulk Power	0.3	"	0.02	"
Generating Including Reserve, Operation and Interconnections (R,O,I)	0.1	"	0.02	"
Generation Only DSCR*	3.6	"	----	

*DSCR = Dependence on Supplemental Capacity Resources

Looking at the second column of numbers, the frequency of loss of load from the customer viewpoint shows 0.26 occurrences per year from all causes, 0.20 occurrences from the distribution system, and 0.02 occurrences per year from area supply, the bulk power system and the generation and interconnection system. Since generation only has no effect on the customer, it is not possible to calculate a customer viewpoint number.

The system components used in Exhibit 2.5 are based on the reliability calculation model suggested by Endrenyi (1978) [2-17]. For example, Chapters 7, 8 and 9, "Generating Capacity Reserve Evaluation", "Operating Reserve Evaluation", and "Interconnected Systems" describe the calculations for generation. Chapter 10 treats "Bulk Power System Reliability", Chapter 11 is "Area Supply System Reliability" and Chapter 12 is "Distribution System Reliability".

The components of the utility system are as follows.

Distribution. Distribution networks deliver energy from the area supply stations to the customers. They generally have much simpler layouts than the transmission networks. Most are radial arrangements, and the components involved in the supply of the customer are in a series connection.

Area Supply. Area supply consists of the transformer stations supplying the load in a given area, and the transmission lines feeding stations. Also included are the station apparatus such as buses, circuit breakers, disconnect switches, transformers and relay equipment. The systems feed the distribution networks. However, where the distribution system was series connected, the area supply system is a network with a great diversity of equipment to consider. Sub-transmission is another name for this supply.

Bulk Power. The bulk power system is defined as the generation system and the high-voltage transmission network extending to point of load transfer or to lower-voltage levels. The entire system, including many companies and perhaps power pools, are included. It does not involve a great variety of station components as do area supply and distribution. However, there is a great number of buses, lines and generators to consider.

Generation Including Reserve, Operation, and Interconnections. The generation system includes considerations of reserve, including emergency operating procedures, operating requirements including spinning reserve and standby equipment, and interconnections including tie line and generating units outside of the system of interest. Contractual agreements are often a consideration.

Generation Only Considering Dependence on Supplemental Capacity Resources. This system component treats generation as an isolated factor which has allowed many reliability indices to be calculated. However, these indices do not refer to loss of load or customers interrupted, but rather dependence on for supplemental capacity resources.

2.3.3 Expected Energy Not Supplied

Exhibit 2.6 contains the amplification of expected energy not supplied from the system and customer viewpoint. The system 1677 MWh is made up of 207 MWh for distribution, 600 MWh for area supply, 720 MWh for bulk power and 150 MWh from generation and ties. The "generation only" number of 3170 MWh dependence on supplemental capacity resources is shown because it is the easiest to calculate. In the second column, the customer viewpoint of 3.13 kWh expected energy not supplied is the sum of 2.04 kWh from the distribution system, 0.41 kWh for area supply, 0.54 kWh for bulk power and 0.14 kWh for generation and interconnection.

Exhibit 2.6

EXPECTED ENERGY NOT SUPPLIED

Notice -- Illustrative Example Only

	YEARLY SYSTEM VIEWPOINT		YEARLY CUSTOMER VIEWPOINT	
All Causes	1677	MWh	3.13	kWh
Distribution	207	"	2.04	"
Area Supply	600	"	0.41	"
Bulk-Power	720	"	0.54	"
Generation Including Reserve, Operation and Interconnections (R,O,I)	150	"	0.14	"
Generation Only DSCR	3170	"	----	

2.3.4 Hourly Loss Of Load Expectation

Exhibit 2.7 amplifies the system and customer viewpoints of hourly loss of load expectation (HLOLE). The system number of 1068 hours/year is made up of 1035 hours due to distribution problems, 30 hours due to area supply causes, 2.4 hours due to bulk-power system causes and 1.0 hours due to generation and interconnection causes. "Generation only" is shown because it is the easiest to calculate and resulted in 22 hours dependence on supplemental capacity resources. In the second column, the customer expects 0.92 hours made up of 0.60 hours from distribution, 0.12 hours from area supply 0.16 hours from bulk power and 0.04 hours from the generation and interconnection system.

Exhibit 2.7

HOURS LOSS-OF-LOAD EXPECTATION

Notice -- Illustrative Example Only

	YEARLY SYSTEM VIEWPOINT	YEARLY CUSTOMER VIEWPOINT
All Causes	1068 Hours	0.92 Hours
Distribution	1035 "	0.60 "
Area Supply	30 "	0.12 "
Bulk Power	2.4 "	0.16 "
Generation Including Reserve, Operations and Interconnections (R,O,I)	1.0 "	0.04 "
Generation Only DSCR	22 "	----

2.3.5 Expected Customers Not Supplied

The expectation of 66,600 customers not supplied during a year, Exhibit 2.3 is modified in Exhibit 2.8. Distribution contributes 13,600 customers not supplied during a year, area supply 20,000 customers, and bulk-power 18,000 customers, and generation and interconnection 15,000 customers per year. The "generation only" quantity cannot be calculated per customers since they are unaffected by the dependence on supplemental capacity resources. Therefore, although "generation only" is the easiest to study, it supplies no information about customers.

Exhibit 2.8

EXPECTED CUSTOMERS NOT SUPPLIED

Notice -- Illustrative Example Only

	YEARLY SYSTEM VIEWPOINT		YEARLY CUSTOMER VIEWPOINT	
All Causes	66,600 Customers		0.26 Occurrences	
Distribution	13,600	"	0.20	"
Area Supply	20,000	"	0.02	"
Bulk Power	18,000	"	0.02	"
Generation Including Reserve, Operations and Interconnections (R,O,I)	15,000	"	0.02	"
Generation Only DSCR	------		-----	

Notice in the second column, the customer viewpoint, the expected number of customers not served per year is the same as the frequency of loss of load, i.e., the quantities in Exhibit 2.8 match those of Exhibit 2.5.

Further research is needed on Customers Not Supplied to make the fundamental definitions viable.

2.3.6 Additional Reliability Indices

Two additional indices can be calculated using the four primary indices already presented, average duration and conditional expected load not supplied, see Table 5-9 in Section 5.

Duration. The average duration of a loss of load occurrence, Exhibit 2.9, is the ratio of frequency, Exhibit 2.5 and hours, Exhibit 2.7. For example, for the "all causes" quantity, the average duration is calculated as:

$$\frac{1068 \text{ hours}}{350 \text{ occurrences}} = 3.1 \text{ hours/occurrence}$$

The ratio for the distribution system is:

$$\frac{1035 \text{ hours}}{345 \text{ occurrences}} = 3 \text{ hours/occurrence}$$

In a similar manner, 6 hours average duration was computed for area supply, 8 hours average duration for bulk-power and 10 hours average duration for generation and ties. The "generation only" number, the easiest to calculate, is 6.0 hours average duration of the dependence on supplemental capacity resources.

The customer viewpoint is calculated as 3.5 hours average duration of loss-of-load due to all causes, 3 hours due to distribution, 6 hours due to transmission, 8 hours due to bulk power and 2 hours due to generation and interconnection. Generation and interconnection blackout effects can be rotated, explaining the difference between the system viewpoint, 10 hours, and the customer viewpoint, 2 hours.

Using rotating blackouts each customer experiences only 2 hours of interruption even though the entire system experiences a 10 hour period of generation shortage.

Exhibit 2.9

AVERAGE DURATION OF LOSS OF LOAD

Notice -- Illustrative Example Only

	SYSTEM VIEWPOINT		CUSTOMER VIEWPOINT	
All Causes	3.1	Hours Per Occurrence	3.5	Hours Per Occurrence
Distribution	3	"	3	"
Area Supply	6	"	6	"
Bulk Power	8	"	8	"
Generation Including Reserve, Operations and Interconnections (R,O,I)	10	"	2	"
Generation Only DSCR	6.0	"	-	

The conditional expected load not supplied, Exhibit 2.10, is the power involved in a loss of load given that an outage has occurred. The "all causes" value of 1.6 is the result of dividing the expected energy not supplied, Exhibit 2.6, by the hours loss of load expectation, Exhibit 2.7:

$$\frac{1677 \text{ MWh}}{1068 \text{ hours}} = 1.6 \text{ MW/occurrence}$$

The expected load not supplied is very small due to the distribution system which has only 0.2 megawatts during an occurrence. The transmission system has 20 MW when an outage occurs, the bulk power system has 300 MW when an outage occurs and the generation and interconnection system has 150 MW load not supplied when an outage occurs. The "generation only" need for supplemental capacity resources is 145 MW when a shortage of capacity occurs.

Exhibit 2.10

CONDITIONAL EXPECTED LOAD NOT SUPPLIED

Notice -- Illustrative Example Only

	SYSTEM VIEWPOINT	CUSTOMER VIEWPOINT
All Causes	1.6 MW Per Occurrence	3.4 kW Per Occurrence
Distribution	0.2 "	3.4 "
Area Supply	20 "	3.4 "
Bulk Power	300 "	3.4 "
Generation Including Reserve, Operations and Interconnection (R,O,I)	150 "	3.4 "
Generation Only DSCR	145 "	---

2.3.7 Summary of Look Ahead Illustration

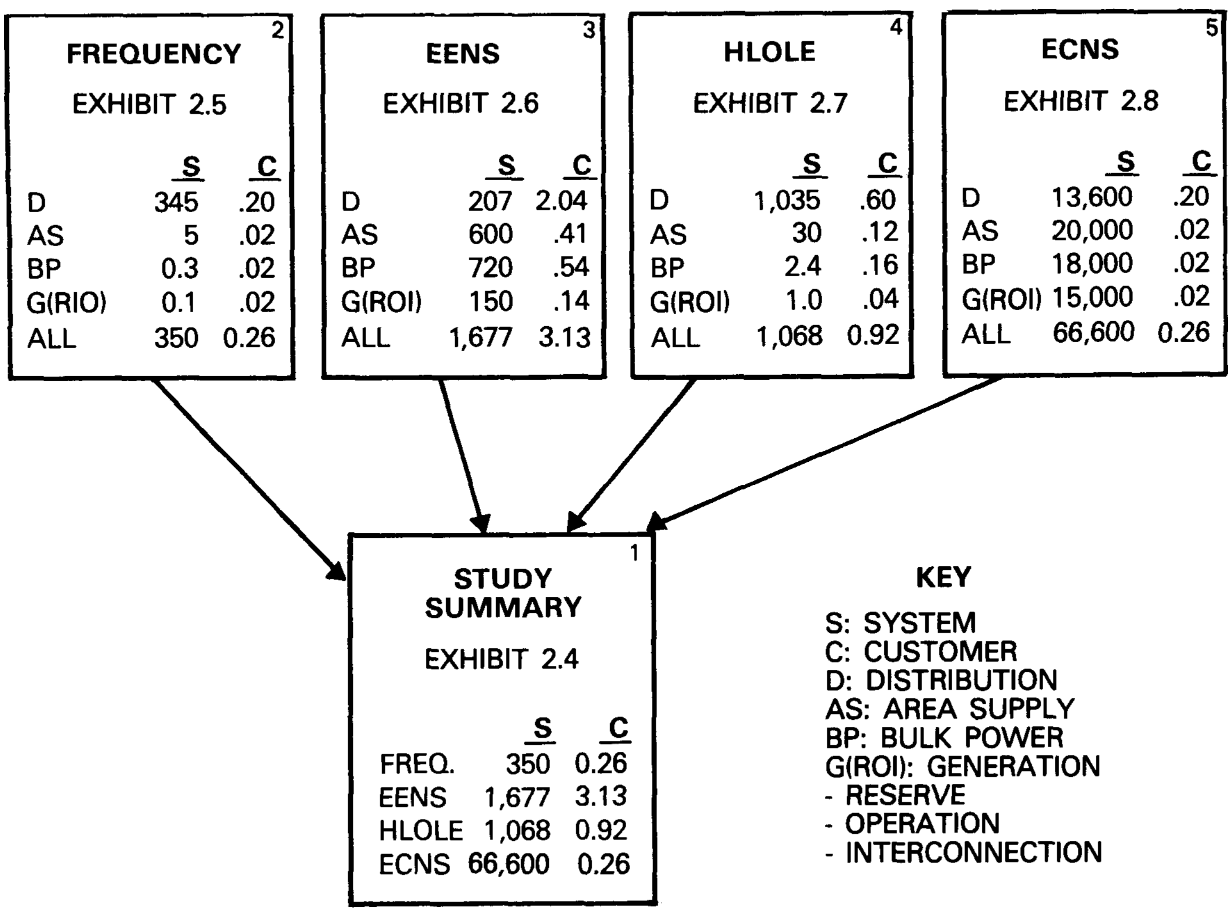
An illustration has been sketched of the type of results that may be obtained in the future when calculations become affordable. These expected values refer to the loss of load (lamp), the load not supplied (meter) and the customers not supplied (meter). The following paragraphs will discuss the structure for computing the indices.

2.4 COMPUTATIONAL STRUCTURE

With the sketch of the final results presented in the previous section, we now turn our attention to the structure for calculating such results. A pictorial summary of the reliability indices just presented is shown in Exhibit 2.11. Frequency, EENS, HLOL, and ECNS results are shown leading into the study summary. The numerical values will guide us through the structure.

Exhibit 2.11

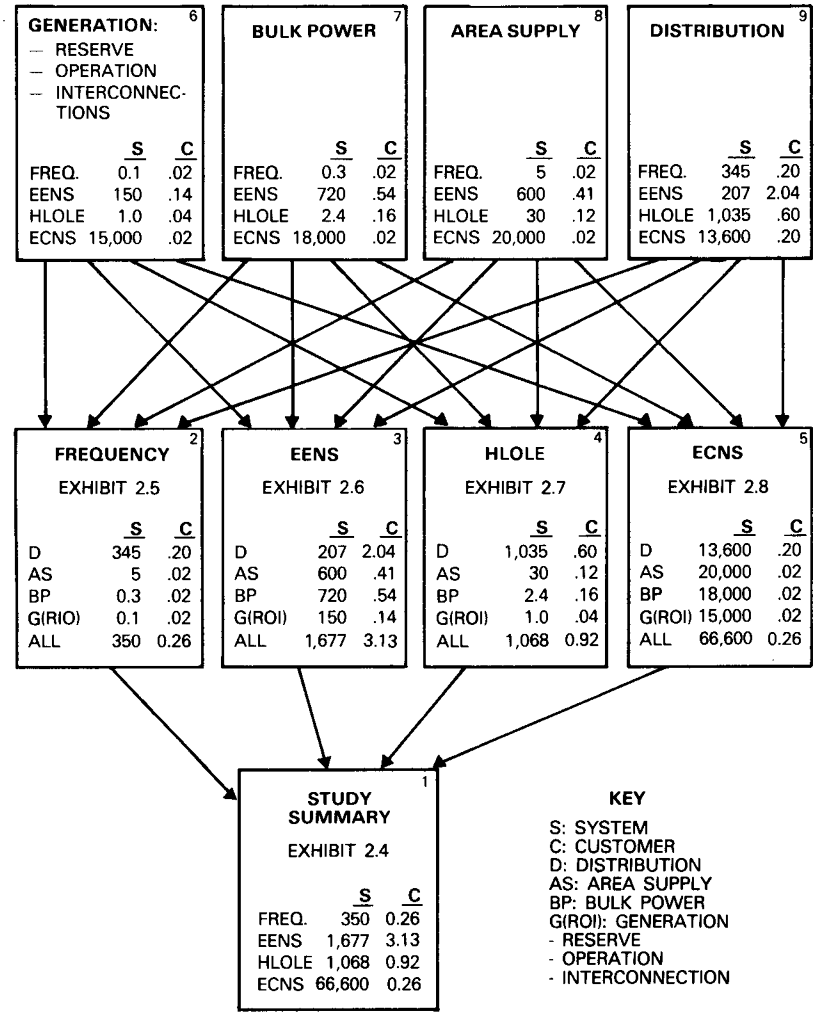
STUDY SUMMARY RELATED TO FOUR BASIC RELIABILITY INDICES



Having started with the desired results, we now move backward to identify the calculation procedure. First, the indices are gathered by calculation model rather than by type of index, Exhibit 2.12. For example, the frequency index 350 is shown in block 2.5 as made up of four components, the generation being 0.1 which is shown in block 2.9.

Exhibit 2.12

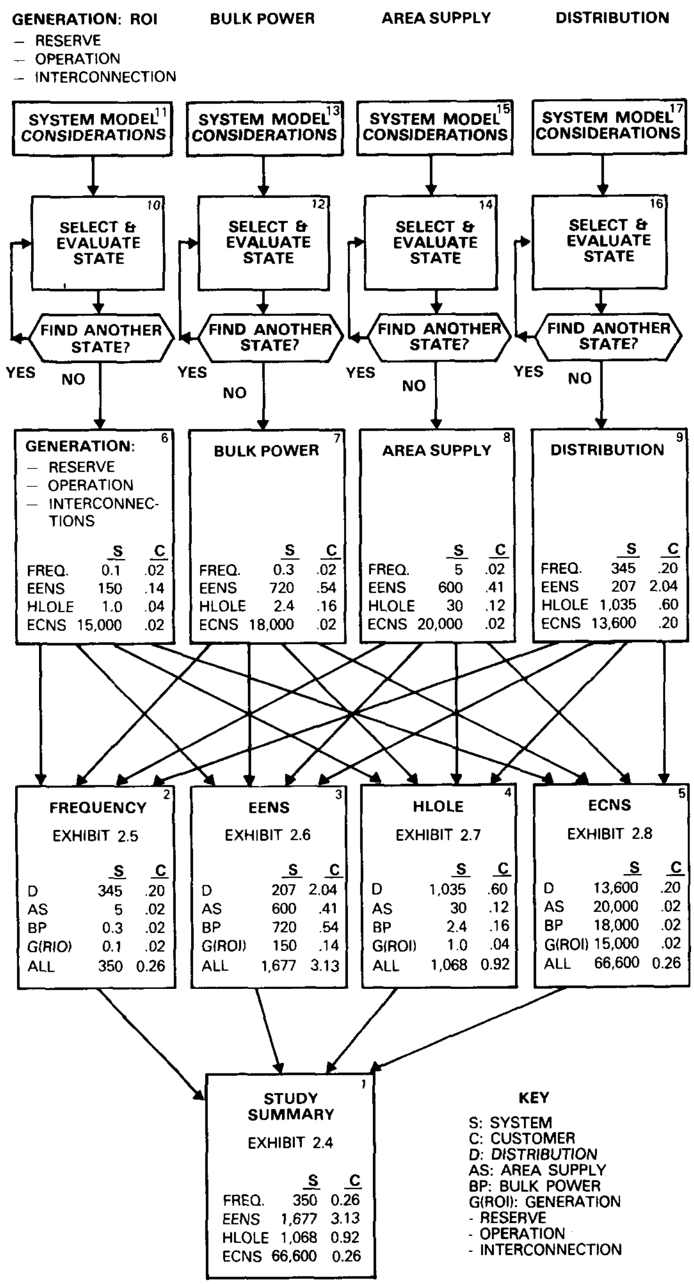
RESULTS BY SYSTEM COMPONENT AND THEN RELIABILITY INDEX



The overview from calculation procedures to final summary is completed in Exhibit 2.13. Each calculation procedure starts with system model considerations and follows with repetitive calculations. In each repetitive calculation a state or states are selected and evaluated. If another state is to be studied, the calculation is repeated. The idea, simple in theory but expensive in practice, requires "sketches" of results rather than sample calculation results.

Exhibit 2.13

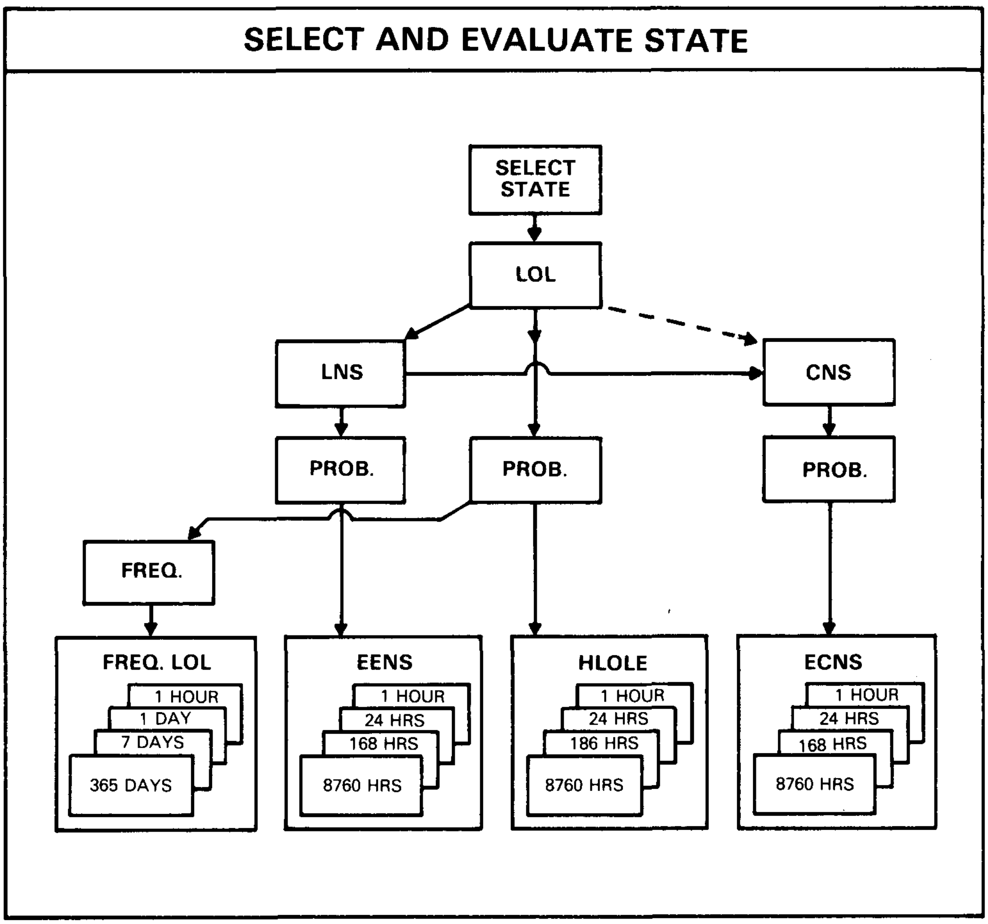
OVERVIEW CALCULATION STRUCTURE



Each of the four system components will be studied in the same general calculation procedure, but differing significantly in the models considered and the types of state evaluations. The general procedure for selecting and evaluating states is shown in Exhibit 2.14. It begins with the selection of a state followed by the loss of load (LOL) evaluation for pass or fail. The procedure continues with load not served (LNS) for magnitude evaluation. If the state is a pass, then there is no magnitude evaluation needed; however, if the state is a failure, then the magnitude of the failure and its location must be identified. (At this point, further research is needed because Load Not Supplied is the basic parameter for Generation, Bulk Power and Area Supply (IEEE-PES-PROSD, 1978) [2-26] while Connected Load Interrupted is the basic parameter for Distribution (IEEE-PES-PROSD, 1975) [2-25]. In the same manner, the customers not served (CNS), if this is a failed state, must be evaluated by identifying how many customers are not supplied and where these customers are located.

Exhibit 2.14

GENERAL PROCEDURE FOR SELECTING AND EVALUATING STATES



After evaluating LOL, LNS, and CNS, the procedure moves to the probability phase. The probability evaluation for LOL is easiest to accomplish and results in an addition to the hourly loss of load expectation (HLOLE). The calculation method now available for state-space evaluations are covered in Chapter 4 of Endrenyi (1978) [2-17]. The probability calculation for load not supplied (LNS), more difficult, results in an addition to the expected energy not supplied (EENS) calculation. In the same manner, the probability associated with customers not supplied results in an addition to the expected customers not supplied (ECNS). Frequency of Loss of Load requires even more difficult calculations, see the description in Endrenyi, pages 53-55 and pages 72-84 (1978) [2-17].

Before leaving this discussion of the general procedure, consider that a "one load" result is the easiest to study, but even one load may require many thousands of calculations. Theoretically though, one could move ahead to one day or 24 hours, seven days or 168 hours, and ultimately, 365 days or 8760 hours.

Section 5 develops the fundamental background to show that Frequency, Hourly Loss of Load Expectation, Expected Energy Not Supplied, and Expected Customers Not Supplied are primary reliability indices. Section 3 discusses and illustrates applying the general evaluation procedure of Exhibit 2.14 to the generation system. Section 4 discusses the evaluation of the bulk power system and the LOL, Loss of Load, calculation methods and examples.

2.5 GENERATION ONLY, A SPECIAL CASE

The "generation only" values shown in, Exhibits 2.5 to 2.10, require the same calculation theory as the other four system components, see Exhibit 2.15. The "generation only" expectation results for one year were 3.6 occurrences of loss of load, 3170 MWh unserved energy and 22 hours of loss of load. These values do not enter directly into any complete system reliability study. However, they can be calculated, as will be shown in Section 3, and they are a relative indicator of improving or deteriorating reliability of the generation system.

The next paragraphs will indicate the significant differences in the calculation proceedings. Exhibit 2.16 illustrates the general procedure, noting that customers not supplied are not related to the Dependence of Supplemental Capacity Resources.

Exhibit 2.15

"GENERATION ONLY" CALCULATION PROCEDURE AND RESULTS RELATED TO STUDY SUMMARY

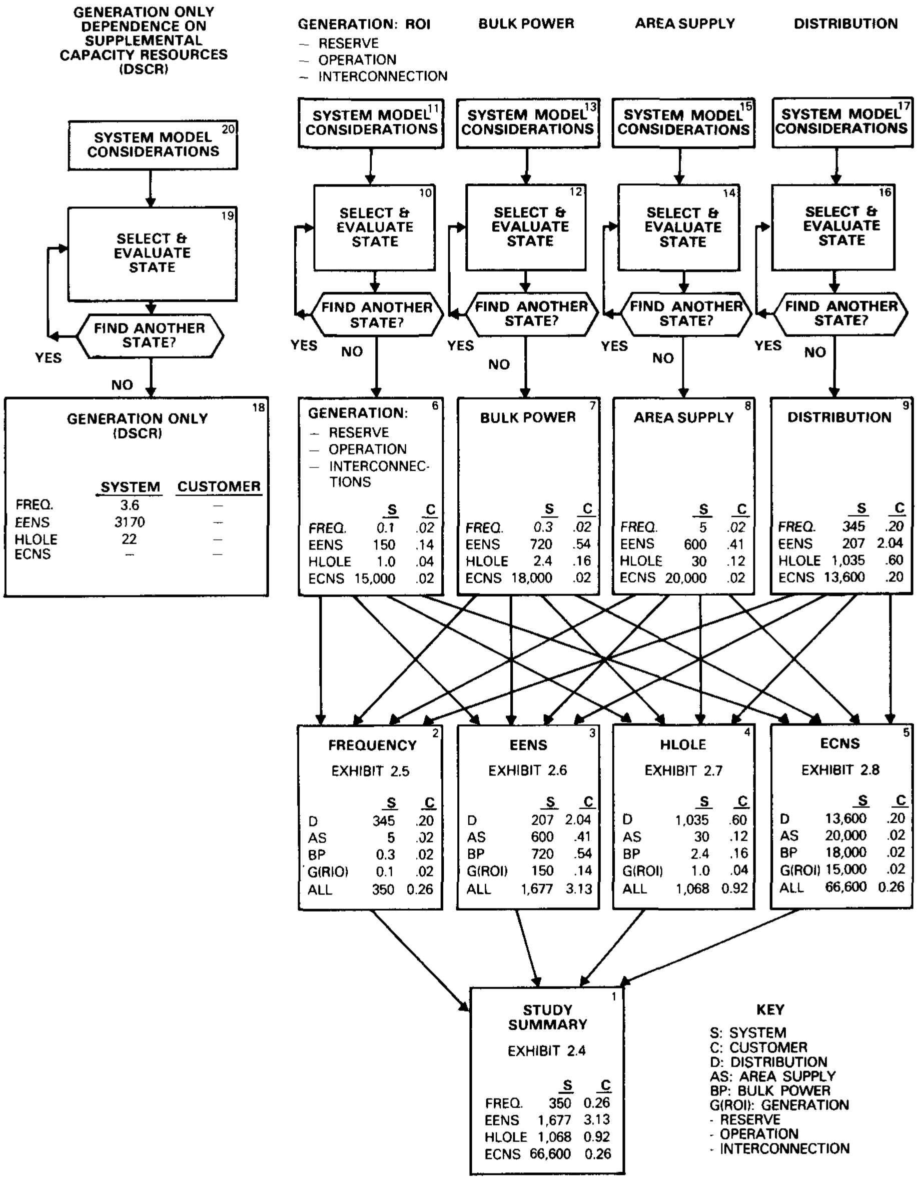
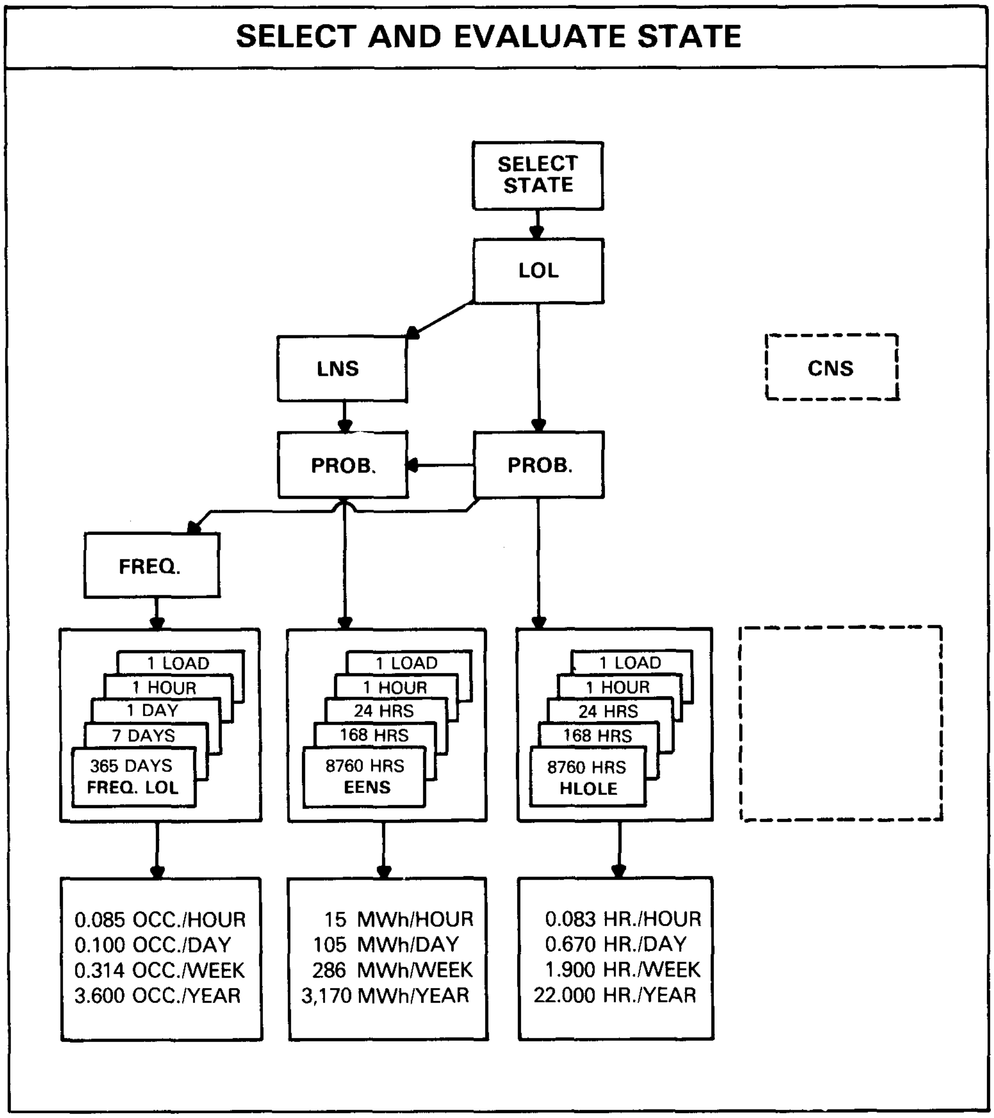


Exhibit 2.16

CALCULATION PROCEDURE FOR GENERATION ONLY,
DEPENDENCE ON SUPPLEMENTAL CAPACITY RESOURCES



2.5.1 Summary of Calculation Procedure

Exhibit 2.17 brings the "generation only" calculation theory to practice. In this case using capacity outage tables, one practical method of calculating reliability indices for generation. LOL and Probability calculations result in a table of distributions of outage probabilities, necessary for hourly loss of load expectation calculations. LOL plus LNS and Probabilities result in two tables, the distribution of outage probabilities and the distribution of energy not supplied, MWh/h. LOL, Probability and Frequency result in two tables, the distribution of probabilities and a distribution of frequency. These tables display the results of enumerating all combinations of load and generation outages.

Exhibit 2.17

"GENERATION ONLY" DESCRIPTIVE CALCULATION PROCEDURE

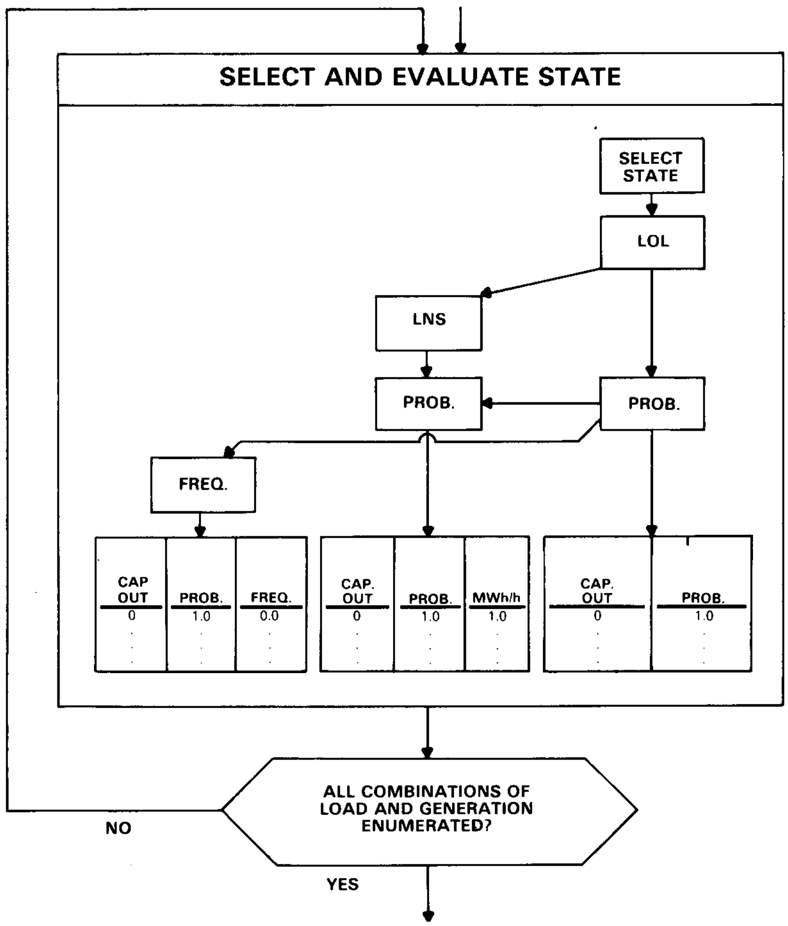
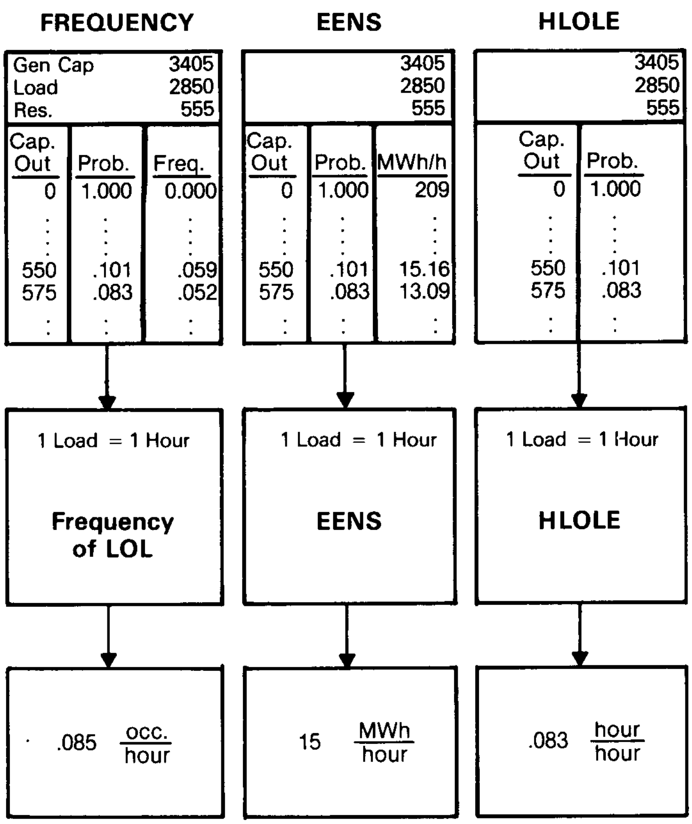


Exhibit 2.18 completes the detailed example with values in the tables for the peak hour of the year. Section 3 and Appendix C detail the calculation procedures to arrive at the Frequency of LOL, 0.085 occurences; EENS, 15MWh; and HLOLE, 0.083 hours. With these tables available, each load is studied by a table lookup. However, there are some limitations which are discussed further in Section 3-2, such as energy limited units including pumped hydro, pondage hydro, wind, solar and operating policies (commitment).

Exhibit 2.18

EXAMPLE CALCULATION BASED ON IEEE-PES RELIABILITY TEST SYSTEM

32 TWO-STATE UNITS REQUIRING 4,294,967,304 STATE ENUMERATIONS



2.5.2 Generation Indices in the Literature

In this section 12 indices will be introduced by a brief summary and a numerical example. The analytical expression and calculation method for each index will be described in Section 3. In order to give a feel for the numerical value of the indices, the IEEE-PES Reliability Test System[2-24] has been used.

1. Percent Reserve. The percent reserve method (Billinton, 1970)[2-5] is a ratio of the total installed generating capacity minus the annual peak load forecast divided by the peak load forecast and results in a value like 19.5%.

2. Largest Units. The largest units method compares the total installed generating capacity less the annual peak load, the reserve, to the largest installed units on the system. For example, a system with a reserve of 555 MW and two large 400 MW units would be expressed as having the largest unit plus 155/400 of the second 400 MW unit.

3. Loss-of-Load Expectation (Probability) - 365 Days (LOLE-365). The loss-of-load probability method, using 365 days per year (or an approximation such as 250 or 260 days per year), results in loss of load expectation (LOLE) expressed in Days (Billinton, 1970, p. 98)[2-5]. Using the sample system, the loss of load probability method results in a reliability index of 3.00 days per year. The inverse is used, 0.33 years per day termed the Index of Service Reliability (ISR).

4. Hourly Loss of Load Expectation - 8760 Hours (HLOLE - 8760). The loss of load probability method, when computed over the 8760 hours calculates the expected number of hours with generation capacity less than load in a year. The reliability index has dimension of hours per year. For the sample system the index has an expected value of 21.9 hours with generation shortages per year. This hourly calculation was discussed by Billinton[2-5] and is a calculation byproduct of stochastic production costing methods based on the Baleriaux-Booth method [2-10].

5. Probability of Positive Margin (POPM). The probability of positive margin method (Mabuce, et al., 1972) [2-29] uses the loss-of-load probability calculation method for one hour, the peak hour of the year. The answer, however, is expressed as a probability of success rather than the probability of failure. A system with a failure probability of 0.083. has a success probability of 1 - 0.085 = 0.915, POPM.

The probability of Positive Margin, as used by the MAIN reliability council, (MAIN Guide 6, 1978) [2-30] includes the uncertainty of annual peak load. This additional refinement does not change the basic meaning of the reliability index, the probability of serving the peak load hour of the year successfully.

6. Quality (Q-365). The quality index based on 365 days per year takes the daily probability of positive margin (POPM) and multiplies the 365 numbers together to arrive at a probability of no generating capacity shortages. The resulting index is a probability number and hence has no dimension. For example, for the sample system the quality index for the 365 days is 0.067. There is a 6.7% probability of no capacity shortage during the 365 days. This can be compared with the one day measure, POPM, shown above as 0.915 probability of success. Even though each day will have better than 90% probability of success, serving 365 days has only a 6.7% chance of success. A problem with the index is that a small value, near zero, does not indicate whether the risk is high on many days or just one day. Additional information such as mean and standard deviation of daily probabilities (Patton, 1975) [2-38] must be examined for this index to have much meaning.

7. Quality - 8760 Hours (Q-8760). This quality index was developed in California (Markel, et al., 1976) [2-32]. Q-8760 is calculated by continuing the procedure used in Q-365 days per year by multiplying together the probability of no generating capacity shortages each hour of year. For the sample system the value turned out to be 1.0 x E-10, a very small probability of success. Statisticians state [2-32] that this calculation method is not appropriate for hourly loads that are not independent on one another. Therefore, this value has little meaning and the 365 value of .067 is closer to the truth.

8. Probability of Loss-of-Load -- 365 Days (PLOL-365). The probability of loss-of-load method calculated over 365 days is one additional step beyond the quality measure (Q-365). The probability of failure equals 1.0 minus

the probability of success. The probability of failure (reliability index) for the example is 0.933. This is one minus the quality index calculated above. It is interesting to compare this probability of failure index with the one day failure index, LOLP-1, calculated as (0.083). There is an 8% probability of failure on the peak day and a 93% probability of some failure during the year.

9. Expected Energy Not Supplied (EENS). The Expected Energy Not Supplied, is computed by summing all the probability of shortages times the megawatt amounts of shortage to arrive at megawatt-hours. A recent publication by the EEI-System Planning Committee (1977, p. 10) [2-16] comments that this is the unserved energy only if a power system truly operated to lose load only when capacity is short, not considering such items as spinning reserve, interruptible loads, voltage reduction and help from interconnections. For the sample system the Expected Energy Not Supplied in a year is 3170 MWh.

   The same input data is used as was required for the LOLP for 8760 hours.

10. Conditional Expected Load Not Supplied (XLNS). The Conditional Expected Load Not Supplied, also termed Expected Loss of Load (XLOL), is defined as the expected value of capacity deficiency given that load not supplied exists. This index is computed by Billinton in a discussion of J.T. Day, et al., (1972) [2-14] and IEEE-PES-PROSD (1978) [2-26] as:

$$\text{XLNS} = \text{XLOL} = \frac{\text{Expected Energy Not Supplied (ENNS), MWh}}{\text{Hourly Loss of Load Expectation (HLOLE), hours}}$$

   For the sample system this would give

$$\text{XLNS} = \text{XLOL} = (3170\ \text{MWh})/(22.0\ \text{Hours}) = 144\ \text{MW}$$

11. Frequency of Loss of Load (FLOL). The frequency calculations require that the average repair time for each generating unit and a duration for each load be specified. With this additional input data it is possible to calculate the number of generation shortage occurrences in a year (Billinton, Ringlee and Wood, 1973) [2-7]. The frequency calculation method using 8760 hours per year (Ayoub and Patton, 1976) [2-3] requires the input of all hourly load forecasts for a year and the average repair time for each generation unit. For the sample system the result is 3.6 Occur.

12. Duration of Loss of Load (DLOL). The duration or average duration of loss of load is the ratio of Hourly Expected Loss of Load Expectation (HLOLE) and Frequency of Loss of Load (FLOL) [2-3]. The result is the conditional expected number of hours duration of a shortage of generation, given there is a shortage. For the sample system:

$$\text{Duration} = \frac{\text{HLOLE}}{\text{FLOL}} = \frac{22.0 \text{ hours}}{3.6 \text{ occur.}} = 6.1 \frac{\text{hours}}{\text{occur.}}$$

Additional Indices. By merging together several of the generation reliability indices and system statistics, such as peak demand and annual energy, it is possible to calculate additional reliability indices. The number of megawatt-hours per occurrence of a generation outage may only be calculated by rationing the EENS to Frequency. For the sample system dividing the megawatt-hours by the occurrences resulted in 3170/3.6 = 881 MWh/Occurrence.

System-Minutes is a reliability index used by Ontario Hydro (Electrical World, September 15, 1979, pp. 124-125)[2-41]. It is the ratio of Expected Energy Not Supplied to forecast peak demand. For the sample system the result is

3170 MWh/2850 MW = 1.1 hrs
= 66.7 system-minutes

2.5.3 Relationships

The 12 indices from the literature include the 3 primary reliability indices used in the Study Summary, Exhibit 2.4.

Frequency is #11
Expected Energy Not Supplied is #9
Hourly Loss of Load Expectation is #4

Expected customers not supplied could not be determined from a generation-only study.

The Loss of Load Expectation (LOLE or LOLP) index is a surrogate for Frequency when the necessary transition rates for generation and loads are not available. Note the similarity between 3.6 occurrences, Frequency, and 3.0 days, LOLE, for the sample system. Frequency and LOLE track each other, with Frequency always being larger.

2.5.4 Capacity Less Than Load Is Not "Load Not Supplied"

Generation reliability index considerations use capacity less than load as a test for failure. These calculations assume that the amount of "load not supplied" is the load minus capacity when the capacity is less than the load. This may not be an exact description of the amount of load not served for a particular hour. It is rather a harbinger of the load not being served. The following example will illustrate the point that even though capacity is greater than load, load may not be served. The converse is also true. On certain systems, those with interconnections, capacity can be less than load without having load not served.

We will use the example from the IEEE Reliability Test System. Assume two generation units are not available.

Installed capacity		3375 MW
Load		2850 MW
Units out	400 MW	
	100 MW	
Total outage		500 MW
Margin		25 MW

The state of this system at peak load results in a margin of 25 MW, but let us look further. Assume that the system is isolated. The spinning reserve rules are shown in Table 2.1.

Table 2.1

SPINNING RESERVE RULES

Largest Unit Size must be spinning, i.e., running and synchronized.

400 MW for the example system.

Second Largest Unit Size must be able to start in 30 minutes.

350 MW for the example system.

The system with 25 MW margin of reserve does not meet the requirements for 400 + 350 = 750 MW of spinning plus 30-minutes reserve. The system is short 725 MW:

25 - 400 - 350 = -725 MW

To calculate the amount of load not supplied due to this 25 MW margin requires knowledge of the <u>emergency operating procedures</u> (EOP). The emergency operating procedures for this example are shown in Table 2.2.

Table 2.2

EMERGENCY OPERATING PROCEDURES

1. In emergency state #1 the 30 minute-start-up reserve is allowed to go to zero. The system is in emergency state 1.

   In emergency state 1, the remaining margin is -375 MW.

   25 - 400 - 350 + 350 = -375 MW

2. In emergency state #2 the voltage is dropped 5%. This reduction appears to will be assumed to produce a 3% reduction in the system load.

   Emergency state 2 results in -290 MW margin.

   0.03 * 2850 MW   85 MW

   25 - 400 - 350 + 350 + 85 = -290 MW

3. In emergency state #3 appeals are made to customers resulting in 5% to 10% load reduction. This assumes that the generation outages occurred sufficient hours ahead of time to allow appeals to be made.

   Assuming that 200 MW results from customer appeals than the margin is -90 MW.

   25 - 400 - 350 + 350 + 85 + 200 = -90 MW

4. In emergency state #4 the spinning reserve is moved toward zero.

   Emergency state 4 removes the requirement that 400 MW of spinning reserve be maintained. Therefore, the resulting margin returns to a positive value, 310 MW in this example.

   25 - 400 - 350 + 350 + 85 + 200 + 400 = 310 MW

5. In emergency state #5 load are disconnected.

   This example did not require any load disconnections.

How much load was not served in this case of 25 MW margin?

1. None.
2. 200 MW. (Due to customer appeals)
3. 285 MW. (Due to customer appeals and voltage reductions)

For "generation only -- DSCR" the answer is "None." Generation including reserve, operation, and interconnections would be more likely to recognize "285" as the load not served.

2.6 GENERATION INCLUDING RESERVE, OPERATION AND INTERCONNECTIONS

Generation, including reserve, operation and interconnections, includes both the Generation-Only considerations plus the following topics listed with a significant reference.

- Emergency operating procedures (Moisan and Kenney, 1976) [2-35].
- Operating considerations such as spinning reserve, units with quick-start abilities, the spinning reserve of pumped hydro units, and maintaining a spinning reserve capacity among the operating generating units (Patton & Hogg, 1979) [2-37].
- Interconnections considered in loss of load probability calculation (Billinton and Jain, 1973) [2-6] (Pang and Wood, 1975) [2-36].

These significant references make a beginning toward evaluating loss of load in two or more areas considering only the generation system, but with operating rules and help from neighboring utilities.

Comparing the sketch of results for generation only, the frequency of loss of load of 3.6 occurrences during a year compares with the frequency for generation including reserve, operation and interconnection of 0.1 occurrences. The major factor affecting the results are the interconnections to neighboring utilities, the emergency operating procedures and the spinning reserve and security considerations. Also the dependence on supplemental capacity resources may occur 22 hours during a year, see Exhibit 2.7, which all generation capabilities are exhausted and loss of load occurs on the average of one hour during a year when.

2.7 BULK POWER EVALUATION

Bulk power and area supply transmission planners will find little help from the research results so far. Even the simplest reliability index, hourly loss-of-load expectation appears too formidable and expensive to evaluate except on 50 bus systems or less. Refer to the works of George Marks (1978) [2-33] and Paul Dandeno, et al. (1977) [2-12]. The evaluation methods in Europe tend toward linear programming methods such as capacitive transshipment (Glover et al., 1974) [2-22] and Ford-Fulkerson (1962) [2-18]. This includes the Belguim method (Baleraux, et al., 1974) [2-4], the French models (Auge, et al., 1972) [2-2] and the Italy (Manzoni, et al., 1979) [2-31]. Whether Monte Carlo methods are used such as the Italians, or state selection methods are used, such as PCAP (Dandeno, Jorgensen, Puntel and Ringlee, 1977) [2-12], the problem is still too large to be solved for real planning problems. The deterministic methods, summarized in Section 4, are used in all parts of the industry for reliability considerations. The research for this report did not uncover any easy, rapid way of evaluating reliability indices for real power systems. Endrenyi (1978) [2-17] in Chapter 10 presents a review of the simplifying assumptions being considered for the bulk power index question.

The area supply system reliability is contrasted with bulk power in that a bulk power system is defined as the composite of the generation system and the high-voltage transmission network extending to the points of load transfer to low-voltage levels. The entire system, including companies, power pools, and even several regions of a country, may be considered. In contrast, the area supply studies will consider the reliability of station components and the sub-transmission system. Thus, station apparatus such as buses, circuit breakers, disconnect switches, transformers, PTs and CTs, the systems under study usually end at the secondary buses: low-voltage switching devices may or may not be considered. The best introduction to area supply system reliability is found in Chapter 11 of Endrenyi (1978) [2-17]. For the remainder of this discussion, we will focus our attention on the bulk power system reliability.

2.7.1 Deterministic Reliability Indices

Five deterministic indices used in bulk power system planning will be discussed below.

Maximum Load Not Supplied. This index is the largest amount of MW curtailed due to a particular set of contingencies.

Maximum Energy Not Supplied. This index, similar to the maximum load curtailed index, considers energy rather than power.

Minimum Load Supplying Capability. The Load Supplying Capability (LSC) [2-21] of a power system is defined as the maximum system load that can be supplied with no generation or transmission line overloaded. For example, a study of single and double line outages on a system with 38 lines shows a load of 2000 MW of load can be served without any element overloading in any case. A load of 2001 MW causes a line overload in at least one case.

Minimum Simultaneous Interchange Capability. The Simultaneous Interchange Capability (SIC) of a power system is defined as the maximum power that can be transferred for a given system state [2-28].

Maximum Line Flow. This index is the largest flow on a particular transmission circuit due to generation and transmission contingencies.

The discussion of the deterministic bulk power transmission reliability indices continues in Section 4.

2.7.2 System Probabilistic Reliability Indices

The indices judged most appropriate for further development are listed below.

Loss of Load Probability (LOLP) - Annual Peak Load. Loss of load probability for the peak load of the year, also termed probability of loss of load, needs only the evaluation of LOL. Power flow studies only identify lines overloaded or voltages out of limits, not where and how much, information that will be needed when we move to evaluating Load Not Supplied. The selection and evaluation of a state shown in Exhibit 2.14 for LOL followed by probability calculation results in the evaluation of LOLP, HLOLE for one hour. A not unreasonable illustration for the IEEE-PES Reliability Test System is 0.008 probability for the peak load hour.

Loss of Load Expectation (LOLE). The Loss of Load Expectation uses LOL evaluation of the peak load of one day or up to 365 days. A not unreasonable illustration for the IEEE-PES System is 0.2 days during a year. LOLE approximates the primary reliability index of frequency of loss of load.

Hourly Loss of Load Expectation (HLOLE). The Hourly Loss of Load Expectation is a primary reliability index for evaluating one hour or up to 8760 hours per year. A not unreasonable illustration for the IEEE-PES System is 2.4 hours during a year.

Frequency of Loss of Load (FLOL) The Frequency of Loss of Load can evaluate from one hour to 8760 hours per year. It must consider both the probability of events and the transition into and out of success and failure states. A not unreasonable illustration for the IEEE-PES System is 0.3 occurrences during a year. This is a primary reliability index whose general calculation procedure is illustrated in Exhibit 2.14.

Expected Energy Not Supplied (EENS). The Expected Energy Not Supplied is a primary reliability index requiring the evaluation of both LOL and Load Not Supplied to evaluate one hour or up to 8760 hours. A not unreasonable illustration for the IEEE-PES System is 720 MWh.

Bulk Power Interruption Index (BPII). The Bulk Power Interruption Index (PROSD, 1978) [2-26] is defined as the ratio of annual load interruption to annual peak load. It can be computed by ratioing the Conditional Expected Load Not Supplied (XLNS) to the annual peak load. For illustration:

BPII = XLNS/Peak Load = 300 / 2850 = 0.11 MW/MW per year.

Bulk Power Energy Curtailment Index (BPECI). The Bulk Power Energy Curtailment Index (PROSD, 1978) [2-26] is the ratio of the annual energy curtailment to the annual peak load. It can be computed from the primary index Expected Energy Not Supplied divided by the annual peak load. For example:

BPECI = EENS / Peak Load = 720 MWh/ 2850 MW = 0.25 MWh/MW.

Expressing this in minutes becomes 0.25 * 60 = 15 MW-min./MW during a year.

Average Number of Curtailments Per Load Point. This index is the expected number of loss of load occurrences at an average load bus during a year (Billinton et al, 1979) [2-8]. In a similar manner, all reliability indices calculated for the system can also be calculated for a load point as Billinton illustrates.

Aspects of bulk power reliability evaluations are further discussed in (Billinton and Kumar, (1980) [2-9] and Gallyas and Endrenyi (1980) [2-19].

2.7.3 Calculation Procedure Considerations

The calculation procedure in Exhibit 2.13 for the bulk-power system reliability evaluation will be discussed further in this subsection. The first step is "system model considerations", referred to as block 13 in Exhibit 2.13.

