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LIGHT-FRONTS APPROACH TO ELECTRON-POSITRON PAIR PRODUCTION IN ULTRARELATIVISTIC HEAVY-ION COLLISIONS

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We solve, in an ultrarelativistic limit, the time-dependent Dirac equation describing electron-positron pair production in peripheral relativistic heavy-ion collisions using light-front variables and a light-fronts representation, obtaining nonperturbative results for the free pair-creation amplitudes in the collider frame. Our result reproduces the result of second-order perturbation theory in the small charge limit while nonperturbative effects arise for realistic charges of the ions.

1 Introduction

In heavy-ion collisions at energies near the Coulomb barrier, quasi-bound molecular states are formed with binding energies which dive into the negative-energy continuum, resulting in a resonance which decays into an electron-positron pair [1]. In contrast, for *ultrarelativistic* heavy-ion collisions at peripheral impact parameters, the ions execute straight-line trajectories and reside near each other for only a very short time. The high charge of the individual ions and the strong Lorentz contraction combine to produce fields sufficient for electromagnetic pair production through a qualitatively different process. Large cross sections for electromagnetic pair production in these collisions were theoretically predicted [2] and experimentally observed [3,4].

In lowest-order perturbation theory, the amplitudes for pair production in heavy-ion collisions are calculated from two-photon exchange diagrams [5]. The quantum field theoretical treatment of this process was reduced to a classical-field approach [6]. Experimental observations of free pair production in the energy range around ten GeV per nucleon (in the collider reference frame) are mostly in good agreement with second-order perturbation theory [3]. For lower energies of a few GeV per nucleon (collider frame), experimental results for free and bound-free pair production show deviations from the predictions obtained from two-photon exchange diagrams [4]. This is likely due to two-center Coulomb effects [7].

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In the near future, larger ultrarelativistic energies above one hundred GeV per nucleon (collider frame) will be available. New nonperturbative effects may become important at colliding-beam accelerators such as the Relativistic Heavy-ion Collider (RHIC) at Brookhaven and possibly the Large Hadron Collider (LHC) at CERN. These nonperturbative effects (or the absence thereof) are the subject of our present work. One of the interesting aspects of this physics is the opportunity to study nonperturbative QED over a continuous range of charges and collision energies. For heavy, fully-stripped ions, the effective coupling constant $(Z\alpha)$ is not small $(Z\alpha \sim 0.6)$.

In a nonperturbative treatment, starting from the QED Lagrange density operator, the Euler-Lagrange equations of motion for the quantum fields are equivalent, under physical assumptions, to the one-particle Dirac equation interacting with classical, electromagnetic fields [9]. Calculations of probabilities and correlations can then be reduced to solving the two-center time-dependent Dirac equation, which describes the dynamics of an electron in the classical field of two relativistically moving charges. In this paper, the Dirac equation is expressed in a light-fronts representation, and hence in a simplified form appropriate to ultrarelativistic energies. In this limit, an exact solution for the Dirac equation is obtained describing free electron-positron pair-production in the collider frame for small impact parameters [10].

2 An ultrarelativistic limit for the time-dependent Dirac equation

Consider a collision between two ions with charges Z_A and Z_B and velocities $\beta\hat{z}$ and $-\beta\hat{z}$, respectively, moving parallel to each other at an impact parameter of $2\vec{b}$. An external-field approach to the influence of these ions on the vacuum is appropriate for peripheral impact parameters (i.e. $b > R_{\mathrm{NUC}}$), heavy ions, and ultrarelativistic energies. The two-center Dirac equation for an electron in the field of these ions is given by:

$$i\frac{\partial}{\partial t}|\Phi(\vec{r},t)\rangle = \left[\hat{H}_0 + \hat{H}_A(t) + \hat{H}_B(t)\right]|\Phi(\vec{r},t)\rangle \,, \tag{1}$$

where $|\Phi(\vec{r},t)\rangle$ is the Dirac spinor wave function of the electron, $\hat{H}_0$ is the free Dirac Hamiltonian and $\hat{H}_A(t)$ and $\hat{H}_B(t)$ are each the interaction with one ion,

$$\hat{H}_{A/B} \equiv \frac{-Z_{A/B}\alpha(\mathrm{I}_4 \mp \beta\breve{\alpha}_z)}{\sqrt{(\vec{r}_\perp \mp \vec{b})^2/\gamma^2 + (z \mp \beta t)^2}} \,. \tag{2}$$

