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WORKSHOP

"Can RHIC be used to test QED?"

Workshop organized by, and Proceedings edited by
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Upton, Long Island, New York 11973

April 20-21, 1990

DEPARTMENT OF PHYSICS AND RHIC PROJECT

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ACTUAL AGENDA

"Can RHIC be used to test QED ?"
April 20-21, 1990
Snyder Seminar Room, AGS
Bldg. 911, 35 Lawrence Drive

Friday, April 20, 1990

08:30		Coffee
09:00	S. Ozaki and T. Ludlam	Welcome and Introduction
09:30	S. Brodsky	Theoretical Overview
10:45	Break	
11:15	B. Müller	Higgs Production in RHI collisions
12:30		Lunch
2:00	G. Couture	$W^{\pm}$ Boson Magnetic Moment in RHI collisions
2:30	M. Strayer	|
3:30	G. Baur	|
4:30	Break	| Lepton Pair Production in RHI collisions
5:00	W. Scheid	|

7:00 Buffet and Welcome Party

Saturday, April 21, 1990

08:30		Coffee
9:15	C. Bottcher	Higher order QED effects
10:15	Break	
10:45	S. Datz	Experimental Program at CERN
11:30	R. Vane	Data from CERN EMU-4 (Parnell)
11:40	A. Belkacem	Experimental Program at LBL
12:30	Lunch	
1:30	*Discussion*: Lepton pair production	Discussion Leader: L. McLerran
2:30	*Discussion*: Higgs production and other issues	Discussion Leader: J. Smith
3:30	Meeting Ends	

Time allotted assumes 10 minutes for questions.

Organizing committee:
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M.J. Tannenbaum SAPIN@BNLDAG

CAN RHIC BE USED TO TEST QED

Workshop Summary

Mirek Fatyga
Mark Rhoades-Brown
Michael Tannenbaum

The two day workshop entitled "Can RHIC be Used to Test QED" took place on April 20 - 21 at Brookhaven National Laboratory. It was attended by approximately 50 physicists from both the U.S. and Europe. Although most of the attendees were theorists, a large portion of the second morning was devoted to pending experiments and the experimental difficulties associated with strong field electromagnetic phenomena. The workshop was remarkably multidisciplinary in nature, attracting elementary particle, nuclear and atomic theorists and experimentalists.

We note that at RHIC, fully stripped Au ions will be accelerated to beam energies of 100Gev/u in a collider mode. At these energies the S-matrix for single $e^{+}+ e^{-}$ pair production violates unitarity bounds, thus implying that multiple pair production will occur. The theoretical language for investigating this phenomena is contained within the so-called virtual two photon representation of the heavy ion electromagnetic fields.

Three general subjects were addressed during the workshop. These subjects were:

1. To understand the validity of the best available descriptions of $e^{+} e^{-}$ pair production in peripheral heavy ion collisions, especially for the domain where this process is known to be non-perturbative.

2. To understand the prospects for using relativistic heavy ions to produce Higgs Bosons or Weak Bosons (Z_0, W+, W-). This production mechanism proceeds through the virtual two photon representation of the heavy ions, and is considered a reason for accelerating heavy ions in both the LHC and the SSC.

3. To study the interference mechanisms between the two processes for hadron production in peripheral heavy ion collisions. These two processes are two photon exchange and two pomeron exchange.

$e^+ e^-$ Pair Production in Heavy Ion Collisions

In addition to the fundamental questions regarding $e^+ e^-$ pair creation and non-perturbative QED, it is important for both detector design and collider performance to understand the pair creation mechanism during the heavy ion beam crossing. The created pairs could be a source of background in the detectors at heavy ion colliders. Of equal importance to the detector background is the so-called capture problem, associated with a heavy ion capturing one of the produced electrons in an atomic orbital. Changing the charge state of the heavy ion by one unit will eventually cause the ion to be lost from the beam. This mechanism is a major component in determining the luminosity lifetime in a heavy ion collider.

As discussed by Brodsky and Strayer, the $e^+ e^-$ pair production cross sections in the lowest order perturbative limit was shown to be 33.7K barns for Au beams at top energy in RHIC. This number came from both exact Monte Carlo evaluation of the perturbative matrix element, and by applying the analytic formula due to Racah (Nuovo Cimento 14 (1937) p. 70). Since the workshop, it has been shown (C. Bottcher, private communication) that the Racah formula and Monte Carlo calculations for $e^+ e^-$ pair production agree extremely well for Lorentz gamma values spanning nearly 4-orders of magnitude (beginning at γ=2).

Strayer showed that with modern computers, the multidimensional integrals associated with pair creation can be efficiently evaluated using a million or so Monte Carlo points. Detailed Monte Carlo calculations with nuclear form factors showed that the form factors act to suppress the p_T momentum dependance of the differential cross sections $d\sigma/dp_T$. The total cross section for electron pair creation is not affected by the nuclear form factor. Calculations for the single e^+ or e^- distribution $d\sigma/dp_T$ showed that most of these particles are produced in the forward (or backward) direction along the beam axis, with the perpendicular component of the singles momentum falling several orders of magnitude as p_T grows from 0 to 100Mev. Thus, most of the singles or pairs may be considered soft in nature. Further calculations of the energy distributions and the angular distributions of the singles and pairs are expected to be performed at the forthcoming RHIC workshop (July 1990). Strayer also showed his calculations for mu-pair production, including the interesting result that for invariant mass values M<4Gev, the coherent differential cross section $d\sigma/dydM$ from beam crossing is larger than the Drell Yan background.

For a certain class of detectors at RHIC, the total integrated cross section may not be the most relevant quantity. If the detector gates on a limited range of impact parameters, the pair production probability for these impact parameters might be more relevant. Further studies are needed on this impact parameter question.

The fact that $e^+ e^-$ pair production at RHIC violates unitarity was clearly shown by Brodsky and Baur, the latter stating that the probability for producing a single pair at an impact parameter corresponding to the Compton wavelength is 2.5 for Au beams at top RHIC energies. This result clearly suggests that higher order QED effects, most notably multiple pair production, could be observed at RHIC. Estimates for the number of $e^+ e^-$ pairs per ion interaction varied greatly. Brodsky, using an argument based on counting the number of equivalent photon-photon interactions per unit of rapidity interval for each heavy ion, gave an upper limit of 2000 pairs per interaction at RHIC. Baur, using an analogy of exciting harmonic oscillators by a time dependent force, schematically sums multiple pair diagrams to arrive at a Poisson distribution for the probability of producing pairs of multiplicity N at a given impact parameter. This technique suggests only a few pairs are produced, but after intense discussion, it was suggested (Muller) that the accuracy of neglecting the time ordering in the perturbation series needs to be explored in more detail.

Two theories for understanding non-perturbative QED phenomena were presented. Bottcher described a promising technique for non-perturbative electron pair production, based on the light cone representation of the Dirac equation. This high energy representation takes advantage of the weak coupling between the longitudinal and transverse momenta of the produced pair. For Au beams at RHIC, calculations based on this method indicate a factor of two reduction in the total electron pair cross section from the perturbative result. In contrast, Schied described a coupled-channels calculation for non-perturbative $e^+ e^-$ pair creation. This calculational technique is difficult to implement at RHIC energies, but at lower energies ($\gamma=10$), two orders of magnitude enhancement over perturbative estimates were claimed for the pair production cross section. Enhancements of this order were also shown for the electron capture cross section.

The striking mismatch between theories for the effects of higher order QED was not resolved at the workshop. The light cone approach is probably not applicable at lower energies (γ=10), and the coupled channels technique is technically difficult to implement at higher energies. However, it is important for both RHIC luminosity lifetime questions, and heavy ion collider detector design, to address the questions on "just how important are higher order QED effects?". As discussed by McLerran, this will require evaluation of higher order diagrams, including box diagrams, as well as detailed comparison with experimental data.

Datz and Belkacem described their experimental procedures for measuring pair production and electron capture at CERN (S beams) and Berkeley (U beams) respectively. Although these fixed target experiments are at energies considerably below what will be achieved at RHIC, they will be an essential component in helping to understand some of the above theoretical discrepancies. Within two years, Au beams will become available at the AGS (γ=10), and thus could provide another set of experimental data.

After the workshop, it was suggested (Fatyga) that the impact parameter dependence of the non-perturbative $e^+ e^-$ production could be studied through a coincidence experiment of $\mu^+ \mu^-$ pairs with $e^+ e^-$ pairs. It is expected that perturbation theory will be accurate for the production of the $\mu^+ \mu^-$ pairs. This concept will be further developed in the July 1990 RHIC workshop.

Producing Higgs Bosons and Weak Bosons in Heavy Ion Colliders

Mueller, Couture, Brodsky, Strayer and Smith discussed in detail the γ-γ physics that would be accessible at the proposed heavy ion colliders (RHIC, plus heavy ions in LHC and SSC). Of particular interest was the probability of producing a Higgs particle or W+ W- pairs via the virtual two photon spectrum of relativistic heavy ions. This mechanism is out of the question at RHIC, because the nuclear size imposes a cut off in the energy of the virtual photon spectra at 3Gev. At LHC or SSC this cutoff extends to 100Gev or 250Gev respectively.

Much of the focus of discussion at the workshop was the then recent preprint of Cahn and Jackson (LBL 28592, 1990). In contrast to the earlier work of Drees, et al (Phys. Lett B223 [1989] 454), Cahn and Jackson used a cutoff in impact parameter space to exclude the finite size of the nucleus. Drees, et al, included this effect through an elastic form factor. Cahn and Jackson showed that for Pb-Pb collisions, production of a Higgs Boson with an intermediate mass of 100Gev is reduced by a factor of 0.14 from Drees' estimate for SSC energies, and reduced a factor of 0.037 at LHC energies. These are discouraging rates, and the workshop left on a pessimistic note that the production rate for Higgs Bosons, using heavy ions, is too low to be experimentally useful.

Since the meeting, preliminary calculations on the Higgs, as shown by Strayer, have been completed. This preprint (Wu, et al, ORNL preprint ORNL/CCIP/90/02) suggests that the results of Cahn and Jackson based on the Weissacker-Williams method were too pessimistic. The Weissacker-Williams method assumes the transverse momentum of the produced pair is zero, an assumption that is not realistic in this case. Using a Monte Carlo evaluation of the impact parameter dependence of the Higgs production matrix elements, Wu, et al, show that the original calculations of Drees, et al, are reduced by only a factor of 1.9 at LHC energies and a factor of 1.4 at SSC energies, if the constraint that the heavy ions remain intact is imposed. At this time the prospects for producing intermediate mass Higgs Bosons from coherent two photon production appears more promising than at the time of the workshop. At the SSC, Wu, et al, now calculate up to 500 Higgs a year, if a luminosity of 10^{28} cm^{-2} sec^{-1} can be achieved. We note that this value of the luminosity is nearly two orders of magnitude beyond initial RHIC operation values.

Hadron Production from Peripheral Heavy Ion Collisions

Mueller and Brodsky discussed the hadron resonances available through so-called γ-γ physics. Of particular interest at RHIC energies are the η_0, f(1270) and $\eta_c(2980)$ resonances. As indicated by Brodsky, heavy ion colliders make good γ-γ factories, particularly in the low mass region. It was also pointed out by Brodsky that a heavy ion collider can be very competitive with $e^+ e^-$ machines, if the luminosity can be increased to 10^{28}.

As part of this discussion, Mueller and Brodsky raised the question of a background to photon photon reactions from Double Pomeron Interactions. For intermediate nuclear A and Z values, the γ-γ and p p amplitudes are comparable, resulting in an interference that needs to be taken into account. For very heavy nuclei, the γ-γ interactions will dominate. Thus, an experiment designed to measure hadron resonance production as a function of the mass and charge of the colliding nuclei could, in principle, study this interference effect. This ability to conduct such a set of comprehensive measurements using the same apparatus and minimal change to the experimental acceptance is a unique capability of RHIC.

Conclusion

The Relativistic Heavy Ion Collider at Brookhaven will be a machine offering some special capabilities in the areas which were discussed during this workshop. One should be able to do comparison measurements of various electromagnetic phenomena with a variety of beams, thus varying the strength of electromagnetic fields in a well-controlled manner. These measurements can be done with a single apparatus and at a fixed value of the Lorentz γ, eliminating many systematic errors due to acceptance corrections or different beam energies. As an example, gold beams can be used to study non-perturbative phenomena in $e^+ e^-$ pair production, with lighter beams providing a reference measurement in which no strong non-perturbative corrections are expected. A search for possible anomalies in the production of hadrons ($q \bar{q}$ pairs) by strong electromagnetic fields can be conducted in a similar manner.

A study of electromagnetic phenomena in extremely peripheral collisions of relativistic heavy ions can become a rich and exciting field that will complement studies of central collisions.

THEORETICAL OVERVIEW

Talk presented by:

S. Brodsky

QED + Relativistic Heavy Ion Collisions

Theoretical Overview

Stan Brodsky

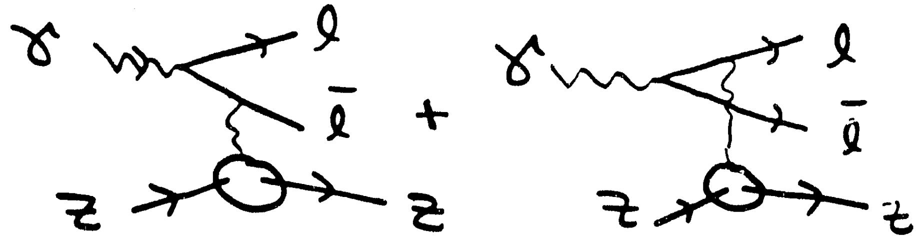
4-20-90

Conventional

Unconventional

* Lepton Pair Production
* Higgs Production
* Double Pomeron + $\gamma\gamma$ Interference
* Nuclear Effects : q, g propagation

Bethe - Heitler Lepton Pair Production: $\gamma Z \to \ell \bar{\ell} Z$



pt-like nucleus

$$\sigma = \frac{28}{9} \frac{Z^2 \alpha^3}{m_\ell^2} \left[\log \frac{2k}{m_\ell} - \frac{109}{42} \right]$$

$$\frac{d\sigma}{dx_1} = \frac{2 Z^2 \alpha^3}{m_\ell^2} \left[x_1^2 + x_2^2 + {}^2/_3 x_1 x_2 \right] \cdot \left[2 \log \frac{k}{m} x_1 x_2 - 1 \right]$$

$$x_1 = \frac{E_1}{k}, \quad x_2 = \frac{E_2}{k} \qquad x_1 + x_2 = 1$$

$$k = E^\gamma_{lab}$$

Factorization:

$$\sigma_{\gamma Z \to \ell \bar{\ell} Z} = \int_0^1 dz \; G_{\gamma/Z}(z) \; \sigma_{\gamma\gamma \to \ell\bar{\ell}}(zs)$$

$\sigma^{TOT}_{\gamma Pb \to e^+e^- Pb}$

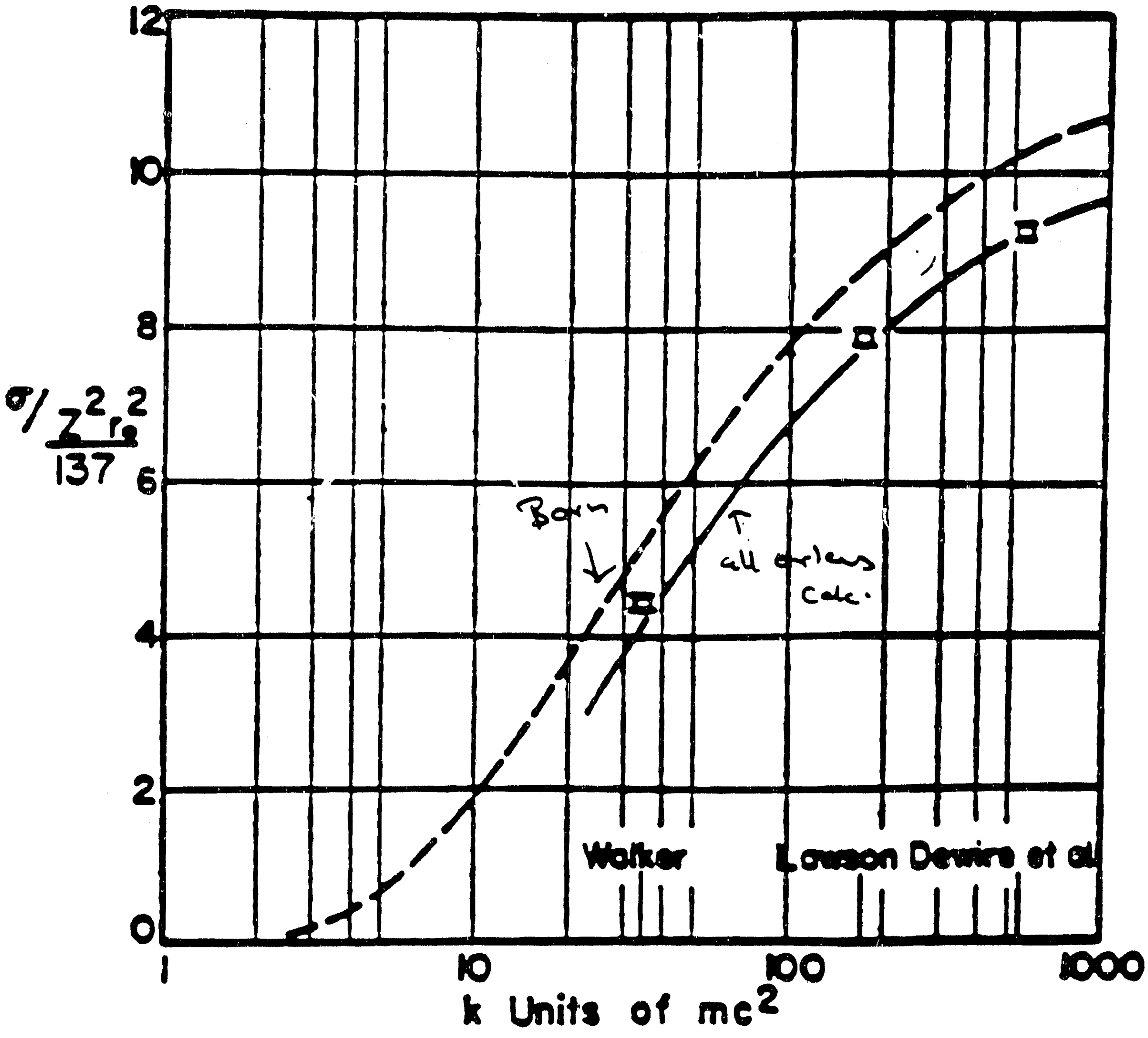
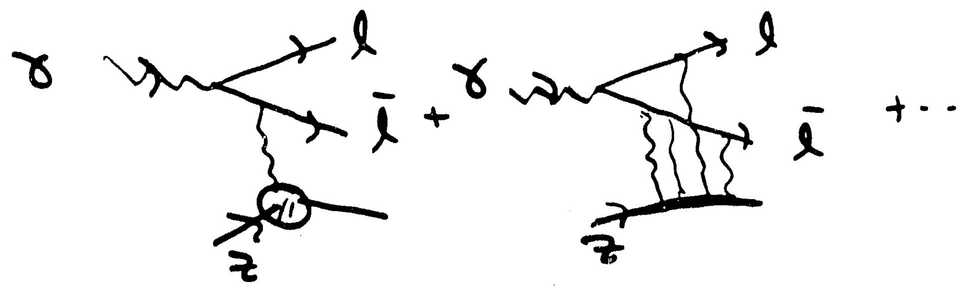


FIG. 2. Total cross section for pair production in Pb. Dotted curve, Born approximation. Solid curve, theory of this paper. Measured points are indicated.

Theory: Davies
Bethe
Maximon

Higher Order Coulomb Corrections to Pair Production



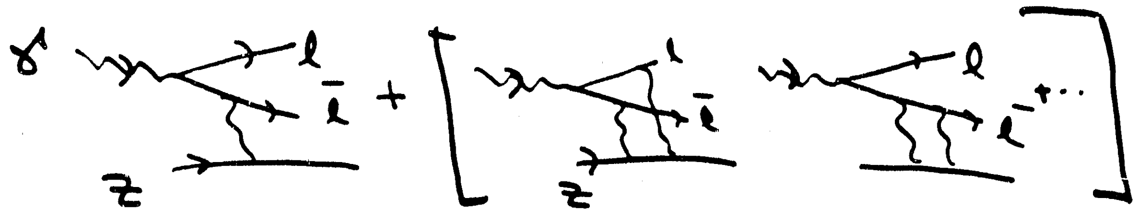
Bethe - Maximon - Davies $\qquad k \gg m_l$

$$\sigma = \frac{28}{9} \frac{Z^2\alpha^3}{m^2} \left[\log \frac{2k}{m} - \frac{109}{42} - f(Z\alpha) \right]$$

$$f(Z\alpha) = (Z\alpha)^2 \sum_{n=1}^{\infty} \frac{1}{n(n^2+(Z\alpha)^2)}$$

* Distribution in x_1, x_2 unchanged : Eikonal valid
* No odd terms in $Z\alpha$ $\qquad \left(\theta_l \sim \frac{m_l}{E_l} \right)$
* $\log \frac{k}{m}$ term from $\int \frac{db}{b}$ only appears in Born
* $f(Z\alpha)$ from $b \sim O(\frac{1}{m_e})$, $q \sim O(m_e)$
* Assumes $F_Z(Q^2) = 1$ $\qquad \left[m_l R_A \ll 1 \right]$

Second Born Corrections to Pair Production



interference odd in Z
$\ell \to \bar{\ell}$

$$d\sigma = d\sigma_0 \left[1 \pm \pi \frac{Z\alpha \sin \frac{\theta_\ell}{2}}{1+\sin\frac{\theta_\ell}{2}} \right]$$

Valid for $F_Z(Q^2) \cong 1$ $\quad \begin{cases} Q_\perp \cong E_\ell \theta_\ell \ll \frac{1}{R_N} \\ Q_\perp^2 > M_\ell^2 \end{cases}$

Not important for σ_{TOT} $\quad \left(\theta_\ell \lesssim \frac{m_\ell}{E_\ell} \right)$

Asymmetry in $p_\perp^{e^-} \leftrightarrow p_\perp^{e^+}$

$$\epsilon(\delta) = \frac{N_+(\delta) - N_-(\delta)}{N_+(\delta) + N_-(\delta)}$$

$$\delta = E_{e^-} - E_{e^+} \quad , \quad \theta_+ = \theta_-$$

calculations in SJB + Gillespie, Phys. Rev.
see also Dalitz + Jennie

$\epsilon(\delta) = \dfrac{N_+ - N_-}{N_+ + N_-}$ $\gamma Z \to e^+e^- Z$ second Born Interference

$N_\pm$: positron less p / electron less p Ting, et al

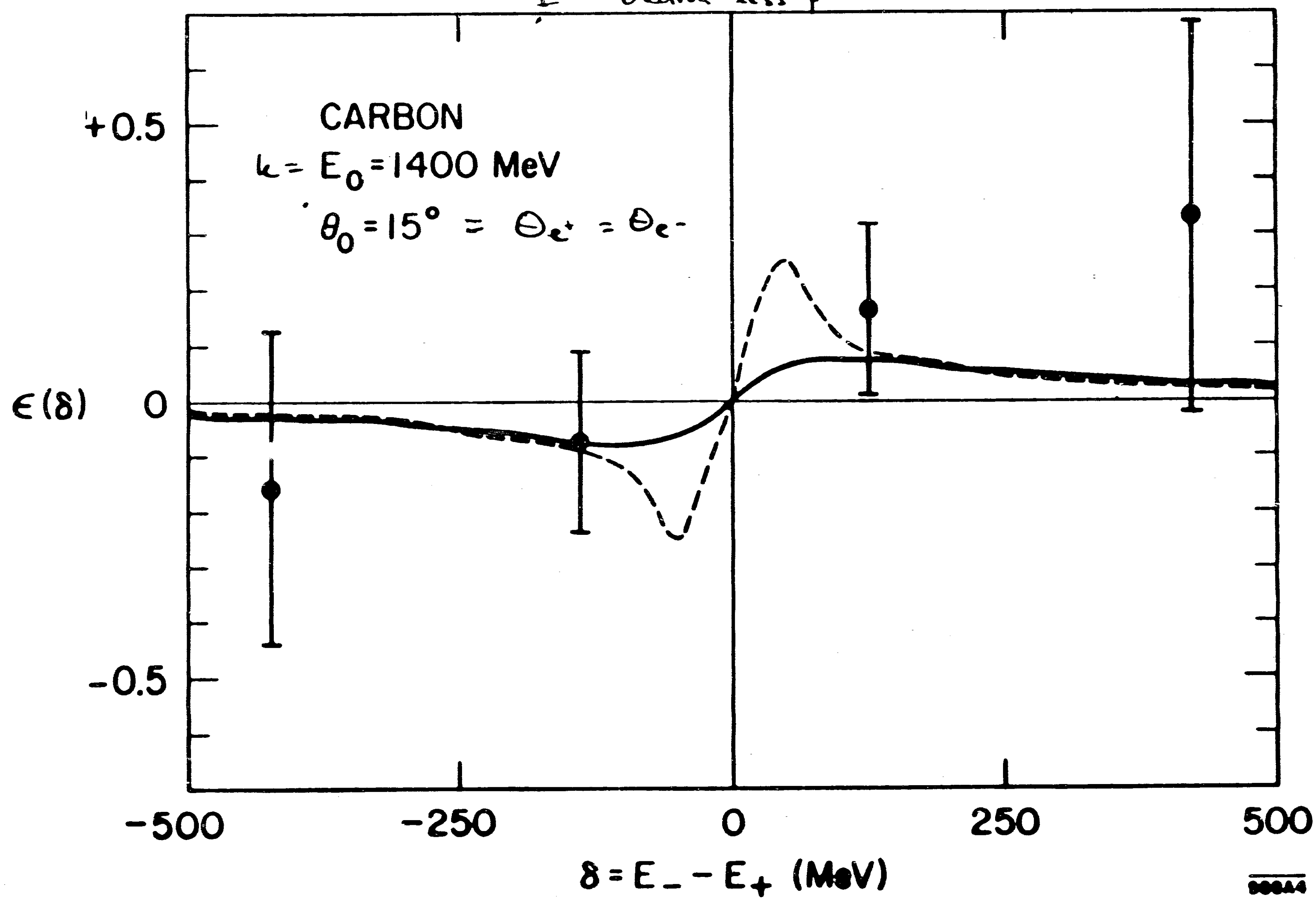
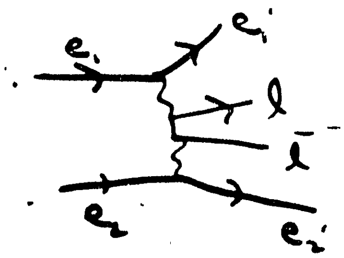


Fig. 4

Calculation by DJB + J. Gillespie

Lepton Pair Production



$$\sigma_{ee \to l^+ l^- e e'} = \frac{28}{27} \frac{\alpha^4}{\pi} \frac{1}{m_l^2} \left[l^3 - A l^2 + B l + C \right]$$

$$l = \log \frac{2 p_1 \cdot p_2}{m_1 m_2} \quad , \quad l_\mu = \log \frac{m_\mu^2}{m_e^2}$$

$$A = \frac{178}{28} \; ; \quad B = -\frac{1}{28} \left[42 l_\mu^2 + 196.4 l_\mu + \frac{7}{75} + 21\pi^2 + 152 \right]$$

$$C = \frac{1}{28} \Big[14 l_\mu^3 + 184.8 l_\mu^2 + \left(1109 + \frac{31}{150} - 7\pi^2\right) l_\mu + \left(36 + \frac{7}{15}\right)\pi^2 + \frac{51403}{18} - 189\,\zeta(3) - \frac{12748}{125} \Big]$$

* Assumes $s \gg m_\mu^2$

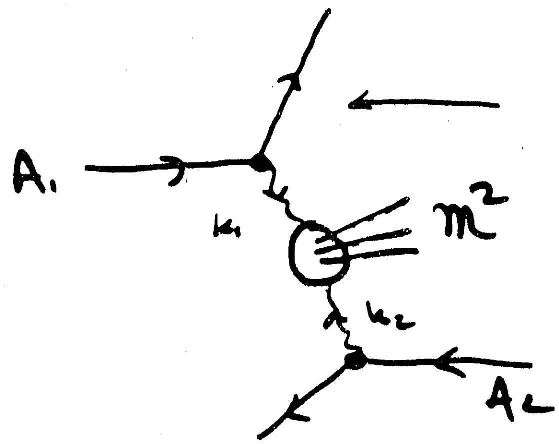
* Calculation by
  - Baier + Fadin
  - Kuraev + Lipatov
  - Bonneau + Martin

$(386 \text{ fm})^2$

Double Equivalent Photon Approximation

F. Low

SJB, Kinoshita, Terazawa



$$Z^2 \frac{\alpha}{\pi} \int d^2k_\perp \frac{F_A^2(k_\perp^2)(\vec{\epsilon}_\perp \cdot \vec{k}_\perp)^2}{\left[k_\perp^2 + x^2 M^2\right]^2}$$

$$x_1 = \frac{k_1^+}{P_1^+} = \frac{k_1^0 + k_1^z}{P_1^0 + P_1^z}$$

$$x_2 = \frac{k_2^-}{P_2^-} = \frac{k_2^0 - k_2^z}{P_2^0 - P_2^z}$$

$$\sigma = \int_0^1 dx_1 \int_0^1 dx_2 \; G_{\gamma/A}(x_1)\, G_{\gamma/A}(x_2)\, \sigma_{\gamma\gamma}(x_1 x_2 s)$$

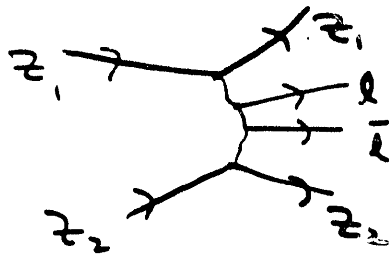
assume $k_1^2, k_2^- \ll$ ...

$$\tau = \frac{m^2}{s} = x_1 x_2$$

$$y = \frac{1}{2} \ln \frac{x_1}{x_2}$$

$$\frac{d\sigma}{dy\, d\tau} = x_1 G_{\gamma/A_1}(x_1)\, x_2 G_{\gamma/A_2}(x_2)\, \sigma_{\gamma\gamma}(x_1 x_2 s)$$

Lepton Pair Production in Heavy Ion Collisions



$\left\{ \begin{array}{c} \text{G. Racah} \\ \text{Nuovo Cimento } \underline{14}, 70 \\ 1937 \end{array} \right.$

$$\sigma_{Z_1 Z_2 \to l \bar{l} Z_1 Z_2}$$

$$= \frac{28}{27 \pi m_l{}^2} (Z_1 \alpha)^2 (Z_2 \alpha)^2 \left[l^3 - A l^2 + B l + C \right]$$

$$A = \frac{178}{28} \simeq 6.36$$

$$B = \frac{1}{28} (7\pi^2 + 370) \simeq 15.7$$

$$C = -\frac{1}{28} \left[348 + \frac{13}{2}\pi^2 - 21\,\zeta(3) \right] \cong -13.8$$

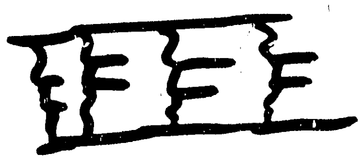
$$l = \log \frac{2 p_1 \cdot p_2}{M_1 M_2}$$

* Assumes Z_1, Z_2 point-like (okay for $l = e$)

Neglects F.S.I. & Nuclei

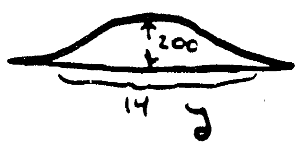
Remarks on Kbn cross section

1. May be limiting rate for $A_1 A_2$ colliding beam luminosity

2. Larger than geometrical since 

   range is controlled by γ exchange

3. Many pairs formed in same collision



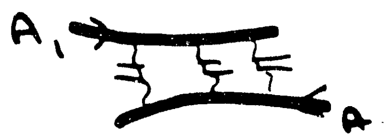
$\times\, G_{\gamma/A}(k) = \frac{dN_\gamma}{dy}$ # γ's / unit rapidity

if all γ's convert to e^+e^- upper bound $\Rightarrow$ 3000/event - probably less!

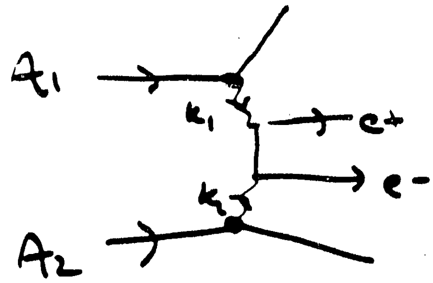
$\frac{Z^2\alpha}{\pi} \log \frac{2p_1 \cdot p_2}{M^2} = 200$

$$\sigma^{(2)} = \frac{[\sigma^{(1)}]^2}{\sigma_{TOT}} \sim \sigma^{(1)} .$$

4. $\sigma_{TOT} \neq \sum_{n=1}^{n_{max}} \sigma^{(n)}$: not correct AGL cutting rule



Electron Pair Production in Heavy Ion Collisions



$$\sigma_{\gamma\gamma \to e^+e^-} \sim \frac{\pi\alpha}{m_e^2}$$

near threshold

$$\left.\begin{array}{l} -k_1^2 = x_1^2 M_1^2 + \vec{k}_{\perp 1}^2 \\ -k_2^2 = x_2 M_2^2 + \vec{k}_{\perp 2}^2 \end{array}\right\} < m_e^2$$

$$G_{\gamma/Z}(x) = \frac{1}{x}\,\frac{Z^2\alpha}{\pi} \int_0^{k_\perp^2 < m_e^2} \frac{dk_\perp^2\, k_\perp^2}{(k_\perp^2 + x^2 M^2)^2}\, F_A^2(k_\perp^2 + x^2 M^2)$$

For $x^2 M^2 << \frac{1}{R_A^2}$ indep of nuclear size, spin

$\therefore$ Racah (pt. like formula valid) $\quad x_1 x_2 \cong \frac{4m_e^2}{s}$

$$\sigma_{Z_1Z_2 \to Z_1Z_2 e^+e^-} = \frac{28\,(Z_1\alpha)^2 (Z_2\alpha)^2}{27\,\pi\, m_e^2}\left[\ell^3 \ldots\right]$$

54.4 bn $\quad$ 628

$\gamma = E/M = 100$, $\ell = \log(4\gamma^2) \sim 11$

* $\sigma_{AuAu \to AuAu\, e^+e^-} \sim 3.4 \times 10^4$ bn !
* $\sigma_{geom} = 2\pi (R_1 + R_2)^2 \sim 12$ bn

$\boxed{\text{Limitation on Luminosity of Heavy Ion Colliders}}$

$$\sigma_{A_1 A_2 \to e^+e^- A_2 A_2} \sim 10^4 \text{ bn}$$

Probably much less

$\Rightarrow$ Every event $\sim 10^3$ e^+e^- pairs ? Few?

$k_\perp \sim \frac{1}{2}$ MeV , $|k_z| < 50$ MeV

$y \sim \frac{1}{2} \log \frac{x_1}{x_2}$ central rapidity

$\Rightarrow$ # interactions $= \mathcal{L} * \sigma = \frac{10^{28}}{\text{cm}^2 \text{sec}} \times 10^{-20} \text{cm}^2$

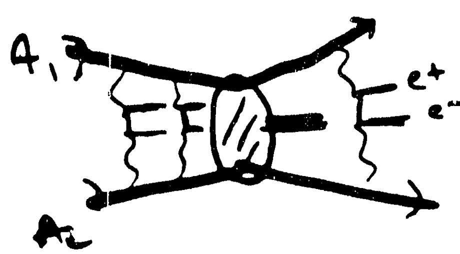
$= 10^9$ interactions/sec

$\boxed{\text{requires fill of } 10^8/\text{sec to maintain } \frac{10^{28}}{\text{cm}^2 \text{sec}} \mathcal{L}}$

Ion energy loss from e^+e^- pairs ~ 10 GeV

compared to $\gamma A \sim 2 \cdot 10^4$ GeV

not serious



refocus?

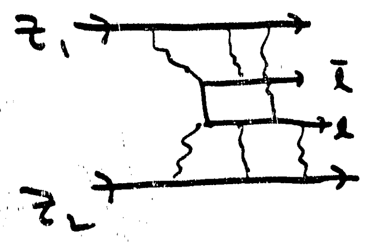
transverse deflection

atomic capture

– coalescence

most critical

$\boxed{\text{Higher Born corrections to } Z_1 Z_2 \to \ell\bar{\ell}\, Z_1 Z_2}$



<u>Guess</u> from Bethe-Maximon analysis

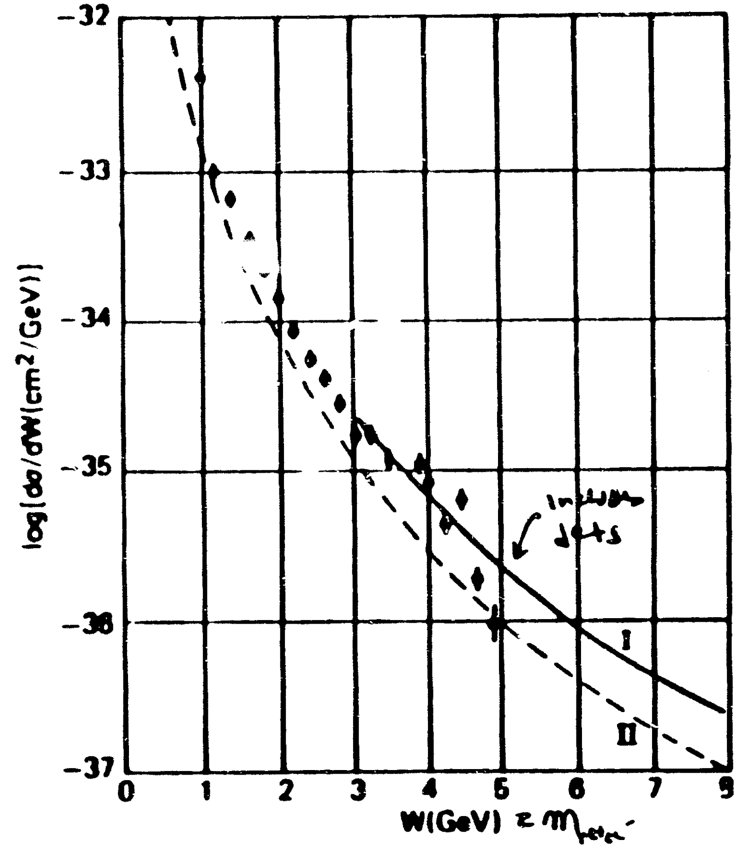
* non-log term $\quad b_1 \sim O(\frac{1}{m_\ell})\ ,\ b_2 \sim O(\frac{1}{m_\ell})$

$$F(Z_{TOT}\alpha) = (Z_{TOT}\alpha)^2 \sum_{n=1}^{\infty} \frac{1}{n(n^2 + Z_{TOT}^2\alpha^2)}$$

$$\left\{ \begin{array}{l} \text{e.g.} \quad Z_1 = Z_2 = 79\ ,\ Z_{TOT} = 158\ ,\ Z_{TOT}\alpha = 1.19 \\ \qquad F(1.15) \cong 0.57 \end{array} \right.$$

Valid for $\quad Z_1 Z_2 \to e^+ e^- Z_1 Z_2$

Not relevant for $\ell^+\ell^- = \mu^+\mu^-$, $\quad F_A(m_\mu^2 R_A^2)$ small.



$\frac{d\sigma}{dW}(pp \to pp\,\mu^+\mu^-)$

$s = 2500$ GeV^2

bkgd to Drell-Yan

expt: $s = 50$ GeV^2 BNL

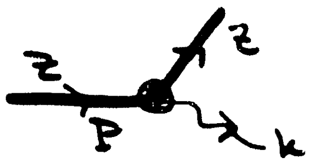
J.H. Christenson, et al (1970)

$pU \to \mu^+\mu^- X$

Fig. 8. The cross-section $d\sigma/dW$ for two-photon pair production in pp-collisions at $s = 2500$ GeV^2. Curve I is the cross section taking into account jets (fig. 1 and fig. 7) and curve II is the contribution of the fig. 1 diagram. The experimental points [30] at $s = 50$ GeV^2 are given for comparison.

calc: Budnev, et al.

$\boxed{\text{Derivation of Photon Distribution from Coherent Source}}$



$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

$$y = \log x \quad \text{(rapidity)}$$

$$\text{M.E. } |\epsilon \cdot J|^2$$

$$G_\gamma(x) = \frac{dN}{dx}(\gamma/Z) = \frac{Z^2 \alpha}{\pi} \frac{1}{x} \int dk_\perp^2 \frac{k_\perp^2 \, F^2(k_\perp^2 + x^2 M^2)}{[k_\perp^2 + x^2 M^2]^2}$$

$$|\text{denom}|^2$$

Here $k^2 = k^+ k^- - k_\perp^2 \equiv -Q^2$

$$= -\frac{x^2 M^2}{1-x} - k_\perp^2 \cong -(x^2 M^2 + k_\perp^2)$$

$$\therefore \quad G_\gamma(x) = \frac{Z^2 \alpha}{\pi} \frac{1}{x} \int_{x^2 M^2} \frac{dQ^2}{Q^2} F^2(Q^2) \left[1 - \frac{x^2 M^2}{Q^2}\right]$$

Grabiak, et al

Note: $F^2(Q^2)$ vanishes rapidly for $Q^2 > \frac{1}{R_A^2}$

$\therefore$ $G_{\gamma/x}(x)$ negligible for

$$xM > \frac{1}{R_A}$$

Fraction of nucleon momentum

$x_N = \frac{k^+}{P_N^+}$:

$$x_N > \frac{1}{M_N R_A} \sim 0.03 \text{ for Au}$$

$$G^{\text{coherent}}_{\gamma/Z_1}(x) = \frac{Z_1^2\alpha}{\pi}\frac{1}{x}\int_{x^2M^2}^{\infty}\frac{dQ^2}{Q^2}F_Z^2(Q^2)\left(1-\frac{x^2M^2}{Q^2}\right)$$

Assume $\quad F_Z(Q^2) = \exp\left(-Q^2/Q_0^2\right) \qquad Q_0 \sim \frac{1}{R_A}$

small x

$$\boxed{G^{\text{coh}}_{\gamma/Z}(x) = \frac{Z^2\alpha}{\pi}\frac{1}{x}\left[\ln\frac{1}{(xMR_A)^2} - 0.18\right]}$$

$R_A = 1.2\, A^{1/3}$ fm.

* This result is not valid for A_1A_2 collisions if we wish the nuclei to stay intact

Must exclude $b > 2R$ to avoid disruption

Cahn + Jackson

$$G^{\text{coherent - intact}}_{\gamma/Z}(x) = \frac{2Z^2\alpha}{\pi x}\left\{\frac{x}{x_0}K_0\left(\frac{x}{x_0}\right)K_1\left(\frac{x}{x_0}\right) - \frac{1}{2}\left(\frac{x}{x_0}\right)^2\left[K_1^2\left(\frac{x}{x_0}\right) - K_0^2\left(\frac{x}{x_0}\right)\right]\right\}$$

small x

$$\boxed{G^{\text{intact}}_{\gamma/Z} \simeq \frac{Z^2\alpha}{\pi x}\left[\ln\frac{1}{(xMR)^2} - 2.15\right]}$$

$$G^{\text{coherent}}_{\gamma/Z_1}(x) = \frac{Z_1^2\alpha}{\pi}\frac{1}{x}\int_{x^2M^2}^{\infty}\frac{dQ^2}{Q^2}F_Z^2(Q^2)\left(1-\frac{x^2M^2}{Q^2}\right)$$

Assume $\quad F_Z(Q^2) = \exp\left(-Q^2/Q_0^2\right) \quad Q_0 \sim \frac{1}{R}$

small x $\quad \boxed{G^{\text{coh}}_{\gamma/Z}(x) = \frac{Z^2\alpha}{\pi}\frac{1}{x}\left[\ln\frac{1}{(xMR_A)^2} - 0.18\right]}$

$R_A = 1.2\,A^{1/3}$ fm.

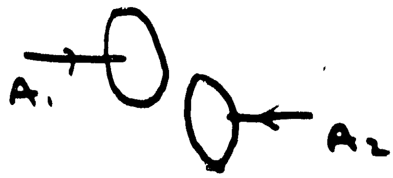
* This result is not valid for $A_1 A_2$ collisions if we wish the nuclei to <u>stay</u> <u>intact</u>

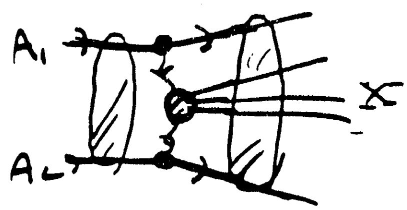
Must exclude $b > 2R$ to avoid disruption

$$G^{\text{coherent - intact}}_{\gamma/Z}(x) = \frac{2Z^2\alpha}{\pi x}\left\{\frac{x}{x_0}K_0\left(\frac{x}{x_0}\right)K_1\left(\frac{x}{x_0}\right) - \frac{1}{2}\left(\frac{x}{x_0}\right)^2\left[K_1^2\left(\frac{x}{x_0}\right) - K_0^2\left(\frac{x}{x_0}\right)\right]\right\}$$

Cahn + Jackson

small x $\quad \boxed{G^{\text{intact}}_{\gamma/Z} \simeq \frac{Z^2\alpha}{\pi x}\left[\ln\frac{1}{(xMR)^2} - 2.15\right]}$





Impact repn.

$$\sigma_{el} = 2\pi \int b\,db\, |1 - S(b)|^2$$

$$S(b) = e^{-\chi(b)}$$

$$M(b) = M_0(b)\, e^{-\chi(b)}$$

$$\simeq M_0(b) \begin{cases} 1 & b > R \\ 0 & b < R \end{cases}$$

Require $b \geq R_1 + R_2$ to avoid disruption

Cahn + Jackson

Calculation could be improved!

C + J limit each distribution separately.

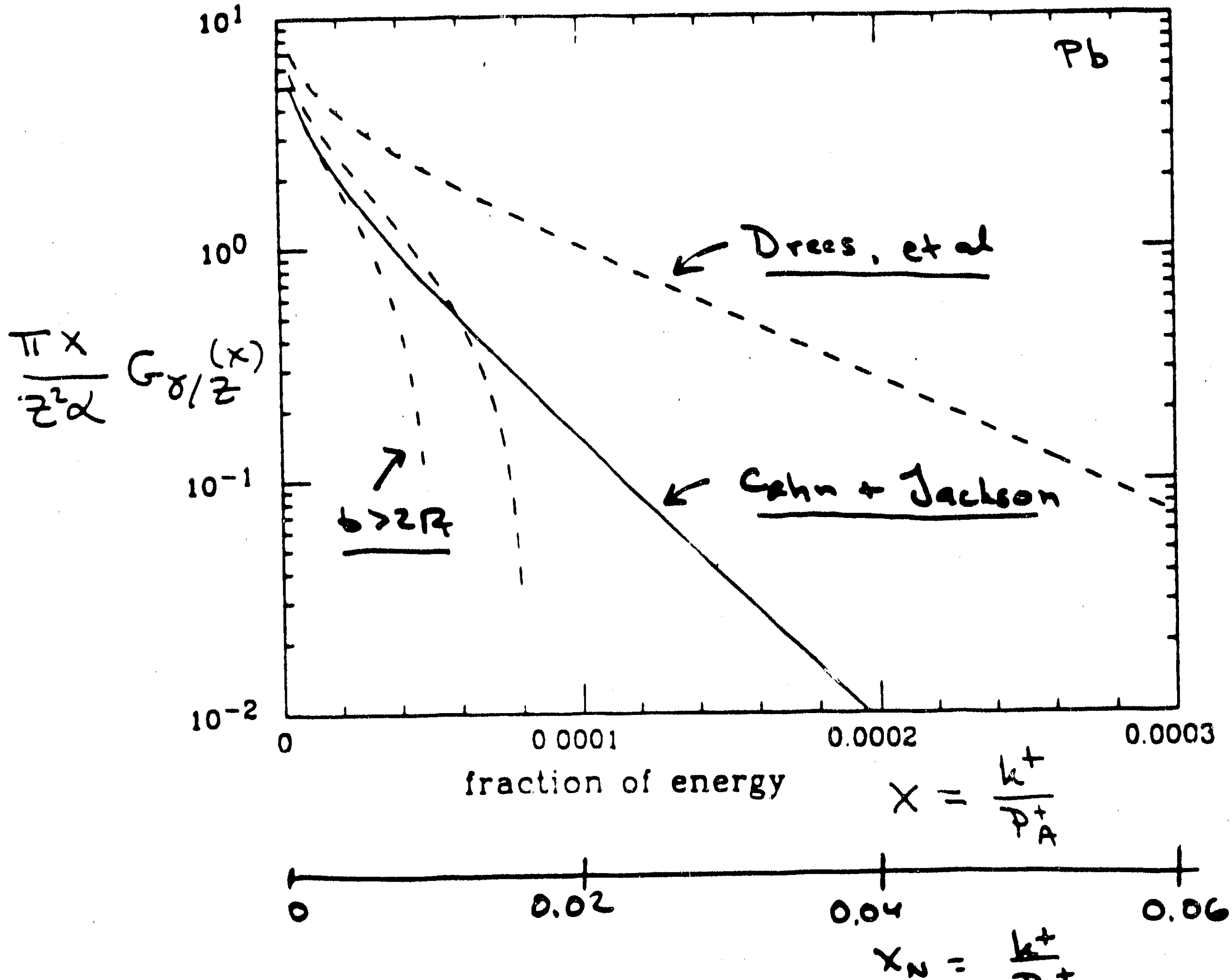


Figure 1: Various approximations to the equivalent photon flux from a beam of fully ionized Pb. The quantity $(\pi x/Z^2\alpha)f$ is shown as a function of x, the fraction of the ion's momentum given to the photon. The result of Drees et al. [3], Eq. (8), is shown as the upper dashed curve. The lower dashed curve is obtained from the DEZ model by setting the limit for the Q^2 integration to $1/b^2_{min} = (1/2R)^2$, as given in Eq. (18), with $\lambda = 1$. The result of Papageorgiu, [2], Eq. (10), is shown as a dotted curve. The full classical result, Eq. (12), is shown as a solid curve. The low-momentum approximation to the full classical result, Eq. (16), is shown as a dashdot curve.

Comparison of $\gamma\gamma$ Production

at e^+e^- and $Z_1 Z_2$ colliders

$$R = \frac{\frac{d\sigma}{dm^2 dy}(Z_1 Z_2 \to Z_1 Z_2 X)}{\frac{d\sigma}{dm^2 dy}(e^+e^- \to e^+e^- X)} \cong \frac{Z_1^2 Z_2^2 \left[\log \frac{1}{\tau M_N^2 R_A^2} - \gamma\right]^2}{\left[\log \frac{S_{e^+e^-}}{m_e^2} - 1\right]^2}$$

$$\tau = \frac{m^2}{S_{NN}} \qquad \gamma = \begin{cases} 2.15 & \text{intact} \\ 0.18 & \end{cases}$$

Typical: $S_{e^+e^-} \sim (30\text{ GeV})^2 \qquad \left[\log \frac{S_{e^+e^-}}{m_e^2} - 2.2\right]^2 = 350$

$Z = 79 \qquad \tau = \frac{10\text{ GeV}^2}{(900\text{ GeV})^2} \qquad \left[\log \frac{1}{\tau M_N^2 R_A^2} - \gamma\right]^2 \sim \begin{cases} 3.8 \\ 15 \end{cases}$

$$R = \begin{cases} 4 \times 10^5 & \text{intact} \\ 1.7 \times 10^6 & \text{total} \end{cases}$$

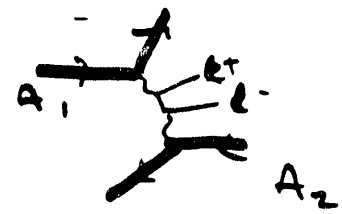
Double Tagged: excellent $\gamma\gamma$ Factory

study $\gamma\gamma \to \mu\bar{\mu}$, $B\bar{B}$, $C=+$

$m_{\gamma\gamma}$ limited by $m_{\gamma\gamma} < \frac{\sqrt{s}}{R_A M_N} \sim \frac{E_N}{35}$

Massive pair production

$$\boxed{\text{Estimate of } \ell^+\ell^- \text{ pair production in Heavy Ion Collisions}}$$



$$A_1 A_2 \to \begin{cases} \ell^+\ell^- A_1 A_2 \\ \ell^+\ell^- X \end{cases}$$

nuclei stay intact

allow breakup

Assume $m_{\ell^+\ell^-} R_A \gg 1$

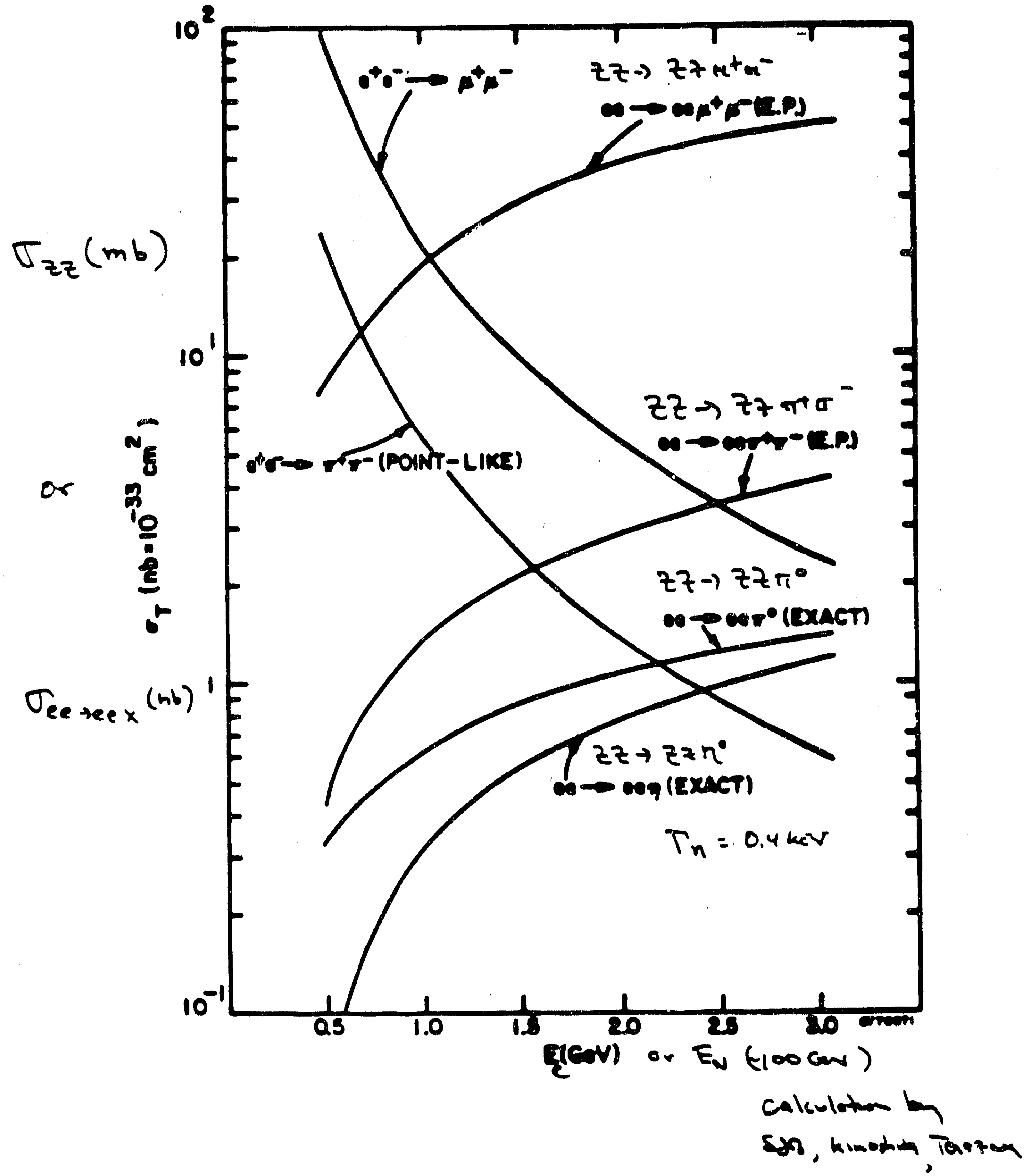
$$\sigma = \int_0^1 dx_1 \int_0^1 dx_2 \, G_{\gamma/A_1}(x_1) \, G_{\gamma/A_2}(x_2) \, \sigma(x_1 x_2 S)$$

$$\sigma(m^2 > m^2_{\ell^+\ell^-}) = \frac{28}{27} \frac{Z_1^2 Z_2^2 \alpha^4}{\pi m^2_{\ell^+\ell^-}/4} \left[\log \frac{1}{(\bar{x}_1 M_1 R_1)^2} - \gamma_1\right] \left[\log \frac{1}{(\bar{x}_2 M_2 R_2)^2} - \gamma_2\right] \log \frac{\hat{s}_{12}}{m^2_{\ell^+\ell^-}}$$

(rapidity integ.)

$$\gamma_i = \begin{cases} -2.15 & \text{intact} \\ -0.18 & \text{breakup from FSI + ISI} \end{cases}$$

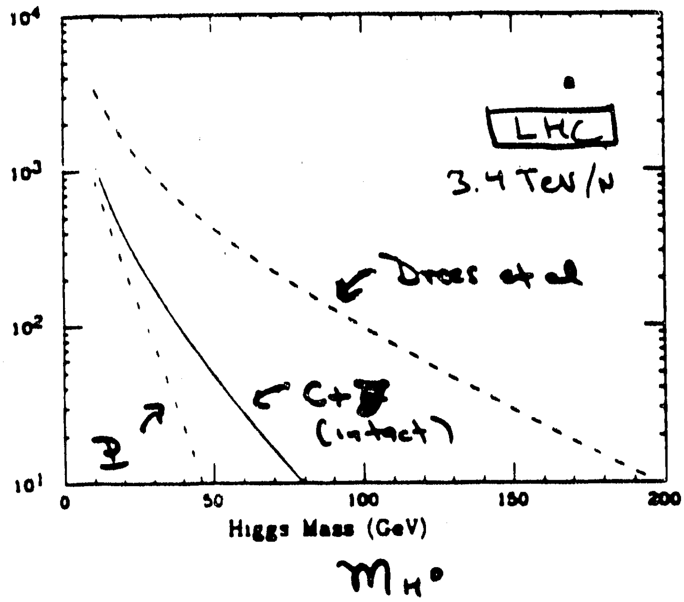
$$\hat{s}_{12} = x_1 x_2 S$$



10^2
10^1
1
10^{-1}
σ_{zz} (mb)
or
$\sigma_{ee \to ee x}$ (nb)
σ_T (nb = 10^{-33} cm^2)
$e^+e^- \to \mu^+\mu^-$
$zz \to zz\,\mu^+\mu^-$
$ee \to ee\mu^+\mu^-$ (E.P.)
$e^+e^- \to \pi^+\pi^-$ (POINT-LIKE)
$zz \to zz\,\pi^+\pi^-$
$ee \to ee\pi^+\pi^-$ (E.P.)
$zz \to zz\pi^0$
$ee \to ee\pi^0$ (EXACT)
$zz \to zz\eta^0$
$ee \to ee\eta$ (EXACT)
T_n = 0.4 keV
0.5
1.0
1.5
2.0
2.5
3.0
E(GeV) or E_N (×100 GeV)
calculation by

Higgs Production in Pb Pb Collisions

σ(pb)

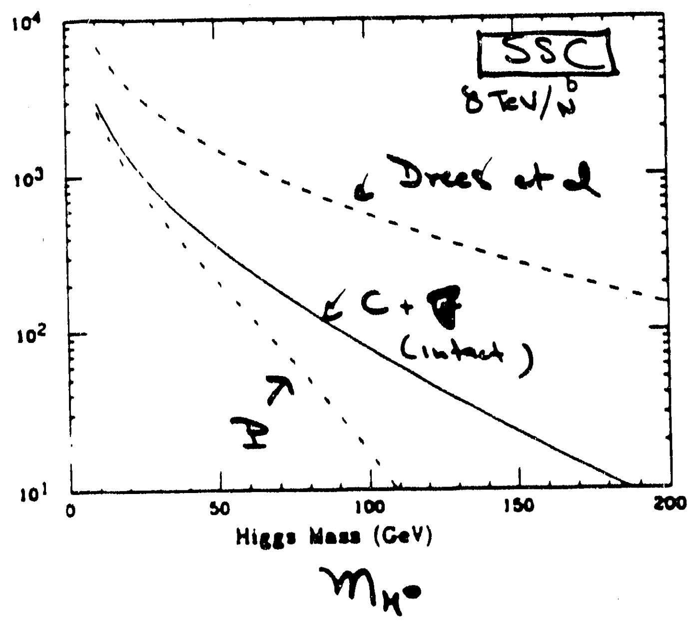


$\sigma(A+A \to H_0\, AA)$
intact

or

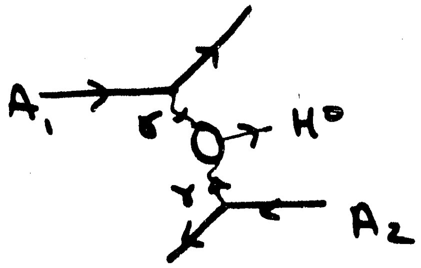
$\sigma(A\, A \to H_0\, X)$

σ(pb)



Calculation by
Cahn + Jackson
Drees et al
Papageorgiu

Figure 2: Various approximations to the cross section for Higgs boson production at (a) the LHC using Pb - Pb collisions with 3.4 TeV per nucleon and (b) at the SSC with 8 TeV per nucleon. The result of Drees et al. [3], Eq. (8), is shown as the dashed curve. The result of Papageorgiu, [2], Eq. (10), is shown as a dotted curve. The full classical result, Eq. (12), is shown as a solid curve. The low-momentum approximation to the full classical result, Eq. (16), is shown as a dashdot curve.



Grabiack
Drees et al
Cahn + Dawson
Papageorgiou
Phys. rev. $\underline{D40}$, 92 (1990)

$$\sigma_{\gamma\gamma\to H^0} = \frac{8\pi^2}{m_H^3}\,\Gamma(H\to\gamma\gamma)\,\delta(s_{\gamma\gamma}-m_{H^0}^2)\,m_H^2$$

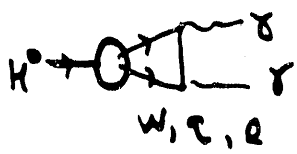
Low
SOS, Kinoshita,
Tanaka

$$\sigma_{A_1A_2\to H_0 A_1A_2} = \int_\tau^1 \frac{dx}{x}\, G_{\gamma/A_1}(x)\, G_{\gamma/A_2}\!\left(\frac{\tau}{x}\right) \frac{8\pi^2\Gamma}{m_H^3}\,\tau$$

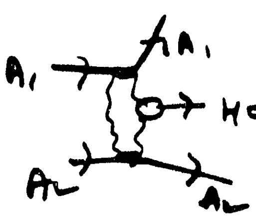
$$\Gamma_{H\to\gamma\gamma} = \frac{\alpha^2 G_F m_H^3}{128\pi^3\sqrt{2}}\left[7-\frac{4}{3}\sum_f Q_f^2 C_f\, I\!\left(\frac{m_f^2}{m_H^2}\right)\right]^2$$

Ellis
Gaillard
Nanopoulos

$$C_f = \begin{cases} 3 & \text{quarks} \\ 1 & \text{leptons} \end{cases}$$

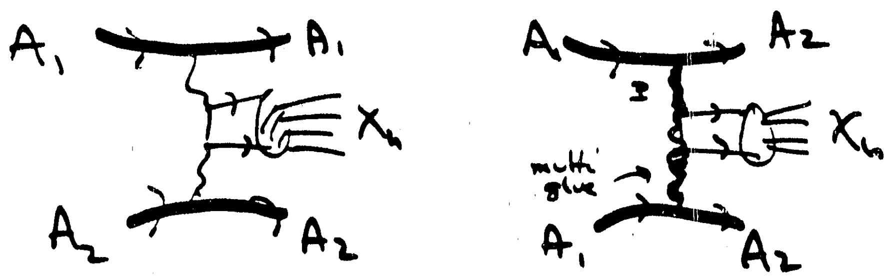


Also $$\Gamma_{H\to gg} = \frac{\alpha_s^2 G_F m_H^3}{128\pi^3\sqrt{2}}\left|\sum_q I\!\left(\frac{m_q^2}{m_H^2}\right)\right|^2 C'$$



Background to Photon-Photon Reactions from Double-Pomeron Interaction

$$A_1 + A_2 \to A_1 + A_2 + X_h$$



From optical theorem: $\mathcal{M} \sim g_{\mathbb{P}NN}\, A^{2/3}\, F_A(t)$

$$\sigma_{A_1A_2\to A_1A_2X} \sim \int d^2k_{\perp 1} \int d^2k_{\perp 2}\; F^2_{A_1}(k^2_{\perp 1} + x_1^2 m_1^2)\, F^2_{A_2}(k^2_{\perp 2} + x_2^2 m_2^2)$$
$$g^4_{\mathbb{P}NN}\, A_1^{4/3} A_2^{4/3}\, \sigma_{\mathbb{P}\mathbb{P}\to X}(x_1 x_2 s)$$

$$\sigma^{\mathbb{P}\mathbb{P}}_{A_1A_2\to A_1A_2X} \sim A_1^{2/3} A_2^{2/3}\, \sigma^{\mathbb{P}\mathbb{P}}_{pp\to ppX}$$

compare

$$\sigma^{\gamma\gamma}_{A_1A_2\to A_1A_2} \sim Z_1^2 Z_2^2\, \sigma^{\gamma\gamma}_{pp\to ppX}$$

similar kinematics range of X:

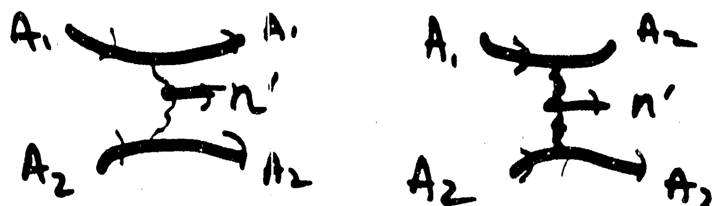
For Au Au
$$\sigma^{\mathbb{P}\mathbb{P}}_{AA} \sim 10^3\, \sigma^{\mathbb{P}\mathbb{P}}_{pp}$$
$$\sigma^{\gamma\gamma}_{AA} \sim 10^8\, \sigma^{\gamma\gamma}_{pp} \quad \text{dominates}$$

Notes on Double Pomeron Contributions

* 1. For intermediate A, Z

$\gamma\gamma$ and PP amplitudes are comparable and interference needs to be taken into account.

* 2. Excellent example η' production



study interference of amplitudes!

$\sigma_{\gamma\gamma} \sim Z^4$ $\qquad$ $\sigma_{PP} \sim A^{4/3}$

$$\Gamma_{\eta' \to \gamma\gamma} \sim 2\% \;\; \Gamma_{\eta' \to X(gg)}$$

Thus

$$\sigma^{\gamma\gamma} \text{ and } \sigma^{PP}$$

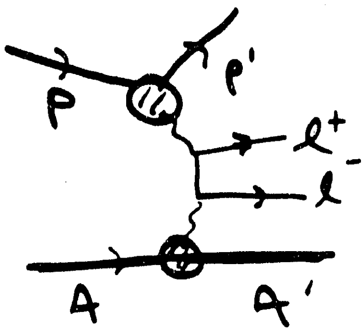
will be comparable at some $A_1 A_2$

* 3. ISR measurement of $\sigma^{PP}_{\alpha\alpha \to \alpha\alpha X}$ Cavasinni et al Z.Phys. 28 [illegible] (1985)

X: 2 chgd tracks $|\eta| < 1$

$$\frac{\sigma_{\alpha\alpha}}{\sigma_{pp}} = \frac{131\,\mu b}{11.7\,\mu b} = 4^{1.74} \quad (\text{faster than } A^{4/3})$$

Tests of Color Transparency in $pA \to \ell\bar{\ell}\, p'A'$



$t = (p'-p)^2$

At low t

ISI + FSI $\Rightarrow$ destroy A'

$b > R_p + R_A$.

At high t, proton has small color dipole moment

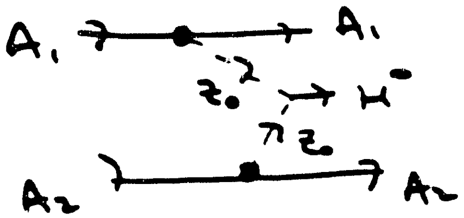
ISI + FSI eliminated!

Test from study of A dependence

First calc: Joyes et al

Coherent Z^0 Production

See also Cahn
Dawson



$$A_1 A_2 \to A_1 A_2 H^0$$

$$\frac{dN}{dx}(A \to Z^0 A) = (Z')^2 \frac{\alpha}{\pi} \int \frac{dk_\perp^2 \; k_\perp^2 \; F_A^2(k_\perp^2 + x^2 m^2)}{[k_\perp^2 + x^2 m^2 + M_Z^2]^2}$$

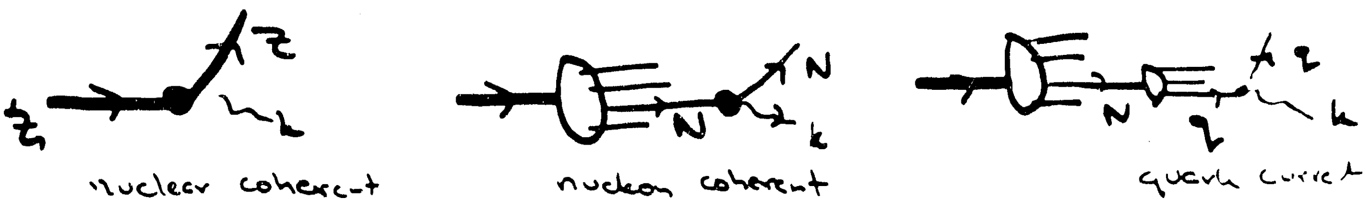
$$k_\perp^2 < R_A^{-2}$$

Suppression factor

$$\frac{\frac{dN}{dx}(A \to Z^0 A)}{\frac{dN}{dx}(e \to Z^0 e)} = (Z')^2 \frac{\left[\frac{1}{R_A^2 M_Z^2}\right]^2}{\log s/M_Z^2}$$

$$\sim \frac{10^4 \; 10^{-7}}{10} \sim 10^{-4}$$

Multiple Coherent + Incoherent γ sources



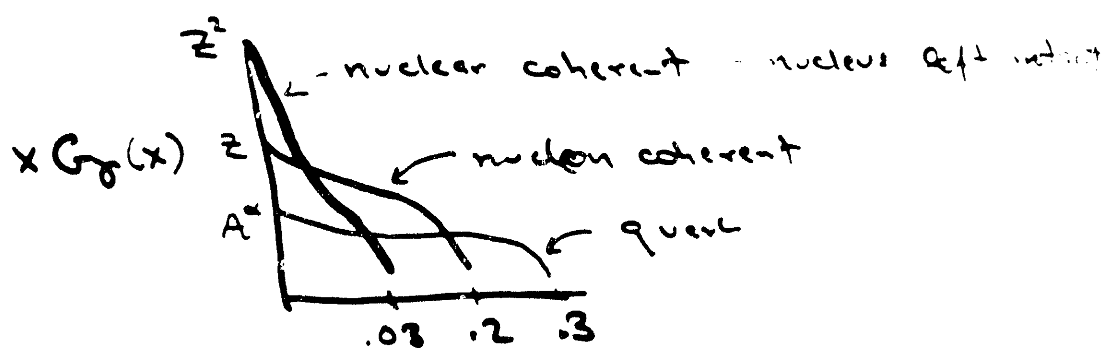
$$G_\gamma(x) = \frac{Z^2\alpha}{\pi}\frac{1}{x}\int_{x^2M_Z^2}\frac{dQ^2}{Q^2}F_Z^2(Q^2)\left[1-\frac{x^2M_Z^2}{Q^2}\right]$$

convolution with $G_{N/A}$ & $G_{q/A}$

$$+\,Z\frac{\alpha}{\pi}\frac{1}{x}\int_{x^2M^2}\frac{dQ^2}{Q^2}F_p^2(Q^2)\left[1-\frac{x^2M_p^2}{Q^2}\right]$$

$$+\sum_q e_q^2\frac{\alpha}{\pi}\frac{1}{x}\int_{x^2m^2}^{s}\frac{dQ^2}{Q^2}$$

"Shadowing" + ...



Observed photon distribution is sum of these contributions.

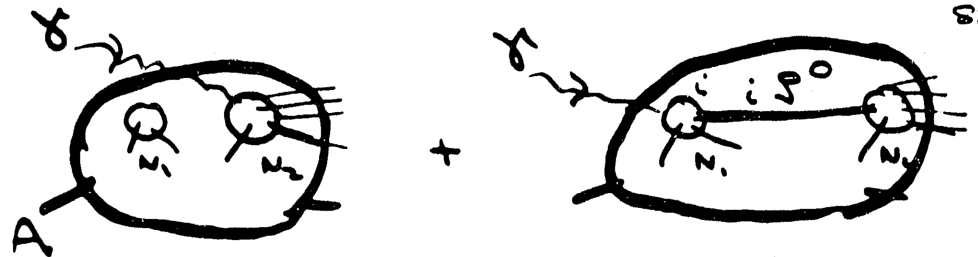
"Shadowing" of Photon Distributions

Recall $\sigma_{\gamma A} \sim A^{.8}\,\sigma_{\gamma N}$ surface nucleons

high energy

SJB + PUMPLIN

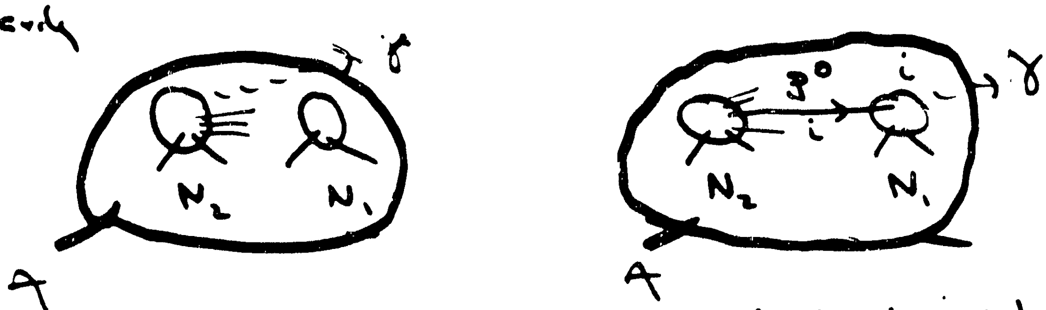
SJB + Close + Gunion



destructive interference on N_2

interior shadowed

Similarly



destructive interference on N_2

Thus only surface nucleons contribute to $\underline{G_{\gamma/A}^{\text{incoherent}} \sim A^{0.8}\, G_{\gamma/N}}$

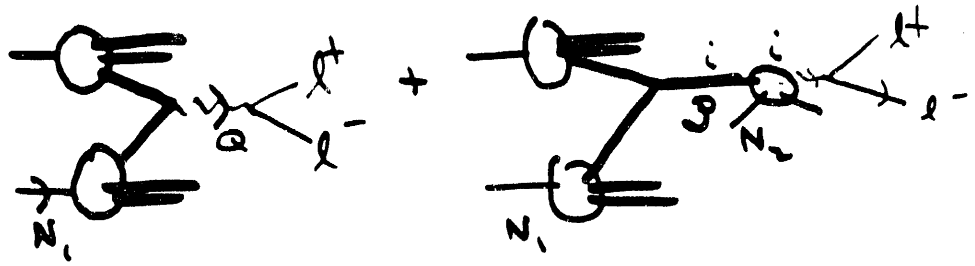
In general

$$\underline{G_{\gamma/A}(x) \sim \sigma_{\gamma A}\left(s \sim \frac{\mu^2}{x}\right)}$$

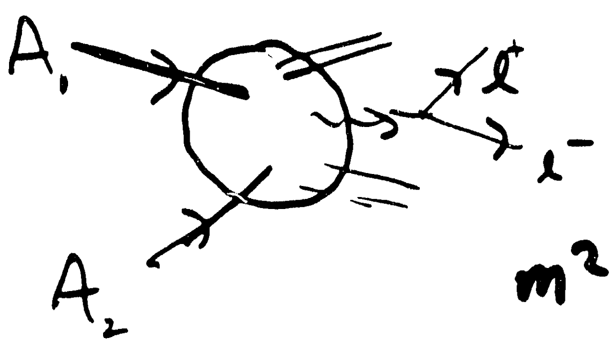
GPS

Low mass lepton-pairs from Drell-Yan reaction shadowed

$Q^2 \lesssim 1\,\mathrm{GeV}^2$



destructive interference of flux from interior nucleon

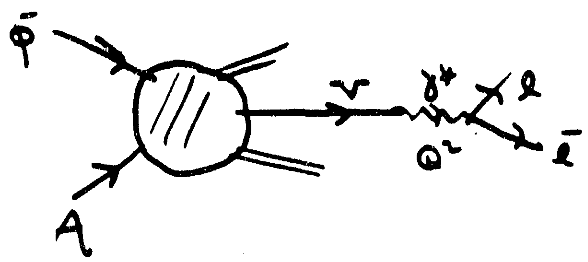


$m^2 < 1\,\mathrm{GeV}^2$

$$\therefore \quad \sigma_{AA \to \ell\bar{\ell} X} \propto \left(A_1^{1/3} + A_2^{1/3}\right)^2$$

surface
not volume

Shadowing in Drell-Yan at low Q^2



$\tau \sim \dfrac{E_{\gamma^*}}{Q^2 - M_V^2}$

very long at $Q^2 \sim M_\rho^2$

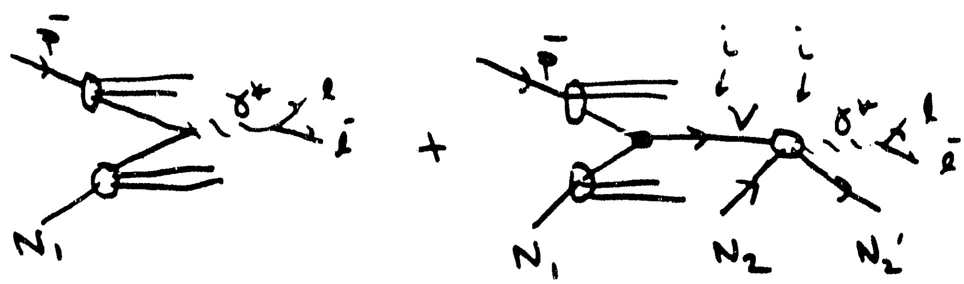
$$\sigma_{DY} \sim \sigma_{\rho A} \propto A^{2/3}$$

∴ Low mass pairs only involve back surface
do not probe inside of nuclei!

Shadowing even stronger than $\sigma_{\gamma^* A}$ or $\sigma_{\gamma A}$

Microscopic view:

Destructive Interference!

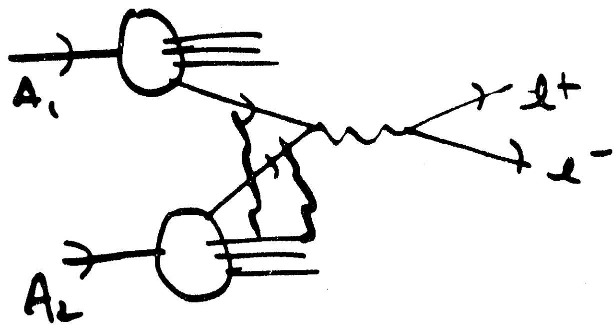


Nucleons N_1 inside A not effective!

∴ Low p_T γ
and Low invariant mass $\ell\bar{\ell}$
do not probe interior of
nuclear reactions

Pumplin + SJB
Gribov
Yennie + Gottfried

Drell-Yan in $A_1 A_2$ collisions



Predict $\quad \dfrac{d\sigma}{dx_1 dx_2} \sim A_1^{\alpha(x_1)} A_2^{\alpha(x_2)} \dfrac{d\sigma_{NN \to \bar{\ell}\ell}}{dx_1 dx_2}$

shadowing $\quad \alpha < 1 \quad$ low $x < 0.1$

+ antishadowing $\quad \alpha > 1 \quad x \sim 0.15$ $\qquad$ SJB + Hung Jung Lu

Predict growth of $\langle Q_\perp^2 \rangle \sim A_1^{1/3} + A_2^{1/3}$

from ISI

Heavy Quark Production

* Gluon Fusion $\quad gg \to c\bar{c}\,,\ b\bar{b}$

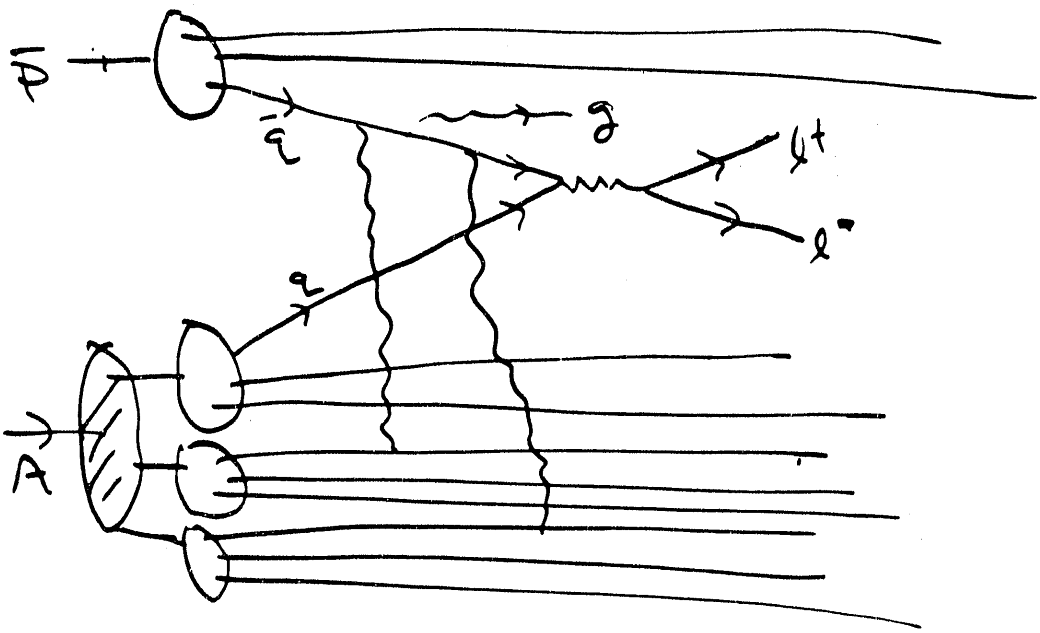
* Intrinsic charm : large x_F J/ψ ..

$\dfrac{d\sigma}{dx_F} \sim A_1 A_2^{2/3}$ behavior

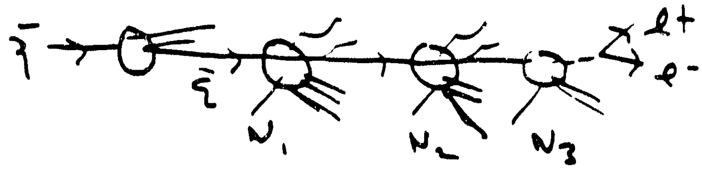
Nuclear Effects in the Initial State

Drell-Yan $\bar{p}A \to \mu^+\mu^- X$

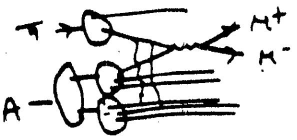
Bodwin, SJB, Lepage



1) Expect usually induced radiation Energy loss $\frac{dz}{z}$



2) Expect $\delta\langle k_\perp^2 \bar{q}\rangle_A \sim A^{1/3}$ Collision Broadening



Elastic collisions
broaden $Q_\perp$ distribution

Not confirmed
by FNAL - CP

Energy dep?.

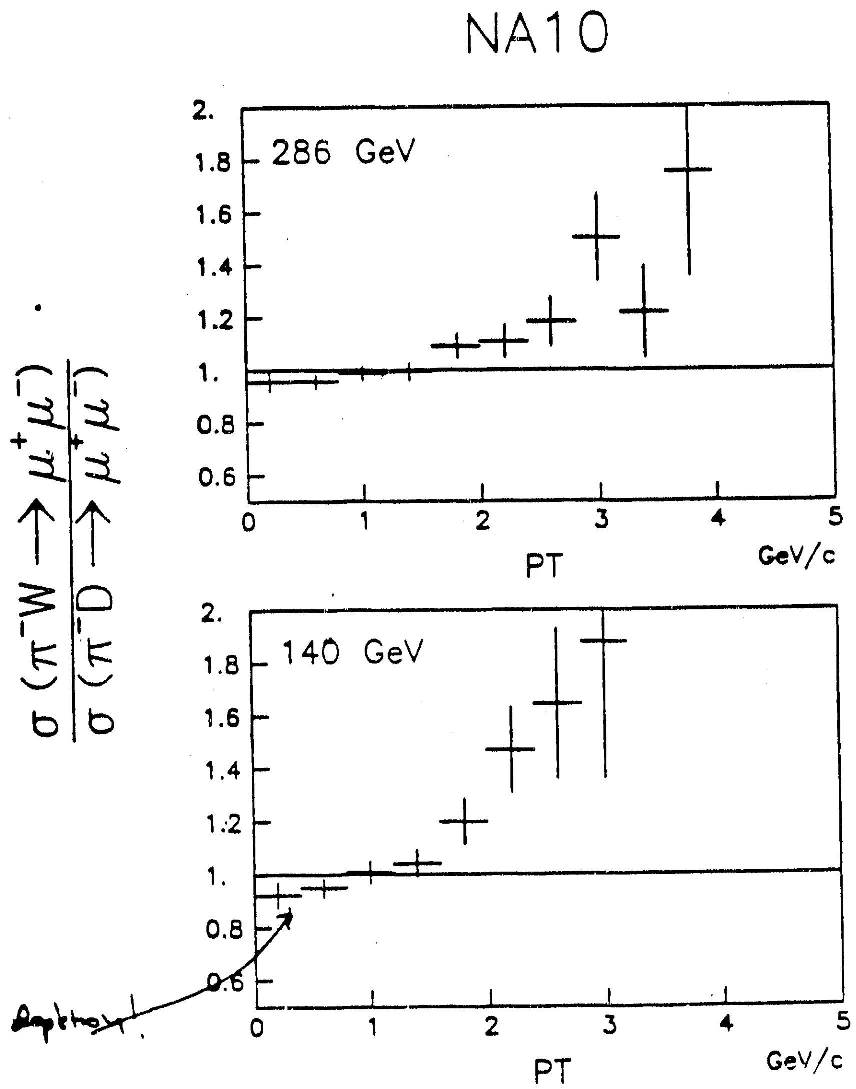


FIGURE 2

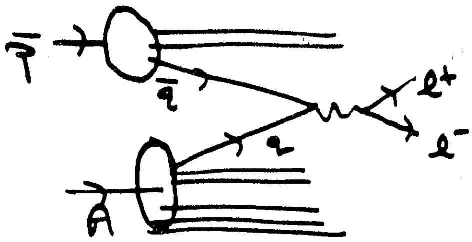
BBL
Michael
Chiapetta + Pirner

$\langle p_T^2 \rangle$ increases
with A

Non-additive
nuclear effect

Drell-Yan Factorization and Initial State Interactions

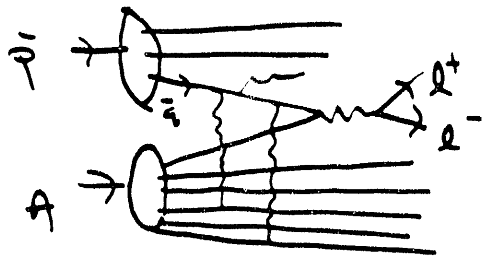
Bodwin - Lepage SJB



$$\frac{d\sigma}{dx_a\,dx_b} = G_{\bar{q}/\bar{p}}(x_a)\, G_{q/A}\, d\sigma(x_a x_b s)$$

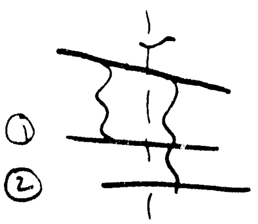
$\uparrow$ independent of A

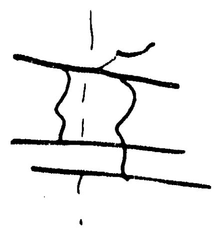
Initial State Bremsstrahlung destroys D.Y. Factorization



Radiation $\frac{dz}{z}$

lowers moment of $\bar{q}$





Two classical processes interfere if $\Delta P_L L_A < 1$

$\psi(p_1+\Delta p_1, p_2)$ $\qquad$ $\psi(p_1, p_2+\Delta p)$

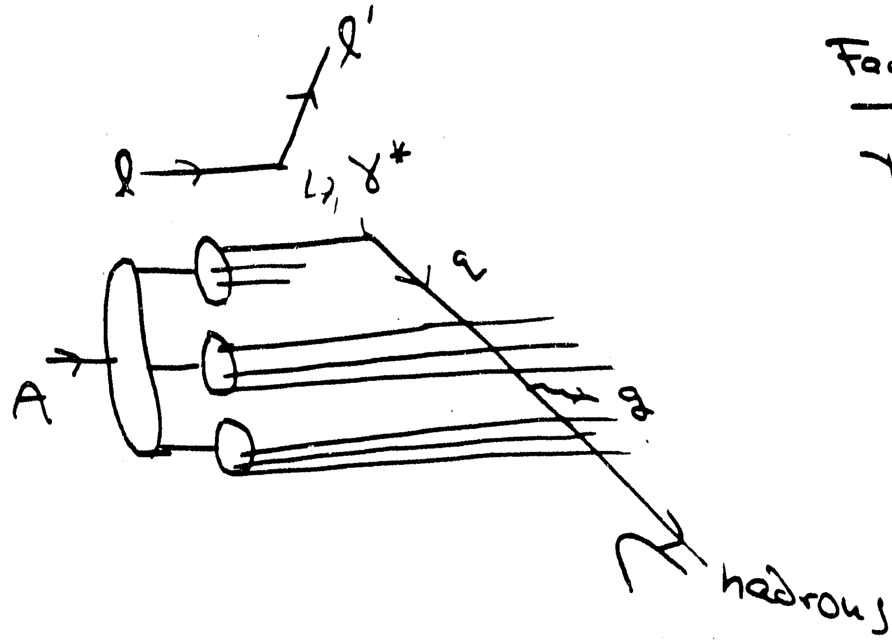
$$\Delta P_L = \frac{m_{g\bar{q}}^2 - m_{\bar{q}}^2}{E_{\bar{q}}} \quad : \ast \boxed{E_{\bar{q}} > \Delta m^2 L_A}$$

$\boxed{\text{Target-length condition for no radiation!}}$

But Elastic collisions allowed! $\frac{d\sigma}{dQ^2\,dQ_\perp^2}$ does not factorize

Bodwin
Lepage
SJB

$\boxed{\text{Study hadronization inside nuclei}}$



Factorization:

$D_{\pi/q}(z, Q)$

independent of A

Formation zone principle:

$\mu^2 = \Delta m^2$
$= m_{q\bar{q}}^2 - m_q^2$

No induced radiation if

$$\boxed{E_q > \mu^2 L_A}$$

Study onset, nuclear medium effects

But: Elastic scattering allowed

$$\boxed{\langle k_\perp^2 \rangle_A \sim A^{1/3}}$$

σ_{el}^{qN}
σ_{el}^{gN}

BBL: QCD + QM interference.

