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EFFECTIVE INTERACTIONS AND ELEMENTARY EXCITATIONS
IN QUANTUM LIQUIDS

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Abstract

The effective interactions which provide a wavevector and frequency dependent restoring force for collective modes in quantum liquids are derived for the helium liquids by means of physical arguments and sum rule and continuity considerations. A simple model is used to take into account mode-mode coupling between collective and multiparticle excitations, and the results for the zero-temperature liquid ^{4}He phonon-maxon-rotor spectrum are shown to compare favorably with experiment and with microscopic calculation. The role played by spin-dependent backflow in liquid ^{3}He is analyzed, and a physical interpretation of its variation with density and spin-polarization is presented. A progress report is given on recent work on effective interactions and elementary excitations in nuclear matter, with particular attention to features encountered in the latter system which have no counterparts in the helium liquids.

1. Introduction

In strongly coupled quantum liquids, such as the electron and helium liquids and nuclear matter, the determination of the effective interactions between quasiparticles and the way in which these, in turn, determine the elementary excitation spectrum, has been a problem of central and continuing concern. It is a problem to which Gerry Brown has made a number of central contributions, and one which I have enjoyed discussing with him at length during the nearly three decades that we have known one another. While our direct interaction has been comparatively weak, in that we have not yet written a paper together, we have published independently on parallel lines (as in the RPA description of excitations in nuclear matter) and have had a continuing strong exchange interaction, which has brought Kevin Bedell, Khandker Quader, and Jochim Wambach, all of whom first worked with Gerry in Stony Brook, to Urbana, and which took Chris Pethick, who was a postdoctoral fellow in Urbana, to Copenhagen.

In my talk today, I shall describe briefly some recent developments in our understanding of effective interactions and elementary excitations in quantum liquids. The basic approach to quantum liquids I shall be using is polarization potential theory, a minimalist, phenomenological, post-RPA, post-Feynman, post-Landau theory in which physical arguments, constraints from static measurements, sum rules, and continuity arguments are combined to reduce markedly (at times to zero) the parameters required to specify the effective quasiparticle interaction and calculate the elementary excitation spectra. It provides a unified and quantitative account of excitations and transport in the helium liquids[1] and has been extended as well to electron liquids[2].

I shall begin by discussing the key physical effects which must be taken into account in any quantitative description of excitation spectra in quantum liquids, and show how these are dealt with for the helium liquids using polarization potential theory. I shall show how an explicit account of spin-dependent backflow thus provides new insight into the physical origin of the quasiparticle mass in ^{3}He, and describe the convergence of the polarization potential approach and microscopic calculations (based on a variational ground state wavefunction) of the phonon-maxon-roton spectrum of ^{4}He. I discuss briefly recent results on spin-polarized ^{3}He, before concluding with an account of work in progress with Khandker Quadar and Jochim Wambach on an extension of polarization potential theory to nuclear matter.

2. Effective Interactions: Pseudopotentials and Backflow

There are two key physical effects which must be described in order to obtain a quantitative account of the wavevector and frequency dependent restoring forces responsible for collective behavior in quantum liquids.

- Short-range correlations associated with strong particle interaction, zero-point motion, and, for Fermi systems, the Pauli principle
- Backflow

In polarization potential theory, the short-range correlations are incorporated into the theory by introducing a frequency-independent configuration space pseudopotential. Backflow is described as a frequency (and wavevector) dependent restoring force whose strength in the long wavelength limit is determined for Bose systems by the quasiparticle effective mass, while for Fermi systems, it is determined by the average quasipair energy. Let us consider these effects separately.

2.1 A Pseudopotential Description of Effective Interactions

As a particle moves in the liquid it induces density fluctuations which act back on it, as well as on the other particles. These fluctuations, whose mean wave-vector and frequency dependence is described by $\langle\rho(q\omega)\rangle$, give rise to polarization fields which both screen external longitudinal fields and act to provide a restoring force for collective behavior. Quite generally the fluctuating scalar potential which gives rise to this restoring force can be written in the form

$$\varphi_{pol}(q,\omega) = f^{s}(q,\omega)\ \langle\rho(q\omega)\rangle; \qquad [1]$$

its coupling to the density fluctuations, ρ_q^+, is $\rho_q^+ \varphi_{pol}(q\omega)$. In the RPA, $f^s(q,\omega) = V_q$, the Fourier-transform of the bare particle interaction; in Landau theory, $f^s(q,\omega)$ is taken to be momentum and frequency independent, and corresponds to the $\ell = o$ component of Landau's phenomenological interaction between quasiparticles on the Fermi surface. In polarization potential theory, $f^s(q,\omega) = f_q^s$, a phenomenological frequency-independent, but momentum-dependent, restoring force. The latter is calculated by considering the physical behavior of its Fourier transform, a non-local configuration space pseudopotential, $f_s(r)$, which describes an effective interaction between particles in the liquid,

$$f_q^s = \int d^3r \; f^s(r) e^{i\underset{\sim}{q}\cdot\underset{\sim}{r}} \; . \qquad [2]$$

At low temperatures, f_o^s, the spatial average of $f_s(r)$, is known from measurements of conpressibility and (for Fermi systems) the specific heat.

How is $f^s(r)$ related to the bare atom-atom potential? The major physical effects which distinguish liquid helium from gaseous helium are at most three-fold:

o Short-range correlations which enable the helium atoms to avoid, insofar as possible, sampling their mutual very strong short range ($\lesssim 2.7 A^{-1}$) repulsive interaction

- Zero point motion
- Quantum statistical correlations (eg., the Pauli principle for ^{3}He atoms)

Since all three physical effects are predominantly short-range, it follows that at long distances, where liquid correlations are not important, $f^s(r)$ should be identical to the bare atom-atom potential. For the helium liquids, Aldrich and Pines[3] describe the short range behavior of the pseudopotential $f^s(r)$ with the aid of a simple physical argument, illustrated in Fig. 1, namely that the positional correlations brought about by the strong short-range repulsion between the atoms prevent the atoms from sampling the full consequences of that interaction. Hence, the almost hard-core repulsion of range $\sim 2.68\text{A}^{-1}$, found in the bare interaction, must go over to a soft-core repulsion, of range r_c, which they characterize by an r^8 potential of strength a,

$$f^s(r) = a\ (1-r/r_c)^8 \qquad r \lesssim r_c \qquad [3]$$

For a given choice of r_c, a simple fitting function is then used to join this part of the potential to the attractive long-range part, taken to be identical to the bare interaction for $r > r_c$. Because f^s_o is known, for a given choice of r_c, a is uniquely determined.

For liquid ^{4}He, $f^s(r)$ describes the effective interaction between ^{4}He quasiparticles; AP took r_c to be the range of the bare atom-atom repulsive interaction. For liquid ^{3}He, $f^s(r)$ is the spin-symmetric average of the effective interaction between parallel and anti-parallel spin quasiparticles,

$$f_3^s(r) = \left[f^{\uparrow\uparrow}(r) + f^{\uparrow\downarrow}(r)\right]/2 \qquad [4]$$

To the extent that the effects of zero point motion and the Pauli principle are small compared to those of the short-range, "hard" core, correlations in determining the ^{3}He pseudopotentials, at a given density the shape and range of $f^{\uparrow\uparrow}(r)$, $f^{\uparrow\downarrow}(r)$, and $f^s(r)$ should be quite similar. The differences in the pseudopotentials can be characterized by changes in r_c, and will arise as a result of zero point motion and the Pauli principle. A good idea of the relative importance of the latter is obtained by comparing, <u>at the same density</u>, the spatial averages of the three pseudopotentials.

As may be seen in Table 1, the spatial average of $f_o^{\uparrow\downarrow}$, 46.5K, is some 70% larger than f_o^s, and is, in fact, quite close to the spatial average (42.8K) of the direct interactions between antiparallel spin ^{3}He quasiparticles in a background of ^{4}He at this density; it thus reflects primarily the increased zero point motion of ^{3}He atoms compared to ^{4}He in liquids of the same density. Comparison of $f_o^{\uparrow\downarrow}$ with $f_o^{\uparrow\downarrow}$ shows that by comparison, the Pauli principle acts to reduce $f_o^{\uparrow\downarrow}$ by some 3%. From a fit to the transport properties of ^{3}He calculated from the corresponding pseudopotentials, Bedell, Hess, Hsu, and Pines (in preparation) find that on going from ^{4}He to ^{3}He the greater zero point motion of the ^{3}He atoms not only leads to an enhanced core radius, but also implies that the correlation-induced screening of the short-range repulsion interaction is somewhat less effective, since $a_{\uparrow\downarrow}$ is increased to 55.7K. The combined effects of the Pauli principle and zero point motion lead to a slightly larger value of r_c, and a very slightly reduced value of $a_{\uparrow\uparrow}$, compared to the corresponding values for anti-parallel spin ^{3}He quasiparticles. These results are also given in Table 1. The hierarchy of physical effects on the various restoring forces is thus clearly established; short range potential-induced correlations, which are similar for the various atom-atom interactions, dominate, followed by the influence of zero point motion, with Pauli principle effects coming in a distant third.

