LA-UR -90-3458

CONF 9011128--1

Los Alamos National Laboratory is operated by the University of California for the United States Department of Energy under contract W-7405-ENG-36.

LA-UR--90-3458

DE91 002321

TITLE: QED THEORY OF EXCESS SPONTANEOUS EMISSION NOISE

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SUBMITTED TO: Will be published by the Society of Photo-Industrial Engineers in the proceedings of the Laser Noise Symposium, November 4-5, 1990, Boston, Massachusetts.

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QED Theory of Excess Spontaneous Emission Noise

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ABSTRACT

The results of a quantum-electrodynamical theory of excess spontaneous emission noise in lossy resonators will be presented. The "Petermann K factor" does not enter into the spontaneous emission rate of a *single atom* in the cavity. The QED theory allows different interpretations of the K factor, and we use this fact to justify semiclassical analyses and to provide in one example a simple derivation of K in terms of the amplification of the quantum vacuum field entering the resonator through its mirrors.

1. INTRODUCTION

The quantum limit to the linewidth of a laser arises physically from spontaneous emission. According to the Scawlow-Townes formula, this lower limit to the linewidth is given by

$$\delta\nu = \frac{\overline{N}_2}{\Delta N_t} \frac{h\nu(4\pi\delta\nu_c)^2}{P_{\rm out}} , \tag{1}$$

where ν and $P_{\rm out}$ are the laser frequency and output power and $\delta\nu_c$ is the cavity bandwidth (HWHM). In terms of the cavity quality factor Q, $\delta\nu_c = \nu/2Q$; for a resonator of length L and mirror amplitude reflectivities r_1 and r_2, $\delta\nu_c = (4\pi)^{-1}[(c/2L)\log(r_1r_2)^{-1}]$, and

$$\delta\nu = \frac{\overline{N}_2}{\Delta N_t} \frac{h\nu}{P_{\rm out}} \left(\frac{c}{2L}\right)^2 [\log r_1 r_2]^2 . \tag{2}$$

ΔN_t is the threshold population inversion ($= \overline{N}_2 - \overline{N}_1$) and $\overline{N}_2$ is the steady-state upper-level population. Ordinarily $\delta\nu$ is too small to be of practical concern or even to be observable. In diode lasers, however, the quantum limit *is* of practical interest, mainly because they have very small lengths L compared with other lasers, and often small mirror reflectivities. Typically $\delta\nu \approx 10-100$ MHz in diode lasers.

Petermann [1] concluded from classical arguments that in gain-guided lasers, where the transverse gain variation serves to confine the field laterally, the spontaneous emission into a laser mode is enhanced by an "astigmatism parameter" K. This factor multiplies the usual Schawlow-Townes linewidth. For the longitudinal modes of a standing-wave laser with mirror reflectivities $r_1 = r_2 = r$, the Petermann factor is [2]

$$K = \left(\frac{1-r^2}{2r\log r}\right)^2 , \tag{3}$$

i.e., the linewidth due to spontaneous emission noise is

$$\delta\nu' = K\delta\nu = \frac{\overline{N}_2}{\Delta N_t} \frac{h\nu}{P_{\rm out}} \left(\frac{c}{2L}\right)^2 \left(\frac{1-r^2}{r}\right)^2 . \tag{4}$$

Typically $r \cong 1$ and $\log r^2 \cong 1 - r^2$, in which case $\delta\nu' \cong \delta\nu$, but in general we should replace $\delta\nu$ by $\delta\nu' > \delta\nu$.

In this paper I will discuss the modification of the Schawlow-Townes formula from the standpoint of QED. Only the main physical ideas will be presented; more detailed calculations will be published elsewhere. The main points are:

(1) Contrary to what is usually implied or stated, the spontaneous emission rate of a single atom in a lossy cavity is not enhanced by the K factor; it *is* enhanced, but by the Q factor, not the K factor. The K factor arises when we consider the spontaneous emission from atoms in a gain medium.

(2) $\delta\nu'$ (or $\delta\nu$) can be ascribed in different ways to the amplification of vacuum field fluctuations and to (many-atom) incoherent spontaneous emission. This fact can be used to justify a semiclassical approach.

(3) The difference between $\delta\nu'$ and $\delta\nu$ lies in the fact that quantum noise is amplified, and that this amplification in a lossy system cannot in general be derived under the approximation of uniformly distributed loss.

2. SINGLE-ATOM EMISSION IN A LOSSY CAVITY

Several years ago Cook and the present author [3] solved the problem of an atom in a lossy, multimode cavity consisting of two flat mirrors. The analysis was fully quantum-mechanical, beginning with the Hamiltonian for the initially excited source atom, the quantized radiation field, and all the atoms making up the dielectric mirrors. For simplicity only modes propagating in the two directions normal to the mirrors were included in the analysis, and the problem was reduced to the solution of a delay differential equation. To the best of our knowledge this was the first treatment of a two-level atom in a multimode cavity. More recently the same results were obtained in a different manner by Feng and Ujihara. [4]

One result of this theory is that the spontaneous emission rate of an atom inside a lossy, single-mode cavity is

$$\gamma_{sp} = -(\log r)^{-1}\gamma_o(2\sin^2 k_o z_o), \tag{5}$$

where γ_o is the "free space" emission rate into the allowed modes when the mirror separation $L \to \infty, \omega_o = k_o c$ is the frequency of the field, and z_o specifies the position of the atom relative to the nearest mirror. If we average over $\sin^2 k_o z_o$ for different positions of the atom in the cavity, then $\gamma_{sp} \to \gamma_{avg} = -(\log r)^{-1}\gamma_o \sim Q\gamma_o$.

