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FATIGUE RELIABILITY OF WIND TURBINE COMPONENTS[1]

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ABSTRACT

Fatigue life estimates for wind turbine components can be extremely variable due to both inherently random and uncertain parameters. A structural reliability analysis is used to quantify the probability that the fatigue life will fall short of a selected target. Reliability analysis also produces measures of the relative importance of the various sources of uncertainty and the sensitivity of the reliability to each input parameter. The process of obtaining reliability estimates is briefly outlined. An example fatigue reliability calculation for a blade joint is formulated; reliability estimates, importance factors, and sensitivities are produced. Guidance in selecting distribution functions for the random variables used to model the random and uncertain parameters is also provided.

INTRODUCTION

The cost-effective production of electricity with a wind-driven generator depends heavily on the reliability of the entire wind turbine system. System reliability in turn depends on the frequency of component failures. Because wind turbine structures have inherently oscillatory loading due to their own rotation and due to atmospheric turbulence, cumulative fatigue damage to structural components is an endemic problem.

Estimating the rate of fatigue damage is complicated by the presence of inherent randomness and parameter uncertainty. We call parameters uncertain if we don't know their exact value, but have estimates that could be improved with additional information. Random quantities, however, are inherently variable and can only be described in a probabilistic or statistical manner. Structural response levels and stress concentration factors are examples of uncertain quantities. Instantaneous wind speeds and material fatigue properties are good examples of random parameters. Random and uncertain parameters can both be described by their probability distributions.

The typical fatigue question is: "How long will this component last?" Unfortunately, fatigue life calculations are very sensitive to small changes in the input parameters. The answer to this question is ill-defined when there is randomness or uncertainty in the parameters describing the loading, structural response, or material properties. This aspect of the fatigue problem is illustrated by the parameter study of a vertical axis wind turbine (VAWT) blade joint in Reference 1. When varying the input quantities over a reasonable range of possible values, fatigue lives were calculated to be somewhere between six months, in a worst case combination of conditions, and six hundred years, for a benign combination of parameters.

Is fatigue life estimation a hopeless problem? Perhaps, if the desired answer is the actual fatigue life of all components. But this level of detail is not necessary. A more useful expression of the fatigue problem might be: "Will the component last long enough?" Because there is almost never a direct "yes" or "no" answer, the basic question should be expanded to: "How likely is it that the component will last long enough to be safe and economically effective?" This formulation lends itself very well to a structural reliability approach, where random and uncertain parameters can be included in the analysis.

STRUCTURAL RELIABILITY

Structural reliability analysis is a tool for predicting the effect of randomness and uncertainty on the performance of a structure. Performance is generally defined as the ability of a structure to withstand its environment for an economical period of time. The inputs to the analysis include descriptions of the probability distributions of variable parameters, as well as fixed parameters and a quantifiable failure criterion. The outputs include the estimated probability of failure (which is usually described by a reliability index for comparison purposes), the relative *importance* of each of the random variables, and measures of the *sensitivity* of the reliability to all of the input quantities.

A detailed description of the mechanics of the structural reliability calculations is not possible in the limited space provided here, but a brief overview is included. An extensive description can be found in Reference 2. The reliability is estimated by the following four steps:

1. *FORMULATION:* The first part of this step is to define a failure criterion. In most structural applications this takes the form of a failure state func-

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tion, $G(\underline{X})$, such as the difference between strength or resistance, R, and load, L.

$$G(\underline{X}) = R - L \tag{1}$$

$G(\underline{X})$ is positive when the structure is safe and negative when it has failed. R and L are functions of components of the vector $\underline{X}$, which are random variables. Figure 1 shows a two dimensional example of a failure state function. All of the calculations needed to determine strength and load are formulated as if all of the parameters in $\underline{X}$ are known. The second part of this step is to define the relative likelihood of all possible values of the random variables. The probability distribution function (pdf) supplies this information. There is more discussion on selecting the pdf later.

2. *TRANSFORMATION:* A transformation between each of the random variables and uncorrelated, unit variance, zero mean, normally distributed random variables must be determined. The transformation between a single random variable, X, and a standard normal random variable, U, is illustrated in Figure 2. Probability levels of the input cumulative density function (cdf), $F(X)$ in Figure 2, and the standard normal cdf, $\Phi(U)$ are equated. If the random variables in the vector $\underline{X}$ are correlated, they must be transformed by successively conditioning on all the previously transformed variables to produce uncorrelated standard normal variates in the vector $\underline{U}$. The calculations then proceed in standard normal space, which is also called U-space.

