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Time Series Analysis by the Maximum Entropy Method

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ABSTRACT

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The principal subject of this report is the use of the Maximum Entropy method for spectral analysis of time series. The classical Fourier method is also discussed, mainly as a standard for comparison with the Maximum Entropy method. Examples are given which clearly demonstrate the superiority of the latter method over the former when the time series is short. The report also includes a chapter outlining the theory of the method, a discussion of the effects of noise in the data, a chapter on significance tests, a discussion of the problem of choosing the prediction filter length, and, most importantly, a description of a package of FORTRAN subroutines for making the various calculations. Cross-referenced program listings are given in the appendices. The report also includes a chapter demonstrating the use of the programs by means of an example. Real time series like the lynx data and sunspot numbers are also analyzed.

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INTRODUCTION

In the past, the spectral analysis of time series has for the most part utilized two classical approaches - the power spectrum method of R. B. Blackman and J. W. Tukey (1959) and the periodogram. The Blackman and Tukey approach involves the estimation of the autocorrelation function. A shortcoming of the estimator used is its assumption of a zero extension of the given data. On the other hand, the periodogram method assumes a periodic extension of the data, the period being equal to the length of the time series. John P. Burg (1968) has presented a new technique of time series analysis which seeks to minimize assumptions about the unavailable data. The method is called maximum entropy spectral analysis (which will be referred to as MESA throughout this report). MESA emerged in an effort to achieve better resolution from short time series.

A great amount of research on MESA has been done by T. J. Ulrych. His recent publications (1972b, 1973, 1974) indicate the wide extent to which MESA can be utilized. Ulrych's contention is that the classical Fourier transform methods are inferior to MESA because of their inability to analyze low frequency variations.

MESA and Fourier analysis are compared in this report. Generally speaking, MESA is superior for short time series, but it has one major disadvantage shared also by older methods. The problem involves the determination of the length of the vector of filter coefficients used for estimating the autocorrelation function. In an attempt to solve this problem, T. J. Ulrych (1975) suggested the use of the Akaike final

prediction error (FPE), (Akaike 1969), as applied to autoregressive decomposition. A brief critical discussion of this idea is also presented here in Chapter IV.

As much as possible, the mathematical presentation of time series analysis is simplified in this report. The derivation of the maximum entropy spectrum, however, employs theorems in complex analysis. Although the mathematics becomes rigorous in some parts, it is hoped that the many examples presented will impart a fair understanding of the methods discussed to less mathematically inclined readers.

Computer programs (written in FORTRAN) which calculate both the Fourier sample spectrum and maximum entropy spectrum plus some other useful information are described and explained in Chapter V. An example is presented in Chapter VI to illustrate the use of these programs. Program listings are given in the appendices.

CHAPTER I

THE DISCRETE FOURIER TRANSFORM METHOD

Consider the functions $y_1(t)$, $y_2(t)$, $y_3(t)$ in Figures 1.1-1.3, respectively.

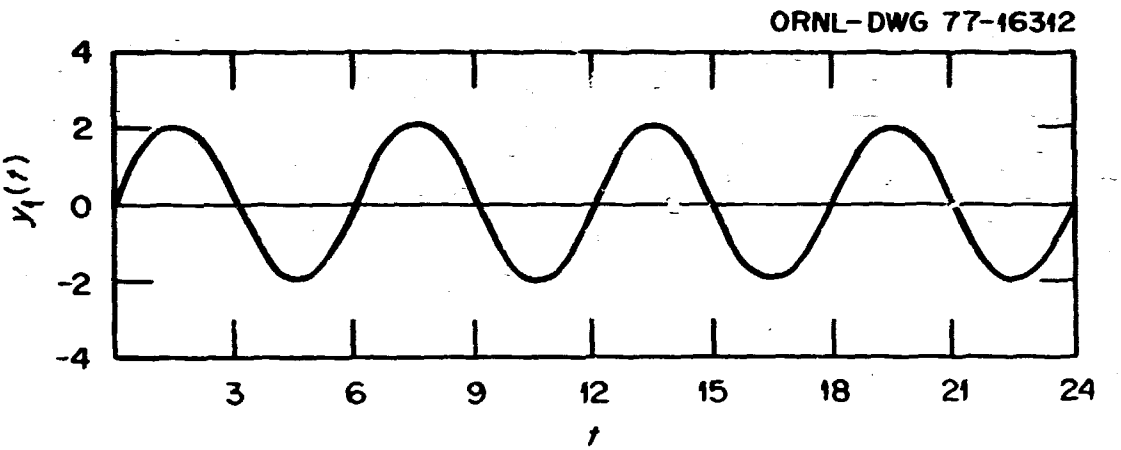

Fig. 1.1 The function $y_1(t)$: amplitude = 2, period = 6 years.

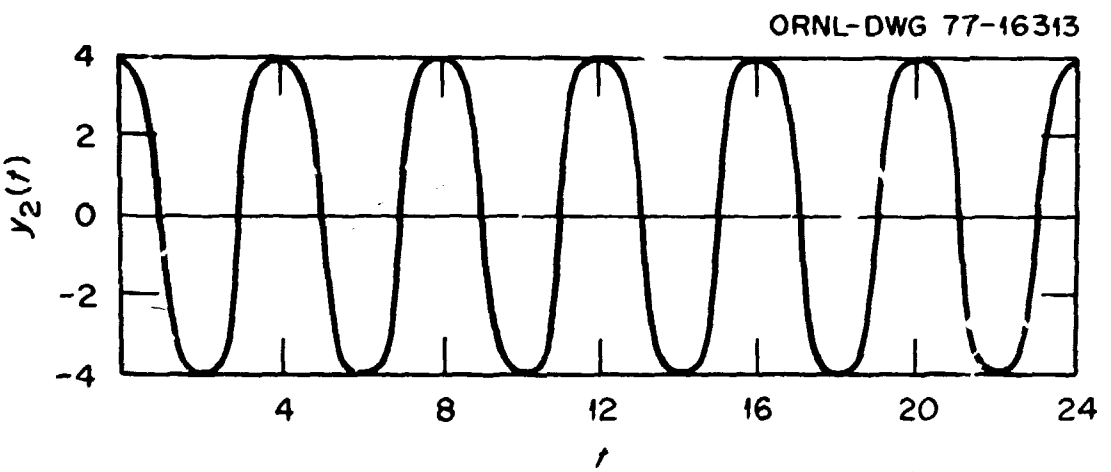

Fig. 1.2. The function $y_2(t)$: amplitude = 4, period = 4 years.

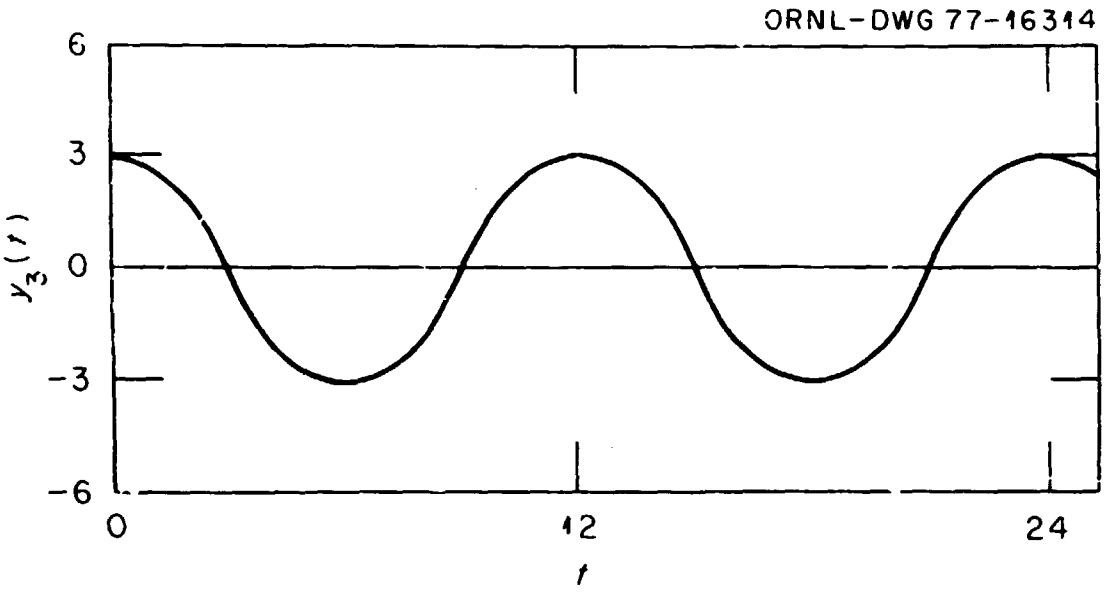

Fig. 1.3. The function $y_3(t)$: amplitude = 3, period = 12 years.

The sum of $y_1(t)$, $y_2(t)$, $y_3(t)$ produces a new function $y(t)$ (Fig. 1.4).

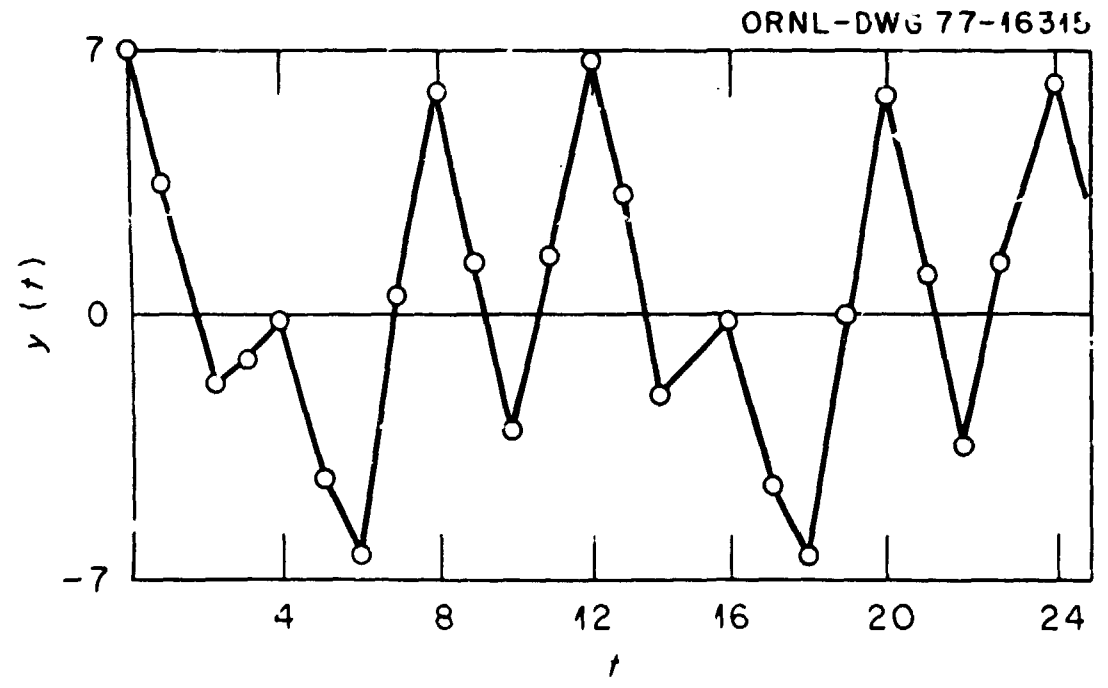

Fig. 1.4. The new function $y(t) = y_1(t) + y_2(t) + y_3(t)$, sampled at intervals Δt = 1 year.

Since y(t) is the sum of three functions whose periods are known, the analysis of its periodic components is clear. In practice, one is usually faced with the inverse problem, i.e., the analysis of a time series like y(t) in order to determine what periodic components are present. This situation leads to the utilization of the Fourier transform method.

Although many natural processes are truly harmonic, in that they can be expressed exactly as a sum of sinusoidal terms, it is more often the case that a natural process will also contain aperiodic components. Such processes can still be analyzed by harmonic methods. In fact, any aperiodic function of finite length can be approximated to any desired degree of accuracy by a sum of sinusoids, provided one includes enough sinusoidal terms in the approximating function. In most real problems one is limited to a finite, discrete time series obtained by sampling the process at say n equally spaced points in time. If the process is denoted by x(t), and if the sampling is arranged so that the sampling interval is one unit of time, i.e., $\Delta t = 1$, with the first sample point being taken at $t = 0$, then the time series can be written x(0), x(1), x(2), ..., x(n-1), or as is done more commonly, $x_0, x_1, x_2, \ldots, x_{n-1}$. It can be shown that the terms of this series can be represented exactly by a finite Fourier expansion of the form

$$(1.1)\quad x_j = a_0 + \sum_{0<k<n/2} \left(a_k \cos \frac{2\pi jk}{n} + b_k \sin \frac{2\pi jk}{n}\right) + (-1)^j a_{n/2},$$

$j = 0,1,2,\ldots,n-1,$

where the $a_{n/2}$ term is included only if n is even. The Fourier coefficients are given by

$$a_0 = \frac{1}{n}\sum_{j=0}^{n-1} x_j \quad , \tag{1.2}$$

$$\left.\begin{aligned} a_k &= \frac{2}{n}\sum_{j=0}^{n-1} x_j \cos\frac{2\pi jk}{n} \quad , \qquad (1.3)\\ b_k &= \frac{2}{n}\sum_{j=0}^{n-1} x_j \sin\frac{2\pi jk}{n} \quad , \qquad (1.4) \end{aligned}\right\} \quad 0 < k < \frac{n}{2}$$

$$a_{n/2} = \frac{1}{n}\sum_{j=0}^{n-1} (-1)^j x_j \quad , \qquad \text{if n is even} \tag{1.5}$$

Equation (1.1) expresses the time series x_j in terms of the component frequencies $f_k=k/n$ and can also be written in terms of the equivalent periods $T_k=1/f_k=n/k$ as

$$x_j = a_0 + \sum_{0<k<n/2} \left(a_k \cos\frac{2\pi j}{T_k} + b_k \sin\frac{2\pi j}{T_k}\right) + (-1)^j a_{n/2} \quad . \tag{1.6}$$

The Fourier spectrum of the time series is the discrete function represented by the set of points (f_k, F_k), k=0,1,...,[n/2], where [n/2] is taken to be n/2 if n is even and (n-1)/2 if n is odd, and F_k is the Fourier amplitude defined by

$$F_k = (a_k^2 + b_k^2)^{1/2}$$

For each of the frequencies f_k one can also define an associated phase by

$$(1.8) \qquad \phi_k = \tan^{-1}(a_k/b_k) \quad .$$

The expansion (1.6) can be written in terms of the Fourier amplitudes and phases as

$$(1.9) \qquad x_j = a_0 + \sum_{0<k<n/2} F_k \sin\left(\frac{2\pi j}{T_k} + \phi_k\right) + (-1)^j a_{n/2} \quad .$$

Again, this expansion is exact for all the sample points x_0, x_1, ..., x_{n-1}, and if n is large enough one might reasonably hope that Equation (1.9) would give a good approximation to the underlying process x(t) at other values of t also. That is, for any value of t in the interval $0 < t < (n-1)$, or even for t outside that interval, one might hope that

$$(1.10) \qquad x(t) \doteq a_0 + \sum_{0<k<n/2} F_k \sin\left(\frac{2\pi t}{T_k} + \phi_k\right)$$

without too much error.

The function (f_k, F_k) is the discrete Fourier spectrum of the data set x_0, x_1, ..., x_{n-1}. We note that continuous functions x(t) have analogous continuous Fourier spectra, but we do not treat such cases here since we are concerned only with finite, discrete time series. For the continuous case the reader is referred to Jenkins and Watts (1968). In computing a discrete Fourier spectrum one hopes that

the result approximates the continuous Fourier spectrum of the underlying process x(t). The approximation is not limited to the discrete frequencies $f_k = k/n$ but can be calculated for intermediate values of f by the formula

$$F(f) = \sqrt{a(f)^2 + b(f)^2} \quad , \tag{1.11}$$

where

$$a(f) = \frac{2}{n} \sum_{j=0}^{n-1} x_j \cos 2\pi jf \quad , \tag{1.12}$$

$$b(f) = \frac{2}{n} \sum_{j=0}^{n-1} x_j \sin 2\pi jf \quad . \tag{1.13}$$

We note in passing that some writers make a distinction between the continuous function F(f) and the discrete function (f_k, F_k), calling the latter a "line spectrum" and the former a "sample spectrum". Whatever it is called, the continuous function F(f) hopefully approximates the Fourier spectrum of x(t). The approximation fails to exactly represent the desired spectrum because the time series $x_0, x_1, \ldots, x_{n-1}$ cannot exactly represent the process x(t). The error arises for two reasons:

(1) The time series is restricted to a finite interval of time, so we do not know what the process x(t) is like outside that interval, and

(2) The time series represents the process only at a finite number of points in the interval, adjacent points being separated by the sample spacing Δt.

The latter restriction limits the highest frequency that is resolvable - one cannot expect to isolate components with frequencies whose corresponding periods are smaller than the sampling interval. In fact it is necessary to have at least two samples per cycle of the highest frequency (shortest period) that can be resolved. The frequency

$$f_N = 1/(2\Delta t) \tag{1.14}$$

is called the Nyquist frequency and is the highest frequency that can be resolved with a sample interval of Δt. The situation is actually worse than it appears on the surface because, if x(t) does have important harmonic components at frequencies greater than the Nyquist frequency, then these components will distort the Fourier spectrum values at lower frequencies. This is because the approximation cannot distinguish between the frequency f and the frequency $(1/\Delta t - f)$. Thus, if $\Delta t = 1$, then the Nyquist frequency is 0.5 and if x(t) has a harmonic component with frequency f = 0.75, the estimate of the transform will be distorted at f = 0.25. This phenomenon is called aliasing and must be kept in mind in designing a sampling procedure. One should always try to choose Δt small enough so that f_N exceeds the frequencies of all the important harmonic components in the process.

The first restriction, i.e., the restriction to a finite time interval, produces distortions in the calculated spectrum at the low frequencies (long periods). Obviously one cannot expect to resolve frequencies corresponding to periods much longer than the total time interval represented by the sample. Most users of the Fourier method do not trust the estimate at frequencies lower than that corresponding to

periods of at most one half the total time interval, and very conservative users demand that the total interval exceed a given period by a factor of 4 or more before they will trust the Fourier estimate at the corresponding frequency. The Fourier technique can detect the presence of long period components (components with periods comparable to the total length of record), but it cannot accurately pinpoint the frequency nor can it distinguish between two or more long period components at frequencies that are relatively close together.

It is convenient to analyze the effect of a truncated record in terms of a function called a data window. Consider a process x(t) defined on the time interval $(-\infty, +\infty)$.

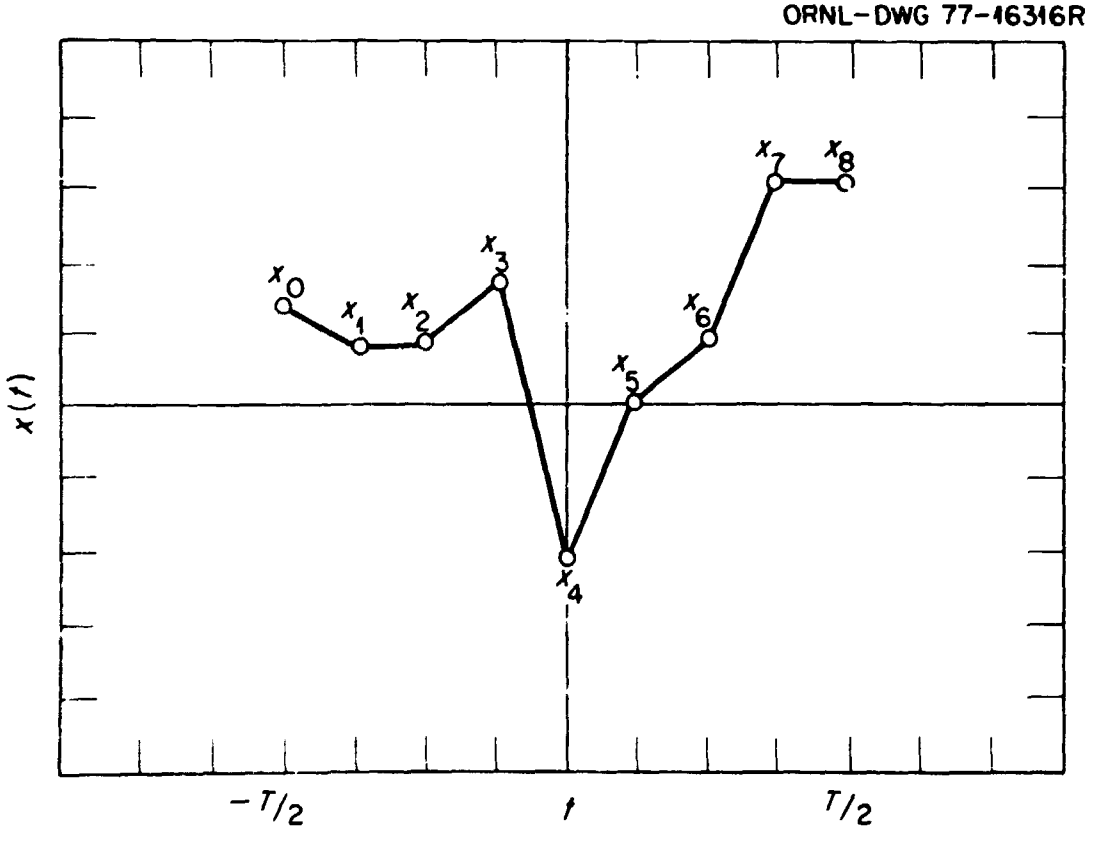

Fig. 1.5. The observed function $x_0(t)$, $\Delta t = 1$.

In general, one can only observe within a finite interval of length T, say [-T/2, T/2] as in Figure 1.5. The available data are then confined to the interval [-T/2, T/2]. The rectangular data window for the interval [-T/2, T/2] is defined by

$$w(t) = \begin{cases} 1, & \text{if } -T/2 \leq t \leq T/2, \\ 0, & \text{otherwise.} \end{cases} \tag{1.15}$$

The observed function corresponding to the available data is represented by

$$x_0(t) = x(t)\, w(t). \tag{1.16}$$

We temporarily ignore the effect of the discrete sampling and concentrate on the effect of applying the rectangular data window, i.e., the effect of truncating the record. Subjecting the observed function $x_0(t)$ to Fourier analysis will result in a transform $\mathcal{F}_{x_0}$ of x_0 which is really a smoothed version of the Fourier transform $\mathcal{F}_x$ of the original signal x(t). That is, if we let

$\mathcal{F}_{x_0}$ = Fourier transform of x_0,
$\mathcal{F}_x$ = Fourier transform of x, and
$\mathcal{F}_w$ = Fourier transform of w,

then it can be shown that

$$\mathcal{F}_{x_0}(f) = \int_{-\infty}^{\infty} \mathcal{F}_x(f')\, \mathcal{F}_w(f - f')\, df' . \tag{1.17}$$

The Fourier transform of the truncated, observed time series is said to be the convolution of the desired Fourier transform with the Fourier transform of the data window. This latter function is called the spectral window and can be shown to be

$$\mathcal{F}_w(f) = T \frac{\sin 2\pi ft}{\pi ft} \quad . \tag{1.18}$$

Applying the data window (1.15), which is sometimes called the boxcar window, necessarily distorts the Fourier transform. It can be shown that $\mathcal{F}_{x_0}(f)$ is necessarily smoother than $\mathcal{F}_x(f)$. A common practice, therefore, is to change the shape of the data window in order to reduce the distortion in the transform. This is usually done by rounding off the corners of the boxcar window, thus damping the contribution of the data at the two ends of the record. The idea is to design a window which decreases the smoothing and hence increases the resolution of the transform. This procedure is not without its own shortcomings. Spectral windows which give good resolution of the spectral peaks are often unstable to changes in the input data. On the other hand, those spectral windows that are stable with regard to changes in the data usually give poor resolution.

A more complete discussion of data windows and their corresponding spectral windows is given by Box and Jenkins (1970). We close our discussion of the subject by noting that using any data window involves some assumption about the data outside the available record (e.g., the boxcar window assumes a periodic extension of the data with period equal to the length of the record). Furthermore, using any window, other than

the boxcar window, distorts the data that are available. As we shall see in the next chapter, the Burg maximum entropy method is motivated by an attempt to overcome these two difficulties.

As an illustration of some of the ideas incorporated in this chapter, we present a very simple example. The number of points sampled is $n = 42$. The function sampled was the sum of a straight line trend and a sinusoid. The linear trend was included in the example because real time series data frequently contain trend components.

The data points in Figure 1.6 are generated from the equation

$$x_i = \int_{i-1}^{i} h(t)dt, \quad i = 1, 2, \ldots, 42,$$

where

$$h(t) = 7 + t + 5 \cos \frac{2\pi t}{6}$$

The linear trend, $x_T(t) = 7 + t$, was first removed by performing a straight line, least squares fit to the data and subtracting out the result. The function subjected to Fourier analysis was then $x(t) - x_T(t)$. It is necessary to remove the trend line because Fourier analysis would interpret it as a component with infinite period. The spectrum of the detrended function is shown in Figure 1.7.

The highest peak in Figure 1.7 occurs at $f = 0.1675$ with a corresponding period of $P = 5.97$, which is a good estimate of $P = 6$, the true period of the function. The other smaller peaks are called sidelobes and do not represent significant spectral components. They are one of the results of truncating the record. The width of the main peak also depends on the record length and gives an idea of the uncertainty in the estimate of the peak frequency.

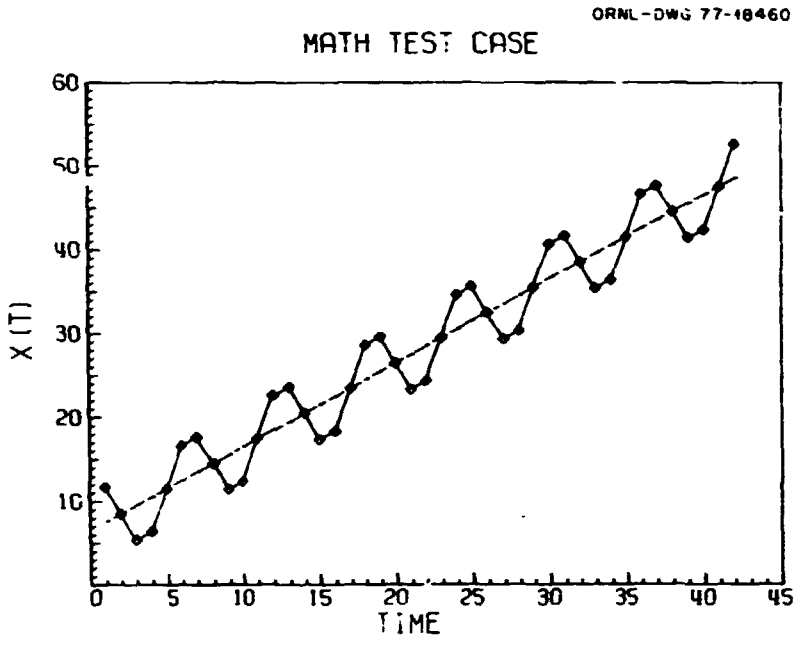

Fig. 1.6. Data x(t) are histogram yearly integrals of $h(t) = 5 \cos \frac{2\pi t}{6} + 7 + t$.

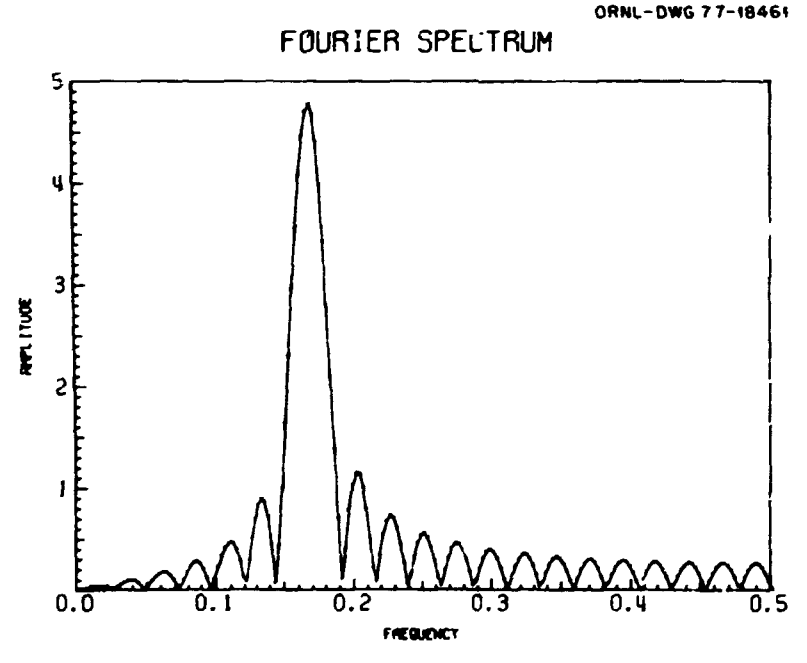

Fig. 1.7. Fourier Spectrum of detrended x(t).

CHAPTER II

MAXIMUM ENTROPY SPECTRAL ANALYSIS

A. A Brief Explanation of the Entropy Function.

In English, when one expresses a simple, grammatically correct, logical statement, his choice of words is governed by probability laws. For example, if a statement starts with "The mountain", one cannot just append any English phrase to have the statement make sense. "The mountain sees" or "The mountain my" do not make sense. However, "The mountain is big" is a perfectly logical statement. Thus, a person's choice of words is strongly dependent upon the logical structure of the language, and hence there is a probability that certain words are more likely than other words to follow a phrase like "The mountain". The freedom of choice in selecting a message is what is scientifically termed as information or entropy.

The word "information" must not be confused with its ordinary meaning. In the physical sciences, the entropy or information associated with a situation is a measure of the degree of randomness in the situation. Entropy is expressed in terms of probabilities - for example, the probability of getting to a certain stage in the process of forming a message and when in that stage the probability that certain words will be chosen next (Shannon and Weaver 1949).

How then can one measure the entropy or information of a process? In particular, the processes treated here are Markoff processes; that is, stochastic processes in which the probabilities depend on previous

events. Shannon and Weaver (1949) have shown that such a measure for information can best be estimated using logarithms of the probabilities.

Suppose $p_1, p_2, \ldots, p_n$ are the known probabilities of occurrence of a set of discrete events. Then

$$(2.1) \qquad I = I(p_1, p_2, \ldots, p_n) = -K \sum_{i=1}^{n} p_i \log_a p_i \quad ,$$

where K is a positive constant, and a is the chosen base for the logarithm, is called the entropy of the set of probabilities $p_1, p_2, \ldots, p_n$. Similarly, the entropy of a continuous distribution with density distribution function p(x) is defined by

$$(2.2) \qquad I = I(p(x)) = -\int_{-\infty}^{\infty} p(x) \log p(x)\, dx \quad .$$

Assume for example that there are two possibilities with probabilities p and q = 1-p. Then the entropy is

$$(2.3) \qquad I = I(p) = -p \log_e p - (1-p) \log_e (1-p) \quad ,$$

where for convenience the natural logarithm is used, and the constant K is taken to be 1. A plot of I in Equation (2.3) appears in Figure 2.1.

I is maximized when q = 1-p, that is $q=p=\frac{1}{2}$. In a physical situation this is the case of greatest uncertainty. In tossing a coin, for

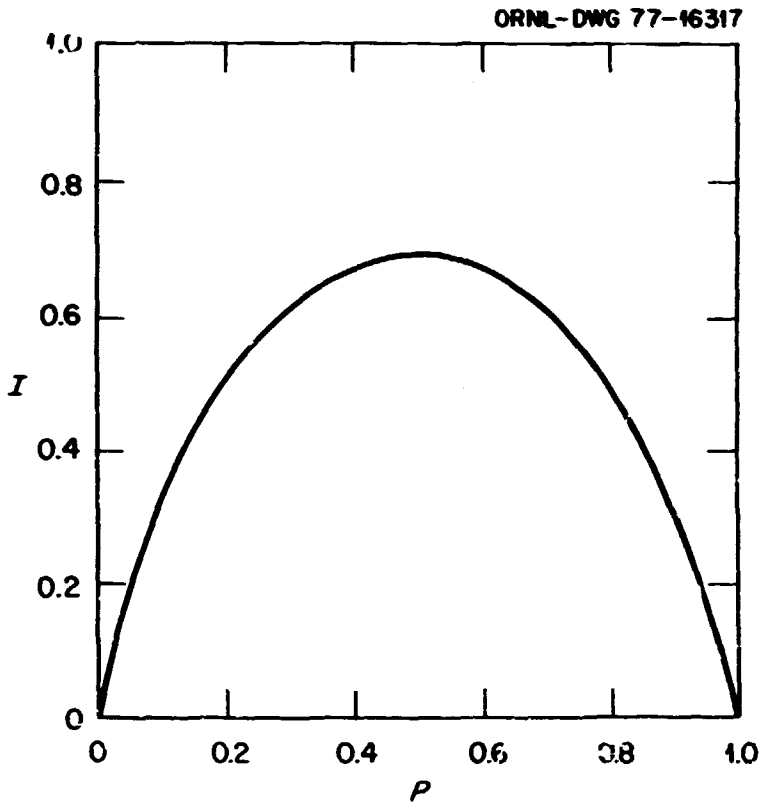

Fig. 2.1. The entropy I as a function of the probability p when there are two possible outcomes to an event.

example, out of 1000 tosses, it is likely that heads will appear about 500 times and tails about 500 times, but the uncertainty for any given toss is maximized. For a biased coin the uncertainty for any given toss is less because if one simply predicted the most likely result on each toss, then one would be right more than 500 times out of 1000. The more uncertain the event is, the greater the freedom of choice in predicting the outcome. The function I in Equation (2.3) is 0 when p=0, q=1 or q=0, p=1. That is I assumes a minimum of 0 when the event is certain to happen. In this case, one has little freedom of choice in making a prediction. In all other cases, I assumes a positive value.

The logarithm is used in the definition of entropy since it is mathematically more convenient to handle. With a logarithmic definition, information is additive, as would be expected. Assume for example that there are two independent events with probabilities p_1 and p_2. Then the probability that both events happen is $p_1 p_2$. The measured information or entropy of the two events is

$$I = -K \log_a(p_1 p_2) = I_1 + I_2 \quad , \tag{2.4}$$

where

$$I_1 = -K \log p_1 \quad , \quad I_2 = -K \log p_2 \quad . \tag{2.5}$$

The most common logarithmic bases employed in the definition of entropy are 2, 10 and e. The natural logarithm is the most convenient for use in mathematical analysis and is the one we adopted here.

From the preceding discussions, we note that I is indeed a reasonable measure of entropy. To maximize I is to maximize the uncertainty in the event. The constraints involved depend on the probabilities $p_1, p_2, \ldots, p_n$ with $0 \leq p_i \leq 1$, $i = 1,2,\ldots,n$ and $\sum_{i=1}^{n} p_i = 1$.

In time series analysis, the relevant probabilities are not known. Instead, one is faced with the analysis of the power spectrum. In this case, the entropy is proportional to

$$I = \int_{-\infty}^{\infty} \ln P(f)\, df \quad , \tag{2.6}$$

where P(f) is the power spectrum (see Burg 1975, Shannon and Weaver 1949).

In looking at the power spectrum of a time series, one asks how much of the original information has been lost or gained. The degree of uncertainty is measured in the entropy function I of Equation (2.6).

B. Maximum Entropy Spectral Analysis (MESA)

The basic assumption in MESA is that the stationary time series under analysis is the most random or least predictable. The classical approach involves the estimation of the power spectrum from known values of the autocorrelation function ρ_k, $|k| \leq M$, where M is less than or equal to the number of observations in the given time series. The standard assumption is that $\rho_k = 0$ for $|k| > M$, but usually a weighting function is also introduced and multiplied by ρ_k and the result is Fourier transformed. Given a set of autocorrelation function values ρ_k with the imposed condition that the power spectrum be non-negative definite, there usually exist infinitely many power spectra which will be consistent with the given data. Maximum entropy spectral analysis is based on the idea of choosing the spectrum which corresponds to the most random time series whose autocorrelation function agrees with given values.

C. Statement of the Problem

Find a real positive function P(f) which maximizes the entropy function

$$I = \int_{-f_M}^{f_M} \ln P(f)\, df \tag{2.7}$$

under the constraints

$$(2.8) \qquad \rho_k = \int_{-f_M}^{f_M} P(f) \exp(i2\pi fk\Delta t)\, df \quad ,$$

where $-M \leq k \leq M$, Δt is the sampling interval, and ρ_k the autocorrelation with time lag $k\Delta t$, $i = \sqrt{-1}$ and $f_M = \frac{1}{2\Delta t}$ is the Nyquist frequency.

D. Solution to the Problem

Wiener has shown that the power spectrum $P(f)$ is the discrete Fourier cosine transform of the autocorrelation ρ_k. So $P(f)$ has a Fourier series representation

$$(2.9) \qquad P(f) = \frac{1}{2f_M} \sum_{k=-\infty}^{+\infty} \rho_k\, e^{-i2\pi fk\Delta t} \quad .$$

Substituting (2.9) into (2.7), we get

$$(2.10) \qquad I = \int_{-f_M}^{f_M} \ln\left(\frac{1}{2f_M} \sum_{k=-\infty}^{+\infty} \rho_k\, e^{-i2\pi fk\Delta t}\right) df \quad .$$

Since the ρ_k's are known for $-M \leq k \leq M$, expression (2.10) will be maximized with respect to the ρ_k's, with $|k| > M$. Setting the appropriate derivatives to zero gives

$$(2.11) \qquad \frac{\partial I}{\partial \rho_\ell} = \int_{-f_M}^{f_M} \frac{\frac{1}{2f_M}\, e^{-i2\pi f \ell \Delta t}}{\left[\frac{1}{2f_M} \sum_{k=-\infty}^{+\infty} \rho_k\, e^{-i2\pi f k \Delta t}\right]} df$$

$$= \frac{1}{2f_M} \int_{-f_M}^{f_M} \frac{e^{-i2\pi f \ell \Delta t}}{P(f)}\, df$$

$$= 0, \quad |\ell| > M \quad .$$

But $[P(f)]^{-1}$ has a Fourier expansion so that

$$(2.12) \qquad [P(f)]^{-1} = \sum_{m=-\infty}^{+\infty} \lambda_m\, e^{-i2\pi f m \Delta t} \quad .$$

Thus (2.11) becomes

$$(2.13) \qquad \frac{\partial I}{\partial \rho_\ell} = \frac{1}{2f_M} \int_{-f_M}^{f_M} \left(\sum_{m=-\infty}^{+\infty} \lambda_m\, e^{-i2\pi f m \Delta t} \right) e^{-i2\pi f \ell \Delta t}\, df$$

$$= \frac{1}{2f_M} \int_{-f_M}^{f_M} \sum_{m=-\infty}^{+\infty} \lambda_m (\cos 2\pi f \Delta t m - i \sin 2\pi f \Delta t m) \times (\cos 2\pi f \Delta t \ell - i \sin 2\pi f \Delta t \ell)\, df$$

$$= 0 \quad \text{if} \quad |\ell| > M \quad .$$

Because of the orthogonal properties of sines and cosines, (2.13) implies that $\lambda_m = 0$ for $|m| > M$.

From (2.12) and the preceding, we get

$$P(f) = \frac{1}{\sum_{m=-M}^{M} \lambda_m e^{-i2\pi fm\Delta t}} \quad . \tag{2.14}$$

Thus (2.8) becomes

$$\rho_k = \int_{-f_M}^{f_M} \frac{e^{i2\pi fk\Delta t}}{\sum_{-M}^{M} \lambda_m e^{-i2\pi fm\Delta t}} \, df \quad . \tag{2.15}$$

Let $z = e^{-i2\pi f\Delta t}$. Under this transformation, $dz = -i2\pi\Delta t e^{-i2\pi f\Delta t} \, df$, $df = -dz/i2\pi\Delta tz$ and (2.15) changes to

$$\rho_k = \frac{1}{2\pi i\Delta t} \oint \frac{z^{-k-1}}{\sum_{m=-M}^{M} \lambda_m z^m} \, dz \quad , \tag{2.16}$$

where the contour integral around the unit circle $|z| = 1$ is computed in the counterclockwise direction.

Now $\sum_{m=-M}^{M} \lambda_m z^m$ must be real and positive for $|z| = 1$ since we require $P(f)$ to be real and positive. So

$$\sum_{m=-M}^{M} \lambda_m z^m = \left(\sum_{m=-M}^{M} \lambda_m z^m \right)^* \tag{2.17}$$

$$= \sum_{m=-M}^{M} \lambda_m^* z^{*m} = \sum_{m=-M}^{M} \lambda_m^* (1/z)^m \quad ,$$

where * denotes complex conjugation and $z\,z^* = 1$.

The expanded form of the left hand side of (2.17) is $\lambda_0 + \lambda_1 z + \ldots + \lambda_M z^M + \lambda_{-1} z^{-1} + \ldots + \lambda_{-M} z^{-M}$, whereas the right hand side is $\lambda_0^* + \lambda_1^* z^{-1} + \ldots + \lambda_M^* z^{-M} + \lambda_{-1}^* z + \ldots + \lambda_{-M}^* z^M$. By equating the coefficients in (2.17), we have the relationship

$$\lambda_j = \lambda_{-j}^* \quad , \quad j = 1,2,\ldots,M, \tag{2.18}$$

and

$$\sum_{m=-M}^{M} \lambda_m z^m = \lambda_0 + \lambda_1 z + \ldots + \lambda_M z^M + \lambda_1^* z^{-1} \ldots + \lambda_M^* z^{-M} \quad . \tag{2.19}$$

By the fundamental theorem of Algebra $\sum_{m=-M}^{M} \lambda_m z^m$ has 2M roots and furthermore is factorable in the form

$$\sum_{m=-M}^{M} \lambda_m z^m = (b_0 + b_1 z + \ldots + b_M z^M)\,x\,(b_0 + b_{-1} z^{-1} + \ldots + b_{-M} z^{-M}) \tag{2.20}$$

$$= (b_0 + b_1 z + \ldots + b_M z^M)\,x\,(b_0 + b_1^* z^{-1} + \ldots + b_M^* z^{-M}) \quad .$$

Factoring b_0 from (2.20) leads to

$$(2.21) \qquad \sum_{m=-M}^{M} \lambda_m z^m = b_0^2 (1 + a_1 z + \ldots\ldots + a_M z^M) \times (1 + a_1^* z^{-1} + \ldots\ldots + a_M^* z^{-M})$$

$$= (b_0^2 / 2 f_M \Delta t) \left(\sum_{m=0}^{M} a_m z^m \right) \left(\sum_{m=0}^{M} a_m^* z^{-m} \right) ,$$

where $b_0 > 0$, $a_0 = 1$, $a_i = b_i / b_0$, $a_i^* = b_i^*/b_0$, $i = 1, 2, \ldots, M$ and $2 f_M \Delta t = 1$.

