LA-7611-MS
Informal Report

UC-34c
Issued: January 197

Noneikonal Effects in the Spin-Dependent Proton-Nucleus Interactions

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NONEIKONAL EFFECTS IN THE SPIN-DEPENDENT

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by

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ABSTRACT

Simple expressions are derived for the non-eikonal corrections to the Glauber diffraction approximation for the proton-nucleus scattering amplitudes, with the spin-dependence of the proton-nucleon amplitudes taken into account. As an example, we study the numerical importance of these corrections for elastic p-^{58}Ni scattering at 800 MeV. The polarization and the spin rotation, which are sensitive to the relative phase of the amplitudes, are at small angles changed by typically 5-10%.

High-precision experiments on proton-nucleus scattering have recently been performed[1] at medium energies. It is hoped that because of the high precision of the data, they may be used for studies of effects like the difference between the proton and neutron density distributions in nuclei, nuclear correlations, and also for extracting parameters of the proton-nucleon scattering amplitudes. Therefore, very accurate methods of analysis are called for. It is the purpose of the present paper to present such methods.

The diffraction approximation by Glauber[2] has proven to be very successful in the interpretation of high-energy hadron-nuclear interactions[3] and is also a remarkably good first approximation at medium energies (on the order of 1 GeV). However, finite-energy corrections are needed here, the deviation from eikonal propagation being the most important effect that has to be corrected for.[4]

Various modifications to the diffraction approximation have recently been proposed.[4-9] With more accurate approximations to the free-wave projectile propagator than the eikonal one, these modified multiple-scattering theories are believed to be more reliable methods of data analysis at medium energies. To order 1/k in an expansion of the free-wave propagator in reciprocal powers of the projectile momentum, these methods[4-8] give practically the same result for the corrections that come from the region of nonoverlap of the ranges of interaction of the projectile with the target nucleons. But they differ by their treatment of the regions of overlap. These treatments of the overalp regions correspond in momentum space to different off-shell extrapolations of the projectile-nucleon t-matrix, which in turn is intimately connected with the treatment of the reflection terms (terms describing mutliple scattering of the projectile from the same target nucleon).

A systematic way of correcting the Glauber amplitudes at medium energies is the eikonal expansion procedure of Wallace.[5] This is based on a similar off-shell extrapolation of the proton-nucleon t-matrix as that used in the original work by Glauber, namely upon the assumption that the t-matrix is generated by a local potential. Unfortunately, the resulting expressions fc. the noneikonal corrections become rather lengthy and difficult to evaluate in the case of a realistic nuclear density and a spin-dependent proton-nucleon t-matrix.

On the other hand, it has been shown[10] that the particular assumptions adopted for the off-shell dependence of the projectile-nucleon amplitude have little effect on the noneikonal corrections, provided that the off-shell amplitude decreases sufficiently fast (e.g., exponentially) as a function of momentum transfer squared. In particular to order 1/k the noneikonal corrections obtained in the model proposed by Bleszynski and Jaroszewicz[7] are quantitatively almost indistinguishable from those of Wallace.[5] There are simple reasons for this. The dominant contributions to the corrections come from the nonoverlap region where the off-shell dependence of the projectile-nucleon t-matrix is irrelevant, and to order 1/k the reflection terms vanish in this region. One may thus use a much simpler procedure than that of Wallace[5] to obtain the finite-energy corrections to the diffraction approximation. The procedure we have adopted is the same as that of Ref. 7 and is in spirit similar to the approach by Gottfried.[4] It is based on the following approximations. (1) The motion of the nucleons in the target nucleus is, during the collision process, negligible (the "frozen-nucleus" approximation); (2) the projectile-nucleon t-matrix is local (this is different than assuming a local potential); (3) the reflection term can be neglected; and (4) for the free-wave propagator we use a more accurate approximation than the eikonal one. For $k \to \infty$, the approximations 1-4 lead to the high-energy results of Glauber.[10]

In the approach specified above and for a spin-zero nucleus, we have found very simple expressions for the proton-nucleus elastic-scattering amplitudes. These contain all corrections to order $1/k$ with respect to the leading Glauber amplitudes and allow for spin-dependent proton-nucleon interactions. All previous calculations of such corrections are valid only for spin-independent interaction.

