RLO-2250T4-225

DOE/ER/70004-225

OITS 98

September, 1978

PARITY VIOLATION AND THE MASSLESSNESS OF THE NEUTRINO

Philip D. Mannheim [(a)]

Institute of Theoretical Science, University of Oregon,

Eugene, Oregon 97403

and

Dept. of Physics and Astronomy, Tel-Aviv University,

Tel-Aviv, Israel [(b)]

We propose that the weak interaction be obtained by gauging the strong interaction chiral flavor group. The neutrinos are then four-component spinors. We allow pairs of right-handed neutrinos to condense into the vacuum. This produces maximal parity violation in both the quark and lepton sectors of the weak interaction, keeps the neutrinos massless, and also leads to the conventional Weinberg mixing pattern. Our approach also in principle provides a way of calculating the Cabibbo angle.

(a) Research supported in part by U.S. Energy Research and Development Administration under contract No. AT (45-1) - 2230.

(b) Summer Visitor.

It is now evident that flavor is important in both strong and weak interactions. It is thus rather natural to identify the flavor groups of the two interactions, particularly given the success of the quark lepton analogy. Indeed, [1-4] recent analysis of the group $SU(4)_L \times U(1)$ as a candidate gauge theory of weak interactions has shown the group to be in as much agreement with experiment as its more familiar $SU(2)_L \times U(1)$ subgroup. Since the group unites the four left-handed quarks (u, d, s, c) into one multiplet while uniting the four left-handed leptons (ν_e, e, μ, ν_μ) into another it is particularly well-suited for studying the structure of flavor breaking. In Refs. 1-4 it was shown that spontaneous breakdown of flavor leads to a theory of Cabibbo mixing, a superweak theory of CP violation, and a theory of muon-number violation. Since these latter violations are very small the theory still awaits experimental verification.

While the $SU(4)_L \times U(1)$ theory is both interesting and viable it is rather unsatisfying [as is $SU(2)_L \times U(1)$] since parity violation is assumed ab initio. Moreover it is a very asymmetrical theory in that it puts left-handed particles of different flavors into common multiplets while requiring their right-handed counterparts to be singlets. Consequently we shall remove this asymmetry by introducing right-handed quartets of the same quarks and leptons so that the group is now $SU(4)_L \times SU(4)_R \times U(1)_V$. [5] Thus the weak interaction is now the local gauge extension of the full chiral flavor group with the current algebra currents generating both the strong and weak interactions. Further we now have four-component neutrinos and the weak interaction is parity conserving in the symmetry limit. It is the main point of this paper that the spontaneous breakdown mechanism used below to obtain parity violation necessitates that the neutrinos stay massless.

Though our interest in parity violation derives from gauging the chiral flavor group, the question of the origin of parity violation in weak interactions is of course of interest in its own right. While the development of the V - A theory

DISCLAIMER

established the fact of parity violation it did not identify its origin. Though a two-component neutrino requires parity violation, parity violation does not require a two-component neutrino. Also even with a two-component theory there is no a priori reason why parity should be violated in processes which do not involve neutrinos. Consequently we should entertain the possibility that the neutrino is a four-component spinor and look for theories in which its masslessness is related to parity violation. The nice feature of the chiral flavor theory is that in it the condition that the neutrino remain massless produces maximal parity violation even in the non-leptonic sectors of the theory.

Before proceeding to develop the spontaneous breakdown pattern we make a general remark about symmetry breaking. While the symmetry group itself is specified by the representation content of the matter fields, there is a priori complete arbitrarity in picking the representations which provide the breaking direction, with simplicity being the only apparent criterion. We therefore need a principle to know how to specify the Higgs multiplets. The most suggestive principle, even though it is not yet completely understood, is dynamical symmetry breaking, with the theory then choosing its own Higgs multiplets. We shall therefore use dynamical symmetry breaking as a guide to picking the appropriate Higgs multiplets, though what we shall present here will only be a phenomenology.

The most obvious representation to use to break the symmetry is the quark mass term which belongs to $(4, \bar{4}) \oplus (\bar{4}, 4)$. However since the quark mass matrix can be diagonalized in the canonical (u,d,s,c) basis, we find immediately that the gauge bosons are also diagonal in a canonical basis specified by W_i^L, W_i^R $(i = 1,..15)$ and W_0. In the notation of Refs. 1-4 we introduce (for convenience in the following) the states

$$
\begin{aligned}
W_\pm &= \left[(W_1+W_{13}) \mp i\,(W_2-W_{14})\right]/2 \\
X_\pm &= \left[(W_1-W_{13}) \mp i\,(W_2+W_{14})\right]/2 \\
U_\pm &= \left[(W_4+W_{11}) \mp i\,(W_5-W_{12})\right]/2 \\
V_\pm &= \left[(W_4-W_{11}) \mp i\,(W_5+W_{12})\right]/2 \\
R &= \left[W_3+W_8/\sqrt{3} - W_{15}\sqrt{2}/\sqrt{3}\right]/\sqrt{2}
\end{aligned}
\tag{1}
$$