Test at NMC PETRA

$\boxed{\text{Study Quark, Gluon Interactions in Nuclei}}$

$$A_1 A_2 \to \ell \bar{\ell} X$$
$$\to Q \bar{Q} X$$
$$\to \text{Jet} + \text{Jet } X$$

1. Growth of $\langle k_\perp^2 \rangle$ from ISI, FSI

2. Absence of induced radiation due to formation zone

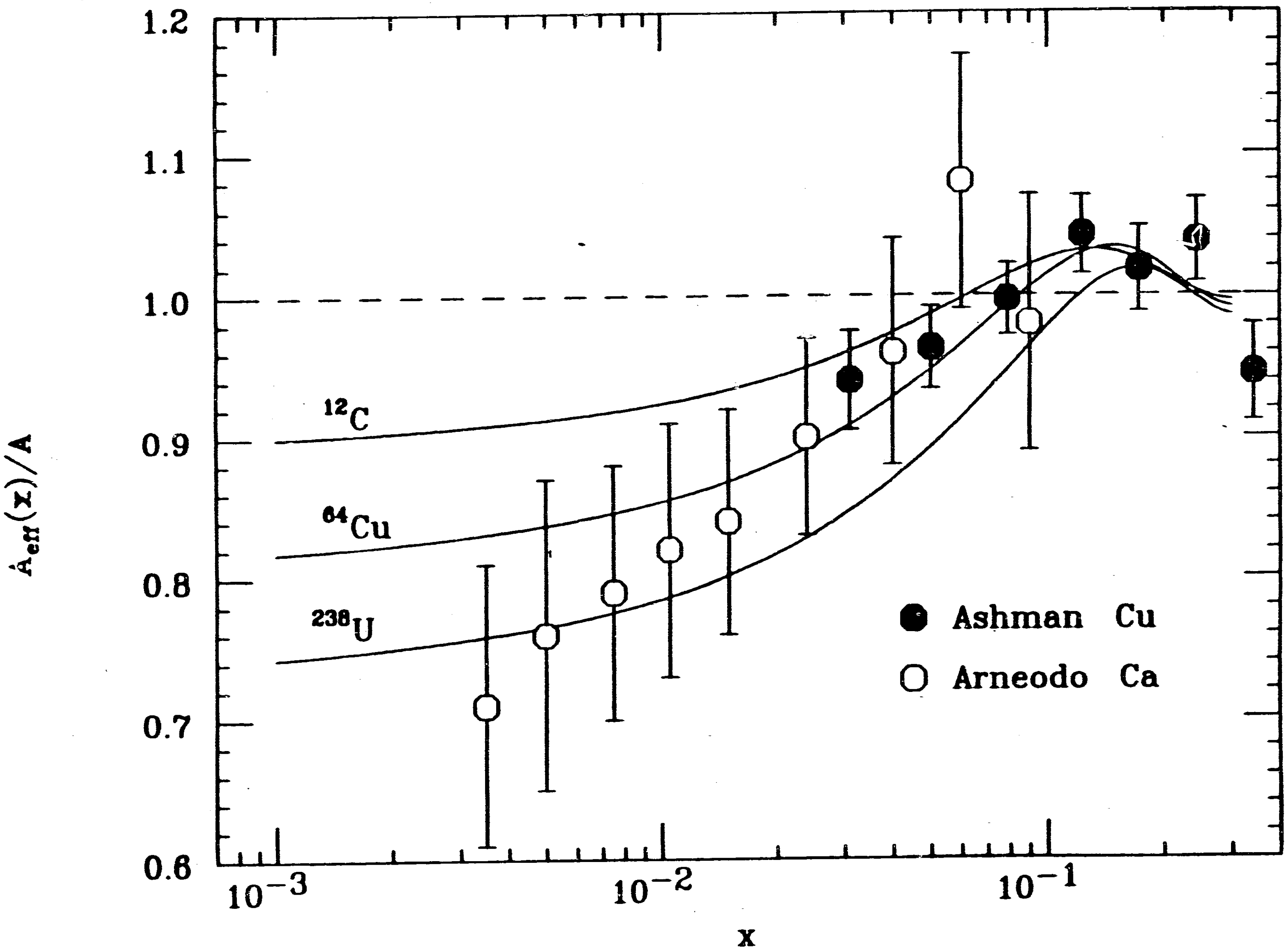
$$E_q > (\Delta M^2) L_A$$

Initial and Final State in A_1, A_2

SJB
Bodwin
Lepage

FIGURE 4

SJB + 1H..



Shadowing + Anti-shadowing

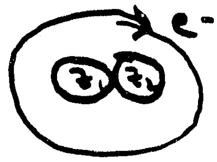
$\Rightarrow$ Detailed information on $\sigma_{\bar{q}N}(s)$

$\boxed{\text{GSI Phenomena}}$

1. U + Th , U + Ta collisions at 6 MeV/N

$\beta_{lab} \cong 0.1$ (Lab K.E.)

Coulomb Barrier stops nuclei



2. D-eqn $-\frac{Z\alpha}{r}\rho_N(r)$ [combined nucleus]

$Z_1 + Z_2 > 169$: $E_{1s} < -mc^2$

$\gamma \to \underset{\substack{\uparrow \\ \text{contin.}}}{e^+} e^-_{1s}$ spontaneous decay

3. Predict: spontaneous + dynamical e^+ production

$\sigma \sim Z_{TOT}^{22}$

Peiper + Greiner (69)
Gershtein + Zeldovich (69
Popov + Marinov

review: BJB + Mohr (1977)

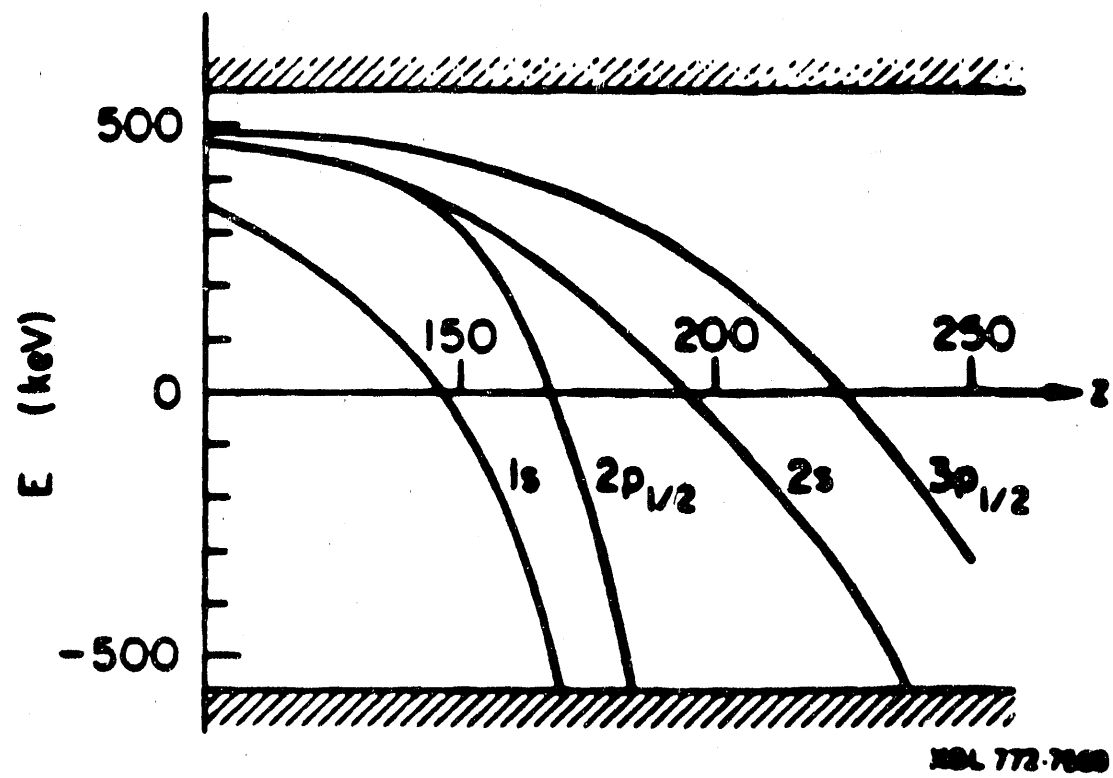


Figure A.13

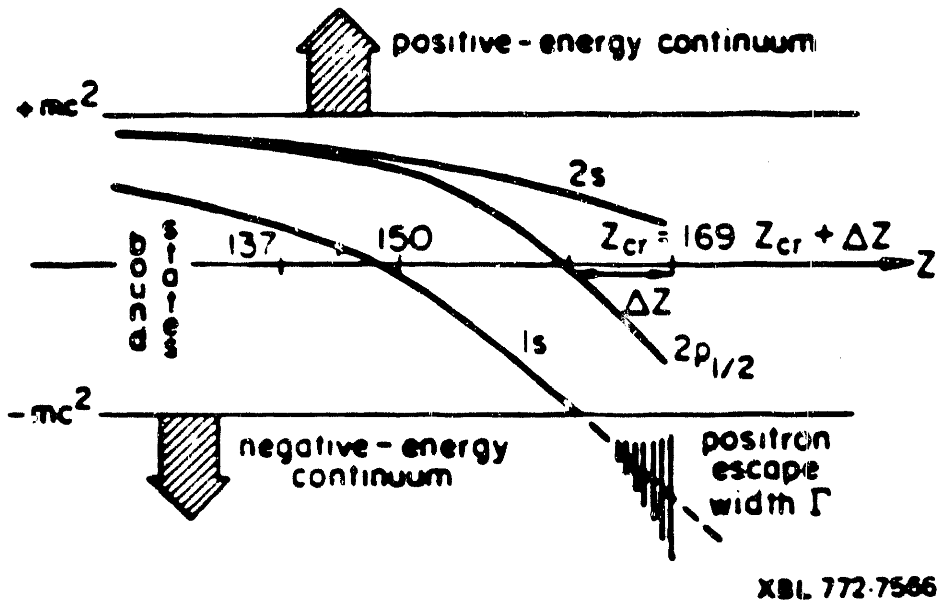

positive - energy continuum
+mc^2
2s
bound states
137
150
Z_{cr} = 169 Z_{cr} + ΔZ
Z
ΔZ
1s
$2p_{1/2}$
-mc^2
negative - energy continuum
positron escape width Γ
XBL 772-7566

Figure A.21

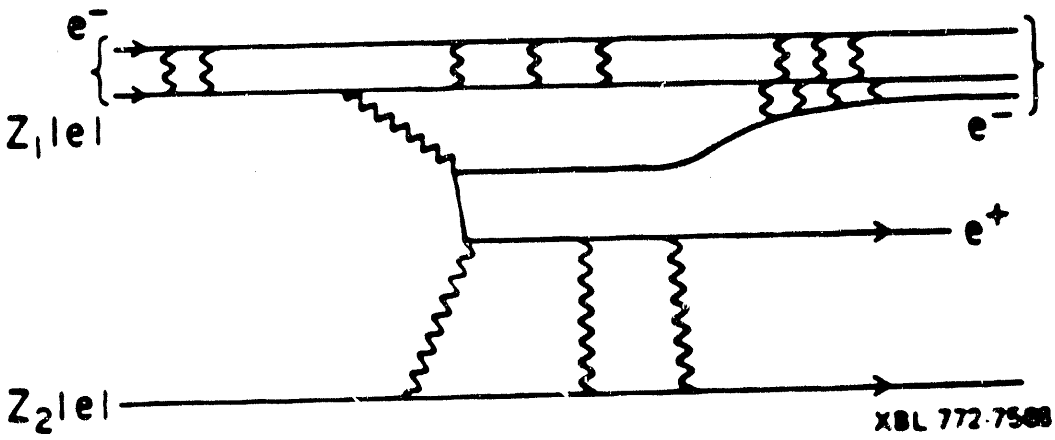


Figure A.26

GSI Expts $\left\{ \begin{array}{l} \text{EPOS} \\ \text{Orange} \end{array} \right.$

coincident pairs; distinct energies

EPOS: $T_{e^+} + T_{e^-}$ (keV)

608 ± 7	620 ± 8	
not reproduced: ~~760 ±~~	748 ± 8	not back to back $E_+ - E_- \cong 150$ keV
809 ± 8	806 ± 8	
$^{238}U + ^{232}Th$	$^{238}U + ^{181}Ta$	

no Z^{22}

Orange: $\left.\begin{array}{l} \sim 810 \text{ keV} \\ \sim 600 \text{ keV} \end{array}\right\}$ $\begin{array}{l} U + U \\ Pb + Pb \end{array}$

$M_{pair} \sim 1.6$ MeV

$\delta\left(\frac{\vec{p}^2}{2m_{pair}}\right) \sim 8$ keV $\quad \sqrt{\delta p^2} \sim 90$ keV

compatible with nuclear source?

$\frac{\hbar}{R_A} \sim 10$ MeV

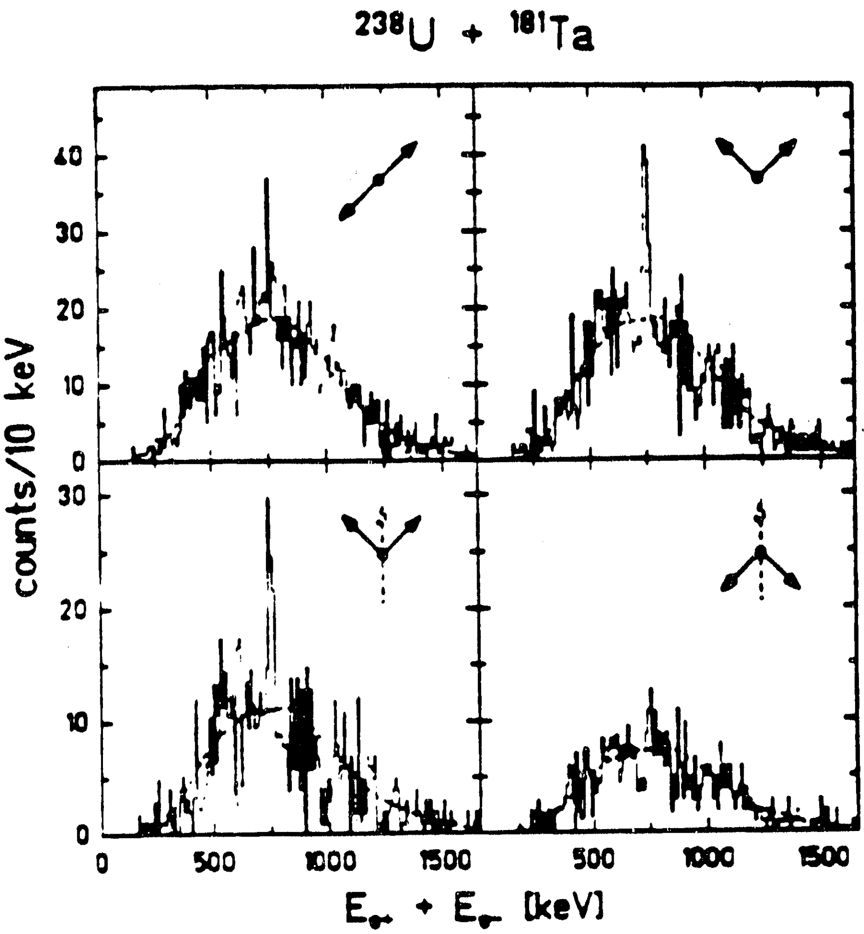


Figure 7. : Sum-energy spectra of $^{238}U + ^{181}Ta$ collisions for the 748-keV line with beam energies, wedge-shaped lepton energy window, and time-of-flight gates as in fig 5a2. The upper part displays the hemisphere correlation as in fig. 6. The lower part contains complementary subgroups of the equal hemisphere correlation data (right up) with the emission of the pair either forward (left down) or backward (right down) with respect to the beam as indicated.

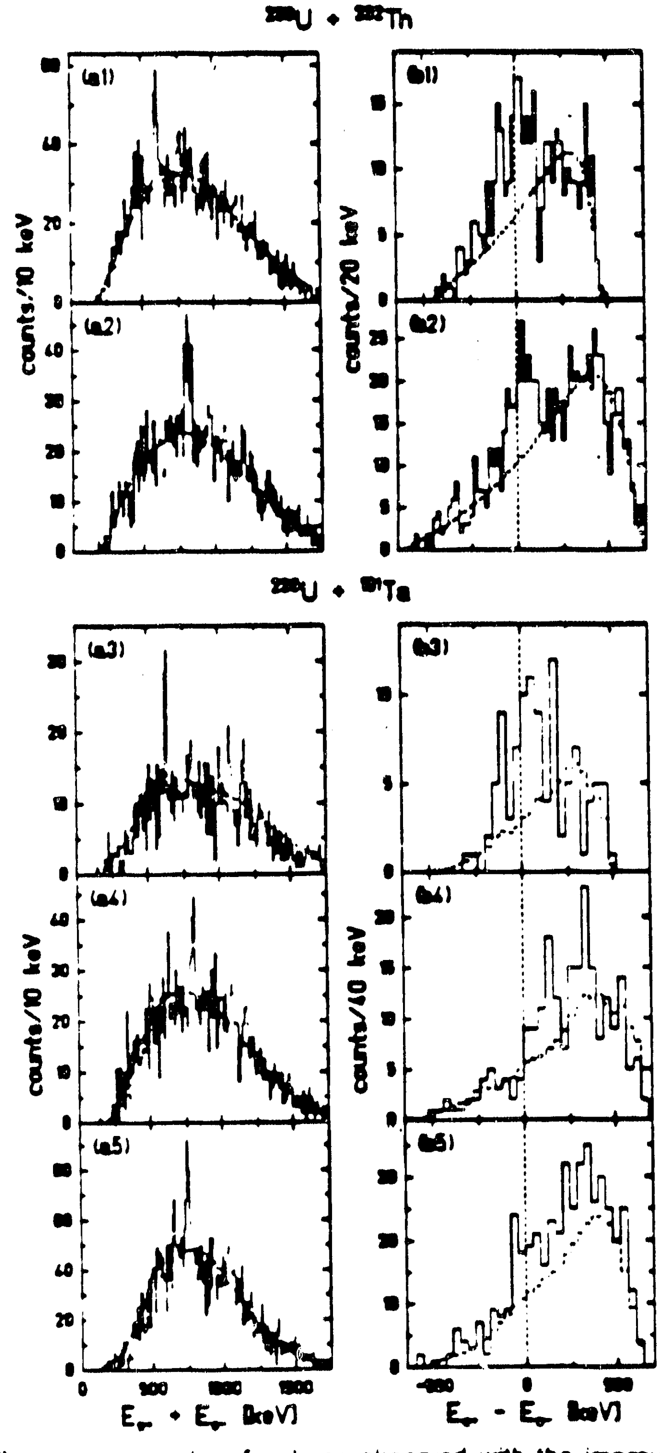


Figure 5. : The sum-energy line family as observed with the improved set of high-resolution Si(Li) detectors for ^{238}U + ^{232}Th (up) and ^{238}U + ^{181}Ta (down). The sum spectra (left) together with the associated difference spectra (right) are obtained for beam energies shown in table 1 and two-dimensional lepton energy and time-of-flight gates (see fig 3) as described in the text.

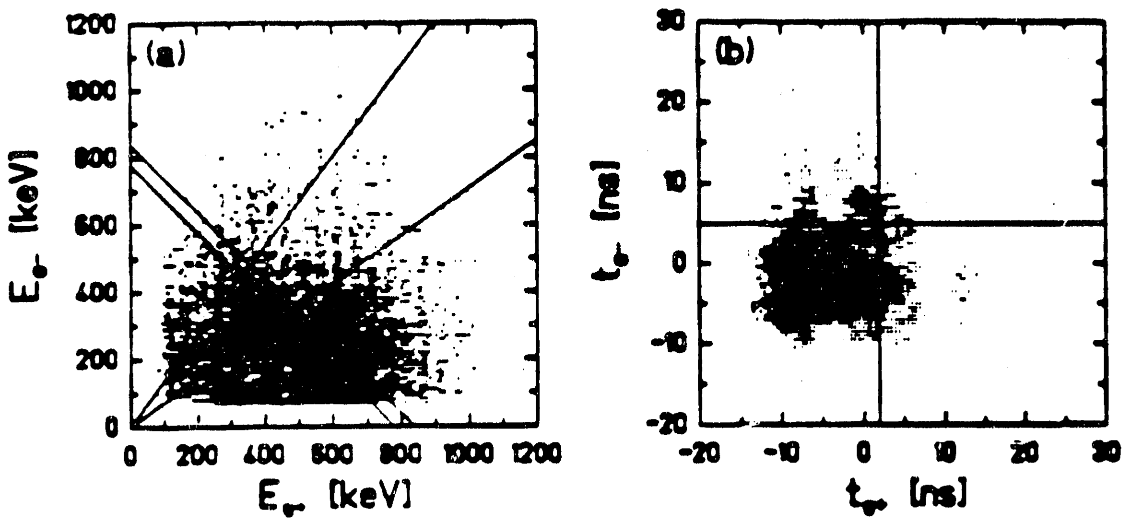


Figure 3. : Monte Carlo simulation of a two-body decay of a particle of 1.8 MeV rest mass into an electron-positron pair together with background electron-positron coincidences from atomic and nuclear processes. The initial intensity of the two-body decay contributes 3% to the total positron yield. The calculation fully accounts for transport and detector features of the EPOS set-up.

(a) Two-dimensional event distribution as function of the lepton kinetic energies. The wedge cut 'W' and the difference energy cut 'S' are indicated.

(b) Two-dimensional event distribution vs the electron and positron transport time for events within the wedge cut 'W'. (t=0 corresponds to the mean transport time (~ 8 ns) of the background lepton-events).

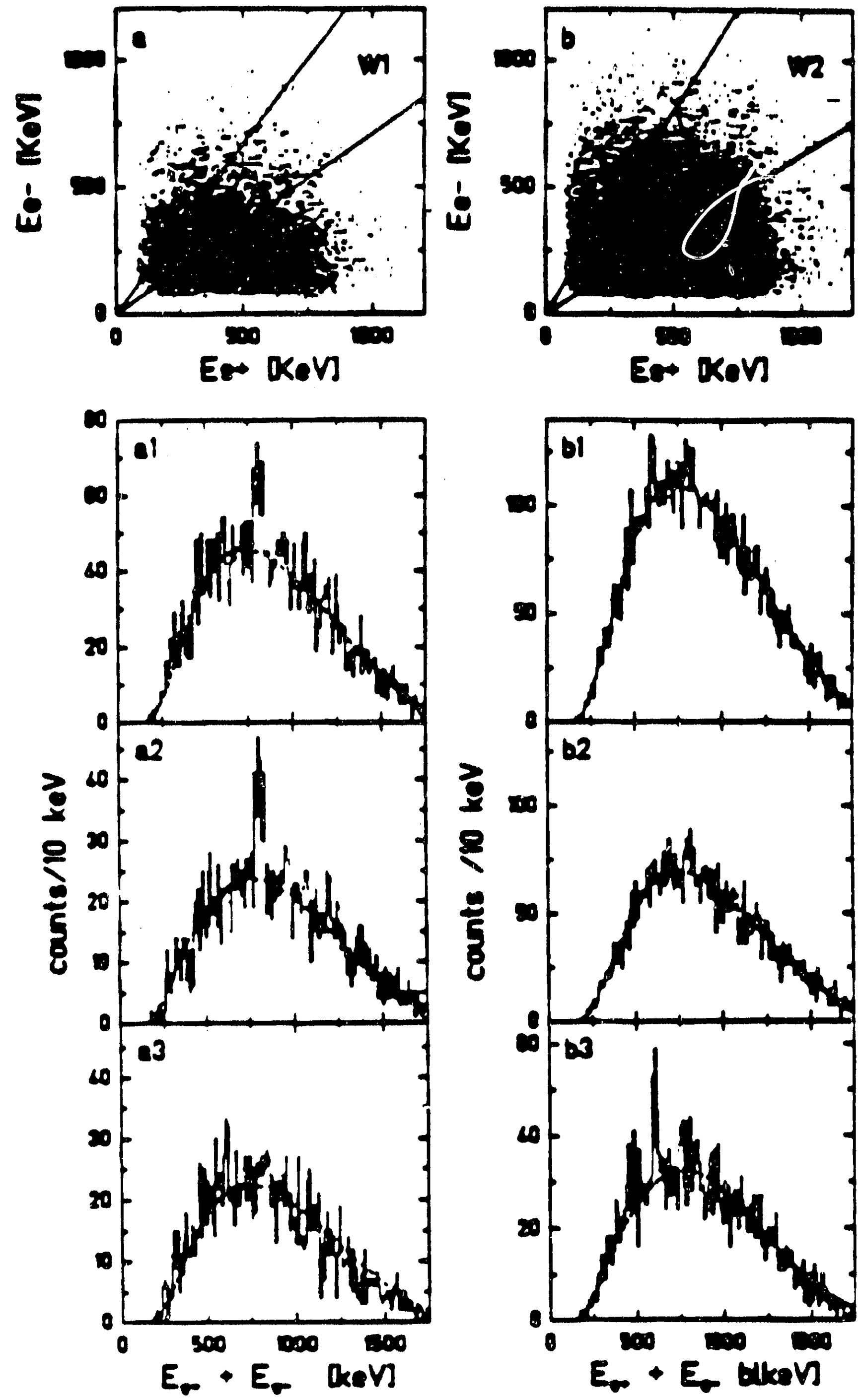


Figure 4. : Sum energy spectra of coincident electron-positron emission in ^{238}U + ^{232}Th collisions associated with beam-energy ranges of 5.87 to 5.90 MeV/u (left) and 5.86 to 5.90 MeV/u (right) together with the two-dimensional event distributions (up) for the individual lepton energies of the pair. The sum energy spectra in the left and the right panel are gated with the two-dimensional wedgeshaped windows 'W1' and 'W2', respectively. The individual spectra additionally differ in the lepton time-of-flight as described in the text.

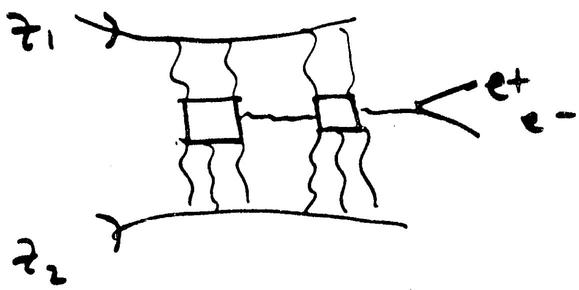
Speculations on New Phase of QED at large $Z\alpha$

1. Celdi + Chodos

2. L. Celenza et al

3. Ng + Kikuchi

4. Fried + Cho [Phys. Rev. D 41, 1488 (1990)]



$\sigma(Z\alpha, \alpha)$

$$\alpha \exp\left(\frac{\rho}{2}\right) \cong \alpha \exp\left(\frac{\pi^2}{2}\right) \cong \frac{139}{137}$$

claims distinct masses

$$M^2_{e^+e^-} = (2m)^2 + \xi_0 (n^2 + \ell^2)^{1/2}$$

M = 1.64 MeV, 1.84 MeV, 2.08 MeV (prediction)

See also Spence + Vary

QED in Relativistic Heavy Ion Collisions

* Racah formula for $\sigma(Z_1 Z_2 \to e^+e^- Z_1 Z_2)$
* Huge cross-sections, # e^+e^- pairs
* Luminosity Limits for Heavy Ion Colliders
* Higher Born corrections
* 2nd Born Interference Asymmetry
* Nuclear Size Effects + E.P.A.
* Higgs, Z_0
* Double Pomeron
* Coherent + Incoherent γ sources; shadowing
* Drell-Yan and $Q\bar{Q}$ phenomena
  - Formation zone
  - Shadowing, Anti-shadowing
  - $\langle k_\perp^2 \rangle_{A_1, A_2}$
* Anomalous QED
  - GSI; New Phenomena

ELECTRO–WEAK PHYSICS AT RHIC

Talk presented by:

B. Mueller

1. Overview ($\gamma\gamma$ exchange)
2. Equiv. photon distrib. of moving charges
3. Selected σ's at RHIC + LHC
   W^+W^-, H^0 production
4. Impact parameter dependence +
   nuclear absorption effects
5. Coherent diffractive scattering (D.P.E.)
6. Conclusions

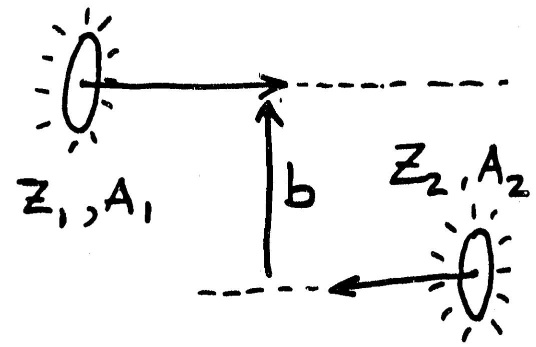
M. GRABIAK (LBL)
W. GREINER (Frankfurt)
J. RAU (Cambridge)
G. SOFF (GSI)

A. J. SCHRAMM (Duke U.)

Work also by Nachtmann & A. Schäfer (Heidelberg)

ELECTRO-WEAK PHYSICS AT (ULTRA-) R.H.I.C.

The basic idea:



Moving Coulomb fields act as ensembles of virtual photons

$$n(\omega)\, d\omega \sim \left(\frac{Ze}{\pi}\right)^2 \frac{d\omega}{\omega}$$

Finite nuclear size imposes cut-off

$$\omega < \omega_0 \sim \gamma R^{-1}$$

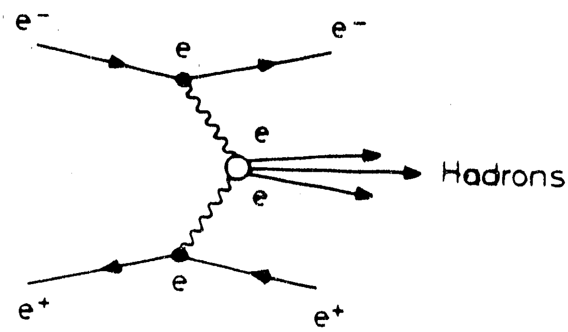
For ^{208}Pb : $R = 7.1$ fm $\rightarrow$ $\omega_0 \sim \gamma \cdot 30$ MeV.

RHIC : $\gamma = 100 \longrightarrow \omega_0 \sim 3$ GeV

LHC : $\gamma = 3500 \longrightarrow \omega_0 \sim 100$ GeV

SSC : $\gamma = 8000 \longrightarrow \omega_0 \sim 250$ GeV

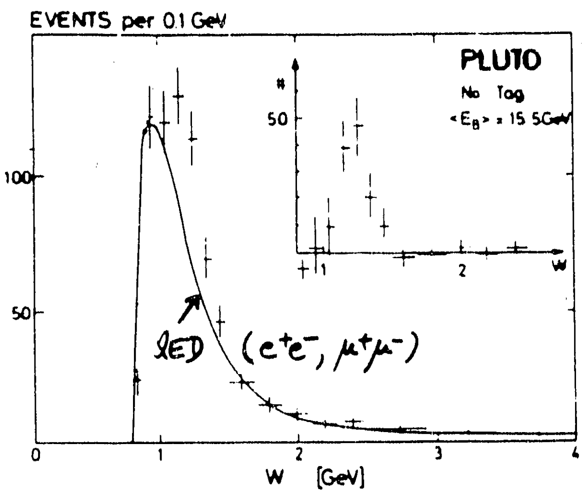
Two-photon physics at e^+e^- colliders
(DESY)



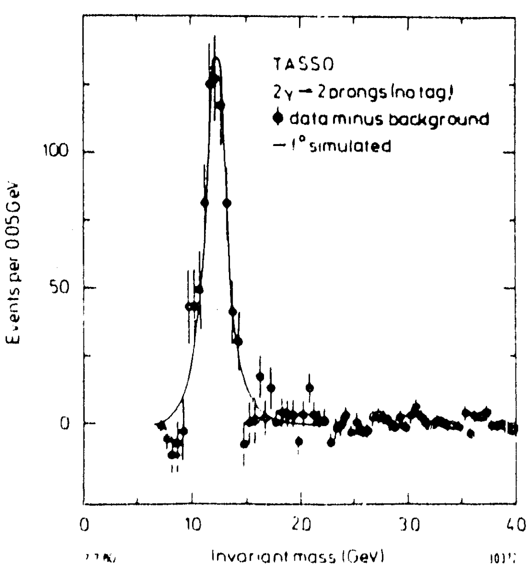
$f_2(1270)$ $\quad J^{PC} = 2^{++}$

$\Gamma_{\gamma\gamma} \sim 25\ \mathrm{keV} \sim \frac{\alpha^2}{\pi^3} M_f$

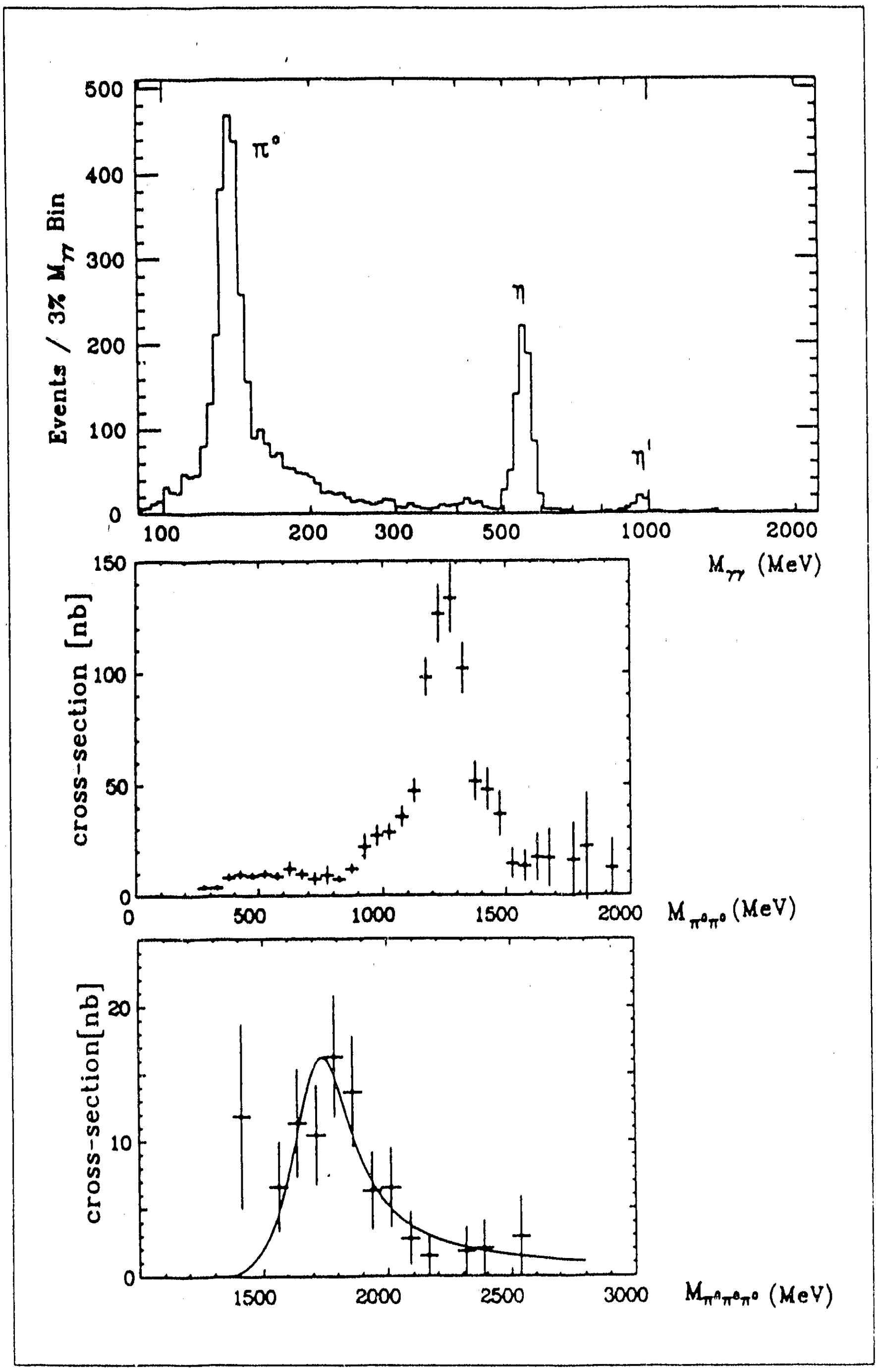
$B_{\pi\pi} \sim 85\%$



- Untagged two prong events from PLUTO plotted versus the pair mass. The solid line shows the QED contribution. The difference between the measured two prong yield and the QED contribution is shown



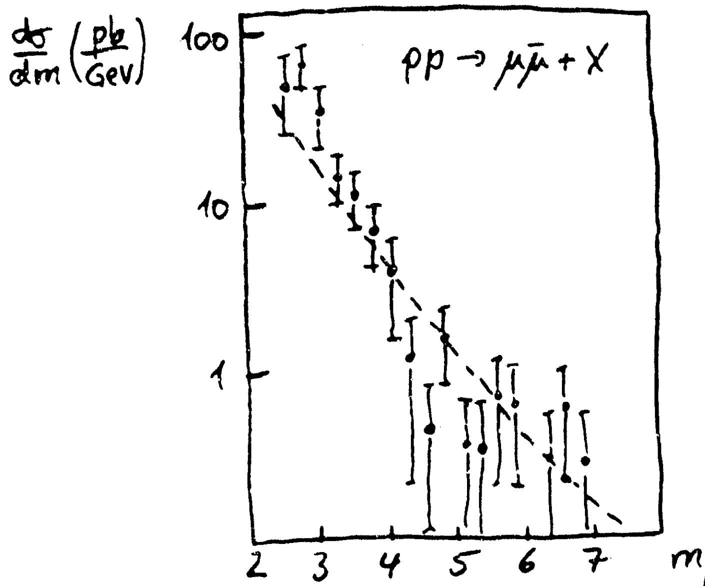
Untagged two prong events from TASSO plotted versus the pair mass with the QED background subtracted. The data are preliminary.

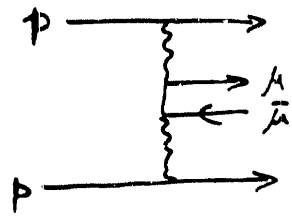


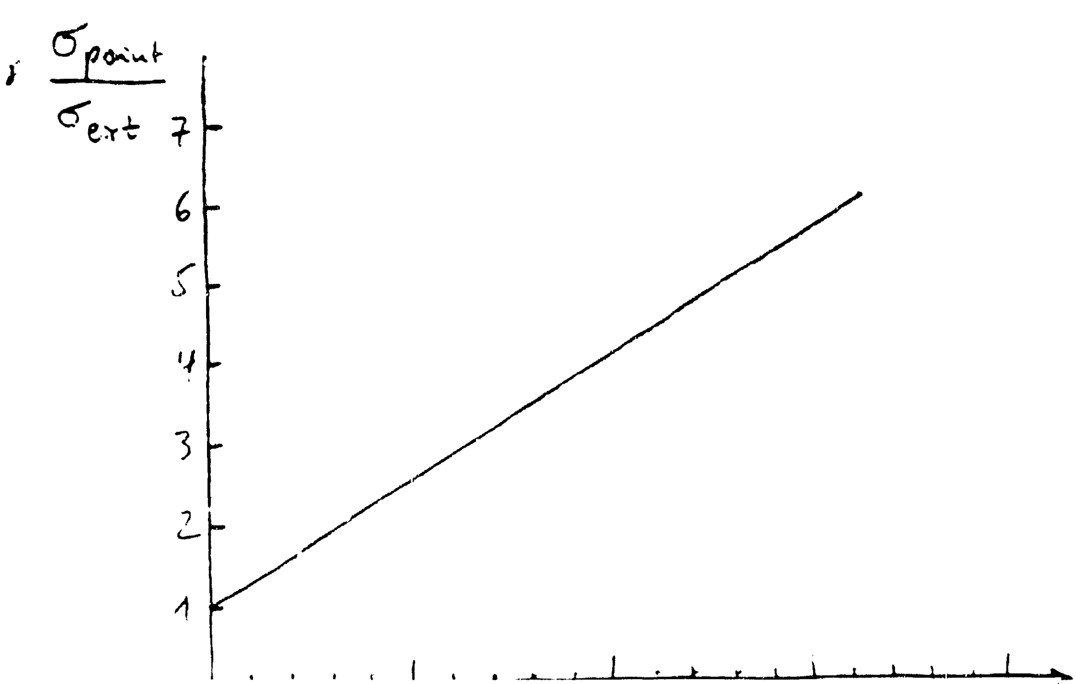
Photon systems produced in photon-photon collisions, as seen by the Crystal Ball detector at the DORIS electron-positron collider. The peak, in the middle plot corresponds to the $f_2(1270)$ resonance, that in the lower plot to the $\pi_2(1680)$ resonance.

Most $\gamma\gamma$ - physics has been done at e^+e^- colliders (SLAC, DESY, ...), but also some work at pp machines.

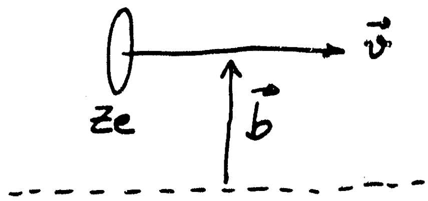
First (?) at CERN - ISR (1980)







Field of a moving charge:



Comoving system:

$$j'^\mu(x') = \rho(|\vec{x}' - \vec{b}|)\, u'^\mu$$

with $u'^\mu = (1, \vec{0})$

Fourier transform:

$$j'^\mu(k') = 2\pi\, \delta(k'^0)\, e^{-i\vec{k}'\cdot\vec{b}}\, Ze\, F(+\vec{k}'^2)\, u'^\mu$$

with $\quad F(\vec{k}'^2) = \frac{1}{Ze}\int_{-\infty}^{\infty} d^3x\, e^{-i\vec{k}'\cdot\vec{x}'}\rho(|\vec{x}'|)$

charge form factor

Lab system $\qquad u^\mu = \gamma(1, \vec{v})$

$$j^\mu(k) = 2\pi\, \delta(k\cdot u)\, e^{ik\cdot b}\, Ze\, F(-k^2)\, u^\mu$$

$$A^\mu(k) = \frac{1}{k^2}\, j^\mu(k) .$$

$\delta(k\cdot u) \longrightarrow u^\mu \approx$ polarization vector.

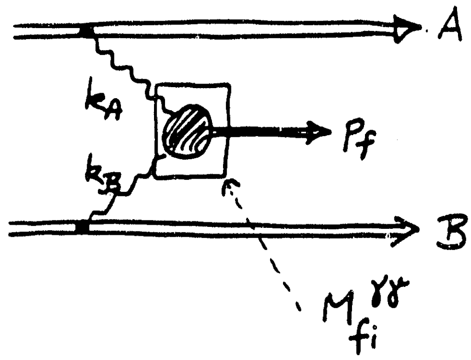
$$A^\mu = Ze\, r(k)\, e^{ik\cdot b}\, u^\mu$$

with $\quad r(k) = 2\pi\, \delta(k\cdot u)\, \frac{F(-k^2)}{k^2}$

$$0 = k\cdot u = \gamma(\omega - k_\parallel v) \quad \Rightarrow$$

$$-k^2 = -\omega^2 + k_\parallel^2 + k_\perp^2 = \frac{\omega^2}{\gamma^2 v^2} + \vec{k}_\perp^2 \approx \frac{\omega^2}{\gamma^2} + \vec{k}_\perp^2$$

Two colliding nuclei



Eikonal approximation

$$\sigma_{AB}^{fi} = \int d^2b \int |S_{fi}(b)|^2 dp_f$$

$$S_{fi}(b) = Z_A Z_B e^2 \int \frac{d^4k_A}{(2\pi)^4} \frac{d^4k_B}{(2\pi)^4} e^{ik_A b} r_A(k_A) r_B(k_B) \delta(k_A + k_B - P_f) \cdot$$
$$\cdot M_{fi}^{\gamma\gamma}(\omega_A, \omega_B)$$

$$|S_{fi}(b)|^2 \sim \cdots \int d^4k_A\, d^4k_A'\, d^4k_B\, d^4k_B' \quad r_A(k_A) r_A(k_A') r_B(k_B) r_B(k_B')$$
$$\cdot \cdots\cdots \quad M_{fi}^{\gamma\gamma}(\omega_A, \omega_B) M_{fi}^{\gamma\gamma}(\omega_A', \omega_B')^*$$

$\delta(k_A \cdot u)$, $\delta(k_A' \cdot u)$ in $r_A(k_A)$, $r_A(k_A')$, etc.

$\hookrightarrow$ k_A, k_A' differ only in transverse components

in particular: $\omega_A = \omega_A'$, $\omega_B = \omega_B'$

$$\sigma_{fi}^{AB} = (Z_A Z_B e^2)^2 \int d^2b \int \frac{d^4k_A}{(2\pi)^4} \frac{d^4k_B}{(2\pi)^4} \frac{d^4k_A'}{(2\pi)^4} e^{i(k_A - k_A')\cdot b} \cdot$$

$$\cdot\, r_A(k_A) r_A(k_A') r_B(k_B) r_B(k_B + k_A - k_A') \sigma_{fi}^{\gamma\gamma}(\omega_A, \omega_B)$$

k_A' - Integration $\qquad \vec{q} \equiv \vec{k}_{A\perp} - \vec{k}_{A\perp}'$

$$\int \frac{d^4k_A'}{(2\pi)^4} e^{i(k_A - k_A')b} r_A(k_A') r_B(k_B + k_A - k_A') =$$

$\int d^2q \quad iqb \quad F(-(k_A - q)^2) \quad F(-(k_B + q)^2)$

Final result:

$$\sigma_{fi}^{AB} = \left(\frac{Z_A Z_B \alpha}{\pi^2}\right)^2 \int d^2b \int d\omega_A d\omega_B \int \frac{d^2q}{(2\pi)^2} e^{iqb} \tilde{n}_A(\omega_A,q)\tilde{n}_B(\omega_B,-q) \cdot \sigma_{fi}^{\gamma\gamma}(\omega_A,\omega_B)$$

with $$\tilde{n}(\omega,q) = \frac{1}{\omega}\int d^2k_\perp \cdot k_\perp^2 \; \frac{F(-k^2)}{k^2} \, \frac{F(-(k-q)^2)}{(k-q)^2}$$

Gives impact parameter dependence.

$$\int d^2b \; e^{iq\cdot b} = (2\pi)^2 \delta(q) \quad ; \quad d^2k_\perp = 2\pi \kappa d\kappa \quad \begin{pmatrix} \kappa^2 = -k^2 \\ = k_\perp^2 + \omega^2/\gamma^2 \end{pmatrix}$$

yields total cross-section

$$\boxed{\sigma_{fi}^{AB} = \int d\omega_A \, d\omega_B \; n_A(\omega_A) \, n_B(\omega_B) \, \sigma_{fi}^{\gamma\gamma}(\omega_A,\omega_B)}$$

with $$n(\omega) = \frac{2Z^2\alpha}{\pi\omega} \int_{\omega/\gamma}^{\infty} d\kappa \; \frac{\kappa^2 - \omega^2/\gamma^2}{\kappa^3} \left(F(\kappa^2)\right)^2$$

"equivalent photon spectrum"

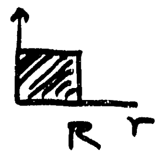
(Weizsäcker-Williams approximation)

<u>Assumptions:</u> (1) Eikonal approx.: $k_A \ll P_A$ (= nucl. mom.)

(2) $$M_{fi}^{\gamma\gamma}\left(k^2 = -k_\perp^2 - \frac{\omega^2}{\gamma^2}\right) \approx M_{fi}^{\gamma\gamma}(k^2 = 0) \qquad \left[\, k^2 \ll \omega^2, k_\parallel^2 \; ! \right]$$

Nuclear elastic form factor: $F_A(\vec{k}^2)$.

measured by elastic electron scattering.

Simplest model: homogeneous charge distribution $\rho(r) =$ 

$$F(\vec{k}^2) = \left(\frac{j_1(|\mathbf{k}| R)}{|\mathbf{k}| R} \right)^2$$

Better: Saxon-Woods distribution or Fermi distribution

$$\rho(r) \sim \frac{1}{e^{+(r-R)/a} + 1}$$

Ellis, Drees, Zeppenfeld have shown that

$$F(\vec{k}^2) = e^{-\vec{k}^2/Q_0^2}$$

is adequate with $Q_0 \sim 60$ MeV/c (^{208}Pb)

M. GRABIAK et al., J. Phys. G 15, L25 (1989)
E. PAPAGEORGIU, Phys. Rev. D40, 92 (1989)
M. DREES et al., Phys. Lett. B 223, 454 (1989)
J. RAU et al., J. Phys. G 16, 211 (1990)

Cross section for neutral state f:

$$\sigma^f_{AA} = \int d\omega_A\, n_A(\omega_A) \int d\omega_B\, n_A(\omega_B) \int ds\, \delta(4\omega_A\omega_B - s)\, \sigma^f_{\gamma\gamma}(s)$$

$$= \frac{4Z^4\alpha^2}{\pi^2} \int \frac{d\omega_A}{\omega_A} \int \frac{d\omega_B}{\omega_B} f\left(\frac{\omega_A}{\omega_0}\right) f\left(\frac{\omega_B}{\omega_0}\right) \int ds\, \delta(4\omega_A\omega_B - s)\, \sigma^f_{\gamma\gamma}(s)$$

$$= \int ds\, \rho(s)\, \sigma^f_{\gamma\gamma}(s)$$

with

$$\rho(s) = \frac{4Z^4\alpha^2}{\pi^2 s} \int d\omega_A \int d\omega_B\, f\left(\frac{\omega_A}{\omega_0}\right) f\left(\frac{\omega_B}{\omega_0}\right) \delta\left(\omega_A\omega_B - \tfrac{1}{4}s\right)$$

$$= \frac{4Z^4\alpha^2}{\pi^2 s} \int_0^\infty \frac{d\omega_A}{\omega_A} f\left(\frac{\omega_A}{\omega_0}\right) f\left(\frac{s}{4\omega_A\omega_0}\right)$$

$$= \frac{8Z^4\alpha^2}{\pi^2 s} \int_{x_0}^\infty \frac{dx}{x} f(x)\, f\left(\frac{x_0^2}{x}\right), \qquad x_0 = \frac{\sqrt{s}}{2\omega_0} = \frac{R\sqrt{s}}{2\gamma}$$

For single neutral particle X^0:

$$\sigma^{X^0}_{\gamma\gamma}(s) = \frac{1}{4s} |\mathcal{M}_{\gamma\gamma}|^2 \frac{M_X \Gamma_X}{(s - M_X^2)^2 + M_X^2\Gamma_X^2} \sim$$

$$\sim \frac{\pi}{4s} |\mathcal{M}_{\gamma\gamma}|^2\, \delta(s - M_X^2)$$

2-photon decay width:

$$\Gamma_{\gamma\gamma} = \frac{1}{32\pi M_X} |\mathcal{M}_{\gamma\gamma}|^2$$

i.e. $$\sigma^{X^0}_{\gamma\gamma}(s) \sim \frac{8\pi^2 M_X \Gamma_{\gamma\gamma}}{s} \delta(s - M_X^2) = \frac{8\pi^2 \Gamma_{\gamma\gamma}}{M_X} \delta(s - M_X^2)$$

$\gamma\gamma$ - physics accessible at RHIC :

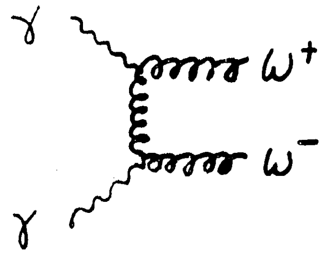
final state	σ
e^+e^-	HUGE
$\mu^+\mu^-$	360 mb
$\tau^+\tau^-$	5.3 μb
π^0	6.3 mb
η^0	1.5 mb
f (1270)	1.3 mb
η_c (2980)	8 μb

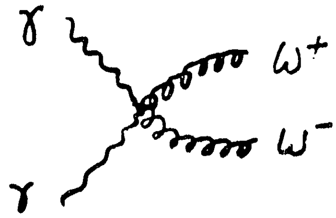
$\gamma\gamma$ - physics accessible at LHC/SSC :

final state	σ_{LHC}	σ_{SSC}
π^0	66 mb	90 mb
η_c	1.16 mb	1.95 mb
η_b	1 μb	2 μb
H (100 GeV)	0.15 nb	1 nb
W^+W^-	7.3 nb	146 nb

Electroweak processes:

$\gamma\gamma \to W^+W^-$



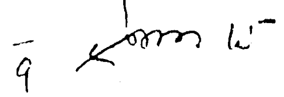


$$\sigma_{\gamma\gamma}^{W^+W^-}(s) = \frac{4\pi\alpha^2}{s}\sqrt{1-\frac{4M_W^2}{s}}\left\{\frac{s}{M_W^2}\left(2+\frac{3M_W^2}{2s}+\frac{6M_W^4}{s^2}\right)+\right.$$

$$\left.+\frac{6\left(M_W^2 s - 2M_W^4\right)}{s^2\sqrt{1-4M_W^2/s}}\ln\frac{1-\sqrt{1-4M_W^2/s}}{1+\sqrt{1-4M_W^2/s}}\right\}$$

$$\sigma_{UU}^{WW} = \begin{cases} 7.3 \text{ nb} & (\text{LHC}) \\ 146 \text{ nb} & (\text{SSC}) \end{cases}$$

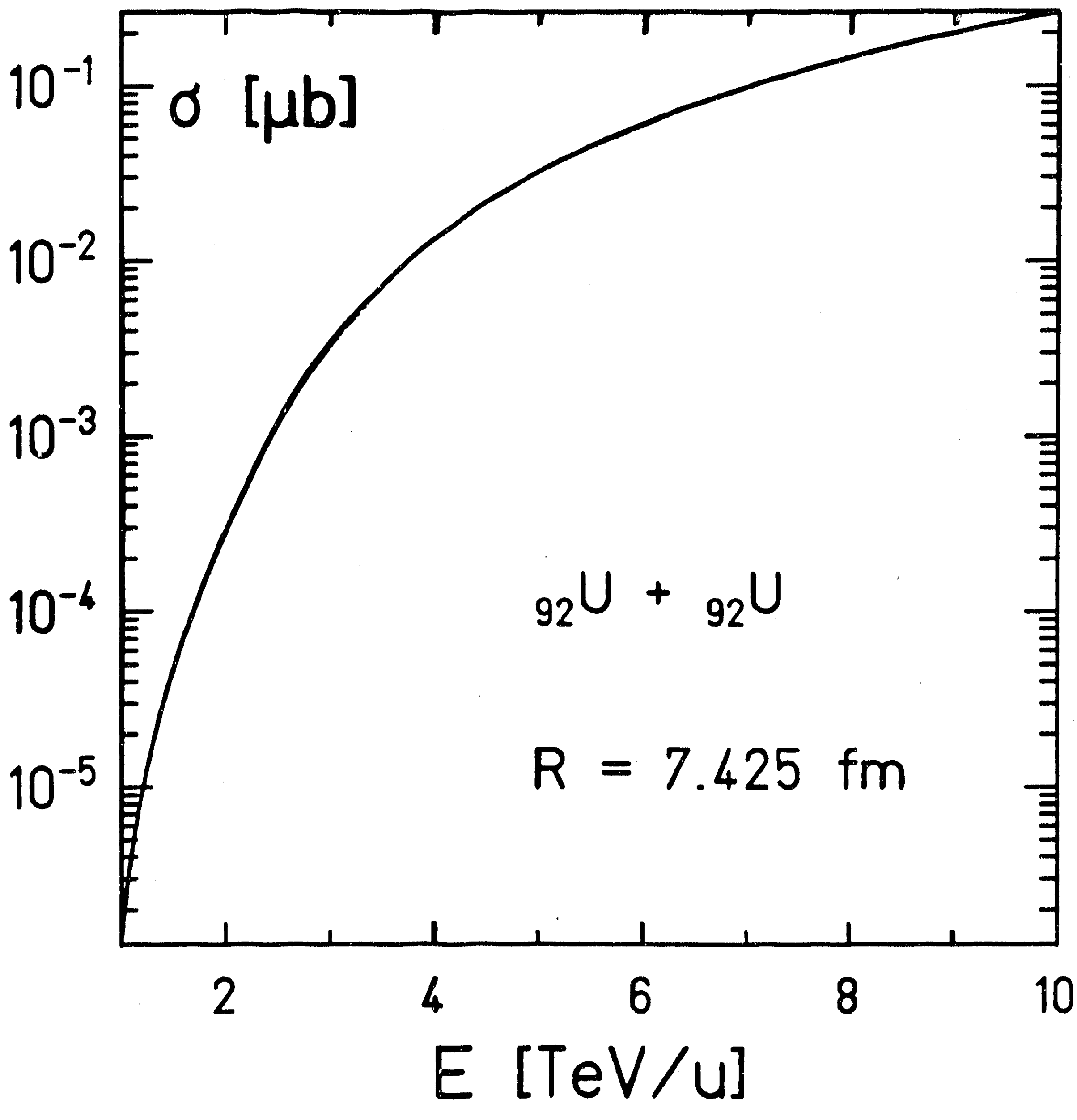
Compare to: $\sigma_{e^+e^-}^{WW} < 10$ pb for $\sqrt{s} < 10$ TeV



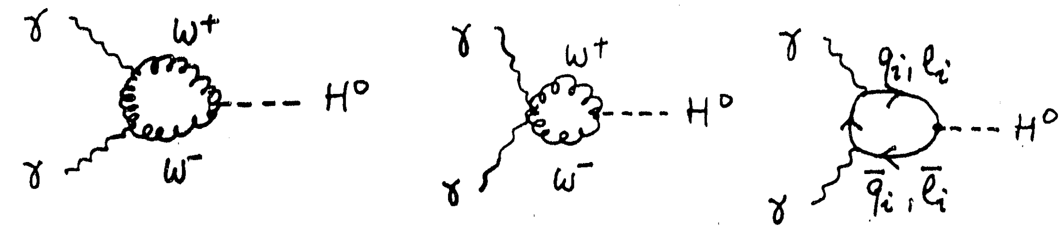
$\sigma_{pp}^{WW} \sim 10$ nb at SSC ($\sqrt{s} = 40$ TeV)

from $q\bar{q} \to W^+W^-$

$A + A \rightarrow \gamma + \gamma \rightarrow \omega^+ \omega^-$



$\gamma\gamma \longrightarrow H^0$



Couplings $q\bar{q}H$, $\ell\bar{\ell}H$, W^+W^-H
are proportional to m_q, m_ℓ bzw. M_W .

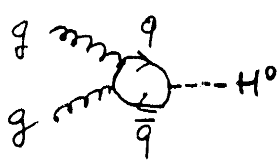
$\rightarrow$ W^+W^-H and $t\bar{t}H$ dominate.

$$\Gamma_{\gamma\gamma}^{H^0} = \frac{\alpha^2}{8\pi^3} \frac{G_F}{\sqrt{2}} m_H^3 \left| I_W + I_q + I_\ell \right|^2$$

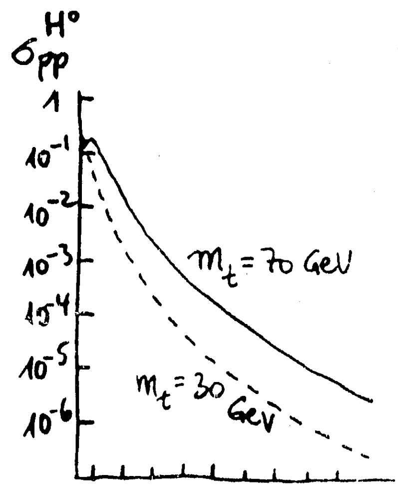
Ellis
Gaillard
Nanopoulos

$$\sigma_{UU}^{H^0} = \left\{ \begin{array}{ll} 0{,}76 \text{ nb} & (\text{LHC}) \\ 2{,}58 \text{ nb} & (\text{SSC}) \end{array} \right\} \quad \begin{array}{l} m_H = 50 \text{ GeV} \\ m_t = 50 \text{ GeV} \end{array}$$

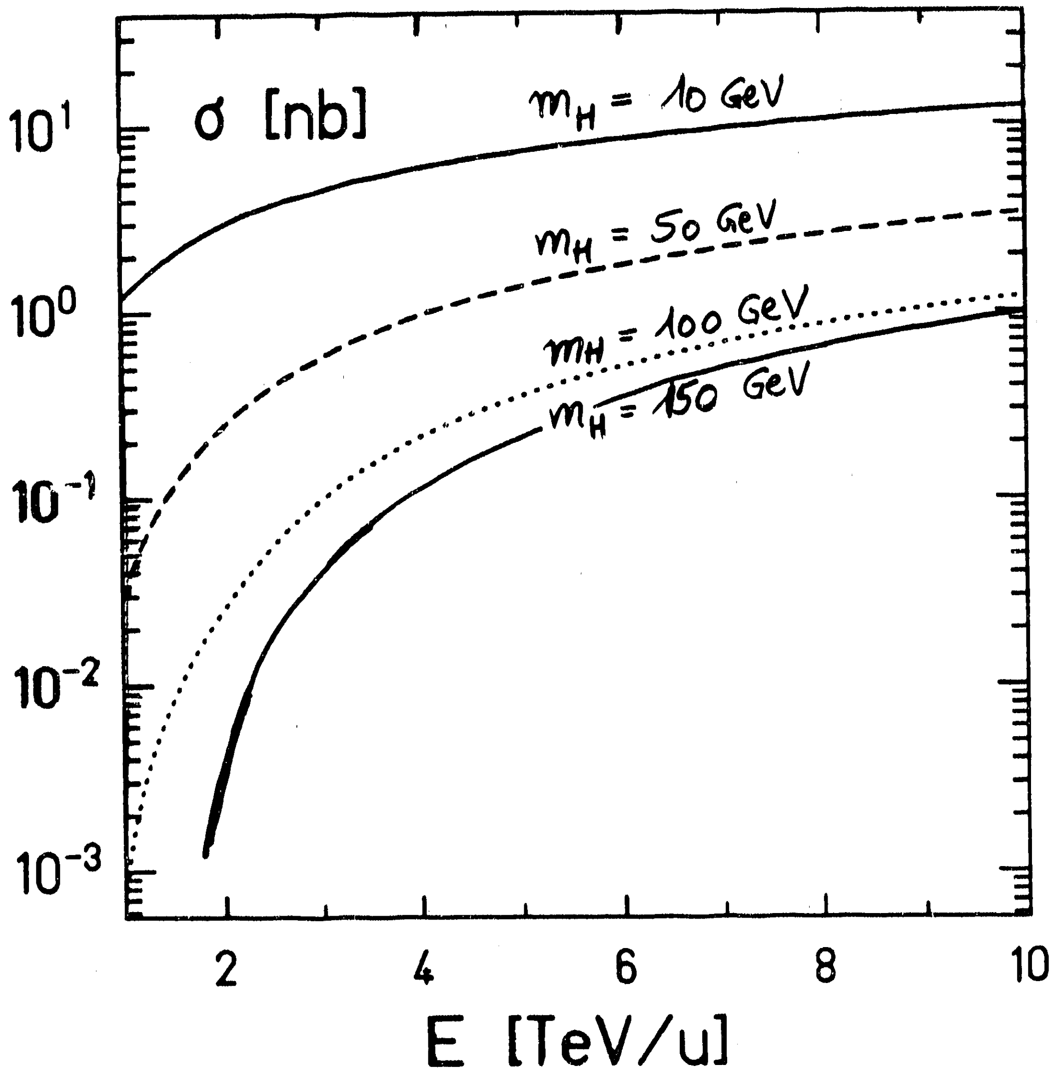
Compare to: $gg \rightarrow H^0$ in pp collisions



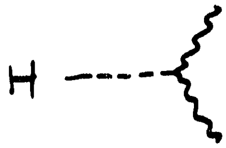
$\sigma_{pp}^{H^0} \lesssim 1$ nb at SSC energies.



$A + A \rightarrow \gamma + \gamma \rightarrow H^0$



Higgs decay and detection:



H-Coupling to other particles $\propto$ mass

$\rightarrow$ H decays into heaviest available particles

For $m_H < 2m_W, 2m_t$: $H \rightarrow b\bar{b}$

This causes major background from $gg \rightarrow b\bar{b}$

$$\sigma_{gg\rightarrow b\bar{b}}(s) \longrightarrow \frac{\pi \alpha_s^2}{3s}\left[\ln \frac{s}{m_b^2} - \frac{7}{4}\right]$$

$$\sigma_{gg\rightarrow H}(s) = \frac{G_F \pi m_H^2}{32\sqrt{2}}\left(\frac{\alpha_s}{\pi}\right)^2 \left|\sum_i I_i\right|^2 \delta(s - m_H^2)$$

Energy (inv. mass) resolution Δm

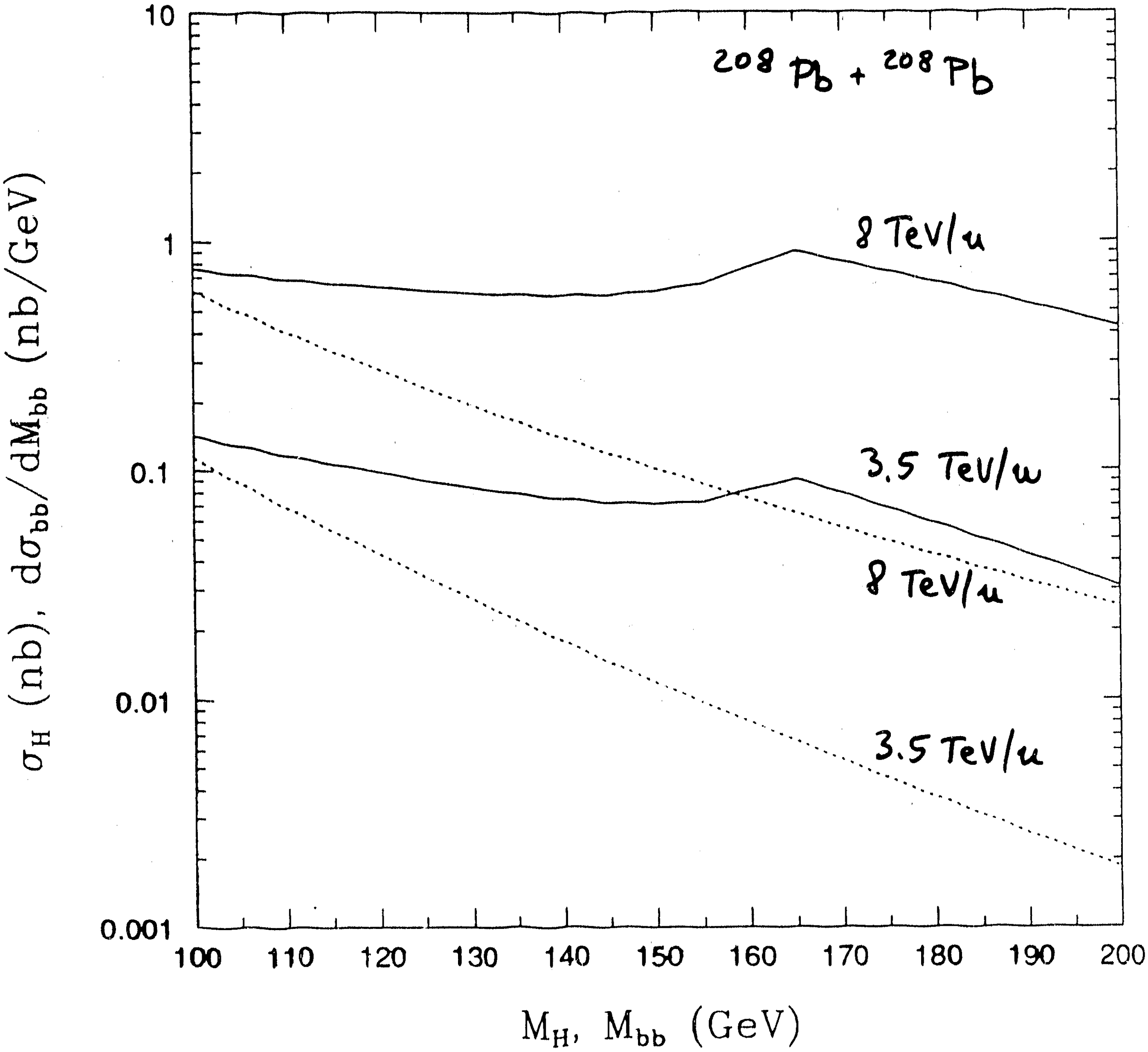
$$\frac{\int \sigma_{gg b\bar{b}}\, dm}{\int \sigma_{ggH}\, dm} = \frac{\pi^2 64\sqrt{2}\,\Delta m}{3 G_F m_H^3} \frac{\left[\ln \frac{m_H^2}{m_b^2}\right]}{\left|\sum_i \tilde{I}_i\right|^2} \approx \frac{280\,\Delta m}{\mathrm{GeV}} .$$

Impossible to achieve $\Delta m \sim 1$ MeV at 150 GeV.

Different in electromagnetic ($\gamma\gamma$) production:

$$\frac{\int \sigma_{\gamma\gamma b\bar{b}}\, dm}{\int \sigma_{\gamma\gamma H}\, dm} = \frac{\pi^2 8\sqrt{2}\,\Delta m}{27 G_F m_H^3} \frac{\left(\ln \frac{m_H^2}{m_b^2}\right)}{\left|\sum_i I_i\right|^2} \approx \frac{0.175\,\Delta m}{\mathrm{GeV}} .$$

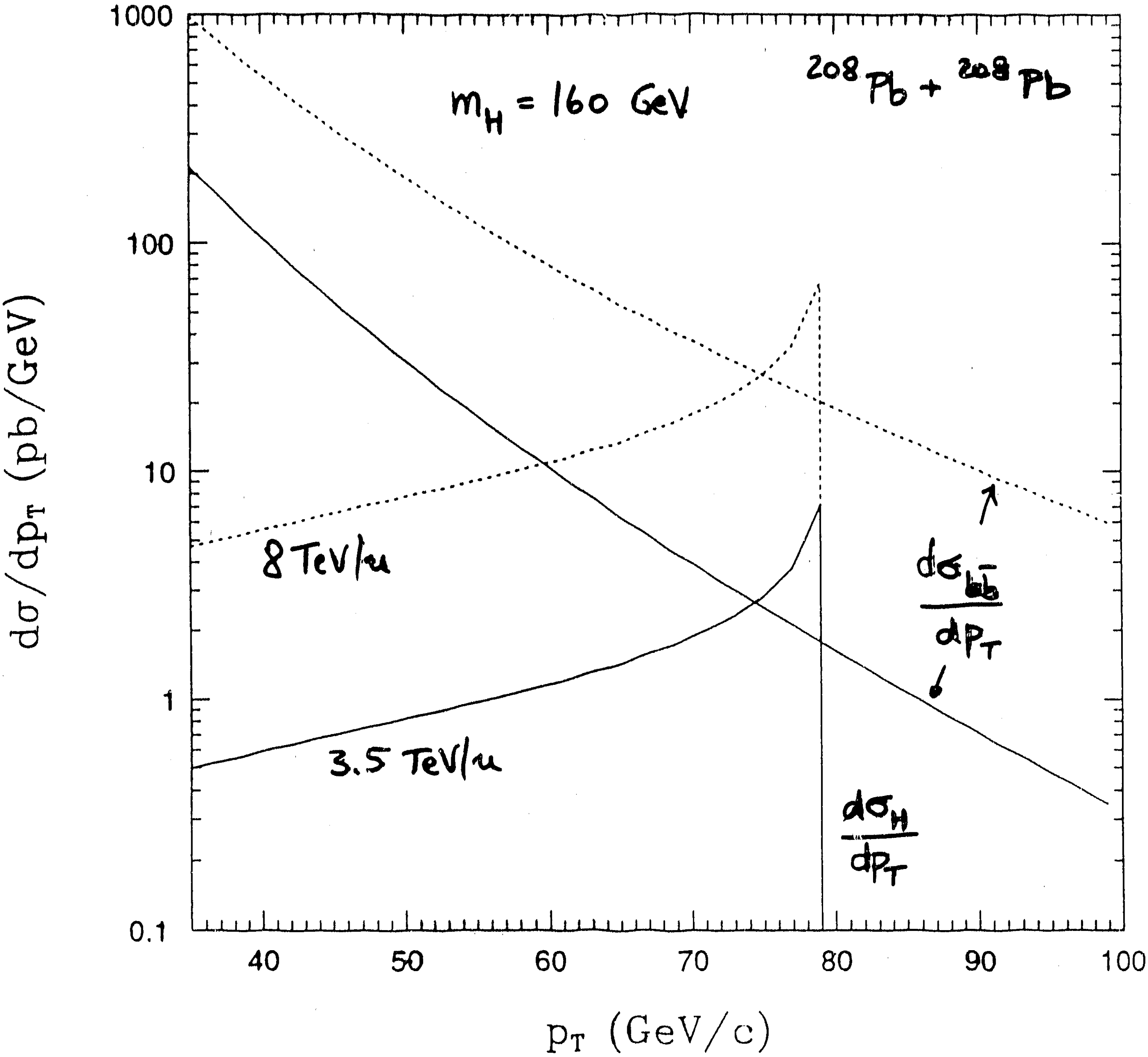
$\Delta m \sim 3$ GeV can be achieved!



—— $\sigma(\gamma\gamma \to H)$ ..... $d\sigma_{b\bar{b}}/dM_{b\bar{b}}$

—— Higgs
---- $b\bar{b}$
$\Big\}$ by $\gamma\gamma$ production

upper: 8 TeV/u
lower: 3.5 TeV/u.



$\gamma\gamma \to H, b\bar{b}$

m_H = 160 MeV

· · · · 8 TeV

—— 3.5 TeV

Impact parameter dependence hard to do:

$$\sigma_{AB} = \left(\frac{Z_A Z_B \alpha}{\pi^2}\right)^2 \int d^2b \int d\omega_A \, d\omega_B \int \frac{d^2q}{(2\pi)^2} e^{iqb} \cdot$$

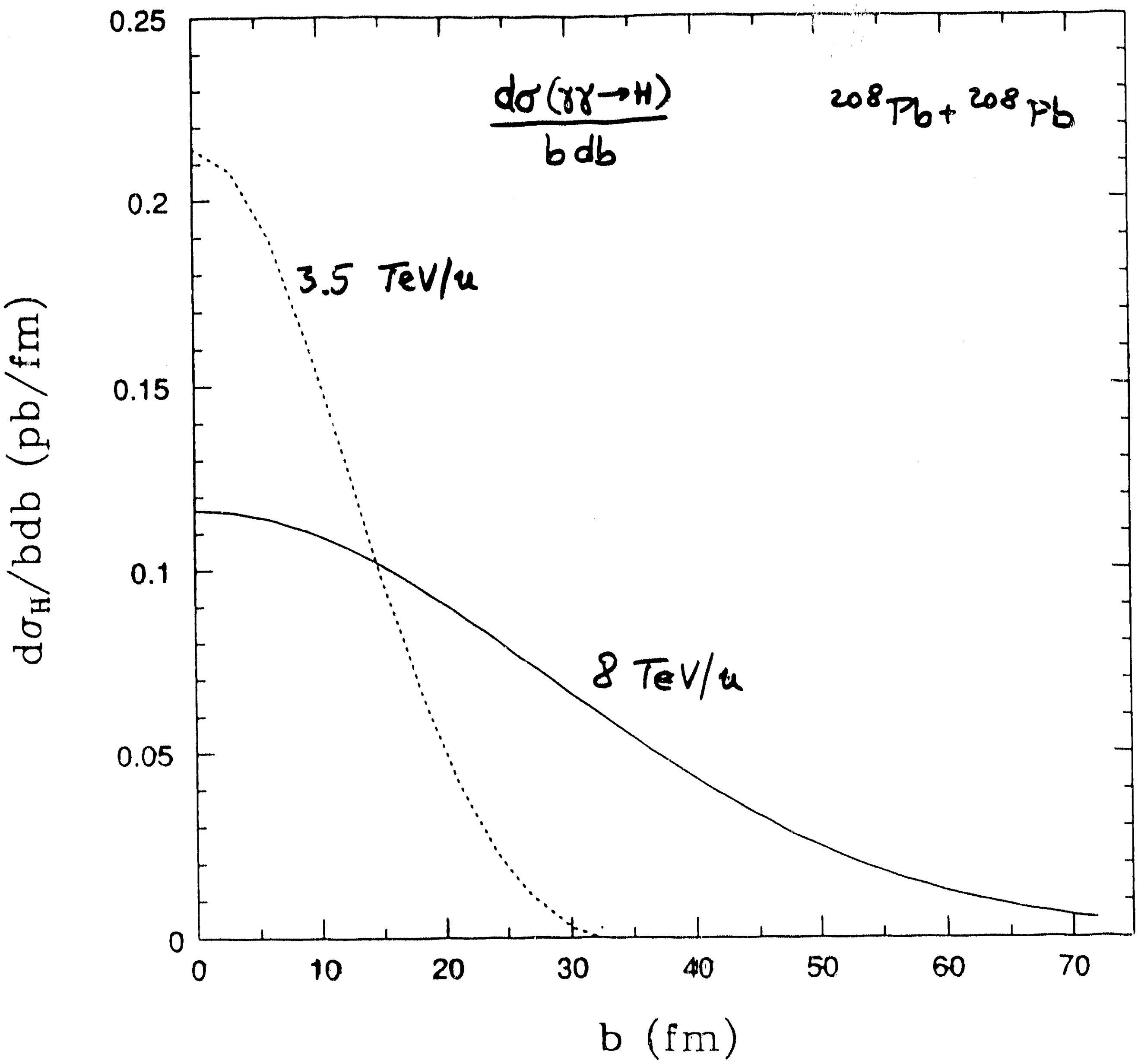
$$\cdot \, \tilde{n}_A(\omega_A, q) \, \tilde{n}_B(\omega_B, q) \, \sigma_{\gamma\gamma}(\omega_A, \omega_B)$$

$$\tilde{n}(\omega, q) = \frac{1}{\omega} \int d^2k_\perp \, k_\perp^2 \, \frac{F(-k^2)}{k^2} \, \frac{F(-(k-q)^2)}{(k-q)^2}$$

$$-k^2 = k_\perp^2 + \omega^2 \gamma^2 \, .$$

Approximation: $\frac{1}{k_\perp^2 + \omega^2\gamma^2} \approx \frac{1}{\omega^2\gamma^2} e^{-k_\perp^2/\omega^2\gamma^2}$

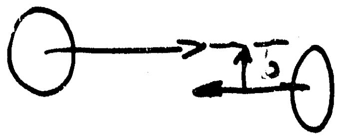
good in total σ to about 30%.



m_H = 150 GeV

······ E/A = 3.5 TeV

— E/A = 8 TeV



2 nuclei colliding at b, what is $P_{el}(b)$?

Glauber approximation:

elastic scattering amplitude

$$f(b) = e^{i\chi(b)}$$

where
$$\chi(b) = \int_{-\infty}^{\infty} dz\, U(b,z)$$

$$U(b,z) = i\sigma_{abs} \int d^3r' \rho_1(r')\, \rho_2(r-r')$$

with $\vec{r} = (z, \vec{b})$

and σ_{abs} is the absorptive N-N cross-section.

$$\chi(b) = i\sigma_{abs} A_1 A_2 \int \frac{d^2k_\perp}{(2\pi)^2} F_1(k_\perp^2)\, F_2(k_\perp^2)\, e^{ik_\perp b}$$

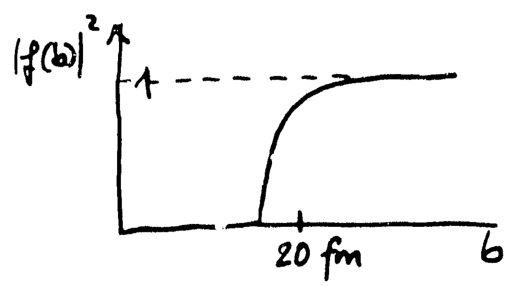
For $F_A(\vec{k}^2) = e^{-\vec{k}^2/Q_0^2}$, $A_1 = A_2 = A$:

$$\chi(b) = \frac{i}{8\pi} \sigma_{abs} A^2 Q_0^2\, e^{-\frac{1}{8}Q_0^2 b^2}$$

and

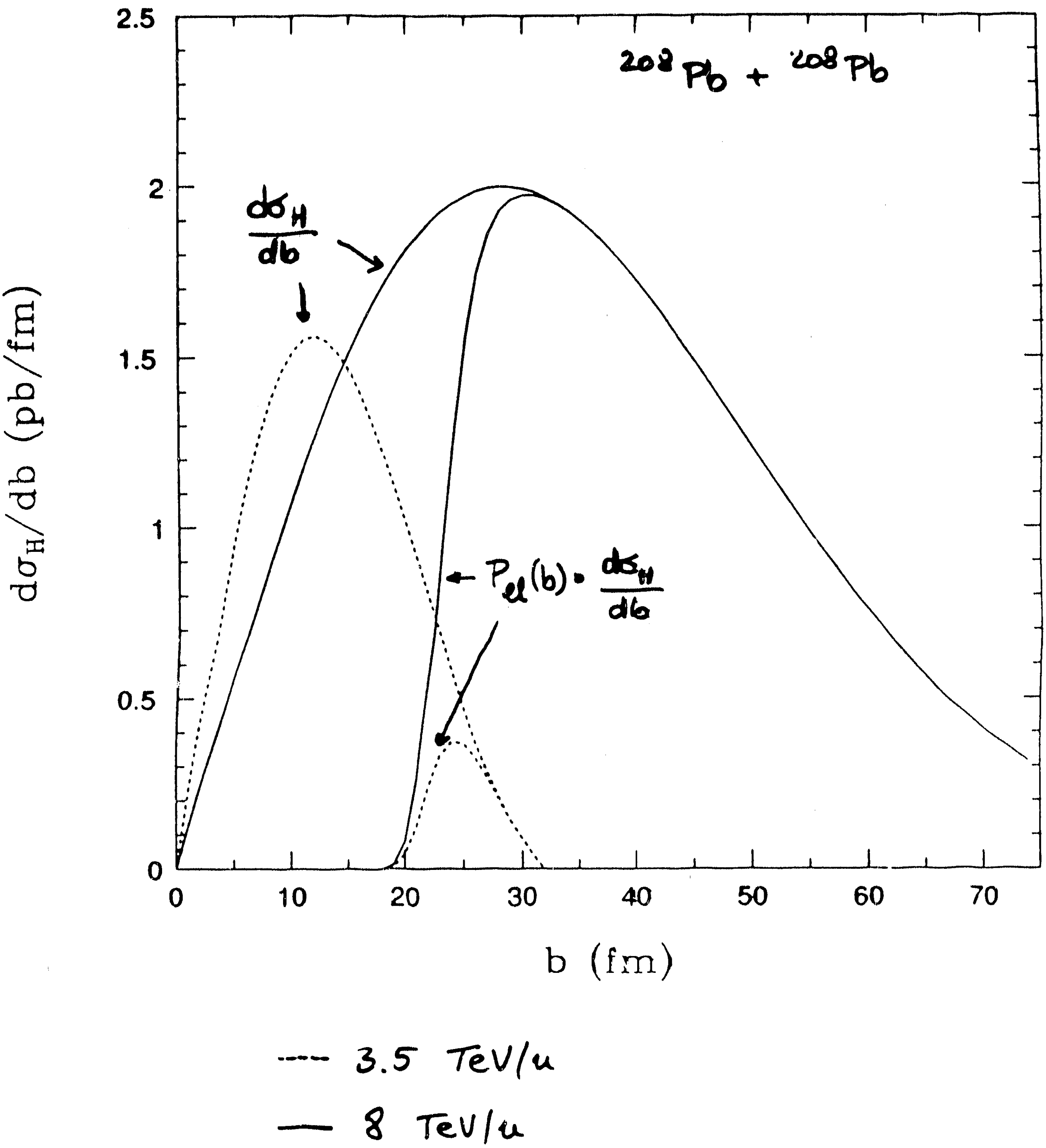
$$|f(b)|^2 = e^{-2\,\mathrm{Im}\,\chi(b)} =$$

$$= e^{-\frac{\sigma_{abs}}{4\pi} A^2 Q_0^2\, e^{-\frac{1}{8}Q_0^2 b^2}}$$



for $^{208}Pb + {}^{208}Pb$

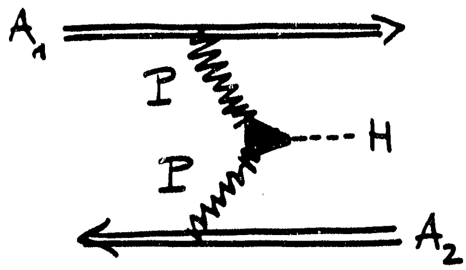
$Q_0 = 60$ MeV/c



$^{208}Pb + ^{208}Pb$
$\frac{d\sigma_H}{db}$
$P_{el}(b) \cdot \frac{d\sigma_H}{db}$
$d\sigma_H/db$ (pb/fm)
b (fm)
---- 3.5 TeV/u
— 8 TeV/u

Background due to <u>strong</u> <u>elastic</u> scattering ?

yes: Coherent double-pomeron exchange.



Two main problems :

① How does pomeron $\mathbb{P}$ Coherently couple to <u>nuclei</u> ?

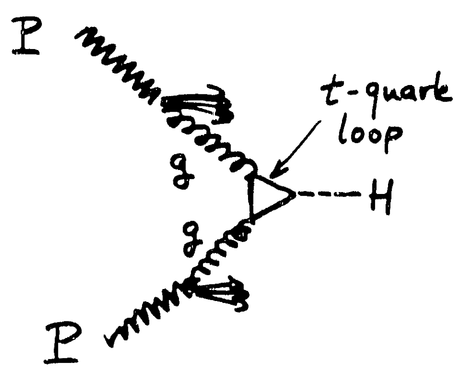
② How does $\mathbb{P}\mathbb{P} \to H$?

① Studied at ISR for d, α nuclei.
[Donnachie & Landshoff, Nucl. Phys. <u>B244</u>, 322 (84)]

$\mathbb{P}$–A coupling is additive, with nuclear elastic form factor $F_A(-k^2)$.

Additivity of $\mathbb{P}$–q coupling is related to additivity of q-q total cross section.

② Most likely : $\mathbb{P}\mathbb{P} \to gg \to H$ (+ anything)



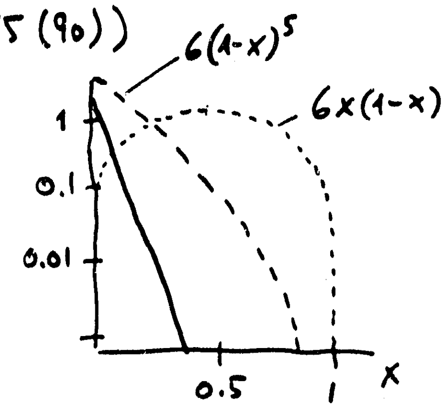
$$\sigma^{\text{incl}}_{\mathbb{P}\mathbb{P}\to H}(\tilde{s}) = \int \frac{d\bar{x}_1}{\bar{x}_1}\frac{d\bar{x}_2}{\bar{x}_2} G_{\mathbb{P}}(\bar{x}_1)\, G_{\mathbb{P}}(\bar{x}_2)\cdot \sigma_{ggH}(\bar{x}_1,\bar{x}_2,\tilde{s})$$

Do we know gluon structure function of the "pomeron", $G_P(x)$? Not well!

Exptl. information: Jet production in diffractive $N\bar{N}$ scatt.
R. Bonino et al., Phys. Lett. B211, 239 (88) – UA8
favors soft structure function $G_P(x) \sim (1-x)^5$.

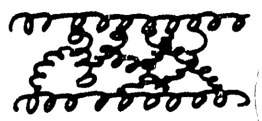
Theory is not unique:

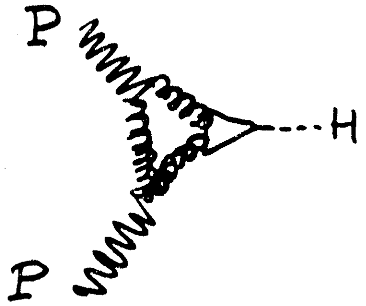
Bartels & Ingelman (P.L. B235, 175 (90))
find even much softer $G_P(x)$



Donnachie & Landshoff
(N.P. B303, 634 (88))
Claim $G_P(x) \sim 0.2\, x(1-x)$,
and there are no valence gluons.

In addition, there will be an exclusive reaction $PP \to (gg) \to H$, since the QCD-pomeron is thought to be a complicated 2 (+x) gluon state

P ∿∿∿∿ = 

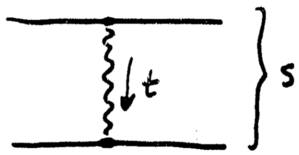


Nachtmann + Schäfer find:

$$\frac{\sigma_{PPH}}{\sigma_{PP\to H+X}} \approx 10^{-3} - 10^{-4}.$$

Range of the pomeron

Regge theory: elastic scattering amplitude



$$A_\ell(s,t) \sim \frac{s^\ell}{t - M_\ell^2}$$

assume linear "Regge trajectory"

$$\ell = \alpha'(0)\, M_\ell^2 + \alpha(0) .$$

or $$M_\ell^2 = \frac{\ell - \alpha(0)}{\alpha'}$$

Then $\sum_\ell A_\ell(s,t)$ can be resummed to

$$A_R(s,t) \sim \frac{(s/m^2)^{\alpha(t)}}{e^{\pi\alpha(t)} + \eta}$$

where $\eta = \pm 1$ is the "signature" factor.

Regge trajectories exist for all internal quantum no.s

Dominant at small t for high c.m. energy s is R. traj. with largest $\alpha(0)$ $\longrightarrow$ Pomeranchuk traj. or "pomeron" ($\eta = -1$).

Formally $A_R(s,t) \longleftrightarrow$ exchange of many diff. particles

Hence, interaction in coordinate space is a coherent superposition of many Yukawa forces.

Pomeron "propagator"

$$D_P(t) \sim \frac{1}{s} A_P(s,t) \sim e^{-\frac{\pi}{2}\alpha_P(t)} \frac{(s/m^2)^{\alpha_P(t)-1}}{\sin \frac{\pi}{2}\alpha_P(t)}$$

For spacelike momenta $\vec{k}$: $t = -\vec{k}^2$, i.e.

$$\alpha_p(t) \longrightarrow \alpha_p(0) - \alpha_p' \vec{k}^2 = 1 + \epsilon - \alpha_p' \vec{k}^2$$

$$D_p(t) \sim \frac{e^{-\pi/2\, \alpha_p(-\vec{k}^2)}}{\sin\frac{\pi}{2}\alpha_p(-\vec{k}^2)}\, e^{\ln(s/m^2)(\epsilon - \alpha_p' \vec{k}^2)}$$

For large $\alpha_p' \cdot \ln\frac{s}{m^2}$, fall-off $D(-\vec{k}^2) \xrightarrow{|\vec{k}|\to\infty} 0$ is dominated by second part. Variation of first factor is negligible. Then pomeron distribution in nucleus A:

$$n_p(r) = \frac{1}{(2\pi)^3}\int d^3k\, e^{i\vec{k}\cdot\vec{r}}\, D_p(-\vec{k}^2)\, F_A(\vec{k}^2)$$

$$= \frac{(s/m^2)^\epsilon}{(4\pi\alpha_p' \ln s/m^2)^{3/2}} \int d^3r'\, \rho_A(r') \exp\left[-\frac{(r-r')^2}{4\alpha_p' \ln(s/m^2)}\right]$$

"Range" of the distribution

$$r_0^2 \approx \left[\alpha_p' \ln(s/m^2)\right]^{-1}$$

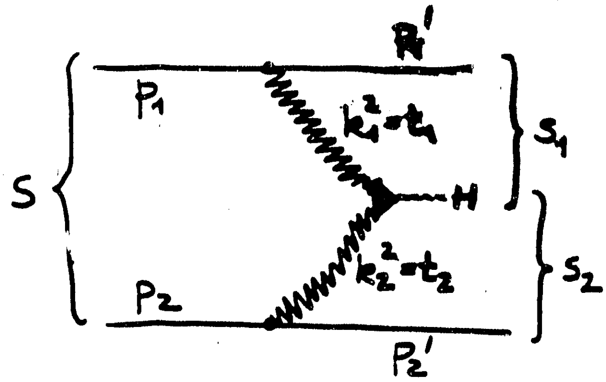
For $\sqrt{s} = 7\cdot A$ TeV, $A = 208$, $m = 1$ GeV, $\alpha_p' = 0.25\ \mathrm{GeV}^{-2}$

$$r_0 \approx 0.4\ \mathrm{fm}.$$

[Compare to range of "last" neutron, ~ 0.8 fm.]