In order to determine the corrected proton-nucleus scattering amplitudes, let us expand the free-wave propagator in powers of $1/k$, where $\vec{k}$ is the average of the initial and final momenta for the overall scattering. We retain only the first two terms;

$$\tilde{G} = G_{eik} + \delta G \quad . \tag{1}$$

The eikonal propagator G_{eik} and the first-order correction are given by

$$\langle \vec{r}\,' | G_{eik} | \vec{r} \rangle = \frac{-i}{2k} \, \theta(z' - z) \, e^{ik(z'-z)} \delta^2(\vec{b}\,' - \vec{b})$$

and

$$\langle \vec{r}\,' | \delta G | \vec{r} \rangle = \frac{1}{4k^2} \left\{ \delta(z' - z) + (z' - z)\theta(z' - z) \left(\frac{\Delta^2}{4} + \nabla_b^2 \right) \right\} \cdot e^{ik(z'-z)} \, \delta^2(\vec{b}\,' - \vec{b}) \quad , \tag{2}$$

in which $\vec{\Delta}$ is the overall momentum transfer, and where $\vec{r} = \left\{ \vec{b}, z \right\}$ and similarly $\vec{r} = \left\{ \vec{b}\,', z' \right\}$.

The proton-nucleus t-matrix operator for scattering by a nucleus of mass number A may then be expressed in terms of the proton-nucleon t-matrix* operators t_j as

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*The relation between the laboratory frame amplitude f and the t-matrix elements is

$$\langle \vec{p}\,' | t | p \rangle = - 4\pi \, f \, (\vec{p} - \vec{p}\,') \quad .$$

$$T = \sum_{j=1}^{A} t_j + \sum_{k \neq j}^{A} t_j \tilde{G} t_k + \sum_{\substack{\ell \neq k \neq j \\ \ell \neq j}}^{A} t_j \tilde{G} t_k \tilde{G} t_\ell + \text{term of order } A \quad . \tag{3}$$

The pp(pn) t-matrix operator involves 5(6) independent amplitudes. Fortunately, for the elastic scattering of protons by spin-zero nuclei with $A \gtrsim 10$, only those amplitudes are important that are independent of the spins of the target nucleons.[11] We thus decompose the proton-nucleon t-matrix operator as

$$\langle \vec{p}\,' | t_j | \vec{p} \rangle = - 4\pi \, e^{i(\vec{p}-\vec{p}\,')\vec{r}_j} \left\{ A_j(\vec{p} - \vec{p}\,') + \vec{\sigma}\cdot\hat{n} |\vec{p} - \vec{p}\,'| C_j(\vec{p} - \vec{p}\,') \right\} \quad , \tag{4}$$

where the index j labels the target nucleons, and $\hat{n} = \hat{k} \times (\vec{p} - \vec{p}\,')/|\vec{p} - \vec{p}\,'|$. (Spins are quantized along $\hat{k}$.) In configuration space we have

$$\langle \vec{r}\,' | t_j | \vec{r} \rangle = \delta^3 (\vec{r}\,' - \vec{r}) \; t_j \; (\vec{r} - \vec{r}_j) \quad , \tag{5}$$

in which

$$t_j(\vec{r} - \vec{r}_j) = - \frac{1}{2\pi^2} \int d^3q \left\{ A_j(\vec{q}) + i\vec{\sigma}\hat{n}_b C(\vec{q}) \frac{d}{db} \right\} e^{-i\vec{q}(\vec{r}-\vec{r}_j)} \quad , \tag{6}$$

and where $\hat{n}_b = \hat{k} \times \hat{b}$, with $\vec{b}$ being the component of $\vec{r}$ that is orthogonal to $\hat{k}$. A profile operator γ_j can be defined in terms of the t-matrix operator by

$$\int_{\infty}^{\infty} dz \; t_j(\vec{r} - \vec{r}_j) = - 2 \, ik \;\; \gamma_j \;\; (\vec{b} - \vec{s}_j) \quad . \tag{7}$$