3. *APPROXIMATION:* Because the transformation to U-space can be quite complicated (although it is usually quite simple to accomplish numerically), the failure state function in U-space, $g(\underline{U})$, cannot ordinarily be written in closed form. The boundary between the failed and safe regions is found by selecting values for $\underline{U}$, transforming to $\underline{X}$, and evaluating $G(\underline{X})$, which equals $g(\underline{U})$. Gradient search methods are used to find the most likely failure point, also known as the design point, which is the point where $g(\underline{U}) = 0$ is closest to the origin, as shown in Figure 3. The probability of failure is the area under the joint multi-dimensional standard normal pdf in the failed region. A good approximation for small probabilities of failure is obtained by fitting a tangent line to the failure state function at the design point. This approximation is called the first order reliability method (FORM). A second order reliability method (SORM) is obtained by fitting a parabola to the failure state function, as shown in Figure 3. The direction cosines of the vector $\underline{\alpha}$ are measures of the *importance* of each of the random variables. A small direction cosine means that the probability of failure is relatively unaffected by the associated random variable. The example shown in Figure 3 illustrates two random variables with roughly equal importance.

4. *COMPUTATION:* Calculation of the failure probability and importance factors is made tractable by the symmetry of standard normal space. The distance from the origin to the design point, β, is sufficient information to calculate the FORM probability of failure, $P_f = \Phi(-\beta)$. The SORM estimate is based on β and the curvatures at the design point. The accuracy of the computation is checked by comparing the estimates. Because the probability of failure is often a very small number (at least

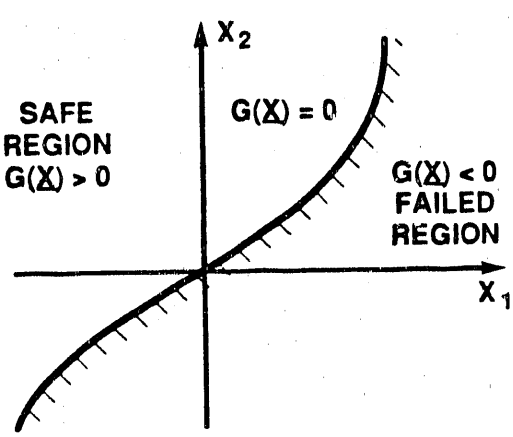

Figure 1: Failure state function.

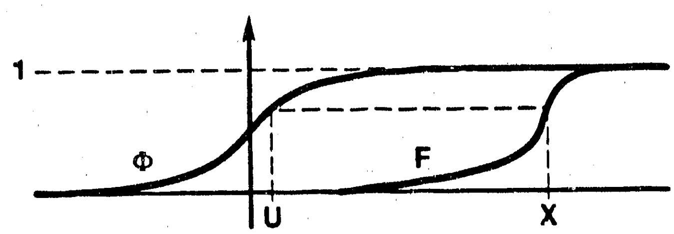

Figure 2: Transformation between a standard normal variate, U, and the physical variate, X.

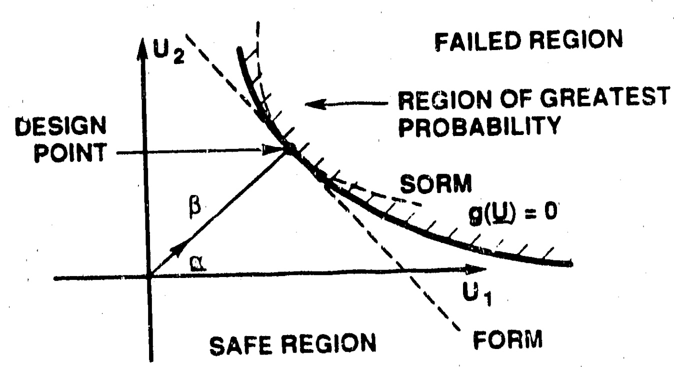

Figure 3: Failure state function in standard normal space (U-space).

for designs approaching an acceptable level of reliability), a reliability index, β^*, is often used. This index is the number of standard deviations from the mean in an equivalent one-dimensional formulation, or equivalently, the inverse normal cdf of one minus the probability of failure: $\beta^* = \Phi^{-1}(1 - P_f)$. In FORM, $\beta^* = \beta$, while in SORM β^* is slightly different than β. Sensitivities are calculated by numerically evaluating the partial derivatives of the reliability index with respect to any or all input parameters, including the means and standard deviations of the random variables, as well as the fixed parameters.

