

APPLICATION OF GALERKIN'S METHOD FOR CALCULATING BOILING WATER REACTOR LIMIT-CYCLE AMPLITUDE USING THE LAPUR FEEDBACK-TRANSFER FUNCTION AND THE POINT-KINETICS EQUATIONS

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ABSTRACT

This paper describes a technique for calculating boiling water reactor (BWR) behavior during steady-state limit-cycle oscillations. An approximate solution is obtained from the application of Galerkin's method to a BWR dynamic model consisting of the point-kinetics equations and the LAPUR-calculated power-to-reactivity feedback-transfer function. The approximate-solution technique is described, and comparisons of approximate solutions with numerical results and measured data are given.

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## INTRODUCTION

Boiling water reactor (BWR) limit-cycle oscillations have been extensively studied to develop a better understanding of the phenomenon, predict whether a given set of reactor conditions will be stable, and estimate the reactor response during the oscillations. The analysis tools for modeling BWR dynamics have been primarily either linear frequency-domain or nonlinear time-domain computer codes. Frequency-domain codes such as the LAPUR code are limited to predicting BWR dynamics in the linear range [1]. Thus, these codes cannot estimate reactor response during limit-cycle oscillations, a nonlinear phenomenon. These codes can predict the onset of instability and quantify the degree of stability for stable systems. Frequency-domain codes can be used to examine the effect of reactor parameters such as flow, power, and power shape on stability and predict the stability boundary [2,3]. Because these codes are linear, they are generally less computation intensive than nonlinear time-domain codes and therefore require less computer resources.

Nonlinear time-domain codes are typically large codes that model BWR behavior by using a multinodal representation in both space and time. These codes predict BWR dynamic behavior in the linear and nonlinear regions and can calculate reactor transients. The main disadvantage of the nonlinear, multinodal, time-domain codes is the large amount of computer resources they require. Other disadvantages include difficulty and expense in debugging and checking models and the possible effect of nodalization on code results [4].

This paper describes a method for estimating steady-state BWR dynamic behavior in the nonlinear region using the thermal hydraulic model from the LAPUR code combined with the point-kinetics equations. It will be shown that this model and these equations can be used to form an equation describing the normalized neutron population during limit-cycle oscillations. An approximate solution to this equation is obtained by using Galerkin's method, a weighted residual technique. The approximation can be made accurate to any desired degree—the price for additional accuracy is increased complexity in the calculation of the approximate solution. In any event, the calculation of steady-state limit-cycle behavior using the approximate-solution technique is fast compared to nonlinear time-domain codes. Note that this technique is limited by the assumptions inherent in the point-kinetics equations and is valid for only relatively low amplitude limit-cycle oscillations because of the linear thermal hydraulics model.

The development of the nonlinear equation for the normalized neutron population from the point-kinetics equations and the LAPUR feedback (power-to-reactivity) transfer function will be presented, followed by a brief discussion of weighted residual methods and the application of Galerkin's method to the solution of the normalized neutron population equation. This discussion will be followed by results showing convergence of the approximate solution to a numerical (exact) solution. The paper concludes by comparing the approximate solution calculated for the LaSalle event to measured data.

## DEVELOPMENT OF THE NORMALIZED NEUTRON POPULATION EQUATION

A block diagram of the BWR dynamics is shown in Fig. 1. The neutron dynamics form the forward-transfer function relating power to reactivity. The feedback loop describes the effect of power on reactivity and consists of fuel heat generation and its transfer to the coolant, the channel hydraulics and its effect on reactivity through the void coefficient of reactivity, and the effect of fuel temperature on reactivity through the Doppler coefficient. The forward-transfer

function is modeled by using the point-kinetics equations, and the feedback loop is modeled by using the power-to-reactivity transfer function calculated by the LAPUR code. These two models can be used to form an equation for the normalized neutron population during limit-cycle oscillations.