Let $P_M = 2 f_M / b_0^2$. Then $P_M > 0$ and Equation (2.14) becomes

$$(2.22) \qquad P(f) = \frac{1}{\sum_{m=-M}^{M} \lambda_m z^m}$$

$$= \frac{P_M \Delta t}{\left(\sum_{m=0}^{M} a_m z^m \right) \left(\sum_{m=0}^{M} a_m^* z^{-m} \right)} .$$

The autocorrelation values in (2.16) simplify into

$$(2.23) \qquad \rho_k = \frac{P_M}{2\pi i} \oint \frac{z^{-k-1}}{\left(\sum_{m=0}^{M} a_m z^m \right) \left(\sum_{m=0}^{M} a_m^* z^{-m} \right)} dz ,$$

$-M \leq k \leq M$. Replacing k by k-j with $j \geq 0$ in (2.23) gives

$$(2.24) \qquad \rho_{k-j} = \frac{P_M}{2\pi i} \oint \frac{z^{-k+j-1}}{\left(\sum_{m=0}^{M} a_m z^m\right)\left(\sum_{m=0}^{M} a_m^* z^{-m}\right)} dz$$

Multiplying (2.24) by a_k^* and forming summations result in

$$(2.25) \qquad \sum_{k=0}^{M} a_k^* \rho_{k-j} = \frac{P_M}{2\pi i} \oint \frac{z^{j-1} \sum_{k=0}^{M} a_k^* z^{-k}}{\left(\sum_{m=0}^{M} a_m z^m\right)\left(\sum_{m=0}^{M} a_m^* z^{-m}\right)} dz$$

$$= \frac{P_M}{2\pi i} \oint \frac{z^{j-1}}{\sum_{m=0}^{M} a_m z^m} dz \quad , \quad j \geq 0 \quad .$$

What needs to be done at this point is to simplify the contour integral on the right hand side of (2.25). To do so requires Cauchy's famous theorems in complex variables theory (see Nehari 1968).

Theorem A: Let f(z) be analytic in a region R and on its boundary C. Then:

$$(2.26) \qquad \oint_C f(z)\, dz = 0 \quad .$$

Theorem B: Cauchy's Integral Formula. If f(z) is analytic inside and on the boundary C of a simply-connected region R and a any point inside C, then

$$f(a) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{z-a}\, dz \quad . \tag{2.27}$$

For j = 0, Equation (2.25) becomes

$$\sum_{k=0}^{M} a_k^* \rho_k = \frac{P_M}{2\pi i} \oint \frac{1}{\sum_{m=0}^{M} a_m z^m} \frac{dz}{z} \quad . \tag{2.28}$$

Let $f(z) = 1/\sum_{m=0}^{M} a_m z^m$. Then f(z) is analytic inside and on the unit circle $|z| = 1$. Thus by Cauchy's integral formula,

$$\frac{P_M}{2\pi i} \oint \frac{1}{\sum_{m=0}^{M} a_m z^m} \frac{dz}{z} = P_M f(0) = P_M \quad , \tag{2.29}$$

since $f(0) = a_0 = 1$.

For $j \geq 1$,

$$\frac{P_M}{2\pi i} \oint \frac{z^{j-1}}{\sum_{m=0}^{M} a_m z^m}\, dz = \frac{P_M}{2\pi i} \oint g(z)\, dz, \tag{2.30}$$

where $g(z) = z^{j-1} / \sum_{m=0}^{M} a_m z^m$, a function which is analytic inside and on the unit circle $|z| = 1$. By Theorem A, $\oint g(z)\, dz = 0$. So

$$(2.31) \qquad \frac{P_M}{2\pi i} \oint_j \frac{z^{j-1}}{\sum_{m=0}^{M} a_m z^m} \, dz = 0, \quad j \geq 1 .$$

By combining (2.25), (2.29) and (2.30), we get

$$(2.32) \qquad \sum_{k=0}^{M} a_k^* \rho_{k-j} = \begin{cases} P_M & \text{if } j = 0 , \\ 0 & \text{if } j \geq 1 . \end{cases}$$

Or similarly, since $\rho^*_{k-j} = \rho_{j-k}$, (2.32) becomes

$$(2.33) \qquad \sum_{k=0}^{M} \rho_{k-j} a_k = \begin{cases} P_M & \text{if } j = 0 , \\ 0 & \text{if } j \geq 1 . \end{cases}$$

The matrix equation corresponding to (2.33) is

$$(2.34) \qquad \begin{bmatrix} \rho_0 & \rho_1 & \cdots & \rho_M \\ \rho_{-1} & \rho_0 & \cdots & \rho_{M-1} \\ \vdots & & & \vdots \\ \rho_{-M} & \rho_{-M+1} & \cdots & \rho_0 \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ \vdots \\ a_M \end{bmatrix} = \begin{bmatrix} P_M \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

The elements a_i, $i = 0,1,2,\ldots,M$ with $a_0 = 1$ are normally called filter or reflection coefficients and Equation (2.34) is the equation for designing an M + 1 term prediction error filter with mean square error P_M. The meaning of these terms will become clearer in the next section. The matrix in (2.34) is a Toeplitz matrix.

Briefly reviewing the results of this section, we see that the power spectrum which maximizes (2.7) subject to (2.8) is the one given in (2.22). We can rewrite (2.22) in the following way:

$$P(f) = \frac{P_M \Delta t}{\left| 1 + \sum_{m=1}^{M} a_m z^m \right|^2} \tag{2.35}$$

$$= \frac{P_M \Delta t}{\left| 1 + \sum_{m=1}^{M} a_m e^{-i2\pi f m \Delta t} \right|^2}$$

since $z = e^{-i2\pi f \Delta t}$ and $a_0 = 1$. The terms P_M and a_m, $m = 1, \ldots, M$ are given by the solution to the matrix equation (2.34). Once these are known, the power spectrum $P(f)$ can be computed for values of the frequency f in the range $-\frac{1}{2\Delta t} \leq f \leq \frac{1}{2\Delta t}$.

E. Predictive Filtering and the Mean Square Error P_M.

As in Chapter I, let the given real time series be $x_0, x_1, \ldots, x_{n-1}$. Assume further that the mean of the process has been approximated and subtracted from the x_i's. The mathematical derivations in the preceding sections do not directly involve the x_i's. Instead, the autocorrelation function values if known provide the basic information

in calculating the power spectrum P(f). However, by working with the x_i's directly, one can arrive at the same matrix equation (2.34). The idea is to apply the filter coefficients a_i, $i = 1, \ldots, M$, with $M < n$ to the time series through a linear prediction relationship.

Let the time series $x_0, x_1, \ldots, x_{n-1}$ be written in the form $x_0, x_1, x_2, \ldots, x_{t-M}, x_{t-M+1}, \ldots, x_{t-1}, x_t, \ldots, x_{n-1}$. Consider the expression

$$\sum_{s=0}^{M} a_s x_{t-s} = x_t - (-a_1 x_{t-1} - \ldots - a_M x_{t-M}) , \tag{2.36}$$

where $a_0 = 1$. Furthermore let

$$\hat{x}_t = \sum_{s=1}^{M} (-a_s) x_{t-s} . \tag{2.37}$$

Then

$$e_t = x_t - \hat{x}_t = \sum_{s=0}^{M} a_s x_{t-s} \tag{2.38}$$

is actually the error in predicting x_t from the previous values $x_{t-1}, \ldots, x_{t-M}$ by means of the prediction coefficients $a_1, a_2, \ldots, a_M$. The error e_t, therefore, has to be kept close to a minimum for a good prediction.

The mean square error is defined to be

$$E\{e_t^2\} = E\{(x_t - \hat{x}_t)^2\} = E\{(\sum_{s=0}^{M} a_s x_{t-s})^2\} \tag{2.39}$$

where E stands for the expectation operator. Now

$$E\{e_t^2\} = E\{[x_t - \sum_{s=1}^{M} (-a_s) x_{t-s}]^2\}$$

$$= E\{x_t^2\} - 2 \sum_{s=1}^{M} (-a_s) E\{x_t x_{t-s}\} \tag{2.40}$$

$$+ E\{[\sum_{s=1}^{M} (-a_s) x_{t-s}]^2\} .$$

Since the x_i's are assumed to be given, we would like to minimize the mean square error with respect to a_j, $j = 1, \ldots, M$.

$$\frac{\partial E\{e_t^2\}}{\partial a_j} = 2E\{x_t x_{t-j}\} + 2 \sum_{s=1}^{M} (-a_s) E\{x_{t-s} (-x_{t-j})\} \tag{2.41}$$

$$= 0 .$$

By definition, the autocorrelation is

$$\rho_t = E\{x_s x_{s-t}\} , \tag{2.42}$$

for real x values. So (2.41) becomes

$$2\rho_j + 2 \sum_{s=1}^{M} (a_s)\, \rho_{j-s} = 0 , \tag{2.43}$$

or

$$\rho_j + \sum_{s=1}^{M} a_s\, \rho_{j-s} = 0, \; j = 1, \ldots, M . \tag{2.44}$$

We thereby have the system of equations

$$\begin{aligned}
&\rho_1 + a_1 \rho_0 + \ldots\ldots + a_M \rho_{1-M} = 0 \\
&\rho_2 + a_1 \rho_1 + \ldots\ldots + a_M \rho_{2-M} = 0 \\
&\quad\vdots \qquad\qquad\qquad\quad \vdots \qquad\quad \vdots \\
&\rho_M + a_1 \rho_{M-1} + \ldots\ldots + a_M \rho_0 = 0 .
\end{aligned} \tag{2.45}$$

The above system of equations shows that each ρ_j, j=1, ..., M is a linear combination of the others. To complete the set, one adds the equation

$$\rho_0 + a_1\rho_1 + \ldots\ldots + a_M\rho_M = E\left(e_t\, x_t\right) \tag{2.46}$$

which follows from (2.38) since

$$E\{e_t\, x_t\} = E\left\{\sum_{s=0}^{M} a_s\, x_{t-s}\, x_t\right\}$$

(2.47)

$$= \sum_{s=0}^{M} a_s\, E\{x_t\, x_{t-s}\} = \sum_{s=0}^{M} a_s\, \rho_s \ .$$

We can actually show that P_M whose value appears in Equation (2.33) can be expressed as

(2.48) $$P_M = E\{e_t\, x_t\} = E\{e_t^2\} \ .$$

That is, P_M is just the least mean square error in the linear prediction. From (2.33) and (2.47), it follows that

(2.49) $$P_M = \sum_{s=0}^{M} a_s\, \rho_s = E\{e_t\, x_t\} \ .$$

To show that $P_M = E\{e_t^2\}$ we first note that from (2.49) it follows that (2.46) can be rewritten with the quantity P_M on the right hand side and when this equation is appended to the system (2.45) the result is identical to the system (2.34). This follows from the fact that $\rho_{-s} = \rho_s$. Equation (2.34) is pre-multiplied by the row vector $[1\ a_1\ a_2\ \ldots\ a_M]$ to get

$$(2.50)\qquad [1\ a_1\ a_2\ \cdots\ a_M]\begin{bmatrix} \rho_0 & \rho_1 & \cdots & \rho_M \\ \rho_{-1} & \rho_0 & \cdots & \rho_{M-1} \\ \cdot & & \cdots & \cdot \\ \cdot & & \cdots & \cdot \\ \rho_{-M} & \rho_{-M+1} & \cdots & \rho_0 \end{bmatrix}\begin{bmatrix} 1 \\ a_1 \\ \cdot \\ \cdot \\ a_M \end{bmatrix} = P_M ,$$

from which it follows that

$$(2.51)\qquad \begin{aligned} P_M &= \sum_{v=0}^{M}\sum_{s=0}^{M} a_v\, \rho_{v-s}\, a_s \\ &= \sum_{v=0}^{M}\sum_{s=0}^{M} a_v\, E\{x_{t-s}\, x_{t-v}\} a_s = E\left\{\left(\sum_{s=0}^{M} a_s\, x_{t-s}\right)^2\right\} . \end{aligned}$$

Applying (2.39) gives

$$(2.52)\qquad P_M = E\left\{\left(\sum_{s=0}^{M} a_s\, x_{t-s}\right)^2\right\} = E\{e_t^2\} .$$

For a fixed M and set of M filter coefficients, every x_t in the set $\{x_0, x_1, \ldots, x_{n-1}\}$ can be predicted linearly as long as one stays within the set of x - values. We can think of this as the process in Diagram 2.1 with $a_0 = 1$. The above process is called filtering. In particular, it is called forward filtering, x_t being predicted from its M previous values. Backward filtering, on the other hand, is described in Diagram 2.2.

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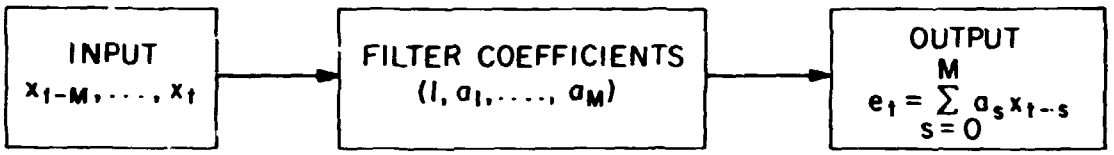

Diagram 2.1. Forward filtering.

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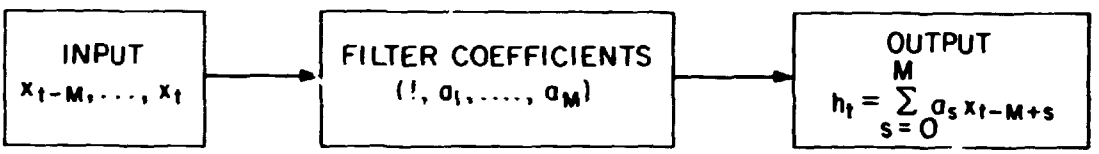


Diagram 2.2. Backward filtering.

In Diagram 2.2, x_{t-M} is predicted from its M future values x_{t-M+1}, ..., x_t. The error in the prediction is h_t and

$$h_t = \sum_{s=0}^{M} a_s \, x_{t-M+s}$$

$$= a_M x_t + a_{M-1} x_{t-1} + \ldots + a_1 \, x_{t-M+1} + x_{t-M} \quad . \tag{2.53}$$

Let

$$\hat{x}_{t-M} = \sum_{s=1}^{M} (-a_s) x_{t-M+s} \quad . \tag{2.54}$$

Then

$$E\{h_t^2\} = E\{(x_{t-M} - \hat{x}_{t-M})^2\}$$

$$= E\left\{\left[x_{t-M} - \sum_{s=1}^{M} (-a_s)\, x_{t-M+s}\right]^2\right\} \quad . \tag{2.55}$$

By taking the partial derivative of $E\{h_t^2\}$ with respect to a_j, $j = 1, \ldots, M$ and equating the results to 0, we get

$$\frac{\partial E\{h_t^2\}}{\partial a_j} = \frac{\partial}{\partial a_j}\left[E\{x_{t-M}x_{t-M}\} - 2\sum_{s=1}^{M} (-a_s)\, E\{x_{t-M+s}x_{t-M}\}\right.$$

$$\left. + E\left\{\left[\sum_{s=1}^{M} (-a_s)\, x_{t-M+s}\right]^2\right\}\right] \tag{2.56}$$

$$= 2E\{x_{t-M}x_{t-M+j}\} + 2\sum_{s=1}^{M} (-a_s)\, E\{(x_{t-M+s})(-x_{t-M+j})\}$$

$$= 0 \quad .$$

By applying the definition of the autocorrelation function to (2.56), the following formula is obtained:

$$\rho_j + \sum_{s=1}^{M} a_s\, \rho_{j-s} = 0, \; j = 1, \ldots, M \quad . \tag{2.57}$$

Equation (2.57) is exactly Equation (2.44). This implies that forward and backward filtering lead to the same matrix equation and that

$$P_M = E\{e_t^2\} = E\{h_t^2\} \tag{2.58}$$

for all possible t.

The following simple example will help illustrate the ideas described in this section. Suppose the time series is the set of values $A = \{x_0, x_1, x_2, x_3\}$. Let M = 2. The set A has two M + 1 ordered subsets: $A_1 = \{x_0, x_1, x_2\}$ and $A_2 = \{x_1, x_2, x_3\}$. Denote the filter coefficients by $a_0 = 1, a_1, a_2$. Then both x_0 and x_1 can be predicted from their future values by backward filtering and

$$\begin{aligned} \hat{x}_0 &= (-a_1)\, x_1 + (-a_2)\, x_2 , \\ \hat{x}_1 &= (-a_1)\, x_2 + (-a_2)\, x_3 , \end{aligned} \tag{2.59}$$

where $\hat{x}_0$, $\hat{x}_1$ are the predicted values of x_0, x_1 respectively. The backward filtering errors are:

$$\begin{aligned} h_0 &= x_0 - \hat{x}_0 = x_0 + a_1 x_1 + a_2 x_2 , \\ h_1 &= x_1 - \hat{x}_1 = x_1 + a_1 x_2 + a_2 x_3 . \end{aligned} \tag{2.60}$$

On the other hand, x_2 and x_3 are predicted by forward filtering and

$$\hat{x}_2 = (-a_1)\, x_1 + (-a_2)\, x_0 \ ,$$
$$\hat{x}_3 = (-a_1)\, x_2 + (-a_2)\, x_1 \ , \tag{2.61}$$

where $\hat{x}_2$, $\hat{x}_3$ are the predicted values of x_2 and x_3 respectively. The forward filtering errors are:

$$e_2 = x_2 - \hat{x}_2 = x_2 + a_1 x_1 + a_2 x_0 \ ,$$
$$e_3 = x_3 - \hat{x}_3 = x_3 + a_1 x_2 + a_2 x_1 \ . \tag{2.62}$$

Briefly summarizing the discussion in the preceding pages, in predicting a point in a time series, either from its future or past values, one should minimize the mean square error P_M with respect to the filter coefficients a_j. In the next section, we discuss an algorithm (developed by J. P. Burg, 1975) which determines the values of the a_j's and P_M in Equation (2.34).

F. The Burg Algorithm for Estimating P_M and the Filter Coefficients

J. P. Burg's algorithm for finding the filter coefficients and P_M in Equation (2.34) depend on both forward and backward filtering, i.e., the points in the time series are predicted from their previous and

future values. The solution to Equation (2.34) is based on a modified version of the Levinson algorithm for solving a Toeplitz matrix equation. Since the algorithm is an iterative procedure, the filter coefficients $(1, a_1, a_2, \ldots, a_M)$ will be written as $(1, a_{1M}, a_{2M}, \ldots, a_{MM})$.

Starting with a one-point filter ($a_{00} = 1$), we have

$$[\rho_0]\,[1] = [P_0] \qquad \text{or} \qquad \rho_0 = P_0 \tag{2.63}$$

So the value of P_0 is determined by the autocorrelation ρ_0. The two-point filter has the matrix equation

$$\begin{bmatrix} \rho_0 & \rho_1 \\ \rho_1 & \rho_0 \end{bmatrix} \begin{bmatrix} 1 \\ a_{11} \end{bmatrix} = \begin{bmatrix} P_1 \\ 0 \end{bmatrix} \tag{2.64}$$

Assuming that a_{11} and P_1 have been determined (the actual computations are described in the next section) the three-point filter is obtained from the preceding equation by means of the following extension:

$$\begin{bmatrix} \rho_0 & \rho_1 & \rho_2 \\ \rho_1 & \rho_0 & \rho_1 \\ \rho_2 & \rho_1 & \rho_0 \end{bmatrix} \left\{ \begin{bmatrix} 1 \\ a_{11} \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} 0 \\ a_{11} \\ 1 \end{bmatrix} \right\} = \begin{bmatrix} P_1 \\ 0 \\ \Delta_1 \end{bmatrix} + c_1 \begin{bmatrix} \Delta_1 \\ 0 \\ P_1 \end{bmatrix} \tag{2.65}$$

where it is required that

$$\Delta_1 + c_1 p_1 = 0 \quad . \tag{2.66}$$

We note that the vectors $c_1 \begin{bmatrix} 0 \\ a_{11} \\ 1 \end{bmatrix}$ and $c_1 \begin{bmatrix} \Delta_1 \\ 0 \\ P_1 \end{bmatrix}$ are just multiples of the reverses of $\begin{bmatrix} 1 \\ a_{11} \\ 0 \end{bmatrix}$ and $\begin{bmatrix} P_1 \\ 0 \\ \Delta_1 \end{bmatrix}$ respectively. The reason for also applying the filter in reverse is that each point in the time series $x_0, x_1, \ldots, x_{n-1}$ is predicted forward and backward. The three-point filter then is $(1, a_{12}, a_{22})$ where

$$\begin{aligned} a_{12} &= a_{11} + c_1 a_{11} \quad , \\ a_{22} &= c_1 \quad , \quad P_2 = P_1 + c_1 \Delta_1 \quad . \end{aligned} \tag{2.67}$$

Extending the matrix equation in (2.65) leads to:

$$\begin{bmatrix} \rho_0 & \rho_1 & \rho_2 & \rho_3 \\ \rho_1 & \rho_0 & \rho_1 & \rho_2 \\ \rho_2 & \rho_1 & \rho_0 & \rho_1 \\ \rho_3 & \rho_2 & \rho_1 & \rho_0 \end{bmatrix} \left\{ \begin{bmatrix} 1 \\ a_{12} \\ a_{22} \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ a_{22} \\ a_{12} \\ 1 \end{bmatrix} \right\} = \begin{bmatrix} P_2 \\ 0 \\ 0 \\ \Delta_2 \end{bmatrix} + c_2 \begin{bmatrix} \Delta_2 \\ 0 \\ 0 \\ P_2 \end{bmatrix} \tag{2.68}$$

where in this case the four-point filter is $(1, a_{13}, a_{23}, a_{33})$ with

$$a_{13} = a_{12} + c_2 a_{22} ,$$

$$a_{23} = a_{22} + c_2 a_{12} , \quad a_{33} = c_2 . \tag{2.69}$$

Again it is required that $\Delta_2 + c_2 P_2 = 0$.

Continuing in this manner, we get the general form:

$$\begin{bmatrix} c_0 & c_1 & \cdots & c_M \\ c_1 & c_0 & \cdots & c_{M-1} \\ c_2 & c_1 & \cdots & c_{M-2} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ c_M & c_{M-1} & & c_0 \end{bmatrix} \begin{bmatrix} 1 \\ a_{1,M} \\ a_{2,M} \\ \cdot \\ \cdot \\ a_{M,M} \end{bmatrix} = \begin{bmatrix} P_M \\ 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \tag{2.70}$$

valid for any $M < n$.

Furthermore, for $i = 1, \ldots, M-1$,

$$a_{i,M} = a_{i,M-1} + c_{M-1} a_{M-i,M-1} ,$$

$$a_{M,M} = c_{M-1} , \quad P_M = P_{M-1} + c_{M-1} \Delta_{M-1} , \tag{2.71}$$

$$\Delta_{M-1} + c_{M-1} P_{M-1} = 0 .$$

Now since $\Delta_{M-1} = - c_{M-1} P_{M-1}$, then

$$
\begin{aligned}
P_M &= P_{M-1} + c_{M-1} \Delta_{M-1} \\
&= P_{M-1} - c_{M-1}^2 P_{M-1} \\
&= P_{M-1} (1 - c_{M-1}^2) \\
&= P_{M-1} (1 - a_{MM}^2) \, .
\end{aligned} \tag{2.72}
$$

The least mean square error P_M can only be estimated in this algorithm. Let us go back to the time series $\{x_0, x_1, \ldots, x_{n-1}\}$ and assume that there are L possible subsets of the form $\{x_{t_j-M}, x_{t_j-M+1}, \ldots, x_{t_j}\}$, $j = 1, \ldots, L$. By applying the filter $(1, a_{1,M}, \ldots, a_{M,M})$ in a forward manner to each of the x_{t_j}, $j = 1, \ldots, L$, we get

$$
\begin{aligned}
& x_{t_j} + a_{1,M} x_{t_j-1} + \ldots + a_{M,M} x_{t_j-M} \\
&= x_{t_j} + (a_{1,M-1} + c_{M-1} a_{M-1,M-1}) x_{t_j-1} \\
&\quad + (a_{2,M-1} + c_{M-1} a_{M-2,M-1}) x_{t_j-2} \\
&\quad + \ldots + c_{M-1} x_{t_j-M} \, ,
\end{aligned} \tag{2.73}
$$

$$j = 1, \ldots, L \, .$$

Equation (2.73) gives the error in predicting x_{t_j} from its previous M values. Since there are L equations of the form (2.73), the mean square error P_M can be estimated by the squared averages

$$P_{1M} = \sum_{j=1}^{L} \gamma_j \Big[x_{t_j} + (a_{1,M-1} + c_{M-1}\, a_{M-1,M-1})\, x_{t_j-1} + \ldots + (a_{M-1,M-1} + c_{M-1}\, a_{1,M-1})\, x_{t_j-M+1} + (c_{M-1}\, x_{t_j-M}) \Big]^2 , \tag{2.74}$$

where the γ_j's are weights such that $\sum_{j=1}^{L} \gamma_j = 1$ and $\gamma_j \geq 0$.

The terms in (2.74) can be re-grouped using the quantities

$$\begin{aligned} \alpha_j &= a_{M-1,M-1}\, x_{t_j-M+1} + \ldots + a_{1,M-1}\, x_{t_j-1} + x_{t_j} , \\ \beta_j &= x_{t_j-M} + a_{1,M-1}\, x_{t_j-M+1} + \ldots + a_{M-1,M-1}\, x_{t_j-1} , \\ j &= 1, \ldots, L . \end{aligned} \tag{2.75}$$

which are seen to be a forward prediction error and a backward prediction error respectively. The new expression is then

$$P_{1M} = \sum_{j=1}^{L} \gamma_j (\alpha_j + c_{M-1}\, \beta_j)^2 . \tag{2.76}$$

If on the other hand, the filter $(1\ a_{1M} \ldots a_{MM})$ were applied backward, then

$$
\begin{aligned}
& x_{t_j-M} + a_{1,M}\, x_{t_j-M+1} + a_{2,M}\, x_{t_j-M+2} + \ldots + a_{M,M}\, x_{tj} \\
&\quad = x_{t_j-M} + (a_{1,M-1} + c_{M-1}\, a_{M-1,M-1})\, x_{t_j-M+1} \\
&\qquad + (a_{2,M-1} + c_{M-1}\, a_{M-2,M-1})\, x_{t_j-M+2} \\
&\qquad + \ldots + (a_{M-1,M-1} + c_{M-1}\, a_{1,M-1})\, x_{t_j-1} \\
&\qquad + c_{M-1}\, x_{t_j} .
\end{aligned}
\tag{2.77}
$$

As was noted earlier, the backward and forward filters should produce the same mean square error P_M. So P_M can also be estimated by

$$
\begin{aligned}
P_{2M} &= \sum_{j=1}^{L} \gamma_j \Big[x_{t_j-M} + (a_{1,M-1} + c_{M-1}\, a_{M-1,M-1}) x_{t_j-M+1} \\
&\qquad + (a_{2,M-1} + c_{M-1}\, a_{M-2,M-1}) x_{t_j-M+2} \\
&\qquad + \ldots + c_{M-1}\, x_{t_j} \Big] \\
&= \sum_{j=1}^{L} \gamma_j (\beta_j + c_{M-1}\, \alpha_j)^2 .
\end{aligned}
\tag{2.78}
$$

Thus one class of estimates for P_M is

$$\hat{P}_M = \frac{1}{2}(P_{1M} + P_{2M}) \tag{2.79}$$

$$= \frac{1}{2} \sum_{j=1}^{L} \gamma_j \left[(\alpha_j + c_{M-1}\, \beta_j)^2 + (\beta_j + c_{M-1}\, \alpha_j)^2 \right]$$

where it is required that the γ_j be specified.

G. Andersen's Version of the Burg Algorithm for Estimating Coefficients

In applying the above algorithm, one would start with an estimate of P_0. Then at each stage M, the quantify $\partial \hat{P}_M / \partial a_{M1}$ is set equal to zero. For M = 0, P_0 is estimated by

$$\hat{P}_0 = r_0$$

$$= \left(\frac{1}{n-1}\right) \sum_{j=0}^{n-1} x_j x_j \,. \tag{2.80}$$

where r_0 is an estimate of the autocorrelation ρ_0.

For M = 1, the set $\{x_0, x_1, \ldots, x_{n-1}\}$ would have n-1 subsets: $\{x_0, x_1\}$, $\{x_1, x_2\}$, ..., $\{x_{n-2}, x_{n-1}\}$ and the average power is:

$$\hat{P}_1 = \frac{1}{2} \frac{1}{n-1} \sum_{j=0}^{n-2} \left[(x_j + a_{11}\, x_{j+1})^2 + (x_{j+1} + a_{11}\, x_j)^2 \right] , \tag{2.81}$$

where the weights are $\gamma_j = \frac{1}{n-1}$, $j = 0, \ldots, n-2$ (see N. Andersen, 1974). Equating the partial of $\hat{P}_1$ with respect to a_{11} to zero, we get

$$\frac{\partial \hat{P}_1}{\partial a_{11}} = \frac{1}{2(n-1)} \sum_{j=0}^{n-2} \left[2(x_j + a_{11} x_{j+1}) x_{j+1} \right.$$

(2.82)

$$\left. + 2(x_{j+1} + a_{11} x_j) x_j \right] = 0 .$$

So

$$a_{11} = -2 \sum_{j=0}^{n-2} x_j x_{j+1} \Big/ \sum_{j=0}^{n-2} (x_j^2 + x_{j+1}^2) . \tag{2.83}$$

Since a_{11} is known at this point, one solves for $\hat{P}_1$ by using (2.72),

$$\hat{P}_1 = \hat{P}_0 (1 - a_{11}^2) = r_0(1 - a_{11}^2) . \tag{2.84}$$

For M=2, the set $\{x_0, x_1, \ldots, x_{n-1}\}$ has n-2 subsets with M + 1 = 3 consecutive elements in each subset: $\{x_0, x_1, x_3\}$, $\{x_1, x_2, x_3\}$, $\ldots$, $\{x_{n-3}, x_{n-2}, x_{n-1}\}$. The resulting average power is:

$$\hat{P}_2 = \frac{1}{2}\frac{1}{n-2}\sum_{j=0}^{n-3}\Big[(x_j + a_{12}\,x_{j+1} + a_{22}x_{j+2})^2 + (x_{j+2} + a_{12}\,x_{j+1} + a_{22}\,x_j)^2\Big]$$

$$= \frac{1}{2}\frac{1}{n-2}\sum_{j=0}^{n-3}\Big\{[x_j + (a_{11} + a_{22}\,a_{11})\,x_{j+1} + a_{22}\,x_{j+2}]^2 + [x_{j+2} + (a_{11} + a_{22}\,a_{11})\,x_{j+1} + a_{22}\,x_j]^2\Big\}, \tag{2.85}$$

since $a_{12} = a_{11} + a_{22}\,a_{11}$ from (2.67). Thus

$$\frac{\partial \hat{P}_2}{\partial a_{22}} = \frac{1}{2}\frac{1}{n-2}\sum_{j=0}^{n-3}\Big[2(x_j + a_{11}\,x_{j+1} + a_{22}\,a_{11}\,x_{j+1} + a_{22}\,x_{j+2})(a_{11}\,x_{j+1} + x_{j+2}) + 2(x_{j+2} + a_{11}\,x_{j+1} + a_{22}\,a_{11}\,x_{j+1} + a_{22}\,x_j)(a_{11}\,x_{j+1} + x_j)\Big] \tag{2.86}$$

$$= 0\,.$$

The value for a_{22} is the following form:

$$a_{22} = -2 \sum_{j=0}^{n-3} (a_{11}\, x_j\, x_{j+1} + a_{11}^2\, x_{j+1}^2 + a_{11}\, x_{j+1}\, x_{j+2} + x_j\, x_{j+2}) \Big/ \sum_{j=0}^{n-3} (x_j^2 + 2a_{11}\, x_j\, x_{j+1} + 2a_{11}^2\, x_{j+1}^2 + 2a_{11}\, x_{j+1}\, x_{j+2} + x_{j+2}^2) \quad (2.87)$$

Since a_{22} is known, $\hat{P}_2$ is given by the expression:

$$\hat{P}_2 = \hat{P}_1(1 - a_{22}^2) \quad (2.88)$$

Continuing in this manner, at the M^{th} stage, there will be n-M subsets of $\{x_0, x_1, \ldots, x_{n-1}\}$ and they are of the form: $\{x_0, x_1, \ldots, x_M\}$, $\{x_1, x_2, \ldots, x_{M+1}\}$, $\ldots$, $\{x_{n-M-1}, x_{n-M}, \ldots, x_{n-1}\}$. The average output power is then

$$\begin{aligned} P_M &= \frac{1}{2}\frac{1}{n-M} \sum_{j=0}^{(n-1)-M} \left[(x_j + \sum_{k=1}^{M} a_{k,M}\, x_{j+k})^2 + (x_{j+M} + \sum_{k=1}^{M} a_{k,M}\, x_{j+M-k})^2 \right] \\ &= \frac{1}{2}\frac{1}{n-M} \sum_{j=0}^{(n-1)-M} \left[(\sum_{k=0}^{M} a_{k,M}\, x_{j+k})^2 + (\sum_{k=0}^{M} a_{k,M}\, x_{j+M-k})^2 \right] . \end{aligned} \quad (2.89)$$

where $a_{0,M} = 1$. From (2.71), we recall that $a_{i,M} = a_{i,M-1} + a_{MM}\, a_{M-i,M-1}$, $i = 1, 2, \ldots, M-1$. If we impose the added condition that $a_{0,M} = 1$ and $a_{i,M} = 0$ for $i > M$, then (2.71) holds for all M and (2.89) simplifies to:

$$\hat{P}_M = \frac{1}{2}\frac{1}{n-M}\sum_{j=0}^{(n-1)-M}\left\{\left[\sum_{k=0}^{M}(a_{k,M-1} + a_{MM}\, a_{M-k,M-1})\, x_{j+k}\right]^2 + \left[\sum_{k=0}^{M}(a_{k,M-1} + a_{MM}\, a_{M-k,\,M-1})\, x_{j+M-k}\right]^2\right\} \tag{2.90}$$

$$= \frac{1}{2}\frac{1}{n-M}\sum_{j=0}^{(n-1)-M}\left[\left(\sum_{k=0}^{M} a_{k,M-1}\, x_{j+k} + a_{MM}\sum_{k=0}^{M} a_{M-k,M-1} x_{j+k}\right)^2 + \left(\sum_{k=0}^{M} a_{k,M-1}\, x_{j+M-k} + a_{MM}\sum_{k=0}^{M} a_{M-k,M-1}\, x_{j+M-k}\right)^2\right] .$$

Let

$$p_{jM} = \sum_{k=0}^{M} a_{k,M-1}\, x_{j+k} = \sum_{k=0}^{M} a_{M-k,M-1}\, x_{j+M-k} , \tag{2.91}$$

$$q_{jM} = \sum_{j=0}^{M} a_{M-k,M-1}\, x_{j+k} = \sum_{k=0}^{M} a_{k,M-1}\, x_{j+M-k} ,$$

$$j = 0, 1, \ldots, (n-1) - M .$$

Then

$$\hat{P}_M = \frac{1}{2}\frac{1}{n-M}\sum_{j=0}^{(n-1)-M}\left[(p_{jM} + a_{MM}\, q_{jM})^2 + (q_{jM} + a_{MM}\, p_{jM})^2\right] . \tag{2.92}$$

By setting $\partial\hat{P}_M/\partial a_{MM} = 0$, we get the value for a_{MM}:

$$a_{MM} = -2\sum_{j=0}^{(n-1)-M} p_{jM}\, q_{jM} \Big/ \sum_{j=0}^{(n-1)-M} (p_{jM}^2 + q_{jM}^2) . \tag{2.93}$$

The recursion formulas for the p_{jm} and q_{jm} are derived from (2.71) and (2.91) They are:

$$p_{jM} = p_{j,M-1} + a_{M-1,M-1}\, q_{j,M-1} , \tag{2.94}$$

$$q_{jM} = q_{j+1,M-1} + a_{M-1,M-1}\, p_{j+1,M-1} .$$

The starting values for the equations in (2.94) for M=1 are:

$$p_{j1} = x_j , \quad q_{j1} = x_{j+1} , \quad j = 0, 1, \ldots, n-2 . \tag{2.95}$$

The equations in (2.94) and (2.95) are implemented in the computer program which is discussed later in this report.

CHAPTER III

APPLICATIONS OF MESA AND FOURIER ANALYSIS

A. Comparative Study of MESA and Fourier Analysis.

EXAMPLE 3.1.

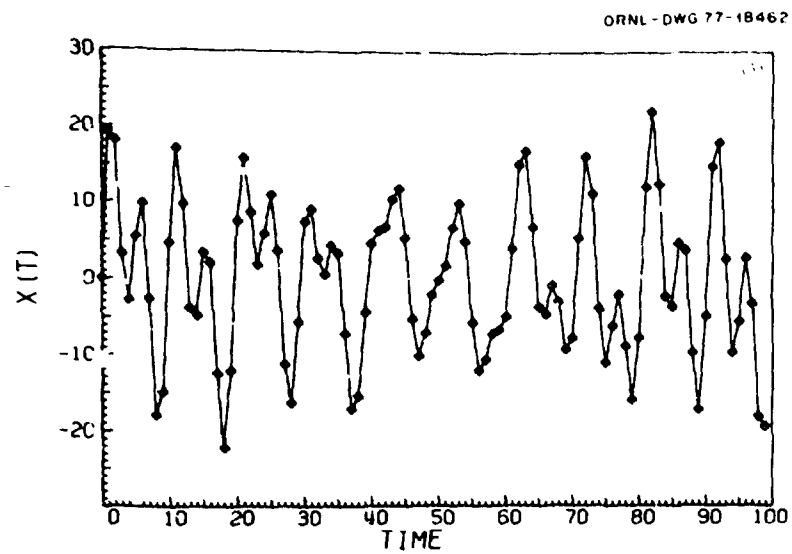
To study the effectiveness of the two methods, consider a mathematical test case consisting of sine functions. The equation is:

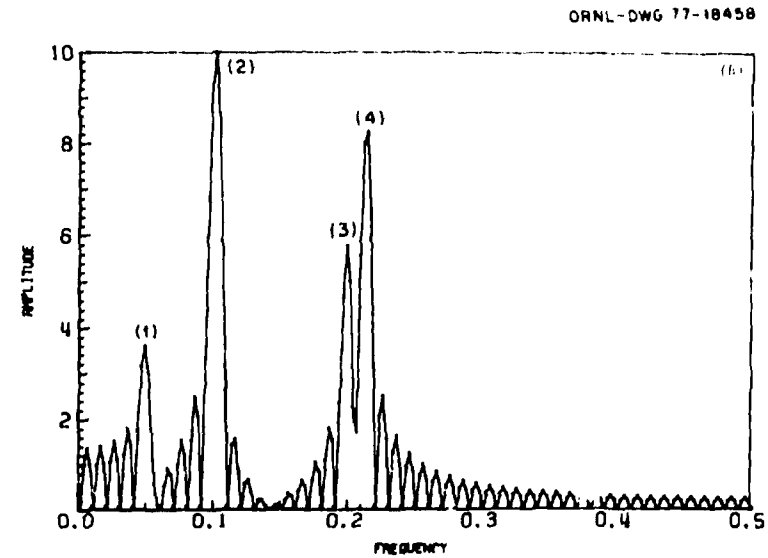
$$x_t = 3 \sin(.05(2\pi t)) + 10 \sin(0.1(2\pi t)) + 5 \sin(0.2(2\pi t)) + 8 \sin(0.21(2\pi t)),$$

$$t = 0,1\ldots, 99.$$

No trend in the time series is removed in the analysis. The discretized function x_t and its spectra are shown in Figures 3.1A, 3.1B, and 3.1C. By examining the equation for x_t, one can tell that the true periods are at the frequencies: $f = 0.05$, $f = 0.1$, $f = 0.2$, $f = 0.21$, corresponding to periods: $P = 20$, $P = 10$, $P = 5$ and $P = 4.76$ respectively. Table 3.1 gives a listing of the periods as predicted by both forms of analysis.

As one can infer from Figures 3.1B, 3.1C and Table 3.1, MESA gives somewhat better predictions of the frequencies of the periodic components. The number of filter coefficients M used in MESA for this particular problem is 51.





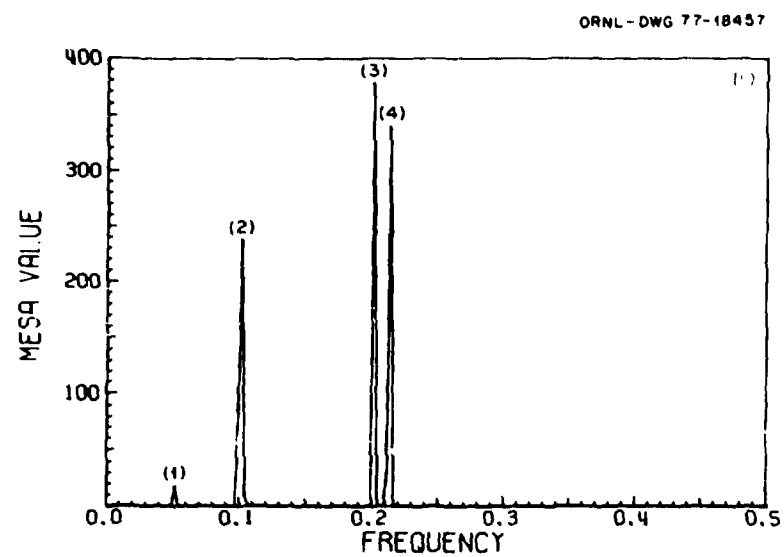


Fig. 3.1. A. Mathematical test case of four sinusoids (100 data points).
B. Fourier spectrum.
C. Maximum entropy spectrum with M = 51.

TABLE 3.1.

