SIMPLIFIED INELASTIC ANALYSIS PROCEDURE TO EVALUATE A BUTT-WELDED ELBOW END

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ABSTRACT

In a thin-walled piping network, the end of an elbow welded to a straight pipe constitutes one of the highly stressed cross-sections that require structural evaluation. Explicit rules are not provided in the ASME Code for structural evaluation of the elbow ovalization and fabrication effects at the welded end. This paper presents a conservative semi-analytical procedure that can be used with simplified inelastic analysis to evaluate the elbow cross-section welded to the straight pipe. The concept of carry-over factors is used to obtain ovalization stresses or strains at the elbow end. The stresses introduced by material and geometric nonuniformities in the fabrication process are then added to the ovalization stresses to complete structural evaluation of the girth butt-welded elbow joint.

INTRODUCTION

A Class 1 elevated temperature piping system that does not satisfy the ASME Code rules [1]* requires a detailed inelastic analysis to comply with the ASME Code Case N-47 [2] criteria. While current piping system analysis methods conservatively predict higher stresses and strains at the middle of elbow, they exclude two effects that are essential to predict structural response at the end of elbow. These are: a) stiffening of an elbow due to contiguous straight pipes and b) girth butt-weld shrinkage. The elastic stress indices specified in the ASME Code include effects due to pipe mismatch and weld profile. These indices do not include the girth butt-weld shrinkage effects, which are significant in Liquid Metal Fast Breeder Reactor (LMFBR) thin-walled piping. Rodabaugh and Moore [3] present design formulae for the stress indices at the girth butt-welded section of a straight pipe. This paper uses these formulae, and presents a conservative procedure to evaluate the structural adequacy of a butt-welded elbow joint.

To evaluate the welded elbow end in a piping system, it is necessary to understand the overall behavior of an elbow subjected to in-plane and out-of-plane loadings. The primary deformation mode of a thin-walled elbow is by ovalization of the circular cross-section. For example, due to in-plane bending, maximum ovalization occurs at the middle of elbow. Through-the-wall

*Numerals in brackets designate references at the end of this paper.

hoop bending stresses induced due to this ovalization decrease toward the ends because of the restraints provided by straight pipes welded to the curved elbow. Current simplified inelastic analysis methods neglect ovalization in straight pipes. To compute stresses at the end of an elbow, the concept of carry-over factors is used. A carry-over factor is defined as a factor by which the stresses in the elastic range (or strains in the inelastic range) at the most highly stressed section of an elbow are multiplied to obtain ovalization effects at the elbow end.

The purpose of this paper is to present a semi-analytical procedure to evaluate the elbow cross-section welded to a straight pipe. This procedure does not require additional inelastic analyses and is shown to be conservative. This simplified procedure is justified for piping systems that satisfy the ASME Code Case N-47 buckling strain factors. A rationale is presented to show that the calculated ovalization stresses at the end of an elbow adequately represent the normal, upset, and emergency conditions experienced by a piping system. The proposed method was utilized for a 24-inch hot-leg piping system in the Clinch River Breeder Reactor Plant (CRBRP). This method differs from the conventional low temperature thick-walled piping design, wherein carry-over effects are excluded by assuming the end of elbow stress to be the same as the middle of elbow stress and the welded joint effects are evaluated separately using the indices for joints in straight pipe.

NONLINEAR ELBOW RESPONSE

A detailed nonlinear shell analysis of a piping system is expensive and time consuming when compared with elastic analysis. Therefore, simplified methods are utilized to evaluate structural adequacy of thin-walled LMFBR piping systems and to comply with the ASME Code rules. In general, the piping system analysis methods exclude large deformation effects that are important in computing the collapse load of a structural component. In this section, the nonlinear collapse response of an elbow structure that is subjected to an in-plane bending moment is examined in an attempt to justify the small deformation assumption currently utilized in piping system analysis.

Figure 1 shows an analytical moment-rotation curve of the CRBRP 24-inch hot-leg elbow subjected to an in-plane bending moment. The collapse moment in the figure is defined as the maximum moment that the elbow can support without geometric or material instability; the deformations increase without bound at collapse load. Nonlinear collapse analysis, which includes both material and geometric nonlinearities, was performed using the doubly-curved shell elements of the MARC program [4]. Figure 2 shows the geometrically symmetric finite element idealization of the elbow structure. The finite element mesh selection was based upon an earlier convergence study of a thin-walled elbow used in the Fast Flux Test Facility (FFTF) [5].

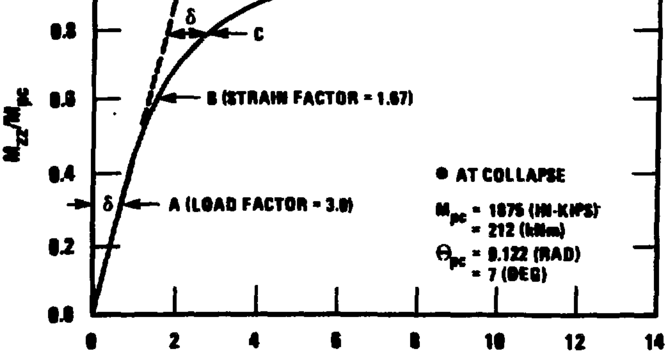

Figure 1. Moment-Rotation Curve for CRBR 24 Inch Elbow (In-Plane Moment)

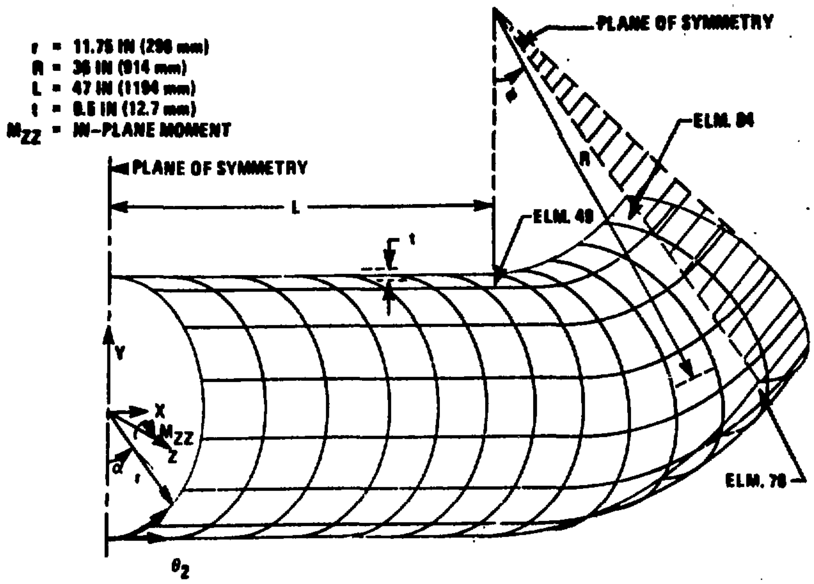

Figure 2. Finite Element Idealization of 24 Inch CRBR Elbow (Symm. Model for In-Plane Loading)

