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THE SOFT PHOTON THEOREM FOR BREMSSTRAHLUNG

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ABSTRACT

We review this theorem and discuss the possible importance of the second term in the expansion of the cross section in powers of the photon momentum, especially for radiation from particles coming from the decay of resonances.

INTRODUCTION

The quantum mechanical amplitude for the bremsstrahlung process depicted in Fig. 1 is written $\epsilon_\mu M^\mu$, where ϵ_μ is the polarization of the photon and M^μ is the matrix element of the electromagnetic current J^μ between the initial and final interacting states. M^μ depends on the momenta and spins of the N incident and emergent particles.

If one has a model for the interaction of the particles as well as a knowledge of their electromagnetic moments, then M^μ can be calculated. This has been done for both nucleon-nucleon and pion-nucleon bremsstrahlung in the energy region below the pion

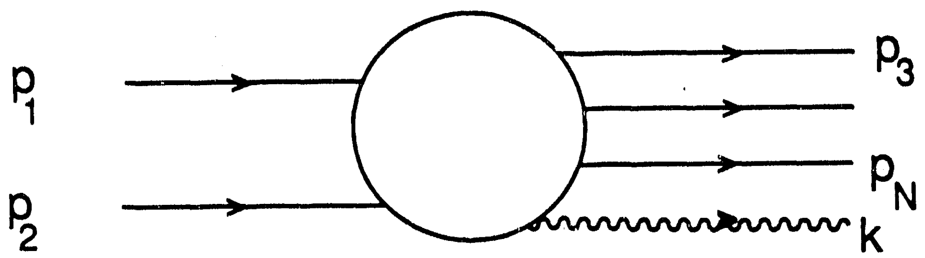

Fig. 1. Kinematics for the bremsstrahlung process.

production threshold. The goal of the nucleon-nucleon experiments has been to distinguish between different potentials that fit the non- radi-tive data equally well; for this purpose the hardest photons are the most useful. The primary motivation for the pion-nucleon experiments has been to learn the magnetic moment of the $\Delta(1232)$ resonance.

For soft photons it is possible to relate the radiative process *directly* to the non-radiative process, without knowing the details of how the particles interact. This is the content of the soft photon theorem.

SOFT PHOTON THEOREM

A schematic statement of the theorem originally proven by Low [1] is as follows. If the matrix element M^μ is expanded in powers of the photon momentum k,

$$M^\mu = \frac{A^\mu}{k} + B^\mu + O(k), \tag{1}$$

then *both* A^μ and B^μ can be calculated from knowledge of the *physical*, i.e., on-shell, T-matrix for the non-radiative process illustrated in Fig. 2; derivatives of T also appear in B^μ. The momenta p_i' in Fig. 2 differ from the p_i in Fig. 1 by $O(k)$. (More on this later.) Hereafter we shall refer to the non-radiative T-matrix and cross section by calling them the "hadronic" T-matrix, T^h, and the hadronic cross-section, σ^h, respectively.

Here are some important remarks about the theorem.

1. Although it is physically reasonable that for sufficiently soft photons the bremsstrahlung process does not reveal anything new about the hadronic interaction, the surprising result is that *two* leading terms can be determined from on-shell information. This is quite significant because the second term, even if small compared to the leading term, does not vanish at $k \to 0$.

2. There is clearly an analogous statement for the bremsstrahlung *cross section*, σ^γ, since it is bilinear in M^μ. For example, the bremsstrahlung cross section summed over the two states of photon polarization is proportional to

$$-kM \cdot M^* = -\frac{A \cdot A^*}{k} - 2Re\ A \cdot B^* + [-kB \cdot B^* + O(k)], \tag{2}$$

