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Prediction of Forces and Moments on Finned Bodies at High Angle of Attack in Transonic Flow

William L. Oberkampf

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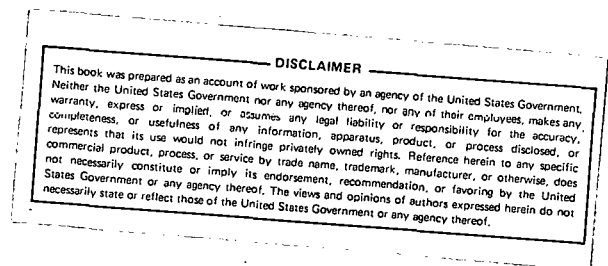
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Prediction of Forces and Moments on Finned Bodies at High Angle of Attack in Transonic Flow

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Abstract

This report describes a theoretical method for the prediction of fin forces and moments on bodies at high angle of attack in subsonic and transonic flow. The body is assumed to be a circular cylinder with cruciform fins (or wings) of arbitrary planform. The body can have an arbitrary roll (or bank) angle, and each fin can have individual control deflection. The method combines a body vortex flow model and lifting surface theory to predict the normal force distribution over each fin surface. Extensive comparisons are made between theory and experiment for various planform fins. A description of the use of the computer program that implements the method is given.

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Contents

Nomenclature	9
Summary	11
Introduction	13
Aerodynamic Analysis	13
Body Flow Field	14
Lifting Theory	17
Roll Moment	35
Normal Force and Pitch Moment	36
Side Force and Yaw Moment	37
Panel Loads	37
Results and Discussion	39
Panel Loads	39
Roll Moments	45
Control Deflections	49
Conclusions and Recommendations	52
References	53
APPENDIX A--Structure and Listing of Computer Program	55
APPENDIX B--Use of the Computer Program	71

Illustrations

Figure

1 Coordinate System and Schematic of Body Vortices	15
2 Cross-Flow Plane Flow Model	16
3 General Planform Fin	17
4 Fin Oriented Coordinates ξ, η	18
5 Normal Force Distribution Over Fin Surface for $\Lambda_{te} = \Lambda_{te} = 60^\circ$	20
6 Normal Force Distribution Over Fin Surface for $\Lambda_{te} = 60^\circ$ and $\Lambda_{te} = 20^\circ$	21
7 Normal Force Distribution Over Fin Surface for $\Lambda_{te} = \Lambda_{te} = 0^\circ$	22
8 Normal Force Distribution Over Fin Surface for $\Lambda_{te} = 60^\circ$ and $\Lambda_{te} = 0^\circ$	24
9 Normal Force Distribution Over Fin Surface for $\Lambda_{te} = 40^\circ$ and $\Lambda_{te} = -40^\circ$	25
10 Surface Normal Vector and Sign Convention for Fin Control Deflection	26
11 Fin Surface Cylindrical Coordinates	27
12 Geometry for Determining Effective Aspect Ratio	28
13 Normal Force Coefficient vs α for Rectangular Planform ($A_e = 1$)	30
14 Normal Force Coefficient vs α for Diamond Planform ($A_e = 1$)	30
15 Normal Force Coefficient vs α for Clipped Delta Planform ($A_e = 1.23$)	30
16 Normal Force Coefficient vs α for Delta Planform ($A_e = 1.46$)	31
17 Normal Force Coefficient vs α for Rectangular Planform ($A_e = 2$)	31
18 Normal Force Coefficient vs α for Arrow Planform ($A_e = 2$)	31

Illustrations (Cont)

19	Normal Force Coefficient vs α for Delta Planform ($A_e = 2.31$).....	31
20	Normal Force Coefficient vs α for Clipped Arrow Planform ($A_e = 2.61$).....	32
21	Normal Force Coefficient vs α for Trapezoidal Planform ($A_e = 3$).....	32
22	Normal Force Coefficient vs α for Delta Planform ($A_e = 4$).....	32
23	Control Interference for Pitch and Roll Control (from Ref. 24).....	32
24	Control Interference for Individual Panel Deflection	33
25	Deflection Interference Coefficient vs a/b_0	34
26	Induced Angle of Attack Due to Rolling Speed	34
27	Roll Damping Moment Coefficient vs a/b_0	35
28	Components of the Fin Normal Force	37
29	Sign Convention for Panel Normal Force, Hinge Moment, and Root Bending Moment	38
30	Fin Planforms Used for Comparison of Theory and Experiment	40
31	Windward Panel Normal Force vs Roll Angle for Configuration A ($M_\infty = 0.8$)	41
32	Windward Panel Normal Force vs Roll Angle for Configuration A ($M_\infty = 1.22$)	41
33	Leeward Panel Normal Force vs Roll Angle for Configuration A ($M_\infty = 0.8$)	41
34	Leeward Panel Normal Force vs Roll Angle for Configuration A ($M_\infty = 1.22$)	42
35	Windward Fin Root Bending Moment vs Roll Angle for Configuration A ($M_\infty = 0.8$)	42
36	Windward Fin Root Bending Moment vs Roll Angle for Configuration A ($M_\infty = 1.22$)	43
37	Leeward Fin Root Bending Moment vs Roll Angle for Configuration A ($M_\infty = 0.8$)	43
38	Leeward Fin Root Bending Moment vs Roll Angle for Configuration A ($M_\infty = 1.22$)	43
39	Windward Fin Hinge Moment vs Roll Angle for Configuration A ($M_\infty = 0.8$)	44
40	Windward Fin Hinge Moment vs Roll Angle for Configuration A ($M_\infty = 1.22$)	44
41	Leeward Fin Hinge Moment vs Roll Angle for Configuration A ($M_\infty = 0.8$)	44
42	Leeward Fin Hinge Moment vs Roll Angle for Configuration A ($M_\infty = 1.22$)	45
43	Induced Roll Moment vs α_b for Configuration A ($M_\infty = 0.8$)	45
44	Induced Roll Moment vs α_b for Configuration A ($M_\infty = 1.22$)	46
45	Spanwise Fin Loading for $\alpha_b = 20^\circ$, $\phi = 20^\circ$ Configuration A ($M_\infty = 0.8$)	46
46	Induced Roll Moment vs α_b for Configuration B.....	46
47	Induced Roll Moment vs α_b for Configuration D.....	47
48	Roll Damping Moment vs α_b for Configuration E.....	47

Illustrations (Cont)

49	Roll Damping Moment vs α_b for Configuration F.....	48
50	Steady-State Roll Rate vs α_b for Configuration E (4° fin cant).....	48
51	Interference Lift Ratio vs a/b_o (from Ref. 24).....	49
52	Pitch Control Force vs α_b for Configuration B ($\phi = 0^\circ, \delta_1 = \delta_3 = 10^\circ, \delta_2 = \delta_4 = 0^\circ$).....	50
53	Pitch Control Force vs α_b for Configuration C ($\phi = 0^\circ, \delta_1 = \delta_3 = 10^\circ, \delta_2 = \delta_4 = 0^\circ$).....	50
54	Pitch Control Force vs α_b for Configuration D ($\phi = 0^\circ, \delta_1 = \delta_3 = 10^\circ, \delta_2 = \delta_4 = 0^\circ$).....	50
55	Pitch Control Force vs α_b for Configuration B ($\phi = 45^\circ, \delta_1 = \delta_2 = \delta_3 = \delta_4 = 10^\circ$).....	50
56	Pitch Control Force vs α_b for Configuration C ($\phi = 45^\circ, \delta_1 = \delta_2 = \delta_3 = \delta_4 = 10^\circ$).....	51
57	Pitch Control Force vs α_b for Configuration D ($\phi = 45^\circ, \delta_1 = \delta_2 = \delta_3 = \delta_4 = 10^\circ$).....	51
58	Roll Control Moment vs α_b for Configuration B ($\phi = 0^\circ, \delta_1 = -\delta_3 = 10^\circ, \delta_2 = \delta_4 = 0^\circ$).....	51
59	Roll Control Moment vs α_b for Configuration C ($\phi = 0^\circ, \delta_1 = -\delta_3 = 10^\circ, \delta_2 = \delta_4 = 0^\circ$).....	52
60	Roll Control Moment vs α_b for Configuration D ($\phi = 0^\circ, \delta_1 = -\delta_3 = 10^\circ, \delta_2 = \delta_4 = 0^\circ$).....	52

Nomenclature

a	Body radius
A_e	Aspect ratio of fins based on exposed planform area, Eq (19c)
A_p	Planform aspect ratio of fins based on extension through the body b^2/S_{fp}
b	Fin span
b_o	Fin semispan
B	Root bending moment (Figure 29)
C_ℓ	Roll moment coefficient ($\ell/q_\infty S_b d$)
$C_{\ell p}$	Roll damping moment coefficient derivative $(\partial C_\ell / \partial p)_{p=0}$
C_m	Pitch moment coefficient ($m/q_\infty S_b d$)
C_n	Yaw moment coefficient ($n/q_\infty S_b d$)
C_n	Local normal force coefficient of fin, Eq (14)
C_N	Local normal force coefficient of fin due to angle of attack, Eq (33)
C_{N_α}	Normal force coefficient derivative of fin $(\partial C_N / \partial \alpha)_{\alpha=0}$
C_b	Panel root bending moment coefficient $[B/q_\infty(b_o - a) S_f]$
C_h	Panel hinge moment coefficient $(H/q_\infty c_r S_f)$
C_p	Panel normal force coefficient $(N/q_\infty S_f)$
c_r	Root chord of fin
C_y	Side force coefficient $(F_y/q_\infty S_b)$
C_z	Normal force coefficient $(F_z/q_\infty S_b)$
d	Body diameter
F_x, F_y, F_z	Forces in the x,y,z directions, respectively (Figure 1)
H	Panel hinge moment (Figure 29)
$\vec{i}, \vec{j}, \vec{k}$	Unit vectors in the x,y,z directions, respectively
I_d	Fin deflection interference coefficient, Eq (39)
I_p	Rolling-motion interference coefficient, Eq (44)
k_B	Interference lift ratio (Figure 51)
K_p	Potential flow lift coefficient, Eq (34h)
K_v	Vortex flow lift coefficient, Eq (34i)
L_s	Length of the vortex sheet, Eq (6a)
$\ell_{,m,n}$	Roll, pitch, and yaw moments, respectively (Figure 1)
M	Mach number
\vec{n}	Unit normal vector for fin surface

N	Normal force
N_v	Total number of vortices in cross-flow plane
p	Dimensionless roll speed $(\dot{\phi} b_o / U_\infty)$
q	Dynamic pressure $(1/2 \rho U^2)$
r_c	Radius of vortex core
r, ϕ	Polar coordinates in cross-flow plane
r_1, ϕ_1	Coordinates of primary body vortex
s	Complex position in the cross-flow plane $(y + i z)$
S_b	Frontal area of missile body (πa^2)
S_f	Exposed planform area of fin, Eq (19d)
S_{fp}	Planform area of two fins including extension through the body, Eq (40b)
u, v, w	Velocities in the x,y,z directions, respectively (Figure 1)
U	Magnitude of velocity
U_c	Freestream cross-flow velocity, $U_\infty \sin \alpha_b$
v_c, w_c	Cross-flow velocities in the y,z directions, respectively, Eq (2)
\vec{V}	Flow field velocity $(u \vec{i} + v \vec{j} + w \vec{k})$
x, y, z	Body Cartesian coordinates (Figure 1)
y_v, z_v	Location of primary body vortex in cross-flow plane
α	Angle of attack
α_ℓ	Local angle of attack
α_s	Stall angle of attack of the fin, Eq (35b)
Γ_p	Strength of the primary body vortex
Γ_s	Strength of vortex sheet
δ_j	Control deflection of the j'th fin (Figure 10)
σ	Geometric scaling factor for fin, Eq (20a)
Λ	Sweep angle of fin
ξ, η	Fin-oriented surface coordinates, Eq (18)
ρ	Fluid density
ϕ	Roll angle (Figure 10)
$\dot{\phi}$	Roll rate (rad/s)

Subscripts:

b	Missile body
e	"Effective" value of argument
ℓe	Leading edge
te	Trailing edge
∞	Freestream conditions

Superscripts:

\cdot	Derivative with respect to time
$\mathbf{\Delta}$	"Effective" value of argument

Summary

The present investigation is concerned with the theoretical prediction of fin forces and moments on bodies at high angle of attack in subsonic and transonic flow. The body is assumed to be a circular cylinder with a streamlined nose and only cruciform fins (or wings) as attached lifting surfaces. The fins are assumed to be planar and have straight leading, trailing and tip chord edges. The leading edge can have arbitrary sweep back and the trailing edge can be swept back or forward. The body can have an arbitrary roll (or bank) angle and each fin can have arbitrary control deflection. The vortices shed from the body are assumed to be symmetrically located with respect to the angle of attack plane and of equal strength but opposite rotational sense. The highest angle of attack of the body for which the body vortices remain symmetric depends on the nose fineness ratio, body fineness ratio, and Mach number; normally this angle is near 25° . The body flow field model and lifting theory use some empirical data, but the user of the method need not provide any additional data. A description of the construction and use

of the program FINLOAD, which implements the method, is given.

Extensive comparisons are made between predicted results and experimental measurements. Included in the comparisons are: panel normal force, root bending moment, induced roll moment, nonlinear roll damping moment, pitch (or yaw) control forces, and roll control moment. The force and moment predictions are compared with experimental data for six different fin geometries; these include delta, clipped delta, and rectangular planforms. Predictions for wing alone normal force characteristics are compared with data for rectangular, delta, clipped delta, diamond, arrow, clipped arrow, and trapezoidal planforms. Extensive discussions are given that explain the underlying aerodynamic causes of fin force and moment nonlinearities and how these are related to fin geometry. Good agreement between the present method and experiment is obtained, except for control force and moment where the agreement could be characterized as "fair."

Prediction of Forces and Moments on Finned Bodies at High Angle of Attack in Transonic Flow

Introduction

The aerodynamics of missiles and bombs at high angle of attack has become increasingly important for modern design requirements. Examples of this are high maneuverability of air-to-air-missiles and tolerance of large disturbances at store separation. At high angle of attack, a body of revolution sheds two symmetric vortices from the leeward side of the body, and these grow in strength along the length of the body. The body's attached lifting surfaces are immersed in this vortex wake flow and, consequently, the surface pressure distributions are significantly changed from the potential flow case. This, in turn, causes nonlinearities in the forces and moments produced by the lifting surfaces. These nonlinearities have been known to cause serious flight stability and controllability problems in vehicle flight dynamics.

Attempts at predicting the forces and moments produced by lifting surfaces in a symmetric body-vortex wake have met with moderate success. Very early work was done by Mello and Sivier¹ for a cruciform-fin missile in supersonic flow. References 2 and 3 report reasonably successful work for incompressible and supersonic flow, respectively, but only rectangular fin planforms were considered. The approach taken was to calculate the body-flow field using a vortex-modeling technique and then use this as input to a lifting theory.^{2,3} The most extensive work on the subject has been achieved by Nielson and his associates.^{4,7} They have attacked the very difficult problem of a general canard-fin-body configuration in transonic and supersonic flow with both symmetric and asymmetric body vortices and canard vortices. Their approach is a combination of slender-body theory, Deffenbaugh's^{8,9} method for the impulse flow analogy, and data base experimental input for fin (or wing) alone characteristics.

The present approach uses the body flow field model developed in Reference 10 and combines it with an approximate lifting surface theory to predict

fin forces and moments. The lifting surface theory and the equations for fin forces and moments are developed in detail. The applicability and accuracy of the present method is evaluated by extensive comparisons of theory and experiment. The purpose of the present work is to develop a force and moment predictive method that is applicable to a wide variety of fin planforms over the subsonic and transonic speed range.

Aerodynamic Analysis

The general approach to the aerodynamics of the problem is to calculate the body flow field and then calculate the forces and moments of attached lifting surfaces exposed to this flow field. This approach is clearly based on the assumption that the body flow field is not significantly affected by the flow induced by lifting surfaces. This assumption implies that the present analysis is not appropriate for missile configurations in which the fin-root chord is a large portion of the length of the missile body (e.g., the Phoenix air-to-air missile (AIM-54A)). The present analysis also assumes that there is only one set of lifting surfaces (wings or fins) and that it is arranged in a cruciform configuration. The present approach could be applied to a two- or three-fin configuration by making appropriate modifications to the lifting theory.

This present approach naturally divides the analysis into two areas: the body flow field and the prediction of lifting surface forces and moments. The model of the body flow field was developed previously in References 2 and 10. For completeness, however, the model and the associated computational procedures are described in this report. The prediction of lifting surface forces and moments is described in two phases. First, the lifting theory for calculating the normal force distribution and the

total normal force of the lifting surface in uniform approach flow is described. Second, the model of the body flow field and the lifting theory are combined to yield a method for predicting forces and moments of attached lifting surfaces.

Body Flow Field

The flow field of a circular cylindrical body at high angle of attack is dominated by the presence of body vortices and their associated feeding sheets. Figure 1 shows the coordinate system and a schematic of the body vortex wake flow. These vortices increase in strength as the angle of attack or body length increases. To model this complex separated flow, the flow is divided into the cross-flow components (v_c and w_c) and the axial flow component, $U_\infty \cos \alpha_b$. The local flow velocity can then be expressed as:

$$\vec{V} = U_\infty \cos \alpha_b \vec{i} + v_j \vec{j} + w_c \vec{k} \quad (1)$$

It is assumed that the steady, three-dimensional, body flow field can be divided into (1) constant axial-flow component and (2) the two-dimensional, potential, flow about a circular cylinder with vortices in the wake and their associated image vortices inside the cylinder. Essentially all of the vorticity is located inside the vortex cores of the primary body vortices and the vortex feeding sheets connecting the body boundary layer separation points and the primary body vortices. Figure 2 shows the primary and sheet vortices in the cross-flow plane. Using this model, the cross-flow velocity components can be written as

$$v_c = \left[\frac{-2U_c a^2 y z}{(y^2 + z^2)^2} + \frac{\Gamma_p}{2\pi} \sum_{j=1}^4 (-1)^j \frac{z - z_j}{(y - y_j)^2 + (z - z_j)^2} \right. \\ \left. + \frac{2\Gamma_s}{\pi(N_v - 4)} \sum_{j=5}^{N_v} (-1)^j \frac{z - z_j}{(y - y_j)^2 + (z - z_j)^2} \right] [1 - e^{-C}] \quad (2a)$$

$$w_c = \left\{ U_c \left[1 + \frac{a^2(y^2 - z^2)}{(y^2 + z^2)^2} \right] - \frac{\Gamma_p}{2\pi} \sum_{j=1}^4 (-1)^j \right. \quad (2b)$$

$$\left. \frac{y - y_j}{(y - y_j)^2 + (z - z_j)^2} - \frac{2\Gamma_s}{\pi(N_v - 4)} \sum_{j=5}^{N_v} (-1)^j \frac{y - y_j}{(y - y_j)^2 + (z - z_j)^2} \right\} [1 - e^{-C}]$$

where

$$C = 1.254 [(y - y_v)^2 + (z - z_v)^2] / r_c^2$$

$$y_v, z_v = \begin{cases} y_1, z_1 & y \geq 0 \\ y_4, z_4 & y < 0 \end{cases}$$

$U_c = U_\infty \sin \alpha_b$ is the free-stream cross-flow velocity

a = body radius

Γ_p = strength of a primary body vortex

Γ_s = strength of a vortex sheet

y_j, z_j = location of the j 'th vortex

N_v = total number of vortices in the cross-flow plane

r_c = vortex core radius

The dependence of Γ_p , Γ_s , y_1 , z_1 , and r_c on angle of attack and body length is taken from experimental measurements.

The exponential term in Eqs (2a) and (2b) was included to model the solid body type rotation in the cores of the primary body vortices. This method of approximating the vortex cores yields a cross-flow velocity field that is continuous, whereas simply imposing a solid body rotation onto a potential flow field does not.

Using the assumption of a symmetric vortex wake, the relationship between the locations of all of the vortices in the cross-flow plane becomes

$$\left. \begin{aligned} y_j &= -y_{j+1} = a^2 y_{j+1} / (y_{j+1}^2 + z_{j+1}^2) \\ z_j &= z_{j+1} = a^2 z_{j+1} / (y_{j+1}^2 + z_{j+1}^2) \end{aligned} \right\} \text{ for } j = 2, 6, 10, \dots, N_v - 2 \quad (3a)$$

$$y_j = -y_{j-3} \text{ and } z_j = z_{j-3} \text{ for } j = 4, 8, 12, \dots, N_v \quad (3b)$$

With these equations the location of all of the vortices can be related to the location of the vortices external to the body in the positive (y, z) quadrant; i.e., $j = 1, 5, 9, \dots, N_v - 3$.

The location of the right-handed vortex sheet, s_s = $r_s e^{i\phi}$, is given as

$$s_s = [a \cos(\pi \phi / 2 \phi_1) + (r_1 + r_c) \sin^2(\pi \phi / 2 \phi_1) / \\ \{1 + (r_1 + r_c)(\phi_1 - \phi) / a\}] e^{i\phi} \quad \phi_{ss} \leq \phi \leq \phi_1 \quad (4)$$

ϕ_{ss} is defined as the angle at which the radial location of the sheet achieves a value of 1.01a; therefore, ϕ_{ss} is defined by the equation

$$\cos(\pi \phi_{ss}/2 \phi_1) + (r_1 + r_c) \sin^2(\pi \phi_{ss}/2 \phi_1) / [a + (r_1 + r_c)(\phi_1 - \phi_{ss})] = 1.01. \quad (5)$$

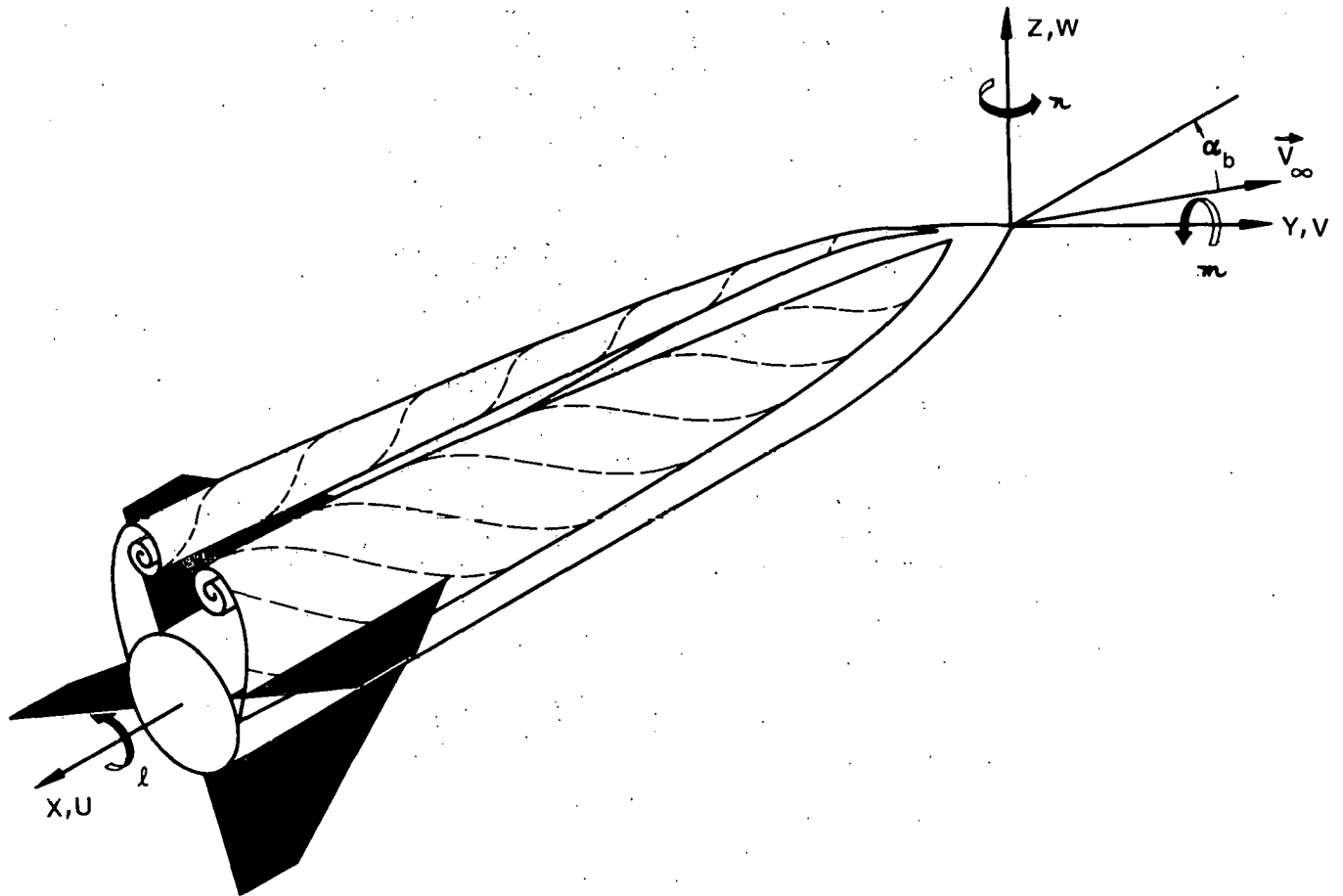
$$L_s = \int_{\phi_{ss}}^{\phi_1} \left[r_s^2 + \left(\frac{dr_s}{d\phi} \right)^2 \right]^{1/2} d\phi \quad (6a)$$
$$\begin{aligned} \frac{dr_s}{d\phi} = & -(a \pi / 2 \phi_1) \sin (\pi \phi / 2 \phi_1) + [(r_1 + r_c) \pi / \phi_1] \\ & \sin (\pi \phi / 2 \phi_1) \cos (\pi \phi / 2 \phi_1) \{1 + (r_1 + r_c) \cdot \\ & (\phi_1 - \phi) / a\} + \{(r_1 + r_c)^2 / a\} \sin^2 (\pi \phi / 2 \phi_1) \} / \\ & \{1 + (r_1 + r_c) (\phi_1 - \phi) / a\}^2 . \end{aligned} \quad (6b)$$


Figure 1. Coordinate System and Schematic of Body Vortices

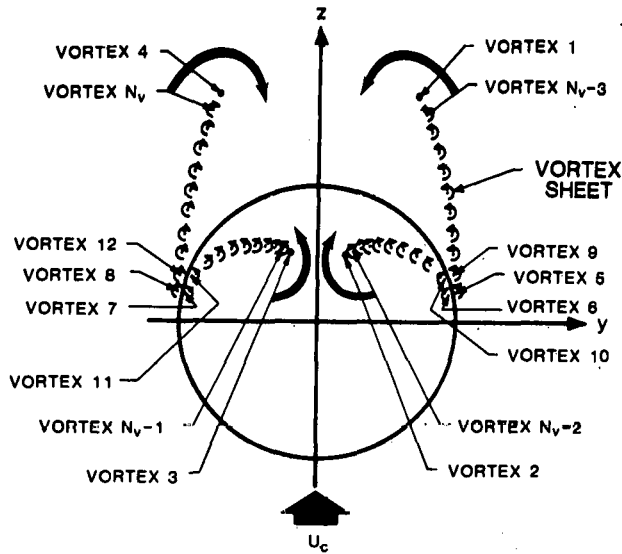


Figure 2. Cross-Flow Plane Flow Model

The first vortex in the sheet (vortex 5) is located at the point where the sheet leaves the body so that $r_5 = 1.01a$ and $\phi_5 = \phi_{ss}$. The angular position for vortices 5, 9, 13, ..., $N_v - 3$ is found from

$$\left(\frac{j-5}{N_v-4} \right) L_s = \int_{\phi_{ss}}^{\phi_j} \left[r_s^2 + \left(\frac{dr_s}{d\phi} \right)^2 \right]^{1/2} d\phi \quad (7)$$

Once the angular position is known from Eq (7), then from Eq (4) one obtains the radial location:

$$r_j = a \cos(\pi \phi_j / 2 \phi_1) + (r_1 + r_c) \sin^2(\pi \phi_j / 2 \phi_1) / \{ 1 + (r_1 + r_c) (\phi_1 - \phi_j) / a \} \quad (8)$$

The experimental inputs required by the theory will now be discussed. The total strength of all the vorticity, Γ_t (primary vortex and feeding sheet), in each half-plane of the wake is taken from the experimental data of Grosche¹²

$$\Gamma_t / (\pi d U_\infty) = 0.35(x/a - 6) \alpha_b^2 \quad \text{for } x/a > 6 \quad (9)$$

This equation represents the data of Grosche for $7^\circ \leq \alpha_b \leq 20^\circ$ and $5 \leq x/d \leq 13$ for incompressible flow. No extensive body-vortex wake surveys have been conducted in compressible subsonic flow.

The division of vorticity between the primary vortex and the feeding sheet is taken from a correlation of data given in Reference 11. This correlation is represented by

$$\frac{\Gamma_p}{\Gamma_t} = 1 - 0.15(x/a) \alpha_b + 0.008(x/a)^2 \alpha_b^2 \quad (10)$$

where $\Gamma_t = \Gamma_p + \Gamma_s$. Although the data from Reference 11 are for supersonic flow, it is reasonable to assume that the ratio of primary vortex strength to total vorticity is the same in subsonic and supersonic flow.

The location of the primary vortex is taken from the experimental data of Grosche,¹² Tinling and Allen,¹³ and Fidler, Nielsen, and Schwind.¹⁴ The experimental data for the location of the right-hand primary vortex (vortex 1) in polar coordinates are approximated by

$$\phi_1 = 74^\circ \quad (11)$$

$$r_1/a = 0.70 + 0.06 \sqrt{M_\infty + 1} (x/a + 6) \sqrt{\alpha_b}$$

These equations incorporate the moderate increase in radial location of the vortex center with Mach number measured by Tinling and Allen.

The radius of the vortex core is taken from the data of Reference 11. A fit of the data for angles of attack of 10° and 15° and body lengths from 7 to 14 calibers is given by

$$r_c/a = 0.030(x/a) \sqrt{\alpha_b} \quad (12)$$

The computational procedure for the body flow field model will now be discussed. The order of calculation is as follows:

- (a) α_b and x are set.
- (b) Γ_t is calculated from Eq (9).
- (c) Γ_p and Γ_s are calculated from Eq (10).
- (d) ϕ_1 and r_1 are calculated from Eq (11).
- (e) r_c is calculated from Eq (12).
- (f) ϕ_{ss} is computed from Eq (5) by increasing ϕ_{ss} from 0° in increments of $0.01\phi_1$ until Eq (5) is satisfied. Recall that $\phi_{ss} = \phi_5$ and $r_5 = 1.01a$.
- (g) L_s is calculated from Eq (6) by Simpson's Rule of numerical integration.
- (h) ϕ_j for $j = 9, 13, \dots, N_v - 3$ is calculated by increasing ϕ_j from ϕ_{j-4} in increments of $0.01\phi_1$ until the integral equation is satisfied. The integral is evaluated by Simpson's Rule. In the present work, N_v is set at 44; that is, 10 vortices in each sheet.
- (i) r_j for $j = 9, 13, \dots, N_v - 3$ is calculated from Eq (8).
- (j) y_j, z_j for $j = 1, 5, 9, \dots, N_v - 3$ are calculated from ϕ_j, r_j using the polar to Cartesian transformation.
- (k) y_j, z_j for all remaining vortices are calculated from Eq (3).
- (l) v_c, w_c are calculated from Eq (2).

Lifting Theory

Various lifting theories were considered for use with the present flow model for the prediction of forces and moments produced by fins. The criteria by which a lifting theory was chosen was that the theory must be able to consider very nonuniform, rotational, approach flow and it must include fin stall and poststall characteristics. These criteria quickly limited the possible theories to strip theory. In strip theory it is assumed that the normal force on a chordwise strip of fin can be calculated by using the local dynamic pressure and angle of attack of the strip, independent of adjacent chordwise strips. Significant elements included in the present lifting theory are the following: normal force distribution over the lifting surface depends upon fin aspect ratio and leading and trailing edge sweep; individual control deflection of each fin is allowed; fin-fin interference due to both control deflection and rolling rate is included; normal force depends upon free-stream Mach number; and nonuniform approach flow alters the effective leading edge sweep.

a. Local Normal Force

The local normal force on a differential element of the fin surface is written as (see Figure 3)

$$dN = C_n q dx dr \quad (13)$$

where C_n is the local normal-force coefficient and q is the local dynamic pressure (including that due to missile rolling speed). The local normal force coefficient C_n is composed of three separate functions: (1) the normal force due to the local angle of attack of the differential element, (2) the local chordwise distribution, and (3) the local spanwise distribution. Assuming a product form of the function, one has

$$C_n = \sigma C_n(\alpha_\ell) C(x,r) S(r) \quad (14)$$

where σ is a geometric scaling factor, $C_n(\alpha_\ell)$ is the local normal force coefficient due to local angle of attack, and $C(x,r)$ and $S(r)$ are the chordwise and spanwise normal force distributions, respectively, for uniform approach flow.

For arbitrary planform fins, it greatly simplifies matters if C and S are written in terms of appropriate fin-oriented coordinates. To determine the appropriate fin-oriented variables, the x coordinate of the leading and trailing edges of the fin is written as

$$\begin{aligned} x_{\ell e} &= x_1 + (x_2 - x_1)(r - a)/(b_o - a) \\ x_{te} &= x_3 + (x_4 - x_3)(r - a)/(b_o - a) \end{aligned} \quad (15)$$

where x_1 , x_2 , x_3 , and x_4 are defined in Figure 3, and b_o is the semispan of the fin. x_2 , x_3 , and x_4 can be related to the leading edge sweep $\Lambda_{\ell e}$, and the root chord of the fin c_r as

$$\begin{aligned} x_2 &= x_1 + (b_o - a) \tan \Lambda_{\ell e} \\ x_3 &= x_1 + c_r \\ x_4 &= x_3 + (b_o - a) \tan \Lambda_{te} \end{aligned} \quad (16)$$

Substituting these equations into Eq (15), one obtains

$$\begin{aligned} x_{\ell e} &= x_1 + (r - a) \tan \Lambda_{\ell e} \\ x_{te} &= x_1 + c_r + (r - a) \tan \Lambda_{te} \end{aligned} \quad (17)$$

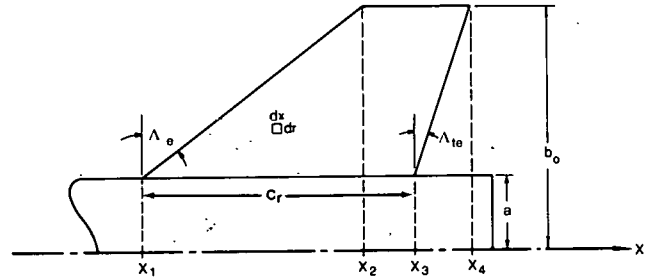


Figure 3. General Planform Fin

Using the boundaries of the fin planform (i.e., $x_{\ell e}$, x_{te} , a , and b_o) as scaling variables, a convenient set of fin-oriented coordinates are

$$\begin{aligned} \xi &= (x - x_{te})/(x_{\ell e} - x_{te}) \\ \eta &= (r - a)/(b_o - a) \end{aligned} \quad (18)$$

where ξ is the chordwise variable and η is the spanwise variable (Figure 4).

b. Normal-Force Distribution

Experimental data¹⁵⁻¹⁹ for normal-force distribution over the surface of a number of fin planforms was studied in order to arrive at general expressions for $C(\xi)$ and $S(\eta)$. After devising and testing a considerable number of expressions for the chordwise and spanwise distributions, the following equations were adopted

$$C(\xi) = \sqrt{\xi} \exp[\xi^2/\sqrt{\cos \Lambda_{\ell e}}] \quad (19a)$$

$$S(\eta) = (1 + \eta^{2A_s}) \sqrt{1 - \eta^2} \quad (19b)$$

where

$$A_e = 2 (b_o - a)^2 / S_f \quad (19c)$$

$$S_f = \frac{1}{2} (b_o - a) [2 c_r / (b_o - a) - \tan \Lambda_{te} + \tan \Lambda_{le}] \quad (19d)$$

A_e is the exposed aspect ratio; that is, the aspect ratio of the lifting surface formed by eliminating the body and placing the root chords of two adjacent fins together. S_f is the planform area of a single fin. Equations (19a) and (19b) have been shown to give valid results for exposed aspect ratios from 0.5 to 5, leading edge sweep from 0° to 80° , and trailing edge sweep from -60° to 80° .