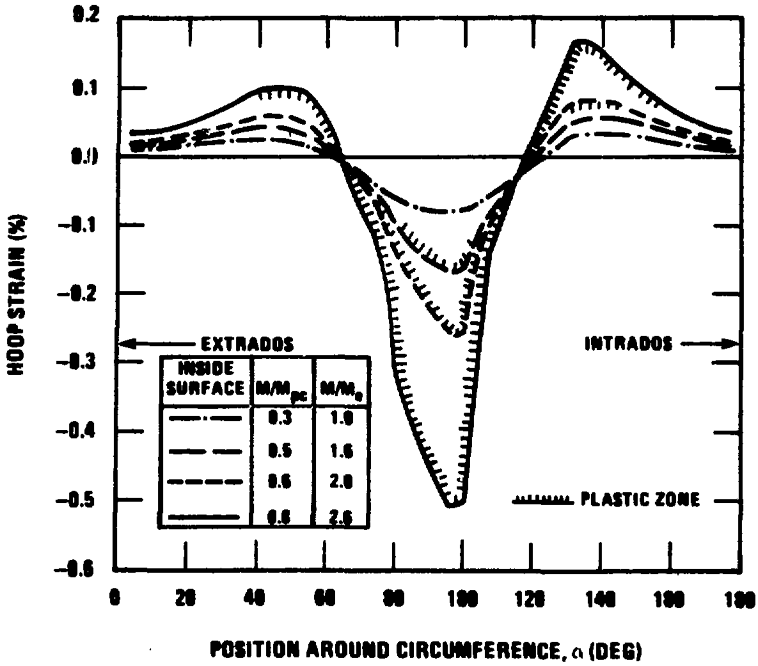

Figure 3. Nonlinear Hoop Strain Response at Middle of Elbow (In-Plane Moment)

Based upon the details of collapse analysis presented in [5], certain characteristics of the elbow deformation behavior can be observed from the moment-rotation curve shown in Fig. 1 and strain distribution shown in Fig. 3 for the CRBRP elbow. Qualitatively, the elbow deformation behavior can be categorized as follows:

- <u>Linear Elastic</u>: The elbow response is linear elastic when loaded up to 30% of the plastic collapse moment M_{pc} (point A in Fig. 1). This applied moment of 0.3 M_{pc} does not cause yielding in the elbow; it is designated as elastic moment M_e in Figure 3.

- **Nearly Linear Elastic-Plastic:** As the moment increases, first the inside and then the outside surfaces near the crown (90° from the extrados) become plastic. Plastic regions initiated at the middle cross-section spread to the end cross-section. The moment deformation behavior up to about 60% of the collapse moment (point B in Fig. 1) is designated as elastic-plastic nearly linear behavior, because the plastic zones at the inside and outside surfaces are confined predominantly around the crown (circumferential angle 30°, as shown in Fig. 3). Furthermore, the spread of plastic regions is constrained by the elastic core within the elbow wall, and the elastic regions at intrados and extrados of the elbow. Since the plastic regions are confined at the crown, the response variables, such as strains and deformation throughout the elbow structure, can be extrapolated from elastic analysis in regions where response is elastic. The error in this extrapolation procedure increases near plastic regions close to the crown at the middle elbow section (see Table 1).

TABLE 1 EVALUATION OF ELASTIC EXTRAPOLATION PROCEDURE UP TO $M = 0.6\ M_{pc}$ (IN-PLANE MOMENT)

Circumferential[(a)] Location (Deg)	Surface Location	Ratio of[(b)] Response Variables — Middle of Elbow: Eff. Stress σ_e	Middle of Elbow: Hoop Strain ϵ_θ	Ratio of[(b)] Response Variables — End of Elbow: Eff. Stress σ_e	End of Elbow: Hoop Strain ϵ_θ
	Inside	2.08	2.07	2.22	2.25
13	Middle	2.03	1.53	2.04	2.04
	Outside	2.13	2.14	2.20	2.21
	Inside	2.28	2.30	2.21	2.40
39	Middle	1.84	1.63	2.11	2.01
	Outside	2.25	2.27	2.19	2.25
	Inside	2.13	0.18	2.14	2.07
64	Middle	2.22	2.24	2.12	2.08
	Outside	1.95[(c)]	3.55[(c)]	2.15	2.03
	Inside	1.17[(c)]	3.03[(c)]	2.21[(c)]	2.28[(c)]
90	Middle	3.10	3.67	2.11	2.01
	Outside	1.56[(c)]	2.68[(c)]	2.34	2.36
	Inside	1.69	1.15	2.27	2.56
116	Middle	2.11	2.49	2.17	2.11
	Outside	1.60[(c)]	1.58[(c)]	2.23	2.31
	Inside	2.21[(c)]	2.33[(c)]	2.11	2.14
141	Middle	1.92	1.88	1.93	1.96
	Outside	2.23[(c)]	2.51[(c)]	2.10	2.26
	Inside	1.95	1.96	2.46	2.44
167	Middle	1.69	3.50	1.35	3.00
	Outside	2.15	2.10	2.41	2.47

NOTES: (a) Circumferential location 90° is the crown of the elbow, extrados is at 0°, and intrados is at 90°.

(b) Ratio of applied moment to the elastic moment is 2.01. The ratios listed in Table 1 are the response variables predicted by nonlinear analysis at $M = 0.6\ M_{pc}$ to the response variables predicted from the elastic solution at $M = 0.3\ M_{pc}$.

(c) Plastic strains present at these locations.

- Elastic-Plastic with Slight Geometric Nonlinearity: At higher load levels, the plasticity initiated at the crown spreads around the middle and end elbow cross-sections, as well as into the straight pipe attachments. The nonlinear geometric effects, although noticeable at load levels above 0.6 M_{pc} (60% of the collapse moment M_{pc}), are not significant [5] until the applied load reaches about 0.8 M_{pc}. This load level, represented as point C in Fig. 1, designates the start of significant geometric nonlinear behavior. This justifies the current piping design practice of performing only small displacement inelastic analysis to meet various deformation limits specified in Code Case N-47; the additional benefit is the reduction in computational costs.

- Elastic-Plastic Substantially Nonlinear: Beyond 80% of the collapse moment, the elbow behavior is highly nonlinear. This behavior can be investigated only if large displacement effects are included in the analysis. Small displacement analysis is unacceptable to model the elbow behavior at loads approaching collapse loads.

The above load level demarcations are qualitative because changes in deformation behavior, with respect to the designated applied moment levels, are gradual and overlapping. The purpose of these designated load levels (points A, B, and C in Fig. 1) is to provide guidance to a designer in making engineering judgments regarding the deformation behavior of a piping network. For example, if an elbow structure complies with the Code Case N-47 [2] strain-controlled buckling factor of 1.67, then the maximum operating load on the elbow, under level C conditions, would be 60% of the collapse load. Thus, at operating load levels, the assumption of small displacements for the full piping system analysis is adequate because large displacement effects are insignificant up to 60% of the collapse load.

To confirm the qualitative designations of elbow behavior at various load levels, Fig. 3 presents the hoop-strain distribution around the middle elbow cross-section. Up to $M = 0.60\ M_{pc}$, the hoop strain increases nearly in the same proportion as the increase in elastic moment, except in plastic regions near the crown. At higher load levels, the spread of plastic zones is extensive, hence it is not possible to estimate the hoop strain by extrapolating the elastic results. Table 1 presents the ratios of the predicted hoop strain and effective stress at 0.60 M_{pc} to the elastic hoop strain and effective stress at 0.30 M_{pc}, at two elbow cross-sections: middle and elbow end. The ratio of actual inelastic predictions to the elastic extrapolation is in reasonable agreement with the applied moment ratio, $M/M_e = 2.01$, except at locations of plastic flow around the elbow crown (90° from extrados). For example, the effective stress is substantially less than 2.0 and the maximum hoop strain prediction around 90° location is significantly higher than 2.0, the ratio of extrapolated elastic values. This conservatism in strain extrapolation is not excessive as long as the plastic flow at the end elbow section is small. But more importantly, if the total (elastic + plastic) strains are computed at the center of the elbow by a simplified method, and if elastic carry-over factors are obtained from a detailed shell analysis, then it is possible to conservatively extrapolate the strains, but not the stresses, from the middle to the end of the elbow.

Based upon the foregoing discussion it may be concluded that:

- A small deformation analysis of a piping system adequately predicts the structural response when the operating load levels satisfy the buckling strain factor of 1.67 ($M = 0.6\ M_{pc}$).