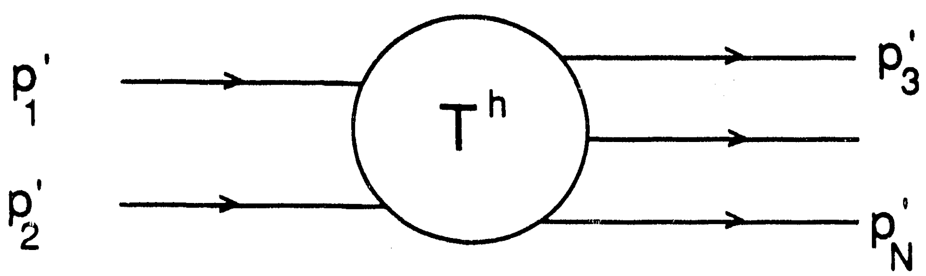

Fig. 2. The non-radiative, or "hadronic", T-matrix, T^h.

so the first two terms are indeed determined by the physical hadronic T-matrix. Part of the third erm is also, but there are additional contributions that involve off-shell information.

The proof of the theorem begins by writing down the part of M^μ that is *singular* at $k = 0$. This comes only from diagrams in which an external leg radiates, and involves T^h at an off-shell point. This is then expanded about a nearby on-shell point and results in contributions to both A^μ and B^μ in Eq. (1). Gauge invariance is sufficient to completely determine the contribution of the *non-singular* part of M^μ to B^μ.

When the singular and non-singular terms are combined [1], A^μ is proportional to (a kinematic factor multiplying) the hadronic T-matrix, T^h, evaluated at the chosen on-shell point. B^μ contains two types of terms: derivatives of T^h, $\partial T^h/\partial p_i$, and also magnetic moments multiplying T^h. At this point one has the stated result that the first two terms in the expansion of the bremsstrahlung matrix element in powers of k are completely determined by the *on-shell* hadronic T-matrix and its derivatives, as well as the charges and magnetic moments of the initial and final particles. Off-shell information first appears in the term $O(k)$ in Eq. (1).

The matrix element in Eq. (1) involves initial and final particles of definite (but arbitrary) polarization and the cross-section calculated from it does also. If the spins of all the particles are summed (averaged) over, giving the *unpolarized* bremsstrahlung cross section, $d\sigma^\gamma$, then there is a very interesting result first proved by Burnett and Kroll [2] for spin-0 and spin-1/2 particles, and generalized by Bell and van Royen [3] to arbitrary spin. It says that the first two terms in the expansion of $d\sigma^\gamma$ (unpolarized) in powers of k depend only on the *unpolarized* hadronic cross section, $d\sigma^h$, and its derivatives. Furthermore, all magnetic moments drop out of the second term.

Although this result can be written for an arbitrary photon polarization, we show it for the case that the two states of photon polarization are also summed over, first giving the square of the matrix element,

$$M \cdot M^* = \alpha \sum_{i=1}^{N} \frac{\pm Q_i p_i}{k \cdot p_i} \cdot \sum_{j=1}^{N} \left[\frac{\pm Q_j p_j}{k \cdot p_j} + Q_j D_j \right] \sum_{spins} |T^h|^2 + O(k). \tag{3}$$

D_j in Eq. (3) is a gauge invariant combination of derivatives,

$$D_j \equiv \frac{p_j}{k \cdot p_j} k \cdot \frac{\partial}{\partial p_j} - \frac{\partial}{\partial p_j}, \tag{4}$$

and the $\pm$ sign goes with a final (initial) particle, whose charge is Q_i in units of the proton charge; α is the fine structure constant. The bremsstrahlung cross section is given by

$$d\sigma^\gamma = -M \cdot M^* d\rho, \tag{5}$$

where $d\rho$ contains the differential number of final states for both the particles and the photon, as well as the flux factor, and leads to

$$d\sigma^\gamma = -\alpha \sum_i \frac{\pm Q_i p_i}{k \cdot p_i} \cdot \sum_j \left[\frac{\pm Q_j p_j}{k \cdot p_j} + Q_j D_j \right] d\sigma^h \frac{d^3 k}{4\pi^2 \omega} + O(k). \tag{6}$$

D_j in Eq. (6) acts only on the square of the hadronic T-matrix, not on the density of states or flux factors.

