Computational Procedures for Determining Parameters in Ramberg-Osgood Elastoplastic Model Based on Modulus and Damping Versus Strain

Tzou-Shin Ueng

Jian-Chu Chen

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RAMBO: A Computer Code for Determining Parameters in Ramberg-Osgood Elastoplastic Model Based on Modulus and Damping Versus Strain

ABSTRACT

A computer code, RAMBO, is developed for obtaining the values of parameters in the Ramberg-Osgood elastoplastic model based on data of shear modulus and damping ratio at various shear strains. The basis and procedures for finding the parameters for the best fit of the data or relations definig modulus and damping ratios versus shear strain are given in this report. The Ramberg-Osgood relationship is rearranged so that the results can best fit data of both modulus and damping ratio. Constraints of data in the model are also discussed.

INTRODUCTION

The Ramberg-Osgood equation has been proposed to describe the nonlinear hysteretic constitutive relation of the one-dimensional elasto-plastic behavior of many materials (Jennings, 1964, 1965). It has been used by many researchers to model the dynamic soil behavior (e.g., Idriss, et al., 1978, Pyke, 1979). It is also implemented as one of many material types in the computer codes, DYNA3D (Whirley and Hallquist, 1991) and NIKE2D (Engelmann and Hallquist, 1991). It has also recently been implemented in NIKE3D. Four parameters are needed in the Ramberg-Osgood equation to describe the hysteretic elasto-plastic behavior of a material. These parameters are generally obtained based on the test data or typical relations in terms of modulus and damping ratios versus strain. Due to the arrangement of the variables in the Ramberg-Osgood equation, it is not a simple and straightforward task to obtain the values of these parameters which give the best fit of the given data. The computer program, RAMBO, is developed to facilitate this task.

RAMBERG-OSGOOD EQUATION

The backbone (monotonic loading) strain-stress relation of the Ramberg-Osgood elastoplastic model can be expressed by:

$$\frac{\gamma}{\gamma_y} = \frac{\tau}{\tau_y}\left(1 + \alpha\left|\frac{\tau}{\tau_y}\right|^{r-1}\right) \tag{1}$$

where γ = shear strain,

τ = shear stress,

γ_y = reference shear strain,

τ_y = reference shear stress,

α = constant ≥ 0, and

r = constant ≥ 1.

For unloading and reloading, according to Masing's rule, the relation becomes:

$$\frac{\gamma - \gamma_o}{2\gamma_y} = \frac{\tau - \tau_o}{2\tau_y}\left(1 + \alpha\left|\frac{\tau - \tau_o}{2\tau_y}\right|^{r-1}\right) \tag{2}$$

where γ_o = shear strain at point of stress reversal, and

τ_o = shear stress at point of stress reversal.

The values of γ_y, τ_y, α, and r are to be determined according to the material properties.

HYSTERESIS PROPERTIES

The dynamic hysteretic properties, such as dynamic soil behavior, are commonly given in terms of equivalent linear (secant) modulus, G_o, and equivalent critical damping ratio, β (Fig. 1). They are usually presented as a function of shear strain as shown in Figs. 2 and 3. The shear modulus at a very low strain, G_{max}, is also available based on the shear wave velocity data which can be obtained, for example, from a seismic survey.

By rearranging Eq. 2, the secant modulus for the backbone curve can be expressed as:

$$G_o = \frac{\tau}{\gamma} = \frac{\tau_y}{\gamma_y}\left(\frac{1}{1 + \alpha\left|\frac{\tau}{\tau_y}\right|^{r-1}}\right) \tag{3}$$

For a very small strain, i.e., $\gamma \rightarrow 0$ and $\tau \rightarrow 0$, since $r > 1$,

$$(G_o)_{\gamma=0} = G_{max} = \frac{\tau_y}{\gamma_y} \tag{4}$$

Then the backbone relation can be rewritten as:

$$\frac{\gamma}{\gamma_y} = \frac{\tau}{G_{max}\gamma_y}\left(1 + \alpha\left|\frac{\tau}{G_{max}\gamma_y}\right|^{r-1}\right) \tag{5}$$

Therefore, besides G_{max}, there are three parameters, γ_y, α, and r, to be determined for the Ramberg-Osgood model.

DETERMINATION OF PARAMETER VALUES

Values of α and r

Substituting $\tau = G_o\,\gamma$ and rearranging Eq. 5, we obtain

$$\frac{G_{max}}{G_o} - 1 = \alpha\left|\frac{G_o\gamma}{G_{max}\gamma_y}\right|^{r-1}, \tag{6}$$

$$\log\left(\frac{G_{max}}{G_o} - 1\right) = \log\alpha + (r-1)\log\left(\frac{G_o\gamma}{G_{max}\gamma_y}\right) \tag{7}$$

If the value of γ_y is assumed as commonly done in the literature, then based on the dynamic soil data of G/G_{max} versus shear strain such as those shown in Fig. 2, we can plot Eq. 7 on Fig. 4. The values of α and r can be determined from the intercept and the slope, respectively, of the best fit straight line. RAMBO uses the ordinary least-squares method in finding the best fit straight line.