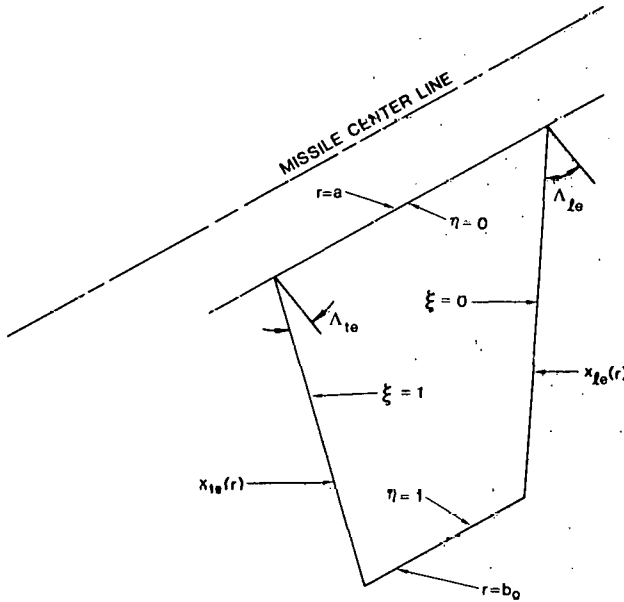


Figure 4. Fin-Oriented Coordinates ξ, η

The normal force distribution over the surface of the fin for uniform approach flow is then provided by the product of Eqs (19a) and (19b). This expression was compared qualitatively with experimental data by means of a three-dimensional computer graphics routine, DISSPLA.* This routine provided a means of visualizing the normal force distribution over the surface of the fin. Shown in Figures 5 through 9 are sets of three-dimensional perspective plots of typical planforms that were examined. Figure 5 shows a

sequence of untapered fins with $\Lambda_{le} = \Lambda_{te} = 60^\circ$ for $A_e = 0.5, 1$, and 3 . The view in the perspective plot is from behind and above the fin surface, looking upstream and slightly inboard. The graphics routine uses a rectangular area over which the surface function is defined. Consequently, the regions that show a surface value of zero are not part of the fin planform. For example, on Figure 5 the trailing edge of the fin is located at the junction of the nonzero and zero surface values. Another point to note on the perspective plots is a "spike" character at the leading edge of the planform. This characteristic is not inherent in the equations but simply is a result of the mesh size on the surface and the zero value of the surface just ahead of the leading edge.

Figure 6 shows a sequence of fins with $\Lambda_{le} = 60^\circ$, $\Lambda_{te} = 20^\circ$ for $A_e = 0.5, 1$, and 2.92 . In Figure 6 the taper ratio decreases until in Figure 6c an arrow wing is achieved. Note in this sequence of figures that as the leading edge becomes longer the normal force loading reflects the very high loading near the leading edge caused by the increasing strength of the leading edge vortex.

Figure 7 shows a rectangular planform for $A_e = 0.5, 1$, and 3 . For the low aspect ratio planform (Figure 7a), note the increase in normal force near the tip chord due to the tip vortex increasing in strength along the tip chord. For the high aspect ratio planform (Figure 7c), it can be seen that the spanwise load distribution nears the classical elliptic loading.

Figure 8 presents the loading for $\Lambda_{le} = 60^\circ$, $\Lambda_{te} = 0^\circ$, and $A_e = 0.5, 1$, and 2.31 . Figures 8a and 8b show clipped deltas, and Figure 8c shows a delta planform. When the distribution for the delta planform is compared with experimental data,¹⁸ it is seen that the empirical equation models the data except near the leading edge.

Figure 9 shows a trapezoid planform with decreasing taper ratio, $\Lambda_{le} = -\Lambda_{te} = 40^\circ$ and $A_e = 0.5, 1$, and 2.38 . The planform with $A_e = 2.38$ (Figure 8c) has a taper ratio of zero and is, therefore, a diamond planform.

The geometric scaling factor in Eq (14) can now be determined. σ is evaluated by the requirement that the integrated average of the assumed normal force distribution over the surface of the fin must be unity; i.e., the assumed normal force loading must be normalized. Therefore, one may write

$$q_\infty \iint_{\text{fin}} C_n dA = q_\infty C_N S_f$$

*DISSPLA is a graphics of software package available from Integrated Software Systems Corporation, San Diego, CA.

Substituting C_n from Eq (14), one has

$$\sigma = \int_a^{b_0} \int_{x_{te}}^{x_{te}} C(x,r) S(r) dx dr = S_f .$$

Transforming to the fin coordinates ξ, η (Eq 18) and solving for σ , one obtains

$$\sigma = \frac{S_f}{(b_0 - a) \int_0^1 \int_0^1 \Omega(\eta) C(\xi) S(\eta) d\xi d\eta} \quad (20a)$$

where

$$\Omega(\eta) = c_r - (b_0 - a) (\tan \Lambda_{te} - \tan \Lambda_{te}) \eta \quad (20b)$$

and $C(\xi)$ and $S(\eta)$ are given by Eq (19).

c. Local Angle of Attack

Referring back to Eq (14), C_N depends on the local angle of attack of the chordwise strip. The local angle of attack is calculated by using the unit normal vector of the fin surface, \vec{n} , and the total velocity, \vec{V} (Figure 10). The geometric local angle of attack can be shown to be

$$\alpha_\ell = \sin^{-1} [\vec{n} \cdot \vec{V} / (|\vec{n}| |\vec{V}|)] . \quad (21)$$

The surface normal vector of the fin depends on the roll angle ϕ and the control deflection of each fin δ_j , $j = 1, 2, 3, 4$. Let the sign convention of the control deflection of each fin be as follows: positive control deflection of Fins 1 and 3 produces a positive normal force, i.e., a pitch down maneuver; positive control deflection of Fins 2 and 4 produces a positive side force, i.e., a yaw left maneuver (see Figure 10). The sign convention for the surface normal vector, however, is such that the vector always points in the counterclockwise direction. Referring to Figure 10, the surface-normal vector is

$$\vec{n} = \sin(\Delta \delta_j) \vec{i} - \cos(\delta_j) \sin \phi \vec{j} + \cos(\delta_j) \cos \phi \vec{k} \quad (22)$$

where

$$\Delta = \cos \phi / |\cos \phi| ;$$

Δ simply provides the sign of δ_j which is consistent with the above-mentioned sign convention.

The local total velocity (i.e., the velocity of the fluid relative to the chordwise strip) is composed of two types of terms: the fluid velocity relative to the fixed coordinate system and the velocity of the fixed coordinate system relative to the spinning chordwise strip. Therefore, using Eq (1) it can be written

$$\vec{V} = U_\infty \cos \alpha_b \vec{i} + (v_c + \dot{\phi} r \sin \phi) \vec{j} + (w_c - \dot{\phi} r \cos \phi) \vec{k} \quad (23)$$

where v_c and w_c are given by Eq (2) and $\dot{\phi}$ is the roll rate of the missile.

The axial location at which the flow model is evaluated is calculated from the pin planform characteristics. The axial location is chosen as the average quarter chord location of the root and tip chords; i.e.,

$$x_v = (x_1 + 0.25 c_r + x_2 + 0.25 c_t) / 2$$

where x_v is the axial location of the vortex model and c_t is the length of the tip chord. Rewriting this equation, one has

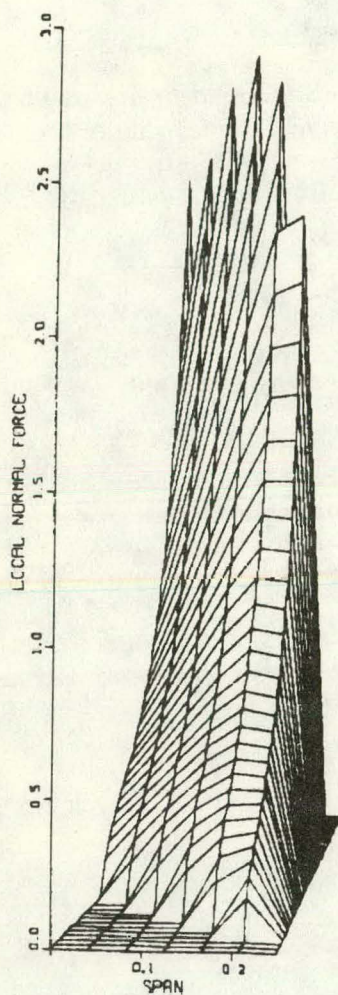
$$x_v = x_1 + (0.25 c_r + 0.25 c_t + (b_0 - a) \tan \Lambda_{te}) / 2 .$$

This axial location is used in Eqs (9) through (12).

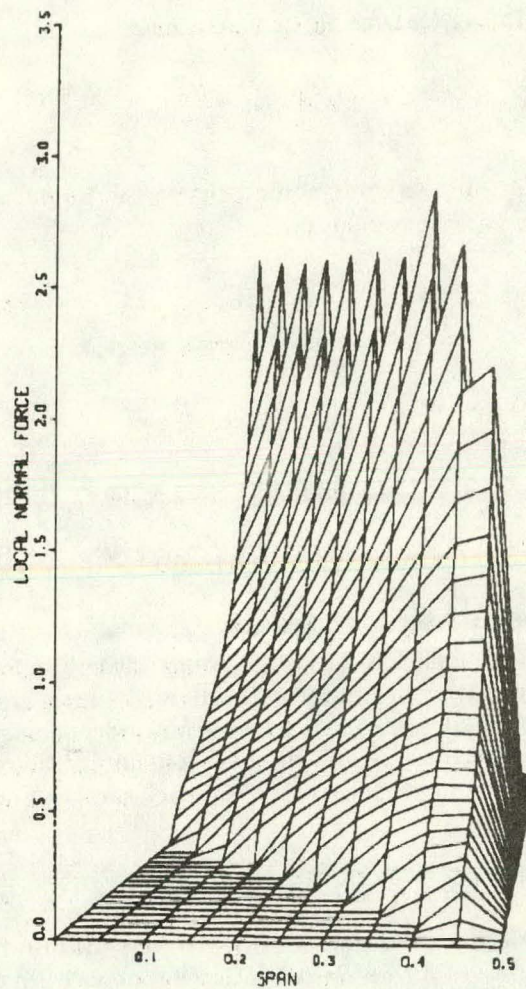
The geometric local angle of attack can now be calculated by substituting Eqs (22) and (23) into Eq (21). Interference between fins, however, will alter α_ℓ for control deflection and a rolling missile. These interference effects are considered in the paragraph entitled Roll Moment.

d. Effective Aspect Ratio

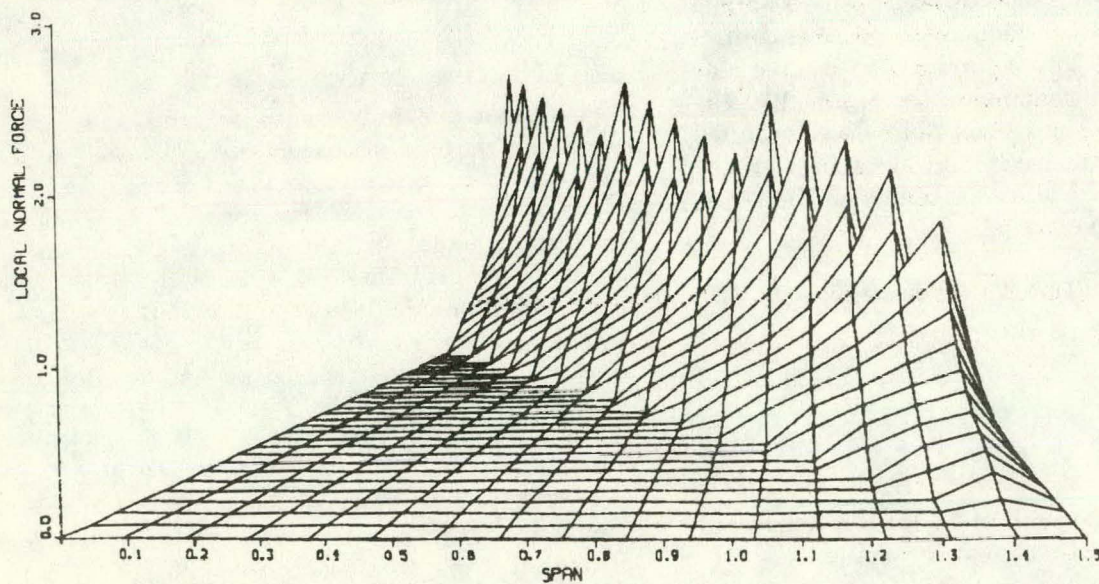
Consider an aerodynamic effect that occurs when the lifting surface is attached to the missile body. If the body is at high angle of attack, the angle in the plane of the fin between the approach flow and the fin leading edge can vary significantly, depending on the roll angle of the body; that is, the fin is yawed with respect to the approach flow for various roll angles around the body. This yaw angle results in an effective change in the leading and trailing edge sweep and effective aspect ratio of the fin. A simple example of this is to consider Fin 4 at a roll angle of 0° with the body at angle of attack α_b (see Figure 10). The effective leading edge sweep of Fin 4 at $\phi = 0^\circ$ then is approximately $\Lambda_{te} - \alpha_b$. If $\Lambda_{te} = 0^\circ$, then the fin at this roll angle would actually be swept forward.



a) $A_e = 0.5$

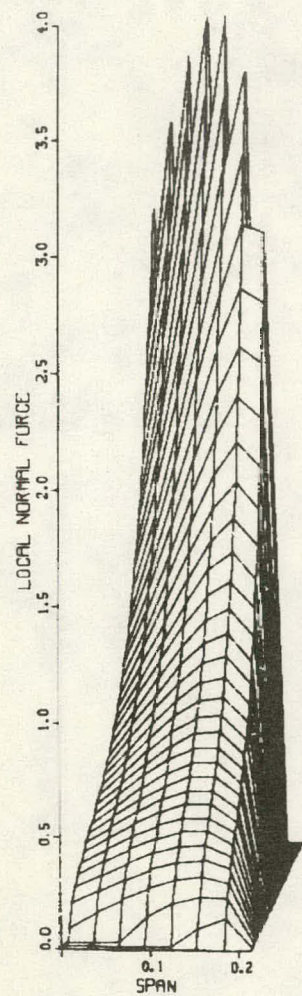


b) $A_e = 1.$

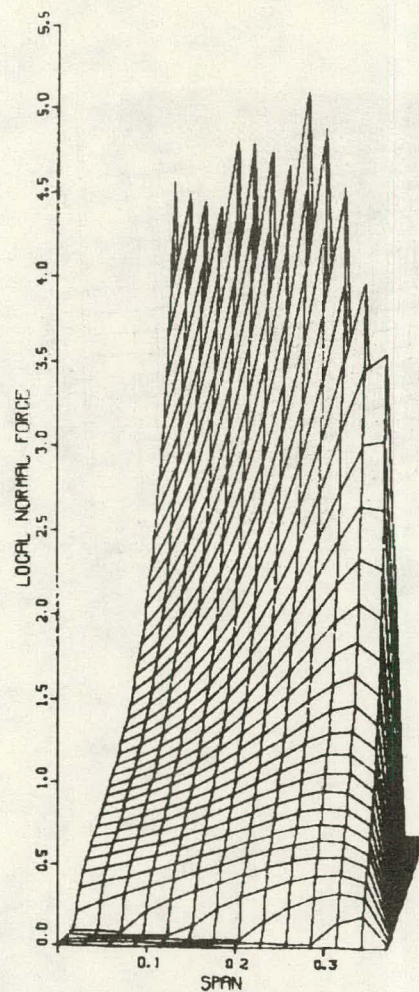


c) $A_e = 3.$

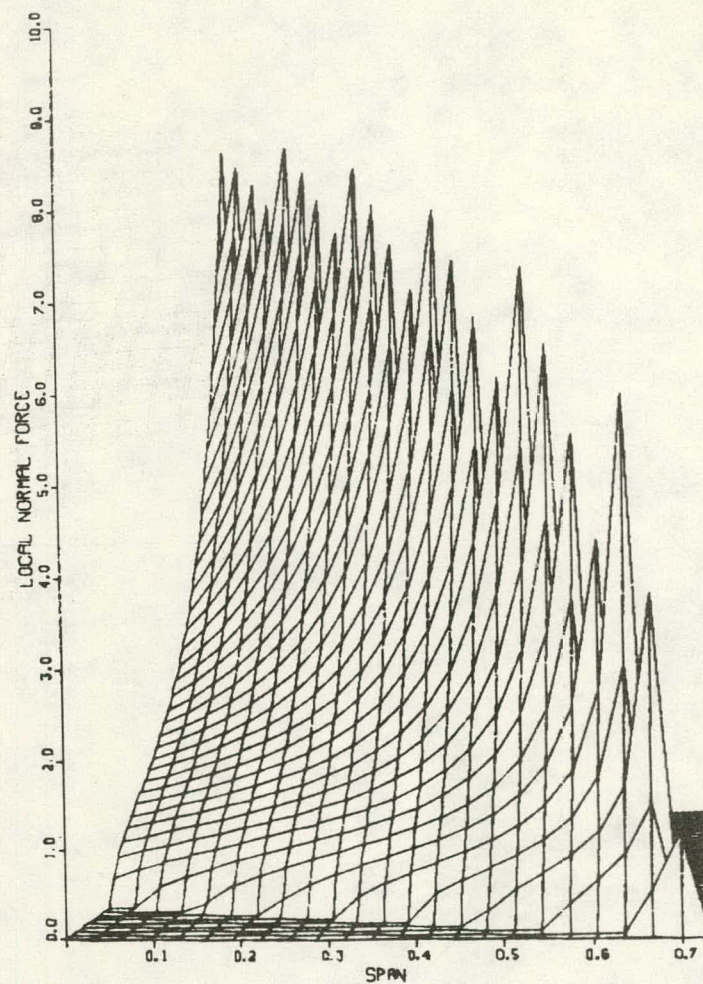
Figure 5. Normal Force Distribution Over Fin Surface for $\Lambda_{le} = \Lambda_{te} = 60^\circ$



a) $A_e = .5$

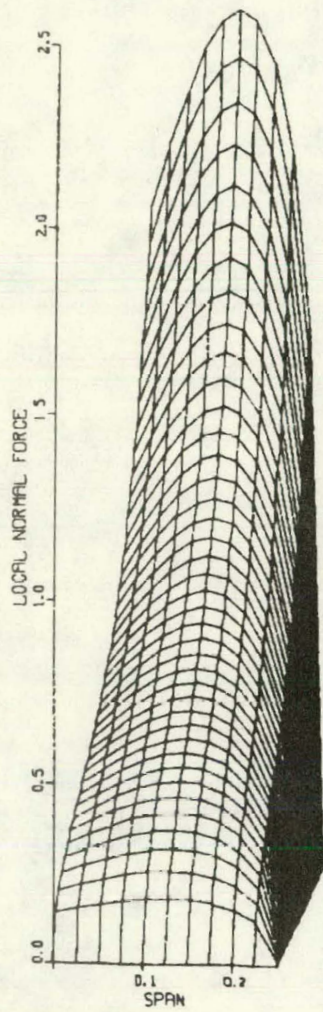


b) $A_e = 1.$

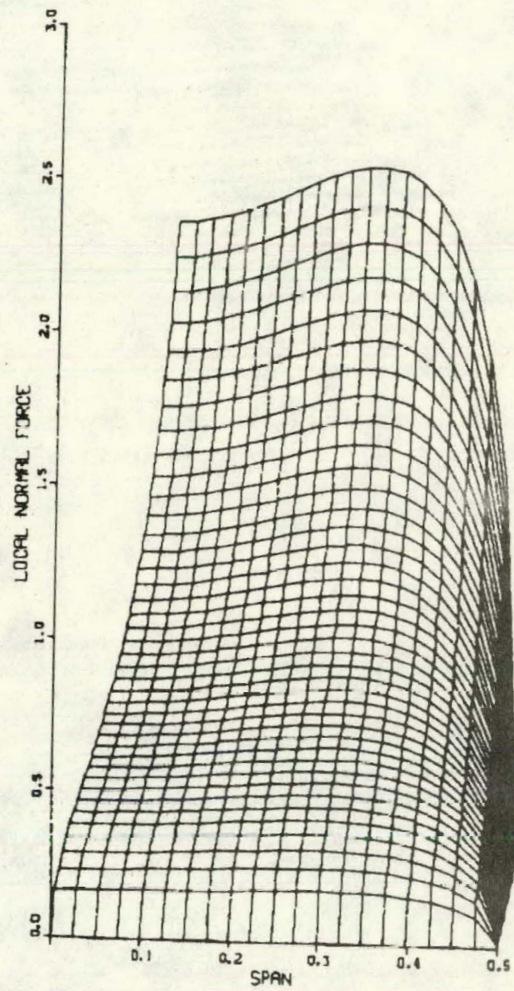


c) $A_e = 2.92$

Figure 6. Normal Force Distribution Over Fin Surface for $\Lambda_{le} = 60^\circ$ and $\Lambda_{te} = 20^\circ$

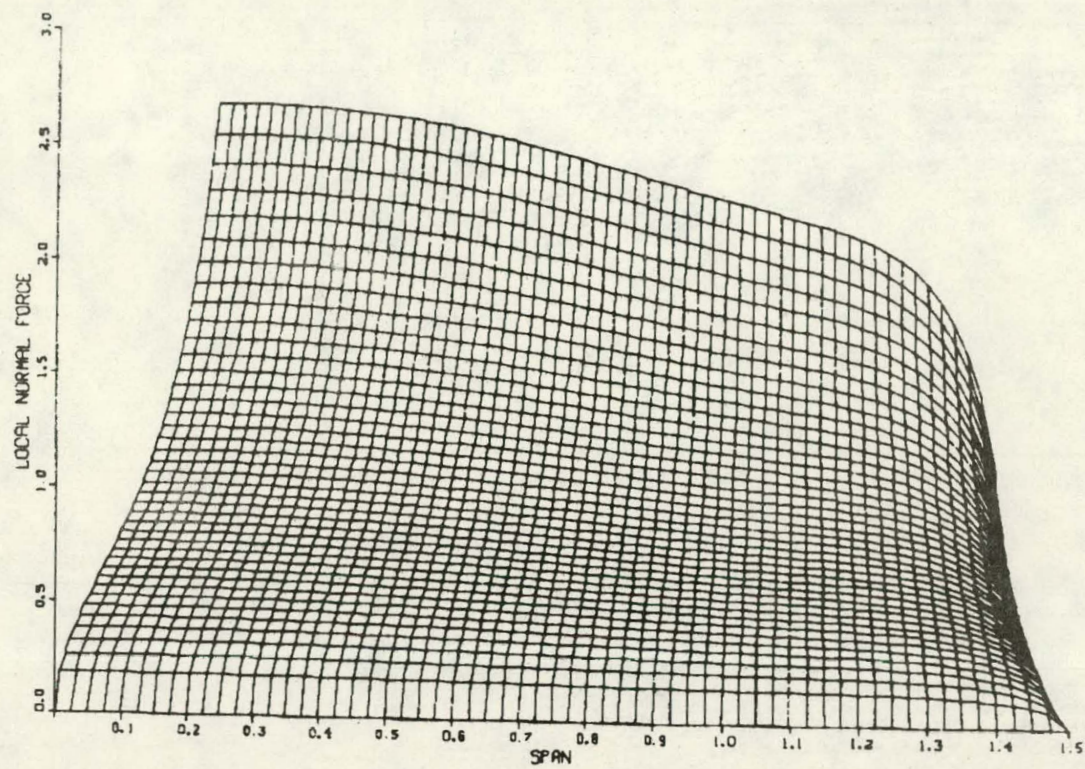


a) $A_e = .5$



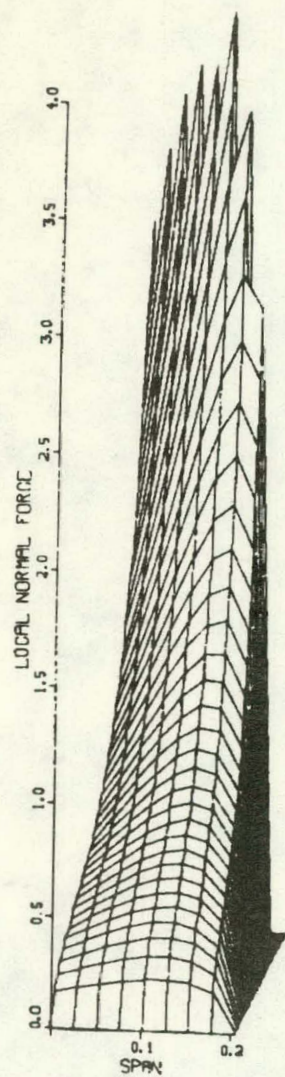
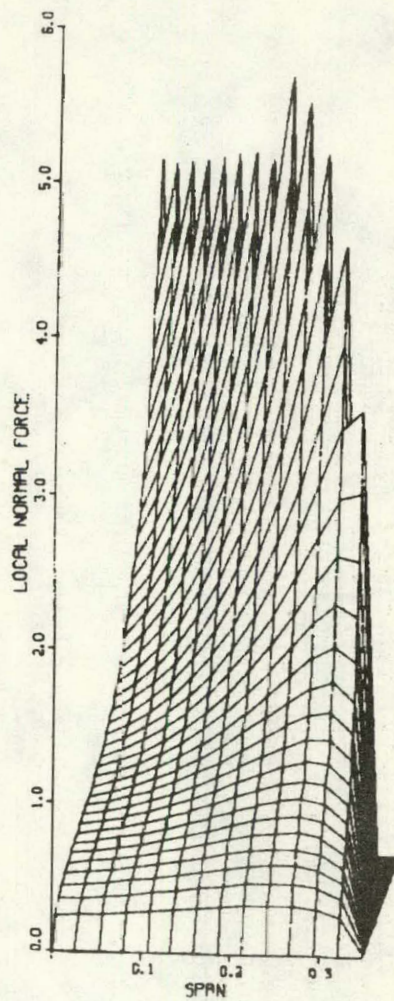
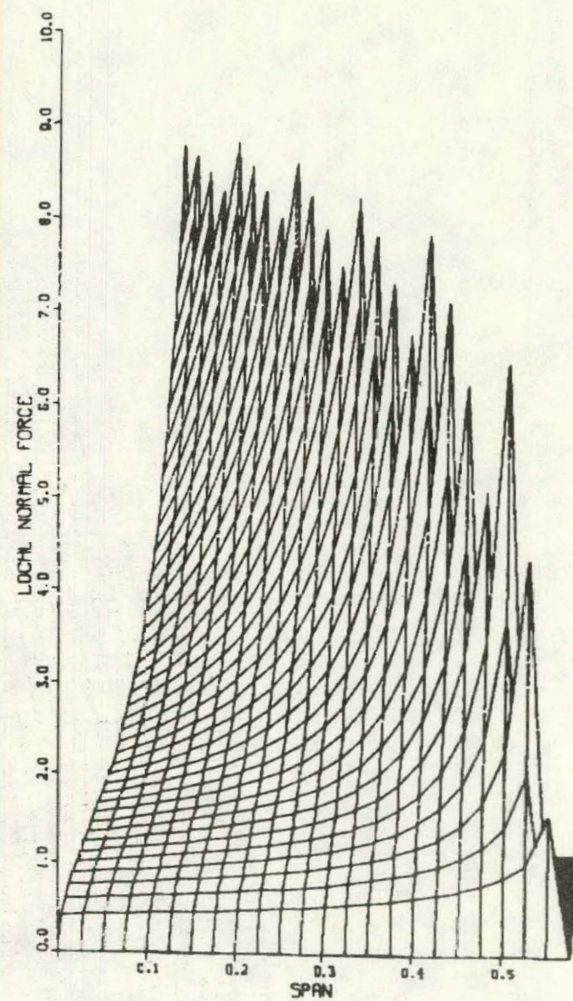
b) $A_c = 1$

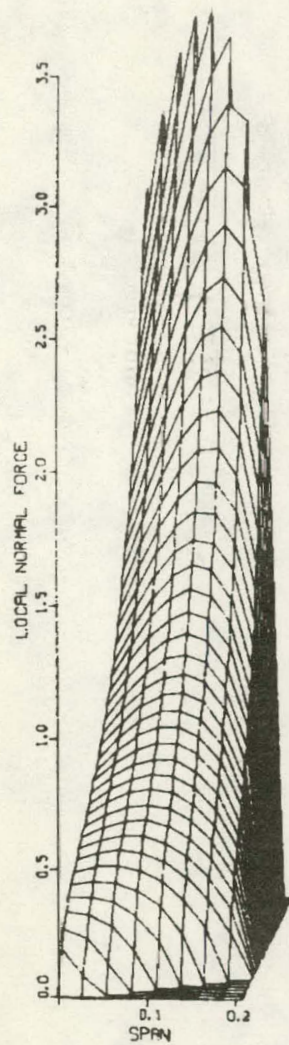
Figure 7. Normal Force Distribution Over Fin Surface for $\Lambda_{\ell e} = \Lambda_{te} = 0^\circ$



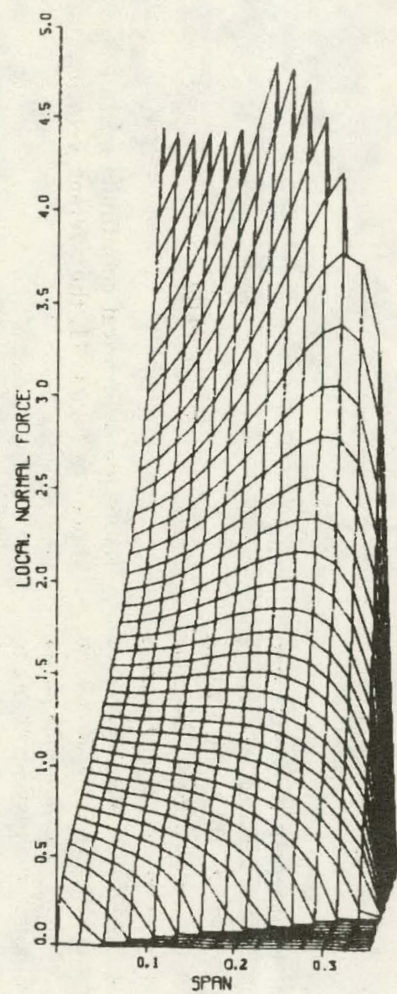
c) $A_e = 3.$

Figure 7. (Concluded)

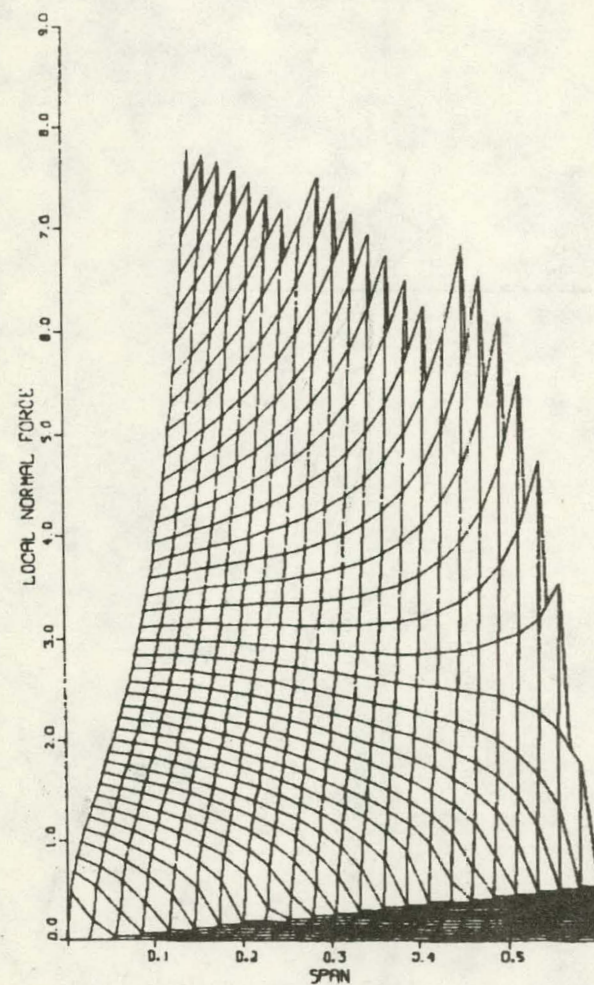
a) $A_e = .5$ b) $A_e = 1.$ c) $A_e = 2.31$ Figure 8. Normal Force Distribution Over Fin Surface for $\Lambda_{le} = 60^\circ$ and $\Lambda_{te} = 0^\circ$



a) $A_e = .5$



b) $A_e = 1.$



c) $A_e = 2.38$

Figure 9. Normal Force Distribution Over Fin Surface for $\Lambda_{le} = 40^\circ$ and $\Lambda_{te} = -40^\circ$