- The strain components at the elbow end can be conservatively predicted using elastic carry-over factors and total strains predicted at central elbow section from simplified inelastic analysis.

CARRY-OVER FACTORS

Simplified inelastic analysis methods currently used for thin-walled LMFBR piping systems do not account for the stiffening effects of straight pipes welded to the elbow. An analytical model, such as the elbow pipe-bend finite element of the MARC program [4], assumes constant ovalization along the length and omits certain thin-shell deformation modes such as warping. Consequently, the simplified pipe-bend model is more flexible and the ovalization stresses due to applied moment are higher than the stresses computed from a doubly-curved thin shell analysis as illustrated in Fig. 4. These results show that the pipe-bend analysis substantially overpredicts the experimental results, whereas the shell analysis is in closer agreement with the experiments. These differences are magnified in the creep range, where the response is sensitive to stresses at the start of creep hold time [6]. This discrepancy is due to the assumption (in the pipe-bend model) that ovalization remains constant for an in-plane bending moment; whereas in an actual elbow structure the ovalization decreases from the middle to the end of the elbow. Consequently, to calculate the end of elbow stresses from simplified analysis, it is necessary to use the concept of carry-over factors. A carry-over factor, as defined here, is a factor by which the maximum value of a stress component at the most highly ovalized section of the elbow would be multiplied to obtain the stress at the same circumferential location at the end of elbow.

Elbow ovalization causes through-the-wall bending in hoop direction: The hoop stress variation at the inside surface varies as shown in Fig. 5. The maximum ovalization stress due to in-plane bending occurs near the crown of the elbow; hence, the hoop stress at the crown is plotted in Fig. 5 with respect to the axial distance. The ovalization effects attenuate away from the middle cross-section, and the hoop stress at three diameters from the elbow end is very small.

Carry-over factors vary with: a) elbow geometry, b) type of loading, c) the principal stress directions (hoop and axial), and d) the location of the stress point (at inside or outside surface) around the circumference. The shape of the stress distributions around the elbow also changes at sections away from the middle of the elbow. Instead of providing complex carry-over formulae, two values suggested in this paper for hoop and axial stress components are deemed adequate for elbows subjected to in-plane and out-of-plane loadings.

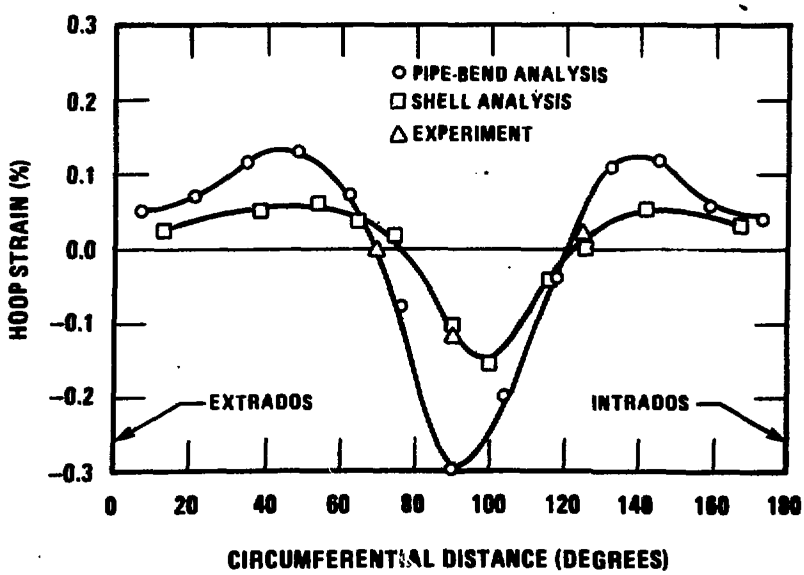

Figure 4. Hoop Strain Distribution Around Central Cross-Section Battelle Elbow Creep Test. M_{ZZ} = 843. t = 0 Hr. [6]

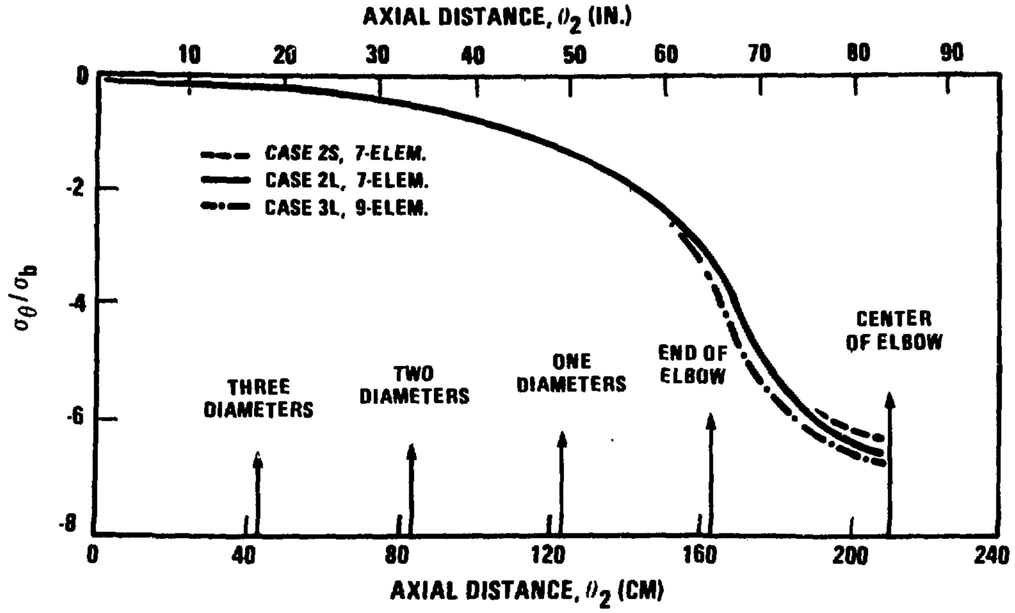

Figure 5. Axial Variation of Normalized Hoop Stress at Elbow Crown [5] (In-Plane Moment)

In-Plane Bending

The maximum ovalization occurs at the middle cross-section; hence, to calculate the carry-over factors, normalized hoop and axial stress distributions around the middle and the end cross-sections are plotted in Figs. 6 and 7, respectively. The normalization is with respect to the elastic beam stress (σ_b = Mr/I). Table 2 presents the elastic carry-over factors at the most highly stressed locations. The inelastic carry-over factors were also calculated, but only the elastic carry-over factors are of interest to the designer.

TABLE 2 CARRY-OVER FACTORS (IN-PLANE MOMENT)

	Carry-Over Factors[a]			
Stress Component	Location[b]	Based Upon[c] Max. Value of Stress	Location[b]	Based Upon[d] Max. Value of Stress
Hoop	100°-Out	0.48	141°-In	0.42
Axial	106°-In	0.55	80°-Out	0.42
Effective[e]	100°-Out	0.54	90°-In	0.48

NOTES: (a) Carry-over factor is equal to stress at end to the stress at the most highly stressed section within the elbow at the same circumferential location.
(b) Circumferential location in degrees is measured from the extrados at the inside (In) or at the outside (Out) surface for the largest carry-over factors.
(c) Designates the maximum value of stress (without regard for algebraic sign) at either the inside or the outside surface.
(d) Designates the maximum value of the stress with opposite algebraic sign when compared with the maximum stress.
(e) For effective stress, the largest carry-over factors at the inside and outside surfaces are presented.