The point-kinetics equations, with no independent neutron source term, are

$$\frac{dN(t)}{dt} = \frac{[\rho'(t) - \beta]}{\Lambda} N(t) + \sum_{m=1}^M \lambda_m C_m(t), \text{ and} \quad (1)$$

$$\frac{dC_m(t)}{dt} = \frac{\beta_m}{\Lambda} N(t) - \lambda_m C_m(t), \quad (m = 1, 2, 3, \dots, M), \quad (2)$$

where  $N(t)$  = weighted neutron population,

$C_m(t)$  = weighted  $m$ th precursor group population,

$\rho'(t)$  = reactivity,

$\beta_m$  = effective delayed-neutron fraction of precursors from the  $m$ th precursor group,

$M$  = number of precursor groups used in the equations,

$\lambda_m$  = effective decay rate of the  $m$ th precursor group,

$\Lambda$  = neutron reproduction time,

$\beta = \sum \beta_m$  = total delayed-neutron fraction.

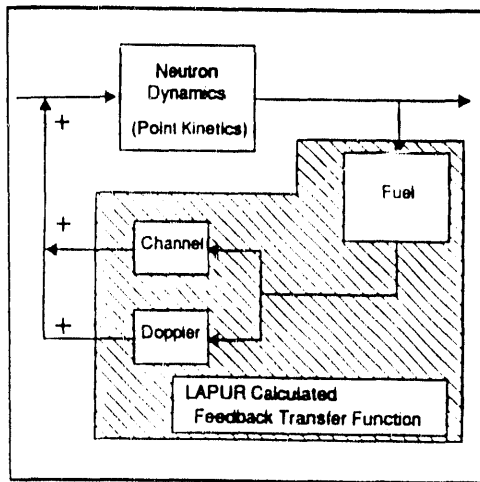


Figure 1. Block diagram of BWR dynamics

Equations (1) and (2) use effective values of the coefficients to take into account the spatial and energy distribution of the neutrons and precursors within the reactor. Thus, Eqs. (1) and (2) assume a certain neutron distribution in space and energy. The nonlinearity in Eq. (1) formed by the product of  $n(t)$  and  $\rho'(t)$  is the main cause of the BWR limit cycle.

These equations can be normalized, yielding

$$\frac{dn(t)}{dt} = \frac{n(t)}{\Lambda} [\rho(t) - \beta] + \frac{\rho(t)}{\Lambda} + \sum_{m=1}^M c_m(t) \lambda_m, \text{ and} \quad (3)$$

$$\frac{dc_m(t)}{dt} = -\lambda_m c_m(t) + \frac{\beta_m n(t)}{\Lambda}, \quad (m = 1, 2, 3, \dots, M), \quad (4)$$

where the normalization is given by

$$n(t) = \frac{N(t) - N_0}{N_0}, \quad c_m(t) = \frac{C_m(t) - C_{m0}}{N_0}, \quad \rho(t) = \rho'(t) - \rho_0,$$

and  $N_0$  and  $C_{m0}$  are the steady-state values of  $N(t)$  and  $C_m(t)$ . Thus,  $n(t)$  and  $c_m(t)$  are the deviation of the weighted neutron and precursor populations from their steady-state values normalized to the steady-state neutron population.

Next, since  $n(t)$  is periodic during limit-cycle oscillations, it can be written as

$$n(t) = R + \sum_{n=1}^{\infty} \left[ P_n \text{SIN}(n\omega t) + Q_n \text{COS}(n\omega t) \right], \quad (5)$$

where  $\omega$  is the fundamental limit-cycle oscillation frequency, and  $R$ ,  $P_n$ , and  $Q_n$  are undetermined constants.

After substituting Eq. (5) for  $n(t)$ , Eq. (4) is solved for  $C_m(t)$ , yielding

$$C_m(t) = \frac{\beta_m R}{\Lambda \lambda_m} + \sum_{n=1}^{\infty} \left[ \frac{\beta_m (n\omega Q_n + \lambda_m P_n)}{\Lambda (\lambda_m^2 + n^2 \omega^2)} \text{SIN}(n\omega t) + \frac{\beta_m (\lambda_m Q_n - n\omega P_n)}{\Lambda (\lambda_m^2 + n^2 \omega^2)} \text{COS}(n\omega t) \right]. \quad (6)$$

The feedback-transfer function calculated by LAPUR is used with Eq. (5) to form an expression for  $\rho(t)$  in terms of  $n(t)$ . If the neutron population undergoes a simple harmonic oscillation in time,  $n(t) = P \text{SIN}(\omega t)$ , the reactivity can be written as the product of  $n(t)$  and the frequency-response function  $H(\omega)$  obtained from the feedback-transfer function:

$$\rho(t) = |H(\omega)| P \text{SIN}[\omega t + \phi(\omega)], \quad (7)$$

where  $|H(\omega)|$  is the magnitude of the frequency-response function and  $\phi(\omega)$  is the phase of the frequency-response function at a frequency of  $\omega$  rad/s.