Equation (6) can be rewritten in terms of the velocities of the particles as

$$d\sigma^\gamma = \alpha \sum_i \frac{\pm Q_i \boldsymbol{n} \times \boldsymbol{v}_i}{1 - \boldsymbol{n} \cdot \boldsymbol{v}_i} \cdot \sum_j \left[\frac{\pm Q_j \boldsymbol{n} \times \boldsymbol{v}_j}{1 - \boldsymbol{n} \cdot \boldsymbol{v}_j} + k Q_j \boldsymbol{n} \times \boldsymbol{D}_j \right] d\sigma^h \frac{d^3k}{4\pi^2 \omega^3} + O(k), \quad (7)$$

with $\boldsymbol{n} = \boldsymbol{k}/k$. As a function of the angle θ between $\boldsymbol{n}$ and $\boldsymbol{v}$, the factor $(\boldsymbol{n} \times \boldsymbol{v})/(1 - \boldsymbol{n} \cdot \boldsymbol{v})$ is maximized at $\cos\theta = v$. For non-relativistic motion, soft photons are emitted primarily at right angles to the velocity of an initial or final particle, whereas for highly relativistic motion they are emitted primarily along the velocity.

It is necessary to say a word about the choice of on-shell point at which T^h or σ^h is evaluated. This is the question of how to relate the momenta p_i' in the hadronic process (Fig. 2) to the momenta p_i in the bremsstrahlung process (Fig. 1). If the second term in the brackets (the one containing the derivatives D_j) in Eq. (6) or (7) is neglected, then it is irrelevant what choice is made because the difference between σ^h evaluated at two points with momenta differing by $O(k)$ is itself $O(k)$, and therefore contributes in the same order as the D_j terms. If, on the other hand, the terms in D_j are kept then it could make a difference.

The key to establishing the connection follows from the fact that the unpolarized cross section, σ^h, does not depend on all N momenta of the incoming and outgoing particles, but only on $3N - 10$ independent *scalars* that can be constructed from them. The first step is to choose a particular set of scalars, u_α. For two particles in and two particles out ($N = 4$), for example, there are just two independent scalars, which could be taken to be $u_1 = p_1' \cdot p_2'$ and $u_2 = p_1' \cdot p_3'$. Equations (3), (6), and (7) follow from the prescription that σ^h is evaluated at the same values of these scalars that they have in the bremsstrahlung process, i.e., for

$$u_\alpha(p_1', \ldots, p_N') = u_\alpha(p_1, \ldots, p_N), \quad (8)$$

which is a set of $3N - 10$ equations. The meaning of the derivatives that appear in these equations is

$$\frac{\partial}{\partial p_j} \equiv \sum_\alpha \frac{\partial u_\alpha}{\partial p_j} \frac{\partial}{\partial u_\alpha}. \quad (9)$$

Comparing Eqs. (3), (6), or (7) with Eq. (2) shows that the terms $|A|^2/k$ and $2Re A \cdot B^*$ are present, but $k|B|^2$ is not. It is not possible to obtain the latter just from a knowledge of the unpolarized hadronic cross section; it is necessary to also know the phase of T^h, as well as magnetic moments.

VALIDITY OF THE EXPANSION

It is well known that the leading term in the expansion of σ^γ, $O(1/k)$, is just the same as that in classical radiation theory, once radiated intensity is converted to number of photons [4]. It is seen above that the second term, $O(1)$, does not contain any additional factors of $\hbar$ and therefore is also classical.

The classical criterion for just keeping the leading term is

$$\omega\tau = \frac{kb}{v} \ll 1, \tag{10}$$

where τ is the duration of the collision, b is the range of the force, and v is the particle velocity. From Eqs. (3), (6), or (7) it is seen that the ratio, R, of the second term in the expansion to the first goes like

$$R \approx \frac{k}{F^h} \cdot \frac{\partial F^h}{\partial p_j},$$

where

$$F^h \equiv \sum_{spins} |T^h|^2. \tag{11}$$

If this ratio is not small one must wonder about the validity of the expansion. Indeed, since $ReA \cdot B^*$ can have either sign the absence of the terms $k|B|^2$ could even lead to a negative cross section.

