MASTER

Phenomenology of Quark Mixing and the Kobayashi-Maskawa Model

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I. INTRODUCTION

In this talk I shall discuss the status of the determination of
the quark mixing matrix in the Kobayashi-Maskawa[1] model and its
phenomenological implications. I am very happy to give the talk
here, since physicists at CESR are making important contributions
in this area. Much of the talk[2] is reviewing current work on
the subject. Some new results of mine on the CP violation effects in
exclusive and inclusive decays of bottom, charm and strange particles
are also given.

II. THE MIXING MATRIX

In the K-M model, assuming the existence of the yet to be dis-
covered top quark t, there are three doublets, $(u,d')_L$, $(c,s')_L$ and
$(t,b')_L$, where $(d',s',b') = (d,s,b)V$. V is a 3×3 unitary matrix
$V^+ V = 1$. In general for n doublets, the number of physically sig-
nificant parameters in V is equal to the number of parameters for an
$n \times n$ unitary matrix minus the relative phases of the doublets, i.e.,
$n^2 - (2n - 1)$. An orthogonal matrix can be characterized by
$\frac{1}{2}n(n - 1)$ angles, thus the rest of the parameters $[n^2 - (2n - 1)] - \frac{1}{2}n(n - 1) = \frac{1}{2}(n - 1)(n - 2)$ has to be characterized by phases. For
$n = 2$, V can be characterized by an angle θ_c and no phase. For $n = 3$,
V is characterized by three angles and one phase
The V matrix is parametrized by Kobayashi and Maskawa[1] as

$$V = \begin{pmatrix} V_{ud} & V_{cd} & V_{td} \\ V_{us} & V_{cs} & V_{ts} \\ V_{ub} & V_{cb} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_1 & -s_1 c_2 & -s_1 c_2 \\ s_1 c_3 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} \\ s_1 s_3 & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} & c_2 s_3 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \quad (2.1)$$

It is this complexity in V that provides the CP violation. Thus,
the salient feature of the K-M model is that <u>the CP violation effect</u>
<u>is tied with the nonvanishing of some of the matrix elements in the</u>
<u>third row or third column, which means that the b and the t flavored</u>
<u>particles must have pure hadronic decays</u>. Models with CP violation
coming from the Higgs couplings, by having more Higgs doublets than
the standard $SU(2)_L \times U(1)$ model, have no such correlation. Actually
in many of these models, the b-flavored particles have only semi-
leptonic decays though this is not imposed on by any first prin-
ciples.[3]
Since the model is designed to provide CP violation, some of
the parameters must be determined from the CP violation of the K_L,
K_S system which, so far, is still the only experimentally estab-

lished system having CP violation. The four parameters of the V matrix have been so far determined from four sets of experimental informations. The $0^+ \to 0^+$ nuclear β decay rates comparing to that of μ decay (assuming no effects from the mixing of the leptons) determines $|V_{ud}|$, and the hyperon semileptonic decays determines $|V_{us}|$. The results of Shrock and Wang's analysis[4] in '78 are $|V_{ud}| = .9737 \pm .0025$, $|V_{us}| = .219 \pm .003$, and $|V_{ud}|^2 + |V_{us}|^2 = .996 \pm .004$. The important point of the result is that the central value of $|V_{ud}|^2 + |V_{us}|^2$ is less than one, indicating that the old Cabibbo theory was not exactly true and there is "leakage" from the first two doublets. It allows the third doublet to decay, i.e., the b can decay into u.

The constraint the other two parameters V_{cs}, V_{cd} we use the two sets of experimental informations, i.e., the K_L, K_S mass difference and the CP violation parameter $|\epsilon|$. To remind you[5] about the parameter ϵ, consider the mass matrix of $|K^0>$ and $|\bar{K}^0>$ states:

$$M = \begin{pmatrix} M_{11} - i\Gamma_{11}/2 & M_{12} - i\Gamma_{12}/2 \\ M_{21} - i\Gamma_{21}/2 & M_{22} - i\Gamma_{22}/2 \end{pmatrix}. \tag{2.2}$$

where M_{ij}, Γ_{ij} are transition matrix elements from virtual and physical intermediate states respectively and can be complex numbers. CPT implies $M_{11} = M_{22}$, $\Gamma_{11} = \Gamma_{22}$, Hermiticity $M_{ij} = M_{ji}^*$, $\Gamma_{ij} = \Gamma_{ji}^*$, and CP invariance $M_{ij} = M_{ji}$, $\Gamma_{ij} = \Gamma_{ji}$. Thus CP invariance with CPT and hermiticity implies that all M_{ij}, Γ_{ij} are real. Therefore, imaginary parts M_{ij}^I and Γ_{ij}^I gives CP violation. After diagonalizing the mass matrix M, one obtains the eigenstates $|K> = (1 + \epsilon)|K^0> - (1 - \epsilon)|\bar{K}^0>$, and $|K_L> = (1 + \epsilon)|K^0> + (1 - \epsilon)|\bar{K}^0>$, where

