

2

Clouf - 820925--Vol. 1
NUREG/ CP-0034
Vol. 1

RECEIVED BY 16 AUG 26 1982

Proceedings of the Topical Meeting on Advances in Reactor Physics and Core Thermal Hydraulics

Held at Kiamesha Lake, NY
September 22-24, 1982

Sponsored by
Northeastern New York Section of the American Nuclear Society

Reactor Physics Division of the American Nuclear Society

Thermal Hydraulics Division of the American Nuclear Society

Proceedings printed by
U.S. Nuclear Regulatory
Commission

DO NOT MICROFILM
COVER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

MASTER

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

The views expressed in these proceedings are not necessarily those of the U. S. Nuclear Regulatory Commission.

The submitted manuscript has been authored by a contractor of the U.S. Government under contract. Accordingly the U.S. Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or allow others to do so, for U.S. Government purposes.

Available from

GPO Sales Program
Division of Technical Information and Document Control
U.S. Nuclear Regulatory Commission
Washington, DC 20555

Printed copy price: \$12.00

and

National Technical Information Service
Springfield, VA 22161

CONF-820925 -- Vol. 1

Proceedings of the Topical Meeting on Advances in Reactor Physics and Core Thermal Hydraulics

Held at Kiamesha Lake, NY
September 22-24, 1982

Manuscript Completed: July 1982
Date Published: August 1982

**Sponsored by
Northeastern New York Section of the American Nuclear Society**

Reactor Physics Division of the American Nuclear Society

Thermal Hydraulics Division of the American Nuclear Society

Proceedings printed by
U.S. Nuclear Regulatory Commission
Washington, D.C. 20555

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

Peg/wtp

ABSTRACT

Technical papers presented at the ANS Topical Meeting on Advances in Reactor Physics and Core Thermal Hydraulics, September 22-24, 1982, at Kiamesha Lake, N.Y. are included in these Proceedings. Reactor physics, core thermal hydraulics, and the interactions between core physics and thermal hydraulics are covered both for thermal reactors and for fast breeders. There are sessions on current challenges in these areas, on measurement and analysis of fast reactor physics parameters, on coupled core physics and thermal-hydraulics analysis, on in-core fuel management, on nodal and homogenization methods in reactor physics, on core thermal hydraulic and nuclear instrumentation, and on validation of fast reactor thermal hydraulic methods. In addition there are sessions on reactor theory, on measurement and analysis of thermal reactor physics parameters, on validation of thermal reactor thermal hydraulics methods, and on development and utilization of differential and integral nuclear data.

The conference brings together experts from many countries to present papers and exchange ideas on these closely related core physics and thermal hydraulics aspects of fission reactors.

PLANNING COMMITTEE MEMBERS

General Chairman: R.C. Block

Program Chairman: M.R. Mendelson

Program Committee:

P.B. Abramson

L.E. Hochreiter

R.C. Block

S.W. Kitchen

R.L. Crowther

M.J. Ohanian

H.B. Currin

S.N. Purohit

D.J. Diamond

R.R. Rippel

A.M. Erdman

P.J. Turinsky

F. Feiner

C.R. Weisbin

N.C. Francis

M. Podowski

O.C. Jones

R.T. Lahey, Jr.

D.R. Harris

Registration Chairman: T.F. Ruane

Publications Chairman: D.R. Harris

Public Information Chairman: E.R. Wagner

Finance Chairman: J. L. Bear

Hotel and Transportation Arrangements: R.C. Rohr

Best Paper Award: D.R. Harris and N. Todreas

Guest Activities: Rita Block

Student Liaison: B. Malaviya

TABLE OF CONTENTS

CURRENT CHALLENGES IN REACTOR PHYSICS AND CORE THERMAL HYDRAULICS SESSION 1

P.B.Hemmig(DOE) and R.T.Lahey,Jr. (RPI), Chairmen

1. PHYSICS ASSESSMENTS OF LMFBR INTEGRAL PARAMETERS
M.J.Lineberry, H.F.McFarlane and P.J.Collins
(ANL-Idaho)..... 1
2. SODIUM-COOLED FAST REACTOR THERMAL-HYDRAULICS
Neil E. Todreas (MIT)..... 20
3. CURRENT CHALLENGES IN LWR PHYSICS R.R.Lee (MAC),
O.Ozer (EPRI), W.R.Cobb (SAI)..... 36
4. ADVANCES IN LWR THERMAL HYDRAULICS Owen C. Jones,
Jr. (RPI)..... 55

MEASUREMENT AND ANALYSIS OF FAST REACTOR PHYSICS PARAMETERS SESSION 2

M.J.Lineberry (ANL-Idaho), Chairman

1. SODIUM-VOID REACTIVITY IN LMFBRs: A PHYSICS
ASSESSMENT C.L.Beck, M.J.Lineberry,
H.F.McFarlane and D.N.Olsen (ANL-Idaho)..... 78
2. CALCULATED AND MEASURED REACTIVITIES IN THE
U9 CRITICAL ASSEMBLIES R.W.Schaefer and
R.G.Bucher (ANL)..... 93
3. NEUTRONIC ANALYSIS OF SIMULATED LMFBR FUEL
SLUMP EXPERIMENTS G.Henneges and W.Maschek
(KfK)..... 108
4. EXTRAPOLATION OF SMALL SAMPLE DOPPLER
REACTIVITY MEASUREMENTS R.E.Kaiser,H.A.Harper,
C.L.Beck and S.G.Carpenter (ANL-Idaho)..... 118
5. AN ASSESSMENT OF PHYSICAL CHARACTERISTICS OF
HETEROGENEOUS CONFIGURATIONS WITH VARIABLE
FERTILE VOLUME W.Scholtyssek (KfK), G.Dagusan,
G.Humbert (CEN), M.Martini (ENA), B.Ruelle (CEN),
F.Kappler (KfK), G. Norvez (Edf)..... 133

TABLE OF CONTENTS CONTINUED

6. CONTROL-ROD PARAMETRICAL STUDIES IN THE FRAMEWORK OF THE PRE-RACINE AND RACINE PROGRAMS G.Humbert, B.Ruelle, G.Daguzan (CEN), A.Stanculescu (Interatom), F.Kappler, W.Scheltyssek (KfK), D.Bouscavet (EdF), M. Martini and U. Broccoli (ENEA).....	148
7. CONTROL-ROD CALCULATION METHODS AND UNCERTAIN- TIES FOR A POWER LMFBR DESIGN M. Salvatores, C. Giacometti, G.Palmiotti (CEN-Cadarache), V.Rado (ENEA), F.Lyon (CEN-Cadarache).....	162
8. VALIDATION OF ALTERNATIVE METHODS AND DATA FOR A BENCHMARK FAST REACTOR DEPLETION CALCULATION B.J.Toppel, C.H.Adams, R.D.Lawrence, H.Henryson II and K.L.Derstine (ANL).....	177
9. DETERMINATION OF THE CORE POWER DISTRIBUTION IN THE CLINCH RIVER BREEDER REACTOR J.A.Lake, S.K.Varner and R.V.Rittenberger (Westinghouse).....	194

COUPLED CORE PHYSICS-THERMAL HYDRAULIC ANALYSIS

SESSION 3

O.C.Jones (RPI), Chairman

1. THE INTERACTION OF NEUTRONICS AND THERMAL- HYDRAULICS IN THE EVALUATION OF BWR STABILITY J.K.Park, G.C.Park, M.Becker, M.Podowski and R.T.Lahey Jr. (RPI).....	209
2. THE IMPORTANCE OF MOMENTUM DYNAMICS IN BWR NEUTRONIC STABILITY P.J.Ostaduy (ORNL) and M.J.Ohanian (U of Florida).....	215
3. BWR CORE RESPONSE TO FLUCTUATIONS IN COOLANT FLOW AND PRESSURE, WITH IMPLICATIONS ON NOISE DIAGNOSIS AND STABILITY MONITORING J.H.Blomstrand and Sven-Ake Andersson (ASEA-ATOM).....	220
4. NEUTRONICS ANALYSIS OF BWR PARTIAL SCRAMS WITH BORON INJECTION B.Chexal, W.Layman, J.Power (EPRI/NSAC), R.Pickering (SCI), R.Grow and C.Simonson (NAI).....	236

TABLE OF CONTENTS CONTINUED

5. ANALYSIS OF NEUTRONIC SPACE-TIME EFFECTS IN
LARGE LMFBR's WITH THERMAL-HYDRAULIC FEED-
BACK USING FX2-TH NUCLEAR REACTOR KINETICS
CODE AND MODE EXPANSION OF THE NEUTRON FLUX
SHAPE FUNCTION B.S.Yarlagadda (ANL)..... 254

REACTOR PHYSICS OF IN-CORE FUEL MANAGEMENT-I

SESSION 4

E.E.Pilat (YAEC), Chairman

1. RECENT BWR FUEL MANAGEMENT REACTOR PHYSICS
ADVANCES R.L.Crowther, S.P.Congdon,
B.W.Crawford, C.M.Kang, C.L.Martin, A.P.Reese,
P.J.Savoia, S.R.Specker and R.Welchly (GE)..... 271
2. THE INTERACTION BETWEEN CORE ANALYSIS
METHODOLOGY AND NUCLEAR DESIGN: SOME PWR
EXAMPLES B.M.Rothleder (SAI) and W.J.Eich (EPRI).. 286
3. APPLICATION OF STATIC AND TIME-DEPENDENT
GENERALIZED PERTURBATION THEORY METHODS
IN LWR PHYSICS ANALYSES J.R.White and
M.L.Williams (ORNL)..... 297
4. DEVELOPMENT AND VALIDATION OF CORE PHYSICS
METHODS FOR IN-CORE FUEL MANAGEMENT OF PWR's
J.M.Aragones (Polytechnic U-Madrid), C.Ahnert
and J.Gomez-Santamaria (JEN-Madrid),
I.R.Olavarria (Iberduero S.A.-Bilbao)..... 315
5. URANIUM RECYCLE IN PWRs M.Darrouzet (CEN-
Cadarche), A.Simon (CEN-Saclay) and
M.Soldevila (EdF)..... 333

NODAL AND HOMOGENIZATION METHODS FOR REACTOR PHYSICS ANALYSIS

SESSION 5

M.Becker (RPI) Chairman

1. THE APPLICATION OF NODAL METHODS TO LIGHT
WATER REACTORS P.J.Finck, C.L.Hoxie,
H.S.Khalil, D.K.Parsons and A.F.Henry (MIT)..... 348
2. EXTENSIONS OF THE NODAL EXPANSION METHOD
BASED ON THE APPLICATION OF FINITE TECHNIQUES
M.A.Robinson and R.A.Rydin (U of Virginia)..... 364

TABLE OF CONTENTS CONTINUED

3. RESPONSE MATRIX NODAL METHODS FOR THERMAL AND FAST REACTOR APPLICATIONS	M.Becker (RPI).....	377
4. A NODAL METHOD FOR DIRECT INTRA-NODE DISTRIBUTION CALCULATION	Yung-An Chao and J.A.Penkrot (Carnegie-Mellon U).....	384
5. A LATTICE HOMOGENIZATION THEORY FOR COARSE MESH DIFFUSION ANALYSIS	Ren-Tai Chiang (GE).....	391
6. VALIDATION STUDIES OF THE ENDF/MC2-2/SDX, CELL HOMOGENIZATION PATH	R.D.McKnight, R.N.Hwang, C.G.Stenberg and R.N.Blomquist (ANL).....	406

CORE THERMAL HYDRAULIC AND NUCLEAR INSTRUMENTATION SESSION 6

R.S.Booth (ORNL), Chairman

1. A REVIEW OF PROPOSED INSTRUMENTATION FOR MEASUREMENT OF INADEQUATE CORE COOLING IN LWRs	P.G.Bailey and Mati Merilo (EPRI).....	424
2. LOSS-OF-FLUID TEST (LOFT) IN-CORE THERMAL-HYDRAULIC MEASUREMENT	D.B.Engelman, D.J.Hanson, H.E.Knauts and M.L.Russell (EG&G Idaho).....	440
3. MONITORING PWR REACTOR VESSEL LIQUID WITH SPNDs DURING LOCAs	J.P.Adams (EG&G Idaho).....	453
4. INVESTIGATION OF THE AXIAL VOID PROPAGATION VELOCITY PROFILE IN BWR FUEL BUNDLES	G.Kosaly, J.M.Fahley (U of Washington), K.Behringer (Swiss Federal Inst. for Reactor Research) and R.D.Crowe (U of Washington).....	468
5. USE OF CONDUCTANCE PROBE METHOD TO MEASURE TWO PHASE FLOW PATTERNS	Gao Huahun, Tu Chuanjing (Xian Jiaotong U-China).....	479
6. SEPARATE EFFECTS TESTS IN SUPPORT OF BWR THERMAL/HYDRAULIC MONITORING	R.W.Albrecht, R.D.Crowe, D.J.Dailey, H.J.Khan (U of Washington) K.Kotani (Hitachi) and S.C.Swoope (U of Washington).....	487

TABLE OF CONTENTS CONTINUED

7. ACCURACY OF POWER PEAKING MONITORING USING FIXED INCORE DETECTOR SYSTEMS C.T.Rombough, B.A.Guthrie, R.E.Clark and H.D.Warren (B&W).....	498
8. METHOD OF MONITORING SUBCRITICAL REACTIVITY DURING CORE REFUELING P.A.Ombrellaro and R.A.Harris (HEDL).....	506
9. ENERGY-DEPENDENT PLANT PROTECTION SYSTEM FOR THE TREAT UPGRADE REACTOR S.K.Bhattacharyya, W.C.Lipinski, R.R.Rudolph and D.C.Wade (ANL).....	518

VALIDATION OF FAST REACTOR THERMAL HYDRAULIC METHODS

SESSION 7

N.E.Todreas (MIT), Chairman

1. A VALIDATION STUDY OF THE COBRA-WC COMPUTER PROGRAM FOR LMFBR CORE THERMAL-HYDRAULIC ANALYSIS E.U.Khan and J.M.Bates (BNWL).....	533
2. ANALYSIS OF IN-CORE COOLANT TEMPERATURE DISTRIBUTIONS WITHIN FFTF INSTRUMENTED FUEL ASSEMBLIES C.W.Hoth (HEDL).....	559
3. THE EFFECTS OF RADIAL HEAT FLUX GRADIENTS AND FLOW REGIMES ON THE PEAK SODIUM TEMPERA- TURE RISE IN WIRE WRAPPED ROD BUNDLES F.C.Engel, R.A.Markley (Westinghouse) and A.A.Bishop (U of Pittsburgh).....	566
4. EFFECTS OF PIN BUNDLE DISTORTIONS ON LMFBR FUEL ASSEMBLY COOLANT TEMPERATURE DISTRI- BUTIONS K.H.Chen, S.Kaplan, T.L.Kurtz and J.D.Stephen (GE).....	583
5. VALIDATION OF SSC USING THE FFTF NATURAL CIRCULATION TESTS W.C.Horak, J.G.Guppy and R.J.Kennett (BNL).....	604
6. CRAB-II: A COMPUTER PROGRAM TO PREDICT HYDRAULICS AND SCRAM DYNAMICS OF LMFBR CONTROL ASSEMBLIES AND ITS VALIDATION M.D.Carelli, L.A.Baker, J.M.Willis, F.C.Engel and D.Y.Nee (Westinghouse).....	618

TABLE OF CONTENTS CONTINUED

REACTOR THEORY

SESSION 8

N.C.Francis (KAPL) Chairman

1. A SIMPLE FORMALISM FOR THE CALCULATION OF
THE EFFECT OF SODIUM ON NEUTRON LEAKAGES
IN A FAST REACTOR B. Benoist (CEN-Saclay)..... 644
2. DEVELOPMENT AND VALIDATION OF A METHOD TO
TREAT LARGE INTERNAL VOIDS IN REACTOR DESIGN
CALCUATIONS R.M.Lell, R.B.Pond and
S.K.Bhattacharyya (ANL)..... 657
3. A THEORETICAL METHOD OF ANALYSIS OF NONLINEAR
REACTOR DYNAMICS M. Podowski (RPI)..... 671
4. SYMMETRIES APPLIED TO TRANSPORT AND
DIFFUSION EQUATIONS M.Makai (KFKI-Hungary)..... 684
5. SIMULATION OF NEUTRON SPECTRA IN SLICES
USING SPACE AND ENERGY DEPENDENT BUCKLINGS
K.N.Grimm and D.MENEGETTI (ANL)..... 699
6. BACKGROUND CROSS-SECTION METHOD FOR EPITHERMAL
SPECTRUM CALCULATION IN THERMAL REACTORS
Yung-An Chao and S.Martinez (Carnegie-Mellon U)..... 714
7. IMPLEMENTATION AND APPLICATION OF RCP SYSTEM
AT BNL L.D.Eisenhart and M.Todosow (BNL)..... 722

COUPLED CORE PHYSICS - THERMAL HYDRAULICS METHODS

SESSION 9

A.F.Henry (MIT), Chairman

1. LIGHT WATER REACTOR COUPLED NEUTRONIC AND
THERMAL-HYDRAULIC CODES D.J.Diamond (BNL)..... 736
2. VALIDATION OF RECORD/PRESTO AGAINST GAMMA
SCAN AND EXPERIMENTAL VOID LOOP DATA
S.Borresen and H.K.Naess (Scandpower)..... 752

TABLE OF CONTENTS CONTINUED

3. TITAN: AN ADVANCED THREE DIMENSIONAL NEUTRONICS/ THERMAL-HYDRAULICS CODE FOR LIGHT WATER REACTOR SAFETY ANALYSIS D.P.Griggs, M.S.Kazimi and A.F.Henry (MIT).....	766
4. THE NEUTRON KINETICS AND THERMAL-HYDRAULIC TRANSIENT COMPUTATIONAL MODULE OF THE NEPTUNE SYSTEM: CHRONOS A.Kavenoky and J.J.Lautard (CEN-Saclay).....	781
5. SIMULATION OF SELF-LIMITING POWER TRANSIENTS IN ENRICHED NUCLEAR REACTOR CORES USING A THERMODYNAMIC NON-EQUILIBRIUM AXIAL FLOW MODEL FOR THE WATER COOLANT A.W.Dalton (AAEC).....	793
6. RASP: AN INTEGRATED REACTOR ANALYSIS SUPPORT PACKAGE G.S.Lellouche, L.Agee, W. Eich and J. Naser (EPRI).....	807

REACTOR PHYSICS ASPECTS OF IN-CORE FUEL MANAGEMENT-II

SESSION 10

D. R. Harris (RPI) Chairman

1. SOME REACTOR PHYSICS EFFECTS RELATED TO RECENT TRENDS IN IN-CORE FUEL MANAGEMENT A. Jonsson, M.W.Crump, P.H.Gavin, S.F.Grill, R.A.Loretz and W.B.Terney (CE).....	822
2. REACTOR PHYSICS AND IN-CORE FUEL MANAGEMENT FOR SWEDISH REACTORS M.Edenius (Studsvik of America).....	835
3. NEACRP LMFBR BENCHMARK CALCULATION INTERCOM- PARISON FOR FUEL BURN-UP PRELIMINARY ANALYSIS G.Palmiotti, M.Salvatores and B.Ruelle (CEN- Cadarache).....	865
4. IMPROVEMENT OF LINEAR REACTIVITY METHODS AND APPLICATION TO LONG RANGE FUEL MANAGEMENT R.A.Woehlke and B.L.Quan (YAEC).....	880
5. EFFECTS OF TRANSITION TO LUMPED BURNABLE POISON EIGHTEEN-MONTH RELOAD CYCLES ON THE AGREEMENT OF MEASURED AND PREDICTED CORE PHYSICS PARAMETERS J.J.Woods, A.R.Long, D.D.Shelburne and G.E.Hanson (B&W).....	889

TABLE OF CONTENTS CONTINUED

MEASUREMENT AND ANALYSIS OF THERMAL REACTOR
PHYSICS PARAMETERS

SESSION 11

B.A.Zolotar (TDC), Chairman

1. COMPARISONS OF CALCULATED AND MEASURED PARAMETERS FOR A PU-FUELED LWHCR LATTICE
R.Chawla, K.Gmur, H.Hager, E.Hettergott,
J.M.Paratte, R.Seiler and D. Stahel
(Swiss Federal Inst. for Reactor Research)..... 902
2. MONTE CARLO ANALYSIS OF BWR BYPASS HEATING,
GAMMA TIP RESPONSE, AND GAMMA REDISTRIBUTION
FACTORS M.J.Hebert (YAEC), D.R.Harris (RPI),
D.M.Kapitz, and E.E.Pilat (YAEC)..... 914
3. STATISTICAL METHODS USED IN QUANTIFYING THE
ACCURACY OF EPRI-NODE-P AT OCONEE L.H.Flores
(Duke Power Company)..... 924
4. EXPERIMENTAL QUALIFICATION OF THE CALCULATED
SUBCRITICALITY IN HIGH DENSITY FUEL STORAGE
A.Santamarina (CEN-Cadarache)..... 937

VALIDATION OF THERMAL REACTOR THERMAL HYDRAULIC
METHODS

SESSION 12

L.E.Hochreiter (Westinghouse), Chairman

1. ASSESSMENT OF THE COBRA-TF SUBCHANNEL
CODE J.M.Kelly, R.J.Kohrt and M.J.Thurgood
(BNWL)..... 944
2. INDEPENDENT ASSESSMENT OF THE TRAC-PD2 CODE
F.Odar (NRC)..... 960
3. REFLOOD CODE VERIFICATION FOR SAFETY ANALYSIS
J.Chiou, L.E.Hochreiter, S.R.Rodd and
M.Y.Young (Westinghouse)..... 982
4. PROPAGATION OF REWETTING FRONT DURING BOTTOM
REFLOOD--AN ANALYSIS K.D.Kimball, (EDS
Nuclear) and R.P.Roy (Arizona State U)..... 999

TABLE OF CONTENTS CONTINUED

5. TRAC ANALYSIS OF THE EFFECT OF INCREASED ECC SUBCOOLING ON THE REFLOOD TRANSIENT IN THE SLAB CORE TEST FACILITY T. Smith (LANL).....	1018
6. POSTTEST TRAC-PD2/MOD1 PREDICTIONS FOR FLECHT SEASET TEST 31504 C.P.Booker (LANL).....	1033
7. VERIFICATION OF THE THREE-DIMENSIONAL ANALYSIS CAPABILITY OF THE TRAC ACCIDENT ANALYSIS CODE F. Motley (LANL).....	1044
8. SIMULATION OF SMALL-BREAK LOCA TESTS WITH NOTRUMP N.Lee (Westinghouse).....	1058

DEVELOPMENT AND UTILIZATION OF DIFFERENTIAL AND INTEGRAL NUCLEAR DATA

SESSION 13

F. Feiner (KAPL), Chairman

1. CROSS SECTION NEEDS: DETERMINATION AND STATUS C.R.Weisbin, D.Gilai, G.deSaussure and R.T.Santoro (ORNL).....	1069
2. VERIFICATION OF NUCLEAR CROSS SECTION PROCESSING CODES D.E.Cullen (IAEA) and W.L.Zijp (ECN), R.E.MacFarlane (LANL).....	1078
3. ²³⁸ U SELF-INDICATION RATIO MEASUREMENT IN THE RESONANCE REGION J.T.Yang, J.L.Munoz-Cobos, G.deSaussure, R.B.Perez and J.H.Todd (ORNL).....	1084
4. FISSION CROSS SECTION MEASUREMENTS OF ²⁴⁴ Cm, ²⁴⁶ Cm and ²⁴⁸ Cm+ C.R.S.Stopa, H.T.Maguire Jr., D.R.Harris, R.C.Block (RPI), R.E.Slovacek (KAPL) J.W.T.Dabbs (ORNL), R.J.Dougan, R.W.Hoff and R.W. Lougheed (LLNL).....	1090
5. FAST INTEGRAL CROSS SECTIONS FOR ²⁴¹ Am and ²⁴³ Am R.A.Anderl, N.C.Schroeder and Y.D.Harker (EG&G Idaho).....	1105

TABLE OF CONTENTS CONTINUED

6. SENSITIVITIES AND UNCERTAINTIES OF A LARGE
LMFBR USING ADJUSTED CROSS-SECTION LIBRARY
ORACLE-1 J.H.Marable, D.G.Cacuci, C.R.Weisbin,
and J.R.White (ORNL), D.Biswas, J.M.Kallfelz
(Georgia Inst. of Tech), C.L.Cowan and
R.Protsik (GE)..... 1120
7. FAST REACTOR DATA TESTING OF ENDF/B-V AT
ORNL R.Q.Wright, W.E.Ford III, J.L.Lucius
and C.C.Webster (ORNL)..... 1135

FOREWORD

The 1982 topical meeting on Advances in Reactor Physics and Core Thermal Hydraulics is the first topical meeting to be jointly sponsored by the Reactor Physics and Thermal Hydraulics divisions. A major purpose of this joint venture, aside from airing the technical issues within each area of expertise, is to stimulate and encourage better communication between these two fundamental but intimately related disciplines. Too often, the working Reactor Physicist or Thermal Hydraulicist is concerned only with the immediate problems within his own field, and does not (or is not able to) step back and view his work in the context of the requirements and limitations of the other discipline. Thus, in a joint meeting of this type, it is hoped that new insights will be gained, and better focused development programs will result.

When the program committee first considered the idea of a Reactor Physics/Thermal Hydraulics Cosponsorship, an immediate concern was that the number of papers would be so overwhelming, and that they would cover such a broad range of topics, that it might not be possible to provide a meaningful focus for the meeting. Using the 1980 Saratoga topical meeting on thermal hydraulics as a guide, it became apparent that the developmental aspects of both disciplines could not possibly be covered in one meeting. For this reason the thermal hydraulics sessions were limited to validation of methods, whereas the reactor physics sessions included both development and validation of methods. Even with this limitation, a large number of papers were accepted for this meeting (about 85), forcing the program committee to arrange triple concurrent sessions.

The papers are split about 60% on reactor physics and 40% on thermal hydraulics. A significant percentage of the papers (about 22%) originated outside of the United States. Among the approximately 45 papers dealing strictly with reactor physics, there is strong representation in four fields of specialization: measurement and analysis of physics parameters (13 papers), reactor theory and methods development (13 papers) reactor physics aspects of fuel management (11 papers), and the development, use, and testing of nuclear data (8 papers). About one-third of these papers deal with fast reactors. There are about 16 papers dealing with validation of thermal hydraulic methods; these are split nearly equally between fast reactor methods (7 papers), and thermal reactor methods (9 papers). Two sessions (11 papers) are devoted to the interaction of neutronic and thermal hydraulic effects in reactor analysis codes and models. Finally, a session on new developments in core instrumentation features about 6 papers dealing with thermal hydraulic phenomena, and two papers dealing with monitoring of power distributions and subcriticality.

The papers span such a broad spectrum of activities that it does not appear to be possible to characterize trends. Suffice it to say that progress appears to be ongoing in all of the areas mentioned, despite what are sometimes severe handicaps due to cutbacks in funding and dwindling supply of fresh young talent. Although today's scene is still one of vigorous technical development, it is with some trepidation that we anticipate the results of a similar assessment in ten years time.

The program for this meeting was planned by the members of the Technical Program Committee (listed elsewhere), with input from a number of sources - members of the New York Northeastern Section of the ANS, members of the Reactor Physics and Thermal Hydraulics Division of the ANS, Colleagues of the General and Technical Program Chairman at RPI and KAPL, paper reviewers, and session chairman. The latter carried the lead responsibility of arranging for invited speakers. We are indebted to all of these participants for their cheerful cooperation and expert guidance.

MR Mendelson
Program Chairman

ACKNOWLEDGEMENTS

The efforts of the speakers and their colleagues are gratefully acknowledged. Also acknowledged are the many persons who provided expert reviews for the papers contributed to the meeting. We are grateful to the U.S. Nuclear Regulatory Commission for publishing these Proceedings.

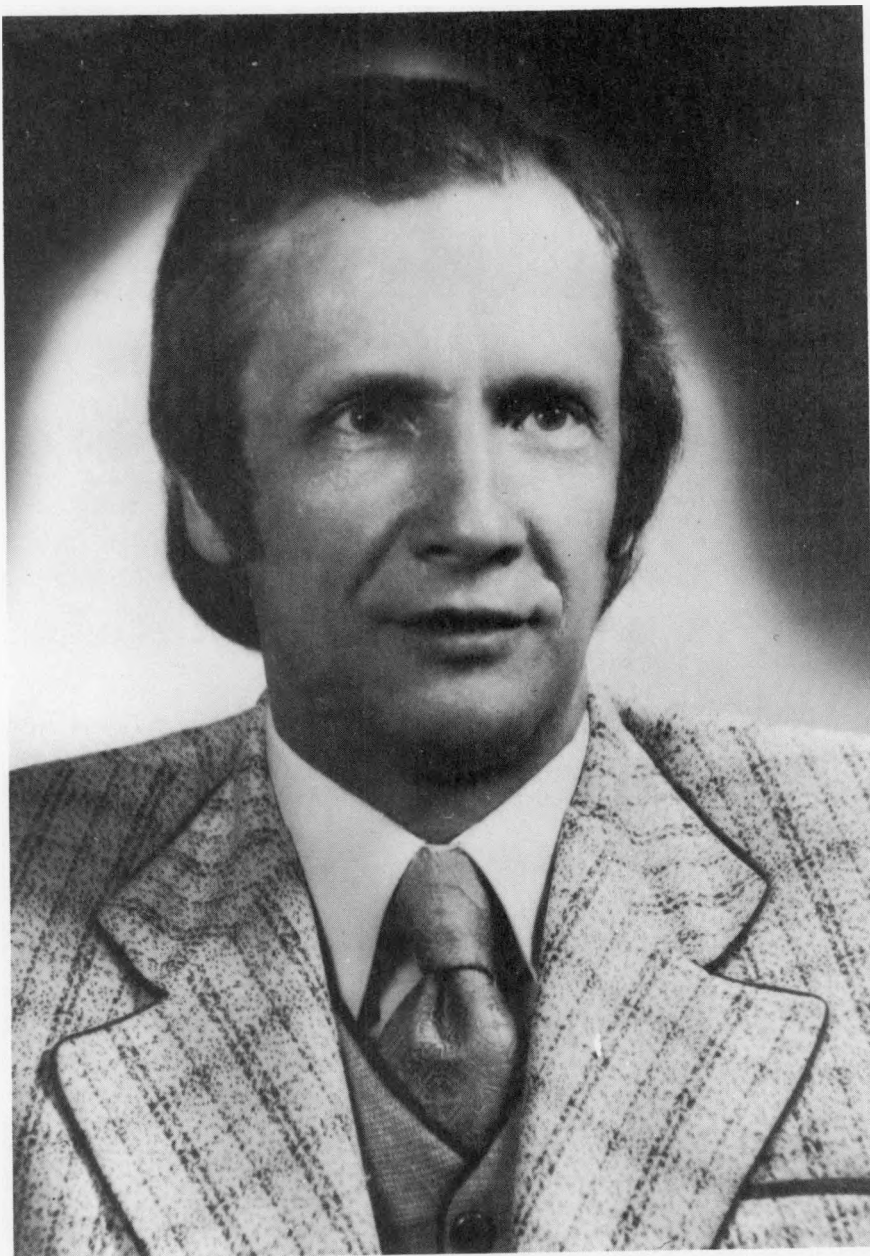
DEDICATION

It is the traditional task of the General Chairman to review the technical content of the program and to thank all of the people who have made the meeting successful. I have read Dick Mendelson's review and I can not express myself any better, so I wholeheartedly support and agree with all of Dick's comments and will not repeat them here. I would like, along with Dick, to thank all of the people who have worked long and hard in preparing for this meeting and, in addition, I also thank all who came to Kiamesha Lake to participate in the program and discussions.

I will take the Chairman's privilege to dedicate this meeting to one man who could not attend and participate in the discussions but whose presence is felt by many of us. We dedicate this ANS Topical Meeting on Advances in Reactor Physics and Core Thermal Hydraulics to Dr. Robert A. Dannels, a very special person who passed away last year. Bob Dannels, a Fellow of the American Nuclear Society, had served our Society in many capacities. He was Chairman and a member of the Executive Committee of both the Reactor Physics Division and of the Mathematics and Computations Division. In addition, Bob chaired the ANS committee which produced the ANSI standard for nuclear data. I first met Bob when he served both as a member of the Cross Section Evaluation Working Group and later as Acting Director of the National Nuclear Data Center at Brookhaven National Laboratory; I learned to appreciate and respect his technical competence and his ability to inspire colleagues to work hard towards achieving the goals he set forth. From his colleagues at Westinghouse I heard only glowing descriptions about Bob's abilities both technically and administratively; he was one of the best liked managers there.

Far and above Bob's technical credits were his marvelous personality, his keen wit and the fact that he was a most feeling and warm human being. We missed you at this meeting, Bob, but your presence was felt throughout.

Robert C. Block
General Chairman



Robert A. Dannels

PHYSICS ASSESSMENTS OF LMFBR INTEGRAL PARAMETERS

by

M.J. Lineberry, H.F. McFarlane, and P.J. Collins
Argonne National Laboratory
Applied Physics Division
P.O. Box 2528
Idaho Falls, ID 83401

ABSTRACT

Recent progress in assessing the status of measurement and analysis of key fast reactor integral parameters is reviewed. The relationship between the parameters (^{238}U capture, fission rates, sodium-void reactivity, Doppler reactivity coefficient, small sample reactivity, and criticality) and typical design and safety parameters in LMFBRs is explained. Major discrepancies between calculations with ENDF/B-Version IV data and the critical experiments are examined, and recent progress in resolution of some long-standing discrepancies is presented.

INTRODUCTION

During the past two years intensive effort has been put into assessing fast critical experiments and analyses that provide key tests of LMFBR designs. Since the broad status of fast reactor physics was surveyed by LeSage within the past two years^{1,2}, it seems appropriate here to narrow the focus to important new results and issues that have emerged since then.

When the ANL Physics Assessments Program¹ was first introduced, control rod worth was the only LMFBR physics parameter for which the evaluation was essentially complete. Except for confirmation of the worth of CRBR control systems in ZPPR-11, there has been little new work in that area beyond that summarized by LeSage and by McFarlane et al.³ Since then the major activities have been assessments of sodium-void reactivity, criticality, ^{238}U capture, and more recently, small sample worths. A start has been made on assessment of the Doppler coefficient and power distributions, and on detailed documentation and error evaluation of key critical experiment measurement techniques.

Defining the relationship between analysis of critical experiments and LMFBR design and safety evaluations is an important part of the assessment of each physics parameter. As much pertinent information as can be obtained, including such foreign experience as can be gleaned from the literature, is put into each assessment. Where necessary, improved methodologies are developed and suggested for using critical experiment and other data to reduce uncertainties in LMFBR design and safety parameters.

The following sections of this paper survey recent evaluations of six of the important LMFBR physics parameters: ^{238}U capture, power distributions, criticality, small sample reactivity, sodium void reactivity, and

^{238}U Doppler effect. The latter four topics are discussed in more detail in other papers presented at this meeting.^{4,5,6,7}

^{238}U CAPTURE RATE

One of the more persistent discrepancies in U.S. fast reactor physics has been the overprediction, by up to 10%, of the ^{238}U capture rate (^{28}c) relative to the ^{239}Pu fission rate (^{49}f) in typical LMFBF spectra. This discrepancy impacts such fundamental design parameters as the reactivity and power distribution changes during a burnup cycle. In Table I results of calculations with ENDF/B-Version IV cross sections are compared to integral measurements in several critical assemblies. The C/E values are centered around 1.07-1.08.

^{238}U capture is a difficult issue. Roughly half the captures in LMFBF spectra are in the resolved resonance range and that portion of the unresolved range where self-shielding is important. Therefore measurement, interpretation, and multigroup processing of ^{238}U capture cross section data over the key energy range is complex. Measurement of the integral capture rates in critical assemblies, and conversion of the data into a quantity to be compared with calculated capture rates, can be difficult, particularly in the plate-type critical assemblies that dominate the data base. In spite of these problems, significant advances have been made in the last two years in at least four areas:

- (i) integral measurement methodology,
- (ii) basic cross section data and their interpretation,
- (iii) processing multigroup data and reactor calculations, and
- (iv) reconciling the ^{238}U capture discrepancy with other observed discrepancies.

TABLE I. Comparison of Calculated (C) and Experimental (E) Ratios of $^{28}\text{c}/^{49}\text{f}$ in LMFBF Cores

Core ^a	Type	Mean C/E	Number of Results	Standard ^b Deviation
ZPR-6/7	Oxide benchmark	1.072	1	---
ZPR-9/31	Carbide	1.066	1	---
ZPPR-4/1 IC	350 MWe Homogeneous	1.048		
OC	Mockup	1.080		
ZPPR-9 IC	700 MWe Homogeneous	1.086	96	0.012
OC	Benchmark	1.098	32	0.016
ZPPR-10A IC	700 MWe Homogeneous	1.071	50	0.011
OC	Mockup	1.082	17	0.012
ZPPR-11B Fuel Zones	350 MWe Heterogeneous	1.084	31	0.014
ZPPR-11C Fuel Zones	350 MWe Heterogeneous	1.076	31	0.010

^aIC = inner core, OC = outer core.

^bStandard deviation of the C/E distribution indicates the spread in C/E results. Absolute uncertainties are 1-2% due to experimental calibrations.

(i) integral measurements

At ZPPR, integral ^{238}U capture rates are determined indirectly by gamma-counting the decay of ^{239}Np from an irradiated foil. Counting statistics are typically on the order of 1% or better, leaving only systematic errors as a major issue. During the last two years, some knowledge of the potential magnitude of systematic errors has been gained by intercomparison of results from ANL's critical experiment program with results from three independent sources.

First, a comparative measurement was conducted by the critical experiment group at ZPR-6/9, and the cross section group at the Fast Neutron Generator (FNG) facility.⁸ Foils were activated in a thermal column and capture rate determined separately on Ge(Li) detection systems. The final result was that the measurement techniques agreed to better than 1%. Thus, for the limited conditions of this experiment, no significant bias in measurement technique was discovered, and it appears that no major portion of the capture rate discrepancy is due to different measurement techniques applied to integral experiment and cross section measurements.

Secondly, a comparative measurement was conducted between critical experiment groups at ZPPR and the ZEBRA facility in the U.K. Prior to that, the ZEBRA-8 null zone experiments had been analyzed in the U.S. with ENDF/B-IV, and the C/E for $^{28}\text{C}/^{49}\text{f}$ was about 1.03 for the several assemblies⁹, or about 4-5% lower than the discrepancy observed in calculation of U.S. experiments. Preliminary results of the experimental comparison indicated that the U.S. value for the measured ^{28}C rate was only $1.2 \pm 0.9\%$ lower than the U.K. value. The difference in C/E values of 4-5% is therefore apparently not explained by differences in measurement technique.

Thirdly, the ZPPR Ge(Li) detection system has been compared with a direct mass spectrometer measurement of ^{238}U capture.¹⁰ Results indicated that if the Ge(Li) system were to be calibrated by the mass spectrometer result, integral measurements would be raised (and C/E lowered) by $2.0 \pm 0.5\%$.

The ZPPR gamma-counting system (since 1972) and the ZPR-6/9 system (since 1975) have been calibrated by a ^{243}Am source whose intensity is determined by absolute alpha counting. In addition to the mass spectrometer calibration discussed, other measurements that provide potential calibrations of the gamma counting system involve counting a standard ^{203}Hg source, a known activation from a thermal column irradiation, or activation determined by radiochemistry. Using any of these alternative calibrations, the Ge(Li) - ^{243}Am measurements would be raised by the following amounts:

^{203}Hg	$+(1.9 \pm 1.3)\%$
Thermal column	$+(1.8 \pm 1.1)\%$
Radiochemistry	$+(0.1 \pm 0.7)\%$

The conclusion on integral measurement techniques is the following: substantial evidence has been accumulated that suggests the $^{28}\text{C}/^{49}\text{f}$ ratio measured in critical experiments by the $\text{Ge(Li)} - ^{243}\text{Am}$ technique should be raised, but by no more than 2%. If the mass spectrometer result were accepted as the calibration, the resultant C/E discrepancy outlined in Table I would be reduced to the 1.05-1.06 range.

(ii) cross section data and interpretation

Over the last two years the ENDF/B-V era has begun, and the preliminary evidence is that the calculated $^{28}\text{C}/^{49}\text{f}$ ratios will be reduced compared to those of ENDF/B-IV. For example, it appears that in ZPR-6/7 the C/E is reduced by almost 2%, although in other assemblies and in blankets the improvements appear to be less pronounced. While the trend to lowered $^{238}\text{U}(n,\gamma)$ evaluated cross sections improves the capture rate discrepancy, it is worthwhile to note that most basic cross section data in the fast neutron range are already higher than the evaluated data, and there is little technical rationale for carrying this trend further.¹¹

A recent development is an extension of the resolved resonance range upward in energy, from 4 to 6 keV.¹² It was found that the strength functions of the resonances in this range were significantly lower than those determined from treating this range as unresolved in ENDF/B. While this effect would apparently increase the $^{28}\text{C}/^{49}\text{f}$ discrepancy (provided the total capture cross section is unchanged), the trend toward extending the resolved resonance range is clearly welcome. If it were possible to resolve resonances up to perhaps 20 keV, then the remaining unresolved resonance range would have no significant self-shielding effects.

With respect to interpretation of basic capture data, de Saussure and Perez point out in a recent review¹³ that, although significant progress has been made, the ENDF/B formalism for treating the unresolved range is theoretically unjustified and logically inconsistent. Their conclusion was that although the models will reproduce the evaluated infinitely-dilute cross section, there is no reason to expect correct values of the self-shielding factors. These observations could impact the overall ^{238}U capture rate in an LMFBR spectrum, but the magnitude is as yet undetermined.

(iii) processing multigroup data, reactor calculation

Calculations of ^{238}U capture in power reactors or critical assemblies generally use multigroup cross sections. Elaborate processing of the basic capture cross section data is necessary to derive multigroup cross sections which preserve the total reaction rate in the heterogeneous unit cells. This is generally more of an issue in critical experiment analysis where resonance self-shielding and spatial heterogeneities are more pronounced.

During the late 1970's Wade performed a comprehensive series of benchmark calculations¹⁴ in which the multigroup processing method in use at Argonne was compared against results from VIM Monte Carlo calculations, the latter utilizing a representation of cross sections that is continuous in energy. The result of these comparisons is that, to about the 1% level

of uncertainty and perhaps better, the multigroup calculation of ^{238}U capture cross section in heterogeneous critical assembly unit cells, is confirmed by the more rigorous Monte Carlo calculation. The most recent comparison, by McKnight et al.⁴ as part of their eigenvalue studies (although the ^{238}U capture rates are not shown in the paper referenced), indicates that for both the ZPR-6/7 unit cell and for the whole ZPR-6/7 assembly, VIM and multigroup finite-difference results agree to better than 0.7%.

A remaining issue in critical experiment analysis is transforming either the measurement or the calculation into equivalent quantities. In the standard experiment, ^{238}U capture rates are measured by activation of a foil located at a point in the heterogeneous cell. The quantity most easily calculated is the cell-averaged ^{238}U capture rate. In the ZPR-6/7 unit cell, the correction to translate from the activation of a foil adjacent to a standard ZPPR Pu-U-Mo fuel plate to a value equivalent to the cell-average capture rate, is on the order of -10%. Measurements of the in-cell ^{238}U capture distribution are done routinely to derive this correction, but they inevitably suffer a drawback: the fuel plate itself contains ^{238}U , and it is not possible to routinely measure a fuel-plate-average value. Although Monte Carlo calculations done in the 1970's have confirmed the measured cell-average factor to 1% or better, the presence of a -10% correction naturally raises questions when the overall discrepancy is roughly half that large.

(iv) reconciliation of the $^{28}\text{C}/^{49}\text{f}$ discrepancy

The $^{28}\text{C}/^{49}\text{f}$ discrepancy is qualitatively consistent with the underprediction of k_{eff} observed in analysis of critical assemblies. Determination of quantitative consistency is the subject of sensitivity analysis.

At least until recently, the discrepancies were not entirely consistent. In Table II, results of data adjustment studies are summarized.¹⁵ In these studies, the U.S. $^{28}\text{C}/^{49}\text{f}$ bias was based on the $\text{Ge}(\text{Li}) - ^{243}\text{Am}$ measurements, and the uncertainty in the U.S. k_{eff} biases was taken to be about 0.003. The important point is that while the $^{28}\text{C}/^{49}\text{f}$ discrepancy in U.S. (ZPR) experiments was reduced from 7% to 2%, it was not eliminated. The suggestion is that not all of the capture discrepancy could be attributed to errors in the cross section data.

Recently, additional adjustment calculations have been done which factor in the latest information. The question posed was: what happens if the $^{28}\text{C}/^{49}\text{f}$ measurements were put on a mass spectrometer basis (lowering the ZPR C/E values by 2%) and the U.S. k_{eff} C/E values reflect the bias found by McKnight et al.⁴ in the Monte Carlo studies of ZPR-6/7? The results of this adjustment are shown in Table III. Although at this stage the results must be viewed as preliminary, and the alterations to the C/E's for $^{28}\text{C}/^{49}\text{f}$ and k_{eff} cannot yet be recommended, the results are nonetheless of some interest for they indicate that the "revised" discrepancies in $^{28}\text{C}/^{49}\text{f}$ and k_{eff} are essentially consistent.

TABLE II. Results of Data Adjustment Studies

		Number of Results	ENDF/B-IV		Adjusted	
			<u><C/E></u>	<u>σ^a</u>	<u><C/E></u>	<u>σ^a</u>
k_{eff}	ZEBRA	7	0.979	0.010	0.997	0.007
	ZPR ^b	4	0.987	0.003	1.000	0.001
$^{28}C/^{49}f$	ZEBRA	6	1.030	0.010	0.988	0.004
	ZPR	2	1.069	---	1.021	---
$^{28}C/^{25}f$	A11	2	1.033	---	0.994	---
$^{28}f/^{49}f$	A11	8	0.975	0.019	0.975	0.017
$^{28}f/^{25}f$	A11	2	1.004	---	1.001	---
$^{25}f/^{49}f$	A11	8	1.000	0.023	0.999	0.024
Reduced χ^2	A11		10.6		1.02	

^aStandard deviation of distribution. Input standard deviations are 0.2% to 0.7% for k_{eff} and 2% to 3% for reaction rate ratios.

^bZero Power Reactor; either from ZPPR, ZPR-6, or ZPR-9.

TABLE III. Data Adjustment with Latest Information

		Number of Results	ENDF/B-IV		Adjusted	
			$\langle C/E \rangle$	σ^a	$\langle C/E \rangle$	σ^a
k_{eff}	ZEBRA	7	0.979	0.010	0.998	0.007
	ZPR	4	0.981	0.003	0.999	0.001
$^{28}C/^{49}f$	ZEBRA	6	1.030	0.010	0.992	0.004
	ZPR	2	1.048	---	1.007	---
$^{28}C/^{25}f$	A11	2	1.033	---	1.000	---
$^{28}f/^{49}f$	A11	8	0.975	0.019	0.975	0.017

^aStandard deviation of distribution. Input standard deviations are 0.2% to 0.7% for k_{eff} and 2% to 3% for reaction rate ratios.

It is known that ENDF/B-V will reduce both the $^{28}\text{c}/^{49}\text{f}$ and the k_{eff} discrepancies in the LMFBR systems. A detailed study of the consistency of integral parameter discrepancies with this data set or its impact on other parameters, such as power distributions, has not yet been addressed in detail.

Finally, there is the issue of how the $^{28}\text{c}/^{49}\text{f}$ discrepancy should impact design parameters. Best estimates of desired parameters can be developed by bias factors where appropriate, or by sensitivity analysis/data adjustment. Burnup reactivity and breeding ratio estimates are an example where the latter approach is particularly useful. Recent calculations indicate a best-estimated breeding ratio for typical LMFBRs reduced ~4% (with no adjustment of enrichment) from that calculated with ENDF/B-IV. A somewhat larger reduction (5.5%) was found by the ORNL group in studies of a 1200 MWe homogeneous reactor.¹⁶

POWER DISTRIBUTION

Optimization of the power distribution in LMFBR design plays an essential role in plant economics and safety. Maintaining a low peak-to-average power distribution during the burnup cycle allows a higher average burnup per subassembly, while allowing an optimum power rating for the reactor. Power limits are usually set by temperature limits on materials, so that for economics and operational safety it is preferable for designs to avoid having regions of high relative power. To assure that limits are not exceeded, final designs must consider not only nominal predictions of the power distribution, but also their accuracy. Manufacturing tolerances, engineering uncertainties and measurement uncertainties must be among the considerations. Since the limits apply even to individual pellets, target accuracies for predictions of nominal peak-to-average power are not more than a few percent¹⁷ in order that they not add substantially to the required design margin.

To establish accuracies for power distributions in LMFBRs, calculational methods are tested against low-power measurements in critical assemblies. Direct thermal measurements of power distributions are not feasible in critical facilities such as the ZPPR, nor would they be particularly desirable from the standpoint of testing calculations. Rather, the major components of the power distribution are measured and analyzed.

For an LMFBR, most of the power results from fission of the plutonium fuel, with the remainder coming from fast fission in depleted uranium and heating from gamma rays that originate in fission, inelastic scattering, and capture processes. In the CRBR, for example, 84% of the power in the fuel results from plutonium fission (^{49}f), 10% from gamma-ray heating and the remaining 6% from ^{238}U fission (^{28}f). Fresh blankets have a 60/40 split between ^{28}f and gamma-ray heating, while end-of-cycle blankets produce 50% of their power from fission of plutonium that builds in over the burnup cycle.

Table IV shows the isotopic contribution to fission in each of the reactor zones of the CRBR at the beginning-of-life. The table helps to illustrate what the essential measurements are in establishing the power

distribution. Measurement of ^{49}f establishes 80% of the total reactor power. The fission rate distributions of the other fissile isotopes are not much different from that for ^{239}Pu on a per atom basis. The remaining requirements are then to measure the ^{28}f distribution and the gamma heating distribution.

TABLE IV. Approximate CRBR* Fission Rate Contribution by Isotope and Zone

Isotope	Isotopic Fission by Zone, % of Reactor Total				Reactor Sum
	Core	Internal Blanket	Radial Blanket	Axial Blanket	
^{239}Pu	79.5	--	--	--	79.5
^{240}Pu	2.6	--	--	--	2.6
^{241}Pu	1.7	--	--	--	1.7
^{235}U	0.4	0.8	0.6	0.2	2.0
^{238}U	5.7	5.3	2.5	0.5	14.1
Zone Sum	90.0	6.2	3.1	0.7	100.0

*ANL calculation of an approximate BOL reactor model.

Power distribution measurements in ZPPR include fission rates in ^{239}Pu and ^{235}U foils as well as fission and capture rates in ^{238}U foils. Gamma-ray heating distributions are made with CaF_2 and LiF TLDs. Special measurements are used to convert the foil reaction rates to cell-average values that can be readily compared with reactor calculations. For ^{49}f and ^{25}f , these adjustments are on the order of 1%; they are typically 5% for ^{28}f and 10% for ^{28}c . Absolute fission rates are determined for each isotope. The calculation is normalized to the measurements such that the sum of the calculated ^{49}f rates at the measured points in the core is set equal to the sum of the measured ^{49}f rates. Experimental uncertainties on relative fission rates in the core are typically 3/4%, 1% and 2% for ^{235}U , ^{239}Pu , and ^{238}U , respectively. Uncertainties are slightly larger in most regions of the blanket, and only become very large in regions where the fission rate is insignificant. Absolute fission rates for the three isotopes have been verified to the 1% level through international and national laboratory intercomparisons.¹⁸ In general, the measurement techniques have been refined over many years to a level such that they do not contribute significantly to the uncertainty in predictions of LMFBR power distributions.

The principal recent developments in calculations of power distribution are the increasing use of three-dimensional diffusion theory modeling and the completion of several whole-core continuous-energy Monte Carlo benchmark calculations. Three-dimensional modeling is now the standard technique for most critical experiment analysis, and LMFBR design studies are making more use of three-dimensional capabilities that include burnup codes¹⁹ and recently developed high-efficiency hex-z nodal diffusion methods.²⁰

In a companion paper⁴, some results of full-core VIM Monte Carlo calculations of benchmark critical assemblies are presented. A VIM calculation of ZPPR-11B, an engineering mockup of the CRBR, has also been done. Comparisons against the VIM results indicate that there may be small but potentially significant deficiencies in the normal deterministic methods for calculating k_{eff} and power distributions.

VIM comparisons for cell calculations have helped to verify MC²-II/SDX¹⁴ processing of cell cross sections. However, recent analyses of complex critical assemblies have tried to account for the effects of different adjacent cell types, either through multicell modeling^{21,22} or by modeling variations in leakages to different adjacent cell types in single cell calculations. In both cases different cross section sets for the same unit cell type are applied according to location in the model.

Comparisons of calculated power distributions against critical experiments can be classified according to core type: (1) homogeneous cores of CRBR-size and smaller, (2) larger homogeneous cores, and (3) heterogeneous cores. All results given here are based on calculations with ENDF/B Version IV data.

Three types of reaction rate discrepancies which affect calculated power distributions are common to all three core classifications: (1) an underprediction of $^{28}\text{f}/^{49}\text{f}$ on the order of 5% in the core, (2) a dip in reaction rate C/E's of 2% for ^{49}f and ^{25}f and of 5% for ^{28}f near the axial core/blanket interface, and (3) a gradual deterioration of all reaction rate C/E's with penetration into the blankets. The axial discrepancy has too small an effect on the peak-to-average ratio to have motivated improved modeling of the interface effects. The discrepancy for ^{28}f deep in the blankets is marked, but is accompanied by experimental uncertainties of tens of percent. This discrepancy is not significant because little power is generated by ^{28}f in the far reaches of the blanket.

The larger homogeneous cores, in the 700-1000 MWe size range, exhibit a overprediction of ^{49}f and other reactions in the outer core relative to the core center. The magnitude of the error is dependent on the core size, the relative enrichment split, and the eigenvalue spectra, and is as large as 5%.

Heterogeneous cores represent the most complex case, with several discrepancies extant. For BOL mockups, radial C/E discrepancies of a few percent to 10% have been observed, depending on the arrangement of core and blanket. As with other assemblies, ^{28}f is underpredicted relative to ^{49}f in the core, but C/E discrepancies of up to 25% are observed between adjacent core and blankets with typical diffusion theory methods. Much of this large discrepancy is removed by transport calculations, and accounting for internal core/blanket interfaces has been found to be necessary in cell cross section processing.

The impact of these comparisons on detailed design calculations varies according to core type. For the smaller homogeneous cores, it may be adequate to accept the standard calculations as providing a good representation of the power distribution, although minor corrections may be made for axial effects and for the $^{28}\text{f}/^{49}\text{f}$ discrepancy. From Table IV note that a

5% problem in the latter is less than a 1% problem in the total power. For the larger homogeneous cores, the radial misprediction of the power distribution is more serious, because it can mean a several percent error in local power density. Here, as in the heterogeneous cores where the problem is more pronounced, it may be desirable to produce radially dependent bias factors for detailed design calculations. It should be noted that the extent of these radial mispredictions is correlated with the power distribution sensitivity to ^{28}c , and its misprediction as discussed in the previous section of this report. For example, when the multigroup cross sections are adjusted to reduce to ^{28}c and k_{eff} discrepancies, about 50% of the radial reaction rate discrepancy is eliminated.

The 25% overprediction of ^{28}f in the internal blankets of heterogeneous cores may seem to be more serious than it actually is with respect to design implications. This relative misprediction is only true for fresh blankets, which are overcooled initially. Power in the blankets only approaches design margins near the end-of-life for blanket subassemblies. In critical assemblies, such as ZPPR-7C and -11C that simulate EOC conditions, the discrepancies between core and blanket ^{28}f are reduced to 7-10%. The uncertainty in blanket power resulting from the ^{28}f contribution is then on the order of that due to uncertainty in the ^{49}f contribution. The latter arises because of the ^{28}c discrepancy, i.e. the Pu content of the blanket is uncertain because it builds in through ^{28}c .

The remaining fission contribution to the reactor power comes from ^{235}U and the plutonium isotopes other than ^{239}Pu . Principally because of convenience, extensive measurements of ^{25}f are made in critical assemblies to supplement the ^{49}f measurements, and ^{25}f has a spatial distribution calculated with the same accuracy as ^{49}f in the core zones. Typically, ^{25}f is overpredicted by 3% relative to ^{49}f , but the impact on predicted power distribution is insignificant. Measurements of fission in ^{240}Pu and ^{241}Pu have been made with fission chambers in critical assemblies, both relative to other fission rates and as radial and axial traverses. Such measurements are probably useful to better than 10%, adequate for most LMFBR compositions. For LWR Pu composition, about 10% of the core fissions occur in ^{241}Pu . A 10% uncertainty in ^{41}f implies a 1% uncertainty in power, but since the ^{41}f distribution follows that for ^{49}f , the effective uncertainty contribution is much smaller.

Gamma heating measurements in ZPPR critical assemblies have relative accuracies of better than 5%. They are useful in verifying shapes of the gamma power component. Recent attempts at absolute normalizations to the core ^{49}f rate have been unsatisfactory, and it has been necessary to rely on more basic data for the gamma normalization. Ties between calculational methods for gamma production and transport and critical experiment measurements are presently somewhat tenuous. Fortunately, gamma heating is only 10% of the total power, and the spatial distribution of gamma-ray energy deposition reasonably tracks the ^{238}U fission distribution in the reactor. The sources of most gamma rays are in the fuel, and mean-free-paths are short in that high-Z material. Therefore, gamma-ray heating does not contribute excessively to power distribution uncertainties.

CRITICALITY

The critical configuration is the most fundamental property of a reactor. Even in fast-reactors with a substantial inherent spontaneous fission source (^{240}Pu), the critical state can be determined to a precision of about 10^{-7} . The physical characterization of the reactor, e.g. composition, position of materials, and temperature distributions, is much less precise. Once pin and subassembly designs are fixed, criticality enters the final design process through specification of the fuel enrichment. The design requirement typically is that the reactor be capable of maintaining full power for a specified length of time, without excessive reactivity hold-down requirements on the control system. In other words, the fuel enrichment at the end of the specified burnup cycle should be just adequate for the reactor to be critical at full power.²³

In the U.S., critical experiments have been used with great success in helping to predict the critical configurations of fast reactors. Initial critical configurations of the Fermi, SEFOR, EBR-II, and FFTF reactors were all predicted to within estimated uncertainties.^{24,25,26,27} Foreign experience with LMFBR criticality has also been satisfactory, including reactors in the 250-350 MWe size range.^{28,29,30}

For critical assembly mockups of CRBR-size and larger mixed-oxide LMFBRs, k_{eff} is typically underpredicted by about 1.5% when ENDF/B Version IV data are used in a standard MC²-II/SDX/ARC-System calculational procedure. Besides the detailed one-dimensional cell modeling used to obtain multigroup cross sections, such procedures generally use three-dimensional reactor modeling in diffusion calculations. Neutron streaming in the plates is modeled by modifying the diffusion coefficients according to the prescriptions of Benoist³¹ or Gelbard.³² For most assemblies, a transport correction is also calculated. For such small assemblies as Big-10³³, U95, and ZPPR-12, the same calculational procedures produce slight overestimates of the measured k_{eff} .

The significant recent development in the assessment of fast reactor criticality has been the completion of VIM continuous-energy Monte Carlo calculations for four critical assemblies. These calculations provide independent benchmark tests for the normal deterministic procedures for calculating eigenvalues, reaction rates, and leakages. Elsewhere in these proceedings, McKnight et al.⁴ report on the detailed comparisons of the results for three vastly different assemblies: a large assembly of enriched uranium and iron, a small mixed-oxide assembly, and a large dilute mixed-oxide (LMFBR inner core composition) assembly. A fourth VIM calculation was done for ZPPR-11B, an engineering mockup critical configuration for the CRBR. To date, comparisons between the ZPPR-11B VIM results and results from the standard calculational procedure have not been as extensive as those reported by McKnight. The ZPPR-11B configuration, a heterogeneous core with 15 control rod positions, is much more complex than the three benchmark assemblies. Consequently, corrections to the standard calculations are more difficult to generate, and are more approximate at this stage.

The major conclusion drawn from the McKnight study is that the deterministic methods overpredict k_{eff} relative to VIM by about 0.5% as shown in Table V. The study also places an uncertainty of 0.3 - 0.5% on the k_{eff} derived by the deterministic path, whereas the VIM statistics were about 0.2% for k_{eff} . Furthermore, as was found in earlier comparisons of cell calculations, both calculation paths yield comparable values for the major cross sections. The ZPPR-11B results appear to be consistent with these conclusions.

There are two potential impacts from these results. First, sensitivity analyses or data adjustment studies can depend crucially on the magnitude of both the k_{eff} bias and its uncertainty. Most of the benchmark data now used in such studies rely on deterministic calculations of k_{eff} . Secondly, enrichment specifications for power reactors could be impacted, but first a similar deterministic-Monte Carlo comparison would have to be made.

Recent critical assembly measurements have examined physics parameters as a function of fissile and fertile content of the core. The variations were 93% enriched uranium, 9% enriched uranium, plutonium, and plutonium plus depleted uranium in four separate benchmark assemblies. It is expected that critical eigenvalue analysis of these cores will help isolate effects due to major isotopes. In addition, neutron streaming and cell heterogeneity, which affect k_{eff} differently in critical assemblies than in power reactors, were studied in detail in ZPPR-12. Those measurements are not yet analyzed, but they are of particular interest because the ZPPR-12 core composition closely matches the BOL fuel composition of the CRBR.

Finally, the change from ENDF/B Version IV to Version V has a significant effect.³⁴ For U/Pu LMFBR-type critical assemblies, the calculated eigenvalue increases by about 1%.

TABLE V. k_{eff} Predictions with
Deterministic and Monte Carlo Methods

Assembly	$(k_D - k_{VIM})^a$
ZPR-6/7	$+0.0062 \pm 0.0021^b$
ZPR-9/32	$+0.0072 \pm 0.0016$
ZPR-9/34	$+0.0047 \pm 0.0026$
ZPPR-11B	$+0.0043 \pm 0.0009$

^a k_D determined with multigroup calculations, corrected for higher-order effects (e.g. transport, streaming, etc.) of known significance.

^bUncertainty is from the Monte Carlo calculation only.

SMALL SAMPLE REACTIVITY

The worths of small samples of basic reactor materials are among the fundamental parameters of reactor physics. Other than criticality, no parameter is easier to measure. The measurement consists of the addition of a small mass of material such as ^{239}Pu to the core, and a direct, high-precision determination of the change in system reactivity that results. The standard calculational method is equally simple--application of first order perturbation (FOP) theory to a situation intended specifically to fit it. (In a typical experiment a 30 g Pu sample is added to a core of 1500 kg Pu.) However, this simple procedure gives rise to one of the most disconcerting problems in fast reactor physics--the central worth discrepancy.

Simply stated, the central worth discrepancy is the consistent over-prediction of the worth of materials added to the reactor core: 10-20% for fissile samples, 5-15% for ^{10}B , and a much wider range for materials with a strong scattering component. More properly, this discrepancy should be called a material worth discrepancy, as the overprediction trend is observed throughout the core. The discrepancy has little direct adverse effect on LMFBR design, in that calculations of such worths are used primarily to make minor adjustments to more basic design parameters. However, its existence does raise some troublesome questions about calculational methods that may be applied to more fundamental quantities, for example FOP methods that are used in safety calculations for sodium-void and Doppler reactivity.

The central worth discrepancy is not universal in fast reactors. For example, it is not observed for small, hard-spectrum cores such as Jezebel, Godiva, and Big-10 at Los Alamos³³; nor is it observed in many foreign LMFBR programs. However, in all LMFBR-type critical assemblies built in the U.S., the problem has persisted regardless of core size, enrichment, type of fissile material, changes in ENDF/B libraries, evolution of analysis codes and computing capabilities, and refinement of measurement techniques. Recently however, progress has at last been made in understanding and reducing the discrepancy.

During the past year, Smith investigated^{35,36} a fundamental flaw in the application of FOP calculations to assemblies composed of arrays of heterogeneous unit cells, as are all LMFBR-type critical assemblies. In the preparation and collapse of cross sections for the homogenized cell, eigenvalue and reaction rates of the heterogeneous cell are preserved. Later these cross sections are used to calculate an adjoint flux for the reactor, which is used in the FOP analysis of the small sample worths. However, the adjoint flux thus calculated does not preserve the properties of the heterogeneous intra-cell adjoint flux. Specifically, the cross sections that preserve eigenvalue and reaction rates cannot simultaneously preserve the properties of the spatial and energy distributions of the heterogeneous adjoint flux. Since the adjoint flux does not have a flat spatial distribution, the worth of a sample within the cell depends on its actual location, i.e. whether it spans the cell, is adjacent to a fuel plate, or near the cell boundary. Secondly, the adjoint energy distribution is mispredicted because inappropriate cross sections are used in its calculation, causing a

further error in FOP calculated worth. Significant corrections to classical FOP worths of up to -10% have been demonstrated relative to FOP worths calculated in the usual way. Special experiments, either modifying the location of the sample in the cell or modifying the cell heterogeneity, have verified the trends predicted by this development.

Smith's discoveries have been incorporated into work discussed in another paper at this meeting, in which Schaefer and Bucher report⁵ some initial results from a systematic approach to attacking the central worth and other long-standing discrepancies. That program involves starting with a core in which discrepancies are small or minimal, and gradually evolving to an assembly of LMFBR spectrum and composition. Starting with discrepancies of up to 10%, they have been able to remove essentially any discrepancy between calculation and measurement of fissile sample worths by applying corrections for the adjoint flux and for other approximations.

Further work is still required to understand fully the historical discrepancy in light of these recent developments. However, much of that historical discrepancy has been eliminated, and experiments have been completed that, when analyzed in the future, may lead to better understanding or reduction of the residual discrepancy. Besides significant variations in core composition and neutron spectrum, these experiments incorporate measurements that can be well-modeled while using more exact calculational methods. C/E values from those calculations can be compared against values from traditional FOP analysis. Preliminary results from these efforts are promising, although ties to past experiments are still incomplete.

SODIUM VOID REACTIVITY

In another report in these proceedings, Beck et al.⁶ present an assessment of the status of sodium void reactivity in LMFBRs. The evaluation is based on analysis of a wide range of critical experiments in many different types of LMFBR mockups. Using three dimensional exact perturbation theory and 28 group cross sections processed from ENDF/B Version IV data as a reference method, 95 experiments were analyzed that included measurements in zones with many different characteristics: small, large, high enrichment, low enrichment, around control rod positions and inserted rods, in axial blankets, negative sodium coefficients, positive sodium coefficients, and rings in heterogeneous cores. The resultant distribution of C/E values had a mean value of 1.12 and a standard deviation of 0.20. When grouped into smaller subsets of data from similar physical environments, the mean C/E's ranged from about 1.07 to 1.16 and standard deviations were typically 0.10. These results represent a marked improvement over historical comparisons of U.S. experiments.

The only significant corrections to the reference calculations were for transport and mesh-size effects in small zones around control rods and in heterogeneous cores. Corrected C/E's for voided central core zones from many critical assemblies had a standard deviation of only 4.3% compared with only 5.9% before corrections. Some effects of first-order-perturbation (FOP) methods, less accurate than the reference technique, were also

investigated. Errors of 10-20% in specific components were found for some FOP approximations, although there was some net cancellation in the total reactivity. In one example cited to compare FOP results with the reference results, the C/E distribution expanded from an standard deviation of 0.20 to 0.44, while the calculated reactivity went from a conservative overprediction ($C/E = 1.12$) to an underprediction ($C/E = 0.94$).

Two component bias factors were fit to the subsets of data. By applying individual bias factors for the positive non-leakage component and the negative leakage component, sodium-void reactivity could be predicted to about 5% accuracy in dominately positive zones. Where significant corrections were not required, non-leakage bias factors fell in the relatively narrow range of 0.85 - 0.90.

A methodology for applying the results of the sodium void study to LMFBRs has been suggested, but as yet only primitive estimates of uncertainties have been applied in U.S. fast reactor safety studies. More realistic descriptions of the initial phase of accident scenarios can be provided if less pessimistic uncertainties are added to the input sodium-void reactivity coefficients that are used in safety codes. Confidence in the sodium coefficients used in LMFBR safety analysis will require the use of an improved methodology for biased predictions and uncertainties.

DOPPLER REACTIVITY EFFECT

Confidence in predictions of the Doppler reactivity effect is a necessary part of fast reactor design. During startup, the excess reactivity must be sufficient to overcome the cold-to-hot temperature defect, which is due predominately to the Doppler effect. Negative Doppler feedback is also a key mitigating factor in most accident scenarios. In safety studies of hypothetical accidents, assurances often must be given that the real magnitude of the Doppler effect exceeds the value input to the calculations. Consequently, the nominal magnitude is reduced by some multiple of the uncertainty in the value. Since about 1970, all fast reactor studies have relied on analysis of the SEFOR experiments³⁷ for estimating uncertainty in calculated Doppler coefficients.

As important as SEFOR data are in verifying the prediction of Doppler reactivity in LMFBRs, there is room for more extensive use of heated-sample Doppler reactivity data obtained in critical experiments. The critical assembly measurements are of interest because:

- (i) they have been performed in a variety of core environments--for example in a range of core sizes, and in both conventional and heterogeneous systems,
- (ii) spatial variations in Doppler coefficients can be studied in detail, and
- (iii) Doppler effects can be determined individually for different materials or isotopes.

Unfortunately, a significant amount of work remains before full utilization of the critical assembly data will be possible. One problem is that consistent calculation methods and cross section data have not been applied to an appreciable portion of the data base.

For the relatively small number of past experiments that have been recalculated with a consistent approach, some promising results have emerged, and they are reported in another paper at this conference.⁷ For the benchmark assemblies in the homogeneous core program (ZPPR-2, ZPR-6 Assembly 7, and ZPPR-9), the average C/E on central-core Doppler effect was 0.92. Using the same general calculational approach, and the same ENDF/B-IV cross section file, the two SEFOR cores had Doppler effect C/E's of about 0.98. However, the methods and models have not yet been made fully consistent, and many physical differences exist between the effects measured in the two reactors.

For the CRBR engineering mockup program, Doppler effect C/E values, as a function of radius, ranged from 0.77 to 0.93 in the beginning-of-life core. The spread of over 20% would represent an unacceptable state of affairs, as it would imply an uncertainty in the range of $\pm 10\%$. But this heterogeneous core, unlike the homogeneous cores cited above, had a significant error in calculated flux shape that impacted all spatial parameters. That portion of the variation in Doppler C/E due to spatial flux shape errors was removed, approximately, by use of foil activation results. This approach essentially removed all the variation in Doppler effect C/E, consistent with an uncertainty of about $\pm 5\%$, and therefore suggested a new way of providing Doppler coefficient information from critical assembly data. Since heated-sample Doppler effect experiments are time-consuming relative to foil activation measurements, significant spatial detail in the Doppler effect can be provided by Doppler measurements at a few locations coupled with foil activations at a host of locations. While such an indirect inference of the spatially-dependent Doppler coefficient with foils will increase the uncertainty, the penalties are not large and it appears that local Doppler effect can be determined in this way to an uncertainty of $\pm(5-7\%)$.

Future work in the Doppler effect first involves consistent recalculation of a number of past heated-sample Doppler coefficient measurements. Once that is done, an attempt will be made to reconcile calculational methods differences between the SEFOR benchmark results and the critical assembly results, so that they can be compared on a consistent basis.

REFERENCES

1. L.G. LeSage, "Status of Fast Reactor Physics, " Proc. of the Conf. on 1980 Advances in Reactor Physics and Shielding, Sun Valley, Idaho, Sept. 14-19, 1980, American Nuclear Society, LaGrange Park, IL, pp. 51-69 (1980).
2. L.G. LeSage and R.D. McKnight, "Discussion of Integral Experiment C/E Discrepancies," Proc. of the Intl. Conf. on Nuclear Cross Sections for Technology, Knoxville, TN, Oct. 22-26, 1979, National Bureau of Standards Special Publication 594, Washington, pp. 297-306 (1980).
3. H.F. McFarlane et al., "ZPPR Studies of Control Rods in Large Homogeneous LMFB's," Proc. of the Conf. on 1980 Advances in Reactor Physics and Shielding, Sun Valley, Idaho, Sept. 14-19, 1980, American Nuclear Society, LaGrange Park, IL, pp. 546-555 (1980).
4. R.D. McKnight et al., "Validation Studies of the ENDF/MC²-2/SDX Cell Homogenization Path," these proceedings.
5. R.H. Schaefer and R.G. Bucher, "Calculated and Measured Reactivities in U9," these proceedings.
6. C.L. Beck et al., "Sodium-void Reactivity in LMFBs: A Physics Assessment," these proceedings.
7. R.E. Kaiser et al., "Extrapolation of Small Sample Doppler Reactivity Measurements," these proceedings.
8. G.J. DiIorio and W.P. Poenitz, "Accurate ²³⁸U Neutron Capture Rate Determination," accepted for publication in Nuclear Instruments and Methods.
9. P.J. Collins and M.J. Lineberry, "The Use of Cross Section Sensitivities in the Analysis of Fast Reactor Integral Parameters," Proc. Seminar-Workshop Theory and Application of Sensitivity and Uncertainty Analysis, Oak Ridge National Laboratory Report ORNL/RSIC-42, pp. 13-23 (1979).
10. J.M. Gasidlo et al., "A Comparison of Gamma Counting and Mass Spectrometer Measurements for ²³⁸U Capture," Trans. Am. Nuc. Soc., 41, p. 574 (1982).
11. W.P. Poenitz et al., "Measurements of the ²³⁸U(n,γ) Cross Section at Thermal and Fast Neutron Energies," Nucl. Sci. Eng., 78, pp. 239-247 (1981).
12. P.S. Meszaros and D.K. Olsen, "Resolved Resonance Parameters for ²³⁸U from 4 to 6 keV," Trans Am. Nuc. Sci., 41, p. 560 (1982).

REFERENCES (cont.)

13. R.B. Perez et al., "On the ENDF/B Unresolved Resonance Region Formalism Representation," Trans. Am. Nucl. Soc., 39, pp. 883-885 (1981).
14. D.C. Wade, "Monte Carlo-Based Validation of the ENDF/MC²-II/SDX Cell Homogenization Path," Argonne National Laboratory Report, ANL-79-5, (1979).
15. P.J. Collins, Argonne National Laboratory, private communication.
16. J.H. Marable et al., "Uncertainty in the Breeding Ratio of a Large LMFBR: Theory and Results," Nucl. Sci. Eng., 75, No.1, pp. 30-55 (1980).
17. P. Greebler and B.A. Hutchins, "Significance of Integral Parameters in Design and Performance of Fast Breeder Reactors," New Developments in Reactor Physics and Shielding, CONF-720901, 1972, pp. 928-939.
18. D.M. Gilliam et al., "U-238 Capture/U-235 Fission Cross Section Ratio in the ISNF," NBS Reactor: Summary of Activities July 1978 to June 1979, National Bureau of Standards Technical Note 1117, pp. 189-194 (1980).
19. B.J. Toppel, ANL, unpublished information.
20. R.D. Lawrence, "A Nodal Interface Current Method for Multigroup Diffusion Calculations in Hexagonal Geometry," Trans. Am. Nucl. Soc., 39, pp. 461-462 (1981).
21. M.J. Lineberry et al. "Physics Studies of a Heterogeneous Liquid Metal Fast Breeder Reactor," Nucl. Tech., 44, pp. 21-43 (1979).
22. P.T. Choong et al., "Cell Modeling Influence on ZPPR Criticality Analysis," Trans. Am. Nucl. Soc., 39, pp. 461-462 (1981).
23. J. Lake, "Fuel Enrichment Requirements for the Clinch River Breeder Reactor," Proc. of the Conf. on 1980 Advances in Reactor Physics and Shielding, Sun Valley, Idaho, Sept. 14-19, 1980, American Nuclear Society, LaGrange Park, IL, pp. 480-493 (1980).
24. M.L. Batch and R.E. Mueller, "Physics Measurements in the Fermi Reactor," Proc. of the Intl. Conf. on Physics of Fast Reactor Operation and Design, London, June 1969, pp. 299-305.
25. A.B. Reynolds and S.L. Stewart, "Application of the SEFOR Critical Experiments at ZPR-3 to SEFOR," Proc. Intl. Conf. on Fast Critical Experiments and Their Analysis, Argonne, IL, Oct. 10-13, 1966, Argonne National Laboratory Report ANL-7320, p. 569 (1966).

REFERENCES (cont.)

26. W.B. Lowenstein, "The Utilization of Critical Assemblies for Fast Reactor Design," op. cit., p. 543.
27. R.B. Rothrock et al., "FFTF Initial Fuel Loading, Preanalysis, and Comparison with Preliminary Results," Proc. of the Conf. on 1980 Advances in Reactor Physics and Shielding, Sun Valley, Idaho, Sept. 14-19, 1980, American Nuclear Society, LaGrange Park, IL, pp. 280-292 (1980).
28. Clauzon et al., "Experimental Physics Results at the Startup of Phenix," Proc. of the Intl. Conf. on Fast Reactor Power Stations, London, (1974).
29. V.V. Orlov et al., "Comparison of the Calculated Basic Physical Characteristics of the BN-350 Reactor with Their Measured Values," op. cit.
30. D.C.G. Smith and R.C. Wheeler, "Reactor Physics Measurements of Some Key Parameters of the PFR," op. cit.
31. P. Benoist, "Streaming Effects and Collision Probabilities in Lattices," Nucl. Sci. Eng., 34, p. 285 (1968).
32. E.M. Gelbard, "Anisotropic Neutron Diffusion in Lattices of the Zero Power Plutonium Reactor Experiments," Nucl. Sci. Eng., 54, p. 327 (1974).
33. G.E. Hansen and H.C. Paxton, "A Critical Assembly of Uranium Enriched to 10% in Uranium-235," Nucl. Sci. and Eng., 72, pp. 230-236 (1979).
34. R.D. McKnight, "ENDF/B-V Content and Performance in Fast Reactor Systems," Trans. Am. Nucl., 28, pp. 735-736 (1978).
35. K.S. Smith, "The Effects of Intracell Adjoint Flux Heterogeneity on the FOP Reactivity Calculations," Trans. Am. Nucl. Soc., 41, p. 618 (1982).
36. K.S. Smith, "The Effects of Intracell Adjoint Flux Heterogeneity on the FOP Reactivity Calculations," to be published in Nucl. Sci. and Eng.
37. L.D. Nobel et al., "Results of SEFOR Zero Power Experiments," General Electric Company Report GEAP 13588, March 1970.

SODIUM-COOLED FAST REACTOR THERMAL-HYDRAULICS

Neil E. Todreas

Massachusetts Institute of Technology

Cambridge, Massachusetts 02139

ABSTRACT

The thermal-hydraulics of LMFBR core performance under nominal and operational transient conditions are reviewed. Two predominant flow regimes are identified - forced convection and mixed convection. It is concluded that forced convection behavior is relatively well-predicted although improvements in the prediction of temperatures of irradiated ducts and of severely distorted bundles are needed. Mixed convection behavior has not yet been well studied. Methods to predict the onset of mixed convection, turbulence and recirculation are needed.

INTRODUCTION: CORE THERMAL-HYDRAULIC CONSIDERATIONS

The role of thermal-hydraulic phenomena in liquid metal fast breeder reactor performance has been described in several recent reviews. These reviews of Costa, Grand and Martin (1981), Fauske (1980), and Jones and Bankoff (1977) have covered respectively the areas of both the core and system, degraded core behavior, and reactor safety. It is appropriate at this conference to focus on thermal-hydraulics related to LMFBR core performance under nominal and operational transient conditions.

The proper thermal and hydraulic description of the core is necessary to insure that fuel temperatures are properly maintained and to provide necessary temperature input for the structural analysis of core components. The thermal-hydraulic/neutronic interaction in LMFBR's is far less significant than that in light water reactors since under normal conditions the coolant is single phase and the LMFBR core restraint system is designed to minimize fuel displacement.

Core thermal-hydraulic predictions are needed in two distinct regimes, forced convection and mixed convection, for at least three assembly types; fuel, blanket and absorber. The wire-wraps employed as fuel pin spacers in U.S., French and Japanese designs exert a profound effect on assembly design as does the severe radial power skew across blanket and absorber assemblies which are up to 3.2:1 and 2.1:1 (max to min) respectively. LMFBR blanket assemblies experience large changes in power over their design lifetime. Since the assembly flowrate is fixed, significant temporal changes in bulk coolant temperature occur. This is the basis for the unique LMFBR characteristic of large interassembly heat transfer rates particularly between blanket and neighboring assemblies which requires analysis tools capable of modeling a whole core configuration under steady/transient and forced/mixed convection conditions.

Extensive out-of-pile heated and unheated pin experiments have been conducted in support of the development of the required analysis tools. The development strategy can be best presented by reference to the fundamental conservation equations governing the sodium behavior in the rod bundle assemblies. Neglecting the energy increase due to viscous dissipation and the volumetric heat source (rod power enters as a boundary condition) we have

$$\text{Continuity: } \nabla \cdot \vec{v} = 0 \quad (1)$$

$$\text{Momentum: } \rho \frac{Dv_z}{Dt} = \frac{\partial p}{\partial z} + \mu_e \nabla^2 v_z + B \quad (2)$$

$$\text{Energy: } \rho c_p \frac{DT}{Dt} = \kappa_e \nabla^2 T \quad (3)$$

where B is the body force which equals $-\rho g$ for upflow.

If all physical properties are considered to be constant except the coolant density in the gravity term in the momentum equation, it is apparent that the energy equation is linked to the continuity and momentum equations only by the velocity which appears in the substantial derivative term, i.e.

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z} \quad (4)$$

To determine the temperature field from equation (3), the velocity field and the effective thermal conductivity, κ_e , which includes both molecular and turbulent effects must be provided. However, it is with respect to the handling of the velocity field that the solution procedures for the forced and mixed convection regimes drastically differ.

FORCED CONVECTION REGIME

In forced convection the velocity field can be taken independent of the resulting temperature field. Hence, two strategies emerge for determining this velocity field.

- 1) Solve the continuity and momentum equations directly (characteristic of the COBRA family of codes)
- 2) Develop empirical correlations for the axial and crossflow velocity components (characteristic of the ENERGY family of codes)

In both strategies a physical understanding of the crossflow and turbulence characteristics of the flow in the wire wrapped assembly is necessary.

In the first strategy this requires development of a multidimensional resistance model of a wire wrap and a turbulent mixing model. Although attempts have been made to assess such models (Basehore and George, 1979), they have involved testing of several input models simultaneously. Existing approaches for the key input parameter, the wire model, are still crude and do not produce crossflows which have been measured in scale models as Figure 1 demonstrates. Nevertheless, this strategy is necessary for a new geometry upon which no experimental data exists or for a distorted geometry.

The second strategy, while viable only for the standard pin array geometry, has proven useful because LMFBR wire wrapped bundle designers have almost universally adopted the concept (Figure 2) of a single, counter-clockwise wrap per pin characterized by the set of geometric ratios $-P/D$ and H/D .

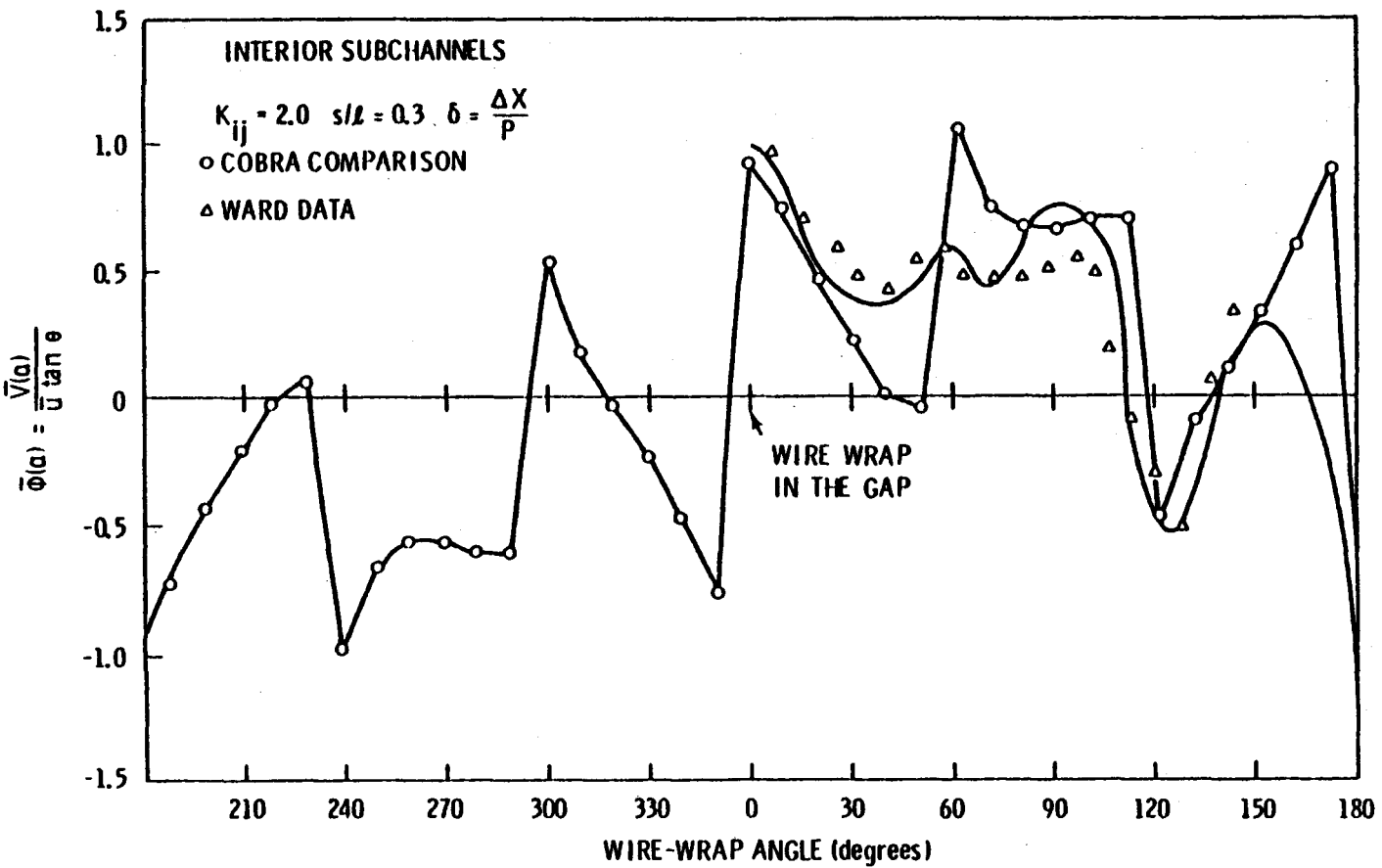


Figure 1. Demonstration of the Inadequacy of a Typical Wire-Wrap Model (taken from Basehore and George, 1979)

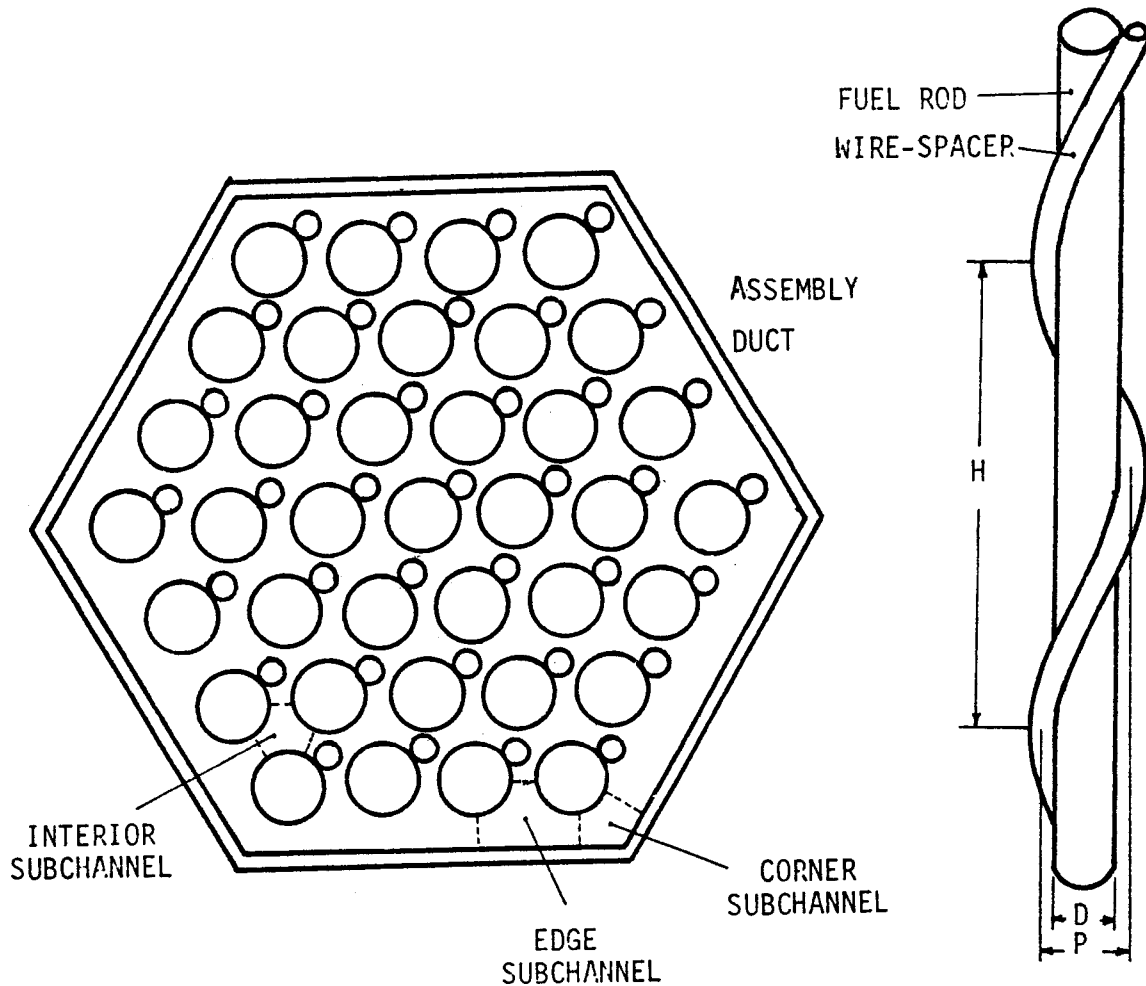


Figure 2. Typical LMFBR Wire-Wrapped Assembly and Rod Configuration

In this strategy, the axial velocities are correlated in terms of the bundle geometry. The cross-velocities are also recognized as different in the edge and interior of the bundle. In the edge region the wire establishes a swirl velocity which is correlated. In the interior region, over each lead length the net crossflow within a subchannel is zero but net energy transfer can occur between subchannels when a radial power gradient exists because the enthalpies transported by each crossflow stream are different. This net energy transfer is modeled as a diffusive process in terms of a constant (an eddy diffusivity) and the lateral temperature gradient. In turbulent flow this wire-induced energy transfer process dominates molecular and turbulent forced convection conditions (Hawley et al, 1980). When applied to a normal geometry bundle in the range of the input correlations, this second strategy gives equivalent results to the first strategy for a fraction of the computational effort.

Representing the density variation with temperature as

$$\rho = \rho_0 (1 - \beta (T - T_0)) \quad (5)$$

and nondimensionalizing equations 1, 2 and 3 obtain

$$\text{Continuity: } \nabla \cdot \vec{v}^* = 0 \quad (6)$$

$$\text{Momentum: (upflow)} \quad \frac{D^* v_z^*}{Dt^*} = - \frac{\partial p^*}{\partial z^*} + \frac{1}{Re_D} \nabla^{*2} v_z^* - \left(\frac{Gr_L}{Re_D^2} \left(\frac{D}{L} \right)^3 - \frac{1}{Fr_D} \right) \quad (7)$$

$$\text{Energy:} \quad \frac{D^* T^*}{Dt^*} = \frac{1}{Re_D Pr} \nabla^{*2} T^* \quad (8)$$

where the dimensionless physical properties are assumed to be 1 for constant properties and turbulent effects are not included.

Additionally, the length scale for the Grashof number is taken as L and for the Re as D on physical grounds. There are four dimensionless groups appearing in equations (7) and (8):

$$\begin{aligned} Re_D \equiv \text{Reynolds number} &= \frac{\text{inertial forces}}{\text{viscous forces}} \\ &= \frac{\rho_o U^2 / D}{\mu_o U / D^2} = \frac{\rho_o D U}{\mu_o} \end{aligned} \quad (9)$$

$$Pr \equiv \text{Prandtl number} \equiv \frac{\text{mass eddy diffusivity}}{\text{thermal eddy diffusivity}} = \frac{\mu_o c_{po}}{k_o} \quad (10)$$

$$\begin{aligned} Gr_L \equiv \text{Grashof number} &= \frac{(\text{buoyancy forces})(\text{inertial forces})}{(\text{viscous forces})^2} \\ &= \frac{g \beta_o^2 (T - T_o) L^3}{\mu_o^2} \end{aligned} \quad (11)$$

$$Fr_D \equiv \text{Froude number} = \frac{\text{inertial forces}}{\text{gravity forces}} = \frac{\rho_o U^2 / D}{\rho_o g} = \frac{U^2}{gD} \quad (12)$$

The combination of Gr and Re in equation (7) is the Richardson Number

$$R_i \equiv \frac{Gr_L}{Re_D^2} \equiv \frac{\rho_o g \beta \Delta T}{\rho_o U^2 / D} \left(\frac{L}{D} \right)^3 = \frac{\text{buoyancy forces}}{\text{inertial forces}} \quad (13)$$

and the product of Re and Pr in equation 8 is the Peclet Number

$$Pe \equiv Re_D Pr = \frac{\text{convection heat transfer}}{\text{conduction heat transfer}} \quad (14)$$

For the high velocity which is characteristic of forced convection, the Richardson number is low and the Froude number is high. Hence, solutions of

equations (7) and (8) are characterized only by the Reynolds and Peclet numbers, i.e.:

$$v^* = f(Re) \quad (15)$$

$$T^* = g(Pe, Re) \quad (16)$$

This is demonstrated by Figure 3 where the subchannel axial velocity normalized by average bundle velocity, i.e., the flow split, is correlated simply with Reynolds number for a given geometry. The antisymmetric curves for the two channel types are required by continuity. The saturation behavior at high Reynolds number signals fully turbulent flow. A remarkable feature is the extensive transition zone between the laminar and turbulent regimes. Further, the onset of fully turbulent flow is much higher ($Re \approx 12,000$ vs 2100) in a wire wrapped bundle than the circular tube presumably due to the joint effect of the wire and the three subchannel shapes which comprise the bundle. The accuracy of existing correlations for the laminar and transition regions is poorer than in the turbulent region. This reflects the paucity of laminar flow split data upon which to base a general correlation.

In addition to this flow split behavior, there are several other important bundle characteristics under forced convection.

- 1) Temperature field for both normal and distorted geometries
 - . clad hot spot temperature for pin integrity assurance
 - . assembly duct temperature for core restraint system design, and
 - . assembly exit temperature for thermal distortion and thermal stripping analyses of above-core structures.
- 2) Pressure drop and flow distribution among assemblies.
- 3) Interassembly heat transfer.

The radial location of the hot channel in a bundle with a radial power gradient is moved inboard from the edge channel due to the overcooling of the edge channels. Figure 4 illustrates such a typical radial temperature profile in the blanket assembly. For sodium, the Prandtl number is fixed so that such temperature solutions are functions only of Reynolds number, geometry, and boundary conditions. Hot rod temperatures are calculated from the hot channel temperatures by application of a global heat transfer correlation which is again a function of Peclet number, geometry and boundary conditions. Hot rod temperature predictions for normal configurations are accurate to 5-10°F based on comparison with out-of-pile heated pin tests. There is a tendency, presently unresolved, toward larger errors in prediction of duct wall temperatures based on comparison with sodium cooled out-of-pile test results. Both rod and duct predictions include uncertainties due to distortions of the heated pins. The utility of current modeling efforts to predict local spatial variations in clad and duct temperatures may be dominated by the effect of irradiation-induced pin bowing and swelling. Additionally, prediction methods for coolant temperatures in severely distorted pin bundles are not yet well developed. Such methods require the iterative application of thermal and structural analysis techniques since these effects are synergistic.

While the pressure field established within these bundles is complex, suitable correlations for bundle and subchannel friction factors exist which can be used in assessing pressure loss. However, because of the high power

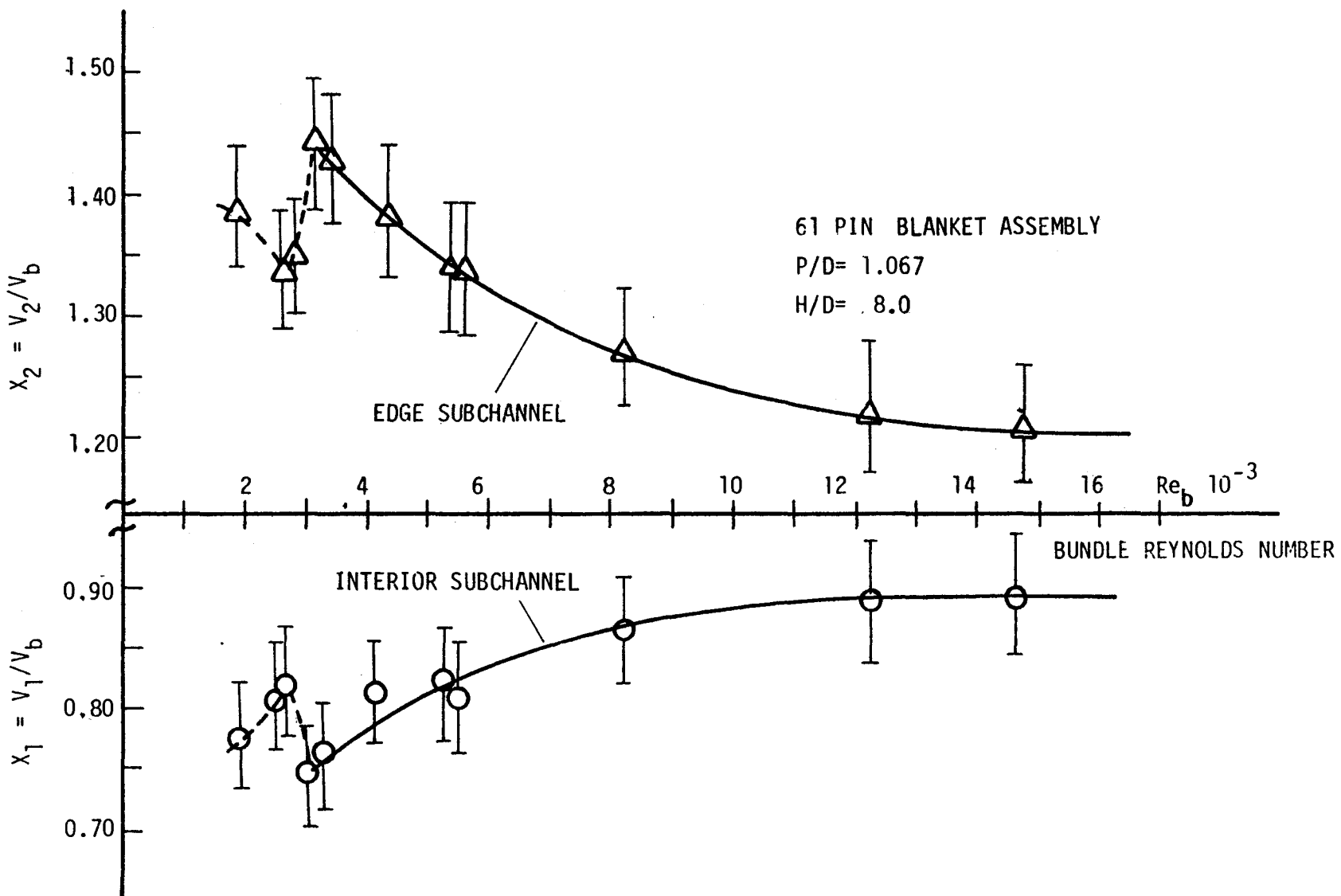


Figure 3. Normalized Subchannel Velocity in a Blanket Assembly
 (Taken from Greene and Todreas, 1981)

density and use of ducted assemblies, development of the proper orificing strategy is an important consideration in LMFBR design (Carelli et al, 1977). With suitable pressure drop correlations and mechanical and thermal design limits in hand, it is important at the start of design to develop an orificing strategy which will insure that sufficient flow is directed to the various types of core assemblies.

Finally, because of the significant differences in pin powers between assemblies at interfaces among core, blanket and absorber assemblies, LMFBRs can experience significant interassembly heat transfer. Calculation of this effect requires calculation of a substantial region of the core. Codes utilizing both solution strategies exist for this computation, although those adopting the second approach are far more economical.

MIXED CONVECTION

It is this flow regime that recent LMFBR thermal-hydraulic research has emphasized. The most obvious evidence of the onset of mixed convection is a dramatic flattening of the coolant radial temperature profile. For example, if the Richardson number is increased by 300%, the 860°F temperature peak of Figure 4 reduces to 850°F as illustrated in Figure 5. This occurs because of lateral convection from the cold (right) to the hot (left) side of the bundle due to density differences initially established by the imposed power skew. Figure 5 also includes a forced convection calculation which does not include provision to model this lateral convection. This temperature flattening is beneficial, since it reduces the hot channel characteristic relative to the bundle average behavior. However, the overall average behavior must itself be maintained satisfactorily. In particular for severely degraded flow conditions, the density differences might be sufficient to induce flow recirculation as sketched in Figure 6, thereby creating the potential for undercooling of local areas.

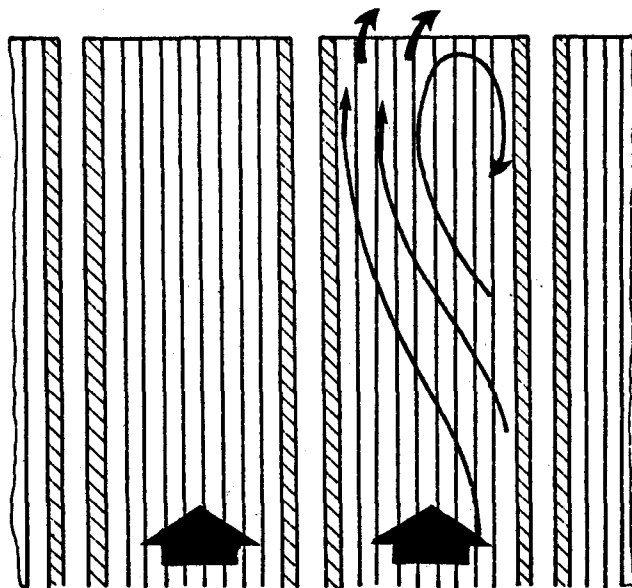


Figure 6. Local Flow Reversal in Rod Assembly

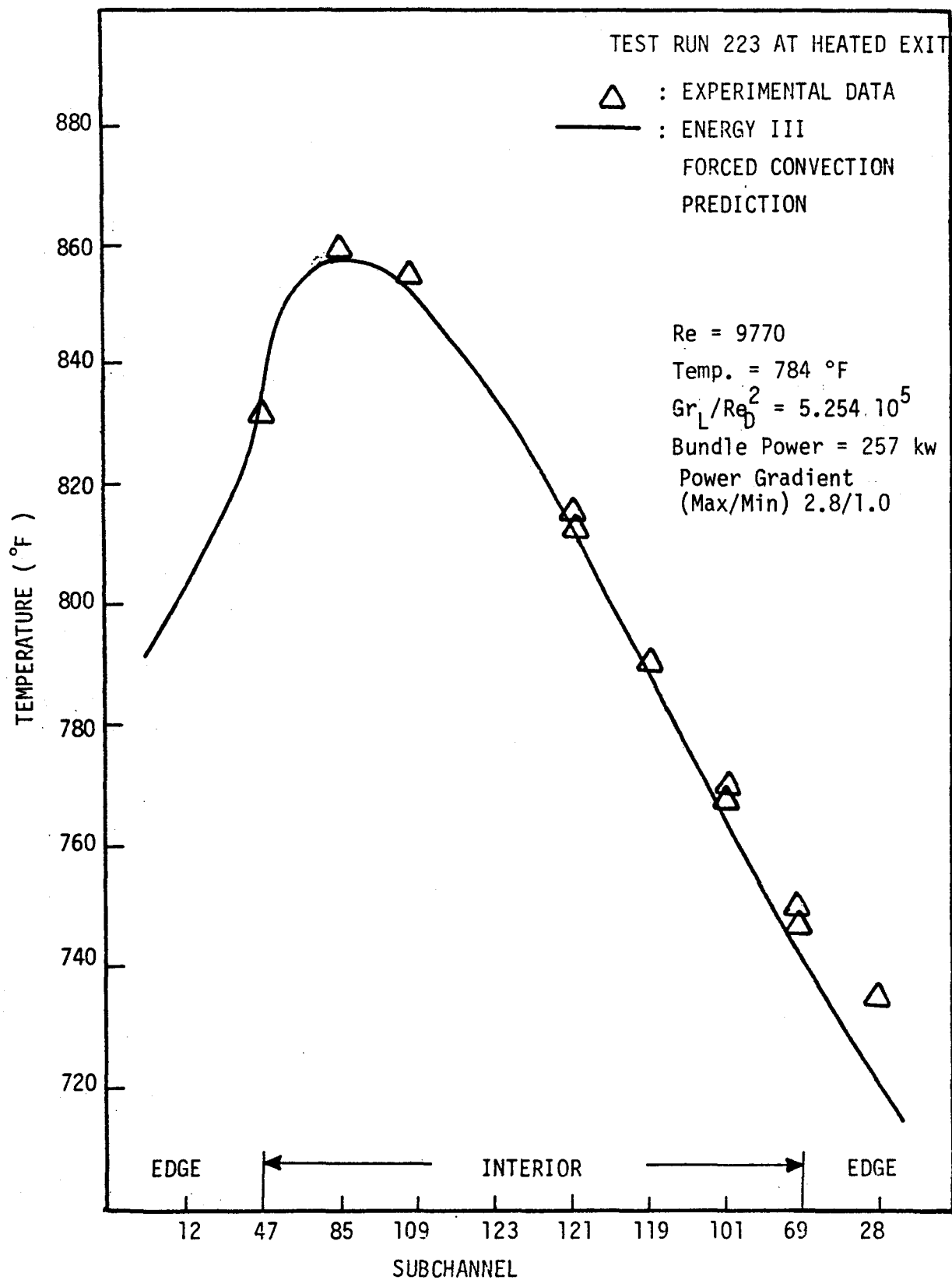


Figure 4. Blanket Assembly Outlet Temperature Profile under Forced Convection (Taken from Greene and Todreas, 1981)

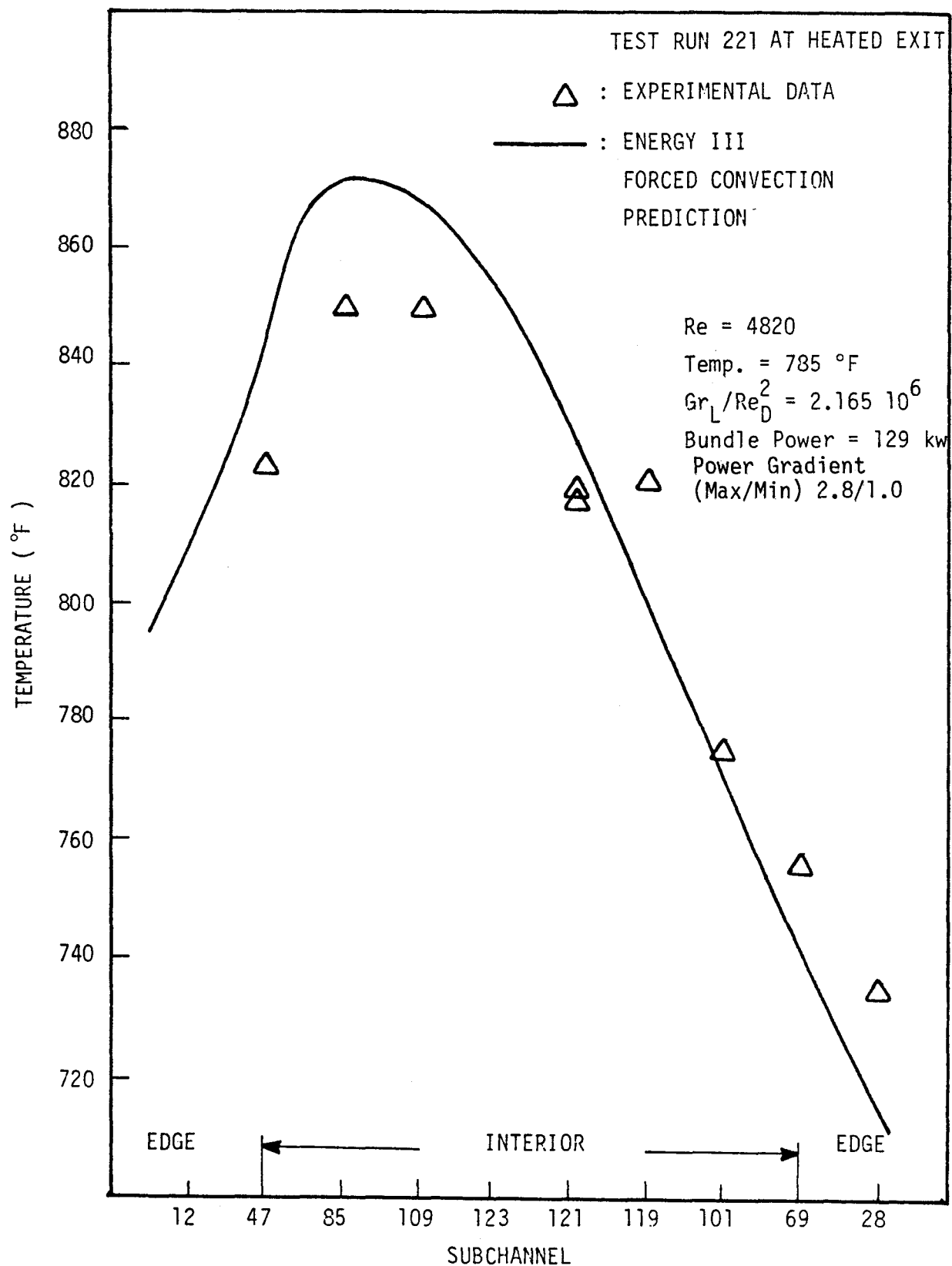


Figure 5. Blanket Assembly Outlet Temperature Profile under Mixed Convection (Taken from Greene and Todreas 1981)

In the mixed convection region the velocity scale is coupled to the temperature field. Simultaneous or at least iterative solution of the set of conservation equations is required. In contrast to the forced convection case, only this single solution strategy exists for mixed convection. The agenda for mixed convection research has two goals:

- 1) Calculation of rod bundle temperature and velocity fields.

Here we must deal with the full momentum and energy equations written as equations (7) and (8). Since our tools are lumped parameter approaches, we must input parameters such as friction factors, conduction shape factors, and ultimately heat transfer coefficients. However, forced convection forms of these parameters are not suitable. For example, consider the friction factor. Under mixed convection conditions the subchannel velocity profile changes as illustrated in Figure 7 from that of a centerline peak to a near-wall region peak due to the buoyant effect near the heated rod. Since the friction factor correlates the wall shear stress which is defined as proportional to the axial velocity radial gradient at the wall, i.e.

$$f \propto \tau_w \propto \frac{\partial v}{\partial r}$$

it is obvious that forced convection correlations are inappropriate to the mixed convection case.

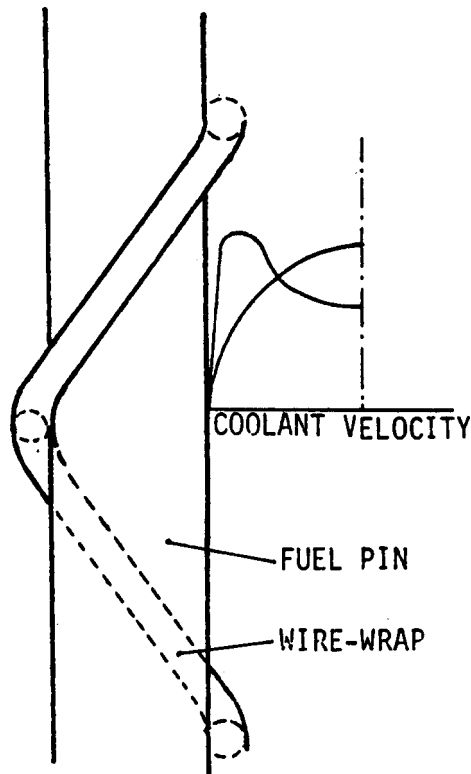


Figure 7. Local Buoyancy Effects

- 2) Establishment of criteria for the onset of mixed convection, recirculation and turbulence.

The incentives for developing these criteria differ. It is desirable to predict the onset of mixed convection to provide a basis for deciding when the simpler forced convection computational codes must be replaced. In many LMFBR transients, although the flow coasts to the laminar regime, the resulting bundle velocity from the loop-core natural circulation mode is sufficient to maintain the rod array at forced convection conditions. Definition of the onset of recirculation criterion is of interest since it establishes a region which we desire to avoid. Finally, the transition to turbulence is of interest because it signals when turbulence characteristics must be modeled in the governing conservation equations whether through simple diffusivity models or more sophisticated turbulence parameter models.

The dimensionless parameters which appear in the governing conservation equations are relevant to both these goals since they establish the form in which the results will appear. In mixed convection the velocity scale, rather than being imposed, is that generated by the balance between buoyant and inertial forces over the length L at which the temperature difference ΔT exists, yielding

$$\frac{\rho U^2}{L} = \rho g \beta \Delta T \quad \text{or} \quad U^2 = g \beta \Delta T L \quad (17)$$

Let us further take the appropriate dimensionless parameter for free convection as the Reynolds number based on the above free convection velocity, i.e.

$$Re_{free} = \frac{\rho U L}{\mu} = \frac{\rho}{\mu} (g \beta \Delta T)^{1/2} L^{3/2} \quad (18)$$

which when squared equals the Grashof number of equation 11. Finally, if we recall that the Richardson number is the ratio of free convection to forced convection effects (squared for convenience) we have

$$R_i = \frac{Re_{free-L}^2}{Re_{forced-D}^2} = \frac{g \beta \Delta T L^3 / \nu^2}{U^2 D^2 / \nu^2} = \frac{Gr_L}{Re_D^2} \quad (19)$$

where it is important to note that physically the length scale in the Grashof number has been taken as L , whereas in the Reynolds number it is D .

Returning to equation 7 we can deduce that a dominant dimensionless ratio is Gr_L / Re_D^2 . This follows from noting that the second term RHS of equation 7 contains $1/Re_D$. For high Re_D or physically large inertia to viscous force

ratio, the second term (the viscous effect) is negligible and the dominant ratio is Gr_L/Re_D^2 . For low Re_D , multiplying equation 7 by Re_D , the dominant ratio is Gr_L/Re_D . Therefore we can expect to see mixed convection results correlated in terms of Gr_L/Re_D^n , where n is between 1 and 2 and is itself a function of Reynolds number.

Present accomplishments toward the goals of mixed convection research are necessarily limited since efforts have only recently been initiated. Regarding the first goal, subchannel friction factors, but only under idealized conditions of fully developed flow in isolated subchannels, have been developed by Wang et al (1981). The numerical code BODYFIT by Chen et al (1980) is capable of computing fine mesh interacting subchannel behavior under mixed convection conditions. If a suitable wire description can be implemented in this code, the necessary mixed convection input parameters for the lumped parameter codes can be determined.

Regarding the second goal, limited empirical results exist for specific geometries and coolants. For the onset of mixed convection for blanket geometry with a range of power skews, Engel et al (1982) have deduced a criterion based on extensive measurement of $Gr_D/Re_D (D/L)=0.04$. No proposals have been made for recirculation although the results of Namekawa et al (1981) suggest that the phenomena may not occur in LMFBR geometries for practical power gradients. Results on the transition to turbulence have not been reported. Bates and Khan (19) have proposed an empirical correlation for the effects of turbulent mixing between channels in a square array water test. However, all this work is limited and not yet encompassed in a physical description of the phenomena. Such descriptions and general criteria logically arising from them are needed.

CONCLUSION

This review has focused on core-related thermal-hydraulics of LMFBR's under single phase conditions. Recent reviews of other important aspects of LMFBR thermal-hydraulic behavior were cited.

Two regimes of performance were stressed: forced convection and mixed convection. For each, the dominant dimensionless parameters in which results are expressed were presented.

Forced convection behavior is relatively well predicted. The striking bundle characteristics are a wide transition region between laminar and turbulent flow and significant interassembly heat transfer among adjacent core blanket and absorber assemblies. Key areas of effort remaining are better temperature predictions for duct walls in normally-irradiated bundles and severely distorted bundles.

Mixed convection behavior has not yet been well studied. The striking bundle characteristic is a beneficial flattening of radial temperature profiles due to lateral convection induced by buoyant effects. However, prediction of the criteria for the onset of mixed convection, the bundle performance when transition to turbulence occurs and the criteria for flow recirculation are not available.

Finally, it is also vital to extend our efforts to include the interaction of the core with the lower and upper plena since these plena establish the core boundary conditions. This effort is just starting and should be important for both forced and mixed convection conditions.

ACKNOWLEDGEMENTS

The support of the Department of Energy which has sponsored the author's work in this area is acknowledged. The assistance of Apostolos Efthimiadis as well as Shih-Kuei Cheng and Paul Symolon in reviewing and preparing this paper are greatly appreciated.

REFERENCES

THERMAL AND HYDRAULIC ASPECTS OF NUCLEAR REACTOR SAFETY. VOLUME 2. LIQUID METAL FAST BREEDER REACTORS, Editors, O. C. Jones, Jr and S. G. Bankoff, Published by American Society of Mechanical Engineering, NY, 1977.

Basehore, K. L. and George, T. L., "COBRA-IV Comparison with the WARD 11:1 Scale LMFBR Air Flow Tests," PNL-2923, Battelle NW, March 1979.

Bates, J. M. and Khan, E. U., "Investigation of Combined Free and Forced Convection in a 2x6 Rod Bundle During Controlled Flow Transients," AIChE Symposium Series, Ht. Transfer Mtg, Orlando, p 215, 1980.

Carelli, M. D., et al, "An Optimized Method for Orificing LMFBR Cores," ANS Transactions, Vol. 26, p 437, 1977.

Chen, Brian C-J., et al, "BODYFIT-1FE: A Computer Code for Three-dimensional Steady-state/Transient Single-phase Rod-bundle Thermal-Hydraulic Analysis," NUREG/CR-1874, ANL-80-127, 1980.

Chan, Y. N. and Todreas, N. E., "Turbulent Flow Split Model and Supporting Experiments for Wire Wrapped Core Assemblies," COO-2245-56TR (Rev. I), 1980.

Costa, J., Grand, D. and Martin, R., "Fast Reactor Thermal-Hydraulics," ANS Vol. 38, p 25, 1981.

Engel, F. C., and Markley, R. A., "The Effects of Radial Heat Flux Gradients and Flow Regimes on the Peak Sodium Temperature Rise in a Wire Wrapped Rod Bundle," To be published at ANS Conference to be held at Kiamesha Lake, NY, September 1982.

Fauske, H. K., "LMFBR Thermal-Hydraulics," Proc. ANS/ASME/NRC Intl. Topical Mtg on Nuclear Reactor Thermal-Hydraulics, Saratoga Springs, NY, NUREG/CP-2214, Vol. 3, p 1724, 1980.

Hawley, J. T., Chan, Y. N. and Todreas, N. E., "Input Parameters to Codes which Analyze LMFBR Wire Wrapped Bundles," COO-2245-17TR, Rev. II, 1980.

Greene, T. E. and Todreas, N. E., "WARD Blanket Assembly Pre-Test Predictions, by SUPERENERGY," N.E.D., M.I.T., COO-2245-40TR, Rev. I, April 1981.

Namekawa, F., et al, "Out-of-Pile Experiments for Natural Circulation Decay Heat Removal in LMFBR Fuel Assembly-Power Skew Effects upon Intra-Assembly

Hydraulic Characteristics," Paper in DECAY HEAT REMOVAL AND NATURAL CONVECTION IN FAST BREEDER REACTORS, Editors, A. K. Agrawal and J. G. Guppy, Hemisphere Publishing Corporation, NY, p 27, 1981.

Wang, S. F., Todreas, N. E. and Rohsenow, W. M., "Subchannel Friction Factors for Bare Rod Arrays under Mixed Convection Conditions," Paper in DECAY HEAT REMOVAL AND NATURAL CONVECTION IN FAST BREEDER REACTORS, Editors, A. K. Agrawal and J. G. Guppy, Hemisphere Publishing Corporation, NY, p 95, 1981.

NOMENCLATURE

B	Body force	km-m/s^2
c_p	Specific heat capacity	J/(kg K)
D	Diameter of a pin	m
f	Friction factor or function (eqn. 15)	-
g	Gravity acceleration or function (eqn. 16)	m/s^2
H	Pitch of the helical wire	m
k	Thermal conductivity	w/m K
L	Length of the channel	m
p	Pressure	N/m^2
P	Pitch of the rod array	m
r	Radial position	m
T	Mean temperature	K
U	Mean axial velocity	m/s
t	Time	s
v	Velocity	m/s
x	Flow split	-

Greek

β	Volume expansion coefficient	$(\text{degree})^{-1}$
ΔT	Temperature difference	K
α	Thermal diffusivity (molecular)	m^2/s
κ_e	Effective thermal conductivity (molecular and turbulent)	w/m K
ν	Kinematic viscosity	m^2/s
μ_e	Effective dynamic viscosity (molecular and turbulent)	kg/m-s

Greek

ρ	Density	kg/m^3
τ	Shear stress	N/m^2
μ	Dynamic viscosity	kg/m-s

Dimensionless Groups

Fr_D	Froude number, $\text{Fr}_D = U^2/gD$
Gr_L	Grashof number, $\text{Gr} = g\beta\Delta T L^3/\nu^2$
Pe	Peclet number, $\text{Pe} = UD(k/\rho c_p) = UD/\alpha$
Pr	Prandtl number, $\text{Pr} = \nu/\alpha = \mu c_p/k$
Re_D	Reynolds number, $\text{Re} = U D/\nu$
Ri_l	Richardson number, $\text{Ri} = g\beta\Delta T L^3/U^2 D^2$

Subscripts

b	Bundle
e	Effective (molecular plus turbulent)
o	Inlet value
w	At the wall
x,y	Lateral directions
z	Axial direction
1,2	Interior and edge subchannels

Superscripts

*	Dimensionless value
$\rightarrow$	Vector

CURRENT CHALLENGES IN LIGHT WATER REACTOR PHYSICS

By

R. R. Lee	Management Analysis Company 11095 Torreyana Road San Diego, CA 92121
O. Ozer	Electric Power Research Institute 3412 Hillview Avenue Palo Alto, CA 94303
W. R. Cobb	Science Applications, Inc. 1503 Johnson Ferry Road, Suite 100 Marietta, GA 30062

ABSTRACT

This paper identifies several areas where improvements and continued development of reactor physics methods and tools will find important application in support of utilities with operating reactors. Increased use of analysis tools will be an important contributor in reaching desired levels of safety and performance for operating reactors. Methods developers must put more emphasis on the end usefulness of code results to speed acceptance and use of analysis tools within utility organizations.

INTRODUCTION

It is our intent in this paper to focus on the areas of light water reactors (LWR) physics that require continuing development and improvement to meet current needs in reactor design and operation support. Our emphasis will be on the needs of the utility organization user. The developer of reactor design methods and operation support tools must recognize the unique characteristics of this growing user community as it will, in many cases, change the emphasis and direction of methods work.

An increased use of analysis within the utility for core design and operation support, operator training and development of plant design modifications can have an important positive impact on reactor safety and plant economic performance and, we believe, is essential if the industry is to reach desired performance levels.

Most utility organizations have a limited theoretical orientation. This requires that the method developer put increased attention on how his product will be used in producing useful results. For example, the developer needs to be concerned not only with producing a power distribution to a specified accuracy, but also with the use of the power distribution in evaluation and prediction of core behavior. It is equally important to develop the end use along with the computer codes in order to obtain a useful utility application in a short time frame.

Our purpose here is to focus the method developer's attention on the motivation of the utility user. The value to be attained from application of computer codes and models in an operating reactor environment is the most important consideration.

After discussing the characteristics of the utility environment, five areas of importance to meeting the overall utility needs will be covered.

- 1) Incorporation of the best available cross-section data into LWR design codes.
- 2) Improvements to computer codes and models to achieve desired levels of accuracy and user efficiency.
- 3) Experimental data needs for testing models of new fuel configurations.
- 4) Coupling of physics calculation results to thermal-hydraulic and transient calculations to improve calculated reactor operating margins.
- 5) Use of physics models in reactor core operation monitoring to improve reactor performance.

UTILITY ENVIRONMENT

The environment at a utility is sufficiently different from that traditionally encountered by methods and computer code developers that we believe it worthwhile to highlight some of these differences. The code developer at a reactor vendor or National Laboratory is generally working with a very sophisticated user within his own organization; there are opportunities to maintain a strong dialogue between the developer and the user.

The utility environment will generally put little emphasis on development activities. The preference will be to bring in computer codes from outside, making only minor modifications to meet specific requirements. The lack of continuing contact with the developer imposes an additional burden on the developer to be aware of how his code will be used before he starts his work.

There are two distinct groups of users at a utility. We are not talking here about the capabilities of individuals, but rather about the primary day-to-day concerns of the individual in doing his job and to a lesser extent, his training. Any individual could work in either group and will frequently move from one to the other over a period of time.

The first group is the operating plant site engineers whose primary focus is on plant operating performance and evaluation of core conditions. Computer codes and the plant models will generally not be set up, maintained or upgraded by this group. They need code results that can be obtained quickly and they must be in a form that allows immediate use in decisions affecting plant operation.

The second group consists of design analysts working somewhat removed from day-to-day plant operations. They are concerned primarily with fuel cycle management and long-term core performance evaluation. Their dependence on out-

side vendors ranges from limited and specialized support to situations where virtually all analysis is done outside the utility. The trend is clearly to internalize as much of the analysis work as possible within the utility. Wide success will depend on the availability of fully developed tools that can be used with very limited continuing dependence on the developer.

Two important characteristics of the utility environment that impact the approach taken by the code developer are:

- Absence of sophisticated theoretical capability within the organization coupled with, generally, limited experience in direct use of large computer codes.
- Need for two levels of model complexity and accuracy to meet the different requirements of the two utility user groups discussed above.

The limited utility experience requires that the codes be easy to use with simplified, automated input features and extensive checking for input data consistency by the code. The user must be given a clear understanding of the code capabilities and limitations. Each code will be part of a larger integrated system. For example, the reactor physics codes must interface with core thermal-hydraulic and transient analysis codes, as well as codes used for processing plant measured data. In general, insufficient attention has been given to the interfaces to provide code systems that are viable in the utility environment.

The need for two levels of model sophistication can be viewed as an opportunity for cooperation rather than competition between proponents of a particular approach. An example of this in the physics area is continuing differences over the need for one or two energy group representations in three-dimensional nodal codes. A strong argument can be presented that both are needed; effort to tie the two approaches together is needed to fully meet utility needs.

The plant site engineers need a fast running on-line code that can be used to quickly evaluate current core conditions and predict core power distribution behavior under a well defined set of conditions (e.g., changes in control rod positions, core flow and temperature, etc.). It is feasible to fine tune and adjust the core model using measured data or more precise off-line calculations periodically. Code accuracy is of secondary importance in this case.

The design analysts, on the other hand, need a code that predicts core power distribution with high accuracy for situations where benchmark data may not be available. Code running time, while still important, is not the primary concern. Detailed cross-section representations over wide ranges of core operating parameters are important for safety analysis of abnormal conditions. The use of a two group model to fine tune one group model provides the link for cooperation between these two approaches.

In the following sections we will show a few examples to indicate how some of the user needs identified above can be met. This is by no means intended to be an exhaustive list, but rather is taken from the experience of the authors as an indication of how new development activities might be defined and justified.

CROSS-SECTION DATA

Two developments which occurred during the past few years can be expected to significantly impact the status of nuclear data utilized in LWR applications. These developments consist of:

- The release of the ENDF/B-V library which contains revisions to high sensitivity data types such as the resolved resonances of U-238, the thermal energy range cross sections for U-235 and the "nu-bar" values for all the major fissile nuclides.
- The development of various full-range, continuous energy Monte Carlo programs utilizing the above library as a data base. Programs such as SAM-CE¹, MCNP², and others can now represent the ENDF/B data with as much accuracy as desired. Variable energy tabulations containing $\sim 10^5$ data points are commonly used to represent the resolved resonance region of major fissile fertile and structural nuclides. The unresolved resonance range is represented in terms of a probability table approach which is equivalent in accuracy to the representation used on the original ENDF/B files. The combinatorial geometry capability inherent in programs such as SAM-CE permits for an exact representation of the geometries involved in benchmark experiments.

This Monte Carlo capability presents an opportunity for testing the adequacy of nuclear data without being subjected to uncertainties due to methods approximations. Conversely, by utilizing data files derived from the same source in the much faster, but less rigorous deterministic programs, the Monte Carlo capability can be used to assess the adequacy of various methods approximations without concern about data uncertainties.

A. STATUS OF THE ENDF/B-V DATA LIBRARY

A general test of the adequacy of ENDF/B data files is obtained from the analysis of a series of experiments defined as "thermal benchmarks" by the Cross-Section Evaluation Working Group. These experiments include simple geometry homogeneous critical solutions fueled by uranium or plutonium nitrate as well as various water moderated U-metal, UO₂ and mixed oxide critical lattices.

Monte Carlo analysis of the above benchmarks with ENDF/B-V data indicates the new library to be an improvement over earlier versions. Eigenvalues for U-metal and UO₂ fueled critical lattices which were too low by as much as 1 percent are now within 0.2 to 0.4 percent of the measured values in most UO₂ systems primarily because of the higher "nu-bar" values in ENDF/B-V.

Sample Monte Carlo calculations obtained by Schmidt and Rose³ of Brookhaven National Laboratory (BNL) are shown on Table 1.

In U-235 fueled high-leakage homogeneous criticals a tendency to overpredict the eigenvalues is no longer evident with the harder U-235 fission spectra introduced in ENDF/B-V.

Nevertheless a few problem areas remain:

- The long-standing discrepancy between measured and calculated resonance capture rates in U-238 (as measured by ρ_{28} , the epithermal-to-thermal capture ratio) has been reduced, in some cases to a point where differences between calculations and measurements are comparable to the sum of statistical and experimental uncertainties. Moreover another BNL⁴ study of isolated-rod resonance integrals gave results in agreement with experiments (within the experimental uncertainty). Yet in all critical lattice analyses, the calculated resonance capture rates are higher than the measured values. Based on sensitivity analyses, it appears that a small (2 to 3 percent) decrease in the U-238 resonance integral will bring about excellent agreement in eigenvalues as well as in reaction rates. Although the low energy U-238 resonance parameters are now well defined, the uncertainty in the parameterization of the experimental data in the resolved-to-unresolved transition region may provide justification for such a modification.
- The U-235 epithermal alpha value (ratio of capture to fission cross sections) is low with respect to integral data⁵. The resonance range cross sections for this isotope are the same as in earlier versions of ENDF/B. A new evaluation taking into account existing measurements of the fission resonance integral is needed. In the low-thermal energy range, uncertainties in the shape of the cross section could not be adequately resolved by a non-linear least-squares analysis⁶ carried out for ENDF/B-V. Since the energy dependence of the alpha value in this region may significantly impact the calculations of temperature dependent effects, high accuracy direct capture measurements may be necessary to reduce uncertainties.
- The eigenvalues of all Pu-H₂O homogeneous criticals are overpredicted by more than one percent. This overprediction, which was already present with ENDF/B-IV data, did not change appreciably with the new library. A detailed study by Hardy & Ullow⁷ using ENDF/B-IV data could not isolate a single source for the discrepancy. Unfortunately, large experimental uncertainties, particularly in isotopic content, result in a large scatter of the calculated values and make the analysis of trends very difficult. Cleaner benchmark experiments may be required to resolve this discrepancy.

Monte Carlo eigenvalues calculated for UO₂-PuO₂-H₂O critical lattices are in good agreement with experiments³.

Experiment	Parameters	Measured Value	Calculated ENDF/B-V	Values ENDF/B-IV
CSEWG Benchmark #18 (VO ₂ lattice BAPL-1) (tight pitch)	Keff	1.0	.996 ±.002	.990 (errors same as
	ρ 28	1.39 ±.01	1.44 ±.01	1.45 ENDF/B-V)
	δ 25	0.084 ±.002	0.0848 ±.0006	0.0841
	δ 28	0.078 ±.004	0.0759 ±.0006	0.0738
	C*	--	0.821 ±.004	0.822
CSEWG Benchmark #20 (VO ₂ lattice BAPL-3) (loose pitch)	Keff	1.0	.998 ±.002	.993
	ρ 28	0.906 ±.01	.920 ±.01	.944
	δ 25	0.052 ±.001	.0532 ±.001	.0523
	δ 28	0.057 ±.003	.0550 ±.001	.0518
	C*	--	.660 ±.003	.667
CSEWG Benchmark #6 (U-metal lattice TRX-1) (tight pitch)	Keff	1.00	.994 ±.002	.984
	ρ 28	1.320 ±.021	1.370 ±.01	1.390
	δ 25	.0987 ±.001	.1004 ±.001	.0999
	δ 28	.0946 ±.004	.1001 ±.001	.0994
	C*	.797 ±.008	.8017 ±.004	.807
CSEWG Benchmark #31 (mixed oxide lattice, UL266)	Keff	1.0002	.9986 ±.0016	

TABLE 1. COMPARISON OF MEASURED AND CALCULATED
PARAMETERS FOR H₂O MODERATED CRITICAL LATTICE EXPERIMENTS

B. UTILIZATION OF ENDF/B-V DATA FOR LWR ANALYSIS

ENDF/B-V data has been utilized in conjunction with continuous energy Monte Carlo programs for the analysis of spent fuel storage rack geometries, giving excellent agreement with benchmark experiments.

Most LWR core physics design programs require the ENDF/B data files to be pre-processed into a simplified form consisting of multi-group cross sections and resonance range shielding factors tabulated as a function of temperature and potential cross section. This procedure introduces a source of uncertainty. The selection of an appropriate energy structure or number of tabulation points represents a balance between accuracy and calculational efficiency. Although it is possible to produce a processed library that will give very accurate results for a specific application (per site, fuel type, enrichment, temperature range, etc.), the preparation of an optimum library for a wide range of applications represents a challenging problem.

The ENDF/B-V library has been processed for use in the EPRI supported core physics programs EPRI-CELL, CPM and CASMO. Various vendor organizations have also converted sections of the library (in full or in part) for use on their internal systems.

The EPRI programs are presently being benchmarked against a range of experiments including hot and cold criticals, PWR and BWR assembly mock-up experiments. Initial results obtained with the processed libraries were not as good as those obtained with Monte Carlo. It has subsequently been possible to narrow the gap with Monte Carlo, partly by optimizing the library structure and partly by improved treatments for homogeneous effects in the design programs. The improvement in the parameters calculated by the PIN-CELL program EPRI-CELL as the analyzed models are improved to better account for energy dependent intermediate resonance factors, resonance interference effects and an optimized fuel escape probability calculation is shown on Table 2 for two infinite lattice calculations.

Analysis of PWR and BWR mockup experiments with the assembly programs CASMO and CPM has shown the processed ENDF/B-V library to give adequate results over experimental temperatures ranging from 20° C to 245° C¹⁰. Excellent agreement can be obtained by adjusting the U-238 multi-group cross sections downward by three percent over the entire resonance energy range. Although it can be argued that such an adjustment is necessary to compensate for methods as well as data inadequacies, adjustments in general should be avoided since even though they will improve agreement with benchmark measurements used or libraries for the adjustment, uncertainties in calculations of un-benchmarked configurations or configurations for which benchmark information is not available will, in general, be larger.

Experiment	Parameters	Original E-Cell	Modified E-Cell	Monte Carlo (% SD)
Low Enrichment	K _{oo}	1.1397	1.1459	1.1471.14
VO ₂ Lattice NB1	ρ 28	1.388	1.352	1.363 (.6)
Typical PWR	K _{oo}	1.3353	1.3390	1.3424 (.26)
Fuel Lattice NB4	ρ 28	2.684	2.658	2.654 (.6)

TABLE 2.
COMPARISON OF EPRI-CELL CALCULATIONS WITH MONTE CARLO

COMPUTER CODE AND MODEL IMPROVEMENTS

Three-dimensional nodal models have reached the point where fuel assembly average powers can be predicted with an accuracy comparable to fine mesh pin-by-pin calculations in PWRs and well within the experimental measurement accuracy in both PWRs and BWRs ^{11,12,13}. The three-dimensional nodal model will become an increasingly important part of reload core design and safety evaluation. Further attention to computer running time and code user interface is needed.

The actual solution time for determining the power distribution in most nodal codes has been reduced to the extent that it is no longer limiting in the modern nodal codes. More time consuming is the preparation and handling of the large amount of cross-section data, which must allow for variation in all important properties such as exposure, moderator density, fuel temperature, and control, as well as the historical values of these properties. Of no less importance is the large amount of "coefficient" data required by the solution algorithm, which in some cases limits the physical size of the problem which can be solved on computers without virtual memory. Thus it appears that one of the principal challenges regarding nodal codes is with the data preparation and handling. Experience has shown that straightforward tabulation of cross-section data versus the many independent variables causes excessive execution times so that a more machine efficient treatment is of high priority. Sophisticated memory management during solution is also of high priority to avoid limitations on problem size.

In the BWR, the direct coupling between sophisticated core thermal-hydraulic models and the neutronics model should improve measurement calculation comparisons for fuel assembly average power distributions. Further, including full treatment of the core and bundle thermal hydraulics effects will allow a true coupled simulation over the entire power-flow operating range. This coupling will also free the user from the burdensome tasks of re-inputting data to his simulation model each time the core thermal hydraulic conditions change.

There remains a disturbing necessity within the use of nodal methods on BWRs to require user normalization to measured or operational data for acceptable results. Typical normalizing parameters include albedos, kernel adjustment factors, etc. Perhaps the greatest challenge is to refine the nodal methodology sufficiently to remove the need for any normalization. Promising efforts to this end have been undertaken by Al Henry at MIT in the QUANDRY code, although not without the penalty of significant additional data handling requirements. A more plausible near-term solution is to develop analytically based procedures for determining these various normalization factors, and to demonstrate their validity by benchmarking over a wide class of BWR applications. Once verified, these analytical procedures should be included within the nodal code, and the freedom for creative normalization disabled.

In the PWR, where a stronger neutronics coupling exists between fuel assemblies, a pin-by-pin fine mesh representation of the core is necessary. The traditional axially averaged two-dimensional core representation is inadequate in a reload core because the individual fuel bundle thermal conditions are a strong function of

axial position and the fuel depletion relationship between fresh and burned fuel changes with axial position. Both the assembly averages and the peak pin to assembly average power distribution will change with axial position. An example of the change in peak pin to bundle average pin power between the core top and mid-plane is shown in Figure 1 for typical reload core fuel management patterns. Differences of three percent are present in the hot assembly with maximum differences of six percent present.

The three-dimensional nodal information can be used in two ways: 1) to define planar properties for a two-dimensional fine mesh calculation at the elevation of the axial peak or 2) to define boundary conditions for a two-dimensional fuel assembly calculation of the hot fuel assembly at the appropriate axial location. The latter approach offers the best promise for a low cost production tool and is being pursued by several organizations ^{14,15}.

MODEL VERIFICATION FOR MODIFIED FUEL CONFIGURATIONS

The growing incentive to improve the capacity of fuel storage facilities at existing reactor sites has led to reexamination of the use of more closely-packed storage arrays with neutron-absorbing materials for reactivity control. A further step involving disassembly of fuel rods for closer packing is also being considered. Although most arrays of these types appear to have little potential for producing a criticality hazard, it is necessary to quantify the degree of subcriticality, particularly in abnormal arrangements in which bundle locations are left empty, fresh fuel is inadvertently mixed with the spent fuel, or boron dilution accidents occur. Since methods of computing reactivity have been validated primarily for lattice arrays typical of normal fuel assembly geometries, there is little experimental basis for judging the capability of theoretical methods to predict the reactivity of many of the potential storage arrays. In addition to the need for experimental verification, it may be necessary to develop new codes and theoretical methods which can describe these more unusual fuel configurations with sufficient accuracy.

In the last five years or so, a number of new experimental programs have been started to provide a data base which can be used to support the design of various means of spent fuel storage. Among these, some of the most useful measurements have been performed by Babcock & Wilcox (B&W) for both storage of fuel bundles and for consolidated clusters of fuel rods. In the former case, various means have been examined for separating the fuel bundles, including water gaps of several thicknesses, stainless steel sheets in an egg-crate configuration, borated aluminum sheets and arrays of B_4C filled pins. Some 21 different benchmark cores have been loaded with 2.46 % enriched UO_2 fuel rods and brought critical primarily on soluble boron in the moderator. The assemblies are admittedly small, involving only nine fuel assemblies in a square array, and it will be necessary to use theoretical means to extend this information to larger arrays, different fuel enrichments, and different compositions and weights of separating materials between the fuel bundles. Nonetheless, this collection of data represents an important addition to the experimental basis for the design of fuel storage racks.

FIGURE 1

COMPARISON OF TOP AND MID PLANE PDQ PEAK PIN
TO BOX AVERAGE PIN POWER

MID

TOP-MID

CYCLE:2

D 1	D 2
1.6330	1.4606
0.0112	0.0079

D 3	D 4	D 5	B 6	D/ 7
1.6786	1.3393	1.2301	1.1404	1.0932
0.0189	0.0236	0.0238	0.0258	0.0225

D 8	B/ 9	C 10	B 11	D/ 12	B 13
1.5879	1.1803	1.0899	1.0850	1.1038	1.0493
0.0258	0.0297	0.0189	-0.0086	0.0061	0.0150

D 14	D/ 15	C+ 16	D/ 17	B 18	B 19	D/ 20
1.5880	1.1116	1.1529	1.1329	1.1680	1.1434	1.1016
0.0258	0.0304	-0.0172	-0.0301	-0.0437	-0.0190	-0.0222

D 21	D/ 22	C+ 23	C 24	B 25	B 26	C+ 27	B 28
1.6788	1.1804	1.1530	1.2458	1.1087	1.0943	1.1178	1.0750
0.0188	0.0298	-0.0170	-0.0673	-0.0262	0.0139	-0.0273	-0.0137

D 29	C+ 30	D/ 31	B 32	C 33	C 34	C 35	C 36
1.3392	1.1001	1.1325	1.1066	1.0929	1.1058	1.1176	1.1009
0.0238	0.0187	-0.0302	-0.0261	-0.0130	-0.0089	-0.0208	-0.0159

D 37	B 38	B 39	B 40	C 41	C 42	B 43	C 44
1.2302	1.0848	1.1878	1.0932	1.1054	1.1738	1.0595	1.1085
0.0238	-0.0085	-0.0438	0.0145	-0.0090	-0.0277	0.0015	-0.0131

D 45

1.6331

0.0111

B 46	D/ 47	B 48	C+ 49	C 50	B 51	C 52	B 53
1.1404	1.1037	1.1431	1.1173	1.1188	1.0592	1.1304	1.1317
0.0258	0.0063	-0.0189	-0.0271	-0.0214	0.0023	-0.0147	-0.0085

D 54

1.4607

0.0078

D/ 55	B 56	D/ 57	B 58	C 59	C 60	C 61	B 62
1.0934	1.0491	1.1015	1.0749	1.1010	1.1084	1.1313	1.0425
0.0224	0.0152	-0.0222	-0.0137	-0.0158	-0.0130	-0.0086	0.0065

MID

TOP-MID

CYCLE 4

F 1	F 2
1.6544	1.3931
0.0145	0.0172

F 3	F 4	F 5	E 6	E/ 7
1.7000	1.3893	1.2009	1.0744	1.0859
0.0313	0.0285	0.0282	0.0205	0.0268

F 8	F/ 9	E 10	D 11	D/ 12	F/ 13
1.5993	1.1534	1.1478	1.1011	1.1000	1.0985
0.0312	0.0356	-0.0126	-0.0325	-0.0138	-0.0072

F 14	F/ 15	E/ 16	D 17	F/ 18	D* 19	D* 20
1.5994	1.1350	1.0822	1.0896	1.1023	1.0920	1.1439
0.0310	0.0363	-0.0108	0.0305	0.0015	-0.0073	-0.0314

F 21	F/ 22	E/ 23	E/ 24	E/ 25	D 26	E 27	D 28
1.6989	1.1530	1.0821	1.0907	1.0794	1.1620	1.1561	1.0784
0.0315	0.0362	-0.0090	-0.0239	0.0031	-0.0363	-0.0292	-0.0091

F 29	E 30	D 31	E/ 32	D 33	E 34	D/ 35	E 36
1.3690	1.1478	1.0678	1.0762	1.0737	1.1327	1.0568	1.0981
0.0278	0.0114	0.0305	0.0052	0.0013	-0.0239	0.0035	-0.0034

F 37	D 38	F/ 39	D 40	E 41	D/ 42	E 43	D/ 44
1.2002	1.1041	1.1024	1.1649	1.1360	1.0559	1.1302	1.0532
0.0272	-0.0359	0.0110	-0.0374	-0.0243	-0.0081	-0.0183	-0.0007

F 45

1.6557

0.0138

E 46	D/ 47	D* 48	E 49	D/ 50	E 51	D* 52	E 53
1.0811	1.1059	1.0915	1.1626	1.0622	1.1313	1.0741	1.0992
0.0123	-0.0148	-0.0013	-0.0308	-0.0050	-0.0193	0.0064	-0.0083

F 54

1.3973

0.0189

E/ 55	F/ 56	D* 57	D 58	E 59	D/ 60	E 61	B+ 62
1.0755	1.1014	1.1633	1.0732	1.1033	1.0402	1.0947	1.0499
0.0293	-0.0092	-0.0346	-0.0119	-0.0026	0.0001	0.0041	0.0010

A more recent series of experiments, again by B&W, have been performed with the same fuel clustered in tight-packed clusters of pins in contact in both triangular and square lattices. In this series of measurements, the critical configuration was a square 5 x 5 array of clusters. The data have not been published as yet so it is too early to say what problems may be involved in analyzing this data. Nonetheless, it should prove to be particularly interesting since most of the methods normally employed for analysis, even those based on Monte Carlo techniques, depend on the use of various equivalence principles to represent the resonance shielding of U-238 in lattice arrays, and these equivalence principles have frequently been represented by approximations which are known only in the range of normal reactor lattice pitches and concentrations. How these methods will perform in the fuel storage configuration with tighter packing and larger water channels remains to be seen.

We understand that these experimental programs are largely completed and it now becomes important for the reactor physics community to study these data to see if the results are adequate for the verification of theoretical design methods and whether further experiments are necessary.

The appearance of these data in the literature emphasizes an important point for reactor physics in the 1980s and beyond, which is easy to overlook. The capability to perform criticality and other LWR physics experiments in the U.S. has decayed almost to the point of extinction--only a very few laboratories capable of producing reliable data of direct significance to the LWR power program continue to operate, and these are being funded on a relatively short-term basis without consideration being given to the vital need for long-term support of both skilled people and adequate technical facilities. One of the most important initiatives needed from the reactor physics community during the next decade is a concerted and effective campaign to preserve the remaining technical capabilities we have in facilities like those at the B&W laboratory.

INTEGRATION OF PHYSICS, THERMAL-HYDRAULIC AND TRANSIENT CALCULATIONS

The strong effect of core thermal-hydraulic properties on the power distribution has required a direct incorporation of the thermal-hydraulic model in the neutronics calculations. Improvements in this area will consist of incorporation of improved thermal-hydraulic calculations within the physics models as well as more direct use of design-specific physics data in the thermal margin evaluation. This will require improved data transfer and accountability between computer codes and simplified, yet more flexible, code user input and control procedures.

The use of point kinetics has been shown to have less conservatism than expected in some LWR transients where there is a strong spatial variation in the coupling between the neutronic and thermal-hydraulic core behavior. One- and two-dimensional kinetic models are coming into wider use to overcome these difficulties. In other cases, three-dimensional space-time calculations coupled with the appropriate thermal-hydraulic models offer promise to remove conservatism from the analysis of selected transients and to provide additional margin for reactor operation.

Changes in core reactivity, the reactivity worth of control rods during a scram, and changes in core power density during transients typically have been evaluated using quasi-static neutronics calculations. Such quasi-static calculations overestimate the severity of transients since they fail to account for the effects of the delayed neutrons which serve to help to preserve the pre-transient neutron flux shapes.

A simple example of the benefits of space time kinetics is illustrated in Figure 2 which compares static and space time calculated reactivity worths as a function of control rod insertion. The static calculation assumes that the neutron flux shape during a control rod insertion is the critical flux shape at each rod position. Control rod worth is greater for small insertions in the space time calculation because the delayed neutron precursors are distributed according to the initial neutron flux shape. The neutron precursors, which increase in importance as the neutron population decreases after the trip, provide a source of neutrons which tends to tilt the neutron flux shape towards the control rods compared to static methods and, hence, which leads to greater reactivity insertion at intermediate rod positions. This increase in rod worths at intermediate insertions serves to shut the reactor down more rapidly and, hence, reduces the severity of transients.

The overall conservatism inherent in conventional static methods is also illustrated in Figure 2 for the loss of flow accident. After the initiation of the event, DNB decreases due to the coastdown of coolant flow. The margin to DNB continues to degrade until the plant protection system detects the loss of flow and initiates a control rod scram. The DNBR reaches a minimum as the control rods begin to shut down the reactor, reducing hot channel heat flux. The space time calculations predict a much higher minimum DNBR than the static results. This is due partially to the greater control rod worth at intermediate insertions, which serves to reduce the hot channel heat flux more rapidly before coolant flow and, hence, DNBR have decreased as significantly, and partially to the space time calculation of the axial heat flux profile.

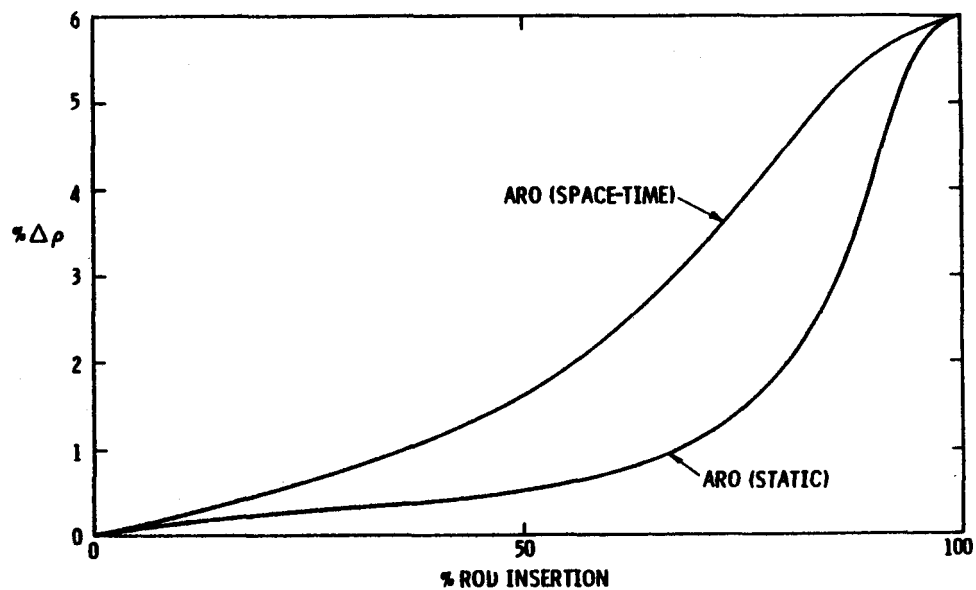
This use of kinetics on a routine basis will require improvements in numerical solution techniques and use of coarse mesh nodal methods to obtain reasonable running times. Extensive validation of these models using plant transient data will also be needed to justify taking credit for the margin reductions.

CORE OPERATION MONITORING

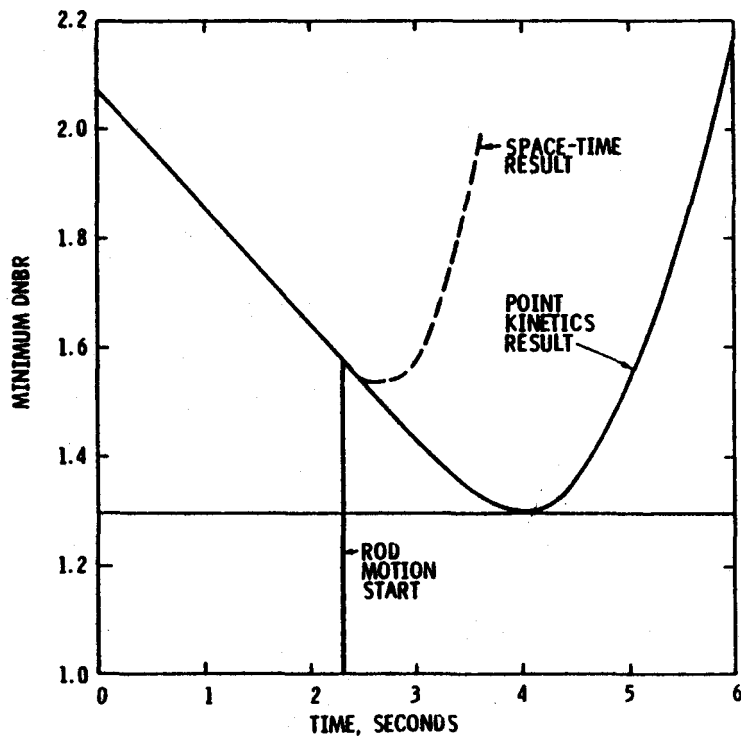
The calculation of PWR core power distributions has reached the point where calculation accuracy is comparable to the accuracy of in-core power distribution measurement systems. The difference between calculation and measurement, observed through statistical parameters, then becomes a very sensitive indicator of changes in core characteristics that affect the neutron distribution. For example, crud deposition caused by changes in hydrogen overpressure can be detected as a shift in the statistical evaluation of measured-calculated power distribution differences before it becomes apparent through the normally monitored power peaking parameters. Use of an on-line statistical evaluation can provide an early indication of problems of this sort and would allow correction before power production must be curtailed.

FIGURE 2

REACTIVITY INSERTION vs ROD INSERTION FOR A SCRAM FROM ALL RODS OUT (ARO)
USING STATIC AND SPACE-TIME METHODOLOGY



LOSS OF COOLANT FLOW
FOUR PUMPS
MINIMUM DNBR vs TIME



An example of the power distribution effects of crud deposition resulting from oxygen ingress into the primary coolant is shown in Figure 3. The substantial power reductions over a three month period starting in early November resulted from a shift in the core power distribution that caused peaking factors to exceed established limits.

Normal monitoring procedures detected a power distribution shift on October 22. An observed core pressure drop increase implied crud deposition in the core. By October 29 a source of oxygen ingress was identified and isolated but the peaking factors continued to increase forcing a power reduction. The time history of the power distribution parameters shown in Figure 3 indicate that a sensitive core monitoring program might have alerted operating personnel to the problem five or six days earlier. The additional time to isolate and correct the problem might have averted an expensive power reduction.

In the BWR, off-line power/distribution calculations have been used to diagnose problems with the core or its instrumentation. Perhaps the most notable example was the observation and examination of erroneously assymmetric TIP readings in several cores. Several BWRs have chosen to replace fission chambers with gamma tips to avoid costly derates resulting from the sensitivity of fission chamber signals to TIP tube location.

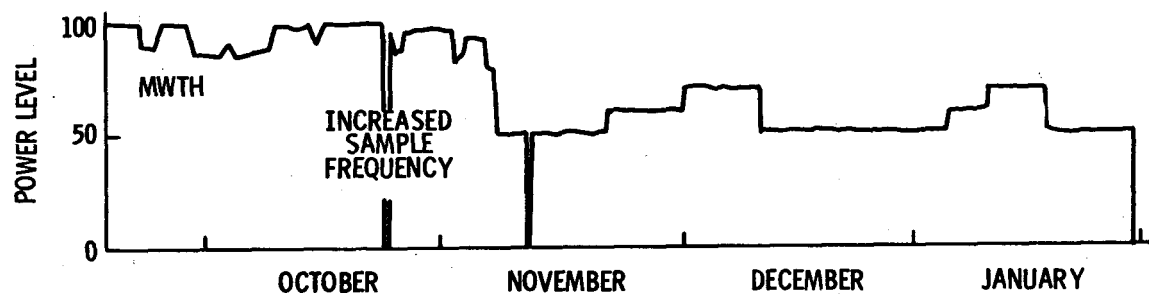
Comparison and forced matching of measured and calculated power distribution can be used to identify the nuclear characteristics of a core anomaly and assess the potential operation significance. This application will require on-line fast running nodal codes whose accuracy is comparable to the best design codes, and development and demonstration of statistical techniques for evaluation of measured data. Some effort in this direction has already been reported ^{16,17}.

With the recent advancements in the so called "super mini-computers", a definite emphasis on moving the reactor simulation models to the reactor site has evolved. At least two BWR fuel vendors are offering (or are planning to offer) reactor site based mini-computer systems with full three-dimensional nodal simulation capability. Also, EPRI has developed the Hybrid Power Shape Monitoring System (PSMS) which employs the NODE-B and THERM-B physics and thermal hydraulics models. Within this system are the capabilities to:

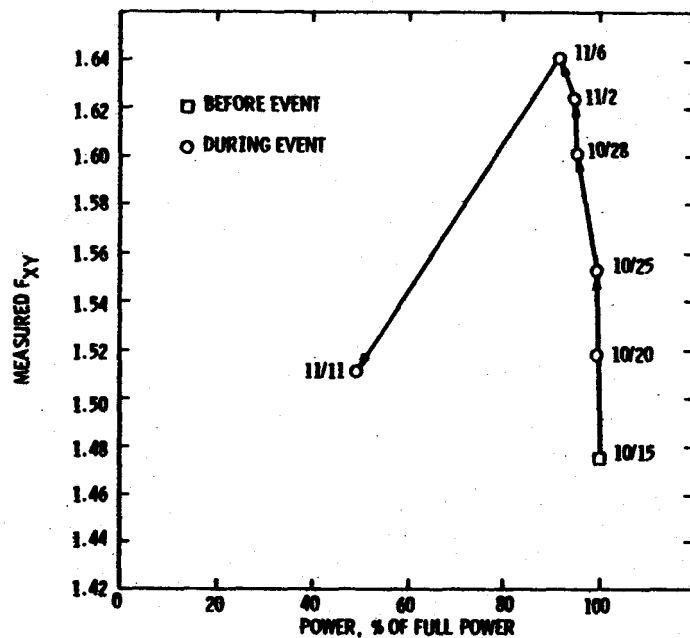
1. Predict power distributions independent of plant measurements.
2. Predict power distributions forced to plant measurements.
3. Trend the comparison between measured and calculated data, and feedback or discard systematic differences.

Moving such detailed analysis capabilities to the plant site presents new challenges to the code and methods developer, and also reinforces the need to meet some of the challenges mentioned earlier. The reactor site engineer is, in a real sense, a completely different class of user than the typical reactor analyst. His differing motivations, training and experience dictate that the requirements for normalizing simulation models by adjustment of input parameters be over-

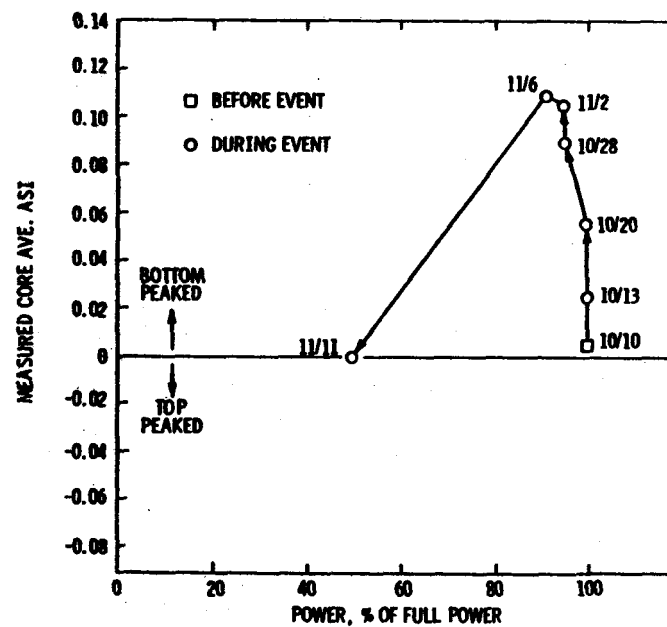
FIGURE 3
POWER LEVEL HISTORY



MEASURED PLANAR PEAKING FACTOR F_{XY} vs POWER AND TIME



MEASURED AXIAL SHAPE INDEX (ASI) vs POWER AND TIME



come. The new challenges to be met are in the areas of response time and presentation of the analysis results. Results must be translated from the abstract reference of computer and program terminology and cast into the more realistic terminology of the plant itself. To be used as an operations tool, the system must allow timely and accurate evaluations of many possible alternatives, with both input directions and output results using familiar terms to the reactor site engineer. The latest version of the PSMS has made significant progress in meeting these challenges, including a response time of as low as 15 seconds for specific analyses utilizing approximations available with the PSMS. However, as systems such as the PSMS achieve more in-plant usage, further necessary refinements will undoubtedly be identified.

With the current increasing emphasis on use of nodal models for prediction of operational maneuvers, additional data from detailed measurements during typical maneuvers will be required for model validation. Partial or complete flux maps during load follow and Xenon transient for the PWR are desirable, and for the BWR, maps during ascension to power will be required.

CONCLUSION

During the 1980s the most significant increases in use of reactor physics methods and computer codes will be in the utilities with operating reactors. We have tried to identify some of the unique characteristics of this environment. A careful look at the real needs of the utilities is the appropriate starting point to define new methods and code development effort. This requires a collaboration and communication between the user and developer, that has not yet taken place, to reach the necessary level of understanding. If the computer codes are to receive wide usage and contribute to improved reactor performance the level of understanding between user and developer must increase.

ACKNOWLEDGEMENT

The sections on model verification for modified fuel configurations and use of spatial kinetics in transient analysis rely heavily on an earlier paper¹⁸ prepared by R.L. Hellens and N.L. Shapiro with the first author. Their contributions are gratefully acknowledged.

REFERENCES

1. H. Lichtenstein, M. O. Cohen, H. Steinberg, E. S. Troubetzkoy, M. Beer "The SAM-CE Monte Carlo System for Radiation Transport and Criticability Calculations in Complex Configurations" (Revision 7.60) EPRI CCM-8. 1980.
2. LASL Group X-G "MCNP-A General Monte Carlo Code for Neutron and Photon Transport" LA-7396 M, Revised November, 1979.
3. E. Schmidt, P. Rose, "Progress in SAM-CE Monte Carlo Capability for Validating Nuclear Data and Reactor Design Codes", to be published as an EPRI NP report.
4. E. Schmidt "Direct Monte Carlo Estimates of Shielded Resonance Interperals", Symposium proceedings on "Thermal Reactor Benchmark Calculations, Techniques, Results and Applications", May 1982. To be published as an EPRI NP report.
5. J. Hardy "ENDF/B Data Testing Results for Thermal Reactor Benchmarks", Ibid.
6. B. R. Leonard, Jr., D. A. Kottwitz, J. K. Thompson "Evaluation of the Neutron Cross Section of U-235 in the Thermal Energy Region", EPRI NP-167, February, 1976.
7. J. Hardy, Jr., J. J. Ullo "Monte Carlo Analysis of PU - H₂O and VO₂-PUO₂-H₂O Critical Assemblies with ENDF/B-IV Data", WAPD-TM-1480, April, 1981.
8. L. D. Eisenhart, M. Todosow "Analysis of Critical Experiments with Monte Carlo and ENDF/B-V Data", Symposium proceedings on "Thermal Reactor Benchmark Calculations, Techniques, Results and Applications", May, 1982. To be published as EPRI report.
9. M. L. Williams, R. Q. Wright, J. Barhen, W. Rothenstein and B. Toney "Benchmarking of Epithermal Methods in the Lattice Physics Code EPRI-CELL", Ibid.
10. E. Johansson "Benchmarking of an ENDF/B-V CASMO-CPM Nuclear Data Library". To be published as EPRI NP report.
11. W. B. Terney, R. P. Bandera, T. G. Ober, "Qualification of C-E's 3-D Spatial Diffusion Code for PWR Analysis," Trans. Am. Nucl. Soc., 30, 1227 (1978).

12. M. R. Wagner, K. Koebke, and H. J. Winter, "A Nonlinear Extension of the Nodal Expansion Method," Proc. 1981 Topl. Mtg. Advances in Mathematical Methods for the Solution of Nuclear Engineering Problems, FRG, April 27-29, 1981.
13. G. Greenman, K. Smith, and A. F. Henry, "Recent Advances in an Analytic Nodal Method for Static and Transient Reactor Analysis," Proc. Topl Mtg. Computational Methods in Nuclear Engineering, Williamsburg, Virginia, April 23-25, 1979.
14. R. L. Hellens and A. Jonsson, "Recent Improvement in PWR Physics," Proc. 1980 Topl. Mtg. Advances in Reactor Physics and Shielding, Sun Valley, Idaho, September 14-17, 1980.
15. A. Jonsson, S. Grill, and J. R. Rec, "Nodel Imbedded Calculation for the Retrieval of Local Power Peaking from Coarse Mesh Reactor Analysis," Proc. 1981 Topl. Mtg. Advances in Mathematical Methods for the Solution of Nuclear Engineering Problems, Munchen, FRG, April 27-29, 1981.
16. C. M. Kang and R. L. Crowther, "Adoptive Method for Three-Dimensional Boiling Water Reactor Simulation," Trans. Am. Nucl. Soc., 38, 340 (1981).
17. B. Frogner, A. Ipaktchi, and A. B. Long, "Model Calibration Methods for the BWR Hybrid Power Shape Monitoring System," Proc. 1981 Topl. Mtg. Advances in Mathematical Methods for the Solution of Nuclear Engineering Problems, Munchen, FRG, April 27-29, 1981.
18. R. R. Lee, R. L. Hellens and N. L. Shapiro, "Future Directions for Thermal Reactor Physics Methods Development," Trans. Am. Nucl. Soc., 39, 921 (1981).

ADVANCES IN LWR THERMAL HYDRAULICS

Owen C. Jones, Jr.
Professor of Nuclear Engineering
Rensselaer Polytechnic Institute
Troy, N.Y. 12181

ABSTRACT

Clearly the entire field of LWR thermal hydraulics encompasses the entire fields of multiphase flow and heat transfer--a technical review of which is impossible in this short a space. So rather than attempt this, the author has taken an overview perspective of thermal hydraulic nuclear safety methods as they have developed over the last decade--centered on the development of advanced computational systems and their associated physics. Discussions of both the drift flux model and the two-fluid model are given along with the accompanying formulative process descriptions required. The advances which have played a role in these models are delineated and further advances required identified.

INTRODUCTION

The growth in the field of nuclear thermal hydraulics parallels closely the growth in the fields of two-phase flow and boiling heat transfer. The explosive increase in the number of references published each year following what Bergles⁽¹⁾ calls the point of "significant literature generation" (not to be confused with the generation of significant literature) parallels closely the developmental and safety activities associated with nuclear power generation. This is, of course, because the latter has grown mainly due to design and safety analysis needs in this area.

The first book in the field was apparently published by Kutateladze and Styrikovich⁽²⁾ in the Soviet Union in 1958. This was followed in the U.S. by a monograph by L.S. Tong⁽³⁾ in 1965 and contained a synopsis of the numerous correlations, mostly empirical, in use at that time. Tong was followed closely by S.L. Soo's⁽⁴⁾ text dealing with the hydrodynamics of particles and suspensions. In this same time period summer courses on two-phase flow and boiling heat transfer were being given at MIT and Dartmouth. The results from the latter were Graham Wallis' text on One Dimensional Two-Phase Flow⁽⁵⁾ and John Collier's recently revised book entitled Convective Boiling and Condensation.⁽⁶⁾ Both have become unsurpassed standards receiving wide use and reference which continues today. During the same period of time a definitive text on annular flows came out of the work Hewitt was undertaking in the U.K., written in collaboration with Hall Taylor.⁽⁷⁾ Govier and Aziz⁽⁸⁾ also published their book on The Flow of Complex Mixtures in Pipes in this time period.

Following these first publications there was a four-year hiatus before the start of a rash of publications which continues today. Hsu and Graham's⁽⁹⁾ book was next. Jones and Bankoff⁽¹⁰⁾ and Lahey and Moody⁽¹¹⁾ were the first to publish books directed toward nuclear design and safety problems in the same year as additional books by Butterworth and Hewitt,⁽¹²⁾ and by Hahne and Grigull⁽¹³⁾

appeared. A stream of others have followed including those by Ginoux,⁽¹⁴⁾ Hewitt,⁽¹⁵⁾ Delhaye, Giot, and Reithmuller,⁽¹⁶⁾ Bergles,⁽¹⁷⁾ and Jones.⁽¹⁸⁾

Obviously, a comprehensive discussion on advances in reactor thermal hydraulics would cover the fields of boiling heat transfer and multiphase flows. There is neither the time nor the space herein in which to provide such a detailed summary. Indeed for this detail the reader is directed to the many fine references previously mentioned.

Over the majority of the past decade, many of the advances related to LWR thermal hydraulics have occurred due to work aimed at developing improved computational capabilities for accident analysis, and have focused intensive although somewhat misguided emphasis on the large break loss-of-coolant accident (LOCA). More recently the emphasis has shifted to small break LOCA's due to the Three Mile Island accident. For this reason the balance of this paper will focus on the improvements which have accompanied the development of large and complex computer codes for nuclear safety analysis. And while improvements in numerical methods for solving these computer codes have been substantial, these will not be discussed herein.

HISTORICAL PERSPECTIVE

An excellent review of LWR safety analysis was developed by Jackson et al. in the last chapter dealing with the design basis accident of LWR's in Reference 18. That material is presented in considerable detail which will not be repeated herein.

The first two public large break LOCA system codes*, FLASH and RELAPSE, were released in 1966. Both used virtually identical one-dimensional, homogeneous equilibrium models meaning that the liquid and vapor phases were coupled having equal velocities and temperatures. The fuel model was for plate geometry with a simplified nominal/hot channel core but without fuel thermal calculations or metal heat conduction. Very simple convective heat transfer and nucleate or film boiling correlations with surface-temperature-dictated transitions. These were essentially two-equation models meaning one equation each expressing mass and energy conservation for the flowing mixture.

During this same time the first version of COBRA was released in 1967. Designed specifically to analyze steady state core thermal hydraulic behavior as opposed to whole systems, COBRA-1 was a three-equation homogeneous equilibrium model (the third equation being the momentum equation). Crossflow was calculated simply through transverse resistance.

The situation with the system codes remained relatively unchanged but for minor improvements for almost a decade. FLASH-2 had lumped heat conduction added in 1967 whereas the RELAP series added a variable bubble rise model in 1968 and improved heat transfer correlations in 1970. COBRA-II was released

* See Reference 18, Chapter 13 for reference to these computer codes.

three years following COBRA-I also in 1970 with added correlations for sub-cooled boiling and transverse mixing as well as lateral heat conduction. A transient capability was made available in COBRA-III (1971) and a full lateral momentum equation replaced the Δp resistances in the 1973 COBRA-IIIC version which also included an implicit transient fuel conduction model.

In 1973 the NRC began an initiative which was to have a profound and lasting effect on the direction nuclear safety analysis was to take. Driven by the increasing pressure of public concern and intervention a massive public relations effort was mounted in an effort to demonstrate that commercial nuclear steam supply systems were safe under the maximum credible accident situations. The NRC would show the public that the technology was available or could be developed to accurately calculate the full sequence of events a nuclear power system would encounter during a LOCA.

It was immediately determined that all existing codes were inadequate because they only accounted for homogeneous flow with thermal equilibrium. The three-equation model was judged inadequate since it was held that an additional mass conservation equation was required as a minimum in order to adequately describe two-phase flows with differing phase velocities. And a code without momentum was unthinkable. RELAP4/MOD3 was released in 1975 with momentum flux added and an improved, distributed metal heat condition capability. One year later MOD5 appeared with slip added empirically and included the first reflood model. COBRA-IV was also released with new pressure boundary capability and the ability to handle flow reversal and recirculation. But more importantly, three new code development efforts were initiated: RELAP5, THOR, and TRAC. All were to employ new technology. Each was to provide full nonhomogeneous, non-equilibrium two-phase flow models and the best available constitutive modeling. THOR was to be a lumped parameter, fast running code based on the drift flux model developed to provide scoping calculations of reactor accidents. RELAP5 and TRAC were to be a detailed, but possibly slower, codes based on the two-fluid model, RELAP5 one dimensional and TRAC three dimensional, to calculate detailed behavior of severe accident situations. More about these later.

Continued improvements were undertaken in RELAP through the release of MOD7 in 1980 but no new versions of FLASH followed the release of FLASH-4. The RELAP-series had become the industry standard. Improved fuel pin models using gap conductance based on FRAP, improved heat transfer correlations and logic, automated renodalization for reflood calculations, and the addition of non-equilibrium calculations for the emergency core coolant injection mixture.

But these were to be stopgap measures. The real advances were to come through the work efforts involved in the advanced LOCA codes; RELAP5, THOR, and TRAC. These will be described in the next section.

FORMULATIVE ADVANCES

The first major advances came with the formalization of the conservation equations for two-phase flows. The work of Vernier and Delhay⁽¹⁸⁾, Delhay⁽²⁰⁾, Ishii^(21,23,24), Kocamustafaogullari⁽²²⁾, and Delhay and Achard⁽²⁵⁾ are noteworthy in that they represent the evolution of development of the field equations along with appropriate time and/or space averaging methods. Beside

the fact that this development is incomplete due at least to improper closure of the thermal energy equation, ⁽²⁶⁾ two facts are clearly evident:

1. Nonvanishing covariance terms between many of the variables arise from the averaging processes which are almost universally ignored in solution techniques; (c.f. Lahey⁽²⁷⁾).
2. Virtually no data exist from which experimental estimates of these correlations can be obtained, nor does it appear that any analytical estimates have been made, with the exception of the Zuber-Findlay C_o . ⁽²⁸⁾ representing the spatial covariance between the mixture volume flux and the vapor volume fraction.

Nevertheless, these much simplified but nonetheless quite complex sets of field equations are currently much in favor. Much money and manpower has been applied to the use of these equations in computer models for transient reactor analysis.

Consideration of the nature and extent of the field description of the flow requires consideration of the complexity and degree of freedom one wishes to deal with in solution to the problems at hand. The following generalizations apply:

1. Dynamic conservation of global mass requires specification of the mixture continuity equation or its equivalent and allows one to determine mixture homogeneous velocities as a minimum around the system.
2. Dynamic conservation of global energy via the first law of thermodynamics requires specification of the mixture energy equation or its equivalent allowing equilibrium single-phase temperatures or equilibrium qualities to be determined as a minimum around the system described.

These were the basic field equations utilized in the early LOCA codes and represent a relatively simple and computationally efficient means of arriving at quasi-static system behavior.

3. Conservation of momentum for purposes of dynamic pressure determination in and throughout the system requires specification of the mixture momentum equation or its equivalent.
4. Dynamic conservation of mass with different phase velocities requires specification of a second mass conservation equation, usually for one of the two phases, or its equivalent.
5. Dynamic conservation of momentum with different phase momenta (velocities) requires specification of a second momentum equation or its equivalent.
6. Dynamic conservation of energy with different phase temperatures requires specification of a second energy equation or its equivalent.

These generalizations are outlined in Table 1. Since phase velocity represents both a phase mass flux per unit mass and a phase momentum flux per unit mass, the additional degree of freedom entailed by allowing unequal phase velocities results in the requirement of two additional phase equations: continuity and

Velocity	E	E	U	U
Pressure	F	V	V	V
Temperature	E	E	E	U
Number of Field Equations	2	3	5	6

E = equal F = fixed U = unequal V = variable

Table 1. Field equation or equivalent requirement depending on detail desired in modeling.

momentum or their equivalent. Decoupling the phase temperatures from one another requires the addition also of a second energy equation or its equivalent. It has thus become common practice to talk in terms of an "N-equation model" referring to the various possibilities of field equation combinations. Of course, correlations can be used to replace field equations such as a correlation for relative velocity or "drift flux" resulting in, say, a four-equation model. Adding a third field such as boron or dissolved gas, or splitting one phase into separate parts such as liquid film and liquid drops, would result in one, two, or three additional field equations depending on the degree of freedom allowed the new field. For instance, boron tracking with identical temperatures and local velocities requires only an additional mass conservation equation which becomes the balance equation for the boron.