We have also found the following identities useful in evaluating Eq. (3).

$$\sum_{(\text{all permutations of } z_i)} \theta(z_1 - z_2) \; \ldots\ldots \; \theta(z_{n-1} - z_n) = 1 \quad , \tag{8a}$$

$$\left[t_i(\vec{r} - \vec{r}_i), \; t_k \; (\vec{r} - \vec{r}_k) \right] = 0 \quad , \tag{8b}$$

and

$$\int d^2b\, e^{i\vec{\Delta}/2\cdot\vec{b}}\, f_1(\vec{b}) \left(\frac{\Delta^2}{4} + \nabla_b^2\right) f_2(\vec{b})\, e^{i\vec{\Delta}/2\cdot\vec{b}} = -\int d^2b\, e^{i\vec{\Delta}\cdot\vec{b}} \vec{\nabla}_b f_1(\vec{b}) \cdot \vec{\nabla}_b f_2(\vec{b}) \quad . \tag{8c}$$

(The last relation[10] holds for f_1 and f_2 such that $f_1(\vec{b})\, f_2(\vec{b})$ vanish for $b \to \infty$.

Now inserting Eqs. (1), (2), and (5) into Eq. (3) and using the identities, Eqs. (8), we find for the t-matrix operator for an A-body target that

$$\langle \vec{k}_f | T | \vec{k}_i \rangle = -2ik \int d^2b\, e^{i\vec{\Delta}\cdot\vec{b}}\, \tilde{\Gamma}(\vec{b};\vec{r}_1, \ldots\ldots, \vec{r}_A) \quad , \tag{9}$$

where $\vec{r}_1, \ldots, \vec{r}_A$ ($\vec{r}_i = \{\vec{s}_i, z_i\}$) are nucleon coordinates. We decompose the profile operator $\tilde{\Gamma}$ as

$$\tilde{\Gamma}(\vec{b};\vec{r}_1, \ldots, \vec{r}_A) = \Gamma(\vec{b};\vec{s}_1, \ldots, \vec{s}_A) + \delta\Gamma\ (\vec{b};\vec{r}^1, \ldots, \vec{r}_A) \quad , \tag{10}$$

in which the familiar

$$\Gamma(\vec{b};\vec{s}\ , \ldots, \vec{s}_A) = 1 - \prod_{j=1}^{A} \left[1 - \gamma_j(\vec{b} - \vec{s}_j)\right] \tag{11}$$

depends only on transverse coordinates. The correction owing to noneikonal propagation can be written

$$\delta\Gamma(\vec{b};\vec{r}_1, \ldots, \vec{r}_A) = \frac{i}{2k} \sum_{m\neq n}^{A} \omega_{mn}(\vec{b};\vec{r}_m,\vec{r}_n) \prod_{j\neq m,n}^{A} \left[1 - \gamma_j(\vec{b} - \vec{s}_j)\right] \quad . \tag{12}$$

With $\vec{r} = \{\vec{b},z\}$, $\vec{r}\,' = \{\vec{b},z'\}$, ω_{mn} can be written as

$$\omega_{mn}(\vec{b};\vec{r}_m,\vec{r}_n) = \int_{\infty}^{\infty} dz \left\{ t_m(\vec{r} - \vec{r}_m)\ t_n\ (\vec{r} - \vec{r}_n) + \left[\vec{\nabla}_b\ t_m(\vec{r} - \vec{r}_m) \cdot \int_{\infty}^{z} dz'\ z'\ \vec{\nabla}_b\ t_n(\vec{r}\,' - \vec{r}_n) + (m \leftrightarrow n)\right] \right\} \quad . \tag{13}$$

The Eqs. (12) and (13) are the main results of this paper. It should be stressed that they were derived with spin-dependent proton-nucleon amplitudes and without any assumptions about the form of the target density. The results are valid for different proton-proton and proton-neutron t-matrices and for different proton and neutron density distributions in the target nucleus.