$$\epsilon = i(M_{12}^I - \frac{i}{2}\Gamma_{12}^I)/(M_{12}^R - \frac{i}{2}\Gamma_{12}^R) , \tag{2.3}$$

where the superscripts I, and R stand for imaginary and real parts respectively. The parameter ϵ can be measured by measuring

$$\eta_{+-} \equiv <\pi^+\pi^-|H_w|K_L>/<\pi^+\pi^-|H_w|K_S> = \epsilon + \epsilon' ,$$

and $\tag{2.4}$

$$\eta_{00} = <\pi^0\pi^0|H_w|K_L>/<\pi^-\pi^-|H_w|K_S> = \epsilon - 2\epsilon' ,$$

where $\epsilon' = \sqrt{2}\, e^{i(\delta_2 - \delta_0 + \pi/2)} \mathrm{Im}(A_2/A_0)$

The δ_2 and δ_0 are respectively the I = 2, I = 0 phase shifts of the $\pi\pi$ scattering amplitudes. The real part of the off diagonal matrix element is related to the eigenvalues M_S, M_L, Γ_S, Γ_L of the mass matrix M by $M_{12}^R = \frac{1}{2}(M_L - M_S)$, $\Gamma_{12}^R = \frac{1}{2}(\Gamma_S - \Gamma_L)$, where M_S, Γ_S, M_L, Γ_L are the mass and width of K_S, K_L respectively. The strategy here is to take $\Gamma_{12} = \frac{1}{2}\, 7.4 \times 10^{-15}$ GeV and $\Gamma_{12} \approx 0$ from experiment and calculate M_{12}^R, M_{12}^I from Fig. (2.1), which involves the mixing matrix.
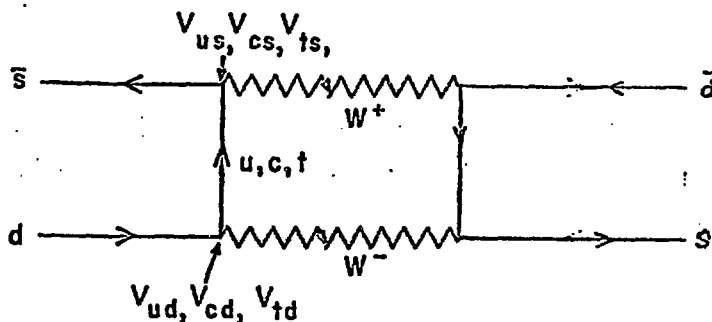
Fig. (2.1)

Fig. (2.1). The box graph for calculating
the $K^0 - \bar{K}^0$ transition matrix

The imaginary part $M_{12}^I \propto s_1 s_2 s_3 s_\delta$ is directly from the complexity in the V_{ij}'s. Comparing the calculated M_{12}^R, M_{12}^I with experimental numbers $M_{12}^R = -\frac{1}{2} \times 3.52 \times 10^{-15}$ and

$$|\varepsilon| = |M_{12}^I| / \sqrt{(M_{12}^R)^2 + (\frac{1}{2}\Gamma_{12}^R)^2} = 2 \times 10^{-3} ,$$

we thus obtained V_{cs} and V_{cd}. There is one warning in calculating M_{12}: after abstracting all the known weak interaction information from Fig. (2.1), one still needs to estimate a strong interaction matrix element $\mathcal{M}_{12} = \langle \bar{K}^0 | [\bar{s}\gamma_\mu(1-\gamma_5)d][\bar{s}\gamma^\mu(1-\gamma_5)d] | K^0 \rangle$. Here the uncertainty can be as big as a factor of two from two different methods of calculations.[7,8] Another uncertainty is that we cannot fix the quadrants in which the angles θ_2, θ_3 and δ of Eq. (2.1) fall in; only $\zeta \equiv \mathrm{sign}(\tan\theta_2 \cdot \tan\theta_3 \cdot \cos\delta)$ matters. The results are rather insensitive to the t quark mass. As an example we give one of the central[9] values of the V matrix determined in Ref. 8.

$$V = \begin{pmatrix} .97 & -.22 & -.046 \\ .22 & .85 - .66 \times 10^{-3}i & .48 + 3.2 \times 10^{-3}i \\ .068 & -.48 + 2.1 \times 10^{-3}i & -.88 - 1.0 \times 10^{-3}i \end{pmatrix} \begin{matrix} d \\ s \\ b \end{matrix} \quad (2.5)$$

It is interesting to observe that the magnitude of the matrix element is the largest on the diagonal and decreases as the element moves away from the diagonal, i.e., there are flavor mixings but they like to keep the original identity. In physical terms, quarks decay in a cascade fashion. The b particles will prominently decay into charm particles, then charm to strange. This is now supported by experiment from CESR.[10] The t particles will decay mainly into b particles.