In order to proceed from here to see where advances have been or must be developed, a brief tutorial relative to the differences between the "drift-flux" and "two-fluid" models must be included.

The Drift-Flux Model

Kinematic relations. The major advantage of using the drift-flux model is that it is developed based on averaging properties of the flow stream across a region of flow, generally the area of the flow passage. As such, its applicability is to averaged duct behavior and its utility is in the use only of global parameters describing the thermofluid dynamic behavior. Since the area-averaging accounts for some smoothing of the heterogeneous nature of two-phase flow⁽³⁴⁾ time averaging has not been explicitly undertaken in development of the appropriate field equations and the short time statistical nature of the flow is thus ignored.

As it has come to be used, the term "drift-flux model" is used to denote any model which has its roots in a characterization of the relative velocity between gas and liquid as described by Zuber and Findlay,⁽²⁸⁾ but having its origins more properly identified with early work of Wallis.^(29,30) By writing

the relative velocity (v_r) between the gas (v_v) and the liquid (v_l) in terms of the volume fluxes of the two (j 's) and the volume fraction of the vapor (α) one obtains

$$v_r \equiv v_v - v_l = \frac{j_v}{\alpha} - \frac{j_l}{(1-\alpha)} \quad (1)$$

The drift flux (j_{vl}) is the definition of the result one achieves when clearing fractions. Thus,

$$j_{vl} \equiv \alpha(1-\alpha) v_r = (1-\alpha) j_v - \alpha j_l \quad (2)$$

If the total volume flow rate (Q) per unit area (A) is denoted by the total volume flux $Q/A \equiv j_v + j_l$, then it is easily seen that the drift flux j_{vl} is the volumetric flux of a component relative to a surface moving at the average volume flux, viz:

$$j_{vl} = \alpha(v_v - j) \quad (3)$$

The reason the drift flux has become so useful is due to its limiting behavior as seen by Eq. (2), vanishing at both concentration limits of all water ($\alpha=0$), and all vapor ($\alpha=1$), and is otherwise positive for vertical upflow. In addition, the relative velocity can normally be characterized in terms of the void fraction (α), the characteristic discussions of the system (either duct or bubble/drop sizes), and thermophysical properties of the fluid. Thus j_{vl} can be correlated with relative ease in terms of $\alpha(1-\alpha)v_r$.

On the other hand, the right-hand side of Eq. (2) for a particular combination of flow rates is a linear function of α having limits of $j_{vl} \rightarrow +j_v$ as $\alpha \rightarrow 0$ and $j_{vl} \rightarrow -j_l$ as $\alpha \rightarrow 1$. The intersection of these straight lines with the $\alpha(1-\alpha)v_r$ correlation represents the operating point of the system thereby yielding the local void fraction, α .

The major contribution of Zuber and Findlay⁽²⁸⁾ was to formalize the averaging processes to provide simple workable methods for predicting the void fraction through characterization of the relative velocity or its equivalent in global terms. Thus, writing the volume flux of the vapor in terms of the total Q/A and the drift flux yields

$$j_v = \alpha j + j_{vl} = \alpha j + \alpha V_{vj} \quad (4)$$

where V_{vj} (the upper case used for historical reasons) is termed the "drift" velocity^{vj} and represents the velocity of the gas relative to the volume flux (Q/A) of the mixture.

The key to the simplicity of the method is seen when Eq. (4) is area-averaged over the flow to yield

$$\frac{\langle j_v \rangle}{\langle \alpha \rangle} = C_o \langle j \rangle + \bar{V}_{vj} \quad (5)$$

The averaging operators are defined as

$$\langle f \rangle \equiv \frac{1}{A} \int_A f dA \quad \text{and} \quad \bar{f} = \frac{\int_A \alpha f dA}{\int_A \alpha dA} = \frac{\langle \alpha f \rangle}{\langle \alpha \rangle} . \quad (6)$$

Comparing the result of averaging Eq. (4) with Eq. (5) one immediately sees that

$$C_o = \frac{\langle \alpha j \rangle}{\langle \alpha \rangle \langle j \rangle} \quad (7)$$

and is one of the covariance terms previously mentioned. This covariance represents the only one to which any attention has been paid to date. While C_o does not represent a new parameter its reciprocal having been previously introduced by Armand⁽³¹⁾ in the Soviet Union and by Bankoff⁽³²⁾ in the U.S., its formal development by Zuber and Findlay marked the turning point in modern analysis upon which most later advances have been founded. Further, the development pointed to the two different mechanisms yielding an effective difference between phase velocities: void-velocity profile effects; local relative velocities. C_o represents the average effect of the former while $\bar{v}_{vj}$ represents the averaged^o effect of the latter.

The key to the success and utility of the drift flux model lies in the observation that for a given geometry and thermodynamic state, both C_o and $\bar{v}_{vj}$ have been found to be constant in most flow regimes, varying only from one flow regime to another. Thereby being quite suitable for experimental observation and correlation. This is because they can be determined quite easily by simply plotting experimental values of $\langle j_v \rangle / \langle \alpha \rangle$ vs. $\langle j \rangle$ from which the slope (C_o) and intercept ($\bar{v}_{vj}$) may be readily determined. Conversely, to enable the area-averaged void fraction $\langle \alpha \rangle$ to be determined locally, one need only specify the flow rates and flow regime and then determine C_o and $\bar{v}_{vj}$ based on the correlation for that regime. Thus

$$\langle \alpha \rangle = \frac{\langle j_v \rangle}{C_o \langle j \rangle + \bar{v}_{vj}} . \quad (8)$$

It is important to note here that the correlation of the void-weighted drift velocity is the same as correlating the area-averaged drift flux per unit averaged void fraction since from Eq. (4) and Eq. (3)

$$\bar{v}_{vj} = \frac{\int_A \alpha v_{vj} dA}{\int_A \alpha dA} = \frac{\int_A j_{vj} dA}{\int_A \alpha dA} = \frac{\langle j_{vj} \rangle}{\langle \alpha \rangle} . \quad (9)$$

Thus, the correlation of $\bar{v}_{vj}$ is an implicit correlation of the "averaged" relative velocity, effectively specifying the mechanical coupling between the phases. Referring back to Table 1 shows that a 5-equation model would thus become a 4-equation model, and was the basis for the original concept which led to the developmental effort on the THOR code. The TRAC code adopted the drift flux model for loop components resulting in a 5-equation model, with

two continuity and energy equations, and one momentum equation. However, the more recent version of TRAC, TRAC-PF1, ⁽³⁸⁾ has abandoned this in favor of a full two-fluid model in the one-dimensional components in order to better handle countercurrent stratified flows.

The actual form of the field equations used in the "drift flux" model depends on the developer. Lahey ⁽²⁷⁾ has described the formulation in the form of volume fluxes and void fractions whereas for the THOR code ⁽³³⁾ the development was in terms of the mass fraction of the vapor (c) and mass flux (G) of the mixture and could be more correctly termed a "diffusion velocity" model.*

Boundary and initial conditions. The drift flux model represents a 4-equation, one-dimensional or quasi-one-dimensional model where the major dependent variables are the phase volume fluxes, the phase temperatures, the pressure, and the void fraction. All these appear in time derivatives so that the initial state ($j_{l0}, j_{v0}, T_{l0}, T_{v0}, p_0, \alpha_0$) must be specified. This is usually done by assuming a steady-state condition exists prior to the start of a transient. Further, these same variables appear only in first derivative terms so only a single boundary condition need be specified for each variable at physical boundaries and also, through matching or jump conditions at the junctions of macroscopic flow volumes. (Diffusion of momentum is generally specified empirically through a D'Arcy-like friction term while thermal diffusion and viscous dissipation are neglected completely in fluids. Correlations relate j_v to j_l and T_v to T_l so the solution vector becomes (j_v, T_v, p, α) or equivalent.

Constitutive relations. It is taken for granted that both thermal and caloric equations of state must be specified. Further, the two major constitutive relations must be specified

1. Drift flux or other relationship coupling the velocity of one phase to another--eliminates the need for the second momentum equation.
2. Constitutive expression relating the temperature of one phase to that of the other or the temperatures of each to a third (saturation).

In addition, other constitutive equations for secondary, nonderivative parameters, must also be specified to provide closure for solving the set of field equations with appropriate initial and boundary conditions. These include:

3. The global rate of vapor generated per unit time, usually on a per unit volume basis--vapor source term;
4. The global friction coefficient usually expressed as a perimeter-averaged friction coefficient and a two-phase multiplier averaged over the volume of interest;

* Unfortunately, no final formal report has been issued summarizing the extensive technology developed during this program.

5. The perimeter-averaged heat flux from the wall usually expressed as through a single- or two-phase heat transfer coefficient and suitably averaged over the volume of interest.

In many circumstances, the advances to date have emphasized improved modeling for certain of these variables taking expected differences between flow regimes or heat transfer regimes into account. Therefore, additional correlations for the following tertiary variables must be provided:

6. Flow regime boundaries;
7. Heat transfer regime boundaries.

For the former, boundaries between regions of bubbly, slug, churn, annular, mist annular, mist, inverted annular, and single-phase-two-phase conditions must all be provided. For the latter, the boundaries between subcooled nucleate boiling, fully developed nucleate boiling, convective boiling or evaporation, and liquid deficient evaporation, must be specified.

Beside the constitutive specifications identified above, certain specific phenomena must also be modeled separately such as quench front velocity, or upper plenum mixing. Furthermore, some of these aforementioned correlations must be specified separately between one geometry and another.

In all cases, it is readily seen that for the drift flux model, only global parameters are specified through constitutive relationships--parameters which, by-and-large, can be measured or inferred from global measurements such as flow rates, pressures, void fractions, etc. This is the obvious power of the drift flux method, in spite of the large numbers of correlations required as previously specified. Because of this, substantial efforts have gone into developing such correlations and many advances have been made and much remains to be accomplished on this front.

The Two-Fluid Model

As the name implies, the two-fluid model tracks liquid and vapor phases separately. The model is much more akin to a standard thermo-fluid mechanical formulation in that the equations used are very much like those used to describe single-phase motion. Differences exist, however, due to the averaging methods used and the assumptions explicitly or implicitly invoked. The basic difficulty which arises is that interfacial matching conditions enter the picture requiring specification of interfacial transfers of mass, momentum, and energy, quantities which have virtually no experimental basis upon which to develop expressions, and quantities which in themselves seem very difficult to determine in the general case.

The basis of the two-fluid model currently used is that developed by Ishii^(21,23) based on a short time average. That is, taking the basic, well-known conservation equations and time averaging them over periods short with respect to system transients but long with respect to inherent phasic fluctuations. This may, however, prove to be an impossible situation but such possibilities are ignored.

Depending on the assumptions used (Table 1) two-fluid models will generally be either 5-equation or 6-equation models for the two major phases of liquid and vapor. RELAP5⁽³⁵⁻³⁷⁾ uses only the total energy equation while using the sum and difference of the two phasic continuity equations and similarly the sum and difference of the two phasic momentum equations thereby yielding one each for the mixture and for the difference between vapor and liquid. One phase is always assumed to be at saturation. TRAC^(38,39) currently uses a 6-equation model having separate phasic mass, momentum, and energy equations. For the PWR version (TRAC-PF1⁽³⁸⁾) an additional continuity equation for tracking non-condensable gas has been included. The COBRA-TF model^(40,41) adds a third field so as to separately track liquid film and liquid drops. This 8-equation model thus has three phasic continuity equations, three phasic momentum equations, and two phasic energy equations--the two liquid fields are assumed to be in thermal equilibrium with each other, but not necessarily with the vapor or at saturation.

Boundary and initial conditions. One of the main features in the one- or multi-dimensional formulations of two-fluid models, as in the drift flux model, is the elimination of second order terms. Thus, the solution vector is generally specified initially, $(v_{l0}, v_{v0}, T_{l0}, T_{v0}, p_0, \alpha_0)$. Boundary values must also be specified for these values at physical boundaries as well as at nodal boundaries through matching conditions. It should be noted that while an instantaneous solution to the multiregion two-phase flow problem would require the use of appropriate matching conditions at the bounding surfaces between regions, the interfaces, that the short term averaging has folded these boundary conditions into the equations. However, they still must be satisfied through appropriate specification of interfacial transfers as constitutive relations.

Constitutive relations. In addition to the usual state equations, the following major constitutive relations must be specified:

1. Interfacial momentum transfer generally expressed through local interfacial drag or friction coefficients, thereby coupling the velocity fields at interfaces;
2. Interfacial (interfield) mass and energy transfer (coupled through the interfacial energy balance and the assumption of local thermodynamic equilibrium on an interface) which is expressed through interfacial heat transfer coefficients, in order to couple the fluid temperature fields at interfaces. Interfield mass transfer rates must also be specified when a droplet field is included, thereby requiring entrainment and deposition of droplets from the liquid film field to be specified;
3. Interfacial area densities which, together with the interfacial transfer terms, allow the total interfacial transfers to be calculated.

In addition to these primary constitutive relationships which couple the fluids at their interfacial boundaries, there are other relations which are generally required to provide closure. These include

4. Wall friction coefficients are required to calculate the frictional pressure loss due to wall-fluid interaction in place of the "no-slip" condition eliminated with elimination of second order momentum terms;
5. Wall heat transfer coefficients are required to calculate the rate of heat transfer to or from solid boundaries of both the heat generating type (fuel elements) and nongenerating boundaries;
6. Flow regime and heat transfer regime boundaries are required to determine the appropriate correlation to use in virtually all five of the above regime-sensitive constitutive packages.

Numerous special purpose correlations are also required in special circumstances, some of which are enumerated below:

7. Countercurrent flow limitations such as required to predict the onset of core cooling from upper plenum spray cooling or upper head injection;
8. Quench front initiation and propagation rates;
9. Fuel rod gap conductance.

From the above, it is seen that while some of the modeling requirements for the two-fluid model are similar to those for the drift flux model, the major difference lies in the need to predict interfacial transfers and interfacial area densities--quantities for which little experimental evidence exists and for which direct measurement appears exceedingly difficult. Measurement and prediction of the interfacial transfers represents the problem and the challenge for two-fluid models, and the direction for further advances in thermal hydraulics of LWR's in the near future.

CONSTITUTIVE ADVANCES

Without doubt, the most significant advances in LWR thermal hydraulics have been in addressing and treating nonequilibrium--both mechanical and thermal. These have allowed the advanced models to be developed which exist today in the form of transient reactor analysis codes. These codes, as well as the THOR code which was not completed due to technical and funding difficulties, lead to substantial needs for development of accurate constitutive relations. Some of these needs remain unfilled today, except on the most rudimentary levels, due to many reasons, not the least of which is the lack of definitive experimental basis on which to formulate adequate conditions. This is especially true in the area of interfacial geometry and interactions.

Flow Regimes

One major factor which has changed in the last decade is the recognition of flow regime effects on various physical interactions and resulting correlations based on specific flow regimes. However, despite the substantial number of reported works delineating the various flow regimes, the regime descriptions currently utilized are generally quite rudimentary. In addition, different

flow regimes exist side by side for use in the same component but for different purposes. (38,39,42,43) This is reasonable if the differences exist for a valid technical reason such as the difference between heated and unheated walls, (40) but seems unjustified under other circumstances. Clearly more work is required first to objectively determine flow regimes, (34,49) and then to incorporate more reasonable descriptions of these regimes in the computational framework.

Mechanical Nonequilibrium

As pointed out earlier, the drift flux model really got its start with the publication of the work of Zuber and Findlay (28) in 1965 who observed that the void-weighted vapor velocity ($\bar{v}_v = \langle \alpha v \rangle / \langle \alpha \rangle$) when plotted against void fraction yielded straight lines in particular flow regimes. The slope of these straight lines was the void-volume flux covariance they termed C_0 while the intercept was the void-weighted drift velocity ($\bar{v}_{vi} = \langle \alpha v \rangle / \langle \alpha \rangle$). They found that the values for C_0 and $\bar{v}_{vi}$ both remained relatively well fixed for a given class of geometry and in a given flow regime. Ishii et al (44) later provided detailed recommendations for annular flows whereas Kroeger (45) estimated the pressure effect on C_0 for high speed flows and more recent estimates for inverted annular flows have been suggested. (33) But it is found that substantial differences exist between differing geometric situations such as downcomers. (46) Indeed, the problem of countercurrent flow in such situations as downcomers and core-upper plenum interfaces are areas which are still receiving considerable attention. (47) The difficulty here is that the correlation coefficients C_0 and $\bar{v}_{vi}$ seem to vary, in some cases substantially, from one geometry to the next, and even between one physical state and the next and no general rules governing these changes has yet been suggested. And the question of geometric scaling is a serious question yet to be answered satisfactorily. (46)

For the two-fluid formulation the phase velocities are primary dependent variables calculated through field equations which have incorporated the fluid-fluid interfacial interactions as well as the fluid-wall momentum transfers in the form, say,

$$(-1)^k \frac{C_i}{\alpha_k \rho_k} v_r |v_r| \quad \text{and} \quad - \frac{C_{wk}}{\alpha_k \rho_k} v_k |v_k| \quad k = \begin{cases} 1=l \\ 2=v \end{cases} \quad (10)$$

where ρ is the density and v_r is the relative velocity ($v_r = v_v - v_l$).

For the interfacial shear coefficient (C_i) the only known information is that provided by Wallis (5) for annular flows, and this has been incorporated into the advanced codes. Otherwise, there appears to be no direct information available so that concoctions of one form or another have been adopted based on spherical drag for single bubble or droplet elements and even linear interpolation between flow regimes where the flow pattern calls for such computations to be made. In one instance (40) an additional interfacial drag is calculated due to interfaces between computational cell boundaries computed as changes in void fraction between cells and for which an interfacial friction factor of 0.08 has been assigned. In this writer's opinion, such formulations, however whimsical they appear, are probably as good as can be expected until experiments and analysis provide better information. One wonders that the computed results for

pressure loss are as accurate as they appear to be and whether the interfacial terms are really important in the final results. An approximate check shows $C_{f1}/C_{f2} \sim 0 [\alpha/(1-\alpha)\phi_{lo}^2]$ for bubbly flow where ϕ_{lo}^2 is the two-phase friction multiplier. At 70 bar this ratio at 30% voids is about 0.1 and decreases monotonically as $\alpha \rightarrow 0$.

Wall shear coefficients are taken without modification from one of several global correlations currently existing for calculating two-phase frictional pressure losses except that the wall area fraction are arbitrarily assigned to each phase. Obviously, the use of two-phase friction multipliers determined experimentally based on a homogeneous model do not apply for use in a relative velocity model. The errors associated with this use, however, are unknown and may simply fall within the expected error associated with the multiplier itself.

Finally, although it is well known that transient accelerations lead to substantial variations in wall shear as well as particle drag coefficients, and that virtual mass effects can also play an important role, these have not been addressed within the framework of existing computational methods although Drew et al.⁽⁵⁰⁾ have recently analyzed virtual mass effects in two-phase flows.

Thermal Nonequilibrium

Conditions where thermal nonequilibrium can occur in LWR systems include: subcooled boiling; post-CHF; prenucleation decompression; critical flows with inlet subcooling; ECC injection; reflood heat transfer. While all these conditions may be encountered in an accident situation, subcooled boiling can occur under steady state or maneuvering transients and is important in hydrodynamic stability considerations. It has thus been the subject of innumerable investigations. Post-CHF heat transfer has also received wide study due to the concern regarding fuel integrity under accident situations. The status of investigations regarding thermal nonequilibrium was reviewed by Jones and Saha in 1977.⁽⁵¹⁾ Experimental and analytical studies in the U.S. relative to thermal nonequilibrium in various situations have been recently undertaken at Northwestern University,⁽⁶³⁾ relative to condensation processes in cocurrent and countercurrent flows, Lehigh University, Atomic Energy of Canada Ltd., and Brookhaven National Laboratory relative to post-CHF,⁽⁶⁰⁻⁶²⁾ and at Brookhaven relative to flashing flows,^(53,56,58) among others.

Nonequilibrium thermal differences between the temperatures of each phase are specified generally through correlation of the vapor generation rate. In the drift flux model this vapor generation rate is based on global parameters whereas in the two-fluid model the phase change rate is based on interfacial mass and heat transfer. Thus

$$\Gamma_v = h_{il} \frac{A_i}{V} (T_l - T_s) + h_{iv} \frac{A_i}{V} (T_v - T_s) \quad (11)$$

where both the interfacial area per unit volume (A_i/V) and the interfacial heat transfer coefficients on the liquid side (h_{il}) and on the vapor side (h_{iv}) must be separately specified. It is generally assumed that the interface temperature T_s is at saturation relative to the local pressure.

Interfacial area densities have been initially modeled by Saha, ⁽⁵²⁾ and in more detail recently by Wu, Abuaf, and Saha. ⁽⁵³⁾ Very rudimentary values are included in RELAP5 ^(36,37) and TRAC ^(38,39) whereas more advanced modeling is included in COBRA-TF ^(40,41). More detailed modeling has been recently undertaken by Ishii et al. ⁽⁴⁸⁾ for the four basic flow regimes including dispersed (droplet or bubbly), slug, churn turbulent, and annular. They also indicate that there are but limited data available upon which to base a correlation. Since the droplet concentrations in annular dispersed flow are very important, not only since they carry substantial area but also for their cooling ability in post-dryout and for precursory cooling in reflood, Ishii et al. have placed special emphasis on entrainment and on droplet sizes of entrained droplets. The resulting correlation ⁽⁴⁸⁾ represents a major recent advance in this area.

Similar to the case of interfacial shear coefficients, measurements of interfacial heat transfer coefficients appear virtually nonexistent and the values used in the codes are a mixture of untested relationships adopted from single particle correlations or simply assumed with no apparent reference or justification. Clearly this is an area which needs improvement.

From a practical global viewpoint, there have been numerous advances in the last decade to allow the actual nonequilibrium temperatures, qualities, and/or void fractions to be predicted. These advances are much too numerous to be addressed here and, indeed, one would have to review the entire field of boiling heat transfer. These methods have, by and large, been enumerated elsewhere. ^(6,51)

It has been shown unequivocally that the process of nonequilibrium vapor generation is a relaxation process. ^(51,54) Heating, cooling, pressurization, or decompression all set up relaxation potentials through which the fluid changes phase due to heat transfer to the interfaces and resulting mass transfer from the interfaces. As such they are all path-dependent processes requiring an initial value or initial condition for complete and proper determination. As a result, only in a limited, coincidental group of circumstances can either the relaxation potential (liquid-vapor temperature difference) or the rate of mass and heat transfer be specified as point functions. It is therefore exceedingly important that inception criteria be thoroughly and accurately detailed: the point of net vapor generation in subcooled boiling; ⁽⁵⁵⁾ the location of the critical heat flux as the onset for post-dryout heat transfer; and the point of flashing inception in decompressing flows. ⁽⁵⁶⁻⁵⁸⁾ Furthermore, since the relaxation potential at a given point is dependent on the balance between the relaxation process and the forcing process integrated over the path from the inception point, any point correlation for a local heat transfer coefficient must ultimately be correlating implicitly, not only the local heat transfer, but also simultaneously the inception point and the past history (path effects).

For instance, critical heat flux correlations are known to perform poorly in prediction of the location of CHF in all but the most simple, uniform heat flux situations. It is not surprising, then, that recent correlations for local heat transfer in post-dryout conditions have errors representing many tens or even hundreds of degrees in wall superheat. ^(59,60) Even a simple empirical improvement taking into account the CHF location separately resulted in almost a 50% improvement in the scatter ⁽⁶¹⁾ and a relatively coarse estimate for the

path process resulted in a wall temperature prediction in post-CHF having an RMS error of only 8°C to 12°C .^(54,62)

Further advances in thermal nonequilibrium must rely on improved estimates for interfacial area densities, interfacial heat fluxes, and proper specification of the inception criteria. Ingenuity, cleverness, and accuracy must be utilized to design and conduct experiments and analysis to identify these important interfacial characteristics.

Critical Flows

Much work has been historically undertaken regarding the prediction of critical two-phase two-component and single-component flows through various geometries. Excellent summaries have been previously prepared by Bouré⁽⁶⁴⁾ and Saha,⁽⁶⁵⁾ and represent well the current technology regarding separate effects predictive analysis. Both authors stress the need for further understanding of the nonequilibrium processes involved. What is obvious is that despite the many excellent works, there is no general method of predicting critical two-phase flows as there is with single-phase flows and numerous empiricisms abound for the various cases of short vs. long pipes, subcooled inlet, etc.

One major advance has occurred which has done much to replace the previous methodology where large computational systems are involved. (Of course the relatively simple, separate effects models will continue to find wide usage where quick calculations are required.) This is the direct calculation of choking by inversion of the determinant of the coefficient matrix of the partial differential field equations describing the flows. It is emphasized here, however, that the accurate use of these methods can only be successful if the interfacial transfer terms are adequately understood,⁽⁶⁴⁾ especially true when these transfer terms involve differentials. It is entirely inadequate, for instance, to analyze equilibrium flows and then draw accurate conclusions regarding such things as the virtual mass coefficient.⁽³⁹⁾

Another recent result which may represent a substantial advance in this area is the observation that flashing inception of subcooled liquids flowing through restricted geometries is relegated to a zone relatively close to the minimum area.⁽⁵⁸⁾ The practical effect of this observation is to turn this problem into a single-phase flow problem: subcooled liquid inlet; single-phase to on throat; a well-defined flashing inception pressure at the throat;⁽⁵⁶⁾ a single-phase discharge coefficient. Critical flows of subcooled liquids through nozzles have thus been correlated within the expected accuracy of the discharge coefficients and throat pressure prediction.⁽⁵⁸⁾

Further advances in accurate, generalized predictions of critical flows require better correlations of the nonequilibrium phase change rates and improved formulations for calculating relative velocities between gas and liquid in high speed flows. Especially important is accurate modeling of the interfacial transfer terms involving differential laws as these feed directly and significantly into the prediction of critical flow rates.⁽⁶⁴⁾

DISCUSSION AND CONCLUSIONS

What has become obvious through the preceding discussion, which has dealt exclusively with phenomena and methods rather than applications, is that the prediction of hydrodynamic behavior of two-phase flows, through the utilization of the two-fluid model, has become a product of the field and boundary specification, and the constitutive formulation of the microphenomena involved--the interfacial momentum transfer (shear), the interfacial heat transfer, and the extent to which adequate physical descriptions are given these processes. But here we find an interesting anachronism. The application of microphenomena has been a relatively selective application. It has been directed towards the non-differential and first order terms only whereas the terms which usually become second order through application, for instance, of the Newtonian and Fourier hypotheses have been either neglected, or maintained zeroth or first order in their approximations. (COBRA-TF^(40,41) models the molecular and turbulent diffusion terms but can not use sufficiently fine nodalization to match the wall boundary microconditions.) Thus, wall-fluid interactions have been maintained in the macroscopic sense so that the concepts of friction and heat transfer coefficients are maintained in the global sense.* This is really a fortuitous result but is certainly beneficial since, had second order closure been required, computer running times would have been astronomical. As it is now they are just very large. But certainly, some deficiencies result from this mixture, most likely those which result in profiles transverse to the flow direction. The inability of the TRAC-PD2 code to accurately compute severe multidimensional void profiles in two-dimensional configurations,⁽⁶⁶⁾ may be a result of this mixture and the lack of any real approximation of the turbulent momentum and heat transfer processes.

Calculation of particular thermal hydraulic phenomena has thus become a matter of general application of the combination of microscale and macroscale models. Countercurrent flow and flooding, for instance becomes grist for the two-fluid mill (although the BWR version of TRAC apparently has installed a separate CCFL model of the Wallis type⁽³⁹⁾). Computation of the progress of the fuel temperatures through a reactor system blowdown becomes a result of the heat transfer coefficients applied during various portions of the computed hydrodynamic process (although the critical heat flux condition is still explicitly determined by correlation). The progress of reflood and rewet become results, for instance, of the flow and heat transfer regime mapping (for instance inverted annular flow at low qualities), entrainment precursory cooling due to the amount of liquid droplets (very important), fine nodalization near the rewet front and a locally detailed fuel pin conduction model for rewet front velocity (rather than an explicit correlation for this velocity), and of countercurrent liquid fallback at the upper plenum-core interface (generally interfacial momentum transfer).

Thus, the real advance of the last decade has been the progression from explicit calculation of various events by way of correlations to implicit calculation of the event sequence based on application of process and field modeling of the flows. One can argue whether or not the invoking of such detail, especially when based on transfer phenomena which are poorly understood

* Even in the present sense, microscale modeling is not truly microscopic in a geometric sense but rather is directed to flow-cell regions of the flow which may be of the same size order as components.

and for which an evidential data base is largely lacking, is really an advance. Nevertheless, this is the direction chosen, largely because reasonable results were available at a much lower stage of evolution in the computational framework than for drift flux models. The latter are much more heavily dependent on a higher order of completion of global formulations requiring much more early emphasis on modeling for adequate results. This is clearly evidenced by the abandonment of the drift flux model in TRAC-PF1⁽³⁸⁾ in favor of the two-fluid model for the one-dimensional components. It was obviously easier to build in a horizontal flow regime description for the two-fluid model than to accurately analyze the range of conditions encountered to directly calculate relative velocities through the drift flux formulation. Further major advances are required in measuring and modeling the microphenomena of interfacial structure and transfer.

However, all is not "peaches and cream." The fertilizer in the two-field rose bed is the fact that the finer nodalization required in the two-fluid models makes the result much more dependent on the user and his art of choosing the proper nodal model. Code assessment efforts at various national laboratories have clearly shown that different code users can get very different results depending on the nodalization used.⁽⁶⁶⁾ Further, there is an obvious and substantial difference between the calculation of results where no data exist, and where data exist in the same circumstance as evidenced by pretest and post-test comparisons.⁽⁶⁶⁾ Make no mistake. Use of these methods is as much an art as a science. It is not clear that these tools can yet be used to believably calculate where we want to go if we haven't been there before (although the hot leg injection results⁽⁶⁷⁾ indicate that progress may be being made in this area). Clearly either this sensitivity must be eliminated or else a corps of computational Rembrandts must be trained if these codes are to achieve widespread effective use.

The future direction for LWR thermal hydraulics lies in improvements in the two-fluid model. Much more emphasis must be placed on an adequate determination of the interfacial characteristics. Experiments must be designed and undertaken. Analysis must be completed based on these results. These will not be either easy or short term--or else they would have been already accomplished. The question of scaling must be addressed in relation to actual reactor components relative to experimental results. Also, a broad range of reactor operational and upset conditions should be addressed to insure computational capability and credibility exist in all areas, not just in design basis accidents. In addition, emphasis has shifted already and will continue to shift more towards analysis of degraded core situations and much experimental and analytical work will be required here. But we should not mistake this shift in emphasis for a completion of the previously emphasized area. The politics of R&D funding are notoriously fickle with seemingly little care regarding the adequacy of solutions to yesterday's problems. But the credibility of nuclear safety analysis methods depends on utilization of believable, accurate, well implemented physics. Otherwise, advanced methods remain nothing more than highly complex, semi-empirical computer fits to system behavior where the many adjustable coefficients are merely buried layers deeper than in stand-alone correlations.

Finally, this writer would normally like to say that emphasis should be placed on the thermal hydraulics required to get better fuel utilization--higher power levels in existing reactors with higher burnup. However, it is

not likely that this will happen in today's sociopolitical climate unless a massive public relations campaign is undertaken to give the nuclear industry a much needed facelift. To paraphrase a recent spokesman on the international scene, it may not be so much as transfusing a patient in intensive care as building his coffin. Clearly there are people in some quarters already gathering the lumber. This situation must be turned around if there are to be continued advances in any area nuclear and therefore, continued advance in LWR thermal hydraulics may, in fact, be dependent on heroic measures towards the resuscitation and recovery of the nuclear industry.

REFERENCES

1. Bergles, A.E., "Two-Phase Flow and Heat Transfer, 1956-1980," in Two-Phase Flow and Heat Transfer in the Power and Process Industries, A.E. Bergles et al., Hemisphere, New York, 1981.
2. Kutatladze, S.S., and Styrikovich, M.A., Gidravlika Gaso-Zhidkostiyykh Sistem (Hydraulics of Gas-Liquid Systems), Gosudarstvennoye Energeticheskoye Izdatel'stvo, Moscow, 1958. (Translation prepared by the Technical Information Center, Wright-Patterson AFB F-TS-9814/V (NP-Tr-550) Sept., 1960).
3. Tong, L.S., Boiling Heat Transfer and Two Phase Flow, John Wiley and Sons, New York, 1965.
4. Soo, L.S., Fluid Dynamics of Multiphase Systems, Blaisdell, Waltham, Mass., 1967.
5. Wallis, G.B., One Dimensional Two Phase Flow, McGraw Hill, New York, 1969.
6. Collier, J.G., Convective Boiling and Condensation, McGraw Hill, New York, 1st Ed. 1972, 2nd Ed., 1981.
7. Hewitt, G.F., and Hall-Taylor, N.S., Annular Two-Phase Flow, Pergamon Press, Oxford, 1970.
8. Govier, G.W., and Aziz, K., The Flow of Complex Mixtures in Pipes, Van Nostrand Reinhold, New York, 1972.
9. Hsu, Y.Y., and Graham, R.W., Transport Processes in Boiling and Two Phase Systems, Hemisphere/McGraw Hill, New York, 1976.
10. Jones, O.C., Jr., and Bankoff, S.G., Thermal and Hydraulic Aspects of Nuclear Reactor Safety, Vol. 1: Light Water Reactors, Amer. Soc. Mech. Eng., New York, 1977.
11. Lahey, R.T., Jr., and Moody, J.F., The Thermal-Hydraulics of a Boiling Water Nuclear Reactor, Amer. Nucl. Soc., Hinsdale, Illinois, 1977.
12. Butterworth, D., and Hewitt, G.F., Two Phase Flow and Heat Transfer, Oxford University Press, 1977.

13. Hahne, E., and Grigull, U., Heat Transfer in Boiling, Hemisphere, York, 1977.
14. Ginoux, J.J., Two-Phase Flows and Heat Transfer with Application to Nuclear Reactor Design Problems, Hemisphere, New York, 1978.
15. Hewitt, G.F., Measurement of Two-Phase Flow Parameters, Academic Press, London, 1978.
16. Delhaye, J.M., Giot, M., and Reithmuller, M.L., Two-Phase Flows in Nuclear Reactors, Hemisphere/McGraw Hill, New York, 1979.
17. Bergles, A.E., et al., Two-Phase Flow and Heat Transfer in the Power and Process Industries, Hemisphere, New York, 1981.
18. Jones, O.C, Jr., Nuclear Reactor Safety Heat Transfer, Hemisphere/McGraw Hill, New York, 1981.
19. Vernier and Delhaye, J.M., "General Two-Phase Flow Equations Applied to the Thermodynamics of Boiling Nuclear Reactors," Energie Primaire, 4, No. 1, 1978.
20. Delhaye, J.M., "Equations Fondamentales des Ecoulements Diphasiques," CEN-G report CEA-R-3429, 1968.
21. Ishii, M., "Thermally Induced Flow Instabilities in Two-Phase Mixtures in Thermal Equilibrium," Ph.D. thesis, Georgia Inst. Tech., 1971.
22. Kocamustafaogullari, G., "Thermo-Fluid Dynamics of Separated Two-Phase Flow," Ph.D. thesis, Georgia Institute of Technology, 1971.
23. Ishii, M., Thermo-Fluid Dynamic Theory of Two-Phase Flow, Eyrolles, Paris, 1975.
24. Ishii, M., "One-Dimensional Drift Flux Model and Constitutive Equations for Relative Motion Between Phases in Various Two-Phase Flow Regimes," Argonne report ANL-77-47, 1977.
25. Delhaye, J.M., and Achard, J.L., "On the Use of Averaging Operators in Two-Phase Flow Modelling," in Thermal Hydraulic Aspects of Nuclear Reactor Safety, O.C. Jones, Jr., and S.G. Bankoff, Eds., Amer. Soc. Mech. Engrs., New York, 1977.
26. Bergles, A.E. et al., "Suggestions for Further Research and Urgent Problems in Two-Phase Flow and Heat Transfer," in Two Phase Flows and Heat Transfer, III, Kakac, S., and Veziroglu, T.N., Eds., Hemisphere, 1976.
27. Lahey, R.T., Jr., "Chapter 8-Nuclear System Safety Modelling," in Nuclear Reactor Safety Heat Transfer, O.C. Jones, Jr., Ed., Hemisphere/McGraw Hill, New York, 1981.
28. Zuber, N., and Findlay, J.A., "Average Volume Concentration in Two-Phase Flow Systems," Trans. Amer. Soc. Mech. Eng., J. Heat Trans., 87, pp. 453-461, 1965.

29. Wallis, G.B., "Some Hydrodynamic Aspects of Two-Phase Flow and Boiling," Proc. Int. Heat Trans. Conf., Amer. Soc. Mech. Eng., New York, pp. 319-340, 1961.
30. Wallis, G.B., "A Simplified One Dimensional Representation of Two-Component Vertical Flow and its Application to Batch Sedimentation," Proc. Interaction Between Fluids and Particles, pp. 9-16, IChE, London, 1962.
31. Armand, A., "The Resistance During Movement of a Two-Phase System in Horizontal Pipes," AERE-trans-828, 1985. (From *Izvestiya Vsesoyuznogo Teplotekhnicheskogo Institute No. 1*, pp. 16-23, 1946.)
32. Bankoff, S.G., "A Variable Density, Single-Fluid Model for Two Phase Flow with Particular Reference to Steam/Water Flow," Trans. Amer. Soc. Mech. Eng., J. Heat Trans., 82, p. 265, 1960.
33. Wulff, W., et al., "THOR-1 (PWR): A Computer Code for Predicting the Thermal Hydraulic Behavior of Nuclear Reactor Systems," BNL-NUREG-29781, 1981.
34. Jones, O.C., Jr., and Zuber, N., "The Interrelation Between Void Fraction Fluctuations and Flow Patterns in Two-Phase Flow," Int. J. Multiphase Flow, 2, pp. 273-306, 1975.
35. Jackson, J.F., et al., "Chapter 13-LWR System Safety Analysis," in Nuclear Reactor Safety Heat Transfer, Jones, O.C., Jr., Ed., Hemisphere/McGraw Hill, New York, 1981.
36. Ransom, V.H., "RELAP5 Program Status and Future Plans," presentation documentation, private communication, January, 1982.
37. Ransom, V.H., et al., "RELAP5, Status and Application Experience," presented at the 9th WRSR Info. Mtg., Oct. 26-30, 1981.
38. Liles, D.R., and Mahaffy, J.H., "TRAC-PF1: An Advanced Best Estimate Computer Program for Pressurized Water Reactor Analysis," private communication, January, 1982.
39. Spore, J.W., et al., "TRAC-BD1: An Advanced Best Estimate Computer Program for Boiling Water Reactor Loss-of-Coolant Accident Analysis," EG&G Idaho report EGG-2109, October, 1981.
40. Thurgood, M.J., et al., "COBRA-TF Equations and Constitutive Models," private communication, June, 1981.
41. Stewart, C.W., et al., "Improvements to the COBRA-TF (EPRI) Computer Code for Steam Generator Analysis," EPRI Report NP-1509, September, 1980.

42. Rohatgi, U.S., and Saha, P., "Constitutive Relations in TRAC-PLA," BNL-51258, August, 1980.
43. Rohatgi, U.S., Jo, J., and Neymotin, L., "Constitutive Relations in TRAC-PD2," BNL report to be issued, January, 1982.
44. Ishii, M., Chawla, T.C., and Zuber, N., "Constitutive Equation for Vapor Drift Velocity in Two-Phase Annular Flow," AICHE J., 22, pp. 283-289, 1976.
45. Kroeger, P.G., "Modelling of the Pipe Break," in Development of a Computer Code for Thermal Hydraulics of Reactors (THOR), Second Quarterly Report, BNL-50455, April, 1975.
46. Block, J.A., and Schrock, V.E., "Emergency Cooling Water Delivery to the Core Inlet of PWR's During LOCA," in Thermal Hydraulic Aspects of Nuclear Reactor Safety, Vol. 1, Light Water Reactors, Jones, O.C., Jr., and Bankoff, S.G., Eds., Amer. Soc. Mech. Eng., New York, 1977.
47. Thomas, D.G., and Combs, S.K., "Measurement of Two-Phase Flow at the Core Upper Plenum Interface Under Simulated Reflood Conditions," presented at the 9th WRSR Info. Mtg., Gaithersburg, Md., October 26-30, 1981.
48. Ishii M., et al., "Phenomenological Modelling of Two-Phase Flow in Water Reactor at ANL," presented at the 9th WRSR Info. Mtg., Gaithersburg, Md., October 26-30, 1981.
49. Jones, O.C., Jr., and Zuber, N., "Slug Annular Transaction with Particular Reference to Narrow Rectangular Ducts," in Two-Phase Momentum Heat and Mass Transfer in Chemical, Process, and Energy Engineering Systems, Durst, F., et al. Ed., Hemisphere Pub. Corp., New York, 1979.
50. Drew, D., Cheng, L., and Lahey, R.T., Jr., "The Analysis of Virtual Mass Effects in Two-Phase Flow," Int. J. Multiphase Flow, 5, 1979.
51. Jones, O.C., Jr., and Saha, P., "Non-Equilibrium Aspects of Water Reactor Safety," in Thermal Hydraulics of Nuclear Reactor Safety, Vol. 1, Light Water Reactors, Jones, O.C., Jr., and Bankoff, S.G., Eds., Amer. Soc. Mech. Eng., New York 1977.
52. Saha, P., in "Reactor Safety Programs: Quarterly Progress for the Period April-June 1977," BNL-50683, p. 145, 1977.
53. Wu, B.J.C., Abuaf, N., and Saha, P., "A Study of Nonequilibrium Flashing of Water in a Converging-Diverging Nozzle, Volume 2--Modeling," BNL-51317, 1981.
54. Jones, O.C., Jr., "Towards a Uniform Viewpoint of Thermal Nonequilibrium," Nucl. Eng. Des., 1982, in press.

55. Saha, P., and Zuber, N., "Point of Net Vapor Generation and Vapor Void Fraction in Subcooled Boiling," Proc. Fifth Int. Heat Trans. Conf., Vol. IV, pp. 175-179, 1974.
56. Jones, O.C., Jr., "Flashing Inception in Flowing Liquids," Trans. ASME, J. Heat Trans., 102, pp. 439-444, 1980.
57. Alamgir, M., and Lienhard, J.H., "Correlation of the Pressure Undershoot During Hot-Water Depressurization," Trans. ASME, J. Heat Trans., 103, pp. 52-55, 1981.
58. Abuaf, N., Jones, O.C., Jr., and Wu, B.J.C., "Critical Flashing Flows in Nozzles with Subcooled Inlet Conditions," in Polyphase Flow and Transport Technology, Bajura, R.A., Ed., Amer. Soc. Mech. Eng., New York, 1980.
59. Groeneveld, D.C., "Post-Dryout Heat Transfer at Reactor Operating Conditions," proc. Nat'l Topical Mtg. on Water Reactor Safety, Amer. Nucl. Soc., 1973.
60. Chen, J.C., Sundaram, R.K., and Ozkaynak, F.T., "Phenomenological Correlation for Post-CHF Heat Transfer," NUREG-0237, 1977.
61. Groeneveld, D.C., and Delorme, G.G.J., "Prediction of Thermal Non-Equilibrium in the Post Dryout Region," Nucl. Eng. Des., 36, pp. 17-36, 1976.
62. Jones, O.C., Jr., "Liquid Deficient Cooling in Dispersed Flows: A Non-Equilibrium Relaxation Model," BNL-NUREG-50639, 1977.
63. Bankoff, S.G., Kim, H.J., and Lee, S.C., "Countercurrent Flow of Steam and Cold Water in Inclined Rectangular Channel," presented at the 9th WRSR Info. Mtg., Gaithersburg, Md., October 26-30, 1981.
64. Bouré, J.A., "The Critical Flow Phenomenon with Reference to Two-Phase Flow and Nuclear Reactor Systems," in Thermal and Hydraulic Aspects of Nuclear Reactor Safety, Vol. 1, Light Water Reactors, Jones, O.C., Jr. and Bankoff, S.G., Eds., Amer. Soc. Mech. Eng., New York, 1977.
65. Saha, P., "Review of Two-Phase Steam Water Critical Flow Models with Emphasis on Thermal Nonequilibrium," BNL-50907, 1978.
66. Odar, F., "Status of Code Assessment," present at the 9th WRSR Info. Mtg., Gaithersburg, Md., October 26-30, 1981.
67. Rothe, P., "Flow Reversal Studies During Hot Leg Injection," presented at the 9th WRSR Info. Mtg., Gaithersburg, Md., October 26-30, 1981.

NOMENCLATURE

Symbols

A = area
c = mass concentration
C = shear coefficient
C_o = Zuber-Findlay distribution parameter
h_o = heat transfer coefficient
g = volume flux (superficial velocity)
p = pressure
Q = volume flow rate
T = temperature
v = velocity
V = volume
V_{vj} = drift velocity

α = void fraction
Γ = vapor source

Subscripts and Operators

i = interface
k = phase-k (1=liquid, 2=vapor)
ℓ = liquid
o = initial state
r = relative (gas less liquid)
s = saturation
v = vapor
vℓ = vapor with respect to liquid

< > = area average
- = void weighted area average

SODIUM-VOID REACTIVITY IN LMFBRs: A PHYSICS ASSESSMENT*

C.L. Beck, M.J. Lineberry, H.F. McFarlane, and D.N. Olsen

Argonne National Laboratory
Applied Physics Division
P.O. Box 2528
Idaho Falls, Idaho 83401

ABSTRACT

An extensive recalculation of sodium-void measurements from the U.S. critical experiment program has been completed. Three-dimensional diffusion-perturbation methods and ENDF/B-Version IV data in 28 groups were used throughout this study. The results of the calculations establish the degree of confidence in the prediction of sodium-void worths in typical LMFBR environments, and further demonstrate the usefulness of a two-component fitting process for biasing calculations in order to produce better predictions of void worths. Measurements in ninety-five zones were analyzed, resulting in a mean C/E (calculation/experiment) ratio of 1.12 ± 0.20 before fitting, and 0.99 ± 0.09 after fitting. Bias factors for both leakage and non-leakage components were in the range 0.85 - 0.92 for a variety of design parameter variations, i.e. a number of different subsets of the data base. These results may be used to extrapolate from the critical assembly data to general LMFBR application and to estimate the uncertainty of the predicted values.

INTRODUCTION

This paper summarizes recent work to put the analysis of past sodium-void experiments from the U.S. critical experiment program on a common basis. This is a necessary prelude to full utilization of the data in LMFBR design, safety analysis, and licensing. In this study, a selected group of critical experiments provides the basis for testing calculation methods and models. The data selected, a subset of the numerous measurements done over the last dozen years, provides sodium-void information in a range of reactor environments -- for example, size, enrichment, heavy-metal volume fraction, heterogeneous and homogeneous arrangement, etc. A consistent calculational approach is chosen which incorporates models and methods judged by previous experience to predict, as accurately as feasible, the sodium-void worths measured in plate-critical assemblies. Results are statistically analyzed to determine quantitatively the level of confidence and to identify trends. Finally, issues in extrapolating these results to LMFBR safety analysis are explored briefly.

Analysis of hypothetical accidents (CDAs) is an important part of the fast reactor safety program. The need to maintain integrity of the containment during CDAs places constraints on the energy release that can be tolerated. Calculation of energy releases is complex and depends on many factors, but sodium-void reactivity is perhaps the most important single element.

*Work performed under the auspices of the U.S. Department of Energy.

There is no simple relation between whole-core sodium-void reactivity and CDA energetics. It is well known, though, that the relationship is very non-linear. The U.S. consensus appears to be that sodium-void reactivities up to perhaps 2\$ are acceptable. This is based on the assumption that prior to fuel melting, Doppler and other feedbacks will control about 1\$. For acceptable consequences, super prompt critical is the limit, thus the 2\$ value. On the other hand, it appears that if void reactivities are about 4\$ or greater, then highly-energetic transient-overpower (LOF/TOP) events, driven by an unprotected loss of flow, cannot be precluded.¹ This large variation in energetics over a relatively small range of sodium-void reactivities makes accurate knowledge of such reactivities important.

Confidence in sodium-void calculations has increased significantly since the last detailed U.S. survey by Hummel and Okrent.² Following the discovery in 1959 that loss of sodium could increase system reactivity³, sodium-void was a persistent issue of the 1960s, but at that time little confidence could be placed in its prediction. Calculations made by different laboratories varied widely. Predictions in one example ranged from -1\$ to +2\$ for the reactivity of the same voided configuration.⁴ Critical experiments provided only limited insight during this period because they were in small or uranium-fueled assemblies. Computational methods were fairly primitive by present standards.

Progress has come as a result of several improvements. In the critical experiments area, the data base has been greatly expanded by the addition of a large number of sodium-void reactivity experiments on a variety of core configurations and sizes in ZPPR, ZPR-6, and ZPR-9.⁵⁻⁸ Evolution of computer technology made extensive application of more powerful methods feasible, for example, three-dimensional exact diffusion perturbation calculations are routine. The differential cross section data, as characterized by the various versions of ENDF/B, have improved significantly. To form the basis for this assessment, an extensive recalculation of many of the integral measurements was undertaken to capitalize on these improved methods and cross section data.

EXPERIMENTAL DATA BASE

Not all of the data from critical experiments programs have been included in this assessment. The data base includes only zone-type void-worth measurements from the U.S. program, and then only a subset which was deemed sufficient to characterize sodium-void worth in typical LMFBR environments.

Zone voiding worths are determined by measuring the change in reactivity of the assembly after the removal of sodium from a zone. Measurements in ZPPR have used three techniques: 1) the position of a calibrated B_4C shim rod, 2) period measurements, and 3) subcritical source multiplication.

The list of experiments shown in Table I was chosen to contain modern data which were measured with current and generally consistent experimental techniques, and to scope a range of parameters of interest, e.g. heavy metal volume fraction and enrichments, isotopic variations, core size,

TABLE I. Selected Data Base of Sodium-Void Experiments
in a Range of Assembly Types

Assembly Designation	Assembly Description	Description of Void Zones	Number of Measurements
ZPR 6/7	LMFBR Oxide Benchmark; one-zone core	Small, central	1
ZPPR-2	LMFBR Oxide Benchmark; two-zone core	Central	1
ZPPR-3/3, -5A, -5B	CRBR-size homogeneous cores; BOL and BOL with control	Extensive radial and axial steps	31
ZPPR-7A, -11E, -11F	CRBR-size heterogeneous cores; benchmark, EOC (no fission products) and BOL	Extensive radial and axial steps	28
ZPPR-9, -10A, -10B	700 MWe two-zone homogeneous cores; benchmark, BOL and BOL with control	Central zones radial and axial steps	34

design, geometry variations and control rod environments. Uncertainties on the measured reactivity changes are seldom greater than 2%.

Notably missing from the measurements in Table I is information about sodium-void reactivity in large heterogeneous cores and in pin zones. The pin zones have different leakage characteristics than plate zones with the same composition, and testing of the relative capabilities to predict sodium-void reactivity in the two environments is essential in the extrapolation to an LMFBR. Measurements in some pin zones have been done, but consistent analysis has not yet been applied. Larger heterogeneous core data are required because such cores dominate current U.S. LMFBR design thinking. With problems already noted in the smaller heterogeneous cores, confidence in sodium-void reactivity predictions will require experimental confirmation.

REFERENCE METHOD AND MODELS

The reference method of calculation for this study was diffusion theory in 28 energy groups, generally in three dimensions, with anisotropic diffusion coefficient modifiers used to account for neutron streaming. ENDF/B-IV cross section data are used throughout the studies. However, the values of β_{eff} that were used to convert the calculated results to the dollar units of the measurements are based on ENDF/B-V delayed neutron data.

The DIF3D/VARI3D diffusion-perturbation codes⁹ were used to calculate fluxes and reactivity. Exact perturbation (EP) calculations were used for

generating reactivity changes of the void steps. The perturbation was taken as the change from flooded unit-cell to voided unit-cell cross-section data, and therefore changes in effective group constants of all materials were included. The EP method involved using the reference (flooded) real flux and a perturbed (voided) adjoint flux in the perturbation calculation. The principal reason for selecting this technique instead of simply taking the difference between the eigenvalues is that a listing of perturbation worths by components (e.g., leakage, absorption, etc.) is made available.

Because of the cost of three-dimensional calculations, a relatively coarse mesh structure was used (~ 5 cm). This corresponds, in the radial plane, to one mesh per ZPPR matrix position.

Cross section preparation involved several steps in generating effective cross sections which could be used in the homogeneous whole-reactor calculations. While somewhat different models were used for the conventional and heterogeneous core designs, both resulted in effective cross sections which were corrected for resonance and spatial heterogeneity typical of the plate drawers of the critical assemblies. Each drawer type was processed for both the sodium flooded and sodium voided condition using the MC²-2/SDX codes. For the conventional homogeneous core designs, the processing was done for infinite arrays of the cells, and a constant buckling was applied to force $k_{eff} = 1$. For blanket cells, no buckling was applied.

This method of cross section preparation was found to be inadequate for the heterogeneous core assemblies. The problem was the numerous core and internal blanket interfaces that influence effective group constants, and some modification to the basic procedure was necessary to account for these interface effects. One such modification used an iterative buckling procedure. First, a group- and region-dependent buckling was generated for each cell type in each reactor region, averaged over all occurrences of that cell in each region. These bucklings were generated in a three-dimensional reactor calculation using the cross sections as discussed in the previous paragraph as a first iteration. One flux calculation was run for the sodium-flooded reference, and one for the fully-voided case. These bucklings were then applied to each cell type in SDX calculations instead of using the single critical buckling value.

Corrections for neutron streaming effects in the plate cells were incorporated at the 28-group level by applying directional diffusion coefficients that were calculated using a prescription based on methods developed by Benoist.¹⁰ Details of the method used at ANL are given by Wade in Ref. 11.

MODEL AND METHODS CONSIDERATIONS

A listing of most of the important considerations for a typical sodium-void reactivity calculation are shown in Table II. Most of these topics have been studied in the past, and methods and models have evolved to reflect the conclusions of those efforts. For that reason, most of the topics will not be discussed extensively. Instead the discussion will concentrate on the areas where differences exist between the reference method chosen for this study and the most commonly applied methods in LMFBR design activities.

In the area of cross-section data, the current U.S. LMFBR program uses a mix of ENDF/B Versions IV and V. The impact of changing from one to the other is small, typically less than 6%. This has not been true of earlier versions. Using Version IV as the reference of 1.0, the void-worth predictions were as low as 0.6 for the Version I and 0.8 for Version III data. Also, the importance of properly accounting for the resonance effects and spatially averaging of cross sections for the specific reactor structure has long been recognized, as is demonstrated by the wide application of codes such as MC²-II/SDX and SPHINX. Group structures with 20 or more groups of about 0.5 lethargy width are generally applied in void worth analysis.

Apparently first noted in the German SNEAK program, neutron streaming effects have been found to be very important in plate-type assemblies, particularly in high-leakage environments.¹² The leakage term for sodium-void worths is increased in magnitude by about 30% when Benoist-type corrections are applied to typical ZPPR assemblies. While not as important in the power reactor, it may still approach the 10-15% level.¹³ Work is currently under way in analysis of the ZPPR assembly 12 program to study the adequacy of the streaming predictions for both pin and plate environments.

Transport effects have been studied for a number of the ZPPR sodium-void measurements. For the cases studied, significant impacts occur only in the heterogeneous configurations (as high as the 10% range) and near inserted control rods, i.e., within 5-10 cm, where transport effects can be as large as 20%. For a set of 16 zones in the central blanket and first fuel ring of the heterogeneous core assemblies ZPPR-7A, -8A, -8B and -8C, calculations with a fine-mesh rz model in diffusion theory gave a mean calculated to experimental ratio of 1.12 ± 0.20 while equivalent transport-corrected S_4P_0 calculations gave 1.02 ± 0.10 . The result was a significant improvement in the C/E distribution, particularly between axial voiding steps and between adjacent core and blanket voiding steps.

With respect to choice of three versus two dimensions, some surprising results have been observed in ZPPR analysis, even in assemblies where rz models were expected to be fairly good. In large homogeneous cores like ZPPR-10A and -10B, with many local heterogeneities, xyz + rz differences of 10-30% were noted. For comparison, the smaller-sized ZPPR-5A and -5B had only 3 and 10% differences, respectively. The conclusion is that as the cores become larger, the calculated flux shape becomes significantly more sensitive to modeling error, and the need for precise modeling increases.

The most important difference which currently exists between the reference method of this study and the design level sodium void worth predictions is in the area of EP and first-order perturbation (FOP) methods applications. With FOP, predictions are less accurate than those that result from exact calculation of every step of a typical void experiment, but they are also fast and inexpensive. More importantly, the safety analysis codes like those of the SAS¹⁴ family traditionally have used reactivity coefficients that are essentially first-order estimates. Therefore, it is of some interest to examine the impact of FOP calculations. However, it must be noted that there exists some variation in how FOP methods are applied to sodium-void calculations.

Appreciable loss of sodium in a reactor region causes both the real and adjoint flux spectra to harden, and the spatial distribution to flatten. Thus, FOP calculations will generally underestimate the positive non-leakage term and overestimate the negative leakage term. It is of some interest to note that these errors will generally reinforce, producing a net result that is too negative. The magnitude of these effects depends on the specific core, but in general the effects on each term are on the order of 10%.

In the most elementary FOP calculations, the void perturbation is taken as simple removal of sodium, i.e., the effect on the cross sections of other isotopes is neglected. Because the non-leakage and leakage components are often in delicate balance, this simplification can result in substantial errors. For example, a 20% decrease in calculated core void reactivity has been noted when the perturbation is modeled by simple sodium removal. Cross sections particularly affected by removal of sodium include a decrease in ^{238}U capture because of increased resonance self-shielding, and reduced scattering in structural materials because of modification to the real flux fine structure between 0.1 and 1.0 MeV. To avoid these difficulties, the EP calculations used infinite-medium voided-cell cross sections to model the void effect in any zone. This also introduces some error, but principally for small zones.

Another approximation that is typically found in FOP analysis is in the choice of the differential diffusion coefficient.⁵ Because the altered transport cross section appears in the denominator of the diffusion coefficient, the differential diffusion coefficient is not proportional to changes in sodium density. Linearity is an important property of operators in safety codes such as SAS, so second order terms are typically dropped from the δD expression. Calculations with the approximate δD expression result in a smaller leakage term. Since the magnitude of leakage effects tends to be overestimated because of other approximations, there may be some beneficial cancellation of errors, but this has not been demonstrated in general.

Another problem in FOP analysis of sodium-void reactivity results from the use of a reactor adjoint flux that does not preserve the spatial and energy-dependent properties of the heterogeneous adjoint flux in the unit cell, as discussed in Refs. 15, 16, and 17. Smith¹⁸ estimates that this effect can reduce the sodium-void reactivity calculated for LMFBR-type critical assemblies by 10-15%. No such effect occurs in the reference EP calculations used for this study.

To provide an example of the relative performance of FOP and EP calculations, a comparison of predicted void worths from equivalent calculations was made for the step-worth measurements of ZPPR-5A, 5B, 11E and 11F. The mean value for the ratio of calculation to experiment (C/E) for the FOP method was 0.94 ± 0.44 compared to 1.12 ± 0.20 for EP. In this example, the calculations differed only in that the adjoint flux was not recalculated for the perturbed condition.

TABLE II. Important Model and Method Considerations in Sodium-Void Analysis

Issue	Comment
<u>Calculation Modeling Effects</u>	
- three dimensions versus two; explicit representation of boundaries	Important with control rods present; importance grows with core heterogeneity and system size.
- mesh spacing	Generally a small (<2%) effect at the ~5 cm mesh spacing level, except near control rods.
<u>Global Methods Effects</u>	
- transport versus diffusion calculations	Small except near control rods in conventional cores. Often 10% or more effect in CRBR heterogeneous cores and within 5-10 cm of control rod singularities.
- exact perturbation versus first order	10-20% effects are common.
<u>Cross Section Effects</u>	
- ENDF/B version	Historically, large variations noted. Versions I to IV changed from 0.6 to 1.0. Change from IV to V is small.
- neutron streaming	Important in plate-critical assemblies (leakage term changes of up to 30%); of lesser importance but can be significant in power reactors.
- asymptotic versus non-equilibrium spectra	Most important in heterogeneous cores. Also an issue in small zones.
- resonance self-shielding	Historical issue.
- spatial weighting	Historical issue especially important in critical experiment analysis.
- group structure	~20 groups generally accepted.

CLB-A6

RESULTS

The reference calculational method--basically coarse-mesh three-dimensional diffusion theory-- was selected after it was demonstrated to predict sodium-void reactivity considerably better than historical techniques. However, it is also of interest to compare the reference results against those obtained when corrections were made for the major deficiencies, i.e., transport and mesh effects. Results were calculated for a few central core zone measurements where corrections could be easily applied. In Table III, C/E values are shown for that select group for reference method and fully corrected calculations. Significant corrections are observed only for the heterogeneous core, ZPPR-7A, and near control rods ZPPR-5B and -10B. For the six results for which corrections are available, the mean reference value and root-mean-square (RMS) deviation of the C/E distribution changes from 1.130 ± 0.067 to 1.174 ± 0.051 . The improvement is shown in the lower RMS deviation while the average corrected C/E value is somewhat farther from unity.

The corrected results have been grouped into subsets in an attempt to discern any trends in the data with variation of design parameters. The average C/Es and RMS deviations are shown in Table IV. The average C/Es vary only from 1.15 to 1.18, so that no significant trends emerge.

The broader data base analyzed for this assessment includes 95 measurements which contain many negative as well as positive void worth zones. The calculations are with the reference method only, as it is neither necessary nor feasible to apply transport and mesh corrections to all cases. Here the approach has been to apply a two-component fit in addition to computing the statistical parameters of the C/E distributions. The fitting process selects a correlated set of bias factors (β and γ) for the calculated leakage (L) and non-leakage (N) reactivity components by minimizing the weighted variance between the measured reactivities and the sums of the biased components.

$$V = \sum_i \omega_i (\beta N_i + \gamma L_i - E_i)^2 \quad (1)$$

This approach is one that has been applied with reasonable success to other studies of sodium void reactivity.^{6,13,19} Bias factors are applied in order to eliminate systematic errors in calculational methods. Since net sodium void reactivity is a balance between contributions from physically different processes, separate bias factors are determined for the non-leakage and leakage components. Covariance matrices, that allow uncertainties in biased reactivities to be estimated systematically, are also produced in the fitting processes. Observing that variations in C/E are much larger than can be explained by the precision of the measurements, a non-linear fitting process has been applied so that "random" calculational errors are reflected in the covariance matrices. Such random calculational errors arise through different sensitivities of individual zone reactivities to approximations in the methods and the models.

Bias factors and their covariance matrix can be applied to an additional calculation of the sodium-void effect to yield a most probable reactivity and its uncertainty. For example, the data used in the fit could be from a

TABLE III. The Effect of Transport and Mesh Corrections on C/E Values for Sodium-Void Reactivities in Central Core Zones

Core	Zone	Calculated Worth, (reference method) £/kg	C/E Ratios		Uncertainty due to Measurement
			Reference Calculation	Corrected Calculation ^a	
ZPPR-5A	Central 120 drawers 0-304.8 mm	0.89	1.193	1.209	±2%
ZPPR-5B	Central 120 drawers ±304.8 mm	1.24	1.050	1.117	±2%
ZPPR-7A	Fuel Ring 1 + 2 ±304.8 mm	0.86	1.053	1.142	±1%
ZPPR-9	Central 97 drawers ±204 mm	1.03	1.094	1.107	±1%
ZPPR-10A	Central 181 drawers ±204 mm	0.91	1.158	1.169	±1%
ZPPR-10B	Central 181 drawers ±204 mm	0.74	1.105	1.200	±1%

^aCorrections are for transport (S_4) and finite mesh size.

CLB-A10

TABLE IV. C/E Results from Corrected Central Sodium-Void Reactivities
Organized According to Core Characteristics

Zone Combinations	Cores	Av. C/E ($\pm$ RMS deviation)
All	All	1.174 ($\pm$ 0.051)
With Central CRPs	ZPPR-3/3, 5A, 10A	1.166 ($\pm$ 0.045)
With Central CRs	ZPPR-5B, 10B	1.159 ($\pm$ 0.059)
Benchmark cores	ZPR-6/7, ZPPR-7A, 9	1.174 ($\pm$ 0.073)
2300% Core Volume	ZPPR-3/3, 5A, 5B, 7A	1.152 ($\pm$ 0.043)
Heavy metal vol. fraction of 0.32	ZPR-6/7, ZPPR-3/3, 5A, 5B, 7A	1.172 ($\pm$ 0.058)
Heavy metal vol. fraction of 0.40, 4500% core volume	ZPPR-9, 10A, 10B	1.159 ($\pm$ 0.047)
Lower enrichment	ZPR-6/7, ZPPR-5A, 9, 10A	1.184 ($\pm$ 0.062)
Higher enrichment	ZPPR-3/3, 5B, 7A, 10B	1.150 ($\pm$ 0.040)

CLB A-9

series of small zones, and the results could be applied to a calculation of whole-core reactivity. Here, the fit has been applied to all data and then to selected subsets which emphasize certain design characteristics. Results are summarized in Table V, where biased calculated values are denoted by a "p".

Some trends can be identified in the data, both before and after application of the bias factors. Half of the individual C/E values fall within the range of 1.02 - 1.22 before being biased. Most values that fall outside the range of 1.0 - 1.3 require significant corrections to the calculations, e.g., those from heterogeneous cores, near control rods, outer core zones, etc.

The C/E distribution for all zones had a mean value of 1.12 with a weighted standard deviation of 0.20 that reduced to a P/E of 0.99 ($\pm$ 0.09)* after the calculations were biased. A subset of 62 inner-core values from zones distributed both axially and radially in the inner core region, had a mean value of 1.12 ($\pm$ 0.10) before bias factors were applied, and 0.99 ($\pm$ 0.07) afterwards.

Some problems remain for high radial-leakage zones and axial blankets as can be seen by the C/E comparisons in Table V. It is expected that additional high-leakage data and more extensive calculations will be required

*Numbers in parentheses are weighted standard deviations of the C/E distributions.

TABLE V. Factors and Average C/Es for the Critical Experiment Data Base

Zones Included in Fit	No. of Meas.	Bias Factors ^a		Before Fit		After Fit ^c	
		Leakage	Non-Leakage	Mean C/E ^b	Standard Deviation ^b	Mean P/E ^b	Standard Deviation ^b
All	95	0.897 ± 0.017	0.890 ± 0.012	1.118	0.195	0.990	0.090
Fuel	62	0.904 ± 0.022	0.896 ± 0.009	1.120	0.095	0.993	0.074
910 mm high conventional cores	23	0.853 ± 0.025	0.858 ± 0.014	1.159	0.091	0.994	0.067
1017 mm high cores	27	0.932 ± 0.025	0.889 ± 0.009	1.123	0.091	0.995	0.057
High Enrich- ment	15	0.927 ± 0.028	0.933 ± 0.010	1.071	0.101	0.994	0.067
Low En- richment	13	0.917 ± 0.022	0.873 ± 0.016	1.159	0.091	0.994	0.063
Axial Blankets	10	0.544 ± 0.139	-0.133 ± 0.35	1.117	0.230	0.985	0.145
Outer Core	6	0.777 ± 0.021	0.806 ± 0.031	1.395	0.125	1.000	0.049
Het core BOL	14	1.007 ± 0.076	0.986 ± 0.028	1.025	0.100	0.996	0.060
Het core EOC	13	1.028 ± 0.044	0.89 ± 0.019	1.098	0.424	0.997	0.053

^aError estimates are for purposes of comparison only.

CLB-A11

^bIn generating statistical parameters, C/E and P/E values are weighted by the square of the reciprocal of the estimated uncertainty in the ratio.^cUsing 2 component method, P is the predicted worth based on a biased calculated worth.

to resolve the situation satisfactorily. Data already taken in ZPPR-12 should help in this regard. For the majority of the data base, both the leakage and non-leakage factors are in the range 0.85 to 0.92. Values for the heterogeneous cores and axial blankets are more consistent with this range when transport corrections are applied.

CONCLUSIONS

Use of these results in safety analysis is a longer-term objective of this work. One approach is to supply biased reactivity coefficients as input to safety codes like those of the SAS family. With respect to the sodium-void effect, the major issues in applying the results are:

- use of critical experiment analysis to generate bias factors for specific calculational methods,
- assignment of an uncertainty based on ability to predict sodium-void reactivity in critical assemblies,
- determination of the additional uncertainty that arises in accounting for physical differences between the critical assemblies and the power reactor,
- incorporation of biased values and uncertainties in safety codes.

In general, bias factors for most subsets of the data base could be determined to within the 1-4% range. Application of the bias factors to the calculations improved the comparisons with experiments such that the standard deviations of most P/E distributions were 5-9%, compared to initial values of 10-20%. Error estimates for individual reactivity predictions varied substantially on a percentage basis, depending on the relative contribution of the non-leakage and leakage components. For example, individual zones of high positive reactivity, dominated by the non-leakage term, typically had estimated errors on net reactivity of 4-5%. For zones where the components were more nearly balanced, estimated errors for the net reactivity were as much as an order of magnitude larger. However, on an absolute basis, estimated errors did not vary much within any given subset of the data. These properties of the biased calculations are particularly useful in safety studies, where often it is desirable to place a reasonable limit on maximum positive reactivity.

There are physical differences between critical assemblies and operating reactors that cannot be treated experimentally. Sodium-void reactivity is significantly impacted by these differences, the two most important being presence of fission products and elevated, non-uniform temperatures. In most cases these effects are included explicitly when making sodium-void reactivity predictions for the reactor. In developing an uncertainty estimate, however, additional uncertainty must be added to that resulting from analysis of a selected set of critical experiments. For example, in their assessment of sodium-void status in the UK, Butland, et al.¹³ estimated this additional uncertainty to increase the estimated uncertainty on the calculated non-leakage bias factor for CDFR by about 4%.

A major unresolved issue of this study is how to apply these results in safety analyses. While the methodology for development of spatially-dependent bias factors and covariance matrices for sodium-void reactivity is reasonably well defined, the means of incorporating bias factors and uncertainty estimates in safety analyses has not been addressed. Historically, FOP calculations have been used in safety analyses. If that trend continues in the future, resolution of the many potential differences between FOP and the reference EP method must be achieved if bias factors and uncertainties are to be applied.

In summary, the uncertainty in predicting sodium-void reactivity in critical assemblies is about 3-7% for zones with significant positive reactivity. Allowing for higher uncertainty in high-leakage zones, the whole-core void effect in critical assemblies appears to be predictable to 5-10%. A major critical assembly issue in application of these data to LMFBFR studies is consistent analysis of measurements in pin zones. For application to a particular reactor design, such as the heterogeneous Large Development Plant reactor, additional mockup experiments probably could reduce the uncertainty in predictions to the lower end of the 5-10% range. If the uncertainties in accounting for physical differences between the critical experiments and power reactor are about 4%, uncertainties in whole-core sodium-void reactivity could be less than 10%. Achievement of such low uncertainty estimates requires that the same calculational methodology be employed, and that sensitivity studies on the LMFBFR model agree with Butland's¹³ 4% estimate for the CDFR.

REFERENCES

1. R. Avery and J.E. Cahalan, "Significance of Physics Parameters in LMFBR Safety," *Proc. 1980 Advances in Reactor Physics and Shielding*, Sun Valley, Idaho, Sept. 14-18, 1980, American Nuclear Society, LaGrange Park, IL, pp. 83-96 (1980).
2. H.H. Hummel and D. Okrent, *Reactivity Coefficients in Large Fast Power Reactors*, American Nuclear Society, LaGrange Park, IL, pp. 82-132 (1970).
3. J.B. Nims and P.F. Zweifel, "Preliminary Report on Sodium Temperature Coefficients in Large Fast Reactors," Atomic Power Development Associate Report APDA-135 (1959).
4. D. Okrent, "Summary of Intercomparison Calculations Performed in Conjunction with Conference on Safety, Fuels, and Core Design in Large Fast Power Reactors," *Proc. Conf. on Safety, Fuels, and Core Design in Large Fast Power Reactors*, Oct. 11-14, 1965, Argonne, IL, Argonne National Laboratory Report ANL-7120, pp. 3-23 (1965).
5. A. Travelli and J.C. Beitel, "Analysis of Sodium-Void Experiments in the FTR Engineering Mockup Critical Assembly," *Proc. Natl. Topical Mtg. on New Developments in Reactor Physics and Shielding*, Sept. 12-15, 1972, Kiamesha Lake, NY, Report CONF-720901, 2, U.S. Atomic Energy Commission, pp. 739-767 (1972).
6. C.L. Beck, et al., "On the Extrapolation of ZPR Sodium Void Measurements to the Power Reactor," *Proc. Intl. Mtg. on Fast Reactor Safety and Related Phys.*, Chicago, Oct. 5-8, 1976. U.S. Energy Research and Development Administration Report CONF-761001, II, pp. 512-520 (1976).
7. P.J. Collins, et al., "Experimental Studies of 350-MW(e) Heterogeneous LMFBR Cores at ZPPR," *Proc Intl. Symposium Fast Reactor Physics*, Aix-en-Provence, Sept. 24-28, 1979, IAEA, Vienna, 2, pp. 57-67 (1979).
8. M.J. Lineberry, et al., "Experimental Studies of Large Conventional LMFBR Cores at ZPPR," *ibid.*, 1, pp. 187-204.
9. D.R. Ferguson and K.L. Derstine, "Optimized Interaction Strategies and Data Management Considerations for Fast Reactor Finite Difference Diffusion Theory Codes," *Nucl. Sci. Eng.*, 64, No. 2, p. 593 (1977).
10. P. Benoist, "Streaming Effects and Collision Probabilities in Lattices," *Nucl. Sci. Eng.*, 34, p. 285 (1968).
11. D.C. Wade, "Monte Carlo-based Validation of the ENDF/MC²-II/SDX Cell Homogenization Path," Argonne National Laboratory Report ANL-79-5 (1979).
12. E. Fischer, et al., "An Investigation of the Heterogeneity Effect in Sodium Void Reactivity Measurements," *Proc. Intl. Symposium on Physics of Fast Reactors*, Oct. 16-19, 1973, Tokyo, Power Reactor and Nuclear Fuel Development Corp., 1, pp. 945-963 (1973).

REFERENCES (Cont.)

13. A.T.D. Butland, et al., "An Assessment of Methods of Calculating Sodium Voiding Reactivity in Plutonium Fuelled Sodium-Cooled Fast Reactors, *Proc. Intl. Symposium on Fast Reactor Physics*, Aix-en-Provence, Sept. 24-28, 1979, IAEA, Vienna, 1, pp. 281-306 (1979).
14. D.R. Ferguson, et al., "Analysis of Hypothetical LMFBR Whole-Core Accidents in the USA," *Proc. Intl. Top. Mtg. Nuclear Power Reactor Safety*, Brussels, Oct. 16, 1978, European Nuclear Society/American Nuclear Society, pp. 1608-1620 (1978).
15. K.S. Smith, "The Effects of Intracell Adjoint Flux Heterogeneity on FOP Reactivity Calculations," *Nucl. Sci. Eng.* (in publication).
16. K.S. Smith, "The Effects of Intracell Adjoint Flux Heterogeneity on FOP Reactivity Calculations," *Trans. Am. Nucl. Soc.*, 41, p. 618 (1982).
17. R.W. Schaefer and R.G. Bucher, "Calculated and Measured Reactivities in the U9 Critical Assemblies," these proceedings.
18. K.S. Smith, private communication.
19. A.R. Baker and P.J. Collins, "Studies of a Mockup of the Prototype Fast Reactor on ZEBRA," *Proc. Intl. Conf. on Phys. of Fast Reactor Operation and Design*, June 24-26, 1969, London, British Nuclear Energy Society Publication, pp. 213-220 (1969).

CALCULATED AND MEASURED REACTIVITIES IN THE U9 CRITICAL ASSEMBLIES*

R. W. Schaefer and R. G. Bucher
Argonne National Laboratory
9700 South Cass Avenue
Argonne, Illinois 60439

ABSTRACT

A set of three benchmark fast critical assemblies, designated U9, had a 9% enriched uranium core composition and three degrees of cell heterogeneity. An extensive series of reactivity experiments was performed in U9, which was aimed at diagnosing the bias between measurement and calculation. Both the measurement and calculation techniques include substantial modifications of the standard ANL methods. The experiments clearly show a trend of increasing calculational error with increasing cell heterogeneity using conventional analysis, and the effectiveness of an adjoint correction in removing this trend. The ratio of calculated to measured worth (C/E) for small material samples near the core center is near unity for most samples. Perturbations of the normal core environment, caused by the measurement technique, are shown to affect sample worth C/E's by only about 1%. An alternative determination of fuel worth, largely independent of the small sample approach, also has a C/E near unity. These U9 results are a major contribution to a database for reevaluating the longstanding central worth discrepancy.

I. INTRODUCTION

One of the longstanding problems from LMFBR critical experiment programs is the central worth discrepancy,^{1,2} the consistent bias between calculated and measured reactivity of a small material sample located near the center of the assembly. The ratio of calculated to measured worth (worth C/E) for fissile samples near the center of ZPR assemblies typically varies from 1.1 to 1.2.² This discrepancy is large compared to typical C/E discrepancies for such quantities as critical eigenvalue and reaction rate ratios.

The central worth discrepancy and several other key discrepancies recently have been under systematic investigation at ANL. This has involved both a calculational methods validation effort³⁻⁵ and an experimental program, the Diagnostic Cores Program.^{6,2} What has been learned from reactivity experiments in the U9 assemblies of this program is the subject of this paper.

U9 refers to a set of three small, plate critical assemblies, each of which had a core cell average composition of 9% enriched uranium plus the matrix and drawer steel. The three U9 assemblies differ in the distribution of the ²³⁵U fuel within the unit cell, i.e., in cell heterogeneity. The three assemblies, in order of increasing cell heterogeneity, are designated the dispersed, reference, and clumped assemblies. The reactivity induced by redistributing the fuel in the cell was measured. Small sample worths were measured in each of the unit cell types, giving the impact of cell heterogeneity on small sample worth C/E's.

It is almost always necessary, in measuring small sample worths, to distort the unit cell in the immediate vicinity of the sample. Unit cell material is moved to make room for the sample, and hardware is introduced to hold the sample

*Work performed under the auspices of the U. S. Department of Energy.

and transport it in and out of the reactor. These alterations affect the flux and adjoint distributions in space and energy, and therefore they affect the sample worth. It is usually necessary to simplify the distortions in the computational model. Historically, these distortions were believed to be unimportant,⁷ but the impact of the simplifications on worth C/E's is being reexamined.

In U9, the distortions associated with the small sample measurement technique were relatively small, and they were examined in special, diagnostic experiments. There was also an alternative measurement of U(93)* worth, the fuel thickening experiment, that entailed no distortion and was almost perfectly suited to the calculational methods. Thus, the impact of the distortions on worth C/E's from U9 can be well defined.

The remainder of the paper is divided into five sections. In Section II there is a description of the assemblies. The measurement techniques, described in Section III, were precise enough to quantify even small effects, and they required much less unit cell distortion than the traditional technique. Likewise, the analysis section, Section IV, includes a description of significant improvements over the standard methods. In Section V, we present results from the several kinds of reactivity experiments mentioned earlier. The major conclusions drawn from the experiments are summarized in the final section.

II. THE U9 ASSEMBLIES

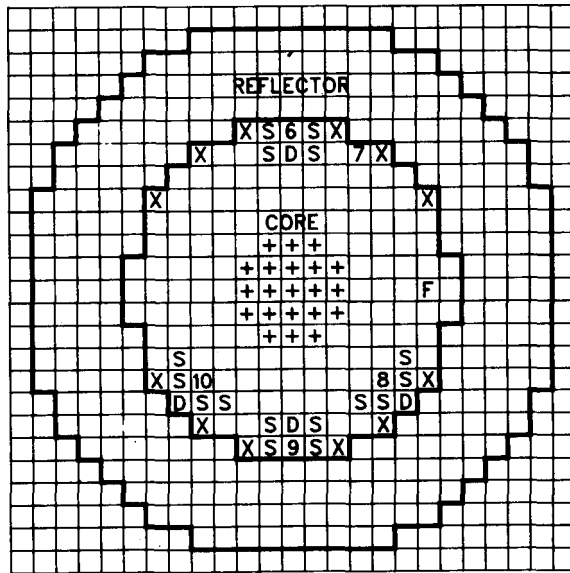
The three U9 assemblies were constructed in the ZPR-6 split-table critical facility.⁸ Cross sectional views of the stationary half (S-half) and moveable half (M-half) matrix loadings are shown in Fig. 1. Each box indicates a drawer loaded with uranium plates.

This one pair of diagrams shows the loadings for each of the three assemblies. The reference assembly had no central zone. The plate loading in the central 21 drawers was changed over a 30.5 cm length to create the dispersed fuel zone assembly. The clumped fuel assembly was created from the reference by 1) rearranging the plates within the core drawers, and 2) replacing core with reflector in drawers labeled "X".

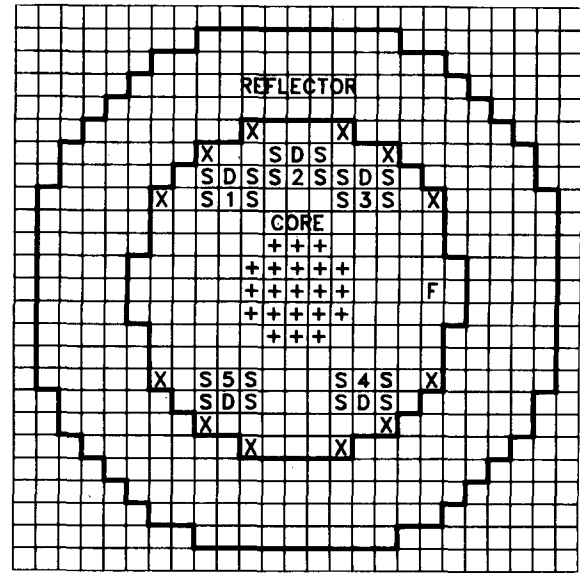
The irregular core boundary is approximately cylindrical, with a radius of 41.0 cm in reference and dispersed assemblies and 39.5 cm in the clumped assembly. The core height was 76.4 cm. Reflector drawers were filled with U(0.2) and the reflectors were effectively infinite, 24.2 cm radial and 83.0 cm axial.

Control islands were the only significant complication in the assemblies. Control islands are the clusters of nonstandard drawers in Fig. 1, which were present to meet operational safety requirements. Their location near the core periphery provided a clean central area for experiments, but it also lowered the central power by about 1%. Also, the control island drawer arrangements and locations were different in the two halves. This produced a 4% axial power tilt over the core height toward the M-half.

*U(x) refers to uranium with x weight percent ²³⁵U.



S-half

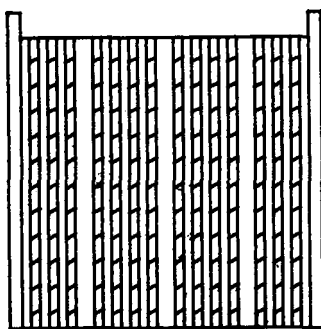


M-half

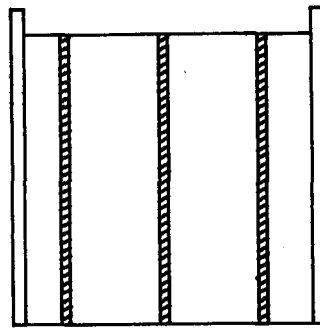
X: reflector drawer in clumped assembly, otherwise core
 +: zone drawer in zoned assemblies, otherwise standard core
 F: FAR during small sample measurements, otherwise core
 number, S, D: control-related drawers

Fig. 1. Cross Section of U9 Matrix Loadings

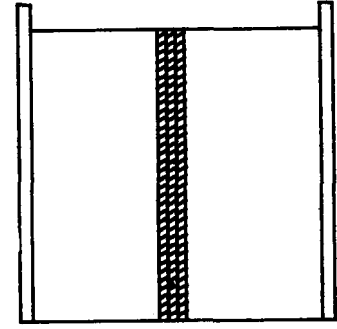
Cross-sectional views of the dispersed, reference, and clumped core drawer loadings (unit cells) are depicted in Fig. 2. The overall cell cross section was 5.52 cm square and the plate cross section was 5.08 cm square. The heavily and lightly shaded columns are 0.16 cm wide U(93) and U(20) fuel, respectively. The end columns are the matrix plus drawer steel, and the remainder is U(0.2). The horizontal matrix and drawer steel is not shown. Representative plate and cell average atom densities are given in Table I.



Dispersed



Reference



Clumped

Fig. 2. U9 Core Unit Cells

TABLE I. Typical Plate and Region Atom Densities^a in U9

Isotope	²³⁴ U	²³⁵ U	²³⁶ U	²³⁸ U	Fe	Cr	Ni	Mn	¹⁶ O
U(93) Plate	0.423	43.170	0.203	2.473	-	-	-	-	-
U(20) Plate	0.057	9.364	0.061	37.170	-	-	-	-	-
U(0.2) Plate	-	0.098	-	45.820	-	-	-	-	-
Homogenized Core	0.033	3.482	0.016	35.173	6.691	1.891	0.821	0.148	0.241
Homogenized Reflector	-	0.084	-	39.401	6.873	1.912	0.830	0.151	0.244

^a Atoms/cm³ × 10⁻²¹

The average composition of a control island was essentially the same as the core composition. The control rod, designated by a number in Fig. 1, was a drawer with nine columns of U(93) rather than three. The drawers in Fig. 1 labeled "S" had two fuel columns each, and "D" drawers had a single fuel column. The balance of each drawer loading was U(0.2). There was one additional operational control rod, Rod No. 7 in Fig. 1. It had the reference unit cell loading and was surrounded by standard core drawers. Control island drawers and Rod No. 7 were not altered in fuel clumping or dispersing operations.

The U9 core neutronics was governed largely by ²³⁵U and ²³⁸U cross sections in the 10 keV to 10 MeV range. Since this is above most of the uranium resonance range, cell heterogeneity was characterized principally by broad group in-cell flux and adjoint variations.

III. MEASUREMENT TECHNIQUES

Small material samples were inserted into the core using a drawer oscillator that provided a minimally perturbed environment for the samples. One sample simply was slipped between plates in a standard cell loading in the drawer oscillator. Most samples were placed in a narrow cavity transverse to the drawer oscillator's plate loading. Distortions existed only in the latter case: the 0.32 or 0.64 cm wide cavity, the ~2 gm steel U-channel at the cavity bottom that maintained the gap, the ~1 gm steel springs that centered the sample in the gap, and the clad around plutonium and boron samples. Distortions from the foreign material would be reflected in nonseparability of worths of the net sample and this material, which was present in with-sample and without-sample oscillations.

Small sample worths were measured with the drawer oscillator using the sample-oscillation, reactivity-difference technique.⁹ A calibrated servo control rod, referred to as the FAR, held the reactor power constant while samples were oscillated in and out of the core in the drawer oscillator. To establish FAR positions, the drawer was halted with the sample at the desired in-core position and again with the sample at a point outside the assembly. The difference in FAR positions was a reactivity signal. In a separate startup the

entire process was repeated, stopping the drawer at the same positions but without the sample in the drawer, and another signal was obtained. The net sample worth was the difference between signals with vs without the sample present. This reactivity difference technique eliminates potential problems, such as nonequilibrium precursor concentrations in the oscillating drawer, and nonuniformity in the oscillator drawer's loading.

FAR calibration uncertainty dominates the net sample worth uncertainty of ± 0.015 inhours (one standard deviation). The FAR calibration experiment used inverse kinetics analysis of the reactor power history resulting from low frequency ramp oscillations of the rod.¹⁰ When relative rather than absolute worths were desired, it sometimes was possible to reduce the uncertainties by avoiding the contribution from the FAR.

The positive period method was used for some measurements. The excess, i.e., supercriticality of the assembly with all (fueled) control rods fully inserted, was measured by this method, and the fuel clumping and fuel thickening worths were obtained from changes in excess. Positive periods also were used, in conjunction with the drawer oscillator, to measure the worth of 30 cm long U(93) foils and to measure the flux distortion effect of the FAR. Effective delayed neutron parameters, computed with ENDF/B Version V delayed data, were used to interpret the power histories. The standard concerns with the positive period method were addressed: 1) correction for slight power drift in the pretransient state, 2) establishment of equilibrium precursor concentrations between period measurements, and 3) data acquisition after the asymptotic period is established and before there is significant temperature feedback. For the change in assembly excess, factors included in the uncertainty estimates are: 1) reproducibility of control rod positions, 2) reproducibility of the M-half closure, and 3) accuracy of the excess adjustment to a common assembly temperature.

For the fuel thickening experiment an unresolved issue is the error in assuming separability of operational control rod worths. Before a thickening step, the core boundary was adjusted so that the excess was held on one rod. Full insertion of this fuel-bearing rod produced the positive period. A second rod was needed to help offset the thickening reactivity addition. The two rods were as far apart as possible to minimize interaction. Reactivities from inserting each rod individually were added to obtain the excess. There is some evidence suggesting that the nonseparability error is about a percent and this uncertainty is not included in the results.

IV. ANALYSIS METHODS AND MODELS

A basically standard procedure was followed in calculating reactivities for U9. First, homogenized broad group cross sections were generated for each cell type. Then, a homogenized assembly model that closely approximates the as-built configuration was constructed. For fuel clumping and thickening experiments, the reactivity was large enough to be determined from the difference in eigenvalue of two configurations. For small material samples, the flux and adjoint for the relevant configuration were used in first order perturbation theory (FOP) calculations. Adjustments to FOP worths were made to account for a number of effects.

Broad group cross sections were generated from ENDF/B Version IV data using the processing code sequence ETOE-II, MC²-2, SDX in nearly the manner described in Ref. 3. Cell homogenization was done at an intermediate (181) group level using slab cell models. Cell average cross sections were produced for each of the cell types in the U9 series: dispersed, reference, clumped, fuel thickened, and the three control island drawer types. Collapse from 181 to 20 groups was done separately for each assembly (including a fuel thickened zone assembly) using completely analogous spherical assembly models. In addition to cell average cross sections, broad group cross sections were produced for different plate types in the cell and for a nonperturbing detector spanning the cell.³ This last type was used for cell-spanning small samples.

Calculations were done with RZ and XYZ assembly models. The principal weakness of the RZ model is the neglect of the control islands, which change the core power distribution. XYZ models were used to include every cell type explicitly. The FAR, which actually was present only in small sample worth measurements, is difficult to model and was ignored in all calculations.

Worths based on XYZ models were obtained from a coarse mesh diffusion theory calculation, to which mesh and transport corrections were applied. Group- and region-dependent bucklings from the XYZ eigenvalue calculations were supplied to XY and Z models. These lower dimensional models were used to obtain corrections to ~2 cm mesh spacing and S_4 transport (with transport-corrected P_0 scattering). The transport correction to an FOP central worth was produced with diffusion perturbation theory and the S_4 scalar flux and adjoint. This is inconsistent in principle, but is adequate because the reactivity's leakage component was very small. Corrected XYZ solutions were obtained for small sample worths in the reference assembly, for axial worth traverses, for fuel clumping, and for fuel thickening worths.

The natural unit of calculated reactivity is $(k - k_0)/k$, where k and k_0 are the perturbed and unperturbed eigenvalues, respectively. Conversion of this reactivity to inhours was calculated using ENDF/B Version V delayed neutron data.

Computation of the adjoint flux with homogenized cross sections that were produced by the conventional flux weighting procedure has long been known to be incorrect in principle.^{11,12,4} However, it has been determined only recently that this deficiency in the adjoint results in a several percent error in FOP calculations of $(k - k_0)/k$ for ZPR unit cells.¹³ It also affects the conversion to inhours, and thereby impacts fuel clumping and fuel thickening reactivity calculations as well. The prescription of Ref. 13, which is based on cell calculations, was used to generate adjoint correction factors to the inhour conversion and FOP worths. The prescription was extended to handle the dispersed zone assembly, where there are two core unit cell types.

The adjoint corrections for U9 were done at the broad group level. There remains an adjoint distortion from the cross section collapse in energy that is substantial only for scattering materials.⁴ The space and energy collapse errors also may not be completely separable for scatterers. The U(0.2) worth C/E in U9 may be affected because the net scatter component of its worth is unusually large in these assemblies. Table III of Ref. 13 indicates that correction at the broad group level is adequate for fissile and absorber materials.

There were a number of adjustments to the FOP worths in addition to the adjoint correction.

- In computing the worth of a sample that did not span the unit cell, a correction from the cell average flux to the foil location flux was made. (The in-cell adjoint variation is built into the adjoint correction factor.)
- A sample size correction factor, that accounts for self-shielding and self multiplication in the sample at the broad group level, was computed with the SARCASM code.¹⁴
- A simple perturbation theory prescription based on a one-group spherical model,¹⁵ was used to account for the flux and adjoint depression caused by the presence of a small cavity around cell-spanning small samples. Although the cavity in U9 is not well approximated by this model, the corrections were so small that they were considered adequate.
- FOP worth calculations at ANL in the past used the neutron emission spectrum of the core composition in the fission source term of all fissionable worth samples. Had this approximation been made in U9, only the Pu worth would have been affected significantly, 1.6%. A similar error occurs for ²³⁵U samples in a Pu-fueled core.¹⁶

V. RESULTS

Small Sample Central Worths

Sample descriptions and measured worths in each U9 assembly are given in Table II. Except for the cell samples, all are uniform materials, 5.08 cm square (the Pu meat is 4.70×4.32), that sample the entire cell cross section (other than matrix tubes). A unit cell sample is a 0.32 cm wide slice of the plate cell cross section, and its reactivity can be thought of as the analog of a small cavity worth in a homogeneous core. All these worths are for samples in the central drawer, axially 4.45 cm into the S-half. This position is near enough to center to avoid significant flux gradients and also avoids potential distortions at the interface of the matrix halves. The worths are as-measured, i.e., there are no adjustments for the cavity or sample size effect.

The C/E results for the cell-spanning samples, shown in Table III, exhibit an important trend with unit cell heterogeneity. With the standard computation of the adjoint, there is a clear trend of increasing C/E with increasing heterogeneity. When the mistreatment of the adjoint is corrected (Section IV), this trend is removed, and most C/E's are within a few percent of unity. The corrected C/E's from the clumped assembly tend to be lower than C/E's from the other assemblies, but, overall, the results are greatly improved by the adjoint correction.

A further reduction in C/E, of about 2%, results when the control islands are accounted for in the calculations. This is the principal difference between the XYZ calculation (last column in Table III) and the RZ reference assembly calculation. This effect should be about the same in all three assemblies.

Additional observations can be made from Table III. The unit cell sample C/E without the adjoint correction shows no trend with cell heterogeneity. This is because in-cell heterogeneity effects for this sample were addressed at

TABLE II. Sample Descriptions and Measured Near-Center Reactivities

Name	Net Sample			Clad gm, Type	Cavity Width (cm)	Measured Worth ^a in Each Assembly			
	Main Components (w/o)	Mass (gm)	Width (cm)			Dispersed (Ih)	(Ih/kg)	Reference (Ih/kg)	Clumped (Ih/kg)
U(93)	93.1 ²⁵ U, 5.6 ²⁸ U	21.37	0.05	-	0.32	2.897	135.5 ± 0.7	134.6 ± 0.7	141.8 ± 0.7
U(93)-C	93.1 ²⁵ U, 5.6 ²⁸ U	21.37	0.05	18, SST	0.64	2.867	134.1 ± 0.7	134.2 ± 0.7	139.9 ± 0.7
Pu(93)	93.5 ⁴⁹ Pu, 4.5 ⁴⁰ Pu, 1.7 Al	10.86	0.04	18, SST	0.64	2.343	215.8 ± 1.4	215.9 ± 1.4	223.4 ± 1.4
U(0.2)	99.8 ²⁸ U, 0.2 ²³⁵ U	72.75	0.17	-	0.32	-0.551	-7.57 ± .21	-7.71 ± .21	-8.33 ± .21
B(96)	96.2 ¹⁰ B, 3.8 ¹¹ B	0.544 ^b	0.01	3, Al	0.32	-0.952	-1750 ± 38 ^b ± 28	-1775 ± 42 ^b ± 33	-1855 ± 39 ^b ± 28
SST	70.8 Fe, 18.3 Cr, 8.5 Ni	46.39	0.24	-	0.32	-0.440	-9.48 ± .32	-9.40 ± .24	-10.36 ± .32
Fe	100.0 Fe	93.58	0.48	-	0.64	-0.805	-8.60 ± .16	-	-9.04 ± .16
C	100.0 C	19.71	0.48	-	0.64	-0.727	-36.9 ± 0.8	-39.0 ± 0.8	-41.7 ± 0.8
Cell	9.2 U(93), 90.8 U(0.2)	150.11	0.32	-	0.32	-	-	5.71 ± 0.10	6.53 ± 0.1
Cell-D	44.8 U(20), 55.2 U(0.2)	150.83	0.32	-	0.32	-0.911	6.04 ± .10	-	-

^aTo convert to more common units, the (adjoint corrected) calculated values of 345 Ih/\$ and $\beta_{eff} = 7.01 \times 10^{-3}$ may be used.

^bBoron mass is uncertain by 1.5% because of fabrication problems. Upper reactivity uncertainty includes this, lower uncertainty does not.

TABLE III. Near-Center Worth C/E's for Small Samples

Sample Name	Typical C/E Uncertainty	Sample Worth C/E in Each Assembly						
		Standard Calculation			Calculation Including Adjoint Correction			
		RZ			RZ			XYZ
		Dispersed	Reference	Clumped	Dispersed	Reference	Clumped	Reference
U(93)	0.005	1.033	1.059	1.092	1.016	1.033	1.006	1.014
U(93)-C	0.005	1.042	1.060	1.104	1.025	1.035	1.018	1.016
Pu(93)	0.007	1.029	1.045	1.093	1.012	1.022	1.013	1.003
U(0.2)	0.028	1.056	1.063	1.082	1.037	1.029	0.970	1.023
B(96)	0.021	0.949	0.941	0.952	0.934	0.930	0.916	0.919
SST	0.034	1.090	1.129	1.168	1.068	1.084	1.024	1.067
Fe	0.020	1.095	—	1.223	1.073	—	1.064	—
C	0.024	1.187	1.188	1.286	1.163	1.133	1.100	1.133
Cell ^a	0.017	0.992	1.008	0.975	0.979	1.013	0.993	0.971

^aUnit cell for each assembly.

the cross section generation level. A second observation is that the adjoint corrected worth C/E's are lower for boron than for fissile samples by about 9%. Recently proposed improvements to the boron cross sections in ENDF¹⁷ probably would reduce this disparity substantially. A third point is that the worth C/E's are about 1% lower for Pu(93) than for U(93) measured analogously (sample U(93)-C). The disparity would have been 1.6% more if a historical calculational problem had not been corrected (Section IV). Finally, the C/E's for scattering materials generally have the poorest C/E's, which, at least in part, is due to the conventional cross section energy collapse procedure (Section IV).

The ability to treat in-cell effects was tested further by measuring sample worths at different locations in the clumped fuel unit cell. A 30.5 cm long, U(93) foil's worth was measured first adjacent to the fuel column and then adjacent to the drawer side. The worth ratio from a cell calculation with transport FOP is 1.078, which is close to the measured ratio of 1.070 ± 0.003 . However, when the standard homogenized adjoint was used, the calculated result was poor, 1.033.

Diagnostic Experiments

There were experiments aimed at quantifying the contribution to worth C/E discrepancies from a number of potential sources. Six effects are discussed and then the implications of the combined effects are considered.

1) It is assumed that foreign material, introduced into the unit cell by the measurement technique, does not distort the net sample's worth. The accuracy of the assumption was checked experimentally. Steel was placed around a normally bare U(93) sample to simulate the clad of the Pu sample, and its worth was measured as the difference of cladged sample and empty clad worths. Making this separability assumption gave essentially the same result as explicitly measuring the bare U(93), a difference of $0.3\% \pm 0.7\%$.

2) Small sample worth calculations ignore the flux distortion at the sample location caused by the presence of the polyethylene FAR (Section III) located about 30 cm away. The FAR distortion of a U(93) sample's reactivity at core center was measured by positive period and was found to be negligible, $0.33\% \pm 0.21\%$. The mass of the polyethylene blade varies linearly over the core height, which could distort a measured axial worth profile. The measured effect on the ratio of U(93) worths at 21.6 cm in M-half vs S-half was $1.2\% \pm 0.8\%$, indicating a marginally significant distortion of axial worth profiles.

3) The effect of the cavity, normally present during measurements of cell-spanning samples, was evaluated. The measured ratio of U(93) worth in a negligible cavity to the worths in cavities used routinely is 1.010 ± 0.003 for a 0.32 cm wide cavity and 1.017 ± 0.003 for a 0.64 cm wide cavity. It is not surprising that values from the simple prescription used to adjust calculated worths, 1.002 and 1.004, are in error and that the impact on worth C/E's is small (Section IV).

To provide a benchmark for testing more sophisticated calculations, the U(93) worth was measured using a large range of widths of the 5.08 cm square cavity. These data are shown in Fig. 3. The sample spanned the cavity width only in the two smallest cavities. The 0.05 cm cavity width was the same as the sample thickness and there was no U-channel spacer. There was not enough room

for centering springs in the 0.16 cm gap and the four foils comprising the sample warped, spanning the gap. For the larger cavity widths, springs centered and compressed the foils.

4) The calculated sample size correction factors used for Table III are less than 1%, except 1.1% for plutonium and up to 5% for iron and steel. Size effect calculations were tested by measuring cell-spanning samples of different thicknesses. The results in Table IV suggest that there may be a slight miscalculation, but the effect on Table III results is negligible.

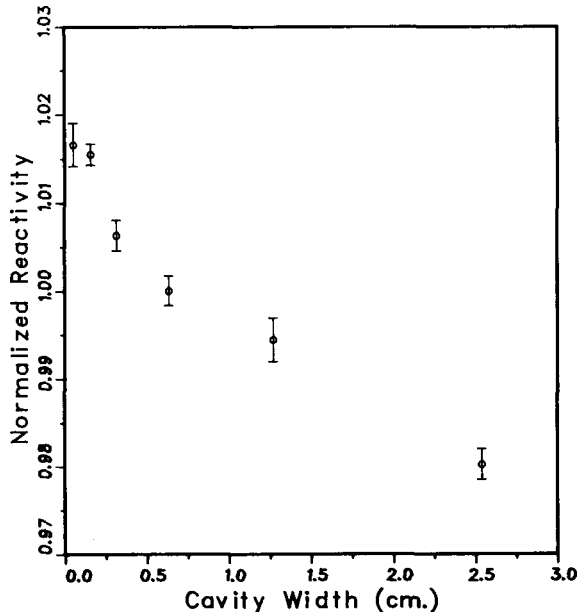


Fig. 3. Cavity Size Effect

TABLE IV. Worth Ratio for Pairs of Samples - Size Effect

Material	Two Sizes (gm/cm ²)	Ratio of Specific Worths	
		Calculated	Measured
Pu(93)	1.110, 0.535	1.010	1.014 ± 0.007
U(93)	2.857, 0.410	1.017	1.026 ± 0.008
U(93)	2.857, 0.828	1.014	1.024 ± 0.006

5) The measured small sample worths in inhours were observed to be insensitive to the delayed neutron data. The difference between interpreting the signals with effective delayed parameters based on ENDF/B Version IV vs Version V was well within the measurement uncertainty.

An error in the delayed neutron data would enter worth C/E's through the conversion of calculated $(k - k_0)/k$ to inhours. Combining the noise measurement of beta-effective with an all-rod drop measurement of effective delayed neutron lifetime,

$$\sum_i (\beta_{\text{eff}}^i / \beta_{\text{eff}}) / \lambda^i,$$

produces a measured value for a quantity that approximates the conversion factor to within a few percent. The C/E on this approximate conversion factor is 1.02 ± 0.02 .

6) Since the numerator of the reactivity expression has the local flux and adjoint while the denominator involves an integral over the assembly,¹⁸ flux and adjoint spatial shape errors affect central worth C/E's. There were axial

traverse measurements in the reference assembly for U(93) and U(0.2) samples and for a ^{252}Cf neutron source. The drop in worth and source importance (adjoint¹⁹) across the core was severe; from center to 30 cm axially it was a factor of 2 for the source importance, a factor of 3 for U(93) worth, and a factor of 5 for U(0.2) worth. Considering this, the shape mispredictions are modest. The C/E drops continuously with distance from center and by 30 cm it is down $4\% \pm 1\%$ for importance, $7\% \pm 1\%$ for U(93) worth. (The C/E data for U(0.2) are too uncertain to show any trend.) Reaction rate traverses also were done, and their C/E's show a comparable behavior.

These diagnostic experiments have identified several small problems. The C/E for the conversion to inhours suggests that there is possibly a delayed data error tending to make calculated worths slightly too high. The error in the flux and adjoint distributions also tends to make calculated central worths too high, by up to a few percent. The error in the calculated cavity distortion factor makes the calculated worths too large by about 1%. There appears to be no significant problem from the three other issues examined. All the identified errors tend to make the C/E's too large, yet the fissile worth C/E's, when adjusted to account for control rods, are very near unity. Thus, it appears that there are some unidentified, compensating error sources which are a few percent in magnitude.

Fuel Thickening

In the fuel thickened zone experiment, the clumped unit cell's fuel column was thickened by 5.3% throughout a 30.5 cm long central zone of drawers. Because of the large reactivity addition, it was necessary to reduce the core radius before creating a 9 drawer zone and again before expanding the zone to 21 drawers. To minimize the complexity, the core boundary drawers converted to reflector were not near control islands. The 9 drawer zone, added to a 159 drawer core, had a measured worth of 93.7 ± 0.3 lh and a C/E of 0.948 ± 0.003 . The 9 to 21 drawer zone expansion worth, in a 155 drawer core, was 109.3 ± 0.3 lh and the C/E was 0.980 ± 0.003 .

Since calculational and measurement techniques for fuel thickening worth were different from those for small sample worths, comparison of the two results provides a consistency check. The C/E's for the thickened zone are slightly lower than for the central, cell-spanning U(93) sample (Table III), which is consistent in light of the axial shape error found in the small sample traverses. A direct comparison of zone and small sample calculational methods was also made. The worth of the 30.5 cm long U(93) foil adjacent to the drawer oscillator's fuel column has a C/E of 0.98 when calculated either as a single drawer zone or as a small sample (FOP). Good agreement between fuel thickening and small sample worth C/E's is not obtained without the adjoint correction. In fact, it was this comparison, in a different diagnostic assembly, that stimulated recognition of the adjoint problem's magnitude.

Fuel Clumping

The reactivity induced by altering the unit cell also was evaluated. For the dispersing of fuel, different plates were involved and the signal was too small (3 lh) relative to the mass change uncertainty. Conversion from the reference to the clumped fuel configuration did not have these problems. There was a large reactivity change, involving only plate rearrangement, that was

accomplished in two steps, a full height 21 drawer zone and complete clumping. Unfortunately, the fuel clumping was always accompanied by a core boundary change. To obtain the clumping reactivity C/E, we used a C/E for the boundary change worth that is based on three boundary change measurements during the fuel thickening experiment. The C/E used is 0.96 with a conservatively large uncertainty of ± 0.03 . Clumping fuel in the central 21 drawers has a calculated worth of 98.7 lh and a C/E of 0.85 ± 0.03 . Similarly, clumping fuel in a 161 drawer core, in all drawers except control islands and Rod No. 7, has a calculated worth of 315.9 lh and a C/E of 0.83 ± 0.03 .

Although the C/E discrepancy is large, the practical consequences apparently are not serious. Since one percent in the eigenvalue is about 500 lh, the C/E corresponds to only a 0.1% error in the eigenvalue of the clumped assembly relative to the reference assembly eigenvalue. Another point is that the clumping experiment was an extremely sensitive test of heterogeneity calculations; it involved a major disruption of the unit cell without any mass change. In the less severe case of fuel clumping, the homogenization procedure worked well.

VI. CONCLUSIONS

The existence of a cell heterogeneity-related problem in small sample worth C/E's was clearly demonstrated experimentally. Standard first order perturbation theory (FOP) calculations of central worths of a set of samples spanning the unit cell yielded C/E's that increase progressively with increasing cell heterogeneity (Table III). With the most heterogeneous cell, the C/E's are comparable to those observed historically in uranium-fueled assemblies.

Results were greatly improved by correcting the adjoint flux used in the FOP calculations. It is well known that cell homogenization by the conventional flux weighting procedure does not preserve the adjoint in principle.^{11,12,4} It was only recently that the magnitude of the problem for our plate unit cells was determined, and a correction procedure was developed.¹³ Applying this correction removes the progressive C/E increase with cell heterogeneity, and yields C/E's near unity for most samples (Table III).

Corrected small sample worth C/E's were obtained for a variety of sample types. One category is a uniform material that spanned the unit cell's cross section. For this type, fissile worth C/E's are near unity. As has been observed historically, the worth C/E's are lower for boron and higher for scattering materials than they are for fissile samples. The C/E's for the cell-spanning samples are consistent with results from a critical assembly at Los Alamos with a similar composition, BIG 10.²⁰ A second sample type is the unit cell sample, which is a thin slice of the plate cell cross section. Unit cell worth C/E's are comparable to those from cell-spanning fissile samples. Because in-cell effects for this sample are addressed at the cross section generation level, there is essentially no trend in the unit cell worth C/E with cell heterogeneity, even without an adjoint correction. The last sample type is the U(93) foil located at either of two in-cell positions, at the fuel column or away from it. Worth C/E's for both positions were comparable to those for the same material spanning the cell, but only when adjoint corrections were applied. Thus, for a variety of small sample experiments, corrected C/E's are near unity (Table III).

An alternative determination of U(93) worth was the fuel thickening experiment. The perturbation consisted of thickening the unit cell's fuel column by 5% in a central zone of the clumped assembly. Both the measurement and calculation techniques were substantially different from those used for small sample worths. Thus, the fact that C/E's for this experiment are comparable to those for small fissile samples enhances confidence in the methodologies.

The U9 program included a large number of reactivity and related experiments that made possible an in-depth understanding of small sample worth C/E's in these assemblies. Most of the reactivity experiments have been mentioned. The possibility of delayed neutron data errors contributing to worth C/E's was examined by combining measurements of beta-effective and effective delayed neutron lifetime. Flux and adjoint spatial shape errors, which affect central worths, were evaluated using reaction rate traverses, a neutron source importance traverse, and sample worth traverses. The ability to treat several sources of distortions in the flux and adjoint at the sample, arising from the measurement technique, was also investigated. Some problems were identified but they are small compared to the historical central worth discrepancy.

The U9 results represent a significant step toward understanding the historical central worth discrepancy, but there are difficulties in relating U9 results to this longstanding problem. One difficulty is that the small sample worth measurement technique used in U9 entailed less distortion of the unit cell environment than the technique used in most LMFBR assemblies. Experimental evidence available so far, which is not conclusive, suggests that the extra distortions do not affect worth C/E's by more than a few percent. A second factor is that calculations for earlier assemblies did not include an adjoint correction. This appears to be about a 5% effect for fissile samples in LMFBR assemblies.¹³ Finally, the U9 spectrum and composition are not typical of the LMFBR. However, U9 is part of a series of benchmark assemblies that covers a wide composition and spectrum range bracketing the LMFBR characteristics. The results from this series will be a valuable database for reassessment of the central worth discrepancy.

VII. ACKNOWLEDGEMENTS

A number of experimenters at ZPR contributed data used in this paper. D. M. Smith assisted with the material worth experiments. E. F. Bennett made the beta-effective measurement, and G. J. DiIorio measured the delayed neutron lifetime. Fuel clumping worth was measured by R. B. Pond and J. A. Morman. Reaction rate traverses were measured by G. J. DiIorio, J. A. Morman, and W. R. Robinson. Calculations of sample size effects were done at ZPPR by P. J. Collins.

VIII. REFERENCES

1. W. W. Little, Jr. and R. W. Hardie, "Discrepancy Between Measured and Calculated Reactivity Coefficients in Dilute Plutonium Fueled Fast Criticals," Nucl. Sci. Eng., 36 p. 115 (1969).
2. L. G. LeSage and R. D. McKnight, "Discussion of Integral Experiment C/E Discrepancies," Proc. Inter. Conf. on Nuclear Cross Sections for Technology, Knoxville, TN, NBS Special Pub. 594 (1979).

3. D. C. Wade, "Monte Carlo-Based Validation of the ENDF/MC²/SDX Cell Homogenization Path," ANL-79-5, Argonne National Laboratory (1979).
4. D. C. Wade and R. G. Bucher, "Conservation of the Adjoint Neutron Spectrum by use of Bilinear-Weighted Cross Sections and Its Effect on Fast Reactor Calculations," Nucl. Sci. Eng. 64, p. 517, (1977).
5. R. D. McKnight et al, "Validation Studies of the ENDF/MC²/SDX Cell Homogenization Path," these proceedings.
6. L. G. LeSage, "Status of Fast Reactor Physics," Proc. Conf. 1980 Advances in Reactor Physics and Shielding, Sun Valley, ID (1980).
7. E. M. Bohn, "The Central Worth Discrepancy in Three Fast Reactor Benchmark Critical Assemblies," ANL-75-14, Argonne National Laboratory (1975).
8. W. G. Davey and W. C. Redman, Techniques in Fast Reactor Critical Experiments, Gordon and Breach Science Publishers, New York, p. 32 (1970).
9. W. K. Foell, Small-Sample Reactivity Measurements in Nuclear Reactors, American Nuclear Society Monograph, Hinsdale, IL, p. 49 (1972).
10. C. E. Cohn, et al, "Computer-Aided Calibration of a Fine Auto Rod," Nucl. Applications, 7, p. 342 (1969).
11. R. B. Nicholson, "Cross Section Averaging Schemes for Group Collapsing and Cell Homogenization in Neutron Transport Calculations for Critical Assemblies," ANL-7610, p. 488, Argonne National Laboratory (1970).
12. R. G. Palmer et al., "A Study of Several Methods for Analysis of Critical Assembly Heterogeneity," Nucl. Sci. Eng., 50, p. 229, (1973).
13. K. S. Smith, "The Effects of Intracell Adjoint Flux Heterogeneity on FOP Reactivity Calculations," Nucl. Sci. Eng., (To be published).
14. P. J. Collins and R. G. Palmer, "Calculated Size Effects for Reactivity Perturbation Samples in ZPPR," ANL-7910, p. 247, Argonne National Laboratory (1972).
15. G. E. Hansen and C. Maier, "Perturbation Theory of Reactivity Coefficients for Fast-Neutron Critical Systems," Nucl. Sci. Eng., 8, p. 532 (1960).
16. K. S. Smith, Private Communication (1981).
17. R. D. McKnight, Private Communication (1982).
18. A. F. Henry, Nuclear-Reactor Analysis, p. 321, M.I.T. Press, Cambridge, MA (1975).
19. W. K. Foell, op. cit., p. 119.
20. G. E. Hansen and H. C. Paxton, "A Critical Assembly of Uranium Enriched to 10% in Uranium-235," Nucl. Sci. Eng., 72, p. 230 (1979).

NEUTRONIC ANALYSIS OF SIMULATED LMFBR FUEL SLUMP EXPERIMENTS

G. Henneges, W. Maschek
Institut für Neutronenphysik und Reaktortechnik
Kernforschungszentrum Karlsruhe
Postfach 3640
D-7500 Karlsruhe
Germany

Abstract

Based on LMFBR accident studies at KfK a number of safety-relevant core configurations were defined for the SNEAK 12A critical assembly. They simulate compacted and diluted regions resulting from expulsion of materials from portions of the core. Transport calculational methods were necessary to give a satisfactory interpretation. In the course of these evaluations two different 2-dimensional transport options available at KfK were tested and compared with experimental results. Direct eigenvalue results with the S_4P_0 approximation in RZ-geometry predict the reactivity changes fairly well while first order perturbation transport theory leads to deviations which increase with the amount of fuel compacted but stay well below 20 % if only moderate quantities of fuel are involved.

Introduction

Hypothetical core disruptive accidents (HCDAS) - though their low probability - have played an outstanding role in Liquid Metal Fast Breeder Reactor (LMFBR) safety evaluations. Since the fast reactor core is not assembled in its most reactive configuration, redistribution processes of the core materials during core disruption, caused e.g. by a LOF accident (loss of flow with failure to scram), have the potential of leading to severe nuclear power excursions.

Early studies of fast reactor accidents emphasized the analysis of energetic excursions where accident initiation e.g. by sodium boiling processes (LOF) ended by a violent core disassembly on the milliseconds timescale. However later on it was realized that the initiation phase of an accident need not terminate energetically. Instead an alternate accident progression path was realized where sufficient negative neutronic feedbacks prevent a violent disassembly but lead to a gradual melting and core disruption and the formation of pools of molten/boiling fuel and steel on a secondstimescale. This accident phase which involves a grossly disrupted core where substantial motion of molten fuel has occurred was called transition phase. Though the transition phase is energetically benign, redistribution and compaction processes of the fuel mobilized by the primary neutronic burst during the initiation phase may result in a new critical configuration (recriticality) which involves a secondary nuclear excursion. Thus also during the mild meltdown accident phase there exists the potential for energetic disassembly of the already disrupted core configuration.

The initiation and transition phase of an accident are usually analyzed with large mechanistic accident codes like SAS3D /1/ and SIMMER /8/.

Due to redistribution processes of the molten materials alternating very dense and highly dilute (voided) regions occur with a complete change of the neutron spectrum and leakage properties compared to the steady state reactor, making the calculation of the reactivity state progressively difficult.

Therefore, based on the above analyses neutronically interesting core configurations were studied in critical assembly experiments at KfK. The scenarios of material relocation which were investigated can be summarized as follows:

After sodium voiding and dryout in the high-powered channels of the core clad and fuel motion starts leading to a redistribution of the material towards the axial blanket regions. After fuel pin breakup and local redistribution the fuel/steel mixture moves away from core mid-plane with further fuel pin breakup and finally comes to rest in the axial blankets where freezing processes may temporarily trap the mobile core material (slump-out).

Gravity or a lack of dispersive pressure forces can also lead to recompaction processes (slump-in) of the mobile fuel/steel material after the intact core geometry is destroyed. The detailed knowledge of the reactivity state here is even more important than for the slump-out conditions as fuel compaction leads to an increase in reactivity /2/.

Experiments

Idealized configurations typical for the accident sequence described above were investigated in SNEAK 12A. This assembly has a single core zone, fuelled with 20 % and 35 % enriched uranium plates as shown in Fig. 1. It is surrounded by a radial blanket built of depleted uranium.

The first part of the experimental program included the reactivity effects of cavities, streaming channels, and steel relocation. Some of the results are given in /3/. The present paper is concerned with fuel relocation which causes the most significant reactivity changes.

Even though the selected core configurations represent accident-caused material redistributions only in an idealized way they will help to assess the validity of transport code methods in use at KfK. It should however be kept in mind that the material configurations in critical assemblies like SNEAK 12A usually exaggerate the material distributions given by accident code simulation with SIMMER or SAS3D. The material compactions and dilutions there are much more smeared out and smoother without density steps. Thus the deviations between calculation and experiment (C/E values) will be smaller in actual accident material configurations given by computer codes.

For this reason in SNEAK 12 also more prototypic accident conditions with more moderate core distortions have been modelled.

All changes of the material arrangement were performed in the central 16 elements corresponding to a zone of 24.56 cm diameter. The reactivities were measured against the case where these elements had the normal arrangement of fuel but were voided of sodium. For the large slump-in case only 12 elements were modified in order to avoid the unloading of too many elements at the core periphery.

All material movements were in axial direction and symmetrical to the core midplane. In some cases intermediate steps were loaded by modifying only one or 4 elements before going to the full number of 16 (or 12).

Fig. 2 shows the sequence of configurations built by compacting fuel material either towards the core midplane or towards the axial core boundary.

In one slump-out case only the depleted uranium oxide plates were moved while the enriched uranium metal plates were kept in place. Although this does not simulate any accident situation the analysis of this case will give some information on the reactivity contributions of the different materials and on the source of discrepancies appearing between measurements and calculations of the more realistic cases.

Fig. 3 gives the plate arrangement of each cell used. The fuel and material inventory per element was kept the same in each step.

The reactivity effects were measured, as far as possible, by compensation with calibrated control rods. The calibration was repeated in each fuel displacement configuration where core size was changed. The reactivity scale based on the inverse kinetic method was converted to the absolute unit of $\Delta k/k$ using a value for the delayed neutron fraction which was derived by the data and method proposed by E.A. Fischer in /6/.

The loss of reactivity due to large fuel slump-out could not always be compensated by adding edge elements. Where subcritical states exceeded about 60 $\%$ the modified source multiplication method (MSM) was applied to determine the reactivity.

Due to the small spontaneous fission yield in an uranium fuelled core it is necessary to add an external n-source. The position had to be off-centre. It is indicated by S in Fig. 1. Four BF_3 detectors are located outside the radial blanket at positions D1 to D4 also shown in Fig. 1. The neutron count rates were measured within an accuracy of better than 0.1 %.

The results given for these measurements so far are uncorrected for changes in source and detector efficiency as the calculations for this correction which involve a point source in a three-dimensional geometry have not yet been accomplished.

The reactivities are derived from the count rates of the detector located opposite the AmBe -source which was placed in the core midplane, 28 cm off the core center. On the basis of previous experience the correction for this position should be less than 3 %.

The reactivity increase in the large slump-in experiments had to be compensated by unloading edge elements. The reactivity worth of these edge elements was taken into account when determining the reactivity changes caused by the slump-in steps.

The worth of these edge elements varied between 14 and 20 β .

This procedure introduces additional errors because the reactivity worth of each edge element is only known with an uncertainty of $\pm 0.5 \beta$. The maximum number of edge elements removed was 17. In this case the uncertainties may sum up to several cents.

The calibration of SNEAK control rods which had to be performed for each step may also introduce additional uncertainty in the order of 1 β .

Calculational Methods

All reactivity worths were calculated using transport techniques available at KfK. The codes used were the S_N -program SNOW /4/ and in connection with it the transport perturbation program TP2 /5/. Data basis for the calculations was the KFKINR001 cross section set with 26 energy groups. Isotropic cross sections for homogenized material mixtures were used.

The different configurations were modelled in RZ geometry. The transport codes used were run in the S_4P_0 -approximation.

The large slumps were also calculated with the RZ diffusion theory code DIXY /9/ to see if the nonconservative behavior found in other laboratories /7/ could be reproduced at KfK.

Results

Table 1 and 2 summarize the fuel slump-out and slump-in experiments performed in SNEAK 12A, the measured reactivities and the corresponding results obtained from eigenvalue transport calculations and from first order perturbation calculations.

The S_4 transport calculations for all cases agree with the measurements within 12 % and $C - E < 16 \beta$.

The perturbation calculations show distinct deviations which increase with the amount of fuel compacted. However for the small fuel relocations and for the large fuel movement in only 4 elements (amount of fissile material compacted <10 kg) the discrepancies stay well below 20 β .

The deviations are due to the fact that the increased multiplication in the compacted region is not taken into account by perturbation theory.

Calculations with the diffusion code DIXY resulted for the large slump-out case in an overestimation of 43 % and for the large slump-in case in an underestimation of 25 % with respect to the corresponding transport calculations. These deviations are both in the non-conservative direction.

Summary

Core configurations with compacted and diluted regions were studied in the SNEAK 12A critical assembly. Only transport calculational methods could reproduce the reactivity effects observed in experiments. Two different 2-dimensional transport methods were tested and compared with experimental

results. First order perturbation theory leads to discrepancies which increase with the amount of fuel involved but stay in the range below 20 % for moderate amounts of materials moved.

Direct transport calculations reproduce the experimental values much better confirming the conclusion in /7/ that the qualitative agreement with experiment is good. They slightly underestimate all reactivity changes which is in the nonconservative direction for the slump-in cases. The increased multiplication in the compacted region leads to a nonlinear behaviour with respect to increased number of perturbed elements. This effect is quite well reproduced by direct transport calculations.

References

- /1/ J. F. Cahalan, D. R. Ferguson et. al., "A Preliminary Users Guide to Version 1.0 of the SAS3D-LMFBR Accident Analysis Computer Code - ANL (July 1977)
- /2/ W. Maschek, et al., "Recriticality Considerations and Core Material Redistribution in the Reactor Vessel of SNR-300 as a Consequence of Loss of Flow Transients in the Mark 1A Core, Int. Meeting on Fast Reactor Safety Technology, Seattle 1979
- /3/ F. Helm, G. Henneges, S. Synetos, "Critical Experiments on Accident Related Configurations for Fast Reactors", ANS-Winter Meeting 12/81, San Francisco
- /4/ E. Günther, W. Kinnebrock, "SNOW - ein zweidimensionales S₂-Programm zur Lösung der Neutronentransportgleichung in Platten- und Zylindergeometrie", KfK 1826 (1973)
- /5/ K. Kobayashi, "TP2 - A Computer Program for the Calculation of Reactivity and Kinetic Parameters by the Two Dimensional Transport Perturbation Theory", KfK 2387 (1979)
- /6/ E.A. Fischer, "Integral Measurements of the Effective Delayed Neutron Fraction in the Fast Critical Assembly SNEAK", Nucl. Sci. Eng. 62 (1977)
- /7/ R. Curtis et. al, "The Use of Benchmark Criticals in Fast Reactor Code Validation", IAEA-OECD Int. Symp. on Fast Reactor Physics, Aix-en-Provence, France 9/1979
- /8/ L. L. Smith (ed.), "SIMMER-II, A Computer Program for LMFBR Disrupted Core Analysis", LA 7515 /NUREG CR 0453, 10/78
- /9/ W. Höbel, "Numerical Methods Used in the Two-Dimensional Neutron Diffusion Program DIXY", Computers, Fast Elliptic Solvers and Applications (Schumann U., Ed.), Advance Publications (1977)

Table 1 Results of the Determination of the Worths of Configuration Changes
Sodium-Void and Slump-out Experiments

Experiment	Number of Perturbed Elements	Reactivity Worth (β)		
		Exp.	Direct Transport ^a	First Order Perturb. ^b
1. Sodium Void Full Core Height	4	- 5.9	- 4.7	- 4.3
	16	- 24.9	- 19.8	- 18.2
2. Small Fuel Slump-Out ^c	4	+ 0.9	-	- 1.1
	16	+ 5.2	-	- 4.4
3. Large Fuel Slump-Out ^c	4	- 47.1	-	- 60.1
	16	-107.2	- 98.2	-230.7
4. Fuel Slump-Out into Blanket	16	-397.0	-391.0	-466.7
5. Only UO ₂ Slump-Out	4	+ 7.7	-	+ 9.4
	16	+ 25.4	+ 22.6	+ 34.3

^a SNOW eigenvalue results, S_4

^b TP2 calculations

$$\beta_{\text{eff}} = 0.007627$$

^c Reactivity changes for cell slump-out and slump-in cases are given in relation to the Na-void case

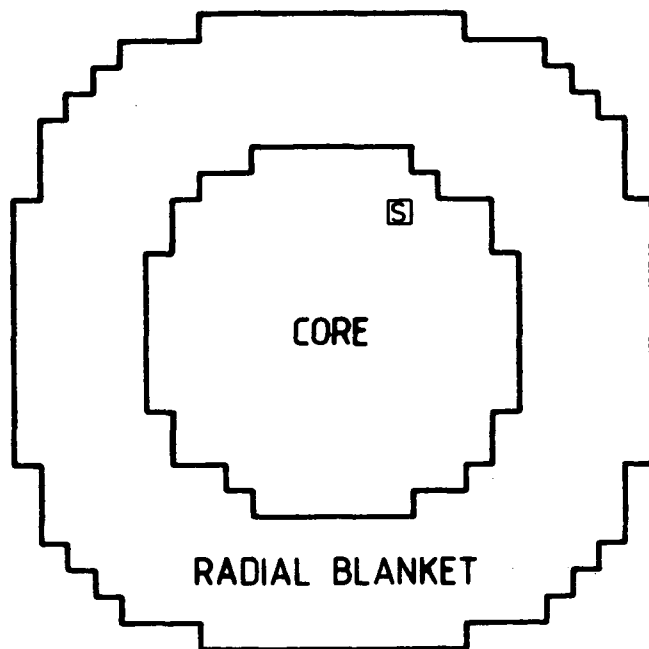
Table 2 Results of the Determination of the Worth of Configuration Changes
Slump-in Experiments

Experiment	Number of Perturbed Elements	Reactivity Worth (ρ)		
		Experiment	Direct Transport ^a	First Order Pert. ^b
1. Small Fuel Slump-In ^c	1	+ 0.9	-	-
	4	+ 4.5	-	+ 1.4
	16	+ 18.1	-	+ 5.1
2. Large Fuel Slump-In ^c	1	+ 17.3	+ 15.1	-
	4	+ 76.5	+ 67.0	+ 60.2
	12	+243.7	+227.7	-
	16	-	+337.7	+230.1

a,b,c see Table 1

D2

D3



Na
U20%
Na
U35%
UO ₂

CORE CELL

Na
UO ₂

AXIAL BLANKET CELL

CORE HEIGHT 80.2 cm

CORE RADIUS 38.8 cm

D1

D4

115.

KfK

FIG. 1

SNEAK 12 A

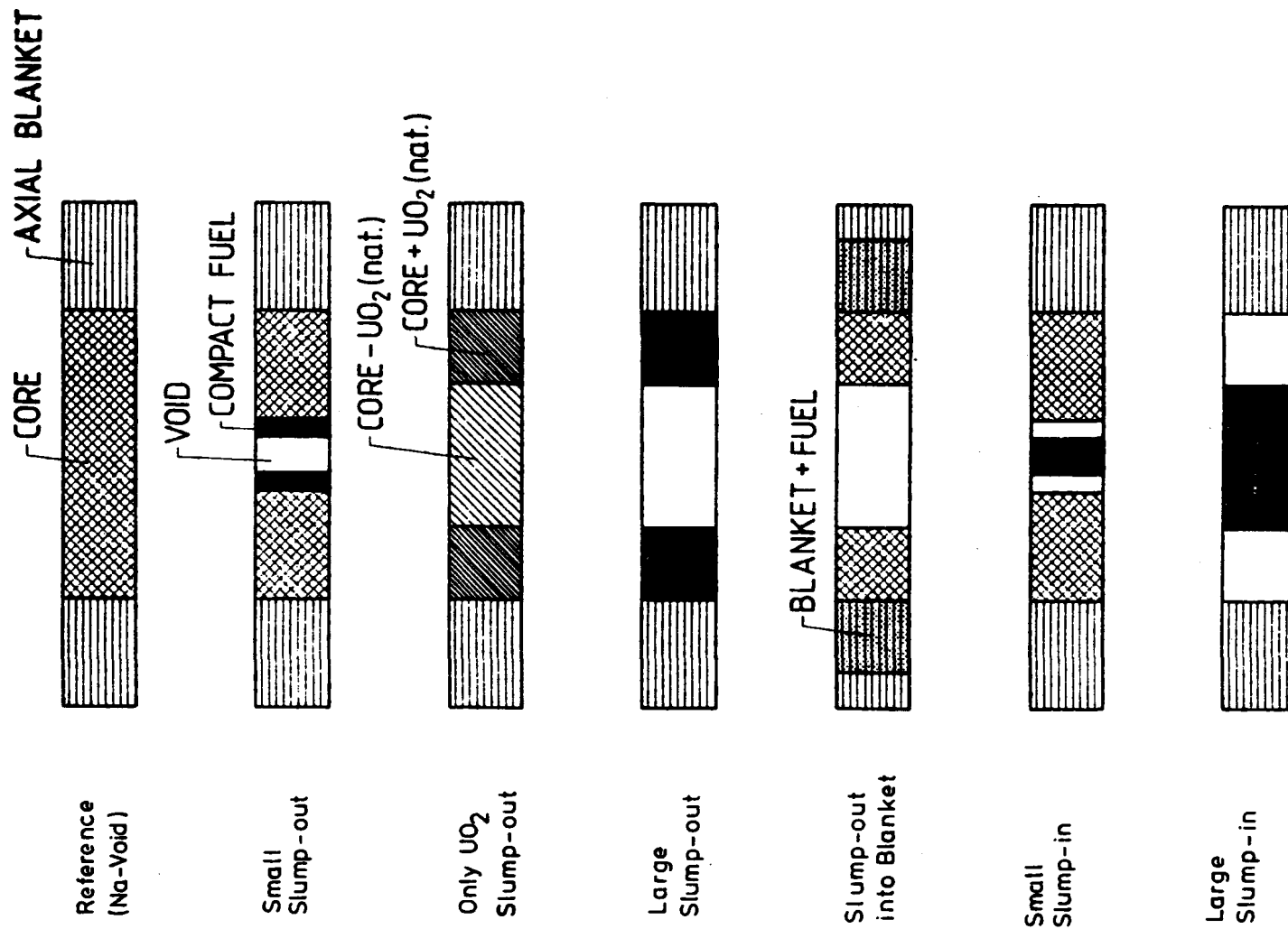


FIG. 2

FUEL REDISTRIBUTION

K&K

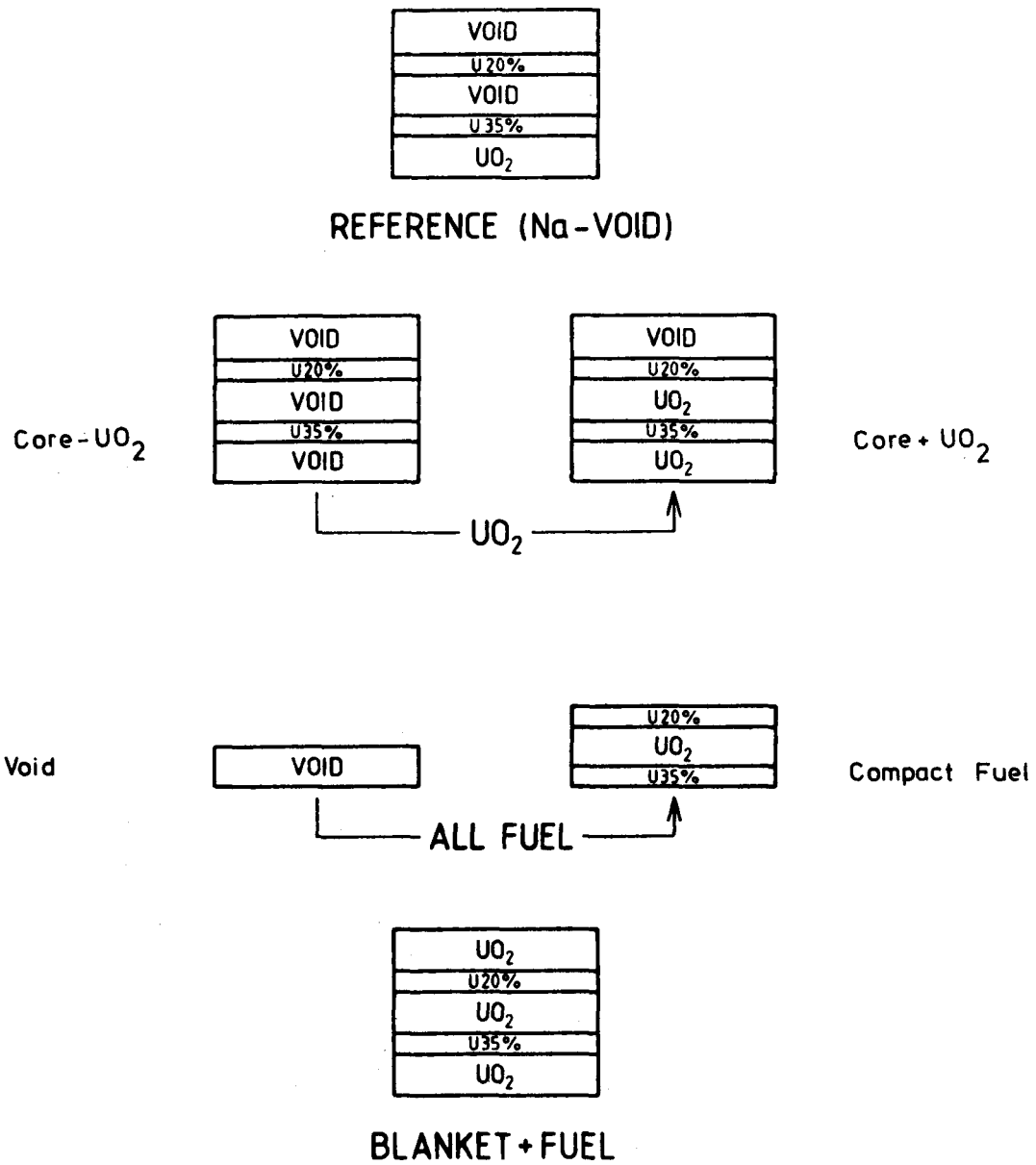


FIG. 3

KIK

EXTRAPOLATION OF SMALL SAMPLE DOPPLER REACTIVITY MEASUREMENTS

R.E. Kaiser, H.A. Harper, C.L. Beck and S.G. Carpenter

Argonne National Laboratory
Applied Physics Division
P.O. Box 2528
Idaho Falls, ID 83401

ABSTRACT

An application of ^{238}U small sample Doppler measurements to the confirmation of LMFBR design calculations is proposed. The method uses the Doppler measurements in conjunction with $^{238}\text{U}(n,\gamma)$ data to predict the radial behavior of the Doppler reactivity C/E, and includes a description of azimuthal variations by using the measured values to normalize a calculated map. The result is a spatial map which can be used, along with data from the SEFOR experiments, to test design level calculations of Doppler reactivity.

1.0 Introduction

The negative reactivity feedback provided by the ^{238}U Doppler effect is an important consideration in an LMFBR in three areas: specifications of enrichment and control requirements; predictions of kinetic behavior of the core; and the prediction of the energy yield of transients. There is strong motivation not only to know the accuracy with which the Doppler effect can be calculated, but to reduce the uncertainties, if possible, by including more and newer information in the analysis.

The principal benchmark used by U.S. LMFBR designers to confirm Doppler reactivity calculations is the SEFOR experiments. The extensive body of small sample Doppler effect measurements performed in Argonne's ZPPR and ZPR-6 and -9 has not been effectively utilized because design groups are generally not prepared to calculate the details of a critical assembly measurement. This paper endeavors to use small sample measurements to infer an estimated Doppler map which can be used as a benchmark to test calculations of Doppler reactivity. Because the Doppler effect can only be measured in a few places in the critical facility, additional information is necessary to interpolate between measurement locations and to account for spatial variations.

The specific measurements studied were those performed in phases B, C and E of ZPPR assembly 11, the last of the CRBR engineering mockup series. Phase B was a representation of the beginning-of-life core, and phases C and E of the end-of-cycle core in the sodium-flooded and sodium-voided configurations respectively. Figure 1 is an interface diagram for Assembly 11B showing the locations of the internal blankets. In 11C and 11E, fuel was added to the blankets and core enrichment was reduced, simulating end-of-cycle conditions.

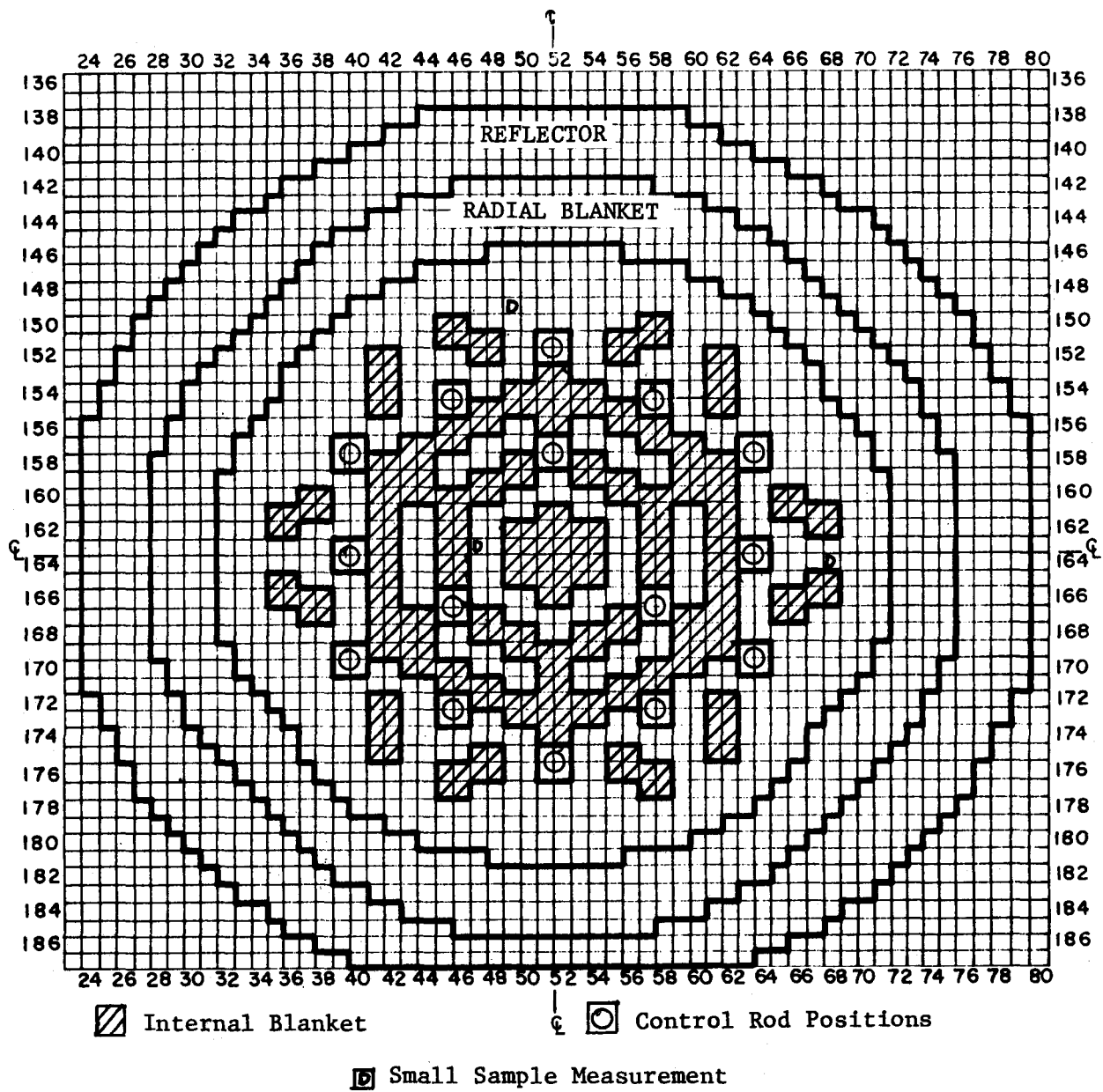


Fig. 1. ZPPR-11B Configuration.

2.0 Small Sample Doppler Studies

Small sample Doppler measurements have been performed at Argonne for many years.¹ A substantial data base exists, consisting of measurements using fissile and fertile samples in a wide variety of reactor environments. In recent years, attention has focused upon ^{238}U as the prime contributor to core Doppler effect. With the advent of heterogeneous designs, spatial variations have also increased in importance.

2.1 Small Sample Measurement Technique

The small sample measurements performed at ANL use a cylindrical UO_2 sample, sealed in Inconel, 305 mm long by 25.4 mm in diameter, with a 3 mm hole down the center to accommodate thermocouples. A typical natural UO_2 sample contains about 1 kg of ^{238}U . The sample can be heated, under vacuum, by a heating element coiled around the Inconel capsule. Reactivity measurements at ZPPR are performed by oscillating the sample in and out of the reactor axially, in a specific matrix location. For this purpose, the sample is mounted in a long oscillator rod and connects to a central vacuum tube. Cooling air is blown through the Doppler rod in order to remove excess heat and minimize heat loss to the surrounding core. To further reduce errors due to heat loss from the sample, a secondary heater, external to the sample, is used to dump heat to the cooling air. The system is regulated so that total output from the two heaters remains constant as the sample temperature increases. Thus the heat loss to the surrounding core is not only small but is designed to be invariant with sample temperature.² The Doppler effect is determined by subtracting the sample worth at the reference temperature from that determined at a higher temperature. The maximum temperature used in such experiments is 1100K.

2.2 Calculations

Analyses of the small sample Doppler reactivity measurements utilize 28 group cross section libraries based on ENDF/B Version IV data. Multigroup cross section data for the reactor regions are prepared through MC²-II/SDX to account for the spatial and resonance heterogeneities of the plate loadings for each drawer type. Two dimensional diffusion-perturbation calculations in xy geometry are done to generate flux distributions and sample worths. In the xy models, each drawer type is explicitly represented. Group- and region-dependent leakage corrections are applied in these calculations to account for the axial dimension. Full core models are required, because the Doppler sample mechanism is a sufficiently large perturbation that flux shifts are observed for the off-center measurements. For this reason, the Doppler sample and mechanism are also explicitly included in the reactor calculation.

The calculation of Doppler worth for the sample is based on first order perturbation theory. Since the sample is explicitly modeled in the reactor calculation, the perturbation is defined as the change from reference temperature to elevated temperature cross sections for the sample. In order to normalize the perturbation denominator and integrate over sample length, the axial flux shape is represented as $\phi(xyz) = \phi(xy) \cos B_z z$, where B_z is a constant buckling value chosen so that an xy calculation with

constant buckling gives the same k_{eff} as the reference case with group- and region-dependent values.

Preparation of the Doppler sample cross section data is based on a pin cell model. A two region pin cell consisting of the Doppler sample at the center surrounded by structural steel, is processed in MC²II/SDX for each temperature to produce resonance self-shielded cross sections in an intermediate (156) group structure. For group collapse to the 28 group level, a diffusion calculation is done for a one-dimensional cylindrical model consisting of sample, structure and reactor core. Cross sections are collapsed for each sample temperature.

2.3 Comparison of Measurement with Calculation

The comparisons of the calculations with experiment are shown in Table I for the three phases studied. The position in fuel ring 1, (matrix position 163-47, see Fig. 1) was adjacent to the first blanket ring. The two positions in fuel ring 4 were radially outward from one blanket island (matrix position 149-49) and between two adjacent blanket islands (matrix position 164-68) for positions 4a and 4b respectively.

Several points can be made about the C/E values quoted in Table I. First, there appears to be a significant radial variation, with generally higher C/Es in fuel ring 4 than in fuel ring 1. There is also a small azimuthal variation in the sodium-voided configuration evidenced by different C/Es for positions 4a and 4b, which are at almost the same radial distance from core center. A significant difference is also observed between phases B and C, which have substantially different fuel loading, and between phases C and E which are sodium flooded and sodium voided versions of the end-of-cycle core, respectively.

3.0 Interpolation of Small Sample Results

Small sample measurements of ^{238}U Doppler reactivity at ZPPR are restricted in number by the physical size of the equipment and by the time required to perform the measurements. If greater spatial detail is required for Doppler information, methods must be devised to interpolate between measurements. One such method is presented below.

3.1 Radial Interpolation Using Reaction Rates

In examining other measurements performed in assembly 11, it is apparent that the tendency toward increased C/E with radius is common to most of them. In particular the reaction rate measurements shown in Tables II and III exhibit this behavior.

The extent of the increase in C/E with radial position is not only consistent among different experiments in the same phase, but also varies predictably between phases. Of those experiments for which C/Es are known in more detail, the $^{238}\text{U}(n,\gamma)$ reaction rate has been selected for use in radial interpolation of the ^{238}U Doppler reactivity measurements. Table IV lists the values of C/E for Doppler measurements, $(\text{C/E})^2$ for $^{238}\text{U}(n,\gamma)$ measurements, and the ratio of the two values for ZPPR-11B and 11C core

TABLE I. Small Sample ^{238}U Doppler Reactivity C/Es in Phases B, C, and E of ZPPR-11

Temp, K	C/E Values ^a		
	Phase B ^b	Phase C ^c	Phase E ^d
Fuel Ring 1 (FR1)			
500	0.950 ± 0.068	0.771 ± 0.022	0.900 ± 0.030
650	0.736 ± 0.026	0.800 ± 0.016	0.800 ± 0.015
800	0.739 ± 0.021	0.814 ± 0.012	0.808 ± 0.012
950	0.753 ± 0.018	0.836 ± 0.011	0.817 ± 0.011
1100	0.791 ± 0.017	0.820 ± 0.008	0.801 ± 0.008
Fuel Ring 4a (FR4a)			
500	0.910 ± 0.075	0.897 ± 0.030	0.880 ± 0.031
650	0.974 ± 0.063	0.859 ± 0.021	0.940 ± 0.023
800	0.901 ± 0.038	0.870 ± 0.017	0.976 ± 0.020
950	0.945 ± 0.039	0.855 ± 0.014	0.957 ± 0.016
1100	0.933 ± 0.035	0.881 ± 0.011	0.910 ± 0.014
Fuel Ring 4b (FR4b)			
500	- -	0.862 ± 0.038	0.852 ± 0.035
650	- -	0.808 ± 0.022	0.908 ± 0.026
800	- -	0.906 ± 0.021	0.886 ± 0.019
950	- -	0.933 ± 0.019	0.872 ± 0.016
1100	- -	0.883 ± 0.015	0.865 ± 0.014

^aUncertainties on C/E values are based on measurement precision only, reference temperature = 300K.

^bBeginning-of-cycle, sodium flooded core.

^cEnd-of-cycle, sodium flooded core.

^dEnd-of-cycle, sodium voided core.

TABLE II. ZPPR-11B: Summary of Radial Reaction Rate Distribution Analysis

Zone ^a	$^{239}\text{Pu}(n, f)$				$^{235}\text{U}(n, f)$				$^{238}\text{U}(n, \gamma)$			
	No.	Mean	C/E	σ	No.	Mean	C/E	σ	No.	Mean	C/E	σ
CB	3	0.992	0.012		5	1.030	0.007		4	1.041	0.009	
F1	5	0.947	0.004		7	0.980	0.010		5	1.029	0.008	
B1	2	1.004	0.016		8	1.032	0.012		4	1.041	0.011	
F2	5	0.960	0.003		8	1.013	0.008		5	1.049	0.003	
B2	3	1.025	0.025		13	1.054	0.022		7	1.074	0.016	
F3	5	0.996	0.014		11	1.043	0.022		5	1.072	0.008	
B3	2	1.059	0.005		6	1.100	0.020		3	1.099	0.013	
F4	16	1.030	0.016		47	1.081	0.017		16	1.119	0.019	
RB	15	1.052	0.026		27	1.073	0.038		15	1.103	0.017	
Ratios												
F2/F1		1.014				1.034				1.019		
F3/F1		1.052				1.064				1.042		
F4/F1		1.088				1.103				1.087		

^aCB = center blanket, B1 = blanket ring 1, F1 = Fuel Ring 1, etc.

KIA-12

TABLE III. ZPPR-11C: Summary of Radial Reaction Rate Distribution Analysis

Zone ^a	²³⁹ Pu(n,f)				²³⁵ U(n,f)				²³⁸ U(n,γ)			
	No.	Mean C/E	σ		No.	Mean C/E	σ		No.	Mean C/E	σ	
CB	3	0.999	0.003		5	1.038	0.013		4	1.054	0.008	
F1	5	0.987	0.014		7	1.019	0.008		5	1.055	0.011	
B1	2	1.016	0.003		8	1.031	0.008		4	1.061	0.004	
F2	5	0.995	0.009		7	1.022	0.011		5	1.074	0.016	
B2	3	1.011	0.005		9	1.040	0.007		6	1.067	0.006	
F3	6	1.006	0.011		14	1.037	0.013		7	1.062	0.012	
B3	2	1.031	0.019		5	1.054	0.009		3	1.077	0.004	
F4	15	1.004	0.010		40	1.043	0.011		15	1.089	0.018	
RB	14	0.998	0.039		23	1.012	0.041		14	1.060	0.032	
Ratios												
F2/F1		1.008				1.003				1.018		
F3/F1		1.019				1.018				1.007		
F4/F1		1.017				1.024				1.033		

^aCB = center blanket, B1 = blanket ring 1, F1 = Fuel Ring 1, etc. KiA-13

TABLE IV. Values of Average C/E for ²³⁸U Doppler Measurements Compared with Average (C/E)² for ²³⁸U(n,γ) Foil Measurements in Fuel Rings of ZPPR-11B and -11C

Fuel Ring	$\overline{C/E}$, Doppler React.	$(\overline{C/E})^2$, $^{238}\text{U}(n,\gamma)^a$	$\frac{\overline{C/E}, \text{Doppler React.}}{(\overline{C/E})^2, ^{238}\text{U}(n,\gamma)}$
<u>ZPPR-11B</u>			
1	0.768 ± 0.026	1.059 ± 0.011	0.725 ± 0.028
4a	0.930 ± 0.026	1.252 ± 0.027	0.742 ± 0.037
		Ave. Ratio = 0.731 ± 0.033	
<u>ZPPR-11C</u>			
1	0.818 ± 0.007	1.113 ± 0.016	0.735 ± 0.017
4a	0.871 ± 0.010	1.186 ± 0.025	0.734 ± 0.027
4b	0.890 ± 0.012	1.186 ± 0.025	0.750 ± 0.017
		Ave. Ratio = 0.738 ± 0.028	

^aValue quoted represents the average of all foils in that zone, and is squared for comparison with the reactivity $\overline{C/E}$. KiA-3

measurements. The C/E for the foil measurements is squared for the purpose of comparison with a reactivity C/E, as reactivity is proportional to flux squared (in a one-group approximation). The ratios of the Doppler C/E to the square of the $^{238}\text{U}(n,\gamma)$ C/E are well within uncertainties for fuel rings 1 and 4, indicating that the foil data provide a means of interpolating for proper Doppler C/E values in fuel rings 2 and 3. Although the foil C/E values vary more or less continuously with radius, the variations are within statistical uncertainties over a limited range such as the width of a fuel ring. Tables II and III show the $^{238}\text{U}(n,\gamma)$ C/E data averaged azimuthally throughout each fuel ring. Standard deviations of the C/E distributions are generally less than 2% compared with experimental statistics of about 1%. This level of variation permits interpolation on a ring by ring basis rather than for individual locations.

Using the relationship between $^{238}\text{U}(n,\gamma)$ reaction rates and Doppler reactivity described above, Doppler effect C/Es can be determined for radial positions by assuming the ratio of C/E in Table IV to be constant for all fuel rings. The similarity in the ratios for the two phases supports this assumption. The average value for ZPPR-11B, 0.731 ± 0.033 , was used to determine the C/E values for fuel rings 2 and 3. Because fuel rings 1 and 2 are thin, the C/E value for either measurement does not vary significantly across them. Rings 3 and 4, however, are thicker, and show significant C/E variation across the ring. For this reason, rings 3 and 4 were divided into zones 3, 4, and 5, where zone 4 consists of the outmost portion of fuel ring 3 and the innermost portion of fuel ring 4. Thus, zone 4 constitutes a transition zone between fuel rings 3 and 4, and uses a C/E value which is the average of those determined for the fuel rings. The values presented in Table V provide additional information about the spatial variation of the Doppler C/Es which is based upon C/E values for both $^{238}\text{U}(n,\gamma)$ and Doppler reactivity measurements. It is obvious at this point that the assumption of a radially constant C/E for the Doppler reactivity would introduce additional errors into the biasing of a calculation of whole core Doppler.

TABLE V. Values of ^{238}U Doppler C/E Used in Determination of Doppler Map in ZPPR-11B

Fuel Zone	$^{238}\text{U}(n,\gamma)$ (C/E) ²	Ratio Doppler C/E:Foil (C/E) ²	Doppler C/E ^a
1	1.059 ± 0.011	0.725 ± 0.028^b	0.768 ± 0.026^b
2	1.100 ± 0.004	0.731 ± 0.033	0.804 ± 0.033^c
3	1.149 ± 0.011	0.731 ± 0.033	0.840 ± 0.035^c
4	- -	- -	0.885 ± 0.044^d
5	1.252 ± 0.027	0.742 ± 0.037^b	0.930 ± 0.026^b

^aUncertainties on the Doppler C/Es are zone average values.

^bAverage of measured values.

^cInterpolated values.

^dTransition value, average of zones 3 and 5.

3.2 Determination of an Estimated Doppler Map

Given sufficient data from $^{238}\text{U}(n,\gamma)$ reaction rate and Doppler reactivity measurements, a complete core map of the ^{238}U Doppler effect could be inferred. In fact, there is no guarantee that mispredictions of Doppler reactivity and $^{238}\text{U}(n,\gamma)$ reaction rates will be similar at locations (such as control rods) where severe changes of neutron spectrum occur. However, experience on ZPPR Assembly 3³ indicates that this problem will be minor. While the referenced results were obtained with ENDF/B-III cross sections and a different calculational approach, they are internally consistent and show only minor variations in C/E near control rods and control rod positions.

In order to extend the available information, a calculation of point-wise Doppler reactivity was performed and used to account for azimuthal variations, using the ring-by-ring C/Es derived from the $^{238}\text{U}(n,\gamma)$ and small sample Doppler measurements.

3.2.1 Doppler Map Calculational Model

The calculational model chosen for prediction of azimuthal variations was selected so as to be convenient for LMFBR design teams to reproduce. Cross sections were evaluated assuming homogeneous unit cells, and a reactivity map was obtained from first-order perturbation theory, using a two-dimensional xy model. Real and adjoint fluxes were calculated using the approach outlined in Section 2.2.

3.2.2 Verification of the Mapping Approach

In order to verify the similarity between the heterogeneous unit cell calculations used in determining Doppler reactivity C/Es and the homogeneous unit cell calculation on which the map is based, two comparisons were made. First, Fig. 2 illustrates the fraction of Doppler reactivity produced in each energy group for the calculations at an outer core ring position. Though they are not identical, the comparison shows that the actual experimental sample is sensitive to a energy distribution similar to that in the homogeneous model. In addition, the specific Doppler reactivity in t/g per degree Kelvin was compared at two locations using the homogeneous unit cell calculation, the calculation of the small sample measurements, and the measured sample, with the results shown in Table VI. In both calculations, only the uranium was heated, and the contribution from ^{235}U was negligible. The values from the homogeneous calculation are seen to lie close to or within the range of the measured and calculated small sample values. It was therefore assumed that a detailed, estimated Doppler reactivity map could be generated from the homogeneous calculation using C/E values determined from the experiment and its calculation.

3.2.3 Inference of a Doppler Reactivity Map

The estimated Doppler reactivity map was obtained by dividing the calculated Doppler worth at a specific matrix position by the C/E value appropriate to that zone, as listed in Table V. The map represents the ^{238}U Doppler reactivity resulting from an increase in temperature from 300K to

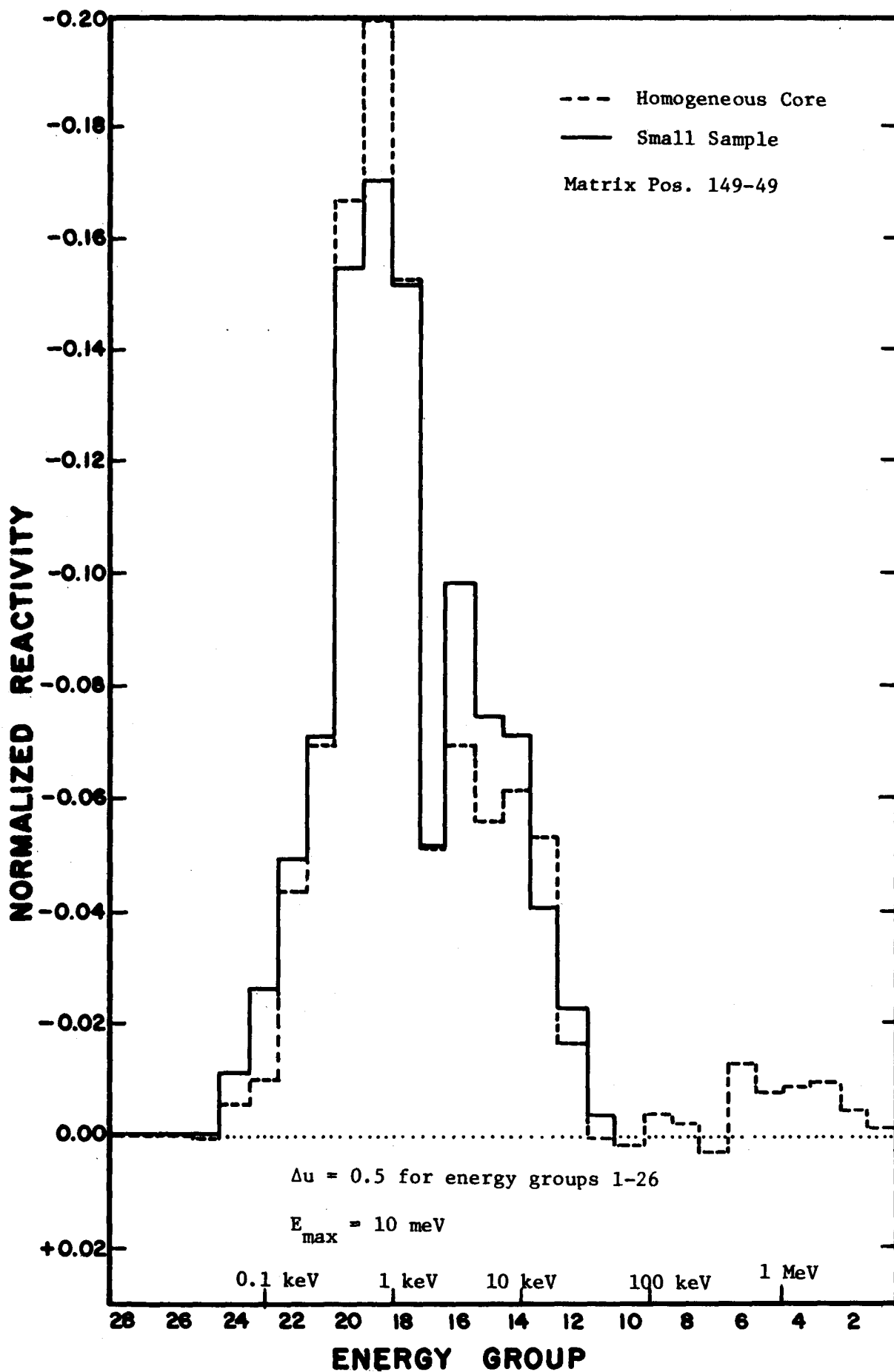


Fig. 2. Energy group variations of Doppler reactivity calculations.
126.

TABLE VI. Specific Reactivity Comparison for ^{238}U Doppler Map, Sample Calculation, and Sample Measurement

Location	Specific Reactivity, $\delta/g\text{ K}^a$		
	Measured Sample ^b	Calculated Sample ^b	Homogeneous Calc. (Map)
FR4a	$-(6.959 \pm 0.006) \times 10^{-8}$	-5.823×10^{-8}	-7.220×10^{-8}
FR1	$-(4.464 \pm 0.012) \times 10^{-8}$	-3.670×10^{-8}	-4.005×10^{-8}

^aCents per kilogram per degree Kelvin.

^bRefers to natural U Doppler sample containing 1.086 kg ^{238}U .

1000K, and is presented as Fig. 3. The values quoted are based on integration over the full core height, where the axial distributions of reactivity are based on a cosine-squared distribution.

Corrections for the crystalline binding of the material are often necessary in determining effective resonance cross sections. This effect is temperature dependent and becomes negligible when thermal oscillation energies are sufficient to overcome the binding forces completely. The errors due to the influence of crystalline binding are negligible for the Doppler reactivity values in the map. In effect, the small sample measurements are being adjusted by a ratio of calculations, both of which would be subject to a correction. However, since the material in the small samples, like that in the reactor, is UO_2 , and the base temperatures of both calculations are identical, the corrections should be similar in magnitude.

3.2.4 Uncertainties in the Doppler Reactivity Map

The uncertainties quoted for the map values are assumed to arise from several sources. First is the uncertainty in the Doppler/Reaction Rate ratio used in the interpolation. This value includes uncertainties from both measurements as well as from the averaging process, and should include most sources of error arising from experimental precision.

Calculational uncertainties applicable to the small sample C/Es include those associated with cross section averaging and modeling. Based on a comparison between the standard equivalence theory approach and a more exact integral transport approximation, 2% is estimated for this contribution. The principal difficulties with modeling are lack of representation of axial blankets and assumption of cosine shaped flux profiles. Comparing rz and xy analyses of Doppler measurements in simple benchmark cores indicates a $\pm 2\%$ modeling error. The values in the Doppler reactivity map are based on a calculation integrated over the full 36 in. core height and adjusted by measurement C/Es which involve an integration over the central 12 in. To estimate the uncertainty arising from this mismatch, the integral of a cosine-squared function, was evaluated over both intervals, using an appropriate extrapolated height. A function which showed a 10% variation at the position of the core/blanket interface induced less than 1.2% error in the ratio of the two integrals. Even though the modeling and axial integrations are not completely independent, as a conservative assumption, the RMS average of the calculational uncertainties was determined

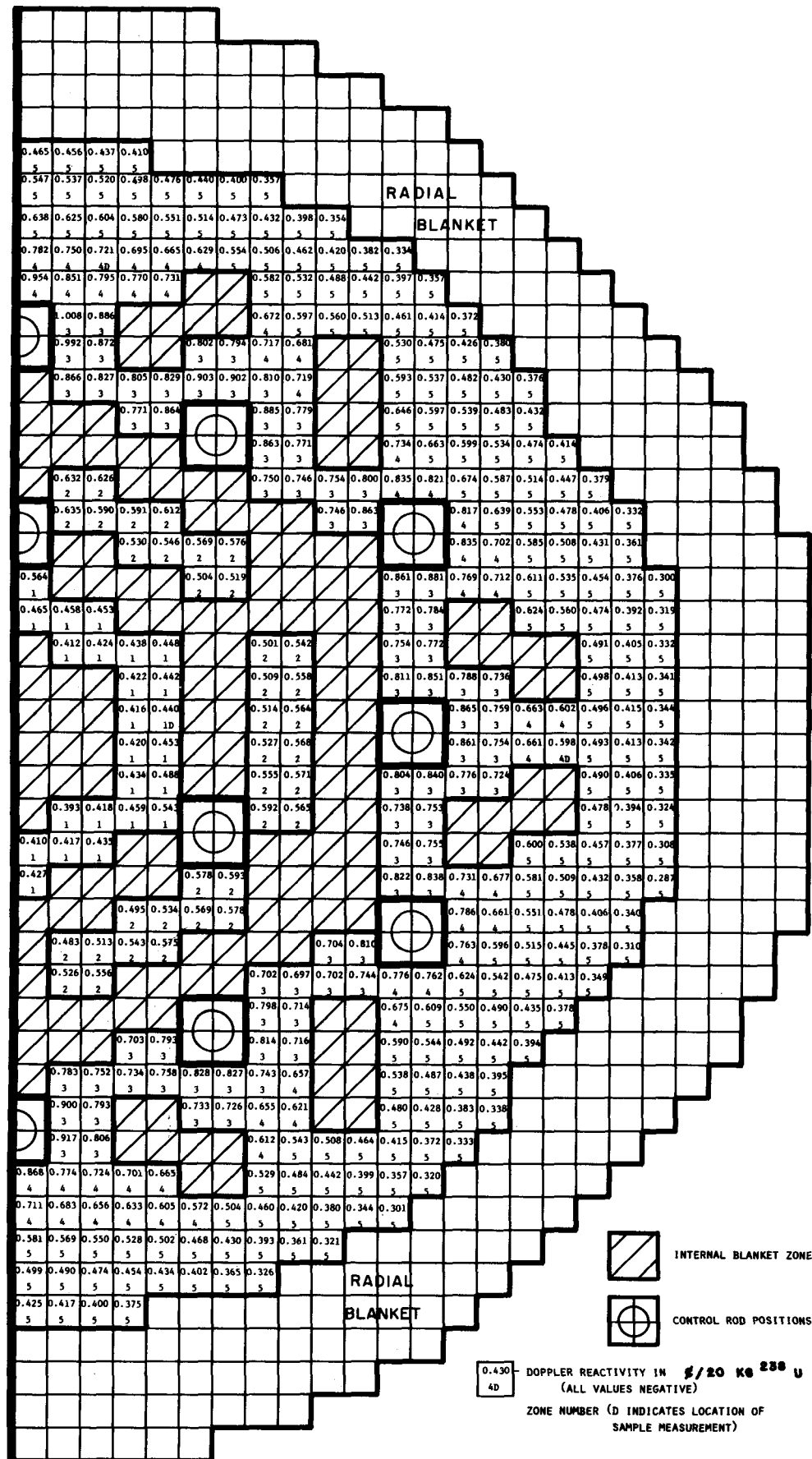


Fig. 3. ²³⁸U Doppler Reactivity in Core Regions of ZPPR-11B For Temperature Increase From 300K to 1000K.

to be 3.1%. This value is based on errors that are non-correlated between the small sample and map calculations, and therefore are not eliminated in the ratio.

A further small contribution is based on the fact that the C/E value appears to increase monotonically with radius, and the map is based on zone averages. This results in a slightly higher uncertainty for the interpolated C/E values in fuel zones 2, 3, and 4. The zone average uncertainties applicable to the map reactivity values, in percent, are 5.0, 6.6, 6.6, 7.2, and 4.8 for the fuel zones one through five, respectively. These values are an RMS combination of the 3.1% calculational uncertainty, the uncertainty of the Doppler reactivity C/E used in that fuel zone, and the uncertainty due to variation of C/E within that fuel zone. Additional uncertainty arising from azimuthal averaging within a ring has already been included in the zone average Doppler C/Es, as described in Section 3.1.

3.2.5 Utilization of the Map

The pointwise Doppler reactivity data that comprise the map are calculations divided by an appropriate C/E, and can be treated as experimentally based values. An LMFBR designer using these data should calculate his own map, using the prescription provided in Section 3.2.1, and then compare the two to identify potential problems. Pointwise C/E values thus derived (C = designers map, E = this map) would indicate any discrepancies which might arise in the design calculation.

The question of application of a crystalline binding correction will not affect the map values as it cancels in the two calculations. However, in using the map, one must recognize that, because of the normalization to experiments, it is characterized by a base temperature of 300K. When a designer performs a calculation of the map to produce a bias factor, a correction for crystalline binding should be included. The result will be a bias factor that is independent of this effect.

4.0 SEFOR Doppler

The Southwest Experimental Fast Oxide Reactor (SEFOR) was designed and operated to obtain integral experimental data about the Doppler coefficient. SEFOR continues to be central to Doppler verification so it is of some interest to apply the same methods and data base as used in the small sample calculations in order to examine the differences.

Several laboratories^{4,5,6,7} were involved in evaluating the SEFOR experiments. C/E results varied between 1.00 and 1.07 when calculations were compared with the consensus experimental values.

As part of this study, a benchmark calculation was done for SEFOR Cores I and II using the model described by the Cross Section Evaluation Working Group (CSEWG) as fast reactor benchmark number 14 with number densities suggested by General Electric.⁴ Some of the number densities required correction to the volume fractions presented by Noble.⁷ C/E results were consistent with the other laboratories. The heavy element resonance treatment in the SEFOR calculations used methods consistent with the small sample

calculations. However, the diluent material resonances in the SEFOR core are given a self-shielding correction which is consistent with the actual composition, as opposed to a smoothed, infinitely dilute approximation in the small-sample case. An exact perturbation method is used in the SEFOR reactivity calculation.

5.0 Small Sample - SEFOR Comparison

If the small sample Doppler measurements are to be used in conjunction with SEFOR data to confirm LMFBR design calculations, the differences between them should be understood and explained. SEFOR was a homogeneous reactor benchmark, not representative of current commercial reactor designs. The small sample data can be obtained in any reactor configuration, but do not constitute a whole core measurement.

The C/E values for benchmark core small sample measurements in Table VII are 4% to 8% lower than for SEFOR. There are several factors such as modeling, transport corrections for flux depression at the sample location, and leakage corrections through the almost-empty matrix tube which are yet to be made, and which may provide small changes to the small sample C/Es.

A correction for crystalline binding should be made to the small sample data, as it already exists in the SEFOR results. Even though both SEFOR and small sample measurements involve oxide fuel materials, the base temperatures are different, 677K and 300K respectively. This will require a more significant correction to the small sample measurements than to the SEFOR data, and will further reduce the calculated small sample Doppler coefficients. The magnitude of this effect has yet to be determined.

TABLE VII. Comparisons of SEFOR and ^{238}U Small Sample Doppler C/Es

Measurement	Calculation Type	C/E
ZPPR-2	rz-FOP	0.912 ± 0.014^a
ZPPR-9	rz-FOP	0.935 ± 0.007^a
ZPR-6/7 ⁸	rz-FOP	0.903 ± 0.004^a
SEFOR, Core I	rz-Exact Pert.	0.989 ± 0.120^b
SEFOR, Core II	rz-Exact Pert.	0.978 ± 0.120^b

^aUncertainty is based on measurement precision.

^bUncertainty is experimental uncertainty, including crystalline binding contribution.

6.0 Conclusions

Small sample Doppler reactivity measurements can provide valuable information with which to confirm design calculations of whole core Doppler. Used in conjunction with low-energy reaction rate results, the small samples

can be used to supply data on the radial variation of C/E in large heterogeneous cores. Used in conjunction with a calculated map of Doppler reactivity, they can add detail about the azimuthal behavior of C/E. Because the spatial variations in heterogeneous LMFBR designs can be difficult to predict, this type of information, together with the SEFOR results, should enable designers to test methods and assign uncertainties.

Certain difficulties remain to be resolved. Small sample C/Es are somewhat lower than the C/Es for SEFOR, and will be further reduced when crystalline binding is addressed. Some authors⁶ recommend a correction to SEFOR, while others⁹ contend it is insignificant. In any case, if SEFOR and small sample Doppler data are to be used together, both must use the same type of crystalline binding correction, even if one is very small. Other refinements to the determination of C/Es should be made in an effort to put both experiments on the same basis.

The consistency observed between the radial variations in the Doppler C/Es and the $^{238}\text{U}(n,\gamma)$ C/Es is encouraging in that it implies that the spatial variation of the Doppler reactivity is calculated consistently with ^{238}U capture. Past experience with small sample Doppler reactivity C/Es in the vicinity of control rods and control rod positions³ supports the assumption of small azimuthal variations. It is therefore concluded that the small sample Doppler data can provide valuable additional information when utilized in the manner described.

REFERENCES

1. C.E. Till, "Fast Neutron Doppler Effect", Proc. Nat'l Topical Meeting on New Developments in Reactor Physics and Shielding, CONF-720901 (Sept 1972).
2. J.M. Gasidlo, R.E. Kaiser, D.P. Pruett, and J.C. Young, "Effectiveness of the Dual Heater Temperature Control System in the ZPPR Reactivity Doppler Mechanism," Applied Physics Division Annual Report, Argonne National Laboratory, ANL-7910 (June 1971).
3. R.E. Kaiser, J.M. Gasidlo, and R.W. Goin, "Influence of Control Rods and Sodium-Filled Control Channels on the ^{238}U Doppler Effect," Trans. Am. Nucl. Soc., 18, 310 (June 1974).
4. D.D. Freeman, "SEFOR Experimental Results and Applications to LMFBR's", General Electric Co. Report GEAP-13929 (1973).
5. H.G. Bogensberger, et al., "Analysis of SEFOR Experiments", Kernforschungszentrum Karlsruhe Report KfK-2095 (1975).
6. R.A. Harris, "Analysis of the Doppler Constant of Cores I and II of SEFOR", Hanford Engineering Development Laboratory Report HEDL-TME-73-42 (May 1973).
7. L.D. Noble, et al., "Results of SEFOR Zero Power Experiments", GEAP-13588, March 1970.
8. R.B. Pond, Argonne National Laboratory, private communication.
9. A.T.D. Butland, "An Assessment of Methods of Calculating Doppler Effects in Plutonium-Fueled Sodium Cooled Fast Reactors", Proc. Int'l Symp. on Fast Reactor Physics, Aix-en-Provence, Sept. 24-28, 1979, IAEA, Vienna, Vol. I, pp. 3-19 (1980).

AN ASSESSMENT OF PHYSICAL CHARACTERISTICS
OF HETEROGENEOUS CONFIGURATIONS WITH
VARIABLE FERTILE VOLUME

W. Scholtyssek*, G. Daguzan**, G. Humbert**, M. Martini***,
B. Ruelle**, F. Kappler*, G. Norvez****

* Kernforschungszentrum, Karlsruhe, R.F.A.

** Centre d'Etudes Nucléaires, Cadarache, FRANCE

*** ENEA, Casaccia, ITALY

**** EDF, Clamart, FRANCE

ABSTRACT

In the present paper, the RACINE program is presented. This program is made in the framework of the collaboration of CEA-ENEA-DEBENE and is devoted to the study of the physics aspects of heterogeneous cores on the basis of a parametric approach (fertile amount, fertile ring position, thickness etc). Experimental results are presented, together with the analysis performed both with CEA and DEBENE methods.

INTRODUCTION

Since 1976, an important experimental program is performed on the MASURCA critical facility at Cadarache, aiming at the investigation of physics and safety parameters of large fast breeder reactors, especially in view of the heterogeneous core concept [1].