The equivalent critical damping ratio, β, for a hysteresis loop with the tip at (γ_o, τ_o) can be expressed as:

$$\beta = \frac{\Delta E}{2\pi\tau_o\gamma_o} = \frac{2\alpha\,(r-1)}{\pi\,(r+1)} \left(\frac{G_o}{G_{max}}\right)^r \left(\frac{\gamma_o}{\gamma_y}\right)^{r-1} \qquad (8)$$

where ΔE = energy dissipation in one loading cycle. Since the values of all parameters are determined as discussed previously, β can then be computed without further information. That is, once the backbone relation is defined, the damping value is also determined. However, the value of β computed from Eq. 8 may not necessarily fit well with the soil damping data obtained in the tests. The difference may be significant in some cases. For a better fit of both modulus and damping data, further considerations on damping data are made:

Substituting Eq. 6 in Eq. 8, we obtain

$$\beta = \frac{2(r-1)}{\pi\,(r+1)} \left(1 - \frac{G_o}{G_{max}}\right), \text{ or} \qquad (9)$$

$$\frac{G_o}{G_{max}} = 1 - \frac{\beta\pi(r+1)}{2(r-1)} \qquad (10)$$

Substitute Eq. 10 in Eq.7, then

$$\log\left[\frac{\beta\pi(r+1)}{2(r-1) - \beta\pi(r+1)}\right] = \log\alpha + (r-1)\log\left(\left[1 - \frac{\beta\pi(r+1)}{2(r-1)}\right]\frac{\gamma}{\gamma_y}\right) \qquad (11)$$

Using only the damping data, Eq. 11 can also be plotted on the same figure on which Eq. 7 is plotted using modulus data (i.e., Fig. 4). Thus, a best fit straight line, and values of α and r, can be found for data including both modulus and damping data.

Value of Reference Strain, γ_y

Examining the Ramberg-Osgood equation, we found that γ_y is a parameter for determining the point of maximum curvature. It will affect the shape of backbone curve and the hysteresis loop. For materials, such as soils, it is very difficult to determine the value of γ_y from the strain-stress relation. In the literature, γ_y was commonly assigned a value either arbitrarily or related to the shear strength. Depending on the type of soils and the loading conditions, the reference stress, τ_y, usually ranges approximately from 0.6 to

0.9 of the shear strength of the soils. Then γ_y can be calculated by multiplying τ_y with G_{max} according to Eq. 4.

Since the value of γ_y affects the modulus and damping, it is conceivable that it can also be determined based on the modulus and damping data. An iteration procedure is possible:

1) Assume a value for γ_y and obtain the values of α and r by plotting the data according to Eqs. 7 and 11.
2) Compute γ_y according to Eq. 8 from the given modulus and damping data, and obtain an average value of γ_y.
3) Compare the new value of γ_y with the previous value. Repeat steps 1 and 2 if the difference is too great.

According to Eq. 8, when a set of modulus and damping data is given, then α, r, and γ_y are not independent. That is, if the value of γ_y changes, we can still get the same modulus and damping results by changing the values of α and r. Therefore, the iteration procedure may give a γ_y without any physical meaning.

COMPUTATIONS OF MODULUS, DAMPING, AND HYSTERESIS LOOP

Once the parameters, α, r, and γ_y are available, the modulus ratio, G_o/G_{max}, at any given strain can be obtained according to Eq. 6 by iteration using the Newton-Raphson method. The damping ratio at that strain can then computed according to Eq. 9.

The backbone curve is obtained according to Eq. 1 also by iteration using the Newton-Raphson method. Then the strain-stress hysteresis loop can be obtained by Eq. 2.

DATA CONSTRAINTS IN RAMBERG-OSGOOD MODEL

Since $0 \le \frac{G_o}{G_{max}} \le 1.0$, then according to Eq. 10,

$$0 \leq \frac{\beta\pi(r+1)}{2(r-1)} \leq 1.0 \qquad (12)$$

The Ramberg-Osgood model calls for $r \geq 1.0$, that is,

$$\frac{r+1}{r-1} \geq 1.0, \qquad (13)$$

Then based on Eq. 12,

$$\beta \geq 0\text{, and } \frac{\beta\pi}{2} \leq 1.0 \text{ or } \beta \leq 0.637 \qquad (14)$$

According to Eqs. 9 and 13,

$$\beta \leq \frac{2}{\pi}\left(1 - \frac{G_o}{G_{max}}\right),\text{ or } \frac{G_o}{G_{max}} \leq 1 - \frac{\beta\pi}{2} \qquad (15)$$

For input data of moduli and damping ratios not satisfying the aforementioned constraints (Eqs. 12 through 15), warnings are given and the data will be ignored in the computations.

When $G_o = G_{max}$, i.e., $G_o/G_{max} = 1.0$, there is no hysteresis damping, i.e., $\beta = 0$. Theoretically, $G_o \neq G_{max}$ unless $\gamma_o = 0$. However, we commonly have data of $G_o/G_{max} = 1.0$ and $\beta \approx 0.5$ to 2.0% at a very low shear strain (≤ 0.0001%). The damping at a very low strain may include damping other than the hysteresis damping modeled by the Ramberg-Osgood equation. If a better fit of damping ratios at low strains is desired, a minimum damping ratio can be assigned. Then RAMBO will subtract the minimum damping from the data, and find the parameter values accordingly. The minimum damping should be added in the computation of damping ratios using the Ramberg-Osgood model.