Higgs production by D.P.E.

Kinematics:



$$p_i = (E_i\,,\ p_{i\parallel}\,,\ \vec{0}\,)$$

$$p_i' = (E_i'\,,\ p_{i\parallel}'\,,\ -\vec{k}_{i\perp})$$

$$k_i = (\omega_i\,,\ k_{i\parallel}\,,\ \vec{k}_{i\perp}\,)$$

$$\left.\begin{aligned} \omega_i &= E_i - E_i' \equiv x_i E_i \\ k_{i\parallel} &\approx \frac{E_i}{p_{i\parallel}} x_i E_i \end{aligned}\right\} \ \omega_i^2 - k_{i\parallel}^2 \approx -x_i^2 M_i^2$$

$$-k_i^2 \equiv -t_i = x_i^2 M_i^2 + k_{i\perp}^2$$

$$s_1 = (p_1 + k_2)^2 \approx x_2 s + M_1^2$$

$$s_2 = (p_2 + k_1)^2 = x_1 s + M_2^2$$

volume element:

$$\frac{dx_1}{x_1} d^2k_{1\perp}\ \frac{dx_2}{x_2} d^2k_{2\perp} = \pi^2\, dt_1\, dt_2\, \frac{ds_1}{s_1}\,\frac{ds_2}{s_2}$$

Inclusive diffractive Higgs production

$$\sigma_{AA}^{H}(s) = \int dx_1\, dx_2\, f_{\mathbb{P}}^{A}(x_1)\, f_{\mathbb{P}}^{A}(x_2)\, \sigma_{pp\to H}^{incl}(x_1 x_2 s)$$

Pomeron distribution in nucleus A (coherent)

$$f_{\mathbb{P}}^{A}(x) = \frac{1}{16\pi x} \int_{-\infty}^{-x^2M^2} dt\, |\beta_{\mathbb{P}}^{A}(t)|^2\, |D_{\mathbb{P}}(t)|^2$$

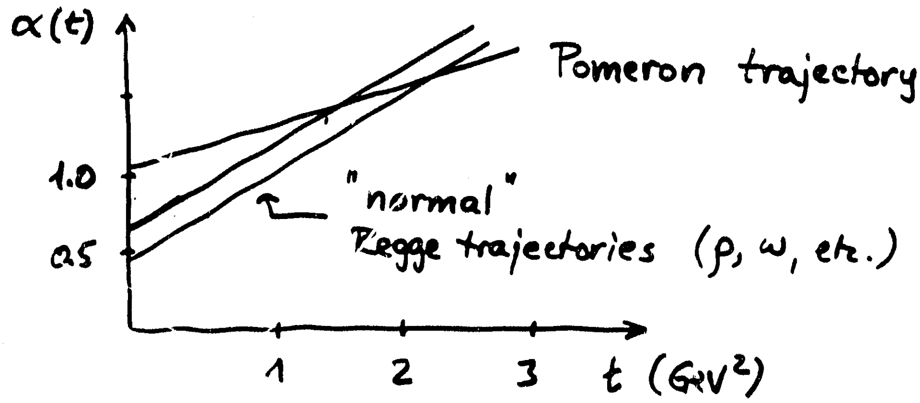
$$f_{I\!P}^{A}(x) = \frac{1}{16\pi x} \int_{-\infty}^{-x^2M^2} dt \, |\beta_{I\!P}^{A}(t)|^2 \, |D_{I\!P}(t)|^2$$

Pomeron "propagator":

$$D_{I\!P}(t) = \frac{(s/m^2)^{\alpha_{I\!P}(t)-1}}{\sin \frac{\pi}{2}\alpha_{I\!P}(t)} \, e^{-i\frac{\pi}{2}\alpha_{I\!P}(t)} \qquad (m = 1\ \text{GeV})$$

with Regge trajectory of pomeron

$$\alpha_{I\!P}(t) = \alpha_{I\!P}(0) + \alpha_{I\!P}' t$$
$$\simeq 1.08 + 0.25\ \text{GeV}^{-2} \cdot t$$



$\alpha_{I\!P}(0) \approx 1 \longrightarrow$ pomeron act like $(C=+1)$ photon.

Pomeron coupling:

$$\beta_{I\!P}^{N}(t) = \beta_0 \, e^{\frac{1}{2}R_0^2 t}$$

$$\beta_0 = \sqrt{\sigma_{NN}} \approx 6.3\ \text{mb}^{1/2}$$
$$R_0^2 = 6.5\ \text{GeV}^{-2} = (0.5\ \text{fm})^2$$

<u>Assume</u>:

$$\beta_{I\!P}^{A}(t) = A \, \beta_0 \, F_A(-t)$$

[o.k. for d, α, but for ^{208}Pb ?]

With $\quad F_A(-t) = e^{t/Q_0^2}$, $\quad Q_0 \approx 60$ MeV/c

we can approximate $\quad \alpha_{\mathbb{P}}(t) \to \alpha_{\mathbb{P}}(0) = 1 + \epsilon$.

Then:

$$\sigma_{AA}^{H} \cong \frac{(A\beta_0 Q_0)^4}{4(16\pi)^2} \int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\left(\frac{x_1x_2s^2}{m^4}\right) e^{-2(x_1^2+x_2^2)M^2/Q_0^2} \cdot \sigma^{incl}_{\mathbb{P}\mathbb{P}\to H}(x_1x_2s)$$

$$\sigma^{incl}_{\mathbb{P}\mathbb{P}\to H}(\tilde{s}) = \frac{\pi^2\Gamma(H\to gg)}{8M_H\tilde{s}} \int \frac{dz}{z} G_{\mathbb{P}}(z)\, G_{\mathbb{P}}\left(\frac{m_H^2}{z\tilde{s}}\right)$$

A - scaling: $\quad$ with $\quad Q_0 \sim \frac{1}{R} \sim A^{-1/3}$

$$\sigma_{AA}^{H} \cong A^{8/3} \sim (A_1A_2)^{4/3}$$

[This does not yet include absorption effects!]

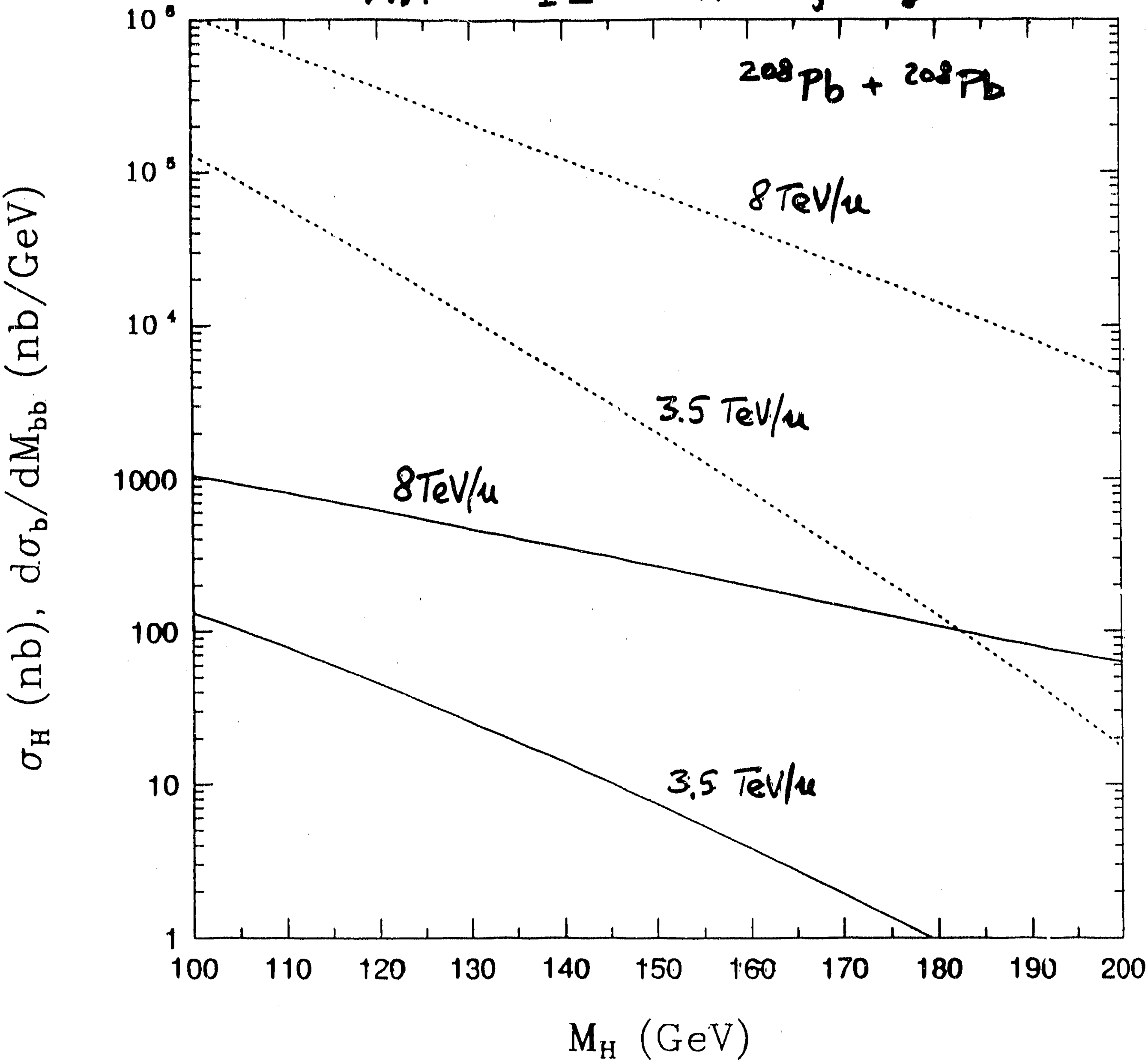
Impact parameter dependence (as for $\gamma\gamma$):

$$\frac{d\sigma_{AA}^{H}}{bdb} \sim e^{-\frac{1}{4}Q_0^2b^2}$$

$$\int bdb\; P_{el}(b)\, \frac{d\sigma_{AA}^{H}}{bdb} \approx \sigma_{AA}^{H} \cdot 2\Big(\underbrace{\frac{\sigma_{abs}Q_0^2A^2}{4\pi}}_{10^{-6}\text{ for } \sigma_{abs}=30\text{ mb},\ A=208}\Big)^{-2}$$

Altogether: $\quad \sim \dfrac{(R_1^2+R_2^2)^2}{R_1^2R_2^2} = \dfrac{(A_1^{2/3}+A_2^{2/3})^2}{A_1^{2/3}A_2^{2/3}}$!?

AA → PP → H + anything

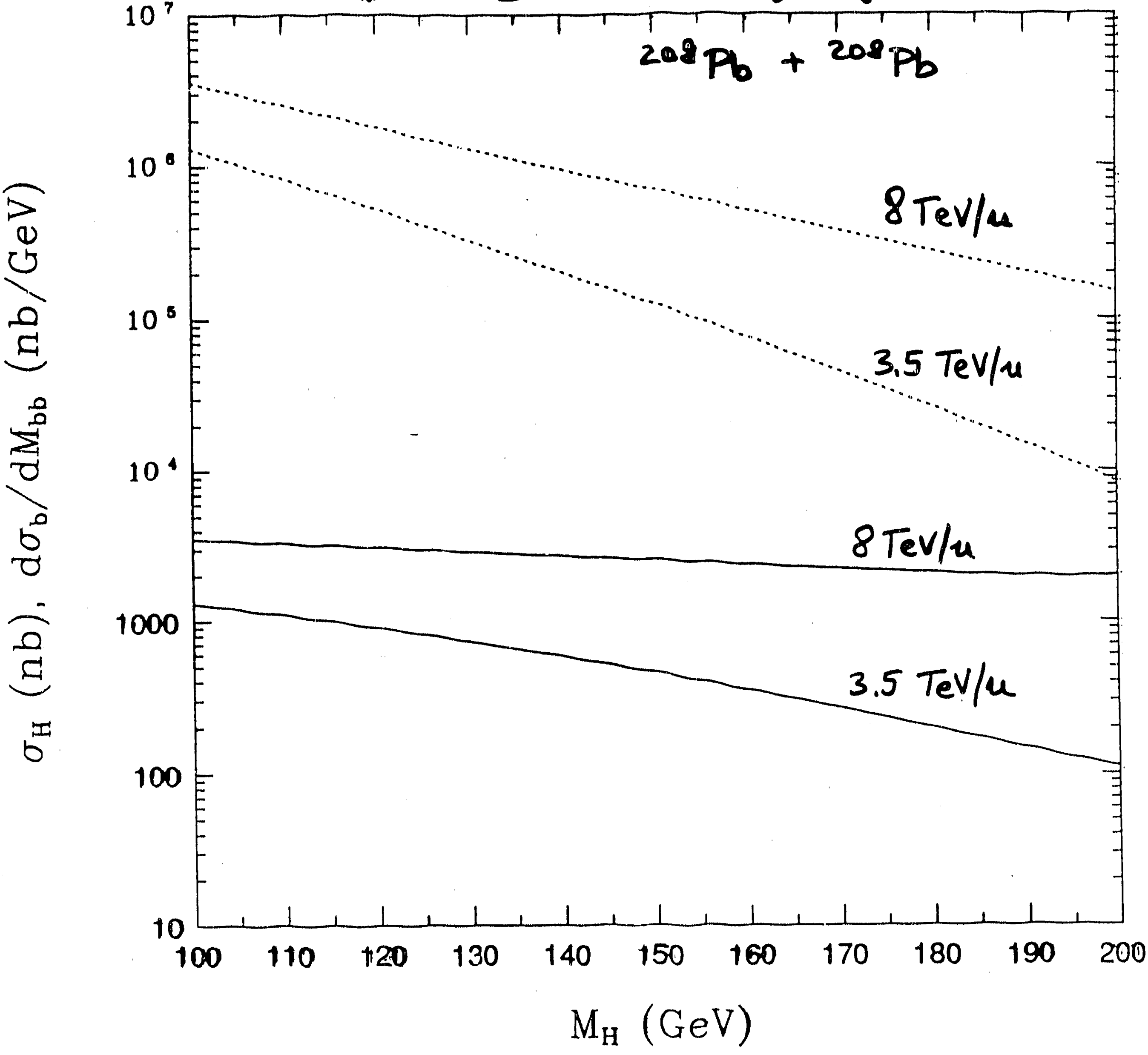


— σ_H ····· $d\sigma_{b\bar{b}}/dM_{b\bar{b}}$

$$G_P(x) = 6(1-x)^5$$

Pomeron à la Berger et al.
$G_P(x) = 6(1-x)^5$

AA → PP → H + anything

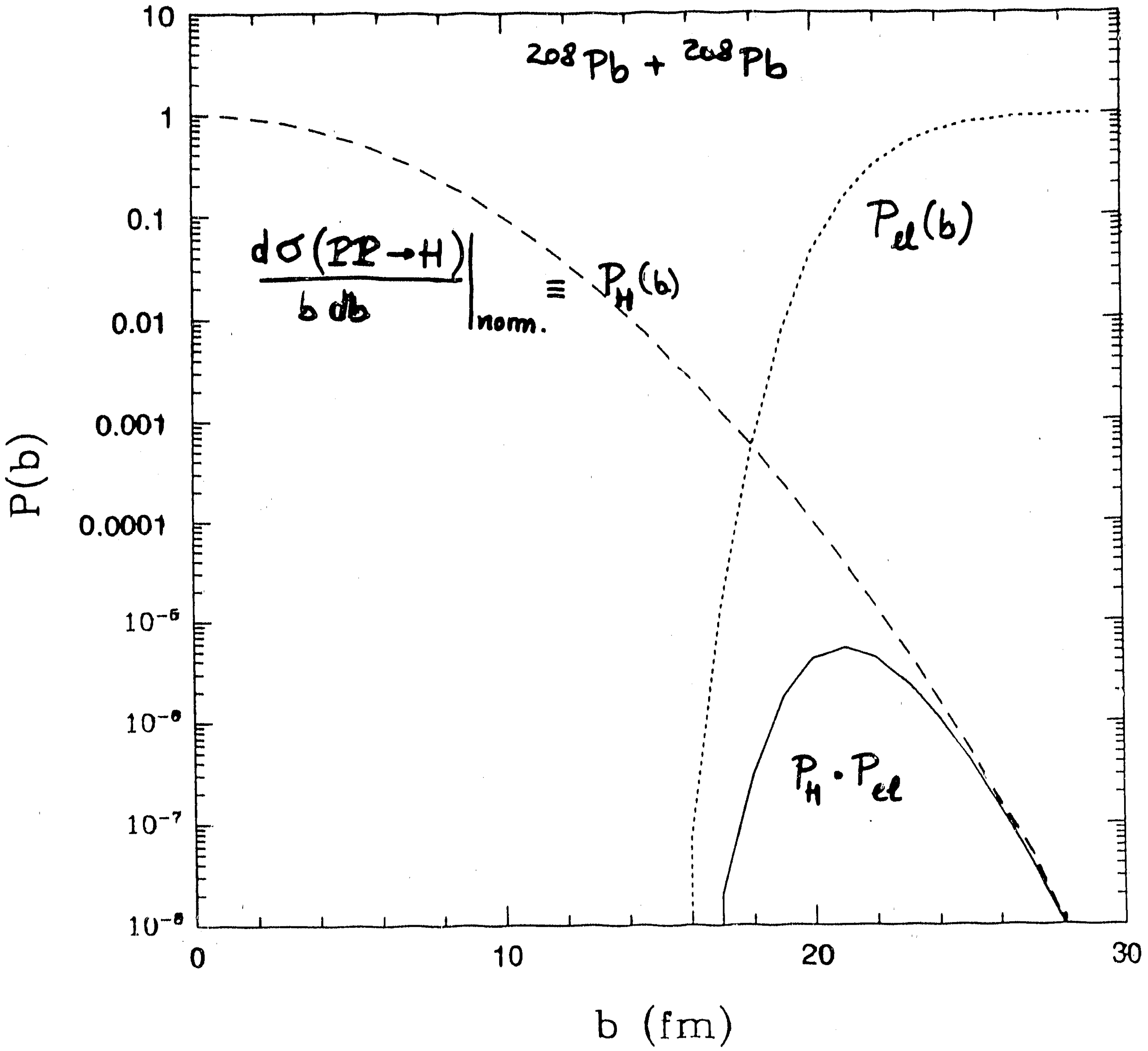


— σ_H ..... $d\sigma_{b\bar{b}}/dM_{b\bar{b}}$

$$G_P(x) = 6x(1-x)$$

Pomeron à la Berger et al.

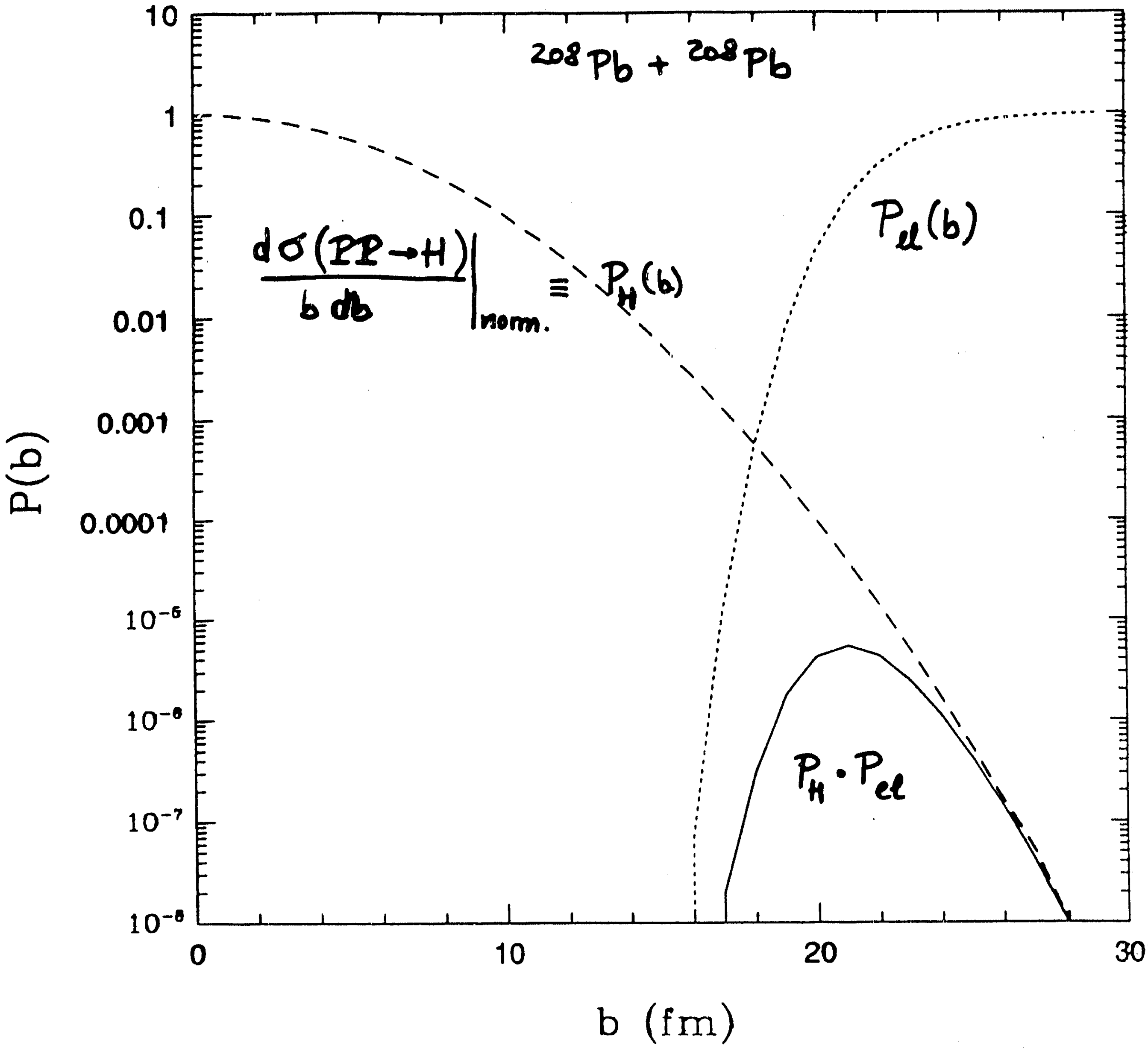
$G_P(x) = 6x(1-x)$



$\sigma_{abs}^{NN} = 10$ mb.

$f_A(\vec{k}^2) = e^{-\vec{k}^2/Q_0^2}$

$Q_0 = 60$ MeV/c



$\sigma_{abs}^{NN} = 10$ mb.

$f_A(\vec{k}^2) = e^{-\vec{k}^2/Q_0^2}$

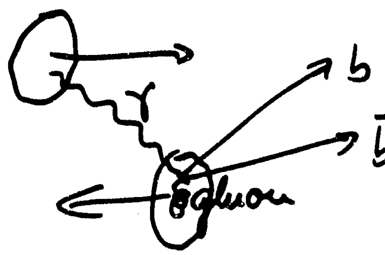
$Q_0 = 60$ MeV/c

Conclusion:

- b-dependence of σ_{AA}^{H} must be studied more carefully!

- Inelastic scattering effects?

  main electromagnetic background



photon-gluon fusion.

must be calculated.

$$\omega_{max}^{beam} = \gamma\,\omega_{max}^{cm} = \gamma^2 \frac{1}{R} \sim (3500)^2\, 30\ \mathrm{MeV} \sim 300\ \mathrm{TeV}.$$

(at RHIC: $\omega_{max}^{beam} \sim (100)^2\, 30\ \mathrm{MeV} \sim 300\ \mathrm{GeV}$.

W–BOSON MAGNETIC MOMENT IN RELATIVISTIC HEAVY–ION COLLISIONS

Talk presented by:

G. Couture

W-Boson Magnetic Moment
in
R. H. I. Collisions.

I) Definition of K

II) Consider the Process

$$Pb + Pb \rightarrow \gamma\gamma \rightarrow W^+ W^-$$

* γ- spectrum
  - effect of nuclear charge form factor
  - effect of impact parameter

* sensitivity of the process to K
  - How well one could measure K

III Conclusions

I. DEFINITION OF κ

IN AN effective $\mathcal{L}$ FORMALISM:

* $\gamma W^+ W^-$ is PARAMETRIZED BY 7 FORM FACTORS

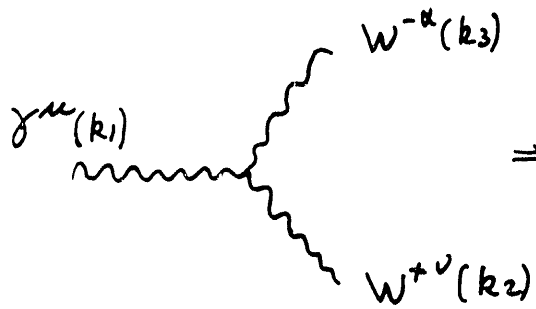
* CP NOT BROKEN AND OPERATORS OF DIMENSION 4 OR LESS

$\Rightarrow$ 2 FORM FACTORS ARE LEFT

$\Rightarrow$ SET ONE OF THEM TO 1 FOR NORMALIZATION $\Rightarrow$

$$\mathcal{L} = -ieA^\mu (W^{-\mu\nu} W_\nu^+ - W^{+\mu\nu} W_\nu^-)$$

$$+ie\underline{\kappa} F^{\mu\nu} W^{+\mu} W^{-\nu} \qquad \text{"As in the S.M."}$$



$$\Rightarrow -ie\{(\underline{\kappa}k_1 - k_2)^\alpha g^{\mu\nu} + (k_2 - k_3)^\mu g^{\nu\alpha} + (k_3 - \underline{\kappa}k_1)^\nu g^{\mu\alpha}\}$$

* in the S.M., $\kappa = 1$

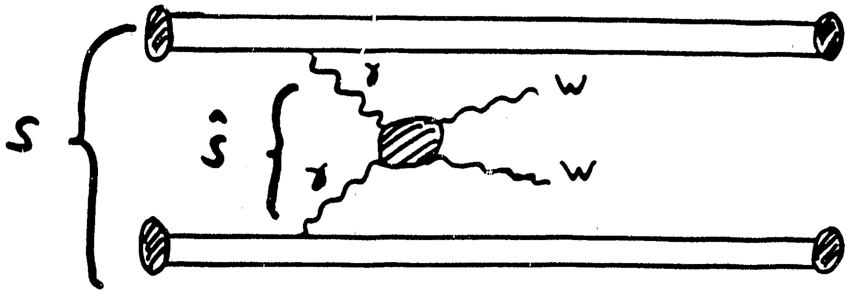
* AS IT IS: $\mathcal{M} = (1+\kappa)\left(\frac{e}{2M_W}\right)$ AND $Q = -\frac{e\kappa}{M_W^2}$

MEASURING κ ~ MEASURING $(g-2)_W$

I Consider

$$Pb + Pb \to \gamma\gamma \to W^+ W^-$$

$$\sigma = \int dx_1\, dx_2\, f(x_1)\, f(x_2) \int d\hat{s}\, \delta(x_1 x_2 S - \hat{s})\, \hat{\sigma}(\gamma\gamma \to W^+ W^-)\, \underline{\underline{Z_1^2 Z_2^2}}$$



* Coherent γ emission
* Use E.P.A.

__Advantage:__ $\gamma W^+ W^-$ vertex enters twice in the amplitude $\Rightarrow$

one term $\sim \kappa^4$ in σ

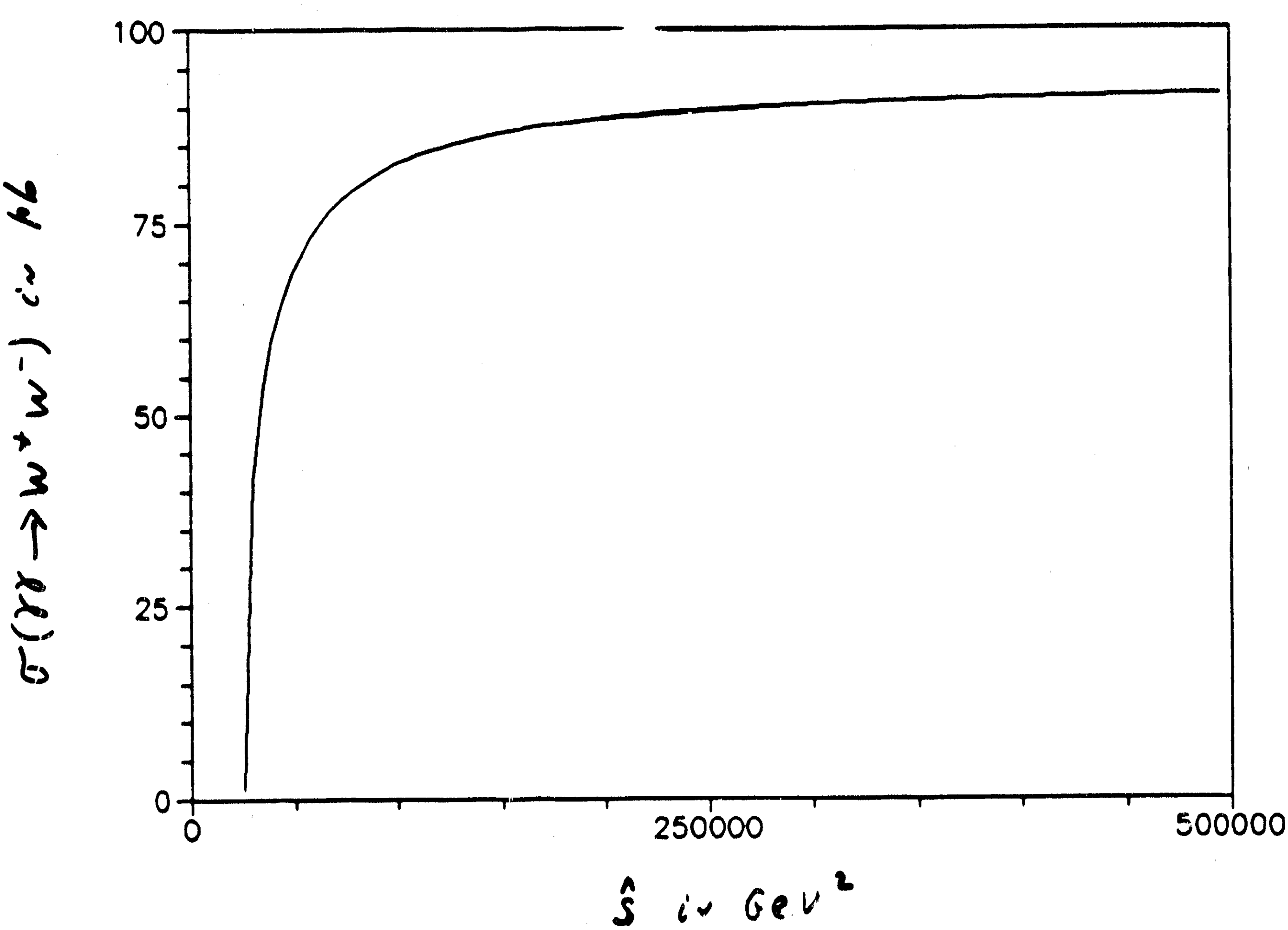
Rough estimate: Δ in $\kappa \Rightarrow \sim 2\Delta$ in σ

A) $$\hat{\sigma}(\gamma\gamma \to W^+ W^-) = \frac{4\pi\alpha^2}{\hat{s}} \sqrt{1-4R}$$

$$\left\{ \frac{1}{R}\left(2 + 3/2\, R + 6R^2\right) + \frac{6R - 12R^2}{\sqrt{1-4R}} \ln\left(\frac{1-\sqrt{1-4R}}{1+\sqrt{1-4R}}\right) \right\}$$

$$R \equiv \frac{M_W^2}{\hat{s}}$$

Note: $\hat{\sigma} \longrightarrow \dfrac{8\pi\alpha^2}{M_W^2} \simeq 93\ \text{pb}$ as $\hat{s} \to \infty$


100
75
50
25
0
0
250000
500000
$\hat{s}$ in GeV^2
$\sigma(\gamma\gamma \to W^+W^-)$ in pb

γ- SPECTRUM

~~If~~ $x \equiv \frac{E_\gamma}{E_{ion}}$ And M = MASS OF THE ION

$$f(x) = \frac{\alpha}{\pi} \int_{x^2 M^2}^{\infty} dq^2 \frac{|F(q^2)|^2}{q^2 x} \left\{ 1 - \frac{x^2 M^2}{q^2} \right\}$$

with $F(q^2) = \frac{4\pi}{q} \int_0^\infty r\, \rho(r) \sin(qr)\, dr$

= F.T. OF NUCLEAR CHARGE.

A) $\rho(r) = \theta(R - r)$

$$\left\{ \begin{array}{l} \text{SOFF, RAU,} \\ \text{GRABIAK,} \\ \text{MUELLER, GREINER} \end{array} \right.$$

$$F(q^2) = 3 \frac{j_1(qR)}{qR}$$

$$f(x) \propto -\left\{ 1 + \frac{x^2}{5} \right\} \ln(x) + \sum_{n=0}^{\infty} c_n x^{2n}$$

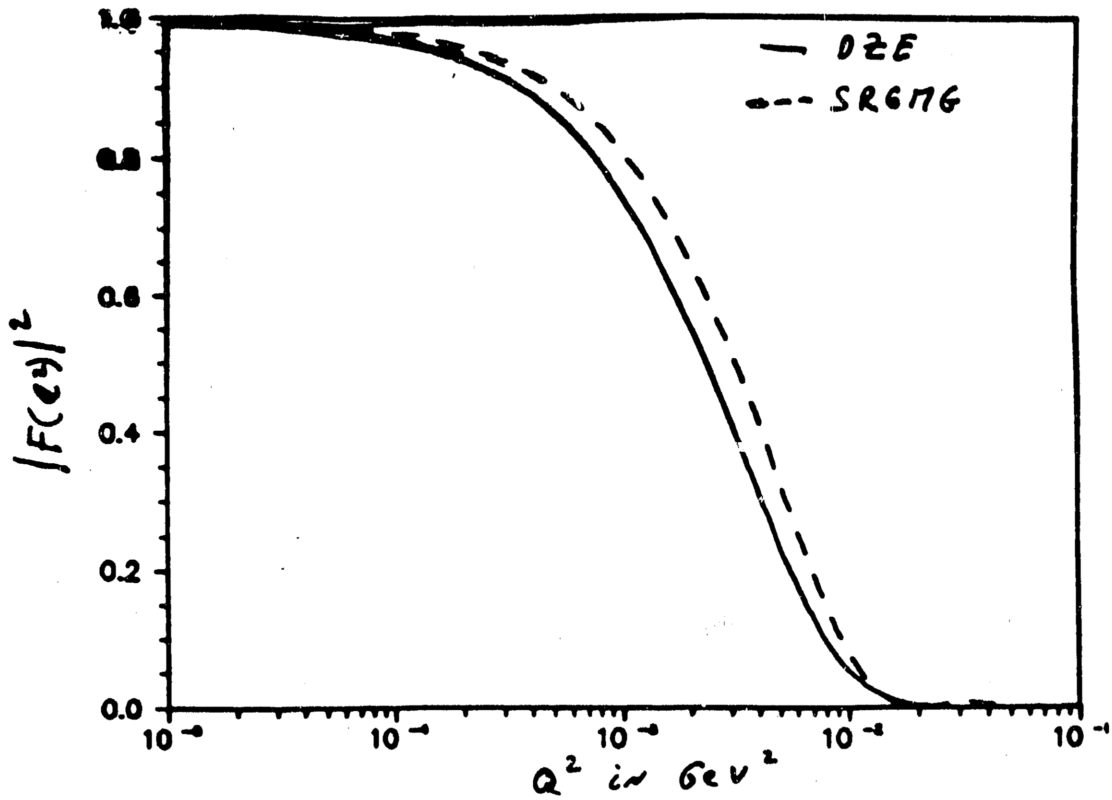
---

b) $\rho(r) = \rho_0 \left[1 + \exp\left(\frac{r - c}{a} \right) \right]^{-1}$ c, a ARE FIT WITH DATA $\left\{ \begin{array}{l} \text{DREES} \\ \text{ELLIS} \\ \text{ZEPPENFELD} \end{array} \right.$

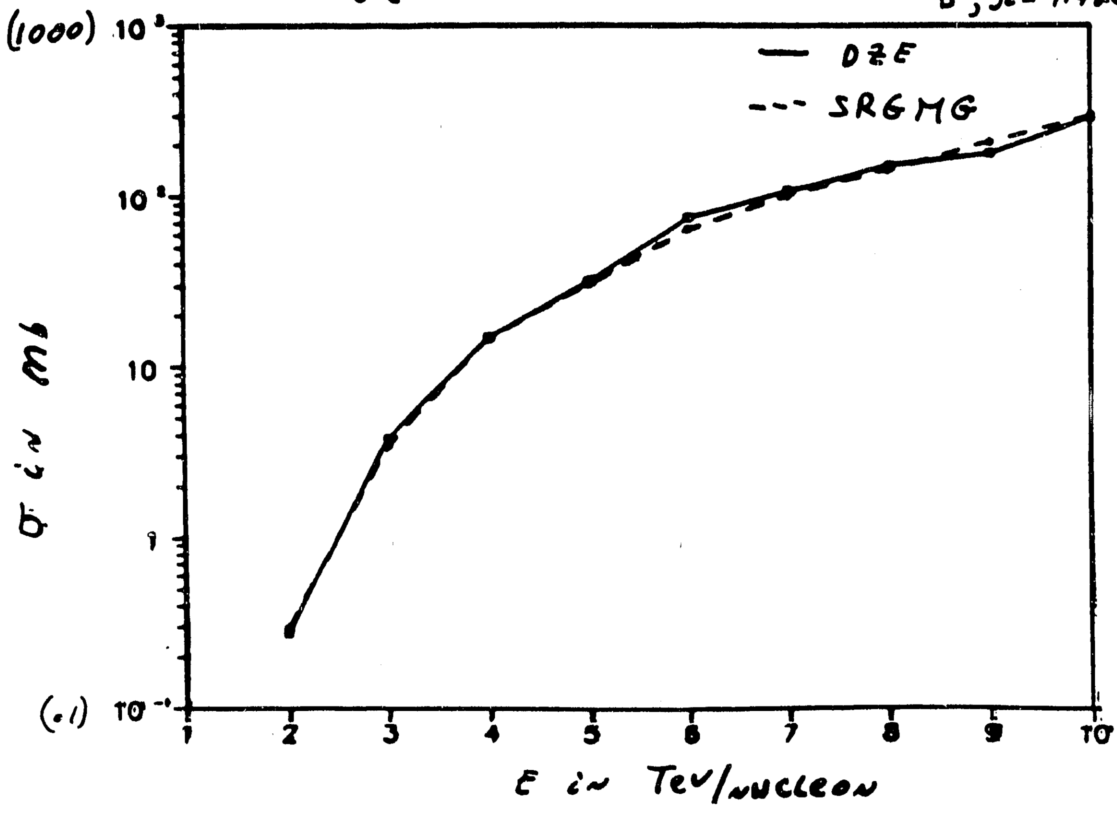
$$|F(q^2)|^2 = \exp(-q^2/Q_0^2)$$

$$f(x) = \left(\frac{\alpha}{\pi} \right) \left(\frac{1}{x} \right) \left\{ (1 + Y^2)\, E_i(Y^2) - \exp(-Y^2) \right\} \qquad Y^2 \equiv \frac{x^2 M^2}{Q_0^2}$$

$$E_i(Y) = \int_Y^\infty \frac{e^{-t}}{t}\, dt$$



$\sigma(U+U\to\gamma\gamma\to W^+W^-)$ ^{238}U, $R = 7.425$ fm



effect of impact parameter

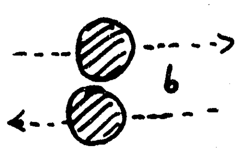
Up to now, the two nuclei are allowed to collide

If one wants a clean signal one must require

that the nuclei do not physically collide.

Therefore: As far as the nuclei are concerned, the collision is classical; with impact parameter $b \geq 2R$

CAHN, JACKSON, MARCH 90



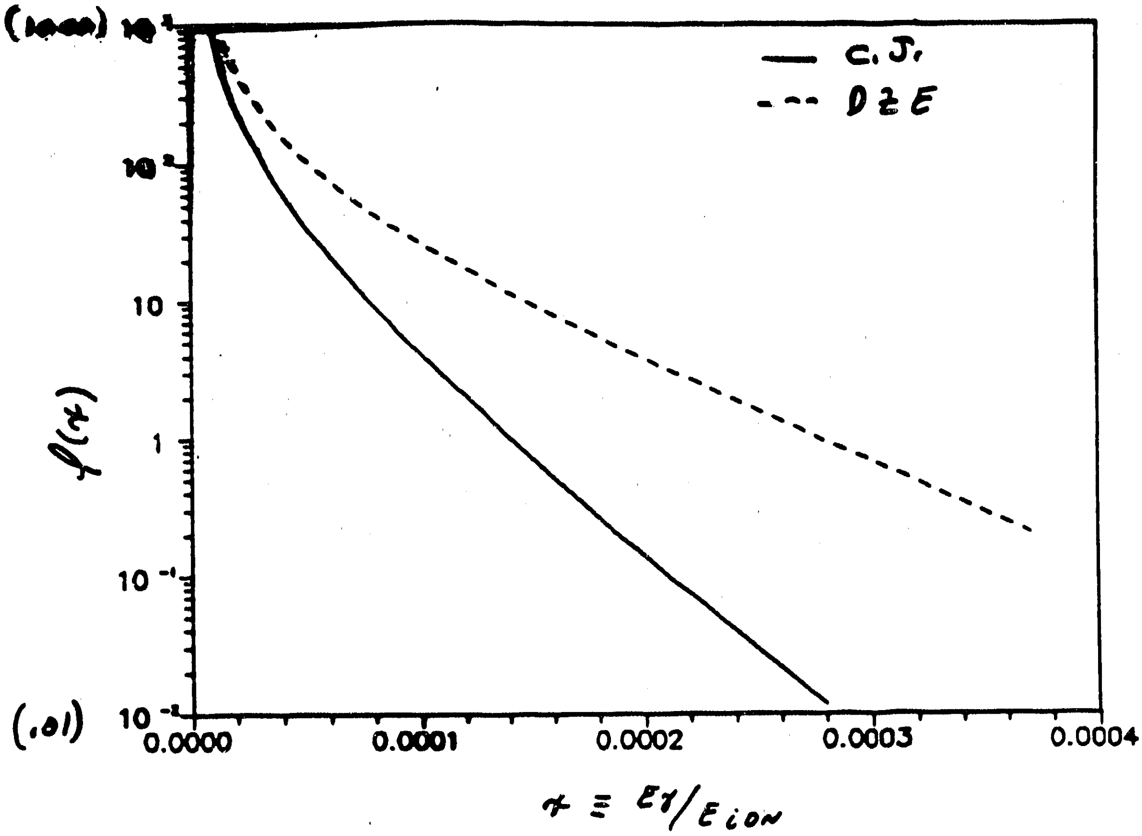
(ALSO SOFF, RAU, MÜLLER, GREINER J. PHYS G16, 211, 1990)

Then

$$f(x) = \frac{2\alpha}{\pi x} \left\{ \rho K_0(\rho) K_1(\rho) - \tfrac{1}{2} (\rho)^2 \left[K_1^2(\rho) - K_0^2(\rho) \right] \right\}$$

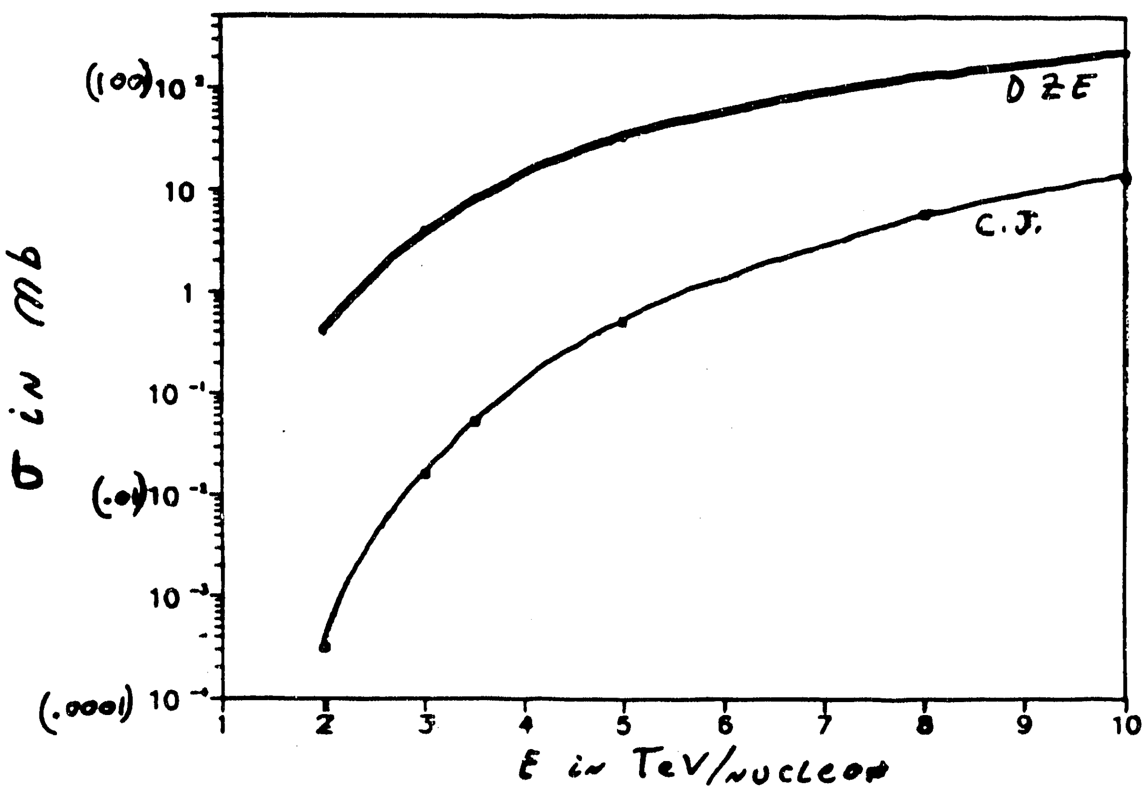
$$\rho = x * b_{min} M$$ { J.D. JACKSON "CLASSICAL E.M.", 2nd ED p 723 }

This reduces greatly the spectrum ... and cross-section



C. J.
D Z E
(1000) 10³
10²
10
1
10⁻¹
(.01) 10⁻²
0.0000
0.0001
0.0002
0.0003
0.0004
f(x)
x ≡ Eγ/E ion

σ(Pb + Pb → γγ → W⁺ W⁻)



D Z E
C. J.
(100) 10²
10
1
10⁻¹
(.01) 10⁻²
10⁻³
(.0001) 10⁻⁴
1
2
3
4
5
6
7
8
9
10
σ in mb
E in TeV/nucleon

Consider:

$$Pb + Pb \to \gamma\gamma \to W^+ W^- \quad (W^- \to \mu^- \bar{\nu}_\mu,\; W^+ \to e^+ \nu_e)$$

$$Pb\, Pb \to \gamma\gamma \to e^+ \nu_e \mu^- \bar{\nu}_\mu \Rightarrow$$

* 13 Feynman diagrams
* Spinor technique
* Monte Carlo integration
  → 8° cut on $e^+\mu^-$
  not on $\nu_e, \bar{\nu}_\mu$

* Possible background from

$$\gamma\gamma \to \tau^+ \tau^- \quad (\tau^- \to \mu^- \nu \bar{\nu},\; \tau^+ \to e^+ \nu \bar{\nu})$$

"can overcome" by:

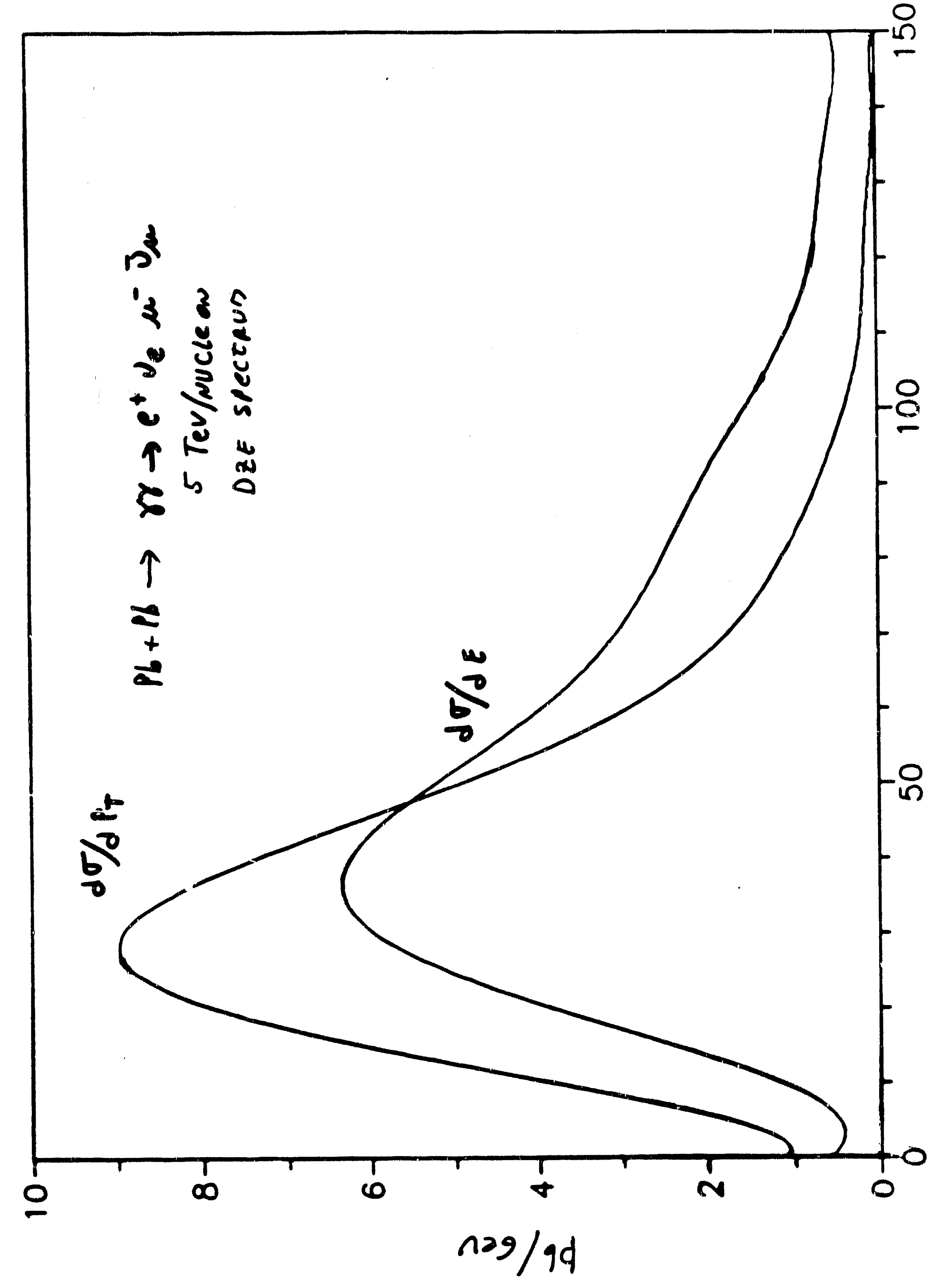
1) invariant mass of $e^+\mu^-$ pair
2) Angle between $e^+\mu^-$

so in principle one can obtain a clean sample of

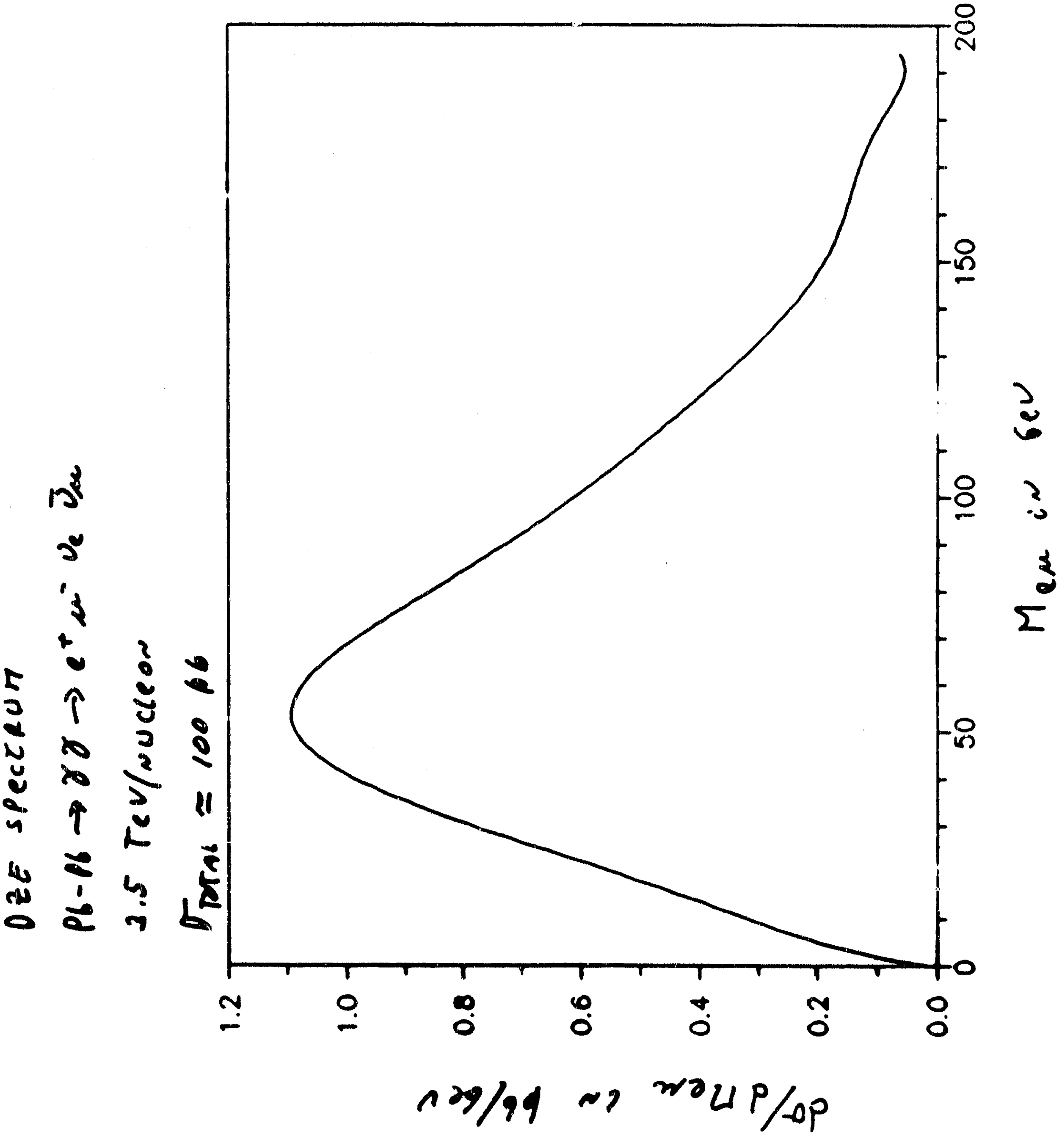
$$Pb + Pb \to \gamma\gamma \to W^+ W^- \to e^+ \mu^- + \text{missing}$$
$$\to e^- \mu^+ + \text{missing}$$

* because of all the integrations:

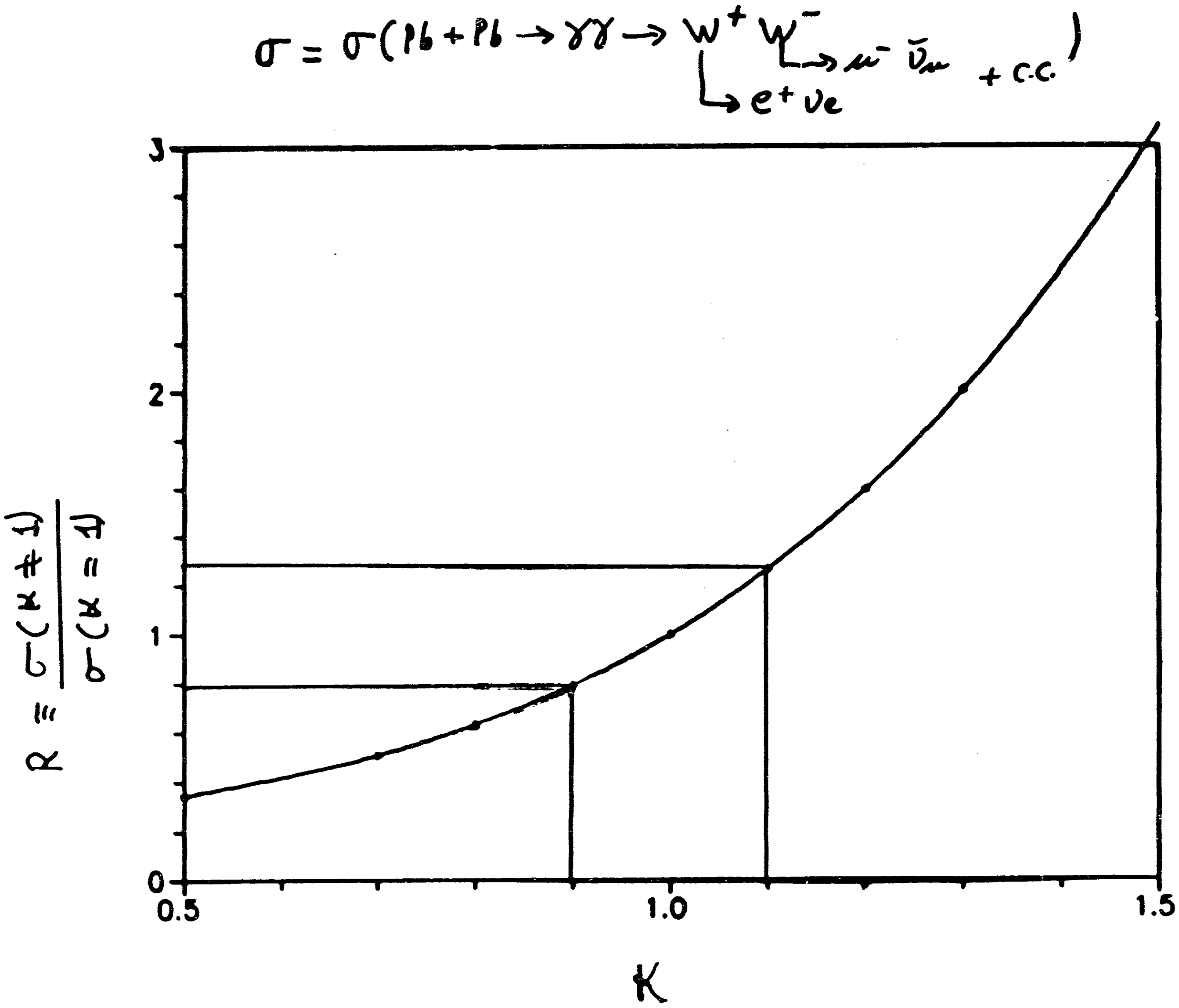
the distributions change GLOBALLY.



Pb + Pb → γγ → e⁺ ν_e μ⁻ ν̄_μ
5 TeV/nucleon
DZE spectrum
dσ/dp_T
dσ/dE
GeV
pb/GeV
0
50
100
150
0
2
4
6
8
10



Pb-Pb → γγ → e⁺μ⁻ νe ν̄μ
3.5 TeV/nucleon
σTOTAL ≈ 100 pb
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
50
100
150
200
dσ/dMeμ in pb/GeV
Meμ in GeV



$\sigma = \sigma(Pb + Pb \to \gamma\gamma \to W^+ W^-)$, $W^+ \to e^+ \nu_e$, $W^- \to \mu^- \bar{\nu}_\mu$, $+ c.c.$
$R \equiv \frac{\sigma(\kappa \neq 1)}{\sigma(\kappa = 1)}$
0
1
2
3
0.5
1.0
1.5
κ

Consider the signal:

$\sigma_T \equiv \sigma(Pb + Pb \to \gamma\gamma \to e^+ \nu_e \mu^- \bar{\nu}_\mu) +$
$\sigma(Pb + Pb \to \gamma\gamma \to e^- \bar{\nu}_e \mu^+ \nu_\mu)$

E (TeV/nucl.)	σ_T DZF pb	σ_T C.J. pb
3.5	200	1.3
5.0	800	14
8.0	3300	145
10.0	5600	240

Let:
1 YEAR ~ 10^7 sec

$10^{28} cm^{-2} sec^{-1} * 1$ YEAR $= 0.1\ pb^{-1}$

$\int \mathcal{L} dt$	E in TeV/nucleon				
	3.5	5.0	8.0	10.0	
0.01 pb^{-1}	* − 1.23	0.81 − 1.13	0.91 − 1.07	0.93 − 1.05	⟨K⟩ DZE
	* * *	* * *	* * *	~0.5 − ~1.2	⟨K⟩ C.J.
0.1 pb^{-1}	0.89 − 1.09	0.95 − 1.04	0.97 − 1.02	0.98 − 1.02	
	* * *	* * *	0.87 − 1.1	0.92 − 1.06	
1.0 pb^{-1}	0.96 − 1.03	0.98 − 1.02	0.99 − 1.01	0.994 − 1.00	
	* * *	0.86 − 1.10	0.96 − 1.03	0.97 − 1.02	

1σ

*** → no bound in the range 0 − 2 could be obtained.

By comparison: σ_{WW} at SSC ~ 0.1 nb : here 8.9 nb C.J
135 nb DZE

CONCLUSIONS

* At the energies considered, the charge distribution of the nucleus (i.e. $\rho(r)$) does not appear to be important... as long as $\langle R \rangle$ is the same.

* The requirement that the nuclei do not collide reduces the signal by orders of magnitude when done à la Cahn-Jackson.

* At 3.5 TeV/N (LHC) WW physics is very difficult to do if one requires a "clean signal" (i.e. ions missing each other)

* At SSC-type energies, heavy ions produce much more W pairs than usual pp or $p\bar{p}$.

* A 5% measurement on κ is not out of reach at SSC via heavy ions, even when requiring a clean signal.

* Future $(g-2)_\mu \Rightarrow |\Delta\kappa| \sim 0.04$ (currently: $|\Delta\kappa| \sim 0.8$) (Renard et al)

LEPTON PAIR PRODUCTION FROM RELATIVISTIC HEAVY–ION COLLISIONS

Talk presented by:

M. Strayer

Lepton Pair Production From Relativistic Heavy–Ion Collisions

C. Bottcher M.R. Strayer & J.S. Wu

A.K. Kerman *MIT*

M.J. Rhoades–Brown *BNL*

" Physics of strong Fields" ed. W. Greiner Vol 153 (1987)

Proceedings of the Second Workshop On Experiments & Detectors for the RHIC (1987)

Nucl. Inst & Meth. B31 (1988) ; Phys. Rev. D39 (1989)

Phys. Letts. 237B (1990) ; J. Phys. G (1990)

ornl 1

Coherent Production of Pairs

Classical Field Model $2-\gamma$ Limit

Compare to Feynman Theory

Monte Carlo Integration

Nuclear Form Factors – "off–shell" Photons

distributions – y, M, P_t

Spin Effects - Yang's Theorem

Enhanced Electroweak couplings W–Pairs, Higgs

ornl 2

Classical Field Model

$$\mathcal{L}_{int} = \mathbf{e} : \overline{\Psi}(\mathbf{x})\gamma^{\mu}\,\Psi(\mathbf{x}) : \mathbf{A}_{\mu}(\mathbf{x}) \;+\; \mathbf{J}^{\mu}(\mathbf{x})\;A_{\mu}(x)$$

Classical field

heavy–ion current

$$-\,\partial\bullet\partial\;\mathbf{A}^{\mu} + \partial^{\mu}\,(\partial\bullet\mathbf{A}\,) \;=\; \mathbf{J}^{\mu}$$

Maxwell's Eqn

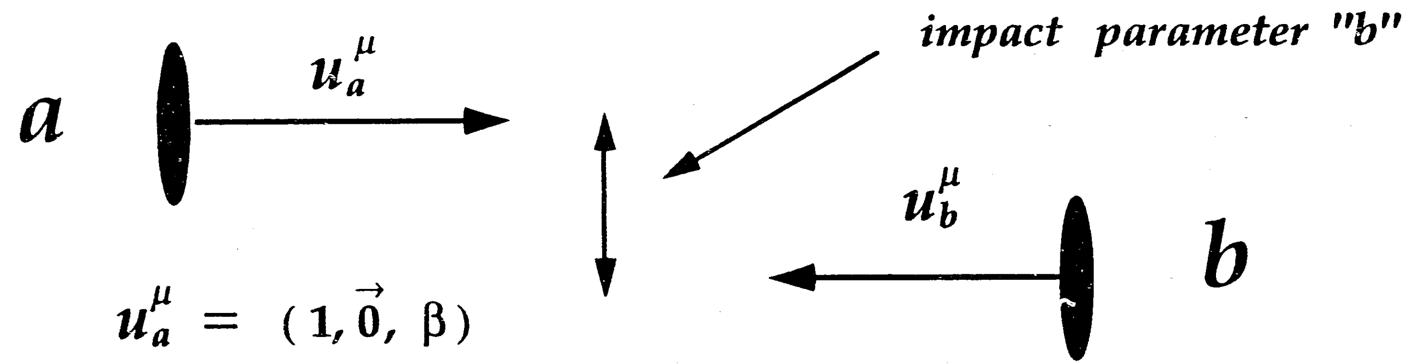
$$\mathcal{S} \;=\; \mathbf{T}\Big\{\exp\Big[\;\mathbf{i}\int \mathbf{d}^{4}\mathbf{x}\,\mathcal{L}_{\mathrm{int}}(\mathbf{x})\;\Big]\Big\}$$

vacuum state

$$\sigma \;=\; \sum_{\mathbf{k}_1\mathbf{k}_2}\int \mathbf{d}^{2}\mathbf{b}\;\left|\,<\mathbf{k}_1\,\mathbf{k}_2\,\right|\,\mathcal{S}^{(2)}\,\left|\,\mathbf{o}>\,\right|^{2}$$

lepton pair state

Classical Fields



$$u_a^\mu = (1, \vec{0}, \beta)$$

$$u_b^\mu = (1, \vec{0}, -\beta)$$

Rest Frame of a & b

Nuclear Form Factor

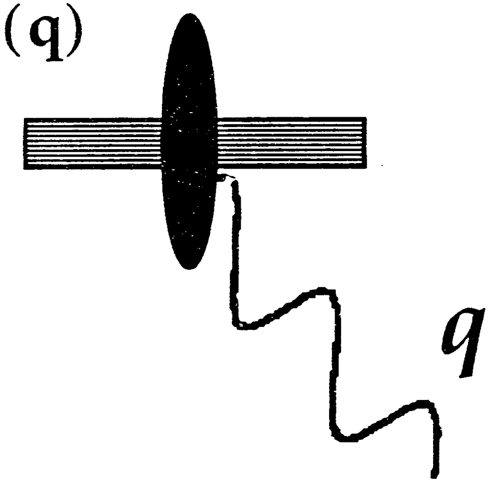
$$A^0_{a,b}(q) = Q_{a,b}\, 2\pi\delta(q^0)\, \frac{F_{a,b}(-q^2)}{-q^2}\, e^{-i\, q_\perp \cdot (\pm b/2)}$$

Classical Fields

$$\mathbf{A}^{\mu}(\mathbf{q}) = \mathbf{A}^{\mu}_{\mathrm{a}}(\mathbf{q}) + \mathbf{A}^{\mu}_{\mathrm{b}}(\mathbf{q})$$

$$\mathbf{A}^{\mu}_{\mathrm{a,b}}(\mathbf{q}) = \mathbf{Q}_{\mathrm{a,b}}\ u^{\mu}_{a,b} \left[\frac{\mathrm{F}_{\mathrm{a,b}}(-\mathbf{q}^2)}{-q^2}\right] 2\pi\, \delta(\mathbf{q}^0 - (\pm\beta)\, \mathbf{q}^z)\, \mathbf{e}^{-i\, \mathbf{q}_{\perp} \cdot (\pm \mathbf{b}/2)}$$

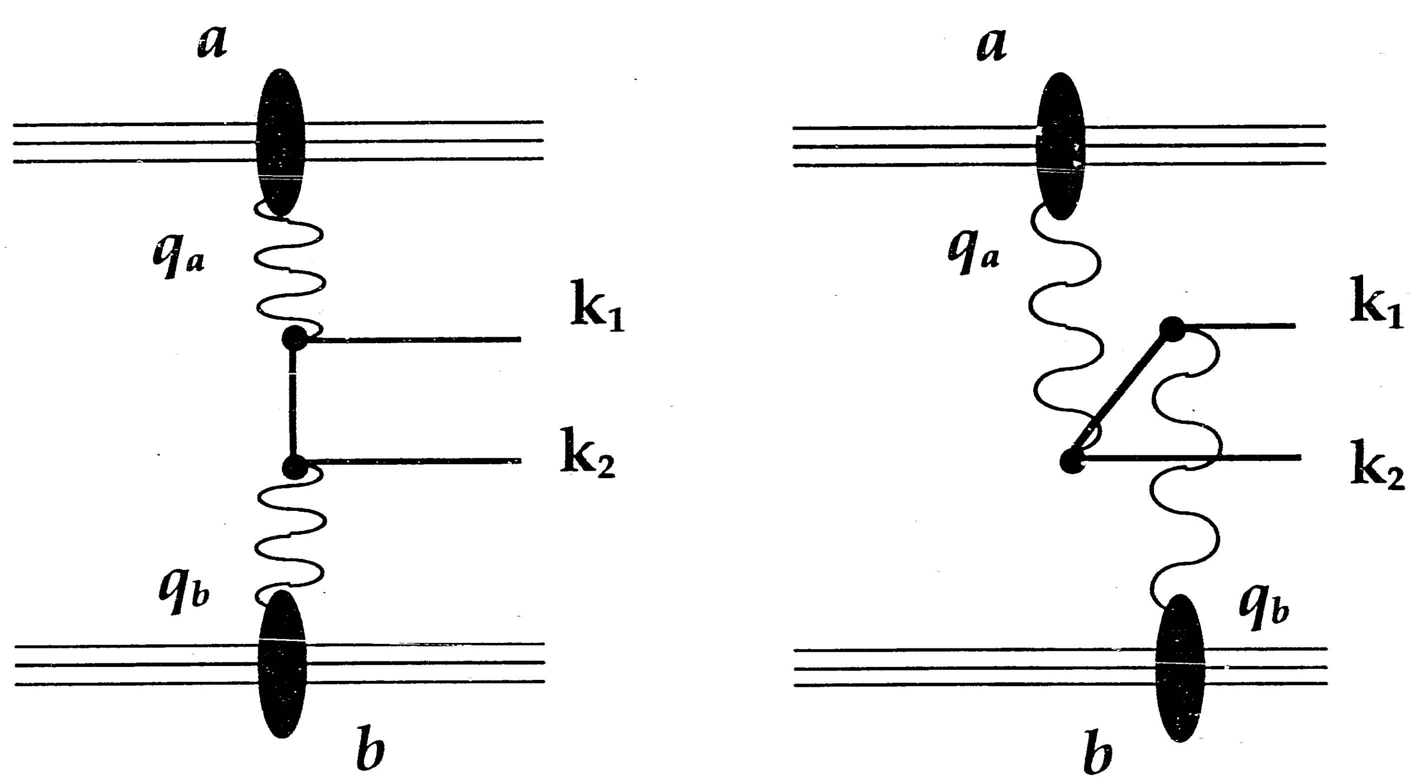
$\mathbf{A}^{\mu}_{\mathrm{a}}(\mathbf{q})$



relates frequencies in the field to the "boost"

Lowest order solution contains two photons

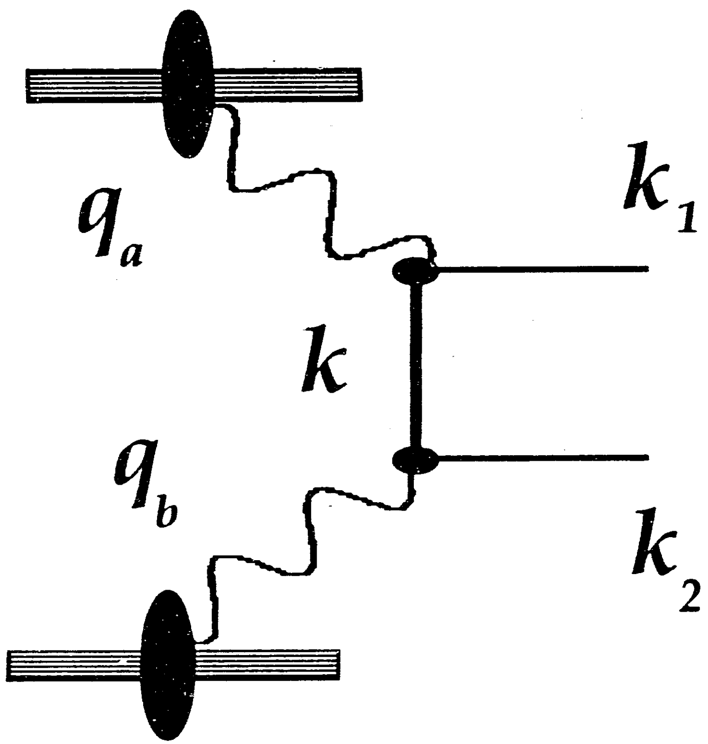
Gauge Invariant



Cross section

$$\sigma = e^4 \sum_{s_1 s_2} \int \frac{d^3k_1\, d^3k_2}{(2\pi)^6\, 2E_1\, 2E_2}\, d^2b \left| \int \frac{d^4q_a\, d^4q_b\, d^4k}{(2\pi)^{12}} (2\pi)^8\, \delta^4(q_a+k-k_1)\, \delta^4(q_b-k-k_1) \right.$$

$$\left. \times\ \bar{u}(k_1 s_1)\, \not{A}_a(q_a) \frac{1}{\not{k}-m} \not{A}_b(q_b)\, \upsilon(k_2 s_2) \ +\ \text{cross term} \right|^2$$

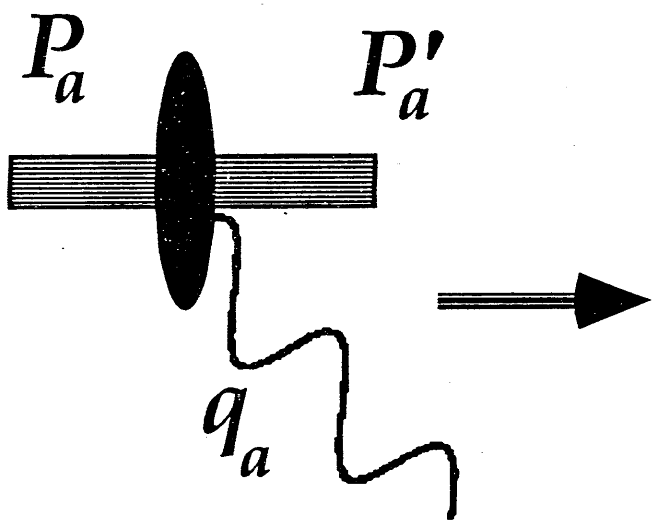


reduces to an
8-d integration

Feynman Diagrams

- Evaluate the cross section expression in terms of explicit Feynman rules for the two–photon diagrams

- For Spin–0 Nuclei:



$$Q_a F_a(-q_a^2)\ (P'_a + P_a)_\mu \ \frac{-ig^{\mu\nu}}{q_a^2}$$

Feynman cross section

lepton states *heavy–ion states*

$$\sigma = \left[\frac{1}{\beta\, 2E_a\, 2E_b} \right] \int \frac{d^3k_1\, d^3k_2}{(2\pi)^6\, 2E_1\, 2E_2}\; \frac{d^3p'_a\, d^3p'_b}{(2\pi)^6\, 2E'_a\, 2E'_b}\, (2\pi)^4 \delta^4(q_a + q_b - k_1 - k_2)$$

$$\times \sum_{s_1 s_2} \left| \mathcal{M} \right|^2$$

$$\mathcal{M} = ie^2\, Q_a\, Q_b\, \frac{F_a(-q_a^2)}{q_a^2}\, \frac{F_b(-q_b^2)}{q_b^2} \times$$

$$\bar{u}(k_1 s_1) \left[(\not{p}'_a + \not{p}_a) \frac{1}{\not{k} - m} (\not{p}'_b + \not{p}_b) + \text{crossed term} \right] v(k_2 s_2)$$

Feynman Diagrams

Assume $\quad P'_a \sim P_a$

$$\text{RHIC} \sim 10^{-4}$$

$$(P'_a + P_a)^\mu \Rightarrow 2\,E_a\,u_a^\mu$$

$$q_a^0 - \beta\, q_a^z = \frac{\upsilon_a}{\gamma} \approx 0 \qquad \upsilon = \sqrt{M^2 + \vec{q}^{\,2}} - M$$

$$q_b^0 + \beta\, q_b^z = \frac{\upsilon_b}{\gamma} \approx 0$$

Cross section

$$\sigma = \left[\frac{e^4 Q_a^2 Q_b^2}{4\beta^2} \right] \int \frac{d^3k_1\, d^3k_2}{(2\pi)^6\, 2E_1\, 2E_2} \frac{d^2 q_\perp}{(2\pi)^2} \left[\frac{F_b(-q_b^2)}{q_a^2} \frac{F_b(-q_b^2)}{q_b^2} \right]^2 \times$$

$$\left| \bar{u}(k_1 s_1) \left[\not{u}_a \frac{1}{\not{k} - m} \not{u}_b + \not{u}_b \frac{1}{\not{k}' - m} \not{u}_a \right] v(k_2 s_2) \right|^2$$

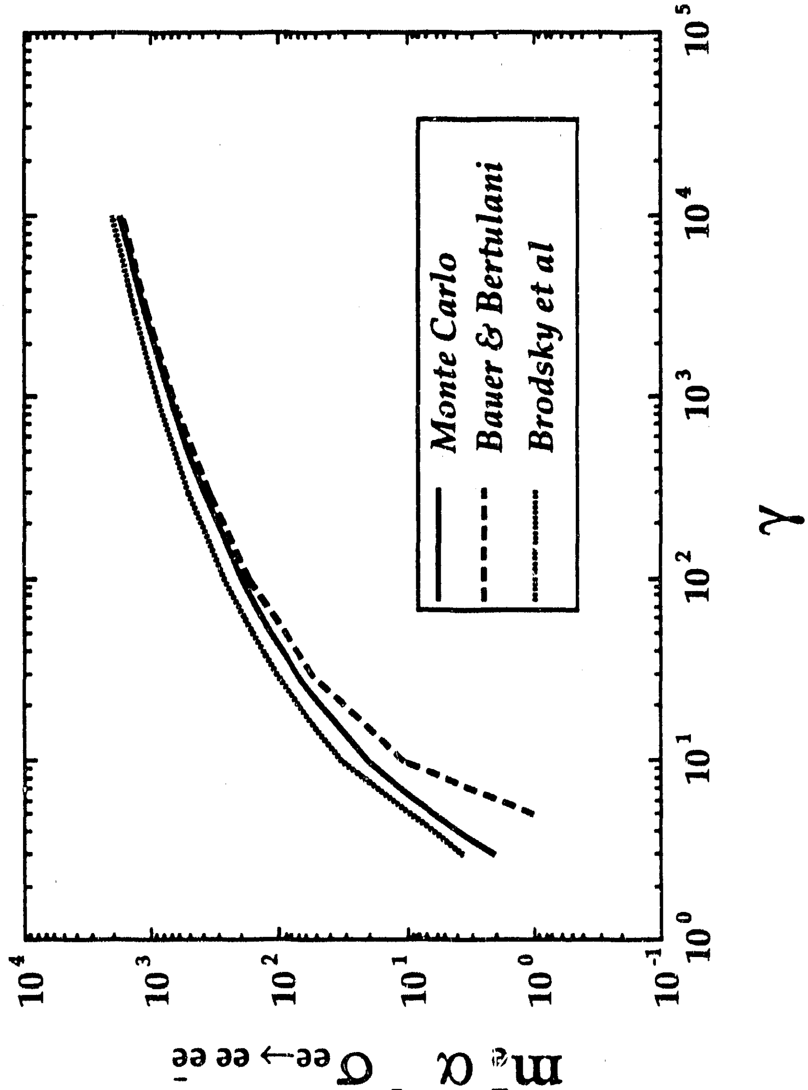
$$k = k_1 + q_a$$

$$k' = k_2 - q_b$$

$$q_a^0 = (P^0 + \beta P^z)/2$$

$$q_b^0 = (P^0 - \beta P^z)/2$$

$$P = k_1 + k_2$$



Monte Carlo
Bauer & Bertulani
Brodsky et al
$m_e^2 \alpha^{-4} \sigma_{ee \to ee\, ee}$
γ
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
10^{5}

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Monte Carlo Integration

Map 8 integration variables onto Monte Carlo points

$$\sigma \Rightarrow \int_0^1 \prod_{j=1}^{8} dx_j \; \Re(\{x_j\})$$

Distributions in y, M, P_t by binning require a large number of points

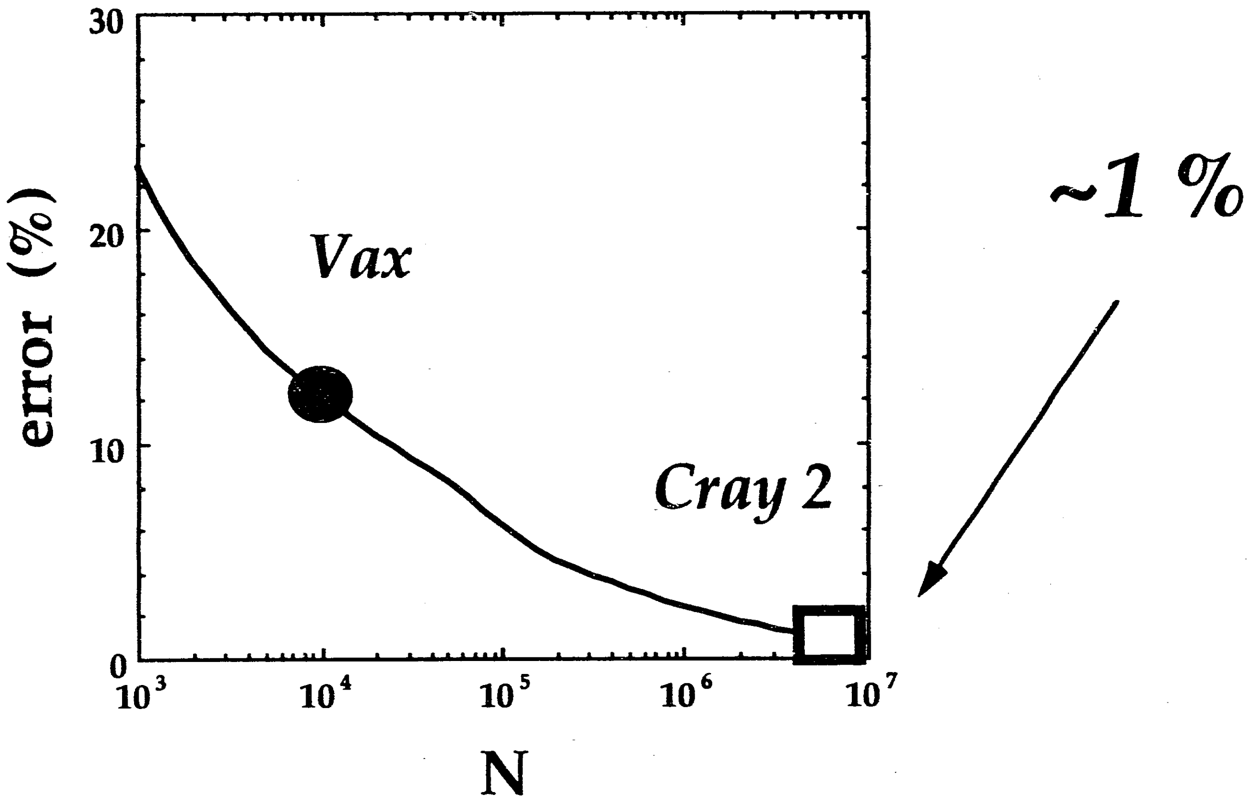
$$N \sim 10^6 - 10^8$$

$$\Delta^2 = \frac{1}{N^2\sigma^2} \sum_n \left[\Re_n^2 - \sigma^2 \right]$$

error

Monte Carlo Integration

3 minutes



Conclusions

Reduction of two-photon Feynman theory

$$q^2/s \sim 0$$

8–dimensional Monte Carlo integration is fast & efficient

However, it is "as easy" to simply evaluate the Feynman diagrams without approximation

Form Factor

Distributions in pair y, M, P_t

$|F(-q^2)|^2 \equiv$ *Probability of emitting a photon of momentum q from a nucleus which remains in its ground state*

$$d\sigma \sim F(-q_a^2)^2 \; F(-q_b^2)^2$$

Suppression of the coherent cross section depending on the momentum transfer to nucleus