The special case of an independent-particle model for the target nucleons is of great practical interest. If we further assume that the single-nucleon densities $\rho_j(\vec{r}_j)$ and the proton-nucleon amplitudes $A_j(\vec{q})$ and $C_j(\vec{q})$ have spherical symmetry, more explicit results will be relatively simple. For the purpose of giving such results, let us define

$$G_k(r) = \frac{1}{(2\pi)^2 ik} \int d^3r_k \rho_k(r_k) \int d^3q\, e^{-i\vec{q}\cdot(\vec{r}-\vec{r}_k)} A_k(q) \quad ,$$

$$H_k(r) = \frac{1}{(2\pi)^2 ik} \int d^3r_k \delta_k(r_k) \int d^3q\, e^{-i\vec{q}\cdot(\vec{r}-\vec{r}_k)} C_k(q) \quad , \tag{14}$$

$$q_k(b) = \int_{-\infty}^{\infty} dz\, G_k(r),$$

$$h_k(b) = \int_{\infty}^{\infty} dz\, H_k(r), \tag{15}$$

and

$$S_k(\vec{b}) \equiv 1 - \int d^3r_k \quad \rho_k(r_k)\ \gamma_k(\vec{b} - \vec{s}_k)$$

$$= 1 - g_k(b) - i\, \vec{\sigma}\cdot\hat{n}_b \frac{d}{db} h_k(b) \quad . \tag{16}$$

With Z(N) protons (neutrons) all having the same density $\rho_p(\rho_n)$, the correction $\delta\Gamma$ averaged over the nuclear density becomes

$$\langle\delta\Gamma\rangle = \frac{i}{2k} \Big[Z(Z-1)\ \omega_{pp}(\vec{b})\ S_p^{Z-2}(\vec{b})\ S_n^N(\vec{b})$$

$$+ 2ZN\ \omega_{pn}(\vec{b}) S_p^{Z-1}(\vec{b})\ S_n^{N-1}(\vec{b}) + N(N-1)\ \omega_{nn}(\vec{b})\ S_p^Z(\vec{b})\ S_n^{N-2}(\vec{b}) \Big] \quad . \tag{17}$$

The decomposition

$$\omega_{k\ell}(b) = \alpha_{k\ell}(b) + i\ \vec{\sigma}\cdot\hat{n}_b\ \beta_{k\ell}(b) \tag{18}$$

gives for the spin-dependent and spin-independent parts

$$\alpha_{k\ell}(b) = \int_{\infty}^{\infty} \Big(dz \left(1 + b \frac{d}{db}\right) \left[G_k(r)\; G_\ell(r)\right] - \left[\frac{d}{db} H_k(r)\right]\left[\frac{d}{db} H_\ell(r)\right]$$

$$- \left[b^{-1} H_k(r) \frac{d}{db} H_\ell(r) + \left[\left(1 + b \frac{d}{db}\right) H_k(r)\right] \frac{d^2}{db^2} H_\ell(r) + (k \leftrightarrow \ell)\right] \Big)$$

and

$$\beta_{k\ell}(b) = \int_{\infty}^{\infty} dz \left\{ \frac{d}{db}\left[G_k(r)\; H_\ell(r)\right] + b \frac{d}{db}\left[G_k(r) \frac{d}{db} H_\ell(r)\right] + (k \leftrightarrow \ell)\right\} . \quad (19)$$

The proton-nucleus scattering amplitudes in the nucleus laboratory frame may be defined by the decomposition

$$F(\Delta) + \vec{\sigma}\cdot\hat{n}\, G(\Delta) = \frac{ik}{2\pi} \int d^2b\, e^{i\vec{\Delta}\cdot\vec{b}} \langle \Gamma + \delta\Gamma \rangle \quad , \quad (20)$$

where $\hat{n} = \hat{k} \times \hat{\Delta}$. A decomposition of $\langle\delta\Gamma\rangle$ into spin-independent and spin-dependent parts thus furnishes the corrections to the eikonal amplitudes $F_{eik}(\Delta)$ and $G_{eik}(\Delta)$. This evaluation is straightforward but results in lengthy expressions, which we shall not reproduce here.