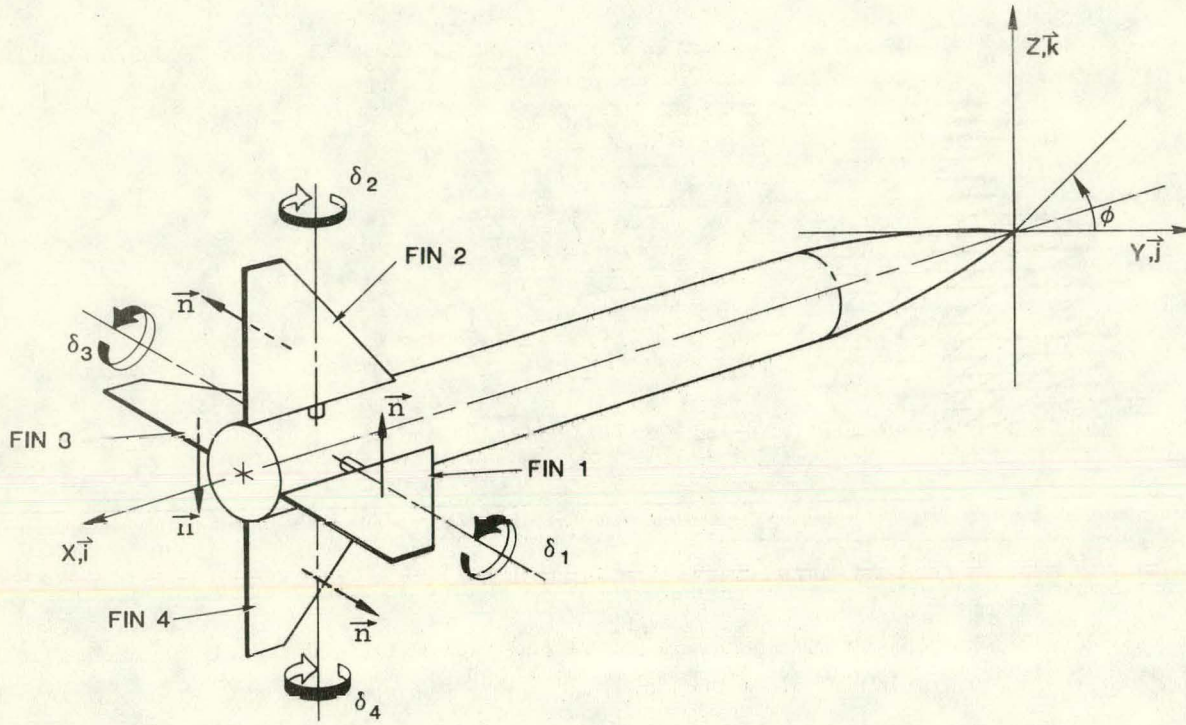


Figure 10. Surface Normal Vector and Sign Convention for Fin Control Deflection

The effective leading edge sweep, $\hat{\Lambda}_{le}$, and effective trailing edge sweep, $\hat{\Lambda}_{te}$, are derived by relating the average flow velocity along the fin to the leading and trailing edge unit vectors, respectively. The most convenient coordinate system to use is cylindrical coordinates with the orthogonal unit vectors located in the plane of the fin (see Figure 11). Let \bar{v}_c and \bar{w}_c be the average cross-flow velocity components along the exposed semispan of the fin at a given roll angle; then,

$$\bar{v}_c = \frac{1}{b_0 - a} \int_a^{b_0} v_c(r, \phi) dr$$

and

$$\bar{w}_c = \frac{1}{b_0 - a} \int_a^{b_0} w_c(r, \phi) dr$$

where v_c and w_c are given by Eq (2). Rewriting these equations in terms of fin-oriented coordinates, one has

$$\bar{v}_c = \int_0^1 v_c(\eta, \phi) d\eta \quad (24a)$$

and

$$\bar{w}_c = \int_0^1 w_c(\eta, \phi) d\eta \quad (24b)$$

Using the cylindrical coordinate unit vectors \bar{e}_x, \bar{e}_r , shown in Figure 11, the average velocity along the leading edge is

$$\bar{V} = \bar{u} \bar{e}_x + \bar{v}_r \bar{e}_r$$

where

$$\bar{u} = U_\infty \cos \alpha_b$$

$$\bar{v}_r = \bar{v}_c \cos \phi + \bar{w}_c \sin \phi$$

Since the sweep angle is measured from a line perpendicular to the approach velocity, the unit vector perpendicular to the average velocity is

$$\bar{V}_\perp = \bar{v}_r / \sqrt{\bar{u}^2 + \bar{v}_r^2} \bar{e}_x + \bar{u} / \sqrt{\bar{u}^2 + \bar{v}_r^2} \bar{e}_r \quad (25)$$

The leading edge unit vector is

$$\bar{e}_{le} = \sin \Lambda_{le} \bar{e}_x + \cos \Lambda_{le} \bar{e}_r \quad (26)$$

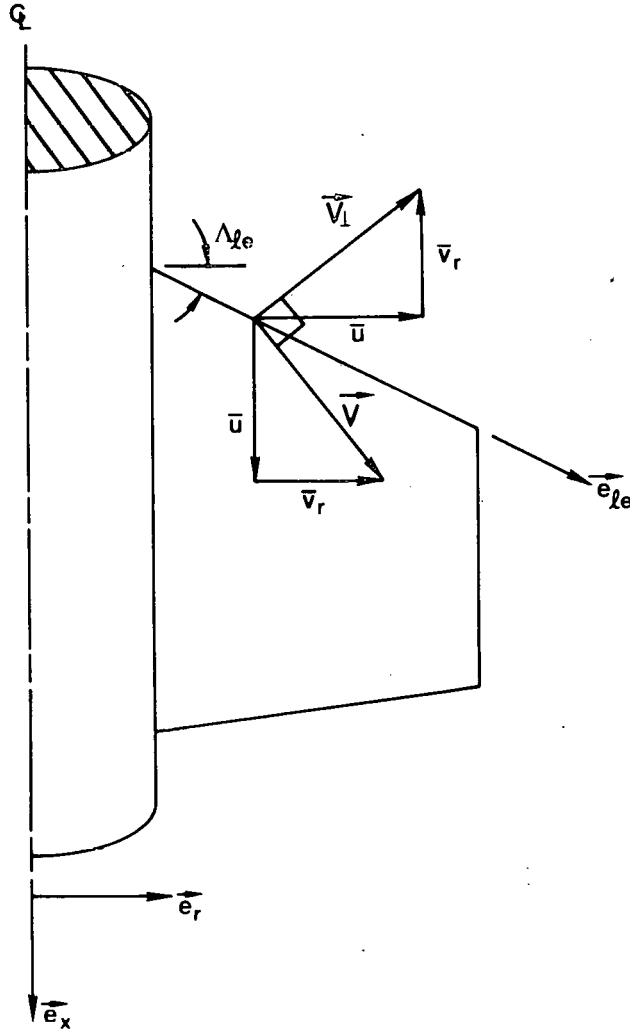


Figure 11. Fin Surface Cylindrical Coordinates

The effective leading edge sweep angle is the angle between \vec{V}_\perp and $\vec{e}_{\theta e}$. Using the scalar product, one obtains

$$\hat{\Lambda}_{\theta e} = \cos^{-1}(\vec{V}_\perp \cdot \vec{e}_{\theta e})$$

Substituting in Eqs (25) and (26),

$$\hat{\Lambda}_{\theta e} = \cos^{-1}[(\bar{u} \cos \Lambda_{\theta e} - \bar{v}_r \sin \Lambda_{\theta e}) / \sqrt{\bar{u}^2 + \bar{v}_r^2}] \quad (27)$$

This expression presents difficulties in evaluation because the arc cosine function is double valued. This problem can be nicely circumvented by noting that the argument of the arc cosine function suggests $\hat{\Lambda}_{\theta e}$ might be split into two angles. Let

$$\hat{\Lambda}_{\theta e} = \Lambda_{\theta e} + \epsilon_{\theta e} \quad (28)$$

where $\epsilon_{\theta e}$ is the deviation of the effective leading edge sweep from the geometric leading edge sweep. Now Eq (27) can be written as

$$\begin{aligned} \cos(\Lambda_{\theta e} + \epsilon_{\theta e}) &= \frac{\bar{u}}{\sqrt{\bar{u}^2 + \bar{v}_r^2}} \cos \Lambda_{\theta e} \\ &\quad - \frac{\bar{v}_r}{\sqrt{\bar{u}^2 + \bar{v}_r^2}} \sin \Lambda_{\theta e} \end{aligned}$$

Using the trigonometric identity for the cosine of a sum, one has

$$\cos \epsilon_{\theta e} \cos \Lambda_{\theta e} - \sin \epsilon_{\theta e} \sin \Lambda_{\theta e} =$$

$$\frac{\bar{u}}{\sqrt{\bar{u}^2 + \bar{v}_r^2}} \cos \Lambda_{\theta e} - \frac{\bar{v}_r}{\sqrt{\bar{u}^2 + \bar{v}_r^2}} \sin \Lambda_{\theta e}$$

In matching terms, one notes that two mathematically equivalent expressions can be written for $\epsilon_{\theta e}$. The one involving the cosine function, however, suffers from the same difficulty mentioned above; therefore use

$$\sin \epsilon_{\theta e} = \bar{v}_r / \sqrt{\bar{u}^2 + \bar{v}_r^2}$$

Substituting this into Eq (28), one finally obtains

$$\hat{\Lambda}_{\theta e} = \Lambda_{\theta e} + \sin^{-1}(\bar{v}_r / \sqrt{\bar{u}^2 + \bar{v}_r^2}) \quad (29a)$$

Using exactly the same procedure, the effective trailing edge sweep is given by

$$\hat{\Lambda}_{te} = \Lambda_{te} + \sin^{-1}(\bar{v}_r / \sqrt{\bar{u}^2 + \bar{v}_r^2}) \quad (29b)$$

The effective leading and trailing edge sweep caused by high angle of attack results in an effective aspect ratio of the fin. This is significant since the local normal force coefficient due to angle of attack, $C_N(\alpha_e)$, will now reflect effective changes in fin geometry due to apparent yaw of the fin. To derive an expression for \hat{A}_e , begin with the definition of A_e

$$A_e = 2(b_o - a)^2 / S_f$$

where S_f is the exposed planform area of a fin. Writing this in terms of fin semispan, root chord, and leading and trailing edge sweep, one has

$$A_e = 4/[2c_r/(b_o - a) - \tan \Lambda_{\theta e} + \tan \Lambda_{te}] \quad (30)$$

Referring to Figure 12, it can be seen that the exposed fin semispan and root chord also effectively change. It can be shown that

$$\widehat{b_o - a} = (b_o - a) \cos \hat{\Lambda}_{le} / \cos \Lambda_{le} ; \quad (31a)$$

$$\hat{c}_r = c_r \cos \Lambda_{te} / \cos(\hat{\Lambda}_{le} - \Lambda_{le} + \Lambda_{te}) . \quad (31b)$$

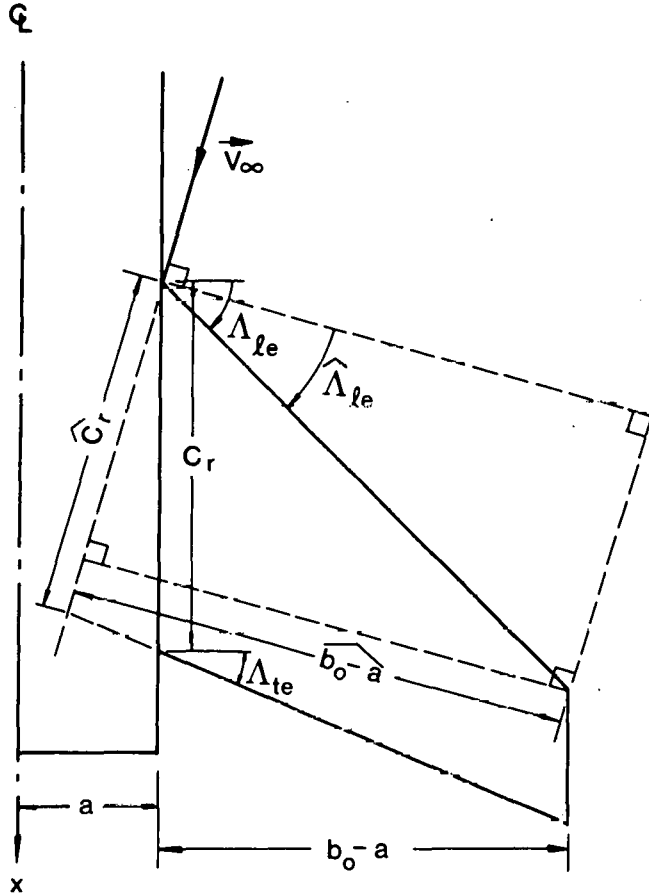


Figure 12. Geometry for Determining Effective Aspect Ratio

Using Eqs (30) and (31), the effective aspect ratio is written as

$$\hat{A}_e = \frac{4}{\frac{2c_r \cos \Lambda_{te} \cos \Lambda_{le}}{(b_o - a) \cos \hat{\Lambda}_{le} \cos(\hat{\Lambda}_{le} - \Lambda_{le} + \Lambda_{te})} - \tan \hat{\Lambda}_{le} + \tan \Lambda_{le}} \quad (32)$$

e. Expression for $C_N(\alpha_e)$

The functional dependence of C_N on the local angle of attack is given by an empirical expression based on lifting surfaces in uniform approach flow. Other investigators (for example, Reference 6) have used experimental data for $C_N(\alpha_e)$ directly in their

analysis. This is rarely an appealing approach because it requires vast amounts of data to construct a data base sufficiently general to address general planforms over a large range of Mach numbers. Also, this is not possible in the present approach because the effective geometry of the fin (discussed earlier) changes with the character of the nonuniform approach flow.

As the local angle of attack on a missile fin can be approximately 40° to 50° when the angle of attack of the body is 25° , the expression for $C_N(\alpha_e)$ must include prestall, stall, and poststall characteristics. The prestall expression for $C_N(\alpha_e)$ is taken from the work of Lecat and Rietschlin.²⁰ Their analysis is based on Polhamus suction analogy.²¹ They extended the original work of Polhamus on delta wings to include arbitrary planforms and transonic flow. Their analysis is not repeated here, but their equations are given in the present nomenclature.

Their expression for $C_N(\alpha)$ is

$$C_N(\alpha) = K_p \sin \alpha \cos \alpha + K_v \sin^2 \alpha . \quad (33)$$

Note that this equation shows C_N continually increasing with α and, as a result, is not appropriate for angles near or greater than the stall angle. Although Lecat and Rietschlin state their method is applicable to subsonic and supersonic flow, the present analysis is considered appropriate for subsonic flow and only supersonic flow with subsonic leading edges.

The calculation procedure for K_p and K_v is as follows:

- (1) Calculate $\widehat{b_o - a}$, \hat{c}_r , and $\hat{\Lambda}_e$ from Eqs (31) and (32).
- (2) Calculate the distance from x_1 to the aftmost point of maximum span, c_r^*

$$c_r^* = \hat{c}_r + (\widehat{b_o - a}) \tan \hat{\Lambda}_{le} \quad (34a)$$

- (3) Calculate the sweep of the semispan diagonal, γ .

$$\gamma = \tan^{-1} [\hat{c}_r / (\widehat{b_o - a}) + \tan \hat{\Lambda}_{le}] \quad (34b)$$

- (4) Calculate the ratio of planform area to rectangular reference area, S_f/S_f^*

$$S_f/S_f^* = 2/[\hat{A}_e \tan \gamma] \quad (34c)$$

- (5) If $M_\infty > 1$, calculate the complement of the Mach angle, γ_M .

$$\gamma_M = \cos^{-1}(1/M_\infty) \quad (34d)$$

- (6) Calculate the planform parameter, p^* .

$$p^* = \begin{cases} S_t/S_i^* & \text{for } M_\infty \leq 1 \\ \frac{S_t/S_i^*}{1 - [\tan \gamma_M / (2 \tan \gamma)]} & \text{for } M_\infty > 1 \end{cases} \quad (34e)$$

- (7) Calculate the planform angle, ψ .

$$\psi = \begin{cases} \tan^{-1} (2p^* \tan \gamma) & M_\infty \leq 1 \\ \tan^{-1} [2p^* (\tan \gamma - \tan \gamma_M)] & M_\infty > 1 \end{cases} \quad (34f)$$

- (8) Calculate the compressibility factor, β .

$$\beta = \begin{cases} \sqrt{1 - M_\infty^2} & M_\infty \leq 1 \\ \sqrt{M_\infty^2 - 1} & M_\infty > 1 \end{cases} \quad (34g)$$

- (9) Calculate the potential flow lift coefficient, K_p .

$$K_p = \frac{4\pi}{\tan \psi + \sqrt{\tan^2 \psi + (\sin^2 \psi / p^2) + 4\beta^2}} \quad (34h)$$

- (10) Calculate the vortex flow lift coefficient, K_v .

$$K_v = [K_p - (K_p^2 \tan \psi) / 4\pi] \sqrt{1 + \tan^2 \psi} \quad (34i)$$

An empirical expression was devised for the normal force coefficient, $C_N(\alpha_t)$, based on the expression of Lecat and Rietschlin, Eq (33). The new expression is more general since it applies both at stall and beyond stall. This expression was constructed after examining and comparing a large number of trial expressions with experimental data^{10,15-23} for a wide variety of planforms. This expression is

$$C_N(\alpha_t) = \begin{cases} (K_p \sin \alpha_t \cos \alpha_t + K_v \sin^2 \alpha_t) [1 - (1-\mu)(\alpha_t/\alpha_s)^2] & 0 \leq \alpha_t \leq \alpha_s \\ f(A_e, \Lambda_{te}) C_N < \alpha_s > + [A_s \cos \Lambda_{te} C_N < \alpha_t - \alpha_s >] / 10 & \alpha_s < \alpha_t \leq 2\alpha_s \\ f(A_e, \Lambda_{te}) C_N < \alpha_s > + [A_s \cos \Lambda_{te} C_N < \alpha_t >] / 10 & 2\alpha_s < \alpha_t \end{cases} \quad (35a)$$

where the symbol $< >$ is used to denote functional dependence in situations where the standard symbol, which is a set of parentheses, would be confusing. Also,

$$\mu = 0.9 - 0.2 A_e \sin 2\Lambda_{te}$$

$$\alpha_s = 38[1 + 0.02(A_e - 2)^4][1 + 1.1(A_e - 1)^{2.5} \cos^2 \Lambda_{te}]$$

$$[1 + [6(A_e - 1)^2 \sin^4 \Lambda_{te}] / e^{\Lambda_s}] / [\cos \Lambda_{te} + 1]^{2(A_e - 1)} \quad (35b)$$

$$f(A_e, \Lambda_{te}) = 1 - 0.35 \left\{ 1 + \frac{1}{8} e^{\Lambda_s \sin^2 \Lambda_{te}} \sin [(A_e - 2.1)\Lambda_{te}] \right\} / [1 + 0.5(A_e - 1)^3] \quad (35c)$$

and K_p and K_v are given by Eq (33).

As can be seen, $C_N(\alpha_t)$ is composed of three angle of attack ranges: zero angle up to the stall angle, stall up to twice the stall angle, and angles larger than twice the stall angle. For angles less than α_s , the expression of Lecat and Rietschlin²⁰ (Eq (33)) has been modified for α_t near α_s . Equation (35a) for $\alpha_t < \alpha_s$ shows that as α_t approaches α_s the normal force coefficient increases more slowly. This characteristic is consistent with experimental measurements. The parameter μ represents the portion of C_N predicted by Lecat and Rietschlin which remains at $\alpha_t = \alpha_s$. The expression for α_s , although rather lengthy, gives a good estimate of the stall angle (in degrees) for planforms with aspect ratios from 1 to 5 and leading edge sweep from 0° to 70° .

$f(A_e, \Lambda_{te})$ represents the portion of $C_N(\alpha_s)$ existing for $\alpha_t = \alpha_s^+$. Note that μ , α_s , and $f(A_e, \Lambda_{te})$ show no dependence on free-stream Mach number. This is not necessarily a reflection of the physics, but simply an admission of lack of data.

$C_N(\alpha_t)$ as predicted by Eq (35) was compared with data for a very wide variety of planform geometries in order to determine its range of applicability. Figures 13 through 22 show typical comparisons of Eq (35) with experimental data for incompressible flow. The figures are placed in order of increasing aspect ratio from 1 to 4. The leading edge sweep angle varies from 0° to 70° ; planform geometries include: rectangular, diamond, clipped delta, delta, arrow, clipped arrow, and trapezoidal. The comparisons are good for all planforms except that for the aspect ratio 3 trapezoid (Figure 21) and rectangular planforms of $A_e \geq 3$ (comparisons not shown). For these types of planforms (i.e., high aspect ratio with small leading edge sweep), the normal force past stall is significantly overestimated by the present expression.

f. Fin - Fin Interference

Two types of fin - fin interference are included in the present lifting theory. The first type is caused by control deflection of the fins, and the second type by the rolling motion of the missile. When a control deflection is input to a fin, the lifting flow field of that fin induces an angle of attack on the adjacent fins. This induced angle of attack causes the adjacent fins to generate forces and moments which depend

on the magnitude and direction of the control input. Rolling motion interference occurs for a similar reason except that the angle of attack of a fin is generated by the angular velocity of the fin relative to the oncoming stream. Control deflection interference will be considered first and then rolling motion interference will be analyzed. Results of slender-body theory are used to evaluate both types of interference considered here.

Figure 23 (taken from Reference 24) shows the interference flow fields and pressures induced on adjacent fins for two types of control input: positive pitch control and positive roll control with horizontal fins. For the pitch control, it is seen that a negative pressure coefficient is produced on both sides of the top fin and a positive pressure coefficient is generated on both sides of the bottom fin. For roll control (i.e., differential deflection), a negative pressure is generated on the top right and bottom left sides of the vertical fins, and a positive pressure is generated on the top left and bottom right sides of the vertical fins. In the present analysis these induced pressures are included by considering the adjacent fins to be at an effective deflection angle.

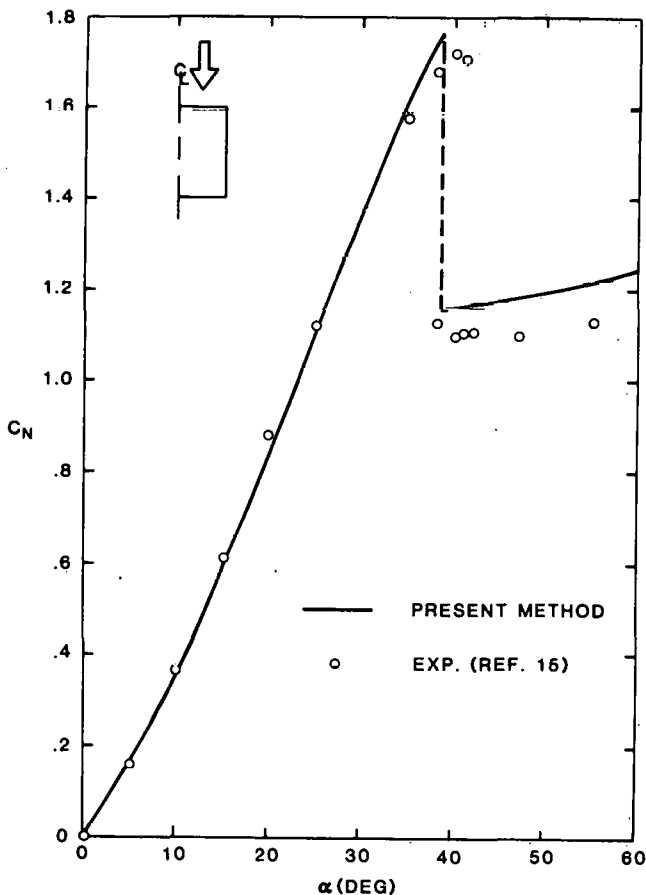


Figure 13. Normal Force Coefficient vs α for Rectangular Planform ($A_e = 1$)

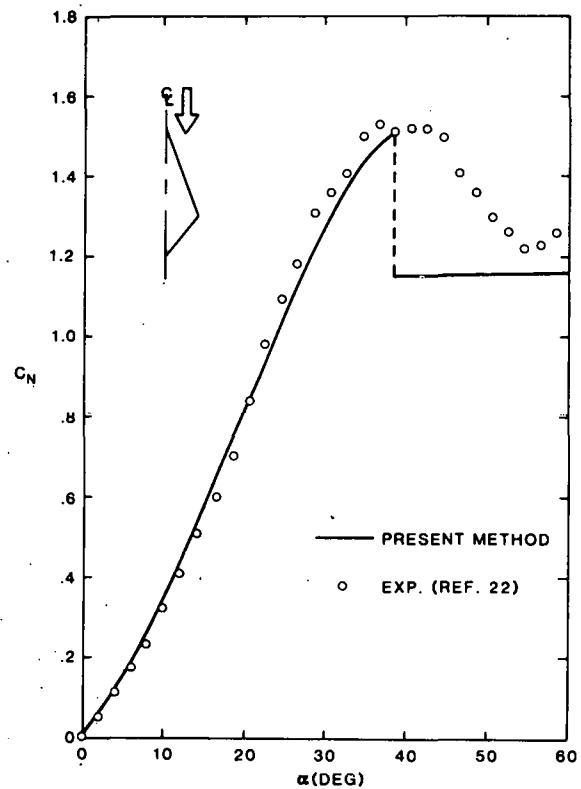


Figure 14. Normal Force Coefficient vs α for Diamond Planform ($A_e = 1$)

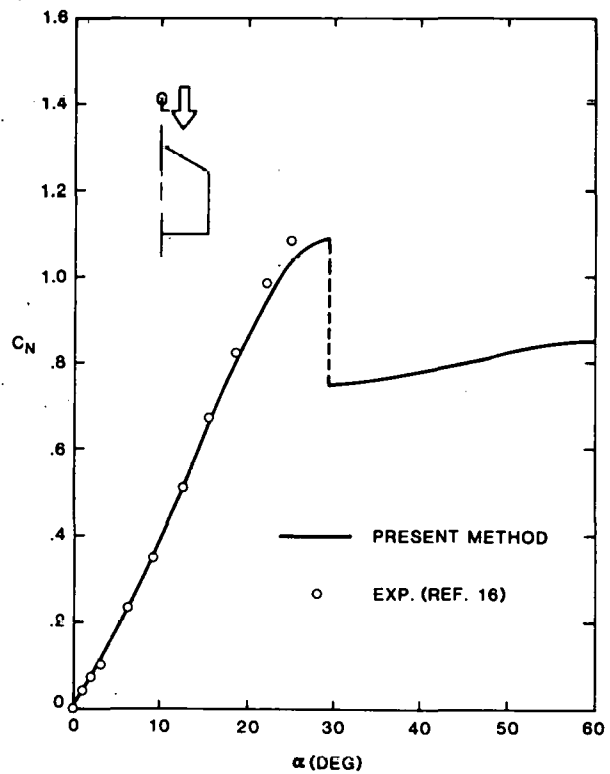


Figure 15. Normal Force Coefficient vs α for Clipped Delta Planform ($A_e = 1.23$)

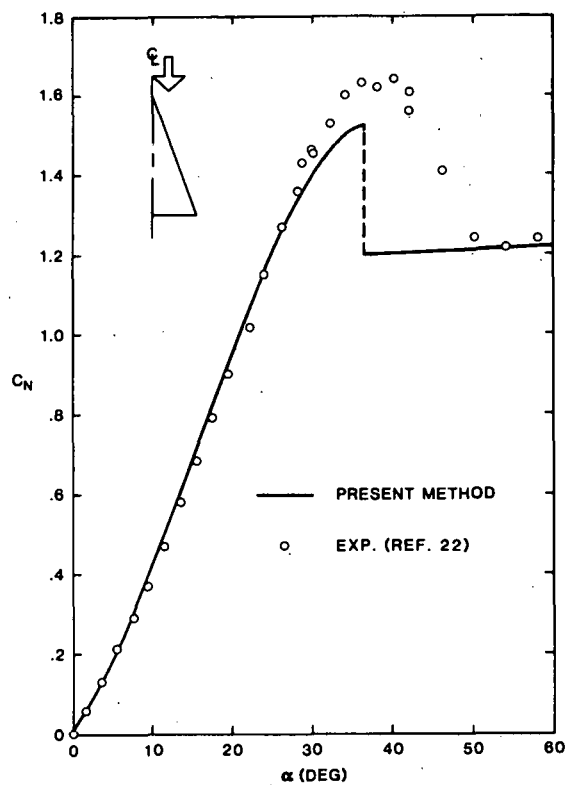


Figure 16. Normal Force Coefficient vs α for Delta Planform ($A_e = 1.46$)

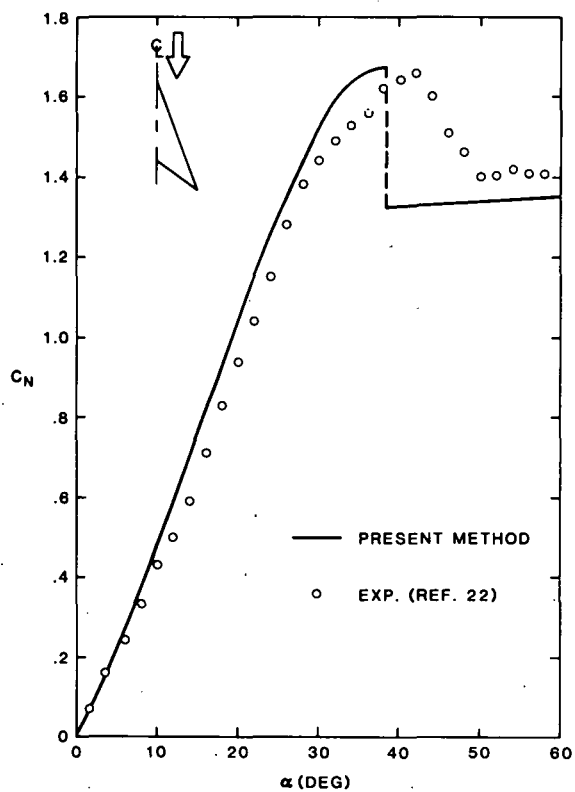


Figure 18. Normal Force Coefficient vs α for Arrow Planform ($A_e = 2$)

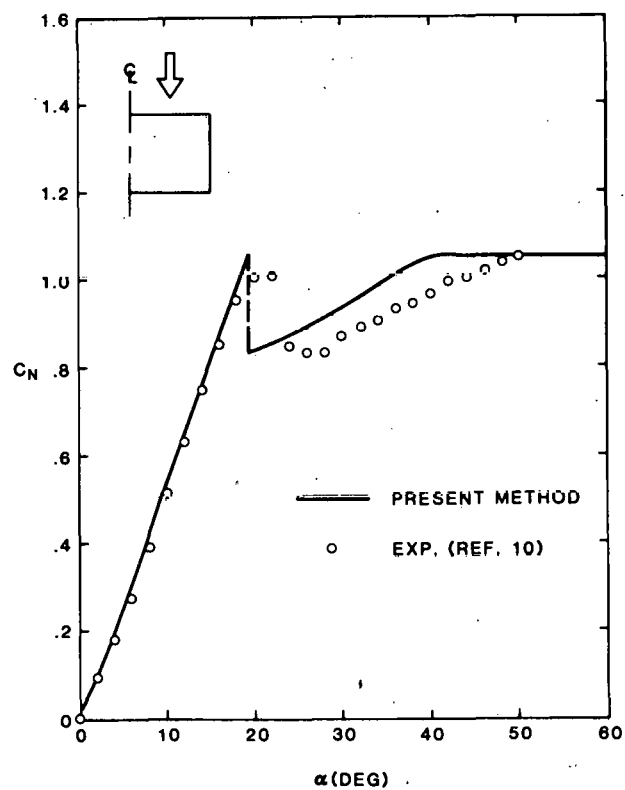


Figure 17. Normal Force Coefficient vs α for Rectangular Planform ($A_e = 2$)

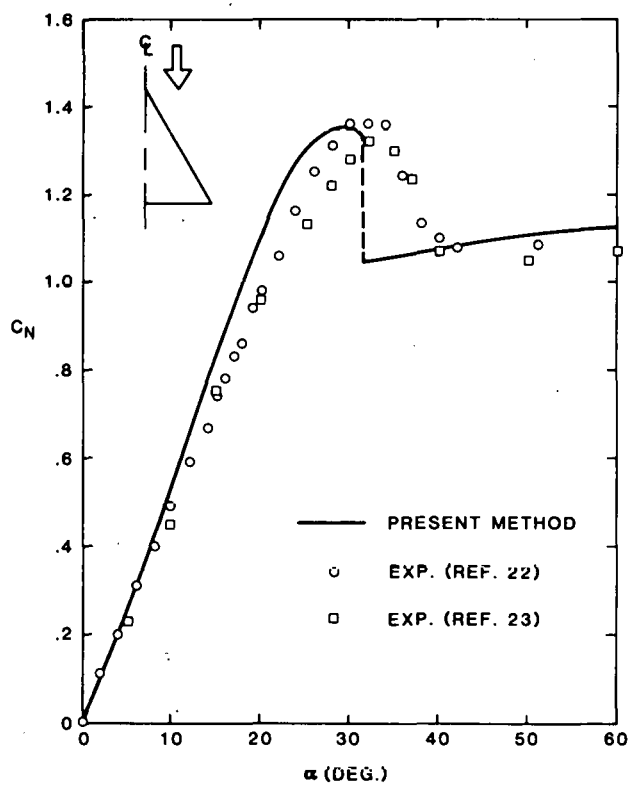


Figure 19. Normal Force Coefficient vs α for Delta Planform ($A_e = 2.31$)

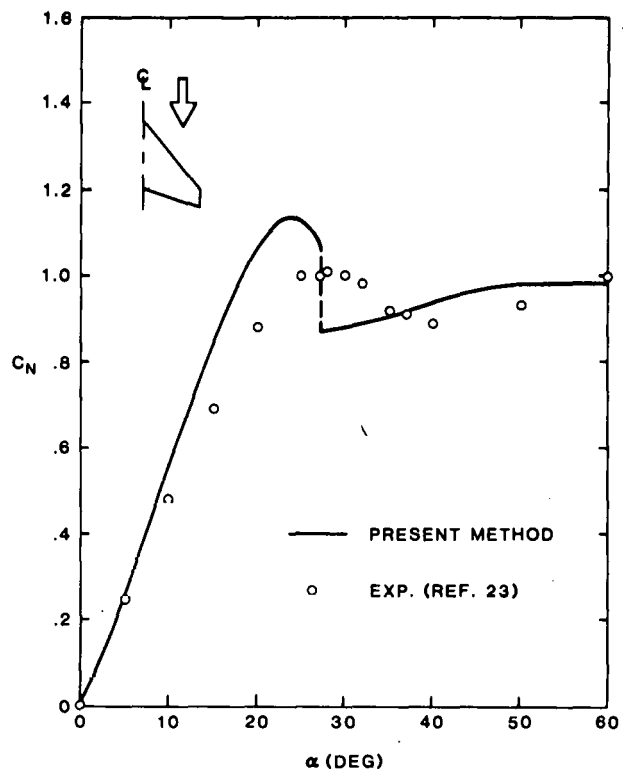


Figure 20. Normal Force Coefficient vs α for Clipped Arrow Planform ($A_e = 2.61$)

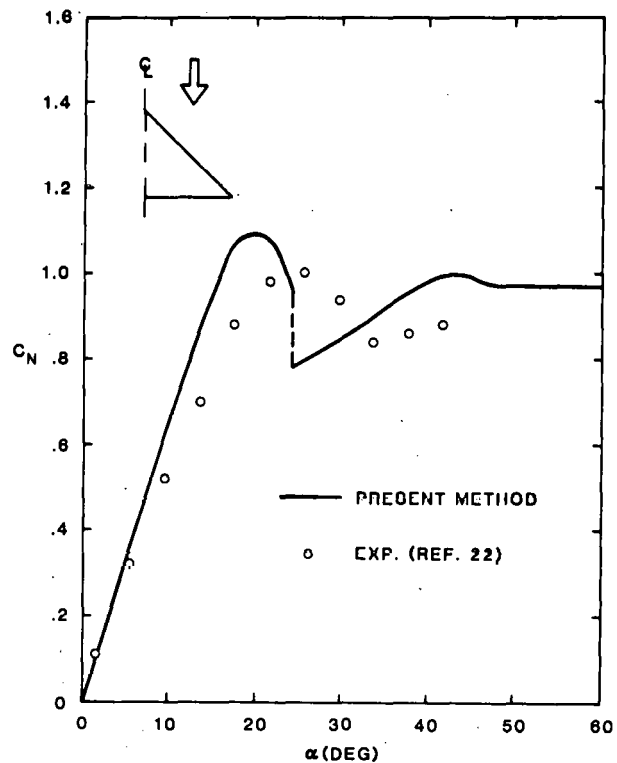


Figure 22. Normal Force Coefficient vs α for Delta Planform ($A_e = 4$)