The momentum dependent pseudopotentials for the effective interactions in ^{3}He and ^{4}He are compared in Fig. 2 with those calculated by Hsu and Pines[4] for ^{3}He-^{4}He mixtures. Note that the major differences occur at wavevectors $\lesssim 1\text{A}^{-1}$. The increase with q seen at long wavelength is quite a general phenomenon, and reflects the changed interplay between the repulsive and attractive parts of the pseudo potentials as one goes to higher moments of $f^s(r)$. Thus if we write:

$$f_q^s = f_o^s(1 + q^2 f_2^s + \ldots)$$

then

$$f_o^s = 4\pi \int_0^\infty dr\ r^2 f^s(r) \qquad [5a]$$

$$f_2^s = \frac{-2\pi}{3} \int_0^\infty dr\ r^4 f^s(r) \ . \qquad [5b]$$

In the r^2 moment of $f^s(r)$, the short range repulsive part of the pseudopotential wins out over the long range attractive part, but in the r^4 moment the situation is reversed. The physical consequence of this behavior is <u>anomalous</u> dispersion of the zero sound modes in both ^{4}He and ^{3}He.

Finally, the density dependence of the various pseudopotentials is straightforward to obtain. For ^{4}He there are no free parameters. Since f_o^s increases with density, the corresponding increase in the core height means that as the density increases the short range correlations in liquid ^{4}He are somewhat less effective in reducing the strength of the repulsive part of the interaction. For pure ^{3}He, where both the increased zero point motion of the ^{3}He atoms and the Pauli principle influence both r_c and a, the change of these parameters with density involves some degree of arbitrariness; Bedell et al. choose the density variation of the respective a's, and of $\delta(\equiv r_c^{\uparrow\uparrow} - r_c^{\uparrow\downarrow})$, to be such that the pseudopotentials go over, at svp for pure ^{3}He, to those previously found by Aldrich and Pines to provide the best fit to neutron scattering experiments, while at pressures greater than 21 atm, the results scale with those found for pure ^{4}He at similar densities. The resulting parameters are given in Table 2. Note that the influence of the Pauli principle is greatest at svp, and that it becomes weaker as the system density increases.

Backflow

In addition to inducing density fluctuations, as a particle moves in the liquid it induces current fluctuations, which act back on it, thereby changing its effective mass. As Feynman and Cohen first emphasized, these backflow effects are quite important in ^{4}He; for a hard sphere moving in a classical liquid the induced current, or backflow, takes a dipolar form, and acts to increase the mass of the sphere by 50%. In polarization potential theory for liquid ^{4}He, the induced current fluctuations, $\langle \underset{\sim}{J}(q\omega)\rangle$, give rise to a vector field,

$$\underset{\sim}{A}_{pol}(q,\omega) = f_q^V \langle \underset{\sim}{J}(q\omega)\rangle \qquad [6]$$

which couples to the current fluctuations, $\underset{\sim}{j}_q$, by a coupling, $\underset{\sim}{j}_q \cdot \underset{\sim}{A}_{pol}(q\omega)$. Since backflow is a purely longitudinal phenomenon, particle conservation links the induced current and density fluctuation according

$$\langle \underset{\sim}{q} \cdot \underset{\sim}{J}(q\omega)\rangle = \omega \langle \rho(q\omega)\rangle \ . \qquad [7]$$

The resulting effect of backflow is to replace $\varphi_{pol}(q\omega)$, in Eq. (1), by a wavevector and frequency dependent quantity.

$$\varphi_{pol}(q\omega) = \left[f_q^s + (\omega^2/q^2)\ f_q^v\right] \langle \rho(q\omega)\rangle \ . \qquad [8]$$

For liquid ^{4}He, the strength of the backflow potential is simply related to the quasiparticle effective mass,

$$m_q^* = m + Nf_q^v. \qquad [9]$$

For ^{3}He, on the other hand, m_q^* is the average single quasiparticle-quasihole effective mass, which reduces to the usual quasiparticle mass in the long wavelength limit. Thus one has

$$m^* = m + Nf_o^v \equiv m\left[1 + F_1^s/3\right] \qquad [10]$$

where F_1^s is the dimensionless spin-symmetric $\ell = 1$ component of Landau's interaction between quasiparticle on the Fermi surface.

Absent the effects of zero point motion and the Pauli principle, we would expect the backflow coefficients, f^v_q, to be essentially the same in liquid ^{3}He and ^{4}He at the same density; to the extent that the short-range correlations are mainly responsible for backflow (as they are for f^s_o), one can then get f^v_o for ^{4}He from the corresponding quantity for ^{3}He at the same density, and one would expect a rather similar fall-off of the backflow field strength, f^v_q, with increasing momentum transfer. For ^{3}He, specific heat experiments tell us what Nf^v_o is as a function of density; AP assumed that at the same density, Nf^v_o would be the same for ^{4}He; they determined the q dependence of m^*_q in phenomenological fashion, from their best fit to the phonon-maxon-roton spectrum of ^{4}He, and then further assumed that the q dependence of the average quasipair mass in ^{3}He would be similar. Typical expressions for Nf^v_q for ^{4}He are shown in Fig. 3.

It is instructive, and indeed necessary for spin-polarized ^{3}He, to separate out the spin-dependence of the backflow terms in ^{3}He. To do so, one simply replaces $\underset{\sim}{j}_q \cdot \underset{\sim}{A}_{pol}(q\omega)$ by the expression:

$$\sum_{\sigma\sigma^1} j_{q\sigma}\, h_{\sigma\sigma^1}(q)\, \langle J_{\sigma^1}(q\omega)\rangle \,, \qquad [11]$$

in which case we could write:

$$m^*_{q\uparrow} = m + N_\uparrow h_{\uparrow\uparrow}(q) + N_\downarrow h_{\uparrow\downarrow}(q) \qquad [12]$$

with a similar expression for $m^*_{q\downarrow}$. For an unpolarized system, one then has $N_\uparrow = N_\downarrow = N/2$, and $f^v_q = (h_{\uparrow\uparrow} + h_{\uparrow\downarrow})/2$; on the other hand, for a fully spin polarized system, one has $N_\uparrow = N$, and

$$\tilde{m}_\uparrow = m + N\tilde{h}_{\uparrow\uparrow}(q) \qquad [13a]$$

$$\tilde{m}_\downarrow = m + N\tilde{h}_{\uparrow\downarrow}(q) \qquad [13b]$$

so that by using microscopic calculations or physical models, one can proceed to follow the spin polarization dependence of the backflow potentials. Note, further, that for the unpolarized system, $h_{\uparrow\uparrow}(o)$ and $h_{\uparrow\downarrow}(o)$ can be determined directly from a knowledge of F^s_1 and the spin-antisymmetric Landau parameter, F^a_1. Representative results, which are extremely sensitive to the value of F^a_1, at two pressures are given in Table 3. There one sees that the backflow potential for antiparallel spin particles is considerably larger than that for those with parallel spin: the former reflects the combined influence of the short-range, potential-induced correlations and zero point motion, and increases rapidly with density; the latter takes into account the Pauli principle as well, and is much smaller because the Pauli principle causes particles of parallel spin to spend more time in the attractive region of the effective particle - particle interaction. Indeed, the weak density dependence of the latter suggests that under pressure the particles of parallel spin experience backflow effects which are nearly the same as those applicable at svp; in other words, the Pauli principle prevents particles of parallel spin from sampling the rapid rise in backflow potential brought on by the comparatively ineffective screening.

The influence of the Pauli principle on backflow is even more striking in spin-polarized ^{3}He. For the fully polarized system at svp, both microscopic calculations and sum rule arguments yield an up-spin effective mass, $\tilde{m}_\uparrow \cong 0.84m$; thus, at a fixed density the backflow potential, $h_{\uparrow\uparrow}(q)$, <u>decreases</u> with increasing spin polarization, to the point that it vanishes, and then becomes negative in the limit of complete spin polarization, as the Pauli principle causes the up spin quasiparticles to sample more of the attractive interaction between quasiparticles than the repulsive interaction.

3. Density-density Response Function

The density fluctuation excitation spectrum is specified by the density-density response function, $\chi(q,\omega)$ defined as the linear response of the system to an external field, $\varphi_{ext}(q,\omega)$, coupled to its density fluctuations, ρ_q^+. If we consider instead the linear response of the system to the sum of φ_{ext} and the polarization potential, $\varphi_{pol}(q,\omega)$, specified by Eq. [8], we obtain

$$\chi^c(q\omega) = \frac{\chi_{sc}(q\omega)}{1-\left[f_q^s + (\omega^2/q^2)\ f_q^v\right] \chi_{sc}(q\omega)} \qquad [14]$$

where $\chi_{sc}(q\omega)$ is the system response to an external potential <u>plus</u> $\varphi_{pol}(q,\omega)$. The poles of $\chi^c(q\omega)$ yield the dispersion relation for the density fluctuation excitation modes; one has therefore

$$1 = \left[f_q^s + (\omega_q^2/q^2) f_q^v\right] \chi_{sc}(q,\omega_q) \qquad [15]$$

where ω_q is the collective mode excitation energy. The identification of m_q^* with the average quasipair mass follows from the sum rule definition of the latter,

$$\frac{\int_0^\infty d\omega\ S_{pair}(q\omega)\ \omega}{\int_0^\infty d\omega\ S_{pair}(q\omega)} = \frac{Nq^2}{2m_q^*} \qquad [16]$$

where S_{pair} is the single quasiparticle-quasihole pair contribution to the dynamic form factor, $S^c(q\omega) \equiv -(1/\Pi)\ \mathrm{Im}\ \chi^c(q,\omega)$.