The system model considerations for the bulk-power system are:

- Generating units including MW limits, VAR limits and location.

- Transmission lines including electrical constants (R,X,B), and several current limits per line.

- Interconnections including transmission lines between systems, generating units located outside the service area and operating rules.

- Generation dispatch information including costs and operating constraints.

- System hourly loads for the study year.

- Stability data for buses, transmission lines, transformers, relays.

- The types of events to consider, such as outages of single generating units, pairs of generating units, single transmission lines, pairs of lines, generating units and lines at the same time, etc.

- The probabilities of the considered events.

These are the general categories of data that are included in the system model considerations. Section 2.7.4 expands on transmission line capacity considerations.

The next block down in Exhibit 2.13, block 12, is the selection and evaluation of a state. The general procedure is shown in Exhibit 2.14. The components of the general procedure for a bulk-power system are shown in Table 2.3, with reference given for both the static and dynamic cases. The first step in the procedure, Exhibit 2.14, is to select the state. The goal of state selection is to identify only the most critical network states for further evaluation (Irisarri, et al., 1979)[2-27]. In the stability column of Table 2.3, both the before states and a change must be defined. Concordia in 1976 [2-11] describes the problem.

Table 2.3

BULK-POWER RELIABILITY EVALUATION COMPONENTS

(Based on the General Procedure in Exhibit 2.14)

	STATIC	DYNAMIC
Select State	- One State (Irisarri, Sasson, and Levner, 1979) [2-27]	Before and Change States (Concordia, 1976) [2-11]
Evaluate LOL -- Loss of Load	- Economic Dispatch (Happ, 1977) [2-23]	- Same
	- AC Power Flow (Stott, 1974) [2-43] (Dopazo et al, 1975) [2-15]	- Transient Stability (Anderson & Fouad, 1977) [2-1]
	- Redispatch (Stott and Marinho, 1979) [2-44]	- Redispatch (Unknown)
	- Transfer Limits (Landgren and Anderson, 1973) [2-28]	- Transfer Limits (Unknown)
	- Load Supplying Capability (Garver, Van Horne and Wirgau, 1979) [2-21]	
Evaluate LNS -- Load Not Supplied	- Assign bus-load reductions bus-voltage out-of-limits line overloads and relay actions (Stott and Marinho, 1979) [2-44]	- Assign Relay Actions during Solution Bus-Voltage Out-of-Limits Line overloads after solution (Unknown)

Evaluate - CNS -- Customers Not Supplied	Assign bus-load reductions to customer outages (Unknown)	- Same
Evaluate - Prob. & Freq.	Evaluate probability and frequency (Endrenyi, 1978, pp. 72-84) [2-17]	- Same

The loss of load evaluation involves economic dispatching, ac power flow calculations and redispatching logic. It may also involve transfer limit logic and load supplying capability logic (see Section 2.7.6 for LSC discussion). As if the static case were not hard enough, a dynamic case requires even more difficult calculations, including both steady-state and transient stability calculations. All of these calculations must be made for each state to determine failure or success as far as loss of load is concerned.

Studying the load not supplied (LNS), static or stability cases, requires assigning the bus voltages out of limits, line overloads, and relay actions to buses. Bus-load reduction studies to identify the load megawatts not supplied during the hours of loss of load are discussed in Section 2.7.4.

To identify the customers not served (CNS) requires assigning the bus-reductions to customer outages. This is not a straightforward procedure. For example, we may know that 10 MW load must be reduced at a certain bus. This is only 20% of the total load of the bus. Which group of customers should be interrupted? Further discussion is contained in Section 2.7.5.

The last steps in Exhibit 2.14 are to calculate the probability and frequency values. These procedures are described by Endrenyi (1978, pp. 72-78) [2-17]. The last step is to decide if another state needs to be evaluated, a complicated decision also discussed by Endrenyi.

2.7.4 Transmission-Line Capacity -- A Stumbling Block

Reliability View. Most reliability papers and discussions begin with the words "-- given the transmission-line capacity --- " and quickly move into the reliability problem.

System Planning View. Bulk-power system planners know that capacity is not an input quantity for power flow calculations or stability calculations. Capacity is a secondary input, used to analyze power flow and stability calculations. Also, one of the most difficult questions in planning is "What is the capacity of a transmission-line?"

An introduction to the capacity quantification is contained in the Department of Energy report, "Factors Influencing Electric Utility Expansion, Vol. II", (1977, p. 5-25 and 5-64) [2-20] and will be briefly summarized with the aid of Figure 2.1. The acceptable current loading on a transmission circuit is discussed using the following ideas. Imagine an electric current meter attached to a transmission circuit. As the current flow or "loading" increases, the pointer approaches the first of four areas designated on the meter, marked (1) normal, preferred, or economic line loading. Experience has shown that loading in this range will be close to the quantity referred to as Surge Impedance Loading. For information on how surge impedance loading is calculated, see the paper by St. Clair (1953) [2-40].

	(1)	(2)	(3)	(4)
0	1.0			

Current Loading, Amperes
Per Unit of Surge Impedance Loading

(1) Normal, preferred, economic line loading.
(2) Emergency -- 8 to 24 hour -- line loading.
(3) Short-Term Emergency -- 1/4 to 2 hour -- line loading.
(4) Relay settings to protect against equipment damage.

Figure 2.1 Transmission-line electric current measurement scale.

As the current loading increases, the second area of the meter is approached (2) referred to as the emergency -- 8 to 24 hour -- line loading. In this area the losses are significant, but will be tolerated for reasonable periods of time.

As the line loading continues to increase, the area of short-term emergency line rating (3) is approached. This is usually the 1/4 to 2 hour rating. The transmission line is experiencing significant heating and its continuation for an appreciable time period would cause the conductor to sag and flashover to some obstruction. The thermal rating of lines is discussed by M.W. Davis (1977) [2-13].

The final area approached by the current meter is relay settings (4) to protect against short-circuit currents. These settings are known with the most certainty of any of these four designations. Even this area is fuzzy because of the problems of accurately measuring relay settings as discussed in Mason (1956) [2-34].

Summary. Both the reliability and the system planning disciplines need the "capacity" quantity. Workers in both disciplines admit the difficulty in quantifying "capacity". Capacity is a major research topic that Detroit Edison (Davis, 1977) [2-13] and others are studying. Therefore for this report on reliability indices the quantity "capacity" is assumed to be well-defined.

2.7.5 Evaluating Load Not Supplied

Evaluating load not supplied requires each line flow greater than the line capacity be allocated to one or more bus-load reductions. Also, bus voltage-out-of-limits must be converted to bus-load reductions.

Background. Kirchhoff's first and second laws are solved with real and imaginary values of voltage and current. The calculation procedure uses economic dispatch, alternating current power flow solution methods, and redispatching to minimize load-not-supplied at the most economic or uncritical buses.

Is Transmission-Line Overloaded a "Load Not Supplied?". An alternating current power flow calculation shows a transmission-line overload. Is load not supplied?

1. Remembering the fuzzy nature of line ratings, see Section 2.7.4, if the line flow is just slightly larger than the normal capacity, then the line is not considered overloaded. Some utility planners may claim that even having the emergency rating slightly overloaded is still not worth considering further. For this discussion, let's assume that even a small flow greater than the emergency capacity is considered a line overload.

2. Redispatch the generation to attempt to reach a "no system overload" condition.

3. Reschedule the reactive power sources, capacitors, synchronous condensers, and static var equipment.

4. Reschedule phase-shifters if available.

5. Reconfigure the transmission network to avoid the overload. This may involve either opening the overloaded line, opening a line affecting the overload, or some other reconfiguration.

6. Open the overloaded line to drop radial load.

7. Load shed at one or more buses. This load shedding may involve phone calls to industrial customers, or disconnecting feeders to the most uncritical customers. Usual system planning studies will select load shedding to minimize the megawatts interrupted.

Steps 6 and 7 would be considered Load Not Supplied and allow megawatts at specific locations to be identified.

2.7.6 <u>Load Supplying Capability Analysis</u>

Transmission network reliability calculations are hampered because of the inability to define a "capability." One possible transmission network capability measure is "Load Supplying Capability" (Garver, Van Horne and Wirgau, 1979) [2-21]. Figure 2.2 will help understand the quantity being calculated.

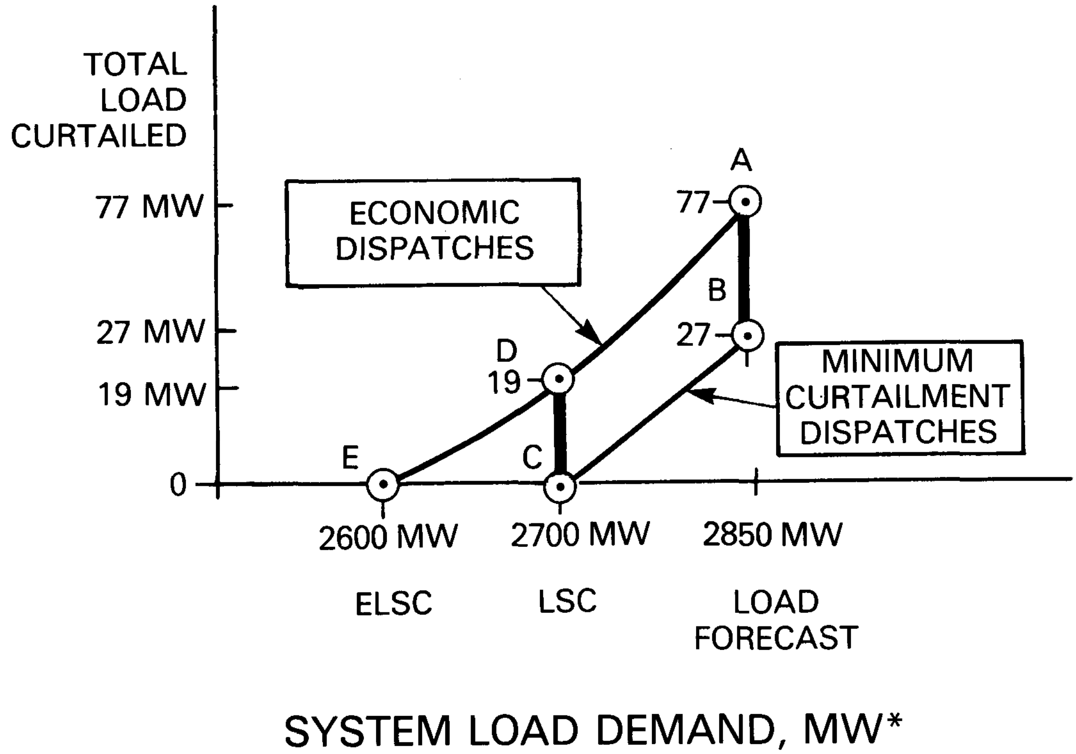


Figure 2.2 Load supplying capability analysis for one generation-transmission-load state.

2600 MW, Economic Load Supplying Capability, ELSC

2700 MW, Load Supplying Capability, LSC

2850 MW, Load State Being Studied

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*System Load Demand is power demanded but not necessarily supplied. For example, at (A), 2850 MW is demanded and 2778 MW is supplied.

In Figure 2.2, total load not supplied is plotted against system load demand, both in megawatts. Point (A) shows the result of a single ac power-flow calculation, with one additional step. Given a certain load demand, termed "Load Forecast", if any equipment is overloaded, then an additional step assigns equipment overloads to bus-load reductions. Once this new step is completed, the total load not supplied is a sum of the individual bus-load reductions. For example, a power-flow result with two lines overloaded, one with 10 MW and one with 20 MW, may result in Bus A assigned a load reduction of 35 MW and Bus B assigned a load reduction of 42 MW. The total of 77 MW becomes the total load not supplied assigned to (A) in Figure 2.2.

By redispatching the generation, Point (B), a lower total of load not supplied is identified. To obtain Point (B), additional computer logic searches out the best generators to turn on and those to turn off in order to serve the load demand with a minimum load curtailment and no concern for economics. For example, the redispatched power-flow result may show a single line overloaded at 12 MW. Using the bus-load assignment logic may result in Bus C load being reduced by 27 MW. This reduced total load curtailment is plotted at (B).

By using the Load Supplying Capability method, Point (C) is identified. At this new load level, referred to as LSC, the maximum system load is served without any facilities being overloaded. Finding this load level using a linear programming method is discussed in Garver, Van Horne, and Wirgau, 1979 [2-21].

Given the "LSC load demand", 2700 MW on Figure 2.2, it is of interest to economically dispatch the generation system and identify the amount of total load curtailed. Using the dc power-flow calculation, augmented with the ability to assign line overloads to bus-load reductions, identifies Point (D). The total load curtailed for the LSC load demand with the units dispatched economically is 19 MW.

A final question of interest is: "At what load level can the system operate economically and not overload any facilities?" This is indicated by Point (E) in Figure 2.2. For example, at the 2600 MW load demand, the system is just able to supply the demand economically without overloading any facilities.

Five Analysis Programs. Figure 2.3 correlates the five points with five analysis programs used in the calculations.

ACTIVITIES

ANALYSIS PROGRAMS	X	A	B	C LSC	D	E ELSC
1. ECONOMIC DISPATCH	1	1	–	–	1	1
2. AC POWER FLOW	2	2	2	2	2	2
3. REDISPATCH (MIN. O.L.)	–	–	3	3	–	–
4. OUTAGES	–	4	4	0*	4	0*
5. NEW LOAD (LSC)	–	–	–	5	–	5

* 0 = ZERO BUS-LOAD OUTAGES, BY DEFINITION

Figure 2.3 Five analysis programs related to six activities. Results of activities A through E are plotted in Figure 2.2.

The following five methods are referred to in Figure 2.3.

1. Economic Dispatch, Refer to Happ, 1977 [2-23].
2. AC/DC Power Flow, Refer to Stagg and ElAbiad, 1968 [2-42].
3. Redispatch to Reduce Line Overloads, Refer to Stott& Marinho, 1979 [2-44].
4. Allocate Line Overloads to Bus-Load Reductions, Refer to Stott & Marinho, 1979 [2-44].
5. Raise or Lower System Load and Redispatch to Approach or Reduce Line Overloads, Refer to Garver, Van Horne and Wirgau, 1979 [2-21].

The first activity, X, is a standard power-flow study, programs #1 and #2. The results are "line overloads" which can be added together but have little meaning when plotted on a figure such as Figure 2.2.

Activity A results in point (A), in Figure 2.2.

Activities B and C, points (B) and (C), use redispatching, program #3, to minimize overloaded lines. If there is any line-overload remaining, then this generation-transmission-load state is classified as a failure, a "loss of load."

Activity B, point (B), uses program #4 to identify bus overloads. This program result is required to do a calculation of Expected Energy Not Served.

Activity C, point (C), identifies the maximum load for a success. By definition the Load Supplying Capability program, #4, selects the new load demand so that the assigned bus outages are zero.

Activity D duplicates activity A, but for the Load-Supplying-Capability demand.

Activity E, point (E), uses the combination of Economic Dispatch, #1; AC Power Flow, #2; and New Load, #5, to identify the maximum load for a successful economic dispatch.

Capability, Generation Vs. Bulk-Power

Figure 2.4 shows the simple concept of generation supplied vs. generation demanded. Normally this is a proportional curve, where each MW is supplied. However, in generation reliability studies we are interested in the point where demand exceeds supply. This is shown as a load demand of 1000 MW. Beyond this demand is loss of load. The amount of load not supplied is the demand above 1000 MW. It is important to note that the installed capacity identifies when the loss of load region begins and it is also a simple method of calculating the amount of load not supplied. This is not the case when considering transmission and generation together, Figure 2.5.

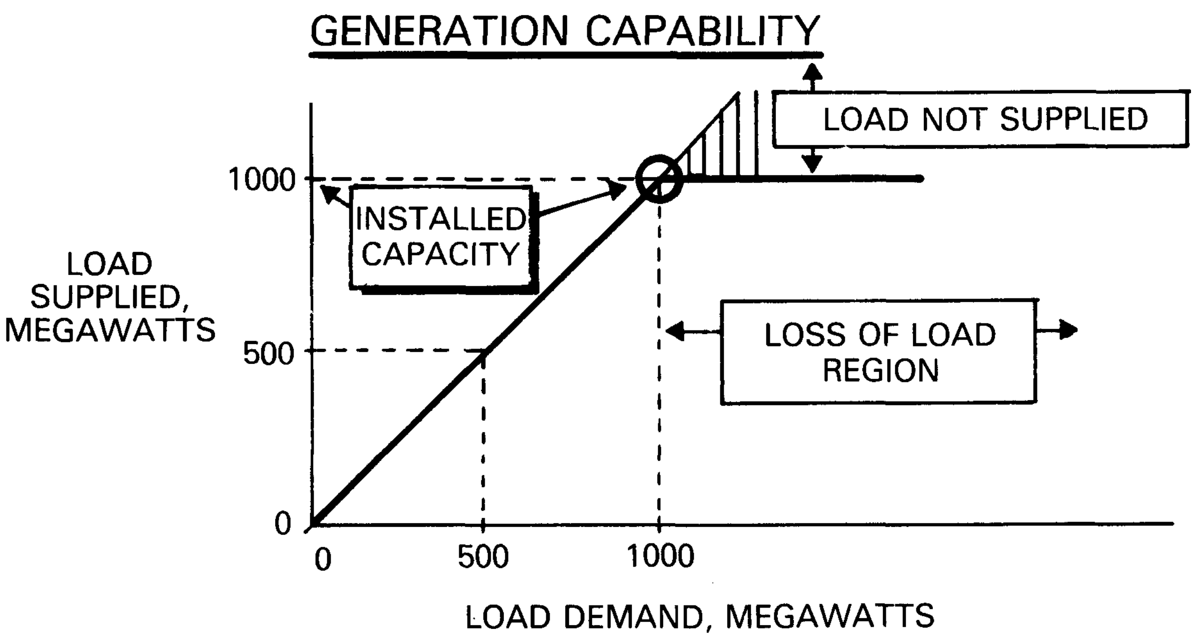

Figure 2.4 Loss of Load and Load Not Supplied Related to Installed Generation Capacity

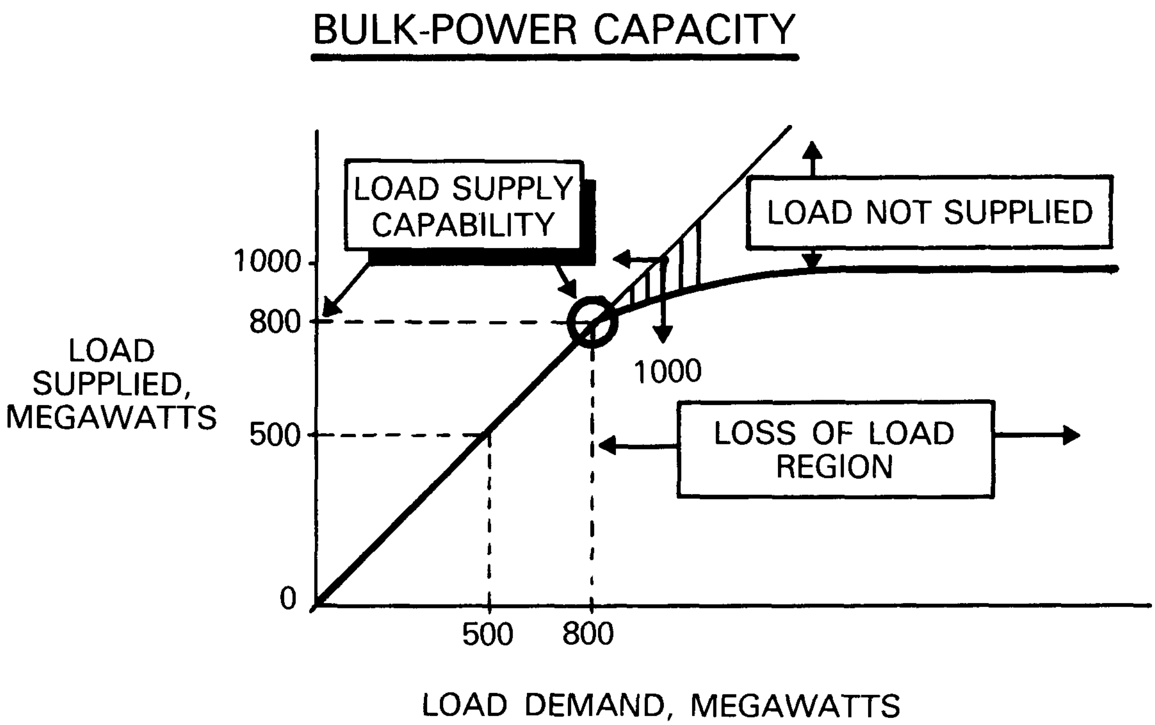

Figure 2.5 Loss of Load and Load Not Supplied Related to Load Supplying Capability

Figure 2.5 considers a bulk-power system where the proportionality of load supplied to load demand continues up to 800 MW. At this point the loss-of-load region begins and load not supplied is the difference between supply and demand. The breakpoint in this proportionality is termed "Load Supplying Capability" and denotes the beginning of the loss of load region. In this sense it is like the installed capacity for the generation system. However, calculating the load not supplied at any demand above 800 MW is not a simple task. Simply subtracting 800 from the load demand does not give a useful estimate of the load not supplied, this is where the Load Supplying Capability differs from the Installed Capacity of generation.

2.8 CONCLUDING REMARKS

Section 2 begins by defining system status indicators, aids in conceptualizing reliability indices. Four primary reliability indices then are introduced with the common ability that they can be used with all four electric utility system components: generation, bulk power, area supply, and distribution. A general computational structure is identified, followed by a description of the specific computational considerations for generation only, generation with reserve, operation and interconnection considerations, and finally, the bulk power system.

2.8.1 Hierarchies of Indices

Table 2.4 presents application and calculation hierarchies summarizing our present state of understanding.

Table 2.4

COMPARISON OF HIERARCHIES

SYSTEM PLANNING APPLICATION HIERARCHY	CUSTOMER APPLICATION HIERARCHY	CALCULATION HIERARCHY
Expected Customers Not Supplied	Frequency of Loss of Load	Hourly Loss of Load Expectation
Frequency of Loss of Load	Hourly Loss of Load Expectation	Expected Energy Not Supplied
Expected Energy Not Supplied	Expected Energy Not Supplied	Frequency of Loss of Load
Hourly Loss of Load Expectation	Expected Customers Not Supplied	Expected Customers Not Supplied

It appears from our discussions with regulators and system planners that Expected Customers Not Supplied (ECNS) is the most important reliability index, but it is the most difficult to calculate for three of the four power system components: generation, bulk power, and area supply. It is almost perverse that expected customers not served would be the most difficult to calculate and therefore appears last in our calculation hierarchy.

Customers are most concerned about the Frequency of Loss of Load and then Duration, calculated by HLOLE/FLOL.

The easiest of the indices to evaluate is the Hourly Loss of Load Expectation (HLOLE) index. However, even its calculation is so burdensome that a major portion of the results comes from knowledge, experience and judgement of the persons doing the reliability calculations. These were used in preparing the numerical sketch used in Exhibits 2.4 through 2.15.

2.8.2 Further Research Needs

Several areas for further research were identified, including the following five items.

1. Conceptualizing reliability indices for the internal system status indicators as discussed with the visualization of a reliability center, Exhibit 2.1, is needed research.

2. Conceptualizing the reliability indices for individual load points or buses, Exhibit 2.2, needs research. More status indicators may be appropriate.

3. Combining the reliability index results of system component studies into a combined index needs further research. For example, combining study results from two parts of a distribution system must recognize the overlapping outage possibility. This problem is more difficult when combining components such as distribution and area supply result, area supply and bulk power system results or bulk power and generation results.

4. Visualizing the customers not supplied in Exhibit 2.4 assumed definition and calculating procedures. These procedures do not now exist for generation, bulk power and area supply. Fundamental definitions and outlining of computation methods are necessary.

5. Energy not supplied assumes that demand not supplied is identifiable. This is the view in generation, bulk power and area supply, but not for distribution. In distribution, the view is of connected load rather than demand. These two viewpoints require further research before reconciliation of these viewpoints is possible.

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Section 3

COMPUTATION OF GENERATION RELIABILITY INDICES

3.1 ANALYTICAL AND COMPUTATIONAL EQUATIONS

Introduction

The various generation reliability indices in use in the power industry were discussed in Section 2. This section describes the analytical formulas used to evaluate each of these indices and shows the basic relationships between the indices. Also developed are the more computationally efficient algorithms which are actually used in determining the indices. The basic assumptions inherent in the indices are also covered in this section of the report.

There are numerous papers and texts which discuss in great detail the theory of generation system reliability calculations. The objective of this section, and related Appendices, is to present the results of these references using a common nomenclature so that comparisons can be made between indices and between the various techniques used to calculate these indices. Examples are presented for each index so that the reader can easily follow all of the details of the calculations. Important terms are included with each equation, and a complete set of terms is listed at the end of this section in Table 3.3. While this nomenclature differs in some instances from that used in Section 5, it was decided that when taking a conventional look at the indices, the more conventional nomenclature should be used.

Background

The term that is common to all probabilistic generation reliability indices is p(x), the probability of exactly x MW of capacity on outage. The cumulative probability of x MW or more on outage,

$$P(x) = \sum_{X=x}^{C} p(X) \qquad (3\text{-}1)$$

where C is the total installed capacity, is often used to reduce the calculation time required for the probabilistic indices. Appendix A shows how the cumulative table, P(x), can be constructed directly using a recursive formula without having to construct the exact table, p(x), first. In order to draw comparisons between indices, both the exact and cumulative probabilities of outage are used throughout this section.

Generation Data - Since the generation units on a system are in discrete megawatt sizes, and since the available outage data on generation units consists of discrete megawatts on outage at some discrete probability, then it follows that the probability of various system capacity outage states is itself a discrete function. Therefore the proper technique to use when determining the probability of being within a range of system states, from a to b MW on outage, is to sum the exact probabilities over this range, and not integrate over the range. There are certain techniques which approximate the exact and cumulative outage tables with continuous functions. When these methods are used then integration is acceptable.

Load Data - In utility applications the type of load data that is recorded is the hourly integrated load. These are discrete hourly load values equal to the system energy requirements plus losses for a particular hour. The daily peak load used in some reliability calculations is the largest value of the 24 hourly integrated loads within a day.

The load data required for reliability calculations can be viewed in two different manners, giving rise to different interpretations of the calculation techniques. For example, if the twenty daily peak loads in a month are specified then they can be viewed as either the twenty individual peaks corresponding to the twenty separate days, or they can be interpreted as the distribution of possible peak loads occurring on any one of the weekdays within the month. If they are viewed as loads on separate days then the probability of a loss of load on each day can be determined and the expected number of weekdays within the month with loss of load at time of peak is simply the sum of the individual probabilities.

$$E[\text{loss of load on peak}] = \sum_{i=1}^{20} P(\text{loss on peak}_i) \qquad (3\text{-}2)$$

If the loads are viewed as the distribution of possible peaks occurring on any day, then the probability of a loss of load on one day is the weighted sum of the probability of a loss of load on any of the peaks. Since this distribution is constant for all of the weekdays within the month the expected number of days with loss of load is

$$E[\text{loss of load on peak}] = 20[P(\text{loss on any day})]$$

$$= 20 \left[\sum_{i=1}^{20} \left(\frac{1}{20} \right) P(\text{loss on peak}_i) \right] \qquad (3\text{-}3)$$

Both equations 3-2 and 3-3 yield the same numerical results, even though the manner in which they were developed differs. The latter method has been expanded in some cases by assuming that the 20 peaks are samples from a continuous distribution. The probability of a loss of load on any one day is then found by integrating over this distribution. The former method, however, is more intuitively appealing and much easier to visualize when trying to learn and understand the calculations involved. For this reasion, Section 3 will assume that each peak load corresponds to a separate day, and that indices can be calculated from each day and then combined to produce interval results.

3.1.1 Deterministic Indices

There are two deterministic indices commonly used in the utility industry. These are percent reserve and the number of largest units.

Percent Reserve. Percent reserve can be calculated in two ways; as a percent of peak load or as a percent of installed capacity. The figure as a percent of peak load is most common, but it is important to specify which index is being referred to in order to avoid confusion when comparing systems.

The two formulas used are:

$$\% \text{ Reserve} = \frac{C-L}{L} \qquad (3\text{-}4)$$

and

$$\% \text{ Reserve}_C = \frac{C-L}{C} \qquad (3\text{-}5)$$

where C = total installed capacity
and L = annual peak load.

Largest Units - The largest units index requires that the reserve capacity be equal to the sum of the capacity of a given number of the largest units on the system. This method is similar to a contingency criteria used in transmission planning. Expressed analytically the criteria is:

$$R = C - L \geq \sum_{i=1}^{N} C_i \qquad (3\text{-}6)$$

where R = system installed reserves

N = largest units criteria

and C_i = capacity of the ith unit, with $C_1 > C_2 > \ldots > C_N$

3.1.2 Probabilistic Indices

Generation reliability indices based on probability mathematics can be divided into two main categories:

1. Those concerned with the probability of a generator outage existing, r.
2. Those concerned with the failure rate, λ, and the repair rate, μ, of a generator.

The two methods are related through the equation:

$$r = \frac{1/\mu}{1/\lambda + 1/\mu} \qquad (3\text{-}7)$$

For example, if a generator experiences a 5 day outage every 95 days then

$$\lambda = 1/95 = .01053$$

$$\mu = 1/5 = .2$$

and $$r = \frac{5}{95 + 5} = .05$$

The cycle time, T, is

$$T = 1/\mu + 1/\lambda = 5 + 95 = 100 \text{ days}$$

and the frequency of failure, f, is defined as the reciprocal of the cycle time

$$f = 1/T = 1/100 = .01 \text{ failures/day}$$

The first method involves the construction of a system capacity outage distribution, p(x), commonly referred to as the exact probability outage table. The second method involves the construction of two tables involving the frequency of capacity states with greater than and less than x MW on outage in addition to the exact probability outage table. Later it is shown how the final combined results of the two frequency tables can be determined directly from a single frequency table, f(x).

In a probabilistic analysis of generation systems the general approach used is to construct an outage table, p(x), which is the probability of x MW on forced outage. This table is valid as long as the generation system does not change for reasons other than forced outages, i.e. units installed or retired or on planned maintenance. The "periods" used in the equations in this report refer to these periods of constant available capacity.

Loss of Load Expectation (LOLE) [3-1]

This value, which is commonly, but incorrectly, called Loss of Load Probability (LOLP), measures the expected number of days that capacity is less than the daily peak load.

If p(x) is the probability of exactly x MW on outage then:

$$\text{LOLE (1 load)} = \sum_{x=C-L}^{C} p(x) \qquad (3\text{-}8)$$

where C = Total installed capacity not on maintenance
and L = Daily peak load.

Combining equations 3-1 and 3-8 gives

$$\text{LOLE(1 load)} = P(C-L) \qquad (3\text{-}9)$$

The LOLE for one load is an actual probability, and is referred to as P(L;t) in Section 5.

Loss of Load Expectation is generally calculated over a period of time rather than for just a single load. The two types of loads used are the daily peak loads and the hourly loads. The former is the most common method, with the resulting LOLE being expressed in days per period. When hourly loads are used the result is in hours per period. Generally the results are summed over a number of periods and expressed on an annual basis. It is important to recognize that daily and hourly loads give rise to completely different interpretations of LOLE. Using daily peaks produces an estimate of the number of outages per year. Using hourly loads determines the total duration of outages within a year. Under no circumstances is it correct to convert between LOLE in hours/year and LOLE in days/year by using a factor of 24 hours/day.

<u>LOLE - Daily Peak Load Model</u> - When daily peak loads are examined then equation 3- 8 can be expanded to:

$$\text{LOLE(period)} = \sum_{j=1}^{n} \sum_{x=C-L_j}^{C} p(x) \text{ daily peaks} \qquad (3\text{-}10)$$

$$= \sum_{j=1}^{n} P(C-L_j) \text{ daily peaks} \qquad (3\text{-}11)$$

where:

n = number of days in the period

L_j = peak load on day j

and C = total installed capacity not on planned maintenance.

The dimensions of LOLE are daily peaks. However, this has traditionally been shortened to just "days". In addition, the interval over which the calculation takes place is usually indicated by dividing the dimension by the length of the interval. For example, the dimensions of Equation 3-11 would usually be expressed as "days/period." This convention will be used throughout the remainder of this section.

These period values of LOLE can then be summed over all the periods in a year to give:

$$\text{LOLE(annual)} = \sum_{i=1}^{m} \sum_{j=1}^{n_i} \sum_{x=C_i-L_{i,j}}^{C_i} p_i(x) \text{ days/year} \qquad (3\text{-}12)$$

$$= \sum_{i=1}^{m} \sum_{j=1}^{n_i} P_i(C_i - L_{i,j}) \text{ days/year} \qquad (3\text{-}13)$$

where

m	=	number of periods in the year
n_i	=	number of days in period i
$L_{i,j}$	=	peak load on day j of period i
C_i	=	total installed capacity not on planned maintenance in period i
$p_i(x)$	=	exact capacity outage table for period i
$P_i(x)$	=	cumulative capacity outage table for period i.

Another technique which has been used (Patton)[3-2] is to define the loads as a cumulative probability distribution $P_L(\ell)$, where:

$$P_L(\ell) = \text{Probability (Load} \geq \ell). \tag{3-14}$$

Equation 3-9 can then be modified to give:

$$\text{LOLE(annual)} = \sum_{i=1}^{m} n_i \sum_{x=0}^{C_i} p_i(x) P_{Li} (C_i - x) \text{ days/year} \tag{3-15}$$

As in equation 3-13, equation 3-15 also eliminates one of the summations. However, for large systems the number of days in a period, n_i, will be much less than the capacity, C_i, so that equation 3-13 is a more efficient algorithm.

If the load distribution, $P_L(\ell)$, and the exact capacity outage table, p(x), can be approximated by continuous analytical functions then the inner summation of equation 3-15 can be changed to an integration for x going from zero to C_i. Although this method involves some loss of accuracy, a direct solution of the integral will greatly reduce the amount of computation time required.

<u>LOLE - Hourly Load Model</u> - In the literature the terms LOLE and LOLP are used interchangeably to denote either the expected number of days or hours per year of capacity shortages. In order to avoid confusion, this report will use LOLE to refer to the expected number of days with insufficient capacity at time of daily peak and HLOLE (Hourly Loss of Load Expectation) to refer to the expected number of hours of insufficient capacity.

Equations 3-10 and 3-11 need to be modified only slightly to determine the hourly loss of load expectation for a period:

$$HLOLE(period) = \sum_{j=1}^{n} \sum_{k=1}^{24} \sum_{x=C-L_{j,k}}^{C} p(x) \text{ hours/period} \qquad (3\text{-}16)$$

$$= \sum_{j=1}^{n} \sum_{k=1}^{24} P(C-L_{j,k}) \text{ hours/period} \qquad (3\text{-}17)$$

where

$L_{j,k}$ = load for hour k of day j.

The annual calculations of HLOLE are a similar extension of equations 3-12 and 3-13:

$$HLOLE(annual) = \sum_{i=1}^{m} \sum_{j=1}^{n_i} \sum_{k=1}^{24} \sum_{x=C_i-L_{i,j,k}}^{C_i} p_i(x) \text{ hours/year} \qquad (3\text{-}18)$$

$$= \sum_{i=1}^{m} \sum_{j=1}^{n_i} \sum_{k=1}^{24} P_i(C_i-L_{i,j,k}) \text{ hours/year} \qquad (3\text{-}19)$$

When a cumulative load distribution is used the resulting equation is:

$$HLOLE(annual) = 24 \sum_{i=1}^{m} n_i \sum_{x=0}^{C_i} p_i(x)P_{L_i}(C_i-x) \text{ hours/year} \qquad (3\text{-}20)$$

where $P_{L_i}(C_i-x)$ is the probability distribution of the hourly loads within period i. HLOLE is sometimes[3-2] divided by the number of hours in the year to obtain a per unit value between zero and one which can be called the loss of load probability, LOLP. It is a probability of capacity deficiency in the sense that it is the expected number of hours of capacity deficiency divided by the total number of hours. However, to avoid confusion we will not use this definition of LOLP anywhere else in the report.

Probability of Positive Margin (POPM)[3-3]

The index POPM is defined as the probability of sufficient capacity available to meet the annual peak load. The basic expression used is

$$POPM = 1 - \sum_{x=C-L}^{C} p(x) \quad (3\text{-}21)$$

$$= 1 - P(C - L) \quad (3\text{-}22)$$

where L = annual peak load.

Although load uncertainty is an option on any of the reliability indices, it is generally included in the expression for POPM. Therefore equation 3-22 can be modified to give

$$POPM = 1 - \int_{L_{min}}^{L_{max}} p_{load}(L)P(C-L)dL \quad (3\text{-}23)$$

where $p_{load}(L)$ is the exact probability density function for the annual peak load and L_{min} and L_{max} are the minimum and maximum values of peak load.

Quality (Q)[3-4]

The quality index is calculated by multiplying together either the 365 daily or 8760 hourly probabilities of no capacity outages for the year. Therefore the general expression for Q is

$$Q = \prod_{i=1}^{m} \prod_{j=1}^{n_i} \left(1 - \sum_{x=C_i-L_{i,j}}^{C_i} p_i(x)\right) \quad (3\text{-}24)$$

$$= \prod_{i=1}^{m} \prod_{j=1}^{n_i} \left(1 - P_i(C_i-L_{i,j})\right) \quad (3\text{-}25)$$

Where m is the number of periods and n_i is the number of loads within a period. When the probabilities of independent events are multiplied together the result is also a probability. However, since it is highly questionable as to whether the daily and hourly loads are independent of each other, the quality index is not an actual probability. Nevertheless, it is an index which has been found useful in some utility planning operations.

Probability of Loss-of-Load (PLOL)

The index PLOL, Probability of Loss-of-Load, is the compliment of the quality index, Q. As with Q, PLOL may be calculated on either a daily or hourly basis. The equation used is:

$$PLOL = 1 - Q$$

$$= 1 - \prod_{i=1}^{m} \prod_{j=1}^{n_i} \left(1 - \sum_{x=C_i-L_{i,j}}^{C_i} p_i(x)\right) \tag{3-26}$$

$$= 1 - \prod_{i=1}^{m} \prod_{j=1}^{n_i} \left(1 - P_i(C_i-L_{i,j})\right) \tag{3-27}$$

where the variables are the same as in equation 3-24.

Expected Energy Not Supplied (EENS)[3-1]

The index EENS, also known as the Loss of Energy Probability (LOEP), is defined as the expected amount of energy not supplied due to generation outages. For a given capacity outage state the unserved energy is the capacity deficiency times the probability of being in that state;

$$\text{Unserved energy} = [x - (C - L)]\ p(x) \tag{3-28}$$

where x = MW on outage

C = installed capacity

L = load

$p(x)$ = probability of x MW on outage

For a given hourly load the unserved energy is then summed over all of the capacity states which result in a capacity deficiency:

$$\text{Unserved Energy} = \sum_{x=C-L}^{C} (x-(C-L))\ p(x)\ \text{MWh} \tag{3-29}$$

This unserved energy can then be summed over all of the hourly loads in a period of constant maintenance, and all of the periods in a year to give:

$$\text{EENS(annual)} = \sum_{i=1}^{m} \sum_{j=1}^{n_i} \sum_{k=1}^{24} \sum_{x=C_i-L_{i,j,k}}^{C_i} (x-(C_i-L_{i,j,k}))\ p_i(x) \text{ MWh/year.} \qquad (3\text{-}30)$$

Figure 3.1 shows graphically the area equal to the unserved energy for each capacity outage state. If we let R = C-L = reserves then it can be seen from either Figure 3.1 or equation 3-29

$$\text{Unserved Energy (R)} = \sum_{x=R}^{C} (x-R)\ p(x) \text{ MWh}$$

$$= \sum_{x=R+1}^{C} (x-R)\ p(x) \text{ MWh,} \qquad (3\text{-}31)$$

since an outage of exactly R MW would result in the available capacity being exactly equal to the load, so that there would be no unserved energy.

Figure 3.2 shows the same area of unserved energy, but now the area has been divided into vertical strips rather than the horizontal strips of Figure 3.1. The height of a vertical strip at point x is

$$\text{height (x)} = \sum_{X=x}^{C} p(X) \qquad (3\text{-}32)$$

which, from equation 3-1 is P(x), the cumulative capacity outage table. From Figure 3.2 we can then see that the unserved energy can be expressed as

$$\text{Unserved Energy (R)} = \sum_{x=R+1}^{C} P(x) \text{ MWH.} \qquad (3\text{-}33)$$

Each term in the summation in equation 3-31 involves a table lookup, (p(x)), a subtraction, (x-R), and a multiplication, ((x-R)p(x)). In equation 3-33, however, only a table lookup is required, once the cumulative outage table has been constructed. Since the cumulative table can be constructed directly, without first constructing the exact table, then equation 3-33 is the most computationally efficient manner to calculate unserved energy.

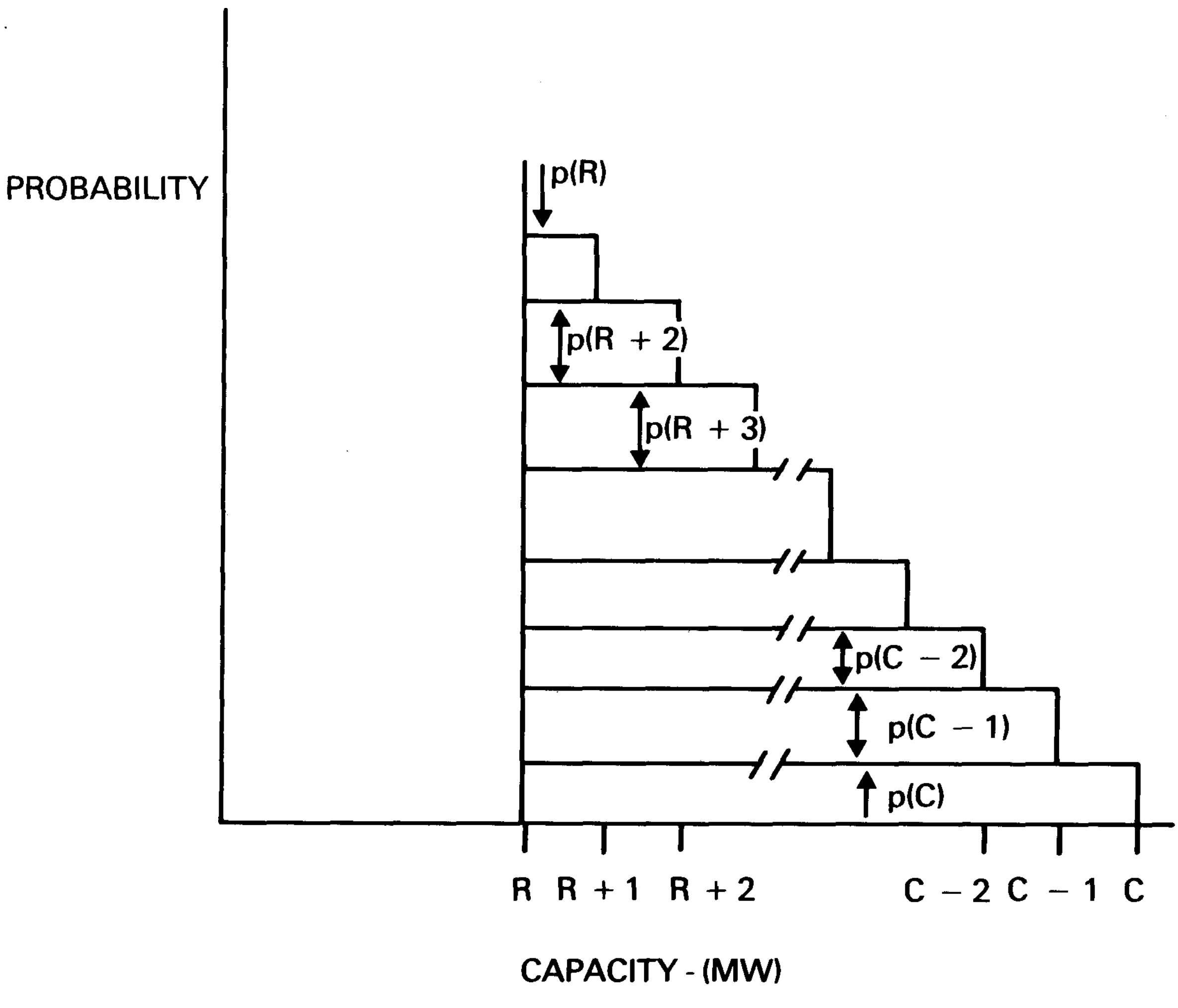

Figure 3-1. Unserved Energy Using Exact Probabilities

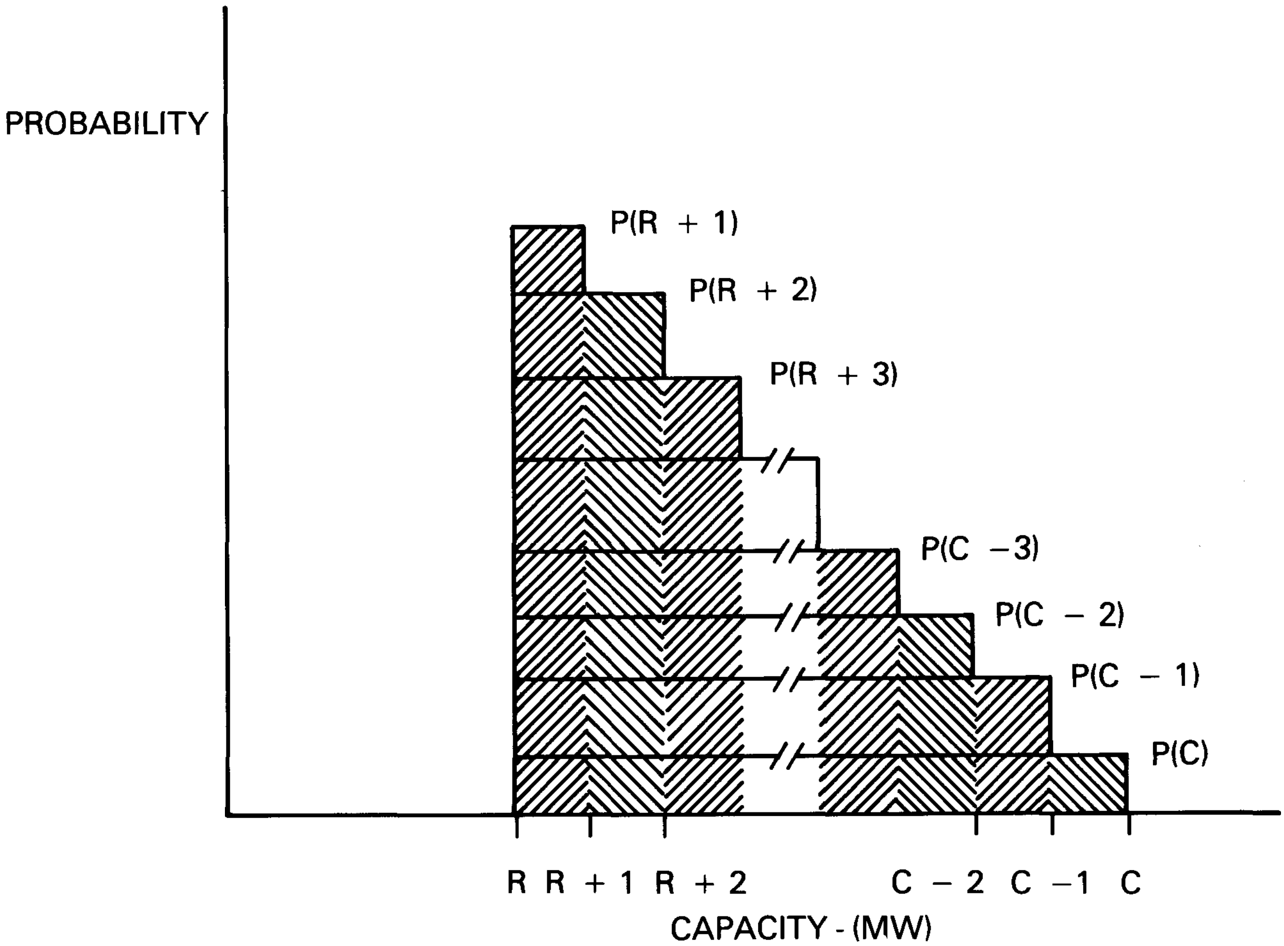

Figure 3-2. Unserved Energy Using Cumulative Probabilities

Figure 3.3 demonstrates another advantage of equation 3-33. Using the method of equation 3-31, knowledge of the unserved energy when reserve = R does not simplify the calculation of unserved energy when reserves = R', since the length of each horizontal strip has changed. Using equation 3-33, however, the unserved energy when reserves = R' is

$$\text{Unserved Energy } (R') = \sum_{x=R'+1}^{C} P(x)$$

$$= \sum_{x=R'+1}^{R} P(x) + \sum_{x=R+1}^{C} P(x) \qquad (3\text{-}34)$$

or

$$\text{Unserved Energy } (R') = \text{Unserved Energy } (R) + \sum_{x=R'+1}^{R} P(x) \qquad (3\text{-}35)$$

Therefore once the unserved energy has been calculated for one value of load during a period of constant maintenance, then the unserved energy can be calculated for other values of load by simply adding on a few more terms.

The annual value of Expected Energy Not Supplied (EENS) is then

$$\text{EENS(annual)} = \sum_{i=1}^{m} \sum_{j=1}^{n_i} \sum_{k=1}^{24} \sum_{x=C_i-L_{i,j,k}+1}^{C_i} P_i(x) \text{ MWh/year} \qquad (3\text{-}36)$$

Equation 3-36 assumes that a 1 MW step is used in building the capacity outage table. If the step size, ΔX, is not 1 MW then equation 3-36 should be modified to

$$\text{EENS(annual)} = \Delta X \sum_{i=1}^{m} \sum_{j=1}^{n_i} \sum_{k=1}^{24} [\sum_{x=C_i-L_{i,j,k}}^{C_i} P_i(x)-\varepsilon] \text{ MWh.} \qquad (3\text{-}37)$$

where ε is a correction term needed when the load does not correspond exactly to an entry in the outage table.

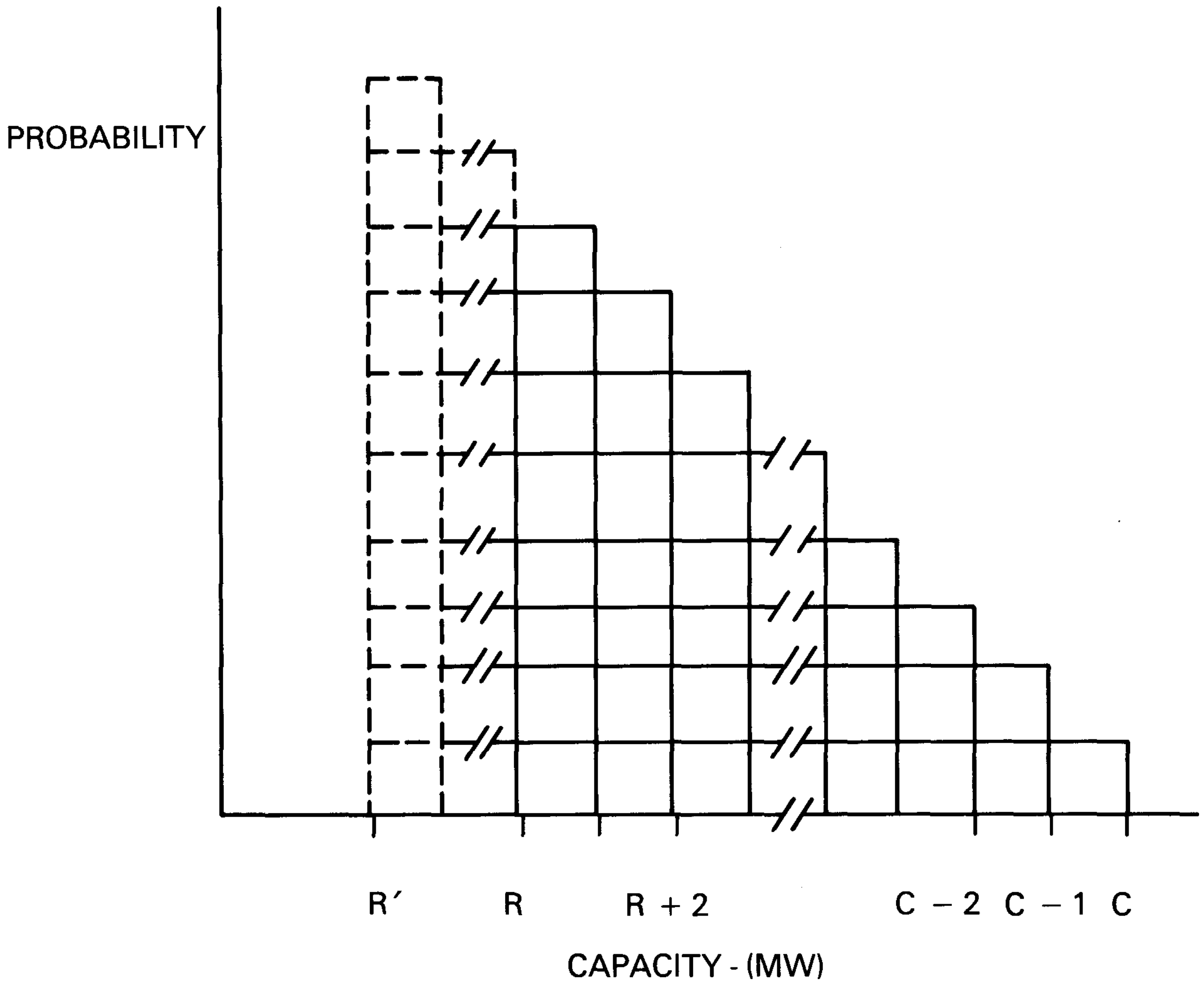

Figure 3-3. Modifying Unserved Energy for Different Reserve Margins

$$\epsilon = ((C-L)/\Delta X-N)P((N+1)\Delta X) \qquad (3\text{-}38)$$

where $N = \text{integer} = [\frac{C-L}{\Delta X}]$

For large systems or small step sizes ϵ is generally negligible compared to the summation term of equation 3-37 and can therefore be dropped. For the remainder of this report ϵ will not be considered in the equations for EENS.