In order to investigate the relative importance of the $\langle\delta\Gamma\rangle$ term, we have calculated this noneikonal correction for the scattering of 800-MeV protons by various nuclei. The remainder of the paper is devoted to the presentation of some results for ^{58}Ni. We have used

$$A(q) = \frac{k\;\sigma(i + \alpha)}{4\pi} \exp(-\tfrac{1}{2}\beta \vec{q}^{\,2})$$

and

$$C(q) = \frac{k\;\lambda(1 + \alpha_C)}{4\pi} \exp(-\tfrac{1}{2}\beta_C \vec{q}^{\,2}) \quad ,$$

with σ_{pp} = 47.3 mb, σ_{pn} = 38.0 mb, α_{pp} = 0.056, α_{pn} = −0.20, β_{pp} = 0.18 fm^2, β_{pn} = 0.24 fm^2, $\lambda_{pp} = \lambda_{pn}$ = 0.8 fm^3, $\alpha_{Cpp} = \alpha_{Cpn}$ = −1.0, $\beta_{Cpp} = \beta_{Cpn}$ = 0.6 fm^2, (cf. Ref. 11) and three parameter density distributions

$$\rho(r) = \rho_o(1 + wr^2/c^2)/\left\{1 + \exp[(r-c)/a]\right\}$$

with $w = -0.14$, $a = 0.42$ fm, and $c = 4.34$ fm for protons and $w = 0.14$, $a = 0.42$ fm, and $c = 4.20$ fm for neutrons.

The calculation of the diffraction approximation contributions $F_{eik}(\Delta)$ and $G_{eik}(\Delta)$ is similar to that of Ref. 11. We include the effects resulting from the coupling of the proton with the nuclear Coulomb field like in Ref. 11.

Let us stress here that the aim of our calculation was to check the importance of the noneikonal corrections to the Glauber model, and we did not attempt to find the best fit to the data. In fact in this calculation, we have used the parameters of A and C amplitudes which were found in Ref. 11 to give the reasonable Glauber-model fit to the polarization at small momentum transfers.

Figure 1 shows the importance of the noneikonal corrections for the polarization

$$P(\Delta) = \frac{2\mathrm{Re}[F(\Delta)\ G^*(\Delta)]}{|F(\Delta)|^2 + |G(\Delta)|^2} \tag{21}$$

and for the spin-rotation function[12]

$$Q(\Delta) = \frac{2\mathrm{Im}[F(\Delta)\ G^*(\Delta)]}{|F(\Delta)|^2+|G(\Delta)|^2} \quad . \tag{22}$$

At small angles the corrections are relatively small (<10% in the region up to the first maximum of the polarization). However, at larger angles they become more important and are particularly significant around the diffraction minima. For lighter, as well as for heavier nuclei, similar results are obtained. The relative importance of the corrections appears to have only a weak A-dependence.

A significant part of the corrections to $P(\Delta)$ and $Q(\Delta)$ is simply due to a change in the relative phase of $F(\Delta)$ and $G(\Delta)$. To a rough approximation, this relative phase is given by the relative phase of $A(0)$ and $C(0)$. Consequently, the importance of the noneikonal corrections to either of $P(\Delta)$ or $Q(\Delta)$ depends upon the relative phase of $A(0)$ and $C(0)$, which at the moment is not very well known.[11] Clearly these corrections should be included in a careful analysis of the data, whether the aim is to determine the proton-nucleon spin-orbit amplitude or to study the neutron density distribution.[11]

In conclusion, let us point out a few advantages of the formalism presented here. It does not require the potential formulation and is technically much simpler than the methods of Ref. 5. It is also very flexible with respect to other corrections, like those owing to correlations, charge exchange, and the