EXAMPLE: VAWT BLADE JOINT

1. Formulation

The fatigue life of wind turbine components can be calculated by summing the damage accumulated during all phases of wind turbine operation. The required information includes definitions of the wind speed distribution, stress response levels as functions of wind speed, and material damage as functions of stress level [3]. Focusing on normal operation (which takes up the vast majority of the time) and neglecting transients, a simplified fatigue life calculation can be approximated by

$$D = \int_0^\infty \int_0^\infty \frac{N_t P(S|V) P(V)}{n_f(S)} dS dV \qquad (2)$$

where

V	=	wind speed
S	=	stress amplitude
N_t	=	total number of applied cycles
$P(V)$	=	pdf of wind speed
$P(S\|V)$	=	pdf of stress given wind speed
$n_f(S)$	=	cycles to fail at stress amplitude S

When the damage summation reaches one, failure is predicted and the number of applied cycles is defined to be the number of cycles to failure; $N_t = N_f$. The time to failure equals the number of cycles divided by the frequency of cycles; $T_f = N_f / f_0$. Equation 2 can then be solved for the time to failure.

$$T_f = \left[f_0 \int_0^\infty \int_0^\infty \frac{N_t P(S|V) P(V)}{n_f(S)} dS dV \right]^{-1} \qquad (3)$$

There is an important difference between the pdf's in Eqs. 2 and 3 and the pdf's of the random variables in $\underline{X}$, which may include parameters of $P(S|V)$ and $P(V)$. The distributions of stress amplitudes and wind speeds describe quantities that vary continually during the lifetime of the component. Over the several years required for an economic lifetime, all likely values of wind speed and stress amplitudes will occur. The fatigue calculation integrates over these pdf's. There is therefore almost no variability in fatigue lifetime due to the fluctuations in instantaneous wind speed and stress amplitude. However, the parameters described by random variables in $\underline{X}$ do not vary over time, but are either uncertain or inherently unknown and described statistically. The pdf's describing these random variables are not distributions of values that will occur at some time, but are distributions of possible values, only one of which is actually realized. The random variables in $\underline{X}$ are the ones that lead to variability in fatigue life.

The fatigue reliability of wind turbine components can be formulated with a failure state function that is based on the difference between T_f and a specified target lifetime, T_t.

$$G(\underline{X}) = T_f - T_t \qquad (4)$$

The above limit state function can, in general, be evaluated by numerically integrating the empirical functions derived from test data that represent the pdf's of wind speed and stress amplitudes. The material fatigue properties, $n_f(S)$, are also obtained by a best fit to fatigue test data.

Equation 4 can be integrated analytically if a few simplifying assumptions are made:

- $P(V)$ is a Weibull distribution with mean $\bar{V}$ and shape parameter α_v.
- $P(S|V)$ is described by a Rayleigh distribution characterized by the standard deviation, or root mean square (RMS) of the stresses at a given wind speed.
- The stress RMS increases linearly with wind speed.
- $n_f(S)$ is a straight line on a log-log plot; $n_f(S) = CS^{-\hat{b}}$, where C is the fatigue coefficient and $\hat{b}$ is the fatigue exponent.
- The mean stress effect is modeled by a Goodman correction in $n_f(S)$.
- The operating speed is constant (therefore, f_0 is constant also).
- The turbine always operates (i.e., no cut-out), a conservative assumption.
- The fatigue damage due to transients is neglected, a nonconservative assumption.

While as much detail should be used in describing the operating conditions as is available, this list of assumptions is a good starting point for a reliability analysis early in the design process. These simplifications are appropriate for the sake of this example and in the initial design phase, and lead to a closed form solution for the time to failure.

$$T_f = \left\{ f_0 C \left(\frac{\sqrt{2} M K \bar{V}}{(1 - S_m/S_u)(1/\alpha_v)!} \right)^{\hat{b}} \left(\frac{\hat{b}}{2} \right)! \left(\frac{\hat{b}}{\alpha_v} \right)! \right\}^{-1} \quad (5)$$

where

M = the slope of RMS stress vs. wind speed
K = the stress concentration factor
S_m = the mean stress
S_u = the ultimate strength
$(\cdot)!$ = the factorial, which is defined by the Gamma function, $\Gamma(\cdot + 1)$, for noninteger arguments.