For planning purposes the annual unserved energy can be divided by the total energy under the load curve to obtain a normalized value called the Percentage Energy Loss (PEL).[3-5] The energy under the load curve is

$$\text{Total Load Energy} = \sum_{i=1}^{m} \sum_{j=1}^{n_i} \sum_{k=1}^{24} L_{i,j,k} \text{ MWh} \qquad (3\text{-}39)$$

so that the percentage energy loss is

P.E.L. = EENS/Total Load Energy

$$= \frac{\sum_{i=1}^{m} \sum_{j=1}^{n_i} \sum_{k=1}^{24} \sum_{x=C_i-L_{i,j,k}}^{C_i} (x-(C_i-L_{i,j,k}))p_i(x)}{\sum_{i=1}^{m} \sum_{j=1}^{n_i} \sum_{k=1}^{24} L_{i,j,k}} \times 100 \qquad (3\text{-}40)$$

$$\frac{\Delta X \sum_{i=1}^{m} \sum_{j=1}^{n_i} \sum_{k=1}^{24} \sum_{x=C_i-L_{i,j,k}+1}^{C_i} P_i(x)}{\sum_{i=1}^{m} \sum_{j=1}^{n_i} \sum_{k=1}^{24} L_{i,j,k}} \times 100 \qquad (3\text{-}41)$$

or, if the annual load factor is available

$$PEL = \frac{\Delta X \sum_{i=1}^{m} \sum_{j=1}^{n_i} \sum_{k=1}^{24} \sum_{x=C_i-L_{i,j,k}+1}^{C_i} P_i(x)}{(24 \sum_{j=1}^{m} n_j)(L_{peak})(L.F.)} \times 100 \qquad (3\text{-}42)$$

where L_{peak} = annual peak load
and L.F. = annual load factor.

Since the percentage energy loss is generally quite small it is sometimes per unitized and subtracted from one and the new value is defined as the Energy Index of Reliability (EIR) [3-1, 3-5]

EIR = 1-PEL/100

$$= 1 - \frac{\sum_{i=1}^{m} \sum_{j=1}^{n_i} \sum_{k=1}^{24} \sum_{x=C_i-L_{i,j,k}}^{C_i} (x-(C_i-L_{i,j,k}))p_i(x)}{\sum_{i=1}^{m} \sum_{j=1}^{n_i} \sum_{k=1}^{24} L_{i,j,k}} \quad (3\text{-}43)$$

$$= 1 - \frac{\Delta X \sum_{i=1}^{m} \sum_{j=1}^{n_i} \sum_{k=1}^{24} \sum_{x=C-L_{i,j,k}+1}^{C_i} P_i(k)}{\sum_{i=1}^{n} \sum_{j=1}^{n_i} \sum_{k=1}^{24} L_{i,j,k}} \quad (3\text{-}44)$$

Expected Loss-of-Load (XLOL)[3-6]

The Expected Loss-of-Load index, XLOL, is defined as the expected magnitude of a capacity deficiency given that a capacity deficiency exists. This index may be calculated for each interval, and a weighted average value can be determined for the year. The expression for the annual average is:

$$XLOL = \frac{\text{Expected Energy Not Supplied (MWh/yr.)}}{\text{Hourly Loss-of-Load Expectations (Hr/yr.)}}$$

$$= \frac{\sum_{i=1}^{m} \sum_{j=1}^{n_i} \sum_{k=1}^{24} \sum_{x=C_i-L_{i,j,k}}^{C_i} (x-(C_i-L_{i,j,k}))p_i(x)}{\sum_{i=1}^{m} \sum_{j=1}^{n_i} \sum_{k=1}^{24} \sum_{x=C_i-L_{i,j,k}}^{C_i} p_i(x)} \text{ MW} \quad (3\text{-}45)$$

$$= \frac{\Delta X \sum_{i=1}^{m} \sum_{j=1}^{n_i} \sum_{k=1}^{24} \sum_{x=C_i-L_{i,j,k}+1}^{C_i} P_i(x)}{\sum_{i=1}^{m} \sum_{j=1}^{n_i} \sum_{k=1}^{24} P_i(C_i-L_{i,j,k})} \text{ MW} \tag{3-46}$$

For large systems XLOL is roughly equal to the characteristic system slope-m defined by Garver.[3-7]

<u>Frequency and Duration (f&d)</u>

The Loss of Load Expectation (LOLE) considers only the probability of generator outages. An extension of this is a method which considers both the frequency and duration of generator outages. The equation for the frequency, or the number of occurrences of capacity deficiency within a period of time is given by [3-2]

$$f = \sum_{\text{all } x} p(x) \{(\rho_+(x)-\rho_-(x))P_L(C-x) + F_L(C-x)\} \quad \text{occurrences/period} \tag{3-47}$$

where

$p(x)$	=	exact probability of having x MW on outage
$\rho_+(x)$	=	effective departure rate from an exact capacity state x to states having less capacity on outage
$\rho_-(x)$	=	effective departure rate from an exact capacity state x to states having more capacity on outage
$P_L(C-x)$	=	cumulative probability of load being C-x MW or greater
$F_L(C-x)$	=	frequency of the state of load greater than or equal to C-x.

The terms $\rho_+(x)$ and $\rho_-(x)$ are developed recursively by adding the units one at a time using the equations

$$\rho_+(x) = \frac{p'(x)(1-r)\rho'_+(x) + p'(x-c)r(\rho'_+(x-c) + \mu)}{p(x)} \tag{3-48}$$

and

$$\rho_-(x) = \frac{p'(x)(1-r)(\rho'_-(x) + \lambda) + p'(x-c)\ r\rho'_-(x-c)}{p(x)} \tag{3-49}$$

where c = capacity of unit being added

λ = average forced outage occurrence rate of unit being added

= 1/(mean time to failure), dimensions = 1/period

μ = average forced outage restoral rate of unit being added

= 1/(mean time of repair), dimensions = 1/period

r = forced outage rate of unit being added

= $(1/\mu)/((1/\mu)+(1/\lambda))$

and the prime quantities are the values before the addition of the new unit. In equation 3-48 and 3-49 the primed terms are zero if x is less than c since states having negative capacity cannot exist. In addition, it must be assumed that the times to failure and repair are exponentially distributed with means of $1/\lambda$ and $1/\mu$ respectively.

The expected duration of an outage, d, is the ratio of the expected number of hours of capacity deficiency, HLOLE, to the expected frequency of occurrence, f.

Therefore the expected duration of outage is

$$d\ (\text{period}) = \frac{\text{HLOLE (period)}}{f\ (\text{period})}\ \text{hours/occurrence} \tag{3-50}$$

Combining equation 3-16 and 3-47 provides.

$$d\ (\text{period}) = \frac{\sum_{j=1}^{n}\sum_{k=1}^{24}\sum_{x=C-L_{j,k}}^{C} p(x)}{\sum_{\text{all }x} p(x)\{(\rho_{+}(x)-\rho_{-}(x))P_L(C-x) + F_L(C-x)\}}\ \text{hours/occurrence} \tag{3-51}$$

Equation 3-47 can be extended to a year by summing the frequencies in each period.

$$f\ (\text{annual}) = \sum_{i=1}^{m}\sum_{\text{all }x} p_i(x)\{(\rho_{+i}(x)-\rho_{-i}(x))P_{Li}(C_i-x)+F_{Li}(C_i-x)\}\ \text{occurrences/year} \tag{3-52}$$

The expected duration for the year would then be

$$d\ (\text{annual}) = \frac{\text{HLOLE (annual)}}{\text{f (annual)}}\ \text{hours/occurrence}$$

$$= \frac{\sum_{i=1}^{m} \sum_{j=1}^{n_i} \sum_{k=1}^{24} \sum_{x=C_i-L_{i,j,k}}^{C_i} p_i(x)}{\sum_{i=1}^{m} \sum_{\text{all } x} p_i(x)\{(\rho_{+i}(x)-\rho_{-i}(x))P_{Li}(C_i-x) + F_{Li}(C_i-x)\}}\ \text{hours/occurrence} \tag{3-53}$$

Equation 3-47 gives one way in which the frequency of capacity shortages can be calculated for a period of constant maintenance. This equation can be rewritten as

$$f = \sum_{\text{all } x} p(x)(\rho_+(x)-\rho_-(x))P_L(C-x) + \sum_{\text{all } x} p(x)F_L(C-x) \tag{3-54}$$

where the first term is the frequency of generation outages times the probability of the load, and the second term is the probability of generation outages times the frequency of the load.

If we define f(x) as the frequency of generation outages of exactly x MW, then the first term of equation 3-54 can be written as

$$\text{term one} = \sum_{\text{all } x} f(x)\ P_{\text{load}}(C-x) \tag{3-55}$$

If f(x) is defined on a daily basis, and hourly integrated loads are used, then equation 3-55 can be rewritten as

$$\text{term one} = \frac{1}{24} \sum_{k=1}^{24} \sum_{x=C-L_k}^{C} f(x)$$

$$= \frac{1}{24} \sum_{k=1}^{24} F(C-L_k)\ \text{occurrences/day} \tag{3-56}$$

where

$$F(x) = \sum_{X=x}^{C} f(X) \tag{3-57}$$

$$= \sum_{X=x}^{C} p(X)(\rho_+(X) - \rho_-(X)) \text{ occurrences/day} \tag{3-58}$$

In Appendix A the recursive formula used to build F(x) directly is derived from equations 3-48, 3-49, and 3-58.

The second term of equation 3-54 includes the cumulative frequency of the load, $F_L(C-x)$. However, if the frequency of outages is expressed on a daily basis, then only the probability of the load is important, not the frequency. In other words, if a system has a load model which peaks twice a day, and the system has a capacity shortage on both peaks but not between peaks, then should one outage be counted or two? If the number of outages is desired, then the frequency of the load, $F_L(C-x)$, is important and therefore must be included in the calculations. However, if the desired quantity is the number of days with outages, then the frequency should be

$$F_L(C-x) = 1 \text{ when } L \geq C-x \tag{3-59}$$

$$0 \text{ when } L < C-x$$

where L is the peak load for the day. The second term of equation 3-54 can then be written as

$$\text{second term} = 0 \sum_{x=0}^{C-L} p(x) + 1 \sum_{x=C-L}^{C} p(x)$$

$$= \sum_{x=C-L}^{C} p(x) = P(C-L) \text{ occurrences/day} \tag{3-60}$$

where P(C-L) is the cumulative probability of generation outages as defined in equation 3-1.

If equations 3-56 and 3-60 are substituted into equation 3-54, then the frequency of capacity shortages, f, becomes

$$f = P(C-L) + \frac{1}{24} \sum_{k=1}^{24} F(C-L_k) \quad \text{occurrences/day} \qquad (3\text{-}61)$$

where L = daily peak load

L_k = hourly integrated load.

and, since the "period" referred to in equations 3-47 is now one day, the units of λ and μ are "1/days."

Equation 3-61 can then be summed over all of the days within a period of constant maintenance, and over all of the periods within a year to obtain

$$f \text{ (annual)} = \sum_{i=1}^{m} \sum_{j=1}^{n_i} \left[P_i(C_i - L_{i,j}) + \frac{1}{24} \sum_{k=1}^{24} F_i(C_i - L_{i,j,k}) \right] \quad \text{occurrences/year} \qquad (3\text{-}62)$$

where

$L_{i,j}$ = daily peak load on day j of period i

$L_{i,j,k}$ = hourly integrated load of hour k on day j of period i.

$P_i(x)$ = cumulative probability table for period i

and

$F_i(x)$ = cumulative frequency table for period i

where P(x) and F(x) are developed recursively in Appendix A.

Equation 3-53 defined the duration of outages on an annual basis as

$$d \text{ (annual)} = \frac{\text{HLOLE (annual)}}{f \text{ (annual)}} \quad \text{hours/occurrence} \qquad (3\text{-}53)$$

Substituting equations 3-19 and 3-62 produces

$$d \text{ (annual)} = \frac{\sum_{i=1}^{m} \sum_{j=1}^{n_i} \sum_{k=1}^{24} P_i(C_i - L_{i,j,k})}{\sum_{i=1}^{m} \sum_{j=1}^{n_i} \left[P_i(C_i - L_{i,j}) + \frac{1}{24} \sum_{k=1}^{24} F_i(C_i - L_{i,j,k}) \right]} \quad \text{hours/occurrence} \qquad (3\text{-}63)$$

Prior to Patton's work[3-2] using hourly loads, the load model used was a square wave, with the magnitude equal to the peak load for "e" hours and a magnitude of zero for "24-e" hours per day, (Ringlee and Wood),[3-8] where "e" was defined as the "effective duration of peak load." With this simplification in the load model equation 3-62 can be reduced to

$$f_e(\text{annual}) = \sum_{i=1}^{m} \sum_{j=1}^{n_i} [P_i(C_i-L_{i,j}) + \frac{e_{i,j}}{24} F_i(C_i-L_{i,j})] \text{ occurrence/year} \quad (3\text{-}64)$$

where

$e_{i,j}$ = effective duration of peak load for day j of period i

and the subscript on "f_e" is to denote that hourly loads are not being used. The equation for the duration can be likewise to simplified to

$$d_e(\text{annual}) = \frac{\sum_{i=1}^{m} \sum_{j=1}^{n_i} e_{i,j} P_i(C_i-L_{i,j})}{\sum_{j=1}^{m} \sum_{i=1}^{n_i} [P_i(C_i-L_{i,j}) + \frac{e_{i,j}}{24} F_i(C_i-L_{i,j,k})]} \text{ hours/occurrences} \quad (3\text{-}65)$$

The Multilevel Exposure Factor (MLEF) load model proposed by Billinton and Singh[3-5] was essentially an intermediate level of detail between the single effective duration of peak load and an actual hourly representation. In this model multiple loads, (generally four or five) were used to represent the daily load variations, with each load assigned a duration. Incorporating this logic into equation 3-64 would result in

$$f_{ek}(\text{annual}) = \sum_{i=1}^{m} \sum_{j=1}^{n} [P_i(C_i-L_{i,j}) + \frac{1}{24} \sum_{k=1}^{k_{i,j}} e_{i,j,k} F_i(C_i-L_{i,j,k})] \quad (3\text{-}66)$$

occurrence/year

where

$k_{i,j}$ = number of load levels being used for interval i, day j.

L_i = kth load level on day j of interval i.

$e_{i,j,k}$ = duration of load $L_{i,j,k}$

and

$$\sum_{k=1}^{k_{i,j}} e_{i,j,k} \leq 24 \text{ for all i and j.} \qquad (3\text{-}67)$$

If $k_{i,j} = 24$

and $e_{i,j,k} = 1$

for all i, j and k then equation 3-66 reduces to equation 3-62. Also, if $k_{i,j}=1$ then equation 3-66 reduces to equation 3-64.

If the load model is known in greater detail than just the hourly integrated loads then equation 3-66 can still be used. The load model for each day can be broken into hundreds of levels and their corresponding durations if this level of detail is required. In the limit, this approaches the Continuously Varying Exposure Factor (CVEF) load model discussed by Billinton and Singh. In practical applications, the load model should be broken down no finer than the step size used in constructing the probability and frequency tables.

Conclusion

Table 3.1 lists the algorithms used to calculate all of the generation indices discussed in this section. Although these formulas are useful for comparing the indices, many of them are rather time consuming to perform. Table 3.2 shows the more computationally efficient methods of determining the indices. These equations produce the same results as those presented in Table 3.1, but the terms have been rearranged in such a manner as to drastically reduce the computation time required. Table 3.3 lists all of the nomenclature used throughout this section.

TABLE 3.1

Analytical Formula of Indices

Index	Algorithm	Equation No.
% Reserve	$\frac{C-L}{L}$	(3-4)
% Reserve_C	$\frac{C-L}{C}$	(3-5)
Largest Units	$R = C-L > \sum_{i=1}^{N} C_i$ MW	(3-6)
LOLE	$\sum_{i=1}^{m} \sum_{j=1}^{n_i} \sum_{x=C_i-L_{i,j}}^{C_i} p_i(x)$ days/year	(3-12)
HLOLE	$\sum_{i=1}^{m} \sum_{j=1}^{n_i} \sum_{k=1}^{24} \sum_{x=C-L_{i,j,k}}^{C} p(x)$ hours/year	(3-16)
POPM	$1 - \sum_{x=C-L}^{C} p(x)$	(3-21)
Q	$\prod_{i=1}^{m} \prod_{j=1}^{n_i} (1 - \sum_{x=C_i-L_{i,j}}^{C_i} p_i(x))$	(3-24)
PLOL	$1 - \prod_{i=1}^{m} \prod_{j=1}^{n_i} (1 - \sum_{x=C_i-L_{i,j}}^{C_i} p_i(x))$	(3-26)
EENS	$\sum_{i=1}^{m} \sum_{j=1}^{n_i} \sum_{k=1}^{24} \sum_{x=C_i-L_{i,j,k}}^{C_i} (x-(C_i-L_{i,j,k}))p_i(x)$ MWh/year.	(3-30)

Index	Algorithm	Equation No.
XLOL	$\dfrac{\sum_{i=1}^{m}\sum_{j=1}^{n_i}\sum_{k=1}^{24}\sum_{x=C_i-L_{i,j,k}}^{C_i}(x-(C_i-L_{i,j,k}))p_i(x)}{\sum_{i=1}^{m}\sum_{j=1}^{n_i}\sum_{k=1}^{24}\sum_{x=C_i-L_{i,j,k}}^{C_i}p_i(x)}$ MW	(3-45)
f	$\sum_{i=1}^{m}\sum_{\text{all } x} p_i(x)\{(\rho_{+i}(x)-\rho_{-i}(x))P_{Li}(C-x)+F_{Li}(C-x)\}$ occurrences/year	(3-52)
d	$\dfrac{\sum_{i=1}^{m}\sum_{j=1}^{n_i}\sum_{k=1}^{24}\sum_{x=C_i-L_{i,j,k}}^{C_i}p_i(x) \text{ hours/occurences}}{\sum_{i=1}^{m}\sum_{\text{all } x} p_i(X)\{(\rho_{+i}(x)-\rho_{-i}(x))P_{Li}(C_i-x)+F_{Li}(C_i-x)\}}$	(3-53)

TABLE 3.2

Computational Equations for Indices

Index	Formula	Equation No.
% Reserve	$\frac{C-L}{L}$	(3-4)
% Reserve_C	$\frac{C-L}{C}$	(3-5)
Largest Units	$R = C-L \geq \sum_{i=1}^{N} C_i$	(3-6)
LOLE	$\sum_{i=1}^{m} \sum_{j=1}^{n_i} P_i(C_i-L_{i,j})$ days/year	(3-13)
HLOLE	$\sum_{i=1}^{m} \sum_{j=1}^{n_i} \sum_{k=1}^{24} P_i(C_i-L_{i,j,k})$ hours/year	(3-19)
POPM	$1 - P(C-L)$	(3-22)
Q	$\prod_{i=1}^{m} \prod_{j=1}^{n_i} (1-P_i(C_i-L_{i,j})$	(3-25)
PLOL	$1 - \prod_{i=1}^{m} \prod_{j=1}^{n_i} (1-P_i(C_i-L_{i,j}))$	(3-27)
EENS	$\Delta X \sum_{i=1}^{m} \sum_{j=1}^{n_i} \sum_{k=1}^{24} \sum_{x=C_i-L_{i,j,k}+1}^{C_i} P_i(x)$ MWH	(3-36)
XLOL	$\dfrac{\Delta X \sum_{i=1}^{m} \sum_{j=1}^{n_i} \sum_{k=1}^{24} \sum_{x=C_i-L_{i,j,k}+1}^{C_i} P_i(x)}{\sum_{i=1}^{m} \sum_{j=1}^{n_i} \sum_{k=1}^{24} P_i(C_i-L_{i,j,k})}$ MW	(3-46)

Index	Formula	Equation No.
f	$\sum_{i=1}^{m} \sum_{j=1}^{n_i} [P_i(C_i - L_{i,j}) + \frac{1}{24} \sum_{k=1}^{24} F_i(C_i - L_{i,j,k})]$ occurrences/year	(3-62)
d	$\dfrac{\sum_{i=1}^{m} \sum_{j=1}^{n_i} \sum_{k=1}^{24} P_i(C_i - L_{i,j,k})}{\sum_{i=1}^{m} \sum_{j=1}^{n_i} [P_i(C_i - L_{i,j}) + \frac{1}{24} \sum_{k=1}^{24} F_i(C_i - L_{i,j,k})]}$ hrs/occurrence	(3-63)

TABLE 3.3

NOMENCLATURE

c	=	capacity of a unit
C	=	total installed capacity not on planned maintenance (MW)
d	=	expected duration of capacity deficiencies
e	=	effective duration of peak load
f	=	expected frequency of capacity deficiencies
f(x)	=	frequency of the state of exactly x MW on outage
F(x)	=	frequency of the state of x MW or more on outage
$F_L(\ell)$	=	frequency of the state of load greater than or equal to ℓ MW
subscript i	=	variation of index for the maintenance periods in a year
subscript j	=	variation of index for the days with a period of constant maintenance
subscript k	=	variation of index for the hours in a day
K	=	number of load levels modeled within a day
L	=	total system load (MW)
m	=	number of maintenance periods in a year
n	=	number of days within a maintenance period
N	=	largest unit criteria
p(x)	=	probability of exactly x MW of capacity on outage
P(x)	=	cumulative probability of x or more MW of capacity on outage
$p_L(\ell)$	=	probability of a load of exactly ℓ MW
$P_L(\ell)$	=	cumulative probability of a load of ℓ MW or greater
r	=	forced outage rate of a unit
R	=	C-L = system reserve
λ	=	average forced outage occurrence rate of a unit=1/(mean time to failure)
μ	=	average forced outage restoral rate of unit = 1/(mean time to repair)
$\rho_+(x)$	=	effective departure rate from an exact capacity state x to states having less capacity on outage
$\rho_-(x)$	=	effective departure rate from an exact capacity state x to states having more capacity on outage
Σ	=	denotes a summation of terms. For example:

$$\sum_{i=1}^{3} a_i = a_1 + a_2 + a_3$$

π = denotes a product of terms. For example:

$$\prod_{i=1}^{3} a_i = a_1 * a_2 * a_3$$

3.2 REFINEMENTS OF CALCULATIONS

The remainder of this section discusses some of the refinements to the calculations and demonstrates the calculation of the indices on a small example system.

The reliability indices discussed in the previous pages can be refined to include:

1. Maintenance scheduling
2. Load uncertainty
3. Multi-state units
4. Margin states and emergency operating procedures
5. Energy limited generating capacity.
6. Intermittent generating devices.

These refinements, while producing significant numerical differences in the results, do not alter the basic interpretation of the indices.

Maintenance Scheduling - All of the equations for the calculation of the indices first determine the value of the index for various periods of constant available capacity, and then combine these values to obtain an annual number. Although certain methods may ignore the impact of maintenance, it is an extremely important factor which can cause significant changes in the numerical value of an index, particularly when maintenance is performed during the annual peak. This factor becomes increasingly important when there are a number of large, base load units on a system which sometimes require up to a couple of months for planned overhauls and refueling.

There are three basic ways in which maintenance is generally scheduled. They are

1. Levelized Reserve. This is the most common method used in reliability and production costing applications. The basic philosophy is to plan for equal megawatts of available reserves throughout the year.

2. Levelized Risk. This method, which was first proposed by Garver,[3-9] is similar to the method of levelizing reserves. The major difference is that instead of using actual loads and unit capacities it uses effective loads and unit effective capacities. This method accounts for not only the size of the units but also their forced outage rates. This then recognizes that there is a greater impact on the system when there are two 400 MW units scheduled for simultaneous maintenance than when a single 800 MW unit is removed, given that their forced outage rates are the same.

3. Actual Maintenance Practices. On an actual utility system the maintenance schedules are continually being revised to take into account other unit forced outages, the uncertainty of when the peak load will occur, the limitations on personnel available to perform the required maintenance and a number of other detailed factors. This is a very time consuming process and is usually not done in long range reliability studies.

Load Uncertainty - With the exception of POPM, all of the calculations presented so far have made an extremely important, and most times incorrect, assumption that all of the loads are known. Load uncertainty arises from two main areas.

1. Weather uncertainty. Portions of the load are very sensitive to weather conditions. Extremely cold or hot days will cause the heating or cooling loads on the system to rise sharply. Due to the uncertainty of long range weather forecasts these factors cannot be accounted for very far ahead of time.

2. Load Growth Uncertainty. The other major factor in load uncertainty is due to the difficulty in predicting the long range system load growth. This load growth is tied to the economic growth of an area, the success of conservation efforts, the number of conversions to electric heating and other factors which can be estimated, but not with a high degree of accuracy.

Once these two factors have been examined a combined load forecast uncertainty can be developed. This uncertainty is usually expressed as a probability of obtaining various annual peak loads for the year. An expected value of any index can then be determined by calculating the value of the index for all possible peak loads and weighing them by the corresponding probability of the peak load occurrence,

$$E(INDEX) = \int_{L_{min}}^{L_{max}} INDEX(L)\, p_L\, (L)dL \qquad (3\text{-}68)$$

where

INDEX(L)	=	value of INDEX for a peak load of L MW.
L_{min}	=	minimum expected value of annual peak.
L_{max}	=	maximum expected value of annual peak.
$p_L(L)$	=	exact probability of an annual peak of L MW.

Multi-State Units - The generating units on a system will generally exist in a number of possible capacity states other than completely up or down. Appendix A shows how this fact can be considered when constructing the probability outage table, P(x). Although some work has been done on the impact of partial outages on the frequency table, F(x), this factor is usually not considered due to the excessive amounts of data required to describe the transitions between the various capacity states. In practice either a capacity derating or an equivalent forced outage rate (or some combination of the two) is generally used to approximate the unit as having only two states. In any event, once the outage tables have been constructed, using either two state or multi-state unit representations, the indices are calculated in the same manner.

Margin States and Emergency Operating Procedures - The equations presented for the calculation of the indices examined the probability of the MW on outage, x, being greater than or equal to the installed reserves, R = C-L. This can be extended to the idea of "margin states" to identify the probability of being within 100 MW or 200 MW, etc. of an outage, or the probability of having an outage of greater than 100 MW or 200 MW, etc. The equations are modified to calculate the probability of the MW on outage being greater than R_i, where

$$R_i = C - L - M_i \qquad (3\text{-}69)$$

where M_i = positive or negative value of margin state.

In this way, for example, a program can calculate a value of LOLE for each margin state, so that the expected number of days the system is in a given margin state can be determined. In practice, the calculation of indices for a number of margin states can be performed for very little increase in the cost of the calculations. In most programs the bulk of the time is spent in processing data, scheduling maintenance and in constructing the probability and frequency tables. Once this has been accomplished, only an additional table look-up is required for each margin state for each load.

The Emergency Operating Procedure[3-10] (EOP) concept is an extension of the use of margin states. In EOP each margin state is identified as a particular operating action. Some actions will decrease the available reserves, for example, requiring that the system cover the two largest units for spinning reserve. Other actions will increase the available reserves thru emergency purchases, voltage reductions, customer appeals, etc. After all of the emergency actions have been taken, then load will be disconnected. In this way an index such as LOLE can estimate the number of voltage reductions or customer appeals and the actual number of times that load had to be interrupted. The EOP analysis can be used with any of the probabilistic indices. In practice the value of the margin states may be a fixed number of megawatts, a certain percentage of the load, or some function of the available units, depending upon what is being represented. The EOP analysis can be useful in developing a physical feel of how the system will react under different load forecasts and load shapes.

Energy Limited Generating Capacity - Energy limited generating capacity such as hydro electric power, energy storage or limited fuel units, can be modeled in two separate manners. In the first representation the amount of energy available is known, and is less than enough energy required to operate the unit at maximum output for an entire period. In the second category there is not enough energy available, and the available energy is described by a probability distribution.

Units in the first category are generally handled deterministically. They are scheduled so as to shave the highest loads as much as their energy will allow.

The modified loads are then used in a standard probabilistic evaluation with the remainder of the units on the system. When only the daily peaks are being examined, as in LOLE, it is often assumed that these units are available at full capacity at the time of daily peak.

The second category basically repeats the analysis performed in the first category for each expected value of energy availability. The expected value of the index is then determined by weighing the various results by their corresponding probability. When a number of these types of units are on the system the calculations can become quite time consuming and expensive. Therefore some type of lumping of units or other approximations are generally made.

Intermittent Generating Devices - In the past few years increased attention has been given to intermittent generating devices such as photovoltaic and wind generators. Many people feel that these devices should only be used in modifying the dispatch of the remainder of the system and that they have no effect on system reliability. Two approaches have been used, however, to quantify the system reliability impact of these devices. The first step in both of the approaches is to determine the hourly output of these units using meteorological data.

The first approach then uses this output to develop an equivalent forced outage rate which is in turn used to convolve the unit into the capacity outage table. This method, unfortunately, does not take into account the time-of-day dependence of the output, which therefore invalidates the results of the analysis. An extension of this method [3-11] groups the output first by months and then by time-of- day within the month. For each month this method then develops 24 sets of equivalent capacities and forced outage rates corresponding to the 24 hours in a day. This is the most exact, and most valid, analysis technique proposed. Unfortunately, this technique can cause an excessive increase in the amount of time required to perform a system reliability analysis.

The second approach which is used [3-12] is much simpler and produces results comparable to the exact analysis described above provided system penetration of the intermittent devices is small (less than 20%). This approach uses the intermittent output to modify the load, and then performs a standard reliability analysis on this modified load. This is similar to the approach commonly used for energy limited devices, as was described previously.

3.3 SUMMARY

This section has presented many of the commonly used analytical expressions for the determination of generation system reliability indices. These expressions are summarized in Table 3.1. The more computationally efficient algorithms which are actually used for the calculation of the indices were then derived from the basic analytical expressions. While many of these efficient algorithms have appeared in other references, the expressions for Expected Energy Not Supplied (EENS) and Frequency were developed under this project. These expressions represent considerable computer savings over the algorithms currently in use by the utility industry. All of the computational algorithms have been summarized in Table 3.2.

In addition to presenting the analytical and computational algorithms a detailed "hand" example is performed in Appendix B. A three unit system is analyzed for a one week period showing each step of the calculations from constructing the outage tables to determining the indices. This appendix will allow those readers unfamiliar with the methods to fully understand each phase of the calculations.

Appendix C completes the example of the calculations. A small time-sharing program is listed which will calculate the indices for a one week period. The results of this program are given for the peak week of the IEEE Reliability Test System [3-13]. Also shown is a method of estimating the amount of capacity required to improve the system reliability to a desired level.

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Section 4

COMPUTATION OF BULK TRANSMISSION RELIABILITY INDICES

4.1 INTRODUCTION

Transmission system reliability indices used by the power industry were introduced in Section 2. The objective of this section is to detail the computational techniques and analytical relationships necessary to compute the indices. The discussion of the indices is preceded by an overview of the failure effects analysis (FEA) commonly used for computation of the indices. The computation of both deterministic and probabilistic indices are discussed in the last portion of this section. An example of the transmission indices is given in Appendix D and calculation of combined generation transmission LOLE is illustrated in Appendix E.

In describing the indices, a common nomenclature is used. Important terms are defined with each equation the first time they are used, and a complete list of the nomenclature is included at the end of the section.

4.1.1 Background

The methods which have been proposed to study transmission reliability generally are procedures based on a failure effects analysis (FEA) similar to the approach shown in Figure 4.1. FEA methods have been chosen because of the lack of analytical methods to compute indices directly and the ability of the simulation methods which are used in the FEA to accurately model the behavior of the power system. Simulation methods allow the user to determine the level of accuracy desired and the assumptions pertinent to his particular application. In contrast, analytic methods, when available, would likely require assumptions to be made in order to make the mathematics tractable. The analytic approach would calculate the reliability indices without actually simulating the events. In concept, the index would be calculated based on the structure and properties of the elements directly. The major drawback to FEA methods is that the computational requirements are high, and therefore the number of contingencies which can be studied must be limited.

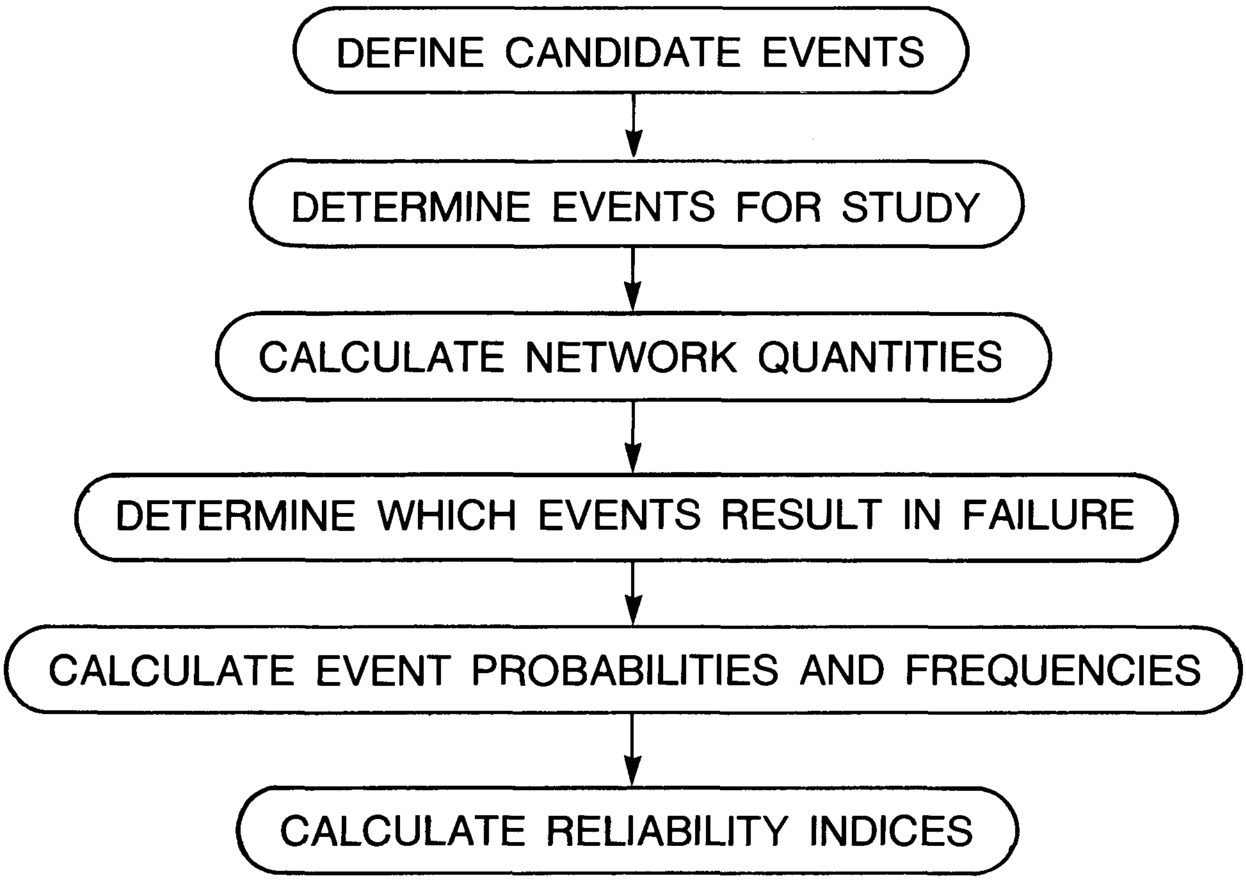

Figure 4.1 FEA of Power Systems

4.1.2 Index Requirements

The basic probability quantity used to calculate the probabilistic transmission indices is p_i, the probability the system is in the state of having specified lines out of service and the remainder available. Note the contrast between this approach and that used for generation system indices in Section 3. The generation indices use P(x), the probability that there are x MW or more on outage. This arises from the fact that the generation state capacity can be found by simply summing the capacities of the available units. This fact has allowed the recursive technique to be applied to the calculation of the cumulative state probabilities. However, in the transmission system it is not correct to sum the line capacities for the available lines to arrive at a measure of state capacity. This is due to the fact that the line flows are governed by Kirchoff's and Ohm's Laws and the line flows cannot be controlled independently of each other. It also follows that the effect on system capacity of adding or removing a line is dependent on the state of the other lines in the network. Therefore, no recursive method is possible for transmission networks, since a recursive method will only work if adding or subtracting a component will have an equal impact regardless of the state of the remaining components.

The second requirement for the index calculations is a determination of loss of load events and a measure of the load not supplied for these events. Again, it is interesting to contrast the transmission system calculations with the generation system. The load not supplied for a particular generation state can be found by subtracting the load demand from available generation. In the transmission system loss of load events and load not supplied must be calculated by solving a large set of simultaneous equations and by invoking specific rules on load shedding. This is necessary to determine when a transmission event results in loss of load and to allocate the loss of load to particular buses. The rules for determining the allocation can be as straightforward as curtailing the load equally at all buses or as complicated as the individual desires. However, some method must be specified and each contingency is evaluated separately. It is also necessary to evaluate each new load demand level separately to accurately assess load not supplied due to the fact that the distribution of the load to the buses may change as load level changes.

The data used in the solution of the network equations may differ in form from that used in the reliability calculations. The most important difference is in the load models. The reliability calculations tend to treat load as a MW quantity. However, in most power system computer programs, loads can be modeled as constant current, constant impedance, constant power or some combination. Thus, the MW value of the load is not always known beforehand.

Similarly, the generator outputs are not specified beforehand. Often generation buses are required to hold a given voltage level, thus modifying the generator output (within limits) based on the system conditions. Also, the use of swing buses in a power flow results in an uncertainty in the actual values for real and reactive power before the simulation is executed. This is a basic difference between the transmission reliability and generation reliability modeling. In generation reliability studies, the generators are treated as fixed MW sources rather than variable depending on the remainder of the network.

The load data most commonly used for the transmission reliability calculations is the system load in MW and the fraction of the system load at each bus. Although it is not necessarily input to computer programs in this form, the load must be apportioned to the buses. Additional load levels other than peak could be studied, but the fact that most methods would require the solution to be repeated entirely results in relatively few studies of alternate load levels. However, it should be recognized that there can be significant risk of loss of load at loads other than peak. The contingencies causing the loss of load could be different from those at peak load which suggests that a philosophy of treating the peak load as an "umbrella" case may not be valid.

4.2 FAILURE EFFECTS ANALYSIS (FEA)

Each of the steps in Figure 4.1 involve several intermediate calculations and can include several levels of detail to produce the desired results. This six step procedure is general in nature and applies to either a state enumeration or minimum cut set approach. The FEA method can be used for either steady state or dynamic power system analysis although the applications have used a steady state approach due to the additional computational burden of including dynamics. The FEA approach has the potential of being a very accurate and powerful tool for reliability analysis if advances are obtained in computer technology which will allow more complete analysis of the power system. In particular, array processors could result in power flow and stability calculations efficient enough to allow multiple runs at a reasonable cost. Each of the steps is described below.

4.2.1 Define Candidate Events

The first step in the FEA is to define the types of events which will be studied and evaluated in the later steps. The choice of the type of events which will be studied depends on the analysis technique which will be used for the system.

For example, if a power flow technique will be used to evaluate the states, then the candidate events will consist of transmission line and generating unit outages. If a stability calculation will be used to identify failures, then the candidate events would consist of faults and time dependent sequences of events on the power system. In summary, the type of events to be studied depends on the amount and detail of data available and the type of system model which will be used.

4.2.2 Determine Events for Study

This step involves determining which of the candidate events will actually be evaluated. Since the possible number of states is extremely large (2^n states for a system with n components), it has been found to be necessary to develop methods which will give useful indices without exhaustively enumerating all possible states. A simulation method, such as Monte Carlo simulation, can be used to select events for further study on a random basis. Other methods which apply only to steady state calculations have been suggested to rank possible outage states in order of severity in order to study only the most severe outage cases.

The ranking methods which have been proposed,[4-1, 4-2, 4-3, 4-4] are based on calculating a performance index for the system and then finding which contingencies will degrade the system performance index the most. Contingency ranking is not a problem unique to reliability analysis and it is anticipated that there will be additional developments in this area. The problem with these methods is that the index that is used to rank the contingencies is not a reliability index. The index used must be easily computed and the change in the index must be able to be found easily. Reliability indices do not fit this requirement. Therefore, the ranking is not necessarily in the proper order for use in reliability calculations.

Most commonly, the events to be studied are determined manually by the person using the program. The states are selected based on the user's knowledge of the power system, experience, and judgement. This is especially true for selection of cases involving a time simulation of the network since no automatic procedures are available.

4.2.3 Solve Network Equations

The method chosen for solving the power system equations is critical to the reliability procedure since both the accuracy and interpretation of the index are dependent on the network solution. In the most general case, the network equations are a set of complex, nonlinear, simultaneous equations which are driven by the values of generation, load and voltage at the buses. Since these bus quantities change continuously with time, the network results are also time varying. However, detailed power system models are rarely used for reliability calculations for several reasons.

The major deterent to modeling the power system in great detail is the excessive computer time required. Reliability calculations require analysis of hundreds or even thousands of states and therefore each solution must be reasonably efficient. As discussed earlier, each change in load level and each change in network topology must be evaluated separately. Determination of loss of load events requires nearly a complete solution of the power system equations for each new state. The resolution of this problem appears to be in the area of improved computer technology such as array processing.

Another problem with detailed modeling is a lack of meaningful data. Transmission data is not generally available in sufficient detail to justify the use of extremely accurate calculation procedures. Data could include information on component (relays, transformers, bus sections, breakers, etc.) connections, outage rates and dependencies between components.

A third difficulty with the use of detailed modeling is the interpretation of the results. Many of the calculations require human judgement to interpret results and it is not always possible to review the number of cases included in a reliability calculation to judge the results. This difficulty can be improved by automatic methods to summarize results, but the judgement of the user cannot be completely replaced.

The large variety of techniques available to perform the solution of the equations causes considerable confusion. All of the commonly used approaches have strengths and weaknesses which have been discussed at length in the literature (references in Stott-1974).[4-5] Most involve making assumptions, either in the solution or in the data input. The nonlinear (ac) power flow is the most common technique due to the wide availability and understanding of the method.

Since contingency analysis is the major part of the computing requirements for index calculations, the power system equations are often linearized for use in the calculation of the contingency power flow to reduce the calculations. The three methods usually used are the linear (dc) power flow, distribution factors power flow or fast decoupled power flow.[4-5]

Having performed the power flow calculations, several additional calculations can then be performed. Transfer limits between systems or areas of the system can be found from a series of power flow calculations. Another calculation available is the Load Supplying Capability (LSC)[4-6] of the network. Both of these procedures measure the capability of the power system to transfer power from generation to load considering static limits. Stability results methods are also used to compute transfer limits as limited by the dynamic performance of the network.

Particular Solution Methods

The network solution methods, listed below, have been described in detail in the references indicated. Therefore, no effort will be made here to discuss the details of the calculations.

(1) AC Power Flow (Stott, 1974)[4-5]
(2) DC Power Flow (Stott, 1974)[4-5]
(3) Distribution Factor Power Flow (Stott, 1974)[4-5]
(4) Capacitated Network Flows (Garver, 1970)[4-7]
(5) Transient Stability Calculation (Adibi, et al., 1974)[4-8]
(6) Dynamic Stability Calculation (Adibi, et al., 1974)[4-8]
(7) Probabilistic Power Flow (Dopazo, 1975)[4-9]

The method chosen must suit the index and application. For example, if an evaluation of the system impact of adding a new circuit is desired, then a large set of contingencies would likely be desired. In that case, an efficient algorithm such as the dc power flow, distribution factor power flow, or capacitated network flow could be used to allow analysis of a large number of cases. If the effect of a change on a smaller set of events is desired, then the ac power flow could be used. The stochastic power flow would be used if a study of load uncertainty or generator output uncertainty was required for a few outage cases. If the impact of a change on a particular event was required, then the stability methods could be utilized to provide a more precise estimate of the actual load not supplied due to a particular sequence of events.

The stability calculations would also be chosen if the time dependent behavior of the system was to be modeled. The particular stability technique chosen would depend on the length of time to be modeled and the detail required. In general, the longer the time period to be studied, the more detailed the calculation procedure will have to be. The time period is considered to be the window of any particular event. For example, a series of faults occurring over a 20 second to two minute period would require a dynamic stability calculation to be performed which would include the power plant response, protective system action, and load changes on the network. At this time, no one seems to be using a technique such as this for calculation of reliability indices. The application of these methods tends to be investigation of particular events rather than a reliability evaluation of the system.

Generation Dispatch

Prior to solving the system, a generation schedule to serve the load must be specified.[4-10] This generation schedule is then input into a network solution method to determine the flows on the transmission lines. The generation can be dispatched economically, some modification of an economic dispatch, or dispatched arbitrarily to serve the load and then if overloads occur, redispatched to alleviate overloads. Therefore, the initial generation schedule may be of little concern if it will be modified at a later time. However, this approach can run into trouble since an infeasible generation dispatch can lead to solution problems in the power flow calculation.

Another area often ignored is whether or not the new dispatch schedule obtained from redispatching will be feasible. While the base case may have no overloads and the outage case may have no overloads when the generation is redispatched, the trajectory between the two points may be infeasible. This case will then be characterized as a success state, where in reality, it will be a failure on the system.

The most popular method for redispatching generation is linear programming (LP) [see references in Stott-1978].[4-11] This approach is reliable, fast, flexible, easy to implement, and requires little computer storage. Several standard algorithms are available which can be implemented directly or modified to solve the particular problem. The LP approach is attractive for the network-constrained rescheduling since the MW flows can be linearized with good accuracy and the LP is computationally attractive.

The rescheduling problem can be formulated in two ways:

(1) Find a new schedule which relieves overloads independent of cost.

(2) Find a new schedule with no overloads (or minimum overloads) which is lowest cost.

The first approach is usually taken for reliability studies since cost of operation is not of primary concern in a reliability evaluation. Thus, the problem is stated as:

$$\text{Minimize } \Sigma\, \Delta P_i \qquad i = 1, \text{ number of buses}$$

where ΔP is the incremental change in real power at bus i. This change can be either a generation MW or a load MW if load shedding is required to alleviate overloads. Other actions such as HVDC control and phase shifter control can also be modeled as equivalent generation changes.

This objective is subject to three types of constraints:

(1) Power balance equation (load + losses = generation)
(2) Generation Limits
(3) Transmission Limits

The first constraint is usually handled implicitly, and the second set of constraints can be handled explicitly or implicitly in the LP code. The third set of constraints require a linearization of transmission flows. Those can be handled by several methods which are described elsewhere (Stott 1974).[4-5] The result is then a new generation schedule minimizing or alleviating network overloads. The amount of load shedding, if desired, will also be minimized.

4.2.4 Determine Which Events Result in Failure

In order to classify events as success or failure, it is convenient to formulate the problem recognizing the five operating states[4-12] which define the power system conditions. The five states, as shown in Figure 4.2, are the normal, alert, emergency, extreme emergency, and restorative states. The states are defined by system currents and voltages, which must not exceed maximum levels based on the equipment limitations and the power system operating guidelines.

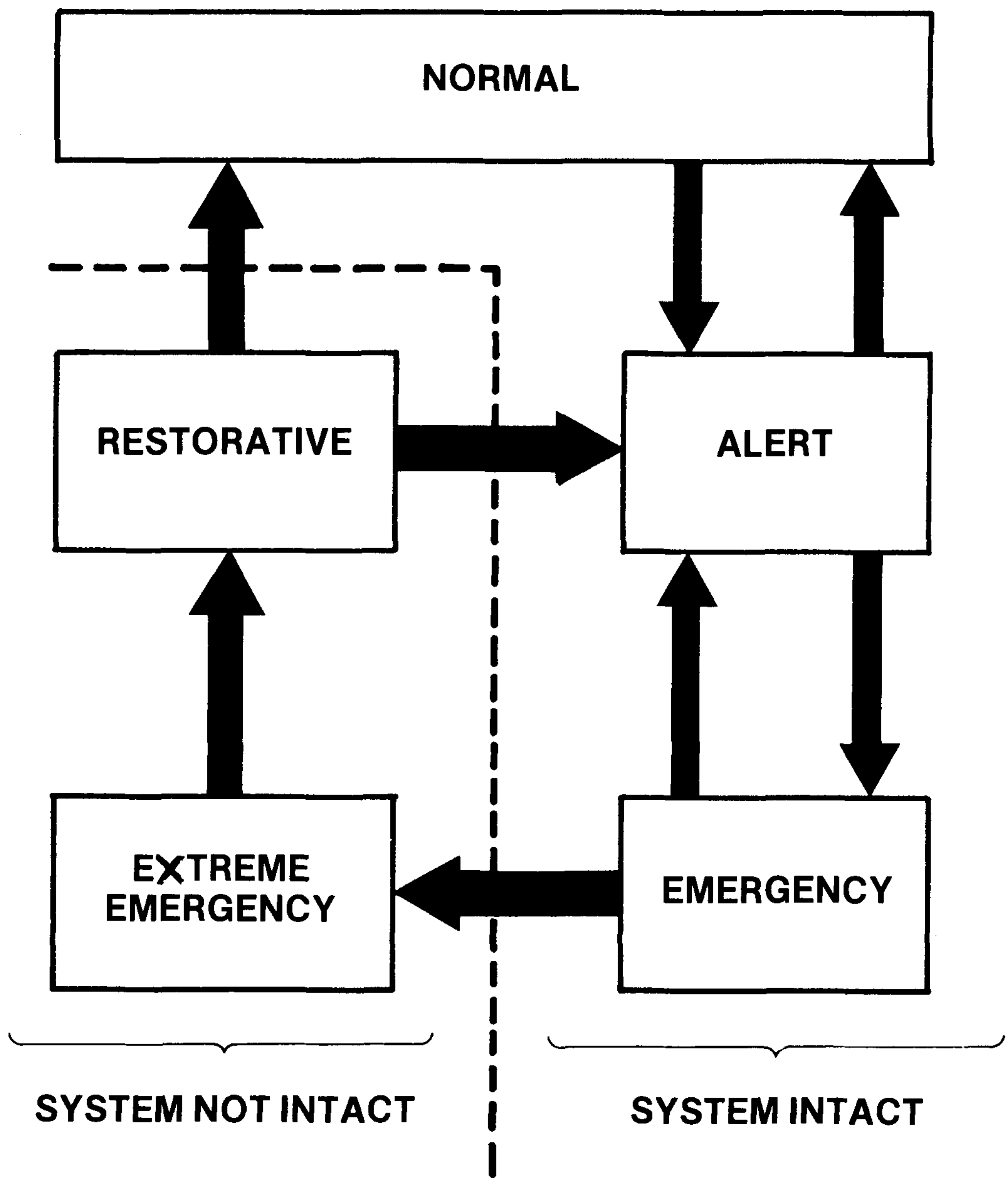

Figure 4.2 System Operating States

In the normal state all equipment and operating constraints are within limits indicating that the generation is adequate to supply the load (total demand), with no equipment overloaded. In the normal state there is sufficient margin such that the loss of any elements, specified by some criteria, will not result in a limit being violated. The particular criteria, such as all single elements, will depend on the planning and operating philosophy of a particular utility.

If a system enters a condition where the loss of any element in the criteria will result in a current or voltage violation, then the system state is in the alert state. The alert state is similar to the normal state in that all constraints are satisfied, but there is no longer sufficient margin to withstand an outage. The system can enter the alert state by the outage of equipment, by a change in generation schedule, or a growth in the system load. From the alert state, corrective actions can restore the system to the normal operating state.

If a contingency occurs or the generation and load changes before corrective action can be taken, the system will enter the emergency state. No load is curtailed in the emergency state, but equipment or operating constraints have been violated. If control measures are not taken in time to restore the system to the alert state, the system will transfer from the emergency state to the extreme emergency state.

In the extreme emergency state, the equipment and operating constraints are violated and load is not supplied. The extreme emergency state is the state in which there is load not supplied according to the definitions presented. However, it should be noted that this definition cannot be enforced rigidly. Since an appropriate corrective action from the emergency to alert state could be some action such as load shedding or a voltage reduction (which is actually a form of load shedding), the system would not strictly be in the extreme emergency state. If may be convenient to define an extreme emergency as uncontrolled load curtailment rather than all load curtailment. Figure 4.2 shows that when the systems makes the transition from emergency to extreme emergency, the system is no longer intact. This implies that islanding and/or load curtailment has occurred.

To transfer out of the extreme emergency state, the system must enter the restorative state to reconnect load and resynchronize the network. The loop can then be closed by either entering the alert state or the normal state. It is important to keep in mind that this is a conceptual diagram and the state definitions are imprecise and subject to judgement. However, the framework does provide a useful means of providing guidance in defining power system failure conditions and determining failure for reliability computations.

Since load is not supplied only in the extreme emergency state, it is the events that result in entering the extreme emergency state that tends to be of interest for reliability calculations. All five states are rarely modeled, but in concept the definition of reliability could be modified to include transitions to other states.

The preceding discussion involved using the concept of margin to define the power system states. Margin may be considered to be either a system or element quantity and can be based on either static or dynamic considerations. For example, the margin could be in terms of MW of load before an overload occurs. This could either be total system load or load at a particular location.

The margin can be defined in terms of steady state limits or in terms of margin to stability. The steady state margin would be calculated based on the limits of the lines, generators and transformers. The stability margin calculation includes generator dynamics and the limits on the amount of energy that system can absorb during a fault.

The margin concept could also be used to define additional power system indices. Indices based on margin could be defined and calculated by redefining failure. The new indices would not be reliability indices in the context of this report since they are not based on load not supplied, but would be a measure of the robustness and flexibility of the system.

4.2.5 Calculate Event Probabilities and Frequencies

In order to calculate the probabilistic indices, estimates of the event probabilities and frequencies are required. If sufficient data is available, the probabilities and frequencies of the events can be calculated to include all dependencies and correlations between events.[4-13] However, the assumption commonly made is that events are independent, thus simplifying the mathematics. It should be noted that this assumption may have a significant effect on the value of the indices. Assuming independence usually results in an estimate of probability which is low for the multiple outages. Since these outages tend to contribute heavily to the index, the reliability calculation is optimistic.

Line outage parameters are used to construct the probability and frequency parameters which are required. Two types of parameters are used.

Type 1. $P(A_i)$ - The probability that event A_i occurs, Event A_i consists only of lines out of service.

Type 2. p_i - The probability that the system is in the state of having lines defined by i in service and all others out of service.

The Type 1 probabilities are used to calculate indices if the minimum cut set approach is used and Type 2 probabilities are used with the state enumeration approach. [4-14]

4.2.6 Calculate Reliability Indices

The indices can now be calculated based on the results of the previous five steps of Figure 4.1. As discussed in Section 5, reliability indices are parameters of the stochastic process $\{Z(t)\}$. These parameters can be considered to be either deterministic or probabilistic. The deterministic indices record a maximum value associated with the process or the number of times an event occurs or other similar statistics. The key property of the deterministic indices is that they record information in the form of "raw" data and do not summarize the entire process.

In contrast, the probabilistic indices reflect the entire process and summarize characteristics of the process. The probabilistic indices characterize $\{Z(t)\}$ by the probability law associated with it.

For purposes of clarity the event $Z(t)$ will be abbreviated to Z for this section. In addition Z will be subscripted to identify particular events and buses at which there is loss of load. The indices will be presented as system indices, however the "system" can be defined to be one bus, an area or the entire system.

4.2.6.1 Deterministic Indices

Due to lack of significant data and a lack of computer programs for calculating probabilistic transmission indices, deterministic indices have been used to a greater extent. Several deterministic indices which have been used are discussed below. The indices are defined assuming that not all possible contingencies are evaluated. This approach was chosen since it is not feasible to study all combinations and the resulting equations are general.

Maximum Load Not Supplied. This index involves determining the greatest amount of MW not supplied due to the contingencies studied. Mathematically the index can be written as:

$$\text{Max Load Not Supplied} = \text{Max} \left[\sum_{k=1}^{NB} Z_{k1}, \sum_{k=1}^{NB} Z_{k2}, \ldots \sum_{k=1}^{NB} Z_{km} \right] \qquad (4\text{-}1)$$

where Z_{ki} = Load curtailed at bus k during transmission state i.