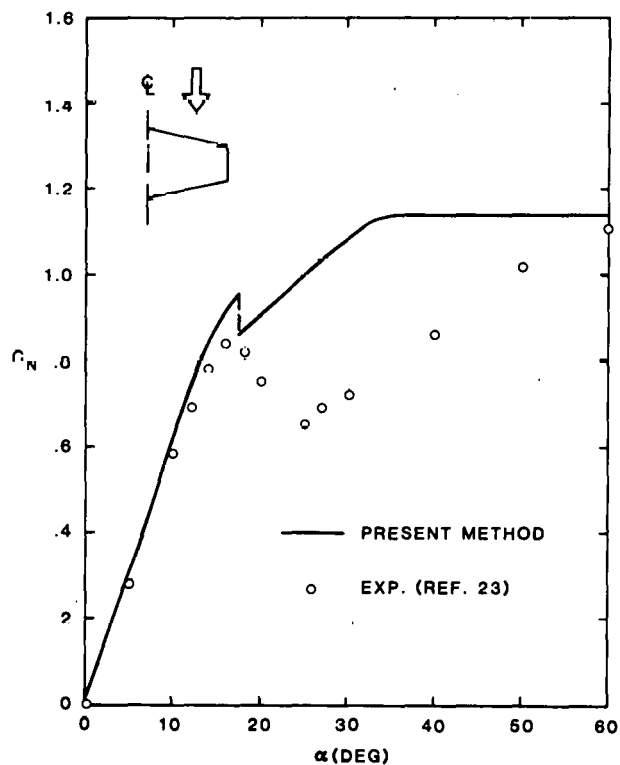


Figure 21. Normal Force Coefficient vs α for Trapezoidal Planform ($A_e = 3$)

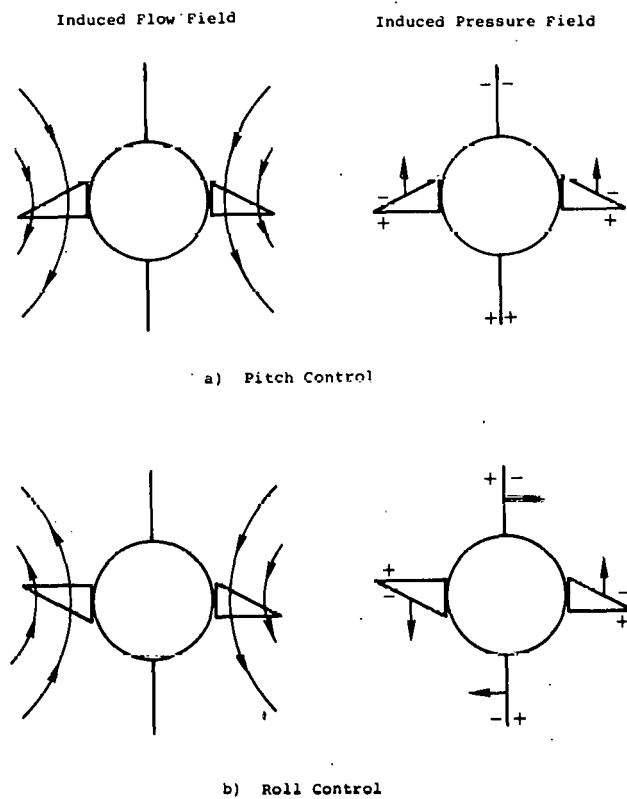


Figure 23. Control Interference for Pitch and Roll Control (from Ref. 24)

Consider the induced pressures on adjacent fins for individual deflection of each fin. Shown in Figure 24 is the induced pressure and effective deflection force for positive deflection of each fin. If we let I_d be the deflection interference coefficient, the following equations describe the interference depicted in Figure 24.

$$\begin{aligned} \text{for } \delta_1 > 0 \quad \begin{cases} \delta_{e_2} = I_d \delta_1 \\ \delta_{e_3} = 0 \\ \delta_{e_4} = -I_d \delta_1 \end{cases} & \quad \text{for } \delta_2 > 0 \quad \begin{cases} \delta_{e_1} = I_d \delta_2 \\ \delta_{e_3} = -I_d \delta_2 \\ \delta_{e_4} = 0 \end{cases} \\ \text{for } \delta_3 > 0 \quad \begin{cases} \delta_{e_1} = 0 \\ \delta_{e_2} = -I_d \delta_3 \\ \delta_{e_4} = I_d \delta_3 \end{cases} & \quad \text{for } \delta_4 > 0 \quad \begin{cases} \delta_{e_1} = -I_d \delta_4 \\ \delta_{e_2} = 0 \\ \delta_{e_3} = I_d \delta_4 \end{cases} \end{aligned} \quad (36)$$

Summing all of the effective control deflections given in Eq (36) and including the actual control deflection, one obtains

$$\begin{aligned} \delta_{e_1} &= \delta_1 + I_d(\delta_2 - \delta_4) \\ \delta_{e_2} &= \delta_2 + I_d(\delta_1 - \delta_3) \\ \delta_{e_3} &= \delta_3 + I_d(\delta_4 - \delta_2) \\ \delta_{e_4} &= \delta_4 + I_d(\delta_3 - \delta_1) \end{aligned} \quad (37)$$

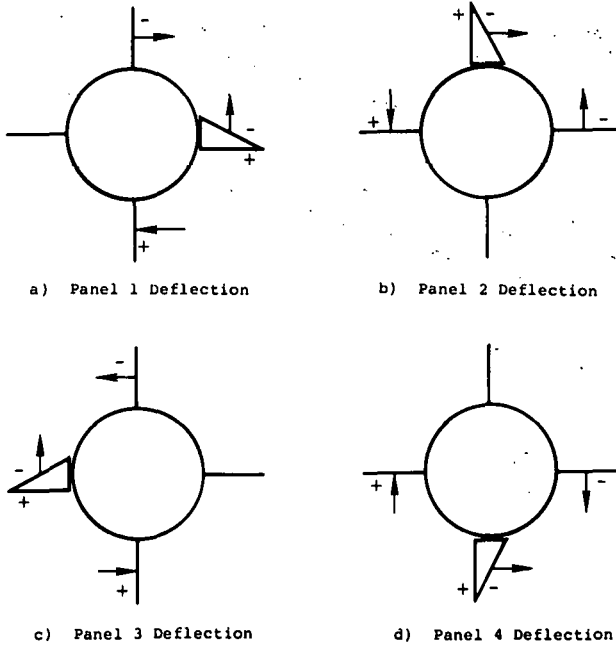


Figure 24. Control Interference for Individual Panel Deflection

I_d is evaluated by using the slender body theory results of Adams and Dugan.²⁵ They derived results from the roll moment coefficient derivative, C_{ℓ_b} , vs a/b_o for differential deflection of two fins. They further showed results for the roll moment coefficient induced on the vertical fins because of differential deflection of the horizontal fins. Assuming the induced angle of attack of the upwash and downwash of the deflected fin on the undeflected fin is constant along the span, one can write

$$\frac{[C_{\ell_b}]_{\text{induced}}}{[C_{\ell_b}]_{\text{deflected}}} = \frac{\delta_e}{\delta} \quad (38)$$

This equation can be proven, given the stated assumption, by writing the roll moment of the induced and deflected fin in double integral form. Noting that the ratio shown in Eq (38) is for differential deflection of two fins, one has

$$I_d = \frac{1}{2} \frac{[C_{\ell_b}]_{\text{induced}}}{[C_{\ell_b}]_{\text{deflected}}} \quad (39)$$

Figure 25 plots I_d vs a/b_o , using the results of Adams and Dugan for $[C_{\ell_b}]_{\text{induced}}$ and $[C_{\ell_b}]_{\text{deflected}}$; it should be noted that they use the planform aspect ratio of the fins, A_p . In the present nomenclature

$$A_p = b^2/S_{fp} \quad (40a)$$

where S_{fp} is the planform area of two fins, including their imaginary extension through the body. It can be shown that

$$S_{fp} = 2 S_f + a[2 c_r + a(\tan \Lambda_{te} - \tan \Lambda_{te})] \quad (40b)$$

I_d , as given in Figure 25, is used in Eq (37) to calculate the effective control deflection of each fin.

Now consider the case of fin - fin interference due to rolling motion. The strategy of this derivation is to determine the interference coefficient by matching the present roll moment formulation for zero angle of attack of the body to that obtained by slender body theory. Assuming $\alpha_b = 0$ and the rolling speed is $\dot{\phi}$, the differential roll moment can be written (Figure 26)

$$d\ell = 4 r C_n \frac{1}{2} \rho_{\infty} [U_{\infty} + (\dot{\phi} r)]^2 dx dr$$

Assuming the rolling speed is small compared to U_∞ and substituting in C_n from Eq (14), one has

$$d\ell = 4 q_\infty r \sigma C_n(\alpha_e) C(x, r) S(r) dx dr \quad (41a)$$

where α_e is the effective local angle of attack along the span. Adams and Dugan²⁵ showed that the spanwise loading induced on one fin by the other fins was roughly quadratic along the span. In the present analysis, this induced loading will be simply approximated by a linear spanwise function; therefore,

$$\alpha_e = -\frac{\dot{\phi} r}{U_\infty} + I_p \frac{\dot{\phi} r}{U_\infty} \quad (41b)$$

where I_p is the rolling motion interference coefficient. The first term in Eq (41b) is the local angle of attack caused by rolling speed and the second is the reduction in local angle of attack caused by fin - fin interference.

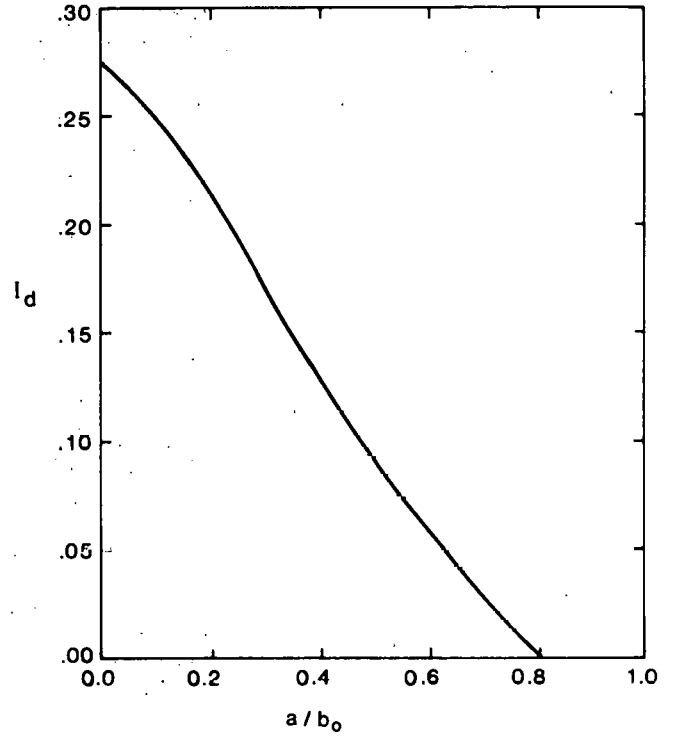


Figure 25. Deflection Interference Coefficient vs a/b_0

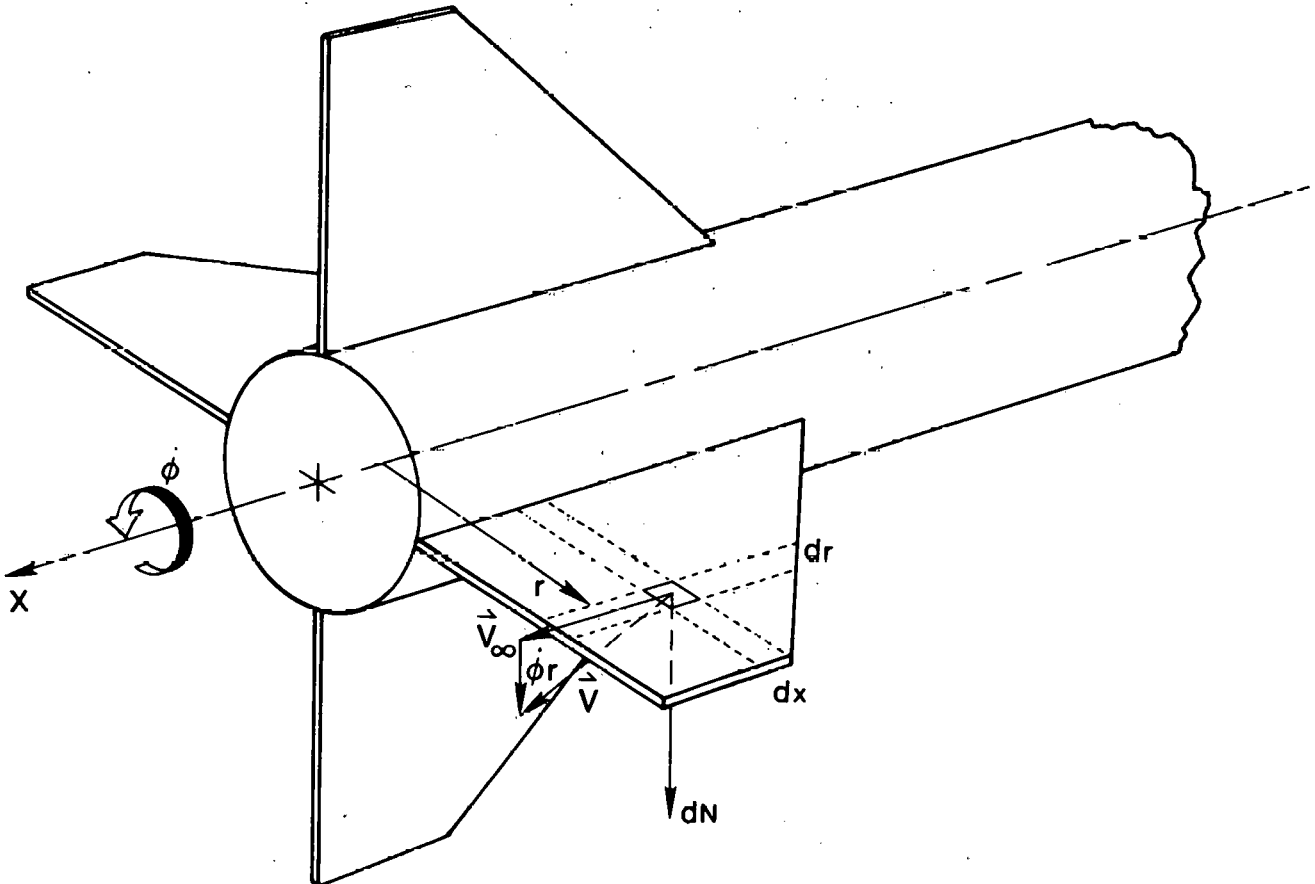


Figure 26. Induced Angle of Attack Due to Rolling Speed

Integrating over the surface of the fin, Eq (41a) becomes

$$\ell = 4 \sigma q_{\infty} \int_a^{b_0} \int_{x_{te}}^{x_{ie}} r C_N(\alpha_e) C(x, r) S(r) dr dx. \quad (42)$$

Assuming α_e is small and using Eq (41b), Eq (42) can be written

$$\ell = -4 \sigma q_{\infty} C_{N_a} (1 - I_p) \frac{\dot{\phi}}{U_{\infty}} \int_a^{b_0} \int_{x_{te}}^{x_{ie}} r^2 C(x, r) S(r) dx dr.$$

Transforming coordinates (x, r) to fin coordinates (ξ, η) and nondimensionalizing with respect to $q_{\infty} S_b d$, one obtains

$$C_{\ell} = -\frac{2}{\pi} \sigma C_{N_a} \frac{(b_0 - a)}{a^3 b_0} (1 - I_p) \frac{\dot{\phi} b_0}{U_{\infty}} \int_0^1 \int_0^1 \mathfrak{N}^2(\eta) \Omega(\eta) C(\xi) S(\eta) d\xi d\eta \quad (43a)$$

where

$$\mathfrak{N}(\eta) = a + (b_0 - a)\eta. \quad (43b)$$

Taking the partial derivative of Eq (43) with respect to p , where $p = \dot{\phi} b_0 / U_{\infty}$, and solving for I_p , one finally obtains

$$I_p = 1 + \frac{C_{\ell_p}}{\frac{2}{\pi} \sigma \frac{(b_0 - a)}{a^3 b_0} C_{N_a} \int_0^1 \int_0^1 \mathfrak{N}^2(\eta) \Omega(\eta) C(\xi) S(\eta) d\xi d\eta} \quad (44)$$

C_{ℓ_p} is computed from the results of Adams and Dugan (see Figure 27). C_{N_a} is obtained by numerically differencing Eq (35) for $\alpha_e \rightarrow 0$.

Roll Moment

The above described body flow field and lifting theory will now be applied to the prediction of the roll moment produced by cruciform fins. If one desired, the present analysis could be applied in a straightforward manner to two or three fin configurations. Consider the missile at angle of attack α_b , roll angle ϕ , and roll rate $\dot{\phi}$. The roll moment produced by a differential surface element on each of the four fins is

$$d\ell = q_{\infty} r \sum_{j=1}^4 C_{n_j} (U_j / U_{\infty})^2 dx dr$$

where j refers to the j 'th fin and U_j is the magnitude of the velocity on the j 'th fin. Substituting C_n from Eq (14) and integrating over the fin surfaces, one has

$$\ell = \sigma q_{\infty} \int_a^{b_0} \int_{x_{te}}^{x_{ie}} r C(x, r) S(r) \sum_{j=1}^4 C_{N_j} (U_j / U_{\infty})^2 dx dr.$$

Transforming to fin coordinates (ξ, η) and nondimensionalizing by $q_{\infty} S_b d$, one obtains the roll moment coefficient

$$C_{\ell} = \frac{\sigma(b_0 - a)}{2 \pi a^3} \int_0^1 \int_0^1 \mathfrak{N}(\eta) \Omega(\eta) C(\xi) S(\eta) \sum_{j=1}^4 C_{N_j} (U_j / U_{\infty})^2 d\xi d\eta \quad (45)$$

where $\mathfrak{N}(\eta)$ is given by Eq (43b), $\Omega(\eta)$ is given by Eq (20b), $C(\xi)$ is given by Eq (19a), and $S(\eta)$ is given by Eq (19b).

U_j is found by taking the magnitude of the local velocity given in Eq (23):

$$U_j = \sqrt{U_{\infty}^2 \cos^2 \alpha_b + (v_q + \dot{\phi} r \sin \phi)^2 + (w_q - \dot{\phi} r \cos \phi)^2}. \quad (46)$$

Note that if ϕ appears without the subscript j (as in the above equation), the angle ϕ is measured from the y axis for the particular fin under consideration.

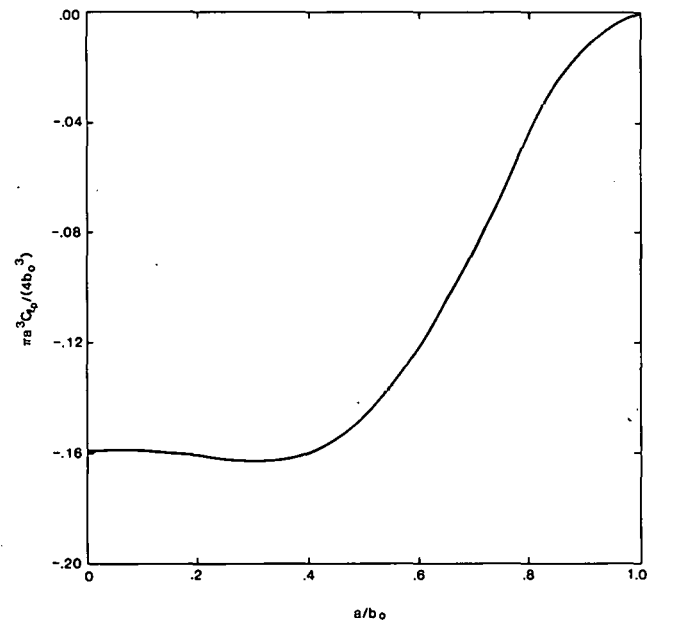


Figure 27. Roll Damping Moment Coefficient vs a/b_0

C_{N_j} depends upon the local effective angle of attack α_{e_j} . To determine this angle, the effective surface normal vector must be found taking into account control deflection and rolling motion interference. Using \vec{n} from Eq (22), the effective fin deflection angle δ_{e_j} from Eq (37), and noting that the rolling motion interference affects the \vec{i} component of the normal vector, one can write*

$$\vec{n}_{e_j} = [\sin(\Delta \delta_{e_j}) + I_p \dot{\phi} r / U_\infty] \vec{i} - \cos(\delta_{e_j}) \sin \phi \vec{j} + \cos(\delta_{e_j}) \cos \phi \vec{k}.$$

Substituting \vec{n}_{e_j} and \vec{V} into the equation for α_e , Eq (21), one obtains the local effective angle of attack.

$$\alpha_{e_j} = \sin^{-1} \{ [U_\infty \cos \alpha_b [\sin(\Delta \delta_{e_j}) + I_p \dot{\phi} r / U_\infty] - (v_{c_j} + \dot{\phi} r \sin \phi) \cos(\delta_{e_j}) \sin \phi + (w_{c_j} - \dot{\phi} r \cos \phi) \cos(\delta_{e_j}) \cos \phi] / (|\vec{n}_{e_j}| U_\infty) \} \quad (47)$$

where

$$|\vec{n}_{e_j}| = 1 + 2 \sin(\Delta \delta_{e_j}) I_p \dot{\phi} r / U_\infty + (I_p \dot{\phi} r / U_\infty)^2.$$

Normal Force and Pitch Moment

The equations for the normal force and pitch moment produced by the fins will now be considered. The differential fin force in the z direction produced by the four fins is (see Figure 28)**

$$dF_z = dN_1 \cos \delta_1 \cos \phi_1 - dN_2 \cos \delta_2 \sin \phi_2 - dN_3 \cos \delta_3 \cos \phi_3 + dN_4 \cos \delta_4 \sin \phi_4.$$

Writing the right side of this equation as a summation and substituting dN_j from Eq (13), one obtains

$$dF_z = q_\infty \sum_{j=1}^4 C_{N_j} (U_j / U_\infty)^2 \cos \delta_j \cos \phi_j dx dr.$$

*Reference 10 accounted for rolling motion interference by way of \vec{j} and \vec{k} components of the velocity. That approach is not considered correct because it also changes the magnitude of the velocity. In fin - fin interference the magnitude of the approach velocity to the fin is not changed, but only the angle with which it approaches the fin.

**Note that the sign of dN_j is determined by its angular relationship to the surface normal vector; that is, positive dN_j is in the counterclockwise sense.

Substituting in C_n from Eq (14) and integrating over the surface of each fin, one has

$$F_z = \sigma q_\infty \int_a^{b_0} \int_{x_{te}}^{x_{le}} C(x, r) S(r) \sum_{j=1}^4 C_{N_j} (U_j / U_\infty)^2 \cos \delta_j \cos \phi_j dx dr.$$

Transforming to fin coordinates (ξ, η) and nondimensionalizing by $q_\infty S_b$, one obtains the normal force coefficient due to the fins:

$$C_z = \frac{\sigma(b_0 - a)}{\pi a^2} \int_0^1 \int_0^1 \Omega(\eta) C(\xi) S(\eta) \sum_{j=1}^4 C_{N_j} (U_j / U_\infty)^2 \cos \delta_j \cos \phi_j d\xi d\eta. \quad (48)$$

The pitch moment produced by the fins is composed of two types of terms: the first term is caused by the normal force just derived and the second by the $\sin \delta_j$ component of the normal force on the fins (Figure 28). The second term produces a pure moment (i.e., a couple) on the missile body and is usually much smaller in magnitude than the first term. The differential pitch moment due to both terms from all four fins is

$$dm = -x dN_1 \cos \delta_1 \cos \phi_1 + r dN_1 \sin(\Delta \delta_1) \sin \phi_1 + x dN_2 \cos \delta_2 \sin \phi_2 + r dN_2 \sin(\Delta \delta_2) \cos \phi_2 + x dN_3 \cos \delta_3 \cos \phi_3 - r dN_3 \sin(\Delta \delta_3) \sin \phi_3 - x dN_4 \cos \delta_4 \sin \phi_4 - r dN_4 \sin(\Delta \delta_4) \cos \phi_4.$$

Writing the right side of this equation as a summation and substituting dN_j from Eq (13), one obtains

$$dm = -q_\infty \sum_{j=1}^4 C_{N_j} (U_j / U_\infty)^2 [x \cos \delta_j \cos \phi_j - r \sin(\Delta \delta_j) \sin \phi_j] dx dr.$$

Substituting in C_n from Eq (14) and integrating over the surface of each fin, one has

$$M = -\sigma q_\infty \int_a^{b_0} \int_{x_{te}}^{x_{le}} C(x, r) S(r) \sum_{j=1}^4 C_{N_j} (U_j / U_\infty)^2 [x \cos \delta_j \cos \phi_j - r \sin(\Delta \delta_j) \sin \phi_j] dx dr.$$

Transforming to fin coordinates (ξ, η) and nondimensionalizing by $q_\infty S_b d$, one obtains the pitch moment coefficient due to the fins:

$$C_m = \frac{-\sigma(b_o - a)}{2\pi a^3} \int_0^1 \int_0^1 \Omega(\eta) C(\xi) S(\eta) \sum_{j=1}^4 C_{N_j} (U_j/U_\infty)^2 \cdot [\mathcal{X}(\xi, \eta) \cos \delta_j \cos \phi - \mathcal{Y}(\eta) \sin(\Delta \delta_j) \sin \phi] d\xi d\eta \quad (49a)$$

where

$$\mathcal{X}(\xi, \eta) = x_l + c_r + (b_o - a) \tan \Lambda_{te} \eta - \Omega(\eta) \xi. \quad (49b)$$

Side Force and Yaw Moment

The derivation of the side force and yaw moment is very similar to that of the normal force and pitch moment and, consequently, is not given. The side force coefficient and the yaw moment coefficient are given by:

$$C_y = \frac{-\sigma(b_o - a)}{\pi a^2} \int_0^1 \int_0^1 \Omega(\eta) C(\xi) S(\eta) \cdot \sum_{j=1}^4 C_{N_j} (U_j/U_\infty)^2 \cos \delta_j \sin \phi d\xi d\eta \quad (50)$$

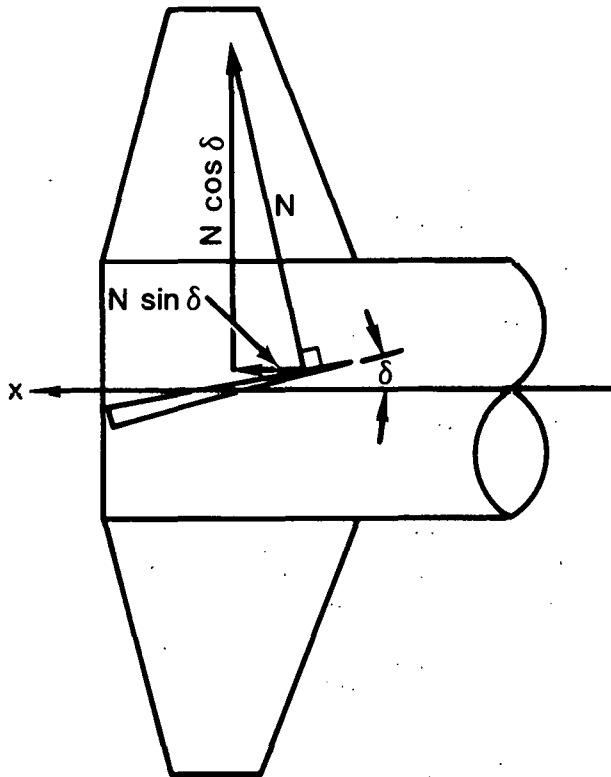


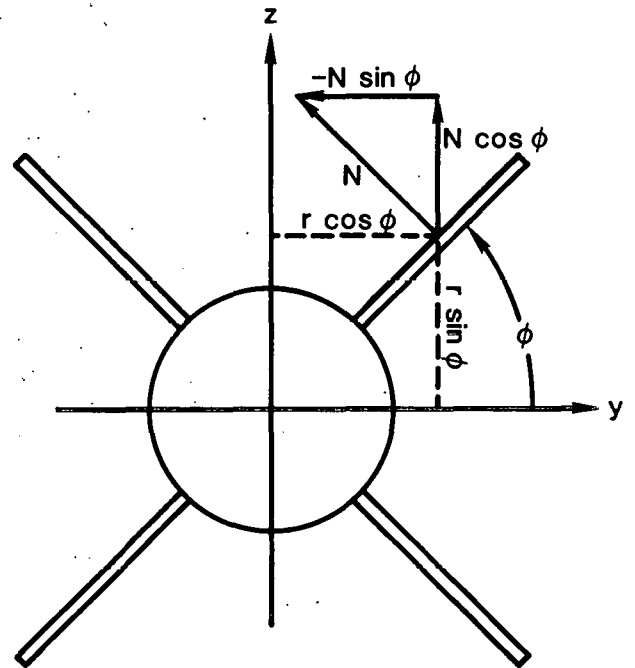
Figure 28. Components of the Fin Normal Force

and

$$C_n = \frac{-\sigma(b_o - a)}{2\pi a^3} \int_0^1 \int_0^1 \Omega(\eta) \sum_{j=1}^4 C_{N_j} (U_j/U_\infty)^2 [\mathcal{X}(\xi, \eta) \cos \delta_j \sin \phi + \mathcal{Y}(\eta) \sin(\Delta \delta_j) \cos \phi] d\xi d\eta. \quad (51)$$

Panel Loads

The panel loads on the fin surfaces are essentially the same forces and moments derived previously (Roll Moment, Normal Force and Pitch Moment, and Side Force and Yaw Moment) except that they are oriented in a fin coordinate system. Figure 29 shows the coordinate system and sign convention for the panel normal force (N), the panel hinge moment (H), and the panel root bending moment (B). The calculation of panel loads is useful for two reasons: first, they provide the most physically meaningful force and moments with which to compare theoretical predictions and experiment because they do not contain the geometry components of the roll angle and the fin-deflection angle. Second, they directly provide the loads needed by the structural designer and the actuator torques needed by the guidance and control designer.



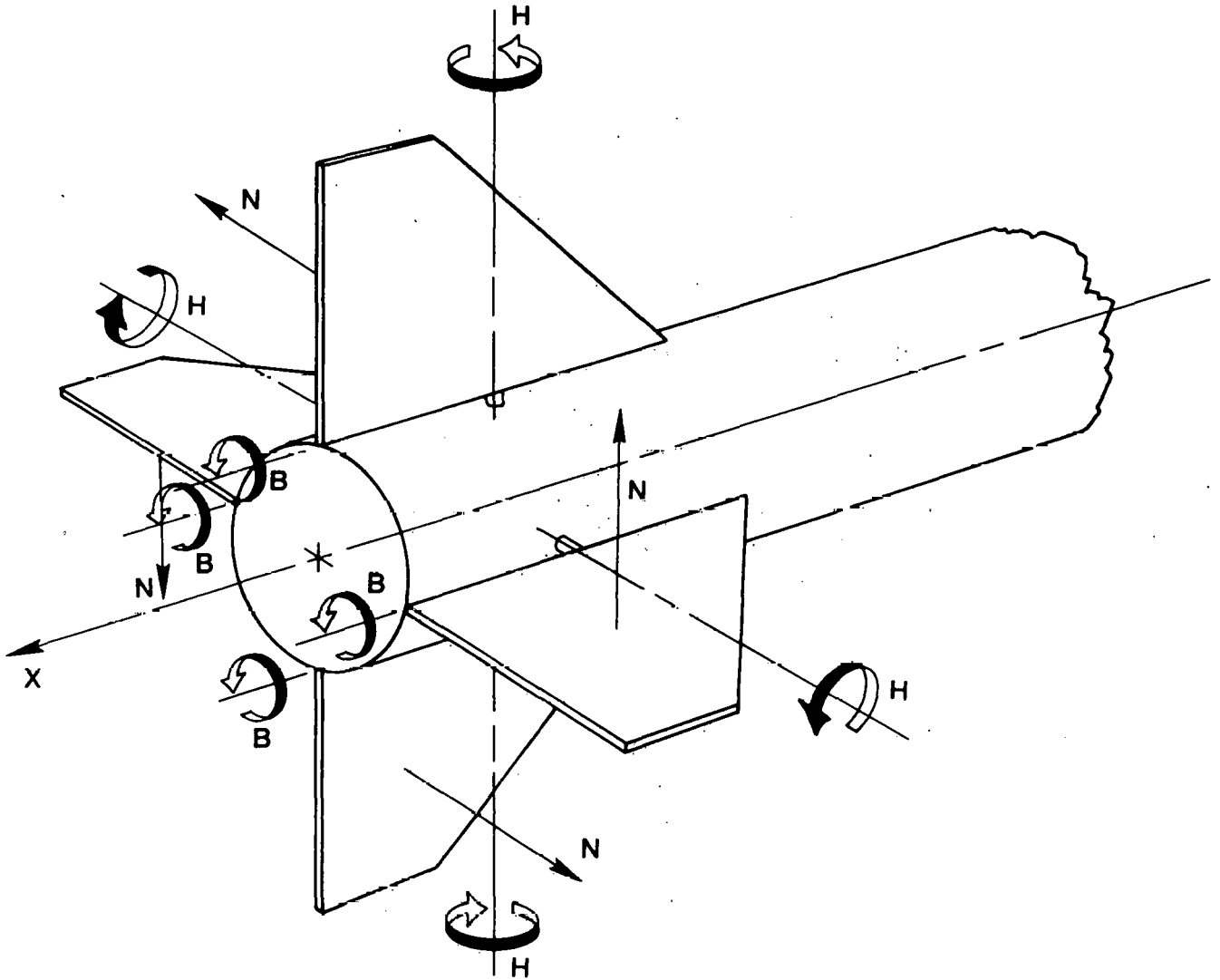


Figure 29. Sign Convention for Panel Normal Force, Hinge Moment, and Root Bending Moment

As the derivation of the panel loads is very similar to the previous derivations, it will only be sketched very briefly. Using Eq (13), the normal force on a fin panel caused by a differential surface element is

$$dN = q_{\infty} C_n (U/U_{\infty})^2 dx dr.$$

Proceeding as before, one obtains the panel normal force coefficient, $N/q_{\infty} S_f$:

$$C_p = \frac{\sigma(b_o - a)}{S_f} \int_0^1 \int_0^1 \Omega(\eta) C(\xi) S(\eta) C_n (U/U_{\infty})^2 d\xi d\eta. \quad (52)$$

Recall that positive normal force points in the direction of the counterclockwise rotational sense.

The panel hinge moment due to a differential surface element is

$$dH = -q_{\infty} (x - x_h) C_n (U/U_{\infty})^2 dx dr$$

where x_h is the axial location of the reference line for the hinge moment. Proceeding as before, one obtains the panel hinge moment coefficient, $H/q_{\infty} c_r S_f$:

$$C_h = \frac{-\sigma(b_o - a)}{c_r S_f} \int_0^1 \int_0^1 [\bar{x}(\xi, \eta) - x_h] \Omega(\eta) C(\xi) S(\eta) C_n (U/U_{\infty})^2 d\xi d\eta. \quad (53)$$

The panel root bending moment due to a differential surface element is

$$dB = q_{\infty}(r-a)C_n(U/U_{\infty})^2 dx dr.$$

Proceeding as before, one obtains the panel root bending moment coefficient, $B/q_{\infty}(b_o - a)S_f$:

$$C_b = \frac{\sigma(b_o - a)}{S_f} \int_0^1 \int_0^1 \eta \Omega(\eta) C(\xi) S(\eta) \cdot C_N(U/U_{\infty})^2 d\xi d\eta. \quad (54)$$

Results and Discussion

The results computed and presented in this investigation were dictated solely by the requirement to validate and determine the bounds of the present theory. Consequently, no results are given without experimental measurements. The theory is evaluated by comparing predictions and measurements for six different fin planforms. The detailed geometry and reference from which the data were taken is given in Table 1. The planforms are shown in Figure 30. It can be seen that the aspect ratios range from 1.0 to 2.0 and the leading edge sweep angles range from 0° to 67.4° .