FREQUENCY COMPONENTS OF THE SINE PROBLEM
OF EXAMPLE 3.1 (100 data points)

Frequencies at which true periods occur		Frequencies predicted by MESA - 51 filter coefficients used	Frequencies predicted by Fourier analysis
(1)	0.05	0.0500	0.0475
(2)	0.10	0.1000	0.1000
(3)	0.20	0.2000	0.1975
(4)	0.21	0.2125	0.2125

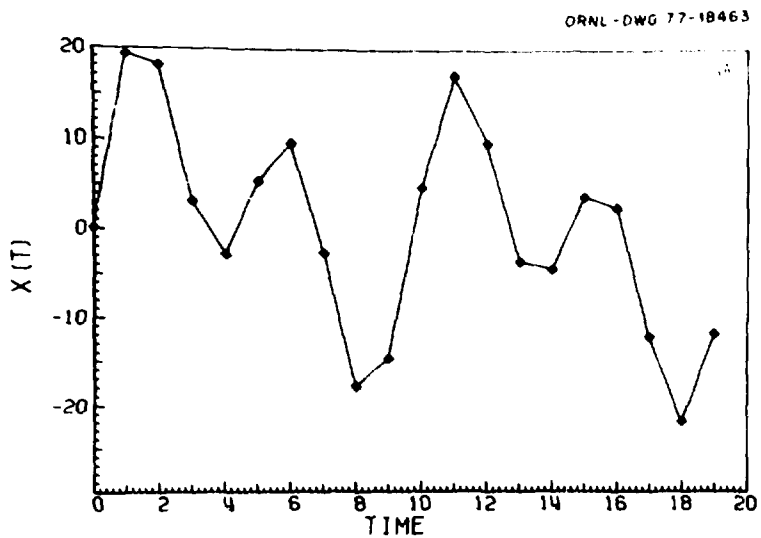
EXAMPLE 3.2

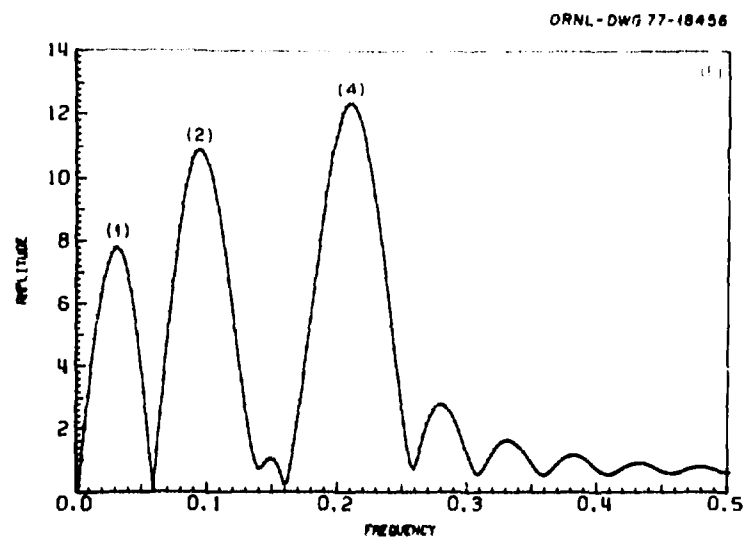
What happens in the spectrum of the time series x_t in Example 3.1 if the number of sampled points is reduced? Suppose only 20 points are used, say at t = 0, 1, ..., 19. The shorter time series x_t is shown in Figure 3.2A and its spectra in Figures 3.2 B and 3.2 C. Table 3.2 lists the frequencies for both spectra.

TABLE 3.2.

FREQUENCY COMPONENTS OF THE SINE PROBLEM OF
EXAMPLE 3.1 (20 data points)

Frequencies at which true periods occur		Frequencies predicted by MESA - 13 filter coefficients used	Frequencies predicted by Fourier analysis
(1)	0.05	0.0475	0.0300
(2)	0.10	0.0975	0.0925
(3)	0.20	0.1925	
(4)	0.21	0.2075	0.2075





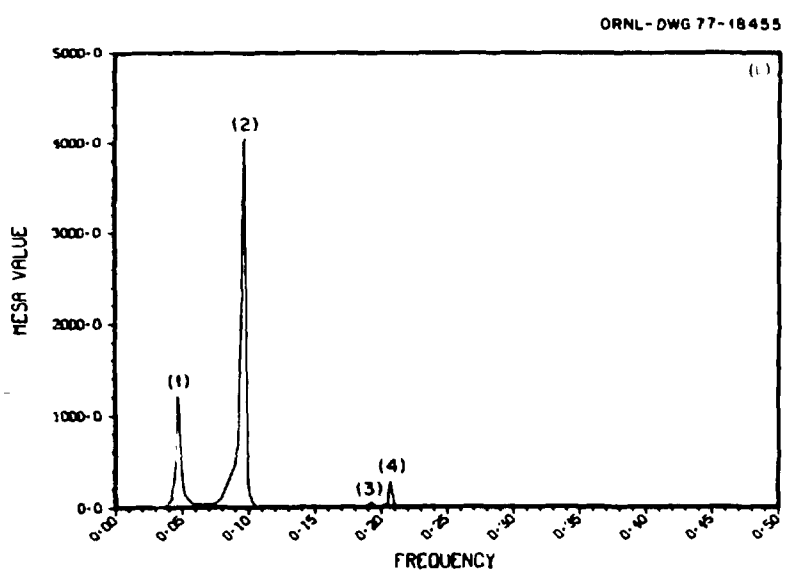


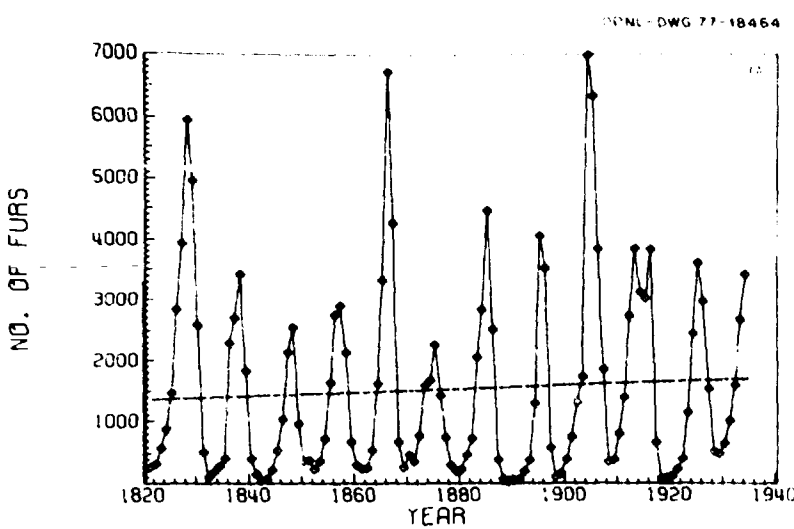
Fig. 3.2. A. Mathematical test case of four sinusoids (20 data points).
B. Fourier spectrum.
C. Maximum entropy spectrum with M = 13.

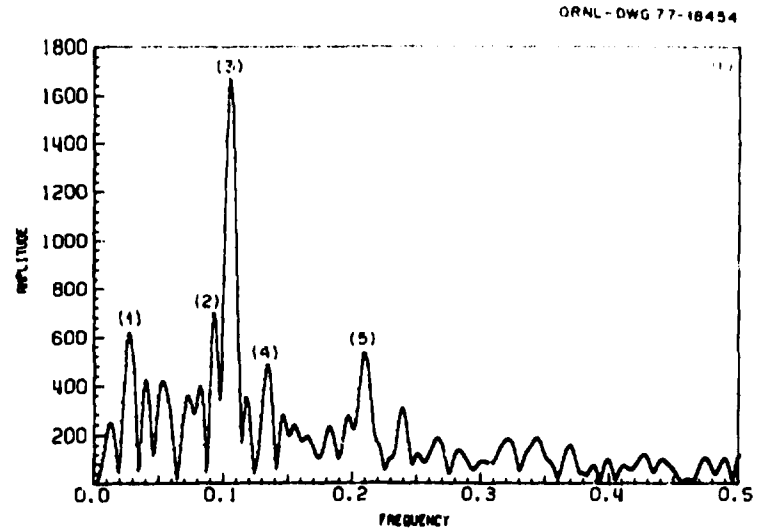
Since the number of data points is relatively small, one certainly can expect shifts in the spectral peaks. Such shifts are present in both Figures 3.2B and 3.2C, but the frequencies predicted by MESA are consistently as accurate or more accurate than those predicted by Fourier analysis. The true peak at f = 0.2 is missing in the Fourier spectrum, but is approximated by f = 0.1925 in the MESA spectrum. The fact that MESA is able to distinguish between two very close frequencies indicates one strong advantage this method has over Fourier analysis. One should also take note that the longest period P = 20 is approximated by P = 21.053 by MESA, but is given as P = 33.33 by the Fourier spectrum.

So far the applications we have been considering involve mathematical test cases, where the periods of the functions are known in advance. In the following three examples, we present real time series.

EXAMPLE 3.3

Figure 3.3A gives a plot of the MacKenzie River region lynx fur returns of the Hudson Bay Company from 1821-1934 (Elton and Nicholson 1942), a period covering 114 years. It is evident that a definite pattern exists. In fact, as one closely examines the points, a period of roughly 10 years emerges. Indeed, both Fourier and MESA spectra bring out this dominant period as can be seen in Figures 3.3B and 3.3C. Both lynx spectra agree quite closely in their estimates of the period of the major peak and of the two more important minor peaks, corresponding to periods of roughly 38 and 11 years. The cycle of about 38 years seems





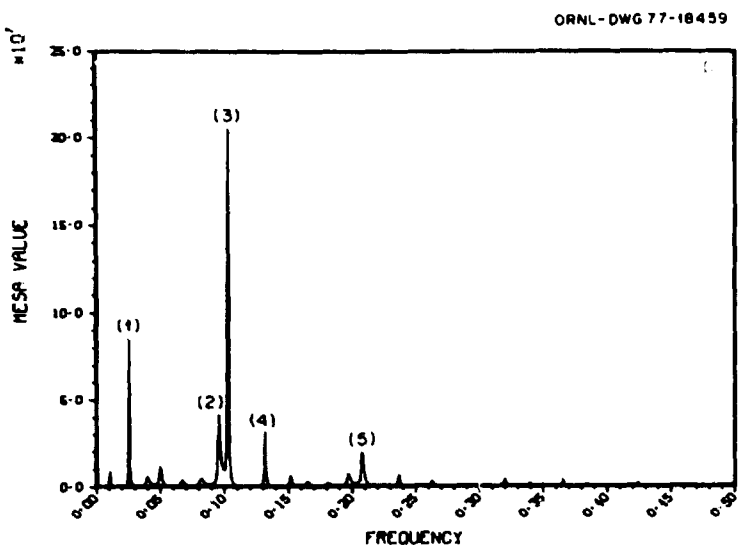


Fig. 3.3. A. Lynx fur returns (1821 - 1934), (Hudson Bay Company data; Straight line represents the linear trend).
B. Fourier spectrum.
C. Maximum entropy spectrum with M = 69.

to be a harmonic of 9.66 years, the major period, for 9.66 x 4 = 38.64 years.

TABLE 3.3

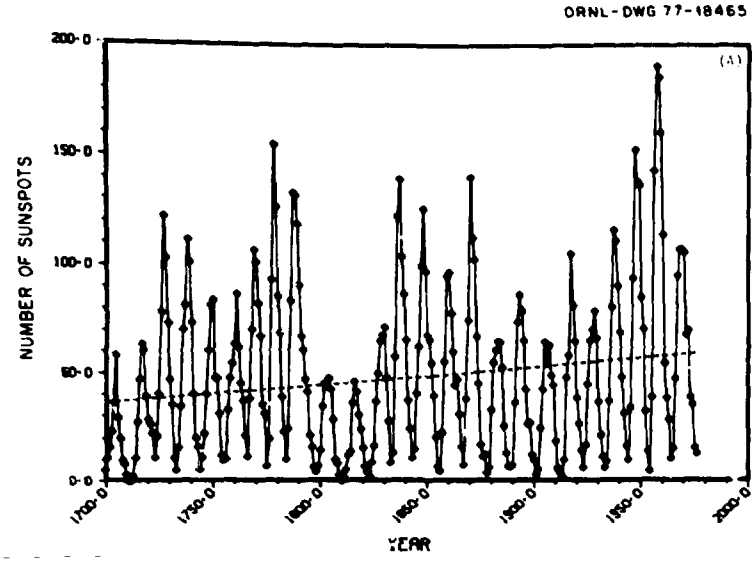
FREQUENCY COMPONENTS OF LYNX DATA

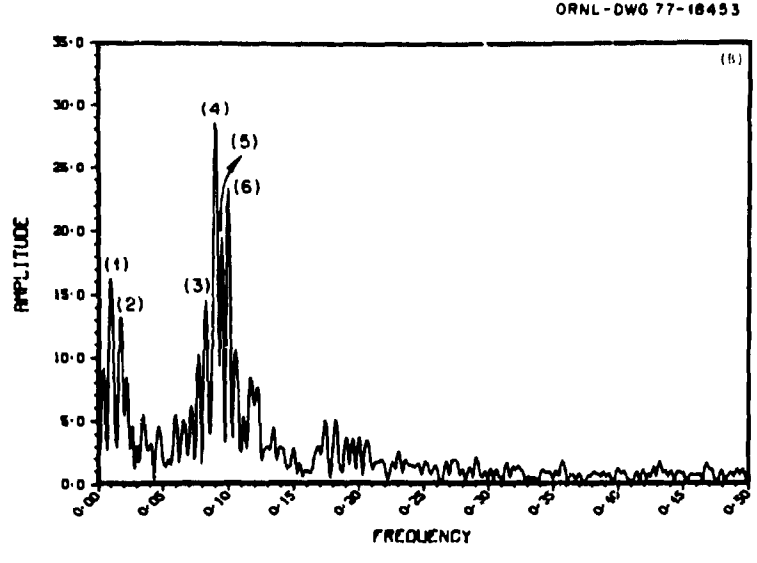
Major frequencies predicted by MESA - 69 filter coefficients used	Corresponding periods	Major frequencies predicted by Fourier analysis	Corresponding periods
(1) 0.0258	38.65	0.0258	38.65
(2) 0.0959	10.42	0.0918	10.89
(3) 0.1035	9.66	0.1035	9.66
(4) 0.1310	7.63	0.1327	7.54
(5) 0.2086	4.79	0.2078	4.81

EXAMPLE 3.4

Another well-studied time series is the Zurich sunspot record from 1700 to 1976 (see Figure 3.4A) as given by M. Waldemeier. The sunspot time series shows a strong periodicity of 11 years. There also seems to be a period of about 100 years if one divides the interval 1700-1976 into roughly 1700-1800, 1800-1900, 1900-1976. Table 3.4 gives a listing of the more dominant peaks in the spectra (Figures 3.4B and 3.4C).

The 99.83 year - period appears to be a harmonic of 9.983, since 9.983 x 10 = 99.83. Several papers have been written [see for example (Currie 1972), (Cole 1973)] in an attempt to explain the different periods (other than those appearing in Table 3.4) in the sunspot spectrum. Some authors have claimed that certain periods are the results of beat





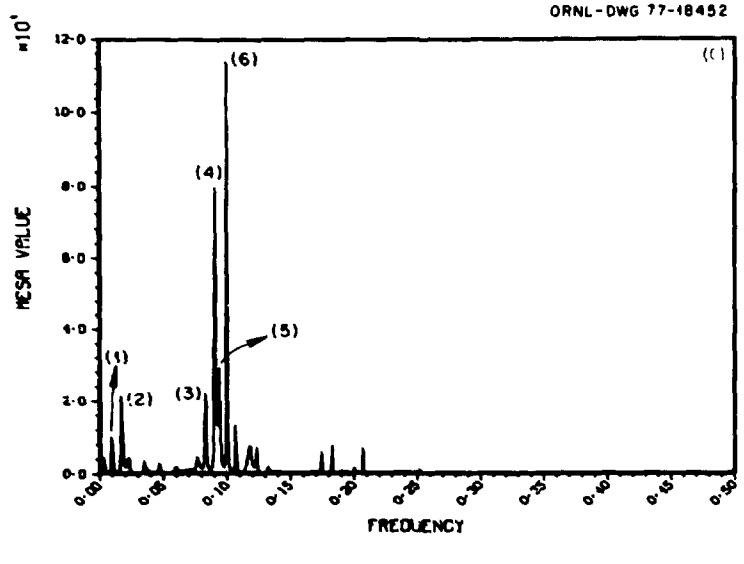


Fig. 3.4. A. Waldemeier Sunspot data (1700 - 1976) (Straight line represents the linear trend).
B. Fourier spectrum.
C. Maximum entropy spectrum with M = 160.

TABLE 3.4

FREQUENCY COMPONENTS OF SUNSPOT DATA

Major frequencies predicted by MESA - 160 filter coefficients used	Corresponding periods	Major frequencies predicted by Fourier analysis	Corresponding periods
(1) 0.01001	99.83	0.01001	99.83
(2) 0.0175	57.05	0.0175	57.05
(3) 0.0834	11.98	0.0826	12.10
(4) 0.0909	10.99	0.0901	11.09
(5) 0.0943	10.60	0.0952	10.51
(6) 0.1001	9.98	0.1001	9.98

frequencies (Wolff 1975). It is commonly accepted that the cycle of about 11 years is a real one.

In the sunspot and lynx records, the number of available data points is more than 10 times the length of the most dominant periods (the MESA estimates of which are 9.66 years in the lynx and 10.99 in the sunspots). Both MESA and Fourier analyses are in agreement with regard to the frequency estimates in the lynx spectrum and the sunspot spectrum. The performance of Fourier analysis is comparable to that of MESA in cases where there are long records available for the time series under study.

EXAMPLE 3.5

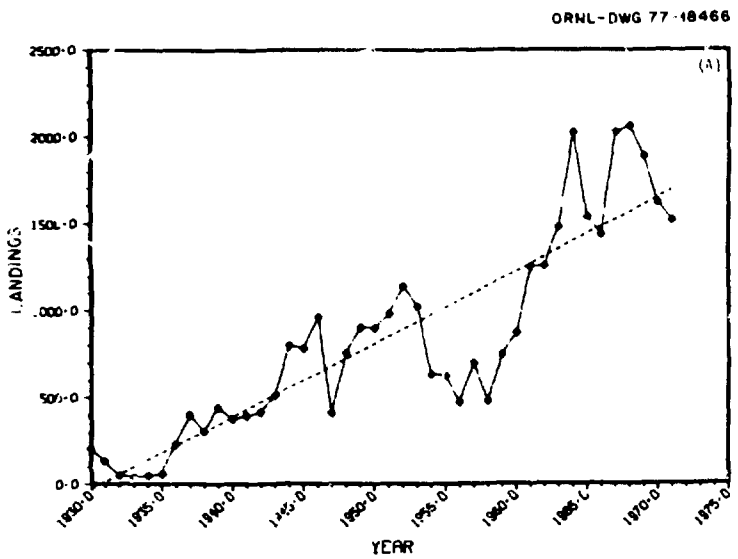
We now consider a shorter time series consisting of 42 data points - the annual commercial landings of the striped bass from the Middle Atlantic region in the years 1930 - 1971 (Van Winkle et al., in press) (see Figure 3.5A). A summary of the frequency components is listed in Table 3.5.

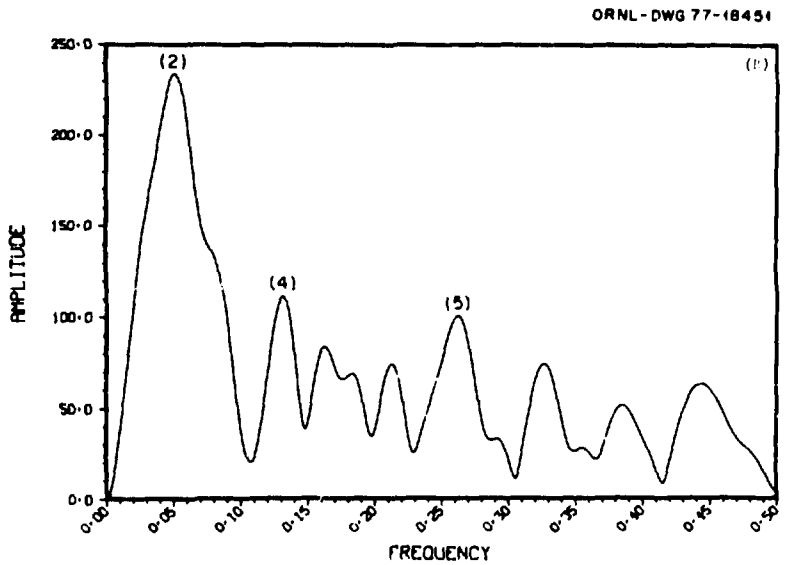
TABLE 3.5

FREQUENCY COMPONENTS OF MIDDLE ATLANTIC STRIPED BASS COMMERCIAL LANDINGS

Major frequencies predicted by MESA - 26 filter coefficients used	Corresponding periods	Major frequencies predicted by Fourier analysis	Corresponding periods
(1) 0.0250	39.93		
(2) 0.0550	18.15	0.0509	19.64
(3) 0.0793	12.61		
(4) 0.1302	7.68	0.1310	7.63
(5) 0.2671	3.74	0.2621	3.82

The spectra in Figures 3.5B and 3.5C differ in the frequency interval (0.00 - 0.10). In the Fourier spectrum, only one peak at f = 0.0509 appears within the interval. A lot of unresolved power is also present. In the same interval the MESA spectrum has three peaks at f = 0.0250, f = 0.0550, and f = 0.0793. The frequency f = 0.2621 with corresponding period P = 3.815 in Figure 3.5B [denoted peak (5)] appears to be a harmonic of f = 0.1310, since 2 x 0.1310 = 0.2620.





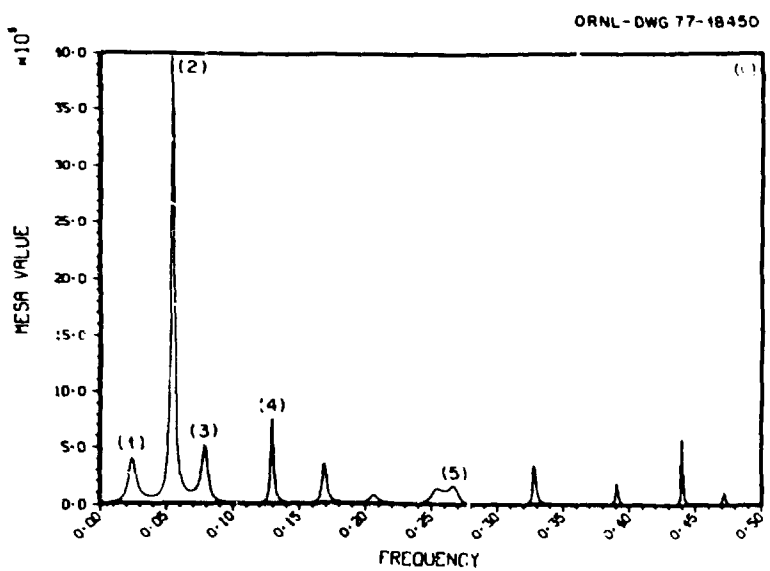


Fig. 3.5. A. Middle Atlantic striped bass yearly landings (1930 - 1971) (Straight line represents the linear trend).
B. Fourier spectrum.
C. Maximum entropy spectrum with M = 26.

B. The Effect of Noise

To compare the performance of the two methods when there is noise in the time series, we consider the sine function of Example 3.1, but utilize only the first 20 points (from t = 0 to t = 19). There are two major reasons for doing so. First of all, the longest period of the sine function is at frequency f = 0.05, corresponding to a period of P = 20. In Fourier analysis, the longest period predicted is the length of the time series. Secondly, we would like to see how well MESA and Fourier analysis perform whenever the length of available data is relatively short. We take note that when the number of points is 100, the predictions given by both methods are in fairly good agreement (see Table 3.1). When the number of points decreases to 20, the difference between the MESA spectrum and the Fourier spectrum can easily be detected (see Table 3.2). The equation under study here is:

$$x_t = 3 \sin(0.05(2\pi t)) + 10 \sin(0.1(2\pi t)) + 5 \sin(0.2(2\pi t)) + 8 \sin(0.21(2\pi t)) + R[t; N(0, 6.5\,\alpha)],$$

$t = 0, 1, \ldots, 19$, where

$R[t; N(0, 6.5\,\alpha)]$ = a random number drawn for each time t from a normal distribution $N(0, 6.5\,\alpha)$;

$N(0, 6.5\,\alpha)$ = the normal distribution with mean 0 and standard deviation $6.5\,\alpha$;

6.5 = $\frac{1}{4}$ (3 + 10 + 5 + 8), the average of the amplitudes;

α = the noise level on a scale of 0.0 to 1.0.

TABLE 3.6

FOURIER ANALYSIS: FREQUENCY COMPONENTS OF EXAMPLE 3.1
(20 data points at different noise levels)

Noise Level (α)	Frequencies (f) of the three highest peaks		
	0.05	0.10	0.2100
0.00	0.0300	0.0925	0.2075
0.05	0.0300	0.0925	0.2075
0.10	0.0300	0.0925	0.2075
0.20	0.0275	0.0925	0.2075
0.30	0.0275	0.0900	0.2075

Table 3.6 gives the estimated Fourier frequencies for various noise levels. The peak at f = 0.05 is approximated by f = 0.03 when α = 0.0, 0.05 and 0.10. As α increases to 0.20 and 0.30, the peak shifts even farther away from the true one. In the sine function under study, the largest amplitude is 10 and it belongs to the sinusoid with period at f = 0.10. This sinusoid is least affected by the increase in noise. The frequency estimate of f = 0.10 is f = 0.0925 for the first four levels of α and only drops to f = 0.09 when α increases to 0.30. The third peak of the sine function is at f = 0.2000 and does not appear in Table 3.6 at any noise level. This is because the true peak at f = 0.2100 is very close to the true peak f = 0.2000. As a result, there is a lot of unresolved power at f = 0.2075, at all levels of α in Table 3.6 (see Figure 3.2B for α = 0.00).

In Table 3.7, the results from MESA are given at different values of the number of filter coefficients M. For M = 3 and α = 0.0, 0.05, 0.10, 0.20, 0.30, one obtains a very poor spectral representation indicating only one of the four frequencies. As M takes the values 4, 5, 6, 7, at least two of the periods are resolved at α = 0.0, 0.05 and 0.10. As the level of noise goes up, the frequencies undergo greater shifts for these values of M. Moreover, when α = 0.30, spurious peaks appear at f = 0.4100 (for M = 6) and f = 0.3525 (for M = 7). Using M = 8, 9, 10, 11 results in a spectrum with three of the four frequencies of x_t at low levels of noise. As M takes the values 12 - 19 and when the noise level stays relatively low, all four frequencies are estimated but only with the addition of spurious peaks as well.

We notice that using too small a value of M results in low resolution, and using too high a value of M results not only in high resolution but also in the appearance of several spurious peaks. Furthermore, for both high levels of noise and large values of M, the number of spurious peaks tends to be high.

Relatively good spectra are obtained for values of M in the range 8 - 14. Although spurious peaks do appear, MESA still gives better estimates than Fourier analysis. Also the problem of sidebands in Fourier analysis is absent from the MESA spectrum (see Figures 3.1B, 3.1C, 3.2B and 3.2C).

Figures 3.6A, 3.6B and 3.6C illustrate the MESA spectrum of x_t at α = 0.0, α = 0.10, and α = 0.30 respectively. The value of M for these plots is 10. Because of the linear plot scaling, the peaks at

TABLE 3.7

MESA: FREQUENCY COMPONENTS OF EXAMPLE 3.1
(20 data points at different noise levels)

Number of filter coefficients (M)	Noise Level (%)				
	0%	5%	10%	20%	30%
3	0.1925	0.1900	0.1900	0.1900	0.1775
4	0.0850	0.0850	0.0850	0.1950	
	0.2050	0.2050	0.2075		
5	0.0925	0.0900	0.0825	0.0875	0.1000
	0.2075	0.2075	0.2050	0.2075	0.2075
6	0.0550				
	0.0975	0.0900	0.0875	0.0825	0.0850
	0.2075	0.2075	0.2050	0.2050	0.2050
					0.4100*
7	0.0950	0.0925	0.0900	0.0850	0.0825
	0.2075	0.2075	0.2050	0.2050	0.2025
					0.3525*
8	0.0475	0.0550			
	0.0950	0.0950	0.0900	0.0900	0.0850
	0.2075	0.2075	0.2050	0.2050	0.2025

*Denotes frequencies at which spurious peaks occur.

TABLE 3.7: Cont'd.

Number of filter coefficients (M)	Noise Level (α) 0%	5%	10%	20%	30%
9	0.0525	0.0525			
	0.0950	0.0950	0.0950	0.0900	0.0900
	0.2075	0.2075	0.2050	0.2050	0.2025
					0.3275*
					0.4475*
10	0.0475	0.0500	0.0500		
	0.0950	0.0950	0.0950	0.0925	0.0900
	0.2075	0.2075	0.2050	0.2050	0.2025
					0.3125*
					0.4025*
11	0.0500	0.0475	0.0500	0.0525	
	0.0950	0.0950	0.0950	0.0950	0.0925
	0.2075	0.2075	0.2050	0.2050	0.2025
				0.3100*	0.3075*
				0.4050*	0.4025*
12	0.0425	0.0475	0.0575		
	0.0700*				
	0.0950	0.0950	0.0975	0.0975	0.0925

*Denotes frequencies at which spurious peaks occur.

TABLE 3.7: Cont'd.

Number of filter coefficients (M)	Noise Level (α) 0%	5%	10%	20%	30%
	0.1875*				
	0.2075	0.2075	0.2050	0.2050	0.2025
	0.2200*				
			0.3125*	0.3125*	0.3075*
			0.4100*	0.4075*	0.4025*
13	0.0475	0.0475	0.0475		
	0.0975	0.0950	0.0950	0.0975	0.0900
	0.1925				
	0.2075	0.2075	0.2050	0.2050	0.2025
	0.2200*				
			0.3150*	0.3100*	0.3050*
			0.4125*	0.4000*	0.3950*
14	0.0450	0.0500	0.0500		
	0.0950	0.0950	0.0950	0.0950	0.0925
	0.1925				
	0.2075	0.2075	0.2050	0.2050	0.2025
			0.3175*	0.3100*	0.3050*
			0.4125*	0.4000*	0.3950*

*Denotes frequencies at which spurious peaks occur.

TABLE 3.7: Cont'd.

Number of filter coefficients (M)	Noise Level (α) 0%	5%	10%	20%	30%
15	0.0475	0.0450	0.0525		
	0.0800*				
	0.0975	0.0975	0.0950	0.0950	0.0975
		0.1875			
	0.2075	0.2075	0.2050	0.2050	0.2025
	0.2225*				
			0.3175*	0.3125*	0.3075*
			0.4125*	0.4000*	0.3950*
16					0.0350*
	0.0500	0.0425	0.0525		
	0.0900*				
	0.0975	0.0975	0.0950	0.0950	0.0875
					0.1225*
	0.1875	0.1875			
	0.2075	0.2075	0.2050	0.2050	0.2025
					0.2675*
			0.3150*	0.3125*	0.3125*
			0.4125*	0.4000*	0.3975*
				0.4750*	0.4750*

*Denotes frequencies at which spurious peaks occur.

TABLE 3.7: Cont'd.

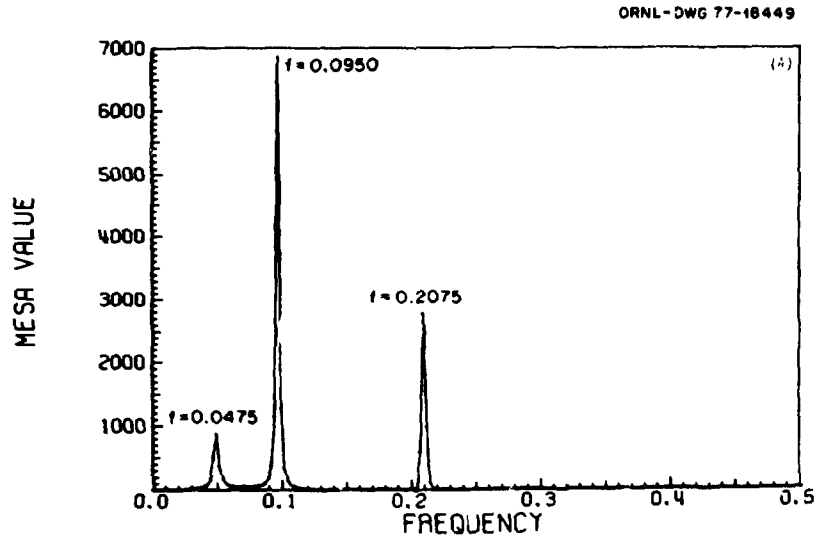
Number of filter coefficients (M)	Noise Level (α)				
	0%	5%	10%	20%	30%
17	0.0500	0.0425	0.0575	0.0550	0.0475
		0.0850*			
	0.0975	0.0975	0.0950	0.0950	0.0900
		0.1850	0.1600		
	0.2075	0.2075	0.2050	0.2050	0.2025
					0.2725*
			0.3150*	0.3150*	0.3150*
			0.4125*	0.4000*	0.3975*
				0.4750*	0.4750*
18	0.0325*		0.0375*		
	0.0500	0.0425	0.0650	0.0550	0.0425
	0.0875*	0.0825*			0.0900*
	0.0975	0.0975	0.0975	0.0975	0.0950
					0.1300*
		0.1800	0.1600		
	0.2075	0.2075	0.2050	0.2050	0.2025
	0.2225*				
				0.2675*	0.2725*
		0.3250*	0.3100*	0.3150*	0.3150*

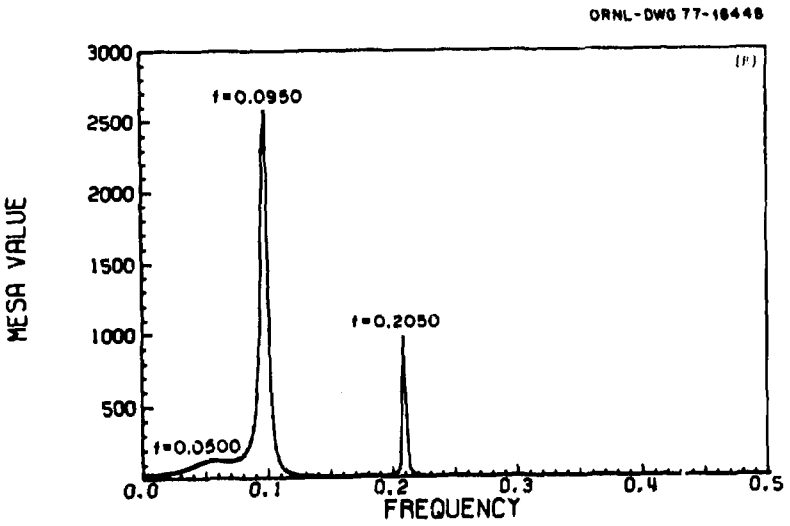
*Denotes frequencies at which spurious peaks occur.

TABLE 3.7: Cont'd.

Number of filter coefficients (M)	Noise Level (α) 0%	5%	10%	20%	30%
			0.3350*	0.4000*	0.3975*
		0.4150*	0.4125*	0.4750*	0.4750*
19	0.0325*		0.0375*	0.0300*	0.0300*
	0.0500	0.0425			
	0.0875*	0.0825*	0.0650*	0.0625*	0.0800*
	0.0975	0.0975	0.0975	0.0950	0.1000
				0.1350*	0.1425*
	0.1800	0.1800	0.1600		
	0.2025*				0.2025*
	0.2075	0.2075	0.2050	0.2050	
	0.2250*				
				0.2700*	0.2725*
		0.3250*	0.3100*	0.3150*	
		0.4125*	0.4125*	0.4725*	0.4725*

*Denotes frequencies at which spurious peaks occur.





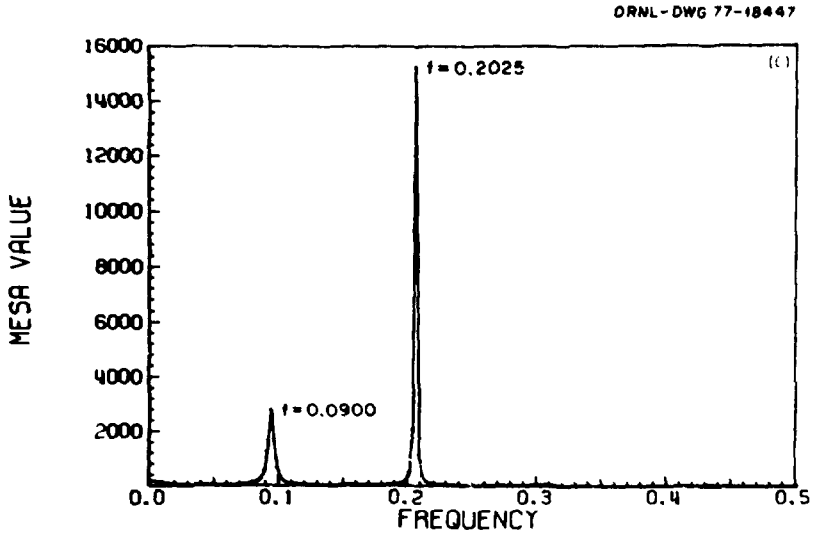


Fig. 3.6. Maximum entropy spectrum of four sinusoids (20 points).
A. M = 10, α = 0.0.
B. M = 10, α = 0.10.
C. M = 10, α = 0.30.

0.3075 and 0.4025 in Figure 3.6C cannot be seen, but do show up when a logarithmic scale is used.

The preceding studies gives some indications of the advantages of MESA over Fourier analysis. Below are the major points to be considered in choosing between the two methods.

Fourier Analysis	MESA
1. Assumes a zero extention of the data and uses data windows.	1. Makes no assumptions outside the given data and no data windows are used.
2. The longest period predicted is the length of available data.	2. Can predict periods longer than the length of available data.
3. Sideband effects are produced resulting from power leakage.	3. No sidebands are produced.
4. Frequency components very close to each other cannot be resolved.	4. Can resolve frequency components very close to each other.

Another basic difference between the MESA spectrum and the Fourier spectrum is that in the former, the power in the spectral peak is proportional to the area under the curve, whereas in the latter, the power is proportional to the height of the peak.

CHAPTER IV

GUIDELINES IN SPECTRAL STUDIES

A. The Akaike Final Prediction Error (FPE).

The determination of the number of filter coefficients (M) which produces the best spectrum is one of the major unsolved problems in MESA. As a general rule, using a very small value for M results in a spectrum with low resolution; on the other hand, a very large value of M, which is close to the total number of data points n results in spurious peaks in the spectrum. A common practice is to use values of M close to half the total number of data points.

T. J. Ulrych (1975) suggested the use of the Akaike final prediction error criterion. This criterion is expressed in terms of the mean square error P_M of the prediction filter used to estimate the MESA spectrum. The idea is to get the least value for P_M at which point the error in the prediction is minimum.

Again our time series will be $x_0, x_1, \ldots, x_{n-1}$. The final prediction error is defined by:

$$\text{FPE}(M) = Q \cdot \hat{P}_M \quad , \tag{4.1}$$

where $\hat{P}_M$ is the expression in Equation (2.92) and

$$Q = \begin{cases} \dfrac{n + M}{n - M} & , \text{ if the time series is not detrended.} \\ \dfrac{n + M + 1}{n - (M + 1)} & , \text{ if the mean is removed.} \\ \dfrac{n + M + 2}{n - (M + 2)} & , \text{ if the series is linearly detrended.} \end{cases} \tag{4.2}$$

To implement the FPE criterion, one computes Equation (4.1) for all $M \leq n-1$. The value of M which gives the smallest FPE(M) determines the number of filter coefficients to use. Table 4.1 lists the FPE values of the sine function in Example 3.1. The FPE values are given for M = 1, ..., 97. They indicate a final prediction error function which attains a minimum at M = 77.

TABLE 4.1

FPE VALUES FOR EXAMPLE 3.1 (100 data points)

Number of filter coefficients M	FPE	Number of filter coefficients M	FPE
1	6.67667D-01	21	4.69691D-12
2	2.93505D-01	22	8.43425D-13
3	1.65996D-01	23	7.30391D-13
4	8.86949D-01	24	2.63503D-14
5	2.94595D-01	25	9.82543D-15
6	1.92259D-01	26	6.92129D-15
7	9.21715D-02	27	4.30912D-15
8	1.60844D-03	28	3.76044D-15
9	7.56116D-04	29	1.37826D-15
10	2.98365D-04	30	9.00214D-16
11	2.36250D-04	31	9.20203D-16
12	4.74525D-06	32	4.59301D-16
13	1.09780D-06	33	2.44462D-16
14	5.55949D-07	34	2.46991D-16
15	3.85160D-07	35	2.12235D-16
16	6.04607D-08	36	1.97891D-16
17	2.05121D-08	37	1.08859D-16
18	3.47970D-09	38	1.09643D-16
19	3.10793D-09	39	8.10464D-17
20	1.06768D-11	40	7.90130D-17

TABLE 4.1: Cont'd.

Number of filter coefficients M	FPE	Number of filter coefficients M	FPE
41	7.69202D-17	71	6.31891D-18
42	6.86446D-17	72	6.05030D-18
43	6.86533D-17	73	5.54125D-18
44	6.74952D-17	74	5.77307D-18
45	6.87912D-17	75	5.96710D-18
46	3.21866D-17	76	5.85930D-18
47	2.15421D-17	77	4.79736D-18
48	2.16067D-17	78	5.04358D-18
49	2.01605D-17	79	5.31335D-18
50	2.06381D-17	80	5.38276D-18
51	2.07332D-17	81	5.68238D-18
52	1.95511D-17	82	5.97227D-18
53	1.64197D-17	83	5.76230D-18
54	1.05644D-17	84	6.00796D-18
55	1.01181D-17	85	6.44054D-18
56	1.02351D-17	86	6.10300D-18
57	9.43625D-18	87	5.21797D-18
58	9.29254D-18	88	5.61166D-18
59	9.39571D-18	89	6.01996D-18
60	9.39461D-18	90	6.24087D-18
61	8.66899D-18	91	6.85661D-18
62	7.06601D-18	92	7.35459D-18
63	7.21889D-18	93	7.65730D-18
64	7.07270D-18	94	7.14762D-18
65	7.22471D-18	95	8.58823D-18
66	7.42477D-18	96	1.06101D-17
67	7.28808D-18	97	1.39827D-17
68	6.61568D-18		
69	6.18102D-18		
70	6.39689D-18		

The spectrum for M = 77 (Figure 4.1) shows the peaks at f = 0.05, 0.10, 0.20, and 0.2125. These are the same frequencies predicted with M = 51 (see Figure 3.1C). Although a minimum FPE is obtained for M = 77, we studied several spectra for M in the interval (50, 80). The FPE

assumes a relatively flat valley in this interval. Hence, very good estimates of the true peaks are produced. In this case the FPE does not seem to be a very discriminating criterion.