INPUT DATA

Input file name: **grbeta.in**

All strains and damping ratios are in fractions.

1st card - Title

2nd card (I5, 4E15.0) - data fitting control parameters

Column	Variable	Description
1 - 5	ng	number of strains at which the values of G/G_{max} and damping ratio to be input (maximum 50).
6 - 20	gmlow	lower limit of strain fitting range. 0 – gmlow = the lowest input strain
21 - 35	gmup	upper limit of strain fitting range. 0 – gmup = the highest input strain
36 - 50	gmyld	yield shear strain, if available. This can be estimated based on shear strength of the soil. 0 – gmyld will be found by iteration
51 - 65	btmin	minimum damping ratio. This is for better fitting at very low shear strains. It can be left blank in most cases.

3rd card (2E15.0) - stress and hysteresis loop. The unit of stress is the same as that of input Gmax. This card can be a blank card. Then the result will just give the parameter values of α, r, and γ_y.

Column	Variable	Description
1 - 15	G_{max}	maximum shear modulus if output of yield shear stress and hysteresis strain-stress loop are desired.
16 - 30	gmhyst	shear strain at the tip of the hysteresis loop. 0 – no hysteresis loop output

4th card (E15.0, 2F10.0) - input data points of G/G_{max} and damping ratio at various strains

Column	Variable	Description
1 - 15	gamma	shear strain
16 - 25	Gratio	G/G_{max}
26 - 35	beta	damping ratio

Repeat for G/G_{max} and damping ratio at other strains.

OUTPUT

Output file name: **rambo.out**

The following results are given in the output:

(1) Input G/G_{max} and damping ratio versus strain, maximum shear modulus, and the fitting range of shear strain,

(2) The values of parameters, i.e., α, r, γ_y, and τ_y, in Ramberg-Osgood model to best fit the data of modulus and damping within the fitting range of shear strain,

(3) Relations of G/G_{max} and damping ratio versus shear strain according to the Ramberg-Osgood model using the given parameter values,

(4) Variance and standard deviation of the data with mean values according to the Ramberg-Osgood relation, and

(5) Strain-stress hysteresis loop for a given strain at the tip.

EXAMPLE

The relations of G/G_{max} and damping ratio versus shear strain of a silty sand are shown in Figs. 5 and 6 based on the test results in cyclic triaxial and resonant column tests. The maximum shear modulus, G_{max}, is estimated to be 32,900 psi based on the results of resonant column tests at very low strains (< 10^{-4}%). To find the parameters in Ramberg-Osgood model for the best fit of these data using RAMBO, an input data file, **grbeta.in**, is created as shown below:

```
LAW's test results, Sample S-26, depth=103.6 ft, conf. pressure=120 psi
  17            1.e-4            1.e-2
        32900.          0.001
      4.00E-07      0.9990      0.0092
      1.00E-06      0.9983      0.0095
      2.30E-06      0.9971      0.0097
      4.90E-06      0.9905        0.01
      1.03E-05      0.9769      0.0109
      2.05E-05      0.9553      0.0128
      3.32E-05      0.9350       0.016
      7.79E-05      0.8548      0.0197
      1.06E-04      0.8240      0.0226
      2.20E-04      0.7899       0.032
      4.60E-04      0.6900      0.0428
      6.90E-04      0.6157      0.1098
      1.46E-03      0.4491      0.1248
      2.25E-03      0.3481      0.1609
      3.89E-03      0.2056      0.2076
      7.14E-03      0.1283      0.2202
      1.74E-02      0.1215        0.23
```

With a fitting shear strain range between 10^{-4} to 10^{-2}, the results are given in the output file, **rambo.out**, shown below. Fig. 7 shows the comparison between the data and the Ramberg-Osgood model using the parameters obtained by RAMBO. The fittings are reasonably good for both modulus and damping. A hysteresis loop based on the Ramberg-Osgood model is also plotted on Fig. 8.

```
Ramberg-Osgood Equation parameters ---
LAW's test results, Sample S-26, depth=103.6 ft, conf. pressure=120 psi

Input modulus and damping curves -

     strain          G/Gm      damping
 -------------- --------- -----------
   0.4000E-06      0.999      0.009
   0.1000E-05      0.998      0.009
   0.2300E-05      0.997      0.010
   0.4900E-05      0.990      0.010
   0.1030E-04      0.977      0.011
   0.2050E-04      0.955      0.013
   0.3320E-04      0.935      0.016
   0.7790E-04      0.855      0.020
   0.1060E-03      0.824      0.023
   0.2200E-03      0.790      0.032
   0.4600E-03      0.690      0.043
   0.6900E-03      0.616      0.110
   0.1460E-02      0.449      0.125
   0.2250E-02      0.348      0.161
   0.3890E-02      0.206      0.208
   0.7140E-02      0.128      0.220
   0.1740E-01      0.122      0.230

maximum shear modulus =   32900.00


 fitting range for strain =    0.1000E-03 to     0.1000E-01


Ramberg-Osgood parameters

 alpha =      0.8499
 R =     2.2822
 yield shear strain =       0.4652E-03
 yield shear stress =        15.30
```