The fact that the emission rate in a lossy cavity is enhanced by the cavity Q factor was first noted by Purcell nearly half a century ago. [3] Recently this prediction has been confirmed experimentally. [5-7] The point here is that γ_{sp} is enhanced by Q, not by K. Moreover, the actual emission rate in any kind of cavity depends on the position of the atom [equation (5) exemplifies this], and this fact is not accounted for by a spatially independent K factor.

3. HEURISTIC DERIVATION OF K FACTOR

First I will give a simple heuristic argument based on the idea that *the K factor can be associated with the amplification of the vacuum field entering the resonator from the outside.*

Consider again a resonator with mirror reflectivities $r_1 = r_2 = r$ and let t be the amplitude transmission coefficient. Let $z = 0$ and $z = L$ specify the positions of the mirrors. Suppose we are interested in the linewidth of the laser radiation exiting the resonator at $z = L$. This fundamental linewidth is associated with spontaneous emission, or equivalently with the fluctuating vacuum field at the exit plane. The vacuum field is *amplified* by the gain medium. Its amplification factor is obtained by multiplying the vacuum field amplitude propagating into the cavity at $z = L$ by (1) the transmission coefficient t; (2) the gain factor G associated with propagation from $z = L$ to $z = 0$; (3) the reflection coefficient r for the mirror at $z = 0$; (4) the gain factor G associated with propagation from $z = 0$ to $z = L$; and (5) the transmission coefficient t for the mirror at $z = L$. The overall multiplication factor is thus $tGrGt = G^2t^2r$, or t^2/r since $Gr = 1$ according to the threshold (and gain clamping) condition for laser oscillation. The power amplification coefficient is then $t^4/r^2 = (1-r^2)^2/r^2$. This is the last factor on the right-hand side of equation (4). [The vacuum field entering the resonator from the mirror at $z = 0$ also contributes to the fluctuating phase and linewidth at $z = L$, and it is easily shown that its amplification involves the same factor t^2/r.] This implies the K factor of equation (3).

Things are only a little more complicated if we consider, for instance, an unstable resonator. In this case we must deal with the transverse field profile $\phi(\mathbf{x})$ of the vacuum field entering the cavity. Then the K factor involves not only the amplification factor associated with the magnification, but also the factor $\int |\phi(\mathbf{x})|^2 d^2x$, where the integration is over the plane of the feedback mirror (in the case of one-sided output coupling). Now the modes $\phi(\mathbf{x})$ are "reversed" modes in the sense that they propagate *into* the resonator from the outside. These are in fact the so-called *adjoint* modes of the resonator.[8] This will be discussed elsewhere; the point here is simply that we can in general ascribe the K factor to the amplification of the vacuum field entering the resonator from the outside.

The story here is very similar to a recent theory of spontaneous emission near a phase-conjugating mirror. [9] There the vacuum field fluctuations are amplified by the phase-conjugating mirror and cause the spontaneous emission rate to be enhanced. Basically what happens is that the vacuum field fluctuations are "promoted" to real, thermal-like fluctuations that can be measured, absorbed, and can stimulate emission of radiation from an atom. Alternatively, what we are talking about here is just amplification of spontaneous emission.[9,10] In the following section I briefly discuss this interplay of vacuum fluctuations and spontaneous emission. This is an old idea [11,12] that has been found useful in many different contexts, most recently in theories of superfluorescence, squeezing, and Raman scattering.

4. SUMMARY OF QED THEORY

In the space allotted I cannot sensibly describe any details of the QED theory of excess spontaneous emission noise. I will simply give a summary of the main results and defer details to a longer paper.

Let a be the annihilation operator associated with a single laser mode. For the correlation function $\langle a(t)a(t+\tau)\rangle$ we obtain

$$\langle a(t)a(t+\tau)\rangle = \overline{n}e^{-(C/2\overline{n})\tau}e^{-i\omega_o\tau} \ , \tag{6}$$

where C is the spontaneous emission rate into the mode, $\overline{n}$ is the average intracavity photon number, and ω_o is the field frequency, which is assumed to be equal to the central frequency of the gain transition. Equation (6) implies the linewidth

$$\delta\nu = C/2\overline{n} = \frac{c\sigma\overline{N}_2}{2\overline{n}} = \frac{\overline{N}_2}{\Delta N_t}\frac{cg_t}{2\overline{n}} \ , \tag{7}$$

where σ is the stimulated emission cross section. Since $P_{\text{out}} = (cg_t)h\nu\overline{n}$, we recover the Schawlow-Townes formula (1) *when the gain and intracavity intensity are assumed to be spatially uniform.*