The expression, [14], provides a formal basis for a unified theory of elementary excitations in superfluid ^{4}He and normal ^{3}He. In it, the role played by frequency and wavevector dependent restoring forces is essentially decoupled from the effects of statistics or temperature; since we have argued that at the same density the corresponding values of f_q^s and f_q^v will be quite similar, the effects of statistics or temperature come in only through $\chi_{sc}(q\omega)$. To the extent that we have described all the consequences of particle interaction by f_q^s and f_q^v, then one has $\chi_{sc} \cong \chi_o^*$, where χ_o^* is the non-interacting single quasiparticle or quasipair response; thus AP take for

$$^4\mathrm{He}: \quad \chi_{sc}(q\omega) \cong \chi_o^*(q\omega) = \frac{Nq^2/m_q^*}{\omega^2 - (q^2/2m_q^*)^2} \qquad [17]$$

$$^3\mathrm{He}: \quad \chi_{sc}(q\omega) \cong \chi_L^*(q\omega) \qquad [18]$$

where the expression, [17], represents the excitation of single quasiparticles from the condensate, while in [18], $\chi_L^*(q\omega)$ is the Lindhard response function for quasipairs of mass m_q^*. To the extent that the collective modes in ^{3}He possess a frequency large compared to the single pair energies appearing in $\chi_L^*(q,\omega)$, this expression is well approximated by its high frequency limit, $Nq^2/m_q^* \omega^2$, and the differences in the collective mode energies for ^{3}He and ^{4}He are determined by their mass difference, (including the influence of zero point motion on f_q^s).

The expressions [17] and [18] are, in fact, exact only in the long wavelength limit (where, for example, [15] and [16] yield the usual Landau result); at finite wavevectors, multiparticle excitations begin to play a significant role, and their influence must be taken into account if one hopes to obtain a quantitative account of the excitation spectrum. We consider these now.

4. The Two-Branch Spectrum, and Mode-Mode Coupling

Quite generally, for both Bose and Fermi systems, one has two modes of excitation out of the ground state, and hence two branches to the excitation spectrum. For Bose systems, the multiparticle branch corresponds to states characterized by the excitation of two or more quasiparticles from the condensate,[5] while for Fermi systems it corresponds to states in which two or more quasipairs are excited from the filled Fermi sea ground state.[6] The branches can easily be distinguished in the long wavelength limit, since in this limit the frequency of the single quasiparticle or quasipair branch vanishes, while the frequency of the multiparticle or multipair branch is finite. In polarization potential theory, we describe the presence of the two branches by letting $\chi_{sc}(q\omega)$ contain contributions from both; thus we write

$$\chi_{sc}(q\omega) = \alpha_q \chi_o^*(q\omega) + (1 - \alpha_q) \chi_m(q\omega) \qquad [19]$$

where the strength of the single quasiparticle or quasipair response function contribution to χ_{sc} is now measured by vertex renormalization function, $\alpha_q(\leq 1)$, and $\chi_m(q\omega)$ is the multiparticle or multipair response

Sum rule considerations,[5,6] show that at long wavelengths,

$$\lim_{q\to 0} \alpha_q = 1 - \alpha_2 q^2 + \ldots ;$$

while to the extent that the characteristic frequencies which enter into $\chi_m(q\omega)$, $\bar{\omega}_m(q)$, are large compared to those of the collective modes we seek to calculate, we can write:

$$\chi_m(q,\omega) \cong \chi_m(q,0) = - \frac{Nq^2/m_q^*}{\langle\omega_m^2(q)\rangle} \qquad [20]$$

For ^{4}He, one can obtain $\langle\omega_m^2(q)\rangle$ directly from neutron scattering experiments, while for ^{3}He, one can estimate it from Raman scattering experiments. To the extent one knows $\langle\omega_m^2(q)\rangle$ the influence of the second, multiparticle or multipair, branch on the collective mode energies is contained in the phenomenological parameter, α_q. Since in all cases $\bar{\omega}_m(q) > \omega_q$, the effect of mode-mode coupling will be to depress the collective mode energy, ω_q.

5. The Phonon-Maxon-Roton Spectrum of ^{4}He

The first major success of polarization potential theory came in its quantitative account of the phonon-maxon-roton spectrum of ^{4}He. AP showed that if the momentum dependence of f_q^v was fixed by requiring that

$Nf_q^V \sim 1.8m$ in the vicinity of the roton minimum, and α_q and $\chi_m(q,o)$ were chosen to provide a fit to the excitation spectrum at svp, and were further assumed not to vary with density, then <u>with no free parameters</u> one could get an excellent fit to the measured excitation spectrum for $q \lesssim 2.1\ A^{-1}$ at pressures up to 25 atm. (Recall that f_q^s is fixed by the pressure dependence of the first sound velocity.) A striking verification of the correctness of their pseudopotential was provided by measurements of anomalous dispersion as a function of pressure. At long wavelengths, the phonon dispersion relation takes the form:

$$\lim_{q \to o} \omega_q = sq\ (1 + \omega_2 q^2 + \omega_3 q^3 + \ldots\ ) \qquad [21]$$

where

$$\omega_2 = \frac{1}{2}\left[f_2^s - \alpha_2 \frac{m^*}{2m} + (4mm_o^* s^2)^{-1}\right] , \qquad [22]$$

α_2 is given by Eq. [19] and $\omega_3 = (\pi^2/24f_c^s) \lim_{q \to o} \left[f^s(r)r^6\right]$. One sees in Eq. [22] that the positive values of f_2^s, characteristic of the AP pseudopotential, bring about anomalous dispersion (a positive value of ω_2) while mode-mode coupling, here represented by α_2 (>0), acts to oppose it. As the density increases above its svp value, f_2^s must decrease in a directly calculable way, since the repulsive part of $f^s(r)$ increases [in a fashion fixed by $s(\rho)$], while the attractive part is unchanged; hence, since AP find $\alpha_2(\rho) \cong \alpha_2(\text{svp})$, anomalous dispersion decreases with increasing pressure. Calculation of the wavevector, q_c, at which anomalous dispersion ends, requires only a knowledge of f_q^s and α_q; AP found excellent agreement with the direct determination of this quantity (as a function of pressure) by Dynes and Narayanamurthi.[7]

In recent work, Coffey and I (manuscript in preparation) have examined anew the polarization potential calculation of the phonon-maxon-roton spectrum. Our reexamination was prompted by two recent developments: Stirling's extraordinarily accurate measurements of the excitation spectrum in the vicinity of the roton minimum,[8] and improved microscopic calculations of the excitation spectrum by Manousakis and Pandharipande.[9] By making use of the experimentally determined values of $\langle\omega_m^2(q)\rangle$ in Eq. [20], we reduced the number of free phenomenological parameters to one, the multiparticle vertex correction, α_q. (Recall that f_q^s is set by the physical arguments presented above, f_o^v by the effective mass in ^{3}He at the same density, and f_q^v by its fall-off in the vicinity of the roton minimum.) As shown in Fig. 4, we were able to obtain an excellent account (~0.1% accuracy) of Stirling's experimental results for the roton excitation spectrum, thereby confirming the AP description of a roton as a ^{4}He quasiparticle, of effective mass ~2.8m_o, moving in a weakly momentum-dependent attractive self-consistent field, Nf_q^s, of strength ~-2K, produced by the other quasiparticles.

In our calculations we neglected the dynamic consequences of mode-mode coupling (since we replace $\chi_m(q,\omega)$ by $\chi_m(q,o)$ in Eq. [20]; this approximation should work very well for rotons, and for long wavelength phonons ($q \lesssim 0.5\ A^{-1}$), but for shorter wavelength phonons and maxons, whose energies lie closer to the multiparticle spectrum, it might be expected to be less accurate. A comparison of our results with experiment, given in Fig. 5, shows that this is the case. Also shown in that Figure is the variational calculation of Manousakis and Pandharipande[9] in which the same physical effects (short-range correlati ack-flow, and static mode-mode coupling) are taken into account. F wavevectors up to $\sim 1.3\ A^{-1}$, the two theories give nearly identical results; it may therefore be plausibly argued that the Manousakis/Pandharipande calculations provide a microscopic justification for the polarization potential approach. It is likely that the reason we do considerably better than Manousakis and Pandharipande is that our calculations incorporate a significant reduction in the strength of the backflow potential for $1.5\ A^{-1} \lesssim q \lesssim 2.4\ A^{-1}$.