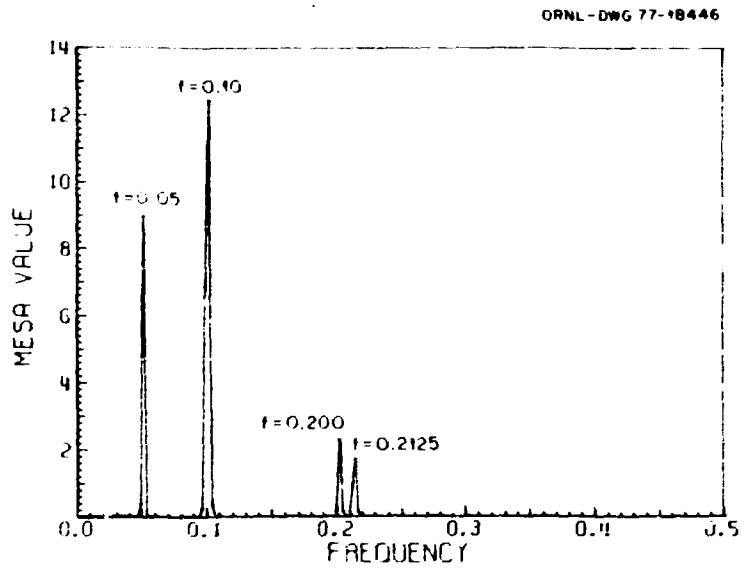
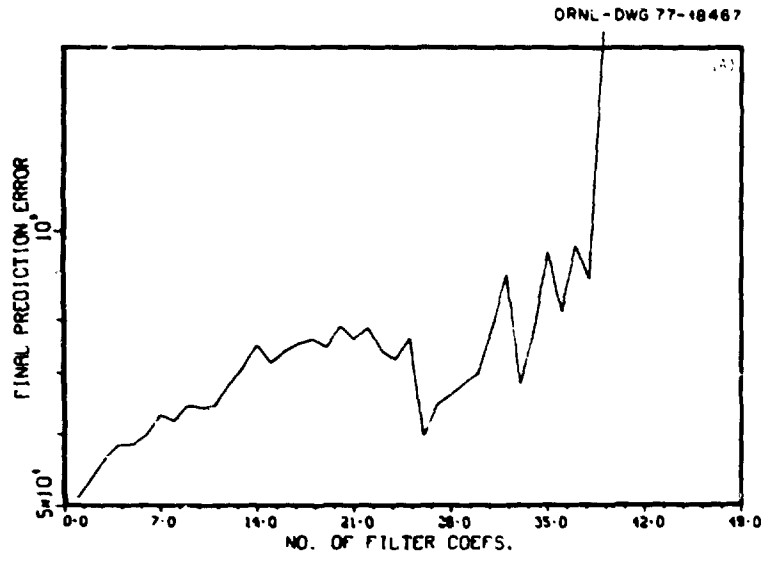
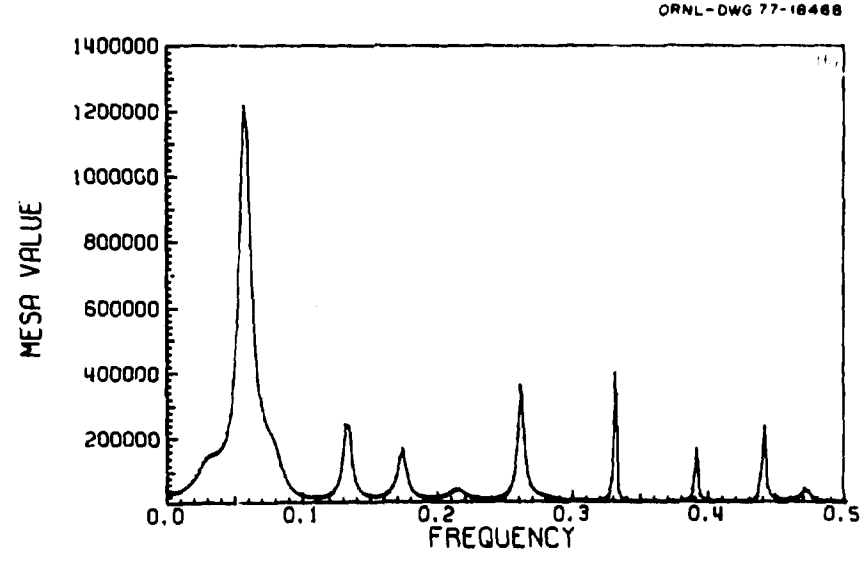


Fig. 4.1. Maximum Entropy Spectrum of Example 3.1 with M=77.

As another example of the unreliability of the minimum FPE criterion, we have plotted the FPE for the Middle Atlantic striped bass landings (Example 3.5) in Figure 4.2A. The FPE minimum is at M = 1. We have already noted that using very small values of M results in a poorly resolved spectrum. However, as one examines the plot for increasing values of M, a major numerical difference is noted between M = 25 and M = 26. The FPE function increases at almost every step before M = 25 and then takes a sudden drop at M = 26. It is interesting to see what happens to the spectrum as M goes from 25 to 26.





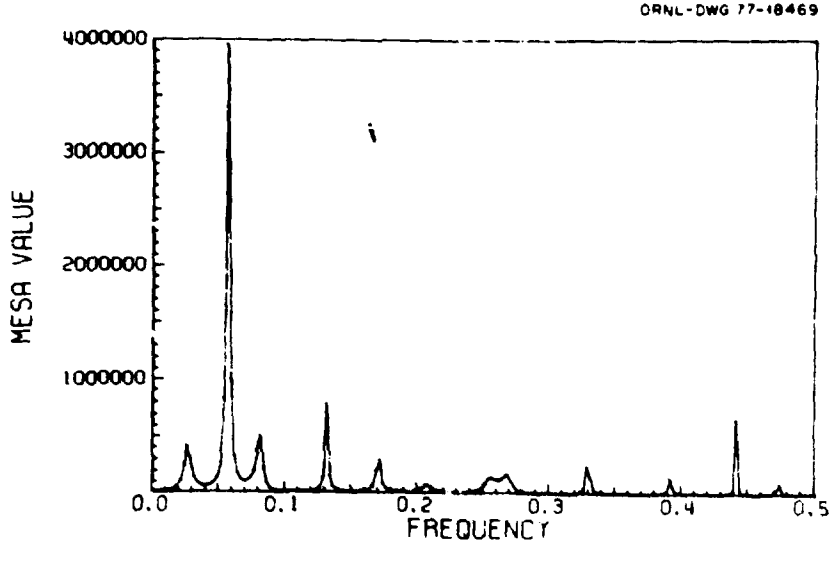


Fig. 4.2. A. FPE of the Middle Atlantic striped bass landings.
B. Maximum entropy spectrum with M = 25.
C. Maximum entropy spectrum with M = 26.

Figure 4.2B shows the spectrum for M = 25 and Figure 4.2C for M = 26. A major change occurs in the frequency interval (0.0, 0.1) as M changes from 25 to 26. Instead of just one major peak within this interval when M = 25, three appear when M = 26. It appears that the three peaks are brought into 'focus'.

For many of the other time series that we have analyzed, the same kind of behavior occurs - that is, when the FPE takes a sudden drop in value, the spectrum produces sharper peaks. Whether, of course, the other two peaks that come into 'focus' are significant is another question.

It is also good practice, when deciding what value of M to choose, to refer to the Fourier spectrum of the time series as a guide in determining what value of M would give a good MESA spectrum.

3. Fisher's Test Statistic - The Periodogram Test

The periodogram is one of the earliest forms of spectral estimates. It can actually be derived from the sample spectrum in Equations (1.11), (1.12), (1.13). Using the same notation as in Chapter I, the periodogram values FF(f) are given by the formula:

$$FF(f) = \frac{n}{2}\left[a(f)^2 + b(f)^2\right] \quad , \tag{4.3}$$

where $f = k/n$, $k = 1, \ldots, m$ if $n = 2m$. If $n = 2m + 1$, Equation (4.3) applies for $f = k/n$, $k = 0, 1, \ldots m$. Since we are using discrete values for f, (4.3) can be simplified into:

$$FF_k = \frac{n}{2}\left[a_k^2 + b_k^2\right] . \tag{4.4}$$

One of the questions often considered is whether the maximum value of FF_k is significantly above the noise level. With Fisher's test, we proceed the following way to answer the question. Let

FF_L = the largest periodogram ordinate, and

$\sum_{k=1}^{m} FF_k$ = the sum of the periodogram ordinates.

The test statistic used is

$$g = FF_L \Big/ \left[(1/m) \sum_{k=1}^{m} FF_k\right] . \tag{4.5}$$

Fisher (1929) showed that for $g > 0$, the probability that the largest of m normalized terms $(FF_i / \sum_{k=1}^{m} FF_k,\ i = 1, \ldots, m)$ should exceed g is

$$P_r\left\{m^{-1} < g\right\} = m\,(1-g)^{m-1} - \frac{m\,(m-1)\,(1-2g)^{m-1}}{2} + \ldots + (-1)^{m-1}\,\frac{m!}{s!\,(m-s)!}\,(1-sg)^{m-1} \tag{4.6}$$

where s is the largest integer less than 1/g. Table 4.2 gives the 1 and 5 percentage points for the distribution of ξ as given by W. Fuller (1976).

TABLE 4.2

PERCENTAGE POINTS FOR THE RATIO OF LARGEST PERIODOGRAM ORDINATE TO THE AVERAGE

Number of ordinates	Probability of larger value 0.05	0.01
2	1.950	1.990
3	2.613	2.827
4	3.072	3.457
5	3.419	3.943
6	3.697	4.331
7	3.928	4.651
8	4.125	4.921
9	4.297	5.154
10	4.450	5.358
15	5.019	6.103
20	5.408	6.594
25	5.701	6.955
30	5.935	7.237
40	6.295	7.663
50	6.567	7.977
60	6.785	8.225
70	6.967	8.428
80	7.122	8.601
90	7.258	8.750
100	7.378	8.882
150	7.832	9.372
200	8.147	9.707
250	8.389	9.960
300	8.584	10.164
350	8.748	10.334
400	8.889	10.480
500	9.123	10.721
600	9.313	10.916
700	9.473	11.079
800	9.612	11.220
900	9.733	11.344
1000	9.842	11.454

To illustrate Fisher's test, the lynx data will be used. The sample spectrum (Figure 3.3B) exhibits the tallest peak corresponding to a period of 9.66 years. Recall that the straight line trend was subtracted from the data before calculating the spectrum. The null hypothesis is

$$H_0: \quad x_t = \mu + \beta t + e_t \quad .$$

The alternative hypothesis is

$$H_A: \quad x_t = \mu + \beta t \quad + A \sin \left[(2\pi t / 9.66) + \phi \right] \quad + e_t \quad ,$$

where e_t is the stochastic error term.

The frequencies used for the sample spectrum are $f = i/114$, $i = 1,2,\ldots, 57$. Then

$$FF_L \Big/ \left[(1/57) \sum_{k=1}^{57} FF_k \right] = (1.42571 \times 10^8) \Big/ \left[(1/57)(2.79734 \times 10^3 \right]$$

$$= 29.0511$$

From Table 4.2, the 1% point for this value of ξ is about 8.20. So $29.055 > 8.20$ and H_0 is rejected. One concludes that the 9.66 period is statistically significant.

C. The Cumulative Periodogram Test

In analyzing a real time series like the lynx data, our normal course is to remove the linear trend in the data before analysis. That is, we really are fitting the model

$$x_t = \mu + \beta t + e_t \quad .$$

The series that then undergoes analysis is $\hat{e}_t$, the estimated error. Thus a question that one might ask is whether the $\hat{e}_t$'s under study comprise just white noise.

It can be shown (Box and Jenkins 1970) that the power spectrum $P(f)$ for white noise has a constant value $2\sigma_e^2$ (σ_e^2 is the variance for e) over the frequency interval $[0.0, 0.5]$. Thus the integrated spectrum for white noise is

$$(4.7) \qquad PP(f) = \int_0^f P(v)\, dv = 2\,\sigma_e^2\, f \quad ,$$

with f defined in the interval $[0.0, 0.5]$.

Dividing both sides of (4.7) by σ_e^2 gives the equation

$$(4.8) \qquad PP(f) \,/\, \sigma_e^2 = 2\, f \quad .$$

Thus the plot of $PP(f)/\sigma_e^2$ against f is a straight line joining (0,0) to (0.5, 1). For test data, the integrated spectrum can be approximated by the cumulative periodogram CP_k:

$$(4.9) \qquad CP_k = \sum_{i=1}^{k} FF_i \Big/ \sum_{j=1}^{m} FF_j \ , \ k = 1, \ . \ . \ ., \ m,$$

which is compared with the white noise straight line.

To determine the significance of the deviations of the cumulative periodogram of the test data from the theoretical straight line joining (0,0) to (0.5,1), limit lines can be drawn on each side of the theoretical line, using the Kolmogorov - Smirnov test (Box and Jenkins 1970). If indeed the series e_t were to comprise white noise, then the cumulative periodogram would cross these limit lines with a stated probability. It should be pointed out however, that the probabilities are only approximated. The limit lines in the frequency interval [0.0, 0.5] are given by the equations:

$$(4.10) \qquad y = 2 f \pm K_\alpha / \sqrt{q}$$

where

$$(4.11) \qquad q = \begin{cases} (n-2) / 2 \ , \text{ if } n \text{ is even} \\ (n-1) / 2 \ , \text{ if } n \text{ is odd,} \end{cases}$$

f is the frequency in the interval [0.0, 0.5], n is the total number of data points, α the stated probability.

The cumulative periodgram of a truly random series would then lie outside the limit lines of Equation (4.10) for an approximate fraction α of the time. Table 4.3 lists K_α versus α. This table is adapted from Box and Jenkins (1970).

TABLE 4.3

VALUES OF α AND K_α NEEDED FOR CUMULATIVE PERIODOGRAM TEST

Probability α	K_α
0.01	1.63
0.05	1.36
0.10	1.22
0.25	1.02

Figure 4.3 is the cumulative periodogram of the lynx data after the linear trend is removed. In this case n = 114. One can see that the periodogram has certainly crossed the limit lines, $y = 2f \pm K_{0.05}/\sqrt{q}$, where $q = \sqrt{(114-2)/2} = 7.48$. Our conclusion therefore is that e_t is not white noise and the errors are not independent. The model $x_t = \mu + \beta t + e_t$ is thus inadequate.

D. The Integrated Maximum Entropy Spectrum

In the sample spectrum in Fourier analysis, the power density in a peak (i.e., the strength of contribution of the frequency f) is directly related to the peak amplitude. In the maximum entropy spectrum, the power density is directly related to the area under the spectral peak [see Lacoss (1971)]. So in order to judge the contribution of a spectral peak in MESA, the area under the peak has to be computed. This

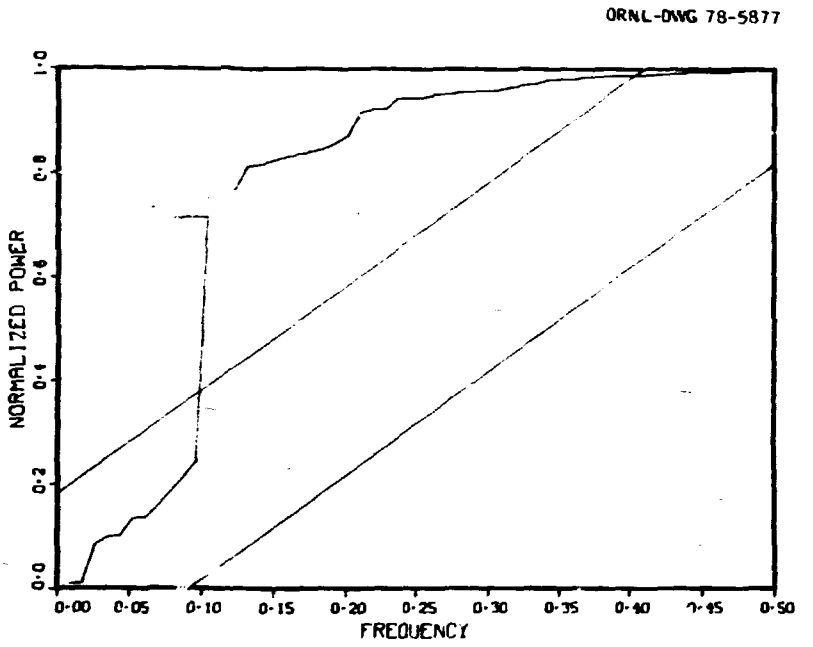


Fig. 4.3. Cumulative periodogram of the Lynx Data.

Solid Curve - Cumulative periodogram.
Dashed lines - Limit lines $y = 2f \pm K_{0.05} / \sqrt{q}$.

suggests the use of an integrated spectrum obtained from the convolution of the MESA spectrum with a rectangular window of height unity and width equal to the effective width of the peaks in the MESA spectrum. The heights of the peaks in the integrated spectrum will then be proportional to the corresponding power densities.

The maximum entropy spectrum as given by Equation (2.35) is

$$P(f) = \frac{P_M \Delta t}{\left| 1 + \sum_{m=1}^{M} a_m e^{-i2\pi f m \Delta t} \right|^2} \qquad (4.12)$$

The integrated spectrum IP(v) will then be of the form

$$IP(\nu_i) = \int_{f_i}^{f_{i+1}} P(f)\, df, \tag{4.13}$$

where $f_{i+1} - f_i$ is the bandwidth of integration and

$\nu_i = (f_{i+1} + f_i)/2$.

To illustrate the above ideas, consider the spectrum P(f) in Figure 4.4A. By choosing an appropriate width of integration, one can find the areas under the two spectral peaks. Figure 4.4B shows the estimated integral of P(f) where

$$\begin{aligned} IP(\nu_i) &= \int_{f_i}^{f_{i+1}} P(f)\, df. \\ IP(\nu_j) &= \int_{f_j}^{f_{j+1}} P(f)\, df. \end{aligned} \tag{4.14}$$

For an M - length prediction error filter, only M/2 + 1 discrete frequency spectral components can be obtained (Jensen and Ulrych 1973). Hence to obtain an estimate of the bandwidth of integration we take the Nyquist frequency f_M and divide it by M/2 + 1. Once the integrated spectrum is computed, the height of a peak would then be proportional to the power density.

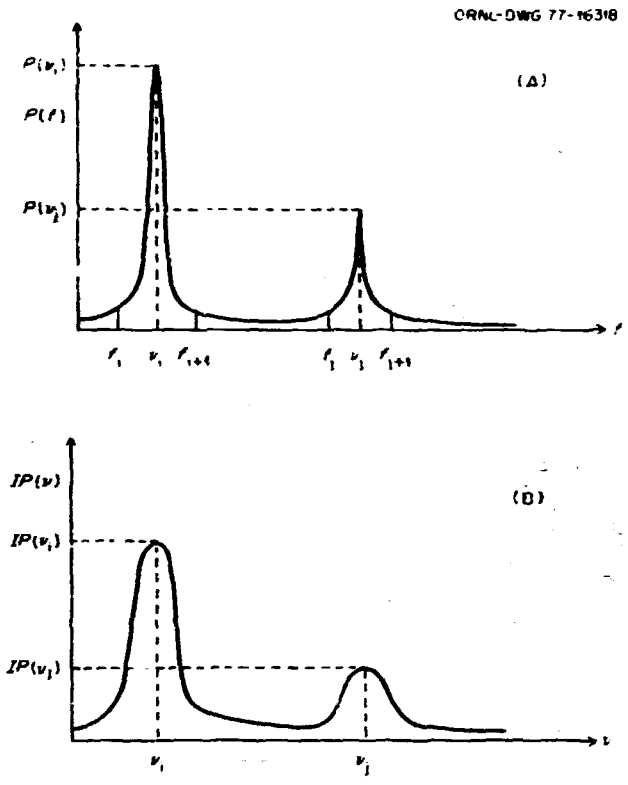


Fig. 4.4. (A) A sample maximum entropy spectrum. (B) The integrated maximum entropy spectrum.

The integrated MESA spectrum for a given time series is very similar in appearance to the Fourier spectrum for the same series, but it is not possible to easily judge whether or not a peak in the integrated MESA spectrum is significantly above the noise level because there does not yet exist a MESA analogue of Fisher's test for the periodogram. The reason is that at the present time no one knows the statistical distribution of integrated spectral estimates for random time series. It is, however, possible to obtain a significance test by a rather cumbersome Monte-Carlo technique which has been described by Jensen and Ulrych (1973). Essentially what one does is to generate a large number

of random time series having the same number of sample points and the same sample spacing as the series being studied. One then calculates the integrated MESA spectrum of each of these random series using the same number of filter coefficients and the same integration window width used to construct the integrated spectrum of the original sample. One can then use this collection of integrated spectra to empirically construct a table giving, as a function of peak height, the probability of obtaining that peak height by chance simply by counting the number of spectra having peaks that high or higher and dividing by the total number of spectra used in the process. The quality of the approximate probabilities obtained in this manner will, of course, improve as more random time series are used in the procedure.

CHAPTER V

THE COMPUTER PROGRAM AND ITS USE

A. Explanation of the Subroutines

Essentially, the program consists of the following routines:

(1) Main program - which initializes the data values and other parameters needed in spectral analysis.

(2) Subroutine TRANS1 - which provides different calls to SPECTR.

(3) Subroutine SPECTR - which does spectral analysis using both Fourier and MESA methods. In turn, SPECTR calls:

(a) Subroutine DETRND - a routine to be provided by the user if detrending is done.

(b) Subroutine SFT - computes the Fourier sine and cosine terms needed to produce the sample spectrum.

(c) DCADRE - a built-in IMSL subroutine used in computing the integrated spectrum if it is desired.

(d) Double Precision Function SPCVAL - computes the function used in integrating the spectrum.

(e) Subroutines DOPLT, DOPLT1, DOPLT2, DOPLT3, DOPLT4, DOPLT5, DOPLT6, DOPLT7, DOPLT8, DOPLT9, SINGLE, GPHBGN, GPHBG1, GPHEND, ANGTIC - perform the necessary plotting using the DISSPLA routines.

1. The Main Program

The parameters which are given initial values are read in this section of the program and are transferred to TRANS1 and SPECTR by the common statement:

```
COMMON/SPECBL/DELTA, YEAR(600), Y(600), XD(600), VMAX, Z(600), V(600),
              S(600), FPE(600), XDIFF(600), DATE(600), X(600),
              ACS(600), BSN(600), AMP(600), ASF(600), PERIOD(600),
              NPTS, NFILT, NV, MCOUNT(600), IPRINT, IPLOT, ITREND,
              IFPEPL, IINTSP, IAUTCV, IYWACV
```

Table 5.1 lists in alphabetical order the variables which are read, their dimensions, types and what they represent. Currently, the dimensions allow a maximum of 600 data points to be analyzed by SPECTR.

TABLE 5.1

LIST OF VARIABLES READ IN THE MAIN PROGRAM

Variable	Dimension	Type	Explanation
DATE	600	REAL*8	time array (times at which sample values occur).
DELTA		REAL*8	sampling interval.
IAUTCV		I*4	indicator for option to plot MESA estimate of autocovariance function.
IFPEPL		I*4	indicator for option to prin'/plot final prediction error.
IINTSP		I*4	indicator for option to compute integrated MESA spectrum.
IPLOT		I*4	indicator for option to plot spectra.

TABLE 5.1. Cont'd.

Variable	Dimension	Type	Explanation
IPRINT		I*4	indicator for option to print output.
ITREND		I*4	indicator for option to detrend data.
IYWACV		I*4	indicator for option to compute and plot Yule-Walker estimate of the autocovariance function.
MCOUNT	600	I*4	contains the array of numbers which correspond to the number of filter coefficients desired in each MESA spectrum.
NAM	14	Real*4	title for the data set.
NFILT		I*4	total number of maximum entropy spectra to be calculated with different values of the number of filter coefficients.
NPTS		I*4	number of data points in time series.
NV		I*4	number of frequencies desired in output; should not exceed a maximum value of 599.

TABLE 5.1: Cont'd.

Variable	Dimension	Type	Explanation
VMAX		Real*8	maximum frequency for the output spectrum; has a maximum value of 0.5, the Nyquist frequency.
XLBL	2	Real*4	label for time axis (x-axis) used in plotting.
Y	600	Real*8	time series to be analyzed.
YLBL	2	Real*4	label for time series data (y-axis) used in plotting.

The indicators IAUTCV, IFPEPL, IINTSP, IPLOT, IPRINT, ITREND, IYWACV should be initialized to values of 0 or 1. In each case, 1 is a positive response and 0 is a negative response.

The variables NAM, XLBL, YLBL, which are read in the main program, appear in the common block BLKLBL. These are mainly used for identification of data and plots. BLKLBL is shared by the Main program, TRANS1, SPECTR and DOPLT. The form of the statement is

COMMON/BLKLBL/NAM(14), XLBL(2), YLBL(2) .

The input cards, all of which are read in the main program, must have the following formats. The formats appear in the order the cards are read.

(1) Input card on first READ statement.

XLBL	YLBL	DELTA	NPTS	NFILT	NV	VMAX
$	$					
2A4 2X	2A4 2X	E10.0	I5	I5	I5	E10.0

XLBL and YLBL must terminate with a dollar sign.

(2) Input card on second READ statement.

NAM
$
14A4

NAM must also end in a dollar sign.

(3) Input cards for third READ statement.

MCOUNT(1)	MCOUNT(2)	MCOUNT(NFILT)
I5	I5	I5

(4) Input cards for fourth READ statement. This set contains the time series to be analyzed.

DATE(1)	Y(1)
DATE(2)	Y(2)
.	.
.	.
.	.
DATE(NPTS)	Y(NPTS)
F4.0	F10.0

(5) Input card for the fifth READ statement.

IPRINT	IPLOT	ITREND	IFPEPL	IINTSP	IAUTCV	IYWACV
I5	I5	I5	I5	I5	I5	I5

A flowchart of the main program is given in Figure 5.1.

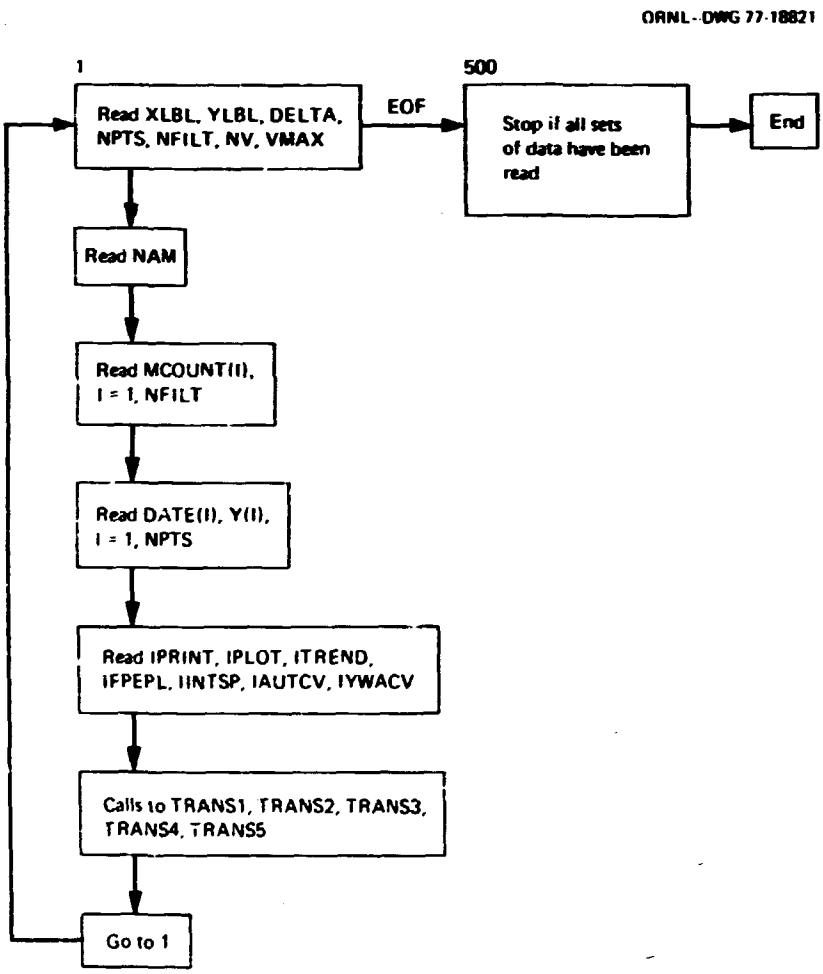


Fig. 5.1. Flowchart of the Main Program. Numbers (e.g., 1 and 500) refer to statement numbers in the program.

2. SUBROUTINE TRANS1

The variables in this subroutine are those that appear in the common area SPECBL. This subroutine allows the user to perform transformations on the raw data (e.g., take logarithms) before doing spectral analysis. Up to five different spectra with five different transformations are allowed.

The basic statements in TRANS1 are the following:

(1) SUBROUTINE TRANS1

Optional user supplied statements for desired transformation on data.

CALL SPECTR

.

.

.

RETURN

(2) ENTRY TRANS2

Optional user supplied statements for second desired transformation.

CALL SPECTR

.

.

.

RETURN

(3) ENTRY TRANS3

Optional user supplied statements.

CALL SPECTR

.

.

.

RETURN

(4) ENTRY TRANS4

Optional user supplied statements.

CALL SPECTR

.

.

.

RETURN

(5) ENTRY TRANS5

Optional user supplied statements.

CALL SPECTR

.

.

.

RETURN

If the user wants to analyze only the input data, he need not supply additional statements and any of the 4 additional entries can be effectively deactivated by omitting CALL SPECTR.

An example of data transformation using the second Entry call (i.e., ENTRY TRANS2) is the following:

```
      ENTRY TRANS2
      DO 20 I = 1, NPTS
      Y(I) = DLOG(Y(I))
20    CONTINUE
      CALL SPECTR
      RETURN
```

In the preceding statements, the natural logarithm of the input data Y(I), I=1, ..., NPTS is taken and the result is stored in Y(I) again. Thus when SPECTR is called, the logarithmically transformed data are analyzed. The program listing for SUBROUTINE TRANS1 given in Appendix III gives the configuration of statements needed to first analyze the original data and then its logarithm.

3. SUBROUTINE SPECTR

The main function of SPECTR is to perform the following:

(1) Detrend the input data by calling DETRND if desired;

(2) Compute:

   (a) the Yule-Walker estimates of the autocovariance function if desired.

   (b) the Fourier spectrum.

   (c) the periodogram ordinates and their sum.

   (d) the cumulative periodogram.

   (e) the maximum entropy spectrum.

(f) the autocorrelation function resulting from the maximum entropy computations.

(g) the integrated maximum entropy spectrum if desired.

(h) the final prediction error if desired.

(3) Print and plot computed values.

A flowchart of subroutine SPECTR appears in Figure 5.2. Table 5.2 lists the the parameters in subroutine SPECTR which appear in common blocks on DIMENSION statements. This list includes the variables which must appear in the calling program and the arrays whose dimension the user might want to alter if he desires to make changes in the programs. Variables that are only used internally in SPECTR or in subroutines called by SPECTR are not included in the list.

4. SUBROUTINE DETRND

DETRND is called by SPECTR with the statement:

CALL DETRND (NPTS, DATE, YEAR, Y, XDIFF, X, SFPE)

where

NPTS - number of data points, i.e., length of time series.

DATE - time array that must not be destroyed.

YEAR - user-evaluated time values; these are to be initialized in DETRND.

Y - input time series values; these values must not be destroyed.

XDIFF - array containing the detrended data; computed in DETRND.

X - array containing the trend of the data; computed in DETRND.

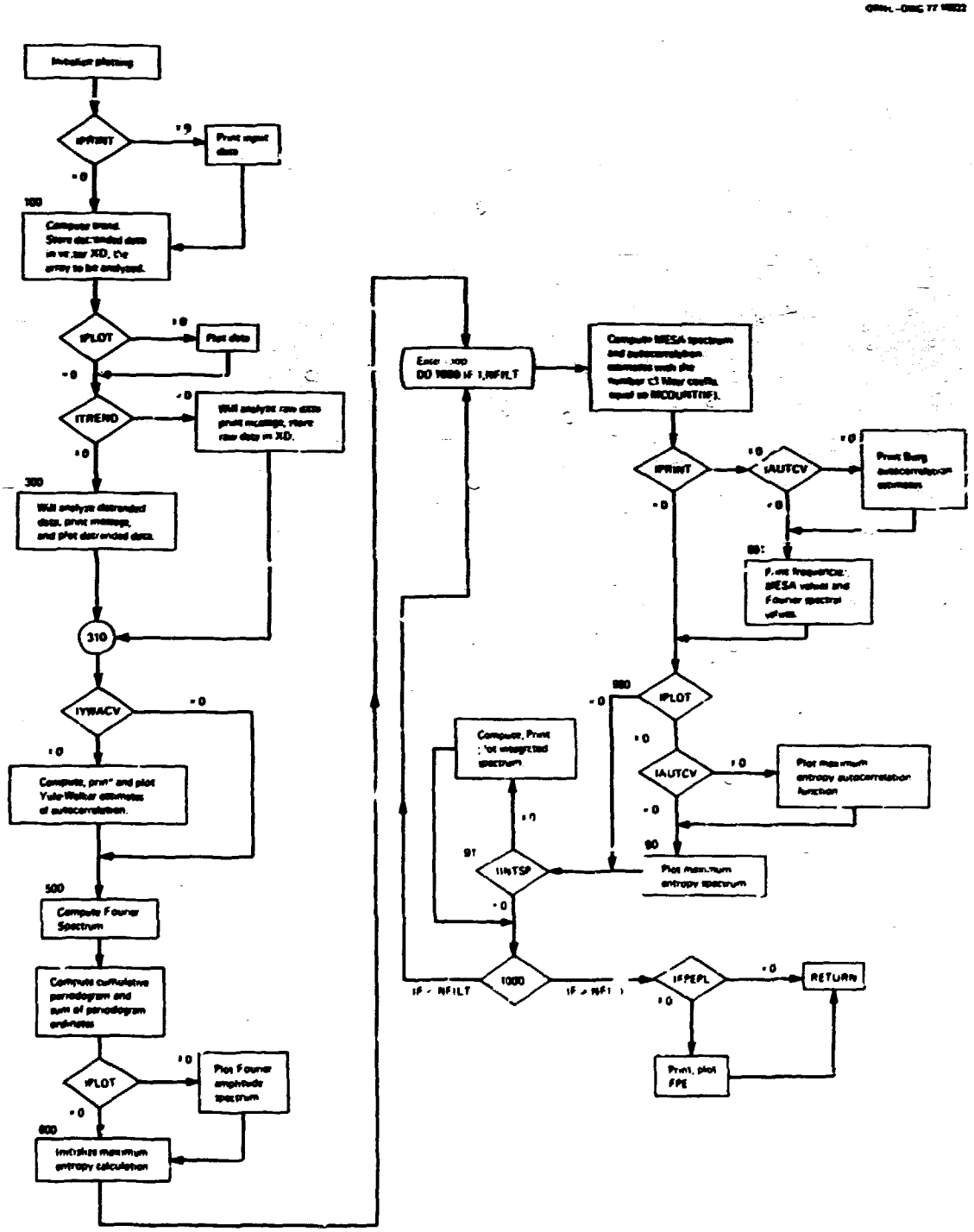


Fig. 5.2. Flowchart of Subroutine SPECTR. Numbers (e.g., 100 and 300) correspond to statement numbers in the program.

TABLE 5.2

LIST OF PARAMETERS IN SUBROUTINE SPECTR

Variable	Dimension	Type	Explanation
A	600	Real*8	Vector of working storage.
ACFN	600	Real*8	Vector of working storage.
ACS	600	Real*8	Vector of working storage.
AMP	600	Real*8	Vector of working storage.
ASP	600	Real*8	Vector containing Fourier power spectral values.
B	600	Real*8	Vector of working storage.
BSN	600	Real*8	Vector of working storage.
CUMPER	300	Real*8	Array of cumulative periodogram values.
DATE	600	Real*8	Time values for input series.
DELTA		Real*8	Sampling interval ('delta T').
FPE	600	Real*8	Array containing final prediction error.
FPER	300	Real*8	Array of periodogram frequencies.
IAUTCV		I*4	Autocovariance plotting flag (Burg estimate).
IFPEPL		I*4	Final prediction error flag.
IINTSP		I*4	Integrated spectrum flag.

TABLE 5.2: Cont'd

Variable	Dimension	Type	Explanation
IPLOT		I*4	Plotting flag.
IPRINT		I*4	Printing flag.
ITREND		I*4	Detrending flag.
IYWACV		I*4	Yule-Walker autocovariance flag.
MCOUNT	600	I*4	Vector of prediction error filter lengths to use.
NAM	14	I*4	Title for the data set.
NFILT		I*4	Number of prediction error filter lengths to try.
NPTS		I*4	Number of data points in time series.
NV		I*4	Number of frequencies at which spectrum is calculated.
PERDG	300	Real *8	Array of periodogram values.
PERIOD	600	Real*8	Vector of working storage. Upon return to the main program, this vector contains the periods corresponding to the frequencies in V.
PPER	300	Real*8	Array of periodogram periods corresponding to FPER.

TABLE 5.2: Cont'd

Variable	Dimension	Type	Explanation
S	600	Real*8	Maximum entropy spectral values are stored in this array.
STATE	2	Real*4	Label for dependent variable (y - axis)
V	600	Real*8	Output vector of frequencies at which spectrum is calculated.
VMAX		Real*8	Maximum frequency at which spectrum is to be calculated.
X	600	Real*8	Upon returning from DETRND (if detrending is done), it should contain the trend of the input data. It is later used as a vector of working storage.
XD	600	Real*8	If detrending is done, it should contain the detrended data; if not, it is the original data. Spectral analysis is applied on XD.
XDIFF	600	Real*8	Upon returning from DETRND, it is the detrended data. It is later used as a working vector.

TABLE 5.2: Cont'd

Variable	Dimension	Type	Explanation
Y	600	Real*8	Input time series values.
YEAR	600	Real*8	Vector of working storage (can be used as time variable in DETRND).
YTIME	2	Real*4	Label for the time axis.
Z	600	Real*8	Prediction error filter coefficients.

SFPE - number of components in the trend of the time series to be removed. For example, if the mean is removed, SFPE = 1; if the intercept and slope are to be removed, SFPE = 2, and so on. SFPE is given the value 0 in SPECTR if detrending is not done.

Should the user choose to analyze the original data without detrending, the dummy subroutine in Table 5.3 may be used.

If the mean of the input data is to be removed, i.e., the model $x_t = \mu + e_t$ is to be fitted, then the subroutine in Table 5.4 will be appropriate to use.

Table 5.5 contains a detrending subroutine in which the linear trend of a time series is eliminated. A least-squares routine BLSQ is called to perform the fitting of the straight line to the data. A write-up of BLSQ appears in Appendix II. SUBROUTINE BLSQ is available from disk at ORNL (see, for example, control card for "BROOKS.LOAD. MODULES" in Tables 6.3 and 6.4).

Table 5.3. Dummy subroutine DETRND

Statement number columns 1-5	Continuation column 6	Fortran statement columns 7-72
		SUBROUTINE DETRND(NPTS,DATE,YEAR,Y,XDIFF,X,SFPE)
		IMPLICIT REAL*8(A-H,O-Z)
		DIMENSION DATE(1), YEAR(1), Y(1), XDIFF(1), X(1)
		RETURN
		END

Table 5.4. A detrending subroutine for removal of the mean of a time series

Statement number columns 1-5	Continuation column 6	Fortran statement columns 7-72	Explanation
		SUBROUTINE DETRND(NPTS,DATE,YEAR,Y,XDIFF,X,SFPE)	
		IMPLICIT REAL*8(A-H,O-Z)	
		DIMENSION DATE(1), YEAR(1), Y(1), XDIFF(1), X(1)	
		SFPE = 1.0	Since only the mean is removed, SFPE = 1.
		AVRAGE = 0.0	
		DO 100 I = 1, NPTS	
		AVRAGE = AVRAGE + Y(I)	Compute the mean of the Y array.
100		CONTINUE	
		AVRAGE = AVRAGE / NPTS	
		DO 300 I = 1, NPTS	
		X(I) = AVRAGE	Store the trend of the data in X array.
		XDIFF(I) = Y(I) - X(I)	Store the detrended data in XDIFF array.
300		CONTINUE	
		RETURN	
		END	

Table 5.5. A detrending subroutine for the removal of the linear trend in a time series

Statement number columns 1-5	Continuation column 6	Fortran statement columns 7-72	Explanation
		SUBROUTINE DETRND(NPTS,DATE,YEAR,Y,XDIFF,X,SFPE)	
		IMPLICIT REAL*8(A-H,O-Z)	
		DIMENSION DATE(1),YEAR(1),Y(1),XDIFF(1),X(1),	
	1	A(601,3),IRUN(601),COEF(2),COEF2(2),T(2,2),T1(2,2),	
	2	VAR(2,2)	
		NROW = 601	NROW is the row dimension of the least squares matrix A.
		NLSCOL = 2	NLSCOL is the number of coefficients of the linear fit.
		NT = 2	NT is the row dimension of the matrices T and T1, which are computed in the least squares routine BLSQ.
		SFPE = NLSCOL	SFPE = NLSCOL since there are two linear coefficients - the intercept and slope - to be removed.
		DO 50 I = 1,NPTS	
		YEAR(I) = I - 0.5	Initialize the time variable
		A(I,1) = 1.0	Set up the least squares matrix.
		A(I,2) = YEAR(I)	

Table 5.5. (continued)

Statement number columns 1-5	Continuation column 6	Fortran statement columns 7-72	Explanation
50		CONTINUE	
		CALL BLSQ(A,Y,COEF,RESID,IRUN,T,T1,NPTS,NLSCOL,	
	1	NROW,NSLCOL)	Call the least squares routine BLSQ
			to perform the linear fit.
		DO 80 I = 1,NLSCOL	Compute the coefficients of the linear
		SUM = 0.0	fit using values returned from BLSQ.
		DO 70 J = 1,NLSCOL	
		SUM = SUM + T1(I,J) * COEF(J)	
70		CONTINUE	
		COEF2(I) = SUM	
80		CONTINUE	
		DO 90 I = 1,NLSCOL	
		COEF(I) = COEF2(I)	COEF(1) contains the intercept of the time series. COEF(2) contains the slope of the trend line.
90		CONTINUE	
		DO 100 I = 1,NPTS	
		X(I) = A(I,NLSCOL + 1)	The trend of the data comes back in
			A(I,NLSCOL + 1) and is stored in X array.

Table 5.5. (continued)

Statement number columns 1-5	Continuation column 6	Fortran statement columns 7-72	Explanation
		XDIFF(I) = Y(I) - X(I)	The detrended data is stored in XDIFF
100		CONTINUE	array.
		RETURN	
		END	

SUBROUTINE BLSQ returns (in the array T1) the information needed to compute the variance - convariance matrix of the coefficients (i.e. the slope and intercept) and these values could be stored in the array VAR which is not used in the preceding program (Table 5.5). A SUBROUTINE DETRND which does add this final step is given in the program listings in Appendix IV.

It should be obvious from studying Table 5.5 and Appendix II how to remove more complicated detrending functions using BLSQ or even a non-linear least squares program if needed.

5. SUBROUTINE SFT

The computation of the factors

$$\sum_{j=0}^{n-1} x_j \cos 2\pi jf \quad , \qquad \sum_{j=0}^{n-1} x_j \sin 2\pi jf \quad ,$$

from Equations (1.12) and (1.13) respectively, which are used in the evaluation of the sample Fourier spectrum, is done in SFT. Subroutine SFT is called by SPECTR with:

```
CALL SFT(XD, ACS, BSN, V, KF1, NYR, PI)
```

The corresponding arguments in SFT are given by:

```
SUBROUTINE SFT(X, A, B, V, K, N, PI)
```

These arguments are described in Table 5.6.

6. DOUBLE PRECISION FUNCTION SPCVAL

The double precision function SPCVAL computes for any frequency the maximum entropy spectral value P(f) as given by Equation (2.35). SPCVAL is used in the integrated maximum entropy spectrum calculations. It is the function which is integrated by the IMSL subroutine DCADRE, which is described in Appendix I. The IMSL subroutine DCADRE is available from disk at ORNL (see, for example, control card for "JDAIMSL1" in Tables 6.3 and 6.4). Table 5.7 lists the important parameters in SPCVAL.