On the other hand, *R being small does not mean that the second term is negligible.* Although the percentage that the second term contributes vanishes as $k \to 0$, its absolute value is non-zero there and may be comparable with the size of the possible discrepancy between theory and experiment that is being discussed at this workshop.

If $d\sigma^h$ is known at only one kinematic point then only the leading term of $d\sigma^\gamma$ can be calculated. But if $d\sigma^h$ is known at enough points to be able to compute the required derivatives in Eq. (6) or (7), the second term of $d\sigma^\gamma$ can also be calculated. A theoretical model of $d\sigma^h$ can, of course, also be used.

If the only scale in the problem is the overall size of the system then the ratio in Eq. (11) just gives back Eq. (10). But R can be larger either if σ^h is suppressed or $\partial\sigma^h/\partial p_j$ is enhanced. The former situation can be the result of a symmetry [5], or else an accidental cancellation of amplitudes. The latter can arise from a resonance, which we now consider.

A. Radiation from Resonance Decay

We are interested in a resonance that decays into one or more charged particles, which then radiate. If none of the products of a particular decay branch is strongly interacting, as in $\rho \to \mu^+\mu^-$ or $\eta \to e^+e^-\gamma$, then the momenta of the particles in the final state carry the information about the resonance and $\partial\sigma^h/\partial p_j$ will be large (when j refers to one of the charged particles from the decay). For an hadronic decay, such as $\rho^+ \to \pi^+\pi^\circ$, this will be true only if final state interactions do not significantly smear out the momenta, i.e., move the particles off resonance.

As an example suppose a resonance created in the collision decays into two particles with momenta p_a and p_b. Choosing $s = (p_a + p_b)^2$ to be one of the scalar variables for describing the hadronic T-matrix, F^h in Eq. (11) will contain a factor $[(s-m^2)^2+(m\Gamma)^2]^{-1}$, where m and Γ are the mass and width of the resonance (if $m \gg \Gamma$). Assuming that this factor represents the dominant dependence of F^h on p_a and p_b,

$$\frac{\partial F^h}{\partial p_a} = \frac{\partial F^h}{\partial p_b} \simeq -2(p_a+p_b)\frac{2(s-m^2)}{[(s-m^2)^2+(m\Gamma)^2]}F^h. \qquad (12)$$

From Eqs. (9) and (12) it is seen that on the wings of the resonance the second term in the square bracket in Eq. (6) is approximately given by

$$(Q_aD_a+Q_bD_b)F^h \approx \pm\frac{2}{m\Gamma}\left\{Q_a\left(\frac{p_a}{k\cdot p_a}k\cdot(p_a+p_b)-(p_a+p_b)\right)+(a\leftrightarrow b)\right\}F^h, \qquad (13)$$

where the $+(-)$ sign refers to the low (high) side of the resonance. Evaluating this expression in the $a-b$ center of mass system and going over to the velocity notation of Eq. (7), the second term in that square bracket becomes

$$k\left(Q_a\boldsymbol{n}\times\boldsymbol{D}_a+Q_b\boldsymbol{n}\times\boldsymbol{D}_b\right)F^h \approx \pm\frac{2k_c}{\Gamma}\left(\frac{Q_a\boldsymbol{n}_c\times\boldsymbol{v}_a}{1-\boldsymbol{n}_c\cdot\boldsymbol{v}_a}+\frac{Q_b\boldsymbol{n}_c\times\boldsymbol{v}_b}{1-\boldsymbol{n}_c\cdot\boldsymbol{v}_b}\right)F^h, \qquad (14)$$

where $\boldsymbol{n}_c = \boldsymbol{k}_c/k_c$ is the direction of the photon in the $a-b$ center of mass system.

Equation (14) should be compared with the leading term in the square bracket in Eq. (7), which is

$$\sum_j \frac{\pm Q_j\boldsymbol{n}_c\times\boldsymbol{v}_j}{1-\boldsymbol{n}_c\cdot\boldsymbol{v}_j}F^h. \qquad (15)$$

On average, one would expect Eq. (14) to be smaller than Eq. (15) by approximately the factor $2k_c/\Gamma$, but for some kinematics there could be significant departures up or down.