The first phase PRE-RACINE [2] was done in collaboration between ENEA and CEA devoted to the study of elementary nuclear properties of a central internal fertile zone.

In 1979, DEBENE joined the program for the second phase RACINE [3]. The fissile stock accumulated at MASURCA allows to investigate critical assemblies with diameters of up to 2.2 m, having one or more fertile zones, and also control-rod configurations of significant antireactivity.

The main objectives are :

1) to test calculational methods, codes and cross-sections, currently in use for core design studies, on geometrically simple configurations, in which parameters typical for heterogeneous cores are varied, e.g. the inner fertile volume fraction, the degree of coupling between fissile zones or their relative importance ;

2) to test and improve design and optimization studies, e.g. with respect to power form factor, minimization of fissile inventory or reactivity of Sodium voided regions ;

3) to measure design parameters in configurations matching as closely as possible large power reactor conditions. This concerns mainly control-rod reactivity and interaction, and power distribution measurements in configurations with partly and completely inserted rods.

The present paper describes experiments and analysis work performed with respect to the first two objectives. Another contribution to this conference [4] concerns control-rod studies.

CONFIGURATIONS AND EXPERIMENTAL TECHNIQUES

1) Description of the configurations :

The assemblies designed for the envisaged parametrical studies have simple, cylindrical geometries, depicted schematically in Fig. 1.

The inner core zones are loaded with Pu with an enrichment of nominally 19% in Pu/(Pu+U). In the outer U zone fuel of 22% enrichment is used.

The various fuel types used -U235 and Pu239 in rodlet and platelet form- necessitated a careful design of the cells to arrive at closely matching neutronic properties.

Fig. 2 shows the basic cells built either with MASURCA type rodlets or SNEAK platelets.

The principal configurations studied up to the end of 1981 are :

- in PRE-RACINE :
 - . PRE-RACINE 2 : a clean-core without inner fertile zones, of the basic fissile composition used throughout the program ;
 - . PRE-RACINE 1 : a core with a central fertile island of 15 cm radius.
- in RACINE :
 - . RACINE 1A : a core with a central fertile island ($r = 15$ cm), and a fertile ring of 10 cm thickness. The ring position was chosen to minimize the radial power form factor ;
 - . two variants, RACINE 1B, 1C, had the fertile ring moved 5 cm inwards and outwards respectively.

A configuration with an 18 cm thick fertile ring, leading to reduced coupling between the fissile zones as compared to RACINE 1A, will be investigated later in the program.

All configurations have the same core height of 91 cm. They are axially and radially surrounded by UO₂ or UO₂/Na blankets and a steel reflector. Table 1 presents some characteristic data of the cores.

	PRE-RACINE		RACINE 1A	RACINE "Thick ring"
	2 Clean core	1 Central Island		
Critical radius/cm	54	65	90	110
Fissile volume/l	850	1150	2025	2900
Fertile/Fissile Vol. ratio %	0.0	5.6	12.8	≈ 20
Critical mass/kg Pu9 equivalent	550	750	1200	1800

Table 1 : Properties of PRE-RACINE and RACINE configurations

Experiments performed systematically and in a reproducible manner in each configuration concern :

- critical mass ;
- reaction rate measurements, to obtain information on :
 - . global power distribution,
 - . breeding gain,
 - . cell fine structure and spectral indices ;
- reactivity measurements of Sodium-voided regions.

The MASURCA experimental techniques are described in [2] and [5]. They are briefly recalled in the following.

2) Reaction rate measurement techniques :

Global distribution of U5, U8, Pu9 fission are obtained with cylindrical ionization chambers, equipped with the isotopes of interest. The chambers are moved in a radial channel close to the core mid-plane, or in axial channels, which can be placed in any core position. In some cases, the B10 (n,α) reaction and the gamma-field were also measured.

In-cell measurements and ²³⁸U capture distributions are performed by the foil activation technique. Special demountable rodlets are available, allowing to place the foil detectors inside the fissile and fertile components of the cells.

Fission indices are obtained from activity ratios of foils irradiated in cell components and in or close to standard fission chambers.

C8/F5 is obtained by CEA by normalizing to the known thermal ratio : Foils are irradiated in parallel in MASURCA and in the thermal column of HARMONIE. Absolute U²³⁸ capture measurements were made by DEBENE by comparing the Np²³⁹ activity of irradiated foils to that of an Am²⁴³ standard source.

3) Sodium voiding :

Voided zones are created by replacement of Sodium filled cans

by identical empty ones. Void zones had mostly ring shape and extended over full core height. For special configurations voids with varying heights were build to obtain information on the axial reactivity variations. In RACINE 1A, excentric island shaped voids were also investigated.

For small zones, void reactivity was determined by the rod compensation technique, with the control-rod being calibrated by inverse kinetics. More important reactivity changes were determined by the modified source multiplication technique.

ANALYSIS OF THE EXPERIMENTS

1) Calculations methods :

The experiments are analysed independently by CEA and DEBENE, using their own cross-sections and fast reactor code systems.

CEA interpretation is based on the CARNAVAL IV system. 25 group heterogeneity corrected cross-sections are obtained with the cell code HETAIRE, using a 2 zoned cylindrical model for the fissile and internal fertile rodlet cells. The fissile cells are treated in the asymptotic mode. For the internal fertile cells, the leakage spectrum of the neighbouring fissile zone is taken as the source spectrum and an appropriate global buckling is used to account for leakage.

Criticality calculations are performed in R-Z diffusion theory, in 25 energy groups. Corrections applied for geometrical approximations concern cylinderization and rodlet end cap effects. The transport correction is made with 25 group homogeneous cross-sections in 2D R-Z geometry.

Reaction rate distributions are calculated in 25 group diffusion theory on the basis of a 2-dim X-Y model. To obtain power distributions, the reaction rate distributions are normalized by the calculated spectral indices in core centre. The same calculated indices are used to normalize the experimental reaction rate distributions for U235, U238 and Pu239 fission.

Zone voiding reactivity was determined by eigenvalue differences and exact perturbation computations. Directional diffusion coefficients, generated according to the Benoist method, have been used to account for streaming effects created by alignment in the axial direction of rodlets of the same material. The spatial calculations were performed with the diffusion approximation in R-Z geometry using 25 energy groups.

All void configurations are recalculated by a simplified method, which can be utilized for project studies based on :

- six group cross-sections collapsed using ϕ and $\phi+$ functions
- first order perturbation theory.

The total differences between reactivity effects calculated using 25 and 6 energy groups (the so-called project method) are, on the average, of the order of 10%.

DEBENE calculations are performed in the KAPROS system [6]. Cell-heterogeneity corrected cross-sections are prepared from KFK/INR 001 group constants [7] with the code GRUCAL and the collision probability code KAPER, that provides also anisotropic diffusion coefficients. Fissile and internal fertile cells are corrected for heterogeneity, the rodlet cells being described in a two zone cylinder model. Leakage is accounted for by introducing direction and energy dependent bucklings obtained from a preliminary core calculation with homogeneous cross-sections. Radial and axial blankets are regarded as homogeneous media.

Critical mass calculations are performed in R-Z geometry, in 26 energy groups, using the diffusion code DIXY. The cylinderization correction is deduced by comparing a 1-D cylinder calculation with a 2-D X-Y calculation. The transport effect is obtained individually in r and z direction. Axially, the transport correction is calculated for each zone in plate geometry, the whole core axial correction is obtained by application of proper weighting factors. Comparison with 2-D transport calculations showed good agreement, so that the combined 1-D transport correction method seems to be adequate also for heterogeneous cores.

Reaction rate distributions are calculated from the fluxes obtained by the R-Z diffusion criticality calculation. The experimental values are corrected for X-Y geometry, and the transport effect is obtained by comparing 1-D diffusion and transport calculations in few energy groups.

The Na void results were calculated in exact perturbation theory, with real and adjoint fluxes obtained from 26 group R-Z diffusion calculations. Heterogeneity corrected cross-sections for voided and non voided zones and anisotropic diffusion constants were used.

2) Critical mass calculations :

In Table 2, criticality calculation results for PRE-RACINE and RACINE configurations are presented. An older MASURCA core, MASURCA 3B, being a pure Uranium core with the same cell R1 as is used in the outer regions of RACINE configurations, is included for comparison purposes.

		PRE-RACINE		RACINE 1A	MASURCA 3B (U235)
		2 Clean Core	1 Central fer- tile Island		
CARNAVAL IV	Keff Dif.	.9904	.9923	.9923	.9957
	Δk Transp.	+ .0042	+ .0040	+ .0043	+ .0043
	C - E 1)	- .0046	- .0028	- .0044	- .0010
KFK/INR 001	Keff Dif.	1.0055	1.0064	1.0031	1.0091
	Δk Transp.	+ .0069	+ .0060	+ .0063	+ .0065
	C - E 1)	+ .0132	+ .0133	+ .0100	+ .0170

1) Diff. + ΔK transport

TABLE 2

The difference between CARNAVAL IV and KFKINR 001 of about 1.5% already observed in earlier calculations, is found again in the present case. CARNAVAL under-estimates criticality by about 0.4%, while KFKINR gives results between 1 and 1.7% high.

The difference in the transport correction is due to the fact, that this correction is partly taken into account already in the cell calculation by the CEA method. Appart fom this, it is found that the transport effect on the multiplication factor does practically not depend on the core heterogeneity as long as the same fissile composition is used in the different configurations.

The variations of the multiplication factor can be correlated with the U235 content in the core. This is demonstrated in Fig. 3, where C-E differences are depicted as a function of the weight of the Uranium zone in the respective configuration. A SNEAK Pu core with nuclear properties not too different from those of the fissile composition in RACINE fits well into the picture, a SNEAK Uranium core however is in strong disagreement. This point is under study. With respect to the RACINE interpretation, this means that the "U235-bias" has to be carefully eliminated in order to get eventually the proper "core-heterogeneity-bias".

3) Reaction rate distribution :

In Fig. 4, the typical radial distributions of U238 and Pu239 fission are depicted. This figure shows experimental values as well as DEBENE calculations in diffusion and transport theory.

For Pu239 fission, the well known effect of higher overprediction with growing radius is again observed. Transport correction is small in fissile zones and in the radial blanket, but reaches significant values in the internal fertile zones.

The U238 fission distribution is strongly modulated by the fertile zones. Diffusion overpredicts in the internal fertile zones, in the outer core parts and in the radial blanket.

Transport correction yields good agreement in the radial blanket, but over corrects in the internal fertile zones.

In table 3, these observations are quantified for several characteristic positions in the core.

TABLE 3

(E-C)/C % for radial fission rate traverses		CARNAVAL IV		KFKINR 001			
		Pu239 Diffusion	U238 Diffusion	Pu239 Diffusion	Pu239 Transport	U238 Diffusion	U238 Transport
Middle of	Central fertile Island	2	0.5	0	- 3	7	- 16
	Inner fissile 1) Ring	0	0	0	0	0	0
	Fertile Ring	1	3.5	2	0.5	6	- 13
Outer fissile ring	*	0	2	2	2.5	3	0
	**	3.5	6	4	4	8	7
	***	5	13	5.5	5.5	9	0
Radial blanket 10 cm		20	0	11	8	17	7

1) Point of normalization

Middle of Pu zone *

" " U zone **

Core blanket interface ***

The agreement between CARNAVAL IV and KFKINR is fairly good for most of the points, with exception for U238 fission in the central fertile island and in the radial blanket. This is certainly due to the differences in the adjustment of the U238 cross-sections in the two sets.

4) Fertile ring position and power form factor :

Two variants of the RACINE reference core, the configurations RACINE 1B and 1C, had the fertile ring moved 5 cm inwards and outwards respectively. The maximum local power variation resulting from this is in the order of 20% (Fig. 5).

Table 4 lists calculated and measured power form factors for the ZONA 1 Pu zone in the configurations (r<65 cm). The agreement is satisfactory.

TABLE 4

$\frac{P_{max}}{P} - 1$ %	RACINE 1A	RACINE 1A	RACINE 1C
Fertile ring position cm	35/45	30/40	40/50
Experiment	0.7	4.4	5.5
Calculation CARNAVAL IV	0.9	4.5	5.0

It is also encouraging to note that an optimal flat power distribution, one of the design objectives for the RACINE 1A reference core, has been properly achieved.

5) Reactivity of Sodium voided zones :

Calculated and experimental values are presented in the following table where similar void configurations of PRE-RACINE and RACINE experiments are compared (CEA calculations).

TABLE 5
Comparison between sodium void reactivity ($10^{-5} \Delta K/K$) for different experiments
(radius in cm)

Experience Void Configu- ration *	PRE-RACINE Clean core 8% Pu40			PRE-RACINE Clean core 18% Pu40			PRE-RACINE Central fertile zone			RACINE		
	Exp.	Calc.	E - C	Exp.	Calc.	E - C	Exp.	Calc.	E - C	Exp.	Calc.	E - C
r = 15.82	- 31± 3	- 17	- 14± 3	- 20± 8	- 13	- 7± 8	120± 5	145	- 25± 5			
r = 29.45	-307±15	- 270	- 37±15	-200±10	- 197	- 3±10	297±10	313	- 16±10			
15.25<r<29.45 **	-305±15	- 285	- 20±15				150±10	190	0±10	121±6	124	- 3±5
19.83<r<29.45	-242±15	- 249	7±15				103±10	89.6	13±10	86±5	90	- 4±5

* Voided zone height is always equal to core height

** Voided zone dimensions in PRE-RACINE (clean core 8% Pu40) are slightly different

The following conclusions can be drawn from this table :

- as it is well know, increasing the Plutonium 240 concentration of the fuel makes the sodium void effect more positive ;
- reactivity worths of sodium-voids are generally calculated too positive by CEA ;
- adjustment factors of non-leakage and leakage components have been calculated : the central component is calculated slightly too positive and this could be due to cross-section uncertainties ; the radial leakage component is not correctly calculated but the associated uncertainty is very large due to the small radial leakage contribution in the considered experiments.

The next table shows a comparison between CEA and DEBENE calculated results for void zones in the inner fissile ring on three different heights in RACINE 1A.

	Exp. $10^{-5} \Delta K/K$	Calc. *)		E - C	
		CEA	DEBENE	CEA	DEBENE
± 25 cm	257 ± 10	252	211	5 ± 10	46 ± 10
Upper and lower 20cm	$- 36 \pm 4$	- 42	- 42	$- 1 \pm 4$	6 ± 4
Full height	204 ± 9	200	162	$- 4 \pm 9$	42 ± 9

*) Diffusion, exact perturbation

DEBENE calculates, as a rule, sodium void reactivity too negative, while CEA calculations confirm the opposite tendency put in evidence previously in the RACINE 1A experiment analysis.

CONCLUSIONS

From the experiment analysis, still being in an early stage considering the most recent configurations RACINE 1A to C, the following observations may be made :

- CEA predictions underestimate the multiplication factors of the PRE-RACINE and RACINE configurations between .25 and .45% ΔK . The results become consistent, if a "U235-bias" is taken into account. Core heterogeneity plays, if at all, only a minor role.
- DEBENE calculations overestimate significantly the multiplication factors. The difference between CARNAVAL IV and KFKINR 001 lies in the order of 1.5%, which is a consequence of the different adjustments of the data bases used. The "U235-bias" deduced from MASURCA cores is, however, much higher than was observed in SNEAK configurations. This point deserves further investigation.
- The transport effect on the multiplication factor seems not to depend on core heterogeneity as long as the same fissile material is used.
- Power distributions are calculated by CEA with deviations of 1-2% or less in PRE-RACINE cores. There are indications that the agreement might be slightly poorer in RACINE, no serious discrepancies were found however.
- DEBENE reaction rate distribution calculations are rather consistent with those obtained by CEA in diffusion theory with the exception of U238 fission in the central fertile island and the radial blanket.
- Transport corrections on Pu239 fission distribution are negligible in fissile zones but reach several percent in internal fertile zones. U238 fission is improved by transport correction in the fissile, but overcorrected in the internal fertile zones.
- The high sensitivity of heterogeneous cores was demonstrated in the 'fertile ring displacement' experiment. Local power va-

riations of up to 20% were observed for a ± 5 cm position variation of the fertile ring. This effect was, however, calculated satisfactorily.

- Na void reactivity is calculated rather consistently by CEA for PRE-RACINE and RACINE cores. Calculation is generally high by $\approx (1 \text{ to } 4) 10^{-4} \Delta K/K$, with the larger discrepancies being found for voids in fertile zones.

- DEBENE calculations of void reactivities are significantly less positive in RACINE 1A than experimental values. This point deserves further investigation.

-o0o-

REFERENCES

- |1| J.C. MOUGNIOT
ANS-ENS Conference
Paris, 1975
- |2| Y.H. BOUGET
ANS Topical Meeting
Gatlinburg, 1978
- |3| P. HAMMER
IAEA Conference on Fast Reactor Physics
Aix-en-Provence, 1979
- |4| G. HUMBERT
This Meeting
Kiamesha-Lake, 1982
- |5| Y.H. BOUGET et al
Physics performances of heterogeneous fast reactor core
concept studied in MASURCA.
ANS, June 1977, Special Session
- |6| G. BUCKEL, W. HOEBEL
Das Karlsruher Programmsystem KAPROS
KFK Reports 2253/2254 (1976).
- |7| E. KIEFHABER
The KFKINR-Set group constants...
KFK Report 1572 (1972).

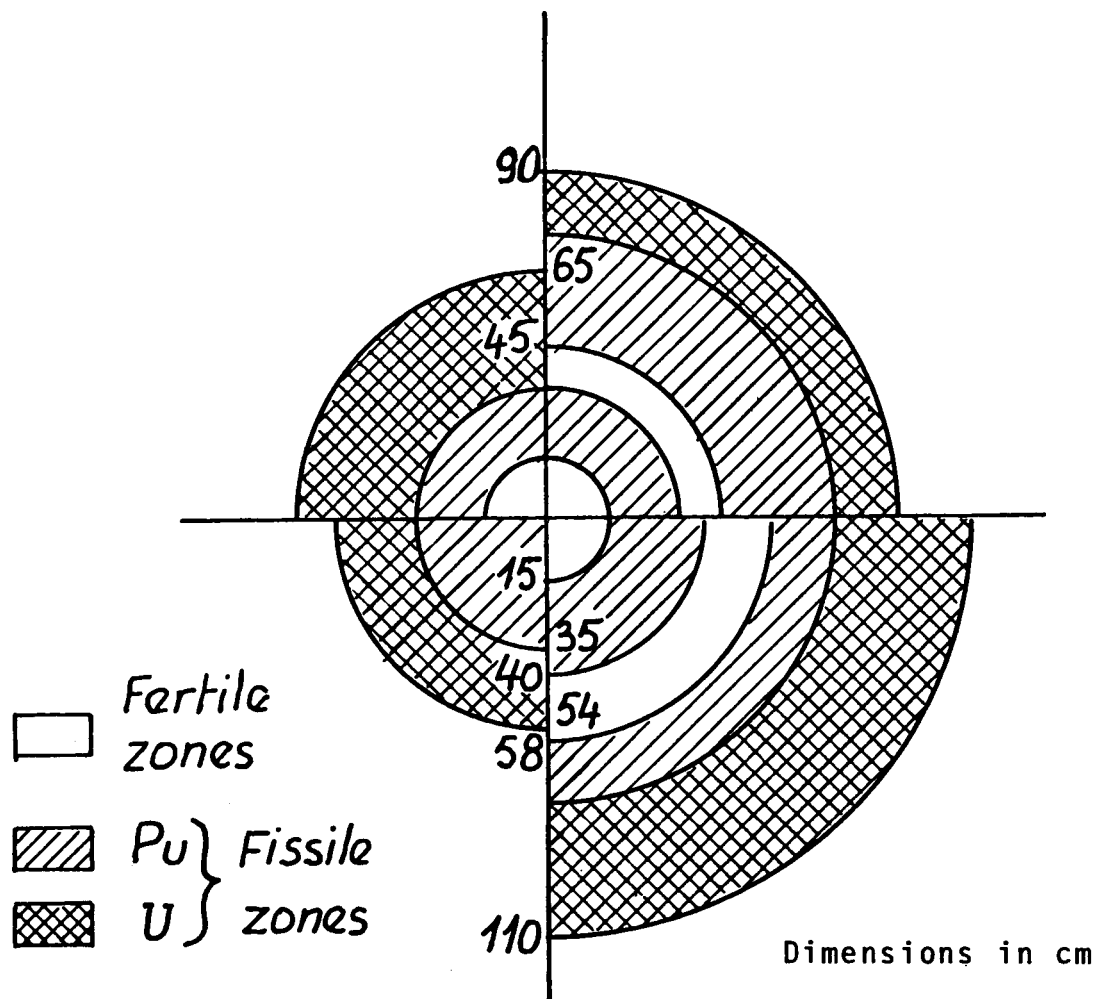


Fig.1 - Basic PRERACINE and RACINE Configurations for heterogeneous core concept studies

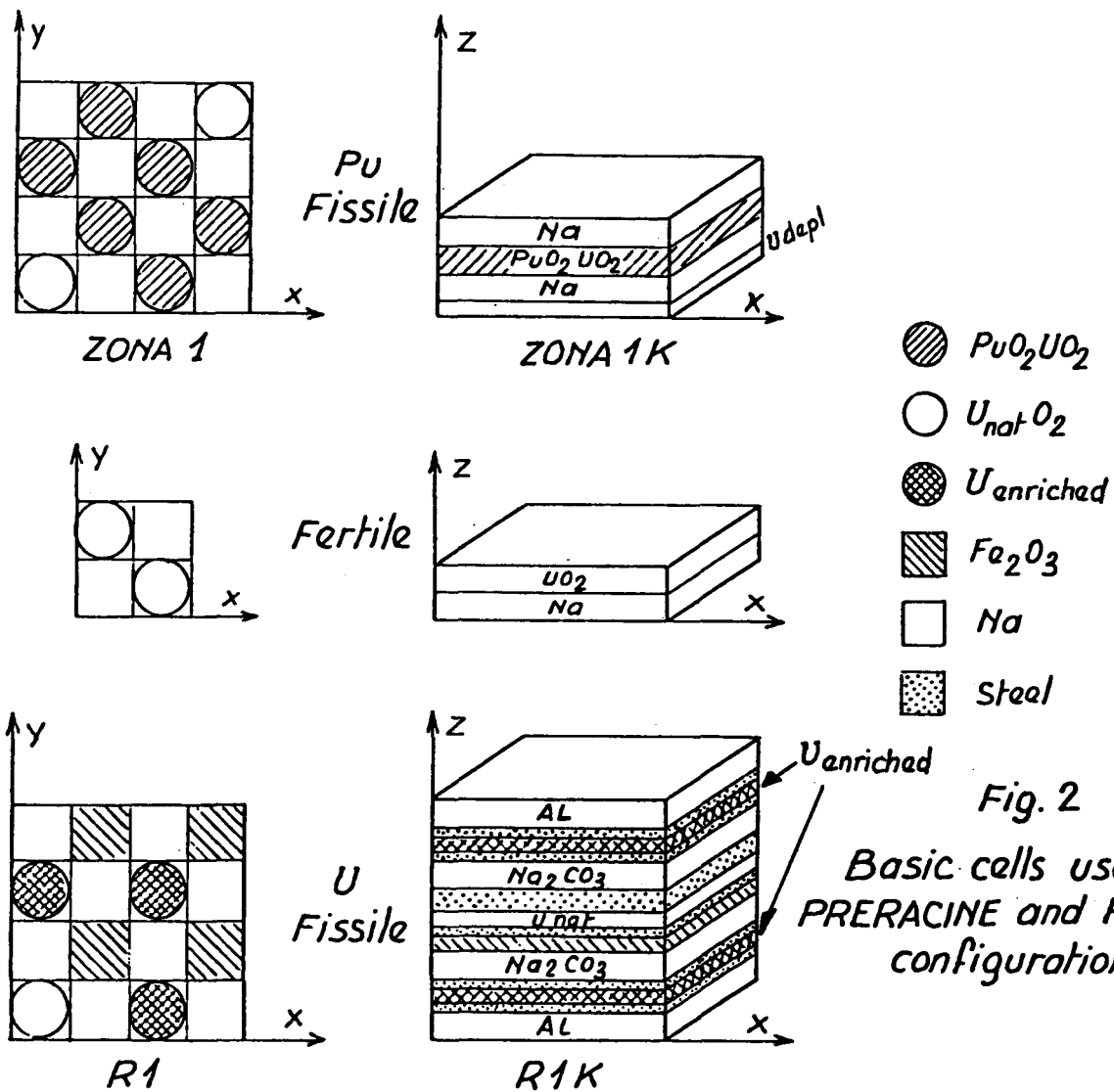
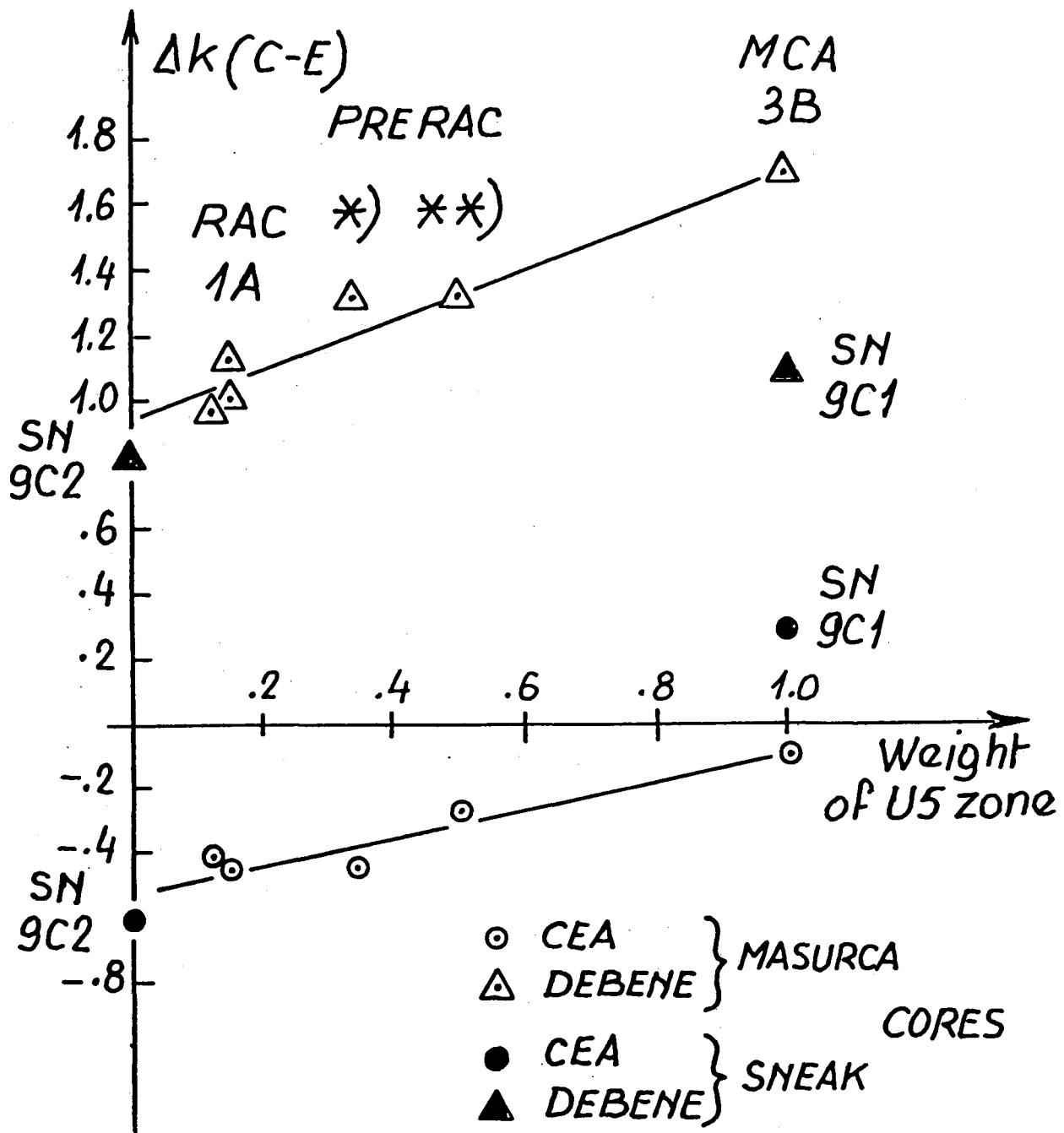


Fig. 2

Basic cells used in
PRERACINE and RACINE
configurations



*) Clean core **) Central fertile island

Fig.3. C-E differences for k_{eff} as a function of the U235 zone weight in the core

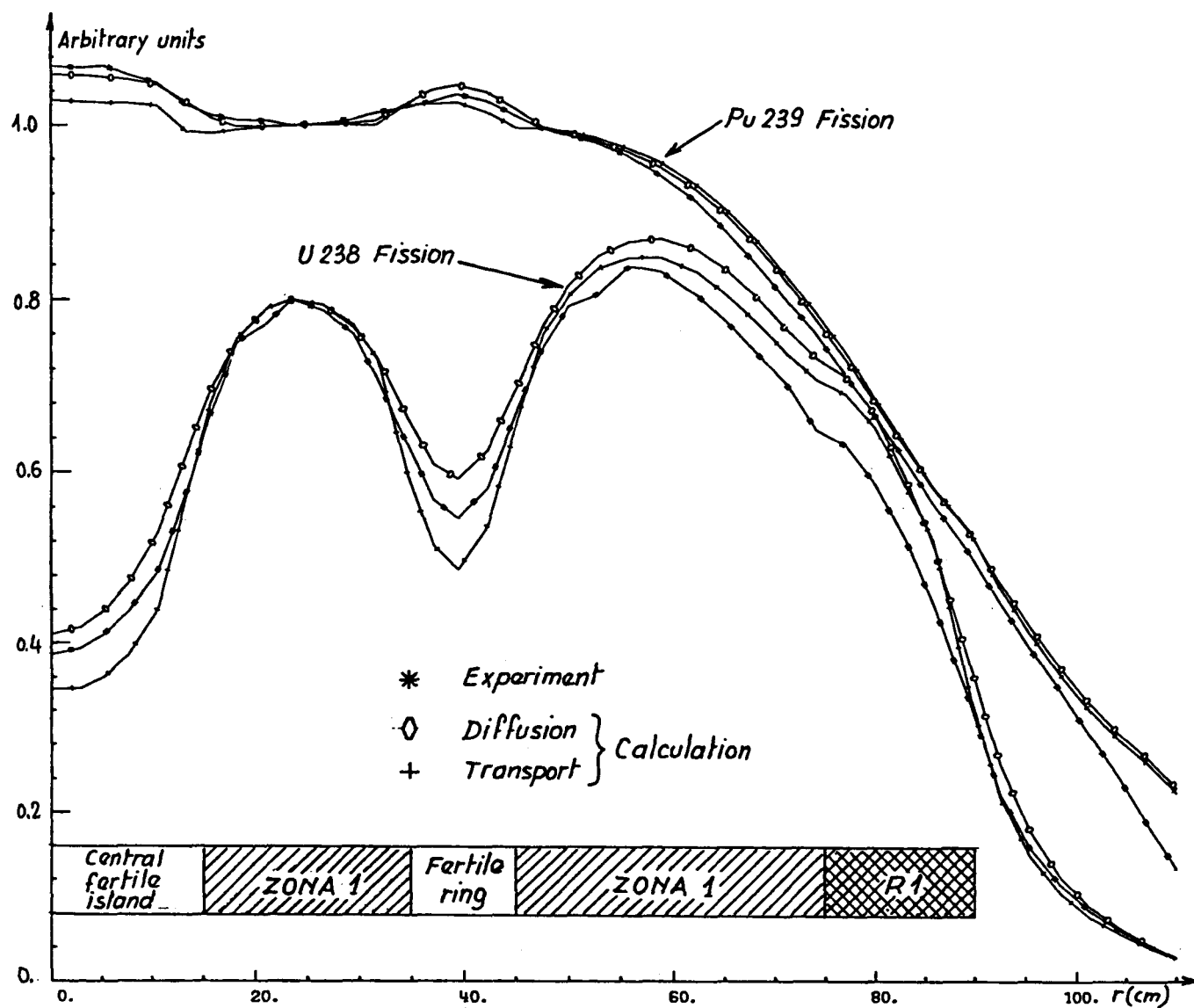


Fig.4. Comparison of experimental and De Be Ne calculated radial fission rate distributions in RACINE 1A

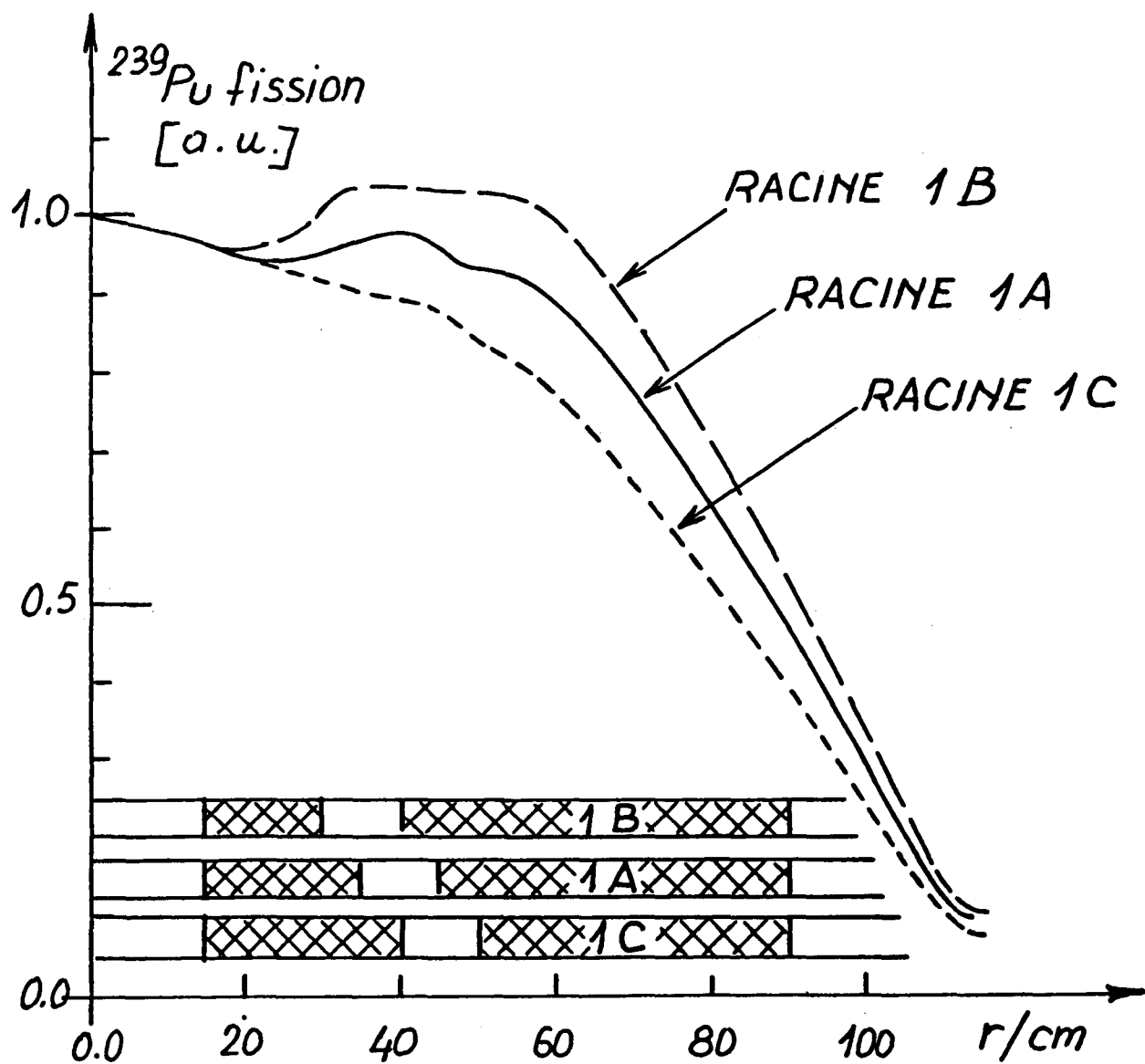


Fig.5. $\text{Pu } 239$ fission distribution in RACINE for various fertile ring positions

CONTROL-ROD PARAMETRICAL STUDIES IN THE FRAMEWORK
OF THE PRE-RACINE AND RACINE PROGRAMS

G. Humbert*, B. Ruelle*, G. Daguzan*,
A. Stanculescu**, F. Kappler***, W. Scholtyssek***,
D. Bouscavet****, M. Martini*****, U. Broccoli*****

- * Centre d'Etudes Nucléaires, Cadarache, FRANCE
** INTERATOM, Bergish-Gladbach, R.F.A.
*** Kernforschungszentrum, Karlsruhe, R.F.A.
**** EDF, Clamart, FRANCE
***** ENEA, Casaccia, ITALY

ABSTRACT

In this paper, a control-rod experimental program is presented. This program, established in the frame of PRE-RACINE and RACINE common DEBENE, Italian and French experiments at MASURCA facility, is still under progress at the moment. The results, limited to single central rod worth are already available. For these experiments, a parametrical approach has been used. The effects of rod worth, varied separately by rod side, boron enrichment and core size, on experiment to calculation relative discrepancy $(E-C)/C$ can be drawn out.

I - INTRODUCTION

The bias factors to be applied to large fast power reactor control-rod design are of major interest. According to a widely accepted point of view, these factors are deduced from experiment to calculation comparisons, in which the various parameters of interest are studied separately and calculations are run with the most accurate methods at hand.

This paper deals with the control-rod part of the experimental program set-up and realized in the frame of the commun Italian-French PRE-RACINE [1] and Italian-DEBENE-French RACINE [2] programs at critical facility MASURCA/CEA Cadarache.

$(E-C)/C$ values and variations versus boron enrichment, rod size and core size are planned to be analyzed for the following involved topics :

- rod efficiency,
- flux shape variations due to the absorbers and relation to rod efficiency,
- rods interactions.

The results related to central-rod worth are now available. The calculation to experiment discrepancies and their associated analysis are given.

II - EXPERIMENTAL PROGRAM

II.1 - The experimental program which is reported started in 1977. In a first phase, the main experiments concerned a single control-rod in PuO₂-UO₂ clean core (PRE-RACINE [1]). The present program includes control-rod studies in a heterogeneous configuration (RACINE [2]). In particular, the control-rod studies are performed in a heterogeneous one-fissile ring configuration with or without a central fertile zone. Moreover, a part of single control-rod program of PRE-RACINE is reanalyzed to study core size effects. A detailed program has been also set up for the study of multiple control-rod configurations with large interactions. This was possible both taking advantage of the expanded fuel inventory of the cooperative program and of the physical characteristics of a heterogeneous core. However, this part of the program, now underway, will not be discussed at the present stage.

The main characteristics of the control-rod program can be summarized as follows :

- continuity between the PRE-RACINE and RACINE programs. The same basic compositions are used in the PRE-RACINE and RACINE configurations ;
- parametrical approach : the central-rod worth is studied with respect to several parameters (size, absorber enrichment, core size...) ;
- use of a heterogeneous configuration to enhance flux tilting effects and interaction factors in the multiple rod experiments.

II.2 - Core and blanket cells description

The description of the cells used in MASURCA cores, covering the range of the PHENIX and SUPER-PHENIX cores neutronic characteristics are given in other papers [1,2,3].

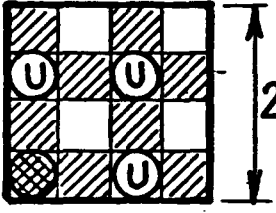
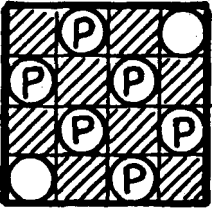
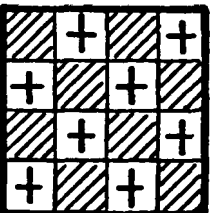
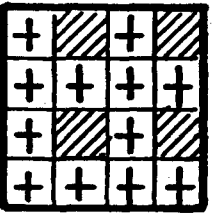
In the PRE-RACINE and RACINE programs, only the medium enrichment cells are used. Platelet type materials, provided from SNEAK, are combined in either Plutonium or Uranium cells of nearly the same characteristics.

In figure I and table I, these descriptions and characteristics are shortly resumed.

TABLE I - Neutronic characteristics of core cells

	$B_m^2 \text{ m}^{-2}$	K^*	D	$\frac{Pu}{U+Pu} \text{ or } \frac{U_5}{U_8+U_5}$	$\frac{\sigma_{F8}}{\sigma_{F5}}$
ZONA 1 (Pu)	14.8	1.61	1.93	18.3%	.0332
ZONA 1 K (Pu) (Platelets)	15.8	1.58	1.79	= 18.3%	.0310
R1 (U)	16.1	1.51	1.75	22.3%	.0345
R1 K (U) (Platelets)	16.0	1.51	1.76	= 22.3%	.0329

(From multizone cells calculations / Carnaval IV)

	Enrichment e $\frac{Pu}{U+Pu}$ or $\frac{U5}{U5+U8}$	
R1 (U)	22,3 %	
ZONA 1 (Pu)	18,3 %	
Axial blanket and in core blanket zone	Depleted U	
Radial blanket	Depleted U	
SUPER PHENIX externe zone	18 %	

- (U) Metallic U 35% (+) Depl. U oxide
 (●) Metallic depleted U (□) Fe₂O₃
 (P) Mixed oxide (UPu)₀₂ 25% Pu (▨) Na
 (○) Natural U oxide

Fig.1

PRE RACINE/RACINE Core and blanket cells

II.3 - Control-rod descriptions

All the rods are measured using as a reference the normal Na rodlets follow along the total core + axial blanket height without any absorber material at the rod position. These followers have the same geometrical cross-sections than the corresponding absorber rods.

Absorbers rods are varied by :

- geometrical cross-section (110 or 280 cm²) for single central rod
- boron enrichment by use of
 - . natural boron B₄C rodlet (or SNEAK boron powder in boxes)
 - . 90% enriched boron B₄C rodlets (or SNEAK boron powder in boxes)
 - . mixed configurations

For the 280 cm² rod, simulating a power reactor rod, some Na is included in the rod component : effect of absorber concentration in the central part of the rod has been studied.

Interactions between rods will be measured for the extreme boron enrichments with rods pairs, and for natural enrichment for more complex configurations. Particular configurations with emphasized flux tilting effects are planned, using natural boron rods.

The various rods are described figure II and table II.

TABLE II - Rods description

Rod N°	Fig. II	ROD	Geometrical cross-section cm ²	Absorber			Steel % V	Na % V	Void % V
				Nature	10B 10B+11B	% V			
01	a	Na follower	112.36				8	90.5	1.5
02	d	Na follower	273.88				8	90.5	1.5
11	a	Natural boron	112.36	B ₄ C (sintered)	20%	85.5	13	0	1.5
12	b	Natural boron	98.32	"	20%	85.5	"	0	1.5
13	c	Enriched boron	"	"	47%	85.5	"	0	1.5
14	b	Enriched boron	"	"	90%	85.5	"	0	1.5
21	e	Enriched boron + Na (heterogeneous)	273.88	"	69%	45	"	41	1
22	f	Enriched boron + Na (homogeneous)	"	"	69%	45	"	41	1
23	f	Natural boron + Na (homogeneous)	"	"	20%	45	"	41	1
31	/	Natural boron	112.36	B ₄ C (powder)	20%	85.5	"	0	1.5
32	/	Enriched boron	112.36	"	90%	85.5	"	0	1.5

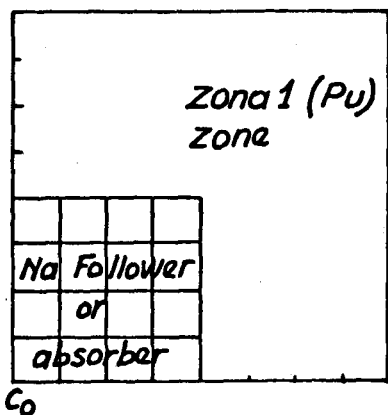


Fig. II a
112 cm² Rod { Na or Nat. B₄C

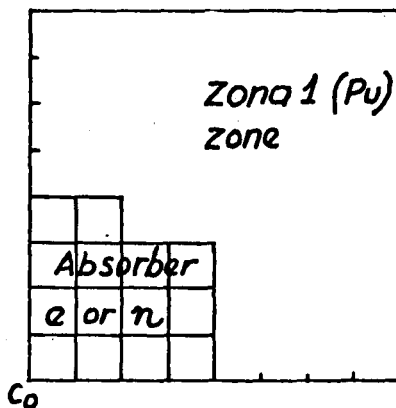


Fig. II b
98 cm² Rod { Nat. B₄C or 90% B₄C

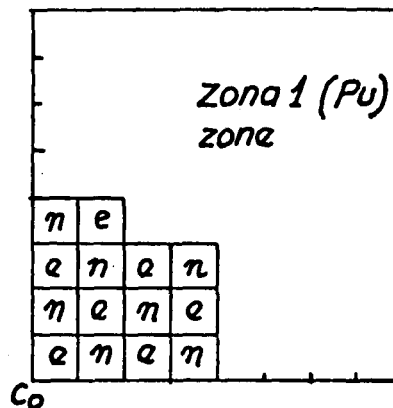


Fig. II c
98 cm² Rod 47% B₄C

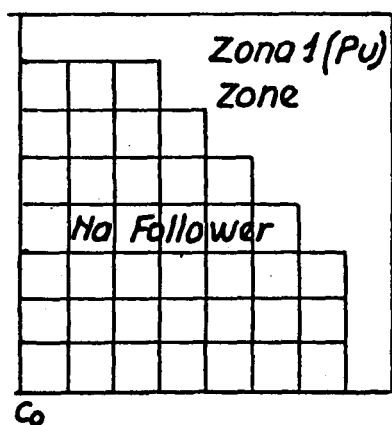


Fig. II d
280 cm² Rod Na

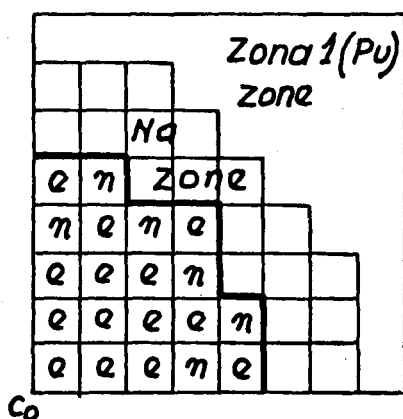


Fig. II e
280 cm² Rod { "Heterog." Na or 69% B₄C

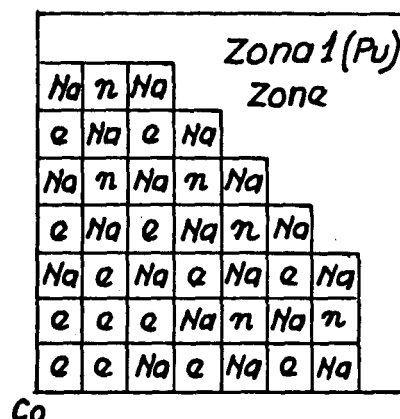


Fig. II f
280 cm² Rod { "Homog" Na or 69% B₄C (e, n) or Nat B₄C (all n)

C₀ = Core Center (NE Masurca subassembly represented)

n = Nat. boron rodlet e = 90% enriched boron rodlet

Fig. II - Central single rods (NE 1/4)

II.4 - Cores description

For our purpose we have to look at the three following steps of the PRE-RACINE/RACINE configurations :

- the PRE-RACINE core, without in core fertile zone, which is a single two zones core with a central single rod position (figure IIIa) ;
- the RACINE 1D core, with a ring fertile zone and a central single rod position (figure IIIb) ;
- the RACINE 1E and following cores, with a central fertile zone and a ring fertile zone, and 12 rod positions in two rings (4 + 8) for multiple rods experiments (figure IIIb).

In these core configurations, the following parameters remain unchanged :

- core fissile and fertile cells used (central fissile zone Pu, ZONA 1 cell, external fissile zone U, R1 cell) ;
- core heigh : 91 cm ;
- axial blanket cell and thickness ;
- radial blanket cell and thickness (18 to 23 cm) (except for PRE-RACINE : radial blanket without Na).

II.5 - Experimental program and measurement technics

Main rod parameters have been studied with the single central rod in the PRE-RACINE program. However, it appeared that some questions concerning core size effect and interaction effects needed more efforts. The previously established multi-rod program in RACINE [2] has been expanded with :

- a single central rod study in the large RACINE core ;
- emphasis on rods interaction and flux tilting measurements aspects.

Table III gives a general survey of the whole program, measurements technics used, and the expected values of the main parameters.

Once more, it should be stressed that the RACINE 1E experiments will not be reported at the present stage.

**TABLE III - General plan of control-rods measurements in the
PRE-RACINE and RACINE program**

Core/(dates)	Rod N° *	Measurements		Attempted results
		Parameters	Technics	
PRE-RACINE (1979) Single Central rod	01	Ref.	Critical radius Variation + Calibrated dri- ving rod (ΔR)	C/E on ΔK versus - rod worth - rod size - core size Rod heterogeneity effect
	11	ΔR		
	12	ΔR		
	13	ΔR	-----	ΔK to $\Delta\phi$ relation
	14	ΔR	Fission chambers	
	02	ΔR ϕ	Radial channel	Max ΔK insertion = 9%
	21	ΔR ϕ	Through the rod	Min ΔK insertion = 4%
	22	ΔR	(ϕ)	
RACINE 1D (End 05/1982) Single Central rod	02	Ref. ϕ	id. (ΔR) + subcritical (ΔK)	
	22	ΔK ΔR ϕ	-----	
	23	ΔK	id. + foils near the rod and inside the rod (ϕ)	
RACINE 1E (Begin 06/1982) Multirod configuration	01 12 Pos.	Ref. ϕ	Subcritical (ΔK) + Critical (id) (ΔR)	C/E on rods interactions versus : - positions - rod worth
	31 (1 to 12 rods)	ΔK	-----	
	32 (1 and 2 rods)	ΔK	-----	ΔK to $\Delta\phi$ relation Max ΔK insertion = 8%
	31 (1 and 2 rods)	ΔR ϕ	Fission chambers Radial Channel	
	32 (1 and 2 rods)	ΔR ϕ	Through the rods + Foils near the rods inside the rods (ϕ)	Interaction factors : 0.1 < f - 1 < 1.0
	31	ΔR ϕ		One rod over 8 : amplification factor > 2
				----- Axial distributions with par- tly inserted rods. Particular flux tilt with partial outer rod-ring

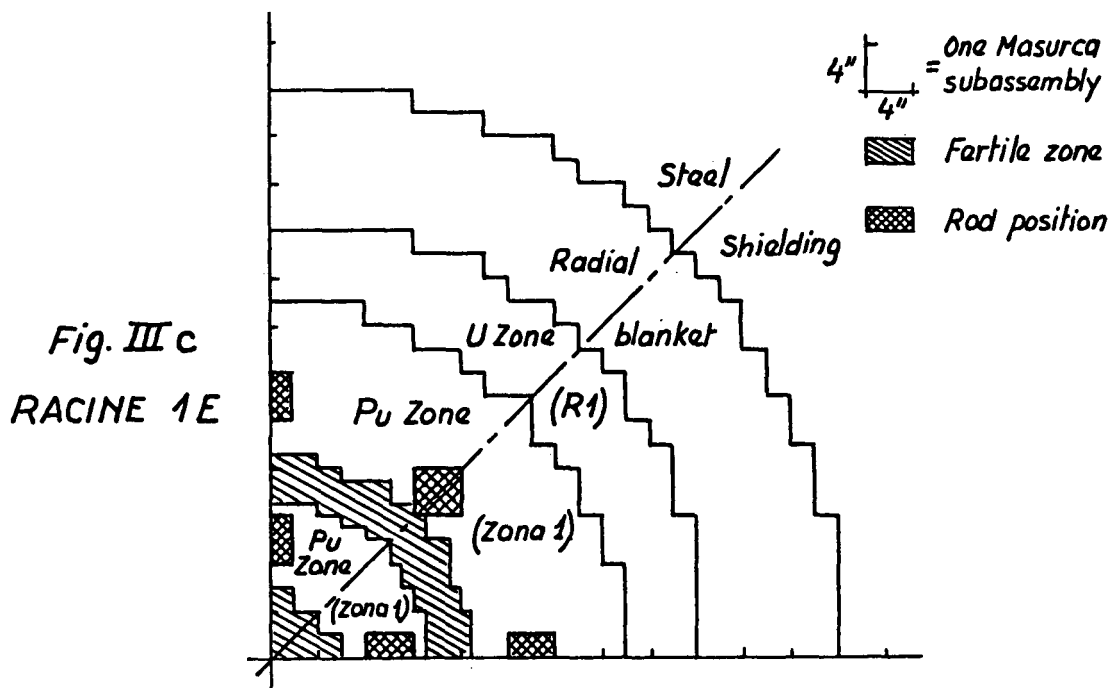
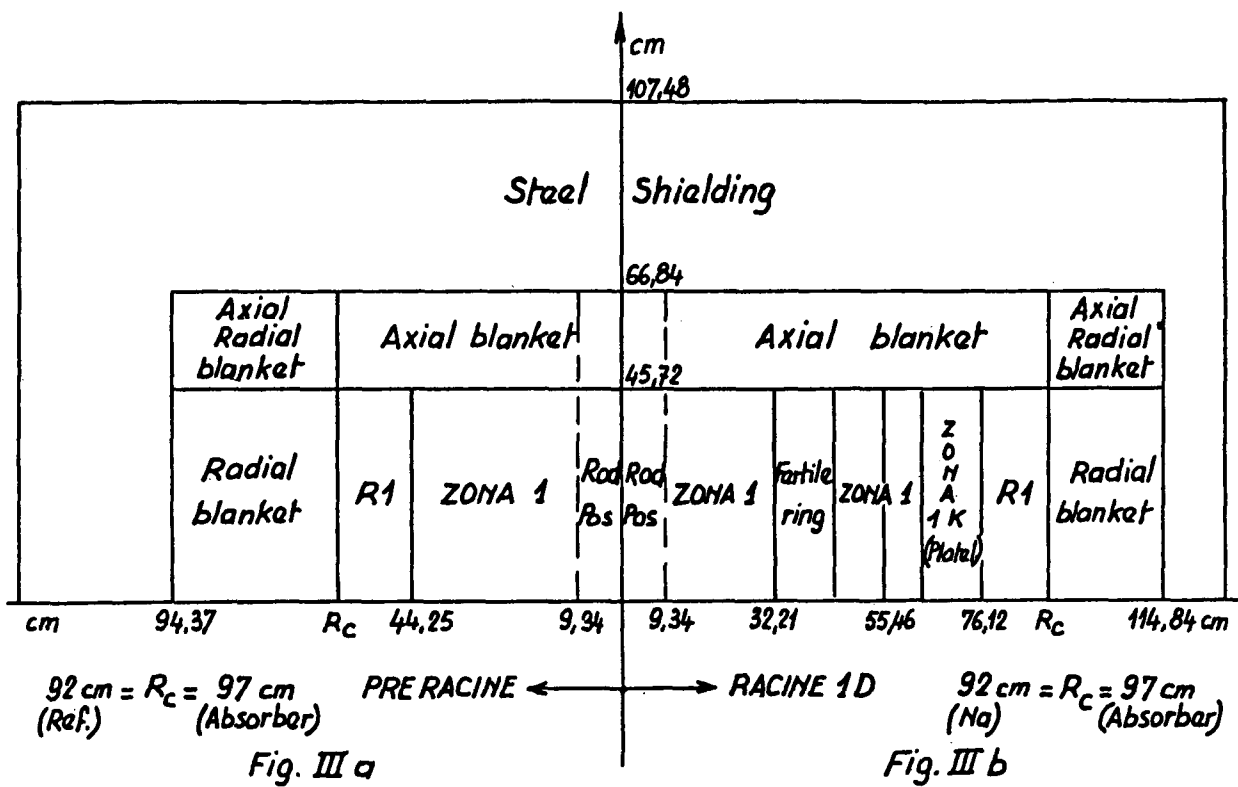


Fig. III . PRE RACINE and RACINE Cores. Rods Positions

III - CALCULATION METHODS, CORRECTIONS AND (E-C)/C VALUES

III.1 - The basic Keff and flux calculations use classical multigroup diffusion theory. Configurations are described in XY geometry. The normal mesh size fits approximately to one MASURCA rodlet (about 1/2") for the core zone. The 25 energy groups approximation of Carnaval IV cross-section set and cell code is used :

- for fissile zones, the so called HH 25 groups cross-sections are issued from a 2 zones cell "HETAIRE" calculation. (Pu isotopes number densities are calculated at the date of rod measurements. Other dates experimental results are corrected of number densities variation effet (41Pu decay)).
- for fertile in core zones, they are issued from a 2 zones cell calculation with the neighbouring core neutron leakage spectrum used as a spectrum for the neutron source ;
- for other zones, including absorbers, an homogeneous cell calculation is performed with a standard spectrum used for the source.

The axial bucklings of the XY calculations are zone and group dependant, issued from a 6 groups RZ calculation.

This basic calculation includes some approximations coming from methods (diffusion theory, mesh size, axial streaming) and from experiment description versus actual experimental conditions (core dissymetries, cell heterogeneity for reaction rates traverses,...).

For some of the approximations, the effect on the (E-C)/C result can be evaluated, with an associated uncertainty, by particular calculations which are described there after.

III.2 - Calculated corrections

III.2.1 - Diffusion to transport theory correction

This correction is to be applied to the reference core value of rod worth of previous diffusion calculation. It is calculated in XY geometry with 6 energy groups homogeneous cross-sections : it is assumed that the main point for this correction is to have an exact representation of the rod/core boundary. Secondary effects, related to the approximation made in this calculation (groups collapsing, axial buckling treatment), are introduced as an uncertain on the correction, after 1D order of magnitude evaluation |4|.

III.2.2 - Mesh size correction

The mesh size used is small : the small related correction is assumed to be, in relative value of the rod-worth, independant of the rod. It has been calculated on the PRE-RACINE central 280 cm² rod using the normal mesh ($\approx 1/2''$) and a double mesh ($\approx 1''$) calculation, by extrapolation to zero mesh size of Keff values with and without absorber.

III.2.3 - Axial streaming corrections

XY geometry has been choosen for the following reasons :

- coherence with design calculations,

- actual rod boundary representation,
- coherence of the whole program :
 - . analysis will include multirod configurations where XY geometry is necessary
 - . radial reaction rates have to be calculated with the exact radial geometry specially near the rod.

It makes necessary an axial streaming correction which is easily calculated for single central rod configurations by 2D(RZ) and 1D(R) (with the same zones and groups dependant axial buckling than XY calculations) 6 energy groups calculations. For multirod configurations, 3D calculations test will be necessary.

III.3 - Other corrections

III.3.1 - Exact number of cells accounting

For cost reasons, only quarter of core calculations are run. This can lead to one or two peripheral cells difference with actual core. A Keff correction is applied to experimental results, using experimental calibrated peripheral cell value.

III.3.2 - Other corrections will be necessary for reaction rates measurements analysis

They will be presented and discussed with these results when available.

III.4 - ΔK analysis with ΔR measurements for rod worth

In previous rod worth experiments analysis, we had assumed that the (E-C)/C values on ΔR could directly be used for transposition to ΔK uncertainties of design calculations. In fact the (E-C)/C value on ΔK at reference core size with Na follower would be the right one. For large rod worth, as it is the case for central rods in RACINE and PRE-RACINE, core size variations are important and induce large variations of rod worth. It is not obvious that (E-C)/C values are ΔR and ΔK can be used as equivalent.

In the present analysis, the ΔK way has been choosen (including use of subcritical measurements in RACINE experiments). It means that :

- for PRE-RACINE ΔR rod worth measurements, a transposition to ΔK (E-C)/C values at reference core size can be assumed,
- a check of this assumption can be made in the RACINE central rod worth measurement : in that case both subcritical and critical ways are used.

In a first time we have tried the following assumption : for a given core and a given single central rod, measured and calculated Keff = f (core radius) curves are parallel straight lines.

In that case it can be written :

$$\left(\frac{E-C}{C}\right)_{\text{on rod worth}} \approx \frac{(k_{E,b}^b - k_{E,ref}^{ref}) - (k_{C,b}^b - k_{C,ref}^{ref})}{k_{C,b}^b \cdot k_{C,ref}^b} \cdot P_C$$

where :

- E means experimental value
- C means calculated corrected value
- b means with absorber inserted
- b means with absorber critical experimental core size
- ref. means without absorber
- ref. means with reference experimental core size

III.5 - Available results : central rod worth (E-C)/C values

At the moment the status of the work is :

- central single rods worth measurements achieved and calculated results available
- reaction rates distributions with central rods analysis under progress
- multi-rod experiments planned for second mid of 82.

So further § of this paper are now restricted to central single rod worth. Table IV and V give calculated corrections and values, (E-C)/C values and uncertainties according to § III.4. Chapter IV introduces discussion on (E-C)/C analysis.

TABLE IV - Calculated $\Delta K/K$ values and corrections (in %) (Reference in k : Na follower situation)

N°	Rod		Reference core Size	XY 25 groups diffusion $\Delta K/K$ %	Calculated corrections %			Corrected $\Delta K/K$ %
	Size cm^2	Boron enrichment			Mesh step	Axial Streaming	Transport *	
12	98	Nat.	PRE-RACINE clean core (without follower) experiment critical	- 4.68	- 0.5±0.2	0.5 ± 0.2	-11.4±1.0	- 4.15±0.07
13	98	47%	"	- 6.90	- 0.6±0.2	0.3 ± 0.2	- 9.6±1.0	- 6.22±0.10
14	98	90%	"	- 8.08	- 0.6±0.2	0.3 ± 0.2	-11.6±1.0	- 7.15±0.14
11	112	Nat.	"	- 5.11	- 0.6±0.2	0.5 ± 0.2	- 7.2±1.0	- 4.73±0.07
11	112	Nat.	PRE-RACINE with Na follower (112 cm^2) experiment critical	- 4.88	- 0.8±0.2	0.5 ± 0.2	- 5.2±1.0	- 4.61±0.07
21	280	69% Hetero. Na	PRE-RACINE with Na follower (280 cm^2) experiment critical	- 8.14	- 0.7±0.2	0.3 ± 0.2	- 7.5±1.0	- 7.50±0.11
22	280	69% Homog. Na	"	- 8.79	- 0.7±0.2	0.3 ± 0.2	- 7.7±1.0	- 8.08±0.12
22	280	69% Homog. Na	RACINE 1D with Na follower (280 cm^2) experiment critical	- 2.88	- 0.5±0.2	0.2 ± 0.2	- 6.4±1.0	- 2.68±0.04
23	280	Nat. Homog. Na	"	- 2.11	- 0.5±0.2	0.0 ± 0.2	- 4.1±1.0	- 2.01±0.03

* For transport correction uncertainty see [4].

TABLE V - Experiment to calculation comparison

N°	Rod		Calculated corrected $\Delta K/K$ %	$\frac{E - C}{C}$ %	COMMENTS
	Size cm ²	Boron en- richment			
12	98	Nat.	- 4.15 ± 0.07	- 1.3 ± 2.0	At PRE-RACINE reference clean core size ($R_c \approx 55$ cm)
13	98	47%	- 6.22 ± 0.10	- 1.7 ± 2.0	
14	98	90%	- 7.15 ± 0.14	- 1.7 ± 2.1	
11	112	Nat.	- 4.73 ± 0.07	- 1.1 ± 2.0	All PRE-RACINE measurements $\Delta R \rightarrow \Delta K$
11	112	Nat.	- 4.61 ± 0.07	- 0.6 ± 2.0	
21	280	69% Hetero. Na	- 7.50 ± 0.11	- 6.5 ± 2.0	At PRE-RACINE with Na follower core size $R_c \approx 56.5$ cm $R_c \approx 58$ cm $R_c \approx 58$ cm
22	280	69% Homog. Na	- 8.08 ± 0.12	- 6.5 ± 2.0	
22	280	69% Homog. Na	- 2.68 ± 0.04	- 7.8 ± 1.5	
22	280	69% Homog. Na	- 2.68 ± 0.04	- 9.7 ± 6.0	RACINE heterogeneous with Na follower $\Delta R \rightarrow \Delta K$ $R_c \approx 92$ cm Subcritical $R_c \approx 92$ cm
23	280	Nat. Homog. Na	- 2.01 ± 0.03	- 7.4 ± 5.0	

IV - COMMENTS ON (E-C)/C VALUES

It can be seen on table V that the discrepancy between experiment and calculation, expressed in % of rod worth, can be considered as independant of rod worth at constant core size and rod geometry.

The general trend to over estimate rod worth can be precised by the following findings :

- over estimation seems to be slightly dependant of core size at constant rod geometry :

- $\approx 6.5\%$ of a 280 cm^2 rod in a 60 cm radius core

- $\approx 8 \%$ for the same rod in a 92 cm radius core

- over estimation is dependant of rod geometry :

- ≈ 1 to 2% for 110 cm^2 rods without Na

- ≈ 6.5 to 9% for 280 cm^2 rods of mixed Na/absorber composition

For this point, at the present stage, it cannot be said wether the effect is due to rod size, presence of sodium, or both. But comparison between rods 21 and 22 in PRE-RACINE indicates that relative position of sodium and absorber as no influence on the discrepancy.

Last, subcritical and critical measurements (analysed in ΔK) seem to be in good agreement.

V - CONCLUSION

At present, a $\pm 13\%$ uncertainty is associated to all control rod design calculations [4]. In this value, the major contribution is due to the experiment to calculation comparison. In fact, starting from the results presented in this paper, a conservative value of $\pm 10\%$ uncertainty (i.e. no bias factor) has been deduced, which is linearly combined with a $\pm 3\%$ method uncertainty.

"The next part of RACINE control rod program, shortly described in the paper, based on rod interactions of the same order of magnitude than in power reactors, should help in reducing substantially the present uncertainties".

We hope that the proposal change is acceptable. Concerning inscription to the meeting and hotel reservations, could you please let us know if special arrangements will be necessary?

REFERENCES :

- |1| Y.H. BOUGET et al.
Experimental study of fast power reactor control rods in
the MASURCA critical assembly
NEACRP Meeting, Aix-en-Provence, 24/28 September 1979
- |2| P. HAMMER
Fast reactor physics at CEA : Present studies and futur
project.
ANS Sun-Valley Meeting, 14/18 September 1980
- |3| W. SCHOLTYSEK et al.
An assessment of physical characteristics of heteroge-
neous configurations with variable fertile volume.
This conference.
- |4| M. SALVATORES et al.
Paper presented at this meeting.

CONTROL-ROD CALCULATION METHODS
AND UNCERTAINTIES FOR A POWER
LMFBR DESIGN

M. Salvatorès*, C. Giacometti*,
G. Palmiotti*, V. Rado**, F. Lyon*

* Centre d'Etudes Nucléaires de Cadarache
Boîte Postale N°1
F.13115 - Saint-Paul-Lez-Durance
France

** ENEA
S.P. Anguillarese, Km 1+300
00060 - Casaccia (Roma)
Italy

ABSTRACT

The present paper summarizes the control-rod calculation methods used at CEA for large fast power reactor design. The standard design method and correction factors due to transport heterogeneity of the control-rod subassembly, number of energy groups and calculation mesh size effects, are defined, with their associated uncertainties.

1. INTRODUCTION

The present paper deals with the methodology developed at CEA for the control-rod configuration calculations. The need of accurate calculations and well defined uncertainties is stated on the basis of design criteria, both safety and operation related, in the case of a large fast power reactor.

A standard calculation method is defined, based on diffusion theory, hexagonal three-dimensional geometry, reduced number of energy groups, and reduced spatial mesh-point grid, with an homogeneous description of the control-rod subassemblies.

Several correction factors have to be applied, in order to improve the standard method with respect to a reference method, which could be defined as a transport three-dimensional calculation, with a large number of energy groups and a detailed description of the heterogeneity of the control-rod subassemblies. For each correction factor, related to a method approximation, an uncertainty is defined, based on physical considerations.

2. DESIGN NEEDS

Control-rod assessment for reactor design is related to the need of calculating the core reactivity in different configurations when total or partial insertion of a part or of the

whole of the control-rod system is envisaged.

The control-rod number and position should be such that :

- . safety requirements are met in all normal or abnormal operation or handling situations during the cycle ;
- . the power distribution has an optimum flat shape ;
- . a sufficient reactivity reserve is obtained to cover the reactivity loss during the cycle.

To define the reactivity needs the following procedure has been followed in the case of the principal control-rod system (SCP) of SUPER-PHENIX 1, as it is shown in Figure 1, where the different calculation steps are indicated :

a) Available reactivity : Point A

The available reactivity at point A is established on the basis of the needed reactivity for the cycle for the reactor operating at nominal power, taking into account the neutronic uncertainties on the critical mass, the reactivity loss per cycle and the fuel fabrication uncertainties.

b) Operation situation : Point F

The control-rod insertion is calculated at nominal power P_n to obtain the compensation of the available reactivity for an optimal power distribution.

c) Control-rod completely inserted : Point B

This point gives the available antireactivity at handling temperature, when all control-rod are completely inserted, starting from the operation position (point F), and the neutron power goes from P_n to zero power, P_0 .

d) Most unfavorable handling configuration : Point C

The available antireactivity is calculated at P_0 at handling temperature for the most unfavorable configuration, defined on the basis of safety requirements.

The calculational method has to be such that all the reactivity of the envisaged situations can be calculated and the associated uncertainties specified, to define the conservative reactivity value at each point of the procedure outlined above (including uncertainties).

3. UNCERTAINTY CONSEQUENCES

The present paper will deal mainly with the calculational method and its associated uncertainties. However, a very short review of the possible uncertainty consequences can be useful to evaluate how far one has to go to reduce these uncertainties.

First of all, let us consider the case in which the control-rod antireactivity is underestimated.

In this case, one has the following consequences :

- . the control-rod number is overpredicted ;
- . in turn, the core enrichment is overpredicted (too many rods in the core) and :
- . the reactivity loss per cycle is slightly higher (core enrichment too high) with respect to what it could have been without underestimation of control-rod worth.

In practice, for a SUPER-PHENIX type power reactor 10% underestimation will have the following effects :

- . about 2 rods in excess ;
- . about 0.44% higher enrichment ;
- . about 0.06 $\Delta K/K$ of reactivity loss increase.

If, on the contrary, the control-rod antireactivity is overestimated, particular arrangements have to be made to avoid :

- a) the cycle reduction (10% overestimation would lead to the reduction of about 120 full power days) ;
- b) safety related consequences.

The "particular arrangements" can be represented by the introduction in the core of some dummy "diluent" subassemblies. In this case however, consequences could be found in the total plant power output or in the power distribution shape. As a theoretical example, 10% underprediction of the control-rod worth would lead to the necessity of the introduction in the core of about 9 "diluent" subassemblies and to the relative loss in the total power output of about 2.4%. These very broad indications are given here as a mean to evaluate possible consequences of control-rod worth misprediction, even if in a real case, the actual scenario due to the misprediction, could be more complicated.

4. REFERENCE CALCULATION METHOD

The proper assessment of the overall uncertainty to be associated to the antireactivity calculations, is related from one side to the uncertainties of the calculation method, and from the other side to the uncertainties of the bias factors derived from critical experiment analysis. In the present paper, we will focus mainly on the calculation method problems. For all design calculation, a "standard design" method is used, related to the need of practical and efficient utilization. Correction factor have then to be applied to take into account special effects, to simulate an exact "reference" method. Both the "reference" and the "standard design" method are summarized in the following table :

	"Reference" Method	"Standard Design" Method	Need for cor- rection fac- tor
Theory	Transport	Diffusion	Yes
N° of energy groups	25 Groups	6 Groups	Yes
Geometry	Hexagonal 3D	Hexagonal 3D	-
Mesh-size	$\Delta x \rightarrow 0$	7 points/subas _y	Yes
Control-rod descript.	Heterogeneous	Homogeneous	Yes
Cross-section prepa- ration formulaire	CARNAVAL - IV	CARNAVAL - IV	-

The four resulting corrections factors will be examined in the following paragraphs. If we define the standard design method result as ρ_D , and the four correction factors as f_T , f_G , f_M , f_H , the corrected ρ_C value is given by :

$$\rho_C = \rho_D f_T f_G f_M f_H$$

All the correction factors are considered to be independent and this hypothesis will also be checked.

As far as cross-section preparation is concerned, B&C and Na data are originated from the 25 energy group formulaire CARNAVAL-IV using an external source mode calculation for an homogeneized control-rod medium. The external source S_i is given, in group i , by the following expression :

$$S_i = (D\phi)_{\text{core}}^i$$

5. MESH-SIZE EFFECTS

The finite mesh-size effect in a K_{eff} calculation results in systematically approximated value.

Finite difference diffusion codes produce K_{eff} values, which need zero-size mesh extrapolation. For 2D-calculations, this extrapolation is obtained using the well-known linear dependence of the K_{eff} on the mesh area.

Let us consider the actual control-rod arrangement in a 2D representation of SUPER-PHENIX 1 (see Figure 2). The 2D representation can be used to deduce zero-size mesh size extrapolations related to full control-rod insertion. This extrapolation can be obtained using two mesh sizes (i.e. 7 or 19 points for each subassembly), and it will vary in general for each control-rod configuration. Since, this corrections can be fairly large (up to 15% for asymmetric control-rod insertion patterns), the correction is actually systematically performed

for each configuration |1,2,3|. For what concerns axial mesh size effects, these effects should be taken into account for partial control-rod insertions. This effect is studied in R-Z geometry to avoid heavy 3D-hexagonal calculations. Only a limited number of cases is considered, i.e. fictitious central control-rod, inner ring or outer ring control-rod schematization and for a limited number of insertion heights.

Finally, it has been verified that the mesh-size correction factor is not dependent on the number of energy groups.

In summary, the mesh-size corrected reactivity for a particular control-rod pattern with partial insertion is obtained in the following way :

$$\rho = \frac{\left(4K_1^{(2)} - K_2^{(2)}\right) f_{Ax}^{(2)} - \left(4K_1^{(1)} - K_2^{(1)}\right) f_{Ax}^{(1)}}{\frac{1}{3} \left(4K_1^{(1)} - K_2^{(1)}\right) \left(4K_1^{(2)} - K_2^{(2)}\right) f_{Ax}^{(1)} f_{Ax}^{(2)}}$$

where $K_1^{(1)}$ and $K_1^{(2)}$ are respectively, the K_{eff} values of the reference, and the perturbed configurations for the two mesh-size, index i, calculations. $f_{Ax}^{(i)}$ is the axial mesh-size factor corresponding to the specific insertion of the i-th control rod configuration and are then calculated for only a limited number of situations, as it was previously mentioned.

It should be noted that the linear extrapolation rule for plane 2D calculations can be affected by the precision of the K_{eff} calculation. K_{eff} definition by balance of productions and losses can improve significantly the results |3|.

In view of the small uncertainties related to the mesh-size corrections, no explicit uncertainty was attributed to the calculation of that effect.

6. ENERGY GROUP STRUCTURE EFFECTS

Energy group structure effects have been studied in two dimensions, in particular with R-Z calculations. At present, a group collapsing algorithm based on region flux weighting has been used, with a large number of zones, in particular close to all interfaces between adjacent regions. Some of the results obtained are shown in table I. The R-Z schematization of SUPER-PHENIX has been subdivided in spatial regions for which six-group collapsed cross-sections were obtained. The group effect is fairly constant for the different configurations and gives a systematically overestimated value of the reactivity of approximately 3.5%. This value has been then chosen for all antireactivity calculations and affected by an uncertainty of $\pm 0.5\% \delta\rho/\rho$. This uncertainty could eventually increase for extreme cases of asymmetric control-rod insertion patterns. To avoid the need of not negligible correction factors and their associated uncertainties, difficult to be systematically assessed, a different collapsing procedure could be adopted, based on a variational principle |4|, which could minimize

the group structure effects. Recent developments and applications indicate that this is actually the case, as it is shown by the results of the table II, from which it is evident that the variational collapsing procedure reduces strongly the group structure effects in all the considered reactivity calculations, with respect to the standard flux weighting procedure (from $-3 \div 4\%$ to about -1%) even if a three-group energy structure is used. This procedure is thus envisaged for future use in design calculations.

As far as transport effects on the group structure effect, they will be considered in a successive paragraph.

7. TRANSPORT EFFECT CORRECTION

Transport effects play a major role in control-rod calculations. With the present available tools, a number of hypothesis has to be made to assess the transport effect corrections and their uncertainties.

The following features have been systematically investigated :

- . dependence of f_T on the energy group structure and on the space-angular mesh-size ;
- . dependence of the transport correction factors f_T on the calculation geometry (1D, 2D, cylindrical or plane X-Y) ;
- . dependence of f_T on the control-rod pattern.

7.1. Group-structure and space-angular mesh-size dependence of f_T

The cylindrical 1D geometry and the ANISN code has been used to investigate these effects. Four test configurations have been used, namely an hypothetical central control-rod ; inner ring inserted ; outer ring inserted ; inner + outer rings inserted simultaneously.

The transport correction factor is defined as :

$$f_T = \frac{(\delta K/KK')_{\text{Transport}} - (\delta K/KK')_{\text{Diffusion}}}{(\delta K/KK')_{\text{Diffusion}}}$$

both diffusion and transport calculation being performed with the same spatial mesh-size and number of groups.

In table III, the results obtained are summarized. The results show that the transport correction is at a very large extent group structure independent. As far as the space mesh-size effect, the details of the comparison show that the transport mesh-size effect is approximately $20 \div 30\%$ of the corresponding diffusion mesh-size effect and of opposite sign for mesh-centered flux diffusion calculation (see table IV), and this result is fairly independent of the control-rod configuration and on the geometry, 2D or 1D (see paragraph 7.2.).

For what concerns S_n effects, they are found to be small for the configurations considered as it can be seen from table IV.

In conclusion, although spatial mesh-size effects have to be carefully taken into account, S_4 calculations seem to be sufficient for most practical cases and the group structure effects are negligible too. On the basis of the previous paragraph results, in connection with variational collapsing procedures, it seems that transport correction factors can be calculated with a very limited number of energy groups.

7.2. Geometry effects

The previous paragraph results are based on cylindrical 1D geometry, and there is an obvious tendency to perform the least possible calculations using 2D transport codes. Moreover, no 2D hexagonal geometry codes have been used up to now, but only the DOT 4 discrete ordinate code, which allows X-Y geometry. However, the X-Y and hexagonal 2D geometry equivalence for control-rod studies, has been verified for several SUPER-PHENIX 1 control-rod patterns. X-Y 2D calculations have been performed for a number of control-rod configurations and the results are compared, when meaningful, to 1D cylindrical calculations. This is the case for example of the central (fictitious) control-rod insertion, and the insertion of both rings. The results are compared in table V.

From the results so far obtained, it seems that the geometry effects do not play a major role, when a comparison between cylindrical and X-Y geometry is possible. However, the main use of the results obtained, is related to the use in hexagonal geometry of the transport corrections obtained both in cylindrical or X-Y geometry. Their use seems reasonable and fairly safe, if one adds a $\pm 1\%$ uncertainty to the f_T values, to take into account geometry effects.

7.3. Configuration dependence of transport corrections

The results obtained in cylindrical geometry, and confirmed by symmetrical configurations calculations in X-Y geometry, shown in table V, indicate that the maximum f_T factors are found for a central control-rod, and that increasingly lower (negative) f_T factors are found for increasingly out-of center control-rod configurations (e.g. lowest negative f_T value is found for the outer ring insertion).

The reason for this tendency seems to be found in the radial gradient effect of the flux. In fact, if one formulates the control-rod insertion antireactivity in the following way :

$$\rho = \frac{1}{F} \langle \phi^* \Delta \Sigma \phi' \rangle_{CR \text{ Volume}} \quad (1)$$

and ϕ' is the flux perturbed by the control-rod insertion, for a central control-rod insertion, one certainly expects :

$$\phi'_{\text{diffusion}} > \phi'_{\text{transport}} \quad (2)$$

(i.e., less flux dip in a diffusion calculation). In the case of an outer ring insertion in 1D cylindrical geometry, the underestimation by diffusion theory of the flux dip is coupled to the overall flux gradient, as it is shown in Figure 3. Across the control-rod region, positive effects, of the type of those related to equation (2), are partly canceled by negative effects, in an outer control-rod region zone.

This type of considerations apply obviously to symmetrical control-rod insertion patterns.

Asymmetric configurations should be studied independently.

In summary, the transport effects study indicated that :

- . transport correction can be made in few groups (< 6) S_4 approximation but with a zero-size spatial mesh size extrapolation, which is about 20% of the corresponding extrapolation effect found in diffusion theory ;
- . cylindrical and X-Y geometry can be safely used for extrapolating to hexagonal geometry, if an uncertainty of about $\pm 1\%$ is added. However this point should need a verification with and hexagonal transport calculation ;
- . the transport correction is configuration dependent. However, the f_T factor calculation in 1D cylindrical geometry for an hypothetical central control-rod insertion, represent an upper limit for all symmetrical control-rod configurations and as such it has been used for safety related studies for SUPER-PHENIX 1.

8. CONTROL-ROD HETEROGENEITY CORRECTION

In the previous paragraphs, we have used an homogeneized description of an actual control-rod (see Figure 4). To take into account the heterogeneous structure effects, different strategies can be used.

In reference [5], it was suggested to use a variational approach to define equivalent heterogeneity corrected cross-sections. The approach followed at CEA has been to take explicitly into account the spatial heterogeneous description of the control-rod.

Several effects should be considered to define a heterogeneity correction factor f_H and its associated uncertainty. The following effects have been studied :

- . f_H factor dependence on diffusion or transport theory ;
- . dependence on geometry (X-Y versus hexagonal) ;
- . dependence on control-rod position.

8.1. Geometry dependence of heterogeneity

The simplest calculational model to test heterogeneity effects is represented by a central control-rod represented in 1D cylindrical geometry (see Figure 4).

This model was first used to verify that the heterogeneity f_H factor, defined as :

$$f_H = \frac{\rho_{\text{Het. descrip.}}}{\rho_{\text{Hom. descrip.}}} - 1$$

is fairly independent of diffusion or transport theory ($f_H = -7.2\%$ in diffusion, and $f_H = -7.5\%$ in transport theory).

A more realistic representation of the control-rod heterogeneity should involve a detailed hexagonal description. The best representation of SUPER-PHENIX 1 control-rod, involves 216 triangles for each subassembly (see Figure 5). Since present finite difference codes preclude the use of local finer description with respect to an overall broad description, the use of the more detailed mesh grid implies a fine mesh grid all over the reactor, i.e. the same base triangle description for each subassembly. On the basis of these considerations, the more accurate description cannot be used routinely. For parametric study purposes, an equivalent control-rod in hexagonal geometry has been defined with the same surface as the reference one (see Figure 6). For the detailed description of this rod, named "star" rod, 24 basic triangles are necessary. To assess the equivalence of the reference rod and of the star rod, a 2D hexagonal cell calculation has been performed with the description of control-rod position surrounded by three fuel subassembly rings. The f_H calculation gave in the two cases, the following results :

$$\begin{aligned} f_H(\text{reference rod}) &= -9.3\% \\ f_H(\text{"star" rod}) &= -10.3\% \end{aligned}$$

From this point on the "star" rod has been tested.

As far as geometry is concerned, 1D cylindrical and 2D hexagonal calculations were compared for the star rod in central core position and the results are shown in table VI. It is evident that the heterogeneity correction is geometry dependent and detailed hexagonal calculations can be necessary for most cases. Since however, a detailed description is at present often unpractical, an uncertainty should be associated to f_H , depending on the calculational model chosen. If the most simplified cylindrical model is chosen, the uncertainty to be associated to the heterogeneity correction for a central control-rod should not be less than $\pm 2\%$.

8.2. Configuration dependence of the heterogeneity correction

In order to study the configuration dependence of the f_H factor, the star rod model was used. Several rod configurations were studied and the results are indicated in table VII. The heterogeneity correction is systematically negative and shows a tendency to be higher at core center. The negative value can be easily shown on the basis of the following simple model for the homogeneous and the heterogeneous calculations :

$$\left\{ \begin{array}{l} \rho_{\text{Hom}} \approx \text{const } \phi^* \sigma_{\text{B}_4\text{C}} N_{\text{hom}} \langle \phi'_{\text{hom}} \rangle \text{Vol.hom} = \text{const } \phi^* \sigma_{\text{B}_4\text{C}} N_{\text{hom}} V_{\text{hom}} \bar{\phi}'_{\text{hom}} \\ \rho_{\text{Het}} \approx \text{const } \phi^* \sigma_{\text{B}_4\text{C}} N_{\text{het}} \langle \phi'_{\text{het}} \rangle \text{Vol.het} = \text{const } \phi^* \sigma_{\text{B}_4\text{C}} N_{\text{het}} V_{\text{het}} \bar{\phi}'_{\text{het}} \end{array} \right. \quad (3)$$

where ρ_{hom} and ρ_{het} are respectively the reactivity due to a control-rod insertion in a homogeneous or heterogeneous model. ϕ^* is the unperturbed adjoint flux, and the reactivity effect is approximated by the effect of B_4C only : $\Delta \Sigma \approx \sigma_{\text{B}_4\text{C}} N_{\text{B}_4\text{C}}$. N_{hom} and N_{het} are the B_4C atom densities in homogeneous and heterogeneous descriptions. These densities are obviously related :

$$N_{\text{hom}} V_{\text{hom}} = N_{\text{het}} V_{\text{het}}$$

where V_{hom} and V_{het} are the control-rod volumes where the B_4C is present : the whole control-rod zone in the case of the homogeneous description and the actual absorber zone in the case of the heterogeneous description.

From the equation (3), one obtains :

$$\rho_{\text{het}} / \rho_{\text{hom}} \approx \bar{\phi}'_{\text{het}} / \bar{\phi}'_{\text{hom}} \quad (4)$$

where $\bar{\phi}'_{\text{het}}$ and $\bar{\phi}'_{\text{hom}}$ are the average perturbed flux values in the zone where the B_4C is present.

Since, $\bar{\phi}'_{\text{het}} < \bar{\phi}'_{\text{hom}}$ due to the higher B_4C concentration in the heterogeneous case, one has negative f_H values. The ϕ' distribution across a central control-rod in the 1D cylindrical model, is shown in Figure 7. However, the radial gradient effects can lower the negative value, due to compensating effects. In Figure 8, we show the ϕ' distribution obtained across a control-rod outer ring zone in a cylindrical model, in a homogeneous and heterogeneous description. This last case was simulated with an inner absorber ring, surrounded by Sodium rings of appropriate surface. If we consider an "inner" and an "outer" control-rod ring zone, we see that the ratio (4) changes sign, going from one zone to the other. This can explain qualitatively the less negative f_H values which are found for the out-of-center control-rod heterogeneity effect in the actual case, to which refers table VII.

In summary, the f_H factor calculated for a central control-rod and applied to a control-rod configuration can be considered an upper limit to this factor. This upper limit should be however affected by a non negligible uncertainty value (of the order of $\pm 2\%$) to take into account geometry dependence, as discussed in paragraph 8.1.

9. EXPERIMENTAL BIAS FACTORS

After having discussed the calculational methods used for the control-rod configuration antireactivity evaluation, and their associated uncertainties, it is worthwhile to recall that the method corrected value ρ_c defined in paragraph 6, is further corrected by a bias factor derived from integral experiments, to which an uncertainty is also associated. The experimental program, performed at MASURCA on a single rod or on a couple of interacting rods in several core configurations and for several B_4C enrichments and control-rod sizes, has provided no evidence for a definite bias factor to be applied to the control-rod calculation of a large LMFBR of the SUPER-PHENIX 1 type, but rather of a supplementary uncertainty, of the order of $\pm 10\%$ which should affect the ρ_c value. This uncertainty derived from the experiment, is at present combined linearly with the uncertainties related to the various method correction factors, discussed in the previous paragraphs. A companion paper, presented at this meeting [6], will discuss the main feature of some significant control-rod experiment performed in MASURCA, and their analysis. Finally, it should be noted that the $\pm 10\%$ uncertainty takes into account the uncertainty of extrapolating from a limited size core critical experiment, to large size core multiple rod configurations in the power reactor. More sensitivity studies are presently underway to improve the extrapolation techniques.

10. CONCLUSIONS

Large fast power reactor control-rod calculations need sophisticated calculational methods. In the present paper, the calculation procedure followed at CEA, in particular for SUPER-PHENIX control-rod antireactivity evaluation, has been described. This procedure consists in the definition of "standard" design method, based on few group, broad mesh size, diffusion calculation using an homogeneous model of the actual control-rod subassembly.

The calculated antireactivities are corrected for the effects of zero-mesh size, large number of groups, transport and heterogeneity. For the last two effects, even if the correction factors could be calculated in principle for each configuration of interest, the standard procedure adopted is at present to use a unique value for each correction factor, which represent an upper limit, in a safety sense. To all the correction factors, an appropriate uncertainty is applied, in particular to take into account simplifications in the geometrical models.

Finally, a supplementary uncertainty, derived from the experimental programs on MASURCA, is linearly combined to the method correction factors uncertainties. Areas for improvement in this field are from one side the use of improved transport and heterogeneity methods, and, from the other side, a better definition of the extrapolation techniques from the experimental results obtained in the critical facilities, to the actual large power reactor configurations.

REFERENCES

- |1| - Y. Kato et al., Nucl. Scien. Eng. 61, 127 (1976)
- |2| - J.C. Estiot, G. Palmiotti, M. Salvatores, D. Honde,
Annals of Nucl. Energy 8, 609 (1981)
- |3| - J. Rowlands, C. Eaton, Proceedings Conf. Fast Reactor
Physics, AIX-EN-PROVENCE, (1979)
- |4| - M. Salvatores, Nucl. Sci. Eng., 57, 340 (1975)
- |5| - J. Rowlands, IAEA-OECD LUGANO Meeting on Homogeneization
Methods (1978)
- |6| - G. Humbert et al., presented at this Meeting.

-oOo-

TABLE I

Configuration	Geometry	25gr.-6gr. 6gr. ϕ weighting
Central CR	R-Z	- 3.0 %
Inner Ring	R-Z	- 3.2 %
Outer Ring	R-Z	- 2.6 %
Inner + Outer Rings	R-Z	- 3.2 %
	Hexagonal	- 3.6 %

Energy group structure dependence of control-rod antireactivity calculations

TABLE II

Configuration	3 group collapsing ϕ weighting	3 group col-lapsing varia-tional weigh-ting
Central CR	- 13. %	0
Inner Ring	- 10.6 %	- 0.1 %
Outer Ring	- 7.7 %	- 0.4 %
Inner + Outer Rings	- 10.4 %	- 1.7 %

Collapsing procedure dependence of energy group structure effects on CR antireactivity calculations

TABLE III

Configuration	M_1		M_∞	
	6 Groups	25Groups	6 Groups	25Groups
Central CR	- 5.1	- 5.4	- 7.7	- 8.1
Inner Ring	- 2.5	- 2.4	- 4.7	- 4.7
Outer Ring	- 0.5	- 0.3	- 1.9	- 1.9
Inner+Outer Rings	- 1.7	- 1.4	- 5.4	- 5.9

Transport effect. f_T values in percentage
 M_1 : 7 points/subassembly calculations
 M_∞ : extrapolated zero mesh-size calculations

TABLE IV

Configuration	$M_\infty/M_1 - 1$		$S_8/S_4 - 1$
	Diffusion	Transport	Transport(M_1)
Central CR	2.2	- 0.7	1.0
Inner Ring	2.0	- 0.4	0.0
Outer Ring	1.3	- 0.4	0.1
Inner/Outer Rings	3.5	- 1.2	0.3

Space-angular mesh effects. Values in percentage on the calculated reactivities.
 M_1 : 7 points/subassembly calculations
 M_∞ : extrapolated zero mesh-size calculations

TABLE V

Configuration	f_T calculation (6groups, S_4 , M_∞)	
	1D Cylindric.	2D X-Y
Central CR	- 7.7	-
Inner Ring	- 4.7	- 5.4
Outer Ring	- 1.9	- 0.6
Inner + Outer Rings	- 5.4	- 8.2

Geometry dependence of transport effect
 f_T percentage values

TABLE VI

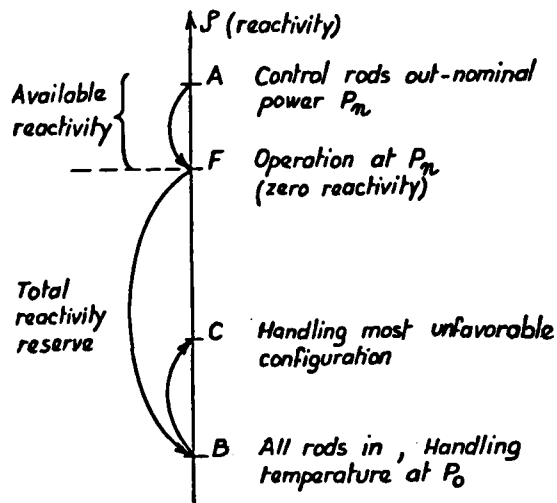
Type of Geometry (same axial B^2)	f_H (Heterogeneity factor)
Cylindrical	- 12.6
Hexagonal	- 13.3

Geometry dependence of heterogeneity correction factor for the "star" rod (see text for details). f_H percentage values

TABLE VII

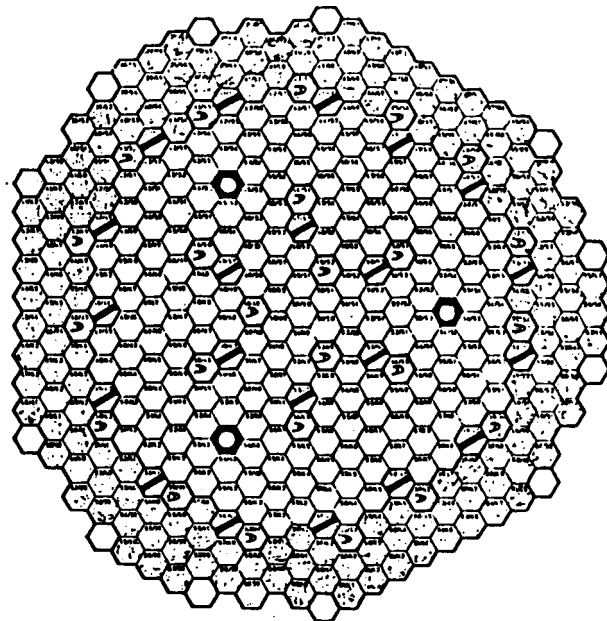
Configuration	f_H
Central CR	- 13.3
Inner Ring Rods	- 5.6
Outer Ring Rods	- 8.0
Inner + Outer Ring Rods	- 10.0
3 Symmetrical Rod of Outer Ring	- 3.1

Configuration dependence of heterogeneity correction factor. f_H percentage values



REACTIVITY NEED ASSESSMENT CALCULATIONAL PROCEDURE

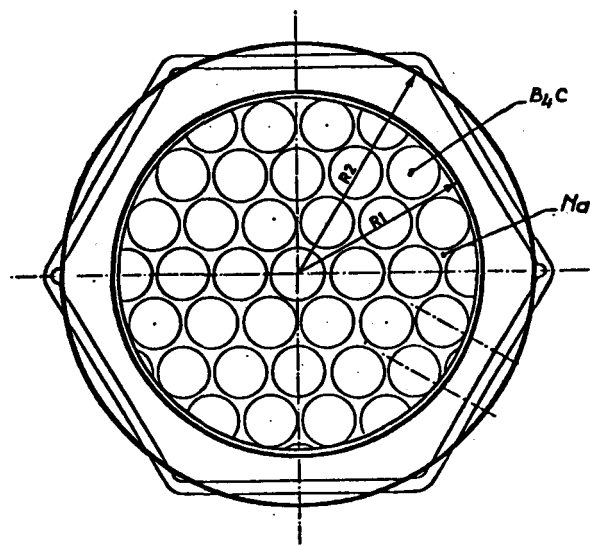
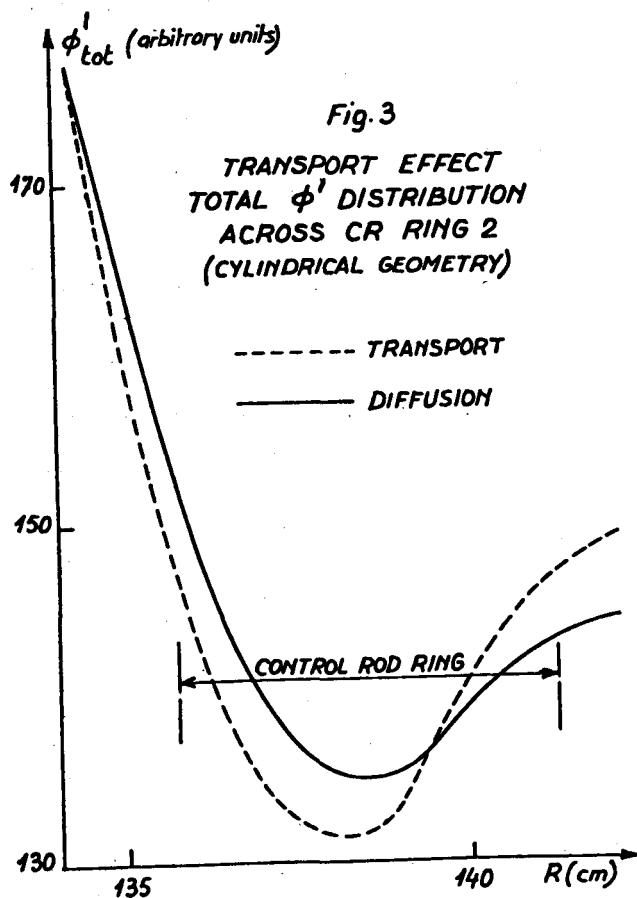
Fig. 1



Subassembly

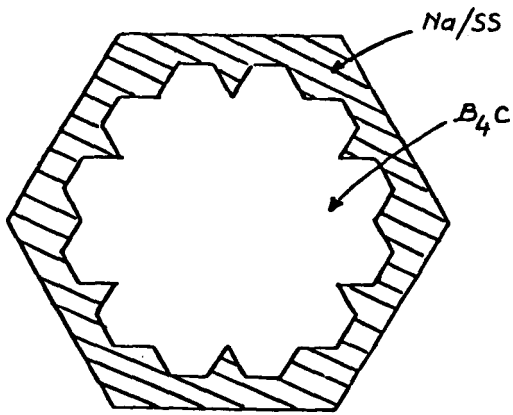
○	Inner fuel	184	} 352
⊗	Outer fuel	168	
⊙	Control rods	21	
⊖		3	
⊕	"Diluent"	24	

Fig. 2 - CONTROL ROD CONFIGURATION



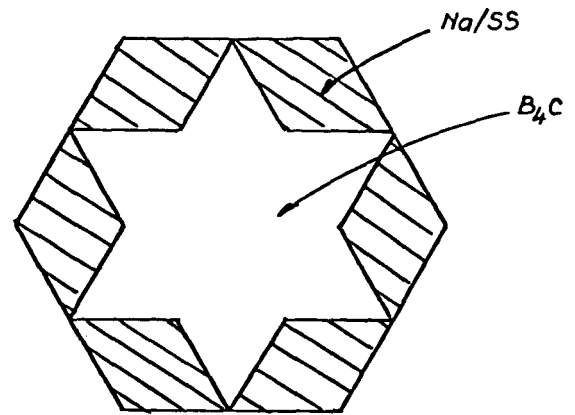
Schematization for 1D cylindrical geometry
 $R1 = 7,5$ cm outer radius of absorber zone
 $R2 = 9,5$ cm outer radius of Na/SS zone

Fig. 4 - HETEROGENEITY EFFECT
SPX1 Control rod



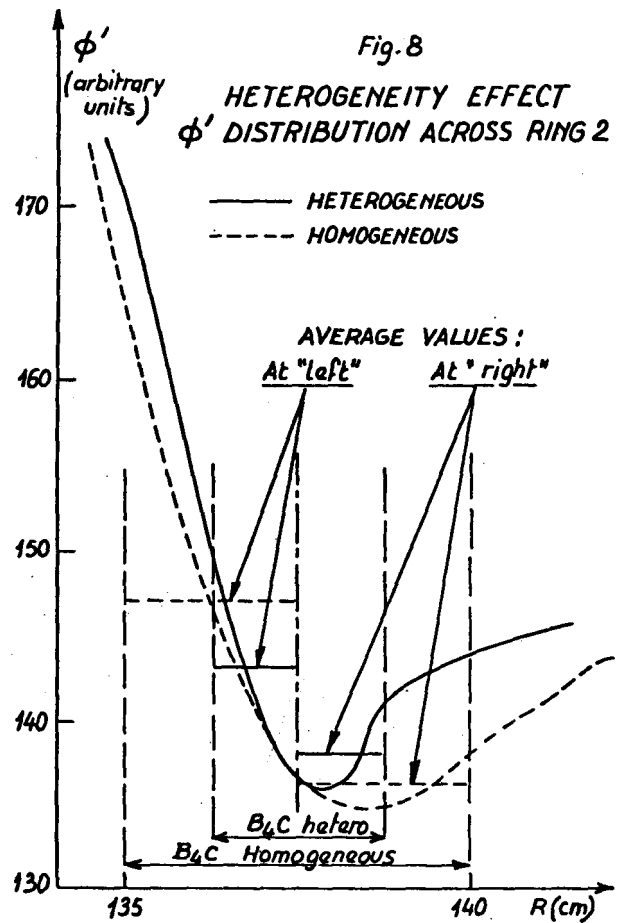
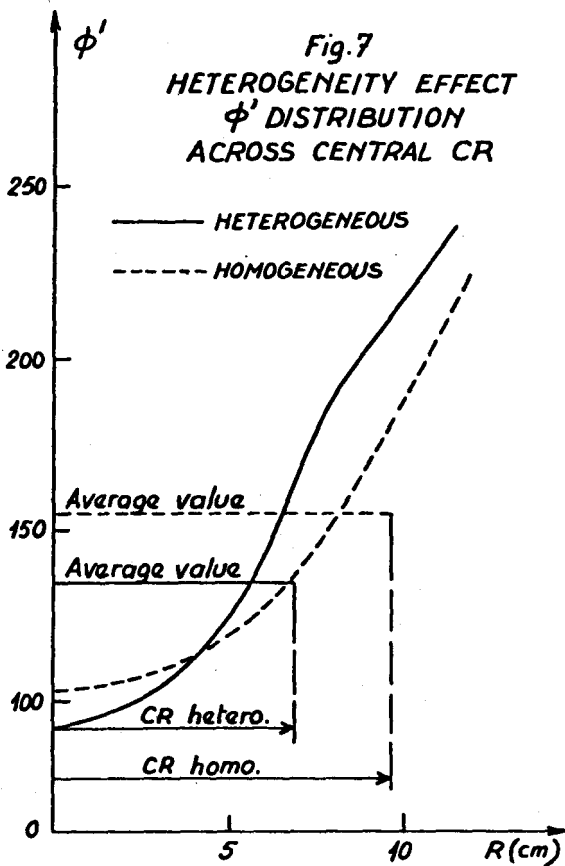
Heterogeneous description in hexagonal:
216 basic triangles

Fig. 5. HETEROGENEITY EFFECTS
SPX 1. Control rod schematization



- Same surface as SPX 1 CR
- 50% B_4C
- Heterogeneous description: 24 basic triangles

Fig. 6. HETEROGENEITY EFFECTS
Star control rod for parametric studies



VALIDATION OF ALTERNATIVE METHODS AND DATA FOR A BENCHMARK
FAST REACTOR DEPLETION CALCULATION*

B. J. Toppel, C. H. Adams, R. D. Lawrence, H. Henryson II, K. L. Derstine
Argonne National Laboratory
Applied Physics Division
9700 South Cass Avenue
Argonne, Illinois 60439

ABSTRACT

Alternative neutronics models, data processing procedures, and data bases have been used to examine two 383.5 day burn cycles for a 1000 MWe heterogeneous LMFBR design developed by the Large Core Code Evaluation Working Group (LCCEWG). The diffusion theory neutronics methods used are finite difference, nodal, and spatial flux synthesis. Four, eight, and twenty-group multigroup cross sections based on ENDF/B-IV and ENDF/B-V data bases are compared. The effect of parked control rods as compared with rods moving during the burn cycles is examined. Various performance parameters are compared including burnup reactivity swings, rod worths, fuel burnup, power splits, and sodium void reactivity effects. The sensitivity of the results to choices in the modeling process are discussed.

INTRODUCTION

Benchmark analysis has traditionally been an important means of qualifying methods, codes and data for reactor physics applications. In this study an attempt is made to quantify the effects of alternative neutronics models, data processing assumptions, and data bases upon the calculated performance characteristics of an LMFBR. The current study differs from the work of other benchmark efforts in that self-consistency among alternative models is emphasized and the impact of both data and methods are addressed in a single benchmark study.

The codes which were used in the study have been developed at Argonne National Laboratory by the authors. This permits a particularly meaningful intercomparison since any bias toward a particular methodology is removed. The three methods which were investigated span the range of those which are generally considered for three-dimensional diffusion-theory reactor depletion studies: finite difference, nodal and spatial flux synthesis. Each has its own advantages and disadvantages. The finite-difference method is well-known and generally considered a standard for such applications but is limited in three-dimensional applications because of long computation times. Although recently developed nodal methods have demonstrated high accuracy in very short running times for light water reactor analyses in Cartesian geometry, relatively little work has been done on the extension of these methods to fast reactor calculations in hexagonal geometry. The spatial flux-synthesis methods are efficient and accurate for fast reactor calculations but it is difficult to assess the accuracy of a particular calculation because of the dependence on the trial function selection.

*Work performed under the auspices of the U.S. Department of Energy

DESCRIPTION OF THE CALCULATIONAL MODEL

The reactor model chosen for the study is a 1000 MWe heterogeneous design which was developed by the Large Core Code Evaluation Working Group (LCCEWG) for benchmark intercomparisons.¹

The benchmark calculations were performed using a three-dimensional model with 60° symmetry in the plane. Fig. 1 shows the arrangement of a typical plane and indicates the placement of the three driver zones, the three internal blanket zones, the radial blanket and shield, and the various control-rod positions. The lattice pitch is 16.33 cm. The fuel batch loading sequence is also indicated.

Figure 2, which is an R-Z representation of the reactor, is presented merely to indicate the axial dimensions of the model. No actual R-Z calculations were performed owing to the necessarily arbitrary nature of such an R-Z representation. The axial mesh used is given in Table 1.

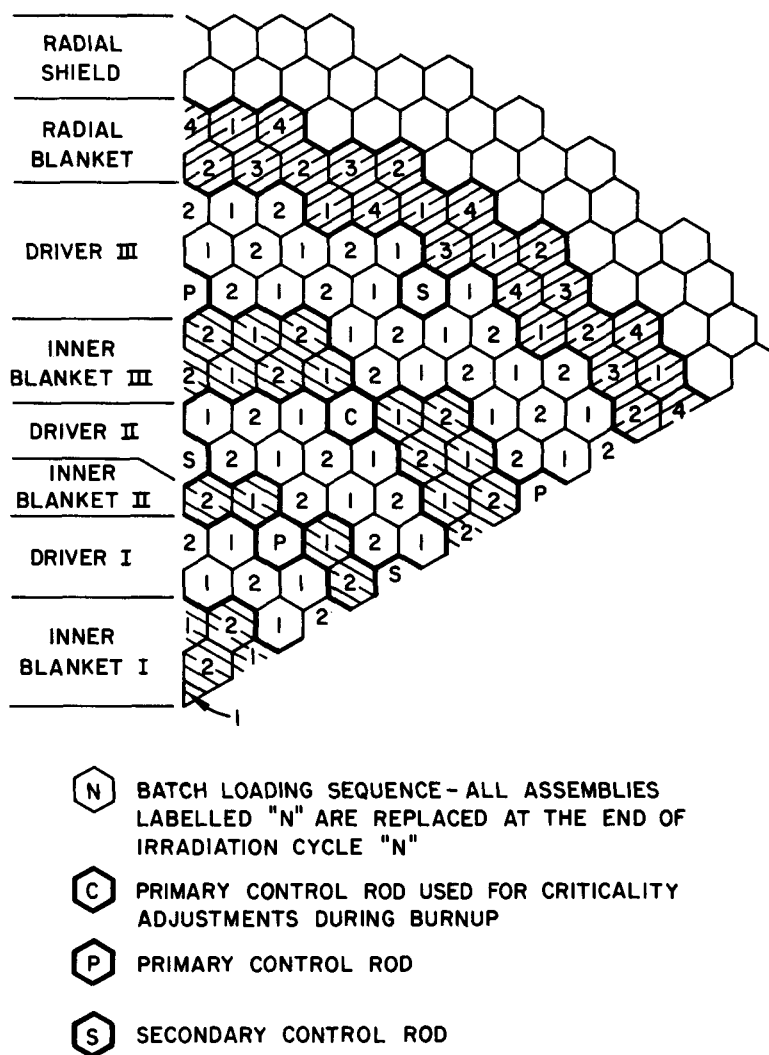


FIGURE 1. SIXTY DEGREE SECTOR OF CORE LAYOUT

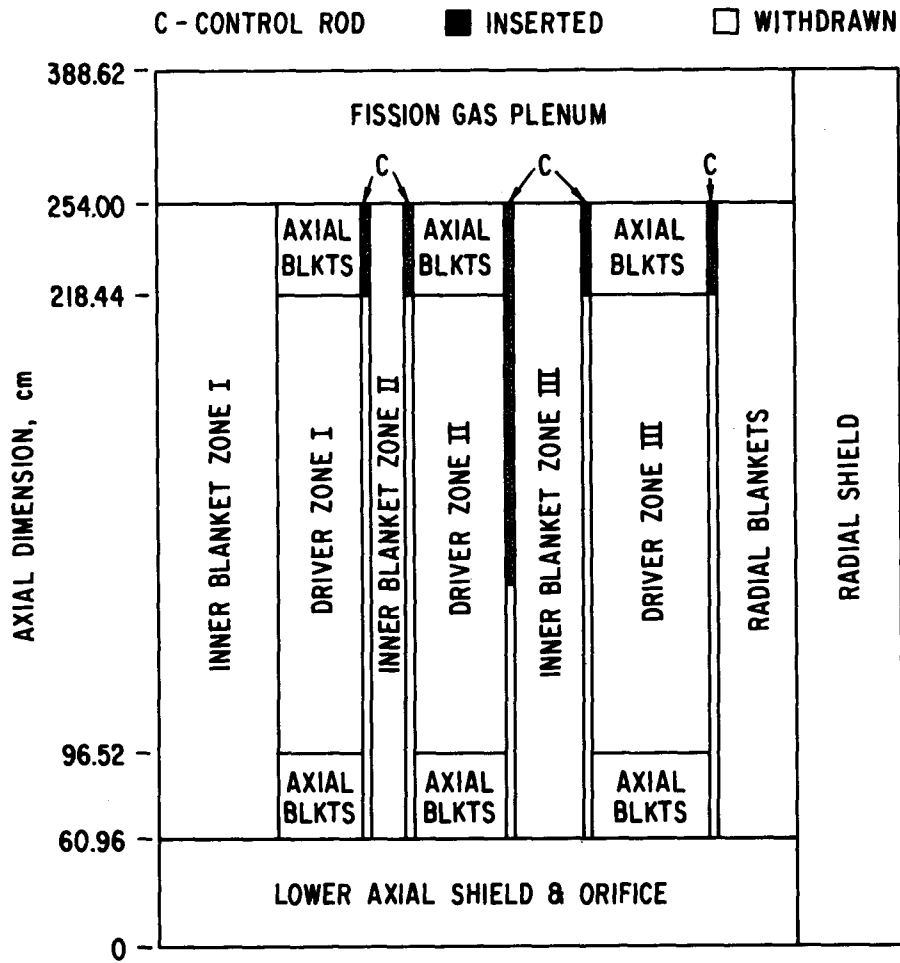


FIGURE 2. REACTOR DIAGRAM INDICATING AXIAL DIMENSIONS

Table 1. Axial Definition Used in the Benchmark Calculations

Axial Range (cm)	Number of Axial Regions	Number of Mesh Intervals	
		Finite Difference and Spatial Synthesis	Nodal
0 - 60.96	1	2	1
60.96 - 96.52	2	6	2
96.52 - 218.44	8	24	11
218.44 - 254.0	2	6	2
254.0 - 388.62	1	4	1

The initial atom densities in the various zones are listed in Table 2.

Standard triangular-mesh finite-difference calculations with six mesh points per hexagon and 42 axial planes were performed using the DIF3D code.² A new nodal scheme for hexagonal-Z geometries which was developed by Lawrence³ and implemented as an option within DIF3D provided the second method. The nodal calculations were performed using one mesh cell (node) per hexagon and 17 axial planes. The axial mesh (~15cm) was dictated by the burnup region definitions and control positions rather than accuracy requirements (which would have allowed a coarser mesh). The third neutronics model used the SYN3D code⁴ to perform spatial flux-synthesis calculations. For the synthesis calculations various two-dimensional trial functions were employed based upon beginning-of-life and end-of-life, rodged and unrodged, buckled eigenvalue calculations in the core plane, and rodged/unrodged fixed source calculations in the axial blanket planes.⁵ For all of the neutronics solutions, a vacuum external boundary condition was assumed.

The various neutronics algorithms were used in the REBUS-3 fast reactor depletion code⁶ to perform a nonequilibrium burnup for two 383.5 day cycles. The simplified fuel burnup chains used for the benchmark analyses are indicated in Figure 3. The half life for ²⁴¹Pu beta decay was taken to be 14.4 years. Control-rod positions were changed at the midpoint of each cycle to approximate the rod motion through the cycle. Four flux calculations were performed for each burn cycle: beginning of cycle (BOC), midcycle with rods at BOC position, midcycle with rod positions modified, and end of cycle (EOC). The primary control rods used for criticality adjustments during burnup (Fig. 1) were positioned at the following heights above the lower surface of the lower axial shield (cm); 172.72 at the beginning of cycle 1 (BOC1); 193.04 at the end of cycle 1 (EOC1); 182.88 at the beginning of cycle 2 (BOC2); and

Table 2. Initial Atom-Densities (atoms/barn-cm)

	Driver	Internal or Radial Blanket	Radial Shield Structure	Control ^a	Na Channel ^b	Axial Blanket	Fission Gas Plenum	Lower Axial Shield
U-235	1.9168-5 ^c	2.9907-5	0.0	0.0	0.0	2.7953-5	0.0	0.0
U-238	8.6912-3	1.3558-2	0.0	0.0	0.0	1.2673-2	0.0	0.0
Pu-239	1.3591-3	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Pu-240	4.0789-4	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Pu-241	2.0392-4	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Pu-242	4.8481-5	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Cr	2.8111-3	2.3756-3	1.3180-2	3.1030-3	1.3980-3	2.8111-3	2.5335-3	1.2280-2
Fe	9.6814-3	8.1994-3	4.5410-2	1.0690-2	4.8170-3	9.6814-3	8.7337-3	4.2290-2
Ni	1.9207-3	1.6266-3	9.0060-3	2.1200-3	9.5520-4	1.9207-3	1.7326-3	8.3900-3
Mo	2.1764-4	1.8431-4	1.0210-3	2.4030-4	1.0820-4	2.1764-4	1.9633-4	9.5070-4
Mn	2.6609-4	2.2528-4	1.2480-3	2.9370-4	1.3240-4	2.6609-4	2.4001-4	1.1620-3
Na	8.2217-3	6.2401-3	2.2370-3	1.0490-3	1.8420-2	8.2217-3	7.8509-3	3.9930-3
O	2.1460-2	2.7176-2	0.0	0.0	0.0	2.1460-2	0.0	0.0
F.P.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
B-10	0.0	0.0	0.0	2.9110-2	0.0	0.0	0.0	0.0
B-11	0.0	0.0	0.0	2.5310-3	0.0	0.0	0.0	0.0
C	0.0	0.0	0.0	8.2700-3	0.0	0.0	0.0	0.0

^aControl assembly inserted

^bControl assembly withdrawn

^cRead as 1.9168×10^{-5}

198.12 at the end of cycle 2 (EOC2). All other primary and secondary control rods were assumed to be parked at the upper axial blanket/driver interface. Fuel shuffling was performed between the first and second cycles as indicated in Fig. 1. The calculations assumed a reactor power of 2740 MWth and a re-fueling interval of 383.5 effective full power days.

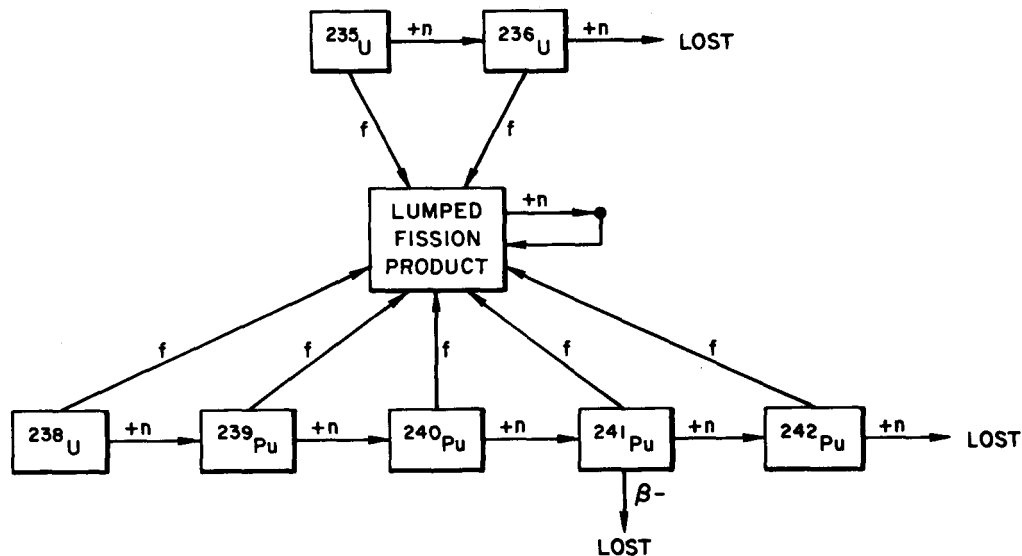


FIGURE 3. FUEL BURNUP CHAINS FOR THE BENCHMARK ANALYSES

Multigroup cross sections were generated using the MC²-2⁷ and MC²-2/SDX codes^{8,9} in 4, 8, and 20 energy group versions. The group structures are indicated in Table 3. Separate cross sections were generated for each driver zone and inner blanket zone, and for the radial blanket, radial shield, axial blankets, lower axial shield, fission gas plenum, and control channels. Pin heterogeneity effects were modeled for all of the driver and blanket fuel assemblies.

Cross sections were generated from both the ENDF/B-IV¹⁰ and ENDF/B-V¹¹ data bases, and also separate cross sections were prepared with sodium voided from the driver and blanket zones for use in the sodium void reactivity worth calculations. A single lumped fission product was used in the depletion calculations with cross sections based upon the ENDF/B-V fission product data files.¹²

DESCRIPTION OF NEUTRONICS METHODS

The reference calculations utilized the 8 energy group cross sections based on the ENDFB-V data. The DIF3D/REBUS-3 calculations used standard triangular-mesh finite difference with six mesh points per hexagon and 42 axial planes as indicated in Table 1. The finite difference equations employed by DIF3D are mesh centered.

The nodal option in DIF3D solves the neutron diffusion equation using 1 mesh cell (node) per hexagonal assembly. The nodal equations are derived using higher order polynomial approximations³ to the spatial dependence of the flux within the hexagonal node. The final equations involve spatial moments of the flux within the node plus face-averaged partial currents across the surfaces

Table 3. Broad Group Energy Structures

Four Group		Eight Group		Twenty Group	
Group	Upper Energy (eV)	Group	Upper Energy (eV)	Group	Upper Energy (eV)
1	1.4191+7*	1	1.4191+7	1	1.4191+7
2	8.2085+5	2	2.2313+6	2	3.6788+6
3	6.7380+4	3	8.2085+5	3	2.2313+6
4	2.0347+3	4	1.8316+5	4	1.3534+6
		5	4.0868+4	5	8.2085+5
		6	9.1188+3	6	4.9787+5
		7	2.0347+3	7	3.0197+5
		8	4.5400+2	8	1.8316+5
				9	1.1109+5
				10	6.7380+4
				11	4.0868+4
				12	2.4788+4
				13	1.5034+4
				14	9.1188+3
				15	5.5309+3
				16	3.3546+3
				17	2.0347+3
				18	1.2341+3
				19	7.4852+2
				20	4.5400+2

*Read as 1.4191×10^7

of the node. The three-dimensional method involves a total of 13 unknowns per group per node: 8 outgoing partial currents (1 on each of the 8 surfaces) plus 5 flux moments (the node-averaged flux, plus one spatial moment in each of the three hex-plane directions and the axial direction). Although the DIF3D finite-difference method using six triangles per hexagon involves only six unknowns per hexagon for each axial mesh interval, the nodal option is considerably faster. This is because the solution of the partial current equations typically requires only 2 inner iterations per group, while the solution of the 6 triangle-per-hexagon finite-difference equations often requires 5 to 10 inner iterations; consequently, for two-dimensional problems, the nodal method requires 2 to 3 times less CPU time than the finite-difference calculation. In addition, for three-dimensional calculations, the higher order axial approximation in the nodal scheme permits the use of an axial mesh which is at least 4 times coarser than that used in typical finite-difference calculations. This decrease in the number of axial planes yields additional factors of 2 to 3 reduction in CPU time, thus giving overall CPU time ratios of between 4 and 9 for three-dimensional problems, depending upon the desired solution accuracy and whether the axial zone boundaries permit full advantage to be taken of the nodal coarse-mesh capabilities. For the reference nodal option DIF3D/REBUS-3, 17 axial planes were used as indicated in Table 1.

One drawback of the nodal approach is the lack of information concerning the spatial distribution of the flux within the node. A simple procedure is used in the nodal option of DIF3D to compute more accurate peak power densities and fluxes than those obtained by sampling only the node-averaged values. In two dimensions, this procedure involves sampling surface-averaged fluxes on the six surfaces of the hexagon; the surface fluxes are readily obtained from the calculated interface partial currents. This procedure is extended to three dimensions by assuming that the flux within the node is separable in the hex-plane and axial directions. Peak values are computed by evaluating this assumed flux shape at several axial elevations within the node. If the computed peak-to-average value in a node is unrealistically high, the separability approximation is abandoned, and the peak value in that node is computed by sampling only the node- and surface-averaged values. This "fixup" has been required only for nodes in control assemblies for the test problems studied to date.

The SYN3D code provides a single-channel, spatial flux-synthesis approximation to the finite difference multigroup neutron-diffusion-theory equations. For three-dimensional calculations, precalculated two-dimensional expansion or trial functions are required. Relatively straight-forward prescriptions for determination of the trial functions have been developed which reduce the uncertainty associated with synthesis calculations.⁵ Nevertheless the arbitrary nature of such trial functions makes it difficult to make a general assessment of the accuracy of a particular calculation.

COMPARISON OF REFERENCE CALCULATIONS

Table 4 compares calculated reactivity swings due to burnup and control rod movement as well as various integral parameters for the three reference methods. Excellent agreement is observed among the three methodologies for power fraction through the cycle, and core peaking factors. Such agreement suggests that the trial functions chosen for the synthesis calculations are appropriate for depletion analysis. The synthesis calculation, however, significantly overpredicts the control rod worth. The Table 4 results also demonstrate a bias in the eigenvalue, burnup reactivity swing, and blanket peaking results between the finite-difference methods and the nodal calculation. The nodal eigenvalues are .3 to .4% lower than the finite-difference results whereas the blanket peaking and burnup are significantly higher. The nodal calculation gives a smaller burnup reactivity swing than the finite-difference results which is related to the plutonium buildup in the internal blankets.

In order to resolve these discrepancies, calculations were made using a previously available four group-cross section set comparing DIF3D/REBUS-3 with 6 and 24 triangles per hexagon and the nodal option DIF3D/REBUS-3 for the first half of the first burn cycle of this benchmark study. Table 5 compares the beginning of cycle 1 (BOC1) and middle of cycle 1 (MOC1) k_{eff} as well as the BOC1 peak power densities. Richardson extrapolation of the finite difference results given in Table 5 shows that the eigenvalues calculated using the nodal option are more accurate than either of the finite-difference calculations. This conclusion is supported by more detailed analysis¹³ of a very similar BOC1 configuration which showed that the zero-mesh corrections to the 6 triangles per hexagon eigenvalue are $\sim 0.4\%$ in the plane and $\sim 0.1\%$ in the axial direction. This same study¹³ also showed that the average fluxes in the inner

Table 4. Comparison of Reference Computational Methods
Using 8 Energy Groups Based on ENDF/B-V

	Finite Difference	Nodal Option	Synthesis ^a
k_{eff} (BOC1)	1.00074	0.99685	1.00054
k_{eff} (BOC1) - k_{eff} (MOC1)	0.0048	0.0042	0.0046
Rod Δk_{eff} (MOC1) ^b	0.0054	0.0056	0.0094
k_{eff} (MOC1) - k_{eff} (EOC1)	0.0021	0.0014	0.0022
k_{eff} (BOC2) ^d	1.00162	0.99851	1.00254
k_{eff} (BOC2) - k_{eff} (MOC2)	0.0025	0.0019	0.0025
Rod Δk_{eff} (MOC2) ^c	0.0033	0.0034	0.0050
k_{eff} (MOC2) - k_{eff} (EOC2)	0.0011	0.0005	0.0012
BOC1			
Power Fraction (%)			
Inner Core	11.9	11.8	11.9
Middle Core	23.8	23.4	23.7
Outer Core	55.2	55.6	55.5
Peak/Average Power			
Inner Core	1.40	1.43	1.38
Middle Core	1.60	1.62	1.62
Outer Core	1.60	1.60	1.62
Total Core	1.62	1.63	1.64
Inner Blankets	3.15	3.82	3.16
Radial Blanket	5.87	7.53	5.81
Breeding Ratio	1.518	1.525	1.518
EOC2			
Average Burnup (MWD/mT)			
Total Core	5.46+4 ^e	5.44+4	5.48+4
Inner Blankets	6.45+3	6.56+3	6.45+3
Peak Burnup (Mwd/mT)			
Total Core	9.83+4	9.85+4	9.70+4
Inner Blanket	2.45+4	2.57+4	2.37+4
Power Fraction (%)			
Inner Core	13.8	13.8	13.3
Middle Core	25.4	25.3	25.3
Outer Core	39.8	39.6	40.4
Inner Blankets	15.2	15.4	15.0
Peak/Average Power			
Inner Core	1.37	1.38	1.37
Middle Core	1.49	1.50	1.48
Outer Core	1.69	1.70	1.67
Total Core	1.80	1.81	1.77
Inner Blankets	3.80	3.92	3.68
Radial Blanket	5.17	6.16	4.99
Breeding Ratio	1.428	1.429	1.430

^aUsing BOC1 + EOC1 trial functions for cycle 1 and BOC2 + EOC2 trial functions for cycle 2.

^bRods moved from 172.72 cm to 193.04 cm.

^cRods moved from 182.88 cm to 198.12 cm.

^dFuel shuffled and rods moved from 193.04 cm to 182.88 cm.

^eRead as 5.46×10^4 .

Table 5. Comparison of Finite Difference Mesh Refinement with the Nodal Option*

	6Δ per hex	24Δ per hex	Nodal Option
k_{eff} (BOC1)	1.00468	1.00126	1.00074
k_{eff} (MOC1)	0.99979	0.99706	0.99648
k_{eff} (BOC1) - k_{eff} (MOC1)	0.00489	0.00420	0.00426
BOC1			
Peak Power Density (watts/cc)			
Total Core	529.1	528.9	530.0
Inner Blankets	80.5	91.1	97.7
Radial Blankets	63.7	74.0	83.8
Neutronics CPU Time**, sec. for BOC1	419.	2192.	79.

*Calculations performed with a previously available 4 group cross section set

**On the IBM 3033

blankets are under-predicted in the 6 triangles per hexagon calculation, thus explaining the smaller breeding ratios and inner blanket burnups and larger reactivity swings calculated using the finite-difference option.

Comparison of the peak power densities given in Table 5 provides an explanation for the differences in the peak/average powers in the inner and radial blankets seen in Table 4. It is clear that the nodal results are more accurate than the finite-difference results simply because the nodal scheme samples the flux shape at the peak location at the core-blanket interface, while the mesh-centered finite-difference method must use successively refined spatial meshes in order to force mesh points closer to this peak location. Good agreement is observed when the finite-difference peak values are computed by sampling the surface fluxes which are readily calculated from the available mesh-centered fluxes.

Finally, the CPU times given in Table 5 demonstrate that the improved accuracy of the nodal option is obtained in significantly reduced CPU times relative to the finite-difference option.

PARKED CONTROL RODS

To assess the impact of the control rod movement during the burnup cycles, two additional nodal option DIF3D/REBUS-3 calculations were run with the eight group cross section set. In the first of these the rods C (Fig. 1) were parked for the entire calculation at a height of 187.96 cm., and for the second, at 218.44 cm., the core-upper axial blanket interface (see Fig. 2).

Table 6 compares the two parked-rod calculations with the reference model calculation involving normal rod movement.

It is obvious from these data that the control rod position assumed for the depletion calculation has very little impact on the computational results. If generally applicable, such a conclusion leads to a considerable simplification

TABLE 6. Comparison of Parked Control Rods with Rod Movement for the Nodal Option Neutronics Method Using 8 Groups and ENDF/B-V

	Reference	Rods Parked at 187.96 cm	Rods Parked at 218.44 cm
k_{eff} (BOC1)	0.99685	1.00060	1.00458
k_{eff} (BOC1) - k_{eff} (MOC1)	0.0042	0.0037	0.0032
Rod Δk_{eff} (MOC1) ^a	0.0056	-	-
k_{eff} (MOC1) - k_{eff} (EOC1)	0.0014	0.0015	0.0014
k_{eff} (BOC2) ^b	0.99851	0.99968	1.00366
k_{eff} (BOC2) - k_{eff} (MOC2)	0.0019	0.0018	0.0016
Rod Δk_{eff} (MOC2) ^c	0.0034	-	-
k_{eff} (MOC2) - k_{eff} (EOC2)	0.0005	0.0006	0.0005
BOC1			
Power Fraction (%)			
Inner Core	11.8	12.0	12.1
Middle Core	23.4	24.6	25.7
Outer Core	55.6	54.3	53.1
Peak/Average Power			
Inner Core	1.43	1.44	1.44
Middle Core	1.62	1.60	1.57
Outer Core	1.60	1.64	1.66
Total Core	1.63	1.63	1.66
Inner Blankets	3.82	3.79	3.80
Breeding Ratio	1.525	1.523	1.525
EOC2			
Average Burnup (MWd/mT)			
Total Core	5.44+4 ^d	5.44+4	5.45+4
Inner Blankets	6.56+3	6.57+3	6.59+3
Peak Burnup (MWd/mT)			
Total Core	9.85+4	9.90+4	9.81+4
Inner Blankets	2.57+4	2.60+4	2.52+4
Power Fraction (%)			
Inner Core	13.8	14.0	13.6
Middle Core	25.3	25.1	25.5
Outer Core	39.6	39.7	39.8
Inner Blankets	15.4	15.4	15.4
Peak/Average Power			
Inner Core	1.38	1.37	1.38
Middle Core	1.50	1.51	1.48
Outer Core	1.70	1.68	1.70
Total Core	1.81	1.82	1.80
Inner Blankets	3.92	3.95	3.83
Breeding Ratio	1.429	1.430	1.431

^aRods moved from 172.77 cm to 193.04 cm.

^bFuel shuffled and rods moved from 193.04 cm to 182.88 cm.

^cRods moved from 182.88 cm to 198.12 cm.

^dRead as 5.44×10^4 .

in LMFBR depletion analysis since one need not determine average control rod positions through the burn cycle but may rely upon calculations which park all control at the EOC position, the core-axial blanket interface. This conclusion might be biased by the fact that the greatest insertion of control in the reference calculation is only 18 inches into the 48 inch core. However, heterogeneous core designs generally have small reactivity swings which in large measure determines the control positions. Thus it would seem that reliable depletion calculations can be performed for heterogeneous fast reactor problems by assuming that control rods are positioned at the core-axial blanket interface throughout the burn.

EFFECT OF MULTIGROUP CROSS SECTION ENERGY DETAIL AND DATA BASE

The nodal option DIF3D/REBUS-3 calculation was repeated using the four-group cross section structure (see Table 3) and with the eight-group structure using the ENDF/B-IV data base. Table 7 compares the reactivity swings due to burnup and rod movement and the various integral parameters with the reference nodal calculation.

It is clear that the four- and eight-group results are in excellent agreement for all parameters. Thus it is possible to significantly reduce computing costs for such depletion analysis by a reduction in the number of energy groups provided that sufficient spatial detail is incorporated into the cross section preparation. As noted above, the cross section sets prepared for the benchmark analysis included spatial collapsing over each reactor zone and thus accounted for variations in the spectral dependence of the flux in detail. If such spatial detail had not been incorporated into the cross-section preparation, equivalent agreement would not have been found. For example, a calculation using only one set of core and internal-blanket cross sections gave a BOC1 eight-group inner-blanket peaking factor of 3.80 whereas the equivalent four-group result was 2.88.

The data of Table 7 also show that there is very good agreement between the results obtained with ENDF/B-IV and ENDF/B-V data. It should be noted, however, that both sets of calculations made use of the same ENDF/B-V lumped fission product. The lower k_{eff} values given in Table 7 with ENDF/B-IV data are typical of dilute LMFBR systems and can be attributed in large measure to increases in $\bar{\nu}$ and σ_f of ^{239}Pu in going to ENDF/B-V. Clearly the k_{eff} bias between the datasets would impact upon enrichment determinations and hence equilibrium cycle depletion analysis unless something were done to account for the effect.

THE INFLUENCE OF TRIAL FUNCTIONS ON THE FLUX-SYNTHESIS RESULTS

Synthesis trial functions were generated using the 2D model shown in Fig. 1, with DIF3D/REBUS-3 performing the two-cycle burnup at a power level of 22.042 MWth, which corresponds to the core average linear power.¹ Functions were saved at BOC1, EOC1, BOC2 and EOC2 for primary rods C inserted and withdrawn. Bucklings of 0.0004 cm^{-1} for rodded calculations and 0.0005 cm^{-1} for unrodded calculations were applied to achieve roughly the critical eigenvalue. Rodded and unrodded BOC1 axial-blanket trial functions were obtained from fixed source calculations of the type described in Reference 5.

Table 8 compares the results of three burnup calculations using SYN3D (with different choices of trial functions) with the calculation which used the finite-difference code DIF3D. The reference SYN3D/REBUS-3 trial-function set

TABLE 7. Comparison of Energy Group Structures and Data Bases
for the Nodal Option Neutronics Method

	ENDFB-V		ENDFB-IV
	8 Groups	4 Groups	8 Groups
k_{eff} (BOC1)	0.99685	0.99841	0.98996
k_{eff} (BOC1) - k_{eff} (MOC1)	0.0042	0.0048	0.0042
Rod Δk_{eff} (MOC1) ^a	0.0056	0.0058	0.0057
k_{eff} (MOC1) - k_{eff} (EOC1)	0.0014	0.0018	0.0013
k_{eff} (BOC2) ^b	0.99851	0.99959	0.99164
k_{eff} (BOC2) - k_{eff} (MOC2)	0.0019	0.0023	0.0018
Rod Δk_{eff} (MOC2) ^c	0.0034	0.0036	0.0035
k_{eff} (MOC2) - k_{eff} (EOC2)	0.0005	0.0009	0.0004
BOC1			
Power Fraction (%)			
Inner Core	11.8	11.8	11.6
Middle Core	23.4	23.4	23.2
Outer Core	55.6	55.7	55.9
Peak/Average Power			
Inner Core	1.43	1.43	1.43
Middle Core	1.62	1.62	1.62
Outer Core	1.60	1.60	1.60
Total Core	1.63	1.63	1.64
Inner Blankets	3.82	3.81	3.86
Breeding Ratio	1.525	1.518	1.528
EOC2			
Average Burnup (MWd/mT)			
Total Core	5.44+4 ^d	5.45+4	5.44+4
Inner Blankets	6.56+3	6.48+3	6.56+3
Peak Burnup (MWd/mT)			
Total Core	9.85+4	9.81+4	9.85+4
Inner Blankets	2.57+4	2.53+4	2.57+4
Power Fraction (%)			
Inner Core	13.8	13.7	13.8
Middle Core	25.3	25.2	25.4
Outer Core	39.6	39.9	39.6
Inner Blankets	15.4	15.2	15.4
Peak/Average Power			
Inner Core	1.38	1.38	1.38
Middle Core	1.50	1.50	1.50
Outer Core	1.70	1.69	1.70
Total Core	1.81	1.80	1.81
Inner Blankets	3.92	3.90	3.92
Breeding Ratio	1.429	1.425	1.432

^aRods moved from 172.72 cm to 193.04 cm.

^bFuel shuffled and rods moved from 193.04 cm to 182.88 cm.

^cRods moved from 182.88 cm to 198.12 cm.

^dRead as 5.44×10^4 .

TABLE 8. Comparison of Trial Functions for the Spatial Synthesis
Neutronics Method Using 8 Energy Groups and ENDF/B-V

Trial Functions Used	BOC1 + EOC1 for Cycle 1 BOC2 + EOC2 for Cycle 2	BOC1 + EOC2	BOC1	Finite Difference
k_{eff} (BOC1)	1.00054	1.00053	0.99989	1.00074
k_{eff} (BOC1) - k_{eff} (MOC1)	0.0046	0.0050	0.0117	0.0048
Rod Δk_{eff} (MOC1) ^a	0.0094	0.0092	0.0095	0.0054
k_{eff} (MOC1) - k_{eff} (EOC1)	0.0022	0.0026	0.0094	0.0021
k_{eff} (BOC2) ^b	1.00254	1.00386	0.99477	1.00162
k_{eff} (BOC2) - k_{eff} (MOC2)	0.0025	0.0030	0.0103	0.0025
Rod Δk_{eff} (MOC2) ^c	0.0050	0.0049	0.0054	0.0033
k_{eff} (MOC2) - k_{eff} (EOC2)	0.0012	0.0019	0.0088	0.0011
BOC1				
Power Fraction (%)				
Inner Core	11.9	12.0	11.4	11.9
Middle Core	23.7	23.7	23.0	23.8
Outer Core	55.5	55.4	56.7	55.2
Peak/Average Power				
Inner Core	1.38	1.38	1.44	1.40
Middle Core	1.62	1.62	1.66	1.60
Outer Core	1.62	1.62	1.58	1.60
Total Core	1.64	1.64	1.64	1.62
Inner Blankets	3.16	3.16	3.18	3.15
Breeding Ratio	1.518	1.518	1.517	1.518
EOC1				
Average Burnup (MWd/MT)				
Total Core	2.79+4 ^d	2.79+4	2.80+4	2.78+4
Inner Blankets	2.63+3	2.61+3	2.34+3	2.62+3
Peak Burnup (MWd/MT)				
Total Core	4.97+4	4.85+4	4.59+4	5.00+4
Inner Blankets	9.28+3	8.93+3	7.91+3	9.84+3
Power Fraction (%)				
Inner Core	13.2	13.1	11.0	13.7
Middle Core	26.4	25.8	23.6	26.4
Outer Core	43.2	43.9	49.3	42.7
Inner Blankets	12.0	11.9	10.3	12.1
Peak/Average Power				
Inner Core	1.36	1.35	1.37	1.36
Middle Core	1.49	1.48	1.56	1.51
Outer Core	1.68	1.65	1.63	1.69
Total Core	1.78	1.74	1.64	1.80
Inner Blankets	3.53	3.42	3.38	3.62
Breeding Ratio	1.469	1.471	1.459	1.466
EOC2				
Average Burnup (MWd/MT)				
Total Core	5.48+4	5.48+4	5.54+4	5.46+4
Inner Blankets	6.45+3	6.46+3	5.56+3	6.45+3
Peak Burnup (MWd/MT)				
Total Core	9.70+4	9.64+4	9.09+4	9.83+4
Inner Blankets	2.37+4	2.37+4	1.85+4	2.45+4
Power Fraction (%)				
Inner Core	13.3	13.3	10.6	13.8
Middle Core	25.3	25.2	22.7	25.4
Outer Core	40.4	40.6	47.5	39.8
Inner Blankets	15.0	15.0	12.4	15.2
Peak/Average Power				
Inner Core	1.37	1.37	1.38	1.37
Middle Core	1.48	1.47	1.56	1.49
Outer Core	1.67	1.67	1.63	1.69
Total Core	1.77	1.76	1.64	1.80
Inner Blankets	3.68	3.67	3.33	3.80
Breeding Ratio	1.430	1.430	1.418	1.428

^aRods moved from 172.72 cm to 193.04 cm.

^bFuel shuffled and rods moved from 193.04 cm to 182.88 cm.

^cRods moved from 182.88 cm to 198.12 cm.

^dRead as 2.79×10^4 .

consisted of six fluxes: rodged and unrodged BOC axial blanket functions plus rodged and unrodged, BOC and EOC core eigenvalue functions within each burn cycle. The core functions in this first set were changed between cycles. The second set also contained six functions, but the core functions were generated at the beginning of the first cycle (BOC1) and the end of the second (EOC2). This set was not changed between cycles. The third set contained four trial functions; rodged and unrodged BOC1 core functions and rodged and unrodged BOC axial blanket functions were used throughout both cycles.

In all three SYN3D/REBUS-3 calculations trial-function zoning was used to reduce running time. Rodged blanket functions were applied only in the top half of the model, and unrodged blanket functions were used only in the lower half. The switch was made at 142.24 cm. Rodged core functions were used in the core, upper blanket and plenum. Unrodged core functions were used in the core, lower blanket and lower shield.

For the most part Table 8 shows the SYN3D/REBUS-3 results to be in good agreement with the DIF3D/REBUS-3 calculation. There is a substantial difference in rod worth for both cycles, but power fractions and core peak-to-average powers tend to agree to within 2%. Peak burnup figures agree well for the core. There are errors as high as 3-4% in some of the peak burnups and peaking factors in the inner blankets.

The differences between SYN3D/REBUS-3 and DIF3D/REBUS-3 in general are somewhat larger for the other two trial-functions sets. Errors are particularly obvious in inner-blanket results and power fractions for the set that only used BOC fluxes (the third set). On the other hand, the reference set, which switches functions between cycles, shows only a small improvement in accuracy over the second set.

REACTIVITY EFFECT OF SODIUM VOIDING

The VARI3D Code¹⁴ was used to analyze the components of the reactivity effect due to voiding sodium from the three core driver zones and the corresponding upper axial blankets. Exact perturbation theory was used in that the real flux was obtained before the sodium voiding and the adjoint solution corresponded to the voided configuration. The atom densities used were generated by the reference 8 group finite difference calculations.

The isotopic concentrations were obtained from the DIF3D/REBUS-3 calculation at EOC2 and will reflect the largest buildup of fission products.

Table 9 compares the components of the Na void reactivity effect at the EOC2 for the 20, 8, and 4 group cross section sets and shows the effect of the ENDF/B-IV vs. ENDF/B-V data bases. Sodium-voided cross sections were used in the appropriate reactor zones for these calculations. For comparison, the BOC1 Na void reactivity effect is also shown in Table 9. The impact of using sodium voided cross sections is also displayed. Using only non-voided cross sections results in about a 28% reduction in the total sodium reactivity.

The data of Table 9 show that fairly accurate sodium void reactivity worths can be calculated using very few energy groups. The difference between the four-group and twenty-group results is less than 40% out of a void worth of ~3.9\$ at EOC2. The large change in void worth over the cycle ~1.5\$, is a consequence of the long fuel residence times, 3 years, for the benchmark model. A very slight decrease in void worth, ~1%, is noted in going from ENDF/B-IV to ENDF/B-V data.

Table 9. Components of the Sodium Void^a $\Delta k/kk^*$ ^b

	Spectral ^c	Total Non-Leakage	Leakage	Total
EOC2				
20 groups ^e	2.187-2 ^d	1.896-2	-5.404-3	1.355-2
8 groups ^e	1.810-2	2.033-2	-7.465-3	1.287-2
4 groups ^e	1.591-2	1.998-2	-7.733-3	1.224-2
8 groups ^f	1.806-2	1.900-2	-7.112-3	1.188-2
8 groups ^g	1.797-2	2.114-2	-8.096-3	1.305-2
BOC1				
4 groups ^e	1.370-2	1.807-2	-1.092-2	7.082-3

^aNa voided from drivers and upper axial blankets

^b $\Delta k = k^* - k$, $k = k_{eff}$ from real non-voided calculation, k^* from adjoint voided calculation

^cGroup-to-group scattering component

^dread as 2.187×10^{-2}

^eUsing the ENDF/B-V data base

^fUsing only non-voided sodium cross sections

^gUsing the ENDF/B-IV data base

SUMMARY AND CONCLUSIONS

In summarizing the results of any computational benchmark study it is necessary to qualify the conclusion with the caveat that they are applicable to the problem studied and it may not be possible to generalize. Nevertheless, the generic nature of the particular problem studied in this paper suggests that many of the results are quite generally applicable.

It was found that each of the three neutronics methodologies considered was capable of providing accurate fast reactor depletion results. Because of their relatively low computational costs, the nodal and synthesis methods make possible routine three-dimensional fast reactor depletion analysis. The nodal method would appear to be the more desirable of the two owing to its accuracy and ease of use.

The benchmark problem results showed that it is possible to obtain accurate depletion results using only four energy groups in the analysis. This conclusion, when coupled with the efficient neutronics methods discussed above, increases the feasibility of routine fast reactor three-dimensional analysis. The accuracy of the few-group results was apparent even for spectrum-sensitive

parameters such as the sodium void effect. To ensure such accuracy considerable care is required in the preparation of such four-group constants. For example, spatial and spectral detail must be retained during the cross section generation.

The benchmark problem results also showed that control-rod position during the depletion cycle had very little impact on problem results. As a consequence of this conclusion, it is possible to perform the depletion analysis with control rods parked at the core-upper axial blanket interface throughout the burn cycle rather than finding an average control-rod position.

It was further found that the change in general purpose database from ENDF/B-IV to ENDF/B-V had very little impact on such integral parameters as reactivity swing, breeding ratio, power peaking, or sodium void worth. There was however a bias in eigenvalue ($\sim 7\%$) between the two data bases.

APPENDIX A

PERFORMANCE COMPARISONS OF NEUTRONICS METHODS

Timing and cost comparisons between methods are always ambiguous, but nonetheless are always of interest. We quote three measures of performance for the 8-group reference calculations: CPU time for the BOC1 neutronics calculation, disk/core data transfers (in buffer loads) for the BOC1 neutronics calculation plus the first burn step, and dollar cost for the BOC1 neutronics calculation plus the first burn step.

Table A1 shows quantitative comparisons of performance. The results are given only relative to the performance of the finite-difference code, not in absolute terms. All SYN3D numbers contain a prorated contribution from the work required to generate the trial functions.

The nodal option in DIF3D currently runs only with all data for all groups core-contained; the finite-difference calculations were run in a mode which kept approximately half of the data for one group core-contained. As run, the nodal option required twice the core storage that the finite-difference option used. 90% of the CPU time required for a burnup calculation is spent in the neutronics module when the neutronics is finite-difference. For the nodal neutronics that figure is 73%. SYN3D/REBUS-3 is able to run efficiently at smaller core-storage allocations than were used, but no effort was made to optimize the runs. The nodal calculation will eventually offer data-management modes that will let it run with smaller core storage.

Table A1. Relative Performance Comparison of Neutronics Methods

Calculation	CPU	Disk/Core I/O	Core Storage	Cost
Finite-Difference	1.00	1.00	1.00	1.00
Nodal	.26	.09	2.23	.35
Flux Synthesis	.54	.21	.88	.39

REFERENCES

1. R. M. Wu, R. P. Omberg, R. Protsik, C. L. Cowan, J. W. Lewellen, ed., "The Large Core Code Evaluation Working Group Benchmark Analysis of a Heterogeneous Fast Reactor," DOE/TIC-2005709, January 1982.
2. D. R. Ferguson and K. L. Derstine, "Optimized Iteration Strategies and Data Management Considerations for Fast Reactor Finite Difference Diffusion Theory Codes," Nucl. Sci. Eng. 64, 593 (1977).
3. R. D. Lawrence, "A Nodal Interface Current Method for Multigroup Diffusion Calculations in Hexagonal Geometry," Trans. Am. Nucl. Soc. 39, 461 (1981).
4. C. H. Adams, "SYN3D: A Single Channel, Spatial Flux Synthesis Code for Diffusion Theory Calculations," ANL-76-21, July 1976.
5. C. H. Adams and W. M. Stacey, Jr., "Flux Synthesis Calculations for Fast Reactors," Nucl. Sci. Eng. 54, 201 (1974).
6. R. P. Hosteny, "The ARC System Fuel Cycle Analysis Capability, REBUS-2," ANL-7721, October 1978.
7. H. Henryson II, B. J. Toppel, and C. G. Stenberg, "MC²-2: A Code to Calculate Fast Neutron Spectra and Multigroup Cross Sections," ANL-8144, June 1976.
8. B. J. Toppel, "The New Multigroup Cross Section Code MC²-2," Proc. Conf. on New Applications in Reactor Mathematics and Applications, CONF-710302 (1971).
9. W. M. Stacey, Jr., B. J. Toppel, H. Henryson II, B. A. Zolotar, R. N. Hwang, and C. G. Stenberg, "A New Space-Dependent Fast-Neutron Multigroup Cross-Section Preparation Capability," Trans. Am. Nucl. Soc. 15, 292 (1972).
10. D. Garber, "ENDF-201, ENDF/B Summary Documentation," BNL 17541 (ENDF-201) 2nd Edition, Oct. 1975.
11. R. Kinsey, "ENDF-201, ENDF/B Summary Documentation," BNL-NCS-17541 (ENDF-201) 3rd Edition, July 1979.
12. J. R. Liaw and H. Henryson II, "Lumped Fission Product Cross Sections from ENDF/B-V for Fast Reactor Analysis," Trans. Am. Nucl. Soc., 38, 657 (1981).
13. R. D. Lawrence, private communication.
14. To be published.

DETERMINATION OF THE CORE POWER DISTRIBUTION IN THE CLINCH RIVER BREEDER REACTOR

J. A. Lake, S. K. Varner, and R. V. Rittenberger
Westinghouse Advanced Reactors Division
Madison, PA 15663 USA

ABSTRACT

The fuel and blanket assembly layout in the CRBRP heterogeneous core is designed to produce a radially flat power distribution which maximizes the total core power with respect to peak linear power limits. The core power distribution is the basis for the demonstration of power-to-melt margins, the determination of flow orificing requirements, and the prediction of temperature and irradiation conditions which are the driving functions behind the fuel and blanket lifetime evaluations. The few-group, diffusion-theory based design methodology and the ENDF/B cross section data are challenged by the unique characteristics and sensitivities of the heterogeneous CRBRP core arrangement. Consequently, an extensive critical experiments program has been performed at ZPPR to assess design bias factors and uncertainties in key integral physics parameters. These parameters include the reactor critical eigenvalue, control rod worth characteristics, isotopic fission and capture rate distributions and gamma heating as they are influenced by fuel/blanket interfaces, core burnup, blanket plutonium accumulation, etc., and core safety characteristics like the sodium void worth distribution. The results of the analyses of these experiments reflect deficiencies in cross section data, cross section processing and/or reactor calculational methodology approximations. These are incorporated in the CRBRP nuclear design uncertainties evaluation along with methods/modeling uncertainties and various engineering and performance tolerances in order to determine the 3σ uncertainty envelope. The resulting maximum peak linear power is 50.2 kW/m (15.3 kW/ft) for fresh-core fuel at beginning of cycle one, and 66.3 kW/m (20.2 kW/ft) for the inner blanket at the end of cycle four (548 effective full power days).

CORE CONFIGURATION AND FUEL MANAGEMENT

The CRBRP heterogeneous core configuration is shown in Figure 1. The first core consists of 156 fuel assemblies containing a single fuel enrichment zone, 82 inner blanket assemblies dispersed heterogeneously throughout the central regions of the core, and 126 radial blanket assemblies. The upper and lower axial blankets are each 36 cm (14 inches) thick. Nine fully enriched (92% B-10) primary control assemblies are located in the 6-row 7 corner and 3-row 4 positions. Six fully enriched secondary control assemblies are located in the 6-row 7 flat positions. The 6-R7C control rods, operated as a bank, serve to control the reactor throughout each cycle. The remaining primary and secondary control rods are normally fully withdrawn for all full-power operations.

The CRBRP fuel management is a batch refueling scheme with a 2-year life-time for fuel and inner blanket assemblies. Annual refueling is planned for all operating cycles. The first cycle is based on operation at 975 MWt for one calendar year at 35% capacity (128 equivalent full power days). The capacity factor is increased to 55% (200 efpd) during the second cycle and to 75% (275 efpd) during the third and subsequent cycles. At the end of the first cycle, 3 inner blanket assemblies in alternating row 6 corner positions are replaced with fresh fuel assemblies for the second burnup cycle. The 3 refueled assemblies are the same enrichment as the initial load. At the end of the second cycle, all of the fuel and inner blanket assemblies are discharged and replaced with a fresh core. Under equilibrium conditions, the fresh fuel batch is burned for one cycle (275 efpd) at which time 6 inner blanket assemblies are replaced with 6 fresh fuel assemblies to supply the additional excess reactivity for the next 275 efpd burnup cycle. The entire fuel and inner blanket batch is replaced at the end of the second cycle and the scheme is repeated in two year intervals. The radial blanket assemblies reside in place for four or five years, respectively, in the first and second radial blanket rows.

FUEL ENRICHMENT AND BREEDING RATIO

The CRBRP employs the plutonium-uranium fuel cycle with mixed oxide fuel and depleted UO₂ blankets. For purposes of design analysis it has been assumed that low Pu²⁴⁰ (~12%) fuel,* similar to that employed in the initial core of FFTF⁽¹⁾, is used throughout the 5-year Demonstration Period. The fuel enrichment specification is based on the requirement that the fissile fuel loadings provide sufficient excess reactivity for lifetime considerations (328 efpd burnup interval for the first core in cycles 1 plus 2 and 548 efpd for equilibrium cores) and uncertainties. The start of life excess reactivity requirement is derived from the net burnup reactivity deficit (two-cycle burnup reactivity depletion minus the mid-term refueling reactivity addition) and includes the statistical sum of uncertainties in criticality determination, burnup reactivity (primarily fission product worth and core conversion ratio uncertainties), cold-to-hot temperature defect, fissile loading and core geometry tolerances, fuel and steel impurity concentrations, modeling approximations and fuel management uncertainties.⁽²⁾ The fuel loading, in turn, is determined by searching the plutonium enrichment** to a value which produces a (biased) k_{eff} of 1.0 plus the required excess reactivity at the start of life with the control rod banks withdrawn. The resulting plutonium enrichment for the first core is 33.6 w/o, and the core fissile (Pu²³⁹ + Pu²⁴¹) loading is 1529 kg. The characteristics of the batch fuel management scheme result in equilibrium fuel enrichments which are only slightly higher than the first-core value.

*0.056 w/o Pu²³⁸, 86.47 w/o Pu²³⁹, 11.75 w/o Pu²⁴⁰, 1.33 w/o Pu²⁴¹, 0.20 w/o Pu²⁴² and ~0.2 w/o decay products.

**Plutonium enrichment is defined as PuO₂ weight fraction in HMO₂.

The breeding ratio for CRBRP is defined as the ratio of the production of fissile plutonium (production of Pu^{239} and Pu^{241} from neutron capture reactions in fertile U^{238} and Pu^{240} , respectively) to the destruction of fissile fuel (neutron absorption reactions in Pu^{239} and Pu^{241}). The quantity of U^{235} up to and including that contained in depleted uranium tails (0.2 atom percent) is not included in this definition as an economically recoverable fissile "fuel". The CRBRP breeding ratio, with low-240 fuel is about 1.27 at the beginning of life (.3 of which is core conversion in the active fuel region), with an equilibrium-average value of 1.22. Recycle LWR plutonium, containing ~25 w/o Pu^{240} , results in a significantly higher breeding gain (~1.27 in equilibrium cycles).

CALCULATIONAL METHODS AND CROSS SECTION DATA

Figure 2 summarizes the CRBRP power distribution calculational sequence. ENDF/B-4 neutron cross section data are processed by LANL to form the 50-energy group LIB-4(3) set of infinitely dilute microscopic cross sections, scattering matrices and temperature dependent self-shielding factors in the Bondarenko format. The cross sections are then resonance self-shielded and temperature corrections are applied in a 2-region heterogeneous pin cell model using Sauer's approximation for a cylindrical fuel or blanket pin in a hexagonal lattice. The neutron cross sections are coupled with ENDF/B-5 gamma yield coefficients. A one dimensional diffusion calculation of the reactor flux spectrum is used to condense the microscopic cross sections to a 9 neutron/4 gamma energy group set. Self-shielding, gamma coupling and group collapse calculations are performed locally with the SPHINX(4) code. The resulting CRBRP region, composition and temperature dependent Master Cross Section Library is used in two-dimensional diffusion calculations with the 2DB(5) code in hexagonal-planar and RZ geometry to determine the radial and axial core power and burnup distributions throughout the first five years of operation. The RZ calculations, employing an effective-volume cylindrical model, result in axial blanket power contributions, normalized axial power shapes for the fuel and blankets, and region- and group-dependent axial leakage (buckling) factors for use in the hexagonal calculations. The hexagonal calculations model the assembly-by-assembly fuel, blanket and control management and burnup distribution. The core power distribution at the beginning and end of each cycle is determined from a fundamental-mode flux calculation with assembly-wise burnup and just-critical row 7 control rod bank insertion using coupled neutron-gamma diffusion techniques. The fuel and inner blanket intraassembly power distributions are calculated by least-squares fitting the 24-mesh-per assembly power density with an 11-term two dimensional polynomial in the POWPIN code. The pin-by-pin plutonium buildup distribution in the radial blanket is determined based on least-squares fitted fluxes, cross sections and gamma heating with the PUMA(6) code. The product of region-dependent normalized radial and axial power shape factors, the region power fraction and the reactor thermal power output determines the fuel and blanket power distribution throughout the core. This power distribution synthesis is discussed in the ensuing paragraphs..

POWER DISTRIBUTION

The synthesis of the three-dimensional power distribution throughout the fuel and blankets is accomplished by the application of normalized radial and axial power peaking factors to the average fuel or blanket power generation as follows:

$$\text{Linear Power (kW/ft)} = \frac{975,000 \cdot \text{Power Fraction}}{\text{No. of Rods} \cdot \text{Length of Rod}} \cdot F_R^N \cdot F_Z^N \quad (1)$$

where 975 MWth is the nominal reactor power level,

Power fraction is the fuel or inner blanket power fraction in the central core region shown in Table 1,

No. of Rods is the number of fuel (217) or blanket (61) rods per assembly times the number of fuel or inner blanket assemblies in the core,

Length of Rod is 92 cm (3.02 feet hot) over the central region of the core in all cases,

F_R^N is the normalized nuclear radial power peaking factor for the fuel or inner blankets,

and F_Z^N is the normalized nuclear axial power peaking factor.

The region power fractions in Table 1 include the distribution of both the isotopic fission rates and gamma heating in the system, determined with the ENDF/B-4 9-neutron/4-gamma group coupled cross section set and 2DB neutron-gamma heating calculations. This capability makes possible an explicit representation of the gamma diffusion from the source zone in the fuel assemblies into the inner and radial blanket zones. The redistribution of power from the fuel to the blanket regions with burnup is evident in the Table 1 values. The power fractions in Table 1 have been corrected using biases from isotopic fission rates and gamma heating measured in the Engineering Mockup Critical (ZPPR-11), and the resulting region powers have been renormalized to maintain the total reactor power. The total power in the fueled zones of the core in Table 1 is slightly less than 975 MWt to account for heating in the radial and axial shielding surrounding the core, and in the primary and secondary control rods.

The normalized radial power factor, F_R^N , defines the local rod power density relative to the average power density in the core region. The radial power distribution reflects the effects of core assembly burnup/buildup, just critical primary control rod bank insertion, and core/blanket interface leakage effects at each time in the reactor operating cycle. The

intraassembly power shape is interpolated from 24-mesh-per-assembly two-dimensional diffusion solutions.

The normalized axial power factor, F_z^N , defines the local axial power density distribution relative to the average power density in the axial channel. Figure 3 shows a typical axial power distribution in the fuel and blankets. The axial peak to average power in the fuel typically decreases with burnup due to preferential fissile depletion at the flux peak near the core midplane and due to flux flattening effects of this same depletion coupled with axial blanket plutonium buildup. In the inner and radial blankets, the increase in the axial peak to average power with burnup results from the peak in the axial distribution of bred plutonium.

The synthesis of the power distribution from two-dimensional radial and axial shape factors according to equation (1) has been compared with an explicit three-dimensional calculation to infer the geometric modeling error in the 2D synthesis technique at beginning of life. This comparison demonstrated good agreement in the resulting power distributions throughout the core, with exception of regions strongly influenced by the partially inserted control rod bank where the agreement was somewhat poorer. The peak power density agreement between the 2D and 3D methods was generally $\pm 2\%$.

POWER UNCERTAINTIES EVALUATION

Experimental verification of the calculational methods and data used to predict the power distribution in CRBRP is provided by the measurements and analysis of the Engineering Mockup Critical experiments (ZPPR-11). CRBRP calculational methods are verified, and design bias factors and uncertainties are derived by direct comparison of calculated and measured ZPPR parameters.⁽⁷⁾ These parameters include isotopic fission and capture rate and gamma heating distributions throughout the fuel and blankets. Table 2 summarizes calculation to experiment ratios and uncertainties for several key isotopic fission and capture rates in ZPPR-11B and C simulating clean beginning and end of life core conditions, respectively. The ZPPR-derived bias factors and uncertainties are applied to the calculation of these same parameters in the CRBRP which have been predicted using the identical calculational tools and data. The regionwise CRBRP isotopic fission rate biases have been nominally included in the core region power fractions in Table 1.

In the fuel, the power uncertainty includes experimental uncertainties (fission and capture rate and gamma heating uncertainties from the Engineering Mockup Critical), methods uncertainties (2D:3D modeling uncertainties, radial and axial power shape determination uncertainties and non-pellet-heating bias), and engineering tolerances (reactor power normalization, fissile content tolerance, reactor criticality uncertainty, and control rod out-of-bank tolerance). Table 3 shows the uncertainty components in the peak kW/ft in a typical, high-power CRBRP fuel assembly at the beginning of life.

The experimental component of the fuel power uncertainty consists of Pu^{239} and U^{238} fission rate and gamma heating uncertainties which are developed from analysis of foil and TLD measurements performed in the Engineering Mockup Critical (ZPPR-11). The statistical deviations in the normalized pointwise calculation-to-experimental ratios are combined with ANL's estimate of the systematic uncertainty in each reaction rate to derive the $\pm 5.5\%$ reaction-rate-weighted 3σ statistical uncertainty in the power. Axial peak-to-average fission rates in ZPPR critical experiments are over-predicted with the standard few-group diffusion theory methods approximately 1% near the peak at the core midplane. The non-pellet heating bias corrects the pellet power density for the fraction of power generated by gamma heating in the stainless steel cladding and duct. The uncertainty in the predicted hot critical state of the reactor (from the uncertainty in k_{eff} bias, cold-to-hot temperature defect, core fissile loading and impurities content tolerance, and burnup reactivity swing uncertainties) results in an uncertainty in the depth of row 7 control rod bank insertion. This, in turn, imposes a space dependent uncertainty on the power distribution. The control rod out-of-bank tolerance of ± 1.5 inches, arising from a combination of core axial alignment and manufacturing tolerances, differential thermal expansion uncertainties, control rod position indicator resolution, and control rod banking band width requirements imposed on the operator, introduces a space-dependent asymmetric power uncertainty throughout the core.

The blanket uncertainties include experimental uncertainties (fission rate and gamma heating uncertainties as well as the uncertainty in the plutonium buildup from neutron capture in U^{238}), methods uncertainties (2D:3D modeling uncertainties, radial and axial power shape uncertainties, and a non-pellet-heating bias to account for the fraction of assembly heat which is deposited in the steel cladding and duct), and engineering tolerances (reactor power normalization, heavy metal and U^{235} tolerances, reactor criticality uncertainty, and control rod-out-of-bank tolerance). Uncertainty components in the peak kW/ft in a typical end-of-life high power blanket assembly are also shown in Table 3.

The experimental component of the beginning of life blanket power uncertainty is dominated by U^{238} fission and gamma heating uncertainties, both of which are nominally overpredicted relative to the principal fissioning isotope, Pu^{239} , in the ZPPR experiments. This overprediction is biased out of the region power fractions in Table 1, and the residual statistical deviations in the normalized calculation-to-experiment ratios are used to derive the reaction-rate-weighted 3σ statistical uncertainty of ± 11 -13% in the fresh inner and radial blankets. The end-of-life blanket power includes a substantial contribution from fission in bred Pu^{239} . An additional bias is included in the power fractions in Table 1 to correct for the nominal 4-7% overprediction of U^{238} capture (Pu^{239} production) in the blankets, and the end of life statistical uncertainty of ± 7 -11% includes contributions from the deviations in C/E ratios for U^{238} capture and Pu^{239} fission as well as U^{238} fission and gamma heating (weighted by their relative contributions to the total power).

The uncertainty in normalized radial power distribution within each inner blanket assembly is $\pm 5\%$ in fresh assemblies with complex spectral and gamma heating gradients, and $\pm 1\%$ in burned assemblies where the assembly-power distribution is relatively flat and dominated by Pu239 fission. Inner blankets are burned with the assembly-average flux which introduces a small ($\pm 1\%$) uncertainty in the end-of-life power associated with the neglect of the radial distribution of Pu239 buildup in the assembly. Radial blanket assembly powers, on the other hand, are predicted with the PUMA code which uses fitted flux, cross section and gamma heating values to calculate the power and burnup in each blanket rod so that the plutonium and burnup distributions are included explicitly. The uncertainty in coarse-mesh PUMA-generated radial blanket power is $\pm 5\%$ at beginning of life and $\pm 3\%$ at end of life (where substantial radial power gradients still exist at the periphery of the core). The fractional heating in the blankets which is gamma heating in the stainless steel cladding and duct does not contribute to the blanket pellet power-to-melt limit, and is therefore biased out of the power densities. The total assembly heat load (coolant outlet temperature), however, includes this steel heating contribution. The reactor power normalization from the plant-side heat balance has an uncertainty of $\pm 1\%$. The blanket pellet dimensional and theoretical density tolerances produce a $\pm 2\%$ (3σ) uncertainty in the power. The specification of U²³⁵ content in the depleted uranium pellet material leads to a $\pm 1\%$ uncertainty in the beginning of life power ($\pm 2\%$ in RB2) and a negligible uncertainty at the end of life. The uncertainty in the prediction of the hot-critical state of the core produces an uncertainty in the depth of control rod insertion which is reflected in a space-dependent power uncertainty throughout the core. The control rod out-of-bank tolerance of ± 1.5 inches introduces an asymmetric, space-dependent power perturbation (uncertainty).

PEAK LINEAR POWER ENVELOPE

Application of radial and axial power peaking factors to core-average power conditions according to equation (1) results in the peak (nominal) linear power distribution in the fuel (at beginning of cycle one) and blankets (at end of equilibrium cycle four) shown in Figure 4. Figure 4 also shows the composite, 3σ power uncertainty for each fuel and blanket assembly. Finally, the peak linear power envelope (3σ plus 15% over full power) for each assembly, which is used for lifetime-limiting analyses like the power-to-melt evaluation, is also shown in Figure 4. The CRBRP peak linear power envelope is 50.2 kW/m (15.3 kW/ft) in the fuel and 66.3 kW/m (20.2 kW/ft) in the blankets.

BURNUP AND FLUENCE

The core-average and peak (nominal) fuel pellet discharge burnups, corresponding to the power conditions discussed above, at end of the 1+2 (core one 328 efpd) cycles are 49.0 and 72.7 MWd/kg, respectively. Equilibrium cycle 3+4 (548 efpd) core average and peak fuel burnups are 75.7 and 109.3 MWd/kg, respectively. Similarly, the average and peak discharge burnups in the inner blanket at the end of these same cycles are 12.4 and 21.7 MWd/kg, respectively.

The first-core peak fast ($E > 0.1$ MeV) fluences in the fuel and inner blankets are 9.1×10^{22} neutrons/cm² and 8.4×10^{22} neutrons/cm², respectively. The corresponding equilibrium core values are 1.4×10^{23} neutrons/cm² for the fuel and 1.3×10^{23} for the inner blanket.

ACKNOWLEDGEMENT

This work was supported by the U.S. Dept. of Energy under contract No. EY-76-C-15-2395. The contributions of the ZPPR staff at Argonne National Laboratory (West) are greatly appreciated.

REFERENCES

1. S. Ramchandran, G. J. Calamai, F. J. Baloh, and S. A. Schellin, "Fuel Pin Enrichment Specifications for FTR Cores 1 and 2," WARD-2171-26, January, 1973.
2. J. A. Lake, "Fuel Enrichment Requirements for the Clinch River Breeder Reactor," in Proceedings of the Conference 1980 Advances in Reactor Physics and Shielding, pp 480-493, American Nuclear Society, LaGrange Park, IL, 1980.
3. R. B. Kidman and R. E. MacFarlane, "LIB-IV, A Library of Group Constants for Nuclear Reactor Calculations," LA-6260-MS, February, 1976.
4. W. J. Davis, M. B. Yarbrough and A. B. Bortz, "SPHINX: A One Dimensional Diffusion and Transport Nuclear Cross Section Processing Code," WARD-XS-3045-17, August, 1977.
5. W. W. Little, Jr. and R. W. Hardie, "2DB* Users Manual," BNWL-831, July, 1968.
6. J. A. Lake, "PUMA - A Technique for LMFBR Blanket Fuel Management Analysis," Trans. Amer. Nucl. Soc. 30, pp 700-701 (1978).
7. R. A. Doncals, J. A. Lake and N. C. Paik, "Use of Integral Data in the Development of Design Methods for Fast Reactors," in Advances in Reactor Physics, pp 191-206, June, 1978.

*The Westinghouse modifications to the 2DB code are proprietary and identified as "except" items under DOE contracts.

TABLE 1
CRBRP REGION POWER FRACTIONS

TIME-IN-LIFE	FUEL	LOWER AXIAL BLANKET	UPPER AXIAL BLANKET	INNER BLANKET CENTER 36"	INNER BLANKET LOWER EXTENSION	INNER BLANKET UPPER EXTENSION	RADIAL BLANKET CENTER 36"	RADIAL BLANKET LOWER EXTENSION	RADIAL BLANKET UPPER EXTENSION
BOC1	.8549	.0072	.0049	.0697	.0051	.0035	.0384	.0035	.0026
EOC1	.8155	.0087	.0057	.0944	.0067	.0042	.0466	.0038	.0030
BOC2	.8184	.0093	.0057	.0926	.0068	.0040	.0466	.0039	.0028
EOC2	.7640	.0115	.0072	.1283	.0090	.0055	.0576	.0044	.0035
BOC3	.8288	.0070	.0048	.0666	.0051	.0034	.0638	.0048	.0035
EOC3	.7549	.0103	.0066	.1153	.0081	.0050	.0772	.0056	.0044
BOC4	.7644	.0112	.0065	.1098	.0081	.0046	.0730	.0056	.0038
EOC4	.7031	.0143	.0082	.1521	.0111	.0062	.0840	.0063	.0046

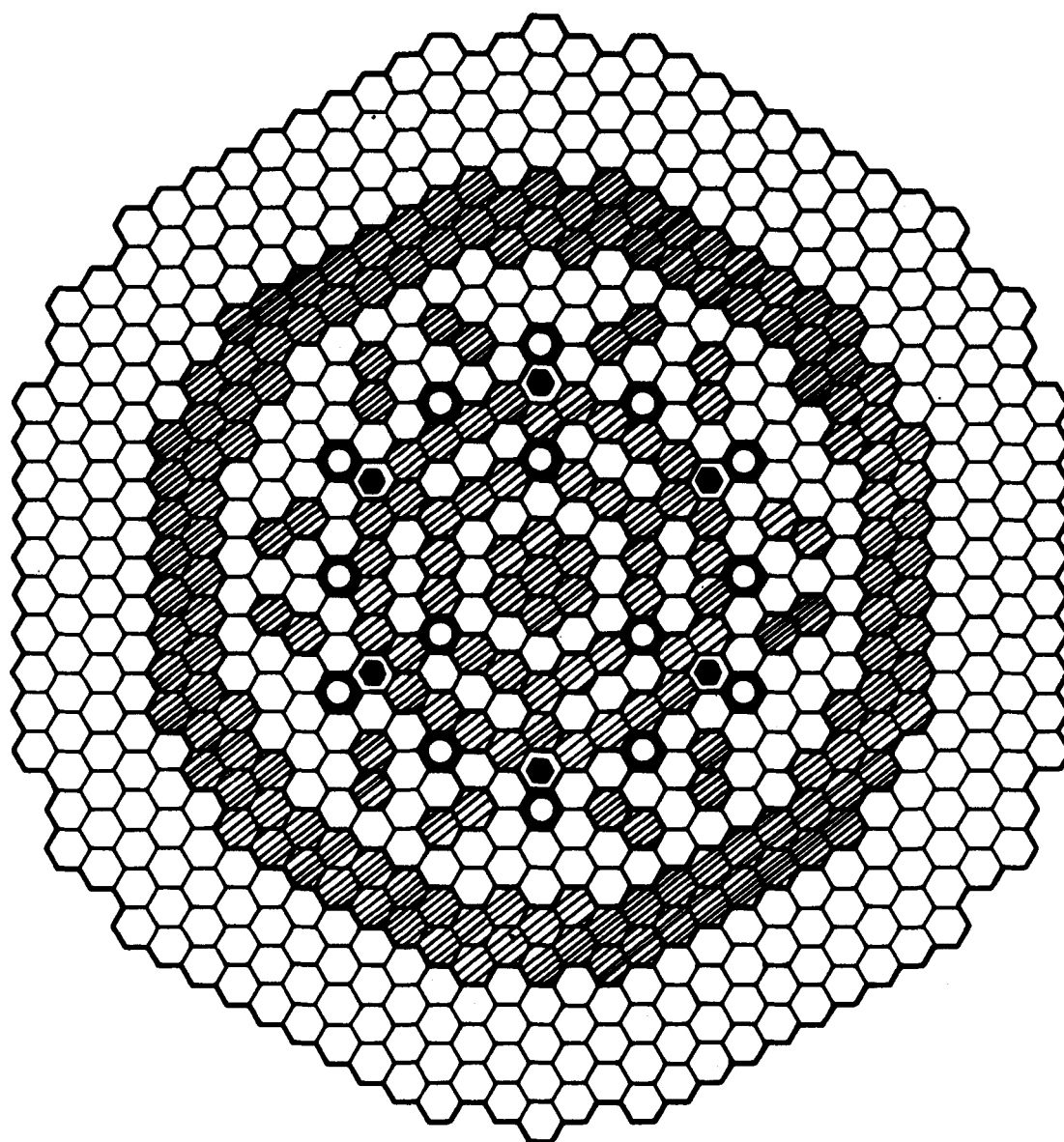
TABLE 2
REACTION RATE CALCULATION TO EXPERIMENT RATIOS

REACTION	ZPPR-11B BEGINNING OF LIFE C/E $\pm 1\sigma$	ZPPR-11C END OF LIFE C/E $\pm 1\sigma$
FUEL		
Pu ²³⁹ (n,f)	1.000 $\pm$.019*	1.000 $\pm$.019
U ²³⁵ (n,f)	1.057 $\pm$.026	1.043 $\pm$.026
U ²³⁸ (n,f)	0.879 $\pm$.034	0.922 $\pm$.034
INNER BLANKET		
Pu ²³⁹ (n,f)	1.014 $\pm$.023	0.989 $\pm$.023
U ²³⁵ (n,f)	1.050 $\pm$.026	1.022 $\pm$.026
U ²³⁸ (n,f)	1.093 $\pm$.041	0.983 $\pm$.032
U ²³⁸ (n,capt)	1.055 $\pm$.025	1.088 $\pm$.025

*Uncertainty includes statistical deviation in foil C/E's plus estimated systematic uncertainty in measurement.