We are using natural units, and applying the conventional notation; $\beta \equiv v/c$, $\gamma \equiv 1/\sqrt{1-\beta^2}$. α is the fine-structure constant, $\breve{\alpha}$ and $\breve{\gamma}^\mu$ are Dirac matrices in the Dirac representation; $\check{\sigma}$ are the Pauli matrices; and I_2, 0_2, I_4, and 0_4 are the 2-dimensional and 4-dimensional unit and zero matrices. We would like to consider Eq. 1 in the ultrarelativistic limit in which $\beta \rightarrow 1$, and $\gamma \gg b, r_\perp$.

Figure 1 shows the familiar form of the interaction energy of an electron with a single component of one ion. The interaction is large near the z-coordinate of the ion with strength proportional to γ, and the width of the interaction is inversely proportional to γ. Note, however, that the long-range "Coulomb tail" is largely independent of γ for large γ.

The interaction of the electron with distant ions should be included in the definition of the asymptotic channel wavefunctions. Here, we consider free-pair production, and are therefore interested in asymptotic channels where $|t| \rightarrow \infty$ and $z \pm t \neq 0$. In this limit, the asymptotic-time Dirac equation is written as

$$i\frac{\partial}{\partial t}|\Psi^\infty(\vec{r},t)\rangle = \left[\hat{H}_0 + \hat{H}_A^\infty(t) + \hat{H}_B^\infty(t)\right]|\Psi^\infty(\vec{r},t)\rangle\ , \tag{3}$$

where $|\Psi^\infty(\vec{r},t)\rangle$ is the asymptotic channel solution, and $\hat{H}_A^\infty(t)$ and $\hat{H}_B^\infty(t)$ are each the interaction of the electron with one distant ion,

$$\hat{H}_{A/B}^\infty \equiv \frac{-Z_{A/B}\,\alpha(I_4 \mp \check{\alpha}_z)}{\sqrt{b^2/\gamma^2 + (z \mp t)^2}}\ . \tag{4}$$

Solutions to Eq. 3 are related to the Dirac plane-waves, $\{|\phi_p(\vec{r},t)\rangle\}$, through multiplication by a space-time dependent phase factor [2,11]

$$|\Phi_p^\infty(\vec{r},t)\rangle = e^{-i\chi(z,t)}|\phi_p(\vec{r},t)\rangle\ , \tag{5}$$

where

$$\begin{aligned}\chi(z,t) \equiv\ & Z_A\alpha \ln\left[-\gamma(t-z) + \sqrt{b^2+\gamma^2(t-z)^2}\right] \\ &+ Z_B\alpha \ln\left[+\gamma(t+z) + \sqrt{b^2+\gamma^2(t+z)^2}\right]\ .\end{aligned} \tag{6}$$

These solutions are the asymptotic channel functions for the collision.

The *exact* transition amplitudes $A_k^{(j)}$ (S-matrix) are then defined as

$$A_k^{(j)} \equiv \lim_{t_f\rightarrow\infty}\langle\Phi_k^\infty(\vec{r},t_f)|\Psi_j(\vec{r},t_f)\rangle\ , \tag{7}$$

where $|\Psi_j(\vec{r},t_f)\rangle$ is the solution to the Dirac equation with the initial condition

$$\lim_{t_i\rightarrow-\infty}|\Psi_j(\vec{r},t_i)\rangle \equiv |\Phi_j^\infty(\vec{r},t_i)\rangle\ . \tag{8}$$

