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A REVIEW OF THE THEORY AND PHENOMENOLOGY OF LEPTON PAIR PRODUCTION

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DISCLAIMER

ABSTRACT: Recent experimental data and theoretical developments on the production of lepton pairs in hadron collisions are reviewed. With emphasis on the interplay between theory and experiment, the relevance of theoretical calculations to the data available at present energies is critically examined. The Drell-Yan mechanism is found to be phenomenologically dominant provided that the parton distribution functions contain effects of gluon radiation in a narrow cone. Explicit QCD perturbative calculations of the non-Drell-Yan type yield results that are apparently important at large transverse momenta, but are contradicted by subsequent data at 400 GeV and below. A consistent picture in the parton model is sketched. Further experiments to probe the basic mechanism are suggested.

CAUTIONARY NOTE: Figures in this preprint that refer to talks presented at the XIIIth Rencontre de Moriond are all preliminary. The final versions of the data have not yet appeared in any papers written by the experimental groups. Those figures are included here in this preprint in order to make this review more useful before it appears in the Proceedings - by which time the data will all have been published. Most of the preliminary figures were copied from hand-drawn view graphs and are not meant for quantitative analysis. Readers who wish to make use of the data should consult the appropriate experimental groups.

PREAMBLE

The contents of this review are influenced strongly by the experimental papers presented during the lepton-pair session and by the recent theoretical development in the framework of QCD. No effort toward completeness has been attempted. On the whole this review covers the development of the subject since summer 1977. For earlier work and for basic kinematics, reference to, for example, the review articles [1] of Shochet (experimental) and Craigie (theoretical) is suggested. Details of important recent data and theoretical investigations relevant to our arguments in this paper are to be found in the articles by Pilcher, Lederman, and Sachrajda, contained in these proceedings.

I. INTRODUCTION

Lepton pair production in hadronic collisions has recently been the subject of intensive investigation both experimentally and theoretically. While many reasons can be given for the high level of interest in the subject, one single underlying theme seems to be that through the detection of dileptons in hadronic collisions one is given a direct access to a picture of the constituent structure of the hadrons. Deep inelastic leptoproduction has been the standard approach for nearly a decade, but it is limited to the nucleons and is hard to reach high-momentum-transfers. Semi-inclusive leptoproduction can provide some more information about the constituents but at the price of involving the quark fragmentation functions. Similar drawback is present in large-p_T inclusive reactions, while in small-p_T processes there are other complications that are not well understood. In lepton pair production many kinematical variables are readily accessible to experimental control, which can therefore be tuned to map out the parton distribution functions. Theoretically, it is also a fertile ground for testing the predictions of quantum chromodynamics (QCD). Indeed, it is the possibility of a high degree of interplay between theory and experiment that has perhaps stimulated the recent activities in this subject.

An inclusive reaction in hadron-hadron collisions in which only a pair of leptons is detected depends in total seven independent variables. They are (1) the initial c.m. energy squared s; (2) the invariant mass of the lepton pair M; (3) the rapidity y or Feynman variable x_F of the lepton pair; (4) the transverse momentum q_T of the lepton pair; (5) the angle θ of the leptons in the lepton rest frame in either the Gottfried-Jackson or helicity coordinate system; (6) nature of the beam particle; and (7) nature of the target. A complete description of the inclusive cross section would require the use of a multidimensional space. However, most of the essential features of the lepton-pair production (LPP) process can be conveyed by the description of how $d\sigma/dMdy$ and $\langle q_T \rangle$ separately depend on M, s, and y. This is a 2 x 3 matrix of data presentation which we shall go through systematically in this review. We shall separately discuss the beam and target dependences first. Angular distribution in θ as well as other topics will be considered at the very end.

Contrasting $d\sigma/dMdy$ against $\langle q_T \rangle$, it should be remarked that the former is an inclusive cross section integrated over q_T and therefore represents the bulk of the cross section. On the other hand, $\langle q_T \rangle$, or more explicitly, $d\sigma/dMdydq_T$, contains information pertaining to the rarer events at large q_T. As is common in hadronic reactions, certain dynamical features that are manifest in rare events are washed out in the integrated results so that the mechanism responsible for the dominant features may be quite different from the physics governing the reactions in some fringe regions of phase space. For that reason we shall separate the discussion of the bulk cross sections from that of the q_T dependence. As it turns out, the former is far less controversial; consequently, we shall be brief. It is the latter that will occupy most of our attention.

Theoretical understanding of the LPP process is not on firm ground because many of the well-tested techniques useful in deep inelastic scattering, such as short-distance expansion and light-cone dominance, [2] are not applicable to LPP. The quark-antiquark annihilation mechanism of Drell and Yan [3] has been amazingly successful even though the reason for its success has been unclear, at least not until very recently. The Drell-Yan mechanism is constructed in the framework of the naive quark-parton model, [4] which in its own right is in need of modification to account for such effects as scaling violation. Some progress has been made in the past few months to elucidate the dominance of the Drell-Yan mechanism in the context of QCD, which is the only candidate theory of strong interaction at the constituent level. While many QCD diagrams are found in perturbative calculations to fall into the general category of Drell-Yan type annihilation process, there are kinematical regions in which they remain distinctly as non-Drell-Yan type diagrams. Since QCD itself, having no intrinsic scale, cannot specify those regions a priori, only phenomenology can determine where certain QCD diagrams are important. It is one of the aims of this review to make that phenomenological analysis and assess to what extent QCD has been tested in LPP.

There are, of course, a number of other models which claim to fit the LPP data. One of them is based on the constituent interchange model [5] (CIM), whose results agree remarkably well with all the data considered, when parameters determined in large-p_T reactions are used. Whether the agreement is maintained in view of the recent changes of the data both in LPP [6] and in large-p_T processes [7] remains to be seen. Although the success of CIM in LPP processes should not be overlooked, it will not be pursued further in this review. Interested readers are referred to Ref. 5). Another model discussed at this meeting [8] is thermodynamic in character and emphasizes the relationship between the production of hadrons and dileptons. Because it de-emphasizes the role played by the virtual photon, one expects it to have difficulty explaining the charge-dependent data, such as the π^+ to π^- beam ratio at high mass LPP. Again, we shall not return to this model in the remainder of this review.

Except for the next section where we shall discuss the theoretical basis for the Drell-Yan mechanism, attempts will be made in all subsequent sections to maximize the interplay between theory and experiment. Not all data will be discussed, obviously. We shall selectively present only the data that are relevant to some theoretical issues. On the whole we shall follow an outline which goes through the 2 x 3 matrix of data presentation mentioned earlier.

II. THE DRELL-YAN MECHANISM

How much can we believe in the Drell-Yan mechanism [3] as the dominant process in LPP? If this is a theoretical question, it can be answered only within the framework of some theory, and QCD is the only one available, which is what will be discussed below. If it is a phenomenological question, then the answer resides in checking whether the parton momentum distributions in LPP assuming the Drell-Yan mechanism are consistent with those in DIS (deep inelastic scattering). This aspect of the quark parton model will be considered in Section VII, after the theoretical and other phenomenological questions about LPP are settled.

Present technology in QCD is inadequate to answer fully the question of Drell-Yan dominance. One outstanding difficulty is the question of interaction among the spectator partons of the two hadrons. There must be such interactions at some level if the final state hadrons are all colour singlets. The initial and final state interactions can ruin Drell-Yan's factorizable form for direct quark-antiquark annihilation. The problem is difficult to handle in QCD because the hadronic wave functions are involved which cannot be treated precisely so far. In Regge theory some statement can be made: [9] all the Pomeron exchange terms cancel. Its relevance to the parton picture is, however, not at all clear. In the literature on partons and QCD all complications related to the spectator interactions have been ignored, as they were in the original work of Drell and Yan. [3]

What one can do in QCD is to calculate perturbatively the gluon corrections to the Drell-Yan process. Some of the low-order diagrams are shown in Fig. 2.1.

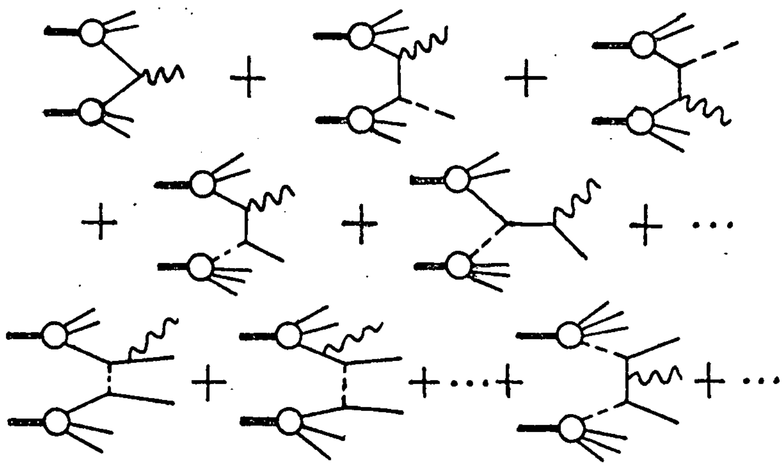


FIG. 2.1

In these diagrams the heavy, light, dashed, and wavy lines represent hadrons, quarks (q) [or antiquarks ($\bar{q}$)], gluons (g), and virtual photons, respectively. The bubbles represent the intrinsic parton distributions in the hadrons, containing all the low Q^2 confinement effects (such as the Fermi motion of the partons) that are not calculable in perturbation theory in QCD. It was shown by Politzer [10] (for the initial partons being $q\bar{q}$) and by Sachrajda [11] (for the partons being qg, qq, and gg) in the leading log Q^2 calculations that two important phenomena occur at high Q^2. One is that the contribution of all the diagrams in Fig. 2.1 to LPP can be put in a factorizable form so that the Q^2 dependence can be absorbed into the hadronic structure function as shown in Fig. 2.2. Thus in the leading log approximation the Drell-Yan

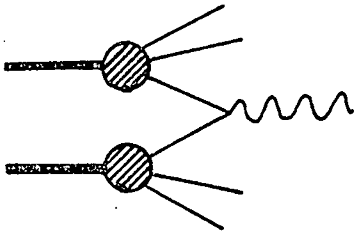


FIG. 2.2

form is recovered.[12] Moreover, the log Q^2 dependence of the structure functions in Fig. 2.2 (indicated by the cross-hatched bubbles) is exactly the same as that for electroproduction (in leading log) with $Q^2 = |q^2|$ in both cases. These features of QCD contribute partially to an explanation of why the Drell-Yan mechanism seems to work better than one would naively expect, as we shall see in the following sections.