for both L and R indices. Then the charged sector of the gauge boson mass matrix is diagonal in the basis specified by

$$
\begin{aligned}
&(W+X)^V, \quad (W-X)^V, \quad (U+V)^V, \quad (U-V)^V \\
&(W+X)^A, \quad (W-X)^A, \quad (U+V)^A, \quad (U-V)^A
\end{aligned}
\tag{2}
$$

where V, A respectively denote vector (L + R) and axial (L - R) fields. The currents associated with, for instance, the vector fields of Eq. 2 are respectively [6]

$$
\begin{aligned}
&\bar{u}\gamma_\lambda d + \bar{\nu}_e\gamma_\lambda e, \quad \bar{c}\gamma_\lambda s + \bar{\nu}_\mu\gamma_\lambda \mu \\
&\bar{u}\gamma_\lambda s - \bar{\nu}_e\gamma_\lambda \mu, \quad \bar{c}\gamma_\lambda d + \bar{\nu}_\mu\gamma_\lambda e
\end{aligned}
\tag{3}
$$

Consequently this basis does not possess quark or lepton universality. Further if we change the basis which diagonalizes the quark mass matrix we find that the quark currents and gauge boson eigenstates rotate together with the quark mass eigenstates so that the interaction remains canonical. Thus in the chiral theory the quark mass matrix cannot provide an origin for the Cabibbo angle; and the $(4, \bar{4}) \oplus (\bar{4}, 4)$ representation can only play a minor role.

It was shown in Ref. 2 that the group structure of SU(4) is such that universality can be maintained if the SU(4) group is broken by adjoint representations. Consequently two left-handed 15 dimensional representations of $SU(4)_L$, η_L and ζ_L, were introduced and an explicit potential was constructed in which $SU(4)_L \times U(1)$ was broken to a subgroup generated by $W_L^{\pm}$, R_L and W_0, the Weinberg-Salam $SU(2)_L \times U(1)$.[7,8] We now simply repeat the work of Ref. 2 by adding two further right-handed 15 dimensional representations of $SU(4)_R$, η_R and ζ_R, and break the theory so that, in a tensor notation,

$$\eta^L = \eta^R = \eta_0 \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \zeta^L = \zeta^R = \zeta_0 \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \qquad (4)$$

This breaks $SU(4)_L \times SU(4)_R \times U(1)$ down to an $SU(2)_L \times SU(2)_R \times U(1)$ subgroup which possesses universality, with the flavor symmetry having been broken spontaneously. Phenomelogical potentials analogous to those of Ref. 2 can be constructed in which the above pattern of breaking is obtained for a range of the parameters in the potential with the parity invariance of the potential allowing the same breaking pattern in the left and right-handed sectors. Dynamically the (15,1) and (1,15) multiplets can be built out of quark and lepton quadrilinears, a point we will discuss again below.

It was further shown in Ref. 4 that a more general breaking pattern is possible, a typical example being

$$\eta^L = \eta^R = \eta_0 \begin{pmatrix} \cos\theta_C & 0 & 0 & -\sin\theta_C \\ 0 & \cos\theta_C & \sin\theta_C & 0 \\ 0 & \sin\theta_C & -\cos\theta_C & 0 \\ -\sin\theta_C & 0 & 0 & -\cos\theta_C \end{pmatrix}$$

$$\zeta^L = \zeta^R = \zeta_0 \begin{pmatrix} \sin\theta_C & 0 & 0 & \cos\theta_C \\ 0 & -\sin\theta_C & \cos\theta_C & 0 \\ 0 & \cos\theta_C & \sin\theta_C & 0 \\ \cos\theta_C & 0 & 0 & -\sin\theta_C \end{pmatrix} \qquad (5)$$

This structure also breaks the theory down to an $SU(2)_L \times SU(2)_R \times U(1)$ subgroup, the one generated by $\tilde{W}_L^{\pm}$, R_L, $\tilde{W}_R^{\pm}$, R_R and W_0. Here

$$\tilde{W}^{+}_{L,R} = W^{+}_{L,R} \cos\theta_C + V^{+}_{L,R} \sin\theta_C \qquad (6)$$

and couple to

$$\bar{u}\gamma_\lambda 1/2\,(1\mp\gamma_5)\,[d\cos\theta_C + s\sin\theta_C] + \bar{c}\gamma_\lambda 1/2(1\mp\gamma_5)\,(-d\sin\theta_C + s\cos\theta_C)$$
$$+ (\bar{\nu}_e\cos\theta_C - \bar{\nu}_\mu\sin\theta_C)\gamma_\lambda 1/2(1\mp\gamma_5)e + (\bar{\nu}_e\sin\theta_C + \bar{\nu}_\mu\cos\theta_C)\gamma_\lambda 1/2(1\mp\gamma_5)\mu \qquad (7)$$