5. Fully Spin-Polarized ^{3}He

Hess, Pines, and Quader (in preparation) have studied effective interactions and elementary excitations in $^3\tilde{He}$, spin-polarized ^{3}He in the fully polarized limit, and I should like to comment briefly on their results. In constructing the polarization potential, $\tilde{f}^{\uparrow\uparrow}(r)$, in this limit, they make use of a simple ansatz for its spatial average, $\tilde{f}_0^{\uparrow\uparrow}$, namely that at a given density the upper limit to the physical effect of spin polarization on $f_0^{\uparrow\uparrow}$ is given by the magnitude of Pauli principle changes on going from $f_0^{\uparrow\downarrow}$ to $f_0^{\uparrow\uparrow}$ for the unpolarized system. Thus

$$\tilde{f}_o^{\uparrow\uparrow} - f_o^{\uparrow\uparrow} \lesssim f_o^{\uparrow\uparrow} - f_o^{\uparrow\downarrow} \equiv 2f_o^a \qquad [23]$$

(Recall that Pauli principle effects are small compared to those of short-range potential induced correlations or zero point motion, and are typically $\sim 2NF_o^a \sim 2K$.) By combining this ansatz with the forward scattering rum rule, and on assuming that only the $\ell = o$ and $\ell = 1$ moments play a role in determining quasiparticle interaction at the Fermi surface, they obtain values for m^*/m and the compressibility which are close to the microscopic calculations of Manousakis et al.[10]

The resulting density fluctuation excitation spectrum depends on the assumed form of the wavevector dependence of the backflow potential. HPQ consider two limits; one in which the effects of spin polarization on m_q^* is assumed to be wavevector independent; one in which this effect is assumed to operate predominantly at long wavelengths, so that by the time one reaches, say, $q \sim 1.8$ A^{-1}, there would be little difference between $\tilde{m}_q^*$ and m_q^*. In the first case the reduction of $\tilde{m}_q^*$ leads to a substantial qualitative change in the spectrum: the onset of Landau damping is much earlier, and no maxon regime is found. In the second, the spectrum is qualitatively the same; the wavevector which characterizes the onset of Landau damping is only slightly reduced.

7. Nuclear Matter

Khandker Quader, Jochim Wambach, and I have been developing a polarization potential theory for nuclear matter, and given Gerry's keen interest in this problem, I should like to give you a progress report on our efforts. Our aim has been the development of a systematic theory of excitation spectra which extends Landau theory by incorporating finite-range effects in the effective quasiparticle interactions which provide the resulting forces for collective modes, and which allows for the likely momentum-dependence of the average single quasipair effective mass.

The basis for the theory is the resemblance between the bare hadron interactions and those between helium atoms. In both cases one has a long range attractive interaction plus an exceedingly strong short range repulsive interaction; in both systems short range correlations in the liquid prevent its constituents from sampling the full consequences of the latter. The construction of the theory follows along the lines of the ^{3}He work of Aldrich and Pines, with, however, certain important modifications made necessary by the more complicated nature of the nuclear problem. First, on the nuclear distance scale the pion-induced interaction between nucleus is not short-ranged, and exchange effects associated with it need to be described explicitly. Second, tensor forces and significant three-body interactions give rise to effects which go beyond allowing for the influence of short-range correlations on the bare two-body interaction. Finally, zero point motion effects are significantly larger in nuclear matter than in ^{3}He, while experiment alone does not serve to fix the six lowest order Landau parameters (four associated with proton-proton and proton-neutron interactions, two from the tensor force contributions) so that a judicious combination of theory and experiment is required to determine these.

The results of our calculations based on the Reid V_6 potential are given in Figures 6-9, and I comment briefly on them.

- The six regularized configuration space pseudopotentials are shown in Fig. 6, where the substantial influence of zero point motion on the range of the repulsion part of the interaction is shown explicitly.
- The corresponding q-dependent pseudopotentials are shown in Fig. 7, while their non-negligible influence on response functions is illustrated by our results for the density-density response function at four different momentum transfers (Fig. 8). Correspondingly large, albeit different, effects are found in the other spin-isospin channels.
- The calculated wavevector dependent static polarizabilities, shown in Fig. 9, display significant structure at wavevectors $\sim 2p_F$ in all but the density channel, reflecting the strong tendency of nuclear matter to form a correlated spin-isospin state.
- A comparison of these calculations with the recent microscopic calculations of Fantoni and Pandharipande (preprint) and Schiavilla et al. (preprint) based on a variational theory of the ground state, shows that when many-body effects are neglected, the two calculations are in rather good agreement. Comparable agreement is found for the pair distribution functions.

8. Concluding Remarks

It is encouraging that where the predictions of polarization potential theory for the excitation spectra of the helium liquids can be compared with experiment, or with the best available microscopic theories, good agreement is found. There was not time in the talk, or space in this written account, to describe its application to excitations and transport properties in ^{3}He, nor to compare its connections with the induced-interaction approach of Babu, Bedell, Brown, Quader, Pfitzner, and Wölfle, and the recent "Gutzwiller" calculation of effective interactions by Anderson, Vollhardt, and Wölfle (preprint). The progress to date on effective interaction and excitations in nuclear matter is likewise encouraging, and Quader, Wambach and I now plan to extend this work to neutron matter and to neutron stars, in the hope that observations of the latter will provide further constraints on the Landau parameters and scattering amplitudes of the theory.

9. Acknowledgments

I should like to take this opportunity to thank my collaborators in this research, C. H. Aldrich, K. Bedell, D. Coffey, D. Hess, W. Hsu, C. J. Pethick, K. F. Quader, and J. Wambach, for helpful and stimulating discussions; to thank the Center for Materials Science (Los Alamos) for its warm hospitality and support during the preparation of this manuscript, and to acknowledge the support of the Department of Energy and of the National Science Foundation (NSF Grant DMR 85-21041 and

PHY86-00377). The sections on the helium liquids are taken, in part, from materials prepared for an invited paper at the October 1986, Banff Conference on Quantum Liquids and Solids, and will appear, in somewhat different form, in the proceedings of that meeting (Can. J. Phys., in press).

Table 1. Comparison of the spatial average, core radius, and core height of the pseudopotentials which describe effective interactions between quasiparticles in helium liquids at a density of 0.0218 A^{-3} (svp for ^{4}He) (from Hsu and Pines).[4]

System	Interaction	Spatial Average (K)	Core Radius (A)	Core Height (K)
^{4}He	^{4}He-^{4}He	$f_o^s = 27.3$	2.68	49.3
^{3}He	$^3He^{\uparrow}$-$^3He^{\downarrow}$	$f_o^{\uparrow\downarrow} = 46.5$	2.773	55.7
^{3}He	$^3He^{\uparrow}$-$^3He^{\uparrow}$	$f_o^{\uparrow\uparrow} = 45.2$	2.780	54.2

Table 2. A comparison, at various densities, of the spatial average, core radius, and core height of the pseudopotentials which describe the effective interactions between quasiparticles responsible for the restoring forces for zero sound and spin density fluctuation excitations in ^{3}He (Bedell et al., in preparation).

Pressure (bar)	$f_o^{\uparrow\downarrow}$ (K)	$f_o^{\uparrow\uparrow}$ (K)	$r_c^{\uparrow\downarrow}$	$r_c^{\uparrow\uparrow}$	$a^{\uparrow\downarrow}$	$a^{\uparrow\uparrow}$
svp	11.8	10.1	3..0	3.03	23.2	20.9
5	22.4	20.8	2.936	2.953	32.0	30.1
10	31.0	29.6	2.878	2.890	39.9	38.2
15	38.7	37.3	2.826	2.833	47.4	45.9
21	46.5	45.2	2.773	2.780	55.7	54.2
34.36	58.8	57.6	2.705	2.712	68.8	67.2

Table 3. Spin-dependent backflow potential parameters for unpolarized ^{3}He.

P (bar)	F_1^a	F_1^s	$Nh_{\uparrow\uparrow}/m$	$Nh_{\uparrow\downarrow}/m$
svp	-0.5	5.3	1.2	2.3
27	-1.0	12.5	1.6	6.8

Legends for Illustrations

Figure 1. Schematic drawing of liquid helium pseudopotentials: a) configuration space, . . . bare interaction, ———— pseudopotential; b) momentum space.

Figure 2. Momentum dependent pseudopotentials which describe the effective interaction between He atoms in very dilute $(x \to 0)$ mixtures of ^{3}He in ^{4}He, compared with the corresponding potentials for pure ^{3}He and pure ^{4}He at the same density [from Ref. (4)].

Figure 3. Backflow potentials in liquid ^{4}He as a function of pressure [from Ref. (3)].

Figure 4. Comparison of the calculated excitation spectrum in the vicinity of the roton minimum with experiment.[8]

Figure 5. Comparison with experiment of the polarization potential and variational [(Ref. 9), shown by triangles] results for the phonon-maxon-roton excitation spectrum; in both theoretical calculations, dynamic mode-mode coupling corrections are neglected.

Figure 6. Nuclear matter configuration space pseudopotentials which describe the effective interaction between protons and protons and protons and neutrons; also shown are the tensor force terms, $f^{\dagger}_{pp}(r)$ and $f^{\dagger}_{pn}(r)$. The dashed lines depict results when zero point motion is neglected. (From Pines, Quader and Wambach (PQW), in preparation).

Figure 7. Nuclear matter momentum space pseudopotentials in various spin-isospin channels (from PQW).

Figure 8. A comparison of the polarization potential results for the density-density response function with those obtained for free particles, and using Landau theory (S^{L}_{oo}) (from PQW).