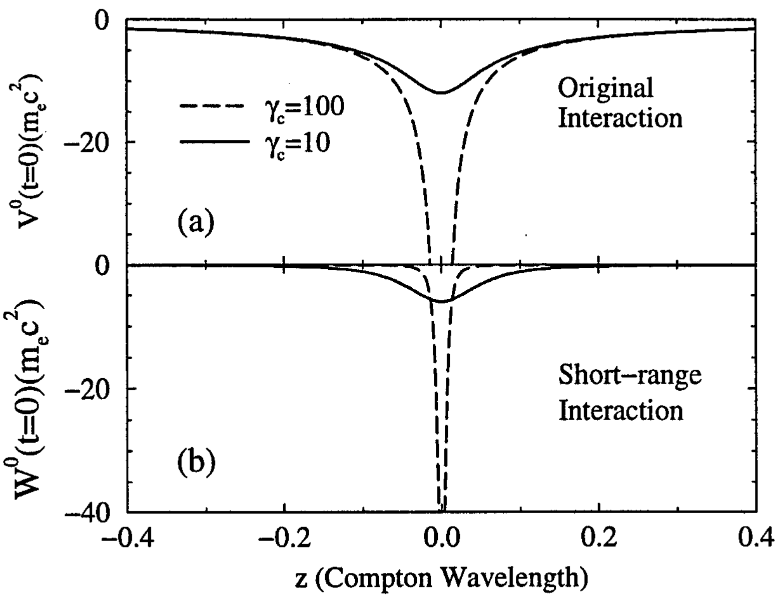

Figure 1: Plotted is (a) the scalar component of the original interaction, V^0, of the electron with a single ion, and (b) the short-range representation, W^0, for the same component of the interaction. This is presented for two different energies $\gamma = 10$ (CERN-SPS energies), and $\gamma = 100$ (RHIC energies), as a function of a narrow range of the z-coordinate for $t = 0$, $\vec{b} = (1,0)$, and $\vec{r}_\perp = (2,0)$.

Using the time-evolution operator, $\hat{U}(t_f, t_i)$, the transition amplitudes may be written in a symmetric form as

$$A_k^{(j)} \equiv \lim_{\substack{t_f \to +\infty \\ t_i \to -\infty}} \langle \Phi_k^{\infty}(\vec{r}, t_f) | \hat{U}(t_f, t_i) | \Psi_j^{\infty}(\vec{r}, t_i) \rangle \ . \tag{9}$$

It is useful to define a new representation for the wavefunction through multiplication by the inverse of the phase factor, $e^{-i\chi(z,t)}$, used in describing the asymptotic solutions, i.e.

$$|\Psi^S(\vec{r}, t)\rangle \equiv e^{+i\chi(z,t)} |\Psi(\vec{r}, t)\rangle \ . \tag{10}$$

We may use this representation in expressing the transition amplitudes, i.e.

$$A_k^{(j)} \equiv \lim_{\substack{t_f \to +\infty \\ t_i \to -\infty}} \langle \phi_k(\vec{r}, t_f) | \hat{U}^S(t_f, t_i) | \psi_j(\vec{r}, t_i) \rangle \ , \tag{11}$$

where $\hat{U}^S(t_f, t_i) \equiv e^{+i\chi(z,t_f)} \hat{U}(t_f, t_i) e^{-i\chi(z,t_i)}$ is the time-evolution operator in the new representation. Equation 11 has the form of a transition amplitude between two asymptotic plane-wave states, i.e. states undistorted by interaction with the ions. Writing the Dirac equation in the new representation,

$$i\frac{\partial}{\partial t}|\Psi(\vec{r},t)\rangle = \left[\hat{H}_0 + \hat{W}_A(t) + \hat{W}_B(t)\right]|\Psi(\vec{r},t)\rangle \ , \qquad (12)$$

where $\hat{W}_{A/B}(t) \equiv \hat{H}_{A/B}(t) - \hat{H}^{\infty}_{A/B}(t)$, one obtains a time-dependent equation of motion with short-range interactions in the beam direction. The asymptotic interaction of the electron with the two ions in the high-energy limit has been included exactly in the phase of the short-range representation wavefunctions. Figure 1 demonstrates the short-range character of the interaction $\hat{W}_{A/B}$.