The calculations referred to above are done by studying low order diagrams in perturbation theory. One could with good reasons ask about the importance of the higher-order gluon-correction terms. Such contributions are hard to calculate even in leading log approximation, since it involves power series in $\bar{g}^2(Q^2) \log Q^2$ where $\bar{g}^2(Q^2)$ is the square of the running coupling constant [13)14)] that varies inversely as $\log Q^2$; thus each order in perturbation theory gives a comparable contribution to the correction terms. Amati, Petronzio and Veneziano [15)] have developed a method for studying the problem, which promises to be able to generalize the Politzer-Sachrajda result to higher orders.

There is then also the question of non-leading log terms. It should be recognized that a power of $\log Q^2$ is developed each time an internal line of a tree diagram (with vertex and propagator insertions) is nearly on mass shell. [16)] It means that for hard gluon radiation the quark and emitted gluon must be nearly collinear. Thus in the leading log approximation all quarks and hard gluons must be restricted to a forward cone with a small half-angle δ.* In other words, the transverse momenta k_T of the partons are

*We shall refer to this in the following as the "narrow" gluon correction.

limited:

$$k_T^2 < \varepsilon Q^2 \qquad (2.1)$$

where ε is a fixed small number. This inequality arises becasue in an integration of dk_T^2/k_T^2 from p^2 to Q^2 it is the lower portion from p^2 to εQ^2 that gives rise to $\log Q^2/p^2$. Indeed, for a related reason, the jet structure in e^+e^- annihilation is also described in terms of cones.[17] Now, if a parton (quark or gluon) in a certain QCD diagram is forced to have a large k_T, such as in a process in which a large q_T dilepton is detected, then one of the internal lines must be far off shell, and one loses a power of $\log Q^2$. Such terms are not included in the leading log computation, and one must calculate separately and explicitly hard-scattering diagrams. There can, of course, still be further gluon corrections to the basic hard-scattering subprocess. Again, in leading log approximation (i.e. collinear gluon emission) of low-order diagrams Sachrajda shows in a separate paper [18] that the effects of narrow gluon corrections to a hard-scattering process are also factorizable and can be absorbed into the distribution (or fragmentation) functions for the initial (or final) partons undergoing the hard collision. The implication for LPP is then that when a detected lepton-pair has a large q_T^2, say, of order Q^2, one should calculate explicitly the Born diagrams, i.e. second to fifth, in Fig. 2.1 with the bubbles cross-hatched as in Fig. 2.2 so that the Q^2 and small k_T dependences due to the narrow gluon corrections can be taken into account. The last three diagrams of Fig. 2.1 are thereby automatically taken into account if the gluons are near mass shell.

In the investigation of the factorization question in perturbation theory [10)11)15)18)] it is the integrated cross section (integrated over k_T of all narrow gluon corrections) that is studied. The resulting Q^2 dependence of the structure function is then found to be the same (in leading log) as for νW_2. It has not been possible to determine what the k_T dependence is of the distribution function $G(x, k_T, Q^2)$ before the integration over k_T. That dependence is, of course, of crucial importance to the calculation of $\langle q_T \rangle$ especially for the direct $q\bar{q}$ annihilation process of Drell-Yan. In the absence of any definitive prediction in QCD and in anticipation of the phenomenology to follow, let me identify two components of parton k_T in $G(x, k_T, Q^2)$, the meaningfulness of which is only a conjecture at this point. One is the intrinsic component $\langle k_T \rangle_o$ usually taken to the roughly 300 MeV/c. It is due to the Fermi motion or binding effects in the hadron, probably independent of Q^2 but may depend on x.[19)20)] The other component,

$\langle k_T \rangle_{narrow}$, is due to hard gluon bremsstrahlung in a narrow cone; it can be Q^2 dependent but at most to the extent of (2.1). How small ϵ is or how narrow the cone is cannot be answered within QCD. An estimate of 10° to 15° by Sterman and Weinberg [17] for jets in e^+e^- annihilation is not unreasonable here. The two components collectively form the hadronic $\langle k_T \rangle_{had}$ which is an estimate of the total transverse spread of the momenta of the partons in a hadron with gluon interactions as probed by deep inelastic scattering. The corresponding distribution function $G(x, k_T, Q^2)$ is to be used in any parton-model calculation that needs a description of the supply of partons in a hadron.

For LPP with "small" values of q_T, i.e. not much greater than $\langle k_T \rangle_{had}$, Drell-Yan picture applies, provided that the above $G(x, k_T, Q^2)$ is used. For significantly "larger" q_T the Drell-Yan mechanism cannot be trusted, so explicit non-Drell-Yan type diagrams must be calculated. Indeed, to test some clean predictions of QCD it is necessary to go outside the hadronic ("small") region. Since QCD in itself does not provide the range of $\langle k_T^2 \rangle_{had}$, only phenomenology can determine where "small" q_T ends and where "large" q_T begins. A significant portion of our discussion later on will be addressed to this problem.

For now we need some theoretical framework in which to begin discussing the data. Since the q_T distribution of the dilepton cross section falls off rapidly with increasing q_T, the bulk of the cross section at present energies is in the region where the hadronic k_T is mainly responsible for the q_T. (Some aspect of the phenomenological conclusion to be reached later on has been used in making the last statement.) Thus we shall adopt as our working hypothesis that for the integrated (over q_T) cross section, $d\sigma/dMdy$, the Drell-Yan mechanism dominates. For $\langle q_T \rangle$, on the other hand, or more particularly for $d\sigma/dMdydq_T^2$, we leave it as an open question to be examined in detail.

For later use let us collect here some equations expressing the Drell-Yan process in various ways. We include the color factor 1/3, and use $G_h^{f,\bar{f}}(p,k,q)$ to denote the distribution function for a quark (or antiquark) of flavor f (or $\bar{f}$) in a hadron h. It depends on the invariants built out of the momenta of the hadron p, parton k, and photon q. [21] The inclusive cross section in the Drell-Yan approximation is

$$\frac{q_0}{d^3q}\frac{d\sigma}{dM^2} = \frac{2\pi\alpha^2}{9M^2}\sum_f e_f^2 \int \left[G_{h_1}^f(p_1,k_1,q)G_{h_2}^{\bar f}(p_2,k_2,q) + (f\leftrightarrow\bar f)\right] \cdot \delta^4(k_1 + k_2 - q)\,\frac{d^3k_1}{k_1^0}\,\frac{d^3k_2}{k_2^0} \tag{2.2}$$

where e_f is the charge of the quark of flavor f in unit of e. If k_T is ignored and $\vec{q}$ integrated, then (2.2) yields

$$\frac{d\sigma}{dM^2} = \frac{4\pi\alpha^2}{9M^4}\int_0^1 dx_1 \int_0^1 dx_2\ \delta(x_1x_2 - \tau)\sum_f e_f^2\left[F_{h_1}^f(x_1,M^2)F_{h_2}^{\bar f}(x_2,M^2)+(f\leftrightarrow\bar f)\right] \tag{2.3}$$

where

$$\tau = M^2/s \tag{2.4}$$

$$x_i = q^2/2p_i\cdot q \tag{2.5}$$

$$F_h^{f,\bar f}(x,M^2) = \int G_h^{f,\bar f}(q^2=M^2)\,d^2k_T \tag{2.6}$$

Finally, if the lepton pair is detected at y = 0, we have

$$\left.\frac{d\sigma}{dMdy}\right|_{y=0} = \frac{4\pi\alpha^2}{9M^3}\sum_f e_f^2\left[F_{h_1}^f(x,M^2)F_{h_2}^{\bar f}(x,M^2) + (f\leftrightarrow\bar f)\right] \tag{2.7}$$

where $x = \sqrt{\tau}$.

III. TARGET AND BEAM DEPENDENCES

Most experiments on LPP are done using nuclear targets. The dependence of the cross section on the atomic number of the target may be parametrized as

$$\frac{d\sigma}{dM} \propto A^{\alpha(M)} \tag{3.1}$$

The variation of $\alpha(M)$ with M is shown in Fig. 3.1 with data from various experiments [22)-27)] using both proton and pion beams. It is evident that for M ≥ 3 GeV the dependence on A is nearly linear, signifying incoherence of scatterings from individual nucleons. Under such circumstances one can admit the possibility of the Drell-Yan mechanism to work since impulse approximation cannot be ruled out. But for M ≤ 3 GeV, α deviates significantl from one, so the nuclear coherence and rescattering effects are not negligible The impulse approximation needed for the parton model is then not justified. Thus the Drell-Yan mechanism should not be applied to LPP with M ≤ 3 GeV; failure of the Drell-Yan formula to account for dσ/dM in that region is therefore not unexpected.[28),29)] In the following we shall not attempt to interpret the data for M ≤ 3 GeV when nuclear targets are used. We await with great interest the results from ISR[30)] in that mass region, especially when the statistics can be improved.

The dependence on the beam type is as dramatic as it is effective in illustrating the utility of the simple quark-parton model and the general validity of the Drell-Yan mechanism. Consider the cross sections for anti-proton vs. proton beams. An antiproton has much more antiquarks at large x than does a proton, so the $\bar{q}q$ annihilation process at large τ (thus involving large x partons) is much more enhanced for the $\bar{p}$ as compared to the p beams. One therefore expects $\sigma(\bar{p})/\sigma(p)$ to increase significantly with τ. The beam dump experiment at CERN SPS with Ω spectrometer verifies this increase despite their poor statistics.[31)] Similar situation is true for σ(π) when compared to σ(p), as shown in Fig. 3.2;[31)32)] a ratio of 330 was reported by Pilcher[26)] for M = 10.5 GeV at p_{beam} = 225 GeV/c.