Figure 9. The calculated wavevector dependent static response functions in four spin-isospin channels (from PQW).

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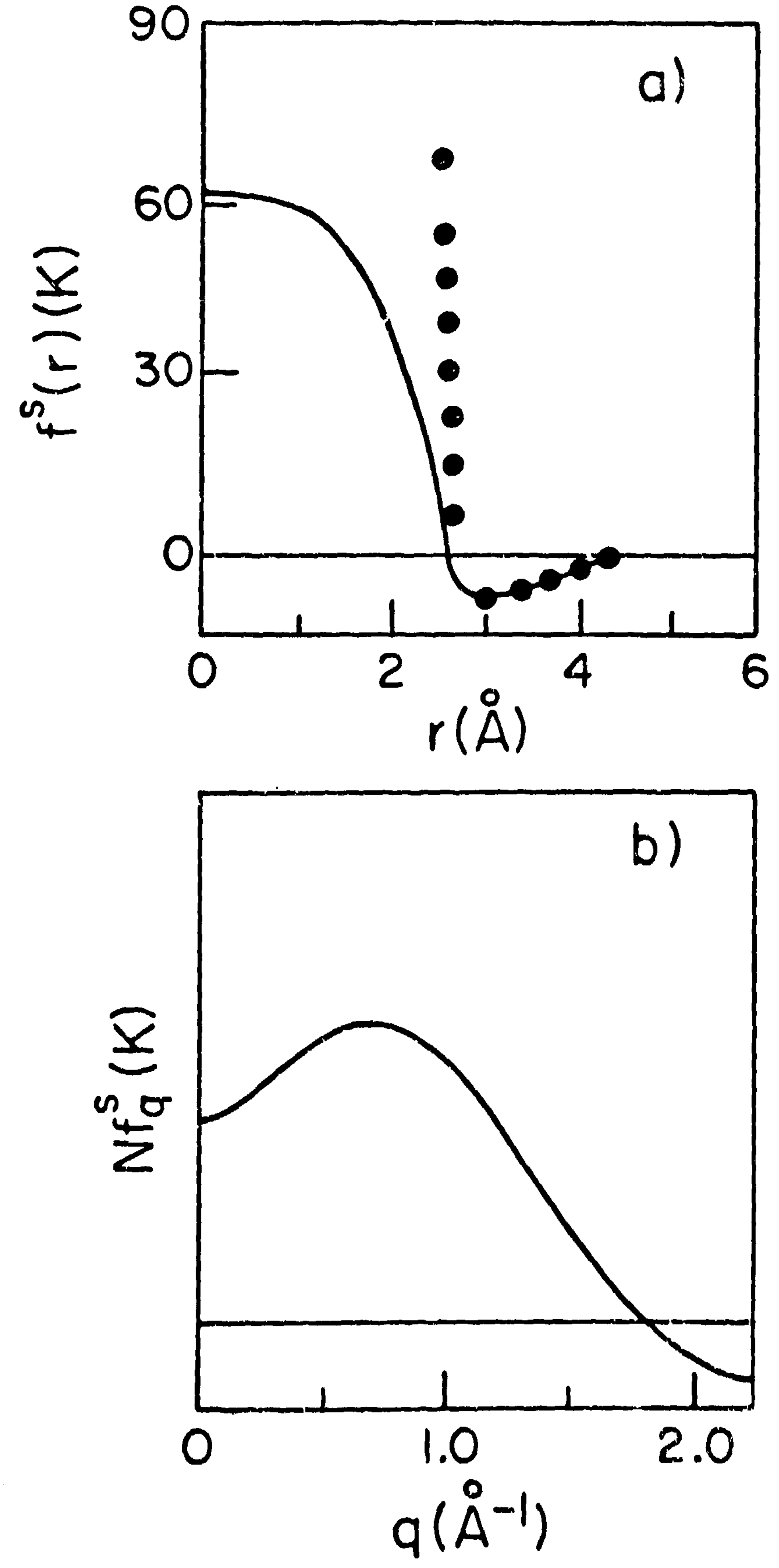

Figure 1

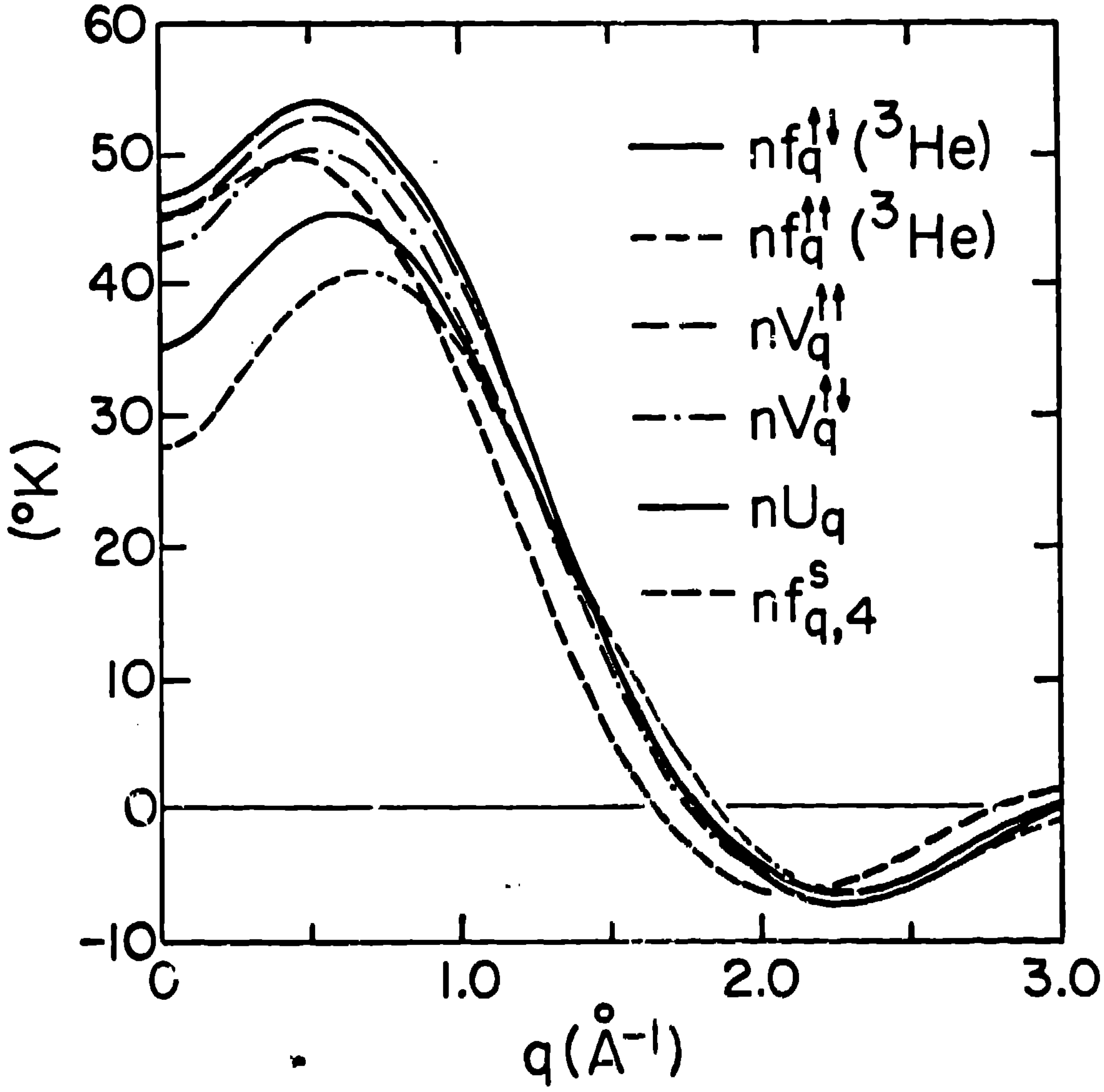
60
50
40
30
20
10
0
-10
(°K)
0
1.0
2.0
3.0
$q(\mathrm{\AA}^{-1})$
$nf_q^{\uparrow\downarrow}$ (^{3}He)
$nf_q^{\uparrow\uparrow}$ (^{3}He)
$nV_q^{\uparrow\uparrow}$
$nV_q^{\uparrow\downarrow}$
nU_q
$nf_{q,4}^s$