The results and comparison with experiment are discussed in three sections: panel loads, roll moments, and control forces and moments. The panel loads presented are panel normal force, panel bending moment, and panel hinge moment. Normal force, pitch moment, side force, and yaw moment predictions could have been compared with experimental measurements, but it is felt that the individual panel loads permit more physical insight into the aerodynamic generation of the forces and moments. Roll moment characteristics presented are the induced roll moment, roll damping moment, and steady-state roll rate. These roll characteristics are of great importance in the dynamic flight stability of finned missiles. Control deflection forces and moments presented are pitch (or yaw) control and roll control for various control deflections and roll angles.

Panel Loads

a. Panel Normal Force

The panel normal force on the windward fin (Fin 4) as a function of roll angle for $\alpha_b = 20^\circ$ for Configuration A at $M_{\infty} = 0.8$ is shown in Figure 31.* Also

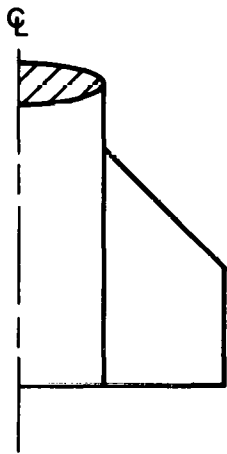
*All panel loads calculated and compared with experimental data are for missile Configuration A.
Configuration A ($M_{\infty} = 0.8$)

shown in Figure 31 is the prediction of Nielsen, Hemsch, and Smith.⁷ As Fin 4 sweeps from $\phi = 0$ to 90° , it moves on the windward side of the body from the angle of attack plane to the (x,y) plane. As can be seen by comparing the present result with experiment, the present method predicts very accurately the loadup of the fin. Above $\phi = 60^\circ$, the present method predicts an increasing normal force, whereas experiment shows a slight drop in normal force. Examining the computer output for spanwise angle of attack and loading reveals that as ϕ increases from zero, the loading increases due to increase in local angle of attack across the span. At $\phi = 40^\circ$, the body upwash near the fin root has increased to the extent that the root chord stalls. As the roll angle increases further, the stall location moves outboard, but the inboard sections begin to add lift as they progress further into poststall. Noting the experimental trend for $\phi > 40^\circ$ and the rise in normal force near $\phi = 80^\circ$ suggests the following explanation of the actual fin loading. The root chord stall is washed outboard due to the spanwise velocity component of the leading edge vortex. This, in turn, decreases the loading along the span such that the total loading decreases; however, as the roll angle increases further and the spanwise location of stall moves outboard, the portion of unstalled span that could be affected steadily decreases. At $\phi = 90^\circ$ it is seen from the present computer results that stall occurs at the very tip of the fin; that is, the entire fin is in poststall. Once this occurs, the fin loading will begin to increase as the angle of attack increases. This hypothesis could be tested quantitatively by examining experimental spanwise pressure distributions on a fin. It could be tested qualitatively by comparing predictions of the present method with experimental data for unswept fins and noting that this phenomenon probably would not occur for unswept geometries.

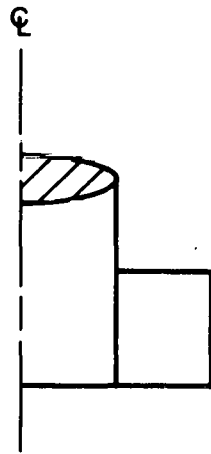
The panel normal force on the windward fin vs roll angle for $\alpha_b = 20^\circ$ at $M_{\infty} = 1.22$ is shown in Figure 32. Good agreement between the present method and experiment is demonstrated in the figure. The trend in panel normal force for $\phi < 50^\circ$ is the same for supersonic flow as the the previous subsonic case. For $\phi > 50^\circ$, however, the experimental data show that stall is delayed to a higher roll angle (i.e., angle of attack) as compared to $M_{\infty} = 0.8$. Figure 32 shows the experimental data still dropping at $\phi = 90^\circ$ which implies, from the previous discussion, that the stall angle of attack has not reached the trip chord. As the present method does not include any dependence of stall angle of attack on Mach number (Eq (35)), this characteristic is not demonstrated by theory.

Table 1. Geometry of Missiles Used for Comparison of Theory and Experiment

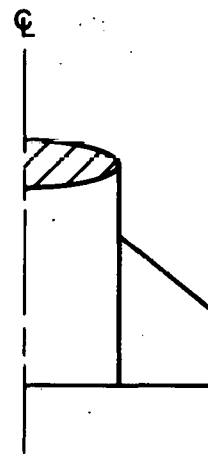
Configuration	b_0/a	c_r/a	x_1/a	$\Delta\ell_e$	Δt_e	x_h/a	A_e	Reference
A	2.4	2.8	17.2	45°	0°	18.4	1.33	27
B	2	1.2	18.1	0°	0°	--	1.67	28
C	2	1.6	17.7	38.7°	0°	--	1.67	28
D	2	2.4	16.9	67.4°	0°	--	1.67	28
E	3	2	18.0	0°	0°	--	2.00	10,29
F	2	2.66	17.34	53.1°	0°	--	1.00	30



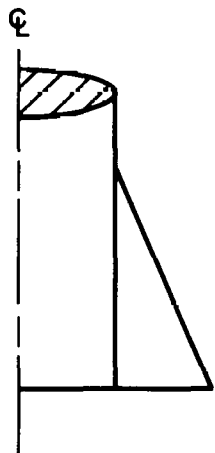
CONFIG. A



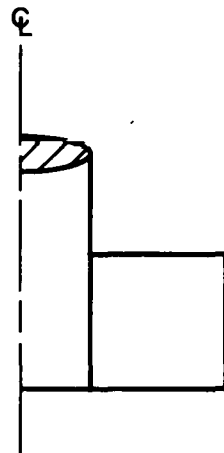
CONFIG. B



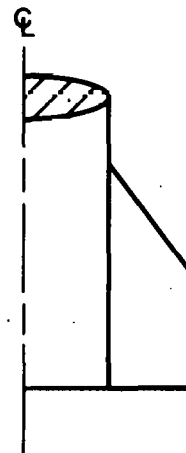
CONFIG. C



CONFIG. D



CONFIG. E



CONFIG. F

Figure 30. Fin Planforms Used for Comparison of Theory and Experiment

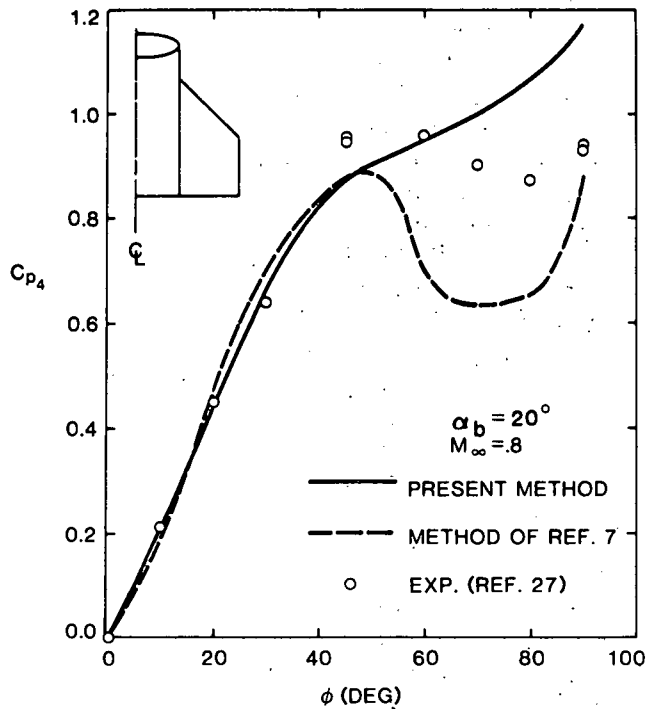


Figure 31. Windward Panel Normal Force vs Roll Angle for Configuration A ($M_\infty = 0.8$)

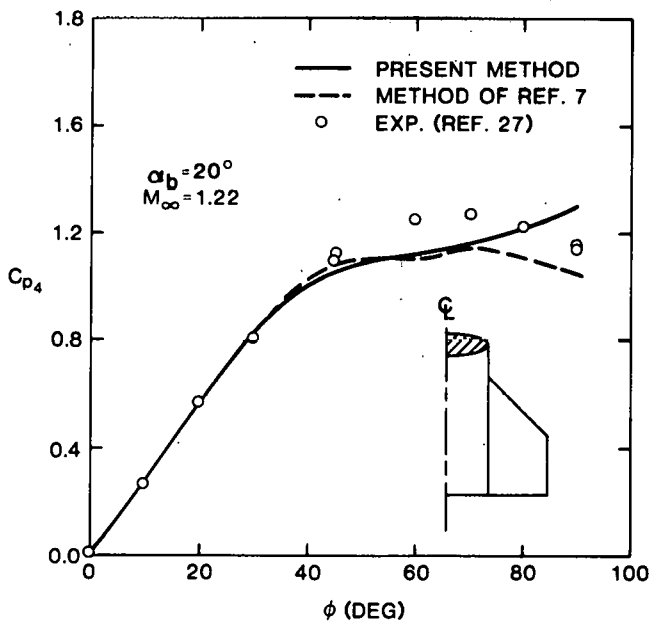


Figure 32. Windward Panel Normal Force vs Roll Angle for Configuration A ($M_\infty = 1.22$)

The panel normal force on the leeward fin vs roll angle for α_b at $M_\infty = 0.8$ is given in Figure 33. Very good agreement between theory and experiment is demonstrated except near $\phi = 0^\circ$. As Fin 1 rolls from $\phi = 0^\circ$, it begins to unload not only because of closer proximity to the angle of attack plane, but also immersion in the body vortex. The inboard portion of the fin unloads much more rapidly than the outboard portion because it is strongly influenced by the vortex feeding sheet. As the fin nears the center of the body vortex it produces essentially no net normal force. The portion inboard of the vortex produces negative force (i.e., in the negative roll moment direction) and the outboard portion produces an almost balancing positive force. This balance is highly dependent on the relation between the radial vortex location and the fin semispan. For example, if the fin had a smaller semispan, the zero load roll angle would be less than that shown in Figure 33.

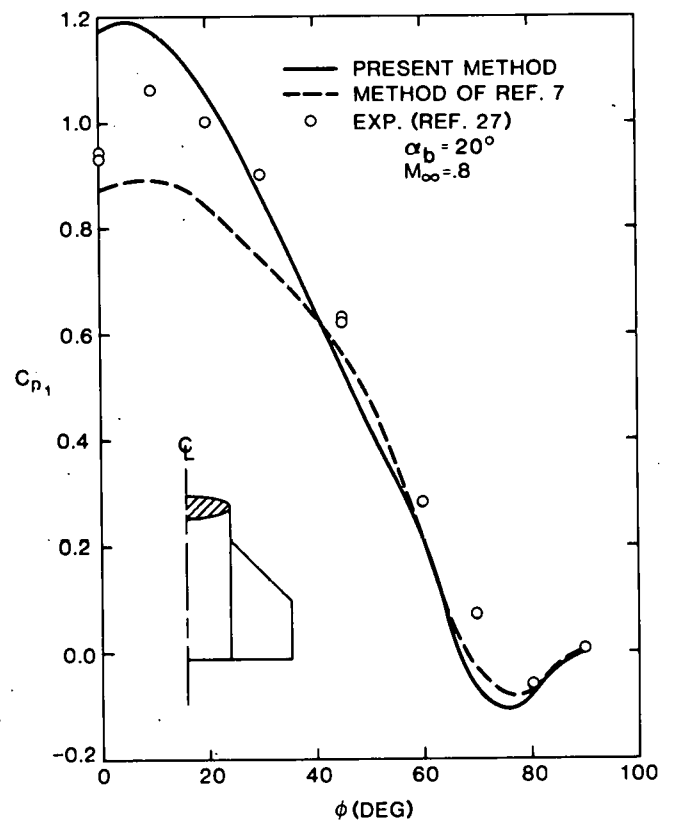


Figure 33. Leeward Panel Normal Force vs Roll Angle for Configuration A ($M_\infty = 0.8$)

Figure 34 shows the same type of comparison as Figure 33, except for $M_\infty = 1.22$. The agreement between theory and experiment is not as good as in the previous comparisons. Note that the experimental roll angle for zero cross-over loading decreased from $\phi = 75^\circ$ for $M_\infty = 0.8$ to $\phi = 60^\circ$ for $M_\infty = 1.22$. This could be caused by the body vortex becoming stronger or the radial location of the vortex increasing, or both. Recalling that the radial location of the vortex increases with Mach number (Eq (11)), one could infer that vortex strength increases significantly with Mach number. This has been suggested by Nielsen in Reference 26.

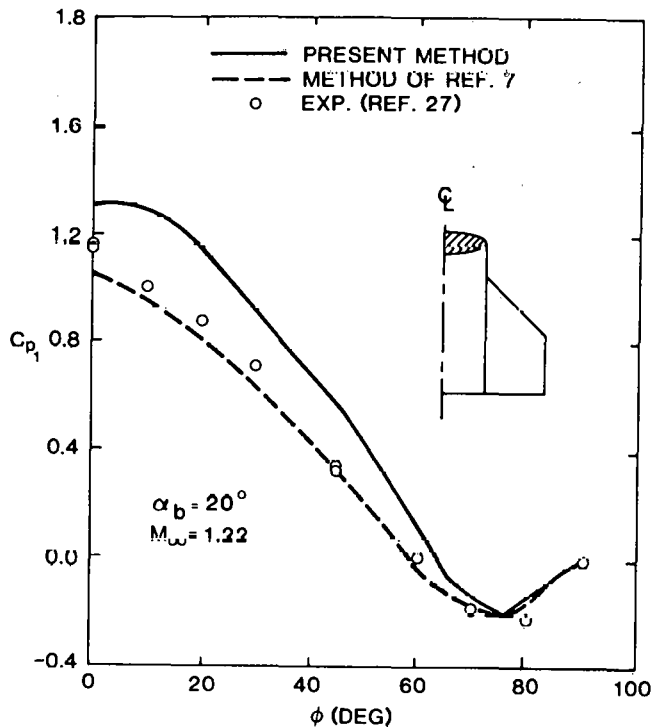


Figure 34. Leeward Panel Normal Force vs Roll Angle for Configuration A ($M_\infty = 1.22$)

b. Panel Bending Moment

Figures 35 and 36 show the panel bending moment for the windward fin vs roll angle for $\alpha_b = 20^\circ$ at $M_\infty = 0.8$ and 1.22, respectively. Generally good agreement is demonstrated between theory and experiment. Good agreement between theory and experiment on the bending moment requires that both the magnitude of the total normal force, C_{p4} , and spanwise pressure distribution are correct. Recall that the theory for panel normal force is in essentially

perfect agreement with experiment (Figures 31 and 32) for $\phi < 40^\circ$. Figures 35 and 36, however, show that the theory slightly underpredicts the bending moment for the same roll angle range. This implies that there is slightly more loading outboard and slightly less loading inboard than predicted by the present method. If bending moment data for other planforms indicated the same trend, the spanwise normal force distribution, $S(\eta)$, given by Eq (19b) could be modified slightly.

The root bending moment for the leeward fin vs roll angle for $\alpha_b = 20^\circ$ at $M_\infty = 0.8$ and 1.22 is shown in Figures 37 and 38, respectively. Excellent agreement between theory and experiment is demonstrated for the subsonic case, and fair agreement is shown for the supersonic case. It is interesting that for $M_\infty = 0.8$ near $\phi = 80^\circ$, the panel normal force is negative (Figure 33) while the bending moment remains near zero (Figure 37). The reason for this is that the reverse flow under the vortex is strong enough to generate a net negative panel force, but the bending moment produced by the outboard positive normal force overcomes the negative bending moment produced by the inboard sections.

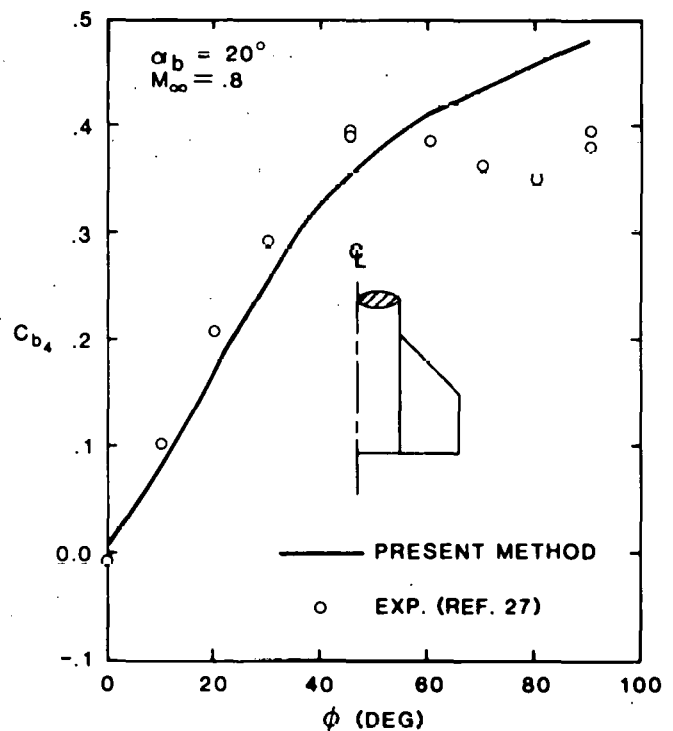


Figure 35. Windward Fin Root Bending Moment vs Roll Angle for Configuration A ($M_\infty = 0.8$)

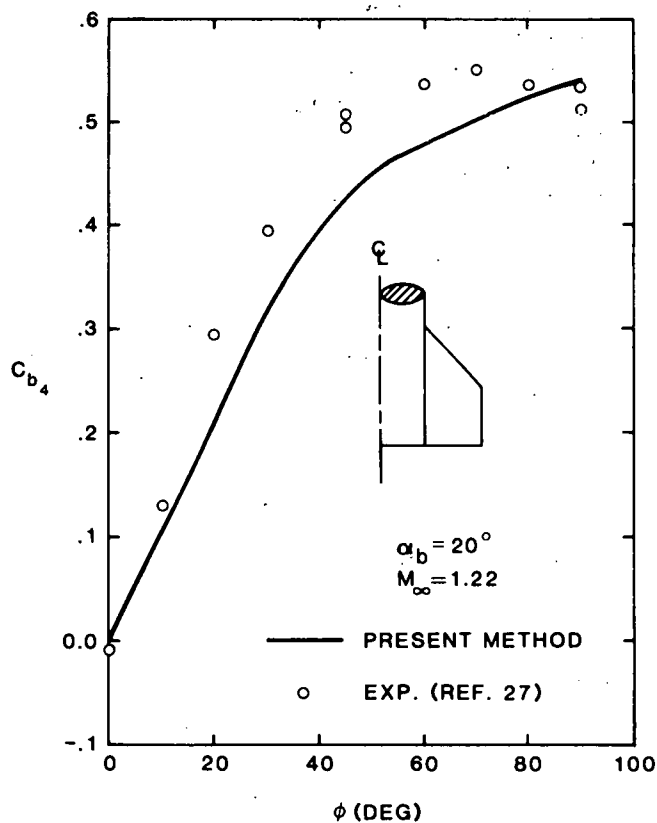


Figure 36. Windward Fin Root Bending Moment vs Roll Angle for Configuration A ($M_\infty = 1.22$)

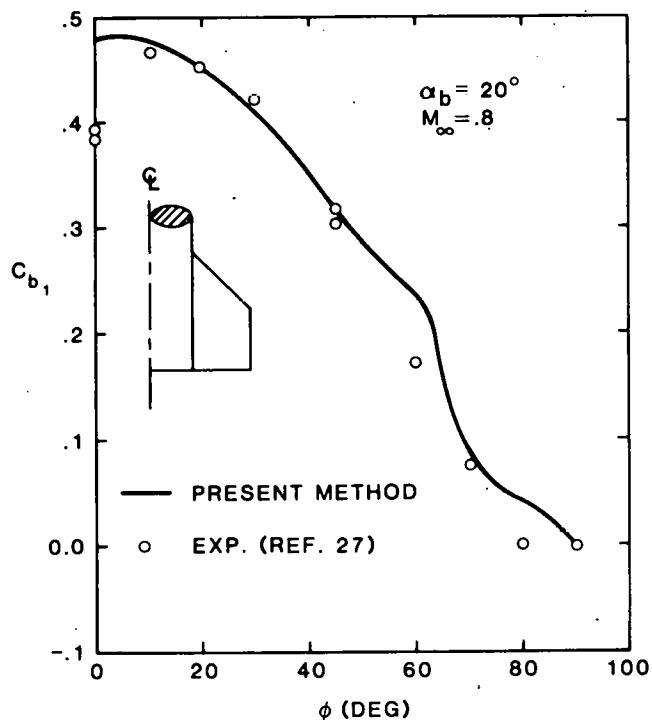


Figure 37. Leeward Fin Root Bending Moment vs Roll Angle for Configuration A ($M_\infty = 0.8$)

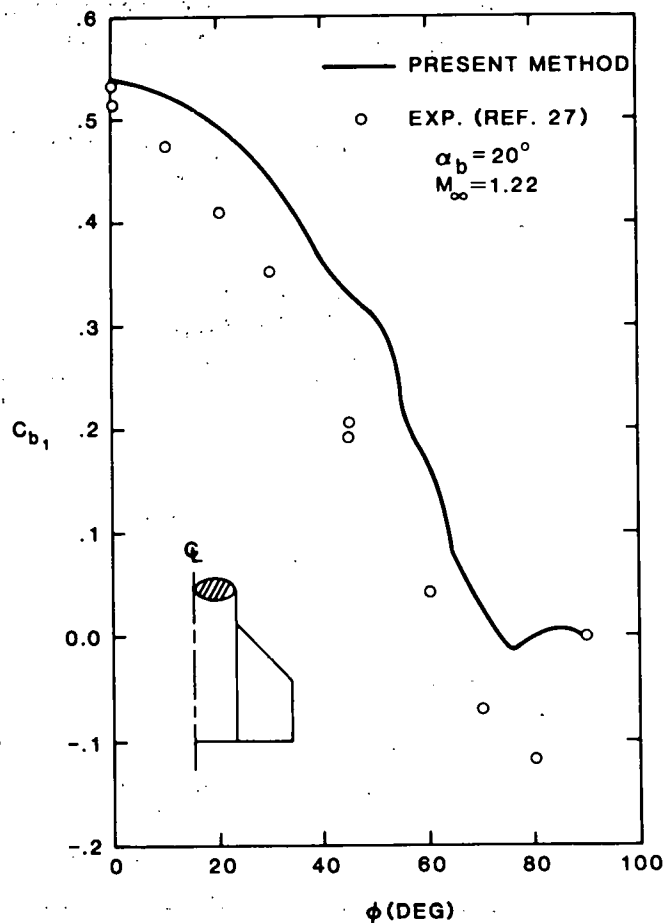


Figure 38. Leeward Fin Root Bending Moment vs Roll Angle for Configuration A ($M_\infty = 1.22$)

c. Panel Hinge Moment

The panel hinge moment for the windward fin vs roll angle for $\alpha_b = 20^\circ$ at $M_\infty = 0.8$ and 1.22 is shown in Figures 39 and 40, respectively. The agreement between theory and experiment appears to be poor, particularly for $M_\infty = 1.22$. One must be aware, however, of the sensitivity of the hinge moment to the location of the hinge line. A better indication of the accuracy of the present method is obtained by a sample comparison of the theoretical and experimental axial location of the fin center of pressure. Using the hinge line, x_h , as the moment reference, one can write

$$x_{cp} C_p q_\infty S_f = H.$$

Solving for the location of the center of pressure, nondimensionalized by the root chord, one has

$$x_{cp}/c_r = C_h/C_p. \quad (55)$$

Applying this equation to both the experimental and theoretical data at $\phi = 90^\circ$ for Fin 4, one has

For $M_\infty = 0.8$

$$\left(\frac{x_{cp}}{c_r}\right)_{exp} = 0.113 \quad \text{and} \quad \left(\frac{x_{cp}}{c_r}\right)_{theory} = 0.023$$

For $M_\infty = 1.22$

$$\left(\frac{x_{cp}}{c_r}\right)_{exp} = 0.129 \quad \text{and} \quad \left(\frac{x_{cp}}{c_r}\right)_{theory} = 0.024$$

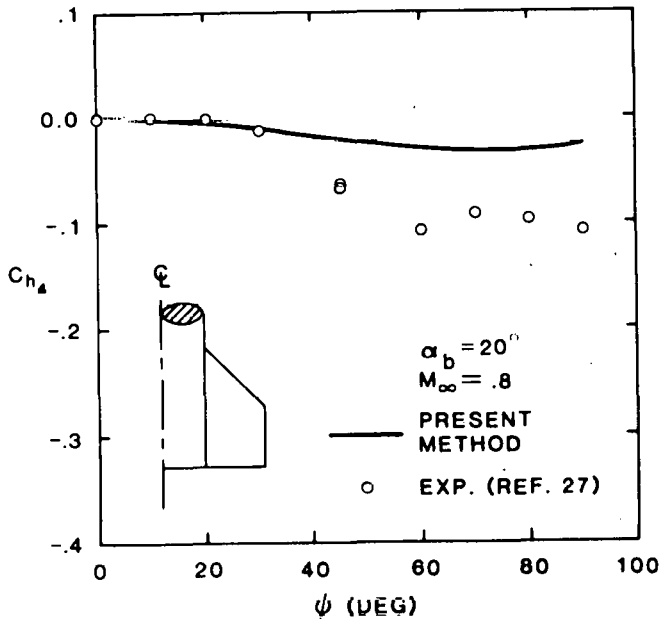


Figure 39. Windward Fin Hinge Moment vs Roll Angle for Configuration A ($M_\infty = 0.8$)

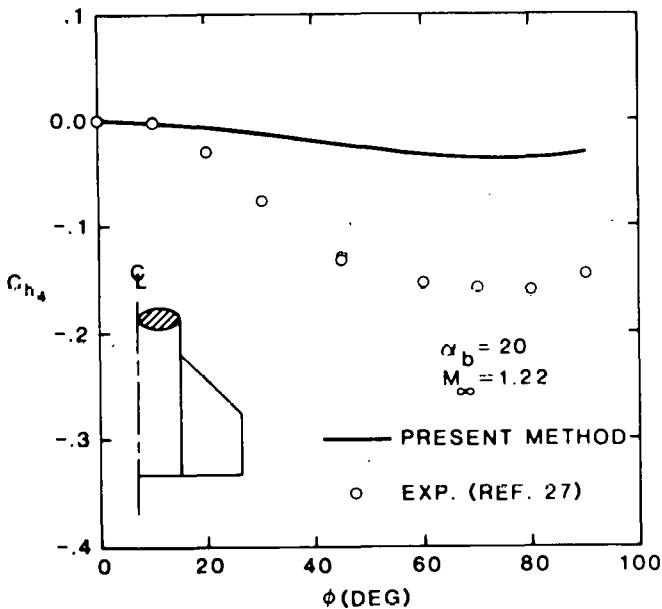


Figure 40. Windward Fin Hinge Moment vs Roll Angle for Configuration A ($M_\infty = 1.22$)

As can be seen by this calculation, the experimental center of pressure is slightly farther aft of the hinge line than predicted by theory. Taking the difference between each pair of ratios, the predicted axial center of pressure is in error only 9.0% and 10.5% of the root chord for $M_\infty = 0.8$ and 1.22, respectively, for the worst agreement between theory and experiment in Figures 39 and 40.

A second point should be made from the axial center of pressure calculation just presented. Note that the experimental data show that the center of pressure moves slightly rearward as the Mach number changes from subsonic to supersonic. This follows the usual trend of lifting surfaces in transonic flow. The predicted center of pressure, however, stays essentially constant with the Mach number; the reason is that the assumed normal force distribution for uniform approach flow (Eq (19)) does not depend upon the Mach number. A slight improvement to the present theory would be to insert Mach number dependence in the chordwise distribution function, $C(\xi)$.

Figures 41 and 42 give the hinge moment for the leeward fin vs roll angle for $\alpha_b = 20^\circ$ at $M_\infty = 0.8$ and 1.22, respectively. Better agreement between theory and experiment is shown for the leeward fin than for the windward fin. This might be somewhat surprising because of the complexity of predicting fin loading for such a nonuniform approach flow. The reason is that for $\phi > 50^\circ$ the panel normal force is small for the leeward fin; therefore, the hinge moment is also small.

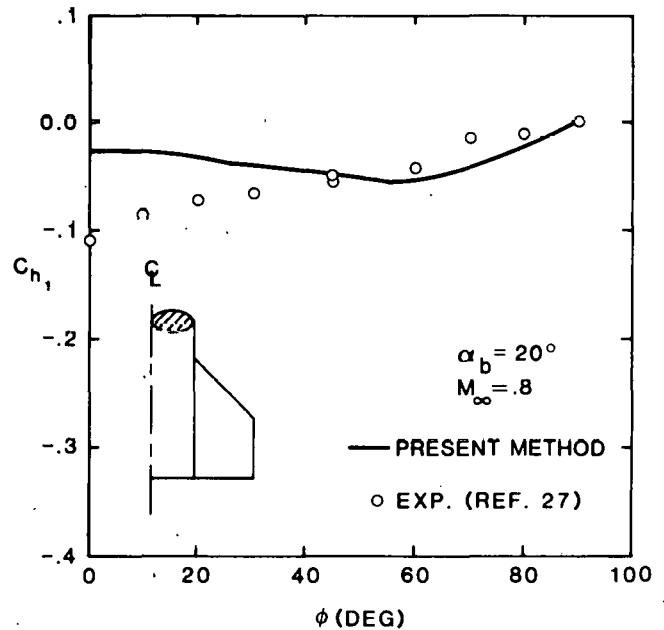


Figure 41. Leeward Fin Hinge Moment vs Roll Angle for Configuration A ($M_\infty = 0.8$)

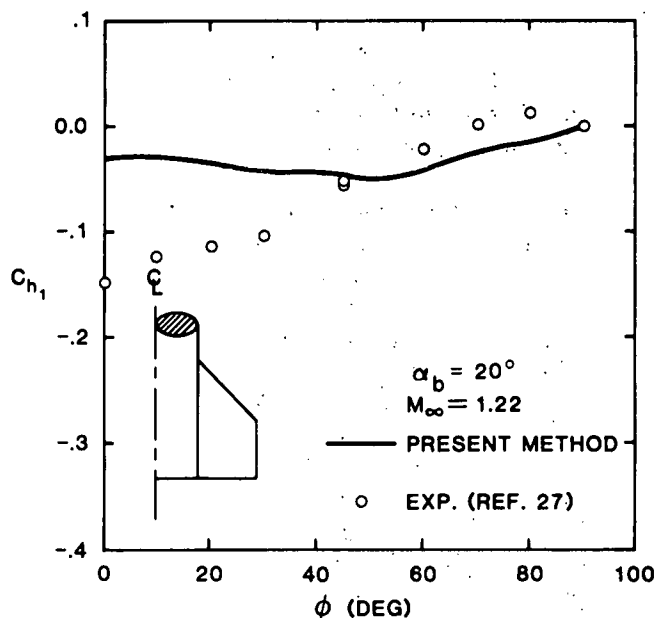


Figure 42. Leeward Fin Hinge Moment vs Roll Angle for Configuration A ($M_\infty = 1.22$)

Roll Moments

a. Induced Roll Moment

Figures 43 and 44 show the induced roll moment vs angle of attack of the body for missile Configuration A for $\phi = 20^\circ$ at $M_\infty = 0.8$ and $M_\infty = 1.22$, respectively. Also shown in the figures is the prediction of Nielsen, Hemsch, and Smith.⁷ Fairly good agreement is observed between the present method and experiment for $M_\infty = 0.8$, but for $M_\infty = 1.22$, the agreement is poor. Note that the induced roll moment is one of the most difficult nonlinear moments to predict because it is composed of the sum and difference of the first moment of four spanwise pressure distributions. The physical explanation of the slightly negative then rapidly increasing positive trend of the induced roll moment with angle of attack is very difficult to delineate because of the many complex aerodynamic phenomena embedded in the present theory. From numerical experiments with the theory, however, certain important elements can be identified: radial location of the body vortex, size of the vortex core, and local stall and poststall along the span of the fin.

To understand the trends plotted in Figures 43 and 44, one must examine the spanwise loading of all the fins. Figure 45 shows the spanwise loading for each fin for $M_\infty = 0.8$. The loading at a given spanwise location shown in Figure 45 is the integrated

value over the local chord. The loading caused by the primary body vortex is clearly seen on Fin 2. The spanwise location of the stall angle of attack can be recognized as a slight drop in the normal force along Fin 3. The loading on Fins 1, 3, and 4 increases steadily as the root chord is approached because of body upwash and the increasing length of the chord. On Fin 1 the loading drops sharply near the root chord because it passes inside the vortex feeding sheet. The roll moment produced by each fin loading shown is: Fin 1, 0.794; Fin 2, -0.031; Fin 3, -0.751; and Fin 4, 0.313. It can be seen that the roll moments produced by Fin 1 and Fin 3 roughly balance. Fin 2 and Fin 4, however, do not balance because the reverse flow loading between the primary body vortex and the root chord on Fin 2 drop (in magnitude) the roll moment on Fin 2 to roughly zero. This results in a large net positive roll moment from Fin 4. By similar reasoning, the small negative induced roll moment for small angles of attack is due to the increased outboard loading on Fin 2 when the vortex is near the body surface.