The short-range Dirac equation, Eq. (12), has a simple ultrarelativistic limit, e.g. $\hat{W}_{A/B}$ has a sharp, delta-function dependence on $(t \pm z)$ [8,10], i.e.

$$\hat{W}_{A/B} \rightarrow (\mathrm{I}_4 \mp \breve{\alpha}_z)\alpha Z_{A/B}\delta(t \mp z)\ln\left[\frac{(\vec{r}_\perp \mp \vec{b})^2}{b^2}\right] \ . \qquad (13)$$

The interaction has zero range in the projectile's direction and a logarithmic behavior in the transverse direction, similar to the potential of a line of charge.

3 The sharp Dirac equation in the light-fronts representation

In this section, Eq. (12), with the limiting form of the interaction in Eq. (13), will be further simplified by changing into light-front variables and by introducing a new representation for the Dirac spinors, the *light-fronts representation.* In terms of light-front variables, space-time and energy-momentum are described by the 4-vectors $(\vec{r}_\perp, \tau_+, \tau_-)$ and $(\vec{p}_\perp, p_+, p_-)$, where $\tau_\pm \equiv (t \pm z)/2$, $p_\pm \equiv E_p \pm p_z$, and $p_+p_- = 1 + p_\perp^2$. The interaction is block-diagonalized by introducing the *light-fronts representation* for the Dirac matrices,

$$\gamma^\mu_{\mathrm{light-fronts}} = \Lambda\gamma^\mu_{\mathrm{Dirac}}\Lambda^\dagger, \ \ \Lambda \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} \mathrm{I}_2 & \hat{\sigma}_z \\ \mathrm{I}_2 & -\hat{\sigma}_z \end{pmatrix} \ , \qquad (14)$$

$$\Lambda\breve{\alpha}_z\Lambda^\dagger = \begin{pmatrix} \mathrm{I}_2 & 0_2 \\ 0_2 & -\mathrm{I}_2 \end{pmatrix} \ , \qquad (15)$$

$$\Lambda\breve{\vec{\alpha}}_\perp\Lambda^\dagger = i\begin{pmatrix} 0_2 & -\breve{\vec{\omega}} \\ \breve{\vec{\omega}} & 0_2 \end{pmatrix}, \ \ \breve{\vec{\omega}} \equiv (-\breve{\sigma}_y, \breve{\sigma}_x). \qquad (16)$$

With this notation, the short-range, two-center Dirac equation in the light-fronts representation in the ultrarelativistic limit is

$$\begin{pmatrix} i\partial_{\tau_+}|G_+\rangle \\ i\partial_{\tau_-}|G_-\rangle \end{pmatrix} = \begin{pmatrix} \delta(\tau_+)B(\vec{r}_\perp) & \hat{h}_0 \\ \hat{h}_0^\dagger & \delta(\tau_-)A(\vec{r}_\perp) \end{pmatrix}\begin{pmatrix} |G_+\rangle \\ |G_-\rangle \end{pmatrix}, \qquad (17)$$

where $|G_+\rangle$ and $|G_-\rangle$ are the upper and lower bi-spinor components of the Dirac wave function in the light-fronts representation, and $\hat{h}_0 \equiv \mathrm{I}_2 - i\breve{\omega}\cdot\hat{\vec{p}}_\perp$,

$$A(\vec{r}_\perp,\vec{b}) \equiv Z_A\alpha\ln\left[\frac{(\vec{r}_\perp-\vec{b})^2}{b^2}\right],\; B(\vec{r}_\perp,\vec{b}) \equiv Z_B\alpha\ln\left[\frac{(\vec{r}_\perp+\vec{b})^2}{b^2}\right]. \qquad (18)$$

The upper and lower bi-spinors are coupled by the free Hamiltonian. Each interacts directly with the external field of one ion and feels the field of the other ion through its coupling to the other bi-spinor.