TABLE 5.6

A LIST OF THE PARAMETERS IN SUBROUTINE SFT

Variable	Dimension	Type	Explanation
A	1	Real*8	The array containing the factors $\sum_{j=0}^{n-1} x_j \cos 2\pi jf$ upon return. It has a maximum dimension of 600.
B	1	Real*8	The array containing the factors $\sum_{j=1}^{n-1} x_j \sin 2\pi jf$ upon return. It has a maximum dimension of 600.
K		I*4	Number of frequencies at which spectrum is calculated.
N		I*4	Number of points in the series under analysis.
PI		Real*8	The real number π.

TABLE 5.6: Cont'd

Variable	Dimension	Type	Explanation
V	1	Real*8	Output vector of frequencies at which spectrum is calculated.
X	1	Real*8	Time series under analysis.

TABLE 5.7

A LIST OF PARAMETERS IN THE FUNCTION SUBROUTINE SPCVAL

Variable	Dimension	Type	Explanation
AMP	600	Real*8	The array containing P_M.
DTIME		Real*8	The time interval Δt.
FREQ		Real*8	The frequency f.
M		i*4	The number of filter coefficients M.
PI		Real*8	The real number π.
SPCVAL		Real*8	The spectral value P(f).
Z	600	Real*8	The array of filter coefficents $a_{i,M}$.

7. SUBROUTINE DOPLT

Subroutine DOPLT is a plotting routine which calls other subroutines primarily from the plotting package DISSPLA (DISSPLA is a universal package that performs the necessary plotting). DISSPLA is available from disk at ORNL (see, for example, control card for "DISSPLA.LOAD" in Tables 6.3 and 6.4). There are nine ENTRY statements, the function of each are listed as follows:

(1) ENTRY DOPLT1 - plots the original data against the time values; if the data are detrended, the trend is also plotted on the same set of axes.

(2) ENTRY DOPLT2 - if detrending is performed, the detrended data are plotted against the time values; the line y = 0 is also included in the plot and serves as a reference line for the residuals. Furthermore, a plot of the residuals against the fitted values or trend is done.

(3) ENTRY DOPLT3 - plots the Yule-Walker autocorrelation function against the lag values if IYWACV = 1.

(4) ENTRY DOPLT4 - plots the Fourier sample spectrum against frequency; it also plots the cumulative Fourier power spectrum against frequency. The 95% Kolmogorov-Smirnov bounds are also plotted on the latter graph.

(5) ENTRY DOPLT5 - plots the Burg autocorrelation function against the number of filter coefficients if IAUTCV = 1.

(6) ENTRY DOPLT6 - plots the maximum entropy spectrum against frequency.

(7) ENTRY DOPLT7 - plots the integrated maximum entropy spectrum against frequency if IINTSP = 1.

(8) ENTRY DOPLT8 - plots the final prediction error against the number of filter coefficients if IFPEPL = 1.

(9) ENTRY DOPLT9 - plots the periodogram and cumulative periodogram as a function of frequency. The 95% Kolmogorov-Smirnov bounds are included in the latter plot.

The other subroutines called by DOPLT after each ENTRY statement are mainly for setting up the axes for plotting. They are:

(1) SUBROUTINE SINGLE - transforms double precision variables into single precision. This is necessary since DISSPLA routines handle only Real*4 information.

(2) SUBROUTINE ANGTIC - controls the number of tick marks and angular position of the numbers on the axes.

(3) SUBROUTINE GPHBGN - sets up the initial x and y axes for plotting according to the range of the data pairs.

(4) SUBROUTINE GPHBG1 - sets up the initial x and y axes for plotting the FPE. This uses logarithmic scaling of the axes, as contrasted with GPHBGN which makes use of linear scaling.

(5) SUBROUTINE GPHEND - plots the specific curve and draws a frame about the picture.

All the necessary parameters for plotting are transferred to Subroutine DOPLT by the two common blocks SPECBL and BLKLBL.

CHAPTER VI

ILLUSTRATION OF THE USE OF THE PROGRAM

A. An Ecological Example

The time series we will consider is the striped bass catch per unit effort data for the commercial fishery in the Hudson River from 1955 - 1975, a series of length 21 years. The set of input cards appears in Table 6.1. The first card contains the labels for the x and y axes, the time interval, the number of points in the time series, the number of times MESA is to be performed, the number of frequencies and the maximum frequency. The second card has the label for the time series data. The third card is the number of filter coefficients used. The time series follows on cards 4 - 24. The last card has all the options to print, to plot, to detrend, to compute the FPE, to compute the integrated spectrum to compute the autocorrelation function from MESA, and to compute the Yule - Walker estimates of the autocorrelation. All these option indicators are given the value 1.

The data were detrended using a SUBROUTINE DETRND which removed a linear trend by calling BLSQ, the least squares procedure (DETRND is in Appendix IV and BLSQ in Appendix II). Spectral analysis was done on both the raw data and the natural logarithm of the raw data. However, only output for the raw data will be presented. The printed output appears in Figure 6.1 and the plots in Figure 6.2A thru Figure 6.2L. The printout on the residual sum of squares, coefficients of the linear fit, and variance-covariance matrix come from DETRND. DISSPLA plot messages are also inserted.

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Table 6.1. Input cards for the Hudson River catch per unit effort data

Card		
1	YEAR ____ $ ___ CATCH ___ $ _____ 1.0 _____ 21 _____ 1 ___ 199 ____ 0.5	
2	HUDSON RIVER CATCH PER UNIT EFFORT$	
3	____ 18	
4	1955 _ 1897.	TIME SERIES DATA
5	1956 _ 2550.	
6	1957 _ 2456.	
7	1958 _ 2479.	
8	1959 _ 3985.	
9	1960 _ 4089.	
10	1961 _ 2372.	
11	1962 _ 1670.	
12	1963 _ 2039.	
13	1964 _ 1378.	
14	1965 _ 1952.	
15	1966 _ 2842.	
16	1967 _ 3595.	
17	1968 _ 3397.	

Table 6.1. (continued)

Card		
18	1969 _ 4852.	TIME SERIES DATA
19	1970 _ 2608.	
20	1971 _ 1743.	
21	1972 _ 1213.	
22	1973 _ 4273.	
23	1974 _ 1429.	
24	1975 _ 1480.	
25	_ _ _ _ 1 _ _ _ _ 1 _ _ _ _ 1 _ _ _ _ 1 _ _ _ _ 1 _ _ _ _ 1 _ _ _ _ 1	

NOTE: '_' indicates a space.

ANALYSIS OF THE ORIGINAL INPUT DATA WITHOUT ANY PRIOR TRANSFORMATIONS

HUDSON RIVER CATCH PER UNIT EFFORTS

YEAR $	C/E $
1.9550D 03	1.8970000D 03
1.9560D 03	2.5500000D 03
1.9570D 03	2.4560000D 03
1.9580D 03	2.4790000D 03
1.9590D 03	3.9850000D 03
1.9600D 03	4.0890000D 03
1.9610D 03	2.3720000D 03
1.9620D 03	1.6700000D 03
1.9630D 03	2.0390000D 03
1.9640D 03	1.3780000D 03
1.9650D 03	1.9520000D 03
1.9660D 03	2.8820000D 03
1.9670D 03	3.5950000D 03
1.9680D 03	3.3970000D 03
1.9690D 03	4.8520000D 03
1.9700D 03	2.6080000D 03
1.9710D 03	1.7830000D 03
1.9720D 03	1.2130000D 03
1.9730D 03	4.2730000D 03
1.9740D 03	1.4290000D 03
1.9750D 03	1.4800000D 03

RESIDUAL SUM OF SQUARES = 2.2522230 07

COEFFICIENTS OF THE LINEAR FIT

INTERCEPT = 2.7189260 03 SLOPE = -1.2681820 01

VARIANCE-COVARIANCE MATRIX

ROW	COLUMN 1	COLUMN 2
1	2.2617159770716380D 05	-1.6164277588586600D 08
2	-1.6164277588586600D 08	1.5398550842615200 03

THE SPECTRAL ANALYSIS WILL BE APPLIED TO THE DETRENDED DATA

YEAR $	C/E $	DETRENDED C/E $
1.9550D 03	1.8970000D 03	-6.1588680 02
1.9560D 03	2.5500000D 03	-1.8980300 02
1.9570D 03	2.4560000D 03	-2.3112120 02
1.9580D 03	2.4790000D 03	-1.9543980 02
1.9590D 03	3.9850000D 03	1.3232620 03
1.9600D 03	4.0890000D 03	1.8399280 01
1.9610D 03	2.3720000D 03	-2.6839390 02
1.9620D 03	1.6700000D 03	-9.5371210 02
1.9630D 03	2.0390000D 03	-5.7203030 02
1.9640D 03	1.3780000D 03	-1.2203680 03
1.9650D 03	1.9520000D 03	-6.3366670 02
1.9660D 03	2.8820000D 03	2.6901520 02
1.9670D 03	3.5950000D 03	1.0386970 01
1.9680D 03	3.3970000D 03	8.8933480 02
1.9690D 03	4.8520000D 03	2.3170610 03
1.9700D 03	2.6080000D 03	8.5782820 01
1.9710D 03	1.7830000D 03	-7.6657580 02
1.9720D 03	1.2130000D 03	-1.2838980 03
1.9730D 03	4.2730000D 03	1.7887880 01
1.9740D 03	1.4290000D 03	-1.0825300 03
1.9750D 03	1.4800000D 03	-9.7888850 02

Fig. 6.1. Standard computer output.

```
PLOT NO.   3 WITH THE TITLE
     RESIDUALS VS. FITTED VALUES#
HAS BEEN COMPLETED.

PLOT TE. BEATS
PLOT 3     12.50.01  THES 13 JUN, 1978    JOB=SLEL    , OBPL  DISSPWR VER 6.0


DATA FOR PLOT

NO. OF CURVES DRAWN  1
-----------------------------
HORIZ. AXIS LENGTH   7.0 INS.
VERT. AXIS LENGTH    5.0 INS.
-----------------------------
HORIZ. ORIGIN 0.2000E 00 VERT.  ORIGIN-0.1500E 00
-----------------------------
HORIZ. AXIS LINEAR
STEP SIZE 0.5000E 02 UNITS/INCH
-----------------------------
VERT.  AXIS LINEAR
STEP SIZE 0.8000E 03 UNITS/INCH
-----------------------------

    ------------------------------------
    . LOCATION OF CURRENT PHYSICAL ORIGIN .
    . X=   1.50      Y=   2.59     INCHES .
    .    FROM LOWER LEFT CORNER OF PAGE   .
    ------------------------------------



YULE-WALKER ESTIMATE OF AUTOCORRELATION FUNCTION


    LAG            AUTOCORR.

 0.0             1.00000D 00
 1.000D 00       2.072630-01
 2.000D 00      -1.052380-01
 3.000D 00      -1.837340-01
 4.000D 00      -1.688130-01
 5.000D 00      -8.560050-01
 6.000D 00      -1.508720-01
 7.000D 00       8.226480-02
 8.000D 00       9.858780-02
 9.000D 00       2.190820-01
 1.000D 01       2.867880-01
 1.100D 01      -7.057590-02
 1.200D 01      -1.867890-01
 1.300D 01       5.222570-02
 1.400D 01      -1.552820-02
 1.500D 01      -1.201860-01
 1.600D 01      -3.052390-02
 1.700D 01       5.378180-02
 1.800D 01      -8.778930-02
 1.900D 01       8.825860-02
 2.000D 01       3.588210-02



PERIODOGRAM


  K      FREQ.       PERIOD      PERIODOGRAM     CUMULATIVE
                                  ORDINATE      PERIODOGRAM


   1   0.04762       21.00000   6.708750 05    0.02979
   2   0.09520       10.50000   7.527130 06    0.36400
   3   0.14286        7.00000   1.698220 06    0.52776
   4   0.19048        5.25000   7.030060 05    0.55897
   5   0.23810        4.20000   2.689110 06    0.67659
   6   0.28571        3.50000   3.525960 06    0.83315
   7   0.33333        3.00000   8.256850 05    0.85205
   8   0.38095        2.62500   3.246380 05    0.86606
   9   0.42857        2.33333   6.987120 05    0.89731
  10   0.47619        2.10000   2.312870 06    1.00000


SUM OF PERIODOGRAM ORDINATES -  2.252220 07

MAXIMUM ORDINATE =  7.527130 06   OCCURRING AT FREQUENCY = 9.523810-02   PERIOD = 1.050000 01

FISHER STATISTIC = 3.342090 00

NUMBER OF ORDINATES =   10
```

Fig. 6.1. (continued).

SUM OF FOURIER ORDINATES = 8.2740190 00

NCORR = 16

AUTOCORRELATION FUNCTION VALUES

LAG	AUTOCORR.
0.0	1.0000000 00
1.0000000 00	2.1501970-01
2.0000000 00	-1.0206580-01
3.0000000 00	-2.1677930-01
4.0000000 00	-1.5189900-01
5.0000000 00	-6.0896050-01
6.0000000 00	-1.2985870-01
7.0000000 00	1.0230890-01
8.0000000 00	2.0425320-01
9.0000000 00	3.0998520-01
1.0000000 01	8.5609720-01
1.1000000 01	-1.6130180-01
1.2000000 01	-3.6101380-01
1.3000000 01	-9.6250570-02
1.4000000 01	-1.6832050-01
1.5000000 01	-3.9043610-01
1.6000000 01	1.1133920-01
1.7000000 01	3.6761020-01
1.8000000 01	2.5062050-01

MAXIMUM ENTROPY AND FOURIER SPECTRA

NYQUIST FREQUENCY = 5.0000-01

RESOLUTION BANDWIDTH = 5.0000-02

FINAL PREDICTION ERROR = 1.1030 07

NCORR= 16	FREQUENCY	PERIOD	MAXIMUM ENTROPY	FOURIER AMPLITUDE
I= 1	0.0	0.0	8.1689160 08	1.2787060-11
I= 2	0.0025126	398.000	8.1892100 08	1.9682880 00
I= 3	0.0050251	199.000	8.2631800 08	7.8156690 00
I= 4	0.0075377	132.667	8.3899350 08	1.6871880 01
I= 5	0.0100503	99.500	8.5752790 08	2.8758400 01
I= 6	0.0125628	79.600	8.8278010 08	4.3897070 01
I= 7	0.0150754	66.333	5.1598680 08	6.1820990 01
I= 8	0.0175879	56.857	5.5887380 08	8.0788950 01
I= 9	0.0201005	49.750	6.1383210 08	1.0140230 02
I= 10	0.0226131	44.222	6.8817810 08	1.2262480 02
I= 11	0.0251256	39.800	7.7850670 08	1.4380270 02
I= 12	0.0276382	36.182	8.9128200 08	1.6829260 02
I= 13	0.0301508	33.167	1.0835900 09	1.8368860 02
I= 14	0.0326633	30.615	1.2442660 09	2.0083290 02
I= 15	0.0351759	28.429	1.5113010 09	2.1588850 02
I= 16	0.0376884	26.533	1.8692680 09	2.2833580 02
I= 17	0.0402010	24.875	2.3496590 09	2.3803380 02
I= 18	0.0427136	23.412	2.9851090 09	2.4506650 02
I= 19	0.0452261	22.111	3.7936680 09	2.4978860 02
I= 20	0.0477387	20.947	4.7389150 09	2.5289600 02
I= 21	0.0502513	19.900	5.6566220 09	2.5585150 02
I= 22	0.0527638	18.952	6.3026770 09	2.5888830 02
I= 23	0.0552764	18.091	6.6702010 09	2.6091320 02
I= 24	0.0577889	17.308	6.8130990 09	2.7527230 02
I= 25	0.0603015	16.583	5.6883180 09	2.9181870 02
I= 26	0.0628141	15.920	5.0576200 09	3.1819630 02
I= 27	0.0653266	15.308	8.5396920 09	3.4170390 02
I= 28	0.0678392	14.741	8.1328120 09	3.7938210 02
I= 29	0.0703518	14.218	1.8816070 09	4.2022730 02
I= 30	0.0728643	13.728	1.6600180 09	4.6500800 02
I= 31	0.0753769	13.267	1.5808750 09	5.1281890 02
I= 32	0.0778894	12.839	1.6007510 09	5.6117090 02

Fig. 6.1. (continued).

I= 33	0.0808020	12.838	3.726+88D 05	6.1103722D 02	
I= 34	0.0829186	12.041	3.973[illegible]D 05	6.5797712D 02	
I= 35	0.0850271	11.706	8.[illegible]D 05	7.0368988D 02	
I= 36	0.0870397	11.371	8.[illegible]D 05	7.8682258D 02	
I= 37	0.0908525	11.056	5.996302D 05	7.8538328D 02	
I= 38	0.0929688	10.757	7.2988768D 05	8.1984812D 02	
I= 39	0.0958778	10.479	8.8676188D 05	8.8925199D 02	
I= 40	0.0979899	10.205	1.3090898D 06	8.7319238D 02	
I= 41	0.1005025	9.950	1.9579738D 06	8.9139868D 02	
I= 42	0.1030151	9.707	3.2898128D 06	9.0373208D 02	
I= 43	0.1055276	9.476	6.1731998D 06	9.1022078D 02	
I= 44	0.1080402	9.256	1.3290618D 07	9.1101368D 02	
I= 45	0.1105528	9.045	2.2161828D 07	9.0639228D 02	
I= 46	0.1130653	8.844	1.5350618D 07	8.9675278D 02	
I= 47	0.1155779	8.652	7.9538898D 06	8.9258938D 02	
I= 48	0.1180905	8.468	8.6862808D 06	8.6887138D 02	
I= 49	0.1206030	8.292	1.1109268D 06	8.8301998D 02	
I= 50	0.1231156	8.122	2.3108798D 06	8.1887708D 02	
I= 51	0.1256281	7.960	1.8527288D 06	7.9267818D 02	
I= 52	0.1281807	7.808	1.5729068D 06	7.6499838D 02	
I= 53	0.1306533	7.658	1.3921938D 06	7.3636308D 02	
I= 54	0.1331658	7.509	1.2673028D 06	7.0711328D 02	
I= 55	0.1356784	7.370	1.1706208D 06	6.7770858D 02	
I= 56	0.1381910	7.236	1.0828008D 06	6.4810788D 02	
I= 57	0.1407035	7.107	9.9897828D 05	6.1836368D 02	
I= 58	0.1432161	6.982	9.2800238D 05	5.8835988D 02	
I= 59	0.1457286	6.862	7.6939138D 05	5.5788388D 02	
I= 60	0.1482412	6.746	6.5306768D 05	5.2651278D 02	
I= 61	0.1507538	6.633	5.6819858D 05	4.9402788D 02	
I= 62	0.1532663	6.525	4.8913708D 05	4.6005708D 02	
I= 63	0.1557789	6.419	3.7038708D 05	4.2831998D 02	
I= 64	0.1582915	6.317	3.0725718D 05	3.8661988D 02	
I= 65	0.1608040	6.219	2.5779398D 05	3.4687688D 02	
I= 66	0.1633166	6.123	2.1987888D 05	3.0516358D 02	
I= 67	0.1658291	6.030	1.9000078D 05	2.6175368D 02	
I= 68	0.1683417	5.940	1.6787688D 05	2.1721518D 02	
I= 69	0.1708543	5.853	1.5033838D 05	1.7265288D 02	
I= 70	0.1733668	5.768	1.3768808D 05	1.3081568D 02	
I= 71	0.1758794	5.686	1.2843028D 05	9.6264928D 01	
I= 72	0.1783920	5.606	1.2218178D 05	8.3048728D 01	
I= 73	0.1809045	5.528	1.1854328D 05	1.0000158D 02	
I= 74	0.1834171	5.452	1.1733438D 05	1.3583028D 02	
I= 75	0.1859296	5.378	1.1853688D 05	1.7857938D 02	
I= 76	0.1884422	5.307	1.2230688D 05	2.2305998D 02	
I= 77	0.1909548	5.237	1.2900158D 05	2.6702288D 02	
I= 78	0.1934673	5.169	1.3926388D 05	3.0927238D 02	
I= 79	0.1959799	5.103	1.5612598D 05	3.4905928D 02	
I= 80	0.1984925	5.038	1.7528388D 05	3.8587198D 02	
I= 81	0.2010050	4.975	2.0529098D 05	4.1935678D 02	
I= 82	0.2035176	4.914	2.4870978D 05	4.4927118D 02	
I= 83	0.2060302	4.854	3.1318308D 05	4.7588218D 02	
I= 84	0.2085427	4.795	4.1268788D 05	4.9793068D 02	
I= 85	0.2110553	4.738	5.7615088D 05	5.1662558D 02	
I= 86	0.2135678	4.682	8.5306858D 05	5.3162688D 02	
I= 87	0.2160804	4.628	1.3670098D 06	5.4302678D 02	
I= 88	0.2185930	4.575	2.3807228D 06	5.5093678D 02	
I= 89	0.2211055	4.523	3.8633028D 06	5.5586768D 02	
I= 90	0.2236181	4.472	4.6857358D 06	5.5671798D 02	
I= 91	0.2261307	4.422	3.8508018D 06	5.5876318D 02	
I= 92	0.2286432	4.378	2.2030568D 06	5.4965038D 02	
I= 93	0.2311558	4.326	1.0558888D 06	5.2139908D 02	
I= 94	0.2336683	4.280	1.0378218D 06	5.3000888D 02	
I= 95	0.2361809	4.238	7.9297518D 05	5.1587288D 02	
I= 96	0.2386935	4.199	6.8235758D 05	4.9780168D 02	
I= 97	0.2412060	4.164	5.8583388D 05	4.7705898D 02	
I= 98	0.2437186	4.103	4.8116588D 05	4.5337938D 02	
I= 99	0.2462312	4.061	4.3781718D 05	4.2706888D 02	
I= 100	0.2487437	4.020	4.0880198D 05	3.9961188D 02	
I= 101	0.2512563	3.980	3.9866688D 05	3.6886278D 02	
I= 102	0.2537688	3.941	3.7497168D 05	3.3901688D 02	
I= 103	0.2562814	3.902	3.6571388D 05	3.7101538D 02	
I= 104	0.2587940	3.868	3.5906668D 05	2.9787888D 02	
I= 105	0.2613065	3.827	3.5388278D 05	2.7170288D 02	
I= 106	0.2638191	3.790	3.8950108D 05	2.6702758D 02	
I= 107	0.2663317	3.755	3.8577188D 05	2.7550938D 02	
I= 108	0.2688442	3.720	3.8305398D 05	2.9691098D 02	
I= 109	0.2713568	3.685	3.8218068D 05	3.2897338D 02	
I= 110	0.2738693	3.651	3.8861598D 05	3.6859888D 02	
I= 111	0.2763819	3.618	3.5188688D 05	4.1277898D 02	
I= 112	0.2788945	3.586	3.6575618D 05	4.5899078D 02	
I= 113	0.2814070	3.558	3.9072578D 05	5.0516188D 02	
I= 114	0.2839196	3.522	4.3206828D 05	5.4934958D 02	
I= 115	0.2864322	3.491	4.9995648D 05	5.9084928D 02	
I= 116	0.2889447	3.461	6.1877998D 05	6.2785978D 02	
I= 117	0.2914573	3.431	8.2299638D 05	6.5953128D 02	
I= 118	0.2939698	3.402	1.2513898D 06	6.8891518D 02	
I= 119	0.2964824	3.373	2.3587198D 06	7.0364818D 02	
I= 120	0.2989950	3.345	6.7126188D 06	7.1502898D 02	
I= 121	0.3015075	3.317	2.5833678D 07	7.1867618D 02	
I= 122	0.3040201	3.289	5.9917108D 06	7.1887598D 02	
I= 123	0.3065327	3.262	1.6822788D 06	7.0236688D 02	
I= 124	0.3090452	3.236	6.9553178D 05	6.8289098D 02	

Fig. 6.1 (continued).

I= 125	0.3115578	3.210	3.6779068 05	6.5502158 02
I= 126	0.3180704	3.144	2.2197218 05	6.2051158 02
I= 127	0.3165829	3.159	1.8639108 05	5.3947108 02
I= 128	0.3199955	3.124	1.0299268 05	5.3258878 02
I= 129	0.3215080	3.109	7.6176688 04	5.0067978 02
I= 130	0.3281206	3.045	5.9670758 04	4.2862888 02
I= 131	0.3266332	3.062	4.6767968 04	3.6547058 02
I= 132	0.3291457	3.038	3.8358288 04	3.0430328 02
I= 133	0.3316583	3.015	3.2290828 04	2.5236068 02
I= 134	0.3341709	2.992	2.7826638 04	1.8112958 02
I= 135	0.3366834	2.970	2.4490688 04	1.2286178 02
I= 136	0.3391960	2.948	2.1983978 04	7.3207938 01
I= 137	0.3417085	2.926	2.0101268 04	5.7669978 01
I= 138	0.3442211	2.905	1.8703598 04	4.0888288 01
I= 139	0.3467337	2.884	1.7695878 04	1.2210658 02
I= 140	0.3492462	2.863	1.7013728 04	1.6250588 02
I= 141	0.3517588	2.843	1.6613038 04	1.9887178 02
I= 142	0.3542714	2.823	1.6468578 04	2.2869298 02
I= 143	0.3567839	2.803	1.6566558 04	2.5288638 02
I= 144	0.3592965	2.783	1.6900338 04	2.6955368 02
I= 145	0.3618090	2.764	1.7488478 04	2.7977998 02
I= 146	0.3643216	2.745	1.8333188 04	2.8325408 02
I= 147	0.3668342	2.726	1.9459408 04	2.8028478 02
I= 148	0.3693467	2.707	2.0890288 04	2.7120108 02
I= 149	0.3718593	2.689	2.2687638 04	2.5676098 02
I= 150	0.3743719	2.671	2.4702328 04	2.3777598 02
I= 151	0.3768844	2.653	2.7160688 04	2.1536378 02
I= 152	0.3793970	2.636	2.9885658 04	1.9101378 02
I= 153	0.3819095	2.618	3.2677268 04	1.6677758 02
I= 154	0.3844221	2.601	3.5461388 04	1.4558168 02
I= 155	0.3869347	2.584	3.7963908 04	1.3111018 02
I= 156	0.3894472	2.568	3.9860818 04	1.2719778 02
I= 157	0.3919598	2.551	4.1017238 04	1.3894388 02
I= 158	0.3944724	2.535	4.1358888 04	1.5189038 02
I= 159	0.3969849	2.519	4.0970158 04	1.8075308 02
I= 160	0.3994975	2.503	4.0071818 04	1.9811528 02
I= 161	0.4020101	2.488	3.8919258 04	2.2164618 02
I= 162	0.4045226	2.472	3.7717758 04	2.4308488 02
I= 163	0.4070352	2.457	3.6679158 04	2.6178678 02
I= 164	0.4095477	2.442	3.5929638 04	2.7588918 02
I= 165	0.4120603	2.427	3.5567638 04	2.8604178 02
I= 166	0.4145728	2.412	3.5672568 04	2.9184738 02
I= 167	0.4170854	2.398	3.6322708 04	2.9328978 02
I= 168	0.4195980	2.383	3.7612278 04	2.9052998 02
I= 169	0.4221106	2.369	3.9669678 04	2.8818278 02
I= 170	0.4246231	2.355	4.2478768 04	2.7896658 02
I= 171	0.4271357	2.341	4.6912898 04	2.6381218 02
I= 172	0.4296482	2.327	5.2740218 04	2.5229558 02
I= 173	0.4321608	2.314	6.0911118 04	2.4182908 02
I= 174	0.4346734	2.301	7.2295898 04	2.3830698 02
I= 175	0.4371859	2.287	8.8585678 04	2.3188828 02
I= 176	0.4396985	2.274	1.1238188 05	2.3863638 02
I= 177	0.4422111	2.261	1.4860368 05	2.4811078 02
I= 178	0.4447236	2.249	2.0603388 05	2.5958798 02
I= 179	0.4472362	2.236	3.0232188 05	2.7963768 02
I= 180	0.4497487	2.223	4.7013568 05	3.0281018 02
I= 181	0.4522613	2.211	7.7016598 05	3.2759318 02
I= 182	0.4547739	2.199	1.2669998 06	3.5268408 02
I= 183	0.4572864	2.187	1.6772268 06	3.7682368 02
I= 184	0.4597990	2.175	1.5078588 06	3.9923268 02
I= 185	0.4623116	2.163	1.2098108 06	4.1915998 02
I= 186	0.4648241	2.151	8.8831378 05	4.3608278 02
I= 187	0.4673367	2.140	6.7889188 05	4.4969888 02
I= 188	0.4698492	2.128	5.5726978 05	4.5967078 02
I= 189	0.4723618	2.117	4.8836098 05	4.6611078 02
I= 190	0.4748744	2.106	4.5478978 05	4.6900028 02
I= 191	0.4773869	2.095	4.4832518 05	4.6884018 02
I= 192	0.4798995	2.084	4.4728068 05	4.6578658 02
I= 193	0.4824121	2.073	4.4588708 05	4.6088698 02
I= 194	0.4849246	2.062	4.3809858 05	4.5366538 02
I= 195	0.4874372	2.052	4.2776198 05	4.4557988 02
I= 196	0.4899497	2.041	4.1292228 05	4.3758768 02
I= 197	0.4924623	2.031	3.9371818 05	4.3029508 02
I= 198	0.4949749	2.020	3.7511118 05	4.2405118 02
I= 199	0.4974874	2.010	3.5525228 05	4.2067238 02
I= 200	0.5000000	2.000	3.3499258 05	4.1936518 02

Fig. 6.1. (continued).

INTEGRATED MAXIMUM ENTROPY SPECTRUM

WIDTH OF INTEGRATION WINDOW = 2.5000E-02

RECORD#= 19	FREQUENCY	PERIOD	NAT. EST. SPEC.	INTEGRATED SPECTRUM	ERROR SQ. (INTEG. SP.)
I= 1	0.0	0.0	0.1669165D 08	0.0	0.0
I= 2	0.0025126	398.000	0.1892150D 08	0.0	0.0
I= 3	0.0050251	199.000	0.2631808D 08	0.0	0.0
I= 4	0.0075377	132.667	0.3899359D 08	0.0	0.0
I= 5	0.0100503	99.500	0.5752780D 08	0.0	0.0
I= 6	0.0125628	79.600	0.8278015D 08	1.2951399D 03	1.8369960D-01
I= 7	0.0150754	66.333	0.1598648D 08	1.3981768D 03	1.9706311D-01
I= 8	0.0175879	56.857	0.5887380D 08	1.5333113D 03	2.2088690D-01
I= 9	0.0201005	49.750	0.1383218D 08	1.7096625D 03	2.4599690D-01
I= 10	0.0226131	44.222	0.8817418D 08	1.9399895D 03	2.8351960D-01
I= 11	0.0251256	39.800	0.7850667D 08	2.2819995D 03	3.3918690D-01
I= 12	0.0276382	36.182	0.9129297D 08	2.6805883D 03	3.9907710D-01
I= 13	0.0301508	33.167	0.1035905D 09	3.1680530 03	4.7522710D-01
I= 14	0.0326633	30.615	0.2442660D 09	3.8637590 03	5.7836990D 00
I= 15	0.0351759	28.429	0.5113010D 09	4.7688620 03	7.0997690D 00
I= 16	0.0376884	26.533	0.9692660D 09	5.9936760 03	8.9698610D 00
I= 17	0.0402010	24.875	0.1798659D 09	7.4811650 03	1.1139610D 00
I= 18	0.0427136	23.412	0.2951800D 09	8.5522710 03	1.2739610D 00
I= 19	0.0452261	22.111	0.7936690D 09	9.8637910 03	2.7383580D-01
I= 20	0.0477387	20.947	0.7389150D 09	1.1009710 04	1.1836290D-01
I= 21	0.0502513	19.900	0.6566220D 09	1.1932670 04	8.9267060D-02
I= 22	0.0527638	18.952	0.3826770D 09	1.2609310 04	6.2249020D-02
I= 23	0.0552764	18.091	0.8702010D 09	1.3028220 04	2.8351360D-02
I= 24	0.0577889	17.306	0.1830990D 09	1.3176270 04	6.6318190D-02
I= 25	0.0603015	16.583	0.6883180D 09	1.3081390 04	2.0886150D-01
I= 26	0.0628141	15.920	0.3576200D 09	1.2635980 04	1.5516690D-01
I= 27	0.0653266	15.309	0.5390920D 09	1.2029960 04	2.8124620D 00
I= 28	0.0678392	14.741	0.1328120D 09	1.1318820 04	5.3750130D-01
I= 29	0.0703518	14.214	0.2234030D 09	1.0499350 04	8.9021180D-02
I= 30	0.0728643	13.728	0.6600380D 09	1.0286640 04	8.5375130D-02
I= 31	0.0753769	13.283	0.2004750D 09	1.0077930 04	1.1298180 00
I= 32	0.0778894	12.839	0.6007170D 09	1.0224070 04	2.8895610 00
I= 33	0.0804020	12.438	0.7258880D 09	1.0776270 04	3.9378780-01
I= 34	0.0829146	12.061	0.9738900D 09	1.1487990 04	7.4776630-01
I= 35	0.0854271	11.706	0.3783860D 09	1.1679660 04	1.7387080 00
I= 36	0.0879397	11.371	0.9632280D 09	1.6752090 04	8.2608830 00
I= 37	0.0904523	11.056	0.4963020D 09	2.2112750 04	9.2275550 00
I= 38	0.0929648	10.757	0.2888160D 09	3.2316510 04	1.8798350 01
I= 39	0.0954774	10.474	0.8676160D 09	5.8119970 04	2.0529400D 01
I= 40	0.0979899	10.205	0.3090890D 06	9.8732490 04	3.2710080 01
I= 41	0.1005025	9.950	0.9579730D 06	1.8759170 05	6.8203200 01
I= 42	0.1030151	9.707	0.2888120D 06	1.7886170 05	6.6893250 01
I= 43	0.1055276	9.476	0.1731940D 06	1.6864900 05	2.6960280 01
I= 44	0.1080402	9.256	0.1290610D 07	1.9620970 05	1.1337730 02
I= 45	0.1105528	9.045	0.2163520 07	2.0015560 05	5.8375010 01
I= 46	0.1130653	8.844	0.5350610D 07	2.0130370 05	9.3183730 01
I= 47	0.1155779	8.652	0.9589890D 06	1.9917500 05	1.2055180 01
I= 48	0.1180905	8.468	0.6862860 06	1.9180600 05	2.5257730 01
I= 49	0.1206030	8.292	0.1109260 06	1.7104160 05	8.0492010 01
I= 50	0.1231156	8.122	0.2108790 06	1.2737120 05	2.5365330 01
I= 51	0.1256281	7.960	0.8527280 06	9.1012980 04	2.7367190 01
I= 52	0.1281407	7.804	0.5729060 06	5.5915780 04	9.5765190-01
I= 53	0.1306533	7.654	0.3921930 06	3.3235230 04	1.6377980 00
I= 54	0.1331658	7.509	0.2673020 06	1.5695570 04	5.2681160 00
I= 55	0.1356784	7.370	0.1706200 06	1.1027690 04	2.3181780 00
I= 56	0.1381910	7.236	0.1082800D 06	2.7188250 03	1.1171820 00
I= 57	0.1407035	7.107	0.8897430 05	2.8375890 03	1.0887710 01
I= 58	0.1432161	6.982	0.8600230 05	2.1704580 03	5.9438880 00
I= 59	0.1457286	6.862	0.6939130 05	1.9235760 03	8.8981180-01
I= 60	0.1482412	6.746	0.5306780 05	1.5689120 03	1.6635570 00
I= 61	0.1507538	6.633	0.4818850 05	1.0660280 03	8.8709700 00
I= 62	0.1532663	6.525	0.4913700 05	1.2588630 03	9.3758860-01
I= 63	0.1557789	6.419	0.3703670D 05	1.0688790 03	8.8609960-02
I= 64	0.1582915	6.317	0.3072570D 05	9.0139170 02	2.5699680D 00
I= 65	0.1608040	6.219	0.2577939D 05	7.5957230 02	1.9819910-02
I= 66	0.1633166	6.123	0.1987880 05	6.8150200 02	8.6724670-02
I= 67	0.1658291	6.030	0.1900097D 05	5.5109880 02	1.5889990-01
I= 68	0.1683417	5.940	0.1676744D 05	0.7919990 02	0.3388180-01
I= 69	0.1708543	5.853	0.1503983D 05	0.2637120 02	9.9766850-02
I= 70	0.1733668	5.768	0.1376800D 05	3.8152810 01	9.8198170-02
I= 71	0.1758794	5.686	0.1288302D 05	3.5819190 01	8.2152690-01
I= 72	0.1783920	5.606	0.1221810D 05	3.3856160 01	3.8612130-01
I= 73	0.1809045	5.528	0.1156320D 05	3.2367870 01	3.8813700-01
I= 74	0.1834171	5.452	0.1713830 05	3.2040190 01	8.3267750-01
I= 75	0.1859296	5.378	0.1053680 05	3.2587130 01	5.5733990-01
I= 76	0.1884422	5.307	0.1230880 05	3.1956170 01	8.0958350-01
I= 77	0.1909548	5.237	0.1900150 05	3.0859990 01	1.2981380 01
I= 78	0.1934673	5.169	0.1392630 05	0.0805950 01	1.7187810-01
I= 79	0.1959799	5.103	0.1612590 05	0.6810630 01	1.3017960-01
I= 80	0.1984925	5.038	0.1720380 05	5.5578780 01	6.3967050-01
I= 81	0.2010050	4.975	0.2052909D 05	6.9975150 01	1.8751010 00
I= 82	0.2035176	4.914	0.2870970 05	9.1707920 01	3.1092280 01
I= 83	0.2060302	4.854	0.3116300 05	1.1505100 02	3.8151280 00

Fig. 6.1. (continued).