Ramberg-Osgood backbone curve and damping -

strain	G/Gm	damping
0.4000E-06	0.9999E+00	0.2481E-04
0.1000E-05	0.9997E+00	0.8025E-04
0.2300E-05	0.9991E+00	0.2331E-03
0.4900E-05	0.9975E+00	0.6126E-03
0.1030E-04	0.9937E+00	0.1574E-02
0.2050E-04	0.9850E+00	0.3729E-02
0.3320E-04	0.9729E+00	0.6728E-02
0.7790E-04	0.9276E+00	0.1801E-01
0.1060E-03	0.8997E+00	0.2493E-01
0.2200E-03	0.8029E+00	0.4903E-01
0.4600E-03	0.6673E+00	0.8275E-01
0.6900E-03	0.5852E+00	0.1032E+00
0.1460E-02	0.4386E+00	0.1396E+00
0.2250E-02	0.3634E+00	0.1583E+00
0.3890E-02	0.2817E+00	0.1786E+00
0.7140E-02	0.2089E+00	0.1967E+00
0.1740E-01	0.1319E+00	0.2159E+00
Variance =	0.1871E-02	0.2907E-03
Std. Dev. =	0.4325E-01	0.1705E-01

strain-stress hysteresis loop

strain	stress
-0.1000E-02	-0.1680E+02
-0.8927E-03	-0.1344E+02
-0.7709E-03	-0.1008E+02
-0.6309E-03	-0.6721E+01
-0.4705E-03	-0.3361E+01
-0.2881E-03	0.4768E-06
-0.8214E-04	0.3361E+01
0.1486E-03	0.6721E+01
0.4052E-03	0.1008E+02
0.6887E-03	0.1344E+02
0.1000E-02	0.1680E+02
0.8927E-03	0.1344E+02
0.7709E-03	0.1008E+02
0.6309E-03	0.6721E+01
0.4705E-03	0.3361E+01
0.2881E-03	-0.4768E-06
0.8214E-04	-0.3361E+01
-0.1486E-03	-0.6721E+01
-0.4052E-03	-0.1008E+02
-0.6887E-03	-0.1344E+02
-0.1000E-02	-0.1680E+02

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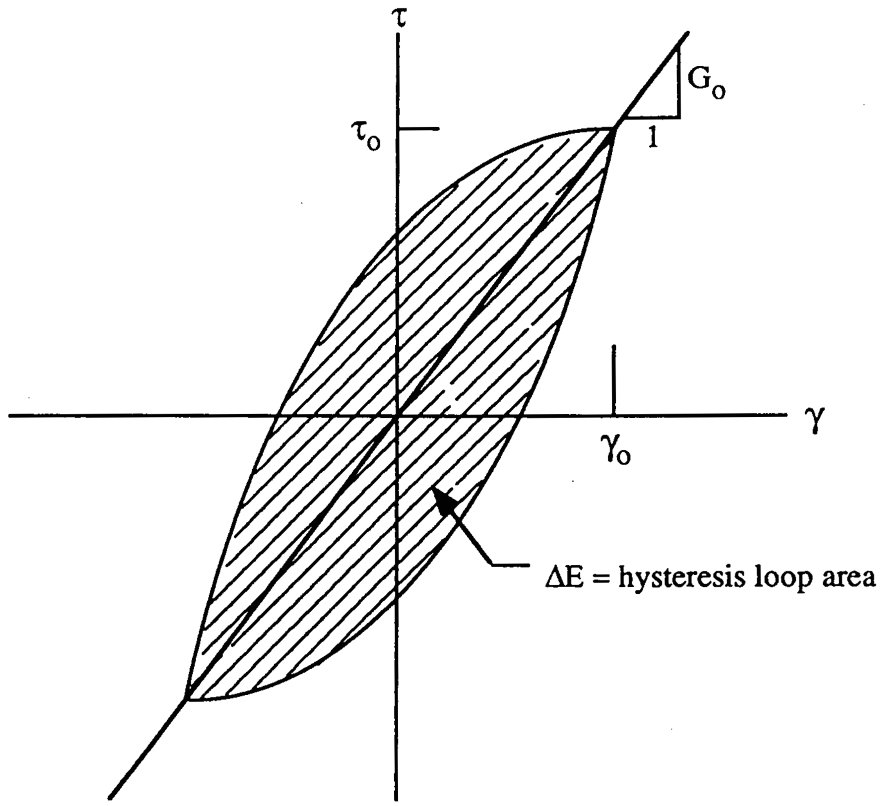