A more useful form of Eq. (7) is obtained by letting the harmonically oscillating normalized neutron population be expressed as the sum of sine and cosine terms:

$$n(t) = P \text{SIN}(\omega t) + Q \text{COS}(\omega t), \quad (8)$$

and letting the frequency-response function be expressed as a complex number:

$$H(\omega) = A(\omega) + iB(\omega), \quad (9)$$

where  $i = \sqrt{-1}$ .

It can be shown that the product of the frequency-response function and  $n(t)$  is given by

$$\rho(t) = [A(\omega)Q + B(\omega)P] \cos(\omega t) + [A(\omega)P - B(\omega)Q] \sin(\omega t) . \quad (10)$$

Extending this concept to the normalized neutron population given by Eq. (5) yields the following expression for the reactivity during limit-cycle oscillations:

$$\rho(t) = A_0 R + \sum_{n=1}^{\infty} \left\{ [A(n\omega)Q_n + B(n\omega)P_n] \cos(n\omega t) + [A(n\omega)P_n - B(n\omega)Q_n] \sin(n\omega t) \right\} , \quad (11)$$

where  $A_0$  is the real part of  $H(\omega)$  for  $\omega = 0$ .

Substituting Eqs. (5), (6), and (11) into Eq. (3) results in a single equation where the unknowns are the constants in the assumed solution for  $n(t)$ . Thus, the solution of the original system of nonlinear differential equations is reduced to determining values for these constants.

If the assumed solution contains a finite number of harmonics, it cannot satisfy Eq. (3) exactly; the resulting error is termed the residual,  $\mathbf{R}$ . Thus, for an approximate solution of  $j$  terms, the result of substituting Eqs. (5), (6), and (11) into Eq. (3) is an equation for the residual:

$$\begin{aligned} \mathbf{R} = & \sum_{n=1}^j \sum_{l=1}^j \left\{ P_l (A_n P_n - B_n Q_n) S_l S_n + [B_n P_l P_n + (A_l + A_n) P_l Q_n - B_l Q_l Q_n] S_l C_n \right. \\ & + Q_l (B_n P_n + A_n Q_n) C_l C_n \left. \right\} \sum_{n=1}^{\infty} \left( \left[ n\omega \left[ \Lambda + \sum_{m=1}^M \left( \frac{\lambda_m \beta_m}{\lambda_m^2 + n^2 \omega^2} \right) \right] - B_n (R + 1) \right] Q_n \right. \\ & + [R(A_0 + A_n) - n^2 \omega^2 \sum_{m=1}^M \left( \frac{\beta_m}{\lambda_m^2 + n^2 \omega^2} \right) + A_n] P_n \left. \right) S_n \\ & + \left( [R(A_0 + A_n) - n^2 \omega^2 \sum_{m=1}^M \left( \frac{\beta_m}{\lambda_m^2 + n^2 \omega^2} \right) + A_n] Q_n \right. \\ & \left. - \left[ n\omega \left[ \Lambda + \sum_{m=1}^M \left( \frac{\lambda_m \beta_m}{\lambda_m^2 + n^2 \omega^2} \right) \right] - B_n (R + 1) \right] P_n \right) C_n \left. \right\} + A_0 R (R + 1) , \quad (12) \end{aligned}$$

where  $C_l = \cos(l\omega t)$ ,  $S_l = \sin(l\omega t)$ ,  $C_n = \cos(n\omega t)$ ,  $S_n = \sin(n\omega t)$ ,  $P_n = P(n\omega)$ ,  $Q_n = Q(n\omega)$ ,  
 $A_n = A(n\omega)$ ,  $B_n = B(n\omega)$ .

## APPLICATION OF GALERKIN'S METHOD

Weighted-residual techniques can be used to solve either linear or nonlinear differential equations [5]. The solution over the interval  $I$  specified by the boundaries of the differential equation is written by using a set of trial functions, each multiplied by an undetermined constant. The set of trial functions should be members of a complete set, and each trial function must satisfy the boundary conditions of the differential equation. A weighted-residual technique is then used to determine the values of the constants so the approximate solution satisfies the differential equation in an average sense.