2. Transformation

Of the nine parameters that appear on the right side of Equation 5, seven are taken to be random variables and two ($\hat{b}$ and S_u) are assumed to be fixed parameters. $\hat{b}$ is fixed because it is not necessary for both $\hat{b}$ and C to be considered random to model variability in fatigue properties. The target lifetime, T_t, is also assumed to be fixed.

The random variables are all assumed to be normally distributed, except C, which has a Weibull distribution based on test data (Ref. 1). More discussion of pdf selection is included later.

To transform a standard normal random variable, U, to a random variable X that is also normally distributed, multiply by the standard deviation of X, σ_X, and add the mean, m_X.

$$X = U\sigma_X + m_X \qquad X \equiv \text{Normal} \quad (6)$$

The Weibull distribution has a different shape than the normal distribution. However, a Weibull variate, X, can be easily created from a standard normal variate, U, in two steps: (1) calculate the cumulative probability level associated with U by evaluating the normal cdf, $\Phi(U)$, and (2) calculate the Weibull variate associated with that probability level by evaluating the inverse Weibull cdf, i.e.,

$$X = \frac{m_X}{(1/\alpha)!} [-\ln(1 - \Phi(U))]^{\alpha} \qquad X \equiv \text{Weibull} \quad (7)$$

The Weibull parameters are the mean, m_X, and the shape factor, α. In most cases, the available information will include the mean and some measure of the spread, such as the coefficient of variation (COV), which is the standard deviation divided by the mean. The COV of a Weibull is related to the shape factor by

$$\text{COV} = \left[\frac{\Gamma(2/\alpha + 1)}{\Gamma^2(1/\alpha + 1)} - 1 \right]^{1/2} \quad (8)$$

A useful approximation is given by

$$\text{COV} \approx 1/\alpha \qquad 0 < \text{COV} < 2 \quad (9)$$

The approximation is exact when the COV is both zero and one, and covers the range of most likely COV values.

The method for transforming other distributions is the same: first calculate the probability level associated with U by evaluating $\Phi(U)$, and then substitute the result into the inverse cdf of the desired distribution. For empirical distributions (not described by a closed form function), the operation is easily done numerically by the same process of matching probability levels, as illustrated in Figure 2.

3. Approximation

Searching for the design point and calculating the probability of failure and importance factors is a difficult task, best accomplished by using existing computer codes. Some of these computer codes even perform the transformation process as well. For this example, a code developed by Rackwitz [4] has been used to approximate the failure region as described above.

4. Computation and Results

The values selected for the mean and COV of the seven random variables are listed in Table 1. The values of the COV are representative of a case where a prototype has already been built and tested, reflecting relatively small uncertainties in structural response parameters. Also, substantial material fatigue testing has been done, resulting in a well characterized S-n curve with relatively little variability.

The three fixed parameters have the following values: T_t = 20 years; $\hat{b}$ = 7.3; S_u = 245 MPa.

By substituting the mean values for all the random variables into Equation 5, the median lifetime is estimated to be 370 years, which seems like a relatively safe buffer for a 20-year design life.

There are, however, combinations of possible values of the random variables that lead to failure in less than the target lifetime. The fatigue reliability of the component is evaluated by estimating the probability that the failure state function (Eq. 4) is negative. For this example, the probability of failure with a target lifetime of 20 years is approximately 2% (1.8% with FORM and 2.2% with SORM), with an associated reliability index of about 2.0 (2.1 for FORM and 2.0 for SORM), i.e., two standard deviations from the mean.

The direction cosines of $\underline{\alpha}$ (see Figure 3), are calculated by the reliability analysis program as a byproduct of the

Table 1: Example Random Variable Parameters

Symbol	Definition	Mean	COV
C	S-n Coefficient	982.	0.10
f_0	Cycle Rate	2.0 Hz	0.20
M	RMS Slope	.45 MPa/(m/s)	0.05
K	Stress Concentration	3.5	0.10
S_m	Mean Stress	25. MPa	0.20
$\bar{V}$	Mean Wind Speed	6.3 m/s	0.05
α_v	Wind Speed Shape	2.0	0.10

solution method. The squares of the direction cosines, which must sum to unity, are a good measure of the percentage of the variability due to each random variable. The importance factors displayed in the pie chart in Figure 4 are the squares of these direction cosines. The fatigue coefficient, C, a material property, is by far the most important source of variability supplying about 55% in this example. The stress concentration factor, K, and the wind speed distribution shape parameter, α_v, have roughly the same importance with about 15% each. The remaining 15% is divided among the other four random variables with the mean stress, S_m, and cycle frequency, f_0, contributing minimally to the overall variability.