TABLE 3
FUEL AND BLANKET PEAK POWER DENSITY UNCERTAINTY COMPONENTS

	FUEL AT BOL		INNER BLANKET AT EOL	
Experimental (ZPPR fission and capture rates)	±5.5% (3σ)		±7.6% (3σ)	
Reactor Criticality	±0.6		±0.2	
Pellet Fissile Content (fuel only)	±5.2		----	
Pellet Heavy Metal Content (blanket only)	----		±2.0	
Control Rod Banking	±3.0		±2.1	
Modeling:				
2D:3D	±2.0 (nonstatistical)		±2.0 (nonstatistical)	
Radial Shape Interpolation	±1.0		±1.0	
Interassembly Plutonium Distribution (blanket only)	----		±1.0	
Axial Shape	-1.0	±1.0	-1.0	±1.0
Reactor Power	±1.0		±1.0	
Non-Pellet Heating Bias	-1.7		-2.0	
TOTAL (upper envelope):	-0.7	±8.3% (3σ)	-1.0	±8.4% (3σ)



 156 FUEL ASSEMBLIES	 6 ALTERNATE FUEL BLANKET ASSEMBLIES
 76 INNER BLANKET ASSEMBLIES	 15 CONTROL ASSEMBLIES
 126 RADIAL BLANKET ASSEMBLIES	312 RADIAL SHIELD ASSEMBLIES

Clinch River Breeder Reactor Core Layout

1544-1

Figure 1

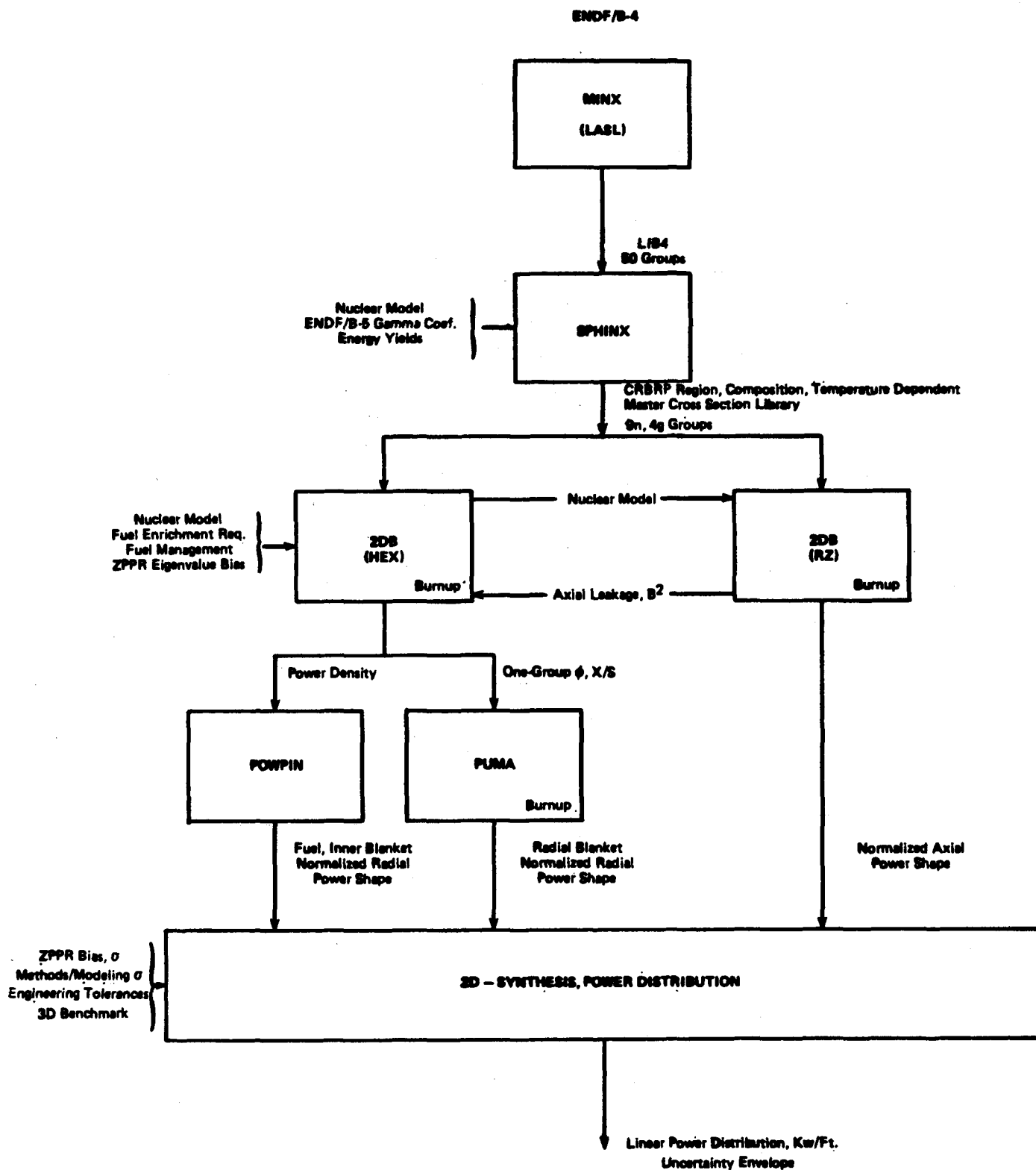


Figure 2. Power Distribution Calculational Method

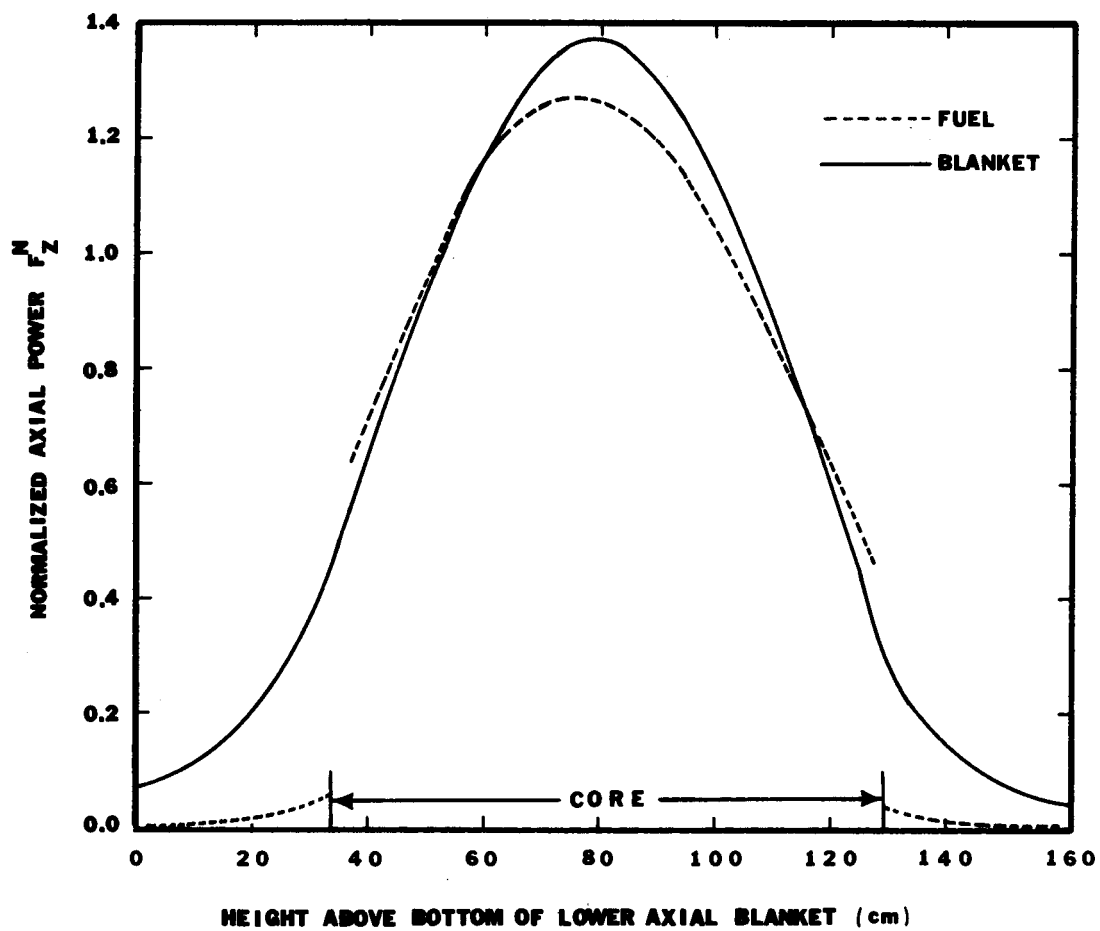


FIGURE 3
NORMALIZED AXIAL POWER SHAPE IN THE FUEL
AT THE BEGINNING OF CYCLE ONE AND IN THE
BLANKETS AT THE END OF CYCLE FOUR



NOMINAL
±3σ (%)
MAXIMUM (3σ + 15% OVERPOWER)

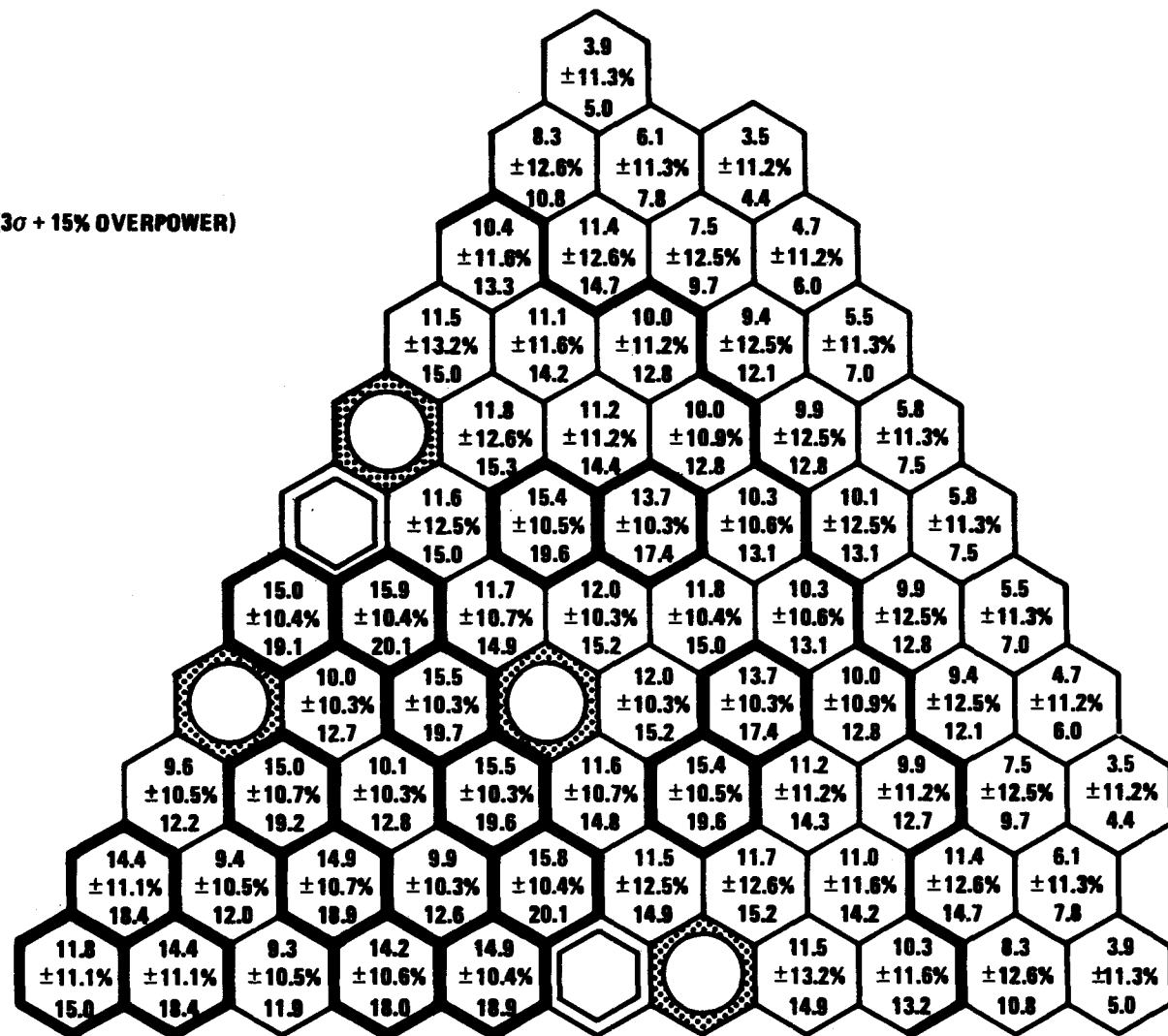


Figure 4. PEAK LINEAR POWER (kw/ft)
Fuel at Beginning of Cycle One Blankets at End of Cycle Four
(End of Cycle Five in Outer Radial Blanket Row)

THE INTERACTION OF NEUTRONICS AND THERMAL-HYDRAULICS IN THE EVALUATION OF BWR STABILITY

J.K. Park, G.C. Park, M. Becker, M. Podowski, R.T. Lahey, Jr.
Department of Nuclear Engineering
Rensselaer Polytechnic Institute

ABSTRACT

Modeling has been performed for the evaluation of the interaction of neutronics and thermal-hydraulics in the analysis of density-wave oscillations. Application of the models show that the interaction is strong, and that stability margin varies considerably with sophistication of neutronics model.

I. INTRODUCTION

Density-wave oscillations constitute a mechanism of potential instability in boiling water nuclear reactors.[1] These oscillations become of increasing concern as innovations in BWR operation, such as flow windowing (ie: increasing the ratio of power level to flow rate), tend to reduce stability margins. Stability with regard to these oscillations have tended to be analyzed as primarily thermal-hydraulic, with highly aggregated and rudimentary treatment of reactor physics.[2,3] In this paper, it is demonstrated that reactor physics interacts in important ways with density-wave oscillations, and that details of the reactor physics representation are of considerable importance in understanding and assessing stability margins.

The reactor physics modeling used in this study has been based on nodal analysis, and in particular, on the types of procedures used in the SIMULATE code.[4] The time-dependent diffusion and precursor equations are Laplace-transformed (in time) yielding a set of "static" equations with complex coefficients. Standard nodal procedures are then applied to these transformed equations. Finally, first order perturbation procedures are applied to yield the transfer functions of interest.

Typically, density-wave oscillations have been analyzed based on the hottest channel. In effect, such analysis presumes that all channels in the reactor are hot channels and that all channels contribute equal void-reactivity feedback. If there were no coupling of reactivity feedback, then the single hot channel would be a reasonable representation, since, for times shorter than recirculation time, channels are hydraulically decoupled. In reality, however, hot channels comprise only a small fraction of the reactor. Thus, BWR stability characteristics change substantially with detail of representation.

This paper will emphasize the reactor physics modeling. Detailed discussion of initial thermal hydraulics, models and of progress in improving thermal hydraulics models, is left to the references.

II. THEORY

The space-dependent kinetics equations are given by,

$$\frac{\partial}{\partial t} \left(\frac{\phi}{v} \right) = \nabla \cdot D \nabla \phi + \frac{1}{k_{\text{eff}}} (1-\beta) v \sum_f \phi - \sum_a \phi + \sum_{\ell=1}^6 \lambda_{\ell} C_{\ell} \quad (1)$$

$$\frac{\partial C_{\ell}}{\partial t} = \frac{1}{k_{\text{eff}}} \beta_{\ell} v \sum_f \phi - \lambda_{\ell} C_{\ell} \quad (2)$$

Small perturbations are taken and products of perturbations are neglected. The resulting equations are Laplace-transformed to yield,

$$-\nabla \cdot D_0 \nabla \delta \phi + \left[\frac{s}{v_0} - \frac{1}{k_{\text{eff}}} (1-\beta_0) v \sum_{f0} + \sum_{a0} \right] \delta \phi - \sum_{\ell=1}^6 \lambda_{\ell} \delta C_{\ell} = M \quad (3)$$

$$M = \left[\frac{s}{v_0} \delta v + \frac{1}{k_{\text{eff}}} (1-\beta_0) \delta(v \sum_f) - \frac{1}{k_{\text{eff}}} v \sum_{f0} \delta \beta - \delta \sum_a \right] \phi_0 + \nabla \cdot \delta D \nabla \phi_0 \quad (4)$$

$$s \delta C_{\ell} = \frac{1}{k_{\text{eff}}} (\beta_{\ell 0} \delta v \sum_f + v \sum_{f0} \delta \beta_{\ell}) \phi_0 + \frac{1}{k_{\text{eff}}} \beta_{\ell 0} v \sum_{f0} \delta \phi - \lambda_{\ell} \delta C_{\ell} \quad (5)$$

Substitution of Eq. 5 into Eq. 3 then yields a single frequency-dependent equation for perturbed flux. To obtain nodal equations, Eqs. 3 and 5 can be integrated over a node. The integration of the diffusion terms yields currents on node surfaces. Nodal equations of the SIMULATE type may then be obtained by relating current to leakage flux L according to

$$J_{nm} = \frac{L_m r_{mn} (1-\rho_{nm}) - L_n r_{nm} (1-\rho_{mn})}{1-\rho_{mn} \rho_{nm}} \quad (6)$$

where J_{nm} is the net current at the interface between nodes n and m and where the other quantities are response matrix nodal parameters defined in the references. The leakage flux is related to the conventional flux by,

$$L = v \sum_f \phi \frac{t}{t-1} \frac{k_{\infty}-1}{k_{\infty}} \quad (7)$$

where t is another nodal parameter. Thus, a nodal formulation developed for static problems applies to the frequency-dependent situation. Some cross-sections (e.g. absorption) become complex and frequency-dependent as do the nodal parameters t .

The perturbed cross-sections in Eqs. 3-5 are obtained in terms of perturbations in void fraction α . Preparation of data provides correlations with exposure and void thereby permitting construction of void coefficients for all relevant data at appropriate position-dependent reference conditions. The perturbations in void result from thermal-hydraulic oscillations induced by externally-

imposed oscillations. In the studies reported here, oscillations are imposed externally on volumetric heat source in fuel $\delta q'''_{\text{ext}}$.

The perturbation $\delta\alpha$ actually results from the net volumetric heat source

$$\delta q''' = \delta q'''_{\text{ext}} - \delta q'''_F \quad (8)$$

where F denotes feedback. In transfer function form, we may write,

$$\delta\alpha = G \delta q''' \quad (9)$$

The feedback heat source results from the perturbation in flux induced by the perturbation in void. This may be written as,

$$\delta q'''_F = H \delta\alpha \quad (10)$$

The perturbation in total power is the sum of the perturbations in heat generation in the reactor. If e is a unit vector and T denotes transpose, then,

$$\delta P = e^T \delta q''' \quad (11)$$

and, similarly,

$$\delta P_F = e^T \delta q'''_F \quad (12)$$

The ratio $\delta P_F / \delta P$, which is the open loop transfer function for power, provides a quantity which may be assessed for stability using the Nyquist criterion.

Thus far, discussion has been in terms of general three-dimensional reactor kinetics. Spatial kinetics models for one and two dimensions may be obtained by integration over dimensions to be eliminated. The integration of diffusion terms is transformed into a surface integral of currents which may be related to nodal fluxes in peripheral nodes through albedoes. In this manner complex frequency-dependent nodal equations are obtained for one-dimensional axial and two-dimensional planar models.

III. APPLICATIONS

To determine the significance of neutronics and thermal hydraulics, analysis was performed on a sequence of problems of differing sophistication of neutron modeling. The problems used data from an operating boiling water reactor.

In all cases considered, the quantity $\delta P_F / \delta P$ is evaluated for stability using the Nyquist criterion. The magnitude of this complex quantity is evaluated when its phase is 180° . The difference between this magnitude and one, measured in decibels (twenty times the logarithm of a quantity) is called the gain margin. Table 1 presents gain margins for various cases.

The first case considered is that of point kinetics based on the hot channel of the reactor. In this case, frequently analyzed on the basis of conservatism, the gain margin is smallest. The second case retains point kinetics, but recognizes that not all channels are hot channels. This case assumes a repeating array of four assemblies including the hot channel. This second case allows for different hydraulic responses in the four channels, leading to a different void reactivity feedback. Recall that if not for the neutronic coupling, the parallel channel oscillations would occur independently in each assembly. The second case demonstrates that it is important not to oversimplify the evaluation of the neutronic feedback.

Case 2, while recognizing differences among assemblies, did not, since point kinetics was used, allow for flux distribution to vary during the oscillation. Case 3 introduces explicit radial (planar) coupling, providing a two-dimensional nodal model. Table 1 shows that the gain margin is larger than in either Case 1 or Case 2.

While there are differences among assemblies, there also are differences among locations within assemblies. In particular, void fraction and void feedback coefficients vary considerably axially. Case 4 again considers a hot channel analysis, but with space-dependent axial one-dimensional nodal kinetics. The gain margin is seen from Table 1 to be greater than for the point kinetics hot channel case.

Since both radial and axial effects were found to yield significant impacts on gain margin, Case 5 was analyzed to study combined radial and axial effects. This case is a three-dimensional nodal model of a checkerboard, i.e. of a repeating array of assemblies including the hot channel. The three-dimensional model yields a gain margin greater than that for either the radial or the axial model.

Case 6 goes beyond the checkerboard to consider radial kinetics for the full core. The gain margin is increased relative to that of the radial kinetics checkerboard, and also relative to the three-dimensional checkerboard.

The results of the cases described above demonstrate that neutronics and thermal-hydraulics interact intimately in connection with density-wave oscillations, and that stability assessment depends strongly on the sophistication of neutronics model used.

Table 1
Gain Margins for Selected Options

Case No.	Option	Gain Margin (db)
1	Point Kinetics - Hot Channel	7.9
2	Point Kinetics - Checkerboard	12.8
3	Radial Kinetics - Checkerboard	16.2
4	Axial Kinetics - Hot Channel	11.0
5	3D Kinetics - Checkerboard	18.5
6	Radial Kinetics - Full Core	20.7

REFERENCES

- [1.] R.T. Lahey, Jr., F. Moody, The Thermal-Hydraulics of a Boiling Water Nuclear Reactor, American Nuclear Society, 1977.
- [2.] L.A. Carmichael, G.J. Scatena, "Stability and Dynamic Performance of the General Electric Boiling Water Reactor", APED-5652, 1969.
- [3.] R.T. Lahey, Jr., G. Yadigaroglu, "NUFREQ, A Computer Program to Investigate Thermo-Hydrodynamic Stability", NEDO-13344, 1973.
- [4.] D.M. VerPlanck, "Manual for the Reactor Analysis Program SIMULATE", YAEC 1158, 1978.

The Importance of Momentum Dynamics
in BWR Neutronic Stability*

P. J. Otaduy

Instrumentation and Controls Division
Oak Ridge National Laboratory
Oak Ridge, Tennessee 37830

M. J. Ohanian

College of Engineering
Department of Nuclear Engineering Sciences
University of Florida
Gainesville, Florida 32611

By acceptance of this article, the
publisher or recipient acknowledges
the U.S. Government's right to
retain a nonexclusive, royalty-free
license in and to any copyright
covering the article.

ANS TOPICAL MEETING

September 22-24, 1982

Kiamesha Lake, New York

*Research sponsored by the Nuclear Regulatory Commission under
contract 40-544-75 under Union Carbide Corporation contract W-7405-eng-26
with the U.S. Department of Energy.

The Importance of Momentum Dynamics in BWR Neutronic Stability

The predominant feedback mechanism to the power generation dynamics of a BWR is the coupling between neutron reactivity and coolant density in the core¹. In modern, large BWR's, two main paths contribute to the coolant density reactivity feedback: (i) a direct power-to-coolant density path which involves the fuel heat transfer and coolant mass and energy dynamics; and (ii) a coolant flow rate-to-coolant density path characterized by the coolant momentum dynamics of the entire flow closed loop formed by the core and recirculation system.

In this paper, we show how the core's neutronic stability (i.e., reactivity-power) is affected by the coolant's momentum dynamics contribution to the density reactivity feedback.

Let

G = reactivity-to-power forward transfer function,

H = power-to-coolant density reactivity feedback transfer function,

H^* = same as H , but omitting the momentum dynamics contribution.

The $1/G$, H and H^* transfer functions calculated² for the Hatch-1 BWR under natural circulation operating conditions (50% power, 32% flow) are shown in the Bode plots of Fig. 1. Note the convenience of plotting the inverse of G when discussing the open loop transfer functions GH and GH^* for stability analysis.

In Fig. 1, the points of intersection M_1 and M_1^* of the magnitude curve of $1/G$ with the magnitude curves of H and H^* correspond to open-loop gains of unity (i.e., 0 db). Likewise, the points of intersection P_2 and P_2^* of the phase curves correspond to open-loop phase shifts of 180 degrees. Consequently, the segments $\overline{P_1Q_2}$ and $\overline{P_1^*Q_1^*}$ between the phase curves along the iso-frequency lines W_1 and W_1^* defined by M_1 and M_1^* represent the phase margin for the core with and without momentum dynamics feedback, respectively. Similarly, the segments $\overline{M_2N_2}$ and $\overline{M_2^*N_2^*}$ between the magnitude curves along the iso-frequency lines W_2 and W_2^* defined by P_2 and P_2^* represent the magnitude margin for the same cases.

Comparison of the margin segment lengths shows that neglecting the closed loop momentum dynamics feedback results in a non-conservative estimate of the core stability. The phase margin decreases from 42 to 22 degrees and the magnitude margin decreases from 13 to 6 db when the close loop momentum dynamics are considered. This destabilizing effect is made evident also by the interaction between the $1/G$ and H , H^* phase curves in Fig. 1 in the 0.3 to 2 Hz frequency range, since if the $1/G$ and the feedback phase curves did not intersect the core would be stable regardless of the feedback's magnitude.

We conclude that the contribution of the core and recirculation momentum dynamics to the density reactivity feedback strongly affects core stability. Stability estimations performed without accounting for this contribution will be non-conservative. Changes in either the core

flow path or the recirculation system dynamic characteristics will affect the power generation stability of the core through this mechanism.

1. R. T. LAHEY, Jr. and J. J. MOODY, "The Thermal-Hydraulics of a Boiling Water Nuclear Reactor," Monograph, American Nuclear Society, LaGrange Park, Illinois (1977).

2. P. J. Otaduy, "Modelling of the Dynamic Behavior of Large Boiling Water Reactor Cores," Ph.D. Dissertation, University of Florida, 1979.

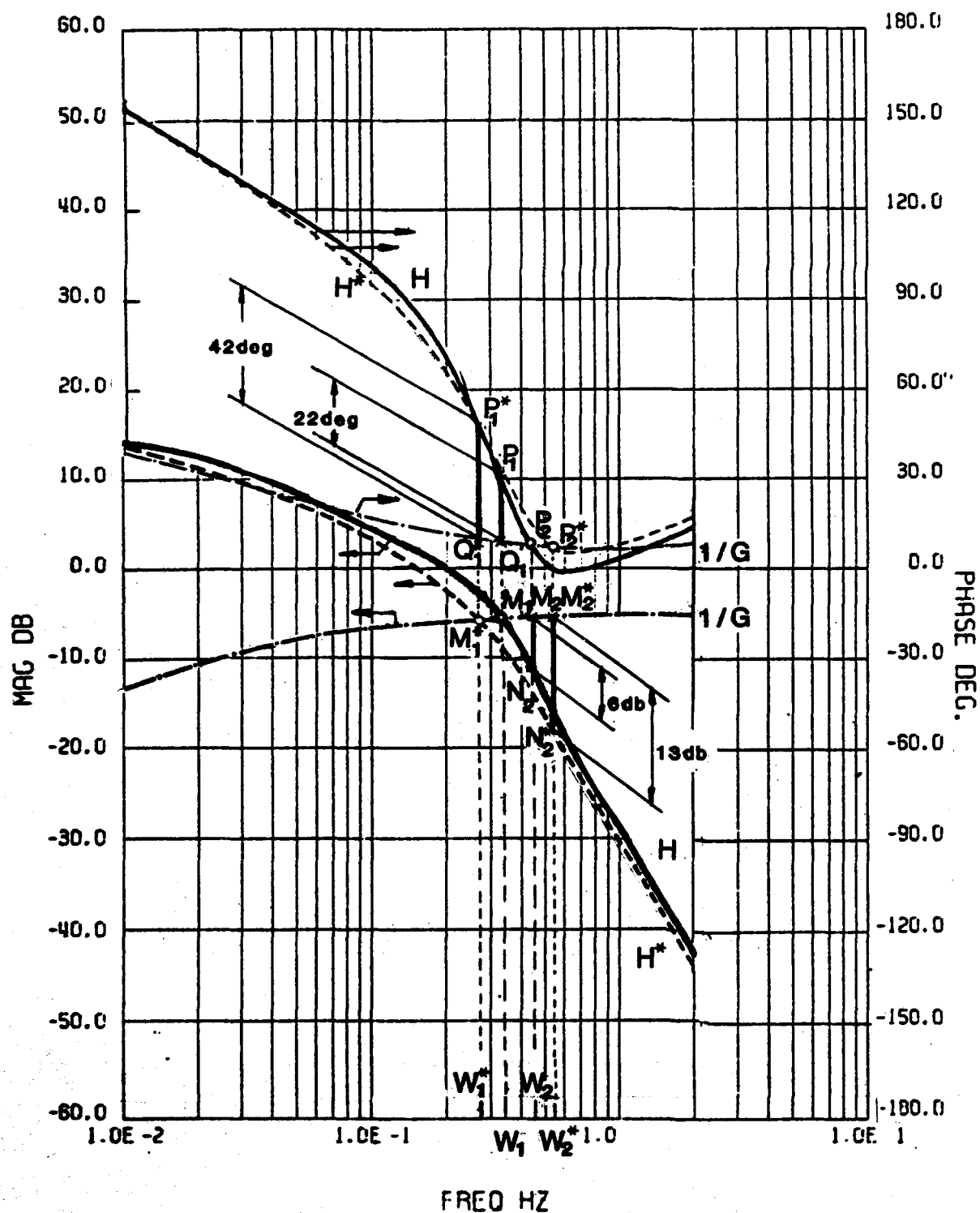


Fig.1. Forward and Density Reactivity Feedback Transfer Functions
with and without Momentum Dynamics for the Hatch-1 BWR at 50%power
32%flow.

BWR CORE RESPONSE TO FLUCTUATIONS IN COOLANT FLOW AND PRESSURE, WITH IMPLICATIONS ON NOISE DIAGNOSIS AND STABILITY MONITORING

By Jan H Blomstrand and Sven-Åke Andersson,
ASEA-ATOM, Västerås, Sweden

ABSTRACT

Reactor dynamic tests, utilising sinusoidal oscillations in pressure and recirculation flow, have been conducted in operating BWRs in Sweden and Finland. Test data recorded, as well as recordings of process noise, have been analysed in terms of dynamic core properties. The results obtained show good qualitative agreement with model predictions of BWR core dynamics. Model studies can often support interpretation of dynamic information obtained from operating plants. Comparisons between model studies, dynamic tests and process noise may also provide improved understanding of test results and noise patterns; in this way it can be demonstrated that some neutron flux noise is caused by noise in coolant flow and steam flow. From reactor test data and noise recordings, core stability parameters have been evaluated by a number of methods. These have been found to provide essentially the same results. The cores investigated were found to be very stable under normal operating conditions. In special operating points, outside the normal operating range, higher decay ratios may occur. The experience indicates that for BWR cores, operated at decay ratios above quarter damping, the stability parameters may be identified from the oscillatory behaviour of the autocorrelation in the time domain of the neutron flux noise.

INTRODUCTION

Dynamic properties of BWR cores are determined by coupled phenomena encompassing core physics, thermal-hydraulics and thermal fuel properties. In more detail, they depend mainly on the mechanical and thermal-hydraulic design of the fuel and the coolant channels, e.g. fuel rod diameter, lattice pitch, bundle size, channel dimensions, inlet orifice, gap areas; the nuclear design, e.g. enrichment, enrichment distribution, means of controlling excess reactivity; the core size, e.g. channel length, number of parallel channels; and the operating conditions, e.g. core power, power distribution, core exposure, coolant flow, core inlet subcooling, pressure. Conditions outside the core also influence the dynamic core properties to some extent, e.g. water level, steam separator flow resistance, water inventories in the down-comer and the lower plenum, recirculation pump characteristics.

The topic of this paper is the dynamic response of BWR cores to small disturbances in recirculation flow and steam dome pressure. The paper also discusses some related topics, such as process noise diagnosis, and the determination of core stability margins via dynamic tests and noise analysis.

Detailed information on core dynamics can be obtained by tests in operating reactors. Systematic perturbations are then introduced in steam dome pressure, for example, and these provoke responses in neutron flux. Data recorded at such tests can normally be evaluated relatively easily in terms of dynamic core properties. However, reactor tests interfere to some extent with plant operation, and their execution usually requires the assistance of specially trained personnel.

In an operating BWR, the core is continuously exposed to small fluctuations in recirculation flow, pressure, etc, of random or pseudo-random nature. Such external disturbances, as well as the boiling process in the core, provoke responses in neutron flux, to an extent and according to a pattern that is governed by the dynamic core properties. The control systems for plant power and reactor pressure operate on signals obtained via the plant instrumentation, and these may include the neutron flux. The fluctuations in the sensed process parameters are fed back via the control systems to the recirculation flow and the pressure, to extents and according to patterns that depend on their detailed design. Although the resulting process noise picture appears complex, it normally contains information on dynamic core properties, for example stability margins. Noise recordings do not interfere with plant operation and can be made at any time without prior arrangements; these are their main attractions. However, process noise is often more difficult to interpret in terms of dynamic properties than data collected from reactor tests.

Results obtained from dynamic tests may be interpreted on their own, the same applies to noise. When both test results and noise analyses are available, they may support each other; for example, the test evaluations may show a dynamic transfer function between two particular parameters similar to that obtained from cross analysis of noise in the same pair of parameters. A comparison may then provide a more complete picture of the processes occurring in the plant than can be obtained from the tests only, or the noise only. This may in turn make it possible to extract additional information from both the test results and the noise. Therefore, it is advantageous to record process noise in connection with dynamic tests, and at the same operating points. Interpretation of results obtained from operating reactors can also be supported by model studies of reactor dynamic characteristics. Models offer some special advantages in this context, since they can predict reactor performance that may not be feasible to test in operating reactors, for example shortcircuiting of certain control systems. They can also investigate how individual parameters influence core dynamics.

In order to provide a base for interpreting test results and noise recordings obtained from operating BWRs, a series of model studies have been performed to assess the dynamic core response of a typical ASEA-ATOM BWR to sinusoidal oscillations in recirculation flow and pressure.

A number of reactor dynamic tests utilising sinusoidal oscillations in steam dome pressure and coolant flow have been conducted in operating ASEA-ATOM BWRs, and process noise has also been recorded on these occasions. The data collected has been evaluated in terms of dynamic core properties, and yielded results in good qualitative agreement with the model predictions. Detailed agreement can not be expected, however, since the plants involved differ in certain respects from the reference plant that was the object of the model studies. Still, the reactor designs are sufficiently similar to permit the model predictions to be used in support of interpretations of measured reactor performance.

Sometimes, reactor tests are conducted - and noise is recorded - in order to study core behaviour in operating points of special interest, for example at high power in combination with reduced recirculation flow. It may then be of interest to interpret measured dynamic core properties in terms of stability parameters. Various methods for evaluating these are discussed and compared. In our experience, the required information may often be extracted from noise recordings alone, as is shown by some examples.

REACTOR DESIGN DETAILS

Dynamic core properties, measured via tests and noise recordings in individual reactors, must be viewed in relation to the corresponding similarities and differences in design and operating conditions between the plants concerned. The same applies to comparisons with model predictions.

So far, all ASEA-ATOM BWRs have utilised essentially the same fuel rod design and the same coolant channel geometry, with regard to lattice type (8x8), lattice pitch, channel length, shroud inner dimensions, inlet orificing, type and number of spacers, etc. Typical fuel and coolant channel design data are given in ref. 1. Thus, the reactor cores considered are similar, from the points of view of thermal fuel properties (time constants, etc) and coolant channel thermal-hydraulics.

Some differences still exist in these respects between the individual reactors. They reflect partly the rated operating conditions of the coolant channels, for example, fuel ratings and void contents; and partly conditions outside the cores, for example, the subcooling at the core inlet and the hydraulic characteristics of the recirculation circuit, including the steam separators and the recirculation pumps. As regards the latter, two design generations of ASEA-ATOM BWRs exist. In the plants of the first generation, these pumps are located in external recirculation loops, whereas the second generation plants utilise internal pumps. Some design data of interest in this context are given below, for the reactors built - and being built - by ASEA-ATOM. The TVO plants are situated at Olkiluoto, Finland; and the remainder are located in Sweden.

Plant	Oskars- hamn 1	Ring- hals 1	Oskars- hamn 2	Barse- beck 1	Barse- beck 2	TVO I	TVO II	Fors- mark 1	Fors- mark 2	Fors- mark 3	Oskars- hamn 3
Customer	OKG	SSPB	OKG	SK	SK	TVO	TVO	FKA	FKA	FKA	OKG
Scope of supply	turn key	nuclear island				turn key	t. k. excl civil works	nuclear island			
Design generation	external recirculation pumps					internal recirculation pumps					
Operational status	in operation									under construction	
In operation since	1972	1974	1974	1975	1977	1979	1980	1980	1981		
Anticipated operation										1985	1985
Plant output, MWe, (net)	440	750	570	570	570	660	660	900	900	1060	1060
Core power, MWth	1365	2270	1700	1700	1700	2000	2000	2700	2700	3000	3000
Number of fuel assemblies	448	648	444	444	444	500	500	676	676	700	700
Fuel rating, kW/kg U	17,4	19,7	21,4	21,4	21,4	22,5	22,5	22,4	22,4	24,1	24,1
Core power density, kW/l	36,2	40,8	44,2	44,2	44,2	46,0	46,0	46,3	46,3	48,6	48,6
Total coolant flow, kg/sec	6900	9260	6070	6070	6070	7800	7800	10400	10400	11400	11400
Feedwater temp, °C	160	180	180	180	180	180	180	180	180	215	215
Steam dome pressure, bars	70	70	70	70	70	70	70	70	70	70	70

To some extent, the design values for the fuel ratings employed in each reactor have influenced core physics. For example, higher coolant flows require thicker fuel channels. This may in turn influence control rod spacings and gap areas, and thereby core moderating ratios. Moreover, most of the information from the operating reactors was extracted during or shortly after commissioning. Thereby, core physics was influenced by the nuclear design of the initial cores, for example the enrichments (which are determined largely by customer requirements on operating periods), and the means employed for controlling the associated excess reactivities. In reactor cores up to and including Barsebeck-1, boron curtains located in the gaps were used for this purpose. In later reactors, these were replaced by burnable absorbers (Gadolinium) dispersed in the fuel, again with absorber arrangements differing between the individual cores (as tailored to customer requirements on operating periods).

MODEL STUDIES

The model studies were performed using the reactor dynamic code BISON, which contains a multizone fuel rod model, one-dimensional axial models of both the coolant channel thermal-hydraulics and the core physics (two-group diffusion theory), a one-dimensional hydraulic model for the external recirculation loop (including the steam separators and the recirculation pumps, etc), and models for the control systems for steam dome pressure, etc. The code is described in more detail in ref. 2.

The model studies comprised analyses of the response in instantaneous core power to sinusoidal oscillations in steam dome pressure and recirculation flow. The pressure oscillations were induced via the steam flow from the dome. The core response to recirculation flow oscillations was studied for both the ideal situation of a constant steam dome pressure, and with account taken of the characteristics of the pressure controller. The studies have investigated how dynamic core properties vary with operating point, especially along the flow control line, and have also included some parametric sensitivity analyses, for example with respect to the void coefficient.

The analyses were performed over a frequency range from about 0.1 Hz up to around 1.0 Hz. This interval adequately covers the core resonance, which is normally located between 0.3 and 0.6 Hz. Results are presented in terms of transfer functions, mostly from the perturbed parameter (flow or pressure) to instantaneous core power, and plotted in Bode diagrams, i.e. gain and phase versus frequency. All gains presented are non-dimensional, relating to the relative changes in the respective parameters, for example, relative response in actual core power in relation to relative perturbation in actual steam dome pressure.

A preliminary design version of one of the internal pump generation plants was chosen as the reference object for the model studies. However, after the analyses were completed, the reactor design was changed in several respects (for reasons unrelated to the analyses), and the results no longer apply in detail even to that reactor. The model studies are still of interest, in view of the comprehensive illustrations that they provide on the general nature of the dynamic response of a modern BWR core to perturbations in coolant flow and pressure. The results lend valuable support to interpretation of measured dynamic core properties and diagnosis of process noise.

MEASUREMENTS IN OPERATING REACTORS

During reactor start-up, excellent opportunities arise to measure the core dynamic response to disturbances in recirculation flow and pressure, in connection with commissioning tests of the corresponding control equipment. Such tests usually involve set point changes, for example sinusoidal oscillations, which produce perturbations in the corresponding controlled process parameters (coolant flow, pressure), provoking in turn responses in neutron flux. Usually, the tests are repeated at various power levels, thereby providing information on how the dynamic core properties vary over the normal operating range of the reactor, for example along the flow control line.

Sometimes, it may be of interest to demonstrate the feasibility of reactor operation at certain power/flow combinations beyond the normal operating range, for example at high core power in combination with reduced recirculation flow, or under natural circulation conditions. The core dynamics in such operating points can be investigated via reactor tests. It is advantageous to conduct these in accordance with the procedures established for the testing of the corresponding control systems, as this minimises the risk for tripping the plant inadvertently.

Tests, involving the use of sinusoidal oscillations, are repeated at a number of discrete frequencies over the frequency interval of interest. Normally, this procedure provides dynamic information of high quality, since at each frequency point studied, the amplitude of the oscillation introduced in the set point for the controlled process parameter can be adjusted until responses of suitable magnitudes are obtained in all the perturbed process parameters (flow, pressure, neutron flux, etc). Such tests have been conducted extensively in ASEA-ATOM BWRs, both during commissioning and on other occasions, and test data has been recorded for subsequent analysis. Process noise is normally recorded in connection with these dynamic tests, and at the same operating points. Sometimes, it is sufficient to record only process noise.

INSTRUMENTATION

The signals to be recorded are obtained from the normal plant instrumentation. For example, neutron flux is monitored via the in-core detectors, these provide information on both local conditions (LPRM) and global conditions (APRM). Steam dome pressure is monitored via pressure sensors. Steam flow is monitored in all the individual steam lines to the turbine; these signals provide information on total steam flow from the vessel. In the external pump generation plants, each recirculation loop is equipped with a flow sensor; from these signals, total recirculation flow through the core (including bypass) is obtained. In the internal pump generation plants, the coolant flows through eight representative channels in the core are monitored via the pressure drops over their calibrated inlet orifices.

RECORDING EQUIPMENT

When reactor tests are conducted, or noise is recorded, a multitude of signals describing the monitored process parameters must be collected, preferably in digital representation to permit the subsequent analysis of the data by digital methods. For this purpose ASEA-ATOM has developed a computerised multichannel data recording system, described in detail in ref. 3. The system was first tried out in Barsebeck-2. Similar systems, with built-in facilities for data analysis, have now been installed permanently in all the operating internal pump generation plants, and are also to be installed in Forsmark-3 and Oskarshamn-3.

The systems have been designed to satisfy a number of demands, some of them posed by the duties required of them, others related to the particular characteristics of the plants where they are to be used. Each system is built up around a central processor - in some plants a Nord 10 CPU with a 40 K memory, in others in a Nord 100 CPU with a 96 K memory - and incorporates one graphic CRT terminal with a hard copy unit, one dual flexible disk unit, one 10 Mbyte disk, and four process interfaces, one for each electrical subdivision (safety train) in the plant.

It is important that the electrical separation between the individual subdivisions in the plant is not violated when signals originating from instrumentation and equipment belonging to the separate subdivisions are connected to the recording system. This is realised by locating one interface within each of the four plant subdivisions, and equipping each (word serial) communication link to the CPU with a galvanic (optical) isolation device. The CPU is normally located in the plant computer room, together with its other peripherals.

The total number of signals that can be connected to the recording system is sufficient for all normal purposes. When a recording system has been installed in a plant, those signals that are of interest to record during the reactor commissioning tests, and throughout the subsequent reactor operation, are connected to the interfaces. The connections are made on a permanent basis, prior to plant commissioning, when the risk is nil for tripping the plant by faulty connections, or shortcircuiting of signals during handling of the connecting cables, etc. The signals connected remain available for recording during the commissioning period of the plant, and indeed throughout its entire lifetime.

The signals that are to be recorded at any particular test, or on any other occasion, are selected by the operator of the recording system, via the CRT terminal. All other communication with the system takes place via the same terminal, in an interactive manner. The recording can be triggered either on a command from the operator of the recording system, or on a remote push button signal, or on a suitable digital input signal from the plant, or if an analogue signal exceeds a pre-set value (the latter options also permit the system to be used for past history recording). The data is normally collected at the fastest scanning rate permitted by the process interfaces and the CPU, and then reduced in the CPU by a digital filtering technique down to a rate suitable for storing. This procedure improves the quality of the recorded data. The data is normally stored on flexible disks.

DATA ANALYSIS

The data analysis can be performed both on the recording system itself, and on other computers. The former option is of interest when test results are needed urgently. The data stored on the flexible disk is then read back into the recording system and analysed there, under the operator's supervision. The analysis results can be displayed on the CRT and plotted on hardcopy. For less urgently needed results, and/or for more detailed analyses, the ASEA Head Office computer may be utilised.

The software packages include programs for qualification of raw data, reconstruction of combined process signals, spectral analyses, including FFT routines for evaluations of cross spectra and coherences, corrections for dynamic properties of sensors, Fourier inversion, plotting of results in both the time and frequency domains, etc.

Transfer function gains obtained from Fourier analyses are presented here non-dimensionally, as relative response (normally in neutron flux), in relation to relative perturbation (in pressure, for example).

CORE RESPONSE TO COOLANT FLOW FLUCTUATIONS

Fig. 1 shows the calculated response in instantaneous core power to sinusoidal oscillations in recirculation flow, at three operating points along the flow control line. These model studies assumed a constant steam dome pressure. Extra studies of the influence of the pressure controller have shown that normal pressure feed-back does not alter the general nature of these transfer functions, but may influence gains and phases locally to a small extent.

Fig. 2 shows examples of measured responses in neutron flux (APRM) to forced oscillations in recirculation flow, at three operating points along the flow control line. These results have been obtained from commissioning tests of the flow control systems in one internal pump generation plant, with control operated in a purely feed-forward mode (neither coolant flow nor neutron flux feed-back). Because of built-in restrictions in the solid-state frequency converters supplying power to the pump motors, forced oscillations could be introduced into the recirculation flow up to a frequency of about 0.25 Hz, i.e. not quite up to core resonance frequencies. Over the frequency range covered by the tests, the transfer functions show good qualitative agreement with the model study predictions (the design power density of the fuel in the plant where these tests were conducted is 93.4% of that in the reference plant for the model studies).

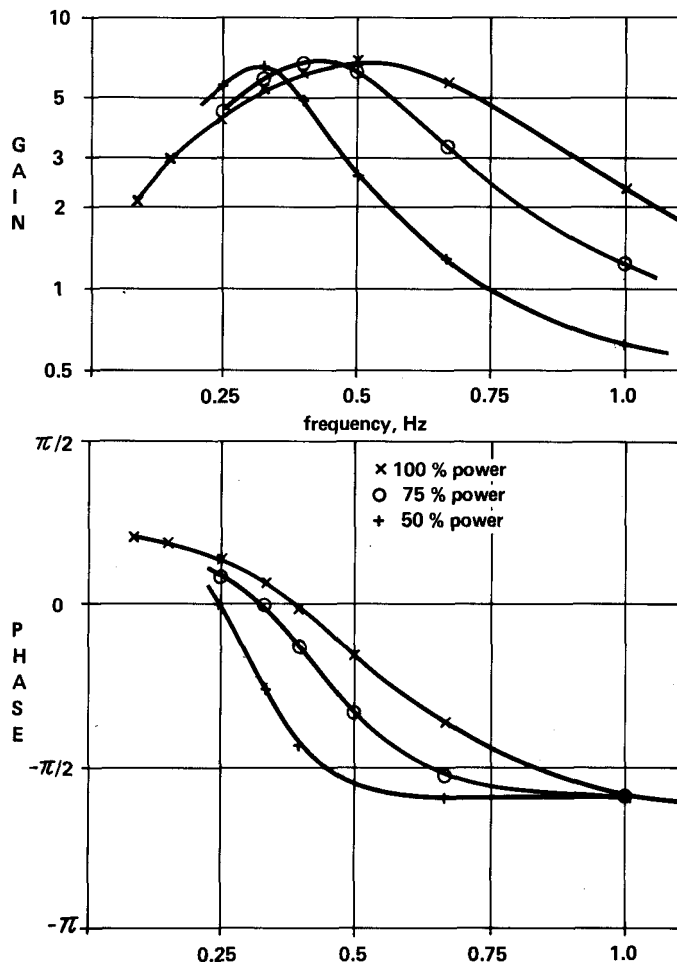


Fig. 1. Calculated core power response to coolant flow oscillations

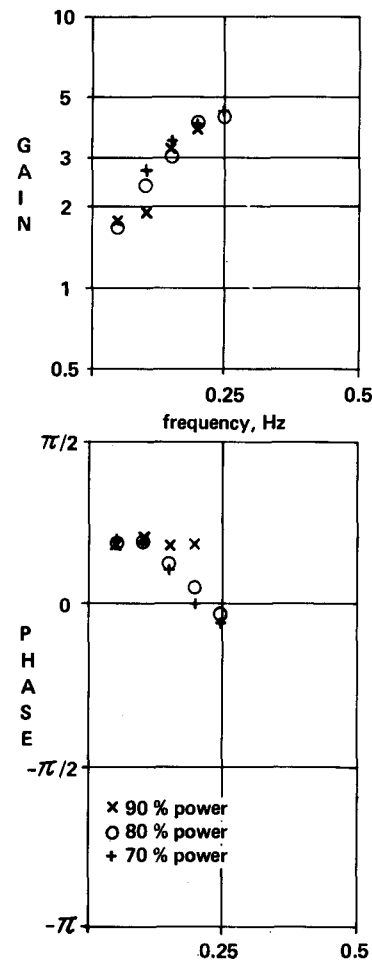


Fig. 2. Measured neutron flux response to coolant flow oscillations

Although the test frequencies did not reach the core resonance, one can still notice how the phase lead decreases at frequencies above 0.15 Hz, when core power is reduced along the flow control line. Comparison with fig. 1 indicates that this feature is a manifestation of a corresponding reduction in core resonance frequency. This example illustrates how model predictions can support interpretations of reactor dynamic test results.

It should be possible to extend measurements of this transfer function to higher frequencies, by making use of the small fluctuations that occur naturally in the recirculation flow in a BWR. Any systematic neutron flux response to these can be expected to show up in cross analyses of noise in neutron flux (APRM) and recirculation flow signals (provided that the latter noise can be monitored reasonably accurately). Figs 3 a-b show examples of such transfer functions; they have been evaluated from noise recordings in one of the plants where the recirculation pumps are located in external loops. The results refer to two operating points along the flow control line. For reasons connected to the improvised conditions under which these recordings were made (the recording system described above was not available then), the absolute magnitudes of the transfer function gains are uncertain. Nevertheless, figs 3 a-b are of interest, as they show phases and gains that vary with frequency in line with the model predictions in fig. 1. The core resonance shows up in both transfer functions, and the observed resonance frequency varies with operating point in accordance with the model predictions. (In the plant from which these results originate, the design power in the fuel is 89% of that to which the model studies refer). For resolution purposes, the noise-evaluated transfer functions in fig. 3 have been plotted in a linear frequency scale. To facilitate comparisons with figs 1 and 2, the same scale was also used there.

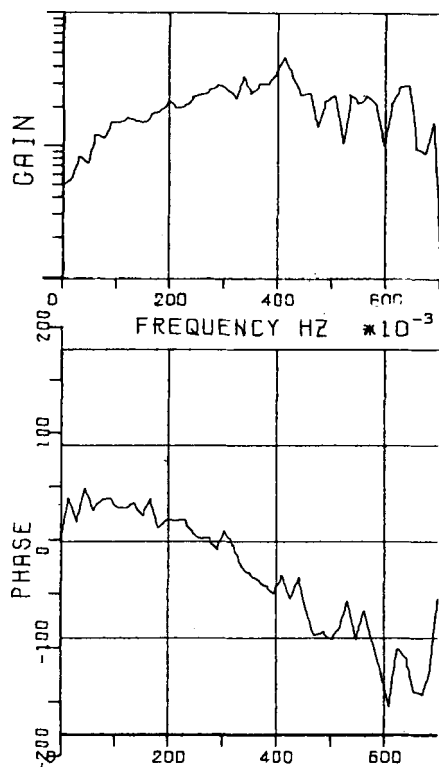


Fig. 3a. Transfer function from coolant flow to APRM, obtained from noise recorded at 80% power

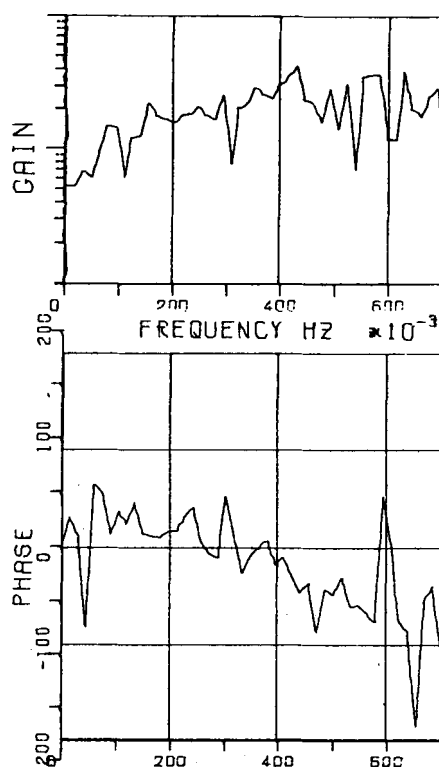


Fig. 3b. Transfer function from coolant flow to APRM, obtained from noise recorded at 100% power

Comparisons with the model studies (fig. 1) and the dynamic test results (fig. 2) show that the transfer functions of fig. 3 describe a situation in which part of the neutron flux noise is a response to the recirculation flow noise. This example illustrates how model studies and/or results from dynamic reactor tests can support interpretation of observed noise patterns. Ref. 4 presents cross analyses of noise, recorded in a BWR-4 plant, showing phase leads at lower frequencies between total core flow and an APRM signal, similar to those in figs 1-3.

CORE RESPONSE TO PRESSURE FLUCTUATIONS

Results from model predictions of the response in instantaneous core power to sinusoidal pressure oscillations are shown in fig. 4, at three operating points along the flow control line.

In reactor dynamic tests, pressure perturbations are introduced via the pressure control system. The changes in steam flow from the reactor vessel permitted by the turbine valve movements are so fast that the core can be exposed to controlled disturbances up to frequencies well above the core resonance; a detailed mapping of the core resonance features becomes then possible. Pressure disturbance tests constitute, therefore, a good means for studying BWR core dynamics; this method was used, for example, in the Peach Bottom-2 stability tests, reported in refs 5, 6, 7. A substantial number of reactor tests utilising sinusoidal pressure oscillations have been conducted in operating ASEA-ATOM BWRs. The information obtained on dynamic core properties shows good qualitative agreement with the model predictions of fig. 4; for example, fig. 5 illustrates measured transfer functions from pressure to neutron flux in two operating points along the flow control line (these results originate from one of the external pump generation plants, in which the design power density of the core is 89% of that to which the model studies refer).

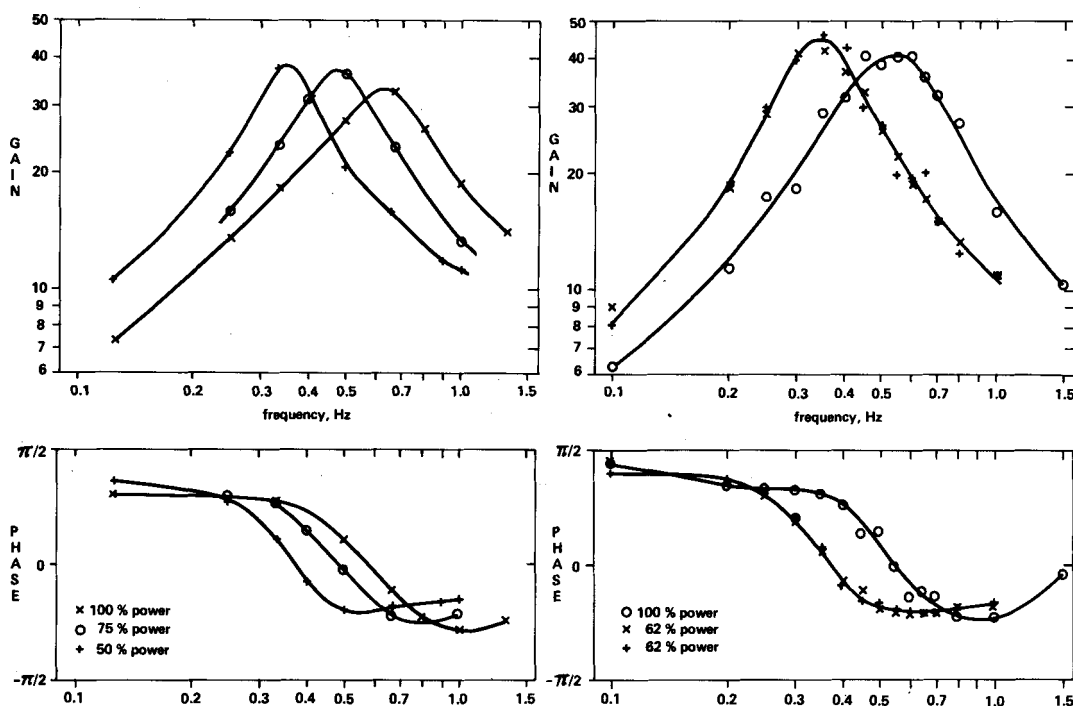


Fig. 4. Calculated core power response to pressure oscillations

Fig. 5. Measured neutron flux response to pressure oscillations

The model studies have also comprised sensitivity analyses, for example, of the manner in which the void coefficient influences core dynamics. Some results are shown in fig. 6. They indicate that stronger void-reactivity couplings, i.e. more negative void coefficients, increase the gain in the calculated transfer function from pressure to instantaneous core power, and produce steeper phase shifts around the core resonance. (Similar trends were also found in the calculated transfer functions from coolant flow to core power.)

Fig. 7 shows transfer functions from pressure to neutron flux, obtained from tests conducted under similar operating conditions in two initial cores that were similar in many respects (fuel rod design, coolant channel thermal-hydraulics, burnable absorber arrangement, power density, etc), but differed with respect to void coefficient by about a factor of 1.3 (according to global core physics calculations), mainly because of small differences in moderation and enrichment. Comparison with fig. 6 suggests that the observed differences in dynamic core properties are largely void coefficient related. This example illustrates how model studies can support interpretation of test results, and indicates that it may be possible to identify the void coefficient in a BWR core by the gain in the measured transfer function from pressure to neutron flux.

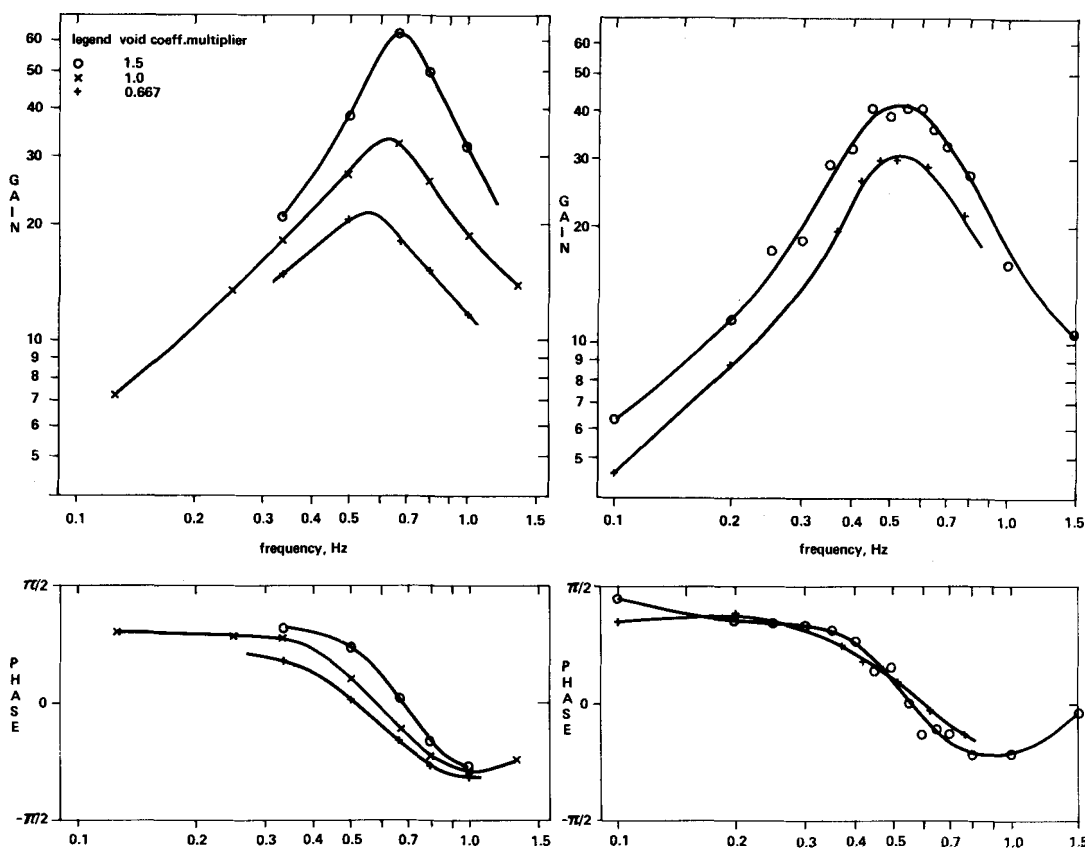


Fig. 6. Calculated void coefficient sensitivity of core power response to pressure oscillations

Fig. 7. Measured neutron flux response to pressure oscillations in cores with different void coefficients

Figs 4-7 show that in the frequency range below the core resonance, both calculated and measured transfer functions behave approximately as $\text{const} \cdot j\omega T$, with a phase lead of nearly 90° and gain roughly proportional to frequency. These characteristics signify that the core differentiates perturbations in pressure, a behaviour brought about by thermal feed-back from the fuel. Concurrently, the steam dome integrates perturbations in steam flow from the reactor vessel, i.e. the dynamic transfer function from total steam flow to pressure behaves approximately as $-(j\omega T_{\text{dome}})^{-1}$, where T_{dome} represents the time constant for the dome, and the minus sign reflects that steam flow is defined as positive in the direction out of the vessel. In the combined processes that forward total steam flow perturbations to the neutron flux, via the steam dome pressure, the integrating properties of the dome approximately compensate the differentiating properties of the fuel, giving a resulting transfer function from total steam flow to neutron flux of a very simple nature.

This trend, seen in the model studies, is confirmed by pressure oscillations tests in the operating reactors. For example, fig. 8a shows a transfer function from total steam flow to neutron flux, evaluated from tests conducted in one of the internal pump generation plants, at an operating point combining the minimum permitted recirculation flow with a relatively high core power. Around and below the core resonance this transfer function resembles closely that of an ideal second order dynamic system, but with a 180° phase shift, for the reasons explained above, and approaches 180° in the lower frequency range. At frequencies above the core resonance, higher oscillatory modes are noticeable, influencing particularly the phase. The corresponding transfer function from pressure to neutron flux is shown in fig. 8b.

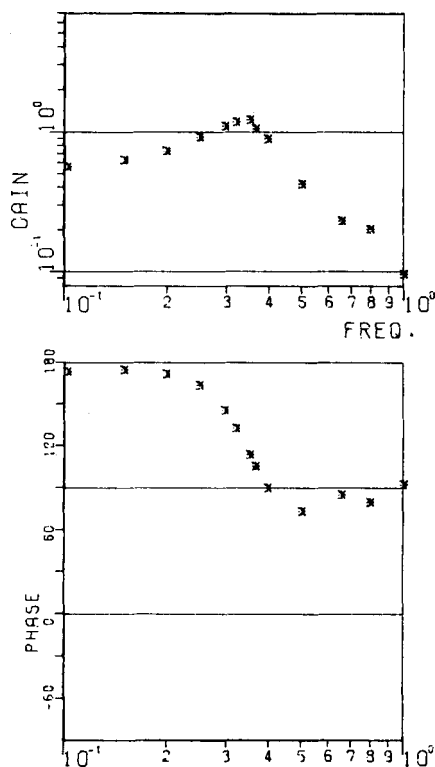


Fig. 8a. Measured neutron flux response to oscillations in total steam flow, at minimum coolant flow

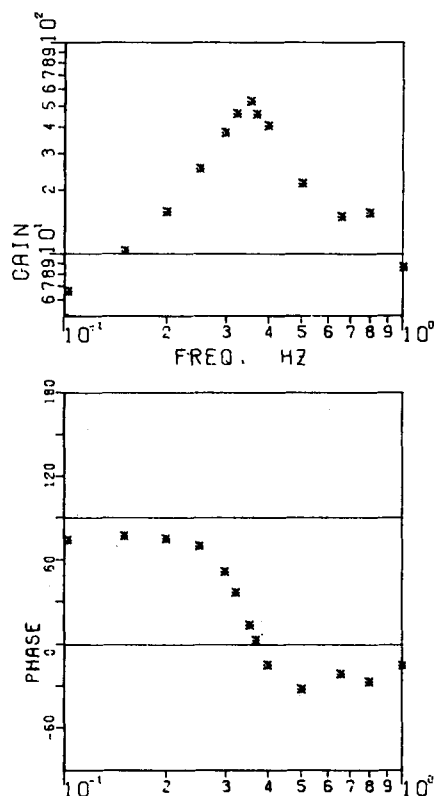


Fig. 8b. The corresponding neutron flux response to pressure oscillations

Because of the particular characteristics of the transfer function from total steam flow to neutron flux illustrated in fig. 8a, turbine valve movements (normally ordered via the pressure control system) cause fast responses in the neutron flux, with time delays determined by the sound propagation through the steam lines and the phase properties of the transfer function from steam flow to neutron flux. The steam flow induced neutron flux noise is superimposed on noise originating from other sources (for example the boiling process, recirculation flow fluctuations). However, cross analyses of noise in total steam flow and neutron flux do not produce transfer functions of the type shown in fig. 8a, because of the process couplings operating in the opposite direction; for example, the pressure controller counteracts, via the steam flow, sensed fluctuations in pressure, and sometimes also in neutron flux. Still, the neutron flux response to steam flow noise can be demonstrated in a simple manner, taking advantage of the fact that the slower steam flow fluctuations are essentially "mirrored" in the neutron flux (because of the $\sim 180^\circ$ phase shift in the transfer function at lower frequencies, see fig. 8a). Normally, this mechanism may be somewhat difficult to perceive in noise plots, since the steam flow induced neutron flux noise is largely drowned in noise originating from other sources. Therefore, clear demonstrations of steam flow induced neutron flux noise must make use of noise patterns recorded under more unusual circumstances, with relatively pronounced steam flow noise.

During a trial test of control parameter settings in the pressure controller in one plant, a temporary situation arose when the reflections of the steam flow fluctuations in the neutron flux noise could be observed rather clearly. Fig. 9 shows a coplot of noise histories in total steam flow (plotted "upside down", for clarity), and neutron flux.

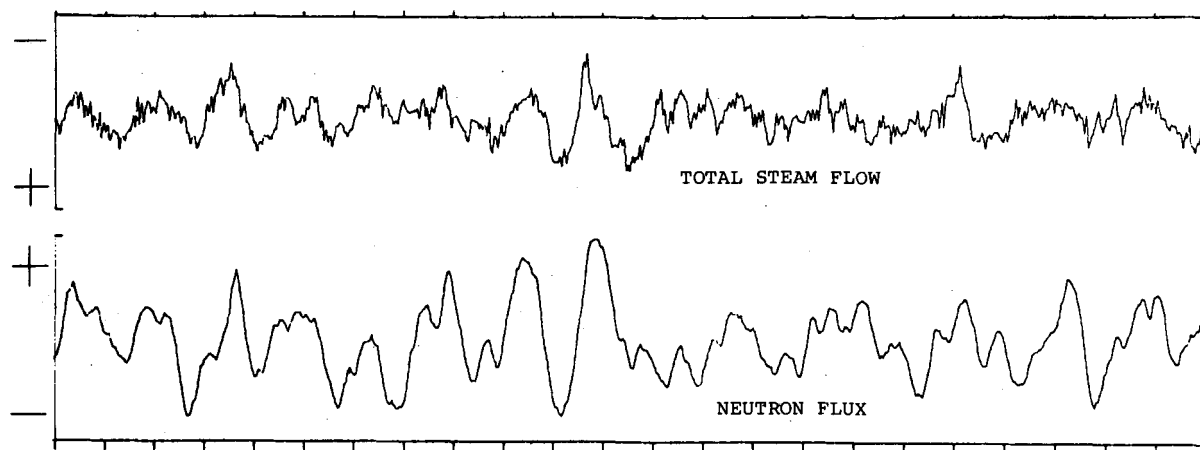


Fig. 9. Coplot of noise histories in total steam flow and neutron flux (APRM)

This example illustrates how the basic features of a transfer function, arrived at via tests (and model studies), can be used to explain an observed noise pattern. At the same time, it must be emphasized that this figure was specially selected to provide a particularly clear illustration of the process of interest. In "normal" BWR noise patterns, this coupling is less visible.

CORE STABILITY

Several motives exist for measuring dynamic core properties in operating BWRs, one of them is the determination of core stability margins. A number of methods can be applied for the evaluation of the core stability parameters (decay ratio, resonance frequency).

By straightforward Fourier inversion of a measured transfer function from the frequency domain into the time domain, the time-dependence of the neutron flux response to a Dirac pulse in the perturbing parameter (for example pressure) can be reconstructed. As an example, figure 10 shows the Fourier inverted transfer function from pressure to neutron flux illustrated in figure 8b. A core resonance frequency of about 0.34 Hz and a decay ratio around 0.27 can be identified from the oscillatory behaviour of this response (disregarding the transients appearing in the first oscillation period). The same procedure can be applied to transfer functions from recirculation flow to neutron flux, such as those shown in figure 3 a-b, obtained from cross analyses of process noise.

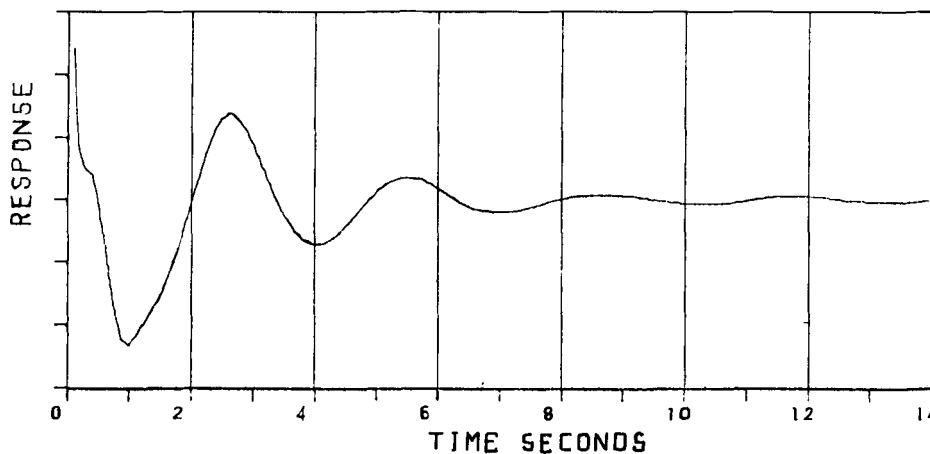


Fig. 10. Fourier inversion of the transfer function shown in fig. 8b.

When a measured transfer function clearly resembles the response properties of an ideal second order dynamic system, the core stability parameters may be identified already in the frequency domain. Such direct identification can often be applied to transfer functions from total steam flow to neutron flux, like the one shown in fig. 8a. The identification may be realised by fitting the experimentally obtained transfer function to the analytical expression describing the dynamics of an ideal second order system:

$$G = \text{const} \cdot (1 + 2j\zeta f/f_0 + f^2/f_0^2)^{-1}$$

to provide values for the "characteristic frequency" f_0 and the "damping coefficient" ζ (a similar procedure was used in ref. 5). Allowance may have to be made for the influence of higher oscillatory modes, and small time delays. From an identified value of ζ the "decay ratio" D is obtained according to

$$D = \exp(-2\pi\zeta / \sqrt{1-\zeta^2})$$

Under favourable circumstances, the stability parameters may even be estimated directly from the measured transfer function, utilising the following properties of G : at the characteristic frequency $f = f_0$ the phase shift is 90° , the slope of the phase curve versus (f/f_0) is $-1/\zeta$, and the gain in the same frequency point exceeds that at frequency zero by a factor $1/2\zeta$.

This procedure may be illustrated for the transfer function given in figure 8a, for example; one obtains a characteristic frequency of ~ 0.35 Hz and a damping coefficient of about 0.22, corresponding to a decay ratio around 0.25 (quarter damping). These results agree well with those obtained in fig. 10 by Fourier inversion of the corresponding transfer function shown in fig. 8b.

Interpretation of the transfer functions shown in figs 3 a-b, 5 and 7 in terms of stability parameters, along the lines described above, yield decay ratios well below quarter damping. Such generous stability margins are representative for the BWR cores investigated, under normal operating conditions. In these situations, the core resonance can be barely perceived in the neutron flux noise.

When operating points move towards more flow-starved conditions, the core resonance narrows and the decay ratio rises accordingly, as illustrated by figs 8 and 10. Fig. 11a shows the autospectrum of neutron flux noise recorded in the same operating point, illustrating that around quarter damping conditions, the core resonance becomes clearly visible in the noise. In consequence, an oscillation resembling a damped cosine appears in the corresponding autocorrelation in the time domain, as illustrated in fig. 11b. It is noteworthy that this autocorrelation displays about the same oscillation period and decay ratio as were identified for the core in this operating point, from the figs 8 and 10.

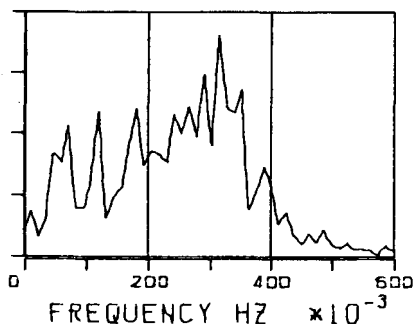


Fig. 11a. Autospectrum of APRM noise recorded near quarter damping conditions

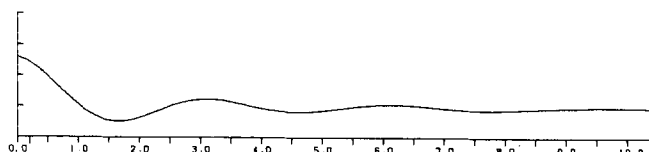


Fig. 11b. The corresponding autocorrelation in the time domain

When stability margins decrease further, the core resonance shows up with increasing clarity in the neutron flux noise. For example, figs 12 a-b and 13 a-b illustrate neutron flux noise autospectra and autocorrelations corresponding to decay ratios just above 0.5 (half damping) and near 0.75 ("three quarter damping"), respectively. These noise recordings were made during special tests of reactor performance at operating points with exceptionally high power/flow ratios, well outside the normal operating region. The autocorrelations strongly resemble damped cosine oscillations (with the possible exception of the first half period), and stability parameters can be readily identified. For example, fig. 13b can be interpreted in terms of an oscillation frequency of ~ 0.31 Hz and a decay ratio of about 0.74.

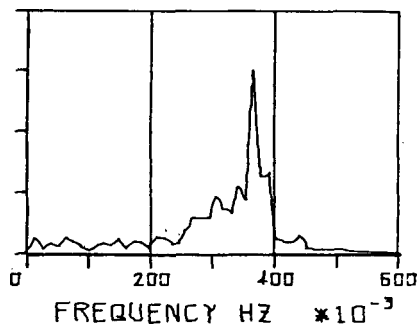


Fig. 12a. Autospectrum of APRM noise recorded just above half damping conditions

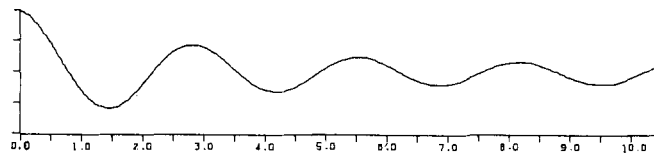


Fig. 12b. The corresponding auto-correlation in the time domain

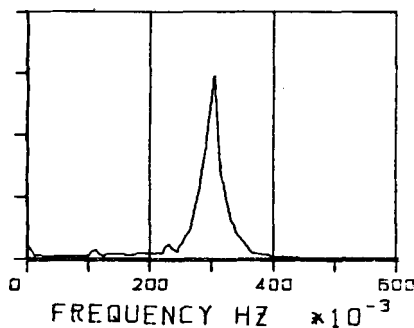


Fig. 13a. Autospectrum of APRM noise recorded near "three-quarter damping" conditions

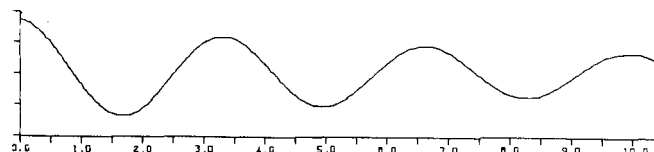


Fig. 13b. The corresponding auto-correlation in the time domain

These parameters, identified from the neutron flux noise, agree well with those obtained by Fourier inversion of the corresponding transfer function from pressure to neutron flux, measured in the same operating point, see fig. 14. Here, a resonance frequency of 0.33 Hz and a decay ratio of 0.74 can be identified.

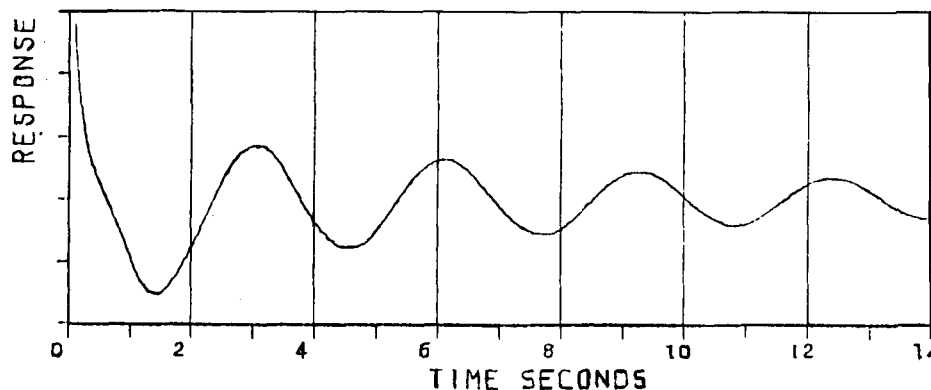


Fig. 14. Fourier inversion of a transfer function from pressure to APRM, measured in the same operating point where the noise in fig. 13 was recorded.

These results suggest that in situations when core decay ratios rise above quarter damping, the stability parameters can be identified via the oscillatory behaviour of the autocorrelation in the time domain of the neutron flux noise, with a precision that improves as the stability margins narrow. Since noise recordings do not interfere with plant operation, and only relatively small efforts are required for deducing the desired stability parameters, noise offers an attractive means for monitoring BWR core stability in operating points with reduced stability margins.

APPLICATIONS

Data collected from dynamic tests conducted in operating BWRs in Sweden and Finland, as well as noise recordings from the same plants, form an extensive data base that contains a wealth of information on dynamic core properties in modern BWRs. The data has been collected along the lines described in this paper, and the same applies to the procedures followed in the interpretation of the test data and the noise recordings in terms of dynamic core properties and stability parameters.

Several incentives exist for operating BWRs at reduced recirculation flows; this saves both neutrons and pumping power, and improves both operational flexibility and Xenon override capability. In all operating ASEA-ATOM BWRs, the recirculation flow window at full power has been increased from 10%, see ref. 8, to 25% à 40%, with corresponding increases at reduced power. The procedures described here have been employed to demonstrate adequate core stability in the extended operating regions. They have also been utilised in tests, aimed at probing dynamic core properties in special operating points with reduced stability margins, for example those reported in ref. 9.

ACKNOWLEDGEMENTS

The authors express their thanks to ASEA-ATOM, and its US subsidiary, AROS Corporation, Pa, for the permission to present this paper. We are grateful to Messrs S. Andersson, L-E. Häll, E. Karlsen, C. Kvarnström, O. Nylund, C-H. Palmgren, S. Olsson, U. Sandberg, S. Steinholtz and Y. Waaranperä for valuable advice and assistance.

REFERENCES

1. B. Almgren, F. Hetzler, S. Junkrans, A. van Santen, F. Segerberg, N E I, 15, 992 (1970).
2. ASEA-ATOM Technical Information 9, Computer Programmes for LWR Design and Fuel Management Evaluation.
3. S-Å. Andersson and J. H. Blomstrand, "A Mobile Computerised Multi-channel Data Recording System for Dynamic Testing and Noise Analysis", IAEA Specialist Meeting on Control System Commissioning and Dynamic Model Validation, Harrogate (1979).
4. W. H. Sides, M. V. Mathis, C. M. Smith, Prog. Nucl. Energy, 1, 119 (1977).
5. L. A. Carmichael and R. O. Niemi, "Transient and Stability Tests at Peach Bottom Atomic Power Station Unit 2 at End of Cycle 2," EPRI NP-564 (June 1978).
6. J. E. Wood, Trans. A N S, 30, 217 (1978)
7. L. A. Carmichael, B. W. Crawford, R. Kankus, J. Kooney, R. B. Linford, R. O. Niemi, R. N. Whitesel, Trans. A N S, 30, 218 (1978).
8. E. Tenerz and Ö. Bernander, Trans. A N S, 33, 801 (1979).
9. Y. Waaranperä and S. Andersson, Trans. A N S, 39, 868 (1981).

NEUTRONICS ANALYSIS OF BWR PARTIAL SCRAMS
WITH BORON INJECTION

B. Chexal, W. Layman, J. Power (EPRI/NSAC)
R. Pickering, (SCI)
R. Grow, C. Simonson (NAI)

ABSTRACT

This article summarizes a reactor neutronics analysis for a transient where some number of control rods do not insert. The analysis is applicable to a typical BWR-4 plant. It focuses on the power distribution in the core and on reactor criticality both with and without standby liquid boron injection. The work is directed towards providing better guidance to the operator regarding boron injection during partial control rod insertion incidents in boiling water reactors.

INTRODUCTION

Prior to TMI, reactor operator actions during partial failure-to-scrum events had been given only minor attention. Scram systems were designed to provide diverse, redundant, automatic and reliable service. The failure of a number of redundant components and instruments was thought to be an improbable, if not impossible event. Procedures to deal with such an occurrence did not exist at many operating plants.

The scram system did contain some status indicators but had few failure diagnostic instruments. Trouble-shooting for a degraded scram system was left to maintenance personnel rather than to the plant operators. The time needed for an operator to resolve a failure-to-scrum situation had not been studied extensively.

In general the analysis did not focus on the operators' abilities or limitations relative to a wide variety of system and component anomalies and on the time available for alternative action.

The Browns Ferry-3 event of July 24, 1980 focused attention on anticipated transient without scram events caused by partial failure of control rods to insert. The event itself had mild consequences but its ramifications went beyond Browns Ferry. It resulted in reevaluations of a number of related safety matters. These identified the need for better operator guidance, and also showed a number of other scram system anomalies. Failures are occurring more frequently than had been predicted and they are more complex than previously envisioned. Some failures may result in such high temperature and high radiation conditions in the reactor building that immediate operator action will be needed to shut the plant down.

The work is directed towards providing better operator guidance with regards to the mitigating actions during partial control rod insertion incidents in BWRs. For the current operating plants, reactor operator actions are now the most efficient and effective means of maintaining operating parameters within specified limits. This paper describes the analytical models developed to perform calculation of power distribution in the core and reactor criticality for different plant operating conditions.

SYSTEM DESCRIPTION

The reactor system which was used in this analysis was a typical BWR-4 rated at a 3293 Mwt. The reactor core consists of an array of fuel assemblies cooled by water and steam. Its nuclear energy is controlled by 185 control rods and their associated mechanical-hydraulic drive system. Each independent drive enters the core from the bottom, and can position accurately its associated control rod during normal operation and yet exert approximately ten times the force of gravity to insert the control rods during the scram mode of operation. The control rods occupy alternate spaces between fuel assemblies and may be withdrawn into the guide tubes below the core during plant operation. The rods are coupled to drive assemblies mounted entirely within housings attached to the bottom head of the reactor vessel.

The control rod drive system makes changes in core reactivity by incrementally positioning neutron-absorbing control rods within the reactor core in response to operator actions. The system is also capable of quickly shutting down the reactor (scram) in emergency situations by rapidly inserting all withdrawn control rods into the core in response to manual or automatic signals.

ANALYSIS

This section describes the EPRI-NODE-B computer program, decay heat and boron injection model developed during the course of this analysis, and the EPRI-NODE-B model of the plant.

EPRI-NODE-B Computer Program

The EPRI-NODE-B computer program (References 2, 3) provides a complete global reactor analysis capability by simultaneously solving for the core effective multiplication factor, the three-dimensional core power distribution, core coolant flow and void distribution, xenon distribution and fuel exposure distribution. The program accurately accounts for the effects of partially inserted full-length control rods, full or part length burable poisons, and up to 13 different fuel assembly types. EPRI-NODE-B has the capacity to represent the core with up to 24 axial nodes and arrays of up to 30 x 30 assemblies in the XY plane.

The program iterates to account for the interaction between power distribution and core nuclear properties which depend on coolant flow and coolant void distributions, fuel temperature distribution and xenon distribution. For each space node the time dependence of xenon following changes in power level and/or changes in power distribution is computed. The program also computes the fuel exposure distribution and accounts for fuel shuffling and fresh fuel insertion for burnup cycle calculations. Individual steps may be stacked for either xenon transient or fuel cycle burnup calculation.

The particular version used in this study is implemented on a PRIME-750 computer as part of the EPRI Power Shape Monitoring System (PSMS) (Reference 4).

Decay Heat Model

The standard EPRI-NODE-B computer code does not include a decay heat model. Such a model was necessary since the transient decay heat could invalidate the EPRI-NODE-B criticality and power distribution results at low power conditions. The computer code was modified for this study to include a transient decay heat simulation model.

The decay heat model implemented in the EPRI-NODE-B code is based on Reference 5. The model fits the nominal ANS decay power curve as a sum of 23 exponential decay terms.

$$f(t) = \sum_{i=1}^{23} \alpha_i \exp(-\lambda_i t), \text{ MeV/fissions-sec}$$

where α_i , λ_i = constants, given in Table I (Reference 5) for U^{235} fission

t = Time after shutdown, seconds

The decay power from products of each fissioning nuclide, $P_d(t, T)$ is then approximated by:

$$P_d(t, T) = \sum_{n=1}^M \frac{P_n}{Q} \sum_{i=1}^{23} \frac{\alpha_i}{\lambda_i} \exp(-\lambda_i t_n) [1 - \exp(-\lambda_i T_n)]$$

where n = Index to identify the periods of constant fission rate in a histogram representation of the fission rate

P_n = Power from fissioning of nuclide during reactor operation period 'n'

Q = Total recoverable energy associated with one fission of nuclide

T_n = Periods of constant fission rate

$$T = \sum_{n=1}^M T_n$$

t = Time after shutdown, seconds

$$t_1 = t; t_2 = t + T_1; t_m = t + \sum_{n=1}^{m-1} T_n$$

For a nominal value of Q of 200 MeV/fissions, the sum

$$\sum_{i=1}^{23} \frac{\alpha_i}{\lambda_i} / Q = 0.0659$$

equals the steady state decay heat fraction at any power.

Boron Injection Model

The boron-coolant mixing model was added to account for the effect of changes in boron concentration in the BWR core by including soluble poison terms in the reactivity equation. The explicit functional equation for boron number density and effect on reactivity was developed using CASMO (Reference 6). The methodology used here is very similar to that employed in PWR modeling and the results obtained for boron reactivity are very similar to those found in PWR models.

The effect of soluble boron was studied by introducing varying boron concentrations at beginning of life and with different conditions: cold, hot zero power, rodged and unrodged, and hot full power at 3 different void conditions. Also, the most abundant fuel type was depleted to end of cycle, and branch calculations with varying boron were made to determine the change in boron reactivity with exposure.

The base model was generated using the fuel assembly types present in other EPRI work. The model for the third cycle was used, because it contains a representative selection of once and twice burned fuel together with fresh assemblies. Figure 1 shows the core map.

The modifications to EPRI-NODE-B include changes for temperatures less than saturation and changes for the reactivity effects of soluble boron in both rodged and unrodged nodes.

For temperatures $T < T_{sat}$ at zero power conditions, the following equations were added:

Migration area:

$$M^2 = C_1 + C_2T + C_3T^2 \quad (1)$$

Unrodded multiplication factor:

$$k_{\infty NR} = C_4 (1 + C_7T + C_{10}T^2) \quad (2)$$

Rodded multiplication factor:

$$k_{\infty R} = C_6 (1 + C_9T + C_{12}T^2) \quad (3)$$

The reactivity effects of soluble boron were added using the following equations:

Boron nodal number density:

$$N_B = \rho_{H_2O} C_n C_B (.01102 \times 10^6), \quad (4)$$

where

ρ_{H_2O} = water density,

C_B = boron concentration in ppm,

$C_n = C_{24}$ for unrodded node, where

C_{24} is the H_2O volume fraction,

$C_n = C_{25}$ for rodded node, where

C_{25} includes the
decrease in soluble boron worth in the presence
of a control rod

Boron reactivity:

$$\delta\rho_B = (C_{49} + C_{56} \alpha + C_{58} \alpha^2) N_B + (A_{49} + A_{56} \alpha + A_{58} \alpha^2) (B + CE + DE^2), \quad (5)$$

where

$$\delta\rho_B = (k_1 - k_2)/k_1 k_2, \quad (6)$$

k_1 = nodal $k_{\infty NR}$ or $k_{\infty R}$,

k_2 = nodal k_{∞} with boron present,

α = nodal void fraction,

E = exposure.

As with the base case, the constants in these expressions were evaluated using CASMO (Reference 6) calculations. The exposure-dependent term was derived from branch calculations from a depletion case for the predominant fuel type. The remaining terms were evaluated at beginning of life for each fuel type present.

EPRI-NODE-B Core Model Description

The load map of the 764 bundle is shown in Figure 2. The core is composed of three bundle types, initial-load low enrichment fuel and high enrichment bundles from reloads 1 and 2. All fuel bundles are 8 x 8 fuel. Three enrichments of the fuel are identified below.

	<u>Fuel Types</u>	<u>Enrichment (wt. % UO₂)</u>
1. Reload #2 Fuel (144 Bundles)	8 x 8	2.65
2. Reload #1 Fuel (208 Bundles)	8 x 8	2.65
3. Initial Fuel Load (412 Bundles)	8 x 8	2.19

Selection of Input Conditions

Initial Conditions

The data used for the initialization of the core were taken from a typical BWR-4 high power operating state prior to the event. The core average exposure was approximately 12,700 Mwd/t.

The following thermal-hydraulic parameters were used for the initial steady state calculations.

Reactor Power.....	3219 MWt (98.5%)
Total Core Flow.....	101×10^6 lbm/hr
Feedwater Flow.....	13.00×10^6 lbm/hr
Reactor Vessel Pressure.....	1020 psia
Core Inlet Subcooling.....	23.2 Btu/lbm

Boron Concentration Buildup

The boron concentration buildup rate has been calculated for a typical 251" reactor at 15% core flow and 86 GPM liquid boron flow. This buildup rate is shown in Figure 3.

Control Rod Scram Group Assignment

A typical 764 fuel assembly BWR-4 plan contains 185 control rods. Each control rod is electrically assigned to one of four groups as shown in Figure 4. In the half-scrum cases analyzed in this report it was assumed that either groups 2 and 4 failed to scram (electrical failure) or control rods in the west half failed to scram (hydraulic failure).

RESULTS

The effects of partial control rod insertion and boron injection on three different types of transient conditions were investigated.

Loss of Feedwater Heater

The first transient involves the loss of feedwater heaters which represents the most limiting transient relative to reactivity control. In this event, loss of a critical group of feedwater heaters reduces the reactor coolant feedwater temperature (decreased 65°F) which produces an increase in core inlet subcooling leading to an increase in core power. Following the transport delay through the feedwater lines (neglected in this analysis) and the time constant for cool-down of the heater tubes, average fuel surface heat flux rises to a maximum value of 106% which is lower than the flux scram set-

point. No fuel reaches boiling transition, even if the plant was initially at thermal operating limits. The reactor is conservatively assumed to be on manual recirculation flow control, therefore, core inlet flow remains at 100%. Had the reactor been on automatic flow control, core inlet flow would have changed to decrease the severity of the transient. Figure 5 shows the response of this event. The water level remains within the normal control range throughout the transient.

If the power reaches 108% of rated power a high power alarm occurs. For this analysis, it is assumed that attempts will be made to bring the power down by inserting rods. If this is not successful, manual scram will be initiated. Recent industry decisions will incorporate SLCS time logic actuation that will develop a full boron injection after 2 minutes. However, in this analysis, manual scram is also assumed to fail. If all attempts to insert control rods fail within the two minute period, the automatic start of boron injection will begin through the jet pump instrument lines. By about 35 minutes the power has decreased below 1%. Recirculation flow remains active during boron injection, providing mixing and dispersion throughout the primary system.

If half of the control rods are assumed to scram (i.e., groups 1 and 3) the power is rapidly reduced as shown in Figure 6.

In both cases, the fuel remains well within thermal safety limits throughout the transient.

The loss of feedwater heating reduces core inlet enthalpy resulting not only in an increase in thermal power level, but a shift in power shape as well. If the fuel is operating at or near the preconditioned envelope in any region of the core, either of these power changes may cause pellet clad interaction.

Total loss of feedwater heating would result in a scram of the reactor on high Average Power Radiation Monitor (APRM) values, and probably no fuel failures due to the short duration of the overpower. However, smaller feedwater heating loss events which would not cause a scram, or extended duration events which were not corrected could lead to fuel failures.

Failure of Control Rods of 2 out of 4 Electrical Groups to Scram

This transient involves a scram of half the control rods resulting from an electrical failure from full power. In this event, the core is assumed to be operating at full power for effectively an infinite time, thereby maximizing the effects of decay heat.

The initial case investigated was a scram with half of the control rods (groups 1 and 3) failing to scram. In this event, the core power is rapidly reduced as shown in Figure 7.

In the second case, a complete failure to scram is assumed but the SLCS is assumed to operate with a 60 second delay and an assumed boron injector rate as shown in Figure 3. In this event the core power is gradually reduced as shown in Figure 9.

In either the half scram case or the delayed SLCS actuation, the fuel remains well within thermal limits.

Failure of Control Rods in West Half of the Core to Scram

A third analysis was performed to evaluate the effect of a failure to scram of the control rods in west half of the core due to a hydraulic failure at 100% power and flow. This event can result from the blockage of the scram discharge header volume.

The initial condition was a typical BWR-4 operating at essentially full power and flow. The assumption was made that the control rods in east half of the core scrammed and the control rods in west half of the core did not move.

Two different conditions were analyzed:

1. The recirculation pumps were not tripped and the core flow continued at 100% flow.
2. The recirculation pumps were tripped with the corresponding reduction in core flow.

Because of the delay before the boron solution reaches the core inlet, the effects of the boron are delayed for 60 seconds.

The core bundle power distributions both axially and radially for both cases at 9 seconds after scram are shown in Figures 10-15. As illustrated by the Figures, the core power is shifted to the partially uncontrolled portion of the core in the east half.

The long term effect of the boron is to reduce gradually the power generation of the core until only the decay power is a significant contribution to total power.

From the results we conclude that the local assembly power level in condition one was limited to approximately 60% of rated full power and in condition two was limited to 41% of rated full power. The maximum nodal power in condition one was approximately 69% of rated full power and for condition two was approximately 49% of rated full power.

Since in condition one, the flow was maintained at full rated flow, the fuel was well within its design capability.

Under condition two, the flow is estimated to be reduced to approximately 41% of rated flow. Under these conditions the thermal margins would not be degraded to the point of approaching the safety limits. Therefore, under both conditions, the plant would maintain adequate thermal margins.

An initial condition of no control rod in the core, which may exist at certain points in the cycle, would result in more severe peaking in the uncontrolled portion of the core during the transient possibly resulting in more severe thermal conditions.

CONCLUSIONS

1. The transient resulting from the loss of feedwater heater corresponding to a decrease of feedwater temperature of 65°F can easily be handled by a scram of only 2 out of the 4 electric groups. Even if all the control rods fail to scram, the core can be shut down through the use of the SLCS without violating thermal safety limits as long as the uniform mixing of

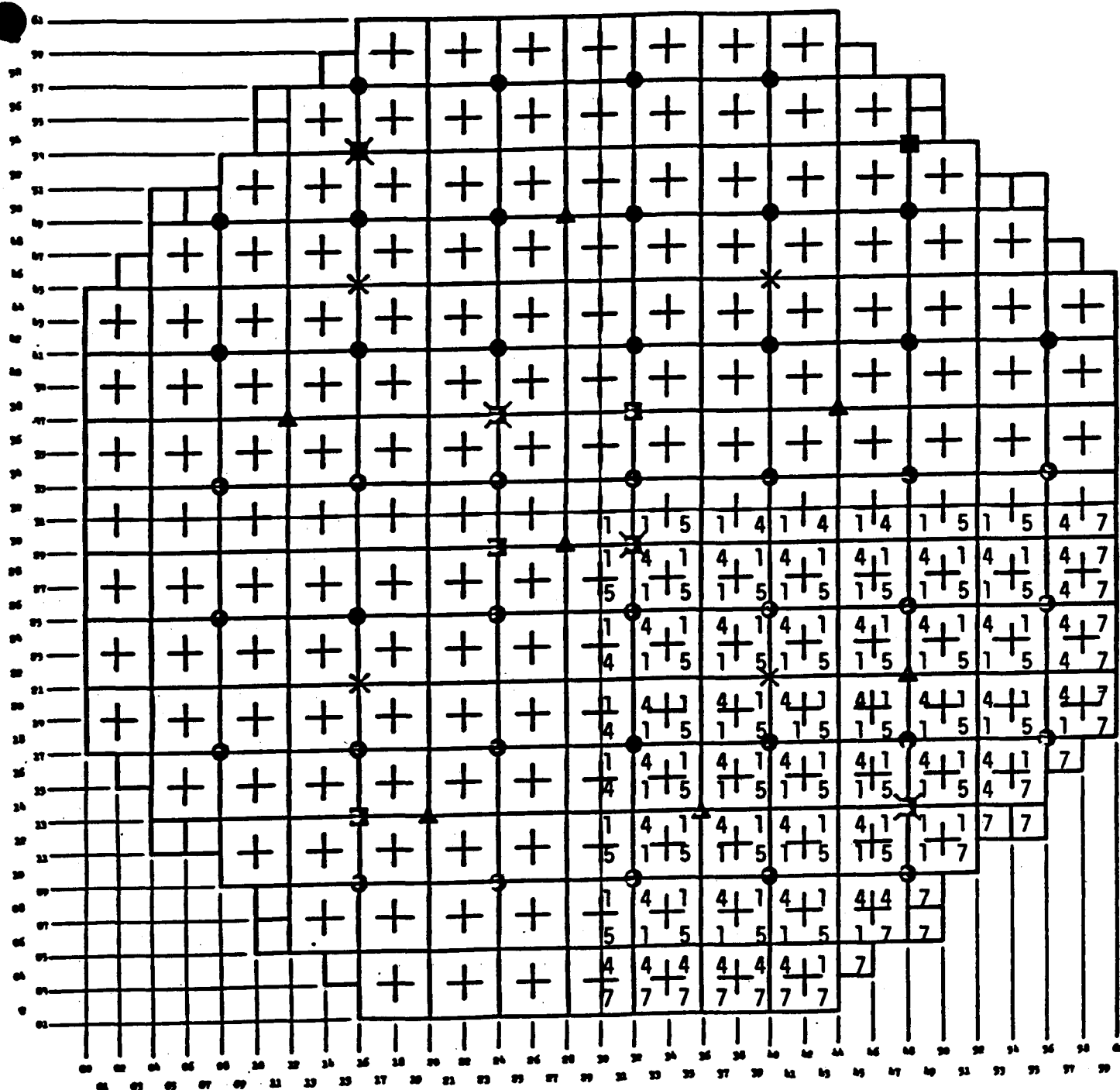
boron occurs. If the recirculation pumps are left on energy released during this transient would be significantly increased. A loss of feedwater heating reduces core inlet enthalpy resulting in not only an increase in thermal power level but also a shift in power shape. If the fuel is operating at or near the preconditioned envelope in any region of the core, either of these power changes can result in pellet clad interactions.

2. If the core is operating at 100% full power, the plant can be successfully shutdown by a scram of 2 out of the 4 electrical groups plus the SLCS. Even if all of the control rods fail to scram, the core can be shut down by the SLCS without violation of thermal limits providing uniform mixing of the boron occurs. However without scram the amount of energy released during such a transient would be significantly increased.
3. If the core is operating at 100% full power in a typical operating control rod pattern and the control rods in west half or east half of the core fail to scram, the core can be successfully shutdown with the SLCS without violating the plant safety margins. However, such a failure to scram with no control rods initially in the core would present a more severe condition.

References

1. Neutronics Analysis of Browns Ferry 3 Partial Control Rod Insertion, Electric Power Research Institute/Nuclear Safety Analysis Center Report, NSAC-33, June 1981.
2. EPRI-NODE-B, Advanced Recycle Methodology Program System Documentation, Part II, Chapter 15, Electric Power Research Institute, EPRI CCM-3, January 1976.
3. EPRI-NODE-P, Advanced Recycle Methodology Program System Documentation, Part II, Chapter 14, Electric Power Research Institute, EPRI CCM-3, January 1976.
4. Power Shape Monitoring System, (PSMS), Electric Power Research Institute, EPRI Np-1660, Final Report - February 1981.
5. A Revised ANS Standard For Decay Heat From Fission Products, Nuclear Technology, Vol. 46, December 1979.
6. CASMO, A Fuel Assembly Burnup Program, Users' Manual, AF-RF-76-4158, 1978.

TYPICAL BWR-4 CYCLE 3 CORE POSITION MAP



Fuel Type 1 - 2.19 w/o, initial core
 4 - 2.19 w/o, reload 1
 5 - 2.65 w/o, reload 2
 7 - 2.19 w/o peripheral core location

FIGURE 1

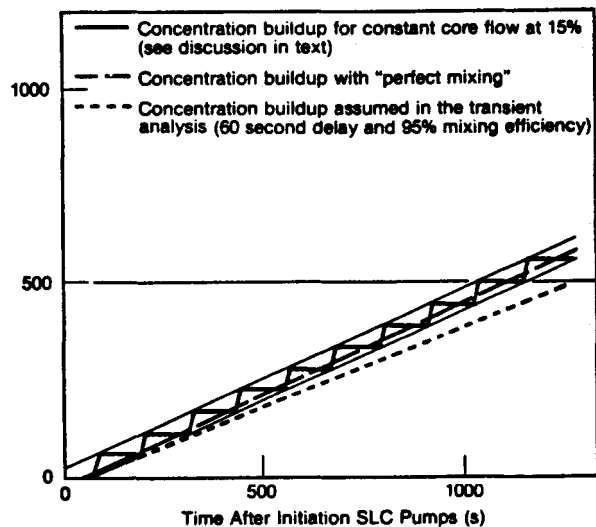
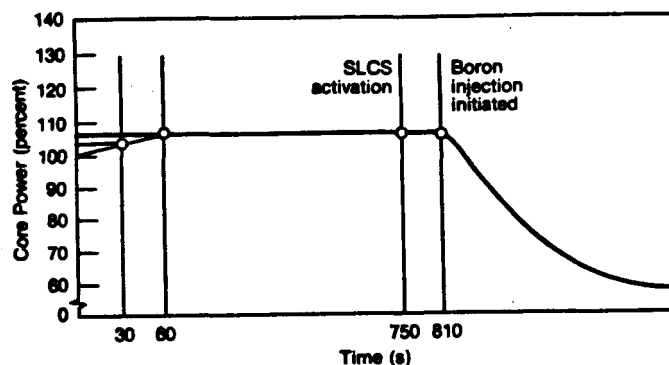


Figure 3. Boron concentration buildup in core water for jet pump injection liquid boron flow of 86 gpm.



Time (s)	Power (percent of initial power)
1150	35.0
1486	5.8

Figure 5. Loss of feedwater heater—no scram.

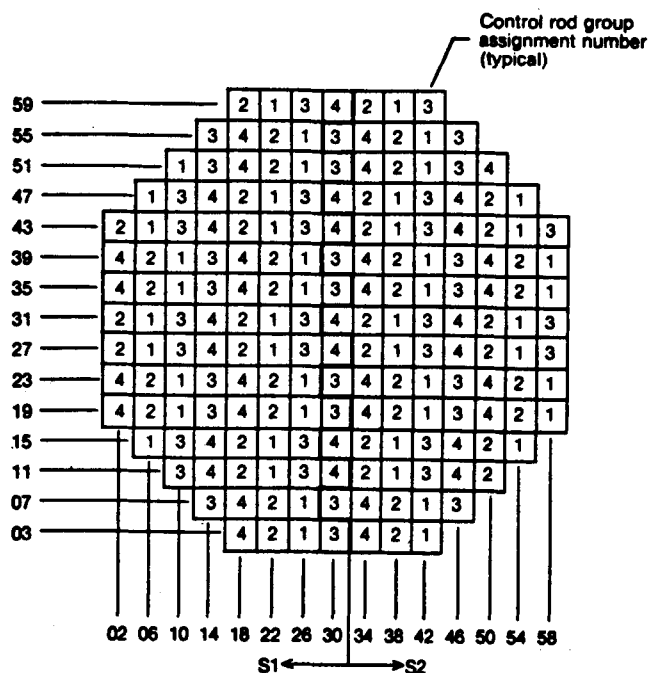
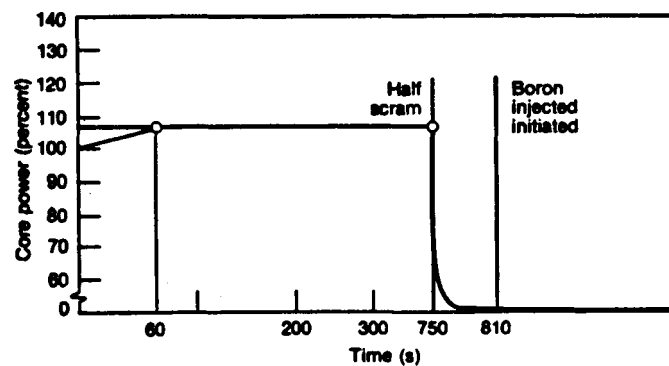


Figure 4. Control rod scram group assignment.



Time (s)	Power (percent of initial power)
759	5.5
768	5.3
1166	2.5

Figure 6. Loss of feedwater heater—half scram.

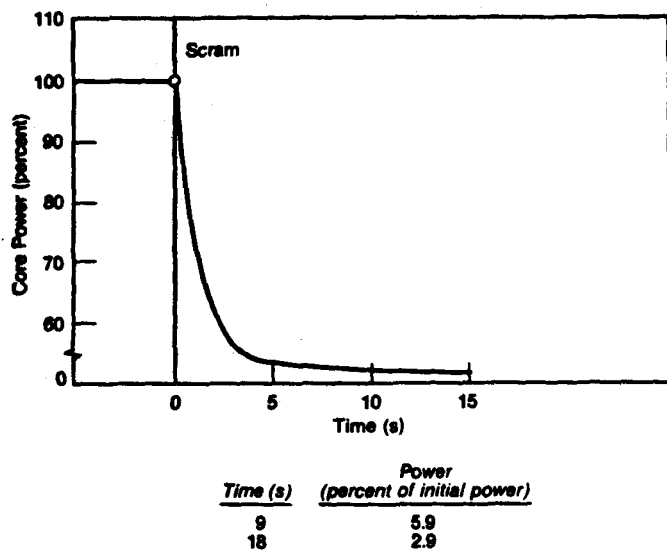


Figure 7. Half-scam from full power.

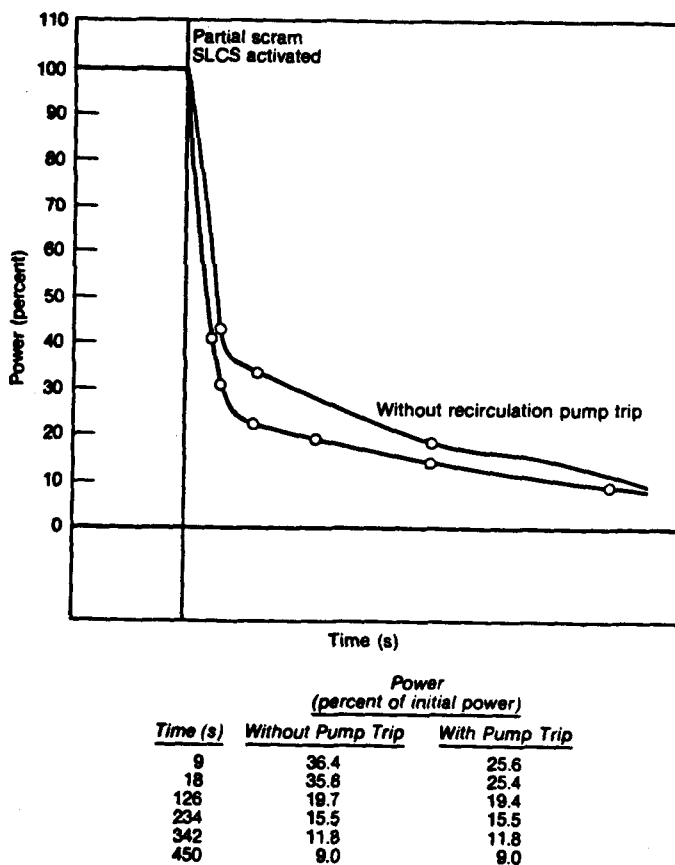


Figure 9. Half-scam from full power with and without recirculation pump trip.

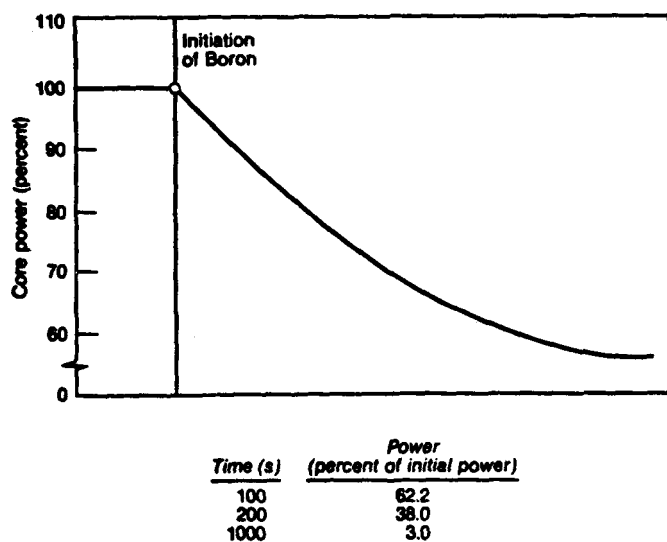


Figure 8. No-scam from full power with Boron injection.

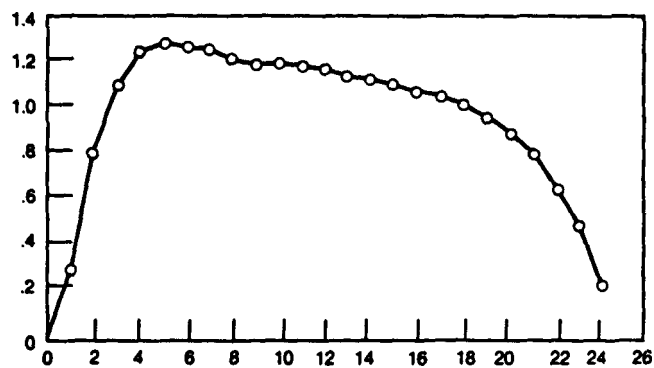


Figure 10. Core average axial power distribution 100% power.

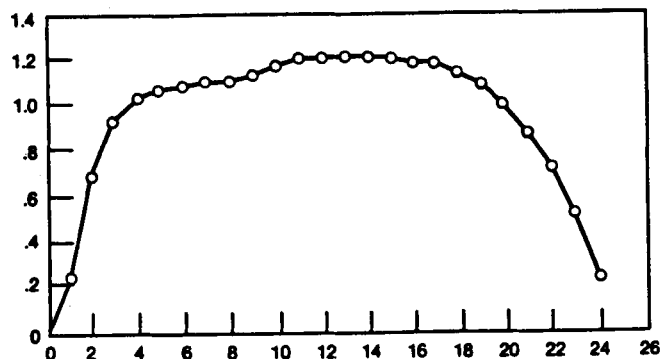


Figure 11. Core average axial power distribution at 9 seconds with recirculation pump trip.

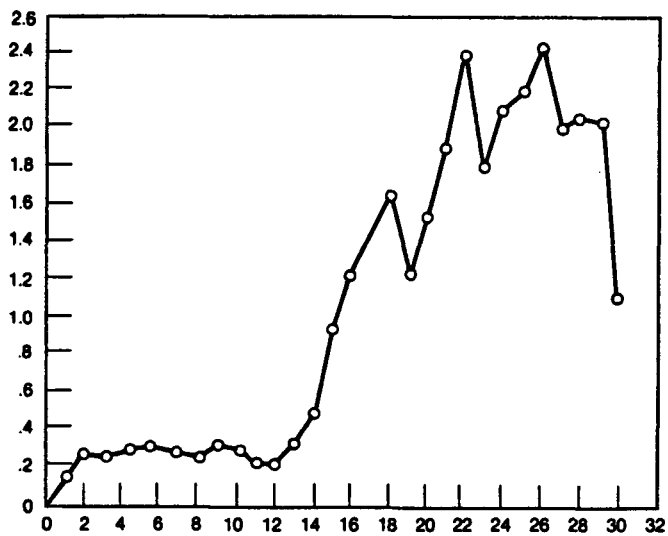


Figure 14. Mid-plane radial power distribution with recirculation pump trip.

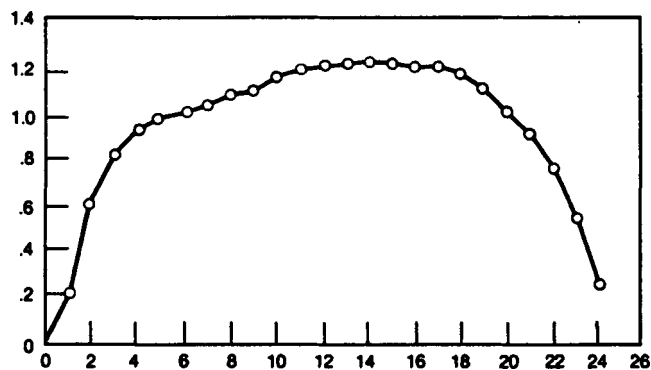


Figure 12. Core average axial power distribution at 9 seconds without recirculation pump trip.

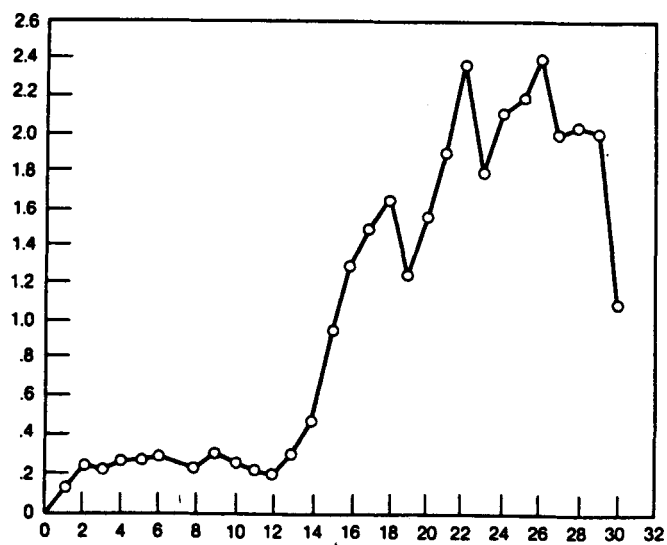


Figure 15. Mid-plane radial power distribution without recirculation pump trip.

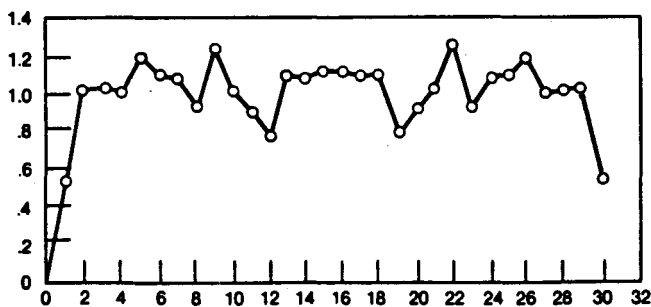


Figure 13. Mid-plane radial power distribution, 100% power.

ANALYSIS OF NEUTRONIC SPACE-TIME EFFECTS IN LARGE LMFBF's
WITH THERMAL-HYDRAULIC FEEDBACK USING FX2-TH NUCLEAR REACTOR
KINETICS CODE AND MODE EXPANSION OF THE NEUTRON FLUX SHAPE FUNCTION

B. S. Yarlagadda
Argonne National Laboratory
Applied Physics Division
9700 South Cass Avenue
Argonne, Illinois 60439

ABSTRACT

For a delayed critical transient in an LMFBF, the dependence of the local flux tilt on the degree of heterogeneity in the core configuration was examined. Using a mode expansion of the neutron flux shape function, decoupling the resulting mode kinetics equations and utilizing the prompt jump approximation, an expression for the i th harmonic local flux tilt was developed. According to this expression, the i th harmonic local flux tilt $\tau_i(\vec{r}, t)$ is inversely proportional to the sum of the eigenvalue separation between the i th harmonic and the fundamental mode and the delayed neutron fraction $(\frac{1}{k_i} - \frac{1}{k_0} + \beta)$ and

is directly proportional to a time dependent factor $T_i(t)$ and a space dependent factor $G_i(\vec{r})$. The modified eigenvalue separation $(\frac{1}{k_1} + \frac{1}{k_0} + \beta)$ and $G_i(\vec{r})$ are

known when the eigenvalues and eigenfunctions of the unperturbed reactor are available. Thus if $T_i(t)$, which depends upon the perturbation, varies little in comparison with the modified eigenvalue separation from one core configuration to another, a simple measure of the i th harmonic local flux tilt is feasible. This flatness criterion for $T_i(t)$ has been investigated by inserting 2¢/sec ramp reactivity in three core configurations. During the transient, the shape function was calculated with the FX2-TH code and $\tau_i(\vec{r}, t)$ and $T_i(t)$ were extracted using mode expansion of the shape function. The results of this analysis are presented

INTRODUCTION

The flux tilt or the change in the neutron flux shape function may be significant during some delayed critical transients in Large Heterogeneous Liquid-Metal Fast Breeder Reactors¹ (LMFBF's). The shape function may be calculated accurately using the two-dimensional FX2-TH code² which takes into account thermal-hydraulic feedback effects. However, during the design stage when several reactor configurations with differing degrees of heterogeneity and differing fuel compositions may be under consideration, transient analysis studies with the FX2-TH code could be too expensive. Hence, a simple measure of the flux tilt (albeit approximate) would be useful in the transient analysis of heterogeneous fast breeder reactors. It has been shown previously^{3, 4, 5} that, in the asymptotic steady state, such a measure can be provided by the fundamental to the first harmonic eigenvalue separation if the asymptotic neutron flux may be approximated by a linear combination of the fundamental and the first harmonic eigenfunctions of the unperturbed reactor. During the transient, however, the expansion of the neutron flux shape function in terms of the fundamental and the first harmonic eigenfunctions alone may not be adequate. Hence, it is assumed that the shape function may be approximated

by an expansion in terms of the fundamental mode and few lower harmonics.

The flux tilt $\tau(\vec{r}, t)$ is defined such that i 'th mode contribution to it is equal to the product of the time dependent coefficient $b_i(t)$ in the expansion of the shape function and a space dependent term $G_i(\vec{r})$ which depends upon the fundamental and i th harmonic eigenfunctions. Following the methods of Wade and Rydin³ and Kaplan⁴ and using suitable approximations, it is shown in the Appendix that $b_i(t)$ is inversely proportional to a quantity which is the sum of the i th harmonic eigenvalue separation from the fundamental and the effective delayed neutron fraction (β) with a time dependent proportionality factor $T_i(t)$. This modified eigenvalue separation reduces to the asymptotic form when the i th harmonic eigenvalue separation is much larger than β . The modified eigenvalue separations and $G_i(\vec{r})$ are essentially known once the modes of the unperturbed steady state reactor are evaluated. If the variation of $T_i(t)$ in going from one reactor configuration to another is relatively smooth in comparison with the corresponding variation in the eigenvalue separation or $G_i(\vec{r})$, a simple measure of the local flux tilt may be feasible. Since evaluation of $T_i(t)$ in the presence of space dependent thermal-hydraulic feedback requires knowledge of the shape function itself, the following empirical procedure was adopted. After evaluating the modes of the initial steady state reactor, at full power, the transient was followed using the FX2-TH code. The modal flux tilt $\tau_i(\vec{r}, t)$ was evaluated during the transient and $T_i(t)$ was extracted from it by using a suitable normalization for the mode functions used in the evaluation of $G_i(\vec{r})$.

THEORY

The mode representation of a neutron flux shape function may be written as

$$\phi(\vec{r}, E, t) = \sum_i b_i(t) \psi_i(\vec{r}, E) \quad (1)$$

where $\psi_i(\vec{r}, E)$ are the eigenfunctions of the unperturbed reactor in the initial steady state and $b_i(t)$ are the coefficients of the expansion.

Let the local power density in a small volume ΔV around the point $\vec{r}$ be

$$P(\vec{r}, t) = \frac{1}{\Delta V} \int_{\Delta V} M(\vec{r} + \vec{r}', E, t) \phi(\vec{r} + \vec{r}', E, t) d\vec{r}' dE \quad (2)$$

and local relative power density be

$$R(\vec{r}, t) = \frac{P(\vec{r}, t)}{P(\vec{r}, 0)} \quad (3)$$

where $M(t) = M_0 + \delta M_0(t)$ is the fission operator.

Correspondingly, the total power and the relative total power are given by

$$P(t) = \int_{\text{Reactor}} M(\vec{r}, E, t) \phi(\vec{r}, E, t) d\vec{r} dE \quad (4)$$

and

$$R(t) = \frac{P(t)}{P(0)} \quad (5)$$

Then, the local flux tilt in a small volume ΔV at point $\vec{r}$ may be defined as

$$\tau(r, t) = \frac{R(\vec{r}, t)}{R(t)} - 1. \quad (6)$$

On substitution of the expressions for $\phi(\vec{r}, E, t)$, $R(\vec{r}, t)$ and $R(t)$, Eq. 6 gives

$$\begin{aligned} \tau(\vec{r}, t) &= \sum_i \frac{b_i(t)}{b_0(t)} G_i(\vec{r}, t) - [1 - G_0(\vec{r}, t)] \\ &= \sum_i \tau_i(\vec{r}, t) - [1 - G_0(\vec{r}, t)] \end{aligned} \quad (7)$$

where

$$G_i(\vec{r}, t) = \frac{\int_{\Delta V} M(\vec{r} + \vec{r}', E, t) \psi_i(\vec{r} + \vec{r}', E) dE d\vec{r}'}{\int_{\Delta V} M_0(\vec{r} + \vec{r}', E) \psi_0(\vec{r} + \vec{r}', E) dE d\vec{r}'} \quad (8)$$

An equation for the expansion coefficient $b_i(t)$ is derived in the Appendix and is given by

$$b_i(t) = \frac{T_i(t)}{(\frac{1}{k_i} - \frac{1}{k_0} + \beta)} \quad (9)$$

where k_0 and k_i are the multiplication constants in the fundamental mode and the i th mode respectively, $T_i(t)$ is a time dependent factor depending on the mode dependent transition reactivity and precursor concentrations (see Eqs. A-14, A-17 and A-30 in the Appendix), and β is the effective delayed neutron fraction. For the delayed critical transients under consideration in this study, where the net reactivity inserted is not large, the coefficient of the fundamental mode $b_0(t)$ in the expansion of the shape function may be assumed to be close to unity and since $M(\vec{r}, E, t)$ and $M_0(\vec{r}, E, 0)$ may not differ significantly from each other the time dependence in $G_i(\vec{r}, t)$ may be ignored. Hence the i th harmonic local flux tilt may be written as

$$\tau_i(\vec{r}, t) \approx \frac{T_i(t)}{(\frac{1}{k_i} - \frac{1}{k_0} + \beta)} G_i(\vec{r}) \quad (10)$$

METHODOLOGY

Two-dimensional time-dependent multigroup diffusion equations are solved in FX2-TH code² by use of the improved quasistatic method.⁶ The thermal-hydraulic model option used in this study solves the heat conduction equations in a one-dimensional fuel pellet, gap, clad and coolant configuration. The code takes into account feedback effects from changes in both the average fuel and the average coolant temperatures. The neutronics solutions are based on an eight energy group cross section set for four temperatures and on six delayed neutron family data derived from the ENDF/B-IV data files.

Due to limitations of the FX2-TH code, it is assumed that boiling does not occur at any point in the transient, that there is no distortion of geometry and that fuel does not expand. The transient modeled for this study is due to withdrawal, at power, of a single control rod located in the farthest ring from the center of the reactor. The calculations were done in triangular geometry with half core symmetry and the half-core was divided into about 60 thermal-hydraulic regions.

For modal analysis of the neutron flux shape function during the transient, eigenfunctions of the initial steady state reactor are needed. A macroscopic cross section file for the materials in each thermal-hydraulic region was generated using the converged steady state fuel and coolant temperature distributions obtained in the steady state option of the FX2-TH code. With the half core symmetry in triangular geometry, these cross sections were used in the RINGIT code⁷ to calculate the fundamental mode and the harmonics of the initial steady state reactor. The RINGIT code makes use of a random guess for the initial real and adjoint neutron flux distributions and calls the DIF3D code⁸ to calculate both the real and adjoint eigenfunctions for each harmonic. When a harmonic has converged to the desired accuracy, that harmonic is stripped from the flux guess and the code will proceed to the calculation of the next harmonic using the stripped flux guess.

CORE CONFIGURATIONS

The three core configurations used in this study are shown in Fig. 1. The size of the reactors was 1000 MW(electric) and the degree of heterogeneity ranged from homogeneous, through tightly coupled heterogeneous to loosely coupled heterogeneous. In the tightly coupled core, one row of internal blankets separates the core regions and in the loosely coupled core, two rows of internal blankets separate the core regions. Both the heterogeneous cores have a blanket at the core center. All cores were enrichment zoned, two zones in the case of the homogeneous configuration and three zones in the case of the heterogeneous configurations. The general reactor data and the fuel and blanket pin descriptions on which the assembly descriptions are based are given in Table I.

DISCUSSION OF RESULTS

For each core configuration, the initial steady state neutron flux, fuel and coolant temperatures were calculated with the steady state option in the FX2-TH code. Four lower harmonics besides the fundamental mode were calculated at the steady state for the configurations and they were numbered sequentially in the direction of increasing eigenvalue separation. A schematic diagram of the nodal lines and the eigenvalue separation from the fundamental mode of each harmonic is given in Table II. In the three configurations considered here,

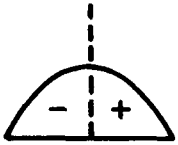
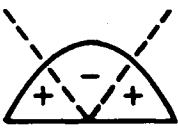
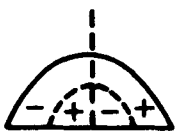
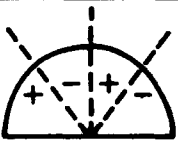
TABLE I
Design Characteristics

<u>General Reactor Data</u>			
Reactor power, MW(electric)		1000	
Reactor power, MW(thermal)		2740	
Thermal efficiency, %		36.5	
Core height, cm		101.6 (40 in.)	
Axial blanket thickness, cm		38.1 (15 in.)	
Reactor outlet temperature, °C		499 (930°F)	
Reactor ΔT , °C		156 (280°F)	
<u>Fuel Pin Design Data</u>		<u>Blanket Assembly and Pin Design</u>	
Fuel material	Pu-UO ₂	Fuel material	UO ₂
Fuel/clad bond	Helium	Fuel/clad bond	Helium
Pin o.d., mm	7.4 (0.290 in.)	Duct outside flat-to-flat, cm	15.34 (6.040 in.)
Cladding thickness, mm	0.35 (0.014 in.)	Duct wall thickness, mm	3.58 (0.141 in.)
p/d ratio	1.186	Number of pins	127
Wire spacer diameter, mm	1.35 (0.053 in.)	Pin o.d., mm	11.94 (0.470 in.)
Plenum length, cm	122 (48 in.)	Cladding thickness, mm	0.38 (0.015 in.)
Fuel smeared density, % TD	90	p/d ratio	1.070
Linear power, W/cm		Fuel smeared density, % TD	90
Nominal peak	459 (14.0 kW/ft)		
Average	295 (9.0 kW/ft)		
<u>Fuel Assembly Design</u>		<u>Blanket Volume Fractions</u>	
Number of pins	271	Fuel (smeared)	0.5592
Assembly pitch, cm	16.03 (6.310 in.)	Structure	0.1656
Duct outside flat-to-flat, cm	15.34 (6.040 in.)	Sodium	0.2752
Duct wall thickness, mm	3.58 (0.141 in.)		
Interassembly gap, mm	6.86 (0.270 in.)		
Nozzle-to-nozzle Δp , kPa	620 (90 psi)		
Peak coolant velocity, m/s	7.9 (26.0 ft/s)		
<u>Core Volume Fractions</u>			
Fuel (smeared)	0.4238		
Structure	0.1969		
Sodium	0.3793		

the first and second harmonics have angular nodes and the third harmonic has a radial node. The fourth harmonic in the loosely coupled core has one angular and one radial node while the fourth harmonic in both tightly coupled and homogeneous cores has angular nodes only. For each harmonic, the eigenvalue separation decreases as the degree of heterogeneity increases. For example, the eigenvalue separation of the first harmonic in the loosely coupled core is smaller approximately by factors of 1.5 and 3.0 than the first harmonic eigenvalue separation in the tightly coupled core and the homogeneous core respectively. The contribution of each harmonic to the local flux tilt was calculated at chosen intervals during the transient.

An overpower transient was initiated in each core configuration by the withdrawal of a single control rod at a rate adjusted so as to insert 2¢/sec ramp reactivity. Due to thermal-hydraulic feedback, the net reactivity insertion rate was lower in each core configuration as shown in Fig. 2. This decrease can be accounted for by the large negative reactivity contribution from the Doppler feedback and a relatively smaller positive contribution from the coolant density feedback. The net reactivity inserted after 15 sec. was 6.0¢, 6.9¢ and 7.8¢ respectively in the homogeneous core, the tightly coupled core and the loosely coupled core. The relative reactor power is shown in Fig. 3 as a function of time for each core configuration. The time dependent local

TABLE II. Eigenvalue Separations in the Three Configurations

Harmonic i	Schematic Diagram of Nodal Lines in Half Core	Loosely Coupled Core		Tightly Coupled Core		Homogeneous Core	
		$\left(\frac{1}{k_i} - \frac{1}{k_0}\right)$	$\left(\frac{1}{k_i} - \frac{1}{k_0} + \beta\right)$	$\left(\frac{1}{k_i} - \frac{1}{k_0}\right)$	$\left(\frac{1}{k_i} - \frac{1}{k_0} + \beta\right)$	$\left(\frac{1}{k_i} - \frac{1}{k_0}\right)$	$\left(\frac{1}{k_i} - \frac{1}{k_0} + \beta\right)$
1		0.0098	0.0133	0.01465	0.0182	0.0309	0.0345
2		0.0254	0.029	0.0431	0.0467	0.0720	0.0756
3		0.0435	0.0471	0.09163	0.0952	0.1004	0.104
4		0.0739	0.0775				
4				0.1018	0.1053	0.133	0.1363

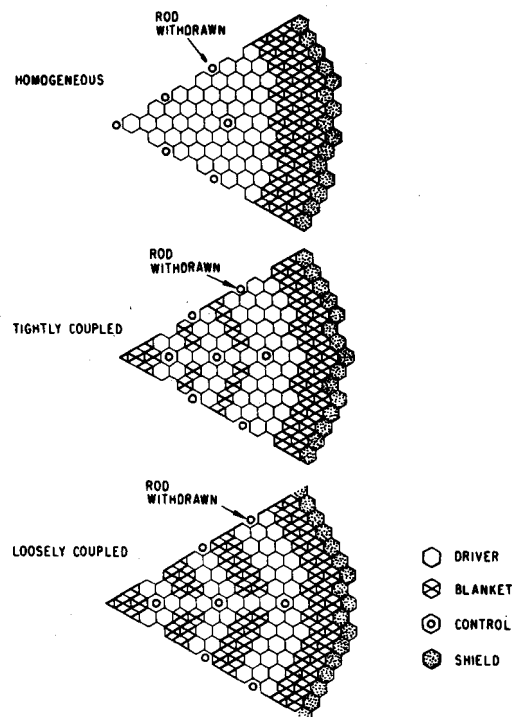
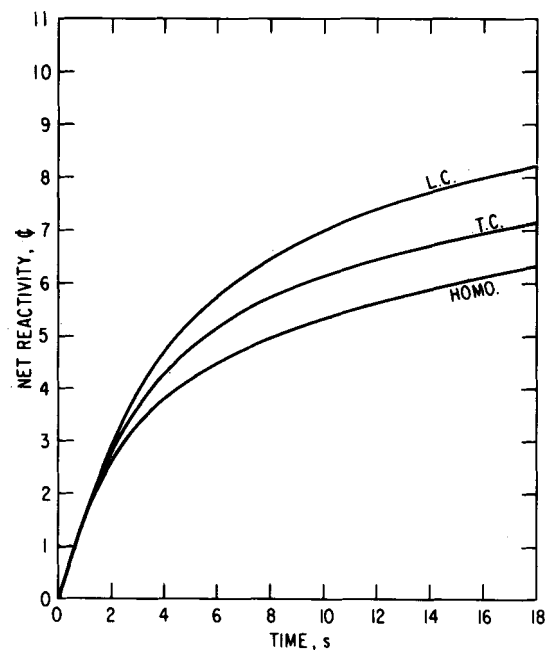
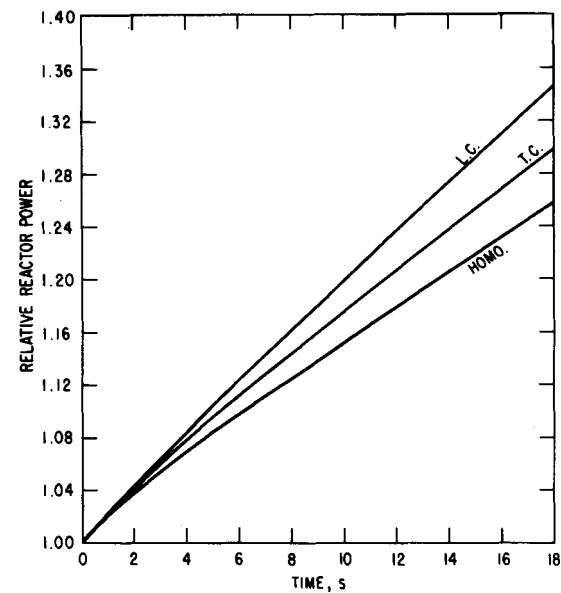


FIG. 1 1000 MWE CORE CONFIGURATIONS

FIG. 2 TIME RESPONSE OF THE NET REACTIVITY AS PREDICTED BY SPACE-TIME KINETICS FOR A 2 β /SEC. RAMP REACTIVITY INSERTIONFIG. 3 TIME RESPONSE OF THE RELATIVE REACTOR POWER AS PREDICTED BY SPACE-TIME KINETICS FOR A 2 β /SEC. RAMP REACTIVITY INSERTION

flux tilt and the modal contributions to it were calculated with the available mode functions and the space and time dependent neutron flux shape function calculated during the transient.

For displaying mode analysis results, the local flux tilt along an axis at a specified time and the local flux tilt at a specified location as a function of time were considered. The chosen axis passed through the core center and through the center of the assembly which contained the withdrawn control rod. The time chosen for displaying the flux tilt along the axis was 15 sec. since it corresponded to the time at which the transient was terminated in the loosely coupled core according to the 30% overpower constraint. The chosen location in each core configuration to display the time dependence of the local flux tilt was the point where the power density was maximum and it was found that this location was about three assemblies away from the withdrawn control rod. The local flux tilt at the location of the peak power density was designated as τ_{peak} .

The flux tilt along the chosen axis at 15 sec. as calculated with the FX2-TH code and the harmonic contributions to the flux tilt are shown in Figs. 4, 5 and 6 respectively for the loosely coupled core, the tightly coupled core and the homogeneous core. In each configuration, the flux tilt along the axis is a maximum at the location of the withdrawn control rod and at this point the flux tilt is not represented well by the four harmonics. About three assemblies away and beyond in all directions from the rod location, the lower four harmonics represented the local flux tilt to within $\pm 20\%$ of the FX2-TH value. The flux tilt along the axis is not symmetric but skewed towards the right half and first harmonic alone cannot account for either the magnitude or the shape of the flux tilt along the axis. The contributions of the third and fourth harmonics to the local flux tilt are significant only around the core centre. The second harmonic is the major contributor to the neutron flux pile-up in the right half since it is a symmetric mode while the first harmonic is an antisymmetric mode.

The time dependence of the local flux tilt at the location of peak power density $\tau_{\text{peak}}(t)$, as calculated with FX2-TH code and the harmonic contributions to it are shown in Figs. 7, 8 and 9 respectively for the loosely coupled core, the tightly coupled core and the homogeneous core. In each core configuration, after about 2 sec. into the transient τ_{peak} and harmonic contributions to it vary linearly with time. In each configuration, the first and second harmonics are the major contributors to τ_{peak} . In the loosely coupled core, the lower four harmonics account for approximately 90% of the τ_{peak} throughout the transient. In the tightly coupled core, τ_{peak} is well represented by the four harmonics during the transient. In the homogeneous core, the first and second harmonics are enough to account for τ_{peak} during the transient and inclusion of third and fourth harmonic contributions overpredicts the τ_{peak} .

For the transient considered, the time dependence of the total flux tilt $\tau(\vec{r}, t)$ and the i th harmonic flux tilt $\tau_i(\vec{r}, t)$ is independent of normalization of the mode functions but $T_i(t)$ and $G_i(\vec{r})$ which are related to it through Eq. (10) are dependent on the normalization of the fundamental mode and the i th harmonic eigenfunction. The normalization of the mode functions was chosen such that $G_i(\vec{r})$ was unity at the left most point on the axis through the center of the core and the withdrawn control rod. The values of $T_i(t)$ derived using this normalization at $t=15$ sec. are shown in Table III for the three configurations.

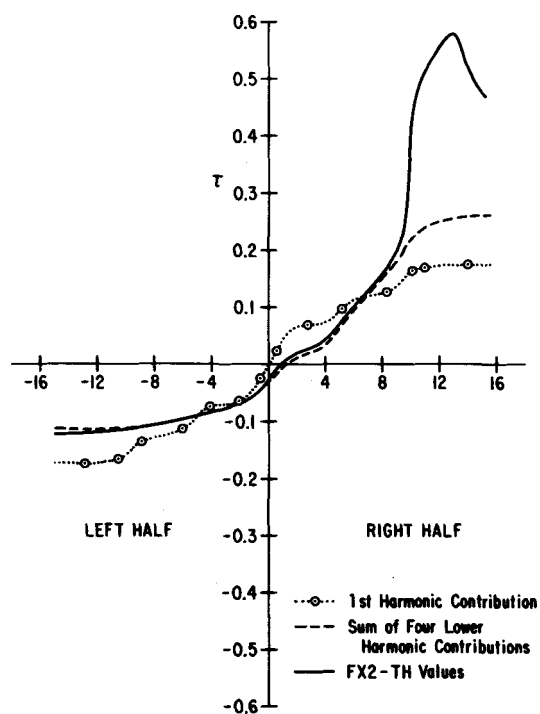


FIG. 4 FLUX TILT τ ALONG THE CHOSEN AXIS AT 15 SEC. IN THE LOOSELY COUPLED CORE FOR A 2¢/SEC. RAMP REACTIVITY INSERTION

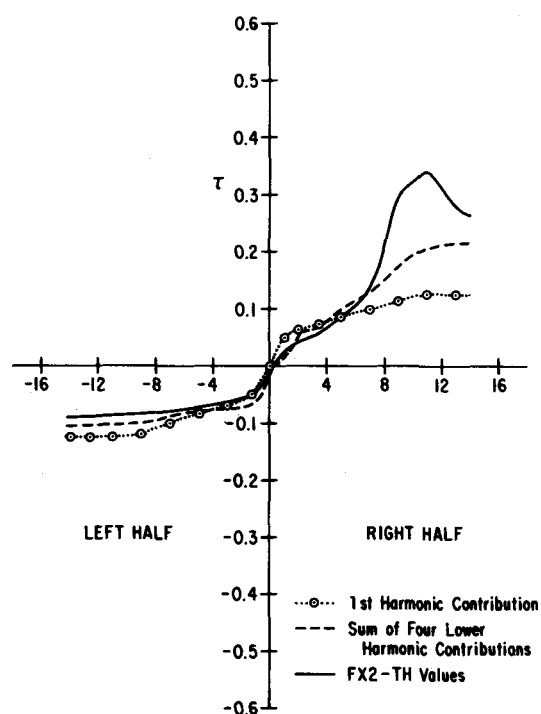


FIG. 5 FLUX TILT τ ALONG THE CHOSEN AXIS AT 15 SEC. IN THE TIGHTLY COUPLED CORE FOR A 2¢/SEC. RAMP REACTIVITY INSERTION

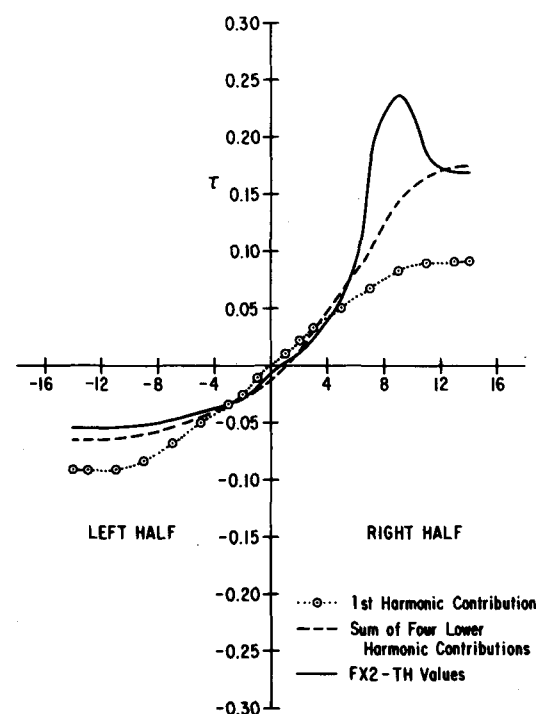


FIG. 6 FLUX TILT τ ALONG THE CHOSEN AXIS AT 15 SEC. IN THE HOMOGENEOUS CORE FOR A 2¢/SEC. RAMP REACTIVITY INSERTION

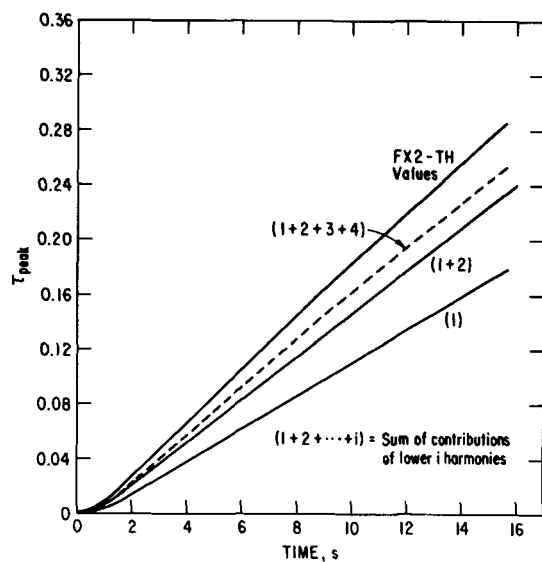


FIG. 7 TRANSIENT FLUX τ_{PEAK} AT THE LOCATION OF THE PEAK POWER DENSITY IN THE LOOSELY COUPLED CORE FOR A 2%/SEC. RAMP REACTIVITY INSERTION

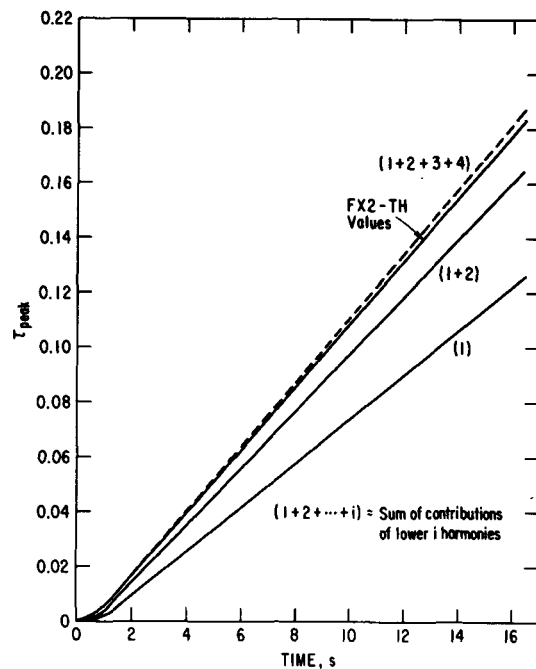


FIG. 8 TRANSIENT FLUX τ_{PEAK} AT THE LOCATION OF THE PEAK POWER DENSITY IN THE TIGHTLY COUPLED CORE FOR A 2%/SEC. RAMP REACTIVITY INSERTION

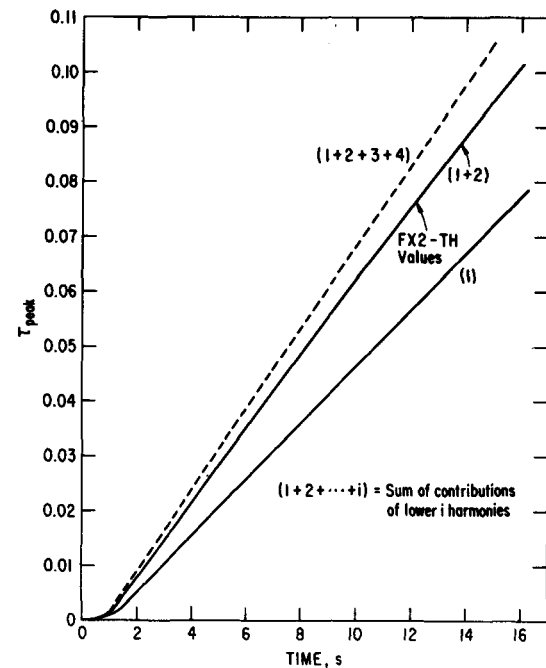


FIG. 9 TRANSIENT FLUX τ_{PEAK} AT THE LOCATION OF THE PEAK POWER DENSITY IN THE HOMOGENEOUS CORE FOR A 2%/SEC. RAMP REACTIVITY INSERTION

Table III

$T_i(t)$ Values in three configurations at $t=15$ sec for $2\beta/\text{sec}$
 Ramp Reactivity Insertion ($G_i(\vec{r})$ are normalized such that they are unity
 at the left hand edge of the axis which passes through the withdrawn control rod)

Harmonic i	$T_i(t)^a$ at $t = 15$ sec		
	Loosely Coupled Core	Tightly Coupled Core	Homogeneous Core
1	2.35×10^{-3}	2.28×10^{-3}	3.11×10^{-3}
2	1.7×10^{-3}	2.175×10^{-3}	1.6×10^{-3}
3	6.5×10^{-4}	7.18×10^{-4}	10.7×10^{-4}
4	$0.8 \times 10^{-3}^b$	3.48×10^{-3}	3.88×10^{-3}

^aThese values are based on normalization used for $G_i(\vec{r})$.

^b4th harmonic in loosely coupled core is of a different type than the 4th harmonic in tightly coupled and homogeneous cores (see Table II).

Even though i th harmonic quantity $T_i(t)$ is not constant between configurations, it can be seen that its variation between them is not large. A variation in $T_i(t)$ of about 65% occurred for the third harmonic in the homogeneous core in comparison with the corresponding value in the loosely coupled core. It is to be noted that the fourth harmonic in the loosely coupled core is of a different type from the fourth harmonic in other configurations and hence cannot be compared with them.

CONCLUSION

For the three LMFBR core configurations considered and for the delayed critical transient studied in them, the variation of the perturbation dependent quantity $T_i(t)$ for the i th harmonic at any time t from one core configuration to another was not found to be large. Hence, for the same transient initiated in a new core configuration, whose heterogeneity falls in the range considered here, the local flux tilt may be estimated harmonic by harmonic when the eigenfunctions and eigenvalues of the new configuration in the unperturbed state are available. The flatness criterion for $T_i(t)$ between configurations cannot be guaranteed, in general, for all possible transients and must be ascertained before the relationship between the i th harmonic local flux tilt and its modified eigenvalue separation from the fundamental is applied.

ACKNOWLEDGEMENT

The author is thankful to H. Henryson II, S. F. Su, Y. Orechwa, R. D. Lawrence, K. L. Derstine and C. H. Adams for valuable discussions during the course of this work.

REFERENCES

1. S. F. Su, Y. Orechwa and H. Henryson II, Nucl. Technol., 52, 370 (1981).
2. R. A. Shober, T. A. Daly and D. R. Ferguson, "FX2-TH: A Two-Dimensional Nuclear Reactor Kinetics Code with Thermal-Hydraulic Feedback," ANL-78-97, Argonne National Laboratory (1978).
3. D. C. Wade and R. A. Rydin, "An Experimentally Measurable Relationship Between Asymptotic Flux Tilts and Eigenvalue Separation," Dynamics of Nuclear Systems, University of Arizona Press, Tucson, Arizona (1972).
4. S. Kaplan, Nucl. Sci. Eng., 9, 357 (1961).
5. Y. Orechwa, Argonne National Laboratory, private communication.
6. K. O. Ott and D. A. Meneley, Nucl. Sci. Eng. 36, 402 (1969).
7. C. H. Adams, Argonne National Laboratory, private communication.
8. D. R. Ferguson and K. L. Derstine, Nucl. Sci. Eng. 64, 593 (1977).
9. R. A. Rydin, "Nuclear Reactor Theory and Design," University Publications, Blacksburg, Virginia, USA, (1977).
10. J. J. Duderstadt and L. J. Hamilton, "Nuclear Reactor Analysis", John Wiley and Sons, Inc., NY (1976).
11. R. Goldstein and L. M. Shotkin, Nucl. Sci. Eng. 38, 94 (1969)

APPENDIX

SPACE-TIME KINETICS EQUATIONS FOR THE λ -MODE EXPANSION COEFFICIENTS

The time-dependent multigroup kinetics equations⁹ are

$$V \frac{\partial \phi(\vec{r}, t)}{\partial t} + L\phi(\vec{r}, t) - (1-\beta) x_p F^+ \phi(\vec{r}, t) - \sum_{i=1}^6 \lambda_i (x_{di} C_i(\vec{r}, t)) = Q(\vec{r}, t) \quad (A-1)$$

and

$$\frac{\partial}{\partial t} (x_{di} C_i(\vec{r}, t) + \lambda_i x_{di} C_i(\vec{r}, t) = \beta_i x_{di} F^+ \phi(\vec{r}, t) \quad i = 1, 6 \quad (A-2)$$

Here ϕ is a column vector of neutron group flux, $C_i(\vec{r}, t)$ is the concentration of the precursors of type i at $\vec{r}$ and time t , V is a diagonal matrix with elements $\frac{1}{v_g}$ where v_g is the neutron speed in group g , x_p is a column vector of the prompt neutron spectrum, x_{di} is a column vector of the i th type delayed neutron spectrum, F^+ is a row vector with elements $v\sigma_{fg}$ where

ν is the number of neutrons emitted in fission and Σ_{fg} is the macroscopic fission cross section in group g , L is the loss operator, λ_i is the decay constant of the i th type precursors and β_i is the corresponding delayed neutron fraction.

Consider the unperturbed reactor in steady state with $k_{eff} = k_0 = 1$ and with the unperturbed flux $\phi_0(\vec{r}, 0)$ being given by a constant multiple of the fundamental mode $\psi_0(\vec{r})$ of the λ -mode eigenvalue problem.

$$L_0 \psi_n = \frac{1}{k_n} M_0 \psi_n \quad (A-3)$$

where L_0 is the time independent loss operator, M_0 is the corresponding production operator and k_n is the eigenvalue of the n th λ -mode.

Let perturbations $\delta L_0(\vec{r}, t)$ and $\delta M_0(\vec{r}, t)$ be introduced in the reactor such that

$$L(\vec{r}, t) = L_0(\vec{r}) + \delta L_0(\vec{r}, t) \quad (A-4)$$

and

$$M(\vec{r}, t) = M_0(\vec{r}) + \delta M_0(\vec{r}, t). \quad (A-5)$$

Defining

$$M = \chi_p F^+(\vec{r}, t) \quad (A-6)$$

and

$$\chi_{di} = \chi^i \chi_p \quad (A-7)$$

where χ^i is a diagonal matrix with elements $\chi_{g,g}^i = \chi_{di,g}/\chi_{p,g}$, the kinetics equations may be written in the form.

$$\begin{aligned} \nu \frac{\partial \phi(\vec{r}, t)}{\partial t} + L\phi(\vec{r}, t) - \left[(1-\beta) + \sum_{i=1}^6 \beta_i \chi^i \right] M\phi(\vec{r}, t) \\ + \sum_{i=1}^6 \beta_i \chi^i M\phi(\vec{r}, t) - \sum_{i=1}^6 \lambda_i \chi_{di} C_i(\vec{r}, t) = Q(\vec{r}, t) \end{aligned} \quad (A-8)$$

and

$$\frac{\partial}{\partial t} (\chi_{di} C_i(\vec{r}, t)) + \lambda_i \chi_{di} C_i(\vec{r}, t) = \beta_i M\phi(\vec{r}, t) + \beta_i (\chi^i - I) M\phi(\vec{r}, t) \quad (A-9)$$

for $i=1, 6$

where I is the unit matrix.

It can be seen that in the steady state at $t=0$ and for $Q=0$, the kinetics equations (A-8 and A-9) reduce to the λ -mode eigenvalue problem with $k_0=1$ if the difference between the prompt and delayed neutron Spectra may be neglected (i.e., $\chi^i=I$).

Expanding the perturbed flux $\phi(\vec{r},t)$ in λ -modes of Eq. (A-3) we have

$$\phi(\vec{r},t) = \sum_j A_j(t) \psi_j(\vec{r}) \quad (A-10)$$

Selective substitution of Eq. (A-10) for $\phi(\vec{r},t)$ in Eqs. (A-8 and A-9), multiplying the resulting equations on the left by the adjoint function ψ_n^* , integrating over the whole volume and noting $k_0=1$ gives ^{3,4}

$$\begin{aligned} \sum_j \Lambda_{nj} \frac{\partial A_j(t)}{\partial t} + \left(\frac{1}{k_n} - \frac{1}{k_0} + \beta \right) A_n(t) - \sum_{i=1}^6 \lambda_i C_{ni}(t) \\ = \rho_n(t) - \sum_{i=1}^6 \beta_i S_{ni}(t) + Q_n(t) \end{aligned} \quad (A-11)$$

and

$$\frac{\partial C_{ni}(t)}{\partial t} + \lambda_i C_{ni}(t) - \beta_i A_n(t) = \beta_i \delta_{ni}(t) + \beta_i S_{ni}(t) \quad \text{for } i=1,6 \quad (A-12)$$

where use is made of the inner product notation $\int x^T y d\vec{r} = (x,y)$ with x^T being the transpose of x and

$$\Lambda_{nj} = \frac{(\psi_n^*, V \psi_j)}{(\psi_n^*, M_0 \psi_n)}, \quad (A-13)$$

$$\rho_n(t) = \frac{(\psi_n^*, [-\delta L_0 + \delta M_0 - \sum_{i=1}^6 \beta_i (\chi^i - I) \delta M_0] \phi(\vec{r},t))}{(\psi_n^*, M_0 \psi_n)}, \quad (A-14)$$

$$S_{ni}(t) = \frac{(\psi_n^*, \chi^i \delta M_0 \phi)}{(\psi_n^*, M_0 \psi_n)}, \quad (A-15)$$

$$\delta_{ni}(t) = \frac{(\psi_n^*, (\chi^i - I) M_0 \phi)}{(\psi_n^*, M_0 \psi_n)}, \quad (A-16)$$

$$C_{ni}(t) = \frac{(\psi_n^*, \chi_{di} C_i(\vec{r}, t))}{(\psi_n^*, M_0 \psi_n)}, \quad (A-17)$$

and

$$Q_n(t) = \frac{(\psi_n^*, Q(\vec{r}, t))}{(\psi_n^*, M_0 \psi_n)} \quad (A-18)$$

For $t \rightarrow \infty$, if an asymptotic steady state is reached such that $\rho_n(\infty)$ and $S_{ni}(\infty)$ are independent of time and if the difference between the prompt and delayed neutrons may be neglected, we get the asymptotic value for $A_n(\infty)$ which is

$$A_n(\infty) = \frac{\rho_n(\infty)}{(\frac{1}{k_n} - \frac{1}{k_0})} \quad (A-19)$$

DECOUPLED QUASISTATIC SYNTHESIS APPROXIMATION

In the quasistatic⁶ approximation, we may represent $\phi(\vec{r}, t)$ exactly as

$$\phi(\vec{r}, t) = N(t) \psi(\vec{r}, t) \quad (A-20)$$

where $N(t)$ is the amplitude function and $\psi(\vec{r}, t)$ is the shape function which is constrained by

$$\int_{\text{Reactor}} \psi^*(\vec{r}, 0) \vee \psi(\vec{r}, t) = 1.0 \text{ for all } t. \quad (A-21)$$

In the quasistatic synthesis approximation, the shape function is written as

$$\psi(\vec{r}, t) = \sum_j b_j(t) \psi_j(\vec{r}) \quad (A-22)$$

All coefficients $b_j(t)$ are not independent since $\psi(\vec{r}, t)$ obeys Eq. (A-21). Hence, knowledge of $N(t)$ and $b_j(t)$ where $j > 0$ is equivalent to knowing all $A_j(t)$ in Eq. (A-10) for the perturbed flux.

We decouple the mode equations by setting $\Lambda_{nj} = 0$ for $n \neq j$ in Eq. (A-11). This is a reasonable approximation in fast reactor systems where neutron generation time (Λ_{00}) is of the order of 10^{-7} . Further, we consider systems with no external source ($Q_n(t)=0$) and neglect small terms $\beta_i S_{ni}(t)$ and $\delta_{ni}(t)$ in Eqs. (A-11 and A-12). With these approximations, the mode kinetics equations can be written as

$$\Lambda_{nn} \frac{\partial A_n(t)}{\partial t} + \left(\frac{1}{k_n} - \frac{1}{k_0} + \beta \right) A_n(t) - \sum_{i=1}^6 \lambda_i C_{ni}(t) = \rho_n(t) \quad (\text{A-23})$$

and

$$\frac{\partial C_{ni}(t)}{\partial t} + \lambda_i C_{ni}(t) - \beta_i A_n(t) = 0 \text{ for } i = 1, 6 \quad (\text{A-24})$$

when a small perturbation is introduced in a critical reactor such that $[b_j(t)/b_0(t)] \ll 1$ and $b_0(t) \approx 1.0$, it can be shown that Eqs. (A-23 and A-24) become the usual point kinetics equations for the amplitude function $N(t)$ with $n=0$. When the inserted reactivity is small ($\rho_0 < 0.5\%$), prompt jump approximation^{10, 11} may be used to evaluate $A_n(t)$. In the prompt jump approxima-

tion, we set $\Lambda_{nn} \frac{\partial A_n(t)}{\partial t} = 0$ and the mode kinetics equations become

$$\left(\frac{1}{k_n} - \frac{1}{k_0} + \beta \right) A_n(t) - \sum_{i=1}^6 \lambda_i C_{ni}(t) = \rho_n(t) \quad (\text{A-25})$$

and

$$\frac{\partial C_{ni}(t)}{\partial t} + \lambda_i C_{ni}(t) - \beta_i A_n(t) = 0 \text{ for } i = 1, 6 \quad (\text{A-26})$$

Formally, we may write Eq. (A-25) in the form

$$\begin{aligned} A_n(t) &= \frac{\rho_n(t) + \sum_{i=1}^6 \lambda_i C_{ni}(t)}{\left(\frac{1}{k_n} - \frac{1}{k_0} + \beta \right)} \\ &= \frac{T'_n(t)}{\left(\frac{1}{k_n} - \frac{1}{k_0} + \beta \right)} \end{aligned} \quad (\text{A-27})$$

The coupled set of equations (A-25 and A-26) can be solved analytically in the one delayed neutron family assumption.

For $n > 0$ and $i = 1$,

$$\frac{dA_n(t)}{dt} + \frac{\lambda}{\alpha_0} A_n(t) = \frac{\lambda \rho_n(t) + \frac{d\rho_n}{dt}}{(\frac{1}{k_n} - \frac{1}{k_0}) \alpha_0} \quad (\text{A-28})$$

where

$$\alpha_0 = 1 + \frac{\beta}{(\frac{1}{k_n} - \frac{1}{k_0})}$$

and λ and β are one delayed neutron family parameters. Using the integrating factor $e^{\frac{\lambda t}{\alpha_0}}$, the solution of Eq. (A-28) may be written as

$$\begin{aligned} A_n(t) &= \frac{e^{-\frac{\lambda t}{\alpha_0}}}{(\frac{1}{k_n} - \frac{1}{k_0} + \beta)} \left[B + \int_0^t dt' e^{\frac{\lambda t'}{\alpha_0}} \left[\lambda \rho_n(t') + \frac{d\rho_n(t')}{dt'} \right] \right] \\ &= \frac{T'_n(t)}{(\frac{1}{k_n} - \frac{1}{k_0} + \beta)} \end{aligned} \quad (\text{A-29})$$

where B is the integration constant.

From observation of Eq. (A-27) and Eq. (A-29), it can be seen that $A_n(t)$ is inversely proportional to the sum of the n th mode eigenvalue separation from the fundamental and the effective delayed neutron fraction.

Since $A_n(t) = N(t) b_n(t)$, the expansion coefficient $b_n(t)$ is given by

$$\begin{aligned} b_n(t) &= \frac{T'_n(t)}{N(t)} \frac{1}{(\frac{1}{k_n} - \frac{1}{k_0} + \beta)} \\ &= \frac{T_n(t)}{(\frac{1}{k_n} - \frac{1}{k_0} + \beta)} \end{aligned} \quad (\text{A-30})$$

RECENT BWR FUEL MANAGEMENT REACTOR PHYSICS ADVANCES

BY

RL CROWTHER, SP CONGDON, BW CRAWFORD, CM KANG, CL MARTIN
AP REESE, PJ SAVOIA, SR SPECKER, R WELCHLY
GE-NED

Invited Paper

ABSTRACT

Improvements in BWR fuel management have been under development to reduce uranium and separative work (SWU) requirements and reduce fuel cycle costs, while also maintaining maximal capacity factors and high fuel reliability¹⁻⁴. These improvements have included low leakage refueling patterns in which last cycle, minimum reactivity fuel is moved to the core periphery; design with natural uranium blanket fuel assemblies in initial cores; tailoring of initial core design to optimally match the transition reload cycle requirements^{5,6}; and application of the Control Cell Core to mitigate power peaking adjacent to partly inserted control blades, to provide increased pellet-cladding-interaction(PCI) margins, to mitigate power peaking increases with application of increased enrichment, extended burnup fuel, to increase the fuel cycle gains achievable via flow control and axial burnup shape optimization spectral shift, to simplify reactor operations and fuel cycle analyses, and to increase operating capacity factors⁸⁻¹¹.

Improved reactor physics methods are playing an increasingly important role in making such advances feasible. The improved design, process computer and analysis methods both increase knowledge of the thermal margins which are available to implement fuel management advances, and improve the capability to reliably and efficiently analyze and design for fuel management advances. Gamma scan measurements of the power distributions of advanced fuel assembly and advanced reactor core designs, and improved in-core instruments also are important contributors to improving 3-D predictive methods and to increasing thermal margins. Interactive color graphics fuel and control management methods have recently been developed and have great potential for further improvement of BWR performance and fuel cycle costs by making more comprehensive and complex coupled fuel and control management feasible.

This paper is an overview of the recent advances in BWR reactor physics fuel management methods, coupled with fuel management and core design advances. The reactor physics measurements which are required to confirm the predictions of performance of fuel management advances also are summarized.

BWR FUEL MANAGEMENT IMPROVEMENTS BEING IMPLEMENTED OR CONSIDERED

BWR fuel management improvements which are being implemented or are under consideration have been reviewed in recent papers^{1-6,17-23}. Some of the reactor physics considerations in design for improved BWR fuel cycles also were reviewed²⁴. For completeness, the key design alternatives will be summarized herein.

BWR design improvement alternatives which improve conversion ratio, reduce neutron leakage, and reduce parasitic neutron absorption in structural materials and control absorbers have been under consideration for many years²⁶⁻²⁸. However, fuel cycle resource and economic changes in the last few years have greatly increased the incentive to consider such improvement alternatives, which have potential to improve nuclear efficiency and increase the discharge burnup without increasing initial enrichment requirements, thereby reducing the separative work unit (SWU), uranium and fuel assembly requirements needed to achieve the desired cycle length between refuelings. Utility desires for longer, 18 to 24 month, cycle lengths between refuelings also have increased the incentive to consider fuel cycle improvement alternatives.

Improvement alternatives which have been considered include:

- o Axial and radial blankets;
- o Low leakage fuel management with last cycle lowest reactivity fuel located on the core periphery;
- o Axial shaping of burnable absorber;
- o Spectral shift via recirculation flow control;
- o Spectral shift via axial burnup shape optimization;
- o Reconstitution and reinsertion of highest enrichment discharge fuel rods;
- o Refueling pattern optimization;
- o EOC feedwater temperature reduction;
- o EOC power coastdown;
- o Once-through partial thorium cycle;
- o Improved fuel assembly design to improve nuclear, thermal, hydraulic, mechanical, licensing, reliability or other characteristics;
- o Extended discharge burnup via increased initial fuel enrichment and/or via application of other improvement alternatives.

Many of these improvements are being applied at operating BWRs and others are under evaluation or development. Such BWR improvement alternatives are retrofitable to existing BWRs and have been under evaluation for potential application to both currently operating and future reactors. Evaluations have considered operating cycle lengths from twelve to twenty-four months.

The benefits and impacts of such fuel cycle improvements are dependent on couplings between the improvement alternatives, and on individual utility and power reactor constraints and economic groundrules. Each utility operates power reactors in a different environment, which can change the optimization of improvement application. Therefore, the capability to reliably evaluate

the coupled effects of many fuel cycle improvements applied in concert and to realistically include unique utility and power reactor constraints and economic or other conditions in the analyses is key to making reliable determinations of the incentives for, and feasibility of, improvement alternative application to any particular power reactor. Examples of some of the interdependencies follow.

If fuel cycle improvements adversely impact thermal or other margins, mitigating design features must be introduced. Furthermore many of the BWR improvement alternatives are highly coupled with one another, and both the impacts of, and the incentives for, the improvement are dependent on which other improvements have been introduced. Consideration of these interdependencies is complex and dependent on the capability and accuracy of the available design and analysis methods.

Design improvements which affect thermal margins directly couple with fuel management improvements. Examples are gamma TIP detectors⁷, which increase thermal operating margins, and the Control Cell Core (CCC)⁸⁻¹¹, which improves thermal and pellet-clad-interaction margins, improves fuel reliability, and simplifies reactor operations and fuel cycle analysis by keeping low reactivity fuel adjacent to inserted control blades and minimizing control blade movement.

The Control Cell Core⁸⁻¹¹ has been observed to mitigate the adverse effects of extended burnup and other fuel cycle improvements on thermal, licensing and operating margins^{2,4}. The CCC design avoids insertion of control blades adjacent to fuel containing burnable absorbers or adjacent to high reactivity fuel, maintains core octant symmetry, simplifies reactor operations and reactor analysis, increases margins for application of pellet-clad-interaction operating management recommendations (PCIOMR), increases the operating capacity factor, increases thermal operating margins, and improves fuel reliability. When fuel cycle improvements such as spectral shift via axial burnup shape optimization, axial burnable absorber shaping, or increased enrichment-extended burnup fuel are applied the CCC design tends to inherently mitigate the adverse effects on thermal and licensing margins or to increase the fuel cycle gains which can be achieved through application of the improvement²⁵.

PCI resistant fuel¹⁷ and improved incore detectors⁷ are other coupled BWR improvements which have potential to increase the practicality of applying the fuel cycle improvement alternatives or which can increase the gains from the fuel cycle alternatives. For example improved gamma TIP detectors which are now being applied at many BWRs have been observed to increase thermal margins by approximately 5% through reduction of instrumentation reading asymmetries. This improvement in margin can be applied to increase the fuel cycle gains that are achievable from improvements such as spectral shift or refueling pattern optimization. PCI resistant fuel potentially can be applied to either increase operating limits or to increase the practicality of reliably achieving extended burnups.

Improvement alternatives also may couple with one another to mitigate some of the adverse performance and/or reliability impacts of design for extended burnup. For example, spectral shift reduces the amount of reactivity that must be provided by control absorbers to control excess reactivity.

Therefore, spectral shift couples well with introduction of increased fuel enrichments to extend burnup or to achieve greater refueling cycle lengths. Such interactions between improvement alternatives are very important in establishing the overall practicality of achieving the ultimate potential BWR fuel cycle savings.

METHODS OF BWR FUEL MANAGEMENT ANALYSIS

The relationship between available thermal margins and fuel management capability creates a direct coupling between fuel management improvements and design and operations analysis reactor physics methods accuracy and capability. The complexity of the coupled fuel and control management also makes the reliability, comprehension, and degree of optimization of BWR fuel management highly dependent on the efficiency and ease of use of the fuel management methods.

Reliable evaluation of BWR fuel management alternatives requires 3-D fuel cycle simulation. Analyses with simplified 1-D methods are sometimes used to scope the fuel cycle and determine approximate refueling batch sizes required to achieve the desired refueling intervals. Multicycle 3-D evaluations are applied to compare alternative fuel management schemes.

In the past, it usually has been possible to separate within cycle evaluations from multicycle evaluations by converging directly on consistent power-exposure or "Haling" EOC states¹². However, application of improvement options such as BWR spectral shift via axial burnup shape optimization¹³ requires detailed simulation of control rod patterns within the cycle. This increases the incentive to apply simplified control rod patterns via the Control Cell Core and emphasizes the need for fast and efficient methods of developing optimal critical control rod patterns.

The 3-D methods which have been used in BWR simulations were reviewed in recent papers¹⁴⁻¹⁵. These methods employ simplified coarse mesh nuclear models and emphasize a complete and balanced description of the reactor system and the nuclear-thermal-hydraulic-reactor instrumentation-fuel mechanical-fuel and control management couplings.

Detailed multicycle 3-D simulations with rod pattern simulation within each refueling cycle have been the basis for recent BWR fuel cycle improvement evaluations which have led to conclusions that, given reliable fuel performance, fuel cycle costs for one of the Browns Ferry units potentially can be reduced by 20 percent, representing a 226 million dollar 10 year savings, and that there is potential to reduce uranium and SWU requirements by 29 and 24 percent.⁴ Comparison of improvement alternatives is sensitive to the degree to which control rod patterns and refueling patterns are optimized, since incomplete optimization of one alternative relative to another can distort the true reactivity, burnup, thermal margin and economic differences. Detailed 3-D, coupled rod pattern--refueling pattern optimization simulations have been made more practical by several recent developments; 1) Improvements in optimization methods; 2) The CCC simplification of control rod patterns; 3) Improvements in the capability and computer use efficiency of 3-D simulation methods; and 4) Development of efficient interactive 3-D simulation methods.

The 3-D methods used in BWR simulation include several alternative control rod pattern and refueling pattern optimization modules. However, fully automated optimization methods frequently lag behind the evolution of complex design and licensing criteria and constraints. As a result, interactive color graphics, 3-D simulation, using intelligent remote terminals, is being applied to optimize control rod patterns. Figure 1 is a gray scale edit of the color power distribution. Control rods can be moved with a light pen with rapid turn around on the revised 3-D results. Up to a factor of 5 time reduction for optimization of rod patterns within a cycle has been observed from application of the color graphics interactive methods.

Such methods have been extended to more complex refueling pattern optimization and an interactive color graphics refueling module is in final testing. Figure 2 is a gray scale edit of the discharge pattern for discharge of the periphery followed by discharge of the minimum reactivity fuel during refueling of an 1100 MWe BWR/4. Figure 3 is the move pattern for movement of the lowest reactivity, non discharge fuel to the periphery.

Historical effects of local control rod and steam void history can impact thermal margins and reactivity, and accurate representation is important in 3-D fuel management simulations. This is particularly the case in evaluation of cold, one-control rod-out shutdown margins, which have impact on refueling patterns and fuel management optimization. Reactivity in the cold state is sensitive to conditions in the top of the core which are strongly dependent on steam void history.

Each change in refueling pattern has to be comprehensively verified for independent withdrawal of each control rod in the cold state at several points within an operating cycle. Recent improvements in the efficiency and accuracy of one-withdrawn-control rod worth methods have led to a 20 to 30% reduction in computer time required to analyze fuel management alternatives. Accurate 2-D/1-D synthesis methods have been developed to make it feasible to rapidly evaluate the cold shutdown worth of all of the control rods in the core.

BWR spectral shift with flow control, EOC feedwater temperature reduction and EOC power coastdown are analyzed either by stepwise variation of the burnup and automated adjustment of the flow, inlet temperature or power to maintain criticality, or by stepwise variation of the flow, inlet temperature and power and use of consistent power-exposure iterations to converge on the burnup increment. The accuracy of the simulation is sensitive to step size.