Equivalent linear modulus = G_o

Damping ratio = $\beta = \Delta E /(2\pi\ \tau_o\ \gamma_o)$

Figure 1. Equivalent linear modulus and damping ratio of soils

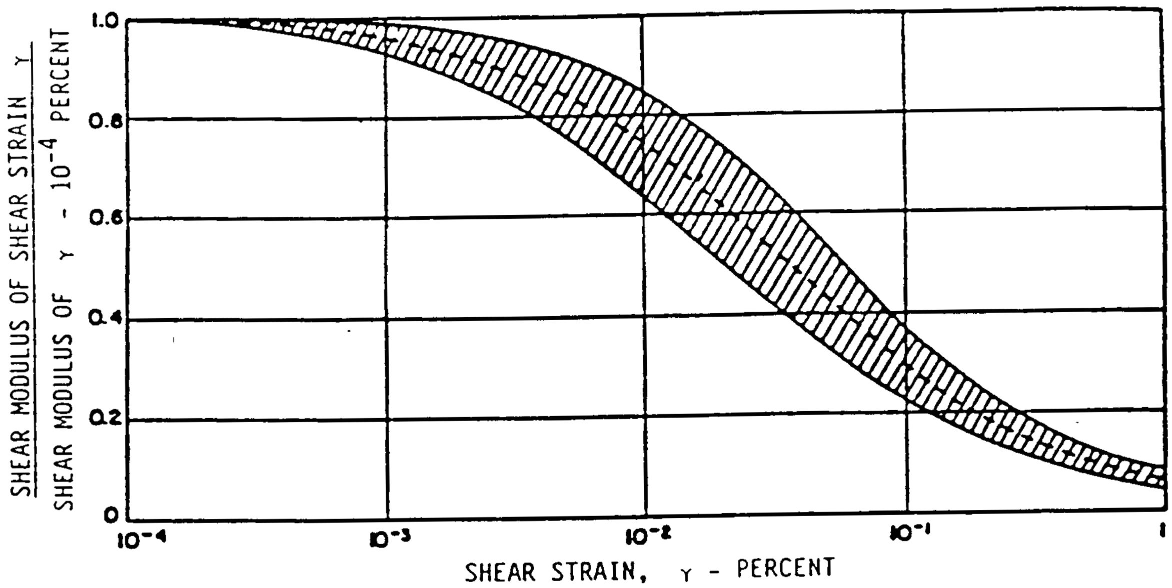

Figure 2. Variation of Shear Modulus with Shear Strain for Sands (Seed and Idriss, 1970)

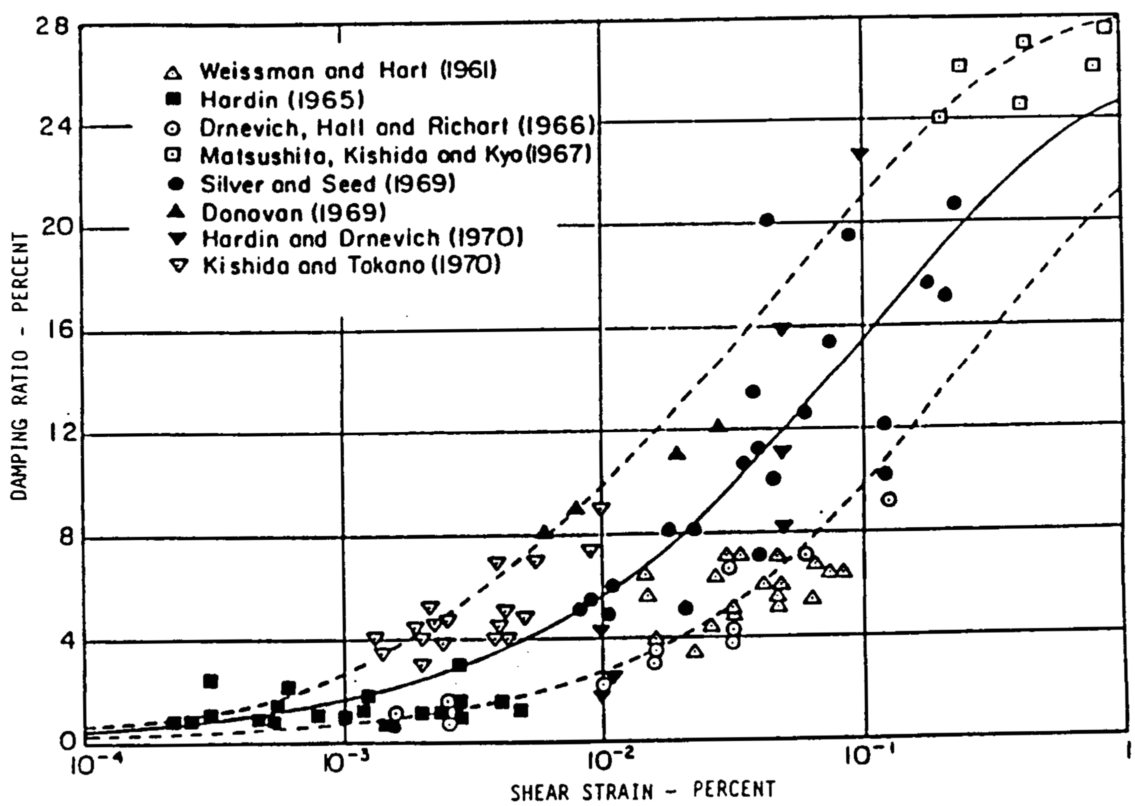

Figure 3. Variation of Damping Ratio with Shear Strain for Sands (Seed, et al., 1984)