NB = Number of buses in system.

m = Number of contingencies studied.

The actual magnitude of the load curtailed depends on the solution method chosen. In order to avoid confusion, the type of calculation used and assumptions included must be stated. For example, it is important to specify if generation redispatch is considered in determining if the events result in loss of load. Some methods such as the Load Supplying Capability (LSC) method, described later in this section, allow this index to be calculated on a system basis, without explicitly requiring the summation of Z over the number of buses.

Maximum Energy Not Supplied. This index is similar to the previous one, except that energy rather than power is of interest.

$$\text{Max Energy Not Supplied} = \text{Max} \left[\sum^{NB} Z_{k1}D_{k1}, \sum_{k=1}^{NB} Z_{k2}D_{k2}, \ldots \sum_{k=1}^{NB} Z_{km}D_{km} \right] \qquad (4\text{-}2)$$

where D_{ki} = Duration of loss of load at bus k due to contingency i.

The computational requirements are the same as the previous index, except that the duration of outage is required. This is a very difficult quantity to calculate since a load model as a function of time for each bus is required. More readily available would be average or historical values for duration which could be used.

Minimum Load Supplying Capability [4-6]

The Load Supplying Capability (LSC) of a power system is defined as the maximum system load that the system can supply with no line overload. The LSC is calculated by varying the dispatch on the generators and raising the loads at each bus until no more power can be dispatched without overloading some transmission line. This point at which the transmission system is at the limit is termed the LSC of the system. A requirement for this calculation is that the manner in which the bus loads increase must be specified. The manner in which they increase will affect the result since the distribution of load will affect the line flows and thus the point at which the transmission becomes limiting.

The LSC can be calculated using standard optimization theory. The calculation can be written to be solved by a general purpose linear program (LP) code as:

$$\text{Maximize} \sum_{k=1}^{N} P_{gk}$$

Subject to:

Line Flows < Transmission Limits

Generator Outputs < Generator Limits

where P_{gk} = Real Power Generation at bus k

The line flow constraints can be modeled by either the non-linear or linearized power flow equations. In order to reduce the computer requirements, a linearized power flow calculation is used. It is in the line flow constraint equations that the distribution of loads is included in the calculation.

If the LSC calculation is repeated for each contingency studied, the results give an indication of the relative severity of the events and thus the reliability of the network. Mathematically this index can be expressed as:

$$\text{Min LSC} = \text{Min } [LSC_1, LSC_2, \ldots LSC_m] \tag{4-3}$$

where LSC_i = Load Supplying Capability for contingency i.

Minimum Simultaneous Interchange Capability [4-15]

The Simultaneous Interchange Capability (SIC) of a power system is defined as the maximum power that can be imported to a particular system or transferred between areas of a system for a given system state. This can be calculated using linear programming in a manner very similar to LSC. The main difference is that the load is fixed for the SIC calculation and the amount of power transferred from the interconnected utilities is maximized. If this is repeated for all contingencies of interest, the minimum SIC is a measure of the reliability of the system.

$$\text{Min SIC} = \text{Min}\ [\text{SIC}_1, \text{SIC}_2, \ldots \text{SIC}_m] \qquad (4\text{-}4)$$

where SIC_i = Simultaneous Interchange Capability for contingency i.

In order to actually calculate SIC, the LP problem is formulated as follows.

$$\text{Maximize} \sum_{i=1}^{I} P_{Ai}$$

Subject to

Line flows < Transmission Limits

$$\sum_{i=1}^{I} P_{Ai} + P_{gA} = \text{Load}$$

where

P_{Ai}	=	Power imported by A from system i
I	=	Number of interconnections
P_{gA}	=	Power Generated Internally by System A

The line flows are commonly modeled by the linearized network equations. The idea of this calculation is to reduce the power generated in one's own system in order to maximize the power transfer.

Maximum Line Flow

This index is an indication of the impact of contingencies on the power flow of a particular circuit. This index helps give a planner an indication of the size necessary for a new line. This index is written as:

$$\text{Max Flow on line } j = \text{Max}\ [\text{Flow}_{j1}, \text{Flow}_{j2}, \ldots \text{Flow}_{jm}] \qquad (4\text{-}5)$$

where Flow j_i = Flow on line j during contingency i

The flows can be calculated by any of the available power flow methods listed earlier. The decision as to method depends on accuracy required and modeling detail necessary. In addition it is useful to know how many contingencies resulted in a certain flow or greater for planning purposes.

4.2.6.2 Probabilistic Indices

Loss of Load Probability [4-13] - The loss of load probability for the transmission system (LOLP) measures the probability of loss of load due to transmission system outages. This calculation is usually performed at the peak load of the year.

$$LOLP = \sum_{i \varepsilon L} p_i \quad \text{days/day} \tag{4-6}$$

where p_i = Probability that the transmission system is in state i.

$i \varepsilon L$ = All states i resulting in a Loss of Load event L.

Conceptually the LOLP calculation can be extended to LOLE for any set of loads or distribution of loads by using the equation (4-7). As discussed in Section 3, the load model assumed does not change the form of the equation.

$$LOLE = \sum_{k=1}^{NL} \sum_{i,k \varepsilon L} p_i \quad \text{days/day} \tag{4-7}$$

where NL = Number of loads to be studied.

$i,k \varepsilon L$ = All transmission states i at load k resulting in a loss of load event L.

This period calculation would require a significant amount of computation if the system quantities had to be completely recalculated. However, by making the assumption that the load at each bus remains a constant fraction of system load, the LSC procedure described earlier could be used in this calculation. The difference between LSC and system load represents load not supplied in a manner similar to installed capacity minus load in a generation reliability analysis. Therefore, with this assumption, the computational burden is reduced to a point where the calculation is feasible for multiple loads. By adopting a linear model of the flows (dc power flow), the calculations are further reduced allowing a greater number of events and loads to be included.

The calculation can also be entered to provide HLOLE, a basic index, for the transmission network. While the HLOLE calculation is not feasible due to computational realities, the index is included for completeness.

$$HLOLE = \sum_{k=1}^{8760} \sum_{i,k \varepsilon L} p_i \text{ hours/year} \quad (4\text{-}8)$$

Frequency of Loss of Load (FLOL) [4-16] - The frequency of loss of load due to transmission outage states can be defined as:

$$FLOL = \sum_{i \varepsilon L} p_i \sum_{j \varepsilon S} \lambda_{ij} \quad (4\text{-}9)$$

where $j \varepsilon S$ = all states j resulting in no loss of load on the system (success).

λ_{ij} = transition rate from failure state i to success state j.

Expected Energy Not Supplied (EENS) [4-17]

The EENS is defined as the expected energy not served due to transmission system outages. For a given load level the unsupplied energy is the amount of shortage for each outage times the probability of the outage, summed for all outages.

$$EENS = \sum_{i \varepsilon L} Z_i \, p_i \text{ MwH} \quad (4\text{-}10)$$

This quantity can then be summed over all desired loads to find the unserved energy per period.

$$EENS = \sum_{k=1}^{NL} \sum_{i,k \varepsilon L_k} Z_i \, p_i \text{ Mwh/period} \quad (4\text{-}11)$$

The calculation and interpretation of the index are related similarly to the previous examples. The procedure for determining the magnitude of the loss of load (Z_i) dictates the interpretation. If power system dynamics were included in the simulation, then the index would have a dynamic significance. Here also, the LSC procedure can be used to make the mathematics tractable.

Bulk Power Interruption Index (BPII) [4-17]

The BPII is similar in concept to the Load Interruption Index used for distribution reliability analysis. This index is the ratio of total load not supplied to annual peak load.

$$BPII = \frac{1}{L_{MAX}} \sum_{k=1}^{NL} \left(\sum_{i,k \epsilon L} p_i \sum_{j \epsilon S} \lambda_{ij} \right) Z_i \quad MW/MW\text{-}yr \qquad (4\text{-}12)$$

where L_{MAX} = peak load of year

The calculation of the BPII requires that enough load levels be studied to adequately define the reliability of the system.

Bulk Power Energy Curtailment Index (BPECI) [4-17]

The BPECI is an extension of the BPII. The BPECI relates annual energy not supplied to peak load. This can be calculated using the EENS.

$$BPECI = \frac{EENS}{L_{MAX}}$$

$$= \frac{1}{L_{MAX}} \sum_{k=1}^{NL} \quad S\Sigma_{i,k \epsilon L} \; Z_i \, p_i \quad MwH/MW\text{-}yr \qquad (4\text{-}13)$$

4.3 EXAMPLE CALCULATIONS

An example of transmission reliability indices is presented in Appendix D. The indices are calculated on a 6 bus system for a 5 day load model. The calculations are based on single transmission contingencies. The FEA procedure is followed for the calculations. The LSC is used to determine failure events and independence is assumed for the probability calculations.

Of particular interest are the probabilistic indices since they are not generally used within the industry. It must be remembered that the indices are based on single contingencies. The probability for the single contingencies adds to .933. In other words, .07 of the probability is not considered and therefore an error is present in the calculations. This error will be discussed later. The LOLP for the transmission system is shown to be .0997 which represents the risk at the peak day. All the risk results from the fact that 5 line outage cases result in failure. For purposes of these calculations failure is defined as events which result in loss of load. The loss of load is based on the LSC calculations which has been described previously. Extending the calculation to LOLE for the 5 day load period results in an LOLE of 1.409.

The frequency calculation shown in Appendix D is much more difficult. Even for this small system the calculations are very cumbersome and are only calculated for the peak load of the period. The ease of this calculation is also misleading because with only single line outages considered the calculation complexity is reduced because of the low number of possible transitions. The frequency is .012 occurrences/hour using the peak load only.

Using the LSC to approximate load not supplied the EENS index is calculated. The results are 84.06 MW hours for the 5 loads. The bulk power interruption index and bulk power energy interruption index are also calculated as ratios of other indices.

As previously mentioned, since the probabilities of the events considered did not sum to 1.0, there is an error present in the calculation. The calculation as formulated will result in an optimistic value of LOLE. For example, the LOLE of 1.4 assumes that all events not considered are success events. As shown in the appendix the opposite assumption, that all events not considered are failures, can also be calculated. In this case the worst possible LOLE is 1.74 days per 5 days. Thus it is possible to bound the true answer.

In Appendix E the LOLE is calculated for the combined transmission-generation system. The purpose of this calculation is to illustrate how the LOLE will change when calculated for the entire system as compared to when it is calculated for the generation and transmission systems individually. The principle result of the appendix shows that LOLE can be calculated for the combined system under a variety of assumptions of outage cases to study. As shown in the appendix the bounds between the optimistic, pesimistic estimates of LOLE narrow as more events are considered in the calculation. It is very interesting to compare the results of the combined calculation to those of the separate calculations. The results indicate that it may not be possible to calculate the reliability of the system independently and then combine LOLE results to determine a system index. The reason is that states are misclassified when one or the other of the generation or transmission systems is assumed to be perfect.

4.4 DISTRIBUTION RELIABILITY [4-13]

The computation of the reliability indices for distribution systems can be expressed in terms similar to that for transmission systems. The FEA approach can be applied to the distribution system in a manner very similar to that for transmission networks. The primary difference between transmission and distribution systems is size. Distribution systems are less complex and contain fewer elements. This fact allows for a more complete analysis of the system with more of the possible outage states evaluated. In addition, since distribution systems tend to be primarily radial, much of the analysis can be done using continuity checks without having to solve the equations describing the network performance. This will reduce the computational requirements significantly; thus making a more complete analysis feasible from a computer resources standpoint.

The same indices can also be defined. As previously discussed, the indices are based on an indication of loss of load and load not supplied. Therefore, the definition of failure in the distribution system is compatible with the remainder of the power system. The calculation of the event probabilities and frequencies is often more detailed for distribution system analysis. The components of the system are usually defined in greater detail than for transmission system calculations. For example, devices such as breakers, fuses, reclosers, sectionalizers and disconnect switches are identified and the failure modes associated with these devices are identified.

The unique characteristic of distribution system reliability calculations is that failures can be related to specific customers. This allows customer indices, such as described in Section 2, to be calculated. In addition, a great variety of modeling detail is possible when the indices are computed. Weather models and switching errors, for example, can be implemented in the calculation of the indices. These considerations lead to a requirement that the models for actual computation be quite sophisticated.

4.5 SUMMARY

This section has presented a conceptual method for calculating transmission reliability indices. The method is based on a failure effects analysis which provides the necessary information for the actual index calculation. The advantage of the FEA approach is that it is flexible enough to accommodate a variety of assumptions and varying degrees of accuracy. In concept all the indices calculated could be based on either static or dynamic considerations since the index computations are based on an indication of loss of load events and measures of load not supplied rather than quantities related to particular solution methods. The actual index calculations are illustrated on a sample system to demonstrate how the computational equations are used to arrive at actual numerical results.

Of the indices presented, only the deterministic indices are commonly calculated. A review of the NERC publications demonstrates that the deterministic indices are the easiest to explain and calculate using tools available today. The probabilistic indices which are compatable with existing generation indices will most likely gain acceptance as programs become available and results can be calibrated.

Table 4.1

Computational Equations for Indices

Index	Formula	Equation No.
Max Load Not Supplied	$= \text{Max} \left[\sum_{k=1}^{NB} Z_{k1}, \sum_{k=1}^{NB} Z_{k2}, \ldots \sum_{k=1}^{NB} Z_{km} \right]$	(4-1)
Max Energy Not Supplied	$= \text{Max} \left[\sum_{k=1} Z_{k1} D_{k1}, \sum_{k=1} Z_{k2} D_{k2}, \ldots \sum_{k=1} Z_{km} D_{km} \right]$	(4-2)
Min LSC	$= \text{Min} [LSC_1, LSC_2, \ldots LSC_m]$	(4-3)
Min SIC	$= \text{Min} [SIC_1, SIC_2, \ldots SIC_m]$	(4-4)
Max Flow on line j	$= \text{Max} [Flow_{j1}, Flow_{j2}, \ldots Flow_{jm}]$	(4-5)
LOLP	$= \sum_{i \& L} p_i$ days/day	(4-6)
LOLE	$= \sum_{k=1}^{NL} \sum_{i,k \epsilon L} p_i$ days/period	(4-7)
HLOLE	$= \sum_{k=1}^{8760} \sum_{i,k \epsilon L} p_i$ hours/year	(4-8)
FLOL	$= \sum_{i \epsilon L} p_i \sum_{j \epsilon S} \lambda_{ij}$	(4-9)
EENS	$= \sum_{i \epsilon L} Z_i p_i$ MwH	(4-10)
EENS	$= \sum_{k=1}^{NL} \sum_{i,k \epsilon L} Z_i p_i$ MWh/period	(4-11)
BPII	$= \frac{1}{L_{MAX}} \sum_{k=1}^{NL} \left(\sum_{i,k e L} p_i \sum \lambda_{ij} \right) L_i$ MW/MW-yhr	(4-12)
BPECI	$= \frac{EENS}{L_{MAX}}$	(4-13)
	$= \frac{1}{L_{MAX}} \sum_{k=1}^{NL} \sum_{i,k \epsilon L} Z_i p_i$ MWh/MW-yr	

Table 4.2

Nomenclature

Z_{ki}	=	Load curtailed at bus k during transmission state i.
NB	=	Number of buses in system.
m	=	Number of contingencies studied.
D_{ki}	=	Duration of loss of load at bus k due to contingency i.
LSC_i	=	Load Supplying Capability for contingency i.
SIC_i	=	Simultaneous Interchange Capability for contingency i.
$Flow_{ji}$	=	Flow on line j during contingency i.
p_i	=	Probability that the transmission system is in state i.
NL	=	Number of loads to be studied.
$i \epsilon L$	=	All states i resulting in loss of load event L.
$i,k \epsilon L$	=	All transmission states i at load k resulting in a loss of load event L.
$j \epsilon S$	=	All states j resulting in no loss of load on the system (success).
λ_{ij}	=	Transition rate from failure state i to success state j.
L_{max}	=	Peak load of year.

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Section 5

PROBABILITY MODELS FOR POWER SYSTEM RELIABILITY INDICES

5.1 INTRODUCTION

One important objective of this project is to express each of the reliability indices in analytical form and to illustrate structural relationships between the indices in analytical terms. This objective is addressed, from different viewpoints, in several sections of this report.

In this section, the analytical statement of indices is accomplished by formulating definitions of power system reliability indices in terms of probability laws of random phenomena. The definitions are general in the sense that they do not depend on any particular probability law, such as the exponential distribution; and they are not restricted to any particular portion of the power system, such as generation. The second part of the objective, structural relationships between indices, is achieved in this section by basing all reliability indices on a single random phenomenon, the load (demand) not supplied at any point in time, denoted $Z(t)$.

Load not supplied is a numerical-valued quantity (MW), whose observed value is governed by probability laws. In probability theory, such random phenomena are modeled as random variables. Furthermore, the observed value of $Z(t)$ evolves with time, again in a random manner. A random variable whose observed value is a function of time is called a stochastic process. Thus, from a probability viewpoint, $Z(t)$ is a random variable, and the collection (set) of random variables $\{Z(t);t>0\}$ is a stochastic process. NOTE: In subsequent discussion, the more compact notation $\{Z(t)\}$ is frequently used to denote the stochastic process $\{Z(t);t>0\}$. Similar notation is used for other stochastic processes. In such cases, it is always understood that the possible range for t is all positive real numbers t.

In this section, the most commonly used power system reliability indices are defined in terms of the probability laws of random variables which are all derived from the stochastic process $\{Z(t);t>0\}$. Several results are obtained from this approach. First, the stochastic process $\{Z(t)\}$ provides a unified mathematical structure for power system reliability indices. Second, it is possible to show mathematical relationships between indices. Third, the indices can be organized according to the type of information required about the probability law of the stochastic process $\{Z(t)\}$ in order to calculate each index. Fourth, the mathematical relationships provide a basis for uniform nomenclature and mathematical symbols to describe the various indices. Finally, a basis for assessing the need for new reliability indices is provided.

5.1.1 Reliability Indicators

The basic point of view in this discussion is that reliability indices are parameters (constants) which describe some characteristic (such as expected value) of a probability law. Probability laws, in turn, are mathematical models for random phenomena. Therefore, the first step in applying probability theory to formulate reliability indices is to define the empirical observations (random phenomena) which are to be modeled. In this section, these empirical observations are called reliability indicators. This is a fundamental term which is used throughout this section.

A reliability indicator is an empirical (observable) characteristic of a power system (like load not supplied) which is important for describing the "reliability" of the system. Roughly speaking, reliability is the degree to which a power system successfully supplies load demand. However, no attempt is made here to precisely define reliability itself. Rather, in this discussion, a characteristic is "important" if its probability law forms the basis for one or more reliability indices. Therefore, the definition of a reliability indicator is:

> Reliability Indicator - A characteristic of a power system whose probability law forms the basis for one or more reliability indices.

With this definition, a reliability index can be defined as:

> Reliability Index - A parameter of the probability law of a reliability indicator.

Thus, the basic viewpoint in this section can be restated as the following premise:

> Every reliability index implies the existence of one or more reliability indicators, whose probability laws form the basis for the reliability index.

The explicit recognition of the empirical characteristics (reliability indicators) which underlie each of the commonly used power system reliability indices is considered to be one of the most important results of this section. This recognition emphasizes that the reliability indicators are the fundamental characteristics which describe the observed reliability of a system. Reliability indices do not in general completely specify the probability law of the associated reliability indicator. Thus, one potential area for new indices is to consider additional parameters of the probability laws of the reliability indicators now in use.

A second important result of this section is the organization of reliability indicators in a logical framework based on how the reliability indicator is observed. Reliability indicators are classified into three fundamental categories - point reliability indicators, interval reliability indicators, and duration reliability indicators. A point reliability indicator is based only on the condition of the system at the observation time, whereas an interval reliability indicator is based on the cumulative performance of the system over a period of time which terminates at the observation time. Load not supplied at time t is an example of a point reliability indicator, while energy not supplied over (0,t) is an example of an interval reliability indicator. Duration reliability indicators are based on the duration of time between specified occurrences, such as the duration of time between beginning and end of a period involving loss of load. Point and interval indicators are both defined for every time t, but are distinguished by the amount of "history" which is considered. By contrast, duration reliability indicators are defined only at the point where the specified event occurs.

The distinction between three categories of reliability indicators is important because subsequent discussion shows that existing computation methods (such as those described in Section 3) are actually methods for calculation of the probability laws for the point reliability indicators and a single parameter (expectation) of the duration reliability indicators. By contrast, the reliability indices most commonly used in planning are in fact parameters (specifically, the expectation) of the probability laws of interval reliability indicators. Fortunately, the expectation of the interval reliability indicators can be computed in terms of the expectation of the point and duration reliability indicators. However, other characteristics of the probability laws of the interval reliability indicators, such as higher moments or percentiles, cannot be derived solely from knowledge of the corresponding characteristics for the probability laws of the point and duration reliability indicators.

5.1.2 Reliability Indices

The second step in application of probability theory to the definition of reliability indices is to specify probability laws for the reliability indicators. The reliability indices are developed as an integral part of this step because reliability indices are regarded here as parameters of the probability law of one, or more, of the reliability indicators. Each of the reliability indicators gives rise to at least one reliability index, namely the expectation (expected value) of the numerical value determined by the reliability indicator (or the probability of the event defined by the reliability indicator). Additional reliability indices can be derived by considering some function of the expectation reliability indices.

A summary of the probability concepts needed to define the reliability indices is given in Appendix F. The discussion of probability theory is necessarily compact. For those wishing further details on the probability concepts given, the two texts by Parzen (5-1)(5-2) are recommended. These two texts in fact provided the motivation for many of the ideas presented, and references to specific pages in these texts are cited at several points in the discussion. For example, Chapter 1 of Reference (5-1) gives a discussion of the notion of a random phenomenon and of the view that probability theory is the study of mathematical models for random phenomena. This chapter provided the motivation for the idea of a reliability indicator as a random phenomenon.

The remainder of Section 5 is organized according to the two major topics, reliability indicators and reliability indices. Each major topic is further discussed in terms of point, interval, and duration reliability indicators. Sections 5.2 through 5.4 define the reliability indicators. Point reliability indicators are defined in Section 5.2, interval reliability indicators are defined in Section 5.3, and duration reliability indicators are defined in Section 5.4.

Sections 5.5 through 5.9 discuss the probability laws, and associated reliability indices, for each reliability indicator. Section 5.5 gives an overview of the reliability index discussion, and Section 5.9 gives a summary of the reliability indices and the relationships between indices. Sections 5.6, 5.7, and 5.8 discuss probability laws for point, duration, and interval reliability indicators, respectively. Finally, Section 5.10 discusses possible new reliability indices from the viewpoint of the framework constructed in preceeding sections.

5.2 POINT RELIABILITY INDICATORS

This section defines two reliability indicators in terms of the system condition at a specific time, without regard to its condition at previous times. Section 5.2.1 defines "load not supplied". This is the most fundamental reliability indicator, because all the remaining reliability indicators can be derived from observation of load not supplied through time. Section 5.2.2 defines the second point reliability indicator, "loss of load", as the event that load not supplied is greater than zero. The distinction between load not supplied (a numerical value) and loss of load (an event) is a key concept, upon which all of the remaining discussion is built. The physical distinction between the two point reliability indicators is illustrated in Section 5.2.3 by means of a conceptual instrument called a "point reliability status monitor".

Since loss of load is an event, it is inherently non-numerical. In subsequent development of interval and duration reliability indicators and reliability indices, it is convenient to have a numerical representation of the loss of load event. This is accomplished by defining the "indicator function of loss of load" in Section 5.2.4. The indicator function is not physically a distinct reliability indicator from the loss of load event, since both indicators describe the same phenomenon. However, the indicator function plays a central role in defining the interval and duration reliability indicators which are based on loss of load, and in deriving the expectation of these reliability indicators.

Section 5.2.5 gives a discussion of terminology and symbols. The names of the two point reliability indicators provide a basis for a unified structure of names for all other reliability indicators as well as for all the reliability indices.

5.2.1 Load not supplied

The objective in a power system is to supply load demand. If load demand is completely satisfied, the system is successful. If load demand is not completely satisfied, the system is failed. The amount (magnitude) of load demand which is not supplied at time t is called load not supplied, denoted here by Z(t). Thus,

$$Z(t) \equiv \text{Load not supplied, MW}$$

The observed quantity is power, denoted by Z. The symbol t denotes the specific time at which the observation is made, relative to a reference time t=0. The reference time t=0 is a mathematical reference. In applications, t=0 can be any point in real (calendar) time-past, present, or future. For this discussion the units for power are assumed to be megawatts (MW) and the units for time are hours. Thus, the statement that Z(10)=100 means that Z=100 MW at t=10 hours.

Load not supplied is a reliability indicator, because the probability law of Z(t) forms the basis for reliability indices. Such indices are discussed in Section 5.6. Mathematically, Z(t) is a random variable, and the collection of random variables $\{Z(t);t>0\}$ is a stochastic process. A particular observation of Z(t) over time can be displayed graphically as shown in Figure 5.1. Such a graph is called a sample record (or sample function).

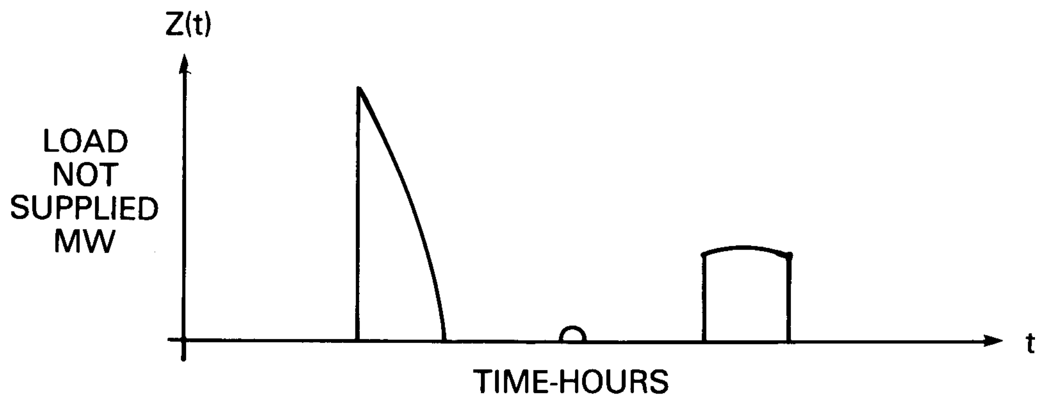

Figure 5.1 Sample record of {Z(t)}.

During periods when load demand is fully supplied, $Z(t)$ is equal to zero. Therefore, while $Z(t)$ measures the magnitude of failure, it does not measure the magnitude of success. Accordingly, it might seem more appropriate to use the term "unreliability indices" rather than "reliability indices" to describe quantities based on the probability law of $Z(t)$ or other random variables derived from $\{Z(t)\}$. However, in keeping with existing practice, the term reliability indices is used in this report.

Margin versus load not supplied

A more general approach would be to define "margin" rather than load not supplied as the basic random phenomenon, where margin is defined as

$$M(t) = \text{Load Supply Capability} - \text{Load Demand}$$

Load not supplied could then be defined in terms of margin by

$$\begin{aligned} Z(t) &\equiv 0 \quad , \quad M(t)>0 \\ &\equiv -M(t), \quad M(t)\leq 0 \end{aligned}$$

That is, $Z(t)$ is the amount of "negative margin", expressed as a positive number.

The advantage of defining margin as the basic reliability indicator is that margin measures magnitude of success as well as magnitude of failure. Further, computational models for reliability generally can be described in terms of submodels for load supply capability and load demand. However, it turns out that all the reliability indices discussed in this section can be based on $Z(t)$. Therefore, it suffices to take $Z(t)$ as the basic reliability indicator in this discussion.

5.2.2 Loss of load

The random variable $Z(t)$ measures the magnitude of load not supplied. In order to classify the system as success or failure, it is actually only necessary to know if $Z(t)$ is greater than zero. Thus, the interval $[Z(t)>0]$ constitutes "failure". In power system reliability evaluation, the term "loss of load" is often used to describe this interval. Mathematically, the interval $[Z(t)>0]$ is an event defined by the random variable $Z(t)$. The symbol L is used in this section to denote this event. Thus

$L \equiv [Z(t) > 0]$

$\quad = \text{Loss of load}$

The relation between event L and the sample record of {Z(t)} is shown graphically in Figure 5.2.

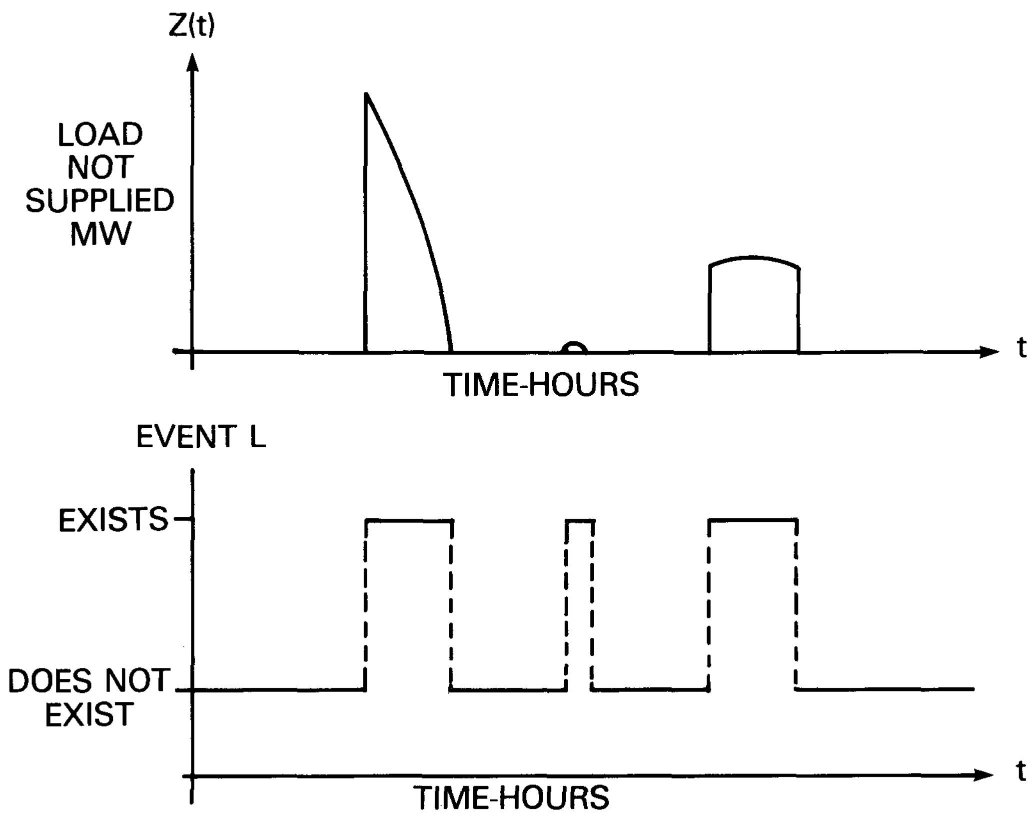

Figure 5.2 Relation between event L and sample record of {Z(t)}.

Reference to Figure 5.2 illustrates that while the graph of event L versus time can be constructed from the sample record of {Z(t)}, it is not necessary to know the numerical value of Z(t) in order to know that event L exists. It is only necessary to know that Z(t)>0. This is a very important point, because as a practical matter it may be much more difficult to determine the value of Z(t) than to determine that Z(t)>0. This is particularly true in the transmission system, where the amount of load not supplied (curtailed) depends on the "rules" assumed for supplying load to each load supply point (bus). Event L is defined in terms of Z(t) in this section in order to provide a unified structure of reliability indicators and reliability indices. It would be possible to start with event L itself as the basic reliability indicator. All the reliability indicators based on event L could then still be defined in the same way. Most of the reliability indicators defined subsequently are in fact based on event L. The only exception is the interval reliability indicator, energy not supplied.

5.2.3 Point reliability status monitor

Two basic ways to describe the reliability status at a point in time have been described.

Z(t) - load not supplied (MW)
L - loss of load (event)

One way to visualize the physical distinction between these two reliability indicators is to imagine an instrument panel which monitors the status of each indicator as shown in Figure 5.3.

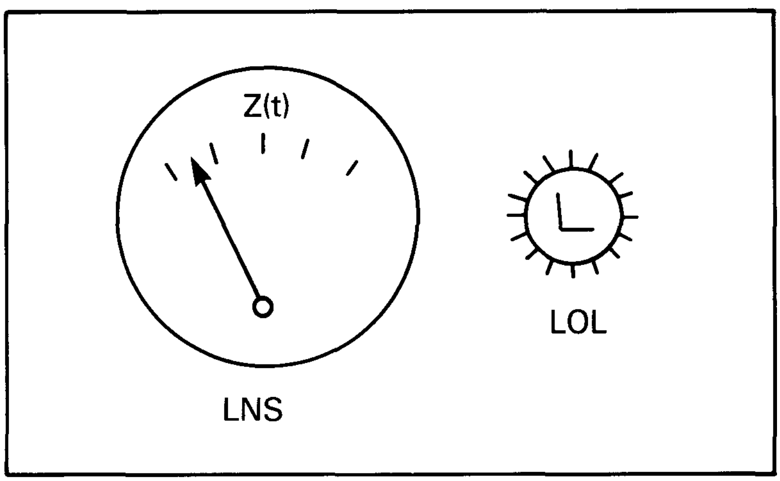

Figure 5.3 Point Reliability Status Monitor

The panel has two indicators. The LNS meter indicates the value of Z(t). A meter is needed because Z(t) is the magnitude of load not supplied. By contrast, whether or not event L exists can be indicated by a light, called the LOL light. Since the status monitor shows the reliability status of the system at a single point in time, it is called a point reliability status monitor.

5.2.4 Indicator function of loss of load

Several additional reliability indicators can be derived from observation of Z(t) through time. Some of these reliability indicators are based only on event L. That is, the magnitude of Z(t) is not considered. In order to define such reliability indicators, it is convenient to first define a function I(t) as follows,

$$
\begin{aligned}
I(t) &\equiv 0, \quad \text{if event L does not exist at time t} \\
&\equiv 1, \quad \text{if event L does exist at time t}
\end{aligned}
$$

Since I(t)=1 means that event L exists while I(t)=0 means that event L does not exist, the function I(t) is called the <u>indicator function of loss of load</u>. A similar function can be defined for any other event. However, in the present discussion, only the indicator function of L is needed, and hence I(t) is subsequently called simply the "indicator function" with the understanding that it is the indicator function of loss of load. The indicator function I(t) is a random variable and {I(t); t>0} is a stochastic process.

The indicator function is related to Z(t) as follows,

$$
\begin{aligned}
I(t) &= 0, \quad Z(t) = 0 \\
&= 1, \quad Z(t) > 0
\end{aligned}
$$

The sample record of {I(t)} can therefore be constructed from the sample record for {Z(t)}. This is illustrated in Figure 5.4.

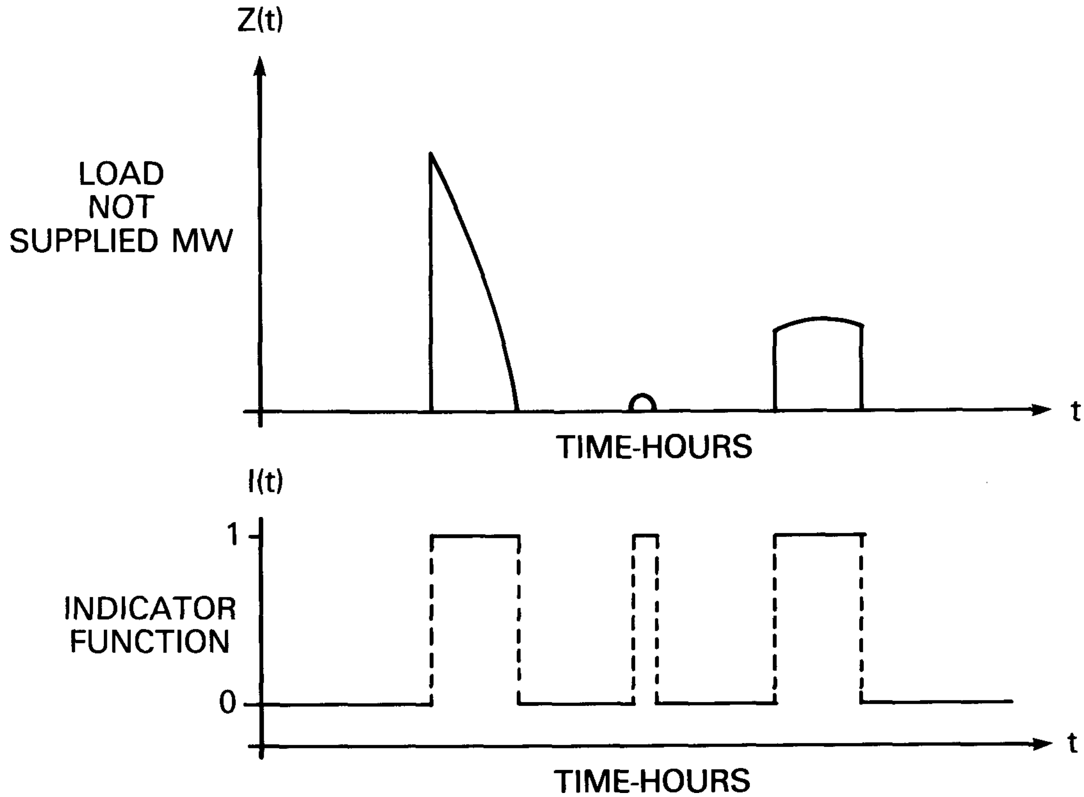

Figure 5.4 Relation between indicator function I(t) and load not supplied Z(t).

The indicator function is a mathematical construction which converts observation of whether or not event L occurs to a numerical-valued observation. Therefore, event L and variable I(t) both represent the same random phenomenon, and the indicator function is not a separate reliability indicator. However, the indicator function plays a central role in the definition of the interval reliability indicators based on event L, and related reliability indices. For example, I(t) provides a basis for the formal definition of the interval reliability indicators, loss of load time and loss of load occurrences. The definition of loss of load time in terms of the indicator function is then used to prove the validity of the procedure for calculating expected loss of load time by addition or integration of loss of load probability values for individual time points. Further, I(t) provides a mathematical basis for definition of conditional expected load not supplied. Finally, the stochastic process {I(t)} provides a basis for the formal definition of duration reliability indicators, from which the frequency and duration reliability indices are then obtained.

5.2.5 Unified terminology

All of the interval and duration reliability indicators described subsequently are derived from observation over time of one of the two point reliability indicators, loss of load or load not supplied. These names can therefore be used as "roots" for building names for the interval and duration reliability indicators. The reliability indices are in turn defined in terms of the probability laws of the reliability indicators. Thus, the two roots can also be used in naming the reliability indices. As a result, a unified structure of terminology is obtained.

The initials of the two point reliability indicators are

LOL = loss of load

LNS = load not supplied

Initials for each reliability indicator and reliability index subsequently discussed could also be stated. However, these initials would not correspond with those in common use by the industry. Therefore, no attempt is made to use the initials for each name as symbols in the equations in this Section. Rather, mathematical symbols are used, such as P(L;t) for the probability of event L. In Section 5.9 a summary of the uniform structure of names is given, and these names are compared with the present industry convention. In addition, comments on industry terminology are included with the discussion of each reliability indicator and reliability index.

The other sections of this report use the industry convention for each reliability index. The purpose of other sections is to discuss various aspects of the present application of reliability indices to power systems. It would be more difficult to understand these sections if unfamiliar terminology were used. By contrast, the purpose of this section is to develop a unified mathematical structure which emphasizes how the various indices are related. For this purpose, a uniform and structured terminology is useful.

It is not the intent of this section to propose new names for currently used reliability indicators and indices. Such industry conventions are established by continued usage. However, if studies such as this project suggest more logical terminology than that in current use, the new terminology may gradually come into use.

5.3 INTERVAL RELIABILITY INDICATORS

This section defines four additional reliability indicators in terms of the cumulative observation of point reliability status over the interval (0,t). These are called interval reliability indicators. The interval reliability indicators can all be derived from the sample record of load not supplied. Thus, referring back to Figure 5.1, it is evident that three ways to summarize the sample record of Z(t) over (0,t) would be by the quantities

1) Number of occurrences of Z(t)>0
2) Total length of time that Z(t)>0
3) Total area (energy) enclosed by the sample record and the abscissa.

These are three fundamental "dimensions" of interval reliability status. The time that Z(t)>0 can be observed either continuously or at specific time points (e.g., peak load times). Therefore, the second dimension gives rise to two interval reliability indicators. As a result, four interval reliability indicators are defined.

The first two of the three dimensions (number and time) do not depend on the magnitude of Z(t). Therefore, the corresponding interval reliability indicators can be defined in terms of the indicator function I(t), and the reliability indicators are named using the root "loss of load", giving the names "loss of load occurrences" and "loss of load time". The third dimension (energy) does involve the magnitude of Z(t), and this reliability indicator is therefore named "energy not supplied", using the root "not supplied" from load not supplied.

Section 5.3.1 describes the interval reliability indicators based on loss of load time, first for continuous time and then for specific time points. In order to distinguish the two reliability indicators based on loss of load time, the names "loss of load hours" and "loss of load points" are used. Section 5.3.2 describes loss of load occurrences. Only continuous time observation is considered. Energy not supplied is defined in Section 5.3.3. In Section 5.3.4, the conceptual reliability status monitor is expanded to include both the point and interval reliability indicators. This monitor serves to illustrate the physical dimension of the various indicators by use of meters, lights, and counters. The status monitor also serves to emphasize how all of the reliability indicators are derived from observation of the basic quantity load not supplied, observed over time.

5.3.1 Loss of load time

One way to describe the cumulative observation of reliability status is to consider the total amount of time in the interval (0,t) that event L exists. This is equivalent to the amount of time that $I(t)=1$, as can be seen by comparison of Figures 5.2 and 5.4.

There are two ways to define the cumulative observation. First, the value of $I(t)$ can be observed continuously over the interval (0,t). A second way is to observe $I(t)$ only at specific points in time, $0<t_1<t_2 \ldots t_n<t$. The name "loss of load hours" is subsequently used to describe the interval reliability indicator which results from continuous observation. When $I(t)$ is observed only at specific time points, the "amount of time" event L occurs is a pure number, and the name "loss of load points" is used.

Loss of load hours

When the value of $I(t)$ is observed continuously, the amount of time in (0,t) that $I(t)=1$ is called loss of load hours. It can be defined mathematically as the integral of the indicator function. Thus

$$T_H(t) \equiv \int_0^t I(y)dy$$

$$\equiv \text{Loss of load hours}$$

Loss of load hours is an interval reliability indicator. It is numerical-valued, and therefore $T_H(t)$ is a random variable and $\{T_H(t);\ t>0\}$ is a stochastic process. Figure 5.5 shows how the sample record of $\{T_H(t)\}$ is related to the sample record of $\{I(t)\}$.

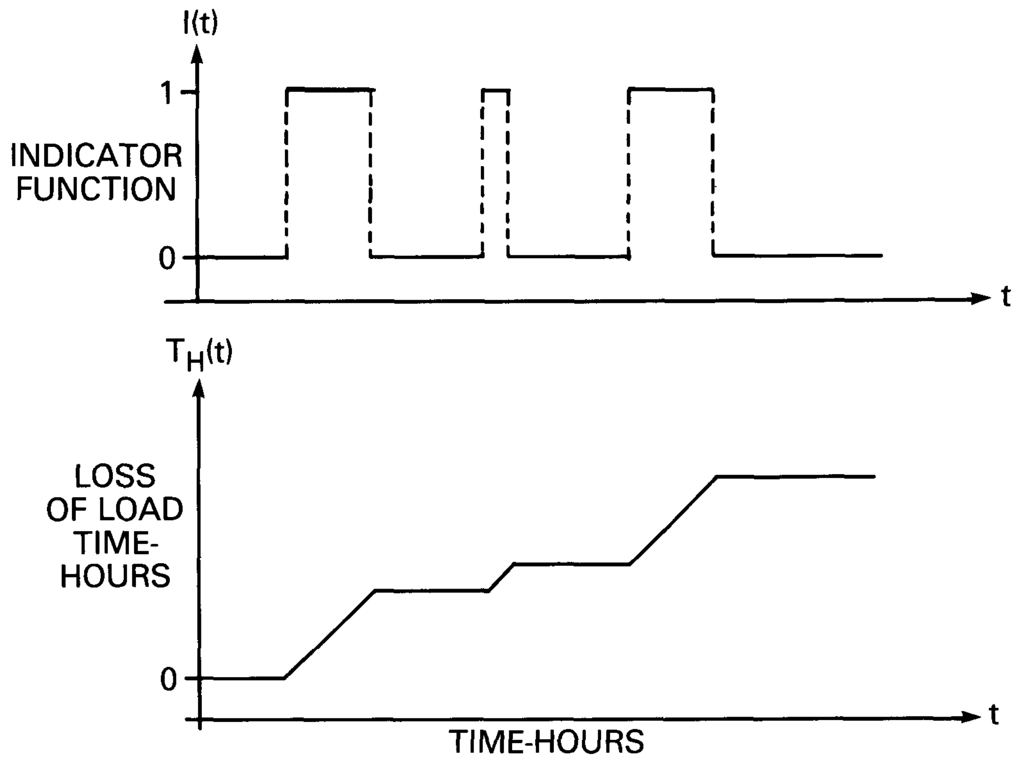

Figure 5.5 Integration of I(t) to obtain $T_H(t)$

The dimension of $T_H(t)$ is time. Thus, in the name loss of load hours, the phrase "loss of load" is an adjective, which describes the kind of time being measured. In the power industry the quantity $T_H(t)$ is often called "hourly loss of load". This name does not clearly reflect the fact that the quantity being measured is time. In fact, the name hourly loss of load would appear to refer to a magnitude of load loss, observed hourly, which is incorrect. It is for this reason that the name loss of load hours is used in this section as a name for the interval reliability indicator $T_H(t)$.

There is another industry convention which can lead to confusion, namely the manner in which the units for $T_H(t)$ are stated. The problem arises from the fact that the quantity $T_H(t)$ involves <u>two</u> numbers, the period of observation t and the observed value of $T_H(t)$. For example, if the sample record of Figure 5.5 covers 168 hours and the total amount of time represented by the three loss of load intervals is 13 hours, the correct mathematical statement of this observation is

$T_H(168) = 13$ hours.

Since 168 hours = 1 week, another way to state this result would be

$T_H(1 \text{ week}) = 13$ hours

From a mathematical viewpoint, this is a less satisfactory way of stating the result, because the defining equation for $T_H(t)$ assumes that both time measurements are in the same units. However, as long as the units on both sides of the statement are given, the correct result is conveyed; and from a practical viewpoint, the second notation is more meaningful than the first, since it explicitly states that the observation interval (0,t) was one week.

A third way to write the previous observed result is

$T_H = 13$ hours/week

From a mathematical viewpoint, this is incorrect. On the other hand, it may be "understood" that this notation is a convention for expressing the result $T_H(1 \text{ week}) = 13$ hours. The convention is that the denominator of the dimension is the time interval considered. That is t=1 unit of the kind of time in the denominator of dimension ratio. Such a convention is convenient because it puts both of the time unit names on the right hand side of the statement, while the name of the quantity observed is on the left hand side. The disadvantage of the convention is that it can be misinterpreted to suggest that T_H is a time rate of accumulation of loss of load hours, applicable to any time period. Under this interpretation, the following statements would be equivalent:

$T_H = 13$ hours/week

$T_H = \frac{13}{7}$ hours/day

$T_H = \frac{13}{168}$ hours/hour

However, the second two statements are not correct, because they do not preserve the identification of the observation period of one week.

Loss of load points

The second way to observe the "amount of time" that event L exists is by observing the reliability status of the system only at specific time points. An interval reliability indicator can then be defined as the <u>sum</u> of the values of the indicator function I(t), evaluated at the specific observation times $t_1, t_2, \ldots t_n$. This interval reliability indicator is called <u>loss of load points</u>. The symbol $T_p(t)$ is used to denote loss of load points. Thus

$$T_p(t) \equiv \sum_{i=1}^{n} I(t_i)$$

$$\equiv \text{Loss of load points}$$

Figure 5.6 shows how $T_p(t)$ is generated from the indicator function I(t). The sample record of I(t) shows three intervals of time when the loss of load event existed. The first loss of load interval included two observation time points. The second loss of load interval did not include any observation time points. The third loss of load interval included two observation time points. Thus, for the example in Figure 5.6, $T_p(t)$=4 at the end of the observation period.

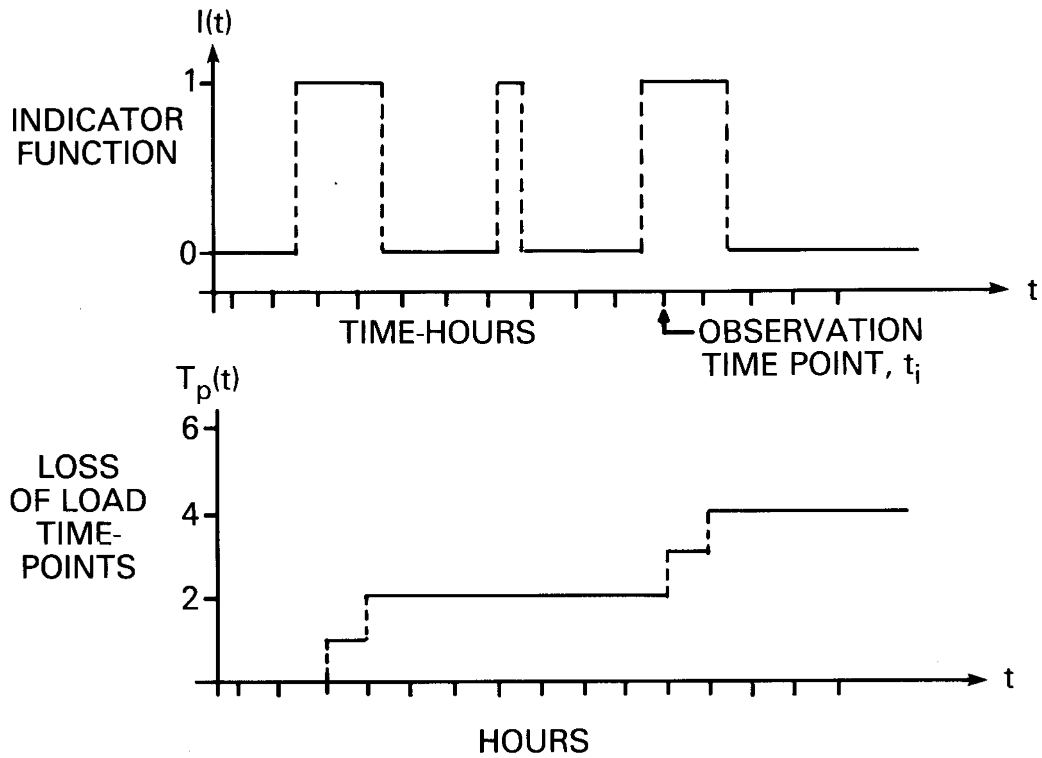

Figure 5.6. Summation of $I(t_i)$ to obtain $T_p(t)$

The name used here for $T_P(t)$ is loss of load points. In applications, the time points chosen are generally the times at which peak load demand occurs, such as times of daily peak load. In this case a more descriptive name for $T_P(t)$ would be loss of load peaks. However, there is a widespread practice in the industry to use the word "days" rather than "peaks" to describe the quantity $T_P(t)$ when the t_i are daily peaks. The convention is that "day" means "daily peak". This convention can lead to misinterpretation because "day" is more commonly understood as a unit for measuring continuous time, like hour, minute, or year. Hence, when $T_P(t)$ is stated in days, there is a possibility that someone not familiar with the convention would interpret the result as $24T_P(t)$ hours, which is not correct. Rather, $T_p(t)$ is mathematically a pure number (dimensionless).

As with $T_H(t)$, it is also common to show the units for t on the right hand side of the statement of $T_P(t)$. For example, if event L is observed on 10 daily peaks over a period of one year, the result would commonly be stated as

$$T_P = 10 \text{ days/year}$$

Here, both the convention "days" for "daily peaks" and statement of the period of observation as the denominator of the dimension ratio have been used. A more correct way to state this result would be

$$T_P(1 \text{ year}) = 10 \text{ daily peaks}$$

5.3.2 Loss of load occurrences

Another way to describe the cumulative observation of event L is to consider the "number of occurrences" of event L in the interval (0,t). An occurrence (of loss of load) is the start of an interval when I(t)=1. Thus, <u>loss of load occurrences</u>, denoted by N(t), is defined by

$$\begin{aligned} N(t) &\equiv \text{Number of times } I(t) \text{ changes from 0 to 1 in } (0,t) \\ &\equiv \text{Loss of load occurrences} \end{aligned}$$

Figure 5.7 shows how N(t) is generated from the sample record of {I(t)}. For each time t, N(t) is a random variable, and {N(t);t>0} is a stochastic process.

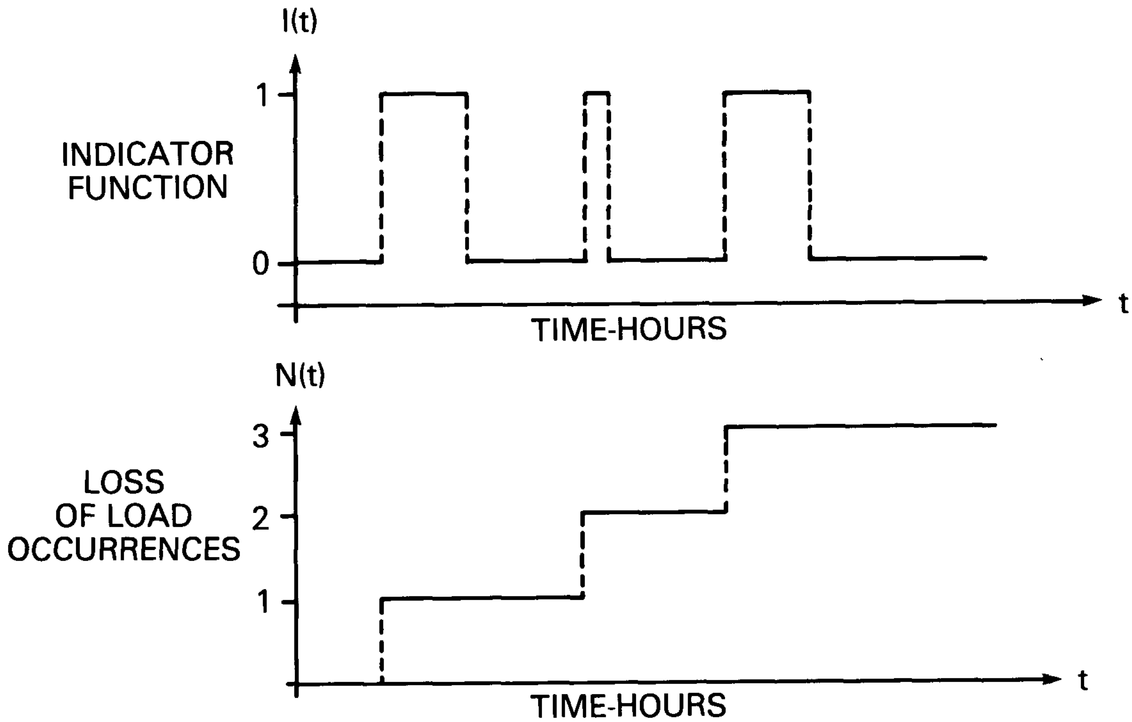

Figure 5.7 Derivation of N(t) from Occurrences of I(t)=1.