Sensitivities are calculated by varying each input parameter slightly and estimating the partial derivative by dividing the change in the reliability index by the change in the parameter. All input parameters can be varied in this manner, including the fixed parameters, as well as the parameters of the distributions of the random variables (i.e., means and COV's). Sensitivities may be normalized in a number of ways; here the derivative is divided by the value of the parameter. With this normalization, the change in the reliability index is estimated by multiplying the fraction that a parameter changes by the normalized sensitivity.

Figure 5 shows the calculated sensitivities for the three fixed parameters and the means and COVs of the five most important random variables in this example. The clearly dominant parameter is the fatigue exponent, $\hat{b}$, which should come as no surprise to fatigue analysts. The reason is seen by examining Equation 5 where $\hat{b}$ appears as an exponent on most of the other variables. The other significant result is that the means of the random variables exhibit at least four times as much sensitivity as the COVs. This is especially important because the mean value is usually much easier to estimate than the spread quantified by the COV. One need not despair, therefore, at obtaining a reasonable estimate of the component reliability when there is only limited information on the spread in the possible values of the random and uncertain parameters.

Although the assumptions made to simplify the calculations for this example are much more restrictive than using the actual data, as in Reference 1, the results are consistent. Both the mean lifetime and probability of lives below 20 years agree with the range of values obtained in the Reference 1 parameter study.

DEFINING THE RANDOM VARIABLES

Perhaps the greatest impediment to more popular use of reliability and other probabilistic methods in everyday engineering practice lies in the difficulty of selecting the distributions of random variables. In structural reliability analysis, the selection of random variable pdf's is most important in applications where very low probabilities of failure are required, such as off-shore platforms, dams, bridges, and other very expensive, one-of-a-kind, life-critical structures. Much higher probabilities of failure are likely to be economically acceptable in wind turbine applications where hundreds and perhaps thousands of individual machines are involved. The shape of the pdf becomes less important as the random variables are evaluated at higher probability levels.

The selection of the pdf is somewhat different for random variables that are inherently random than for random variables that describe parameter uncertainty. Inherently random parameters can often be measured and.

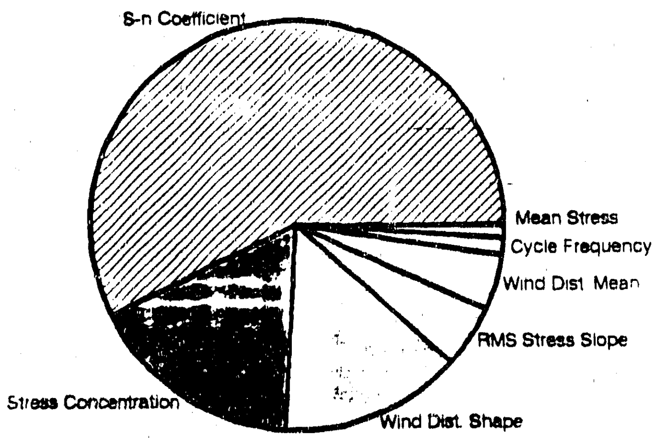

Figure 4: Importance factors from the example reliability analysis.