Equation (17) has no discontinuities in the transverse direction. It is therefore useful to Fourier transform its solution with respect to $\vec{r}_\perp$. Two mixed bi-spinors wave-functions, $|g_\pm(\vec{q}_\perp;\tau_+,\tau_-)\rangle$, are then defined by

$$|G_\pm(\vec{r}_\perp,\tau_+,\tau_-)\rangle \equiv \int d\vec{q}_\perp e^{i\vec{r}_\perp\cdot\vec{q}_\perp}|g_\pm(\vec{q}_\perp;\tau_+,\tau_-)\rangle. \qquad (19)$$

$|g_+\rangle$ and $|g_-\rangle$, like $|G_+\rangle$ and $|G_-\rangle$, are coupled by the free Hamiltonian.

Off the light fronts, i.e. for $\tau_+ \neq 0$ and $\tau_- \neq 0$, the wave function satisfies the free Dirac equation and Eq. (17) reduces to two coupled equations for the mixed bi-spinors $|g_\pm(\vec{q}_\perp;\tau_+,\tau_-)\rangle$.

$$i\frac{\partial}{\partial\tau_+}|g_+\rangle = (\mathrm{I}_2 - i\breve{\omega}\cdot\vec{q}_\perp)|g_-\rangle,\quad i\frac{\partial}{\partial\tau_-}|g_-\rangle = (\mathrm{I}_2 + i\breve{\omega}\cdot\vec{q}_\perp)|g_+\rangle. \qquad (20)$$

A solution to Eqs. (20) is given by the Dirac plane waves, which in the light-fronts representation are

$$\begin{pmatrix}|F_+^p\rangle\\|F_-^p\rangle\end{pmatrix} \equiv \Lambda|\chi_p(\vec{r},t)\rangle,\;\; |F_\pm^p\rangle \equiv \int d\vec{q}_\perp e^{i\vec{r}_\perp\cdot\vec{q}_\perp}|f_\pm^p(\vec{q}_\perp;\tau_+,\tau_-)\rangle, \qquad (21)$$

$$|f_\pm^p(\vec{q}_\perp;\tau_+,\tau_-)\rangle = \delta(\vec{q}_\perp-\vec{p}_\perp)e^{-i(\tau_-p_++\tau_+p_-)}|\Gamma_\pm^p\rangle, \qquad (22)$$

where the bi-spinors, $|\Gamma_\pm^p\rangle$, satisfy the simple relation $|\Gamma_-^p\rangle = \frac{\mathrm{I}_2-i\breve{\omega}\cdot\vec{p}_\perp}{p_+}|\Gamma_+^p\rangle$.

It is standard procedure in wave-mechanics to form piece-wise solutions by satisfying continuity relations at the boundaries between free regions. A δ-function singular interaction at a light front results in a discontinuity in the electron wave function which is given by a space-dependent phase shift [8,10]. For our case of Eq. (17), the discontinuity is given by

$$|G_+(\tau_+=0^+)\rangle = e^{-iB(\vec{r}_\perp,\vec{b})}|G_+(\tau_+=0^-)\rangle, \qquad (23)$$

$$|G_-(\tau_-=0^+)\rangle = e^{-iA(\vec{r}_\perp,\vec{b})}|G_-(\tau_-=0^-)\rangle. \qquad (24)$$

Due to this phase-shift, the transverse momentum is not conserved and the Fourier components of Eq. (19) are mixed when the light fronts are crossed,

$$|g_\pm(\vec{q}_\perp;\tau_\pm = 0^+)\rangle = \int d\vec{p}_\perp Q_{Z_{B/A}}(\vec{p}_\perp - \vec{q}_\perp, \mp\vec{b})|g_\pm(\vec{p}_\perp;\tau_\pm = 0^-)\rangle, \quad (25)$$

where the transverse momentum transfer distribution is given by

$$Q_Z(\vec{\kappa},\vec{b}) \equiv \frac{1}{(2\pi)^2}\int d\vec{r}_\perp\, e^{i\vec{r}_\perp\cdot\vec{\kappa}}\left[\frac{(\vec{r}_\perp - \vec{b})^2}{b^2}\right]^{-i\alpha Z}. \quad (26)$$

Continuity is recovered in the limit $Z \to 0$, as $Q_Z(\vec{\kappa},\vec{b}) \to \delta(\vec{\kappa})$. The properties of the distribution $Q_Z(\vec{\kappa},\vec{b})$ for finite charge are considered elsewhere [10].