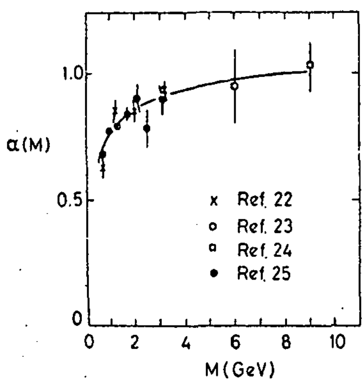


FIG. 3.1

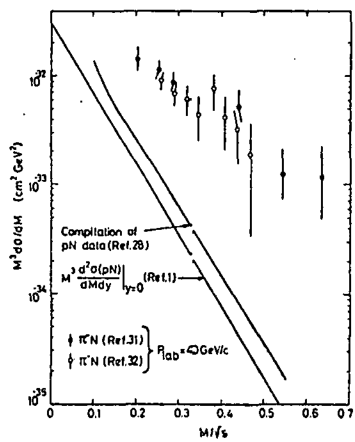


FIG. 3.2

Another aspect of the quark-parton model (QPM) concerning the charges of the constituents is also tested in LPP if one examines the $\sigma(\pi^+)/\sigma(\pi^-)$ ratio for isoscalar targets. The valence quarks of π^+ are $u\bar{d}$, while those of π^- are $d\bar{u}$. The annihilation of the antiquarks with the corresponding quarks in a nucleon at large x should lead to a ratio of the squares of the charges e_d^2/e_u^2. Thus one expects $\sigma(\pi^+)/\sigma(\pi^-)$ to approach 1/4 as $\tau \to 1$. This is verified by the recent data [25),26),31)] as shown in Fig. 3.3.

One must conclude here that the Drell-Yan mechanism is operative for the major part of the integrated cross sections.

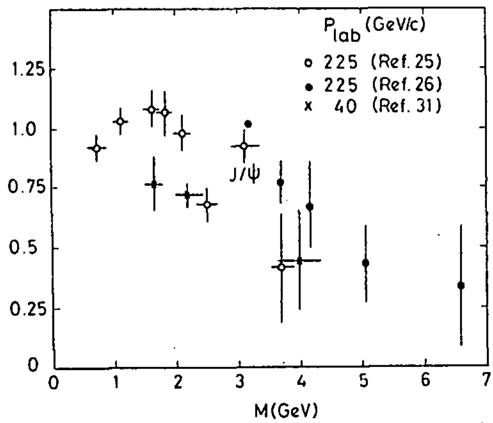


FIG. 3.3

IV. THE INCLUSIVE CROSS SECTION dσ/dMdy

We now consider the bulk of the cross section dσ/dMdy (integrated over q_T) and discuss its dependences on M, s and y in that order. As we have argued in Section II, for the integrated cross section we shall think in the framework of the Drell-Yan process whenever necessary. We shall see that the data offer no critical test of the model and can be readily accommodated by it.

At fixed y, particularly at y = 0, the dependence of the cross section on M has been well measured up to M ≈ 14 GeV both for proton [6),27)] and pion beams. [26)] Because of the unknowns contained in (2.7), the data by themselves cannot provide a precise check on the validity of the Drell-Yan process. In fact, the CFS data [6] have been used in conjunction with νW_2 to determine the antiquark distribution; depending on what is used, a number of fits are possible. [6),33),34)] Based on the q and $\bar{q}$ distributions of the proton so determined, one could then use the CP data [26] to map out the parton distributions of the pion, a result that will surely be forthcoming before long.

The dependence on the beam energy has also been studied by both the CP and CFS groups. [24),27)] Fig. 4.1 shows the result for proton beam at y = 0. [27] To exhibit scaling the data at y = 0.2 have been plotted in terms of the dimensionless quantity $M^2 d\sigma/d\sqrt{\tau}dy$ versus $\sqrt{\tau}$, as in Fig. 4.2. One sees that the data satisfy scaling amazingly well - in fact, almost too well for comfort. According to (2.7) and disregarding the small value of y, the quantity plotted should be a function only of $\sqrt{\tau}$, or x, except for the scaling violating M^2 dependences (hence s dependences for fixed τ) of the parton distribution functions, which are not negligible for the range of s explored. However,

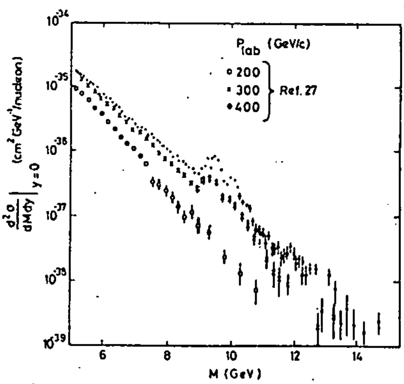


FIG. 4.1

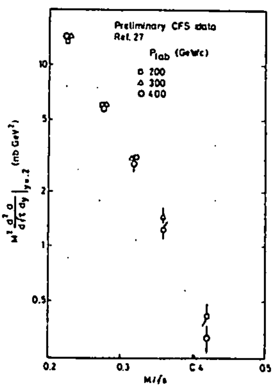


FIG. 4.2

we know from νW_2 probed in deep inelastic leptoproduction [35] that the scaling violation of the structure function has a pivotal point at x ≈ 0.2 and that for x > 0.2 it decreases with increasing Q^2. Qualitatively, this feature is preserved in the data shown in Fig. 4.2 if one identifies $x = \sqrt{\tau}$, although an exact parallel is not expected since the quark and antiquark distributions (having different behaviours of scaling violation at large x) appear multiplicatively in LPP but additively in DIS (deep inelastic scattering).

There are also some data from ISR [36] at $\sqrt{s}$ from 28 to 62 GeV. Although the error bars are large, their values for $M^3 d\sigma/dMdy$ at y = 0 for $\sqrt{\tau}$

from 0.08 to 0.25 are in general agreement with the CFS data. Thus on the whole one can say that scaling seems to work rather well in LPP.

The y or x_F dependence of the cross section for various bins of M values greater than 3 GeV is now also known from the CP and CFS experiments. [26),27)] Because of the extra $\bar{q}$ content in the pion, the x_F dependences are expected to be quite different for the π and p induced cross sections. The new CP data ought to be able to put enough constraints on the Drell-Yan formula (2.2) to determine the parton distribution functions for the pion.

In using the large x_F data one must be careful about kinematical limitations since the annihilating quarks and antiquarks may not originate from separate hadrons as in the usual Drell-Yan picture, but from the same hadron. [37)] In that case the $q\bar{q}$ annihilation process is then closely related to the $q\bar{q}$ recombination process [38)] responsible for the production of mesons at large x_F. [39)-42)] The ratio of lepton-pair to π^0 or ρ^0 production at large x_F with nucleon target should then provide a direct clue to the recombination function.

The CFS data also show the y distribution for both positive and negative values of y. The asymmetry shown in Fig. 4.3 for $\sqrt{\tau}$ between 0.2 and 0.25 is at first sight larger than is anticipated from the use of nuclear targets.

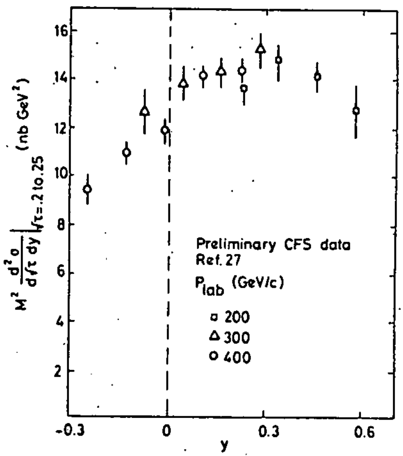


FIG. 4.3

However, a quick estimate can be made if, for extreme simplicity, we assume that all quark distributions are alike in protons as well as in neutrons, and similarly for all antiquark distributions. Since no asymmetry can arise when the beam proton strikes a proton in the target, we need only consider the pn collision. Counting the squares of the charges only, on the proton side uud leads to (4+4+1)/9, while on the neutron side udd leads to (4+1+1)/9. Assuming equal proportions of p and n in the target nucleus, we see that the excess on the beam side is 3 out of 18, which is not negligible. This estimate is extremely crude but is sufficient to infer that the observed asymmetry presents no puzzle. A quantitative calculation in the Drell-Yan picture is straightforward.

V. TRANSVERSE MOMENTUM OF THE LEPTON PAIR

There are two aspects about the average transverse momentum $\langle q_T \rangle$ of the dileptons that are noteworthy. The CFS data [6)] at 400 GeV shown in Fig. 5.1 serve to illustrate the situation. First, the value of $\langle q_T \rangle$ is about 1.2 GeV/c for $M \geq 4$ GeV, a value much higher than is naively expected. Secondly, over the same range in M it is remarkably flat except for the Υ point a phenomenon that needs explanation. The data have stimulated a great deal of theoretical activities because model calculations can be made on the subject with predictions that can be compared with data. There have also been hopes that QCD can be tested. Since there are a number of methods of treating the

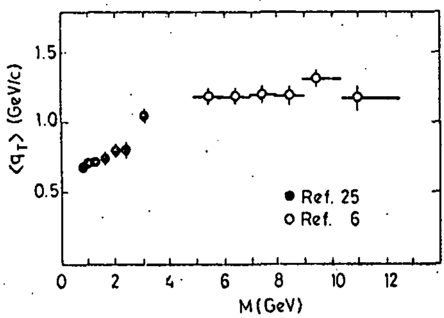


FIG. 5.1

problem even within the context of QCD, I shall organize the discussion by first identifying two distinct approaches, commenting on the difficulties in each, and then making a conclusion based on phenomenological fact. In order to be relevant to the data available now or forthcoming, it is important to bear in mind what is calculable and what is not, and to distinguish the two at present energies.

A. BACKDOOR APPROACH

This is the approach along the avenue of space-like q^2. Calculate the average parton transverse momentum $\langle k_T\rangle$ and then infer the dilepton's $\langle q_T\rangle$. Ref. 43)-46) are examples of considerations along this line. Schematically, the steps followed are:

$$QCD \xrightarrow{(a)} DIS \xrightarrow{(b)} R \xrightarrow{(c)} \langle k_T^2\rangle \xrightarrow{(d)} \langle q_T^2\rangle$$

It is in the last step that the Drell-Yan mechanism is assumed and the backdoor crossed. In Ref. 46) steps (b) and (c) are short-circuited. We remark now on each of the steps.