Reconstitution and re-insert of high enrichment discharge fuel rods from the center of BWR fuel assemblies requires individual fuel rod management and complicates the 3-D simulation. Also, as efforts continue to improve the accuracy and comprehension of 3-D BWR simulations, there is incentive to reliably predict local fuel pin power and other properties. Techniques have recently been developed to rapidly estimate 3-D fuel pin power distribution and such methods will be of increasing importance in the optimization of advanced fuel management improvements and in including global tilts in fuel pin history evaluations. Progress is being made in improving the accuracy of the fuel pin power estimations and to display the 3-D results so that they can be easily assimilated and applied in design and operations optimization. Figure 4 is a gray scale edit of the color representation of the fuel pin

Figure 1

GRAY SCALE EDITS OF COLOR POWER
DISTRIBUTION--CORE WIDE & LOCAL BLOWUP

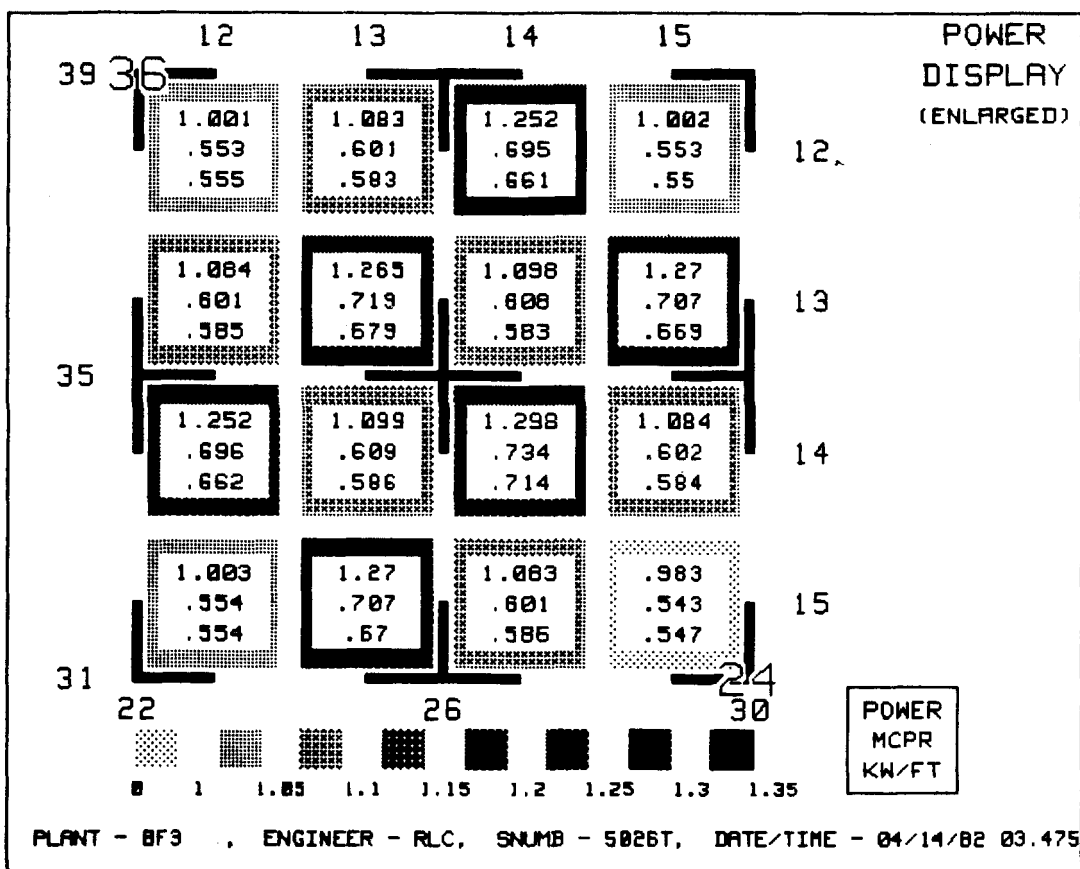
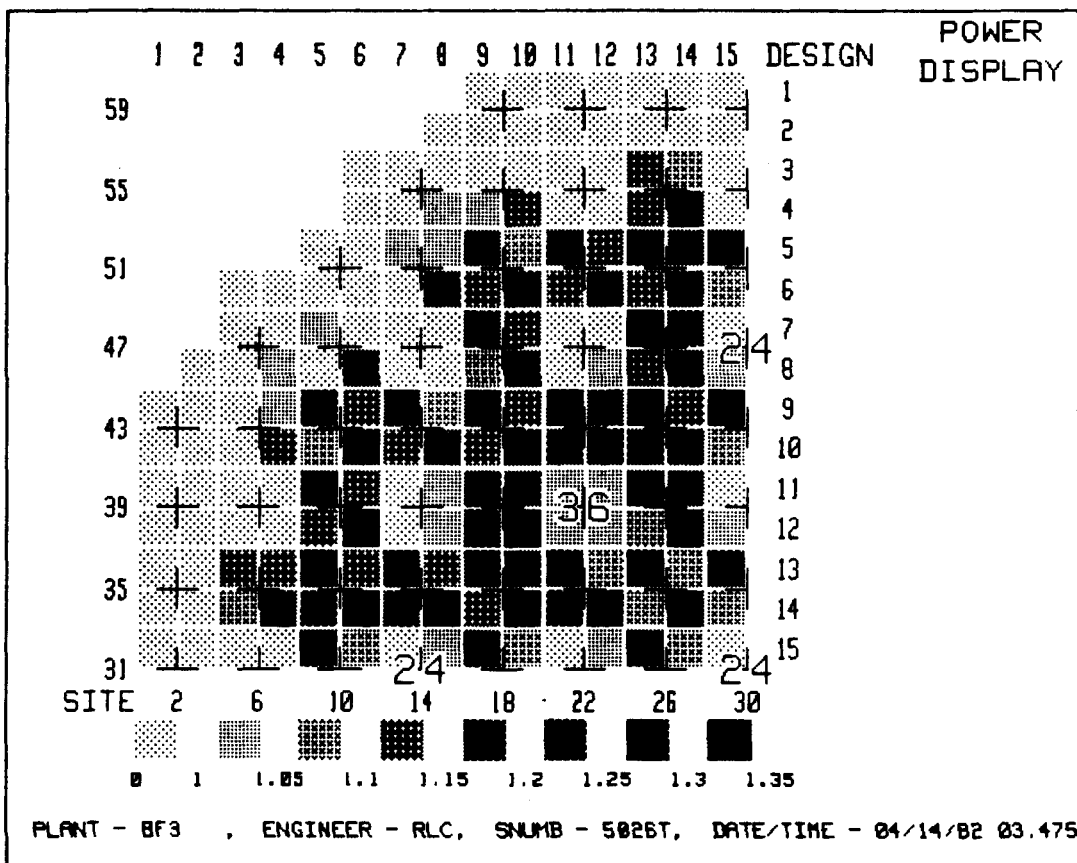
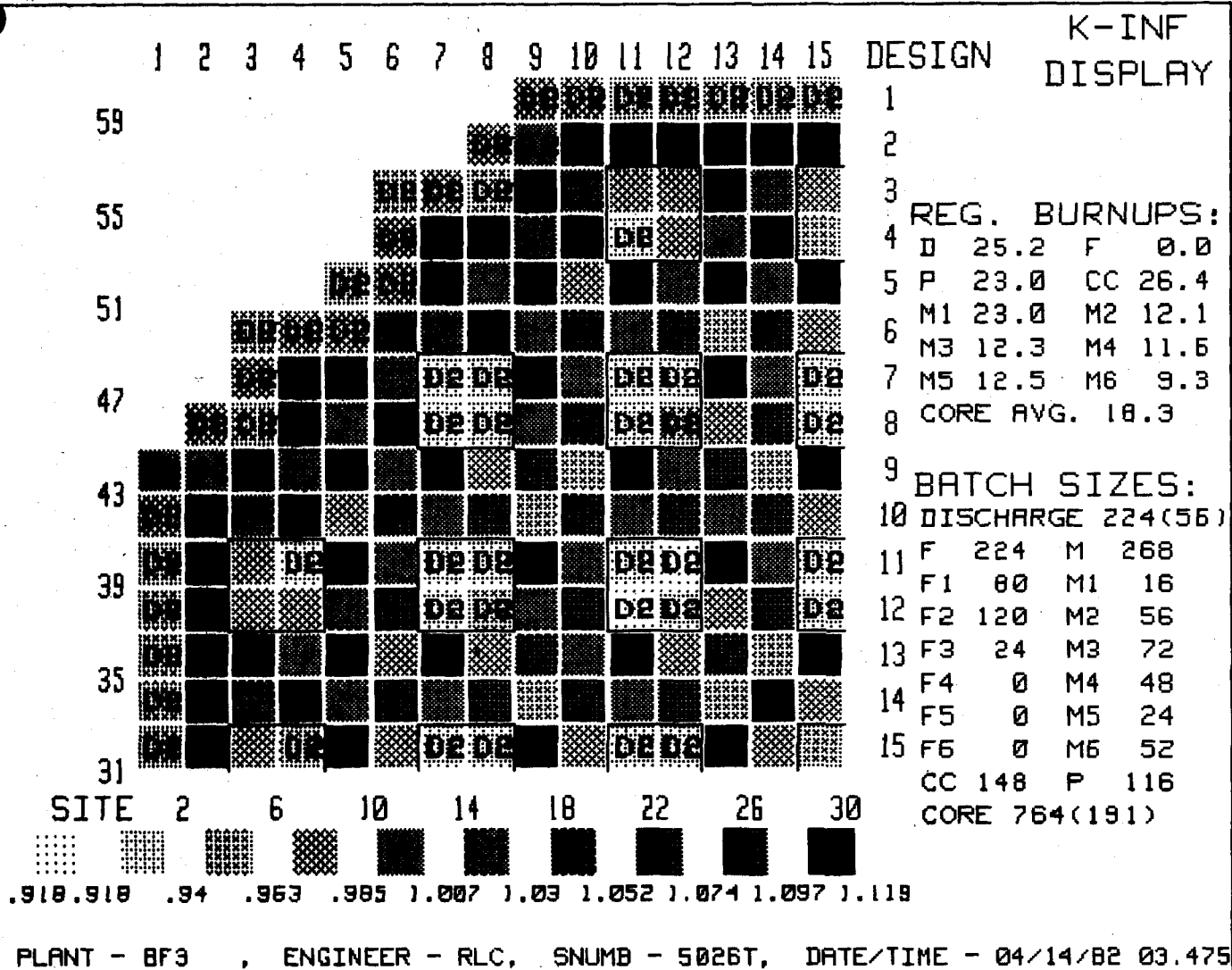


Figure 2

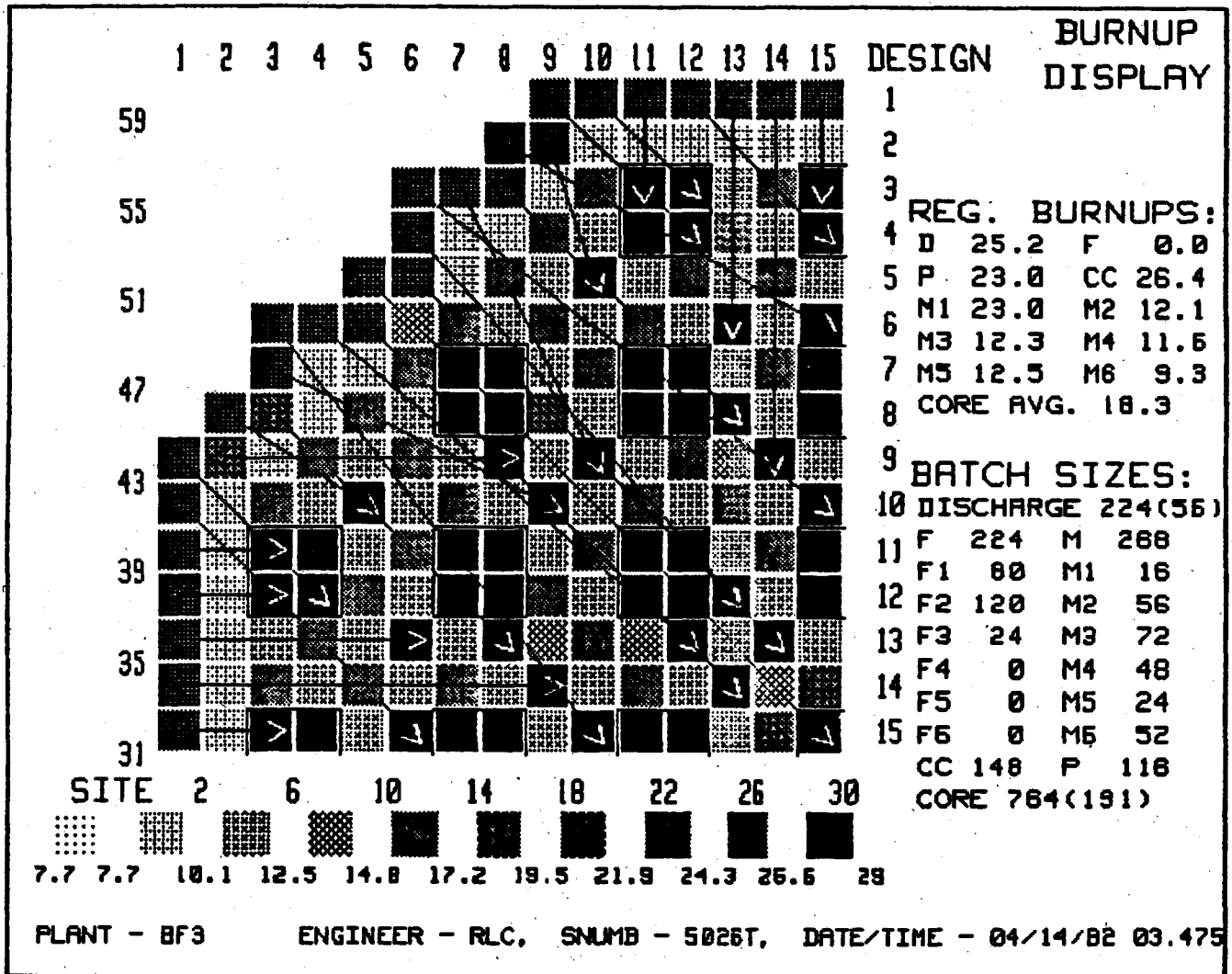
GRAY SCALE EDIT OF MINIMUM REACTIVITY DISCHARGE PATTERN



...AVERAGE CHANNEL K-INFINITY

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1									.99	.97	.96	.96	.95	.96	.96
2								.97	1.00	1.12	1.12	1.12	1.12	1.12	1.12
3					.96	.97	.95	1.12	1.05	.97	.97	1.11	1.04	.97	
4				.96	1.12	1.12	1.04	1.10	.93	.96	1.03	1.10	.94		
5			.96	.98	1.12	1.01	1.10	.98	1.11	1.02	1.11	1.02	1.11		
6		.96	.97	.98	1.06	1.04	1.10	1.03	1.10	1.02	1.10	.96	1.09	.97	
7		.97	1.12	1.12	1.04	.93	.93	1.11	1.02	.92	.92	1.11	1.02	.93	
8	.97	.95	1.12	1.00	1.10	.93	.93	.99	1.10	.92	.94	.98	1.10	.94	
9	.99	.99	1.12	1.04	1.10	1.03	1.11	.98	1.10	.96	1.11	1.02	1.09	.96	1.11
10	.97	1.12	1.05	1.10	.98	1.10	1.02	1.10	.96	1.09	1.02	1.09	1.02	1.09	.97
11	.96	1.12	.98	.93	1.11	1.02	.92	.92	1.11	1.02	.93	.92	1.11	1.02	.92
12	.96	1.12	.97	.97	1.03	1.10	.92	.94	1.02	1.09	.92	.94	.97	1.10	.93
13	.95	1.12	1.11	1.03	1.11	.96	1.11	.98	1.09	1.02	1.11	.97	1.10	.96	1.11
14	.96	1.12	1.04	1.10	1.02	1.09	1.02	1.10	.96	1.09	1.02	1.10	.96	1.11	.98
15	.96	1.12	.97	.94	1.11	.97	.93	.94	1.11	.97	.92	.93	1.11	.98	.94

Figure 3
GRAY SCALE EDIT OF MOVE PATTERN-MINIMUM REACTIVITY
NON DISCHARGE FUEL TO PERIPHERY



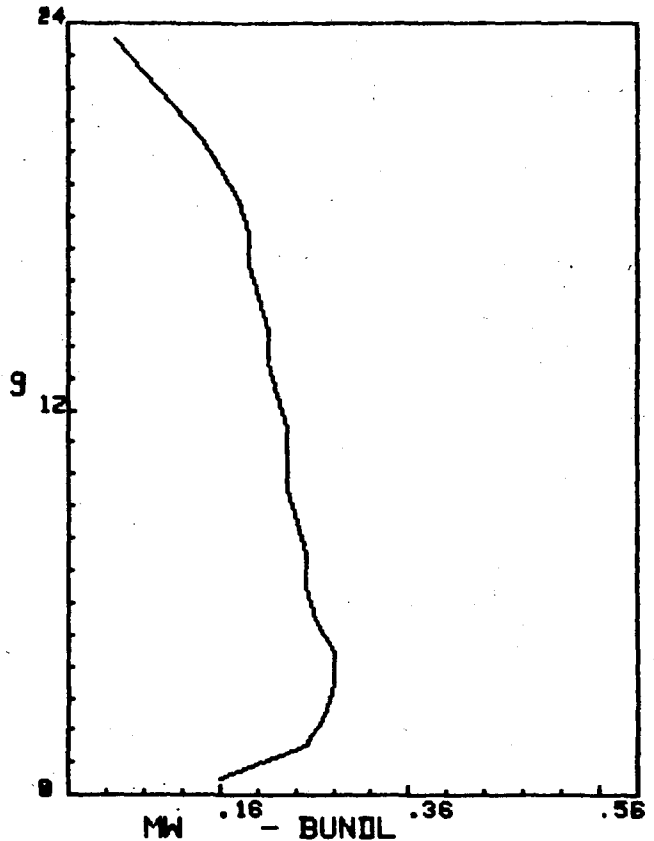
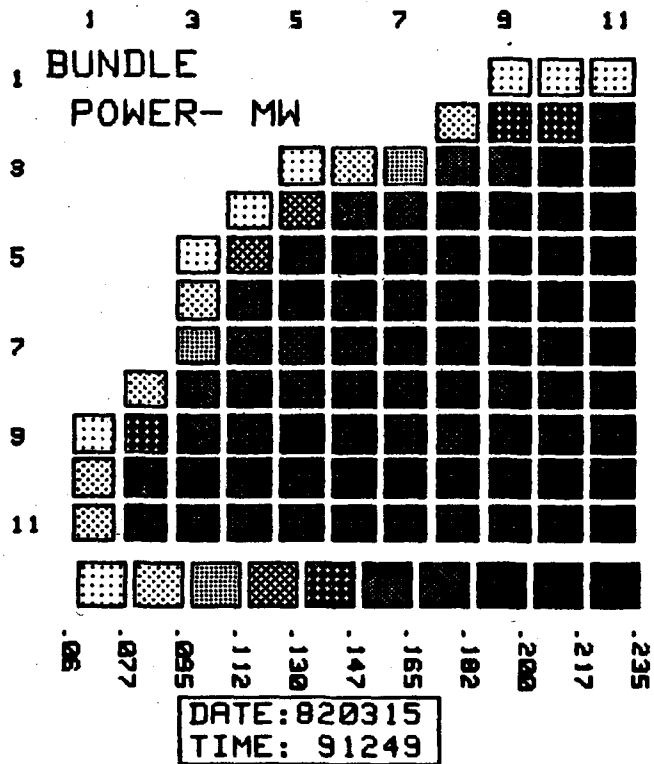
...BURNUP, GWD/T

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1									23.7	22.4	22.4	25.8	23.8	24.4	22.9
2								23.0	22.4	7.8	8.3	8.7	9.2	9.2	9.1
3						23.3	24.0	22.0	8.9	15.1	23.9	22.9	10.7	15.8	24.2
4						23.5	8.1	9.5	16.2	10.6	27.8	21.3	17.5	11.5	24.0
5					25.5	24.3	9.5	18.8	11.2	20.2	11.7	18.0	12.2	18.4	12.1
6		23.5	23.4	24.4	13.6	15.9	11.0	17.8	11.9	18.0	12.0	24.9	12.2	22.9	
7		23.9	8.1	9.5	15.9	27.1	27.5	11.9	18.1	28.5	28.3	12.3	18.1	27.9	
8		22.3	21.9	9.5	19.1	11.0	27.3	27.1	20.9	12.0	28.2	25.6	24.6	12.2	26.0
9	23.6	21.2	8.9	16.3	11.2	17.8	11.9	20.2	12.1	22.8	12.4	18.6	12.5	24.4	12.4
10	23.0	7.8	15.2	10.7	22.9	11.9	18.1	12.0	22.8	12.5	18.5	12.4	18.9	12.5	19.8
11	22.4	8.3	23.9	27.6	11.7	18.0	28.6	28.2	12.4	18.5	27.3	29.0	12.5	18.2	28.4
12	22.5	8.8	24.2	21.2	17.8	12.0	28.2	25.6	18.6	12.5	29.0	26.3	23.5	12.2	27.5
13	23.8	9.2	10.7	17.5	12.2	24.9	12.3	24.4	12.6	18.9	12.5	23.5	12.4	21.4	12.4
14	24.3	9.3	15.8	11.5	18.1	12.2	18.1	12.2	24.4	12.5	18.1	12.2	21.6	12.6	21.5
15	22.8	9.1	23.9	25.7	12.1	22.8	28.0	26.2	12.4	19.8	28.4	27.5	12.4	21.7	24.2

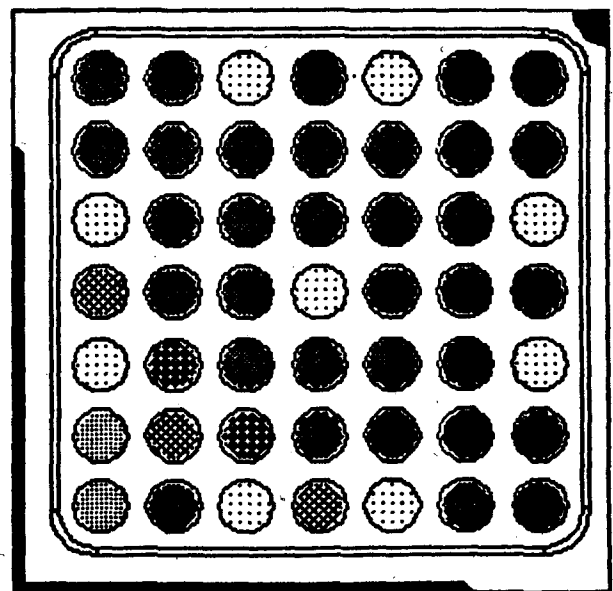
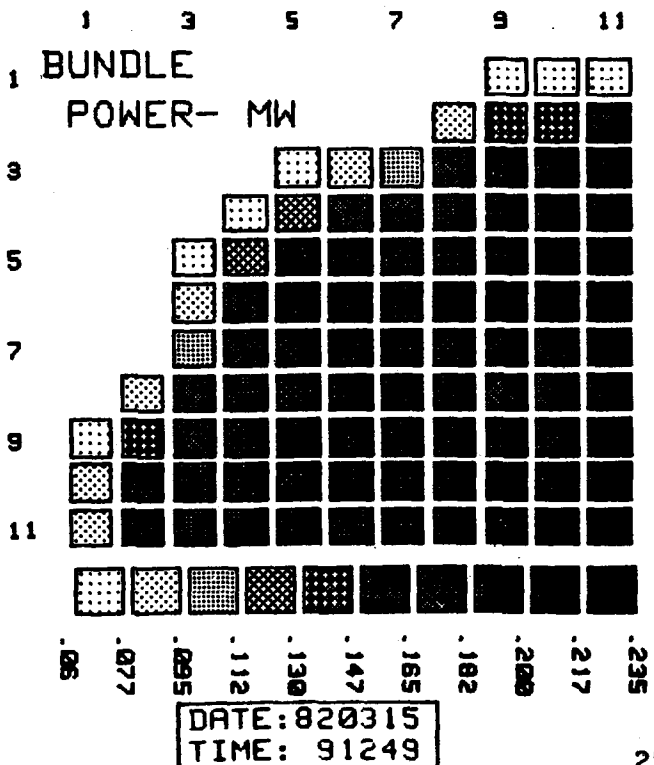
FIGURE 4
GRAY SCALE POWER GLOBAL/LOCAL DISTRIBUTIONS

4

POWER DISTRIBUTION



POWER DISTRIBUTION



power distribution for the peak power plane of a fuel assembly with the axial power shape shown. A light pen was used to select both the fuel assembly and the axial position at which the pin power distribution was to be displayed.

MEASUREMENTS REQUIRED TO VALIDATE FUEL CYCLE ADVANCES

Although many of the BWR fuel management and core design advances have great promise for utility fuel cycle cost reduction, utility priorities are on assuring reliable power supply and maintaining high capacity factors.⁴ Therefore, very careful and detailed benchmark confirmatory measurements, similar to those that have been used to qualify nuclear methods¹⁶ and fuel performance methods¹⁷⁻²², are essential to assure the reliability of fuel management and core design improvements.

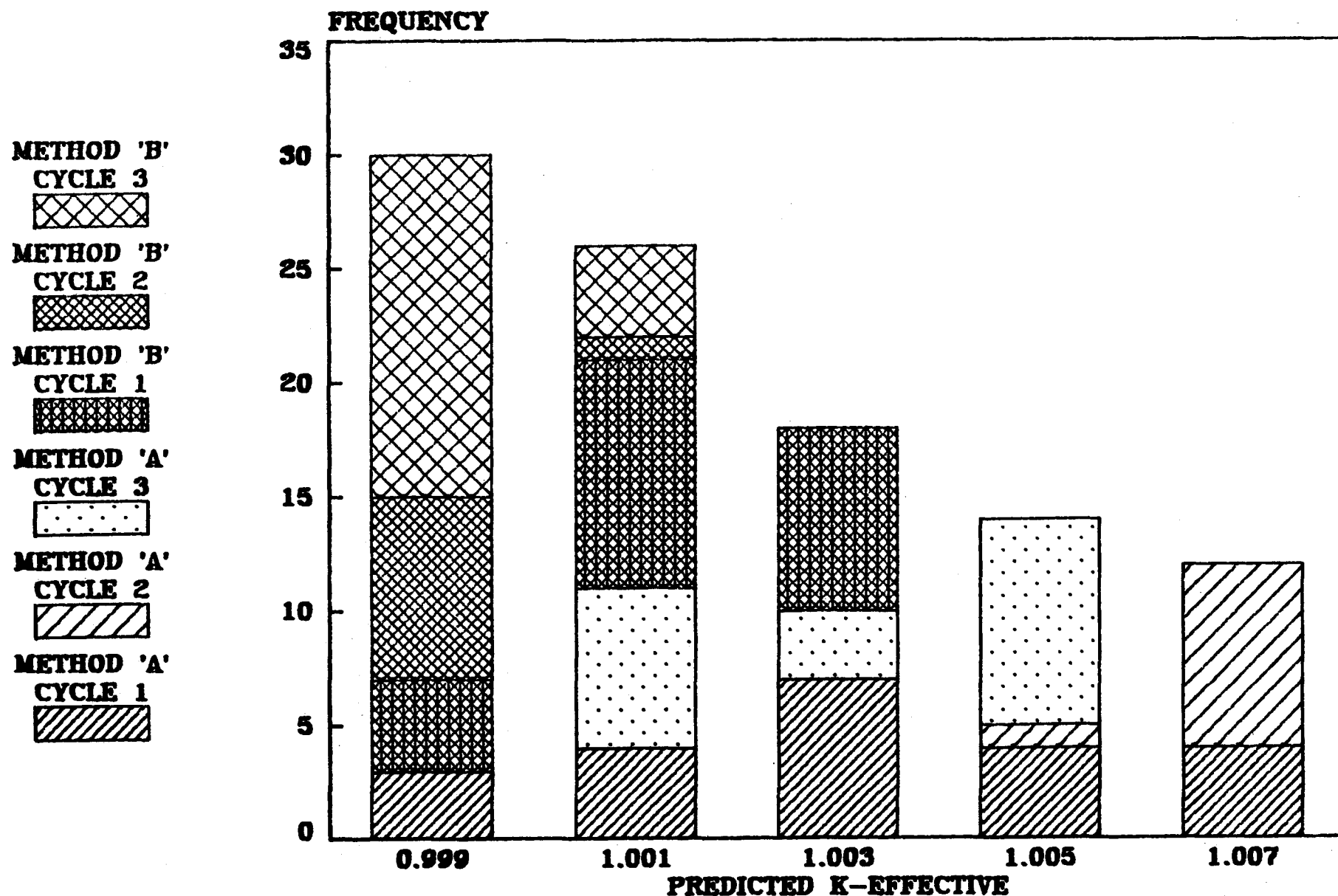
BWRs are instrumented with both fixed and traversing in-core detectors. Therefore, considerable information on the performance of fuel management advances can be obtained from detailed logging of data available from power plant instruments and from comparisons of predictions with the data. Comparison of measured and predicted criticality as a function of burnup also aids in assessing the reliability of fuel and control management predictions. Figure 5 is a comparison of predicted reactor criticality for the Hatch-1 reactor through the first three cycles of operation, using two different coupled fuel assembly design/3-D simulation methods. It is apparent that significant improvements in predictive accuracy were achieved by the physical and nuclear data improvements introduced in Design Methods Set B. The latter method incorporates improved models for representation of the two dimensional thermal neutron spectrum within a BWR fuel assembly, has improved coarse mesh nuclear and thermal-hydraulic models, utilizes ENDF/B-V cross sections, and includes other physical model improvements.

In many instances, the normal power plant data does not provide sufficient detail to comprehensively assess the accuracy of predictions or to determine the causes of poor predictions. For example, the in-core instruments are located between fuel assemblies and do not directly measure the global or the local fuel rod power distributions. Non-destructive and destructive measurements of heavy element and fission product isotopic compositions and burnup provide additional detail needed to assure reliability of predictions and to provide the high degree of reliability assurance that is desirable to give confidence that fuel cycle advances will perform as predicted and will not depreciate reactor performance or capacity factors.

Power distributions, isotopic compositions, spatial burnup and conversion ratio, reactivity, cold shutdown flux shapes, fuel performance and physical property, transient response and coupled nuclear-thermal-hydraulic stability measurements are desirable to confirm the performance of fuel management advances²⁴. It has been concluded that it is desirable to concentrate such measurements on one reactor system, if possible, so as to minimize the overall measurement expense and to provide the advantageous diagnostics from comprehensive coupled measurements. For example, if inaccuracies were observed in predicted performance for a fuel management improvement alternative, gamma scan measurements of power distributions and the other coupled destructive and non-destructive measurements would be

FIGURE 5. HATCH-1 CRITICALITY PREDICTIONS

281.



invaluable in determining the cause of the inaccuracy. Coupled fuel performance measurements of for example, isotopic composition and fission gas release are sensitive functions of the power distribution. The power distribution benchmark measurements help assure that the inputs to the fuel performance prediction comparisons with experiment are reliable.

Although, comprehensive coupled measurements on a single power reactor provide the best data at minimum cost, utilities are selective in improvement introduction and it frequently is not possible to conduct such measurements. Hence, validation usually requires coupled evaluation of measurements from a variety of power reactors, operating under different conditions.

Following are examples of some of the BWR validation measurements related to fuel cycle advances, which are either planned, underway or recently completed:

o Montecello Extended Burnup Program

- Simulated one-stuck-control rod cold criticals and fast and thermal flux wire measurements,
- Global and local gamma scans to determine power distribution,
- Heavy element isotopic composition measurements,
- Gadolina isotopic composition measurements,
- Axial and local burnup measurements,
- Profilometry, corrosion, fission gas pressure and metallurgical.

o Quad Cities-1 Advanced Fuel and Extended Burnup Program

- Global and local gamma scans to determine power distributions(at end of every cycle of fuel irradiation),
- Heavy element isotopic composition measurements,
- Gadolinium isotopic composition measurements,
- Fission product absorber isotopic composition measurements,
- Axial and local burnup measurements,
- Annular pellet fuel rod neutrography,
- Simulated one-stuck-control rod cold criticals and fast and thermal flux wire measurements,
- Profilometry, corrosion, fission gas pressure and metallurgical.

o High Enrichment Discharge Fuel Rod Reconstitution and Reinsert Demo

- Global and local gamma scans to determine power distribution.

o Advanced Control Blade Program

- Critical experiments on advanced control blade materials,
- Neutrographic measurements of power reactor control blades,
- Isotopic measurements on power reactor control blades.

o Spectral Shift Via Axial Burnup Shape Optimization

- Power reactor reactivity measurements,
- Cycle energy measurements,
- Spatial incore detector measurements.

o Control Cell Core

- Global gamma scans to measure power distribution,
- Fuel pin gamma scans to measure local power distribution, including the effects of control rod history in the control cells,
- Reactivity, incore detector and cycle energy measurements,
- Coupled effects of axial burnup shape optimization spectral shift,
- Coupled effects of axial gadolina shaping in reload fuel,
- Coupled effects of refueling cycle length,
- Relative startup times and operational simplicity,
- Relative capacity factor,
- Relative fuel reliability.

The preceding tabulation is not all inclusive, but gives insight as to the types of measurements required to validate fuel cycle advances. Comparison of current and improved design methods with such data is a monumental task which limits the frequency with which improvements can be introduced in methods of analysis. Space is insufficient to go into detail on the comparisons of methods with this data; however, additional detail will be provided in two forthcoming publications^{25,29}.

CONCLUSIONS

In conclusion, there has been significant reactor physics progress in advancing BWR fuel management and core design. Improved reactor physics measurements and methods have played an important role in this advancement. Considerable challenge remains to obtain the data necessary to test the reliability of the predicted improvements, to provide any needed additional methods accuracy improvements, and to provide the design and operations simulation methods capability required for reliable assessment of, and introduction of, additional new BWR fuel management improvements that are under consideration.

REFERENCES

1. P Wei, RL Crowther, LE Fennern, PJ Savoia, SR Specker, RM Tilley, DB Townsend, RA Wolters, "BWR Uranium Utilization Improvement Potential", GEAP-24965 (June 1980).
2. RL Crowther, PJ Savoia, P Wei, "Advanced BWR Fuel Cycles". ANS Topical Conference on "Technical Bases for Nuclear Fuel Cycle Policy", Newport, Rhode Island, (Sept. 1981).
3. US Department of Energy, Report of the Nonproliferation Alternative Systems Assessment Program. Volume IX: Reactor and Fuel Cycle Description", DOE/NE-001/9, (June 1980).

4. PD Brown, JR Ratliff, OW Taylor, RL Crowther, GC Hopkins, CL Kunz, PJ Savoia, DB Townsend, P Wei, D Yee, "Impact of Extended Burnup on the BWR Fuel Cycle", ANS Topical Conference on "LWR Extended Burnup", Williamsburg, VA (April 1982).
5. SR Specker, RL Crowther, JM Haun, "Optimization of the BWR for Once-Through Fuel Cycles", Trans. ANS 30,277 (Nov. 1978)
6. RL Crowther, PJ Savoia, P Wei, RA Wolters, "BWR Fuel Management Improvements for Once-Through Fuel Cycles", (Invited Paper), Trans. ANS 33,396-97 (Nov. 1979)
7. SP Congdon, RL Crowther, CM Kang, RB Linford, JP Neissel, GR Parkos, "BWR In-Core Nuclear Instrumentation", Trans. ANS 35,559 (Nov. 1980)
8. SR Specker, LE Fennern, RE Brown, KL Stark, RL Crowther, "The BWR Control Cell Core Improved Design", Trans. ANS 30,336-38 (Nov. 1978)
9. LE Fennern, RL Crowther, PJ Savoia, SR Specker, KL Stark, "Improved BWR Control Cell Core Design Concept", Trans. ANS 33,642-45 (Nov. 1979)
10. TG Piascik, et. al., "BWR Operating Experience at Millstone I with the Control Cell Core Improved Design", Trans. ANS 32,706 (June 1979)
11. "Control Cell Nuclear Reactor Core", U.S. Patent No. 4,285,769 (1981)
12. RL Crowther, "Methods of Fuel Management Analysis--An Overview", ANS Topical Conference on "Mathematical Models and Computational Techniques for Analysis of Nuclear Systems", Ann Arbor, Michigan (April 1973)
13. GC Hopkins, et. al., "BWR Spectral Shift", GEAP-25391 (April 1981).
14. RL Crowther, SP Congdon, BW Crawford, CM Kang, A Leonard, CL Martin, CT McDaniel, LA Schwager, RA Wolters, "Three Dimensional Simulation of Large Power Reactors", (Invited Paper), Presented at International ENS/ANS Topical Meeting on "Advances in Mathematical Methods for the Solution of Nuclear Engineering Problems", Munich, Germany (April 1981)
15. CM Kang and RL Crowther, "Adaptive Method for Three-Dimensional Boiling Water Reactor Simulation", Trans. ANS 38,340 (June 1981)
16. RL Crowther, CM Kang, GR Parkos, RA Wolters, "Feedback of Reactor Operating Data to Nuclear Methods Development", invited paper presented at ANS Topical Conference on Advances in Reactor Physics, Gatlinburg, Tennessee, CONF-780401 (1978)
17. HS Rosenbaum, "Demonstration of Fuel Resistant to Pellet Cladding Interaction, Phase I-Final Report", GEAP-23773-2, (March 1979). Also, "Phase 2-Fourth Semiannual Report", GEAP-25163-4 (March 1981)
18. "Monticello Nuclear Generating Plant Extended Exposure Fuel Program", NEDO-24202, (July 1979)

19. "Quad Cities 1 Nuclear Generating Plant Extended Burnup Fuel Program", NEDO-24855, (August 1978)
20. JE Gonser, TC Rowland and DK Dennison, "Boiling Water Reactor Fuel Rod Performance Program Third Interim Site Explanation of the Lead Test Assembly Fuel at Peach Bottom 2", NEDC-25440, (July 1981).
21. HH Klepfer and GA Roupe, "Fuel Programs to Improve Nuclear Plant Operations and Economics", Nuclex 81, Basel, Switzerland (Oct. 1981)
22. WE Baily, RB Elkins and RA Proebstle, "BWR Fuel Performance", Paper presented at Sun Valley Topical Meeting, (Aug. 1981)
23. RL Crowther, GC Hopkins, CL Kunz, PJ Savoia, P Wei (GE); PD Brown, JR Ratliff (TVA), "Advances in BWR Extended Burnup Fuel Cycles" ENS/ANS International Conf. on "New Directions in Nuclear Energy with Emphasis on Fuel Cycles", Brussels, (April 1982).
24. RE Brown, RL Crowther, LE Fennern, PJ Savoia, DB Townsend, P Wei, RA Wolters, "Reactor Physics Considerations in Design For Improved BWR Fuel Cycles", Trans. ANS, 33, 802-3 (Nov. 1979)
25. RL Crowther, GC Hopkins, CL Kunz, PJ Savoia, P Wei, "Advances in BWR Design For Extended Burnup" To Be Published. ANS, Wash., D.C. (Nov. 1982).
26. RL Crowther and DL Fischer, "Nuclear Characteristics of Large Advanced Boiling Water Reactors" Proc. of ANS Topical Conference on Nuclear Performance of Power Reactor Cores, San Francisco (Sept. 1963).
27. RL Crowther and LK Holland, "Control Management and Automation of the Boiling Water Reactor", Nuclex 69, Basel, Switzerland (Oct. 1969).
28. PG Aline, RO Brugge, WR Clancey, RL Crowther, DL Fischer, CP Ruiz, "Fuel Management and Isotopic Composition Prediction and Experiment in Light Water Power Reactors", Proceedings of International Conference on "The Physics Problems in Thermal Reactor Design", 297, London (1967).
29. SP Congdon, BW Crawford, RL Crowther, SR Specker, "BWR Fuel Management Optimization and Validation Through Operating Data"(invited paper). To be Published. ANS, Wash., D.C. (Nov. 1982).
30. PJ Savoia, P. van Dieman (GE); T Tanzawa, K. Sakaruda (NIAG-Japan); R. Yoshioka (Toshiba-Japan); "Critical Experiments on Advanced BWR Control Blade Materials" Trans. ANS 41,583 (June 1982).

THE INTERACTION BETWEEN CORE ANALYSIS METHODOLOGY AND NUCLEAR DESIGN: SOME PWR EXAMPLES

B.M. ROTHLEDER
Science Applications, Inc., McLean, VA

W.J. Eich
Electric Power Research Institute, Palo Alto, CA

Acknowledgement

The authors are especially grateful to Mr. George R. Poetschat of G.R.P. Consulting, Inc. for programming the PSEUDAX methodology and for providing a comprehensive discussion of its programming functions. The authors are also especially grateful to Dr. Laurence D. Eisenhart of Brookhaven National Laboratory for his invaluable advice and suggestions concerning the structuring of PSEUDAX so as to make it a compatible modification of PDQ7/HARMONY.

ABSTRACT

The interaction between core analysis methodology and nuclear design is exemplified by PSEUDAX, a major improvement related to the Advanced Recycle Methodology Program (ARMP) computer code system, still undergoing development by the Electric Power Research Institute. The mechanism of this interaction is explored by relating several specific nuclear design changes to the demands placed by these changes on the ARMP system, and by examining the meeting of these demands, first within the standard ARMP methodology and then through augmentation of the standard methodology by development of PSEUDAX.

INTRODUCTION

Changes in nuclear core design are necessitated by the desire either to achieve generic improvements (e.g., tighter margins) or to meet specific requirements (e.g., modified cycle length.) To model these changes, standard core analysis methodology often must be augmented by improved techniques which then become part of the standard methodology. Examples of this interaction between nuclear design and core (or fuel management) analysis methodology relevant to PWR cores will be discussed.

Among utilities and related organizations, PWR core analysis is performed principally with an XYZ nodal simulator (e.g., EPRI-NODE-P¹ or SIMULATE²) coupled with an XY representation of the quarter-core using PDQ7/HARMONY.^{3,4} The XYZ nodal simulator has the following characteristics:

- 1 or 4 nodes per assembly in the XY plane, each node homogenized,
- 12 to 24 nodes per assembly axially (in Z), each node homogenized,

- one-group or group-and-a-half energy structure,
- temperature (moderator and fuel) and xenon feedbacks,
- no explicit representation of baffle or reflector.

Characteristics of the XY PDQ7/HARMONY representation are

- 1 mesh square per pin cell (fine mesh), each mesh homogenized,
- explicit thimble or lumped absorber locations, each thimble or lumped absorber homogenized,
- explicit baffle and reflector,
- two energy macro-groups,
- uniform average core temperature (moderator and fuel).

The nodal simulator is used, typically, to determine

- boron letdown,
- assembly XY power,
- axial power shapes,
- reactivity worths and coefficients.

The PDQ7/HARMONY is used to determine

- nodal simulator normalization,
- XY pin power peaks for loading (shuffling) patterns and for stuck rod and related analyses.

To evaluate the limitations of this nodal/PDQ analysis methodology, we postulate as "ideal" the possible but, for utilities, impractical gedanken calculation performed by full-core PDQ7/HARMONY having as components XYZ geometry, fine mesh, and temperature and xenon feedbacks. These ideal components become the nuclei about which augmentations to the standard methodology may be constructed.

Several nuclear design changes introduced over the past several years have resulted in augmentations now being developed. These changes are described below.

- Use of XY PDQ7/HARMONY to determine XY pin power peaks in the absence of axial coupling and feedback has long been of some concern relative to advanced cycle fuel loading pattern determination and relative to stuck rod and other accident analyses. More recently, tighter margins on enthalpy rise peaking have focussed attention on the accuracy of PDQ7/HARMONY pin power peaks during normal operation. One solution in the past has been the imposition of conservatism on calculated local peaking. Elimination of this conservatism will result in improved core performance by allowing operation at higher power levels.
- Introduction of the burnable absorber gadolinium, to improve core performance for current designs with

tight margins, will necessitate axial zoning of some form to control axial power shifting.^{5,6,7,8} Such axial zoning may well be incompatible with reliance on an average moderator and fuel temperature single plane XY PDQ7/HARMONY for local peaking analysis.

- Establishment of a short first cycle, required by some utilities (e.g., Texas Utilities Services for Comanche Peak Unit 1), has necessitated use of a relatively low region 1 enrichment. This has caused the first cycle XY power peak to occur in a peripheral assembly, and has caused a large hot-zero-power to hot-full-power power shift.⁹ This behavior enhances the importance of including axial coupling and feedback effects in local XY power peaking calculations (PDQ7/HARMONY).

These nuclear design changes were confronted initially using standard core analysis methodology coupled with innovative formulations of the problems to be solved. In the ensuing discussion, this initial confrontation will be examined first. Remaining unaccounted for, however, have been the effects of axial coupling and feedback in the XY pin power analysis. This has led to a major augmentation to the standard core analysis methodology. This augmentation, called PSEUDAX, is now under development and will be examined secondly in the ensuing discussion. PSEUDAX is constructed around a nucleus of the ideal components: XYZ geometry, fine mesh, and temperature feedback. Its goal is to produce XY pin powers from PDQ7/HARMONY which reflect axial coupling and feedback phenomena with sufficient accuracy to make more realistic assessments of local peaking effects for fuel shuffling patterns, for potential gadolinium designs, for power distributions such as those found in the Comanche Peak plant, for stuck rod conditions, etc.

SOME NUCLEAR DESIGN PROBLEMS AND THEIR SOLUTIONS

An Old Problem and Its Solution: Fuel Loading Patterns and Pin Power Peaking

When loading patterns are established in PWRs beyond first cycle, partially burned assemblies must be repositioned to achieve a minimum hot channel factor.*¹⁰ For a 121-assembly PWR (a small core) which typically retains two-thirds of its cycle n charge for cycle $n+1$, the total number of possible arrangements of partially burned assemblies in cycle $n+1$ is an astronomical $81!$ or $\sim 10^{120}$, but quarter-core symmetry reduces this to $\sim 10^{26}$ — still astronomical. Since the value of $\hat{F}_{xy}$ must be essentially a minimum, one must judiciously choose a tractable number of arrangements from among the 10^{26} possible such that trial-and-error may be used to determine which arrangements will lead to an $\hat{F}_{xy}$ value convincingly near the minimum. The principles for making these judicious choices are described in the following discussion.

* The hot channel factor is defined as the maximum pin power in the XY plane ($\hat{F}_{xy}$).

During fuel management between cycles, retained assemblies may be rearranged according to any of four mapping schemes: (1) translational mapping, in which two assemblies are interchanged within the same quadrant; (2) vertical mapping, in which quarter-core symmetric assemblies are interchanged across the vertical; (3) horizontal mapping, in which quarter-core symmetric assemblies are interchanged across the horizontal; and (4) diagonal mapping in which quarter-core symmetric assemblies are interchanged diagonally. In XY depletion codes, an assembly is necessarily and sufficiently represented by an $n \times n$ array of isotopic number densities, hence it is these number densities which are mapped.

Since fuel management design calculations are usually performed on a quarter-core basis, the full-core mapping schemes must be simulated; these full-core to quarter-core simulations are exact, except when full-core symmetry boundaries bisect assemblies. Translational mapping remains unchanged in this simulation. Vertical mapping is simulated by a pointwise vertical reflection (VR) of number densities within an assembly of the quarter-core. Horizontal mapping is simulated by a similar pointwise horizontal reflection (HR). Diagonal mapping is simulated by a similar pointwise 180° rotation (R-180). When rearranging assemblies, the following transformation rules should be noted.

(VR)(VR)	= (HR)(HR)	= (R-180)(R-180) = 0
(HR)(VR)	= (VR)(HR)	= R-180
(HR)(R-180)	= (R-180)(HR)	= VR
(VR)(R-180)	= (R-180)(VR)	= HR

Power peaking in advanced cycles will occur near the least burned corners of the least burned assemblies. These "hot channel corners" must be located to minimize their power peaking. Primary locations adjoin the baffle or adjoin the center assembly. Secondary locations adjoin the fresh fuel at the core periphery with the hot channel corners pointing outward from the core center. All primary and secondary locations cannot be used in the same cycle, since the result would be an amassing of low burnup assemblies and consequently severe power peaking. Tertiary locations for these hot channel corners may be anywhere in the core interior provided that each corner is surrounded by a set of three highest burnup corners to offset the power peaking potentialities of the hot channel corner. The essential characteristics of the primary, secondary, and tertiary locations are that they are either in regions of low neutron importance or in regions of depressed power.

The value of $\hat{F}_{xy}$ will be decreased additionally by gross power flattening over the entire core. This may be achieved by avoiding arrangements where low and high burnup assemblies are separately amassed in regions of high neutron importance. In so doing, the assemblies should be oriented so that the burnup gradients of contiguous assemblies oppose each other, and relatively low burnup corners are counterbalanced by relatively high burnup corners adjacent to them. At this point approximate eighth-core symmetry should result.

Another factor important in determining an acceptable assembly loading pattern in advanced cycles is the fresh fuel enrichment. If conventional fuel management (out-in) is used, a high fresh fuel enrichment will draw

power toward the outside of the core, while a low enrichment will push power nearer to the core center. If low-leakage fuel management is used, the principles discussed in the foregoing still apply, but different techniques would evolve during application.

In practice, the foregoing principles are applied using the XY PDQ7/HARMONY model described earlier. This model lacks axial coupling and feedback, hence there is no guaranty that the loading pattern so chosen would produce a minimum $\hat{F}_{XY}$ in the "real core", i.e., in the results from its simulation by the impractical gedanken calculation described previously. The need for a "best model" to determine $\hat{F}_{XY}$ stems from the following factors:

1. $\hat{F}_{XY}$ is not measured,
2. No functional relationship exists between $\hat{F}_{XY}$ and loading pattern,
3. The value of $\hat{F}_{XY}$ set as a technical specification is being met in many current cores with little or no margin.

The role of PSEUDAX in producing this "best model" will be discussed further on.

A Recent Problem and Its Solution: Use of Gd_2O_3 as a Burnable Absorber

As an alternative to the use of lumped burnable absorbers in PWR cores, distributed burnable absorbers are being introduced, taking the form of Gd_2O_3 mixed with UO_2 in selected fuel rods (as is currently done in BWR cores). Studies have been performed examining total substitution of lumped burnable absorbers by Gd_2O_3 ^{5,6}, and examining a hybrid design wherein Gd_2O_3 augments the normal lumped burnable absorber complement in order to achieve a more negative moderator temperature coefficient (MTC)^{7,8}. Because of the complex manner in which gadolinium depletes, careful attention must be paid to the peaking factors and power distributions resulting from its incorporation in a given design. Specifically, one must examine the enthalpy rise peaking factor ($F_{\Delta H}$), the global peaking factor ($\hat{F}_Q$), the XY pin power peaking factor ($\hat{F}_{XY}$), and the axial offset ($AO = (P_U - P_L)/(P_U + P_L)$, where P is power, U refers to the upper half-core, L refers to the lower half-core.) Axial zoning patterns relative to Gd_2O_3 have a major effect upon AO and $\hat{F}_Q$ and a lesser but non-negligible effect upon $\hat{F}_{\Delta H}$. The effect of axial zoning on $\hat{F}_{XY}$ is also significant, although subtle in its manifestation. An additional factor, axial normalization of the nodal simulator^{2,6,8,11}, introduces a new complexity.

In general, two techniques may be used to establish an axial zoning pattern: "mutual reinforcement" and "homogeneous nonuniformity." Mutual reinforcement consists of adjusting top-of-core and bottom-of-core gadolinium loadings such that they reinforce each other in shifting power with depletion in a desired manner. Homogeneous nonuniformity uses alternation of axial gadolinium concentrations to reduce expectable local power increase consequent to depletion. Older designs that have relatively

large peaking and axial offset margins may dispense with these techniques and zone axially by merely absencing Gd_2O_3 from the upper and lower 15 to 45 centimeters of the fuel stack. Newer designs with tight margins must resort to more elaborate axial zoning and as a result may achieve better core performance in general. Specifically, the hybrid design referred to previously has the following properties relative to a reference (Westinghouse 157-assembly) Cycle 1 design:

1. The least negative moderator temperature coefficient is 40% more negative than that of the reference design.
2. The enthalpy rise peaking factor ($\hat{F}_{\Delta H}$) is increased by only 1% over that of the reference design.
3. The global peaking factor ($\hat{F}_Q$) is decreased by 5% over that of the reference design.
4. The axial offset band is tighter than that of the reference design by 1% in axial offset units.
5. Control rod worth at beginning-of-cycle is greater than that of the reference design by 6% (absolute).
6. At end-of-cycle, the properties of the hybrid design become indistinguishable from those of the reference design.

For the hybrid design, the value of $\hat{F}_{xy}$ determined from quarter-core XY PDQ7/HARMONY calculations was always within technical specifications except at beginning-of-cycle 1 at zero xenon.¹¹ Although this is an unrealistic full-power operating condition, judicious substitution of a few offending Gd_2O_3/UO_2 rods by UO_2 rods would lower the $\hat{F}_{xy}$ at this condition enough to meet the technical specifications. During depletion of the hybrid design, the $\hat{F}_{xy}$ showed significant migration from the core exterior to the core interior. This is not surprising in light of the manner in which gadolinium depletes and in view of the axial power shifting observed, even after axial zoning.^{6,8}

The optimism shown in the foregoing discussion concerning Gd_2O_3 as a burnable absorber is tempered somewhat by the complexity introduced by axial normalization of the nodal simulator. In the absence of gadolinium (the reference design), the assumption of total reflection top and bottom has proved to be adequate based on the comparison of axial shapes produced by a reference-point XYZ PDQ7/HARMONY with explicit reflectors (but without thermal feedback) with those axial shapes of the XYZ nodal simulator (without thermal feedback, by option).¹¹ In the reference design, the effect of omitting thermal feedback, though significant, is small enough (6% or less on the axial peak and 6% or less in axial offset units) to be superposed by inclusion after normalization. In the hybrid design, axial albedos must be adjusted to values other than those representing total reflection because of the non-uniform axial loading of the gadolinium. Furthermore, the shifting (oscillating) in axial power brought about by the depletion characteristics of the gadolinium necessitate adjusting or

improving the axial albedos during depletion. In the hybrid design, the effect on the axial parameters of omitting thermal feedback is a factor of from 3 to 6 greater than that in the reference design and is greatly oscillatory with depletion.¹¹ Under these circumstances, thermal feedback may not be superposed after normalization without introduction of an unacceptably large error. Therefore, the reference to which the nodal simulator must be normalized axially for the hybrid (or any gadolinium) design must include not only explicit top and bottom reflectors, but also must examine the effects of thermal feedback for possible inclusion.

Since the effect of thermal feedback on the axial parameters is so much greater in the presence of gadolinium than in its absence, one would expect this condition to obtain for $\hat{F}_{xy}$ also, although to a much lesser degree. In another sense, the absence of axial coupling and feedback from the XY PDQ7/HARMONY, having already cast doubt on loading patterns chosen for cores not containing gadolinium, would be expected to cast greater doubt on values of $\hat{F}_{xy}$ so determined for cores with gadolinium. Indeed this is the case. In the reference design, the interior-to-exterior shift in power brought about by ignoring feedback and axial coupling is no greater than 12% during first cycle, while the corresponding parameter for the hybrid design is 17%.¹¹ The role of PSEUDAX in introducing axial coupling and feedback to $\hat{F}_{xy}$ calculations will be discussed shortly.

Additional Problems

A value of $\hat{F}_{xy}$ determined without axial coupling or feedback is not sufficiently accurate, not only in relation to the two problems just discussed, but also when it is to be calculated for stuck rod conditions or for any conditions in which a skewed power distribution or a tilted (but symmetric) power distribution obtains. An example of the latter condition is that found in the Comanche Peak core, referenced previously. PSEUDAX's role in these considerations will be discussed in the sections following.

A NEW SOLUTION: PSEUDAX

Background

A PSEUDAX calculation is a PDQ7/HARMONY case which has, as input, assemblywise powers, sources, and moderator temperatures defined as truth and serving as target values for the PDQ7/HARMONY solution. These target powers, sources, and temperatures may be specific plane (e.g., peak plane) values from a nodal analysis, average plane values from a nodal analysis, or any set of values reflecting axial coupling and feedback (e.g., data from non-neutronics codes or from measurements.) PSEUDAX PDQ7/HARMONY will modify its solution by adjusting cross sections which vary with power (i.e., fuel temperature) on a pointwise basis using feedback about the imposed average power and by adjusting cross sections which vary with moderator temperature on an assemblywise basis. These adjustments will be made with reference to target powers, sources, and temperatures, the pointwise power and source effects being determined by interpolation based on assembly gradients. Thus, the PSEUDAX PDQ7/HARMONY will produce a solution which will include substantially the effects of axial coupling and feedback while retaining a fine mesh description.

Since the PSEUDAX PDQ7/HARMONY can use target data from non-neutronics codes such as those used for nuclear safety analysis, it can be expected to find additional application in the area of coupled nuclear core and safety calculations. Work currently in progress, not related to PSEUDAX, will provide such coupling independently by linking the ARMP system ¹ to RETRAN ¹² and VIPRE ¹³. This linkage and more specific details of improvements in RETRAN and VIPRE are discussed in a paper prepared concurrently for this meeting. ¹⁴

PSEUDAX Development

PSEUDAX was developed in 3 sequential stages: Assembly Target PSEUDAX, XY Feedback PSEUDAX, and Matched Power PSEUDAX. At present, Matched Power PSEUDAX is in the process of being tested by implementation using the Comanche Peak core model.

Assembly Target PSEUDAX

The Assembly Target PSEUDAX methodology assumes that the XY PDQ7/HARMONY results are improved by using better cross section representation, e.g., cross section dependence on power (fuel temperature) and moderator temperature. The nodal assembly powers and moderator temperatures may be obtained from a radial slice of an XYZ nodal calculation and are input to PDQ7/HARMONY as dummy nuclide densities. The target nodal assembly power and moderator temperature are different for each assembly but are uniform within an assembly. The nodal assembly powers and moderator temperatures are used to obtain macroscopic cross section modifications. This is an improvement over the normal ARMP PDQ7/HARMONY, which does not use power and moderator temperature in the cross section fitting. The iteration segments then calculate the eigenvalue and the flux distribution. Finally the pinwise powers are obtained. The preliminary results of Assembly Target PSEUDAX showed that the PDQ7/HARMONY assembly powers were closer to the target powers but went only partway toward duplicating these powers. The fact is that the XY solution in PDQ7/HARMONY will not necessarily match the XY edit of an XYZ flux solution.

XY Feedback PSEUDAX

The XY Feedback PSEUDAX methodology was implemented to perform a pinwise power feedback calculation normalized to the nodal (target) assembly powers. These nodal assembly powers are input as in the Assembly Target PSEUDAX. The cross section modification and iteration segments are performed initially with uniform intra-assembly powers as in the Assembly Target PSEUDAX. At the end of the iteration, PDQ7/HARMONY calculates pinwise powers. These pinwise powers are normalized back to the target assembly powers. Then the pinwise powers are fed back to the cross section segment for another iteration pass, this time with pinwise powers. The feedback convergence is obtained when the calculated powers from two feedback iterations are within a specified tolerance. Feedback iterations are defined as those in which the macroscopic cross sections are changed. These feedback iterations should not be confused with the inner and outer

iterations used to converge the fluxes. The feedback options under discussion, however, still do not fully reproduce the assembly average nodal target powers.

Matched Power PSEUDAX

The concept of Matched Power PSEUDAX is a combination of XY Feedback PSEUDAX and pinwise fixed source PDQ7/HARMONY calculations. This combination assumes that a fixed source PDQ7/HARMONY iteration based on pinwise source values, where the source values are determined from the pinwise power distribution in each assembly normalized to the input nodal average assembly powers, will produce the desired PDQ7/HARMONY pinwise power distribution with the average powers in each assembly matching the nodal powers. The present method of the PDQ7/HARMONY fixed source problem requires the user to select the fixed source option and to supply fixed source values by composition and group as input data. During the cross section calculations, the nu-sigma-fission term for each mesh block is replaced by the input source values for this composition in each group. The iteration segments then perform the fixed source calculation to determine the flux distribution based on this input source. The program then calculates the integration edits, but it skips the buckling, k-effective, and k-infinite calculations, and it edits the source terms in place of the nu-sigma-fission terms. Note that since the normal PDQ7/HARMONY fixed source method is limited to fixed source values by composition, there is no method for pinwise source input.

The Matched Power PSEUDAX requires a pinwise capability. A pinwise fixed source method can be implemented by assuming that the pinwise fixed source is a function of the CHI (fission source fraction) values in each group and the pinwise power. The pinwise power is obtained in the same manner as in the XY Feedback PSEUDAX by performing feedback iterations. Once the desired pinwise source is obtained, the flux and power distributions will be used for edits and for the pinwise depletion. For the fixed source portion of the calculation, the nu-sigma-fission values are converted to source in each mesh block as follows:

$$\text{SOURCE}(\text{group}) = \text{CHI}(\text{group}) * \text{Relative Power}$$

A normal fixed source calculation is then performed. The PSEUDAX routine will convert the pinwise powers into pinwise power distributions normalized to the nodal target powers in each assembly. These powers will be compared to the pinwise relative powers used in the cross section calculation, and if these values are within a specified tolerance, the feedback iterations will be terminated. If the powers are not converged, the new normalized powers will be inserted into the concentration file and the calculation will be fed back to the cross section routine for another sweep through the Matched Power PSEUDAX iteration scheme.

PSEUDAX Applications

The PSEUDAX methodology will enable XY PDQ7/HARMONY to include the effects of axial coupling and feedback in its fine mesh representation of a single plane. The source of the axial coupling and feedback effects may be

a nodal analysis, a non-neutronics analysis, or a set of measurements. PSEUDAX will provide pin peaking factors of greater accuracy than practicably obtainable hitherto, while still retaining practicability. PSEUDAX will thus find application in loading pattern determination for advanced cycles, power distribution analysis of gadolinium-containing cores, stuck rod analysis, and power distribution analysis of skew-loaded cores (either tilt-loaded or asymmetric-loaded).

CONCLUSIONS

Within the standard framework of the Advanced Recycle Methodology Program¹ — a widely used PWR (and BWR) core analysis computer code system — the problem of determining advanced cycle loading patterns has been solved by applying transformation rules for mathematical arrays to a set of physical conditions determined by the behavior of reactor materials in a neutron flux, and the problem of axial power shape control and its effect on XY power distribution in gadolinium-loaded PWR cores has been solved by using zoning techniques which take advantage of the physical behavior of gadolinium in a PWR neutron spectrum. Both of these problems, along with several others, continue to be analyzed, within the standard core analysis framework, by methods which, for reasons of practicability, omit the components of axial coupling and feedback from the XY pin power determinations. New methodology in the form of PSEUDAX is being developed to correct for this omission while still retaining practicability. Thus, the essential results and accuracy of an XYZ PDQ7/HARMONY with pinwise detail and all feedback effects are obtained at minimal cost through application of the PSEUDAX methodology.

REFERENCES

1. Advanced Recycle Methodology Program, System Documentation, CCM-3 Electric Power Research Institute (Sept. 1977).
2. D.M. Ver Planck, "Manual for the Reactor Analysis Program SIMULATE," YAEC 1158, Electric Power Research Institute (Aug. 1, 1978).
3. W.R. Cadwell, "PDQ-7 Reference Manual," WAPD-TM-678, Bettis Atomic Power Lab. (Jan. 1967).
4. R.J. Breen, O.J. Marlowe, and C.J. Pfeifer, "Harmony: System for Nuclear Reactor Depletion Computation," WAPD-TM-478, Bettis Atomic Power Lab. (Jan. 1965).
5. B.M. Rothleder and W.J. Eich, "Global Effects of Axially Zoned Gadolinium in PWR Cores," Trans. Am. Nucl. Soc., 35, 97(1980).
6. B.M. Rothleder, "Feasibility of Using Gadolinium as a Burnable Poison in PWR Cores," EPRI NP-1663, Electric Power Research Institute (Feb. 1981).
7. B.M. Rothleder, "Reduction of Moderator Temperature Coefficient by Gadolinium Substitution for PWR Application," Trans. Am. Nucl. Soc., 39, 616 (1981).

8. B. M. Rothleder, "Reduction of Moderator Temperature Coefficient by Gadolinium Substitution for PWR Application," EPRI NP-2191, Electric Power Research Institute (Jan. 1982).
9. W.J. Eich, et al., "ENDF/B-V-Based Few Group PWR Baffle/Reflector Constants," Trans. Am. Nucl. Soc. 39, 907(1981).
10. B.M. Rothleder, "Power Reactor Fuel Management Beyond First Cycle Operation," Trans. Am. Nucl. Soc., 14, 89(1971).
11. B.M. Rothleder, "Gadolinium in PWR Cores: Local Effects," Electric Power Research Institute, to be published.
12. "RETRAN-02: A Program for Transient Thermal-Hydraulic Analysis of Complex Fluid Flow Systems," EPRI NP-1850, Electric Power Research Institute (May 1981).
13. VIPRE Code Manuals, EPRI CCM, Electric Power Research Institute, to be published.
14. G. Lellouche, et al., "RASP: An Integrated Reload Analysis and Safety Program," published in these Proceedings.

APPLICATION OF STATIC AND TIME-DEPENDENT GENERALIZED
PERTURBATION THEORY METHODS IN LWR PHYSICS ANALYSES*

J. R. White and M. L. Williams
Oak Ridge National Laboratory
Oak Ridge, TN 37830

By acceptance of this article, the publisher or recipient acknowledges the U.S. Government's right to retain a nonexclusive, royalty-free license in and to any copyright covering the article.

ABSTRACT

This paper examines the validity and potential usefulness of generalized perturbation theory (GPT) as a tool in the physics analysis of light water reactor (LWR) systems. Special emphasis is placed on the ability of the time-dependent methodology to quantitatively relate the end-of-cycle state of a realistic PWR model to variations made at earlier times in the cycle. Comparisons between static and time-dependent GPT results are made and the physics implications of the observed differences are briefly discussed. The paper also outlines several generic advantages and disadvantages of perturbation methods, and it is concluded that GPT methods represent an attractive alternative to standard brute-force methods for many types of LWR physics analyses. However, there are some types of problems frequently encountered in LWR core calculations for which first-order perturbation theory breaks down, and these are also examined and briefly discussed.

INTRODUCTION

Generalized Perturbation Theory (GPT) is a technique used to estimate changes in various reactor "response" parameters arising from changes in the core material configuration or in the cross-section data utilized in the reference computational model. It differs from the usual k_{eff} perturbation theory in that it cannot only be applied to eigenvalue perturbations but also to perturbations in more general responses such as reaction rate ratios, power density ratios, axial power offset, reactivity worths, reactivity coefficients, etc.. This general analysis technique has been extensively developed over the years and has been successfully applied in many areas.¹⁻⁴ More recent theoretical and computational developments in time-dependent perturbation theory have greatly extended the potential applications of these "static" GPT methods, particularly with respect to fuel depletion problems.⁵⁻⁸

The new DEPTH-CHARGE⁸ modules of the VENTURE⁹ system represent the formal implementation of time-dependent GPT for application to realistic multidimensional, multigroup, and multicycle burnup analyses. The DEPTH-CHARGE system contains both nuclide density perturbation and cross-section sensitivity formulations, with the more-familiar static GPT formulation contained as a special case of the time-dependent methodology. This newly-developed calculational capability represents an attractive alternate analytical and computational tool for certain types of reactor physics and design optimization studies.

Although there is a fair amount of applications experience with time-dependent perturbation methods, most of this knowledge has been directed

*Research sponsored by the U.S. Department of Energy under contract W-7405-eng-26 with the Union Carbide Corporation.

toward the analysis of fast-reactor systems.¹⁰⁻¹² Therefore, the primary purpose of the present study is to examine the overall applicability of GPT methods as a tool for the physics analysis of thermal reactor systems. The main subject areas addressed in support of this goal include:

1. the accuracy of first-order perturbation methods (both static and time-dependent) for "reasonable" perturbations to the reference system,
2. the comparison of "static" and "time-dependent" sensitivity coefficients, and
3. the physics implications of the sensitivity information.

In addition, several of the benefits and potential applications, as well as the present limitations of GPT methods, are indicated throughout the remainder of this paper.

As a final introductory remark, it should also be noted that a time-dependent perturbation formulation specifically designed for LWR nodal analysis has been implemented into the SIMULATE code.¹³⁻¹⁴ However, the present GPT capability in SIMULATE has not been as extensively developed and tested as that in the DEPTH-CHARGE/VENTURE system. The VENTURE system, on the other hand, was primarily designed for fast-reactor analysis and has some limitations for routine LWR physics applications (mainly in the treatment of burnup-dependent cross sections). Since our primary goal here is the demonstration of a perturbation methodology rather than a definitive analysis seeking final results for a particular LWR design, we have chosen the DEPTH-CHARGE system as the GPT analysis tool for this study. Of course, the implication here is that if these methods were to be widely used for LWR physics analysis, then the appropriate perturbation capability would need to be implemented and evaluated in standard LWR physics codes (like SIMULATE or PDQ).

METHODOLOGY

The 2-D LWR model utilized in this study corresponds to the midplane of a 1000-MW PWR based on the B&W TMI-1 design.¹⁵ The beginning-of-cycle (BOC) model has three different enrichment assemblies and three different burnable poison loadings, arranged with 1/8-core symmetry as shown in Fig. 1. Representative four-group microscopic cross sections were generated with EPRI-CELL, and these data were assumed to be constant in all subsequent burnup calculations.

The 18-month cycle-1 operating history for this TMI-1 model was simulated in 5 burnup steps (75 full power days/step). A four-step soluble boron letdown curve was imposed and the nuclide transmutation during each step was calculated using assembly-averaged fluxes. This depletion calculation served as the reference case for the sensitivity study and provided the "unperturbed" BOC and EOC power distributions presented in Fig. 1.

For the purpose of this study, the power distribution was also characterized by the six coarse regions shown in Fig. 1. This was done to reduce the number of integral parameters used to represent the overall power profile. However, it should be noted that the choice of these particular regions was arbitrary, and only represents one of many alternate methods of describing the power distribution. In fact, if more detail is desired, each assembly power could be treated as a response-region (at a somewhat increased cost since each response requires a separate adjoint calculation).

Fig. 1. Core Map Showing Assembly Locations and Zone/Region Power Distributions

W = Position #
X = Assembly Type
A = 2.06 w/o
B = 2.747 w/o
C = 2.747 w/o; 0.047 gms Boron/in
D = 2.747 w/o; 0.054 gms Boron/in
E = 2.747 w/o; 0.062 gms Boron/in
F = 3.05 w/o; 0.054 gms Boron/in
G = 3.05 w/o
Y = BOC Power Fraction
Z = EOC Power Fraction
R = Region #

Region Power Fractions

Region	BOC	EOC
1	1.103	1.079
2	1.200	1.066
3	1.139	1.107
4	0.873	0.910
5	1.074	1.098
6	0.727	0.852

WX	R
Y	
Z	

				29G	6		
				0.52			
				0.68			
				26A	5	27F	6
				0.95		0.78	
				1.04		0.99	
				28G	6		
				0.49			
				0.60			
				22A	5	23C	5
				1.17		1.09	
				1.06		1.15	
				24A	6	25G	6
				0.95		0.80	
				1.03		0.88	
				16A	2	17C	2
				1.21		1.22	
				1.02		1.11	
				18A	3	19D	3
				1.15		1.02	
				1.06		1.15	
				20F	4	21G	4
				0.85		0.54	
				1.05		0.62	
				9A	1	10E	2
				1.14		1.17	
				1.03		1.09	
				11A	2	12D	3
				1.21		1.19	
				1.03		1.13	
				13A	3	14E	4
				1.13		1.00	
				1.08		1.10	
				15G	4		
				0.82			
				0.80			
1E	1	2E	1	3A	2	4D	2
1.01		1.09		1.17		1.21	
1.14		1.12		1.02		1.10	
5A	3	6D	3	7B	4	8G	4
1.21		1.19		1.36		0.96	
1.05		1.16		1.16		0.82	

Using data from the reference burnup calculation, a comprehensive set of adjoint computations were performed with the DEPTH-CHARGE/VENTURE system. The responses chosen for this study included both BOC and EOC k_{eff} and region-averaged power ratios for the six regions defined in Fig. 1 (total of 14 adjoint cases, 7 static and 7 depletion runs). These responses should allow a detailed analysis and characterization of the BOC and EOC states for this reactor model.

The numerical output of the adjoint solutions is a set of time-dependent derivatives that describe how the system response will react to a specified nuclide or cross-section perturbation. The nuclide derivatives,

$$\underline{N}_z^*(t) = \frac{dR(t_f)}{dN_z(t)},$$

are nuclide-, space-, and time-dependent functions describing the rate of change of the response at time t_f with respect to the nuclide density vector $\underline{N}$ in zone z at time t . The components of the nuclide vector correspond to the atom densities of the various nuclides in the reactor, and the elements of the $\underline{N}^*$ vector are the respective nuclide "importances." Similarly, the nuclear data derivatives,

$$\beta^*(t_i) = \frac{dR(t_f)}{d\beta(t_i)},$$

are nuclide-, set-, time-, and energy-dependent functions describing the rate of change of the final-time response with respect to the cross-section and decay data utilized during time step i in the reference burnup calculation. The discrete time notation t_i is utilized to indicate that the microscopic data are assumed to be fixed during each burnup interval. The functions $\underline{N}_z^*$ and β^* are often called the "sensitivity coefficients" for the response to changes in the nuclide density vector and nuclear data, respectively.

In practice, the nuclide derivatives of interest are also discrete functions of time, since one is only allowed to "perturb" the reference calculational model at the beginning of each time step (e.g., assembly shuffling or burnable poison loading at BOC, soluble boron or control rod adjustments during the cycle, etc.). Using these discrete derivatives, one can easily estimate the change in the response $R(t_f)$ due to a given nuclide (or cross-section) perturbation ΔN with the evaluation of a simple integral expression of the form

$$\Delta R(t_f) = \sum_i \sum_z \sum_j (\underline{N}_{jz}^* V_z \Delta N_{jz})_i,$$

where the ijz subscripts refer to the time, nuclide, and space variables, respectively. The above expression is accurate to second order and assumes the familiar linear relationship between the perturbed variable and perturbed response. This allows the superposition of multiple perturbations. It should also be emphasized that the "importance functions" are independent of the specific perturbation and, therefore, only need to be calculated once for each response of interest. Thus many different design and operational variations to the reference model can be investigated once the appropriate adjoint functions have been determined - this is the real cost advantage of any perturbation theory method.

At this point it is important to emphasize the simple relationship of the above time-dependent formulation to the more-familiar concepts of static perturbation

theory. To distinguish between static and time-dependent perturbation theory, one need only specify the time at which the perturbation is made. If $t=t_1=t_f$ in the above expressions for the nuclide and cross-section derivatives, the familiar static sensitivity theory results. If, however, t and t_1 are less than t_f , the derivatives become time-dependent functions and thus relate the change in the final-time response to perturbations made prior to t_f . In particular, time-dependent perturbation theory can provide a quantitative relationship between the BOC and EOC states of our PWR model (by letting $t = \text{BOC}$ and $t_f = \text{EOC}$). Such a capability can be a very useful analysis tool, especially in reload studies where many burnup calculations are required to determine a consistent loading pattern that gives acceptable core conditions throughout the burnup cycle.

Up to now we have intentionally omitted many details concerning the theoretical and computational aspects of the time-dependent GPT methods under consideration in this study. These aspects of the technique are thoroughly documented elsewhere,⁵⁻⁹ and will not be repeated here. However, in order to understand some of the results to follow, it is important that the reader realize that the present development treats the nonlinear coupling between the neutron and nuclide fields encountered in depletion analyses in a rigorous and consistent manner that is accurate to second order. Three adjoint equations (for the flux shape, flux normalization, and nuclide densities) are required to fully account for this neutron/nuclide interaction. This rigorous treatment allows one to address the "direct effects" of a given perturbation as well as the "indirect effects" caused by the perturbation's influence on the flux distribution and overall normalization. A simple example of this is that a localized perturbation will have a "local" direct effect as well as a "global" indirect effect due to the local perturbation's effect on the global flux distribution, which may vary as a function of time.

Lastly, it should be noted that once a reference calculational setup has been performed within the VENTURE system, the actual preparation of the adjoint computations within the DEPTH-CHARGE codes is quite trivial. Also, each adjoint case has a comparable computer cost to the reference static and/or depletion problem. The reader is referred to Ref. 8 for further details of the adjoint computational procedure.

COMPARISON OF GPT WITH DIRECT CALCULATION

As indicated previously, a comprehensive set of nuclide and cross-section sensitivity information has been generated for the PWR model at hand. Our goal here is to present some of this data and to discuss some of the conclusions and implications of these results.

Tables 1 and 2 present a comparison of GPT results with direct calculations for the BOC and EOC regional power density responses. These data show results predicted with first-order perturbation theory versus "exact" results for both "local" and "global" perturbations made to the reference model at BOC (see the footnote at the bottom of Table 1 for a description of the "local" assembly interchanges and "global" variations). There are several important observations to be made here.

First, we see that for the "local" perturbations (Cases 1-3), good agreement for the BOC responses is uniformly achieved only for Case 1. Although only a few assemblies were interchanged in Cases 2 and 3, these perturbations resulted in

Table 1. Comparison of GPT Results with Direct Calculation
for the BOC Power Ratio Responses

Region		$\% \Delta R_n / R_n$ ($R_n = PD_n / PD_{ave}$ at BOC)				
n		Case 1	Case 2	Case 3	Case 4	Case 5
1	Direct	12.33	- 40.80	49.14	- 4.44	- 11.42
	GPT	13.33	- 32.00	29.14	- 3.62	- 10.85
2	Direct	8.58	- 30.00	10.56	- 2.92	- 7.67
	GPT	9.47	- 22.35	8.37	- 2.60	- 6.74
3	Direct	- 1.32	- 9.31	- 6.23	- 0.07	- 0.51
	GPT	- 1.42	- 5.06	- 4.00	- 0.21	- 0.21
4	Direct	- 17.87	13.86	- 4.47	4.18	10.82
	GPT	- 20.13	13.46	- 0.28	3.69	9.10
5	Direct	6.33	10.06	- 11.55	- 0.79	- 1.54
	GPT	7.13	6.27	- 8.92	- 0.66	- 0.90
6	Direct	3.44	56.12	- 16.51	1.51	4.54
	GPT	3.96	35.73	- 11.92	1.56	4.05

Note: Case 1: In Zone 7 replace assy. B with assy. A.
Case 2: In Zones 20 and 27 replace assy. F with assy. G.
Case 3: In Zones 1, 2, 10, and 14 replace assy. E. with assy. A.
Case 4: Uniform 10% increase in ^{238}U density.
Case 5: Uniform 10% increase in moderator density (not in reflector).

Table 2. Comparison of GPT Results with Direct Calculation
for the EOC Power Ratio Responses

Region		$\% \Delta R_n / R_n$ ($R_n = PD_n / PD_{ave}$ at EOC)				
n		Case 1	Case 2	Case 3	Case 4	Case 5
1	Direct	- 2.41	9.55	- 18.26	1.48	.19
	GPT	- 2.59	9.63	- 17.73	1.43	.27
2	Direct	- 1.22	6.57	- 2.44	1.50	- 1.50
	GPT	- 1.30	6.05	- 2.91	1.49	- 1.88
3	Direct	0.81	0.63	3.07	0.61	- 1.46
	GPT	0.79	0.46	2.59	0.65	- 1.44
4	Direct	0.99	- 2.09	- 4.40	- 1.62	1.57
	GPT	1.15	- 1.97	- 3.45	- 1.68	1.89
5	Direct	- 0.64	- 3.46	6.28	0.39	- 0.52
	GPT	- 0.73	- 3.56	6.21	0.44	- 0.47
6	Direct	0.35	- 7.75	6.10	- 1.54	2.45
	GPT	0.25	- 6.32	6.09	- 1.51	2.53

large variations in the power profile that were not well-predicted with first-order GPT. Thus, it appears that "static" first-order perturbation theory is not very accurate for the analysis of PWR power distributions that result from "realistic" assembly interchanges. However, perturbations which result in 20-40% changes in the reference power distribution are not usually acceptable anyway; thus static GPT methods could be used to easily and inexpensively eliminate these unacceptable configurations.

In contrast to the BOC responses, Table 2 indicates that changes in the EOC power distribution due to BOC assembly interchanges are generally well-predicted with first-order time-dependent GPT. Thus, as indicated earlier, depletion perturbation theory does indeed provide an accurate quantitative means of relating parameters at the EOC to variations made at earlier times in the cycle, even for large perturbations for which the static method breaks down!

Considering now the "global" variations to the system (Cases 4 and 5), we see that both static and time-dependent GPT results are in relatively close agreement with direct calculations. This is an important result since it tends to validate any subsequent data sensitivity analyses or global reactivity coefficient computation that may be performed, since these types of variations to the reference system are typically uniform in nature. The ^{238}U density perturbation in Case 4, for example, could represent an uncertainty in the actual density or cross-section data, and the moderator density change in Case 5 could be that associated with a coolant temperature change. Either of these cases would represent a nearly uniform variation to the reference reactor model.

A comparison of GPT results with direct calculation for the BOC and EOC k_{eff} responses is presented in Table 3. Here we see that the estimates of a "global" response parameter behave similarly to the prediction of the "local" regional power responses. Thus, we may conclude that for large "local" perturbations, BOC responses are not usually well-predicted (Cases 2 and 3), but that the effects

Table 3. GPT Versus Direct Calculation
for the k_{eff} Response

Case		BOC $\frac{\% \Delta K}{K}$	EOC $\frac{\% \Delta K}{K}$
1	Direct	- 0.24	- 0.18
	GPT	- 0.28	- 0.17
2	Direct	1.17	0.21
	GPT	0.65	0.28
3	Direct	0.69	- 0.84
	GPT	0.49	- 0.84
4	Direct	- 2.29	- 1.29
	GPT	- 2.36	- 1.32
5	Direct	0.91	0.81
	GPT	0.97	0.87

of "global" variations are usually estimated quite accurately at BOC. Furthermore, all the perturbations examined here were effectively accounted for at the EOC. This summary gives some general overall guidelines on the use of GPT methods for LWR applications, as follows:

1. GPT estimates of "static" responses due to large local perturbations should be used only in a qualitative sense,
2. GPT estimates of "static" responses due to realistic global system variations may be used quantitatively, and
3. GPT estimates of "time-dependent" responses (i.e., responses at some time following the given system variation) are accurately predicted and may be used to quantitatively determine the final-time state of the system without performing a detailed depletion analysis.

COMPARISON OF "STATIC" AND "TIME-DEPENDENT" SENSITIVITY DATA

An obvious question raised from the results of the previous subsection is: "Why does time-dependent GPT generally perform better than the static case?" This question is easily answered by comparing the absolute magnitudes of the deviation in the BOC and EOC regional power responses for a given BOC variation. This comparison indicates that for the same BOC perturbation the change in the final-time response is considerably less than the observed change in the initial static response. This suggests that the EOC responses can be more accurately predicted with perturbation theory methods since the standard assumption inherent in these methods is that the perturbations are "small" and result in minor variations in the system state variables, such that the second-order terms are small.

The above observation is consistent with similar effects observed in previous fast-reactor studies¹⁰⁻¹² and in the SIMULATE work,¹³ and is due mainly to the self-correcting nature of the depletion process. For example, highly non-uniform flux distributions tend to burn the peaked locations at a faster rate, thereby smoothing the overall distribution. In the present case, the reference power profile at EOC was quite uniform and even though large perturbations were introduced at the BOC for Cases 1-3, they had nearly died away by the EOC, again leaving fairly uniform distributions. Thus, the sensitivity of the EOC distribution to variations made at the BOC tends to be relatively small and linear in nature. It is this linearity that permits the use of the time-dependent method in a wider range of applications relative to static GPT.

Although the above arguments are well-known and easily understood in a qualitative sense, the ability to quantify these sensitivities with depletion perturbation theory is a new and potentially useful capability. Figures 2 and 3, for example, illustrate the detailed time- and space-dependent nature of the sensitivity functions for several important nuclides in assembly 9. These profiles represent the rate of change of the EOC responses P1/PT and P4/PT (Figs. 2 and 3, respectively) due to a nuclide density change in assembly 9. Here the P1/PT response is defined as the total power generated in region 1 divided by the total reactor power. The sensitivity data has been normalized to unity at the EOC "static" state. Also, it should be noted that the observed discontinuities are due to the discretization of the depletion process and would be smoothed out as more depletion steps were included. The particular plots given here were constructed from data for only six discrete time points.

of "global" variations are usually estimated quite accurately at BOC. Furthermore, all the perturbations examined here were effectively accounted for at the EOC. This summary gives some general overall guidelines on the use of GPT methods for LWR applications, as follows:

1. GPT estimates of "static" responses due to large local perturbations should be used only in a qualitative sense,
2. GPT estimates of "static" responses due to realistic global system variations may be used quantitatively, and
3. GPT estimates of "time-dependent" responses (i.e., responses at some time following the given system variation) are accurately predicted and may be used to quantitatively determine the final-time state of the system without performing a detailed depletion analysis.

COMPARISON OF "STATIC" AND "TIME-DEPENDENT" SENSITIVITY DATA

An obvious question raised from the results of the previous subsection is: "Why does time-dependent GPT generally perform better than the static case?" This question is easily answered by comparing the absolute magnitudes of the deviation in the BOC and EOC regional power responses for a given BOC variation. This comparison indicates that for the same BOC perturbation the change in the final-time response is considerably less than the observed change in the initial static response. This suggests that the EOC responses can be more accurately predicted with perturbation theory methods since the standard assumption inherent in these methods is that the perturbations are "small" and result in minor variations in the system state variables, such that the second-order terms are small.

The above observation is consistent with similar effects observed in previous fast-reactor studies¹⁰⁻¹² and in the SIMULATE work,¹³ and is due mainly to the self-correcting nature of the depletion process. For example, highly non-uniform flux distributions tend to burn the peaked locations at a faster rate, thereby smoothing the overall distribution. In the present case, the reference power profile at EOC was quite uniform and even though large perturbations were introduced at the BOC for Cases 1-3, they had nearly died away by the EOC, again leaving fairly uniform distributions. Thus, the sensitivity of the EOC distribution to variations made at the BOC tends to be relatively small and linear in nature. It is this linearity that permits the use of the time-dependent method in a wider range of applications relative to static GPT.

Although the above arguments are well-known and easily understood in a qualitative sense, the ability to quantify these sensitivities with depletion perturbation theory is a new and potentially useful capability. Figures 2 and 3, for example, illustrate the detailed time- and space-dependent nature of the sensitivity functions for several important nuclides in assembly 9. These profiles represent the rate of change of the EOC responses $P1/PT$ and $P4/PT$ (Figs. 2 and 3, respectively) due to a nuclide density change in assembly 9. Here the $P1/PT$ response is defined as the total power generated in region 1 divided by the total reactor power. The sensitivity data has been normalized to unity at the EOC "static" state. Also, it should be noted that the observed discontinuities are due to the discretization of the depletion process and would be smoothed out as more depletion steps were included. The particular plots given here were constructed from data for only six discrete time points.

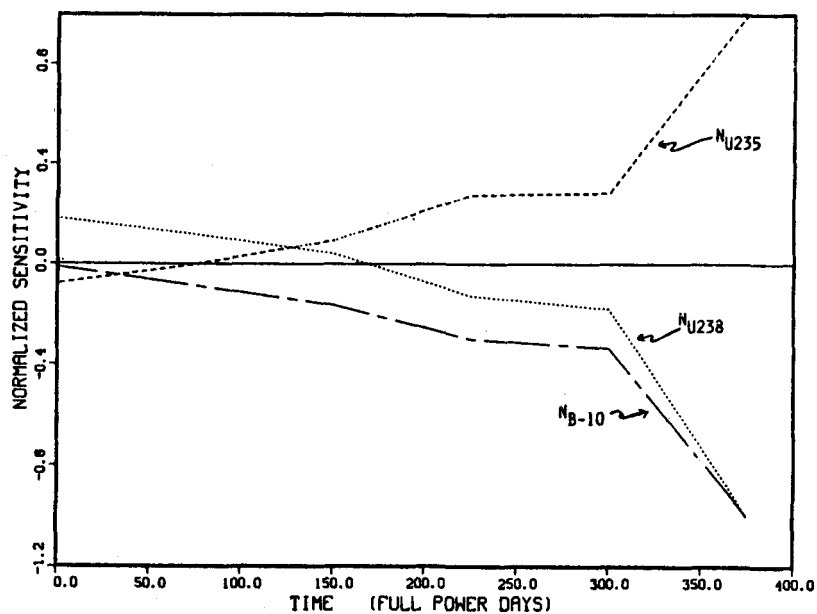


Fig. 2. Time-dependent zone 9 nuclide sensitivity profiles for the EOC P1/PT response.

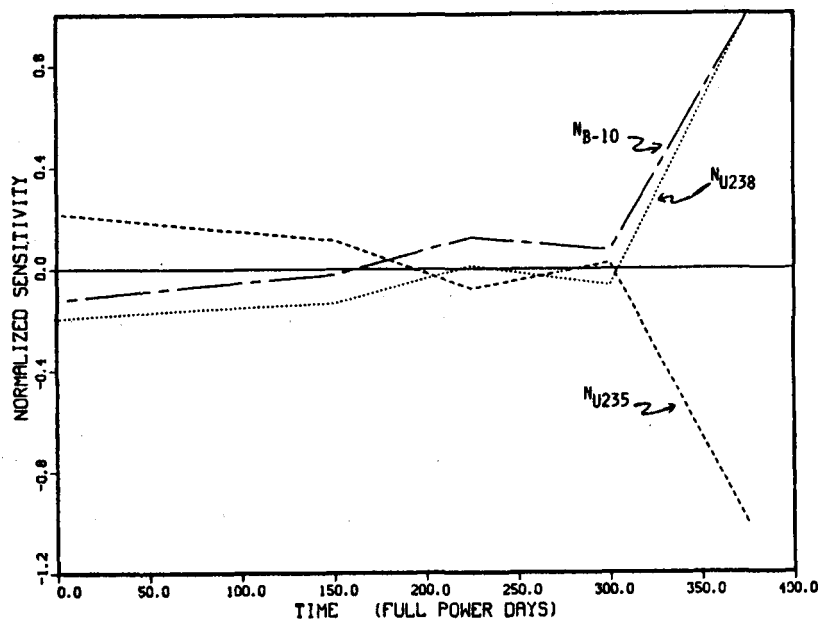


Fig. 3. Time-dependent zone 9 nuclide sensitivity profiles for the EOC P4/PT response.

Several of the previous statements relative to the comparison of static and time-dependent GPT and their relationship to the overall depletion process can be justified through the analysis of the data in Figs. 2 and 3. First, we see that in only one time interval (75 full power days), the sensitivities for the EOC responses have fallen to less than 30% of their initial EOC "static" values and continue to remain small for the remainder of the cycle. Of course, combinations of many nuclide variations can add in various ways, possibly changing the relative importance of the overall static and time-dependent character of the response. However, as observed in Tables 1-3, the combined time-dependent effects are typically much smaller than the static effects and therefore, are more accurately treated with linear perturbation theory methods.

Detailed analysis of Figs. 2 and 3 can also provide considerable insight into the space-time interactions that may be involved in a given problem. Recalling that assembly 9 is included within region 1, we see from Fig. 2 that the addition of fissile material close to the EOC state increases the P1/PT response at EOC and that the addition of a neutron absorber decreases the response, as expected. However, these effects are reversed if one considers inserting additional ^{235}U and ^{238}U at the BOC. In the ^{235}U case, the increased burnup rate at the BOC in the central regions of the core and corresponding decreased burnup in the outer regions are enough to shift the power distribution to the outer regions at the EOC, accounting for the small negative sensitivity at the BOC. The same kind of argument can be made to account for the positive BOC ^{238}U sensitivity to the EOC P1/PT response, with proper consideration given to the buildup of ^{239}Pu during the 18-month irradiation period.

Since a perturbation in assembly 9 has no direct effect on the power production in region 4, the sensitivity profiles in Fig. 3 for the P4/PT response result totally from the indirect effects associated with assembly 9's effect on the flux and burnup distribution. Basically, we see that the roles of the fissile and absorber materials are just reversed from that seen for the P1/PT response. A "static" increase in the ^{235}U density in assembly 9 increases the P1/PT response, but it decreases the P4/PT ratio due to the shift in the flux distribution to the core center. After a certain burnup period, however, the initial ^{235}U perturbation has altered the relative burnup rates throughout the core such that the EOC P4/PT response is increased relative to the reference configuration.

The above illustrations are representative of the kind of detailed interactions that are an inherent part of any burnup analysis. The strong coupling of the space- and time-dependent neutron and nuclide fields complicates the overall process, making intuitive reasoning sometimes impossible. Perturbation theory is a tool that can be utilized to gain some insight into these detailed processes, and its predictive capability can be used to efficiently and economically evaluate many of the options open to the core-design or reload analyst.

One last topic of interest in the comparison of static and time-dependent GPT is the area of cross-section sensitivity and uncertainty analysis. As indicated previously, an "error" or uncertainty in the nuclear data utilized in a computational analysis manifests itself as a global-type perturbation to the reference system. Thus, data sensitivities are typically integrated over the reactor volume and therefore do not exhibit the complicated spatial variations that are observed for the nuclide density sensitivity functions. However, much

information is lost in the integration procedure, making the interpretation of data sensitivities somewhat more difficult, especially when analyzing sensitivities for locally defined responses.

Table 4 presents a comparison of static and time-integrated sensitivity coefficients for the EOC k_{eff} , P1/PT, and P4/PT responses. The static sensitivities represent the percent change in the EOC response due to a one percent uniform variation in the group cross sections utilized in the EOC flux-eigenvalue calculation. The time-integrated coefficients, however, reflect the effect of the cross sections being altered throughout the entire burnup calculation. Note that group-dependent cross-section variations can also be treated with little additional complexity.

Table 4. Comparison of Static and Time-Integrated Data Sensitivity Coefficients

Data Parameter	EOC k_{eff}		EOC P1/PT		EOC P4/PT	
	Static	Time- Integrated	Static	Time- Integrated	Static	Time- Integrated
^{235}U σ_c	- 0.058	- 0.076	0.331	- 0.005	- 0.201	- 0.023
	σ_f	0.213	0.196	- 0.908	- 0.060	0.514
	ν	0.490	0.498	- 2.372	- 0.016	1.349
^{238}U σ_c	- 0.271	- 0.175	- 0.193	0.131	0.167	- 0.174
	σ_f	0.043	0.045	0.033	- 0.000	- 0.021
	ν	0.068	0.068	0.065	- 0.005	- 0.038
	σ_s	- 0.001	- 0.001	- 0.197	0.013	0.119
^{239}Pu σ_c	- 0.106	- 0.146	- 0.413	0.081	0.238	0.036
	σ_f	0.198	0.176	0.947	- 0.093	- 0.594
	ν	0.383	0.376	1.627	- 0.180	- 0.951
^{240}Pu σ_c	- 0.051	- 0.013	- 0.411	0.075	0.224	- 0.039
^{241}Pu σ_c	- 0.008	- 0.010	- 0.102	- 0.046	0.055	0.035
	σ_f	0.028	0.022	0.378	0.072	- 0.209
	ν	0.052	0.051	0.675	0.201	- 0.357
Macro σ_c	- 0.061	- 0.056	- 0.072	0.004	0.063	- 0.001
	σ_s	0.217	0.144	- 0.781	- 0.036	0.533

There is a large amount of useful information contained in Table 4, but only a few of the major points will be emphasized here. First, we make the obvious statement that in many cases there is a considerable difference in the results from static and time-dependent GPT analyses, especially for the regional power ratio responses, indicating that the more comprehensive time-dependent methodology should be utilized in future data sensitivity analyses for operating power

reactors. These differences are due to the complicated self-correcting coupling of the neutron and nuclide fields, and can be accurately treated only with "coupled" GPT methods.⁵

Also noteworthy is the fact that the variation in the static and time-integrated sensitivity data is usually such that the time-integrated value has a lower absolute magnitude than the static value (an exception to this is for capture in the fissile nuclides for the EOC k_{eff} response). This observation can usually be justified by considering the physics of the specific situation. For the EOC k_{eff} response, for example, the approximate 35% decrease in the time-integrated ^{238}U capture sensitivity is simply a result of the increased ^{239}Pu production associated with an increase in the ^{238}U capture rate. At the EOC, the additional neutron loss rate due to an increase in ^{238}U is partially offset by greater neutron production from the additional ^{239}Pu ; the end result being that the EOC k_{eff} is less sensitive to the ^{238}U capture data than is the "static" reactivity.

Figure 4 presents the actual time-dependent sensitivity profiles of the above three responses to the ^{238}U capture cross section. As just discussed, the profile for the k_{eff} response shows the effect of the steadily increasing ^{239}Pu inventory with burnup due to the increased ^{238}U capture rate (note that the burnup interval referred to here is relative to the EOC state). The other responses have profiles that are more difficult to interpret, but the observed shapes are the combined result of slight shifts in the flux shape and subsequent redistribution of the ^{239}Pu density field associated with the given ^{238}U capture perturbation. It appears that a depletion interval of 75 full power days is long enough to obtain near-equilibrium conditions following the data variation. Thus, we see a relatively slowly varying profile at larger burnups (relative to the EOC state) for the power ratio responses.

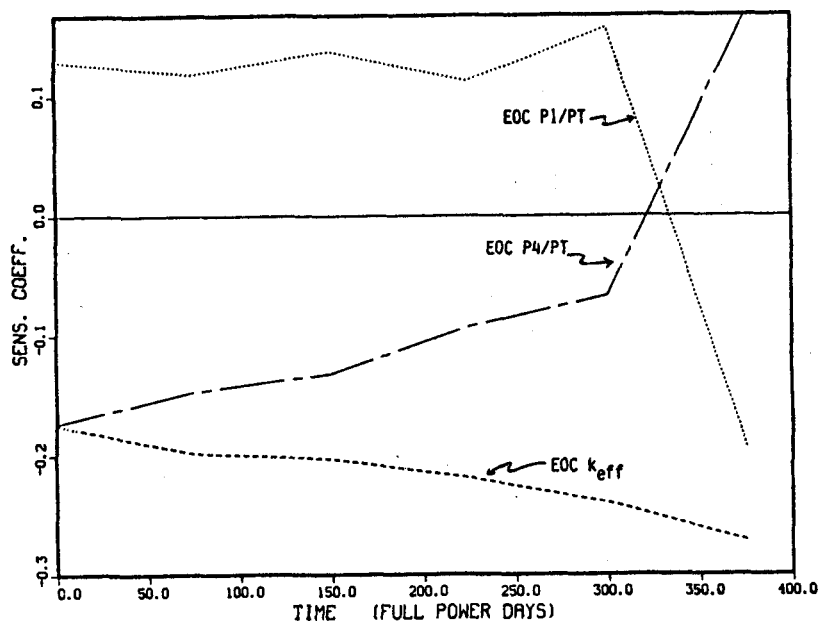


Fig. 4. Cumulative ^{238}U capture cross-section sensitivity profiles for three EOC responses.

As indicated previously, many of the above phenomena are generally well known to the LWR physics analyst, thus we are not really trying to present new ideas here. The real point to be emphasized from the above examples is the ability of GPT methods to quantify these concepts and to provide a detailed understanding of the physics involved in some complicated, but realistic, analysis situations.

POTENTIAL APPLICATIONS AND PRESENT LIMITATIONS

In the previous section it was noted that the sensitivity data generated in this study can be a very useful design aid, even without further manipulation. In fact, qualitative sensitivity data, whether generated by GPT methods or by the direct recalculation of many "forward" problems, are routinely and extensively utilized as an analysis tool to determine what system parameters are important, how they affect core performance, and where a more detailed treatment may be required. However, it is felt that static and time-dependent nuclide and data sensitivities can also be effectively utilized as a systematic and quantitative design analysis and optimization tool. Thus, the purpose of this section is to briefly outline where GPT methods might be utilized as such a computational tool. In addition, we will also indicate some of the present limitations of the method as applied to LWR analysis.

To address this topic we note that there are at least four generic ways in which sensitivity-based methods could be applied:

1. Obtain a more detailed understanding of the physical system through investigation of the important sensitivities.
2. Utilize the sensitivity data in cross-section and design uncertainty analyses.
3. Perform parametric design studies using GPT sensitivity estimates.
4. Utilize the predictive capability of GPT methods in computer-automated design optimization studies.

The first item has been discussed previously in some detail, and its potential benefits should be apparent. The application of GPT methods to data uncertainty analyses (Item 2) is a well-established field for fast-reactor analyses.¹⁻⁴ Although some work has been done in this area for LWR applications,¹⁶⁻¹⁷ the full capability of these methods has not been extensively utilized for thermal reactor systems. The recent introduction of time-dependent GPT methods has also greatly extended the range of application of static methods to include the investigation of a wide variety of burnup-dependent response functionals. As indicated previously, a comparison of static and time-dependent sensitivities generally shows a decreased magnitude for the major nuclide reactions for the time-dependent cases, thus indicating a reduced data-related integral parameter uncertainty associated with the final-time or time-integrated responses. The implications noted here were not analyzed in detail in this study (for example, k-reset effects³⁻⁴ were not considered), but they do open up a potentially interesting and informative area of research.

The application of GPT methods in parametric design studies (Item 3) is quite straightforward and can be a very powerful and efficient analysis tool. Questions like "What would happen if the fuel loading pattern was changed slightly, or if the number or enrichment of the new assemblies were varied, or if the burnable poison loadings were altered, etc.," could be answered (approximately) in a very efficient and cost-effective manner if a library of sensitivity data (such as that generated in this study) were available.

The last application noted above is the use of GPT methods as a driving force for a design optimization package that could provide an estimate of the "optimum" design consistent with a set of user-defined system constraints. The basic components of such a package would be 1) a library of sensitivity data for the main system performance parameters, 2) a linear programming or other optimization scheme to automate the selection of the "optimum" design, and 3) a driver module to control the overall calculation, define system constraints, and generally handle any interfacing requirements.

Although several authors have attempted to exploit the advantage of GPT methods for design optimization,¹⁸⁻²⁰ most of the presently developed methods require many simplifying assumptions and treat only relatively simple reactor models. This required simplification is due in part to the overall complexity of the problem to be solved, and also to the present limitations of first-order GPT methods. In this context, the major limitations of present perturbation methods are their 1) complexity, 2) limited accuracy, and 3) ability to accurately treat only integral responses. It is the combination of these three limitations that presently restricts the widespread use of GPT methods. Two out of three of these negative attributes could be tolerated in many cases, but the addition of the third component has a serious impact on the applicability of GPT methods for use in comprehensive design optimization.

To justify the above statements, consider the following arguments:

1. If the generation of the sensitivity data were trivial and inexpensive, one would not be concerned with the number of responses treated and the number of times that the sensitivities were recomputed.
2. If the perturbation results were accurate for any combination and magnitude of the variations to the system, then a single set of sensitivities for each response would be valid for the life of the core (and any similar core design), and would be well worth the initial calculational cost.
3. If perturbation methods could treat distribution functions (flux, power, burnup, etc.), the number of responses would be dramatically reduced, also greatly decreasing the cost and time associated with a comprehensive analysis.

In the present study, six responses were used to characterize the overall power distribution. If a more detailed analysis were to be performed, however, one might consider a single response per assembly, or four responses per assembly, etc., thereby increasing the complexity and cost of such an analysis. The fact that present time-dependent methods can only treat responses at a given point in time (integral over a delta function in time) or averaged over time is also a major disadvantage. In the present case, the BOC and EOC responses were treated separately, and core conditions between these two points were not considered since this would have increased the number of responses (and adjoint computations) to be considered. Thus, there is a real need for a perturbation method that can economically treat responses that are functions of space and/or time, and some work is currently being performed in this general area.²¹⁻²³

Our above statements are somewhat contradictory. First, we illustrated the validity and potential usefulness of GPT methods for LWR physics analysis and now we state that they have some major disadvantages that affect the realistic application of the methods. Both arguments are true and a compromise is usually made when a specific application is treated. For detailed optimization studies, where many variables and performance parameters are important, present GPT

methods are usually not appropriate (to the authors' knowledge there is no method at present that solves this problem). However, if the geometric complexity of the core is reduced and other simplifying assumptions are imposed (as is done in most analysis situations), then GPT methods again become a viable tool for the analysis and optimization of the simplified mathematical model, with the relationship to the more complex situation inferred through direct computations, experience, intuition, etc.

SUMMARY

The present work has sought to demonstrate the validity and potential usefulness of static and time-dependent perturbation theory as a tool in the analysis of LWR systems. The previous sections of this paper have described the major results and conclusions obtained in this study. It has been shown that time-dependent GPT methods can relate the EOC state of a realistic PWR model to variations made at earlier times in the cycle, and that the prediction of EOC responses is generally more accurate than the estimates of static functionals, due mainly to the smoothing nature of the depletion process. Even in the worst cases, however, static GPT was capable of providing useful qualitative information about the system. It was also indicated that a considerable amount of detailed physics information could be obtained from GPT analyses and that these data can provide invaluable guidance in the analysis of LWR fuel management and reload design studies. The comparison of static and time-dependent data sensitivities was also briefly mentioned, and it was noted that the more comprehensive time-dependent methodology allows one to determine the effect of the data uncertainties throughout the depletion cycle rather than just at the initial static state. Lastly, several generic application areas and general limitations of perturbation methods were outlined.

In addition, it should be noted that generalized perturbation theory methods are capable of treating a more comprehensive set of responses and perturbations than that analyzed in this study. For this analysis, the multiplication factor and power distribution were chosen as the representative responses of interest and an interchange of assemblies was used as a typical perturbation to the system. Other LWR performance parameters that may be estimated using GPT methods include the moderator temperature and Doppler coefficients, control rod worths, axial power offset, shutdown reactivity margin, or any parameter that can be written as a functional (or ratio of functionals) of the system variables (flux, nuclide densities, and cross sections). Similarly, a more complete list of typical perturbations include moderator temperature or density, control rod insertion/withdrawal, soluble boron density, burnable poison loadings, reactor power level, fuel enrichment, etc., as well as fuel assembly shuffling. Thus, GPT methods can treat many of the response/perturbation pairs of interest in LWR's with an accuracy which is dependent on whether or not the response is a static or final time functional and on the local/global nature of the perturbation.

In addressing the above topics, many questions were answered relative to the validity and overall applicability of GPT methods as a tool for the physics analysis of thermal reactor systems. From the results obtained here, we can conclude that GPT methods are a viable option, and we recommend that further efforts be made to implement the appropriate methodology into existing LWR physics codes. An efficient and economical initial point for continuing this effort would be to further develop and test the capability already initiated by ORNL within the SIMULATE code. With this capability, the LWR analyst could get hands-on experience in the realistic application of GPT methods.

REFERENCES

1. C. R. Weisbin et al., "Sensitivity and Uncertainty Analysis of Reactor Performance Parameters," Advances in Nucl. Sci. and Tech., Plenum Press, New York (1982).
2. C. R. Weisbin et al., Eds., Proc. Sem.-Workshop, "Theory and Application of Sensitivity and Uncertainty Analysis," ORNL/RSIC-42, Oak Ridge, Tenn. (1979).
3. J. H. Marable and C. R. Weisbin, "Uncertainties in the Breeding Ratio of a Large LMFBR," Proc. of the Am. Nucl. Soc. Topical Meeting, "Advances in Reactor Physics," CONF-780401, Gatlinburg, Tennessee (1978).
4. J. M. Kallfelz et al., "Design and Sensitivity Analysis of a CDS-type LMFBR Heterogeneous Core," Proc. of the Am. Nucl. Soc. Topical Meeting, "1980 Advances in Reactor Physics and Shielding," Sun Valley, Idaho (1980).
5. M. L. Williams, "Development of Depletion Perturbation Theory for Coupled Neutron/Nuclide Fields," Nucl. Sci. Eng., 70, 20 (1979).
6. J. R. White and T. J. Burns, "Development and Verification of Multicycle Depletion Perturbation Theory," Trans. Am. Nucl. Soc., 34, 824 (1980).
7. E. Greenspan et al., "Time-Dependent Generalized Perturbation Theory for Coupled Neutron-Nuclide Problems," Nucl. Sci. Eng., 73, 210 (1980).
8. J. R. White, "The DEPTH-CHARGE Static and Time-Dependent Perturbation/Sensitivity System for Nuclear Reactor Core Analysis," ORNL/CSD-78 (1981).
9. D. R. Vondy et al., "The Bold VENTURE Computation System for Nuclear Reactor Core Analysis, Version III," ORNL-5711 (1981).
10. J. R. White et al., "On the Implementation, Verification, and Application of Multicycle Depletion Perturbation Theory," Proc. of the Am. Nucl. Soc. Topical Meeting, "1980 Advances in Reactor Physics and Shielding," Sun Valley, Idaho (1980).
11. J. R. White, "Burnup Effects in Data Sensitivity Studies," Trans. Am. Nucl. Soc., 35, 587 (1980).
12. D. Biswas, et al., "Depletion Analysis and Sensitivity Study of PHENIX Fuel Irradiation Experiments," Trans. Am. Nucl. Soc., 38, 673 (1981).
13. D. M. VerPlanck, "Manual for the Reactor Analysis Program SIMULATE," EPRI Report for RP710-2 (1978).
14. S. M. Bowman, "The Development of Depletion Perturbation Theory for a Reactor Nodal Code," ORNL/TM-7589 (1981).
15. Three Mile Island, Unit 1, Final Safety Analysis Report, Babcock & Wilcox Co.
16. R. L. Childs et al., "Sensitivity and Uncertainty Analysis for the Mixed Oxide Thermal Lattice U-L212," EPRI-NP-1248 (1979).

17. J. M. Ryskamp et al., "Sensitivity of Light Water Reactor Fuel Cycle Parameters and Costs to Uncertainties in Thermal Nuclear Data and Methods," Nucl. Sci. Eng., 77, 285 (1981).
18. J. O. Mingle, "In-Core Management via Perturbation Theory," Nucl. Tech., 27, 248 (1975).
19. D. D. Ebert et al., "The Development of Manuevering Strategies Using Optimal Control Theory," Nucl. Sci. Eng., 69, 398 (1979).
20. D. Rozon and A. Hebert, "The Application of Generalized Perturbation Theory and Mathematical Programming to Equilibrium Refueling Studies of a CANDU Reactor," Nucl. Sci. Eng., 78, 211 (1981).
21. D. G. Cacuci, "Sensitivity Theory for Nonlinear Systems. I. Nonlinear Functional Analysis Approach," J. Math. Phys., 22, (12) (1981).
22. D. G. Cacuci, "Sensitivity Theory for Nonlinear Systems. II. Extensions to Additional Classes of Responses," J. Math. Phys., 22, (12)(1981).
23. A. Gandini, "On the Standard Perturbation Theory," Nucl. Sci. Eng., 79, 426 (1981).

DEVELOPMENT AND VALIDATION OF CORE PHYSICS METHODS FOR IN-CORE FUEL MANAGEMENT OF PWR's.

J.M. Aragonés*, C. Ahnert**, J. Gómez-Santamaría***, I.R. Olavarria***.

* Dept. Nuclear Energy; UNIVERSIDAD POLITECNICA; Madrid.

** Div. Reactor Physics; JUNTA DE ENERGIA NUCLEAR; Madrid

*** Div. Nuclear Fuel; IBERDUERO S.A.; Bilbao

ABSTRACT

Under the objective of developing and experiencing a proper core physics calculating capability to perform the in-core fuel management, to evaluate the nuclear core design and startup tests and to support the reactor operation and licensing, a system of neutronic codes and related methods and procedures, has been developed and validated against both the nuclear design results, provided by the reactor vendor and the startup and operating measurements, from a PWR core of 900 MWe started-up in Spain in 1981.

1. OBJECTIVES AND METHODOLOGY.

The paper summarizes the joint work carried out by a Spanish electric utility (IBERDUERO), owner of a nuclear power plant with twin PWR units of 900 MWe in advanced construction, and the Spanish Nuclear Research Institute (JEN), under the direction and consulting of a University professor (formerly leader of the Reactor Theory and Methods Development group of the JEN).

The objective of this work was to develop and experience the proper core physics calculating capability for the utility staff, to be able to:

- a. perform the in-core fuel management analysis
- b. evaluate the nuclear design and startup tests.
- c. support the plant operation and licensing

This capability is to be applied for the first and reload cycles of PWR cores. Developments are planned to progress in scope to include other thermalhydraulics and safety analysis areas.

A detailed presentation of the objectives and general methodology of the core physics calculating capability for the nuclear reactor utilities, was included in the papers presented at the annual meetings of the Spanish Nuclear Society (Refs. 1 and 2).

The specific methodology followed in this work, was structured in the following steps:

- a. Selection of the basic computer codes, methods and nuclear data.
- b. Selection of the specific application models and procedures.
- c. Testing and validation of the calculating capability.
- d. Feedback of application experience to further develop and improve the efficiency and accuracy of the calculating codes, methods and procedures.

For the selection of the basic computer codes, methods and nuclear data, priority was given to generality, completeness and demonstrated accuracy of available and experienced codes, and also to the consistency and improvement potential of

the code system. This led to the selection of the WIMS-D code (3) for the cell and assembly calculations in bundle geometry, because it allows an accurate and detailed treatment of the spectral and spatial neutron transport effects between the fuel cells, burnable absorber and control rod cells of PWR assemblies, with elaborated and efficient methods for the calculation of the resonance absorption, fuel cells smearing, explicit multigroup transport in 1 D geometries, B_1 corrected leakage, and isotopic burnup with explicit fuel evolution per rod, burnable absorber depletion and fission products, among other effects. For the core calculations, three alternative models were selected. including 1) a detailed (one mesh point per fuel rod) two group XY core model using the diffusion theory code VENTURE (4), with macroscopic cross sections depending on the cell types; 2) a nodal core simulator of the FLARE (5) type improved to allow an explicit calculation of the transport kernels and albedoes, so that a more accurate 3D source calculation, with 2 x 2 nodes per assembly, provides more detailed power distributions, with all the relevant nuclear and thermalhydraulic local feedback effects included; and 3) a regionwise core simulator CICLON (6), with approximate balance of energy and reactivity along reload cycles that provides fast and complete fuel management neutronic balances for the evaluation of refueling strategies.

For the selection of specific models and procedures to be applied in the core physics calculations, we applied an evolutive methodology, where each step was tested and validated as independently as possible, looking for the adequate approximations in the geometric and material configurations, transport order, energy groups, absorber cells corrected diffusion, reflector treatment, nuclear and thermalhydraulic feedbacks and other effects.

For the testing and validation of our calculations, we rely ultimately on the analysis of the nuclear design results provided by the reactor vendor (WESTINGHOUSE) and of the measured startup and operation data available from a twin PWR (ALMARAZ plant), that started-up in 1981. Once that our detailed models (WIMS + VENTURE) were validated, we have used them extensively for the testing and improvement of our more simple models (3D core simulator and CICLON). Of course, we have also autotested our detailed models for selecting the number of mesh points, transport order, energy groups and other modelling options available in WIMS and VENTURE.

The feedback of the validation and application experience to further develop and improve the efficiency and accuracy of the codes, methods and procedures, has been done, and will proceed, in a continuous and consistent way. This exciting interrelationship between the research and development and the application works has been very fruitful, since it has provided a thorough learning of the difficulties and capabilities in core physics calculations and analysis. This learning is more essential for both, the research and the utility staff, than the efficiency of starting with a fully developed calculating capability, where every procedure is established, so that the know-how is available, but the know-why, and all the experience behind it, is out of discussion and thus out of sight and understanding.

In this paper, we will discuss first our implementations and developments in core physics methods and codes for in-core fuel management and operation support of PWR's, emphasizing the models and approximations more than the mathematics and computing. Then, we will summarize our applications for validation and experience in the startup tests and operation of PWR cores.

2. CORE PHYSICS CODES AND METHODS FOR PWR's

The core of pressurized water reactors is made up of fuel assemblies of identical configuration, in a very regular square lattice of fuel rods and guide-thimbles, but with fuel enrichments varying from one assembly to the neighbours, in a checkerboard loading pattern, and with the clusters of fixed burnable absorber rods and movable control rods located in alternate fuel assemblies, in a closely checkerboard pattern also. The outer core assemblies are, in general, of the highest fuel enrichment, configuring a single row of assemblies between the checkerboard core and the thick core shroud and the water reflector. Thus, there is an important coupling between fuel assemblies within the core, of a regular checkerboard kind, due to the spectral interactions and the spatial flux shaping between neighbour assemblies, and there are very large spatial and spectral flux gradients in the peripheral assemblies. The power distributions in large cores are rather sensitive to the details of these interactions among assemblies with alternate neutronic properties and in the core-reflector interface. These effects have to be considered in the whole core calculations, either by fewgroup fine-mesh methods or by corrected homogenization methods for the 2-group diffusion constants or for the 1-group multiplication factor, transport kernels and albedoes.

Within a PWR fuel assembly all the fuel rods are of the same enrichment and the pitch of the lattice is nearly equal to the pitch between fuel rods of contiguous assemblies. In the PWR designs with guide-thimbles in place of single fuel rods, the effects of the associated water cells, burnable absorber and control rods on the spectrum of the fuel cells, is of second order and can be approximated by a supercell or cluster model of the fraction of the fuel assembly associated to each water or absorber cell. Thus detailed calculations of the whole fuel assemblies are not required for standard PWR core calculations, although some superassembly calculations, modelling the contiguous quarters of different fuel assemblies in the checkerboard pattern, can be required to compute the effects of the interassembly couplings. These considerations led to the developments of our code system and related methods and procedures for PWR core physics, which are presented in the following.

2.1. PWR Cell and fuel assembly calculations (WIMS-TRACA code).

We use the WIMS-TRACA code (7) for the modelling of all the kinds of fuel assemblies in PWR Cores, as well as for a radial core model with detailed reflector regions. WIMS-TRACA is our improved version of the WIMS-D (3) and WIMS D-4 (8) codes originated in Winfrith, and implemented in the UNIVAC-1100/80 computer at JEN.

The most relevant characteristics of the WIMS-TRACA code for PWR application are the following:

A basic cross section library in 69 groups for all fuels, materials and fission products required in thermal reactors, that has been tested and corrected very thoroughly and includes temperature and dilution dependent multigroup resonance integrals, temperature dependent thermal scattering matrices including bounding effects for Hydrogen in water by the Nelkin model and for other moderators, and nuclear data for isotopic burnup, as well as the relevant multigroup cross sections and slowing-down parameters. In the D-4 version, the library can be stored on disk or on central memory, speeding considerably the execution by

reducing I/O times.

- A resonance self-shielding treatment which includes the relevant effects, such as those of bundle geometry (Dancoff), equivalences for heterogeneities, interaction among different resonance, temperature and dilution dependence of resonance integrals and within group flux attenuation due to higher resonances.
- A cluster modelling of fuel assemblies, where the central fuel or absorber rod is represented explicitly and the surrounding fuel or water cells are smeared is homogenized rings, using the "Spectrox" flux disadvantage factors calculated for the equivalent fuel cell with fuel, clad, coolant and the associated outer moderator.
- A multigroup transport calculation by S_N or collision probabilities methods where the user can select the transport order, group number and partition and spatial mesh, without limitations other than computing time and memory. This provides a very accurate flux calculation, even for heavily absorber rods or hollow burnable absorber rods with water or void inside.
- A fundamental mode B_1 spectrum calculation, to include leakage effects, for the homogenized supercell or cluster. In the WIMS-TRACA code, the leakage corrected flux spectrum is used to collapse the cross sections and diffusion coefficients in few groups, over selected regions of the cluster (for different fuel cells and absorber cells if spectrum interaction is taken into account), as well as for the whole supercell or cluster.
- A isotopic burnup calculation of the Uranium and Plutonium chains, with 34 fission products explicitly, and of burnable absorbers in time steps selected by the user, for fuel spectrum and spatial transport recalculation, and a number of intermediate steps, for the numerical integration of the burnup equations with critical buckling search recalculation. The user can select the specific power and time step at each burnup step, as well as the change in nuclear densities (i.e. Boron or Xenon concentration) and temperatures. This is very useful to follow a variable Boron and power density program with burnup, as well as to change the Boron or Xenon concentration at intermediate steps, without burnup, to compute their reactivity worths at various burnups. The user can select as many burnable materials as desired, thus allowing a non-uniform burnup within the central fuel rod or for each annulus of rods in the supercell cluster.

The WIMS-TRACA code requires rather complex input data sets for every type of fuel assembly or radial core, including the geometrical dimensions and material densities (which vary with the temperatures) for the different annulus and rods of the cluster. It also requires the corrected Dancoff factors that take into account the effects of the fuel rod vacancies in the guide-thimble cells on the neighbor fuel rods. For the control rods of Ag-In-Cd, since the WIMS library has infinitely diluted resonance cross sections for some of these nuclides that are also fission products, the effective self-shielded resonance absorption cross sections have to be also input to WIMS for correcting this effect, which according to our calculations changes the multiplication factor of a rodged assembly in a few percent. The self-shielded resonance absorption cross sections in the multigroup transport partition of WIMS, for Ag-In-Cd rods in PWR assemblies, have been calculated using the

detailed space-energy cross section and flux dependent code HETRES, developed by one of us. (Ref. 9).

For efficient application, to save input data preparation time and to reduce errors and inconsistencies, the PREWIM code (10) has been developed for the generation of the complete input data sets required by WIMS-TRACA, for the different types of PWR assemblies and the radial core model with reflectors.

The main characteristics of the PREWIM code for PWR assemblies are the following:

- PREWIM generates the complete data sets, for WIMS-TRACA, for every type of PWR, fuel assembly including:
 - a) Clean assemblies with water in guide-thimbles. If there are T guide thimbles in the whole assembly a fraction $1/T$ is modelled in the super-cell bundle with the water cell at the center.
 - b) Control rod assemblies. If there are C control rods per assembly, a fraction $1/C$ of the assembly is modelled with the control rod at the center explicitly treated.
 - c) Burnable absorber assemblies with B burnable rods per assembly are modelled by a supercell bundle, including $1/B$ of the assembly around the burnable rod at the center.
 - d) A simplified radial core model with an average enrichment at the center, the outer core annulus with the peripheral assemblies and the reflector regions explicitly cylindricalized conserving the thickness of the core shroud.
- The reference configuration and modelling options are stored on a permanent binary file for future cases. Standard configuration and material data are included in PREWIM as default values, but every option can be changed by input.
- For specific conditions, the only data required as input for PREWIM to generate the complete WIMS-TRACA data sets are the temperatures, Boron concentrations and burnup steps (only 2 card per case).
- PREWIM calculates the dimensional expansion and material densities and the Dancoff and Bell factors with effects of fuel vacancies and inter-assembly gaps.
- PREWIM provides flexibility for assembly modelling in the reference case, and minimum input for change cases, with capability for multiple stacking of cases in a single run for different temperatures, Boron concentrations, burnup steps and assembly types. PREWIM requires very reduced preparation and running time, while providing consistency for changes in conditions and low error chances.

To process the few-group homogenized cross sections for each fuel or absorber cell type or for the whole assembly, the POSWIM utility code has been developed. POSWIM selects and orders the material (cell) cross sections produced by WIMS-TRACA for different assembly types and conditions and generates the complete few-group library for the diffusion codes CITATION (11) or VENTURE (4). POSWIM corrects the few-group diffusion coefficients for absorber cells (burnable or control), to be used in diffusion calculations with one mesh point per cell, by preserving both reaction rates and fluxes in the absorber cell and 4

neighbor fuel cells, including also the mesh size effects in the finite difference formulation of the diffusion term in CITATION or VENTURE. POSWIM generates also the core homogenized few-group cross sections and the radial group buckling, from the WIMS-TRACA radial core model with detailed reflectors, for use in the axial diffusion core model to calculate the axial bucklings.

The PREWIM + WIMS - TRACA + POSWIM system of codes can be run in a single job, with multiple executions, for every type of fuel assembly, radial core and axial core model of a PWR in any given condition or set of conditions. For a large PWR (900 MWe) with 9 types of fuel assemblies, the total running time on the UNIVAC-1100/80 of the whole job for a single condition of Boron and temperatures including the radial and axial core calculations, is about 10 minutes with the WIMS library stored on memory. For each fuel assembly, the total running time is from 30 to 45 seconds per condition or burnup step.

We tested our WIMS-TRACA model for different PWR fuel assemblies of 17 x 17 rods with 24 control rod guide-thimbles and 1 instrumentation guide-thimble per assembly, by comparing the K_{∞} of the whole assembly with the calculation by the diffusion theory code CITATION (11) of the detailed quarter of assembly in XY geometry, using 2 points per cell and 8 group cross sections generated with WIMS-TRACA. The results are given in the following table:

Table 1. Comparison of the WIMS-TRACA supercell cluster model and detailed CITATION fuel assembly calculations.

enrichment	B.P. rods	control rods	K_{∞} (WIMS-TRACA)	K_{∞} (CITATION)
3.1 w/o	0	0	1.1708	1.1705
2.6 w/o	12	0	1.0069	1.0078
2.6 w/o	16	0	0.9716	0.9754
2.6 w/o	20	0	0.9381	0.9421
3.1 w/o	12	0	1.0632	1.0640
3.1 w/o	16	0	1.0289	1.0324
2.1 w/o	0	24	0.6958	0.7086

These differences are lower than the effects of spectral and leakage interaction among fuel assemblies in the center of the core, with a checkerboard pattern, and in the peripheral assemblies neighbor to reflectors. Thus, we conclude that the WIMS-TRACA cluster model gives a good accuracy for the fuel assembly calculation of PWR's.

2.2. PWR core detailed calculations (VENTURE-TRACA code).

In PWR cores, the axial neutron flux distribution is similar in the different fuel assemblies, which allows a simple separation of first order synthesis between the axial and radial (X-Y) flux. Thus, the global PWR core calculation can be approximated by a detailed X-Y diffusion calculation in few-groups, using axial bucklings provided by an axial diffusion calculation using core homogenized cross sections and radial bucklings.

Since there are a large number of core configurations and conditions at BOL, HZP to be analyzed for the reactor startup tests, where the assemblies of the same type are homogeneous, we selected a finite difference diffusion

theory code to perform the detailed core calculations, with enough accuracy to be a reference to validate the whole calculating system and to correct the simplified 1 group core simulator.

If one point mesh per fuel rod is used in the finite difference diffusion codes, about 140 x 140 points are required for the XY quarter core of large PWR's. Our CITATION code (11) required more than the 256.000 words of memory available for direct addressing in the UNIVAC-1100/80, even using as much I/O storing as allowed, so that we implemented the VENTURE code (4), which has a more efficient memory and I/O management. Our VENTURE-TRACA version (12), implemented in the UNIVAC 1100/80 after a large effort that required developments of assembler subroutines for dynamic control memory utilization and for direct I/O access, that reduces total running time to less than 1/3 compared with the standard FORTRAN I/O version, only requires 160.000 words for data storage in memory of the same problem, so that it was selected for PWR detailed core calculations, using the following procedures:

- 2 group macroscopic cross sections depending on cell type provided by WIMS-TRACA + POSWIM. The diffusion constants for absorber cells are transport and mesh corrected in POSWIM. The reflector constants are flux-weighted in the radial core model of WIMS-TRACA for the steel core shroud and the water reflector.
- Regular fine-mesh with one point per cell in the core, two points in the shroud and 5 or 10 in the reflector, using a void or absorber material for the extra points. The mesh in the shroud and reflector is the same as the one used there in the radial core model.
- Chebychev acceleration and double error mode extrapolation on outers, with 3 or 4 inners per outer using overrelaxation on inners.
- Group dependent axial bucklings from a simplified axial core model are used in the XY detailed calculation. From this, core homogenized cross sections and radial bucklings are computed and input to recalculate the axial core. The effective multiplication factor from this second axial core calculation differs from the XY one in only a few pcm, demonstrating the convergence of the first order synthesis.

Two auxiliary codes were developed for use with the detailed VENTURE-TRACA code. The MAPA code generates the fine-mesh zone mapping from an assembly-wise core description and the cell mapping of each assembly type by octant. The POTENCIA (or POSVEN) code computes the normalized rod powers from the power density interface file produced by VENTURE, and also the relative assembly or quarter of assembly averaged powers and the maximum relative rod power per assembly or quarter.

The VENTURE-TRACA XY detailed core model is used for every core condition and configuration at BOL-HZP, for different Boron concentrations, moderator and isothermal temperatures and for different rod insertion patterns. The accuracy found in the validation work, to be discussed afterwards, allows the use of this detailed model as the reference for other simplified models, according to the methods and procedures that will be summarized in the next paragraph.

2.3. PWR core 3D simulator (SIMULA-3)

A simplified 3D core simulator is very useful for actual core operation analysis by electrical utilities, because, after it has been adequately adjusted, it provides very complete results with accuracy enough for analysis of the operating core conditions throughout the whole cycle, including off-nominal conditions where the axial-radial separation is not accurate, as well as Xenon transients, coolant temperature changes, power level changes and control rod insertions or withdrawals.

Our 3D PWR core simulator, SIMULA-3 (13), has been developed starting from the FLARE (5) - type code NUTRIX (14), available at JEN since 1972. The following developments have been done in SIMULA-3.

- The source iteration method has been changed to a successive line elimination-substitution method over axial lines, using source over-relaxation with a factor $\beta = 1.7$ and about 8 source iterations per void iteration. For a source convergence of 10^{-5} , this method requires only from 1/5 to 1/10 of the iterations needed with the point-Jacobi method and less than 1/2 of these needed with the Gauss-Seidel or Chebyshev source acceleration.
- Various formalism of the transport kernels have been programmed as options in SIMULA-3. The one that provides the best accuracy for one node per quarter of assembly, is the following:

$$W_{lm}^h = \frac{2}{1 + \frac{f_1 M_1^2}{f_m M_m^2}} \cdot \frac{M_1^2}{K_{\infty 1} \cdot X^2} ; W_{lm}^v = \frac{2}{1 + f_{lm} \frac{M_1^2}{M_m^2}} \cdot \frac{M_1^2}{K_{\infty 1} \cdot Z^2} \quad |1|$$

where W_{lm}^h and W_{lm}^v are the neutron transport kernels between node 1 and node m in the horizontal and vertical directions, M_1^2 and M_m^2 are the migration areas of nodes 1 and m, X and Z are the node width and height, $K_{\infty 1}$ is the infinite multiplication factor of node 1, f_1 and f_m are the transport correction factors of nodes 1 and m in the horizontal direction and f_{lm} is the transport correction factor from node 1 to m in the vertical direction.

The neutron balance equation for each node 1 is written as:

$$\sum_m [W_{lm} S_1 - W_{ml} S_m] + \frac{S_1}{k_1} = \frac{1}{k_{eff}} S_1 \quad |2|$$

where K_{eff} is the actual multiplication factor of the reactor core. Equation |2| is used for source iteration and K_{eff} is computed at each iteration from source multiplication or overall neutron balance. For nodes in the core periphery, radial and axial albedoes are used in the first term of |2|.

- The nodal K_{∞} and M^2 are calculated as a function of the nodal moderator density U, Boron concentration B, exposure E and relative power density SP, using polynomial factorized correlations with coefficients depending on fuel type. The effects of moderator density on Boron, Doppler and Xenon worths and those of exposure on Boron and Xenon worths are treated through suitable factors.
- The Boron search has been improved and the Boron worth is computed. An

axial fuel variation is allowed for part length burnable absorber rods or control rods insertions. A dynamic memory management improves efficiency of the code. Quarter and octant core representations with reflected or rotational symmetries on node centerline or boundaries have been implemented also in SIMULA-3, among other developments.

The SIMULA-3 code is completed with the auxiliary codes MELON (15, 16), AJUSTA (17) and CONCON (18).

The MELON code computes the coefficients of the correlations of K_{∞} and M^2 as a function of moderator density, Boron Concentration, exposure and relative power density (Doppler and ^{235}Xe worth), using least squares fits to sets of assembly averaged K_{∞} and M^2 provided by the user (as calculated with WIMS-TRACA in this application). Consistency of the fits of the different worths and effects is provided and the correlation coefficients by fuel type are punched in the SIMULA-3 format.

The AJUSTA code computes the optimum mixing factors of the transport kernels with different formalisms, by searching the minimum square root deviations of the nodal sources, as calculated by a radial-axial simplified nodal method, relative to reference radial and axial power distributions provided as input. The multivariable Newton-Raphson iterative search is used for up to 3 free variables (mixing factors), using second order difference formulas for the derivatives involved. The reference radial and axial power distributions are provided to AJUSTA by the POTENCIA code (discussed in 2.2) from detailed core diffusion calculations. Different transport kernel formalisms can be selected to find out the optimum one in a range of core conditions. After the optimum transport kernels are computed, using the internal nodes, the effective albedoes of the core periphery are calculated explicitly from the reference nodal powers at the outer nodes.

The CONCON code calculates explicitly the transport correction factors of the nodal 1-group transport kernels of eq. [1], in the XY planes and axial direction, from detailed core diffusion calculations. CONCON reads the geometry and flux interface files produced by VENTURE-TRACA, as well as the cross sections, and calculates the averaged 1-group absorptions, diffusion D_1 and source S_1 in each node, as well as the total net leakages L_{1m} at all the nodal interfaces (internal and external), using the fine-mesh finite difference formalism of VENTURE-TRACA. By inserting [1] in the first terms of [2] and equalizing them to the total net leakages at the interfaces of nodes 1 and m, the ratios of transport correction factors are given by:

$$\frac{f_1}{f_m} = \frac{M_m^2}{M_1^2} \frac{D_1 - L_{1m} X^2/2}{D_m - L_{1m} X^2/2} ; D_1 = \frac{M_1^2}{K_{\infty 1}} S_1 \quad [3]$$

Although, rigorously, [3] results in effective transport correction factors by node depending on the X, Y or Z directions, we have found that the X and Y factors are nearly the same, so that we use an average for the horizontal direction of the nodal transport correction f_1 , and the ratio $f_{1m} = f_1/f_m$ directly for the vertical direction, since this ratio is practically constant (and close to unity) for all the internal interfaces in the axial direction, but significantly different for the interfaces between the bottom or top two nodes. We have also found that the horizontal transport correction factors f_1 are nearly the same and thus can be averaged over the nodes of the same type for quarter of assembly nodes.

Physically these transport correction factors, include all the effects of node size and heterogeneity, as well as the spectral variation of leakage between nodes in the 1-group nodal transport approximation.

CONCON also calculates the actual K_{∞} and M^2 of each node, simply by the ratios of source or diffusions to absorptions as calculated from the 2-group detailed diffusion theory calculations. Then CONCON computes the correction factors for K_{∞} and M^2 relative to the values computed by WIMS-TRACA with the fundamental mode material buckling leakage approximation. These correction factors for K_{∞} and M^2 include not only the spectral interaction between assemblies but also the detailed flux distribution by cell within each node. It is found, for ckekerboard loading patterns in large PWR cores, that the correction factors for K_{∞} and M^2 are nearly the same for the nodes of the same type, so that their averages by node type are used in SIMULA-3. Their variation with the Boron concentration or moderator density is negligible and they come closer to unity as the fuel assemblies with burnable absorbers deplete with burnup. For the rodged nodes the K_{∞} , M^2 and nodal transport correction factors are significantly different from unity and those of the neighbor nodes also change because of the effect of the control rod.

CONCON also calculates the "exact" albedoes in the core boundary from the detailed diffusion leakages. The degree of detail in the SIMULA-3 adjustment, using the CONCON explicitly calculated correction and albedoes factors from the detailed diffusion solutions of VENTURE-TRACA, has been thoroughly investigated for large PWR cores at BOL in the first cycle. Using the "physical" and "exact" correction factors, the differences in the nodal powers (with quarter of assembly nodes) between the nodal method of SIMULA-3 (Eqs. [1] and [2]) and the detailed diffusion calculations of VENTURE-TRACA, are reduced from maximum relative differences of about 7 to 10 percent, for unrodged and rodged cores respectively, when unity correction factors are used, to less than 3 to 5 percent, when CONCON correction factors averaged by node type are used. The corresponding RMS deviations range from less than 1 to 2 percent. Using node-wise correction factors the differences are very small, so that they are not necessary, except if accurate power distributions are needed in rodged core configurations. The "physical" and "exact" albedoes calculated by CONCON vary very little with Boron concentration, rod insertion power level and burnup, and vary in a very consistent way with the moderator density of the core and reflectors, that has been also introduced in SIMULA-3.

As a consequence of these developments and the systematic validation work carried out, the SIMULA-3 core simulator has become a very efficient and much improved accurate calculating tool for complete analysis of PWR cores in operating conditions. Furthermore, the physically found correction factors, introduced through the CONCON code from detailed diffusion calculations, have provided a richful learning on the PWR core physics problems, very useful for the evaluation of the design analysis and startup and operating measurements, as well as for the development work of more-rigorous and complete methods for core simulation using nodal few group approximations.

2.4. PWR cycles calculations (CICLON)

The CICLON code (6) is available since 1976 for approximate energy and reactivity balance of the reload cycles of PWR cores. The core state models include various options of detail, from region or bath-wise models to assembly wise 2D nodal models, as thoroughly discussed in reference 19. It provides

a fast and complete neutronic calculation of fuel management problems for PWR successive reload cycles using different reloading strategies. CICLON has been extensively used for fuel management studies of PWR cores in the past years. A recent application was presented in Ref. 20.

3. VALIDATION OF PWR CORE PHYSICS METHODS:

Our system of PWR core physics codes and methods has been extensively applied to the analysis of the first core of a large PWR (900 MWe). After some comparisons with other codes and results, presented in Refs. 20 and 21, we performed a complete analysis of the nuclear design for the first core of a 900 MWe PWR, comparing our results with those provided by the reactor vendor (Westinghouse). From these comparisons, published in Refs. 22 and 23, we found a very good overall agreement between our detailed methods and our explicitly adjusted core simulator and the latest vendor calculations. Assembly average and peak rod powers at BOL-HZP for various rod configurations, agreed within 1% RMS and less than 3% maximum relative difference. Differential and integral rod worths agreed within 5% in relative difference. Boron worth agreed within 2%. Moderator and isothermal temperature coefficients agreed within 1 pcm/°F. Doppler temperature coefficient difference was about 0.15 pcm/°F at full power and about 0.35 pcm/°F at zero power. Doppler only power coefficient was nearly the same at full power, as well as the total power coefficient throughout the power range. Assembly averaged powers and burnups along the cycle burnup agreed within about 3% RMS and 5% maximum relative difference, axial power distributions were within 5% of maximum difference at EOL, and the differences in the critical Boron concentrations were within ± 40 ppm throughout the first cycle.

Ultimate validation of our core physics codes and methods has been recently reassured through the extensive comparison of calculated results with the measured data available from the startup tests (24) and operating measurements of the Almaraz nuclear power plant, a 900 MWe PWR unit started up in 1981. The more significant results of this validation are presented in the following.

3.1. Critical Boron concentrations and control bank worths at BOL, HZP

The calculated and measured results for the end-point critical Boron concentrations and control bank worths, for the different control rod banks in successive insertion, in the BOL, HZP condition are given in the next table 2.

Table 2. Critical Boron concentrations and control bank worths at BOL-HZP

Control banks inserted	Critical Boron Conc.(ppm)		Control bank worth (pcm)	
	Measured	Calculated	Measured	Calculated
ARO	1327	1325	—	—
D	1183	1190	1424	1364
D+C	1068	1075	1192	1162
D+C+B	880	885	1918	1920
D+C+B+A	771	775	1196	1159
ARI-H14*	585	580	7641	7673

* All control rods inserted less the most reactive control rod.

All of these calculations were performed with the WIMS-TRACA and detailed diffusion VENTURE-TRACA codes, without any kind of correction. The calculated control bank worths were obtained from the Boron concentration differences and the average Boron worth between the fully withdrawn and inserted conditions.

3.2. Isothermal temperature coefficient at BOL-HZP

The isothermal temperature coefficient at BOL-HZP was calculated with WIMS-TRACA and VENTURE-TRACA, considering two extra isothermal temperatures at $\pm 20^{\circ}\text{F}$. The results are compared in Table 3 with the values measured in the startup tests.

Table 3. Isothermal temperature coefficient.

Core configuration	α_{iso} (pcm/ $^{\circ}\text{F}$)	
	Measured	Calculated
ARO	-0.98	-2.14
D+C	-7.78	-9.17
D+C+B+A	-14.4	-13.8

3.3. Peak rod powers at BOL-HZP

From the previous detailed core calculations, the peak rod powers relative to core average rod power, are the following:

Table 4. Peak rod powers.

Core configuration	$\frac{F^N}{\Delta H}$	
	Measured	Calculated
ARO	1.43	1.39
D	1.44	1.43
D+C	1.67	1.67

3.3. Differential and integral control bank worths at BOL-HZP

The differential and integral control bank worths versus insertion were calculated for the successive insertion of the control-rod banks, using the SIMULA-3 code with transport correction factors and albedoes calculated by CONCON from the detailed VENTURE-TRACA diffusion results. Figures 1 to 6 include the measured and calculated curves as a function of the bank insertion.

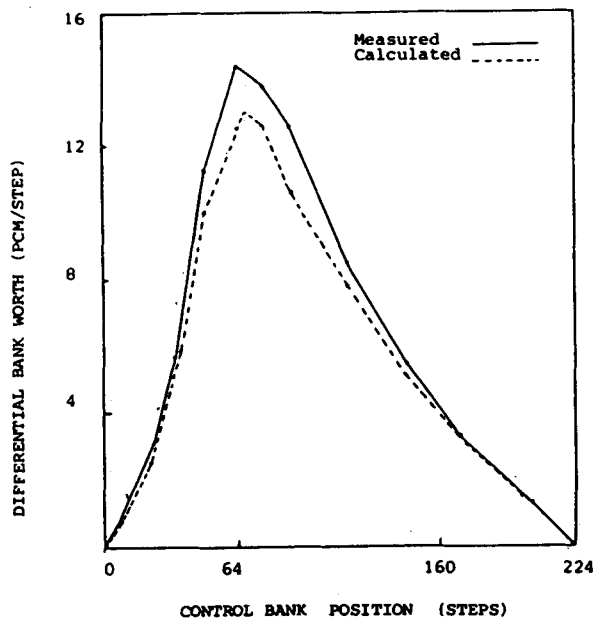


Figure 1.- Differential Control Bank Worth at BOL, HZP for Bank D.

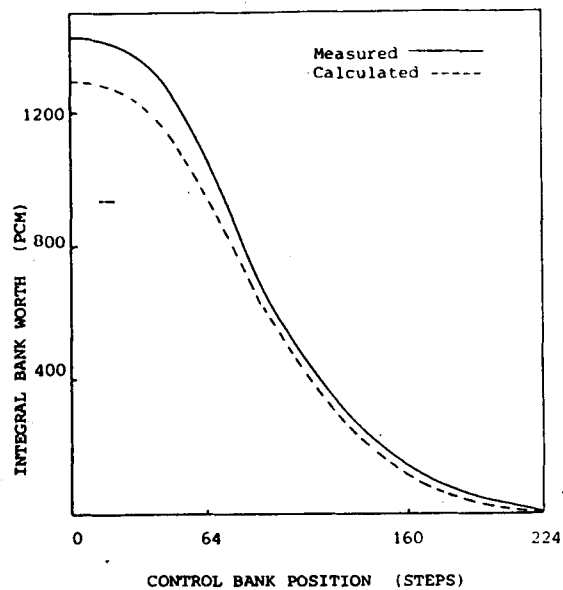


Figure 2.- Integral Control Bank Worth at BOL, HZP for Bank D

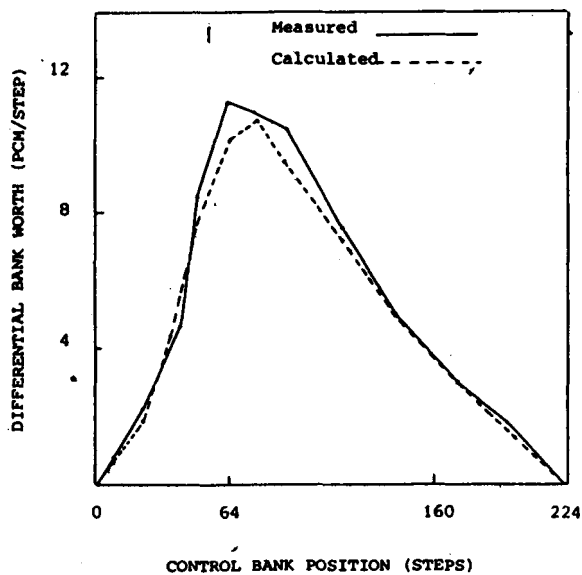


Figure 3.- Differential Control Bank Worth at BOL, HZP, D in, for Bank C

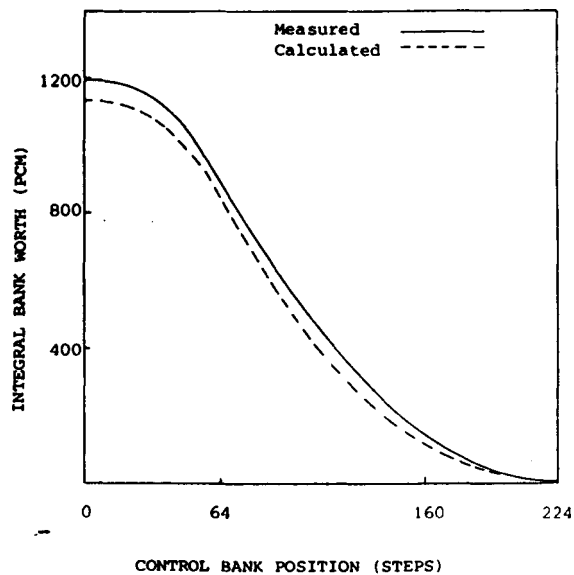


Figure 4.- Integral Control Bank Worth at BOL, HZP, D in, for Bank C

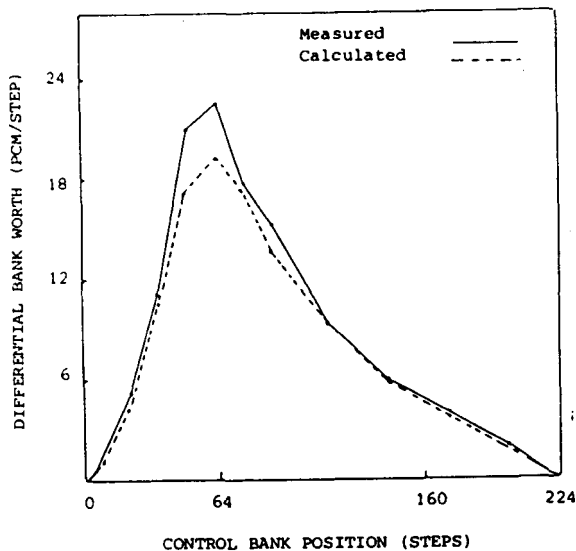


Figure 5.- Differential Control Bank Worth at BOL, HZP, D+C in, for Bank B

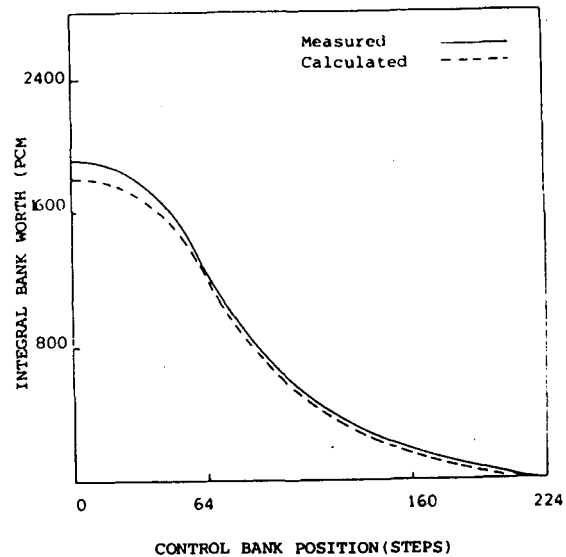


Figure 6.- Integral Control Bank Worth at BOL, HZP, D+C in, for Bank B

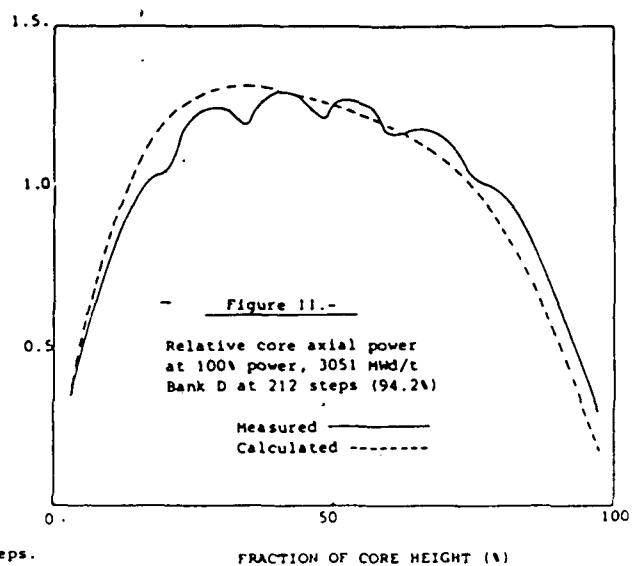
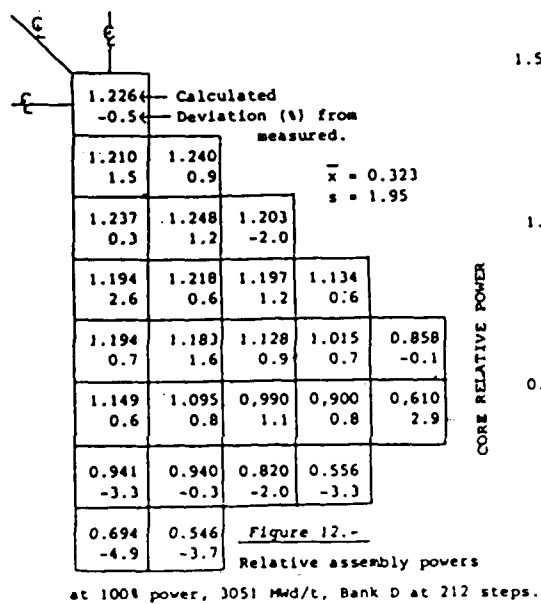
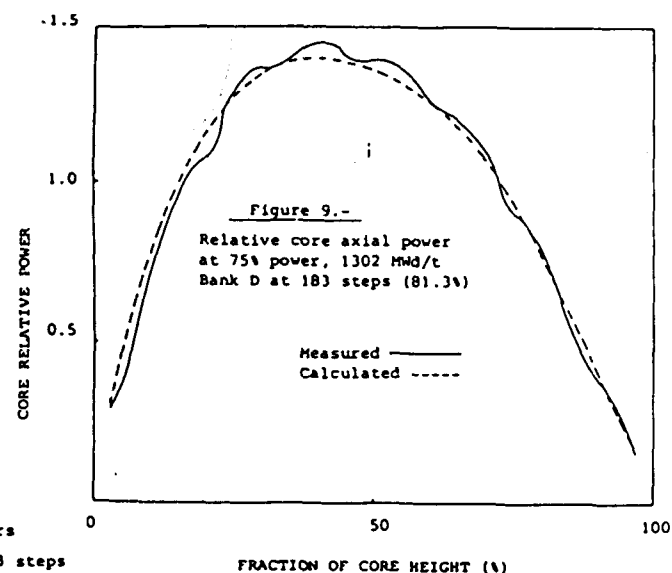
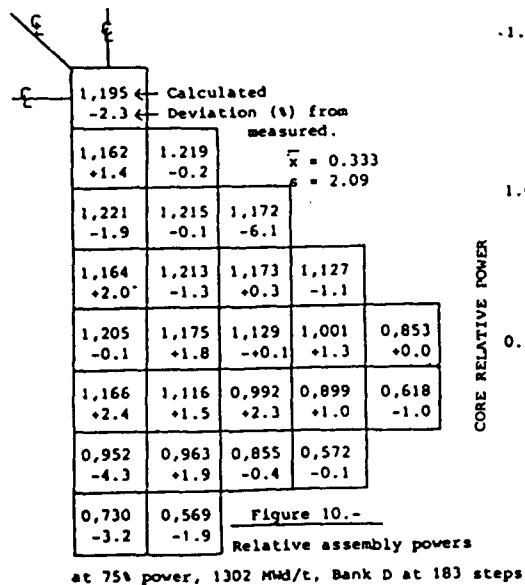
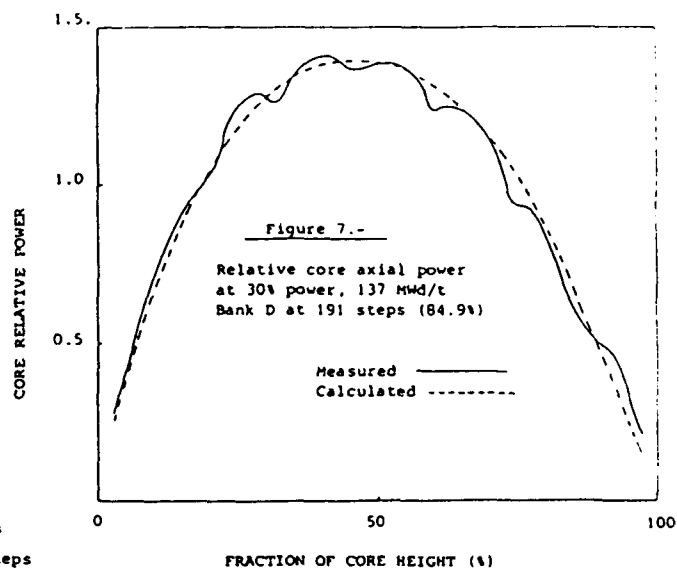
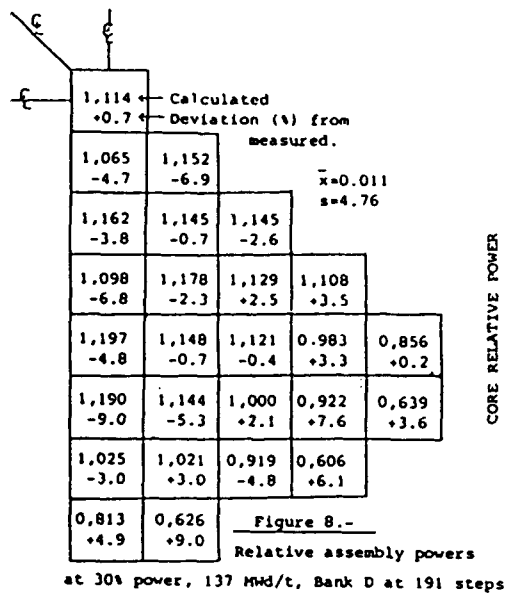
3.4. Radial and axial power distributions at various powers and burnups

Using the SIMULA-3 core simulator, the power program during the first burnup period of the Almaraz PWR core was followed, calculating the critical Boron concentrations at the intermediate steps and the corresponding relative assembly power distribution and the relative core average axial distribution. The measured and calculated critical Boron concentrations are compared in Table 5, where the control bank insertion worths have been corrected.

Table 5. Critical Boron at various powers and burnups.

Burnup (MWd/t)	Power (%)	Critical Boron (ppm)	
		Measured	Calculated
137	30	1065	1057
300	51	984	978
648	75	932	907
1302	75	971	903
2423	96	866	813
2448	96	813	812
3051	100	811	766

The measured and calculated radial and axial power distributions for selected powers and burnups are compared in figures 8 to 12.



3.5. Doppler only power coefficient

The Doppler only power coefficient was calculated at BOL using the SIMULA-3 code for various powers. The results are compared in figure 13 with the measured values at the conditions included in table 5 and figures 8 to 12.

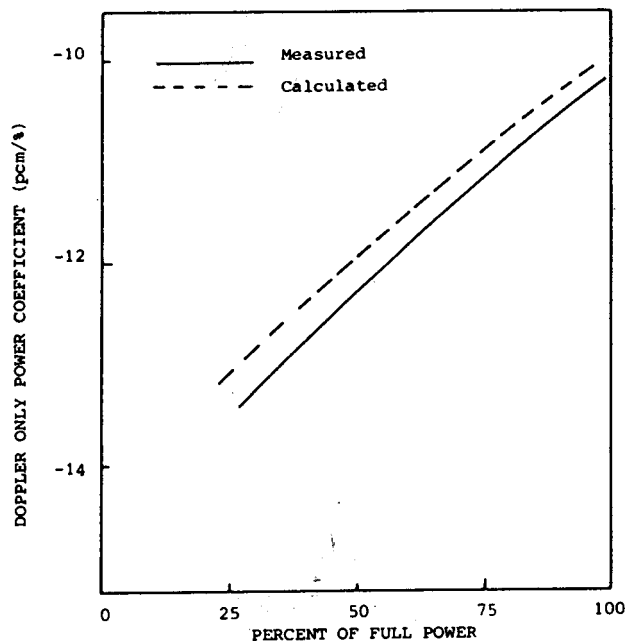


Figure 13.- Doppler only power coefficient versus power level at BOL.

ACKNOWLEDGMENT

C.N. ALMARAZ is acknowledged for providing the measured data of the startup tests and early operation of the Almaraz I nuclear power plant.

REFERENCES

1. J.M. ARAGONES, C.AHNERT, "Métodos de Cálculo Neutrónico de Reactores Térmicos", V Reunión Anual, Soc. Nucl. Española, Barcelona (1979).
2. J.M. ARAGONES, "Cálculos Neutrónicos del Núcleo de PWR para Soporte de la Evaluación del Diseño y de la Operación", VI Reunión Anual, Soc. Nucl. Española, Madrid (1980).
3. J.R. ASKEW, F.J. FAYERS, P.B. KEMSHELL, "A General Description of the Lattice Code WIMS", Journal Brit. Nucl. Energy Soc., 5, 564 (1966).
4. J.R. VONDY, T.B. FOWLER, C.W. CUNNINGHAM, "VENTURE, A Code Block for Solving Multigroup Neutronic Problems Applying the Finite Difference Diffusion Theory Approximation to Neutron Transport", ORNL-5062, Oak Ridge Nat. Lab. (1975).
5. D.L. DELP et al, "FLARE, A Three-Dimensional Boiling Water Reactor Simulator", GEAP-4598, General Electric Co (1964).

6. J.M. ARAGONES, "CICLON, A Neutronic Fuel Management Program for PWR's Consecutive Cycles", JEN-338, Junta de Energía Nuclear (1976).
7. C. AHNERT, "Programa WIMS-TRACA para Cálculo de Elementos Combustibles. Manual de Usuario y Datos de Entrada", JEN-461, Junta de Energía Nuclear (1980).
8. M.J. HALSALL, "A Summary of WIMS D-4 Input Options", AEEW-M-1327, Atomic Energy Establishment, Winfrith (1980).
9. J.M. ARAGONES, Nucl. Sci. Eng., 68, 281 (1978).
- 10 J.M. ARAGONES, "Programa PREWIM para generación de los Datos Completos para WIMS-TRACA en los Elementos Combustibles de Reactores PWR", JEN report to be published, Junta de Energía Nuclear (1982).
- 11 T.B. FOWLER, D.R. VONDY, G.W. CUNNINGHAM, "CITATION: Nuclear Reactor Core Analysis Code", ORNL-TM-2496, Rev. 2, Oak Ridge Nat. Lab. (1971).
- 12 C. AHNERT, "El Sistema de Códigos de Teoría de Difusión en Multigrupos VENTURE-TRACA. Información a sus Usuarios", JEN/TCR/A 10-80. Junta de Energía Nuclear (1980).
- 13 J.M. ARAGONES, "SIMULA-3. A Three Dimensional PWR Core Simulator with Improved Transport Kernels and Neutronic Correlations", JEN report to be published (1982).
- 14 Y.S. KIM, "NUTRIX. A Digital Computer Program for Three-Dimensional Analysis of Time-Dependent Operating Reactors", NUS-657, Rev. 1, NUS Corporation (1972).
- 15 C. AHNERT, J.M. ARAGONES, "MELON-1, A Program to Calculate the NUTRIX Correlations for K^∞ and M^2 ", NUS-NA-338, NUS Corporation (1971).
- 16 I. R. OLAVARRIA, "Programa MELON-3 para Cálculo de los Coeficientes de las Correlaciones del Programa SIMULA-3", JEN/TCR/A 04-82, Junta de Energía Nuclear (1982)
- 17 J.M. ARAGONES, "Programa AJUSTA para Ajuste Optimo de los Parámetros del Simulador Nodal de PWR", JEN report, to be published (1982).
- 18 J.M. ARAGONES, C. AHNERT, "Programa CONCON para Determinación Explícita de los Factores de Corrección del Transporte del Simulador Nodal de PWR", JEN report to be published (1982).
- 19 J.M. ARAGONES, J.M. MARTINEZ-VAL, M.R. CORELLA, "Métodos y Programas de Cálculo para Gestión del Combustible en PWR", JEN-338, Junta de Energía Nuclear (1976); and Nucl. Technology, 34, 398 (1977).
- 20 I.R. OLAVARRIA, "Resultados de los Cálculos Neutrónicos de Quemado y Recarga de Combustible de un Reactor PWR de 900 MWe", VI Reunión Anual, Soc. Nucl. Española, Madrid (1980).
- 21 J.G. SANTAMARIA, "Resultados de los Cálculos Neutrónicos de los Elementos Combustibles y del Núcleo a Principio de Vida de un Reactor PWR de 900 MWe", VI Reunión Anual, Soc. Nucl. Española, Madrid (1980).

22. J.G. SANTAMARIA, I.R. OLAVARRIA, J.M.ARAGONES, C. AHNERT, "Cálculos Neutrónicos del Núcleo de un PWR de 900 MWe a Principio de Vida y Potencia Cero", VII Reunión Anual, Soc. Nucl. Española, Madrid (1981)
23. I.R. OLAVARRIA, J.G.SANTAMARIA, J.M:ARAGONES, C.AHNERT, "Cálculos Neutrónicos del Núcleo de un PWR de 900 MWe a Potencia en el Primer Núcleo", VII Reunión Anual, Soc. Nucl. Española, Madrid (1981).
24. J.M. CALLEJA, L.G. ACUYO, "Pruebas Físicas del Reactor de C.N.ALMARAZ Unidad I", VII Reunión Anual, Soc. Nuclear Española, Madrid (1981).

URANIUM RECYCLE IN PWRs

M.DARROUZET ⁽¹⁾, A.SIMON ⁽²⁾, M.SOLDEVILA ⁽³⁾

- (1) Commissariat à l'Energie Atomique. Dept. des Réacteurs à Eau. CEN/CADARACHE-B.P. n° 1-13115 SAINT-PAUL-LEZ-DURANCE (France)
- (2) Commissariat à l'Energie Atomique - Dept. d'Industrialisation et d'Essais - CEN/SACLAY - 91191 GIF-sur-YVETTE - Cédex
- (3) Electricité de France - Direction des Etudes et Recherches - 1 Av. du Général de Gaulle - 92141 CLAMART (France)

ABSTRACT :

Uranium recycling allows a better use of the natural resources, but the accumulation of some uranium isotopes (232, 234 and 236) in the spent fuel results in neutronic and radioprotection problems :

. The parasitic absorption of U 234 and U 236 which requires an over enrichment of the fuel,

. The activity of the spent fuel to be reenriched due to some daughters of U 232.

Moreover, a choice has to be made between two reenrichment strategies : must reprocessed uranium be introduced into the natural uranium cycle, or in a specialized one ?

The first part of this paper is devoted to a brief analysis of these problems.

In the second part, we present the neutronic characteristics of a PWR core loaded with fuel made from reprocessed uranium. These results are compared to those related to a standard fuel loaded core ; they were obtained by means of cell and core calculations. In the discussion, the importance of some phenomena (self shielding, spectral variation) is pointed out.

1. INTRODUCTION

In 1990 there will be about 60,000 MW (e) installed in pressurized water reactors (PWRs) in France. By then, these reactors will have produced about 7,500 t of irradiated fuel to be reprocessed, at a rate of 27 t per GW (e) and per year. This value is to be compared to the known reserves of natural uranium in France which amount to 100,000 t.

The U235 content in the irradiated fuel is greater than for natural uranium.

These remarks show clearly the advantages of uranium recycle : it allows a better use of natural resources.

Uranium recycling could be carried out in two ways :

1. In gas-graphite reactors, but it would be a partial solution because the number of reactors concerned is small. With the assumption that all of them are loaded with reprocessed uranium, they would only use about 500 t per year ; that is, the production of 20 PWRs.

2. In PWRs, after re enrichment in U235.

The present paper describes the studies performed jointly at CEA and EDF in such a way. The importance of some neutronics phenomena such as cross sections self shielding or spectral variation is pointed out. The results presented were obtained by means of a complete calculation method including a transport code for cell calculations (APOLLO) and a diffusion code for core calculations.

This theoretical study is part of a general program which includes experimental studies in order to validate the nuclear data of light uranium isotopes (U232, U234, U236).

2. PROBLEMS CONNECTED WITH URANIUM RECYCLE

In the first part of this section, we analyse the consequences of the accumulation of new uranium isotopes in the spent fuel from the neutronic and radioprotection standpoint.

In the second part, we examine the different strategies for uranium re enrichment in a gaseous diffusion plant.

2.1. Neutronics and radioprotection

The accumulation of new uranium isotopes (U232, U234, U236) in the irradiated fuel introduces some penalties. The predicted isotopic composition of this fuel (33,000 MWd/t ; 3 years cooling time) is shown in table 1 hereafter.

	Natural U (%)	Enriched U (%)	Reprocessed U (%)
U 232	0	0	$1.1 \cdot 10^{-7}$
U 234	0.0055	0.03	0.02
U 235	0.72	3.25	0.91
U 236	0	0	0.42
U 238	99.275	96.72	98.65

TABLE 1

* U 234 and U 236 are absorbing isotopes. They modify the neutron balance by poisoning the reactor. The microscopic absorption cross section and resonance integral are given in table 2 hereafter.

	σ_a thermal (barn)	I (barn)
U 235	680.8 ± 1.3	144 ± 6
U 234	100.2 ± 1.5	630 ± 70
U 236	5.2 ± 0.3	365 ± 20

TABLE 2 /1/

U 234 is present in natural uranium ; its concentration is about 50 ppm. It increases during the enrichment process and reaches 310 ppm in the standard PWR fuel. This concentration then decreases during the irradiation, because of (n,γ) reactions leading to U 235.

On the other hand, U 236 is built-up during the fuel irradiation by (n,γ) reactions on U 235. Taking into account the resonant absorption, the self shielding increases with the concentration ; then the absorption rate per atom decreases.

Whatever the strategy adopted for the re enrichment may be, the concentration of these isotopes increases during the process. This results in an over enrichment in U 235 in order to compensable the parasitic absorption. This aspect will be developped in sections 5 and 6.

* Uranium 232, although its content in the reprocessed fuel is very small (about $10^{-7}\%$ of the uranium) and does not play any part in the neutronic balance, is the main source of trouble in the fuel cycle : Two daughters of this isotope, Bi 212 and Tl 208 are very hard γ emitters.

The decay chain of U 232 is shown in fig. 1. Its half life is 72 years and that of Th 228 is 1.9 years. All the following elements have very short half lives (< few days).

The resulting activity of the spent fuel is a function of the cooling time ; it reaches the maximum value in 10 years.

Since U 232 is a light isotope of uranium, its concentration increases during the enrichment process. Difficulties may occur in the following operations :

- . Transport and storage of UF₆,
- . Enrichment : risk of Th 228 accumulation and deposits
- . Above all, fuel fabrication.

Nevertheless it is possible to filter out all or part of Th 228 during chemical process or UF₆ transfer.

The formation of U 232 is complicated ; the main ways are the following

- . decay of Pu 236
- . decay of U 234 followed by two neutronic captures.

The corresponding basic neutronic data are inaccurate and several parameters must be taken in account. A sensitivity study was carried out /2/. It will be completed by experimental measurements of U 232 and Pu 236 concentrations in irradiated fuel.

2.2. Reenrichment strategies

After the reprocessing, three main options can be defined about the future of the recovered uranium.

Option 1 : simple storage of uranyl nitrate (open cycle)

Option 2 : reenrichment in a plant made to enrich natural uranium working with two feeds (reprocessed and natural)

Option 3 : reenrichment in a small capacity plant specially built to enrich reprocessed uranium.

Option 1 is generally and temporarily chosen but it is not politically and economically justifiable except if the quantities of stored reprocessed uranium are small and (or) if the cost of natural uranium is particularly low.

In fact, in the future the problem will be in order to finish the uranium cycle, must reprocessed uranium be introduced into the natural uranium cycle, into the conversion plant (option 2) or on the contrary, is it necessary to create a second and specialized cycle for the uranium?

The answer to this question is not easy : it depends on many technical commercial and political factors ; among them,

. The combustion rate of irradiated fuel which determines the amounts of U 232, 235 and 236.

. The cooling time of fuel before reprocessing which determines the amount of U 232.

. The annual flow of reprocessed uranium. As far as the number of reactors is small, it is obvious that option 2 and especially option 3 cannot be economically justified.

. The market cost of natural uranium. The lower the cost of natural uranium, the less the advantage of recycling is obvious.

. The production units of the fuel cycle already working.

More particularly, the capacity of these units to physically protect workers against contamination and irradiation linked to the presence of traces of fission products, transuranic elements, and artificial isotopes of uranium. If it does not exist, option 2 must be abandoned and only option 3 can be considered.

However the ratio of natural uranium flow to the reprocessed uranium flow, asymptotic to a value of about 6.5 for a small closed yard of reactors, near equilibrium, can strongly penalize from an economic point of view every part of the cycle (conversion, enrichment, manufacture of fuel) if these units use processes with a large scale effect. With regard to enrichment let us quote the method of gaseous diffusion, hence the interest of other methods allowing the construction of small capacity plants with acceptable economic conditions such as ultracentrifugation or chemical methods which are not yet available for industrial projects.

The manufacturer of a gaseous diffusion plant, according to option 2, risks facing difficult material evolution and commercial problems : for instance, if a customer gives natural uranium to be enriched will he agree to receive a product with U 236 and U 232 in it?

The isotopic compositions of reprocessed uranium reenriched in U 235 to the standard value of 3.25% (without over-enrichment due to the presence of U 234 and U 236) are given in table 3 for options 2 and 3.

	Reprocessed U (%)	Reenriched U	
		Option 2 (*) (%)	Option 3 (%)
U 234	$1.1 \cdot 10^{-7}$	$0 - 1.1 \cdot 10^{-7}$	$7 \cdot 10^{-7}$
U 234	0.02	0.03 - 0.04	0.09
U 235	0.91	3.25	3.25
U 236	0.42	0 - 0.26	1.14
U 238	98.65	96.72 - 96.45	95.52

TABLE 3

(*) The marked values correspond to ratios of massic flow of reprocessed uranium to natural uranium from 0 to 0.15 which respectively represent the start of a yard of reactors and the working at equilibrium.

Between these two values, about twenty years distant, the concentrations can be considered as linear function of the ratio of the flows.

In conclusion, the most realistic strategy is probably a stepping in the time, with the augmentation of available quantities of reprocessed uranium, of the three options in the order given before, with as an end option 3 if there is no technical obstacle to this problem.

So the answer to this problem necessitates studying the possibility of recycling uranium. This feasibility and the determination of the necessary over-enrichment in U 235 is presented later in this paper considering the extreme case of option 3 which corresponds to the most important concentrations in artificial uranium isotopes.

3. PROBLEM FORMULATION

The unknown part of the problem is the U 235 content of the reenriched fuel which is necessary to reach in a PWR core performances similar to those of the standard fuel (natural uranium enriched in U 235 at 3.25%).

The reenriched fuel is supposed to be introduced into a standard fuel loaded core at the end of an equilibrium cycle (reference cycle).

The successive reloads consist of 52 (100%) re enriched fuel assemblies, in order to reach a new equilibrium cycle which should meet the following conditions.

- a. Radial form factor (F_{xy}) ≤ 1.435
- b. Reactivity life time as close as possible to that of the reference cycle.

These characteristics have been determined in both cases by means of the calculation method presented in section 5.

4. REACTOR DESCRIPTION

The reactor concerned in our calculations was that of the Tricastin plant which belongs to the CP1 family. It is a three loop pressurized water reactor the electrical power of which is 920 MW. The operating temperature and pressure of the primary circuit are respectively 306.3°C and 155b.

The core consists of 157 fuel RCC assemblies arranged in several regions depending on their burn-up. After each irradiation cycle, 52 assemblies are discharged and replaced by fresh assemblies which are loaded at the core periphery. The remaining burnt assemblies are arranged following a checkerboard pattern in the central region of the core.

The fuel assemblies are composed of 264 fuel rods, 24 guide tubes and one instrumentation tube arranged in a 17 x 17 square array. The rod pitch is 1.26 cm. The guide tubes provide channels for the motion of control rods as well as for the burnable poisons used during the first cycle.

5. CALCULATION METHOD

The calculation of reactor nuclear characteristics is based on the use of a series of computer codes which form the standard EdF calculation method for PWR'S. This method has been qualified by many experimental comparisons. A brief description of it is given hereafter.

5.1. Cell calculations

Cell calculations are performed in infinite medium by the APOLLO code /3/ which solves the multigroup Boltzmann's equation using the collision probabilities method. The interaction between the different types of cells is treated by the ROTH formalism. The geometrical representation of the assembly, which characterizes six types of cell depending on their composition and relative location is shown in fig. 2.

The 99 group library used allows the self shielding of U 236 absorption cross section to be taken in account : as will be seen later, its effect on the reactivity proved very important for the considered concentrations. Fission cross sections of U 236 are also included.

The APOLLO code generates 2 groups homogeneized microscopic and macroscopic sections for each type of cell as a function of burn-up.

5.2. Assembly calculations

The nuclear constants determined by APOLLO are then input to an assembly calculation of the spatial flux and the power distribution by means of a two dimensionnal diffusion code (finite differences method). The reactivity feedback effects of variations in the moderator and fuel temperature are taken into account. This code provides two group homogeneized sections for the assembly as a function of burn-up to be used in the last step.

5.3. Core calculations

A finite elements diffusion code has been used to determine the two dimensionnal power distribution (X-Y) in a quarter of core and the critical boron concentration for each time step. The reactivity feed back effects are included in this code. More over, it performs the analysis of reloading patterns for an equilibrium cycle. In all the core calculations performed, the end of cycle corresponds to a critical boron concentration of 10 ppm.

6. RESULTS

6.1. Characteristics of the reference core

The calculation method presented above has been used to determine the characteristics of the standard fuel loaded core equilibrium cycle. The results are summarized in the following table 4.

	Average core burn-up (MWd/t)	Critical boron concentration (ppm)	Boron efficiency (pcm/ppm)	Fxy
BOC Xenon equilibrium	10060	1130	-7.9	1.31
EOC	20460	10	-8.7	1.21

TABLE 4

The cycle length is about 10,400 MWd/t, or 272 efpd

6.2. Characteristics of the re enriched fuel loaded core

In order to avoid a large amount of computations, the U 235 content of the re enriched fuel was determined in two steps :

a. An approximate value was obtained by cell calculations in such a way that the reactivity should be equal to that of the standard fuel at a burn-up of 20 GWd/t which is close to the average burn-up of a PWR core at the end of the equilibrium cycle.

b. The exact value was determined by an interpolation between the results of two core calculations.

The isotopic composition of the reenriched fuel equivalent to the standard fuel is given in table 5.

	Standard Fuel	Equivalent fuel
U 234	0.03%	0.099%
U 235	3.25%	3.55%
U 236	-	1.19%
U 238	96.72%	95.16%

TABLE 5

With regard to the standard fuel, the over-enrichment required to compensate the effects of U 234 and U 236 is 0.3%.

The characteristics of the associated core equilibrium cycle are given in table 6 below.

	Average core burn-up (MWd/t)	Critical boron concentration (ppm)	Boron efficiency (pcm/pmm)	Fxy
BOC Xenon equilibrium	10020	1185	-7.4	1.31
EOC	20410	10	-8.1	1.20

TABLE 6

The cycle length of the reference core is clearly respected : the variation is about 10 MWd/t.

The value of Fxy stays below its limit and is very close to the reference one ; the power distribution is therefore regular.

The increase of the initial boron concentration and the corresponding decrease of its efficiency with regard to the reference core are due to the spectrum hardening ; this phenomenon and other points of some importance are discussed out of the cell calculation results in the next section.

7. DISCUSSION

7.1. Discrimination between U 234 and U 236

The contributions of both isotopes U 234 and U 236 to the total absorption rate in the re enriched fuel assembly are given in table 7 as a function of burn-up together with their one group average cross section.

	U 234		U 236	
Burn-up	σ_a (b)	relative abs. (%)	σ_a (b)	relative abs. (%)
0	16.8	0.5	4.7	1.8
326Wd/t	16.0	0.3	4.6	2.1

TABLE 7

If we define the contribution of each isotope to the over-enrichment as $\frac{\text{U 234 (resp. U 236) abs. rate}}{\text{U 234 abs. rate} + \text{U 236 abs. rate}}$, we obtain a 80% contribution for U 236 at a zero burn-up.

7.2. Spectrum hardening

The one group average cross section of boron is a good indicator of the change in the flux spectrum (Table 8);

Burn-up	Boron σ_a (b)	
	Standard fuel	Reenriched fuel
0	68.9	64.4
32 GWd/t	63.1	59.1

TABLE 8

The decrease of σ_a is about 6% with regard to the standard fuel.

This phenomenon is due to

- the increase of U 235 concentration, which represents 80% of the effect.
- the presence of U 236 and U 234 which are thermal poisons.

7.3. Self shielding of U 236 absorption cross section

The importance of an accurate treatment of U 236 resonant absorption is shown in table 9 which summarizes the results of two cell calculations differing from one another in the self shielding of U 236 cross section. These results are related to the equivalent fuel previously defined.

Burn up = OMWd/t	With self shielding	Without self shielding
k_{∞}	1.21810	1.20668
σ_a (U 236)	4.7 b	7.0 b
relative abs. rate	1.8%	2.7%

TABLE 9

The reactivity gain is about 800 pcm. The relative absorption rate and σ_a of U 236 are reduced by more than 30% when taking into account self shielding.

If this effect had been neglected, the isotopic composition of the equivalent reenriched fuel would have been that shown in table 10 below :

	With self shielding	Without self shielding
U 234	0.099%	0.106%
U 235	3.55%	3.78%
U 236	1.190%	1.25%
U 238	95.161%	94.854%

TABLE 10

It may be seen that the over enrichment required with regard to the standard fuel would have been about 0.55% instead of 0.3% in the present case.

7.4. Isotopic composition of the burnt fuels

The isotopic composition of the burnt equivalent fuel was calculated in EOC conditions, i.e. at 32 GWd/t. The results are presented in Table 11 together with those related to the standard fuel.

Burn-up = 32 GWd/t	Standard fuel	Reenriched fuel
U 234	0.02%	0.07%
U 235	0.97%	1.19%
U 236	0.40%	1.54%
Pu (all isotopes)	1.03%	1.06%

TABLE 11

It can be seen that the uranium contained in the spent equivalent fuel is fully degraded.

The small increase of the plutonium amount is a result of the spectrum hardening through U 238 epithermal capture. Its isotopic composition is also affected by the presence of a large amount of U 236 as shown in table 12.

Burn-up = 32 GWd/t	Standard fuel	Reenriched fuel
Pu 238	1.80%	4.48%
Pu 239	57.64%	57.81%
Pu 240	22.13%	20.58%
Pu 241	13.43%	12.82%
Pu 242	5.00%	4.31%

TABLE 12

CONCLUSION

This theoretical study shows the feasibility of uranium recycling in PWRs from the neutronic point of view.

The main result is the following : the parasitic absorptions of 1.2% in U 236 and 0.1% in U 234 require a 0.3% overenrichment in U 235 with regard to the standard fuel.

Our calculations have shown the importance of some physical phenomena such as the self-shielding of U 236 cross sections.