I= 84	0.2095827	0.795	0.1269780 05	2.0778400 04	1.0598570 01
I= 85	0.2110553	0.794	0.7815630 05	3.1384900 04	1.5820390 01
I= 86	0.2135670	0.682	0.5308850 05	0.1257180 34	2.7944430 00
I= 87	0.2160000	0.629	0.1670030 06	0.7765820 34	1.3499170 01
I= 88	0.2185530	0.575	2.3497225 06	5.1613150 04	0.0288010 00
I= 89	0.2211055	0.523	0.9633020 06	5.3810950 04	2.1184850 01
I= 90	0.2236181	0.872	0.5857950 06	5.4877090 04	2.9253030 00
I= 91	0.2261307	0.822	0.8540810 06	5.4892290 04	2.6133830 01
I= 92	0.2286832	0.178	2.2030560 06	5.3628590 04	1.2225870 00
I= 93	0.2311558	0.326	1.8558895 06	5.0322750 04	2.6204220 00
I= 94	0.2336683	0.280	1.0376210 06	6.3640900 04	1.5016500 01
I= 95	0.2361809	0.238	7.9297510 05	3.3632100 04	0.4258080 00
I= 96	0.2386935	0.199	6.8235750 05	2.8389720 04	1.0561020 00
I= 97	0.2412060	0.166	5.8543390 05	1.8827085 04	6.9980760-01
I= 98	0.2437186	0.193	6.9116580 05	1.8899490 04	2.0177770 00
I= 99	0.2462312	0.061	6.3781710 05	1.2751930 34	9.9864730-01
I= 100	0.2487837	0.029	0.0880199 05	1.1369370 04	0.2855120-01
I= 101	0.2512563	0.000	3.9846605 05	1.0893820 04	2.5778050-01
I= 102	0.2537588	0.001	3.7497105 05	9.8897610 03	1.4886670-01
I= 103	0.2562814	0.002	3.6571300 05	9.8747120 03	0.0095480 00
I= 104	0.2587940	0.000	3.5906660 05	9.1868305 03	2.9566940 00
I= 105	0.2613065	0.027	3.5388270 05	8.9895750 03	2.2566910 00
I= 106	0.2638191	0.000	3.4550105 05	9.8639330 03	2.7046790 00
I= 107	0.2663317	0.055	3.4537100 05	8.9051280 03	2.3813930 00
I= 108	0.2688842	0.026	3.4205390 05	8.8219760 03	3.1528050 00
I= 109	0.2713568	0.005	3.4218060 05	8.9390010 03	1.3266620-01
I= 110	0.2738693	0.051	3.4881500 05	9.2038700 03	2.5109390-01
I= 111	0.2763819	0.018	3.5188880 05	9.7803980 03	5.5535200-01
I= 112	0.2788945	0.006	3.6575610 05	1.0592810 04	1.4502900 00
I= 113	0.2814070	0.056	3.9072570 05	1.2276060 04	3.1117690 00
I= 114	0.2839196	0.022	4.3206820 05	1.5571980 04	6.2184670 00
I= 115	0.2864322	0.001	4.9995600 05	2.0123950 04	8.1827250 00
I= 116	0.2889447	0.061	6.1877990 05	5.4875770 04	3.7818010 00
I= 117	0.2914573	0.031	8.2299630 05	1.0066350 05	9.9359230 01
I= 118	0.2939698	0.002	1.2513880 06	1.0794550 05	6.5896610 01
I= 119	0.2964824	0.073	2.3587180 06	1.0669650 05	6.9060830 00
I= 120	0.2989950	0.305	6.7126190 06	1.0992390 05	2.6991560 01
I= 121	0.3015075	0.317	2.5836670 07	1.0915800 05	3.2069510 01
I= 122	0.3040201	0.289	5.8917100 06	1.0782990 05	6.9862120 01
I= 123	0.3065327	0.262	1.6822990 06	1.0557290 05	5.3893370 01
I= 124	0.3090452	0.238	6.9551740 05	1.0166330 05	2.0966230 01
I= 125	0.3115578	0.219	3.4779060 05	9.1371400 04	1.8867290 01
I= 126	0.3140704	0.198	2.2197210 05	5.1355730 04	6.8681170 00
I= 127	0.3165829	0.169	1.4639100 05	1.3787480 04	2.1385500 00
I= 128	0.3190955	0.138	1.0299260 05	6.0708920 03	2.2825570 00
I= 129	0.3216080	0.109	7.6176680 04	3.5030270 03	0.1195830-01
I= 130	0.3241206	0.085	5.8678750 04	2.3182590 03	1.1793990-01
I= 131	0.3266332	0.062	4.6787960 04	1.6625200 03	0.5213990-01
I= 132	0.3291457	0.039	3.8350280 04	1.2667120 03	1.8809950-01
I= 133	0.3316583	0.015	3.2290020 04	1.0094080 03	0.7900020-02
I= 134	0.3341709	2.992	2.7826430 04	0.3832710 02	0.8901330-02
I= 135	0.3366834	2.970	2.4090980 04	7.1107030 02	2.8618620-02
I= 136	0.3391960	2.948	2.1981970 04	6.2251100 02	1.9682820-01
I= 137	0.3417085	2.926	2.0101280 04	5.5822810 02	1.2295780-01
I= 138	0.3442211	2.905	1.8703590 04	5.1169920 02	8.0182310-02
I= 139	0.3467337	2.888	1.7695470 04	4.7471120 02	5.4565690-02
I= 140	0.3492462	2.883	1.7013320 04	4.5651500 02	3.8730060-02
I= 141	0.3517588	2.883	1.6613030 04	4.4332950 02	2.8572220-02
I= 142	0.3542714	2.883	1.6468570 04	4.3803580 02	2.1625770-02
I= 143	0.3567839	2.883	1.6566550 04	4.3999410 02	3.2200960-03
I= 144	0.3592965	2.883	1.6900330 04	4.4992250 02	1.0867810-02
I= 145	0.3618090	2.868	1.7388870 04	4.6875660 02	3.1775830-03
I= 146	0.3643216	2.785	1.8333180 04	4.9754260 02	2.2865120-01
I= 147	0.3668342	2.726	1.9659800 04	5.3784850 02	2.2095110-02
I= 148	0.3693467	2.707	2.0890280 04	5.7583280 02	8.0200280-02
I= 149	0.3718593	2.693	2.2667630 04	5.9782340 02	1.4058780-01
I= 150	0.3743719	2.691	2.4702320 04	6.8677450 02	1.2516370-01
I= 151	0.3768844	2.695	2.7166680 04	6.9955130 02	4.8218820-02
I= 152	0.3793970	2.696	2.9885650 04	7.5376490 02	1.6762780-01
I= 153	0.3819095	2.699	3.2877260 04	8.0668300 02	1.3258280-01
I= 154	0.3844221	2.691	3.5861390 04	8.5552570 02	2.0902120-02
I= 155	0.3869347	2.688	3.7983000 04	8.9787910 02	2.7078780-01
I= 156	0.3894472	2.564	3.9860810 04	9.3191810 02	3.5811080-02
I= 157	0.3919598	2.551	4.1017230 04	9.5650240 02	3.4568990-02
I= 158	0.3944724	2.515	4.1358880 04	9.7126850 02	5.9203140-02
I= 159	0.3969849	2.519	4.0970150 04	9.7665100 02	2.6817920-01
I= 160	0.3994975	2.501	4.0071810 04	9.7393990 02	3.8326070-01
I= 161	0.4020101	2.849	3.8930250 04	9.6531810 02	9.6926800-02
I= 162	0.4045226	2.872	3.7717750 04	9.5375340 02	6.6170930-02
I= 163	0.4070352	2.857	3.6679150 04	9.4274160 02	2.2868990-02
I= 164	0.4095477	2.882	3.5929630 04	9.3599160 02	2.0871860-02
I= 165	0.4120603	2.827	3.5567630 04	9.3719150 02	3.7384810-01
I= 166	0.4145729	2.812	3.5672580 04	9.5000960 02	3.6165570-02
I= 167	0.4170854	2.798	3.6322700 04	9.7831070 02	8.8229800-02
I= 168	0.4195980	2.783	3.7612720 04	1.0267220 03	1.5262170-01
I= 169	0.4221106	2.769	3.9669870 04	1.0814820 03	2.5853100-01
I= 170	0.4246231	2.755	4.2678780 04	1.2114800 03	8.3880830-01
I= 171	0.4271357	2.781	4.6912880 04	1.3710730 03	5.7835200-02
I= 172	0.4296482	2.727	5.2780230 04	1.6030070 03	1.0796600-01
I= 173	0.4321608	2.318	6.0911110 04	1.9863860 03	2.1084260-01
I= 174	0.4346734	2.301	7.2295690 04	2.4704700 03	8.2321580-01
I= 175	0.4371859	2.287	8.9585670 04	3.3068060 03	8.0526060-01
I= 176	0.4396985	2.278	1.1238180 05	4.6981850 03	9.2953860-01
I= 177	0.4422111	2.261	1.4860160 05	7.0855700 03	2.4620160 00

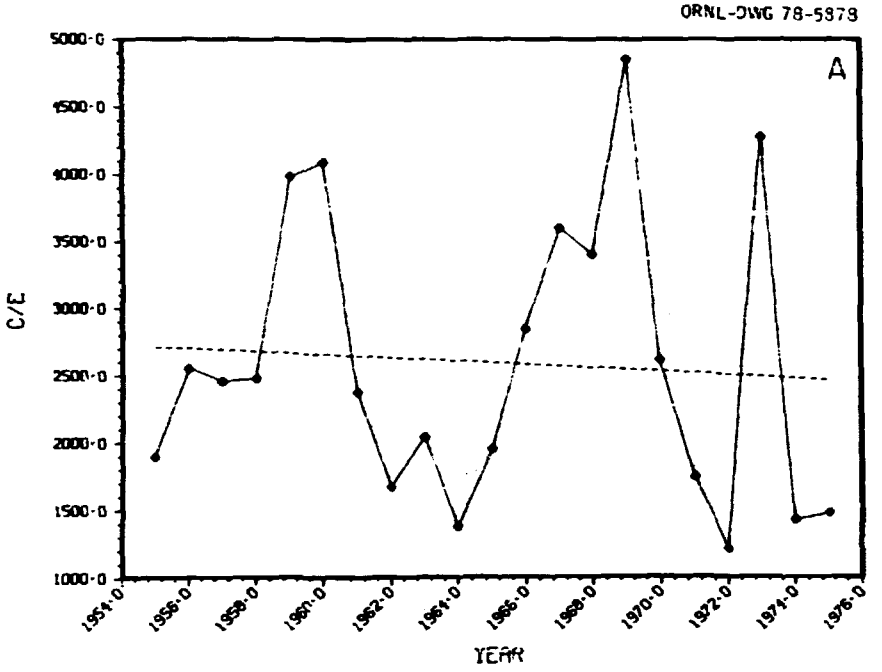
(continued)

I= 178	0.8887236	2.289	2.0609380 05	1.0615260 06	2.8002990 00
I= 179	0.8872362	2.234	7.0202180 05	1.0080480 06	8.2698090 00
I= 180	0.8897887	2.223	8.7013560 05	1.8005200 06	8.9619700-01
I= 181	0.8922613	2.219	7.7016590 05	2.0819710 06	8.9751370-01
I= 182	0.8947739	2.199	1.2069090 06	2.2986980 06	3.0827990 00
I= 183	0.8972660	2.187	1.6772260 06	2.3189600 06	1.7279520 01
I= 184	0.8997980	2.175	1.5978580 06	2.3825040 06	8.8269140 00
I= 185	0.9023116	2.163	1.2098100 06	2.8005270 06	2.2630820 00
I= 186	0.9048281	2.151	8.8831370 05	2.3633150 06	1.8278980-01
I= 187	0.9073367	2.140	6.7889180 05	2.2280560 06	7.2515980-01
I= 188	0.9098492	2.128	5.5726970 05	1.9702710 06	6.1982940 00
I= 189	0.9123618	2.117	6.8836090 05	1.6867050 06	7.2982220 00
I= 190	0.9148788	2.106	8.5878970 05	1.5036460 06	2.3197020-01
I= 191	0.9173869	2.095	8.8832510 05	1.8607590 06	2.0236700 00
I= 192	0.9198995	2.084	8.6728060 05	1.5831360 06	3.8753810 00
I= 193	0.9224121	2.073	5.1588700 05	1.9230020 06	7.8093200-01
I= 194	0.9249286	2.062	6.0409850 05	2.6819980 06	1.4901810 01
I= 195	0.9274372	2.052	7.5776390 05	3.8759860 06	2.0946720 01
I= 196	0.9299497	2.041	1.0292220 06	0.0	0.0
I= 197	0.9324623	2.031	1.5373810 06	0.0	0.0
I= 198	0.9349749	2.020	2.5513110 06	0.0	0.0
I= 199	0.9374874	2.010	4.4525220 06	0.0	0.0
I= 200	0.5000000	2.000	5.9989250 06	0.0	0.0

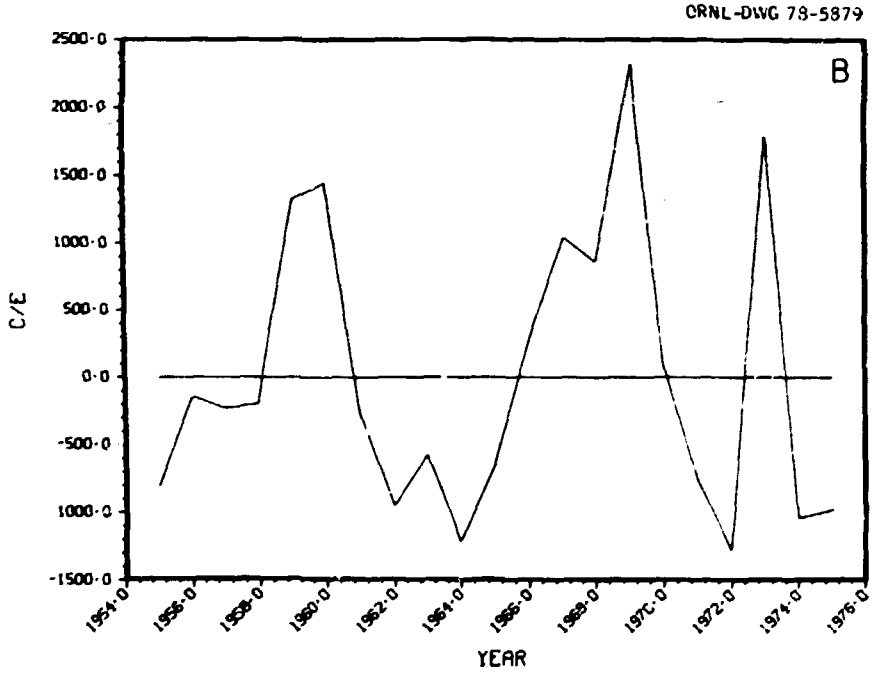
FINAL PREDICTION ERROR

I	NUMBER OF FILTER COEFFICIENTS	FINAL PREDICTION ERROR
1	2	1.3638800 06
2	3	1.8679190 06
3	4	1.5753000 06
4	5	1.7316700 06
5	6	1.0635700 06
6	7	1.1832100 06
7	8	1.2585900 06
8	9	1.3729300 06
9	10	1.1436700 06
10	11	1.2879000 06
11	12	1.3103900 06
12	13	1.5152100 06
13	14	1.8073300 06
14	15	2.2138800 06
15	16	2.7763700 06
16	17	3.7868500 06
17	18	5.5756400 06
18	19	1.1030300 07

Fig. 6.1. (continued).

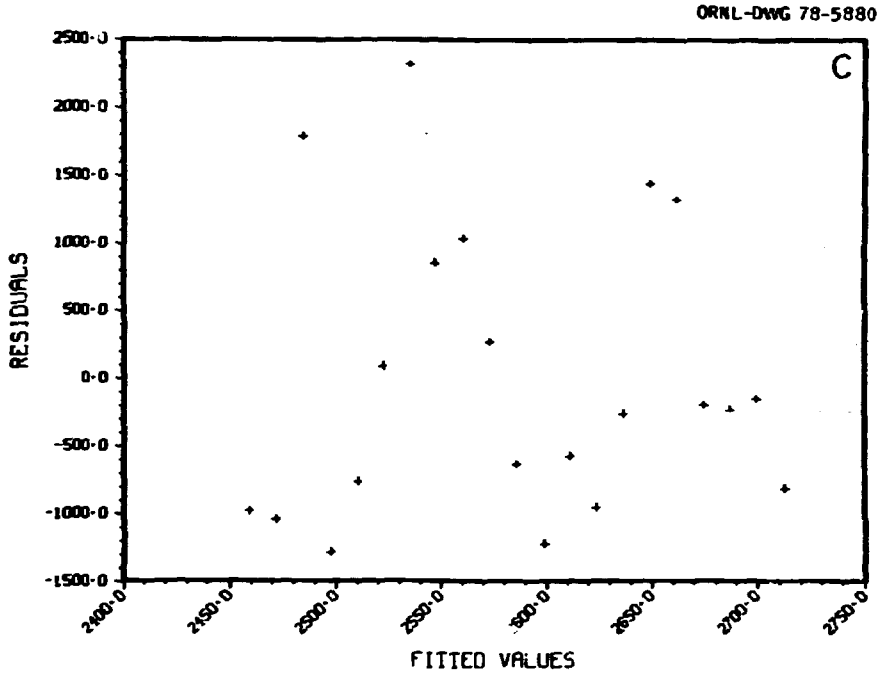


A. Hudson River catch per unit effort data; dahsed line represents the linear trend (Source for the time series: Texas Instruments, Inc., 1977, p. 76, Table 7.2-1).

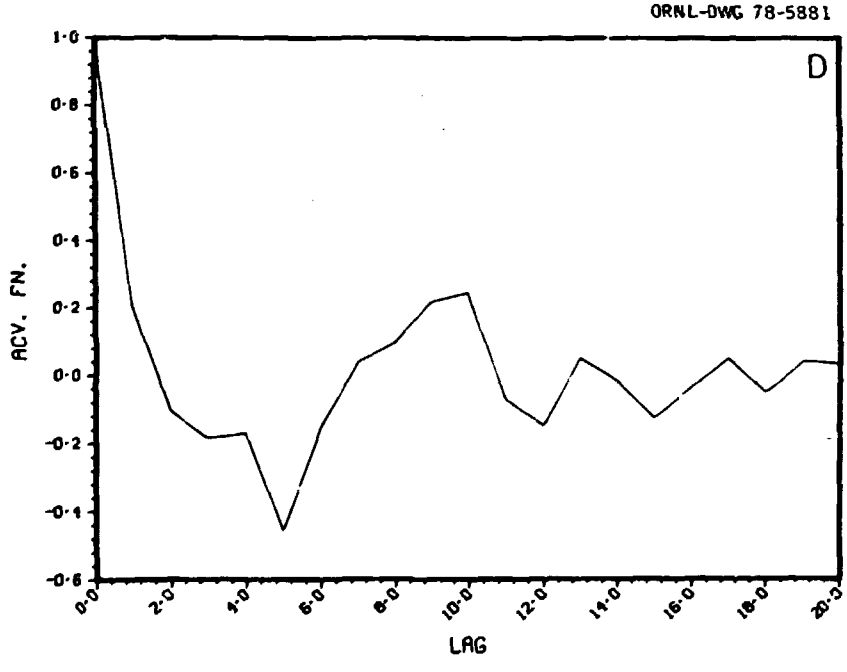


B. Detrended Hudson River catch per unit effort; solid line represents the axis y = 0.

Fig. 6.2. Analysis of the Hudson River catch per unit effort data.

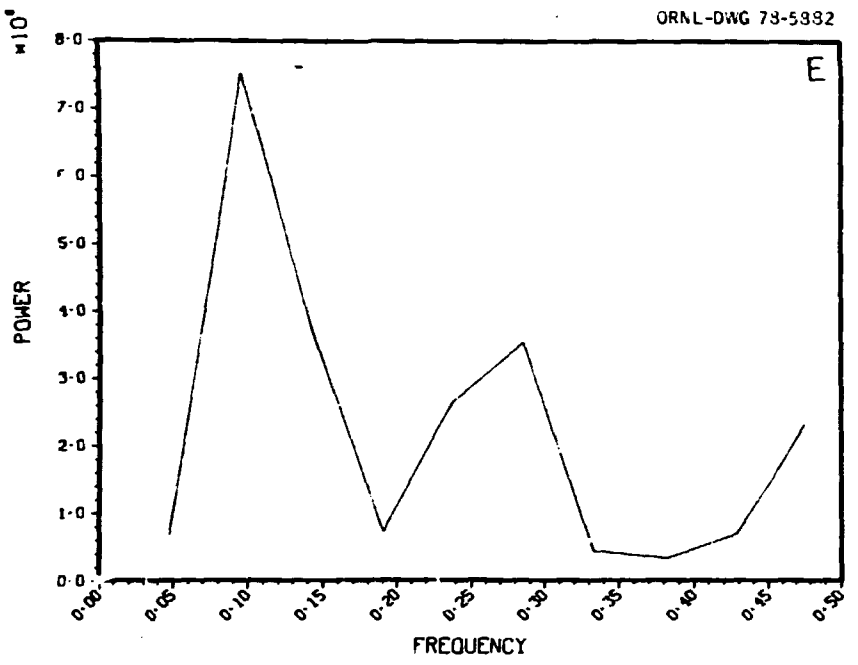


C. Residuals versus fitted values.

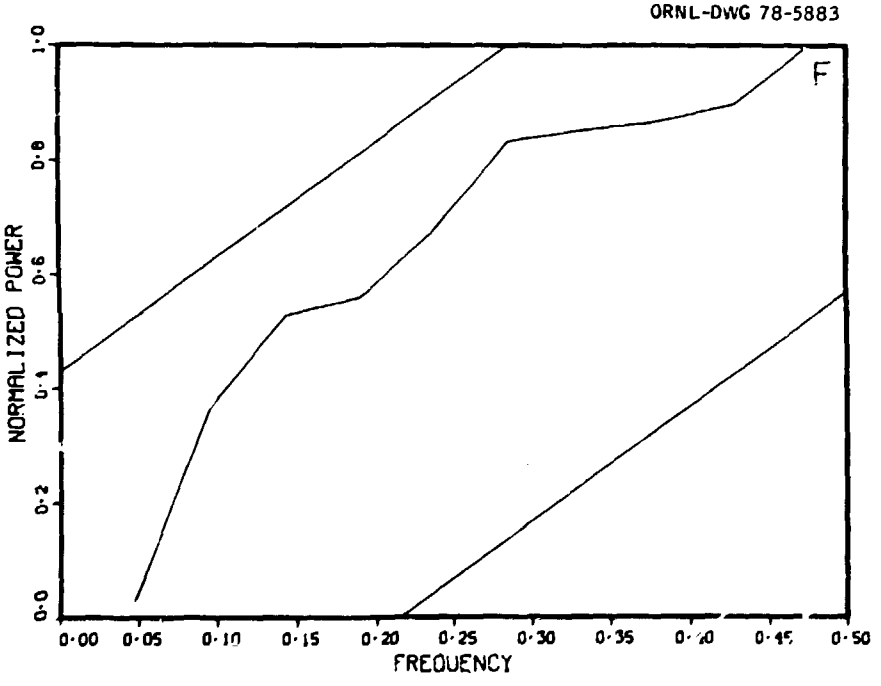


D. Yule - Walker autocorrelation function.

Fig. 6.2. (continued).

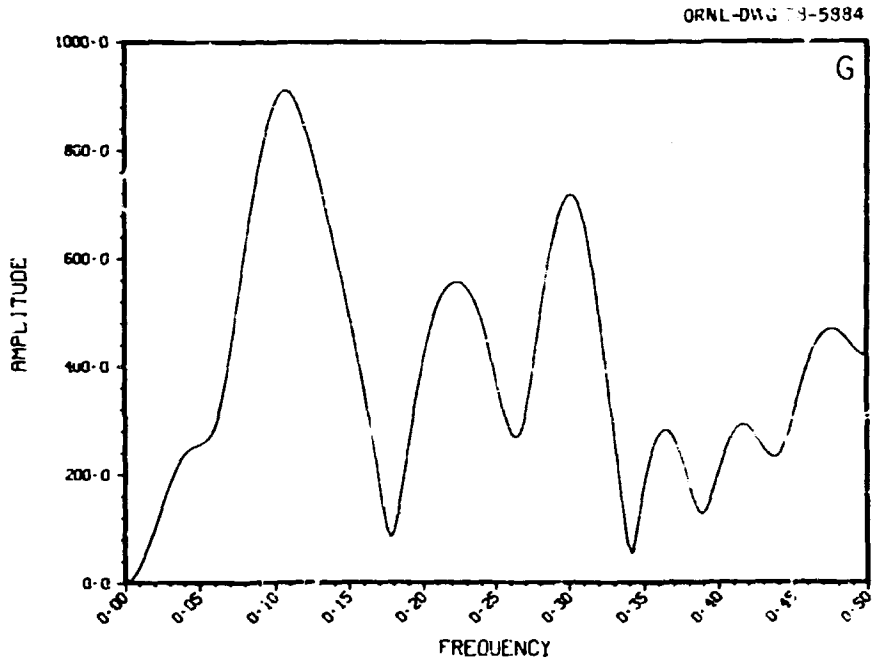


E. Periodogram.

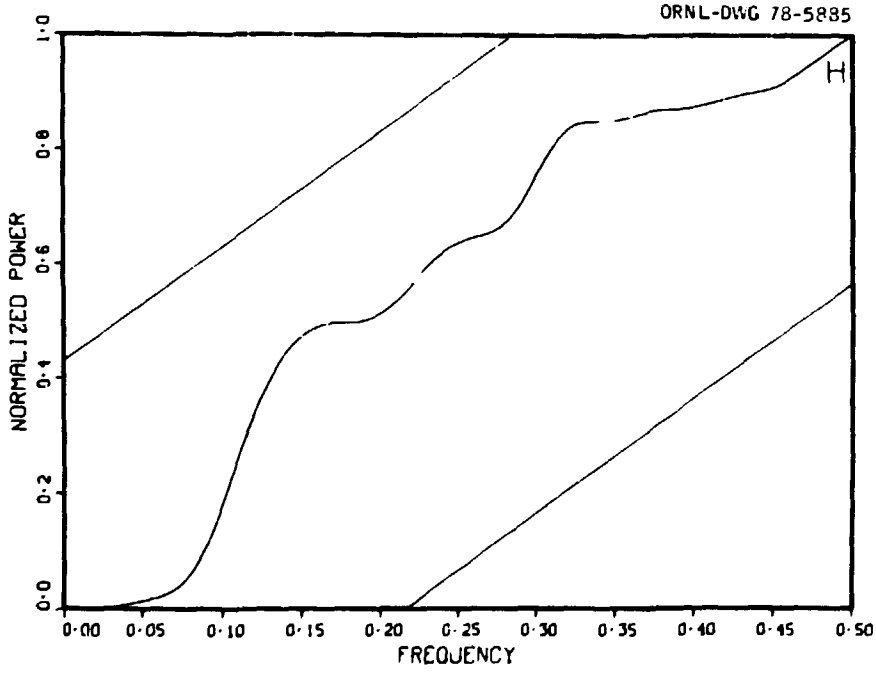


F. Cumulative periodogram.

Fig. 6.2. (continued).

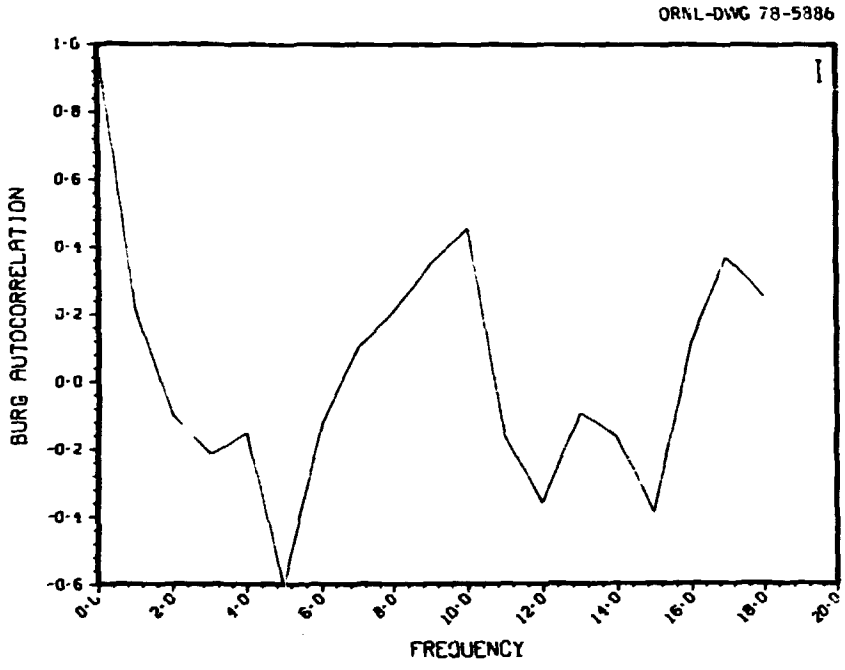


G. Fourier amplitude spectrum.

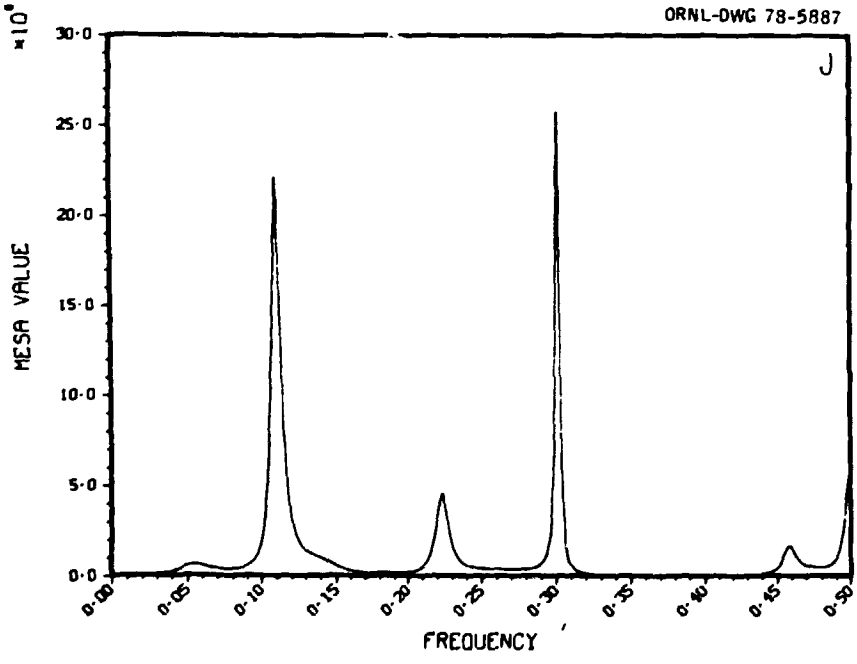


H. Cumulative Fourier power spectrum.

Fig. 6.2. (continued).

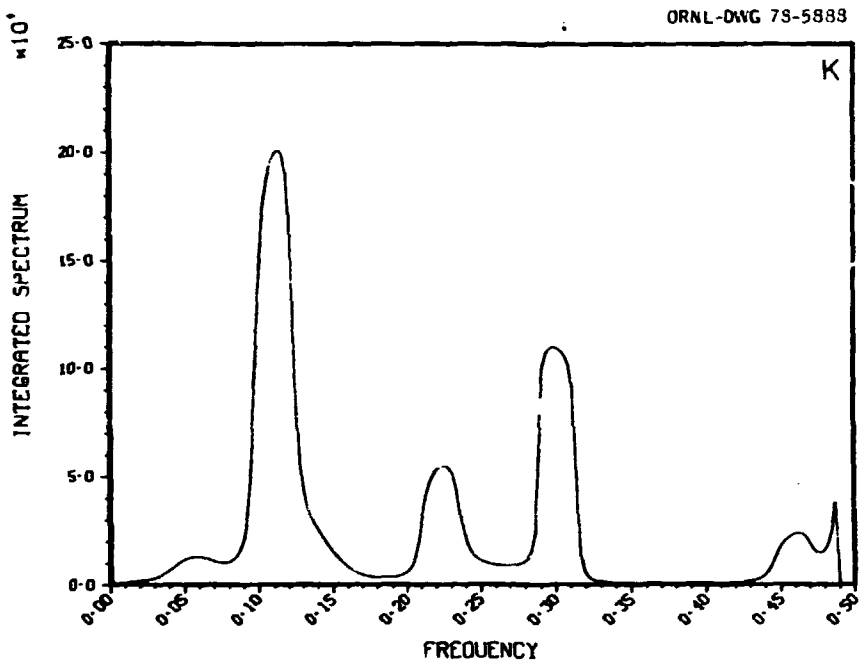


I. Burg autocorrelation estimate.

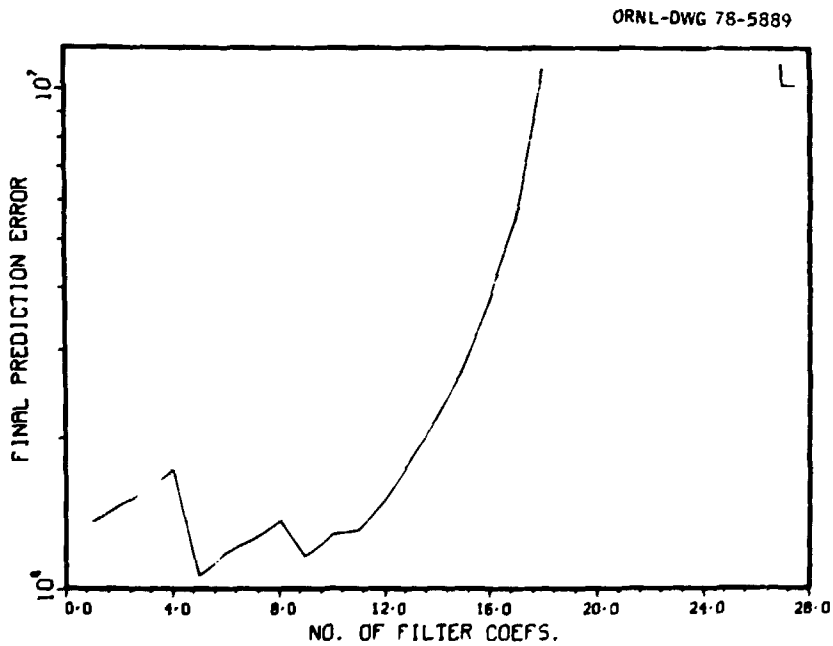


J. Maximum entropy spectrum with M = 18.

Fig. 6.2. (continued).



K. Integrated maximum entropy spectrum.



L. Final prediction error

Fig. 6.2. (continued).

B. Setting up the Program for Execution

The time it takes to run the program depends on many factors, primarily the length of the time series, the number of frequencies used, the number of times MESA is performed and the use of other options. Table 6.2 lists a few examples indicating the time and core used. Although the list is short, it is hoped that it will serve as a guide in the classification of a program run. All the data were linearly detrended.

For users at ORNL, Table 6.3 is a typical setup of the job control cards for a Class C job with disks used for plotting. Similarly, Table 6.4 is a setup for a class E job with the use of tapes in plotting. The job initials BLKL (wherever they appear) should be changed to that of the user's.

The inclusion of the following cards is explained:

1. "//_DD_DSN = SYS2.DISSPLA,DISP = SHR ..... "gives access to the plotting routines.
2. "//_DD_DSNAME = BROOKS.LOAD.MODULES ....." gives access to the BLSQ.
3. "//_DD_DSNAME = JDAIMSL1.REF 10276 ....." gives access to DCADRE.

Table 6.2. List of programs runs; time and core used.

Number of points in time series	IPRINT	IPLOT	ITREND	IFPEPL	IYWACV	IINTSP	IAUTCV	Number of frequencies	Mcount values	Logarithmic transform analyzed	Computer time and core used
21	yes	yes	yes	no	yes	no	no	600	8-17	yes	1.04 minutes 286 K
21	yes	yes	yes	yes	no	no	no	600	19	yes	23.90 seconds 286 K
31	yes	yes	yes	no	yes	yes	no	501	15,16	no	41.88 seconds 280 K
42	yes	yes	yes	no	no	no	no	600	40	yes	28.37 seconds 320 K
70	yes	yes	yes	no	no	no	no	201	30	no	11.35 seconds 280 K
114	yes	yes	yes	no	no	no	no	600	69	no	21.26 seconds 286 K
260	yes	yes	yes	no	no	no	no	600	109 107	no	35.46 seconds 290 K
277	yes	yes	yes	no	no	no	no	600	160	yes	49.78 seconds 320 K
277	yes	yes	yes	yes	no	no	no	600	275	yes	1.06 minutes 320 K

Table 6.3. Job setup for disk plotting

```
//BLKL JOB (16064),'ENV_2001_X-10_KIRK'
//*CLASS_CPU91=5M,REGION=540
//_EXEC_FORTHCLG,PARM.FORT='XREF',
//_PARM.GO='TIME=5.0,EU=-1,DUMP=-I,SO=51',REGION.GO=540K
//FORT.SYSIN_DD_*
.
.
.
source deck
.
.
.
/*
//LKED.SYSLIB_DD
//_DD
//_DD
//_DD_DSN = SYS2.DISSPLA,DISP=SHR
//_DD_DSNAME=BROOKS.LOAD.MODULES,UNIT=2314,
//_VOLUME=SER=ZZZZZZ,DISP=SHR
//_DD_DSNAME=JDAIMSL1.REF10276,UNIT=2314,VOL=SER=CADPK1,DISP=SHR
//LKED.PLOTSUBS_DD_DSN=JGSPLOTH,DISP=SHR
//LKED.SYSIN_DD_*
_INCLUDE_PLOTSUBS
/*
```

Table 6.3. (continued)

```
//GO.FT49F001 DD UNIT=IN2OU2,DISP=(NEW,KEEP),
//  SPACE=(3208,40,RLSE),DSN=PLOTOO.BLKL,
//  DCB=(RECFM=VS,LRECL=3204,BLKSIZE=3208)
//GO.FT05F001 DD *
.
.
.
data cards
.
.
.
/*
//
```

Table 6.4. Job setup for tape plotting

```
//*NOSEQCARD
//BLKL_JOB_(16064),'ENV_2001_X-10_KIRK'
//*CLASS_CPU91=5M,REGION=540,LINES=40,CARDS=10,SPECIAL=TAPE
//*PLOT_ _NUMBER=30
//_EXEC_FORTHCLG,PARM.FORT='XREF',
//_PARM.GO='TIME=5.0,EU=-1,DUMP=I,SO=51',REGION.GO=540K
//FORT.SYSIN_DD *
.
.
.
source deck
.
.
.
/*
//LKED.SYSLIB_DD
// DD
// DD
//_DD_DSN=SYS2.DISSPLA,DISP=SHR
//_DD_DSNAME=BROOKS.LOAD.MODULES,UNIT=2314,
//_VOLUME=SER=ZZZZZZ,DISP=SHR
//_DD_DSNAME=JDAIMSL1.REF10276,UNIT=2314,VOL=SER=CADPK1,DISP=SHR
//GO.PLOTTAPE_DD_UNIT=TAPE7,LABEL=(,NL),DISP=OLD,VOLUME=SER=15
//GO.FT05F001_DD_*
```

Table 6.4. (continued)

```
.
.
.
data cards
.
.
.
/*
//
```

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APPENDICES

APPENDIX I

DESCRIPTION OF FUNCTION DCADRE*

```
    FUNCTION   DCADRE  (F,A,B,AERR,RERR,ERROR,IER)

DCADRE--------S/D------LIBRARY 1-----------------------------------------

    FUNCTION            - INTEGRATE F(X) FROM A TO B, USING CAUTIOUS
                          ADAPTIVE ROMBERG EXTRAPOLATION.
    USAGE               - FUNCTION DCADRE (F,A,B,AERR,RERR,ERROR,IER)
    PARAMETERS   DCADRE - ESTIMATE OF THE INTEGRAL OF F(X) FROM A TO B.
                 F      - A SINGLE-ARGUMENT REAL FUNCTION SUBPROGRAM
                          SUPPLIED BY THE USER.  F MUST BE DECLARED
                          EXTERNAL IN THE CALLING PROGRAM.
                 A,B    - THE TWO ENDPOINTS OF THE INTERVAL OF
                          INTEGRATION.  (INPUT)
                 AERR   - DESIRED ABSOLUTE ERROR IN THE ANSWER.  (INPUT)
                 RERR   - DESIRED RELATIVE ERROR IN THE ANSWER.  (INPUT)
                 ERROR  - ESTIMATED BOUND ON THE ABSOLUTE ERROR OF THE
                          OUTPUT NUMBER, DCADRE.
                 IER    - ERROR PARAMETER
                          WARNING ERROR(WITH FIX) = 64 + N
                            N = 1 IMPLIES THAT ONE OR MORE SINGULARITIES
                              WERE SUCCESSFULLY HANDLED.
                            N = 2 IMPLIES THAT, IN SOME SUBINTERVAL(S),
                              THE ESTIMATE OF THE INTEGRAL WAS ACCEPTED
                              MERELY BECAUSE THE ESTIMATED ERROR WAS
                              SMALL, EVEN THOUGH NO REGULAR BEHAVIOR
                              WAS RECOGNIZED.
                          TERMINAL ERROR = 128 + N
                            N = 3 -- FAILURE DUE TO INSUFFICIENT
                              INTERNAL WORKING STORAGE.
                            N = 4 -- FAILURE.  THIS MAY BE DUE TO TOO
                              MUCH NOISE IN THE FUNCTION (RELATIVE
                              TO THE GIVEN ERROR REQUIREMENTS) OR
                              DUE TO AN ILL-BEHAVED INTEGRAND.
                            N = 5 INDICATES THAT RERR IS GREATER THAN
                              0.1, OR RERR IS LESS THAN 0.0, OR RERR
                              IS TOO SMALL FOR THE PRECISION OF THE
                              MACHINE.
    PRECISION           - SINGLE/DOUBLE
    REQD. IMSL ROUTINES - UERTST
    LANGUAGE            - FORTRAN
-------------------------------------------------------------------------
```

*Reprinted from IMSL LIBRARY 1. Reference Manual. Edition 4 (Fortran IV) S/370 - 360, 1975.

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FUNCTION DCADRE(F,A,B,AERR,RERR,ERROR,IER)

Purpose

DCADRE attempts to solve the following problem: Given the name F of a real function subprogram, two real numbers A and B, and two non-negative numbers AERR and RERR, find a number DCADRE such that

$$\left| \int_A^B F(x)dx - DCADRE \right| \quad \max \left(AERR, RERR * \left| \int_A^B F(x)dx \right| \right)$$

Algorithm

This routine uses a scheme whereby DCADRE is computed as the sum of estimates for the integral of F(x) over suitably chosen subintervals of the given interval of integration. Starting with the interval of integration itself as the first such subinterval, cautious Romberg extrapolation is used to find an acceptable estimate on a given subinterval. If this attempt fails, the subinterval is divided into two subintervals of equal length, each of which is considered separately.

See reference: de Boor, Carl, "CADRE: An algorithm for numerical quadrature", Mathematical Software (John R. Rice, Ed.), New York, Academic Press, 1971, Chapter 7.

Programming Notes

1. DCADRE can, in many cases, handle jump discontinuities and certain algebraic discontinuities. See reference for full details.

2. The relative error parameter RERR must be in the interval [0,0.1]. For example, RERR=0.1 indicates that the estimate of the integral is to be correct to one digit, whereas RERR=10 calls for four digits of accuracy. If DCADRE determines that the relative accuracy requirement cannot be satisfied, IER is set to 133 (RERR should be large enough that, when added to 100.0, the result is a number greater than 100.0).

3. The absolute error parameter, AERR, should be nonnegative. In order to give a reasonable value for AERR, the user must know the approximate magnitude of the integral being computed. In many cases it is satisfactory to use AERR=0. In this case, only the relative error requirement is satisfied in the computation.

4. We quote from the reference, "A very cautious man would accept DCADRE only if IER is 0 or 65. The merely reasonable man would keep the faith even if IER is 66. The adventurous man is quite often right in accepting DCADRE even if IER is 131 or 132". Even when IER≠0, DCADRE returns the best estimate that has been computed.

APPENDIX II

DESCRIPTION OF SUBROUTINE BLSQ*

Identification

Linear least squares solution.

Purpose

ALSQ and BLSQ are double precision FORTRAN IV subroutines to solve the linear least squares problem by applying the Household'der reduction to the least squares matrix [1]. Both programs use the same technique in solving the least squares problem; however, BLSQ returns additional information useful in statistical applications. Both programs have subordinate entries, ALSQ1, BLSQ1, which enable the user to perform additional fits at very little cost once the least squares matrix has been reduced.

Method

The linear least squares problem to be solved may be formulated as follows: given n x m ($m < n$) matrix A of rank m and an n-vector y find an m-vector b such that

$$||Ab - y||^2 = \min. \qquad (1)$$

The programs, in effect, use the Householder reduction of the matrix A to triangular form to find an n x n orthogonal matrix H such that

---

*Reprinted from Westley and Watts, The Computing Technology Center Numberical Analysis Library. CTC - 39. (Oct. 6, 1970) pp. 383-386.

$$HA = \begin{bmatrix} T \\ 0 \end{bmatrix} , \tag{2}$$

where T is an m x m upper triangular matrix and "0" is an (n - m)xm null matrix. If we let U be the n x m matrix consisting of the first m columns of H^T (in order) and V be the n x (n - m) matrix consisting of the remaining columns and consider the new least squares problem of minimizing $\|Uc - y\|^2$, then

$$c = U^T y \tag{3}$$

and

$$b = T^{-1}c. \tag{4}$$

ALSQ performs the Householder reduction (2) and uses equations (3) and (4) to calculate b. BLSQ calculates and returns the vector c and the matrices T and T^{-1} which are useful in statistical applications. The vector b may subsequently be calculated by the user from equation (4). Both programs return the approximating vector Ab = Uc and the residual sum of squares $\|Ab - y\|^2$.

Under the usual statistical assumptions of regression analyses, it can be shown that the number of runs in $V^T y$ minus one has the binomial distribution with parameters n - m - 1 and .5. If the problem being solved involves fitting continuous functions of a real variable, a statistical test based on the above may be expected to be powerful when the

fit inadequately represents the data. Because of this, BLSQ returns the number of runs in $V^T y$.

Usage

The calling sequence for ALSQ is:

CALL ALSQ(A, Y, B, R2, N, M, NA),

where

A is a doubly subscripted double precision array containing the least squares matrix. The elements of A are altered by the program.

Y is a singly subscripted, double precision array containing the vector to be fit. The elements of Y are unchanged by the program.

B is a singly subscripted, double precision array which upon return contains the coefficients of the fit.

R2 contains, upon return, the double precision residual sum of squares of the fit.

N is the integer number of rows in the least squares matrix.

M is the integer number of columns in the least squares matrix.

NA is the integer first dimension of the array A.

The program requires that the array A be dimensioned so that it will hold one extra row and one extra column in addition to the least squares matrix, i.e., the dimension of A must be at least N + 1 by

M + 1. Upon return from the program, the locations A(I, M + 1) contain the components of the approximating vector Ab (see Method).

A second entry, ALSQ1, allows the user to obtain additional fits without repeating the reduction of the least squares matrix. ALSQ must be called before calling ALSQ1, and the contents of the array A must not be changed between these calls.

The calling sequence is:

CALL ALSQ1(Y, B, R2, K)

where Y, B, and R2 are the same as above and

K is the number of columns of the least squares matrix to fit to the vector Y; e.g., if K = 4, the first four columns of A will be fit to Y returning four coefficients in the array B. K must be less than or equal to M in the call ALSQ.

This entry also returns the approximating vector in the locations A(I, M + 1).

The calling sequence for BLSQ is:

CALL BLSQ(A, Y, C, R2, IRUN, T, T1, N, M, NA, NT),

where A, Y, R2, N, M, and NA are the same as in the calling sequence for ALSQ and

C is a singly subscripted, double precision array which upon return contains the coefficients of the transformed problem given by (3).

IRUN is an integer array whose i^{th} element contains upon return the number of runs in $V^T y$ after i columns of the least squares matrix have been fitted.