No vacuum field fluctuations have entered explicitly into this derivation. But that is only because we have somewhat arbitrarily *chosen* a normal ordering of field operators. Symbolically, we have $a = a_o + \Sigma/(\Delta N_t)^{1/2}$, where a_o is the source-free (vacuum) field operator and Σ represents an atomic lowering operator such that the steady-state expectation values of interest are

$$\langle\Sigma^\dagger\Sigma\rangle = \overline{N}_2 \ , \tag{8}$$

$$\langle\Sigma\Sigma^\dagger\rangle = \overline{N}_1 \ , \tag{9}$$

$$\langle\Sigma^\dagger\Sigma + \Sigma\Sigma^\dagger\rangle = \overline{N}_1 + \overline{N}_2 \ . \tag{10}$$

Here $\overline{N}_1$ and $\overline{N}_2$ are the populations of the lower and upper states, respectively, of the gain transition.

In our normally ordered calculation outlined above, we have only had to contend with

$$\langle a^\dagger a\rangle = \frac{\langle\Sigma^\dagger\Sigma\rangle}{\Delta N_t} = \frac{\overline{N}_2}{\Delta N_t} \ . \tag{11}$$

No vacuum contribution arose because $\langle a_o^\dagger a_o\rangle = 0$. If we work instead with a symmetric ordering of field operators, however, we incur

$$\begin{aligned}\frac{1}{2}\langle aa^\dagger + a^\dagger a\rangle &= \frac{1}{2}\langle a_o a_o^\dagger + a_o^\dagger a_o\rangle + \frac{1}{2}\langle\Sigma\Sigma^\dagger + \Sigma^\dagger\Sigma\rangle/\Delta N_t \\ &= \frac{1}{2}\left[1 + \frac{\overline{N}_1 + \overline{N}_2}{\overline{N}_2 - \overline{N}_1}\right] = \frac{\overline{N}_2}{\Delta N_t} \ ,\end{aligned} \tag{12}$$

since $\langle a_o a_o^\dagger\rangle = 1$. This is the same as (11). Or we can use an anti-normal ordering:

$$\begin{aligned}\langle aa^\dagger\rangle &= \langle a_o a_o^\dagger\rangle + \frac{\langle\Sigma\Sigma^\dagger\rangle}{\Delta N_t} \\ &= 1 + \frac{\overline{N}_1}{\overline{N}_2 - \overline{N}_1} = \frac{\overline{N}_2}{\Delta N_t} \ .\end{aligned} \tag{13}$$

Now, whichever ordering we employ, we should take into account the amplification of the vacuum field by the gain medium. When we do this, we obtain as in the heuristic derivation above the Petermann K factor. The results (11)-(13) above imply that *the Σ-dependent contributions to the linewidth will have the same K factor as the explicit vacuum-field contribution.* That is, the source and vacuum contributions are identical. Since the Σ-dependent contribution really involves propagation of the spontaneously emitted radiation from different portions of the gain medium, and from different atoms, the vacuum contribution turns out to be a lot easier to calculate - as our heuristic derivation demonstrates.

The QED theory outlined above also makes it easy to connect different derivations of the K factor. For instance, Petermann's original argument [1] was essentially classical - there was no vacuum field involved. This corresponds to the normal-ordering approach with the additional feature that the Σ operators are replaced by classical dipole amplitudes. Since the spontaneous emission rate into a given mode is correctly given by classical theory, even if the detailed dynamics (like the dependence on $\overline{N}_2$) is not, Petermann obtained the correct enhancement factor.

Siegman's approach [8] is semiclassical, and so the field is not quantized and there are no vacuum field fluctuations. This corresponds again to normal ordering of field operators. But now the Σ's are operators, so that the dependence on the excited-state probability $\overline{N}_2$ is correctly included. Note that in our approach we have not had to explicitly invoke the non-orthogonality of the resonator modes.

An approach by Ujihara [13] is semiclassical but includes vacuum field fluctuations as classical reservoir noise. Since it deals with classical electric fields, this approach corresponds to a symmetric ordering of $a's$ and $a^\dagger's$ in the QED theory. [14] Thus Ujihara's theory leads to the sum of source and vacuum contributions as in equation (12). It is worth noting that in this work the equality of the source and vacuum contributions to the linewidth noted earlier is obtained by explicit calculation.

The QED theory shows clearly why the K factor arises in lossy systems. When there is large loss there must be large gain to compensate, and when the gain is large we have to account for the amplification of quantum noise. Large gain means that the intracavity fields are not spatially uniform, and we cannot assume that the loss is effectively distributed uniformly throughout the gain medium.

6. ACKNOWLEDGEMENTS

I wish to thank Dr. David Stoler for bringing some of the references on excess spontaneous emission noise to my attention, and Prof. Philip Goldberg for a very helpful discussion of adjoint modes of resonators. Brief discussions with M.-L. Shih, E. M. Wright, and M. Sargent III are also gratefully acknowledged.

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