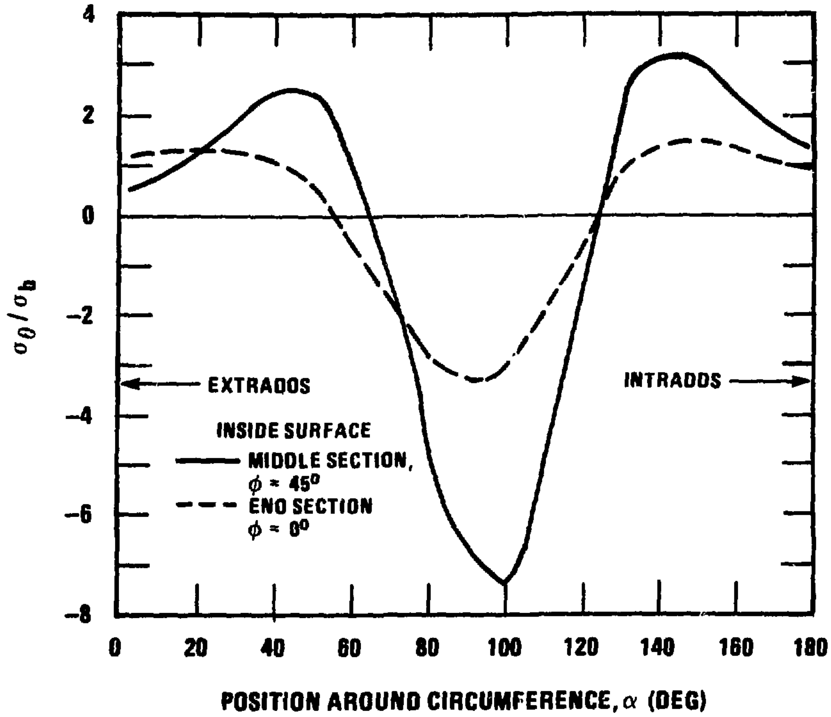

Figure 6. Normalized Hoop Stress Distribution (In-Plane Moment)

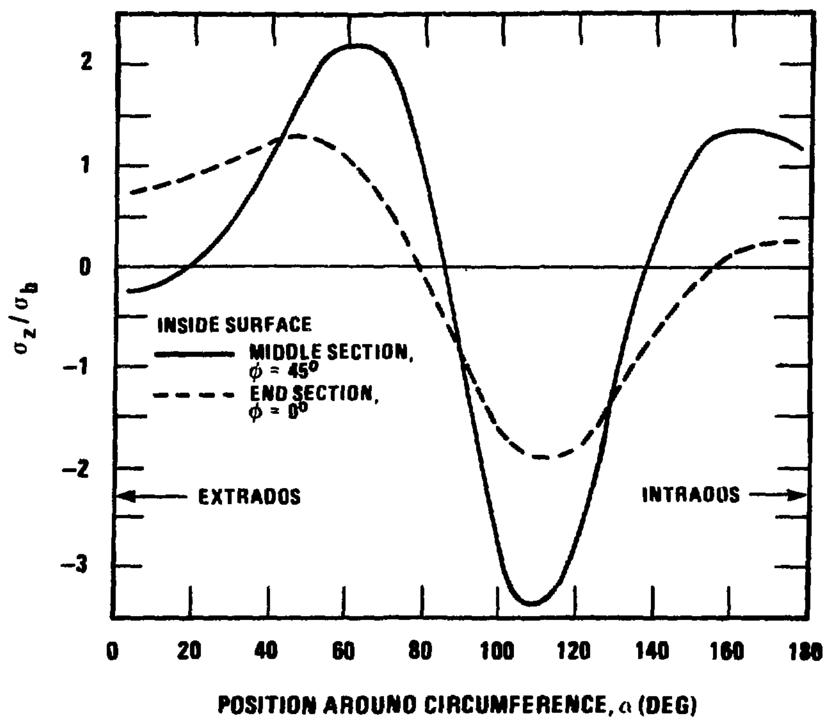

Figure 7. Normalized Axial Stress Distribution (In-Plane Moment)

The peak hoop stress as shown in Figure 6 occurs near the crown, whereas the peak positive and negative axial stresses occur at 60° and 106°, respectively, from the extrados. In the ASME Code type of stress analysis, it is necessary only to show that the highly stressed regions in a structure satisfy the Code criteria; then by implication the lower stressed regions would automatically satisfy the Code criteria. Therefore, for simplicity, carry-over factors associated with peak stresses will be used in the semi-analytical method that will be discussed later.

Out-of-Plane Bending

An out-of-plane moment produces antisymmetric deformations about the plane through intrados and extrados of the elbow. Therefore, a symmetric elbow model with antisymmetric boundary conditions is adequate to predict elbow response to an out-of-plane moment. The geometric model shown in Figure 8 was analyzed using the doubly-curved shell elements of the MARC program. To check validity of antisymmetric assumptions, normalized hoop and axial stress distributions from the MARC analysis are compared in Figures 9a and 9b, respectively, with the full 360° elbow model analyzed with the STAGS finite difference computer program [7]. The antisymmetric boundary conditions are valid only for small deformation analysis, where axial shortening of the straight pipe can be neglected.

Figures 10 and 11 show normalized hoop and axial stress distributions predicted by MARC analysis around four elbow sections: 0°, 30°, 45°, and 90°. The most highly stressed section is around 30° and not at the middle of the elbow, as was the case with the in-plane moment. Consequently, the carry-over factors for out-of-plane moment, presented in Table 3, is with respect to the most highly stressed 30° elbow section. Because the out-of-plane moment contribution to end ovalization is substantially large at $\phi = 0°$ than at $\phi = 90°$, the maximum factors provided in Table 3 are with respect to the elbow end at $\phi = 0°$.

TABLE 3 CARRY-OVER FACTORS (OUT-OF-PLANE MOMENT)

	Carry-Over Factors(a)			
Stress Component	Location(b)	Based Upon(c) Max. Value of Stress	Location(b)	Based Upon(c) Max. Value of Stress
Hoop	126°-Out	0.59	74°-In	0.54
Axial	90°-In	0.77	141°-In	0.37
Effective	116°-In	0.67	74°-Out	0.71

NOTES: For (a), (b), and (c) are refer. Table 2. Shear stresses are excluded.

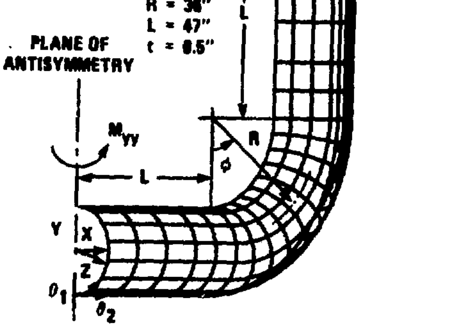

Figure 8. Finite Element Idealization of 24 Inch CRBR Elbow (Anti-Symm. Boundary Conditions for Out-of-Plane Moment)