Figure 2

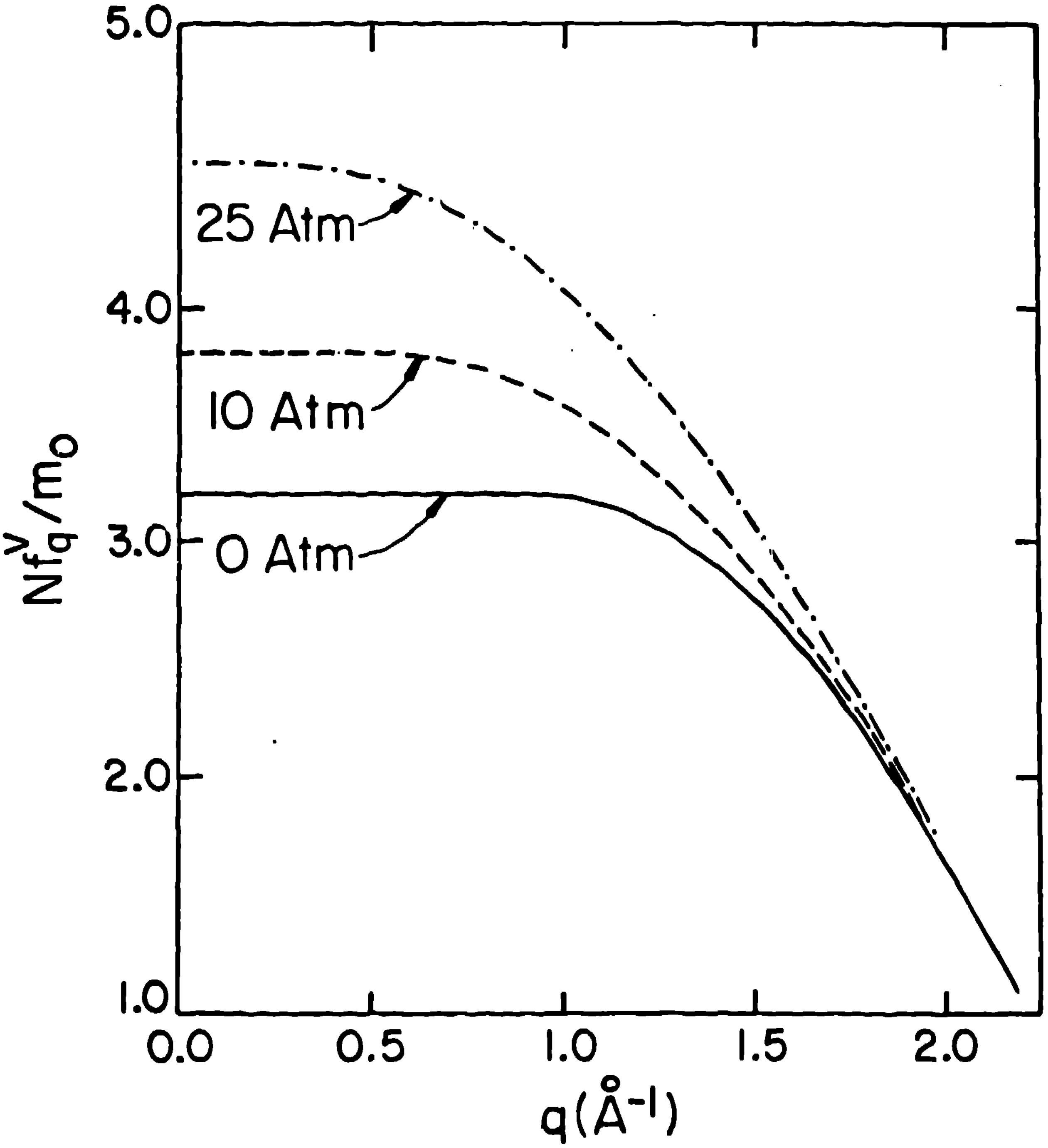

Figure 3

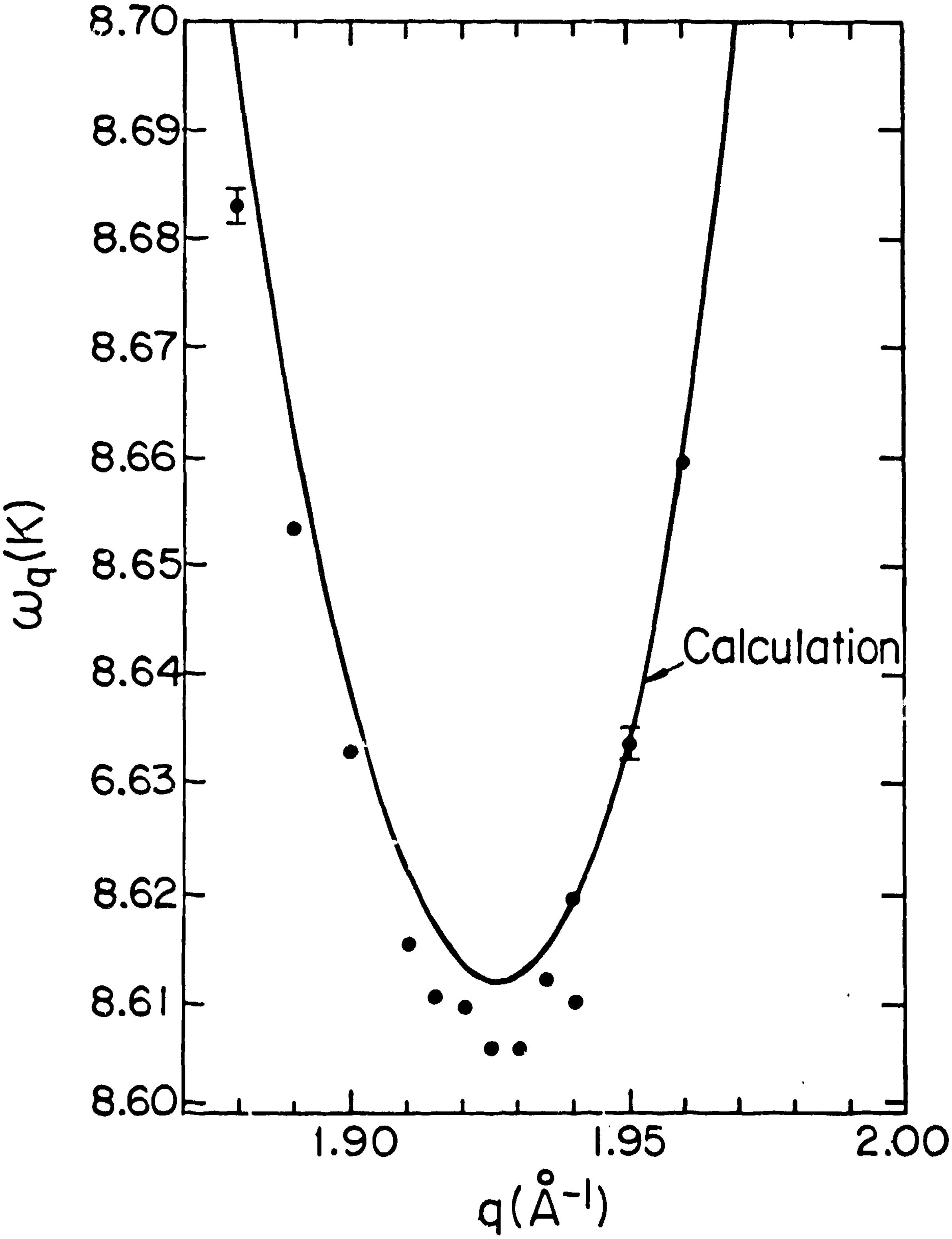
8.70
8.69
8.68
8.67
8.66
8.65
8.64
6.63
8.62
8.61
8.60
$\omega_q(K)$
Calculation
1.90
1.95
2.00
q(Å^{-1})

Figure 4

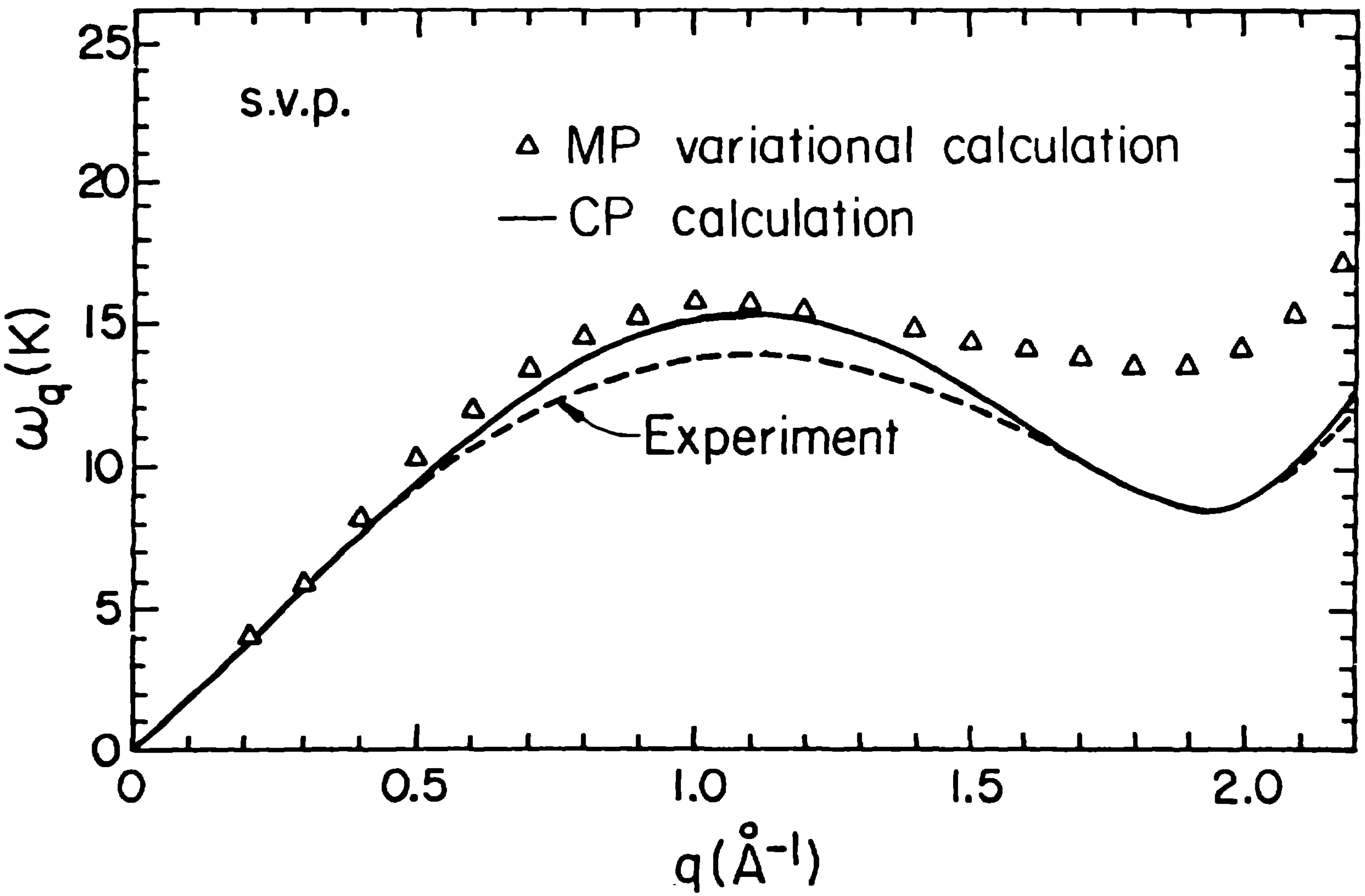
s.v.p.
△ MP variational calculation
— CP calculation
Experiment
ω_q(K)
0
5
10
15
20
25
0
0.5
1.0
1.5
2.0
q(Å^{-1})