Equations for the solution constants are formed by requiring the product of the residual and a weighting function, integrated over the interval  $I$ , to be zero. The number of weighting functions equals the number of trial functions in the approximate solution. Evaluating these integrals results in a set of simultaneous algebraic equations that can be solved for the values of the constants. As the number of trial functions in the approximate solution increases, the residual approaches zero everywhere in  $I$ , and the approximate solution converges to the exact solution. The choice of the weighting functions determines the particular weighted-residual method employed. Although different weighted-residual methods involve various degrees of computational complexity and have different rates of convergence to the exact solution, all converge as additional terms are included in the approximate solution.

In Galerkin's method, the weighting functions are the same as the trial functions. For steady-state (i.e., limit-cycle) oscillations, the obvious choices for the trial functions in the approximate solution are sines and cosines which satisfy the periodic boundary conditions, resulting in the form of the approximate solution for  $n(t)$  given by Eq. (5). Selection of Galerkin's method causes the integrals of the product of the residual and the weighting functions, which are used to form equations for the constants, to consist of only products of trigonometric functions whose arguments are integer multiples of  $\omega t$ . Many of the terms in these integrals are identically zero when integrated over one cycle, while the remaining nonzero terms become constants after the integration. Thus, selection of Galerkin's method for this application results in relatively simple algebraic equations for the constants in the approximate solution. With Galerkin's method, the equations for the constants are given by:

$$\int_{t=0}^{\frac{2\pi}{\omega}} \mathbf{R} \mathbf{SIN}(k\omega t) dt = 0, \quad (k = 1, 2, 3, \dots j), \quad (13)$$

$$\int_{t=0}^{\frac{2\pi}{\omega}} \mathbf{R} \mathbf{COS}(k\omega t) dt = 0, \quad (k = 1, 2, 3, \dots j), \quad \text{and} \quad (14)$$

$$\int_{t=0}^{\frac{2\pi}{\omega}} \mathbf{R} \mathbf{R} dt = 0. \quad (15)$$

Equations (13), (14), and (15) result in  $2j + 1$  equations for  $R$ ,  $P_n$  and  $Q_n$  values.

The final equation needed to solve for  $\omega$  is obtained from the stipulation that the approximate solution satisfy the differential equation in an average sense—equivalent to requiring the residual, integrated over the path taken by  $n(t)$  during one cycle, to be zero:

$$\int_{\text{one cycle}} \mathbf{R} dn = \int_{t=0}^{\frac{2\pi}{\omega}} \mathbf{R} \dot{n} dt = 0. \quad (19)$$

The final equation needed to solve for the constants in the approximate solution is obtained by integrating Eq. (17) after substituting Eq. (12) for  $\mathbf{R}$  and the time derivative of Eq. (5) for  $n(t)$ .

Equations (13) through (16) determine the unknowns in the approximate solution in the following way. The approximate solution and Eq. (12) form a relationship among  $\mathbf{R}$ ,  $t$ , and  $n$ . This relationship can be represented by a line in three-dimensional space with orthogonal coordinates  $t$ ,  $n$ , and  $\mathbf{R}$ . Since the residual is a measure of the error in the approximate solution, requiring the average error over one cycle to be zero is equivalent to requiring the average value of the three-dimensional line to lie in the  $\mathbf{R} = 0$  plane. Equation (15) forces the integral of the projection of the line in the  $\mathbf{R}-t$  plane to be zero. Similarly, Eq. (16) forces the integral of the projection of the line in the  $n-\mathbf{R}$  plane to be zero. Together, these conditions force the average value of the line to lie in the  $\mathbf{R} = 0$  plane, thereby fulfilling the requirement that the approximate solution have an average error of zero over one complete cycle.

Equations (15) and (16) force the average value of the residual over one cycle to be zero. At any point during the cycle, however, the residual may be relatively large, especially so for approximate solutions containing few harmonics. Equations generated from Eqs. (13) and (14) calculate the  $P_n$  and  $Q_n$  values, which minimize the magnitude of the residual. Thus, by satisfying Eqs. (13) through (16), an approximate solution is obtained for which the average value of the residual over one cycle is zero and for which the residual during the cycle is minimized.