Though the central value of the V matrix, Eq. (2.1), has not been challenged by various considerations,[9] it is important to have independent determinations of V_{cs}, V_{cd} in a more model-·independent[2] way similar to the determination of V_{ud}, V_{us}, which had been emphasized in Ref. (2). Recently an attempt has been made in this direction by S. Pakvasa, S. F. Tuan and J. J. Sakurai[11]. First, from the opposite-sign dilepton production due to the charm production in neutrino scattering, i.e. $\nu + d \rightarrow c + \mu^- \rightarrow e^+\mu^-X$, they found $.192 < |V_{cd}| < .34$; second, from $D \rightarrow Ke\bar{\nu}$, they calculated $|V_{cs}| = .66 \pm .33$; third, from the life time of B determined here from CESR they obtained $|V_{cb}| > .01$; and fourth, they used the preliminary result from CESR $|V_{bu}/V_{cb}| > .36$. From these results they obtained some solutions of S_2, S_3, S_δ, which are quite consistent with those from Ref. 8. It is interesting to note that so far we still have not been able to determine separately S_2, S_3, S_δ except that we know $S_2S_3S_\delta \sim 10^{-4}$ and $S_2 \leq .5$. We do not yet know whether the smallness of $S_2S_3S_\delta$ is from the smallness of S_3 or S_δ. If the smallness is from that of S_3, for example we can choose $S_2 \sim .3$, $S_\delta \sim 1$, $S_3 \sim .005$, it will have important implications on the CP violation in the B decay, which I shall elaborate in section IV. Here I must emphasize that the results of Ref. (11) are still very crude and model dependent. Continuous effort in this direction ought to be made, especially by the experimenters. I shall list a few such possibilities:

(1) Obtain V_{cs} from $D \rightarrow \ell\bar{\nu}_\ell X$ (with K), and V_{cd} from $D \rightarrow \ell\bar{\nu}_\ell X$ (without K). It is desirable to study decay rates in $e^+e^- \rightarrow \psi(3770)$ $\rightarrow D\bar{D}$ with one D or $\bar{D}$ explicitly selected from its exclusive decays.

(2) From the results of Ref. (12) $\Gamma(D^+ \rightarrow \pi^+\pi^0)/\Gamma(D^+ \rightarrow \bar{K}^0\pi^+)$ $= \frac{1}{2}|V_{cd}/V_{cs}|^2$, which, in addition, has the nice feature that both final states $\pi^+\pi^0$, $\bar{K}^0\pi^+$ are exotic, thus free from possible complications of final state interactions.

(3) Comparing the decays $b \rightarrow c\overset{+}{W}_{\rightarrow c\bar{s}}^-$ and $b \rightarrow c\overset{+}{W}_{\rightarrow \mu\bar{\nu}_\mu}^-$ ought to give information about V_{cs}.

It is interesting to note that if $V_{ud}V_{us} \neq - V_{cs}V_{cd}$, i.e., if the strangeness neutral current is not cancelled in the first two doublets then the t quark that so far eludes observation is needed. If $|V_{cs}|^2 + |V_{cd}|^2 < 1$, the b flavored particle must decay into charm.

III. CP VIOLATION FROM THE COMPLEXITY IN THE MASS MATRIX

As we have elaborated in the last section, the complexity in the mixing matrix gives rise to the CP violation effect in the K^0 system. The parameter ϵ_K specifies the deviation of K_S, K_L from CP eigenstates. It is Nature's magic that K has a mass so near the 3π threshold so that K_S (mainly goes to 2π) and K_L (mainly goes to 3π) can have such large time differences in life. Such wonder probably will not happen again in D^0, $\bar{D}^0$ system again. It probably will be hard to measure ϵ_c, ϵ_B using the same method as for ϵ_K. As pointed out a few years ago in Refs. (13) and (14), the transition of $D^0 \rightleftarrows \bar{D}^0$ (or $B^0 \rightleftarrows \bar{B}^0$) can give rise to the asymmetry δ of same sign double-lepton final state in $e^+e^- \rightarrow D^0\bar{D}^0X^0$ (or $\rightarrow B^0\bar{B}^0X^0$) $\rightarrow \ell^+\ell^+X^{--}$, $\ell^-\ell^-X^{++}$

is $\delta \equiv (N_{++} - N_{--})/(N_{++} + N_{--}) = 4\mathrm{Re}\varepsilon$, where ε is the CP violation parameter for D^0, or B^0 system. It was estimated[15] to be small, ($\delta \sim 10^{-3}$) for the K-M model, but bigger ($\delta \sim 10^{-2}$) for the Higgs CP violation. Thus a large double charge asymmetry in e^+e^- experiment can rule out the K-M model. However, such a double lepton charge asymmetry has sever contamination form the chain semileptonic decays of quarks.