Loss of load occurrences is a number; it is not a rate of occurrence per unit time, although it may be stated in terms such as "occurrences/year" in reporting observed results. Such a statement reflects the convention described previously for loss of load time. The denominator of the dimension refers to the time period (0,t) over which the number of occurrences N(t) was observed. In the case of loss of load occurrences, this convention does have a serious disadvantage; it makes it very difficult to distinguish loss of load occurrences from loss of load frequency, which does have the true dimension 1/time. Loss of load frequency is defined here as a reliability index, not a reliability indicator. Therefore, further discussion of this point is deferred until the discussion of reliability indices based on N(t), in Section 5.8.4.

5.3.3 Energy not supplied

A third way to describe interval reliability status is to consider the amount of energy not supplied in (0,t), called energy not supplied. It can be defined mathematically as the integral of Z(t). The symbol U(t) is used in this section for energy not supplied. Thus

$$U(t) \equiv \int_0^t Z(y)dy$$

$$\equiv \text{Energy not supplied}$$

If time is measured in hours and load not supplied is measured in MW, the dimension for energy not supplied is MWh. Figure 5.8 shows how U(t) is generated from the sample record of $\{Z(t)\}$. For each time t, U(t) is a random variable; and $\{U(t);t>0\}$ is a stochastic process.

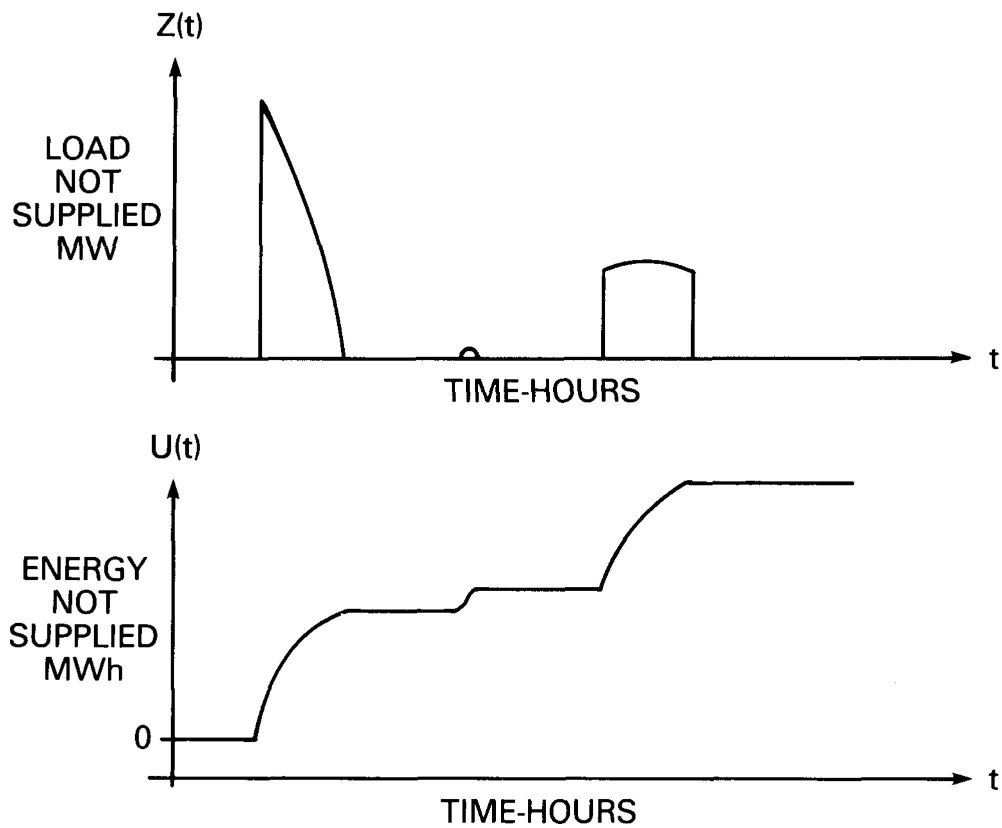

Figure 5.8 Integration of Z(t) to obtain U(t).

5.3.4 Reliability status monitor

Table 5.1 summarizes the four interval reliability indicators which have been defined.

Table 5.1

INTERVAL RELIABILITY INDICATORS

Mathematical Symbol	Name	Definition
$T_H(t)$	Loss of load hours	$T_H(t) = \int_0^t I(y)dy$
$T_P(t)$	Loss of load peaks	$T_P(t) = \sum_{i=1}^{n} I(t_i)$
$N(t)$	Loss of load occurrences	Number of times $I(t)$ changes from 0 to 1
$U(t)$	Energy not supplied	$U(t) = \int_0^t Z(y)dy$

The four interval reliability indicators describe the cumulative observation of $Z(t)$ over the interval $(0,t)$ with respect to three basic dimensions,

Loss of load time, $T_H(t)$ and $T_P(t)$
Loss of load occurrences, $N(t)$
Energy not supplied, $U(t)$

In order to better visualize the physical dimensions of the various reliability indicators, the conceptual reliability status monitor described previously is expanded in Figure 5.9 to include the interval reliability indicators. Energy not supplied and loss of load time are continuous quantities, and they are therefore represented by meters with a scale and pointer. By contrast, loss of load peaks and loss of load occurrences are integers, and they are therefore represented by counters.

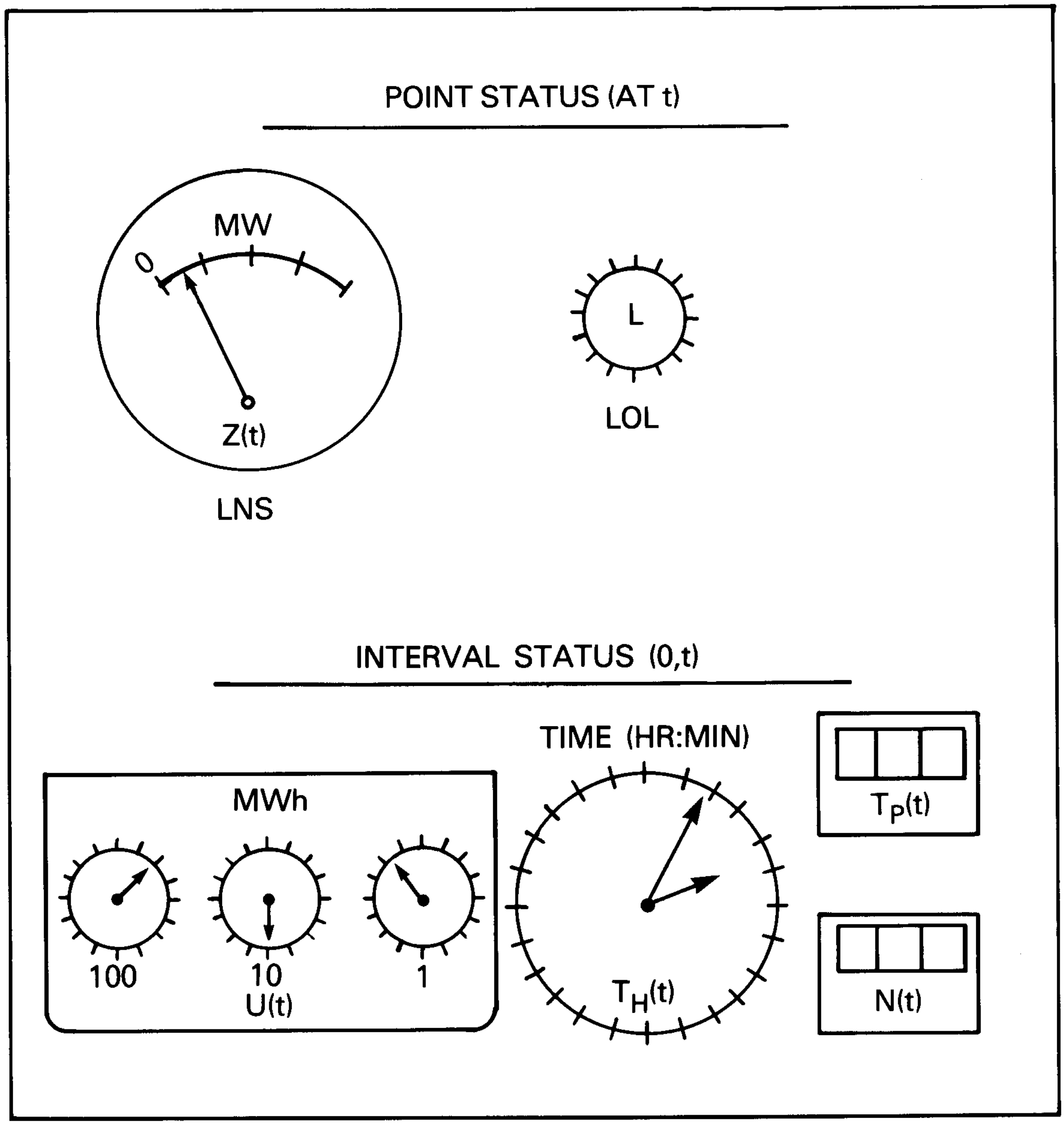

Figure 5.9 Reliability Status Monitor

The Reliability Status Monitor now has six indicators, but all indicators are still driven by the single input Z(t), load not supplied. Whenever Z(t) is positive, the value of Z(t) is shown on the LNS meter (a "watt meter") and the LOL light is ON. The U(t) meter integrates the LNS meter value ("watt-hour meter"), while the $T_H(t)$ meter is a time meter, which runs whenever the LOL light is on. The N(t) counter registers one count every time the LOL light comes on. The $T_P(t)$ counter registers one count every time the LOL light is on at the time of daily peak load.

5.4 DURATION RELIABILITY INDICATORS

This section defines additional reliability indicators in terms of the times between successive changes in value of the indicator function I(t). Reference to Figure 5.4 shows that changes in the value of I(t) from zero to one correspond to the beginning of a period when the loss of load event exists. A change in I(t) from zero to one also marks the end of a period when the loss of load event does not exist. Thus, the times at which the indicator function changes value can be used to define additional reliability indicators based on the length of time between the time points. These are called "duration reliability indicators". Figure 5.10 shows a portion of a sample record for I(t) which includes one complete "cycle" involving a change in I(t) from zero to one (loss of load occurrence), a subsequent change in I(t) from one to zero (end of loss of load period), and finally a second change in I(t) from zero to one.

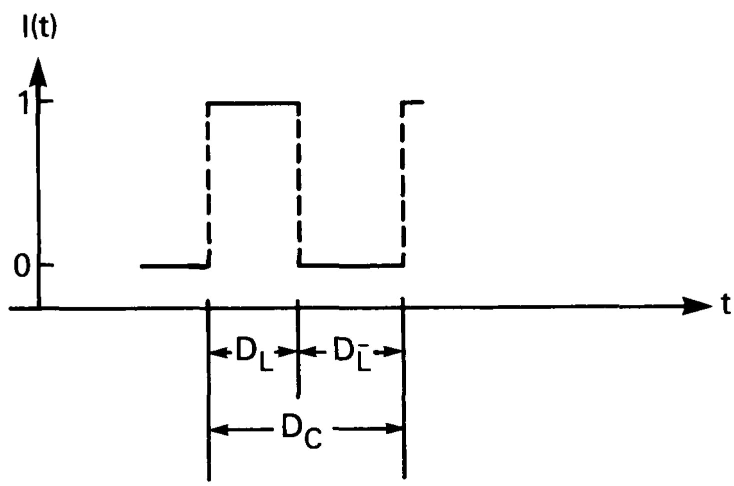

Figure 5.10. Duration reliability indicators defined by I(t).

Figure 5.10 illustrates three duration reliability indicators, which can be defined as follows;

D_L	≡	Time between change in I(t) from 0 to 1 and next change in I(t) from 1 to 0.
$D_{\bar{L}}$	≡	Time between change in I(t) from 1 to 0 and next change in I(t) from 0 to 1.
D_C	≡	Time between successive changes in I(t) from 0 to 1.

Since the duration reliability indicators are defined by the loss of load event (and do not depend on the magnitude of load not supplied), they are named using the root name "loss of load" as follows;

D_L = loss of load duration

$D_{\bar{L}}$ = no loss of load duration

D_C = loss of load cycle duration

The word "cycle" is used to refer to the duration between successive occurrences of loss of load, giving the name "loss of load cycle duration".

It would be possible to define another cycle duration as the time between successive changes in I(t) from 1 to 0. Following the terminology convention used for D_L and $D_{\bar{L}}$, this second cycle duration would be called "no loss of load cycle duration". This second cycle duration is not needed to define any of the reliability indices discussed in this report, and therefore it has not been defined or denoted on Figure 5.10. Since only one cycle duration reliability indicator is needed in this discussion, the name "cycle duration" rather than "loss of load cycle duration" is used subsequently.

In the reliability literature, D_C is often called cycle "time" rather than cycle duration. However, in this section the word "time" is used to describe the interval reliability indicator "loss of load time" which is the total amount of time that event L exists during (0,t). The word "duration" is used to describe reliability indicators based on periods formed from the changes in value of I(t).

The point and interval reliability indicators are defined for each point in time. Thus, for each time t, load not supplied is a random variable Z(t), and the collection of random variables $\{Z(t);t>0\}$ is a stochastic process. By contrast, the duration reliability indicators are not defined for each point in time. For example, observation of the system over time does not produce an observation of D_L for each time t. However, observation over time does produce a sequence of observations of each duration reliability indicator. This is illustrated in Figure 5.11.

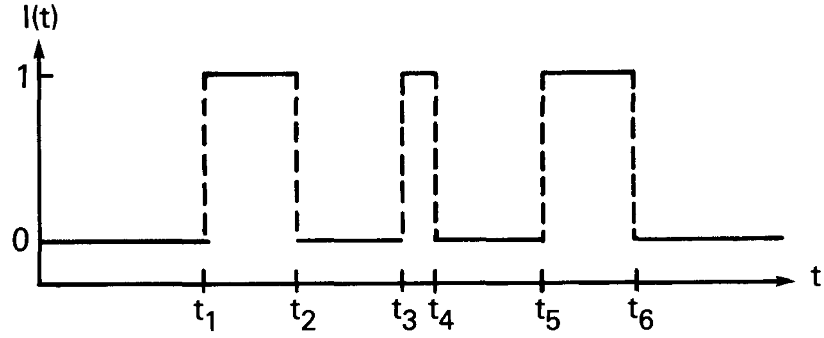

Figure 5.11 Observation of I(t) over time

Figure 5.11 includes three complete observations of loss of load duration,

$$D_L(1) = t_2 - t_1$$
$$D_L(2) = t_4 - t_3$$
$$D_L(3) = t_6 - t_5$$

Each observation of loss of load duration is a random variable and the sequence of observations, $\{D_L(i);i=1,2,...\}$ is a stochastic process. The indexing parameter is "i", the number of changes in I(t) from 0 to 1. In a similar manner, sequences of observations for cycle duration, $\{D_C(i);i=1,2,...\}$ and no loss of load duration, $\{D_{\bar{L}}(i);i=1,2,...\}$ are produced, and each sequence is a stochastic process.

In Figure 5.11, the time t_1 defines the end of a cycle duration, but the beginning of the cycle was not observed. The period $(0,t_1)$ is called an "incomplete" (or censored) observation of cycle duration. Similarly, t_5 defines the beginning of a cycle duration, but the end of the cycle duration was not observed. This is also an incomplete observation. In Figure 5.11, there are no incomplete observations of loss of load duration. However, if the observation period starts or ends when event L exists, then an incomplete observation of loss of load duration results.

5.5 RELIABILITY INDICES

The previous discussion has described eight reliability indicators. Each of the reliability indicators can be regarded as describing the results of observing the outcome of a particular random phenomenon. A random phenomenon is a phenomenon in which the results of an observation cannot be predicted in advance but in which the possible observations are governed by a probability law.

The probability law of a random phenomenon is any rule or function which serves to specify the probability of every event determined by the phenomenon. A parameter of a probability law is a constant (numerical value) which serves to partially or totally specify the probability law. In this discussion, a parameter of the probability law of a reliability indicator is called a reliability index.

In subsequent sections, the probability law of each of the eight reliability indicators is discussed, and reliability indices based on these probability laws are defined. The discussion of probability laws and reliability indices is grouped according to the three categories of reliability indicators. Probability laws based on point reliability indicators are discussed in Section 5.6. These give rise to the following reliability indices

- Loss of load probability
- Expected load not supplied
- Conditional expected load not supplied

Next, the probability laws of the duration reliability indicators are considered in Section 5.7. This discussion gives rise to the additional reliability indices

- Expected duration of loss of load
- Expected cycle duration
- Frequency of loss of load

Finally, the probability laws of the interval reliability indicators are considered in Section 5.8. The resulting reliability indices are

- Expected loss of load points
- Expected loss of load hours
- Expected energy not supplied
- Expected loss of load occurrences

The interval reliability indices are considered last because it turns out that only the expectation of the probability law for each of the interval reliability indicators can be computed using present analytical methods. Furthermore, the expectation of each interval reliability indicator is computed in terms of the expectation of one of the point or duration reliability indicators.

A general discussion of probability laws for random variables is given in Appendix F. In Section 1 of Appendix F, general methods for specifying the probability law of a random variable are described; and Section 2 defines expectation of a random variable, which is a fundamental parameter of the probability law of a random variable, and the basis for most of the commonly used reliability indices.

Section 5.9 gives a summary of all the reliability indices developed in Sections 5.6 through 5.9.

5.6 RELIABILITY INDICES BASED ON POINT RELIABILITY INDICATORS

5.6.1 Probability law of loss of load

Consider the probability law of event L. There are only two possible outcomes: event L occurs (exists at time t) or event L does not occur (does not exist at time t). In this case, the probability function is completely defined by stating the probability that event L exists at time t.

$P(L;t) \equiv$ Probability that event L exists at time t.

The probability P(L;t) is a reliability index. That is P(L;t) is a parameter of the probability law of event L. This illustrates that the probability of an event can itself be considered as a parameter of the probability law for the random pheonomenon associated with the event.

Methods for calculating P(L;t) in terms of other system data are discussed in other sections of this report. The basic approach in all cases is to consider the capability of the system to supply load, and the load demand by customers. If load demand exceeds load supply capability, then load demand cannot be fully supplied, and event L (loss of load) exists.

What name should be given to the reliability index P(L;t)? In probability theory, it is common practice to call P(A) the "probability of event A". This terminology reflects the fact that mathematically, P is a function which assigns a number to each event A on a sample space. Thus, literally, P(A) is "the number assigned to event A by the function P". This is the meaning of the phrase "probability of event A". If this convention is applied, the name for P(L;t) would be "probability of loss of load". That is, P(L;t) is the probability of event L.

In the power industry it is common to call P(L;t) loss of load probability rather than probability of loss of load. From a mathematical viewpoint, this convention is awkward; it is equivalent to referring to P(A) as the "A probability". However, when "A" is replaced by the word name of the event, then the convention is more natural. For example, the probability of success is often called "success probability" and probability of failure is called "failure probability". These examples indicate that, from an application viewpoint, probability is treated as a physical property. That is, for a particular system, P(L;t) is considered to be a unique number, and the object of analysis or observation of the system is to determine the value of P(L;t). In this context, it is quite natural to call P(L;t) loss of load probability. It is the (unique) "amount of probability" (quantity) "possessed by" (property of) the system. Subsequently in this section, the name loss of load probability is used for the reliability index P(L;t).

It has been emphasized that event L is a point reliability indicator. At each point in time, event L either exists or does not exist. A reliability index which is based on the probability law of a point reliability indicator is called a point reliability index. Thus, P(L;t) is a point reliability index, called loss of load probability.

Unfortunately, the name loss of load probability (and mnemonic symbol LOLP) has been widely used to describe another reliability index based on the probability law of the interval reliability status indicator $T_P(t)$, loss of load points. This reliability index is discussed subsequently in Section 5.8.1. In this report, the name loss of load probability is only used to describe the point reliability index P(L;t).

5.6.2 Probability law of the indicator function

Since the indicator function I(t) has only two possible values (0 and 1) it can be modeled as a discrete random variable. Using equation (F-5) of Appendix F, the probability mass function of I(t) is defined by

$$p_I(0) = P[I(t)=0]$$

$$p_I(1) = P[I(t)=1] \qquad (5\text{-}1)$$

In equation (5-1), the probability mass function of I(t) has been denoted by $p_I(i)$ rather than $p_{I(t)}(i)$. The later notation would be more correct, since there is actually a whole family of random variables {I(t)}, indexed by the observation time t. However, the notation $p_{I(t)}(i)$ is cumbersome, and in the present discussion time t is not significant.

The probability that I(t)=1 is just the probability that event L exists. Thus

$$P[I(t)=1] = P(L;t),$$

where P(L;t) is the probability of event L. Furthermore, since I(t) has only two possible values, the probability that I(t)=0 is

$$P[I(t)=0] = 1 - P(L;t)$$

Thus, the probability mass function of I(t) can be written in terms of P(L;t), the loss of load probability

$$p_I(0) = 1 - P(L;t)$$

$$p_I(1) = P(L;t) \qquad (5\text{-}2)$$

Equation (5-2) specifies the probability law of I(t) in terms of the single parameter P(L;t), the probability of event L.

The expected value of I(t) is obtained by application of equation (F-11) in Appendix F, which gives

$$E[I(t)] = P(L;t) \qquad (5\text{-}3)$$

Equation (5-3) shows that the expected value of the indicator function I(t) is the probability of event L. Equation (5-3) plays an important role in calculation of expected loss of load time as discussed in Sections 5.8.1 and 5.8.2.

5.6.3 Probability law of load not supplied

Load not supplied is a point reliability indicator. Methods for calculating the complete probability law of Z(t) in terms of system capacity and load demand data are described in other sections of this report.

One way to determine the probability law of Z(t) is to calculate its distribution function, defined by

$$F_Z(z) = P[Z(t) \leq z] \tag{5-4}$$

Since there is a random variable Z(t) for each time t, the distribution function would most properly be denoted $F_{Z(t)}(z)$. This notation is somewhat awkward, and the more compact notation $F_Z(z)$ is used. In addition to being compact, this notation focuses attention on the fact that the argument of F_Z is a value of power, z, not a value of time, t. That is, F_Z is the distribution function of load not supplied. It specifies the probability that the load not supplied, in MW, is less than or equal to the argument z.

Load not supplied cannot be negative. If the power system has more capacity available than needed to supply load demand, the load not supplied is zero. Thus,

$$F_Z(z) = 0, \; z < 0$$

The distribution function of Z(t) has a jump at z=0. The size of the jump is the probability of the event [Z(t)=0]. Since this event is the complement of the loss of load event (L),

$$P[Z(t)=0] = 1-P(L;t)$$

If $F_Z(z)$ is represented as a continuous function for Z(t)>0, a graph of $F_Z(z)$ would appear as shown in Figure 5.12.

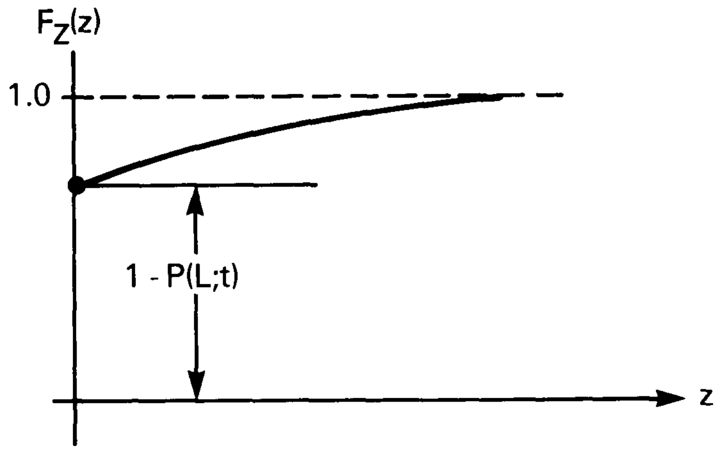

Figure 5.12. Distribution function of Z(t) for $F_Z(z)$ continuous when Z(t)>0.

If $F_Z(z)$ is represented as a discrete distribution function for Z(t)>0, a graph of $F_Z(z)$ would appear as shown in Figure 5.13.

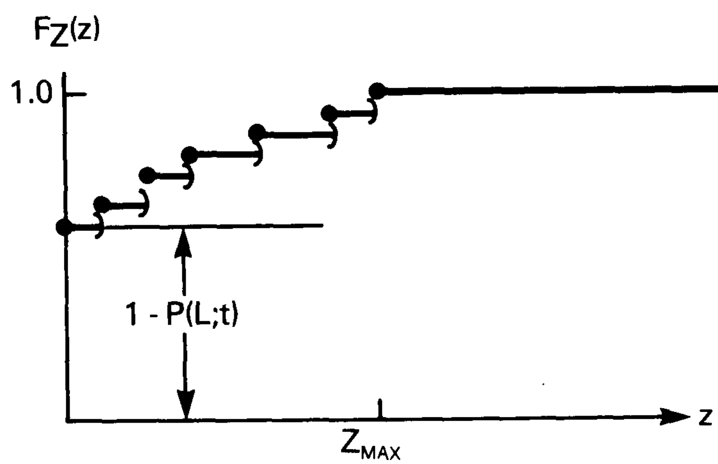

Figure 5.13. Distribution function of Z(t) for $F_Z(z)$ discrete when Z(t)>0.

In this case, the jump points in $F_Z(z)$ beyond z=0 are possible specific values of load not supplied, and z_{max} is the maximum possible value of Z(t).

Whether the discrete or continuous formulation for Z(t)>0 is used in a particular application depends on the models used for capacity and load demand. For purposes of this discussion, it is most convenient to use the discrete formulation of $F_Z(z)$ for z>0. This is because $F_Z(z)$ is necessarily discrete at $z=0$, having a jump equal to $1-P(L;t)$. By assuming $F_Z(z)$ is discrete for $Z(t)>0$, the distribution function $F_Z(z)$ is discrete over its entire range, and $Z(t)$ is then a discrete random variable. If $F_Z(z)$ is represented as a continuous distribution function for $z>0$, then $Z(t)$ becomes a <u>mixed</u> random variable (since it must still be discrete at $z=0$). All of the subsequent discussion still applies, but the expression for such quantities as the expectation of $Z(t)$ are more cumbersome to write. Thus, assume $Z(t)$ is a discrete random variable, with possible values $z_0<z_1<z_2<\ldots<z_m$, where $z_0=0$ and $z_m=z_{max}$, the maximum possible value of load not supplied. The probability law of $Z(t)$ can then be completely specified by its probability mass function

$$p_Z(z_i) = \begin{cases} P[Z(t)=z_i], & i=0,1,2,\ldots m \\ 0 \quad , & \text{otherwise} \end{cases} \qquad (5\text{-}5)$$

The expectation (expected value) of $Z(t)$ is then given by

$$E[Z(t)] = \sum_{i=1}^{m} z_i p_Z(z_i) \qquad (5\text{-}6)$$

Since $Z(t)$ is called "load not supplied" the corresponding name for $E[Z(t)]$ would be <u>expected load not supplied</u>.

Equation (5-6) is the definition of $E[Z(t)]$. In order to compute $E[Z(t)]$ using this equation, it is necessary to multiply each possible value of $Z(t)$ by the corresponding probability of that value, $p_Z(z_i)$. However, the expectation of $Z(t)$ can also be expressed in the following form

$$E[Z(t)] = \sum_{i=1}^{m} (z_i-z_{i-1})\bar{F}_Z(z) \qquad (5\text{-}7)$$

where

$$\bar{F}_Z(z) = 1-F_Z(z)$$

In equation (5-7), multiplication is still required. However, in computation procedures, a fixed "step size" is usually used so that (z_i-z_{i-1}) is a constant Δz. In this case,

$$E[Z(t)] = \Delta z \sum_{i=1}^{m} \bar{F}_Z(z) \qquad (5\text{-}8)$$

Equation (5-8) is more convenient than equation (5-6) for purposes of computing E[Z(t)]. First, the multiplication in equation (5-6) is eliminated. Second, the cumulative function $F_Z(z)$ is often obtainable directly from the computing algorithm used to compute loss of load probability P(L;t), whereas when equation (5-6) is used, the probability mass function $p_Z(z)$ would have to be computed from $F_Z(z)$, by taking differences of successive values. The use of equation (5-8) rather than equation (5-6) to compute E[Z(t)] for a generation system is illustrated in Section 3.

The derivation of equation (5-8) from equation (5-6) is an application of a useful theorem from basic probability theory. To state the theorem, consider first a continuous random variable X, specified by its distribution function $F_X(x)$. Figure 5.14 illustrates a possible graph of $F_X(x)$.

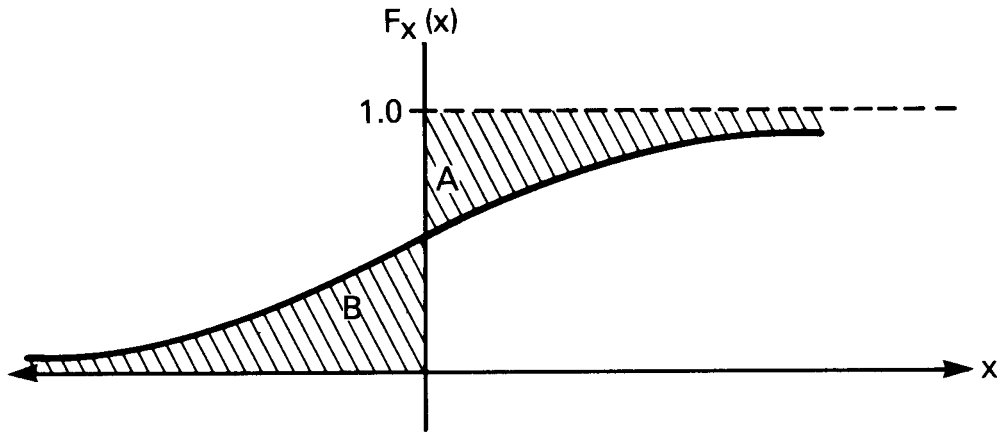

Figure 5.14. Distribution function defines two areas, A and B.

The graph in Figure 5.14 identifies two areas, labeled as A and B. These areas are defined mathematically by

$$A = \int_0^\infty \bar{F}_X(x)dx$$

$$B = \int_{-\infty}^0 F_X(x)dx$$

where again, $\bar{F}_X(x) = 1-F_X(x)$.

The theorem states that the expected value of X is simply the difference between the two areas, A-B,

$$E[X] = A-B \tag{5-9}$$

The definition of the expected value of X is

$$E[X] = \int_{-\infty}^\infty xf_X(x)dx$$

The range of integration can be broken at zero to express E[X] as the sum of two integrals, which have the values A and -B. That is, it can be shown using basic calculus that

$$\int_0^\infty xf_X(x)dx = \int_0^\infty \bar{F}_X(x)dx$$

and

$$\int_{-\infty}^0 xf_X(x)dx = -\int_{-\infty}^0 F_X(x)dx$$

Equation (5-9) then follows directly. The theorem given by equation (5-9) is given in Reference 5-1, pages 211-212, exercise 2.5, entitled: "Geometrical interpretation of the mean of a probability law."

Equation (5-9) applies also if X is a discrete random variable, but some difficulties in notation are encountered if general equations are attempted for areas A and B in the discrete case. However, in most practical applications, the random phenomenon of interest is non-negative ($X \geq 0$). Such is the case with Z(t). For a non-negative random variable X, area B=0. Hence the theorem becomes

$$E[X] = \text{area } A \tag{5-10}$$

If X is a continuous non-negative random variable, equation (5-10) becomes

$$E[X] = \int_0^{\infty} \bar{F}_X(x)dx \qquad (5\text{-}11)$$

Equation (5-11) is frequently used in computing the "mean time to failure" of a non-repairable system in terms of its "reliability function".

If X is a non-negative discrete random variable, with possible values $x_0 < x_1 < x_2 < \ldots < x_m$, where $x_0 = 0$, then equation (5-10) becomes

$$E[X] = \sum_{i=1}^{m} (x_i - x_{i-1}) \bar{F}_X(x_i) \qquad (5\text{-}12)$$

Application of equation (5-12) to Z(t) gives the result previously stated in equation (5-6).

5.6.4 Conditional distribution of load not supplied

For power systems, the loss of load probability P(L;t) is typically a small number, say P(L;t)=0.001. Thus, [1-P(L;t)] is very close to 1.0. Referring back to Figure 5.13, the graph of $F_Z(z)$ jumps to [1-P(L;t)] at z=0; and "area A" is very small if the value of $F_Z(z)$ at z=0 is close to 1.0. The expected load not supplied is the value of this area, expressed in units of z (power). If P(L;t) is small, then this value will be small relative to the values z_1, z_2, ... z_m, which represent possible positive values of Z(t).

As a simple example, suppose that P(L;t)=0.001 and that there are only two possible values of Z(t), 500 MW and 1000 MW. Furthermore, assume that if a loss of load exists either value is "equally likely". With this data, the probability mass function of Z(t) is

$$p_Z(0) = 0.999$$
$$p_Z(500) = 0.0005$$
$$p_Z(1000) = 0.0005$$

Using equation (5-6), the expected value of Z(t) is

$$E[Z(t)] = (0)(.999) + 500(.0005) + 1000(.0005)$$
$$= 0.75 \text{ MW}$$

Thus, the expected load not supplied is 0.75 MW. It is the weighted average of the three possible values, 0, 500, and 1000. However, since almost all the weight (probability "mass") is concentrated at z=0, the expected value is very small compared to the possible positive values, 500 and 1000.

From a practical viewpoint, it is evident that a more useful index would be the expected value of Z(t) "given that Z(t)>0". That is, the expected value should be calculated using only the values $z_i>0$, which in the example are 500 and 1000. As in "unconditional" expectation, the z_i values should be weighted according to their probabilities. However, in order that the weights add to unity, each probability should be divided by the sum of the probabilities for all values of $z_i>0$, which is just [P(L;t)]. Thus, in the example, the "expected value of Z(t), given that Z(t)>0" is

$$\left(\frac{0.0005}{0.001}\right)(500) + \left(\frac{.0005}{.001}\right) 1000 = 750 \text{ MW}$$

This is, intuitively, the value which best "summarizes" the fact that two values of Z(t)>0 are possible, 500 and 1000 MW, and both are equally likely.

In order to give a precise mathematical definition of "conditional expected load not supplied", it is necessary to define the notion of the conditional distribution of a random variable and the conditional expectation of a random variable.

If X and Y are two random variables, the conditional distribution function of X given Y=y is defined by

$$F_{X/Y}(x/y) \equiv P(X \leq x/Y=y) \qquad (5\text{-}13)$$

where $(X \leq x/Y=y)$ is the conditional event that $X \leq x$ given that Y=y. The definition in equation (5-13) is based on Reference 5-1, page 338, equation 11.14.

The basic definition of conditional probability for two events A and B, with P(B)>0, is

$$P(A/B) \equiv \frac{P(AB)}{P(B)} \qquad (5\text{-}14)$$

Using equation (5-14), equation (5-13) can be written

$$F_{X/Y}(x/y) = \frac{P(X\leq x, Y=y)}{P(Y=y)} \tag{5-15}$$

The event in the numerator of equation (5-15) is the joint event that $X\leq x$ and $Y=y$.

Equation (5-15) is meaningful only if $P(Y=y)>0$. Therefore, equation (5-15) is useful only if Y is a discrete random variable. If Y is discrete, then (5-15) is defined for values $y=y_i$ such that $p_Y(y_i)>0$.

Equation (5-15) defines conditional distribution in terms of two random variables. However, by use of the indicator function, the definition can be extended to the situation where the "condition" is an event. Thus, suppose that A is an event defined by the random variable X. For example, suppose A is the event that $X>0$. The indicator function of event A is

$$I_A = 1, \quad X>0$$
$$I_A = 0, \quad X\leq 0$$

The indicator function is a random variable. Thus, equation (5-15 can be applied to define the conditional distribution function of X given $I_A=1$.

$$F_{X/I_A}(x/1) = \frac{P(X\leq x, I_A=1)}{P(I_A=1)}$$

The condition $I_A=1$ is equivalent to the occurrence of event A. Thus, this equation can be used to define the "conditional distribution of X given event A" as follows:

$$F_{X/A}(x) \equiv \frac{P(X\leq x, A)}{P(A)} \tag{5-16}$$

Equation (5-16) can now be applied to define the conditional distribution of load not supplied, given the event loss of load. Thus,

$$F_{Z/L}(z) \equiv \frac{P[Z(t)\leq z, L]}{P(L;t)} \tag{5-17}$$

Event L is defined by Z(t)

$$L = [Z(t)>0]$$

Therefore, the event in the numerator of equation (5-17) can be expressed as

$$\begin{aligned}[Z(t)\leq z,L] &= [Z(t)\leq z,Z(t) > 0] \\ &= [0< Z(t) \leq z]\end{aligned}$$

Using this result in equation (5-17), the conditional distribution of load not supplied becomes

$$\begin{aligned}F_{Z/L}(z) &= 0 &, \quad z<0 \\ &= \frac{F_Z(z) - F_Z(0)}{P(L;t)} &, \quad z\geq 0\end{aligned}$$

Recall that $F_Z(0) = 1-P(L;t)$. Substituting this result and rearranging gives

$$\begin{aligned}F_{Z/L}(z) &= 0 &, \quad z<0 \\ &= 1 - \frac{1-F_Z(z)}{P(L;t)} &, \quad z\geq 0\end{aligned} \qquad (5\text{-}18)$$

Equation (5-18) is the desired result. It expresses the conditional distribution of load not supplied in terms of the unconditional distribution of load not supplied and loss of load probability. Figure 5.15 compares the unconditional and conditional distribution functions for load not supplied. For illustration purposes, the distribution functions all shown as continuous for z>0.

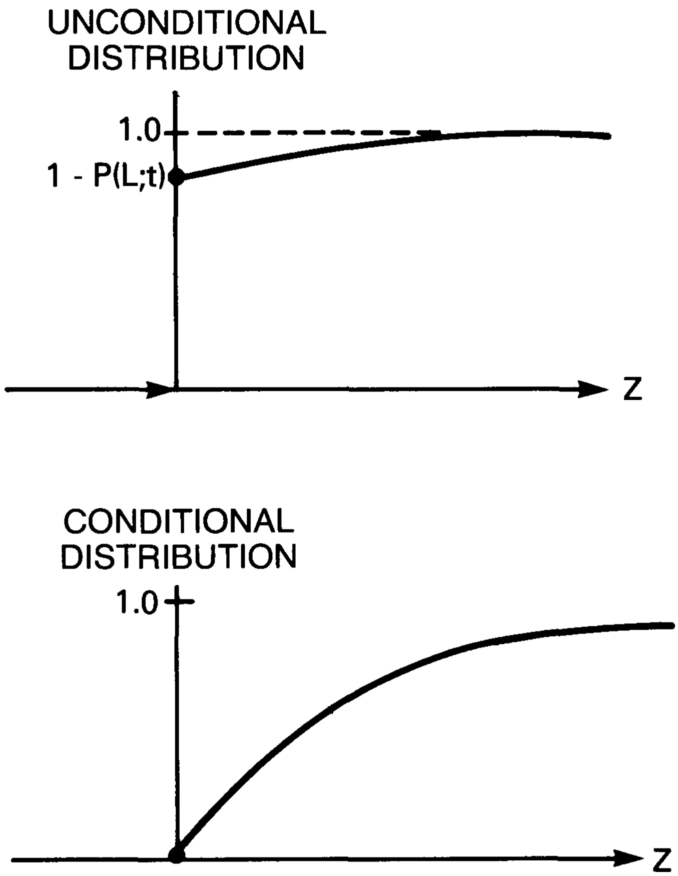

Figure 5.15. Comparison of unconditional and conditional distribution of load not supplied.

The unconditional distribution function has a jump equal to [1-P(L;t)] at z=0. The conditional distribution function has no jump at z=0. For z>0, the two distribution functions are related by the factor P(L;t) in the sense that

$$1 - F_{Z/L}(z) = \frac{1 - F_Z(z)}{P(L;t)}$$

Conditional expectation is defined by taking the expectation with respect to the conditional distribution function. Conditional expectation is defined in Reference 5-1, page 384, equation 7.1. As before, assume now for convenience that Z(t) is a discrete random variable, with possible positive values $0<z_1<z_2<\ldots<z_m$. The conditional expected load not supplied is then defined by

$$E[Z(t)/L] \equiv \sum_{i=1}^{m} z_i \; p_{Z/L}(z_i) \qquad (5\text{-}19)$$

where $p_{Z/L}(z_i)$ is the conditional probability mass function of load not supplied, defined by

$$p_{Z/L}(z) \equiv \frac{P[Z(t)=z,L]}{P(L;t)} \qquad (5\text{-}20)$$

Equation (5-19) is a formal definition of the reliability index conditional expected load not supplied, which was described intuitively by the simple example given earlier. In the example, two possible values of 500 and 1000 MW were given, 500 and 1000 MW. Thus, m=2, z_1=500, and z_2=1000. The statement that both values of Z(t) are "equally likely" specifies the conditional probability mass function as

$$p_{Z/L}(500) = 0.5$$

$$p_{Z/L}(1000) = 0.5$$

Substituting these values into equation (5-19) gives

$$E[Z(t)/L] = 750 \text{ MW}$$

which is the result obtained intuitively.

It is possible to derive a simple relation between the conditional and unconditional expected values of Z(t) as follows. For z>0, the event in the numerator of equation (5-20) is equivalent to the event [Z(t)=z] alone. Therefore,

$$P[Z(t)=z,L] = p_Z(z)$$

For z<0, the numerator of equation (5-20) is zero. Using these results in equation (5-20), and then using equation (5-19), there results

$$E[Z(t)/L] = \frac{1}{P(L;t)} \sum_{i=1}^{m} z_i \; p_Z(z_i) \qquad (5\text{-}21)$$

The unconditional expectation of Z(t) is

$$E[Z(t)] = \sum_{i=0}^{m} z_i \, p_Z(z_i)$$

The first term in the summation is zero, since $z_0=0$ by definition. Hence, the above summation is equal to the summation in equation (5-21). Therefore

$$E[Z(t)/L] = \frac{E[Z(t)]}{P(L;t)} \tag{5-22}$$

Equation (5-22) gives a computational method for obtaining the conditional expected load not supplied directly from unconditional exxpected load not supplied. In the example given earlier, with P(L;t)=0.001, the unconditional expected load not supplied was computed as 0.75 MW. The conditional expected load not supplied is therefore

$$E[Z(t)/L] = \frac{0.75}{0.001} = 750 \text{ MW}$$

which agrees with the result obtained by direct application of equation (5-19).

The loss of load probability P(L;t) and the expected load not supplied E[Z(t)] are fundamental reliability indices because they are each parameters of the probability law of one of the point reliability indicators. By contrast, in view of equation (5-22), conditional expected load not supplied could be called a derived reliability index, because it can be expressed in terms of other reliability indexes which are based on one of the reliability indicators. On the other hand, from a practical viewpoint, it may be more useful to report P(L;t) and the conditional expectation of Z(t). Thus in the previous example

P(L;t) = .001
E[Z(t)] = 0.75
E[Z(t)/L] = 750 MW

The first and third values are more useful in describing the reliability of the system. For this reason, there has been little mention of the index E[Z(t)] in the literature.

A common industry name for the conditional expectation of Z(t) is expected loss of load often denoted by the symbol XLOL. In this discussion a very careful distinction has been made between the two fundamental point reliability status indicators

L = Loss of load - an event

Z(t) = Load not supplied - magnitude (random variable)

Using this terminology, the correct name for E[Z(t)/L] is conditional expected load not supplied.

5.7 RELIABILITY INDICES BASED ON DURATION RELIABILITY INDICATORS

In Section 5.4 three duration reliability indicators were defined by considering the time points at which the indicator function changes value. The three duration reliability indicators are

$D_L(i)$ = Loss of load duration

$D_{\bar{L}}(i)$ = No loss of load duration

$D_C(i)$ = Cycle duration

where i = 1,2,... is the index denoting successive observations of each reliability indicator.

Each observation of loss of load duration is a random variable, and the sequence of observations $\{D_L(i);i=1,2,...\}$ is a stochastic process. Similarly for the other two duration reliability indicators.

The expectation of each duration reliability indicator can be defined as a reliability index. This would give the following symbols and terminology

$E[D_L(i)]$ = expected duration of loss of load

$E[D_{\bar{L}}(i)]$ = expected duration of no loss of load (5-23)

$E[D_C(i)]$ = expected cycle duration

In general, $E[D_L(i)]$ is different for each successive loss of load duration, i=1,2,..., and similarly for $E[D_{\bar{L}}(i)]$ and $E[D_C(i)]$. However, consider the special case where it is assumed that successive durations are independent and identically distributed, denoted IID. That is, the following conditions are assumed.

1) The loss of load durations $D_L(i)$ are IID, with common expectation $E[D_L(i)]=d'_L$;
2) The no loss of load durations $D_{\bar{L}}(i)$ are IID, with common expectation $E[D_{\bar{L}}(i)]=d'_{\bar{L}}$;
3) The sequences $\{D_L(i)\}$ and $\{D_{\bar{L}}(i)\}$ are mutually independent.

It follows that the cycle durations $D_C(i)$ are also IID, with common expectation

$$d'_C = d'_L + d'_{\bar{L}} \tag{5-24}$$

A stochastic process which consists of a sequence of IID random variables is called a renewal process. Thus, under conditions (1), (2), and (3) above, each of the stochastic processes $\{D_L(i)\}$, $\{D_{\bar{L}}(i)\}$, and $\{D_C(i)\}$ is a renewal process. Under these conditions, observation of the process $\{I(t)\}$ generates, alternately, observations from the two renewal processes $\{D_L(i)\}$ and $\{D_{\bar{L}}(i)\}$. Therefore, the process $\{I(t)\}$ itself is called an alternating renewal process.

When $\{I(t)\}$ is an alternating renewal process, the reciprocal of the common expected cycle duration is called loss of load frequency. Thus define

$$f' \equiv \frac{1}{d'_C} \tag{5-25}$$

$= \text{Loss of load frequency}$

The constant d'_C is a reliability index. It is properly called expected cycle duration, because it is the expected value of the duration reliability indicator "cycle duration". However, it is not correct to call the constant f' defined by equation (5-25) the expected frequency of loss of load. This terminology would imply that frequency had been defined as a reliability indicator, whose expected value is f'. As defined by equation (5-25), frequency is simply an alternate way of stating the expected cycle duration.

Equation (5-25) defines frequency under the assumption that {I(t)} is an alternating renewal process. In the models commonly used for power system reliability evaluation, an alternating renewal process is not actually used. For example, system capacity is generally modeled as a Markov process in which individual components are assumed to have two or more states, with constant transition rates between states. For a particular (constant) load demand, the loss of load event L can then be defined in terms of the state space formed by the components. The sequence of loss of load durations resulting from such a model are not IID. However, the models do have the property that the expected loss of load duration approaches a constant value (called the steady state value) as the number of observations increases. Thus, instead of the three conditions for an alternating renewal process, suppose the stochastic processes $\{D_L(i)\}$ and $\{D_{\bar{L}}(i)\}$ have the property that

$$\lim_{i\to\infty} E[D_L(i)\} = d_L$$

$$\lim_{i\to\infty} E[D_{\bar{L}}(i)] = d_{\bar{L}} \qquad (5\text{-}26)$$

The expected cycle time then also approaches a constant

$$\lim_{i\to\infty} E[D_C(i)] = d_C \qquad (5\text{-}27)$$

where

$$d_C = d_L + d_{\bar{L}} \qquad (5\text{-}28)$$

Frequency can then be defined as

$$f = \frac{1}{d_C} \qquad (5\text{-}29)$$

Equation (5-29) is an alternate definition of frequency, which applies when the stochastic process {I(t)} satisfies the conditions in equation (5-26).

The definition of frequency in equation (5-29) is consistent with the definition in equation (5-25) in the sense that if {I(t)} is an alternating renewal process, then the conditions in equation (5-26) hold, since every observation of

D_L has the same distribution and expected value, and similarly for observations of $D_{\bar{L}}$. Thus, if $\{I(t)\}$ is an alternating renewal process

$$d_L = d'_L \tag{5-30}$$

$$d_{\bar{L}} = d'_{\bar{L}}$$

$$d_C = d'_C$$

The converse is not true. That is, the condition in equation (5-26) does not make $\{I(t)\}$ an alternating renewal process.

When loss of load frequency is defined by equation (5-29), the parameters d_L and $d_{\bar{L}}$ can also be considered as reliability indices. These parameters are defined by the limiting condition in equation (5-26). Thus, d_L is literally the "expected loss of load duration under steady state conditions". The parameter d_L is still an expected value. However it is misleading to call d_L simply the "expected loss of load duration", because this implies that the loss of load durations $\{D_L(i)\}$ are IID (that is, that $\{I(t)\}$ is an alternating renewal process). If the word "expected" is used, then the phrase "steady state" should also be used. Thus d_L can be called "steady state expected loss of load duration". In practice, the parameter d_L is often called simply "loss of load duration", and the two reliability indices f and d_L are called "frequency and duration" (of loss of load).

5.8 RELIABILITY INDICES BASED ON INTERVAL RELIABILITY INDICATORS

5.8.1 Expected loss of load peaks

The random variable $T_P(t)$ was defined in Section 5.3.1 as

$$T_P(t) = \sum_{i=1}^{n} I(t_i) \tag{5-31}$$

In equation (5-31), the t_i values are "observation points" in the interval $(0,t)$, with $0<t_1<t_2...<t_n<t$. The t_i values can be any points which satisfy this condition, but in practice the t_i values typically represent the times of daily peak load, and $T_P(t)$ is then sometimes called loss of load peaks.

Equation (5-31) defines $T_P(t)$ as a sum of the random variables $I(t_i)$. Therefore, the observed value of $T_P(t)$ for a particular interval (0,t) is simply the sum of the $I(t_i)$ values observed in this same interval. This was illustrated previously in Figure (5-6). However, it does not follow that the probability law of $T_P(t)$ can be obtained in a simple way from the probability laws of the individual indicator functions $I(t_i)$. In order to express the probability law of $T_P(t)$ in terms of the probability law of the indicator functions $I(t_i)$, it would be necessary to derive first the joint probability law of the random variables $I(t_1)$, $I(t_2) \ldots I(t_n)$. However, the expectation of $T_P(t)$ can be calculated as the simple summation of expected values of the indicator functions $I(t_i)$. Thus,

$$E[T_P(t)] = \sum_{i=1}^{n} E[I(t_i)] \tag{5-32}$$

Equation (5-32) results from a basic probability theorem concerning the expectation of a sum of random variables. Thus, in general, if X_1, X_2, ... X_n are random variables and Y is a random variable defined by

$$Y = X_1 + X_2 + \ldots X_n$$

then the theorem states that

$$E[Y] = \sum_{i=1}^{n} E[X_i] \tag{5-33}$$

This theorem does not require any assumption about the statistical relation between the X_i's. In particular the X_i's need not be independent. Equation (5-33) is a very useful and powerful result, because it shows how to calculate the expectation of Y even though the probability law of Y has not been derived. For a proof of equation (5-33) for two variables, see page 355 of Reference 5-1, equation (2.7). Equation (5-33) itself is stated on page 366 of Reference 5-1, as equation (4.1). Equation (5-32) for the expectation of $T_P(t)$ results from applying the theorem in equation (5-33) to the definition of $T_P(t)$ in equation (5-18).

The expected value of $I(t_i)$ was shown in equation (5-3) to be simply P(L;t). Substituting equation (5-3) into equation (5-32) gives

$$E[T_P(t)] = \sum_{i=1}^{n} P(L;t_i) \tag{5-34}$$

Equation (5-34) is perhaps the most widely used equation in generation system reliability evaluation. In words, it states that the expected value of $T_p(t)$ can be obtained by summing up the probability of event L at each of the observation points t_i.

The fact that the summation on the righthand side of equation (5-34) is a sum of probabilities, together with the tendency to regard equation (5-34) as a definition of the quantity represented by the summation, has resulted in widespread use of the name loss of load probability for the summation. Equation (5-34) shows that the summation is not a probability; it is the expectation (or expected value) of $T_p(t)$. Furthermore, equation (5-34) is not a definition of $E[T_p(t)]$, but is rather a calculation method for $E[T_p(t)]$ based on application of the theorem in equation (5-33), together with equation (5-3). The name "loss of load probability" is more correctly applied to the quantity P(L;t), since this quantity is a true probability - the probability that event L exists at time t. Indeed, early papers on power system reliability did use the name "loss of load probability" to refer to the quantity P(L;t).

Recognizing that the quantity calculated in equation (5-34) is an expectation rather than a probability, the term loss of load expectation, with corresponding mnemonic symbol LOLE, has been used quite commonly in recent years. This is the convention which has been adopted in other sections of this report. However, while loss of load expectation (LOLE) has received considerable acceptance in the technical literature, the long and widespread use of the name "loss of load probability" and, more especially, the symbol LOLP, in verbal discussions and general reports virtually assures the continued use of loss of load probability and LOLP as "names" for $E[T_p(t)]$. In defense of this convention, it can be noted that equation (5-34) does calculate the "accumulated probability" that event L occurs. In this sense, the total phrase "loss-of-load-probability" can be regarded as simply a name for the summation (of probabilities) in equation (5-34). No claim is then being made that the summation is actually the probability of any particular event.

Actually, the term loss of load expectation, while more technically correct than loss of load probability, is itself a poor use of terminology. The word expectation is more properly be placed at the beginning, as "expected loss of load". Further, the quantity whose expectation is being described should be indicated. Loss of load is an event, not a numerical value. The quantity

$E[T_p(t)]$ is the expectation of the number of times that event L is observed. In section 5.3.4, $T_p(t)$ is called loss of load points. The expectation of $T_p(t)$ would then logically be called "expected loss of load points". This is the name which is subsequently used for $E[T_p(t)]$ here.

The comments in Section 5.3.4 about stating the dimensions of $T_p(t)$ apply to $E[T_p(t)]$ as well. The quantity $E[T_p(t)]$ is, mathematically, a pure number, but in stating the results some indication is needed of which specific time points in (0,t) were used. If the calculated value of $E[T_p(t)]$ is 3.4, based on daily peak load points for t=1 year, the correct mathematical statement is

$$E[T_p(1\ \text{year})] = 3.4\ \text{daily peaks}$$

The common industry convention for stating this result is

$$\text{LOLE} = 3.4\ \text{days/year}$$

The convention is that

1) "Days" means daily peak load points.
2) The denominator of the dimension (years) is the period (0,t) used in the calculation.