T is a doubly subscripted, double precision array which upon return contains the upper triangular transformation matrix of equations (2).

T1 is a doubly subscripted, double precision array which upon return contains the inverse of the transformation matrix.

NT is the first dimension of the arrays T and T1.

BLSQ also has a second entry, BLSQ1, to calculate new fits. The same restrictions apply to the use of BLSQ1 as apply to ALSQ1. The calling sequence is:

CALL BLSQ1(Y, C, R2, IRUN, K),

where Y, C, R2, IRUN are the same as in the calling sequence of BLSQ and K is the number of columns of the least squares matrix to fit to the data. Both BLSQ and BLSQ1 return the components of the approximating vector in the locations A(I, M + 1).

Coding Information

Although these programs are coded in double precision arithmetic, they may be easily converted to single precision. Additional accuracy may be obtained in the single precision program by accumulating inner products in double precision. The places where this should be done are signaled by the appearance of the variables SS and PP.

Reference

(1) Golub, G. (1965): "Numerical methods for solving linear least squares problems," Numer. Math. 7, 206-216.

(2) Wampler, R. H., "An Evaluation of Linear Least Squares Computer Programs," J. of Research of NBS, Vol. 73B, No. 2, p. 59.

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[Editor's Note: For an evaluation of ALSQ in single and double precision see (2). Unfortunately, the single precision version reported in (2) did not accumulate inner products in double precision.]

APPENDIX III

PROGRAM LISTING

```
LEVEL 21.8 ( JAN 76 )                          OS/360  FORTRAN H

        COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                           SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
             C
             C
ISN 0002             IMPLICIT REAL*8(A-H,O-Z)
ISN 0003             REAL*4 HAR,XLBL,YLBL
ISN 0004             COMMON /SPECBL/ DELTA,YMAX(600),Y(600),XD(600),
                    1 VMAX,X(600),V(600),S(600),PPE(600),XDIFF(600),
                    2 DATE(600),X(600),ACS(600),BSH(600),AMP(600),ASP(600),
                    3 PERIOD(600),NPTS,NFILT,NV,NCOUNT(600),IPRINT,IPLOT,ITREND,
                    4 IFFEPL,IINTSP,IAUTOV,IYBACV
ISN 0005             COMMON /BLKLBL/HAN(18),XLBL(2),YLBL(2)
             C
             C
             C
             C       INITIALIZE VALUES.
             C
             C  XLABEL = IDENTIFIER (LABEL) FOR TIME AXIS  (A8)
             C
             C  YLABEL = IDENTIFIER FOR TIME SERIES DATA  (A8)
             C
             C  DELTA  = SAMPLING TIME INTERVAL
             C
             C  NPTS   = NUMBER OF DATA POINTS IN TIME SERIES
             C
             C  NFILT  = TOTAL NUMBER OF TIMES MAXIMUM ENTROPY IS
             C           TO BE DONE WITH DIFFERENT VALUES FOR
             C           THE NUMBER OF FILTER COEFFICIENTS
             C
             C  NV     = NUMBER OF FREQUENCIES NEEDED IN OUTPUT
             C
             C  YMAX   = MAXIMUM FREQUENCY FOR OUTPUT SPECTRUM
             C           (SHOULD NOT EXCEED THE NYQUIST FREQUENCY).
             C
             C  HAN    = TITLE FOR THE DATA SET  (A80)
             C           (TITLE PRINTED AT TOP OF THE PLOTS.
             C
             C
             C
             C
             C
ISN 0006         1 CONTINUE
             C
ISN 0007         READ (5,20,END=500) XLBL,YLBL,DELTA,NPTS,NFILT,NV,YMAX
ISN 0008        20 FORMAT (2A4,2X,2A4,2X,E10.0,3I5,E10.0)
ISN 0009         READ (5,21,END=500) HAN
ISN 0010        21 FORMAT (18A4)
             C
             C
             C    READ NCOUNT. NCOUNT IS THE ARRAY OF NUMBERS WHICH CORRESPOND TO THE
             C    NUMBER OF FILTER COEFFICIENTS DESIRED IN THE MAXIMUM ENTROPY.
             C
             C
ISN 0011         READ 40, (NCOUNT(I),I=1,NFILT)
ISN 0012        40 FORMAT (16I5)
             C
             C
```

```
C
C
C
C  READ THE DATA TO BE ANALYZED
C
C  DATE   = TIME ARRAY
C
C  Y      = TIME SERIES TO BE ANALYZED
C
C           THE DATA CONSISTS OF A SERIES OF POINTS
C              ( YEAR(I),Y(I) ), I=1,NPTS.
C
C
ISN 0013        DO 80 I=1,NPTS
ISN 0014        READ 60,DATE(I),Y(I)
ISN 0015     60 FORMAT (F8.0,F10.0)
ISN 0016     80 CONTINUE
C
C
C       READ VALUES WHICH WILL GIVE CERTAIN OPTIONS.
C
C       1 MEANS 'YES'.
C       0 MEANS 'NO'.
C
C       IPRINT = OPTION TO PRINT.
C       IPLOT = OPTION TO PLOT.
C       ITREND = OPTION TO DE-TREND.
C       IFPEPL = OPTION TO PRINT/PLOT FINAL PREDICTION ERROR
C       IINTSP = OPTION TO COMPUTE INTEGRATED SPECTRUM
C       IAUTCV = OPTION TO PLOT AUTOCOVARIANCE FUNCTION
C
C       IYWACV = OPTION TO COMPUTE AND PLOT YULE-WALKER
C                ESTIMATE OF AUTOCOVARIANCE FUNCTION
C
C
ISN 0017        READ 100, IPRINT,IPLOT,ITREND,IFPEPL,IINTSP,IAUTCV,IYWACV
ISN 0018    100 FORMAT (7I5)
C
C
ISN 0019        CALL TRANS1
ISN 0020        CALL TRANS2
ISN 0021        CALL TRANS3
ISN 0022        CALL TRANS4
ISN 0023        CALL TRANS5
C
C
ISN 0024        GO TO 1
ISN 0025    500 CALL DOFEPL
ISN 0026        STOP
ISN 0027        END
```

*****F O R T R A N C R O S S R E F E R E N C E L I S T I N G*****

SYMBOL	INTERNAL STATEMENT NUMBERS					
I	0011	0011	0011	0013	0014	0014
S	0004					
V	0004					
X	0004					
Y	0004	0014				
Z	0004					
EV	0004	0007				
ID	0004					
ACS	0004					
ASP	0004					
ASP	0004					
BSE	0004					
FPL	0004					
HAH	0003	0005	0007			
DATE	0004	0014				
HPTS	0004	0007	0013			
VEAX	0004	0007				
XLBL	0003	0005	0007			
YEAR	0004					
YLBL	0003	0005	0007			
DELTA	0004	0007				
IPLOT	0004	0017				
HPYLT	0004	0007	0011			
ID:PP	0004					
DO:EPL	0025					
IAL:TCV	0004	0017				
IPP:PL	0004	0017				
IIE'SP	0004	0017				
IPRIb/	0004	0017				
ITREND	0004	0017				
IYEACV	0004	0017				
HCOEET	0004	0011				
PERIOD	0004					
TRANS1	0019					
TRANS2	0020					
TRANS3	0021					
TRANS4	0022					
TRANS5	0023					

*****F O R T R A N C R O S S R E F E R E N C E L I S T I N G*****

LABEL	DEFINED	REFERENCES
1	0006	0024
20	0004	0007
21	0010	0009
40	0012	0011
60	0015	0014
80	0016	0013
100	0018	0017
900	0025	0007 0009

```
LEVEL 21.8 ( JUN 7( )                          OS/360  FORTRAN H

         COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                            SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
                C
ISN 0002                SUBROUTINE TRANS1
                C
ISN 0003                IMPLICIT REAL*8 (A-H,O-Z)
ISN 0004                REAL*4 NAM,XLBL,YLBL
                C
ISN 0005                COMMON /SPECBL/ DEL' \,YEAR(600),Y(600),XD(600),
                       1 YEAX,X(600),V(600),S(600),PPE(600),XDIFF(600),
                       2 DATE(600),X(600),ACS(600),BSE(600),AMP(600),ASP(600),
                       3 PERIOD(600),NPTS,NFILF,NV,NCOUNT(600),IPRINT,IPLOT,ITREND,
                       4 IPPEPL,IINTSP,IAUTCV,IYEACV
ISN 0006                COMMON /BLKLBL/NAM(10),XLBL(2),YLBL(2)
                C
                C
ISN 0007                ILOG = 0
ISN 0008                PRINT 10
ISN 0009             10 FORMAT (1H1,'ANALYSIS OF THE ORIGINAL INPUT ',
                       1 'DATA WITHOUT ANY PRIOR TRANSFORMATIONS'/)
                C
ISN 0010                CALL SPECTR
                C
                C
ISN 0011                RETURN
                C
                C
ISN 0012                ENTRY TRANS2
ISN 0013                ILOG = 1
ISN 0014                PRINT 30
ISN 0015             30 FORMAT (1H1,'ANALYSIS OF THE LOGARITHM OF THE ORIGINAL DATA'//)
ISN 0016                DO 50 I=1,NPTS
ISN 0017                IF (Y(I).EQ.0.) Y(I) = 1.
ISN 0019                Y(I) = DLOG(Y(I))
ISN 0020             50 CONTINUE
ISN 0021                CALL SPECTR
ISN 0022                RETURN
                C
                C
ISN 0023                ENTRY TRANS3
                C
ISN 0024                RETURN
                C
                C
ISN 0025                ENTRY TRANS4
                C
ISN 0026                RETURN
                C
                C
ISN 0027                ENTRY TRANS5
                C
ISN 0028                RETURN
                C
                C
ISN 0029                END
```

```
                    ******F O R T R A N   C R O S S   R E F E R E N C E   L I S T I N G******

SYMBOL  INTERNAL STATEMENT NUMBERS
I       0016  0017  0017  0019  0019
S       0005
V       0005
X       0005
Y       0005  0017  0017  0019  0019
Z       0005
BV      0005
XD      0005
ACS     0005
ASP     0005
ASP     0005
BSU     0005
FPR     0005
BAR     0004  0006
DATE    0005
DLOG    0019
ILOG    0007  0013
NPTS    0005  0016
VMAX    0005
XLBL    0004  0006
TEAR    0005
YLBL    0004  0006
DELTA   0005
IPLOT   0005
NFILT   0005
IDIFF   0005
IAUTCV  0005
IYPERL  0005
IINTSP  0005
IPRINT  0005
ITREND  0005
IYVACV  0005
NCOUNT  0005
PERIOD  0005
SPECTR  0010  0021
TRANS1  0002
TRANS2  0012
TRANS3  0023
TRANSM  0025
TRANSS  0027
```

```
******F O R T R A N   C R O S S   R E F E R E N C E   L I S T I N G******

LABEL   DEFINED     REFERENCES
   10      0009     0008
   30      0015     0014
   50      0020     0016
```

```
LEVEL 21.8 ( JUN 78 )                    OS/360  FORTRAN H

         COMPILER OPTIONS - NAME= MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                            SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
                     C
                     C
 ISN 0002                  SUBROUTINE SPECTR
                     C
                     C          MAXIMUM ENTROPY SPECTRAL ANALYSIS SUBROUTINE
                     C
                     C
                     C
                     C       ACS     = VECTOR OF WORKING STORAGE (SAME LENGTH AS V)
                     C                 ONE USE IS TO COMPUTE INTEGRATED SPECTRUM
                     C                   IF THAT OPTION IS CHOSEN.  IF SO, THEN ASP
                     C                   WILL CONTAIN FINAL INTEGRATED SPECTRUM UPON
                     C                   RETURN TO CALLING PROGRAM.
                     C
                     C       ANP     = VECTOR OF WORKING STORAGE (SAME LENGTH AS V)
                     C
                     C       ASP     = FOURIER POWER SPECTRUM OF DATA  (HEREIN CALCULATED)
                     C                   (DIMENSIONED SAME LENGTH AS V)
                     C
                     C       BSN     = VECTOR OF WORKING STORAGE (SAME LENGTH AS V)
                     C
                     C       CUMPER  = CUMULATIVE PERIODOGRAM.
                     C
                     C       DATE    = TIME VALUES FOR INPUT SERIES.
                     C
                     C       DELTA   = SAMPLING INTERVAL ('DELTA T')
                     C
                     C       FPE     = FINAL PREDICTION ERROR (CALC. HEREIN)
                     C                   THIS ARRAY SHOULD HAVE SAME DIMEN. AS MCOUNT.
                     C
                     C       FREP    = PERIODOGRAM FREQUENCIES.
                     C
                     C       IAUTCV  = AUTOCOVARIANCE PLOTTING FLAG (BURG ESTIMATE).
                     C                     IF(IAUTCV.NE.0) PLOT THE AUTOCOVARIANCE FUNCTION
                     C                     IF(IAUTCV.EQ.0) DON'T PLOT
                     C
                     C       IFPEPL  = FPE FLAG
                     C                     IF(IFPEPL.NE.0) PRINT AND PLOT FINAL PREDICTION
                     C                                     ERROR VS. NO. LAGS USED.
                     C                     IF(IFPEPL.EQ.0) DON'T PRINT AND PLOT FPE
                     C
                     C       IINTSP  = INTEGRATED SPECTRUM FLAG
                     C                     IF(IINTSP.NE.0) CALC., PRINT, AND PLOT THE
                     C                                     INTEGRATED SPECTRA.
                     C                     IF(IINTSP.EQ.0) DON'T CALC. INTEG. SPECTRA.
                     C
                     C       IPLOT   = PLOTTING FLAG
                     C                     IF(IPLOT.NE.0) PLOT DATA AND SPECTRA
                     C                     IF(IPLOT.EQ.0) DON'T PLOT
                     C
                     C       IPRINT  = PRINTING FLAG
                     C                     IF(IPRINT.NE.0) PRINT DATA AND SPECTRA
                     C                     IF(IPRINT.EQ.0) DON'T PRINT
                     C
                     C       ITREND  = DETRENDING FLAG
                     C                     IF(ITREND.NE.0) USE DETRENDED TIME SERIES
```

```
C                                TO COMPUTE THE SPECTRA
C                     IF(ITREND.EQ.0) USE RAW DATA
C
C     IYWACV = YULE-WALKER AUTOCOVARIANCE FLAG .
C                IF(IYWACV.NE.0) CALCULATE, PRINT, AND PLOT THE
C                                YULE-WALKER ESTIMATE OF THE
C                                AUTOCOVARIANCE FUNCTION
C                IF(IYWACT.EQ.0) DON'T CALCULATE YULE-WALKER
C                                ESTIMATE
C
C     NCOUNT = VECTOR OF PRED. ERROR FILTER LENGTHS TO USE
C
C     NAM    = TITLE FOR THE DATA SET  (TITLE FOR THE PLOTS)
C                MAX. OF 80 CHARACTERS (10A8) ENDING WITH $
C
C     NFILT  = NO. OF PREDICTION ERROR FILTER LENGTHS TO TRY
C                (LENGTH OF PRED. ERROR FILTER = NUMBER OF LAGS)
C
C     NPER   = NO. OF PERIODOGRAM ORDINATES.(NPER = GREATEST INTEGER
C              .LE. NTA/2).
C
C     NPTS   = NO. OF DATA POINTS IN TIME SERIES.
C
C     NV     = NO. OF FREQUENCIES AT WHICH SPECTRUM IS TO BE CALC.
C
C     PERDG  = PERIODOGRAM VALUES.
C
C     PERIOD = VECTOR OF WORKING STORAGE (SAME LENGTH AS V)
C                UPON RETURN THIS VECTOR CONTAINS THE PERIODS
C                 CORRESPONDING TO THE FREQUENCIES IN V.
C
C     PPER   = PERIODOGRAM PERIODS IN CORRESPONDENCE WITH PPER.
C
C     S      = THE MAXIMUM ENTROPY SPECTRUM.  ONE ELEMENT
C                FOR EACH ELEMENT IN VECTOR V.
C                  THIS ARRAY IS WRITTEN OVER FOR EACH NEW
C                  VALUE OF NCOUNT.  UPON RETURN TO CALLING
C                  PROGRAM, IT WILL CONTAIN THE LAST SPECTRUM
C                  COMPUTED, I.E., THE SPECTRUM CORRESPONDING
C                  THE LAST LAG VALUE IN NCOUNT ARRAY.
C
C     STATE  = LABEL FOR DEP. VARB. (TIME SERIES) AXIS
C                MAX. OF 8 CHARACTERS (A8) ENDING WITH $
C
C     V      = OUPUT VECTOR OF FREQUENCIES AT WHICH SPECTRUM
C                IS CALC. (HEREIN GENERATED).  SHOULD BE
C                 DIMENSIONED TO HAVE AT LEAST (NP+1) ELEMENTS.
C
C     VMAX   = MAX. FREQ. AT WHICH SPECTRUM IS TO BE CALC.
C
C     X      = UPON RETURNING FROM DETREND IS THE TREND OF THE INPUT DATA.
C                IT IS LATER ON USED AS A WORKING VECTOR.
C
C     XD     = IF DETRENDING IS DONE, IT IS THE DETRENDED DATA.
C                IF DETRENDIN IS NOT DONE, IT IS THE ORIGINAL DATA.
C                MESA AND FOURIER ANALYSIS ARE PERFORMED ON XD.
C
C     XDIFF  = UPON RETURNING FROM DETREND IS THE DETRENDED DATA.
```

```
C                  IT IS LATER USED AS A WORKING VECTOR.
C
C       Y       = INPUT TIME SERIES VALUES.
C
C       YEAR    = WORKING VECTOR (CAN BE USED FOR TIME VARIABLE
C                    IN SUBROUTINE DETREND.
C
C       YTIME   = LABEL FOR THE TIME AXIS
C                    MAX. OF 8 CHARACTERS (A8) ENDING WITH $
C
C       Z       = PRED. ERROR FILTER COEFFS. (HEREIN CALC. & RETURNED)
C
C
ISN 0003        IMPLICIT REAL*8(A-H,O-Z)
ISN 0004        REAL*8 HYPSEC
ISN 0005        REAL*8 NAM,YTIME,STATE
ISN 0006        COMMON /SPICBL/ DELTA,YEAR(600),Y(600),XD(600),
               1 VMAX,Z(600),V(600),S(600),PPE(600),XDIFF(600),
               2 DATE(600),X(600),ACS(600),BSN(600),AMP(600),ASP(600),PERIOD(600),
               3 NPTS,NFILT,NV,NCOUNT(600),IPRINT,IPLOT,ITREND,IFPEPL,IINTSP,
               4 IAUTCV,IYBACV
ISN 0007        COMMON /BLKLBL/ NAM(14),YTIME(2),STATE(2)
ISN 0008        COMMON /PRIGSB/ PPER(300),PPER(300),PERDG(300),CUMPER(300)
ISN 0009        EXTERNAL SECVAL
ISN 0010        COMPLEX*16 CSUM,CV
ISN 0011        DIMENSION ACFN(600),A(600),B(600)
ISN 0012        DATA HI/Z4132E3F6A8885A31/
C
C
C INITIALIZE DISSPLA ROUTINES FOR PLOTTING.
C
C
ISN 0013        CALL SOPIT
C
C
C       PRINT INPUT DATA.
C
C
ISN 0014        IF (IPRINT.EQ.0) GO TO 100
ISN 0016        PRINT 10, NAM
ISN 0017     10 FORMAT (1HO,14A8)
ISN 0018     20 PRINT 40,YTIME,STATE
ISN 0019     40 FORMAT (1HO,2X,2A4,12X,2A4)
ISN 0020        PRINT 60
ISN 0021     60 FORMAT (1H )
ISN 0022        DO 80 I=1,NFIS
ISN 0023        PRINT 70 , DATE(I), Y(I)
ISN 0024     70 FORMAT (1H ,1PE11.4,3X,1PE19.6)
ISN 0025     80 CONTINUE
ISN 0026    100 CONTINUE
C
C
C
C  DETREND ORIGINAL DATA BY CALLING SUBROUTINE DETRND.
C
C  SUBROUTINE DETREND MUST NOT DESTROY THE ARRAYS DATE AND CATCH.
C         IT SHOULD SET UP ITS OWN TIME ARRAY IN YEAR.
```

```
C
C
ISN 0027        CALL DETSNL(NPTS,DATE,YEAR,Y,XDIFF,X,SPPE)
ISN 0028        DO 170 I=1,NPTS
ISN 0029        XD(I) = XDIFF(I)
ISN 0030      170 CONTINUE
C
C
C
C     PLOT DATA AND ITS TREND COMPONENT.
C
C
ISN 0031        IF (IPLOT.EQ.0) GO TO 200
ISN 0033        CALL DCPIT1
ISN 0034      200 IF (ITREND.NE.0) GO TO 300
ISN 0036        PRINT 202
ISN 0037      202 FORMAT (1HO///1HO,'THE SPECTRAL ANALYSIS WILL BE ',
               1 'APPLIED TO RAW DATA WITHOUT REMOVING THE TREND'////)
ISN 0038        SPPE = 0.0
ISN 0039      220 DO 240 I=1,NPTS
ISN 0040        XD(I) = Y(I)
ISN 0041      240 CONTINUE
ISN 0042        GO TO 310
C
C
ISN 0043      300 CONTINUE
ISN 0044        PRINT 302
ISN 0045      302 FORMAT (1HO///1HO,'THE SPECTRAL ANALYSIS WILL BE ',
               1 'APPLIED TO THE DETRENDED DATA'////)
ISN 0046        PRINT 303,YTIME,STATE,STATE
ISN 0047      303 FORMAT (1HO//1HO,2X,2A8,12X,2A8,10X,'DETRENDED ',2A8/ )
ISN 0048        DO 305 I=1,NPTS
ISN 0049        PRINT 304, DATE(I), Y(I), XDIFF(I)
ISN 0050      304 FORMAT ( 1H ,1F011.4,8X,1PD14.6,8X,1PD14.6)
ISN 0051      305 CONTINUE
C
C
C  PUT IN PLOT FOR DETRENDED DATA.
C
C
ISN 0052        CALL DCPIT2
ISN 0053      310 CONTINUE
C
C
C  COMPUTE YULE-WALKER ESTIMATES OF AUTOCOVARIANCE
C   FUNCTION IF DESIRED.  LAG VALUES STORED IN X,
C     ACV FUNCTION VALUES STORED IN XDIFF.
C
C
ISN 0054        IF(ITWACV.EQ.0) GO TO 500
ISN 0056        AN = NPTS
ISN 0057        CMEAN=0.0D0
ISN 0058        DO 410 I=1, NPTS
ISN 0059        CMEAN=CMEAN+   XD(I)
ISN 0060        X(I)=(I-1)
ISN 0061        XDIFF(I)=0.0D0
ISN 0062      410 CONTINUE
ISN 0063        CMEAN=CMEAN/AN
```

```
ISN 0064            DO 440 I=1, NPTS
ISN 0065            ILAG=I-1
ISN 0066            NNI = NPTS - ILAG
ISN 0067            DO 430 J=1,NNI
ISN 0068            JLAG=J+ILAG
ISN 0069            XDIFF(I)=XDIFF(I)+((   XD(J)-CMEAN)*(   XD(JLAG)-CMEAN))
ISN 0070          430 CONTINUE
ISN 0071            XDIFF(I)=XDIFF(I)/AN
ISN 0072          440 CONTINUE
ISN 0073            XDF1 = XDIFF(1)
ISN 0074            DO 445 I = 1, NPTS
ISN 0075            XDIFF(I) = XDIFF(I)/XDF1
ISN 0076          445 CONTINUE
                C
ISN 0077            PRINT 450
ISN 0078          450 FORMAT (1H1,'YULE-WALKER ESTIMATE OF AUTOCORRELATION ',
                   1 'FUNCTION'//1H0,4X,'LAG',11X,'AUTOCORR.'/)
ISN 0079            DO 460 I=1,NPTS
ISN 0080            PRINT 455, X(I),XDIFF(I)
ISN 0081          455 FORMAT (1H ,1PE10.3,5X,1PE12.5)
ISN 0082          460 CONTINUE
                C
ISN 0083            CALL DOPLT3
                C
                C
                C     COMPUTE PERIODOGRAM AND FOURIER SPECTRUM.
                C
                C
ISN 0084          500 NYR = NPTS
ISN 0085            KP1 = NY + 1
                C
                C     FIRST THE PERIODOGRAM AND CUMULATIVE PERIODOGRAM.
                C
ISN 0086            PTEMP = NPTS / 2
ISN 0087            NPER = PTEMP
ISN 0088            DO 502 J=1,NPER
ISN 0089            AJ = J
ISN 0090            FPER(J) = AJ / NPTS
ISN 0091            PPER(J) = 1.0D0 / FPER(J)
ISN 0092          502 CONTINUE
ISN 0093            CALL SFT(XI,ACS,BSN,FPER,NPER,NPTS,PI)
ISN 0094            SUMPER = 0.0D0
ISN 0095            TWONYA = 2.0D0 / NPTS
ISN 0096            DO 503 I = 1,NPER
ISN 0097            PERDG(I) = TWONYA * (ACS(I)**2 + BSN(I)**2)
ISN 0098            SUMPER = SUMPER + PERDG(I)
ISN 0099          503 CONTINUE
ISN 0100            DSUM = 0.0D0
ISN 0101            PERMAX = PERDG(1)
ISN 0102            IMAX = 1
ISN 0103            DO 505 I=1,NPER
ISN 0104            DSUM = DSUM + PERDG(I)
ISN 0105            CUMPER(I) = DSUM / SUMPER
ISN 0106            IF (PERDG(I) .LE. PERMAX) GO TO 505
ISN 0108            PERMAX = PERDG(I)
ISN 0109            IMAX = I
ISN 0110          505 CONTINUE
ISN 0111            FISHER = PERMAX / ((1.0D0/NPER) * SUMPER)
```

```
C
C     PRINT THE PERIODOGRAM AND CUMULATIVE PERIODOGRAM.
C
ISN 0112        PRINT 507
ISN 0113      507 FORMAT (1H1,'PERIODOGRAM'///1H0,2X,'K',5X,'FREQ.',
                 1 6X,'PERIOD',5X,'PERIODOGRAM',3X,'CUMULATIVE',/1H ,
                 2 33X,'ORDINATE',4X,'PERIODOGRAM'//)
ISN 0114        PRINT 508, (K,FPER(K),PPER(K),PERDG(K),CUMPER(K),K=1,NPER)
ISN 0115      508 FORMAT (1H ,I4,F10.5,F14.5,1PD14.5,0PF10.5)
ISN 0116        PRINT 509, SUMPER,PERMAX,FPER(IMAX),PPER(IMAX),FISHER,NPER
ISN 0117      509 FORMAT (1H0//1H0,'SUM OF PERIODOGRAM ORDINATES = ',1PD12.5,
                 1 /1H0,'MAXIMUM ORDINATE W ',1PD12.5,'   OCCURING AT ',
                 2 'FREQUENCY =',1PD12.5,'   PERIOD =',1PD12.5,/1H0,
                 3 'FISHER STATISTIC =',1PD12.5/1H0,'NUMBER OF ',
                 4 'ORDINATES = ',I4)
C
C     COMPUTE THE KF+1 FREQUENCIES FOR FOURIER AND MESA SPECTRA.
C
ISN 0118        V(1) = 0.
ISN 0119        V(2) = VMAX / NV
ISN 0120        KFF = KF1 - 2
ISN 0121        DO 510 I=1,KFF
ISN 0122        V(I+2) = V(2) + I*V(2)
ISN 0123      510 CONTINUE
C
C     COMPUTE THE FOURIER SPECTRUM.
C
ISN 0124        CALL SPT(ED,ACS,BSN,V,KF1,NYR,PI)
ISN 0125        DO 520 I=1,KF1
ISN 0126        ASP(I) = (2.*ACS(I)/NYR)**2 + (2.*BSN(I)/NYR)**2
ISN 0127        ASP(I) = DSQRT(ASP(I))
ISN 0128      520 CONTINUE
C
C
C     COMPUTE THE CUMULATIVE FOURIER SPECTRUM.
C
C
ISN 0129        YSUM = 0.
ISN 0130        DO 535 I=1,KF1
ISN 0131        YEAR(I) = NPTS / 2 * (ASP(I)**2)
ISN 0132        YSUM = YSUM + YEAR(I)
ISN 0133      535 CONTINUE
ISN 0134        ZSUM = 0.0D0
ISN 0135        DO 540 J=1,KF1
ISN 0136        ZSUM = ZSUM + YEAR(J)
ISN 0137        BSN(J) = ZSUM / YSUM
ISN 0138      540 CONTINUE
ISN 0139        PRINT 550, YSUM
ISN 0140      550 FORMAT (1H1, 'SUM OF FOURIER ORDINATES = ',1PE19.6)
C
C
ISN 0141        IF (IPLOT.EQ.0) GO TO 600
ISN 0143        CALL DOPLT9(KF1,NPER)
ISN 0144        CALL DOPLT8(KF1)
C
C
C     APPLY THE MAXIMUM ENTROPY METHOD. REFERENCE TO
C         ANDERSEN'S ALGORITHM.
```

```
C
C
ISN 0145      600 M1 = MYR - 1
ISN 0146          SUM = 0.
ISN 0147          DO 620 I=1,MYR
ISN 0148          SUM = SUM + XD(I)**2
ISN 0149      620 CONTINUE
ISN 0150          SUM = SUM/MYR
ISN 0151          AMP(1) = SUM
ISN 0152          XDIFF(1) = AMP(1)
C
C
C  MAXIMUM ENTROPY COMPUTATIONS BEGIN HERE.
C
C
ISN 0153          IFLAG = 0
ISN 0154          DO 1000 IF = 1,NFILT
ISN 0155          IFLAG = IFLAG + 1
ISN 0156          IF (IFLAG.GT.1) GO TO 795
ISN 0158          M = 1
ISN 0159          ACS(1) = XD(1)
ISN 0160          BSN(M1) = XD(MYR)
ISN 0161          DO 640 I=2,M1
ISN 0162          ACS(I) = XD(I)
ISN 0163          BSN(I-1) = XD(I)
ISN 0164      640 CONTINUE
ISN 0165      660 DNUM = 0.
ISN 0166          DDEN = 0.
ISN 0167          NN = MYR-M
ISN 0168          DO 680 I=1,NN
ISN 0169          DNUM = DNUM + ACS(I)*BSN(I)
ISN 0170          DDEN = DDEN + ACS(I)**2 + BSN(I)**2
ISN 0171      680 CONTINUE
ISN 0172          Z(M) = 2 * DNUM/DDEN
ISN 0173          AMP(M+1) = AMP(M)*(1.-Z(M)**2)
ISN 0174          QNUM = M+1
ISN 0175          QDEN = MYR-M
ISN 0176          IF (.TREND.EQ.0) GO TO 690
ISN 0178          QNUM = QNUM+SPPE
ISN 0179          QDEN = QDEN - SPPE
ISN 0180      690 CONTINUE
ISN 0181          IF (QDEN.EQ.0.E0) QDEN=1.0D-5
ISN 0183          SPE(M) = AMP(M+1) * QNUM/QDEN
ISN 0184          IF (M.NE.1) GO TO 760
ISN 0186      700 XDIFF(2) = XDIFF(1) * Z(1)
ISN 0187      720 M = M+1
ISN 0188          M1 = M-1
ISN 0189          DO 730 I=1,M1
ISN 0190          YEAR(I) = Z(I)
ISN 0191      730 CONTINUE
ISN 0192          NN = MYR-M
ISN 0193          DO 740 I=1,NN
ISN 0194          ACS(I) = ACS(I) - YEAR(M-1)*BSN(I)
ISN 0195          BSN(I) = BSN(I+1) - YEAR(M-1)*ACS(I+1)
ISN 0196      740 CONTINUE
ISN 0197          GO TO 660
ISN 0198      760 M1 = M-1
ISN 0199          DO 780 I=1,M1
```

```
ISN 0200          Z(I) = YEAR(I) - Z(M)*YEAR(M-I)
ISN 0201       780 CONTINUE
ISN 0202          SUM = 0.
ISN 0203          DO 790 I=1,M
ISN 0204          SUM = SUM + XDIFF(M+1-I)*Z(I)
ISN 0205       790 CONTINUE
ISN 0206          XDIFF(M+1) = SUM
ISN 0207          GO TO 798
ISN 0208       795 CONTINUE
ISN 0209          M = NCOUNT(IP-1)
ISN 0210       798 CONTINUE
ISN 0211          IF (M.EQ.NCOUNT(IP)) GO TO 820
ISN 0213       800 GO TO 720
ISN 0214       820 DO 860 I=1,KF1
ISN 0215          CSUM = 0.
ISN 0216          DO 840 J=1,M
ISN 0217          EXT = - 2. * PI * J * V(I) * DELTA
ISN 0218          CV = DCMPLX(0.D0,EXT)
ISN 0219          CSUM = CSUM + Z(J)*CDEXP(CV)
ISN 0220       840 CONTINUE
ISN 0221          CSUM = 1.+CSUM
ISN 0222          EX = (CDABS(CSUM))**2
ISN 0223          S(I) = (ABS(M+1) * DELTA) / EX
ISN 0224       860 CONTINUE
ISN 0225          NC = NCOUNT(IP)
                  C
                  C
                  C     PRINT OUT AUTOCORRELATION AND ENTROPY VALUES.
                  C
                  C
ISN 0226          NYFREQ = 1.0D0 / (2.0D0 * DELTA)
ISN 0227          RESBW = NYFREQ/((DFLOAT(NCOUNT(IP))/2.0D0)+1.0D0)
ISN 0228          IF (IFRINT.EQ.0) GO TO 980
ISN 0230          IF (IAUTCV.EQ.0) GO TO 891
ISN 0232          PRINT 869,NCOUNT(IP)
ISN 0233       869 FORMAT (1H1,'NCOUNT = ',I5///)
ISN 0234          PRINT 870
ISN 0235       870 FORMAT (1HO/1HO,'AUTOCORRELATION FUNCTION VALUES'//,
                  1 1HO,'LAG',20X,'AUTOCORR.'/)
ISN 0236          ACT = NCOUNT(IP)+1
ISN 0237          XDF1 = XDIFF(1)
ISN 0238          DO 875 I=1,ACT
ISN 0239          ACFN(I) = XDIFF(I) / XDF1
ISN 0240       875 CONTINUE
ISN 0241          DO 890 I=1,ACT
ISN 0242          X(I) = I-1.
ISN 0243          PRINT 880, X(I), ACFN(I)
ISN 0244       880 FORMAT (1H ,1P2E19.6)
ISN 0245       890 CONTINUE
ISN 0246       891 CONTINUE
ISN 0247          PRINT 895,NYFREQ,RESBW,FPE(NC)
ISN 0248       895 FORMAT (1H1,'MAXIMUM ENTROPY AND FOURIER SPECTRA',///,
                  1 1HO,'NYQUIST FREQUENCY  = ',1PD10.3//,
                  2 1HO,'RESOLUTION BANDWIDTH  = ',1PD10.3///,
                  3 1HO,'FINAL PREDICTION ERROR = ',1PD10.3///)
ISN 0249          PRINT 900,NCOUNT(IP)
ISN 0250       900 FORMAT (1HO,'NCOUNT=',I3,6X,'FREQUENCY',6X,'PERIOD',10X,
                  1'MAXIMUM ENTROPY',7X,'FOURIER AMPLITUDE')
```

```
ISN 0251            DO 960 I=1,KF1
ISN 0252            IF (V(I).EQ.0) GO TO 920
ISN 0254          910 PERIOD(I) = 1./V(I)
ISN 0255            GO TO 930
ISN 0256          920 PERIOD(I) = 0.
ISN 0257          930 PRINT 940,I,V(I),PERIOD(I),S(I),ASF(I)
ISN 0258          940 FORMAT (1H ,'I=',I4,10X,F9.7,6X,F10.3,6X,1PE19.6,6X,1PE19.6)
ISN 0259          960 CONTINUE
                  C
                  C
                  C     PLOT MAXIMUM ENTROPY AND AUTOCORRELATION FUNCTION.
                  C
                  C
ISN 0260          980 IF (IPLOT.EQ.0) GO TO 91
ISN 0262            IF (IAUTCV.EQ.0) GO TO 90
ISN 0264            CALL DOPLT5(MCT,MC,ACFN)
ISN 0265           90 CONTINUE
ISN 0266            CALL DOPLT6(KF1,MC)
ISN 0267           91 CONTINUE
                  C
                  C
                  C  COMPUTE INTEGRATED SPECTRUM IF DESIRED
                  C
                  C
ISN 0268            IF(IINTSW.EQ.0) GO TO 1000
                  C
                  C
ISN 0270            WINDOW = RESBW/2.0D0
ISN 0271            PRINT 991,WINDOW,MCOUNT(IP)
ISN 0272          991 FORMAT (1H1,'INTEGRATED MAXIMUM ENTROPY SPECTRUM',////,
                     1 1HO,'WIDTH OF INTEGRATION WINDOW  = ',1PD10.3,///,
                     2 1HO,'MCOUNT=',I3,6X,'FREQUENCY',6X,'PERIOD',10X,
                     3 'MAX. ENT. SPEC.',7X,'INTEGRATED SPECTRUM',7X,
                     4 'ERROR ED. (INTEG. SP.)',/)
ISN 0273            WINDO2=WINDOW/2.0D0
ISN 0274            SPCVIN = SPCVL1 (AMP,2,DELTA,KF1,M)
ISN 0275            DO 995 I=1,KF1
ISN 0276            VLO=V(I)-WINDO2
ISN 0277            VUP=V(I)+WINDO2
ISN 0278            IF((VLO.GE.0.0D0).AND.(VUP.LE.NYFREQ)) GO TO 992
ISN 0280            A(I) = 0.0D0
ISN 0281            B(I) = 0.0D0
ISN 0282            GO TO 995
ISN 0283          992 CONTINUE
                  C
ISN 0284            A(I)    =DQLADRE(SPCVAL,VLO,VUP,0.0D0,0.001D0,
                     1 ERROR,IER)
ISN 0285            B(I) = ERROR
ISN 0286            IF (IER.EQ.66) GO TO 995
ISN 0288            PRINT 993, I, IER, V(I), A(I), B(I)
ISN 0.49          993 FORMAT (1HO/1HO,'I   =',I4,5X,'IER  = ',I3,5X,'FREQ.  =',
                     1 1PD14.6,5X,'INTEGRAL  =',1PD14.6,5X,'ERROR PARAMETER  =',
                     2 1PD14.6/)
ISN 0290          995 CONTINUE
ISN 0291            DO 997 I=1,KF1
ISN 0292            PRINT 996, I,V(I),PERIOD(I),S(I),A(I),B(I)
ISN 0293          996 FORMAT(1H ,'I=',I4,10X,F9.7,6X,F10.3,6X,1PE19.6,6X,
                     1 1PE19.6,6X,1PE19.6)
```

```
ISN 0294        997 CONTINUE
ISN 0295            CALL DOPLT7(NF1,MC,A)
                C
                C
ISN 0296        1000  CONTINUE
                C
                C
ISN 0297            IF (IPPEFL.EQ.0) RETURN
ISN 0299            PRINT 1010
ISN 0300         1010 FORMAT (1H1,'FINAL PREDICTION ERROR',//1H0,2X,
                    \ 'I',3X,'NUMBER OF FILTER',6X,'FINAL PREDICTION',/1H ,
                    \ 8X,'COEFFICIENTS',11X,'ERROR',//)
ISN 0301            DO 1020 I=1,MC
ISN 0302            I1 = I+1
ISN 0303            PRINT 1015,I,I1,FPE(I1)
ISN 0304         1015 FORMAT (1H ,I3,I15,12X,1PD12.5)
ISN 0305         1020 CONTINUE
                C
                C
                C  PLOT THE FINAL PREDICTION ERROR.
                C
                C
ISN 0306            CALL DOPLT8
ISN 0307            RETURN
ISN 0308            END
```

*****F O R T R A N C R O S S R E F E R E N C E L I S T I N G*****

SYMBOL	INTERNAL STATEMENT NUMBERS																	
A	0011	0280	0284	0288	0292	0295												
B	0011	0281	0285	0288	0292													
I	0022	0023	0023	0028	0029	0079	0039	0040	0040	0048	0049	0049	0089	0058	0059	0060	0060	0061	0064
	0065	0069	0069	0071	0071	0074	0075	0075	0079	0080	0080	0096	0097	0097	0097	0098	0103	0104	0105
	0106	0108	0109	0121	0122	0122	0125	0126	0126	0126	0127	0127	0130	0131	0131	0132	0147	0188	0161
	0162	0162	0163	0163	0168	0169	0169	0170	0170	0189	0190	0190	0193	0194	0194	0194	0195	0195	0195
	0199	0200	0200	0200	0203	0204	0204	0218	0217	0223	0238	0249	0239	0241	0242	0242	0243	0243	0251
	0252	0254	0254	0256	0257	0257	0257	0257	0257	0275	0276	0277	0280	0281	0284	0285	0288	0288	0288
	0288	0291	0292	0292	0292	0292	0292	0292	0301	0302	0303	0303							
J	0067	0068	0069	0070	0089	0090	0091	0091	0135	0136	0137	0216	0217	0219					
K	0114	0114	0114	0114	0114	0114	0114												
M	0158	0167	0172	0173	0173	0173	0174	0175	0183	0183	0184	0187	0187	0188	0192	0194	0195	0198	0200
	0200	0203	0204	0206	0209	0211	0216	0223	0274										
S	0006	0223	0257	0292															
V	0006	0118	0119	0122	0122	0122	0124	0217	0252	0254	0257	0276	0277	0288	0292				
X	0006	0027	0060	0080	0242	0243													
Y	0006	0023	0027	0080	0049														
Z	0006	0172	0173	0186	0190	0200	0200	0204	0219	0274									
AJ	0089	0090																	
AM	0056	0063	0071																
CV	0210	0218	0219																
EX	0222	0223																	
IF	0158	0209	0211	0225	0227	0232	0236	0249	0271										
IT	0302	0303																	
MC	0225	0247	0264	0266	0295	0301													
MI	0188	0189	0198	0199															
MM	0167	0168	0192	0193															
MV	0006	0085	0119																
MX	0145	0160	0161																
PI	0012	0093	0124	0217															
XD	0006	0029	0040	0059	0069	0069	0093	0124	0148	0159	0160	0162	0163						
ACS	0006	0093	0097	0124	0126	0159	0162	0169	0170	0194	0194	0195							
AMP	0006	0151	0152	0173	0173	0183	0223	0274											
ASE	0006	0126	0127	0127	0131	0257													
BSM	0006	0093	0097	0124	0126	0137	0160	0163	0169	0170	0194	0195	0195						
EMT	0217	0218																	
FPE	0006	0183	0247	0303															
IEE	0284	0286	0288																
KPE	0120	0121																	
KP1	0085	0120	0124	0125	0130	0135	0143	0144	0214	0251	0266	0274	0275	0291	0235				
MCI	0236	0238	0241	0264															
MAR	0005	0007	0016																
MMI	0066	0067																	
MTR	0084	0124	0126	0126	0146	0147	0150	0160	0167	0174	0175	0192							
SFT	0093	0124																	
SUM	0146	0148	0148	0150	0150	0151	0202	0204	0204	0206									
VLO	0276	0276	0284																
VUP	0277	0275	0288																
ACPN	0011	0239	0243	0264															
CSUM	0010	0215	0219	0219	0221	0221	0222												
DATE	0004	0023	0027	0049															
DDEN	0166	0170	0170	0172															
DNUM	0165	0169	0169	0172															
DSUM	0100	0104	0104	0105															
FPER	0008	0090	0091	0093	0114	0116													
ILAG	0065	0066	0068																
IMAX	0102	0109	0116	0116															