For a resonance with a very small width, such as the η-meson, the factor $2k_c/\Gamma$ is very large for any photon energy of experimental interest, and hence just using the leading term of the soft photon theorem would be completely unreliable. But even for a resonance with a large width, such as the ρ-meson, the second term in the expansion can be very signficant. In the experiment of Chliapnikov *et al.*[6], for example, the photons were required to have a laboratory momentum greater than $m_{\pi^0}/2$, which is comparable with $\Gamma_\rho/2$. It should be expected, therefore, that if radiation from ρ decay products is important then the bremstrahlung cross section will not be accurately given by the $O(1/k)$ term of the soft photon theorem. Conditions are better in the experiment of Goshow *et al.*[7] where the photon momenta are restricted to smaller values, but the $O(1)$ term in the bremsstrahlung cross section is still not negligible.

We note that there are formulas for σ^γ that differ from Eqs. (6) or (7) by $O(k)$, which may be better suited to a system with resonances [5], but they require knowledge of the phase of T^h.

B. Higher Order Terms

It is apparent that there is no universal rule for deciding whether or not a photon is "soft". Part of the answer is contained in the second term in the expansion of $d\sigma^\gamma$ in powers of k, and we have just seen that a resonance can make this term large even for values of k such that $kb \ll 1$. But this is still not the whole story because the term $O(k)$ in the cross section contains brand new information, not just higher derivations of the hadronic cross section.

Recall that the term B^μ in the bremsstrahlung amplitude, Eq. (1), gets a contribution from $\partial T^h/\partial p_j$ and also from the magnetic moments of the particles, but that the latter information disappears from $ReA \cdot B^*$ in the unpolarized bremsstrahlung cross section. It does show up, however, in $B \cdot B^*$, which is just one part of the term $O(k)$ as shown in Eq. (2). It was mentioned above that to be able to compute $B \cdot B^*$, even for spinless particles, requires a knowledge of the phase of the hadronic T-matrix, not just $|T^h|$. For the nucleon-nucleon system below the pion production threshold this information is known from a complete set of elastic scattering experiments, and the $B \cdot B^*$ contribution to bremsstrahlung has been calculated as well as the two leading terms. We show the results of some calculations that were kindly provided by H. Fearing [8] on very short notice in time for this workshop.

For $pp \to pp\gamma$ the final state is completely specified by five variables, which are here taken to be the laboratory polar and azimuthal angles of the two final protons, (θ_3, ϕ_3) and (θ_4, ϕ_4), and the polar angle of the photon, θ_k. In the experiment of Kitching *et al.* [9] all particles are coplanar with the two protons emerging on opposite sides of the beam, i.e., $\phi_3 = 0°$ and $\phi_4 = 180°$. For the non-radiative process there is a unique value for $\theta_3 + \theta_4$ of approximately 87° at a laboratory kinetic energy $T_L = 280$ MeV — the sum is 90° in the non-relativistic limit — so the extent to which $\theta_3 + \theta_4$ departs from 87° is a measure of how hard the bremsstrahlung photons are.

Figures 3 and 4 show the leading contributions to $d\sigma^\gamma$ for two cases. The first has $\theta_3 = \theta_4 = 40°$, for which the maximum photon energy (in the center of mass system) is $k_c = 42$ MeV. Based upon Eq. (10) one would expect that these photons are moderately soft since $kb/v \lesssim 0.4$. No data were taken here because soft photons are not considered interesting for learning about the nucleon-nucleon interaction. The second case has $\theta_3 = 12°$ and $\theta_4 = 14°$, for which the maximum value of k_c is 134 MeV. Since this is almost the entire available energy in the center of mass system, this radiation strongly perturbs the hadronic kinematics, and hence these photons are "hard".

Examination of Fig. 3 (the "soft" case) reveals some interesting things. The lowest curve, which is just the $|A|^2/k$ term of σ^γ [10], vanishes at $\theta_k = 0°$ and 180° because of the symmetric geometry. [This can be seen directly from Eq. (7).] The near vanishing of the $|A|^2/k$ term at $\theta_k \approx 70°$ seems to be an accidental cancellation.

