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Inclusive Quarkonium Production

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Abstract

In the approximation of zero binding energy, we study the production of the heavy vector quarkonium states V(=ψ, Υ,...) accompanied with gluon (g) jets in γN and e^+e^- collisions. For the rare decay of the Z^0 resonance into Vgg, the rate is found extremely small.

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I. Introduction

The production of heavy quarkonium states (ψ, Υ, ...) is interesting for their clear detection signatures in the experiments and perturbative QCD predictions in the theory. Hereafter, we use V as a generic symbol for all heavy vector quarkoniums including the presumed toponium ζ states made of top quark pair ($t\bar{t}$). In this note, the inclusive processes of V production in γN and e^+e^- collisions are studied (see Fig. 1). We visualize these processes in the high energy limit as $\gamma g \to Vg$ and $e^+e^- \to \gamma^* \to Vgg$ with gluon (g) jets recoiling. The V vertex of the heavy quark pair $Q\bar{Q}$ is related to the wave function at the origin $\Phi(0)$ in the approximation of zero binding energy ($2m_Q = M_V$). Let $\langle V, \vec{p} = 0 | i \rangle$ be the transition amplitude from the initial state i to the final state with V at rest

$$\langle V, \vec{p} = 0 | i \rangle = \int \phi(p) \langle Q_p | \times \langle \bar{Q}_{-p} | i \rangle d^3p \ . \tag{1}$$

Here $\phi(p)$ is wave function of $Q\bar{Q}$ system in the momentum representation. Since $\langle Q_p | \times \langle Q_{-p} | i \rangle$ is a slowly varying function in the neighborhood $p \simeq 0$, the right hand side of Eq. (1) can be approximated as

$$\left(\int \phi(p) d^3p \right) \langle Q_0 | \times \langle Q_0 | i \rangle \ . \tag{2}$$

Therefore,

$$\langle V, \vec{p} = 0 | i \rangle = (2\pi)^{3/2} \phi(0) \frac{\bar{u}}{(2\pi)^{3/2} \sqrt{2m_Q}} \mathcal{T} \frac{v}{(2\pi)^{3/2} \sqrt{2m_Q}}$$

$$= \mathrm{Tr} \left\{ \mathcal{T} v \bar{u} \ (2/M_V)^{\frac{1}{2}} \Phi(0) \right\} \left[(2\pi)^3 2M_V \right]^{\frac{1}{2}} \tag{3}$$

with $\mathcal{F}$ as the matrix from subgraphs other than the V vertex. The last factor in Eq. (3) can be absorbed into the phase space normalization. The polarization $\vec{\varepsilon}$ of V is related to the heavy quark spinors as follows:

$$v(\pm)\ \bar{u}(\pm) = \not{\varepsilon}(\pm)(\not{P}_V+M_V)/\sqrt{2}\ ;$$

$$\left[v(+)\bar{u}(-)+v(-)\bar{u}(+)\right]/\sqrt{2} = \not{\varepsilon}(0)(\not{P}_V+M_V)/\sqrt{2} \tag{4}$$

From Eqs.(3) and (4), the effective coupling between $(Q\bar{Q})$ and V (see Fig.2) is obtained

$$\left[(Q\bar{Q})\to V^{\mu}\right]\ :\ \gamma^{\mu}(\not{P}_V+M_V)\phi(0)M_V^{-1/2} \tag{5}$$

For ψ and Υ, the wave functions at the origin $\phi(0)$ were measured through the leptonic decay widths Γ_{ee}

$$|\phi(0)|^2 = M^2\Gamma_{ee}/(16\pi\alpha^2 e_Q^2)\ . \tag{6}$$

The values of $|\phi(0)|^2$ for ψ and Υ are therefore 0.04 and 0.4 $(\mathrm{GeV})^3$ respectively. For the toponium ζ state, we assume M_ζ = 40 GeV and approximate the wave function value $|\phi_\zeta(0)|^2$ from the calculation based on a Coulomb-like binding potential

$$|\phi_\zeta(0)|^2 = \alpha_s^3 M_\zeta^3/(27\pi)\ , \tag{7}$$

with the strong coupling $\alpha_s \approx 0.3$.