If equation (5-34) is applied for a single day, then t=1 day and n=1, and

$$E[T_p(1\ \text{day})] = P(L;t_1)$$

where $P(L;t_1)$ is the loss of load probability for the single day being considered. This result states that the expected number of loss of load peaks in a one day interval is P(L;t), the loss of load probability for that day. Using this result, the loss of load probability itself is sometimes called LOLE, using the dimension "days/day". The interpretation of this convention is that

1) The numerator in days/day means one daily peak.
2) The denominator means that t=24 hours.
3) The results of equation (5-34) are being applied to a single time point (n=1).

5.8.2 Expected loss of load hours

The random variable $T_H(t)$ was defined in Section 5.3.1 as

$$T_H(t) = \int_0^t I(y)dy \quad \text{hours} \tag{5-35}$$

As in the case of $T_P(t)$, the observed value of $T_H(t)$ over a given interval $(0,t)$ is easy to compute, as illustrated previously in Figure 5.5. By contrast, the probability law of $T_H(t)$ is not easy to obtain. However, the expectation of $T_H(t)$ can be calculated from the expectation of $I(t)$ in a manner analogous to equation (5-32) for $T_P(t)$. The equation for $E[T_H(t)]$ is

$$E[T_H(t)] = \int_0^t E[I(y)]dy \tag{5-36}$$

Equation (5-36) is intuitively an extension of equation (5-32) for continuous time. However, equation (5-36) can also be derived rigorously by application of the mean value theorem of stochastic processes. This theorem is given in Reference 5-2, p. 79, Theorem 3A.

Equation (5-17) can be substituted into equation (5-23) to give

$$E[T_H(t)] = \int_0^t P(L;y)dy \tag{5-37}$$

Equation (5-37) is the analog of equation (5-34) for continuous time. As such it is another basic equation in power system reliability evaluation.

From a computational standpoint, integration must be carried out by summation. If $P(L,t)$ is constant over an interval Δt, then

$$\int_t^{t+\Delta t} P(L;y)dy = [P(L;t)]\ \Delta t$$

If the interval $(0,t)$ is divided into h equal increments, $\Delta t = t/h$, with $P(L;t)$ constant over each increment, then

$$E[T_H(t)] = \Delta t \sum_{i=1}^{h} P(L;t) \tag{5-38}$$

where $P(L;t_i)$ is the constant value of $P(L;t)$ in increment i. If Δt is taken as one hour, then equation (5-38) becomes

$$E[T_H(t)] = \sum_{i=1}^{h} P(L;t_i) \text{ hours} \tag{5-39}$$

In equation (5-39), h is the number of hours in the interval (0,t). Equations (5-34) and (5-39) are very similar. However, equation (5-39) involves observation over continuous time, and therefore the dimension of $E[T_H(t)]$ is hours. By contrast, equation (5-34) involves observation at specific time <u>points</u>. Therefore, the dimension of $E[T_P(t)]$ is number of time points.

Using the unified structure of terminology in this discussion, the correct name for $E[T_H(t)]$ is "expected loss of load hours". In other sections of this report, $E[T_H(t)]$ is called "hourly loss of load expectation" (HLOLE). The word "hourly" is used to distinguish HLOLE from loss of load expectation (LOLE), which is assumed to be based on daily peaks only.

The comments in Section 5.3.3 about stating the dimensions for $T_H(t)$ apply to $E[T_H(t)]$ as well. Thus, if the calculated value of $E[T_H(t)]$ is 13 hours, based on a period t=1 year, the correct mathematical statement is

$$E[T_H(1 \text{ year})] = 13 \text{ hours}$$

The industry convention is to state this result as

HLOLE = 13 hours/year

5.8.3 Expected energy not supplied

Energy not supplied is another one of the interval reliability indicators, defined by

$$U(t) = \int_0^t Z(y)dy \quad \text{MWh} \tag{5-40}$$

The relationship between Z(t) and U(t) is the same as the relationship between loss of load time and the indicator function I(t),

$$T_H(t) = \int_0^t I(y)dy$$

Thus, $T_H(t)$ accumulates the amount of time that event L exists $(I(t)=1)$ while U(t) accumulates the amount of energy not served during this time. As in the case of $T_H(t)$, analytical methods are not currently available for computing the entire probability law of U(t). However, the expectation of U(t) can be computed from the expectation of Z(t) by the equation

$$E[U(t)] = \int_0^t E[Z(y)]dy \qquad (5\text{-}41)$$

Equation (5-41) is an application of the same theorem (mean value theorem of stochastic processes) that was used to derive equation (5-36) relating $E[T_H(t)]$ to $E[I(t)]$. Continuing the parallel development with that for $T_H(t)$, if the interval (0,t) is divided into h equal increments $\Delta t=t/h$ with the probability law of Z(t) constant over each increment, then

$$E[U(t)] = \sum_{i=1}^{h} E_i[Z(t)]\Delta t$$

$$= \Delta t \sum_{i=1}^{h} E_i[Z(t)] \qquad (5\text{-}42)$$

where $E_i[Z(t)]$ is the constant expectation of Z(t) in interval i. If $\Delta t=1$ hour then

$$E[U(t)] = \sum_{i=1}^{h} E_i[Z(t)] \quad \text{MWh} \qquad (5\text{-}43)$$

where h is now the length of the interval (0,t) in hours. Assuming that Z(t) is a discrete random variable, the expected value of Z(t) was given previously in equation (5-6). However, for computational purposes, an alternate equation for E[Z(t)] was given in equation (5-7), or equation (5-8) if the possible values of Z(t) are equally spaced. Those equations can be used in equation (5-43) for computation of expected energy not supplied.

As in the case of other interval reliability indices, the comments about dimensions again apply. The reliability index E[U(t)] involves two dimensions, the dimension of the index itself (MWh) and the period of time over which the computation applies. Thus, if the calculated value of $E[U(t)]$ is 236 MWh, based on a period of one year, the correct mathematical statement is

$$E[U(1\ \text{year})] = 236\ \text{MWh}$$

The industry convention is to state this result as

$$\text{EENS} = 236\ \text{MWh/year}.$$

5.8.4 Expected loss of load occurrences

Loss of load occurrences, $N(t)$, is defined as the number of times the loss of load event "occurs" during $(0,t)$. In terms of the indicator function $I(t)$, a loss of load occurrence time is a time point where $I(t)$ changes value from zero to one, and $N(t)$ is the number of loss of load occurrence times in $(0,t)$.

As with the other interval reliability indicators, the probability law of $N(t)$ cannot in general be computed. Further, it is not even possible in general to compute the expectation of $N(t)$ in terms of the expectation of other reliability indicators, as was the case with loss of load hours and energy not supplied. However, under special conditions, called "steady state conditions", the expected value of $N(t)$ is related in a simple way to the frequency of loss load.

First it is necessary to define what is meant by the "steady state expected value of $N(t)$". Suppose that the total length of the observation period is expressed as $(s+t)$, where s is the first part of the interval and t is the last part. Thus, the observation interval is the interval $(0,s+t)$, consisting of the interval $(0,s)$ plus the interval $(s,s+t)$. The number of loss of load occurrences in the second part of the total interval is denoted $N(t;s)$. It can be expressed as

$$N(t;s) = N(s+t) - N(s) \qquad (5\text{-}44)$$

It should be noted that $N(t;s)$ is a different reliability indicator than $N(t)$. The "steady state expected value of $N(t)$" is defined by

$$E[N(t;\infty] \equiv \lim_{s\to\infty} E[N(t;s)] \qquad (5\text{-}45)$$

It is the parameter $E[N(t;\infty)]$ which is related in a simple way to the frequency of loss of load. If $\{I(t)\}$ is assumed to be an alternating renewal process, then

$$E[N(t;\infty)] = f't \tag{5-46}$$

where f' is the frequency defined by equation (5-25) in Section 5.7. If $\{I(t)\}$ is not an alternating renewal process, but the expected cycle time is constant under steady state conditions, as defined by equation (5-27), then

$$E[N(t;\infty] = ft \tag{5-47}$$

where f is the frequency defined by equation (5-29) in Section 5.7.

In either case, the expected (number of) loss of load occurrences is just the loss of load frequency multiplied by the length of the observation period.

The physical dimension of $E[N(t;\infty)]$ is occurrences (of loss of load). However, as with other interval indices, the time over which the calculation applies must also be stated. A convenient way to do this is by the convention "occurrences/period" where "period" defines the length of the observation interval. For example, if the calculated value of $E[N(t;\infty)]$ over a 1 year period is 3.2 occurrences, the correct mathematical statement using the notation of this section is

$$E[N(1 \text{ year}; \infty)] = 3.2 \text{ occurrences}$$

where the infinity notation refers to the steady state conditions under which the computation was made. The convention is to report this result as 3.2 occurrences/year.

It would seem logical to adopt a mnemonic symbol for $E[N(t;\infty)]$ similar to HLOLE for $E[T_H(t)]$ and EENS for $E[U(t)]$. A symbol such as ELOLO (expected loss of load occurrences) would be natural. In practice, this has not been done. Rather, the name "frequency" is commonly used to report calculated results of $E[N(t;\infty)]$. Thus, the previous result would commonly be stated

Frequency = 3.2 occurrences/year

This means that in practice, the name "frequency" is used to describe two different reliability indices:

1) Frequency = reciprocal of steady state expected cycle duration.
2) Frequency = steady state expected number of occurrences

It can be pointed out that if t=1 time unit in equation (5-46) or (5-47), then the steady state expected occurrences is numerically equal to the reciprocal of the steady state expected cycle duration. However, the calculation time period is often divided into sub-intervals (e.g., maintenance intervals). For each interval i the frequency of loss of load (f_i) is constant. The steady state expected loss of load occurrences is then calculated as

$$E[N(t;\infty)] = \sum_{i=1}^{m} f_i \, m_i \text{ occurrences}$$

where m_i is the duration of interval i and

$$\sum_{i=1}^{m} m_i = t$$

In this case it is clear that the calculated total occurrences is not the reciprocal of the expected cycle time for any one maintenance interval. Yet the name frequency is commonly used to describe such a result.

Actually, the convention is analogous to the convention of using the name loss of load probability (and symbol LOLP) to describe the expected value of $T_p(t)$, loss of load peaks. That is, the models used actually compute P(L;t) for any time t and frequency f under steady state conditions, given the system configuration at time t. Each of these two "point" calculations are then "accumulated" over the total interval of interest, with due account of the changes in system configuration (e.g., maintenance) or load demand pattern. In both cases, the cumulative quantity is physically and dimensionally different than the computed point quantity. However, there is a long-standing industry convention for naming both quantities by the name properly associated with the point quantity. Thus, $E[T_p(t)]$ is commonly called loss of load probability (rather than expected loss of load points, and $E[N(t;\infty)]$ is commonly called frequency rather than expected number of occurrences. It is interesting to note that no such confusion exists for the interval reliability index E[U(t)]. This index is correctly called expected energy not supplied. Yet, in practice, the computation algorithms in use actually compute E[Z(t)], expected load not supplied. This value is accumulated through time to obtain E[U(t)].

These industry conventions may result from a lack of appreciation of the physical phenomena (reliability indicators) which underly each reliability index. One basic objective of this section is to show the distinction and relation between point, duration, and interval reliability indicators, and the associated reliability indices.

5.9 SUMMARY OF RESULTS

5.9.1 Reliability indicators

The basic point of view in this section is that reliability indices are parameters of probability laws of reliability indicators, which are empirical characteristics of a power system. Three types of reliability indicators have been identified.

1) Point reliability indicators
2) Duration reliability indicators
3) Interval reliability indicators

Point and interval reliability indicators are defined for every point in time. Point indicators describe the system condition at the observation time, and interval indicators describe the cumulative performance of the system over the interval (0,t). Duration indicators are observations of the duration of time between specified occurrences. Table 5.2 is a summary of the reliability indicators upon which one or more reliability indices are based.

Table 5.2

RELIABILITY INDICATORS

Symbol	Name
	Point Reliability Indicators
$Z(t)$	Load not supplied
L	Loss of load
$I(t)$	Indicator function (of loss of load)
	Duration Reliability Indicators
$D_L(i)$	Loss of load duration
$D_C(i)$	Cycle duration
	Interval Reliability Indicators
$T_P(t)$	Loss of load points
$T_H(t)$	Loss of load hours
$N(t)$	Loss of load occurrences
$U(t)$	Energy not supplied

The loss of load reliability indicator (L) is an event. The indicator function I(t) is a numerical-valued observation of event L. Thus, event L and the indicator function I(t) represent the same random phenomenon. With this understanding, there are eight distinct reliability indicators listed in Table 5.2.

All of the eight reliability indicators can be derived from a time record of the point reliability indicator, load not supplied. Therefore, load not supplied is considered to be the fundamental reliability indicator.

Each numerical-valued reliability indicator defines both a random variable and a stochastic process. The stochastic process is the collection of random variables defined by the indexing parameter. For point and interval reliability indicators, the indexing parameter is time. For the duration reliability indicators the indexing parameter in the number of loss of load occurrences.

5.9.2 Reliability Indices

The probability law of each reliability indicator forms the basis for at least one reliability index. For numerical-valued reliability indicators, the basic reliability index is the expectation of the reliability indicator. For event L, which is not numerical-valued, the probability of event L is the basic reliability index. Tables 5-3 shows the eight basic reliability indices which correspond to the eight reliability indicators.

Table 5.3
BASIC RELIABILITY INDICES

Reliability Indicators	Reliability Index	Index Name
	Point Reliability Indices	
$Z(t)$	$E[Z(t)]$	Expected load not supplied
L	$P(L;t)$	Loss of load probability
	Duration Reliability Indices	
$D_L(i)$	$E[D_L(i)]$	Expected loss of load duration
$D_C(i)$	$E[D_C(i)]$	Expected cycle duration
	Interval Reliability Indices	
$T_P(t)$	$E[T_P(t)]$	Expected loss of load points
$T_H(t)$	$E[T_H(t)]$	Expected loss of load hours
$N(t)$	$E[N(t)]$	Expected loss of load occurrences
$U(t)$	$E[U(t)]$	Expected energy not supplied

A ninth reliability index is defined by considering the conditional distribution of load not supplied, given loss of load. The expected value of this distribution gives the reliability index.

$E[Z(t)/L]$ = Conditional expected load not supplied

The reliability indices in Table 5.3 are defined without any restriction on the probability law of the associated reliability index other than the assumption that the expected value exists. However, in computation models for planning applications, it is common to compute the reliability indices based on duration reliability indicators under "steady state" conditions. That is, expected loss of load duration and expected cycle time are computed under the assumption that a "large number" of loss of load occurrences have been observed. Specifically, the steady state values of the duration reliability indices were defined in Section 5.7 as follows:

$$d_L = \lim_{i \to \infty} E[D_L(i)]$$

$$d_C = \lim_{i \to \infty} E[D_C(i)]$$

Under steady state conditions, the reciprocal of d_C is called loss of load frequency. Thus frequency of loss of load is defined by

$$f = \frac{1}{d_C}$$

The steady state values d_L and d_C are still expected values. For example, d_L is the "steady state expected loss of load duration". By contrast, frequency as defined in this discussion is not an expected value. Further, if frequency is only defined for steady state conditions, it is not necessary to use the phrase "steady state" in the name. Thus, f is called simply "loss of load frequency", and the two parameters f and d_L could be described together as "the frequency and steady state expected duration of loss of load". In practice, it is common to refer simply to f and d_L as simply "frequency and duration (of loss of load)".

Table 5.4 summarizes all the reliability indices based on duration reliability indicators, including those defined under steady state conditions.

Table 5.4

RELIABILITY INDICES BASED ON DURATION RELIABILITY INDICATORS

Reliability Indicator	Reliability Index	Index Name
$D_L(i)$	$E[D_L(i)]$	Expected loss of load duration
	d_L	Steady state expected loss of load duration
$D_C(i)$	$E[D_C(i)]$	Expected cycle duration
	d_C	Steady state expected cycle duration
	f	Loss of load frequency

5.9.3 Computation of Reliability Indices

Analytical models are available for calculation of loss of load probability, in terms of basic input data on equipment capability (capacity) and load demand. The single parameter $P(L;t)$ completely defines the probability law of the reliability indicator, event L. Similarly, analytical models are available for calculation of the probability law of load not supplied. The reliability index $E[Z(t)]$ can then be computed directly from the defining equation for expectation. Thus, the point reliability indices are computed directly from the probability laws of the associated reliability indicators.

By contrast, it is not generally practical to compute the probability laws of the duration reliability indicators $D_L(i)$ and $D_C(i)$. Neither are models generally available to calculate the expectations $E[D_L(i)]$ and $E[D_C(i)]$ for each loss of load occurrence. Rather, existing methods are directed at the calculation of the steady state reliability indices f and d_L. These are called frequency and duration models.

In practice, the third class of reliability index, those based on the interval reliability indicators are the most important indices because these are ones which are actually of interest in planning. However, analytical methods to compute the probability law of these reliability indicators are not available. Fortunately, the expected value of the interval reliability indicators can be computed from the expected value of the point and duration reliability indicators. These computations are shown in Table 5.5.

Table 5.5

COMPUTATION OF INDICES BASED ON INTERVAL RELIABILITY INDICATORS

Desired Index	Computed Index	Computation of Desired Index
$E[T_P(t)]$	$P(L;t)$	$\sum_{i=1}^{n} P(L;t_i)$
$E[T_H(t)]$	$P(L;t)$	$\int_0^t P(L;y)dy$
$E[U(t)]$	$E[Z(t)]$	$\int_0^t E[Z(y)]dy$
$E[N(t;\infty)]$	f	ft

Table 5.5 is one of the key results of this section. It emphasizes the fact that four reliability indices are of primary interest for applications - the "desired" column of Table 5.5. However, these indices cannot be computed directly, in the sense that the probability law of the underlying random variable (interval reliability indicator) cannot be computed. However, each desired index can be computed in terms of the parameters of other probability laws, which in turn can be computed in terms of known system characteristics. Thus, three fundamental "computed" indices are not presently used in applications. They exist for the sake of computing the interval reliability indices.

The computation equations in Table 5.5 are not definitions of the reliability indices computed by the equations. The reliability indices are expected values, and as such they imply the existence of an underlying random phenomenon (called a reliability indicator) whose expected value is being computed. In Table 5.5, the interval reliability indicator whose expectation is being computed is the quantity inside the brackets in the first column.

5.9.4 Uniform versus industry terminology

Table 5.6 gives a comparison of the name given to each reliability index using the uniform terminology of this section and the name which is in common industry use. For reference, the symbol used for the index in this section is also shown.

Table 5.6

COMPARISON OF UNIFORM TERMINOLOGY AND PRESENT INDUSTRY TERMINOLOGY

Uniform terminology	Symbol	Industry terminology
Loss of load probability	$P(L;t)$	Loss of load probability (Note 1)
Expected loss of load points	$E[T_P(t)]$	Loss of load expectation (Note 1)
Expected loss of load hours	$E[T_H(t)]$	Hourly loss of load expectation (Note 1)
Loss of load frequency	f	Frequency
Steady state expected loss of load duration	$\bar{d}_L$	Duration
Expected load not supplied	$E[Z(t)]$	None
Conditional expected load not supplied	$E[Z(t)/L]$	Expected loss of load
Expected energy not supplied	$E[U(t)]$	Expected energy not supplied (Note 2)
Expected loss of load occurrences	$E[N(t;\infty)]$	Frequency

NOTE 1: The name loss of load probability is often applied to the second and third indices. The first index is then sometimes called "risk".

NOTE 2: In earlier years, the name loss of energy probability was used, with symbol LOEP. The name expected unserved energy (EUE) has also been used in recent years.

As stated in Note 1 of Table 5.6, the name "loss of load probability" is often associated with the interval reliability indices $E[T_P(t)]$ and $E[T_H(t)]$ rather than with $P(L;t)$. Similarly, Table 5.6 shows that the name frequency is commonly used to describe both the index f and the index $E[N(t;\infty)]$.

5.10 NEW RELIABILITY INDICES

One of the objectives of this project was to identify the need for new reliability indices. The unified structure in this section provides a framework for considering possible new indices. The question of possible new indices can be considered with respect to the following three questions.

1) Does the stochastic process $\{Z(t);t>0\}$ completely describe the reliability of a power system (or some point in a power system) or are there other random phenomenon (empirical observations) not described by $\{Z(t)\}$ which describe "reliability"?

2) Are there any additional "reliability indicators" which can be defined in terms of the process $\{Z(t)\}$. If so, the expectation or other parameters of the probability law of these indicators would be possible reliability indicators.

3) What additional parameters of the probability laws of the six reliability indicators defined in this section would be useful?

Consideration of question 1 raises several possibilities. For example, Section 2 describes a list of "internal reliability indicators." See Exhibit 2-1. In general, these indicators are not described by $\{Z(t)\}$. Rather, $\{Z(t)\}$ describes the "external" indicators in Exhibit 2-1. For each light or meter in Exhibit 2-1, a corresponding random phenomenon could be defined. A second possibility in response to question 1 would be to consider the "excess capability" of the system. Thus, when $Z(t)=0$, the load is fully supplied. In general, the system is then capable of supplying more load than the current demand. Thus, a possible generalization of $\{Z(t)\}$ would be to define the margin $M(t)$ by

$$M(t) = C(t) - D(t)$$

where

$C(t) =$ Load supplying capability at time t

$D(t) =$ Load demand at time t.

In terms of margin, Z(t) can be defined as

$$Z(t) = 0, \quad M(t) \geq 0$$
$$Z(t) = -M(t), \quad M(t) < 0$$

Therefore, any reliability index defined by the process {Z(t)} could also be obtained from the process {M(t)}. However, {M(t)} could be used to define additional reliability status indicators based on positive margin.

Question 2 asks whether additional status indicators can be obtained from {Z(t)} itself. It is easy to think of additional "useful" indicators. For example, consider the maximum value of Z(t) in (0,t).

$$Z_{max}(t) = \max \{Z(y); 0 < y \leq t\}$$

This is a random variable just like the other status indicators, such as $T_H(t)$, which were derived from Z(t), and $\{Z_{max}(t); t>0\}$ is a stochastic process. A fundamental reliability index based on $Z_{max}(t)$ would be $E[Z_{max}(t)]$, the expected maximum load not supplied in (0,t). However, while it is easy to define useful indicators and associated reliability indices, it is not easy to compute such reliability indices. In particular, it is not possible to compute $E[Z_{max}(t)]$ in terms of the three "computable indices" in Table 5.7. On the other hand, it may be possible to calculate the value of Z_{max} for a given set of possible contingencies and load demands. Under these conditions, Z_{max} is not a random variable, but it is a single value determined by the set of contingencies and loads. Thus, Z_{max} in this case could be called a deterministic reliability index. This idea is discussed further in Section 4 for the transmission system.

Question 3 asks whether additional reliability indices can be defined using the existing six status indicators based on {Z(t)}. Again, it is easy to formulate useful indices. In general, two categories of additional indices could be considered

1) Higher moments (e.g., variance)

2) Percentiles.

If the entire distribution of a random variable is known, then any specified moment or percentile can be computed. In the case of Z(t), present analytical methods are available to define the whole distribution. However, for the interval status indicators, $T_H(t)$, $T_P(t)$, N(t), and U(t), it is not in general practical to compute the entire distribution functional analytically. Simulation models (e.g., Monte Carlo) models could be used, however. Such models have become much more practical in recent years with increases in computing speeds and available software.

References

5-1 E. Parzen, Modern Probability Theory and Its Applications, Wiley 1960.

5-2 E. Parzen, Stochastic Processes, Holden-Day 1962.

Section 6

EVALUATION OF INDICES

6.1 INTRODUCTION

The work described in this section was directed toward the identification of those attributes, qualities, or features of reliability performance indices which are most important and relevant to consumers, regulatory bodies, and electric utility planners. The work proceeded through three steps or avenues. First, a preliminary list of important reliability performance attributes was developed based on the investigator's experience and judgement and a general understanding of the literature. Second, a survey outline was developed to be used in interviews with regulatory bodies and electric utility personnel to obtain their views on important attributes of reliability indices. The third avenue of investigation was a review of the literature on methods for quantifying the cost and worth of electric service reliability. Here the focus of the investigation was on discovering those attributes of service reliability which are of economic importance and which must be characterized or computed in reliability studies if economic assessments of reliability cost/worth are to be made.

The final result of the investigation reported in this section is a listing and evaluation of the relative importance of the various indices of reliability performance.

6.2 PRELIMINARY INVESTIGATION OF RELIABILITY PERFORMANCE ATTRIBUTES

As a first step in the investigation and determination of those attributes of reliability performance (and reliability performance indices) which are most important and relevant to consumers, regulatory agencies, and utility planners, the investigators listed and discussed attributes as given in the following sections.

6.2.1 Consumers

Consumers are divided into three categories for purposes of analysis and discussion. These categories are: domestic, commercial, and industrial. Domestic consumers are, it is believed, primarily concerned with those attributes of reliability which impact convenience and comfort. Also of importance to domestic consumers are subjective judgements of reliability trends and the efficiency of utility management. In contrast, industrial consumers are probably most concerned with those reliability attributes which directly influence the cost, ease, quality, and quantity of production. The concerns of commercial consumers would seem to fall somewhere between those of domestic and industrial consumers.

Attributes believed to be of general concern to all classes of consumers are listed first; then followed by discussions of concerns which may be unique to a consumer class. Underlying all the attributes listed is the notion that consumers react only to actually experienced reliability performance. Thus, reliability indices must have absolute, measurable, significance to be of value from the consumer's viewpoint. The list of reliability attributes follows in rough order of estimated importance.

(1) Number of interruptions per year. Since consumers may be nonlinearly sensitive to the number of interruptions experienced per year or other time period, this attribute implies the probability distribution of the number of interruptions in a year or other time period as well as the long-term average number of such interruptions. It is also understood that what constitutes an interruption from the consumer viewpoint may be very much a function of the nature of the consumer.

(2) Duration of interruption. The sensitivity of consumers to interruption duration is believed to be very nonlinear and to vary appreciably from one class of consumers to another. Thus, the probability distribution of interruption durations is important as well as the long term average duration.

(3) Energy demanded, but not supplied.

(4) Time of day, day of week, and season of year of interruption.

(5) Forewarning, if any, of interruption.

(6) Geographical extent of interruption.

(7) Reason for interruption. Interruptions whose cause is obvious and outside of control of utility, e.g., storm-caused interruptions, appear to be regarded more tolerantly than other interruptions.

(8) Time since last interruption. It is believed that the irritation with a particular interruption event may well be a function of the time since the last interruption. Thus, bunched interruptions might be regarded as more irritating than evenly spaced interruptions. This attribute probably applies most importantly to domestic consumers whose evaluation of service reliability is primarily subjective.

It is observed in general that consumers' reliability expectations are thought to be highly correlated with past performance. Thus, any significant reduction in reliability performance, regardless of base level, is likely to cause strong negative reactions.

Domestic Consumers. Some comments on the general list of attributes given above follow.

(1) Domestic consumers probably have little sensitivity to very short interruptions (up to one or two minutes) unless the frequency of such interruptions is high.

(2) The sensitivity to interruptions is believed to be very dependent on time-of-day, day-of-week, and season. It has been suggested that interruptions during the prime leisure hours, early evenings and weekends, are viewed much more critically than interruptions at other times. It is believed that housewives can reschedule their activities so that interruptions during normal working hours are much less objectionable than those during leisure hours.

(3) It is believed that unserved energy is not particularly relevant from the point of view of individual domestic consumers.

(4) It is believed that interruption forewarning has little relevance to domestic consumers.

Commercial Consumers. Some comments regarding this class of consumers follow.

(1) It is believed that safety and security are prime concerns of commercial consumers. Hence, such consumers are probably very nonlinearly sensitive to interruption duration.

(2) Commercial consumers would clearly be far more sensitive to interruptions during business hours than at other times.

Industrial Consumers. Some comments regarding this class of consumers follow.

(1) The sensitivity of industrial consumers to interruption duration is very much a function of the industrial process. Some processes are sensitive to very short interruptions, or even voltage dips, which would not create problems for other types of consumers.

(2) Service quality attributes such as voltage regulation, waveforms, and frequency tend to be more important to industrial consumers than to other classes of consumers. This suggests that reliability indices reflecting continuity only may not be sufficient for some industrial consumers.

(3) Energy not supplied is believed to be an important reliability attribute from the point-of-view of most industrial consumers, but such subjective attributes as geographical extent of interruption, reason for interruption, and time since last interruption are probably not as important.

(4) Forewarning of interruption would seem to be more important to industrial consumers than to other types of consumers. However, the Ontario Hydro survey concluded that forwarning was not very significant in its impact on interruption cost.

6.2.2 Utility Planners

The basic desirable attributes of reliability indices from the point-of-view of utility planners are believed to be as follows.

(1) Reliability indices should accurately reflect the influence on system reliability performance of system and load characteristics. The parameters which are important are a function of the level of a system study: generation, transmission, or distribution, but the following is a partial list of those parameters or factors whose influence should be considered.

(a) Number, capacity, and characteristics of generating units including failure and repair rates, start-up failure probabilities, start-up times, and outage postponability distributions;
(b) operating reserve policy including unit commitment policy and other factors which influence the operating duty of generating units;
(c) basic energy limitations for generating units;
(d) planned outage schedules for generators, lines, and other major pieces of equipment;
(e) failure rates and repair times of T&D equipment with special attention to treatment of non-independent failures;
(f) failure and repair characteristics of protective relay and control systems;
(g) network topology and switching schemes including times to perform switching operations;
(h) load cycle shape;
(i) load forecast accuracy.

(2) Reliability indices and methods should be capable of displaying the sensitivity of system reliability performance with respect to planning parameters. This capability or attribute provides guidance in the choice of alternatives in the planning process.

(3) Reliability indices should be physically significant and measurable from historical information. If these conditions are met, it follows that indices are absolutely, as opposed to relatively, significant and the methods for their calculation can be verified against historical performance.

(4) Reliability indices should be meaningful to the public and understandable by, and defensible before, utility commissions. It does not follow

that all useful and valid reliability indices should measure consumer reliability performance. However, it is felt that the best, and ultimately the most useful and defensible, indices for planning purposes should measure consumer performance. Only indices which measure consumer performance in an absolute manner will be useful in economic evaluations of reliability.

6.2.3 Regulatory Agencies

The views of regulatory agencies are expected to fall somewhere between those of consumers and utility planners. That is, regulatory agencies are believed to be concerned about consumers on the average rather than individually and to be concerned about the final effects and justifications of plans rather than the iterative process through which these plans were evolved. Thus, the desired basic attributes of reliability indices are believed to be as follows.

(1) Reliability indices, whether directly indicative of consumer reliability performance or not, should be physically meaningful and measurable from historical information.

(2) Reliability indices which measure consumer reliability performance should be significant in an absolute sense and accurately measure average consumer reliability experience. Measures of performance which are believed to be most significant are: (1) expected unserved energy, (2) average number of interruptions per year, (3) average interruption duration, and (4) the magnitude of interruptions as measured by the number of consumers interrupted, MW interrupted, or geographical extent of interruption.

(3) Reliability indices should be suitable for use in computing the economic impact of interruptions.

6.3 INTERVIEWS WITH UTILITIES AND REGULATORY AGENCIES

It was considered important to obtain a sampling of the views of utilities and regulatory agencies on those attributes of reliability performance and reliability indices which are most important from their respective points of view. Accordingly, an interview format was designed for utilities and for regulatory agencies and a limited number of interviews was conducted. The interview formats and the results of the interviews conducted are presented in the following sections.

6.3.1 Utility Interviews

In interviews with utilities it was desired to obtain the viewpoints of: (1) utility planners and designers; (2) customer service personnel; and (3) personnel who appear before regulatory agencies. It was felt that the views of these personnel would cover the spectrum of possible uses of reliability indices in a utility namely: (1) internal use for system planning and design; (2) use in dealing directly with consumers and in assuring consumer satisfaction; and (3) use in support of hearings before regulatory agencies.

Each utility interview began by a presentation of the preliminary ideas presented in Section 6.2 above to indicate to the interviewees the general nature of the information desired and to provide a vehicle for discussion. Thereafter, the following questions were posed to the utility personnel being interviewed.

A. What attributes of reliability performance are most important and significant to domestic, commercial, and industrial consumers?

B. What reliability indices are most meaningful from consumer point-of-view?

C. What is the relationship of computed reliability indices to actual experience? Is it considered important that indices have absolute significance?

D. What features of reliability indices are most important to planners? What reliability indices are most useful now? What improvements or different indices are needed for the future?

E. What fundamental attributes of reliability performance are most important to the Public Service Commission (PSC)?

F. What present reliability indices does the PSC find most acceptable?

G. What are the desirable attributes or features of reliability indices for use in PSC proceedings? Does the PSC think it important that indices be physically measurable and have close relationship to actual experience?

H. Does the PSC favor economic evaluation of reliability performance? If so, what methods are preferred?

I. What methods of reliability evaluation does the PSC seem to be favoring for the future?

6.3.1.1 Baltimore Gas and Electric Company

An interview was held at the Baltimore Gas and Electric Company on July 9, 1979. In attendance at the meeting were BG&E personnel with responsibilities in the areas of generation and transmission planning, regulatory agency hearings, and energy (consumer) services. Responses to the questions listed above are summarized as follows.

<u>Questions A and B</u>

General comments:

1) The number of consumers affected by an interruption is very important as is the geographical extent of the interruption.
2) Forewarning of interruption is important for all classes of consumers. Here, at least for domestic consumers, the public relations aspect of forewarning is of primary importance.
3) Rapid explanation of an interruption and its expected duration is important. It is believed that consumers who are kept informed will be more tolerant of the situation.
4) It is believed that consumers display a definite threshold of irritation to interruptions. Once a consumer is provoked by a major problem, he is much less tolerant of other minor problems.
5) There are very few complaints to the PSC concerning the number of interruption experienced. Rather, complaints tend to be concerned with the effects of the interruptions, i.e. damage to an industrial process.
6) Unserved energy is not a very meaningful attribute of service reliability from the consumer's viewpoint. Much more meaningful are the frequency and duration of interruptions and the costs they entail.

Domestic consumers:

1) The sensitivity of consumers to interruption is very much a function of whether the consumer is all electric or has gas service as well. Consumers with electric service only are believed most sensitive to interruptions in the winter while consumers with both gas and electric service are most sensitive to summer interruptions. Interruptions in spring or fall are regarded as much less troublesome than those in winter or summer regardless of the type of service.
2) The time since last interruption is regarded as somewhat important. This is related to the psychological impact of the interruption on the consumer.
3) Consumers are non-linearly sensitive to the duration of interruptions. Maximum sensitivity is believed reached for interruption duration of two days or more.

4) Time-of-day of the interruption is regarded as very important with interruptions in the period 5-9 PM regarded most negatively. The time-of-day effect is believed to be somewhat influenced by type of service: electric only or gas and electric.
5) The importance of attributes of service reliability are ranked as follows: (1) time-of-day of interruption, (2) duration of interruption, (3) frequency of interruption.

Industrial consumers:

Service reliability attributes in estimated order of important to industrial consumers are given as follows.

1) Time-of-day, week, or season of interruption. Interruptions during working hours and at times of peak production or dependence on electric service are much more important than are interruptions at other times.
2) Forewarning. Interruption forewarning is very important as it permits orderly shutdown of a process to prevent excessive loss of product and/or damage to equipment.
3) Service quality. Service quality problems including single-phasing and voltage dips and spikes are important. These types of problems cause equipment damage and product losss unique to the industrial consumer.
4) Interruption duration.
5) Time since last interruption.
6) Interruption frequency.
7) Reason for interruption. The industrial consumer is not sensitive to the reason for an interruption-unless the interruption is repeated. Thus, the industrial consumer is motivated by economic rather than psychological considerations.
8) Geographical extent of interruption. The industrial consumer is primarily concerned with his own service.

Hospitals:

It is felt that hospitals would rank service reliability attributes as follows.

1) Interruption duration.
2) Reason for interruption. The reason for the interruption is important to prevent a repetition with consequent danger to patient safety.

3) Interruption frequency.
4) Time since last interruption.
5) All other attributes.

Commercial consumers:

Commercial consumers are a very diverse group making it difficult to generalize the sensitivities of these consumers to the various attributes of service reliability. Many commercial consumers are organized into business groups with potent political and public relations powers. This fact coupled with the fact that commercial consumers serve the public means that interruptions to commercial consumers may have very negative consequences.

Service reliability attributes in estimated order of importance are as follows.

1) Interruption frequency.
2) Interruption duration. The sensitivity to interruption duration is very much a function of the nature of the business.
3) Time-of-day, day-of-week, season-of-year of interruption.

Other factors of secondary and more-or-less equal importance are

a) Forewarning.
b) Geographical extent. Severe reaction is not expected unless commercial consumers in a shopping center or three continuous blocks along a street are interrupted for more than 10 minutes.
c) Reason for interruption. Commercial consumers are tolerant of any interruption if the reason for the interruption is obvious and beyond the control of the utility.
d) Time since last interruption.

Unsupplied energy is not felt to be an important attribute from the viewpoint of commercial consumers.

Question C

The opinion was that planners generally do not need indices that have absolute significance. Rather, what is needed by planners are indices which are sensitive to the right factors and which are capable of reflecting incremental reliability costs. However, it was also felt that utility commissions would favor indices which were absolutely significant and which gave results comparable to historical results. An opinion was expressed that top management tended to have views similar to those of the regulatory commission on this point.

Some limited comparison of historical and computed reliability indices measuring generating capacity adequacy has been made. These comparisons have indicated reasonable agreement between computed and historical results.

Question D

Planners need to have reliability indices that lend themselves to sensitivity analyses of the underlying assumptions and study parameters. That is, reliability indices should be computed using methods which explicitly recognize the factors and parameters which are important in the planning process. This suggests that more detailed reliability models may be required in the future.

It is also very important that reliability indices be explainable and defendable to people outside the reliability engineering field and the system planning department.

The most widely used and accepted index for generation capacity studies is LOLE though frequency and duration indices are used for some special studies. Frequency and duration indices are used exclusively for transmission, substation, and distribution studies where quantitative reliability methods are employed. It is anticipated that more detail will be required in the modeling process in the future.

Question E

The utility commissions are becoming more and more interested and involved in the details of planning studies. That is, the commissions are now less willing to view the utility as a "black box" and only concern themselves with the final output. Rather, the commission wants the control and right of review of planning alternatives and planning criteria. It should be pointed out, however, that the commissions do not want the responsibility of initiating porposals, but the right of review of proposals made by the utilities.

The utility commissions appear to favor indices which can be expressed in readily understandable terms and which have actual physical significance. The amount of load not served and the frequency and duration of curtailment events would fall in this category. It should be noted, however, that the commissions are not presently explicitly requesting these featurs.

Question F

The New Jersey PUC has accepted the LOLE index for generation evaluations. It appears that the Maryland PSC will be similar in point-of-view. Quantitative reliability evaluation indices do not seem to be applied to transmission and distribution facilities.

Question G

The commissions are believed to favor indices which are readily explainable and appealing to the public. They are also believed to favor indices for which some relationship between computed indices and actual historical experience can be demonstrated. The New Jersey PUC, for example, was favorably impressed by a study which showed the relationship between voltage reductions and load reductions and low generating reserves.

Question H

The Maryland PSC has not mentioned economic evaluation of reliability. However, it was felt that the PSC might desire such an evaluation if a practical method was developed.

In a general discussion of economic evaluation of reliability the following points were made.

1) An overall economic index is a good objective, but many never be achieved.
2) A trend toward economic evaluation does exist as evidenced by the over-under capacity evaluation studies which have been made recently.
3) It is believed that the cost of outages may have been included in transmission studies made in Pennsylvania.

Question I

The PSC has not brought up the question of economic evaluation of reliability to date.

6.3.1.2 Cleveland Electric Illuminating Company

An interview was held at the Cleveland Electric Illuminating Company on October 3, 1979 with company representatives with responsibilities in the areas of generation, transmission, and distribution planning; regulatory commission hearings; and energy application (consumer) services in attendance. Responses to the questions listed above are summarized as follows.

Question A

General satisfaction was expressed with the preliminary list of attributes given in Section 6.2.1, but the following specific points were also made.

Domestic consumers:

1) There is little sensitivity to interruptions of short duration as long as the frequency of such interruptions in low.
2) The sensitivity to outage duration is non-linear with an appreciable increase in sensitivity for durations greater than 8 to 10 hours. However, obvious severe weather conditions mitigate sensitivity.
3) Forewarning of interruptions is not very important except for public relations effects (but these PR effects are recognized as important to the company).
4) Interruption duration is believed to be the most important attribute of reliability performance for "normally spaced" interruptions.
5) The bunching of interruptions (time since last interruption) is regarded as important as a psychological effect which bears on the public's perception of utility efficiency. The bunching of interruptions may trigger complaints to the Public Service Commission.

Industrial consumers:

1) Momentary interruptions are very important to many industrial consumers.
2) Forewarning of interruptions is considered important and is routinely done where possible. Care is also taken to minimize momentary outage effects during restoration procedures.
3) It is considered important to keep consumers advised of expected time needed to restore service when an interruption occurs.
4) It is believed that consumers are sensitive to the frequency of interruptions and may well take their experience into account when making load expansion decisions.

5) It is believed that major industrial consumers are primarily concerned with the long-term adequacy of generation and bulk transmission (primarily concerned with energy adequacy).

Commercial consumers:

1) Interruption duration is regarded as very important.

2) Momentary interruptions, as well as sustained interruptions, are important for many.

3) The geographical extent of an interruption is not thought to be very important except for possible impact on public disturbance aspects.

4) Commercial consumers are regarded as more pragmatic and less affected by psychological factors than domestic consumers.

Question B

The consumer reliability indices used by the company are: (1) sustained interruption frequency, (2) momentary interruption frequency, (3) expected sustained interruption duration, and (4) an index formed by the sum of sustained interruption frequency and duration. This last index is used for some relative comparisons.

Question C

It is felt that "absolute" reliability indices are generally preferable. Great pressure does not exist to establish the relationship between computed and historical indices.

Question D

Distribution:

1) Quantitative reliability prediction methods are not presently used in distribution, but may find future application if a data base is established.

2) The primary need for the distribution planner is for methods and indices which will pinpoint the seat of difficulties.

Generation:

1) The indices presently computed and used are the expected number of days per year that supplemental resources including outside purchases are required together with the magnitude of such supplemental requirements. These indices are presently computed using daily peak loads only.
2) Indices which may be used in the future include: (a) expected unserved energy computed using hourly load models, and (b) frequency and duration of shortage events.

Bulk systems (generation and transmission):

1) The approach to bulk system reliability evaluation presently used is to discover those sets of contingencies, if any, which cause overloads or voltages outside limits and which have probability of occurrence greater than a threshold level. That is, a probabilistic index as such is not computed.
2) In the future it is proposed to compute an index which is the probability of trouble considering "all" contingencies.

Questions E-I

The Public Utility Commission staff is primarily concerned with generation adequacy and currently relies on percent reserve as an index of reliability. Currently, the PUC is trying to recognize more factors influencing generation reliability such as unit demonstrated capabilities and unit deratings. In the future, the PUC may move toward more quantitative and realistic reliability indices.

It is believed that the PUC would favor economic evaluation of reliability if suitable methods were available and understood.

6.3.1.3 Pacific Gas and Electric Company

An interview was held at the Pacific Gas and Electric Company on May 8, 1980 with company representatives with responsibilities in the areas of generation, transmission, and distribution planning and regulatory commission hearings. Responses to the questions listed above are summarized as follows.

Questions A and B

The most important indices of reliability performance to consumers are believed to be: 1) interruption duration, and 2) interruption frequency. Interruption duration is believed to be the more important of the two indices to most consumers. However, some industrial consumers are also very sensitive to interruption frequency. The time of occurrence of interruptions is also believed to be important, particularly to domestic consumers.

Question C

The relationship of computed reliability indices to actual experience has not been an issue in the past, but may be important in the future. Thus far indices having only relative significance have proven satisfactory. As an example, generation reliability indices are presently computed on the basis of dry-year hydro conditions - a deliberately conservative approach.

Question D

Presently, the only quantitative reliability index used in generation planning is LOLE computed on a daily peak basis and assuming hydro capacity consistent with dry-year water conditions. The computed LOLE is used in conjunction with additional deterministic criteria to assess required levels of generation reserve.

Transmission and distribution planning is done using deterministic criteria such as "contingency rules".

A primary future concern is uncertainty in basic energy resource availability. Thus, in the future, indices which can properly reflect energy as well as capacity shortages will be favored.

Questions E-G

The regulatory agencies of the State of California are generally passive on the question of reliability indices and seem to be willing to accept whatever indices are proposed by the utility. The present tendancy is to rely on deterministic indices such as per cent reserve. LOLE is also used in assessing generation reserves and is well accepted.

The commissions are primarily concerned with generation and bulk transmission and with the economics of alternatives at those levels. Hence, no great need has been felt to use reliability indices which relate directly to consumer reliability performance.

Questions H and I

The commissions do not presently seem to favor economic evaluation of reliability performance because of concerns with the accuracy and practicality of such methods. However, they feel that economic evaluation of reliability is a basically proper philosophy and might adopt it if practical methods were available.

The commissions seem to be receptive to the use of new modeling techniques and reliability indices which can be shown to be practical and accurate.

6.3.2 Regulatory Agency Interviews

The interview approach to regulatory agencies was considerably different than that used for utilities. The approach was to simply outline the objective of the interview; then proceed with a list of questions. The agency was not briefed on the investigators' preliminary ideas and lists of attributes as were the utilities to avoid biasing the agency responses. The questions asked in the interview are summarized as follows.

A. What fundamental attributes or measures of reliability performance are important in generation, transmission, distribution, and to the ultimate consumer.

B. What reliability indices or performance measures are most acceptable now in the regulatory process?

C. What reliability indices or performance measures would you like to see used in the future?

D. Do you favor or use economic evaluation of reliability performance? If so, what methods are used or favored?

6.3.2.1 Maryland Public Service Commission

An interview with members of the staff of the Maryland Public Service Commission was held on July 10, 1979. Present were the Chief Engineer, the Chief Hearing Examiner, and a representative of the consumer advocates office.

The investigators found the PSC staff in general sympathy with the objectives of the project and appreciative of the opportunity to provide their ideas and input. There seemed to be a general feeling that quantitative reliability indices, mutually understood and agreed upon, would be useful in the regulatory process and beneficial to both the commission and the utilities. The feeling was that quantitative indices would sharpen and focus arguments, help to make the parties' positions clearer, and reduce the amount of "handwaving" and uncertainty in the regulatory process.

Specific comments and points in response to the general questions listed above are summarized as follows.

1) The fundamental measures or attributes of reliability performance which are believed most important for generation are percent reserve, reserve in relationship to largest units on system, percent of peaking generation, intertie capabilities, and LOLE. Transmission effects are important to the PSC and evaluations must include the effects of major transmission line losses. However, quantitative, definitive indices of transmission performance have evidently not been defined by the PSC. The focus of the PSC is on generation and transmission as far as reliability is concerned. The PSC feels that little can be done about distribution reliability.

2) The PSC feels that the measures of service reliability which are most important to ultimate consumers are, in order: interruption duration and interruption frequency. Thus the commission is concerned about the response of the utility to interruptions when they occur. Other factors believed important are the number of consumers interrupted in a single event and the geographical extent of such an interruption. It was felt that consumers might be more tolerant of an interruption if the cause was obvious and beyond the control of the utility if the interruption duration was no longer than one day, but interruptions of longer duration would be regarded very negatively regardless of cause.

3) The past level of service reliability was felt to have an important effect on consumers' perception of reliability adequacy. That is, there is a reliability ratchet effect. It was also felt that consumers have a definite threshold of irritation which, if exceeded, will cause consumers to begin to complain about many minor problems which would other-

wise be overlooked. It was felt that psychological factors closely related to media exposure and political factors have a great effect on consumers perception of reliability.

4) Reliability indices must be credible and explainable in simple terms to the commission and the public. It is important that reliability indices be validated against past history.

5) Reliability indices must show the effects of specific unit and line additions to be useful. The incremental cost of achieving specific reliability improvements is important.

6) It would be useful to be able to estimate the variation in service reliability from one year to the next as well as the expected (average) service reliability. This would be helpful in explaining and understanding reliability performance. "Confidence levels" on estimated reliability performance are important.

7) Economic evaluation of reliability is regarded as the ultimate answer, but reservations exist as to the practicability of this approach.

8) The PSC is interested in the details of the planning process, but is not interested in making the fundamental decisions. They view their role as solely one of approving or disapproving utility plans.

6.3.2.2 California Public Utilities Commission

An interview with members of the engineering staff of the Utilities Division, Electrical Branch of the California PUC was held on May 8, 1980. Responses to the questions posed are summarized as follows.

General Comments

The primary concern at the California PUC is economics and rates. Little detailed attention has been given to system reliability performance or to reliability as seen by the ultimate consumer. The stated reason for this is "reliability performance has been generally good and has not, therefore, wanranted detailed consideration". Some detailed work is now being done on the availability performance of certain coal-fired power plants to encourage availability improvement. However, the motivation for this work is primarily economics through the reduction of use of expensive fuel oil.

Questions A and B

It is believed that consumers are primarily sensitive to interruption frequency and to service voltage levels and secondarily to interruption duration.

The PUC is primarily concerned with generation and bulk transmission and only to a minor degree with distribution.

LOLE is the primary (evidently only) quantitative index used in connection with generation. It is, however, recognized that LOLE does not have great physical significance. Therefore, the PUC would probably favor more physically-based indices such as the frequency and expected duration of capacity shortage events. It was felt that such indices might be more appealing to the public and to the PUC commissioners. That is, physically-based and simple-to-understand indices would be favored.

Quantitative indices have not been used in evaluating transmission proposals and evidently none are being considered. Transmission systems are evaluated using engineering judgement and simple contingency rules.

Question C

The PUC would probably favor use of more physically-based indices such as "frequency" and "duration" in the future.

A concern was also expressed over energy adequacy as well as capacity adequacy in the future. This suggests the future desirability of energy-related indices such as expected unserved energy.

Question D

The PUC is skeptical about the accuracy and value of economic evaluation of the reliability performance. The view was expressed that the public would react very negatively to any reduction of present reliability levels regardless of economics. (The presumption is that an economic approach to reliability evaluation would suggest an optimum reliability level lower than the present level.) Thus, the PUC basically seems to favor a historical experience approach to setting acceptable reliability levels.

6.3.2.3 California Energy Commission

An interview with members of the engineering staff of the California Energy Commission was held on May 9, 1980. Responses to the questions posed are summarized as follows.

General Comment

The jurisdiction of the California Energy Commission is limited to power plant need and siting and those transmission lines needed to connect units into the transmission grid. Thus, the Energy Commission deals almost exclusively with generation and has no direct responsibility for consumer service reliability.

Question A

The fundamental measures of generating system reliability performance believed to be most important are:

1) frequency and expected duration of capacity margin events,
2) expected unserved energy,
3) magnitude of load loss or capacity deficiency.

No opinion was ventured on indices for transmission and distribution since these systems are outside the jurisdiction of the Energy Commission. It is believed that the frequency and duration of interruption events are the most important attributes of reliability from the ultimate consumer's viewpoint. It is further believed that consumers are non-linearly sensitive to interruption duration and that the sensitivity to interruption frequency may be widely variable with some industrial consumers (semi-conductor manufacturers) very sensitive to interruption frequency.

Question B

The present practice in regulatory proceedings is to characterize generation reliability through the use of per cent reserve where such per cent reserve has been calibrated to yield the desired level of LOLE. Thus, the only quantitative measure of reliability which is presently used in proceedings is LOLE. Other measures of performance including the frequency and duration of capacity margin events are often used in internal Energy Commission studies.

The use of per cent reserve in regulatory proceedings apparently stems from the overriding need for a simple, unambiguous, index in such proceedings.

Question C

The Energy Commission staff sees a need for simple, appealing, indices to be used in the regulatory process. In general the commission would favor consumer-oriented indices having absolute significance and which have been validated against historical experience.

The commission believes that improved models capable of computing reliability indices reflecting energy uncertainty will be needed in the future. In particular the Commission would like to be better models for handling "soft" technologies such as solar and wind energy.

The Commission is generally satisfied with reliability indices which reflect steady-state system reliability performance and sees no particular need for indices reflecting dynamic performance. The indices favored by the Commission staff for future use in generation reliability evaluation are:

1) frequency and expected duration of capacity margin and load loss events,
2) expected unserved energy,
3) magnitude of load loss events.

Question D

The Commission is presently skeptical of the accuracy of methods for the economic evaluation of reliability and does not use them.

6.4 ECONOMIC EVALUATION OF RELIABILITY

A literature survey of methods for ascertaining the cost of service interruptions to consumers and incorporating these costs into system reliability analyses has been conducted. The emphasis in the survey was to discover which attributes of reliability performance are believed to be economically important and to discover what reliability indices would be needed to compute the cost of interruptions in planning studies. The survey does not claim to be complete as a great amount of work has been recently done or is in progress in this area. However, it is believed that the survey does provide a reasonable sample of methods for economic evaluation of reliability and the indices required in use of these methods.

The information gleaned from the survey is summarized as follows.

6.4.1 Sources Reviewed

R.B. Shipley, et al. [6-1]

These authors suggested that an estimate of the cost of interruptions could be obtained by dividing gross national product (GNP) by total Kwhr sales to obtain a value or worth of unserved energy expressed in $/Kwhr. The reliability index required in this method is expected unserved energy.

M.L. Telson [6-2,3]

Telson investigates several methods for computing the value of unserved energy expressed in $/Kwhr. The reliability index required is, of course, expected unserved energy.

France, Italy

The utilities of France and Italy have for many years evaluated planning alternatives considering the economic value of expected unserved energy. More recently the French have begun use of a method which recognized that the value of unserved energy, expressed in $/Kwhr, should also be a function of the expected duration of an interruption. The values of $/Kwhr used in France and Italy are primarily "imputed" costs rather than costs obtained by survey or use of economic statistics. That is, the costs were obtained by working backward from existing, and satisfactory, systems to discover the interruption cost implied by these systems. Thus, the French and Italian methods amount to maintaining the same economic level of reliability in the future as was found satisfactory in the past.

Sweden

Possibly the earliest surveys of the cost of interruptions to consumers were carried out in Sweden. The surveys give costs expressed in $/Kwhr of energy unserved and in $/Kw interrupted. Thus, the Swedish method suggests that the cost of interruptions is related to: unserved energy, expected number of interruptions per year or other period (frequency), and the expected magnitude of load lost per interruption. The indices required to implement this method are apparent.

Z.G. Todd [6-4]

Todd presented results obtained from records of consumer complaints at Indianapolis Power & Light Company. He found that consumers were nonlinearly sensitive to both interruption frequency and duration. Recognition of this fact in economic evaluation (Todd assigned no dollar costs to interruption events) would require that the probability distributions of the number of interruptions per year and the durations of interruption events be found as well as the more common expected values of frequency and duration.

Consumers Power Company [6-5]

Consumers Power Company conducted a field survey to investigate the impacts of interruptions on domestic consumers. Two groups of consumers were surveyed: (1) those who had not experienced an interruption at home within one month, and (2) those who had experienced an interruption. A visit was made to the interrupted consumers within 10 days of the event. The investigation focused on the effects of: (1) interruption duration, (2) weather conditions, (3) geographic location, (4) time of day, and (5) season of year. Some conclusions of the investigation were:

1) Interruption duration is nonlinearly important with durations over eight hours considered quite serious compared to shorter durations.
2) Time of day is not particularly important.
3) Season of year is important (clearly this is very much a function of the climatic region).
4) Time since last outage was found to have a significant effect on the perceived seriousness of an interruption.
5) Urban consumers were found to be more sensitive to interruptions than consumers living in rural or suburban areas. This may evidence the "ratchet effect" of reliability performance or may evidence the degree of dependence on electric service.