[the second term can also be written, symbolically, as $\bar{\nu}_e(e\cos\theta_c + \mu\sin\theta_c) + \bar{\nu}_\mu(-e\sin\theta_c + \mu\cos\theta_c)$], and hence possess Cabibbo universality. Thus in theories built on the group SU(4) we find that there are many SU(2) subgroups which could serve as the Weinberg-Salam SU(2), and this freedom is precisely what is needed to introduce a Cabibbo angle, (and also CP violation). This phenomenon is discussed in detail in Ref. 4. Since the (15,1) and (1,15) representations involve independent chiral rotations on the left and right-handed quarks they do not couple to the quark mass (thus producing no parity or flavor violation in the strong interaction), and hence the Cabibbo structure which was obtained above by gauge boson mixing is not removed by rotations of the quark mass matrix. The parameters η_0 and ζ_0 can be made large so that the superheavy gauge boson sector leads to small muon number violations, etc., analogous to the discussion of Refs. 1-4, and will concern us no more here.

So far the theory has Cabibbo universality and is still parity conserving. We now want to break the theory down to the Weinberg-Salam model. We note first that any fermion multilinear built out of powers of $\bar{\Psi}\lambda_i\Psi$ will never break the U(1) fermion number group. The simplest representation that will do so is $\overline{\psi^c}\lambda_i\psi$, a di-fermion. [9] Further, in order that electric charge not be broken, the only available di-fermion is a neutrino pair. The key point is to now notice that whatever the handedness of ψ, only the same handedness for $\overline{\psi^c}$ couples in $\overline{\psi^c}\lambda_i\psi$. Because of Fermi statistics we thus consider the (10,1) and (1,10) representations of $SU(4)_L \times SU(4)_R$. We allow (1,10) to acquire a large expectation value, so that in a tensor basis it behaves as

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

[i.e. $\langle\overline{\nu_e^c}(1+\gamma_5)\nu_e\rangle = \langle\overline{\nu_\mu^c}(1+\gamma_5)\nu_\mu\rangle \neq 0$] while permitting no (10,1) breaking. This gives large masses to the right-handed gauge bosons. Following the notation of Ref. 2, if we denote the gauge couplings of $SU(4)_L$ and $SU(4)_R$ by $g/\sqrt{2}$ and that of U(1) by g', we obtain a heavy neutral boson

$$Z_R = (g\,R_R - g'\,W_0)\,/\,(g^2 + g'^2)^{1/2} \qquad (9)$$

leaving $\bar{W}_L^{\pm}$, R_L, $g'R_R + g\,W_0$ so far massless. [10] Further, since the breaking is in (1,10) only, the vacuum remains invariant to all chiral rotations on the left-handed neutrinos. Hence the neutrinos stay massless.

Finally we now break according to the quark masses and/or electron and muon masses, the last and smallest stage of breaking. This then gives masses to $W_L^{\pm}$ and $R_L - R_R$. We now have to rediagonalize the neutral sector. Defining new eigenstates

$$A = -\left[g'\,(R_L + R_R) + g\,W_0\right] / \,(g^2 + 2g'^2)^{1/2}$$

$$Z_L = \left[-(g^2 - g'^2)R_L + g'^2 R_R + gg'W_0\right] / \left[(g^2+g'^2)\,(g^2+2g'^2)\right]^{1/2} \qquad (10)$$

then leads to a low energy interaction Lagrangian for the neutral sector

$$\mathcal{L} = \frac{-\,g\,g'}{(g^2+2g'^2)^{1/2}}\left\{A^\lambda - \frac{g'}{(g^2+g'^2)^{1/2}}\,Z_L^\lambda\right\} J_\lambda^{e.m.}$$

$$-\,\frac{g}{2}\left\{\frac{(g^2+2g'^2)}{(g^2+g'^2)}\right\}^{1/2} Z_L^\lambda \Big\{\bar{u}\gamma_\lambda u + \bar{c}\gamma_\lambda c - \bar{d}\gamma_\lambda d - \bar{s}\gamma_\lambda s + \bar{\nu}_e\gamma_\lambda\nu_e + \bar{\nu}_\mu\gamma_\lambda\nu_\mu - \bar{e}\gamma_\lambda e - \bar{\mu}\gamma_\lambda\mu\Big\}_L \qquad (11)$$