Form Factor

$$q'_a = (0 , \vec{0}_\perp , +q)$$

$$q'_b = (0 , \vec{0}_\perp , -q)$$

rest frame

$$P = (2\gamma\beta q , \vec{0})$$

$$M^2 = 4 \gamma^2 \beta^2 q^2$$

$$q \geq \frac{2M}{\beta\gamma} \equiv q_{th}$$

$$q_a = \gamma (\beta q , \vec{0}_\perp , +q)$$

$$q_b = \gamma (\beta q , \vec{0}_\perp , -q)$$

boost

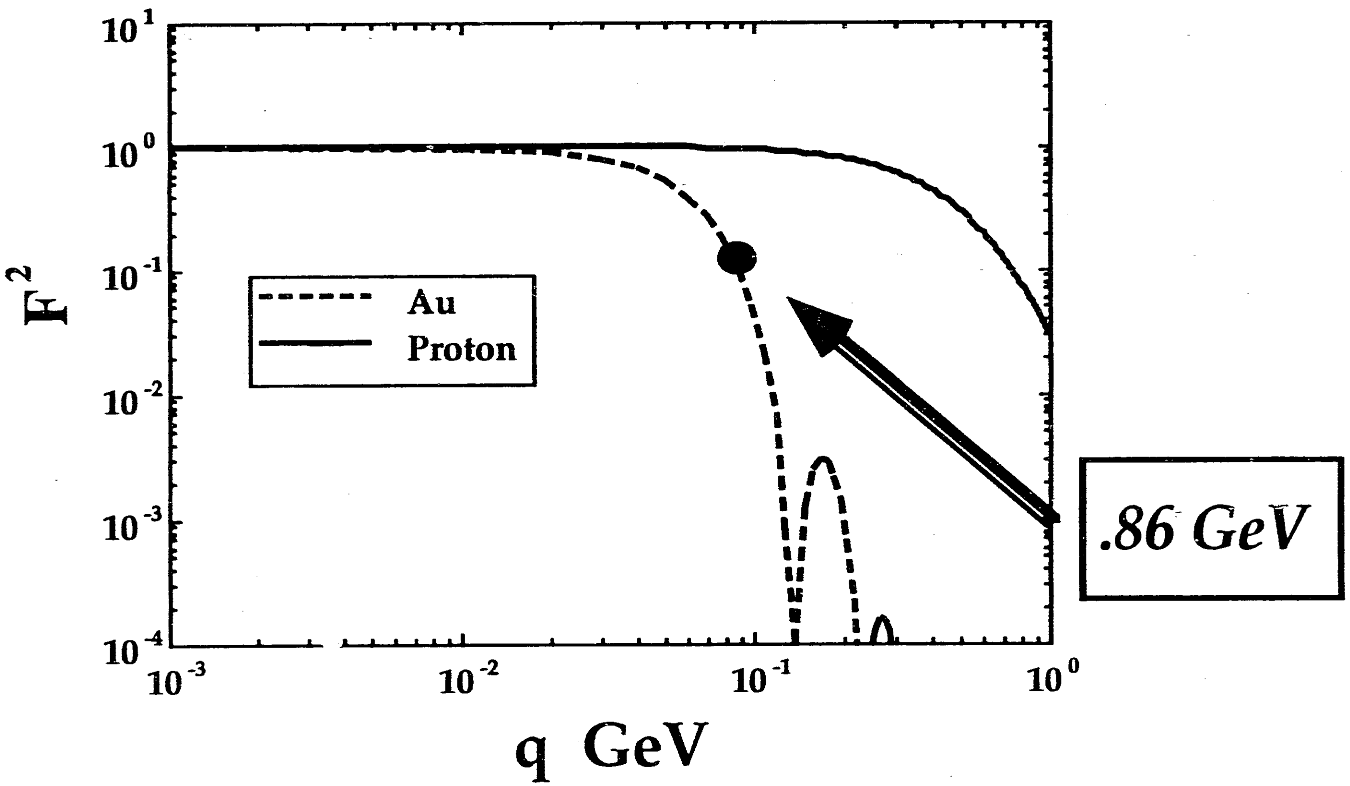
$$| F(q_{th}^2) |^2 \sim 10^{-1}$$

$$\Rightarrow q_{th} \sim \frac{3.60}{R}$$

$$\boxed{\frac{M}{\gamma} \leq \frac{2}{R}}$$

Form Factor

M< 86 GeV RHIC



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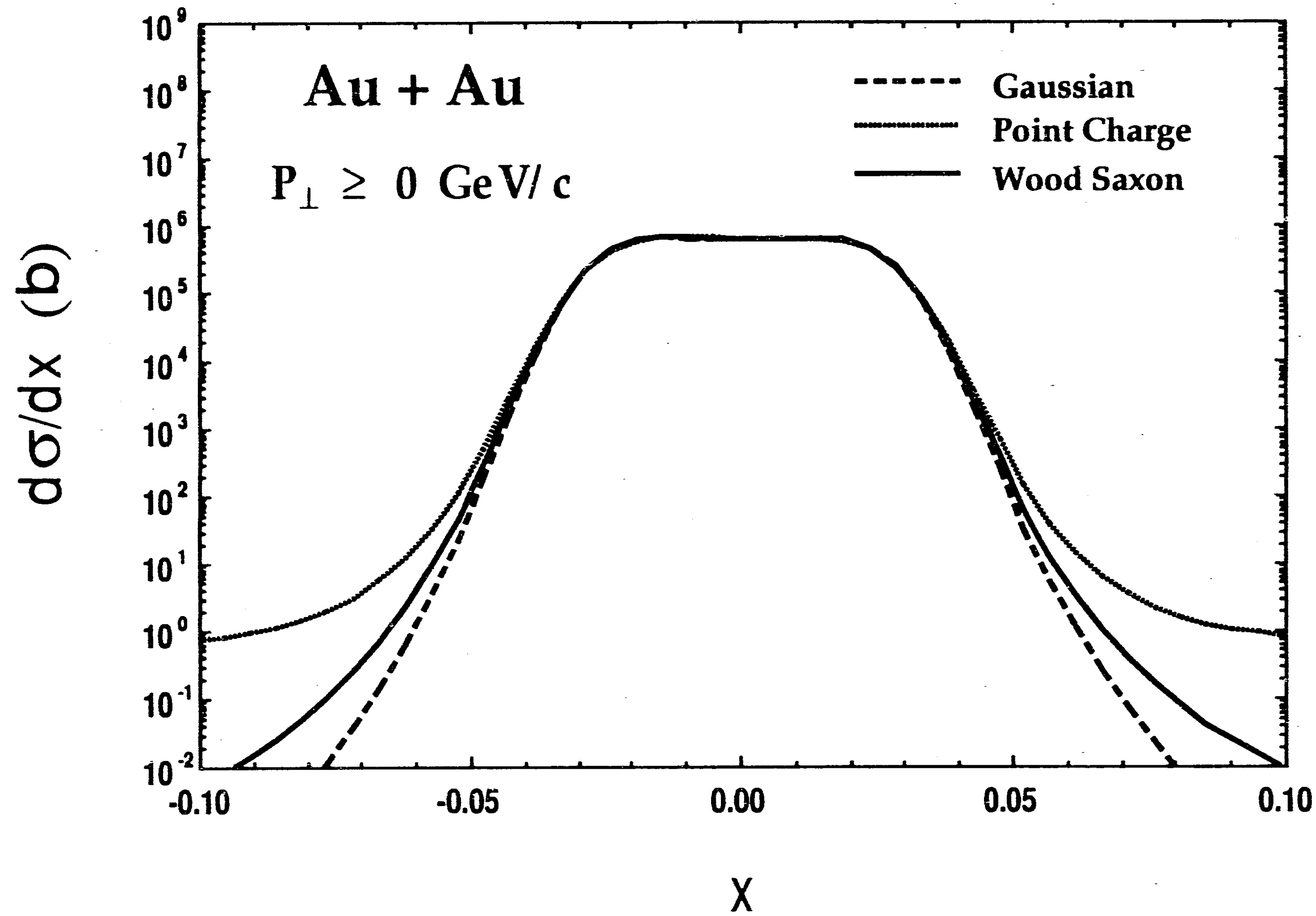
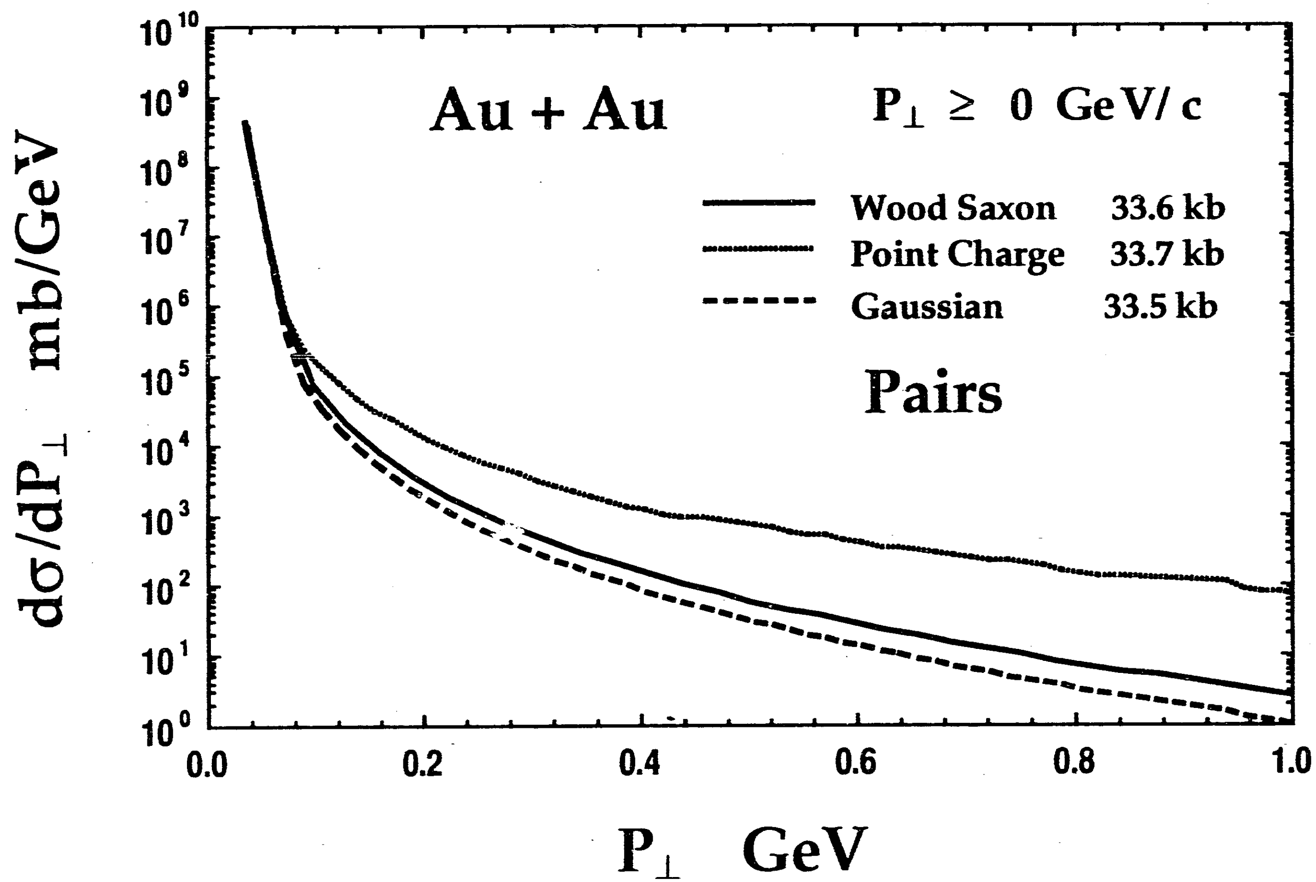
e – Pairs



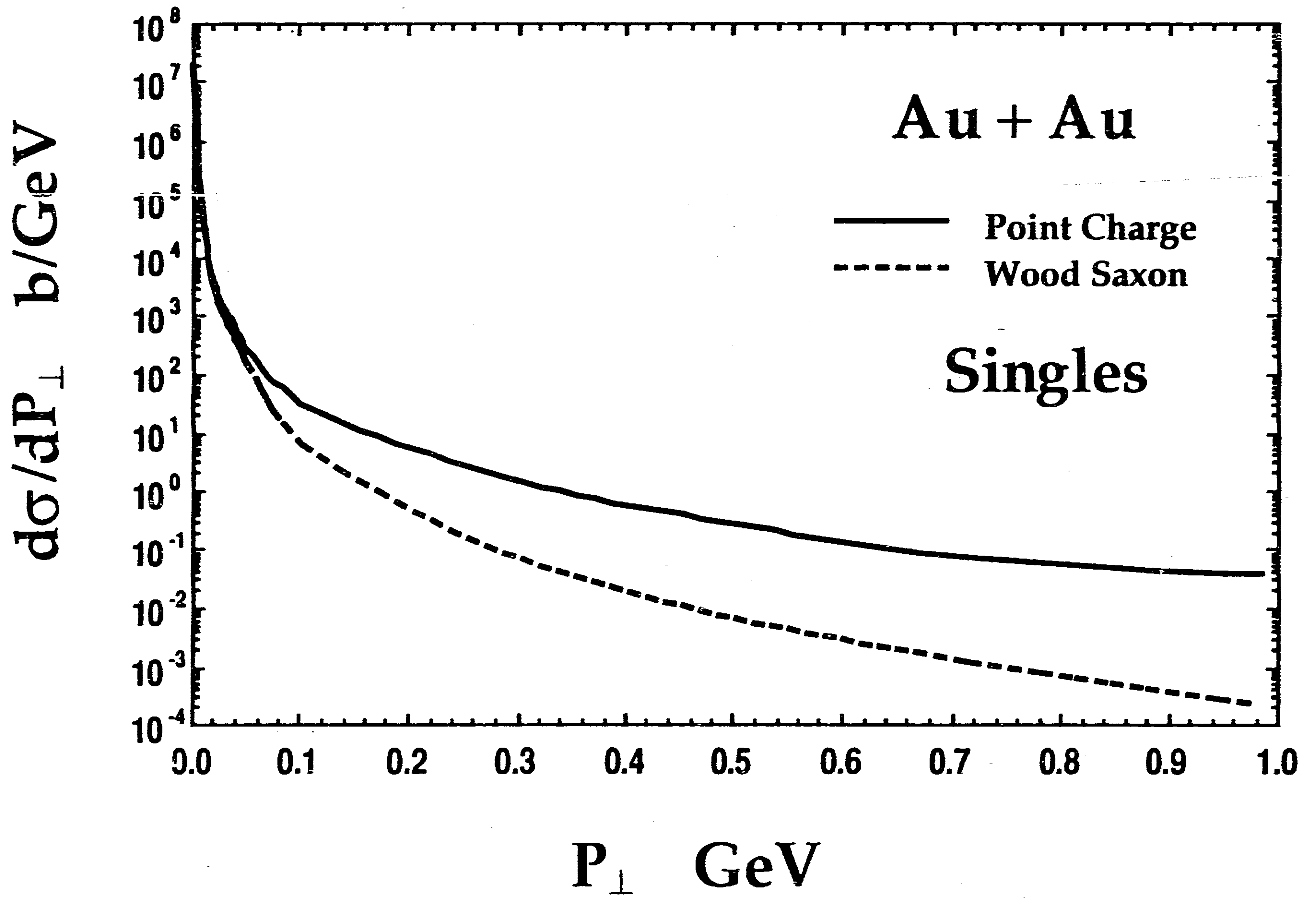
Au + Au
P⊥ ≥ 0 GeV/ c
Gaussian
Point Charge
Wood Saxon
dσ/dx (b)
X
10^9
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10^1
10^0
10^-1
10^-2
-0.10
-0.05
0.00
0.05
0.10

e – Pairs



Au + Au
P⊥ ≥ 0 GeV/ c
Wood Saxon 33.6 kb
Point Charge 33.7 kb
Gaussian 33.5 kb
Pairs
dσ/dP⊥ mb/GeV
P⊥ GeV
0.0
0.2
0.4
0.6
0.8
1.0
10^0
10^1
10^2
10^3
10^4
10^5
10^6
10^7
10^8
10^9
10^10

e – Pairs



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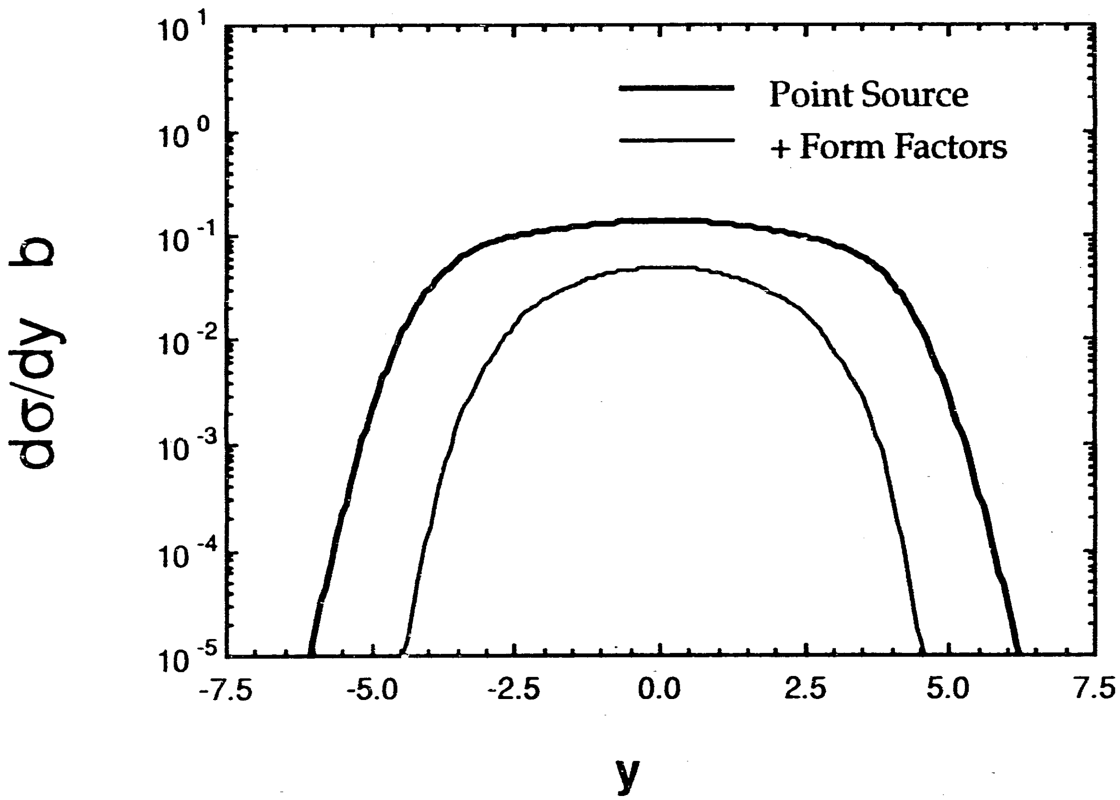
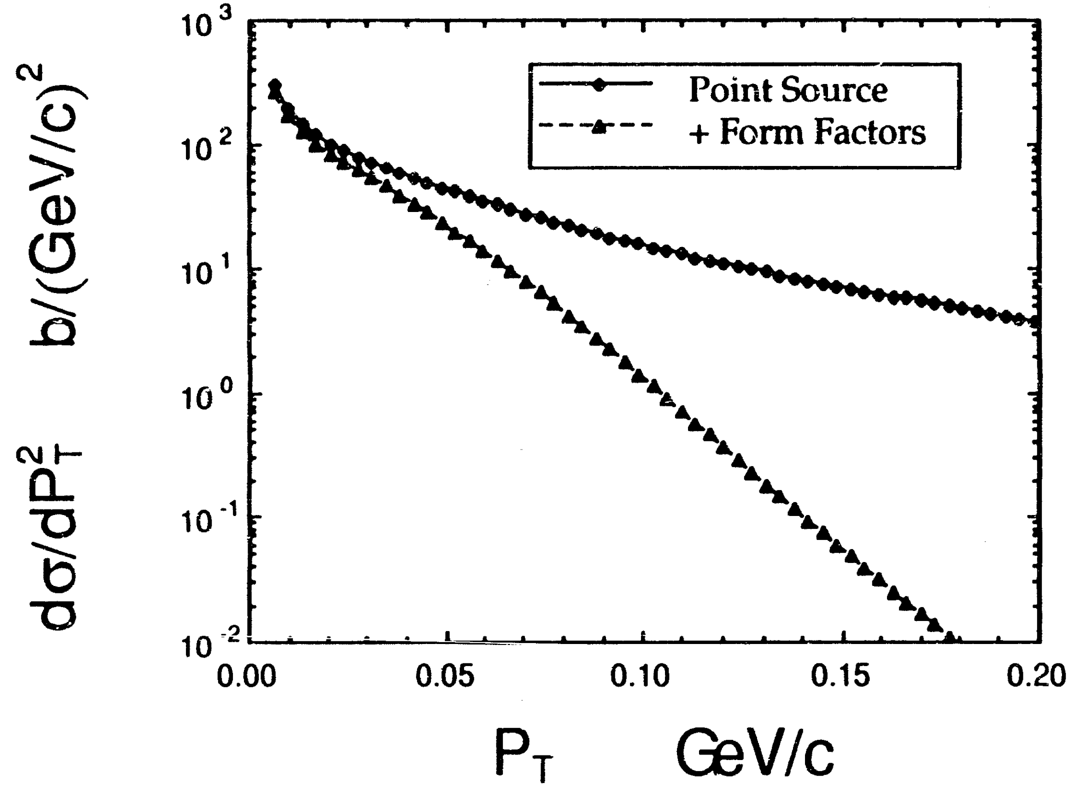
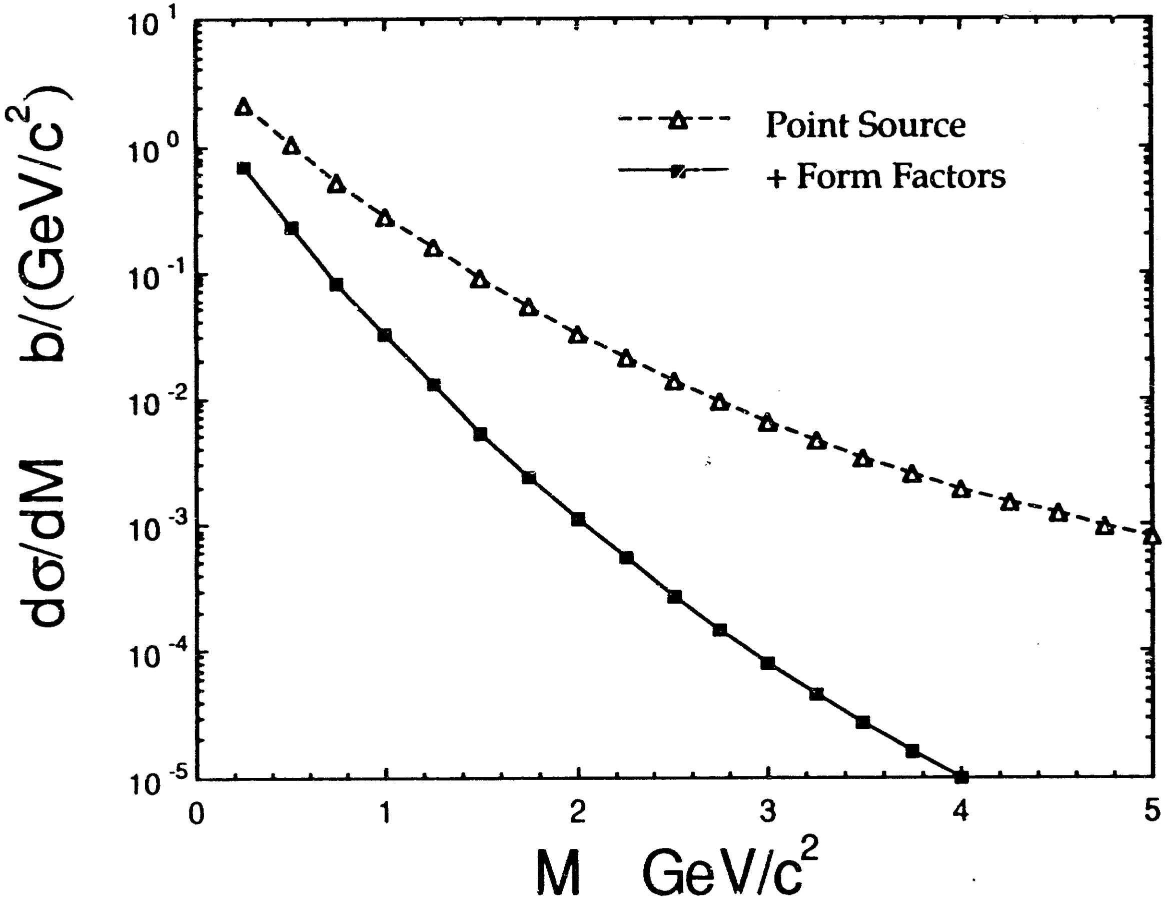
μ–Pairs Au @ RHIC



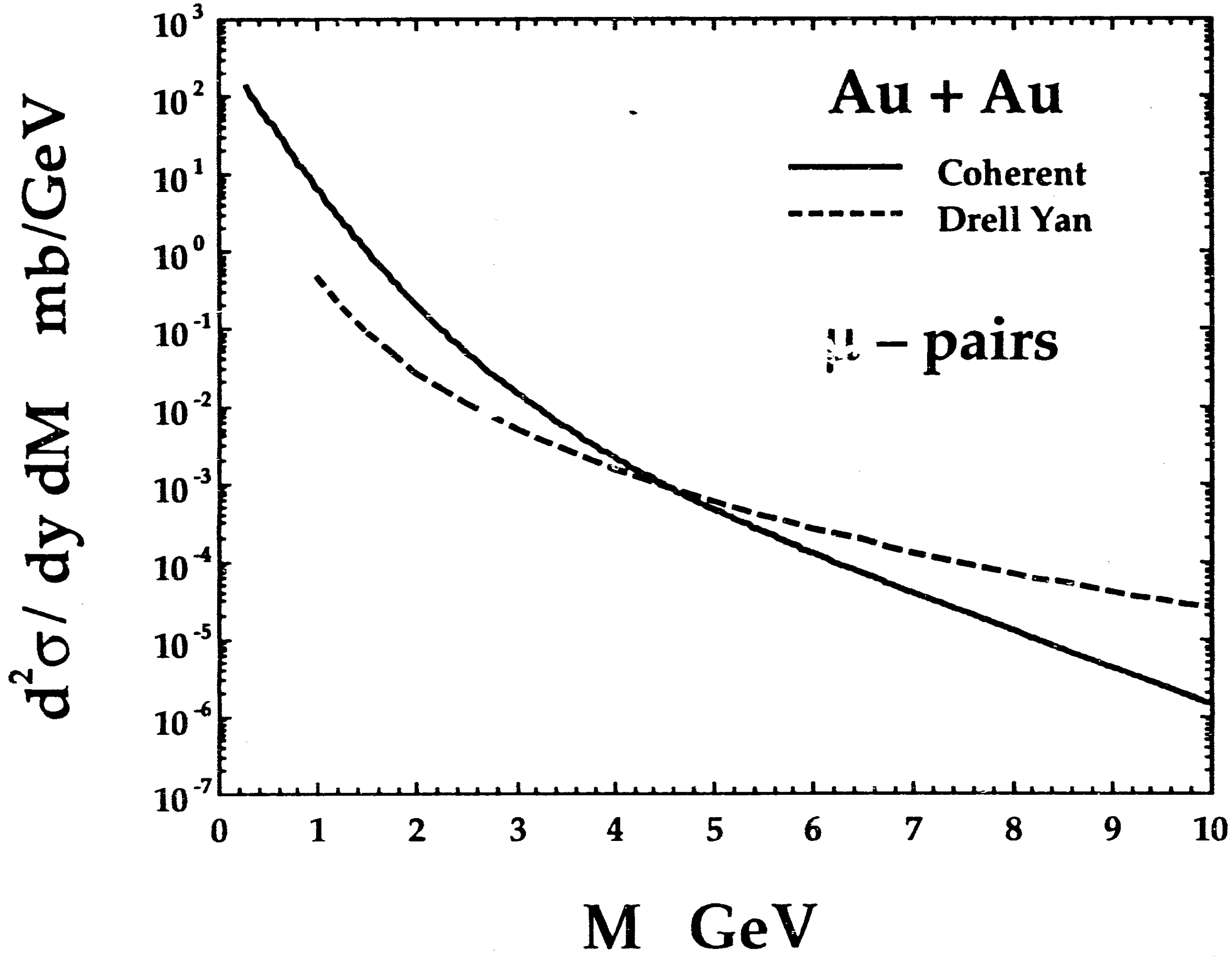
μ–*Pairs* *Au @ RHIC*



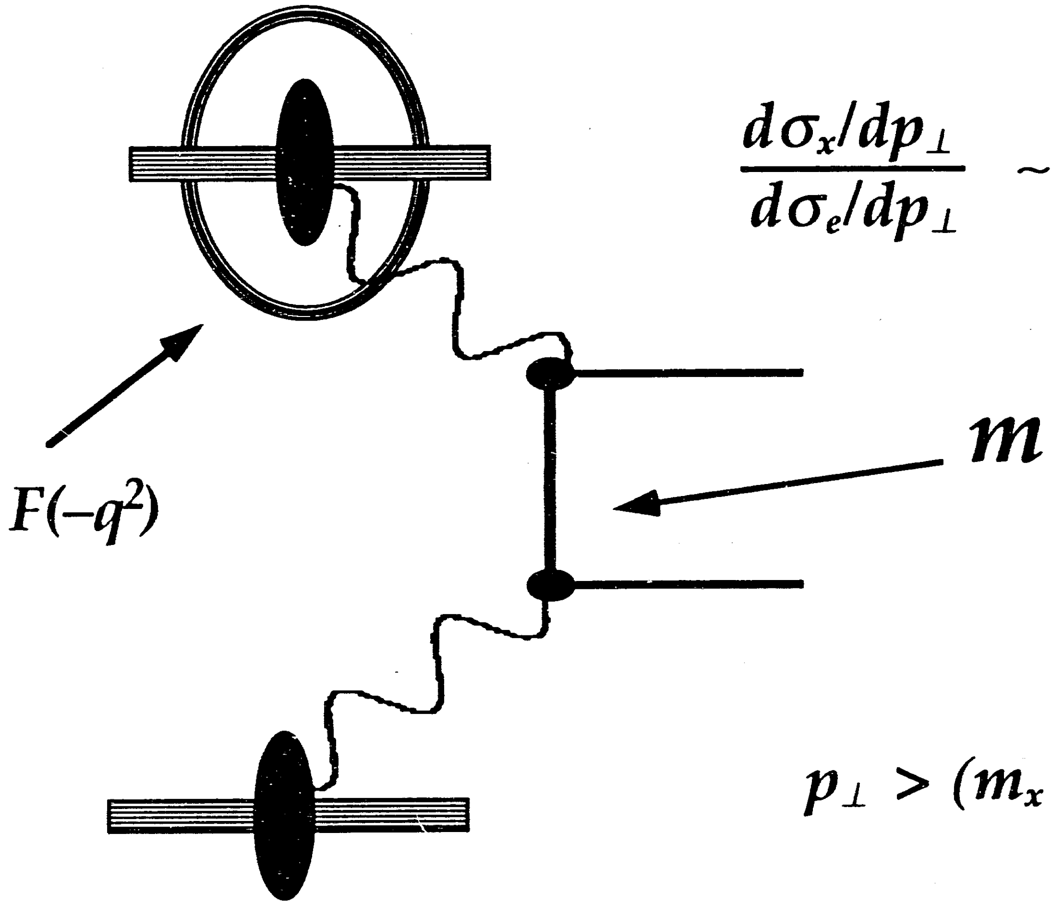
μ–Pairs Au @ RHIC



μ–Pairs Au @ RHIC

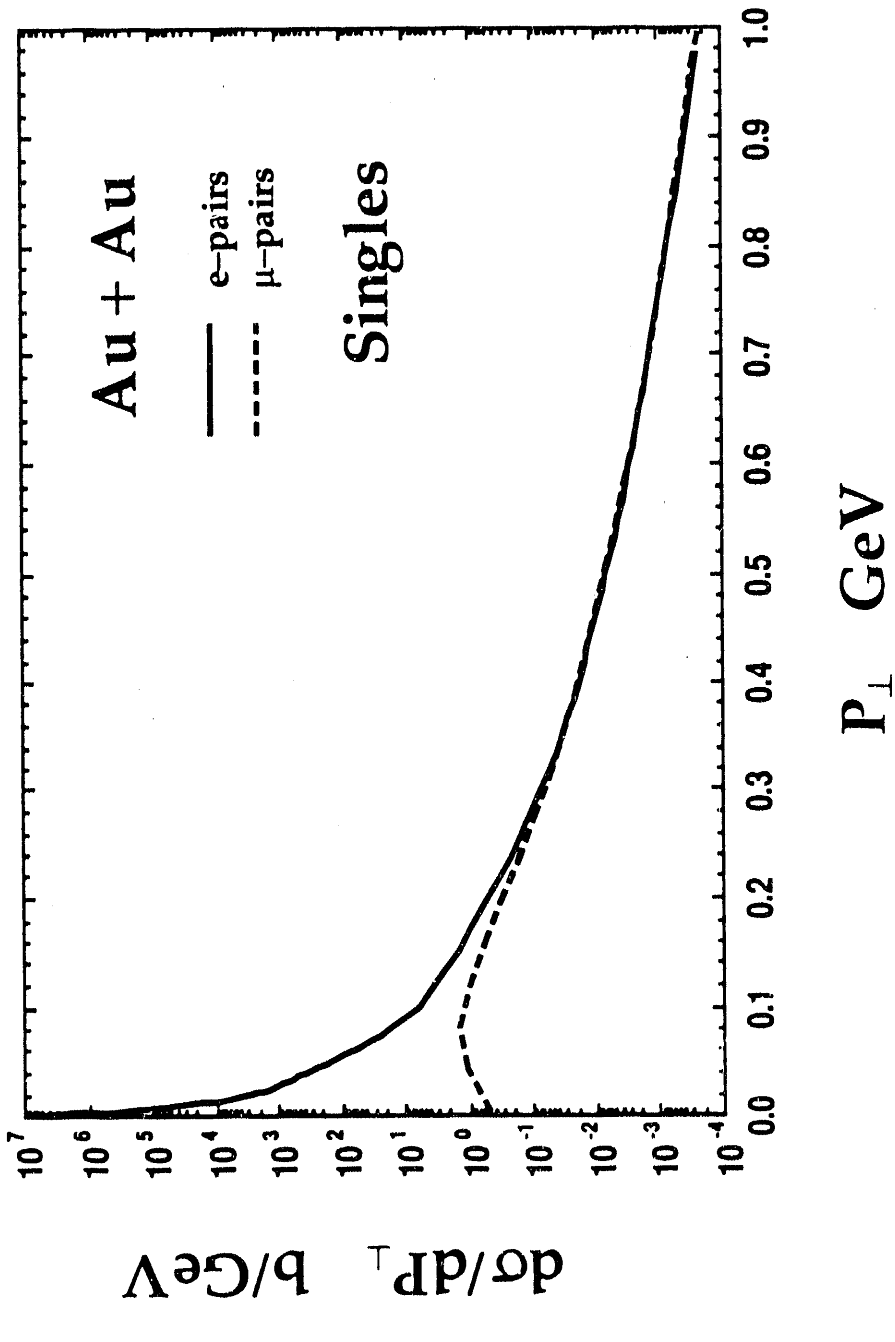


Scaling



$$\frac{d\sigma_x/dp_\perp}{d\sigma_e/dp_\perp} \sim n_c\,(Q_x/e)^4$$

$$p_\perp > (m_x\ ,\ m_e\ )$$


Au + Au
e–pairs
μ–pairs
Singles
$P_\perp$ GeV
$d\sigma/dP_\perp$ b/GeV
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
10^{7}
10^{6}
10^{5}
10^{4}
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}

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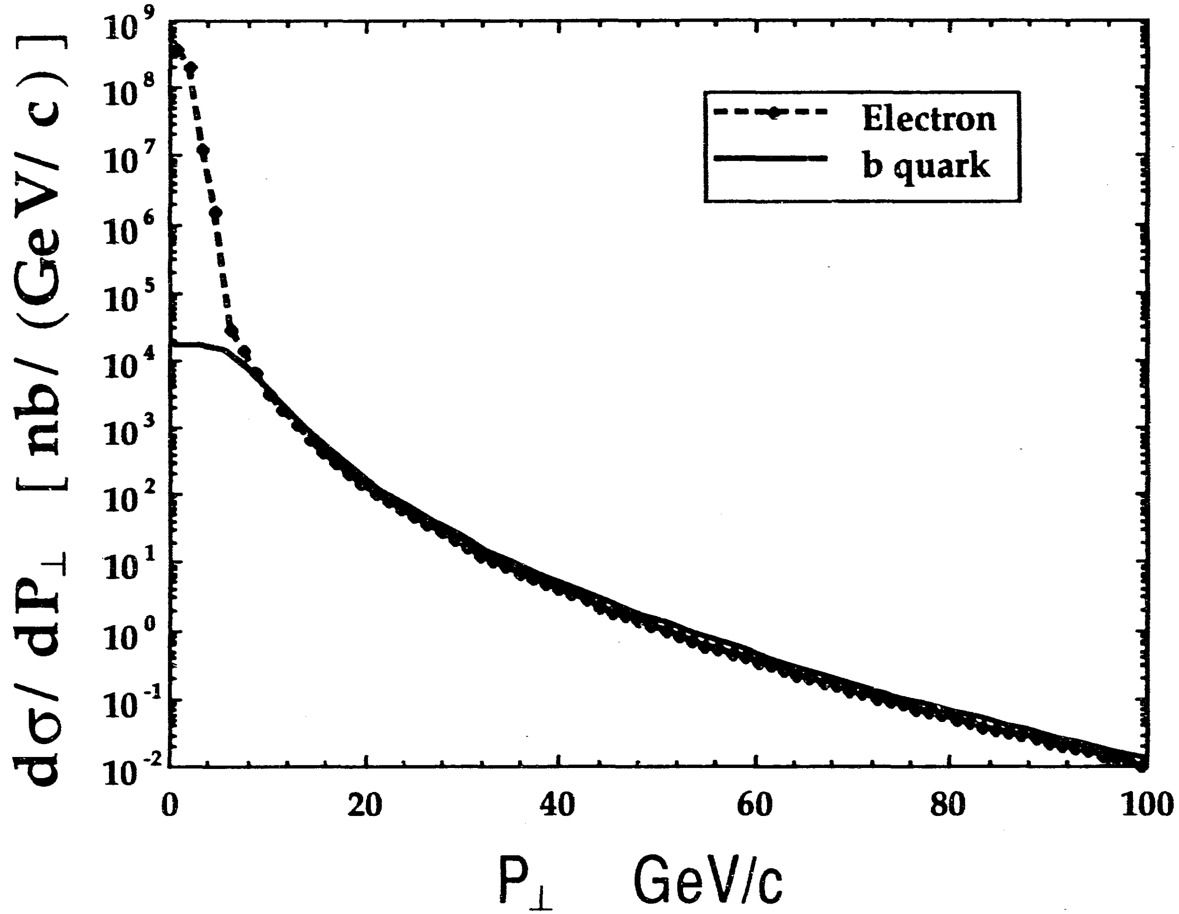

Electron
b quark
dσ/ dP⊥ [nb/ (GeV/ c)]
10^9
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10^1
10^0
10^-1
10^-2
0
20
40
60
80
100
P⊥ GeV/c

Conclusions

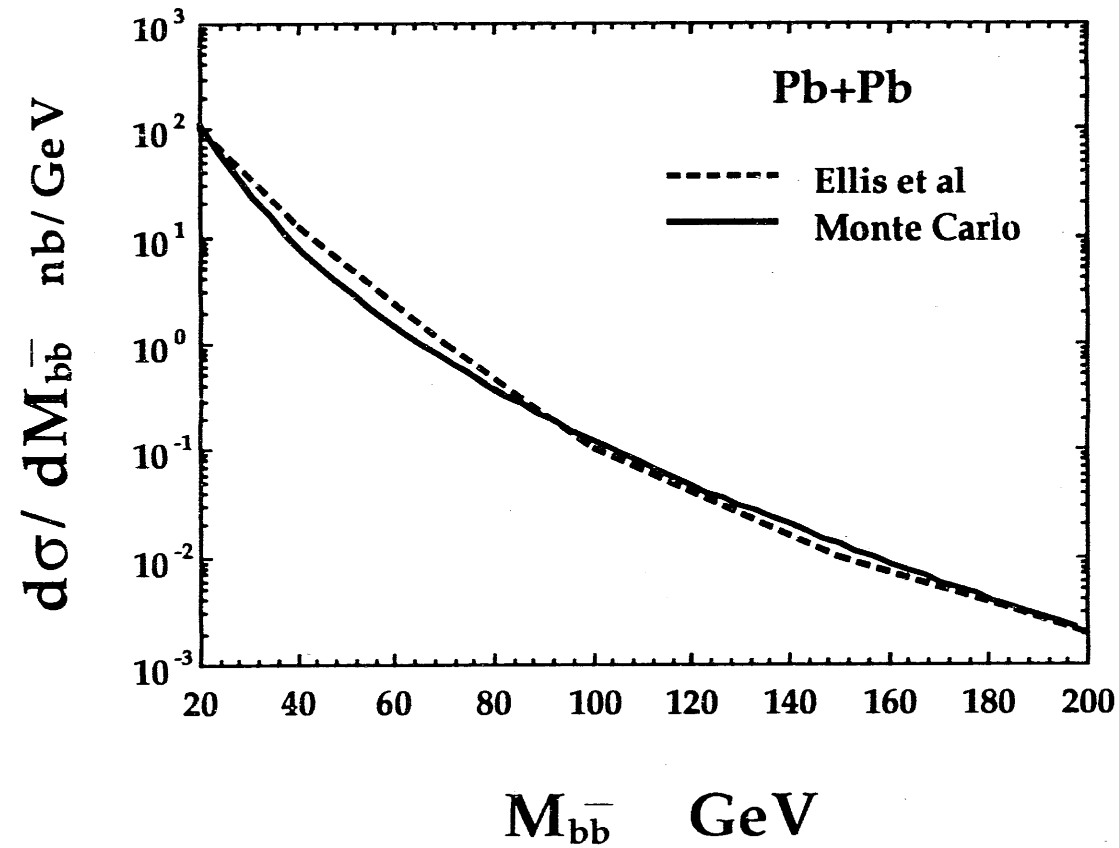
The form factor acts to suppress the coherent cross section depending on the momentum of the photons

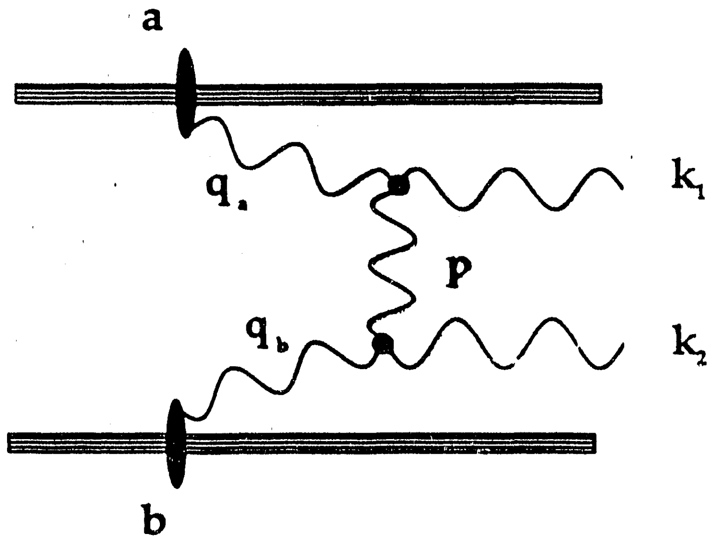
Details of distributions are sensitive to the Form Factor which imposes a "cut–off" of $q \sim 2/R$

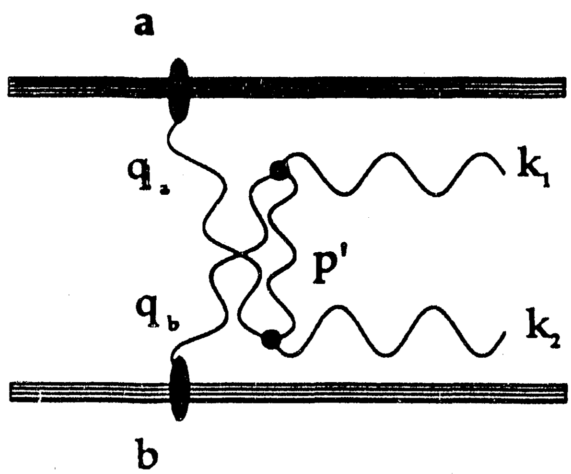
However the high momentum "tails" of the form factor can yield appreciable cross sections at large momenta

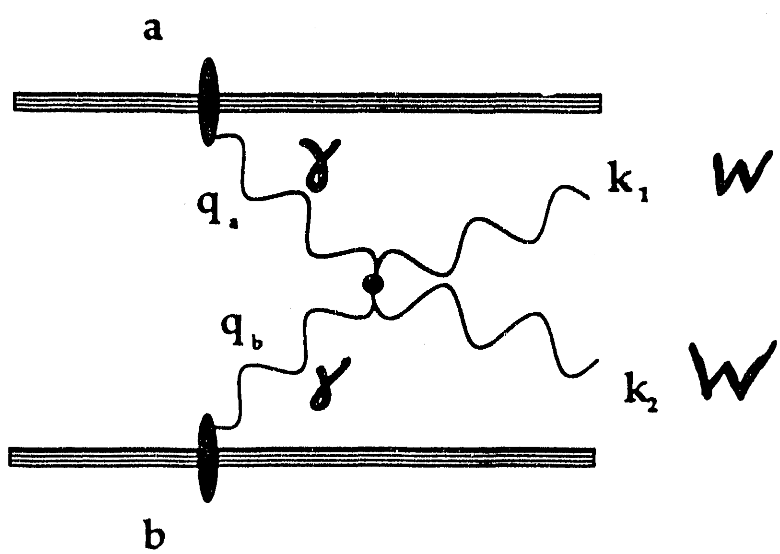
For very heavy pairs the Form Factor cuts–off the P_t distribution thereby "improving" the epa

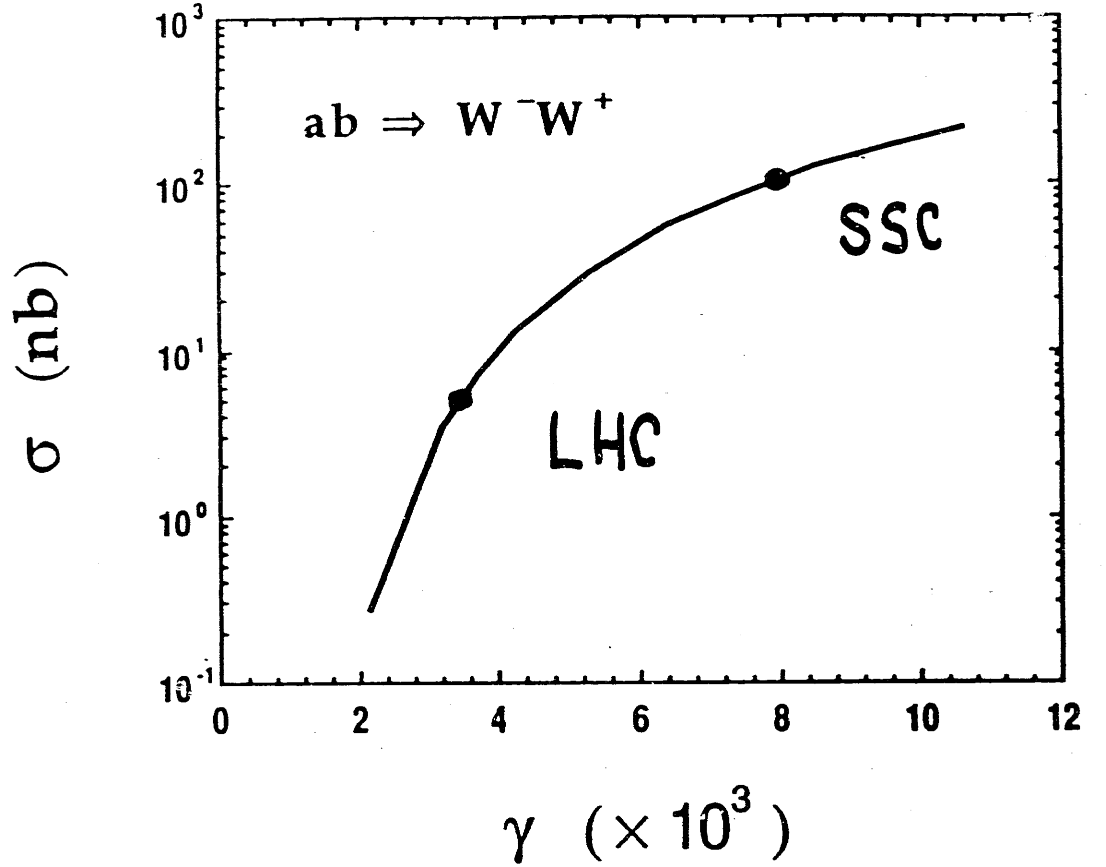
$b\,\bar{b}$ *Pairs* *Pb@LHC*




a
q_a
k_1
p
q_b
k_2
b

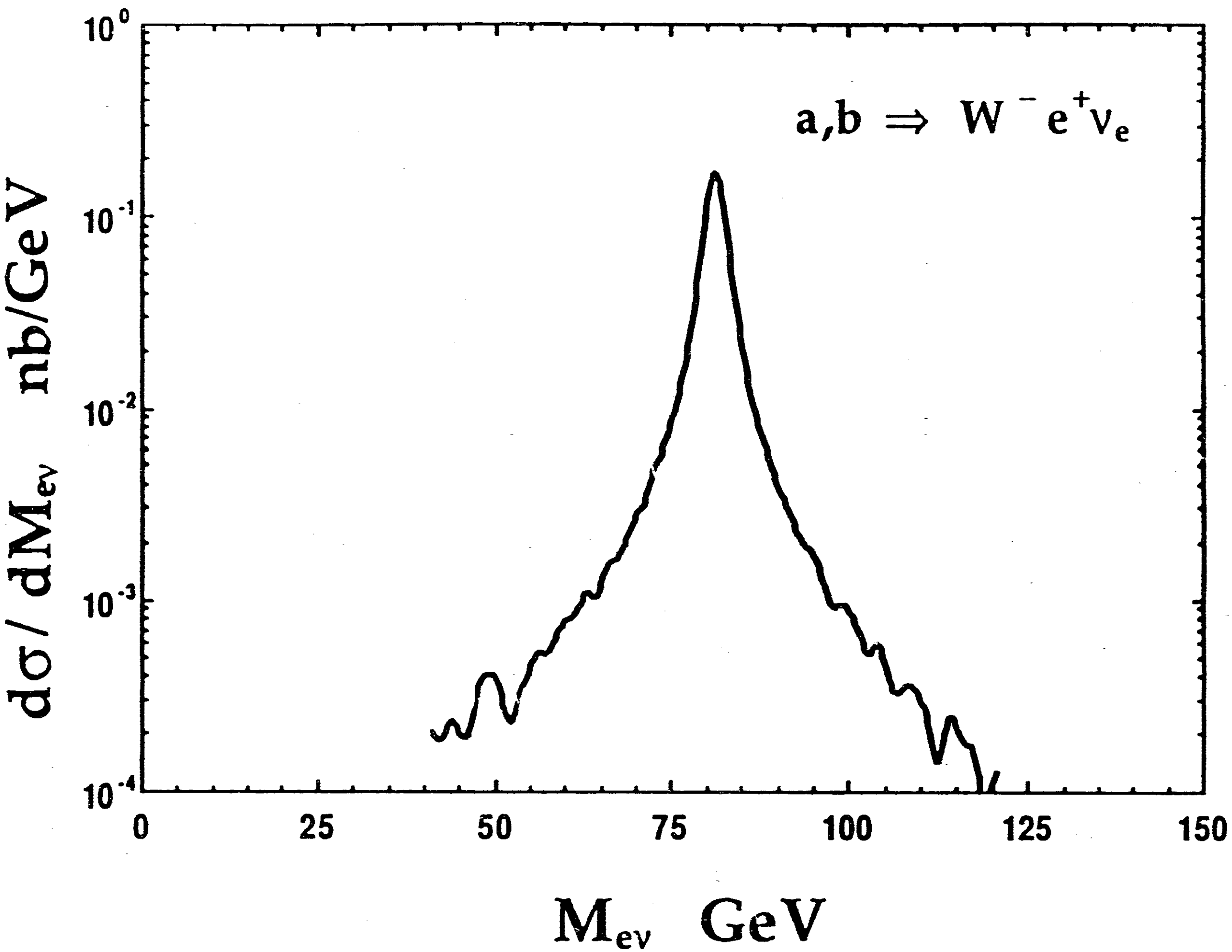

a
q_a
k_1
p'
q_b
k_2
b


a
γ
q_a
k_1
W
q_b
γ
k_2
W
b

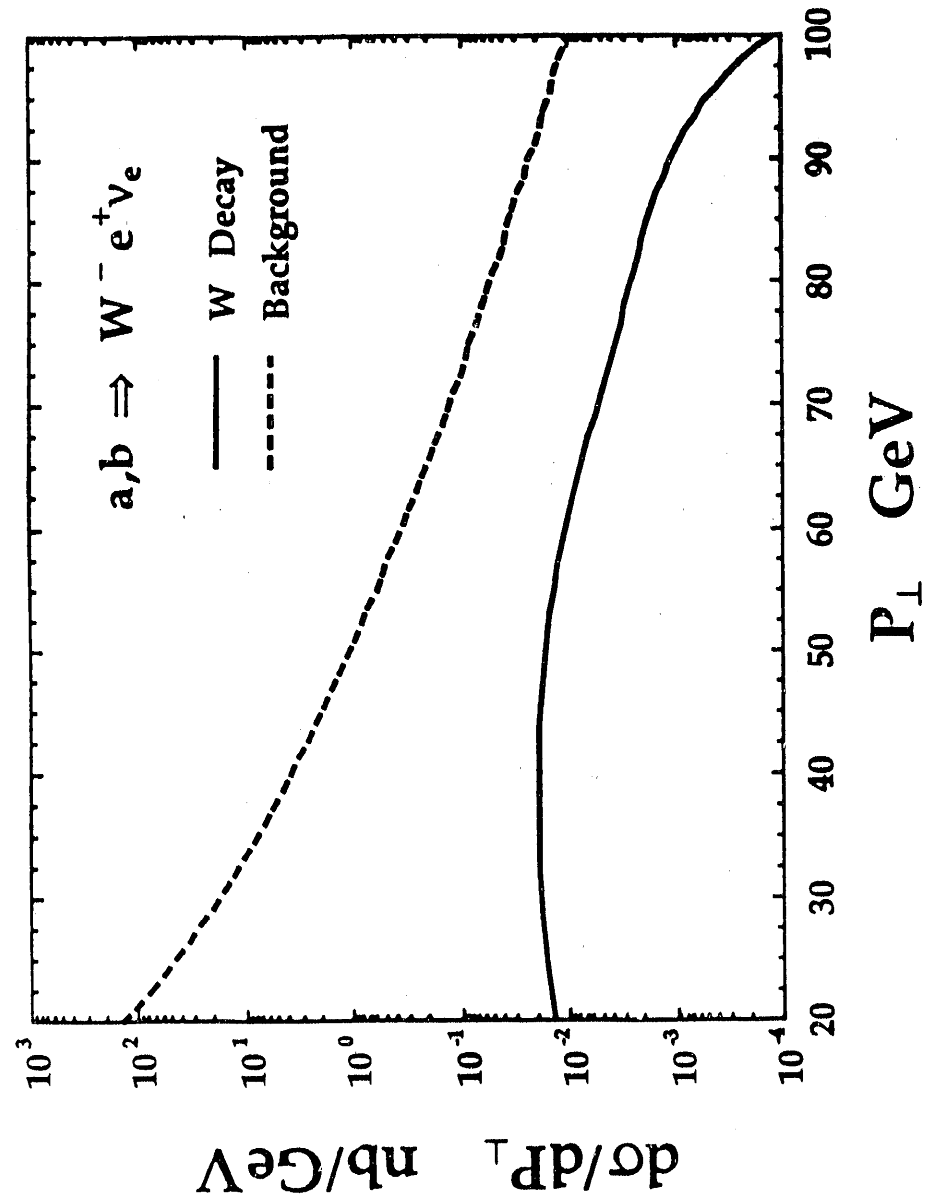


$$\int_{yr} \mathcal{L} \sim \frac{1}{pb}$$

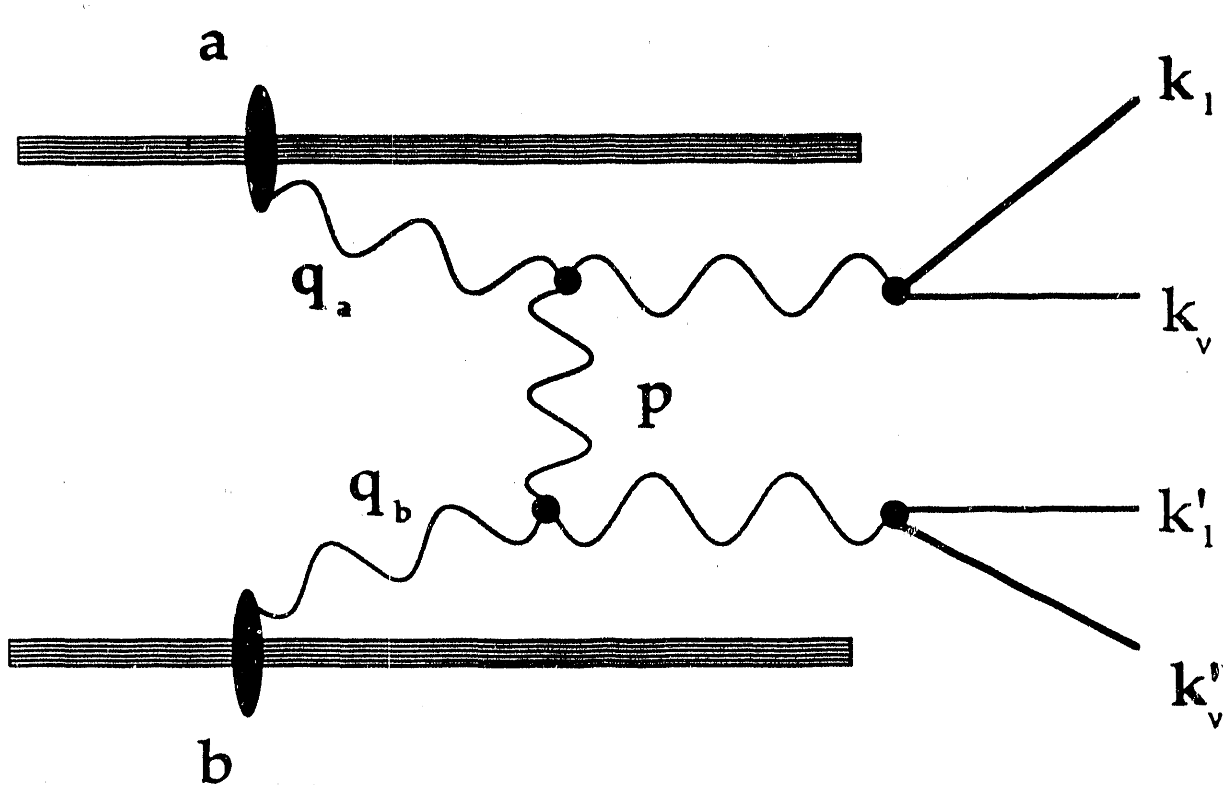
LHC	5 nb	$5\times10^3/yr$
SSC	10^2 nb	$10^4/yr$

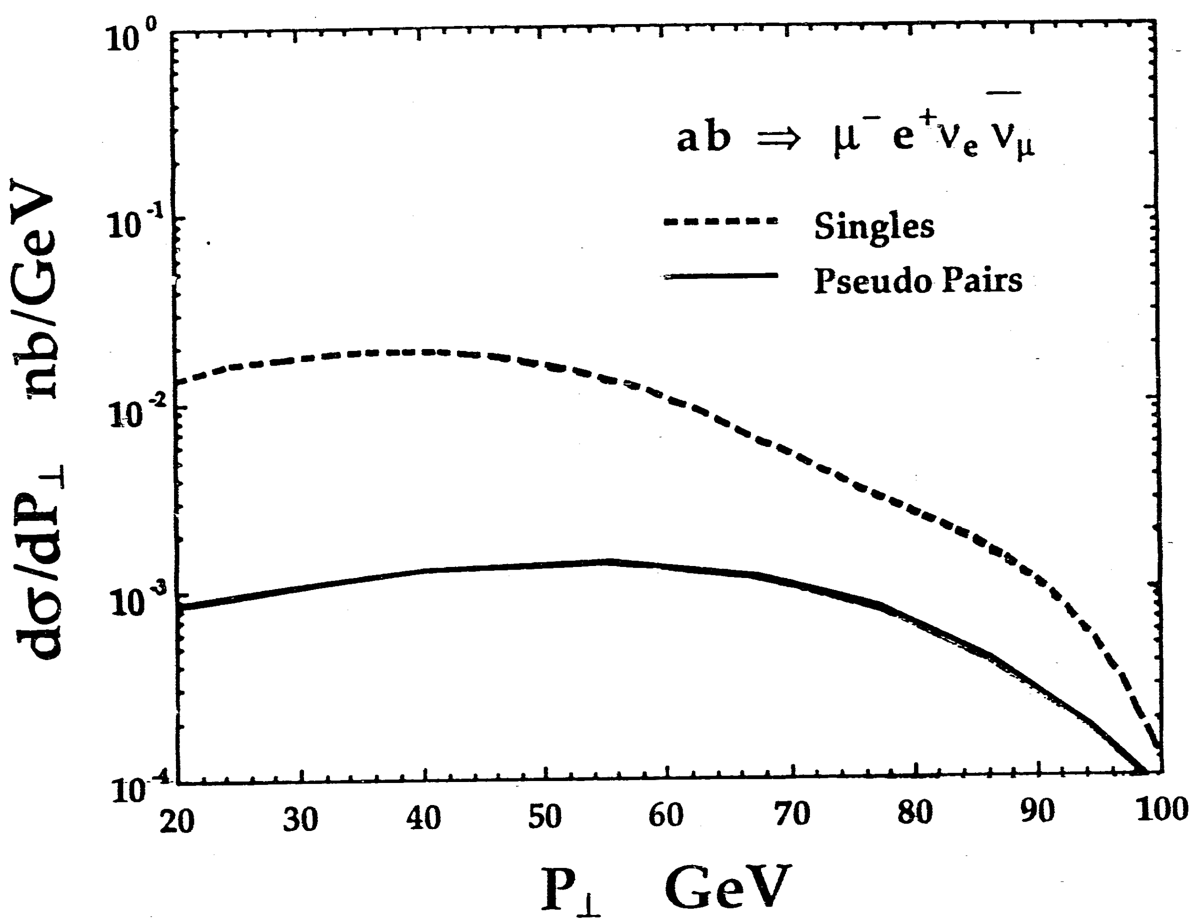


$\mathcal{B}(W \to e\nu) \sim .1$

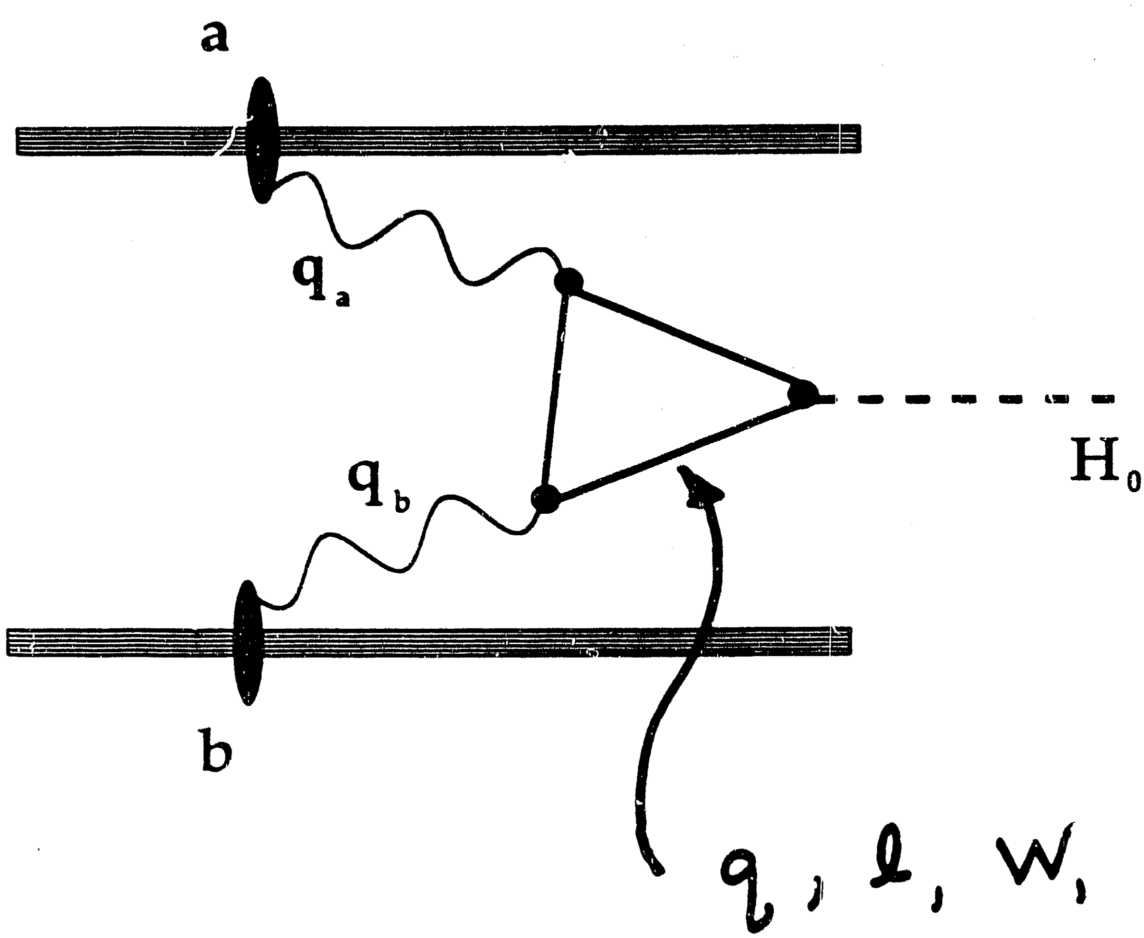


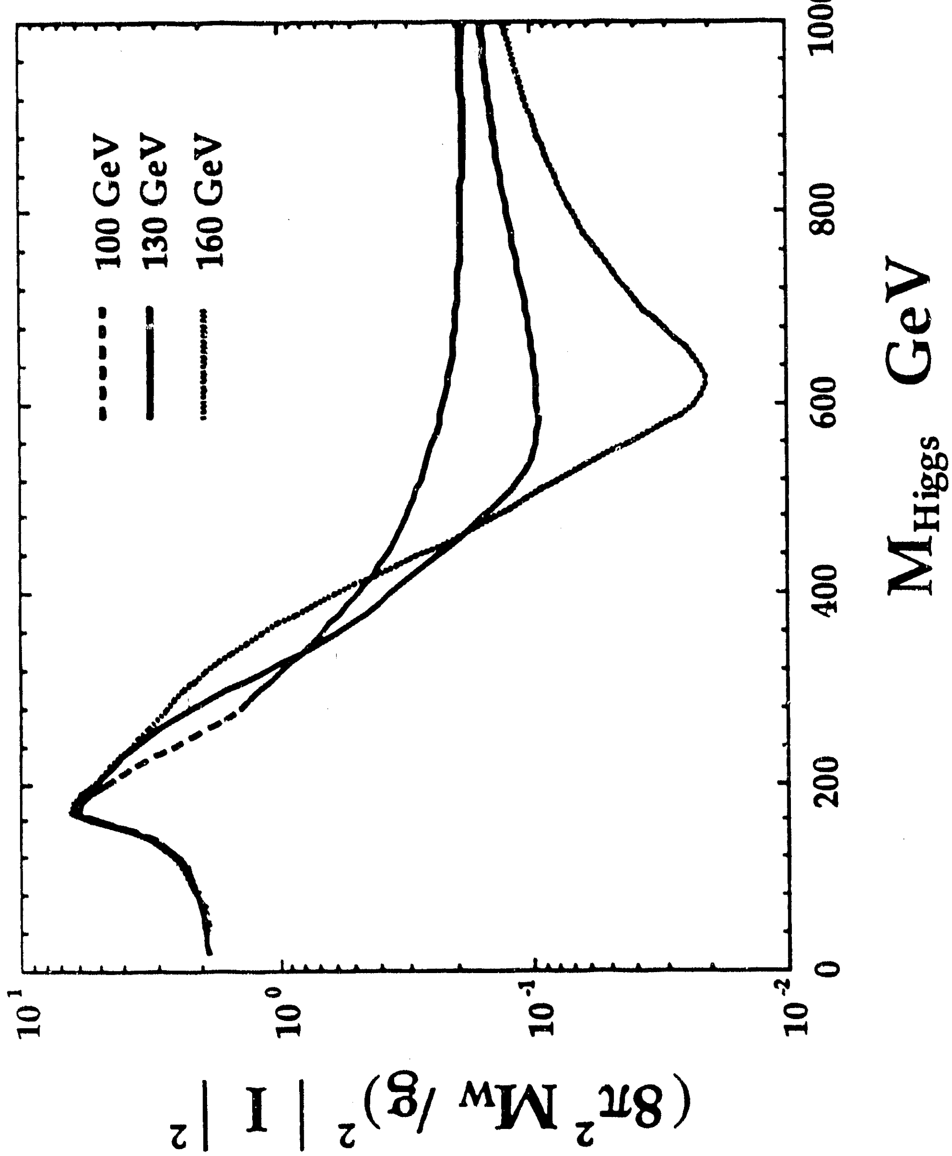
a,b ⇒ W⁻ e⁺ν_e
W Decay
Background
P⊥ GeV
dσ/dP⊥ nb/GeV

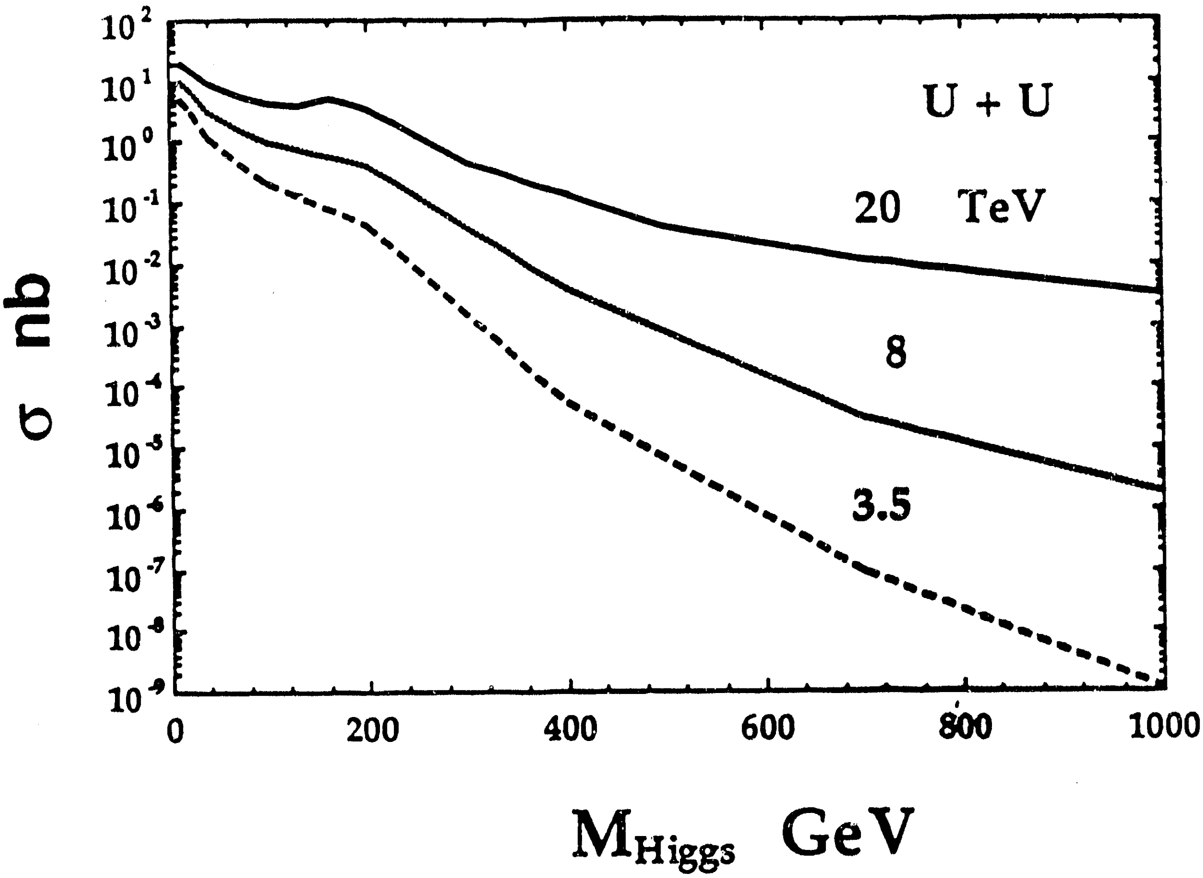

a
k_l
q_a
k_ν
p
q_b
k'_l
k'_ν
b



Background: $b\bar{b}$ decay

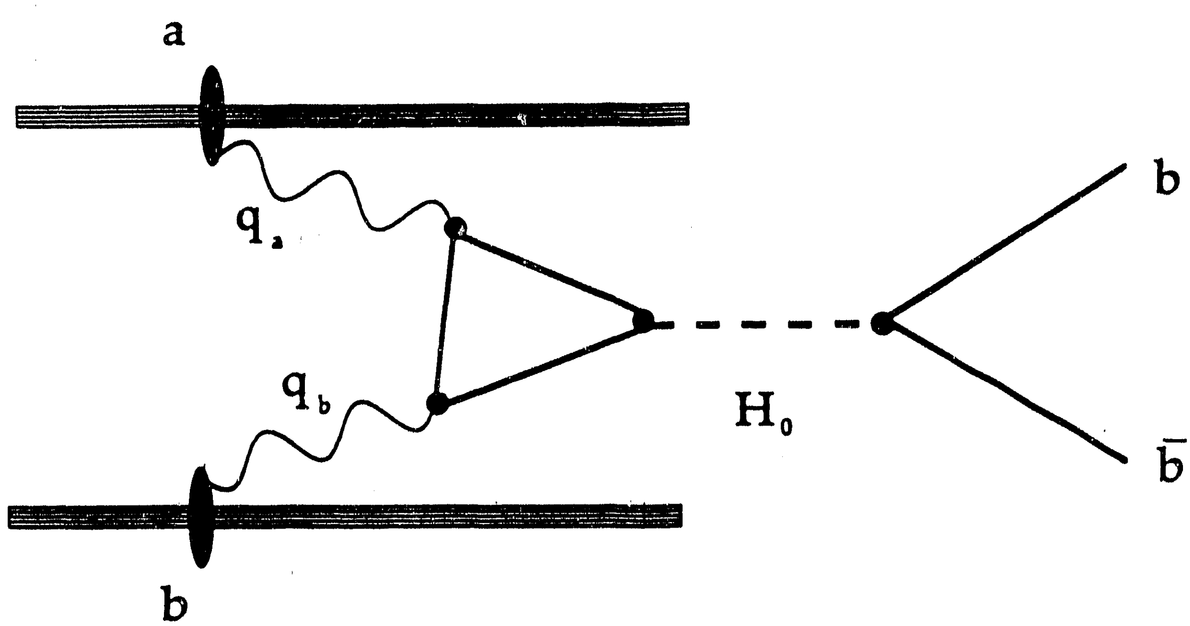

a
q_a
q_b
H_0
b
q, ℓ, W,


100 GeV
130 GeV
160 GeV
0
200
400
600
800
1000
M_{Higgs} GeV
10^{1}
10^{0}
10^{-1}
10^{-2}
$(8\pi^2 M_W / g)^2 \mid I \mid^2$

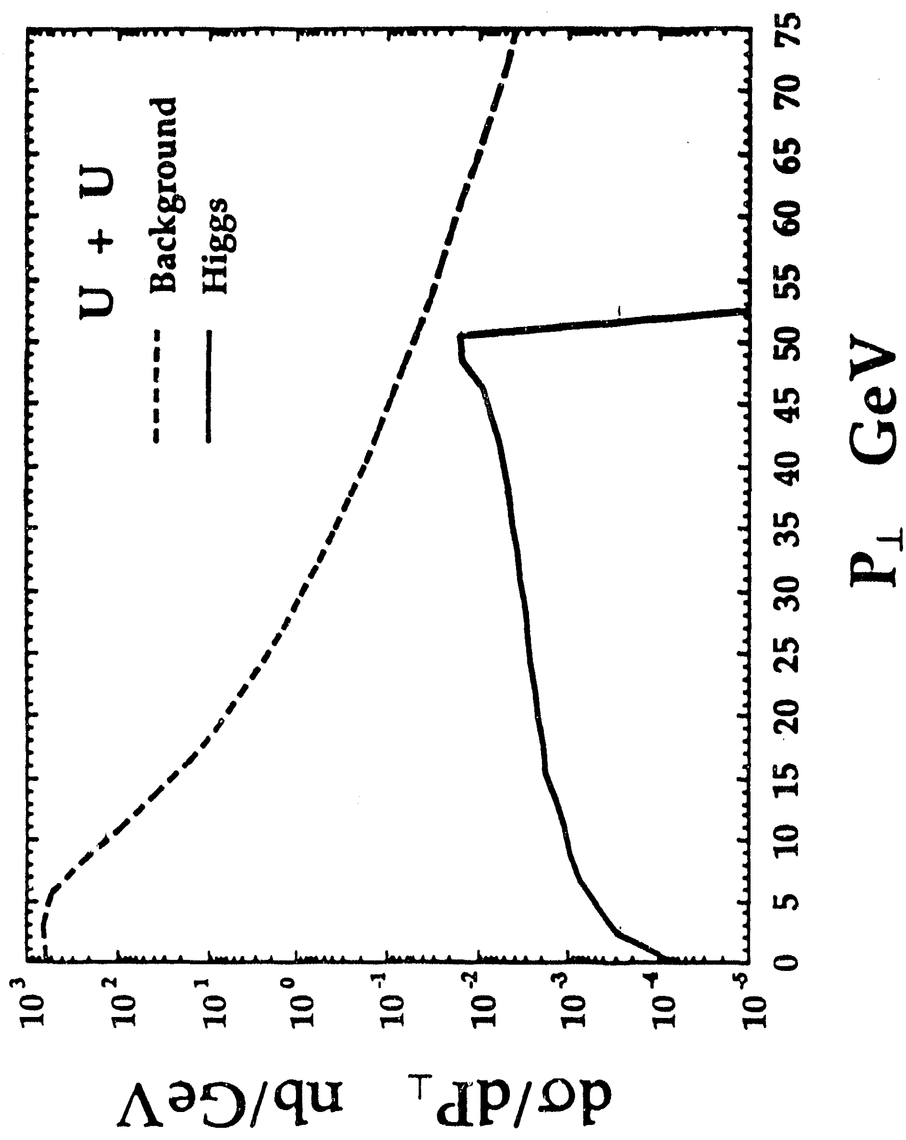


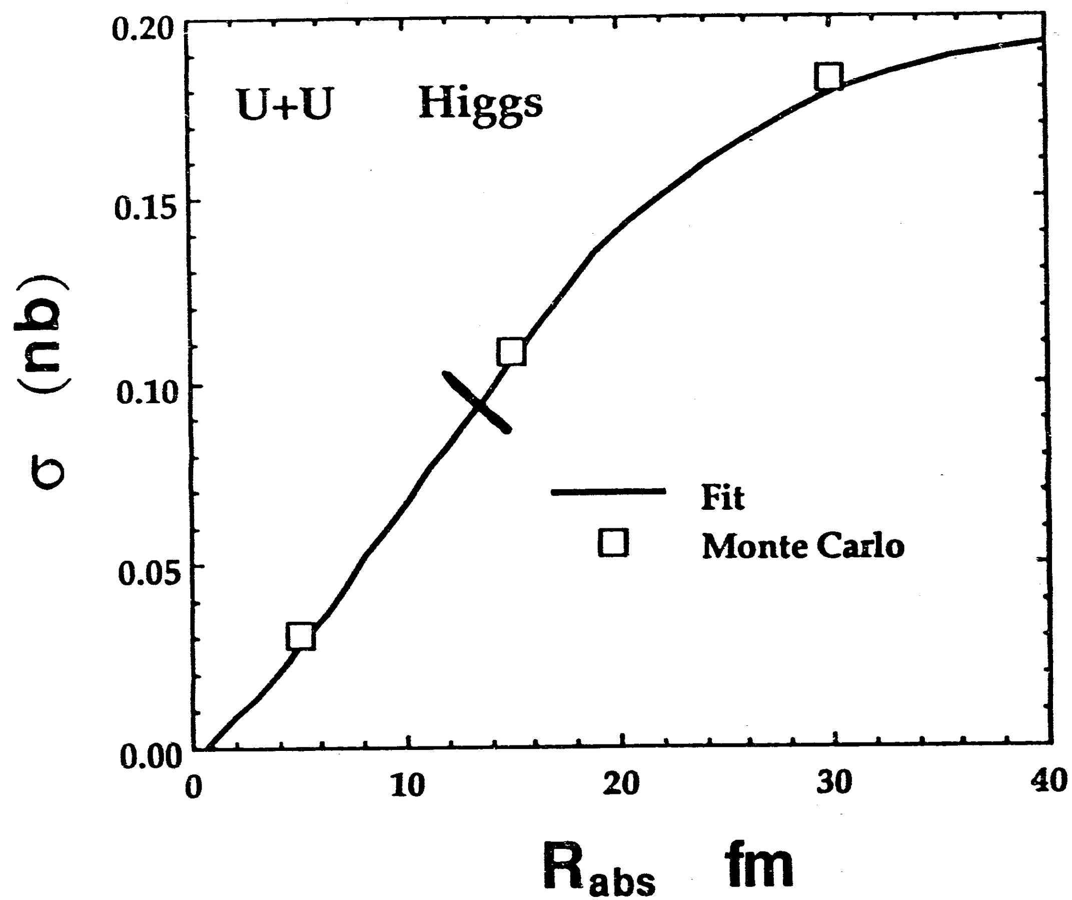
$m_H = 10^2$ GeV

gg	0.5	nb
LHC	0.2	nb
SSC	1	nb

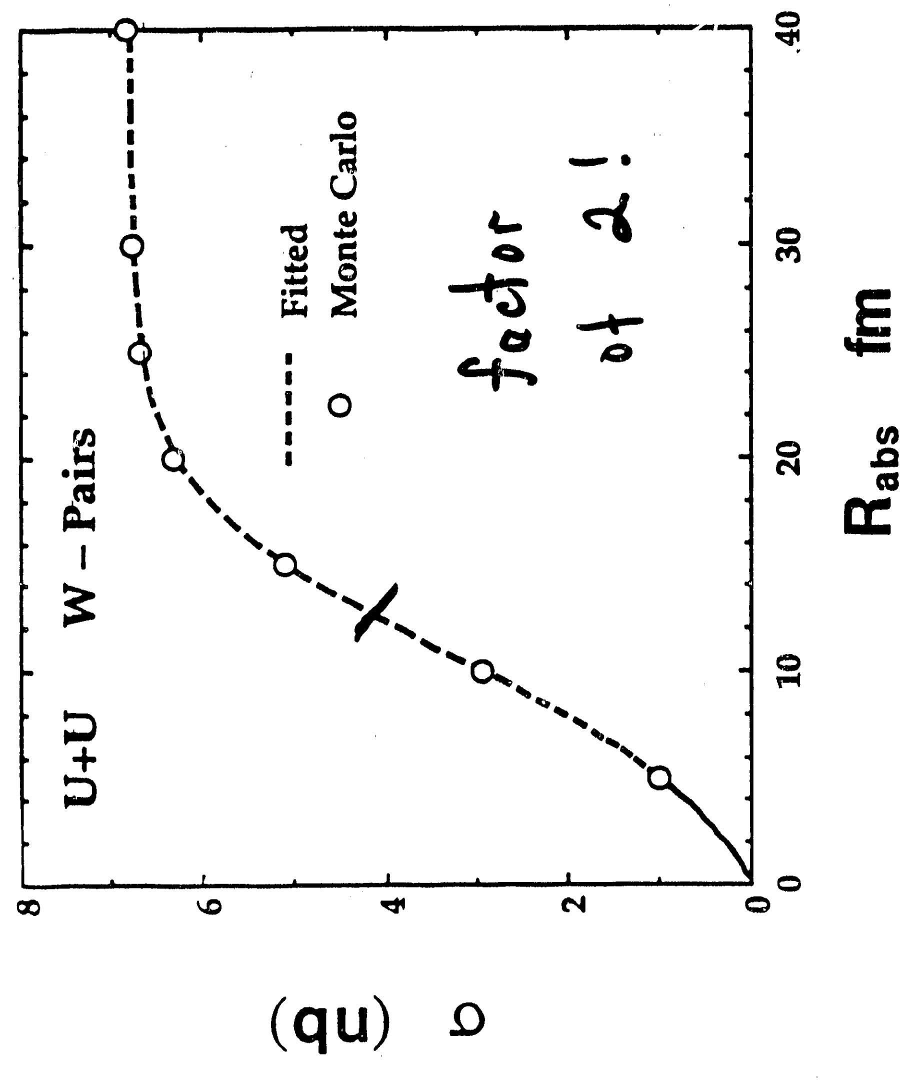


Dreg, Ellis, Zeppenfeld
P.L.