4 A piece-wise solution to the sharp Dirac equation

The trajectories of the ions cut space-time along the light fronts into four regions. A piece-wise solution is defined in each of these regions by $|g_\pm(\vec{q}_\perp;\tau_+,\tau_-)\rangle = |g_\pm^{(i)}(\vec{q}_\perp;\tau_+,\tau_-)\rangle$, where (i)= I for $\tau_+ < 0$ and $\tau_- < 0$, (i)= II for $\tau_+ > 0$ and $\tau_- < 0$, (i)= III for $\tau_+ < 0$ and $\tau_- > 0$, and (i)= IV for $\tau_+ > 0$ and $\tau_- > 0$. In each region, the wave function is continuous and solves the local free Dirac equation. Consider the initial condition of a single plane wave with the quantum numbers j, expressed in light-front variables as $j = \{\vec{j}_\perp, j_+, j_-, s_j\}$. The continuity off the light fronts gives the solution in region I,

$$|g_\pm^{\mathrm{I}}(\vec{q}_\perp)\rangle = \delta(\vec{j}_\perp - \vec{q}_\perp)e^{-i(\tau_- j_+ + \tau_+ j_-)}|\Gamma_\pm^j\rangle\,. \quad (27)$$

The solution in regions II and III is obtained by applying Eqs. (25) for the discontinuities across $\tau_\pm = 0$ and then solving the coupled equations (20) inside each of the intermediate space-time regions. We cross from regions II and III into region IV by applying Eq. (25) again for the discontinuities across $\tau_\pm = 0$ to obtain the solution on the hyper-surfaces adjacent to the light fronts,

$$\begin{aligned}|g_-^{\mathrm{IV}}(\vec{k}_\perp;\tau_- = 0)\rangle &= \int d\vec{q}_\perp \exp\left[-i\tau_+\left(\frac{1+q_\perp^2}{j_+}\right)\right] Q_{Z_A}(\vec{q}_\perp - \vec{k}_\perp, \vec{b}) \\ &\times Q_{Z_B}(\vec{j}_\perp - \vec{q}_\perp, -\vec{b})\left(\frac{1 + i\vec{\omega}\cdot\vec{q}_\perp}{j_+}\right)|\Gamma_+^j\rangle\,, \quad (28)\end{aligned}$$

$$\begin{aligned}|g_+^{\mathrm{IV}}(\vec{k}_\perp;\tau_+ = 0)\rangle &= \int d\vec{p}_\perp \exp\left[-i\tau_-\left(\frac{1+p_\perp^2}{j_-}\right)\right] Q_{Z_B}(\vec{p}_\perp - \vec{k}_\perp, -\vec{b}) \\ &\times Q_{Z_A}(\vec{j}_\perp - \vec{p}_\perp, \vec{b})\left(\frac{1 - i\vec{\omega}\cdot\vec{p}_\perp}{j_-}\right)|\Gamma_-^j\rangle\,. \quad (29)\end{aligned}$$

The integrand of the transition amplitudes, $A_k^{(j)}$, is the time-like component of a conserved 4-vector current density[10]. This *transition current*, $(\vec{J}, J_0)$, is defined by $J_0 \equiv \chi_k^\dagger \, \psi^{(j)}$, and $\vec{J} \equiv \chi_k^\dagger \vec{\alpha} \psi^{(j)}$. An equivalent form for the transition current in terms of light-fronts representation wave-functions includes

$$J_\pm \equiv J_0 \pm J_z \ = \ 2 \ F_\pm^{k\dagger} \ G_\pm \ . \tag{30}$$

We are now in position to use Gauss' theorem on the hyper-surface enclosing region IV to show that $A_k^{(j)}$ can be expressed as the hyper-surface integral over the transition current flowing into region IV[10], i.e.