(a) QCD $\longrightarrow$ DIS

QCD, being an asymptotically free gauge theory, [13),14)] has been successfully applied to DIS where operator product expansion and light-cone ideas are useful. [2)] Scaling violations of the moments of the structure fucntion can be calculated using renormalization group method and they are not inconsistent with the present data, [47)] although one cannot claim on that basis that QCD has been successfully tested.

(b) DIS $\longrightarrow$ R

Here R is

$$R = \frac{\sigma_L}{\sigma_T} = (1 + \frac{\nu^2}{Q^2})\frac{W_2}{W_1} - 1 \equiv \frac{F_L}{F_T} \qquad (5.1)$$

This ratio is zero in the lowest order and requires a calculation of the one-loop corrections to the Wilson coefficients. [48)-50)] Because the latter are related to the moments of the structure functions, an inversion to F_L and F_T themselves results in an integral relation, [51)] which in the lowest order for F_T is

$$F_L(\omega,Q^2) = 4C_2(R)\frac{\bar{g}^2}{16\pi^2}\;\frac{1}{\omega^2}\int_1^{\omega} d\omega'\;\omega'\;F_T(\omega',Q^2) \qquad (5.2)$$

where $\omega = 1/x$, and $C_2(R)$ is the quadratic Casimir operator evaluated in the representation R for the quarks. Eq. (5.2) is given here mainly to show that the relationship between F_L and F_T is not simple. In the limit $\omega \to 1$, the integral is approximated by a linear dependence on $\omega - 1$, and one obtains

$$R \sim C\frac{1 - x}{\log Q^2/\Lambda^2} \quad , \quad x \to 1 \tag{5.3}$$

where the log factor appears because of the running coupling constant $\bar{g}^2$. Among other things the constant C depends on the rate at which F_T vanishes as $\omega' \to 1$. For lack of a more reliable but simple formula, (5.3) has been used as a QCD description of R for the entire range of x from 0 to 1. The value of C is therefore devoid of precise meaning. Depending upon the user of (5.3) or other considerations, various values have been assigned to it: C = 16/25 in Ref. 43), 1/2 in Ref. 45) and later 1/"4" in Ref. 52). The prediction of QCD as expressed in (5.3) is thus unreliable in both normalization and x dependence.

More careful numerical calculations of R have been carried out [50),53)] in the framework of QCD and renormalization group techniques. The results are consistently and significantly lower than the old data [54),55)] particularly at high x. The new data [56)] give an even higher value of R (0.25 ± 0.10 when averaged with the R value using the old data), so that the discrepancy is even worse. To account for the difference Nachtmann[47)] found that a fit of the old data would require the addition of a component attributable to the intrinsic transverse momenta of the partons, which he took to be

$$\langle k_T^2 \rangle_o = 0.5x \ (\text{GeV}/c)^2 \tag{5.4}$$

Note the x dependence. The new data on R would require an even larger $\langle k_T^2 \rangle_o$ which should not vanish at x=0. [*)]

(c) $R \longrightarrow \langle k_T^2 \rangle$

The discussion immediately above refers to a relationship between R and the parton $\langle k_T^2 \rangle$, which in the naive quark-parton model (QPM) is [4)]

$$R = \frac{4\langle k_T^2 \rangle}{Q^2} \tag{5.5}$$

---

*) According to Quirk (private communication) the CHIO group has measured in deep inelastic muoproduction a value of R = 0.5±0.2 at x=0.025 for Q^2 in the range 1-4 $(\text{GeV}/c)^2$.

where the quark confinement effects are included in $\langle k_T^2 \rangle$. Combining with (5.3) one has [52]

$$\langle k_T^2 \rangle = \frac{C}{4} \frac{(1 - x)}{\log Q^2/\Lambda^2} Q^2 + \langle k_T^2 \rangle_o \quad . \tag{5.6}$$

The validity of this expression depends not only on how good (5.3) is as an approximation for R, but also on (5.5) which is good only if $\langle k_T^2 \rangle$ is very small, as is usually assumed in QPM. Indeed, on the basis of (5.5) one ordinarily expects R to vanish at high Q^2. However, if R shows no decisive dependence on Q^2, as is apparent in the data, [56] or depends on Q^2 only very mildly as in (5.3), then $\langle k_T^2 \rangle$ must increase with Q^2, contrary to the assumption in QPM. The validity of (5.5) is then called into question. Indeed, the description of the QPM itself would need serious modification. In QCD the relationship between R and $\langle k_T^2 \rangle$ is not well understood.

(d) $\langle k_T^2 \rangle \longrightarrow \langle q_T^2 \rangle$

In this last step Drell-Yan mechanism is the key link between the parton k_T and the dilepton's q_T. According to the results of Politzer [10] and Sachrajda [11] discussed in Section II, the Drell-Yan formula is made more acceptable if the parton distributions contain the Q^2 dependences due to gluon corrections, same as in DIS. How much k_T is allowed in the same parton distributions as a result is not clear, except that in the leading log calculations only "small" k_T in the narrow cone satisfying (2.1) is included. To apply the parton $\langle k_T^2 \rangle$, as calculated in step (c) above, to the Drell-Yan formalism and then to infer the dilepton's $\langle q_T^2 \rangle$ is a procedure based on the assumption that the calculated $\langle k_T^2 \rangle$ is indeed "small". Diagrams in which some parton propagators are far off shell do not contribute to the leading log terms; their effects on LPP are not factorizable and thus not re-expressible in the Drell-Yan form. If one examines the calculations leading to (5.2) and therefore to the determination of R, [16), 48)-51] one finds that it is precisely the non-leading log terms that are responsible for the answer, i.e. the parton k_T is not restricted to the narrow cone. Hence, it is in principle inconsistent to apply the Drell-Yan picture to the component $\langle k_T^2 \rangle_R$ [the first term on the r.h.s. of (5.6)] obtained from R. However, in practice, it may be loosely regarded as an approximate way of estimating $\langle q_T^2 \rangle$ from renormalization-group-improved perturbative calculation of R.

Overlooking the above reservation we proceed to a discussion of what has been obtained for $<q_T^2>$. The Drell-Yan formula, given in (2.2), involves in general, a rather complicated convolution of the transverse momenta of the annihilating partons over the surface of an ellipsoid. If k_T is infinitesimal (which is not the case at hand), the convolution integral can be simplified and the following approximate relation obtains

$$<q_T^2> \;=\; <k_{1T}^2> \;+\; <k_{2T}^2> \tag{5.7}$$

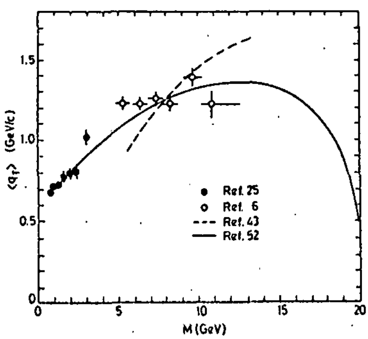
where the two terms on the r.h.s. refer to the $<k_T^2>$ for the two hadrons at the appropriate x values. For the lepton pair at $x_F=0$, and for identical hadrons, it reduces to

$$<q_T^2> \;=\; 2<k_T^2> \tag{5.8}$$

We stress that this formula is valid only for infinitesimal $<k_T>$, and that significant departure from it actually prevails [33] for the kinematical range of available data.

Comparisons with data have been made on the basis of (5.6) and (5.8). In Fig. 5.2 are shown two theoretical curves assuming $<q_T> = <q_T^2>^{\frac{1}{2}}$: the dashed line [43] is for C = 16/25 and $<k_T^2>_o = 0$, while the solid line [52] is for C = 1/4 and $<k_T^2>_0 = (0.3)^2$ $(GeV/c)^2$. The agreement is not outstanding.

FIG. 5.2



As we have seen above, the predictive power of QCD in this approach is weakened by various uncertainties, not the least of which are: (i) the value of C, (ii) the x dependence of R, (iii) the size of $<k_T^2>_o$, (iv) the x dependence of $<k_T^2>_o$, (v) the validity of (5.5), and (vi) the departure from (5.8). If we take seriously Nachtmann's attempt [47] to make up for the difference between the calculated R in Ref. 50) and the data, but do it for the new value, [56] then we would conclude that the part calculable in QCD is overwhelmed by the part due to the hadronic component $\langle k_T^2 \rangle_{had}$ that is not calculable. In that case one might as well fit the data out right. This is not to say that QCD is at fault. It merely means that at present energies the kinematical variables do not extend to the region where the part calculable dominates over that which is not.

Lam and Yan [46)] circumvent the issue of R by generalizing the evolution equation of Altarelli and Parisi [57)] and studying the generation of the parton k_T in the same way that scaling violation is generated by the renormalization group method. This ought to be the proper way of investigating the problem in QCD if the Drell-Yan mechanism is assumed. There are questions of uniqueness of the generalized master equation (particularly about the kernel) that requires further investigation. Assuming a simple form for the kernel and using an intrinsic $\langle k_T \rangle_0$ of 300 MeV/c, [58)] a reasonable fit of the data can be obtained upon applying (5.8) to infer $\langle q_T \rangle$.

It should be mentioned that Soper [59)] also followed the backdoor approach but in a very different way. Working in a renormalizable field theory that is not exactly asymptotically free, he studied the large k_T behaviour of the parton distribution function using light-cone techniques in operator-product analysis. Terms falling off as k_T^{-2} and k_T^{-4} are found, whereupon q_T^{-2} and q_T^{-4} behaviour are inferred for the lepton pair. Whether these behaviours attributed to the Drell-Yan mechanism are distinct from the non-Drell-Yan processes at large q_T is not clear.

B. FRONTDOOR APPROACH

Having reviewed the backdoor approach, it should come as no surprise that one could also approach the subject of $\langle q_T \rangle$ directly, keeping q^2 time-like throughout. By studying some low-order perturbation diagrams, one can, in fact, calculate $d\sigma/dMdydq_T^2$. Comparison with the data on q_T distributions would be a far more stringent test of QCD than that with $\langle q_T \rangle$. Besides, the Born term calculations are clean and not too difficult to do. Although one should bear in mind that cleanliness is no guarantee for relevance, they certainly should be done. The idea did not occur to just one or two, but has led to a multitude of papers, Ref. 60) to 64) being only a few examples of the collection.