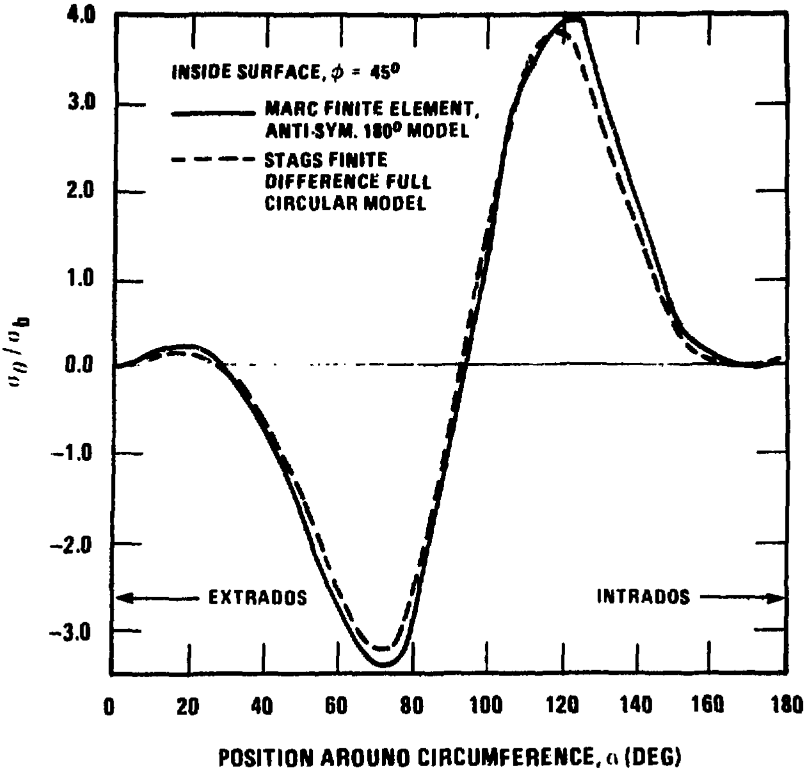

Figure 9a. Comparison of Hoop Stress Distribution (Out-of-Plane Moment)

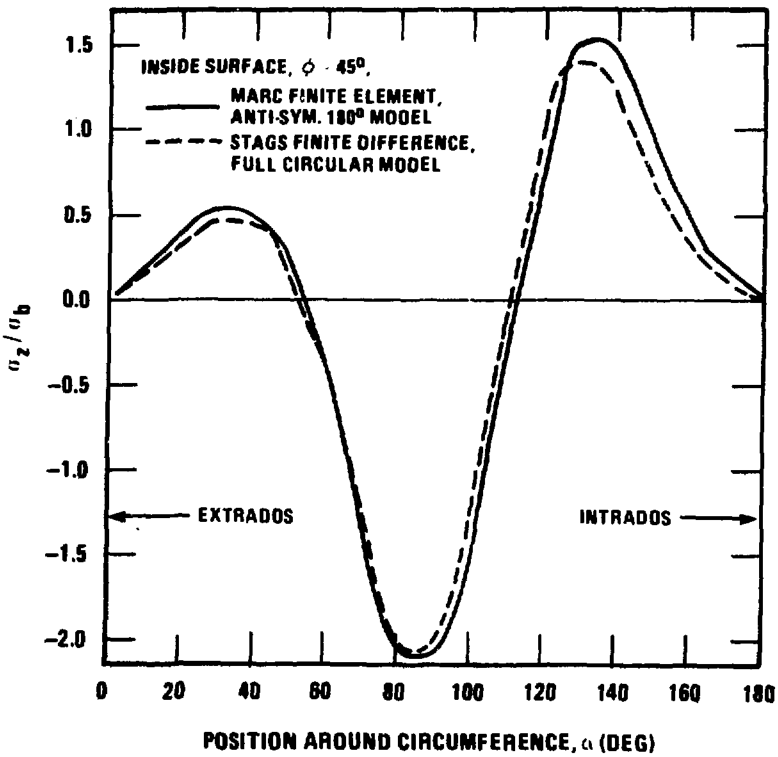

Figure 9b. Comparison of Axial Stress Distribution (Out-of-Plane Moment)

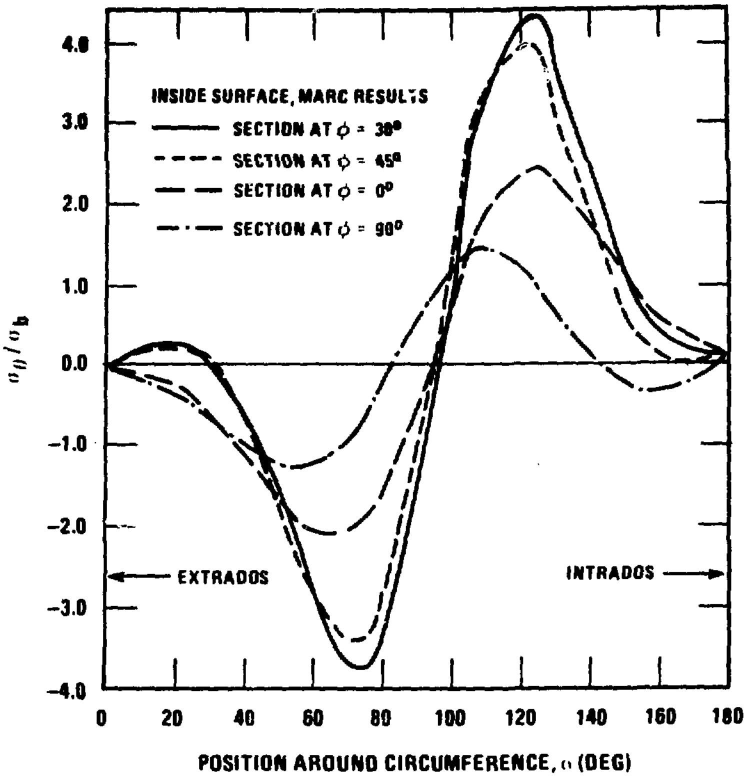

Figure 10. Normalized Hoop Stress Distribution at Sections Along the Elbow (Out-of-Plane)

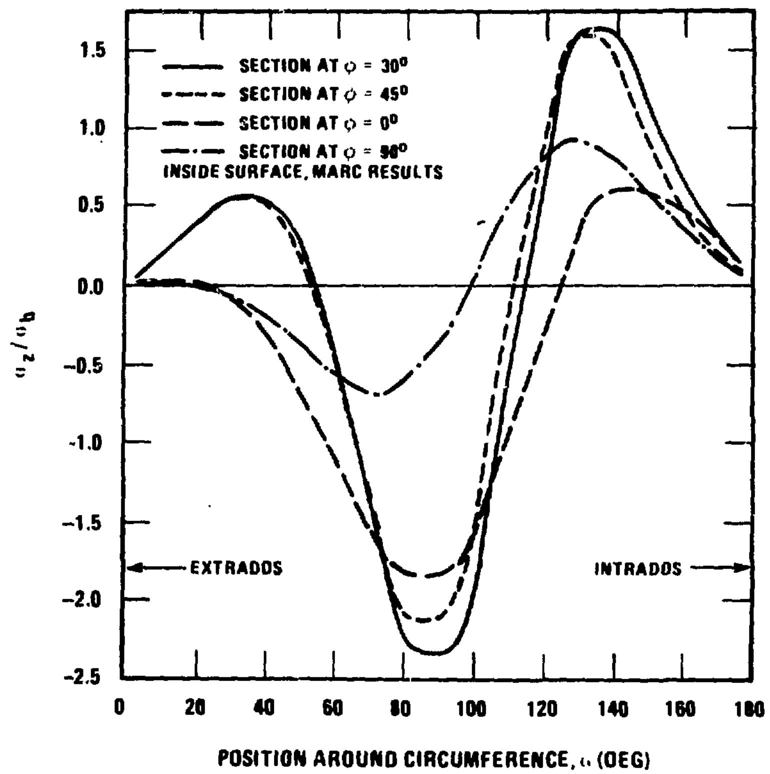

Figure 11. Normalized Axial Stress Distribution at Sections Along the Elbow (Out-of-Plane)

The carry-over factors for out-of-plane moment (Table 3) are larger than those for in-plane moment (Table 2). The maximum and minimum stress component locations are also different for in-plane and out-of-plane moments. In actual design, piping elbows would be subjected to a combination of in-plane and out-of-plane moments and forces. Thus, the realistic stress distributions would be different from the idealized distributions shown in Figs. 6, 7, 10, and 11. Of course, it is not possible to predict the exact stress distribution at the end of an elbow from carry-over factors. However, for ASME Code type stress analysis, a compromise carry-over factor for each of the stress components can be selected from Tables 2 and 3 for the simplified method. For thin piping elbows, carry-over factors of 0.55 and 0.65 in the hoop and axial direction are deemed adequate for design use.