IV. CP VIOLATION IN PARTIAL DECAY RATES

Besides contributing CP violation effects in the mass matrix, the complexity in the mixing matrix can also rise CP violation in the decay amplitudes. There have been many earlier studies[13,16,17,18] on the subject from various points of view. For convenience of discussion, I shall first use the quark-diagram scheme of Ref. (19) to give an overall view and also some new results. I shall comment on the known results where they fit.

The decays of a heavy-quark meson (the bottom, the charm, and the strange) can be described by six independent amplitudes, a, b, c, d, e, f, as shown in Fig. (4.1).
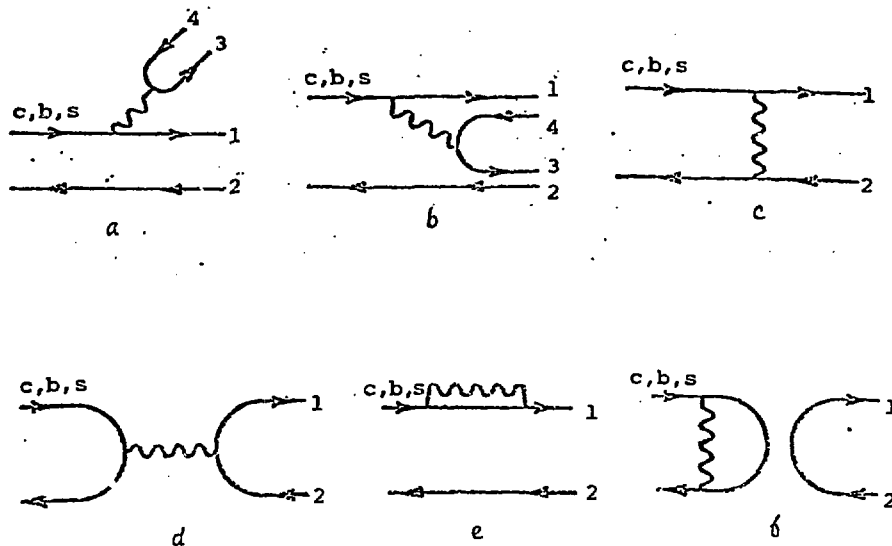

Fig. (4.1)

For a given final state of particles, we need only to add the appropriate $q\bar{q}$ lines (the hairpin quark lines) to each diagram and then project out the given final particles. In Ref. (19) the amplitudes of charm mesons D^0, D^+, F^+ decaying into two pseudo scalar mesons

are given. These diagrams are meant to include all strong interaction effects (the gluon lines), which are, in general, not yet calculable. Thus we do not know the magnitude of each diagram. However, we can classify experimental results using the diagrams. Eventually, we can obtain the sizes and phases of these diagrams from decay rates and CP violation effects, which we shall elaborate.

It was discussed quite some time ago by the authors of Refs. (13) and (14) that, though CPT predicts equal total decay rate for particle and anti-particle, the partial decay rates of particle and anti-particle into CP conjugated final particles can be different if CP is not invariant. The quark-diagram scheme provides an easy way to sort out the decay channels where particle and anti-particle decay rates can be different.

a) CP violation in Charm decay.

In the following we list all the semi-mixing-angle-suppressed decays of D^0, D^+, F^+ into two pseudo mesons, taking from Ref. (19):

$$A(D^0 \to K^- K^+) = V_{us}V_{cs}(a + c + e + 2\delta) - V_{ud}V_{cd}(e + 2\delta), \quad (4.1a)$$

$$A(D^0 \to \pi^- \pi^+) = V_{us}V_{cs}(e + 2\delta) - V_{ud}V_{cd}(a + c + e + 2\delta), \quad (4.1b)$$

$$A(D^0 \to \bar{K}^0 K^0) = \tfrac{1}{2}(V_{us}V_{cs} - V_{ud}V_{cs})(2c + 4\delta),$$

$$A(D^0 \to \pi^0 \pi^0) = \frac{1}{\sqrt{2}} [V_{us}V_{cs}(e + 2\delta) + V_{ud}V_{cd}(b - c - e - 2\delta)], \quad (4.1c)$$