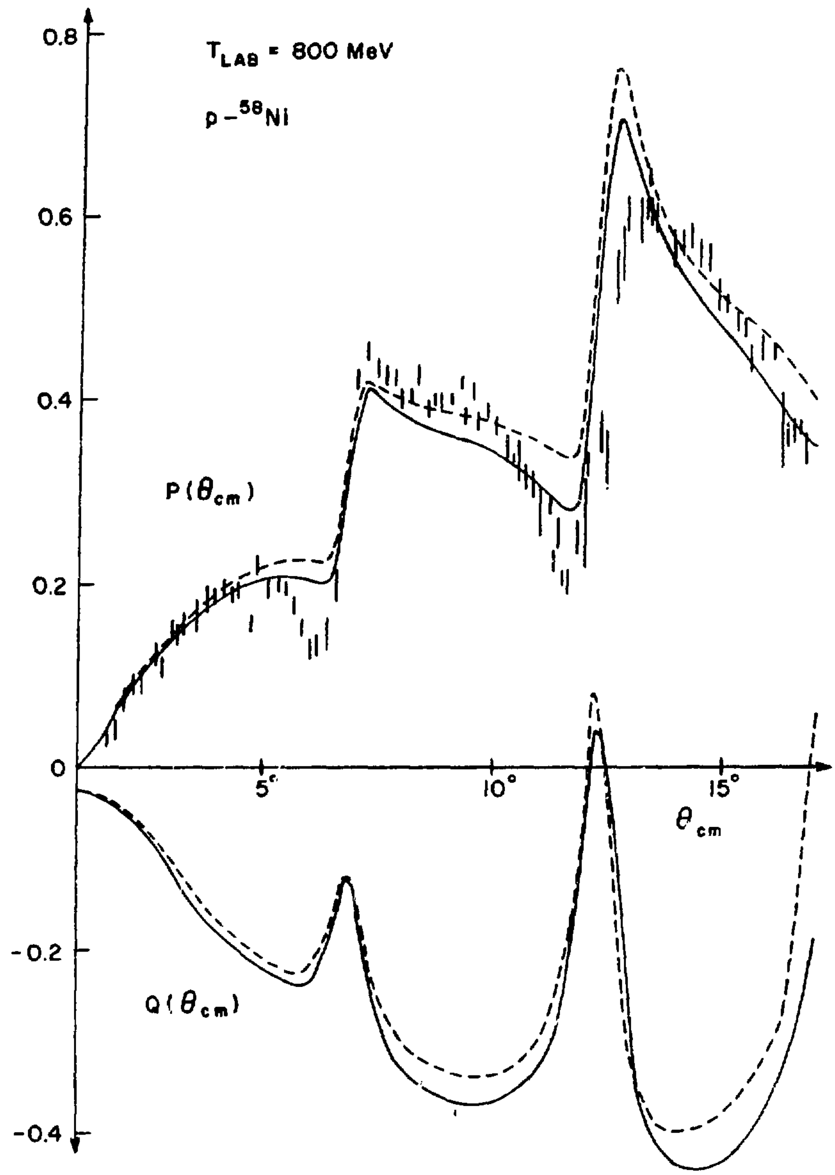

Fig. 1.
The polarization P and the spin-rotation function Q at 800 MeV calculated according to the Glauber formula (broken line) and with the noneikonal corrections (solid line). Data are taken from Ref. 1. Parameters of the NN amplitude and of the target density are given in the text.

intermediate propagation of heavier baryons. In contrast to the KMT approach,[13] the difference between the proton-proton and proton-neutron interactions is treated exactly, and therefore, it is valid also for light nuclei.

ACKNOWLEDGMENTS

This research is supported in part (P.O.) by the U.S. Department of Energy under contract no. EY76-C-02-3064 and through a travel grant (P.O.) from the Norwegian Research Council for Science and Humanities.

Part of this work was done while Per Osland visited the Los Alamos Scientific Laboratory. He expresses his graditude to L. Rosen and H. A. Thiessen for the hospitality extended to him in the MP Division.

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