A FORTRAN computer code was written to calculate the approximate solution. The code forms the simultaneous nonlinear algebraic equations given by Eqs. (13), through (16) and solves for the undetermined constants in the approximate solution. The code input consists of the point-kinetics parameters, the feedback-transfer function, and an initial estimate of the values of the undetermined constants. The initial estimate is used by the numerical routine to solve the set of nonlinear algebraic equations generated by the program. Program output consists of the calculated values of the constants in the approximate solution.

## VALIDATION OF THE APPROXIMATE-SOLUTION TECHNIQUE

A comparison of the approximate solution to a numerical solution was used to check the validity of the approximate-solution technique. The test case is based on a simple phenomenological model (consisting of five equations) that qualitatively predicts BWR dynamics [2]. The equations can be solved numerically to obtain  $n(t)$ , or a feedback-transfer function equivalent to Eq. (9) can be used to form an approximate solution. This test case was selected to eliminate the possibility of errors in the approximate solution because of modeling deficiencies in either the LAPUR thermal hydraulics model or the point-kinetics equations. The phenomenological model is given by the following equations:

$$\frac{dn(t)}{dt} = \frac{\rho(t) - \beta}{\Lambda} n(t) + \lambda c(t) + \frac{\rho}{\Lambda} \quad (17)$$

$$\frac{dc(t)}{dt} = \frac{\beta}{\Lambda} n(t) - \lambda c(t) \quad (18)$$

$$\frac{dT(t)}{dt} = a_1 n(t) - a_2 T(t) , \quad (19)$$

$$\frac{d^2 \rho_\alpha(t)}{dt^2} + a_3 \frac{d\rho_\alpha(t)}{dt} + a_4 \rho_\alpha = kT(t) , \text{ and} \quad (20)$$

$$\rho(t) = \rho_\alpha(t) + DT(t) , \quad (21)$$

where  $T(t)$  = deviation of the fuel temperature from its steady-state value,  
 $\rho_\alpha$  = excess void reactivity feedback,  
 $D$  = Doppler reactivity coefficient,  
 $a_1$ - $a_4$  and  $k$  = empirically determined parameters.

Table I

Phenomenological model  
parameter values

Parameter	Value
$a_1$	$25.04 k \cdot s^{-1}$
$a_2$	$0.23 s^{-1}$
$a_3$	$2.25 s^{-1}$
$a_4$	$6.82 s^{-2}$
$K$	$-4.40 \times 10^{-3} k^{-1} \cdot s^{-2}$
$D$	$-2.52 \times 10^{-5} k^{-1}$
$\beta$	0.0056
$\Lambda$	$4.00 \times 10^{-5} s^{-1}$
$\lambda$	$0.08 s^{-1}$

In this model, Eq. (17) and (18) are the point-kinetics equations for one precursor group, Eq. (19) relates power and fuel temperature, Eq. (20) models the channel hydraulics, and Eq. (21) relates the total reactivity to the void reactivity and the fuel temperature. The parameter  $k$  is proportional to the void reactivity coefficient and the fuel heat transfer coefficient and effectively controls the model's feedback gain and, thus, its stability. It can be shown that the feedback power-to-reactivity transfer function of the model is given by:

$$H(s) = \frac{a_1 (Ds^2 + Da_3s + Da_4 + k)}{(s + a_2)(s^2 + a_3s + a_4)} \quad (22)$$

where  $s$  is the Laplace variable.

Table I shows parameter values that cause a limit cycle in the

phenomenological model. Figure 2 shows  $N(t)/N_0$  for approximate solutions consisting of two, four, six, and eight harmonics over five cycles. The convergence of the solution is apparent.

Equations (17) through (21) were also solved numerically by using the Advanced Continuous Simulation Language (ACSL) program [6]. The steady-state results of this calculation and the eight-harmonic approximate solution are shown in Fig. 3. Comparing the results shows that the approximate solution converges toward the exact solution as additional harmonics are included in the approximation. This convergence implies that the approximate-solution technique will accurately predict steady-state limit-cycle oscillations if the point-kinetics equations and the feedback-transfer function accurately model the processes occurring in the reactor.