$$A(D^0 \to \eta^0 \eta^0) = \frac{1}{\sqrt{2}} [V_{us}V_{cs}(\tfrac{2}{3} c - \tfrac{1}{3} b + \tfrac{1}{6} e + \delta)$$

$$- V_{ud}V_{cd}(\tfrac{1}{6} b + \tfrac{1}{6} c + \tfrac{1}{6} e + \delta)], \quad (4.1d)$$

$$A(D^0 \to \pi^0 \eta^0) = \frac{1}{\sqrt{3}} [V_{us}V_{cs}(-b + e) + V_{ud}V_{cd}(c - e)], \quad (4.1e)$$

and

$$A(D^+ \to \bar{K}^0 K^+) = V_{us}V_{cs}(a + e) - V_{ud}V_{cd}(d + e), \quad (4.2a)$$

$$A(D^+ \to \pi^0 \pi^+) = \frac{1}{\sqrt{2}} V_{ud}V_{cs}(a + b), \quad (4.2b)$$

$$A(D^+ \to \eta^0 \pi^+) = \frac{1}{\sqrt{2}} V_{us}V_{cs}(-2b + 2e) - V_{ud}V_{cd}(a + b + 2d + 2e),$$

$$(4.2c)$$

and

$$A(F^+ \to K^0 \pi^+) = V_{us}V_{cs}(d + e) - V_{ud}V_{cd}(a + e), \quad (4.3a)$$

$$A(F^+ \to K^+ \pi^0) = \frac{1}{\sqrt{2}} [V_{us}V_{cs}(d + e) + V_{ud}V_{cd}(b - e)], \quad (4.3b)$$

$$A(F^+ \to K^0 \eta^0) = \frac{1}{\sqrt{2}} [V_{us}V_{cs}(2a + 2b + d + e) + V_{ud}V_{cd}(b - e)]. \quad (4.3c)$$

For $\bar{D}^0$, D^-, F^- decays, we replace V_{ij} in Eqs. (4.1), (4.2) and (4.3) by V_{ij}^*. That the amplitudes a, b, c, d, e, f do not change in particle and anti-particle decays is a consequence of CP invariance in strong interactions. I have not listed the mixing-angle nonsuppressed and doubly suppressed channels since they have the same decay probability for particle and anti-particles, see Ref. 19.

Typically, the decay amplitudes for particle, anti-particle are of the following form, e.g.,

$$A(D^+ \to \bar{K}^0 K^+) = V_{us} V_{cs} A_1 + V_{ud} V_{cd} A_2, \tag{4.4a}$$

$$\bar{A}(D^- \to K^0 K^-) = V_{us}^* V_{cs}^* A_1 + V_{ud}^* V_{cd}^* A_2, \tag{4.4b}$$

where $A_1 = a + e$, $A_2 = d + e$. For different decays, A_1, A_2 represents the corresponding combination of amplitudes a, b, c, d, e as given in Eqs. (4.1), (4.2) and (4.3). That the partial decay rates of particle and anti-particle can be different in the K-M model is due to the complexity in V_{ij},

$$\Delta_c \equiv \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}} \equiv \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2}, \text{ where } \Gamma \equiv |A|^2, \ \bar{\Gamma} = |\bar{A}|^2,$$

$$= \frac{4\text{Im}(V_{us} V_{cs} V_{ud}^* V_{cd}^*) \, \text{Im}(A_1 A_2^*)}{|A|^2 + |\bar{A}|^2}$$

$$= \frac{4 s_2 s_3 s_\delta \, c_1 c_2 c_3 \, \text{Im}(A_1 A_2^*)}{(|A|^2 + |\bar{A}|^2) s_1^{-2}} \tag{4.5}$$

We divide the demoniator by s_1^2 because both $|A|^2$ and $|\bar{A}|^2$ have a factor of s_1^2. Δ now is again proportional to $s_2 s_3 s_\delta$. The same combination contributes to the CP violation parameter ϵ in K_L decay. In addition to mixing angles and phases, Δ depends crucially on the phases and magnitude of A and $\bar{A}$. Δ is zero if A and $\bar{A}$ have the same phase. Unfortunately we do not have reliable ways to calculate A and $\bar{A}$. Therefore, it is extremely difficult to give an accurate prediction of Δ. The present scheme provides the information about what are the possible channels where particle-anti-particle decay rates can be different.