II. $\gamma N \to VX$

Recently, deep inelastic production $\gamma^* N \to \phi X$ with large recoiling hadron energy was measured in the muon experiment.[1] The parton process $\gamma^* g \to Vg$ described in Fig. 1 (i) would also produce a gluon jet carrying certain amount of energy[2,3] as observed in the experiment. Similar reaction in the crossed channel is known as the ortho-positronium decay into three photons, In textbooks,[4] the corresponding transition probability for $e^+(0)+e^-(0) \to \gamma(k_1)+\gamma(k_2)+\gamma(k_3)$ has been worked out as follows:

$$\langle M^2(e^+e^-\to 3\gamma)\rangle_{\text{spins}} = 16e^6\left(\frac{m_e - k_1^o}{k_2^o k_3^o}\right)^2 + \text{cyclic permutations } (k_1^o, k_2^o, k_3^o) \tag{8}$$

where summing over the spins of the final states and averaging over the spins of the initial states are assumed. With appropriate variable substitutions, we have

$$\langle M^2(\gamma g\to Vg)\rangle_{\text{spins, color}} = \left[\frac{1}{12}\right]\cdot\left[\frac{2|\phi(0)|^2}{M_V}\right]\cdot 16e_Q^2 e^2 g_s^4 \cdot \frac{4M_V^2\hat{s}^2}{(t-M_V^2)^2(u-M_V^2)^2} + \text{cyclic } (\hat{s},t,u) \tag{9}$$

Here, the first factor 1/12 comes from the color average and the second factor $2|\phi(0)|^2/M_V$ is originated from Eq. (3). Therefore,

$$\frac{d\hat{\sigma}(\gamma g\to Vg)}{dt} = (16\pi s^2)^{-1}\langle M^2\rangle_{\text{spins, color}}$$

$$= \frac{128\pi^2}{3\hat{s}^2}\; e^2\alpha\alpha_s^2 M_V|\phi(0)|^2\left\{\frac{\hat{s}^2}{(t-M_V^2)^2(u-M_V^2)^2} + \text{cyclic}(\hat{s},t,u)\right\} \tag{10}$$

After integrating t variable,

$$\hat{\sigma}(\hat{s}) = \frac{128\alpha_s^2\alpha e_Q^2\pi^2}{3M_V^5}\,|\phi(0)|^2\left\{\frac{2}{\gamma^2}\left(\frac{\gamma+1}{\gamma-1} - \frac{2\gamma\ln\gamma}{(\gamma-1)^2}\right) + \frac{2(\gamma-1)}{\gamma(\gamma+1)^2} + \frac{4\ln\gamma}{(\gamma+1)^3}\right\} \tag{11}$$

with $\gamma = \hat{s}/M_V^2$. To give measurable cross sections, the expressions in

Eqs. (10-11) are to be convoluted with the gluon distribution $G(x;x=\hat{s}/s)$ which we choose the simple form

$$G(x) = 3(1-x)^5/x \tag{12}$$

suggested by counting rule and momentum constraints. In the above formalism, the calculation is infrared finite. If this process is a dominant one, we can use Bjorken variable

$$x = t/\left[2M_N(E_\gamma - E_\psi)\right] \tag{13}$$

as the momentum fraction of the incident gluon in the nucleon and, hence, the gluon distrib .tion $G(x)$ could be measured. We compare our calculation with EMC data[1] on $\gamma^* N \to \psi X$ at low $Q^2 (Q^2 = -k_1^2)$. For the case of non-vanishing Q^2, we obtained much lengthy expression for the cross section through the computer program ASHMEDAI. In Fig. 3, our prediction for the cross section of the inelastic ψ production reaches 9nb at E_γ = 200 GeV. The data rise faster than the prediction in the energy range $E\gamma$ = 100-200 GeV. It is likely that more reaction channels,[5] not expected in this model, are opened at high energy. On the other hand, the predicted distribution of the inelasticity $z(=E_\psi/E\gamma^*)$ grees excellently with the data as shown in Fig. 4. For the Q^2 and t dependences, our predicted distributions are flatter than the data (Fig. 5,6). As there are success and failure of this model, we expect future higher statistics experiments could give better understanding of the production mechanism.

III $e^+e^- \to \gamma^* \to Vgg$

We can utilize the above formalism to study a similar process[6] $e^+e^- \to \gamma^* \to Vg(\ell)g(k)$ as shown in Fig. 1(ii). The scaling variables are defined as

$$x_i = 2E_i/\sqrt{s} \qquad i = V,\ell,k$$

$$x_V + x_\ell + x_k = 2 \tag{14}$$

$$\lambda = M_V^2/s \ .$$

Here E_i is the energy measured in the e^+e^- c.m. frame and M_V is the mass of the V meson. The differential cross section after integrating over the angles between the plane of Vgg and the direction of e^+e^- beam is related to the "decay width" $\Gamma(\gamma^* \to Vgg)$ of the virtual photon γ^* as follows

$$d\sigma/(dx_V dx_\ell) = 4\pi\alpha s^{-3/2}\, d\Gamma(\gamma^* \to Vgg)/(dx_V dx_\ell) \tag{15}$$