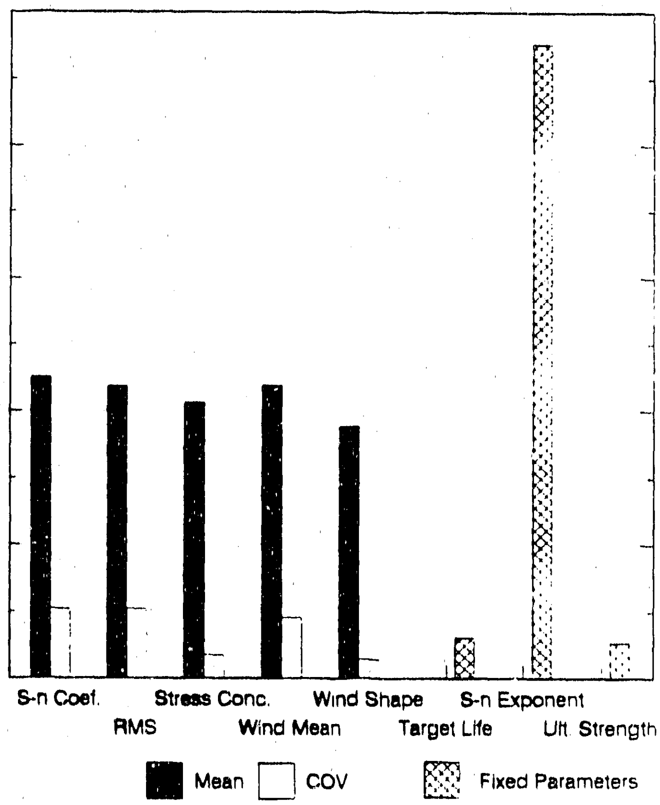

Figure 5: Sensitivities from the example reliability analysis.

over time, data can be gathered and collected into histograms describing the frequency of occurrence of different parameter values. With sufficient data, the histogram can be normalized and used directly as an empirical description of the pdf. With less data, the histogram is useful in describing the overall shape of the pdf, but a better description is obtained by selecting an analytical pdf and using the data to determine the parameters of the analytical function. Without any data, engineering judgment must suffice to describe the range of possible values.

A lack of data is often the case when describing uncertain parameters. The best that can often be obtained are measures of the mean and the spread in the possible values.

A few of the simplest and therefore most useful distributions are listed here:

- **Uniform:** This distribution requires only minimum and maximum values to describe the pdf. All values between these limits are equally likely. Unfortunately, equally likely outcomes with fixed limits are very rare in engineering applications.

- **Triangular:** A slight improvement on uniform, the triangular distribution concentrates more probability near a most likely value and gradually reduces the probability as values increase or decrease, as shown in Figure 6. While easy to define, the triangular distribution is also rarely found in engineering applications.

- **Normal:** The normal, or Gaussian, distribution is the most common pdf found in nature. Its characteristic bell shape is described by only two parameters, the mean and the standard deviation, or equivalently the mean and COV. The normal is symmetric about the mean and is unbounded in either direction. While it may appear unrealistic to allow values of the random variable to go to plus and minus infinity, the normal distribution usually describes extreme values well because the probability of occurrence decays rapidly. Truncation of the distribution is not advised unless there is some rigid constraint. Even then, it may not matter if the probability of exceeding the constraint is lower than the overall probability of failure.

- **Weibull:** The general class of one-sided distributions described by the Weibull pdf covers a wide range and includes some very useful distributions as special cases. The exponential distribution is a Weibull with a COV of unity. The Rayleigh is a Weibull with a COV of just over one half. Figure 7 illustrates the diverse shapes taken by Weibull distributions with COVs ranging from 0.1 to 1.0. As the COV decreases, it begins to resemble a normal distribution, as shown in Figure 8.

- **Log-normal:** The log-normal distribution results when the logarithm of the random variable is normally distributed. The distribution is especially popular in multi-variate applications; the product of log-normal variates is another log-normal variate. Its distribution is skewed toward higher values, which makes it conservative when used for parameters that are more dangerous when large.

Reference 5 provides an extensive table of probability distributions, including most of the above.

It is often tempting to set fixed limits on distributions of random variables. Limits are in general not a good idea because knowledge of the limit implies very specific information, which is usually available only after copious data collection.

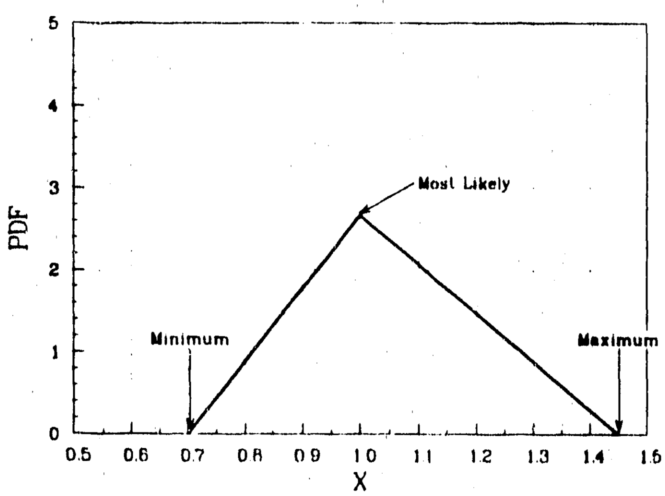

Figure 6: Triangular probability density function (pdf).