$$A_k^{(j)} = 2 \int d\vec{r}_\perp \int_{0+}^{\infty} d\tau_- \ J_+(\tau_+ = 0^+) - 2 \int d\vec{r}_\perp \int_{0+}^{\infty} d\tau_+ \ J_-(\tau_- = 0^+) \ . \tag{31}$$

The amplitudes are finally obtained by integrating over $r_\perp$ and $\tau_\pm$, resulting in the *exact* solution of the sharp Dirac equation off the light fronts,

$$\begin{aligned} A_k^{(j)} = \frac{(2\pi)^3}{i\pi} \Bigg\{ & \int d\vec{p}_\perp \ Q_{Z_B}(\vec{p}_\perp - \vec{k}_\perp, -\vec{b}) \ Q_{Z_A}(\vec{j}_\perp - \vec{p}_\perp, \vec{b}) \\ & \times \frac{\langle \Gamma_+^k | 1 - i\vec{\omega} \cdot \vec{p}_\perp | \Gamma_-^j \rangle}{j_- k_+ - (1 + p_\perp^2) + i\eta(-1)^{\lambda_j}} \\ & - \int d\vec{q}_\perp \ Q_{Z_A}(\vec{q}_\perp - \vec{k}_\perp, \vec{b}) \ Q_{Z_B}(\vec{j}_\perp - \vec{q}_\perp, -\vec{b}) \\ & \times \frac{\langle \Gamma_-^k | 1 + i\vec{\omega} \cdot \vec{q}_\perp | \Gamma_+^j \rangle}{j_+ k_- - (1 + q_\perp^2) + i\eta(-1)^{\lambda_j}} \Bigg\} \ , \end{aligned} \tag{32}$$

where η is an infinitesimal small constant, which can be omitted for pair-production amplitudes corresponding to $E_j < 0$ and $E_k > 0$, i.e. $j_\pm k_\mp < 0$.

The small-charge perturbative-limit of the pair-production amplitude has been previously calculated[5]. To leading order in αZ, the amplitude is given by a sum over two Feynman diagrams, where each diagram describes a two-photon exchange process. This second-order perturbation-theory result, $\mathcal{S}_k^{2\ (j)}$, for the transition amplitude between an initial negative-energy state j and a final positive-energy state k is available in the literature[5]. In the ultrarelativistic limit, $\beta \longrightarrow 1$ and $\gamma \gg 1$, this perturbative result reduces in our notation to

$$\begin{aligned} \mathcal{S}_k^{2\ (j)} = & \int d\vec{p}_\perp \exp[-i\vec{b} \cdot (2\vec{p}_\perp - \vec{j}_\perp - \vec{k}_\perp)] \\ & \times \frac{i8 \ (\alpha Z_A)(\alpha Z_B)}{(\vec{p}_\perp - \vec{k}_\perp)^2 \ (\vec{p}_\perp - \vec{j}_\perp)^2} \frac{\langle \Gamma_+^k | 1 - i\vec{\omega} \cdot \vec{p}_\perp | \Gamma_-^j \rangle}{j_- k_+ - (1 + p_\perp^2)} \end{aligned}$$

$$- \int d\vec{q}_\perp \exp[i\vec{b} \cdot (2\vec{q}_\perp - \vec{j}_\perp - \vec{k}_\perp)]$$

$$\times \frac{i8\ (\alpha Z_A)(\alpha Z_B)}{(\vec{q}_\perp - \vec{k}_\perp)^2\ (\vec{q}_\perp - \vec{j}_\perp)^2} \frac{\langle \Gamma_-^k | 1 + i\check{\omega} \cdot \vec{q}_\perp | \Gamma_+^j \rangle}{j_+ k_- - (1 + q_\perp^2)} . \tag{33}$$

It is interesting to compare the *perturbative* result of Eq. (33) to our *nonperturbative* result of Eq. (32). In the small-charge limit of $\alpha Z \rightarrow 0$, the leading-order perturbative limit for Q_Z can be used [10],

$$Q_Z(\vec{\kappa}, \vec{b}) \rightarrow \delta(\vec{\kappa}) - \frac{i\alpha Z}{\pi} \frac{1}{\kappa^2} \exp[i\vec{b} \cdot \vec{\kappa}] . \tag{34}$$

Direct substitution shows that in this limit the nonperturbative result of Eq. (32) exactly reproduces the perturbative result of Eq. (33).