with

$$\frac{d\Gamma(\gamma^* \to Vgg)}{dx_V dx_\ell} = \frac{128}{9} e_Q^2 \alpha\alpha_s{}^2 s^{-3/2} M_V |\Phi(0)|^2$$

$$\times \left\{ \frac{(2+x_k)x_k}{(2-x_V)^2(1-x_\ell-\lambda)^2} + \frac{(2+x_\ell)x_\ell}{(2-x_V)^2(1-x_k-\lambda)^2} + \frac{(x_V-\lambda)^2-1}{(1-x_k-\lambda)^2(1-x_\ell-\lambda)^2} \right.$$

$$\left. + \frac{1}{(2-x_V)^2}\left[\frac{6(1+\lambda-x_V)^2}{(1-x_k-\lambda)^2(1-x_\ell-\lambda)^2} + \frac{2(1-x_V)(1-\lambda)}{(1-x_k-\lambda)(1-x_\ell-\lambda)\lambda} + \frac{1}{\lambda}\right]\right\} \tag{16}$$

The matrix element above is related to the corresponding matrix element[7] of the crossed channel reaction $V \to \gamma^* gg$ with appropriate variable replacements. The allowed region of the phase space is

$$2\sqrt{\lambda} < 1 + \lambda$$

$$x_\ell \lessgtr \frac{1}{2}\left[2 - x_V \pm (x_V^2 - 4\lambda)^{1/2}\right] \tag{17}$$

Our results are shown in Fig. 7. The cross sections for the inclusive V production are small but not unmeasurable.

The process $e^+e^- \to VX$ can, in principle, allow a study of the final state from two gluon jets over a full range of kinematics $M_x < (\sqrt{s} - M)$. The invariant mass squared of the hadronic recoil system is $M_x^2 = s(1 - x_V + \lambda)$. Figure 8 shows the spectrum of M_x in $e^+e^- \to VX$.

It is natural to consider Z^0 resonance in e^+e^- annihilation as a source for Vgg production. The differential decay rate is given by Eq. (16) with the following substitutions

$$\alpha \to \sqrt{2}\ G_F M_Z^2/\pi$$

$$e_Q \to \frac{1}{4} - |e_Q|\ \sin^2\Theta_W \qquad (18)$$

$$\sqrt{s} \to M_{Z^0}.$$

Here we use the standard electroweak model[8] and set $\sin^2\Theta_W = 0.23$. There is no contribution from the axial vector coupling as a result of charge conjugation. The decay width $\Gamma(Z^0 \to Vgg)$ is very small because of the smallness of the vector coupling in the standard electroweak model. Results are tabulated below:

$$\Gamma(Z^0 \to Vgg)/\Gamma(Z^0 \to \mu\bar{\mu}) = \begin{cases} 1.1 \times 10^{-5} & V = \psi \\ 4.7 \times 10^{-5} & V = \Upsilon \\ 2.2 \times 10^{-5} & V = \zeta \end{cases} \qquad (19)$$

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Figure Captions

Fig. 1. Diagrams for inclusive V productions in (i) the γN collision and (ii) the e^+e^- annihilation.

Fig. 2. Effective coupling between $(Q\bar{Q})$ and V

Fig. 3. The predicted inelastic cross section $\sigma(\gamma N \to \psi X)$ versus E_γ is compared with data extrapolated from $\mu N \to \mu\psi X$.

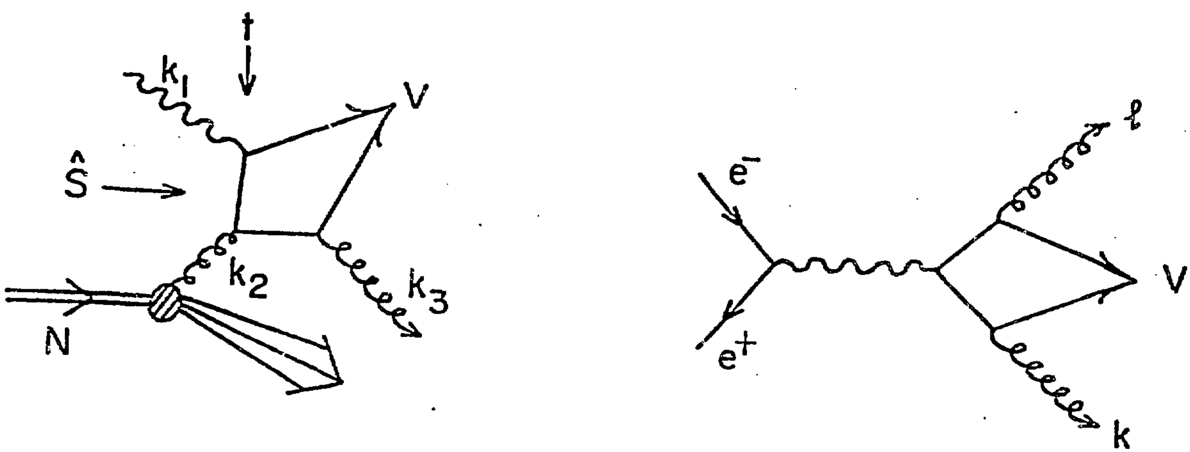
Fig. 4. The predicted z distribution is compared with EMC data.