U + U
Background
Higgs
P⊥ GeV
dσ/dP⊥ nb/GeV
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75
10^3 10^2 10^1 10^0 10^-1 10^-2 10^-3 10^-4 10^-5



$$\sigma_{abs} \sim \frac{1}{2}\,\sigma_{bare}$$


U+U W – Pairs
Fitted
Monte Carlo
factor of 2!
σ (nb)
R_{abs} fm
0
2
4
6
8
0
10
20
30
40

Parton Model Cross Sections

$$\sigma = \int dx\, dx'\, f(x)\, f(x')\, \sigma(x, x')$$

$$f(x) = \left\{\frac{4}{9}\mathrm{Z} + \frac{1}{9}\mathrm{N}\right\} u(x) + \left\{\frac{4}{9}\mathrm{N} + \frac{1}{9}\mathrm{Z}\right\} d(x)$$

	RHIC		SSC	
	μ	τ	μ	τ
$p + p$	14 nb	2 pb	60 nb	13 pb
$Au + Au$	400 μb	400 nb	2 mb	3 μb

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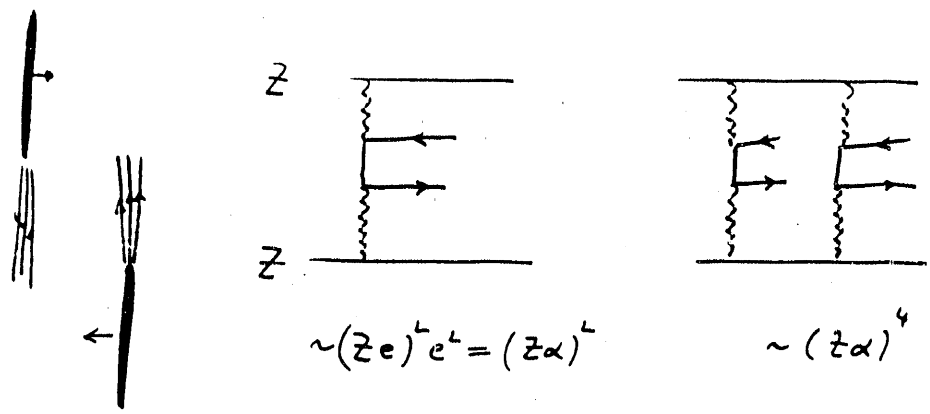
LEPTON PAIR PRODUCTION AT RHIC

Talk presented by:

G. Baur

Lepton Pair Production at RHIC



$\sim (Ze)^2 e^2 = (Z\alpha)^2$ $\qquad$ $\sim (Z\alpha)^4$ $\qquad$ $Z\alpha \lesssim 0.7$

$\gamma = (1-\beta^2)^{-\frac{1}{2}}$

$q = (i\omega, \vec{q}_t, \frac{\omega}{\beta})$

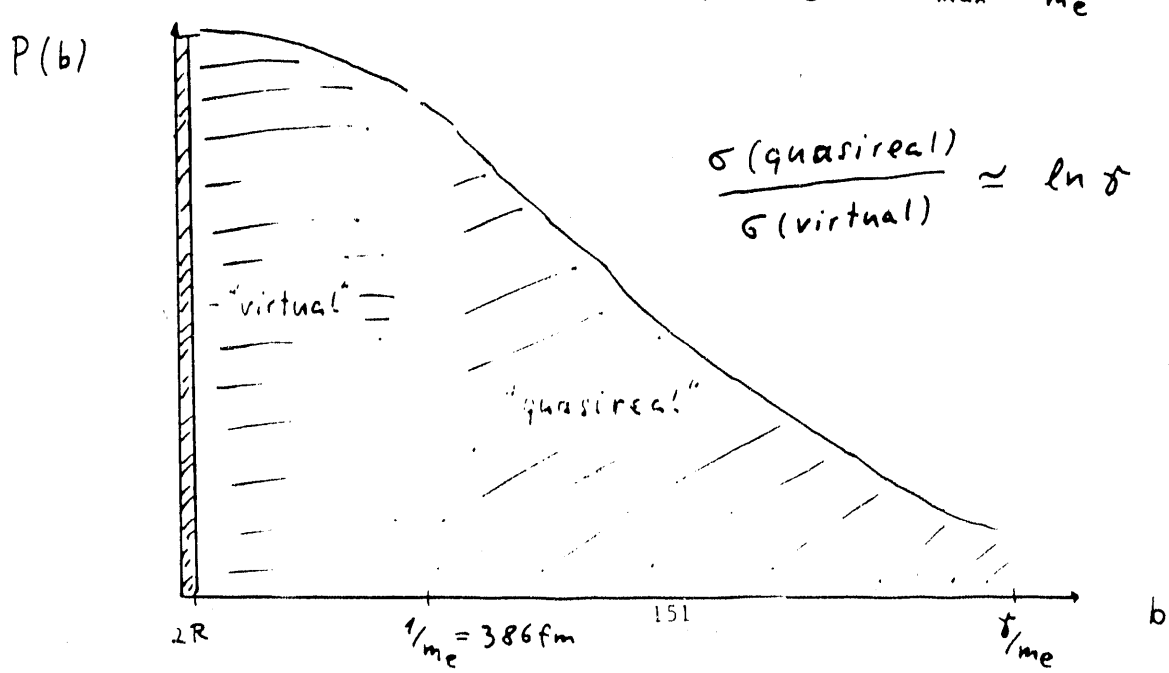
coherency of Z protons:

$|\vec{q}_t| \lesssim \frac{1}{R}$ $\qquad$ $\frac{\omega}{\gamma} \lesssim \frac{1}{R}$

invariant mass W in γ-γ collisions: $\sim 2\omega$

$W_{max} \simeq \frac{2\gamma}{R}$ $\qquad$ $\gamma = 100$: $W_{max} = 6$ GeV $\quad$ ($R = 7.5$ fm)

production of e^+e^- pairs : $W_{min} = 2m_e$ $\qquad$ $b_{max} \simeq \frac{\gamma}{m_e}$



$\frac{\sigma(\text{quasireal})}{\sigma(\text{virtual})} \simeq \ln\gamma$

1. One pair-production

a) e^+e^- pair in lowest order

problems with unitarity

b) energy loss in colliders due to pair production

c) beam loss: $Z + Z \to (Z + e^-)_{K,L,\ldots} + e^+ + Z$

2. A Strong Field Effect:

Multiple pair production

3. γ-γ physics at RHIC

heavy lepton pairs

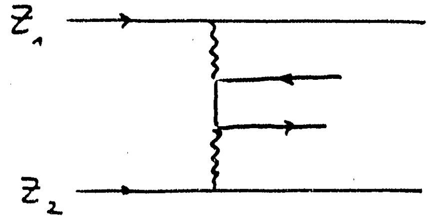
hadronic states (test QCD)

4. Conclusion

1. One pair-production

1^{st} order amplitude:

$$a_{e^+e^-} = \frac{1}{i} \int_{-\infty}^{\infty} dt \; e^{i\omega t} \langle \psi_{e^-} | j_\mu A_\mu | \psi_{e^+} \rangle$$



A_μ ... potential created by projectile Z_1 straight line motion

$\psi_{e^\pm}$ ... wave function in the field of target Z_2. All orders in Z_2

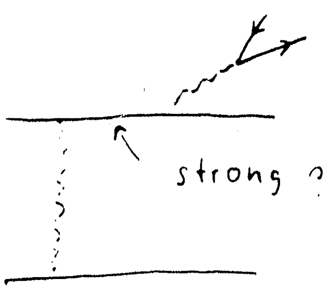
Landau, Lifshitz (1935): γ-γ mechanism $\quad \sigma \sim Z_1^2 Z_2^2$

W. Scheid et al.: Coulomb-Dirac wave functions

C. A. Bertulani, G.B.: Sommerfeld-Maue wave functions,

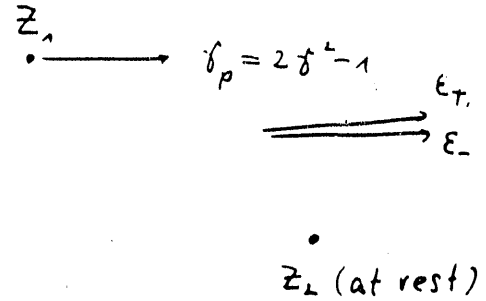
analytical results, based on

Bethe-Maximon theory for $\gamma + Z_2 \to e^+e^- + Z_2$



strong suppression of "bremsstrahlung pair production" due to the large ion mass

S. J. Brodsky, S.C.C. Ting Phys. Rev. 1966



mainly "fast pairs":

$m_e \ll \varepsilon_+, \varepsilon_- \ll \gamma_p m_e$

(angle integrated) <u>probability for pair production</u>

impact parameter $b > \frac{1}{m_e}$, $\omega = \varepsilon_+ + \varepsilon_-$

$$\frac{d^2 P(b)}{d\varepsilon_+ d\varepsilon_-} = \frac{4}{\pi^2} (Z_1 Z_2 \alpha^2)^2 \frac{K_1^2\left(\frac{\omega b}{\gamma_p}\right)}{m_e^2 \gamma_p^2 \omega^2} \left[\varepsilon_+^2 + \varepsilon_-^2 + \frac{2}{3} \varepsilon_+ \varepsilon_- \right] \times$$

$$\times \left[\ln\left(\frac{2 \varepsilon_+ \varepsilon_-}{m_e \omega}\right) - \frac{1}{2} - f(\alpha Z_2) \right]$$

(no "screening")

$$f(\alpha Z) \equiv (\alpha Z)^2 \sum_{n=1}^{\infty} \frac{1}{n(n^2 + (\alpha Z)^2)}$$

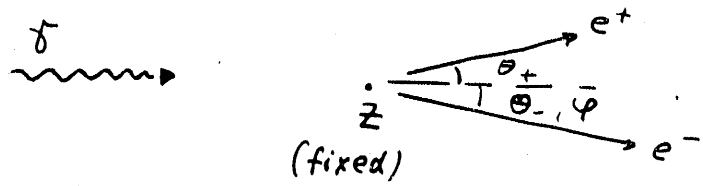
obvious problem: Z_1 and Z_2 treated asymmetrically
(symmetrization (?))

importance of higher order corrections in Z_2 depend on region of phase space;

negligible if momentum transfer $q < m_e$

(see also M. Fatyga, M. Tannenbaum, H. J. Crawford, J. C. Hill, AGS letter of intent, and refs.)

e^+, e^- scattering angles



$\theta_\pm \lesssim \frac{m}{\varepsilon_\pm}$, see Bethe-Maximon theory

transform to collider system: $\theta_\pm^{coll} \lesssim \frac{m}{\varepsilon_\pm^{coll.}}$

$\gamma + \gamma \to e^+ + e^-$:

$$\frac{d\sigma}{d\Omega} \simeq \frac{\alpha^2}{2\,\varepsilon^2\left(\theta^2 + \frac{m^2}{\varepsilon^2}\right)}$$

total probability for pair-production:

$$\frac{1}{m_e} < b < \frac{\gamma_p}{m_e}$$

$$P(b) = \frac{14}{9\pi^2} \left(Z_1 Z_2 \alpha\right)^2 \frac{1}{m_e^2 b^2} \left[\ln^2 \frac{\gamma_p}{2 m_e b} - \dots \right]$$

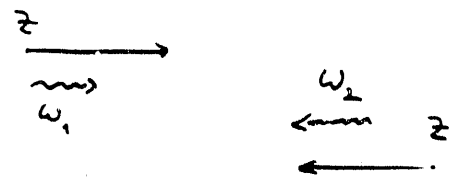
$Z_1 = Z_2 = 92$ $\gamma_p = 20\,000$ $(\gamma \approx 100)$

$$P\left(\frac{1}{m_e}\right) = 2.4$$

$\Longrightarrow$ higher order effects

invariant mass m_{ee} of e^+e^--pair

use equivalent photon method.



$$m_{ee}^2 = 4\omega_1\omega_2$$

$$\frac{d\sigma}{dm_{ee}^2} = J(m_{ee}^2)/m_{ee}^2 \cdot \sigma_{\gamma+\gamma\to e^+e^-}(m_{ee}^2)$$

$$J(m_{ee}^2) = \int d\omega_1\, n(\omega_1)\cdot n\left(\omega_2 = \frac{m_{ee}^2}{4\omega_1}\right)$$

$n(\omega)$... equivalent photon spectrum

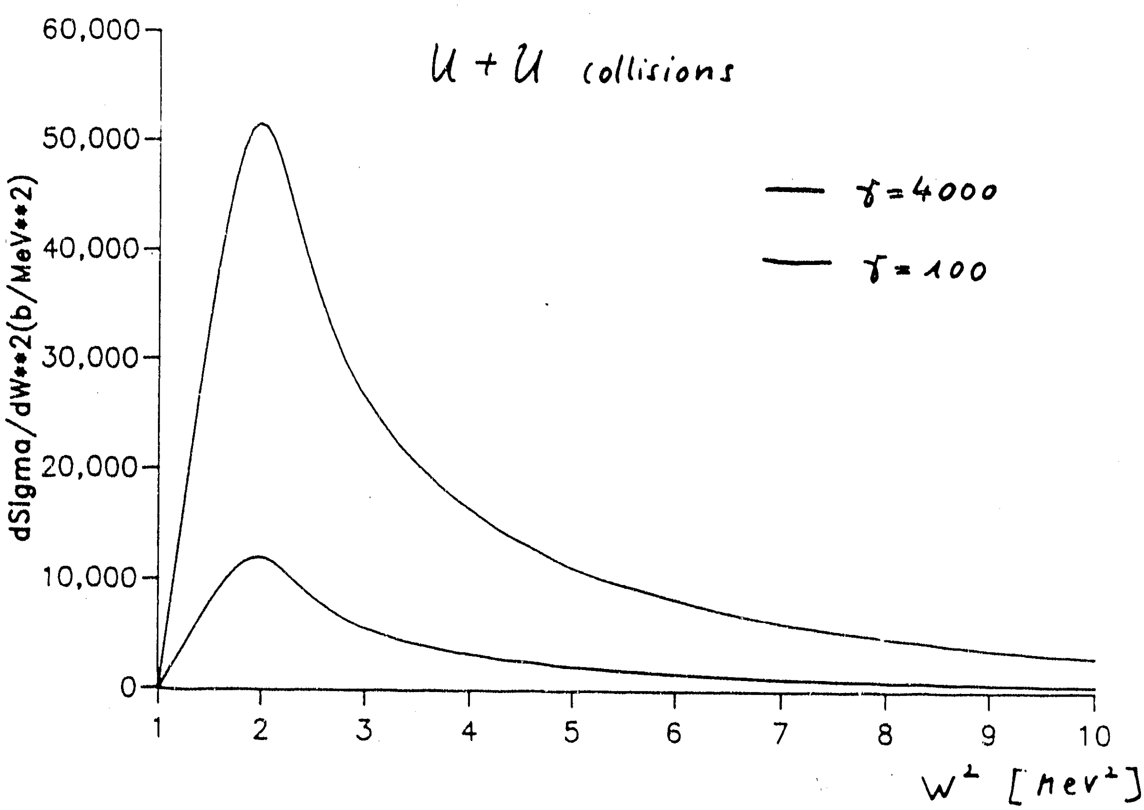
$\left(b_{min} \approx \frac{1}{m_e}\right)$

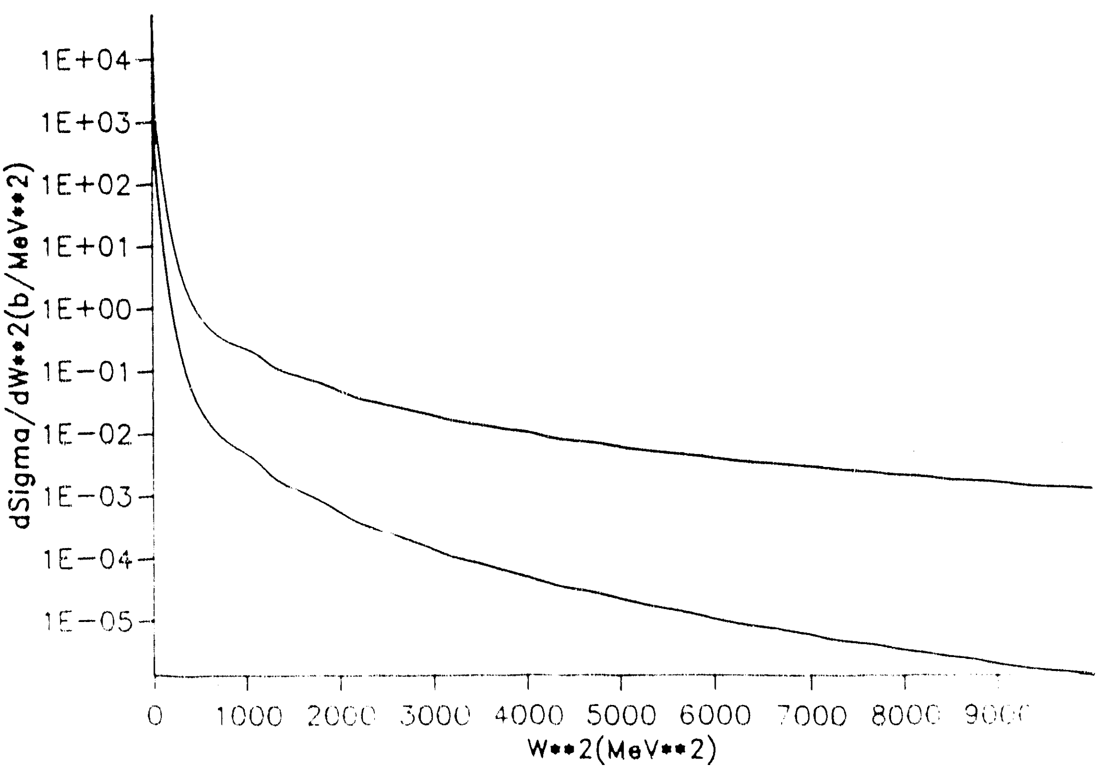
sharp peak just above threshold

G. Alexander et al.

G. B., C. A. Bertulani, L. Gonzaga

e^+e^- invariant mass distributions

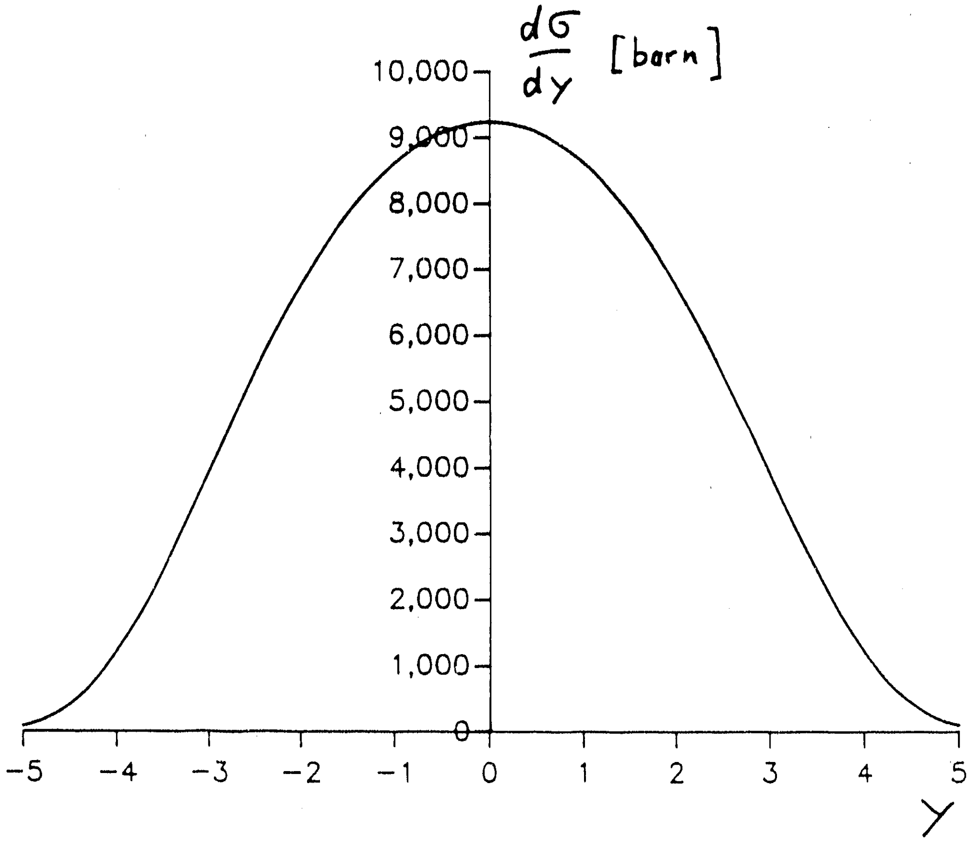




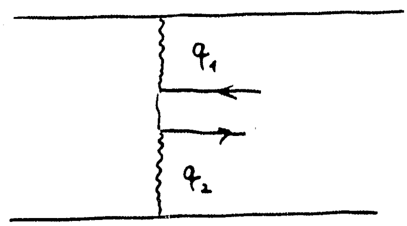
Rapidity distribution of (e^+e^-)-pair. $y \simeq \frac{1}{2} \ln \frac{\omega_1}{\omega_2}$

$U + U \rightarrow U + U + e^+e^-$

$\gamma = 100$



Improvement for $b < \lambda_{el}$ (work in progress)



for $q_1^2, q_2^2 > m_e^2$:

$$"\gamma" + "\gamma" \rightarrow e^+ + e^-$$

$\sigma_{ab}(q_1^2, q_2^2, m_{ee}^2)$ ... known in analytical form

$a, b = T$ and S

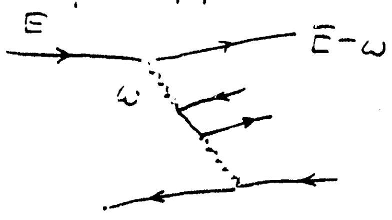
$$\sigma_{Z_1 Z_2 \rightarrow Z_1 Z_2 e^+ e^-} = \sum_{a,b} \rho_{ab} \, \sigma_{ab}(q_1^2, q_2^2, m_{ee}^2)$$

flux of (virtual) photons

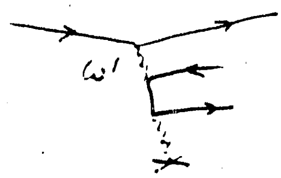
(see e.g. V.M. Budnev et al.)

b) Energy loss in Collider due to e^+e^- pair production

simple approach:



collider frame



one nucleus at rest

$$\frac{d\sigma}{d\omega} = \frac{n(\omega)}{\omega} \cdot \sigma_{Bethe-Heitler}(\omega' = 2\gamma\omega)$$

$$\cong \frac{56}{9\pi}\left(\frac{Z^2\alpha^2}{m_e}\right)^2 \left(\ln^2\gamma - \ln^2\left(\frac{\omega}{m_e}\right)\right)$$

average energy loss:

$$\langle\omega\sigma\rangle \cong 2\int d\omega \cdot \omega \cdot \frac{d\sigma}{d\omega} = \frac{224}{9\pi}(\alpha Z)^4 \frac{\gamma \cdot \ln\gamma}{m_e}$$

even less for muons

influence on beam dynamics is negligible

(in agreement with conclusion by G. Young, RHIC workshop '88

at CERN, LHC: $Z = 82$, $\gamma = 4000$, $L = 10^{28}\ cm^{-2}s^{-1}$, $K = 3$

(D. Brandt)

$$\frac{dE}{dt} = K \cdot L \cdot \langle\omega\sigma\rangle = 7.6 \cdot 10^{11}\ MeV\,s^{-1} = 1.3 \cdot 10^{-2}\ W$$

c) A mechanism for beam loss: $Z + Z \to (Z + e^-)_{1s,2s,\ldots} + e^+ + Z$

H. Gould

W. Scheid et al.

Rhoades-Brown et al.

C. A. Bertulani, G.B.: use Sommerfeld-Maue wave function and a simple K-shell wave function ($Z\alpha \ll 1$)

analytical results:

K-capture into target (at rest)

e^+ - angular distribution:

$$\sim \sin^2\theta_+ \qquad (\varepsilon_+ - m) \ll m$$

$$\sim \frac{\theta_+^2}{\left(\left(\frac{m}{\varepsilon_+}\right)^2 + \theta_+^2\right)^4} \qquad \varepsilon_+ \gg m$$

e^+ - energy-distribution, for $\varepsilon_+ \gg m$

$$\frac{d\sigma}{d\varepsilon_+} = 8 Z_1^2 Z_2^5 \alpha^7 \frac{1}{m\varepsilon_+^2} \ln \frac{\gamma m}{\varepsilon_+}$$

$$\sigma = \frac{33}{20} Z_1^2 Z_2^5 \alpha^7 \frac{1}{m^2} \ln\gamma \qquad (\ln\gamma \gg 1)$$

with equivalent photon method:

input: $\gamma + Z \rightleftarrows e^+ + (e^- + Z)_{K,L,\dots}$ (*)

E. Fermi, Uhlenbeck (1933) Nishina, Tomonaga, Tamaki (1934)

$$\frac{d\sigma}{d\omega} = \frac{n(\omega)}{\omega} \cdot \sigma_\gamma$$ recover previous formula

with $n(\omega) = \frac{2}{\pi} Z_1^2 \alpha \ln \frac{\gamma m}{\omega}$ (low frequency approx.)

could also be improved by more elaborate calculation for the cross section (*). (related to photoelectric effect)

Capture to higher orbits: 2, 4, ...

$$\sigma_{ns} \simeq \frac{\sigma_{1s}}{n^3}$$ (see Bethe, Salpeter)

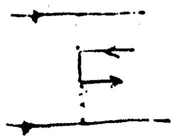
$$\sum_{n=1}^{\infty} \frac{1}{n^3} = \zeta(3) = 1.202\dots$$

i.e. ~ 20% capture to higher orbits

capture: scales with $Z_1^2 Z_2^5 \sim Z^7$ $(Z_1 = Z_2)$

cf. electromagnetic fragmentation via giant resonance excitation
$\sim Z^{10/3}$

2. Multiple e^+e^- Pair Creation: A Strong Field Effect

lowest order  was found to violate unitarity,

under realistic conditions: $P^{(1)}(b = \frac{1}{m_e}) = 2.4 \quad (Z = 92, \gamma = 100)$

$$S = T \exp(-i \int_{-\infty}^{\infty} dt\, V(t))$$

neglect time ordering T, if

interaction time $\frac{1}{\gamma}\frac{b}{c}$ << excitation time $\frac{1}{\omega}$ "sudden approx."

$$\int_{-\infty}^{\infty} dt\, V(t) = \sum_i (u_i B_i^+ + u_i^* B_i) \qquad B_i^+ \equiv a_{i_-}^+ b_{i_+}^+$$

creates (destroys) e^+e^- pairs in state $i \equiv i_- i_+$

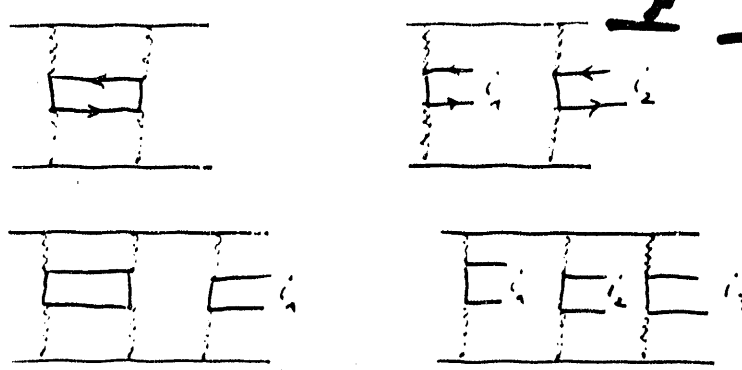
$$P^{(1)}(b) = \sum_i |u_i|^2$$

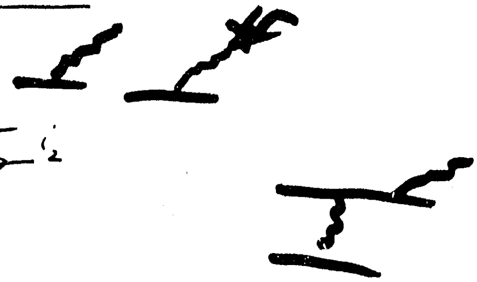
quasiboson approximation for e^+e^--pair

(i.e. neglect Pauli-blocking,

$[B_i, B_j]^+ = \delta_{ij}$

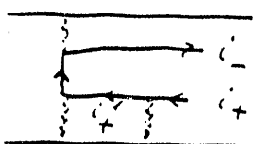
summation of infinitely many diagrams





etc.

neglect: ("rescattering")



etc.

$$Z + Z \to Z + Z + (e^+e^-)_{i_1} + (e^+e^-)_{i_2} + \ldots (e^+e^-)_{i_N}$$

with $e^{A+B} = e^A \, e^B \, e^{-\frac{1}{2}[A,B]}$ (for $[A,B]$ = c-number):

$$\langle i_1 i_2 \ldots i_N | S | 0 \rangle = e^{-\frac{1}{2} p^{(1)}(b)} \, i^N \prod_{K=1}^{N} u_{i_K}$$

$$P(i_1, i_2, \ldots, i_N; b) = e^{-p^{(1)}(b)} \prod_{K=1}^{N} |u_{i_K}|^2$$

probability for multiplicity N:

$$P(N, b) = \frac{e^{-p^{(1)}(b)}}{N!} \left(p^{(1)}(b)\right)^N \qquad \text{Poisson distribution}$$

A new meaning of 1^{st} order calculation

$P^{(1)}(b)$ corresponds to mean value of Poisson distribution

the same phase space distribution as in the 1^{st} order calculation

(analogy to excitation of harmonic oscillators by a time dependent force;
and to infrared catastrophe in QED: emission of very many soft photons, perturbation theory breaks down)

see also related work by E. Papageorgiu, thesis, MPI Munich

Total cross sections for $\gamma \to \infty$

$$\sigma(N) \equiv 2\pi \int_{1/m_e}^{\infty} b\, db\, P(N, b)$$

$$\sigma(1) = \frac{28}{27\pi} \frac{(Z\alpha)^4}{m_e^2} \ln^3 \gamma \qquad \left(b_{max} = \frac{\gamma}{m} \right)$$

$N \geq 2$
$$\sigma(N) = \frac{\pi\, P^{(1)}(b)}{m_e^2\, N(N-1)} \left[1 - e^{-P^{(1)}} \sum_{k=0}^{N-2} \frac{\left(P^{(1)}\right)^k}{k!} \right]$$

$$\simeq \frac{\pi \left(P^{(1)}\right)^N}{m_e^2\, N!\,(N-1)} \qquad P^{(1)} \equiv P^{(1)}\left(\frac{1}{m_e}\right) \ll 1$$

Violation of Froissart Bound

$\sigma < \text{const.}\,(\ln \gamma)^2$ from rather general arguments

subtle, academic (?)

study of asymptotic QED

H. Cheng, T. T. Wu
V. N. Gribov

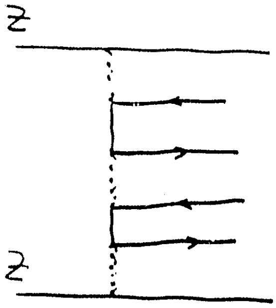
use "main logarithmic approximation" : $\alpha^2 \ln s \sim 1$

Froissart bound is found to be violated

For $s > e^{(137)^2}$ include diagrams which would cure the violation

Another mechanism for $2(e^+e^-)$ pair production

E. Papageorgiu



with $\sigma(\gamma\gamma \to (e^+e^-)(e^+e^-)) \xrightarrow[\text{energy}]{\text{high}} \sigma_0 = \frac{\alpha^4}{m_e^2} \; \frac{175\,\zeta(3) - 38}{36\pi} \simeq 7 \cdot 10^{-30}\, cm^2$

(Cheng, Wu)

$$\sigma\,(Z + Z \to Z + Z + (e^+e^-) + (e^+e^-)) \longrightarrow \frac{8}{3}\, \frac{Z^4 \alpha^2}{\pi^2}\, \sigma_0 \, \ln^4 \gamma$$

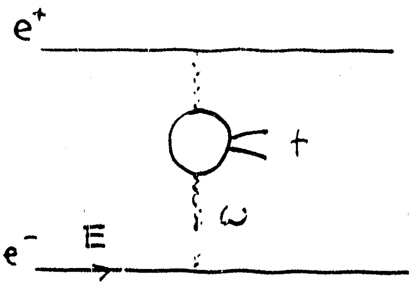
cf. to

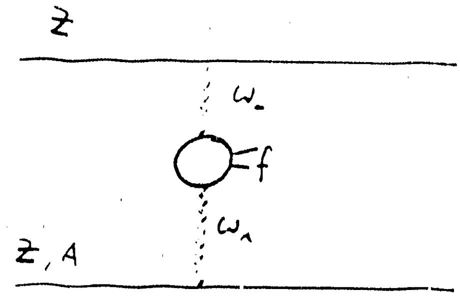
$$\sigma\,(N=2) = \frac{98}{81\pi^3}\, Z^8 \alpha^8 \,\frac{1}{m_e^2}\, \ln^4 \gamma$$

$$\frac{\sigma\,(\text{now})}{\sigma\,(\text{Papageorgiu})} \sim (\alpha Z)^2 Z^2$$

3. γ-γ physics at RHIC

production of heavy leptons and hadrons





at e^+e^--colliders, the fractional energy loss can be large, $x \equiv \frac{\omega}{E}$ $\quad 0 < x < 1$

"virtual" as well as "quasireal" photons

at RHI colliders: $\quad x_{max} = \frac{1}{R} \cdot \frac{\hbar}{m_A c} \ll 1$

only "quasireal" photons (except for $\gamma + \gamma \to e^+e^-$ case)

$$\frac{d\sigma}{dW^2} = Z^4 \; \sigma_{\gamma\gamma\to f}(W^2 = 4\omega_1\omega_2) \cdot J(W^2)$$

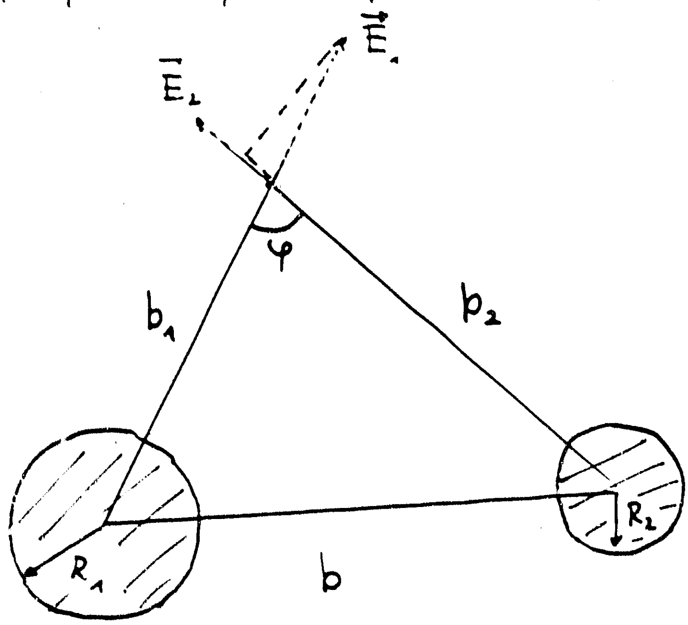
$$J(W^2) \equiv \int \frac{d\omega_1}{\omega_1} F\left(\omega_1, \frac{W^2}{4\omega_1}\right)$$

... the hadronic background can be suppressed by selecting events in which the photons are emitted coherently and the nucleus remains intact in the beam pipe, while the mesons from the γ-γ collision appear in the central region...

W. J. Willis, The Suite of Detectors for RHIC, RHIC workshop '85

Calculation of $J(W^2)$ including strong absorption effects

$N(\omega, b)$ ... equivalent photon spectrum for given impact parameter b



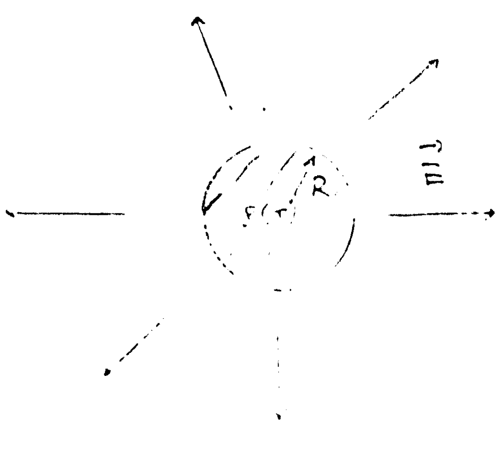
$$J(W^2) = \int \frac{d\omega_1}{\omega_1} F(\omega_1, \frac{W^2}{4\omega_1})$$

$$F(\omega_1, \omega_2) = 2\pi \int_{R_1}^{\infty} db_1 b_1 \int_{R_2}^{\infty} db_2\, b_2 \int_0^{2\pi} d\varphi\; N(\omega_1, b_1) \cdot N(\omega_2, b_2) \cdot \Theta(b - R_1 - R_2)$$

(sharp) cut-off, due to strong absorption (G.B. Phys. Scripta)

a refinement for polarisation effects: $\sigma \cdot J \to \sigma_{\parallel} J_{\parallel} + \sigma_{\perp} J_{\perp}$

photon polarisation parallel perpendicular



$\vec{E}$ field for $r > R$ independent of $\rho(r)$ (for $\rho(r) = 0$, $r > R$)

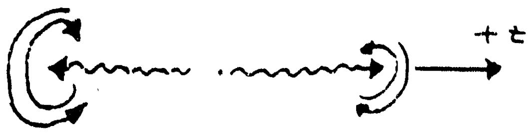
$F(q) = \int e^{i\vec{q}\cdot\vec{r}} \rho(r)\, d^3r$ ... does not enter into calculation

Polarization effects in γ-γ-collisions

$0^+ \longrightarrow \gamma + \gamma$ polarization $\parallel$

$0^- \longrightarrow \gamma + \gamma$ polarization $\perp$

general analysis by L. D. Landau, C. N. Yang (1949)



ψ_{RR}, ψ_{LL}, ψ_{LR}

ψ_{RL} (photon in +z-direction : R

" " −z− " : L)

choose 4 states :

	$\psi_{RR}+\psi_{LL}$	$\psi_{RR}-\psi_{LL}$	ψ_{RL}	ψ_{LR}
R_φ	1	1	$e^{2i\varphi}$	$e^{-2i\varphi}$
P	1	−1	1	1
polarization	$\parallel$	$\perp$	50% $\parallel$, 50% $\perp$	50% $\parallel$, 50% $\perp$

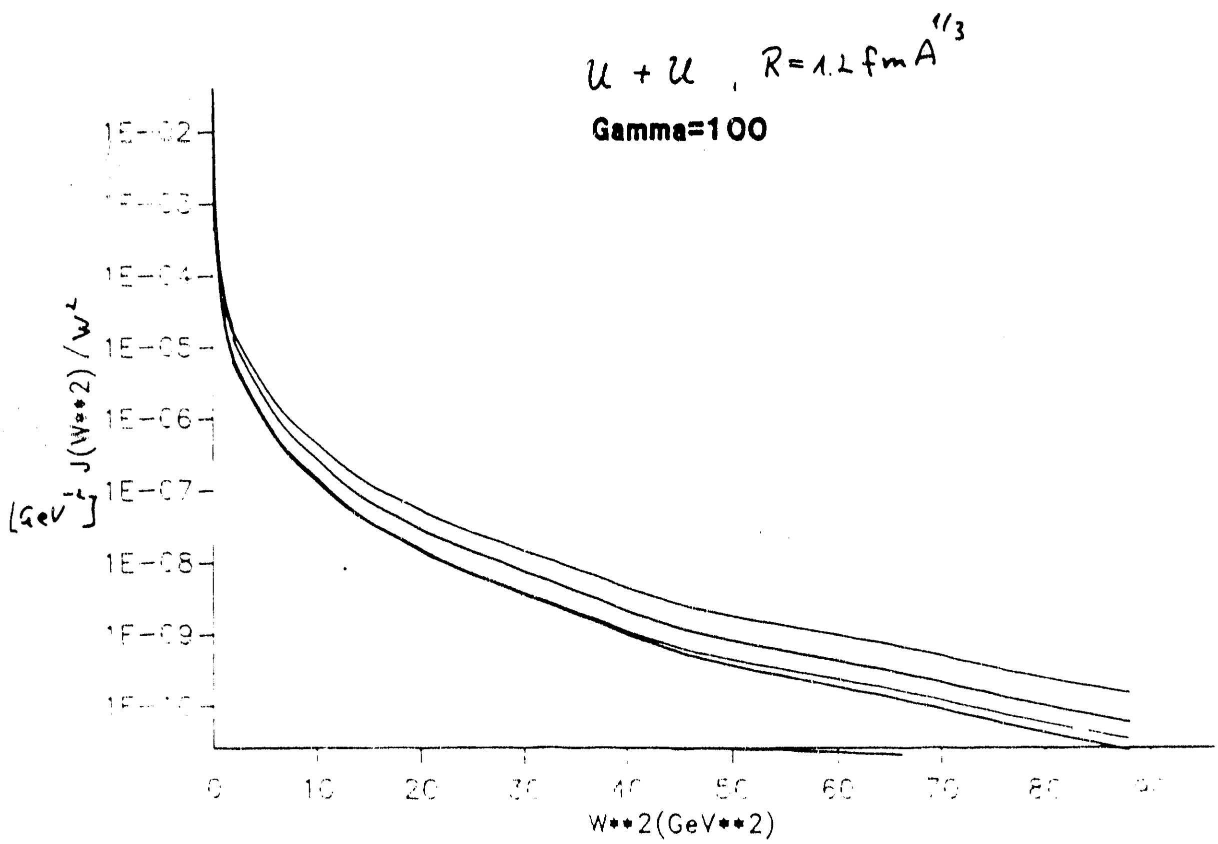
π \ J	0	1	2, 4, 6, ...	3, 5, 7, ...
+	$\psi_{RR}+\psi_{LL}$	—	$\psi_{RR}+\psi_{LL}$ ψ_{RL}, ψ_{LR}	ψ_{RL}, ψ_{LR}
−	$\psi_{RR}-\psi_{LL}$	—	$\psi_{RR}-\psi_{LL}$	—

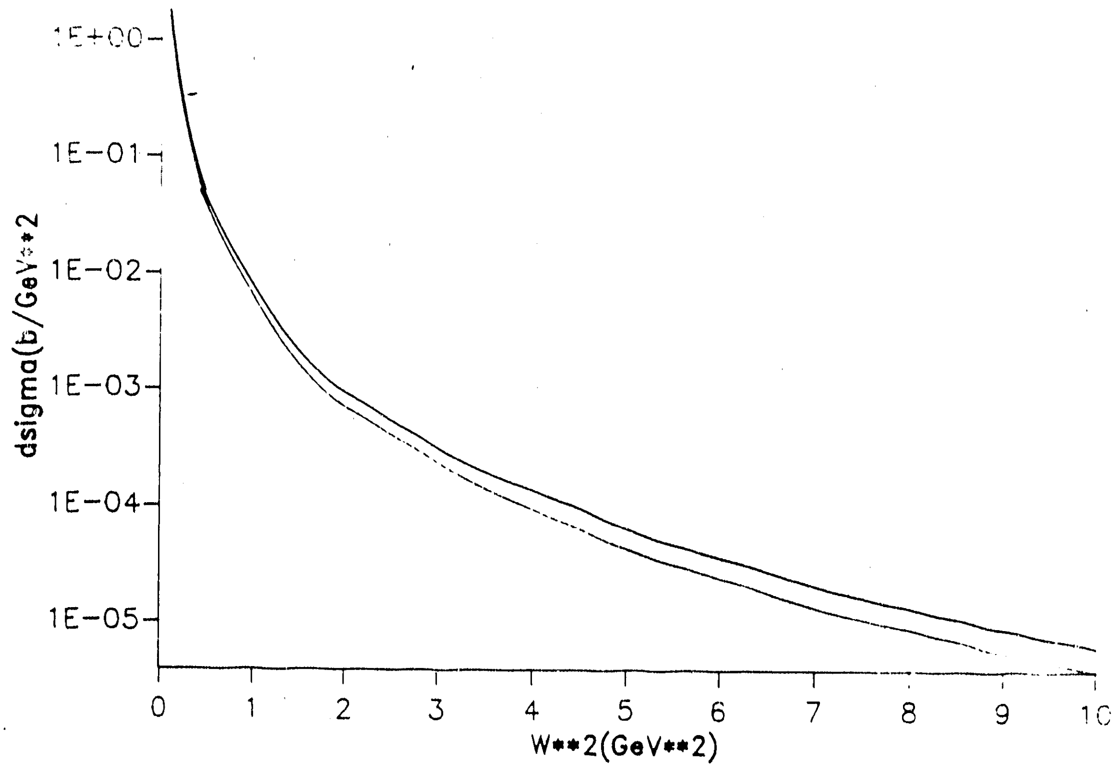
γ-γ-luminosity function $J(W^2)$

—— neglecting strong absorption

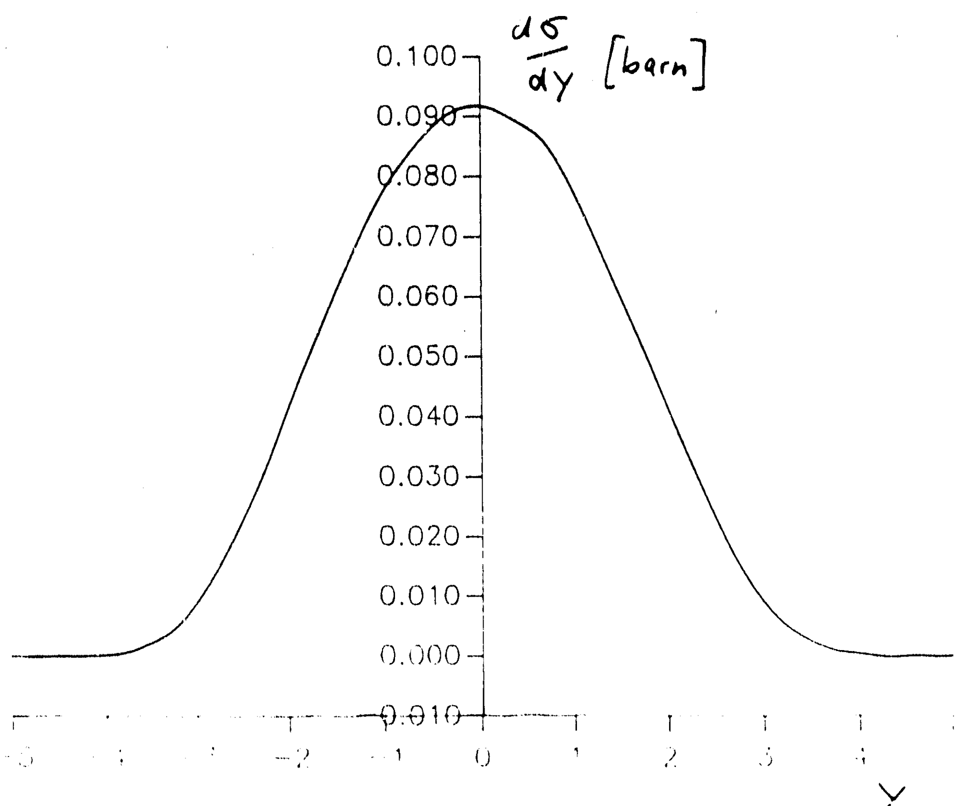
—— including strong absorption (sharp cut-off model)

— $J_{\parallel}$





$U + U \rightarrow \mu^+ + \mu^- + U + U \qquad \gamma = 100$

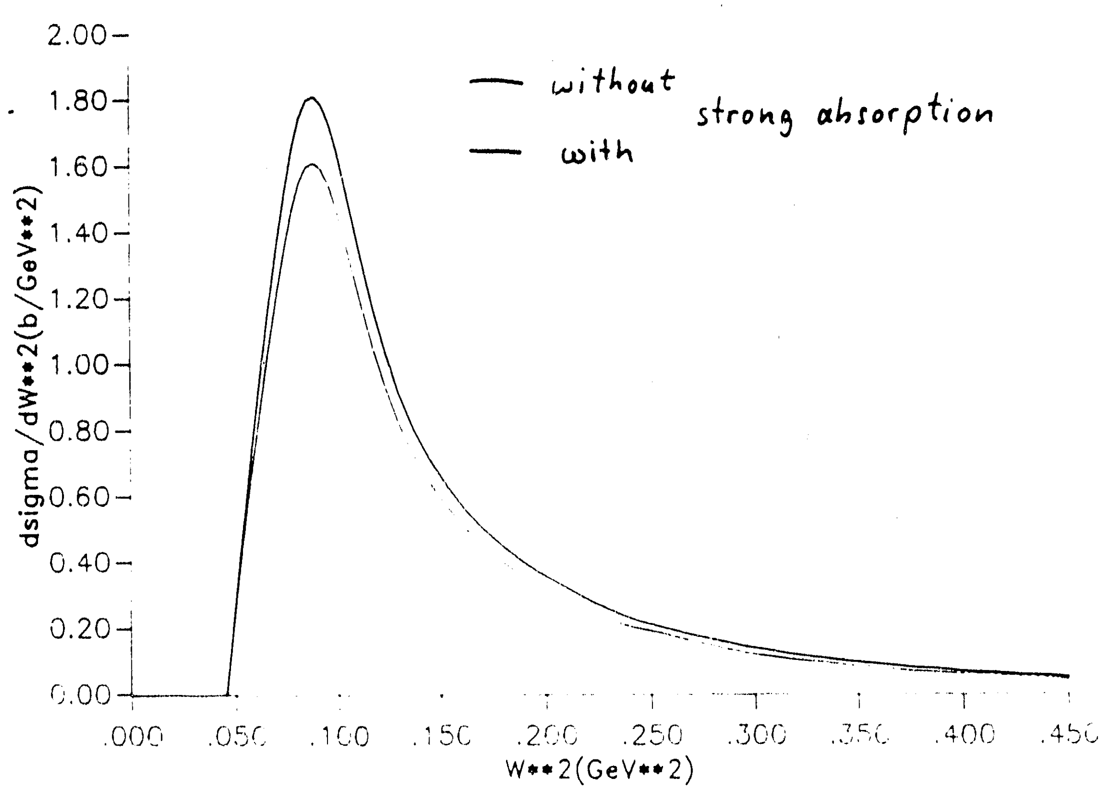


Production of heavy lepton pairs

$$\frac{1}{m_e} \longrightarrow \frac{1}{m_\mu} \quad \text{or} \quad \frac{1}{m_\tau}$$

386 fm 1.86 fm 0.11 fm

U + U $\gamma = 100$



at (heavy ion) LHC, or SSC:

a favourable scenario for medium mass Higgs production: $\gamma + \gamma \to H^0$

E. Papageorgiu; M. Grabiak, B. Müller, G. Soft, W. Greiner et al.; J. Ellis et al.

Can one test QCD at RHIC ?

measure two photon couplings of $C=+1$ hadronic states

$\Gamma(\text{glueball} \to \gamma\gamma)$ should be very small

meson 0^-	mass (GeV)	$\Gamma(R \to \gamma\gamma)$ (keV)	σ $Z=92, \gamma=100$
π^0	0.135	$9 \cdot 10^{-3}$	9 mb
η	0.549	1	3.3 mb
η'	0.958	5	1.3 mb
η_c	2.981	6.3	3.7 μb
η_b	9.4	0.4	0.02 nb

studies with improved statistics (as compared to γ-γ physics at e^+e^- machines) seem possible

e.g. with $L = 10^{27}\ \text{cm}^{-2}\text{s}^{-1}$

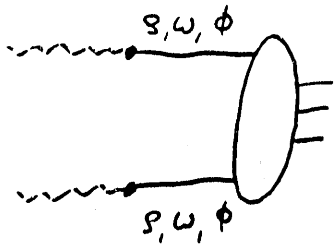
$$\text{rate}(\eta_c) = L \cdot \sigma(\eta_c) = 4 \cdot 10^{-3}\ \text{s}^{-1}$$

$$\text{rate}(\eta') \simeq 1\ \text{s}^{-1}$$

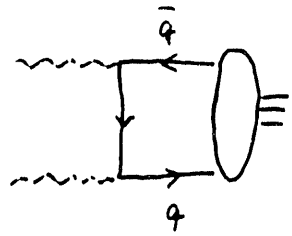
study $\gamma + \gamma \to \eta'$, and interference with pomeron-pomeron $\to \eta'$

(S. Brodsky)

Vector-meson production and total hadronic production by 2 photons



and



$$\sigma(\gamma\gamma \to \text{hadrons}) = \sum_{VV'} \frac{\alpha\pi}{\gamma_V^2} \frac{\alpha\pi}{\gamma_{V'}^2} \, \sigma(VV' \to \text{hadrons}) \qquad \text{VDM}$$

much in known from e^+e^--colliders, yet

"... it seems premature to judge the quality of the models on the basis of the available data." (H. Kolanoski, P.M. Zerwas, 1988)

cross section at RHI colliders

$\sigma(\gamma\gamma \to \text{hadrons}) \approx \text{const} = 300\,\text{nb} \equiv \sigma_0$

$$\sigma_c = \frac{8}{3\pi^2} Z^4 \alpha^2 \sigma_0 \left(\ln \frac{2\gamma}{W_0 R} \right)^4 \quad \ldots \text{ integrated from } W_0 \text{ to } \infty .$$

$Z = 92$, $\gamma = 100$, $W_0 = 1\,\text{GeV}$: $\sigma_c = 2.5\,\text{mb}$

with $L = 5 \cdot 10^{26}\,\text{cm}^{-2}\text{s}^{-1}$ $\Rightarrow$ ~ 1 event/s

4. Conclusions

Intense, high frequency electromagnetic fields

calculational methods well established

hugh e^+e^- cross sections:

- a nuisance for the ring operation
- useful (?), as a beam monitor
- strong fields $\rightarrow$ multiple pair production

new possibilities for γ-γ-physics: up to several GeV invariant mass

$\rightarrow$ QCD tests

photon-photon fluxes can be calculated reliably, including strong absorption

LEPTON PAIR PRODUCTION IN RELATIVISTIC HEAVY–ION COLLISIONS

Talk presented by:

W. Scheid

Lepton pair production in relativistic heavy ion collisions

1. Introduction
2. Cross sections for lepton pair creation in first order perturbation calculation

2.1 Multipole expansion, Coulomb-Dirac wave functions

2.2 Angular distributions for K-shell capture

2.3 Angular distributions of free e^+e^--pairs with continuum distorted wave model

2.4 Muon pair production with inner shell capture in $U^{92+}-U^{92+}$ collisions

3. Coupled channel calculations for electron-positron pair creation
4. Lepton pair production by bremsstrahlung in central heavy ion collisions
5. Summary and conclusions

Ulrich Becker, University Giessen, Stanford

Gustavo Deco
Norbert Grün — University Giessen
Thomas Lippert
Klaus Momberger

Gerhard Soff GSI (Darmstadt)

1. Introduction

We studied atomic processes in relativistic heavy ion collisions:

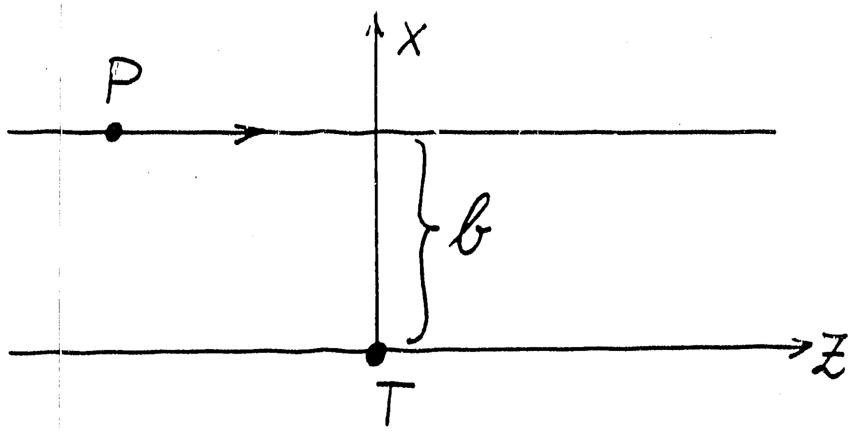
1. Ionisation of K-shell
2. Electron-positron creation including K-shell capture
3. Muon-pair creation with capture

Methods which we used:

1. Classical Monte Carlo trajectory method for ionisation of the K-shell
2. Perturbative methods
   first-order time-dependent perturbation theory with wavefunctions centered at target or functions distorted by target and projectile field
3. Finite-difference method for solving the time-dependent Dirac equation: calculation of ionisation
4. Coupled channel calculations for ionisation and e^+e^--pair creation

Also work on <u>nuclear</u> collisions of heavy ions: Photons and lepton-pairs produced by <u>bremsstrahlung</u> in <u>central</u> heavy ion collisions; calculations in first-order time-dependent perturbation theory.

2. Cross sections for lepton pair creation in first order perturbation theory



Assumptions: straight-line trajectory, constant projectile velocity, no recoil of target nucleus

e.g. $U^{92+} + U^{92+}$ at $E_{lab} = 1\,GeV/n$

$b = 30\,fm$, $\Delta v_T = 4.2 \cdot 10^{-3} c$, scattering angle $0.13°$

in target rest frame:

potential of target charge: $\frac{Z_T e}{r}$

potential of projectile charge (Liénard-Wiechert):

$$\phi_P = Z_P e \gamma / ((x-b)^2 + y^2 + \gamma^2 (z - v_P t)^2)^{1/2}$$

$$\vec{A}_P = \phi_P v_P \vec{e}_z / c \quad ; \quad \gamma = 1/(1 - v_P^2/c^2)^{1/2}$$

Hamiltonian density of e^+e^--field $\hat{\psi}$:

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{int}$$

choice of $\mathcal{H}_0$:

$$\mathcal{H}_0 = \mathcal{H}_T \;, \quad \mathcal{H}_T = \hat{\psi}^+ \left(c\vec{\alpha}\vec{p} + \beta mc^2 - \frac{Z_T e^2}{r}\right)\hat{\psi}$$

$$\mathcal{H}_{int} = \hat{\psi}^+ (-e\phi_P + e\vec{\alpha}\vec{A}_P)\hat{\psi}$$

$\hat{\psi}$ expanded in eigenstates of target H.

$$\hat{\psi} = \sum \varphi_e(\vec{r},t)\, b_e + \varphi_p(\vec{r},t)\, d_p^+$$

$$(c\vec{\alpha}\vec{p} + \beta mc^2 - Z_T e^2/r)\,\varphi_{e,p}(\vec{r},t) = i\hbar \frac{\partial}{\partial t}\varphi_{e,p}(\vec{r},t)$$

electron positron

S-matrix:

$$S = \mathcal{U}(\infty,-\infty) = T\exp\left(\frac{-i}{\hbar}\int_{-\infty}^{+\infty} dt \int d^3x\, \mathcal{H}_{int}(x)\right)$$

$$= 1 - \frac{i}{\hbar}\int_{-\infty}^{\infty} dt \int d^3x\, \mathcal{H}_{int}(x) + \ldots$$

Amplitude for creation of pair in first order:

$$a_{ep} = \langle 0 | d_p b_e \left(-\frac{i}{\hbar}\right) \int_{-\infty}^{\infty} dt \int d^3x\, \mathcal{H}_{int}(x) | 0 \rangle$$

$$= \frac{i}{\hbar}\int_{-\infty}^{+\infty} dt \int d^3x\, \varphi_e^+ (e\phi_P - e\vec{\alpha}\vec{A}_P)\varphi_p$$

2.1 Multipole expansion, exact Coulomb-Dirac wave functions

$(c = \hbar = 1)$

$$a_{ep} = i\gamma Z_p e^2 \int_{-\infty}^{\infty} dt \exp(i(E_e+E_p)t) \langle \psi_e | (1 - v_p \alpha_z)/r' | \psi_p \rangle$$

eigensolutions of H_T: Coulomb-Dirac wave funct.

$$\psi_p = \psi_{\kappa_p, \mu_p}, \quad \psi_e = \psi_{\kappa_e, \mu_e}$$

$$\gamma = 1/(1 - v_p^2)^{1/2}, \quad r' = ((x-b)^2 + y^2 + \gamma^2 (z - v_p t)^2)^{1/2},$$

$$\alpha_z = \begin{pmatrix} 0 & \sigma_z \\ \sigma_z & 0 \end{pmatrix}$$

Fourier transformation of $1/r'$:

$$1/r' = \frac{1}{2\pi^2} \int \frac{d^3 s'}{s'^2} \exp(i\vec{s}'\vec{r}')$$

$$= \frac{1}{2\pi^2 \gamma} \int d^3 s \, \frac{\exp(i\vec{s}(\vec{r} - \vec{R}(t)))}{s^2 - v_p^2 s_z^2}$$

$$\vec{r} = (x, y, z), \quad \vec{R}(t) = (b, 0, v_p t)$$

$$\exp(i\vec{s}\vec{r}) = 4\pi \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} i^{\ell} j_{\ell}(sr) Y_{\ell m}^{*}(\vartheta_s, \varphi_s) Y_{\ell m}(\vartheta, \varphi)$$

$$a_{pe} = \frac{8\pi i Z_p e^2}{v_p} \int_{\bar{q}}^{\infty} \frac{s\,ds}{s^2 - q^2 v_p^2} \sum_{\ell,m} i^{\ell-m} M_{ep}^{(\ell,m)}(s)\, B_{\ell m}(b,q,s)$$

straight-line factors (Amundsen, Aashamar 1981):

$$B_{\ell m}(b,q,s) = Y_{\ell m}(\cos^{-1}(q,s),0)\, J_m(b(s^2-q^2)^{1/2})$$

transition matrix element:

$$M_{ep}^{(\ell,m)}(s) = \langle \psi_e | (1 - v_p \alpha_z) j_\ell(sr) Y_{\ell m}(\hat{r}) | \psi_p \rangle$$

minimal momentum transfer: $q = (E_p + E_e)/v_p$

double-differential probabilities:

$$\frac{d^2 I_b}{dE_p\, dE_e} = \sum_{\kappa_p, \mu_p, \kappa_e, \mu_e} |a_{pe}|^2$$

double-differential cross section:

$$\frac{d^2\sigma}{dE_p\, dE_e} = 2\pi \int_0^{\infty} \frac{d^2 I_b}{dE_p\, dE_e}\, b\, db$$

$$\frac{d^2\sigma}{dE_p dE_e} = 8\pi \left(\frac{Z_p e^2}{v_p}\right)^2 \sum_{\kappa_p,\mu_p \kappa_e,\mu_e q} \int^{\infty} \frac{s\,ds}{(s^2 - q^2 v_p^{-2})^2} \left| M_{ep}^{(s)} \right|^2$$

$$M_{ep}(s) = 4\pi \sum_{\ell} i^{\ell}\, Y^{*}_{\ell,\mu_e-\mu_p}(\cos^{-1}(q/s),0)\, M_{ep}^{(\ell,\mu_e-\mu_p)}(s)$$

Scaling of cross sections with Z_p^2

$v_p \to 1$, $\gamma \to \infty$: $|a_{pe}|^2 = const.$

$$\lim_{v_p \to 1} \frac{d^2\sigma}{dE_p dE_e} \sim \ln \gamma$$

K-shell capture: $\lim\limits_{v_p \to 1} \dfrac{d\sigma}{dE_p} \sim \ln \gamma$

Coulomb - Dirac wave functions

$$\psi_{e,p} = \begin{pmatrix} g_\kappa \, \chi_\kappa^\mu(\vartheta,\varphi) \\ i f_\kappa \, \chi_{-\kappa}^\mu(\vartheta,\varphi) \end{pmatrix}$$

$$\langle \psi_f | (1 - v_\ell \alpha_z) j_\ell(sr) Y_{\ell m}(\hat{r}) | \psi_i \rangle = \sum_{\lambda=1}^{3} A_\lambda(\ell m, \mu_f, \mu_i) d_\lambda F_\ell^{(\lambda)}(s)$$

A_λ geometrical factors

e.g. $A_1(\ell m, \mu_f, \mu_i) = \langle \chi_{\kappa_f}^{\mu_f} | Y_{\ell m} | \chi_{\kappa_i}^{\mu_i} \rangle$

$$F_\ell^{(1)}(s) = \int_0^\infty dr\, r^2 j_\ell(sr) (g_i g_f + f_i f_f)$$

$$F_\ell^{(2)}(s) = \int_0^\infty dr\, r^2 j_\ell(sr) f_f g_i$$

$$F_\ell^{(3)}(s) = \int_0^\infty dr\, r^2 j_\ell(sr) g_f f_i$$

$$\longrightarrow I_\ell(s) = \int_0^\infty dr\, j_\ell(sr)\, r^{\gamma_i + \gamma_f} \exp(-i(p_i + p_f) r)$$
$$\times {}_1F_1(a_i, b_i, 2ip_i r)\, {}_1F_1(a_f, b_f, 2ip_f r)$$

analytical evaluation : Appell functions

$$F_2(\alpha, \beta, \beta', \gamma, \gamma', x, y) = \sum_{n\,m=0}^{\infty} \frac{(\alpha)_{n+m} (\beta)_n (\beta')_m}{(\gamma)_n (\gamma')_m n! m!} x^n y^m$$

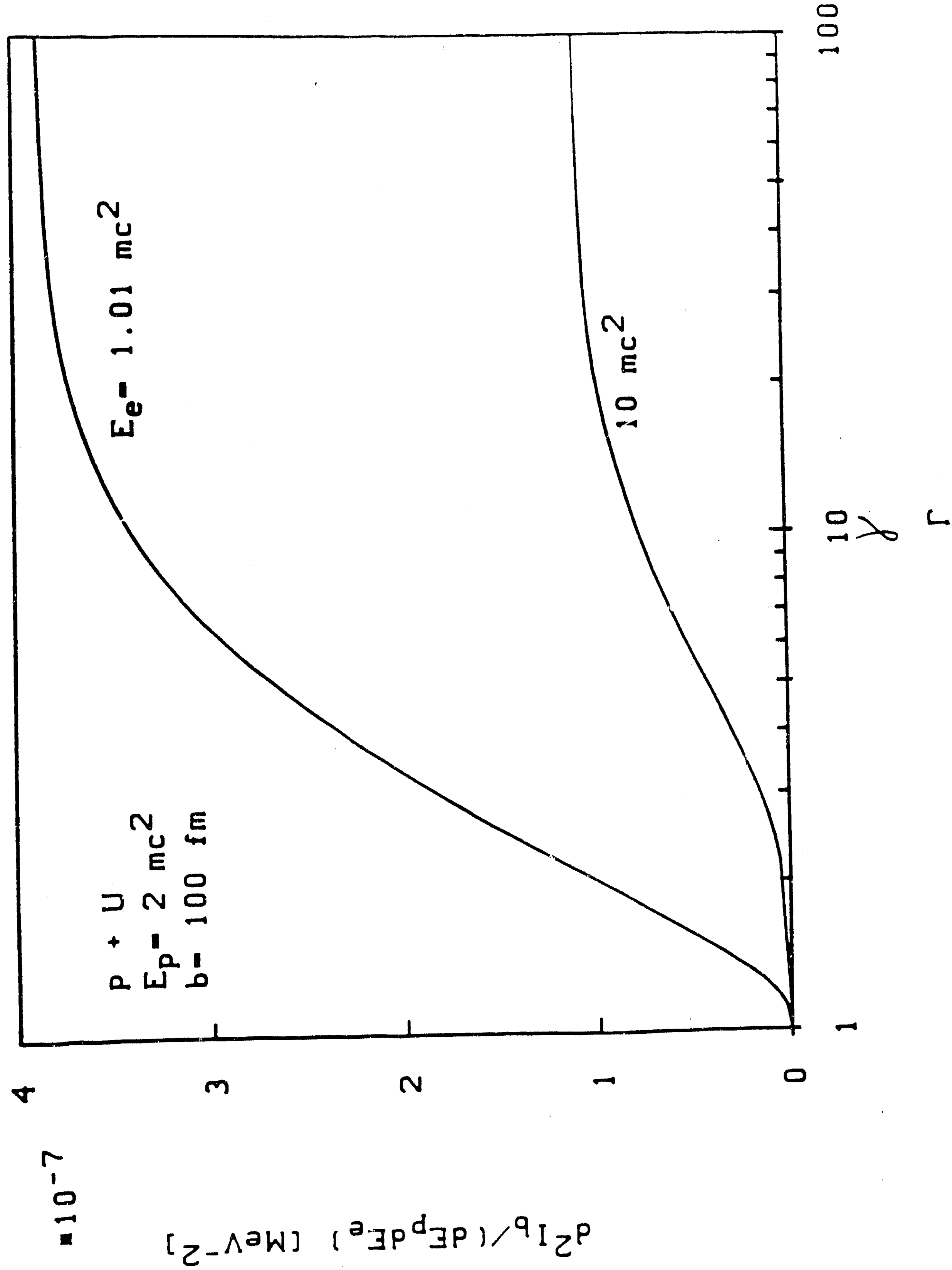

p + U
Ep= 2 mc2
b= 100 fm
Ee= 1.01 mc2
10 mc2
Γ
d2Ib/(dEpdEe) [MeV-2]
*10-7
0
1
2
3
4
1
10
100

Fig. 1

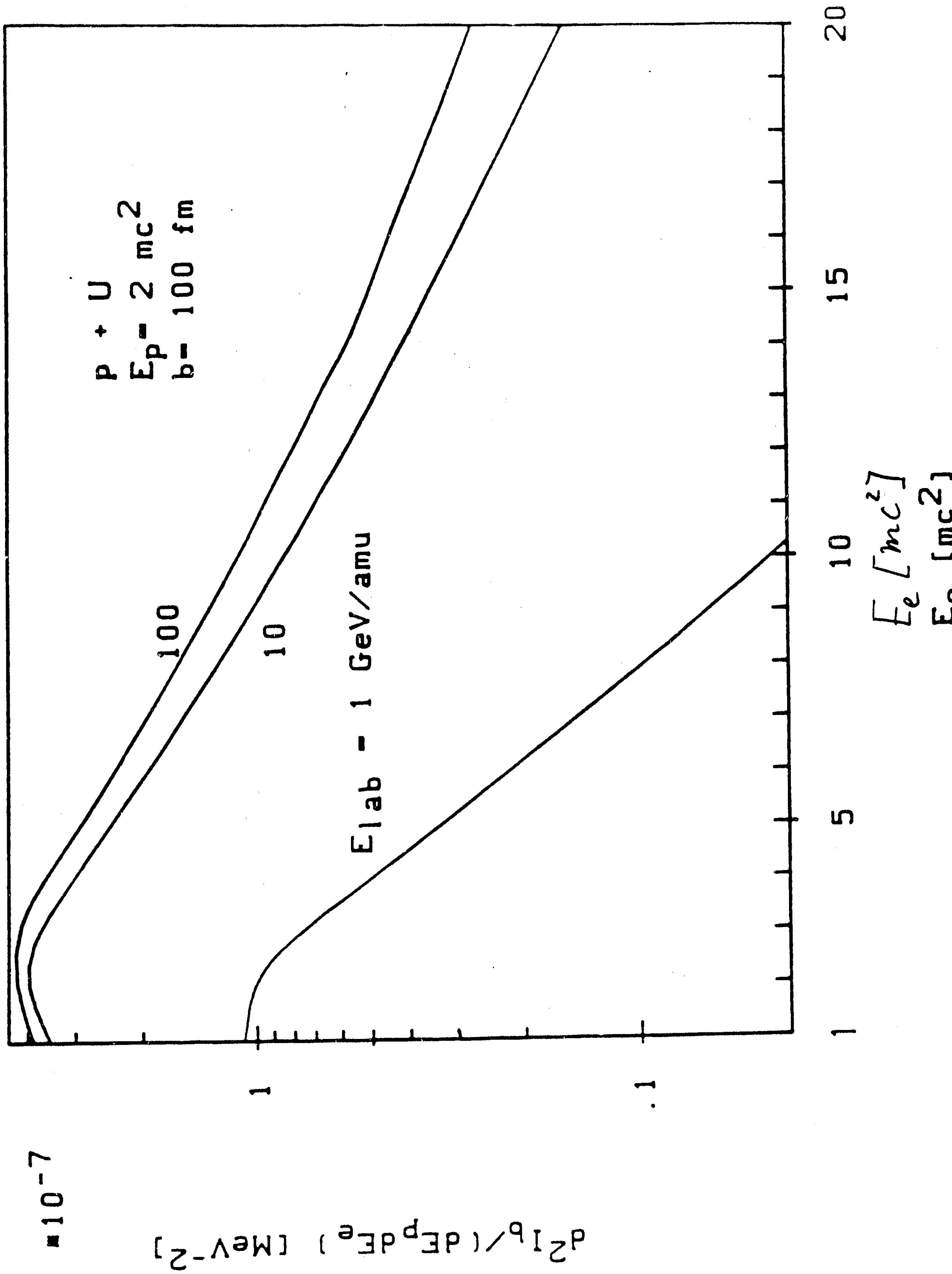
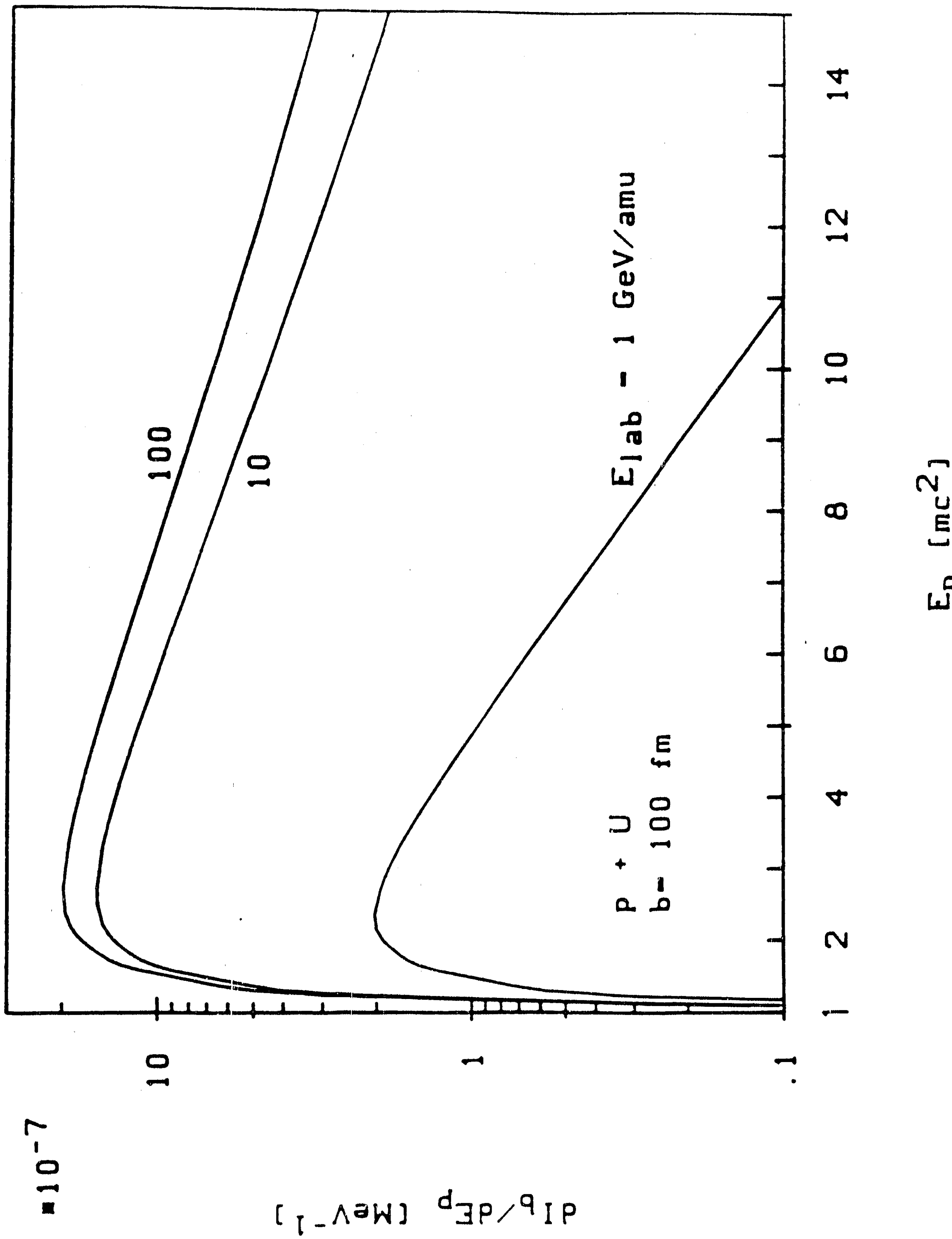

p + U
E_p= 2 mc^2
b= 100 fm
100
10
E_lab = 1 GeV/amu
1
5
10
15
20
E_e [mc^2]
1
.1
*10^-7
d^2I_b/(dE_p dE_e) [MeV^-2]

Fig. 2



100
10
E_lab = 1 GeV/amu
p + U
b= 100 fm
1
2
4
6
8
10
12
14
E_p [mc^2]
10
1
.1
*10^-7
dI_b/dE_p [MeV^-1]

Fig. 3

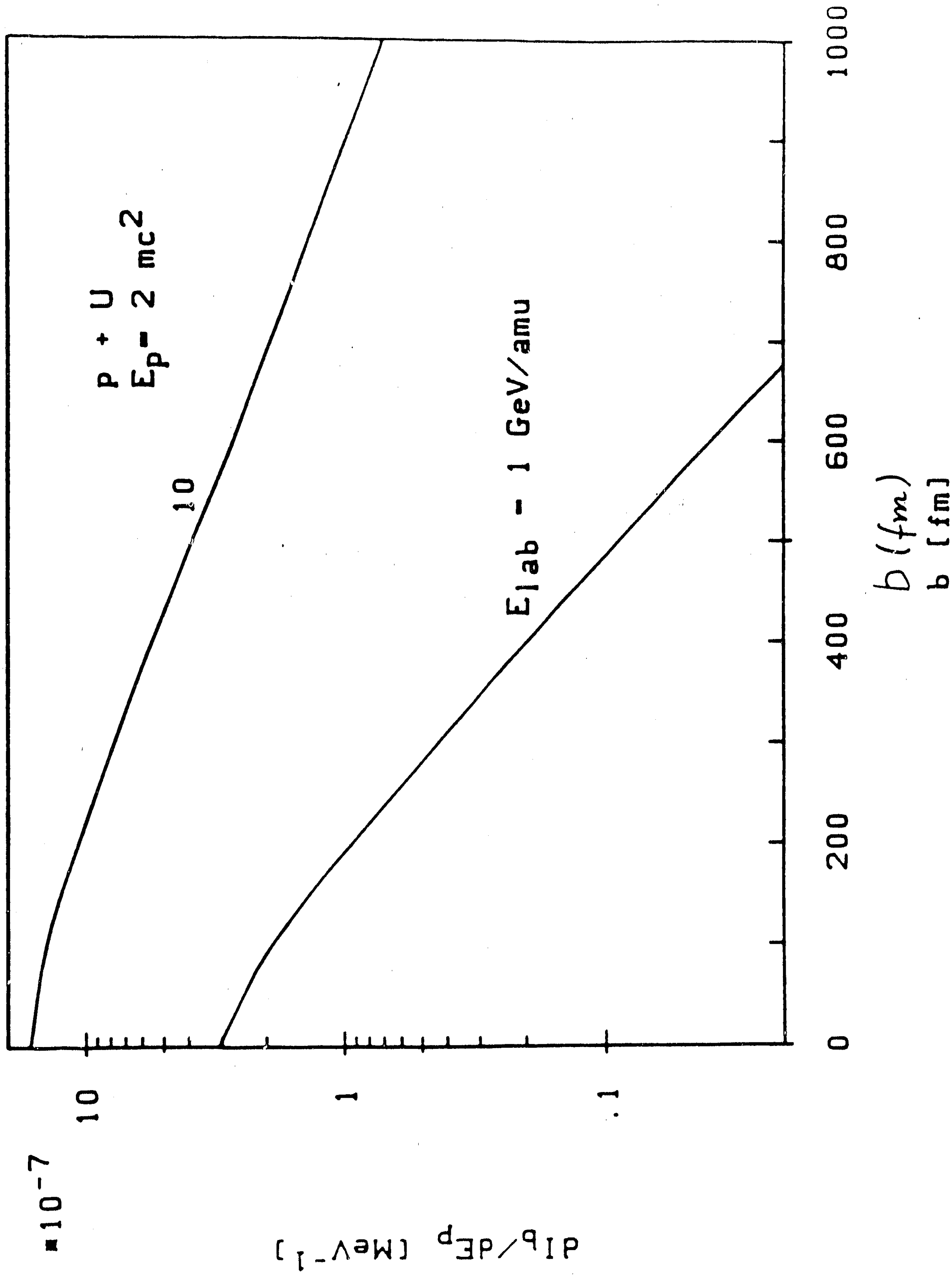

p + U
E_p= 2 mc^2
10
E_lab = 1 GeV/amu
b (fm)
b [fm]
0
200
400
600
800
1000
10
1
.1
*10^-7
dI_b/dE_p [MeV^-1]

Fig. 4

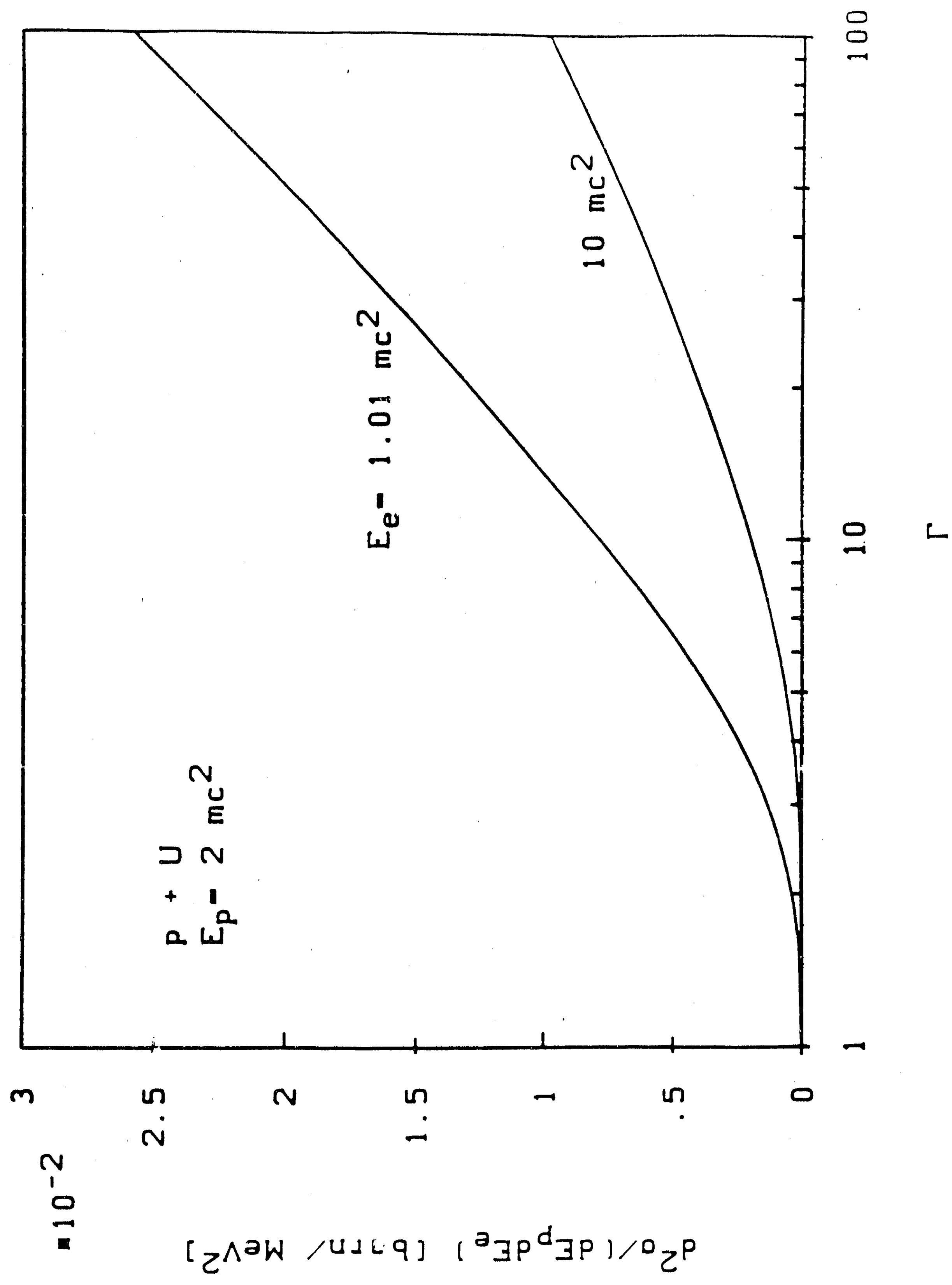
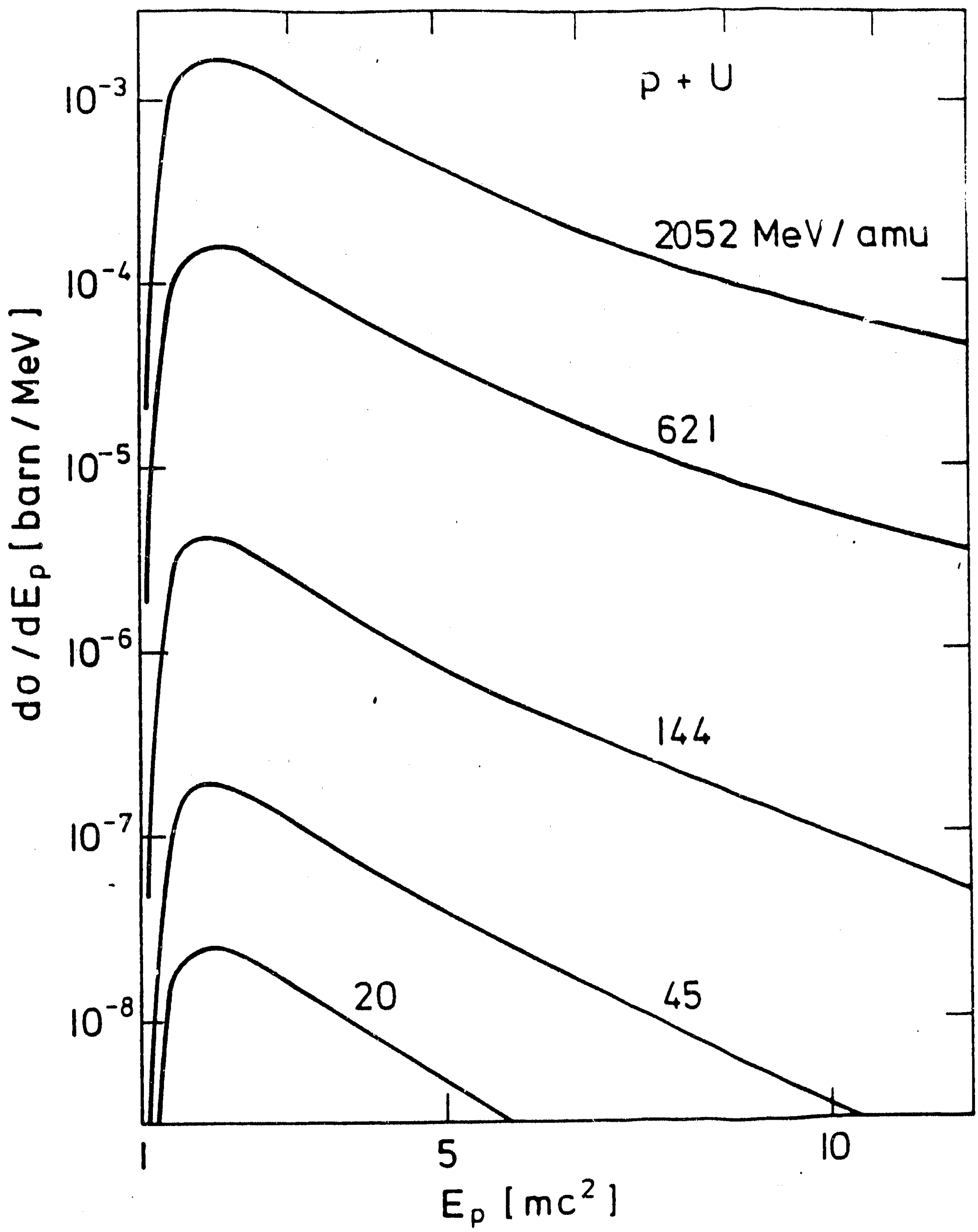

P + U
E_p= 2 mc^2
E_e= 1.01 mc^2
10 mc^2
*10^-2
d^2σ/(dE_pdE_e) [barn/ MeV^2]
0
.5
1
1.5
2
2.5
3
1
10
100
Γ

Fig. 5



p + U
2052 MeV / amu
621
144
45
20
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
$d\sigma / dE_p$ [barn / MeV]
1
5
10
E_p [mc^2]

Fig. 6

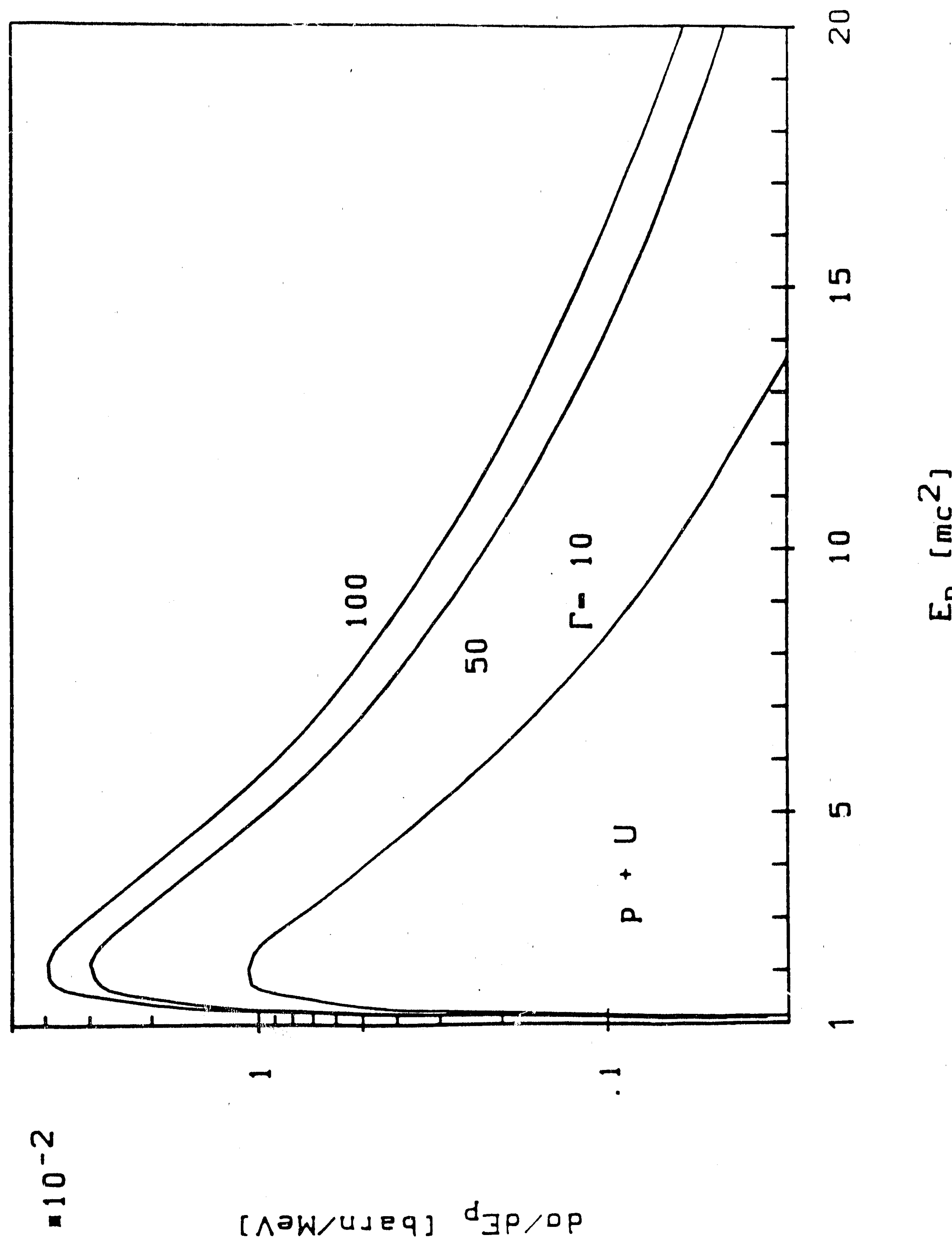
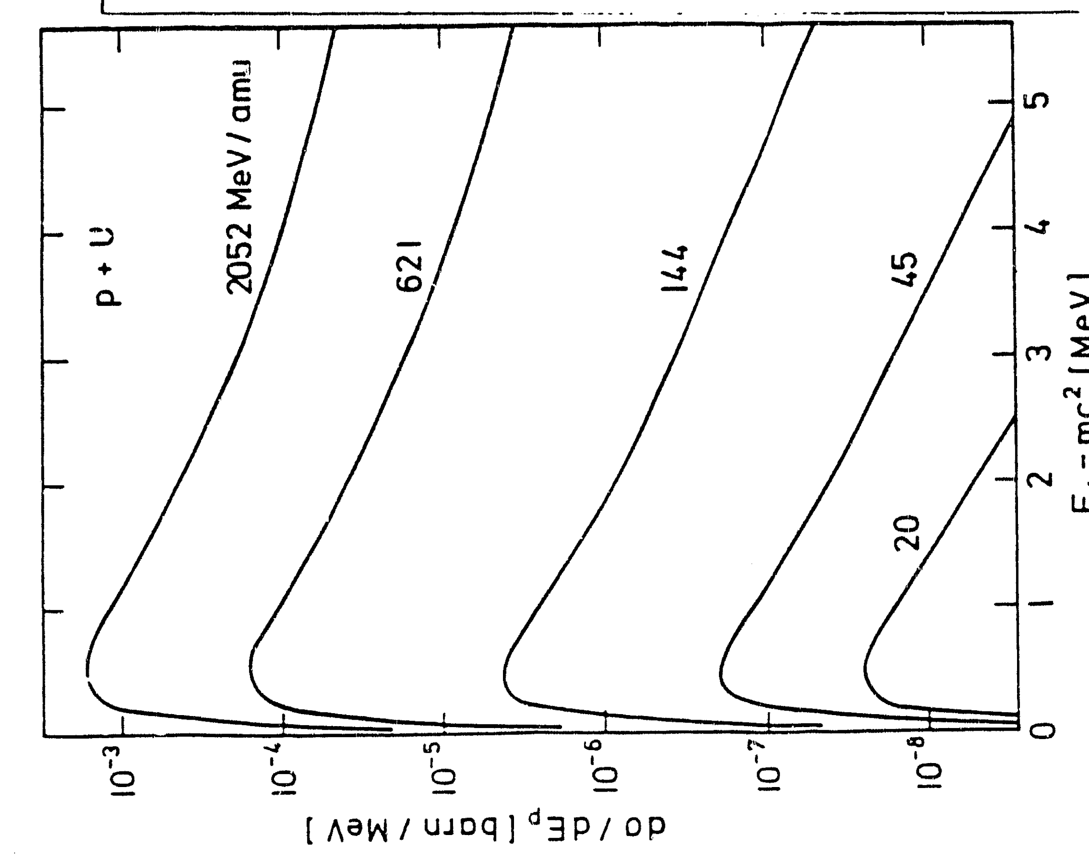
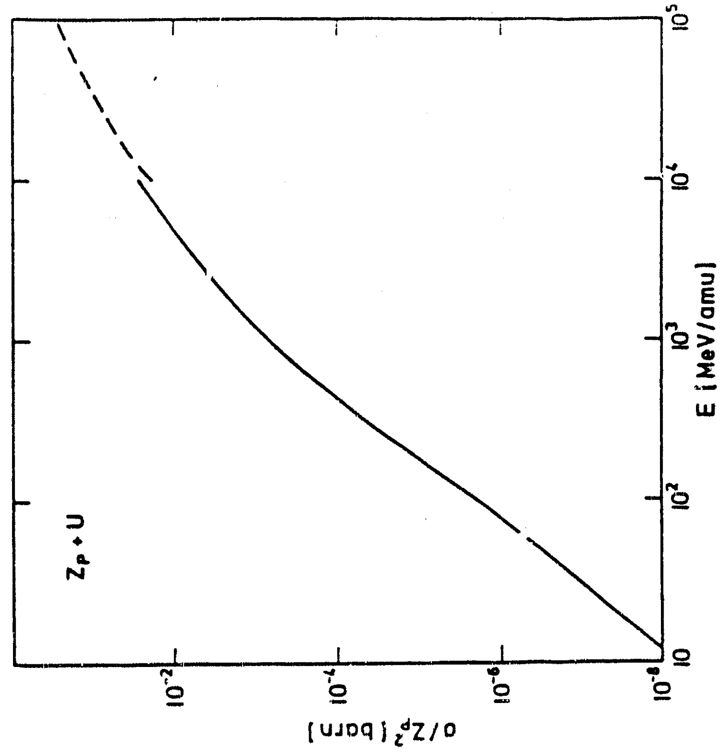


Fig. 7


p + U
2052 MeV/amu
621
144
45
20
E_f − mc² [MeV]
dσ/dE_p [barn/MeV]
10^-3
10^-4
10^-5
10^-6
10^-7
10^-8
0
1
2
3
4
5


Z_p + U
E [MeV/amu]
σ/Z_p² [barn]
10^-2
10^-4
10^-6
10^-8
10
10^2
10^3
10^4
10^5

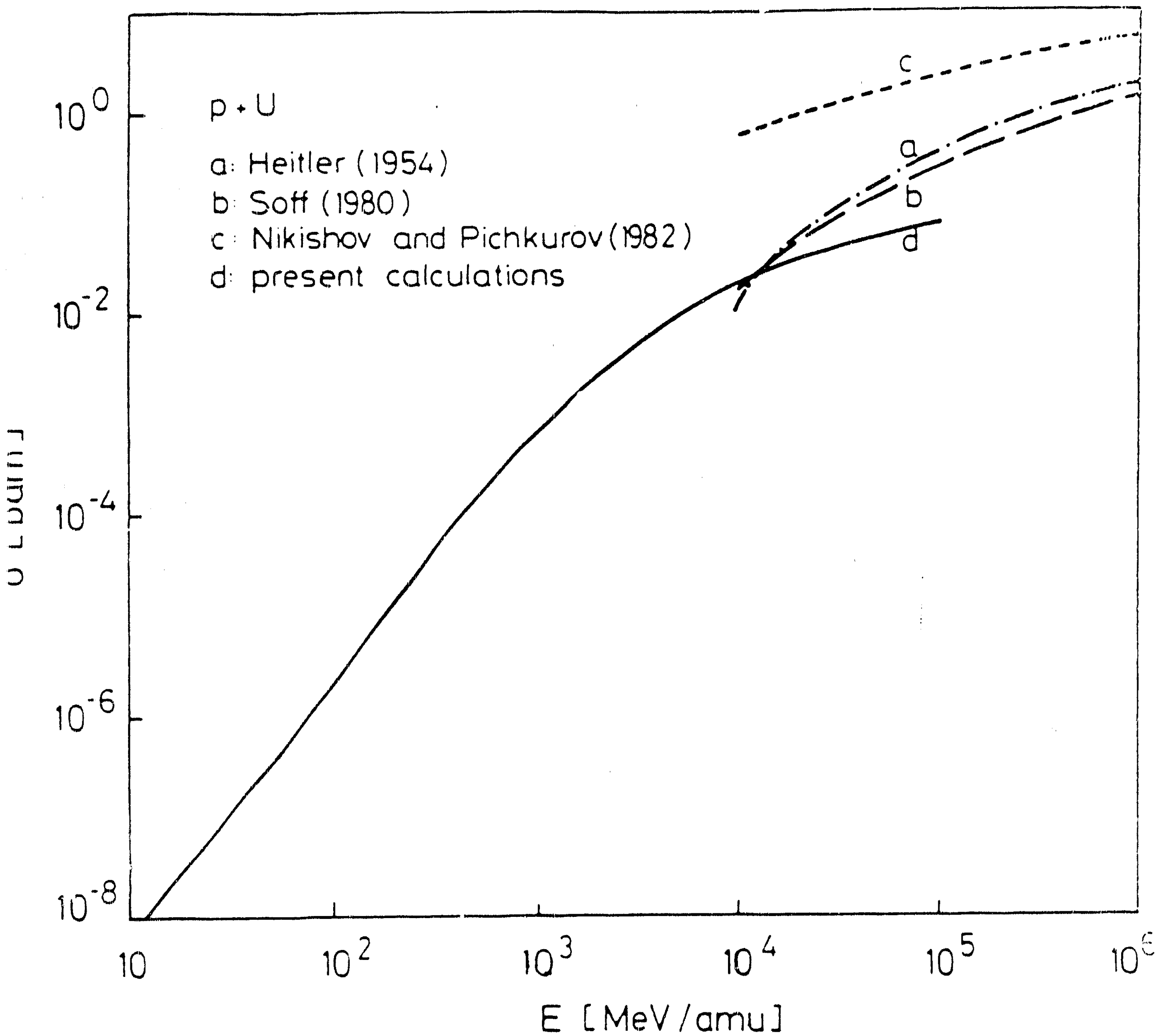

p + U
a: Heitler (1954)
b: Soff (1980)
c: Nikishov and Pichkurov (1982)
d: present calculations
a
b
c
d
10^0
10^-2
10^-4
10^-6
10^-8
10
10^2
10^3
10^4
10^5
E [MeV/amu]