Figure 5

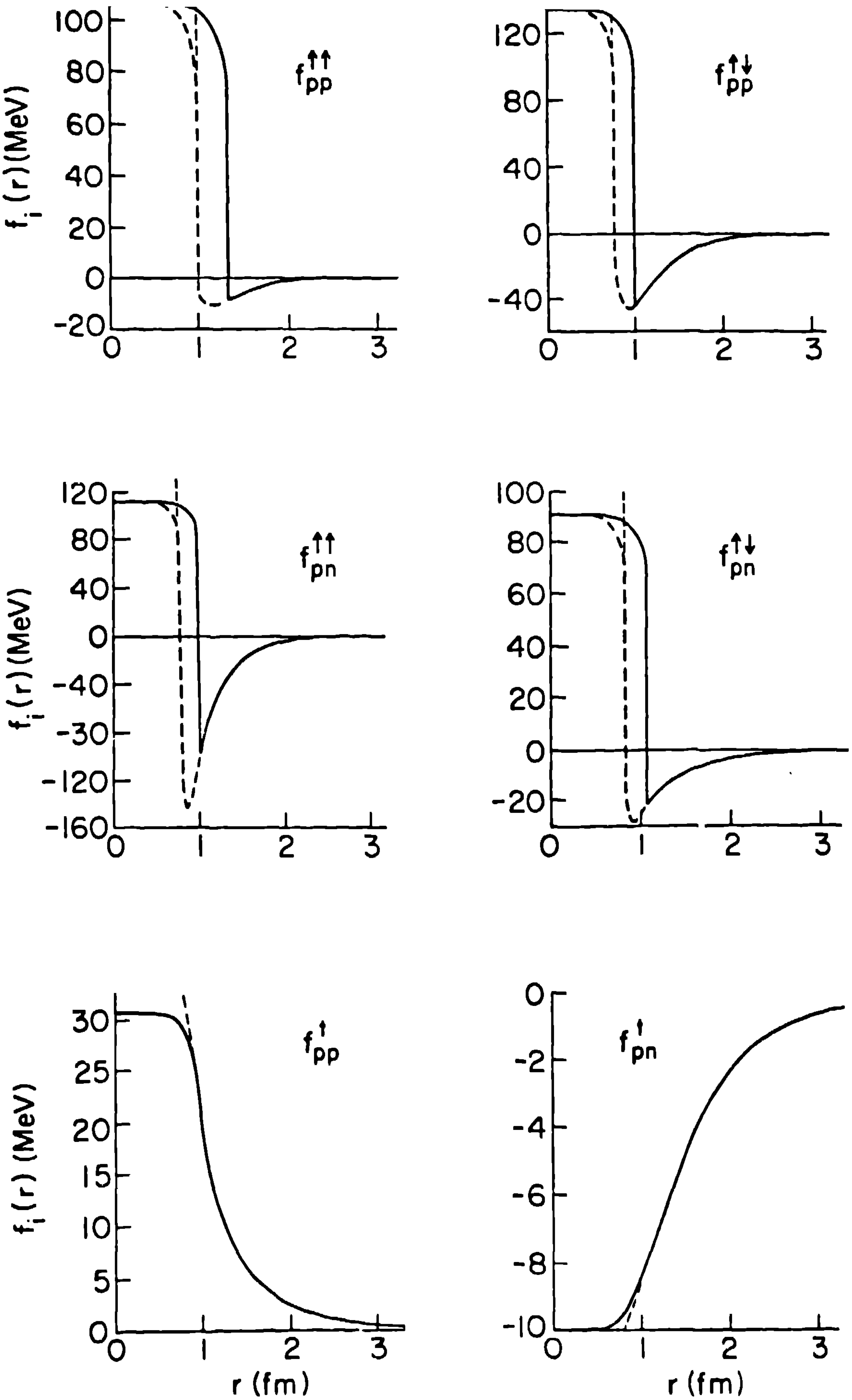
$f_{pp}^{\uparrow\uparrow}$
$f_{pp}^{\uparrow\downarrow}$
$f_{pn}^{\uparrow\uparrow}$
$f_{pn}^{\uparrow\downarrow}$
$f_{pp}^{\uparrow}$
$f_{pn}^{\uparrow}$
$f_i(r)$ (MeV)
$f_i(r)$ (MeV)
$f_i(r)$ (MeV)
100
80
60
40
20
0
-20
120
80
40
0
-40
120
80
40
0
-40
-30
-120
-160
100
80
60
40
20
0
-20
30
25
20
15
10
5
0
0
-2
-4
-6
-8
-10
0
1
2
3
r (fm)
r (fm)