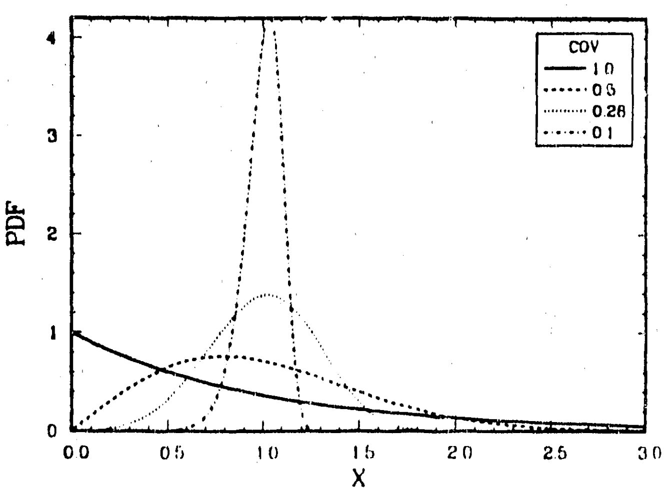

Figure 7: Weibull pdf's with a mean value of one.

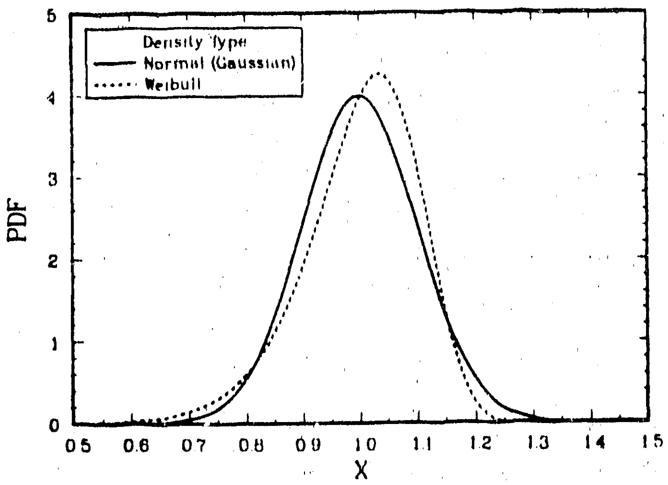

Figure 8: Normal and Weibull pdf's with a mean of one and COV = 0.1.

This list of possible alternatives may make it seem impossible to ever choose an appropriate distribution. However, the desire to use the "right" pdf should be tempered by the fact that you can only supply as much detail as is supported by data. The only available information is often some measure of the central tendency (mean) and the spread (COV). Constraints, such as non-negative values, may also apply. With limited information, it is usually best to use a normal distribution for most parameters and perhaps a Weibull distribution for non-negative parameters. These two distributions cover a wide range of possible behaviors, all of which are defined by supplying just two parameters, mean and COV. Because the normal and Weibull are distributions that appear commonly, they are as likely as any other distributions to be "right." If more information is available, the analyst can seek out the best fit from the long list of candidate pdf's.

Example: Weibull vs Normal

The above example used normally distributed random variables wherever possible for the sake of simplicity. Because some of these parameters are meaningless at negative values (e.g., $\bar{V}$), and the normal pdf allows values from positive to negative infinity, it can be argued that one-sided distributions need to be used. The reliability was again calculated using Weibull distributed random variables for all non-negative parameters. The means and COVs were kept the same as shown in Table 1.

The FORM results are almost identical. The SORM probability of failure changed only slightly, rising from 2.2% to 2.6%, while the reliability index dropped from 2.0 to 1.9. The importance factors are shown in Figure 9 for both the normal and Weibull cases. Again there is little difference except that the importance of the wind speed shape parameter has increased slightly over that of the and stress concentration factor.