Defining

$$e = \frac{gg'}{(g^2+2g'^2)^{1/2}} \quad , \tan\theta_W = -\frac{g'}{(g^2+g'^2)^{1/2}} \qquad (12)$$

we obtain the conventional Weinberg mixing pattern. Also the standard relations $G_F/\sqrt{2} = g^2/8M_W^2$, $M_W = M_Z \cos\theta_W$ follow from the same breaking pattern. Moreover, since there is no breaking in the (10,1) representation we are free to rotate the left-handed neutrinos amongst themselves to remove the angle in Eq. 7, and thus obtain for the current that couples to $\tilde{W}_L^+$.

$$\bar{u}\gamma_\lambda (d_L \cos\theta_C + s_L \sin\theta_C) + \bar{c}\gamma_\lambda (-d_L \sin\theta_C + s_L \cos\theta_C)$$

$$+ \bar{\nu}_e \gamma_\lambda e_L + \bar{\nu}_\mu \gamma_\lambda \mu_L \qquad (13)$$

the usual Cabibbo current. Thus our pattern of breaking produces all the usual phenomenology, with maximal parity violation everywhere in the low energy structure of the theory.

It is important to note that whichever handedness of dineutrinos goes into the vacuum, only the other handedness appears at low energies. Consequently in ordinary laboratory experiments the disappearance of a pair of neutrinos into the vacuum will not be observable, and hence cannot immediately be investigated experimentally. The more likely place to look for such effects would be in astrophysics (perhaps the high temperature normal symmetry limit of the early universe, or the missing solar neutrino flux), though this remains to be studied.

Returning to our motivation of dynamical symmetry breaking we now see the role of the quadrilinears. The fermion bilinears are used up in breaking $SU(2)_L \times SU(2)_R \times U(1)$, and hence we need more representations in order to break

the full chiral group. Quadrilinears are then the next obvious choice.[11] While it remains to be seen whether the pattern of breaking we have described here will ever emerge dynamically, we note that if it does then the orientation of the (15,1) representations relative to the diagonal $(4, \bar{4}) \oplus (\bar{4}, 4)$ (i.e. the Cabibbo angle[4] and CP violation parameters[3], such as in the example of Eq. 5) will then be determined by the potential (analogous to the analysis of Ref. 4). Since dynamical symmetry breaking also determines the parameters that appear in the potential in terms of the basic gauge couplings of the theory, it will then in principle be possible to calculate the Cabibbo angle. However that remains a remote prospect for the moment.

A detailed description of our work will be published elsewhere. I would like to thank Professor Y. Ne'eman and E. Gotsman for the kind hospitality of Tel-Aviv University where part of this work was carried out.

References:

1. N.G. Deshpande, R.C. Hwa and P.D. Mannheim, Phys. Rev. Lett. 39, 256 (1977).
2. N.G. Deshpande, R.C. Hwa and P.D. Mannheim. SU(4) X U(1) Gauge Theory: I. Muon Number Nonconservation, University of Oregon Report OITS-82, to be published.
3. N.G. Deshpande, R.C. Hwa and P.D. Mannheim, SU(4) X U(1) Gauge Theory: II. CP Nonconservation, University of Oregon Report OITS-83, to be published.
4. N.G. Deshpande, R.C. Hwa and P.D. Mannheim, SU(4) X U(1) Gauge Theory: III New Approach to Cabibbo Mixing, University of Oregon Report OITS-84, to be published.
5. The very recent proliferation of quarks and leptons may necessitate the use of groups such as $SU(6)_L \times SU(6)_R \times U(1)$. The group theoretical analysis of this paper will remain valid in the bigger groups
6. The field R^V couples to $\bar{u}\gamma_\lambda u + \bar{c}\gamma_\lambda c - \bar{d}\gamma_\lambda d - \bar{s}\gamma_\lambda s + \bar{\nu}_e\gamma_\lambda \nu_e + \bar{\nu}_\mu \gamma_\lambda \nu_\mu - \bar{e}\gamma_\lambda e - \bar{\mu}\gamma_\lambda \mu$. Thus in terms of the generators F_i^V the electric charge is given by $Q = F_3^V + F_8^V/\sqrt{3} - \sqrt{2}\, F_{15}^V/\sqrt{3} + F_0$, the Gell-Mann Nishijima formula.
7. S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967).
8. A. Salam, in Elementary Particle Theory: Relativistic Groups and Analyticity, edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.
9. Quantities such as $\overline{\psi^c}\lambda_i \psi$ may be obtained from $\bar{\psi}\lambda_i\psi$ by a Pauli-Gürsey transformation. They thus correspond to standard $\bar{\psi}\lambda_i\psi$ bilinears written in the particle-hole basis.
10. The standard Weinberg mixing is obtained by breaking according to the fundamental. Since the dineutrino is a pair of fundamentals it gauges the same way as a fundamental only with twice the couplings (just like a Cooper pair). Hence the pattern is the same.

11. In dynamical symmetry breaking it is natural to use the fermion composites of lowest dimension since they are the most infrared singular.