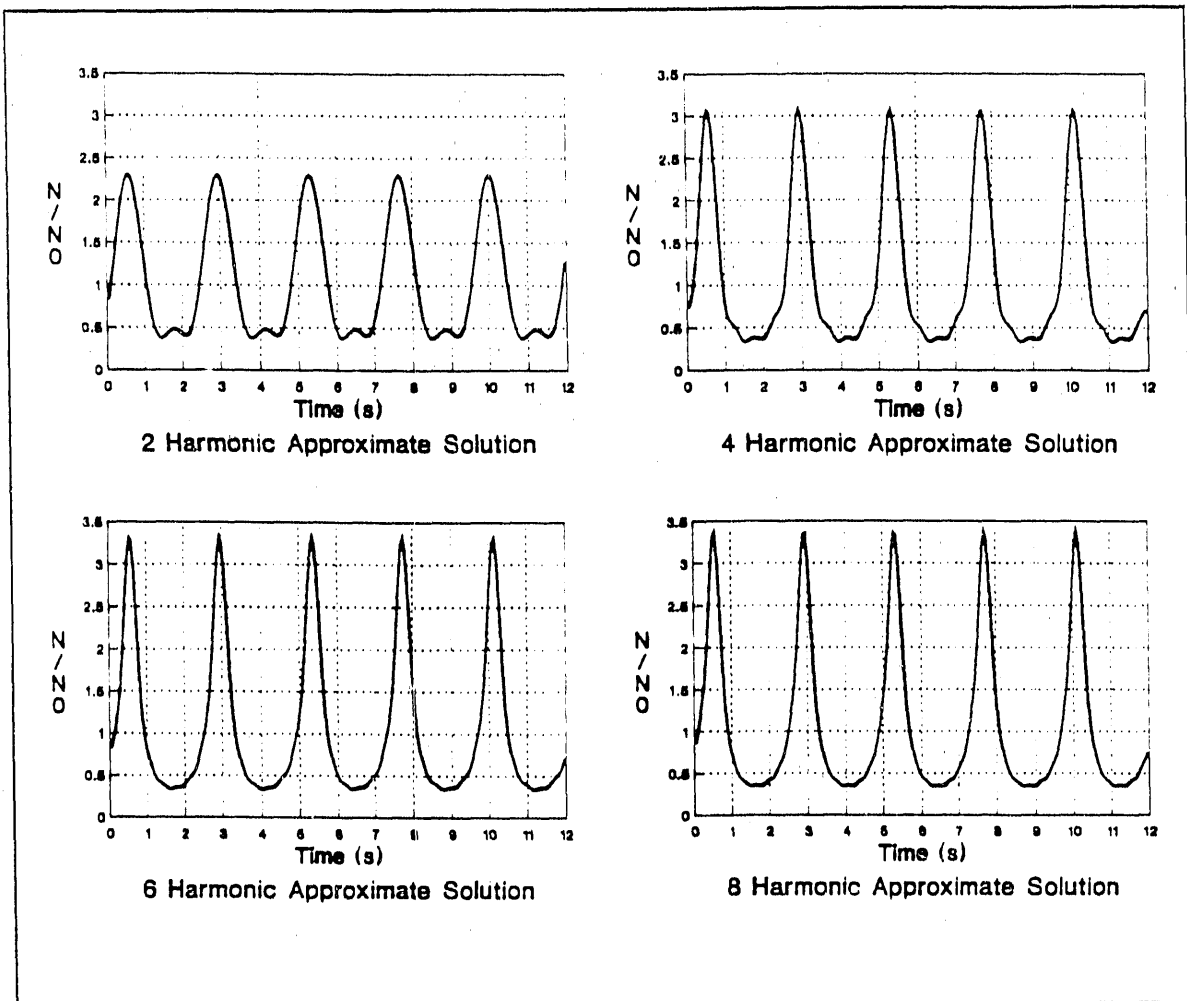


Figure 2. Two-, four-, six-, and eight-harmonic approximate solutions.

#### COMPARISON OF THE APPROXIMATE-SOLUTION TO MEASURED LIMIT-CYCLE DATA

Once an approximate solution for  $n(t)$  has been obtained, the behavior of additional reactor parameters can be calculated by using the appropriate LAPUR-calculated transfer (i.e., frequency-response) functions. If an approximate solution containing  $j$  harmonics has been obtained for  $n(t)$ , a multiplication similar to Eq. (7) by using a frequency-response function relating the neutron population to another parameter will result in an approximate solution of similar form for the new parameter. This solution is valid only if the magnitude of the limit-cycle oscillations is such that the linear thermal hydraulics model that generates these frequency-response functions is valid. This procedure will be applied by using LAPUR input data corresponding to the LaSalle event [7].