The diagrams to be considered are those shown in Fig. 2.1, although in actuality only the calculations to the lowest nontrivial order in $\bar{g}$ have been done, i.e. the second to fifth diagrams of Fig. 2.1. We recall that all the diagrams in that figure can be put in the Drell-Yan form, Fig. 2.2, in the leading log Q^2 approximation, [10),11)] which means "small" k_T and q_T. To put Q^2 dependent distribution functions in the diagrams of Fig. 2.1 would be double counting for small q_T. However, if one is only interested in large q_T, then the distribution functions should include Q^2 and "small" k_T dependences. Calculating in that way for large q_T it is hoped that the second to fifth diagrams of Fig. 2.1 give clean predictions of QCD that can be compared with experiment.

Clearly, the two approaches are distinctively different. The backdoor approach depends on the Drell-Yan mechanism, while the frontdoor approach is mainly interested in the non-Drell-Yan processes. The two are actually complementary. On the other hand, if $\langle k_T^2\rangle_R$ is recognized as basically non-Drell-Yan in origin as we have discussed, then that part of the contribution in the backdoor approach is not unrelated to the calculation in the present approach.

We now examine more closely the issues involved in the perturbative calculations of the non-Drell-Yan processes. What has thus far been done is to assume no k_T for the initial partons of the subprocesses ($q\bar{q}$ annihilation and qg "Compton scattering"), and then to convolute the computed cross sections of the subprocesses with the appropriate parton distribution functions whose x dependences are partly inferred from DIS and partly guessed. All calculations suffer from the trouble at small q_T where, owing to the masslessness of the quarks and gluons, the cross section $d\sigma/dMdydq_T^2$ diverges as $1/q_T^2$, as $q_T \to 0$. It is a reflection of the fact that a massless quark can emit or absorb a massless gluon collinearly without violating energy-momentum conservation. Since in reality the cross section does not diverge, and for reasons already stated regarding Drell-Yan, the result is not to be trusted at small q_T. However, it has been hoped [60)-64)] that the calculation gives a fair description of LPP at large q_T, in both normalization and shape. Fig. 5.3 shows the comparison of the theoretical predictions with data [6)] at y = 0; it is taken from Ref. 63), and is typical of results obtained also by others. [60)-62),64)] The agreement is regarded as a significant and favourable test of QCD. However, this may be an over-optimistic view, as I shall present arguments later to the contrary.

Because of the divergence at $q_T = 0$, computation of $\langle q_T \rangle$ is meaningless. To emphasize the high q_T portion of the distribution one should consider at least the second moment $\langle q_T^2 \rangle$, for which the role of the divergence is

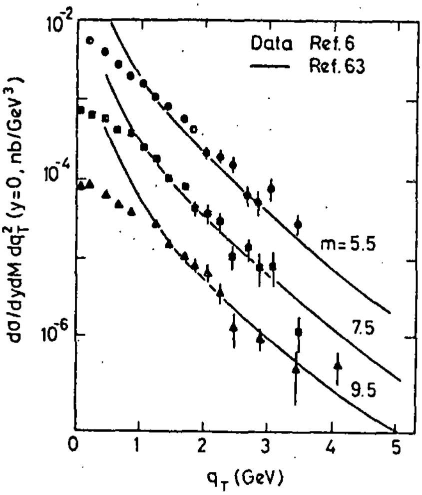


FIG. 5.3

suppressed. The computed result which we denote by $\langle q_T^2 \rangle_{pert}$ turns out to be too small compared to data.[62),63)] To fit the data a hadronic $\langle k_T^2 \rangle_{had}$, discussed in Section II, must be added, i.e.

$$\langle q_T^2 \rangle = 2\langle k_T^2 \rangle_{had} + \langle q_T^2 \rangle_{pert} \tag{5.9}$$

where (5.8) and the idea of Drell-Yan has been used. If $\langle k_T^2 \rangle_{had}$ is allowed to have a dependence on x or Q^2, which is unknown, then perturbative calculation in QCD has no predictive power on $\langle q_T^2 \rangle$. Assuming a constant $\langle k_T^2 \rangle_{had}$ the data in Fig. 5.1 can be fitted using[63)]

$$\langle k_T^2 \rangle_{had} = 0.4\ (\mathrm{GeV}/c)^2 \tag{5.10}$$

This is significantly larger than $\langle k_T^2 \rangle_o = (0.3)^2\ (\mathrm{GeV}/c)^2$ used by Politzer[52)] in connection with Fig. 5.2 or by Lam and Yan.[58)] It is consistent with the conclusion reached toward the end of the previous subsection. Thus we find here the first indication of a phenomenological support for the conjecture we made in Section II that $\langle k_T^2 \rangle_{had}$ includes not only the intrinsic $\langle k_T^2 \rangle_o$ but also a "narrow" component $\langle k_T^2 \rangle_{narrow}$ associated with the scaling violation of the parton distribution. The component calculated through R in the last subsection is over and above $\langle k_T^2 \rangle_{had}$, just like the contribution of the perturbative calculation in the present approach.

Because the two terms on the r.h.s. of (5.9) are comparable in magnitudes at present energies, that procedure of determining $\langle q_T^2 \rangle$ is unsatisfactory, except for the merit of rendering a very quick and rough estimate of the two effects. The narrow divergence at $q_T = 0$ cannot be ignored. It does not get cancelled by contributions from other diagrams as in the case of the "soft" divergences.[65),66)] The only work that has given attention to the problem (by giving a regularization procedure) is described in the second paper of Altarelli, Parisi and Petronzio.[62)] Because the implication of the regularization is relevant to our discussion below, we give here the essence of their procedure.

Let $F_{pert}(q_T) = d\sigma/dq_T^2\big|_{pert}$ be the q_T distribution determined in perturbative calculation. It is singular at $q_T = 0$. The integrated cross section, $\sigma_{pert} = \int_0^\infty F_{pert}(q_T)dq_T^2$, is therefore infinite. However, the true (regularized) distribution $F_{reg}(q_T) = d\sigma/dq_T^2\big|_{reg}$ has no singularity at $q_T = 0$, and the corresponding integrated cross section σ_{reg} is finite. If one believes that the only disease with F_{pert} is at small q_T, and that its large q_T behaviour is a faithful statement of reality, then a procedure of subtracting out the diseased small q_T part ought to yield F_{reg}. Suppose that the realistic, but uncalculable, small q_T component, which arises out of the k_T distribution in the hadron, and which is responsible for the first term on the r.h.s. of (5.9), is described by $f(q_T)$, suitably normalized. Then, consider

$$\int dq'^2_T \, F_{pert}(q'_T) f(q'_T - q_T) - \sigma_{pert} \, f(q_T) \qquad . \qquad (5.11)$$

The first term is a convolution which installs the proper small q_T behaviour but carries the wrong normalization; the second has the same characteristics by construction. The two are both infinite quantities but the difference is finite. If $f(q_T)$ falls off faster than $F_{pert}(q_T)$, then (5.11) has the same large q_T behaviour as $F_{pert}(q_T)$. These are just the properties that one wants to ascribe to the realistic quantities in the combination

$$F_{reg}(q_T) - \sigma_{reg} f(q_T) \qquad . \qquad (5.12)$$

Hence, the identification of (5.11) with (5.12) defines $F_{reg}(q_T)$. This regularization procedure has been used to fit the data with $f(q_T)$ being an arbitrary function. Assuming $f(q_T)$ to be a Gaussian with a mean $\langle q_T^2 \rangle_{had} = 0.8$ GeV/c, corresponding to[67)]

$$\langle k_T \rangle_{had} = 0.66 \text{ GeV/c} \quad , \tag{5.13}$$

APP II [62] obtained a good fit of the data on q_T distributions. [6] The hadronic "small" k_T component needed is very close to that found in (5.10). The goodness of fit of the data has no great significance since an arbitrary function is at one's disposal, but the M dependence of the data seems to be well described by $F_{reg}(q_T)$ without the necessity of invoking an M dependence of $f(q_T)$.

The regularization procedure leads to the conclusion that the non-Drell-Yan type perturbative calculations in QCD can at present energies account for a portion of the q_T effects in agreement with the rough estimate made earlier. In quantitative terms it apparently accounts for approximately half of $\langle q_T^2 \rangle$ and for the part of $d\sigma/dq_T^2$ with $q_T > 2$ GeV/c. If it is true, then one ought to (1) come to terms with the conclusion made toward the end of the previous subsection regarding the Drell-Yan picture and (2) seek other phenomenological tests that are sensitive to the non-Drell-Yan component. The latter will be considered in the next section with results damaging to the conclusion just made. We shall attempt to show that the resolution of the dilemma will at the same time allow a co-existence between the two approaches considered in this section.

Before leaving this discussion, let us locate the loop hole in the argument that led to the conclusion of the regularization procedure, which incidentally is not unique, but is accepted here for argument's sake. The expression in (5.12) contains all the essential ingredients. The first term is the data, and the second is the hadronic component uncalculable in perturbation theory. The difference, identified with (5.11), is then the contribution from the non-Drell-Yan terms calculated. In APP II [62] $f(q_T)$ was assumed to be a Gaussian with an adjustable width. But it could just as well have been a power-law fall-off with a tail resembling the data. The point is that the more the second term of (5.12) resembles the data over a wider range of "small" q_T, the more the non-Drell-Yan contribution is pushed out to "larger" q_T. With an arbitrary function $f(q_T)$ to adjust, the data can be fitted in an infinite number of ways, especially since the parton distribution needed for the calculation of F_{pert} are mostly still adjustable also. At present there is no way a priori to determine where the "small" q_T range stops and the effects of the perturbative calculations emerge. The demarkation may even change with energy. Only phenomenology can give us further clues - at present and future energies.

VI. FURTHER PROPERTIES OF $\langle q_T \rangle$

In the preceding section we have dwelt mainly on the theoretical issues related to $\langle q_T \rangle$. They have not been resolved by the confrontation with data on $\langle q_T \rangle$ or $d\sigma/dq_T^2$ as functions of M. We now bring to bear on the problem other phenomenological facts, viz. their dependences on the beam type, x_F , and s. The data are all very new [26),27)] and provide a timely hint toward a more complete picture.