Fig. 5. The predicted t distribution is compared with EMC data.

Fig. 6. The predicted Q^2 distribution is compared with EMC data.

Fig. 7. Ratio of total $e^+e^- \to VX$ cross section to the pure QED $e^+e^- \to \mu^+\mu^-$ cross section versus $\sqrt{s}$ for the cases V being ψ, T and ζ.

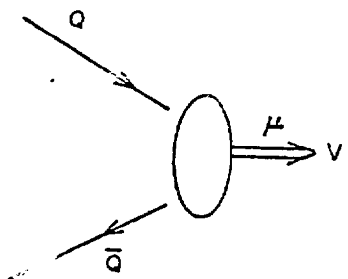
Fig. 8. Hadronic mass spectrum $\sigma^{-1}\, d\sigma/d(M_x/\sqrt{s})$ predicted for the process $e^+e^- \to VX$. The cases shown have $\sqrt{s} = 2M_V$, $3M_V$ and $10M_V$.



(i) $\gamma^* N \longrightarrow VX$

(ii) $e^+e^- \longrightarrow VX$

Figure 1



$$\gamma_\mu (\not{P}_V + M_V)\, \phi(0) / \sqrt{M_V}$$

Figure 2

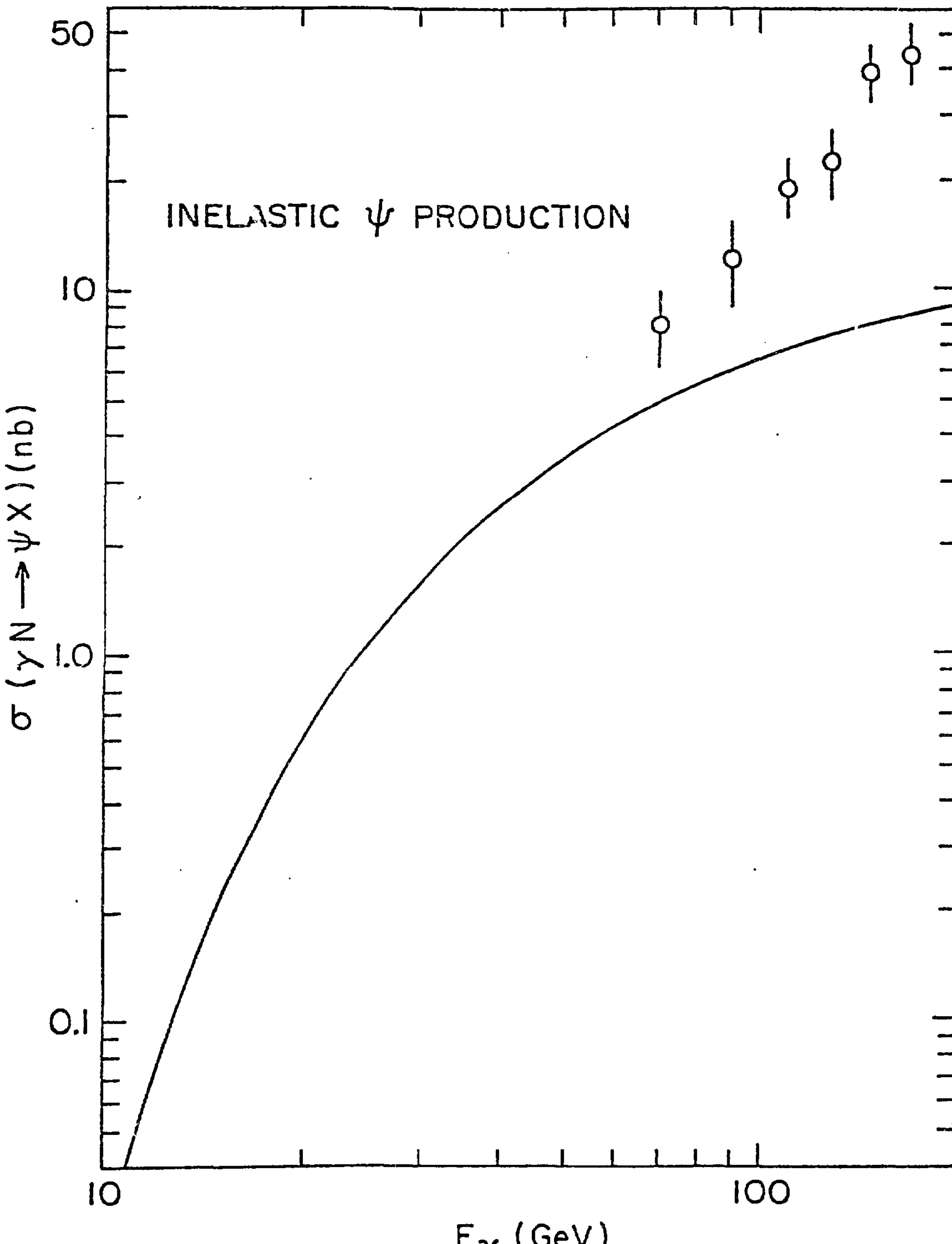


Figure 3

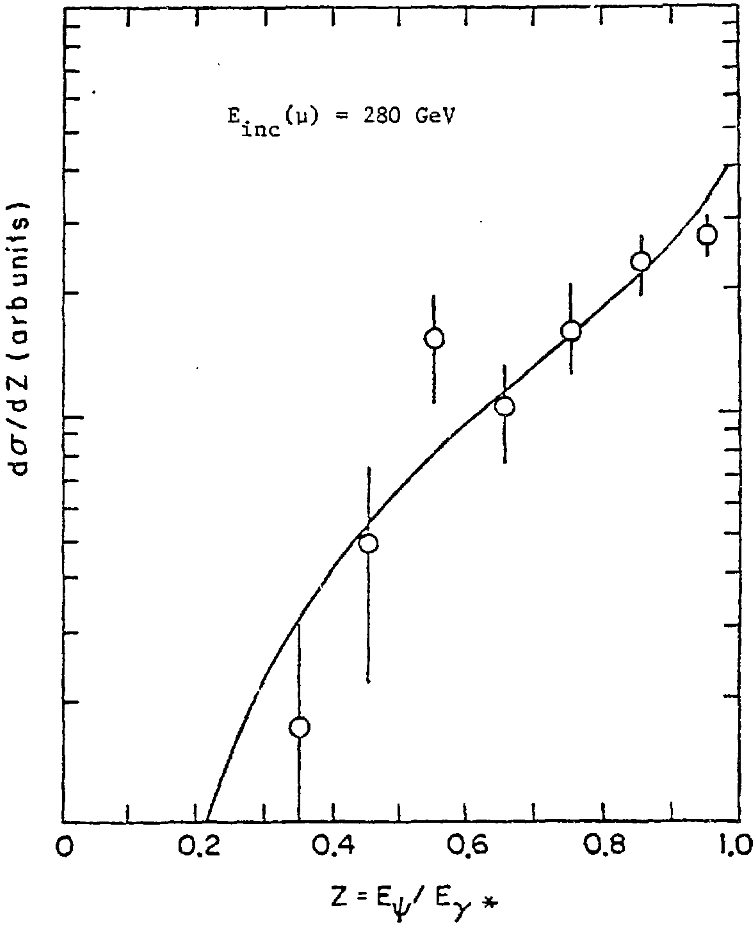
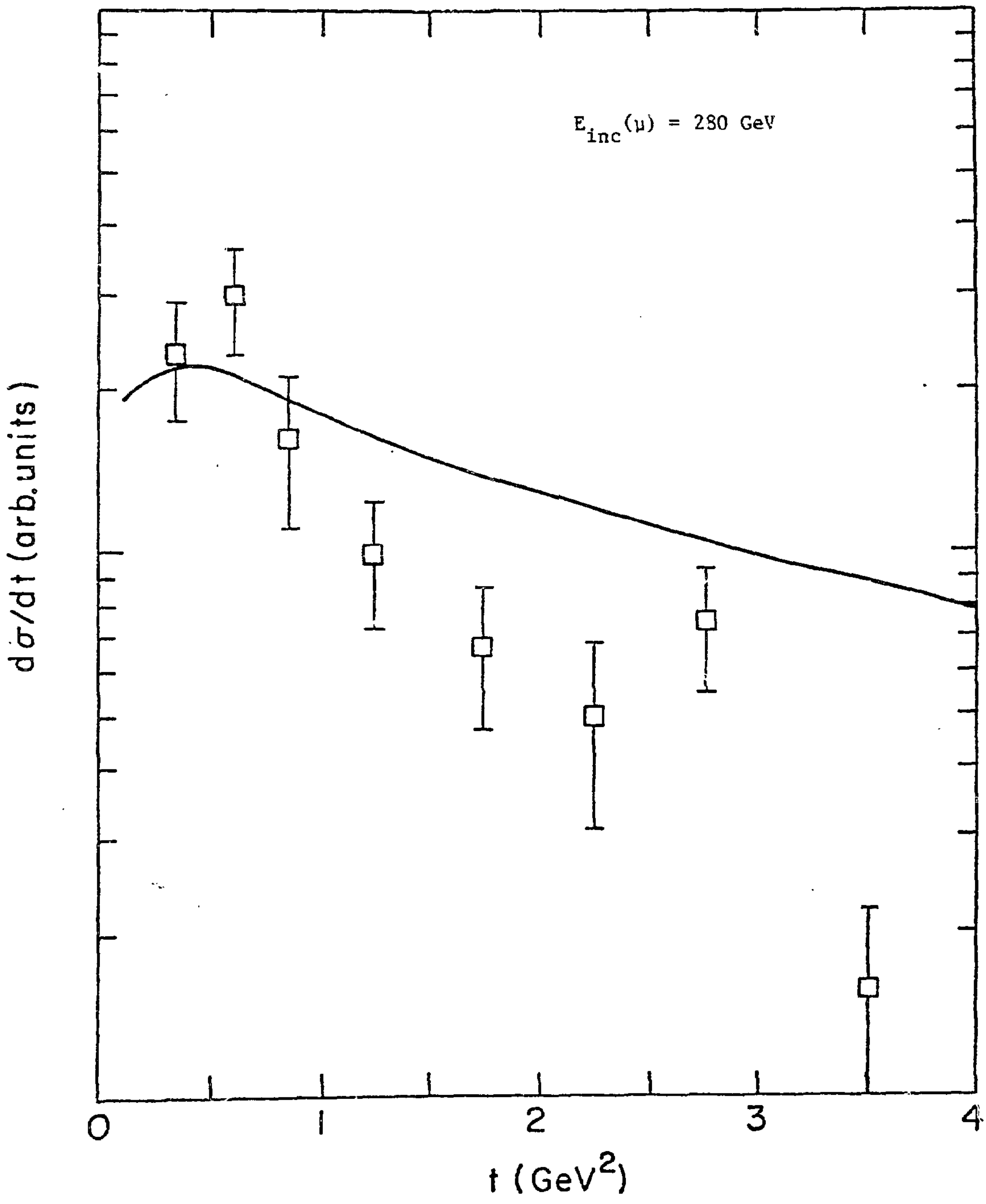