The above discussion pertains to the simplified inelastic analysis of a piping system. This may be contrasted with the simplified elastic piping system analysis, wherein the ASME Code stress intensification factor for in-plane loading is used for both in- and out-of-plane loading. However, the stress intensification factor for out-of-plane moment (Fig. 10) is about four when compared with the stress intensification of seven for in-plane moment (Fig. 6). Thus, the simplified elastic procedure is conservative. Interestingly, the simplified elastic procedure uses carry-over factors of 0.5 and 0.6 for hoop and axial directions, respectively. This procedure is again conservative because actual carry-over factors presented in Table 2 for hoop and axial direction, are less than 0.5 and 0.6, respectively.

Pressure Loading

The pressure loading does not ovalize the original circular elbow cross-section; hence, the elbow response is not affected by the stiffening effects of straight pipes. However, due to the different radii of intrados and extrados, the pressure stresses are not uniform but vary around the elbow section. For all practical purposes, carry-over effects are negligible. Interestingly, as discussed in [8], the initial ovality is reduced in all internally pressurized elbows, because the section becomes less oval due to pressure. This circularization of the elbow would also mitigate some ovalization stresses due to applied moments. However, these effects can be accurately evaluated only by performing a large deformation analysis [8]. In the context of the LMFBR piping system, these beneficial effects as well as any carry-over effects are excluded in the simplified inelastic analysis.

In summary, to calculate ovalization stresses at the end of elbow, carry-over factors of 0.55 and 0.65 in the hoop and the axial directions are suggested for design use. These are simple averages of maximum factors presented in Tables 2 and 3. These factors are slightly conservative for in-plane loading, and slightly unconservative for out-of-plane loading. On the other hand, the maximum ovalization stress intensification in hoop direction due to an in-plane moment (Fig. 6) is nearly twice the hoop stress intensification due to an out-of-plane moment (Fig. 10). In an actual piping system, the ovalization of an elbow end is often constrained by hanger support clamps or component nozzles [7] which would further reduce these analytical ovalization stresses. Additional refinement of carry-over factors using a complex formula is considered unnecessary for practical problems.

FABRICATION EFFECTS DUE TO GIRTH BUTT WELDS

An inelastic analysis of a piping system, whether simplified pipe bend or detailed shell analysis, does not generally include the fabrication effects at the welded junction between an elbow and a straight pipe. To evaluate the adequacy of the welded pipe elbow junctions, it is necessary to determine the material and geometrical nonuniformities introduced in the fabrication process. The nonuniformities that result from fabrication are as follows:

- mismatch at the pipe and elbow junction
- radial girth butt weld shrinkage
- weld condition

To include these effects, it is necessary to compute the actual stress components at the joint. These stresses can then be modified to reflect the fabrication effects. The basic procedure adopted in this paper is to calculate the elastic fabrication stresses at the pipe weld using the stress intensity factor, C, and stress concentration factor, K, specified in Table NB-3681(a)-1 of the ASME Code [1], and add these effects to the elbow end stresses without the weld. The C-indices are used to obtain stresses which are primary and secondary in nature. They are affected by factors that influence the welded joint design such as mismatch, radial weld shrinkage, etc. The K-indices are used to obtain peak stresses for use in fatigue strength calculations. Incidentally, to satisfy the Code Case N-47 inelastic creep-rupture damage requirements, it is necessary to calculate the surface stresses (not linearized stresses) using the K-index.

The predominant loadings, which ovalize piping elbows, are in the form of in- and out-of-plane loadings and deformations due to thermal expansion. In inelastic analysis, it is difficult to separate the stresses due to pressure, nonuniform temperature distributions, and moment loadings. Therefore, C- and K-indices will be determined conservatively, assuming that the stresses are all due to moment loading. These indices will be used to calculate creep-rupture and fatigue damage and strain accumulations at the welded sections of piping elbows.

In the following discussion, specific numerical values are utilized to reflect the fabrication tolerances that are permitted in the CRBRP piping system; for other LMFBR plants, these tolerances may be different but the basic procedure would remain unchanged.

Mismatch Effects

In thin-walled piping, mismatch may be due to pipe offset or due to the elbow thickness and ovality being different from those of the welded straight pipe. For welds not ground flush, the ASME Code specifies that the inside diameters must match within 1/16 inch, and the maximum mismatch at any point must not exceed 3/32 inch. As discussed in [3], the maximum mismatch of 3/32 inch can occur only at isolated points around the periphery of the weld; hence, 1/32 inch is appropriate in the calculation of additional stresses due to mismatch. The C-indices specified in the ASME Code were evaluated by Rodabaugh and Moore [3]. For flush welds between nominally identical pipe walls with thickness greater than 0.237 inch (as against 3/16 inch in the Code), the C_2 index recommended [3] for the moment loading is 1.0. This value of $C_2 = 1.0$ is adequate for the CRBRP 0.5-inch (relatively thick-walled) as-welded pipe.

Radial Weld Shrinkage

The radial girth weld shrinkage effects are not included in the current ASME Code [1]. The pipe wall offset due to weld shrinkage at the junction of elbow and straight pipe introduces through-the-wall bending stress for both pressure and moment loadings, but not for thermal loadings. For simplicity, the additional stresses caused by radial weld shrinkage in piping elbow welds need be computed only for moment loading, because the pressure loading in the CRBRP piping system is small.

Based upon an idealized radial weld shrinkage model, the hoop and axial stresses were calculated in [3] by performing a series of elastic shell analyses of a pipe model. The secondary stresses due to radial weld shrinkage depend upon the exact geometry and details of discontinuity. Therefore, upper bound values of both hoop and axial stresses are presented in [3]. The diameter-to-thickness ratio, D/t, investigated in [3] ranged from 20 to 80; and the radial weld shrinkage-to-thickness ratio, Δ/t, ranged from 0.05 to 2.0. This evaluation, although derived for straight pipes, can be applied to end of elbow to compute weld shrinkage effects.

The distribution of hoop stress at the end of elbow, without radial girth weld, is similar to the distribution at the middle of the elbow (see Figures 6, 7, 10, and 11). For example, the maximum hoop stress due to in-plane moment loading occurs near the crown of the elbow, and the magnitude is nearly twice the maximum axial stress. The additional stresses due to girth weld shrinkage, when added to the elbow end stresses that are based upon carry-over factors, give the following upper bound values:

$$\sigma_z^r = |\sigma_{ze}| + 2.9\ (\Delta/t)|S_{me}| \quad (1a)$$

and

$$\sigma_\theta^r = |\sigma_{\theta e}| + 1.6\ (\Delta/t)|S_{me}| \quad (1b)$$

where $|\sigma_{ze}|$ and $|\sigma_{\theta e}|$ are the absolute values of axial and hoop stresses at the elbow end without radial girth weld; σ_z^r and σ_θ^r are the axial and hoop stresses at the elbow end with radial girth weld; Δ is the radial weld shrinkage; t is the thickness of elbow; $|S_{me}|$ is the maximum axial stress at the elbow end (this stress is the larger of the two axial stresses at the elbow end): maximum axial stress in the pipe due to resultant end moment, or the maximum elbow end stress obtained from the axial carry-over factor. The second term on the right-hand side of Eqs. (1a) and (1b) represents additional stresses [3] due to radial weld shrinkage. It should be emphasized that: a) the upper bound value of weld shrinkage stress is conservative for r/t greater than 10, b) at the inside surface, the weld shrinkage stress is of the opposite sign from σ_{ze}, c) the hoop stress at the inside surface due to weld shrinkage is only 10% of that at the outside surface under moment loading, and d) the maximum $\sigma_{\theta e}$ does not occur at the same location as S_{me}. Thus, Eqs. (1a) and (1b) are conservative.