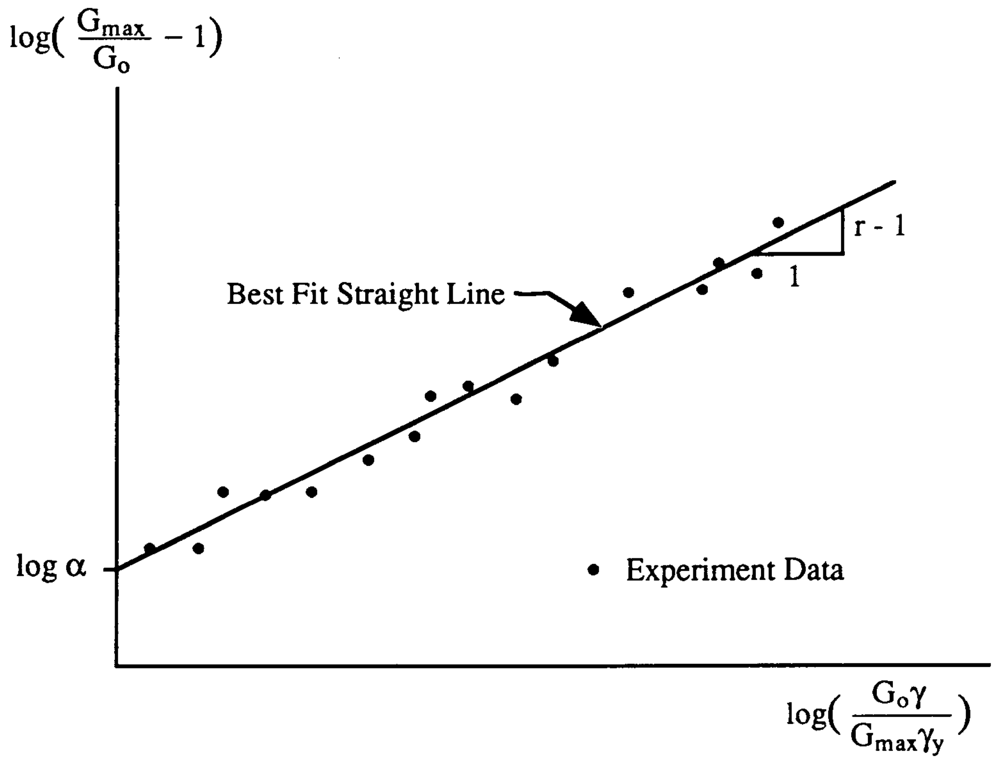

Figure 4. Plot of $\log(\frac{G_{max}}{G_o} - 1)$ vs. $\log(\frac{G_o\gamma}{G_{max}\gamma_y})$