Recognition of the above factors in the economic evaluation of reliability would require, in general, probability distributions of the number of interruptions per year and their durations with breakdowns by season of year and geographic area.

W.T. Miles, et al. [6-6]

These authors conducted a study of the costs of the 1977 New York City blackout for the U.S. Department of Energy. A major conclusion of this study was that the indirect costs of this event were much greater than the direct costs. Direct costs are defined to be losses directly occasioned by the loss of service, while indirect costs are those occasioned by the community's response to the interruption event.

Clearly, indirect costs would be related in some way to such reliability indices as geographic extent of interruption or magnitude of load lost in the interruption. The dollar values translating these indices into indirect costs would clearly be very specific to particular areas.

K.B. Powell [6-7]

Powell describes a method in use at Utah Power & Light Company. Here interruptions are evaluated using values of $/Kwhr of unserved energy plus values of $/Kw interrupted per interruption event. Costs are broken down by major consuming sectors: industrial, agricultural, commercial, and residential and are determined using values of gross state product and Kwhr consumption as well as some limited field survey information.

SRI International [6-8]

The study was a part of the "Large-Scale System Effectiveness Analysis" program funded by the U.S. Department of Energy. A basic concept of the approach which was developed is that all units of electricity do not have the same worth or value; rather worth is specific to the use to which the electric energy is put. Thus, the unit of worth advocated is $/hour of interruption time for a specific use.

Attributes believed to have the greatest influence on the cost of interruptions are:

1) Nature of electricity use,
2) Duration of interruption,
3) Time of day and season of year of interruption,
4) Geographic extent of interruption,

5) Nature (reliability) of previous service including such factors as time since last interruption and average frequency of past interruptions, and
6) Forewarning of interruption.

G.C. Krohm [6-9]

Krohm reports the results of a survey of the costs and consequences of a widespread interruption in Central Illinois in the spring of 1978. The survey was limited to residential consumers and was aimed at discovering the cost of this particular interruption event rather than interruptions in general. Therefore, the report does not deal with reliability indices as such. It is apparent, however, that the following reliability performance attributes were found to be important:

1) Interruption duration,
2) Interruption magnitude - number of consumers and area affected,
3) Season of year and weather conditions at time of interruption, and
4) Degree of dependence on electric service.

It was also found that consumers are relatively tolerant of interruptions caused by natural disasters.

ECON Inc. [6-10]

The ECON study is also a part of the DOE "Large-Scale System Effectiveness Analysis" program. The method which has been developed proposes to use two indices or measures of reliability. The first measure of reliability is expected unserved energy. This index is expected to be dominated by distribution outages and is to be computed separately for each consumer class. The second index in the probability of interruption as a function of the magnitude of interruption. This index reflects all parts of the system and seeks to reflect the nonlinearity of interruption costs with magnitude of interruption. This large-scale interruption index is assumed to apply equally to all consumer classes. Both indices of reliability are to be computed for each load level considered.

The method reported here is also different in that it proposes to use expected consumer surplus as the measure of system worth. Consumer surplus is defined as the amount the consumer would be willing to pay for various amounts of electric energy at varying degrees of reliability less the amount actually paid for service and the direct costs of service interruptions. A basic idea here is that the value of electric service is related to the quality (reliability) of service provided.

Ontario Hydro [6-11,12,13]

Probably the most extensive field survey of consumer interruption costs and factors influencing such costs has been conducted by Ontario Hydro. Some of the most important conclusions of this study are summarized as follows.

1) Interruption costs are very much a function of consumer classes,
2) Interruption costs are nonlinear functions of interruption duration,
3) Advance warning of interruptions can be important for industrial consumers,
4) Residential consumers perceive the worst times for interruptions to be 5-7 PM and 6-8 AM; on Sundays, Mondays, or Fridays; and during the winter.

Interruption costs have been presented by Ontario Hydro in terms of $/Kw interrupted as a function of interruption duration and consumer class. Thus, at a minimum, the reliability indices needed for use in connection with the Ontario Hydro cost figures are: interruption frequency, expected interruption duration, and the expected magnitude of load interrupted. These indices would be needed for each consumer class.

6.4.2 Summary Comments on Economic Evaluation of Reliability

Methods for the economic evaluation of reliability performance require a variety of reliability indices. These indices, in rough order of their frequency of use in the methods surveyed, are as follows:

1) Expected unserved energy,
2) Expected duration of an interruption,
3) Average frequency of interruption,
4) Expected magnitude of load lost in an interruption,
5) Probability distribution of interruption duration,
6) Probability distribution of number of interruptions per year or other time period, and
7) Probability distribution of amount of load lost in an interruption.

Many methods, of course, make use of several of the above indices.

It is also evident that accurate economic evaluation of reliability requires that indices be calculated separately for a range of consumer classes or types of enduse consumption and that indices may be required for different times of day, day of week, season of year, and geographical areas.

One general requirement is implicit in all methods for the economic quantification of reliability performance. This is the requirement that the reliability indices utilized be computed accurately and bear close relationship to actual historical reliability performance. This is necessary because the computed cost of interruptions is usually treated in an absolute way and traded off against other system costs such as owning and operating costs to determine an "optimum" system or level of reliability performance.

6.5 EVALUATION AND RANKING OF RELIABILITY INDICES

The preceding sections have presented lists and discussions of attributes and indices of reliability performance. It now remains to evaluate and rank existing and proposed reliability indices. The general bases for the ranking are:

1) significance and meaningfulness to consumers,
2) measurability from historical records of system reliability performance,
3) usefulness to planners,
4) acceptability to regulatory agencies, and
5) usefulness in economic evaluation of reliability.

The order of listing of the above bases reflects the general weight given the various bases.

A basic conclusion is that the trend in system reliability indices is, and should be, toward indices which have the closest possible relationship to attributes of service reliability which are significant and meaningful to the ultimate consumer. It is believed that this emphasis on consumer-related reliability indices will tend to focus attention on the ultimate goal of system reliability; will tend to unify indices which are used in generation, transmission and distribution systems; and will tend to produce indices which could be used in economic evaluation of reliability.

The second basis for ranking indices, measurability from historical records, is also felt to be very important. Indices which can be measured from historical records will be, by definition, physically significant - an important attribute and closely related to the first ranking basis. Measurability from historical records also implies that computed reliability indices can be verified against historical performance thereby providing a means of checking the accuracy of calculation models. Thus, the use of indices which are physically measurable should foster the use of more accurate modeling methods and should tend to produce index values having greater absolute significance.

The usefulness to planners of an index is very largely the extent to which the index is sensitive to and reflects planning parameters of interest. Planners are often satisfied with indices having only relative significance for choosing between alternatives. However, planners seem to be favoring more physically-based, absolute significant, indices for the future primarily because such indices will be better received outside the planning department.

The factors which make an index acceptable to regulatory agencies are fundamentally similar to those factors of concern to planners and utility management. However, an overriding concern of regulatory agencies is that reliability indices be simple, unambiguous, and intuitively appealing. In the future, regulatory agencies would appear to favor consumer-related, physically-measurable, indices which can or have been varified against historical experience.

The usefulness of reliability indices in making economic evaluations of system reliability does not seem to be an important concern at this time or for the near future. Both planners and regulatory agencies presently view the accuracy of economic evalutions of reliability with great skepticsm. However, most seem to agree that economic evaluation of reliability would be a good approach if practical and accurate methods for such evaluations can be found. Further, indices which are selected on the basis of significance to consumers and physical measurability will also satisfy most if not all, of the requirements of economic evaluation methods.

Using the above bases for ranking, the investigators have arrived at the following ranking of generic reliability indices. Note that, in many cases, a single index is meaningful only if used in conjunction with other indices.

1) Frequency of interruption, load loss, capacity shortage, or other capacity margin event. We here interpret frequency to include all indices which measure the average or expected number of interruptions or outages in a given time period. This includes the "expected number of days of capacity deficiency" as used in traditional generation reliability studies as well as the modern and precisely defined "frequency" indices used in both generation and T&D studies.
2) Expected duration of interruption, load loss, capacity shortage, or other capacity margin event.
3) Expected magnitude of interruption or capacity shortage given such an event.
4) Expected unserved energy in a specified time period. Note this index can be computed if the first three above are known. Note further that this index is, perhaps, the most descriptive single number index which exists.
5) Probability distribution of interruption duration. This index, if it can be called that, is needed where the expected value of interruption duration does not adequately describe the range of possible durations or where consumers are significantly nonlinearly sensitive to interruption duration.
6) Probability distribution of the number of interruptions per time interval. This index is needed where consumers are significantly nonlinearly sensitive to the time between interruptions.
7) Probability distribution of the magnitude of interruption. This index is needed when interruption impact is significantly nonlinear with respect to interruption magnitude.
8) All other indices including, in particular, those indices which express reliability in terms of probability of "successful" or "unsuccessful" operation. These indices are judged to have much less physical significance than those listed above and to be generally less desirable.

There are, of course, many methods for computing the above indices. Some methods for computing given indices contain more detailed modeling than others and thereby give greater accuracy and permit the impact of more planning variables to be evaluated. The methods to be used in given studies obviously should be those which give the required accuracy at least computing cost for the specific application.

Now consider the specific reliability indices presented previously in this report for use in generation, transmission, and distribution systems. Using the above ranking of generic reliability indices as a guide we rank specific indices as follows. Note that, in general, only quantitative probabilistic indices are ranked.

Generation

1) Frequency and expected duration of capacity shortage or other capacity margin events computed using an hourly load cycle, (F&D). Note that the indices of frequency and duration are coupled, and, in general, must be used together for proper interpretation of results.
2) Frequency and expected duration of capacity shortage or other capacity margin events computed using an idealized daily load cycle, (F&D). These indices are given a lower ranking than those above because they are more idealized and less able to reflect planning parameters bearing on load cycle shape.
3) Expected number of hours per year that capacity is less than load or other margin levels, (HLOLE).
4) Expected number of days per year that capacity is less than load or other margin levels, (LOLE). This index, the most widely-used index, is ranked below HLOLE became of its greater idealization and inability to reflect the effects of load cycle shape.
5) Expected magnitude of capacity shortage given such an event, (XLOL). This index must be used in conjunction with one of the above indices or index pairs to be meaninfful.
6) Expected energy not supplied, (EENS). This is perhaps the best single number index and is widely used in Europe. EENS becomes relatively more important when energy resources as well as capacity are uncertain.
7) Probability of not serving annual peak load, (LOLE-1).
8) Probability of serving annual peak load successfully, (POPM). This index is, of course, not fundamentally different from LOLE-1, but is ranked below LOLE-1 because of its probable lack of sensitivity to planning parameters as compared to LOLE-1. That is, POPM will be a number close to unity in typical cases and is likely to exhibit only small percentage variations in response to planning parameter variations even when LOLE-1 varies widely.

9) Probability of loss-of-load on any day of the year, (PLOL). This index, and those below, are judged to be of much lower value than those above. This judgement is based on the lack of physical significance of the index together with the doubtful assumptions made in its calculation.
10) Probability of no load loss on any day of the year, (Q). (Q) is ranked below (PLOL) for the same reason as discussed in (8) above.
11) Probability of loss-of-load on any hour of the year, (PLOL-hourly).
12) Probability of no load loss of any hour of the year, (Q-hourly).

Probability distributions of capacity shortage or load loss duration and of the number of capacity shortage or load loss events per year do not appear in the ranking because they are not calculable today except by Monte Carlo simulation models. These "indices", if available, would fall at about level (7) in the rankings. Similarly, the probability distribution of load loss magnitude given a load loss event is not ranked because it is not routinely calculated. If calculated, this "index" would also fall at about level (7) in the rankings.

Transmission

The ranking of transmission reliability indices is difficult because of the dual roles of transmission: 1) load point supply, and 2) generalized load transfer between areas and systems. Few, if any, indices can adequately reflect both these roles. Therefore, indices applying primarily to each of the two roles will be ranked separately with precedence given to the load supply indices.

Load Supply Indices

Frequency of Interruption. The frequency of interruption events may be useful either on an individual consumer or load point basis or on a system basis depending on the application. System average values which are normalized to a per consumer or per load point basis are judged to be more meaningful than unnormalized values. Thus, frequency and frequency-like indices are ranked as follows.

1) (a) Frequency of interruption events at a given bus or consumer.
   (b) System average interruption frequency index, (SAIFI) - the average number of interruption events per consumer served per year.
   (c) Average number of interruption events per load point per year.
2) Frequency of interruption events on system, unnormalized basis.

3) LOLE (period) - expected number of events resulting in load loss over period. This index is numerically similar to (2), but conceptually less defensible.

4) LOLP (1 load) - expected number of events resulting in load loss at one load level, usually annual peak load. This is judged inferior to (3) because load variations are not considered. The index is believed to have relative significance only.

5) Consumer average interruption frequency index, (CAIFI) - the average number of interruption events per consumer interrupted per year. This index can be observed, with difficulty, but cannot be calculated using probabilistic models. Further, the index is considered conceptually inferior to the more easily computed index SAIFI.

Duration of Interruption

1) (a) Expected duration of interruption event at a given bus or consumer.
   (b) Consumer average interruption duration index, (CAIDI) - average duration of interruptions experienced by consumers on system.

2) Connected load interruption duration index - average duration of interruption events on load rather than consumer basis. This index is similar to (1) (b), but is regarded as somewhat inferior to it.

Magnitude of Interruption

1) Bulk power supply average curtailment per disturbance - the average magnitude of load interrupted given an interruption event.

2) Maximum load curtailed considering a particular set of contingencies. This index is deterministic and judged to be of relative significance only.

Unserved Energy

1) (a) Expected energy not supplied per year due to transmission system problems, (EENS). A possibly preferable form of this index would normalize the unserved energy by dividing by the total energy demanded to provide a more universally comparable index.
   (b) Expected energy not supplied per year for given load points or consumers.
   (c) Bulk power energy curtailment index, (BPECI).

2) Consumer curtailment index - KVA - minutes interrupted per consumer interrupted per year. This index though not strictly an unserved energy index is similar and gives an indication of annual "energy" which is not supplied to those consumers who are interrupted.
3) System curtailment index - KVA - minutes per consumer served per year.
4) Maximum energy not supplied considering a particular set of contingencies. This index is deterministic, dependent on the set of contingencies studied, and judged to be of relative significance only.

Probability Measures

1) (a) Probability of loss of load at a given bus.
   (b) Transmission loss of load expectation - probability or expected number of days per year on which transmission load supply capability would be exceeded.
2) Service unavailability index - ratio of consumer-minutes that service was unavailable to total consumer - minutes demanded.
3) Service availability index - the complement of service unavailability index.

Other Measures. Certain other measures of reliability have been proposed based on the sum of the magnitudes of load which are interrupted in a given time inteval. These indices are judged to have little physical significance. The indices are:

1) Bulk power interruption index, (BPII) - ratio of total annual load interrupted to annual peak load.
2) Load interruption index - ratio of average connected load interrupted per year to toal connected load.
3) Average annual load interrupted per load point served.

Load Transfer Indices

Two general approaches to measurement of the reliability of the generalized load transfer function of transmission networks have been proposed: load supplying capability (LSC) and simultaneous interchange capability (SIC). The two approaches seem complementary rather than competitive and therefore indices based upon them are given equal rank. Those indices recognizing the likelihood of contingencies are judged to be of greater value than deterministic versions. Thus, the ranking is as follows.

1) (a) Expected LSC - the expected load supplying capability of a network considering the range of contingencies which are possible.

   (b) Probability SIC is less than given amount.

2) (a) Minimum LSC - minimum of the maximum loads which can be supplied with no overloading of any line or generator for a given set of contingencies studied.

   (b) LSC - all lines and generators in service.

   (c) Minimum SIC - minimum transfer capability between areas of systems for a given set of contingencies studied.

Distribution

Distribution indices are available both to describe reliability performance at individual load points and to describe system average reliability. Hence, we rank indices for both applications in the following manner.

1) (a) Sustained interruption frequency at individual load points.

   (b) Momentary interruption frequency at individual load points.

   (c) System average interruption frequency index, (SAIFI).

2) (a) Expected sustained interruption duration at individual load points.

   (b) Consumer average interruption duration index, (CAIDI).

3) Expected unserved time per year at individual load points.

4) Probability distribution of sustained interruption duration at individual load points. This "index" provides a useful supplement to (2) (a) particularly where consumers are significantly non-linearly sensitive to interruption duration.

5) Probability distribution of number of interruptions per year. This "index" which can be obtained for no additional effort over that required to obtain interruption frequencies (expectations) offers a useful supplement to the frequency indices.

6) System average interruption duration index, (SAIDI). This index is considered less descriptive than the similar index CAIDI.

7) Consumer average interruption frequency index, (CAIFI). This index is considered less descriptive than the similar index SAIFI.

8) Service unavailability index.

9) Service availability index.

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Section 7

FUTURE WORK

Through the concept of a reliability indicator, Section 5 gives definitions for each basic reliability index in terms of the probability law of a single empirical characteristic. This is in contrast to the approach where a "model" is first stated which relates several characteristics of a power system (such as capacity and load demand) and a reliability index is then viewed as a result of "combining" the variables in the model in some way. Such an approach obscures the fact that the reliability index is in fact related to a single characteristic which, conceptually at least, could be observed on an actual system. The objective in Section 5 is to strip away the "capacity-load" model and focus on describing the essential nature of the reliability indices in terms of fundamental probability concepts. This goal has been accomplished.

One important benefit from this development is that a clear structure of reliability indices emerges, based on the classification of point, duration, and interval reliability indicators. Secondly, a consistent terminology is suggested, and a basis is provided for clarifying some of the conventions which are in common use. Thirdly, by focusing on the reliability indicator itself, a minimum of assumptions about the probability laws of the indicators was required. Only those assumptions were made which were necessary to define the indices or to establish relationships between indices. For example, duration reliability indices (expected loss of load duration and expected cycle duration) were defined without any restrictions on the probability law of the associated reliability indicator other than the existence of an expected value. However, frequency was defined under the assumption that the sequence of cycle durations has a steady state expectation.

Although the development in Section 5 provides a sound probability foundation for the reliability indices, application of the reliability indices in planning (or other areas) does require that the "reliability indicator" be modeled in terms of other characteristics of the power systems which are important in the planning process. The two most fundamental characteristics are capability (capacity) of individual facilities, and load demand. Therefore, a logical extension of the probability models in Section 5 would be the application of these models to various specific areas of the power system in terms of capacity/load demand models.

The computational techniques themselves are well known, and these are summarized in other sections of this report. However, in many cases the limitations and assumptions which underly the particular techniques are not widely appreciated. This is particularly true in the case of frequency and duration models. Furthermore, there is a wide variation in nomenclature and symbols. The structure of reliability indices and nomenclature developed in Section 5 provide a basis for examining existing models, assumptions, limitations, and nomenclature in a consistent manner.

For example, a basic element of the system capacity model is the two-state component model. Algorithms are available for computing the probability and frequency of system capacity levels (generation) and system states or cut sets (transmission and distribution), as described in Sections 3 and 4. What are the minimum assumptions and conditions required in order for the algorithms to apply? On the load demand side, the literature is very weak with respect to the underlying assumptions and practical consequences of these assumptions. In particular, frequency is defined under "steady state" conditions. How can such models be reconciled with the practical reality that load demand follows daily, weekly, and seasonal patterns? In transmission system reliability, the "minimum cut" method is widely used. This method is based on the assumption of a "coherent system". Transmission systems are not necessarily coherent, in the sense that removing a component from service may sometimes "repair" a system which has failed.

In summary, the formulation of power system reliability indices in terms of fundamental empirical characteristics provides a solid foundation. The application of these probability concepts to existing computation methods is a logical next step. The objectives would be to identify and validate the essential assumptions and limitations of the methods, show the relationship between various methods, and relate industry terminology and symbols in a common framework.

Appendix A

RECURSIVE EQUATIONS

For two state generators the exact capacity outage table can be built recursively using the equation

$$p(x) = (1-r)p'(x) + rp'(x-c) \qquad \text{(A-1)}$$

where

- c = capacity of the unit being added to the table
- r = full forced outage rate of new unit
- $p'(x)$ = probability of exactly x MW on outage before the new unit is added to the system
- $p(x)$ = probability of exactly x MW on outage after the new unit is added to the system
- $p'(x)$ = 0 when $x < 0$.

The same equation can be used to build the cumulative capacity outage table, P(x), with the substitution of P for p everywhere in A-1 and the slight change that $P'(x) = 1.0$ when $x < 0$. P(x) is defined as

$$P(x) = \sum_{X=x}^{C} p(X), \qquad \text{(A-2)}$$

or the probability of x MW <u>or</u> <u>more</u> on outage.

When multi-state units are modeled and the cumulative outage table is used, equation A-1 can be expanded to

$$P(x) = (1-\sum_{i=1}^{n} r_i)P'(x) + \sum_{i=1}^{n} r_i P'(x-c_i) \qquad \text{(A-3)}$$

where

c_i = MW on outage

r_i = probability of c_i on outage

and

n = number of outage states.

Equation A-3 may be used for either the exact or cumulative capacity outage table, depending upon the conditions for $x < 0$.

If an outage table is built with all of the units on the system, then throughout the year various units will have to be removed for maintenance. Solving equation A-1 for $P'(x)$ and using the cumulative probabilities produces

$$P'(x) = \frac{P(x) - rP'(x-c)}{1-r} \tag{A-4}$$

where $P(x)$ is still the table with the unit in and $P'(x)$ is the table with the unit removed for maintenance.

Caution - although mathematically correct, equation A-4 can become unstable when r is greater than 0.5. In a similar manner equation A-3 can be solved for $P'(x)$. However, this equation will be numerically unstable under most conditions.

Frequency Table

In equation 3-55 we defined $f(x)$, the frequency of generation outages of exactly x MW, as

$$f(x) = p(x)(\rho_+(x) - \rho_-(x)) \tag{A-5}$$

where $p(x)$ is the exact probability of x MW on outage and $\rho_+(x)$ and $\rho_-(x)$ were defined in equations 3-48 and 3-49. Substituting for $\rho_+(x)$ and $\rho_-(x)$ in equation A-5 gives

$$f(x) = p'(x)(1-r)\rho'_+(x) + p'(x-c)r(\rho'_+(x-c) + \mu)$$

$$- p'(x)(1-r)(\rho'_-(x) + \lambda) - p'(x-c)r\rho'_-(x-c). \tag{A-6}$$

The terms in equation A-6 can then be regrouped as

$$f(x) = (1-r)\ p'(x)(\rho'_+(x) - \rho'_-(x))$$

$$+ r\ p'(x-c)(\rho'_+(x-c) - \rho'_-(x-c))$$

$$- \lambda(1-r)p'(x) + \mu r\ p'(x-c) \qquad \text{(A-7)}$$

substituting from equation A-5 produces

$$f(x) = (1-r)\ f'(x) + rf'(x-c)$$

$$- \lambda(1-r)p'(x) + \mu r\ p'(x-c). \qquad \text{(A-8)}$$

By definition

$$r = \frac{1/\mu}{1/\lambda + 1/\mu} \quad , \qquad \text{(A-9)}$$

so that

$$\lambda(1-r) = \frac{1}{1/\mu + 1/\lambda} = \mu r \qquad \text{(A-10)}$$

and equation A-8 can be written as

$$f(x) = (1-r)\ f'(x) + rf'(x-c)$$

$$- \mu rp'(x) + \mu rp'(x-c) \qquad \text{(A-11)}$$

The cumulative frequency of x MW or greater on outage, F(x), was defined in equation 3-57 as

$$F(x) = \sum_{X=x}^{C} f(X). \qquad \text{(3-57)}$$

Substituting equation A-11 into 3-57 gives

$$F(x) = \sum_{X=x}^{C} [(1-r)f'(X) + rf'(X-c) - \mu rp'(X) + \mu rp'(X-c)] \qquad \text{(A-12)}$$

$$= (1-r) \sum_{X=x}^{C} f'(X) + r \sum_{X=x}^{C} f'(X-c) - \mu r \sum_{X=x}^{C} p'(X) + \mu r \sum_{X=x}^{C} p'(X-c) \qquad \text{(A-13)}$$

Substituting equations 3-1 and 3-57 into equation A-13 produces

$$F(x) = 1-r\ F'(x) + r\ F'(x-c) - \mu r\ P'(x) + \mu r\ P'(x-c) \qquad \text{(A-14)}$$

With equation A-14 the cumulative frequency table can be built from the cumulative frequency and probability tables prior to adding the new unit to the system.

Appendix B

EXAMPLE CALCULATIONS OF GENERATION RELIABILITY INDICES

This appendix presents an example of the use of the equations presented in Table 3.2. A small system is defined and the indices are calculated for a period of one week. Extensions to annual values are also given.

Load Model

Table B.1 shows the daily peak loads in per unit of the weekly peak. Table B.2 gives the corresponding hourly loads in per unit of the daily peak load. These tables, when combined with a weekly peak load of 400 MW, provide the necessary load information for the calculation of all of the generation reliability indices discussed in Section 3.

TABLE B.1

Daily Peak Loads

Day	Per unit of Weekly Peak
1	.85
2	1.00
3	.95
4	.85
5	.75
6	.60
7	.45

TABLE B.2

Hourly Loads

Hour	Per unit of Daily Peak	Hour	Per unit of Daily Peak
1	.70	13	.95
2	.65	14	.95
3	.60	15	.95
4	.60	16	1.00
5	.55	17	1.00
6	.65	18	1.00
7	.75	19	.95
8	.80	20	.90
9	.85	21	.80
10	.90	22	.75
11	.90	23	.75
12	.90	24	.70

Daily Load Factor = 81.5%

Weekly Load Factor = 63.4%

Capacity Model

The capacity model, shown in Table B.3 consists of unit capacities, forced outage rates, and average repair times. This information can then be used to calculate a cumulative capacity outage table and a frequency table.

TABLE B.3

Units in Generating System

Unit	Capability (MW)	Forced Outage Rate	Repair Time (days)
A	100	0.01	2.0
B	150	0.02	2.0
C	200	0.03	2.5

The generating system is modeled by constructing the cumulative probability outage table directly, using the recursive formula given in Appendix A. This method of building in one unit at a time requires that a step size for the table be chosen and that the calculation stop when probability becomes sufficiently small.

The construction of the table with a 50 MW step size is as follows:

1. Add unit 1: c = 100, r = .01

x	$P_{old}(x)$	$P_{new}(x)$
≤0	1.0	1.0(.99)+1.0(.01) = 1.0
50	0	0 (.99)+1.0(.01) = .01
100	0	0 (.99)+1.0(.01) = .01
150	0	0 (.99)+0 (.01) = 0

2. Add unit 2: c = 150, r = .02

x	$P_{old}(x)$	$P_{new}(x)$
≤0	1.0	1.0 (.98)+1.0 (.02) = 1.0
50	.01	.01(.98)+1.0 (.02) = .0298
100	.01	.01(.98)+1.0 (.02) = .0298
150	0	0 (.98)+1.0 (.02) = .02
200	0	0 (.98)+ .01(.02) = .0002
250	0	0 (.98)+ .01(.02) = .0002
300	0	0 (.98)+0 (.02) = 0

3. Add unit 3: c = 200, r = .03

x	$P_{old}(x)$	$P_{new}(x)$
≤0	1.0	1.0
50	0.0298	0.058906
100	0.0298	0.058906
150	0.02	0.049400
200	0.0002	0.030194
250	0.0002	0.001088

300	0	0.000894
350	0	0.000600
400	0	0.000006
450	0	0.000006
500	0	0

Calculation of Frequency Table

The cumulative frequency table can be calculated from the probability tables, the forced outages and repair rates. The table is constructed by convolving units using the recursive formula given in Appendix A in a manner similar to the probability table.

1. Add Unit 1: $c = 100$, $r = .01$, $\mu = 1/2 = .5$

x	$F_{old}(x)$	$P_{old}(x)$	$F_{new}(x)$
≤0	0	1.0	.99(0)-.5(.01)(1.0)+.01(0)+.5(.01)(1.) = 0
50	0	0	.99(0)-.5(.01)(0) +.01(0)+.5(.01)(1.0) = .005
100	0	0	.99(0)-.5(.01)(0) +.01(0)+.5(.01)(1.0) = .005
150	0	0	.99(0)-.5(.01)(0) +.01(0)+.5(.01)(0) = 0

2. Add unit 2: $c = 150$, $r = .02$, $\mu = 1/2 = .5$

x	$F_{old}(x)$	$P_{old}(x)$	$F_{new}(x)$
≤0	0	1.0	.98(0) -.5(.02)(1.0) +.02(0)+.5(.02)(1.0) = 0
50	.005	.01	.98(.005)-.5(.02)(.01) +.02(0)+.5(.02)(1.0) ≑ 0.148
100	.005	.01	.98(.005)-.5(.02)(.01) +.02(0)+.5(.02)(1.0) = .0148
150	0	0	.98(0) -.5(.02)(0) +.02(0)+.5(.02)(1.0) = .01
200	0	0	.98(0) -.5(.02)(0) +.02(.005)+.5(.02)(.01) = .0002
250	0	0	.98(0) -.5(.02)(0) +.02(.005)+.5(.02)(.01) = .0002
300	0	0	.98(0) -.5(.02)(0) +.02(0)+.5(.02)(0) = 0

3. Add unit 3: $c = 200$, $r = .03$, $\mu = 1/2.5 = .4$

x	$F_{old}(x)$	$P_{old}(x)$	$F_{new}(x)$
≤0	0	1.0	0
50	.0148	.0298	.025998
100	.0148	.0298	.025998

150	.01	.02	.021460
200	.0002	.0002	.012192
250	.0002	.0002	.000993
300	0	0	.000802
350	0	0	.000540
400	0	0	.000008
450	0	0	.000008
500	0	0	0

Index Calculations

The equations presented in Table .3.2 can now be combined with the load and capacity data presented to determine the value of each of the indices.

Percent Reserve

$$\% \text{ Reserve} = \frac{450-400}{400} = 12.5\%$$

$$\% \text{ Reserve}_C = \frac{450-400}{450} \; 11.1\%$$

Largest Units

Reserves = 450-400 = 50 MW

Largest unit size = 200 MW

Therefore $\frac{50}{200}$ = 25% of the largest unit is covered.

Loss-of-Load Expectation (LOLE)

Table B.4 presents the daily peak loads, the available reserves for each day and the corresponding probability of a capacity shortage on each day.

TABLE B.4

Probability of Shortage on Daily Peak

Day	Load (MW)	Reserves (MW)	Probability of Shortage
1	340	110	0.049400
2	400	50	0.058906
3	380	70	0.058906
4	340	110	0.049400
5	300	150	0.030194
6	240	210	0.001088
7	180	270	0.000894

The Loss-of-Load Expectation (LOLE) for the week is then the sum of the probabilities of shortage on the daily peaks:

LOLE(week) = .248788 days/week

Hourly Loss-of-Load Expectation (HLOLE)

Table B.5 contains the load, reserve and probability of shortage for each hour of day 1. The summation of the twenty four probabilities of shortage is then the HLOLE for day 1.

HLOLE(day 1) = .731986 hours/day

TABLE B.5

Probability of Shortage for Day 1

Hour	Load (MW)	Reserves (MW)	Probability of Shortage
1	238	212	.001088
2	221	229	.001088
3	204	246	.001088
4	204	246	.001088
5	187	263	.000894
6	221	229	.001088
7	255	195	.030194

8	272	178	.030194
9	289	161	.030194
10	306	144	.049400
11	306	144	.049400
12	306	144	.049400
13	323	127	.049400
14	323	127	.049400
15	323	127	.049400
16	340	110	.049400
17	340	110	.049400
18	340	110	.049400
19	323	127	.049400
20	306	144	.049400
21	272	178	.030194
22	255	195	.090194
23	255	195	.030194
24	238	212	.001088

The ratio of HLOLE to LOLE is sometimes referred to as the equivalent duration of peak load. This value can be used as an estimate of the duration of an outage when frequency calculations are not done. For day 1 the equivalent duration is:

e.d.(day 1) = .731986/.049400
= 14.82 hours

Repeating this analysis for each of the seven days produces the results shown in Table B.6. The value of HLOLE for the week is the sum of the values for the seven days, so that:

HLOLE = 3.803962 hours/week

TABLE B.6

Probability of Shortages

Day	LOLE (days/day)	HLOLE (hours/day)	e.d. (hours)
1	.049400	.731986	14.82
2	.058906	1.010788	17.16
3	.058906	.914552	15.53
4	.049400	.731986	14.82
5	.030194	.374414	12.40
6	.001088	.022902	21.05
7	.000894	.017334	19.39

The equivalent duration of peak load for the week is then

e.d.(week) = 3.803962/.248788
= 15.29 hours/day

Probability of Positive Margin (POPM)

The peak of the week is on day 2. Therefore:

POPM = 1-LOLE (day 2)
= 1-.058906
= .941094

Quality (Q)

The quality index is determined by multiplying together the compliment of each day's LOLE. Table B.7 shows the complements.

TABLE B.7

Calculation of Quality

Day	LOLE (Days/day)	1-LOLE (Days/day)
1	.049400	.950600
2	.058906	.941094
3	.058906	.941094
4	.049400	.950600
5	.030194	.969806
6	.001088	.998912
7	.000894	.999106

The resulting value of quality for the week, using daily peak loads, is:

$$Q(\text{week}) = \prod_{i=1}^{7} (1-LOLE_i)$$

$$= .774614$$

Probability of Loss-of-Load (PLOL)

The Probability of Loss-of-Load is equal to one minus the quality,

$$PLOL = 1-Q$$

$$= 1-.774614$$

$$= .225386$$

Expected Energy Not Served (EENS)

From equation 3-37, the unserved energy for one hour is:

$$EENS(\text{hour } 1) = \Delta X \sum_{x=C-L+1}^{C} P(x) - \varepsilon MWh,$$

where the ε term is included in this example since the step size is large compared to the system size.

If the first load of the week is used (238 MW), then this equation becomes:

$$EENS(hour\ 1) = 50 \sum_{x=213}^{450} P(x) - 12\ P(250)$$

$$= 50\ [.001088 + .000894 + .000600 + .000006 + .000006] - 12\ (.001088)$$

$$= .116644 \text{ MWh}$$

Table B.8 shows the expected unserved energy for each hour of the first day of the week. Summing the twenty four values gives:

EENS(day 1) = 34.081578 MWh

Table B.9 shows the results of this calculation for the entire week. The total unserved energy for the week is:

EENS (week) = 228.4124 MWh.

TABLE B.8

Expected Energy Not Served for Day 1

Hour	Load (MW)	Reserves (MW)	EENS (MWh)
1	238	212	.116644
2	221	229	.098148
3	204	246	.079652
4	204	246	.079652
5	187	263	.063678
6	221	229	.098148
7	255	195	.280670
8	272	178	.793968
9	289	161	1.307266
10	306	144	1.935800
11	306	144	1.935800
12	306	144	1.935800
13	323	127	2.775600
14	323	127	2.775600
15	323	127	2.775600
16	340	110	3.615400
17	340	110	3.615400
18	340	110	3.615400
19	323	127	2.775600
20	306	144	1.935800
21	272	178	.793968
22	255	195	.280670
23	255	195	.280670
24	238	212	.116644

TABLE B.9

Expected Energy Not Served

Day	LOLE (Days/day)	HLOLE (Hours/day)	EENS (MWh/day)
1	.049400	.731986	34.08158
2	.058906	1.010788	80.52168
3	.058906	.914552	63.30018
4	.049400	.731986	34.08158
5	.030194	.374414	13.89716
6	.001088	.022902	1.77767
7	.000894	.017344	.75254

Expected Loss-of-Load (XLOL)

From equation 3-45 the value of XLOL for the week can be determined as:

$$\begin{aligned} XLOL(week) &= EENS(week)/HLOLE(week) \\ &= 228.4124/3.803962 \\ &= 60.0459 \text{ MW} \end{aligned}$$

Frequency and Duration (f&d)

Table B.10 lists the frequency of the available capacity being less than each of the loads on day 1. When these values are combined in equation 4-61 the results are:

$$\begin{aligned} f(\text{day } 1) &= P(450-340) + \frac{1}{24} \sum_{k=1}^{24} F\,(450-L_K) \\ &= .049400 + \frac{1}{24}\,(.315972) \\ &= .062566 \text{ occurrences/day} \end{aligned}$$

The expected duration of the outage is:

$$d(\text{day } 1) = HLOLE(\text{day } 1)/f(\text{day } 1)$$
$$= .731986/.062566$$
$$= 11.6995 \text{ hours/occurrence}$$

This analysis is repeated for the remainder of the week in Table B.11. The resulting frequency for the week is the sum of the daily values:

$$f(\text{week}) = \sum_{i=1}^{7} f\ (\text{day } i)$$
$$= .318052 \text{ occurrences/week}$$

and the expected duration of shortage is

$$d(\text{week}) = HLOLE(\text{week})/f(\text{week})$$
$$= 3.803962/.318052$$
$$= 11.9602 \text{ hours/occurrence}$$

TABLE B.10

Frequency of Capacity Shortages

Hour	Load (MW)	Reserves (MW)	F (Reserves)
1	238	212	.000993
2	221	229	.000993
3	204	246	.000993
4	204	246	.000993
5	187	263	.000802
9	289	161	.012192
10	306	144	.021460
11	306	144	.021460
12	306	144	.021460
13	323	127	.021460
14	323	127	.021460
15	323	127	.021460
16	340	110	.021460
17	340	110	.021460
18	340	110	.021460
19	323	127	.021460
20	306	144	.021460
21	272	178	.012192
22	255	195	.012192
23	255	195	.012192
24	238	212	.000993

TABLE B.11

Daily f&d of Shortages

Day	Peak Load (MW)	f (occur/day)	d (hours/occur)
1	340	.062566	11.6995
2	400	.077184	13.0957
3	380	.075495	12.1141
4	340	.062566	11.6995
5	300	.036746	10.1891
6	240	.001953	11.7284
7	180	.001543	11.2365

Summary of Example

Table B.12 lists each index, its value for the one week examined, and its annual value provided the year contains 52 identical weeks.

TABLE B.12

Calculated Values of Indices

Index	Dimension/ Interval	1 Week Value	Annual Value
% Reserves	%	12.5	12.5
% Reserves_C	%	11.1	11.1
Largest Unit	#	.25	.25
LOLE	days	.2488	12.9370
HLOLE	hours	3.8040	197.8060
equivalent dur. of peak load	hours/day	15.29	15.29
POPM	per unit	.9411	.9411
Q	per unit	.7746	1.71×10^{-6}
PLOL	per unit	.2254	1.000
EENS	MWh	228.4124	11877.4
P.E.L.	%	.5359	.5359
XLOL	MW	60.0459	60.0459
f	occurrences	.3181	16.5387
d	hours/ occurrence	11.9602	11.9602

Appendix C

SAMPLE COMPUTER PROGRAM FOR CALCULATION OF GENERATION RELIABILITY INDICES

This Appendix gives the listing for a simple computer program which calculates the various reliability indices for a one week period. The IEEE reliability test system is used as input data, and the indices are calculated for the peak week of the year.

Page C-4 shows the three data files input to the program. The first file, HYPER, lists the hourly loads for weekdays and weekend days respectively, in percent of the daily peak. The second file, DYPER, specifies the effective duration of peak load used in the approximate f&d calculations, the weekly peak load, and the daily peak loads in percent of the weekly peak. The third file, I3EDATA, specifies for each unit its size, forced outage rate and repair time.

Pages C-5 through C-7 show the interactive data and the sample output. This is then followed by the listing of the program.

The purpose of this appendix is to show an actual example of how the various reliability indices can be calculated using a computer program.

Sample Generation System Expansion

The calculation of a reliability index is only part of a planning study. If the level of reliability indicated by the index is unacceptable then the next question is "How much capacity needs to be added?" The main results for the one week calculation are shown below in Table C.1. Assume that it is desired

TABLE C.1

Index Values for Original System

Index	Value
LOLE	.259
HLOLE	1.945
EENS	286.487
Frequency	.314
XLOL	147.324

to reduce the frequency of interruption to only .2 occurrences/week, or 1 interruption every 5 weeks. A method has been developed to help the planner estimate the amount of capacity required, taking into account the important factors of unit size and forced outage rate. The first step is to determine the effective capacity, as defined by Garver, required to reduce the frequency to .2 occurrences/ week. Using XLOL as an estimate of the characteristic slope-m of the system produces

$$C^* = \text{XLOL} \ln(\text{old risk}/\text{new risk}) \tag{C-1}$$
$$= 147.3 \ln(.314/.2)$$
$$= 66.4 \text{ MW}$$

The next step is find a unit with this effective capacity. Trying an 80 MW unit with a 10% forced outage rate produces

$$C^* = C - \text{XLOL} \ln(1-r + r\, e^{C/\text{XLOL}}) \tag{C-2}$$
$$= 80 - 147.3 \ln(1-.1 + .1\, e^{80/147.3})$$
$$= 69.7 \text{ MW}$$

Installing a unit with these characteristics, and a mean repair time of 2 days, produces the results shown in Table C.2.

TABLE C.2

Index Values for New System

Index	Value	New Value/Old Value
LOLE	.179	.691
HLOLE	1.271	.654
EENS	171.74	.600
Frequency	.219	.700
XLOL	135.07	.917

Although the desired frequency was not quite reached, the few simple hand calculations did put the result in the right neighborhood. If the calculations are based on annual values of the indices the results tend to be even better.

Therefore, by use of these approximations the indices not only tell what the relative reliability of the generation system is, they also indicate how much capacity is required to improve the reliability to a desired level.

HYPER

```
1    67 63 60 59 59 60 74 86 95 96 96 95 95 95 93 94100100100 96 91 83 73 63
2    78 72 68 66 64 65 66 70 80 88 90 91 90 88 87 87 91100 99 97 94 92 87 81
```

DYPER

```
1     6.0
2    2850
3      93  100   98   96   94   77   75
```

I3EDATA

```
10   155.      .04     1.667
11   155.      .04     1.667
12   155.      .04     1.667
13   155.      .04     1.667
14   197.      .05     2.08
15   197.      .05     2.08
16   197.      .05     2.08
17   350.      .08     4.167
18   400.      .12     6.25
19   400.      .12     6.25
100    12.       .02     2.5
110   12.      .02      2.5
120   12.      .02      2.5
130    12.       .02     2.5
135    12.       .02     2.5
140    20.       .1      2.083
150    20.       .1      2.083
160    20.       .1      2.083
170    20.       .1      2.083
180    50.      .01     .833
190   50.       .01    .833
200   50.       .01    .833
210   50.       .01    .833
220   50.       .01     .833
230   50.       .01     .833
240   76.        .02     1.667
250   76.        .02     1.667
260   76.        .02     1.667
270   76.        .02     1.667
280   100.       .04     2.083
290   100.       .04     2.083
300   100.       .04     2.083
```

```
ENTER STEP SIZE, TABLE CUT-OFF, NO. OF UNITS, & CAP. DATAFILE:
?25,10E-10,32,I3EDATA
PRINT CAPACITY OUTAGE TABLES?  1=YES;0=NO...?1
PERFORM RELIABILITY CALCULATIONS?  1=YES;0=NO...?1
   ENTER DATA FILE'S NAME CONTAINING DAILY PEAK PERCENTAGES:  ?DYPER
   ENTER DATA FILE'S NAME CONTAINING HOURLY PERCENTAGES:  ?HYPER
PRINT CONTENTS OF UNIT DATA FILE?  1=YES;0=NO...?0
```

************************ RELIABILITY OUTPUT ************************

TOTAL CAPACITY: 3405.00 MW

CAPACITY OUTAGE TABLES

X	PROBABILITY	UNS. ENERGY	FREQUENCY
0	0.1000000E+01	0.8360000E+01	0.
25	0.7384781E+00	0.7621522E+01	0.1661968E+00
50	0.6222461E+00	0.6999276E+01	0.1780617E+00
75	0.5870243E+00	0.6412252E+01	0.1605588E+00
100	0.5571962E+00	0.5855055E+01	0.1518505E+00
125	0.5134035E+00	0.5341652E+01	0.1487317E+00
150	0.4957344E+00	0.4845918E+01	0.1391578E+00
175	0.4463615E+00	0.4399556E+01	0.1335637E+00
200	0.4228342E+00	0.3976722E+01	0.1198396E+00
225	0.3728109E+00	0.3603911E+01	0.1162487E+00
250	0.3484561E+00	0.3255455E+01	0.1029923E+00
275	0.3352312E+00	0.2920224E+01	0.9174392E-01
300	0.3273661E+00	0.2592858E+01	0.8423651E-01
325	0.3166720E+00	0.2276186E+01	0.7681063E-01
350	0.3119498E+00	0.1964236E+01	0.7140632E-01
375	0.2805587E+00	0.1683677E+01	0.7374333E-01
400	0.2662495E+00	0.1417428E+01	0.6728099E-01
425	0.1877924E+00	0.1229635E+01	0.9364894E-01
450	0.1513658E+00	0.1078269E+01	0.8700124E-01
475	0.1362978E+00	0.9419715E+00	0.7700041E-01
500	0.1257986E+00	0.8161729E+00	0.7058153E-01

525	0.1085593E+00	0.7076136E+00	0.6462354E-01
550	0.1012225E+00	0.6063910E+00	0.5858726E-01
575	0.8284544E-01	0.5235456E+00	0.5238625E-01
600	0.7403128E-01	0.4495143E+00	0.4527403E-01
625	0.5904261E-01	0.3904717E+00	0.4018694E-01
650	0.5161310E-01	0.3388586E+00	0.3432294E-01
675	0.4698266E-01	0.2918759E+00	0.2953204E-01
700	0.4438307E-01	0.2474929E+00	0.2641813E-01
725	0.4066967E-01	0.2068232E+00	0.2303980E-01
750	0.3899449E-01	0.1678287E+00	0.2076232E-01
775	0.3006236E-01	0.1377663E+00	0.1939428E-01
800	0.2596570E-01	0.1118006E+00	0.1661866E-01
825	0.1900738E-01	0.9279326E-01	0.1570752E-01
850	0.1548188E-01	0.7731138E-01	0.1364123E-01
875	0.1324757E-01	0.6406381E-01	0.1171253E-01
900	0.1200583E-01	0.5205798E-01	0.1044645E-01
925	0.9697402E-02	0.4236058E-01	0.8945857E-02
950	0.8657336E-02	0.3370324E-01	0.7777273E-02
975	0.6382362E-02	0.2732088E-01	0.6452781E-02
1000	0.5281849E-02	0.2203903E-01	0.5297846E-02
1025	0.3976328E-02	0.1806270E-01	0.4364246E-02
1050	0.3304868E-02	0.1475783E-01	0.3620327E-02
1075	0.2775856E-02	0.1198198E-01	0.2971017E-02
1100	0.2503713E-02	0.9478263E-02	0.2566163E-02
1125	0.2086054E-02	0.7392209E-02	0.2088182E-02
1150	0.1891854E-02	0.5500355E-02	0.1779162E-02
1175	0.1206441E-02	0.4293915E-02	0.1468058E-02
1200	0.8871459E-03	0.3406769E-02	0.1157900E-02
1225	0.7133893E-03	0.2693379E-02	0.9231830E-03
1250	0.6049339E-03	0.2088446E-02	0.7715699E-03
1275	0.4788181E-03	0.1609627E-02	0.6256922E-03
1300	0.4232868E-03	0.1186341E-02	0.5364000E-03
1325	0.3112062E-03	0.8751344E-03	0.4207627E-03
1350	0.2576097E-03	0.6175247E-03	0.3411471E-03
1375	0.1615449E-03	0.4559798E-03	0.2559041E-03
1400	0.1144685E-03	0.3415113E-03	0.1925284E-03
1425	0.8718562E-04	0.2543257E-03	0.1478298E-03
1450	0.7148989E-04	0.1828358E-03	0.1202787E-03
1475	0.5138676E-04	0.1314490E-03	0.9089661E-04
1500	0.4240967E-04	0.8903933E-04	0.7377048E-04
1525	0.2718738E-04	0.6185195E-04	0.5200675E-04
1550	0.1977837E-04	0.4207358E-04	0.3819008E-04
1575	0.1273497E-04	0.2933861E-04	0.2690013E-04
1600	0.9053027E-05	0.2028558E-04	0.1970981E-04
1625	0.6204129E-05	0.1408145E-04	0.1394267E-04
1650	0.4780938E-05	0.9300516E-05	0.1068052E-04
1675	0.3090414E-05	0.6210101E-05	0.7307662E-05
1700	0.2293267E-05	0.3916834E-05	0.5388205E-05
1725	0.1360706E-05	0.2556128E-05	0.3481201E-05
1750	0.8963053E-06	0.1659823E-05	0.2352180E-05
1775	0.5719449E-06	0.1087878E-05	0.1562980E-05
1800	0.4011530E-06	0.6867248E-06	0.1105614E-05
1825	0.2469847E-06	0.4397400E-06	0.7119357E-06
1850	0.1747568E-06	0.2649832E-06	0.5021240E-06
1875	0.1018227E-06	0.1631606E-06	0.3102878E-06
1900	0.6598839E-07	0.9717218E-07	0.2030184E-06

AT TIME OF PEAK...
===============

PEAK LOAD = 2850
RESERVES = 555.00
HLOLE = 0.082845
FREQUENCY = 0.085028
UNS. ENR. = 14.745548

RELIABILITY CALCULATIONS
========================

DY	◆DAILY PKS◆	◆ LOLE ◆	◆HLOLE◆	◆UNS. ENR◆	◆ FREQ ◆
1	2650.500000	0.030062	0.204325	26.351593	0.036731
2	2850.000000	0.082845	0.665565	105.212343	0.100280
3	2793.000000	0.059043	0.484538	72.415921	0.072330
4	2736.000000	0.046983	0.338190	49.231214	0.057012
5	2679.000000	0.038994	0.247647	32.891639	0.046471
6	2194.500000	0.000713	0.002673	0.250077	0.000869
7	2137.500000	0.000479	0.001653	0.133865	0.000578
WK		0.259120	1.944592	286.486652	0.314272

DY	◆ DUR ◆	◆ MWH/OCC ◆	◆Q-LOLE◆	◆Q-FREQ◆	◆Q-HOURLY◆
1	5.562676	717.412048	0.969938	0.963269	0.813535
2	6.637030	1049.180527	0.917155	0.899720	0.503891
3	6.699021	1001.191605	0.940957	0.927670	0.609569
4	5.931862	863.516670	0.953017	0.942988	0.709190
5	5.329027	707.783226	0.961006	0.953529	0.778140
6	3.076637	287.812168	0.999287	0.999131	0.997330
7	2.860778	231.612942	0.999521	0.999422	0.998348
WK	6.187602	911.587173	0.765711	0.721870	0.137302