Let me first summarize the findings of the last section. In the back-door approach one identifies

$$\frac{1}{2} \langle q_T^2 \rangle = \langle k_T^2 \rangle = \langle k_T^2 \rangle_{had} + \langle k_T^2 \rangle_R \tag{6.1}$$

where $\langle k_T^2 \rangle_R$ is obtained through the QCD calculation of R, and $\langle k_T^2 \rangle_{had}$ is more than the intrinsic $\langle k_T^2 \rangle_o = (0.3)^2\ (\mathrm{GeV/c})^2$. While $\langle k_T^2 \rangle_{had} = \langle k_T^2 \rangle_o$ is assumed in Fig. 5.2, a sizeable difference between them is needed to fit the experimental value of R. In the frontdoor approach one has

$$\langle q_T^2 \rangle = 2\langle k_T^2 \rangle_{had} + \langle q_T^2 \rangle_{pert} \tag{6.2}$$

where $\langle q_T^2 \rangle_{pert}$ is obtained by perturbative calculation of explicit non-Drell-Yan diagrams. It is not clear to what extent $\langle q_T^2 \rangle_{pert}$ can be identified with $2\langle k_T^2 \rangle_R$, but they must in some way be related. Their difference is to be absorbed by the uncalculable $\langle k_T^2 \rangle_{had}$ in the two cases. Phenomenology discussed so far infers that $\langle k_T^2 \rangle_{had}$ is roughly 0.4 $(\mathrm{GeV/c})^2$ in both cases. According to our discussion in Section II, $\langle k_T^2 \rangle_{had}$ can be further decomposed into two components which we express here in a naive additive form as

$$\langle k_T^2 \rangle_{had} = \langle k_T^2 \rangle_o + \langle k_T^2 \rangle_{narrow} \tag{6.3}$$

While $\langle k_T^2 \rangle_o$ is identified with the static property of the hadron at low Q^2, $\langle k_T^2 \rangle_{narrow}$ is associated with the transverse momentum in the parton distribution confined to a "narrow" cone arising from gluon radiation. It is the parton's transverse spread in a jet, whether it be quarks in a hadron or hadrons in a quark (or gluon) jet. $\langle k_T^2 \rangle_{narrow}$ may depend on Q^2 but should satisfy (2.1).

Since the frontdoor approach is more direct and transparent, our discussion on the QCD results in the following will refer only to that type of perturbative calculations. Although the low-order calculation in perturbation theory is unambiguous, the procedure to eliminate the divergence at $q_T = 0$ is not, resulting in an uncertainty in the estimation of the actual contribution from the non-Drell-Yan process. The situation can best be illustrated by the schematic plots of two possibilities shown in Fig. 6.1 (a) and (b). The open regions under the curves represent the contributions

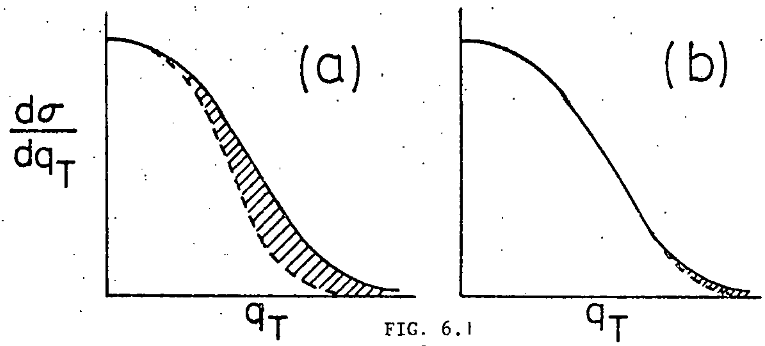


FIG. 6.1

from the hadronic "small component $\langle k_T^2\rangle_{had}$, the part described by $f(q_T)$. The shaded regions represent the effects of the non-Drell-Yan terms at "large" q_T. Data fitted by the overall (solid) curves admit the possibility of $f(q_T)$ being either (a) a Gaussian with smaller $\langle k_T^2\rangle_{had}$, or (b) a powerlaw fall-off with a larger $\langle k_T^2\rangle_{had}$. The conclusions of Ref. 62 to 64 suggest case (a), in which the shaded region contributes a significant fraction (roughly half) of $\langle q_T^2\rangle$. Case (b) is not ruled out, but we proceed with our discussion assuming (a) to be the case.

Consider now the dependence of $\langle q_T\rangle$ on beam type. Pilcher [26)] showed the CP data on π beam at 225 GeV and compared with the CFS data on p beam at 200 GeV. As can be seen from Fig. 6.2, $\langle q_T\rangle^\pi \approx 1.2$ GeV/c while $\langle q_T\rangle^p = 1.0$ GeV/c in the flat region. One does not know whether the difference of 0.2 GeV/c should be attributed to the hadronic $\langle k_T\rangle_{had}$ or the perturbative $\langle q_T\rangle_{pert}$ parts of the beam particles, or both. The difference $\langle q_T^2\rangle^\pi_{pert} - \langle q_T^2\rangle^p_{pert}$ can in principle be calculated if the parton distributions in the pion is known, and if $\langle k_T^2\rangle^\pi_{had} = \langle k_T^2\rangle^p_{had}$. It must be positive because the contributions to $\langle q_T^2\rangle_{pert}$ from the diagrams in Fig. 2.1 are enhanced by the excess antiquark in the pion. Thus the least one can conclude is that $\langle q_T^2\rangle_{pert}$ ought to be larger than $\langle q_T^2\rangle^p_{pert}$, but probably not as much as the entire difference implied by Fig. 6.2 since

since $\langle k_T \rangle^{\pi}_{had}$ may be larger than $\langle k_T \rangle^{p}_{had}$ too.

We now see, for both $\langle q_T^2 \rangle^p$ and $\langle q_T^2 \rangle^{\pi}$, that $\langle q_T^2 \rangle^{p,\pi}_{pert}$ and $2\langle k_T^2 \rangle^{p,\pi}_{had}$ share roughly equal proportions according to the argument above and the conclusion of the perturbative calculations discussed in Section V.-B. We

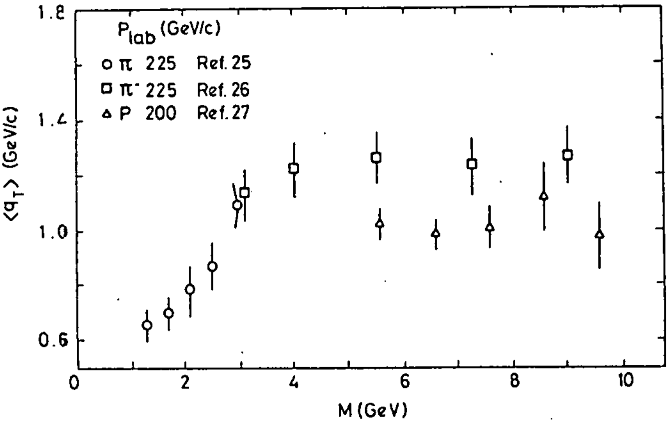


FIG. 6.2

come next to the y or x_F dependence of $\langle q_T \rangle$. Both Lederman [27)] and Pilcher [26)] showed data on that dependence and indicated that within errors they are essentially constant. (See Fig. 6.3.) This is highly significant, especially for the pion data [26)] since they cover a wider range of x_F. The independence of $\langle q_T \rangle$ on x_F contradicts the earlier conclusion that $\langle q_T^2 \rangle_{pert}$ is an important part of $\langle q_T^2 \rangle$ because it has been shown that $\langle q_T^2 \rangle_{pert}$ decreases dramatically with x_F. [63)] This is shown in Fig. 6.4. This behaviour may be understood as

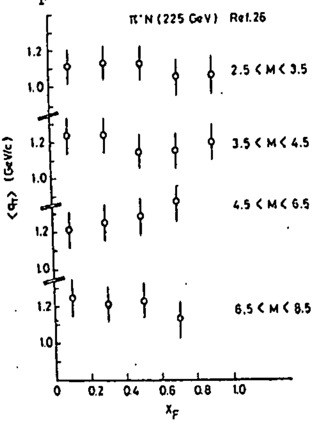


FIG. 6.3

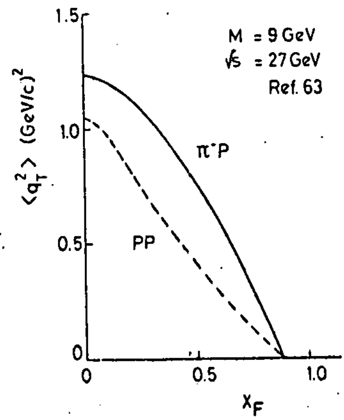


FIG. 6.4

a reflection of the fact that the subprocesses of the lower order non-Drell-Yan diagrams in Fig. 2.1 are all two-body scatterings. The two-body kinematics implies that large-angle scattering (hence large q_T) yields dileptons mainly at small x_F while small-angle scattering (small q_T) leads to dileptons at large x_F. There are complications due to convolution over the parton momenta; however, the dominance of $d\sigma/dMdx_F$ at small x_F makes this physical picture a reasonable interpretation of the numerical result [63] shown in Fig. 6.4.

From the contradiction between Figs. 6.3 and 6.4 one concludes that $\langle q_T^2\rangle_{pert}$ cannot be a significant part of $\langle q_T^2\rangle$. This conclusion would be even more inevitable if Fig. 6.3 were a plot of $\langle q_T^2\rangle$ rather than $\langle q_T\rangle$, although I doubt that the constancy of $\langle q_T\rangle$ does not reflect the same for $\langle q_T^2\rangle$. In order that $\langle q_T^2\rangle_{pert}$ be negligible at present energies, thereby negating the earlier conclusion, it is necessary to regard case (a) in Fig. 6.1 as unrealistic. That is, the "small" q_T component described by $f(q_T)$ need not fall off sharply as in a Gaussian. Assuming that it is damped more slowly like in a power law so that the observed q_T distribution follows closely $f(q_T)$ over most of the q_T range explored, as in case (b) in Fig. 6.1, the region where the perturbative calculations are relevant is then pushed out to higher values of q_T not yet measured. One then has

$$\langle q_T^2\rangle \simeq 2\langle k_T^2\rangle_{had} \tag{6.4}$$

The question is whether this is compatible with the independence on x_F.