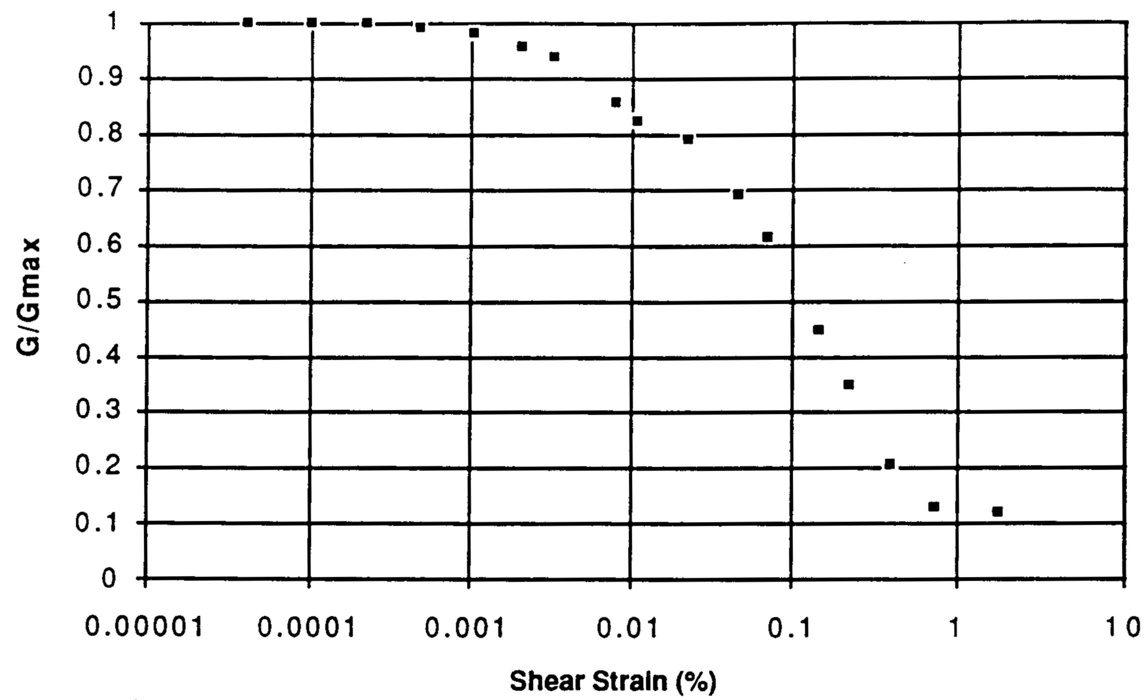

Figure 5. Data of G/G_{max} versus Shear Strain for a Silty Sand

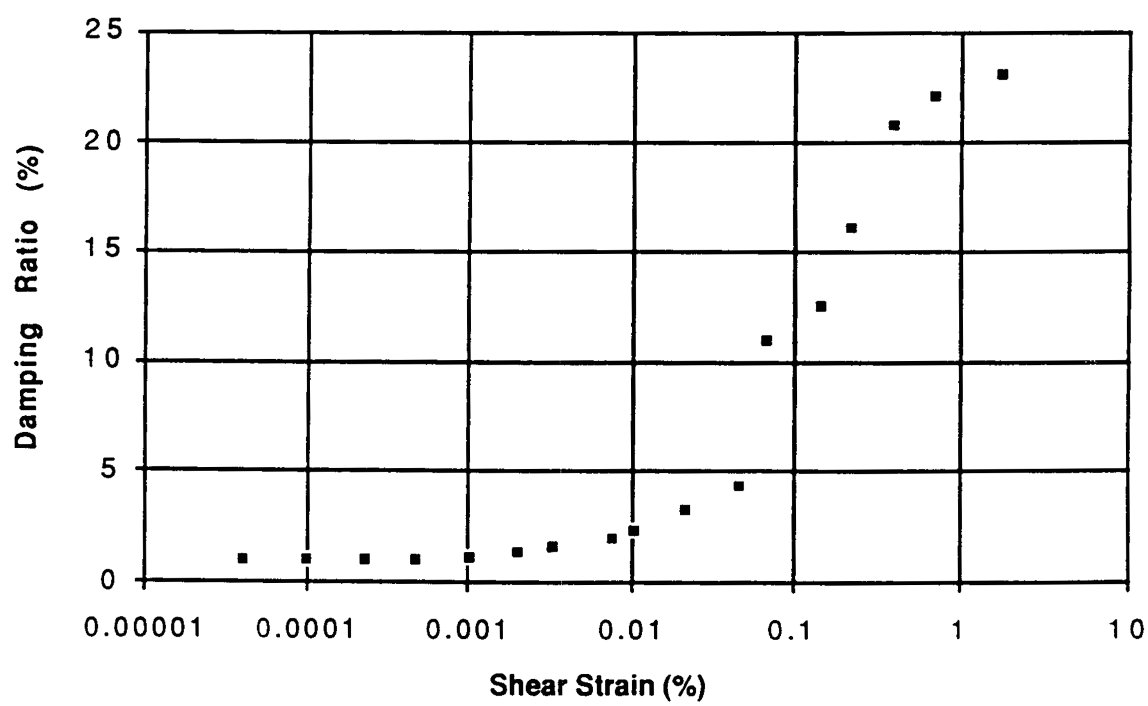

Figure 6. Data of Damping Ratio versus Shear Strain for a Silty Sand

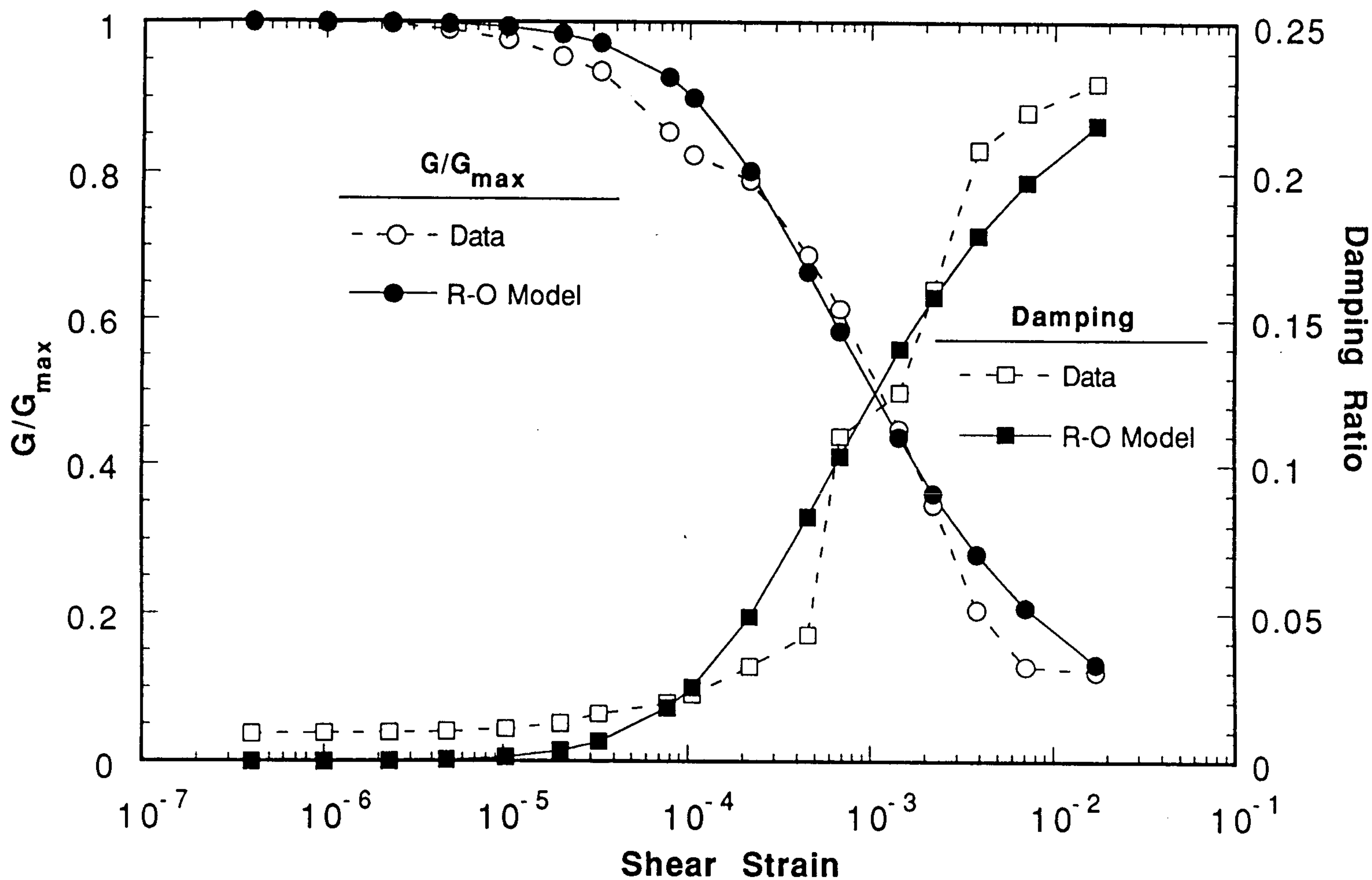