5 Conclusions and Outlook

We have obtained a useful ultrarelativistic limit for the two-center Dirac equation, which allows for an exact solution off the light fronts, i.e. away from the ions. The S-matrix amplitudes were calculated here in the ultrarelativistic limit, assuming γ to be large. No assumption was made on the value of the charge times the fine-structure constant $Z\alpha$. When taking the limit of small $Z\alpha$, we are able to show complete agreement with the ultrarelativistic limit of the expression obtained from standard second-order perturbation theory [5].

In second-order perturbation theory, pair production is described as a two-photon exchange process in which each ion exchanges one photon with a negative-energy electron. The negative energy electron is kicked off its energy shell by the first interaction and then kicked back to the energy shell by the second ion, but this time with a positive energy. The two diagrams that contribute to the amplitude differ in the time order of these photon exchanges, or 'kicks'.

In our work, a very similar physical picture of the pair production as a 'two-kicks' process is obtained in the ultrarelativistic limit within a rather different, and completely *nonperturbative* approach. The electromagnetic fields of the ions are confined to the light fronts by the extreme Lorentz contraction and by the use of the short-range representation of the Dirac equation. As the velocity of the ions approaches the velocity of light, each ion carries with it, perpendicular to its trajectory, a wall of singular electromagnetic interaction. An initial plane wave in the space between the approaching ions acquires a space-dependent phase shift as this singular-interaction wall sweeps past. A single plane wave between the ions gives a distribution of local plane waves in the space behind each ion. Had there been only one ion, no transition would be allowed, i.e. no pairs would be produced. Pairs are produced because, as the ions move past each other, the two phase-shift planes collide. As the ions move apart, the solution in the space between them is determined by the non-trivial boundary conditions on the light fronts. In the perturbative limit of a small coupling constant, the effect of the singular field perpendicular to each ion reduces to a single photon exchange. For realistic charges of the ions, the perturbative linear dependence of the amplitudes on each charge is replaces by nonperturbative, non-trivial phases in the integral representation of the S-matrix, Eq. 32.

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References

1. W. Greiner, B. Müller, and J. Rafelski, "Quantum Electrodynamics of strong fields", (Springer-Verlag, Berlin Heidelberg, 1985).
2. J. Eichler and W.E. Meyerhof, "Relativistic Atomic Collisions", (Academic Press, 1995); J. Eichler, Phys. Rep. **193**, 165 (1990).
3. C.R. Vane, et al., Phys. Rev. Lett. **69**, 1911 (1992); Phys. Rev. A **50**, 2313 (1994); Phys. Rev. A **56**, 3682 (1997).
4. A. Belkacem, et al., Phys. Rev. Lett. **71**, 1514 (1993); Phys. Rev. Lett. **73**, 2432 (1994); Phys. Rev. A **56**, 2806 (1997).
5. C. Bottcher and M.R. Strayer, Phys. Rev. D **39**, 1330 (1989).
6. J.-S. Wu, C. Bottcher, and M. R. Strayer, Phys. Lett. B **252**, 37 (1990).
7. J. Eichler, Phys. Rev. Lett. **75**, 3653 (1995); D. C. Ionescu and J. Eichler, Phys. Rev. A **54**, 4960 (1996).
8. A.J. Baltz, Phys. Rev. Lett. **78**, 1231 (1997).
9. J.C. Wells, et al., Phys. Rev. A **45** 6296 (1992).
10. B. Segev and J.C. Wells, Phys. Rev. A **57**, 1849 (1998).
11. J.C. Wells, B. Segev, and J. Eichler, in preparation.

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