Using Fig. (4.1), we can work out decay amplitudes for higher multiplicty final states and for semi-inclusive decays. Here we list the channels for which particle and anti-particle can have different decay rates:

$$D^\pm \to \bar{K}^0 K^+, \ \eta^0 \pi^\pm, \ K^\pm K^\mp X(s = 0 \text{ states}), \ \eta^0 \pi^\pm X(s = 0 \text{ states}),$$

$$K^\pm X(s = \mp \text{ states}), \ K^+ X(s = \mp \text{ states}), \ \eta^0 X(s = 0 \text{ states}), \text{ etc.}$$

$$\tag{4.6a}$$

$$F^{\pm} \to K^0 \pi^{\pm},\ K^{\pm} \pi^0,\ K^{\pm} \eta^0,\ K^0 \pi^{\pm} X(s = 0 \text{ states}),\ K^{\pm} \pi^0 X(s = 0 \text{ states}),$$

$$K^{\pm} \eta^0 (s = 0 \text{ states}), \pi^{\pm} X(s = \pm 1 \text{ state}),\ K^{\pm} X(s = 0 \text{ states}),\ \text{etc.}$$

$$\text{(4.6b)}$$

$$\left.\begin{array}{c} D^0 \\ \bar{D}^0 \end{array}\right\} \to K^- K^+,\ \pi^+ \pi^-,\ \pi^0 \pi^0,\ \eta^0 \eta^0,\ \pi^0 \pi^0,\ \pi^0 \eta^0,\ \text{and their inclusive states.}$$

$$\text{(4.6c)}$$

Here s denotes strangeness. The inclusive state X for decays of particle and anti-particle are CP conjugated. It is interesting to note from Eq. (4.2b), the mixing-angle-semisuppressed decay $D^{\pm} \to \pi^0 \pi^{\pm}$ has same decay rate, so do D, $\bar{D}^0 \to \bar{K}^0 K^0$.

Here we see a rich variety of channels where one can search for CP violation effects. Needless to say high experimental sensitivity, in the range of ε, is needed in such searches.

b) CP violation in B decays.

The B_{bu}^-, B_{bd}^0, $B_{bs}^0 \to$ ordinary (no charm) particle final states:

We first list the decay amplitudes of the B_{bu}^-, B_{bd}^0, B_{bs}^0 to two ordinary pseudo meson (no charm particle in the final states).

$$A(B_{bu}^- \to \pi^- \pi^0) = \frac{1}{\sqrt{2}} [V_{ub} V_{ud} (a + b + e + d) + V_{cb} V_{cd}\ e], \quad \text{(4.7a)}$$

$$A(B_{bd}^0 \to \pi^+ \pi^-) = V_{ub} V_{ud} (a + c + e + f) + V_{cb} V_{cd}\ (e + f), \quad \text{(4.7b)}$$

$$A(B_{bs}^0 \to \pi^- K^+) = V_{ub} V_{ud}\ a + V_{cb} V_{cd}\ e. \quad \text{(4.7c)}$$

We see that the interference can only come from the loop diagrams e and f, the so called "Penguin" diagrams. The partial decay rates can be different for particle and anti-particle for the following channels:

$$\left.\begin{array}{c} B_{bu}^- \\ B_{bu}^+ \end{array}\right\} \to \pi^{\mp} \pi^0,\ \pi^{\mp} X^0 (s = 0)\ , \quad \text{(4.8a)}$$

$$\left.\begin{array}{c} B_{bd}^0 \\ \bar{B}_{bd}^0 \end{array}\right\} \to \pi^+ \pi^-,\ \pi^+ \pi^- X^0 (s = 0),\ \pi^+ X^- (s = 0),\ \pi^- X^+ (s = 0), \quad \text{(4.8b)}$$

$$\left.\begin{array}{c} B_{bs}^0 \\ \bar{B}_{bs}^0 \end{array}\right\} \to \pi^{\mp} K^{\pm},\ \pi^{\mp} K^{\pm} X^0 (s = 0),\ K^{\pm} X^{\mp} (s = 0),\ \pi^{\mp} X^{\pm}(s = \mp 1)\ . \quad \text{(4.8c)}$$

The difference of partial decay rate in the CP conjugated decays are of the form

$$\Delta \equiv \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}} = \frac{4\,\text{Im}(V_{ub}V_{ud}V^*_{cb}V^*_{cd})\ \text{Im}(A_1A^*_2)}{|A|^2 + |\bar{A}|^2}$$

$$= -2(s_2/s_3)s_\delta c_1 c_2 c_3\ \text{Im}(A_1A^*_2)/[|A|^2 + |\bar{A}|^2]\,(s_1)^{-2}(s_3)^{-2},$$

where $\tfrac{1}{2}[|A|^2+|\bar{A}|^2](s_1)^{-2}(s_3)^{-2} = |c_1A_1|^2 + c_2|[c_1c_2 + (s_2/s_3)c_3 e^{i\delta}]A_2|^2$

$$- 2c_1c_2[c_1c_2 + (s_2/s_3)\,c_3 c_\delta]\cdot\text{Re}(A_1A^*_2). \qquad (4.9)$$