Fig. 8

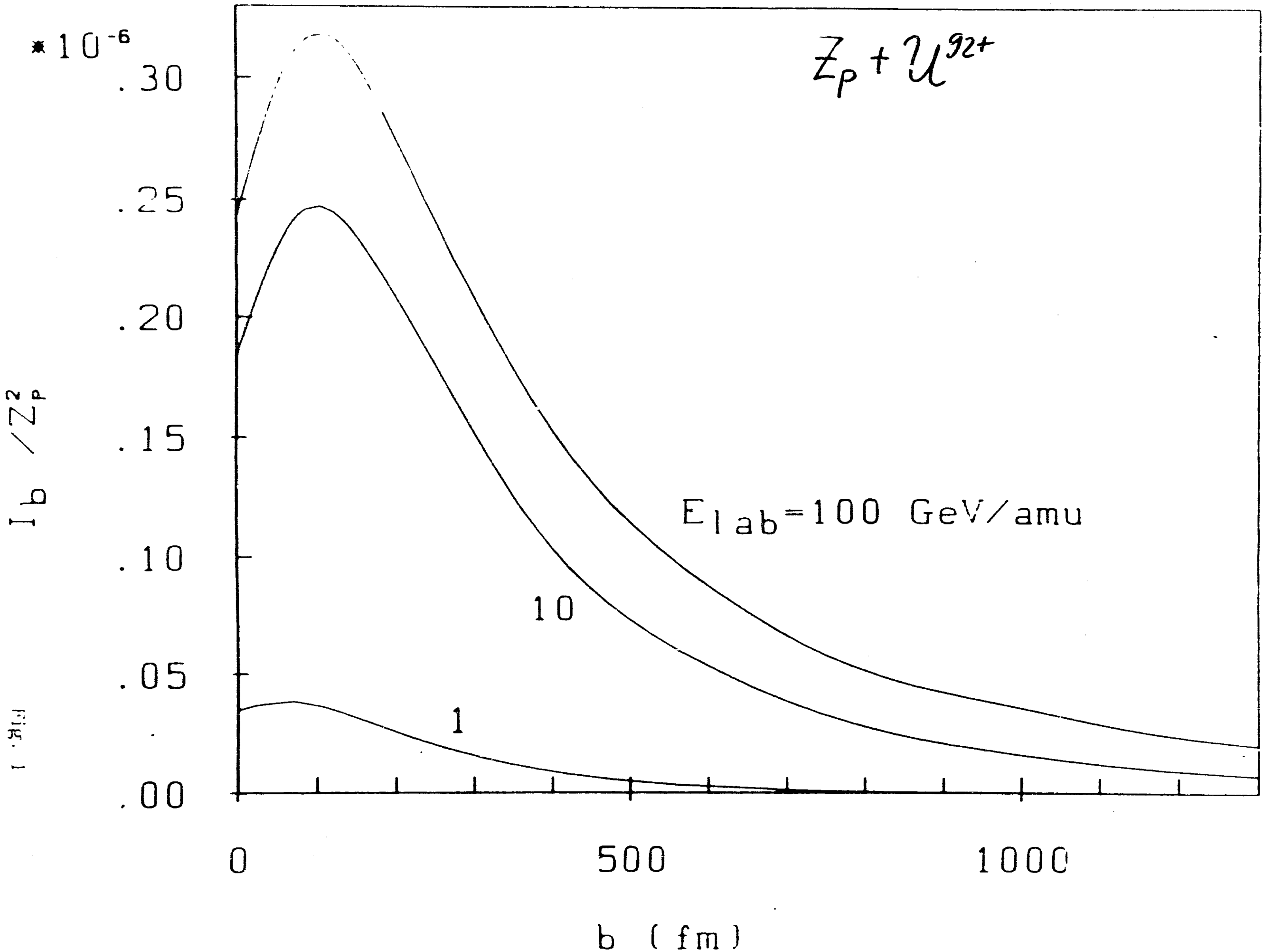

$Z_p + U^{92+}$
E_{lab}=100 GeV/amu
10
1
$*10^{-6}$
.30
.25
.20
.15
.10
.05
.00
I_b / Z_p^2
0
500
1000
b (fm)

Fig. 1

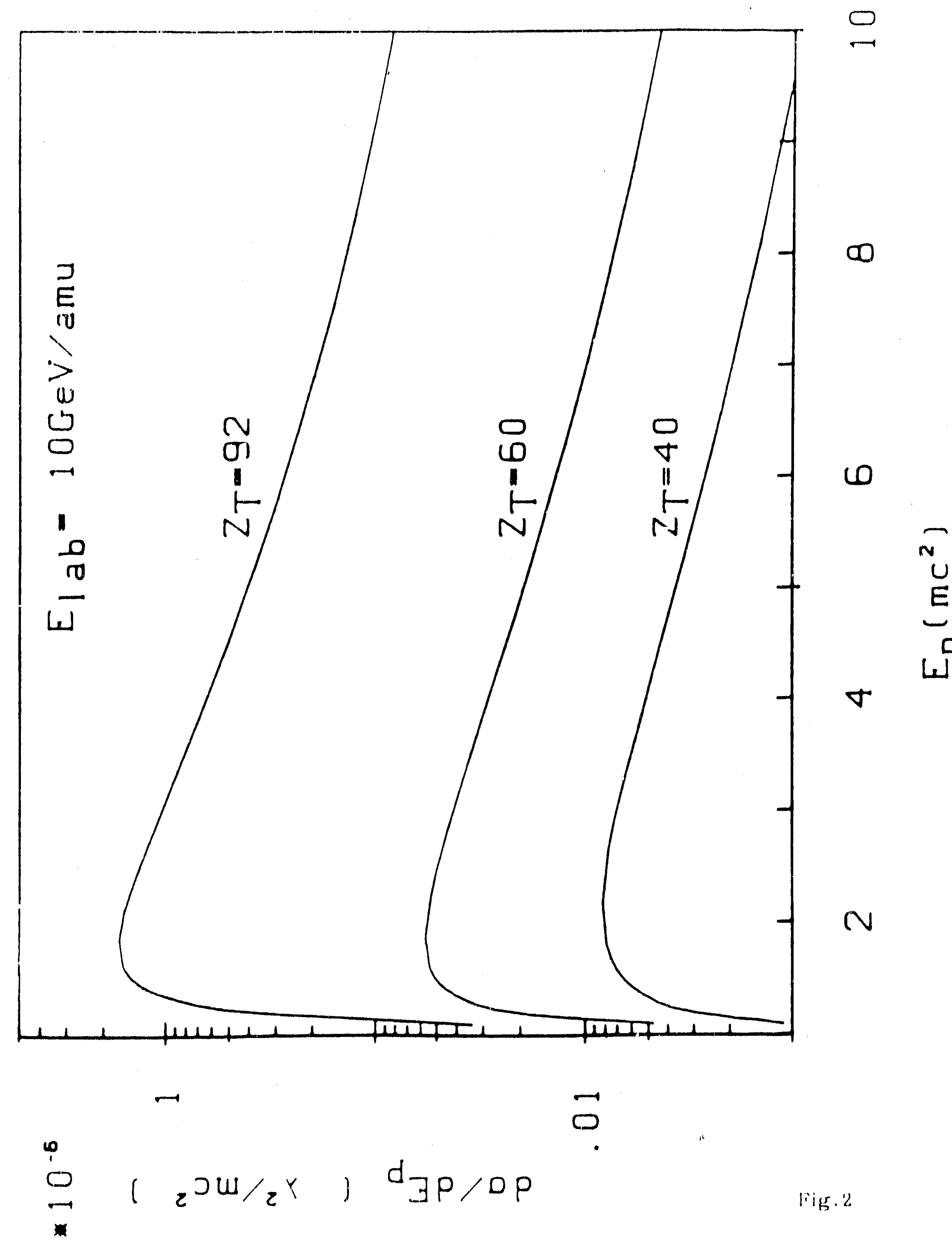

E_lab= 10GeV/amu
Z_T=92
Z_T=60
Z_T=40
E_p(mc²)
2
4
6
8
10
1
.01
dσ/dE_p (λ²/mc²)
*10^-6

Fig.2

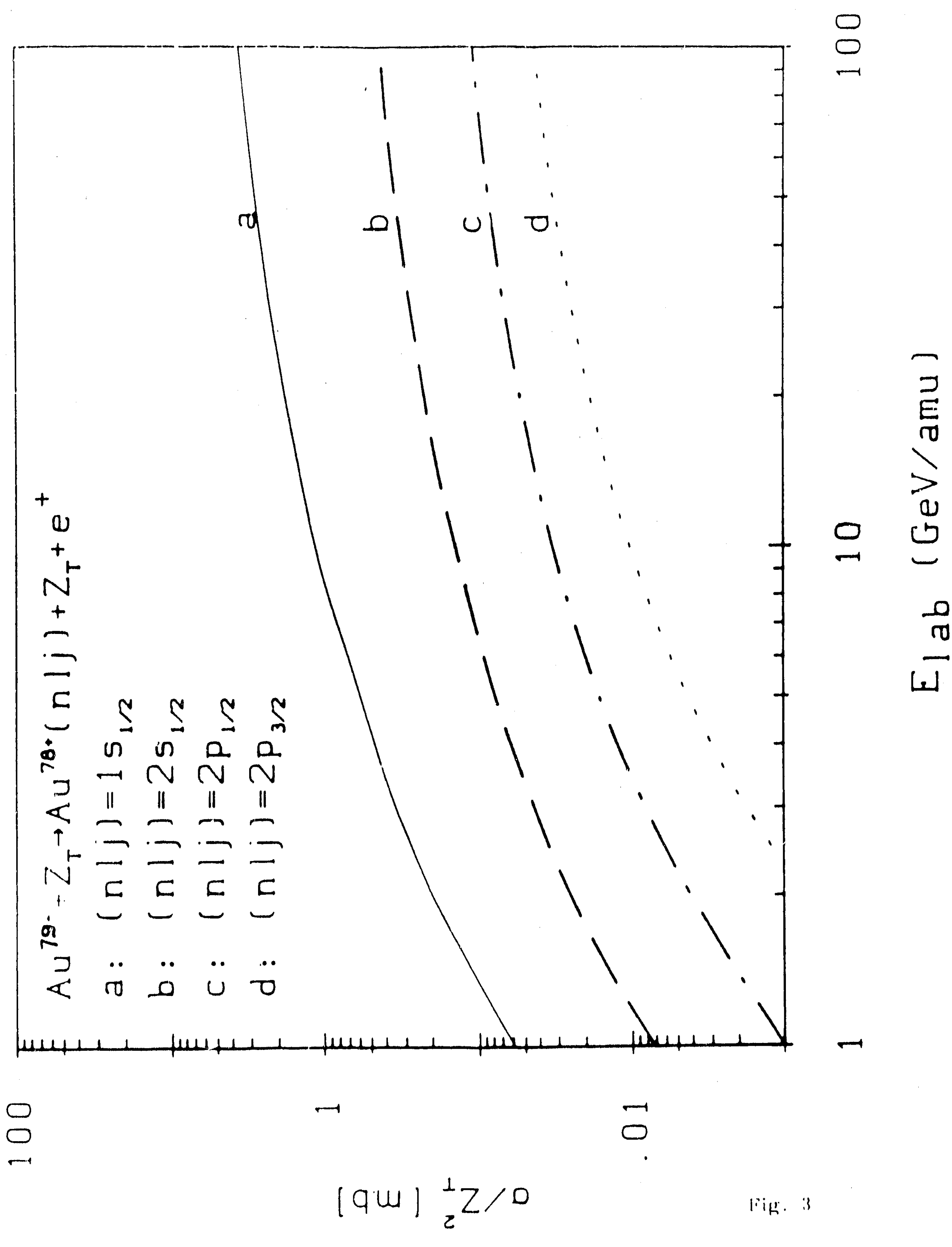
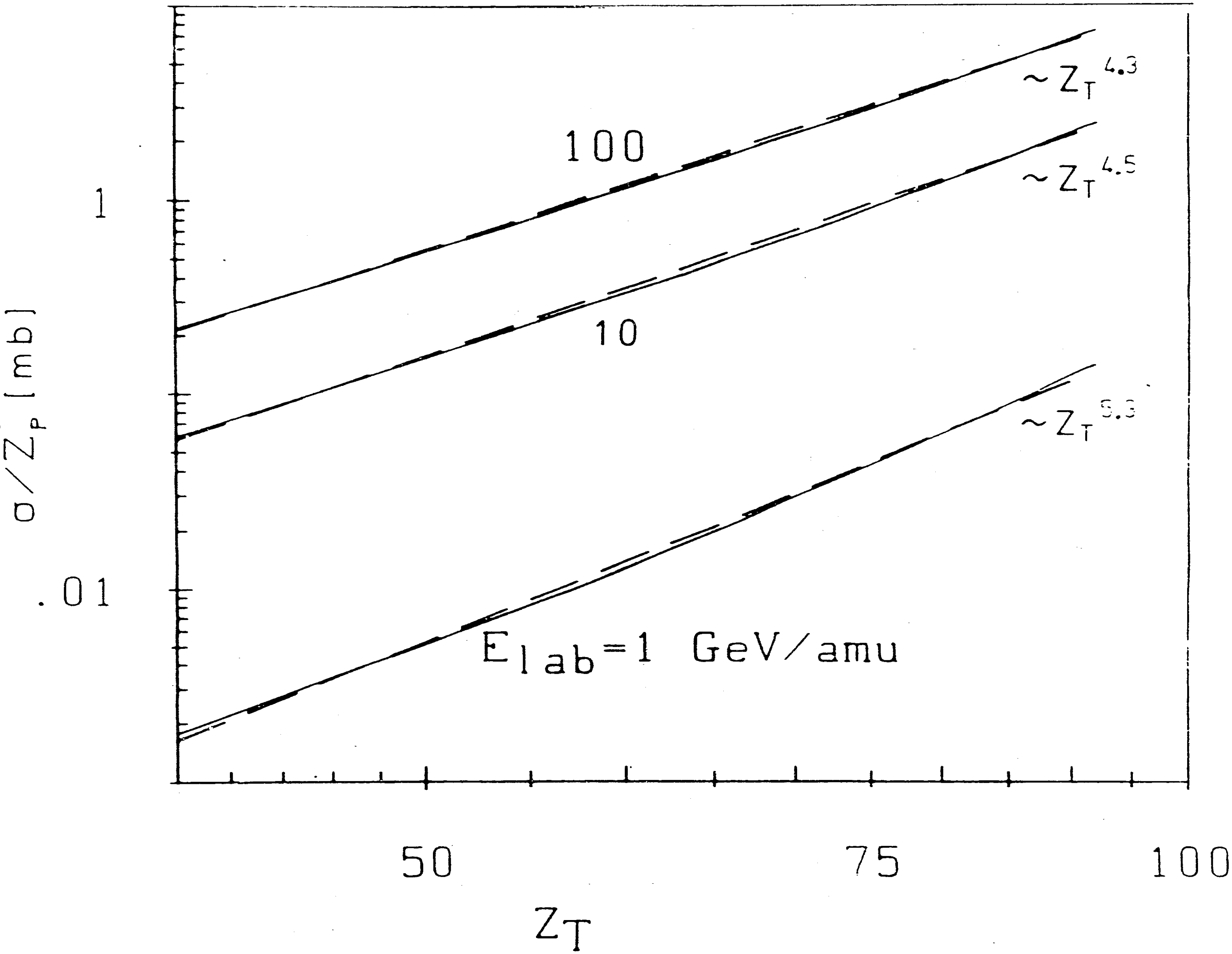

$Au^{79+} + Z_T \rightarrow Au^{78+}(nlj) + Z_T + e^+$
a: $(nlj) = 1s_{1/2}$
b: $(nlj) = 2s_{1/2}$
c: $(nlj) = 2p_{1/2}$
d: $(nlj) = 2p_{3/2}$
a
b
c
d
100
1
.01
σ/Z_T^2 [mb]
1
10
100
E_{lab} (GeV/amu)

Fig. 3



capture into K-shell of target ion

Fig. 4

2.2 Angular distributions for e^+e^--production with inner shell-capture – comparison of Coulomb-Dirac and Sommerfeld-Maue functions

$$\frac{d^2\sigma}{dE_p\, d\Omega_p} = 4\left(\frac{Z_p\alpha}{v_p}\right)^2 \int_q^\infty \frac{s\,ds\,d\varphi_s}{(s^2-(qv_p)^2)^2} \sum_{m_p m_e} \left| \langle \psi_e | (1-v_p\alpha_z) e^{i\vec{s}\vec{r}} | \vec{\psi}_p \rangle \right|^2$$

a) Coulomb-Dirac functions

ψ_e bound state for electron at target

$\vec{\psi}_p$ continuum state for positron within the intervals dE_p and $d\Omega_p$

Landau, Lifshitz 1971, Jakubaßa-Amundsen 1982:

$$\vec{\psi}_p = \sum_{\kappa_p, \mu_p, m_s} i^{l_p} e^{-i\delta_{\kappa_p}} (l_p m_s, \tfrac{1}{2} m_p | j_p \mu_p) Y^*_{l_p m_s}(\Omega_p)\, \psi_{\kappa_p}$$

$$\psi_{\kappa_p} = \begin{pmatrix} g_{\kappa_p}(r)\, \chi^{\mu_p}_{\kappa_p} \\ i f_{\kappa_p}(r)\, \chi^{\mu_p}_{-\kappa_p} \end{pmatrix}$$

Cross section

$$\frac{d^2\sigma}{dE_p\, d\Omega_p} = 2\pi \left(\frac{8\pi Z_p \alpha}{v_p}\right)^2 \int_q^\infty \frac{s\,ds}{(s^2-(qv_p)^2)^2} \sum_{m_e \mu_p \kappa_p \kappa_p'} Z^{(\mu_p)}_{\kappa_p \kappa_p'}(\Omega_p)$$

$$\times \sum_{m, \ell \ell'} i^{\ell-\ell'} Y^*_{\ell m}(\vartheta_s, 0)\, Y_{\ell' m}(\vartheta_s, 0)\, M^{(\ell, m)}_{ep}(s)\, M^{(\ell', m)*}_{ep}(s)$$

$$\vartheta_s = \cos^{-1}(q/s)$$

$$Z^{(\mu_p)}_{\kappa_p \kappa_p'}(\Omega_p) = i^{\ell_p - \ell_p'} \exp(-i(\delta_{\kappa_p} - \delta_{\kappa_p'}))\cdot$$

$$\times \sum_{m_p} (\ell_p\, \mu_p - m_p, \tfrac{1}{2} m_p | j_p \mu_p)(\ell_p' \mu_p - m_p, \tfrac{1}{2} m_p | j_p' \mu_p)$$

$$\times Y^*_{\ell_p \mu_p - m_p}(\Omega_p)\, Y_{\ell_p' \mu_p - m_p}(\Omega_p)$$

b) Sommerfeld-Maue and Darwin wave functions

high-energy approximation to exact continuum state

$$\vec{\psi}_p = N_p \exp(-ipr)\left(1 + i\frac{\vec{\alpha}\vec{\nabla}}{2E_p}\right)A_p^{m_p}\ {}_1F_1\left(i\eta_p, 1, i(pr + \vec{p}\vec{r})\right)$$

$$A_p^{m_p} = \begin{pmatrix} \varphi_{m_p} \\ \frac{\vec{\sigma}\vec{p}}{E_p - 1}\varphi_{m_p} \end{pmatrix}$$

approximation for K-shell wave function by Darwin or Pauli function (non-relativistic app.)

$$\psi_e(\vec{r}) = N_e \begin{pmatrix} \varphi_{m_e} \\ i\frac{\vec{\sigma}\vec{k}}{1+E_e}\varphi_{m_e} \end{pmatrix} \exp(-Z_T\alpha r)$$

$E_e = 1 - (Z_T\alpha)^2/2$, $\underline{\vec{k} = Z_T\alpha\vec{e}_r}$; good app. for $Z_T\alpha \ll 1$

$$\langle\psi_e|(1 - v_p\alpha_z)e^{i\vec{s}\vec{r}}|\vec{\psi}_p\rangle = \gamma^{-2}\langle\psi_e|e^{i\vec{s}\vec{r}}|\vec{\psi}_p\rangle$$
$$+ \frac{v_p^2}{E_p + E_e}\langle\psi_e|(s_x\alpha_x + s_y\alpha_y)e^{i\vec{s}\vec{r}}|\vec{\psi}_p\rangle$$

for $s_z = (E_e + E_p)/v_p$; right hand side has correct behaviour for $\gamma \to \infty$; cross section $\sim \ln\gamma$ for $\gamma \to \infty$.

Becker

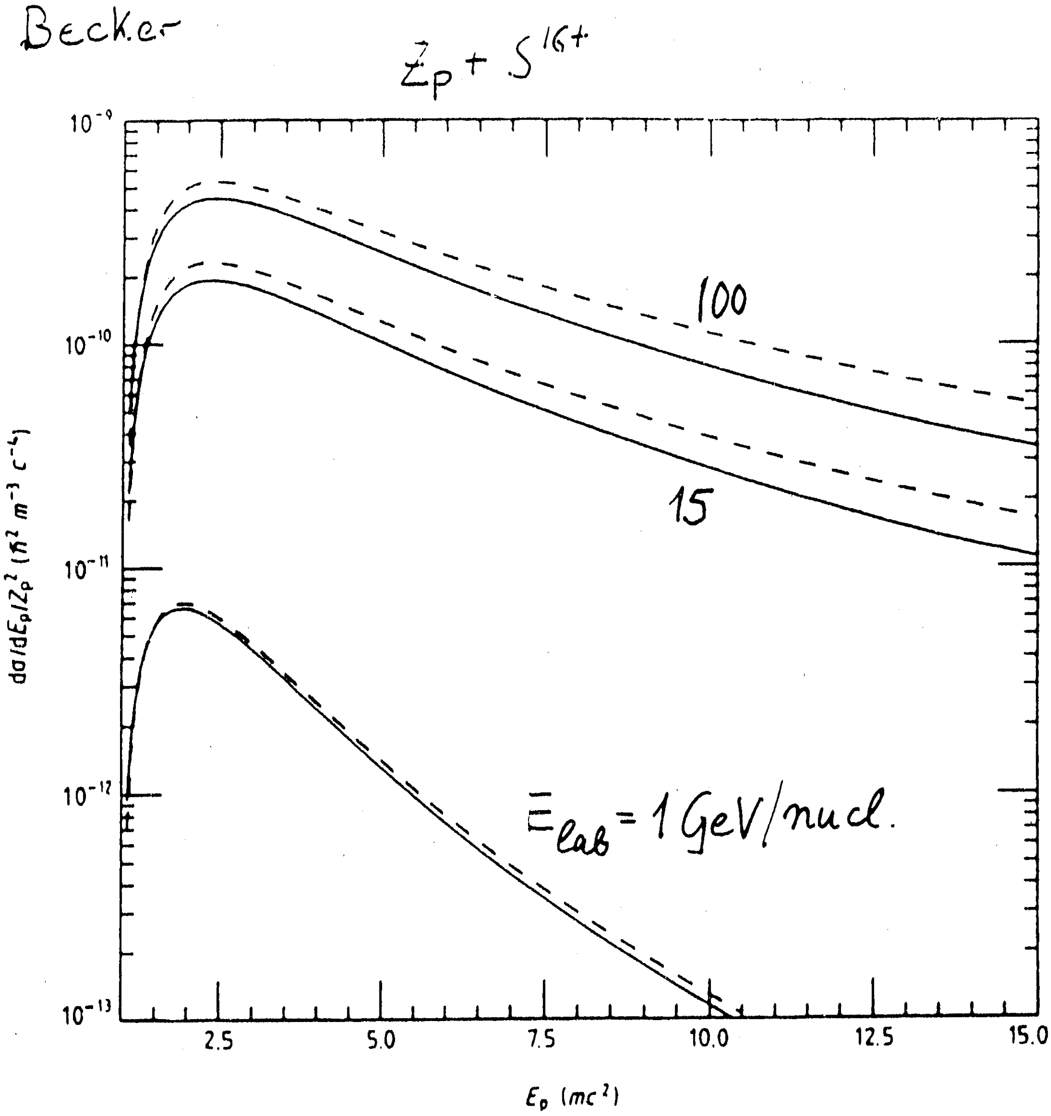


Figure 1. Differential cross section for pair production with simultaneous K-shell capture in $Z_p + S^{16+}$ collisions at $E_{lab} = 1$ (bottom), 15 (middle) and 100 (top) GeV amu^{-1} as a function of the positron energy. Full curves: equation (21); broken curves: equation (20). All quantities in natural units ($\hbar = c = m = 1$; $\hbar^2/m^3c^4 = 2.9$ b keV^{-1}; the kinetic energy of the positron is $(E_p - 1)\ mc^2$).

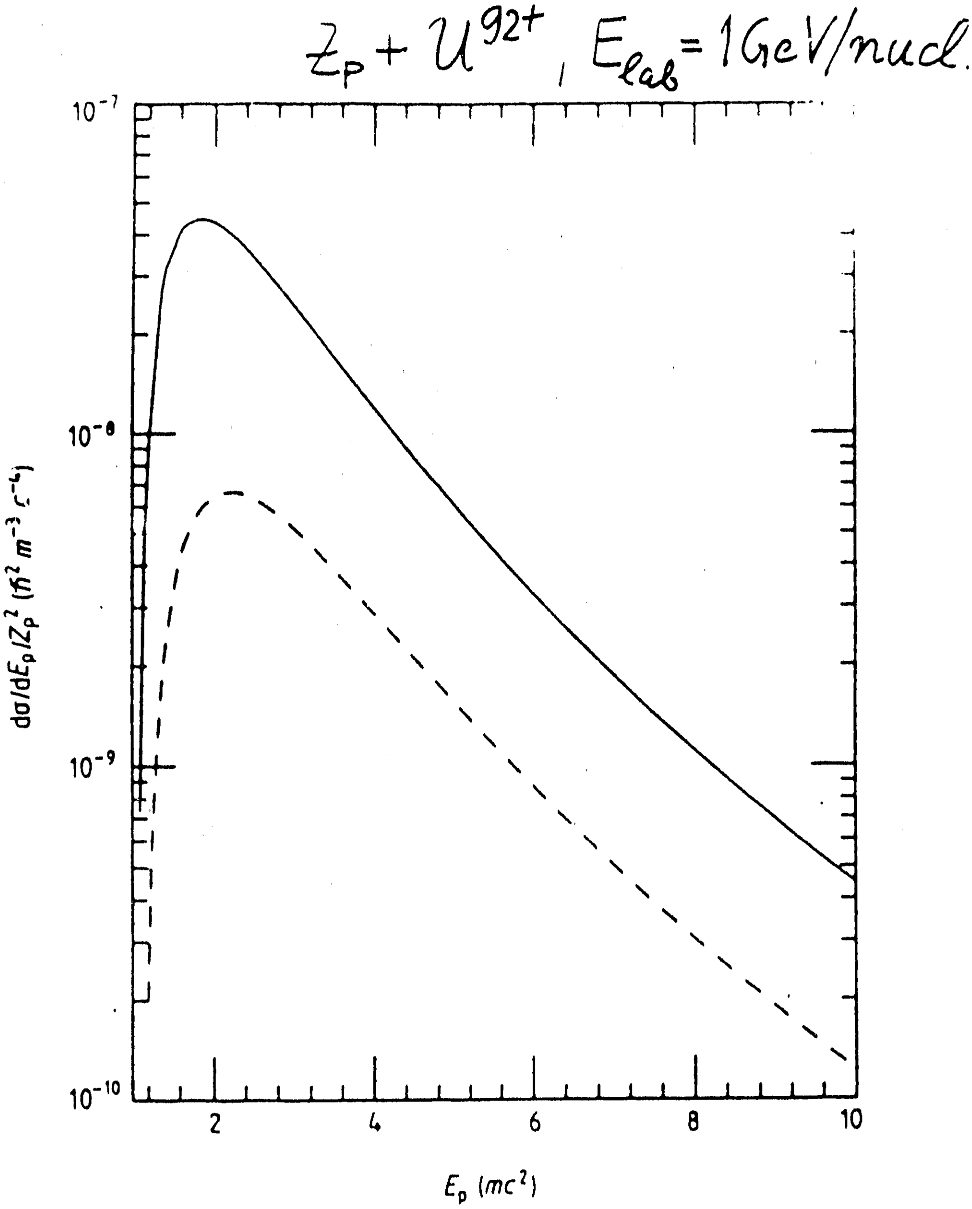


Figure 2. Same as figure 1 at $E_{lab} = 1$ GeV amu^{-1} for U^{92+} target ions.

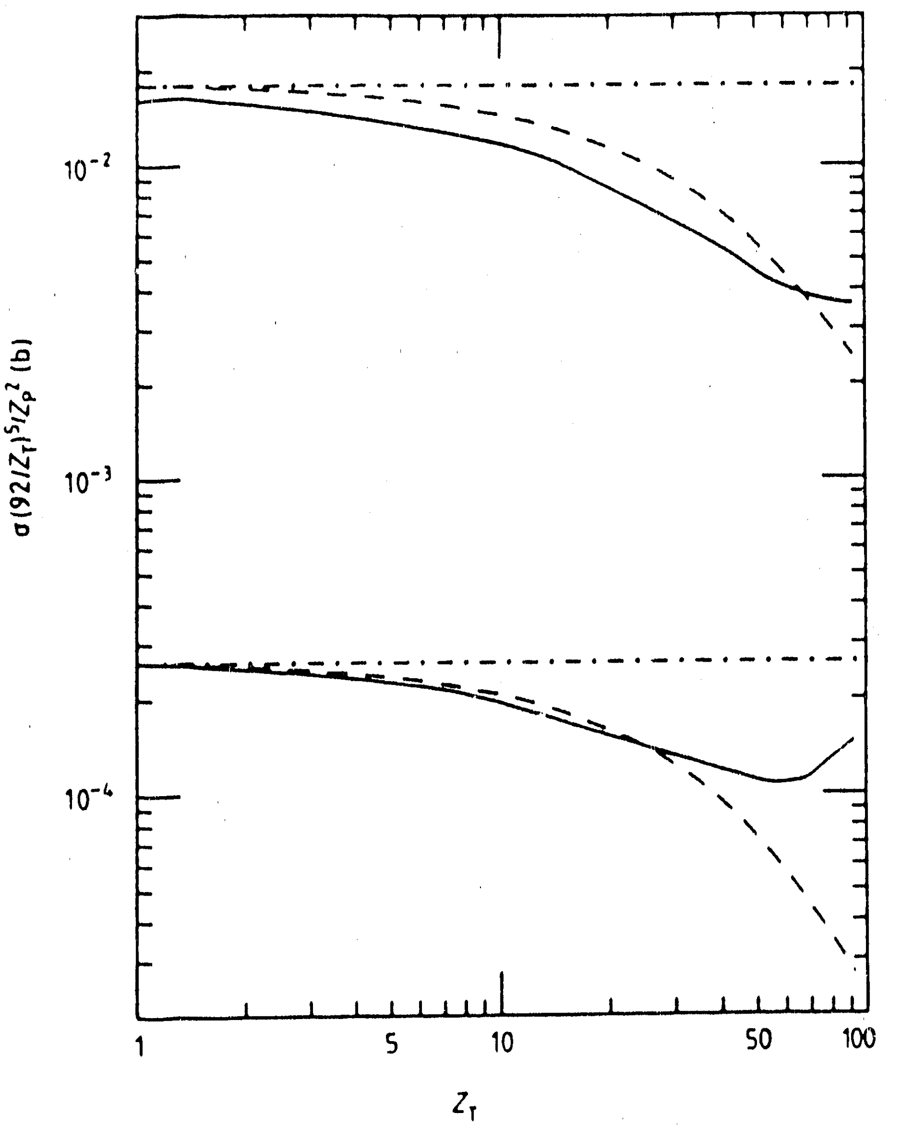


Figure 3. Cross section for pair production with K-shell capture at $E_{lab} = 1$ (bottom) and 15 (top) GeV amu^{-1} as a function of the target charge. Full curves: numerical integration of equation (21); broken curves: equation (20); chain curves: Z_T^{-5} scaling extrapolated over the whole range of target charges (see text).

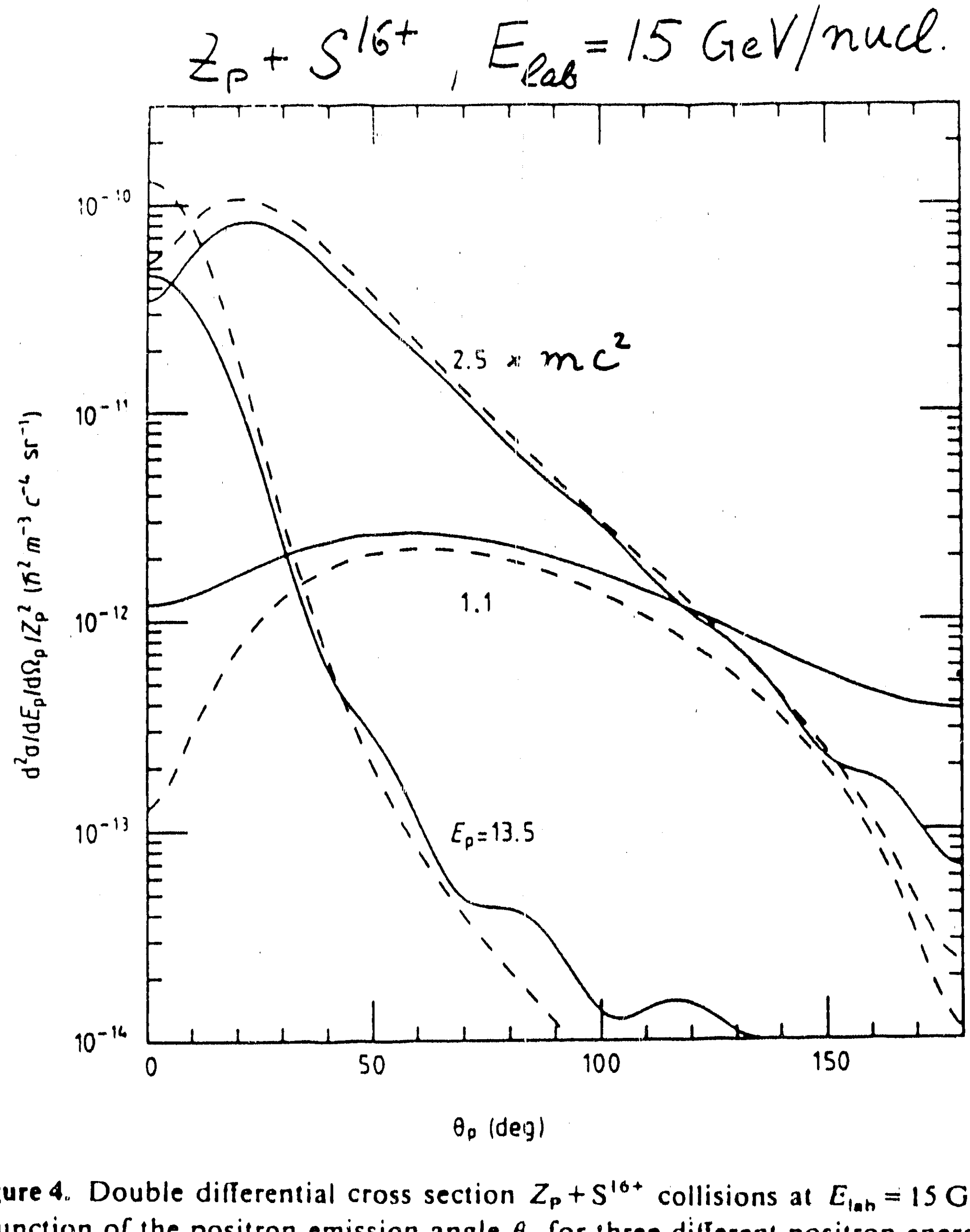


Figure 4. Double differential cross section $Z_p + S^{16+}$ collisions at $E_{lab} = 15$ GeV amu^{-1} as a function of the positron emission angle θ_p for three different positron energies (in units of mc^2). Full curves: equation (10); broken curves: equation (19).

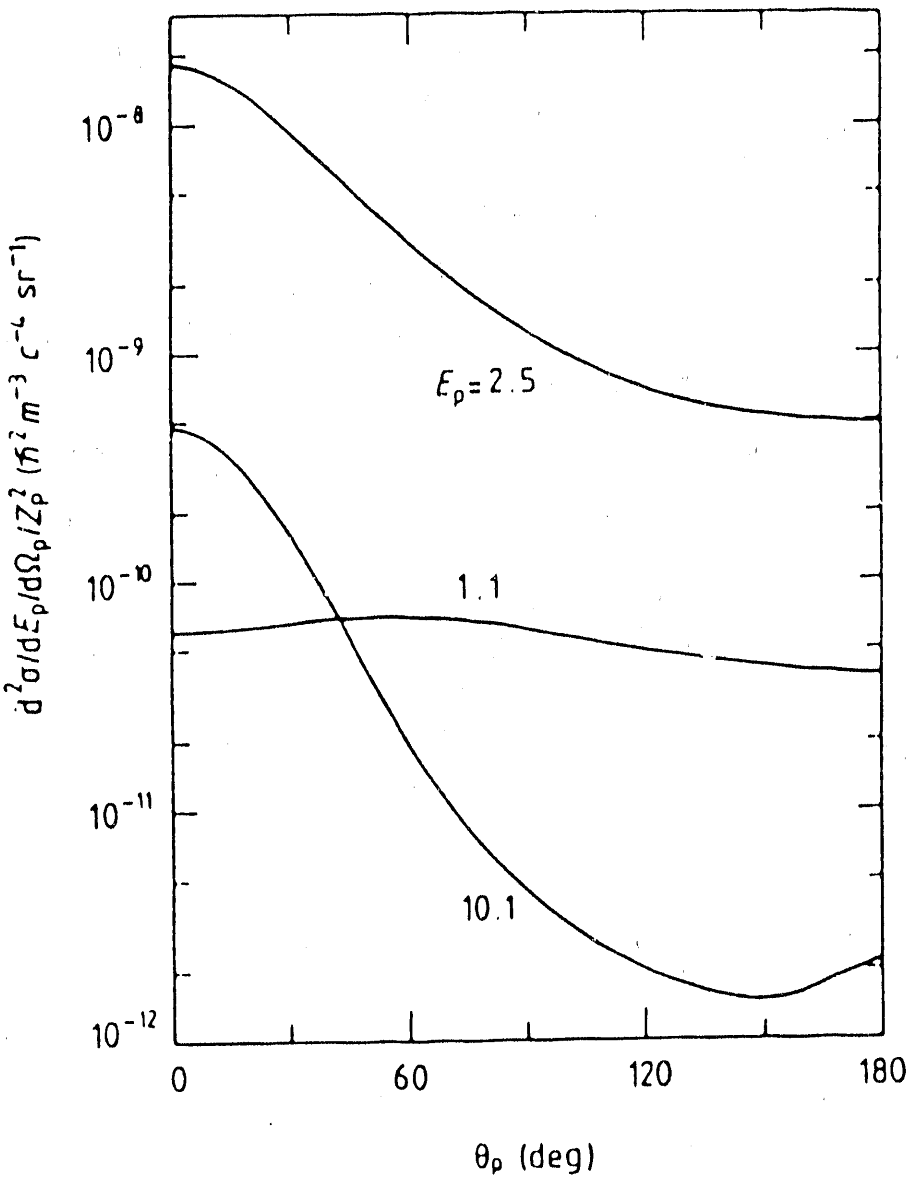


Figure 5. Same as figure 4 for U^{92+} target ions at $E_{lab} = 1$ GeV amu^{-1}.

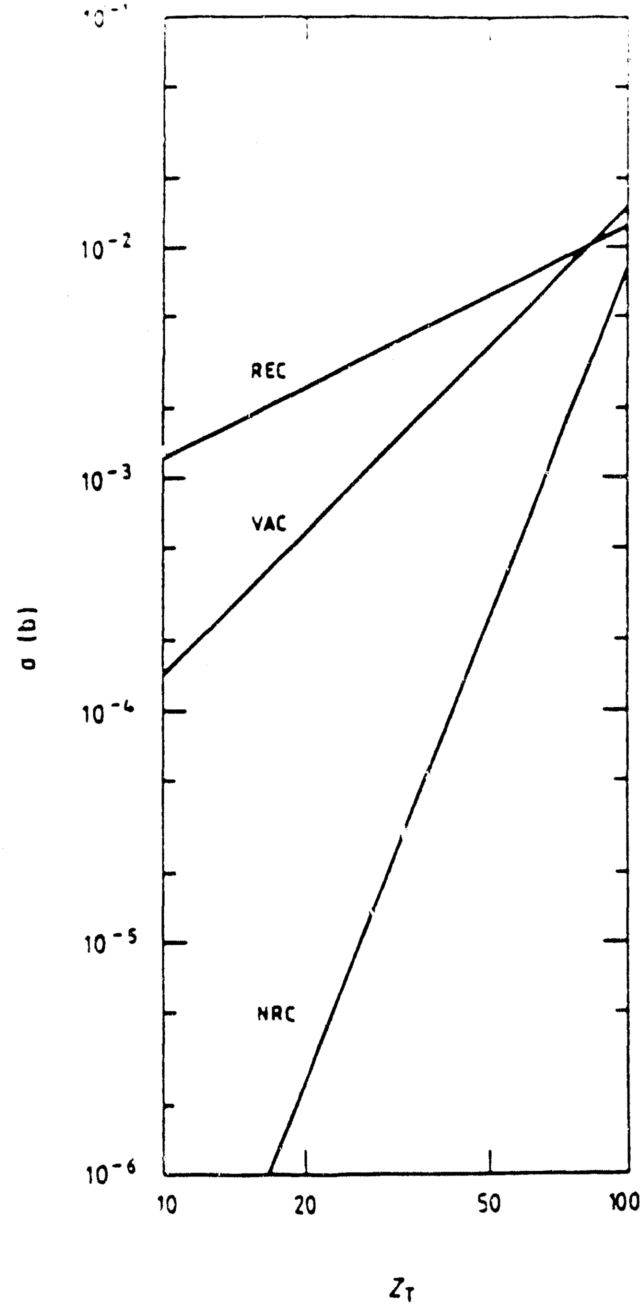


Figure 6. Capture cross sections for 15 GeV amu^{-1} S^{16+} projectiles as a func target charge; REC: radiative electron capture; NRC: non-radiative capture; VA capture from pair production (capture from vacuum). Unlike in the previous fi the projectile captures the electron.

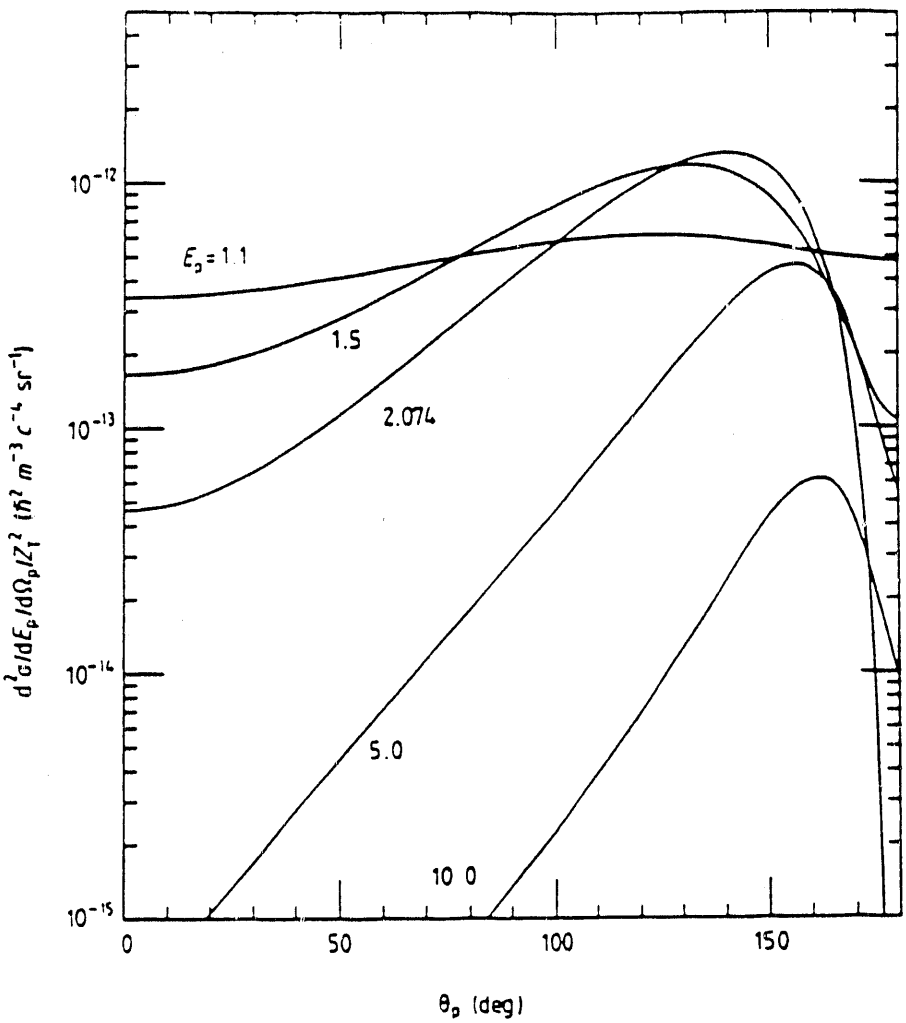


Figure 7. Double differential cross sections in $S^{16+} + Z_T$ collisions at $E_{lab} = 1$ GeV a as a function of the positron emission angle θ_p, measured with respect to the z calculated in the rest frame of the target. The projectile captures the electron and appro along the negative z direction.

2.3 Angular distributions of e^+e^--pairs with continuum distorted wave model

Continuum distorted wave model for direct pair production, Deco 1989/90

$$a_{if}(b) = -i\int_{-\infty}^{+\infty} dt \int d^3r_T\, \Psi_f^{-\dagger}(\vec{r}_T,t)\left(H - i\frac{\partial}{\partial t}\right)\chi_i(\vec{r}_T,t)$$

$$H = c\vec{\alpha}\vec{p} + \beta mc^2 - Z_T e^2/r_T - \gamma(1-\vec{\alpha}\vec{v}/c)Z_P e^2/r_P'$$

χ_i initial negative energy state

Ψ_f^- outgoing exact solution of Dirac equation for electron in a positive continuum state

Ansatz for the initial distorted wave function

$$\underline{\chi_i = L_i \varphi_i}$$

φ_i is solution of target Hamiltonian $c\vec{\alpha}\vec{p} + \beta mc^2 - \frac{Z_T e^2}{r_T}$

L_i describes the effect of the projectile field.

L_i transformed to system of projectile:

$$\tilde{L}_i' = T^{-1}(\exp(-ic^2mt)\, L_i)$$

$$(-ic\vec{\alpha}\,\vec{\nabla}_{r_p'} - V_p' - i\frac{\partial}{\partial t'})\tilde{L}_i' + mc^2\beta\tilde{L}_i'\beta = 0$$

T is boost operator for spinor from projectile to target system.

$$\longrightarrow\quad (H - i\frac{\partial}{\partial t})\chi_i = -ic(\vec{\alpha}L_i - L_i\vec{\alpha})\vec{\nabla}_{r_T}\varphi_i$$

$$+ mc^2(\beta L_i - L_i\beta + L_i - \beta L_i\beta)\varphi_i$$

Solution with Sommerfeld-Maue functions

$$\chi_i = T\mathcal{L}_i T^{-1}\varphi_i$$

$$\mathcal{L}_i = \exp(-\pi\nu_+'/2)\Gamma(1+i\nu_+')\left(1+\underline{\frac{i\vec{\alpha}\,\vec{\nabla}_{r_p'}}{2\gamma_+'c}}\right){}_1F_1(-i\nu_+', 1, i(p_+'r_{p_+}' + \vec{p}_+'\vec{r}_p'))$$

$$\nu_+' = Z_T/v_+'$$

$$1/2\gamma_+'c \;<\; 1/2c \;=\; 1/2\cdot 137 \qquad (c = 137 \text{ in atomic units})$$

$$\varphi_i = \exp(-\pi \nu_+/2)\, \Gamma(1+i\nu_+)\, {}_1F_1(-i\nu_+, 1, i(p_+ r_T + \vec{p}_+ \vec{r}_T))\, \mathcal{U}_+^{\mu}(\vec{r}_T, t)$$

$$\mathcal{U}_+^{\mu}(\vec{r}_T, t) = \frac{\exp(i\epsilon_+ t - i\vec{p}_+ \vec{r}_T)}{(2\pi)^{3/2}} \left(\frac{\epsilon_+ + c^2}{2\epsilon_+} \right)^{1/2} \begin{pmatrix} \frac{c\vec{\sigma}\vec{p}_+}{\epsilon_+ + c^2} \zeta_\mu \\ \zeta_\mu \end{pmatrix}$$

analogous wave function for electron

$$\chi_f = \exp(\pi(\nu_- + \nu_-')/2)\, \Gamma(1+i\nu_-)\, \Gamma(1+i\nu_-')\, \mathcal{U}_-^{\mu}(\vec{r}_T, t)$$
$$\cdot\, {}_1F_1(-i\nu_-, 1, -i(p_- r_T + \vec{p}_- \vec{r}_T))\, {}_1F_1(-i\nu_-', 1, -i(p_-' r_P' + \vec{p}_-' \vec{r}_P'))$$

$$\mathcal{U}_-^{\mu}(\vec{r}_T, t) = \frac{\exp(-i\epsilon_- t + i\vec{p}_- \vec{r}_T)}{(2\pi)^{3/2}} \left(\frac{\epsilon_- + c^2}{2\epsilon_-} \right)^{1/2} \begin{pmatrix} \zeta_\mu \\ \frac{c\vec{\sigma}\vec{p}_-}{\epsilon_- + c^2} \zeta_\mu \end{pmatrix}$$

Approximations:

L_i obtained by using Sommerfeld-Maue functions for Ω_i

χ_f, φ_i approximate Sommerfeld-Maue functions

$$a_{if}(\vec{b}) = -i \int_{-\infty} dt \int d^3 r_T\, W_{\mu_+\mu_-}(\vec{r}_T, \vec{r}_P{}') \exp\left(i\left((\epsilon_+ + \epsilon_-)t - (\vec{p}_+ + \vec{p}_-)\vec{r}_T\right)\right)$$

$$W_{\mu_+\mu_-} = \frac{1}{(2\pi)^6} \int d^3k \int d^3q \exp\left(i(\vec{k}\vec{r}_P{}' + \vec{q}\vec{r}_T)\right) T_{\mu_+\mu_-}(\vec{q}, \vec{k})$$

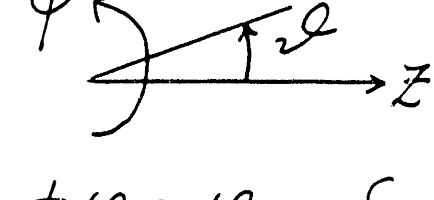
$$\longrightarrow \quad a_{if}(\vec{b}) = \frac{-i}{(2\pi)^2 \gamma v} \int d\eta_x\, d\eta_y \exp(-i\vec{b}\vec{\eta})\, T_{\mu_+\mu_-}$$

Cross section

$$\frac{d^6\sigma}{d\Omega_+ d\Omega_- d\epsilon_+ d\epsilon_-} = \frac{\epsilon_+ \epsilon_- p_+ p_-}{c^4} \frac{1}{2\pi^2 \gamma^2 v^2} \int d\vec{\eta} \left(|T_{\uparrow\uparrow}|^2 + |T_{\uparrow\downarrow}|^2 \right)$$

In addition amplitude multiplied by Sacharov-factor $e^{\pi/v_r}\,\Gamma(1+i/v_r)$ (v_r relative velocity) for attraction between e^+ and e^-.

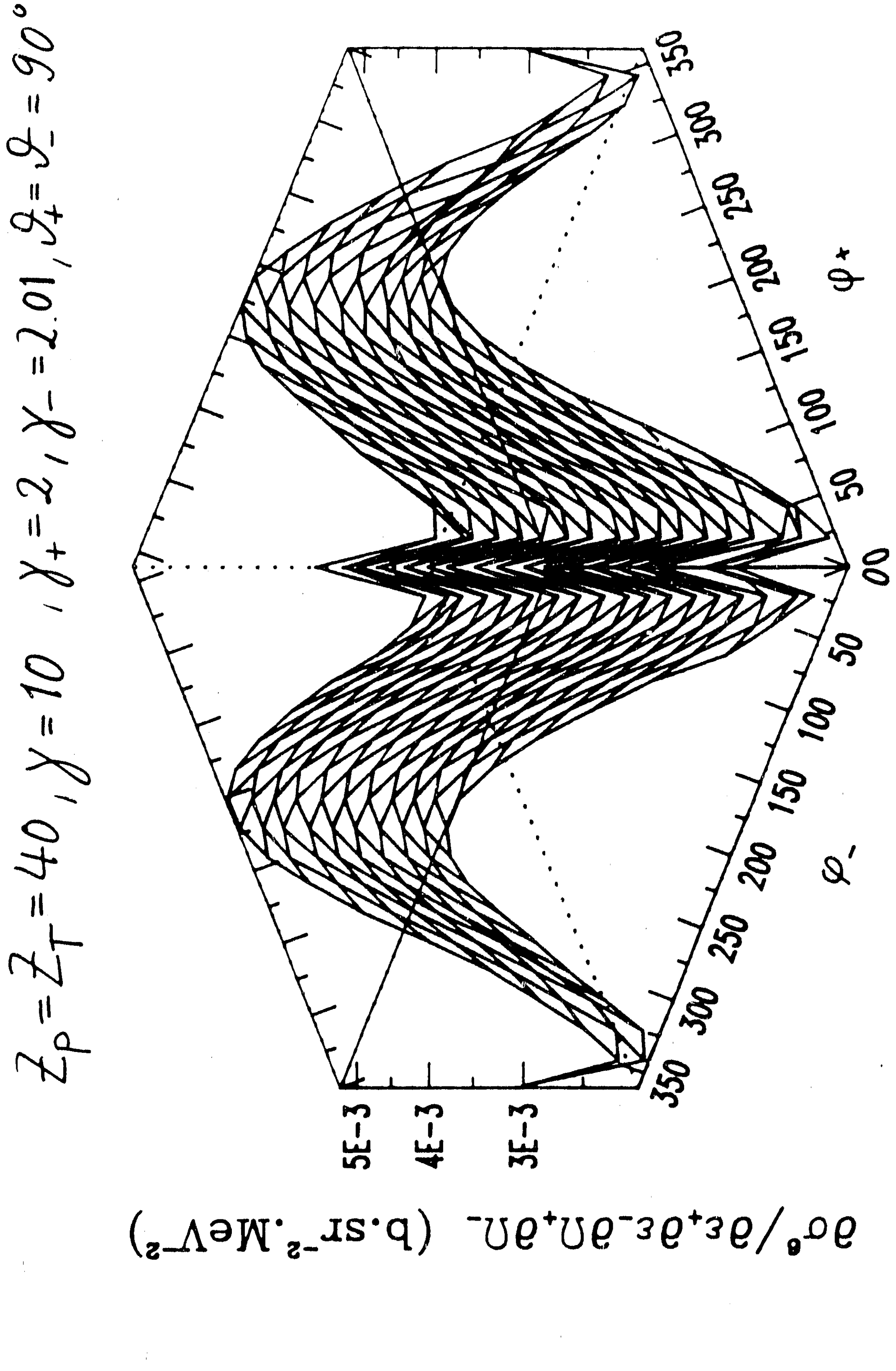
<u>Results for $Z_P = Z_T = 40$, $\gamma = 10$</u>



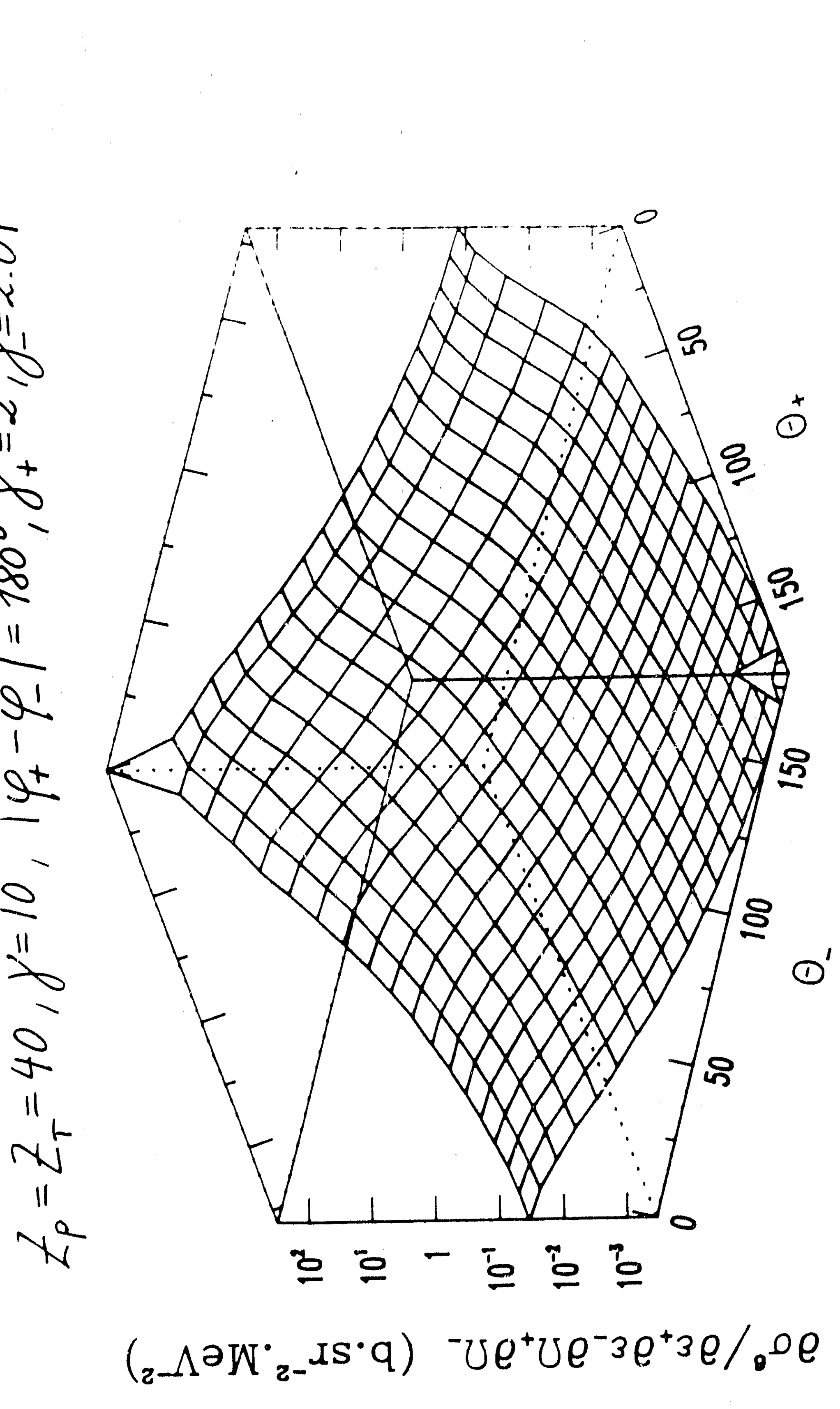
hump at $\varphi_+ \approx \varphi_-$: Sacharov factor

maxima $|\varphi_+ - \varphi_-| \approx 180°$

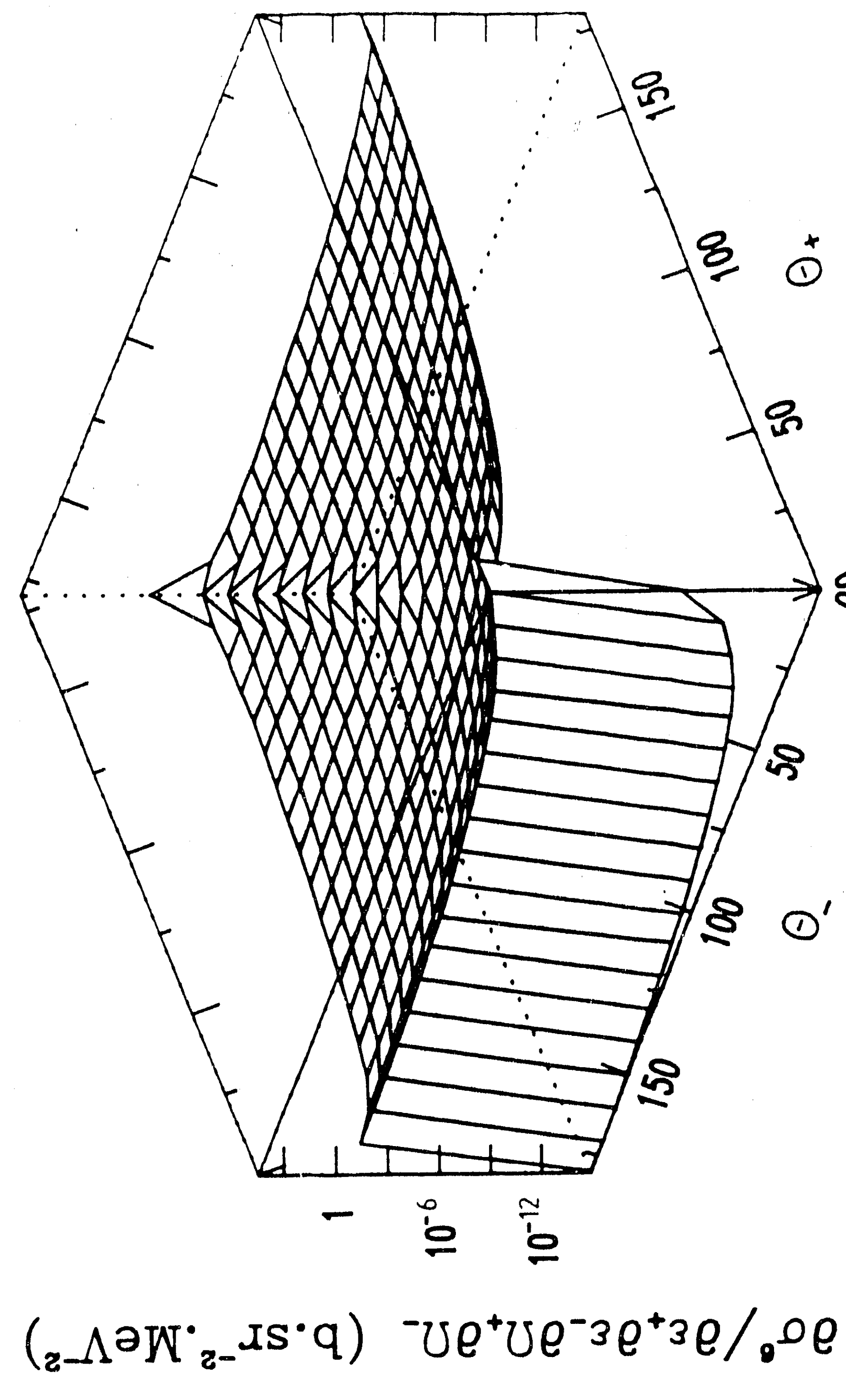
electron – positron pairs created in opposite directions.



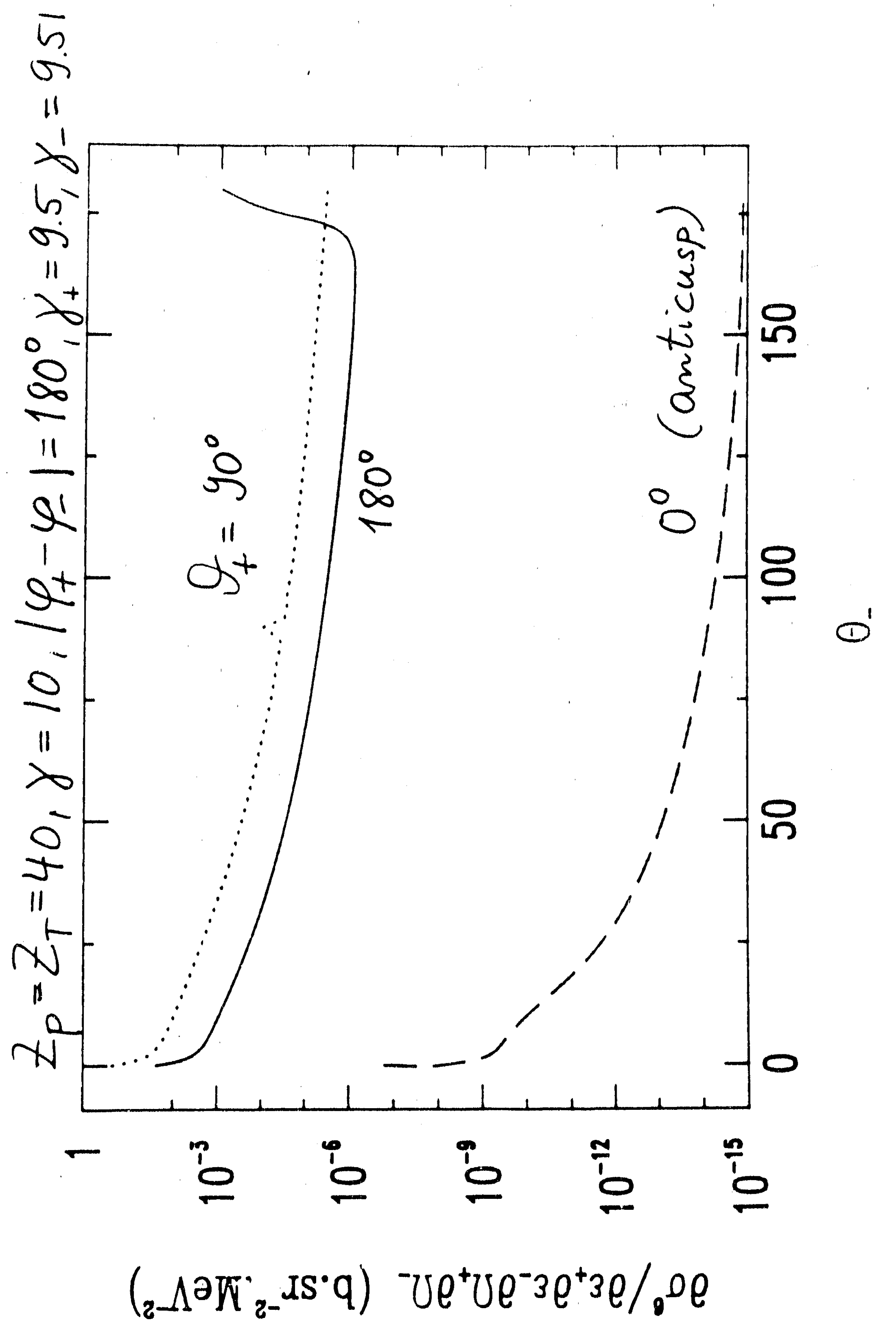
Zp = ZT = 40, γ = 10, γ+ = 2, γ− = 2.01, ϑ+ = ϑ− = 90°
∂σ6/∂ε+∂ε−∂Ω+∂Ω− (b.sr−2.MeV−2)
5E-3
4E-3
3E-3
φ−
350 300 250 200 150 100 50 00
φ+
00 50 100 150 200 250 300 350

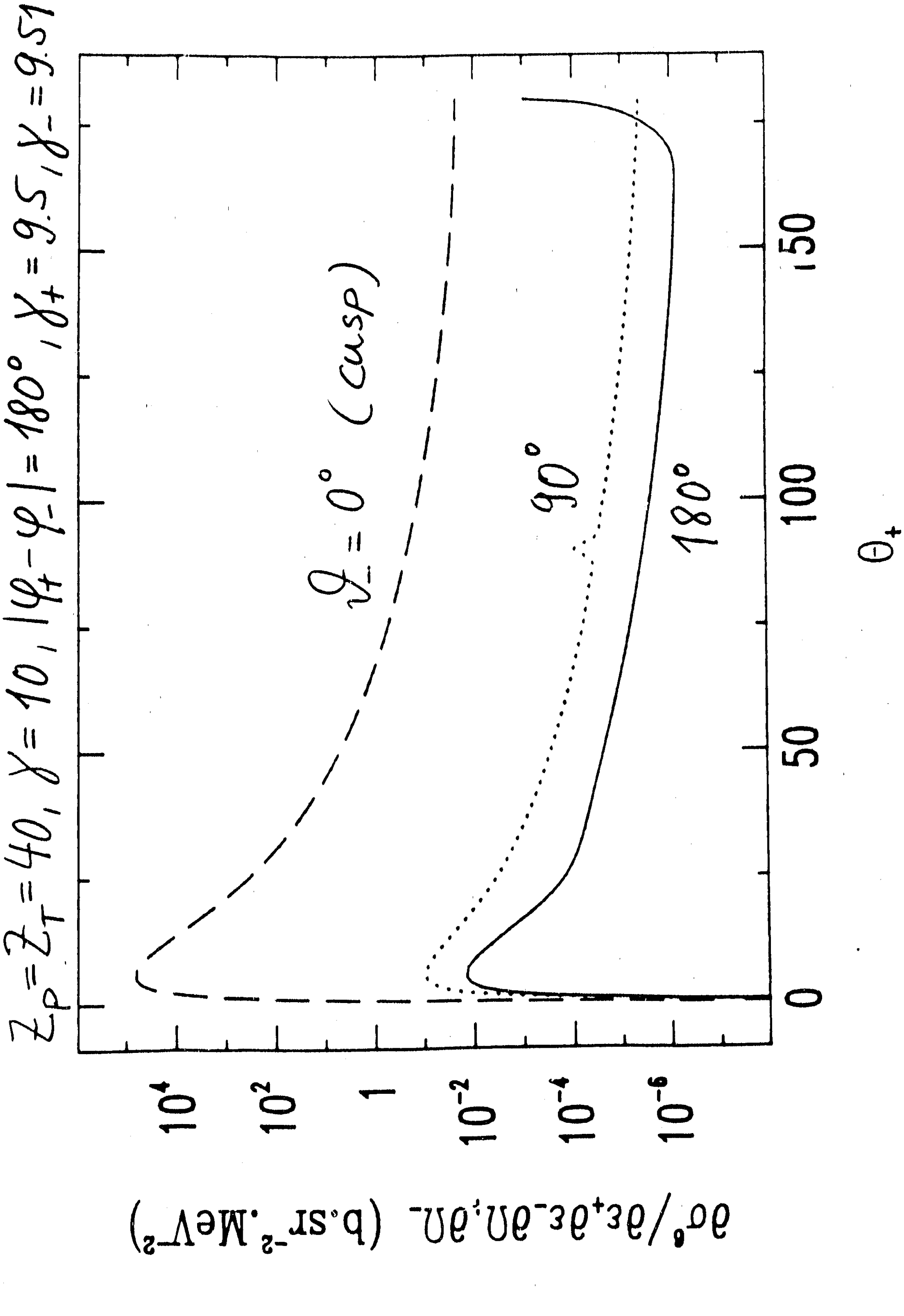


$Z_P = Z_T = 40,\ \gamma = 10,\ |\varphi_+ - \varphi_-| = 180°,\ \gamma_+ = 2,\ \gamma_- = 2.01$
$\partial\sigma^6/\partial\varepsilon_+\partial\varepsilon_-\partial\Omega_+\partial\Omega_-$ (b.sr^{-2}.MeV^{-2})
10^2
10^1
1
10^{-1}
10^{-2}
10^{-3}
0
50
100
150
Θ_-
Θ_+



$\partial\sigma^6/\partial\varepsilon_+\partial\varepsilon_-\partial\Omega_+\partial\Omega_-$ (b.sr^{-2}.MeV^{-2})
1
10^{-6}
10^{-12}
Θ_+
Θ_-
00
50
100
150


$Z_P = Z_T = 40, \gamma = 10, |\varphi_+ - \varphi_-| = 180°, \gamma_+ = 9.5, \gamma_- = 9.51$
$\vartheta_+ = 90°$
180°
0° (anticusp)
0
50
100
150
Θ_-
1
10^{-3}
10^{-6}
10^{-9}
10^{-12}
10^{-15}
$\partial\sigma^6/\partial\varepsilon_+\partial\varepsilon_-\partial\Omega_+\partial\Omega_-$ $(b.sr^{-2}.MeV^{-2})$



$Z_P = Z_T = 40,\ \gamma = 10,\ |\varphi_+ - \varphi_-| = 180°,\ \gamma_+ = 9.5,\ \gamma_- = 9.51$
$\vartheta_- = 0°$ (cusp)
90°
180°
10^4
10^2
1
10^{-2}
10^{-4}
10^{-6}
0
50
100
150
Θ_+
$\partial\sigma^6/\partial\varepsilon_+\partial\varepsilon_-\partial\Omega_+\partial\Omega_-$ $(b.sr^{-2}.MeV^{-2})$

2.4 Muon pair production with inner shell capture in $U^{92+}-U^{92+}$ collisions

First estimate of the cross section

$$\sigma_{\mu^+\mu^-} = (\lambdabar_\mu / \lambdabar_e)^2 \, \sigma_{e^+e^-}$$

$\lambdabar_\mu$ and $\lambdabar_e$ are reduced Compton wave lengths

$$\left(\frac{\lambdabar_\mu}{\lambdabar_e}\right)^2 = \left(\frac{m_e}{m_\mu}\right)^2 = 0.25 \cdot 10^{-4}$$

Example: $U+U$ at 10 GeV/n, K-shell capture

$$\sigma_{e^+e^-} = 19\,b \longrightarrow \sigma_{\mu^+\mu^-} = 4.7 \cdot 10^{-4}\,b$$

Reduced Compton wave length: $\lambdabar_\mu = 1.9$ fm
nuclear radius of U: $R_0 = 7.4$ fm

Finite extension of nuclear charge is important for inner-shell capture of μ^-.

$$\frac{dI_b}{dE_{\mu^+}} = \overline{\sum_{\kappa_{\mu^+}, m_{\mu^+}, m_{\mu^-}}} \cdot |a_{\mu^+\mu^-}|^2$$

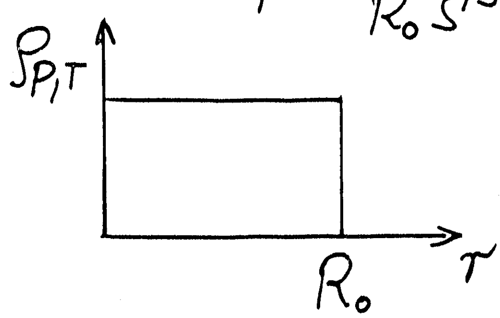
$$a_{\mu^+\mu^-} = \frac{2i}{\hbar v_p} \sum_{l,m} i^{l-m} \int_q^\infty s\,ds\, \tilde{\Phi}_p(s') B_{lm}(b,q,s)$$

$$\times \langle \psi_{\mu^-} | (1 - v_p \alpha_z / c) j_l(sr) Y_{lm}(\hat{r}) | \psi_{\mu^+} \rangle$$

$$q = (E_{\mu^-} + E_{\mu^+})/\hbar v_p \quad , \quad s' = (s^2 - q^2 v_p^2 / c^2)^{1/2}$$

$$\tilde{\Phi}_p(s') = \frac{4\pi}{s'} \int_0^\infty dr\, r \sin(rs')\, \phi_p(r)$$

$$= 4\pi Z_p e^2 \frac{3}{R_0 s'^3} j_1(s' R_0)$$



ϕ_T: target potential used for calculation of $\psi_{\mu^-}, \psi_{\mu^+}$; numerical solutions of Dirac equation

$$U_T^{92+} + U_P^{92+} \rightarrow U_T^{91+}(1s_{1/2}) + U_P^{92+} + \mu^+$$

$E_{lab} = 10\ \mathrm{GeV}/n$, $R_0 = 7.4\ \mathrm{fm}$

Results:

1. differential probabilities $\frac{dI_b}{dE_\mu^+}$ as functions of E_{μ^+} for $b = 14,\ 18.68,\ 28\ \mathrm{fm}$, $|K_{\mu^+}| = 20$ and 40

2. probability I_b as function of impact parameter

3. cross sections

   α) $b_{min} = 0$ $\quad \sigma = 3.7 \cdot 10^{-9}\,b$

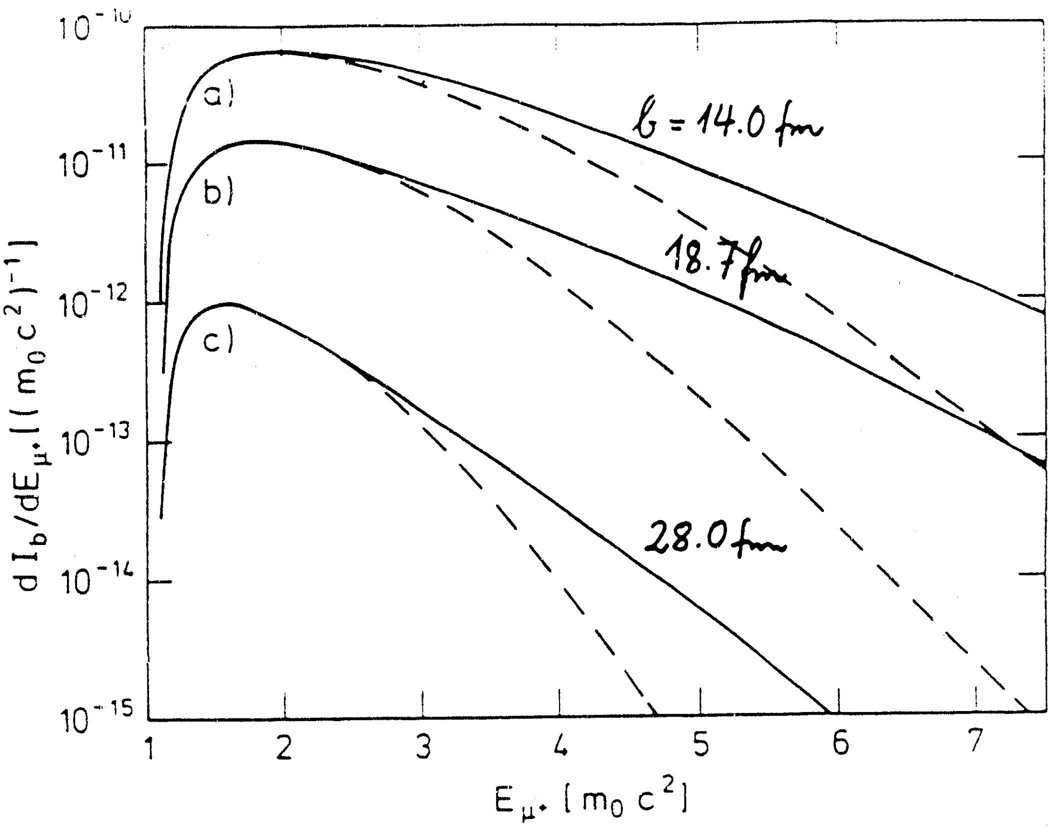
   comparison: point charges $\sigma = 4.7 \cdot 10^{-4}\,b$

   $\left.\begin{array}{l}\text{projectile: point charge}\\ \text{target: extended charge}\end{array}\right\} \sigma = 2.6 \cdot 10^{-8}\,b$

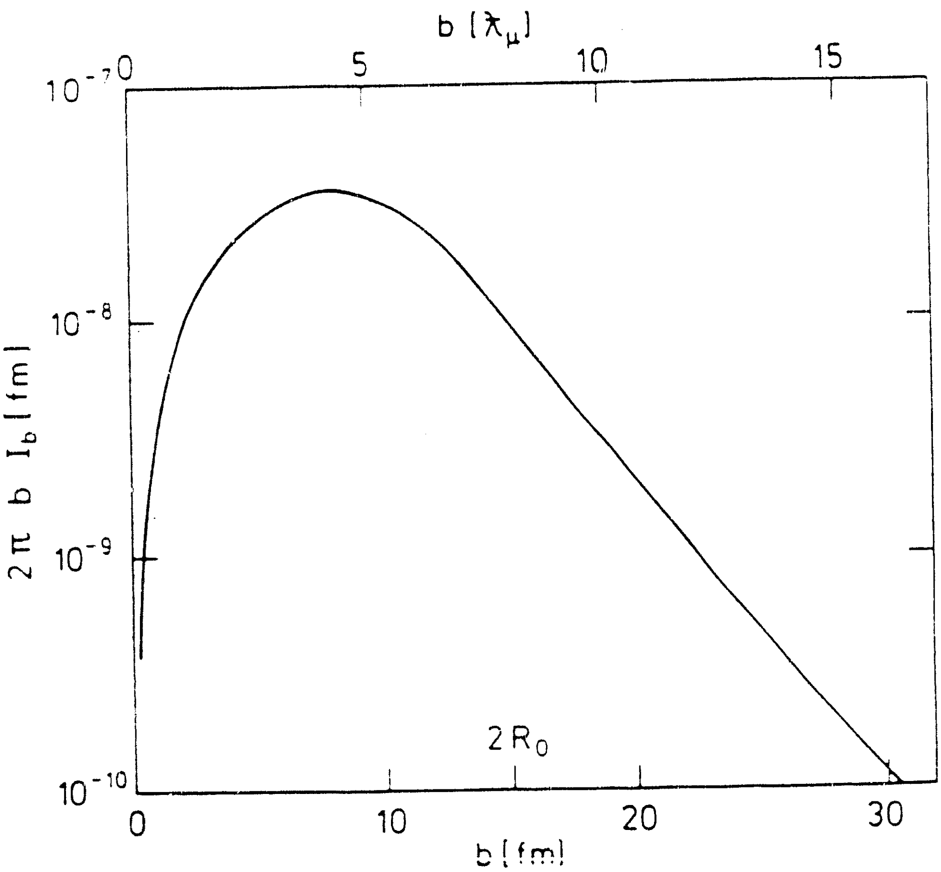
   $\left.\begin{array}{l}\text{projectile extension: reduction } 10^{-1}\\ \text{target extension: reduction } 10^{-4}\end{array}\right\} \underline{\underline{10^{-5}}}$

   β) $b_{min} = 15\ \mathrm{fm}$ $\quad \underline{\sigma = 2.9 \cdot 10^{-10}\,b}$

   $b_{min} = 18\ \mathrm{fm}$ $\quad \underline{\sigma = 1.0 \cdot 10^{-10}\,b}$



10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
$dI_b/dE_{\mu^+}[(m_0c^2)^{-1}]$
a)
b)
c)
b = 14.0 fm
18.7 fm
28.0 fm
1
2
3
4
5
6
7
$E_{\mu^+}[m_0c^2]$



$b\,[\lambda\!\!\!^{-}_{\mu}]$
0
5
10
15
10^{-7}
10^{-8}
10^{-9}
10^{-10}
$2\pi\ b\ I_b\,[fm]$
$2R_0$
0
10
20
30
b [fm]

3. Coupled channel calculations for electron-positron pair creation

Momberger 1990

Question of nonperturbative treatment of electron-positron creation in the framework of semiclassical theory

Possibilities:

1. Perturbation theory of higher orders
2. Solution of coupled time-dependent differential equations of first order for expansion coefficients (= transition amplitudes)
3. Direct numerical solution of Dirac equation by means of finite difference method or finite element method (e.g. Bottcher, Strayer)

here: solution of coupled equations

i) Coupled channel equations and probabilities

electron-positron field

$$\hat{\psi} = \sum_{E_q > E_F} \varphi_q(\vec{r},t)\, b_q + \sum_{E_q < E_F} \varphi_q(\vec{r},t)\, d_q^{+}$$

$$H_D\, \varphi_q(\vec{r},t) = i \frac{\partial}{\partial t} \varphi_q(\vec{r},t)$$

$$H_D = H_T - e^2 \gamma Z_p (1 - v_p \alpha_z)/r'$$

ansatz in terms of eigenfunctions of H_T:

$$\varphi_q(\vec{r},t) = \sum_p a_{pq}(t)\, \chi_p(\vec{r},t)$$

$$H_T\, \chi_p(\vec{r},t) = i \frac{\partial}{\partial t} \chi_p(\vec{r},t)$$

coupled equations:

$$\underline{i\, \dot{a}_{pq}(t) = \sum_{p'} M_{pp'}(t)\, a_{p'q}(t)}$$

$$M_{pp'}(t) = -\langle \chi_p | e^2 \gamma Z_p (1 - v_p \alpha_z)/r' | \chi_{p'} \rangle$$

initial condition: $a_{pq}(t \to -\infty) = \delta_{pq}$

probability for an electron in an unperturbed state with $E_q > E_F$

$$W_q^{(+)}(t\to\infty) = \langle F| \left(\int \chi_q^+ \hat{\psi}\, d^3r\right)^+ \left(\int \chi_q^+ \hat{\psi}\, d^3r\right) |F\rangle_{t\to\infty}$$

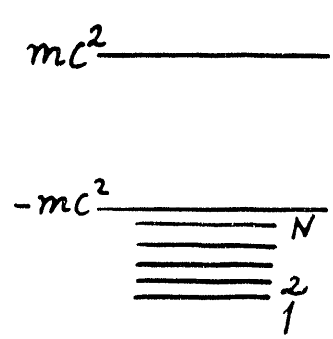
$$= \sum_{E_{q'}<E_F} |a_{qq'}(t\to\infty)|^2$$

probability for a positron in an unperturbed state with $E_q < E_F$

$$W_q^{(-)}(t\to\infty) = \sum_{E_{q'}>E_F} |a_{qq'}(t\to\infty)|^2$$

Simple interpretation of the formalism:
many-particles state represented by Slater determinant; states of negative continuum initially filled

$$\psi = \frac{1}{\sqrt{N!}} \begin{vmatrix} \varphi_1(1) & \varphi_1(2) & \dots & \varphi_1(N) \\ \varphi_2(1) & & & \vdots \\ \vdots & & & \vdots \\ \varphi_N(1) & \dots & & \varphi_N(N) \end{vmatrix}$$



Amplitude for transition from state i to state f

$$A_{i\to f} = \left|\begin{matrix} \langle \chi_1 | \varphi_1\rangle & \cdots & \langle \chi_f | \varphi_1\rangle & \cdots \\ \langle \chi_1 | \varphi_2\rangle & & \langle \chi_f | \varphi_2\rangle & \cdots \\ \vdots & & & \end{matrix}\right|_{t\to\infty}$$

i^{th} column

$$= \left|\begin{matrix} a_{11} & a_{21} & \cdots & a_{f1} & \cdots & a_{N1} \\ a_{12} & & & a_{f2} & & \vdots \\ \vdots & & & \vdots & & \vdots \\ a_{1N} & \cdots & \cdots & a_{fN} & \cdots & a_{NN} \end{matrix}\right|_{t\to\infty}$$

First-order perturbation theory : $A_{i\to f} = a_{fi}(t\to\infty)$

b) Treatment of interaction matrix and wave function:

$$M_{pp'} = -e^2 \gamma Z_p \exp(i(E_p - E_{p'})t) \langle \psi_p | (1 - v_p \alpha_z)/r' | \psi_{p'} \rangle$$
$$= -4 Z_p \alpha \exp(i(E_p - E_{p'})t) \sum_{l,m} \int_0^\infty ds\, M_{pp'}^{(l,m)}(s)\, Z_{lm}(s,t)$$

$$\chi_p(\vec{r},t) = \exp(-iE_p t)\, \psi_p(\vec{r})$$

$$M_{pp'}^{(l,m)} = \langle \psi_p | (1 - v_p \alpha_z)\, j_l(sr)\, Y_{lm}(\hat{r}) | \psi_{p'} \rangle$$

$$Z_{lm} = i^{l-m} \int_{-1}^{1} dx\, \frac{1}{1 - v_p^2 x^2}\, Y_{lm}^*(\arccos x, 0)\, \exp(i s v_p t x) \times J_m(b s \sqrt{1 - x^2})$$

bound and continuum states:

Coulomb-Dirac wave functions

continuum wave functions discretized by means of wave packets

$$\psi_{E_j} = \frac{1}{\sqrt{\Delta E_j}} \int_{E_j - \Delta E_j/2}^{E_j + \Delta E_j/2} dE\, \psi_E(\vec{r}) \exp(i(E_j - E)t)$$

time-dependent packets, travelling outwards for $t>0$

c) Applications

System $Z_p + U^{92+}$ at $E_{lab} = 10\,GeV/n$, $Z_p = 1, 92$
$b = 386\,fm$, $v_p = 0.99636\,c$

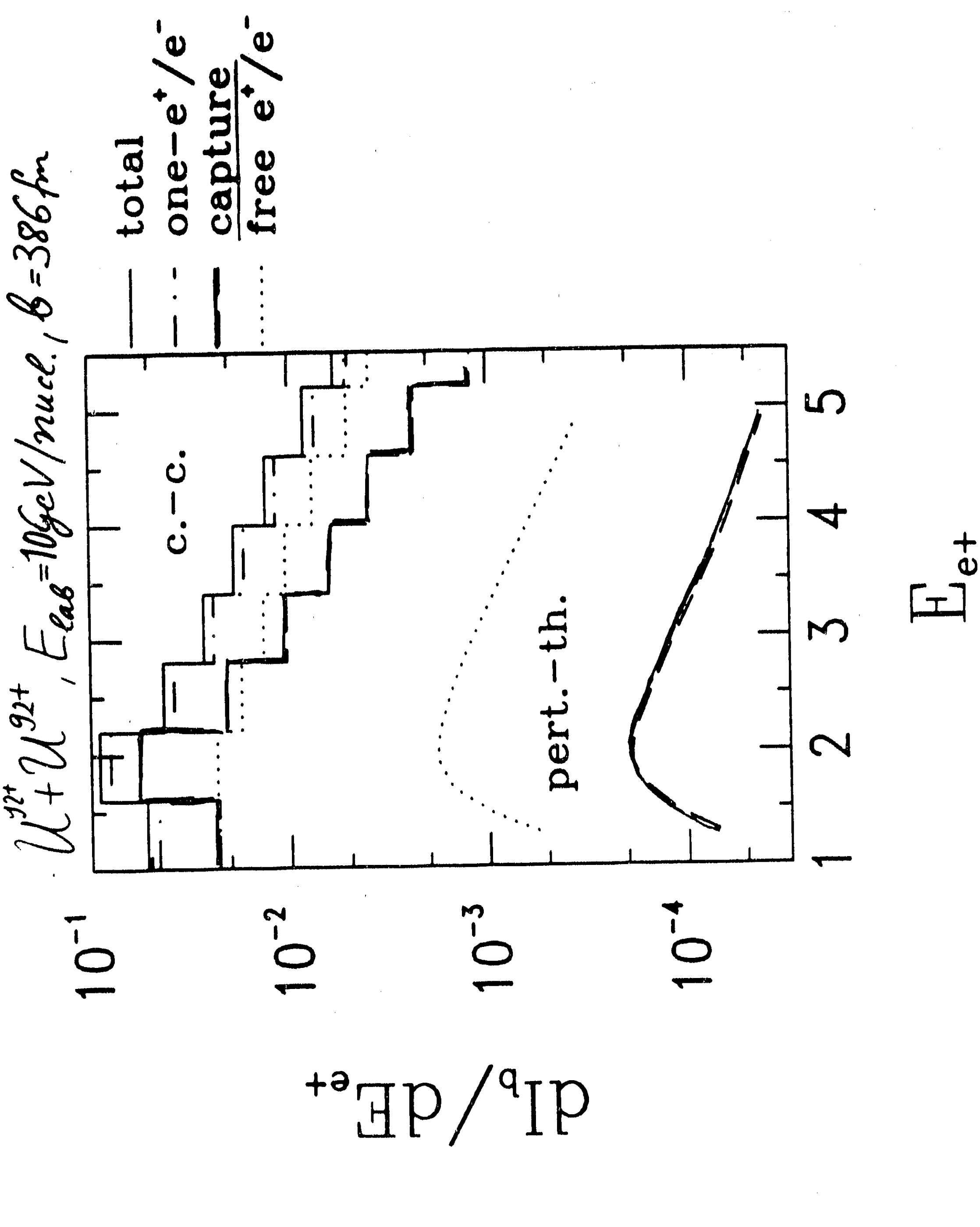
basis states:

bound states: $1s_{1/2}, 2s_{1/2}, 2p_{1/2}, 2p_{3/2}, 3s_{1/2}$
continuum states: $E_j = 1.3 \ldots 7.3\,mc^2$, $\Delta E = 0.6\,mc^2$
$= -1.3 \ldots, -6.1\,mc^2$
angular momenta $s_{1/2}, p_{1/2}, p_{3/2}$

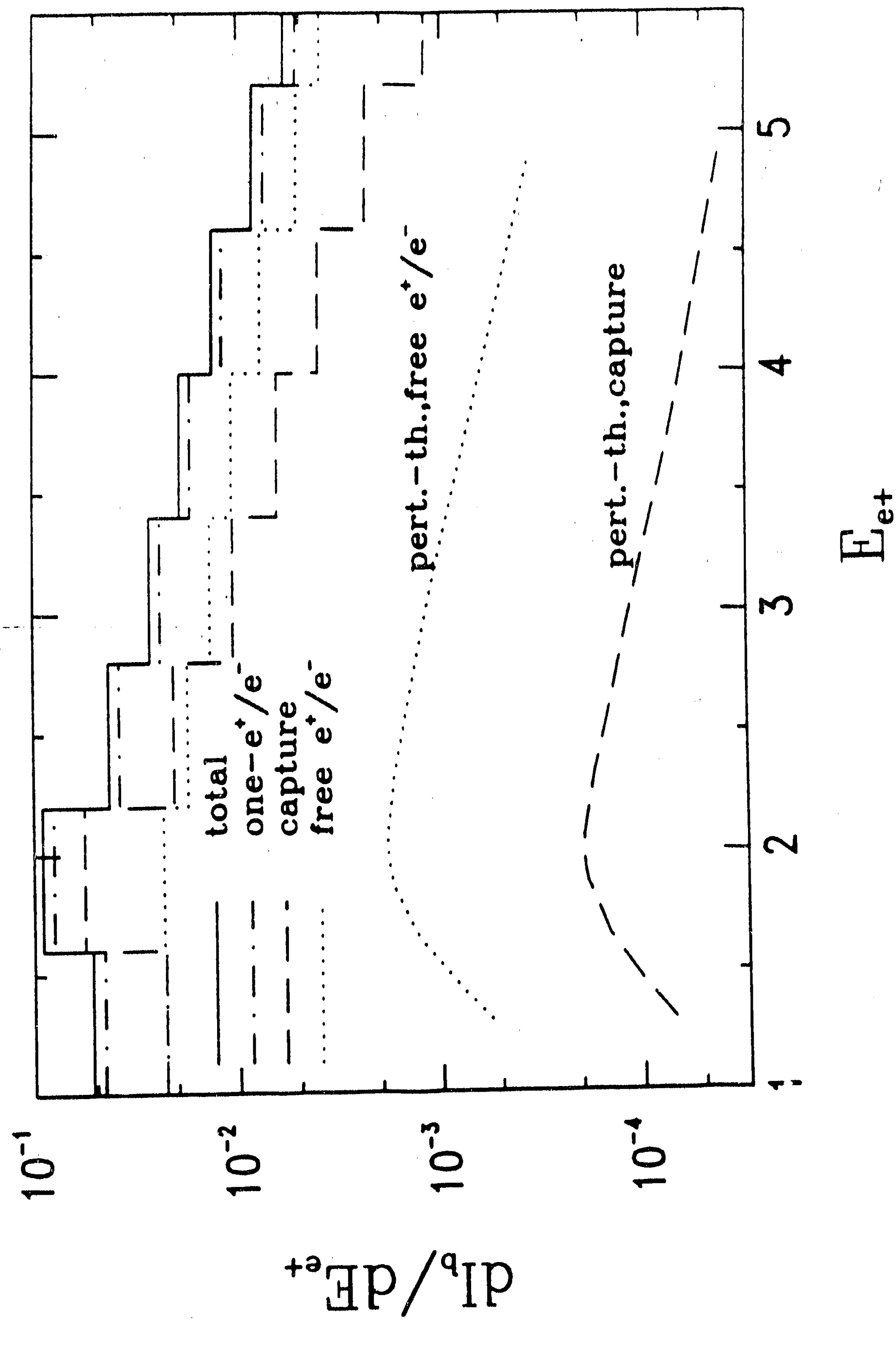
This basis set means 172 coupled equations. From first-order perturbation theory it is known that this basis set is too small with respect to summation of angular momenta.