The difference between normal and Weibull distributions at relatively low COV levels is not very pronounced, as shown in Figure 8. The main difference is that the normal is symmetric while the Weibull is slightly skewed to lower values. Another popular distribution, the log-normal, is skewed toward higher values. The reason for the increase in importance of the wind speed shape over the stress concentration is due to this slight skewing: a large stress concentration produces more damage while a small wind speed shape parameter predicts more high winds.

The effect of changing pdf's is relatively small when COVs are small and probabilities of failure are relatively high. To be safe, a conservative reliability estimate can be created by selecting pdf's that are skewed in the direction of more damaging values.

SUMMARY

Structural reliability analysis is a tool for use at all stages of the design and development process. Complete information is not needed to estimate the probability of premature failure or to assess the most important factors in improving component reliability. As more data is gathered, the reliability estimates can be updated and the direction of further data aquisition can be refined. Once the problem is formulated, and the transformations are

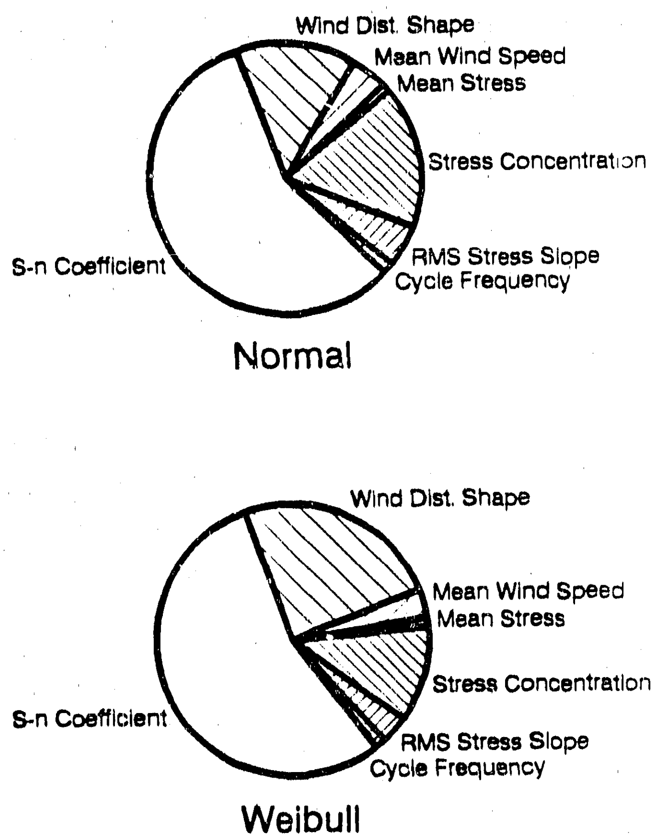

Figure 9: Importance factors using normal distributions and Weibull distributions for all non-negative random variables.

coded, the approximation and computation can be done with updated parameter estimates relatively quickly and easily.

The reliability format adds meaning to fatigue life estimates where small changes in parameter values usually lead to large differences in predicted lives. By supplying relative measures of goodness, the reliability analysis provides a tool for evaluation of competing design alternatives. The additional information provided by importance factors and sensitivities allows wind turbine designers and manufacturers to identify areas where focused effort and design improvement can have the greatest pay-off on enhanced component reliability. These results are also applicable to the larger issue of system economic analysis, when each component reliability estimate is folded into the wind turbine system.

REFERENCES

1. Ashwill, T. D., H. J. Sutherland, and P. S. Veers, "Fatigue Analysis of the Sandia 34-Meter Vertical Axis Wind Turbine," Proc., *Ninth ASME Wind Energy Symposium*, New Orleans, Louisiana, January 14-18, 1990.

2. Madsen, H. O., S. Krenk, and N. C. Lind, Methods of Structural Safety, Prentice-Hall, 1986.

3. Sutherland, H. J., "Analytical Framework for the LIFE2 Computer Code," SAND89-1397, Sandia National Laboratories, Albuquerque, New Mexico, 1989.

4. "User's Manual for Structural Reliability Programs CUTALG - FORM - SORM - SYSREL," Berichte zur Zuverlässigkeitstheorie der Bauwerke, Sonderforschungsbereich (SFB) 96, Laboratorium für den Konstruktiven Ingenieurbau (LKI), Technische Universität München, 1985.

5. Ang, A. H-S. and W. H. Tang, Probability Concepts in Engineering Planning and Design, Volume I, pp. 224-225, John Wiley and Sons, 1975.

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