The evaluation of creep-rupture damage according to Code Case N-47 requires the numerical value of effective stress (σ_{ee}) during creep hold time. Since thin elbows are idealized in two dimensions as plane stress cases, σ_{ee} can be obtained from stress components in Eqs. (1a) and (1b) as follows:

$$\sigma_{ee} = [(\sigma_{ze})^2 + (\sigma_{\theta e})^2 - \sigma_{ze}\ \sigma_{\theta e}]^{1/2} \quad (1c)$$

Equations (1a) to (1c) are valid for linear elastic elbows. For simplicity, the second subscript "e" from here on will designate elbow end stresses due to applied loading, carry-over effects, and stress intensification due to fabrication. Once again, it should be emphasized that the carry-over factors are valid only for strains, and not for stresses, in the plastic regime. Therefore, expressions in terms of component strains are as follows:

$$\epsilon_z^r = |\epsilon_{ze}| + 1/E\ [2.9\ (\Delta/t)\ (S_{me}) - \nu\ (1.6)\ (\Delta/t)\ (S_{me})] \quad (2a)$$

$$\epsilon_\theta^r = |\epsilon_{\theta e}| + 1/E\ [1.6\ (\Delta/t)\ (S_{me}) - \nu\ (2.9)\ (\Delta/t)\ (S_{me})] \quad (2b)$$

and

$$\epsilon_{ee} = 1/(1+\nu)\ [(\epsilon^2_{ze} + \epsilon^2_{\theta e} - \epsilon_{ze}\ \epsilon_{\theta e}) + (\nu/1-\nu^2)\ (\epsilon_{ze} + \epsilon_{\theta e})^2]^{1/2} \quad (2c)$$

The effective stress (σ_{ee}) can be obtained from the uniaxial stress-strain curve as:

$$\sigma_{ee} = E\ \epsilon_{ee} \quad (2d)$$

The nomenclature used in Eqs. (2a) to (2d) is similar to that used earlier in Eqs. (1a) to (1c) except that E and ν are the modulus of elasticity and the Poisson's ratio of the material, respectively.

Weld Condition
In the above two sections gross discontinuity effects were discussed. The peak stresses that arise at girth butt weld are due to local structural discontinuities. The local discontinuities may be due to abrupt weld reinforcement contours, lack of penetration or cracks, undercuts, slag inclusions, and porosity [3]. Thus, the weld condition is primarily responsible for the K-indices specified in the ASME Code. More accurately, the weld condition is used to compute CK-products.

The C- and K-indices in the ASME Code equations (10) to (14) (para NB-3650) guard against fatigue failure. Fatigue test data reviewed in [3] provide direct evaluation of the adequacy of these Code equations. In the Code, the baseline fatigue data are obtained from polished bars and not from typical girth butt weld in straight pipe. Accordingly, fatigue tests performed by Markle, Newman, Iida, and Yazuki [11] have been used in the ASME Code to arrive at a relationship between the girth weld test data and the polished bar test data. These results, summarized in [3], show that the nominal stress amplitude ratio for polished bar data and girth weld test data is about two. At higher stresses, a direct comparison of the two curves is not appropriate; hence, a plasticity correction factor is used to correlate stress intensification factors with stress indices. The overall correlation suggests a relationship $C_2K_2 = 2$. However, for Class I piping, because of the better quality of welds, C_2 of 1.0 and K_2 of 1.8 is specified in the ASME Code for as-welded condition. The test data for flush welds indicate that the C_2K_2-product may be taken as 1.0 for low cycle fatigue. However, the high-cycle fatigue test data suggests a C_2K_2 factor of about 1.2. As a compromise, the product $C_2K_2 = 1.1$ is specified in the ASME Code to account for the scatter in the test data. Therefore, in CRBRP piping system analysis, $C_2 = 1.0$ and $K_2 = 1.1$ is assumed to compute creep-rupture damage and fatigue damage at the welded sections. To reflect the inelastic procedure described here, the allowable fatigue cycles for a specified strain range are obtained from inelastic design curves in Code Case N-47.

The indices recommended in [3] and specified in the ASME Code are based upon straight pipe tests; that is, $C_2 = 1.0$, and weld condition effects are represented by K_2 index. In contrast, when the elbows are welded to straight pipes, C_2 is greater than 1.0. The elbow ovalization produces a biaxial stress field, wherein the peak hoop stress is at least 50% higher than the axial stress. Since the weld discontinuity is in the axial direction, the K_2-index should be applied to axial stress, and a fraction of K_2 may be used for the hoop stress. However, lack of experimental fatigue test data on elbows welded to straight pipes make it difficult to arrive at a conservative general procedure for all loading conditions. Consequently, the creep-fatigue damage in the CRBRP piping system is evaluated (in the next section) by multiplying the effective stress σ_{ee} by the index K_2, although this approach is conservative.

PROCEDURE TO COMPLY WITH INELASTIC CODE CRITERIA

A complex loading condition, as imposed on a piping system , includes in-plane and out-of-plane forces and moments, pressure, and through-the-wall temperature differentials. In the context of simplified pipe-bend analysis of a piping system, it is difficult to treat each of these loading components separately to compute the stress components at the end of an elbow. Therefore, it is necessary to simplify the procedure to evaluate the structural adequacy of a welded joint.

Creep-Fatigue Interaction
The accumulation of creep-rupture and fatigue damage including hold time and strain rate effects for the combination of normal, upset, and emergency conditions are evaluated according to the following Code Case N-47 equation:

$$\sum_{j=1}^{p} \left(\frac{n}{N_D}\right)_j + \sum_{k=1}^{q} \left(\frac{t}{T_D}\right)_k \leq D \qquad (3)$$

where D is the Code allowable creep-fatigue damage; n is the number of applied cycles of loading condition, j; N_D is the number of design allowable cycles of loading condition, j; t is the time duration of the load condition, k; and T_D is the allowable time at a specified effective stress from load, k.

Creep-Rupture Damage. The creep-rupture damage is based upon the effective stress during creep hold time at steady state full-power operation. The loading on the piping elbows is due to thermal expansion and primary pressure. In thin-walled pipes thermal stresses due to through-the-wall radial temperature differential are absent during creep hold time, which simplifies the calculation of the elbow end stresses given by the following equations:

$$\sigma_{ze} = 0.65\ (\sigma_{zm}) + 0.35\ (pr/2t) \quad (4a)$$

$$\sigma_{\theta e} = 0.55\ (\sigma_{\theta m}) + 0.45\ (pr/t) \quad (4b)$$

$$\epsilon_{ze} = 0.65\ (\epsilon_{zm}) + 1/E\ [0.35\ (pr/2t) - \nu\ (0.45)\ (pr/t)] \quad (4c)$$

$$\epsilon_{\theta e} = 0.55\ (\epsilon_{\theta m}) + 1/E\ [0.45\ (pr/t) - \nu\ (0.35)\ (pr/2t)] \quad (4d)$$

where subscripts e and m refer to the end, and the middle or the most highly stressed section of the elbow; and (pr/t) and (pr/2t) are the hoop and axial stressed due to internal pressure p. The first term in these equations represent the total stress or strain at the end of the elbow, which includes contributions due to pressure loading. Since the pressure stresses essentially remain unchanged along the length of the elbow, a portion of these stress components reduced by carry-over factors is added back to the end section, as given by the second term in Eqs. (4a) to (4d). At this point, the radial weld shrinkage stresses, given by Eqs. (1) and (2) are added to obtain the total stresses at the end of the elbow.