The last question cannot be answered in the context of QCD since it is the uncalculable component. However, it arises due to the Drell-Yan mechanism. Recall from Fig. 5.1 that $\langle q_T^2\rangle$ is independent of τ at $x_F = 0$ where $x_1 = x_2 = \sqrt{\tau}$. Thus $\langle q_T^2\rangle$ is insensitive to which x regions of the parton distribution functions that contribute to the formation of the lepton pair. As x_F increases different x_1 and x_2 regions are probed. The insensitivity mentioned above then implies the approximate independence of $\langle q_T^2\rangle$ on x_F.

Comparing (6.4) to (6.1) infers that $\langle k_T^2\rangle_R$ is negligible also. This is not inconsistent with our discussion in Section V.-A. The calculated values [50),53)] of R are small compared to the old experimental values [54),55)] of R, let alone the new one. [56)]

In light of (6.4) the value of $\langle k_T^2\rangle_{had}$ must now be revised upward. This brings us to the question of s dependence. The CFS data [27)] shown in Fig. 6.5

exhibit an increase in $\langle q_T \rangle$ as P_{lab} ranges from 200 to 400 GeV/c. For the flat region the following parametrization is given

$$\langle q_T^2 \rangle = 0.7 + 0.0018s \ (\text{GeV/c})^2 \tag{6.5}$$

If $\langle q_T^2 \rangle_{pert}$ were important, one would naturally associate with it the s dependent term above, since it can be shown from perturbative calculation that [61)-63)]

$$\langle q_T^2 \rangle_{pert} = \alpha_s(Q^2) \ s \ h(\tau, \alpha_s) \tag{6.6}$$

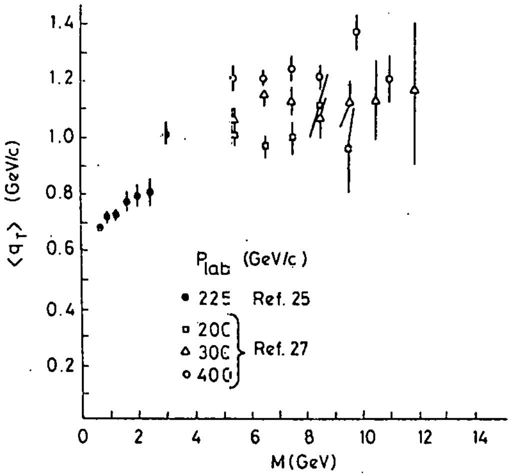


FIG. 6.5

where the scaling function $h(\tau, \alpha_s)$ does not depend on α_s at the lowest non-trivial order. Quantitatively, for the range of values of M currently explored, (6.6) is in the general vicinity of the second term in (6.5). It is unfortunate that this association cannot be made on account of the x_F independence. The burden is then on $\langle k_T^2 \rangle_{had}$ to produce the correct s dependence.

Recalling (6.3) we note that while $\langle k_T^2 \rangle_o$ is some fixed number independent of Q^2, $\langle k_T^2 \rangle_{narrow}$ can depend on Q^2, hence, s for fixed τ. Here we are relying on the conical structure of the jet (partons in a hadron) to allow $\langle k_T^2 \rangle_{narrow}$ to increase with energy, since a conical distribution has no inherent scale. Quantitative results have been obtained in a model study and are in agreement with the data. They will be discussed in the next section.

The conclusion we draw in this section is that the lepton pairs detected so far in hadron-hadron collisions are mainly produced by the direct annihilation of quarks and antiquarks in the parton jets of the incident particles. In consequence, the dileptons have an average $\langle q_T \rangle$ that is independent of x_F. In contrast, the perturbative calculations in QCD essentially study four jet events, two of which are remnants of the incident hadrons, the third is the lepton pair at a large angle, and the last one is the quark or gluon jet recoiling against the dilepton. The $\langle q_T \rangle$ of such processes are naturally sensitive to x_F. The four-jet events will no doubt emerge at high energy and should be looked for to check QCD. However, they constitute an insignificant portion of the events detected now. The situation is analogous to the large-p_T physics for pion production. Distinct QCD features such as p_T^{-4} have not yet appeared.[7)] For $p_T < 5$ GeV/c the parton transverse momentum plays an important part in the shape of the inclusive distribution. Similarly, in LPP one must also focus on the tail of the q_T distribution and study correlation between opposite jets to isolate the simple QCD effects studied in perturbative calculations.

VII. QUARK PARTON MODEL WITHOUT NEGLECTING PARTON k_T

Phenomenology of LPP at present energies has forced us to the view that the Drell-Yan mechanism is dominant and is repsonsible for almost the entire range of q_T measured. Thus the q_T of the dilepton owes its origin to the parton k_T in the distribution functions. The component of k_T due to quark binding cannot be reliably calculated; the other component due to gluon corrections in a narrow cone at high Q^2 should, in principle, be calculable in QCD just like the effects of scaling violation.[46)] However, at present there are no unambiguous results free from adjustable parameters. In the absence of any definitive description of the k_T dependence of the distribution functions $G(x,k_T,Q^2)$ in QCD, the problem of LPP can only be dealt with in a phenomenological investigation in the framework of QPM suitably generalized to account for non-negligible k_T. This, of course, would not be very meaningful if the Drell-Yan mechanism is found not to be dominant. The object of the investigation would then change from making predictions in QCD to checking consistency among all processes to which QPM is relevant and on which data are available. This has been done in Refs. 33) and 68); we give here some of the results of Ref. 33).

Since $G(x,k_T,Q^2)$ includes only the hadronic k_T components indicated in (6.3), the partons are all nearly on shell. It is only when a gluon is emitted with a large k_T that the associated quark goes off mass shell and a power of log Q^2 is lost in the calculation of the structure function. Thus in the generalization of the QPM we continue to use the usual assumptions that partons are on mass shell and impulse approximation applies. However, we let k_T to be non-negligible, while keeping it within the bound of

$$\langle k_T^2\rangle_{narrow} < \varepsilon Q^2 \quad . \tag{7.1}$$

This inequality is, of course, not very precise for our purpose here since ε is undetermined and Q^2 may not be very large. Its origin [cf. discussion following (2.1)] follows from the mathematical properties of divergent integrals as $Q^2 \to \infty$. For finite values of Q^2, ε need not be infinitesimal to keep the parton from going significantly off mass shell. Thus at "low" M^2, say $20(\mathrm{GeV}/c)^2$, $\langle k_T^2\rangle_{had}$ may well be as large as the observed $1\ (\mathrm{GeV}/c)^2$. The generalized QPM has been extended to include that region.

Because k_T is not negligible, a number of new features arise that are absent in the naive QPM. The Bjorken variable x is not necessarily the parton's longitudinal-momentum fraction, which in itself is no longer a Lorentz invariant. A new scaling variable z is needed to describe the transverse degree of freedom. Through z DIS and LPP can be kinematically related and unified by a common parton distribution function. In recognition of the conical structure of the k_T dependence, the distribution function G has been parametrized in terms of a radial scaling variable and an angle. It is found that in terms of a single G function (with appropriate separation of quark and antiquark components) all data on DIS and LPP can be simultaneously fitted; more specifically, they are νW_2, R, $d\sigma/dMdy$ at $y = 0$, and $\langle q_T\rangle$ vs. M.

It is also discovered that significant and interesting departure from (5.8) occurs for a wide range of M where data exist. This is shown in Fig. 7.1. It is a manifestation of the fact that k_T is not negligible and that the convolution in (2.2) over an ellipsoidal surface introduces the discrepancy between $\langle k_T\rangle$ and $\langle q_T\rangle/\sqrt{2}$. Note that where $\langle q_T\rangle$ is flat in accordance to the data, $\langle k_T\rangle$ increases with M, a behaviour reminiscent of Fig. 5.2. Thus in that figure the agreement with data is actually better than meets the eye.

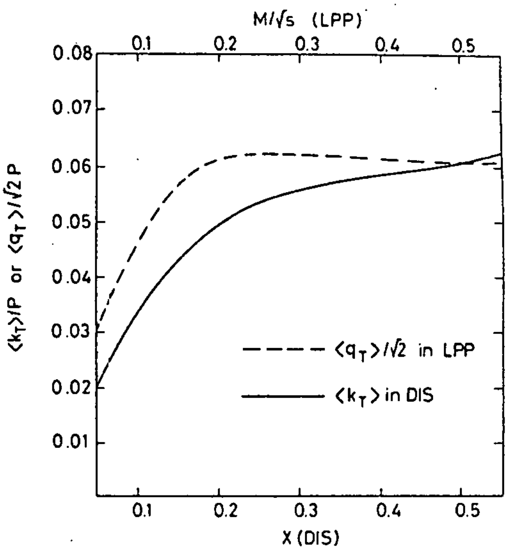


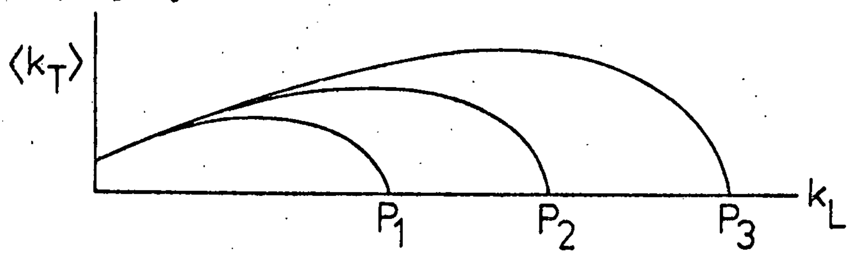
FIG. 7.1

Since the parametrization of G has no scale, the calculated $\langle q_T \rangle$ must increase with $\sqrt{s}$ for fixed τ. It is found that the value in the flat, plateau region behaves as

$$\langle q_T \rangle_{plateau} = 0.042 \sqrt{s} \tag{7.2}$$

at large s. This result was obtained before the data [27)] on energy dependence became known. It agrees with (6.5) rather well.