An experimental program is carried out in order to validate the predicted consequences. It is based on the measurements of :

- . the effective cross section of U 236 by destructive analysis of some UO_2 samples (U 236 content up to 1%) irradiated in a special device in MELUSINE (Sherwood II experiment)

- . the fission rate of U 234, U 236, Np 237 and Pu 238 using fission chambers.

- . the reactivity effect, oscillating samples containing various U 236 concentrations, in several PWR lattices located in the center of the critical facility MINERVE.

The results of these experiments will be available at the beginning of 1983.

REFERENCES

/1/ BNL 325 - 3rd edition

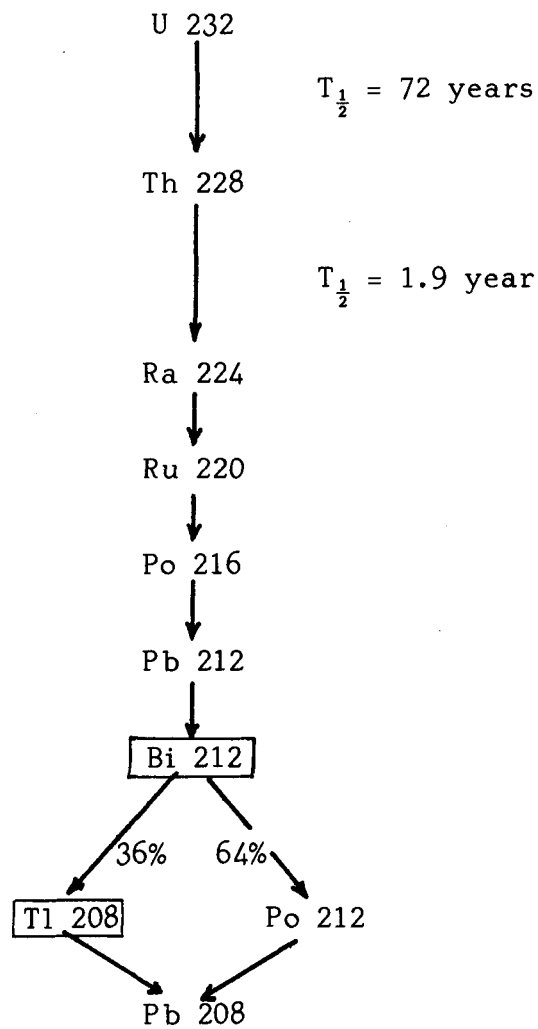
/2/ Conséquences de l'utilisation de l'uranium de retraitement dans les réacteurs à eau.

M. DARROUZET et alter

Note technique CEA/SEN/LPRE/81-148 - Juin 1981

/3/ APOLLO - A general code for transport, slowing down and thermalization in heterogeneous media.

A. KAVENOKY in Proc. Mathematical models and computational techniques for Analysis of nuclear systems, Am Arbor CONF 730414.1.95 (1973)



xxxx γ emitter

Figure 1 - U 232 decay chain

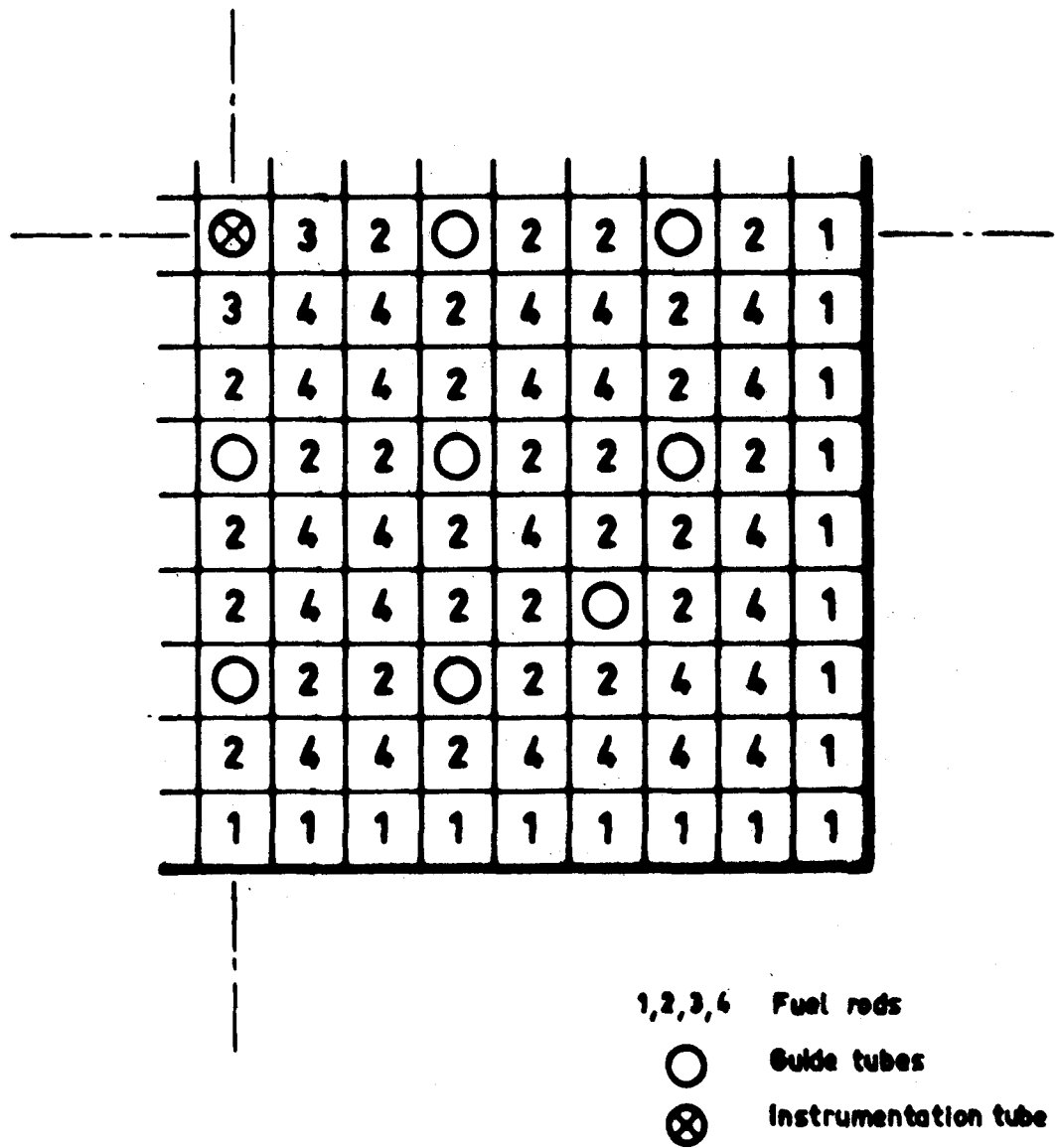


Figure 2 - Assembly representation

THE APPLICATION OF NODAL METHODS TO LIGHT WATER REACTORS

P. J. Finck, C. L. Hoxie, H. S. Khalil,
D. K. Parsons and A. F. Henry
Massachusetts Institute of Technology*

ABSTRACT

Systematic methods for determining node-homogenized group parameters and reconstructing local fuel pin powers using only fine-mesh calculations for isolated fuel assemblies--in some cases extended assemblies or color sets--along with two-group nodal calculations for determining reactor eigenvalue and flux distribution are described. Numerical tests for small two dimensional benchmark problems yield maximum errors in local pin powers at beginning of life of ~ 5% for both PWR's and BWR's.

INTRODUCTION

The determination of neutron densities throughout a reactor by nodal methods requires the resolution of three somewhat separate difficulties. First it is necessary to have a nodal procedure. Next it is necessary to determine the parameters (generally homogenized group-diffusion-theory cross sections) needed for the nodal equations, and, finally, it is necessary to infer local fuel pin power densities from the nodal results. Historically these difficulties were attacked sequentially, and it is only now that progress has been made in all three areas that their close connection can be recognized. Thus the present paper is concerned with the reconstruction of detailed flux shapes throughout heterogeneous assemblies after nodal solutions providing average flux values in each reactor node and on each nodal surface have been carried out. But the reconstruction will require knowledge of the average value of the detailed heterogeneous flux over the faces of the nodes. Thus a nodal procedure and a homogenization procedure that permit making good estimates of these averages must be employed if the reconstruction is to be accomplished. Accordingly in the following section we shall review the characteristics of the homogenization method and nodal model that yield the information needed for our flux reconstruction procedures. The following sections then describe the procedures developed for flux reconstruction and some numerical tests of their accuracy.

ASSEMBLY HOMOGENIZATION AND NODAL METHODS

Systematically derived nodal methods for solving group diffusion equations (1,2,3,4) are generally based on a nodal balance condition and nodal coupling equations. The nodal balance condition is exact and states that the sum of the net leakage rates across the faces of a node for a given neutron energy group equals the difference between the node-volume-integrated production and removal rates.

The nodal coupling equations are what distinguish one nodal approximation from another. They relate the average net current across an interface separating two nodes to the volume-averaged fluxes for those two nodes (and usually--weakly--to other face-averaged nodal currents). For system-

*Work supported by the Electric Power Research Institute.

atically derived nodal methods, the face-averaged nodal fluxes appear during the derivation of the nodal coupling expressions (and are then eliminated).

The implicit presence of these face-averaged fluxes in the nodal coupling expressions permits us to correct these expressions in such a way that they can be made exact.

Thus, we define ideal homogenized reaction cross sections (and diffusion constants) for a node by

$$\bar{\Sigma}_{\alpha g}^k \equiv \frac{\int_{V_k} \Sigma_{\alpha g}(\underline{r}) \phi_g^{\text{het}}(\underline{r}) dV}{\int_{V_k} \phi_g^{\text{het}}(\underline{r}) dV} \quad (1)$$

where $\Sigma_{\alpha g}(\underline{r})$ is the position-dependent group-g cross section for reaction- α , $\phi_g^{\text{het}}(\underline{r})$ is the true group-g flux for the reactor, and the integrals are over V_k , the volume of node k. Then we require that the volume-averaged group-g flux for node-k be given by

$$\bar{\phi}_{gk}^{\text{hom}} \equiv \frac{1}{V_k} \int_{V_k} \phi_g^{\text{het}}(\underline{r}) dV \quad (2)$$

If in addition we require that face-averaged currents be preserved, the basic equations of the nodal model from which the nodal coupling relationship are derived imply unique values for face-averaged nodal fluxes. The difficulty is that if, for example, we consider the nodal face at the interface separating node (k) from node (k-1), the face-averaged homogenized flux implied by the basic equations for node (k) will differ from that implied by the basic equations for node (k-1). If, in order to eliminate these face-averaged fluxes, we assume they are equal, the resultant nodal coupling equation relating the face-averaged current at the interface to $\bar{\phi}_{gk}^{\text{hom}}$ and $\bar{\phi}_{g,k-1}^{\text{hom}}$ will be only approximate.

If instead we define "discontinuity factors" as

$$f_g^{\pm}(U_\ell) = \frac{\bar{\phi}_g^{\text{het}}(U_\ell)}{\bar{\phi}_g^{\text{hom}}(U_\ell \pm)} ; U_\ell = X_i, Y_j, \text{ or } Z_k \quad (3)$$

where $\bar{\phi}_g^{\text{het}}(U_\ell)$ is the (physically real) face-averaged, group-g, heterogeneous flux on a nodal surface at $U = U_\ell$; $\bar{\phi}_g^{\text{hom}}(U_\ell +)$ is the (artificial) face-average flux implied by the basic nodal equation for the node on the plus side of U and $\bar{\phi}_g^{\text{hom}}(U_\ell -)$ the corresponding quantity for the node on the minus side we have

$$f_g^-(U_\ell) \bar{\phi}_g^{\text{hom}}(U_\ell -) = f_g^+(U_\ell) \bar{\phi}_g^{\text{hom}}(U_\ell +) \quad (4)$$

Equation (4) can be used to eliminate the $\bar{\phi}_g^{\text{hom}}(U_\ell \pm)$ from the basic nodal equations and thereby obtain nodal coupling equations (containing the $f_g^{\pm}(U_\ell)$) that will reproduce the $\bar{\phi}_{gk}^{\text{het}}$, face-averaged currents and reactor eigenvalue exactly.

A more detailed sketch of this theory and a description of its close relationship to Koebke's "Equivalence Theory" (5) appears in Reference (6). A complete derivation is given in Reference (7). Note that to find exact values of the $\bar{\Sigma}_{og}^k$ and $f_g^\pm(U_\ell)$, exact values of the true heterogeneous flux throughout the reactor (as well as the global eigenvalue and face-average surface currents) must be known. Hence, in practice, only approximate values of the $\bar{\Sigma}_{og}^k$ and $f_g^\pm(U_\ell)$ will be found. Also note that the $\bar{\phi}_g^{\text{hom}}(U_\ell^\pm)$ are to be determined using the equations of some particular nodal model. Thus "ideal" discontinuity factors correct for any deficiencies in this model as well as for the heterogeneous nature of the nodes. Finally note that the computation of the $\bar{\Sigma}_{og}^k$ and $f_g^\pm(U_\ell)$ need not be restricted to a diffusion theory approximation.

FLUX RECONSTRUCTION PROCEDURES

We wish to construct heterogeneous flux shapes from nodal results without performing any fine-mesh calculations for full heterogeneous core or even for two dimensional cuts through the core at particular elevations. Instead we wish to perform fine-mesh calculations only for two dimensional cuts through reactor assemblies--or, at worst, clusters involving a few assemblies. Accordingly we shall define a group-g "form function" for each assembly making up the reactor as

$$F_g^k(\underline{r}) \equiv \frac{\phi_g^{\text{het}}(\underline{r})}{A_g^{\text{het}}(\underline{r})} ; \quad \underline{r} \in \text{assembly } k. \quad (5)$$

where $A_g^{\text{het}}(\underline{r})$ is a flux shape for the assembly, with all heterogeneities represented explicitly, but obtained using approximate boundary conditions on the surfaces of the assembly. With the $A_g^{\text{het}}(\underline{r})$ precomputed, the $\phi_g^{\text{het}}(\underline{r})$ can be constructed if the $F_g^k(\underline{r})$ can be inferred from the output of the nodal calculation.

A) Computation of Form Functions For PWR's

In order to gain insight about how to find the $F_g^k(\underline{r})$ it is most instructive to plot them for typical situations. Figure (1) shows such plots for a 17-water-hole PWR assembly with assembly functions, $A_g^{\text{het}}(\underline{r})$, determined from eigenvalue calculations for the node with $\underline{J}_g \cdot \underline{n} = 0$ boundary conditions imposed. Reference thermal form functions, $F_g^k(\underline{r})$, for a two-group model have been constructed by performing full core, fine mesh calculations (2D) for several small benchmark cases; thus the form functions shown are exact. Figure (1a) is the reference thermal flux $\phi_2^{\text{het}}(\underline{r})$ for an off-center assembly; (1b) is the corresponding form function; (1c) is the thermal form function corresponding to an infinite checkerboard arrangement of fuel assemblies, and (1d) is for an assembly next to a baffle (Assembly 6 of Figure 2). Corresponding exact fast group form functions do not exhibit the transient exponential behavior near assembly boundaries.

Three conclusions are immediately suggested by these figures: (1) Division of the $\phi_g^{\text{het}}(\underline{r})$ by the $A_g^{\text{het}}(\underline{r})$ removes all irregularities in shape due to the water holes; (2) For interior assemblies the form function appears to be bi-quadratic in the interior of the assembly but with an exponential shape present near the assembly surface. (3) For an assembly adjacent to the core baffle and with the $A_g^{\text{het}}(\underline{r})$ based on $\underline{J}_g \cdot \underline{n} = 0$ boundary conditions, there is a significant distortion in $F_g^k(\underline{r})$ at the corner where there is a 90° wedge of water on the outside of the baffle.

B) Determination of F from its Defining Differential Equation

One approach to finding approximate expressions for the $F_g(\underline{r})$ is to base their mathematical forms on the differential equations which they obey. Thus if $A_g^{het}(\underline{r})F_g(\underline{r})$ is substituted in the defining equations for $\phi_g^{het}(\underline{r})$, and the defining equation for the $A_1^{het}(\underline{r})$ and $A_2^{het}(\underline{r})$ are multiplied by $F_1(\underline{r})$ and $F_2(\underline{r})$, subtraction and rearrangement yields

$$-\nabla \cdot D_1 \nabla F_1 + \left[\left(\frac{1}{\lambda_A} - \frac{1}{\lambda_\phi} \right) v \Sigma_{f1} + \frac{1}{\lambda_A R} v \Sigma_{f2} \right] F_1 - \frac{1}{\lambda_\phi R} v \Sigma_{f2} F_2 = \frac{2D_1}{A_1} \nabla F_1 \cdot \nabla A_1 \quad (6)$$

$$-\nabla \cdot D_2 \nabla F_2 + R \Sigma_{21} [F_2 - F_1] = \frac{2D_2}{A_2} \nabla F_2 \cdot \nabla A_2 ; \quad R \equiv \frac{A_1(x,y)}{A_2(x,y)} \quad (7)$$

where λ_ϕ is the eigenvalue (k_{eff}) of the global solution and λ_A that for the assembly solution (here based on $\underline{J}_g \cdot \underline{n} = 0$ boundary conditions on the assembly surface).

If it were not for the cross-gradient terms on the right hand sides of these equations, they would be analogous to two-group diffusion equations and could be attacked by standard methods. We argue that these cross gradient terms may be neglected since ∇A_1 is small throughout the assembly and ∇A_2 is small except at the boundaries of the 17 water holes. But here ∇F_2 remains fairly small and ∇A_2 has opposite signs on opposite faces so that the term $\nabla F_2 \cdot \nabla A_2$ has a small volume-averaged value over the region of the water holes. Accordingly we shall omit the cross gradient terms in (6) and (7). Moreover since the difference between λ_A and λ_ϕ is rarely more than 0.2, since $v \Sigma_{f1}$ is small compared to $v \Sigma_{f2}$ and since in the interior of the assembly (but not near the edge) F_1 is very close to F_2 , we shall use throughout the assembly values of the group parameters D_g , $v \Sigma_{fg}$ and Σ_{21} (fuel-cell-homogenized) appropriate to the fuel near the surface of the assembly. We shall also assume that the value of R , the ratio of the fast to slow assembly flux, appropriate to that region can be used throughout the assembly. With these assumptions (6) and (7) become two coupled Helmholtz equations with constant coefficients:

$$-D_1 \nabla^2 F_1 + \left[\left(\frac{1}{\lambda_A} - \frac{1}{\lambda_\phi} \right) v \Sigma_{f1} + \frac{1}{\lambda_A R} v \Sigma_{f2} \right] F_1 - \frac{1}{\lambda_\phi R} v \Sigma_{f2} F_2 = 0 \quad (8)$$

$$-D_2 \nabla^2 F_2 + R \Sigma_{21} F_2 - R \Sigma_{21} F_1 = 0 \quad (9)$$

If now accurate (inhomogeneous) boundary conditions for the F_g on the surface of the assembly can be inferred from the nodal solution, the F_g can be found, and good approximations to ϕ_g^{het} can be reconstructed from (5).

Using spatially constant values for the coefficients in (8) and (9) makes it possible to write approximate analytical solutions for those equations. Guided by Figures (1b,c) we do this for each of the four triangles formed when diagonal lines connecting alternate assembly corner points are drawn. Thus for the triangle (1) formed by the lines $x = -h/2$; $-x = \pm y$; $(-h/2 \leq x \leq 0)$ we write the following particular solution of (8) and (9):

$$F_g^{(1)}(x,y) = C s_g [\cos \mu_x^{(1)} + (1/\mu_x^{(1)}) C_x^{(1)} \sin \mu_x^{(1)} x] [\cos \mu_y^{(1)} y + (1/\mu_y^{(1)}) C_y^{(1)} \sin \mu_y^{(1)} y] + \left[\left(\frac{1}{2} + \frac{y}{h} \right) a_2 + \left(\frac{1}{2} - \frac{y}{h} \right) a_3 \right] t_g \exp [-B_2 \left(\frac{h}{2} + x \right)] \quad (10)$$

where B_1^2 and $-B_2^2$ are characteristic roots of the homogeneous equations obtained when $\nabla^2 F_g$ in (8) and (9) is replaced by $-B^2 F_g$; where the assembly has dimensions $(h \times h)$ and where

$$\begin{aligned} [\mu_x^{(1)}]^2 + [\mu_y^{(1)}]^2 &\equiv B_1^2; \quad s_1 = t_1 = 1; \quad s_2 \equiv (D_2 B_1^2 + R \Sigma_{21}) / R \Sigma_{21}; \\ t_2 &\equiv (D_2 B_1^2 + R \Sigma_{21}) / R \Sigma_{21} \end{aligned} \quad (11)$$

In the expression (10) C , $C_x^{(1)}$, and $C_y^{(1)}$, a_2 and a_3 are to be inferred from information provided by the nodal solution. To do so we make use of the volume and face averaged fluxes, $\bar{\phi}_g^{\text{hom}}$ and $\bar{\phi}_g^{\text{het}}(-h/2)$ found directly from the global nodal solution.

C) Determination of Corner Point Fluxes

We also need fluxes at nodal corner points to find the unknowns in (10). These we obtain by assuming that the shapes of the $F_g(x,y)$ on nodal surfaces can be represented by quadratics which in turn can be expressed algebraically in terms of the (known) $\bar{\phi}_g^{\text{het}}(-h/2)$ and the (desired) $\bar{\phi}_g^{\text{het}}(-h/2, \pm h/2)$. Then at each corner point we make use of the relationship suggested by Koebeke and Wagner (8) that the surface integral $\oint \underline{J}_g \cdot \underline{n} \, ds$ about a small square containing a corner point vanishes as the area of the square goes to zero. This condition, along with the assumption that the form functions are quadratic on the surfaces of the assemblies couples group- g fluxes at each corner point to their four nearest neighbors. The resultant equations can be solved iteratively for the corner point values.

Details of this method for approximating the $F_g(x,y)$ are presented in Reference (9). The procedure sketched above is carried out for each of the four triangles into which a given assembly is partitioned. Thus a form function for each triangle can be found, and (5) can be used to reconstruct the $\bar{\phi}_g^{\text{het}}(\underline{r})$ within the assembly. It should be noted however that the solution is at best approximate since the $F_g(x,y)$ and their derivatives are in general discontinuous across the diagonal lines which partition an assembly into triangles.

D) The Use of Extended Assemblies to Reconstruct Pin Power

For assemblies adjacent to the core baffle Figure (1d) shows that Eq. (10) provides a poor representation of the $F_2^{(n)}(x,y)$. Since $\mu_y^{(1)}$ is small in magnitude, $F_2^{(1)}(-h/2,y)$ as implied by Eq. (10) is very nearly quadratic in y , and the form functions for the other surfaces will have similar shapes. Thus they cannot represent the exponential decay shown in Figure (1b), and trying to find a corner point on the core-baffle interface under the assumption that the surface shapes are quadratic will lead to serious error.

One way to circumvent this difficulty is to make use of "extended assemblies" in order to determine the $A_g^{\text{het}}(\underline{r})$ and discontinuity factors for assemblies adjacent to the baffle. Figure (2) shows a two-dimensional PWR benchmark problem that we have used to illustrate and test flux reconstruction schemes for PWR's. Assemblies labeled (A) and (B) both contain 17 control rod water holes but have different fuel enrichments. The dashed zones comprised of regions (1,5), (2,3,4,6,7,8,10,11,12), and (15,16) are "extended assemblies." For these, heterogeneous flux shapes are computed by imposing $\underline{J}_g \cdot \underline{n} = 0$ boundary conditions on all outer surfaces except those at the outer edge of the reflector where zero-returning-current boundary conditions are imposed. Homogenized group parameters, discontinuity factors, and assembly shapes $A_g^{\text{het}}(\underline{r})$

are then edited for each individual assembly making up an extended assembly. The $A_g^{\text{het}}(\underline{r})$ are no longer subject to the $\underline{J}_g \cdot \underline{n} = 0$ boundary conditions on all surfaces, and the corresponding form functions are more accurately represented by formulas like (10).

E) The Use of Color Sets to Reconstruct Pin Power

An alternative procedure for reconstructing heterogeneous flux shapes at the edge of the core is to make use of color sets (10). Color sets are formed by partitioning each assembly into four nodes and then recombining the nodes into assembly sized regions composed of one node from each of four neighboring assemblies. The dashed lines in Fig. (2) indicate color sets. Detailed heterogeneous flux shapes for the color sets are then found by imposing $\underline{J}_g \cdot \underline{n} = 0$ boundary conditions on their surfaces. Such boundary conditions are closer to the correct ones than they are for fuel assemblies, and as a result form functions for color sets do not have the exponential shapes seen in Fig. (1c) at the assembly surfaces. In fact within each node making up a color set (but not within each full color set when one of its nodes is reflector material) the shape of the form functions can be fairly well represented as the product of two quadratics. Accordingly we may adopt such a representation and run global problems using four nodes per assembly and finding homogenized parameters, discontinuity factors and quarter assembly shapes $A_g^{\text{het}}(\underline{r})$ for the nodes from the color set calculations.

EXTENSION OF THE THEORY THROUGH THE USE OF RESPONSE MATRICES

If assembly discontinuity factors based on $\underline{J}_g \cdot \underline{n} = 0$ boundary conditions are used to analyse BWR's, errors in predicted assembly power are generally in the range of a few percent, although for certain rodged assemblies and assemblies next to the reflector, errors of almost 10% can result. Such errors are smaller than those encountered when even the most carefully tuned $1\frac{1}{2}$ group schemes are used. Thus the ADF procedure is probably acceptable if only average assembly powers are desired. However, local pin powers reconstructed from the ADF results are quite inaccurate if the methods described above are used. Accordingly we have developed, primarily for BWR's*, a more accurate, iterative procedure for predicting both local and global power distributions.

The essential assumption of this procedure is that, on the surfaces of an assembly, the shape of the form function can be represented by a quadratic. We then partition that quadratic into three basic quadratics $Q_n(u)$; $n = 1, 2, 3$ having coefficients that are fixed numbers and constructed so that, for example, on the face $y = y_j$; $x_1 \leq x \leq x_{i+1}$

$$\begin{aligned} \phi_g^{\text{het}}(x, y_j) \approx \psi_g^{\text{het}}(x, y_j) [& \phi_g^{\text{het}}(x_1, y_j) Q_1(x) + \phi_g^{\text{het}}(y_j) Q_2(x) + \\ & + \phi_g^{\text{het}}(x_{i+1}, y_j) Q_3(x)] \end{aligned} \quad (12)$$

where $\psi_g^{\text{het}}(x, y_j)$ is the heterogeneous assembly flux shape along the face in question, computed using $\underline{J}_g \cdot \underline{n} = 0$ boundary conditions.

Finally we impose a flux shape of the form $\psi_g^{\text{het}}(x, y_j) \sum_{n=1}^3 a_n Q_n(x)$ on the surface

*The theory has been tested for simple PWR benchmarks (14). Primarily because of the difficulty of interpolating corner points, little improvement over the already accurate ADF results was found.

$y = y_j$; $x_i \leq x \leq x_{i+1}$ (where the a_n are fixed numbers chosen to make this surface flux positive and determine a fine-mesh (inhomogeneous) flux solution throughout the node, editing the nodal reaction rates and face-averaged fluxes for the assembly. Face-fluxes for other groups and other faces are set to zero. Such computations are performed for each group separately, for each face (except when symmetry considerations make the calculation redundant) and for three independent sets of a_n 's. They must be performed for each different kind of assembly as a function of global eigenvalue, temperature and density. From such calculations "flux" response matrices and interpolation formulas can be constructed so that if the corner points and face-averaged fluxes for an assembly along with its temperature, density and the global eigenvalue are available, all reaction rates, face-averaged currents and fluxes as well as detailed flux shapes throughout the node can be determined by a simple matrix multiplication.

With the necessary response matrices precomputed, the following iterative procedure provides a solution for the global problem:

- 1) Find homogenized group parameters and ADF's from a $J_g \cdot n = 0$ assembly calculation.
- 2) Use these to determine a global nodal solution; edit face- and volume-averaged fluxes, and infer corner point fluxes as for the PWR case.
- 3) Use the response matrices based on the latest estimates of eigenvalue, temperatures and densities to update the assembly-homogenized parameters and to determine face-averaged net-currents.
- 4) Use these face-averaged net-currents, the homogenized group parameters, and the volume averaged fluxes along with the basic nodal equations to find face-averaged homogenized fluxes $\Phi_g^{\text{hom}}(U_d^{\pm})$ and determine the discontinuity factors from (3).
- 5) Return to step (2) and continue the iteration to convergence. Then use the response matrices appropriate to the converged conditions to reconstruct pin-powers for any assembly desired.

NUMERICAL TESTING

Figure (3) provides numerical comparisons between the full-core, fine-mesh, two-group solution of the PWR benchmark shown in Figure (2) and nodal solutions for the same problem using extended assembly or color set ADF's. QUANDRY (3) was used for the nodal calculations and either QUANDRY (run with fine-mesh) or the finite-difference code PDQ was used for the reference solution and assembly or color set calculations.

Both the extended assembly and color set ADF's yield results very close to the standard in predictions of reactor eigenvalue and average nodal power. The maximum errors in reconstructed thermal fluxes translate to less than 5% in maximum pin-power error, and that error is for a low power pin next to the baffle. About half of this error is due to inaccuracies in the inferred thermal fluxes for corner points lying on the core-baffle interface.

One very significant result shown by Figure (3) is the fact that, even when the homogenized group parameters for the node are found by averaging over heterogeneous flux shapes obtained from extended assembly or color set calculations, errors in eigenvalue and average nodal powers are significant if unity discontinuity factors (UDF's) are used.

For BWR's, use of ADF's based on $J_g \cdot n = 0$ assembly calculations leads to errors in average nodal power that are marginally acceptable. Table (1) summarizes

the situation for beginning-of-life radial slices of a Vermont-Yankee core at three elevations. Figure (4) shows the geometry of the core assemblies.* The maximum errors in average assembly power are for nodes next to a control rod. Errors in reconstructed flux would be larger.

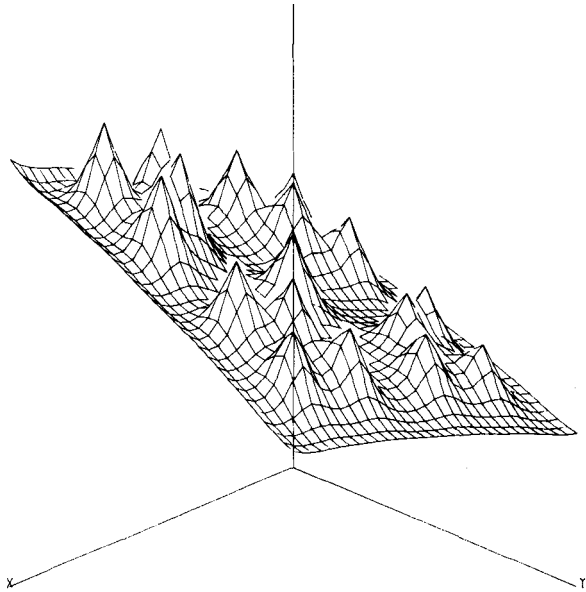
Figure (3) shows the quarter core layout of a more severe test of the theory called the LSHBWR Benchmark problem (12). The fuel assemblies are those of Figure (4); assemblies marked W are water reflector assemblies, and those marked 0, 40 or 70 are fuel assemblies containing 0, 40 or 70% void.

Tables (2) and (3) show how predictions of various approximations differ from those of the numerical standard. Except for the anomalous errors in eigenvalue (which we think may be due to having used fine-mesh QUANDRY for the numerical standard but CITATION (15) for the generation of response matrices) the response results are excellent. The "Q" or "F" designations are to indicate that the quadratic or flat approximations for the shape of the transverse leakage (3) were used in QUANDRY. The apparent insensitivity to that approximation illustrates that discontinuity factors correct both for homogenization errors and for approximations in the nodal model.

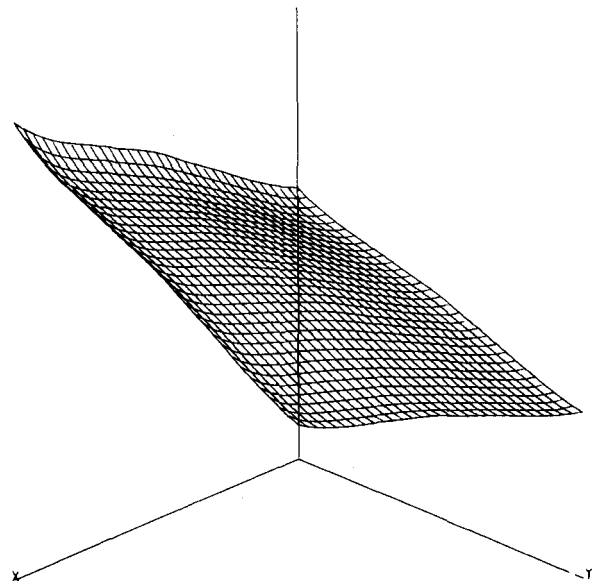
CONCLUSIONS

The methods described in this paper give promise of a reliable, cheap and theoretically well founded procedure for predicting pin powers in light water reactors. There is, however, still work to be done. For example it is essential to extend our testing to cover depletion and fuel shuffling situations. (Preliminary results here have been very encouraging.) Also it would decrease complexity considerably if we could avoid the use of extended assemblies or color sets for PWR analysis. And, of course, it would help the situation for BWR's immensely if we could avoid the use of response matrices and still predict pin powers with errors of less than 5%. Even without these improvements, however, the present scheme provides predictions quite close to those of a full-core fine-mesh, finite-difference procedure, and does so at a very significant reduction in computation costs.

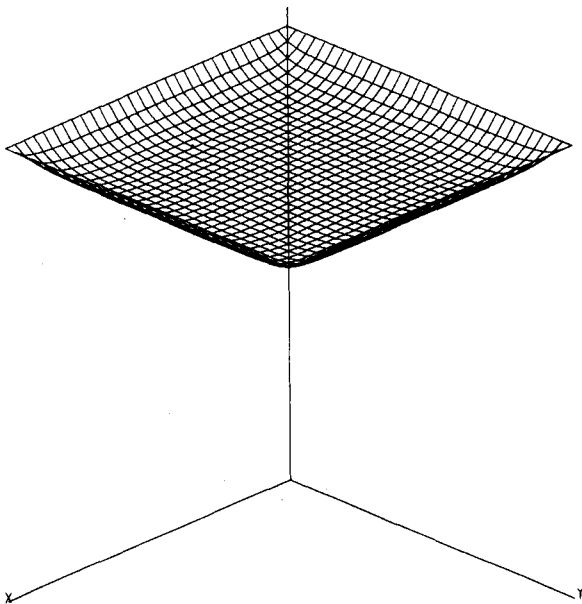
*The PDQ reference calculations and $J_g \cdot n = 0$ assembly calculations for this problem as well as the results of applying SIMULATE (12), a 1.5 group nodal code, to the analysis were provided to us by David VerPlanck of the Yankee Atomic Electric Company.



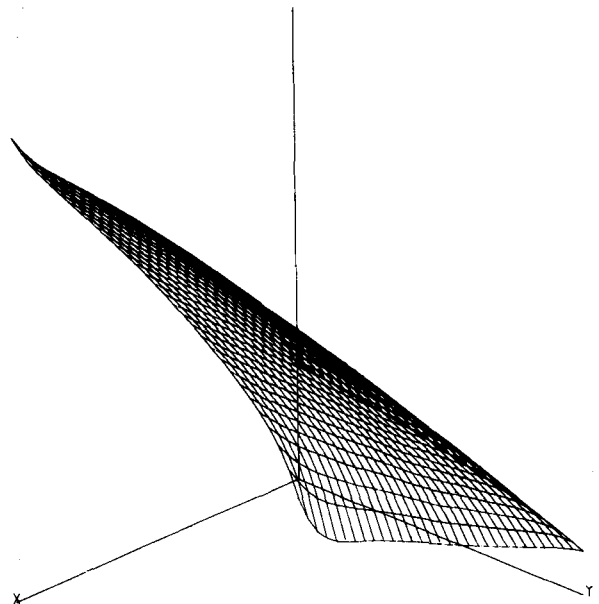
1a: Reference Thermal Flux
Off Center Assembly



1b: Reference Form Function
Corresponding to 1a.



1c: Reference Thermal Form
Function for Checkerboard
Loading.



1d: Reference Thermal Form
Function for Assembly
6 of Figure (2).

Figure 1: PWR Assembly Flux Shapes and Form Functions

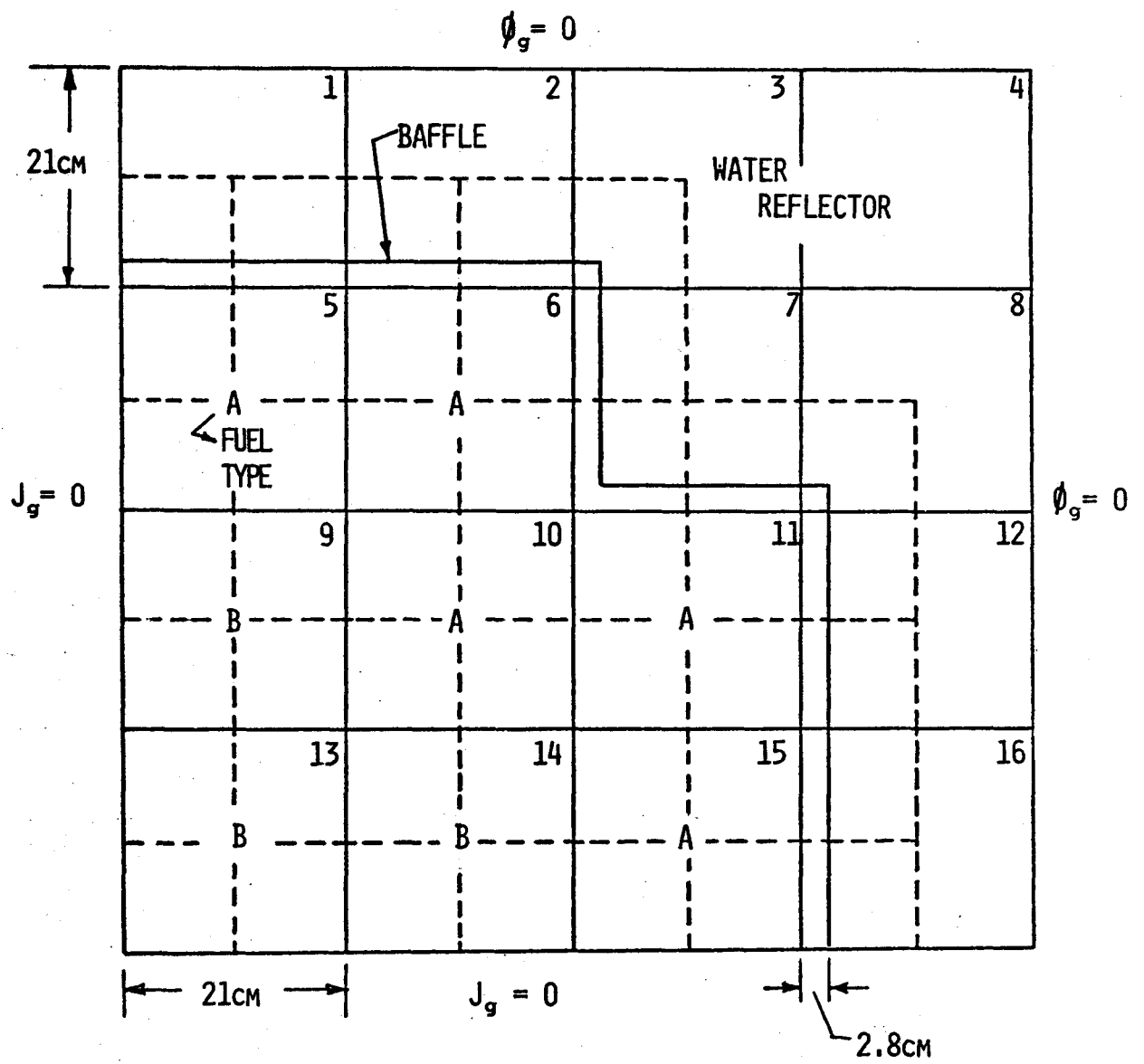


FIG. 2. A PWR BENCHMARK PROBLEM

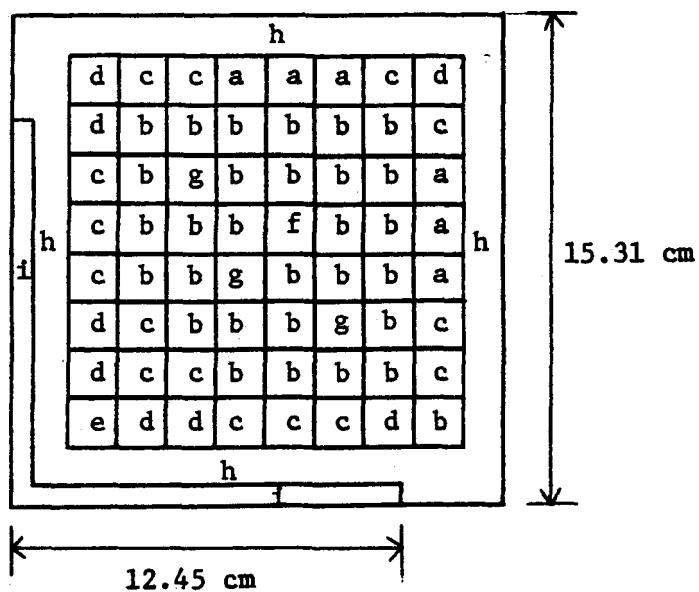
% error in k_{eff}		$\left\{ \begin{array}{l} \text{ADF: } 0.00752 \\ \text{UDF: } 0.178 \end{array} \right.$	
0.8482	0.6097		
0.01	-0.16		
2.17	5.37		
5.84	5.50		
1.2173	1.2062		
0.03	0.02		
-2.01	-1.00		
-3.17	3.36		
1.4435			
0.05			
-2.86			
-1.60			
		Reference Relative Power % error ADF % error UDF max % error, reconstructed thermal flux	

Extended Assembly Results

% error in k_{eff}				$\left\{ \begin{array}{l} \text{ADF: } 0.0480 \\ \text{UDF: } 0.0621 \end{array} \right.$			
0.6574	0.6170	0.5255	0.3569				
-0.92	-0.98	-1.17	-0.98				
4.76	4.92	5.40	12.30				
-3.70	-3.82	-4.33	4.80				
1.0857	1.0316	0.8991	0.6541				
-0.06	-0.16	-0.32	-0.88				
-1.08	-0.93	-0.41	4.99				
-0.77	-1.01	-1.00	-3.40				
1.1524	1.1035	1.2101	0.9745				
0.31	0.14	0.07	-0.33				
-1.34	-1.22	-1.00	-0.08				
0.48	0.33	-0.61	-3.18				
1.3319	1.2825	1.4323					
0.46	0.35	0.26					
-1.64	-1.44	-1.35					
0.67	0.66	0.46					
1.4431	1.3945						
0.55	0.47						
-1.74	-1.69						
0.66	0.51						
1.4973							
0.58							
0.65							

Color Set Results

Fig. 3. Errors in nodal power and reconstructed thermal flux for nodal analysis using extended assembly or color set calculations to determine homogenized parameters and reconstructed fluxes.



Zone	Material
a	2.50 w/o fuel adjacent to zircaloy can
b	2.50 w/o fuel in the interior
c	1.90 w/o fuel
d	1.49 w/o fuel
e	1.18 w/o fuel - corner rod
f	The water rod
g	The gadolinium rods
h	Channel walls and wide and narrow gaps
i	Control rod (if present)

Figure 4: Assembly Configuration for BWR Benchmark

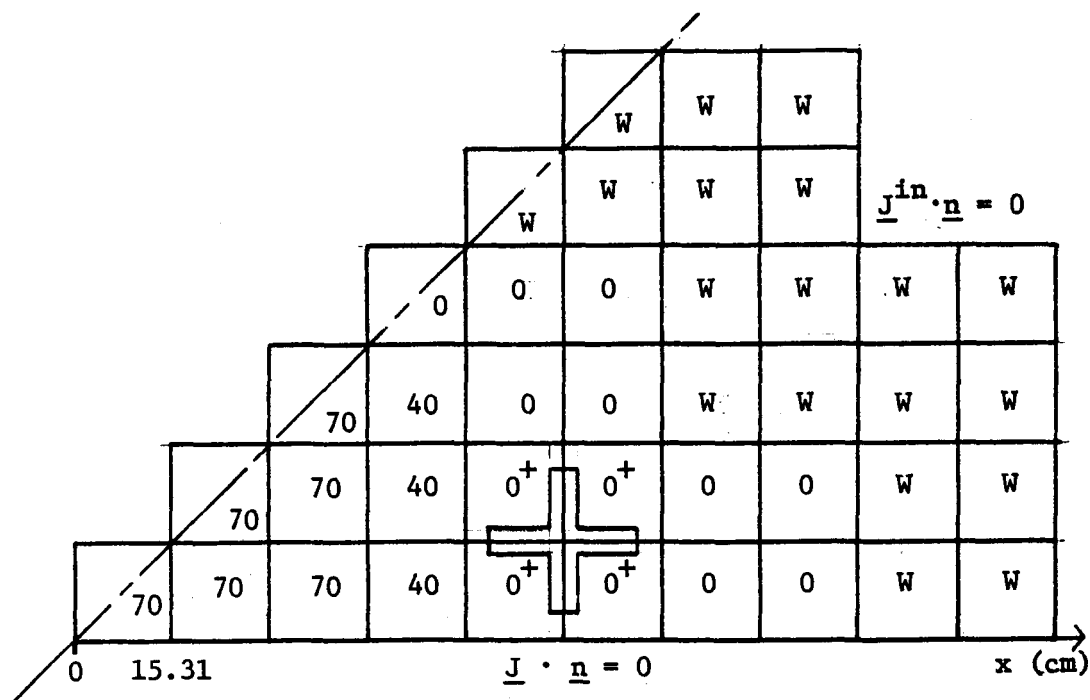


Figure 5: Core Layout for the LSHBWR Benchmark

Table 1

Errors in eigenvalue and nodal powers
found using various nodal approximations

		Elevation 1	Elevation 2	Elevation 3
Reference λ		1.02907	1.02708	1.00333
% error in λ	ADF	0.039	-0.028	-0.060
	UDF	-0.408	0.157	-0.119
	SIMULATE	-0.109	-0.242	-
Max % error in nodal power	ADF	-9.23	-2.49	-2.17
	UDF	25.83	13.75	13.89
	SIMULATE	14.51	6.34	-7.55

Table 2

Errors in Eigenvalue and Power

	<u>Standard flux weighted parameters</u>	<u>ADF Q</u>	<u>RESPONSE Q</u>	<u>RESPONSE F</u>
Error in eigenvalue	- .29%	-.05%	.54%	.54%
Maximum error in power	22.6%	9.61%	1.06%	1.12%
Ave. error in power	5.90%	3.26%	.46%	.46%

Table 3

Maximum errors in reconstructed fluxes

	<u>Node #6</u>	<u>Node #7</u>
Maximum error in reconstructed fast flux in fueled region	-1.89%	-2.85%
Maximum error in reconstructed thermal flux in fueled region	-2.68%	-3.37%

References

1. S. Langenbuch, W. Maurer and W. Werner, "Simulation of Transients with Space-Dependent Feedback by Coarse Mesh Flux Expansion Method", CSNI/NEACRP Specialists' Meeting on New Developments in Three-Dimensional Neutron Kinetics and Review of Kinetics Benchmark Calculations, Garching (Munich), Jan. 22-24, 1975.
2. H. Finnemann and M. R. Wagner, "A New Computational Technique for the Solution of Multidimensional Neutron Diffusion Problems", presented at the International Meeting of Specialists on "Methods of Neutron Transport Theory in Reactor Calculations", Bologna, Italy (Nov. 1975).
3. G. Greenman, K. Smith, and A. F. Henry, "Recent Advances in an Analytic Nodal Method for Static and Transient Reactor Analysis", ANS Proceedings of the Topical Meeting on Computational Methods in Nuclear Engineering, Williamsburg, Virginia, Vol. 1, p. 3-49 (April 1979).
4. R. D. Lawrence and J. J. Dorning, "A Nodal Green's Function Method for Multidimensional Neutron Diffusion Calculations," Nucl. Sci. Eng. 76, 218-231 (1980).
5. K. Koebke, "A New Approach to Homogenization and Group Condensation," paper presented at the IAEA Technical Committee Meeting on "Homogenization Methods in Reactor Physics," Lugano, Switzerland, 13-15 November 1978.
6. K. Smith, A. F. Henry, and R. Loretz, "Determination of Homogenized Diffusion Theory Parameters for Coarse Mesh Nodal Analysis," ANS Topical Meeting, Sun Valley, Idaho, September 1980.
7. K. Smith, "Spatial Homogenization Methods for Light Water Reactor Analysis," Ph.D. Thesis, Department of Nuclear Engineering, M.I.T., Cambridge, MA (June 1980).
8. K. Koebke and M. R. Wagner, "The Determination of the Pin Power Distribution in a Reactor Core on the Basis of Nodal Coarse Mesh Calculations," Atomkernenergie (ATKE) Bd. 30 (1977).
9. C. L. Hoxie, "Application of Nodal Equivalence Theory to the Neutronic Analysis of PWR's," Ph.D. Thesis, Department of Nuclear Engineering, M.I.T. (June 1982).
10. W. J. Eich, "LWR Assembly Reaction Rate Representation," EPRI-NP-1098, Symposium Proceedings: Nuclear Data Problems for Thermal Reactor Application p. 18-4 (June 1979).
11. C. J. Pfeifer, PDQ-7 Reference Manual II, WAPD-TM947(L), Bettis Atomic Power Laboratory (1971).
12. D. M. VerPlanck, "Manual for the Reactor Analysis Program SIMULATE," Yankee Atomic Electric Company (1978).
13. R. A. Loretz, "The Determination of Equivalent Diffusion Theory Parameters by Local Implementation of the Koebke Method," S.M. Thesis, Department of Nuclear Engineering, M.I.T., Cambridge, MA (August 1980).

14. D. K. Parsons, "Application of Response Matrix Methods to PWR Analysis," S.M. Thesis, Department of Nuclear Engineering, M.I.T. (June 1982).
15. T. B. Fowler, D. R. Vondy, and G. W. Cunningham, "Nuclear Reactor Core Analysis Code: CITATION, ORNL-TM-2406, Oak Ridge National Laboratory, 1969.

EXTENSIONS OF THE NODAL EXPANSION METHOD BASED ON THE APPLICATION OF FINITE ELEMENT TECHNIQUES

M. A. Robinson*, R. A. Rydin
University of Virginia

ABSTRACT

In the present study, the inclusion of trial functions in the Nodal Expansion Method (NEM), by means of techniques associated with the Finite Element Method (FEM), is explored. The results of several methods based on these techniques are examined, in terms of the capability to reproduce detailed flux shapes within a mesh box.

INTRODUCTION

One of the objectives of computer aided reactor design is to accommodate the competing constraints on the accuracy and expense of the modelling process. In response to the cost of utilizing techniques that employ a detailed spatial model, numerous alternatives have been developed.¹ In constructing these coarse mesh methods (CMM), the object was to retain the ability to accurately estimate global parameters, such as the eigenvalue and mesh box average fluxes. The spatial detail of the solution was typically sacrificed in favor of a reduction in the number of spatial variables. However, several recently developed CMM attempt to retain both the global accuracy and the spatial detail which are characteristic of fine mesh methods.¹⁻⁶

The Nodal Expansion Method (NEM) is based on the assumption that the spatial dependence of the neutron flux within a mesh box can be represented by a linear combination of space-dependent polynomial trial functions. The expansions employed previously have been defined in terms of the flux at points of support at the mesh box center and surface centerpoints only. (Mesh box and surface average fluxes have also been employed.) These points of support are sufficient to completely specify the expansion coefficients of all basic quadratic methods.² For instance, the coefficient a_{pq} of the trial function $\xi_x^p \xi_y^q$ in the QUAD-SUM method expansion,

$$\phi_2(x,y) = \phi_{ij} (1 + a_{10} \xi_x + a_{20} \xi_x^2 + a_{01} \xi_y + a_{02} \xi_y^2) \quad (1)$$

where $\phi_{ij} = \phi(X_i, Y_j)$ and $\xi_x = (X - X_i)/\Delta X_i$ is a variable in the local coordinate system of mesh box ij with center (X_i, Y_j) , is uniquely defined in terms of the fluxes at the support points shown in Figure 1A. The higher order extensions proposed by Langenbuch, et al^{3,4} and Finnemann, et al^{5,6} employ procedures that do not involve additional support points. Since continuity is imposed only at a single support point on each surface, the flux is discontinuous at all other surface points. Furthermore, neither method explicitly defines the coefficients of multiple odd-order trial functions. Thus, the order of error in the approximation is $O(h^2)$.

*Presently employed at Bettis Atomic Power Laboratory.

ALTERNATE PATTERNS OF SUPPORT BASED ON LAGRANGIAN FINITE ELEMENTS

Certain select Lagrangian finite elements can be modified for use in the NEM. The goal of both the FEM and the NEM is to evaluate the variables in a low order expansion of the dependent variable. In the FEM, local sets of equations, obtained from a variational principle or weighted residual conditions, are collected to form a single large 'stiffness' matrix-vector equation. In the NEM, current continuity relations are used to decouple the system into a banded matrix-vector equation of the standard finite difference form, plus auxiliary tridiagonal matrix-vector equations. This decomposition significantly reduces the computational effort necessary to solve the system, not only by decreasing the size of the largest matrix to be inverted, but also by expressing the system in a form that allows the use of established finite difference solution procedures. To accomplish this decomposition, a support point must be located at the center of the mesh box, since the centerpoint flux variables are used to form the pseudo finite difference equations. Lagrangian elements that contain such a central point of support (biquadratic and biquartic elements, etc.) can be used in the NEM without modification.

Lagrangian elements without support on the mesh box axes (bilinear and bicubic elements, etc.) can be adapted for use in the NEM. In these elements, the addition of the central support point increases the order of the trial functions in the expansion associated with the modified element. For instance, the trial functions 1, x, y, and xy of the Bilinear Lagrangian element are supported at the four corners of the element. If a central support point is included, and continuity is maintained on the mesh box surfaces, then the trial functions associated with the modified bilinear element become 1, x, y, xy, x^2 , x^2y , and x^2y^2 . Since the bilinear and modified bilinear expansions exclude x^2 and y^2 , the order of the error in both expansions is $O(h^2)$.

A new NEM based on the Biquadratic Lagrangian element has been developed. The coefficients a_{ij} of the flux expansion associated with the Biquadratic element (Figure 1B),

$$\phi_4(x,y) = \phi_{ij} \left(\sum_{i'=0}^2 \sum_{j'=0}^2 a_{ij'} \xi_x^{i'} \xi_y^{j'} \right) \quad (2)$$

where $a_{00}=1$, can be defined by a collocation procedure which requires that the flux ϕ_{pq} at each point of support (x_p, y_q) be defined as

$$\phi_{pq} = \int_{V_{ij}} dv \delta(x_p, y_q) \phi(x,y)$$

where V_{ij} is the volume of mesh box ij , and δ is the Dirac delta function. Solving the set of linear equations resulting from collocation at each support point yields the coefficients

$$a_{10} = \frac{\phi_{i+\frac{1}{2}} - \phi_{i-\frac{1}{2}}}{\phi_{ij}}, \quad a_{20} = \frac{2}{\phi_{ij}} (\phi_{i+\frac{1}{2}} + \phi_{i-\frac{1}{2}} - 2\phi_{ij})$$

$$a_{11} = \frac{1}{\phi_{ij}} (\phi_{i+\frac{1}{2},j+\frac{1}{2}} - \phi_{i-\frac{1}{2},j+\frac{1}{2}} - \phi_{i+\frac{1}{2},j-\frac{1}{2}} + \phi_{i-\frac{1}{2},j-\frac{1}{2}})$$

$$a_{21} = \frac{2}{\phi_{ij}} (\phi_{i+\frac{1}{2},j+\frac{1}{2}} + \phi_{i-\frac{1}{2},j+\frac{1}{2}} - \phi_{i+\frac{1}{2},j-\frac{1}{2}} - \phi_{i-\frac{1}{2},j-\frac{1}{2}} - 2 a_{10} \phi_{ij})$$

$$a_{22} = \frac{4}{\phi_{ij}} (\phi_{i+\frac{1}{2},j+\frac{1}{2}} + \phi_{i-\frac{1}{2},j+\frac{1}{2}} + \phi_{i+\frac{1}{2},j-\frac{1}{2}} + \phi_{i-\frac{1}{2},j-\frac{1}{2}} - 4 \phi_{ij} - [a_{20} + a_{02}] \phi_{ij}) \quad (3)$$

where $\phi_{i+\frac{1}{2},j} = \phi(X_{i+\frac{1}{2}}, Y_j)$ and $\phi_{i,j+\frac{1}{2}} = \phi(X_i, Y_{j+\frac{1}{2}})$. The coefficients a_{10} and a_{20} are the same as those of the QUAD-SUM method. The same coefficients can be obtained from the Lagrangian interpolation at the support points of the Biquadratic element.

The integration of the neutron balance equation over the volume of mesh box ij can be performed directly using equation (2),

$$\begin{aligned} & \int_{V_{ij}} (D \nabla^2 \phi(x,y) - \Sigma_a \phi(x,y) + \frac{v \Sigma_f \phi(x,y)}{\lambda}) dV \\ &= 2 D_{ij} \phi_{ij} (a_{20} + a_{02} + \frac{a_{22}}{6}) \\ &+ (\frac{v \Sigma_f \phi_{ij}}{\lambda} - \Sigma_{aij}) \phi_{ij} \Delta X_i \Delta Y_j (1 + \frac{a_{20} + a_{02}}{12} + \frac{a_{22}}{144}). \end{aligned} \quad (4)$$

In order to arrange the balance equation (4) in the standard finite difference form, the surface and corner fluxes must be eliminated in favor of the mesh box center fluxes. The necessary relations are obtained by enforcing current continuity at the surface points of support. For instance, the current continuity condition at $(X_i, Y_{j+\frac{1}{2}})$ can be written as

$$\phi_{j+\frac{1}{2}} = f_{ij} \phi_{ij} (1 - \frac{1}{4} \left[\frac{\phi_{j-\frac{1}{2}}}{\phi_{ij}} \right]) + f_{j+1} \phi_{j+1} (1 - \frac{1}{4} \left[\frac{\phi_{j+3/2}}{\phi_{j+1}} \right]) \quad (5)$$

where f_{ij} and f_{j+1} are dependent on the diffusion coefficients and mesh spacings in mesh boxes ij and $ij+1$. If the flux ratios in the square brackets are known quantities, then the above equation defines the flux at the surface centerpoint $\phi_{j+\frac{1}{2}}$ in terms of the mesh box center fluxes ϕ_{ij} and ϕ_{j+1} . These ratios are obtained from the previous iterate.

Since the current is discontinuous at the corner support points, an 'average' current continuity is enforced. At $(X_{i+\frac{1}{2}}, Y_{j+\frac{1}{2}})$, the continuity condition imposed is

$$\lim_{\epsilon \rightarrow 0} D_{ij} \left. \frac{\partial \phi^{ij}}{\partial x} \right|_{x_{i+\frac{1}{2}}, y_{j+\frac{1}{2}} - \epsilon} + D_{j+1} \left. \frac{\partial \phi^{i,j+1}}{\partial x} \right|_{x_{i+\frac{1}{2}}, y_{j+\frac{1}{2}} + \epsilon}$$

$$= \lim_{\epsilon \rightarrow 0} D_{i+1} \left. \frac{\partial \phi^{i+1,j}}{\partial x} \right|_{x_{i+\frac{1}{2}}, y_{j+\frac{1}{2}} - \epsilon} + D_{i+1,j+1} \left. \frac{\partial \phi^{i+1,j+1}}{\partial x} \right|_{x_{i+\frac{1}{2}}, y_{j+\frac{1}{2}} + \epsilon}$$

where the superscripts denote the mesh box of the flux expansion. Solving the resultant expression for $\phi_{\frac{1}{2}\frac{1}{2}}$,

$$\phi_{\frac{1}{2}\frac{1}{2}} = f_{ij} \phi_{j+\frac{1}{2}} \left(1 - \frac{1}{4} \left[\frac{\phi_{-\frac{1}{2}\frac{1}{2}}}{\phi_{j+\frac{1}{2}}} \right] \right) + f_{i+1} \phi_{i+1,j+\frac{1}{2}} \left(1 - \frac{1}{4} \left[\frac{\phi_{3/2\frac{1}{2}}}{\phi_{i+1,j+\frac{1}{2}}} \right] \right) \quad (7)$$

Employing equation (5) and the relation

$$\phi_{i+1,j+\frac{1}{2}} = \phi_{i+1} \left[\frac{\phi_{i+1,j+\frac{1}{2}}}{\phi_{i+1}} \right],$$

where the ratio in the square bracket is again taken from the previous iterate, equation (7) can be defined in terms of mesh center fluxes and flux ratios obtained from the previous iterate. An analogous expression based on the current balance in the y direction is written. The average of these relations gives $\phi_{\frac{1}{2}\frac{1}{2}}$ as a function of ϕ_{ij} , ϕ_{j+1} , and ϕ_{i+1} . Equations (5) and (7), and the analogous results for the other mesh box surface support points, constitute sufficient conditions for the expression of the integral balance equation (4) in terms of mesh box center fluxes. That is, using these current continuity conditions, equation (4) is reduced to a five-point difference equation in the mesh box centerpoints, which is the finite difference form.

If the mesh box centered variables ϕ_{i+1} and ϕ_{ij} are known, then the current continuity equation (5) can be written in the form of a three-point difference equation in the surface fluxes. The tridiagonal matrix-vector equation formed from the corresponding relations at the surface centerpoints of each mesh box can be solved for the surface center fluxes. In a similar manner, equation (7) can be used to form a three-point relation for the corner fluxes if the surface fluxes are known.

The solution procedure is initiated by making a mesh box center flux guess ϕ_{ij} . From this guess the surface fluxes are obtained by solving the tridiagonal matrix-vector system corresponding to equation (5). The corner fluxes are then computed via the tridiagonal system of equation (7). The matrix-vector system defining the mesh box center fluxes ϕ_{ij} of the second iterate is solved. The cycle thus repeats until convergence is attained. The solution procedure is illustrated in Figure 2.

STATIC CONDENSATION - SUBSTRUCTURING TECHNIQUE

In the FEM static condensation or substructuring technique, weighted residual (WR) conditions are used to eliminate the variables associated with either internal support points, or variables without explicit support. This procedure reduces the size of the global stiffness matrix.⁷

In the NEM, an analogous reduction can be accomplished by means of WR conditions. These conditions are used to define coefficients of high order trial functions in terms of the previously defined coefficients of the linear and quadratic trial functions.⁶ This can be accomplished, in the case of the CUBE-SUM expansion, by requiring that the cubic trial functions be zero at the support points of the QUAD-SUM expansion. Thus

$$\phi(x,y) = \phi_2(x,y) + \phi_{ij} \left(a_{30} \left[\xi_x^3 - \frac{1}{4} \xi_x \right] + a_{03} \left[\xi_y^3 - \frac{1}{4} \xi_y \right] \right) \quad (8)$$

where $\phi_2(x,y)$ is defined by equation (1). Definitions of the linear and quadratic coefficients in equation (8) remain the same as those in the QUAD-SUM expansion (equation (3)). The two additional conditions that are necessary to specify a_{03} and a_{30} are assumed to be of the form

$$\int_{V_{ij}} W R_{ij} dv = 0, \quad (9A)$$

where the residual

$$R_{ij} = D_{ij} \nabla^2 \phi(x,y) - \Sigma_{aij} \phi(x,y) + \frac{v \Sigma f_{ij} \phi(x,y)}{\lambda} \quad (9B)$$

and W is the weight function. The solution procedure is the same as for the Lagrangian Biquadratic NEM without the corner flux computation.

The results of several versions of the NEM using cubic expansions were compared. The coefficients of the cubic trial functions were defined using the moments ($w = \xi_p$, $p = x,y$), the least squares ($w = \partial R_{ij} / \partial a_{30}, \partial R_{ij} / \partial a_{03}$), modified Galerkin ($w = \xi_p^3 - \xi_p/4$, $p = x,y$), and several least squares collocation WR methods. Application of the least squares and the moments cubic NEM produced more accurate results than those of the other WR based methods.²

FLUX INTERPOLATION

Flux interpolation is a technique by which a high order flux expansion is constructed from information obtained from a converged lower order NEM solution.⁸ In this interpolation technique, the fluxes at the points of support generated by a NEM are used as the basis of the interpolation. The coefficients of high order terms, not included in the initial NEM solution, are obtained by enforcing additional continuity and/or WR conditions.

In the present study,² two interpolation procedures which define the coefficients of a biquadratic expansion with cubic terms ($O(h^4)$),

$$\phi(x,y) = \phi_4(x,y) + \phi_{ij} \left(a_{30} \left[\xi_x^3 - \frac{1}{4} \xi_x \right] + a_{03} \left[\xi_y^3 - \frac{1}{4} \xi_y \right] \right), \quad (10)$$

where $\phi_4(x,y)$ is defined by equation (2), were compared. In the more accurate procedure, the surface and mesh box centerpoint fluxes are generated using the CUBE-SUM method as described in the previous section. An estimate of the corner fluxes is obtained by application of the biquadratic current continuity conditions at the corners (c.f. equation (6)). The solution of the resultant tridiagonal system yields an approximation of the corner fluxes. Having acquired estimates for the corner, surface, and mesh box centerpoint fluxes, the cubic coefficient is defined by the moments WR process. The residual R_{ij} in equation (9) is computed using equation (10). For instance with $W=\xi_x$,

$$a_{30} = \frac{\frac{a_{10}}{6} (\Sigma a_{ij} - \frac{v \Sigma f_{ij}}{\lambda}) + \frac{a_{12}}{3} \left[\frac{1}{24} (\Sigma a_{ij} - \frac{v \Sigma f_{ij}}{\lambda}) - \frac{D_{ij}}{\Delta Y_j^2} \right]}{\frac{D_{ij}}{\Delta X_i^2} + \frac{1}{60} (\Sigma a_{ij} - \frac{v \Sigma f_{ij}}{\lambda})}$$

The coefficients a_{10} and a_{12} are specified in terms of the known surface, corner, and centerpoint fluxes. The solution procedure is shown in Figure 3.

RESULTS AND CONCLUSIONS

Variations of the NEM, which incorporate the methods described in the previous sections, have been employed to solve several two-dimensional, one-group model problems.² Flux shapes generated by means of three of these techniques are illustrated in Figures 5 and 6, for two traverses of a problem comprising a 3 x 3 array of square material regions (Figure 4). Each of the nine distinct regions is one of three materials. Zero flux (Z) and reflecting (RB) boundary conditions are each imposed on two of the external surfaces of the problem. The problem was constructed in this manner, in order to investigate the accuracy with which the NEM methods can reproduce a nonseparable flux shape within a mesh box.

The Lagrangian Biquadratic method, the moments CUBE-SUM method, and the moments CUBE-SUM interpolation method were each employed to solve the model problem, using one mesh space per material region ($\Delta x = 21.8$ cm). The utilization of each of these techniques yielded improvements in both the average and pointwise results as compared with the lower order QUAD-SUM method.² All of the methods tested yielded reasonably accurate pointwise values along the mesh box centers (Figure 5). However, flux shapes generated by the QUAD-SUM and CUBE-SUM techniques along traverses that are away from the mesh box center were generally poor approximations of the VENTURE shape. For instance in Figure 6, the CUBE-SUM results both fail to match the external boundary conditions and exhibit large discontinuities on the mesh box surfaces.

The results of the Lagrangian Biquadratic method exhibit an increased capability for the reproduction of a nonseparable flux shape within a mesh box. In mesh box 2, the biquadratic solution approximates the flux change from a predominately odd function of position along the near central traverse (Figure 5), to nearly

an even function along the upper traverse (Figure 6). The biquadratic solution is continuous at all points, including the mesh box surfaces, and matches the external boundary conditions. However, the quadratic expansion is of insufficient order to accurately reproduce the magnitude of the flux in mesh box 2 (Figure 6).

Application of the CUBE-SUM interpolation technique yielded the most accurate average and pointwise results of the methods tested. The discontinuities characteristic of the CUBE-SUM results are largely eliminated in the CUBE-SUM interpolation solution. These results illustrate the necessity to include the effects of both high order one-dimensional trial functions and some multi-dimensional trial functions in the flux expansion, if both accurate global and local parameters are to be obtained.

REFERENCES

1. J. Dorning, "Modern Coarse - Mesh Methods -- A Development of the '70's", ANS Topl. Mtg. Computational Methods in Nuclear Engineering, Williamsburg, Virginia, 1979.
2. M. A. Robinson, "A Critical Analysis and Extension of the Nodal Expansion Method for the Spatial Discretization of the Neutron Diffusion Equation", PhD Dissertation, Dept. of Nucl. Eng. and Eng. Phy., University of Virginia (January 1982).
3. S. Langenbuch, W. Mauer, and W. Werner, Nucl. Sci. Eng., 63, 437 (1977).
4. S. Langenbuch, W. Mauer, and W. Werner, Nucl. Sci. Eng., 64, 508 (1977).
5. H. Finnemann, Proc. Joint NEACRP/CSNI Specialists' Mtg. New Developments in Three-Dimensional Neutron Kinetics, MRR145, 145 (1975).
6. H. Finnemann, F. Bennewitz, and M. R. Wagner, Atomkernenergie, 30, 123 (1977).
7. O. C. Zienkiewicz, The Finite Element Method, 3rd Ed., Maidenhead, Berkshire, England: McGraw-Hill Book Co., 1977.
8. K. Koebke and M. R. Wagner, Atomkernenergie, 30, 136 (1977).

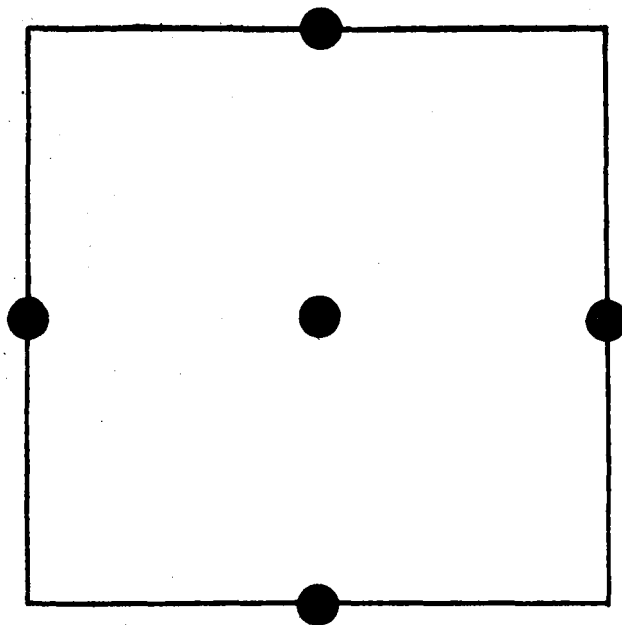


FIGURE 1A. QUAD-SUM points of support

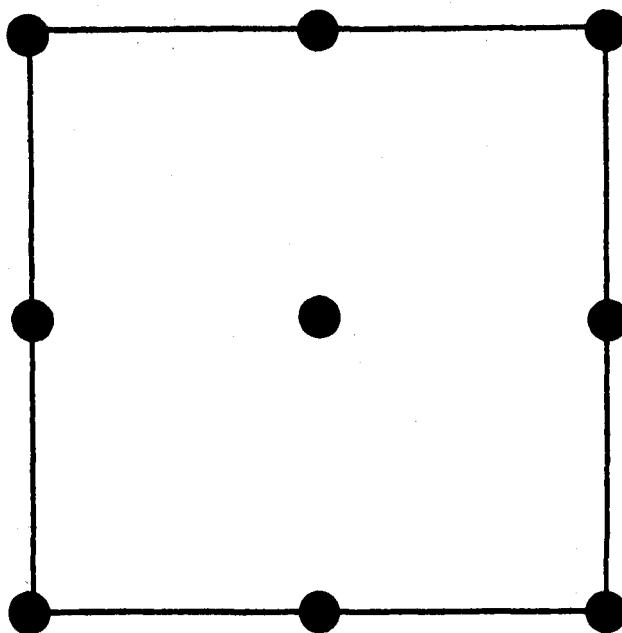


FIGURE 1B. Lagrangian Biquadratic points of support

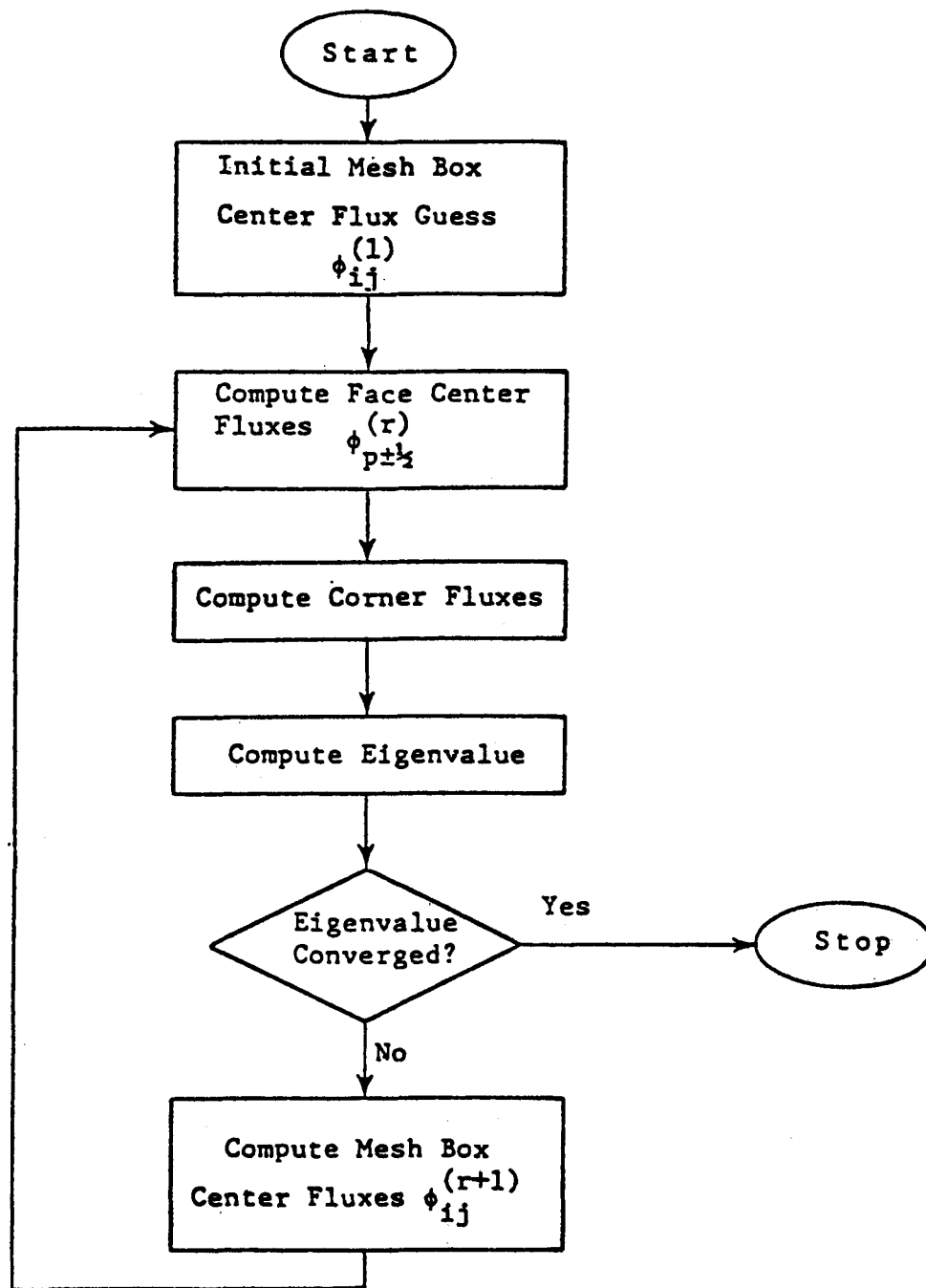


FIGURE 2. LAGRANGIAN BIQUADRATIC SOLUTION PROCEDURE

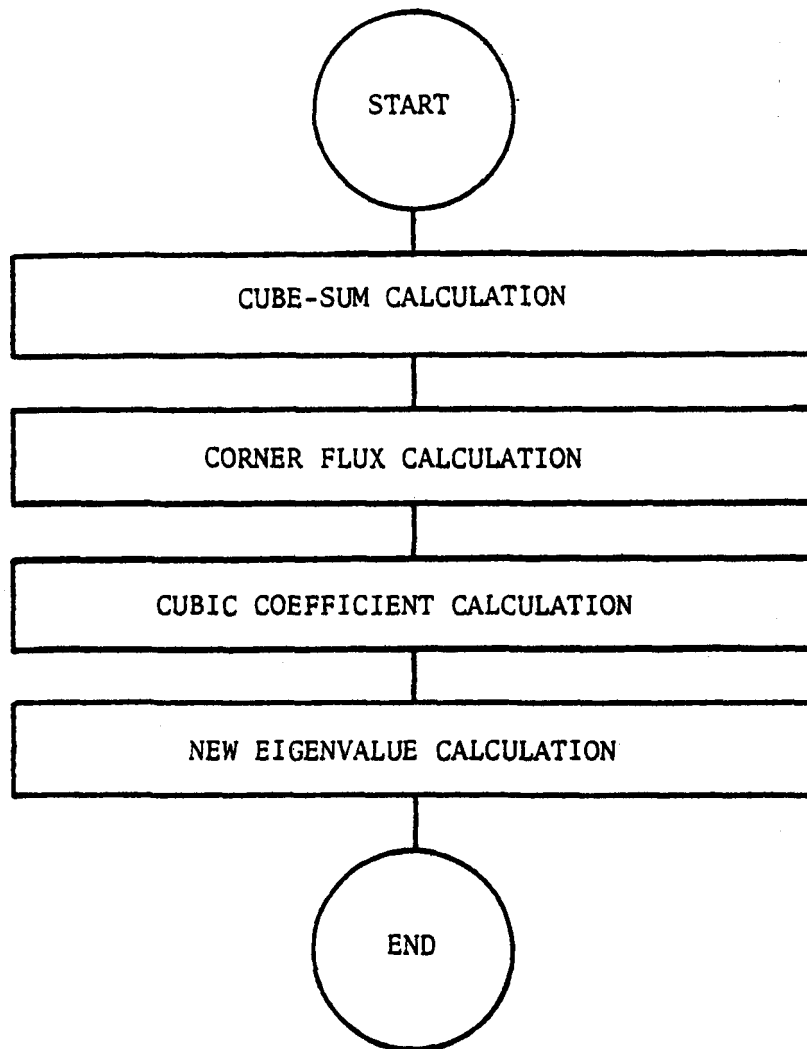
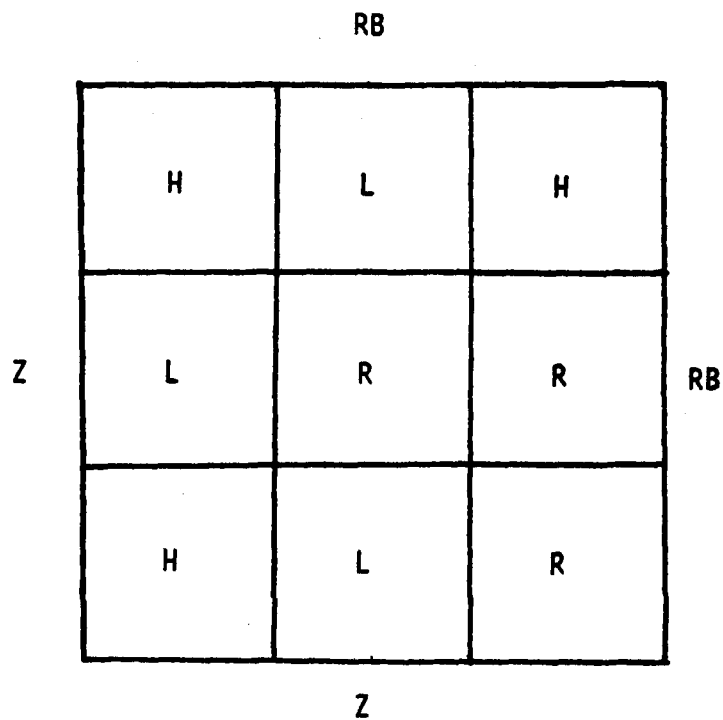
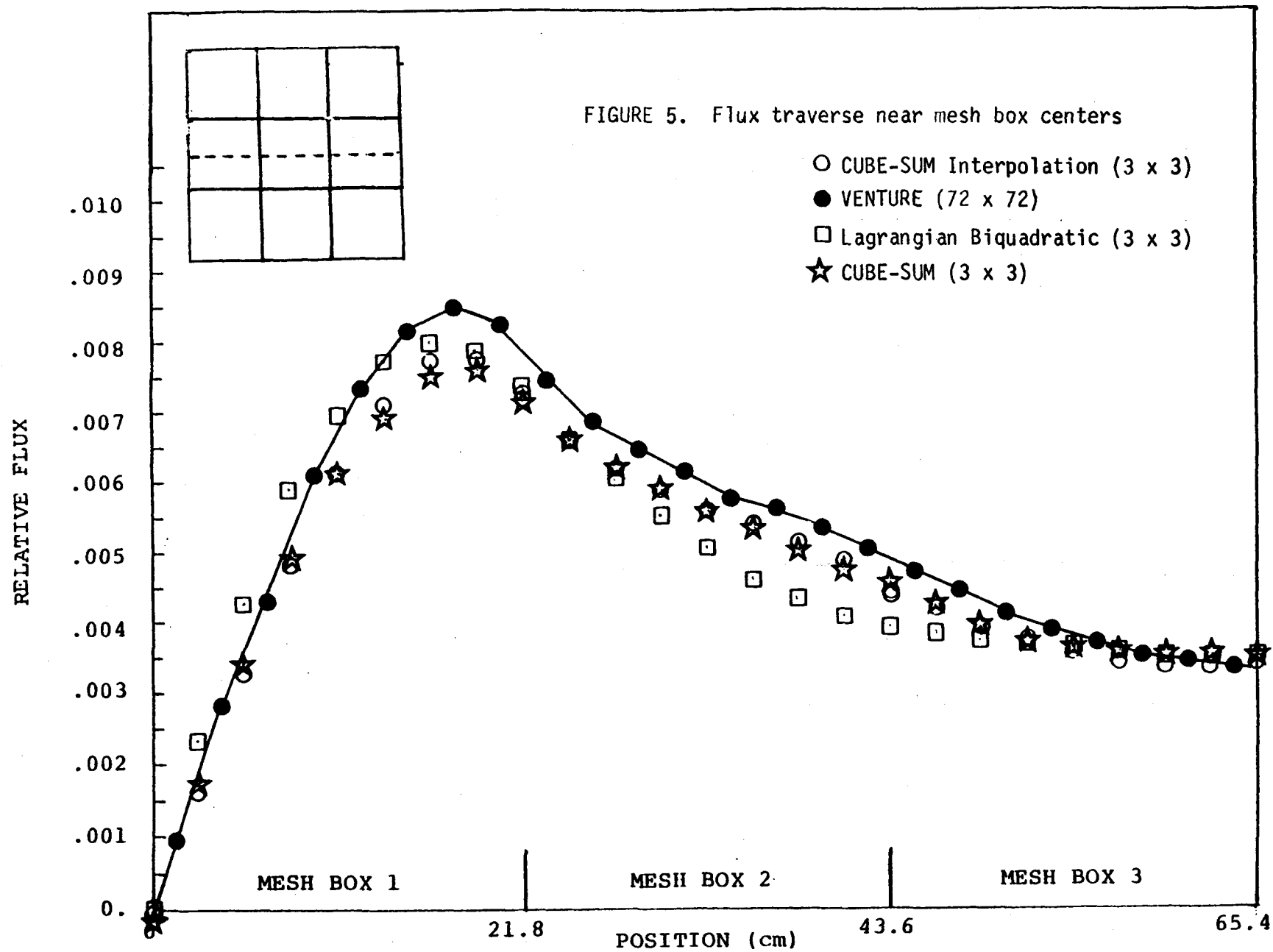


FIGURE 3. CUBE-SUM Interpolation Procedure



<u>Region</u>	<u>D(cm)</u>	<u>$\Sigma_a(\text{cm}^{-1})$</u>	<u>$v\Sigma_f(\text{cm}^{-1})$</u>
L	1.313	.0238	.0230
H	1.313	.0227	.0300
R	1.313	.0155	0.0

FIGURE 4. Model problem definition



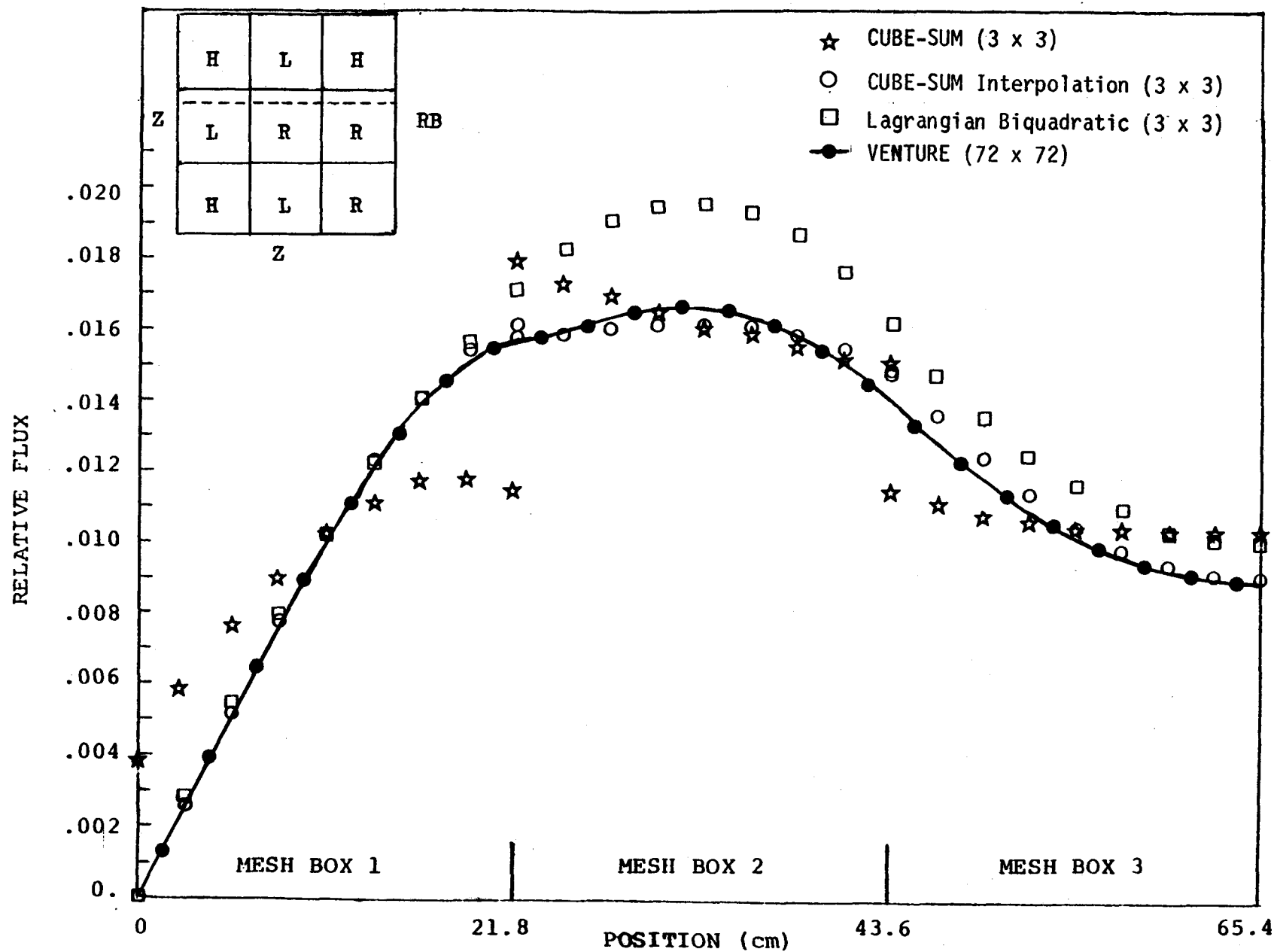


FIGURE 6. Flux plot along a traverse near mesh box edges

RESPONSE MATRIX NODAL METHODS FOR THERMAL AND FAST REACTOR APPLICATIONS

Martin Becker
Department of Nuclear Engineering
Rensselaer Polytechnic Institute

ABSTRACT

Response matrix nodal methods have been applied to both thermal and fast reactor problems. Several common nodal procedures are included as special cases. The general framework has been adaptable to modifying existing options and introducing new ones. For thermal reactors, several options give comparable, acceptable results. Current attention is being devoted to numerical efficiency. For fast reactors, results are sensitive to option used. A reflection-based coarse mesh method has been introduced for fast reactors which has been more successful than other options.

Calculation of power distributions in large power reactors, especially in three dimensions, would be expensive and often impractical if total reliance were to be placed on fine-mesh finite difference solution to the diffusion equation. Accordingly, a class of procedures called nodal methods has been introduced to handle such calculations efficiently. Nodal methods typically involve use of node (the smallest region considered) dimensions equal to an assembly side in the plane or, in the case of pressurized water reactors, to half of an assembly side.

Several nodal methods have been introduced. At RPI, a general response matrix nodal procedure has been introduced that includes individual methods introduced by others as special cases. This was done for several reasons -

- (1) Generality is useful in providing flexibility to the analyst
- (2) Generality promotes understanding of relationships among alternatives
- (3) Generality facilitates development potential through ease of introduction of new options
- (4) Generality permits users to switch methods, e.g. to be compatible with techniques used by different suppliers

This general response matrix procedure has been incorporated in the code SIMULATE,² which now is in use within the utility industry for analysis of light water reactors.

The response matrix procedure is based on considering a node in vacuum and postulating a partial current entering the node through one face u . The responses t_{uv} are then calculated for partial currents leaving through face v . The techniques of the various nodal methods are then used in optional parallel paths to compute the t_{uv} . Thus, except for the computation of the t_{uv} , the analysis is the same regardless of nodal option.

To reduce the complexity of the computation, a separability assumption is made

$$t_{uv} = (1-\rho_u) t_{rv} \quad (1)$$

where ρ , sometimes called "direct reflection," is the difference between

actual reflection and transmission through the opposite face. This type of separability leads to a direct derivation of the heuristically-obtained TRILUX model,³ whose equations are of the form

$$\left[\sum_j \frac{(1-\rho_{ji})r_{ij}}{1-\rho_{ij}\rho_{ji}} + \right] L_i = \sum_j \frac{(1-\rho_{ij})r_{ji}}{1-\rho_{ij}\rho_{ji}} L_j \quad (2)$$

where the L variable, called "leakage flux," is

$$L = t \sum_u (1-\rho_u) j_u^{\text{in}} \quad (3)$$

where j_u^{in} is inward partial current on face u. The separability assumption makes it possible to have one unknown variable per node with nearest neighbor coupling.

The relationship between generality and development potential has been found to be useful. While several options were provided for in the original code, interest developed in the polynomial expansion methods not provided for originally. Capability for incorporating such methods was found to be easy to introduce.⁴

The separability assumption has been found to be rigorous for certain nodal options and under certain conditions. Where the assumption is not rigorously correct, an allocation procedure is used to promote correspondence between the actual and separable transmission. Testing has shown that for light water reactor problems, the separability assumption is reasonable. In fast reactor problems, however, the form of Eq. 1 can lead to significant error. However, a more general separability assumption

$$t_{uv} = b_u tr_v \quad (4)$$

has been found to work well. In essence, the restriction of the form of b_u in Eq. 1 to equal $1-\rho_u$ in order to obtain the TRILUX form of coefficient is not necessary. The basic features of Eq. 2, single unknown per node and nearest neighbor coupling, are preserved, although the forms of the coefficients are modified somewhat.

Experience to date has indicated that in light water reactor application, several nodal options give acceptable results. An example with the fourth order polynomial expansion option is shown in Fig. 1. Models have been used to predict auxiliary quantities - supplemental power corrections (due to thermal neutron "transients") and albedoes. Some adjustments in albedoes have been found to be desirable, but these adjustments typically have been modest and in directions expected by more precise⁵ (but difficult to implement) albedo representations.

Some inaccuracy is associated with homogenization of heterogeneous assembly characteristics to get nodal parameters. This is particularly true for boiling water reactors. Homogenization, for example, typically is based on assembly calculations with zero gradient boundary conditions. In regions with strong gradients the relative flux weighting of discrete zones such as control blades may not correspond to the actual situation. Indeed, there may be more incentive at this point to improve the procedures for obtaining the nodal parameters than to improve the nodal coupling models to improve accuracy

relative to fine mesh homogeneous tests such as the IAEA Benchmark Problem of Fig. 1.

At the present time, effort is in progress to reduce the numerical effort associated with the response matrix nodal procedure. Because of storage considerations and the two-dimensional character of some parameters in Eq. 2, a large number of arithmetic operations typically is associated with Eq. 2 in SIMULATE. A further separability assumption is postulated

$$\frac{(1-\rho_{ji})r_{ij}}{1-\rho_{ij}\rho_{ji}} = P_i(h_{ij}) Q_j \quad (5)$$

where h_{ij} is node dimension. This assumption makes it possible to change variables and obtain an equation of the form

$$\psi_i = \frac{A_i}{\lambda} \left[\sum_{j=1}^4 \psi_j + \sum_{j=5}^6 R_j \psi_j \right] \quad (6)$$

The R_j is the ratio of the P_j evaluated for axial and planar node dimensions. For some nodal options R_j is a single number (i.e. is not node-dependent). Eq. 6 reduces considerably both the number of arithmetic operations and the number of arrays in storage. The assumption of Eq. 5 is satisfied exactly for one option, and for other common options involves only the commonly used assumption⁶

$$\frac{2D_i D_j}{D_i + D_j} \approx \sqrt{D_i} \sqrt{D_j} \quad (7)$$

Initial testing of this change of variables has shown encouraging results.

For application to fast reactors, the response matrix nodal procedure has been extended to multigroup form. Analogous to Eq. 4, a separability assumption is made of the form

$$t_{uv}^{g' \rightarrow g} = b_u^{g' \rightarrow g'} t^{g' \rightarrow g} r_v^g \quad (8)$$

By introducing an additional parameter

$$c^{g' \rightarrow g} = \frac{t^{g' \rightarrow g}}{t^{g' \rightarrow g'}} \quad (9)$$

it is possible to obtain⁷ equations analogous to Eq. 2, with a group leakage flux defined by

$$L^g = t^{g \rightarrow g} \sum_u b_u^{g \rightarrow g} j_u^{g, in} \quad (10)$$

The various nodal options that were used in SIMULATE for study of light water reactor problems have thereby been applied to fast reactor problems.

Experience to date has demonstrated greater sensitivity of accuracy to particular nodal options in fast reactor cases than in thermal reactor cases. This greater sensitivity is attributable to greater differences in characteristics of assemblies in fast reactors than in thermal reactors. Fast reac-

actors include blanket and control assemblies in addition to fuel assemblies. Fig. 2 shows a comparison for a particular test problem.

The greater sensitivity to method for fast reactors may be illustrated in mathematical terms. The modified coarse mesh diffusion option (MCMDDT) involves evaluation of average flux in a node according to (for rectangular geometry)

$$\bar{\phi}_i = a \phi_i + \frac{1-a}{6} \sum_j \phi_{ij} \quad (11)$$

where ϕ_{ij} is the flux at the interface of nodes i and j . For ordinary coarse mesh diffusion theory (CMDT), the parameter "a" is unity. For MCMDDT, "a" was selected empirically⁶ to be .3. In one group analysis, the "t" parameters in SIMULATE for CMDT and MCMDDT differ by a term dependent² on $(k_{\infty}-1)$ as well as on "a". In light water reactors, the variation of k_{∞} (infinite medium multiplication factor) among assemblies is relatively modest. In fast reactors, however, k_{∞} is near one in fuel assemblies, and is near zero in non-fuel assemblies. The mathematical differences among models lead to greater sensitivity in fast reactor problems.

Because of the sensitivity to "a," some empirical study was performed to see if improved results could be obtained. It was found⁸ that best results were obtained for a variety of problems using negative values of "a". Theoretical analysis has led to an algorithm for calculating "a". This algorithm is based on preserving reflection from the node, and the procedure is referred to as reflection-based coarse mesh theory (RBCMT). The superior result in Fig. 2 using RBCMT has been observed consistently over the range of test problems considered.

The procedure for calculating the "a" parameter in RBCMT is straightforward in rectangular geometry. Fast breeder reactor assemblies, in designs considered to date, are hexagonal. A hexagonal geometry response matrix nodal test code has been written. At this time, however, the procedure for calculating "a" does not apply directly to hexagonal geometry. As an interim measure, "a" parameters are being calculated based on equal-area square cross sections. Again, results using the RBCMT values have been found to yield superior results. An explicit algorithm for hexagonal geometry would be desirable.

In summary, response matrix nodal methods have been found to constitute a flexible and accurate approach to analyzing problems of both thermal and fast reactors. The capability to introduce new nodal options readily and the capability to modify existing options easily have been important in making the approach adaptable to new situations.

REFERENCES

1. A. Ancona, M. Becker, M. Beg, D. Harris, A. DaC. Menezes, D. M. VerPlanck, E. Pilat, Nucl. Sci. Eng. 64, 405 (1977).
2. D. M. VerPlanck, "Manual for the Reactor Analysis Program SIMULATE," Yankee Atomic Electric Report YAEC-1158, (1978).
3. L. Goldstein, F. Nakache, A. Veras, Trans. Am. Nucl. Soc. 10, 300 (1967).
4. C. Park, J. K. Park, M. Becker, Trans. Am. Nucl. Soc. 33, 959 (1981).
5. S. Bokharee, M. Becker, Trans. Am. Nucl. Soc. 26, (1978).

6. S. Borresen, Nucl. Sci. Eng. 44, 37 (1971).
7. A. DaC. Menezes, M. Becker, Trans. Am. Nucl. Soc. 33, 781 (1979).
8. J. Anaf, M. Becker, Trans. Am. Nucl. Soc. 39, 890 (1981).

0.7432 0.7447 -0.2016	SIMULATE POWER REFERENCE POWER % DIFFERENCE				
1.2957 1.3042 -0.6512	1.4319 1.4301 0.1243				K _{eff} (Ref) : 1.0296 K _{eff} (SIM) : 1.0309 Power % SD: 0.42 Node Size: 10x10x20cm ³
1.4390 1.4491 -0.6991	1.4683 1.4755 -0.4876	1.4590 1.4657 -0.4561			
1.1984 1.2065 -0.6708	1.3143 1.3114 0.2201	1.3382 1.3425 -0.3169	1.1945 1.1912 0.2766		
0.6117 0.6102 0.2412	1.0647 1.0670 -0.2175	1.1752 1.1780 -0.2393	0.9630 0.9664 -0.3539	0.4767 0.4729 0.8082	
0.9339 0.9329 0.1058	1.0427 1.0349 0.7514	1.0740 1.0709 0.2866	0.9168 0.9087 0.8884	0.6892 0.6891 0.0100	0.5958 0.5942 0.2698
0.9310 0.9327 -0.1872	0.9515 0.9496 0.1989	0.9784 0.9767 0.1721	0.8515 0.8533 -0.2059	0.6076 0.6073 0.0413	
0.7549 0.7520 0.3906	0.7387 0.7340 0.6343	0.6973 0.6957 0.2318			

Fig.1 2-D IAEA Benchmark Problem, Comparison of 1/8 Core Relative Powers Calculated by Fine Mesh Diffusion vs. Fourth Order Polynomial Option in SIMULATE.

CONFIGURATION OF PROBLEM

REFLECTING BOUNDARY CONDITON

NODE SIZE - (10.CM X 10.CM X 10.CM)

F - FUEL MATERIAL

B - BLANKET MATERIAL

C - CONTROL MATERIAL

```

F F F F   F F F F   F F F F   F F F F
F F F F   F F F F   F F F F   F F F F
F F F F   F F F F   F F F F   F F F F
F F F F   F F F F   F F F F   F F F F

C C C C   B B B B   B B B B   F F F F
C C C C   B B B B   B B B B   F F F F
C C C C   B B B B   B B B B   F F F F
C C C C   B B B B   B B B B   F F F F

F F F F   F F F F   B B B B   F F F F
F F F F   F F F F   B B B B   F F F F
F F F F   F F F F   B B B B   F F F F
F F F F   F F F F   B B B B   F F F F

B B B B   F F F F   C C C C   F F F F
B B B B   F F F F   C C C C   F F F F
B B B B   F F F F   C C C C   F F F F
B B B B   F F F F   C C C C   F F F F
    
```

OF GROUPS = 4

K EFFECTIVE

METHOD	KEFF	% ERROR
2DB	1.08810	REF.
CMDT	1.12132	3.05290
MCMDT	1.10916	1.93548
NEM2	1.10956	1.97220
NEM4	1.10318	1.38578
RBCMT	1.08896	0.07896

Fig. 2 Comparison of Results for a Fast Reactor Problem

A NODAL METHOD FOR DIRECT INTRA-NODE DISTRIBUTION CALCULATION

Yung-An Chao and John A. Penkrot*
Carnegie-Mellon University
Nuclear Science and Engineering Division
Pittsburgh, PA 15213

ABSTRACT

A nodal method is developed which calculates intra-node flux distribution directly but solves only nodal equations of conventional structure and simplicity. For a given homogeneous node, the intra-node flux is rigorously and analytically expressed as a sole surface integral of the complete multi-group and multi-dimensional Green's function and the surface fluxes. Assuming parabolic surface flux profiles, nodal equations involving surface fluxes as the only unknowns are derived. Each surface profile is expressed in terms of the flux values at the corners of a surface and the averaged flux value on that surface. For a 2-D (or 3-D) node, there are all together three (or four) unknowns; they are one corner flux and two (or three) surface averaged fluxes. Once these corner and surface averaged fluxes are solved from the nodal equations, the parabolic surface profile is known and the intra-node flux is immediately obtained.

BASIC IDEAS

Recently nodal methods have made some significant progress, and several modern nodal codes are capable of making impressively accurate calculations of eigen values and node-averaged fluxes.⁽¹⁻³⁾ Aside from the question of effectively homogenizing each node, which is taken for granted in nodal methods, there still remains, however, the drawback of how to calculate intra-node flux distributions to the same accuracy, even for homogeneous nodes. Here we suggest a different view to nodal methods and develop a new formulation capable of direct intra-node distribution calculation.

The fact that such coarse-mesh as one per assembly can provide so accurate results on eigen value and node-averaged fluxes is not really surprising. For a homogeneous medium, the exact analytic solution to the diffusion equation is readily available in terms of boundary values. Thus it should not be necessary to rely on fine meshes over a homogeneous assembly to obtain the flux in that assembly. The real problem is therefore not how to solve the diffusion equation within each homogeneous assembly, but how to interface the fluxes valid in the homogeneous assemblies. A diffusion solution in an assembly can be uniquely determined by either specifying the boundary fluxes or the boundary currents (or a combination of the two which is however not very interesting practically). It therefore should be possible to formulate nodal equations in terms of either assembly surface fluxes or assembly currents only. Furthermore, once the surface fluxes or currents are solved from the nodal equations, the intra-node flux of each assembly should be determinable to the same accuracy. From this point of view, the accuracy and power of a nodal method depends on its capability in approximating the surface flux profiles, or surface current profiles.

Modern nodal methods share three common features. One is that they all choose surface currents and node-averaged flux as the unknowns (although according to the above analysis, the node-averaged flux itself is uniquely determinable through the surface currents.) The second is that the diffusion equation is always reduced to one dimension by integrating over other dimensions. That

*Permanent address: Westinghouse Electric Corp., P.O. Box 3912, Pittsburgh, PA 15230.

surface-averaged one-dimensional diffusion equation is then solved analytically to express surface-averaged intra-node flux in terms of the surface currents, from which relation nodal equations are derived. The last and the most important is that each surface current profile (or equivalently, as we will see, the transverse leakage) is approximated by a parabola which extends over the two neighboring surfaces as well. These three common features imply two common limitations on these methods. The use of surface-averaged one-dimensional diffusion equation restrains any direct calculation via the methods to only surface-averaged quantities. And the use of surface currents hinders the approximation to surface profiles in two ways. Each of the corner lines of a cubic node is singular in current continuity and there is really no theoretical basis for assuming the surface current profile continuous across the corner lines. The approximation of one parabola extending over three adjacent surfaces also strains the flexibility of shaping the parabola to fit the actual profile over the middle surface. To alleviate the "clamping" on the profile by the two adjacent surfaces would increase significantly the number of unknown parameters expressing each surface profile and hence complicates the nodal equations. In Ref.(2), it is pointed out that the commonly adopted parabolic approximation to transverse leakage profile (or equivalently, the surface current profile) is not adequate and gives large fitting errors. However since the nodal equation deals with only surface averaged quantities, these large errors in surface profile fit are not reflected in the calculation of eigen values and node averaged fluxes because the surface averaged currents are still adequately approximated. But if one would try to uncover intra-node fluxes by solving diffusion equation for each homogeneous node using these poorly approximated surface current profiles as boundary values, one would expect to find sizable errors.

Since we view the merit of a nodal method as by-passing the unnecessary intra-node meshing and "sewing up" directly at interfaces the trivially obtainable analytic solutions of diffusion equation for each homogeneous node, we might as well take the full advantage of it to directly calculate the intra-node flux which is already implicitly dealt with in the derivation of the nodal equations. To this end we suggest two revisions of the conventional nodal method. One is that to derive nodal equations we do not have to first reduce the diffusion equation to one-dimension and then solve it. Instead we can solve the multi-dimensional homogeneous problem in terms of surface quantities and then derive nodal equations for the surface quantities. Once the surface quantities are solved for from the nodal equations the intra-node flux is also readily obtainable. The other revision is that we use surface flux instead of surface currents to facilitate surface profile approximations. Unlike currents, flux profiles are continuous everywhere in all directions including corners of the nodes. Because node corners are not singular for flux continuity, it suffices to introduce only one corner flux per node to provide each surface profile with the freedom of adapting itself to any parabolic shape without being strained by flux profiles on neighboring surfaces. Unlike conventional nodal methods, node-averaged flux is absent in our nodal equations. Thus the total number of unknowns in our nodal equations remains the same as that in conventional ones. The nodal coupling in conventional methods extends over three nodes along each direction because their surface current profile spans over three nodes. In our method the coupling chain is shortened to the nearest neighbor coupling.

THEORY

For simplicity, we will consider the 2-G, 2-D diffusion equation for a homogeneous square assembly,

$$(\nabla^2 - [A]) |\phi\rangle = -[F] |\phi\rangle \quad (1)$$

where $[A]$ and $[F]$ are the 2x2 absorption and fission matrices divided by the diagonal diffusion coefficient matrix $[D]$, and $|\phi\rangle$ stands for the two component flux vector with the fast and thermal components ϕ_1 and ϕ_2 . In terms of a 2-G and 2-D Green's function satisfying the zero flux boundary condition, the solution of eq.(1) can be expressed as a surface integral of the Green's function and the surface flux profile,

$$|\phi\rangle = [\tilde{G}_1] \int |\phi\rangle_s \nabla \mathcal{G}(\tilde{k}_1) \cdot d\mathbf{s} + [\tilde{G}_2] \int |\phi\rangle_s \nabla \mathcal{G}(\tilde{k}_2) \cdot d\mathbf{s} \quad (2)$$

where $\tilde{k}_1^2$ and $\tilde{k}_2^2$ are the eigen values of the matrix, $[A]-[F]$, the two matrices $[\tilde{G}_1]$ and $[\tilde{G}_2]$ are defined as

$$\begin{aligned} [\tilde{G}_1] &= [\tilde{R}]^{-1} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} [\tilde{R}] \\ [\tilde{G}_2] &= [\tilde{R}]^{-1} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} [\tilde{R}], \end{aligned} \quad (3)$$

with $[\tilde{R}]$ defined as the diagonalizing matrix of $[A]-[F]$, namely,

$$[\tilde{R}] ([A] - [F]) [\tilde{R}]^{-1} = \begin{bmatrix} \tilde{k}_1^2 & 0 \\ 0 & \tilde{k}_2^2 \end{bmatrix} \quad (4)$$

and $\mathcal{G}$ is the 1-G, 2-D Green's function satisfying the zero flux boundary condition,

$$\mathcal{G}(k, x, y; x', y') = \frac{2}{a} \sum_{\text{odd } n} \sin(n\pi x/a) \sin(n\pi x'/a) G(p_n, y; y') \quad (5)$$

where a is the side of the square node, G is the 1-G, 1-D Green's function satisfying zero flux boundary condition,

$$G(p_n, y; y') = \begin{cases} \sinh p_n(y-a) \sinh p_n y' / (p_n \sinh p_n a), & y > y' \\ \sinh p_n y \sinh p_n(y'-a) / (p_n \sinh p_n a), & y < y' \end{cases} \quad (6)$$

and p_n is related to the k of eq.(5) as

$$p_n^2 = k^2 + (n\pi/a)^2 \quad (7)$$

Eq.(5) is of course symmetric in x and y , and can be re-expressed with x and y interchanged.

Applying eq.(2) to the neighboring nodes to calculate the surface currents and impose interface current continuity (flux continuity being automatically satisfied), one readily derives nodal equations involving node surface flux profiles $|\phi\rangle_s$ only. Up to this point no approximation has been made. However the current continuity equation so derived involves infinite series and moreover is supposed to hold at every point on the interface. In general this cannot be satisfied unless the input surface flux profile is exact or contains infinite number of undertermined parameters. By projecting out coefficients of the infinite series one obtains a set of algebraic nodal equations, which, to be exact, will be infinitely many and thus practically useless. Now comes the only approximation

to be made, namely, the parametrization of the profile. Depending on how many parameters for a profile, a corresponding set of finite number of nodal equations can be derived by projecting out coefficients of so many terms in the series. Since the infinite series involves factors of $\sin n\pi x/a$ or $\sin n\pi y/a$, which are orthogonal to each other, the projection is therefore done with these harmonic weightings. Each projected out nodal equation then means physically the continuity of the harmonic components of the interface current.

It is worth a side comment that the above analysis can also be done with surface current profile instead of flux profile as input. The only change will then be the replacement of $\sin n\pi x/a$ (odd n) by $\cos m\pi x/a$ (even m). If only one parameter per surface is chosen to prescribe the surface current profile, and only the lowest $m=0$ (uniform weighting) term is projected out, then the nodal equation will stand for the continuity of the usual surface-averaged flux. In fact it can be shown that if the one parameter per surface is chosen to be the surface averaged current, and the current profile is prescribed as a parabola parametrized by the averaged currents of the three adjacent surfaces, the resulting nodal equations are equivalent to those of ref.(2).

Coming back to the parametrization of the surface flux profile, we allow each surface a parabolic profile of its own. The three parameters for the parabola are two corner fluxes and the ($\sin \pi x/a$ weighted) first harmonic of the flux. For each node there are three nodal quantities to be determined--one corner flux and two first harmonics of the x - and y - surfaces, as shown in Fig.(1) for the n th node.

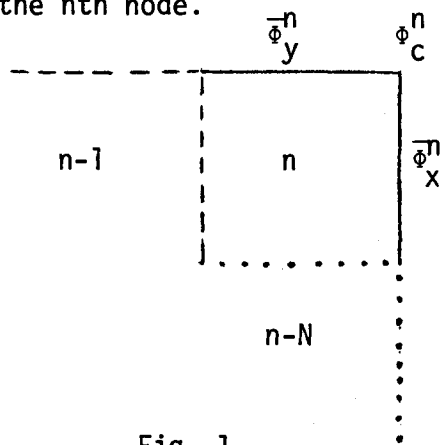


Fig. 1

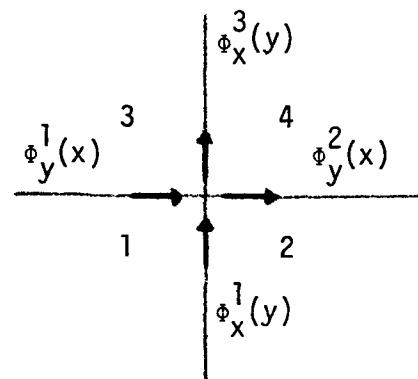


Fig. 2

We need three nodal equations to determine the three unknowns. Two of them can be obtained by projecting out the first harmonic (i.e., $\sin \pi x/a$ weighted) of surface current on the x - and y - surfaces and impose continuity on it. The other can be derived by requiring the overall current conservation at each corner as shown in Fig.(2),

$$(D_1 + D_2) \left. \frac{d\phi_x^1(y)}{dy} \right|_{y=a} + (D_1 + D_3) \left. \frac{d\phi_y^1(x)}{dx} \right|_{x=a} = (D_2 + D_4) \left. \frac{d\phi_y^2(x)}{dx} \right|_{x=0} + (D_3 + D_4) \left. \frac{d\phi_x^3(y)}{dy} \right|_{y=0} \quad (8)$$

This current conservation condition at the corner is physically complimentary to that of the continuity of the first harmonic of the surface current, as the first harmonic is weighted by $\sin \pi x/a$ which peaks at the center and vanishes at the corners.

* That the same nodal equations are derived in those two different ways indicates that the profile approximation made to the transverse leakage implies the same approximation to the surface current.

The three nodal equations can be symbolically expressed as

$$f_s(v; |\bar{\phi}_x^{n-1}\rangle, |\bar{\phi}_x^n\rangle, |\bar{\phi}_x^{n+1}\rangle; |\bar{\phi}_y^{n-N}\rangle + |\bar{\phi}_y^n\rangle, |\bar{\phi}_y^{n+1-N}\rangle + |\bar{\phi}_y^{n+1}\rangle; |\bar{\phi}_c^{n-1-N}\rangle + |\bar{\phi}_c^{n-1}\rangle, |\bar{\phi}_c^{n-N}\rangle + |\bar{\phi}_c^n\rangle, |\bar{\phi}_c^{n+1-N}\rangle + |\bar{\phi}_c^{n+1}\rangle) = 0 \quad (9a)$$

$$f_s(v; \text{same as above except for interchanging } x \text{ and } y) = 0 \quad (9b)$$

$$f_c(|\bar{\phi}_c^{n-N}\rangle, |\bar{\phi}_c^{n-1}\rangle, |\bar{\phi}_c^n\rangle, |\bar{\phi}_c^{n+1}\rangle + |\bar{\phi}_c^{n+N}\rangle; |\bar{\phi}_x^n\rangle, |\bar{\phi}_x^{n+N}\rangle; |\bar{\phi}_y^n\rangle, |\bar{\phi}_y^{n+1}\rangle) = 0 \quad (9c)$$

where f_s contains coefficient matrices that involve matrices $[\tilde{G}_1]$ and $[\tilde{G}_2]$ and eigen values $\tilde{k}_1$ and $\tilde{k}_2$, which in turn depend on the fission multiplicity ν . But f_c of eq.(9c) is independent of ν and depends only on diffusion coefficients. In all three nodal equations the nodes are coupled with nearest neighbors only. As shown in Fig.3, eq.(9a) couples nodes n and $n+1$, eq.(9b) couples nodes n and $n+N$, and eq.(9c) couples the four surfaces crossing at corner n .

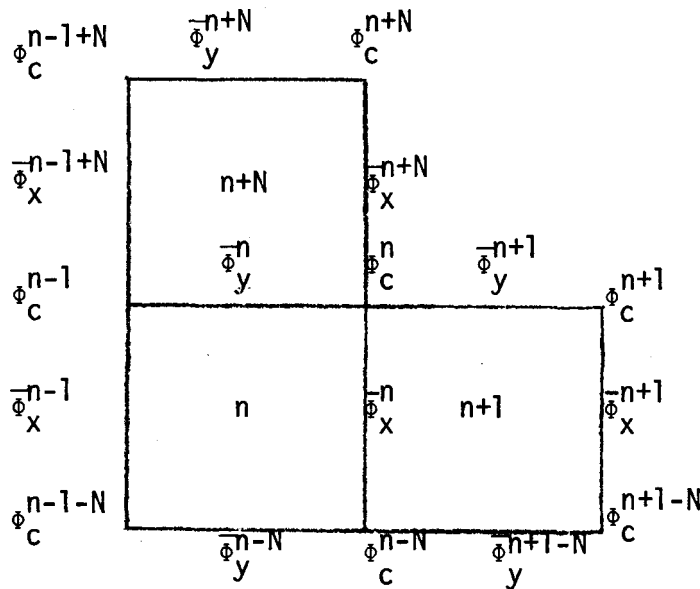


Fig. 3

The nodal equations (9a)-(9c) pose one inconvenience in criticality search, as the ν -dependence is implicit in the matrix elements and there is no explicit fission source term in the equations. To facilitate the criticality search, the following trick is used. If we go back to the original diffusion equation(1) and regard the fission source term as an inhomogeneous term, the flux can then be expressed as

$$|\phi\rangle = [\hat{G}_1] \int |\phi\rangle \nabla \mathcal{G}(\hat{k}_1) \cdot d\underline{s} + [\hat{G}_2] \int |\phi\rangle_s \nabla \mathcal{G}(\hat{k}_2) \cdot d\underline{s} - [\hat{G}_1][F] \int |\phi\rangle_s \mathcal{G}(\hat{k}_1) dvol - [\hat{G}_2][F] \int |\phi\rangle_s \mathcal{G}(\hat{k}_2) dvol \quad (10)$$

where hatted quantities are entirely analogous to the tilted ones in eq.(2). But they involve the diagonalizing matrix and eigen-values of matrix $[A]$, not $[A]-[F]$, and do not depend on the fission multiplicity ν . The hatted quantities are obviously equal to the special values of the tilted ones evaluated at $\nu=0$. It is clear that a repeat of the same analysis as presented before, using eq.(10) instead of eq.(2) will lead to the following three nodal equations as compared to those of eqs.(9a)-(9c),

$$f_S(\nu=0; \text{ same as eq.(9a)}) = \text{some volume integral proportional to } [F] \quad (11a)$$

$$f_S(\nu=0; \text{ same as eq.(9b)}) = \text{some volume integral proportional to } [F] \quad (11b)$$

$$f_C(\text{same as eq.(9c)}) = 0 \quad (11c)$$

where the additional term on the right hand side originates from the (inhomogeneous) fission source term of eq.(10). A comparison of eq.(11) against eq.(9) suggests that eq.(11) can be rewritten as (in short-hand notations)

$$f_S(\nu=0; \phi's) = f_S(\nu=0; \phi's) - f_S(\nu; \phi's) \quad (12)$$

with the right hand side identified as a variant of fission source term. The criticality search can then be performed by introducing an eigen-value $\gamma(\nu)$ to eq.(12) such that

$$f_S(0; \phi's) = \frac{1}{\gamma(\nu)} (f_S(0; \phi's) - f_S(\nu; \phi's)) \quad (13)$$

As ν hits the critical value $\nu_{0,\gamma}$ ($\nu_0=1$, otherwise $\gamma(\nu) \geq 1$ for $\nu \geq \nu_0$ respectively). The criticality search can therefore be performed by determining γ and updating ν iteratively until γ equals one.

We see that the nodal equations themselves are of similar mathematical structure and simplicity as the conventional methods. Eventhough we started with the complete 2-G, 2-D Green's function involving infinite series, nevertheless the infinite series does not appear in the nodal equations. This is due to the particular simplicity made in approximating the surface profiles and the use of orthogonality of the harmonics in the series. Once the surface quantities $\bar{\phi}_x$, $\bar{\phi}_y$ and ϕ_C are obtained from the nodal equations, we can easily construct the parabolic profiles of $|\phi>_S$ and go back to eq.(2) to evaluate in principle the flux at any point inside a node. However this time the infinite series does appear in the calculation. How fast the series converges depends on how far away the point in question is from the boundary of the node. If it is not too close to the boundary, the convergence is fast and a few terms suffice. On the other hand, we do know the flux on the node boundary, that it is not necessary to compute the flux in the very close neighborhood of the node boundary.

PRELIMINARY TESTS

A prototype 2-G, 2-D nodal code based on the theory presented is under development. Some tests of principle have been made with extremely encouraging results. They are briefly outlined as follows.

- 1) Because of the minimal number of unknowns involved and the nearest neighbor nodal coupling, the code translating the simple nodal equations turns out to be very compact in structure.
- 2) A simple example of 10 by 10 identical PWR assemblies of sides of about 21 cm with zero flux boundary condition is taken as a benchmark problem for the proof of principle tests. For this simple example closed-form analytic solution is available for detailed comparisons.

- 3) Without any kind of convergence acceleration techniques employed, the iterations already converge very well (typically in about 10 sec. on PDP10). There does not seem to be any difficulty of convergence.
- 4) The critical eigen value of the inverse of fission neutron multiplicity calculated is 1.32628 versus the exact value of the analytical solution 1.32633. The errors in $\bar{\phi}_x$, $\bar{\phi}_y$ and ϕ_c is of the order of 0.1%.
- 5) To a truncation error of less than 1%, the intra-node computation can be done with only the first one or two terms of the infinite series for points near the center of a node, and with three to four terms for the rest of the node except for the fuel cells right next to the boundary (the assembly considered is of 17x17 fuel cells). This test calculation is done independently of the benchmark problem of 10x10 assemblies. The conclusion holds for the general case of a node with parabolic flux profiles on the four sides of the node.
- 6) Another test done is to compare the accuracy in the surface flux profile approximation we suggested to that in the surface current (or transverse leakage) profile approximation adopted in other modern nodal methods. We take the analytic solution to the problem of a 5x5 assembly with zero flux boundary condition and calculate for each surface segment the exact flux profile, exact current profile, the exact first harmonic of the flux $\bar{\phi}$ and the corner fluxes ϕ_c , and the exact surface average current $\bar{J}$. We then use $\bar{\phi}$ and ϕ_c to construct the approximate parabolic flux profile, and use $\bar{J}$ of three adjacent surfaces to construct the approximate current profile as done in Ref.(1)-(3). The approximate profiles are then compared against the exact ones from the analytic solution. What we found is that the maximum error in the approximated surface flux profile is about .005% near the center of the core and .4% near the boundary, while the maximum error in the approximated surface current profile varies from .1% near the center of the core to .50% at the boundary of the core. Although this is an oversimplified problem, it does demonstrate the point made in the first section of this paper.

This work is supported in part by Westinghouse Electric Corporation. We appreciate very much the helpful discussions with Dr. Y. S. Liu.

REFERENCES

- (1) H. Finnemann, F. Bennewitz and M. Wagner, Atomkernenergie 30, 123, 1977; M. Wagner, H. Finnemann, K. Koebe and H. J. Winter, Atomkernenergie 30, 129, 1977.
- (2) K. S. Smith, "An Analytic Nodal Method for Solving the Two Group, Multi-dimensional, Static and Transient Neutron Diffusion Equations," MIT, Thesis 1979.
- (3) R. D. Lawrence and J. J. Dorning, Nucl. Sci. Eng. 76, 218, 1980.

A LATTICE HOMOGENIZATION THEORY FOR COARSE MESH DIFFUSION ANALYSIS

Ren-Tai Chiang

General Electric Company
175 Curtner Avenue
San Jose, California 95125

ABSTRACT

A lattice homogenization theory is developed for coarse mesh diffusion analysis in a large heterogeneous system composed of two-dimensional, exactly periodic or near-periodic fuel bundles. The development systematically leads to the separation of the solution to the heterogeneous reactor diffusion equation into a "local shape function" (the fundamental unit-bundle diffusion theory eigenfunction) and a "global shape function" (the fundamental solution to the bundle-homogenized reactor diffusion equation). The eigenvalue of the homogenized reactor diffusion equation is shown to preserve the eigenvalue of the heterogeneous reactor diffusion equation. The bilinear-weighted homogenized diffusion coefficient and cross-sections are derived in terms of forward and adjoint solutions to the unit-bundle diffusion equation. Discontinuous interface conditions are used in determination of the solution to the homogenized diffusion equation for large systems composed of near-periodic bundles. Numerical results are given for several fictitious light-water slab reactors composed of exactly periodic or near-periodic fuel bundles.

INTRODUCTION

The nuclear design and analysis of contemporary light water power reactors are normally achieved by use of approximate methods except that the exact but costly Monte Carlo neutron transport method is occasionally used for

benchmark purpose. The approximate methods contain a series of procedures that solve the neutron balance equations in the three structural systems (fuel cell, fuel bundle, and whole core) of a light-water power reactor. The neutron transport theory methods are employed to calculate the fine-group (on the order of 100) cell fluxes and the cell-homogenized broad-group (usually three: fast, epithermal, and thermal) cross-sections. The two-dimensional neutron diffusion theory methods are used to calculate the broad-group bundle fluxes and the bundle-homogenized cross-sections. The three-dimensional neutron diffusion theory methods are used to calculate the core flux and power distribution. In this sequence of calculations, homogenization serves as a "bridge" and the accuracy of the final results depends on the correctness of the homogenization calculations.

Cell homogenization theories¹⁻¹¹ have been developed during the past score by utilizing buckling or asymptotic expansion methods for large heterogeneous systems composed of exactly periodic or near-periodic fuel cells. The present development represents the logical extension of five earlier works^{5,6,8,10,11} in these cell homogenization theories. It extends the development of cell-homogenized diffusion theory to the development of bundle-homogenized diffusion theory.

We consider a large heterogeneous system (such as a light-water power reactor) that is composed of two-dimensional, near-periodic fuel bundles. Our development accommodates slow global spatial variation (due to non-uniform fuel loading and burnup) in addition to rapid periodic lattice variation of the nuclear properties among the bundles. The development systematically leads to, rather than assumes as in Reference 12, the separation of the solution to the heterogeneous reactor diffusion equation into a "local shape function" (which is the fundamental unit-bundle diffusion theory eigenfunction) and a "global shape function" (which is the fundamental solution to the bundle-homogenized reactor diffusion equation). The development also shows that the eigenvalue of the homogenized reactor diffusion equation preserves the eigenvalue of the heterogeneous reactor diffusion equation, and that the homogenized diffusion coefficient and cross-sections are bilinear-weighted in terms of forward and adjoint solutions of the unit-bundle diffusion equation. These bilinear-weighted homogenized parameters are different from the flux-weighted homogenized diffusion coefficient and cross-sections commonly being utilized in thermal power reactor analysis.¹³ Finally, for simplicity but without loss of generality, numerical results are given for several fictitious light-water slab reactors composed of one-dimensional, exactly periodic or near-periodic fuel bundles.

THEORY

To carry out the development, we consider a large, three-dimensional, heterogeneous system composed of two-dimensional, near-periodic, square fuel bundles. The bundle pitch p is on the order of a typical removal mean-free-path $\bar{\lambda}$, which is small compared to the characteristic system dimension R ; hence, the ratio $\epsilon \equiv \bar{\lambda}/R$ is a small parameter. Two spatial-length scales are introduced: one ($\xi_i \equiv x_i/\bar{\lambda}$ $i = 1, 2$) corresponds to local spatial variations within a unit bundle, and the other ($\underline{\xi} \equiv x/R$) accounts for less rapid, global spatial variations. These are formally treated as independent variables, and are introduced into the multigroup diffusion equation along with the dimensionless variables

$$\underline{D}^{\underline{\xi}}(\xi_1, \xi_2) = \underline{D}(\underline{x})/\bar{\lambda}, \quad \underline{\sigma}_{ss}^{\underline{\xi}}(\xi_1, \xi_2) = \bar{\lambda} \underline{\sigma}_{ss}(\underline{x}), \quad \underline{\sigma}_R^{\underline{\xi}}(\xi_1, \xi_2) = \bar{\lambda} \underline{\sigma}_R(\underline{x}), \text{ and}$$

$$|\nu \sigma_f^{\underline{\xi}}(\xi_1, \xi_2)\rangle = \bar{\lambda} |\nu \sigma_f(\underline{x})\rangle \text{ to obtain}$$

$$\begin{aligned} \frac{\partial}{\partial \xi_1} \underline{D}^{\underline{\xi}}(\xi_1, \xi_2) \frac{\partial}{\partial \xi_1} |\phi(\xi_1, \xi_2, \underline{\xi})\rangle + \frac{\partial}{\partial \xi_2} \underline{D}^{\underline{\xi}} \frac{\partial}{\partial \xi_2} |\phi\rangle + \epsilon^2 \nabla_{\underline{\xi}} \cdot \underline{D}^{\underline{\xi}} \nabla_{\underline{\xi}} |\phi\rangle - \underline{\sigma}_R^{\underline{\xi}}(\xi_1, \xi_2) |\phi\rangle \\ + \{ \underline{\sigma}_{ss}^{\underline{\xi}}(\xi_1, \xi_2) + \frac{1}{k_{\text{eff}}} |\chi\rangle \langle \nu \sigma_f^{\underline{\xi}}(\xi_1, \xi_2)| \} |\phi\rangle = 0. \end{aligned} \quad (1)$$

Here, $|\phi(\xi_1, \xi_2, \underline{\xi})\rangle$ is the column vector of group fluxes, $\underline{D}^{\underline{\xi}}$ is the matrix of group diffusion coefficients, $\underline{\sigma}_R^{\underline{\xi}}$ is the matrix of group removal cross-sections, $\underline{\sigma}_{ss}^{\underline{\xi}}$ is the matrix of inter-group scattering cross-sections, $|\chi\rangle$ is the column vector of group fission fractions, $\langle \nu \sigma_f^{\underline{\xi}}|$ is the row vector of group production cross-sections, and the superscript $\underline{\xi}$ represents the parametric dependence upon global variation. The near-periodic spatial dependence of the nuclear parameters leads to the properties that these parameters are periodic in ξ_1 or ξ_2 for each $\underline{\xi}$, for example,

$$\begin{aligned} \underline{D}^{\underline{\xi}}(\xi_1 + \tilde{p}, \xi_2) &= \underline{D}^{\underline{\xi}}(\xi_1, \xi_2) \\ \underline{D}^{\underline{\xi}}(\xi_1, \xi_2 + \tilde{p}) &= \underline{D}^{\underline{\xi}}(\xi_1, \xi_2) \end{aligned}$$

where $\tilde{p} (=p/\bar{\lambda})$ is the dimensionless bundle pitch.

We expand $|\phi(\xi_1, \xi_2, \underline{\xi})\rangle$ and $1/k_{\text{eff}}$ in powers of ϵ :

$$\begin{aligned} |\phi(\xi_1, \xi_2, \underline{\xi})\rangle &= \sum_{n=0}^{\infty} \epsilon^n |\phi_n(\xi_1, \xi_2, \underline{\xi})\rangle \\ 1/k_{\text{eff}} &= \sum_{n=0}^{\infty} \epsilon^n \frac{1}{k_n}. \end{aligned}$$

Substituting them into Equation (1) and setting the coefficients of the various powers of ϵ equal to zero leads to the singular perturbation equations. Here, perturbation means as if an infinite system were perturbed

and reduced to a large finite system.

The ϵ^0 - equation is

$$\begin{aligned} F | \phi_0 (\xi_1, \xi_2, \underline{\zeta}) > = \frac{\partial}{\partial \xi_1} \underline{D}^{\underline{\zeta}} \frac{\partial}{\partial \xi_1} | \phi_0 > + \frac{\partial}{\partial \xi_2} \underline{D}^{\underline{\zeta}} \frac{\partial}{\partial \xi_2} | \phi_0 > \\ - \frac{\sigma_{\underline{\zeta}}}{R} | \phi_0 > + \{ \underline{\sigma}_{ss} + \frac{1}{k_0} | \chi > < \nu \sigma_f^{\underline{\zeta}} | \} | \phi_0 > = 0, \end{aligned} \quad (2)$$

with periodic (bundle) boundary conditions. Physically, Equation (2) is the diffusion equation in a fictitious, exactly periodic, infinite lattice with bundle nuclear properties depending parametrically upon the slow variation of the diffusion coefficients and cross-sections as indicated by $\underline{\zeta}$. The general solution for $| \phi_0 >$ has the form:

$$| \phi_0 (\xi_1, \xi_2, \underline{\zeta}) > = A_0 (\underline{\zeta}) | \psi_0^{\underline{\zeta}} (\xi_1, \xi_2) >, \quad (3)$$

where $| \psi_0^{\underline{\zeta}} >$ is the fundamental eigenvector of the unit-bundle diffusion equation [Equation (2)] corresponding to the fundamental eigenvalue $k_{0,0}^{\underline{\zeta}}$. Physically, $k_{0,0}^{\underline{\zeta}}$ is just the fundamental bundle eigenvalue $k_{\infty,0}^{\text{bundle}}$ of the fictitious, parametrically dependent, piece-wise exactly periodic, infinite lattice. The other higher mode eigenvectors generally contain negative or imaginary components, and therefore are not physical solutions to Equation (2). The coefficient $A_0 (\underline{\zeta})$ is yet to be determined.

The adjoint equation to Equation (2) is

$$\begin{aligned} \frac{\partial}{\partial \xi_1} \{ \frac{\partial}{\partial \xi_1} < \psi_0^{\dagger, \underline{\zeta}} (\xi_1, \xi_2) | \} \underline{D}^{\underline{\zeta}} + \frac{\partial}{\partial \xi_2} \{ \frac{\partial}{\partial \xi_2} < \psi_0^{\dagger, \underline{\zeta}} | \} \underline{D}^{\underline{\zeta}} \\ - < \psi_0^{\dagger, \underline{\zeta}} | \frac{\sigma_{\underline{\zeta}}}{R} + < \psi_0^{\dagger, \underline{\zeta}} | \{ \underline{\sigma}_{ss} + \frac{1}{k_{0,0}^{\underline{\zeta}}} | \chi > < \nu \sigma_f^{\underline{\zeta}} | \} = 0, \end{aligned} \quad (4)$$

with periodic (bundle) boundary conditions. Here, the row vector $< \psi_0^{\dagger, \underline{\zeta}} |$ is the fundamental eigenvector of the adjoint equation.

The ϵ^1 - equation is

$$F | \phi_1 (\xi_1, \xi_2, \underline{\zeta}) > = - \frac{1}{k_1} | \chi > < \nu \sigma_f^{\underline{\zeta}} | \phi_0 >. \quad (5)$$

Since Equation (5) is an inhomogeneous equation to Equation (2) with the same fundamental eigenvalue, the Fredholm alternative theorem indicates that Equation (5) has a solution if and only if the source term is orthogonal to the solution of the equation which is adjoint to Equation (2) [i.e., Equation (4)]. This leads to $1/k_1 = 0$.

The ϵ^2 - equation is

$$\bar{\mathbb{E}} \mid \phi_2(\xi_1, \xi_2, \underline{x}) > = - \nabla_{\underline{x}} \cdot \bar{\mathbb{D}} \nabla_{\underline{x}} \mid \phi_0 > - \frac{1}{k_2} \mid \chi > \nabla \sigma_f \mid \phi_0 > . \quad (6)$$

Since Equation (6) is also an inhomogeneous equation to Equation (2), applying the Fredholm alternative theorem again, truncating the expansion of $1/k_{\text{eff}}$ at the ϵ^2 - order, and transforming back to dimensional variables yields the one-group bundle-homogenized diffusion equation

$$\begin{aligned} \nabla_{\underline{x}} \cdot \bar{\mathbb{D}}(\underline{x}) \nabla_{\underline{x}} A_0(\underline{x}) + \left\{ \frac{1}{k_{\text{eff}}} \bar{\nabla \Sigma}_f(\underline{x}) - \bar{\Sigma}_a(\underline{x}) \right\} A_0(\underline{x}) \\ + \{ \nabla_{\underline{x}} \cdot \underline{G}(\underline{x}) A_0(\underline{x}) - \underline{H}(\underline{x}) \cdot \nabla_{\underline{x}} A_0(\underline{x}) - W(\underline{x}) A_0(\underline{x}) \} = 0 \end{aligned} \quad (7)$$

where

$$\begin{aligned} \bar{\mathbb{D}}(\underline{x}) &= \frac{1}{N^X} \int_{\text{bundle}} d^2 \xi < \psi_0^{\dagger, X} \mid \underline{\mathbb{D}}^X \mid \psi_0^X > \\ \bar{\nabla \Sigma}_f(\underline{x}) &= \frac{1}{N^X} \int_{\text{bundle}} d^2 \xi < \psi_0^{\dagger, X} \mid \chi > \nabla \Sigma_f^X \mid \psi_0^X > \\ \bar{\Sigma}_a(\underline{x}) &= \frac{1}{k} \int_{\infty, X} \bar{\nabla \Sigma}_f(\underline{x}) \\ \underline{G}(\underline{x}) &= \frac{1}{N^X} \int_{\text{bundle}} d^2 \xi < \psi_0^{\dagger, X} \mid \underline{\mathbb{D}}^X \mid \nabla_{\underline{x}} \psi_0^X > \\ \underline{H}(\underline{x}) &= \frac{1}{N^X} \int_{\text{bundle}} d^2 \xi < \nabla_{\underline{x}} \psi_0^{\dagger, X} \mid \underline{\mathbb{D}}^X \mid \psi_0^X > \\ W(\underline{x}) &= \frac{1}{N^X} \int_{\text{bundle}} d^2 \xi < \nabla_{\underline{x}} \psi_0^{\dagger, X} \mid \cdot \mid \underline{\mathbb{D}}^X \mid \nabla_{\underline{x}} \psi_0^X > . \end{aligned}$$

and N^X is a normalization factor defined as

$$N^X = \int_{\text{bundle}} d^2 \xi < \psi_0^{\dagger, X} \mid \psi_0^X > .$$

Equation (7) is a generalization of the standard homogenized diffusion equation because it includes additional new terms (those in the braces). These terms are non-zero due to the global parametric space-dependence of the forward and adjoint eigenvectors that results from the deviation of the spatial dependence of the diffusion coefficients and cross-sections from exact periodicity. However, because the global and local spatial variables are formally treated as independent variables, these additional terms are zero in the interior of bundles and only exist on the interface. Furthermore, since the forward and adjoint eigenvectors on one side are different from those on the other side of the interface, each of the additional terms $\underline{G}(\underline{x})$, $\underline{H}(\underline{x})$, and $\underline{W}(\underline{x})$ has a finite jump on the interface -- these introduce a discontinuity to $\underline{A}(\underline{x})$ on the interface. Therefore, Equation (7) can be represented by the standard multiregion diffusion equation together with the discontinuous interface conditions, which substitute for the additional terms.

For a large system composed of exactly periodic bundles, there is no global spatial variation in diffusion coefficients and cross-sections. Thus, the forward and adjoint eigenvectors do not parametrically depend upon $\underline{x}$ and their derivatives become zero; therefore, the quantities $\underline{G}(\underline{x})$, $\underline{H}(\underline{x})$, and $\underline{W}(\underline{x})$ vanish, and Equation (7) reduces to the conventional homogenized diffusion equation.

The homogenized diffusion coefficient and cross-sections in Equation (7) are bilinear-weighted in terms of the forward and adjoint unit-bundle fluxes -- they are not flux-weighted as those commonly being used in thermal power reactor analysis.¹³ Physically, the adjoint fluxes are known as neutron importance; hence, the bilinear-weighted cross-sections preserve the neutron-importance-weighted reaction rates -- this has a good physical implication. Moreover, the eigenvalue of the homogenized diffusion equation preserves the exact diffusion theory eigenvalue $1/k_{\text{eff}}$ of the heterogeneous reactor to second order in the small parameter ϵ .

The asymptotic solution to the heterogeneous reactor diffusion equation is given by

$$|\phi(\underline{x})\rangle = \underline{A}_0(\underline{x}) |\psi_0^{\underline{x}}(x_1, x_2)\rangle + O(\epsilon). \quad (8)$$

The leading-order solution is the product of a local, rapidly-varying, unit-bundle eigenfunction and a global, slowly-varying, homogenized reactor eigenfunction. Hence, this result justifies the assumption in Reference 12 in which a similar "synthesized" solution of this form was used.

The boundary and interface conditions for $A_0(\underline{x})$ can be established from the corresponding boundary and interface conditions of $|\phi(\underline{x})\rangle$. The free-surface boundary condition requires that $|\phi(\underline{x})\rangle$ vanish on the extrapolated surface:

$$|\phi(\underline{x}^s)\rangle = 0, \quad (9)$$

where $\underline{x}^s$ is any point on the extrapolated surface. Combining Equations (8) and (9) yields

$$A_0(\underline{x}^s) |\psi_0^{\underline{x}^s}(\underline{x}_1, \underline{x}_2)\rangle + O(\epsilon) = 0. \quad (10)$$

Here, since $|\psi_0^{\underline{x}^s}(\underline{x}_1, \underline{x}_2)\rangle$ is the infinite-medium-like bundle eigenfunction and does not equal to zero, this leads to that, with a possible small error on the order of ϵ , $A_0(\underline{x}^s)$ vanishes on the extrapolated surface:

$$A_0(\underline{x}^s) = 0, \quad (11)$$

which is the boundary condition for $A_0(\underline{x})$.

The interface condition of $|\phi(\underline{x})\rangle$ requires that $|\phi(\underline{x})\rangle$ be continuous across the interface:

$$|\phi^{(j)}(\underline{x}^b)\rangle = |\phi^{(k)}(\underline{x}^b)\rangle, \quad (12)$$

where the superscripts j and k represent two regions next to each other, and $\underline{x}^b$ is any point on this j - k interface. Combining Equations (8) and (12) yields

$$A_0^{(j)}(\underline{x}^b) |\psi_0^{(j)}(\underline{x}_1^b, \underline{x}_2^b)\rangle + O(\epsilon) = A_0^{(k)}(\underline{x}^b) |\psi_0^{(k)}(\underline{x}_1^b, \underline{x}_2^b)\rangle + O(\epsilon). \quad (13)$$

Since $|\psi_0^{(j)}(\underline{x}_1^b, \underline{x}_2^b)\rangle$ is different from $|\psi_0^{(k)}(\underline{x}_1^b, \underline{x}_2^b)\rangle$, $A_0(\underline{x})$ must have a discontinuity across the interface as discussed earlier due to the finite jumps introduced by the additional terms in Equation (7). Besides, $A_0(\underline{x})$ is the solution to the one-group, global, homogenized diffusion equation and $|\psi_0^{\underline{x}}\rangle$ is the solution to the multigroup, local, unit-bundle diffusion equation. It is impossible to obtain an interface condition for $A_0(\underline{x})$ that preserves the continuity condition of $|\phi(\underline{x})\rangle$ and yet matches all discontinuous components of $|\psi_0^{\underline{x}}(\underline{x}_1, \underline{x}_2)\rangle$ across the

interface, because these components are group- and local-spatial-dependent. However, we can obtain an interface condition for $A_o(\underline{x}^b)$ that preserves the continuity condition of $|\phi(\underline{x}^b)\rangle$ to the leading order in the small parameter ϵ and matches the discontinuity of the surface-averaged fast component of $|\psi_o^x(\underline{x}_1, \underline{x}_2)\rangle$ across the interface. Here, the choice of the fast component rather than the epithermal or thermal component in determination of the interface condition for $A_o(\underline{x}^b)$ is based on the fact that almost all fission neutrons are born in the fast group, and that the neutron upscattering from epithermal and thermal groups to fast group is negligibly small. Therefore, less error will be produced if we match the discontinuity of the fast component. Consequently, Equation (13) is substituted by

$$A_o^{(j)}(\underline{x}^b) \psi_{01}^{(j)} + O(\epsilon) = A_o^{(k)}(\underline{x}^b) \psi_{01}^{(k)} + O(\epsilon), \quad (14)$$

where $\psi_{01}^{(j)}$ and $\psi_{01}^{(k)}$ are the surface-averaged fast components of $|\psi_o^{(j)}(\underline{x}_1, \underline{x}_2)\rangle$ and $|\psi_o^{(k)}(\underline{x}_1, \underline{x}_2)\rangle$, respectively. Hence, the interface condition for $A_o(\underline{x}^b)$, with a possible small error on the order of ϵ , is given by

$$A_o^{(k)}(\underline{x}^b) = \frac{\psi_{01}^{(j)}}{\psi_{01}^{(k)}} A_o^{(j)}(\underline{x}^b). \quad (15)$$

A similar discontinuous interface condition of this type was used in Reference 13, but our development is different from theirs. Our homogenization theory is a systematic development that leads to the one-group homogenized diffusion equation with bilinear-weighted cross-sections, whereas their homogenization theory is an ad hoc development that leads to the multigroup homogenized diffusion equation with flux-weighted cross-sections.

NUMERICAL RESULTS

For simplicity but without loss of generality, numerical results were given for several fictitious light water slab reactors composed of one-dimensional, exactly periodic or near-periodic fuel bundles. Each fuel bundle contains four cell-homogenized fuel regions, two channel regions, and two water regions, and these regions are located symmetrically with respect to the center line of the bundle as shown in Figure 1. The bundle pitch is 15.24 cm, the half thickness of water gap is 0.692 cm, and the channel thickness is 0.466 cm--these are a one-dimensional representation of a typical BWR fuel bundle. Two types of fuel bundle were used for comparisons: one does not contain the burnable poison gadolinia (called the non-Gd bundle) and the other contains gadolinia (called the Gd bundle); their nuclear properties are listed in Table 1.

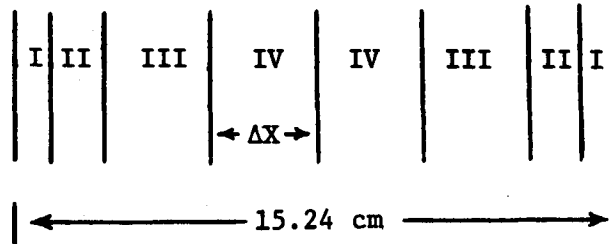
The three-group unit-bundle forward and adjoint diffusion equations were solved, and the group-dependent fluxes were normalized to

$$\sum_{g=1}^3 \sum_{m=1}^M \psi_{og,m} \Delta X_m = 1 ,$$

where the subscript m refers to a region in the bundle, $\psi_{og,m}$ is the g -th group forward or adjoint flux in the m -th region, ΔX_m is the m -th region width, and M is the total number of regions. The homogenized diffusion coefficient and cross-sections were then calculated from the bilinear-weighted expressions. Finally, the one-group homogenized coarse-mesh diffusion equation and the three-group heterogeneous fine-mesh diffusion equations were solved separately.

The results of the homogenized coarse-mesh calculations were compared with the results of the corresponding heterogeneous fine-mesh calculations; the latter were considered as benchmarks. Table 2 summarizes the comparisons for slab reactors composed of six, twelve, and eighteen exactly periodic non-Gd-fuel bundles, respectively. The eigenvalues, which were shown to be preserved to second order in the small parameter ϵ , agree very well in all cases with the errors less than 0.02%. The maximum error in bundle power is 1.05%. Table 3 summarizes similar comparisons for periodic Gd-fuel bundles. The maximum eigenvalue error is 0.17%, which is larger than that of non-Gd-fuel bundles. The maximum error in bundle power is 7.22%, which is

FIGURE 1
FUEL BUNDLE CONFIGURATION



I: Water, II: Channel, III: Fuel 1, IV: Fuel 2
 $\Delta X_I = 0.692$ cm, $\Delta X_{II} = 0.466$ cm, $\Delta X_{III} = \Delta X_{IV} = 3.231$ cm

TABLE 1
NUCLEAR PROPERTIES OF FUEL BUNDLES
 (IVa FOR NON-Gd BUNDLE AND IVb FOR Gd BUNDLE)

<u>Composition</u>	<u>I</u>	<u>II</u>	<u>III</u>	<u>IVa</u>	<u>IVb</u>
D_1	2.3613	1.9946	1.8073	1.8132	1.8278
$\Sigma_{s1,1}$	0.0613	0.0224	0.0334	0.0331	0.0331
$\Sigma_{a,1}$	0.0001	0.0005	0.0032	0.0033	0.0031
$\nu \Sigma_{f1}$	0.0	0.0	0.0046	0.0048	0.0042
Σ_{f1}	0.0	0.0	0.0016	0.0017	0.0015
D_2	0.7707	0.9526	0.6854	0.6792	0.6631
$\Sigma_{s1,2}$	0.1012	0.0370	0.0541	0.0530	0.0541
Σ_{a2}	0.0007	0.0032	0.0246	0.0267	0.0441
$\nu \Sigma_{f2}$	0.0	0.0	0.0115	0.0151	0.0099
Σ_{f2}	0.0	0.0	0.0047	0.0062	0.0041
D_3	0.2278	0.4479	0.3294	0.3362	0.3143
Σ_{a3}	0.0097	0.0068	0.0764	0.0901	0.4873
$\nu \Sigma_{f3}$	0.0	0.0	0.1241	0.1542	0.0187
Σ_{f3}	0.0	0.0	0.0509	0.0633	0.0077

TABLE 2

COMPARISONS FOR SLAB REACTORS
COMPOSED OF PERIODIC NON-Gd-FUEL BUNDLES

	Number of Bundles		
	6	12	18
Eigenvalue (Fine)	1.2412	1.2992	1.3112
Eigenvalue (Coarse)	1.2414	1.2991	1.3111
$\Delta k/k \times 100\%$	0.02%	- 0.01%	- 0.01%
Maximum Error in Bundle Power	0.41%	1.05%	1.03%
Average Error in Bundle Power	0.31%	0.51%	0.41%

TABLE 3

COMPARISONS FOR SLAB REACTORS
COMPOSED OF PERIODIC Gd-FUEL BUNDLES

	Number of Bundles		
	6	12	18
Eigenvalue (Fine)	0.5155	0.5376	0.5421
Eigenvalue (Coarse)	0.5146	0.5371	0.5419
$\Delta k/k \times 100\%$	- 0.17%	- 0.09%	- 0.04%
Maximum Error in Bundle Power	5.40%	6.70%	7.22%
Average Error in Bundle Power	3.21%	2.75%	2.43%

TABLE 4

COMPARISONS FOR SLAB REACTORS COMPOSED OF NEAR-PERIODIC FUEL BUNDLES: (a) EIGHTEEN EXPOSURE-DEPENDENT (0, 10, AND 20 MWD/ST) NON-Gd BUNDLES; (b) NINE Gd AND NINE NON-Gd BUNDLES SIDE-BY-SIDE.

	<u>Cases</u>	
	<u>Exposure-Dependent</u>	<u>Side-by-Side</u> ⁺⁺
Eigenvalue (Fine)	1.1878	1.2868
Eigenvalue (CND*)	1.1843	1.2853
$\Delta k_{ND}/k \times 100\%$	- 0.29%	- 0.12%
Eigenvalue (CWD ⁺)	1.1857	1.2858
$\Delta k_{WD}/k \times 100\%$	- 0.18%	- 0.08%
Error in Bundle Power (CND)) Maximum	6.72%	13.57%
) Average	3.54%	6.91%
Error in Bundle Power (CWD)) Maximum	5.32%	12.45%
) Average	2.33%	5.44%

* CND = Coarse-mesh with no discontinuous interface conditions.

+ CWD = Coarse-mesh with discontinuous interface conditions.

++ Bundle power comparisons were restricted to those with relative power greater than 0.01.

worse than that of non-Gd-fuel bundles. This is probably because the thermal absorption cross-section of the Gd-fuel region is very large and its thermal fission cross-section is very small compared with those of the neighboring non-Gd-fuel region. These large changes in the adjacent cross-sections tend to reduce the accuracy of our method.

For slab reactors composed of near-periodic fuel bundles, the comparisons were given in Table 4. These comparisons were made among the results of the fine-mesh diffusion calculations, those of the coarse-mesh diffusion calculations with no discontinuous interface conditions (CND), and those of the coarse-mesh diffusion calculations with discontinuous interface conditions (CWD). Case (a) simulates an exposure-dependent non-uniform loading in a typical power reactor. That is, the reactor core is symmetric with respect to its center line and, in each half of its core, is composed of three repeating sets of 20 MWD/ST, 10 MWD/ST, and 0 MWD/ST non-Gd-fuel bundles with the 20 MWD/ST bundle in the outer location. After employment of the discontinuous interface conditions, the eigenvalue error reduces from 0.29% to 0.18%, the maximum error in bundle power reduces from 6.72% to 5.32%, and the average error in bundle power reduces from 3.54% to 2.33%. The cross-sections in various regions of the non-Gd bundle at zero exposure have been given in Table 1, but those of the non-Gd bundle at 10 and 20 MWD/ST are not listed because of the space limitation. Case (b) is for a slab reactor composed of nine non-Gd bundles (at zero exposure) and nine Gd bundles side-by-side. After employment of the discontinuous interface conditions, the eigenvalue error reduces from 0.12% to 0.08%, the maximum error in bundle power reduces from 13.57% to 12.45%, and the average error in bundle power reduces from 6.91% to 5.44%.

SUMMARY

A lattice homogenization theory for coarse-mesh diffusion analysis in large heterogeneous systems composed of two dimensional, near-periodic fuel bundles has been developed by introducing two spatial scales and utilizing asymptotic expansion. The product of the fundamental unit-bundle eigenvector and the solution to the homogenized diffusion equation was shown to be the leading order term in an asymptotic expansion of the solution to the heterogeneous reactor diffusion equation. The eigenvalue of the homogenized reactor diffusion equation was shown to preserve the eigenvalue of the heterogeneous reactor diffusion equation to second order in the small parameter ϵ . The homogenized diffusion equation was systematically derived along with the formulation of bilinear weighted homogenized diffusion coefficient and cross-sections, which is different from the commonly used flux-weighting formulation. The additional terms were also obtained due to the small slowly varying deviation of the diffusion coefficients and cross-sections from their rapid exactly periodic variations. Discontinuous

interface conditions were introduced to represent these additional terms, which only exist on the interfaces.

The numerical results justify the preservation of the eigenvalue, and indicate that our method works very well for large systems composed of exactly periodic bundles with no large changes in cross-sections between adjacent regions in each bundle. They also indicate that our method works reasonably well for large systems composed of exposure-dependent, near-periodic bundles with no large changes in adjacent cross-sections -- a case of practical importance in thermal power reactor analysis. For systems with large changes in neighboring cross-sections, the method tends to give poorer results. However, this is the deficiency associated with all perturbation methods, which require that all changes in systems be small.

Acknowledgements

The author acknowledges the useful discussions with S.P. Congdon, R.L. Crowther, and C.J. Shao of the General Electric Company, and J.J. Dorning of the University of Illinois.

REFERENCES

1. P. BENOIST, "Theory du Coefficient de Diffusion des Neutrons Dans un Réseau Comportant des Cavities," CEA-R2278, Centre d'Etudes Nucleaires, Saclay, 1964.
2. V. DENIZ, Nucl. Sci. Eng., 28, 397 (1967).
3. R. A. BONALUMI, Energ. Nucl. (Milan), 18, 395 (1971).
4. E. M. GELBARD, Nucl. Sci. Eng., 54, 327 (1974).
5. E. W. LARSEN, J. Math. Phys., 16, 1421 (1975).
6. E. W. LARSEN, Nucl. Sci. Eng., 60, 357 (1976).
7. R. P. HUGHES, Nucl. Sci. Eng., 67, 85 (1978).
8. R-T CHIANG and J. J. DORNING, Trans. Am. Nucl. Soc., 33, 779 (1979).
9. E. W. LARSEN and R. P. HUGHES, Nucl. Sci. Eng., 73, 274 (1980).
10. R-T CHIANG and J. J. DORNING, Trans. Am. Nucl. Soc., 34, 829 (1980).
11. R-T CHIANG and J. J. DORNING, "A Homogenization Theory for Lattices with Burnup and Non-Uniform Loadings," Advances in Reactor Physics and Shielding, pp. 319-329, Sun Valley, Idaho (1980).
12. C. L. HOXIE and A. F. HENRY, Trans. Am. Nucl. Soc., 39, 905 (1981).
13. K. S. SMITH, A. F. HENRY, and R. A. LORETZ, "The Determination of Homogenized Diffusion Theory Parameters for Coarse Mesh Nodal Analysis," Advances in Reactor Physics and Shielding, pp. 294-308, Sun Valley, Idaho (1980).

VALIDATION STUDIES OF THE ENDF/MC²-2/SDX

CELL HOMOGENIZATION PATH*

R. D. McKnight, R. N. Hwang, C. G. Stenberg, and R. N. Blomquist
Argonne National Laboratory
9700 South Cass Avenue
Argonne, Illinois 60439

ABSTRACT

An extensive validation study of the ENDF/MC²-2/SDX cell homogenization path has been made. The validation procedure has relied on the availability of rigorous Monte Carlo results obtained with the VIM code. The current study has investigated three ZPR benchmark assemblies with rather diverse characteristics. The calculations have included homogeneous zero-leakage cells, critically-buckled heterogeneous cells, and full "as-built" critical assemblies. This study has identified and eliminated several limitations and inconsistencies in these analytical methods and their application. The deterministic methods and Monte Carlo have been shown to agree very well (i.e., generally within 1σ uncertainties) for zero-buckled homogeneous and heterogeneous cell calculations. For the high-buckling cases, there was a bias on leakage-related parameters for both the homogeneous and heterogeneous cell calculations. This bias -- less leakage with MC²-2/SDX relative to VIM -- was consistent with the results observed in the full assembly calculations. MC²-2/SDX eigenvalues for the three ZPR assemblies were 0.5-0.7% δk high relative to VIM. Principal cross sections and neutron spectrum in the full assembly calculations were in good agreement between these methods. The source of these apparent differences in neutron transport or leakage between the deterministic and Monte Carlo calculations of the full assembly must be identified and eliminated.

I. INTRODUCTION

This paper summarizes the results from an extensive Monte Carlo based validation study of the unit cell homogenization prescriptions and the methods and codes used for the analysis of Zero Power Reactor (ZPR) fast breeder reactor critical experiments at Argonne National Laboratory. The present study includes a diverse set of ZPR benchmark assemblies and represents an extension in both scope and detail of earlier validation studies by Wade et al.¹⁻⁵ Results of standard analysis methods based on the ENDF/MC²-2/SDX cell homogenization path⁶⁻⁸ have been compared with results produced by the VIM continuous energy Monte Carlo code.^{9,10} The present study has intercompared these methods not only for unit cells but also for three-dimensional full reactor models. A primary goal of these efforts has been to identify and quantify the methods' biases and uncertainties in the full assembly calculations.

For each ZPR assembly, the validation effort progressed from homogenous zero-leakage tests through critically-buckled heterogeneous cells to calculations of the full "as-built" assembly. For each step in the validation study, eigenvalue, neutron spectrum, broad group cross sections, and reaction rates obtained with deterministic methods were compared to values obtained with the VIM Monte Carlo code. ENDF/B Version IV data were used for all calculations in this

*Work performed under the auspices of the U. S. Department of Energy under Contract No. W-31-109-ENG-38.

study. Sensitivities of results to many user options (e.g., P_n spectrum options in creating the intermediate SDX base library for highly buckled systems, mesh and transport corrections, use of Benoist¹¹ or Gelbard¹² methods for treating anisotropic neutron diffusion or "streaming") were also determined.

This paper is organized in the following way. Section II discusses briefly the standard ZPR analysis methods and codes employed at ANL. The validation procedure is also outlined. Section III describes the principal characteristics and unit cells of the three ZPR assemblies considered in this study. Section IV reviews the unit cell calculations, focusing primarily on the core unit cell of ZPR-9 Assembly 34, the U/Fe benchmark assembly. The full assembly calculations are presented in Section V. Section VI provides a brief summary of results and conclusions.

II. ANALYTICAL METHODS AND VALIDATION PROCEDURE

Fast reactor configurations are constructed by loading the ZPR matrix tubes with drawers which have been loaded with plates and blocks of reactor materials. These plates are approximately 2 in. (5.08 cm) high, vary in width from 1/16 to 2 in. (0.16 to 5.08 cm) and in length from approximately 2 to 12 in. (5.08 to 30.48 cm), and are loaded parallel to the drawers. Furthermore, the unit cell loading is repeated in each drawer, forming planes (interrupted by relatively thin matrix tube walls and drawer bottom) of each material in the unit cell. In the standard ZPR analysis methods, a prescription is used to convert the three-dimensional physical geometry of the plate type drawer loadings into one-dimensional calculational models for the unit cell. Details of this 3D $\rightarrow$ 1D modeling prescription are given in Ref. 1.

The standard ZPR critical assembly analysis methods employed by ANL are diagrammed in Fig. 1. The basic nuclear data (ENDF/B) are processed by the ETOE-2/MC²-2/SDX codes. ETOE-2 reformats the ENDF/B data into libraries for MC²-2 and SDX; MC²-2 is a zero-dimensional ~ 2000 ultra-fine-energy group slowing-down code which is used to produce a ~ 200 fine group library for SDX (which excludes the contributions of selected capture and fission resonances). SDX is a one-dimensional cell homogenization code which collapses in space and energy from the ~ 200 fine group level to produce ~ 10 to 30 broad group cell-averaged cross sections. As depicted in Fig. 1, SDX produces both cell-average and plate cross sections for use in ZPR analysis. "Cell-average" cross-sections include the effects of plate spatial self-shielding and platewise resonance energy self-shielding in such a way that unit cell-average reaction rates are preserved in a calculation in which the unit cell composition is homogenized. "Plate" cross-sections are used in the generation of broad group cell anisotropic diffusion coefficients.

Anisotropic diffusion coefficients are generated using the methods of Benoist¹¹ or Gelbard¹². The resulting broad group parameters are used in multigroup diffusion and S_n transport calculations* of full reactor RZ or XYZ models which employ smeared atom densities to represent the contents of the unit cells.

The validation of these standard ZPR analysis methods was based on comparison with rigorous Monte Carlo solutions obtained with the VIM code. The VIM Monte

*All transport calculations in this study used a P_0 (transport-corrected) in-group elastic scattering cross section and σ_{tr} . No estimates of the effects of higher order scattering moments were included in this study.

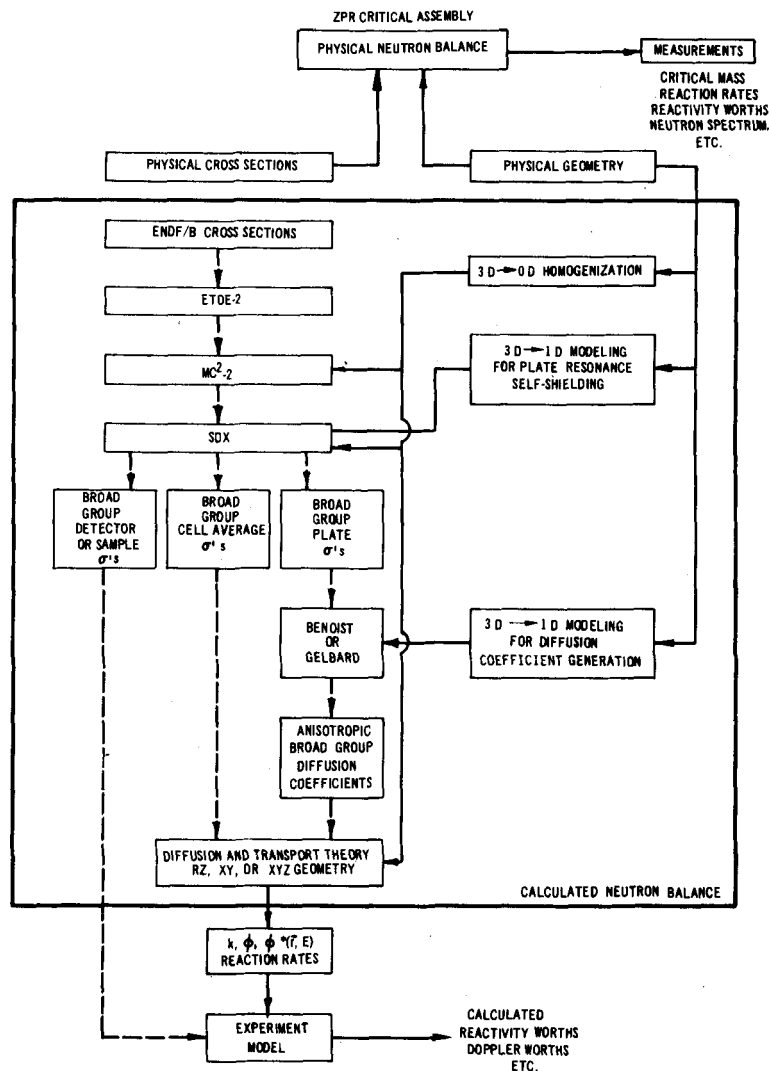


Fig. 1. Diagram of ZPR Critical Assembly Analysis Methods.

Carlo code permits an explicit three-dimensional geometrical representation of the unit cell. Neutron cross sections are derived from the ENDF/B data files and are treated as continuous functions of energy. Given the geometry and composition of the three-dimensional cells and the ENDF/B data, these Monte Carlo methods give an essentially exact solution of the Boltzmann equation and low-variance estimates of the integral parameters of interest.

As diagrammed in Figure 1, these standard methods are used with ENDF/B data to produce calculated values to compare with experimental values measured in the ZPR critical assembly. In the present study, the physical geometries of the unit cells and of the full critical assemblies have been provided in explicit three-dimensional platewise detail to the VIM code together with ENDF/B data to produce "rigorous" or "best" calculated values. That is, the physical geometry of each matrix tube and drawer, every plate loaded in the drawers, and all remaining gaps were defined explicitly. Each assembly was loaded with thousands of drawers and the entire assembly contained tens of thousands of plates. Furthermore, thirteen rectangular parallelepiped regions were required to describe each of the canned ZPR materials, such as sodium or plutonium. For

example, ZPR-6 Assembly 7 contained almost 4000 drawers and over 100,000 plates. The model used in the VIM Monte Carlo calculation of ZPR-6 Assembly 7 defined over a quarter of a million distinct physical regions. Because the goal of this study was to compare calculated values (i.e., to obtain C/C' values not C/E values), some minor simplifications were made in modeling the "as-built" assemblies. Each of the three assemblies included in this study had a single core zone (i.e., a single unit cell loading). However, limitations in the ZPR plate inventory did not allow each core drawer to be loaded with the same plate lengths. For two of these assemblies (ZPR-6 Assembly 7 and ZPR-9 Assembly 34), it was assumed that the plate loadings were identical in all drawers. For the third assembly (ZPR-9 Assembly 32) the plate loadings in each drawer were modeled exactly as loaded in the "as-built" assembly. The following points should be understood. The configurations modeled in the calculations closely retained the principal characteristics of the "as-built" assemblies, including dimensions, composition, fissile mass, and unit cell loadings, i.e., the modeling simplifications were small. And most importantly, the same assemblies were modeled in both the deterministic methods and the Monte Carlo methods, i.e., the models were consistent.

III. DESCRIPTIONS OF THE ZPR SYSTEMS

Three ZPR assemblies were examined — ZPR-6 Assembly 7 and ZPR-9 Assemblies 32 and 34. These assemblies span a wide range of characteristics, including composition, fuel type and enrichment, core volume, spectrum, unit cell heterogeneity, and leakage fraction. Some of the characteristics of these assemblies are summarized in Table I and the core unit cells are diagrammed in Fig. 2.

TABLE I. Summary of Assembly Characteristics

	ZPR-6/7	ZPR-9/32	ZPR-9/34
Core			
Height (cm)	152.56	91.60	183.20
Diameter (cm)	161.36	88.38	125.46
Height/Diameter	0.945	1.036	1.46
Volume (liters)	3120	562	2265
No. of Enrichment Zones	1	1	1
Compositions			
Core	Pu/U/O/Na/SST	Pu/U/O/Na/SST	U/SST
Radial and Axial Blankets	U/SST	U/O/Na/SST	-----
Radial and Axial Reflectors	-----	U/SST	SST/U
Fuel Type	(Pu,U)oxide	(Pu,U)oxide	U metal
Fissile Enrichment (%)	13.39	22.18	93.30
Core Fe Atom Density (atoms/barn/cm)	0.0128	0.0188	0.0705
Critical Mass (kg)	1133.1	335.1	992.0
k_{∞} (for Homogeneous Core)	1.22	1.65	1.53
Core Leakage Fraction	0.22	0.39	0.36
Spectral Index, $^{28}\text{f}/^{25}\text{f}$	0.0205	0.0290	0.0202

ZPR-6 Assembly 7 was a large, uniform, dilute plutonium oxide fueled core¹³ with a single drawer unit cell. It had a relatively clean cylindrical geometry

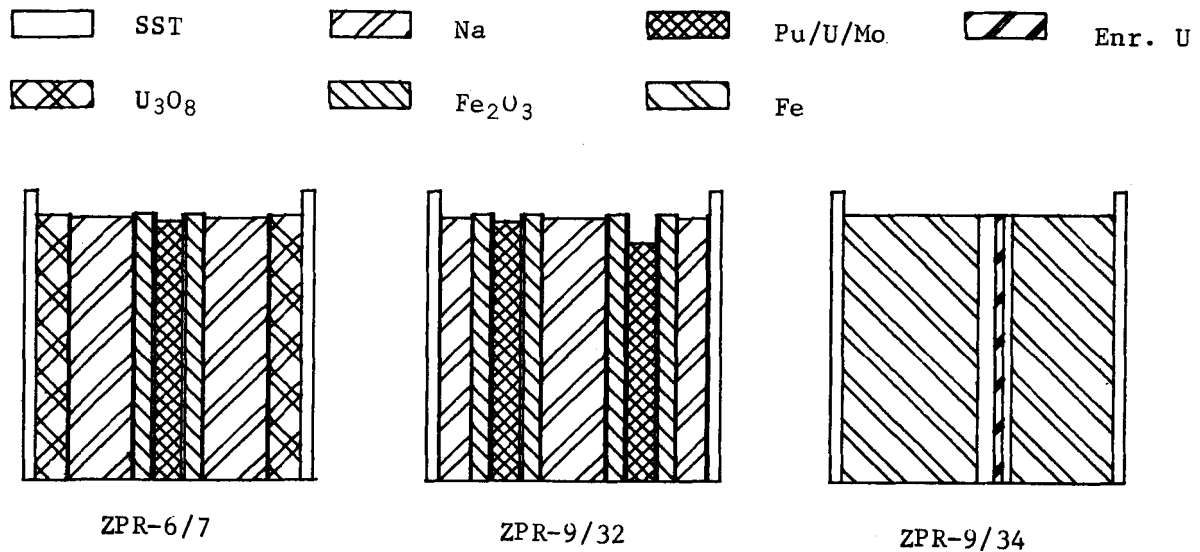


Fig. 2. Core Unit Cell Diagrams.

and a full depleted uranium blanket. It is one of the principal fast reactor data testing benchmark assemblies. The core unit cell had a symmetric loading that included a single column of Pu/U/Mo fuel and two columns of U_3O_8 . In the core of Assembly 7, approximately 84% of the absorptions occurred in ^{239}Pu (~47%) and ^{238}U (~37%); ~82% of the core fissions occurred in ^{239}Pu and ~12% occurred in ^{238}U . The total core leakage fraction was ~22%. Of the three assemblies studied, Assembly 7 had the most typical LMFBR spectrum and composition.

ZPR-9 Assembly 32 was part of the LMFBR Safety-Related Critical Experiments Program.¹⁴ The reference configuration consisted of a small single-zone core with an LMFBR outer-core composition (with $k_\infty \approx 1.65$). The core unit cell was a single drawer cell with two columns of Pu/U/Mo fuel. These two fuel plates had substantially different enrichments (18 vs 26%) and heights (4.928 vs 4.521 cm). In the core of Assembly 32, approximately 87% of the absorptions occurred in ^{239}Pu (~60%) and ^{238}U (~27%); ~87% of the fissions occurred in ^{239}Pu and ~9% occurred in ^{238}U . Of the three assemblies included in this study, Assembly 32 had the hardest neutron spectrum and the largest core leakage fraction (~39%).

ZPR-9 Assembly 34 consisted of a single-zone core¹⁵ of enriched (93% ^{235}U) uranium and iron with stainless steel axial and radial reflectors and an outer radial reflector of depleted uranium. The U/Fe benchmark assembly was part of a series of critical experiments to support the Safety Test Facility (STF) core design efforts. The unusual composition, ^{235}U and iron, and the clean geometry of the core made the assembly useful as a data testing benchmark. The core had a single drawer unit cell, consisting of a single enriched uranium metal fuel plate embedded in an iron cell (with all 5.08 cm plate heights). In the core of Assembly 34, over 95% of the absorptions occurred in ^{235}U (80%) and Fe (15%); and ~99.7% of the fissions occurred in ^{235}U . This assembly had the simplest core unit cell in terms of heterogeneity, and virtually no "streaming" or anisotropic diffusion effects. The core leakage fraction was large (~36%) for Assembly 34 (comparable to Assembly 32).

IV. UNIT CELL STUDIES

The initial eigenvalue comparison between MC²-2/SDX and VIM calculations for ZPR-9 Assembly 34 (the U/Fe benchmark assembly) revealed unacceptably large eigenvalue biases (up to 2.1% δk) in the multigroup methods relative to Monte Carlo. Attention was focused on a series of unit cell calculations for this assembly, resulting in the identification and elimination of several limitations and inconsistencies in these analytical methods and their application.

The high concentration of iron in the ZPR-9/34 assembly caused an unexpected high-energy self-shielding effect in iron. Unlike the other principal nuclides of interest, the elastic scattering data for iron given in the ENDF/B "smooth" files exhibit strong resonance-like fluctuations in the energy region above the upper cut-off point of the resolved resonance region.* The energy range in question is between 60 keV (the upper limit of the resolved energy range for Fe) and 2 MeV. These fluctuations in the Fe elastic scattering cross section have average "half-widths" much smaller than the corresponding ultra-fine group (ufg) width used in the MC²-2 library ($\Delta u = 1/120$) and are given as point-wise data in the "smooth" cross section file. While the VIM cross section library retains the detail as given in ENDF, the data are preprocessed as infinitely dilute ultra-fine group cross sections into the MC²-2 library. Since the self-shielding effects will be significant only for a limited number of systems with high iron concentrations, special iron ufg cross sections with the appropriate self-shielding effect can be incorporated at the MC²-2 library preparation level using the narrow resonance approximation. For the present study, self-shielded iron cross sections corresponding to each assembly studied were added to the MC²-2 library. For the ZPR-9/34 assembly, the effects of the iron self-shielding were quite dramatic (changing the eigenvalue by $\sim 1.6\% \delta k$). The MC²-2/SDX values were in excellent agreement with the VIM results when the iron self-shielding effects were included. This iron self-shielding effect was $\sim 0.2\% \delta k$ in ZPR-9/32 and $< 0.1\% \delta k$ in ZPR-6/7.

The unit cell validation study progressed through four stages:

- Homogeneous Tests with Zero-Leakage
- Homogeneous Tests with Critical-B²
- Heterogeneous Cell with Zero-Leakage
- Heterogeneous Cell with Critical-B².

In each step of the validation study, eigenvalue, neutron spectrum, broad group cross sections and reaction rates obtained with deterministic methods were compared to the VIM Monte Carlo results. For the non-zero leakage cases, two leakage related parameters, $\bar{\lambda}^2$ and $k_\infty - k(\vec{B})$, were also derived. $\bar{\lambda}^2$ is the mean-squared distance traveled from birth to fission. In the MC²-2/SDX calculations, $\bar{\lambda}^2$ has been determined from the expression

$$\bar{\lambda}^2 = - \frac{2}{k_\infty} \frac{\delta k}{\delta B^2},$$

for a small change in buckling, δB^2 , around the zero buckling case. In the VIM calculations of the zero-buckling infinite lattices, a site tape was generated

*The self-shielding effects in iron discussed herein refer to the self-shielding of the high energy elastic scattering cross section. All calculations in this study have included self-shielding of capture resonances of iron (and the other structural materials).

containing birth and death coordinates of each neutron and its fission weight at death. An efficient method developed by Gelbard et al.¹⁶⁻¹⁸ was used to derive Monte Carlo estimates of eigenvalue in an infinite uniform lattice as a function of buckling. (Details of these methods have been outlined by Wade in Reference 1.)