Figure 4



fFigure 5

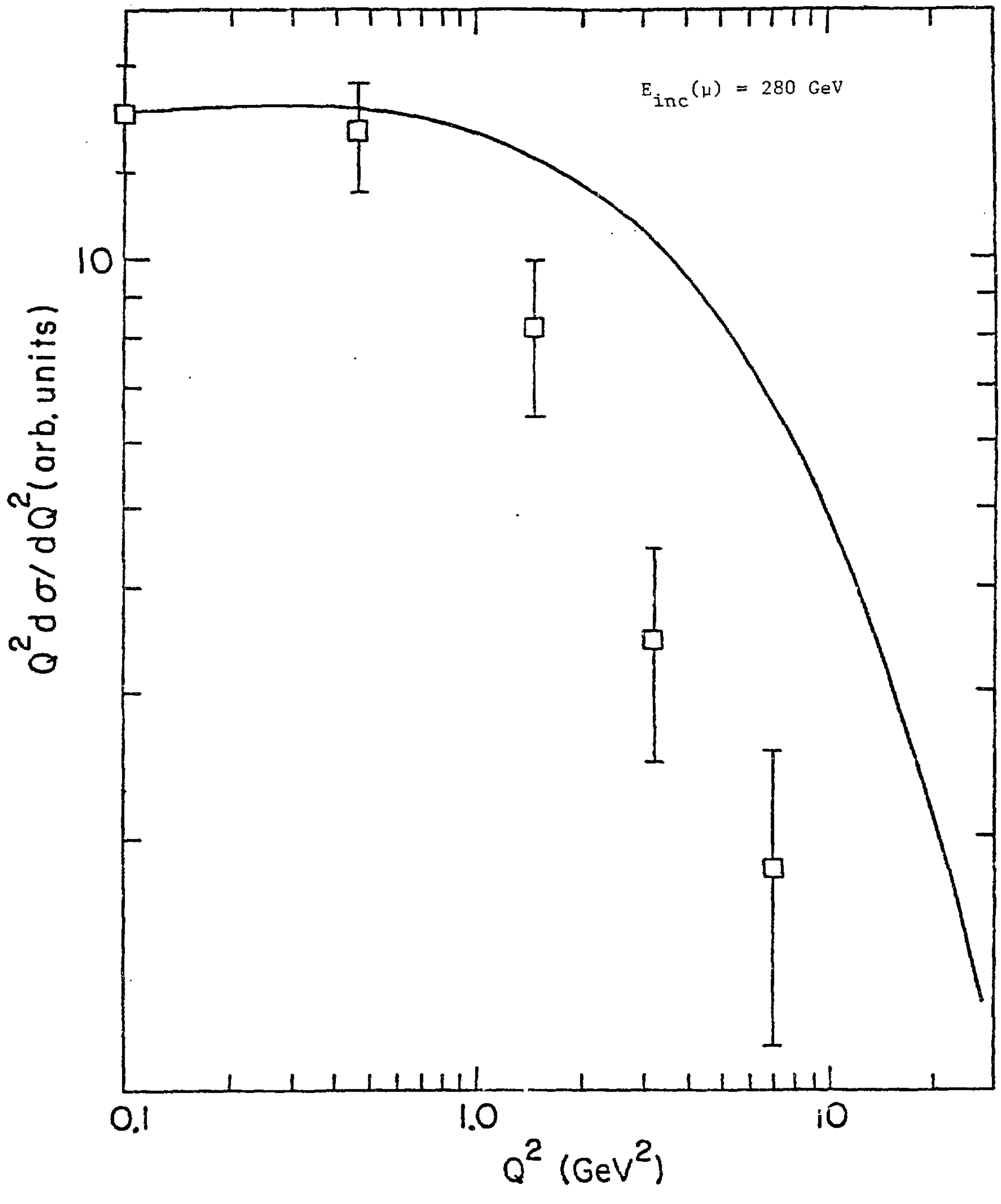


Figure 6

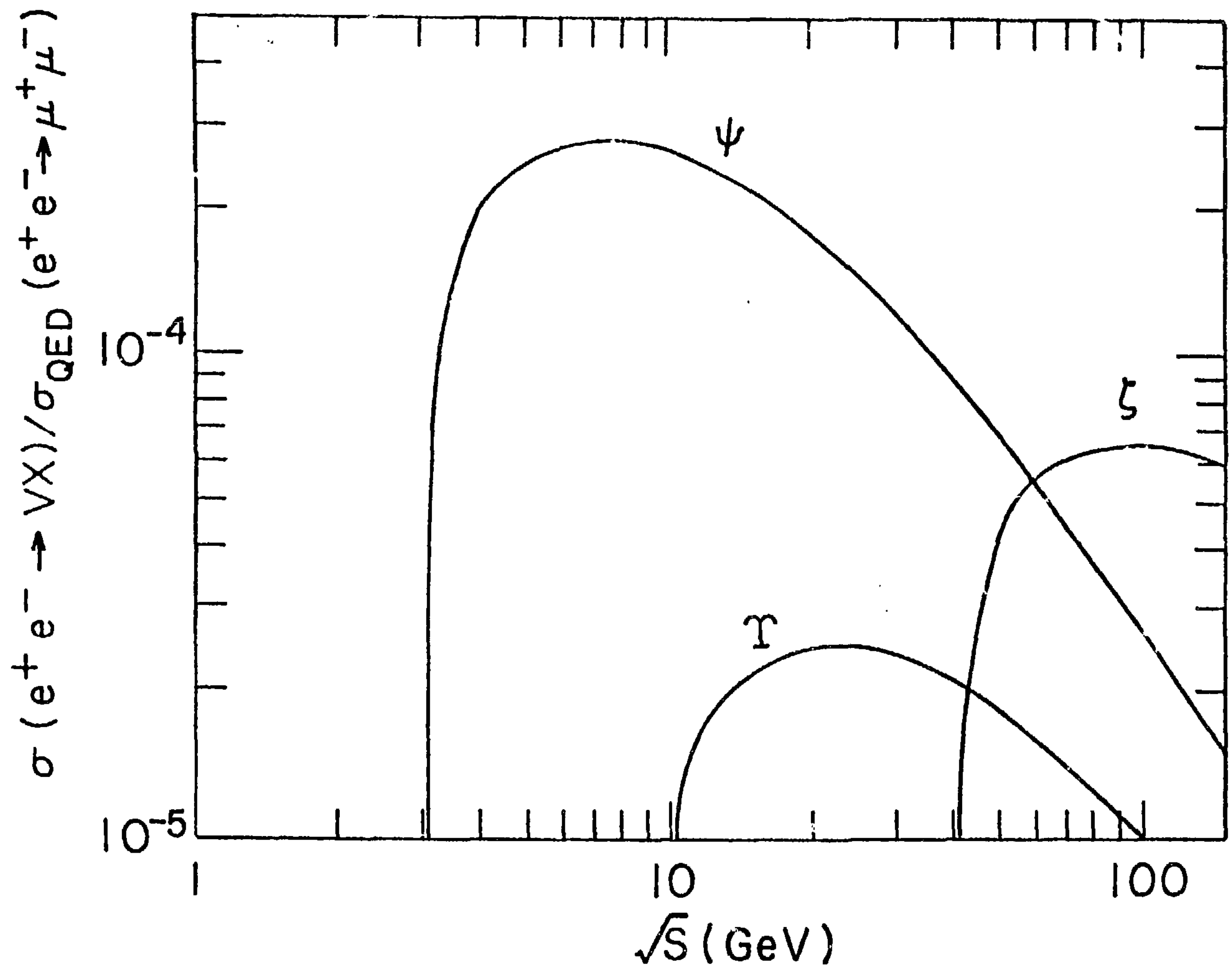