The important thing here is that Δ now is proportional to a factor of $(s_2/s_3)s_\delta$ different from that in charm decays, $s_2 s_3 s_\delta$ which is constraint to be small $\sim 10^{-4}$ by the observed CP violation in K_L decay. From the angle analysis of Ref. (8), we can, in principle, make s_3 very small and s_δ close to unity. For example, we can choose $s_2 = .3$, $s_\delta = 1$ and $s_3 \cong 0.005$, while still being consistent with all existing data, including the recent results of CESR.[10] Therefore, if the phases of A_1, A_2 are favorable, Δ can be large. We see that the study of CP violation in B decays will provide crucial information about the angles, phases, and strength of the amplitudes.

Earlier analysis of Bander, Silverman and Soni[16] estimated different partial decay rates for B and $\bar{B}$ from a time-like single gluon emission diagram.

The B^-_{bu}, B^0_{bd}, $B^0_{bs} \to$ double charm particle final states:

The mixing matrix and amplitude dependences of $B^-_{bu} \to D^0D^+$, $B_{bd} \to D^+D^-$, $B_{bs} \to F^+D^-$ are listed as follows:

$$A(B^-_{bu} \to D^0D^-) = V_{cb}V_{cd}(a + b + e) + V_{ub}V_{ud}(d + e), \qquad (4.10a)$$

$$A(B^0_{bd} \to D^+D^-) = V_{cb}V_{cd}(a + b + e) + V_{ub}V_{ud}\,e, \qquad (4.10b)$$

$$A(B^0_{bs} \to F^+D^-) = V_{cb}V_{cd}(a + b + e) + V_{ub}V_{ud}\,e. \qquad (4.10c)$$

Again we see that there can be particle-antiparticle partial decay rate differences in

$$\left.\begin{array}{c} B^-_{bu} \\[6pt] B^+_{bu} \end{array}\right\} \to D^0D^\pm,\ D^\pm D^0X^0(s=0),\ D^\pm X(c = \mp 1), \qquad (4.11a)$$

$$\left.\begin{array}{c} B^0_{bd} \\[6pt] B^0_{bd} \end{array}\right\} \to D^+D^-,\ D^+D^-X^0(s=0),\ D^+X^-(c=-1),\ D^-X^+(c=\mp 1),\quad (4.11b)$$

$$\left.\begin{array}{c} B^0_{bs} \\[6pt] B^0_{bs} \end{array}\right\} \to F^\pm D^\mp,\ F^\pm D^\mp X^0(s=0),\ F^\pm X^\mp(c=\mp 1),\ D^\mp X^\pm(s=\mp 1,\ c=\pm 1).$$
$$\qquad (4.11c)$$

The partial decay rate is given by the same formula as in Eq. (4.9). Bernabeu and Jarlskog[18] discussed this situation. But only partial rate difference of $B_{b\bar{u}}^{\pm} \to D^0 D^{\pm}$ is predicted since the diagram ℓ was ignored.

The dominant decay channels of $B_{b\bar{u}}$, $B_{b\bar{d}}$, $B_{b\bar{s}}$ are final states with $c = 1$. They, in this model, will in general have the same decay rates between particle and anti-particles, except the case considered in Ref. (17) where the final states can come from both D^0 and $\bar{D}^0$ state of the same B decay. The interference between D^0 and $\bar{D}^0$ provide CP violation effects. They considered the difference of the two decays

$$B^- \begin{array}{c} \nearrow D_0 K_s X^- \\ \searrow \bar{D}_0 K_s X^- \end{array} \longrightarrow K_s K_s X^- \qquad , \qquad (4.12a)$$

$$B^+ \begin{array}{c} \rightarrow D_0 K_s X^+ \\ \searrow \bar{D}_0 K_s X^+ \end{array} \longrightarrow K_s K_s X^+ \qquad . \qquad (4.12b)$$

The rate difference again is of similar form to that of Eq. (4.9).

c) CP violation in the strange particle decay

Besides the CP violation effects in the K_s and K_L decays, we can also ask about partial rate differences: It is well known that $K^{\pm} \to \pi^{\pm}\pi^0$ must have the same decay rates from CPT. Our quark diagram scheme checks with that. We list the decay amplitudes of K into two mesons.

$$A(K^+ \to \pi^+\pi^0) = \frac{1}{\sqrt{2}} V_{us}V_{ud}(a + b), \qquad (4.13a)$$

$$A(K^0 \to \pi^+\pi^-) = V_{us}V_{ud}(a + c + e + 2\delta) + V_{cs}V_{cd}(e + 2\delta), \qquad (4.13b)$$

$$A(K^0 \to \pi^0\pi^0) = \tfrac{1}{2} V_{us}V_{ud}(b + c + e) + V_{cs}V_{cd}\, e . \qquad (4.13c)$$