Some of the results of the unit cell studies for the core cell of ZPR-9 Assembly 34 are summarized in Table II. In the first two sections of Table II, results are presented from MC²-2 calculations obtained both with and without the high energy self-shielding of the iron elastic scattering data. In the homogeneous zero-buckled case, the effect of the Fe self-shielding on the MC²-2 eigenvalue or k_{∞} is relatively small ($\sim 0.2\% \delta k$). There were, of course, large differences in the total cross section of iron, which produced large differences in the calculated neutron spectra. In the comparison with VIM, the inclusion of the iron self-shielding in the MC²-2 calculations produced excellent agreement

TABLE II. Unit Cell Calculations for ZPR-9 Assembly 34

Homogeneous U/Fe Core Cell $B^2 = 0.0$		
	k_{∞}	Δk (Relative to VIM)
VIM $\pm 1\sigma$	1.53135 ± 0.00127	
MC ² -2 without Self-Shielded Fe	1.52914	-0.00221
MC ² -2 with Self-Shielded Fe	1.53116	-0.00019
Homogeneous U/Fe Core Cell $B^2 \approx B_{crit}^2$		
	$\bar{\Sigma}^2 \text{ cm}^2$	$k_{\infty} - k(B^2 = 0.0008)$
VIM $\pm 1\sigma$	3665.77 ± 21.97	0.53387 ± 0.00410
MC ² -2 without Self-Shielded Fe (B_{crit}^2)	3173.07	0.50829
MC ² -2 with Self-Shielded Fe (B_{crit}^2)	3357.84	0.53048
MC ² -2 with Self-Shielded Fe ($B^2 = 0.0$)	3582.95	0.53493
Heterogeneous U/Fe Core Cell $B^2 = 0.0$		
	k_{∞}	Δk (Relative to VIM)
VIM $\pm 1\sigma$	1.53226 ± 0.00197	
MC ² -2(IP1)/SDX(IP1)	1.53037	-0.00189
MC ² -2(CP1)/SDX(IP1)	1.53033	-0.00193
MC ² -2(CP1)/SDX(IP1) with Fine Mesh	1.52959	-0.00267
Heterogeneous U/Fe Core Cell $B^2 \approx B_{crit}^2$		
	$\bar{\Sigma}^2 \text{ cm}^2$	$k_{\infty} - k(B^2 = 0.0008)$
VIM $\pm 1\sigma$	3713.78 ± 23.19	0.53582 ± 0.00411
MC ² -2(CP1)/SDX(IP1) - B_{crit}^2	3568.82	0.52977
MC ² -2(IP1)/SDX(IP1) - B_{crit}^2	3453.89	0.51774
MC ² -2(CP1)/SDX(IP1) - $B^2 = 0.0$	3605.44	0.52823

IP1 and CP1 refer to inconsistent and consistent P1 options in MC²-2/SDX.

(i.e., generally within 1σ uncertainties). The eigenvalue bias ($<0.02\% \delta k$) is much less than the Monte Carlo estimated 1σ ($0.127\% \delta k$); the neutron spectrum agrees well; and the principal cross sections agree within 0.1-0.2% over most of the energy range.

The effects of the iron self-shielding on leakage parameters are very dramatic in the high buckling case. Note the MC²-2 calculation which did not include the self-shielding of iron underestimates $\bar{\lambda}^2$ by almost 15% (relative to VIM) and underestimates the decrease in k_{eff} due to the imposed buckling ($B^2 = B_{\text{crit}}^2$) by almost 5% of its value, i.e., the MC²-2 $k_{\text{eff}}(B^2 = 0.0008)$ is $\sim 2.2\% \delta k$ higher than the VIM estimate. The MC²-2 calculation which did include the self-shielding of iron yields a zero-buckling estimate of $\bar{\lambda}^2$ which is $\sim 2.3\%$ low relative to VIM ($1\sigma \pm 0.6\%$); MC²-2 underestimates the change in the k due to leakage by $\sim 0.6\%$ of its value relative to VIM ($1\sigma \pm 0.8\%$). Alternately, MC²-2 overpredicts $k_{\text{eff}}(B^2 = 0.0008)$ by $\sim 0.34\% \delta k$. Although the agreement of MC²-2 with VIM is much better in this case, the underprediction of $\bar{\lambda}^2$ in these homogeneous cell calculations is symptomatic of difficulties in predicting leakage. As will be discussed in the following section, the MC²-2/SDX calculation of the full ZPR-9 Assembly 34 overpredicted the eigenvalue by about this same amount ($\sim 0.47\% \delta k$). Furthermore, a similar bias was observed in the earlier validation study by Wade.¹ In that study, estimates of $\bar{\lambda}^2$ for a homogeneous GCFR composition ($k_{\infty} \approx 1.43$) obtained from various options of MC²-2/SDX were all low relative to VIM.

In the final two sections of Table II which pertain to the heterogeneous core unit cell, all MC²-2/SDX calculations utilized the self-shielded iron data. For the zero-buckling case, three options in the MC²-2 and SDX cross section collapse codes were compared with the VIM results. Specifically, in the first stage of the cross section collapse, the P1 option of MC²-2 was used in both its consistent and inconsistent (energy loss upon P1 scattering is neglected) forms. In the second stage of this process, SDX collapses σ_{tr} using an inconsistent P1 (ϕ/Σ_{tr}) weighting. The values in Table II indicate these options do not impact the MC²-2/SDX eigenvalue (k_{∞}) calculations, which agree well with the three-dimensional VIM Monte Carlo calculation of the infinite array of core unit cells. The multigroup neutron spectra agree well with the VIM spectrum over the entire energy range, except for the iron window group (21-31 keV) which is $\sim 4\%$ higher. All the principal broad group cross sections agree well over most of the range of interest. There is, however, a consistent bias in the MC²-2/SDX cross sections (which are 0-15% larger than VIM) in the MeV range. Two principal sources of this bias are differences in the treatment of the fission source spectrum and (more importantly) differences in the calculated flux peaking in the fuel plate -- a single 1/16 in. (0.16 cm) enriched (93% ²³⁵U) uranium plate. With regard to the fission spectrum, VIM handles the fission chi matrix as specified in the ENDF/B files. The secondary energy distributions are a function of the incident neutron energies. The MC²-2/SDX codes handle only a chi vector for each isotope which is based on an average fission temperature (derived for a typical fast reactor spectrum). Adjunct calculations indicate the sensitivity of the eigenvalues to the differences between the MC²-2/SDX and VIM fission spectra are negligible ($<0.01\% \delta k$). However, replacement of the multigroup chi with the VIM chi did improve agreement in the neutron spectra in the top two energy groups (6-14 MeV) and the threshold reaction rates. With regard to the calculated flux peaking in the fuel plate, the peaking factors are large (typically 1.2-1.4 in the higher energy groups). The initial SDX calculations specified only one mesh per material region. This

was refined by subdividing the fuel region (with 5 mesh/plate) and the adjacent stainless steel regions (with 4 mesh/plate). This change had a negligible effect on the eigenvalue ($-0.00074 \delta k$). There was a significant (up to 7%) improvement in the agreement of MC²-2/SDX versus VIM in the broad group cross sections in the top 6 or 7 groups.

For the non-zero buckling tests of the heterogeneous core cell, the MC²-2/SDX values of leakage-related parameters exhibit a bias relative to the VIM values as was observed in the homogeneous tests. Additionally, these values are significantly affected by the collapsing spectrum option of MC²-2. With the consistent P1 option of MC²-2, the change in eigenvalue due to leakage is underpredicted by $\sim 0.6\% \delta k$ (relative to VIM). With the inconsistent P1 option of MC²-2, this change in k_{eff} is underpredicted by $\sim 1.8\% \delta k$. Using the consistent P1 spectrum calculation in MC²-2, the SDX zero-buckling estimate of $\bar{k}^2$ is $\sim 3\%$ low (3605.44 cm^2 compared to the VIM estimate of $3713.78 \pm 23.19 \text{ cm}^2$).

One additional observation from the unit cell studies will be made. The unit cell of ZPR-9 Assembly 32 provided the most difficult analytical test for the one-dimensional modeling used with the SDX code. The core unit cell of ZPR-9/32 (which is illustrated in Figure 1) included two metal fuel plates with substantially different heights ($\sim 9\%$) and enrichments (18% vs 26%). The one-dimensional modeling in the MC²-2/SDX treatment was found to mispredict the multi-dimensional effect of the ZPR-9/32 core cell (due to differing heights of the fuel plates). This was reflected in errors in the flux peaking factors in the fuel plates and principally impacted the cross sections in the top 6 or 8 broad energy groups (above $\sim 500 \text{ keV}$). A simple correction* was implemented in SDX which produced cell averaged broad group cross sections in excellent agreement with VIM.

V. FULL ASSEMBLY CALCULATIONS

One of the principal goals of the current work was to extend the validation study to full reactor calculations -- to intercompare the results of standard analysis methods based on the ENDF/MC²-2/SDX cell homogenization path with the results of detailed VIM Monte Carlo calculations for the full critical assembly. As outlined in Section II, the standard ZPR analysis methods employ two- and three-dimensional diffusion theory with RZ and XYZ models of the critical assembly. Additional calculations, e.g., with S_n transport theory, anisotropic diffusion theory, or finer spatial mesh, are used to obtain corrections or refinements. In the present study the reference MC²-2/SDX calculations for the full assemblies utilized two-dimensional S_4 transport theory in 29 broad energy groups ($E_{top} = 14.19 \text{ MeV}$) with RZ models having $\sim 2.5 \text{ cm}$ mesh. Adjunct calculations, utilizing diffusion theory with isotropic diffusion coefficients and with Benoist and/or Gelbard anisotropic diffusion coefficients, various mesh spacings, alternative broad group energy structures, three dimensional diffusion theory, and S_n transport theory with various higher orders of angular quadrature, were performed for each assembly to refine the reference calculations. Some of the details of various eigenvalue calculations for ZPR-6 Assembly 7 based on MC²-2/SDX are displayed in Figure 3.

*The modeling in SDX is one-dimensional. In calculating the flux self-shielding factors or "flux-peaking" factors, SDX takes the volume of each plate (or the entire cell) to be proportional to its thickness. A simple modification to the code allowed the user to input the height of each plate, and the volume of each plate was taken to be proportional to the product of the thickness and height of the plate.

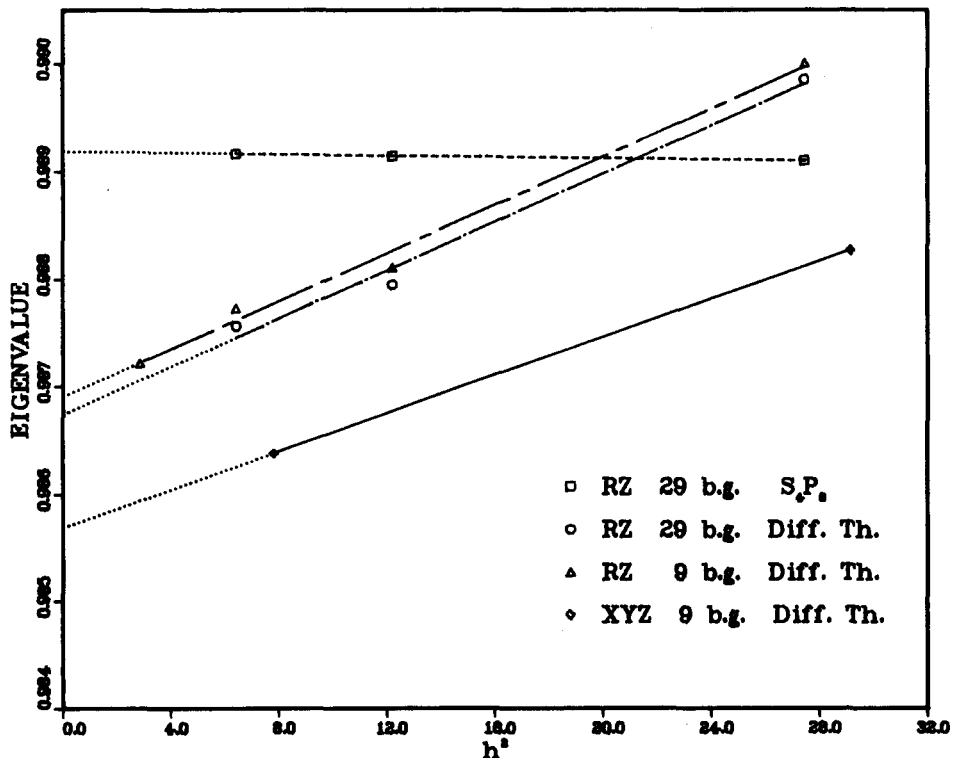


Fig. 3. Extrapolation of ZPR-6/7 Calculated Eigenvalue vs Square of Average Mesh Spacing (k_{eff} vs h^2).

- There is a significant difference in the extrapolations of the eigenvalue to zero mesh spacing between diffusion and transport theory solutions. Consequently, any transport correction (i.e., diffusion theory to transport theory correction) to the eigenvalue must be made consistently with the appropriate mesh correction.
- The eigenvalue differences between the 9 and 29 broad group energy structures were consistent and small ($<0.0002 \delta k$). Further extrapolation of eigenvalue to finer energy group structures was neglected.

The MC²-2/SDX results for ZPR-6 Assembly 7 eigenvalue calculations are summarized in Table III. The mesh correction is almost negligible (because the fine mesh transport theory solution has been used as the reference). For Assembly 7, the largest eigenvalue corrections account for streaming and 2D-to-3D modeling effects.* The streaming correction is obtained as the difference in eigenvalue between diffusion theory solutions with isotropic diffusion coefficients and Benoist anisotropic diffusion coefficients. In the XYZ calculations, the X direction is perpendicular to the plates in the unit cell and the Y and Z directions are parallel to the plates. In XYZ calculations, the Benoist $D_{\perp}$ is used for D_x and the Benoist $D_{\parallel}$ is used for D_y and D_z . In RZ calculations, D_R is set equal to $(D_{\perp} + D_{\parallel})/2$. The close agreement for $\delta k_{streaming}$ obtained by RZ and XYZ calculations confirms the adequacy of this definition of D_R for these relatively clean, simple configurations. Applying all of the corrections summarized in Table III, the MC²-2/SDX eigenvalue for ZPR-6 Assembly 7 is 0.983679.

*All of these corrections can vary significantly depending on the assembly. For example, in Assembly 34, the $\delta k_{streaming}$ correction is very small; in Assembly 32, the $\delta k_{S_4 \rightarrow S_{\infty}}$ is significant.

TABLE III. Summary of MC²-2/SDX k_{eff}
Calculations of ZPR-6 Assembly 7

1. Reference Calculation.
ENDF/B Version IV data; MC²-2/SDX; 29 broad energy groups
RZ Model with ~2.5 cm mesh spacing
Two-Dimensional Transport Theory (S_4P_0)
 $k_1 = 0.989162$
2. Mesh Correction.
Two-Dimensional Transport Theory (S_4P_0) with various mesh spacings

Mesh Spacing, cm	k
5.2396	0.989098
3.4931	0.989142
2.5310	0.989162

These values extrapolate to a value with zero mesh spacing

$$k_2 = 0.989180$$

$$\delta k_{\text{Mesh}} = k_2 - k_1 = 0.000018$$

3. Angular Quadrature Correction.
Two-Dimensional Transport Theory; 29 broad energy groups
RZ Model with ~2.5 cm mesh spacing

Quadrature	k
S_4	0.989162
S_8	0.988591
S_{16}	0.988511

$$\delta k_{S_4+S_\infty} = \delta k_{S_4+S_{16}} = k_{S_{16}} - k_{S_4} = -0.000651$$

4. Streaming Correction.
RZ Model with ~2.5 cm mesh spacing; 29 broad energy groups
Two-Dimensional Diffusion Theory
with Isotropic Diffusion Coefficients
 $k_{\text{RZ,Iso}} = 0.987563$
with Benoist Anisotropic Diffusion Coefficients
 $k_{\text{RZ,Ben}} = 0.984572$
 $\delta k_{\text{RZ,Streaming}} = k_{\text{RZ,Ben}} - k_{\text{RZ,Iso}} = -0.002991$
XYZ Model with ~5.4 cm mesh spacing; 29 broad energy groups
Three-Dimensional Diffusion Theory
with Isotropic Diffusion Coefficients
 $k_{\text{XYZ,Iso}} = 0.988120$
with Benoist Anisotropic Diffusion Coefficients
 $k_{\text{XYZ,Ben}} = 0.985149$
 $\delta k_{\text{XYZ,Streaming}} = k_{\text{XYZ,Ben}} - k_{\text{XYZ,Iso}} = -0.002971$
5. 2D + 3D Correction.
Three-Dimensional Diffusion Theory with ~5.4 cm mesh spacing
29 broad energy groups
 $k_{\text{XYZ}} = 0.988120$
Two-Dimensional Diffusion Theory extrapolated to ~5.4 cm
mesh spacing; 29 broad energy groups
 $k_{\text{RZ}} = 0.989999$
 $\delta k_{2D+3D} = k_{\text{XYZ}} - k_{\text{RZ}} = -0.001879$

Applying all of the above corrections, i.e.,
 $k_1 + \delta k_{\text{Mesh}} + \delta k_{S_4+S_\infty} + \delta k_{\text{XYZ,Streaming}} + \delta k_{2D+3D}$

yields: $k = 0.983679$

This value is to be compared with the VIM Monte Carlo calculated eigenvalue for Assembly 7 of 0.977517 ± 0.002081 ($k_{\text{eff}} \pm 1\sigma$ based on 100,000 neutron histories). The MC²-2/SDX eigenvalue is $0.006162 \delta k$ (or 3.0 standard deviations) higher than the VIM calculation.

The calculations for the other two assemblies exhibit a similar bias. The MC²-2/SDX eigenvalues, including all the corrections discussed previously for the Assembly 7 calculations, are $0.007239 \delta k$ (or 4.7 standard deviations) and $0.004728 \delta k$ (or 1.8 standard deviations) higher than the VIM calculations for Assemblies 32 and 34, respectively. These results are summarized in Table IV. The source of this eigenvalue bias has been investigated by comparing MC²-2/SDX and VIM group- and region-wise edits of cross sections, reaction rates and fluxes. Similar trends were observed in the comparisons for each of these assemblies. Some of these details will be provided for the comparison of MC²-2/SDX versus VIM calculations of ZPR-6 Assembly 7. These Assembly 7 results and conclusions may be considered to apply to all three assemblies; exceptions from the calculations for ZPR-9 Assemblies 32 and 34 will be noted.

TABLE IV. Summary of MC²-2/SDX vs VIM Eigenvalues

	$k_{\text{eff}} \pm 1\sigma$ (VIM) ^a	k_{eff} (MC ² -2/SDX) ^b	$\frac{k_{\text{eff}} \text{ (MC2-2/SDX)}}{k_{\text{eff}} \text{ (VIM)}}$
ZPR-6 Assembly 7	0.977517 ± 0.002081	0.983679	1.0063 ± 0.0021
ZPR-9 Assembly 32	0.995190 ± 0.001552	1.002429	1.0073 ± 0.0016
ZPR-9 Assembly 34	0.981748 ± 0.002600	0.986476	1.0048 ± 0.0026

^aVIM Monte Carlo calculations for three-dimensional detailed platewise models of the full assemblies. Calculations for ZPR-6/7 and ZPR-9/34 are based on 100,000 neutron histories; calculation for ZPR-9/32 is based on 200,000 neutron histories.

^bThese MC²-2/SDX eigenvalues are based on fine mesh two-dimensional S_4 transport theory calculations and include corrections for δk_{Mesh} , $\delta k_{S_4 \rightarrow S_\infty}$, $\delta k_{\text{Streaming}}$ and $\delta k_{2D \rightarrow 3D}$.

Overall neutron balances calculated for Assembly 7 are summarized in Table V. These neutron balances have been normalized such that total production in the assembly equals unity, i.e.,

$$(\text{Fission Production}/k_{\text{eff}}) + \text{Net } (n,2n) \text{ Source} = \text{Absorption} + \text{Leakage} = 1.0$$

for the assembly. The VIM neutron balance was obtained from the track length estimates of the isotopic reaction rates. The track length estimate of the eigenvalue (0.977361 ± 0.002984) is $0.000156 \delta k$ lower than the estimate quoted above (which was based on an average of three VIM estimators). However, the track length k_{eff} is inherently consistent with these reaction rate edits. The neutron balance labelled MC²-2/SDX represents a composite of the results for the RZ model calculated with the 29 broad group MC²-2/SDX generated cross sections.

TABLE V. Neutron Balance Summary for Assembly 7^a

	VIM $\pm 1\sigma^b$	MC ² -2/SDX ^c	$\frac{\text{MC}^2\text{-2/SDX}^b}{\text{VIM}}$
<u>Fission Production</u>			
Core	0.92257 $\pm$ 0.00361	0.93136	1.00953 $\pm$ 0.00391
Axial Blanket	0.01591 $\pm$ 0.00036	0.01634	1.02729 $\pm$ 0.02240
Radial Blanket	0.03605 $\pm$ 0.00063	0.03587	0.99503 $\pm$ 0.01760
Matrix	---	---	---
Total	0.97452 $\pm$ 0.00328	0.98357	1.00929 $\pm$ 0.00337
<u>Net (n,2n) Source</u>			
Core	0.00217 $\pm$ 0.00008	0.00203	0.93479 $\pm$ 0.03620
Axial Blanket	0.00018 $\pm$ 0.00003	0.00025	1.37855 $\pm$ 0.18600
Radial Blanket	0.00055 $\pm$ 0.00006	0.00055	1.00100 $\pm$ 0.11000
Matrix	---	---	---
Total	0.00290 $\pm$ 0.00010	0.00284	0.97847 $\pm$ 0.03550
<u>Capture</u>			
Core	0.39843 $\pm$ 0.00220	0.40199	1.00894 $\pm$ 0.00553
Axial Blanket	0.07549 $\pm$ 0.00118	0.07488	0.99187 $\pm$ 0.01560
Radial Blanket	0.17241 $\pm$ 0.00200	0.16862	0.97802 $\pm$ 0.01160
Matrix	0.00006 $\pm$ 0.00000	0.00006	0.99646 $\pm$ 0.04540
Total	0.64640 $\pm$ 0.00260	0.64555	0.99869 $\pm$ 0.00403
<u>Fission</u>			
Core	0.31643 $\pm$ 0.00124	0.31954	1.00983 $\pm$ 0.00392
Axial Blanket	0.00592 $\pm$ 0.00013	0.00604	1.02074 $\pm$ 0.02170
Radial Blanket	0.01336 $\pm$ 0.00022	0.01327	0.99356 $\pm$ 0.01670
Matrix	---	---	---
Total	0.33571 $\pm$ 0.00112	0.33885	1.00935 $\pm$ 0.00335
Eigenvalue	0.97736 $\pm$ 0.00298	0.98637	1.00922 $\pm$ 0.00305
<u>Leakage</u>			
from Core	0.23125 $\pm$ 0.00532	0.22473	0.97181 $\pm$ 0.02301
from Reactor	0.01789 $\pm$ 0.00284	0.01560	0.87180 $\pm$ 0.15860

^aThis neutron balance is normalized such that total production in the assembly is unity, i.e., (Fission Production/ k_{eff}) + Net (n,2n) Source = Absorption + Leakage = 1.0 for the entire assembly.

^bThe VIM neutron balance was obtained from the track length estimates of the isotopic reaction rates based on 100,000 neutron histories. All uncertainties represent 1- σ intervals and include only the VIM Monte Carlo uncertainties.

^cThe MC²-2/SDX neutron balance was obtained by adjusting the S_4P_0 transport theory values by the ratio of values obtained with anisotropic and isotropic diffusion theory.

To account for both streaming and transport effects, the MC²-2/SDX values in these tables were obtained by multiplying the S₄P₀ transport theory values by the ratio of the values obtained with anisotropic and isotropic diffusion theory, or

$$[MC^2-2/SDX] \equiv S_4P_0 * \frac{\text{Anisotropic}}{\text{Isotropic}}.$$

The MC²-2/SDX calculations (relative to VIM) produce more absorptions in the core -- more capture and more fission (and therefore more net production and larger eigenvalue). Alternately, the increased absorptions in the core result in less leakage from the core (MC²-2/SDX 22.5% vs VIM 23.1%).

The MC²-2/SDX and VIM broad group capture and fission cross sections for ²³⁹Pu and ²³⁸U (which account for ~84% of the absorptions in the core of Assembly 7) are in good agreement. A comparison of the broad group ²³⁸U capture cross section is provided in Table VI. Above group 18 (3.35 keV) agreement is generally within the VIM estimated 1-σ values. The MC²-2/SDX values are generally lower in the resolved range.* The MC²-2/SDX calculated value of c²⁸/f⁴⁹

TABLE VI. Comparison of VIM Monte Carlo
and MC²-2/SDX Values of σ_c^{28}

Group	VIM Monte Carlo	MC ² -2/SDX	$\frac{MC^2-2/SDX}{VIM}$
1	0.0017616 ± 0.0000583	0.0017641	1.0014 ± 0.0331
2	0.0044323 ± 0.0000429	0.0044394	1.0016 ± 0.0097
3	0.011556 ± 0.0000566	0.011510	0.9960 ± 0.0049
4	0.030535 ± 0.000103	0.030645	1.0036 ± 0.0034
5	0.068772 ± 0.000173	0.068680	0.9987 ± 0.0025
6	0.11570 ± 0.00019	0.11608	1.0033 ± 0.0017
7	0.11577 ± 0.00014	0.11578	1.0001 ± 0.0012
8	0.11022 ± 0.00014	0.11008	0.9987 ± 0.0013
9	0.12954 ± 0.00017	0.12979	1.0019 ± 0.0014
10	0.16188 ± 0.00018	0.16194	1.0004 ± 0.0011
11	0.20927 ± 0.00027	0.20894	0.9984 ± 0.0013
12	0.32658 ± 0.00043	0.32689	1.0009 ± 0.0013
13	0.41144 ± 0.00078	0.41041	0.9975 ± 0.0019
14	0.49685 ± 0.00091	0.49600	0.9983 ± 0.0018
15	0.59424 ± 0.00164	0.59496	1.0012 ± 0.0028
16	0.70024 ± 0.00256	0.69420	0.9914 ± 0.0037
17	0.79923 ± 0.00560	0.79432	0.9939 ± 0.0070
18	0.94978 ± 0.01140	0.90018	0.9478 ± 0.0120
19	0.88196 ± 0.01032	0.86654	0.9825 ± 0.0117
20	1.1041 ± 0.01358	1.0726	0.9715 ± 0.0123
21	1.1519 ± 0.02592	1.1090	0.9628 ± 0.0225
22	1.0320 ± 0.04324	0.96954	0.9395 ± 0.0419
23	1.6302 ± 0.06260	1.4358	0.8808 ± 0.0384
24	1.6600 ± 0.15919	1.7016	1.0251 ± 0.0959
25	5.0663 ± 1.68201	4.7218	0.9320 ± 0.3320
26	27.963 ± 0.00011	2.4757	0.0885 ± 4.0×10 ⁻⁶
27	---	0.25955	---
28	---	0.058418	---
29	---	0.54549	---

Note: All uncertainties represent 1-σ intervals and include only the VIM Monte Carlo uncertainties.

*The results are expected to be in better agreement with Monte Carlo in the resolved range if calculated with RABANL,⁷ the hyper-fine-group integral transport theory module of MC²-2. This was demonstrated in calculations for the ZPR-9 Assembly 32 core unit cell.

(the ratio of capture rate per atom in ^{238}U to the fission rate per atom in ^{239}Pu) agrees well with the VIM value. Core-averaged values in Assembly 7 were:

	<u>VIM $\pm 1\sigma$</u>	<u>MC²-2/SDX</u>	<u>$\frac{\text{MC}^2\text{-2/SDX}}{\text{VIM}}$</u>
c^{28}/f^{49}	0.1549 ± 0.0010	0.1542	0.9954 ± 0.0065

The core-averaged real flux spectra from the MC²-2/SDX anisotropic diffusion theory and S_4P_0 transport theory calculations are displayed in Fig. 4 for comparison with the VIM calculated spectrum ($\pm 1\sigma$ error bands). The differences between the Benoist diffusion theory and S_4P_0 transport theory spectra are very small with the S_n spectra slightly harder. There is general agreement between these spectra and the VIM spectrum (i.e., within $\pm 1\sigma$ error bands); however, the VIM spectrum is somewhat harder.

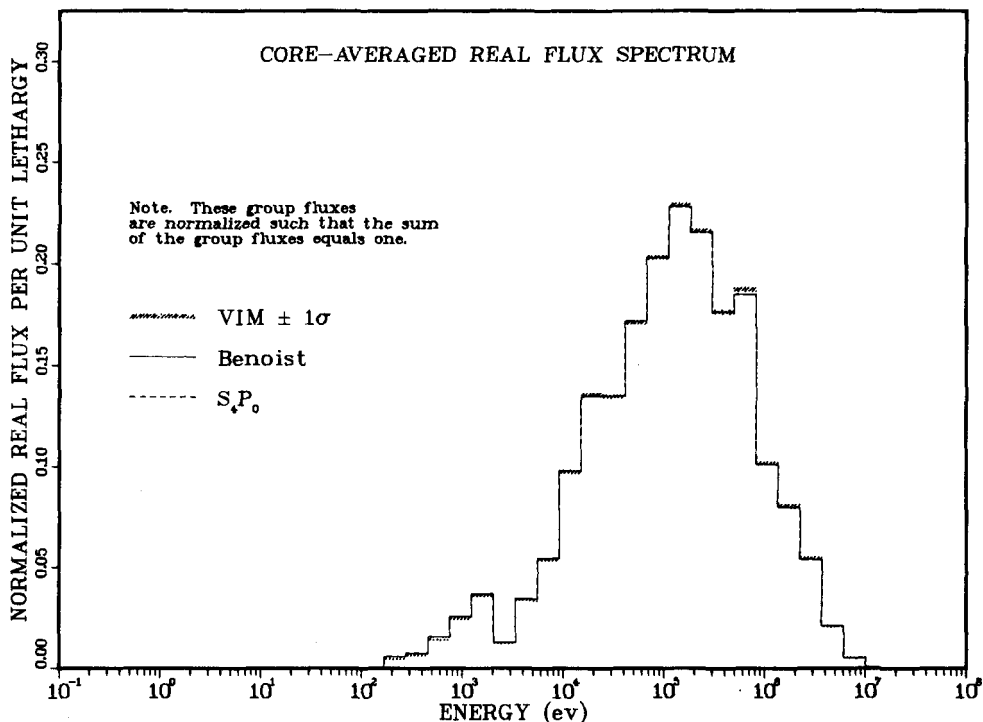


Fig. 4. Core-Averaged Flux Spectrum for ZPR-6 Assembly 7.

The larger core absorption fractions in the MC²-2/SDX calculations relative to VIM do not result from the differences in the capture and fission cross sections (which are in fact slightly smaller in the MC²-2/SDX case) or from the differences in the energy spectra of the core neutron flux. The increase in the MC²-2/SDX absorption rate in the core is the result of larger total flux in the core. That is, there are significant differences between the MC²-2/SDX and VIM calculations of the spatial distribution of the neutron flux. The treatment of the neutron transport in the MC²-2/SDX calculations (both S_n and diffusion theory) produces a higher flux in the core relative to the external regions than produced by the VIM Monte Carlo calculations. Therefore, the major

source of the eigenvalue bias observed for each of these three ZPR assemblies appears to be global effects of neutron transport through the assemblies. This conclusion is consistent with the results discussed in the preceding section. That is, the unit cell studies, both homogeneous and heterogeneous, indicated a similar bias on leakage, namely, less leakage with MC²-2/SDX relative to VIM or equivalently a shorter mean flight to fission, $\bar{\ell}^2$, in MC²-2/SDX than in VIM.

VI. SUMMARY

The present benchmark calculations of three ZPR assemblies with significantly different characteristics have provided an excellent opportunity to study the ENDF/MC²-2/SDX cell homogenization codes. With the inclusion of appropriate modifications discussed above, the agreement between the deterministic methods and Monte Carlo was very good (i.e., generally within 1σ uncertainties).

Initial results revealed a large eigenvalue bias (~2.1% δk) for ZPR-9 Assembly 34 in the multigroup methods relative to Monte Carlo. This error was the result of an unexpected high energy self-shielding effect in the elastic scattering cross sections of iron. The inclusion of the self-shielded iron elastic scattering data in the MC²-2/SDX calculations produced excellent agreement with the VIM results.

In the non-leakage tests for both homogeneous and heterogeneous cells, eigenvalue, neutron spectrum, and broad group cross sections and reaction rates obtained with MC²-2/SDX were in excellent agreement with the VIM results. The biases (MC²-2/SDX vs VIM) ranged from 0.1-0.2% δk in eigenvalue, 0.1-0.2% in principal cross sections in the important energy range, and 0.5-1.0% in broad group cross sections in unimportant (top and bottom) groups for homogeneous zero-buckled systems; and 0.1-0.2% δk in eigenvalue and 0.2-0.5% in principal cross sections in the important energy range, and 1-4% in the broad group cross sections in unimportant energy ranges for the zero-buckled heterogeneous cell. However, comparison of leakage-related parameters in both homogeneous and heterogeneous cells indicated a bias -- less leakage with MC²-2/SDX relative to VIM.

The eigenvalue comparisons for the full assembly calculations were:

	$\frac{k(\text{MC}^2\text{-2/SDX}) - k(\text{VIM})}{+0.0062 \pm 0.0021}$
ZPR-6/7	
ZPR-9/32	+0.0072 ± 0.0016
ZPR-9/34	+0.0047 ± 0.0026

These calculations show a consistent bias -- the MC²-2/SDX eigenvalues being 0.5-0.7% δk high relative to VIM. The apparent source of the bias is also consistent. For each assembly, the MC²-2/SDX calculations (relative to VIM) produce more absorptions in the core and less leakage from the core.

These δk biases are much larger (by an order of magnitude) than the uncertainties in the integral experiments. However, they are comparable to the combined uncertainties in the analysis methods. A full discussion of methods uncertainties is beyond the scope or intent of the present summary. However, a point of reference on these uncertainties would be useful in assessing the impact of

these δk biases. The methods uncertainty in calculated eigenvalue depends on the nature of the system (i.e., critical assembly). For a well-studied system, that is, one which requires little or no extrapolation from a system for which the analytical methods have been validated and for which methods biases have been determined and/or eliminated, the uncertainties will be comparable to the uncertainties in the validation method. In the present study, this would be the precision of the Monte Carlo methods ($\sim 0.2\% \delta k$). For systems with unique characteristics which require extrapolation of the methods beyond the range of their validation, the estimated uncertainties must be increased. The initial MC²-2/SDX calculations for ZPR-9/34 (the U/Fe benchmark assembly), which produced a bias of over 2% δk , illustrate that point.

For the present case, the consistency of the results indicate the potential of an eigenvalue bias of $\sim 0.5\% \delta k$ between these methods. Further studies are planned to investigate these apparent differences in neutron transport or leakage between the MC²-2/SDX codes and Monte Carlo.

VII. ACKNOWLEDGEMENTS

The authors are indebted to other personnel in the Applied Physics Division at ANL which have participated in the continuing program of code development, testing, and validation, and wish to acknowledge the special assistance of P. Collins, E. Gelbard, H. Henryson II, R. Schaefer, B. Toppel, and D. Wade.

VIII. REFERENCES

1. D. C. Wade, "Monte Carlo-Based Validation of the ENDF/MC²-2/SDX Cell Homogenization Path," ANL-79-5 (April, 1979).
2. D. C. Wade, M. J. Lineberry, and R. E. Prael, "Monte Carlo Analysis of ZPR Heterogeneity," Trans. Am. Nucl. Soc., 21, 446 (June, 1975).
3. R. D. McKnight, "Validation Results for the SDX Cell Homogenization Code for Pin Geometry," Nucl. Sci. Eng., 75, 111-125 (1980).
4. D. E. Shank and R. D. McKnight, "Detailed Monte Carlo Calculations for a Hypothetical Model of ZPR-6 Assembly 7," Trans. Am. Nucl. Soc., 38, 668-669 (June, 1981).
5. R. W. Schaefer, "Calculations of Steam Worth in a GCFR Lattice," Trans. Am. Nucl. Soc., 28, 772-773 (June, 1978).
6. B. J. Toppel, H. Henryson II, and C. G. Stenberg, "ETOE-II/MC²-2/SDX Multigroup Cross Section Processing," RSIC Seminar Workshop on Multigroup Cross Sections, ORNL, March 14, 1978.
7. H. Henryson II, B. J. Toppel, and C. G. Stenberg, "MC²-2: A Code to Calculate Fast Neutron Spectra and Multigroup Cross Sections," ANL-8144, (June, 1976).
8. W. M. Stacey, Jr., B. J. Toppel, H. Henryson II, B. A. Zolotar, R. N. Hwang, and C. G. Stenberg, "A New Space-Dependent Fast-Neutron Multigroup Cross-Section Preparation Capability," Trans. Am. Nucl. Soc., 15, 292 (1972).

9. E. M. Gelbard and R. E. Prael, "Monte Carlo Work at Argonne National Laboratory," Proc. of NEACRP Meeting of a Monte Carlo Study Group, ANL-75-2, p. 201, Argonne National Laboratory (1975).
10. R. N. Blomquist, R. M. Lell, and E. M. Gelbard, "VIM -- A Continuous Energy Monte Carlo Code at ANL," A Review of the Theory and Application of Monte Carlo Methods Proceedings of a Seminar-Workshop, Oak Ridge, TN, April 21-23, 1980, ORNL/RSIC-44, p. 31 (August, 1980).
11. P. Benoist, "Streaming Effects and Collision Probabilities in Lattices," Nucl. Sci. Eng., 34, 285 (1968).
12. E. M. Gelbard, et al., "Calculations of Void Streaming in the Argonne Gas-Cooled Fast Reactor Critical Experiments," Nucl. Sci. Eng., 64, 624-637 (1977).
13. C. E. Till, et al., "ZPR-6 Assemblies 6A and 7: Benchmark Specifications for the Two Large Single-Core-Zone Critical Assemblies -- ^{235}U -Fueled Assembly 6A and Plutonium-Fueled Assembly 7 -- LMFBR Demonstration Reactor Benchmark Program," ANL-7910, 86-101 (1972).
14. S. K. Bhattacharyya, D. C. Wade, R. G. Bucher, D. M. Smith, R. D. McKnight, and L. G. LeSage, "A Critical Experimental Study of Integral Physics Parameters in Simulated Liquid-Metal Fast Breeder Reactor Meltdown Cores," Nucl. Technol., 46, 517 (1979).
15. W. R. Robinson, G. J. DiIorio, J. A. Morman, and R. W. Schaefer, "Comparison of Calculations and Integral Measurements for a Uranium/Iron Benchmark Assembly," Trans. Am. Nucl. Soc., 34, 754 (1980).
16. E. M. Gelbard and R. Lell, "Anisotropic Diffusion in the Presence of Voids," Nucl. Sci. Eng., 63, 9 (1977).
17. E. M. Gelbard, "Lattice Eigenvalue as a Function of Buckling: Correction to First Order Perturbation Theory," Trans. Am. Nucl. Soc., 27, 390 (1977).
18. E. M. Gelbard and R. P. Hughes, "Lattice Eigenvalue as a Function of Buckling: Correction to First Order Perturbation Theory," Nucl. Sci. Eng., 70, 262-273 (1979).

A REVIEW OF PROPOSED INSTRUMENTATION FOR
MEASUREMENT OF INADEQUATE CORE COOLING IN LWRs

Patrick G. Bailey and Mati Merilo
Nuclear Power Division
Electric Power Research Institute
Palo Alto, CA 94303

ABSTRACT

Recent events in the nuclear industry have resulted in an increase of attention and emphasis toward the development and use of instrumentation systems to measure the approach to inadequate core cooling conditions in light water reactors. A large number of instrumentation concepts and systems have been proposed for this purpose. Of the systems which have actually been constructed, few have been thoroughly tested. The various concepts and systems are reviewed and their status, capabilities, and limitations briefly discussed. Two instrumentation systems based upon the use of differential pressure measurements and heated junction thermocouples have been developed and partially tested by Westinghouse and Combustion Engineering, respectively. As the NRC has recently recommended the generic approval of these two systems, many utilities are expected to pursue the use of these systems to satisfy NRC requirements.

INTRODUCTION

After the accident at Three Mile Island, Unit 2 (TMI-2), it was recognized that a direct indication of the water level in the reactor vessel during the event may have been helpful to the operators in understanding the event and terminating the accident. As a result, the Nuclear Regulatory Commission (NRC) has defined requirements in NUREG-0660 (1) and in NUREG-0737 (2) for detection of inadequate core cooling (ICC) in Pressurized Water Reactors (PWRs), and has directly specified the need to measure a "coolant level" in Reg. Guide 1.97 (3). Neither the definition of inadequate core cooling nor water level have been clearly defined in these documents although there appears to be an implication that a low water level signifying a core uncover implies inadequate core cooling. This assumption may be consistent with flow stagnation and boil-off transients, such as that assumed in TMI-2, but may not be meaningful for other transient situations, such as cooling by a two-phase mixture in a small-break transient with the cooling pumps running.

A large number of measurement concepts and instrumentation system designs has been suggested for the measurement of parameters that might indicate an approach to an inadequate core cooling condition in PWRs. Of all the systems proposed, few have been tested to ascertain their ability to function properly in the operating environments expected or to measure appropriate system parameters, such as subcooling, saturation, vessel water level, or fuel temperature.

This paper will discuss many of the various concepts and systems which have been proposed for the determination of water level in PWRs. Further information regarding the concepts and systems discussed is summarized in Reference 4. In addition, three instrumentation systems offered by the PWR

vendors will also be presented since they have already been or are about to be installed in commercial operating reactors. These systems are of particular interest to the nuclear utilities, as the NRC has required that each utility (on a plant-by-plant basis) commit to a choice of a system for installation in each PWR to satisfy the NUREG-0737 requirements by the first refueling outage in 1983. The previous deadline of January 1, 1982, specified in NUREG-0737 was extended by the NRC to allow further development and prototype testing of these devices as well as to allow further documentation of the testing results to the utilities.

VARIOUS PROPOSED ICC MEASUREMENT SYSTEMS

A large number of concepts, devices, and instrumentation systems has been proposed for measuring the approach to ICC or water level in PWRs. A summary list of many of these devices is given in Table 1. Of all the devices listed in Table 1, most are in a conceptual stage of development.

A list of criteria proposed for evaluating these instrumentation systems is given in Table 2. From the utility viewpoint, the primary evaluation criteria are the availability, testing, installation and retrofit, and operational effectiveness of the instrumentation. Data quality, reliability, and maintenance are also important factors. The actual cost of the systems is an important consideration and technically should perhaps be considered as the least important factor.

The various concepts and instrumentation systems are separated into three classifications for discussion: (1) nonintrusive, (2) semi-intrusive, and (3) intrusive devices. These classifications are convenient as they describe the relationship of the measured concept to the reactor vessel pressure boundary. This provides a measure of the complexity of the installation and any possible interference between the device and the normal operation of the primary coolant system.

Nonintrusive instruments are defined as being placed completely outside the reactor pressure vessel boundary. Such devices will not in any manner interfere with the operating behavior of the reactor or any plant parameters being measured. Semi-intrusive instruments require some penetration into the pressure vessel boundary and, in general, are not expected to interfere with system operation or other measured parameters. Intrusive instruments have components inside the pressure vessel that may interfere with system operation or measurement of other system parameters.

1. Nonintrusive Measurement Devices

A. Existing Neutron Detectors

Source and power range monitors are located outside the reactor vessel in all reactors to measure the neutron flux during shutdown, power ascension, and normal operation. Usually, as in the case of TMI-2, these detectors are neutron detectors filled with boron-trifluoride gas and are situated at diametrically opposite sides of the core midplane, outside the reactor pressure vessel. While the primary purpose of these detectors is the measurement of power level, these instruments have been used to infer the two-phase mixture level during the TMI-2 accident. The results indicate that such

instrumentation alone cannot indicate unambiguously the two-phase or collapsed liquid level without the interpretation of other instrumentation data (5). As the analysis of TMI-2 indicates, such detectors may be useful in monitoring void trending information but are not useful in uniquely determining a single- or two-phase level of the coolant in the vessel.

B. Externally Placed Neutron Detectors

A measurement system employing assemblies of neutron detector assemblies placed above and below the reactor vessel has been designed and tested, in part, at the Farley Unit One plant. These assemblies measure the total neutron flux at the assembly location using boron-trifluoride neutron detectors surrounded by a polyethylene moderator. A schematic of a typical assembly is shown in Figure 1. A lead plate at the bottom of each assembly was used for gamma attenuation.

Various tests have been performed to assess the usefulness of such a concept to measure water level. A series of drain down and refill tests were performed at Farley Unit One in November 1980 to measure the count rate of four of the assemblies placed above the top of the vessel head as a function of the water level in the vessel four days after a normal plant shutdown. The results from these tests are compared with computer predictions of the test in Figure 2. The results indicate that this concept can detect a slight increase of count rate when the water level dropped to within about four feet from the top of the core. A more detailed description of these tests and results is given in Reference 6.

Additional draindown tests are scheduled at Farley Unit Two to test the effectiveness of measuring both the top and bottom count rates and the top-to-bottom count rate ratios to measure water level using the system illustrated in Figure 3. These tests are scheduled to occur during a normal plant outage in late 1982.

Another concept of this type is the use of an array of fast neutron detectors placed vertically outside the reactor vessel (7). In this arrangement, the bottom detector could serve as a reference point and the count rate data from the other detectors could infer a water level. Fission chambers are proposed to measure the fast neutron flux outside the vessel and the calculations to infer a water level would be performed with an on-line minicomputer.

While these nonintrusive radiation measurement systems can appear very desirable, their ability to infer an accurate water level at times after reactor shutdown still remains to be proven. Difficulties in obtaining data using these systems can arise from a variety of reasons, including: detector saturation from high counting rates, poor counting statistics due to low counting rates, and the inability to measure the desired effects due to interference from other sources of types of radiation.

2. Semi-Intrusive Measurement Devices

A. Self-Powered Neutron Detectors

Self-powered neutron detectors (SPNDs) are generally used in PWR core designs to measure the radial and axial thermal neutron flux distribution within the core. As they are located within the core region, they cannot by themselves be used to detect an approach to inadequate cooling, although they can provide an indication that such conditions have been reached.

B. In-Core Thermocouples

Core exit thermocouples are generally employed in all PWR designs. By themselves, they provide a viable method of monitoring core exit coolant temperatures. When used in conjunction with local pressure measurements, they can be used to infer local subcooling, saturation, and superheat conditions in the core exit region. However, these measurements do not appear to be sufficient by themselves for determining an approach to ICC or directly indicating a water level in the vessel.

C. Gamma Thermometers

Gamma thermometers have recently been employed in many reactors throughout the world to provide a more inexpensive detailed mapping of core power distributions than is generally available from SPND measurements. These gamma thermometers typically employ two thermocouples per measurement location to infer the local power level. A three thermocouple, or dual purpose, gamma thermometer design has been proposed to measure both reactor power and coolant water level throughout the core (8,9). A schematic of this proposed design is shown in Figure 4.

Simple calculations indicate that a ratio of temperature differences ($T_{hot}-T_{mid}/T_{mid}-T_{cold}$) might be used to infer changes in local heat transfer in the core region. Results of these analyses indicate that this temperature ratio may be relatively independent of reactor power and strongly dependent on the local heat transfer coefficient, or water level. A predicted response of this device for a simulated scram and simulated instantaneous loss-of-coolant accident (LOCA) is given in Figure 5. Further research and testing are required to demonstrate the operational effectiveness of this device to measure approaches to ICC in PWRs.

D. Differential Pressure Measurements

Differential pressure (Δp) measurements have been used for many years for monitoring the liquid inventory level in many various applications. Both Babcock & Wilcox and Westinghouse Electric Corporation are proposing the use of such systems to monitor approaches to ICC in the PWR vessel. While the major advantage of this concept is its relative simplicity, there may be difficulties in interpreting the measurements when coolant is being pumped through the system.

Differential pressure measurements have been compared against thermocouple measurements and gamma densitometer measurements in a variety of tests, including small break tests at Semiscale, low- and intermediate-flow tests at

the Oak Ridge National Laboratory (ORNL), and BWR low flow bundle uncover tests at General Electric Company. Typically, these tests all indicate that delta-p measurements can give an accurate measurement of level under no-flow and low-flow rate conditions. At higher flow rates, as observed in the ORNL tests, the levels calculated by the delta-p measurements can be much greater than the actual levels (10). The interpretation of these results is that the frictional pressure drop becomes an important factor at higher flow rates and contributes to the differential pressure readings.

The results of these tests indicate that differential pressure measurements can give very good indications of PWR vessel inventory when the pumps are off. With the pumps on, such measurements give qualitative information which may show important trending information.

3. Intrusive Measurement Devices

A. Conductivity Probes

Conductivity probes have been developed for extensive use in the loss-of-fluid test (LOFT) facility (11). The probes have been used to measure liquid level at various locations in many of the LOFT tests. A typical probe schematic is shown in Figure 6. Results of LOFT tests indicate that these probes can infer the liquid level when a distinct steam-water interface exists and have difficulty inferring a level when two-phase flow is present, due to the continued rewetting of the probe. Therefore, these probes are applicable for some of the LOFT tests but are not yet suitable for PWR applications.

B. Torsional-Wave Acoustic Sensors

A torsional-wave device has been proposed to measure water level in PWR vessels by measuring the time required for a vibrational wave to travel down a flat steel ribbon and reflect on machined notches in the ribbon (12). A schematic of this device is given in Figure 7. Tests have been performed using this type of device at ORNL that indicate accurate measurements can be made of a distinct steam-water interface when corrections for coolant temperature along the ribbon are included. Drawbacks of this system include its untested ability to measure a two-phase froth interface and the effects of noise or vibrational interference in a PWR. Further testing is required to assure the applicability of this design.

C. Time Domain Reflectometry Devices

Three separate concepts have been proposed to measure a distinct water level or steam-water mixture density as a function of depth using the reflected transit times of electrical impulses. Each of these systems is only conceptual at the present time since test results demonstrating the effectiveness or usefulness of these devices are not available.

An electrical transmission design consisting of two parallel conductors separated by an insulator running vertically into the reactor vessel is one proposed concept. In this design, changes in impedance at air- or steam-water interfaces would reflect electrical pulses transmitted down the conductors. From an analysis of the reflected wave pulses, it is proposed that impedance changes as a function of depth, and hence, a water level or water density, may be calculated.

Another concept is that of using frequency-modulated (FM) microwaves to determine a water interface (13). The microwave system would consist of a transmitter, receiver, and waveguide as illustrated in Figure 8. The principle of operation of this device relies on taking the inverse Fourier Transform of the reflected electrical signals to obtain an analysis of the signal strength vs. depth in the vessel as illustrated in Figure 9. From this information, a water level could be estimated. Although the principles upon which this device is based are very sound, the operation and usefulness of this device have not been tested in typical operating reactor conditions. One potential disadvantage of this device is the waveguide penetration that would be required in the vessel head to allow passage of the microwave signals.

Another proposed concept incorporates the use of pulsed lasers through sapphire or yttralex fiber optics light pipes down into the reactor vessel (14). A schematic diagram of this concept is shown in Figure 10. In this concept, the intensity of the reflected light pulses is a measure of the medium surrounding the fiber, so that it may be possible to electronically decipher the reflected signals to determine a liquid level. The use of special optical fibers that can withstand expected reactor environments is crucial. Darkening of the fibers due to radiation exposure and photobleaching techniques of the fibers to counter this effect are not at present fully understood under expected PWR conditions.

D. Floating Source Methods

Various concepts involving the use of a high intensity radiation source floating in the reactor vessel have been proposed. These methods suffer from problems similar to the nonintrusive radiation detectors in Section 1 above. While simple in concept, such devices would be extremely difficult to install and most difficult to monitor and interpret, even after reactor shutdown.

E. Neutron Thermalization Gauges

A device composed of a series of neutron thermalization gauges has also been proposed to be used to detect the presence or absence of coolant axially in the vessel (15). In this device, each gauge would detect neutrons thermalized from Cf-252 sources located near a neutron detector. Limited tests have been performed to show the effectiveness of these devices for various moisture detection applications. However, they have not been tested to show their effectiveness in reactor operating conditions or in saturated steam or two-phase flow conditions.

F. Resistive Thermal Devices

Commercially available resistive thermal devices (RTDs) are used widely by various industries for specific applications. Specially designed RTDs were designed for PWR use and were tested at the Transient Instrument Test Facility at ORNL (16). The results indicate that these RTDs can be effective for measuring the loss-of-coolant in a saturated steam environment. No test results were presented under two-phase flow conditions. Additional testing of these RTD assemblies in representative guide tube environments will be necessary to assess their ability to operate in a representative PWR environment.

G. Heated Junction Thermocouples

The heated thermocouple concept has been extensively used to measure the temperature at various locations in reactor facility tests performed at the Idaho National Engineering Laboratory. The NRC has undertaken a program at ORNL to test the heated thermocouple concept under simulated normal and abnormal PWR operating conditions and to evaluate their use for inadequate cooling measurement devices for nuclear plant applications.

Many heated junction thermocouple (HTC or HJTC) designs have been proposed, developed, and tested. The HTC ORNL I probe design is illustrated in Figure 11. Common to all HJTC probe designs is the use of a splash shield to shield the probes from dispersed droplets during two-phase flow conditions. While the design of the probe and splash shield will affect somewhat the operation of the probe assembly in transient conditions, the operation of all HJTC probes is essentially the same; i.e., to indicate the presence or absence of coolant in the thermocouple region by a measured temperature difference. An illustration of the steady-state temperature differences from the HTC ORNL I probe in liquid and steam is given in Figure 12.

Many various concepts have been suggested that incorporate the use of HJTCs to measure water level. All these designs are discussed in Reference 4. The design proposed by Combustion Engineering is discussed in the next section.

VENDOR PROPOSED INSTRUMENTATION SYSTEMS

The three major PWR vendors have each proposed an instrumentation measurement system to satisfy the NUREG-0737 requirements. These systems are described in detail elsewhere and are included in this paper for completeness (4,17)

A. Westinghouse Reactor Vessel Level Instrumentation System (RVLIS)

The Westinghouse RVLIS design utilizes differential pressure measurements to determine a vessel inventory as illustrated in Figure 13 (13). The system utilizes differential pressure measurements from the top-to-bottom and hot leg-to-bottom of the vessel to determine a representative vessel inventory. A recent review of this system recommends that it receive NRC generic approval to satisfy the NUREG-0737 requirements (18). Although many utilities have committed to the purchase and use of this system and many tests have been conducted in the Semiscale Test Facility, prototype tests to demonstrate the correct operation of the system have not been conducted under both representative pumps-on and pumps-off conditions.

B. Babcock & Wilcox Hot Leg Level Monitoring System (HLLMS)

A system proposed by Babcock & Wilcox determines the approach to inadequate core cooling by measuring the differential pressures between the top of the hot leg, the vessel hot leg exit, and the top of the vessel. A schematic of this system is shown in Figure 14 (19). As seen in the figure, the system is designed to measure the inventory or level in the hot leg and not the coolant level in the vessel. The NRC is examining whether or not this system is acceptable for satisfying the NUREG-0737 requirements.

C. Combustion Engineering HJTC System

Combustion Engineering, Incorporated (C-E) has proposed a system utilizing a series of HJTC probes in a long probe assembly to measure the water level in the upper head region above the upper fuel support plate (20). Schematics of the HJTC probe assembly design and principle of operation are given in Figures 15 and 16. From these figures, it is seen that this design is essentially a differential pressure system with the holes at only the ends of the separator tubes playing the role of pressure taps. The HJTCs then measure the level of the collapsed liquid level in the separator tube. A recent review of this system also recommended that it receive NRC generic approval to satisfy the NUREG-0737 requirements (21).

CONCLUSIONS

The post-TMI requirements to provide measurements of ICC and water level in PWRs have resulted in a great deal of concepts, instrumentation design, and technical work on this subject. A detailed review is given in Reference 4. Of the large number of ideas proposed, only a few have been tested. One conclusion from this review is that it is clear from assessing these systems and devices that none of them will accurately satisfy all of the NUREG-0737 requirements as stated to measure the vessel coolant inventory under all conditions. In particular, with the pumps on, all of these systems will give some indication that coolant is being lost, but the quantification of the amount of coolant remaining is still elusive. For the safe operation of the plant this may not be an important point since the operator is required to take corrective action upon any indication of unexpected loss in inventory.

The NRC staff has participated in many conferences and meetings to review in detail the proposed instrumentation concepts discussed in this paper. The most recent meetings during the time of this writing were held March 31 through April 1, 1982 (22,23). As a result of these meetings, the NRC has concluded that the proposed Westinghouse system and proposed C-E system are acceptable on a generic basis, subject to further exploration of a small number of unresolved issues.

The utilities are required to commit to a choice of an inventory monitoring or water level indicator instrumentation system on a plant-to-plant basis before their respective first refueling outage in 1983. While the systems discussed above appear promising, much concern has been expressed by the ACRS and others regarding the commitment, installation, and use of such systems before they have been fully and adequately tested under all postulated reactor transient and accident conditions (24,25).

ACKNOWLEDGMENTS

The authors would like to thank S. Untermeyer of National Nuclear Corporation, W. Lyman of Westinghouse Electric Corporation, and C. Neuschaefer of Combustion Engineering Inc. for providing detailed information of the water level measurement systems proposed by their respective companies.

REFERENCES

1. NRC Action Plan Developed as a Result of the TMI-2 Accident, U.S. Nuclear Regulatory Commission Report NUREG-0660, Vols. 1 and 2, May 1980.
2. Clarification of TMI Action Plan Requirements, U.S. Nuclear Regulatory Commission Report, NUREG-0737, November 1980.
3. Instrumentation for Light-Water-Cooled Nuclear Power Plants to Assist Plant and Environs Conditions During and Following an Accident, U.S. Nuclear Regulatory Commission Regulatory Guide 1.97 (Task RS 917-4), Revision 2, December 1980.
4. A Review of Proposed Instrumentation for Measurement of Water Level As a Means of Detecting Inadequate Core Cooling in Light Water Reactors, Electric Power Research Institute Special Report, to be published.
5. Analysis of Three Mile Island - Unit 2 Accident, EPRI Report NSAC-80-1, Revision 1, March 1980.
6. Water Level Measurements Using Ex-Core Neutron Detectors at Farley Unit One, Electric Power Research Institute Final Report, NP-2354, April 1982.
7. "A Concept for a Fast Neutron, Non-Intrusive, Liquid Level and Density Gauge for Nuclear Power Reactors," W. Jester et al., USNRC Water Reactor Safety Research Meeting, October 1981.
8. Theoretical Characterization of a Dual Purpose Gamma Thermometer, T. Burns and J. Johnson, Oak Ridge National Laboratory Report ORNL/TM-7567, January 1981.
9. Thermal-Hydraulic Analysis of the Dual-Function Gamma Thermometer, J. Johnson and T. J. Burns, Oak Ridge National Laboratory Report ORNL/TM-8089, December 1981.
10. Advanced Two-Phase Flow Instrumentation Program Quarterly Progress Report for April-June 1981, Oak Ridge National Laboratory Report NUREG/CR-2204, Vol. 2, ORNL/TM-8010, October 1981.
11. Conference minutes, LOFT/Utility Technology Transfer Meeting, Idaho National Engineering Laboratory, October 16-17, 1980.
12. Advanced Two-Phase Flow Instrumentation Program Quarterly Progress Report for January-March 1980, Oak Ridge National Laboratory Report NUREG/CR-1647, ORNL/NUREG/TM-403, September 1980.
13. Summary of EPRI and Utility Sponsored Research in Non-Intrusive Reactor Vessel Water Level Monitoring, Meeting on Reactor Vessel Liquid Level Measurement. U.S. Nuclear Regulatory Commission Report NUREG/CP-0016, Office of Nuclear Regulatory Research, January 1981.
14. D. R. Dunn, Lawrence Livermore National Laboratory, personal communications, 1982.

15. "Demonstration of Reactor Vessel Water-Level Gauge Based on Neutron Thermalization," Science Applications, Inc., personal communication, 1981.
16. Advanced Two-Phase Flow Instrumentation Program Quarterly Progress Report for April-June 1980, Oak Ridge National Laboratory Report NUREG/CR-1768, ORNL/NUREG/TM-414, December 1980.
17. "Some Coolant Inventory Measurement Methods for Pressurized Water Reactors," M. Merilo and P. Bailey, ASME Paper 82-NE-38, Second Joint ASME/ANS Nuclear Engineering Conference, July 25-28, 1982.
18. Adequate Core Cooling Instrumentation Using Differential Pressure for Reactor Vessel Level Measurement, Oak Ridge National Laboratory Report NUREG/CR-2628, ORNL/TM-8269, March 1982.
19. "Discussion of Reactor Vessel Water Level Indicators," Transcription of the Meeting of the U.S. Nuclear Regulatory Commission Meeting, January 8, 1982.
20. Neuschaefer, C. H., "A Reactor Vessel Level Monitoring System, An Aid to the Operators in Assessing an Approach to Inadequate Core Cooling," 1981 Nuclear Science Symposium, October 21-23, 1981.
21. Inadequate Core Cooling Instrumentation Using Heated Junction Thermocouples for Reactor Vessel Level Measurement, Oak Ridge National Laboratory Report NUREG/CR-2627, ORNL/TM-8268, March 1982.
22. Transcription of the Advisory Committee on Reactor Safeguards, Subcommittee on Electrical Power Systems and Emergency Core Cooling Systems, March 31, 1982.
23. Transcription of the Advisory Committee on Reactor Safeguards, 264th General Meeting, April 1, 1982.
24. "ACRS Position on Reactor Vessel Level Instrumentation," C. Mark, USNRC Advisory Committee on Reactor Safeguards, position statement, January 8, 1982.
25. "NRC Staff to Prescribe Training Before Use of Inventory Monitors," Inside N.R.C., April 19, 1982.

Table 1

QUALITATIVE COMPARISON OF PROPOSED ICC INSTRUMENT CONCEPTS

	DEVICE	MEASURES	INTERPRETATION	OPERATIONAL EFFECTIVENESS	DATA QUALITY
1.	Source Range Detector	Thermal Neutron Flux	Local Coolant Density	Poor - Single Measurement Location	Poor - Limited Calibration
2.	Externally Placed Radiation Monitors	Fast or Thermal Flux	Local Coolant Density	Unproven	Unproven
3.	Weighing Vessel	Vessel Weight	Coolant Inventory	Unproven	Unproven
4.	Self-Powered Neutron Detectors	Thermal Neutron Flux	Local Coolant Density	Poor	Unproven
5.	In-Core Thermocouples	Local Coolant Temperature	Local Coolant Phase	Good	Good
6.	Gamma Thermometers (2 TC)	Comparative Local Heat Transfer	Local Coolant Phase	Poor	Fair
7.	Gamma Thermometers (3 TC)	Comparative Local Heat Transfer	Local Coolant Phase	Unproven	Unproven
8.	Delta-P Measurements	Pressure Differences	Density of Coolant	Good, Generally	Good, Pumps Off
9.	Conductivity Probes	Local Electrical Conductivity	Local Coolant Phase	Fair	Fair
10.	Acoustic Sensors	Reflected Vibrational Waves	Water Level	Fair	Fair
11.	TDR Devices	Reflected Electrical Waves	Water Level	Fair	Fair
12.	Microwave Devices	Reflected Microwaves	Water Level	Unproven	Unproven
13.	Floating Source	Neutron Gamma Flux	Water Level	Unproven	Unproven
14.	Neutron Thermalization Gauge	Thermal Neutron Flux	Water Level	Unproven	Appears Good
15.	Resistive Thermal Devices	Comparative Local Heat Flux	Coolant Phase	Good	Good
16.	Heated Thermocouples	Comparative Local Heat Flux	Coolant Phase	Good	Good

Table 2

CRITERIA FOR EVALUATION OF ICC INSTRUMENTATION

1. Operational Effectiveness
 - Requirements
 - Design Concept
 - Operation Regimes
 - Actual Testing
 - Accuracy
 - Operator Displays
2. Data Quality
 - Unambiguous Data
 - Independent Data Measurements
 - In Situ Verification
 - In Situ Calibration
3. Reliability
 - No Spurious Signals
 - Continuous Performance
4. Survival
 - Performs During and After All Transients
 - Satisfies IEEE and NUREG Standards
5. Installation and Retrofit
 - Minimum New Penetrations
 - No Impact on Plant Operation
 - No Impact on Plant Availability
6. Maintenance
 - No Operator Required Action
 - Minimum Servicing
7. Cost

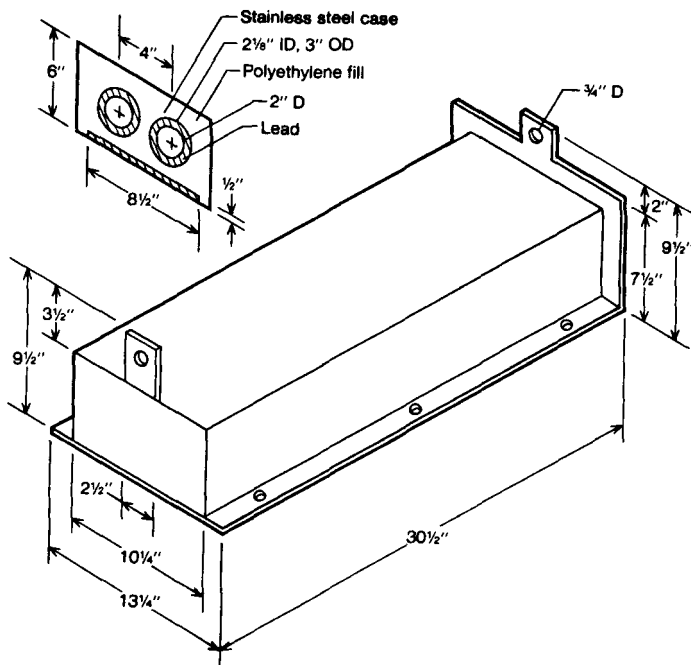


Fig. 1. Farley Neutron Detector Assembly Design

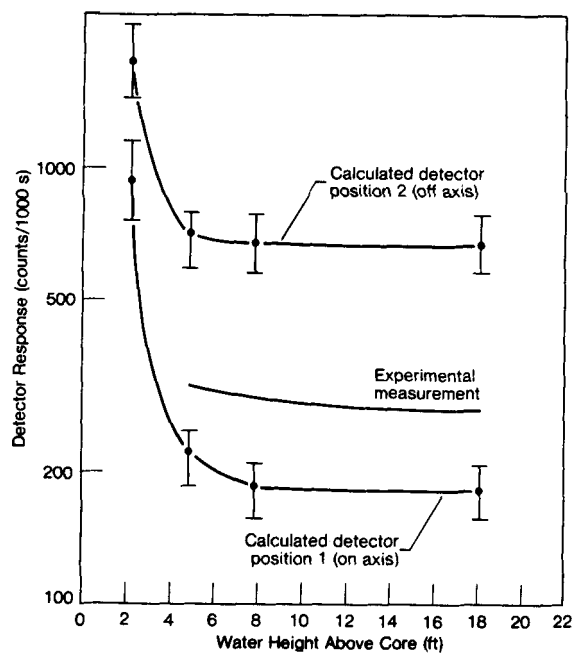


Fig. 2. Farley Unit One Neutron Detector Test Results

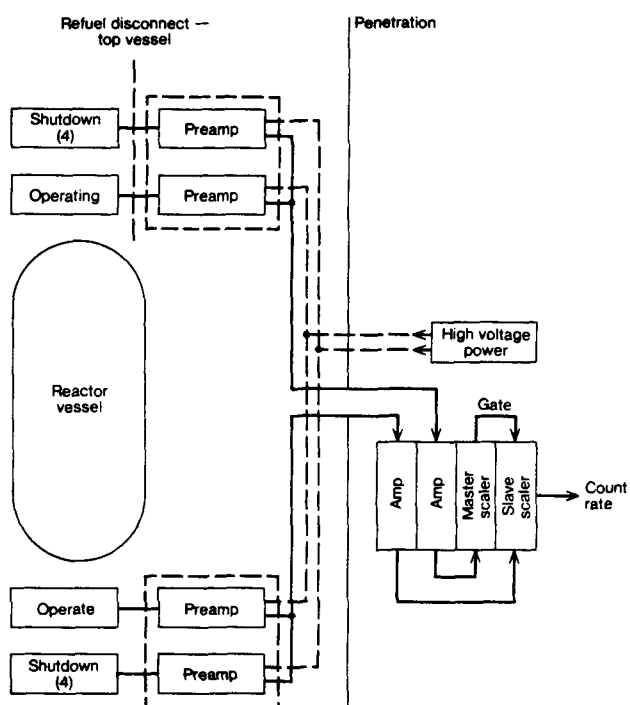


Fig. 3. Non-Intrusive Neutron Detector Measurement System

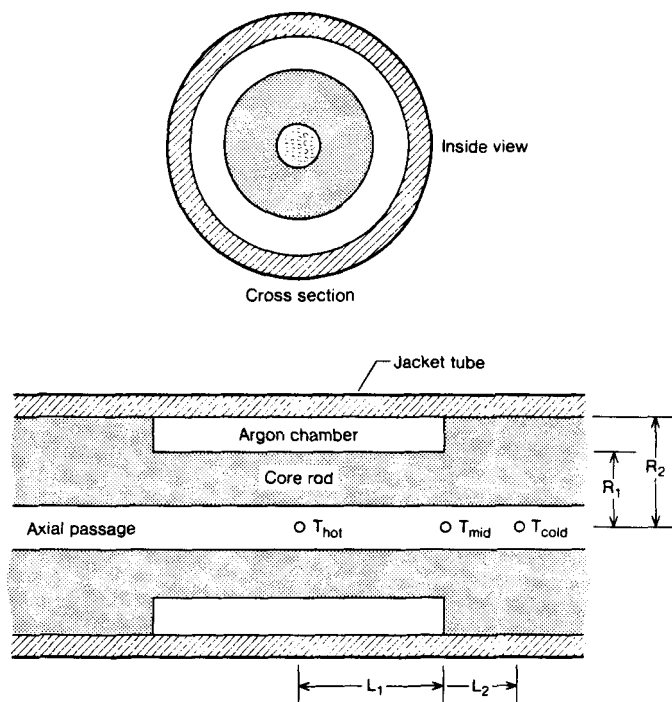


Fig. 4. Schematic of a Dual Purpose PWR Gamma Thermometer

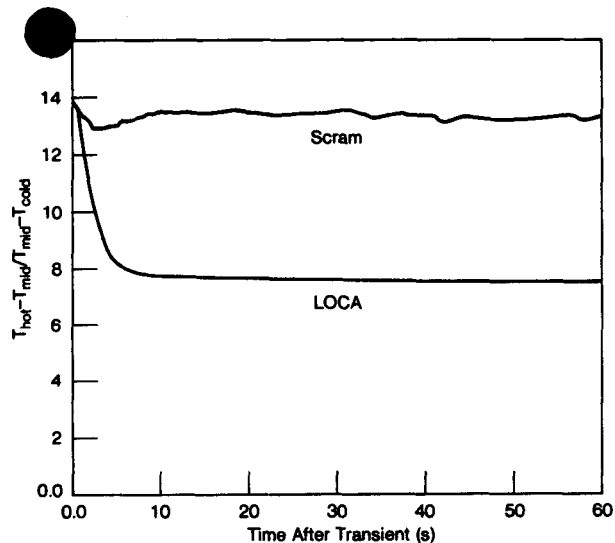


Fig. 5. Calculated Dual Purpose Gamma Thermometer Responses

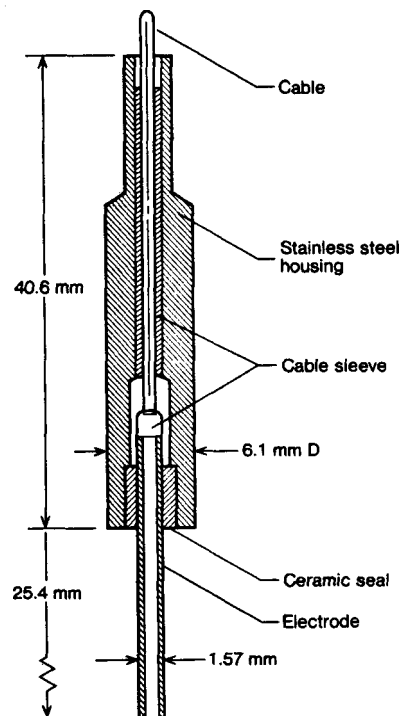


Fig. 6. Schematic of Conductivity Probe

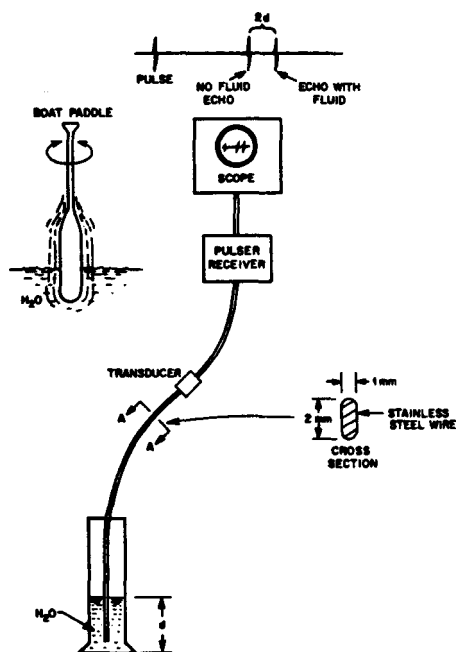


Fig. 7. Schematic of Torsional Wave Acoustic Sensor

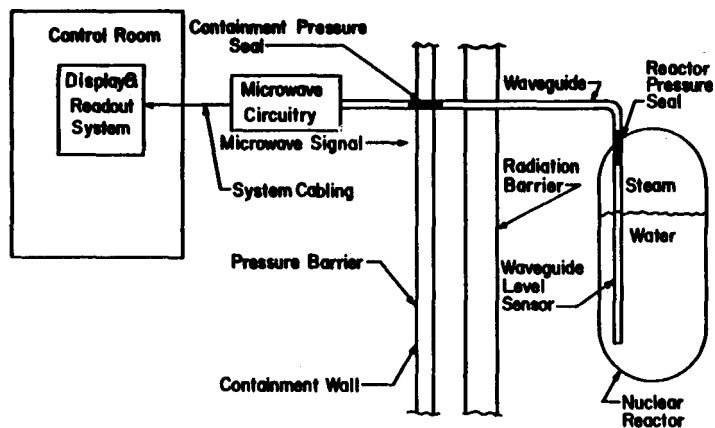


Fig. 8. Schematic of PM Microwave System

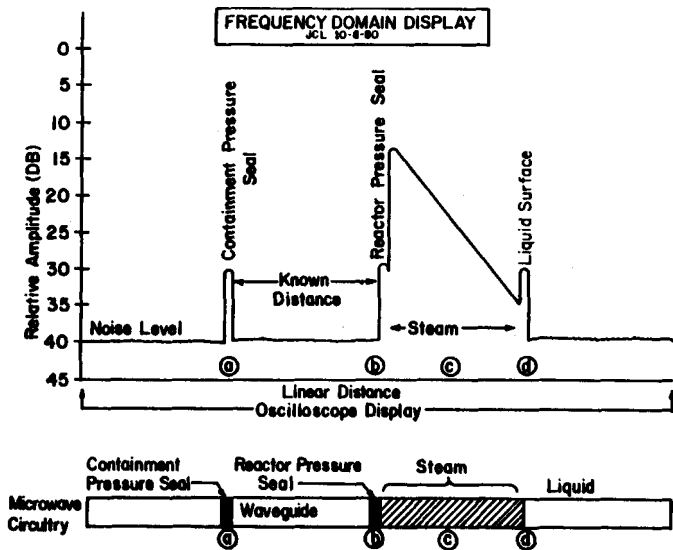


Fig. 9. Conceptual FM Microwave System Frequency Domain Display

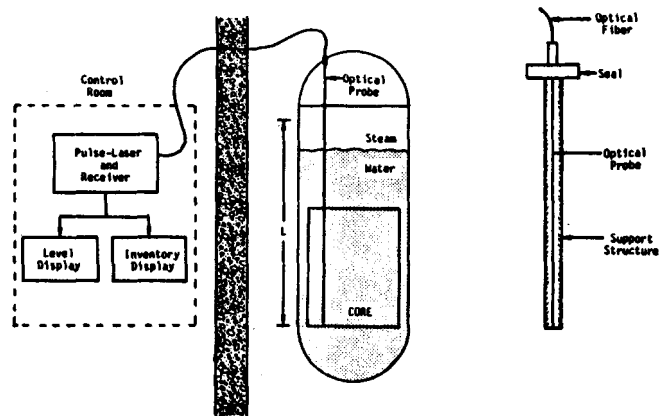


Fig. 10. Schematic of Pulsed Laser Fiber Optics Sensor System

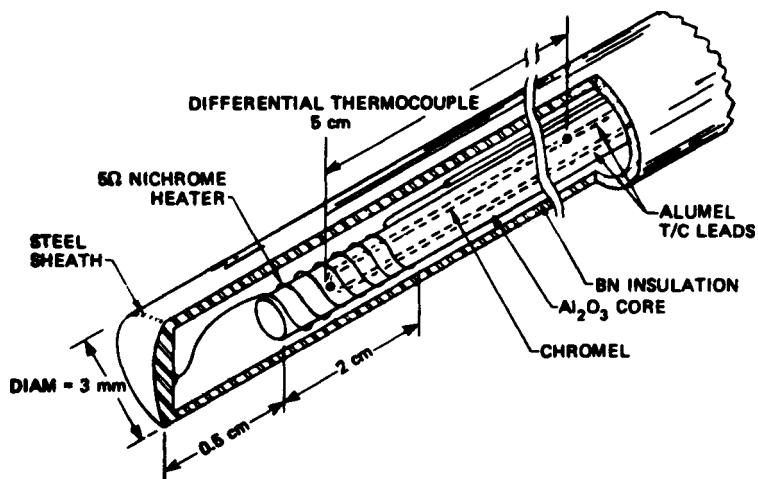


Fig. 11. Schematic of ORNL HTC I Probe Design

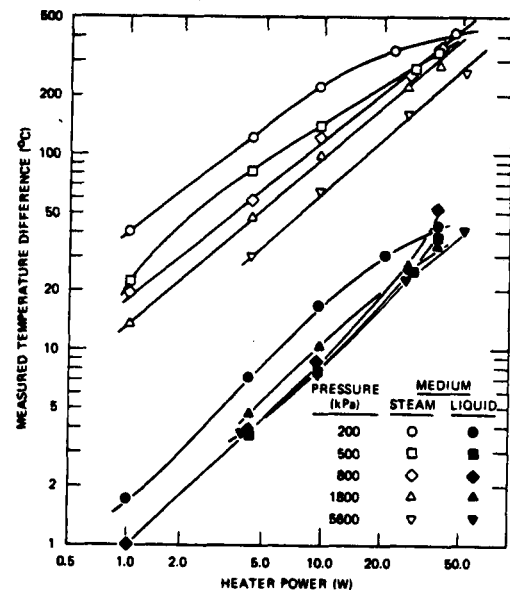


Fig. 12. Steady State Temperature Differences Recorded with HTC ORNL Probe I.

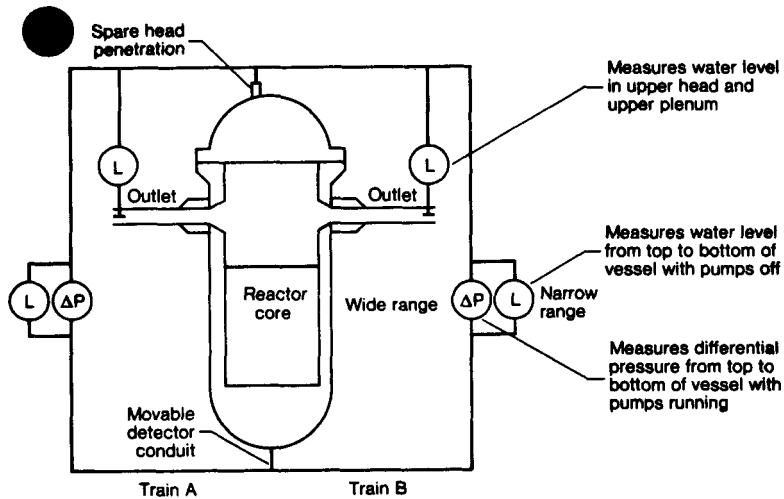


Fig. 13. Schematic of Westinghouse Reactor Vessel Instrumentation System

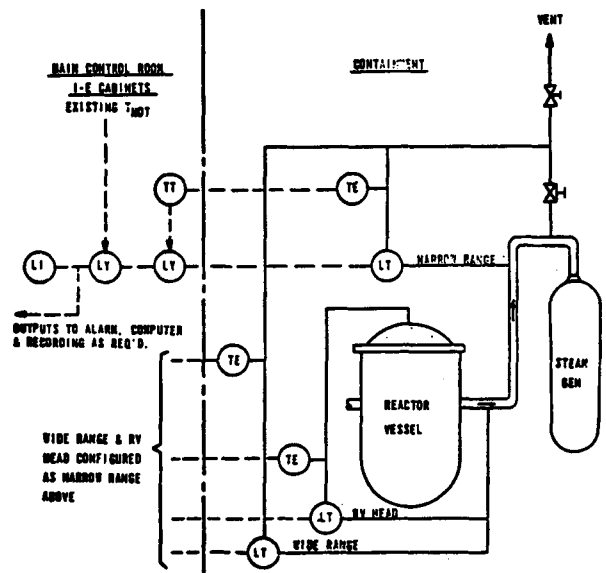


Fig. 14. Schematic of Babcock & Wilcox Hot Leg Level Monitoring System

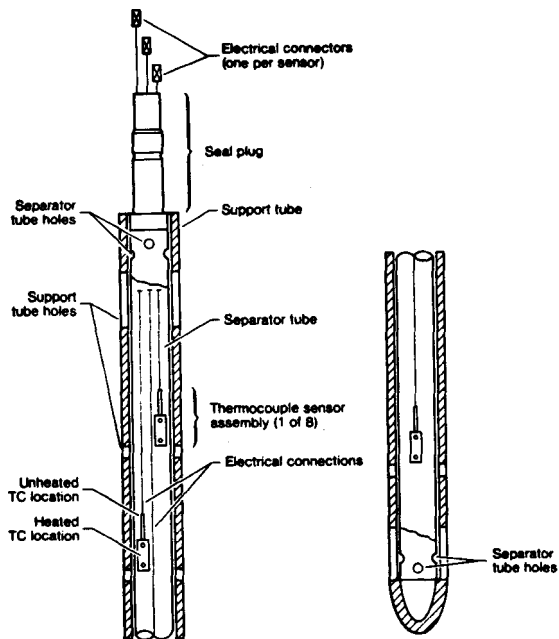


Fig. 15. Schematic of C-E HJTC Probe Assembly Design

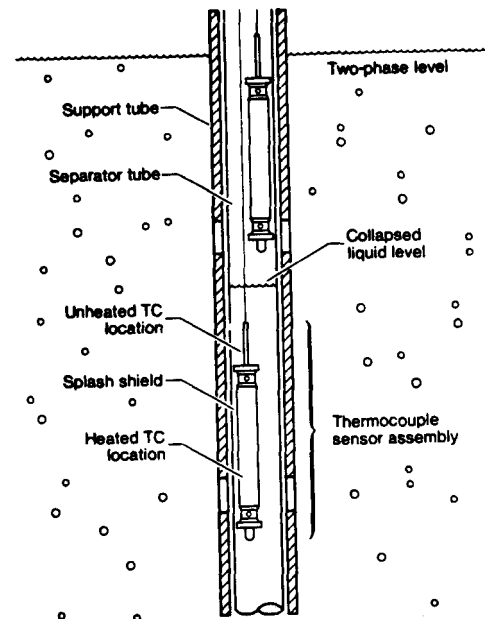


Fig. 16. Schematic of C-E HJTC Probe Principle of Operation

LOSS-OF-FLUID TEST (LOFT)

IN-CORE THERMAL-HYDRAULIC MEASUREMENTS^a

D. B. Engelman, D. J. Hanson, H. E. Knauts, M L. Russell
EG&G Idaho, Inc.

ABSTRACT

The LOFT Project performs experiments to investigate the thermal-hydraulic and fuel response of a reactor system during simulated off-normal occurrences, including large- and small-break loss-of-coolant accidents (LOCAs), operational transients, and anticipated transients without scram (ATWS). The complex phenomena that are of interest in this diverse experimental program are characterized using a large set of state-of-the-art measurement systems. Included in this set are a group of measurements which are specifically designed to characterize the hydraulic and thermal behavior of the nuclear core. These unique core measurements have evolved through many years of research and development at the Idaho National Engineering Laboratory (INEL). This successful evolution was made possible through use of uncommon materials and original processes for the construction and attachment of the devices. In general, the in-core measurements have provided excellent results while showing good resistance to the harsh environment.

INTRODUCTION

LOFT is a 55 MW(t) reactor system designed to provide information on system response during off-normal events; including large- and small-break LOCAs, operational transients, and anticipated transients without scram (ATWS). The core region is heavily instrumented to provide both thermal-hydraulic and fuel response information. In many cases, the instruments are unique and are the result of many years of developmental effort.

The LOFT core has nine fuel bundles, with six of the bundles instrumented, as represented by Figure 1. Approximately one-half of the core is uninstrumented, so that any perturbation caused by the presence of the instruments can be assessed. The instruments are arranged throughout the core in such a pattern as to give information on both axial and radial temperature, liquid level, and mass flow.

Temperature measurements are made throughout the instrumented fuel bundles and include fuel, cladding, guide tube, and fluid temperatures. Due to space limitations, the hydraulic measurements are primarily on the center fuel module (CFM), at the core inlet and core outlet. These measurements include density, velocity, mass flow and fluid temperature. Hydraulic conditions at other positions within the core are determined using calculational techniques.

a. Work supported by the U.S. Nuclear Regulatory Commission, Office of Reactor Safety Research under DOE Contract No. DE-AC07-76ID01570.

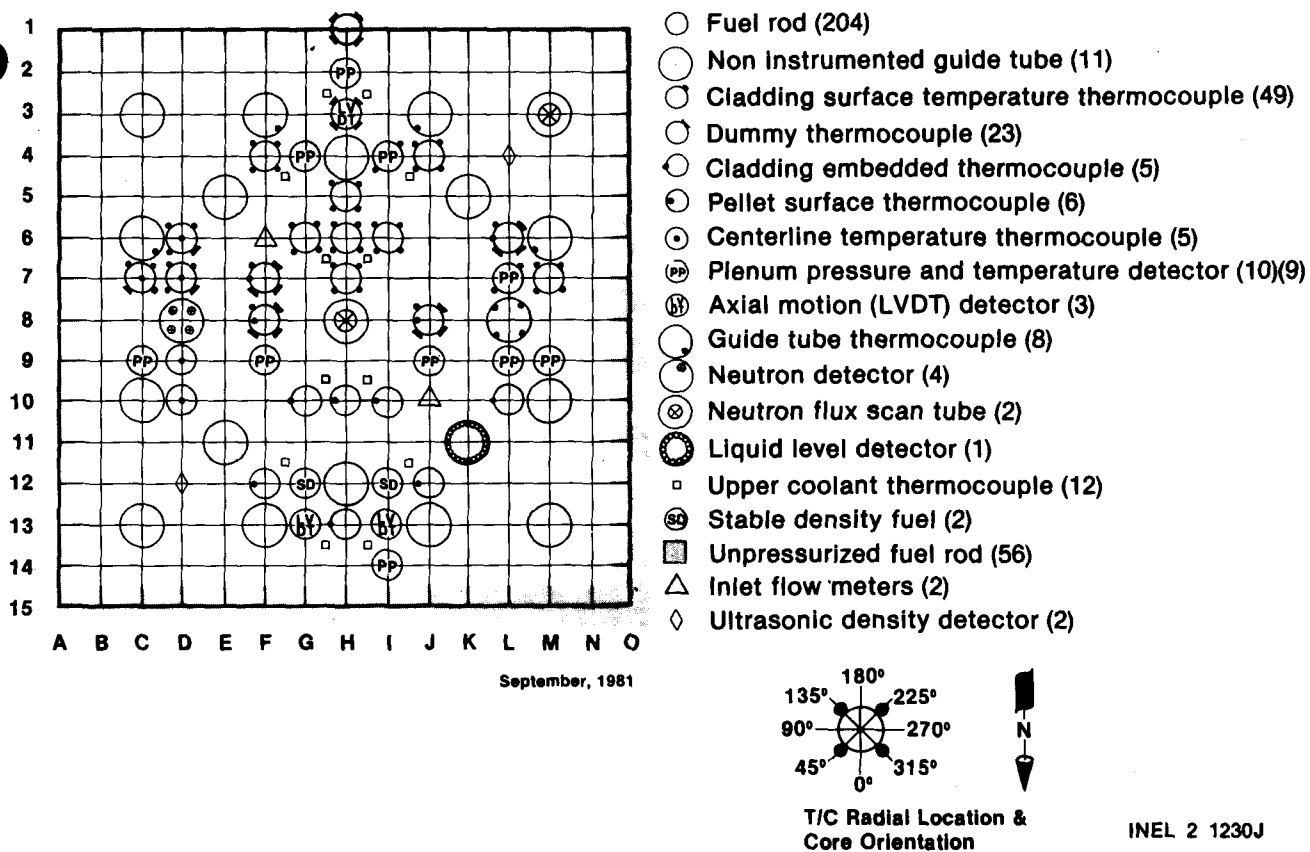


FIGURE 1. LOFT F1 CENTER FUEL BUNDLE MEASUREMENT LOCATIONS

In general, measurements in the core region must survive severe environmental conditions such as high temperatures, pressures, and radiation levels. LOFT operating conditions are typical of large pressurized water reactors (LPWR) as shown in Table 1. Not only must the instruments and instrument cables survive normal operation, but also the more extreme off-normal conditions encountered during accident simulations (experiments). The instruments may be required to survive several thousand hours of normal operation and multiple transients or experiments.

TABLE 1. LOFT COMPARED TO A LARGE PRESSURIZED WATER REACTOR

Parameter	LOFT	LPWR (4 x 4)
Design pressure, MPa	17.2	17.2
Design temperature, K	617	617
MLHGR, kW/m	52	52 + 62
Power, MW	50	3,250
PCS volume, m ³	7.8	356
Power/volume ratio, MW/m ³	6.4	9.1
Number of fuel rods	1,300	39,372
Core length, m	1.68	3.66

HYDRAULIC MEASUREMENTS

General hydraulic conditions are determined at the upper and lower boundaries of the center fuel module (CFM) by measurements of fluid velocity, momentum flux, density, and pressure and temperature. LOFT uses a unique drag-disc turbine instrument combination for velocity and momentum flux. A newly-developed ultrasonic density detector is used for density measurement. There are also a number of pressure taps in the core region; most of these applications use standard transducers and techniques and will not be covered.

Velocity is measured by use of turbines at the center fuel bundle core inlet and outlet. LOFT Turbines (see Figure 2) are mated to a drag-disc unit (for momentum flux information) and mounted within a shroud (see Figure 3), in a drag-disc turbine transducer (DTT). This DTT is then mounted at two places in the CFM core inlet, and one place at the CFM outlet. The core inlet orifices, where the DTTs are mounted, have been sized to allow flow identical to an orifice without instruments. This sizing is accomplished by the use of specially developed orifice inserts.

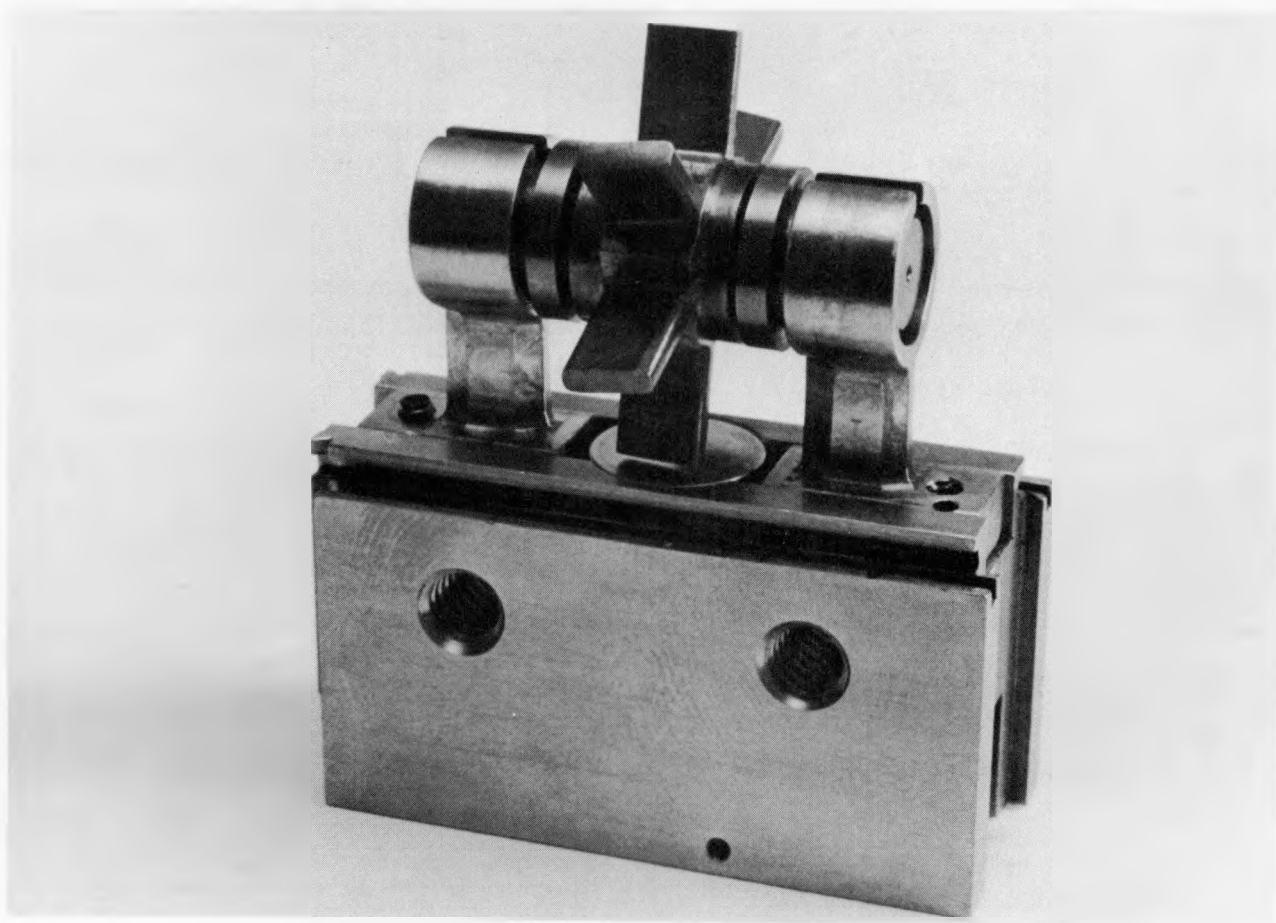


FIGURE 2. ASSEMBLED LOFT TURBINE

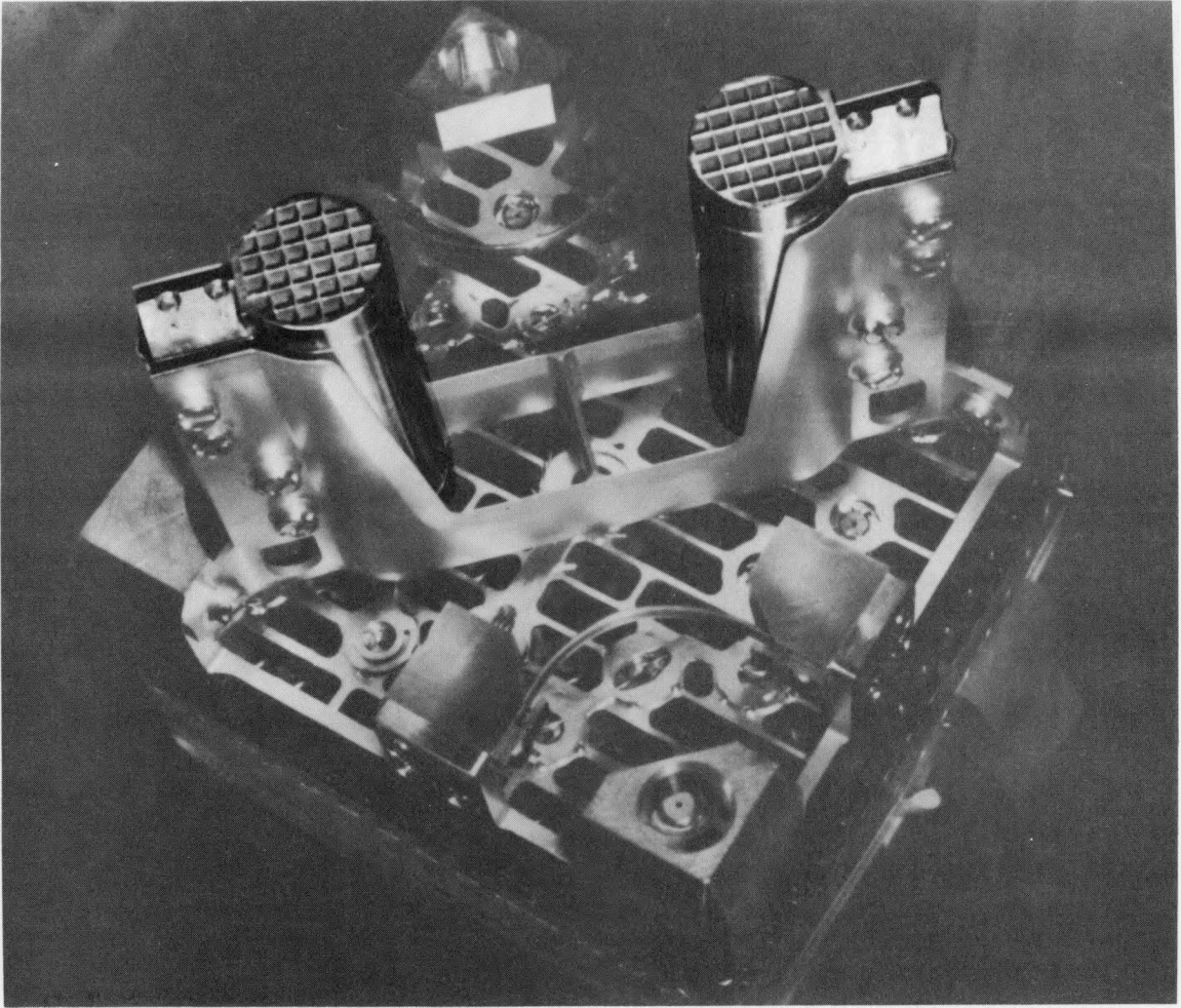
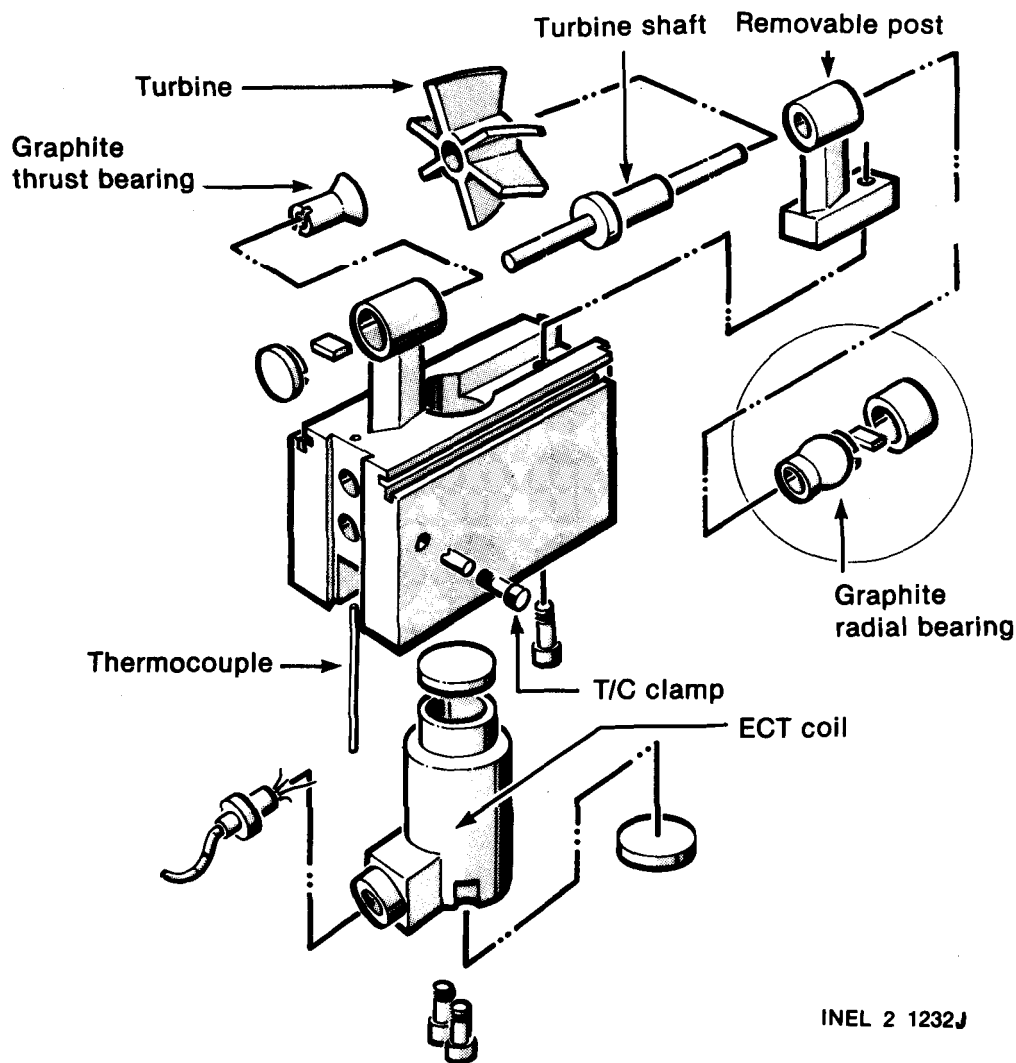


FIGURE 3. LOFT DRAG-DISC/TURBINE TRANSDUCERS INSTALLED IN CENTER FUEL MODULE CORE INLET.

The turbines used in LOFT are the result of many years of experimentation. In the core region, two varieties of turbine are being used with satisfactory results. The older turbine design (see Figure 4), which is used in the CFM presently installed in LOFT, uses two bearings (one radial and one thrust bearing), and a rotating shaft and rotor assembly. An eddy-current coil is used to detect the rotor movement. The bearings are graphite, with a stainless steel rotor and shaft, mounted on stainless steel posts.

Although this turbine design has survived and performed well in this configuration, in other applications, particularly in a rake arrangement in the piping in LOFT, turbines have shown limited lifetime. As a result, a new style turbine (see Figure 5) has been developed. This type of unit has been used through several experiments in the piping application, and has never had a failure--even on multiple-use units. The next scheduled LOFT CFM, and the core outlet region of the present (FI) CFM, use this style turbine.



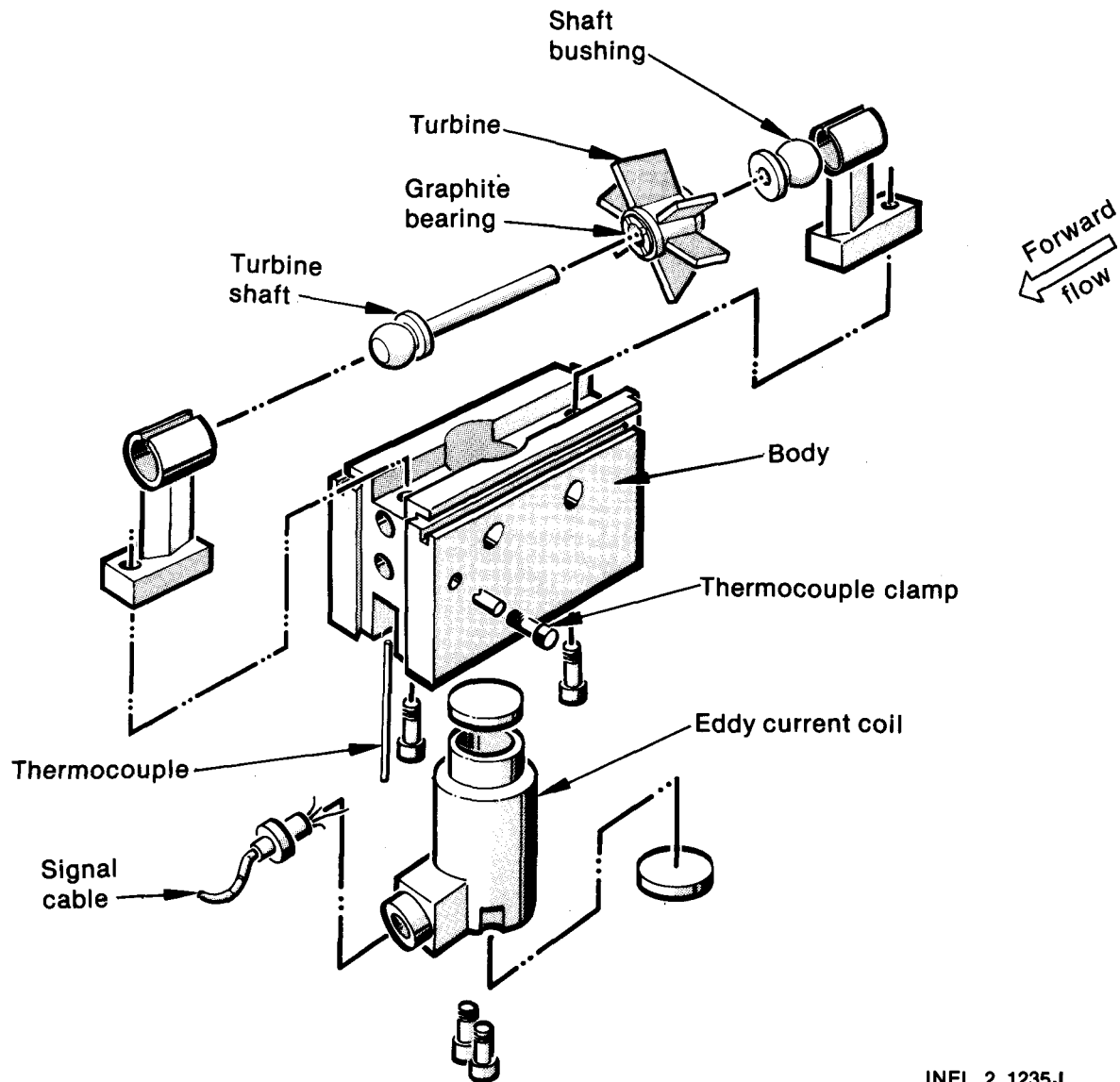
INEL 2 1232J

FIGURE 4. LOFT MOD-II, OLD STYLE TURBINE WITH TWO BEARINGS.

This newer turbine design uses stainless steel mounting posts, a fixed tungsten carbide (or silicon carbide) shaft, and a stainless steel rotor. The rotor has a single graphite bearing, under compression, which acts as both radial and thrust bearing. So far, units of this type have performed excellently; both in LOFT (over 600 hours of run-time at LOFT temperature, pressure, and 35 ft/s velocity) and in other test facilities (over 2000 hours at LOFT conditions and 50 ft/s velocity).

Turbine uncertainty is approximately 6% in single phase water; it is somewhat higher in two-phase flow, depending on flow regime.¹

The LOFT drag-disc unit has been successfully used for several years. It is a modular unit, designed to mate with the turbine or with another drag device. As shown in Figure 6, it consists of a moveable carrier and core assembly mounted with the core in a variable reluctance transducer (VRT), and with a sized disc as a target. This moveable carriage is held by two springs that are sized to provide the appropriate instrument range. The uncertainty of this device is approximately 16%.^{1,4}



INEL 2 1235J

FIGURE 5. NEW STYLE, SINGLE BEARING LOFT TURBINE.

The direct measurement of density poses a significant problem, since there is no easily applied, unambiguous technique which can meet the core environment and space restrictions. Density can be derived from the drag-disc/turbine unit with temperature information from nearby thermocouples. This indirect method is applicable only in those fluid conditions where both turbine and drag-disc units perform well (primarily single phase and very low quality). As a result of a development project at INEL, an ultrasonic technique^{2,3} was chosen to obtain an additional independent density measurement in the core inlet and outlet regions. Figure 7 shows the ultrasonic density detector installed in the core inlet.

The ultrasonic density detector (UDD) detects the transit time of an ultrasonic wave through a sensing bar immersed in the fluid (see Figure 8). The sensing bar is stainless steel, with a width-to-depth ratio of about 4

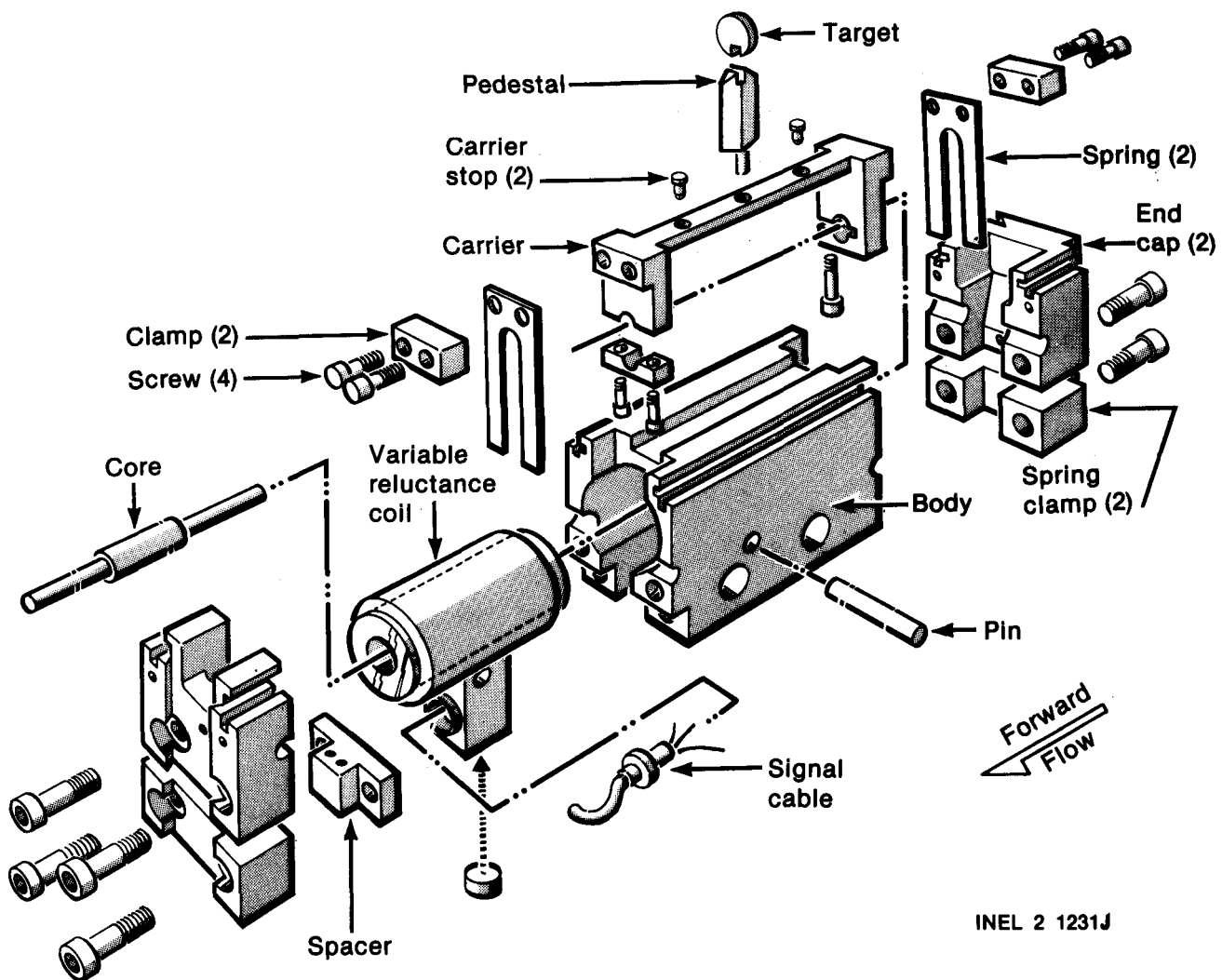


FIGURE 6. UNASSEMBLED LOFT DRAG-DISC UNIT.

to 1. Two magnetostrictive transducers (remanders) are attached to the edge of the bar (see Figure 9). This attachment produces a conversion of an extensional wave, generated by electrically pulsing the sending remander, to a torsional wave which propagates through the sensing bar, back to an extensional wave at the receiving transducer, which then produces a detectable electrical pulse. Differences in the density of the media surrounding the sensor bar produce differences in the propagation velocity of the acoustic pulse. These differences are detected by a phase-shift detection technique.

The UDD must be considered a developmental unit; it has sensitivities to flow and temperature as well as density. Because of this, its uncertainties are high, on the order of 20%. Under certain fluid conditions, it is relatively accurate, with uncertainties on the order of 5% for static, isothermal fluid.⁵

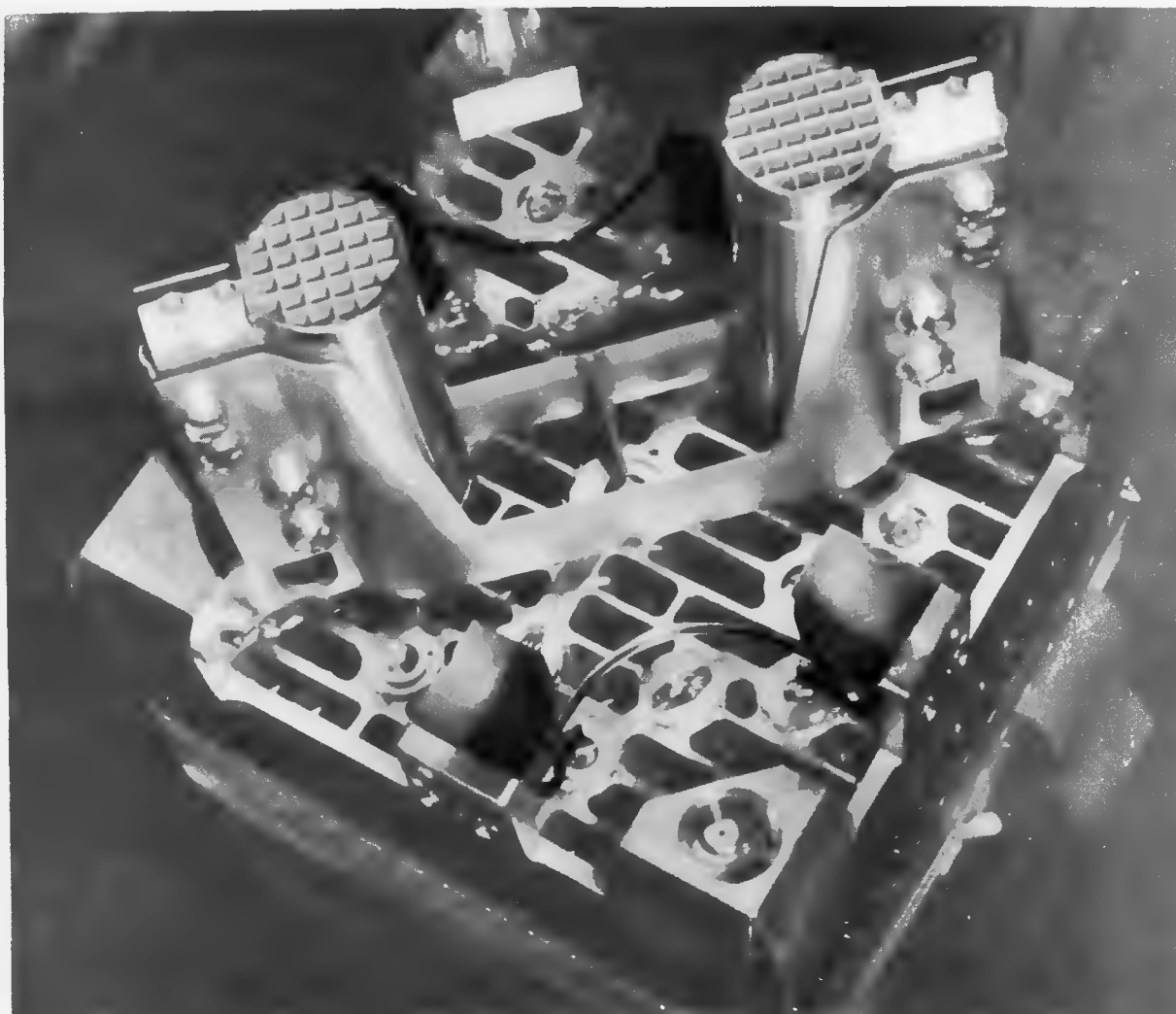


FIGURE 7. LOFT ULTRASONIC DENSITY DETECTORS INSTALLED IN CENTER FUEL MODULE CORE INLET.

THERMAL MEASUREMENTS

LOFT has several types of thermocouples designed to measure both fluid, structural, and fuel behavior. These thermocouples include coolant, structural, and several type of fuel rod cladding and fuel thermocouples. Thermocouples are arranged throughout the core region to give radial and axial measurements of both coolant and cladding temperatures.

Coolant and structural thermocouples are arranged to give a radial temperature distribution at the core inlet and outlet. They also provide data for temperature correction of the mass flow instruments. These thermocouples are standard type K, stainless steel-sheathed instruments, with grounded junctions for fast response. Measurements are made in guide tubes, at points in the upper structure, and in flow channels in the lower and upper core support areas. Operating experience has shown that these thermocouples are extremely reliable.

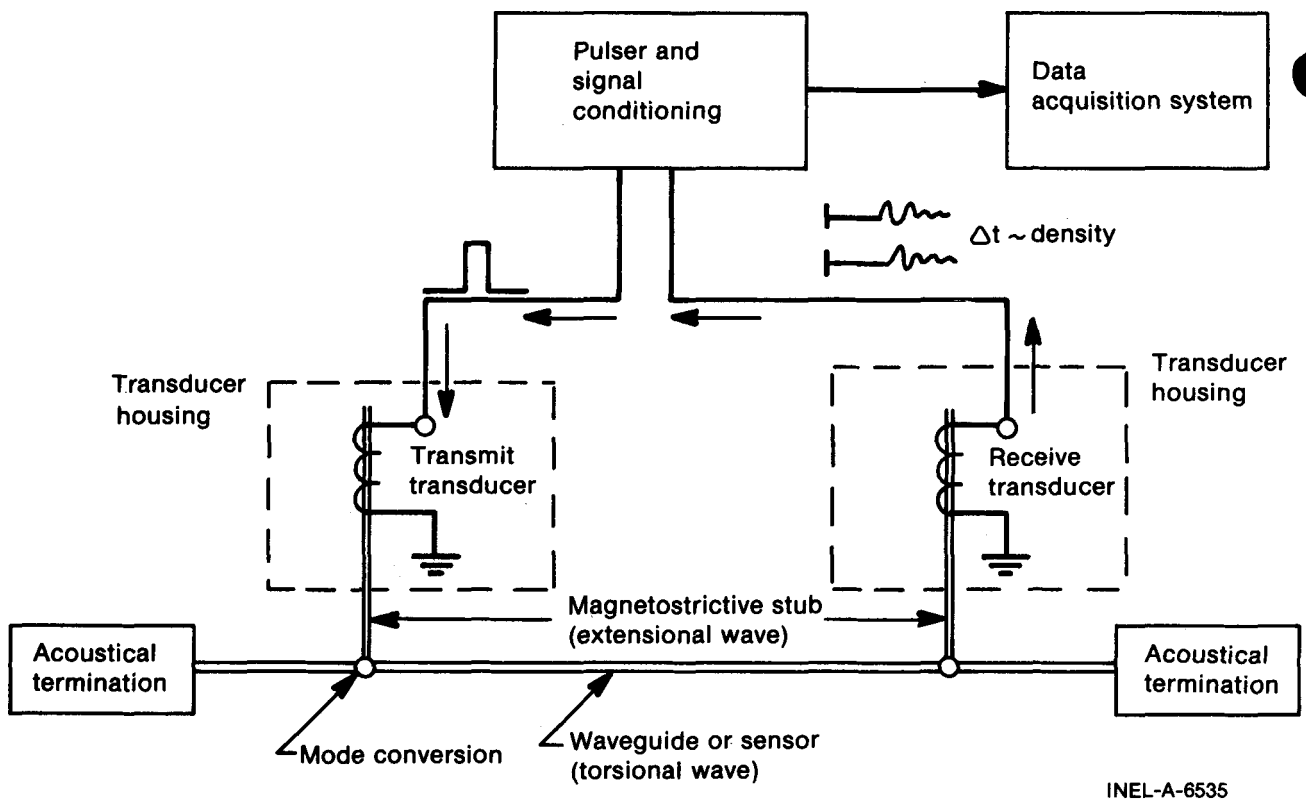


FIGURE 8. BLOCK DIAGRAM OF ULTRASONIC DENSITY DETECTOR SYSTEM SHOWING FUNCTIONAL RELATIONSHIPS

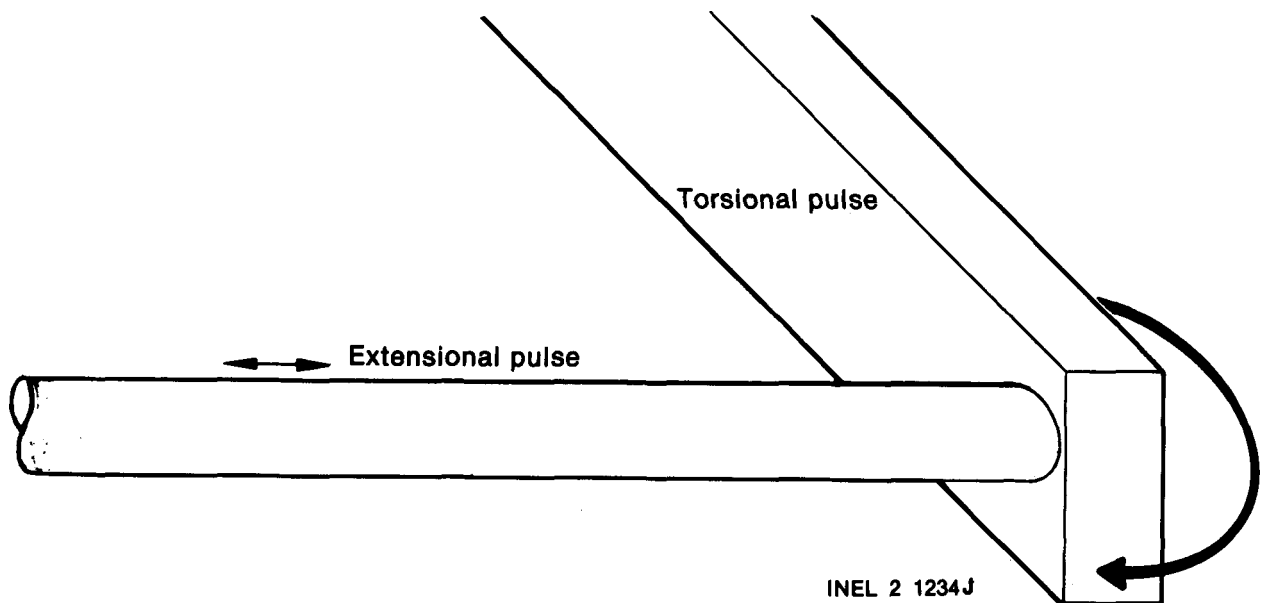


FIGURE 9. ATTACHMENT OF REMANDER TO SENSOR BAR TO PRODUCE TORSIONAL PULSE.

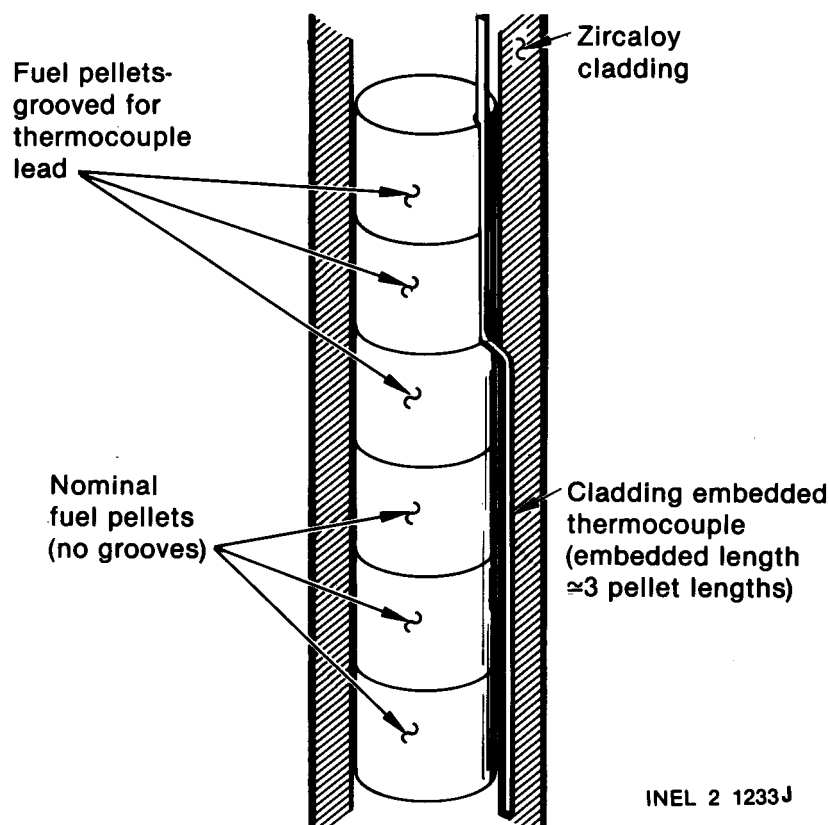


FIGURE 10. LOFT CLADDING EMBEDDED THERMOCOUPLE INSTALLATION.

Fuel rod cladding temperatures are measured by three different thermocouple types: external cladding, cladding embedded, and internal cladding thermocouples. Each thermocouple has been developed to fill a specific measurement requirement.

The externally mounted thermocouples have titanium or zircaloy sheaths, with MgO insulation and type K thermoelements, and are attached to the external surface of the fuel rod using a laser welding technique. These thermocouples have a long history of successful operation in LOFT; however, during certain rapid reflood or two-phase flow conditions, there are questions about the effects of the thermocouple on the cladding temperature.⁶ The uncertainty of the measurement during these conditions is therefore quite high; however, during most other conditions, the measurement uncertainty is about 3%.

Problems in the interpretation of data from the externally mounted cladding thermocouples led to the development of a cladding-embedded thermocouple. This thermocouple (see Figure 10) is routed from the top of the fuel rod through grooves in the periphery of the fuel pellets. The junction end is flattened to 0.25 mm thick by 0.69 mm wide, and is laser-welded into an elongated patch made from a section of cladding. This patch is then welded into an identical slot cut in the fuel rod cladding. Because of material compatibility problems, it is necessary to use zircaloy as the thermocouple sheath material. This makes miniaturization extremely difficult, but is

solved by a carefully controlled series of swaging, drawing, and annealing operations. Above the fuel pellet stack, the thermocouple is routed to a hermetic seal which is used to prevent fission gas release to the exterior of the fuel rod, and to provide a transition to a stainless steel-sheathed cable. This extension cable is then laser welded into a specially designed end cap, and routed out of the reactor. Specially grooved fuel pellets are used up to the point where the thermocouple enters the cladding; normal fuel pellets are used below that point, assuring typical fuel behavior in the vicinity of the junction.

Although the embedded thermocouples have the potential to provide a very accurate measurement, they have potential disadvantages: (1) the attachment weakens the cladding; in tests where cladding integrity is important they are not recommended; (2) because of the miniaturization, they are extremely fragile. For tests where both cladding integrity and an unperturbed coolant flow are important, another thermocouple is being used. An Inconel-sheathed, 0.51 mm diameter thermocouple is attached to the interior surface cladding using a bimetallic weld developed by Battelle Pacific Northwest Laboratories. This thermocouple is routed up through grooves in the pellets, and through the end cap. There are up to three of these internal clad thermocouples mounted in one fuel rod. This design has been successfully used in fuel rods in experiments in the NRU reactor loop in Canada, to temperatures above 1400 K.

There are two types of fuel thermal measurements used in LOFT; surface temperature measurement, and a centerline temperature measurement. The fuel surface measurement uses a type K thermocouple, which is required to operate near or above the quoted limits for this type of thermocouple. The fuel centerline temperature is even higher; a new type of thermocouple was required for this application. A refractory-sheathed, tungsten-rhenium thermocouple was developed for this measurement application.

In order to provide information on heat transfer from the fuel to the cladding and then the coolant, a thermocouple mounted close to the exterior of the fuel is used. This fuel surface or external fuel thermocouple is a stainless steel-sheathed, alumina-insulated type K. It is routed from the top of the rod, like the embedded thermocouple, through grooves in the fuel pellet exteriors. About three pellets above the junction, it is routed away from the cladding, into holes drilled about 75% of the way from the center of the exterior of the pellet. It uses a hermetic seal/transition joint to exit the fuel rod and to prevent fission gas release.

The LOFT Centerline Thermocouple is designed to provide information on fuel rod response and is capable of measuring temperatures up to 2500 K. This thermocouple uses a vapor-deposited, tungsten-augmented rhenium sheath, 1.6 mm in diameter. Development of this sheath material and fabrication method was the key to the successful development of this instrument. This material provides the rigidity missing from pure rhenium sheaths, yet avoids the fragility of most tungsten-rhenium compounds. The insulation material used is hafnia, with thermoelements of tungsten-5% rhenium vs. tungsten-26% rhenium. The thermocouple junction is located in the fuel hot zone, about midway down the fuel rod. The thermocouple cable is installed in holes pressed in the center of the fuel pellet, and exits the fuel at the fuel rod plenum, where the cable terminates in a hermetic seal. This seal, once

again, provides a barrier to release of fission gas from the fuel rod as well as allowing a transition to a less expensive, stainless steel-sheathed, compensated lead cable which exits the rod and reactor.

The hafnia insulation was chosen over other materials because it has excellent electrical resistance and is chemically inert at the elevated temperatures experienced in the fuel; however, it has a high neutron cross section. Evaluations have been performed on the effect on fuel rod power and center-line temperature of the hafnia and the thermocouple in the center of the rod. These studies have shown that the removal of fissile material causes a power depression which is much greater than that caused by the presence of any materials including the hafnia.⁷ The indicated temperature may be as much as 300 K lower than the temperature in a similar, non-instrumented rod. This finding emphasizes the need to miniaturize, as much as possible, any future fuel measurement devices. In the LOFT application, however, analytical techniques have been developed to compensate for the flux depression caused by this instrumentation.

The fuel rod plenum gas pressure is an indicator of fuel rod thermal response, and is important in understanding the mechanical response. Due to LOFT restrictions in the core and reactor system design, it is necessary to measure fuel rod pressure with an in situ transducer. This requirement for installing a transducer inside the fuel rod produces significant restrictions on the size, reliability, and environmental resistance of this transducer.

An eddy-current pressure transducer, which has been used in similar applications at the Sodium Loop Safety Facility (SLSF), is used in LOFT.⁸ It is installed directly in the fuel rod plenum, with the cable exiting through a hermetic seal and specially designed end cap. There is also a thermocouple mounted in the plenum, to be used in conjunction with the pressure transducer. The presence of the transducers in the plenum requires a shortening of the fuel pellet stack in order to produce a typical plenum volume.

These transducers are potentially sensitive to temperature. Hanford Engineering Development Laboratory (HEDL) has developed a temperature-compensation technique⁹ which corrects for temperature transients by exciting the transducer at a much lower frequency than the primary excitation frequency (50 K Hz vs. 500 K Hz). The signal from the low frequency is then separated from the primary signal. This low frequency signal is primarily responsive to temperature effects, rather than pressure. The primary signal can then be compensated for temperature effects. Testing by HEDL has shown a factor of 3 improvement in the uncertainty due to temperature effects.

CONCLUSIONS

The LOFT core has been extensively instrumented to provide information on the thermal-hydraulic behavior of this region during steady state operation and during transients designed to simulate accident conditions. Hydraulic measurements are made at the entrance and exit of the core to provide a greater understanding of the heat transfer and fluid phenomena. Fuel, fuel rod cladding, and core structure thermal response is measured by a variety of thermocouples of numerous radial and axial locations in the core. This

response is used to understand the multidimensional response of the core and to provide reliable information against which computer codes, developed to predict core response to accident conditions, can be assessed.

REFERENCES

1. S. Silverman, LOFT Experimental Measurements Uncertainty Analysis, Volume XIV: LOFT Drag-Disc Turbine Transducer Uncertainty Analysis, NUREG/CR-1069, TREE-1089, November 1978.
2. M. E. Buchanan and R. W. Shurtliff, The Feasibility of Using a Conductivity or Ultrasonic Type Detector to Measure Void Fraction in the LOFT Reactor Core Inlet and Outlet, LTR 141-68, January 1978.
3. A. E. Arave, "Ultrasonic Densitometer Development," NRC Instrumentation Review group meeting, Silver Springs, Maryland, NUREG/CP-007, July 1979.
4. T. M. Mecham, "A Comparison of Mass Flow Calculations Using Drag-Disc/Turbine Transducer (DTT) Data from the Transient Flow Calibration Facility," Advances in Test Measurement, Proceedings of the 27th International Instrumentation Symposium, Indianapolis, Indiana, 1981, 18, Part 2.
5. A. E. Arave, Ultrasonic Density Detector Air-Water Flow Test, LTR 141-91, January 1979.
6. E. L. Tolman and E. W. Coryell, Review of LOFT Cladding Temperatures Response for L2-2 and L2-3: Recommendations for Improved LOFT Fuel Rod Measurements, EGG-LOFT-5244, Project No. P394, October 1980.
7. M. L. Carboneau, Effects of Fuel Centerline Thermocouples on Fuel Rod Steady State Temperatures and Stored Energy, LTR-LO-14-80-071, October 1980.
8. T. R. Billeter, Summary Report of Pressure Sensor for the Loss-of-Fluid Test (LOFT) Reactor, NUREG/CR-1720, HEDL-TME 80-53, February 1981.
9. T. R. Billeter and C. R. Wandling, "Compensation Techniques for Improving Pressure Sensors Accuracy During LOFT LOCA Transient," NRC Instrumentation Review group meeting, Oak Ridge, Tennessee, July 1980.

NOTICE

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, or any of their employees, makes any warranty, expressed or implied, or assumes any legal liability or responsibility for any third party's use, or the results of such use, of any information, apparatus, product or process disclosed in this report, or represents that its use by such third party would not infringe privately owned rights. The views expressed in this paper are not necessarily those of the U.S. Nuclear Regulatory Commission.

MONITORING PWR REACTOR VESSEL LIQUID LEVEL WITH SPNDs DURING LOCAs

J. P. Adams^a
EG&G Idaho, Inc.
P.O. Box 1625
Idaho Falls, Idaho 83415

ABSTRACT

Data from in-core self-powered neutron detectors taken during two nuclear loss-of-coolant accident simulations have been correlated with core moderator density changes. The detector current attenuation has been calculated during blowdown and reflood phases of the simulation. Based on these data, it is concluded that these detectors could be used to monitor reactor vessel liquid level during loss-of-coolant accidents in pressurized water reactors.

1. INTRODUCTION

Since the nuclear accident at Three Mile Island (TMI), interest has been heightened in developing the capability of directly monitoring reactor vessel water level so that the operators can take action to avoid uncovering the core and damaging the fuel. Previously, operators of commercial pressurized water reactors (PWRs) had no direct means to monitor this level and had to infer primary system mass inventory from the pressurizer liquid level. TMI demonstrated the potential unreliability of this method. Since TMI, reactor vessel liquid level measuring devices, including differential pressure transducers and heated thermocouples,¹ have either been installed or are planned for installation in commercial PWRs but these have not yet been verified under loss-of-coolant accident (LOCA) conditions.

One of the program objectives involving the Loss-of-Fluid Test (LOFT) facility is to develop and test new instrumentation and to find new uses for existing instrumentation for commercial PWR applications. Consistent with this objective, in-core self-powered neutron detector (SPND) data from two LOCA simulations L5-1 and L8-2², in which the reactor vessel water level decreased to different elevations in the reactor core, have been correlated with independently measured moderator density variations. The correlation supports previously published conclusions^{3,4} that SPNDs are capable of monitoring PWR reactor vessel liquid levels under LOCA conditions.

This paper discusses the correlation between SPND output and core moderator density variations and specifically reports the results of the analysis of SPND response to liquid level changes measured during the blowdown and reflood phases of the two LOCA simulations. Additionally, the potential application of these detectors to monitor reactor vessel liquid level during small break LOCAs in PWRs is addressed.

a. Work supported by the U.S. Nuclear Regulatory Commission, Office of Nuclear Regulatory Research under DOE Contract No. DE-AC07-761D01570.

2. THE LOFT EXPERIMENTAL FACILITY AND SPND DESCRIPTION

The LOFT facility is a 50-MW(t), PWR system designed to reproduce, both in time and approximate magnitude, the significant thermal and hydraulic events expected in a commercial PWR during a postulated LOCA. Extensive experimental instrumentation, in addition to the normal PWR process instrumentation, is used to characterize temperature, pressure, flow, density, and nucleonics in transients resulting from simulated LOCA events. A schematic diagram of the LOFT facility is shown in Figure 1. Detailed descriptions of the facility and the scaling philosophy for its design are found in References 5 and 6, respectively. Table 1 lists typical operating characteristics in LOFT compared to ZION, a Westinghouse designed commercial PWR.

The primary focus of this paper is on SPNDs which are located in the LOFT core to measure local power density. There were three operating SPNDs in the LOFT core during the two simulations, one located in each of three fuel assemblies as shown in Figure 2. The detectors are centered at a height of 0.66 m (26 in.) above the bottom of the active core, which is the elevation of maximum core power density. Each detector consists of a 0.23 m (9.0 in.) long by 0.19 cm (0.075 in.) diameter cobalt emitter inside an Inconel collector which also forms the sheath. The emitter is separated from the sheath by aluminum oxide insulation. The detector geometry is schematically depicted in Figure 3. Figure 4 is an illustration of a LOFT core guide tube, cut away to show how the SPND is installed.

There are two leads from the detector assembly, a signal lead connected to the emitter and a cable compensation lead. The signals from the two lead wires are independently amplified by a low input bias current ($\sim 10^{-14}$ A) current to voltage amplifier (see Figure 5). The compensation voltage output is subtracted from the main signal output and this difference is again amplified and input into the computerized data collection system. Table 2 lists additional SPND characteristics.

In a previous paper⁴, the response of cobalt emitter SPNDs to a changing water density is discussed in detail. In summary, a decreasing water density (caused, for example, by the water level decreasing past the detector) causes a reduction in the thermal neutron flux and a resultant decrease in the neutron component of SPND current. The same water density decrease causes an increase in γ flux (due to decreased attenuation) and a resultant increase in the γ component of SPND current. Since the γ current is of opposite polarity to the neutron current, these changes act in concert to cause an increase in negative SPND current during decreasing density. The opposite occurs during an increasing water density.

3. LOFT EXPERIMENTS L5-1 AND L8-2

LOFT Experiments L5-1 and L8-2 each simulated the rupture of a single 0.36 m (14 in.) accumulator injection line in a commercial PWR. Prior to initiation of each experiment, the reactor was operating at approximately full power (50 MW) with a maximum linear heat generation rate of 46 kW/m (14 kW/ft). Although the SPNDs are located at the elevation of maximum power density, they are located in peripheral fuel modules and, consequently, the

linear heat generation rate at the SPND locations was approximately 31 kW/m (9.6 kW/ft). Experiment L8-2 was a repeat of L5-1 except that the core uncover was allowed to continue longer in order to deplete more of the primary system mass and obtain higher cladding temperatures. This was done in order to determine whether restart of the primary coolant pumps in highly voided conditions would be effective in cooling the core.

The following core liquid level scenario is common to both experiments.¹¹ After the break was initiated, the primary system depressurized to fluid saturation within 1 s. Power to the primary coolant pumps was turned off at 3.2 s. The water level in the reactor core decreased at an average rate of 2 cm/s (0.7 in./s) under the influence of a slow core boiloff. The level was radially and azimuthally uniform as evidenced by cladding temperatures which departed from saturation (indicating cladding dryout) at the same time for any given elevation. The level reached the elevation of the SPNDs at approximately 125 s after experiment initiation.

The level decrease continued in L5-1 until the level was within 13 cm (5 in.) of the core bottom (at 185 s) at which time the accumulators, and shortly thereafter the low pressure injection system, started injection, refilling the core within 30 s. In L8-2, the level was allowed to drop until the core was completely dry. At approximately 300 s after L8-2 initiation, emergency core cooling injection was initiated, reflooding the core within 12 s.

The maximum fluid temperature reached at the SPND locations during the two experiments was 745 K (882°F) which exceeds the temperature for nominal operation of the SPNDs. Therefore thermionic (non-radiation induced) currents were a possibility. A sensitivity of SPNDs to thermionic currents has been reported for the rhodium detectors installed in the TMI core.⁹ While the results from these rhodium SPNDs cannot be directly applied to the LOFT cobalt SPNDs, it is informative to note the following: (a) the thermionic current phenomenon measured at TMI had a temperature threshold of 775 K (900°F), and (b) when this phenomenon occurred, the TMI SPND output went high off scale. During the two LOCA simulations reported in this paper, the maximum temperature measured at the SPND locations was less than the threshold measured at TMI. In addition, the LOFT SPND output did not exhibit any evidence of an unusual shape as would be expected if the thermionic temperature threshold existed and had been exceeded (see Figures 6 and 7). On the basis of this evidence, the conclusion is reached that the SPNDs responded only to radiation during Experiments L5-1 and L8-2. Superimposed on the L5-1 and L8-2 SPND data in Figures 6 and 7 are data from LOFT Experiment L3-7 which was a small break LOCA simulation. During the time frame shown, the system in L3-7 was approximately steady state. Thus, the L3-7 SPND data closely resemble that which would occur in LOFT during hot standby following a reactor scram. Deviations from L3-7 SPND behavior provide a measure of SPND response to water density changes only.

4. SPND RESPONSE TO CHANGING WATER LEVEL IN L5-1 AND L8-2

After the initial drop in SPND current at reactor scram, the current asymptotically approached zero (due to the decaying radiation fields). When the liquid level approached the detector axial location, the slope of the

detector current reversed (see Figures 8 and 9) and the current magnitude started to increase (i.e. become more negative). The gradual decrease in the SPND current indicated that the region around the detector uncovered in approximately 15-20 s. During this time, the liquid level decreased 11-14 in., compared with the detector length of 9 in. This difference (between level decrease and detector length) is a measure of the region of local sensitivity of the SPNDs and compares well with an independently measured sensitivity range of between 2 and 6 cm (0.8 and 2.4 in.) for boiling water reactor conditions.¹⁰

When the emergency core cooling (ECC) injection was initiated in each experiment and the core was rapidly reflooded, the SPND responded rapidly to the increase in fluid density (Figures 8 and 9). In addition, in L5-1, the core fluid at first was subcooled and then was heated to saturation (hence, less dense) conditions. This post-reflood density change was also detected by the SPNDs as shown in Figure 9. This effect, if present during L8-2, was masked by the more violent thermal and hydraulic conditions which existed during reflood in this experiment (Figure 8).

5. CORRELATION OF SPND RESPONSE AND CORE WATER DENSITY CHANGES DURING BLOWDOWN AND REFLOOD

In an initial attempt to quantify the SPND response to core water density changes during Experiments L5-1 and L8-2, the expected neutron flux was calculated. The kinetic equation was solved as a function of time after a step negative change in reactivity was inserted, representative of the integrated rod worth at reactor scram. Previously measured values for delayed neutron fraction and neutron generation time⁵ were used. Coupled with a thermal neutron flux at full power of 5.073×10^{13} n/cm²s at the beginning of each experiment, the calculated thermal neutron flux at 120 s (just prior to the core liquid level reaching the sensitive volume of the SPNDs) was 2.55×10^{10} n/cm²s. This results in a calculated SPND current of 0.2×10^{-9} A which is approximately 2% of the total measured SPND current at this time. Since this would be the maximum expected neutron current during times of interest in L5-1 and L8-2, the contribution of the neutron current in subsequent calculations is neglected.

Consistent with the approach of Reference 4, the γ -flux attenuation as a function of changing moderator density was calculated using ANISN,¹¹ a γ -transport computer code. For the maximum possible density change in the experiments, the maximum attenuation of the γ -flux was less than 10%. That is, $A(\Delta\rho) \geq 0.90$ where:

$$A(\Delta\rho) = \frac{\phi_f}{\phi_i} \quad (1)$$

and

ϕ_i = the unattenuated γ -flux

ϕ_f = the attenuated γ -flux.

Since the SPND γ -current, I_γ , equals the product of the SPND sensitivity, S_γ , and the γ -flux, ϕ_γ ;

$$I_\gamma = S_\gamma \phi_\gamma. \quad (2)$$

Combining Equations (1) and (2) yields:

$$I_{\gamma f} = A(\Delta\rho) I_{\gamma i}$$

where

$$I_{\gamma i, f} = \text{SPND } \gamma\text{-current corresponding to the unattenuated and attenuated } \gamma\text{-flux, respectively.}$$

However, as can be seen in Figures 8 and 9, the ratio of SPND current prior to and subsequent to reflood is less than 0.6. Therefore, the response of the LOFT SPNDs to the water level changes during Experiments L5-1 and L8-2 cannot be explained by γ -flux attenuation alone. In fact, the SPNDs appear to be more sensitive to liquid level changes than originally estimated, based only on γ -flux attenuation. Therefore, an additional radiation field is postulated that increases the sensitivity of the SPNDs to a varying liquid level. The existing electron flux, made up of Compton and photo-electrons originating from the interaction of the γ -flux with core structural materials, especially the SPND guide tube material, is believed to be responsible for this increase. Since the SPND sheath is grounded, the SPND electron current would consist of those external electrons with sufficient energy to penetrate the sheath, migrate through the aluminum oxide insulation, and stop in the emitter. Thus, the electron current has the same polarity as the γ current and would respond to a changing water density in the same manner (increasing magnitude with decreasing moderator density). Due to the relatively short mean free path of electrons in water, the attenuation of the electron flux as a function of water density would be large.

An empirical correlation between SPND attenuation and water density change has been developed using data from L5-1 and L8-2. Since neither the electron flux nor the SPND sensitivity to this flux is known, no attempt is made to explain this correlation on the basis of existing theory. The relative attenuation coefficient, $A'(\Delta\rho)$ is correlated with the change in water density, $\Delta\rho$. The water density change that occurred during core uncover and reflood in L5-1 and L8-2, in addition to that which occurred as the water saturated following reflood in L5-1 were used in the correlation. $A'(\Delta\rho)$ and $\Delta\rho$ are defined as follows:

$$A'(\Delta\rho) = \frac{I(\rho_f)}{I(\rho_i)}$$

$$\Delta\rho = \rho_f - \rho_i$$

where

I	=	SPND total current
ρ_f	=	higher water density (e.g. prior to core uncover)
ρ_i	=	lower water density (e.g. subsequent to core uncover)

Water densities were calculated for Experiments L5-1 and L8-2 using system pressure (for specific saturated liquid and vapor densities), core decay heat and a calculated average bubble rise velocity using the formalism of Harmathy¹² (for average core void fraction). Superheated vapor densities were calculated from measured system pressure and temperature for the core-uncovered time frames. The results of these calculations are summarized in Table 3.

As can be seen in Table 3, $A(\Delta\rho)$ for Experiments L5-1 and L8-2 is a monotonic function of density differences, and is consistent between the two experiments regardless of whether the density was decreasing (decreasing water level) or increasing (increasing water level). Thus, the LOFT SPNDs consistently demonstrated the ability to detect the liquid level as it passes the SPND. A vertical string of SPNDs in the core could map the liquid level wherein each SPND acts as a discrete liquid level transducer, at least for times after reactor shutdown on the order of the duration of the L8-2 and L5-1 transients.

SPND data from other experiments,^{13,14} in addition to Experiments L5-1 and L8-2, show that the long term decay of the SPND current closely follows the expected decay of the γ -flux as calculated from Reference 15. Therefore, Reference 15 can be used as a basis to extrapolate the results from Experiments L5-1 and L8-2 to times long after reactor scram. Using these data, together with the correlation described above, it is expected that these SPNDs will be sufficiently sensitive to measure a changing water level for at least 4.5 h after reactor shutdown.

6. CONCLUSIONS

The following conclusions are based on the experimental results discussed herein:

1. The LOFT SPNDs were sensitive to water level changes which took place during the blowdown and reflood phases of Experiments L5-1 and L8-2.
2. A consistent correlation has been measured between core water level changes and the SPND output. This correlation was shown to be consistent during the two independent experiments.
3. The correlation has been extrapolated to times representative of small break LOCAs. These detectors are expected to be sensitive to similar water level changes at least 4.5 h after reactor shutdown.

NOTICE

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, or any of their employees, makes any warranty, expressed or implied, or assumes any legal liability or responsibility for any third party's use, or the results of such use, of any information, apparatus, product or process disclosed in this report, or represents that its use by such third party would not infringe privately owned rights. The views expressed in this paper are not necessarily those of the U.S. Nuclear Regulatory Commission.

7. REFERENCES

1. Minutes of a meeting of the Nuclear Regulatory Commission on "Discussion of Reactor Vessel Water Level Indicators" held in Washington, D.C., January 8, 1982.
2. D. B. Jarrell and J. M. Divine, Experiment Data Report for LOFT Intermediate Break Experiment L5-1 and Severe Core Transient Experiment L8-2, NUREG/CR-2398, EGG-2136, November 1981.
3. J. P. Adams and V. T. Berta, "Response of LOFT SPNDs to Reactor Coolant Density Variations During a LOCA Simulation", Transactions of the American Nuclear Society, 1980 Winter Meeting, Washington, D.C., November 16-21, 1980.
4. J. P. Adams and V. T. Berta, "Response of LOFT SPNDs to Reactor Coolant Density Variations During a LOCA Simulation", Nuclear Technology (in press).
5. D. L. Reeder, LOFT System and Test Description (5.5 ft Nuclear Core 1 LOCes), NUREG/CR-0247, TREE-1098, Change 1, September 1980.
6. L. J. Ybarrodo, et al., "Examination of LOFT Scaling" 74-WA/HT-53, Proceedings of the 95th Winter Annual Meeting of the ASME, New York, N.Y., August 1974.
7. Operations and Maintenance Manual for LOFT Self-Powered Neutron Detector Amplifier - OMM 141-17, July 31, 1973.
8. J. P. Adams, Quick Look Report on LOFT Nuclear Experiments L5-1 and L8-2, EGG-LOFT-5625, October 1981.
9. K. H. Ardron and D. G. Cain, "Modelling of the Liquid Level During Uncovering of the TMI-2 Core", Proceedings of the American Nuclear Society and European Nuclear Society Topical Meeting on Thermal Reactor Safety, CONF-800403, pp. 597, April 6-9, 1980.

10. P. Gebureck et al., "Experimental Determination of Local Steam Velocities and Boiling Boundaries in a BWR", Proceedings of the ANS/ASME/NRC International Topical Meeting on Nuclear Reactor Thermal-Hydraulics, Saratoga Springs, N.Y., NUREG/CP-0014, pp. 204, October 1980.
11. "ANISN-W Multigroup One-Dimensional Discrete Ordinate Transport Core with Anisotropic Scattering" RSIC-CCC-255, July 1975.
12. T. Z. Harmathy, "Velocity of Large Drops and Bubbles in Media of Infinite or Restricted Extent," AIChE, 6, 281, 1960.
13. P. D. Bayless, J. B. Marlow and R. H. Averill, Experiment Data Report For LOFT Nuclear Small Break Experiment L3-1, NUREG/CR-1145, EGG-2007, January 1980.
14. D. L. Gillas and J. M. Carpenter, Experiment Data Report for LOFT Nuclear Small Break Experiment L3-7, NUREG/CR-1570, EGG-2049, August 1980.
15. ANS Standard (Proposed Revised) 5.1, August 1977.

TABLE 1. TYPICAL OPERATING CONDITIONS IN LOFT AND ZION REACTORS

Item	LOFT	ZION
Fuel Rod Number	1300.0	39372.0
Fuel Rod Length (m)	1.68	3.68
(ft)	5.5	12.1
Primary System Volume (m ³)	7.63	346.6
(ft ³)	269.5	12240.1
Primary Coolant Flow/Volume ($\frac{\text{kg/s}}{\text{m}^3}$)	25.6	51.7
($\frac{\text{lbm/s}}{\text{ft}^3}$)	1.60	3.23
Power (MW)	36.7	3540.5
Maximum Linear Heat Generation Rate (kW/m)	39.4	39.4
(kW/ft)	12.0	12.0
Peaking Factor	2.34	1.60
Coolant Temperature Rise (K)	32.2	32.2
(°F)	58.0	58.0

TABLE 1. continued

Item	LOFT	ZION
Power/Coolant Volume (MW/m^3)	4.81	10.2
(MW/ft^3)	0.14	0.29
Core Mass Flux ($\text{kg}/\text{s}\cdot\text{m}^2$)	1248.8	3707.3
($\text{lbm}/\text{s}\cdot\text{ft}^2$)	255.8	759.3
Core Mass Flux/System Volume ($\text{kg}/\text{s}\cdot\text{m}^3$)	25.6	51.7
($\text{lbm}/\text{s}\cdot\text{ft}^3$)	1.60	3.23

TABLE 2. LOFT SPND CHARACTERISTICS

<u>Physical Dimensions</u>	
Emitter Diameter	0.19 cm (7.5×10^{-2} in.)
Emitter Length	22.9 cm (9.0 in.)
Collector Diameter	0.35 cm (0.14 in.)
Collector Thickness	0.025 cm (9.8×10^{-3} in.)
Cable Length	7.6 m (25 ft)
<u>Materials of Construction</u>	
Emitter	Cobalt
Collector	Inconel 600
Signal and Compensation Wires	Inconel 600
Detector Insulation	Aluminum Oxide
Cable Insulation	Magnesium Oxide
<u>Nuclear Characteristics</u>	
Detector Neutron Sensitivity	$6.6 \times 10^{-21} \text{A}\cdot\text{cm}^2\cdot\text{s}/\text{n} \pm 9.5\%$
Detector Gamma Sensitivity	$1 \times 10^{-16} \text{A}\cdot\text{h}/\text{R} \pm 6.5\%$
Cable Neutron Sensitivity	$0 \pm 1.14 \times 10^{-24} \text{A}\cdot\text{cm}^2\cdot\text{s}/\text{n}$
Cable Gamma Sensitivity	$0 \pm 1.9 \times 10^{-17} \text{A}\cdot\text{h}/\text{R}$
<u>Electrical Characteristics</u>	
Leakage Resistance at 294 K (70°F)	$\geq 1 \times 10^{13}$ ohms
Impedance (Both Emitter to Sheath and Compensating Lead to Sheath) at 673 K (752°F)	$\geq 2 \times 10^9$ ohms
Short Circuit Current at 616 K (648°F)	
Emitter to Sheath	$\leq 7.0 \times 10^{-11}$ A
Compensating Lead to Sheath	$\leq 6.0 \times 10^{-11}$ A

TABLE 3. LOFT SPND ATTENUATION COEFFICIENTS FOR EXPERIMENTS L5-1 AND L8-2

Density Difference $\Delta\rho = \rho_f - \rho_i$ (kg/m ³)	Attenuation Coefficient $A'(\Delta\rho)$	Applicable Experiment and Time Region
321	0.73	LOCE L5-1 Core uncover
321	0.74	LOCE L8-2 Core uncover
873	0.68	LOCE L5-1 Core reflood after saturation conditions are reached in the core
881	0.55	LOCE L5-1 Core reflood with subcooled liquid in core
952	0.53	LOCE L8-2 Core reflood

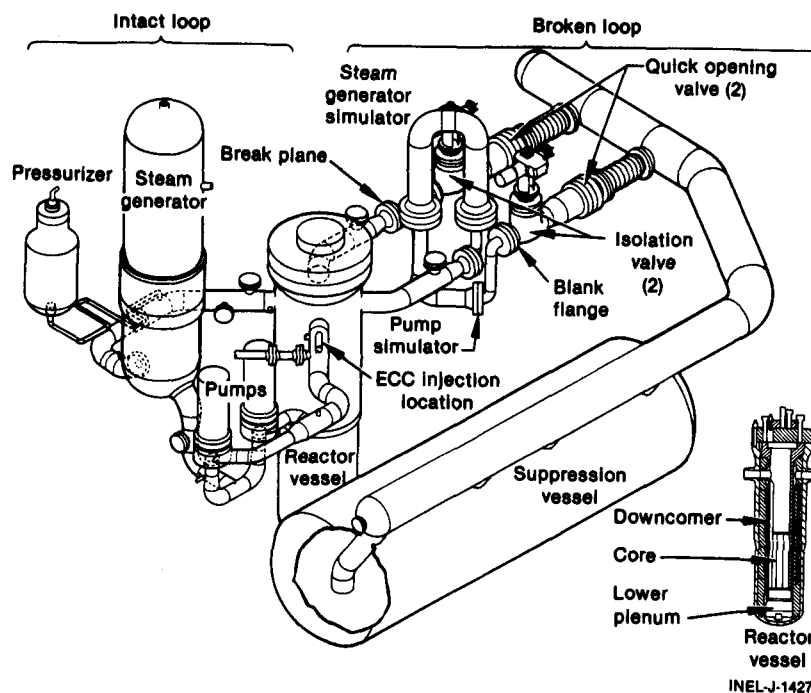


Figure 1; Axonometric projection of the LOFT system

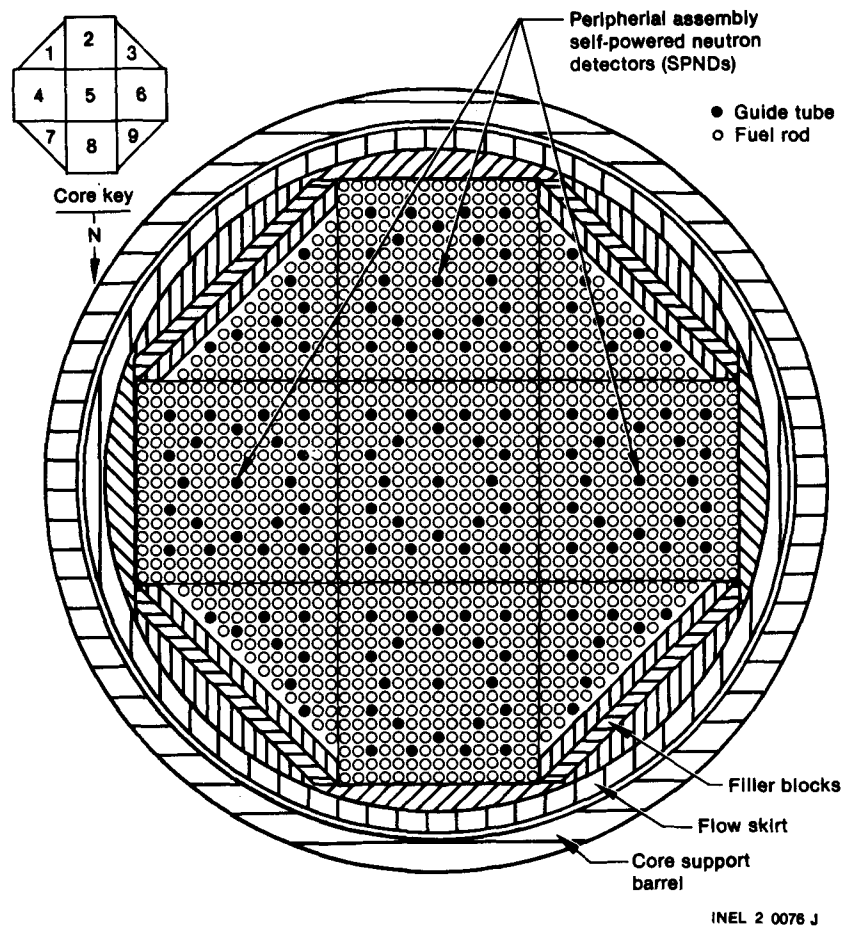


Figure 2: LOFT core map showing the radial location of the SPNDs

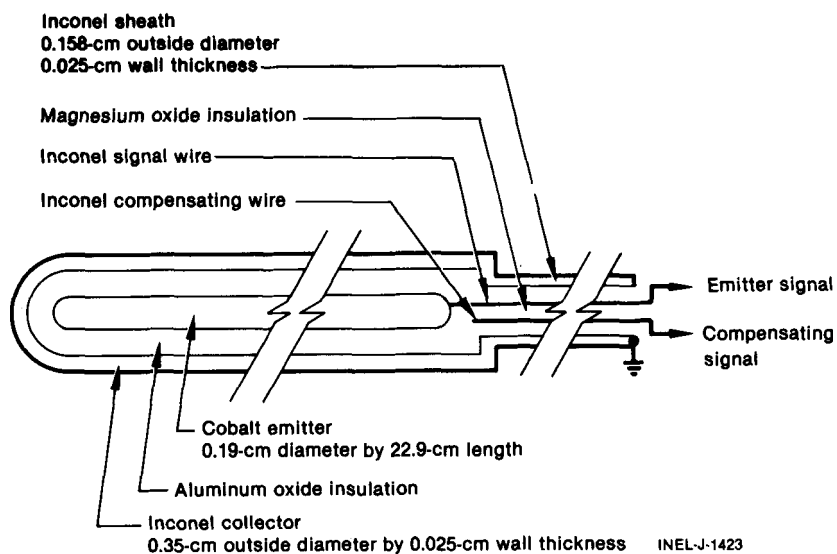


Figure 3: Schematic diagram of LOFT SPND

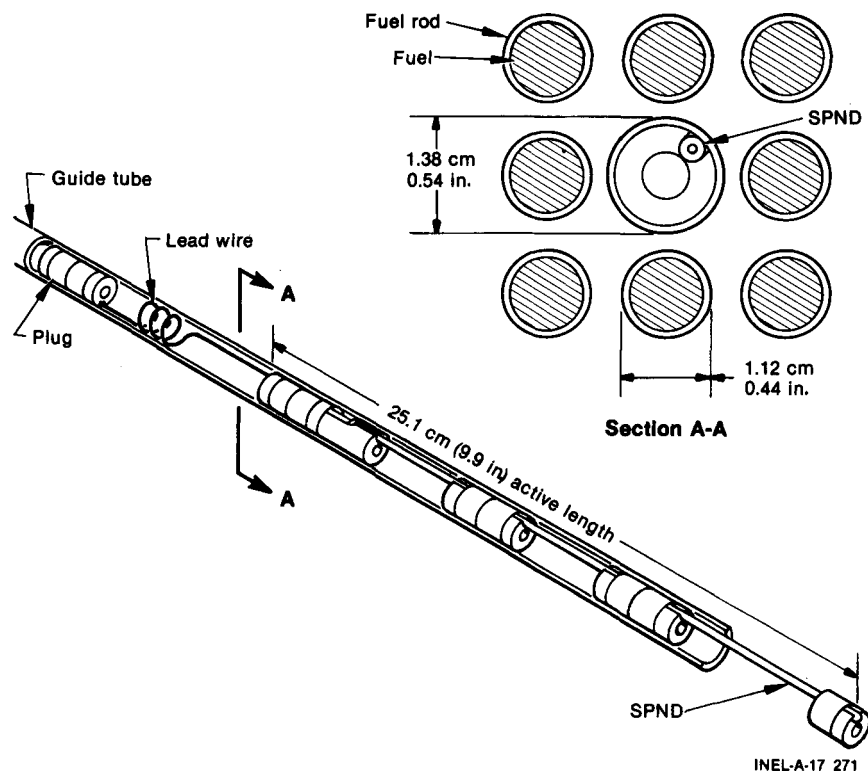


Figure 4: LOFT SPND guide tube assembly

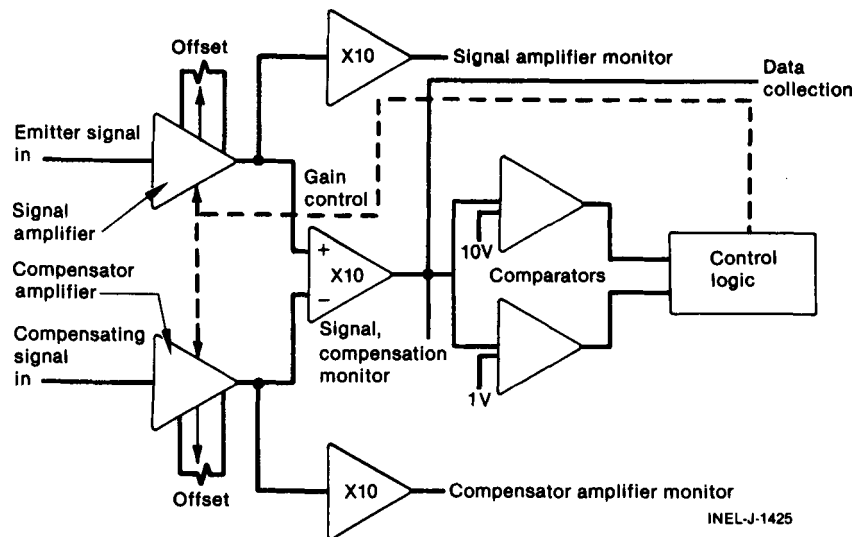
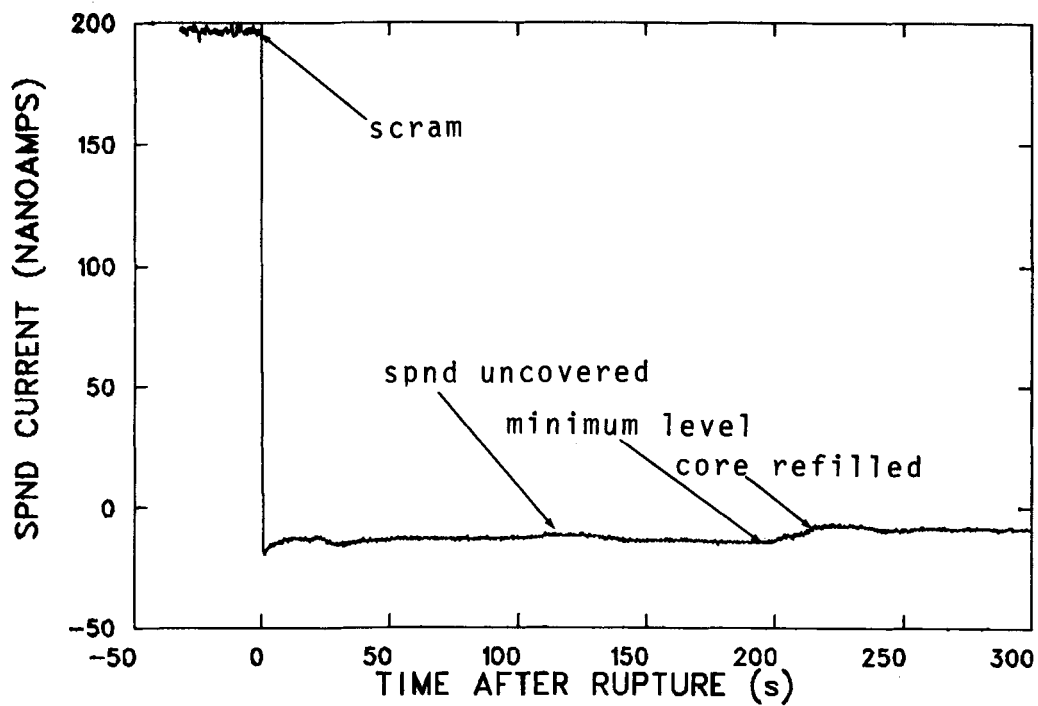
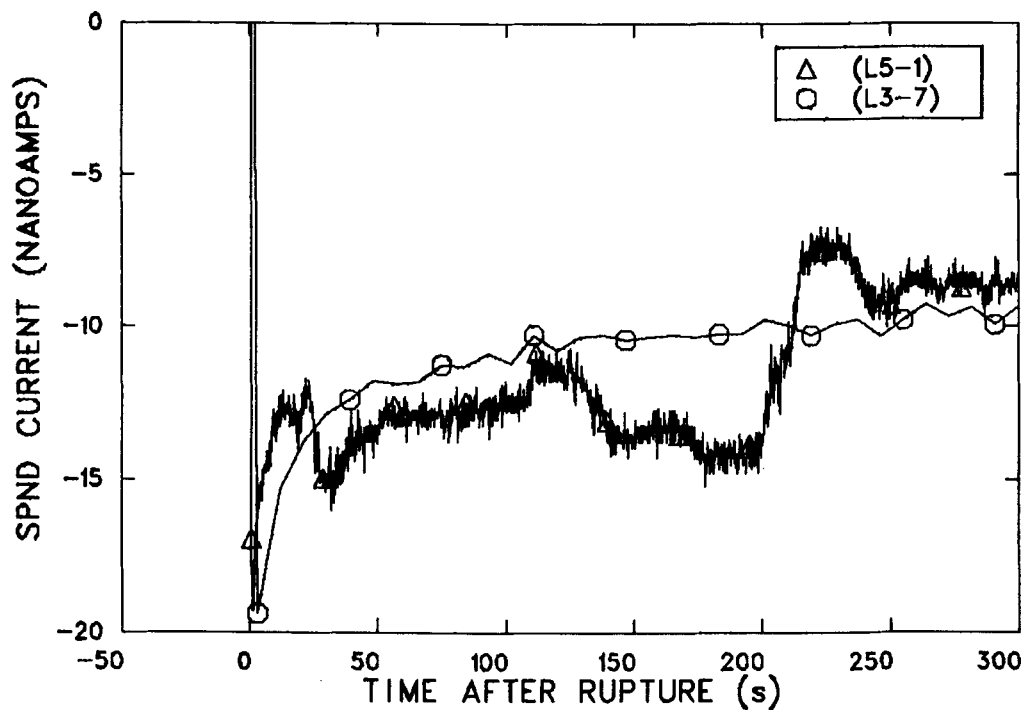


Figure 5: Block diagram of the LOFT SPND amplifier



(a) full scale



(b) expanded scale

Figure 6: SPND response during L5-1 compared to quiescent post-scrum behavior

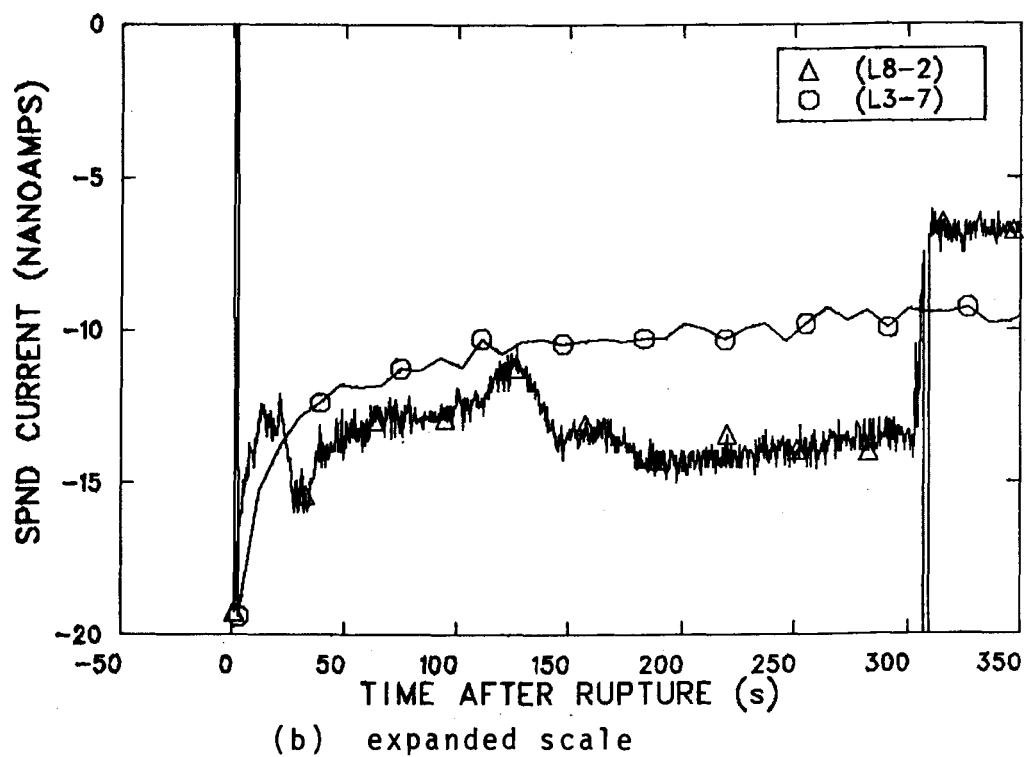
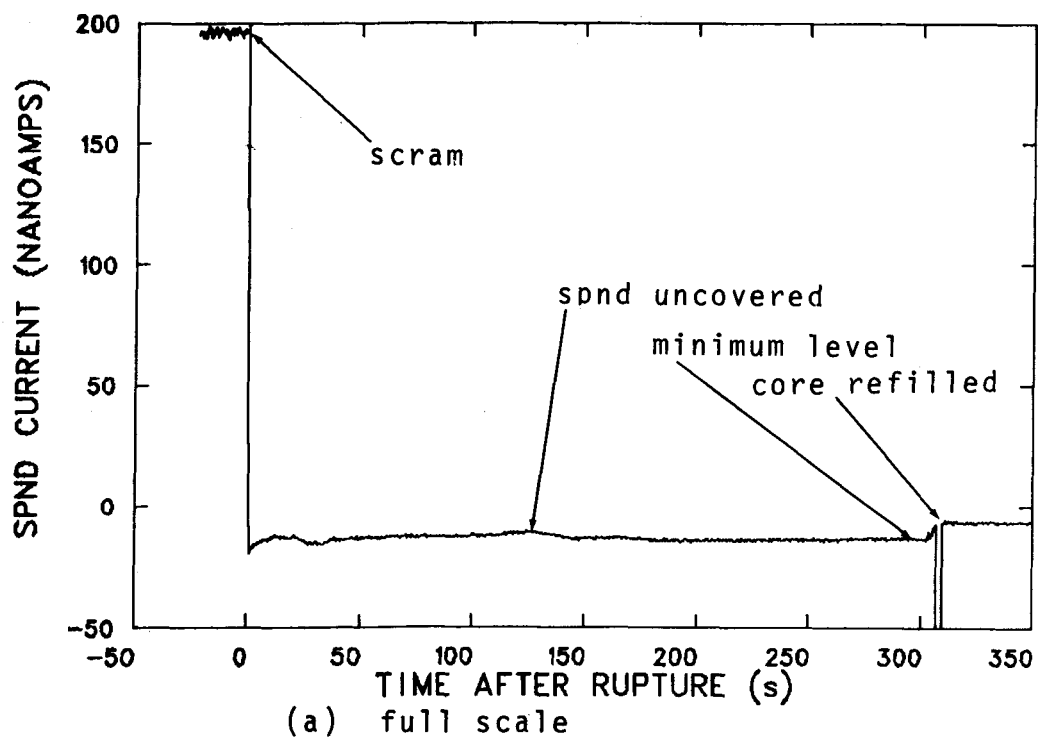


Figure 7: SPND response during L8-2 compared to quiescent post-scram behavior

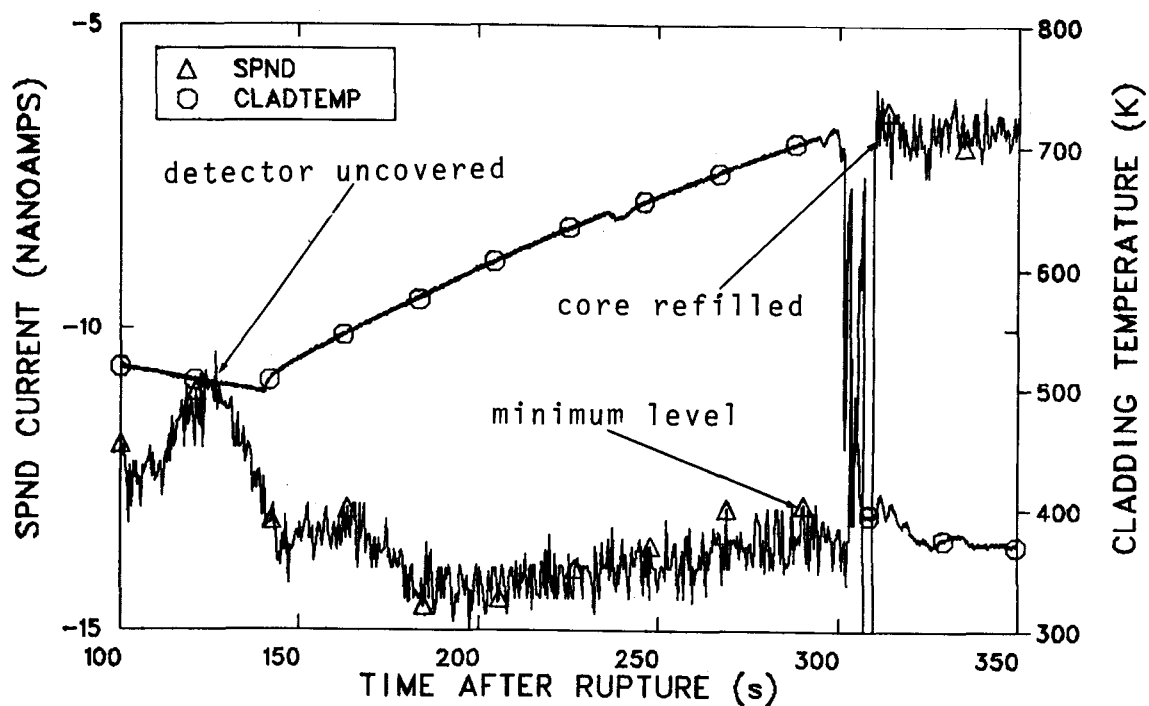


Figure 8: SPND and clad temperature response to core uncover and refill during L8-2

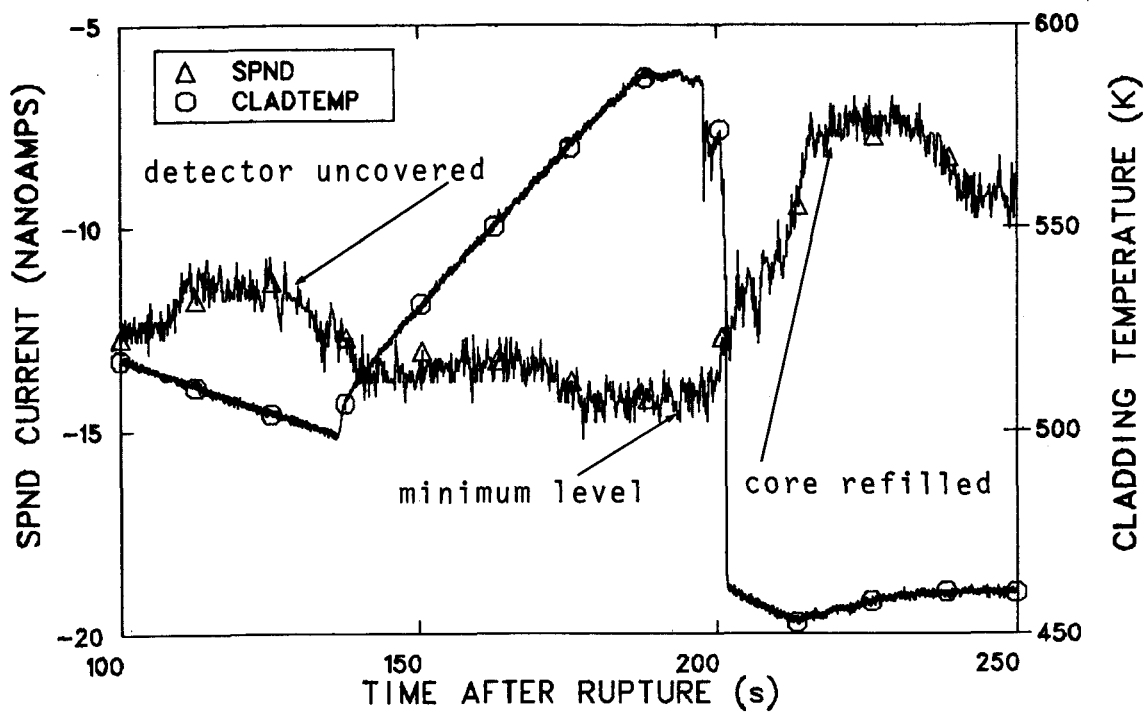


Figure 9: SPND and clad temperature response to core uncover and refill during L5-1

INVESTIGATION OF THE AXIAL VOID PROPAGATION
VELOCITY PROFILE IN BWR FUEL BUNDLES*

G. Kosály⁺, J.M. Fahley⁺, K. Behringer⁺⁺ and R.D. Crowe⁺

⁺Department of Nuclear Engineering
University of Washington
Seattle, WA 98195

⁺⁺Swiss Federal Institute for Reactor Research
5303 Würenlingen, Switzerland

ABSTRACT

The paper describes the method of measuring axial void propagation velocity in the core of an operating BWR. Experimental data obtained from the noise signals of LPRM and TIP detectors are compared to calculated steam velocity and kinematic wave velocity profiles.