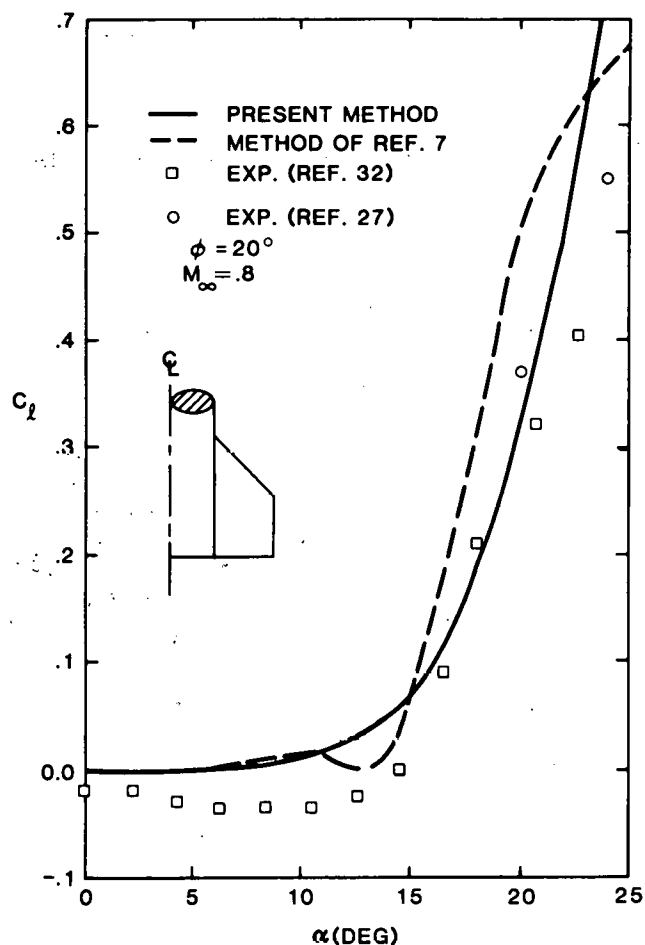


Figure 43. Induced Roll Moment vs α_b for Configuration A ($M_\infty = 0.8$)

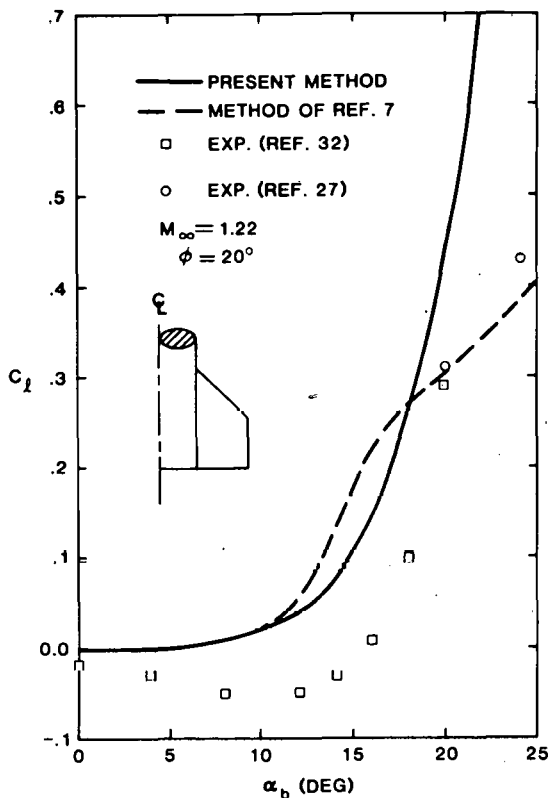


Figure 44. Induced Roll Moment vs α_b for Configuration A ($M_\infty = 1.22$)

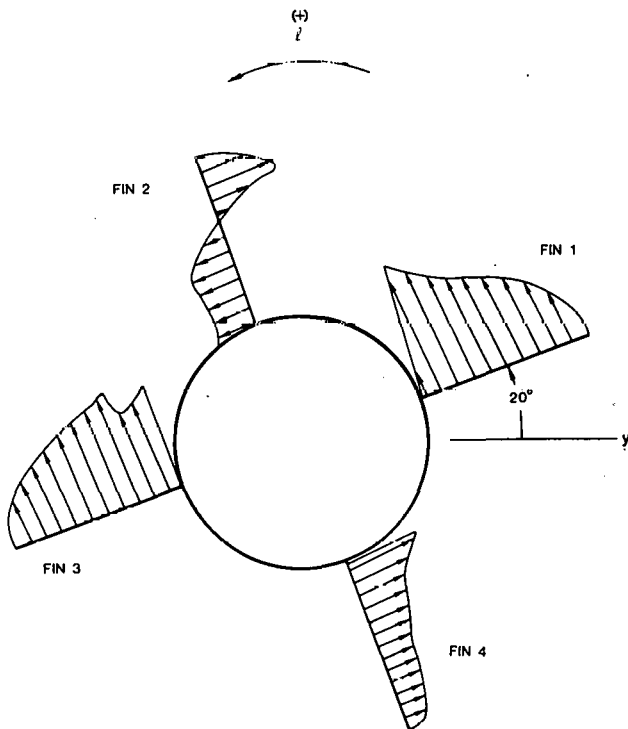


Figure 45. Spanwise Fin Loading for $\alpha_b = 20^\circ$, $\phi = 20^\circ$ Configuration A ($M_\infty = 0.8$)

Figure 46 gives the induced roll moment vs angle of attack for Configuration B for $\phi = 22.5^\circ$ and $M_\infty = 0.7$. Theory and experiment are in very good agreement for the rectangular fin planform. Comparing the induced roll moment coefficient for the clipped delta and rectangular planform (Figures 43 and 46) for $\alpha_b = 20^\circ$, it is seen that* $C_l = 0.325$ and 0.186 , respectively. Recalling the previous explanation given concerning the origin of the large positive induced roll moment, one could ask why C_l for the shorter span rectangular planform is not larger than that for the clipped delta. The reason for the question would be that the reverse flow loading on Fin 2 for the short span fin should produce a relatively larger positive roll moment than would the clipped delta. The reasoning is correct; the paradox, however, is created by the nondimensionalization used in the present analysis and in almost all other analyses. The roll moment coefficient is based on the body cross-sectional area, not on the fin planform area. If one converts the two previous coefficients to coefficients based on fin planform area, one has $[C_l]_{S_f} = 0.347$ and 0.487 , respectively. Now it is clear that the short span rectangular planform is much more efficient at producing an induced roll moment than the longer span clipped delta.

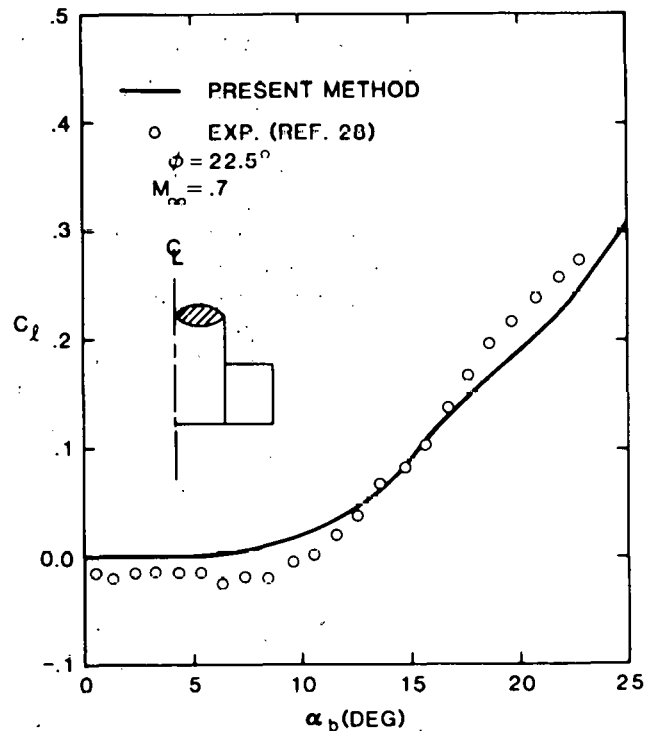


Figure 46. Induced Roll Moment vs α_b for Configuration B

*This comparison ignores the fact that the roll angle and Mach number are not exactly the same.

The induced roll moment vs angle of attack for Configuration D for $\phi = 22.5^\circ$ and $M_\infty = 0.7$ and 1.2 is shown in Figure 47. The theory reproduces the experimental data fairly well, but the change in induced roll moment with Mach number is not predicted accurately for this planform geometry. The reason is probably the effect of Mach number on the spanwise and chordwise pressure distributions, $S(\eta)$ and $C(\xi)$, as mentioned earlier. Concerning the magnitude of the induced roll moment coefficient, a direct comparison can be made between the rectangular planform (Figure 46) and the delta planform (Figure 47) because they both have the same missile body and planform area. The delta planform produces a slightly smaller induced roll moment because of the greatly different spanwise loading and stall angle of attack (compare, for example, Figures 17 and 19).

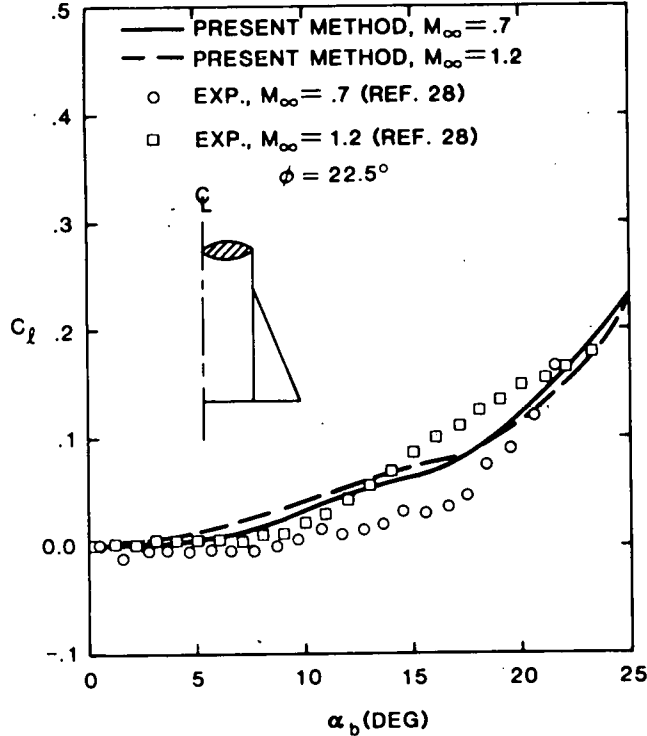


Figure 47. Induced Roll Moment vs α_b for Configuration D

b. Roll Damping Moment

Figure 48 shows the roll damping moment vs angle of attack for Configuration E for $M_\infty = 0.22$. There is excellent agreement between theory and experiment for angles of attack up to 12° , but above that the theory agrees well with one set of experimental data and not the other. The only comment that can be made is that the experimental techniques

of the two investigators differ greatly. Clare³¹ used a roll oscillation technique; Regan²⁹ used the standard rolling-speed decay method.

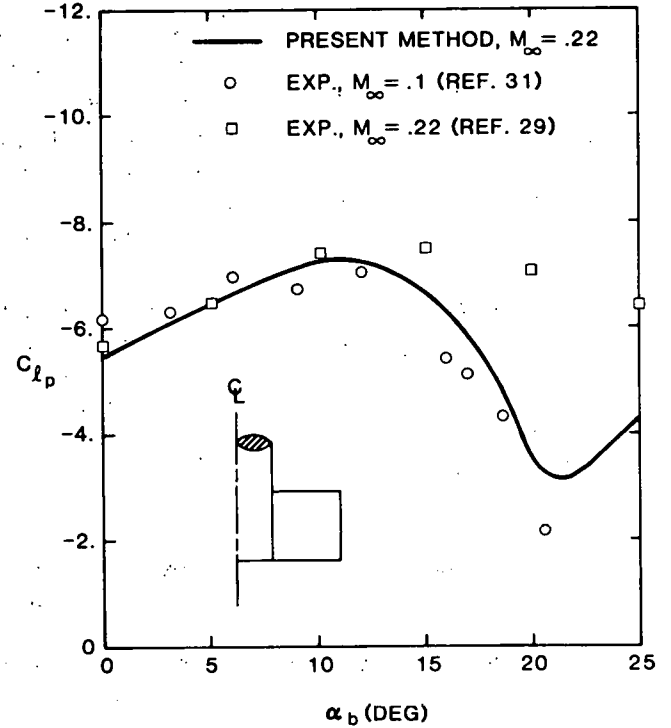


Figure 48. Roll Damping Moment vs α_b for Configuration E.

A comment should be made concerning the present numerical method of calculating dynamic derivatives, such as C_{l_p} . As the missile fins rotate through a 90° roll cycle, the forces and moments continually change for nonzero angle of attack of the body. Therefore, one method of computing rolling motion derivatives is to numerically difference the average value of the force or moment coefficient over a roll cycle; that is,

$$C_{l_p} = \frac{2}{\pi} \left[\int_0^{\pi/2} C_{l_p} |_{p=\Delta p} d\phi - \int_0^{\pi/2} C_{l_p} |_{p=0} d\phi \right] / \Delta p \quad (56)$$

where C_i represents any force or moment coefficient. This computation of C_{l_p} is appropriate for large values of rolling speed. The method described by Eq (56) is one used in the present analysis. It should be mentioned, however, that another method could be used. This method computes the difference between two values of the coefficient at different rolling speeds, but at the same roll angle; that is

$$C_{l_p} |_{\phi=\phi_1} = \left[C_i |_{\phi=\phi_1, p=\Delta p} - C_i |_{\phi=\phi_1, p=0} \right] / \Delta p. \quad (57)$$

This method could be used for very low rolling speeds, such as roll oscillations about a roll trim angle; it would yield, for example, the roll angle variation of the roll damping moment while a missile is oscillating in roll lock-in.

Figure 49 gives the roll damping moment vs angle of attack for Configuration F for $M_\infty = 0.6$ and 1.3. Although the slight drop in C_{ℓ_p} near $\alpha_b = 17^\circ$ is not predicted, the agreement between theory and experiment is generally good over the angle of attack range. Comparing Figures 48 and 49, it is seen that the trend of C_{ℓ_p} with α_b is very different between the two planforms. One might suspect, based on the earlier discussion of induced roll moment, that this different character would be due primarily to the difference in semispan between the two configurations. The roll damping moment for several fin planforms and semispans was computed in order to identify the dominant parameter causing the rapid decrease in C_{ℓ_p} for α_b near 20° ; it was the stall angle of attack of the planform and not fin span. For example, consider the case of holding the planform and span constant (say a rectangular planform of a given span), and varying the aspect ratio so that the stall angle of attack varies. The aspect ratio decreases (and the stall angle of attack increases) as the drop in C_{ℓ_p} at large angle of attack disappears.

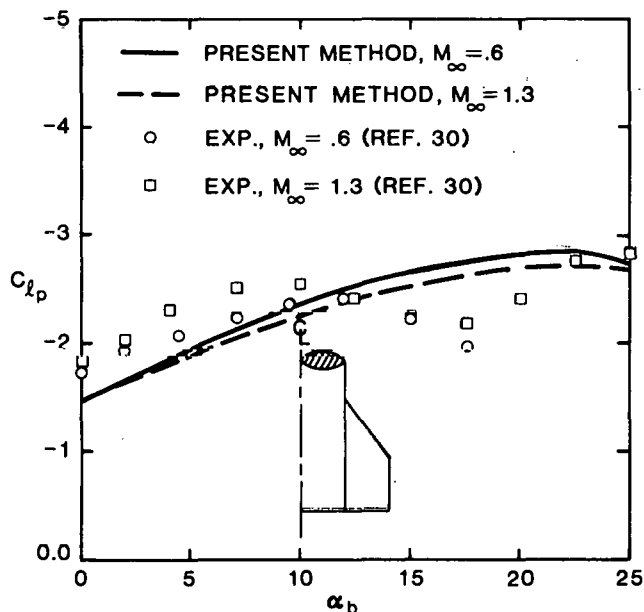


Figure 49. Roll Damping Moment vs α_b for Configuration F.

c. Steady-State Rolling Speed

The steady-state roll rate of a missile whose fins are canted is calculated in a manner similar to C_{ℓ_p} . The steady-state roll rate is defined as the rolling speed at which the roll driving moment balances with the roll damping moment. Therefore, the non-dimensional steady-state roll rate, p_{ss} is the roll rate such that

$$\int_0^{\pi/2} [C_{\ell}]_{p=p_{ss}} d\phi = 0.$$

Only one comparison of theory and experiment is made for steady-state rolling speed because of the lack of data for other planforms. Figure 50 shows the steady-state rolling speed vs angle of attack for Configuration E for a fin cant of 4° (differential fin deflection) and $M_\infty = 0.23$. The theory slightly overpredicts p_{ss} for angles of attack less than 12° , but for $\alpha_b > 12^\circ$ the theory agrees perfectly with one set of data and underpredicts p_{ss} for the other set. The reason the theory predicts roll slowdown is rather difficult to determine. From numerical experiments with the present method it was found that roll slowdown was predicted for each of the planforms shown in Figure 30. One qualitative comment that can be made, however, is that even though the roll damping moment decreases at large angle of attack (Figure 48), the roll driving moment (i.e., fin cant) decreases at a faster rate on every planform investigated.

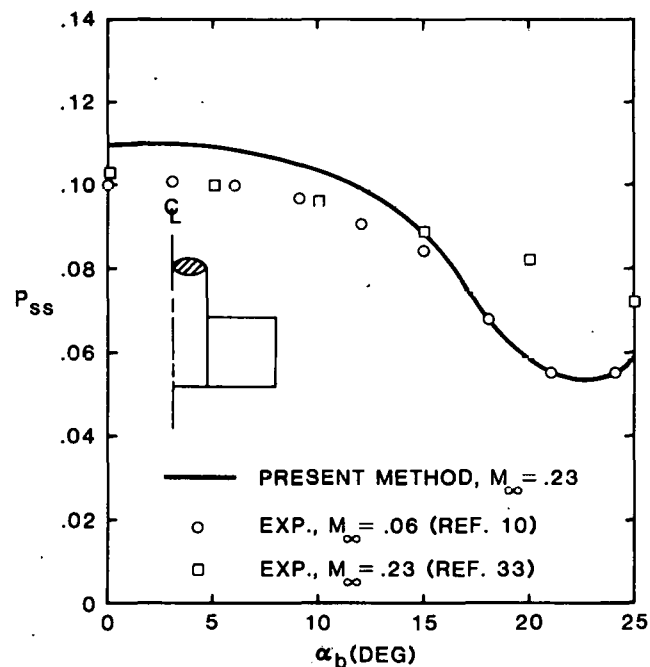


Figure 50. Steady-State Roll Rate vs α_b for Configuration E (4° fin cant)

Control Deflections

a. Pitch (or Yaw) Control

Pitch control force coefficient is defined as the normal force coefficient of the entire missile configuration (body plus fins) with the fins deflected minus the normal force coefficient of the entire missile without the fins deflected; that is,

$$\Delta C_z = [C_{z_{B+f}}]_{\delta_j \neq 0} - [C_{z_{B+f}}]_{\delta_j = 0}$$

Using slender body theory notation, one can write

$$\begin{aligned} \Delta C_z = & [C_{z_{B(f)}}]_{\delta_j \neq 0} + [C_{z_{f(B)}}]_{\delta_j \neq 0} \\ & - [C_{z_{B(f)}}]_{\delta_j = 0} - [C_{z_{f(B)}}]_{\delta_j = 0} \end{aligned} \quad (55)$$

The second and fourth terms are computed in the present analysis, but the first and third are not. The first term can be segregated into two terms:

$$[C_{z_{B(f)}}]_{\delta_j \neq 0} = [C_{z_{B(f)}}]_{\delta_j = 0} + C_{z_{B < \delta_j >}} \quad (56)$$

where the second term on the right side of the equation is the normal force coefficient of the body due to control deflection of the fins. Substituting Eq (56) into Eq (55), one has

$$\Delta C_z = C_{z_{B < \delta_j >}} + [C_{z_{f(B)}}]_{\delta_j \neq 0} - [C_{z_{f(B)}}]_{\delta_j = 0} \quad (57)$$

A simple result of slender body theory is used to estimate $C_{z_{B < \delta_j >}}$. Using the definition of the interference lift ratio, k_B (see Ref. 24, pp 213-218), one has

$$C_{z_{B < \delta_j >}} = k_B [C_{z_{f(B)}}]_{\delta_j \neq 0}$$

Substituting this into Eq (57), one has

$$\Delta C_z = [1 + k_B] [C_{z_{f(B)}}]_{\delta_j \neq 0} - [C_{z_{f(B)}}]_{\delta_j = 0} \quad (58)$$

The term k_B is calculated from slender body theory assuming that the angle of attack of the body is zero. One could use Eq (58) to calculate ΔC_z for any angle of attack and neglect the inconsistency between this and the derivation of k_B . The present analysis, however, chooses the approach of segregating the $\alpha_b = 0$ fin deflection interference and then using this constant value for nonzero α_b . Rewriting Eq (58) according to this approach, we have

$$\Delta C_z = [C_{z_{f(B)}}]_{\delta_j \neq 0} - [C_{z_{f(B)}}]_{\delta_j = 0} + k_B [C_{z_{f(B)}}]_{\delta_j \neq 0, \alpha_b = 0} \quad (59)$$

Equation (59) was used in the present analysis for computing pitch control forces with fin-body interference. k_B is a simple function of a/b_0 and is plotted in Figure 51.

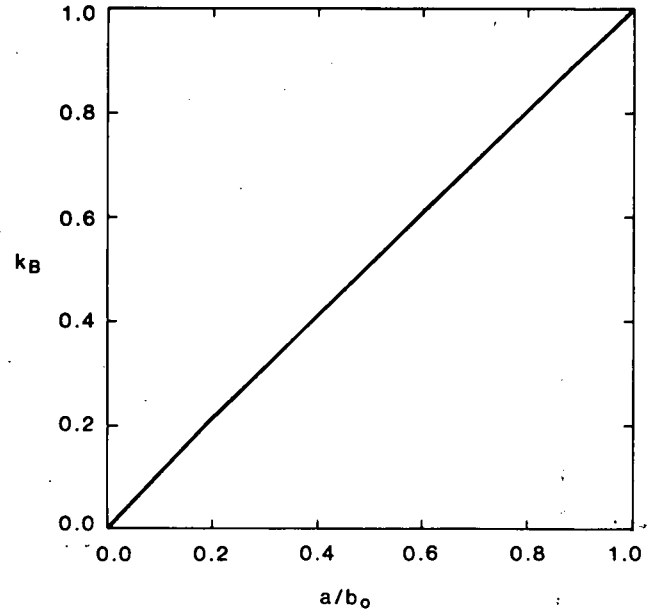


Figure 51. Interference Lift Ratio vs a/b_0 (from Ref. 24)

Figures 52, 53, and 54 show the pitch control force versus angle of attack for $M_\infty = 0.7$ and 1.2 for fin configurations B, C, and D, respectively. The control force shown in these figures is for $\phi = 0^\circ$ and the horizontal panels deflected 10° ; that is, $\delta_1 = \delta_3 = 10^\circ$ and $\delta_2 = \delta_4 = 0^\circ$. The agreement between theory and experiment is generally fair for the three fin planforms and the angle of attack range. It can be seen from these three figures that the control force for $\alpha_b = 0^\circ$ for the three planforms is almost identical. Although the leading edge sweep angles are 0° , 38.7° , and 67.4° , respectively, all three planforms have the same aspect ratio: 1.67. The trend with angle of attack is significantly different for the three planforms. For $\Lambda_{te} = 0^\circ$ (Figure 52), the control force drops off sharply with angle of attack to the extent that for $15^\circ < \alpha_b < 20^\circ$ a positive control input produces a negative control force. Note that this phenomenon is not caused by the body vortex wake, but by the interaction of the nonlinear normal force curve (i.e., fin stall and body upwash). For $\Lambda_{te} = 38.7^\circ$ (Figure 53), ΔC_z is nearly constant for low α_b and then drops off with angle of attack. The experimental data for $M_\infty = 0.7$ show a region of control force reversal

near $\alpha_b = 17^\circ$. For $\Lambda_{te} = 67.4^\circ$ (Figure 54), ΔC_z is almost constant out to $\alpha_b = 14^\circ$ and then shows a slight decrease with angle of attack. Also note the insensitivity of ΔC_z to Mach number. This is due to the low Mach number normal to the leading edge of the planform.

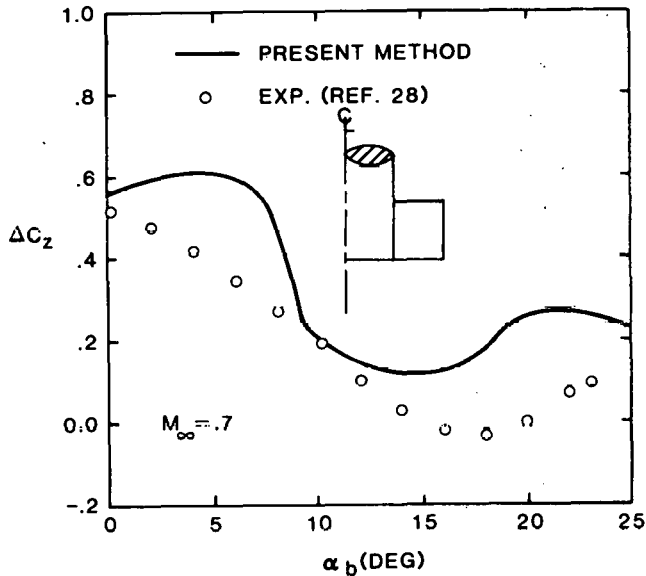


Figure 52. Pitch Control Force vs α_b for Configuration B ($\phi = 0^\circ$, $\delta_1 = \delta_3 = 10^\circ$, $\delta_2 = \delta_4 = 0^\circ$)

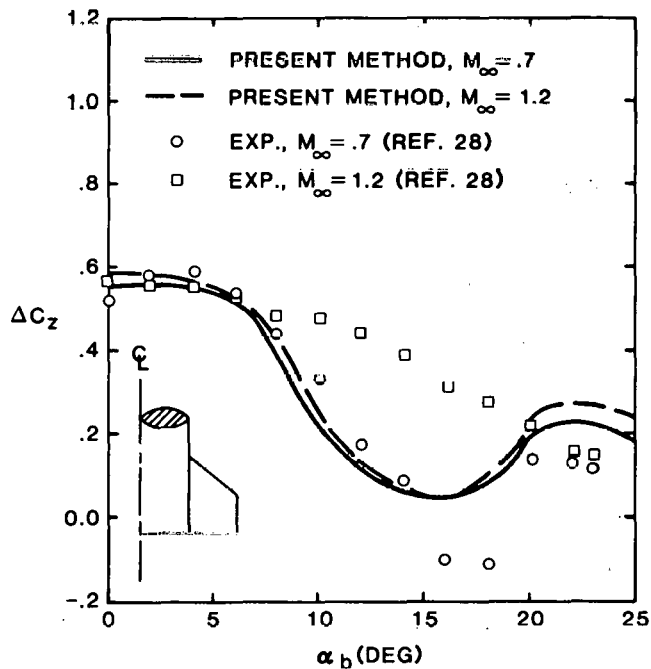


Figure 53. Pitch Control Force vs α_b for Configuration C ($\phi = 0^\circ$, $\delta_1 = \delta_3 = 10^\circ$, $\delta_2 = \delta_4 = 0^\circ$)

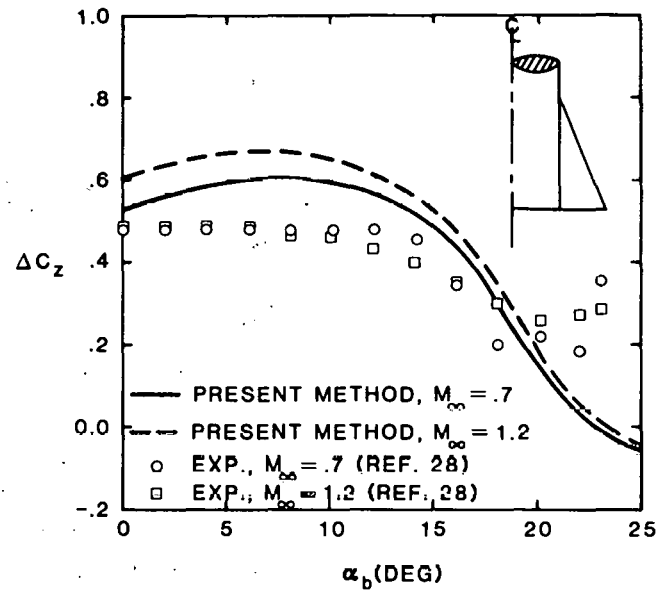


Figure 54. Pitch Control Force vs α_b for Configuration D ($\phi = 0^\circ$, $\delta_1 = \delta_3 = 10^\circ$, $\delta_2 = \delta_4 = 0^\circ$)

Figures 55, 56, and 57 show the pitch control force vs angle of attack for all panels deflected 10° ($\phi = 45^\circ$) for fin planforms B, C, and D, respectively. The agreement between theory and experiment is generally fair for the three planforms over the angle of attack range. Similar trends of ΔC_z vs angle of attack are seen in Figures 55, 56, and 57 as compared to Figures 52, 53, and 54, respectively. For $\phi = 45^\circ$ and $\delta_i = 10^\circ$, however, no control reversals are seen over the angle of attack range. Also, the delta planform (Figure 57) shows that the control effectiveness increases slightly with angle of attack up to $\alpha_b = 12^\circ$ for both Mach numbers.

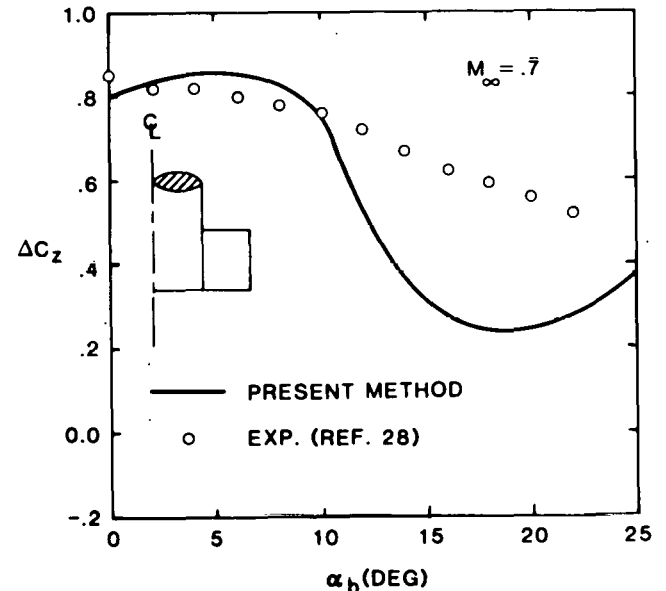


Figure 55. Pitch Control Force vs α_b for Configuration B ($\phi = 45^\circ$, $\delta_1 = \delta_2 = \delta_3 = \delta_4 = 10^\circ$)

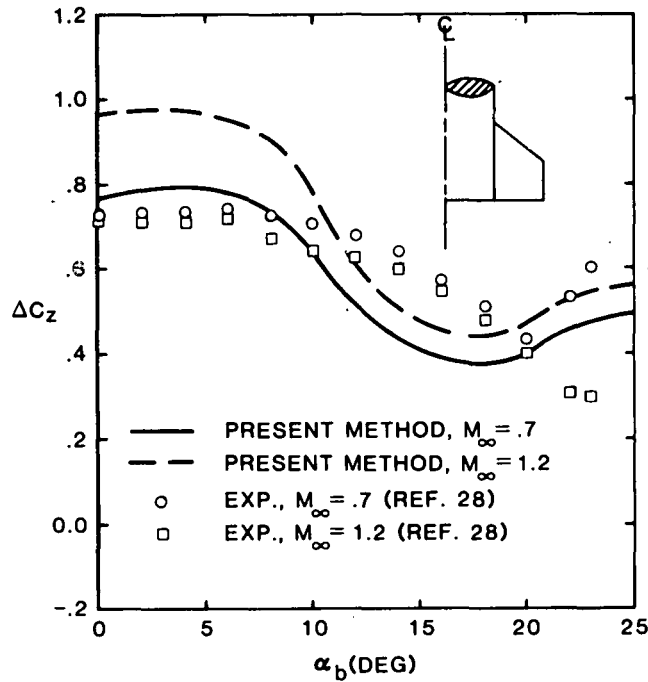


Figure 56. Pitch Control Force vs α_b for Configuration C ($\phi = 45^\circ$, $\delta_1 = \delta_2 = \delta_3 = \delta_4 = 10^\circ$)

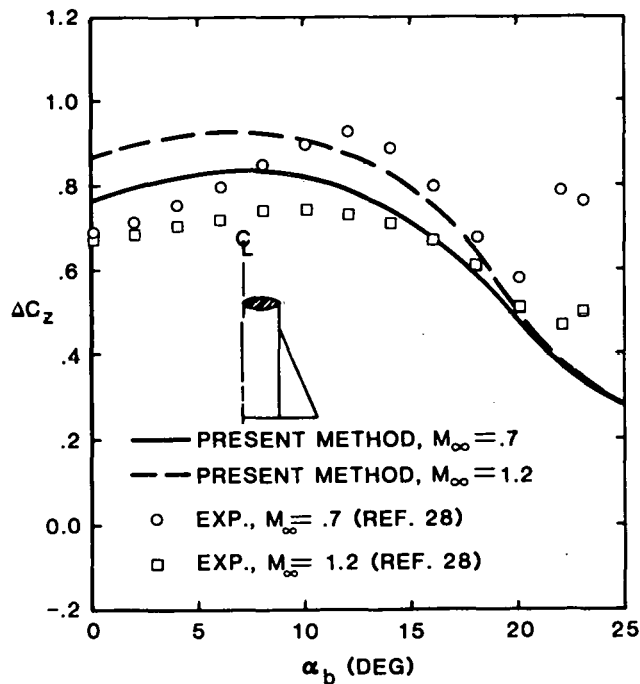


Figure 57. Pitch Control Force vs α_b for Configuration D ($\phi = 45^\circ$, $\delta_1 = \delta_2 = \delta_3 = \delta_4 = 10^\circ$)

b. Roll Control

Figures 58, 59, and 60 show the roll control moment vs angle of attack for $M_\infty = 0.7$ and 1.2 for fin Configurations B, C, and D, respectively. These roll control moments are for $\phi = 0^\circ$ and $\delta_1 = -\delta_3 = 10^\circ$ and $\delta_2 = \delta_4 = 0^\circ$. The theory generally predicts larger roll control moments than experiment, but the theoretical trends with angle of attack are valid. For the rectangular and clipped delta planform (Figure 58 and 59), a large region of roll control reversal occurs for $\alpha_b > 15^\circ$. This region is fairly well predicted by the present analysis. For the delta planform (Figure 60), the theory substantially overpredicts the roll control moment up to $\alpha_b = 20^\circ$ and then a control reversal is predicted. The overprediction near $\alpha_b = 0^\circ$ could be due to an inaccurate spanwise loading or it could be due to a physical characteristic of control deflection not included in the present theory (for example, root chord gap). When a fin panel is deflected for control, a spanwise gap is created at the root chord of the fin. The gap size increases as the fin deflection and root chord length increases. For Configuration D the root chord length is 100% longer and 50% longer, respectively, than configurations B and C. This characteristic would decrease the predicted value of the roll control moment if it were included in the theory.