DY	◆HLOLE-E◆	◆FREQ-E◆	◆ DUR-E ◆	◆ XLOL ◆
1	0.180374	0.034911	5.166696	128.968870
2	0.497073	0.095942	5.180970	158.079821
3	0.354256	0.069089	5.127501	149.453423
4	0.281896	0.054366	5.185183	145.572613
5	0.233967	0.044185	5.295158	132.816593
6	0.004280	0.000944	4.533365	93.547652
7	0.002873	0.000635	4.522548	80.961511
WK	1.554719	0.300072	5.181144	147.324804

```
C                        GENERAL ELECTRIC COMPANY
C               ELECTRIC UTILITY SYSTEMS ENGINEERING DEPARTMENT
C***********    PROGRAM TO CALCULATE HOURLY RELIABILITY INDICES    **********
C
C
      FILENAME DATAF1,DATAF2,DATAF3
      DIMENSION POLD(3000),PNEW(3000),FOLD(3000),FNEW(3000),CUMCAP(3000)
      DIMENSION CAP(3000),FOR(3000),REP(3000)
      DIMENSION DYPK(7),HRPK(7,24)
      DIMENSION HLOLP(7),DYEEU(7),DUR(7),FREQ(7),XLOL(7)
      DIMENSION QLOLP(7),QFREQ(7),QHOUR(7)
      DIMENSION HLOLPE(7),FREQE(7),DURE(7)
      REAL LOLP(7),MWHOC(7),MWHWK
      INTEGER X(3000),DUM,STEP
      INTEGER HRPER(2,24),DYPER(7),WKPK
   11 FORMAT (I3,3(F8.4))
   22 FORMAT (I3,7(I5))
   33 FORMAT (//"   X",15X,"PROBABILITY",9X,"UNS. ENERGY",10X,"FREQUENCY")
   44 FORMAT (I5,13X,E13.7,7X,E13.7,7X,E13.7)
   55 FORMAT (//" TOTAL CAPACITY:",F9.2,1X,"MW")
   66 FORMAT (3X,"=",15X,"===========",9X,"===========",10X,"=========="/)
   77 FORMAT (7(I5))
   88 FORMAT (25(I3))
   99 FORMAT ("==============="//)
  222  FORMAT (" WK",13X,4(3X,F11.6))
  333  FORMAT (" DY",3X,"*DAILY PKS*",5X,"* LOLE *",7X,"*HLOLE*",
&                4X,"*UNS. ENR*",6X,"* FREQ *")
  444  FORMAT (" DY",4X,"*  DUR  *",3X,"* MWH/OCC *",6X,"*Q-LOLE*",
&                 6X,"*Q-FREQ*",5X,"*Q-HOURLY*")
  555  FORMAT (" WK",2X,F11.6,4(3X,F11.6))
  666  FORMAT (" DY",4X,"*HLOLE-E*",6X,"*FREQ-E*",
&                5X,"* DUR-E *",6X,"* XLOL *")
  777  FORMAT (" WK",2X,F11.6,3(3X,F11.6))
  999  FORMAT (I4,10X,F8.4,2(11X,F8.4))
  1111 FORMAT (//)
  3333 FORMAT (/14X,"CAPACITY",13X,"F.O.R.",12X,"REP. RATE ")
  4444 FORMAT (14X,"========",13X,"======",12X,"========="/)
  5555 FORMAT ("  PEAK LOAD = ",I6,
&           /,"  RESERVES  = ",F7.2)
  6666 FORMAT ("  HLOLE      = ",F11.6,/,
&              "  FREQUENCY = ",F11.6,/
&              "  UNS. ENR. = ",F11.6,/)
  7777 FORMAT (" AT TIME OF PEAK..."/,"================"//)
  8888 FORMAT (1X,I1,5(3X,F11.6))
  9999 FORMAT (/)
C
C
C
 10   PRINT,"ENTER STEP SIZE, TABLE CUT-OFF, NO. OF UNITS, & CAP. DATAFILE:"
      PRINT 1111
```

```
      INPUT,STEP,PMIN,NUM,DATAF1
      PRINT,"PRINT CAPACITY OUTAGE TABLES?  1=YES; 0=NO..."
      INPUT,ISWTCH2
      PRINT,"PERFORM RELIABILITY CALCULATIONS?  1=YES; 0=NO..."
      INPUT,ISWTCH
      IF(ISWTCH .NE. 1)  GO TO 95
      PRINT,"   ENTER DATA FILE'S NAME CONTAINING DAILY PEAK PERCENTAGES:  "
      INPUT,DATAF2
      PRINT,"   ENTER DATA FILE'S NAME CONTAINING HOURLY PERCENTAGES:  "
      INPUT,DATAF3
C
 95   X(1) = 0
      HALF = STEP/2.0
      AVCAP = 0.
      LENGTH = 1
      PNEW(1) = 1.0
      FNEW(1) = 0.0
C
      PRINT,"PRINT CONTENTS OF UNIT DATA FILE?  1=YES; 0=NO..."
      INPUT,DUM
   PRINT 1111
   PRINT,"*********************** RELIABILITY OUTPUT ***********************"
      PRINT 1111
      IF(DUM .EQ. 1)  GO TO 98
C
C
C  ****READ (& PRINT) CAPACITY OUTAGE DATA****
      DO 100 L = 1,NUM
      READ (DATAF1,11) DUM,CAP(L),FOR(L),REP(L)
      AVCAP = AVCAP + CAP(L)
 100  CONTINUE
      GO TO 97
C
 98   PRINT,"CAPACITY OUTAGE DATAFILE:  ",DATAF1
      PRINT,"========================="
      PRINT 1111
      PRINT 3333
      PRINT 4444
C
      DO 101 L=1,NUM
      READ(DATAF1,11) DUM,CAP(L),FOR(L),REP(L)
      AVCAP = AVCAP + CAP(L)
      PRINT 999,L,CAP(L),FOR(L),REP(L)
 101  CONTINUE
C
C  ****BUILD CAPACITY OUTAGE & FREQUENCY TABLES****
C
 97   PRINT 55,AVCAP
      PRINT 99
      PRINT 1111
```

```
C
      DO 500 J=1,NUM
      OUT = FOR(J)
      IF (REP(J) .NE. 0.0)  GO TO 91
      RR=0.0
      GO TO 93
 91   RR = 1.0/REP(J)
 93   KAP = (CAP(J)+HALF)/STEP
C  ****NEW TABLE BECOMES OLD TABLE****
         DO 200 K = 1,LENGTH
         POLD(K) = PNEW(K)
         FOLD(K) = FNEW(K)
 200     CONTINUE
      LENGTH = LENGTH+KAP
         DO 300 IX=1,LENGTH
C  ****CHECK SUBSCRIPT'S SIGN--IF > 0, USE FULL FORMULA...
         IF ((IX-KAP) .GT. 0)  GO TO 50
C  ...OTHERWISE, USE DEFINITIONS P(X)=1.0 & F(X)=0.0 WHEN X < 0
         PNEW(IX) = POLD(IX) * (1.0-OUT) + OUT
         FNEW(IX) = (1-OUT)*FOLD(IX)+RR*OUT*(1-POLD(IX))
         X(IX+1) = X(IX) + STEP
         GO TO 300
  50     PNEW(IX) = POLD(IX)*(1.0-OUT) + POLD(IX-KAP)*OUT
         FNEW(IX) = (1-OUT)*FOLD(IX)+RR*OUT*(POLD(IX-KAP)-POLD(IX))
&                   +OUT*FOLD(IX-KAP)
         X(IX+1) = X(IX) + STEP
         IF (PNEW(IX) .GE. PMIN)  GO TO 300
         LENGTH = IX
         GO TO 500
 300     CONTINUE
 500  CONTINUE
C
C  ****CONSTRUCT CUMULATIVE-CUMULATIVE CAPACITY OUTAGE TABLE FOR
C      UNSERVED ENERGY CALCULATIONS****
      DO 600 KK=1,LENGTH
      ISUB = LENGTH + 1 - KK
      CUMCAP(ISUB) = CUMCAP(ISUB+1) + PNEW(ISUB)
 600  CONTINUE
C
      IF(ISWTCH2 .NE. 1)  GO TO 92
C
C  ****PRINT CAPACITY OUTAGE TABLES****
      PRINT 1111
      PRINT," CAPACITY OUTAGE TABLES"
      PRINT," ======================"
      PRINT 33
      PRINT 66
       IF (STEP .GT. 10)  GO TO 46
         DO 601 KK=1,LENGTH,50
         PRINT 44, X(KK),PNEW(KK),CUMCAP(KK+1),FNEW(KK)
```

```
 601     CONTINUE
      GO TO 92
  46  PRINT 44,  (X(KK),PNEW(KK),CUMCAP(KK+1),FNEW(KK),  KK=1,LENGTH)
C
C
C
C   ****CALCULATE RELIABILITY PARAMETERS****
C
 92  IF (ISWTCH .NE. 1)   GO TO 96
     READ(DATAF2,11)  DUM,E
     READ(DATAF2,22)  DUM,WKPK
     READ  (DATAF2,22)  DUM,  (DYPER(I),I = 1,7)
        DO 1200 I = 1,2
        READ(DATAF3,88)  DUM,(HRPER(I,J),J=1,24)
 1200    CONTINUE
C
C   ****CALCULATE PEAK INFO & PRINT IT SEPARATELY****
C
C   ****FIND RESERVES FOR PEAK****
      RES=AVCAP-WKPK
      IRES=RES+.5
      KSUB=IRES/STEP+2.0
      N=IRES/STEP
C   ****CALCULATE HLOLE,FREQ,AND UNS. ENERGY FOR PEAK****
C
      PKLOLE=PNEW(KSUB)
      PKUNS=CUMCAP(KSUB)*STEP  -  (RES-N*STEP)*PNEW(KSUB)
      PKFREQ=PNEW(KSUB)+(1./24.)*FNEW(KSUB)
C
C   ****PRINT PEAK RESULTS****
C
      PRINT 1111
      PRINT 7777
      PRINT 5555,WKPK,RES
      PRINT 6666,PKLOLE,PKFREQ,PKUNS
      PRINT 1111
C
     QWKL=1.0
     QWKF=1.0
     QWKH = 1.0
     TOTLOLP = 0.0
     TOTFREQ = 0.0
     TOTEEU = 0.0
     TOTHLOL = 0.0
     TOTFREQE = 0.0
     TOTDURE = 0.0
     K=1
C
     DO 1000 I = 1,7
     IF(I.GE.6) K=2
```

```
      DYPK(I) = DYPER(I)*.01*WKPK
      RES=AVCAP-DYPK(I)
      IRES=RES+.5
      KSUB = IRES/STEP + 2
       QHOUR(I) = 1.0
C
C
C   ****COMPUTE DAILY LOLP AND SUM****
      LOLP(I) = PNEW(KSUB)
      TOTLOLP = TOTLOLP + LOLP(I)
      FREQPK = FNEW(KSUB)
C
C   ****COMPUTE HOURLY INDICES****
C
         DO 1100 J = 1,24
         HRPK(I,J) = DYPK(I)*.01*HRPER(K,J)
         RES = AVCAP - HRPK(I,J)
         IRES = RES + .5
         N = IRES/STEP
         KSUB = IRES/STEP+2.0
C
C   ****HOURLY LOLP = PNEW(KSUB)****
C   ****SUM HOURLY LOLP FOR THE DAY****
         HLOLP(I) = HLOLP(I) + PNEW(KSUB)
C
C   ****CALCULATE Q-HOURLY****
      QHOUR(I) = QHOUR(I)*(1.0-PNEW(KSUB))
C
C   ****HOURLY FREQ. = FNEW(KSUB)****
C   ****SUM HOURLY FREQ. FOR THE DAY****
C
      SUMFREQ = SUMFREQ + FNEW(KSUB)
C
C   ****UNSERVED ENERGY = CUMCAP(KSUB)*STEP****
C   ****SUM UNSERVED ENERGY FOR THE DAY****
      DYEEU(I) = DYEEU(I) + CUMCAP(KSUB)*STEP-(RES-N*STEP)*PNEW(KSUB)
C
 1100 CONTINUE
C
      SUMFREQ = SUMFREQ/24.0
C
C   ****COMPUTE DAILY FREQUENCY AND SUM****
      FREQ(I) = LOLP(I) + SUMFREQ
      TOTFREQ = TOTFREQ + FREQ(I)
C
C   ****SUM DAILY UNSERVED ENERGY****
      TOTEEU = TOTEEU + DYEEU(I)
C
C   ****SUM HOURLY LOLP****
      TOTHLOL = TOTHLOL + HLOLP(I)
```

```
C
C   ****CALCULATE DURATION FOR THE DAY****
       DUR(I) = HLOLP(I)/FREQ(I)
C
C   ****CALCULATE MWH/OCC****
       MWHOC(I) = DYEEU(I)/FREQ(I)
C
C   ****CALCULATE Q-DAILY FOR THE DAY AND WEEK****
       QLOLP(I) = 1.0 - LOLP(I)
       QWKL = QWKL * QLOLP(I)
C
C   ****CALCULATE Q-FREQ FOR THE DAY AND WEEK****
       QFREQ(I) = 1.0 - FREQ(I)
       QWKF = QWKF * QFREQ(I)
C
C   ****CALCULATE Q-HOURLY FOR THE WEEK****
       QWKH = QWKH * QHOUR(I)
C
C   ****CALCULATE HLOLP WITH E AND SUM****
       HLOLPE(I) = E * LOLP(I)
       TOTHLOLE = TOTHLOLE + HLOLPE(I)
C
C   ****CALCULATE FREQUENCY USING E AND SUM****
       FREQE(I) = LOLP(I) + E/24.0 * FREQPK
       TOTFREQE = TOTFREQE + FREQE(I)
C
C   ****CALCULATE DURATION USING E****
       DURE(I) = HLOLPE(I)/FREQE(I)
C
C   ****CALCULATE EXPECTED LOSS OF LOAD****
       XLOL(I) = DYEEU(I)/HLOLP(I)
C
C   ****RESET SUMFREQ AND COMPUTE NEXT DAY****
       SUMFREQ = 0.0
C
C
 1000 CONTINUE
C
C
C
C   ****COMPUTE TOTAL DURATION****
       TOTDUR = TOTHLOL/TOTFREQ
C
C   ****CALCULATE MWH/OCC FOR THE WEEK****
       MWHWK = TOTEEU/TOTFREQ
C
C   ****CALCULATE TOTAL DURATION USING E****
       TOTDURE = TOTHLOLE/TOTFREQE
C
C   ****CALCULATE TOTAL XLOL****
```

```
      TOTXLOL=TOTEEU/TOTHLOL
C
      PRINT 9999
      PRINT,"RELIABILITY CALCULATIONS"
      PRINT,"========================"
C
      PRINT 1111
      PRINT 9999
      PRINT 333
      PRINT 9999
      DO 962 I=1,7
       PRINT 8888,I,DYPK(I),LOLP(I),HLOLP(I),DYEEU(I),FREQ(I)
 962   CONTINUE
      PRINT 9999
       PRINT 222,TOTLOLP,TOTHLOL,TOTEEU,TOTFREQ
C
C
    PRINT 1111
      PRINT 9999
      PRINT 444
      PRINT 9999
      DO 963 I=1,7
       PRINT 8888,I,DUR(I),MWHOC(I),QLOLP(I),QFREQ(I),QHOUR(I)
 963   CONTINUE
      PRINT 9999
      PRINT 555,TOTDUR,MWHWK,QWKL,QWKF,QWKH
C
C
      PRINT 1111
      PRINT 9999
      PRINT 666
      PRINT 9999
       DO 964 I=1,7
       PRINT 8888,I,HLOLPE(I),FREQE(I),DURE(I),XLOL(I)
 964   CONTINUE
      PRINT 9999
      PRINT 777,TOTHLOLE,TOTFREQE,TOTDURE,TOTXLOL
      PRINT 1111
C
C
 96   STOP
      END
```

Appendix D

CALCULATION OF TRANSMISSION RELIABILITY INDICES

D.1 INTRODUCTION

This appendix presents an example of the calculation of the transmission indices and equations detailed in Section 4. A six bus system is described and the indices are calculated for a five day load model based on single transmission contingencies.

D.2 SYSTEM DESCRIPTION

As shown in Figure D.1, the example system consists of six buses connected by 13 circuits on nine separate rights of way. The generation consists of three units at three different buses. The loads at each bus are shown as a fraction of the total system load. In this example, only active power will be considered, therefore, a linearized load flow will be used to calculate the system quantities.

The bus data for the system is given in Table D.1 and the line data in Table D.2. The indices are calculated for all single line outages.

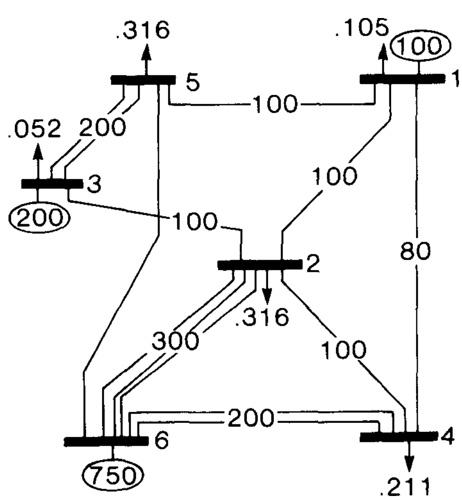

Figure D.1 Sample System

Table D.1 Sample System Bus Data

Bus	Load		Generation (MW)	
No.	Fraction	MW	Capacity	Dispatch
1	.105	80	100	100
2	.316	240	0	0
3	.052	40	200	200
4	.211	160	0	0
5	.316	240	0	0
6	0	0	750	460
Total	1.0	760	1050	760

Table D.2 Sample System Line Data

Line	From	To	Length	Impedance (pu)		
No.	Bus	Bus	(mi.)	R	X	Capacity (MW)
1	1	2	40	.10	.40	100
2	1	4	60	.15	.60	80
3	1	5	20	.05	.20	100
4	2	3	20	.05	.20	100
5	2	4	40	.10	.40	100
6	2	6	30	.08	.30	100
7	2	6	30	.08	.30	100
8	2	6	30	.08	.30	100
9	3	5	20	.05	.20	100
10	3	5	20	.05	.20	100
11	4	6	30	.08	.30	100
12	4	6	30	.08	.30	100
13	5	6	61	.15	.61	100

D.3 FAILURE EFFECTS ANALYSIS (FEA)

The FEA procedure described in Section 4 will be used to illustrate the computation of the indices on the sample system. Each step of the FEA, as given in Figure 4.1 is detailed.

D.3.1 Define Candidate Events

For this example, the candidate events are outages of transmission lines. The events will be considered in the static sense with no consideration of the time dependent behavior of the system.

D.3.2 Determine Events for Study

All single line outages will be evaluated for purposes of this example. Since all single outages are to be evaluated, no ranking of the events is attempted.

D.3.3 Calculate Network Quantities

The power flow will be solved first using the dispatch specified in Table D.1 for all outage cases. The base case power flow is shown in Figure D.2 and the outage case power flow results are summarized in Table D.3. No redispatching was used for these power flows.

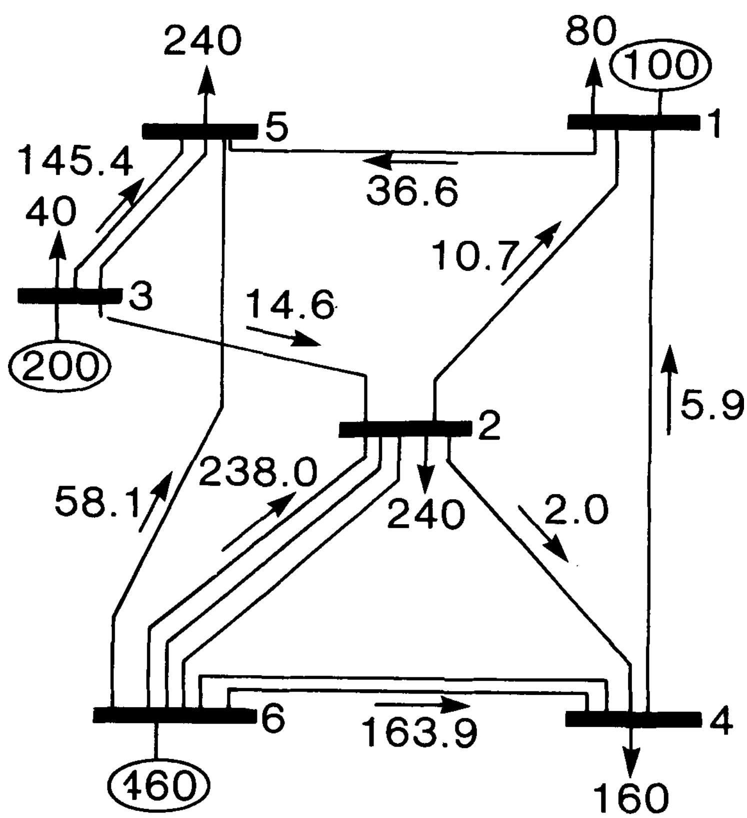

Figure D.2 Sample System Base Case Power Flow

Table D.3 Summary of Sample System Outage Power Flows

Outage No.	From Bus	To Bus	Overloaded Lines From Bus	To Bus	Flow	Overload	
1	1	2		NONE			
2	1	4		NONE			
3	1	5		NONE			
4	2	3		NONE			
5	2	4		NONE			
6	2	6	2	6	211.2	11.2	(on
7	2	6	2	6	211.2	11.2	two
8	2	6	2	6	211.2	11.2	circuits)
9	3	5	3	5	124.2	24.2	
10	3	5	3	5	124.2	24.2	
11	4	6	4	6	125.8	25.8	
12	4	6	4	6	125.8	25.8	
13	5	6		NONE			

D.3.4 Determine Which Events Result in Failure

A second set of power flow cases was executed for the base case and all single line outages. For these cases, complete freedom to redispatch was allowed to relieve any overloaded lines. At the same time, the system load was increased or decreased (maintaining the load fractions at each bus) to the maximum level possible such that no overloads exist and no more generation can be dispatched without overloading some line. This load is termed the Load Supplying Capability (LSC) of the system. The LSC calculation is used to determine the loss of load (LOL) failure events. The LSC determines the load at which there is a failure somewhere in the system.

As shown in Figure D.3, as demand increases, the load supplied will increase proportionally until a limit is reached in the bulk power system. The limit is termed the LSC of the network. As demand increases further, the load supplied will also increase but at a rate slower than demand indicating load not supplied (LNS) on the system. The key point is that the LSC correctly determines when an LOL event occurs.

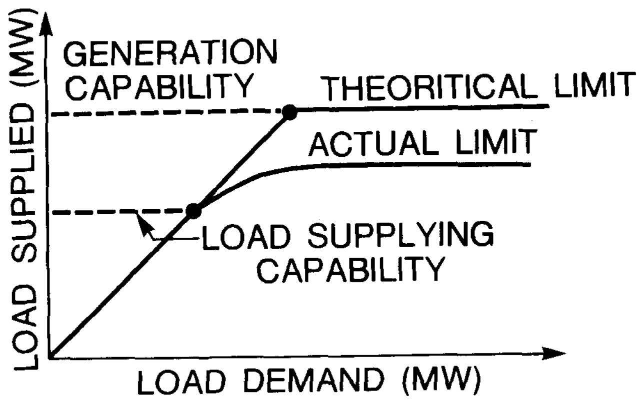

Figure D.3 Load Supplied Vs. Load Demand

The LSC results for all cases are tabulated in Table D.4. It is important to remember that the MW of load not supplied is calculated assuming that the bus load fractions remain constant. This may result in a larger than actual value of unsupplied load in some cases. However, for purposes of illustrating the index calculations, further refinement of these values is not necessary.

Table D.4 LSC Results for Sample System

Outage No.	From Bus	To Bus	LSC	Forecasted Load	Load Not Supplied
0	-	-	875	760	0
1	1	2	867	760	0
2	1	4	865	760	0
3	1	5	879	760	0
4	2	3	879	760	0
5	2	4	874	760	0
6	2	6	736	760	24
7	2	6	736	760	24
8	2	6	736	760	24
9	3	5	803	760	0
10	3	5	803	760	0
11	4	6	649	760	111
12	4	6	649	760	111
13	5	6	791	760	0

D.3.5 Calculate Event Probabilities and Frequencies

Prior to calculation of the indices, the state probabilities corresponding to the contingencies must be found. Table D.6 illustrates the outage data required for the calculation of the probabilistic indices. For each outage, a mean time to repair (MTTR), the mean time between failures (MTBF), and the resulting outage probabilities are shown. The state probabilities are then calculated by multiplying the probability that the line is out times the probability that all other lines are in service.

$$P_{state} = \sum_{\substack{i=\text{elements}\\ \text{on outage}}} P_i \quad \sum_{\substack{j=\text{elements}\\ \text{on outage}}} (1 - P_j)$$

Table D.6 Probability Data

Outage No.	MTTR	MTBF	Outage Probability	State Probability
0	--	--	0.	0.64500212
1	8	200	.04	0.02687509
2	8	133.3	.06	0.04117035
3	8	400	.02	0.01316331
4	8	400	.02	0.01316631
5	8	200	.04	0.02687509
6	8	266.7	.03	0.01994852
7	8	266.7	.03	0.01994852
8	8	266.7	.03	0.01994852
9	8	400	.02	0.01316331
10	8	400	.02	0.01316331
11	8	266.7	.03	0.01994852
12	8	266.7	.03	0.01994852
13	8	133.3	.06	0.04117035
				0.93348881

D.3.6 Calculate Reliability Indices

The indices discussed in Section 4 are calculated using the results of the system solution, failure identification and probability steps of the FEA.

D.3.6.1 Deterministic Indices

The deterministic indices discussed in Section 4 can be calculated based on the results of the two sets of power flows. Even for this small system, a computer program is necessary to complete even the deterministic portion of the example. For example, power flow results for the base case and contingencies is necessary. It is not practical to calculate the power flows by hand or to determine which events result in loss of load using a trial and error procedure, even using a power flow program.

Maximum Flow

The maximum flow encountered on each line during a single contingency is shown in Table D.5. These results were obtained using a set dispatch, with no redispatching allowed and a linearized power flow calculation.

Table D.5
Maximum Flow Results

Line	Maximum Flow MW
1	23.6
2	17.4
3	61.1
4	35.8
5	26.9
6	211.2
7	211.2
8	211.2
9	124.2
10	124.2
11	125.8
12	125.8
13	69.9

Maximum Load Not Supplied

As mentioned previously, the LSC will be used to illustrate indices based on load not supplied. For this system the maximum system load not supplied, using Table D.4, due to a single contingency is 111 MW.

Maximum Energy Curtailed

Calculation of energy curtailment requires knowledge of the duration of outage. If sufficient data was available, in theory, the duration could be calculated based on repair times and load shape in a manner similar to that described for the generation indices. However at this time no methods for calculating expected duration of transmission related outages is known. Therefore, a duration of eight hours for load interruptions due to transmission outages will be used for illustrative purposes. For this system the maximum energy curtailment is due to either contingency 11 or 12. Therefore, a curtailment of 111 MW for 8 hours represents an energy curtailment of 888 MWH.

Minimum Load Supplying Capability

From Table D.4 the minimum LSC for the single contingencies is 649 MW. This corresponds to 85% of the peak load of 760 MW.

D.3.6.2 Probabilistic Indices

The probabilistic indices described in Section 4 can now be calculated using the information in Tables D.4 and D.6. These indices will represent the risk due to single line outage contingencies only. From Table D.6, it is noted that these events represent .933 of the probability for the small system. Therefore, although the bulk of the cases not studied will result in Loss of Load, they have a relatively small probability of occurrence. This is not necessarily true for larger systems.

In order to classify individual events as success or failure, the LSC concept will be utilized. Since the calculation of the indices is independent of the method chosen for identifying failure, the calculations that follow could be implemented for more detailed system models of system failure.

LOLP

The Loss of Load Probability from the single line outages for the peak load can be determined from the data given and equation 4-6. The following line outages are failure events; outages 6, 7, 8, 11, 12.

$$\begin{aligned} LOLP &= P(6) + P(7) + P(8) + P(11) + P(12) \\ &= .01994852 + 0.1994852 + .01994852 + .01994852 + .01994852 \\ &= 0.09974260 \text{ days/day} \end{aligned}$$

If the LSC's are assumed constant for a given period of time, the LOLE can be calculated using equation 4-7. A five day load model is given below with the LOLE results for each load shown and the LOLE indicated.

Load	LOLP (days/day)
400	0.0
760	0.09974260
868	0.23528500
800	0.14091294
900	0.93348881
	LOLE = 1.40942934 days/period

Frequency of Loss of Load (FLOL)

The frequency of loss of load due to single line outages requires that the repair time be specified. This information is contained in Table D.6 as Mean Time To Repair (MTTR). For this sytem, all repair times are assumed to be equal to eight hours. The other quantity required by equation 4-9 is the state probabilities given in Table D.6. The system frequency is found by calculating the outage frequencies associated with the outage of lines 6, 7, 8, 11, and 12, which cause loss of load.

$$FLOL = P(6)(1/MTTR(6)) + P(7)(1/MTTR(7)) + P(8)(1/MTTR(8)) + P(11)(1/MTTR(11)) + P(12)(1/MTTR(12))$$

$$= .01994852(1/8) + .01994852(1/8) + .0994852(1/8) + .01994852(1/8) + .01994852(1/8)$$

$$= .01246783 \text{ occ./hour}$$

This calculation is more difficult in general than indicated by this example. Since only single line outages are considered, only one transition to a success state is possible. If multiple outages are included, the calculation requires that all transitions be examined to determine if the transition results in success or failure.

Expected Energy Not Supplied (EENS)

Using equation 4-10, the Expected Energy Not Supplied can be calculated for this example. Again, the LSC-Load is used to approximate the load not supplied and Table D.6 is used for the probability data.

$$\begin{aligned} EENS &= L(6)\ P(6) + L(7)\ P(7) + L(8)\ P(8) + L(11)\ P(11) \\ &\qquad + L(12)\ P(12) \\ &= (24)(.01994852) + (24)(.01994852) + (24)(.01994852) \\ &\quad + 111(.01994852) + (111)(.01994852) \\ &= 5.86486488 \text{ MWh} \end{aligned}$$

For the five loads used in the previous examples the period calculation is:

Load	EENS (MWh)
400	0
760	5.86486488
868	21.66879907
800	10.22510203
900	46.30583285

EENS = 84.06459883 MWh/period

Bulk Power Interruption Index (BPII)

The BPII is calculated using equation 4-12. Note that this index is just a linear function of the frequency index.

$$\begin{aligned} BPII &= \frac{1}{760}\,(F(6)\ L(6) + F(7)\ L(7) + F(8)\ L(8) + \\ &\qquad F(11)\ L(11) + F(12)\ L(12)) \\ &= \frac{1}{760}\,((.00249357)(24) + (.00249357)(24) + \\ &\quad (.00249357)(24) + (.00249357)(111) + (.00249357)(111)) \\ &= .00096462 \text{ MW/MW} \end{aligned}$$

Bulk Power Energy Curtailment Index (BPECI)

As shown in equation 4-13, the BPEII is the EENS divided by peak load.

$$BPECI = \frac{84.06459883}{760} = .11061131 \text{ MWh/MW-yr}$$

D.3 ADDITIONAL CALCULATIONS

The values shown in the previous sections considered only the contribution to the risk which resulted from the base case and all single line outages. The total probability of the events considered was .93349 (Table D.6). It is of interest to determine the maximum possible value of the index if all events not studied are considered to be failures. This calculation will fix a ceiling on the index since it represents the worst case.

This calculation will be illustrated for the LOLE calculation. For each load level, 1-.93349 of the probability was not considered. The maximum LOLE is calculated by multiplying the probability not considered by five (number of loads) to determine the loss of load expectation for events not considered. This is then added to the calculated value to find the maximum value.

LOLE (max) = LOLE (calculated) + (1-.93349)(No. of loads)
LOLE (max) = 1.40943 + (1-.93349)5
LOLE (max) = 1.74198 days/period

The manner in which this value changes as more of the event probability is included, is shown in Table D.7. The maximum values are tabulated for the base case only, all single line outages, and all single and double line outages. The LOLE calculated for the base case only is equal to the probability sum because one of the five loads evaluated (900 MW) was a failure and the remaining four were success states. Therefore the LOLE is the sum of the LOLP for the four successes (0) plus the LOLP for the failure (.645).

Table D.7 Maximum LOLE

Case	Probability Sum	LOLE Calculated	LOLE Maximum
Base Case only	.64500	.64500	2.41992
All Single Line Outages	.93349	1.40943	1.74198
All Single and Double Line Outages	.99218	1.60797	1.64708

D.4 INTERPRETATION OF RESULTS

Using the values in Table D.7, the interpretation of the indices can be discussed. When less than 100% of the event probability is included in the LOLE calculation, the results tend to be too low. If this number (LOLE (calculated)) is quoted as "the loss of load expectation for the transmission system", an important assumption has been left unstated. Namely, by only counting the contribution to the LOLE from the events studied, the computation has assumed that all events not studied will contribute zero to the actual LOLE. Therefore, the LOLE calculated is the optimistic result which, in general, will be too low. It is necessary to calculate the maximum, or pessimistic, answer in order to bound the actual value.

Appendix E

CALCULATION OF GENERATION AND TRANSMISSION LOLE

E.1 SAMPLE SYSTEM

The six bus, three generating unit, 13 transmission line system, used in Appendix D, will be used to illustrate the calculation of combined generation and transmission LOLE. The transmission data and load data for the five loads was detailed in Appendix D. The generation data is given in Table E.1.

TABLE E.1

Unit No.	Bus No.	Capacity (MW)	Outage Probability
1	1	100	.12
2	3	200	.15
3	6	750	.18

E.2 STATE ENUMERATION APPROACH

In order to calculate the combined LOLE of the generation and transmission systems, a state enumeration approach has been selected because it is a conceptually simple, straightforward approach. The calculations are repeated for several combinations of states to illustrate the impact of neglecting states.

E.3 COMBINATIONS EVALUATED

Using the state enumeration approach, six combinations of states were evaluated. The six combinations which will be referred to as cases are:

Case 1. No generation or transmission elements on outage.

Case 2. No generation outages and all single transmission line outages.

Case 3. No generation outages and all single and double transmission line outages.

Case 4. All generation outages and no transmission line outages.

Case 5. All generation outages, all single transmission line outages and all combinations of generation and single transmission line outages.

Case 6. All generation outages, all single and double transmission lines and all combinations of generation and single and double transmission line outages.

E.4 PROBABILITY CALCULATIONS

The probability of each state is required to determine the LOLE. Using the outage probabilities from Appendix D and assuming independence, the state probabilities are calculated by:

$$P_{state} = \sum_{\substack{i=\text{elements} \\ \text{on outage}}} p_i \quad \sum_{\substack{j=\text{elements} \\ \text{not on outage}}} (1-p_j)$$

If all possible states were evaluated, the state probabilities would sum to 1.0. However, none of the six combinations included all possible outages, and therefore the probability sum is less than 1.0. The actual probability sums for the cases are shown in Table E.2.

TABLE E.2

PROBABILITY SUMS

Case	# of Events	Probability Sum
1	1	.396
2	14	.573
3	92	.609
4	8	.645
5	112	.933
6	736	.992

Obviously, as more states are considered, the probability will increase. Also as states with more elements out of service are considered more will result in failure. These concepts are illustrated in Figures E.1 and E.2. Figure E.1 illustrates the increase in number of failure states as the analysis progresses from single to double to triple contingencies. Also shown is the calculation of the number of total events for each category. In Figure E.2 the decrease in state probabilities as the number of elements on outage increases is shown. The amount of decrease depends on the size of the network and line outage probabilities.

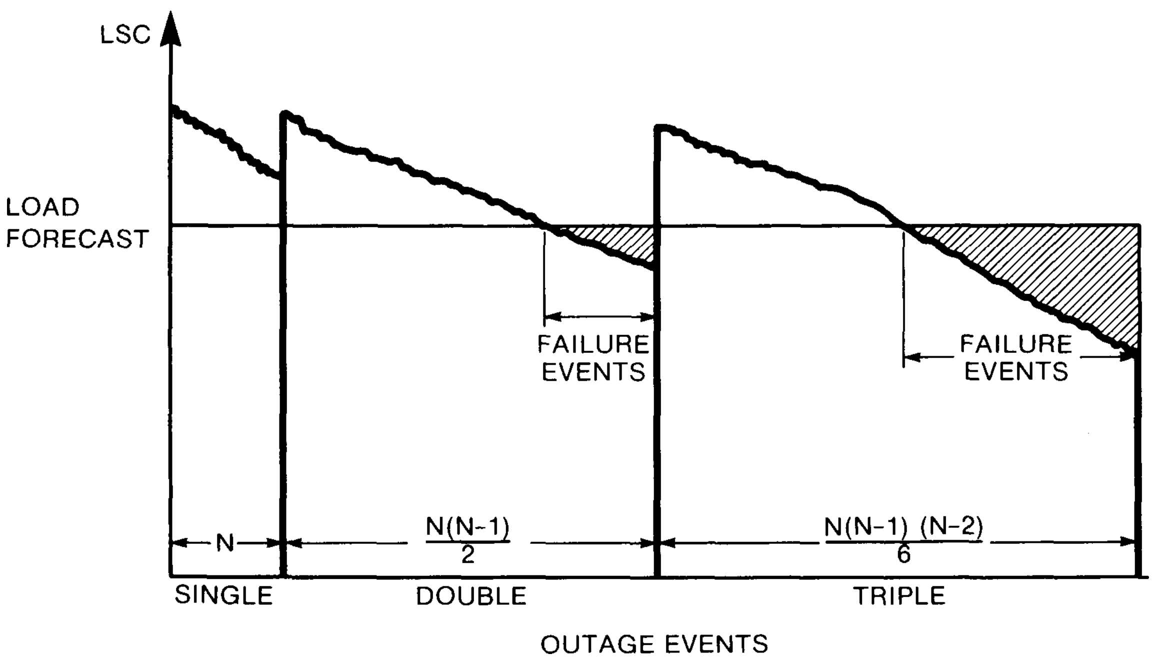

Figure E.1 Failure Events for Outage Cases

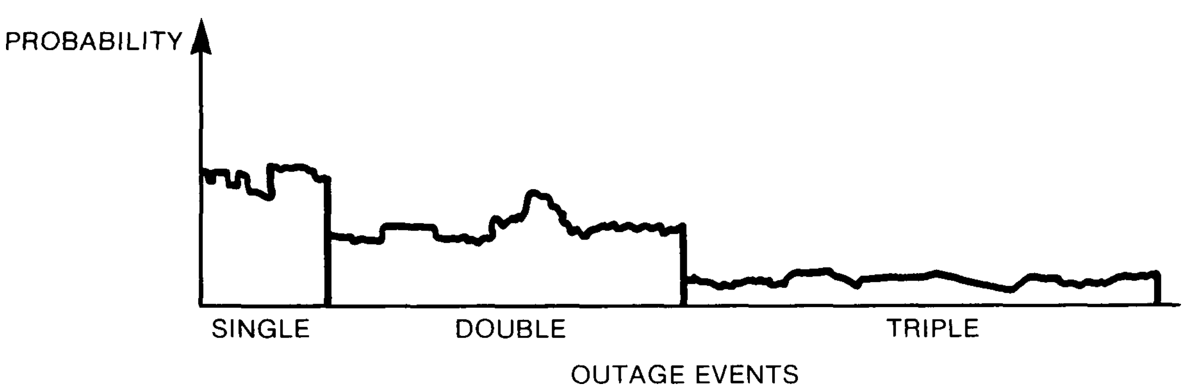

Figure E.2 State Probabilities for Outage Cases

E.5 LOLE CALCULATIONS

By enumerating the necessary states and calculating an LSC for each state, a threshold of failure is defined. The load levels are then checked against the LSC to determine pass or fail. The LOLE is then the sum of the state probabilities for those events resulting in failure. Table E.3 illustrates the pass or fail decision for each of the states included in case 2. Table E.4 shows the results for case 4. As more and more states are considered, the pass fail tables become more and more extensive. For example, case 6 included 736 different states and with five loads resulted in 3,680 combinations.

TABLE E.3

STATE ENUMERATION FOR CASE 2

P = Passed State

F = Failed State

Transmission State	LSC	Loads 400	760	868	800	900
All in	875	P	P	P	P	F
1	867	P	P	P	P	F
2	865	P	P	P	P	F
3	879	P	P	P	P	F
4	879	P	P	P	P	F
5	874	P	P	P	P	F
6	736	P	F	F	F	F
7	736	P	F	F	F	F
8	736	P	F	F	F	F
9	803	P	P	F	P	F
10	803	P	P	F	P	F
11	649	P	F	F	F	F
12	659	P	F	F	F	F
13	791	P	P	F	F	F

TABLE E.4

STATE ENUMERATION FOR CASE 4

Generation State	Available Generation	LSC	400	760	868	800	900
all in	1050	875	P	P	P	P	F
100	950	776	P	P	F	F	F
200	850	655	P	F	F	F	F
750	300	300	F	F	F	F	F
100, 200	750	556	P	F	F	F	F
100, 750	200	200	F	F	F	F	F
200, 750	100	100	F	F	F	F	F
all out	0	0	F	F	F	F	F

The LOLE can now be found for the six different cases listed earlier. The first involved calculation of LOLE considering only the all elements in service state for all five loads. As seen in the first line of either Table E.3 or E.4, only the 900 MW load is a failure state. Therefore, for case 1 the LOLE is equal to the probability of that state.

$$\text{Case 1 LOLE} = \text{Prob}(400) + \text{Prob}(760) + \text{Prob}(865) + \text{Prob}(800) + \text{Prob}(900)$$
$$0 + 0 + 0 + 0 + .3956 = .3956 \text{ days/period}$$

Since the probability of the events considered does not add to 1.0, it is interesting to put an upper bound on the LOLE. This can be done by assuming that all cases not studied are failures. For case 1 this calculation is as follows:

$$\text{Probability of events not studied} = 1 - .3956 = .6044$$

$$LOLE_{MAX} = LOLE_{Calculated} + (.6044)(\text{No. of Loads})$$

$$= .3956 + (.6044)(5)$$

$$= 3.4175 \text{ days/period}$$

Since all of the events not studied do not result in failure, additional cases will show that this maximum LOLE is pessimistic and decreases as more events are included. Table E.5 details the probability sum, calculated LOLE and maximum LOLE for each of the six cases.

TABLE E.5

COMBINED G&T LOLE

Case No.	Description	Probability Sum	LOLE Calculated	LOLE Maximum
1	No Gen Outages No Trans Outages	.396	.39562	3.41753
2	No Gen Outages All Single Line Outages	.573	.86449	3.00167
3	No Gen Outages All Single and Double Line Outages	.609	.98626	2.94345
4	All Gen Outages No Trans Outages	.645	1.455305	3.23029
5	All Gen Outages All Single Line Outages	.933	2.44532	2.77788
6	All Gen Outages All Single and Double Line Outages	.992	2.64205	2.68117

E.6 DISCUSSION OF RESULTS

As in Appendix D, it is necessary to interpret the calculated values of LOLE as optimistic results since the cases not studied, are implicity assumed to be success states. Therefore the actual LOLE of the system is between the calculated and maximum values.

Many of the values in Table E.5 could have been obtained by other methods. For example, the LOLE of the transmission system alone was shown in Appendix D to be 1.4094. The LOLE for case 2 can be obtained from this result by multiplying by the probability that the generation is fully available (assumption in Appendix D).

$$\begin{aligned}\text{Case 2 LOLE} &= (1.4094)(1-.12)(1-.15)(1-.18) \\ &= (1.4094)(.6134) \\ &= .8645 \text{ days/period}\end{aligned}$$

It is also interesting to compare these results to the conventional LOLE for the generation system calculated using the program in Appendix C. For this system the LOLE for generation only is 1.175 days/period. However, since this calculation contains no information about the transmission system, all combinations of generation and transmission could be failures, so the maximum combined LOLE is 5.0. Thus a standard LOLE calculation does not give a great deal of information on the combined G&T LOLE.

The error present in the conventional calculation can be seen from Table E.4. There are six failure states (underlined on Table E.4) which the conventional calculation considers to be success (available capacity greater than load) which the calculation including transmission limits determines as failure.

Also note that the generation and transmission LOLE's cannot be evaluated separately and then combined. For example, from the numbers given above:

$$LOLE_G = 1.175$$
$$LOLE_T = 0.98626$$

and therefore

$$LOLE_G + LOLE_T = 2.161 = LOLE_{GT}$$

APPENDIX F

PROBABILITY LAWS FOR RANDOM VARIABLES

1. SPECIFYING THE PROBABILITY LAW OF A RANDOM VARIABLE

Numerical-valued random phenomenon can be modeled mathematically as random variables. For purposes of this discussion, the terms "random variable" and "numerical-valued random phenomenon" can be regarded as equivalent. This appendix reviews some basic terminology and definitions from basic probability theory. These concepts can be applied to all the numerical-valued reliability indicators. The notation and terminology in the following discussion follows closely that given in Reference 5-1, pp. 151-157. For a thorough discussion of the notion of a random variable, see Reference 1, Chapter 7.

The probability law of a random variable can be specified in several ways. One way is to specify the distribution function. If X is a random variable, the <u>distribution function</u> of X is defined by

$$F_X(x) \equiv P(X < x). \qquad \text{(F-1)}$$

In words, the distribution function of X is defined for any real number x as the probability that the observed value of the random variable is less than or equal to x. The distribution function increases from zero to one as x increases from $-\infty$ to $+\infty$. The distribution function may be constant over some intervals, but it never decreases (as x increases).

As illustrated by equation (F-1), capital letters are commonly used to denote random variables. Equation (F-1) also illustrates that the symbol denoting the random variable is used in two ways. First, the capital letter refers to the phenomenon itself (that is, to all possible values). This is the meaning of the subscript X on F_X in equation (F-1). The symbol F is the general symbol for a distribution function, and X denotes a particular random variable. Second, the capital letter refers to the observed value of the random phenomenon on a particular trial. This is the meaning of the symbol X in the righthand side of equation (F-1). The expression ($X \leq x$) refers to the event that the observed value X of the random phenomenon is less than or equal to the argument x.

As an example, consider the toss of a single six-sided die. The outcome of the toss is a numerical-valued random phenomenon, with possible (realizable) values 1,2,3,4,5,6. Let X be a random variable defined by

X = outcome of tossing a single six-sided die.

Thus "X" is a mathematical symbol for the physical phenomenon "outcome of tossing a single die". If the die is "fair", a reasonable distribution function for X is

$$
\begin{aligned}
F_X(x) &= 0 \quad , \quad x<0 \\
&= \frac{[x]}{6} \quad , \quad 0 \leq x \leq 6 \qquad \text{(F-2)} \\
&= 1 \quad , \quad x>6
\end{aligned}
$$

where

$$[x] \equiv \text{largest integer} < x$$

Equation (F-2) defines a "staircase function" with a "jump" of 1/6 at each integer from 1 to 6. A graph of equation (F-2) is shown in Figure F-1.

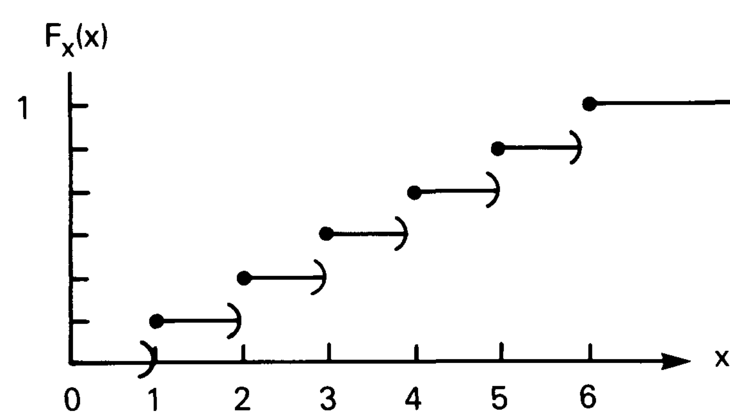

Figure F-1. Distribution function for toss of a single die

Equation (F-2), and Figure F-1, illustrate that the distribution function of a random variable is defined for all real numbers x, even when $F_X(x)$ only increases in jumps.

Random variables are classified as discrete, continuous, or mixed in terms of their distribution functions. If $F_X(x)$ increases only in jumps, so that $F_X(x)$ is constant between jump points, then $F_X(x)$ is said to be a discrete distribution function and X is called a discrete random variable. Figure F-1, defined by equation (F-2), is an example of a discrete distribution function.

If $F_X(x)$ increases continuously from 0 to 1 (no jumps), then $F_X(x)$ is said to be a continuous distribution function, and X is called a continuous random variable.

As a simple example of a continuous random variable, consider the phenomenon of spinning a pointer and observing the point at which the tip of the pointer comes to rest on a circle. Suppose that the circumference of the circle is 6 inches. Let

X = Position of pointer on circle, with respect to some "zero" position.

If the pointer does not favor any particular portion of the circle, a reasonable distribution function for X is

$$
\begin{aligned}
F_X(x) &= 0, \quad x<0 \\
&= \frac{x}{6}, \quad 0 \le x \le 6 \qquad \text{(F-3)} \\
&= 1, \quad x>6
\end{aligned}
$$

Equation (F-3) defines a continuous function, which increases linearly from 0 to 1 as x increases from 0 to 6, as shown in Figure F-2.

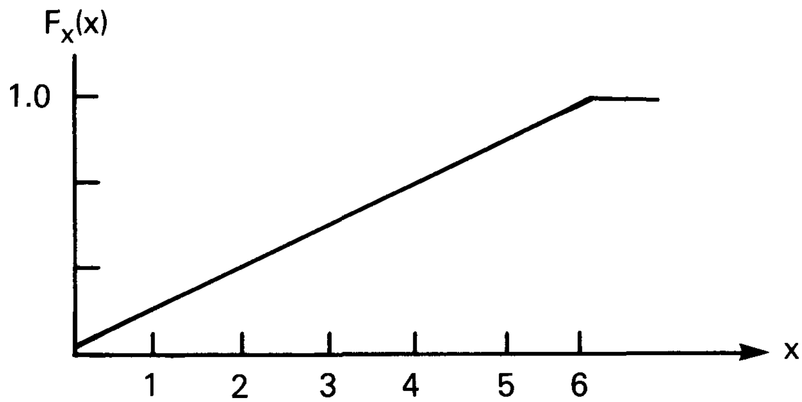

Figure F-2. Distribution function for position of a pointer on a circle.

If the distribution function has both jump points and intervals where $F_X(x)$ increases continuously, then X is said to be a mixed random variable.

The distribution function of a mixed random variable can always be expressed as a sum of a discrete distribution function, $F_d(X)$ and a continous distribution function $F_c(X)$. Thus, if X is a mixed random variable, the distribution function of X can be expressed as

$$F_X(x) = c_1F_d(x) + c_2F_c(x) \qquad \text{(F-4)}$$

where c_1 and c_2 are positive constants such that $c_1+c_2=1$. The constant c_1 is just the sum of all the jumps in $F_X(x)$, and $c_2=1-c_1$.

It should be noted that the terms discrete and continuous as used here refer to the probability law used as a mathematical model for the random phenomenon, not to the physical nature of the possible observed values. Sometimes it is convenient to use a continuous probability law as a model, even though none is discrete. For example, the possible values of capacity outage are physically discrete, but because there are a large number of possible values, it may be convenient to model the probability law of capacity outage as a continuous random variable. Conversely, load demand is physically a continuous phenomenon, since the observed value is not limited to any particular set of values. However, it may be convenient to model load demand as a discrete random variable.

The distribution function can be used to specify the probability law of any random variable. However, other functions can also be used. In particular, the probability law of a discrete random variable can be specified by its probability mass function, and the probability law of a continuous random variable can be specified by its probability density function. These two functions are defined next.

The probability mass function (of a random variable X) is defined by

$$p_X(x) = P(X=x) \tag{F-5}$$

In words, the probability mass function of X is the probability of the event (X=x), that the observed value of X is the particular value x. If $p_X(x)$ is positive for some value x_i, then $F_X(x)$ has a jump at x_i, and $p_X(x_i)$ is the magnitude of the jump.

If X is a discrete rahdom variable, so that $F_X(x)$ increases only in jumps, then the value of $F_X(x)$ at any point is just the sum of all the jumps in $F_X(x)$ up the value x. Therefore, for a discrete random variable

$$F_X(x) = \sum_{x_i<x} p_X(x_i) \tag{F-6}$$

where the x_i values are the jump points in $F_X(x)$. Thus, the probability law of a discrete random variable can be specified by either its distribution function or its probability mass function. For example, a possible distribution function for the toss of a single die was given in equation (F-2). The corresponding probability mass function is

$$p_X(x) = \frac{1}{6}, \quad x = 1,2,3,4,5,6 \tag{F-7}$$

$$= 0, \quad \text{otherwise}$$

Equation (F-7) illustrates that, like the distribution function, the probability mass function is defined for all real numbers x.

The terminology "probability mass function" derives from the fact that $p_X(x)$ can be viewed as a discrete "probability mass" concentrated at the point x.

Thus, the total "probability mass" of 1.0 is distributed along the x-axis in discrete "pieces" $p_X(x_i)$ at the values x_i.

For a continuous random variable, the distribution function has no jumps. In this case, the probability mass function cannot be used to specify the probability law, since $p_X(x)=0$ for all x. However, another function, called the <u>probability density function</u> is defined by

$$f_X(x) = \frac{d}{dx} F_X(x) \qquad \text{(F-8)}$$

The probability density function is defined at those points where the derivative of $F_X(x)$ exists.

If X is a <u>continuous</u> random variable, then $F_X(x)$ is continuous and equation (F-8) can be integrated to give

$$F_X(x) = \int_{-\infty}^{x} f_X(y)dy \qquad \text{(F-9)}$$

Therefore, the probability law of a continuous random variable can be specified by either its distribution function or its probability density function. For example, a possible distribution function for the position of a pointer was given in equation (5-3). The corresponding probability density function is

$$\begin{aligned} f_X(x) &= 0, \quad x<0 \\ &= \frac{1}{6}, \quad 0<x<6 \qquad \text{(F-10)} \\ &= 1, \quad x>6 \end{aligned}$$

Equation (F-10) illustrates that $f_X(x)$ is defined at all real numbers x, except those where the derivative of $F_X(x)$ does not exist. In the example, the derivative of $F_X(x)$ is not defined at x=0 or x=6, because these are "break points" in $F_X(x)$. However, $F_X(x)$ is nonetheless continuous at these points (no jump), and the integral in equation (5-9) is therefore still meaningful. A graph of the probability density function in equation (F-10) is shown in Figure F-3.

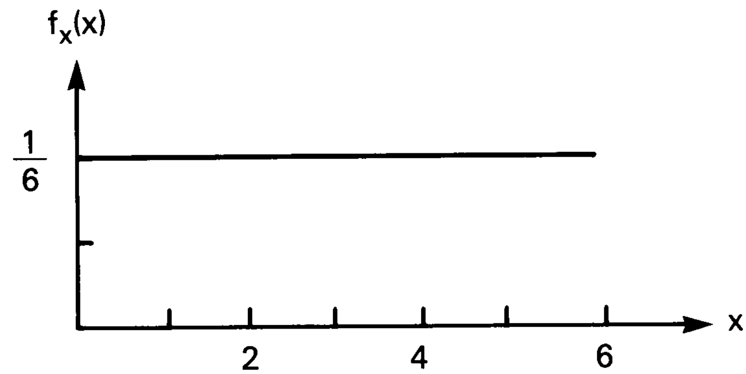

Figure F-3. Probability density function for position of pointer.

The terminology "probability density function" derives again from the "mass" analogy of probability. While there is zero probability assigned to any point x, the "probability mass" associated with a small interval dx is $f_X(x)dx$. Therefore, $f_X(x)$ is the "density" of probability mass at the value X.

The foregoing discussion can be summarized as follows. A random variable is continuous, discrete, or mixed, according to whether the distribution function of the random variable increases continuously, jumps, or both. The probability law of a continuous random variable can be specified by either its distribution function or its probability density function. Given either function, the probability of any event defined by the random variable can be computed. Similarly, the probability law of a discrete random variable can be specified by either its distribution function or its probability mass function. The probability law of a mixed random variable can be specified by a combination of probability laws for a discrete and a continuous random variable.

2. EXPECTATION OF A RANDOM VARIABLE

The probability law of a random variable X can be "summarized" by a single value called the <u>expectation</u> of X.

If X is a discrete random variable, the expectation of X is defined by

$$E[X] = \Sigma\, x_i\, p_X(x_i) \qquad \text{(F-11)}$$

where the sum is over all values x_i such that $p_X(x_i) > 0$.

In equation (F-11), the expectation of X is the weighted sum of the values of X which have positive probability, with weights equal to the assigned probability mass function. If probability is interpreted as a mass, the expectation of X is the center of gravity of the probability mass with respect to the x-axis. For example, equation (F-7) gives the probability mass function for the toss of a single die. The expectation of X is

$$E[X] = \sum_{i=1}^{6} \frac{1}{6}(i)$$

$$= \frac{21}{6}$$

$$= 3.5 \text{ "spots"} \qquad \text{(F-12)}$$

The probability mass function and expectation of X for the single die example is shown in Figure F-4.

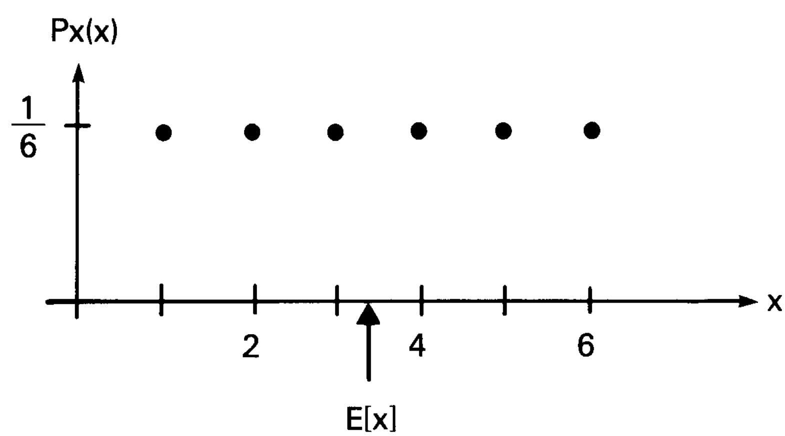

Figure F-4. Probability mass function and expectation for toss of a single die.

Frequently, the term "expected" rather than "expectation" is used, particularly when referring to the actual name of the random phenomenon being considered. Thus, the previous example shows that the expected number of spots resulting from throwing a single die is 3.5. However, the example also illustrates that the expected value of a random variable need not be a possible (realizable) value.

The expectation of a random variable is also often called the <u>mean</u> value, and the mean value of X is denoted by m_X. The mean value is a <u>parameter</u> of the probability law of X.

If X is a <u>continuous</u> random variable, the expectation of X is defined by

$$E[X] = \int_{-\infty}^{\infty} x f_X(x) dx \tag{F-13}$$

Again, if probability is interpreted as a mass, the expectation of X is the center of gravity of the area described by $f_X(x)$ with respect to the x-axis. For example, equation (F-10) gave the probability density function for the position of a pointer on a circle with a circumference of 6 inches. The expectation of X is therefore

$$E[X] = \int_0^6 x\left(\frac{1}{6}\right)dx$$

$$= \frac{x^2}{12}\Big|_0^6$$

$$= 3 \text{ inches}$$

The dimensions for E[X] are the same as for the random variable X.