•••••C R T R A N C R O S S R E F E R E N C E L I S T I N G•••••

SYMBOL	INTERNAL STATEMENT NUMBERS																		
JLAG	0068	0069																	
NPER	0087	0088	0093	0096	0103	0111	0114	0116	0143										
NPTS	0006	0022	0027	0028	0039	0045	0056	0058	0064	0066	0074	0079	0084	0086	0090	0093	0095	0131	
PPER	0008	0091	0114	0116															
QDEN	0175	0179	0179	0181	0181	0183													
QNUM	0174	0178	0178	0183															
SPPE	0027	0038	0178	0179															
YMAX	0006	0119																	
XDP1	0073	0075	0237	0239															
YEAR	0006	0027	0131	0132	0136	0190	0194	0195	0200	0200									
YSUM	0129	0132	0132	0137	0139														
ZSUM	0134	0136	0136	0137															
CDABS	0222																		
CDEXP	0219																		
CMEAN	0057	0059	0059	0063	0063	0069	0069												
DELTA	0006	0217	0223	0226	0274														
DOPLT	0013																		
DSQRT	0127																		
ERROR	0283	0285																	
IFLAG	0153	0155	0155	0156															
IPLOT	0006	0031	0141	0260															
NFILT	0006	0154																	
P DG	0008	0097	0098	0101	0104	0106	0108	0114											
PTEMP	0086	0087																	
RESBW	0227	0267	0270																
STATE	0005	0007	0018	0046	0046														
XDIFF	0006	0027	0029	0049	0061	0069	0069	0071	0071	0073	0075	0075	0080	0152	0186	0186	0204	0206	0237
	0239																		
YTIME	0005	0007	0018	0046															
CUMPER	0008	0105	0114																
DCADRE	0284																		
DCMPLX	0218																		
DETRND	0027																		
DFLOAT	0227																		
DOPLT1	0033																		
DOPLT2	0052																		
DOPLT3	0083																		
DOPLT4	0184																		
DOPLT5	0264																		
DOPLT6	0266																		
DOPLT7	0295																		
DOPLT8	0306																		
DOPLT9	0183																		
FISHER	0111	0116																	
IAUTCV	0006	0230	0262																
IFPEPL	0006	0297																	
IINTSP	0006	0268																	
IPRINT	0006	0014	0228																
ITREND	0006	0034	0176																
IYVACV	0006	0054																	
NCOUNT	0006	0209	0211	0225	0227	0232	0236	0249	0271										
NYFREQ	0006	0226	0227	0267	0278														
PERIOD	0006	0254	0256	0257	0292														
PERMAX	0101	0106	0108	0111	0116														
SPCVAL	0009	0284																	

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*****F C A T R A N C R O S S R E F E R E N C E L I S T I N G*****

SYMBOL	INTERNAL STATEMENT NUMBERS					
SPCVIB	0274					
SPCVL1	0274					
SPECTR	0002					
SUBPER	0094	0098	C098	0105	0111	0116
TBORTA	0095	0097				
WINDOW	0270	0271	C273			
WINDO2	0273	0276	0277			

*****FORTRAN CROSS REFERENCE LISTING*****

LABEL	DEFINED	REFERENCES
10	0017	0016
20	0018	
40	0019	0018
60	0021	0020
70	0024	0023
80	0025	0022
90	0265	0262
91	0267	0260
100	0026	0014
170	0030	0028
200	0034	0031
202	0037	0036
220	0035	
240	0041	0039
300	0043	0038
302	0045	0044
303	0047	0046
304	0050	0049
305	0051	0048
310	0053	0052
410	0062	0058
430	0070	0067
440	0072	0064
445	0076	0074
450	0078	0077
455	0081	0080
460	0082	0079
500	0084	0054
502	0092	0088
503	0095	0096
505	0110	0103 0106
507	0113	0112
508	0115	0114
509	0117	0116
510	0123	0121
520	0128	0125
535	0133	0130
540	0138	0135
550	0140	0139
600	0145	0141
620	0149	0147
640	0164	0161
660	0165	0197
680	0171	0168
690	0180	0176
700	0186	
[illegible]	0187	0213
730	0191	0189
740	0196	0193
760	0198	0188
780	0201	0199
790	0205	0203
795	0208	0156
798	0210	0207
800	0213	
820	0218	0211
840	0220	0216

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*****F O R T R A N C R O S S R E F E R E N C E L I S T I N G*****

LABEL	DEFINED	REFERENCES
860	0228	0214
869	0233	0232
870	0235	0234
875	0240	0238
880	0244	0243
890	0245	0241
891	0246	0230
895	0248	0247
900	025C	0249
910	0254	
920	0256	0252
930	0257	0255
940	025E	0257
960	0259	0251
980	026C	0228
991	0272	0271
992	0283	0278
993	0289	0288
995	0290	0275 0282 0286
996	0293	0292
997	0298	0291
1000	0296	0154 C266
1010	030C	0299
1015	0304	0303
1020	0305	0301

```
LEVEL 21.8 ( JUN 78 )                          OS/360   FORTRAN H

      COMPILER OPTIONS - NAME= MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                         SOURCE,NOEBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
C
C
C
C
C
      SUBROUTINE SPT(X,A,B,V,K,N,PI)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION X(1),A(1),B(1),V(1)
      DO 40 I=1,K
      ASUM = 0.
      BSUM = 0.
      DO 20 J=1,N
      ALPHA = 2.*PI*(J-1)*V(I)
      ASUM = ASUM+X(J)*DCOS(ALPHA)
      BSUM = BSUM+X(J)*DSIN(ALPHA)
   20 CONTINUE
      A(I) = ASUM
      B(I) = BSUM
   40 CONTINUE
      RETURN
      END
```

```
******F O R T R A N   C R O S S   R E F E R E N C E   L I S T I N G******

SYMBOL  INTERNAL STATEMENT NUMBERS
A       0002  0004  0013
B       0002  0004  0014
I       0005  0009  0013  0014
J       0008  0009  0010  0011
K       0002  0005
N       0002  0004
V       0002  0004  0009
X       0002  0004  0010  0011
PI      0002  0009
SPI     0002
ASIN    0006  0010  0010  0013
BSIN    0007  0011  0011  0014
BCOS    0010
DSIN    0011
ALPHA   0009  0010  0011
```

**** FORTRAN CROSS REFERENCE LISTING ****

LABEL	DEFINED	REFERENCES
20	0012	0008
40	0015	0005

```
LEVEL 21.8 ( JUN 74 )                          OS/360  FORTRAN H

         COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                            SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
               C
               C
  ISN 0002           DOUBLE PRECISION FUNCTION SPCVAL (FREQ)
               C
  ISN 0003           IMPLICIT REAL*8 (A-H,O-Z)
  ISN 0004           COMPLEX*16 CSUB,CV
  ISN 0005           DIMENSION ARP(1),Z(1)
  ISN 0006           DATA PI /3.141592653589793238462643383279/
               C
  ISN 0007           CSUB=0.0D0
  ISN 0008           DO 100 J=1,N
  ISN 0009           EXT = -2.0D0*PI*J*FREQ*DTIME
  ISN 0010           CV = DCMPLX (0.0D0,EXT)
  ISN 0011           CSUB = CSUB - Z(J)*CDEXP(CV)
  ISN 0012       100 CONTINUE
  ISN 0013           CSUB = 1.0D0+CSUB
  ISN 0014           EX = (CDABS(CSUB))**2
  ISN 0015           SPCVAL = (ARP(N+1)*DTIME)/EX
               C
  ISN 0016           RETURN
               C
  ISN 0017           ENTRY SPCVL1 (ARP,Z,DTIME,KP1,N)
               C
               C  INITIAL ENTRY FOR DEFINING PARAMETERS IN
               C      ARGUMENT LIST
               C
  ISN 0018           SPCVL1 = 7777777.
  ISN 0019           RETURN
               C
  ISN 0020           END
```

```
                    ******F O R T R A N   C R O S S   R E F E R E N C E   L I S T I N G******

SYMBOL  INTERNAL STATEMENT NUMBERS
J       0000  0009  0011
S       0000  0015  0017
I       0005  0011  0017
CV      0004  0010  0011
RI      0014  0015
PI      0006  0009
ARP     0005  0015  0017
RMT     0009  0010
XP1     0017
CSUB    0004  0007  0011  0011  0013  0013  0014
FREQ    0002  0009
COARS   0014
CDEXP   0011
STIRG   0009  0015  0017
DCMPLX  0010
SPCVAL  0002  0015
SPCVL1  0017  0018
```

*****F O R T R A N C R O S S R E F E R E N C E L I S T I N G*****

LABEL	DEFINED	REFERENCES
100	0012	0008

```
LEVEL 21.8 ( JUN 74 )                    OS/360  FORTRAN H

        COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                           SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
                C
                C
 ISN 0002               SUBROUTINE DCPLT
 ISN 0003               IMPLICIT REAL*8(A-H,O-Z)
 ISN 0004               REAL*8 XORIG,XSTEP,YORIG,YCYCLE,XMAX,YMAX
 ISN 0005               REAL*4 NAM,YTIME,STATE
 ISN 0006               REAL*8 XS(600),YS(600)
 ISN 0007               COMMON /SPECBL/ DTIME,YEAR(600),CATCH(600),XX(600),
                       1 YMAX,Z(600),V(600),S(600),PPE(600),XDIFF(600),
                       2 DATE(600),X(600),ACS(600),BSH(600),AHP(600),ASP(600),PERIOD(600),
                       3 NYA,NPLT,KF,NCOUNT(600),IPRINT,IPLOT,ITREND,IPPEPL,IINTSP,
                       4 IAUTCV,YYMACV
 ISN 0008               COMMON /FLELBL/ NAM(10),YTIME(2),STATE(2)
 ISN 0009               COMMON /PRDGRM/ PPER(300),PPER(300),PERDG(300),CUMPER(300)
 ISN 0010               COMMON /QQZZIP/ XORIG,XSTEP,YORIG,YCYCLE,XMAX,YMAX,IDUM(3)
 ISN 0011               DIMENSION ACSN(600),A(600)
                C
                C
 ISN 0012               CALL CALCHF
 ISN 0013               XLONG = 7.
 ISN 0014               YLONG = 5.
 ISN 0015               RETURN
                C
                C
 ISN 0016               ENTRY DOPLT1
 ISN 0017               CALL GPH3GH(XS,YS,DATE,CATCH,NYA,XORIG,XMAX,YORIG,YMAX)
 ISN 0018               CALL TITLE(NAM,100,YTIME,100,STATE,100,XLONG,YLONG)
 ISN 0019               CALL ANGTIC
 ISN 0020               XORIG = XS(1)
 ISN 0021               XMAX = XS(NYA)
 ISN 0022               CALL GRAF(XORIG,'SCALE',XMAX,YORIG,'SCALE',YMAX)
 ISN 0023               CALL MARKER(5)
 ISN 0024               CALL CURVE(XS,YS,NYA,1)
 ISN 0025               CALL RESET('MARKER')
 ISN 0026               IF (ITREND.EQ.0) GO TO 199
 ISN 0028               CALL SINGLE(XS,YS,DATE,X,NYA)
 ISN 0029               CALL DASH
 ISN 0030               CALL CURVE(XS,YS,NYA,0)
 ISN 0031               CALL RESET('DASH')
 ISN 0032           199 CONTINUE
 ISN 0033               CALL FRAME
 ISN 0034               CALL ENDPL(0)
 ISN 0035               RETURN
                C
                C
 ISN 0036               ENTRY DOPLT2
 ISN 0037               CALL GPHDGH(XS,YS,DATE,XDIFF,NYA,XORIG,XMAX,YORIG,YMAX)
 ISN 0038               CALL TITLE('DETRENDED DATA$',100,YTIME,100,STATE,100,
                       1 XLONG,YLONG)
 ISN 0039               CALL ANGTIC
 ISN 0040               XORIG = XS(1)
 ISN 0041               XMAX = XS(NYA)
 ISN 0042               CALL GRAF(XORIG,'SCALE',XMAX,YORIG,'SCALE',YMAX)
 ISN 0043               CALL CURVE(XS,YS,NYA,0)
 ISN 0044               DO 300 I=1,NYA
 ISN 0045               YS(I) = 0.
```

```
ISN 0046        300 CONTINUE
ISN 0047            CALL CURVE(XS,YS,NYA,0)
ISN 0048            CALL FRAME
ISN 0049            CALL ENDPL(0)
                C
                C
                C     PLOT RESIDUALS AGAINST FITTED VALUES.
                C
                C
ISN 0050            CALL GPHBGN(XS,YS,X,XDIFF,NYA,XORIG,XMAX,YORIG,YMAX)
ISN 0051            CALL TITLE('RESIDUALS VS. FITTED VALUES$',100,'FITTED VALUES$',
                   1          100,'RESIDUALS$',100,7.,5.)
ISN 0052            CALL ANGTIC
ISN 0053            CALL GRAF(XORIG,'SCALE',XMAX,YORIG,'SCALE',YMAX)
ISN 0054            CALL MARKER(3)
ISN 0055            CALL CURVE(XS,YS,NYA,-1)
ISN 0056            CALL RESET('MARKER')
ISN 0057            CALL FRAME
ISN 0058            CALL ENDPL(3)
ISN 0059            RETURN
                C
                C
ISN 0060            ENTRY DOPLT3
ISN 0061            CALL GPHBGN(XS,YS,X,IDIFF,NYA,XORIG,XMAX,YORIG,YMAX)
ISN 0062            CALL TITLE('YULE-WALKER AUTOCORRELATIONS$',100,'LAGS$',100,
                   1 'ACV. FN.$',100,XLONG,YLONG)
ISN 0063            CALL GPHEND(XORIG,YORIG,XMAX,YMAX,XS,YS,NYA)
ISN 0064            RETURN
                C
                C
ISN 0065            ENTRY DOPLT4(KF1)
ISN 0066            CALL GPHBGN(XS,YS,V,ASF,KF1,XORIG,XMAX,YORIG,YMAX)
ISN 0067            CALL TITLE('FOURIER AMPLITUDE SPECTRUM$',100,'FREQUENCYS',100,
                   1 'AMPLITUDES',100,XLONG,YLONG)
ISN 0068            CALL GPHEND(XORIG,YORIG,XMAX,YMAX,XS,YS,KF1)
ISN 0069            CALL GPHBGN(XS,YS,V,BSN,KF1,XORIG,XMAX,YORIG,YMAX)
ISN 0070            CALL TITLE('CUM. FOURIER POWER SPECTRUM$',100,'FREQUENCYS',100,
                   1 'NORMALIZED FCN.$',100,XLONG,YLONG)
ISN 0071            CALL GRAF(XORIG,'SCALE',XMAX,YORIG,'SCALE',YMAX)
ISN 0072            CALL CURVE(XS,YS,KF1,0)
ISN 0073            IF ((2*(NYA/2)).EQ.NYA) QQQ = (NYA - 2) / 2
ISN 0075            IF ((2*(NYA/2)).NE.NYA) QQQ = (NYA - 1) / 2
ISN 0077            RATIO = 1.96 / DSQRT(QQQ)
ISN 0078            XS(1) = 0.0
ISN 0079            YS(1) = RATIO
ISN 0080            YS(2) = 0.5 * (1.0-RATIO)
ISN 0081            XS(2) = 1.0
ISN 0082            CALL CURVE(XS,YS,2,0)
ISN 0083            XS(1) = 0.5 * RATIO
ISN 0084            YS(1) = 0.0
ISN 0085            XS(2) = 0.5
ISN 0086            YS(2) = (1.0 - RATIO)
ISN 0087            CALL CURVE(XS,YS,2,0)
ISN 0088            CALL FRAME
ISN 0089            CALL ENDPL(0)
ISN 0090            RETURN
                C
                C
```

```
ISN 0091            ENTRY DOPLT5(NCT,NC,ACFN)
ISN 0092            CALL GPHBGN(XS,YS,X,  ACFN,NCT,XORIG,XMAX,YORIG,YMAX)
ISN 0093            CALL TITLE(1H ,1,'FREQUENCYS',100,'BURG AUTOCORRELATIONS$',
                   1 100,XLONG,YLONG)
ISN 0094            CALL MESSAG('NO. OF FILTER COEFS. ISS',100,.5,YLONG+.25)
ISN 0095            CALL INTNO(NC,3.5,YLONG+.25)
ISN 0096            CALL GPHEND(XORIG,YORIG,XMAX,YMAX,XS,YS,NCT)
ISN 0097            RETURN
                    C
                    C
ISN 0098            ENTRY DOPLT6(KP1,NC)
ISN 0099            CALL GPHBGN(XS,YS,X,S,KP1,XORIG,XMAX,YORIG,YMAX)
ISN 0100            CALL TITLE(1H ,1,'FREQUENCYS',100,'MESA VALUES$',100,
                   1 XLONG,YLONG)
ISN 0101            CALL MESSAG('NO. OF FILTER COEFS. ISS',100,.5,YLONG+.25)
ISN 0102            CALL INTNO(NC,3.5,YLONG+.25)
ISN 0103            CALL GPHEND(XORIG,YORIG,XMAX,YMAX,XS,YS,KP1)
ISN 0104            RETURN
                    C
                    C
ISN 0105            ENTRY DOPLT7(KP1,NC,A)
ISN 0106            CALL GPHBGN(XS,YS,X,  A,KP1,XORIG,XMAX,YORIG,YMAX)
ISN 0107            CALL TITLE(1H ,1,'FREQUENCYS',100,'INTEGRATED SPECTRUMS',100,
                   1 XLONG,YLONG)
ISN 0108            CALL MESSAG('NO. OF FILTER COEFS. ISS',100,.5,YLONG+.25)
ISN 0109            CALL INTNO(NC,3.5,YLONG+.25)
ISN 0110            CALL GPHEND(XORIG,YORIG,XMAX,YMAX,XS,YS,KP1)
ISN 0111            RETURN
                    C
                    C
ISN 0112            ENTRY DOPLT8
ISN 0113            NYA2 = NYA - 3
ISN 0114            CALL GPHBG1(XS,YS,NYA2,FPE)
ISN 0115            XAX = XXCHG
ISN 0116            XSTEP = FLOAT(NYA) / XAX
ISN 0117            NSTEP = XSIZE
ISN 0118            XSTEP = FLOAT(NSTEP) + 1.
ISN 0119            CALL TITLE(NAN,100,'NO. OF FILTER COEFS.$',100,
                   1 'FINAL PREDICTION ERRORS',100,XLONG,YLONG)
ISN 0120            CALL XTICKS(5)
ISN 0121            CALL YLOG(XORIG,XSTEP,YORIG,YCYCLE)
ISN 0122            CALL CURVE(XS,YS,NYA2,0)
ISN 0123            CALL FRAME
ISN 0124            CALL ENDPL(0)
ISN 0125            RETURN
                    C
                    C
ISN 0126            ENTRY DOPLT9(KP1,NPER)
ISN 0127            CALL GPHBGN(XS,YS,FPIN,PERDG,NPER,XORIG,XMAX,YORIG,YMAX)
ISN 0128            CALL TITLE('PERIODOGRAMS',100,'FREQUENCYS',100,
                   1 'POWERS',100,XLONG,YLONG)
ISN 0129            CALL GPHEND(XORIG,YORIG,XMAX,YMAX,XS,YS,NPER)
ISN 0130            CALL GPHBGN(XS,YS,PPER,CUMPER,NPER,XORIG,XMAX,YORIG,YMAX)
ISN 0131            CALL TITLE('CUMULATIVE PERIODOGRAMS',100,'FREQUENCYS',100,
                   1 'NORMALIZED POWERS',100,XLONG,YLONG)
ISN 0132            CALL GRAF(XORIG,'SCALE',XMAX,YORIG,'SCALE',YMAX)
ISN 0133            CALL CURVE(XS,YS,NPER,0)
ISN 0134            IF ((2*(NYA/2)).EQ.NYA) QQQ = (NYA - 2) / 2
```

```
ISN 0136            IF ((2*(NIA/2)).NE.NIA) QQQ = (NIA - 1) / 2
ISN 0138            RATIO = 1.36 / DSQRT(QQQ)
ISN 0139            XS(1) = 0.0
ISN 0140            YS(1) = RATIO
ISN 0141            XS(2) = 0.5 * (1.0-RATIO)
ISN 0142            YS(2) = 1.0
ISN 0143            CALL CURVE(XS,YS,2,0)
ISN 0144            XS(1) = 0.5 * RATIO
ISN 0145            YS(1) = 0.0
ISN 0146            XS(2) = 0.5
ISN 0147            YS(2) = (1.0 - RATIO)
ISN 0148            CALL CURVE(XS,YS,2,0)
ISN 0149            CALL FRAME
ISN 0150            CALL ENDPL(0)
ISN 0151            RETURN
                C
                C
ISN 0152            END
```

*****C I T R A N C R O S S R E F E R E N C E L I S T I N G*****

SYMBOL	INTERNAL STATEMENT NUMBERS																		
A	0011	0105	0106																
I	0004	0045																	
S	0007	0099																	
T	0007	0066	0069	0099	0106														
X	0007	0028	0050	0061	0092														
Z	0007																		
KP	0007																		
NC	0091	0095	0098	0102	0105	0109													
XD	0007																		
XS	0006	0017	0020	0021	0024	0028	0030	0037	0040	0041	0043	0047	0050	0055	0061	0063	0066	0068	0069
	0072	0078	0080	0082	0083	0085	0087	0092	0096	0099	0103	0106	0110	0114	0122	0127	0129	0130	0133
	0139	0141	0143	0144	0146	0148													
YS	0006	0017	0024	0028	0030	0037	0043	0045	0047	0050	0055	0061	0063	0066	0068	0069	0072	0079	0081
	0082	0084	0086	0087	0092	0096	0099	0103	0106	0110	0114	0122	0127	0129	0130	0133	0140	0142	0143
	0145	0147	0148																
ACS	0007																		
ARP	0007																		
ASP	0007	0066																	
BSN	0007	0069																	
FPE	0007	0114																	
KP1	0065	0066	0068	0069	0072	0098	0099	0103	0105	0106	0110	0126							
NCT	0091	0092	0096																
NAB	0005	0008	0018	0119															
NYA	0007	0017	0021	0024	0028	0030	0037	0041	0043	0044	0047	0050	0055	0061	0063	0073	0073	0073	0075
	0075	0075	0113	0116	0134	0134	0134	0136	0136	0136									
QQQ	0073	0075	0077	0134	0136	0138													
XAX	0004	0010	0115	0116															
YAX	0004	0010																	
ACPN	0011	0091	0092																
DASH	0029																		
DATE	0007	0017	0028	0037															
PPER	0009	0127	0130																
GRAP	0022	0042	0053	0071	0132														
IDUM	0010																		
MPER	0126	0127	0129	0130	0133														
NYA2	0113	0116	0122																
PPER	0009																		
YMAX	0007																		
XMAX	0017	0021	0022	0037	0041	0042	0050	0053	0061	0063	0066	0068	0069	0071	0092	0096	0099	0103	0106
	0110	0127	0129	0130	0132														
YEAR	0007																		
YLOG	0121																		
YMAX	0017	0022	0037	0042	0050	0053	0061	0063	0066	0068	0069	0071	0092	0096	0099	0103	0106	0110	0127
	0129	0130	0132																
CATCH	0007	0017																	
CURVE	0024	0030	0043	0047	0055	0072	0082	0087	0122	0133	0143	0148							
DOPLT	0002																		
DSQRT	0077	0138																	
DTIME	0007																		
ENDPL	0038	0049	0058	0089	0124	0150													
FLOAT	0116	0118																	
FRAME	0033	0048	0057	0088	0123	0149													
INTNO	0095	0102	0109																
IPLOT	0007																		
NPILT	0007																		
NSTEP	0117	0118																	
PERDG	0009	0127																	

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*****F O R T R A N C R O S S R E F E R E N C E L I S T I N G*****

SYMBOL	INTERNAL STATEMENT NUMBERS																			
RATIO	0077	0079	0080	0083	0086	0138	0140	0141	0144	0147										
RESET	0025	0031	0056																	
STATE	0005	0008	0018	0038																
TITLE	0018	0038	0051	0062	0067	0070	0093	0100	0107	0119	0128	0131								
XDIFF	0007	0037	0050	0061																
XLONG	0013	0018	0038	0062	0067	0070	0093	0100	0107	0115	0119	0128	0131							
XORIG	0004	0010	0017	0020	0022	0037	0040	0042	0050	0053	0061	0063	0066	0068	0069	0071	0092	0096	0099	
	0103	0106	0110	0121	0127	0129	0130	0132												
XSTEP	0004	0010	0116	0117	0118	0121														
YLONG	0014	0018	0038	0062	0067	0070	0093	0094	0095	0100	0101	0102	0107	0108	0109	0119	0128	0131		
YORIG	0004	0010	0017	0022	0037	0042	0050	0053	0061	0063	0066	0068	0069	0071	0092	0096	0099	0103	0106	
	0110	0121	0127	0129	0130	0132														
YTIME	0005	0008	0018	0038																
ANGTIC	0019	0039	0052																	
CALCRP	0012																			
COMPER	0009	0130																		
DOPLT1	0016																			
DOPLT2	0036																			
DOPLT3	0060																			
DOPLT4	0065																			
DOPLT5	0091																			
DOPLT6	009C																			
DOPLT7	0105																			
DOPLT8	0112																			
DOPLT9	0126																			
GPNBGN	0017	0037	0050	0061	0066	0069	0092	0099	0106	0127	0130									
GPNBG1	0114																			
GPNEND	0063	0068	0096	0103	0110	0129														
IADTCV	0007																			
IFPEPL	0007																			
IINTSP	0007																			
IPRINT	0007																			
ITBEND	0007	0096																		
ITNACV	0007																			
MARKER	0023	0058																		
NCOUNT	0007																			
NESSAG	0098	0101	0108																	
PERIOD	0007																			
SINGLE	0028																			
XTICKS	0120																			
YCYCLE	0004	0010	0121																	

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*****FORTRAN CROSS REFERENCE LISTING*****

LABEL	DEFINED	REFERENCES
199	0032	0026
300	0046	0048

```
LEVEL 21.8 ( JUN 78 )                          OS/360   FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,XREF
  ISN 0002            SUBROUTINE SIMPLE(XS,YS,XD,YD,N)
                    C
                    C
  ISN 0003            REAL*8 XD(1),YD(1)
  ISN 0004            DIMENSION XS(1),YS(1)
                    C
                    C
  ISN 0005            DO 100 I=1,N
  ISN 0006            XS(I) = XD(I)
  ISN 0007            YS(I) = YD(I)
  ISN 0008          100 CONTINUE
                    C
                    C
  ISN 0009            RETURN
  ISN 0010            END
```

```
                    ******F O R T R A N   C R O S S   R E F E R E N C E   L I S T I N G******

SYMBOL  INTERNAL STATEMENT NUMBERS
I       0005  0006  0006  0007  0007
N       0002  0005
ID      0002  0003  0006
IS      0002  0004  0006
YD      0002  0003  0007
YS      0002  0004  0007
SINGLE  0002
```

LABEL	DEFINED	REFERENCES
100	0008	0005

```
LEVEL 21.8 ( JAN 76 )                    OS/360  FORTRAN H

        COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                           SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOXREF,ID,XREF
ISN 0002            SUBROUTINE SPHERE(XS,YS,X,Y,N,ZORIG,XRAY,YORIG,YRAY)
                    C
                    C
ISN 0003            COMMON /SQUIRE/ XOR,XSTP,YOR,YSTP,XAX,YAX,IDUM(3)
ISN 0004            REAL*8 X,Y
ISN 0005            DIMENSION XS(1),YS(1),X(1),Y(1)
ISN 0006            CALL SINGLE(XS,YS,X,Y,N)
ISN 0007            CALL SCRPL(-1)
ISN 0008            CALL PAGE(9.5,10.5)
ISN 0009            CALL SLRPLT
ISN 0010            CALL LINPLT(XS,YS,7)
ISN 0011            ZORIG = XOR
ISN 0012            YORIG = YOR
ISN 0013            XRAY = XOR + XSTP * XAX
ISN 0014            YRAY = YOR + YSTP * YAX
                    C
                    C
ISN 0015            RETURN
ISN 0016            END
```

*****FORTRAN CROSS REFERENCE LISTING*****

SYMBOL	INTERNAL STATEMENT NUMBERS			
A	0002	0004	0010	
I	0002	0004	0005	0006
I	0002	0004	0005	0006
IS	0002	0005	0006	0010
IS	0002	0005	0006	0010
IAI	0003	0013		
IOR	0003	0011	0013	
IAI	0003	0014		
IOR	0003	0012	0014	
IDUM	0003			
PACK	0000			
ISAI	0002	0013		
ISTP	0003	0013		
ISAI	0002	0014		
ISTP	0003	0014		
SUMPL	0007			
IOFAO	0002	0011		
TOBIO	0002	0012		
SPRDUM	0002			
XLOPLT	0009			
XINPLT	0010			
SIBOLS	0006			

```
LEVEL 21.8 ( JUN 76 )                          OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,XREF
  ISN 0002            SUBROUTINE GPHS01(XS,YS,NYA2,Y)
                    C
                    C
  ISN 0003            REAL*8 Y
  ISN 0004            COMMON /QQBIYR/ XORIG,XSTEP,YORIG,YCYCLE,YAX,YAY,IQUM(3)
  ISN 0005            DIMENSION XS(1),YS(1),Y(1)
  ISN 0006            DO 100 I=1,NYA2
  ISN 0007            XS(I) = I
  ISN 0008            YS(I) = Y(I)
  ISN 0009          100 CONTINUE
  ISN 0010            CALL DCSPL(-1)
  ISN 0011            CALL PAGE(9.5,10.5)
  ISN 0012            CALL HLDPLT
  ISN 0013            CALL YLGPLT(XS,YS,NYA2)
                    C
                    C
  ISN 0014            RETURN
  ISN 0015            END
```

******F O R T R A N C R O S S R E F E R E N C E L I S T I N G******

SYMBOL	INTERNAL STATEMENT NUMBERS				
I	0006	0007	0007	0008	0008
J	0002	0003	0005	0008	
JS	0002	0005	0007	0013	
JS	0002	0005	0008	0013	
JAX	0004				
JAX	0004				
JDUM	0004				
MIA2	0002	0006	0013		
PAGE	0011				
SQRPL	0010				
XORIG	0004				
XSTEP	0004				
YORIG	0004				
GPHDG1	0002				
ELDPLT	0012				
YCYCLE	0004				
YLGPLT	0013				

```
LEVEL 21.8 ( JUN 74 )                          OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
  ISN 0002            SUBROUTINE GPBEND(XORIG,YORIG,XMAX,YMAX,XS,YS,N)
                      C
                      C
  ISN 0003            DIMENSION XS(1),YS(1)
                      C
  ISN 0004            CALL XTICKS(5)
  ISN 0005            CALL YTICKS(5)
                      C
  ISN 0006            CALL XAXANG(45.)
  ISN 0007            CALL YAXANG(0.)
                      C
  ISN 0008            CALL GRAF(XORIG,'SCALE',XMAX,YORIG,'SCALE',YMAX)
                      C
  ISN 0009            CALL CURVE(XS,YS,N,0)
  ISN 0010            CALL FRAME
                      C
  ISN 0011            CALL ENDPL(0)
                      C
                      C
  ISN 0012            RETURN
  ISN 0013            END
```

```
******F O R T R A N   C R O S S   R E F E R E N C E   L I S T I N G******

SYMBOL  INTERNAL STATEMENT NUMBERS
A       0002  0009
XS      0002  0003  0009
YS      0002  0003  0009
GRAF    0000
XMAX    0002  0008
YMAX    0002  0008
CURVE   0009
SUBPL   0011
FRAME   0010
XORIG   0002  0008
YORIG   0002  0008
OPENMD  0002
XAXANG  0006
XTICKS  0004
YAXANG  0007
YTICKS  0005
```

```
LEVEL 21.8 ( JUN 74 )                          OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NAP,NOXREF,ID,TERM
  ISN 0002            SUBROUTINE AUGTIC
                    C
                    C
  ISN 0003            CALL XTICKS(5)
  ISN 0004            CALL YTICKS(5)
  ISN 0005            CALL XAXANG(45.)
  ISN 0006            CALL YAXANG(0.)
                    C
                    C
  ISN 0007            RETURN
  ISN 0008            END
```

```
*****F O R T R A N   C R O S S   R E F E R E N C E   L I S T I N G*****
SYMBOL  INTERNAL STATEMENT NUMBERS
ANGTIC  0002
XAXANG  0005
XTICKS  0003
YAXANG  0006
YTICKS  0004
```

APPENDIX IV

LISTING OF SUBROUTINE DETRND

```
        COMPILER OPTIONS - NAME= MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                           SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
ISN 0002            SUBROUTINE DETREND(NYA,DATE,YEAR,CATCH,XDIFF,X,SPPE)
                    C
                    C
                    C       A      = MATRIX OF DIMENSION NYA*1 BY NLSCOL*1. USED IN BLSQ.
                    C
                    C       CATCH  = INPUT TIME SERIES VALUES.
                    C
                    C       COEF   = VECTOR CONTAINING THE COEFFICIENTS OF THE LINEAR FIT.
                    C
                    C       COEF2  = WORKING VECTOR OF SAME LENGTH AS COEF.
                    C
                    C       DATE   = TIME VALUES FOR INPUT SERIES.
                    C
                    C       NLSCOL = NO. OF COEFFICIENTS OF THE LINEAR FIT.
                    C
                    C       NROW   = ROW DIMENSION OF MATRIX A.
                    C
                    C       NYA    = NO. OF DATA POINTS IN TIME SERIES.
                    C
                    C       X      = LINEAR TREND OF TIME SERIES (HEREIN CALCULATED AND
                    C                  RETURNED).
                    C
                    C       XDIFF  = DETRENDED TIME SERIES (HEREIN CALCULATED AND RETURNED).
                    C
                    C       YEAR   = TIME VARIABLE (HEREIN CALCULATED).
                    C
                    C
                    C
ISN 0003                    IMPLICIT REAL*8(A-H,O-Z)
ISN 0004                    DIMENSION DATE(1),YEAR(1),CATCH(1),XDIFF(1),X(1)
ISN 0005                    DIMENSION A(601,3),IROW(601),COEF(10),T(2,2),T1(2,2),VAR(2,2)
ISN 0006                    DIMENSION COEF2(10)
                    C
                    C
                    C       INITIALIZE VALUES.
                    C
                    C
ISN 0007                    NROW = 601
ISN 0008                    NLSCOL = 2
ISN 0009                    NT = 2
ISN 0010                    SPPE = NLSCOL
ISN 0011                    DO 50 I=1,NYA
ISN 0012                    YEAR(I) = I - .5
ISN 0013                    A(I,1) = 1.0
ISN 0014                    A(I,2) = YEAR(I)
ISN 0015                 50 CONTINUE
                    C
                    C
                    C       CALL LEAST SQUARES ROUTINE.
                    C
                    C
ISN 0016                    CALL BLSQ (A,CATCH,COEF,RESID,IROW,T,T1,NYA,NLSCOL,NROW,NLSCOL)
ISN 0017                    DO 80 I=1,NLSCOL
ISN 0018                    SUM = 0.
```

```
ISN 0019            DO 70 J=1,NLSCOL
ISN 0020            SUM = SUM + T1(I,J)*COEF(J)
ISN 0021         70 CONTINUE
ISN 0022            COEF2(I) = SUM
ISN 0023         80 CONTINUE
ISN 0024            DO 90 I=1,NLSCOL
ISN 0025            COEF(I) = COEF2(I)
ISN 0026         90 CONTINUE
                 C
                 C
ISN 0027            DO 100 I=1,NYA
ISN 0028            Y(I) = A(I,NLSCOL+1)
ISN 0029            RDIFF(I) = CATCH(I) - Y(I)
ISN 0030        100 CONTINUE
                 C
                 C
                 C       PRINT THE COEFFICIENTS OF THE TREND LINE.
                 C
                 C
ISN 0031            PRINT 130, RESID
ISN 0032        130 FORMAT (1H0,'RESIDUAL SUM OF SQUARES = ', 1PE19.6)
ISN 0033            PRINT 150
ISN 0034        150 FORMAT (1H0//1H0,'COEFFICIENTS OF THE LINEAR FIT'//)
ISN 0035            PRINT 200, (COEF(I),I=1,NLSCOL)
ISN 0036        200 FORMAT (1H0/1H0,'INTERCEPT = ',1PD16.6,3X,'SLOPE = ',1PD16.6//)
                 C
                 C
                 C       COMPUTE THE VARIANCE-COVARIANCE MATRIX.
                 C
                 C
ISN 0037            DBULT = RESID / (NYA-NLSCOL)
ISN 0038            DO 400 I=1,NLSCOL
ISN 0039            DO 400 J=1,NLSCOL
ISN 0040            SUM = 0.
ISN 0041            DO 300 K=1,NLSCOL
ISN 0042            SUM = SUM + T1(I,K)*T1(J,K)
ISN 0043        300 CONTINUE
ISN 0044            VAR(I,J) = DBULT * SUM
ISN 0045        400 CONTINUE
ISN 0046            PRINT 500
ISN 0047        500 FORMAT ( 1H0//1H0,'VARIANCE-COVARIANCE MATRIX'//)
ISN 0048            CALL ARRAYD(VAR,NLSCOL,NLSCOL,MT,4H    )
                 C
                 C
ISN 0049            RETURN
ISN 0050            END
```

*****FORTRAN CROSS REFERENCE LISTING*****

SYMBOL	INTERNAL STATEMENT NUMBERS																		
A	0009	0013	0014	0016	0028														
I	0011	0012	0012	0013	0014	0014	0017	0020	0022	0024	0025	0025	0027	0028	0028	0029	0029	0029	0035
	0035	0035	0038	0042	0044														
J	0019	0020	0020	0039	0042	0044													
K	0041	0042	0042																
T	0005	0016																	
X	0002	0004	0021	0029															
BT	0009	0048																	
TI	0005	0016	0020	0042	0042														
BYA	0002	0011	0016	0027	0037														
SUB	0018	0020	0020	0022	0040	0042	0042	0044											
VAR	0005	0044	0048																
BLSQ	0016																		
COMP	0005	0016	0020	0025	0035														
DATE	0002	0004																	
IBUF	0005	0016																	
NROW	0007	0016																	
SPPE	0002	0010																	
YEAR	0002	0004	0012	0014															
CATCH	0002	0004	0016	0029															
COMP2	0006	0022	0025																
DEFLT	0037	0044																	
RESID	0016	0031	0037																
IDIFF	0002	0004	0029																
ARRAYD	0048																		
DETERD	0002																		
BLSCOL	0008	0010	0016	0016	0017	0019	0024	0028	0035	0037	0038	0039	0041	0048	0048				

******F O R T R A N C R O S S R E F E R E N C E L I S T I N G******

LABEL	DEFINED	REFERENCES
50	0015	0011
70	0021	0019
80	0023	0017
90	0026	0024
100	0030	0027
120	0032	0031
150	0034	0033
200	0036	0035
300	0043	0041
400	0045	0038 0039
500	0047	0046

```
LEVEL 21.0 ( JUN 74 )                          OS/360  FORTRAN H

         COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                            SOURCE,EBCDIC,NOLIST,NOSEQ,LOAD,MAP,NOIDIT,ID,INSRT
  ISN 0002           SUBROUTINE ABRAYD (B,NRR,NCC,LVV,NAME)
                   C  PROGRAM AUTHORS  J. DOUPNIK OF COMPUTING TECHNOLOGY CENTER AND
                   C                   L. FELDMAN FORMERLY WITH
                   C  COMPUTING TECHNOLOGY CENTER, UNION CARBIDE CORP., NUCLEAR DIV.,
                   C  OAK RIDGE, TENN.
                   C
                   C  CTC ORD PROGRAM NO. 7005.2
  ISN 0003           DOUBLE PRECISION COLUMN,B(1)
  ISN 0004           DATA COLUMN /8HCOLUMN  /
  ISN 0005           NR = NRR
  ISN 0006           NC = NCC
  ISN 0007           LV = LVV
  ISN 0008           DO 3 NL = 1,NC,5
  ISN 0009           NH = NL + 4
  ISN 0010           IF (NC .GE. NH) GO TO 1
  ISN 0011           NH = NC
  ISN 0012         1 NLN1 = (NL-1)*LV
  ISN 0013           NHN1 = (NH-1)*LV
  ISN 0014           DO 3 LI = 1,NR,50
  ISN 0015           NL = LI
  ISN 0016           NR = NL + 49
  ISN 0017           IF (NR .GE. NR) GO TO 2
  ISN 0018           NR = NR
  ISN 0019         2 PRINT 100,    NAME
  ISN 0020       100 FORMAT (1H0//1H0,A8)
  ISN 0021           PRINT 200,    (COLUMN,J,J=NL,NH)
  ISN 0022       200 FORMAT (4HOROW,3X,5(3X,A8,I5,9X))
  ISN 0023           PRINT 300
  ISN 0024       300 FORMAT (1H0)
  ISN 0025           DO 3 L=NL,NR
  ISN 0026           NNL = NLN1 + L
  ISN 0027           NHN = NHN1 + L
  ISN 0028         3 PRINT 400,    L,(B(J),J=NNL,NHN,LV)
  ISN 0029       400 FORMAT (1H ,I3,3X,1P5D25.16)
  ISN 0030           RETURN
  ISN 0031           END
```

```
                    ******F O R T R A N   C R O S S   R E F E R E N C E   L I S T I N G******

SYMBOL  INTERNAL STATEMENT NUMBERS
B       0002  0003  0030
J       0023  0023  0023  0030  0030  0030
L       0027  0028  0029  0030
LI      0015  0016
LV      0007  0013  0018  0030
MM      0009  0010  0012  0018  0023
ML      0008  0009  0013  0023
MC      0006  0008  0010  0012
MM      0017  0018  0020  0027
ML      0016  0017  0027
MR      0005  0015  0018  0020
LVV     0002  0007
MMM     0029  0030
MML     0028  0030
MCC     0002  0006
MFR     0002  0005
MMM1    0018  0029
MLM1    0013  0029
MARR    0002  0021
ARRAYD  0002
COLMMM  0003  0006  0023
```

```
      ******F O R T R A N   C R O S S   R E F E R E N C E   L I S T I N G******

LABEL    DEFINED    REFERENCES
    1      0013     0010
    2      0021     0018
    3      0030     0008  0015  0027
  100      0022     0021
  200      0024     0023
  300      0026     0025
  400      0031     0030
```