The LaSalle Unit 2 reactor experienced core-wide limit-cycle oscillations on March 9, 1988. The reactor had been operating at approximately 83% power and 75% core flow when a

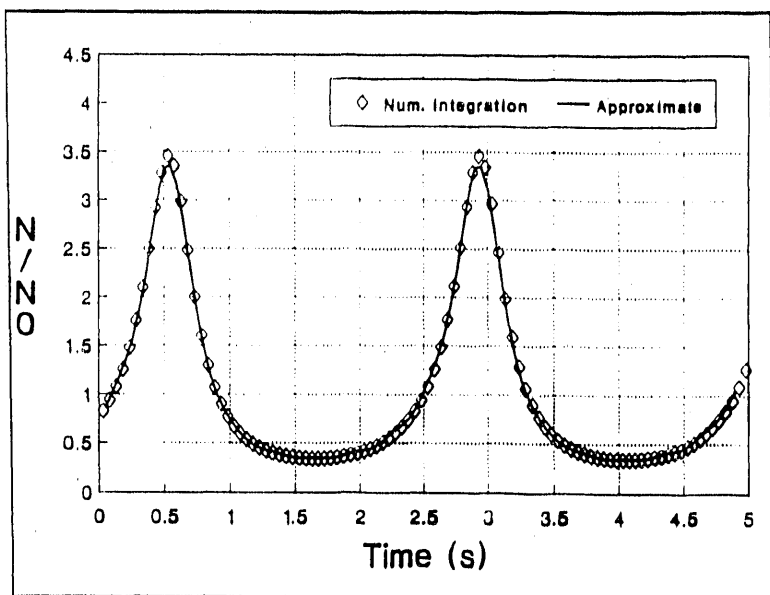


Figure 3. Comparison of numerical and eight-harmonic approximate solution.

slowly oscillating, accounting for the changing peak amplitude of the limit-cycle oscillations. An APRM signal recorded during the event shows the normalized power amplitude variations due to the flow and water level fluctuations (Fig. 4).

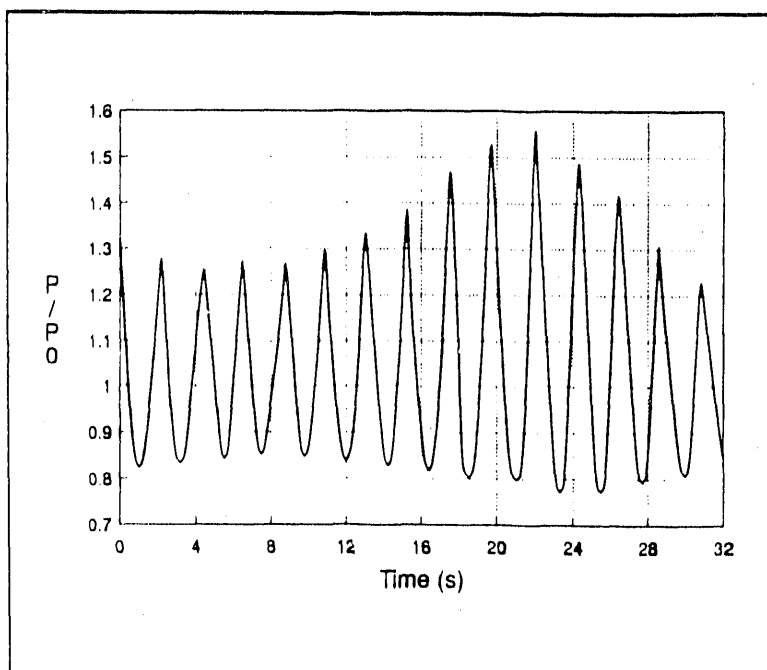


Figure 4. Average power range data taken during the LaSalle event.

technician error caused both recirculation pumps to trip. Core flow then coasted down to natural circulation conditions, approximately 28% of rated core flow, and the reactor power decreased to approximately 43%. Five minutes after the recirculation pump trip, oscillations were observed on the average and local power range monitors (APRM's and LPRM's). The oscillations continued for 2 min until the reactor exceeded the high-neutron-flux trip point and automatically scrammed. The oscillation frequency was  $\sim 0.45$  Hz, and the APRM peak-to-peak oscillation amplitude varied between 20 and 100%. During the event, the total core flow and reactor vessel water level were