For $\bar{K}$ decays, same equations apply except V_{ij} replaced by V_{ij}^*. Here we see that the rate of $K^0 \to \pi^+\pi^-(\pi^0)$ can be different from $\bar{K}^0 \to \pi^+\pi^-(\pi^0)$ and $K^{\pm} \to \pi^{\pm}\pi^{\pm}\pi$ can differ in decay rates. Note that the differences here like in the B → ordinary particle case, come from the interference of the Penguin diagrams. The decay rate difference is again of the form of Eq. (4.5). They are always proportional to $s_2 s_3 s_\delta$, therefore of the same order of value as ϵ, depending on the phase and magnitude of A_1, A_2.

Based on the same quark diagram argument, it is easy to see that $\Lambda(\bar{\Lambda}) \to \pi^- p(\pi^+\bar{p})$ $\Sigma^+(\bar{\Sigma}^-) \to p\pi^0(\bar{p}\pi^0)$, $p\pi^+(\bar{p}\pi^-)$ can have different particle-anti-particle decay rates. The magnitudes of the differences are again proportional to $s_2 s_3 s_\delta$.

We see that the K-M model in our quark diagram formulation gives a systematic way of study the CP violation in partial decay rates. It is of interest to do experiments to check these partial decay rates systematically.

V. THE NEUTRON ELECTRIC-DIPOLE MOMENT

There are three form factors for the neutron, $\langle n|J_\mu^{e.m.}(0)|n\rangle$ $\sim \bar{u}(p')[F_1(q^2)\gamma_\mu - F_2(q^2)\sigma_{\mu\nu}\gamma_\nu + F_3(q^2)i\gamma_5\sigma_{\mu\nu}q^2]u(p)$, where $F_1(0) = 0$ the charge form factor, $F_2(0) = \mu_n$ the magnetic moment and $F_3(0)$ $= d_n$ the electric dipole moment. Again the complexity in V_{ij} can give d_n of the neutron via the diagrams of Fig. (5a) with a photon attached in all possible ways. It was first estimated by Ellis,
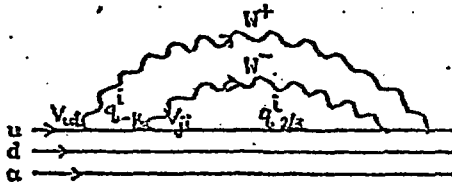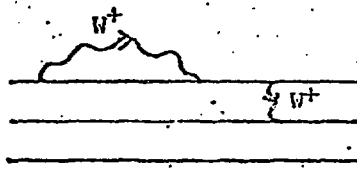

Fig. (5a)　　　　　　　　　　　　Fig. (5b)

Diagrams considered for the neutron electro-dipole moment, where $q^i_{-1/3}$, $q^j_{2/3}$ are the quarks of charge of $-1/3$ and $2/3$ respectively.

Gaillard, Nanopoulos[20] in '76, $d_n \sim 10^{-30}$ cm. Then Shabalin[21] showed that actually the sum of graphs in Fig. (5a) gives $d_n = 0$. Calculations have also been done including strong interations[22] and interquark exchange forces[23] Fig. (5b). The results are quite model dependent but they all give very small d_n in contrast to the result from Higgs CP violation, which is very close to the experimental[24] limit $d_n \le 1.6 \times 10^{-24}$ cm.

VI. CONCLUDING REMARKS

To end the lecture, I would put these challenges to the experimentalists:

(1) "Direct" measurements of V_{cs}, V_{cd}: Inclusive and semi-leptonic decays of charm and B decays, $\Gamma(D^+ \to \pi^+\pi^0)/\Gamma(D^+ \to \bar{K}^0\pi^+)$.

(2) To narrow down alternatives to the K-M model it is crucial to know the B decay properties: Does B decay only semileptonically? Which decay of B is favored $b \to c \to s$ or $b \to u$? For these CESR already have an answer, yes and $b \to c \to s$ respectively. Is there b-changing neutral current, $b \to q \ell\bar{\ell}$, $B \to \ell\bar{\ell}$? Some limits are already given by the CESR Experiment.[10]

(3) CP properties of the charm and the B system: ε', $\dfrac{N^{++} - N^{--}}{N^{++} + N^{--}}$, differences of various partial decay rates of CP related channels.

(4) Better neutron electric dipole moment measurements.

The real challenge that confronts us is the "family" problem. How many generations of quarks are there? How does the mixing come about? What is the origin of CP violation? It is likely that the current distinction between the K-M origin and complex-Higgs origin may turn out to be a superfluous one.

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