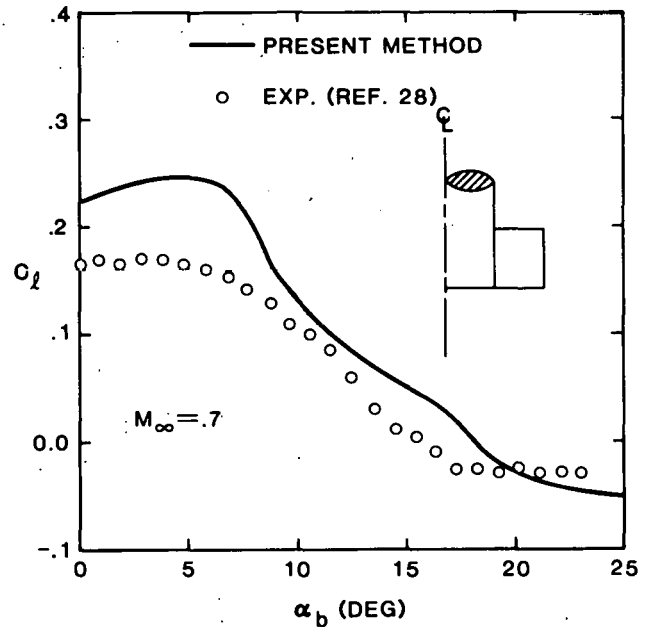


Figure 58. Roll Control Moment vs α_b for Configuration B ($\phi = 0^\circ$, $\delta_1 = -\delta_3 = 10^\circ$, $\delta_2 = \delta_4 = 0^\circ$)

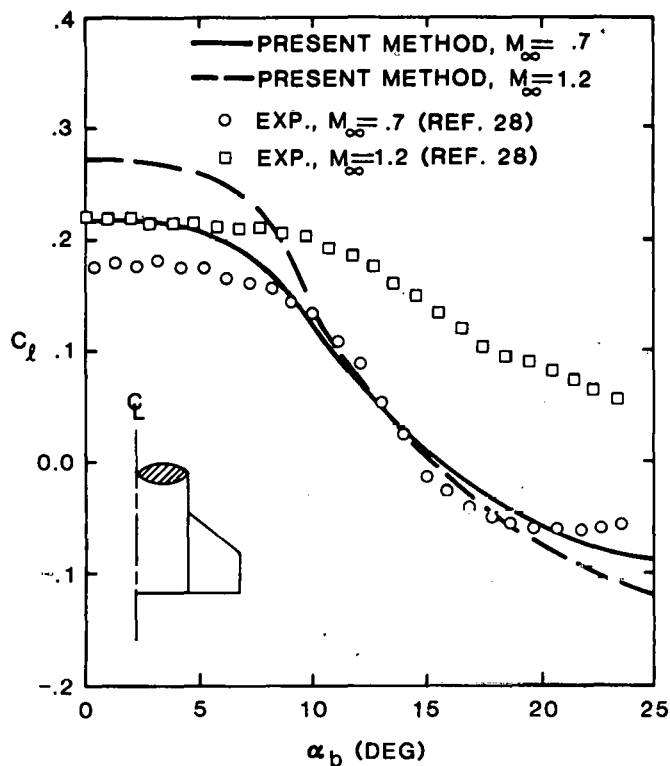


Figure 59. Roll Control Moment vs α_b for Configuration C ($\phi = 0^\circ$, $\delta_1 = -\delta_3 = 10^\circ$, $\delta_2 = \delta_4 = 0^\circ$)

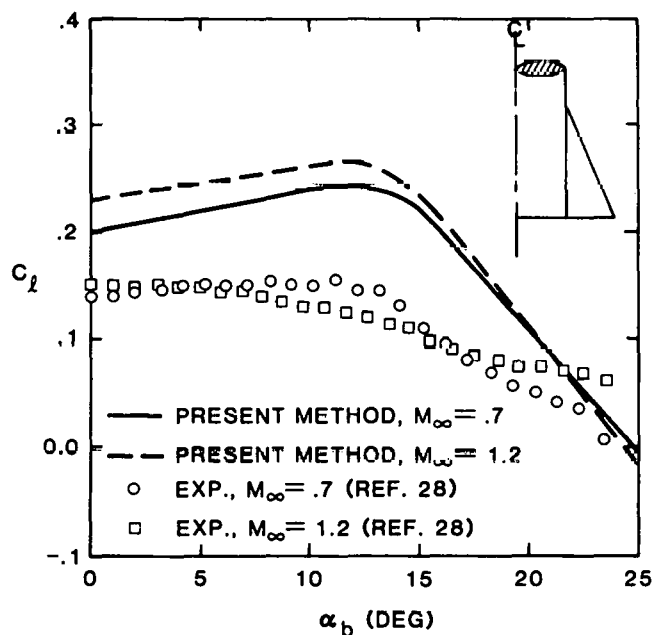


Figure 60. Roll Control Moment vs α_b for Configuration D ($\phi = 0^\circ$, $\delta_1 = -\delta_3 = 10^\circ$, $\delta_2 = \delta_4 = 0^\circ$)

Conclusions and Recommendations

The following conclusions and recommendations are drawn from the present investigation.

1. The spanwise and chordwise pressure distributions are of sufficient accuracy to be used as an aid in estimating root bending and hinge moment structural loads. The hinge moment predictions can also be used as a design aid in sizing control deflection actuators.
2. Evidence was found to suggest that stall near the root chord on highly swept fins washes outboard and, consequently, decreases the outboard loading on the fin. Although this element is not included in the present theory, one might conceive of a method of approximating this phenomenon in the analysis.
3. Certain comparisons between theory and experiment suggest that the stall angle of attack increases with Mach number. If sufficient data could be gathered on this trend, it should be added to the analysis.
4. The results of the present method could be enhanced by improving the empirical data for the body flow field model or by using a more accurate body flow field model. An improved flow model should contain more compressibility effects.
5. In general, short span fins produce larger nonlinear forces and moments (such as induced side force and induced roll moment) than larger span fins. This typically occurs even though the exposed fin area is smaller.
6. The present method could be improved by requiring the basic chordwise and spanwise pressure distributions to be dependent on free stream Mach number. This should be done after sufficient pressure distribution data on wings alone planforms becomes available for various Mach numbers.
7. The nonlinear decrease in roll damping for large angles of attack was found to be due to the stall angle of attack of the fin. The stall angle is primarily determined by the aspect ratio and leading edge sweep.
8. The present method should be coupled to a body force and moment predictive method

so that complete vehicle force and moment predictions are available.

9. The rapid decrease in pitch control force at large angles of attack is predicted by the method.
10. Predicted roll control reversal at large angles of attack generally agrees with experimental data.
11. The present method should prove a valuable tool for flight vehicle designers because of its ability to address general fin planforms.

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APPENDIX A

Structure and Listing of Computer Program

The computer program which implements the theoretical method is constructed in a modular fashion. This design was chosen so that a user could more easily identify any difficulties one might have with the program and make changes. A large number of COMMENT cards also are included in the program to aid the user. The main program and subprogram names, card image lengths, and computer memory lengths are shown in Table A-1.

The structure of the program is depicted in the overview flow chart (Figure A-1). The flow chart shows the main program and every subprogram along with the communication between each of them. However, the flow chart does not show any data transfer by way of COMMON statements. All input and essentially all output is accomplished from the main program.

A complete listing of the computer program is given in Table A-2.

A user could rather easily reduce the computer memory requirements needed to run the program. The majority of memory (88%) is used in two triply dimensioned arrays: VVEL (6, 73, 51) and WVEL (6, 73, 51). These arrays are used to store v and w velocity components calculated in FLOW and transferred to FINLOAD. These arrays store v and w in the cross-flow plane at the axial location of the fins for a given angle of attack of the body. The first subscript sets the maximum number of α_b 's that can be calculated in a

given computer run, the second subscript sets the number of roll angles used to calculate fin loads, and the third subscript sets the number of spanwise locations along the exposed semispan used to calculate fin loads. By changing the program such that only three α_b 's are permitted in one run, a user can reduce computer memory by 22,338 words. The second and third subscripts should not be changed in the program for purposes of conserving memory.

Although the computer run time is discussed in Appendix B, a short discussion is given here concerning a simple modification of the code that, for certain cases, would significantly reduce the run time. The majority of computation time in the execution of the program is consumed in calculating the body flow field in subroutine FLOW. As discussed in the previous paragraph, the v and w velocity components are calculated in a polar grid pattern for an axial location at the fins and for a given α_b . If a user plans to make a number of computer runs for fin planforms with the same semispan and located at the same axial station along the body, the modification should be made. For this case the body flow field calculated in FLOW would be exactly the same for each computer run. Therefore, the flow field stored in arrays U (I), VVEL (I, J, K), and WVEL (I, J, K) should be stored on disc or tape and recalled for later computer runs. In this way the subroutine FLOW will not have to be called each time a computer run is made.

Table A-1. Description of Quantity of Computer Code

Program Element Type	Name	Card Length	Memory Length (Words)
Main Program	FINLOAD	562	48 349
Subroutine Subprogram	IDSUB	22	81
Subroutine Subprogram	IPSUB	37	182
Function Subprogram	CN	25	118
Subroutine Subprogram	CNPREP	53	302
Subroutine Subprogram	FLOW	134	903
Function Subprogram	TBLOOK	29	82
Subroutine Subprogram	SIM	26	82
Blank Common	--	--	16
Labelled Common	--	--	233
	Total	888	50 348 (142 254 ₈)

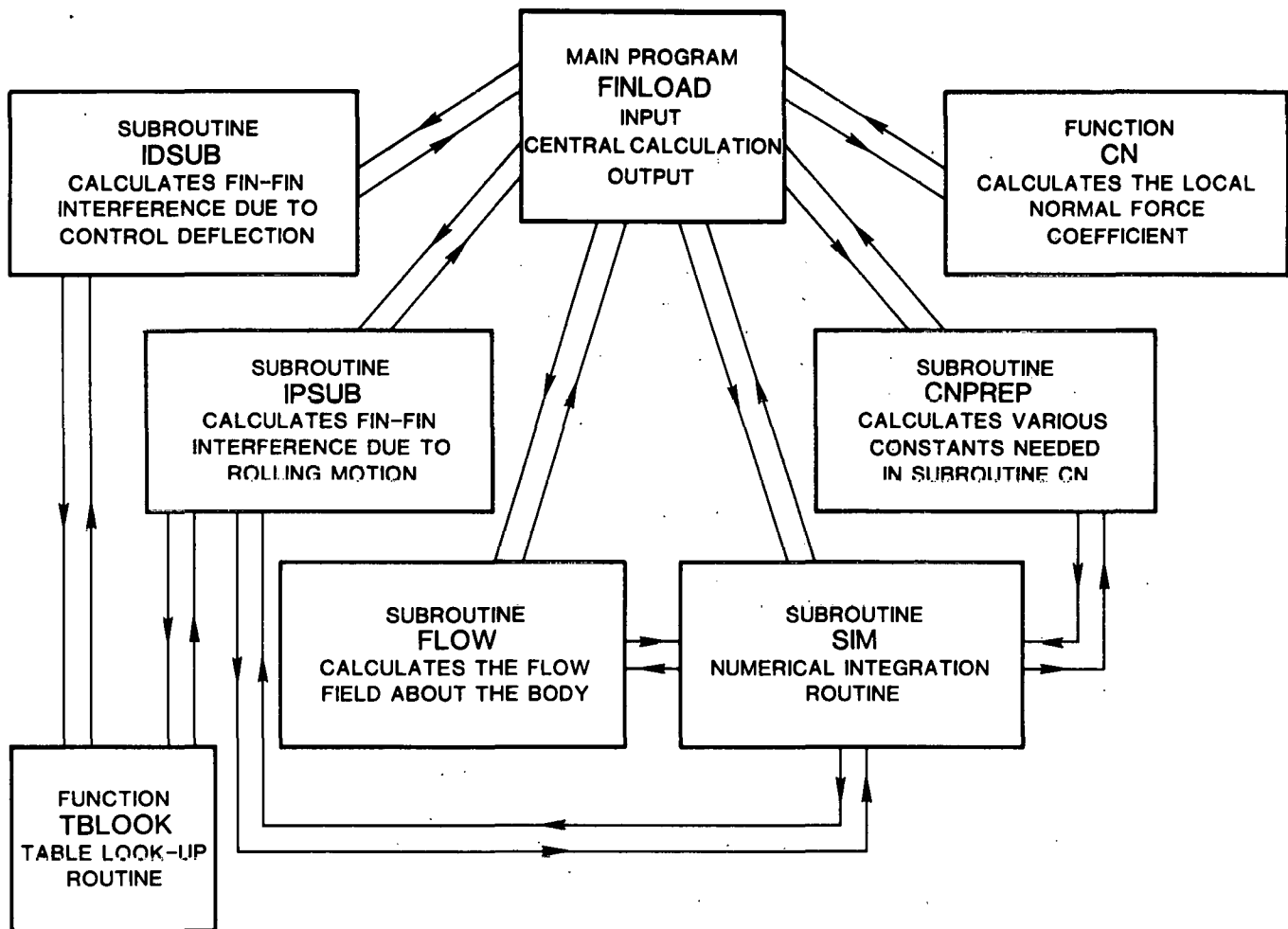


Figure A-1. Overview Flow Chart for Computer Program

Table A-2. Computer Program Listing

C	PROGRAM FINLOAD(INPUT,OUTPUT,TAPE5=INPUT)	FIN	2
C		FIN	3
C	THIS PROGRAM CALCULATES THE FORCES AND MOMENTS PRODUCED	FIN	4
C	BY CRUCIFORM FINS ATTACHED TO A MISSILE IN SUBSONIC AND	FIN	5
C	TRANSONIC FLOW.	FIN	6
C	PROGRAMMED BY W. L. OBERKAMPF (NOV. 1979 - APRIL 1980)	FIN	7
C		FIN	8
	COMMON A,BJA,CR,AE,PI,DETA,COSLE,TANLE,TAYTE,MACH,ALPHA	FIN	9
	COMMON /A/ B0,SF,SIGMA,CNA,INT1,INT2	FIN	10
	COMMON /B/ RAD,XV,IALPHA	FIN	11
	COMMON /C/ UVEL,VSTO,WSTO,SINP,COSP,SINLE,IAB,SINTE,COSTE,	FIN	12
	CBETA,TANGM,GAMLE,GAMTE	FIN	13
	REAL MACH,IP,INT1,INT2,INT3,INT4,INT5,INT6,INT7,INT8,	FIN	14
	1INT9,INT11,INT12,INT13,INT14,INT15,INT16,INT17,INT18	FIN	15
	DIMENSION FCTRL(4),ALPHA(6),ROLLR(6),INT1(51),INT2(51),ZETAS(51),	FIN	16
	1ETAS(51),INT3(51),INT4(51),INT5(51),VVEL(6,73,51),WVEL(6,73,51),	FIN	17
	1VSTO(51),WSTO(51),INT6(51),INT7(51),INT8(51),INT9(51),INT10(51),	FIN	18
	1INT11(51),INT12(51),CSUBL(73),CSUB2(73),CSUBM(73),CSUBY(73),CSUBN(FIN	19
	173),UVEL(6),INT13(51),INT14(51),INT15(51),INT16(51),INT17(51),	FIN	20
	1INT18(51),CSUBP(73),CSUBH(73),CSUBB(73),FCTRL(4),CZAV(6),CHAV(6),	FIN	21
	1CYAV(6),CVAV(6),CLAV(6),CZTOT(19),CHTOT(19),CYTOT(19),CNTOT(19),	FIN	22
	1CLTOT(19),TITLE(10)	FIN	23

Table A-2. Computer Program Listing (Cont)

C		FIN	24
C	BEGIN INPUT OF MISSILE AND FIN GEOMETRY (USE METRIC UNITS)	FIN	25
C	INPUT TITLE FOR MISSILE GEOMETRY (MAXIMUM OF 6) CHARACTERS)	FIN	26
C		FIN	27
C	READ (5,120) (TITLE(I),I=1,10)	FIN	28
C		FIN	29
C	A=BODY RADIUS (METER)	FIN	30
C	B0=SEMI-SPAN (METER)	FIN	31
C	CR=ROOT CHORD (METER)	FIN	32
C	X1=X LOCATION OF LEADING EDGE ROOT CHORD (METER)	FIN	33
C	GAMLE=LEADING EDGE SWEEP (DEG)	FIN	34
C	GAMTE=TRAILING EDGE SWEEP (DEG)	FIN	35
C	XH=X LOCATION OF PANEL HINGE MOMENT REFERENCE	FIN	36
C		FIN	37
C	READ (5,*) A,B0,CR,X1,GAMLE,GAMTE,XH	FIN	38
C		FIN	39
C	END INPUT OF MISSILE AND FIN GEOMETRY	FIN	40
C		FIN	41
C	BEGIN INPUT FOR DESIRED FORCES AND MOMENTS	FIN	42
C	ENTER 1 IF QUANTITY DESIRED; ENTER ZERO IF NOT DESIRED	FIN	43
C	ICZCM=FIN NORMAL FORCE AND PITCH MOMENT	FIN	44
C	ICYCN=FIN SIDE FORCE AND YAW MOMENT	FIN	45
C	ICL=FIN ROLL MOMENT	FIN	46
C	IPANEL=PANEL NORMAL FORCE, HINGE MOMENT AND ROOT BENDING MOMENT	FIN	47
C		FIN	48
C	READ (5,*) ICZCM,ICYCN,ICL,IPANEL	FIN	49
C		FIN	50
C	ENTER ANGULAR PARAMETERS OF FINS AND BODY	FIN	51
C	FCOTRL(I)=CONTROL DEFLECTION OF EACH FIN (DEG); FOUR VALUES	FIN	52
C	IALPHA=NUMBER OF BODY ANGLES OF ATTACK (MAX. OF SIX)	FIN	53
C	ALPHA(I)=BODY ANGLES OF ATTACK (DEG)	FIN	54
C	IROLLR=NUMBER OF BODY ROLL RATES (MAX. OF SIX)	FIN	55
C	ROLLR(I)=BODY ROLL RATES (PHIDOT*B)/U, DIMENSIONLESS)	FIN	56
C		FIN	57
C	READ (5,*) (FCOTRL(I),I=1,4)	FIN	58
C	READ (5,*) IALPHA,(ALPHA(I),I=1,IALPHA)	FIN	59
C	READ (5,*) IROLLR,(ROLLR(I),I=1,IROLLR)	FIN	60
C		FIN	61
C	ENTER FREESTREAM MACH NUMBER; FIN LEADING EDGE MUST BE SUBSONIC	FIN	62
C		FIN	63
C	READ (5,*) MACH	FIN	64
C		FIN	65
C	END INPUT FOR FORCES AND MOMENTS	FIN	66
C		FIN	67
C	BEGIN CALCULATION OF NEEDED CONSTANTS	FIN	68
C		FIN	69
C	PI=ACOS(-1.)	FIN	70
C	RAD=180./PI	FIN	71
C	DETA=1./5J.	FIN	72
C	DZETA=DETA	FIN	73
C	AA=A*A	FIN	74
C	B0A=B0-A	FIN	75
C	GAMLE=GAMLE/RAD	FIN	76
C	GAMTE=GAMTE/RAD	FIN	77
C	SINLE=SIN(GAMLE)	FIN	78
C	COSLE=COS(GAMLE)	FIN	79
C	TANLE=TAN(GAMLE)	FIN	80
C	SINTE=SIN(GAMTE)	FIN	81
C	COSTE=COS(GAMTE)	FIN	82
C	TANTE=TAN(GAMTE)	FIN	83
C	SF=.5*B0A**2*(2.*CR/B0A-TANLE+TANTE)	FIN	84
C	AE=2.*B0A**2/SF	FIN	85
C	CT=CR-B*A*(TANLE-TANTE)	FIN	86
C	XV=X1+.5*(.25*CR+.25*CT+B0A*TANLE)	FIN	87
C	GAMMAM=0.	FIN	88
C	IF(MACH.GT.1.) GAMMAM=ACOS(1./MACH)	FIN	89
C	TANGM=TAN(GAMMAM)	FIN	90
C	IF(GAMLE.GE.GAMMAM) GO TO 9	FIN	91

Table A-2. Computer Program Listing (Cont)

	PRINT 200	FIN	92
	STOP	FIN	93
	9 IF(MACH.LE.1.) BETA=SQRT(1.-MACH**2)	FIN	94
	IF(MACH.GT.1.) BETA=SQRT(MACH**2-1.)	FIN	95
	HCZCM=2HNO	FIN	96
	HCCN=2HNO	FIN	97
	HCL=2HNO	FIN	98
	HPANEL=2HNO	FIN	99
	IF(ICZCM.EQ.1) HCZCM=3HYES	FIN	100
	IF(ICCN.EQ.1) HCCN=3HYES	FIN	101
	IF(ICL.EQ.1) HCL=3HYES	FIN	102
	IF(IPANEL.EQ.1) HPANEL=3HYES	FIN	103
C		FIN	104
C	END CALCULATION OF NEEDED CONSTANTS	FIN	105
C		FIN	106
C	BEGIN CALCULATION OF SIGMA	FIN	107
C		FIN	108
	ETA=0.	FIN	109
	DO 11 J=1,51	FIN	110
	ZETA=0.	FIN	111
	ETAF=(1.+ETA**2)*SQRT(1.-ETA**2)	FIN	112
	TEMP=CR-B)*A*(TANLE-TANTE)*ETA	FIN	113
	DO 10 I=1,51	FIN	114
	ZETA=SQRT(ZETA)*EXP(ZETA**2/SQRT(COSLE))	FIN	115
	INT1(I)=TEMP*ZETA*ETAF	FIN	116
	ZETAS(I)=ZETA	FIN	117
10	ZETA=ZETA+DZETA	FIN	118
	CALL SIM(INT1,DZETA,51,INT2(J))	FIN	119
	ETAS(J)=ETA	FIN	120
11	ETA=ETA+DETA	FIN	121
	CALL SIM(INT2,DETA,51,SIGMA)	FIN	122
	SIGMA=SF/30A/SIGMA	FIN	123
C		FIN	124
C	END CALCULATION OF SIGMA	FIN	125
C		FIN	126
C	BEGIN CALCULATION OF CNA	FIN	127
C		FIN	128
	UVEL(1)=1.	FIN	129
	DO 12 J=1,51	FIN	130
	VSTO(J)=0.	FIN	131
12	WSTO(J)=0.	FIN	132
	IAB=1	FIN	133
	SINP=0.	FIN	134
	COSP=1.	FIN	135
	CALL CNPREP	FIN	136
	CNA=CN(1./RAD)*RAD	FIN	137
C		FIN	138
C	END CALCULATION OF CNA	FIN	139
C		FIN	140
C	CALL FIN-FIN INTERFERENCE SUBROUTINE	FIN	141
C		FIN	142
	FCTRL(1)=FCOTRL(1)	FIN	143
	FCTRL(2)=FCOTRL(2)	FIN	144
	FCTRL(3)=FCOTRL(3)	FIN	145
	FCTRL(4)=FCOTRL(4)	FIN	146
	IF((FCOTRL(1).EQ.0.).AND.(FCOTRL(2).EQ.0.).AND.(FCOTRL(3).EQ.0.))	FIN	147
	C.AND.(FCOTRL(4).EQ.0.)) GO TO 14	FIN	148
	CALL IDSUB(FCOTRL,FCTRL)	FIN	149
C		FIN	150
C	CALL ROLL DAMPING INTERFERENCE SUBROUTINE	FIN	151
C		FIN	152
14	IP=0.	FIN	153
	DO 15 I=1,IROLLR	FIN	154
15	IF(ROLLR(I).NE.0.) GO TO 16	FIN	155
	GO TO 17	FIN	156
16	CALL IPSUB(IP)	FIN	157
17	GAMLE=GAMLE*RAO	FIN	158

Table A-2. Computer Program Listing (Cont)

GAMTE=GAMTE+RAD	FIN	159
CNA=CNA/RAD	FIN	160
PRINT 101,(TITLE(I),I=1,10),A,B,CR,X1,GAMLE,GAMTE,XH,SF,AE,CT,	FIN	161
1 SIGMA,CNA,IP,HCZCM,HCCYN,HCL,HPANEL,(I,FCOTRL(I),I=1,4)	FIN	162
PRINT 102,(ALPHA(I),I=1,IALPHA)	FIN	163
DO 19 I=1,IALPHA	FIN	164
18 ALPHA(I)=ALPHA(I)/RAD	FIN	165
PRINT 103	FIN	166
PRINT 104,(ROLLR(I),I=1,IROLLR)	FIN	167
PRINT 112,MACH	FIN	168
GAMLE=GAMLE/RAD	FIN	169
GAMTE=GAMTE/RAD	FIN	170
DO 19 I=1,4	FIN	171
FCOTRL(I)=FCOTRL(I)/RAD	FIN	172
19 FCTRLE(I)=FCTRLE(I)/RAD	FIN	173
C	FIN	174
C CALL FLOW FIELD CALCULATION SUBROUTINE	FIN	175
C	FIN	176
C CALL FLOW(UVEL,VVEL,WVEL)	FIN	177
IAB=1	FIN	179
C	FIN	179
C BEGIN ANGLE OF ATTACK LOOP	FIN	180
C	FIN	181
32 AB=ALPHA(IAB)	FIN	182
COSAB=COS(AB)	FIN	183
C	FIN	184
C BEGIN ROLL RATE LOOP	FIN	185
C	FIN	186
IRR=1	FIN	187
35 PHID=ROLLR(IRR)/80	FIN	188
PHI=0.	FIN	189
IPHI=1	FIN	190
DPHI=5./RAD	FIN	191
DIPHI=1	FIN	192
ISWTC=1	FIN	193
ISAV=0	FIN	194
C	FIN	195
C BEGIN ROLL ANGLE LOOP	FIN	196
C	FIN	197
40 SINP=SIN(PHI)	FIN	198
COSP=COS(PHI)	FIN	199
ITEMP=1	FIN	200
IF((IPHI.GE.20).AND.(IPHI.LE.37)) ITEMP=2	FIN	201
IF((IPHI.GE.38).AND.(IPHI.LE.55)) ITEMP=3	FIN	202
IF(IPHI.GE.56) ITEMP=4	FIN	203
IF(ITEMP+ISAV.EQ.5) ISAV=0	FIN	204
TEMP2=FCTRLE(ITEMP+ISAV)*SIGN(1.,COS(PHI-ISWTC*(1.E-5)))	FIN	205
SINDE=SIN(TEMP2)	FIN	206
COSDE=COS(TEMP2)	FIN	207
TEMP=FCOTRL(ITEMP+ISAV)*SIGN(1.,COS(PHI-ISWTC*(1.E-5)))	FIN	208
SIND=SIN(TEMP)	FIN	209
COSD=COS(TEMP)	FIN	210
DO 45 I=1,51	FIN	211
VSTO(I)=VVEL(IAB,IPHI,I)	FIN	212
45 WSTO(I)=WVEL(IAB,IPHI,I)	FIN	213
C	FIN	214
C CALL SUBROUTINE WHICH PREPARES CONSTANTS NEEDED IN SUBPROGRAM CN	FIN	215
C	FIN	216
C CALL CNPREP	FIN	217
ETA=0.	FIN	218
C	FIN	219
C BEGIN ETA(R) LOOP	FIN	220
C	FIN	221
DO 69 J=1,51	FIN	222
ZETA=0.	FIN	223
RTEMP=A+B)*A*ETA	FIN	224
XTEMP=CR-80A*(TANLE-TANTE)*ETA	FIN	225
XTE=X1+CR+80A*TANTE*ETA	FIN	226

Table A-2. Computer Program Listing (Cont)

	ETAF=(1.+ETA**2**AE))*SQRT(1.-ETA**2)	FIN	227
	PHIDR=PHID*RTMP	FIN	228
	UL=SQRT(UVEL(IAB)**2+(VSTO(J)+PHIDR*SINP)**2+(WSTO(J)-	FIN	229
	1PHIDR*COSP)**2)	FIN	230
	TEMP1=COSAB*(SINDE+IP*PHIDR)	FIN	231
	TEMP2=(VSTO(J)+PHIDR*SINP)*COSDE*SINP	FIN	232
	TEMP3=(WSTO(J)-PHIDR*COSP)*COSDE*COSP	FIN	233
	VECN=1.+2.*SINDE*IP*PHIDR+(IP*PHIDR)**2	FIN	234
	AL=ASIN((TEMP1-TEMP2+TEMP3)/VECN/UL)	FIN	235
	CNN=CN(AL)	FIN	236
C		FIN	237
C	BEGIN ZETA(X) LOOP	FIN	238
C		FIN	239
	DO 59 I=1,51	FIN	240
	ZETA=SQRT(ZETA)*EXP(ZETA**2/SQRT(COSLE))	FIN	241
	TEMP=XTEMP*ZETA*ETAF*CNN*UL**2	FIN	242
	IF(ICL.EQ.0) GO TO 55	FIN	243
	INT3(I)=RTMP*TEMP	FIN	244
55	IF(ICZCH.EQ.0) GO TO 57	FIN	245
	INT4(I)=TEMP*COSD*COSP	FIN	246
	INT5(I)=TEMP*((XTE-XTEMP*ZETA)*COSD*COSP-RTMP*SIND*SINP)	FIN	247
57	IF(ICYCN.EQ.0) GO TO 58	FIN	248
	INT6(I)=TEMP*COSD*SINP	FIN	249
	INT7(I)=TEMP*((XTE-XTEMP*ZETA)*COSD*SINP+RTMP*SIND*COSP)	FIN	250
58	IF(IPANEL.EQ.0) GO TO 59	FIN	251
	INT13(I)=TEMP	FIN	252
	INT14(I)=TEMP*((XTE-XTEMP*ZETA)-XN)	FIN	253
	INT15(I)=ETA*TEMP	FIN	254
59	ZETA=ZETA+DZETA	FIN	255
C		FIN	256
C	END ZETA(X) LOOP	FIN	257
C		FIN	258
	IF(ICL.EQ.0) GO TO 65	FIN	259
	CALL SIM(INT3,DZETA,51,INT8(J))	FIN	260
65	IF(ICZCH.EQ.0) GO TO 67	FIN	261
	CALL SIM(INT4,DZETA,51,INT9(J))	FIN	262
	CALL SIM(INT5,DZETA,51,INT10(J))	FIN	263
57	IF(ICYCN.EQ.0) GO TO 68	FIN	264
	CALL SIM(INT6,DZETA,51,INT11(J))	FIN	265
	CALL SIM(INT7,DZETA,51,INT12(J))	FIN	266
58	IF(IPANEL.EQ.0) GO TO 69	FIN	267
	CALL SIM(INT13,DZETA,51,INT16(J))	FIN	268
	CALL SIM(INT14,DZETA,51,INT17(J))	FIN	269
	CALL SIM(INT15,DZETA,51,INT18(J))	FIN	270
69	ETA=ETA+DETA	FIN	271
C		FIN	272
C	END ETA(R) LOOP	FIN	273
C		FIN	274
	IF(ICL.EQ.0) GO TO 70	FIN	275
	CALL SIM(INT8,DETA,51,CSUBL(IPHI))	FIN	276
70	IF(ICZCH.EQ.0) GO TO 72	FIN	277
	CALL SIM(INT9,DETA,51,CSUBZ(IPHI))	FIN	278
	CALL SIM(INT10,DETA,51,CSUBH(IPHI))	FIN	279
72	IF(ICYCN.EQ.0) GO TO 73	FIN	280
	CALL SIM(INT11,DETA,51,CSUBY(IPHI))	FIN	281
	CALL SIM(INT12,DETA,51,CSUBN(IPHI))	FIN	282
73	IF(IPANEL.EQ.0) GO TO 74	FIN	283
	CALL SIM(INT16,DETA,51,CSUBP(IPHI))	FIN	284
	CALL SIM(INT17,DETA,51,CSUBH(IPHI))	FIN	285
	CALL SIM(INT18,DETA,51,CSUBB(IPHI))	FIN	286
74	PHI=PHI+DPHI	FIN	287
	IPHI=IPHI+DIPHI	FIN	288
	IF(IPHI.LE.73) GO TO 40	FIN	289
	IF(((FCOTRL(1).EQ.0.).AND.(FCOTRL(2).EQ.0.).AND.(FCOTRL(3).EQ.0.))	FIN	290
	1.AND.(FCOTRL(4).EQ.0.)).OR.(ISWTCH.EQ.-1)) GO TO 79	FIN	291
C		FIN	292
C	BEGIN PREPARATIONS FOR DOUBLE VALUED CONTROL FORCES AND MOMENTS	FIN	293

Table A-2. Computer Program Listing (Cont)

C	PHI=PI/2.	FIN	294
	IPHI=19	FIN	295
	DPHI=PI/2.	FIN	296
	DIPHI=18	FIN	297
	ISWTCH=-1	FIN	298
	ISAV=1	FIN	299
	IF(ICL.EQ.0) GO TO 75	FIN	300
	SAVL19=CSUBL(19)	FIN	301
	SAVL37=CSUBL(37)	FIN	302
	SAVL55=CSUBL(55)	FIN	303
75	IF(ICZCH.EQ.0) GO TO 76	FIN	304
	SAVZ19=CSUBZ(19)	FIN	305
	SAVM19=CSUBM(19)	FIN	306
	SAVZ37=CSUBZ(37)	FIN	307
	SAVM37=CSUBM(37)	FIN	308
	SAVZ55=CSUBZ(55)	FIN	309
	SAVM55=CSUBM(55)	FIN	310
76	IF(ICYCN.EQ.0) GO TO 77	FIN	311
	SAVY19=CSUBY(19)	FIN	312
	SAVN19=CSUBN(19)	FIN	313
	SAVY37=CSUBY(37)	FIN	314
	SAVN37=CSUBN(37)	FIN	315
	SAVY55=CSUBY(55)	FIN	316
	SAVN55=CSUBN(55)	FIN	317
77	IF(IPANEL.EQ.0) GO TO 78	FIN	318
	SAVP19=CSUBP(19)	FIN	319
	SAVH19=CSUBH(19)	FIN	320
	SAVB19=CSUBB(19)	FIN	321
	SAVP37=CSUBP(37)	FIN	322
	SAVH37=CSUBH(37)	FIN	323
	SAVB37=CSUBB(37)	FIN	324
	SAVP55=CSUBP(55)	FIN	325
	SAVH55=CSUBH(55)	FIN	326
	SAVB55=CSUBB(55)	FIN	327
78	GO TO 40	FIN	328
		FIN	329
C		FIN	330
C	END PREPARATIONS FOR DOUBLE VALUED CONTROL FORCES AND MOMENTS	FIN	331
C		FIN	332
	79 CONTINUE	FIN	333
C		FIN	334
C	END OF ROLL ANGLE LOOP	FIN	335
C		FIN	336
C	BEGIN PRIMARY OUTPUT	FIN	337
C		FIN	338
	TEMPH=SIGMA*80A/2./PI/A**3	FIN	339
	TEMPF=SIGMA*80A/PI/AA	FIN	340
	TEMPP=SIGMA*80A/SF	FIN	341
	ATEMP=ALPHA(IAB)*RAD	FIN	342
	IF(ICZCH.EQ.0) GO TO 82	FIN	343
		FIN	344
C		FIN	345
C	BEGIN OUTPUT FOR CZ AND CH	FIN	346
C		FIN	347
	PRINT 105,ROLLR(IRR),ATEMP	FIN	348
	PRINT 105	FIN	349
	DO 80 I=1,19	FIN	350
	PHI=5.*(I-1)	FIN	351
	TEMP1=TEMPF*CSUBZ(I)	FIN	352
	TEMP2=TEMPF*CSUBZ(I+18)	FIN	353
	TEMP3=TEMPF*CSUBZ(I+36)	FIN	354
	TEMP4=TEMPF*CSUBZ(I+54)	FIN	355
	IF((I.NE.19).OR.(ISWTCH.EQ.1)) GO TO 795	FIN	356
	TEMP1=TEMPF*SAVZ19	FIN	357
	TEMP2=TEMPF*SAVZ37	FIN	358
	TEMP3=TEMPF*SAVZ55	FIN	359
795	CZTOT(I)=TEMP1+TEMP2+TEMP3+TEMP4	FIN	360
	80 PRINT 107,PHI,CZTOT(I),TEMP1,TEMP2,TEMP3,TEMP4	FIN	360

Table A-2. Computer Program Listing (Cont)