Figure 7. Comparison of Relations of G/G_{max} and Damping Ratio versus Shear Strain between Input Data and Ramberg-Osgood Model

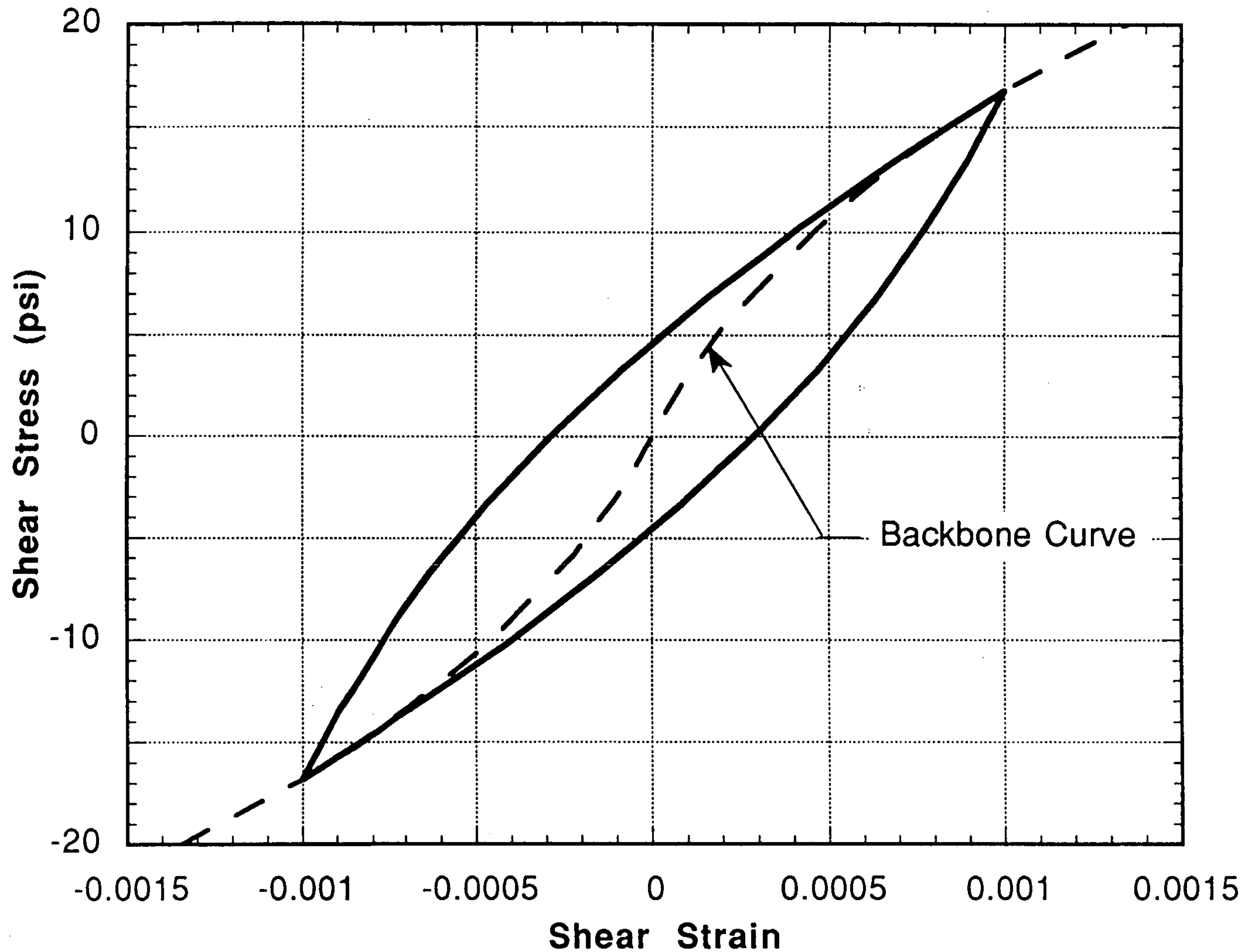

Figure 8. Hysteresis Loop According to Ramberg-Osgood Model