Adding on the term $2ReA \cdot B^*$ to $d\sigma^\gamma$ makes only a small change at any value of θ_k, but the $k|B|^2$ portion of the $O(k)$ term is quite substantial, presumably due to radiation from the large magnetic moments of the protons. A dynamical calculation of σ^γ, using a potential energy (in the Schrödinger equation) that fits σ^h well over a wide energy range, is within 10% of the soft photon calculation that includes the $k|B|^2$ term [8].

For the hard photon case shown in Fig. 4, not surprisingly the $O(1/k)$ term and the

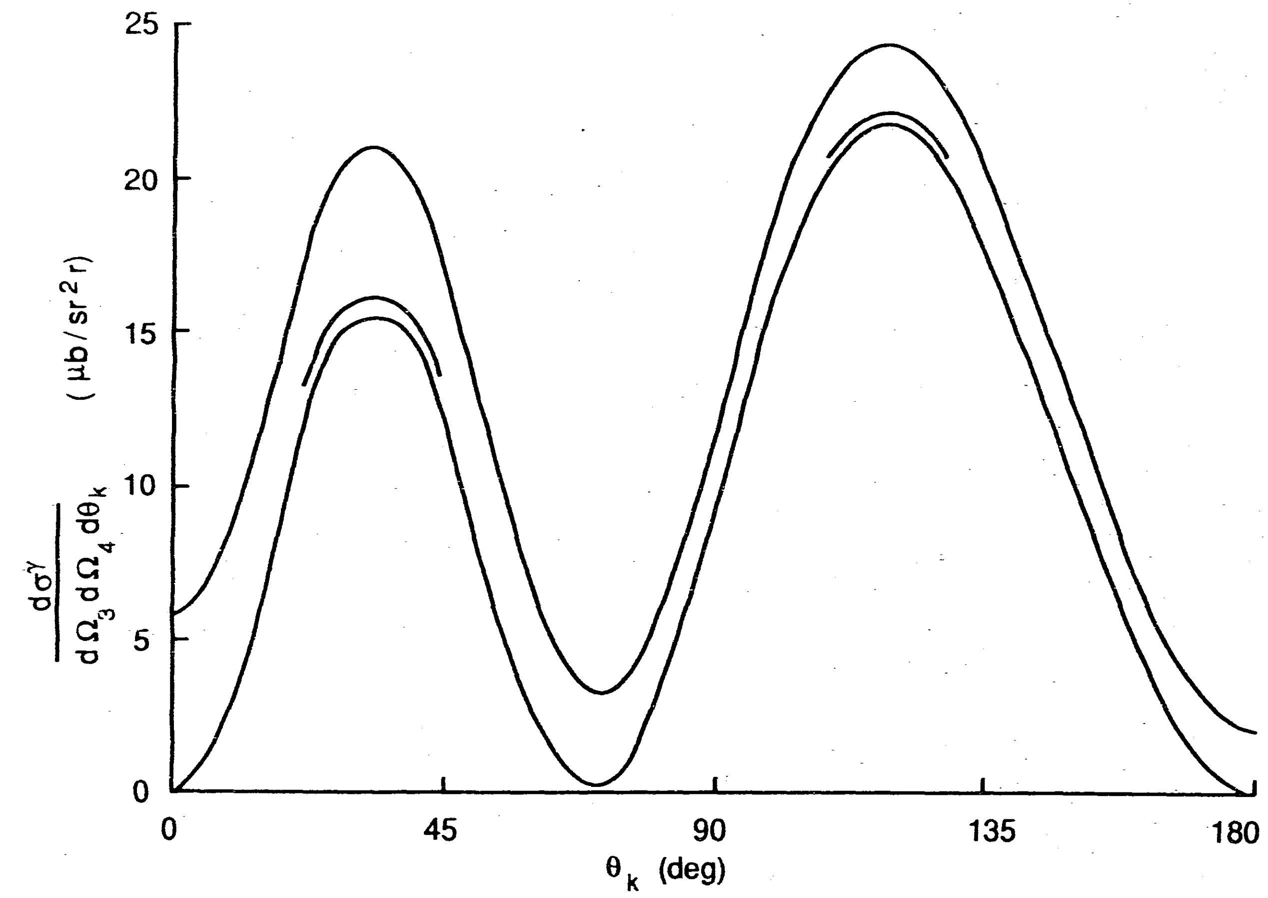

Fig. 3. The unpolarized proton-proton bremsstrahlung cross section vs. the polar angle of the photon, at a laboratory kinetic energy $T_L = 280$ MeV. The two protons emerge on opposite sides of the beam at polar angles $\theta_3 = \theta_4 = 40°$, with all momenta coplanar. The lower curve is the $O(1/k)$ term of $d\sigma^\gamma$ evaluated at a particular kinematic point [10]. The incomplete curve just above (which merges with the lower curve) is the result of adding on the $O(1)$ term, which is small. The upper curve results from also adding on the $k|B|^2$ portion of the $O(k)$ term. The maximum photon energy (at $\theta_k = 0°$) in the center of mass system is $k_c = 42$ MeV. Theoretical calculations by H. Fearing [8].