PRINT 108	FIN	361
DO 81 I=1,19	FIN	362
PHI=5.*(I-1)	FIN	363
TEMP1=-TEMPM*CSUBM(I)	FIN	364
TEMP2=-TEMPM*CSUBM(I+18)	FIN	365
TEMP3=-TEMPM*CSUBM(I+36)	FIN	366
TEMP4=-TEMPM*CSUBM(I+54)	FIN	367
IF((I.NE.19).OR.(ISWTCH.EQ.1)) GO TO 805	FIN	368
TEMP1=-TEMPM*SAVM19	FIN	369
TEMP2=-TEMPM*SAVM37	FIN	370
TEMP3=-TEMPM*SAVM55	FIN	371
805 CMTOT(I)=TEMP1+TEMP2+TEMP3+TEMP4	FIN	372
81 PRINT 107,PHI,CMTOT(I),TEMP1,TEMP2,TEMP3,TEMP4	FIN	373
C	FIN	374
C	FIN	375
C	FIN	376
82 IF(ICYCN.EQ.0) GO TO 85	FIN	377
C	FIN	378
C	FIN	379
C	FIN	380
PRINT 105,ROLLR(IRR),ATEMP	FIN	381
PRINT 109	FIN	382
DO 83 I=1,19	FIN	383
PHI=5.*(I-1)	FIN	384
TEMP1=-TEMPF*CSUBY(I)	FIN	385
TEMP2=-TEMPF*CSUBY(I+18)	FIN	386
TEMP3=-TEMPF*CSUBY(I+36)	FIN	387
TEMP4=-TEMPF*CSUBY(I+54)	FIN	388
IF((I.NE.19).OR.(ISWTCH.EQ.1)) GO TO 925	FIN	389
TEMP1=-TEMPF*SAVY19	FIN	390
TEMP2=-TEMPF*SAVY37	FIN	391
TEMP3=-TEMPF*SAVY55	FIN	392
825 CYTOT(I)=TEMP1+TEMP2+TEMP3+TEMP4	FIN	393
83 PRINT 107,PHI,CYTOT(I),TEMP1,TEMP2,TEMP3,TEMP4	FIN	394
PRINT 110	FIN	395
DO 84 I=1,19	FIN	396
PHI=5.*(I-1)	FIN	397
TEMP1=-TEMPM*CSUBN(I)	FIN	398
TEMP2=-TEMPM*CSUBN(I+18)	FIN	399
TEMP3=-TEMPM*CSUBN(I+36)	FIN	400
TEMP4=-TEMPM*CSUBN(I+54)	FIN	401
IF((I.NE.19).OR.(ISWTCH.EQ.1)) GO TO 835	FIN	402
TEMP1=-TEMPM*SAVN19	FIN	403
TEMP2=-TEMPM*SAVN37	FIN	404
TEMP3=-TEMPM*SAVN55	FIN	405
835 CNTOT(I)=TEMP1+TEMP2+TEMP3+TEMP4	FIN	406
84 PRINT 107,PHI,CNTOT(I),TEMP1,TEMP2,TEMP3,TEMP4	FIN	407
C	FIN	408
C	FIN	409
C	FIN	410
85 IF(ICL.EQ.0) GO TO 87	FIN	411
C	FIN	412
C	FIN	413
C	FIN	414
PRINT 105,ROLLR(IRR),ATEMP	FIN	415
PRINT 111	FIN	416
DO 86 I=1,19	FIN	417
PHI=5.*(I-1)	FIN	418
TEMP1=TEMPM*CSUBL(I)	FIN	419
TEMP2=TEMPM*CSUBL(I+18)	FIN	420
TEMP3=TEMPM*CSUBL(I+36)	FIN	421
TEMP4=TEMPM*CSUBL(I+54)	FIN	422
IF((I.NE.19).OR.(ISWTCH.EQ.1)) GO TO 855	FIN	423
TEMP1=TEMPM*SAVL19	FIN	424
TEMP2=TEMPM*SAVL37	FIN	425
TEMP3=TEMPM*SAVL55	FIN	426
855 CLTOT(I)=TEMP1+TEMP2+TEMP3+TEMP4	FIN	427
86 PRINT 107,PHI,CLTOT(I),TEMP1,TEMP2,TEMP3,TEMP4	FIN	428

Table A-2. Computer Program Listing (Cont)

C		FIN	429
C	END OUTPUT FOR CL	FIN	430
C		FIN	431
C	87 IF(IPANEL.EQ.0) GO TO 89	FIN	432
C		FIN	433
C	BEGIN OUTPUT FOR PANEL LOADS	FIN	434
C		FIN	435
	IF(ICL.EQ.0) PRINT 105,ROLLR(IRR),ATEMP	FIN	436
	PRINT 113	FIN	437
	DO 88 I=1,19	FIN	438
	PHI=5.*(I-1)	FIN	439
	TEMP1=TEMP*CSUBP(I)	FIN	440
	TEMP2=TEMP*CSUBP(I+18)	FIN	441
	TEMP3=TEMP*CSUBP(I+36)	FIN	442
	TEMP4=TEMP*CSUBP(I+54)	FIN	443
	TEMP5=-TEMP*CSUBH(I)/CR	FIN	444
	TEMP6=-TEMP*CSUBH(I+18)/CR	FIN	445
	TEMP7=-TEMP*CSUBH(I+36)/CR	FIN	446
	TEMP8=-TEMP*CSUBH(I+54)/CR	FIN	447
	TEMP9=TEMP*CSUB8(I)	FIN	448
	TEMP10=TEMP*CSUB8(I+18)	FIN	449
	TEMP11=TEMP*CSUB8(I+36)	FIN	450
	TEMP12=TEMP*CSUB8(I+54)	FIN	451
	IF((I.NE.19).OR.(ISWTCH.EQ.1)) GO TO 88	FIN	452
	TEMP1=TEMP*SAVP19	FIN	453
	TEMP2=TEMP*SAVP37	FIN	454
	TEMP3=TEMP*SAVP55	FIN	455
	TEMP5=-TEMP*SAVH19/CR	FIN	456
	TEMP6=-TEMP*SAVH37/CR	FIN	457
	TEMP7=-TEMP*SAVH55/CR	FIN	458
	TEMP9=TEMP*SAVB19	FIN	459
	TEMP10=TEMP*SAVB37	FIN	460
	TEMP11=TEMP*SAVB55	FIN	461
	88 PRINT 114,PHI,TEMP1,TEMP2,TEMP3,TEMP4,TEMP5,TEMP6,TEMP7,TEMP8,	FIN	462
	CTEMP9,TEMP10,TEMP11,TEMP12	FIN	463
C		FIN	464
C	END OUTPUT FOR PANEL LOADS	FIN	465
C		FIN	466
	89 DPHI=5./RAD	FIN	467
	IF(ICZCM.EQ.0) GO TO 93	FIN	468
	CALL SIM(CZTOT,DPHI,19,CZAV(IRR))	FIN	469
	CALL SIM(CMTOT,DPHI,19,CMAV(IRR))	FIN	470
	CZAV(IRR)=CZAV(IRR)*2./PI	FIN	471
	CMAV(IRR)=CMAV(IRR)*2./PI	FIN	472
	93 IF(ICYCN.EQ.0) GO TO 91	FIN	473
	CALL SIM(CYTOT,DPHI,19,CYAV(IRR))	FIN	474
	CALL SIM(CNTOT,DPHI,19,CNAV(IRR))	FIN	475
	CYAV(IRR)=CYAV(IRR)*2./PI	FIN	476
	CNAV(IRR)=CNAV(IRR)*2./PI	FIN	477
	91 IF(ICL.EQ.0) GO TO 92	FIN	478
	CALL SIM(CLTOT,DPHI,19,CLAV(IRR))	FIN	479
	CLAV(IRR)=CLAV(IRR)*2./PI	FIN	480
	92 IRR=IRR+1	FIN	481
	IF(IRR.LE.IROLLR) GO TO 35	FIN	482
C		FIN	483
C	END ROLL RATE LOOP	FIN	484
C		FIN	485
	IF((ROLLR(1).EQ.0.).AND.(IROLLR.EQ.1)) GO TO 99	FIN	486
	IF((ICZCM.EQ.0).AND.(ICYCN.EQ.0).AND.(ICL.EQ.0)) GO TO 99	FIN	487
C		FIN	488
C	BEGIN OUTPUT FOR AVERAGE FORCES AND MOMENTS OVER A CYCLE OF ROLL	FIN	489
C		FIN	490
	PRINT 115,ATEMP	FIN	491
	IF(ICZCM.EQ.0) GO TO 94	FIN	492
	PRINT 116	FIN	493
	DO 93 I=1,IROLLR	FIN	494
	93 PRINT 117,ROLLR(I),CZAV(I),CMAV(I)	FIN	495
	94 IF(ICYCN.EQ.0) GO TO 96	FIN	496

Table A-2. Computer Program Listing (Cont)

PRINT 118	FIN	497
DO 95 I=1,IROLLR	FIN	498
95 PRINT 117,ROLLR(I),CYAV(I),CNAV(I)	FIN	499
96 IF(ICL.EQ.3) GO TO 99	FIN	500
PRINT 119	FIN	501
DO 97 I=1,IROLLR	FIN	502
97 PRINT 117,ROLLR(I),CLAV(I)	FIN	503
C	FIN	504
END OUTPUT FOR AVERAGE FORCES AND MOMENTS OVER A CYCLE OF ROLL	FIN	505
C	FIN	506
END PRIMARY OUTPUT	FIN	507
C	FIN	508
99 IAB=IAB+1	FIN	509
IF(IAB.LE.IALPHA) GO TO 30	FIN	510
C	FIN	511
END ANGLE OF ATTACK LOOP	FIN	512
C	FIN	513
100 FORMAT(1H1)	FIN	514
101 FORMAT (141,/,60X,*F I N L O A D *,/,42X,*PREDICTION OF FORCES AN	FIN	515
10 MOMENTS PRODUCED BY FINS*,/,5X,15A5,/,5X,*BODY RADIUS =*,	FIN	516
1F6.3,* M*,/,5X,*FIN SEMI-SPAN =*,F6.3,* M*,/,5X,*ROOT CHORD =*,	FIN	517
1F6.3,* M*,/,5X,*AXIAL LOCATION OF FIN LEADING EDGE AT THE BODY =*,	FIN	518
1F7.3,* M*,/,5X,*LEADING EDGE SWEEP =*,F6.2,* DEG*,/,5X,	FIN	519
1*TRAILING EDGE SWEEP =*,F6.2,* DEG*,/,5X,	FIN	520
1*AXIAL LOCATION OF HINGE LINE =*,F7.3,* M*,/,5X,	FIN	521
1*EXPOSED SURFACE AREA OF A FIN =*,F7.3,* SQ. M*,/,5X,	FIN	522
1*EXPOSED ASPECT RATIO =*,F6.3,/,5X,*TIP CHORD =*,F6.3,* M*,/,5X,	FIN	523
1*NORMAL FORCE SCALING COEFFICIENT =*,F6.3,/,5X,*NORMAL FORCE COEFF	FIN	524
1ICIENT SLOPE (FIN ALONE, N/Q SFIN) =*,F6.4,* (1/DEG)*,/,5X,	FIN	525
1	FIN	526
1EFFICIENT =*,F5.3,/,5X,*CZ AND CM DESIRED, *A1,/,5X,	FIN	527
1*CY AND CN DESIRED, *A10,/,5X,*CL DESIRED, *A1,/,5X,	FIN	528
1*CP, CH AND CB DESIRED, *A10,4(,/,5X,*CONTROL DEFLECTION FIN*,I2,	FIN	529
1* =*,F6.2,* DEG*),/,5X,*ANGLES OF ATTACK DESIRED (DEG)*)	FIN	530
102 FORMAT (10X,6F7.1)	FIN	531
103 FORMAT (/5X,*ROLL RATES DESIRED (DIMENSIONLESS)*)	FIN	532
104 FORMAT (10X,6F7.3)	FIN	533
105 FORMAT(1H1,/,5X,*ROLL RATE =*,F6.3,* (DIMENSIONLESS)*,5X,	FIN	534
1*ANGLE OF ATTACK =*,F5.1,* DEG*)	FIN	535
106 FORMAT(,/,5X,*PHI*,10X,*CZ FINS*,10X,*CZ FIN1*,10X,*CZ FIN2*,	FIN	536
110X,*CZ FIN3*,10X,*CZ FIN4*,/)	FIN	537
107 FORMAT(F9.1,F16.3,4F17.3)	FIN	538
108 FORMAT(,/,5X,*PHI*,10X,*CM FINS*,10X,*CM FIN1*,10X,*CM FIN2*,	FIN	539
110X,*CM FIN3*,10X,*CM FIN4*,/)	FIN	540
109 FORMAT(,/,5X,*PHI*,10X,*CY FINS*,10X,*CY FIN1*,10X,*CY FIN2*,	FIN	541
110X,*CY FIN3*,10X,*CY FIN4*,/)	FIN	542
110 FORMAT(,/,5X,*PHI*,10X,*CN FINS*,10X,*CN FIN1*,10X,*CN FIN2*,	FIN	543
110X,*CN FIN3*,10X,*CN FIN4*,/)	FIN	544
111 FORMAT(,/,5X,*PHI*,10X,*CL FINS*,10X,*CL FIN1*,10X,*CL FIN2*,	FIN	545
110X,*CL FIN3*,10X,*CL FIN4*,/)	FIN	546
112 FORMAT (/5X,*MACH NUMBER =*,F5.2)	FIN	547
113 FORMAT (,/,5X,*PHI*,3X,*CP FIN1*,3X,*CP FIN2*,3X,*CP FIN3*,3X,	FIN	548
1*CP FIN4*,3X,*CH FIN1*,3X,*CH FIN2*,3X,*CH FIN3*,3X,*CH FIN4*,3X,	FIN	549
1*CB FIN1*,3X,*CB FIN2*,3X,*CB FIN3*,3X,*CB FIN4*,/)	FIN	550
114 FORMAT (F8.1,F9.3,11F10.3)	FIN	551
115 FORMAT (1H1,/,5X,*AVERAGE VALUE OF FORCES AND MOMENTS OVER ONE ROL	FIN	552
11 CYCLE*,/,5X,*ANGLE OF ATTACK =*,F5.1,* DEG*)	FIN	553
116 FORMAT (,/,5X,*ROLL RATE*,10X,*CZ AVERAGE*,10X,*CM AVERAGE*/)	FIN	554
117 FORMAT (F12.3,F20.4,F20.3)	FIN	555
118 FORMAT (,/,5X,*ROLL RATE*,10X,*CY AVERAGE*,10X,*CN AVERAGE*/)	FIN	556
119 FORMAT (,/,5X,*ROLL RATE*,10X,*CL AVERAGE*/)	FIN	557
120 FORMAT (15A6)	FIN	558
200 FORMAT (1H1,10(/),10X,	FIN	559
1*))/10X	FIN	560
1*MACH NUMBER TOO LARGE. SUPERSONIC LEADING EDGE WILL RESULT.*/10X	FIN	561
1*)))	FIN	562
END	FIN	563
SUBROUTINE IDSUB(F,FE)	FIN	564

Table A-2. Computer Program Listing (Cont)

C		FIN	555
C	THIS SUBROUTINE CALCULATES FIN-FIN INTERFERENCE DUE TO CONTROL	FIN	566
C	DEFLECTION	FIN	567
C		FIN	568
	COMMON A,BJA,CR,AE,PI,DETA,COSLE,TANLE,TANTE,MACH,ALPHA	FIN	569
	REAL IDT,MACH,IO	FIN	570
	DIMENSION F(4),IDT(11),DOBT(11),ALPHA(6),FE(4)	FIN	571
	DATA (IDT(I),I=1,11)/.275,.249,.214,.171,.128,.190,.057,.028,	FIN	572
	C.000,.000,.000/	FIN	573
	DOB=A/(80A+A)	FIN	574
	X=0.	FIN	575
	DO 5 I=1,11	FIN	576
	DOBT(I)=X	FIN	577
5	X=X+.1	FIN	578
	ID=TBLOOK(DOB,DOBT,IDT,11)	FIN	579
	FE(1)=F(1)+ID*(F(2)-F(4))	FIN	580
	FE(2)=F(2)+ID*(F(1)-F(3))	FIN	591
	FE(3)=F(3)+ID*(F(4)-F(2))	FIN	582
	FE(4)=F(4)+ID*(F(3)-F(1))	FIN	583
	RETURN	FIN	584
	END	FIN	585
	SUBROUTINE IPSUB(IP)	FIN	586
C		FIN	587
C	THIS SUBROUTINE CALCULATES THE ROLL DAMPING MOMENT INTERFERENCE	FIN	588
C	COEFFICIENT	FIN	589
C		FIN	590
	COMMON A,BJA,CR,AE,PI,DETA,COSLE,TANLE,TANTE,MACH,ALPHA	FIN	591
	COMMON /A/ B0,SF,SIGMA,CNA,INT1,INT2	FIN	592
	REAL IP,INT1,INT2,MACH	FIN	593
	DIMENSION DOBT(11),CLPT(11),INT1(51),INT2(51),ALPHA(6)	FIN	594
	DATA (CLPT(I),I=1,11)/-.159,-.159,-.161,-.163,-.160,-.146,-.121,	FIN	595
	1-.086,-.043,-.013,.000/	FIN	596
	SFP=2.*SF+A*(2.*CR+A*(TANLE-TANTE))	FIN	597
	AP=(2.*B0)**2/SFP	FIN	598
	TEMP=SFP+2.*B0/(PI*A*A+2.*A)	FIN	599
	DOB=A/B0	FIN	600
	X=0.	FIN	601
	DO 5 I=1,11	FIN	602
	DOBT(I)=X	FIN	603
5	X=X+.1	FIN	604
	CLP=TBLOOK(DOB,DOBT,CLPT,11)*TEMP*AP	FIN	605
	DZETA=1./50.	FIN	606
	DETA=DZETA	FIN	607
	ETA=0.	FIN	608
	DO 11 J=1,51	FIN	609
	ZETA=0.	FIN	610
	DO 10 I=1,51	FIN	611
	ZETAF=SQRT(ZETA)*EXP(ZETA**2/SQRT(COSLE))	FIN	612
	ETAF=(1.+ETA*(2.*AE))*SQRT(1.-ETA**2)	FIN	613
	INT1(I)=(A+B0A*ETA)**2*(CR-B0A*(TANLE-TANTE)*ETA)*ZETAF*ETAF	FIN	614
10	ZETA=ZETA+DZETA	FIN	615
	CALL SIM(INT1,DZETA,51,INT2(J))	FIN	616
11	ETA=ETA+DETA	FIN	617
	CALL SIM(INT2,DETA,51,TEMP)	FIN	618
	IP=1.+CLP/(2./PI*SIGMA*B0A/B0/A**3*CNA*TEMP)	FIN	619
	IF(IP.LT.1.) IP=0.	FIN	620
	RETURN	FIN	621
	END	FIN	622
	FUNCTION CN(A)	FIN	623
C		FIN	624
C	THIS SUBPROGRAM CALCULATES THE LOCAL NORMAL FORCE COEFFICIENT	FIN	625
C	GIVEN THE LOCAL ANGLE OF ATTACK	FIN	626
C		FIN	627
	COMMON /D/ KP,KV,AS,SAV1,SAV2,SAV3	FIN	628
	REAL KP,KV	FIN	629
	CN=0.	FIN	630
	IF(A.EQ.0.) RETURN	FIN	631
	ASAVE=A	FIN	632

Table A-2. Computer Program Listing (Cont)

SIGN=A/ABS(A)	FIN	633
A=ABS(A)	FIN	634
IF(A.GT.AS) GO TO 1	FIN	635
CN=(KP*SIN(A)*COS(A)+KV*SIN(A)**2)*(1.-SAV1*A**3)	FIN	636
GO TO 3	FIN	637
1 CNAS=(KP*SIN(AS)*COS(AS)+KV*SIN(AS)**2)*(1.-SAV1*AS**3)	FIN	638
IF(A.GT.2.*AS) GO TO 2	FIN	639
CNAMAS=(KP*SIN(A-AS)*COS(A-AS)+KV*SIN(A-AS)**2)*(1.-SAV1*(A-AS)**3)	FIN	640
CN=SAV2*CNAS+SAV3*CNAMAS	FIN	641
GO TO 3	FIN	642
2 CN=(SAV2+SAV3)*CNAS	FIN	643
3 CN=SIGN*CN	FIN	644
A=ASAVE	FIN	645
RETURN	FIN	646
END	FIN	647
SUBROUTINE CNPREP	FIN	648
C THIS SUBPROGRAM CALCULATES VARIOUS CONSTANTS NEEDED IN THE	FIN	649
C SUBPROGRAM CN	FIN	650
C	FIN	651
COMMON A,B0A,CR,AE,PI,DETA,COSLE,TANLE,TANTE,MACH,ALPHA	FIN	652
COMMON /C/ UVEL,VSTO,WSTO,SINP,COSP,SINLE,IAB,SINTE,COSTE,	FIN	653
CBETA,TANGM,GAMLE,GAMTE	FIN	654
COMMON /D/ KP,KV,ASTL,SAV1,SAV2,SAV3	FIN	655
REAL MACH,MU,KP,KV	FIN	656
DIMENSION VSTO(51),WSTO(51),UVEL(6),ALPHA(6)	FIN	657
UAV=UVEL(IAB)	FIN	658
CALL SIM(VSTO,DETA,51,VAV)	FIN	659
CALL SIM(WSTO,DETA,51,WAV)	FIN	660
VRAV=VAV*COSP+WAV*SINP	FIN	661
GAMLEE=GAMLE+ASIN(VRAV/SQRT(UAV**2+VRAV**2))	FIN	662
GAMTEE=GAMTE+ASIN(VRAV/SQRT(UAV**2+VRAV**2))	FIN	663
IF(ABS(GAMLEE).GT.1.57) GAMLEE=SIGN(1.57,GAMLEE)	FIN	664
IF(GAMLEE.LT.0.) GAMLEE=0.	FIN	665
IF(ABS(GAMTEE).GT.1.57) GAMTEE=SIGN(1.57,GAMTEE)	FIN	666
COSLEE=COS(GAMLEE)	FIN	667
SINLEE=SIN(GAMLEE)	FIN	668
TANTEE=TAN(GAMTEE)	FIN	669
CRE=CR+COSTE/COS(GAMTE-GAMLE+GAMLEE)	FIN	670
B0AE=B0A+COSLEE/COSLE	FIN	671
AEE=4./(2.*CR+COSTE+COSLE/(B0A+COSLEE+COS(GAMTE-GAMLE+GAMLEE))	FIN	672
1 TANTEE=TAN(GAMLEE)	FIN	673
CRS=CRE+B0AE+TANTEE	FIN	674
GAMMA=ATAN(CRS/B0AE)	FIN	675
TANG=TAN(GAMMA)	FIN	676
PS=2./AEE/TANG	FIN	677
IF(MACH.GT.1.) PS=PS/(1.-TANGM/2./TANG)	FIN	678
PSI=ATAN(2.*PS+TANG)	FIN	679
IF(MACH.GT.1.) PSI=ATAN(2.*PS*(TANG-TANGM))	FIN	680
TANPSI=TAN(PSI)	FIN	681
SINPSI=SIN(PSI)	FIN	682
KP=4.*PI/(TANPSI+SQRT(TANPSI**2+SINPSI**2/PS**2+4.*BETA**2))	FIN	683
KV=(KP-KP**2*TANPSI/4./PI)*SQRT(1.+TANPSI**2)	FIN	684
AETEMP=AEE	FIN	685
IF(AETEMP.LT.1.) AETEMP=1.	FIN	686
IF(AETEMP.GT.4.) AETEMP=4.	FIN	687
ASTL=.6632*(1.+02*(AETEMP-2.)**4)*(1.+1.1*(AETEMP-1.)**2.5*	FIN	688
CCOSLEE**2)*(1.+6.*(AETEMP-1.)**2*SINLEE**4/EXP(AETEMP))/(COSLEE+1)	FIN	689
C**2*(AETEMP-1.)	FIN	690
MU=.9-.2*SQRT(AETEMP)*SIN(2.*GAMLEE)	FIN	691
SAV1=(1.-MU)/ASTL**3	FIN	692
SINLEE=ABS(SINLEE)	FIN	693
SAV2=1.-.35*(1.+EXP(AETEMP)/8.*SINLEE*(1./AETEMP)*SIN((AETEMP-2.1	FIN	694
C)*GAMLEE)/(1.+5*(AETEMP-1.)**3)	FIN	695
IF(SAV2.LT..65) SAV2=.65	FIN	696
SAV3=COSLEE**2*AEE/10.	FIN	697
RETURN	FIN	698
END	FIN	699
	FIN	700

Table A-2. Computer Program Listing (Cont)

C	SUBROUTINE FLOW (U,VVEL,WVEL)	FIN	731
C	THIS SUBROUTINE CALCULATES THE FLOW FIELD ABOUT THE BODY	FIN	732
C	FOR UP TO SIX ANGLES OF ATTACK, EVERY FIVE DEGREES IN ROLL	FIN	733
C	ANGLE, AND AT 51 EQUALLY SPACED RADIAL LOCATIONS ALONG THE	FIN	734
C	FIN SEMI-SPAN. A DIFFERENT BODY FLOW FIELD CALCULATION	FIN	705
C	TECHNIQUE CAN BE USED IF THE RESULT IS PROPERLY STORED IN	FIN	735
C	U, VVEL, AND WVEL.	FIN	707
C		FIN	708
C	COMMON A,BJA,CR,AE,PI,DETA,COSLE,TANLE,TANTE,MACH,ALPHA	FIN	739
	COMMON /B/ RAD,X1,IALPHA	FIN	710
	REAL LS,LSHEET,MACH	FIN	711
	DIMENSION YV(44),ZV(44),PHIV(44),RV(44),LS(102),PHISS(102),	FIN	712
	1 VVEL(6,73,51),WVEL(6,73,51),ALPHA(5),U(6)	FIN	713
	NV=44	FIN	714
	BR=BJA/A*DETA	FIN	715
	XV=X1/A	FIN	716
	IAB=1	FIN	717
10	AB=ALPHA(IAB)	FIN	718
	U(IAB)=COS(AB)	FIN	719
	SINAB=SIN(AB)	FIN	720
C		FIN	721
C	BEGIN CONCENTRATED VORTEX CALCULATION	FIN	722
C		FIN	723
	PHIV(1)=74./RAD	FIN	724
	RV(1)=.70+.060*SQRT(MACH+1.)*(XV+6.)*SQRT(AB)	FIN	725
	GT=.35*PI*(XV-6.)*AB**2	FIN	726
	GC0GT=1.-.15*AB*XV+.008*(AB*XV)**2	FIN	727
	GC=GC0GT*GT	FIN	728
	GS=GT-GC	FIN	729
	RC=.030*XV*SQRT(AB)	FIN	730
	IF(RC.LT.1.E-10) RC=1.E-10	FIN	731
	YV(1)=RV(1)*COS(PHIV(1))	FIN	732
	ZV(1)=RV(1)*SIN(PHIV(1))	FIN	733
C		FIN	734
C	END CONCENTRATED VORTEX CALCULATION	FIN	735
C		FIN	736
C	BEGIN VORTEX SHEET CALCULATION	FIN	737
C		FIN	738
	PHISS(1)=3.	FIN	739
	DPHIV=PHIV(1)/100.	FIN	740
	DO 21 N=1,101	FIN	741
	COSDUM=COS(PI*PHISS(N)/(2.*PHIV(1)))	FIN	742
	SINDUM=SIN(PI*PHISS(N)/(2.*PHIV(1)))	FIN	743
	BLANK=1.+(RV(1)+RC)*(PHIV(1)-PHISS(N))	FIN	744
	RSHEET=COSDUM*(RV(1)+RC)*SINDUM**2/BLANK	FIN	745
	DRSDPH=-PI/(2.*PHIV(1))*SINDUM+((RV(1)+RC)*PI*SINDUM*COSDUM*	FIN	746
	1 BLANK/PHIV(1)+(RV(1)+RC)**2*SINDUM**2)/BLANK**2	FIN	747
	LS(N)=SQRT(RSHEET**2+DRSDPH**2)	FIN	748
	IF(RSHEET.LT.1.01) GO TO 20	FIN	749
	GO TO 21	FIN	750
20	LS(N)=0.	FIN	751
	PHIV(5)=PHISS(N)	FIN	752
21	PHISS(N+1)=PHISS(N)+DPHIV	FIN	753
	CALL SIM(LS,DPHIV,101,LSHEET)	FIN	754
	YV(5)=1.01 *COS(PHIV(5))	FIN	755
	ZV(5)=1.01 *SIN(PHIV(5))	FIN	756
	NDUM=NV-3	FIN	757
	N=1	FIN	758
	DO 23 NN=9,NDUM,4	FIN	759
	BLANK2=(NN-5)*LSHEET/(NV-4)	FIN	760
22	N=N+1	FIN	761
	CALL SIM(LS,DPHIV,N,BLANK1)	FIN	762
	IF(BLANK2.GT.BLANK1) GO TO 22	FIN	763
	PHIV(NN)=(PHISS(N)+PHISS(N-1))/2.	FIN	764
	COSDUM=COS(PI*PHIV(NN)/(2.*PHIV(1)))	FIN	765
	SINDUM=SIN(PI*PHIV(NN)/(2.*PHIV(1)))	FIN	766
	BLANK=1.+(RV(1)+RC)*(PHIV(1)-PHIV(NN))	FIN	767
		FIN	768

Table A-2. Computer Program Listing (Cont)

	RV(NN)=COSDUM*(RV(1)+RC)*SINDUM**2/BLANK	FIN	769
	YV(NN)=RV(NN)*COS(PHIV(NN))	FIN	770
23	ZV(NN)=RV(NN)*SIN(PHIV(NN))	FIN	771
	NDUM=NV-2	FIN	772
	DO 24 N=2,NDUM,4	FIN	773
	YV(N)=YV(N-1)/(YV(N-1)**2+ZV(N-1)**2)	FIN	774
	ZV(N)=ZV(N-1)/(YV(N-1)**2+ZV(N-1)**2)	FIN	775
	YV(N+1)=-YV(N)	FIN	776
	ZV(N+1)=ZV(N)	FIN	777
	YV(N+2)=-YV(N+1)	FIN	778
24	ZV(N+2)=ZV(N+1)	FIN	779
C		FIN	780
C	END VORTEX SHEET CALCULATION	FIN	781
C		FIN	782
C	BEGIN VELOCITY COMPONENT CALCULATION	FIN	783
C		FIN	784
	PHI=0.	FIN	785
	IPHI=1	FIN	786
31	PHI=PHI/RAD	FIN	787
	COSP=COS(PHI)	FIN	788
	SINP=SIN(PHI)	FIN	789
	R=1.	FIN	790
	IR=1	FIN	791
31	Y=R*COSP	FIN	792
	Z=R*SINP	FIN	793
	IF(ABS(Y-YV(1)).LT.ABS(Y-YV(4))) GO TO 32	FIN	794
	Y14=YV(4)	FIN	795
	Z14=ZV(4)	FIN	796
	GO TO 33	FIN	797
32	Y14=YV(1)	FIN	798
	Z14=ZV(1)	FIN	799
33	C=1.254*((Y-Y14)**2+(Z-Z14)**2)/RC**2	FIN	800
	V=0.	FIN	801
	W=0.	FIN	802
	DO 34 J=1,4	FIN	803
	CON1=(Y-YV(J))**2+(Z-ZV(J))**2	FIN	804
	V=V+(-1)**J*(Z-ZV(J))/CON1	FIN	805
34	W=W+(-1)**J*(Y-YV(J))/CON1	FIN	806
	VS=0.	FIN	807
	WS=0.	FIN	808
	DO 35 N=5,NV	FIN	809
	CON1=(Y-YV(N))**2+(Z-ZV(N))**2	FIN	810
	VS=VS+(-1)**N*(Z-ZV(N))/CON1	FIN	811
35	WS=WS+(-1)**N*(Y-YV(N))/CON1	FIN	812
	V=	FIN	813
	(-2.*Y*Z*SINAB/(Y**2+Z**2)**2+GC*V/(2.*PI)+	FIN	814
	12.*GS/(PI*(NV-4))*VS)*(1.-EXP(-C))	FIN	815
	W=	FIN	816
	(SINAB*(1.+	FIN	817
	(Y**2-Z**2)/(Y**2+Z**2)**2)-	FIN	818
	1GC*W/(2.*PI)-2.*GS*WS/(PI*(NV-4))*(1.-EXP(-C))	FIN	819
	IF(ABS(V).GT.2.) V=2.*V/ABS(V)	FIN	820
	IF(ABS(W).GT.2.) W=2.*W/ABS(W)	FIN	821
	VVEL(IAB,IPHI,IR)=V	FIN	822
	WVEL(IAB,IPHI,IR)=W	FIN	823
	R=R+DR	FIN	824
	IR=IR+1	FIN	825
	IF(IR.LE.51) GO TO 31	FIN	826
	PHI=PHI+RAD	FIN	827
	PHI=PHI+5.	FIN	828
	IPHI=IPHI+1	FIN	829
	IF(IPHI.LE.73) GO TO 30	FIN	830
	IAB=IAB+1	FIN	831
	IF(IAB.LE.IALPHA) GO TO 10	FIN	832
C		FIN	833
C	END VELOCITY COMPONENT CALCULATION	FIN	834
C		FIN	835
	RETURN	FIN	836
	END	FIN	837
	FUNCTION TBLLOOK(X,AB,ORD,N)	FIN	838
C		FIN	839

Table A-2. Computer Program Listing (Cont)

C	LINEAR INTERPOLATION SUBPROGRAM. ABSCISSA MUST BE IN	FIN	837
C	INCREASING VALUES. LETS YOU KNOW IF VALUE IS OUT OF THE	FIN	838
C	RANGE OF THE TABLE, AND THEN CONTINUES ASSUMING SINTER=1.	FIN	839
C	N IS THE NUMBER OF ENTRIES IN THE TABLE.	FIN	840
C		FIN	841
	DIMENSION AB(1),ORD(1)	FIN	842
	IF(X.LT.AB(1).OR.X.GT.AB(N)) GO TO 5	FIN	843
	NN=N/2	FIN	844
	IF(X.LT.AB(NN)) GO TO 3	FIN	845
	I=NN	FIN	846
	NN=NN+NN/2	FIN	847
	IF(X.GT.AB(NN)) I=NN	FIN	848
	GO TO 4	FIN	849
3	I=1	FIN	850
	NN=NN/2	FIN	951
	IF(X.GT.AB(NN)) I=NN	FIN	852
4	I=I+1	FIN	853
	IF(X.GT.AB(I)) GO TO 4	FIN	854
	TBLOOK=ORD(I-1)+(X-AB(I-1))/(AB(I)-AB(I-1))*(ORD(I)-ORD(I-1))	FIN	855
	RETURN	FIN	856
5	PRINT 1,X	FIN	857
	TBLOOK=1.	FIN	858
1	FORMAT(//30X,	FIN	859
	152HABSCISSA WAS NOT IN THE RANGE OF THE TABLE,ABSCISSA=	FIN	860
	1F10.5,//)	FIN	861
	RETURN	FIN	862
	END	FIN	863
	SUBROUTINE SIM(Y,H,NN,ANS)	FIN	864
C		FIN	865
C	SIMPSON RULE INTEGRATION. INTEGRAND MUST BE EVENLY SPACED.	FIN	866
C	NN IS THE NUMBER OF INTEGRAND POINTS.	FIN	867
C		FIN	868
	DIMENSION Y(1)	FIN	869
	ANS=0.	FIN	870
	IF(NN-2) 2,3,5	FIN	871
2	RETURN	FIN	872
3	ANS=.5*H*(Y(1)+Y(2))	FIN	873
	RETURN	FIN	874
5	II=NN/2	FIN	875
	II=II+2	FIN	876
	IF(NN-II) 6,8,6	FIN	877
6	II=NN-2	FIN	878
	DO 7 J=1,II,2	FIN	879
7	ANS=ANS+ (Y(J)+4.*Y(J+1)+Y(J+2))	FIN	880
	ANS=ANS*H/3.	FIN	881
	RETURN	FIN	882
8	II=NN-3	FIN	893
	DO 9 J=1,II,2	FIN	894
9	ANS=ANS+ (Y(J)+4.*Y(J+1)+Y(J+2))	FIN	885
	ANS=ANS*H/3.	FIN	886
	ANS=ANS+.5*H*(Y(II+2)+Y(II+3))	FIN	887
	RETURN	FIN	888
	END	FIN	889

Use of the Computer Program

Card 1: Input the title of the missile and fin geometry. The title can be any combination of letters and numbers and must be located in columns 1 through 60. Sample card:

[illegible]

.165, .375, .325, 3.8, 30, -20., 3.955

[illegible]

1, 1, 0, 1

[illegible]
$$\sqrt{5, -8, -5, -8,}$$
[illegible]

[illegible][illegible][illegible]

moment coefficient, side force coefficient, yaw moment coefficient, and roll moment coefficient averaged over one 90° roll cycle; if the roll rate is specified as zero, this part is eliminated.

Parts two through five list the forces and moments as a function of roll angle for each angle of attack of the body and each roll rate. Part six lists the forces and moments as a function of roll rate for each angle of attack of the body. The output in part six is used to calculate quantities such as $C_{\dot{p}}$ and steady-state roll rate.

72

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