VELOCITY DETERMINATION IN A BWR CORE USING NEUTRON NOISE ANALYSIS

In order to review the general principle of transit time (velocity) determination via noise analysis, we consider streaming fluid and two detectors (thermocouples, neutron detectors, etc.) located upstream and downstream as indicated in Figure 1.

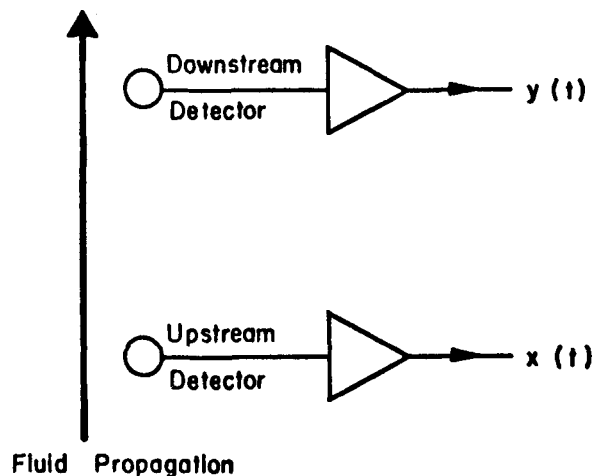


FIGURE 1. DETECTOR ARRANGEMENT FOR MEASURING PROPAGATION TIME IN A STREAMING FLUID

The fluctuating components of the detector signals (noise-signals) are driven by some kind of fluctuation in the fluid (temperature, density, etc.) transported by the stream. In many cases of engineering interest (1) there exists a frequency region where upstream and downstream signals are related to each

* Work sponsored by EPRI under Contract No. RP 1384-2

other by the simple relationship*

$$y(t) = \gamma x(t-T) . \quad (1)$$

Here γ is an attenuation coefficient and T is the transit time of the perturbation between the two detector positions. By the term "perturbation" we mean the fluctuation driving the signals.

Using Eq. (1) the cross-spectrum between the upstream and downstream signals can be written as

$$\text{CPSD}_{x,y}(\omega) = \gamma \text{APSD}_x(\omega) e^{-i\omega T} . \quad (2)$$

Here

$\text{CPSD}_{x,y}(\omega)$ = cross-power spectral density of the signals $x(t)$ and $y(t)$ (complex function of ω).

$\text{APSD}_x(\omega)$ = auto-power spectral density of the signal $x(t)$ (real function of ω).

ω = angular frequency.

According to Eq. (2) the phase-shift of the cross-spectrum ($\phi_{x,y}(\omega)$) reads

$$\phi_{x,y}(\omega) = -\omega T . \quad (3)$$

This means that if the sensors are "viewing" a propagating disturbance, the phase-shift is linear in frequency and the slope of the line is the transit time of the disturbance between the sensors.

In recent years neutron noise experiments have been performed to investigate the information contained in the fluctuating part of the Local Power Range Monitor (LPRM) and Traversing In-Core Probe (TIP) signals (4-10).

Figure 2 shows the axial location of the LPRM ($\sim A, B, C, D$) and the TIP detectors in the instrument tube between four fuel bundles. In the figure the movable TIP detector is shown half-way between LPRM B and LPRM C.

The measurements reported in Refs. (4-10) indicate that the phase-shift between axially displaced incore detector pairs is linear above 2 Hz.

Since for $f > 2$ Hz the neutron noise is driven by the local fluctuations of the void fraction it is concluded that two BWR incore detectors of the same radial position can be considered as upstream and downstream sensors "viewing" the axial propagation of the void disturbances in the neighboring channels. This means that from the linear phase shift of two incore detector signals (LPRM-LPRM, TIP-LPRM, LPRM-TIP) the transit time of the void disturbances between the measuring positions can be evaluated. From the measured transit time and the known distance between the detectors the propagation velocity of the void disturbance can be obtained. Using several different detector positions an axial velocity profile can be determined.

*The discussion of the present report is based on the over-simplified relationship given in Eq. (1). For a more general discussion we refer to Ref. (2).

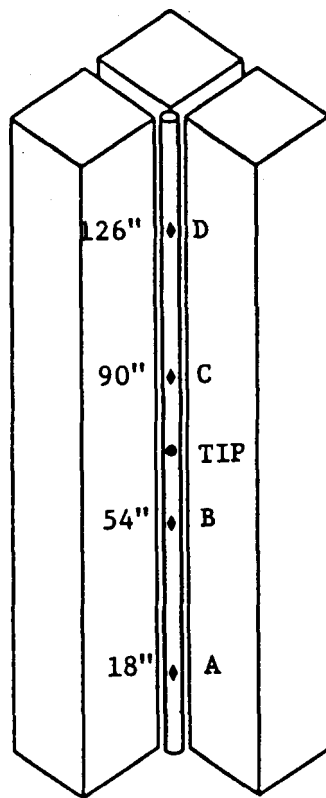


FIGURE 2. AXIAL LOCATIONS OF THE LPRMs, A, B, C AND D IN AN INSTRUMENT GUIDE TUBE. TIP IS SHOWN HALF-WAY BETWEEN B AND C.

The view, that above 2 Hz, BWR neutron noise is driven by local void fraction fluctuations is supported by recent neutronic calculations. These calculations indicate that for frequencies higher than 2 Hz the global (reactivity) component of the neutron noise becomes negligible. The fluctuations of the flux are driven by perturbations close to the detector (11-15). Extensive measurements in a research reactor (16) clearly establish the sensitivity of neutron fluctuations to local void disturbances.

Earlier reports were largely based on experiments performed at low exit qualities. (The exit quality was 0.13 in Ref. (7)). Usually only a few experimental points (~ 9 points) were taken along the axis.

The discussion of the present report is based on a recent measurement performed at the Muhleberg-BWR in Switzerland (8). The four bundles adjacent to the actual instrument tube (LPRM string: 12-13) were highly rated (Exit quality ≈ 0.23). In the measurements, 33 different TIP positions were used resulting in 63 data points. The experimental results were represented as a continuous line fitted through the data points. The fine spatial mesh used in this latter series of measurements makes the results attractive for detailed analysis.

THEORETICAL BACKGROUND AND CALCULATIONAL MODEL

It follows from the foregoing discussion that the transit time measured between two incore detectors is the transit time of the propagation of the void disturbances between the two sensors. The quantity we evaluate from the transit time is the void propagation velocity in the fuel bundles around the instrument guide tube. It was assumed in earlier reports that the measured velocity can be well approximated by the velocity of the steam phase (6-10).

The basis of the present discussion is the one-dimensional drift-flux model of two-phase flow (17). We write the void propagation velocity in a fuel bundle as the velocity of the kinematic wave in the two-phase flow mixture (18-19).*

$$C_k = V_g + \langle \alpha \rangle \langle j \rangle \frac{\partial C_o}{\partial \langle \alpha \rangle} + \langle \alpha \rangle \frac{\partial V_{gj}}{\partial \langle \alpha \rangle} \quad (4)$$

$$V_g = C_o \langle j \rangle + V_{gj} \quad (5)$$

Here

- C_k = kinematic wave velocity
- V_g = steam velocity
- C_o = drift-flux (distribution) parameter
- V_{gj} = drift-velocity
- $\langle \alpha \rangle$ = void fraction averaged over the flow area of a fuel bundle
- $\langle j \rangle$ = superficial velocity averaged over the flow area of a fuel bundle.

Equation (4) shows clearly the relationship between the measured velocity and the velocity of the steam phase. The two velocities are not identical unless the second and the third terms on the RHS of Eq. (4) are negligible (18, 10). The measured velocity is to be identified as the kinematic wave velocity of void propagation.

Steam velocity and kinematic wave velocity profiles have been calculated for the experimental case reported in Ref. (8). Thermal-hydraulic input parameters were taken from the plant-process computer at the date of the measurements. Calculations were performed for each of the four fuel bundles adjacent to the actual instrument guide tube.

The axial dependence of the equilibrium quality ($\langle x_{eq} \rangle$) the flow (true) quality ($\langle x \rangle$) and the void fraction ($\langle \alpha \rangle$) were calculated for each bundle. Flow quality was related to the equilibrium quality using the Levy-model of subcooled boiling (20). Between void fraction and flow quality the Zuber-Findlay relationship (17) was used (c.f. Eq. (5)).

* For a more detailed discussion of the use of Eq. (4) in the interpretation of the neutron noise measurements we refer to Appendix B of Ref. (2).

The calculations were performed with the computer code COBRA-IIIc (21). In the first phase of the work bundle averages were calculated using the subchannel geometry of an 8x8 GE fuel bundle. Because of the known limitations of applying COBRA-IIIc to analyze subchannel behavior in BWR-bundles (22), in the second phase of the work we calculated the total bundle quantities directly. The subchannel-to-subchannel mass transfer in COBRA-IIIc was "turned off" (23) and 1/8 of a total bundle was modeled by a single closed subchannel. The calculational results shown in the present report were obtained by the second method. For the comparison of the two methods and further calculational details we refer to (24).

The results of the calculations depend quite sensitively on the actual choice of the parameters C_0 and V_{gj} . Several authors represented the flow regime dependence of these parameters by relating them to the quality or to the void fraction. In the calculations we used two of the published models: the Ohkawa-Lahey (19) and the Lellouche-Zolotar (25) model.

For the actual case of the fuel bundle of a BWR in normal operation both models give the drift flux parameters as the function of pressure and void fraction. At low values of the Reynolds-number the Lellouche-Zolotar model accounts also for the mass flux dependence of the distribution parameter.

Figures 3 and 4 show the $\langle \alpha \rangle$ -dependence of the drift-flux parameters at a given pressure according to the two models. Both C_0 curves rise rapidly from

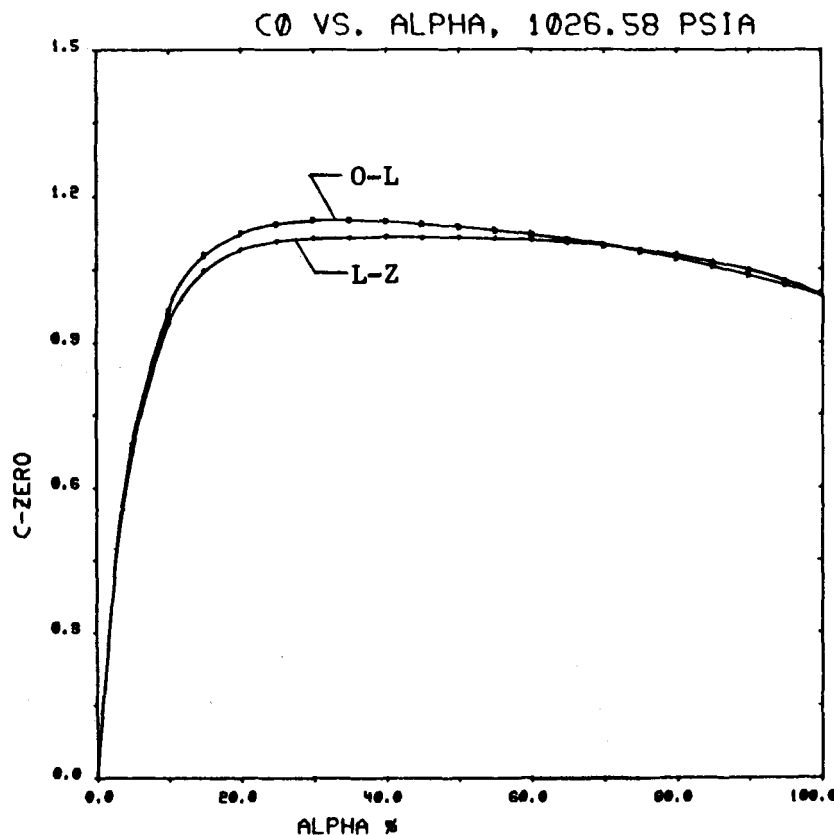


FIGURE 3. THE VOID FRACTION DEPENDENCE OF THE DISTRIBUTION PARAMETER FOR A BWR FUEL CHANNEL (O-L = OHKAWA-LAHEY (19), L-Z = LELLOUCHE-ZOLOTAR (25))

zero to a plateau value in the region of developing bubbly flow. After a plateau-region of relatively small change both curves decrease to unity as $\langle \alpha \rangle$ approaches unity (22). Figure 4 indicates a substantial difference in the two V_{gj} correlations.

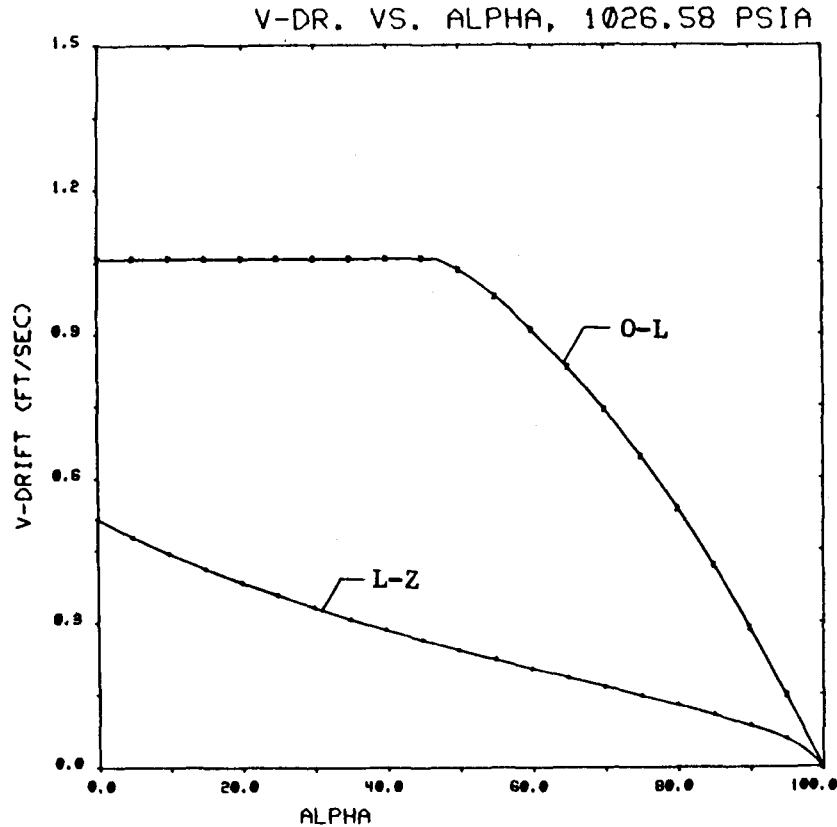


FIGURE 4. THE VOID FRACTION DEPENDENCE OF THE DRIFT-VELOCITY FOR A BWR FUEL CHANNEL (O-L = OKHAWA-LAHEY (19), L-Z = LELLOUCHE-ZOLOTAR (25)).

The aim of the calculations is to compare calculated velocity profiles (C_k and V_g) with the measured curve reported in Ref. (8). Due to the difference in the values of the input parameters the calculations provide different velocity profiles in the four bundles. The measured data are compared to the arithmetic average of the four bundle profiles.*

* More detailed results published in Ref. (2) indicate that the bundle-profiles are sufficiently close to each other to be represented fairly by their arithmetic average.

RESULTS AND DISCUSSION

Figure 5 shows the equilibrium and flow ("true") quality profiles calculated for the experimental case. Like all the other profiles shown in the present paper, the quality profiles represent the arithmetic average of the individual bundle profiles. The equilibrium quality becomes positive at about 19" above core-bottom. All the further results refer to the $x_{eq} > 0$ region.

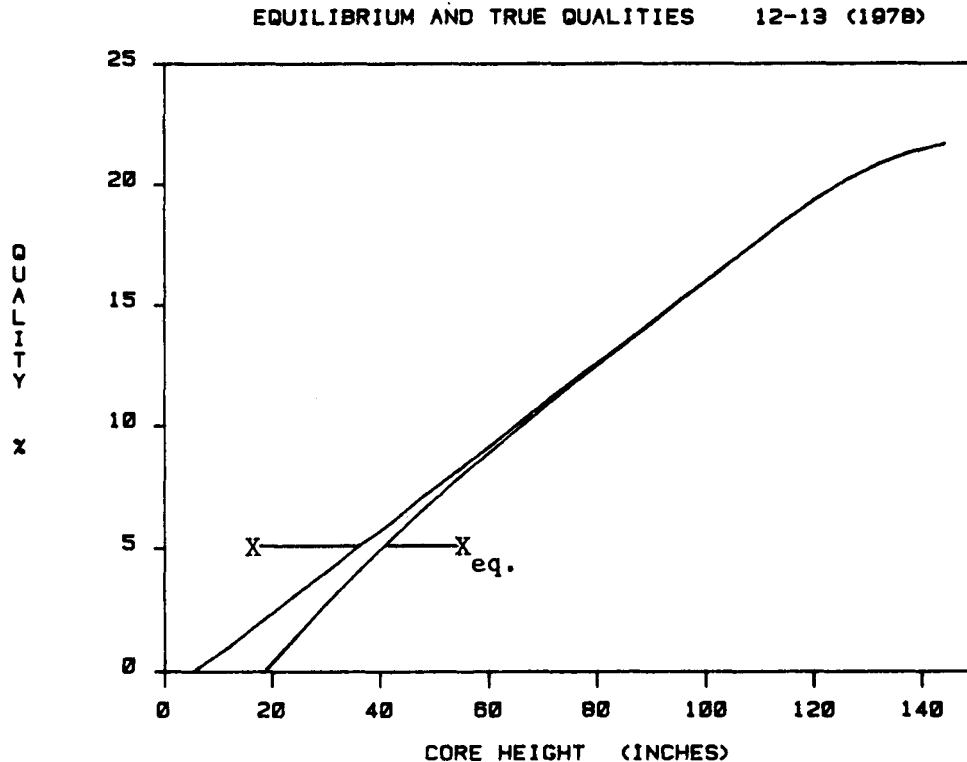


FIGURE 5. CALCULATED AXIAL PROFILES OF THE EQUILIBRIUM AND THE FLOW (TRUE) QUALITY

Figure 6 shows void fraction profiles calculated using the O-L and the L-Z models. The two models predict roughly similar values. The difference in the results is mainly due to the difference in the distribution parameter. In the bulk boiling region the contribution of the drift velocity is less important.

Figures 7 and 8 compare calculated velocity profiles to the experimental data reported in Ref. (8). The figures indicate clearly that in the actual case the measured curve cannot be approximated by the steam velocity profile. Both C_k -profiles are closer to the experimental curve than the corresponding V_g -profiles. The C_k -profile calculated via the Ohkawa-Lahey model is in complete agreement with the experiments in the upper part of the core ($z > 60"$, $x > 0.1$). The C_k -profile calculated using the Lellouche-Zolotar model are reasonably close to the experimental curve but remain always above the data.

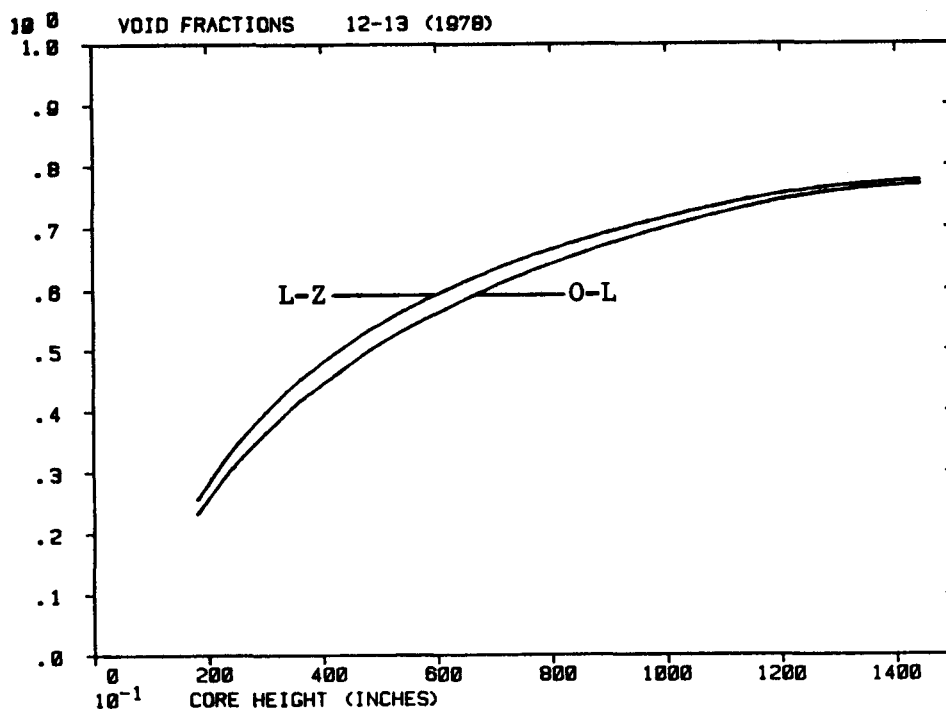


FIGURE 6. CALCULATED AXIAL VOID FRACTION PROFILES.

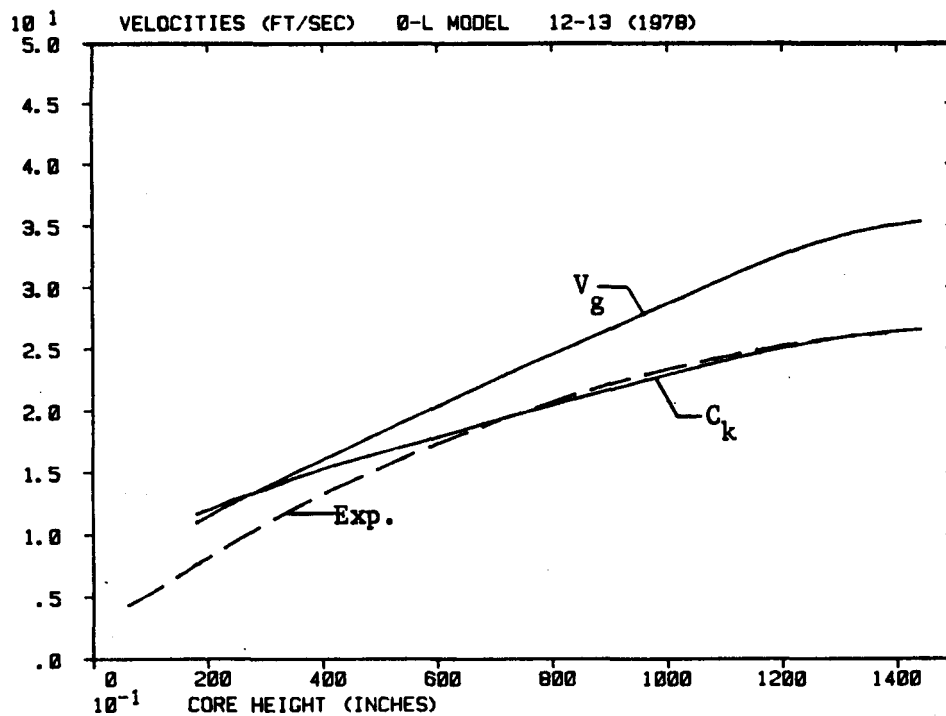


FIGURE 7. COMPARISON OF THE MEASURED VELOCITY DATA TO STEAM VELOCITY (V_g) AND KINEMATIC WAVE VELOCITY (C_k) PROFILES CALCULATED USING THE OHKAWA-LAHEY MODEL.

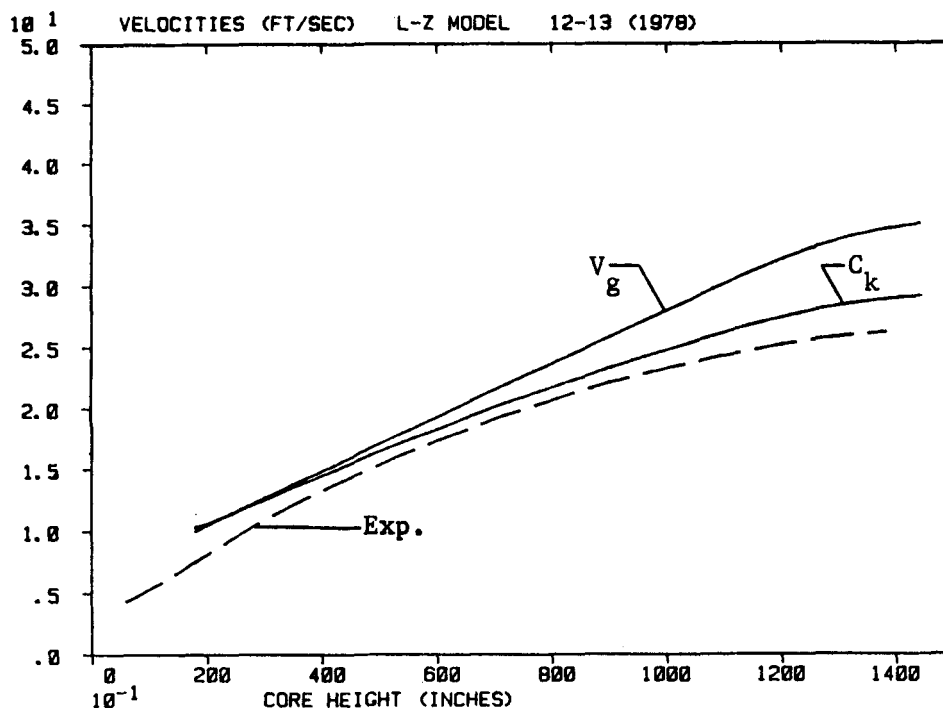


FIGURE 8. COMPARISON OF THE MEASURED VELOCITY DATA TO STEAM VELOCITY (V_g) AND KINEMATIC WAVE VELOCITY (C_k) PROFILES USING THE LELLOUCHE-ZOLOOTAR MODEL.

Discussing the observation that the calculated C_k -profiles tend to remain above the data let us remember that the velocity profiles come from total bundle calculations. It is obvious, however, that the neutron detector "sees" the perturbation at different points of the adjacent bundles with different weights. According to earlier phenomenological predictions (26) and a recent transport theory calculation (15), both the axial and the radial dimensions of the "sensitivity volume" of a neutron detector become quite small above 2 Hz. This means that it is justified to compare each measured velocity value to the calculated values referring to the same axial level. It also means, however, that the flow near to the corner of the bundle will have a relatively strong influence on the results.

Since the flow near to the corner is expected to be slower than at the center of the bundle it seems to be natural that the measured values tend to be lower than the calculated results. This tendency can be clearly seen in the Lellouche-Zolotar results but can also be observed in the Ohkawa-Lahey results in the lower part of the core. In order to discuss further the effects caused by the radial dimensions of the sensitivity volume proper subchannel calculations are needed.

The calculations were based on the thermal-hydraulic input/parameters taken from the plant-process computer. Further studies are needed to determine the sensitivity of the C_k -profiles to possible errors in the input parameters.

It follows from the above discussion that the method has a number of open problems. The aim of the present work was to start to explore the possibility of using in-core detectors as BWR flow sensors. For reducing the method to practice substantial further research work is needed. It would be therefore premature to use the results to decide between different void-quality correlations.

CONCLUSIONS

The discussion confirms that the fluctuating part of the LPRM and TIP signals contain pertinent information about the two-phase flow in the fuel bundles of an operating BWR. The axial velocity profile inferred from the measured phase-shifts can be interpreted as the kinematic wave velocity of the void propagation.

As a general conclusion we maintain that the fluctuating signals of in-core neutron detectors can be used as flow sensors in an operating BWR. For a further discussion of this idea we refer to (16).

ACKNOWLEDGMENT

The authors are indebted to Professor R.T. Lahey, Jr. and Dr. G.E. Dix for their comments concerning the calculation of the measured velocity using the drift-flux model. Thanks are due to Professor R.W. Albrecht and Dr. G.S. Lellouche for their constant interest in the work and for many useful discussions.

REFERENCES

1. J.S. Bendat and A.G. Piersol, "Engineering Applications of Correlation and Spectral Analysis". John Wiley & Sons, Inc., 1980.
2. G. Kosály and J.M. Fahley, "Investigation of Axial Void Propagation Velocity Profiles in BWR Fuel Bundles," (EPRI-Report to be published, 1982).
3. P.G. Bentley and D.G. Dawson, "Fluid Flow Measurement by Transit Time Analysis of Temperature Fluctuations," Trans. of the Soc. of Instrument Techn. (1966).
4. W. Seifritz and F. Cioli, Trans. Am. Nucl. Soc. 17, 451 (1973).
5. D. Wach and G. Kosaly, Atomkernenergie 23, 244 (1974).
6. M.A. Atta, D.N. Fry, J.E. Mott and W.T. King, Nucl. Sci. Eng. 66, 264 (1978).
7. G. Kosály, Lj. Kostic, L. Miteff, G. Varadi and K. Behringer, Nucl. Eng. Design 52, 357 (1979).
8. K. Behringer and R.D. Crowe, Atomkernenergie 38, 47 (1981).
9. D.N. Fry, "In-Core Flow Velocity Profiles During the First Fuel Cycle at Hatch-1-Inferred from Neutron Noise," EPRI-NP-2083, October 1981.
10. N. Naito, Y. Ando, F. Yamamoto and E. Takeuchi, J. Nucl. Sci. Technol. 17, 351 (1980).
11. H. Dam, Atomkernenergie 27, 8 (1976).
12. K. Behringer, G. Kosály and Lj. Kostic, Nucl. Sci. Eng. 63, 306 (1977).
13. K. Behringer, G. Kosály and I. Pazsit, Nucl. Sci. Eng. 72, 304 (1979).
14. F.C. Difilippo, Nucl. Sci. Eng. 80, 211 (1982).
15. G. Kosály and R. Sanchez, "Calculation of the Sensitivity Volume of a BWR In-Core Detector Above 2 Hz," Summary accepted for presentation at the ANS Summer Meeting, Los Angeles, June 6-11, 1982.

16. R.W. Albrecht, R.D. Crowe, D.J. Dailey, M.J. Damborg and G. Kosály, "Measurement of Two Phase Flow Properties Using the Nuclear Reactor Instrument," Paper presented at the 3rd Specialist's Meeting on Reactor Noise (SMORN-III), Tokyo, Japan, October 26-30, 1981.
17. N. Zuber and J.A. Findlay, J. Heat Transfer, ASME Trans. Serv. C. 87, 453 (1965).
18. N. Zuber and F.W. Staub, Nucl. Sci. Eng. 30, 268 (1967).
19. R. Ohkawa and R.T. Lahey, Jr., Nucl. Eng. and Design 61, 245 (1980).
20. S. Levy, "Forced Convection Subcooled Boiling-Prediction of Vapor Volumetric Fraction," GEAP-5157 (1966).
21. D.S. Rowe, "COBRA-IIIc: A Digital Computer Program for Steady State and Transient Thermal-Hydraulic Analysis of Rod Bundle Nuclear Fuel Elements," BNWL-1695, March 1973.
22. R.T. Lahey, Jr. and F.J. Moody, "The Thermal-Hydraulics of a Boiling Water Nuclear Reactor," American Nuclear Society, 1977.
23. A.F. Ansari, R.R. Gay and B.F. Gitnick, "FIBWR: A Steady-State Core Flow Distribution Code for Boiling Water Reactors," EPRI-NP-1923, July 1981.
24. J.M. Fahley, M.S. Thesis in progress, 1982.
25. G.S. Lellouche and B.A. Zolotar, "A Mechanistic Model for Predicting Two Phase Void Fractions for Water-Vertical Tubes, Channels and Rod Bundles," (EPRI in press, 1982).
26. G. Kosály, Progress in Nuclear Energy 5, 154 (1980).

USE OF CONDUCTANCE PROBE METHOD TO MEASURE TWO PHASE FLOW PATTERNS

Gao Huahun Tu Chuanjing

Xi'an Jiaotong University Xi'an, Shaanxi Province

The People's Republic of China

ABSTRACT

A set of double conductance probes is used to measure the flow patterns in two phase horizontal straight and helical flow. Both the visual and conductance probe methods agree moderately well.

I. Introduction

Gas-liquid two phase flows are used widely in the nuclear reactors, rockets, boilers, evaporators and chemical equipments. The requirement for economic designs, optimization of operating conditions and assesement of safety factors leads to the need for quantitative information of the two phase flow and heat transfer. It is often important in analysing two phase flow systems to classify the flow into a number of "flow patterns". It helps in obtaining a qualitative understanding of the flow and ultimately is able to lead to better predication methods (analytical or experimental) for the various two phase flow parameters.

Flow pattern information is usually obtained by visual observation of the flow. The way is to observe the flow in a transparent channel, but the major difficulty is that the picture is often confusing, especially when dealing with the predication of a transition point. The measurement of two phase flow parameters is described by Jones et al.(1), but only a few methods can be directly employed to measure flow patterns. Even with high speed photography, it is often not possible to observe the structure of the flow clearly due to the complex light-refraction paths within the medium. Jones et al.(2) developed an X-ray system for the void measurement and flow pattern characterization. Hsu et al.(3) utilized a hat wire anemometry technique for measuring void distribution and flow pattern characterization. Choe et al.(4) detected the pattern transition based on the pressure fluctuations all of these methods have their disadvantages, for they can not obtain the results directly, or can not measure all the flow patterns, or a transparent channel must be used.

The conductance probe method is the most popular technique for two phase (gas-liquid) flow pattern categorisation. The technique is to insert a needle facing directly to the flow and to measure the current from the tip of this needle, through the two phase flow, to the wall of the channel. The current is displayed on an oscilloscope and the type of response is considered to be representative of the flow pattern. Although this technique has been used by a number of authors (5-8), the flow pattern is defined on the basis of a single probe. Dvora Barnea et al.(9) describes a set double D.C. conductance probes for straight glass tube, with which all flow patterns can be detected. In this work, an improved conductance double probe technique is proposed by us.

II. Experiment

The following contents can be summarized as:

1. We have used D.C. conductance probes as illustrated in Figure 1 and A.C. conductance probes as illustrated in Figure 2. The air and water mixture flows in a horizontal glass tube and the flow patterns are detected by using the double conductance probes (one probe is a flat plate, and the other is a needle).

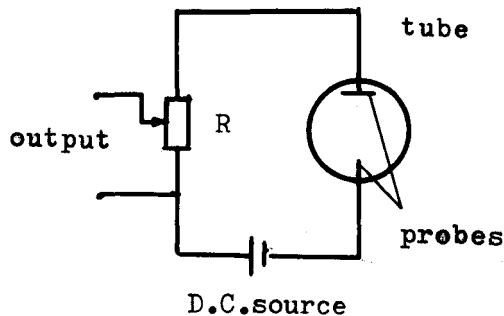


Fig.1. D. C. Conductance probe

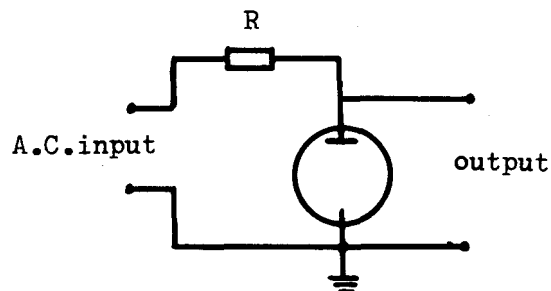


Fig.2. A. C. conductance probe

2. We have used two kinds of probes as illustrated in Figure 3 (probes b and c), with three situations used for kind b (probe a, b and d). Therefore we have used four types of probes (Figures 3a, 3b, 3c, 3d). The D.C. probes are types b and c, the A.C. probes are types a, b, c, d.

Three types of D.C. conductance probes are proposed by Dvora Barnea et al.(9). In our work, the flat electrode is located at the top of the tube (b and c), and is considered more advantageous.

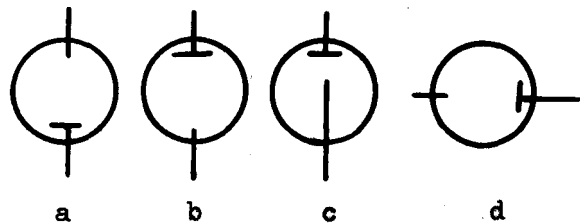


Fig.3. Four different types of probes

3. Detecting the flow patterns in the horizontal straight tube and in the helical tube.

The two phase flow patterns in the vertical and horizontal straight tubes were detected by many authors, but the patterns in the helical tube were observed only by a few authors. The helical tube steam generators are used in many nuclear power stations, such as F B R, H T G R and ship P W R and so on, and are also used in steam boilers and chemical equipments etc.

4. To compare the flow patterns of the horizontal straight glass tube with those of the steel tube.

An experiment is carried out in a water-air system as shown in Figure 4. The probes were placed in a test section, and the material of test section is glass or steel.

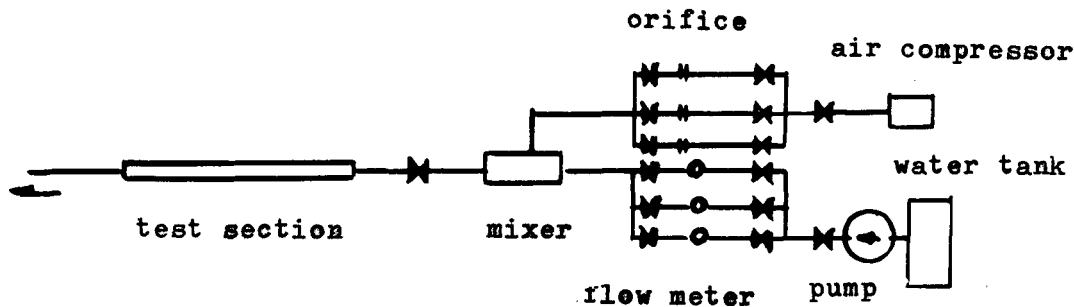


Fig.4. Scheme of experimental loop

The probes are made of copper. The probe for steel tube has a needle electrode, coated with insulating lac apart from 2mm long tip. The surface of the flat electrode, facing to the wall of tube, also is coated with insulating lac.

III. Results

1. The probing system with the D.C. method. The electrical diagram is shown in Figure 1. When the tube is full of water the electric circuit is closed through the water and a maximum output voltage V_{max} is detected. When the tube is empty of water the circuit is open and a minimum output voltage V_0 is detected. Two probes designated as b and c are used, as illustrated in Figure 5. Top line: the tube is full of water, bottom line: the tube is full of air.

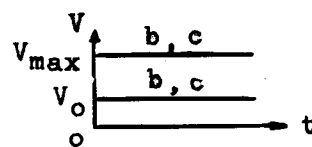


Fig.5. Single phase flow (air or water)

$$W_{ls}=0.3\text{m/s } W_{gs}=0.01\text{m/s}$$

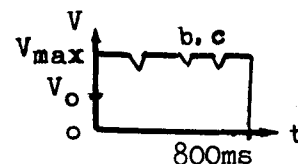


Fig.6. Bubble flow

The traces of Figure 6 obtained by probe b and c correspond to the bubble flow. Superficial velocity of water W_{ls} , superficial velocity of gas W_{gs} .

The traces of Figure 7 correspond to the plug flow. Figure 7a is obtained by probe b and Figure 7b is obtained by probe c. In the trace, the voltage fluctuates intermittently from maximum conductivity, which corresponds to a full liquid, to minimum conductivity, which corresponds to a gas zone.

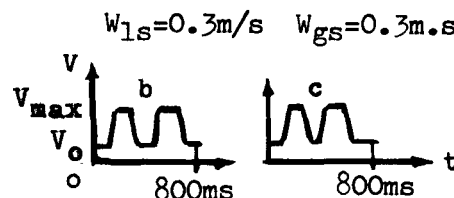


Fig.7. Plug flow

Figure 8 shows that the trace in the slug flow obtained by probe c is similar to that in the plug flow. In the slug flow, adjacent gas zones are separated by a liquid zone which is aerated by gas bubbles. The air bubbles in the liquid slug are indicated by short pulses.

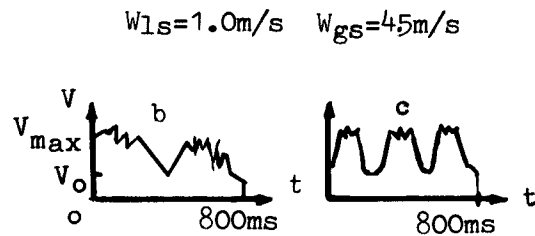


Fig.8. Slug flow

Figure 9 shows the trace of annular flow. The mean output voltage of probe b is proportional to the film thickness. Probe c is almost completely dry as seen from the V_0 voltage output, and it is the same value when the tube is full of gas.

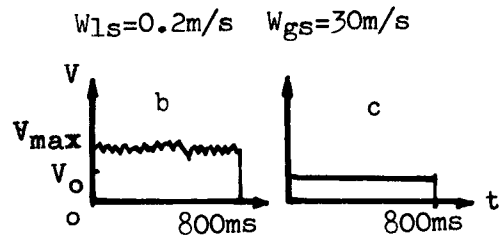


Fig.9. Annular flow

Figure 10 illustrates the type of traces obtained in the dispersed bubble flow by probe b and c. Many dispersed bubbles which hit the probes cause a series of pulses of high frequency. It must be pointed out that the time ordinate is different from picture above mentioned. Here is 100ms, but the picture of above mentioned is 800ms.

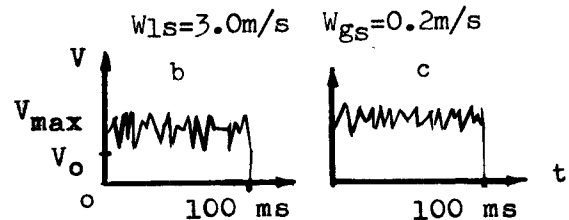


Fig.10. Dispersed flow

Figure 11 shows the traces of stratified flow pattern. The trace is a line detected by probe b or probe c.

In our work the probe system is shown to clearly detect all flow patterns based on an oscillograph trace given by probe b and c. Probe b is more available.

2. The probing system with the A.C method

The electrical diagram is shown in Figure 2. This method is also shown to clearly detect all flow patterns. The result is satisfactory.

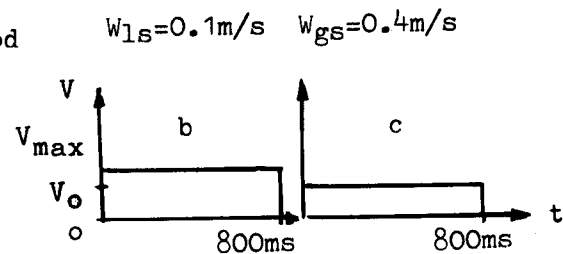


Fig.11. Stratified flow

Figure 12 shows the single phase flow (water) trace with $V=40$ mv when the tube is full of water.

Figure 13 shows single phase (air) trace with $V=90$ mv when the tube is full of gas.

The stratified pattern also has a similar trace, but the voltage is different. It is about between 40-90mv.

Figure 14 shows the trace of annular flow with $V=80$ mv and $U=60$ mv U is fluctuation voltage of envelope curve, in which modulation wave frequency f is high.

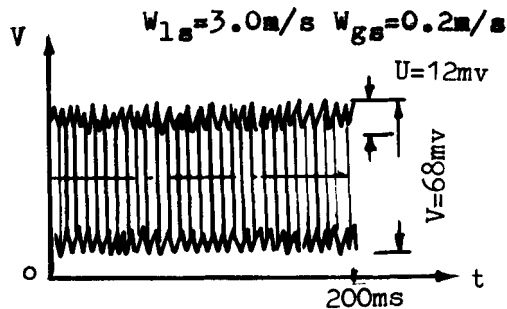


Fig.15. Dispersed flow A.C. probe

Figure 15 shows the trace of dispersed bubble flow with $V=68$ mv and $U=12$ mv in which f is higher.

Figure 16 shows the trace of slug flow with $V=90$ mv $U=15$ mv, in which f is low. The A.C. conductance probe method is also very useful and satisfactory to detect all the flow patterns.

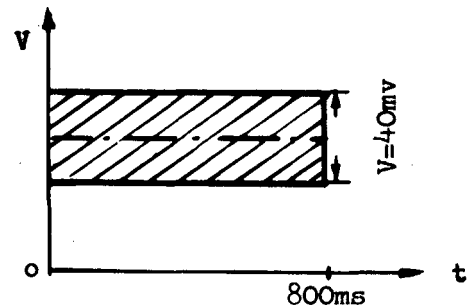


Fig.12. Single phase (water)

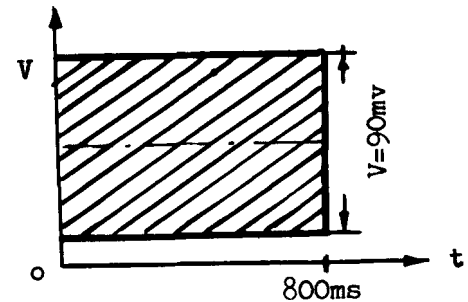


Fig.13. Single phase (air).

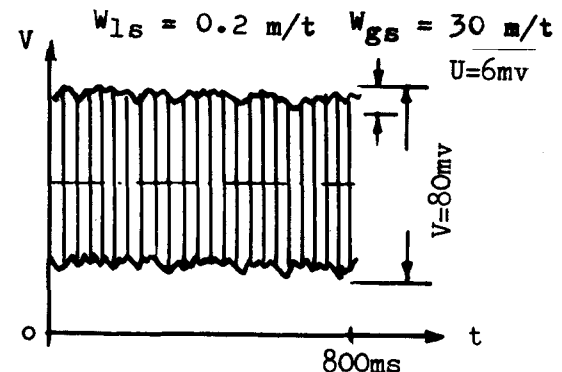


Fig.14. Annular flow A.C. probe

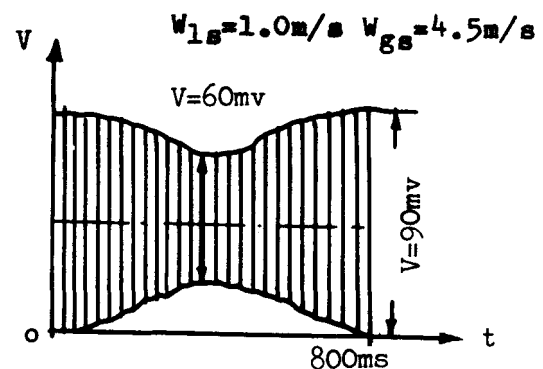


Fig.16. Slug flow A. C. probe

3. To compare the different probes

Table 1 shows the results of comparing four A.C. probes (Figure 3. a, b, c, d)

Table 1 (in mv)

type of A.C. probe flow pattern	a	b	c	d
single phase (water)	V=40	V=40	V=40	V=40
single phase (gas)	V=90	V=90	V=90	V=90
annular flow $W_{ls}=0.2m/s$ $W_{gs}=30m/s$	V=84 Small U	V=80 U=6	V=88 small U	V=75 small U
slug flow $W_{ls}=1.0m/s$ $W_{gs}=4.5m/s$	V=75-50	V=90-60	V=90-60	V=80-60
dispersed flow $W_{ls}=3m/s$ $W_{gs}=0.2m/s$	V=65 U=6	V=68 U=12	V=80 small U	V=70 U=10
stratified flow $W_{ls}=0.11m/s$ $W_{gs}=0.4m/s$	V=80	V=86	V=88	V=90
plug flow $W_{ls}=0.3m/s$ $W_{gs}=0.3m/s$	V=40 U=10	V=40 U=20	V=40 U=20	V=40

The type of prove b is most available, it can be employed clearly detect various flow patterns. In this work, a criterion to determine the transition state between annular and dispersed bubble flow patterns is proposed, as denoted in Table 2.

The visual and conductance probe methods agree moderately well.

Table 2

dispersed bubble flow pattern	V 70mv	V 10mv	the frequency of pulse is high
annular flow pattern	V 70mv	V 10mv	the frequency of pulse in low

4. Flow pattern in the helical tube

The A.C. probes are used. The helical diameter D is 500mm and the tube diameter d is 20 mm with $D/d = 25$. The trace obtained by A.C. probe b in the helical tube is the same as that in the horizontal straight tube.

In the helical tube, the stratified pattern under certain condition can cause a film inversion (10). The film inversion can be detected by a rotating probe b, and following results can be obtained:

$V=80\text{mv}$ obtained by probe b

$V=75\text{mv}$ obtained by probe d
(probe b turns through 90°)

$V=86\text{mv}$ obtained by probe a
(probe b turns through 180°)

When probe b turns through 90° , the needle contacts with the film, then the V value decrease, this means that there is a film inversion.

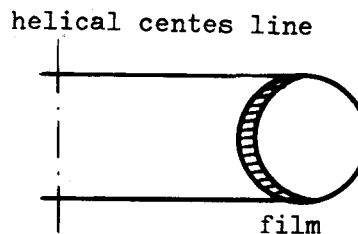


Fig.17. Film inversion

5. Flow pattern characterization in glass and steel tubes

The carbon steel and stainless steel tubes are used. There is no difference to compare the traces between them. Comparing the traces obtained by D.C. or A.C. probes for the glass tube with those obtained by D.C. or A.C. probes for the steel tube shows that there nearly the same results, as illustrated in Figure 6.

In the steel tube the surface of the flat electrode facing to the tube wall is insulated. When the size of the bubble is less than 2mm, it can not be detected, because the small bubble is located at the top of the tube. In the annular flow the sensitivity of probe for the glass tube is larger than the value for the steel tube. There are no differences between them in other flow patterns.

IV. Conclusions

This paper describes a set of double conductance probes for air-water systems.

- 1) Probes can be used with the D.C. method or with the A.C. method.
 - 2) By probes for steel tube and glass tube the almost results are obtained.
 - 3) The type of probe b is the most applicable one.
- The probe remains to be determined in detail.

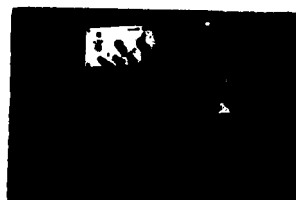
Acknowledgement: The authors wish to thank professor Chen Xuejun for his valuable guide.

REFERENCES

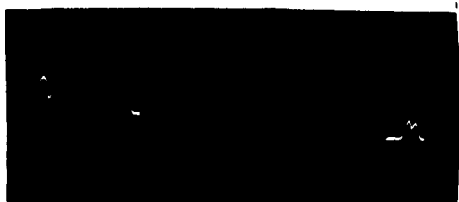
1. Jones, JR. et al,
1976 Int.J. Multiphase Flow 3, 89-116.
2. Jones, JR. et al, 1975 Int.J. Multiphase Flow, 2, 273-306.
3. Hsu Y.Y et al, 1963 ASME winter Meeting, Philadelphia.
4. Chos W.G et al,
1976 Proceeding of Symposium in Two-phase flow and Heat transfer.
5. Solomon J.V., 1962 M.Sc. Thesis, Mech Eng Dept., MIT.
6. Griffith P., 1964 ASME Paper 64-WA/HT-43.
7. Fiori M.P. et al,
1966 Rep. 5382-40, Mech Eng Dept., MIT.
8. Bergles A.E. et al, 1967 J. Heat Transfer, 89, 69-74.
9. Dvora Barnea et al, 1980 Int. J. Multiphase Flow No. 10.
10. Banerjee S.,
1967 AIChE J. Vol. 13 No.1.

APPENDIX

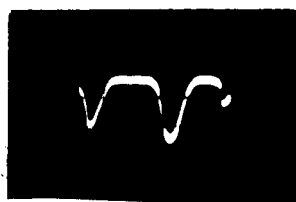
Test Photograph :



Instrument



Bubble flow (D.C.)



Plug flow (D.C.)



Annular (D.C.)



Slug flow (D.C.)



Dispersed bubble(D.C)



Annular flow(A.C)

SEPARATE EFFECTS TESTS IN SUPPORT OF BWR

THERMAL/HYDRAULIC MONITORING*

R.W. Albrecht, R.D. Crowe, D.J. Dailey,
H.J. Khan, K. Kotani,** S.C. Swoope
Department of Nuclear Engineering
University of Washington
Seattle, Washington 98195

* Work supported by NRC

** On leave from Hitachi Energy Research
Laboratory, Hitachi, Ltd., Japan

ABSTRACT

Miniature fission chambers, mounted on a flow section, were used in a small research reactor to characterize two-phase flow. A stochastic description of the structure, dynamics, and propagation is developed. The property of propagation, as measured by neutronic noise analysis, is addressed. The drift flux model is compared to experimental results.

INTRODUCTION

To develop the technology necessary to analyze BWR thermal hydraulics on-line from existing incore sensors (e.g., LPRM and TIP detectors) a comprehensive program must include (1) BWR measurements (2) separate effects tests and (3) neutronic and thermal hydraulic calculation. To promote this effort a series of air-water, two-phase flow experiments was carried out using a small research reactor /1-5/. In parallel to the measurements a theory relating the void fraction fluctuation to the measured neutronic signal was developed /5,6/. It was then possible to separate the effects of the neutronics from the information about the two-phase flow, and study two-phase flow using the reactor as an instrument /6,7/. It has been demonstrated that essentially all the information obtainable from conventional two-phase flow instruments (e.g., densitometry measurements) can be realized by the analysis of neutronic fluctuations. In addition the neutronic noise techniques developed using a small reactor have no basic impediments to application in a large scale BWR, whereas competing instruments have severe restrictions in the harsh environment of an operating commercial power plant. Once the two-phase flow information is available from the reactor instrument, the questions of type of characterization and the relationship to usable thermal hydraulic parameters remains.

One dimensional two-phase flow can be characterized either deterministically or stochastically. The deterministic approach is used in nearly all thermal/hydraulic calculations for LWR's. However, due to the stochastic nature of two-phase flow a complete characterization requires the inclusion of stochastic descriptors. The need for stochastics is demonstrated by the use of constitutive relations and interfacial transfer terms that depend upon the flow structure and dynamics. It is therefore useful to characterize the stochastics in a quantitative manner. Three categories are proposed, these being: propagation, structure, and dynamics. Further, it has been shown that a true stochastic characterization requires ten parameters /7/. However, to deter-

ministically describe the "averaged" properties of a two-phase flow demands only a subset of five stochastic parameters. To begin the discussion of the characterization methodology a description of the three categories and their respective parameters follows.

STRUCTURE

The time averaged structure of a two-phase flow, as characterized by the probability per unit void fraction of being within some small increment of void fraction about a particular void fraction at some point in space (e.g., $p(\alpha(z))d\alpha$) can completely describe the time averaged spatial organization. When values of this probability for all void fraction (α) are determined the probability density function becomes a flow regime description /7,8/. It has been observed that there are two basic types of distribution for two-phase flow, unimodal and bimodal /7,8/. The unimodal case corresponds to the situation where the flow statistics remain constant over a long period of time (e.g., bubble flow). The bimodal case is associated with flow types that have approximately unchanging statistics for some average time and then switch to a second state, which is stable for an average time before it is forced by instabilities to switch back to the original state. This switching results in a probability density function having a distinct peak associated with each void state, and a region between the peaks where the individual distributions overlap. The fraction of the area under each peak is indicative of the portion of the total observation time the flow spends in that state /9/. A large class of two-phase flows exhibit bimodality. Parameterization of the probability density function can be done in two ways. First a direct measurement of the first four moments of the observed density is possible /8/. This approach provides only limited physical insight into the density functions as the shape cannot be reconstructed based upon only four moments. The second possibility is to fit a limited parameter distribution to the density functions /7/. A suggested density is the beta density:

$$p(\alpha) = \lambda B^{-1}(a,b) \alpha^a (1-\alpha)^b + (1-\lambda) B^{-1}(c,d) \alpha^c (1-\alpha)^d \quad (1)$$

where

$$B(a,b) = \int_0^1 \alpha^a (1-\alpha)^b d\alpha$$

The fitted distribution provides all the necessary information to reconstruct the shape and has the advantage that the λ parameter is directly interpretable as the fraction of observation time the flow spends in a single state (see Fig. 1). Knowledge about the fraction of the total observation time a flow spends in each state provides no information about the distribution of the existence time for each state. As a result of the time averaging process this information is lost. So a separate set of dynamic descriptors must be used to address the time statistics of two-phase flow.

DYNAMICS

In order to describe the dynamics of state changes in two-phase flow the existence time for each of the states must be considered. The stochastic nature of the existence time for each state forces the consideration of the

probability per unit time that, given that the flow entered a state at time t , it switches out of the state in dt at $t + \tau$. To form this density function

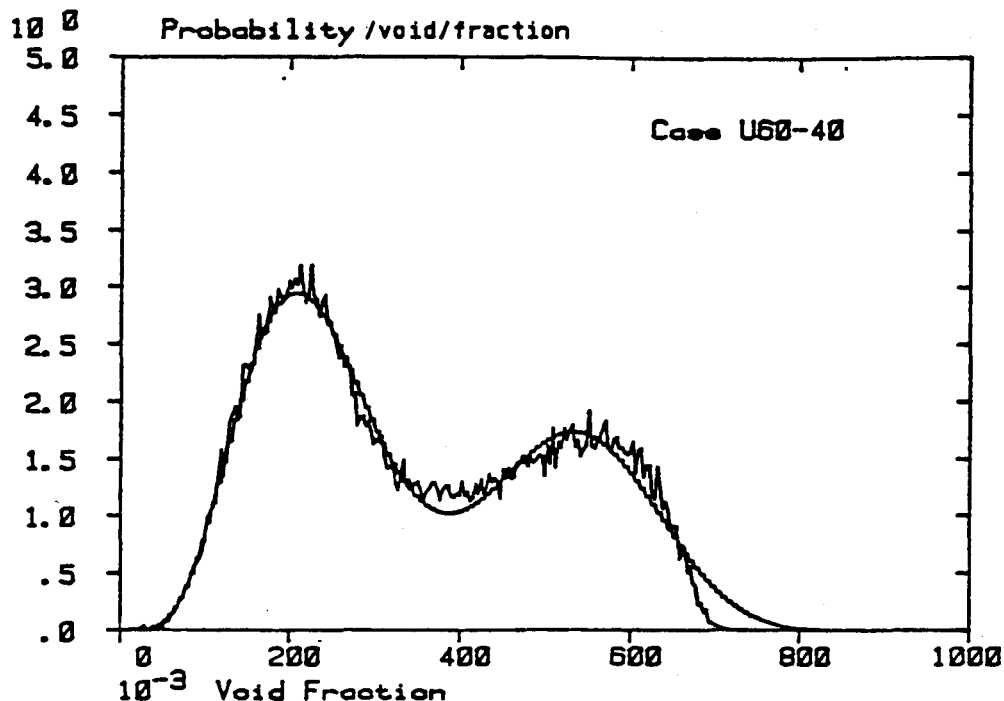


FIGURE 1 : PROBABILITY DENSITY FUNCTION WITH FITTED BETA DENSITY ($a = 3.08$, $b = 19.5$, $c = 13.8$, $d = 12.1$, $\lambda = 0.59$).

it is convenient to introduce a counting functional to define the state changes. The required counting functional will be a series of delta functions of alternating sign and unit height.

$$\eta_b(t) = \begin{cases} 1 & \text{when } \left| \frac{\partial x}{\partial t} \right|_{\max} \text{ and } x(t) > a \\ -1 & \text{when } \left| \frac{\partial x}{\partial t} \right|_{\max} \text{ and } x(t) < b \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

This function thresholds on both the derivative and the original signal. The maximum of the derivative is easily defined; however, values for the signal thresholding must be determined. The previously developed idea of the time average statistics as expressed by the PDF is useful to this end. Since the PDF exhibits two summed distributions, the mean and variance of each mode can be estimated by fitting a limited parameter distribution. Confidence intervals can be estimated for distances away from the mean of each mode and these used to evaluate the constants a and b . In this way the certainty of switching can be quantified and the confidence in the accuracy of the representation of the dynamics can be expressed. Now, using the time between the switches, the intervals of existence for each state can be histogrammed and normalized to become a probability density (see Fig. 2). These density func-

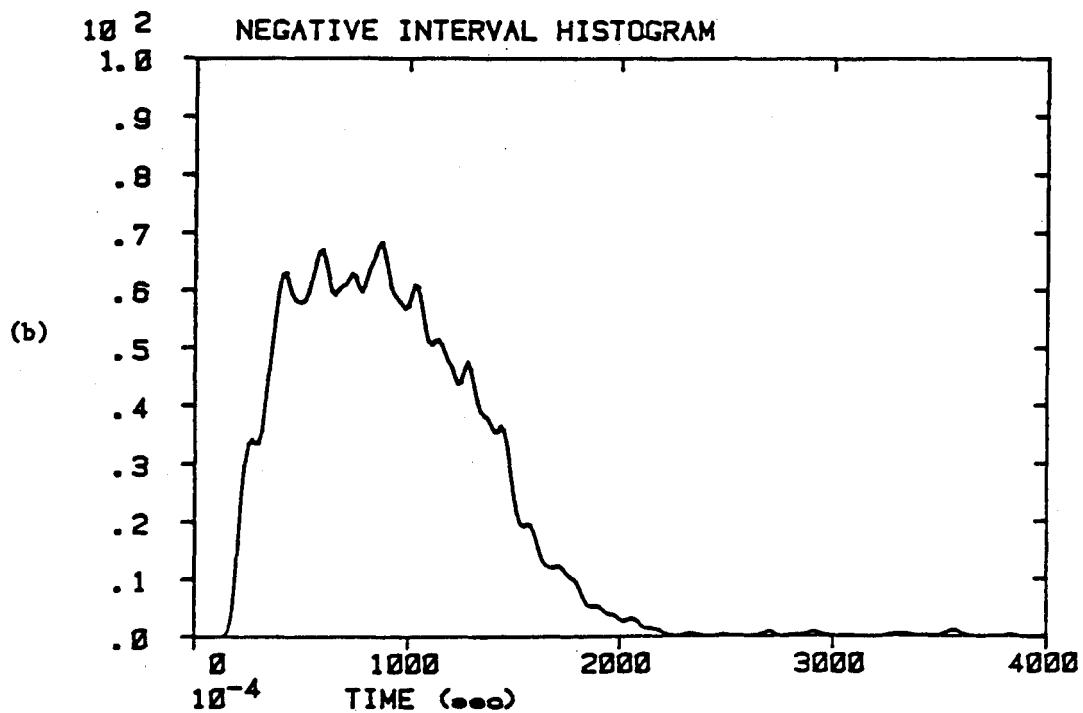
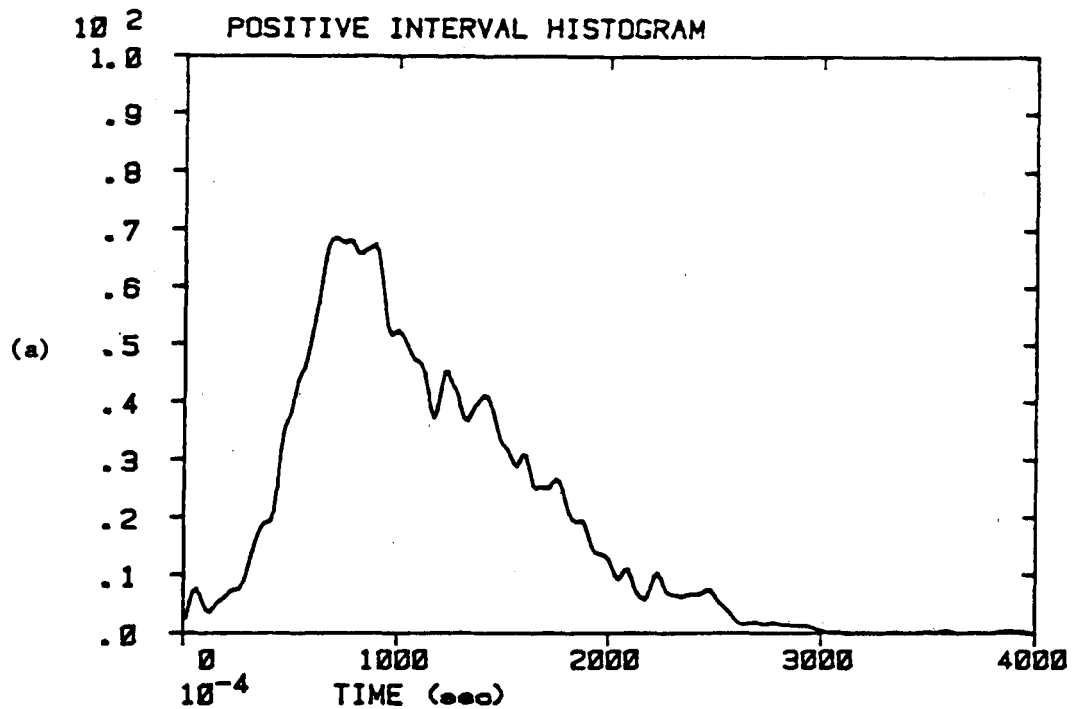


FIGURE 2 : HISTOGRAM OF DURATION TIMES FOR EACH STATE. (POSITIVE IS LOW VOID STATE, NEGATIVE IS HIGH VOID STATE.)

tions provide the required stochastic information about the dynamic property of two-phase flow. It has been suggested that the mean and variance of these densities is an appropriate choice of parameters to quantify these distributions /7/. It is apparent that the properties of structure and dynamics are very much related.

PROPAGATION

The property of propagation is a stochastic variable in the sense that the velocity at which a density perturbation propagates fluctuates. To quantify this fluctuation a probability density function of the time delay (τ) between two detectors of known displacement (d) can be formed. If the variance is small with respect to the mean a single parameter (mean time delay) is sufficient to describe the fluctuation of the propagation. Since the distance between detectors is known this parameter relates to the mean velocity ($v=d/\tau$). The mean value of the velocity is the continuity wave velocity

$$v_{\omega} = \left(\frac{\partial j_g}{\partial \alpha} \right)_j \quad (3)$$

Using the definition of gas velocity $v_g = j_g / \alpha$ and the drift flux model suggested by Zuber-Findley /11/

$$\bar{v}_g = C_o \langle j \rangle + \bar{v}_{gj} \quad (4)$$

where

$\bar{v}_g$ - is the weighted mean gas velocity $\left(\frac{\langle v_g \alpha \rangle}{\langle \alpha \rangle} \right)$

C_o - is the distribution parameter $\left(\frac{\langle \alpha j \rangle}{\langle \alpha \rangle \langle j \rangle} \right)$

$\langle j \rangle$ - is the area averaged volumetric flux

$\bar{v}_{gj}$ - is the weighted mean drift velocity $\frac{\langle v_{gj} \alpha \rangle}{\langle \alpha \rangle}$

and

$$\langle F \rangle \equiv \frac{1}{A} \int_A F(\underline{r}) dA$$

is the area averaging operation. Kosály et al /14/ propose that the velocity measured using neutron noise techniques can be expressed

$$C_k = v_{\omega} = \bar{v}_g + \langle \alpha \rangle \langle j \rangle \frac{\partial C_o}{\partial \langle \alpha \rangle} + \langle \alpha \rangle \frac{\partial \bar{v}_{gj}}{\partial \langle \alpha \rangle} \quad (5)$$

where $\langle \alpha \rangle$ is the area averaged void fraction. This result (C_k) is identified as the kinematic wave velocity /12,15/. It is clear from this relationship how the weighted mean gas velocity is related to the measured velocity, if the

second and third terms on the RHS are negligible the measured velocity is the weighted mean gas velocity. Having measured the volumetric flux $\langle j \rangle$ from the flow rates and the cross sectional area of the pipe and accepting the measured velocity (C_k) as sufficiently close to the gas velocity ($\bar{v}_g$) over some range, an estimate of volumetric concentration ($\langle \alpha \rangle$) is possible. Using $\bar{v}_g$ and $\langle j \rangle$, drift flux parameters can (in principle) be evaluated by linear regression. Thus the deterministic part (mean) of the property of propagation can be addressed by neutron noise techniques, and parameters for a widely used model can be determined.

EXPERIMENTAL RESULTS

In this paper the emphasis is upon the propagation property of the flow. An extensive set of air-water two-phase flow conditions, using a 3/4 in. pipe were examined using the University of Washington Nuclear Reactor. In general, to describe the propagation of the flow, the time delay between two fission chambers near the flow tube is measured. This is done either by the displacement of the peak in the cross correlation function (CCF) or by the phase shift in the cross power spectral density (CPSD):

$$\begin{aligned} \text{CCF} = R_{xy}(\tau) &= \int_{-\infty}^{\infty} x(t) y(t + \tau) dt \\ \text{CPSD} = S_{xy}(\omega) &= F \left\{ R_{xy}(\tau) \right\} \end{aligned} \quad (6)$$

For all conditions the time delays between three pairs of detectors were calculated. Using the knowledge about the displacement of the detectors a set of velocities for all cases and all pairs of detectors was compiled. The CCF technique was used to determine a time dependent history of the time delay. Each delay estimate is based upon ten seconds of data; there are, therefore, many delay estimates that are histogrammed and normalized to become a probability density function. Figure 3 shows an example of the resulting density; it is clear that the standard deviation is small compared to the mean (e.g., $\sigma = 0.017$ and $\bar{\tau} = 0.40$). Because the standard deviation is small only the mean velocity, as determined from detector separation d and the slope of the CPSD phase, is discussed here.

$$\begin{aligned} \tau \text{ (sec)} &= \text{Slope } \phi \left(\frac{\text{deg}}{\text{Hz}} \right) \left(\frac{1 \text{ cycle}}{360 \text{ deg}} \right) \\ v_{\text{exp}} &= d/\tau \text{ (m/sec)} \end{aligned} \quad (7)$$

A first approach to examine all of the velocity data on a single graph is to use a nondimensional form of the drift flux quantities. Since the volumetric fluxes of air and water are controlled in the experiments, the independent axis is the ratio of the gas volumetric flux to the total volumetric flux

$$\langle \beta \rangle = \frac{\langle j_g \rangle}{\langle j_g + j_l \rangle} \quad (8)$$

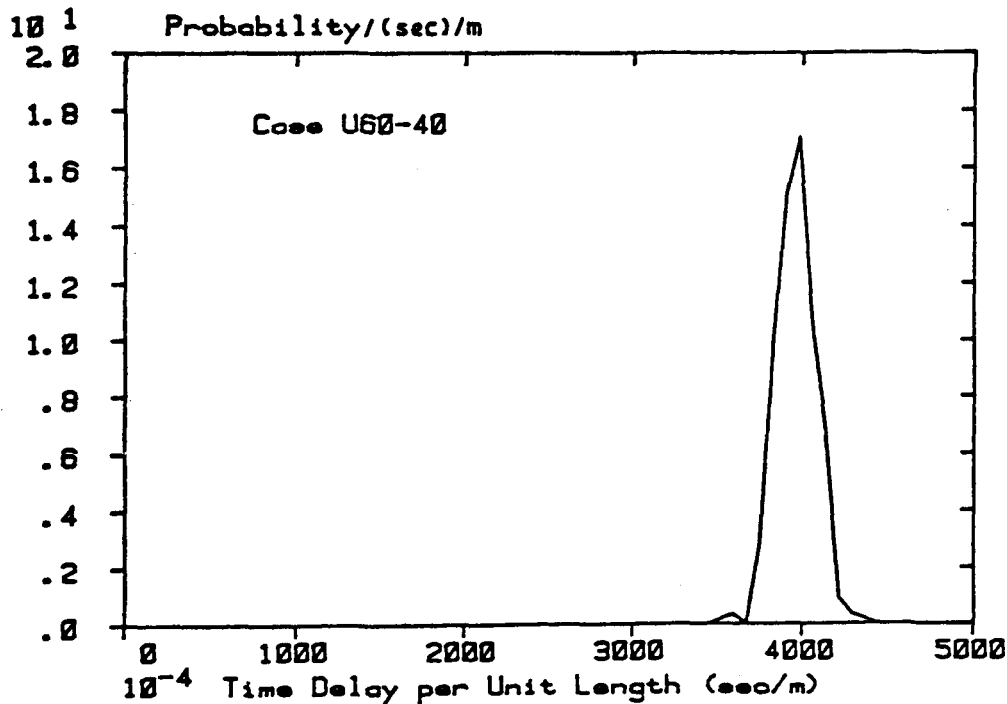


FIGURE 3 : TYPICAL PROBABILITY DENSITY FUNCTION OF TIME DELAY DISTRIBUTION BETWEEN TWO DETECTORS

The dependent variable is the area averaged concentration derived from $\langle \alpha \rangle = \langle j_g \rangle / \bar{v}_g$, where $\bar{v}_g$ is the weighted mean velocity described previously. However, in the experiment, the kinematic wave velocity is actually measured. But if the second and third terms on the RHS of Eq. 5 are small the approximation $\bar{v}_g \approx C_k$ can be used. The non-dimensional form of the drift flux model can be written

$$\langle \alpha \rangle = \langle \beta \rangle \left(\frac{1}{C_0 + \frac{\bar{v}_{gj}}{\langle j \rangle}} \right) \quad (9)$$

where $\left(\frac{1}{C_0 + \frac{\bar{v}_{gj}}{\langle j \rangle}} \right)$ can be identified as the Armand parameter, K, /11/.

It is apparent that when the experimental points approximate a straight line the slope will be K. Under conditions when

$$\frac{\bar{v}_{gj}}{\langle j \rangle} \ll 1$$

the drift velocity is much less than the volumetric flux and the slope can be identified as the inverse of distribution parameter $1/C_0$. Figure 4 shows the neutronically estimated void fractions based upon the velocity as measured between the central pair of detectors and plotted on the $\langle \alpha \rangle$ - $\langle \beta \rangle$ plane. The points fall along a straight line below a value of approximately $\beta = 0.67$.

When the $\beta = 0.67$ line is drawn on a flow map developed from visual observation, for this experimental apparatus, it corresponds well to the annular flow transition (Fig. 5). Previous work for larger diameter pipes suggests the linear approximation may be valid as high as $\beta = 0.8$ /13/. Linear regression below $\beta = 0.67$ yields a value for the Armand parameter, K , of 0.76 with a correlation coefficient of 0.94. If Eq. 10 is valid, this implies a C_0 value of 1.32.

The use of the Armand parameter does not separately consider the relative velocity between the phases, and assumes the ratio $\langle \alpha \rangle / \langle \beta \rangle$ is independent of mass flow rate. The drift flux model proposed by Zuber and Findlay includes the possibility of substantial relative velocity between the phases. Zuber suggests that for churn-turbulent bubbly flow the weighted mean drift velocity of the gas can be written:

$$\frac{\langle \alpha v_{gj} \rangle}{\langle \alpha \rangle} = 1.53 \left[\frac{\sigma g (\rho_l - \rho_g)}{\rho_l^2} \right]^{1/4} \quad (10)$$

The churn-turbulent bubbly flow is characterized by an agitated and unsteady flow pattern which represents a transition between ideal bubble and ideal slug flows /10/. A substantial number of the flows studied using neutron noise techniques fit well into this flow description. In fact the map produced by visual observation shows the majority of the cases examined in this system to be in either the churn or froth regimes rather than ideal bubble or slug flows. On this basis Eq. 10 is applicable to the majority of the data presented here. The use of Eq. 10 is suggested because of the lack of data in the region of nearly zero water flow. Combining Eqs. 4 and 10, the drift flux model is written:

$$\bar{V}_g = C_0 \langle j \rangle + 1.53 \left[\frac{\sigma g (\rho_l - \rho_g)}{\rho_l^2} \right]^{1/4} \quad (11)$$

When a subset of the neutronic measurements is plotted on the C_k vs. $\langle j \rangle$ plane there is a region over which $C_k = \bar{V}_g$ is a good approximation. From the previous discussion this range appears to be limited by a boundary at $\beta = 0.67$; however, the experimental points which can be described as churn or froth flows are below this cutoff value. The set of experimental data and Eq. 11 uniquely define a value for C_0 of 1.22. Figure 6 shows the correspondence between the prediction from Eq. 11, with $C_0 = 1.22$, and the experimental results from the points shown in Figure 5. It is apparent that the data and predictions from the drift flux model are in agreement. It is therefore possible, over some range of experimental conditions, to evaluate the distribution parameter from the drift flux model with relative certainty from neutron flux fluctuation measurements.

It is important to note that the linear behavior in the $C_k - \langle j \rangle$ plane is not exhibited by flows falling above the annular flow transition. The cause of this deviation is the assumption used to estimate the gas velocity. This implies that C_0 and $\bar{V}_{gj}$ are, first, only functions of α , and further, that the change of these parameters with respect to $\langle \alpha \rangle$ is very small. For

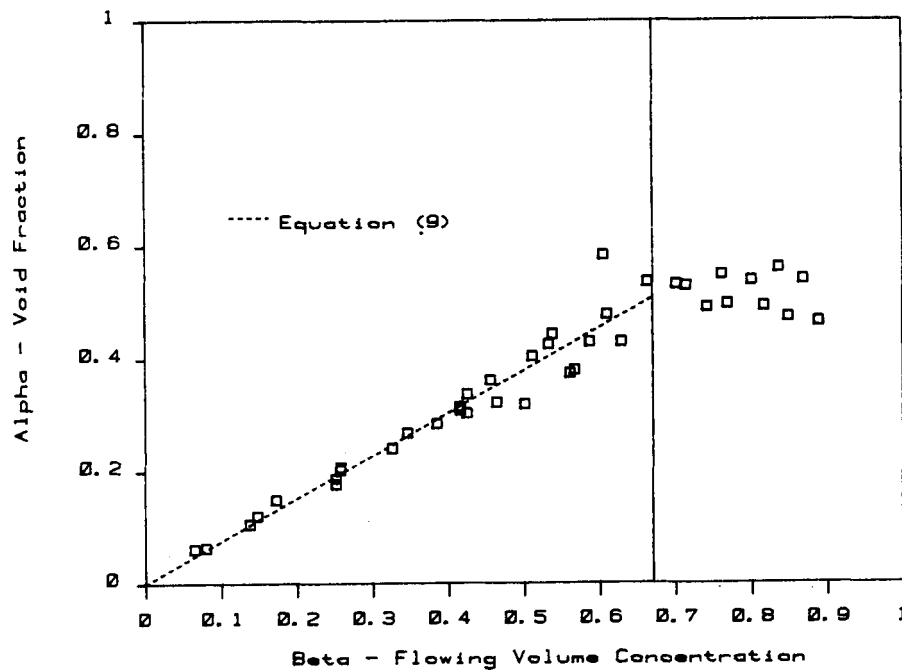


FIGURE 4 : NEUTRONIC RESULTS ON $\langle \alpha \rangle$ - $\langle \beta \rangle$ PLANE

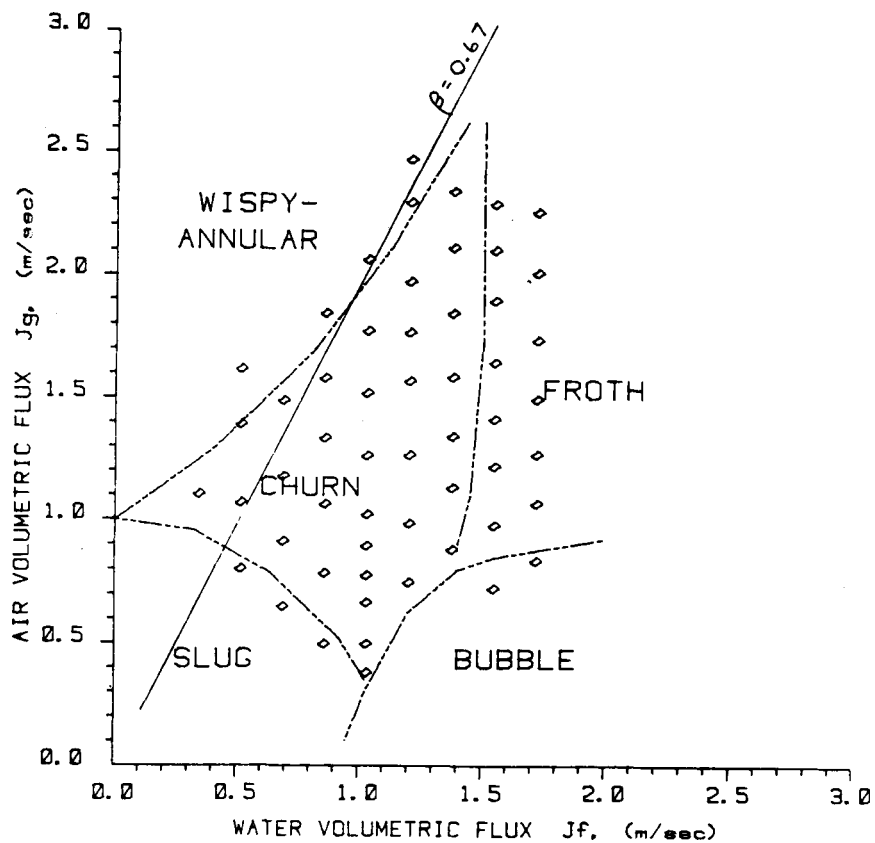


FIGURE 5 : FLOW MAP FROM VISUAL OBSERVATION OF TWO-PHASE FLOW IN 3/4 IN. TUBE

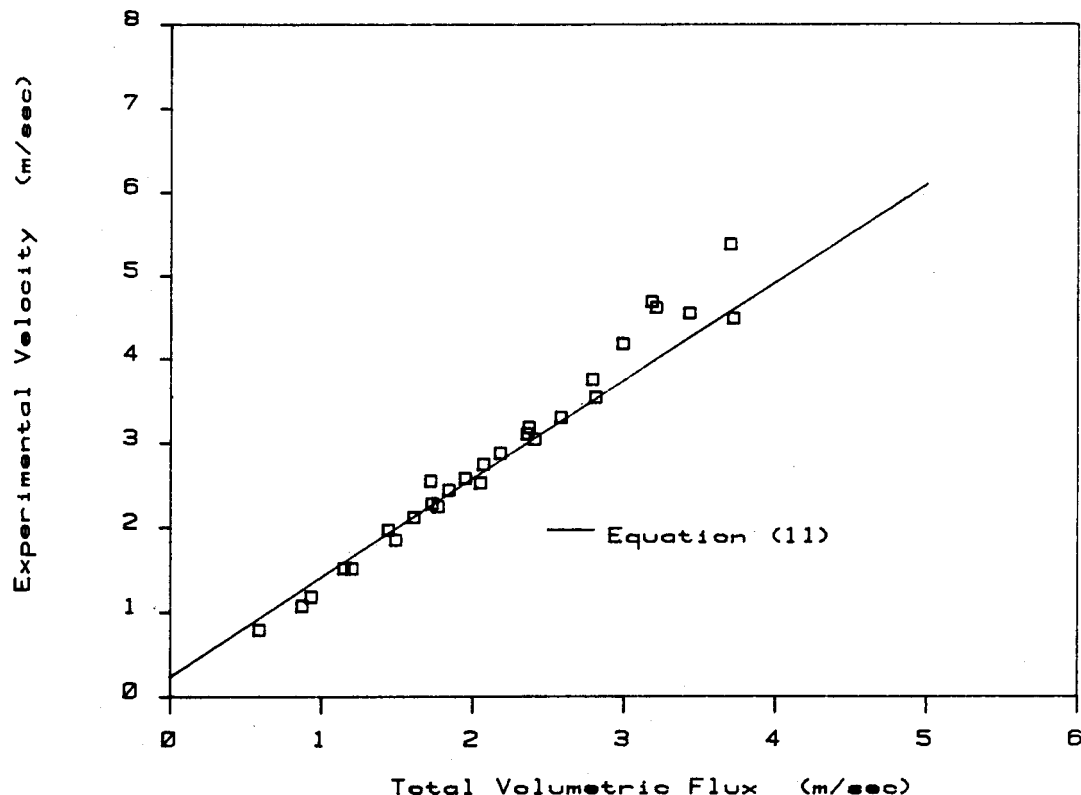


FIGURE 6 : NEUTRONICALLY DETERMINED VELOCITIES VS. TOTAL VOLUMETRIC FLUX $\langle j \rangle$

high void fractions the term

$$\langle \alpha \rangle \langle j \rangle \frac{\partial C_0}{\partial \langle \alpha \rangle}$$

is negative and not negligible [11,14], and therefore the $C_k = \bar{V}_g$ assumption is not valid. To use neutron noise techniques directly, an expression for $C_0(\langle \alpha \rangle)$ is required. Such correlations exist and have been used with some success to compare BWR calculations to measurements [11].

Neutron noise techniques provide a means of measuring two-phase flow quantities in a hostile environment. The property of propagation can be addressed in many cases by a deterministic parameter, the mean value. The mean of the neutronically measured velocities can be related to the weighted mean velocity of the gas phase, and further, classic drift flux parameters can be obtained from experimental results. This provides a demonstration of the usefulness of a deterministic parameter arrived at from neutronic measurements. In addition other stochastic properties of two-phase flow can be addressed. These separate effects tests, using a small reactor, provide a demonstration of the value of neutron noise techniques in the analysis of two-phase flow. This information is valuable in understanding the response of neutron detectors to the two-phase flow conditions found in an operating BWR.

REFERENCES

1. R.D. Crowe, S.W. Eisenhawer, F.D. McAfee, R.W. Albrecht, Progress in

- Nuclear Energy 1 (1977) pg 85.
2. R.W. Albrecht, R.D. Crowe, D.J. Dailey. Trans. ANS 38 pg 645 (1981).
 3. G. Kosály, R.W. Albrecht, S.L. Gubin. Trans. ANS 38 pg 643 (1981).
 4. R.W. Albrecht, R.D. Crowe, D.J. Dailey, G. Kosály. Trans. ANS 39 pg 1003 (1981).
 5. R.W. Albrecht, R.D. Crowe, D.J. Dailey, G. Kosály. Advances in Test Measurement, Vol. 1, p 335 (1981).
 6. G. Kosály, R.W. Albrecht, R.D. Crowe, D.J. Dailey. Presented SMORN III, Tokyo (1981).
 7. R.W. Albrecht, R.D. Crowe, D.J. Dailey, M.J. Damborg, G. Kosály. Presented SMORN III, Tokyo (1981).
 8. O.C. Jones and N. Zuber. International Journal of Multiphase Flow, Vol. 2, 1975.
 9. M.A. Vince, R.T. Lahey. NUREG/CR-1692 (1980).
 10. G.B. Wallis. One Dimensional Two-Phase Flow, McGraw-Hill Book, New York (1969).
 11. N. Zuber, J.A. Findlay. Trans. ASME Heat Transfer, Ser. C, Vol. 87, p 452 (1965).
 12. F.W. Staub, N. Zuber, Nuclear Science and Engineering, Vol. 20 No. 2, (1967) pg 296.
 13. G.E. Smitsaert. "Two component two-phase flow parameters for low circulation rates" ANL-6755 July (1963).
 14. G. Kosály, J. Fahley. "Investigation of axial void propagation velocity profiles in BWR fuel bundles," Report prepared for EPRI (1980).
 15. K. Ohkawa, R.T. Lahey, Jr. Nuclear Engineering and Design, 61, (1980) p 245-255.

ACCURACY OF POWER PEAKING MONITORING USING FIXED INCORE DETECTOR SYSTEMS

C. T. Rombough, B. A. Guthrie, R. E. Clark, H. D. Warren
Nuclear Power Generation Division
Babcock & Wilcox

ABSTRACT

A definitive sensitivity study has quantified the error associated with using a fixed incore detector system to monitor the peak power in a nuclear reactor core. Results of this study show that by using a recently developed curve fit technique, the error is less than 2% (for a peak of 1.5 RPD) for 5 or more fixed detectors in the fuel assembly. This technique incorporates some innovative improvements which minimize the curve-fit error. Grids are realistically simulated in the fitted curves and extrapolation points are determined as a function of the measured signal at each end. With these improvements, a fixed system can produce an accurate representation of the axial flux shape within the assembly on a continuous basis.

INTRODUCTION

The power distribution in operating nuclear power reactors must be monitored periodically to insure that the power peaking is below design limits. These limits are established based on criteria for departure from nucleate boiling (DNB), centerline fuel melt (CFM), and assumptions made in the safety analysis. The measurement accuracy for the power distribution monitoring system being used must be understood so that appropriate conservatism can be applied when comparing the measured results to the design limits. To this end, a comprehensive analytical study has been completed to determine the accuracy in measuring the radial and axial power distribution using a fixed incore detector system having a finite number of detectors positioned along the active fuel height.

Under steady state conditions, a movable system, in which a single detector slowly traverses the length of a fuel assembly, can measure the axial flux shape more accurately than a fixed system because in the fixed system, a mathematical curve is passed through a finite number of measured points. A fixed system, however, does have certain advantages over a movable system and these are:

1. the three-dimensional power distribution can be monitored continuously (i.e., "on-line"),
2. a fixed system gives the reactor operators instantaneous feedback during changing core conditions, and
3. the safety limits which protect the thermal condition of the fuel can be monitored directly resulting in wider operating limits, increased maneuverability and/or possible increases in allowable power levels.

The actual time frame for the measurements depends on the promptness of response of the fixed detectors being used. This varies from less than .01 seconds for Hafnium (prompt) to approximately 1 minute for Rhodium (delayed).

This study examined and quantified the fitting error for different fixed incore detector configurations.

The following parameters were varied:

1. Type of curve fit method used.
2. Number of fixed detectors along the fuel height.
3. Location of the fixed detectors.
4. Length of the detectors.
5. Accuracy of the individual detector signals.

The "detector" can be any device which measures (or infers) the relative power density in the fuel assembly such as a self-powered neutron detector, fission chamber, gamma detector, or gamma thermometer. The error associated with representing a continuous power shape using a fixed number of measurements was determined.

METHODOLOGY

The data base of axial power shapes used in the analysis comprised 580 different curves. These were obtained from 20 independent three-dimensional power distributions, each of which consisted of the axial shapes for each of the 29 1/8-core locations (see Figure 1) in the B&W standard 177-fuel assembly core (i.e., $29 \times 20 = 580$).

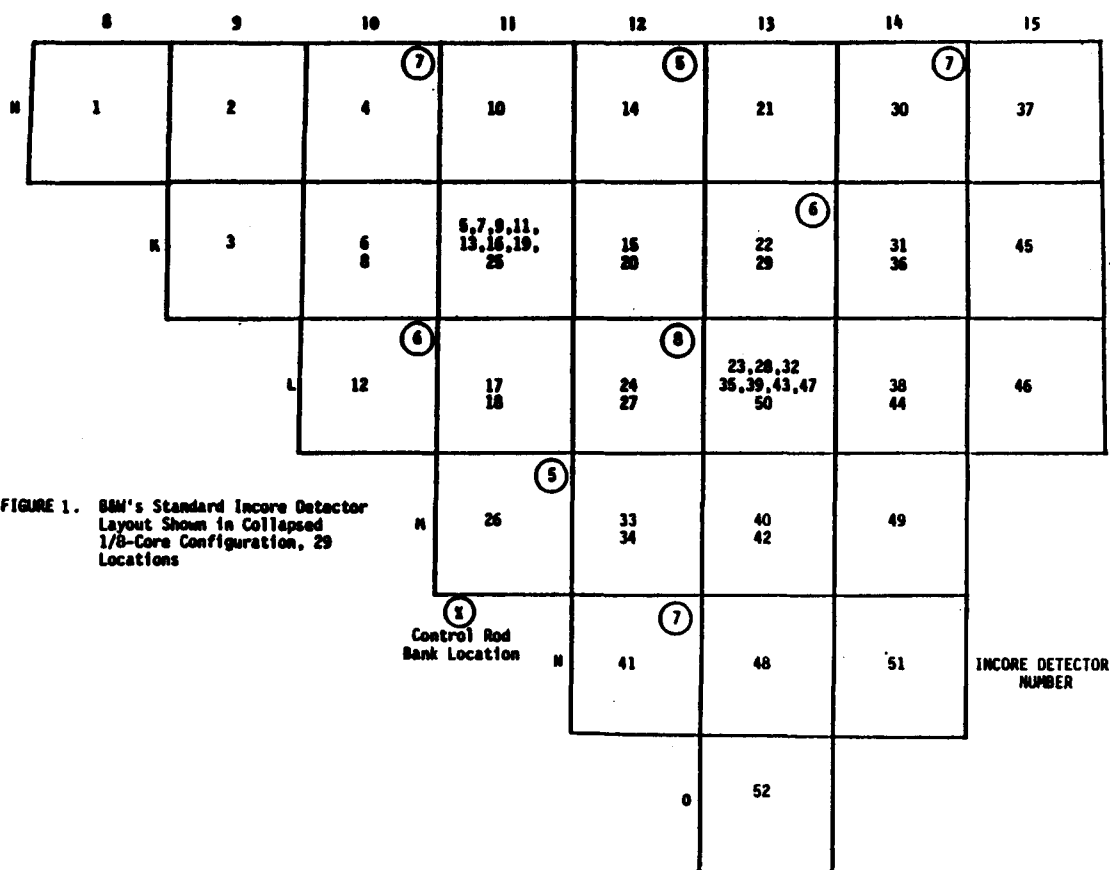


FIGURE 1. B&W's Standard Incore Detector Layout Shown in Collapsed 1/8-Core Configuration, 29 Locations

FIGURE 2
TYPICAL INPUT AXIAL POWER SHAPE

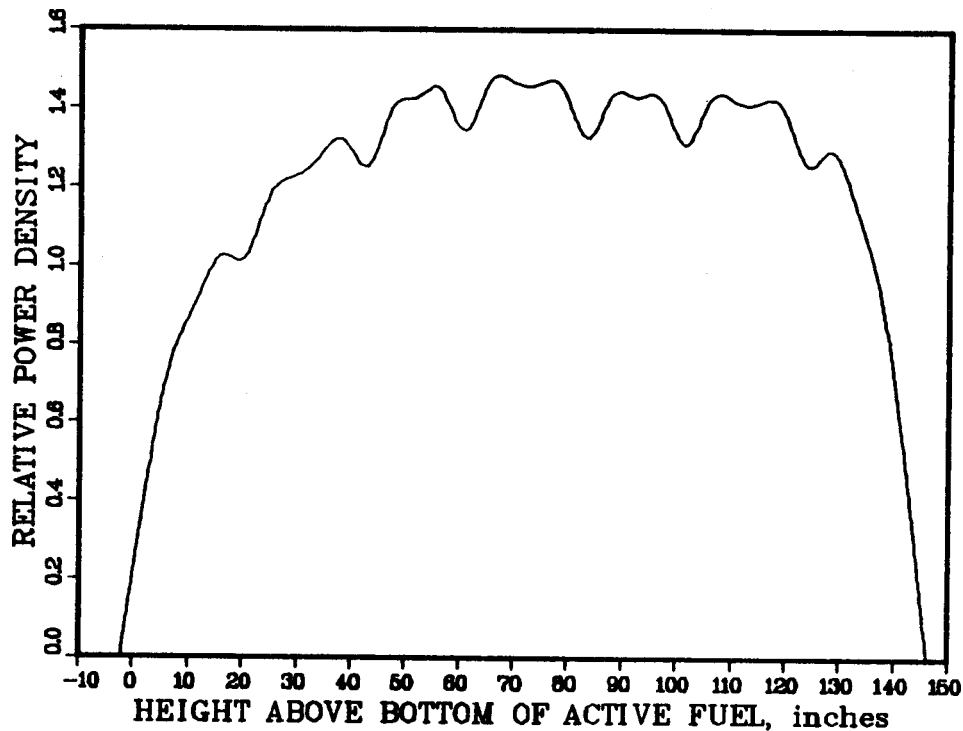


Figure 2 is an example of one of these curves. These shapes were generated using B&W's 3-D nodal code, FLAM3, with 32 axial nodes per fuel assembly (active fuel height=144 inches). For the 20 cases, Group 6 and 7 rod positions were varied from fully inserted to fully withdrawn and the part-length rods (Group 8) were varied from fully inserted to fully withdrawn. As a result, the total peak ranged from 1.4803 to 2.9527, the location of the total peak ranged from 25 to 110 inches from the bottom of the active fuel, and the offset $\left(\frac{\text{Power in Top} - \text{Power in Bottom}}{\text{Total Power}} \times 100 \right)$ ranged

from -60 to +32%. Using movable flux traces from Gadolinium detectors as a basis, an algorithm was developed to account for the effects of fuel assembly grid depressions as shown in Figure 2. This data base should bound all conceivable axial power shapes under both steady-state and severe transient conditions.

The parameters to be determined were 1) the difference between the maximum value of the fitted curve (relative axial peak) and the maximum value from the FLAM3 run (denoted DIFF_p) and 2) the difference between the value of the integral under the fitted curve (relative radial peak) and the integral value from FLAM3 (denoted DIFF_r). These two differences were determined for each of the 580 curves and the standard deviation of the error was defined as

$$\sigma_{\text{peak}}^2 = \sum_{580} (\text{DIFF}_p)^2 / 580 \quad (\text{Relative Axial Peak Error})$$

$$\sigma_{\text{radial}}^2 = \sum_{580} (\text{DIFF}_r)^2 / 580 \quad (\text{Relative Radial Peak Error})$$

These errors are in units of relative power density (RPD).

CURVE FITTING

An accurate axial curve fitting method was developed as a result of this study. Of the mathematical methods investigated, the cubic spline technique (Reference 1) gave the most accurate results. Moreover, several improvements to the basic spline fit were developed which reduced the error further. One of these improvements was a method by which extrapolation points (points at each end of the fuel assembly in which flux is assumed to go to zero) can be calculated "on-line" from the actual measured detector signals. Another improvement was to incorporate the grid depressions into the measured curves using an algorithm based on movable flux traces. The reason this reduces the error is that the effect of grids changes the peak to average ratio differently depending on the location of the peak relative to the grids. The effect of these improvements can be seen in the following results.

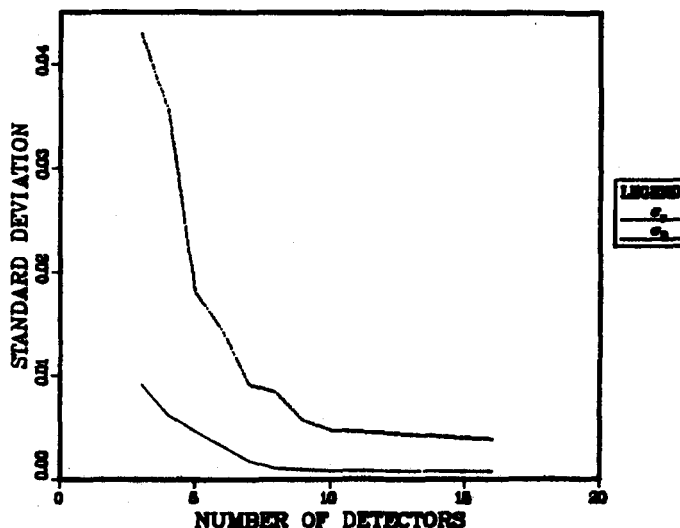
<u>No. of Detectors</u>	<u>Fitting Routine</u>	<u>σ_{peak} (RPD)</u>
5	1) basic spline	.0281
	2) with "on-line" extra- polation points	.0229
	3) with grids in measured curves	.0181

RESULTS

Number of Detectors

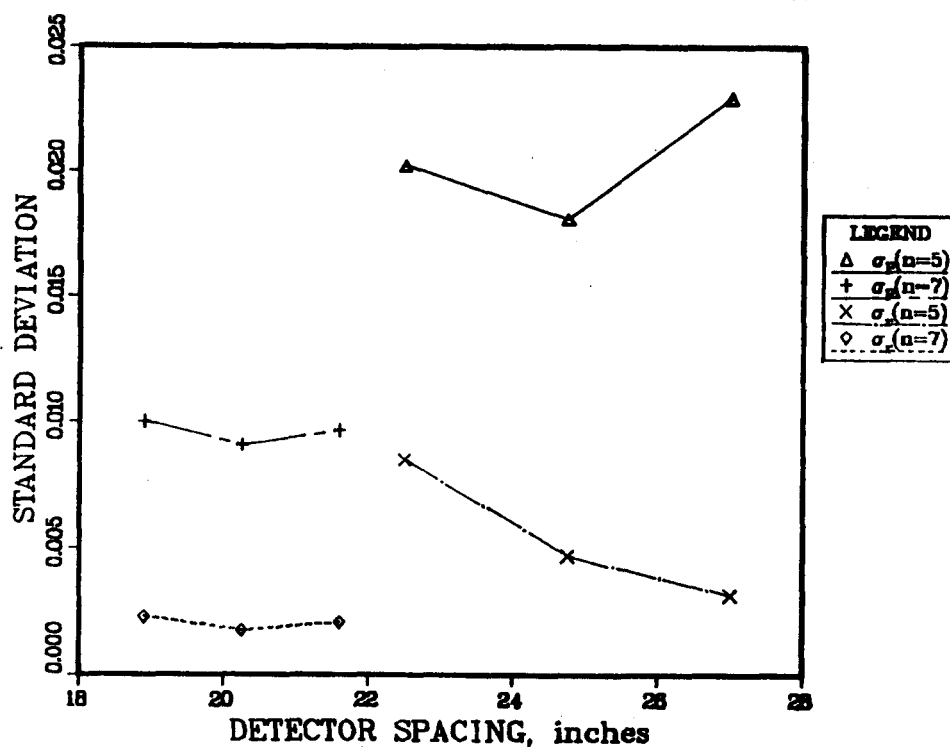
As expected, the most sensitive variable in a fixed detector system is the number of axial locations monitored. The number of measurements along the fuel height was varied from 16 to 3. The results are tabulated below and shown graphically in Figure 3.

FIGURE 3
MEASUREMENT ACCURACY VS NO. OF DETECTORS



<u>No. of Detectors</u>	<u>σ_{peak} (RPD)</u>	<u>σ_{radial} (RPD)</u>
16	.0039	.0008
10	.0048	.0009
9	.0057	.0010
8	.0084	.0011
7	.0091	.0018
6	.0145	.0033
5	.0181	.0040
4	.0356	.0063
3	.0429	.0092

FIGURE 4
MEASUREMENT ACCURACY VS DETECTOR SPACING



Spacing

The spacing of the detectors along the active fuel height was varied for 5 and 7 detectors (see Figure 4). The dependence is weak but the optimum spacing is about 20" for 7 detectors and 25" for 5 detectors.

<u>No. of Detectors</u>	<u>Spacing</u>	<u>σ_{peak} (RPD)</u>
7	Equally-spaced, centered with spacing between detectors (S) = 18.9"	.0100
	S = 20.3"	.0091
	S = 21.6	.0097
5	S = 22.5"	.0202
	S = 24.8"	.0181
	S = 27.0"	.0230

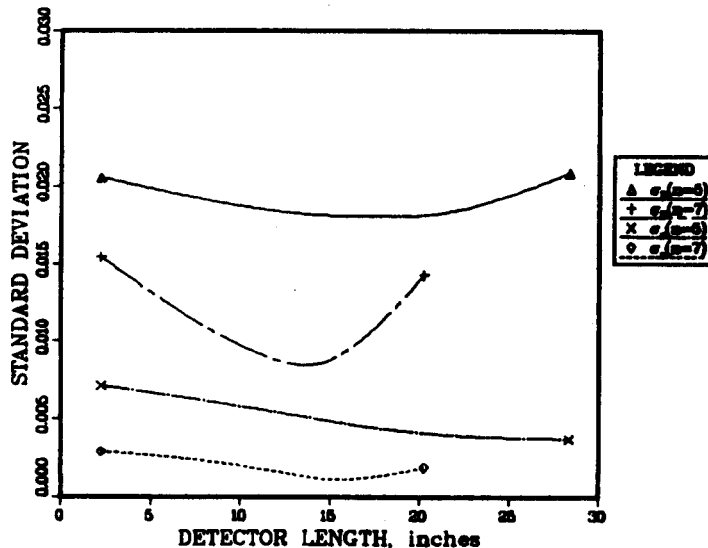
Length

A somewhat surprising result was that the error is relatively insensitive to the length of the measuring devices. It was expected that the shorter the detector, the higher the resolution in measuring the peak (and hence, better accuracy). However, because the location of the peak is just as likely to occur between detectors as it is to occur at the detector, it is the longer detectors which can measure the peak more accurately in these cases. Although not a strong function, the optimum length was found to be that in which about 65% of the active fuel height is covered. Results are shown below and illustrated in Figure 5.

<u>No. of Detectors</u>	<u>Length (inches)</u>	<u>σ_{peak} (RPD)</u>	<u>σ_{radial} (RPD)</u>
7	2.3	.01543	.00290
7	9.5	.01003	.00208
7	11.3	.00908	.00176
7	13.1	.00849	.00141
7	20.3	.01429	.00189
5	2.3	.02051	.00713
5	13.1	.01826	.00523
5	15.8	.01808	.00471
5	18.5	.01805	.00430
5	28.4	.02085	.00371

FIGURE 5

MEASUREMENT ACCURACY VS DETECTOR LENGTH



Signal Uncertainty

The effect of detector signal and processing uncertainty was simulated by allowing the signals to vary randomly (normally distributed with standard deviation = σ_s). This effect is shown below:

<u>No. of Detectors</u>		<u>σ_{peak}</u>	<u>σ_{radial}</u>
N = 7	$\sigma_s = 0$	.0091	.0018
	= 2%	.0244	.0084
	= 4%	.0438	.0165
N = 5	$\sigma_s = 0$	.0181	.0047
	= 2%	.0272	.0108
	= 4%	.0437	.0200

Note that increasing the number of detectors in the assembly becomes an insignificant advantage as the signal uncertainty becomes larger.

CONCLUSIONS

A definitive sensitivity study has quantified the error associated with using a fixed incore detector system to monitor the peak power in a nuclear reactor core. Results of this study show that by using a recently developed curve fit technique, the error is less than 2% (for a peak of 1.5 RPD) for 5 or more fixed detectors in the fuel assembly. This technique incorporates some innovative improvements which minimize the curve-fit error. Grids are realistically simulated in the fitted curves and extrapolation points are determined as a function of the measured signal at each end. With these improvements, a fixed system can produce an accurate representation of the axial flux shape within the assembly on a continuous basis.

The sensitivity study determined that for 5 detectors, the optimum placement is to space the detectors equally along the active fuel height (regardless of grid location) with a spacing of 25 inches between detectors and a length of 18.5 inches per detector. These results have been used to design a fixed incore detector assembly using prompt-responding detectors for the Empire State Electric Energy Research Company (ESEERCO). Because the incore flux would be monitored directly, a Reactor Protection System (RPS) using fixed incore prompt detectors would have significantly fewer observability penalties than the present RPS based on out-of-core detectors.

REFERENCE

1. Ahlberg, J., Nilson, E., and Walsh, J., The Theory of Splines and Their Applications, Academic Press, New York, 1967.

METHOD OF MONITORING SUBCRITICAL REACTIVITY DURING CORE REFUELING

P. A. Ombrellaro and R. A. Harris
Hanford Engineering Development Laboratory
Westinghouse Hanford Company
Richland, Washington U.S.A.

ABSTRACT

The Modified Source Multiplication (MSM) technique, was used to measure core subcritical reactivity and core component reactivity worth during a major refueling of the Fast Flux Test Facility (FFTF) core. Measured results were compared with calculated values as a means of verifying core component changeouts during the refueling. The results demonstrate the accuracy expected from MSM measurements and show that the technique was very effective in monitoring core subcritical reactivity during a refueling in which the neutron sources changed significantly and were not uniformly distributed.

1.0 OBJECTIVES

A reactivity monitoring system, based on the MSM technique⁽¹⁻⁴⁾ for measuring subcritical reactivity was developed for the FFTF. The system was developed for use in monitoring core reactivity during all subcritical operations of the FFTF,⁽⁵⁻⁸⁾ especially during core reloadings. The design goal accuracy of the system is ten percent (one standard deviation) for reactivity measurements in a fully shutdown core.

The reactivity monitoring system was used as part of the normal data collection process for monitoring the progress of a major refueling. The monitoring results were obtained to verify individual component movements, the total reactivity change that occurred after each movement, and to ensure that after each movement the core was more subcritical than -13\$. The latter is a surveillance requirement established for the MSM method.

These measurements made it possible, for the first time, to establish the effectiveness of the MSM technique in a core environment in which the neutron sources resulting from irradiated fuel^(9,10) are changing significantly and not uniformly distributed in the core, as occurs during refueling.

This paper summarizes the results of core subcritical reactivity measurements made during the core refueling and compares these results with calculated values. This is done to demonstrate the accuracy of the MSM measurements and the effectiveness of using the technique to monitor subcritical reactivity and core component changeouts during FFTF core refueling.

2.0 DESCRIPTION

2.1 MSM Equations

The MSM technique is based on the point kinetics equation when applied to a steady state subcritical reactor.⁽²⁾ Subcritical reactivity for core configuration, i , is related to the count rate of a detector near the core by the relation:

$$\rho_i = - \frac{QF_i}{CR_i} \quad (1)$$

where, ρ_i is the static reactivity.

CR_i is the detector count rate.

Q is the detector calibration constant.

F_i is the core configuration factor.

The calibration constant, Q , is a measured quantity⁽¹⁾ which is obtained from a calibration of the detector and F_i is a calculated quantity. Thus the MSM technique corrects for changes in Q by using core configuration factors, F_i . These factors are functions of the position of the control rods and other core components as well as the location and magnitude of the inherent neutron sources. This latter component, the magnitude of the neutron source, is quite significant.

2.2 Control Rod Calibrations for Accommodating Neutron Source Errors

To minimize errors, the core subcritical reactivity monitoring procedures for the FFTF require that all neutron detectors near the core be calibrated at the start of refueling and recalibrated at stages during refueling using a control rod whose worth is known. These recalibrations normalize the F_i factors to known reactivities and thereby normalize out any neutron source errors.

The replacement of burned fuel elements by unirradiated fuel elements during refueling will cause decreases in the effective neutron source in the core. Each replacement will change the total source strength by one to three percent. At the end of a major refueling as much as one-third of all the fuel elements may have been replaced with unirradiated fuel. After refueling, the total neutron source in the core is expected to be reduced by about one-third.

2.3 Requirements of the MSM Method

To guide the component changeout operations during refueling and to ensure that the reactor is always more subcritical than an acceptable limit, certain surveillance requirements have been established for the MSM technique. The most important requirement is to verify that after any core component movement (for example, the insertion of a fuel subassembly into an unoccupied region of the core), the core is more subcritical than a specified reactivity limit, -13% . Compliance with this core subcritical reactivity limit ensures that the reactor will not become critical if the single worst refueling error is made. This error is the inadvertent withdrawal of the primary control rod having the maximum reactivity worth. Protection against this error is important, since there is no control rod scram system operable in the FFTF during core refueling.

In addition, there are two secondary requirements which must also be verified following a component movement. The first is that the difference between the MSM measured and calculated reactivities of the core must be less than or equal to 1.50% . This requirement ensures that the refueling that has taken place up to the current point produced the predicted reactivity change.

The second requirement is that the difference between the measured and predicted values of the reactivity worth of the core component that was just moved be less than or equal to 1.00% . Differences between measurement and prediction exceeding the accuracy of MSM will give an early warning of a possible refueling error.

3.0 REACTOR CORE CONFIGURATION

Figure 1 shows a midplane core mapping of the fuel and reflector subassemblies in the FFTF core. The core consists of a central region of inner driver (low enrichment) fuel surrounded by concentric regions of outer driver (high enrichment) fuel, and a reflector, respectively. There are nine control rods in the core; three of these, indicated by SR, are safety control rods, and six, indicated by CR, are secondary control rods. In addition, the core contains two open test assemblies, which are the Vibration Open Test Assembly (VOTA) and the Absorber Open Test Assembly (AOTA), and seven In-core Shim Assemblies (ICSA). Each ICSA consists of stainless steel pins in a duct. There are three neutron detectors, each called a Low Level Flux Monitor (LLFM), located at the core midplane, 120° apart, in the shield region of the core. The LLFMs are shown as A, B and C in Figure 1, and each is located at a distance of 113 cm from core center.

4.0 REACTIVITY MONITORING DURING REFUELING

From the initial ascent to power to shutdown from power operation just prior to this refueling, the core was operated at various power levels for physics testing. In this time period, the core had accumulated 13.4 Effective Full Power Days (EFPD) of exposure.

During the actual refueling, 21 irradiated fuel components were replaced with unirradiated fuel components and four reflector components were replaced with similar type reflector tests. The locations of these 25 core components are shown in Figure 1. The total neutron source was gradually reduced with fuel component changeouts so that at the completion of the refueling, the total source was reduced approximately 25 percent from its initial value.

The LLFMs were calibrated initially by performing an inverse kinetics analysis of the dropping of a safety rod. The resulting calibration constants were used in MSM to assess core subcritical reactivity for all core states that occurred from the start of refueling to the first recalibration of the LLFMs. Safety rod number 3 (indicated by SR3, trisector 3, in Figure 1) was used for this calibration and all subsequent LLFM recalibrations performed during the refueling.

Because of potential errors, it was judged to be necessary to recalibrate the LLFMs following the insertion of approximately every seventh core component and at the end of the refueling.

Reactivity measurements using MSM were performed before and after every core component movement, including the withdrawal and reinsertion of rod number 3 for recalibration, to ensure that the refueling progressed as planned and the surveillance requirements were met. Every reactivity measurement required count rates measured by the LLFMs and a set of precalculated F_i factors.

Core configuration factors, F_i factors, were precalculated for every core state at which a reactivity measurement would be required per the detailed refueling plan. They were calculated using diffusion theory applied to a two-dimensional, hexagonal model of the core in which each core component was represented as a separate region having a specific material composition and a neutron source when the material was fuel. For these calculations, each core configuration representing a refueling state was modeled as closely as possible to the actual refueling state; each core component was characterized by an appropriate set of isotopic concentrations and a neutron source strength reflecting the amount of exposure it received. When the core state resulted from a fuel component replacement, the new core component was characterized by fresh (unirradiated) fuel isotopic concentrations and a low neutron source strength. The isotopic concentrations of all actinides built up and depleted during fuel component irradiation were calculated with ORIGEN.⁽¹¹⁾ The corresponding neutron source strength was obtained from these isotopic concentrations and the appropriate isotopic neutron yields for spontaneous fission and (α ,n) reactions with the isotopes of oxygen present in the oxide fuel. The total source strength was the sum of contributions of neutron source strengths resulting from every actinide isotope.

The two-dimensional diffusion theory calculations, performed to obtain core configuration factors, also provided the multiplication factors from which were obtained calculated (predicted) core reactivity values. The predicted reactivity of a core state that occurred during the refueling was a normalized reactivity. This normalization was accomplished by equating the calculated reactivity of the prerefueling core state to the reactivity measured at shutdown from the previous power operation.

The difference in the calculated reactivities of the core states that occurred before and after each component movement provided the predicted value of the reactivity worth of the core component just moved.

5.0 RESULTS

Figures 2 and 3 show plots of the measured and predicted reactivities for each core state that occurred during refueling as a function of the refueling step. A step indicates the position in the sequence of refueling events at which the particular core state occurred. Each average value of measured core reactivity given in the measurement curves is shown with a bar above and below its value to indicate the magnitude of its one standard deviation uncertainty.

The calibration of the LLFMs at shutdown immediately following power operation provided a set of Q values which were used in MSM to monitor subcritical reactivity in the shutdown period prior to refueling. However, it was necessary to adjust these calibration constants at the start of refueling to account for neutron source decay.

The shutdown period prior to refueling extended for 92 days; during this time Cm^{242} , Pu^{241} and the isotopes of neptunium decayed so that the total neutron source in the core was reduced. The new adjusted calibration constants were used to monitor the core subcritical reactivity during all core component movements performed up to the time of the first recalibration of the LLFMs, which occurred at steps 16 and 17 in the refueling sequence as shown in Figure 2.

The reactivity measurement at step 0 shows the value obtained for the prerefueling state. Steps 1 and 2 show core reactivity measurements made to measure the reactivity worth of the calibration control rod (rod number 3) in the fully shutdown core. The spike, indicated by the reactivity value -18.57\$ in step 1, shows the value of the reactivity measurement performed after the safety rod was fully withdrawn. This measured rod worth was used to scale the predicted worth of the calibration safety rod at the recalibration states. Four recalibrations of the LLFMs were performed; these occurred in steps 16 and 17, 31 and 32, 45 and 46, and finally, in steps 61 and 62, as shown in Figures 2 and 3.

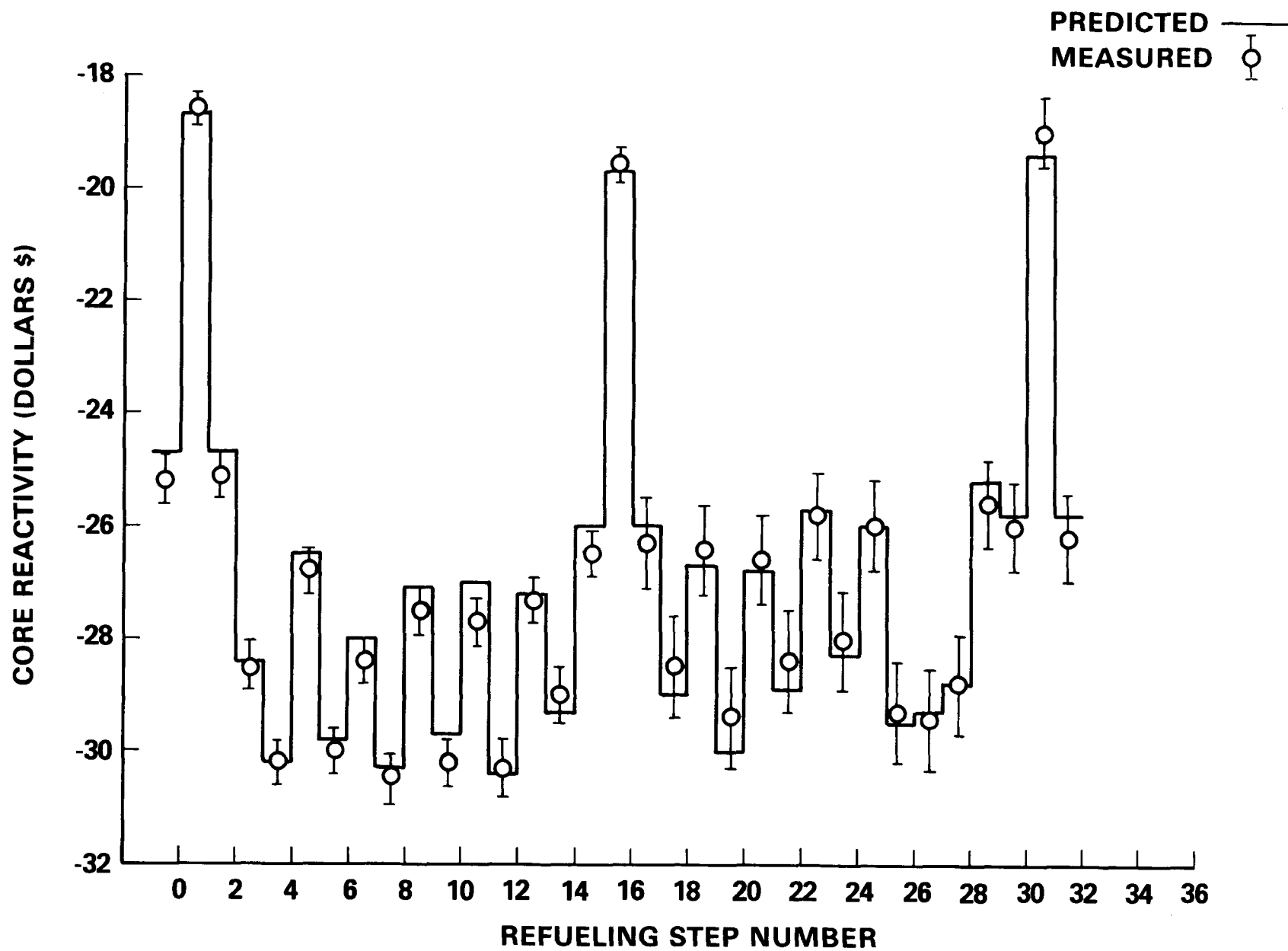


Figure 2. Reactivity Versus Refueling Steps 0 through 32.

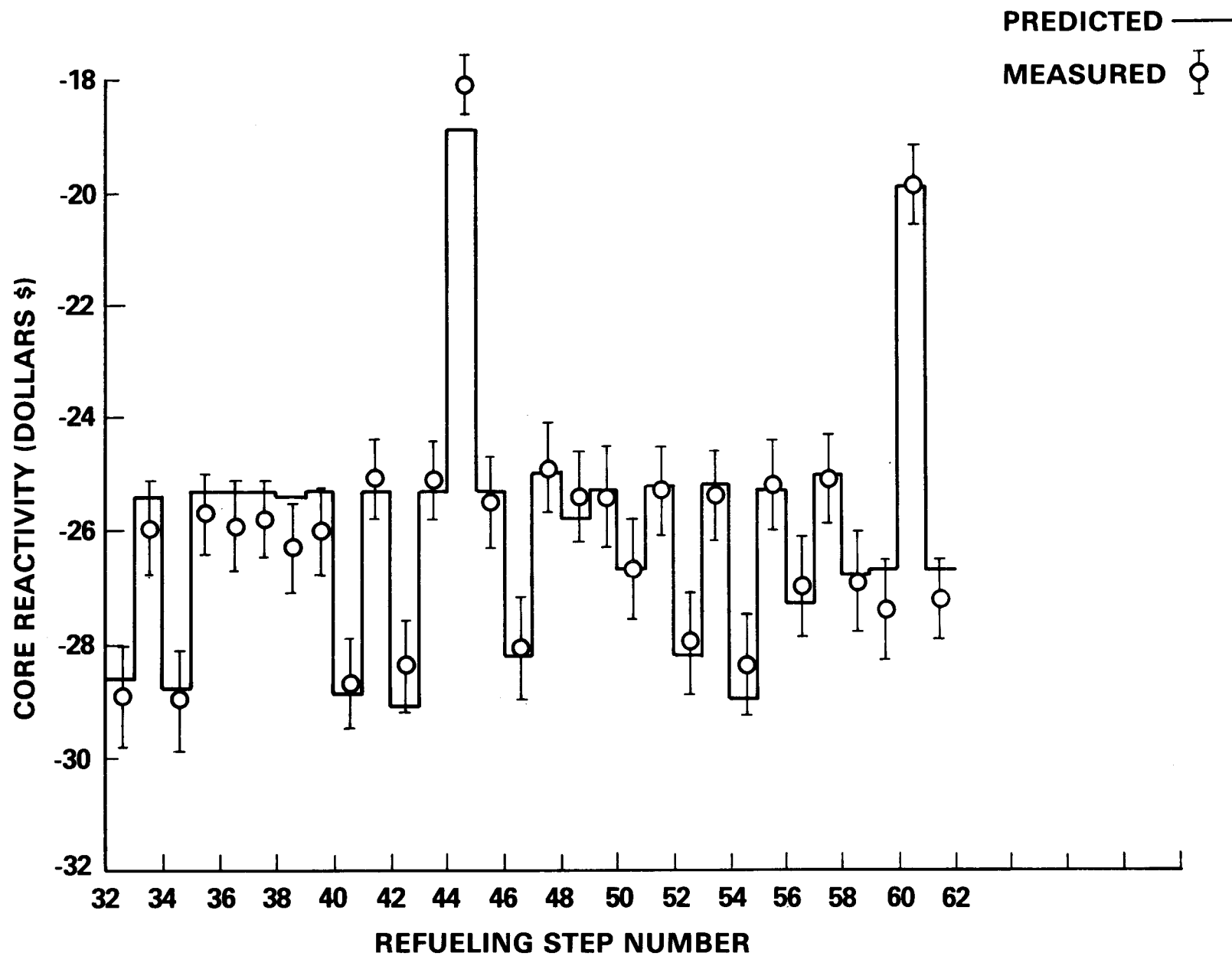


Figure 3. Reactivity Versus Refueling Steps 32 through 62.

Core configuration factors used in the reactivity measurements were precalculated for every refueling step using the fixed source option and four neutron energy groups in the two-dimensional diffusion theory code, 2DBS.⁽¹²⁾ Representative values of these F factors are shown in Table 1. Also shown in the table are predicted values of reactivities and core component reactivity worths derived from these same F factor calculations.

Table 2 shows a few comparisons of the measured and predicted reactivities selected from the data of Figures 2 and 3. Also shown in Table 2 are the measured and predicted values of reactivity worths of a selected number of fuel components.

6.0 SUMMARY AND CONCLUSIONS

Comparison of measured and predicted values of core reactivities and core component worths shows that all surveillance requirements were met during the refueling. In every case, the difference between measured and predicted reactivities was less than 1.50\$. The maximum difference was 0.94\$ (step 39, Figure 3 and Table 2). Also, in every case, the difference between measured and predicted core component worths was less than 1.00\$. The maximum difference was 0.72\$ (step 23 of Figure 2 and Table 2). This application of the MSM technique shows that with the procedures adopted, the measurements are within acceptable limits for reactivity monitoring. Moreover, all surveillance requirements can be met without difficulty.

It has been demonstrated that the MSM technique yields accurate subcritical reactivity assessments even if the buildup of sources may not be predicted accurately. However, this accuracy is a direct result of numerous recalibration maneuvers. Further studies will be conducted to increase the accuracy of predictions and thus lessen the frequency of these required maneuvers.

TABLE 1

F FACTORS AND REACTIVITY PREDICTIONS FOR REFUELING MONITORING

OPERATION STEP NUMBER	OPERATION DESCRIPTION	CORE COINFIGURATION FACTOR "F" AT COMPLETION OF STEP			PREDICTED CORE REACTIVITY $\rho(\%)$ +	PREDICTED REACTIVITY CHANGE IN THIS STEP $\Delta\rho(\%)$ +
		LLFM A	LLFM B	LLFMC		
0	PRECHANGEOUT	1.096	1.120	1.109	-24.69	
1	WITHDRAW CR #3*	1.078	1.131	1.007	-18.72	+5.97
2	INSERT CR #3	1.096	1.120	1.109	-24.69	-5.97
3	REMOVE DFA** FROM POSITION 1303	1.141	1.163	1.080	-28.38	-3.69
9	INSERT UFA*** INTO POSITION 1506	1.083	1.140	1.033	-27.14	+3.14
18	REMOVE DFA FROM POSITION 2507	0.907	1.121	1.060	-29.04	-3.02
30	INSERT UFA INTO POSITION 1711	0.992	1.060	0.751	-25.78	-0.56
31	WITHDRAW CR #3	0.977	1.055	0.684	-19.37	+6.41
32	INSERT CR #3	0.992	1.060	0.751	-25.78	-6.41
44	INSERT UFA INTO POSITION 3507	0.920	0.940	0.687	-25.31	+3.79
47	REMOVE DFA FROM POSITION 3403	0.947	0.899	0.712	-28.23	-2.92
60	INSERT UFA INTO POSITION 2402	0.869	0.891	0.666	-26.67	+0.17
61	WITHDRAW CR #3	0.852	0.893	0.606	-19.9	+6.77
62	INSERT CR #3	0.869	0.891	0.666	-26.67	-6.77

*CR #3 = PRIMARY CONTROL ROD #3

**DFA = IRRADIATED FUEL ASSEMBLY.

***UFA = UNIRRADIATED FUEL ASSEMBLY.

+ REACTIVITY AND COMPONENT WORTH ARE IN UNITS OF DOLLARS (%)

TABLE 2
MEASURED AND CALCULATED REACTIVITIES AND
CORE COMPONENT REACTIVITY WORTHS

OPER STEP NO.	OPERATION DESCRIPTION	AVERAGE CORE REACTIVITY, ρ (\$), MEASURED AT END OF OPERATION		AVERAGE COMPONENT REACTIVITY WORTH, $\Delta\rho$ (\$), MEASURED AT END OF OPERATION	
		MEASURED	PREDICTED	MEASURED	PREDICTED
0.	PRECHANGE OUT	-25.15 $\pm$ 0.36	-24.69		
1.	WITHDRAW CR #3 +	-18.57 $\pm$ 0.27	-18.72	6.57 $\pm$ 0.23	5.97
2.	INSERT CR #3	-25.14 $\pm$ 0.36	-24.69	-6.57 $\pm$ 0.14	-5.97
8.	REMOVE ID-DFA FROM 2401	-30.52 $\pm$ 0.45	-30.28	-2.16 $\pm$ 0.11	-2.30
11.	INSERT ID-UFA INTO 2401	-27.67 $\pm$ 0.40	-27.06	2.48 $\pm$ 0.11	2.67
16.	WITHDRAW CR #3	-19.62 $\pm$ 0.29	-19.84	6.93 $\pm$ 0.12	6.18
17.	INSERT CR #3	-26.26 $\pm$ 0.80	-26.02	-6.80 $\pm$ 0.13	-6.18
20.	REMOVE ID-DFA FROM 1405	-29.42 $\pm$ 0.89	-30.05	-3.02 $\pm$ 0.15	-3.38
23.	INSERT ID-UFA INTO 1405	-25.78 $\pm$ 0.78	-25.66	2.53 $\pm$ 0.13	3.25
26.	REMOVE ID-DFA FROM 2301	-29.32 $\pm$ 0.87	-29.54	-3.32 $\pm$ 0.14	-3.54
28.	REMOVE REFL FROM 1711	-28.75 $\pm$ 0.85	-28.76	0.58 $\pm$ 0.11	0.54
29.	INSERT ID-UFA INTO 2301	-25.63 $\pm$ 0.76	-25.22	3.01 $\pm$ 0.13	3.54
30.	INSERT REFL INTO 1711	-26.11 $\pm$ 0.79	-25.78	-0.54 $\pm$ 0.09	-0.56
31.	WITHDRAW CR #3	-19.02 $\pm$ 0.58	-19.37	6.58 $\pm$ 0.22	6.41
32.	INSERT CR #3	-26.16 $\pm$ 0.76	-25.78	-6.98 $\pm$ 0.21	-6.41
35.	REMOVE OD-DFA FROM 1501	-28.99 $\pm$ 0.86	-28.81	-3.06 $\pm$ 0.14	-3.33
36.	INSERT OD-UFA INTO 1501	-25.72 $\pm$ 0.73	-25.28	3.28 $\pm$ 0.14	3.53
39.	REMOVE REFL FROM 2812	-26.30 $\pm$ 0.75	-25.36	-0.33 $\pm$ 0.08	-0.08
40.	INSERT REFL INTO 2812	-26.03 $\pm$ 0.76	-25.28	0.09 $\pm$ 0.07	0.08
45.	WITHDRAW CR #3	-18.09 $\pm$ 0.52	-18.89	7.03 $\pm$ 0.22	6.42
46.	INSERT CR #3	-25.46 $\pm$ 0.84	-25.31	-7.06 $\pm$ 0.25	-6.42
51.	REMOVE OD-DFA FROM 3604	-26.67 $\pm$ 0.89	-26.65	-1.31 $\pm$ 0.11	-1.34
53.	INSERT OD-UFA INTO 3604	-25.34 $\pm$ 0.84	-25.23	1.33 $\pm$ 0.11	1.42
55.	REMOVE ID-DFA FROM 3304	-28.44 $\pm$ 0.94	-29.02	-3.0 $\pm$ 0.14	-3.81
56.	INSERT ID-UFA INTO 3304	-25.16 $\pm$ 0.83	-25.28	3.29 $\pm$ 0.16	3.74
61.	WITHDRAW CR #3	-19.88 $\pm$ 0.66	-19.9	7.47 $\pm$ 0.26	6.77
62.	INSERT CR #3	-27.12 $\pm$ 0.74	-26.67	-7.45 $\pm$ 0.18	-6.77

+ CR #3 = PRIMARY CONTROL ROD NO. 3

* REACTIVITY AND REACTIVITY WORTH ARE UNITS OF DOLLARS (\$)

7.0 REFERENCES

1. P. A. Ombrellaro, R. A. Harris, J. W. Daughtry and R. A. Bennett, "Evaluation of MSM for Assessing Subcritical Reactivities in FFTF," Trans. Am. Nuc. Soc., 35, p. 526, 1980.
2. D. L. Selby and G. F. Flanagan, "Subcriticality Calculations in Support of the Reactivity Surveillance Procedures Experiments for the FTR Engineering Mockup Facility," ORNL-5061, Oak Ridge National Laboratory, October 1976.
3. J. T. Mihalcz, M. V. Mathis and V. K. Paré, "Reactivity Surveillance Experiments with the Engineering Mock-Up Core of the Fast Flux Test Reactor," Nucl. Sci. Eng., 59, p. 350, 1976.
4. A. R. Buhl, N. J. Ackermann, Jr. and J. C. Robinson, "The Status of Subcritical Reactivity Monitoring for LMFBRs," Trans. Am. Nucl. Soc., 16, p. 289, 1973.
5. "Fast Flux Test Facility," Nuc. Engr. Int'l., August 1972.
6. P. L. Hofmann, "FTR Neutronics: Status, Problems and Development Programs," Proceedings Topical Meeting, New Developments in Reactor Physics and Shielding Calculations, CONF-720901, Kiamesha Lake, NY, September 12-15, 1972.
7. R. A. Bennett et al, "Nuclear Start-up, Testing and Core Management of the Fast Test Reactor (FTR)," Fast Reactor Physics 1979, Proceedings of a Symposium, Aix-En-Provence, IAEA, Vol. I, p. 447, 1980.
8. C. P. Cabell, "A Summary Description of the Fast Flux Test Facility," HEDL-400, Hanford Engineering Development Laboratory, Richland, WA, December 1980.
9. Paride A. Ombrellaro and David L. Johnson, "Calculation of Neutron Source Strength in Fast Flux Test Facility Fuel as a Function of Irradiation," Nuclear Technology, 54, pp. 180-200, 1981.
10. P. A. Ombrellaro and N. E. Petrowicz, "Neutron Source Strength in FFTF Fuel as a Function of Irradiation," Trans. Am. Nuc. Soc., 32, p. 736, 1979.
11. M. J. Bell, "ORIGEN-The ORNL Isotope Generation and Depletion Code," ORNL-4628, Oak Ridge National Laboratory, May 1973.
12. D. R. Marr, "A User's Manual for 2DBS, A Diffusion Theory Shielding Code," BNWL-1291, Pacific Northwest Laboratory, Richland, WA, February 1970.

Energy-Dependent Plant Protection System for the TREAT Upgrade Reactor

S. K. Bhattacharyya, W. C. Lipinski, R. R. Rudolph, and D. C. Wade
Argonne National Laboratory
Argonne, Illinois 60439

ABSTRACT

A major upgrade of the TREAT reactor is under way to enhance significantly its transient-testing capabilities in support of the LMFBR safety program. Complex power-vs-time histories, spanning time intervals of several seconds and including a burst mode, are to be generated for experiments with up to 37, 0.027 in. diameter LMFBR test pin clusters. The need to meet demanding functional requirements within several major constraints has produced a design that has a relatively small margin between operating and damaging temperature conditions for the TREAT fuel clad. It was, therefore, necessary to design a sophisticated plant protection system that would detect reactivity insertion malfunctions soon after initiation and intercede promptly with scram action, to prevent reactor clad damage. A plant protection system with energy-dependent power level trip set points lowered worst-case clad temperatures by 180 °C, as compared with the use of conventional fixed-level trip set points. The multilevel, multilayered protection strategy used in the TREAT Upgrade reactor represents an effective protection strategy for transient reactors.

INTRODUCTION

The TREAT reactor [Ref. 1] has been a major transient test facility for in-pile tests in support of the LMFBR safety research program. In the course of its 22 years of operation, the TREAT reactor has provided valuable data on the phenomenology of LMFBR accident energetics. In recent years, results of TREAT experiments have contributed to the license review process for both the FFTF and CRBR facilities. It was recognized in the early 1970s, however, that it was necessary to enhance the capabilities of TREAT in order to address the evolving issues of LMFBR safety. In particular, the limitation on the size of test clusters that can be tested in TREAT (up to 7 pins) precluded study of the development of transition phase phenomena.

In the course of a detailed study [Ref. 2], it was decided that the optimum strategy for acquisition of additional in-pile transient testing capability was to upgrade the capabilities of the TREAT reactor for near-term tests, and to design and construct a new Safety Test Facility (STF) for tests of longer term. Reevaluation of the U.S. LMFBR program has resulted in an indefinite deferral of the STF. The TREAT Upgrade (TU) reactor is therefore one of the world's major safety test facilities. A set of demanding functional requirements has been imposed on the facility. These requirements refer to both static (test energy deposition, radial intra-test-cluster power profile) and dynamic (transient shapes) parameters.

The necessity of meeting these demanding functional requirements in the TREAT Upgrade reactor has produced a design with a smaller margin between operating and damaging temperatures for reactor fuel assembly cladding, than is currently available in the TREAT reactor. An obvious impact of this

reduced margin in temperature is the reduced time interval between initiation of a reactivity accident and progression to reactor clad damaging conditions. In other words, unless protective mechanisms detect a malfunction and intervene in time, the possibility of fuel clad damage from reactivity accidents must be considered. This paper describes the development of the design of a multilayered reactivity accident protection system for the TREAT Upgrade reactor.

PRINCIPAL FEATURES OF THE TREAT UPGRADE REACTOR

Detailed descriptions of the TREAT Upgrade (TU) reactor have been presented in earlier publications [Ref. 3]. Figure 1 is a perspective view of the reactor, showing the core and some major components. The reactor is operated in both steady state and adiabatic super prompt critical burst modes. The central 11 x 11 fuel assembly array was modified within the 19 x 19 fuel assembly matrix that comprises the TREAT core. The modified core, called the "converter", is fueled with UO_2 -graphite (as in TREAT), clad in Inconel 625, and can be operated at very high temperatures (1000 °C, compared to 400 °C in TREAT). The unmodified outer core, called the "driver", contributes significantly to the negative temperature feedback of the core and to core reactivity.

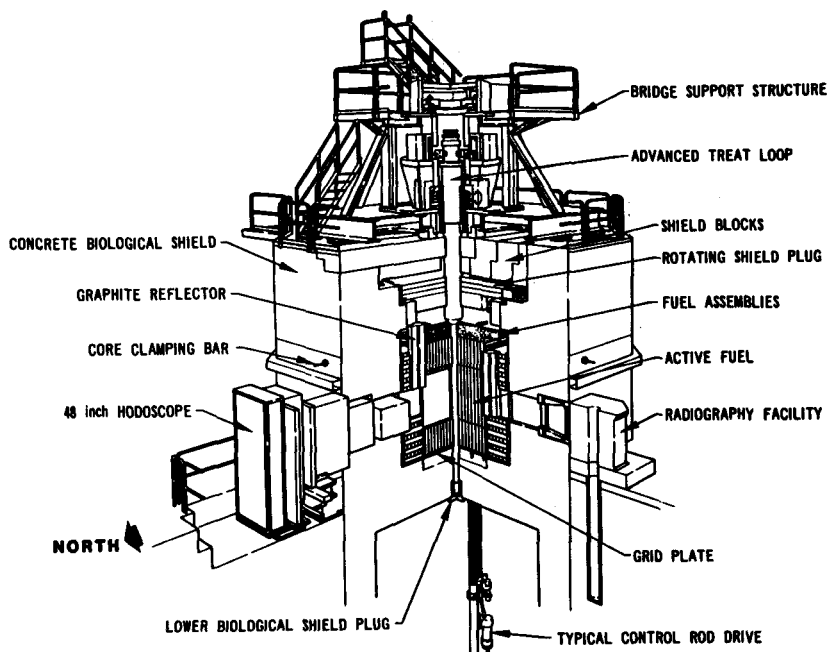


Fig. 1. Perspective View of TREAT Upgrade Reactor

Figure 2 shows a horizontal section of the TU core at the axial midplane. The core features three independent banks of control rods. Super prompt initial bursts are produced by computer-controlled ejection of hydraulically-driven "transient" rods, often at high speed. These transient rods are

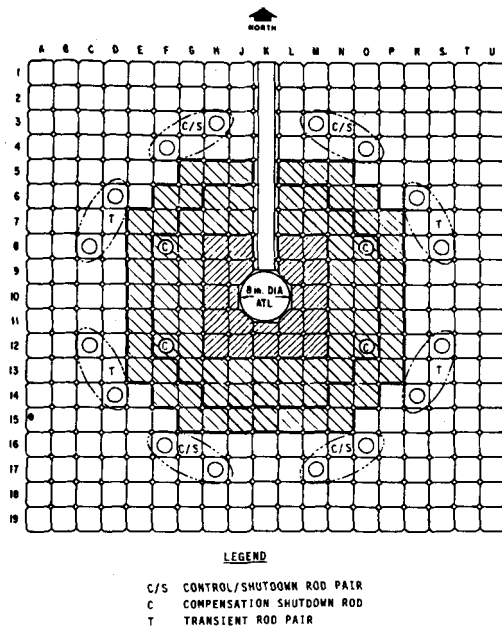


Fig 2. Horizontal Cross Section of TU Core

GENERATION OF REQUIRED TRANSIENT SHAPES

Before discussing the potential reactivity accidents in TREAT Upgrade and their consequences, it is necessary to discuss the transient operations in TU and the Automatic Reactor Control System (ARCS) designed to produce the transient shapes. The transient shape requirements of the TU facility were formally expressed in terms of accident ramp rates in contemporary fast reactor designs. These requirements were translated into parameters usable in the TU design, with the help of the in-pile test user community, by defining a set of six reference transients. All transient experiments expected to be run in TU will be constrained by the envelope of characteristics defined by these reference transients.

The reference profiles of transient power-vs-time consist generally of low-level flattop power operation for several seconds, followed by a super-prompt critical burst and subsequent die-away, with or without control of the shape of the post-burst tail [Fig. 3]. The initial flattop operation preconditions

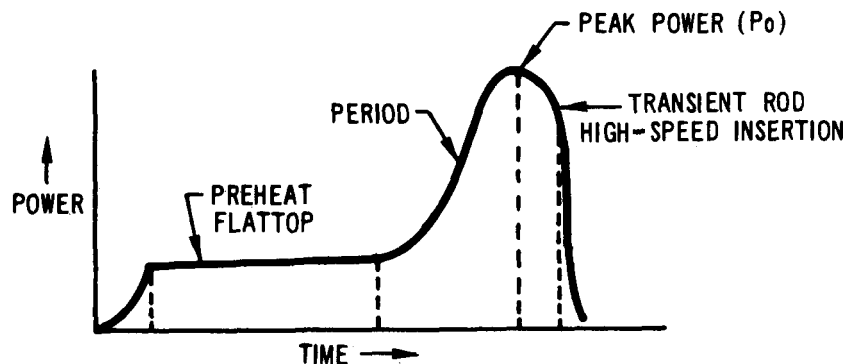


Fig. 3. Characteristic Parameters of Typical Transients

the test fuel to LMFBR operating conditions, and the burst simulates an accidental overpower. The tail of the burst is used to simulate decay heat and coast-down conditions. The three principal parameters that define the reference shapes are the flattop energy, the period of the burst, and the peak power of the burst (usually expressed as a ratio to the flattop power). In addition, the total transient energy and breakdown of its components into the three transient phases (flattop, burst, and post-burst) provide definition of these transients. The reference transients have test fuel characteristics of preheat energies in the range of 720 - 1000 J/g, peak power in the range of 3.5 - 40 times the preheat power, minimum periods ranging from 10s to 0.1s, and total energy requirements ranging from 2500 J/g to 4000 J/g.

It has been demonstrated [Ref. 4] that the point kinetics approximation is applicable to the tightly-coupled TU core. Explicit kinetics analyses using best estimate reactor parameters (Table I) and the TUKIN code [Ref. 5] have

TABLE I
Parameters Used in Analysis

<u>Reactor</u>			
Temperature coefficient of reactivity feedback, $\Delta k/k/^\circ\text{C}$ rise in average driver fuel temperature			-7.78×10^{-5}
Effective neutron generation time, μs			426
Delayed Neutron fraction			7.13×10^{-3}
<u>PPS</u>			
Detector channel time constants, ms			
Linear Power			10^{-2}
Log power (≤ 100 kW)			≥ 0.65
(≥ 1 MW)			0.28
Temperature			10^3
Circuit time constants, ms			
Integrated power			10^{-2}
Period			50
Bistable scram trip delay, ms			1
<u>Rod Banks</u>	<u>Transient</u>	<u>Control</u>	<u>Compensation</u>
Rod drive latch release time, ms	55	65	30
Rod scram time (from latch release to full in), ms ^a	260	205	205
Scram reactivity worth of rod type, \$	3.89	11.00	12.60

- ^aAssumes: 1) delay of 25 ms to account for acceleration to full velocity of 170 in./s for transient rods.
2) constant acceleration of 2892 in./s² for control/shutdown and compensation/shutdown rods.

shown that the reference transients could be produced in TU, and have established the reactivity insertion rate, $\dot{\rho}(t)$, and total reactivity insertion requirement, ρ_{total} , necessary to produce each of these transients. This information was used in the design of the Automatic Reactor Control System (ARCS) that will drive the transient rods to produce the desired transient shapes.

A system diagram of the ARCS is shown in Fig. 4. It includes a process control computer that produces the prescribed power-vs-time trajectory by commands to the transient rod controllers and a process monitor computer that checks the performance of the control computer. Upon sensing an abnormal condition, the ARCS can either stop the motion of the transient rods or issue a scram request to the Reactor Trip System (RTS) of the Plant Protection System (PPS). Figure 4 shows the real-time transient rod and reactor simulators used to verify that there are no hardware failures or software errors in the control and monitor computers before initiation of a transient. The ARCS can be operated with both simulators in the loop or with the actual transient rod systems providing position inputs to the reactor simulator. During the simulations, the reactor is kept subcritical by maintaining the full insertion of the control/shutdown and compensation/shutdown rods. The output of the reactor simulator is used to insert simulated ion chamber currents at the input of the measurement channels.

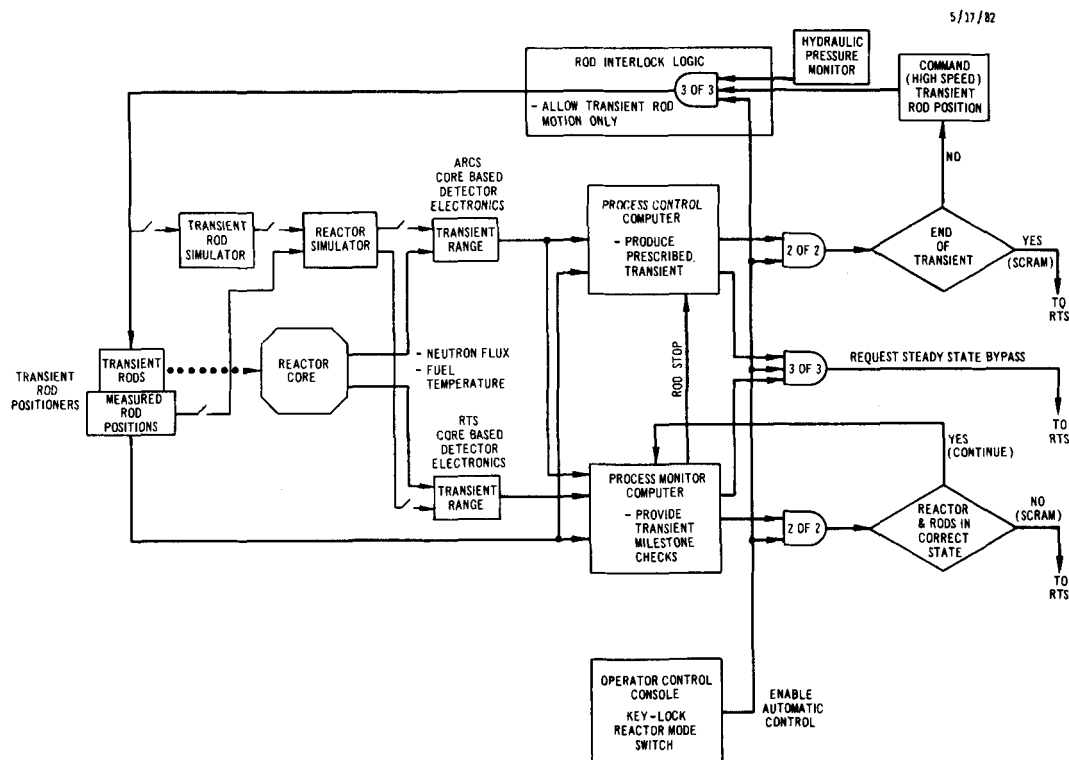


Fig. 4. Automatic Reactor Control System Block Diagram

The machine code statements for the control computer are derived from an off-line process. A high level language is used, which enables a reactor operator to enter the parameters defining the experimenter-specified power-vs-time trajectory, and a specially developed software package translates the experiment input specifications into control computer machine language. An off-line verification is made by a non real-time simulation run with the control statements before the software is loaded into the control computer. The results of this variable sample rate off-line simulation are used for later comparison with the 1 ms fixed sample rate real-time simulation results, to establish that acceptable performance can be obtained. The control computer has as inputs: linear power, log power, period, and transient rod position. The control computer provides command signals to four hydraulically-driven transient rods (see Fig. 2) that can move at speeds of up to 200 in/s.

The monitor computer inputs are: two channels each of PPS transient linear power, log power, period, and integrated power (energy); ARCS linear power, log power, and period, and individual transient rod positions. The monitor computer can request a scram if the control computer causes a deviation from the expected transient. Each of the above functions will be performed three times, using nuclear measurements for the two RTS channels and the linear and log RCS channels.

The monitor computer will calculate the average bank position of the transient rods, compare each rod individually against the average bank position, and issue a scram signal if the deviation exceeds a prescribed value. This function, designed to avoid azimuthal power tilts, will be bypassed at the time the burst is initiated.

The monitor computer includes a timer that will call for reactor trip after a prescribed time limit is exceeded.

The monitor computer will include a transient-dependent sloped boundary between the desired burst trajectory and the PPS trip boundary, to provide a transient rod stop function for experiment assurance in the event that transient rod runaway occurs during the burst phase. The monitor computer will then control the burst in an on-off control mode.

REACTIVITY ACCIDENT SCENARIOS

From the discussion in the preceding section, it is clear that the transient rods must move in a complicated time pattern to produce the desired transient shapes. A typical example of a reactivity insertion time pattern is shown in Fig. 5. If the transient rods are moved erroneously at an unanticipated rate at an unanticipated time, reactivity accidents can result. The consequences of reactivity accidents may be higher clad temperatures than for normal events and, for some classes of low-probability accidents, perhaps clad damage (rendering some fuel assemblies nonreusable), and even clad melting.

The TU fuel assemblies are square in cross section, with chamfered corners. In the assembled core, the chamfered corners form coolant channels. The core is cooled by air in the once-through, open-cycle mode. During the

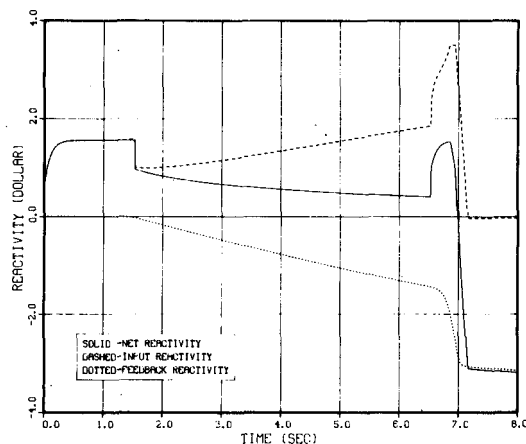


Fig. 5. Time Shape of Reactivity for Typical TU Transient

short times of the transient experiment (several seconds) cooling is ineffective, and the facility behaves adiabatically, with the burst energy stored in the large heat capacity of the graphite matrix. This heating of the graphite causes a negative reactivity feedback (dotted line in Fig. 5) that "turns" the burst power around. The heat is eventually transferred to the clad and is removed by the air coolant over a period of hours.

The spatial and time components of clad heating are complicated. The former is governed by the location of the coolant channels and preferential heat removal from them; the latter by the transient shape (which

determines the direct gamma and neutron heating) and the thermal impedance of the pyrolytic graphite insulation between the clad and the fuel. These spatio-temporal effects lead to large thermal gradients within the clad, and can be postulated to cause eventual clad failure from cumulative creep and fatigue effects. The clad has been designed such that the planned TU transients (approx 1000) can be performed, without causing clad failure. Reactivity accidents leading to temperatures in the upset, emergency, and faulted categories hasten clad failure. In addition, accidents in the emergency category have the potential of causing clad distortion, rendering it un reusable (at temperatures exceeding approx 1060 °C), and accidents in the faulted category can lead to incipient clad melting (solidus temperature of carburized Inconel 625 is 1220 °C). The upper limit of the faulted category is defined by the 1220 °C temperature. Hence, there is a strong incentive to detect malfunctions promptly and terminate reactivity accidents.

Analyzing transients and potential accident scenarios in the power-vs-energy (P-E) plane instead of in the more conventional power-vs-time plane has been found to be very effective. The primary advantage of this scheme is that the power and energy parameters are of direct interest operationally and experimentally, whereas time is of interest only in determining energy. A second advantage of the P-E plane analysis is that an exponential power rise is represented as a straight line ($\tau = E/P$) and this greatly simplifies the conceptual development of the accident analysis method to be discussed next. The six enveloping transients are shown plotted as solid lines in the P-E plane in Fig. 6. Power, in terms of specific power (kW/g) in the test fuel, is the vertical axis and energy, in terms of specific energy (J/g) in the test fuel, is the horizontal axis. Time increases with movement from left to right along the curve.

The three phases for each transient (flat top, burst, and tail) are easily identifiable from this perspective. Requirements for shaping the tail of the burst are met by insertion of the transient rods after the peak of the

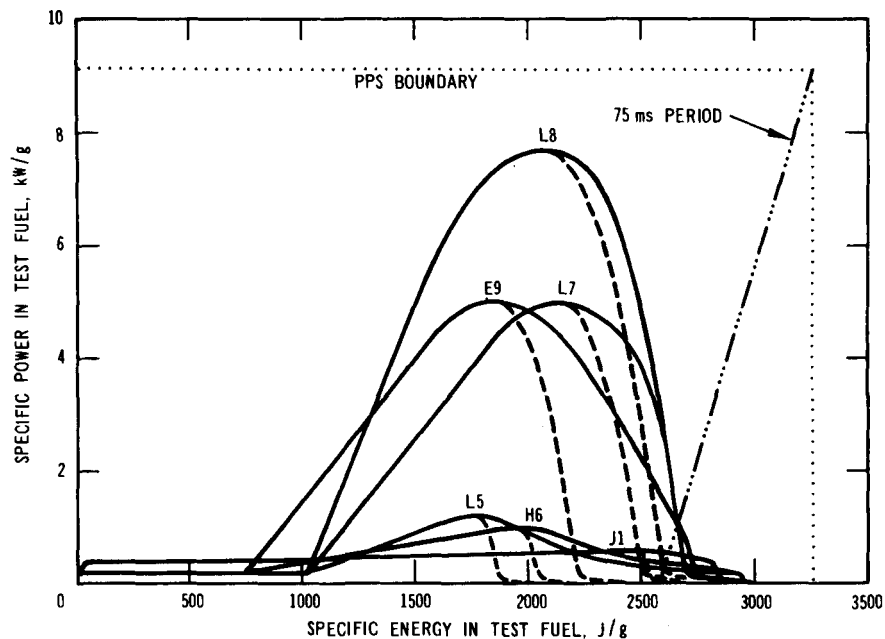


Fig. 6. Enveloping TU Transient Shapes in P-E Plane with Conventional PPS

burst. This insertion, or "clip" can occur as early as the time of the peak power (dashed lines in the figure) or be delayed to meet the maximum functional energy requirement of 3250 J/g (solid post-peak lines). These two post-burst paths define a range of possible post-clip tails for the transients.

CONSEQUENCES OF REACTIVITY ACCIDENTS WITH CONVENTIONAL PPS

The parameters that are typically monitored for protection purposes are reactor power, integrated power, period, and peak clad temperature. The neutronic information is gathered in several detector channels, the principal ones being uncompensated BF_3 ion chambers located in the biological shield. In addition, peak clad temperature is monitored in the hottest assembly through thermocouple measurements in instrumented fuel assemblies and calibration data on the transformation of the measured temperature to a peak clad temperature. The slow response of the thermocouples renders the temperature readings inadequate for protective scram during most transient operations.

The various detection parameters provide an umbrella of protection for the reactor over the full spectrum of possible reactivity insertion accidents. Because of the modes of potential reactivity accidents, it is generally true that each parameter activates optimum protection against classes of accidents within specific ranges of reactivity insertion rates encountered. For example, period trips protect against fast rod runaways while energy trips against slow rod runaways. Occasionally, parameters provide redundant protection; generally, however, each parameter has its own optimum protection regime.

Reactivity insertion accident consequences were analyzed by use of the TUKIN code. The conventional PPS settings are represented in the P-E plane by the boundaries of a power level of 9.0 kW/g and an energy level of 3250 J/g,* representing upper limits of parameter values anticipated in TU operation. Within this protection "box" (Fig. 6), the maximum allowable slope (the maximum inverse period $1/\tau = \dot{P}/\dot{E} = 13.33 \text{ s}^{-1}$) is effective everywhere. This represents a slightly shorter period (75 ms) than the minimum period anticipated in TU operation (100 ms). If the path of any transient event passes through either boundary of this box or exceeds the slope, a PPS-induced scram is initiated.

A major advantage of the P-E plane display method is that the limiting accident scenarios can be determined by inspection. The energy released in an accident is composed of two parts: 1) that energy deposited before the PPS trip is encountered, and 2) that energy generated after the trip. It is clear that the highest energy deposition and, hence, consequences** occur for reactivity accidents that trigger the period, power, and energy trips simultaneously. Scramming on energy assures the highest pre-trip energy and scramming on the highest permissible power and shortest permissible period guarantees the largest post-trip energy deposition. It is also clear that the "worst-case" reactivity accident consequence is path-independent provided that the trips are triggered simultaneously. In the subject analysis of the worst-case consequences, conservatism has been added by disregarding physical limitations of the control system that could preclude generation of a transient trajectory that would simultaneously trigger all three trips.

If limitations on total reactivity and reactivity insertion rates are disregarded, the "maximum" consequence trip point can be reached from any transient, provided the slope (dot-dashed line in Fig. 6) is not exceeded. Temperature consequences for this accident† with the trip set points initially assumed are a peak TU converter fuel temperature of 1356 °C and a corresponding peak clad temperature of 1291 °C.†† The latter is well above the safety

*For the purposes of this discussion, the PPS trip settings are given in terms of the parameters used in the functional requirements, i.e., power and energy deposition in the test fuel. In designing the actual PPS, these settings will be translated into reactor equivalent values.

**Consequences are defined as peak adiabatic fuel or clad temperature.

†In these analyses, the reactivity insertion causing the accident is of undefined origin. To be conservative, the latter part of the insertion is assumed to be by transient rod withdrawal. Therefore, the insertion is allowed to continue for 20 ms after the scram signal has been sent, to account for delays in stopping the rod withdrawal. During this time the rate of reactivity insertion is allowed to increase only to the constant 75 ms period power rise. This disregards any realistic limits on the amount of rate of reactivity expected to be available for the accident.

††The calculated peak converter fuel temperature is 5% higher than the peak clad temperature, as determined from detailed thermal calculations.

limit (1220 °C) for melting of the Inconel 625 clad. Because the PPS trip set points had been chosen on the assumption of a realistic margin above the limits of the transients and the characteristics of the PPS circuits and components are considered state-of-the-art, the only possible changes in the PPS are improvements on the fixed value PPS set point concept.

DEVELOPMENT OF THE TU PPS DESIGN

From a study of the transient power-vs-energy shapes (Fig. 6) it is apparent that the only changes that can be made in the trip boundaries, without affecting the generation of the required transients are at low- and high-energy values. In order to determine the impact of moving the high-energy trip boundaries inward (reducing the size of the protected "box"), a set of isothermperature scram consequence lines was generated (Fig. 7). These lines are determined by tracing the path of simultaneous power and energy trip set points that lead to the same peak fuel temperature. The approximate linearity of these isothermperature lines is a result of the approximate independence between the effects of variations in power and energy on consequences.

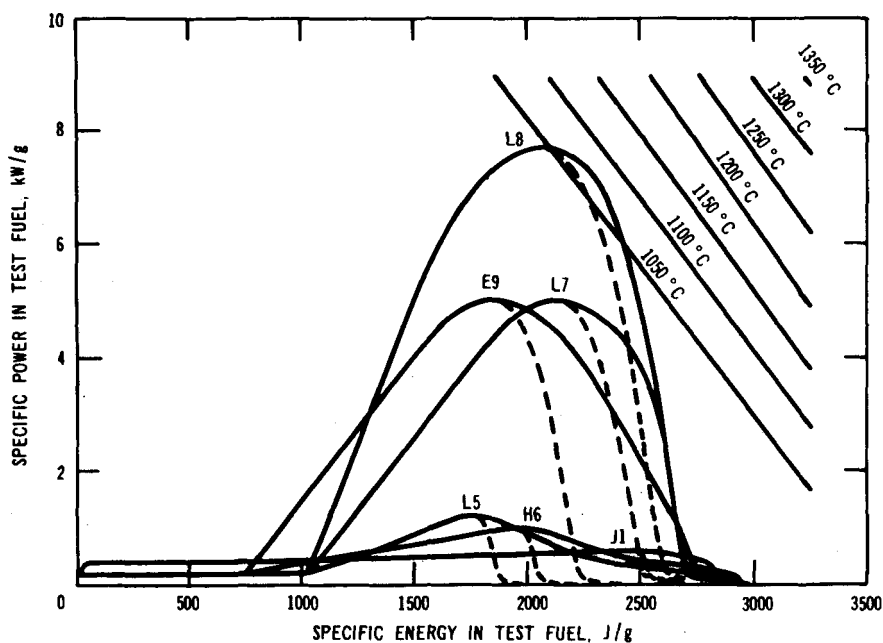


Fig. 7. Isothermperature Scram Consequence Lines

Given the enveloping transient paths and the isothermperature scram consequence lines, the lowest possible fuel temperature for a worst-case malfunction is approx 1075 °C, if a sophisticated PPS were used to provide a trip boundary along the isothermperature line closest to the desired transient path. This temperature represents a 280 °C decrease in peak fuel temperature from that with the fixed value set point protection concept discussed in the preceding section.

Several concepts of post-burst peak reduction of the protection area were explored. An easily-achievable two-step approximation of the isotherm line is shown in Fig. 8. The first "shelf" was selected to decrease the power level to 6.75 kW/g from 9.0 kW/g after 2400 J/g of energy had been deposited in the test fuel. This was followed by a further reduction to 1.0 kW/g at energies above 2800 J/g. For this PPS trip selection, the worst-consequence reactivity accident is a scram at the P-E coordinate (2400 J/g, 9 kW/g), of a transient "running away" on a 75 ms period. For this limiting case accident, the peak fuel temperature is 1165 °C (1110 °C, clad). This represents a 191 °C decrease in the peak fuel temperature (181 °C in the clad), compared with the conventional fixed-setting PPS case. The clad temperature is well below the clad melting temperature, 1220 °C.

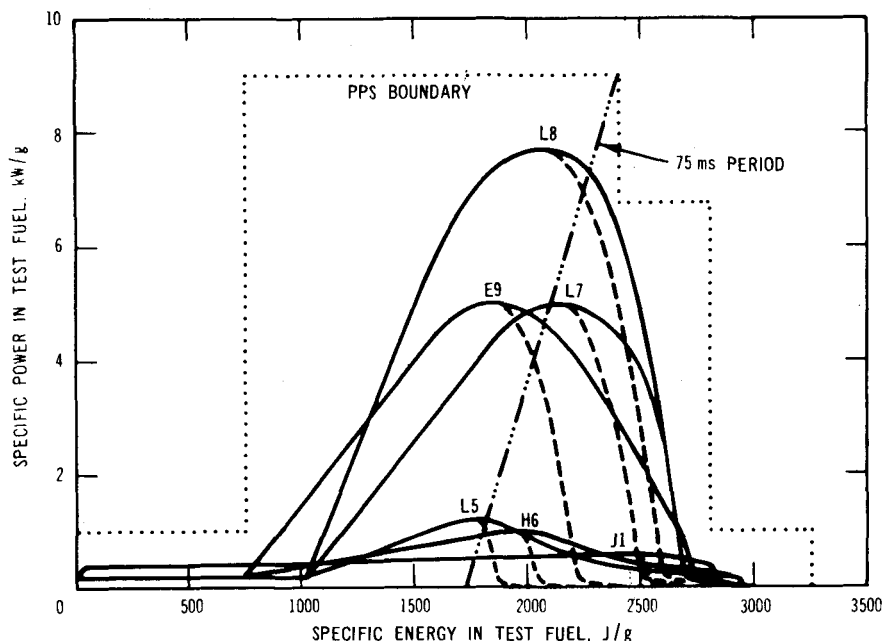


Fig. 8. TU Step Shelf PPS with Enveloping Transients

The computed peak clad temperature for the accident analyzed in the previous section uses a point kinetics reactor model, with best-estimate values for neutronic parameters, state-of-the-art protection system characteristics, and realistic reactor-parameter trip set points. At the present time, the major question in assessing these temperatures concerns the effects of uncertainties in the calculational parameters on the results. Because no critical experiments have been available to support the TREAT Upgrade physics design, there are relatively major uncertainties in the values of several of the key neutronic parameters. This situation will not be improved until the pre-operational testing on the TREAT Upgrade reactor has been completed, parameter values have been measured, and calibration factors have been determined. Computer calculations of the neutronic parameter values have been prudently conservative. However, it is likely that the real values of

these parameters will differ from their assumed values by varying amounts and directions. Similarly, because the PPS is still in the design phase, the characteristics of the system have not been confirmed.

The results of a sensitivity study that was based on uncertainties in PPS characteristics (channel/circuit time constants, rod worths, insertion times, and mechanical delay times) are given in Table II. Failure of a single rod drive to scram is calculated to increase the peak temperatures by no more than 8 °C, and failure of an entire rod bank (4 drives) could increase the consequences by up to 40 °C. A study using random combinations of uncertainties in key parameters showed that there was less than 0.6% likelihood of clad melting for accidents in the class considered here.

With the protection strategy in TU, reactivity accidents initiated late in the transients result in the greatest consequences. Accidents initiated early in the transient can also result in relatively large consequences, because a large amount of reactivity is available for accidental insertion. An initial lowered power trip shelf at 1 kW/g, extending up to an energy deposition of 750 J/g (Fig. 8), effectively reduces the consequences of the accidents in the early phase of the transient.

PPS DESIGN DESCRIPTION

The Plant Protection System design for TU, shown in Fig. 9, provides protection for both steady-state and transient operations. For the steady-state

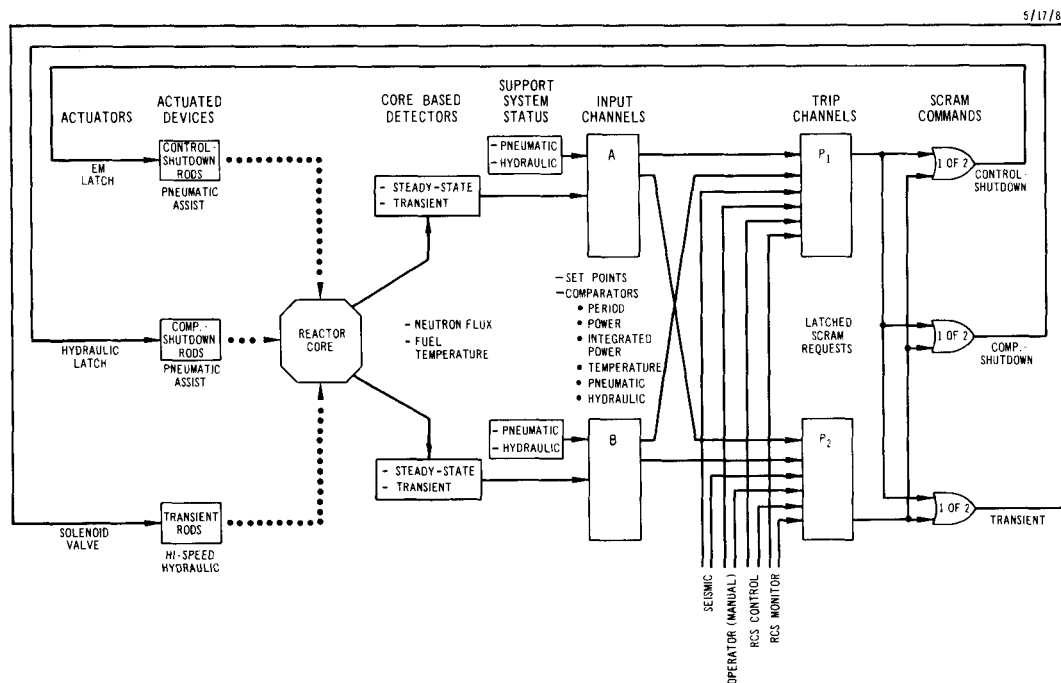


Fig. 9. Plant Protection System Block Diagram

TABLE II

Sensitivity of Peak Clad Temperature to Variations
in PPS Characteristics

Characteristic	Nominal Value	Uncertainty, %	Change in Temperature, °C
PPS Channels/Circuits			
Linear power circuit time constant, ms	10	10	$+2.8 \times 10^{-3}$
Scram logic circuit trip signal delay, ms	1	10	+0.3
Worth of Rod Bank in Scram, %			
Compensation/Shutdown rods	12.62	10	-3.6
Transient rods	3.89	10	-3.2
Control/Shutdown rods	11.00	10	-2.4
Delay in Stopping Transient Rod Drives, ms	20	9.5 ^a	+4.9
Scram Acceleration, in./s ²			
Control/Shutdown rods	2892	10	-1.9
Compensation/Shutdown rods	2892	10	-4.8
Transient rods scram speed, in./s	170	10	-2.9
Scram Delay in Latch Release, ms			
Control/Shutdown rods	65	3.1 ^a	+1.5
Compensation/Shutdown rods	30	6.7 ^a	+2.5
Transient rods	75	2.7 ^a	+2.5

^aAdditional 2 ms assumed.

mode, the trips are set conservatively at a 5 s period, a 150 kW reactor power level, and a 400 °C core peak clad temperature. For the transient mode the trip settings are as described in the preceding section.

The PPS is required to meet the single failure criterion and a one-out-of-two system for scrambling was selected. The specification of PPS components is designed to provide system reliability meeting the requirement that the probability of accidents with consequences exceeding those of the DBA is less than 10^{-7} per year given that 100 transients per year are run. In addition, to the normal neutronic parameter, the scram mechanism supporting systems are monitored, and trips are provided on low pneumatic and hydraulic pressure. Upon scram, the reactor trip system causes high-speed insertion of four compensation/shutdown rods, eight control/shutdown rods yoked in pairs to four rod drives, and eight transient/shutdown rods, also yoked in pairs to four rod drives. The compensation/shutdown rods are driven (60 in. travel) by pneumatic pistons (300 psi on 10 in.²) with fast-release hydraulic latches (30-ms release). The control/shutdown rods are driven by identical pneumatic pistons and released by electromagnets (60-ms release). The transient rods are driven (40 in. travel) by hydraulic pistons (3000 psi and 3.75 in.² piston area) and inserted by deenergizing a solenoid valve (60-ms release).

The PPS is designed to fail safe on loss of power. In addition, seismic trips with a horizontal wave trip setting of 0.04 g are provided to initiate reactor scram. The twelve shutdown rod drives are seismically qualified for rod insertion during the 0.11 g P-wave component and to remain in the core during the 0.22 g S-wave component of the safe shutdown earthquake for the facility.

In order to meet the PPS reliability requirements, a 10-h test interval has been selected. PPS verification is aided by use of a Dedicated Micro-processor Tester (DMT), that inserts simulated ionization chamber currents at the inputs of the nuclear channels, verifies the trip point settings, and prepares a printout of the test results. The operation of the pressure switches is similarly tested by changing the pressure and recording the pressure at which switch actuation takes place. A 10-h window is available for running an experiment starting at the time the PPS test was begun.

The detailed design of the trip set point circuitry for the shelf power trip settings is straightforward in concept and hardware requirements. Integrated power readings are used to identify the energy points at which power trip set point transfers occur. Comparator circuits trigger the change of state of a bistable, activating the appropriate power-level trip set point. For a trip boundary following a limiting isotherm line, signals are summed in an operational amplifier, with appropriate weighting corresponding to the equation of the line. The processed signal essentially acts as a continuous comparator, and triggers a bistable output to seal-in a trip signal until reset.

It should be noted that the top level trip settings (corresponding to the fixed conventional PPS settings) are not disabled during the trip setting transfers. This feature produces a multilevel trip setting over most of the transient. An additional protection level is available for all but the

most demanding transients, in the form of more restrictive transient-dependent settings on period, power, and energy. The energy-dependent trip setting concept for the power level is applied to this transient-dependent case also. The analytical procedure described in the previous section is used to determine the settings for the transient-dependent trips.

SUMMARY AND CONCLUSIONS

Because of the need to meet demanding functional requirements, the TREAT Upgrade reactor design has a high operating temperature. A sophisticated plant protection system, featuring energy-dependent multilevel power trip settings and multilayered protection channels, has been designed to respond promptly in case of reactivity accidents and to keep clad temperatures within acceptable limits. The TREAT Upgrade reactor operation relies on the PPS for core protection. This unique mode of operation for burst facilities is possible because of the improved protection strategy devised for the TU reactor.

REFERENCES

1. G. A. Freund et.al., "TREAT, A Pulsed Graphite-Moderated Reactor for Kinetic Experiments," Proc. Second International Conference on the Peaceful Uses of Atomic Energy, Geneva, Vol 1D, 461 (1958).
2. R. Avery et.al., "Report on Experiment Needs and Facilities Study," ANL/RAS 76-22, Argonne National Laboratory (1976).
3. D. C. Wade et.al., "Core Design of the Upgraded TREAT Reactor," Presented at the ANS Topical Meeting on Fast, Thermal and Fusion Reactor Experiments, Salt Lake City, Utah, April 1982.
4. S. T. Yang, Private Communication, Argonne National Laboratory, 1980.
5. R. R. Rudolph, "TUKIN - Validation and Users Instructions," ANL Internal Report, 1980.