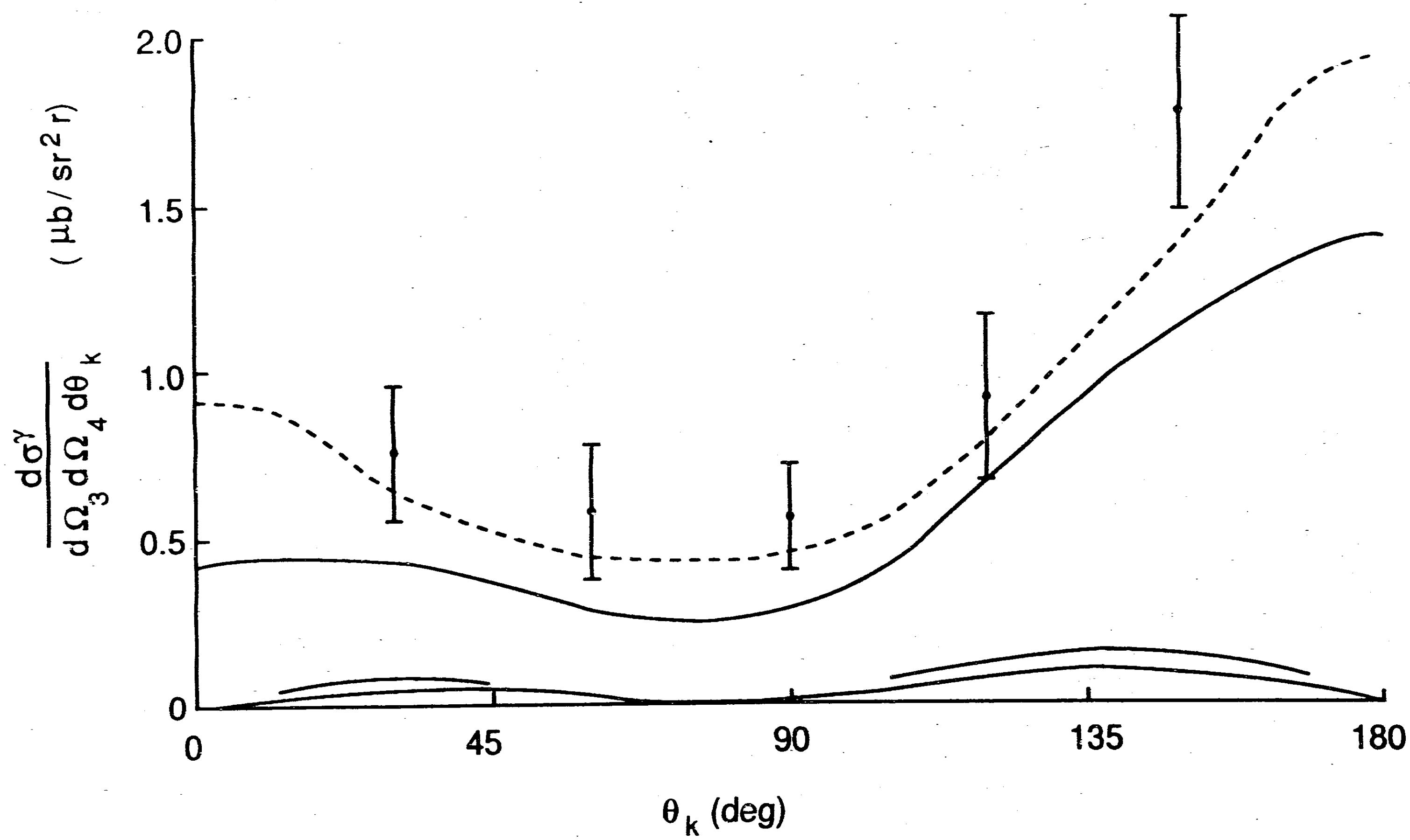

Fig. 4. The same as Fig. 3 except that the two protons emerge at polar angles $\theta_3 = 12°$ and $\theta_4 = 14°$; the maximum photon energy is $k_c = 134$ MeV. The three solid curves [8] have the same meaning as in Fig. 3. The dashed curve is the result of a dynamical calculation using the Bonn Potential [8]. The data points are a sample of those from Reference 9, averaged over two nearby kinematic points.

$O(1)$ term in the expansion of σ^γ do not come anywhere near the experimental data [9]. (Note that the cross section scale in Fig. 4 is down from that in Fig. 3 by a factor of 10; this is a reflection of the larger magnitude of k.) Addition of the substantial $kB \cdot B^*$ term begins to resemble the data, but does not agree with it as well as a dynamical calculation using a potential [8].

SUMMARY

In trying to decide whether or not a given photon energy is low enough to use an approximation to the bremmstrahlung cross section that is based on the soft photon theorem, several issues have been discussed. The first is the size of the parameter kb/v, which arises already in classical radiation theory, where b is the range of the interaction. Even with a small value of this parameter it is clear that if the hadronic cross section varies significantly when the momentum of one of the particles changes by $O(k)$, then this variation must be taken into account. This is just what the $O(1)$ term in the expansion of σ^γ in powers of k does, maintaining gauge invariance in the process. This term becomes especially important if the $O(1/k)$ term is suppressed, or if a resonance is an important part of the hadronic cross section. To evaluate it one only needs to calculate derivatives of σ^h with respect to a chosen set of scalar variables, *and this should certainly be done.* The prescription for doing this is spelled out in Eqs. (6) - (9).