The effective stress, σ_{ee}, at the end of the elbow is now calculated from stress or strain components using Eq. 1c) or (2d). This effective stress has to be multiplied by C- and K-indices, whose values depend upon pipe mismatch and weld condition. For CRBRP piping systems, the stress intensification factor, C = 1.0, and the stress concentration factor, K = 1.1, for flush welds and K = 1.8 for as-welded condition.

Fatigue Damage. The evaluation of fatigue damage requires knowledge of the strain range at the end of elbow. In thin pipes, the maximum and minimum strains generally occur during the largest positive and largest negative through-the-wall radial temperature differentials. Since these thermal stresses are essentially unchanged along the length of the elbow, it is not possible to use the concept of carry-over factor outlined earlier. In general, fatigue damage summation is at least two orders of magnitude lower than the creep-rupture damage summation for the following reasons. The fatigue damage is primarily caused by radial temperature differentials and a few seismic events during thermal transients. If significant fatigue damage is caused by thermal expansion loading, then most probably the piping design would not comply with Code Case N-47 requirements. Conservatively, the strain range at the most highly stressed elbow section can be taken to be the strain range at the end of elbow. In addition, the structural evaluation would have to be based upon increased peak strains due to weld condition. The following equation, which is a modified form of Equation (7) of Code Case N-47, is used to compute the strain intensification, K, due to weld condition.

$$\epsilon_{re} = S^*/S\ K\ \epsilon_{rm} \quad (5)$$

where, ϵ_{re} is the strain range at the elbow end; S^* is stress from isochronous stress-strain curve at zero time and maximum temperature for total strain without the K factor; S is same as S^* but with K factor; ϵ_{rm} is the equivalent strain range at the middle of elbow; and K is the largest stress intensification due to weld condition for the applied loading. K = K_2 for piping, but subscripts to K-index will be deleted in subsequent discussion.

To simplify strain range calculations, the equivalent strain range ϵ_r is used in Equation (5) instead of the strain components specified in Code Case N-47. This assumption is conservative, because in an actual piping system the stress intensification factor may multiply only stress component, whereas in Equation (5) the equivalent strain range ϵ_r is multiplied by K. It may be noted that the stress intensification K is actually the fatigue strength reduction factor which includes geometric "notch" effects obtained from fatigue tests. Therefore, it is not necessary to multiply the strain range by K^2 as is done in Code Case N-47.

Strain Accumulation. The end of the CRBRP elbow is not as highly stressed as the middle of elbow; hence, the plastic strains (without weld effects) are not accumulated at this cross-section. Therefore, the inelastic membrane and bending strain accumulation is only due to creep strains. This membrane and linearized bending strain accumulation is obtained from the highest effective stress at the end of the elbow. Since, the effective stress was computed for creep-rupture damage summation, it is straightforward to compute creep strain accumulation from the creep equation of the material.

For peak strain accumulation, it is necessary to check that no plastic strains have occurred because of stress intensification due to weld condition. Otherwise, plastic strains have to be added to the peak creep strain. For simplicity, instead of calculating strain components, it is judged adequate to compute the effective creep strain from the peak effective stress at creep hold time. It has been observed that a piping system that complies with the Code Case N-47 creep-fatigue as well as buckling limit does not undergo significant plastic strains at the middle of the elbow. Thus, the plastic strains at the middle of the elbow can be directly multiplied by the stress intensification factor to obtain peak plastic strain which can be added to peak creep strain for total strain accumulation.

CONSERVATIVE ASSUMPTIONS IN THE PROPOSED METHOD

The conservative assumptions in the series of steps outlined in this paper are as follows:

- A simplified inelastic analysis, such as a pipe-bend analysis of the piping system, excludes end stiffening effects; hence, the ovalization stresses (or strains) within an elbow under an applied moment are higher than both the experimental strains, and the stresses (and strains) predicted from a shell analysis of the elbow structure (Fig. 4).

- In an actual piping system, clamps or component nozzles near the end of an elbow constrain free ovalization of the end section; consequently, the ovalization stresses would be lower than those estimated by carry-over factors. This clamping effect is excluded in the simplified procedure.

- In the inelastic range, the elastic carry-over factors, when applied to strains, overestimate the actual strains at the end of elbow (Table 1).

- Internal pressure circularizes an oval cross-section at the middle as well as the end of an elbow, thus, mitigating the ovalization stresses at both of these cross-sections. These effects are excluded when the stress is carried-over from the middle to the end of elbow.

- Creep-rupture damage calculations are based upon surface stresses which include stress intensification factor K.

- Fatigue damage strain range calculations are based upon the most highly strained section within the elbow instead of reducing the strain components to the end of the elbow. The stress intensification factor K is used for all strain components, whereas only one strain component experiences the maximum peak strain due to the weld condition.

CONCLUDING REMARKS

Thin-walled LMFBR piping elbows are an order of magnitude more flexible than the straight pipe attachments; hence, it is necessary to include ovalization effects in the final structural evaluation of the welded end of an elbow in a piping system. The ASME Boiler and Pressure Vessel Code [1] does not provide specific guidance to evaluate the end of elbow section welded to a straight pipe. The semi-analytical method proposed in this paper can be used in conjunction with detailed inelastic analysis of a piping system, where the end stiffening effects of straight pipes are excluded from the system analysis. Three basic concepts that are essential in evaluating structural integrity of the welded end of the elbow are as follows:

- Up to 60% of the collapse load the overall elbow behavior is nearly linear, although small plastic zones are present within the elbow.
- The maximum stress (or strain) components at the end elbow section can be obtained using the concept of carry-over factors and the stress distribution at the most highly ovalized section within the elbow.
- Experimental data on girth butt weld and other fabrication effects, which are available in the literature for straight pipes, can be conservatively applied to evaluate the creep-fatigue damage summation and inelastic strain accumulation at the welded end section of an elbow.

These concepts, which are important to capture elbow end effects, are summarized in the following paragraphs.

Up to 30% of the in-plane collapse moment the elbow behavior is linear elastic. Increase in moment beyond the elastic range, initiates wedge shaped plastic zones with the tip moving away from the most highly stressed location within the elbow towards the end of the elbow. More importantly these plastic zones are constrained by elastic regions along the length and around the circumference of the elbow, as well as by the elastic core in the thickness direction. At about 60% of the collapse moment (M_{pc}), the tip of the plastic wedge just reaches the end of elbow. The overall elbow response is nearly linear elastic at 0.6 M_{pc}, and the end elbow section is also elastic. Consequently, at this load level, which corresponds to Code Case N-47 buckling strain factor of 1.67, the strain components can be conservatively extrapolated from the most ovalized section within the elbow to the end of elbow using elastic carry-over factors. Also, small deformation piping system analysis is adequate to predict local inelastic response within the elbow.

The elbow ovalization causes through-the-wall hoop bending which is absent in straight pipes. Only in-plane and out-of-plane loadings or deformations due to thermal expansion of the piping system ovalize the elbow; pressure loading and radial temperature differentials do not produce significant ovalization. The most highly ovalized section is within the elbow, and deformations and stress intensification due to this ovalization attenuate towards the ends. The ASME Code provides stress intensification factors to calculate maximum stresses within the elbow, but no specific guidance is provided to compute maximum stress components at the elbow end section. Therefore, carry-over factors of 0.55 and 0.65 are suggested in this paper to calculate the maximum hoop and axial stress components, respectively, at the elbow end.

The fabrication stresses due to pipe mismatch, radial weld shrinkage, and weld profile are added to the maximum stress components obtained at the elbow end. These stresses are calculated according to the stress intensity factor, C_i and the stress concentration factor, K, suggested by Rodabaugh and Moore [3] for straight pipes.

Finally, a conservative procedure is suggested to compute creep-fatigue damage summation and strain accumulation at the welded end of an elbow in a piping system, to comply with Code Case N-47 criteria.

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