Test of coupled program:

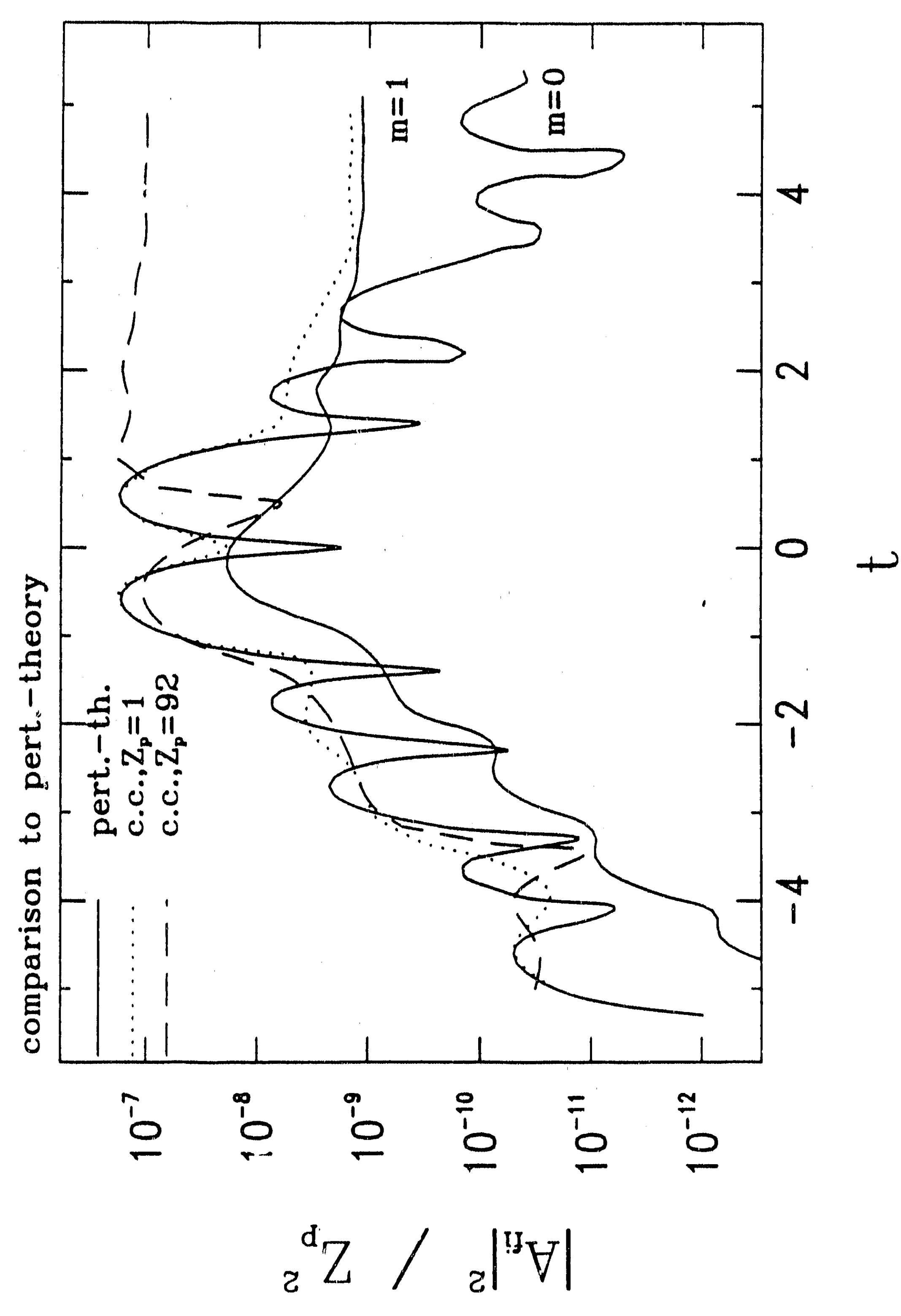
1. time reversal symmetry: $|a_{if}(\infty)|^2 = |a_{fi}(\infty)|^2$
2. Agreement of first-order perturbation theory and coupled channel calculation for $Z_p = 1$



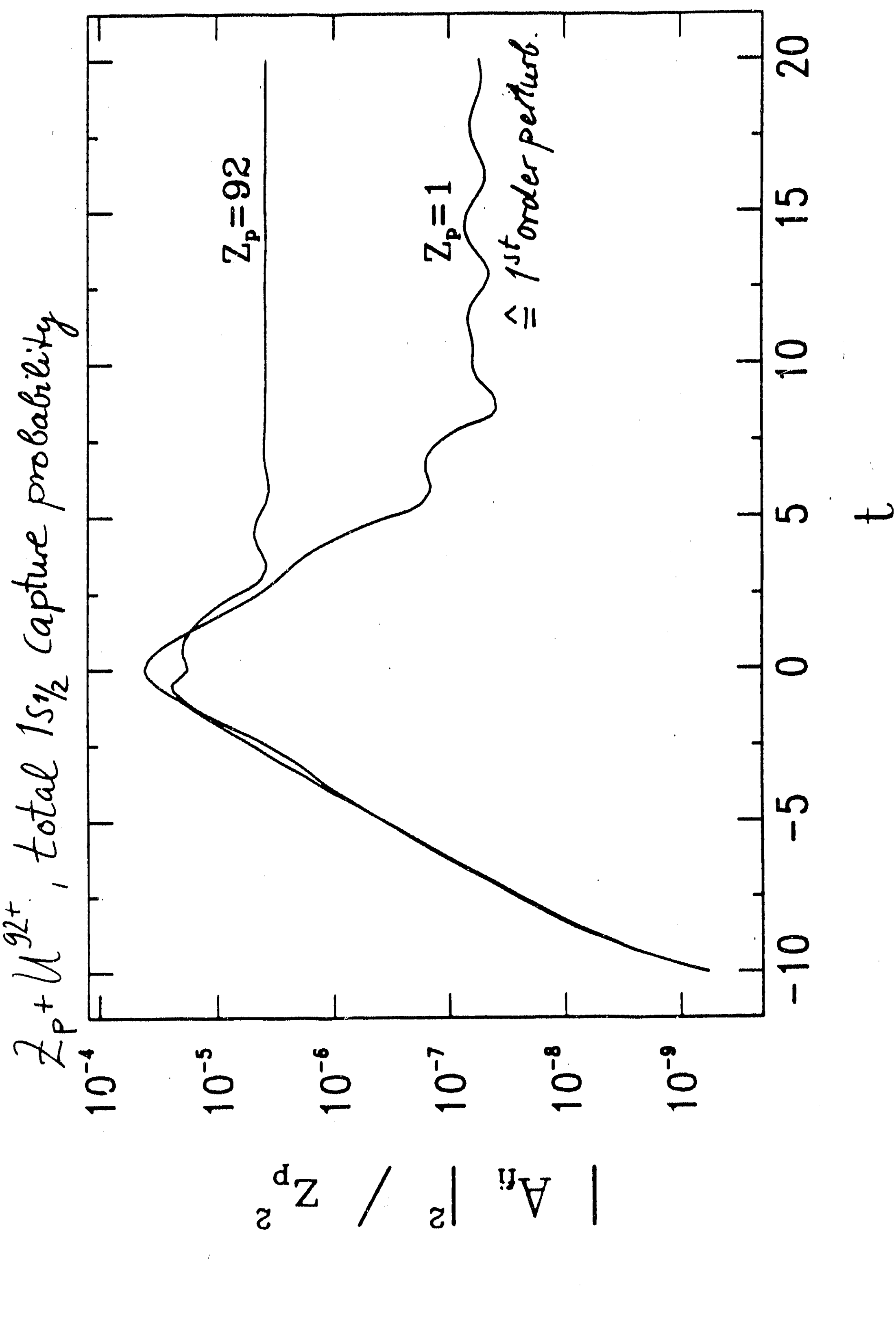
$U^{92+} + U^{92+}$, $E_{lab} = 10 GeV/nucl.$, $b = 386 fm$
total
one-e^+/e^-
capture
free e^+/e^-
c.-c.
pert.-th.
10^{-1}
10^{-2}
10^{-3}
10^{-4}
1
2
3
4
5
E_{e+}
dI_b/dE_{e+}



total
one-e+/e-
capture
free e+/e-
pert.-th.,free e+/e-
pert.-th.,capture
E_{e+}
dI_b/dE_{e+}
1
2
3
4
5
10^{-1}
10^{-2}
10^{-3}
10^{-4}



comparison to pert.-theory
pert.-th.
c.c.,Z_p=1
c.c.,Z_p=92
m=1
m=0
t
$|A_{fi}|^2 / Z_p^2$
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
-4
-2
0
2
4



Z_P + U^92+, total 1s1/2 capture probability
Z_p=92
Z_p=1
≙ 1st order perturb.
10^-4
10^-5
10^-6
10^-7
10^-8
10^-9
|A_fi|^2 / Z_p^2
-10
-5
0
5
10
15
20
t

4 Lepton pair production by bremsstrahlung in central heavy ion collisions

For fixed impact parameter probability for pair creation is constant at large velocities.

$\longrightarrow$ cross section $\sigma \sim \ln \gamma \sim \ln E$

Question: Collisions with stronger rising cross sections for pair creation?

Answer: Nuclear head-on collisions!

In nuclear head-on collisions one studies nuclear matter under extreme conditions.

Study of:

1. equation of state,
2. phase transition of hadronic matter to quark-gluon plasma.

Signatures for quark-gluon plasma at high temperatures:

production of strange particles
J/Ψ suppression by color charge screening
thermal photons
dileptons

Photons and leptons experience only electromagnetic interaction. They penetrate the plasma region without rescattering in contrast to the strongly interacting particles.

Background of photons and lepton pairs:

a) Hard preequilibrium scattering processes producing very massive Drell-Yan pairs and photons of very high energy
b) Production in the hadronic phase at the final stage of the collision process:
$\pi^+ + \pi^- \longrightarrow \ell^+\ell^- + X \;:\; \pi^0 \longrightarrow 2\gamma$

Window for dilepton production from quark-gluon phase in the range of pair masses

$$1\,\mathrm{GeV} \leq Mc^2 \leq 3\text{-}4\,\mathrm{GeV}.$$

We have asked the question of background in the dilepton spectrum produced by bremsstrahlung in the initial stage of reaction.

Calculation of photon and dilepton spectrum produced by bremsstrahlung

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Gale and Kapusta 1987 : estimate of low mass dilepton rate from cascading nucleons

Alexander et al. 1987 : dileptons from coherent two-photon processes by use of equivalent photon approximation

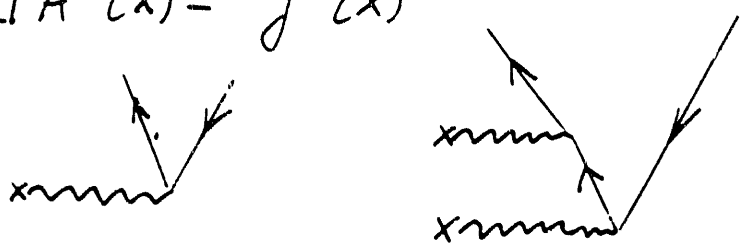
a) Differential probability for pair creation

$$S = T\exp\left(-i\int d^4x\, j_\mu(x) A^\mu(x)\right)$$

$j^\mu(x) = e\bar{\psi}(x)\gamma^\mu\psi(x)$ leptonic current density

$A^\mu(x)$ from classical current density $J^\mu(x)$ of nuclear charged matter

$$\Box A^\mu(x) = J^\mu(x)$$



Here: only first-order perturbation calculation

Second-order terms offer possibility of rather large pair masses without being far off-shell with photon lines; expected less strong off-shell suppression as in first-order calculation.

transition amplitude:

$$a(\vec{p}_+, s_+, \vec{p}_-, s_-) = -\frac{iem\,4\pi}{(2\pi)^3 q^2 (E_+E_-)^{1/2}} \bar{u}(\vec{p}_+, s_+)\gamma^\mu v(\vec{p}_-, s_-) J_\mu(q)$$

$q^\mu = p_+^\mu + p_-^\mu$ four momentum of pair

$q^2 = q_0^2 - \vec{p}^{\,2} = M^2$

$$J_\mu(q) = \int d^4x \, \exp(iq_\nu x^\nu) J_\mu(x)$$

free waves assumed for leptons

differential probability for pair creation

$$E_+E_- \frac{d^6W}{d^3p_+ d^3p_-} = E_+E_- \sum_{s_+ s_-} |a(\vec{p}_+, s_+, \vec{p}_-, s_-)|^2$$

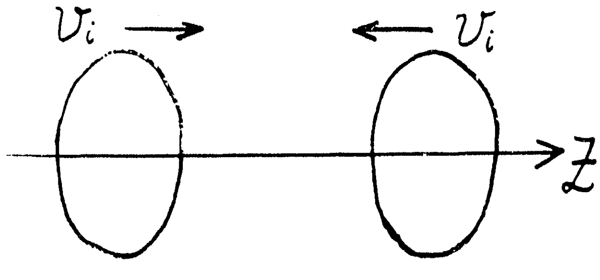
$$= \frac{\alpha}{4\pi^4 M^4} J_\mu^*(q)\, l^{\mu\nu} J_\nu(q)$$

leptonic tensor:

$$l^{\mu\nu} = p_+^\mu p_-^\nu + p_-^\mu p_+^\nu - \tfrac{1}{2} g^{\mu\nu} M^2$$

b) Current density $J^{\mu}(q)$

Assumption: central colliscons of rel. nuclei



only currents in z-direction considered

$$\longrightarrow J^{\mu}(q) = \left(1, \frac{q_0}{q_z}\vec{e}_z\right)\rho(q)$$

$\rho(q)$: Fourier-transformed charge density

$$\frac{d^6W}{d^3p_+ d^3p_-} = \frac{\alpha}{4\pi^4 q^4}\frac{|\rho(q)|^2}{E_+E_-}\Big\{2E_+E_- - \frac{1}{2}q^2$$

$$-\frac{2q_0}{q_z}\left(p_{-z}E_+ + p_{+z}E_-\right) + \frac{q_0^2}{q_z^2}\left(2p_{+z}p_{-z} + \frac{1}{2}q^2\right)\Big\}$$

total mass $M^2 = q^2 = q_0^2 - \vec{q}^{\,2}$

transverse mass of pair $M_t^2 = q_0^2 - q_z^2$

rapidity $\quad y = 0.5 \ln\left((q_0+q_z)/(q_0-q_z)\right)$

$\longrightarrow$ calculation of $\frac{d^2W}{dM^2 dy}$

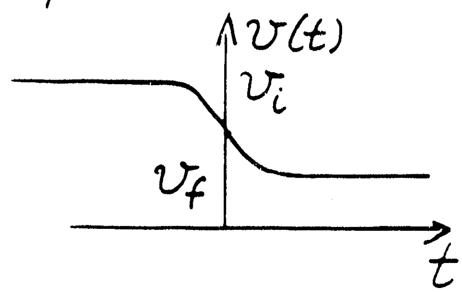
for cylindrically symmetric charge densities
$\rho(q) = \rho(q_0, q_z, q_t)$

$$\frac{d^2W}{dM^2 dy} = \frac{\alpha}{3(2\pi)^2} \frac{1}{M^2 \sinh^2 y} \left(1 - \frac{4m^2}{M^2}\right)^{1/2} \left(1 + \frac{2m^2}{M^2}\right)$$

$$\times \int_{M_t^2 \geq M^2} dM_t^2 \, |\rho(q_0, q_z, q_t)|^2$$

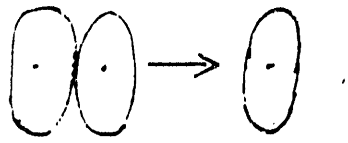
c) Collective model for stopping

Central collisions of equal nuclei;
parametrization of velocity of nuclear
matter-elements:



$$v(t) = \frac{v_i - v_f}{1 + \exp(\lambda t)} + v_f$$

v_i initial velocity, $v_f = \delta v_i$ final velocity
δ transparency parameter $0 \leq \delta \leq 1$
λ deceleration parameter $\sim$ inverse stopping time



$R = R_0 / \gamma_{cm}$ (γ_{cm} in center-of-momentum system)

$$\lambda = \frac{2v_i}{R} = \frac{2c}{R_0}\left(\gamma_{cm}^2 - 1\right)^{1/2}$$

α) point charges Ze

$$\rho(q_0, q_z) = Z\left(\rho_{pt}(q_0, q_z) + \rho_{pt}(q_0, -q_z)\right)$$

$$\rho_{pt}(q_0, q_z) = \frac{e}{\lambda} B\left(\frac{i}{\lambda}(q_0 - v_i q_z), -\frac{i}{\lambda}(q_0 - v_f q_z)\right)$$

$B(x,y) = \Gamma(x)\Gamma(y)/\Gamma(x+y)$ Euler's Beta-function

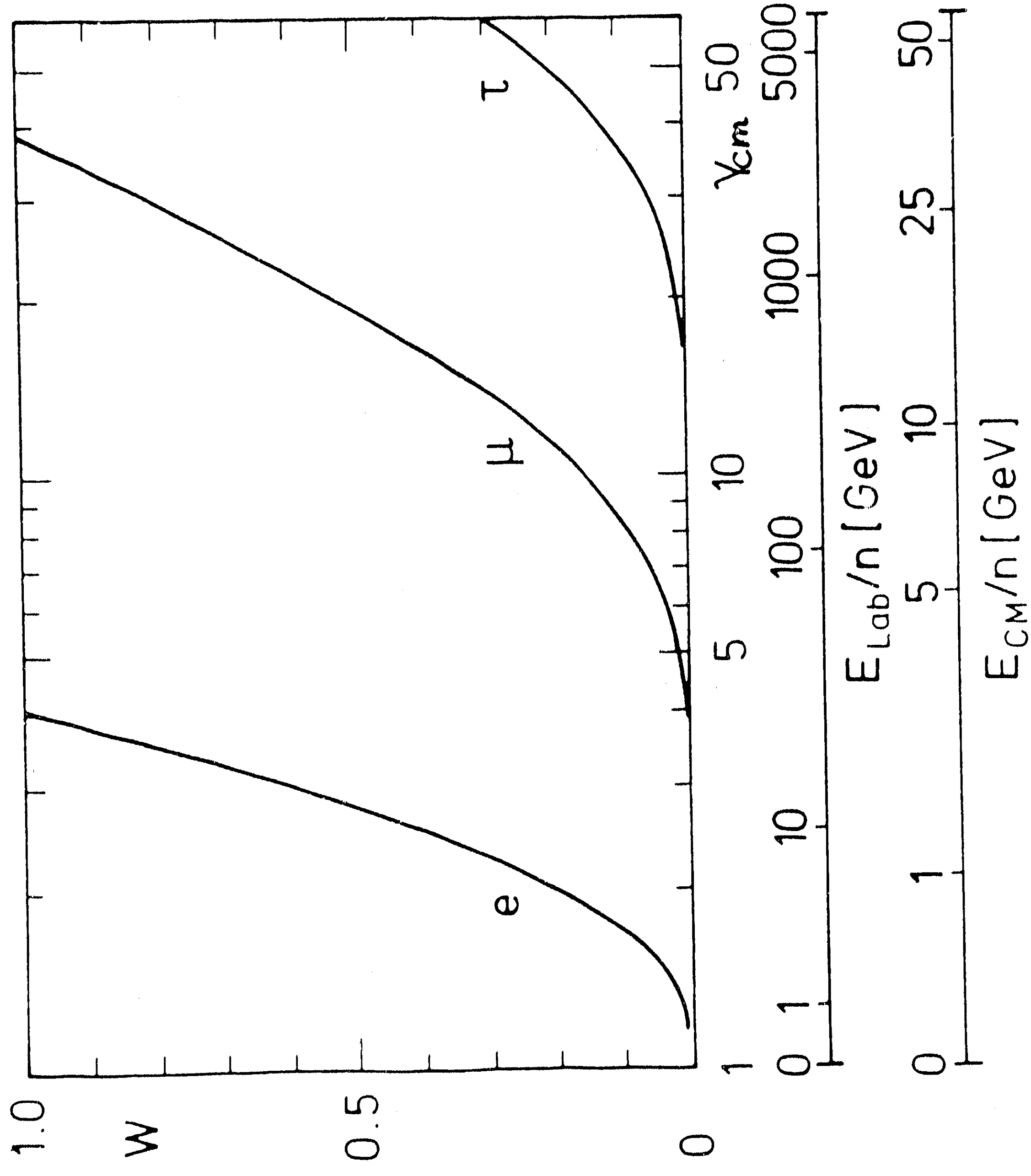
β) extended charge distribution

$$\rho(q_0, q_z) = Z\left(\rho_{pt}(q_0, q_z) + \rho_{pt}(q_0, -q_z)\right) f_\perp(q_t) f_\parallel(q_z)$$

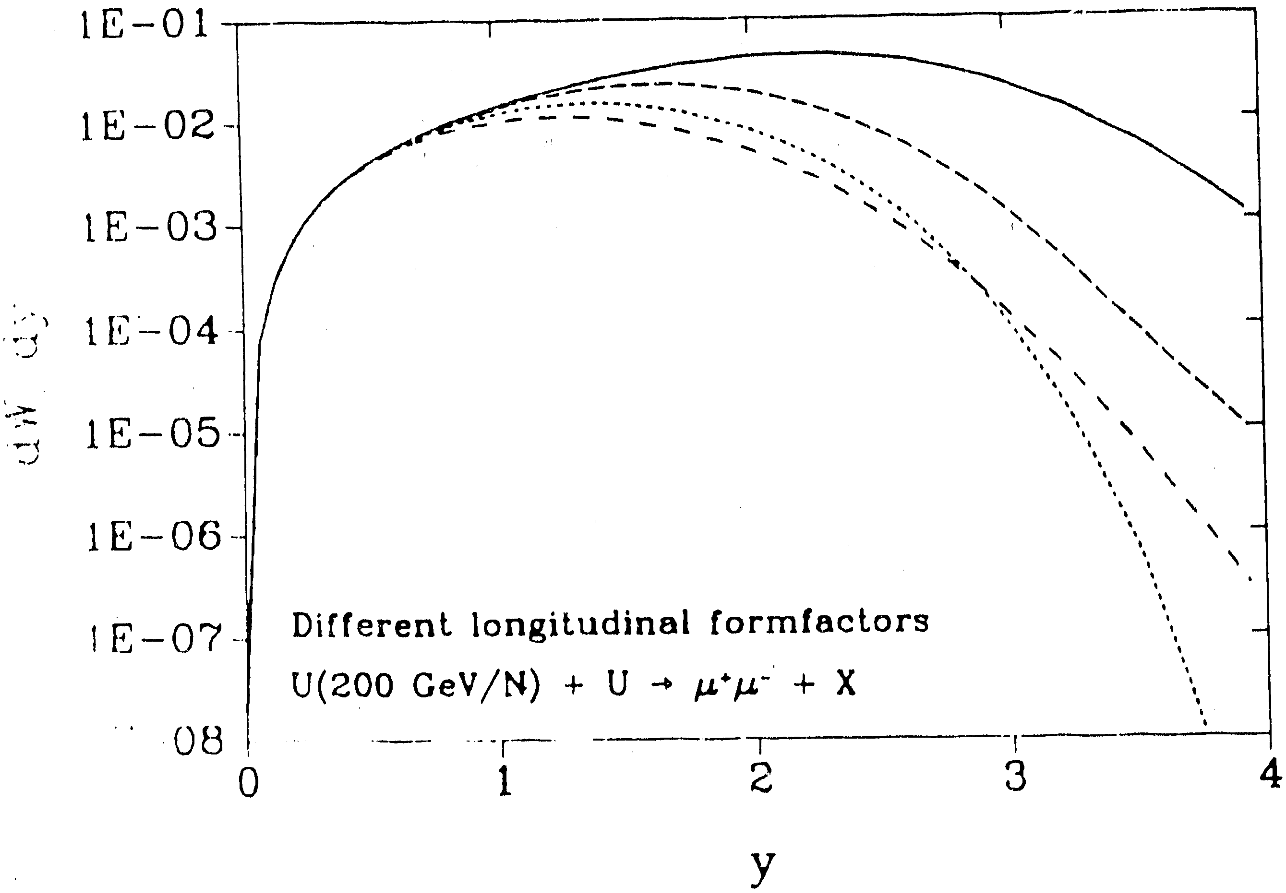
form factors for transverse and longitudinal direction;

$$f_\perp(q_t) = \frac{2}{R_0 q_t} J_1(R_0 q_t)$$

Results for $\mu^+\mu^-$-production in U+U collisions



W
1.0
0.5
0
e
μ
τ
γcm
1
5
10
50
0
1
10
100
1000
5000
E_Lab/n [GeV]
0
1
5
10
25
50
E_CM/n [GeV]



- - **dipole**
····· **gaussian**
---· **const. density**
— **point charge**

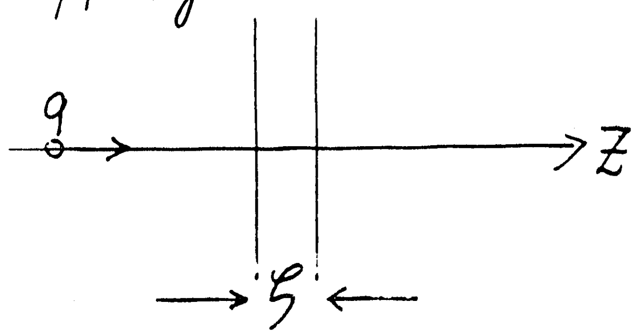
d) pn-collisions as source of bremsstrahlung

α) incoherent summation of contributions from pn-collisions:

$$|\rho(q)|^2 = Z\left(|\rho_{pt}(q_0,q_z)|^2 + |\rho_{pt}(q_0,-q_z)|^2\right)$$

cross section proportional to Z

β) stopping in a certain range ζ



$$\lambda = \frac{2v_i}{\zeta}(1+\delta) \qquad \zeta \approx 0 - 1\,fm$$

γ) kinematical correction and proton formfactor

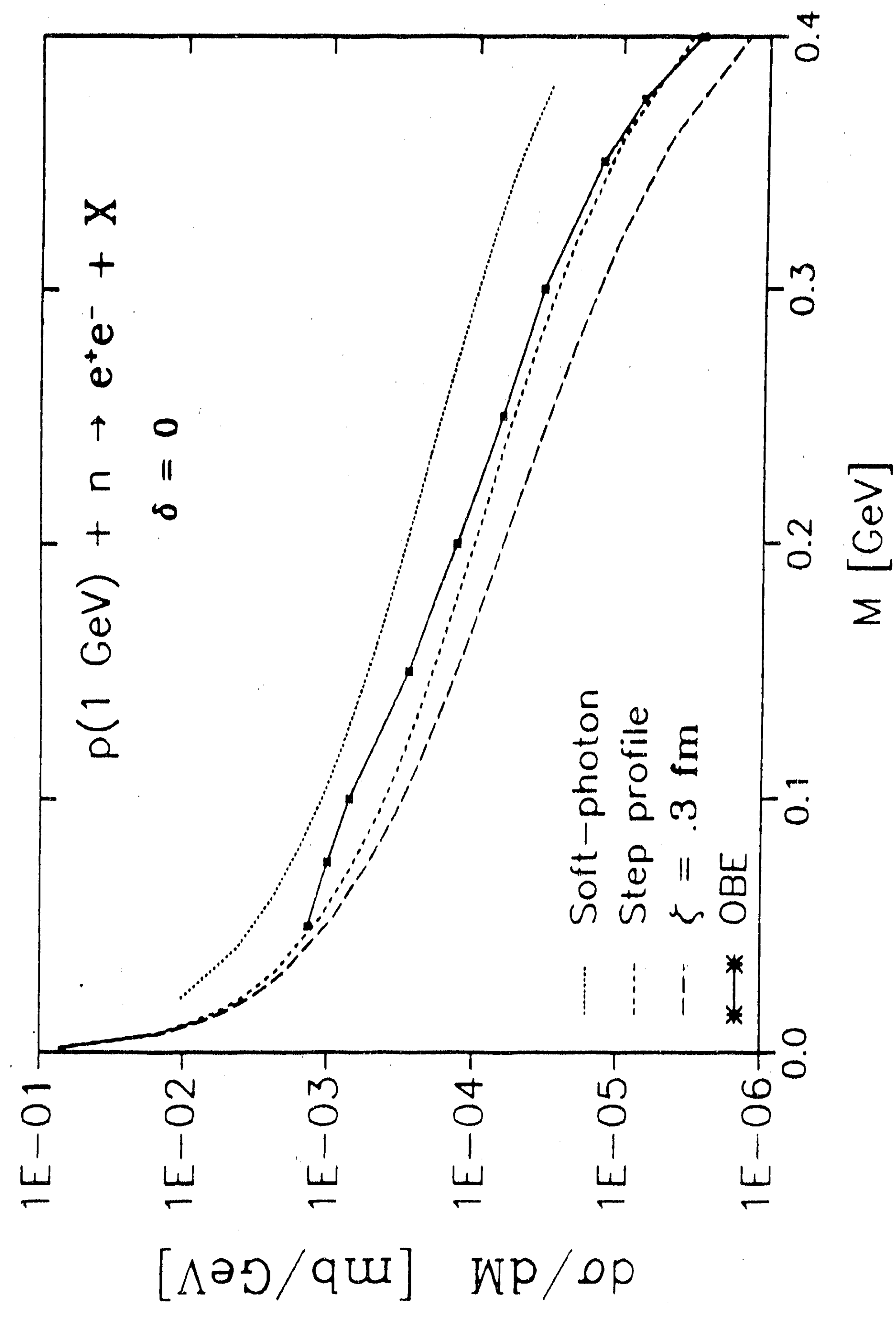
$$|\rho(q)|^2 \longrightarrow |\rho(q)|^2\, K(q_0)\left(F_p(q^2)\right)^2$$

Pair energy is limited: $\bar{q}_0 = 2M_N(\gamma_{cm}-1)$ in NN-CM frame $\longrightarrow K(q_0) = \left(1+\exp\left(\beta\left(\frac{q_0}{\bar{q}_0}-1\right)\right)\right)^{-1}$

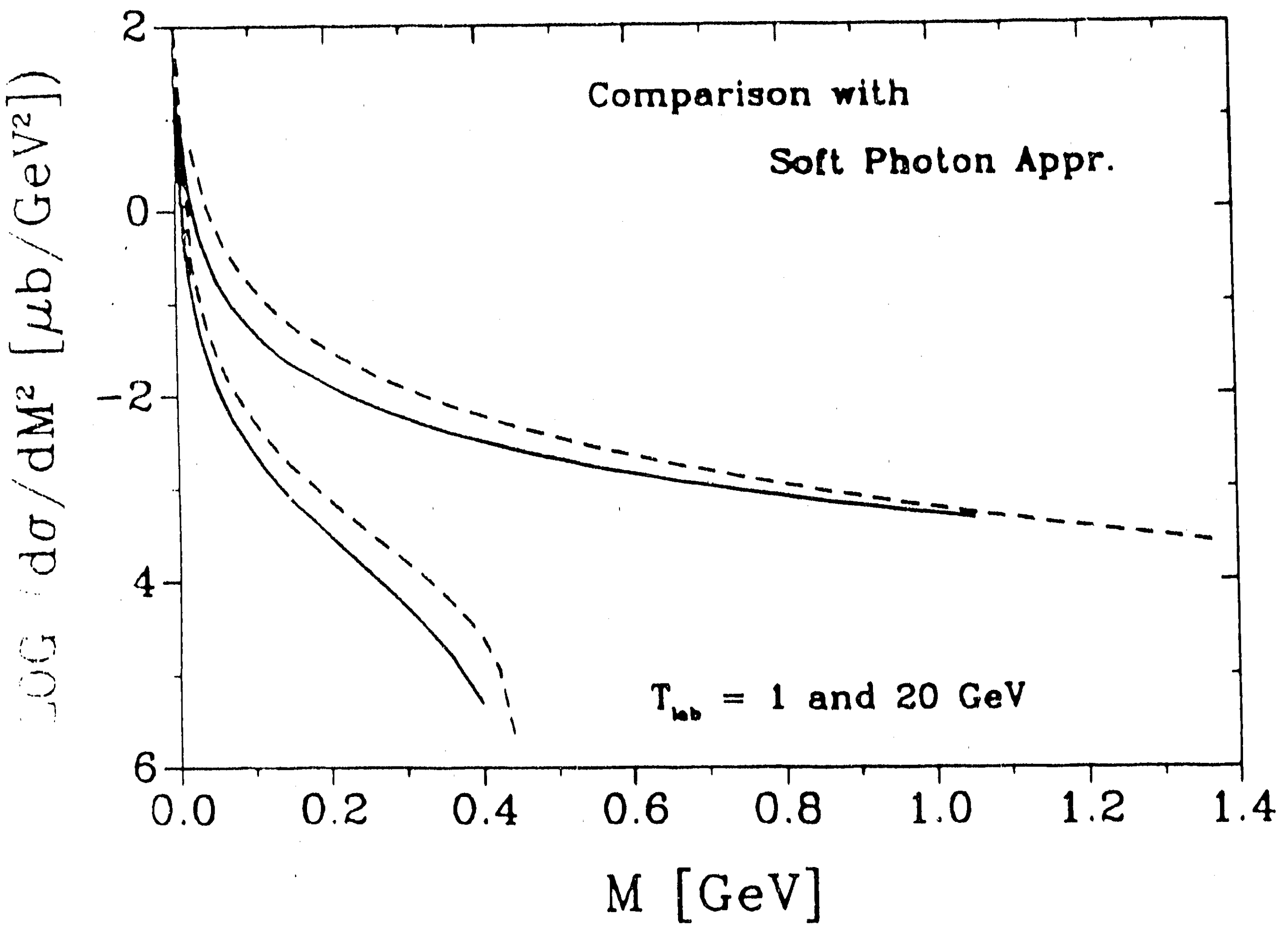
Proton formfactor $F_p(q^2) = 1/(1+q^2/\Lambda^2)^2$; $\Lambda^2 = 0.71\,GeV^2$

Results for pn-collision at $E_{lab} = 1\,GeV$;
Ca(2GeV/n) + Ca in comparison with
Bevalac data.

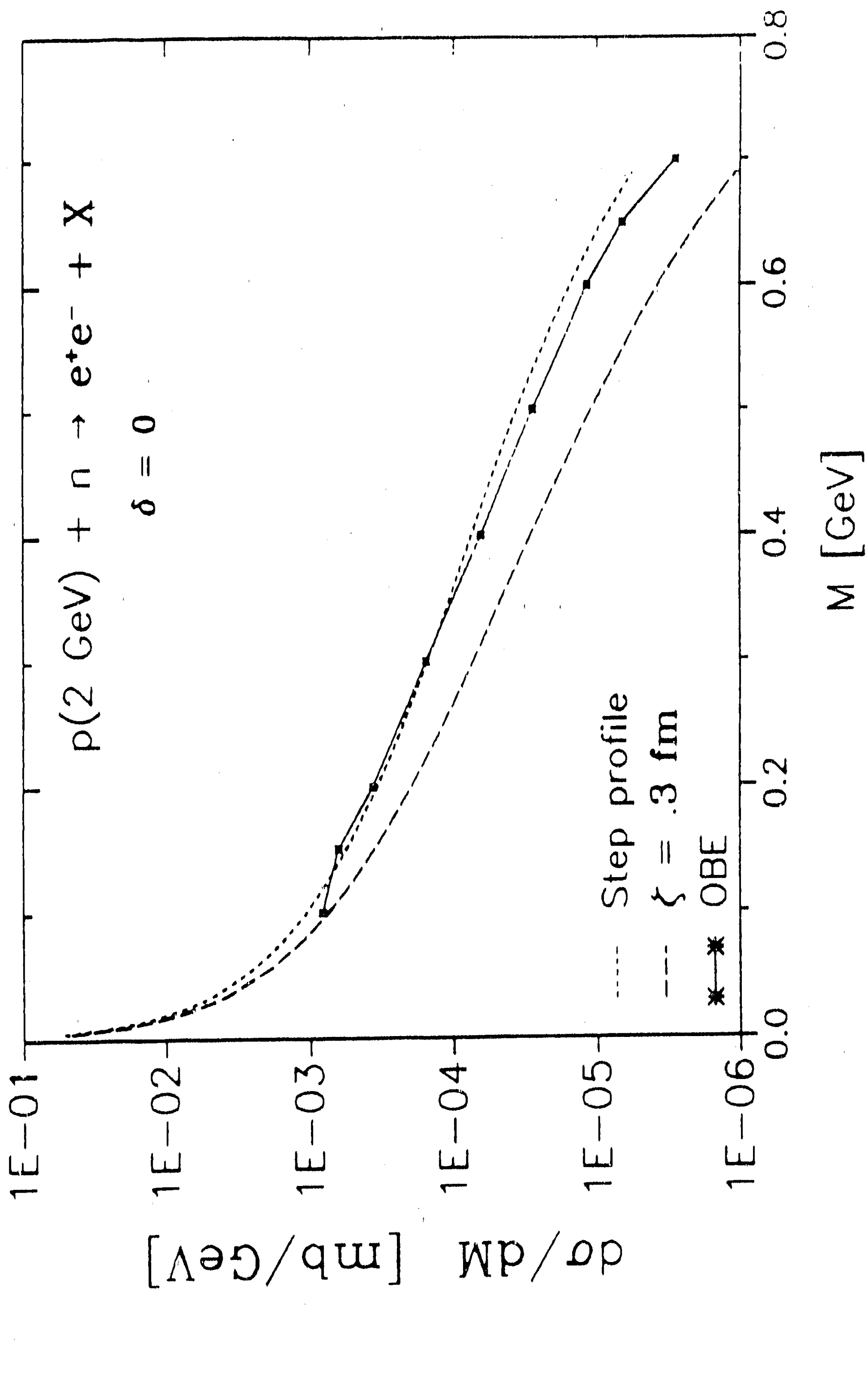
Similar, but more time-consuming calcula-
tions were done with BUU-codes:
e.g. Mosel, Blättel 1989/90

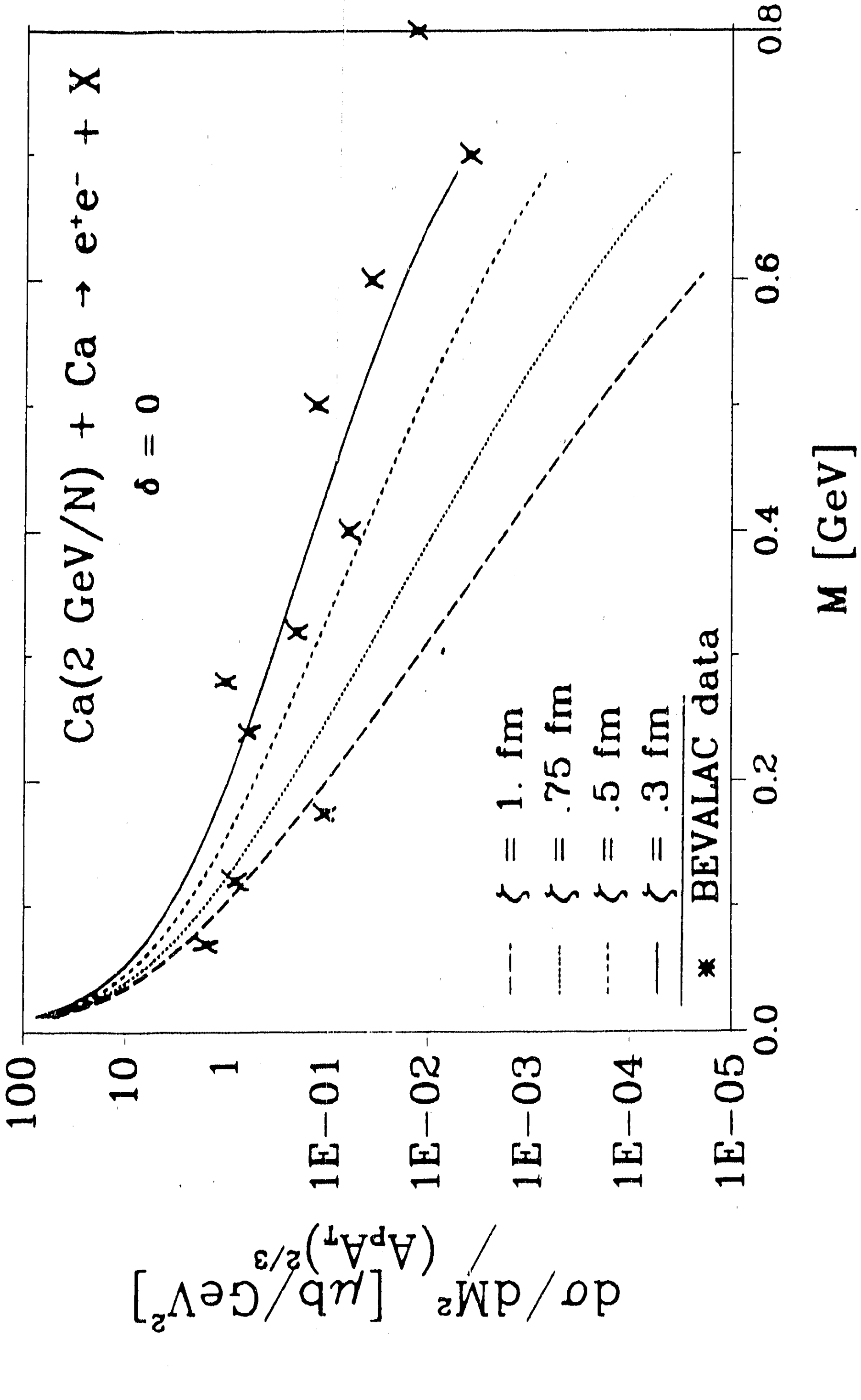


p(1 GeV) + n → e⁺e⁻ + X
δ = 0
Soft−photon
Step profile
ζ = .3 fm
OBE
dσ/dM [mb/GeV]
M [GeV]
1E−01
1E−02
1E−03
1E−04
1E−05
1E−06
0.0
0.1
0.2
0.3
0.4

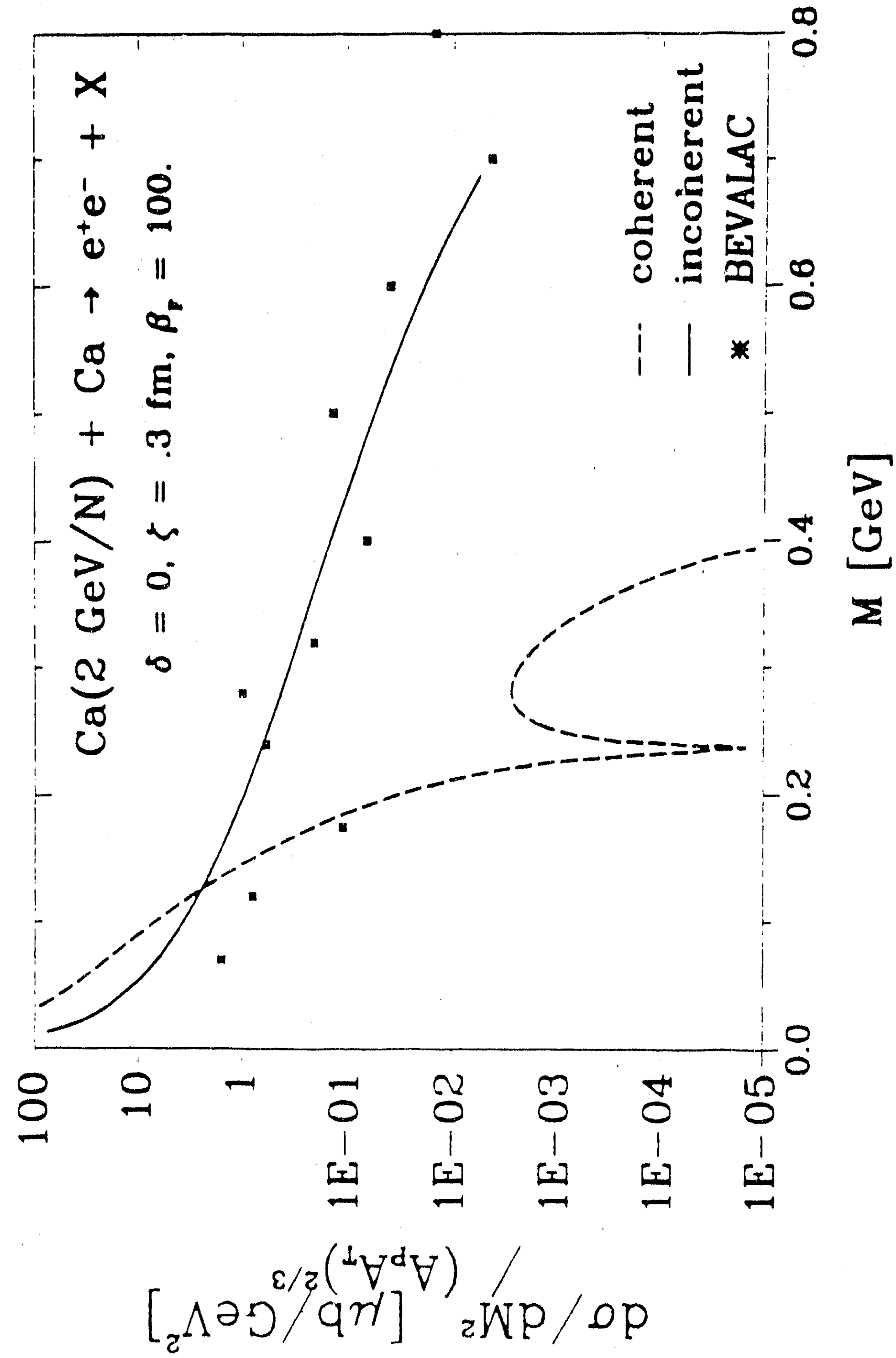


-- **Soft Photon**
— **Bremsstrahlung**

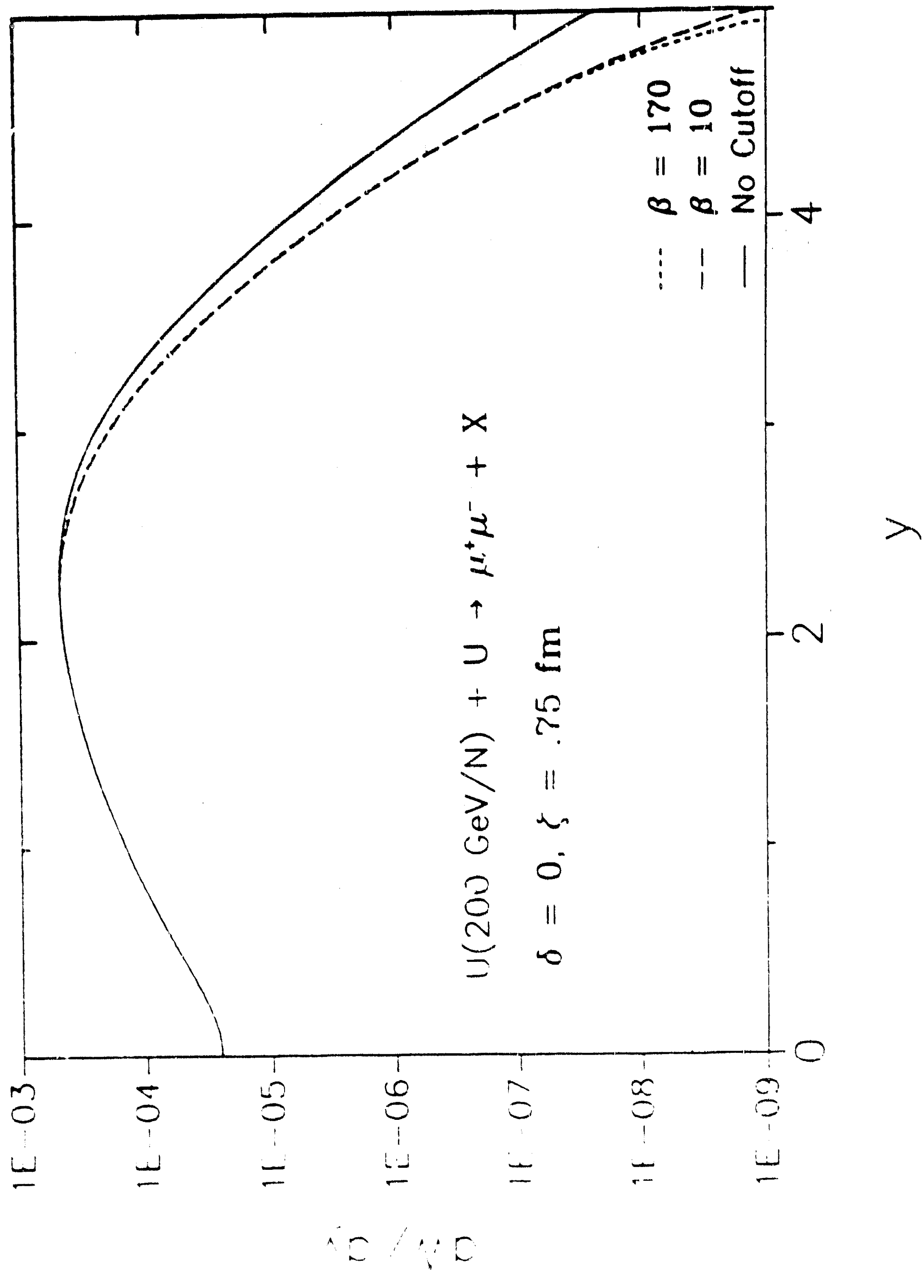

p(2 GeV) + n → e⁺e⁻ + X
δ = 0
Step profile
ζ = .3 fm
OBE
dσ/dM [mb/GeV]
M [GeV]
1E−01
1E−02
1E−03
1E−04
1E−05
1E−06
0.0
0.2
0.4
0.6
0.8



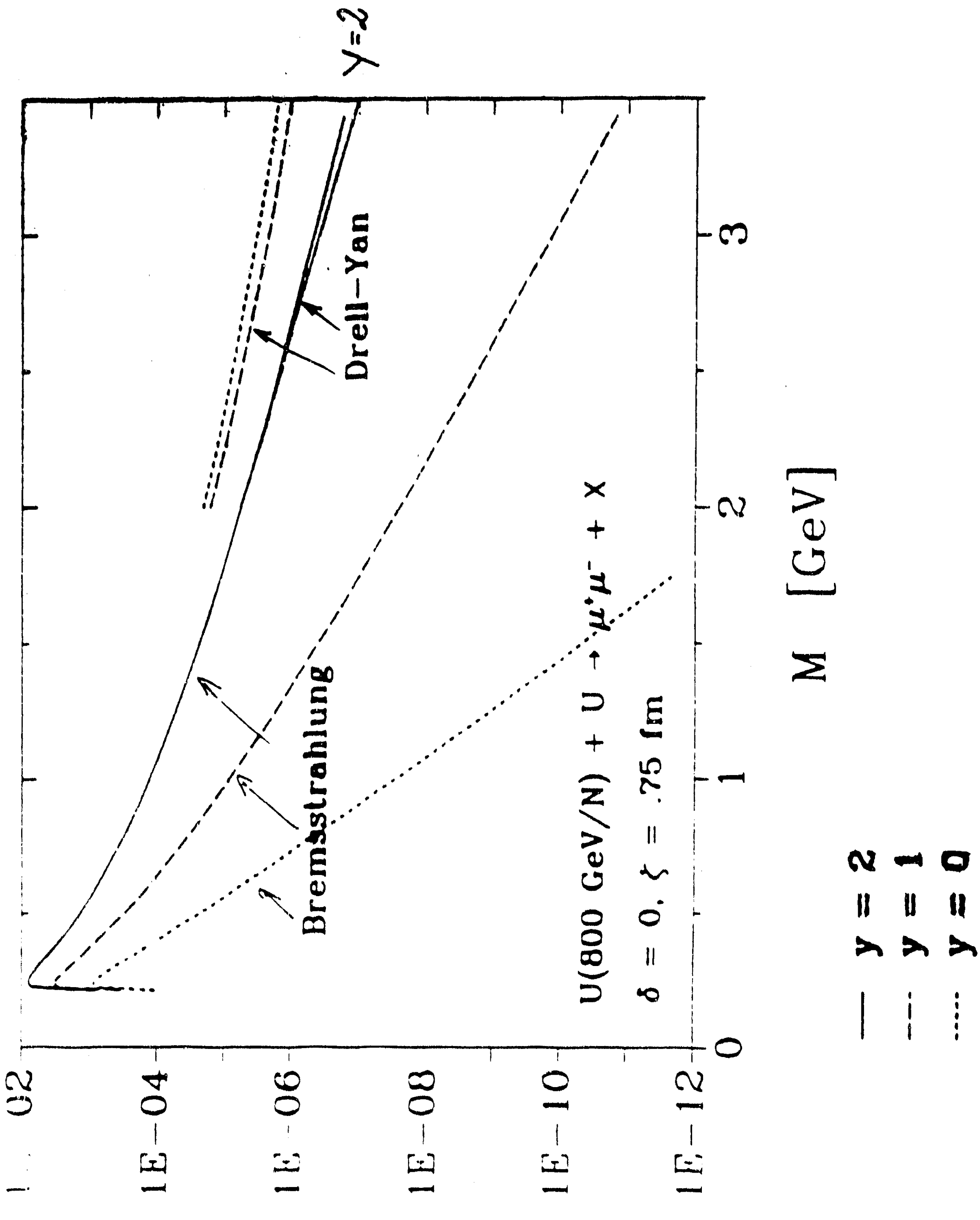
Ca(2 GeV/N) + Ca → e⁺e⁻ + X
δ = 0
ζ = 1. fm
ζ = .75 fm
ζ = .5 fm
ζ = .3 fm
BEVALAC data
M [GeV]
dσ/dM² / (A_P A_T)^{2/3} [μb/GeV²]
100
10
1
1E−01
1E−02
1E−03
1E−04
1E−05
0.0
0.2
0.4
0.6
0.8



Ca(2 GeV/N) + Ca → e+e- + X
δ = 0, ζ = .3 fm, βr = 100.
coherent
incoherent
BEVALAC
M [GeV]
dσ/dM² / (ApAT)^2/3 [μb/GeV²]
100
10
1
1E-01
1E-02
1E-03
1E-04
1E-05
0.0
0.2
0.4
0.6
0.8



U(200 GeV/N) + U → $\mu^+\mu^-$ + X
δ = 0, ζ = .75 fm
β = 170
β = 10
No Cutoff
0
2
4
y
1E-03
1E-04
1E-05
1E-06
1E-07
1E-08
1E-09
dσ/dy



Y=2
Drell–Yan
Bremsstrahlung
U(800 GeV/N) + U → μ⁺μ⁻ + X
δ = 0, ζ = .75 fm
M [GeV]
0
1
2
3
1E−04
1E−06
1E−08
1E−10
1E−12
y = 2
y = 1
y = 0

5. Summary and conclusions

a) Various approximations in first-order perturbation theory

Multipole expansion with Coulomb-Dirac wave functions

Angular distributions for K-shell capture, comparison of Coulomb-Dirac and Sommerfeld-Maue function

Angular distributions with distorted wave model, functions including field of target and projectile

Muon capture by taking finite extension of nuclear charge into account, cross section depressed by 5 orders of magnitude

→ Consequences for the production of elementary particles heavier than electrons.

b) Coupled channel calculations:

probability for electron-positron pairs in U+U-collision larger by 1-2 orders of magnitude, probability for K-shell capture larger by 2-3 orders of magnitude; results are very preliminary; no dependence on impact parameter up to now.

Strong interaction between continuum states.

c) Lepton production in central nucleus-nucleus collisions:

coherent or incoherent process?

leptons treated as free particles; influence of time-dependent charge distribution.

Second-order perturbation theory allows pairs with high pair masses; not yet considered.

NON–PERTURBATIVE ELECTRON PAIR PRODUCTION IN RELATIVISTIC HEAVY–ION COLLISIONS

Talk presented by:

C. Bottcher

Non Perturbative electron Pair Production in Relativistic Heavy-Ion Collisions

C. Bottcher M.R. Strayer & J.S. Wu

A.K. Kerman *MIT*

M.J. Rhoades–Brown *BNL*

" Physics of strong Fields" ed. W. Greiner Vol 153 (1987)

Proceedings of the Second Workshop On Experiments & Detectors for the RHIC (1987)

Nucl. Inst & Meth. B31 (1988) ; Phys. Rev. D39 (1989)

Phys. Letts. 237B (1990) ; J. Phys. G (1990)

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Production of Electrons by Point Nuclei

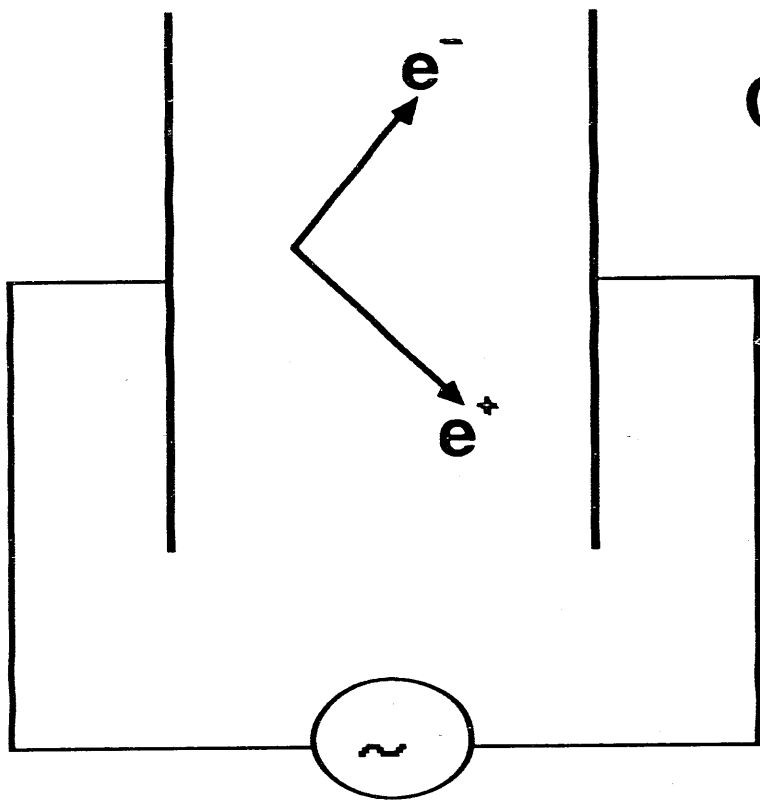
- Physical pictures
- Structure of perturbation theory
- Equivalent Photon Approximation
- • Quasi 1D (Light Cone) Reduction
- Possible Experiments & future directions

Physical Pictures of Pair Production

- **Schwinger model**
- **Two photon diagram**

Non Perturbative Pair Production

Schwinger ("box") Model



$$\sigma = \int_{\text{volume}} R(\mathcal{E}, \Delta t)\, d\tau$$

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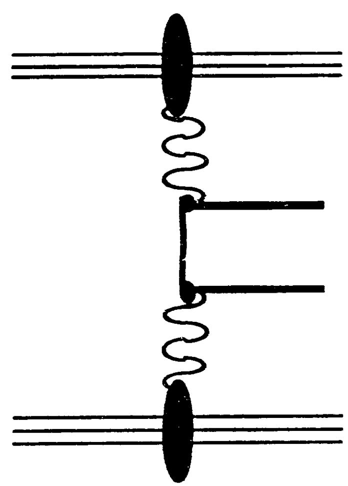
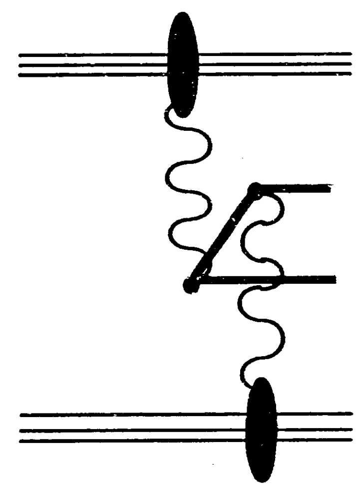
Two photon diagrams



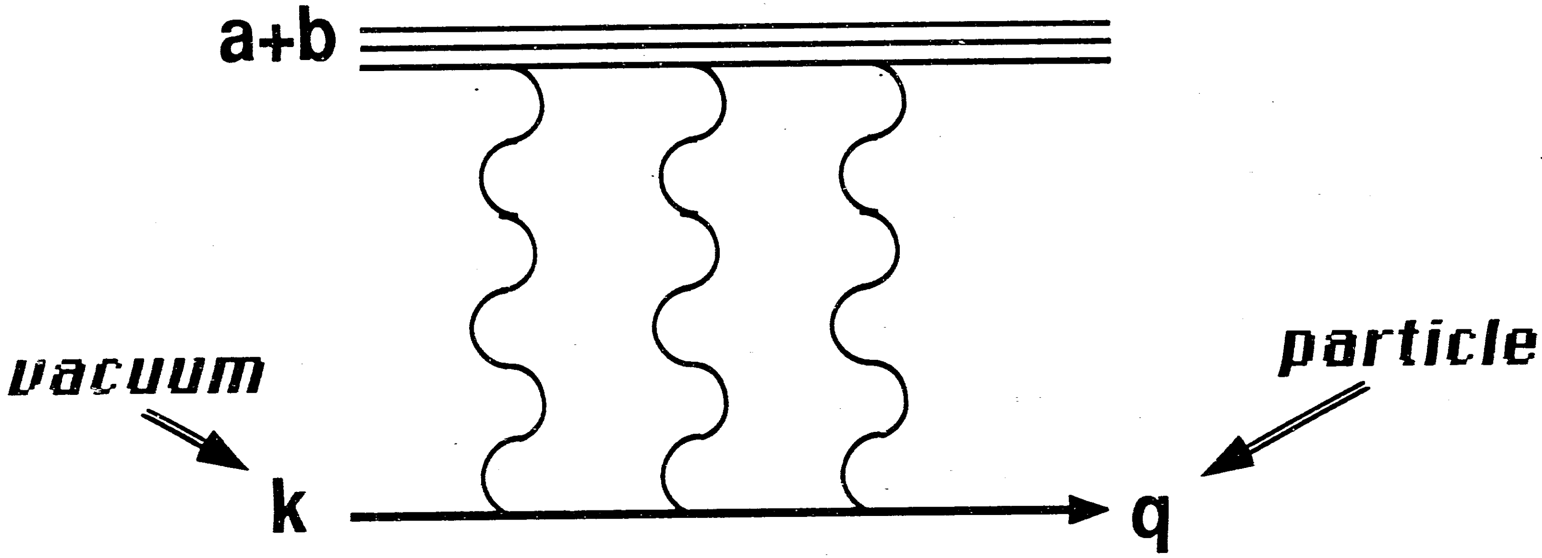
direct
a



exchange
b

Non Perturbative Pair Production

Propagation of Vacuum States:



Single particle Dirac equation...

$$i\frac{\partial \Psi_k}{\partial t} = [\mathcal{H}_0 + \mathcal{V}_a + \mathcal{V}_b]\Psi_k$$

Non Perturbat.ve Pair Production

Two photon diagrams :

e.g. direct amplitude

$$S_{kq}^{direct} = \bar{u}_k \not{A}_a \frac{1}{\not{p}+m} \not{A}_b v_q$$

cross section for pair production

$$\sigma_{pair} = \sum_{k,q} \int d^2b \, |S_{kq}^{dir} + S_{kq}^{exc}|^2$$

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Non Perturbative Pair Production

Semi-classical two-photon expression:

Coulomb interaction $A_a \sim \frac{Z_a}{q^2}\,(1-\beta\alpha_z)$

leads to <u>exact</u> two-photon cross section

$$\sigma_{pair} = \int d^3k\, d^3q\, d^2p\; F_a F_b |\mathcal{T}_{kq}|^2$$

and approximate equiv. phot. expressions

$$\sigma_{equiv} = \int dk_z\, dq_z\, d^2K\, \Phi_a \Phi_b \sigma_{\gamma\gamma}(\omega_a, \omega_b; K)$$

General features of pair production amplitudes:

- **Longitudinal momenta (strong var.)**

$$k_z\ ,\ q_z \leq \gamma m_\perp$$

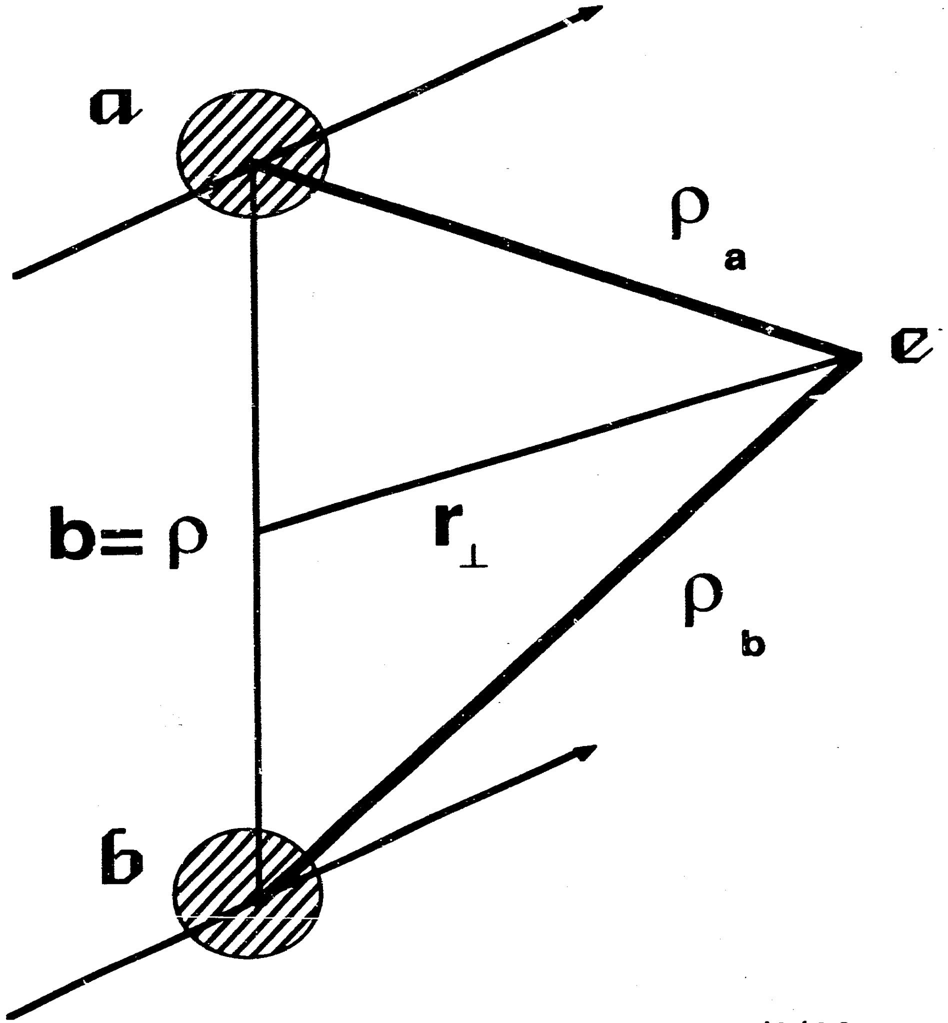
- **Transverse momenta (weak var.)**

$$\mathbf{k}_\perp\ ,\ \mathbf{q}_\perp \sim m$$

- **Longitudinal and transverse momenta weakly coupled**

Non Perturbative Pair Production

Coordinates for heavy ion collision



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... suggests quasi-1D reduction:

In the exact expression ....

$$S_{kq} = \int dt\,dt'\exp(-iE_k t - iE_q t') \times$$

$$<\chi_k|\mathcal{V}_a(\rho_a;zt)\exp[-i(t-t')\mathcal{H}_0]\,\mathcal{V}_b(\rho_b;zt')|\chi_q>$$

... replace ... $\chi_k \longrightarrow u(K_\perp;k_z s_k)\exp(ik_\perp \bullet r_\perp)$

$$\mathcal{H}_0(-i\nabla_\perp) \longrightarrow \mathcal{H}_0(K_\perp)$$

where $K = \frac{1}{2}(k-q)\,,\quad Q = k+q$

$$\rho_{a,b} = \frac{1}{2}(\rho \pm r_\perp)$$

... leading to the reduced amplitude

$$S_{kq} \approx \int d^2 r_\perp \exp(iQ_\perp \cdot r_\perp)\, \mathcal{S}(\rho_a \rho_b K | k_z s_k q_z s_q)$$

... where the effective 1D amplitude

$$\mathcal{S}_{kq}\,(\rho_a \rho_b K) = \int dt\,dt' \exp(-iE_k t - iE_q t')\ X$$

$$\langle u_k | \mathcal{V}_a(\rho_a; zt) \exp[-i(t-t')\mathcal{H}_0]\, \mathcal{V}_b(\rho_b; zt') | v_q \rangle$$

is derivable from the 1D Dirac equation

$$i\frac{\partial \Psi}{\partial t} = [\mathcal{H}_0(K) + \mathcal{V}(r_\perp)]\Psi$$

Light Cone variables

Write Dirac Equation as...

$$i\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}\right)\Psi=$$

$$[(\gamma_0+\alpha\cdot\mathbf{p}_\perp)-(1-\beta\alpha_z)\mathcal{V}_a-(1+\beta\alpha_z)\mathcal{V}_b]\Psi$$

Define... $\tau_\pm=\frac{1}{2}(t\pm z)$, $G_\pm=(1\pm\alpha_z)\Psi$

whence

$$i\frac{\partial G_+}{\partial \tau_+}=W_+G_++h_0G_-$$

$$i\frac{\partial G_-}{\partial \tau_-}=h_0G_++W_-G_+$$

... transforming potentials into phases

$$i\frac{\partial g_+}{\partial \tau_+} = hg_- \ , \ i\frac{\partial g_-}{\partial \tau_-} = hg_+$$

where $h_0 = 1 - i\omega \cdot p_\perp \ , \ \ \omega = z \times \sigma$

$$h = 1 - i\omega \cdot (p_\perp - A_\perp)$$

Interaction --> $A_\perp$ ... essentially vec. pot. in radiation gauge:

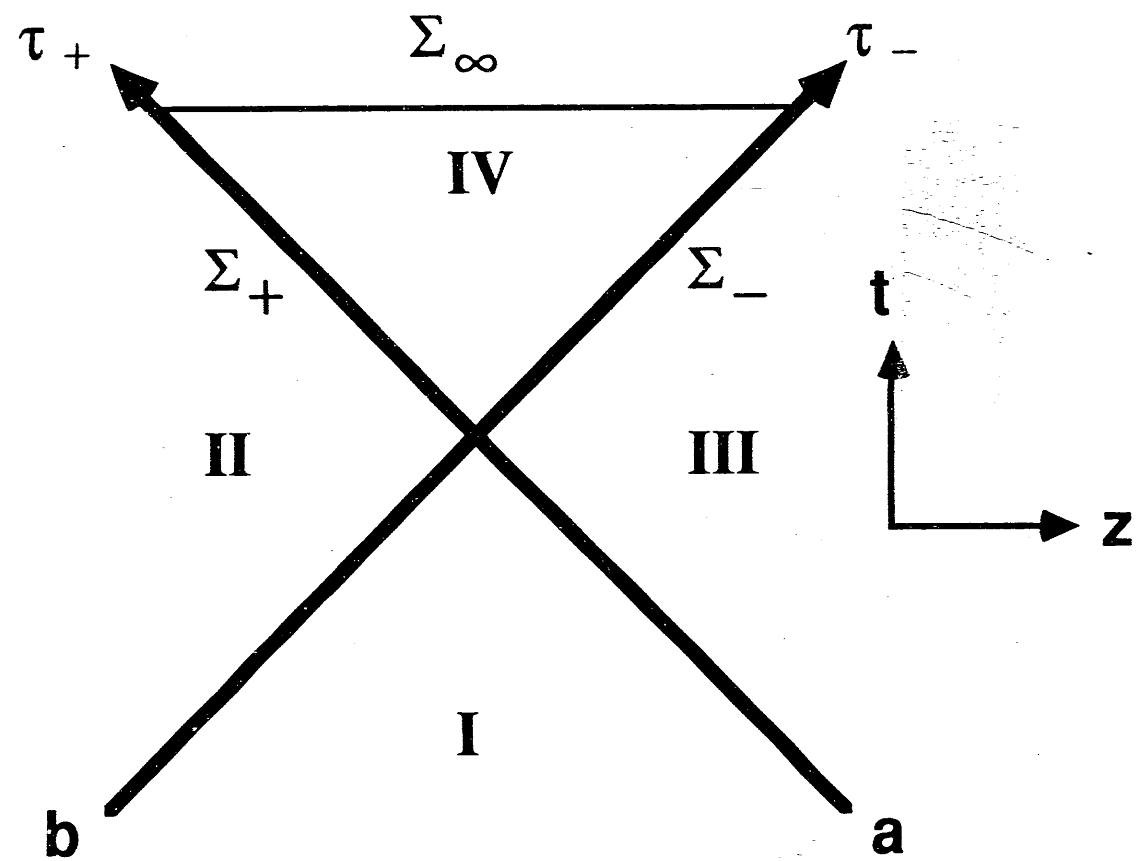
$$A_\perp = \Delta_a \theta\left(\frac{2\gamma\tau_-}{\rho_a}\right) + \Delta_b \theta\left(\frac{2\gamma\tau_+}{\rho_b}\right)$$

$$\theta(x) = \frac{x}{\sqrt{1+x^2}}$$

$$\Delta_a = \frac{C_a \vec{\rho}_a}{\rho_a^2} \ , \ \ \Delta_b = \frac{C_b \vec{\rho}_b}{\rho_b^2}$$

Non Perturbative Pair Production

Geometry of heavy ion collision in light cone picture



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Ansatz to freeze out transverse momentum: $\Psi(\vec{r};t) = \exp(i\vec{K}\cdot\vec{r})\Phi(\vec{K},\vec{r};z,t)$

... leads to interaction matrix (2 X 2)

$$h = \begin{pmatrix} 1 & P_x - iP_y \\ P_x + iP_y & 1 \end{pmatrix} \qquad \vec{P} = \vec{p}_\perp - \vec{A}_\perp \,, \quad \vec{p}_\perp \longrightarrow \vec{K}$$

Note: overall errors are $O\left(\frac{1}{\gamma^2}\right)$

Free Particle Solutions:

$$\varphi_q = \exp(-iK_+\tau_+ - iK_-\tau_-)\,[w\,,\,w^{-1}] \qquad (K_\pm = E_q \pm q_z)$$

w - a 2X2 matrix -
depends upon $p_\perp$ (transverse mom.)

Conservation of current:

$$J_\pm = F_\pm^\dagger G_\pm \longrightarrow \frac{\partial J_+}{\partial \tau_+} + \frac{\partial J_-}{\partial \tau_-} = 0$$

$$\int_\Sigma ds(J_+\nu_+ + J_-\nu_-) = 0$$

Sudden Approximation:

... replace

$$\theta\left(\frac{2\gamma\tau_-}{\rho_a}\right) \longrightarrow \textit{sign}\,(\tau_-)\,,\quad \theta\left(\frac{2\gamma\tau_+}{\rho_b}\right) \longrightarrow \textit{sign}\,(\tau_+)$$

... so that

$$\vec{P} \cong \vec{p}_\perp - \vec{\Delta}_a\,\textit{sign}\,(\tau_-) - \vec{\Delta}_b\,\textit{sign}\,(\tau_+)$$

depending upon which quadrant

Joining conditions: **(follow from diff. eqns.)**

$$\Delta[G_+] = 0 \subset \tau_+\,,\quad \Delta[G_-] = 0 \subset \tau_-$$

Joining to intermediate-time solutions:

$$\psi_{II} = \exp(-iK_{I+}\tau_+ - iK_{II-}\tau_-)\,[w_{II}^2 w_I^{-1}\ ,\ w_I^{-1}]$$

[–]

$$\psi_{I} = \exp(-iK_{I+}\tau_+ - iK_{I-}\tau_-)\,[w_I\ ,\ w_I^{-1}]$$

[+]

$$\psi_{III} = \exp(-iK_{III+}\tau_+ - iK_{I-}\tau_-)\,[w_I\ ,\ w_{III}^{-2} w_I]$$

Note: spinors tilted - no pair production

Expand final-time solution in plane waves:

$$\psi_{IV} = \sum_{\sigma_q} \int dq_z \, S_{kq} \varphi_q$$

$$\varphi_q = \mathcal{N}_q \exp(-iK_{IV+}\tau_+ - iK_{IV-}\tau_-) \, [w_{IV} \, , \, w_{IV}^{-1}]$$

Integrate current around contour Σ :

$$\frac{1}{2}\int_{-\infty}^{+\infty} dz (F_+^\dagger G_+ + F_-^\dagger G_-) = \int_0^\infty d\tau_+ F_-^\dagger G_- + \int_0^\infty d\tau_- F_+^\dagger G_+$$

... where $\varphi_{IV} = [F_+ \, , \, F_-]$ $\psi_{II,III} = [G_+ \, , \, G_-]$

Sudden approximation S matrix:

$$S_{kq} = \mathcal{N}\left\{ \frac{K_{IV-}}{K_{IV-}-K_{II-}} w_{IV}^{-1} w_{II}^{2} w_{I}^{-1} - \frac{K_{IV+}}{K_{IV+}-K_{III+}} w_{IV} w_{III}^{-2} w_{I} \right\}$$

Asymptotic behaviour:

(1) $\frac{d\sigma}{dp_\perp} \sim m_\perp^{-4}$ i.e. convergent

Asymptotic behaviour:

(2) $\frac{d\sigma}{d\kappa} \sim \kappa^{-2} \qquad \kappa = \sqrt{k^2 + q^2}$

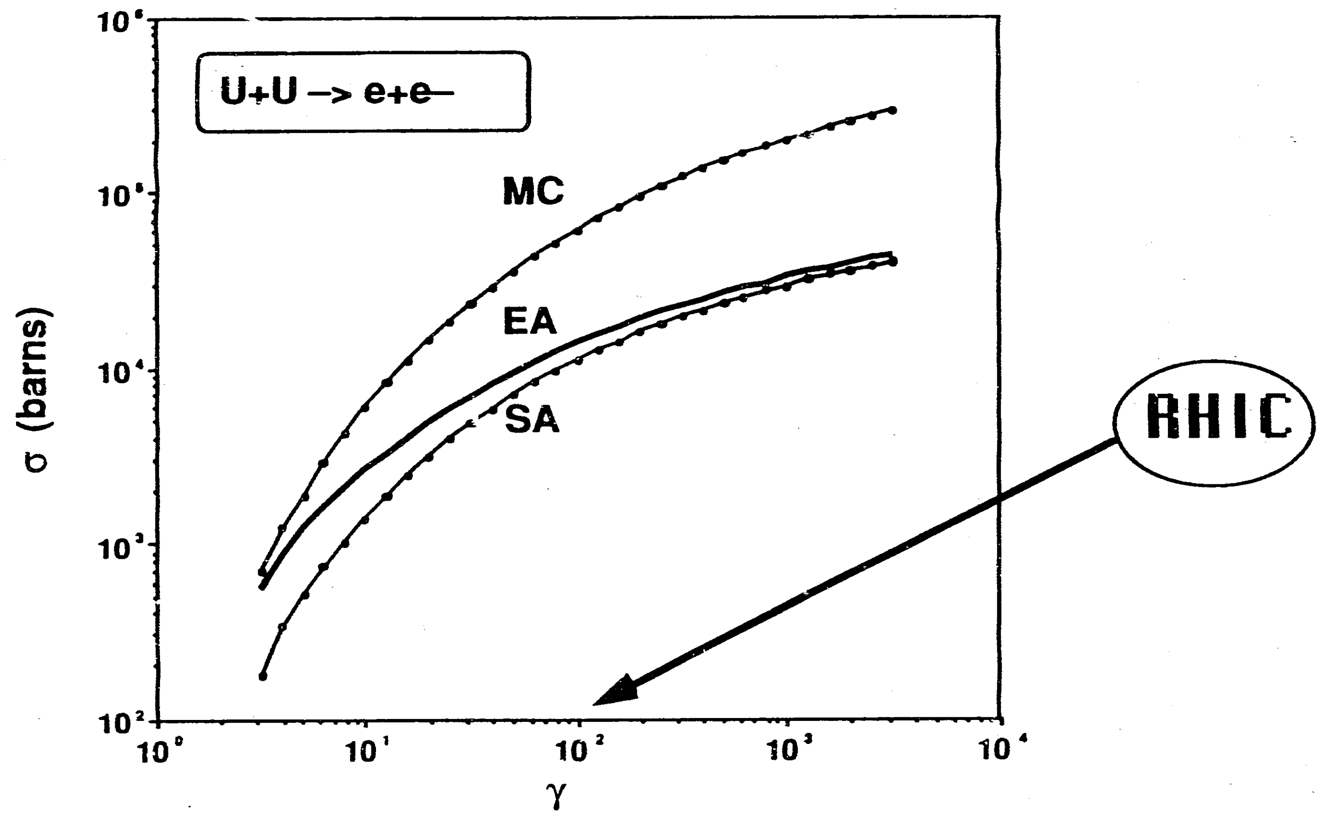
i.e. log divergent – ion velocity <c converts to $\kappa^{-4} \qquad \kappa \geq \gamma$

(3) $\frac{d\sigma}{d\Delta_a} \sim \Delta_a^2 \sim C_a^2 \rho_a^{-2}$

i.e. log divergent – finite step converts to $K_0\left(\frac{\omega\rho}{\gamma}\right) \qquad \rho \geq \gamma/\kappa$

Non Perturbative Pair Production

Total Pair Cross Sections



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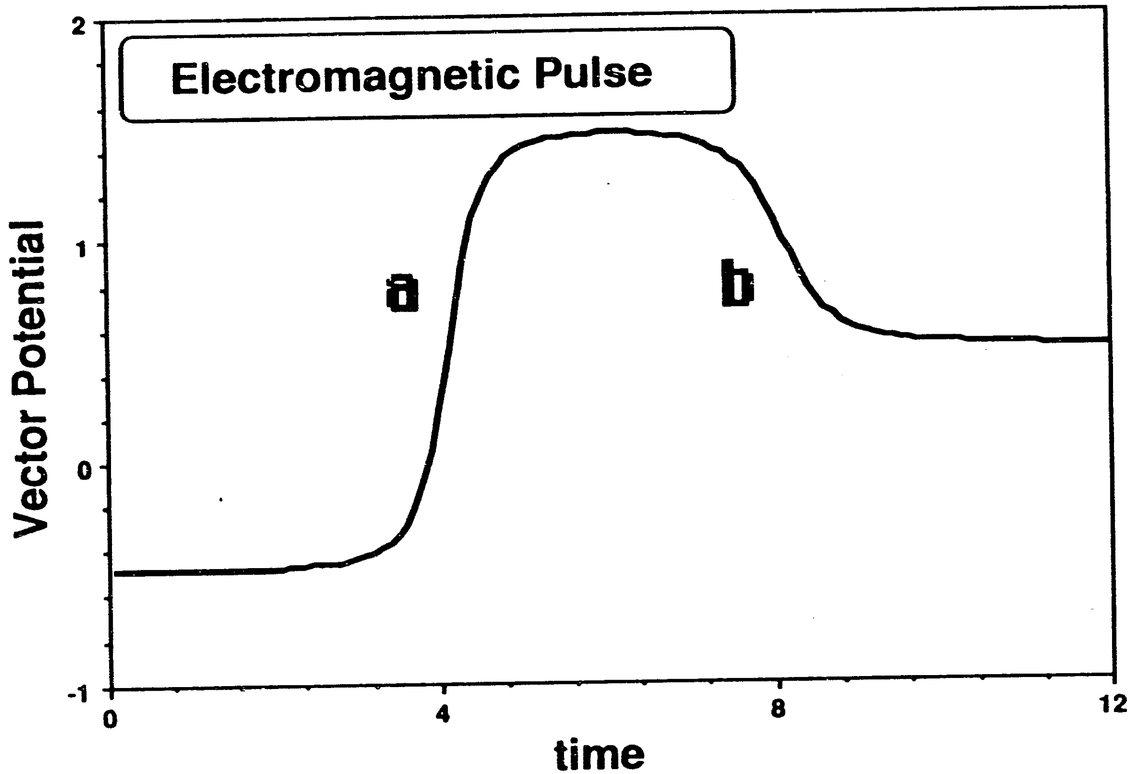
Non Perturbative Pair Production



"Box" model: $S_{kq} = \Omega_a G_{II} \Omega_b - \Omega_b G_{III} \Omega_a$

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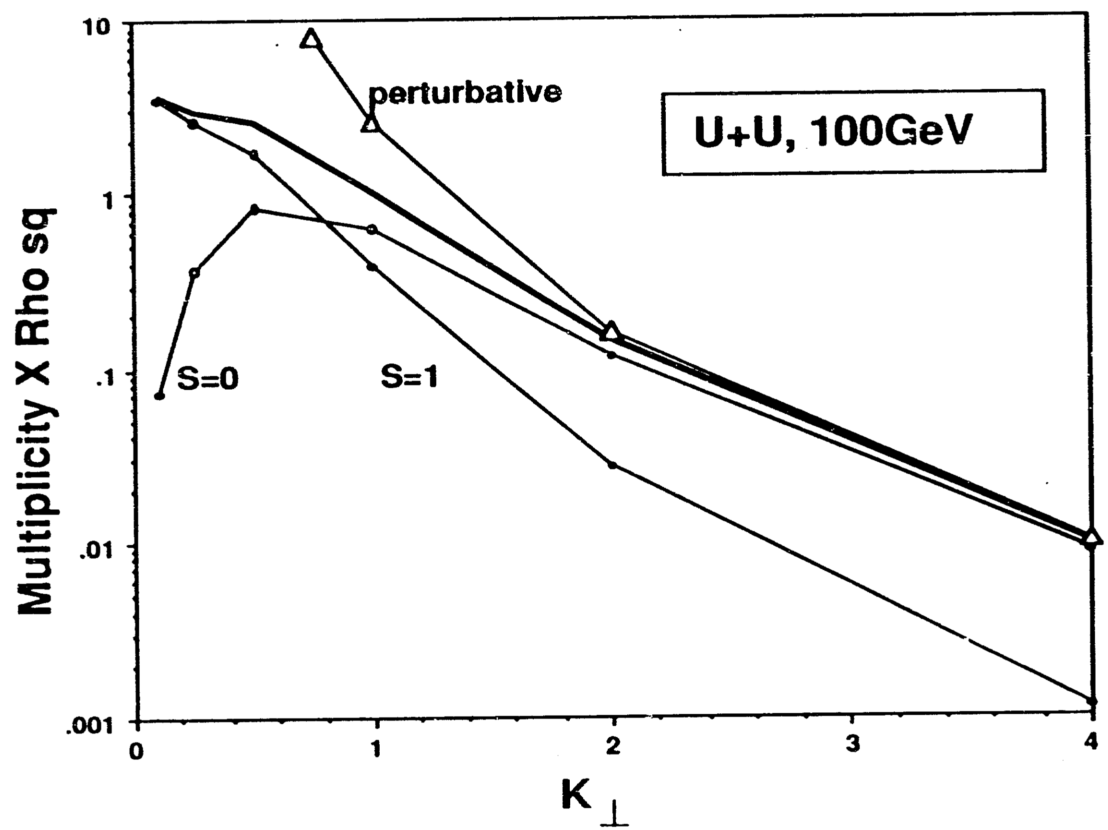
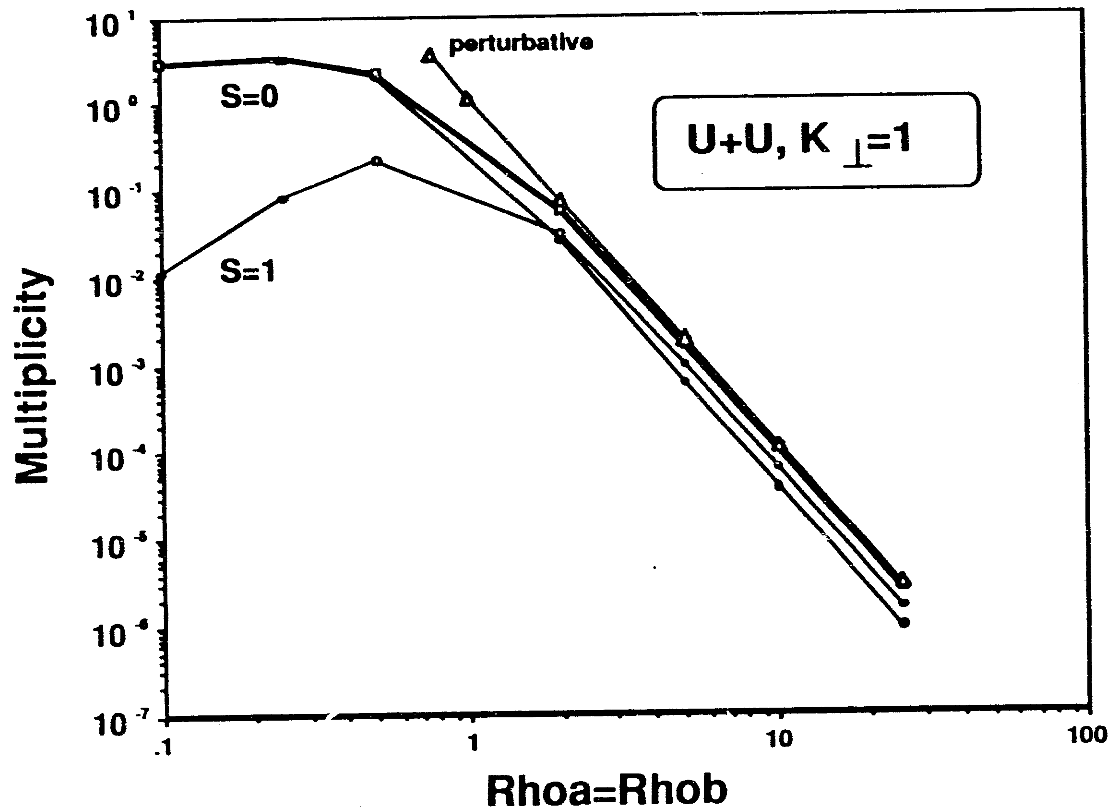
Non Perturbative Pair Production

Deviation from perturb. behaviour (1)



Non Perturbative Pair Production

Deviation from perturb. behaviour (2)



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Probability vs. Multiplicity

multiplicity >1 ?

$$\mu = \int d^3k\, d^3q\, |S_{kq}|^2$$

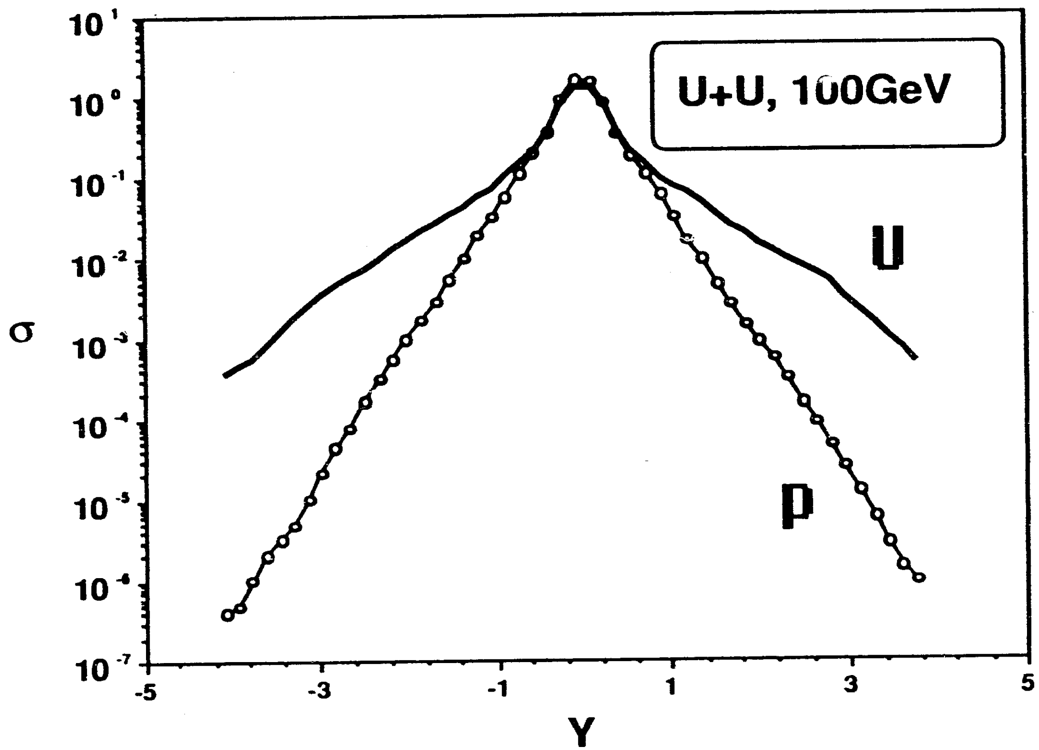
probability <1

$$\mathcal{P}_m = \frac{\mu^m}{m!} \exp(-\mu)$$

$$\sum_{m=0}^{\infty} \mathcal{P}_m = 1; \quad \langle m \rangle = \mu$$

Non Perturbative Pair Production

Rapidity ... U vs. p



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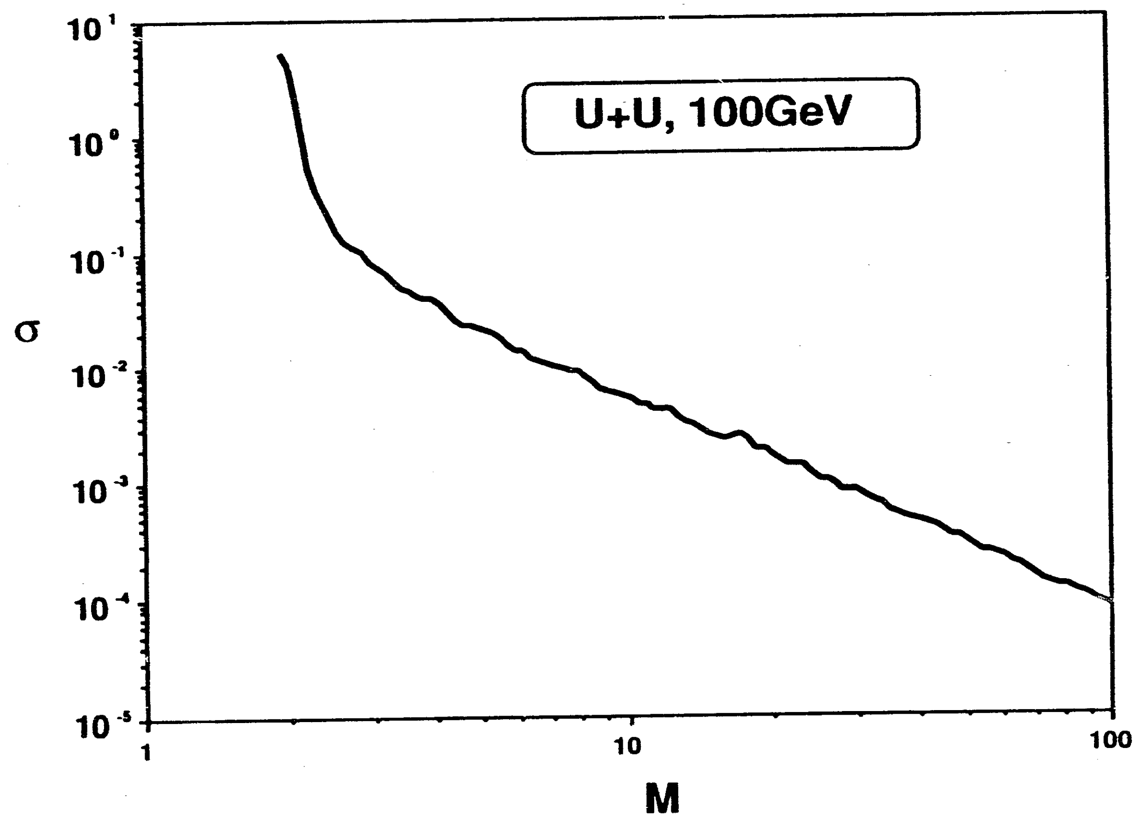
Non Perturbative Pair Production

Pair Mass: U vs. p ... little effect!