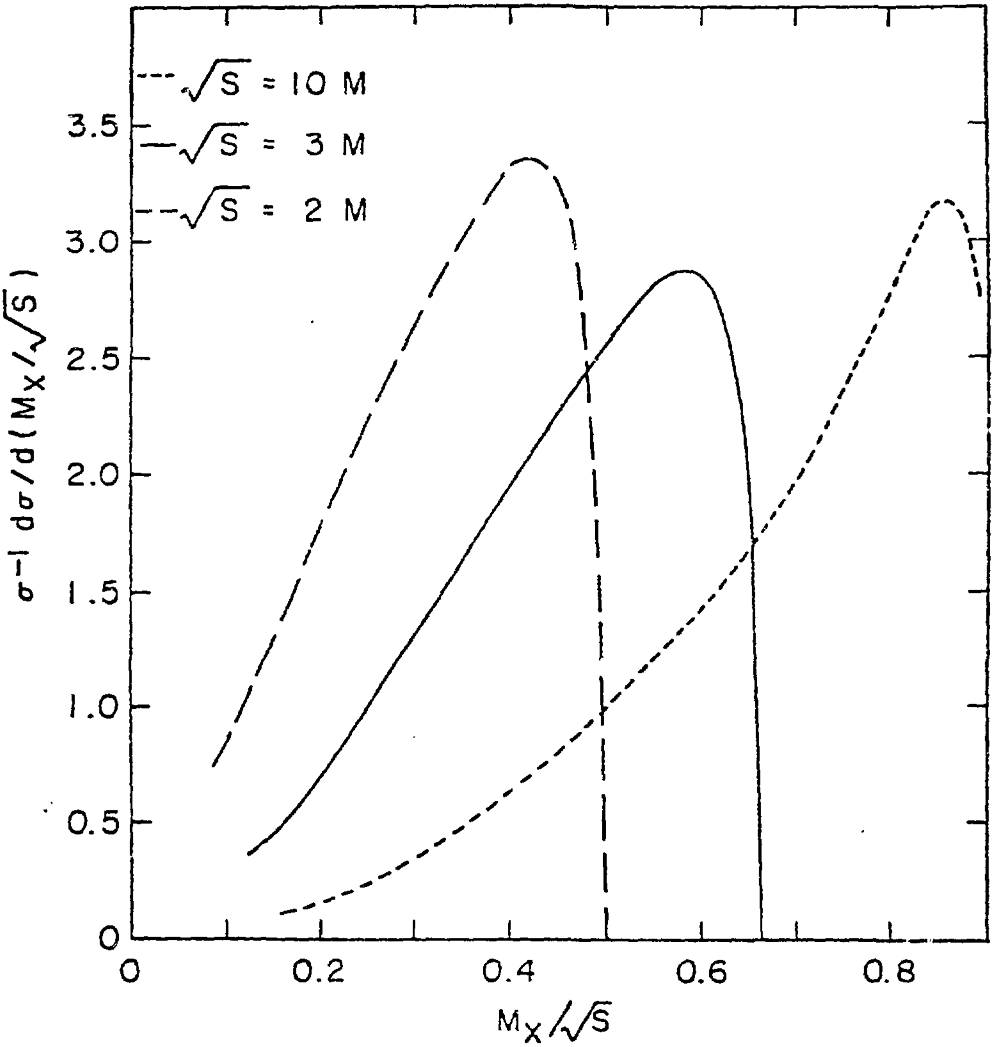


Figure 7



$\sqrt{S}$ = 10 M
$\sqrt{S}$ = 3 M
$\sqrt{S}$ = 2 M
$\sigma^{-1}\, d\sigma / d(M_X/\sqrt{S})$
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
0
0.2
0.4
0.6
0.8
$M_X/\sqrt{S}$

Figure 8