Figure 6

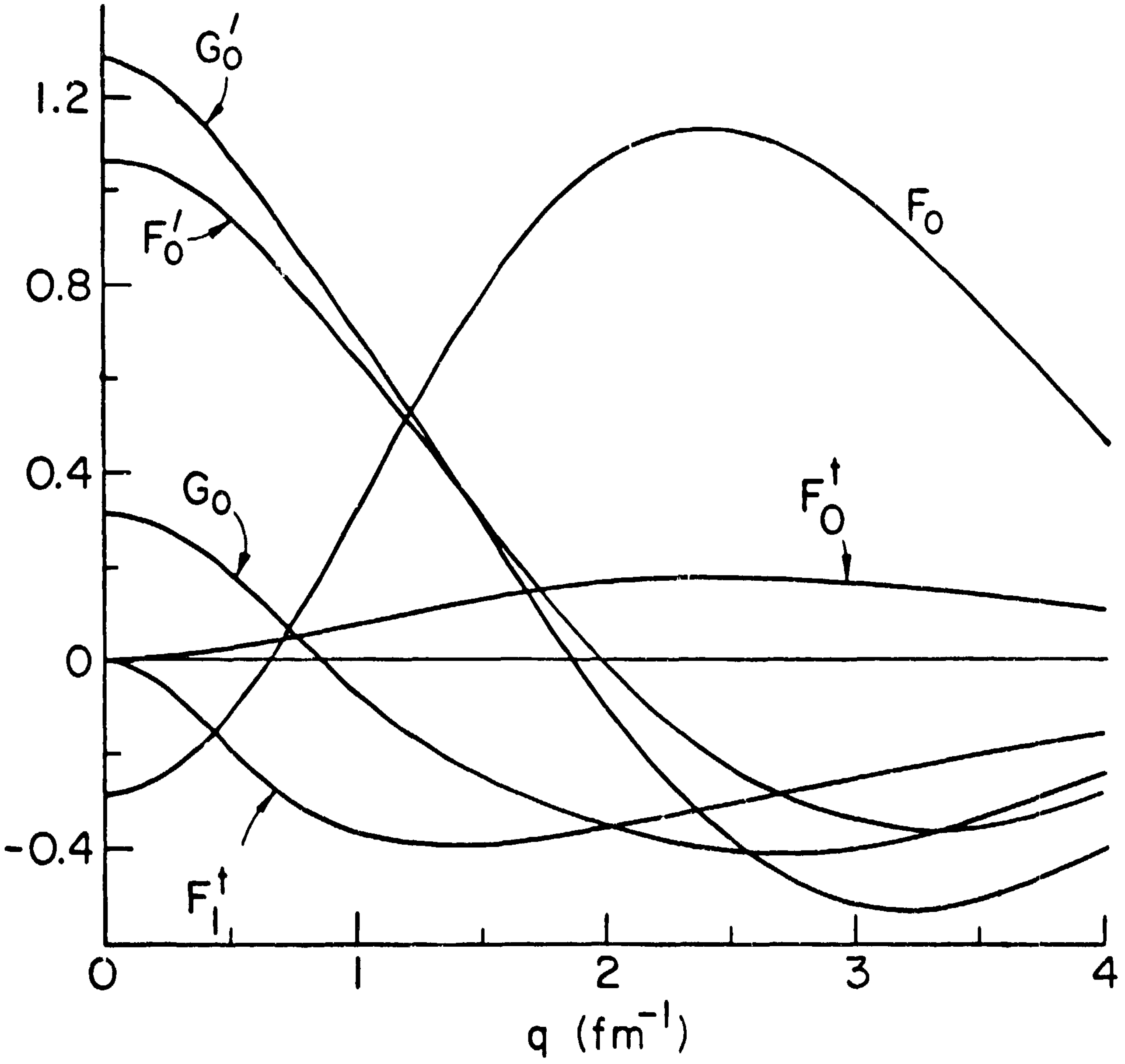

Figure 7

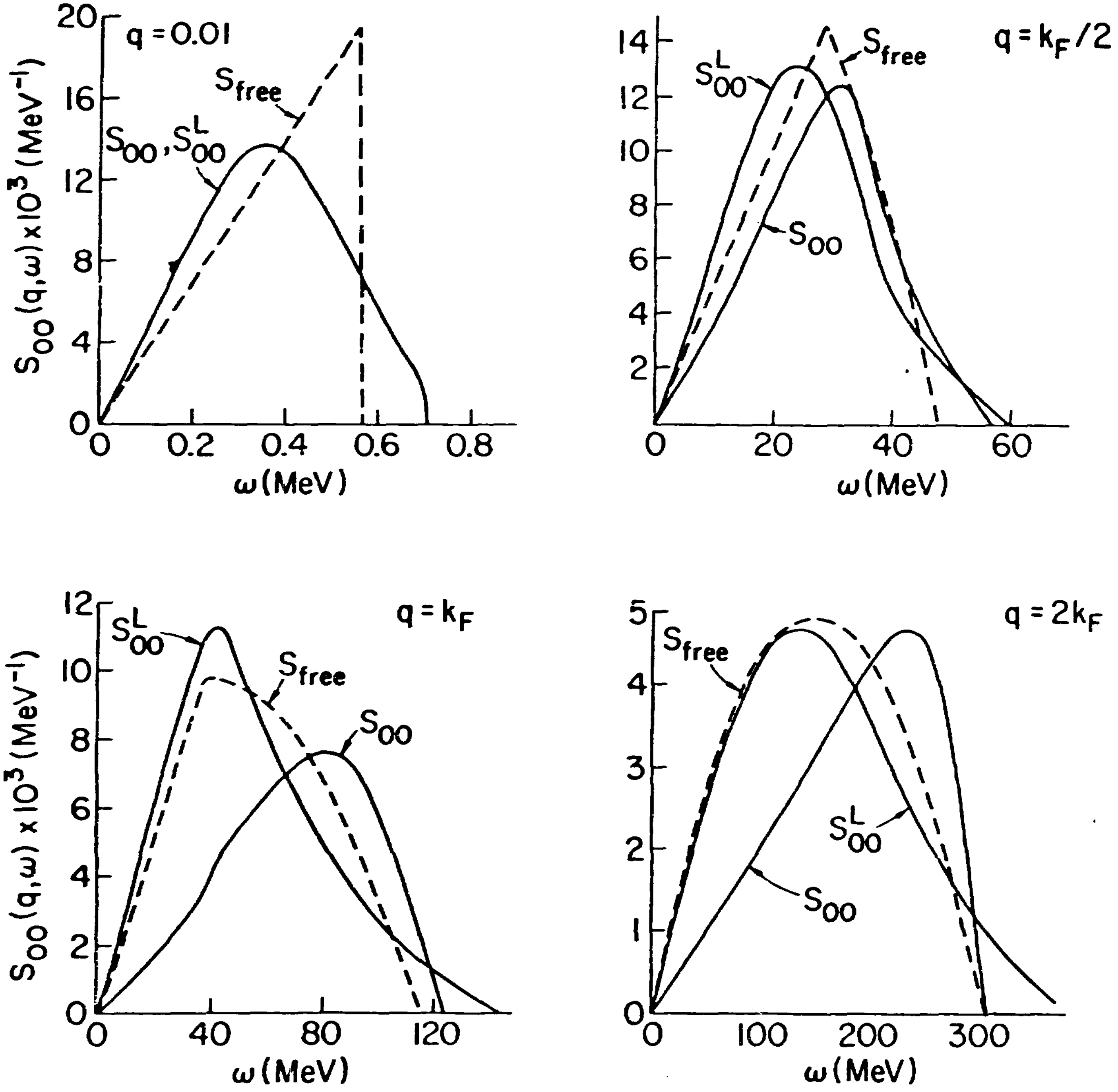
q = 0.01
S_{free}
S_{00}, S_{00}^L
$S_{00}(q,\omega) \times 10^3$ (MeV^{-1})
0
4
8
12
16
20
0
0.2
0.4
0.6
0.8
ω (MeV)
$q = k_F/2$
S_{00}^L
S_{free}
S_{00}
2
4
6
8
10
12
14
0
20
40
60
ω (MeV)
$q = k_F$
S_{00}^L
S_{free}
S_{00}
$S_{00}(q,\omega) \times 10^3$ (MeV^{-1})
0
2
4
6
8
10
12
0
40
80
120
ω (MeV)
$q = 2k_F$
S_{free}
S_{00}^L
S_{00}
0
1
2
3
4
5
0
100
200
300
ω (MeV)

Figure 8

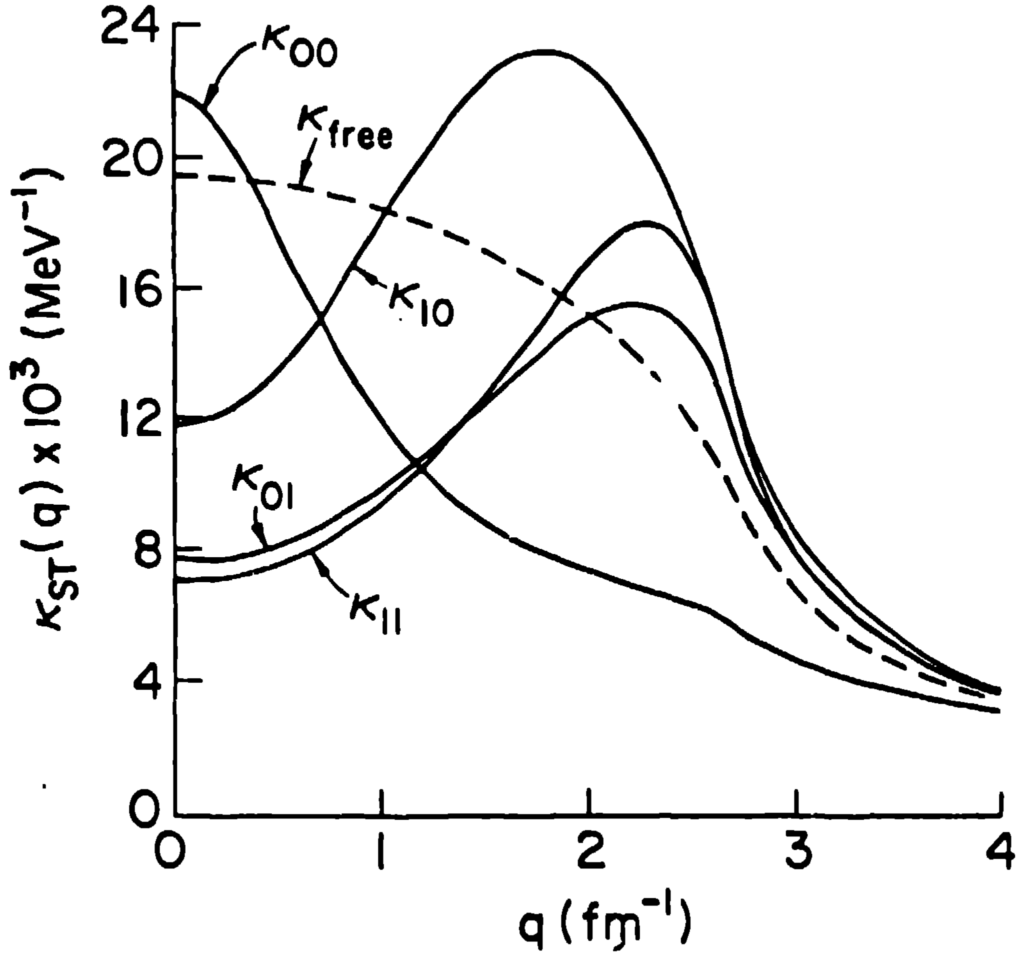
$\kappa_{ST}(q) \times 10^3$ (MeV^{-1})
24
20
16
12
8
4
0
κ_{00}
κ_{free}
κ_{10}
κ_{01}
κ_{11}
0
1
2
3
4
q (fm^{-1})

Figure 9