Insofar as higher order terms in the expansion of σ^γ are concerned, the $k|B|^2$ portion of the $O(k)$ term can only be calculated if one knows the phase of T^h as well as its magnitude. Magnetic moments also contribute here, whereas they cancel out of the term $2ReA \cdot B^*$ in the unpolarized cross section. For the proton-proton bremsstrahlung calculations shown in Figs. 3 and 4, $k|B|^2$ is larger than $2ReA \cdot B^*$. This is presumably due to the facts that (i) electric dipole radiation is suppressed, and (ii) the magnetic moments are large. We would not expect this to happen in most systems.

Going from the hard photon case shown in Fig. 4 to the moderately soft case in Fig. 3 there is a considerable improvement in the accuracy of the leading term ($|A|^2/k$) in the expansion of σ^γ in powers of k. But even with $kb/v \lesssim 0.4$, there are kinematic points [see Fig. 3] where $|A|^2/k$ is suppressed and consequently provides a very poor approximation to the cross section. Near the peak in $d\sigma^\gamma$ at $\theta_k \approx 30°$, corrections to the leading term are approximately 30%.

ACKNOWLEDGMENTS

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[10] The hadronic cross section was evaluated at the average of the initial and final values of s in the bremsstrahlung process, and the average of the two squared momentum transfers, t.

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