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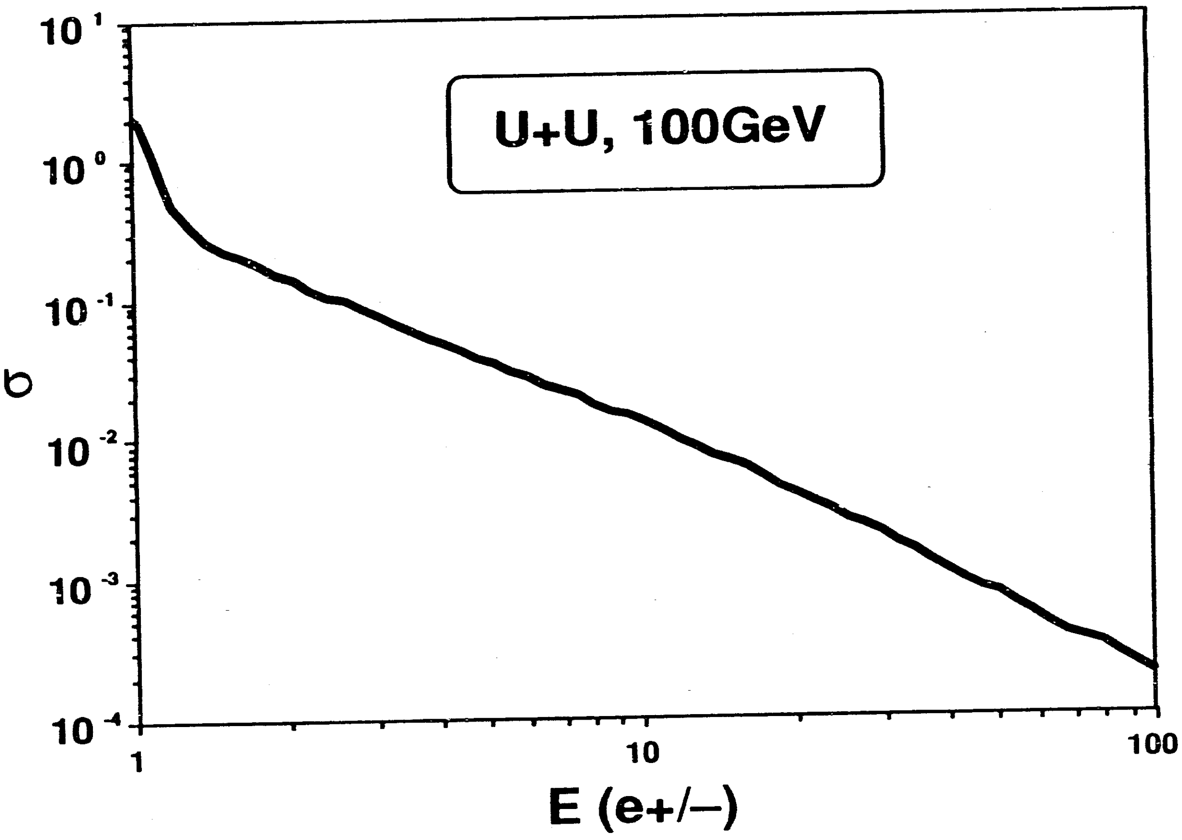
Non Perturbative Pair Production

Singles Energies: **little effect!**



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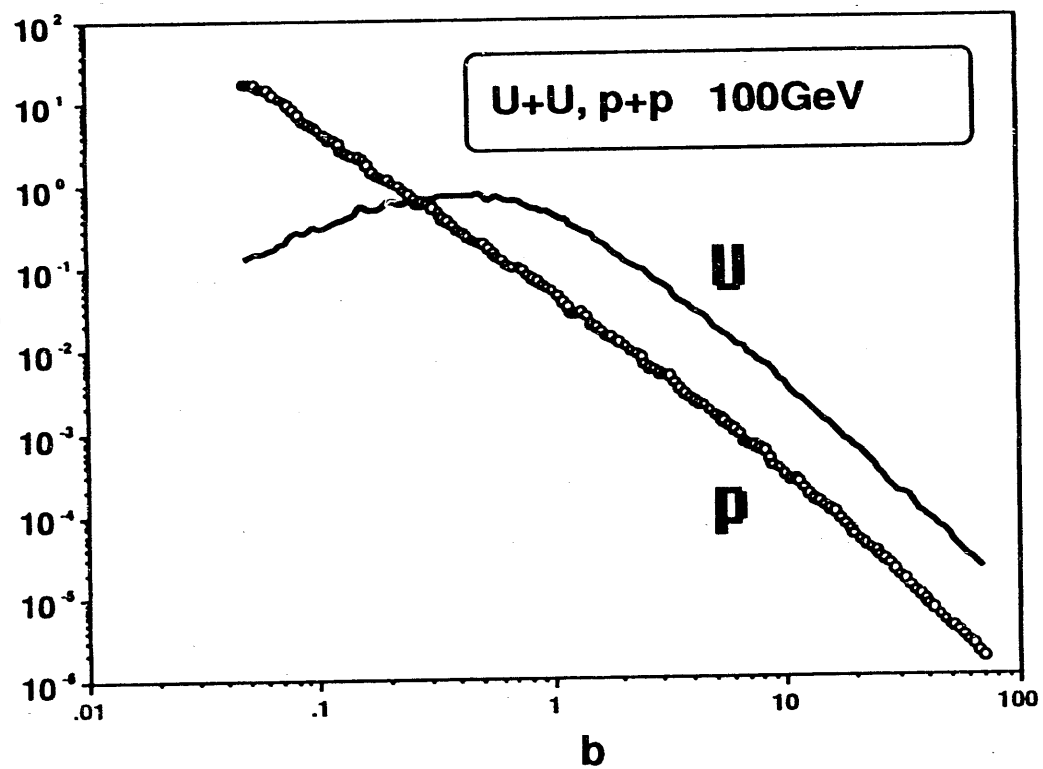
Non Perturbative Pair Production

Impact Parameter: U vs. p



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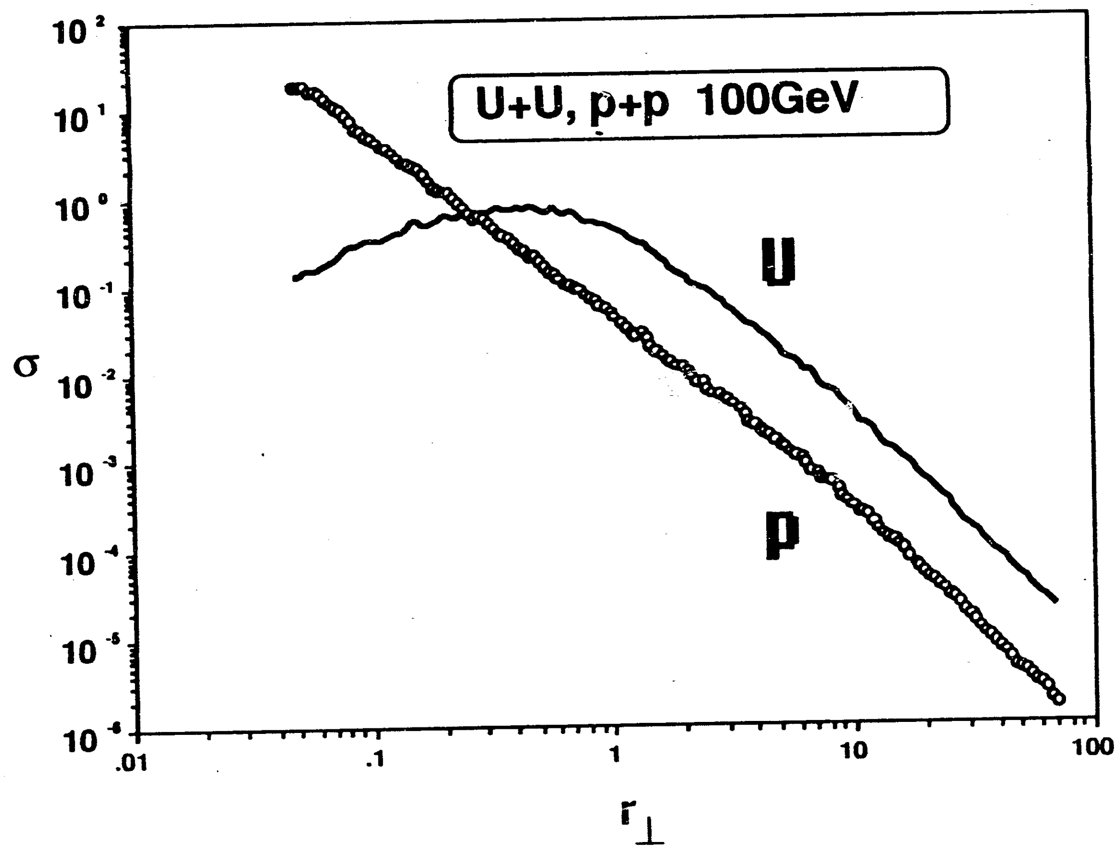
Non Perturbative Pair Production

$r_\perp$ ($Q_\perp$): U vs. p



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Unitarity etc.

In 1D reduction... $\sum_{\sigma_q} \int dq_z |S_{kq}(\vec{K}_\perp \vec{\rho}_a \vec{\rho}_a)|^2 = 1$

Froissart bound... $P(b) = \Phi(\gamma) \exp(-b/R)$

implies $\sigma = \pi[R \ln(\Phi(\gamma))]^2$

Empirical fits... $\sigma_U \cong A(\ln \gamma)^{2.5}$

$\sigma_{100GeV} \cong BZ^4 \left[1 + \left(\frac{Z}{70}\right)^{3.7}\right]^{-1}$

resummation?

Conclusions

- Pair production & capture in relativistic heavy ion collisions exhibit non perturbative effects
- Experimentally detectable for $Z>70$
- Semi-classical Light Cone reduction is a powerful tool - potentially applicable to confined theories

MEASUREMENT OF PAIR PRODUCTION AND CAPTURE FROM THE CONTINUUM IN HEAVY PARTICLE COLLISIONS

Talk presented by:

S. Datz

PROPOSAL SPSC 90-3/P 247

MEASUREMENTS OF PAIR PRODUCTION AND ELECTRON CAPTURE FROM THE CONTINUUM IN HEAVY PARTICLE COLLISIONS

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A. BELKACEM, N. FEINBERG, H. A. GOULD -

LAWRENCE BERKELEY LAB

R. H. SCHUCH -

MANNE SIEGBAHN INSTITUTE

P. HVELPLUND, H. KNUDSEN -

UNIV. OF AARHUS, INST. OF PHYSICS

J. P. GIESE -

KANSAS STATE UNIVERSITY

WHAT - WE WANT TO MEASURE

I. ELECTRON PAIR PRODUCTION DIFFERENTIAL CROSS SECTIONS IN RELATIVISTIC HEAVY-ION COLLISIONS (200 GeV/u SULFUR + Zt= 13,47,79)

$$S^{16+} + Z_T \rightarrow S^{16+} + Z_T + (e^+, e^-)$$

II. CAPTURE CROSS SECTIONS FOR THE PAIR-PRODUCED ELECTRONS

$$S^{16+} + Z_T \rightarrow S^{15+} + Z_T + e^+ \qquad e^- \text{ capture.}$$

$$\hookrightarrow S^{16+} + e^- (\gamma \doteq 200, \theta_L = 0^\circ) \qquad e^- \text{ loss.}$$

MOTIVATION FOR MAKING THESE MEASUREMENTS

- COULOMB (2 PHOTON) PRODUCTION CROSS SECTIONS FOR ELECTRON-POSITRON PAIRS ARE LARGE (Kb) FOR HIGH-ENERGY, HIGH-Z SYSTEMS.

  THERE IS CONCERN THEN THAT PAIR CREATION BY E-M MECHANISMS WILL INTERFERE WITH LEPTON PAIR SIGNATURES FOR OTHER PROCESSES, e.g., ELECTRON PAIRS AS A PENETRATING PROBE OF QUARK-GLUON PLASMA.

- PREPARATION FOR MEASUREMENTS WITH HEAVIER BEAMS AT CERN AND RHIC WHERE CAPTURE OF THE CREATED ELECTRON IS EXPECTED TO BE THE LEADING CHARGE CHANGING MECHANISM. FOR HEAVY-ION STORAGE RINGS THIS WOULD REPRESENT A SIGNIFICANT BEAM LOSS MECHANISM.

- DISAGREEMENTS AMONG CALCULATIONS FOR PAIR PRODUCTION CROSS SECTIONS, ESPECIALLY WITH RESPECT TO ENERGY AND ANGULAR DISTRIBUTIONS OF THE ELECTRONS AND POSITRONS.

  TESTS OF THEORETICAL APPROXIMATIONS IN PERTURBATION EXPANSIONS IN $(Z\alpha)$ AS Z IS INCREASED.

- LARGER DISAGREEMENTS (~X10) AMONG THEORIES FOR ELECTRON CAPTURE OF THE CREATED ELECTRON.

HOW - EXPERIMENTAL METHODS

I. TRANSMISSION EXPERIMENT - PARASITIC WITH WA80 - THIN ($\sim$ 1 mg/sq-cm) TARGETS. use WA80 0° cal. to ID S^{16+}

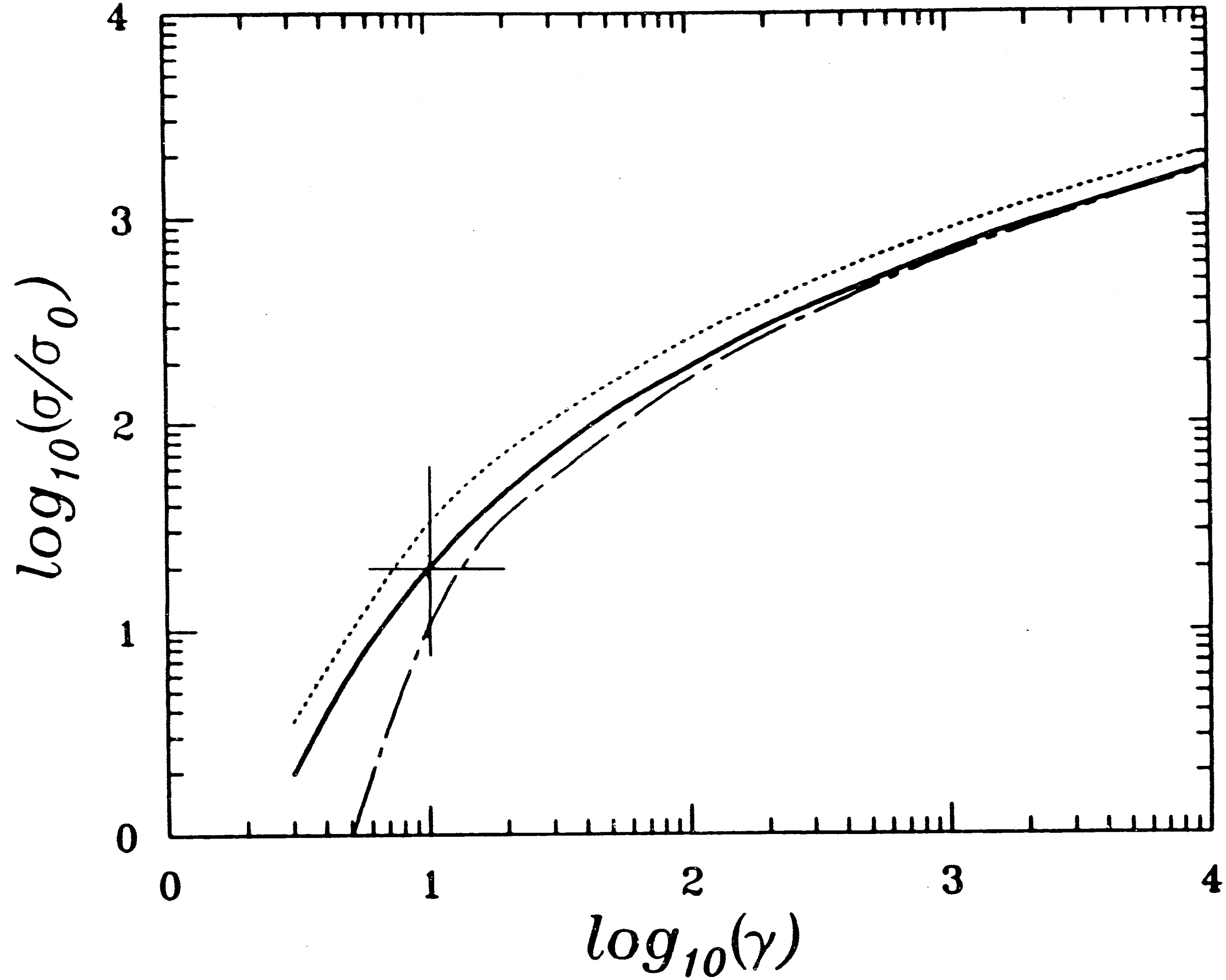
II. LOW-ENERGY (0.5 - 10 MeV) ELECTRONS AND POSITRONS.

A. 180 DEGREE MAGNETIC ANALYSIS OF FORWARD EMITTED, LOW-ENERGY ELECTRONS AND POSITRONS. ($B \leq 0.3T$)
B. COUNTING IN DISCRETE ARRAYS OF SOLID STATE DETECTORS.

III. HIGH-ENERGY (100 MeV) ZERO DEGREE ELECTRONS.

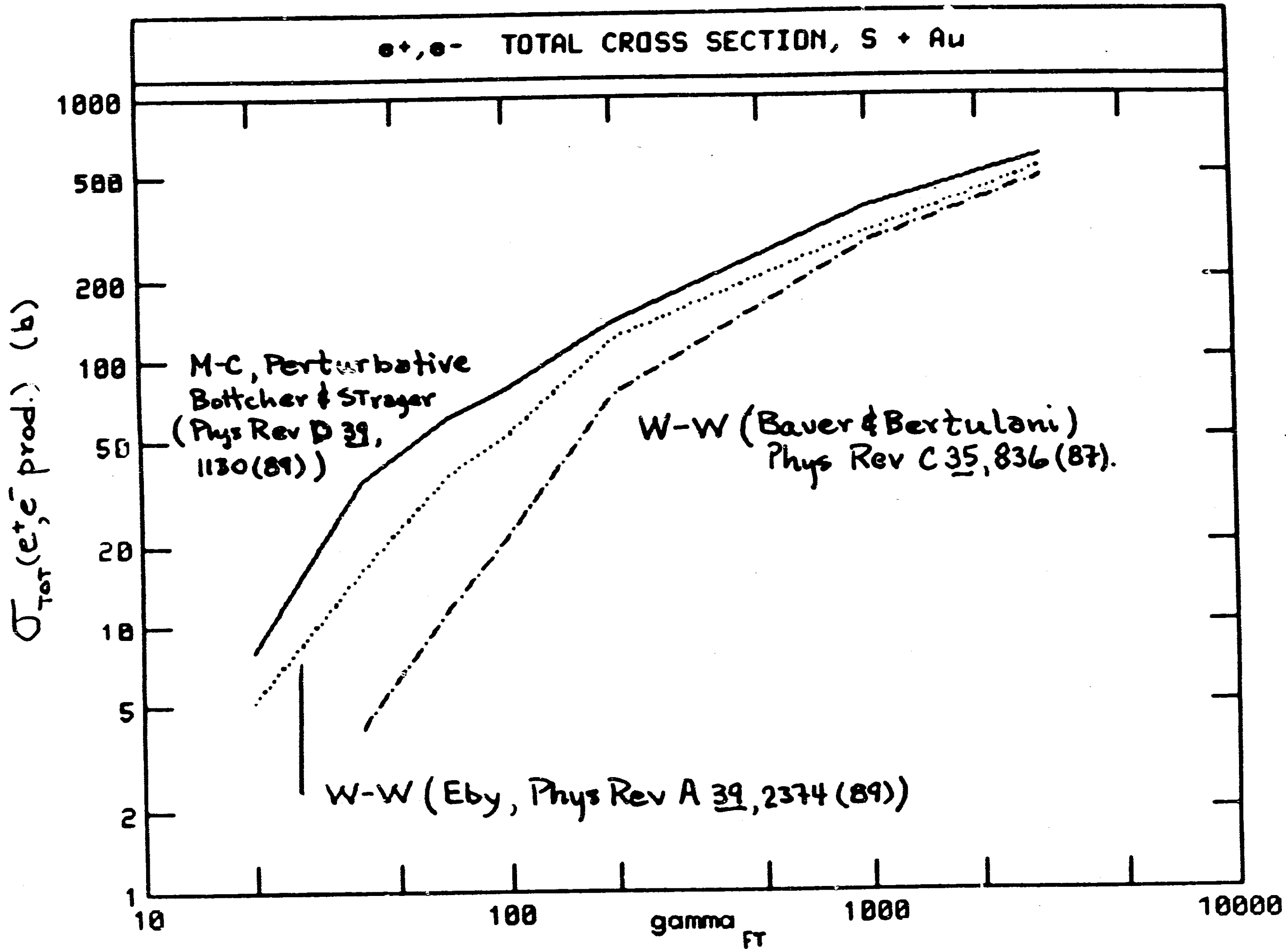
A. MAGNETIC DEFLECTION FROM BEAM AXIS ($\sim$20 DEG). ($B \leq 0.3T$)
B. HIGH-RESOLUTION TRAJECTORY AND FULL ENERGY MEASUREMENTS.

ORNL-DWG 88-12984



$\sigma_0 = \lambda^2 Z_1^2 Z_2^2 \alpha^4$ σ_0 (S-Au) = 6.7 b

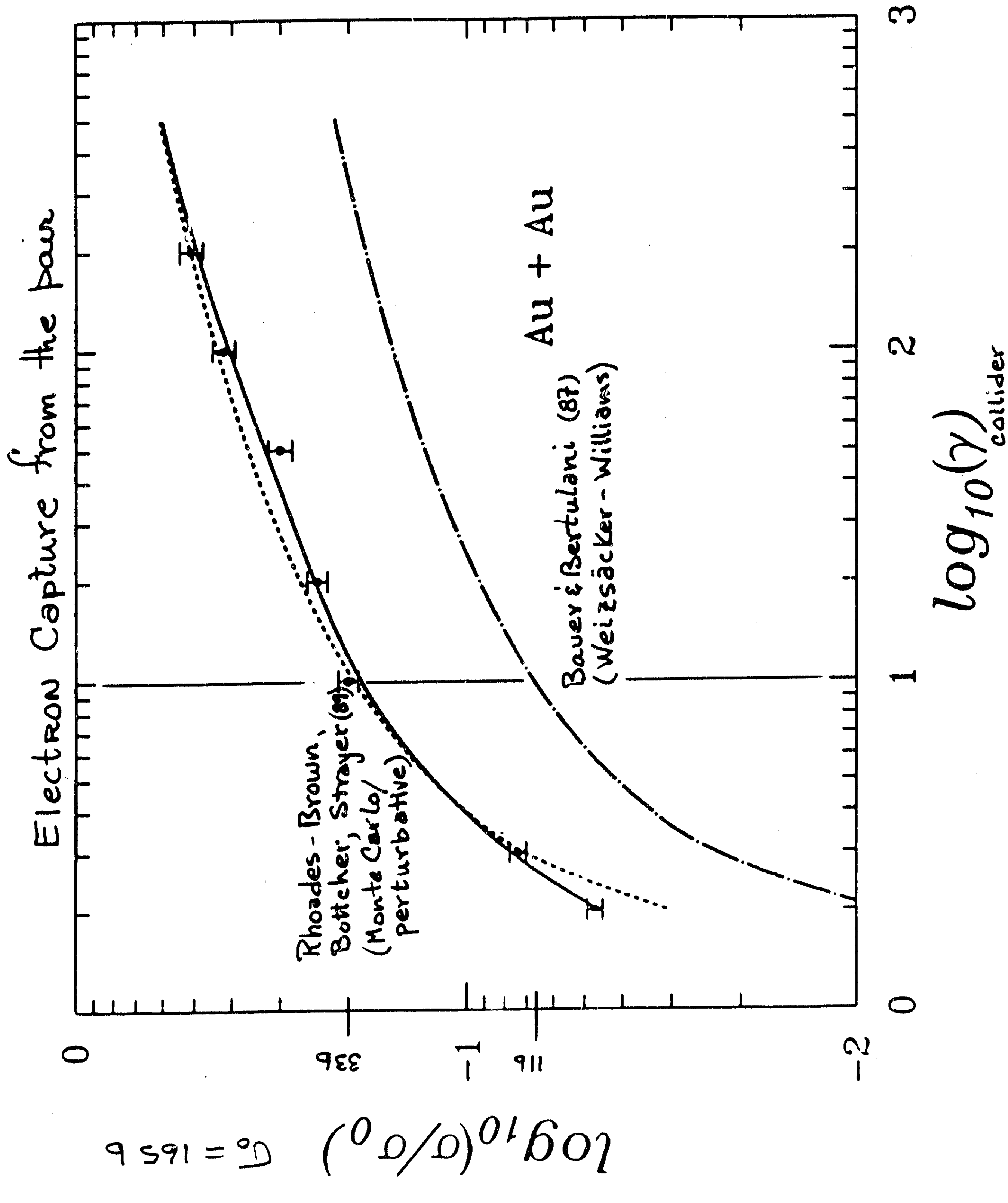
$\lambda = \hbar/mc$ σ " = 85 b

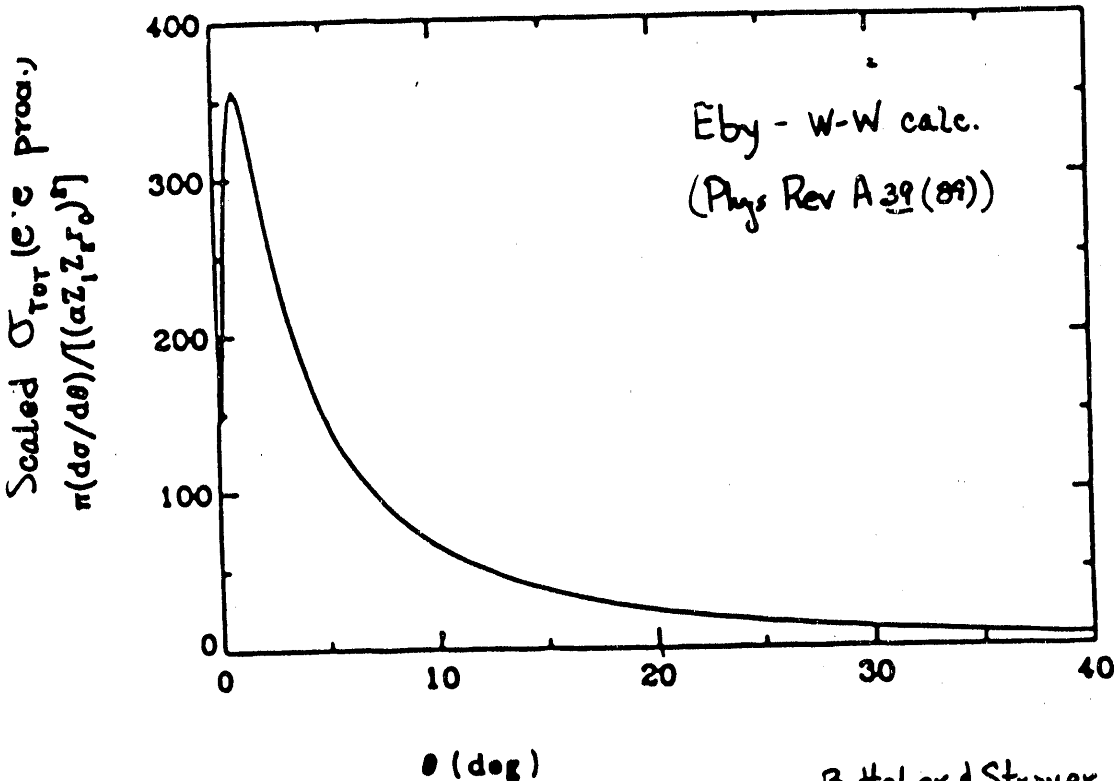


W-W = equivalent photon method.
ie, decompose E-M pulse into equiv. photon spectrum, then calc. pair formation, integrate over impact parameters.

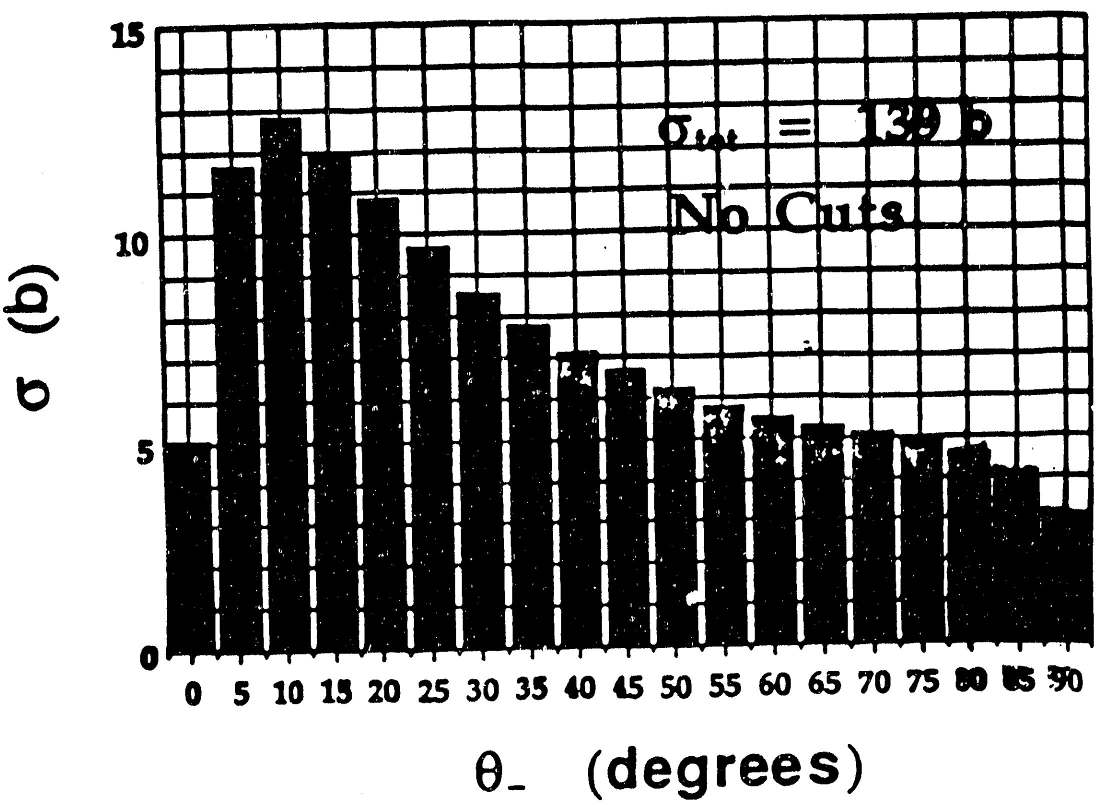
MC-pert. = Monte Carlo integ. eval. of 2-photon diagrams. assuming time-dep. classical fields for the heavy-Ions.

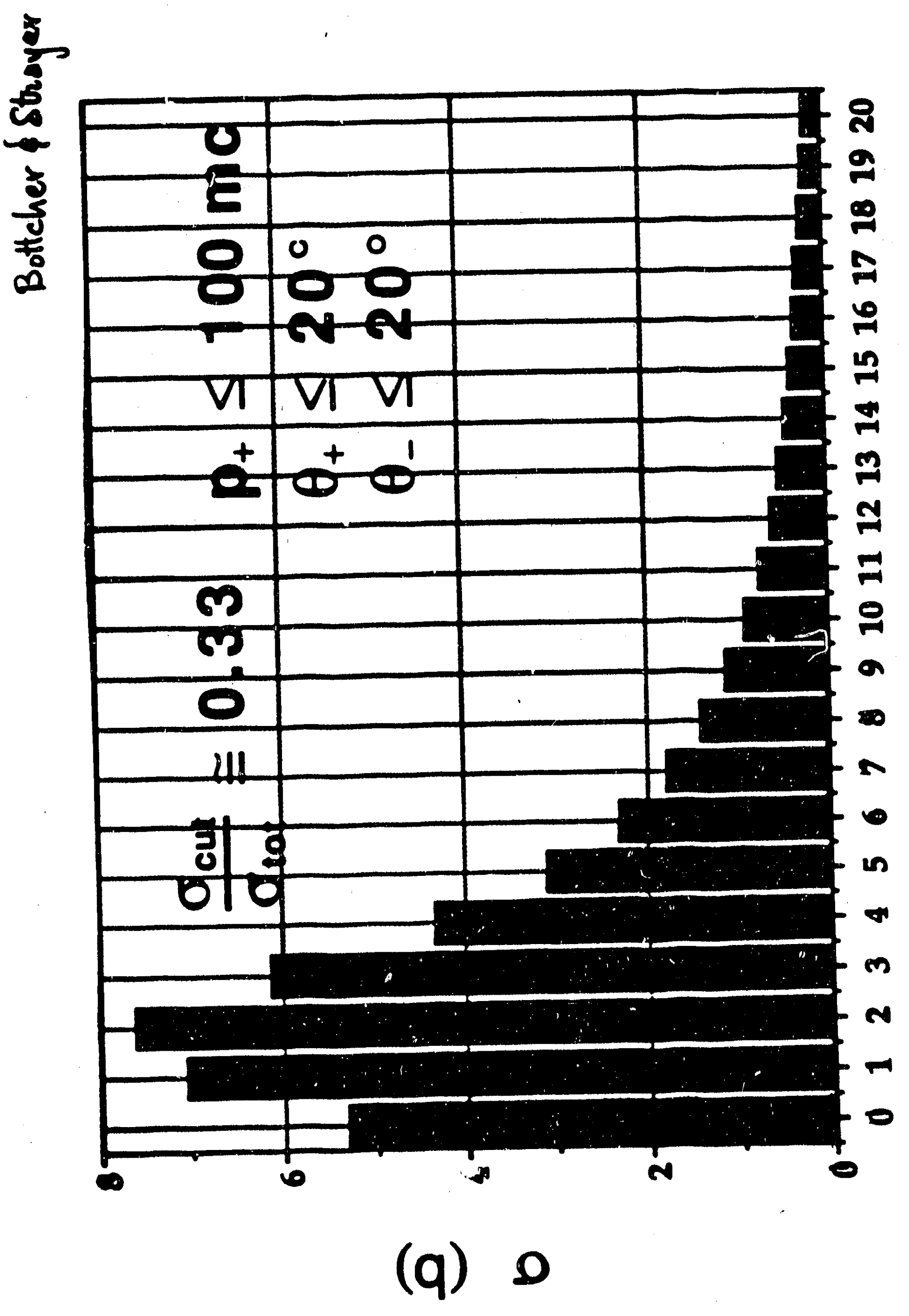
ORNL-DWG 89-7689





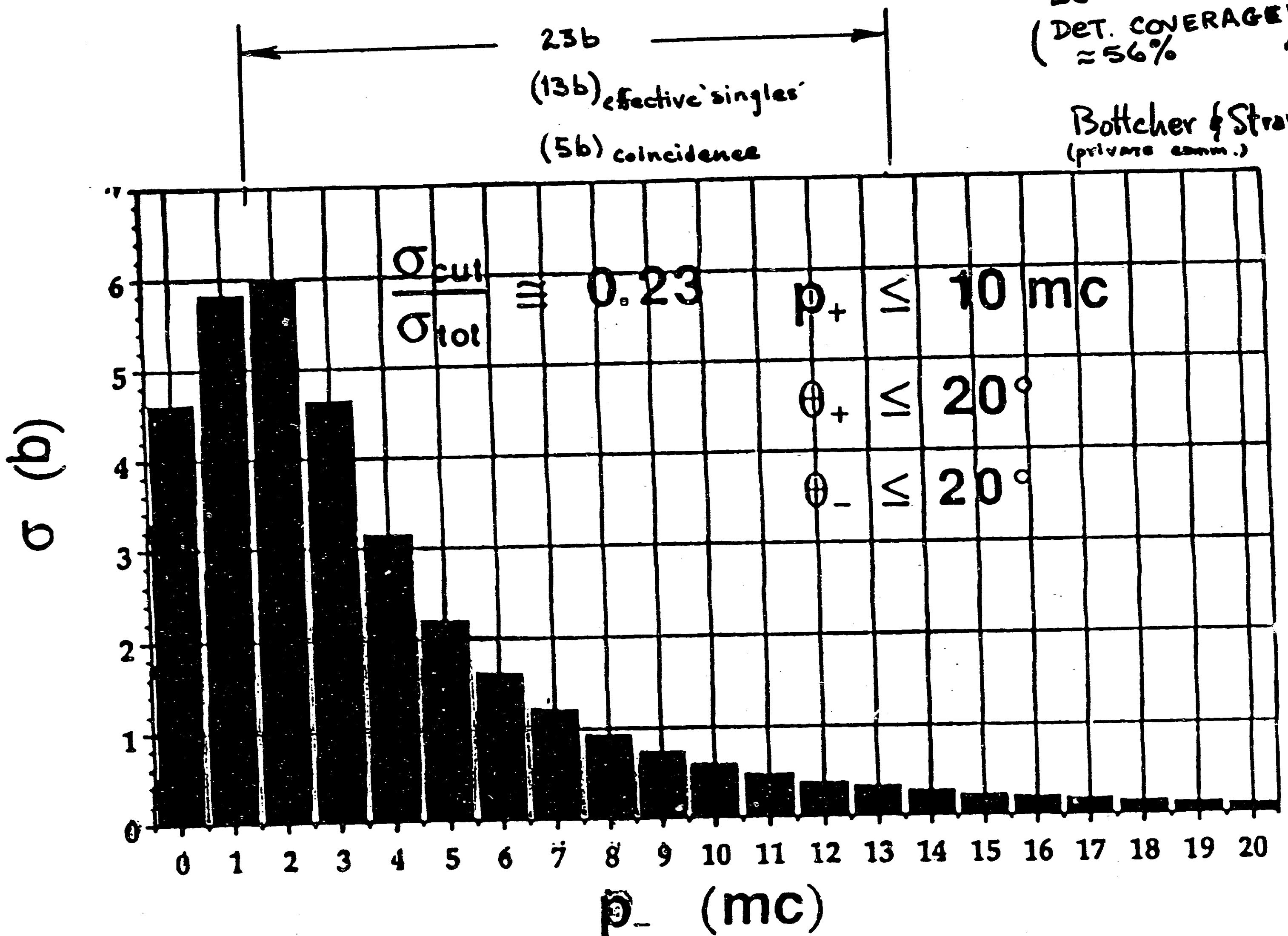
Bottcher & Strayer



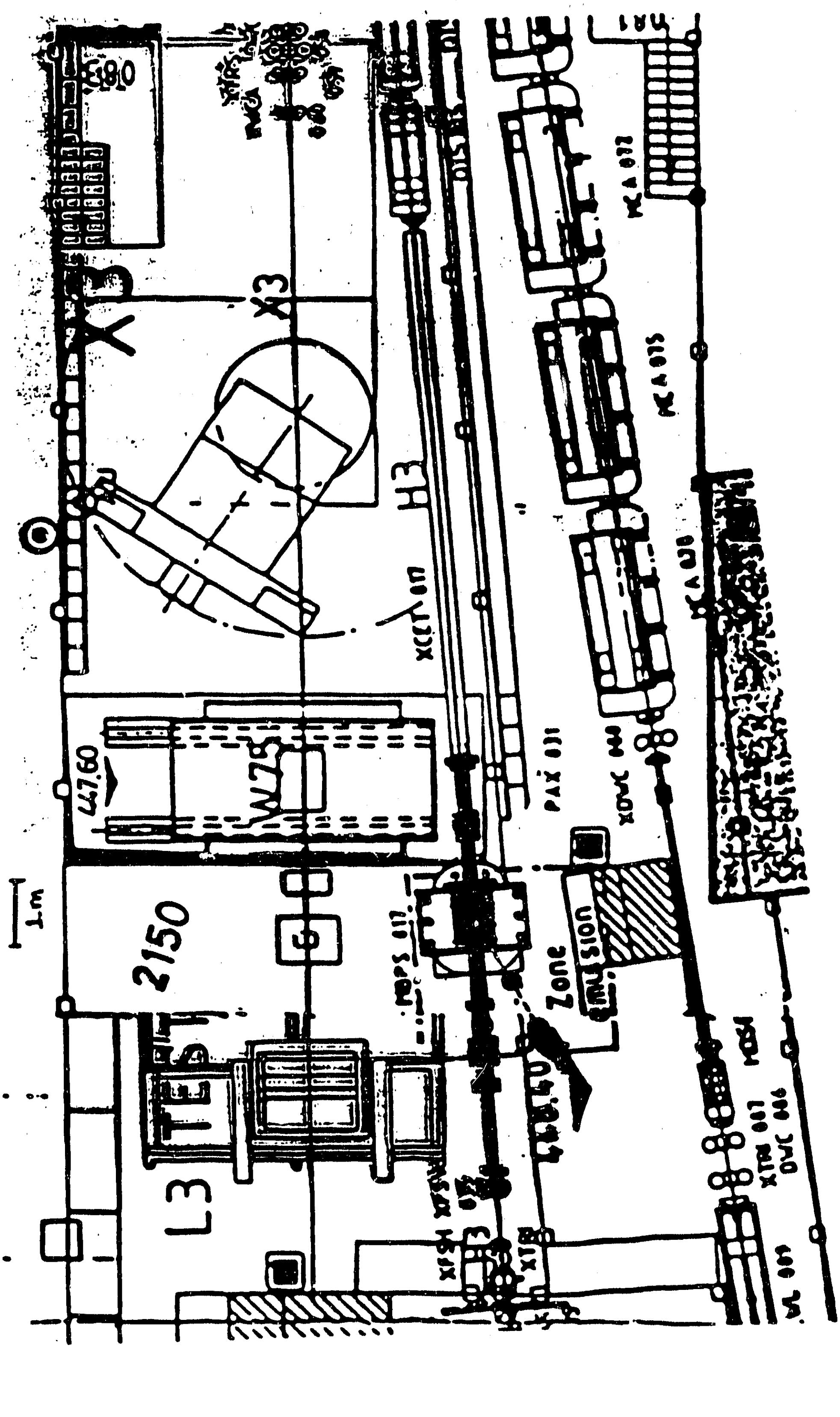

Bottcher & Strayer
$\frac{\sigma_{cut}}{\sigma_{tot}} \cong 0.33$
$p_+ \leq 100$ mc
$\theta_+ \leq 20°$
$\theta_- \leq 20°$
σ (b)
0
2
4
6
8
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
p_- (mc)

$B = 0.18\,T$
$\Delta p_{det} \doteq 1-7\ \frac{MeV}{c}$
$\Delta\theta \doteq 0-20°$
(DET. COVERAGE $\approx 56\%$)

Bottcher & Strayer (private comm.)

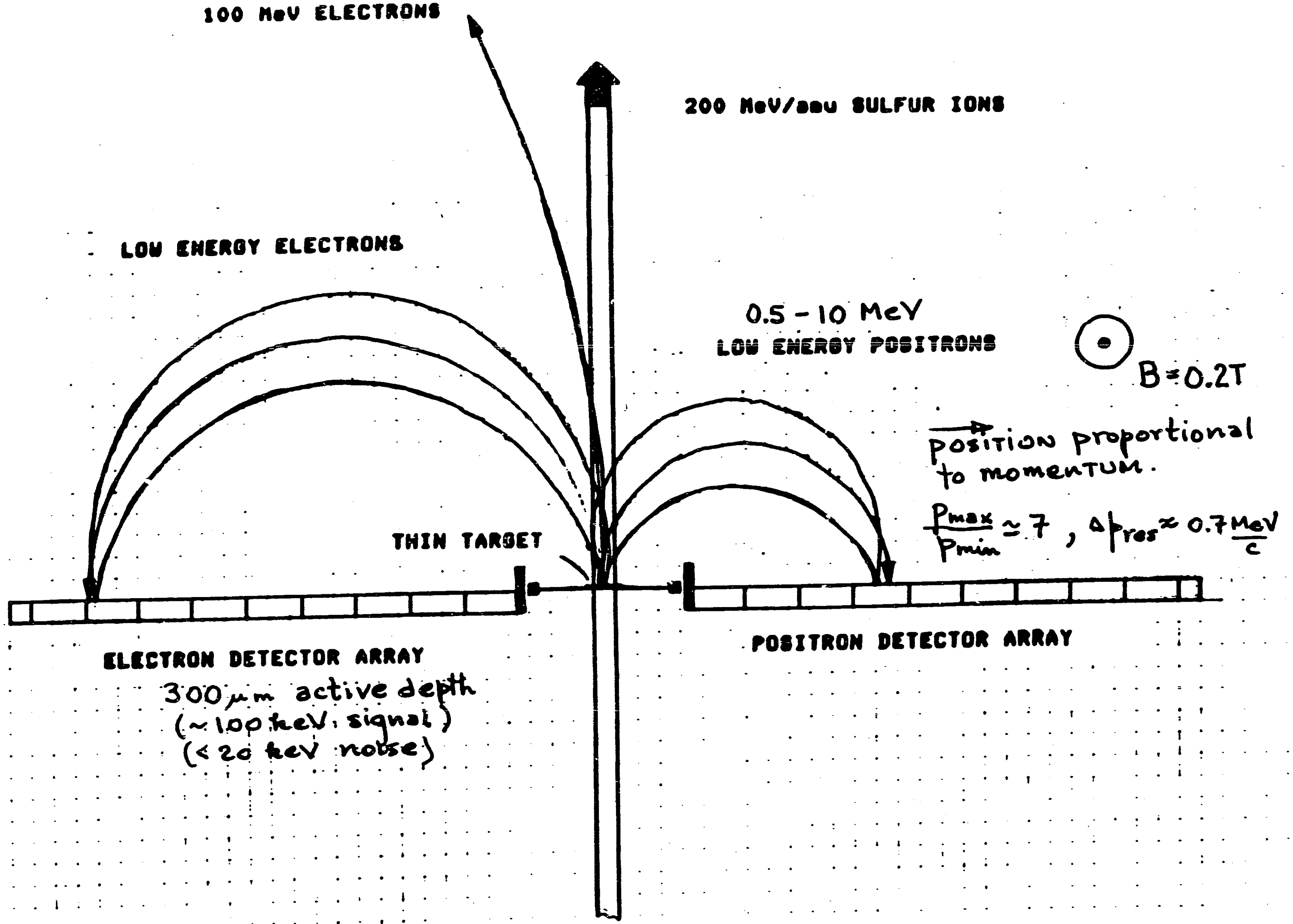


Limits on t_T:
1. MCS $\langle\theta^2\rangle^{1/2} = 8°$ at 0.5 MeV in 1 mg/cm² Au
2. KO singles rate ~ 3×10^3/sec
3. effects on WA80

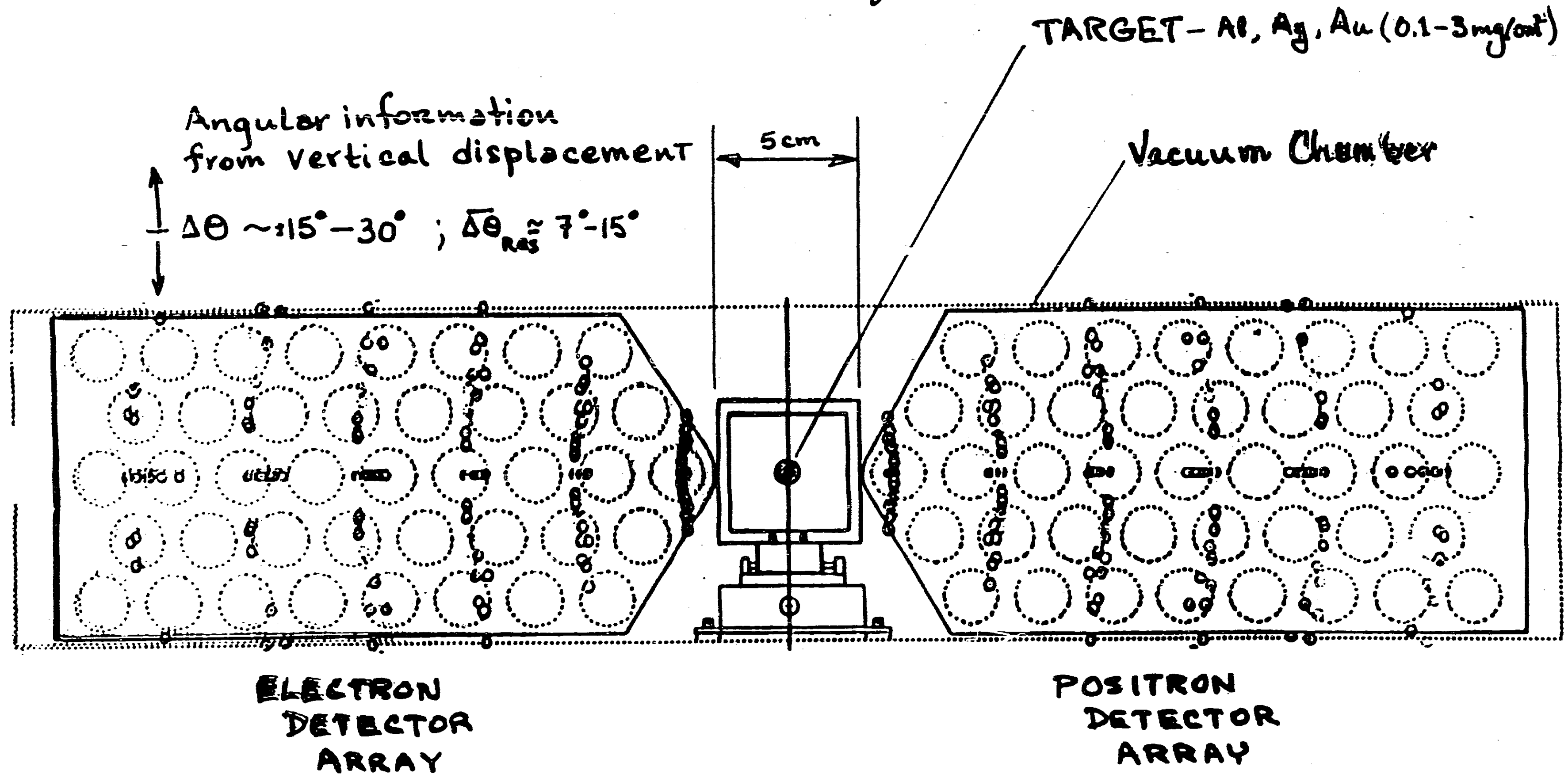


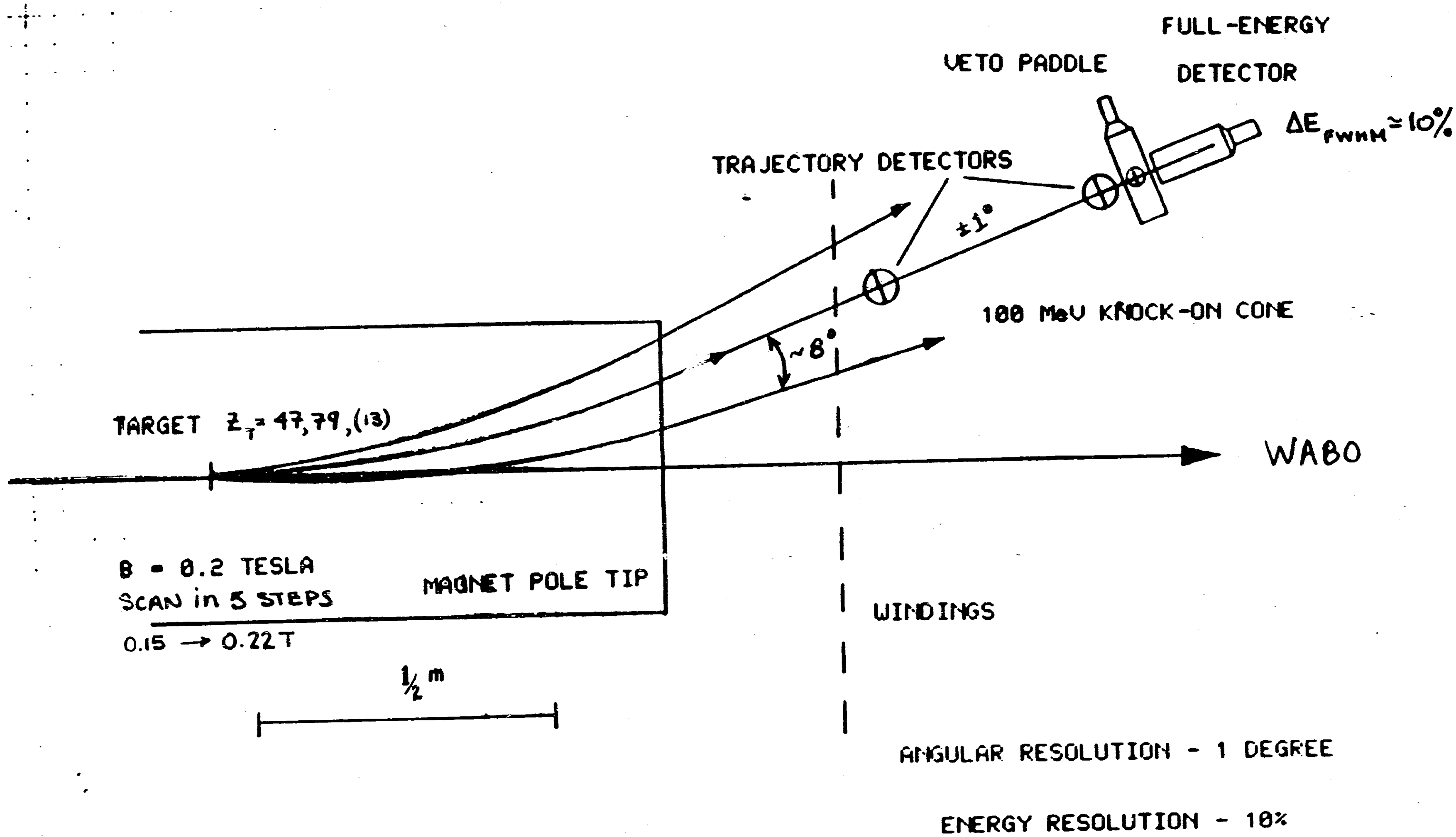
1m
2150
TEST
L3
H3
X3
Zone
47.60

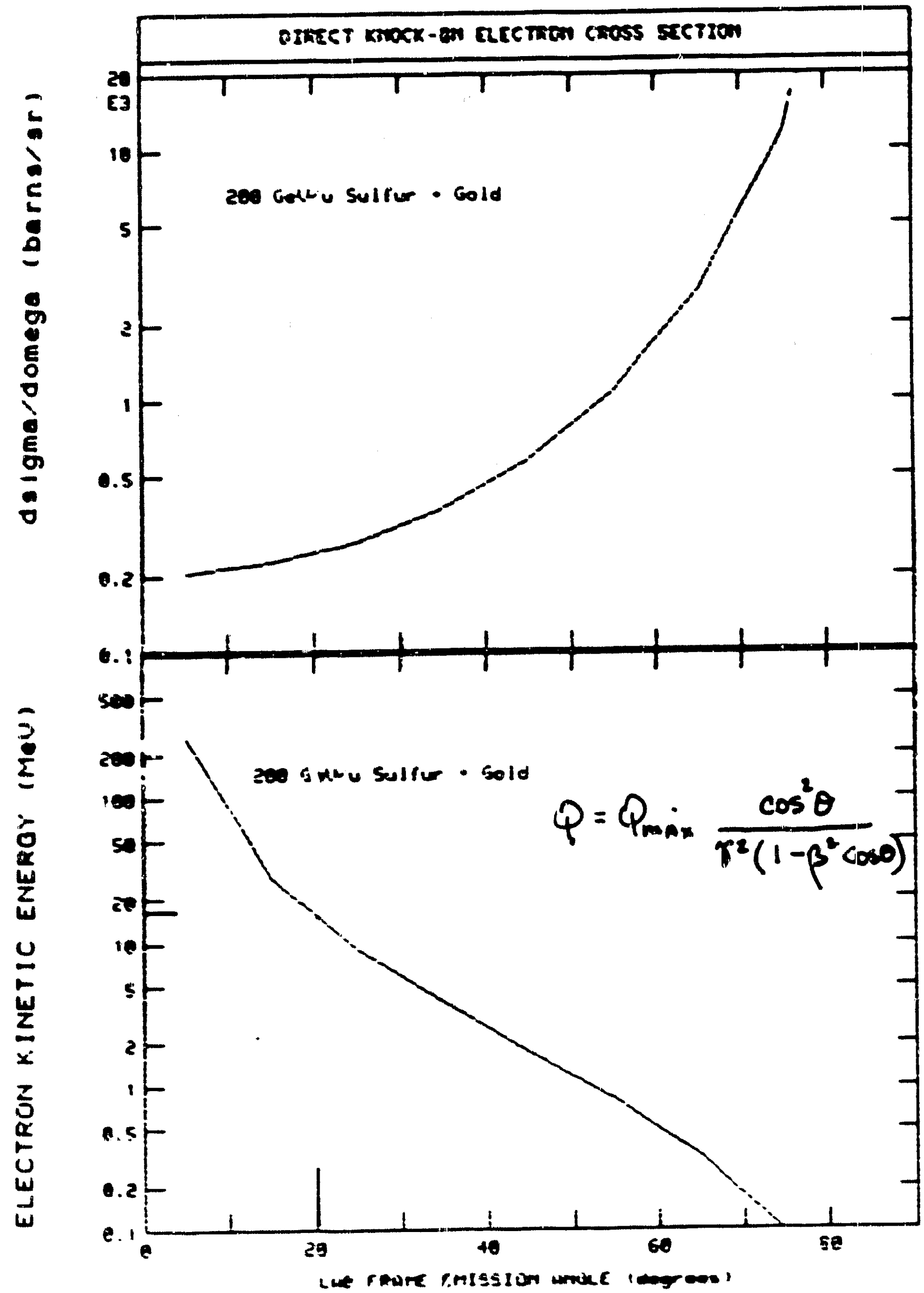
WA80


100 MeV ELECTRONS
200 MeV/amu SULFUR IONS
LOW ENERGY ELECTRONS
0.5 - 10 MeV
LOW ENERGY POSITRONS
B = 0.2T
position proportional to momentum.
$\frac{p_{max}}{p_{min}} \simeq 7$, $\Delta p_{res} \approx 0.7 \frac{MeV}{c}$
THIN TARGET
ELECTRON DETECTOR ARRAY
300 μm active depth
(~100 keV signal)
(< 20 keV noise)
POSITRON DETECTOR ARRAY

End View of Detector Arrays




FULL-ENERGY
DETECTOR
VETO PADDLE
$\Delta E_{FWHM} \simeq 10\%$
TRAJECTORY DETECTORS
±1°
100 MeV KNOCK-ON CONE
~8°
TARGET Z_T = 47,79,(13)
WABO
B = 0.2 TESLA
SCAN in 5 STEPS
0.15 → 0.22 T
MAGNET POLE TIP
WINDINGS
½ m
ANGULAR RESOLUTION - 1 DEGREE
ENERGY RESOLUTION - 10%


DIRECT KNOCK-ON ELECTRON CROSS SECTION
dsigma/domega (barns/sr)
200 GeV/u Sulfur + Gold
ELECTRON KINETIC ENERGY (MeV)
200 GeV/u Sulfur + Gold
$\varphi = \varphi_{max} \frac{\cos^2\theta}{\gamma^2(1-\beta^2\cos\theta)}$
LAB FRAME EMISSION ANGLE (degrees)

CROSS SECTIONS FOR 200 GeV/u SULFUR + GOLD

	sigma (barns/atom)	events/ion (1 mg/sq-cm Au)
Pair Production (e+,e-)		
Perturbative calc.	140	4.3E-4
Nonperturbative calc.	(8 - 130)	(2.5E-5 to 4.0E-4)
Electron Capture		
from the vacuum pair	(0.4 - 2)	(1.E-6 to 8.E-6)
radiative electron capture	1.2E-3	3.7E-9
mechanical capture	1.6E-5	4.9E-12
Electron Loss		
$(S^{15+}(1s) \rightarrow S^{16+} + e^-)$	5.4E+5	0.51 @ 1mg 0.80 @ 3mg
Delta Electron Production (E delta = 0.6 - 6 MeV)		
all angles	7.7E+3	2.4E-2
0 - 40 deg	4.4E+2	1.3E-3
Nuclear Cross Sections (200 GeV/u S + Pb)		
Spallation		
total	8.7	2.7E-5
Zf=15	2.4	7.3E-6
Zf=14	1.5	4.6E-6
Zf=13	0.4	1.2E-6
Electromagnetic		
total	4.6	1.4E-5
Zf=15	2.2	6.7E-6
Zf=14	1.2	3.7E-6
Zf=13	0.4	1.2E-6
Giant Resonances ((n,γ) -> e+,e- pairs)	1.0E-5	3.0E-11

ELECTRON MULTIPLE COULOMB SCATTERING HALF-ANGLES

e- KE(MeV)	rms half-angle (deg)
0.5	8.0
1.0	4.5
2.0	2.5
4.0	1.3
8.0	1.0

EXPERIMENTAL PROGRAM AT LBL

Talk presented by:

A. Belkacem

Experimental Program at LBL

- participation in CERN pairs experiment
  - Ali Belkacem
  - Ben Feinberg* (AFRD)
  - Harvey Gould

  - design, build & operate e⁻ calorimeter to look at capture from pair production

- Bevalac experimental program
  - Ali Belkacem
  - Ben Feinberg* (AFRD)
  - Harvey Gould
  - Walter Meyerhof (Stanford)

  - look for electron capture from pair production (U^{92+} on Au) and measure e^+ angular and energy distribution.

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* moonlighting

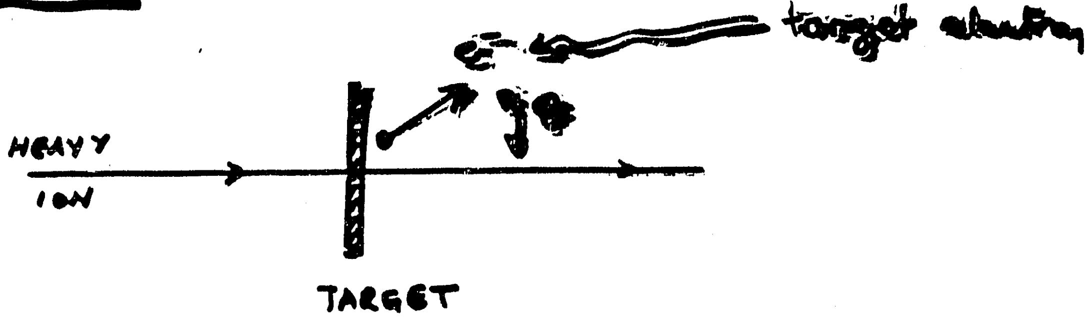
BACKGROUND

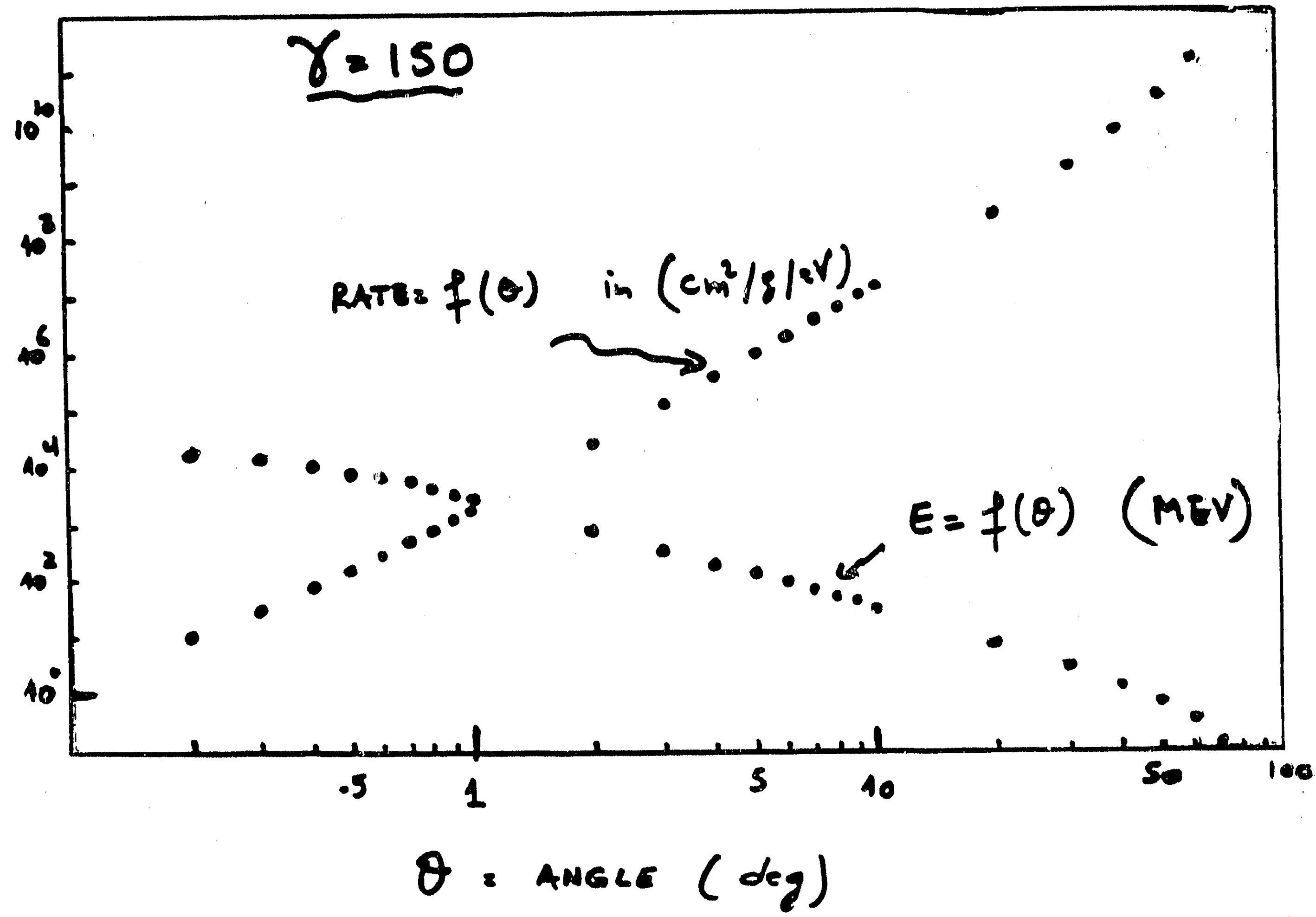
- due to electrons of target

	BEVALAC	CERN
Knock-on electrons ($T_{max} \approx 2\gamma^2 m_e c^2$)	●	●
capture of target electrons	⬤	·
Loss of electron after capture by projectile	●	⬤
Bremmsstrahlung by target electrons	·	●

- Nuclear orrgin ● ●

KNOCK ON ELECTRONS





$$T_{max} \lesssim 2\gamma^2 mc^2 \simeq 20 \text{ GeV}$$

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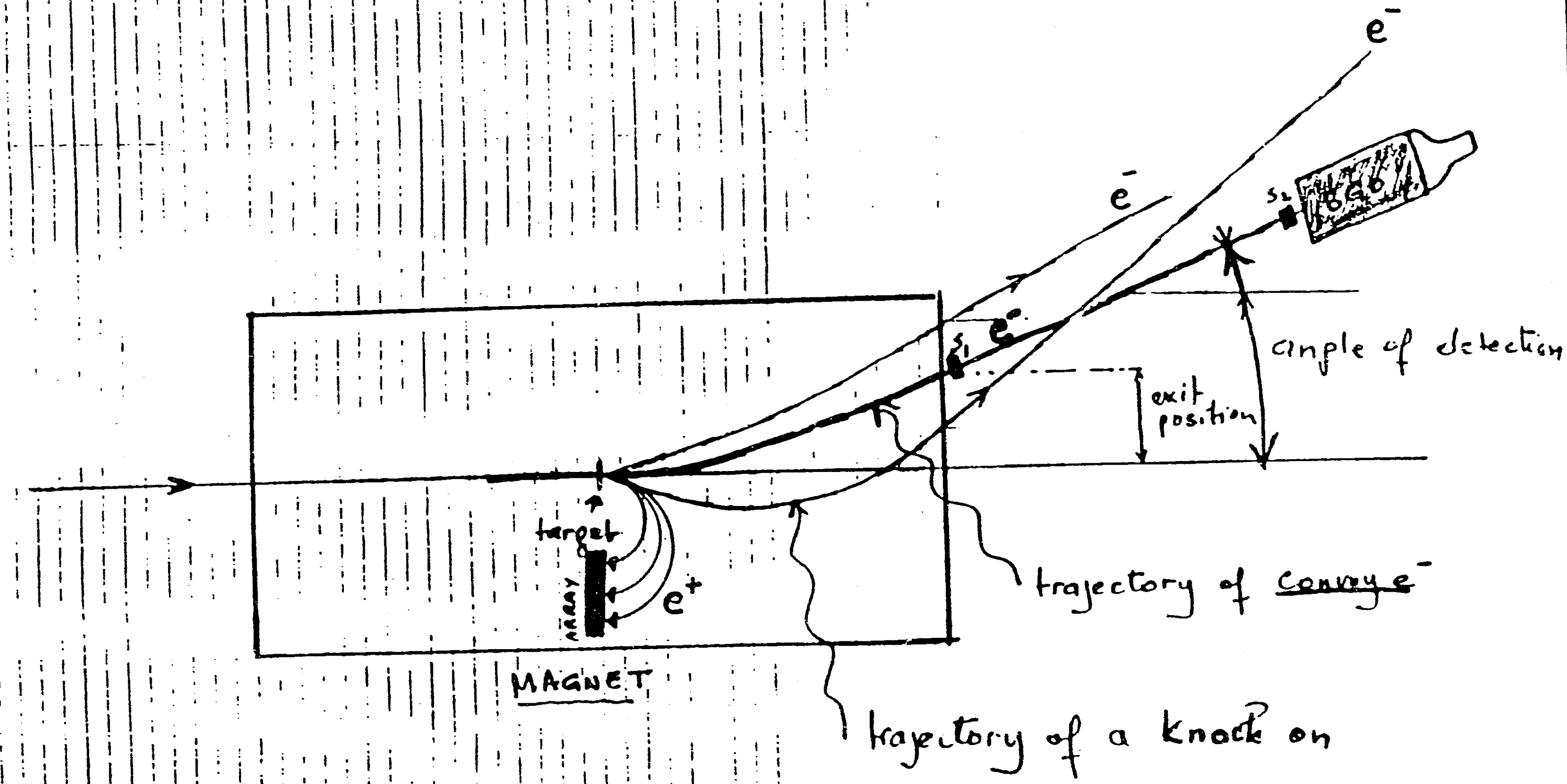
100 MeV e⁻ detecting sketch



- S_1 and S_2 are 2 scintillators which define the angle of detection
- the scintillator S_1 set the exit position necessary to be detected

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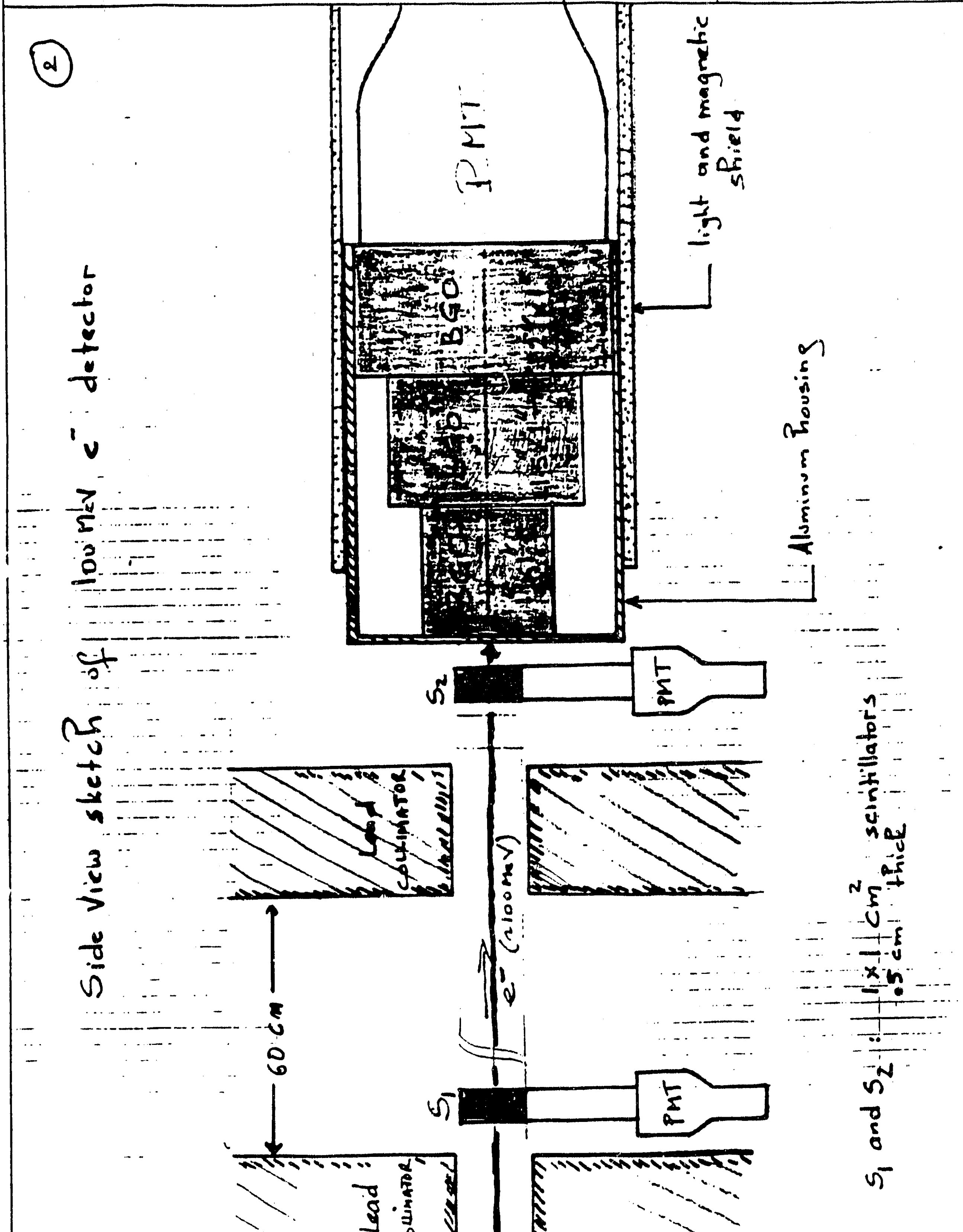
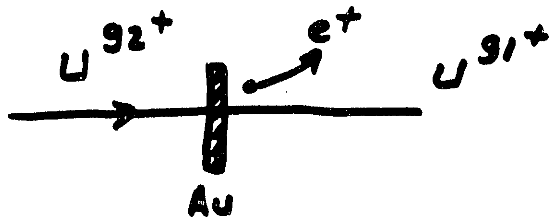


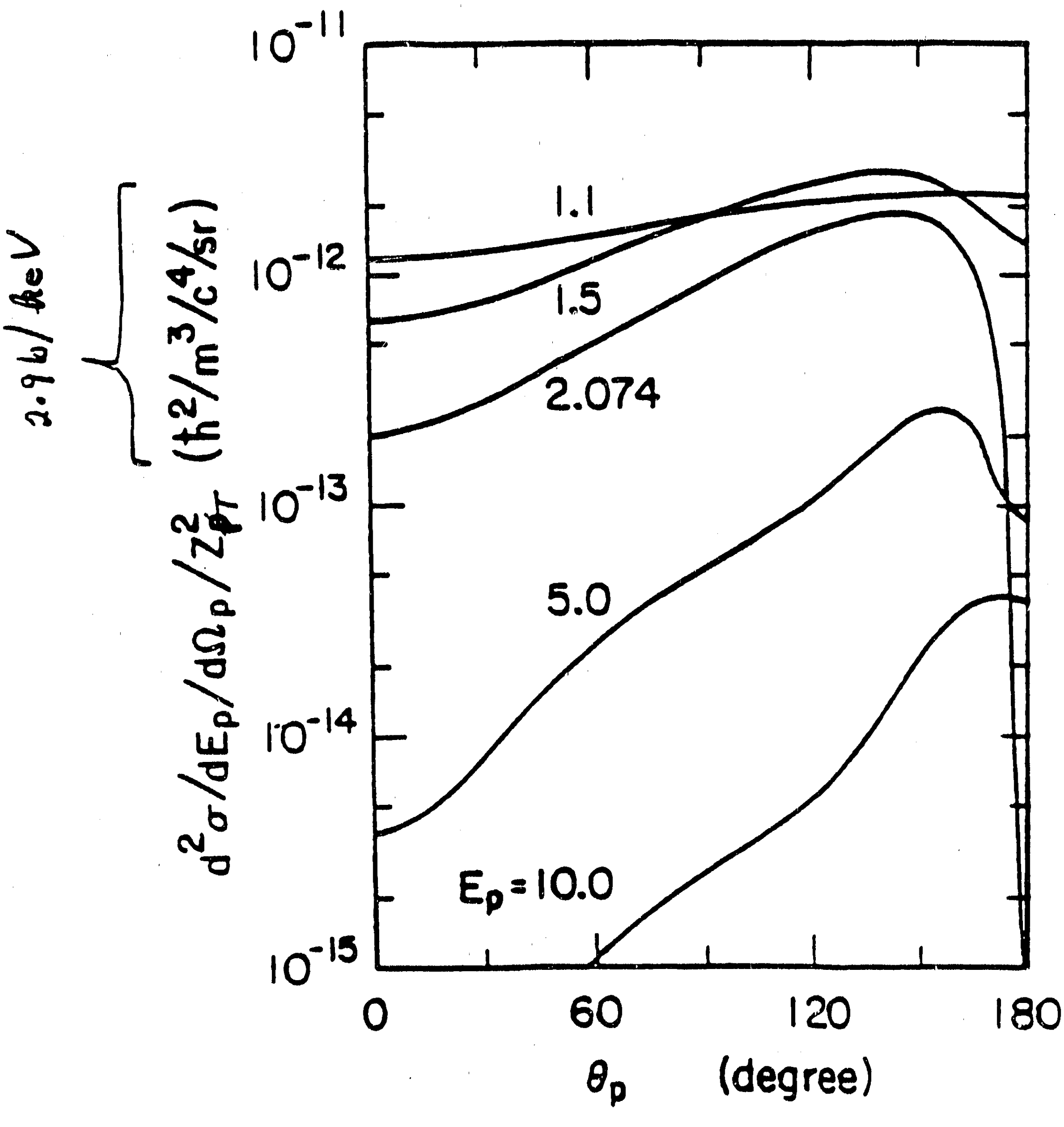
ELECTRON CAPTURE FROM PAIR PRODUCTION
AT BEVALAC ENERGIES
(1-2 GeV/n)



SOME NUMBERS

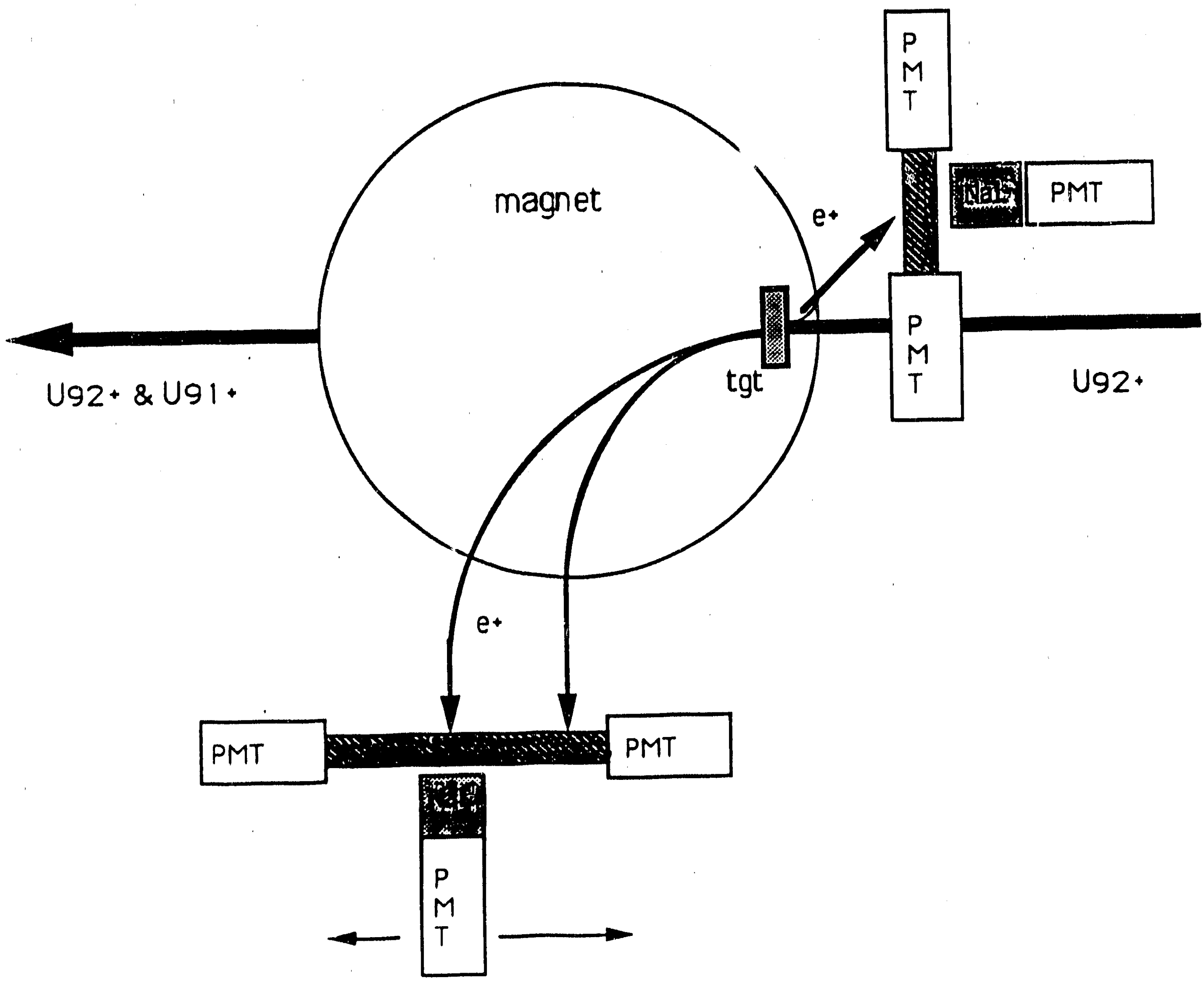
	σ	rate for 1mg/cm Au target/i
CAPTURE FROM pair production	~1 b	1.5×10^{-6}
Free pair production	~ 80 b	1.4×10^{-4}
CAPTURE RATE (REC, ...) TARGET e^-	3.6 kb	5×10^{-3}
Loss rate	20 kb	3×10^{-2}
Double event: free pair + charge changing with capture of a target e^-		8×10^{-7}

FROM U. BECKER, J. PHYS B20, 6563 (1987)



Double differential cross section in $S^{16+} + Z_T$ collisions at E_{lab} = 1 GeV/amu as a function of the positron emission angle θ_p calculated in the restframe of the target. The projectile captures the electron and approaches along the negative z-direction.

BEVALAC PAIR CAPTURE SPECTROMETER
(VERSION 0)



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