Approximate solutions for the normalized reactivity and inlet flow were obtained by multiplying  $n(t)$  by the appropriate frequency-response function. Figure 5 shows  $n(t)$ ,  $\rho(t)$ , and the normalized inlet flow over four cycles. The effects of the higher harmonics, which are prevalent in the solution for  $n(t)$ , are reduced in magnitude in the solutions for  $\rho(t)$  and flow, making them appear more sinusoidal. This reduction is expected since the magnitudes of the power-to-reactivity and power-to-flow frequency-response functions rapidly decrease with increasing frequency. The computer time required for this example, including the time needed to run the LAPUR code, was less than three minutes on a VAX 11/780.

Comparison of the measured data and the approximate solution is somewhat difficult because the actual limit cycle never reached a true steady state. Nevertheless, one can conclude from the overall agreement between the measured data and the approximate solution that the LAPUR thermal hydraulics model and the point-kinetics equations accurately model the LaSalle event. This accurate modeling indicates that the approximate solution technique can accurately estimate actual BWR behavior during limit-cycle oscillations.

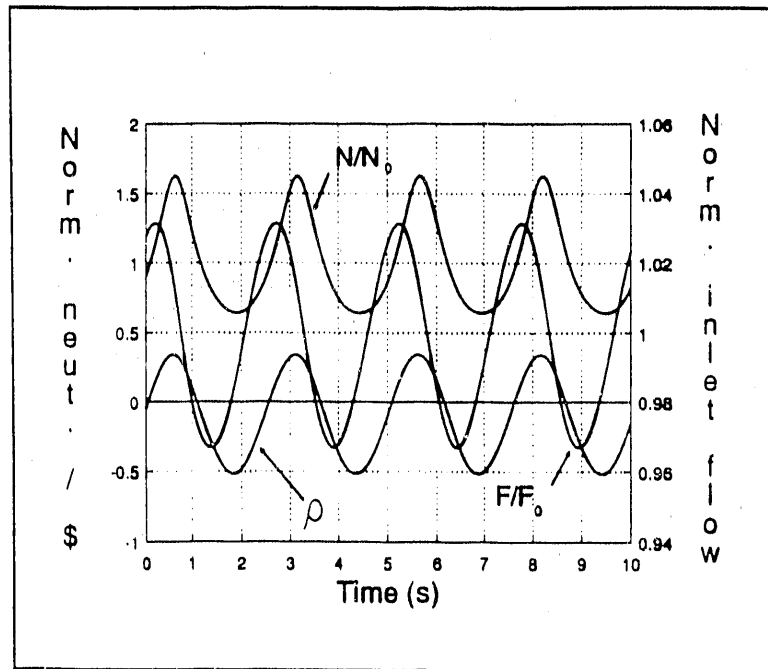


Figure 5. Seven-harmonic approximate solution for the LaSalle event.

## CONCLUSIONS

It has been shown that the combination of the LAPUR-calculated feedback-transfer function and the point-kinetics equations can be used in conjunction with Galerkin's method to calculate BWR limit-cycle behavior. It was shown how the feedback-transfer function can be used to express reactivity as a function of the normalized neutron population and how this expression was substituted into the point-kinetics equations, resulting in a single equation for  $n(t)$ . The application of Galerkin's method to obtain approximate solutions to this equation was then described.

Comparison of the approximate solution with a numerical solution showed that the approximate solution converges toward the numerical (exact) solution as more harmonics are included in the approximation. Thus, it can be concluded that the application of Galerkin's method results in an accurate representation of BWR limit-cycle oscillations if the point-kinetics

equations and the LAPUR-calculated feedback-transfer function accurately model the processes occurring within the BWR during the limit cycle.

Finally, the approximate solution for the LaSalle event was calculated and used to determine neutron, reactivity, and flow oscillations during the event. Comparison of the approximate solution to measured data show good overall agreement. Thus, it has been demonstrated that the approximate solution obtained from the application of Galerkin's method to the combination of the LAPUR-calculated feedback-transfer function and the point-kinetics equations can be used to predict BWR limit-cycle behavior.

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