The conical structure of the parton distribution in a hadron can best be described in the Breit frame for DIS, which is free of the constraint between x and Q^2 in LPP. In that frame $k_L = Q/2$ and $x = k_L/P$, P being the momentum of the hadron. In a schematic drawing that exaggerates the transverse scale, the parton distributions for various values of P ($P_1 < P_2 < P_3$) may look like:



Apart from the kinematical turn-over at high k_L the relationship between $\langle k_T \rangle$ and k_L is basically conical. Plotted against x, $\langle k_T \rangle$ exhibits a "flying" sea-gull effect in that at fixed x it increases with Q. The independence of the dilepton's $\langle q_T \rangle$ on M is to be understood through the difference between $\langle k_T \rangle$ and $\langle q_T \rangle / \sqrt{2}$ as shown in Fig. 7.1. The increase with $\sqrt{s}$ is self-evident. It is anticipated that the same properties are possessed by the jet structure of parton decay into hadrons.

VIII. OTHER TESTS OF THE DRELL-YAN MECHANISM

In previous sections we have discussed how well the Drell-Yan mechanism works in LPP when all available data on the process are collectively taken into account. In this section we mention three possible tests to check the mechanism further. Corresponding experiments to carry out the tests are therefore suggested.

A. SUM RULES

So far in applying the Drell-Yan mechanism some adjustable functions describing the quark and antiquark distributions must be assumed to fit the data. Thus there are always some free parameters in the model. However, if the Drell-Yan formula is strictly correct, exact sum rules can be derived from it that are free from any adjustable parameters.[69] Confrontation with data can then provide a clean test of the dominance of the mechanism.

To be free from the uncertainties related to the distribution functions $G_h^{f,\bar{f}}$ in (2.2), we recall the exact sum rules well-known in leptoproduction

$$\int \frac{d^3k}{k_o} \left[G_{\bar{p}}^{f} - G_{p}^{\bar{f}} \right] = \begin{cases} 2 \\ 1 \\ 0 \end{cases}, \quad \text{for } f = \begin{cases} u \\ d \\ s \end{cases} \qquad (8.1)$$

and other similar ones for different hadrons. In order to make use of them in LPP, we consider the integral (for fixed M^2)

$$I = \int_0^1 \frac{d\tau}{\tau} M^4 \left[\frac{d\sigma}{dM^2}(\bar{h}_1 h_2) - \frac{d\sigma}{dM^2}(h_1 h_2) \right] \qquad (8.2)$$

where h_1 and h_2 stand for the beam and target hadrons, and $\bar{h}_1$ is the antiparticle of h_1. Substituting (2.3) into (8.2), one obtains

$$I = C \sum_f e_f^2 (N^f_{h_1} - N^{\bar{f}}_{h_1})(N^f_{h_2} - N^{\bar{f}}_{h_2}) \tag{8.3}$$

where $C = 4\pi\alpha^2/9$ and $N_h^{f,\bar{f}} = \int G_h^{f,\bar{f}}\, d^3k/k^o$. Now equations such as (8.1) can be used for various h_1 and h_2, yielding the following table for I:

$(\bar{h}_1, h_1)$	$h_2 = p$	$h_2 = n$
$\bar{p}, p$	$\frac{17}{9}C$	$\frac{10}{9}C$
π^-, π^+	$\frac{7}{9}C$	$\frac{2}{9}C$
K^-, K^+	$\frac{8}{9}C$	$\frac{4}{9}C$

These sum rules are independent of the details about the G functions, and of the value of M provided that it is high enough to warrant the Drell-Yan picture.

The integration in (8.2) is over τ for fixed M^2; thus it is effectively over s, which is not easy to do experimentally. If we neglect scaling violation which is not unreasonable in view of Fig. 4.2, (8.2) may be re-expressed in terms of an integral over M^2 for fixed s

$$I = \int_0^s dM^2\, M^2 \left[\frac{d\sigma}{dM^2}(\bar{h}_1 h_2) - \frac{d\sigma}{dM^2}(h_1 h_2)\right] \tag{8.4}$$

Here the two terms inside the square bracket must cancel at small M^2 since they diverge individually. If they do not cancel in the limit $M^2 \to 0$, it means that the observed $d\sigma/dM^2$ differs from the prediction of the Drell-Yan formula in a way which is not invariant under charge conjugation of the beam hadron; in that case the identification of (8.4) with (8.2), and consequently with the values in the above table, is invalid, and one is then forced back to the use of (8.2).

Verification of the sum rules provides an unambiguous affirmation of the applicability of the Drell-Yan mechanism.

B. ANGULAR DISTRIBUTION OF THE LEPTONS

One can also learn about the LPP mechanism by studying the angular distribution of the leptons in the rest frame of the lepton pair relative to some axis.[70)-73)] If that axis is along one of the incident hadrons, then it specifies the Gottfried-Jackson angle θ, which we use here for definiteness. In a Drell-Yan picture if the annihilating quark and antiquark have no transverse momentum relative to the incident hadrons, then the Gottfried-Jackson frame is the same as the c.m. system of the subprocess $q\bar{q} \to \gamma^* \to \mu^+\mu^-$ with the z axis along the initial partons. Hence, one expects

$$\frac{d\sigma}{d\cos\theta} \propto 1 + \cos^2\theta \,, \qquad (k_T = 0) \qquad (8.5)$$

On the other hand, if the parton transverse momenta are nonzero, it can be shown that (8.5) is modified to

$$\frac{d\sigma}{d\cos\theta} \propto 1 + A\cos^2\theta \,, \qquad (k_T \neq 0) \qquad (8.6)$$

where the coefficient A is less than one and depends on other kinematical variables of LPP. It is in the character of A that one hopes to find indications of the nature of the production mechanism.

If in a direct $q\bar{q}$ annihilation of the Drell-Yan type the only transverse momenta of the partons are "intrinsic" and fixed, e.g. $\langle k_T \rangle_0$, then A decreases with increasing M^2 and is therefore nonscaling.[71),72)] On the other hand, if the production mechanism is of the explicitly non-Drell-Yan type that involves hard gluon emission with large k_T, then A scales apart from logarithms[73),74)] i.e. it is a function of x_T and τ. However, there is the intermediate region corresponding to k_T being in the narrow cone, the situation which we have argued to be the predominant one. There is no explicit statement of the behaviour of A in that case. Further work is needed to map out the behaviours of A (in both normalization and shape) as functions of M^2, x_T and τ for all three cases mentioned above. Experiments on the angular distribution remain to be done.

C. HADRON PRODUCTION WITH DILEPTON TRIGGER

Another way to learn about the LPP mechanism is to use LPP as a trigger in studying the production of hadrons at low p_T but large x.[75)] A meaningful prediction about this type of correlation relies on a sensible model for hadron production in the fragmentation region. The parton recombination model[38)] has been successful[39)-42),76)] in giving a quantitative understanding of the meson inclusive distributions at low p_T. In that model a meson at at large x is produced by a fast valence quark of the incident hadron recombining with an antiquark from the sea of the same hadron. Thus if the same fast valence quark is needed for the dilepton trigger, then the meson distribution will obviously be seriously affected.

In Ref. 75), the Drell-Yan mechanism is assumed for LPP and the ratios of the semi-inclusive cross sections for the production of π^+ and π^- are calculated for various beam particles and for various values of τ of the dilepton trigger. The most dramatic feature predicted is in $\sigma(\pi^+)/\sigma(\pi^-)$ vs. τ for π^+p collision. As $\sqrt{\tau}$ increases from 0 to 0.4, the ratio at $x = 0.5$ increases by more than a factor 2. This is to be understood as follows. The π^+ beam particle has a $\bar{d}$ valence quark which is highly efficient in depleting the d valence quark of the target proton in forming a high τ lepton pair. The remaining valence quarks of the proton are both u-type which can readily produce π^+ at large x but not π^-.

Ref. 75) does not address itself to the question of testing the difference between Drell-Yan and non-Drell-Yan mechanisms. In the light of our discussion in Section VI it is not difficulty to extend their argument to provide such a test. Since the non-Drell-Yan processes are expected to be important only at large q_T, one should study, for example, the ratio $\sigma(\pi^+)/\sigma(\pi^-)$ in π^+p collisions mentioned above as a function of q_T for some fixed large x and large τ. As q_T increases, if the "Compton" subprocess $\bar{d} + g \rightarrow \bar{d} + \gamma^*$ at large angle becomes important, the d quark in the proton is not annihilated a good fraction of the time; consequently the ratio should decrease accordingly. The dependence of the ratio on q_T is therefore a good measure of the extent to which the "Compton" subprocess plays an important role. According to perturbative calculations in QCD [62)-64)] it is dominant at high q_T; the proposed experiment would determine how high is "high".

IX. CONCLUSION

We have reviewed the recent data on LPP and used them to examine and interpret the many theoretical papers that have been written on the subject. Our emphasis has been to extract from the theoretical results their physical relevance to the phenomenology of LPP at present energies. On the whole we have found that the Drell-Yan mechanism works extremely well; in fact, we have found no significant conflict with any data. Theoretically, QCD explains why it works better than one should naively expect.

QCD, however, also predicts that there are distinctly non-Drell-Yan processes which should become important at high q_T. Phenomenology has led us to conclude that the kinematical region in which it is dominant has not yet been reached at present energies. Thus we are in the unfortunate situation where what is needed theoretically to describe LPP (i.e. the parton distribution functions) cannot be calculated in QCD by perturbative or other means.

In the absence of meaningful and quantitative QCD predictions about the data available now, one turns to phenomenological analyses of the data in the framework of the QPM. One finds that a consistent picture can be given, in which the partons in a hadron have a jet-like distribution. That is, in addition to an intrinsic k_T component reflecting the binding effects at low Q^2, there is also a "conical" component of k_T the mean value of which grows slowly with the hadron momentum. The latter component is due to hard gluon radiation in a narrow cone, the same mechanism that gives rise to scaling violation. It is also the non-Drell-Yan type effects which can be factorized and absorbed into the distribution functions with the consequence that the Drell-Yan mechanism is thereby restored.

Experiments are suggested to reveal further the basic mechanism for LPP. If they confirm the picture outlined above, then the distribution of partons in a hadron holds the key to most of the properties of LPP, and the urgency for a theoretical understanding of it becomes all the more pressing. But then that is, after all, one of the primary reasons for doing experiments on LPP in the first place.

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