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# CAN MODEL UPDATING TELL THE TRUTH?

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## Abstract

This paper discusses to which extent updating methods may be able to correct finite element models in such a way that the test structure is better simulated. After having unified some of the most popular modal residues used as the basis for optimization algorithms, the relationship between modal residues and model correlation is investigated. This theoretical approach leads to an error estimator that may be implemented to provide an *a priori* upper bound of a model's predictive quality relative to test data. These estimates however assume that a full measurement set is available. Finally, an application example is presented that illustrates the effectiveness of the estimator proposed when less measurement points than degrees of freedom are available.

## Introduction

Finite element updating methods are commonly used for attempting to improve the predictive quality of dynamic models. Modal parameters such as frequencies, mode shapes and damping ratios are identified from test data and provide the *baseline* behavior to be reproduced by the updated model. Starting from a cost function, many optimization algorithms may be used for solving the updating problem and, in general, solutions are obtained but the question often remains whether these solutions *truly* represent the system being analyzed.

This work attempts to address this issue by investigating *a priori* error estimators for the predictive quality of a finite element model relative to test data [1]. We will restrict our discussion to the case where modal residues are used as basis for the optimization. Such residues are defined here as vectors of dimensions equal to the number of unknowns (that is, the size of the finite element mass and stiffness matrices). Furthermore, residues may define a distribution of error. Using residual vectors is particularly interesting for attempting to locate the areas of modeling error in the mesh. The second fundamental assumption required here is that experimental and numerical models have the same size (that is, number of measurements = number of unknowns of the finite element problem). In other words, it is assumed that every degree of freedom of the model is measured or that model reduction is applied in order to condense out any non–measured degree of freedom. Such a highly unrealistic assumption is necessary for establishing results that follow. Clearly, reduced–order experimental models (less measurement points or less identified modes than the total number of degrees of freedom of the model) transform the updating problem into a nonlinear problem. Therefore, results presented below are to be understood as a *best–case scenario*, nevertheless useful for establishing first–order *a priori* estimations of the quality of a computational model.

After having summarized and unified some of the most popular residues used for model updating, error estimators are proposed for measuring the predictive quality of a model relative to test data. These estimators are developed for eigenvalues and mode shapes. Finally, an application example is presented and the issue of incomplete instrumentation is briefly addressed by illustrating the influence of removing sensor measurements on the frequency error estimator.

## Theory of Model Updating

The starting point of any test–analysis correlation is the dynamic equation of equilibrium. When a Finite Element Model (FEM) is used, the vibration

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equation can be written as:

$$[\mathbf{K}]\{\mathbf{\Phi}_j\} = \omega_j^2[\mathbf{M}]\{\mathbf{\Phi}_j\} \qquad (1)$$

where $[\mathbf{K}]$ and $[\mathbf{M}]$ represent the FEM–based stiffness and mass matrices, respectively, and $\omega_j$ and $\{\mathbf{\Phi}_j\}$ denote the $j$th cyclic frequency and associated eigenvector. In the following, a linear, conservative model is assumed because the mathematical representation of damping is still a matter of research to a great extent. In the general case of forced vibration, the equation of equilibrium can be obtained in the frequency domain as:

$$\left([\mathbf{K}] - \omega_s^2[\mathbf{M}]\right)\{\mathbf{u}\} = \{\mathbf{f}\} \qquad (2)$$

where $\{\mathbf{f}\}$ represents the input force at frequency $\omega_s$ and $\{\mathbf{u}\}$ denotes the corresponding output displacements. In the case where the input force is an impulse at one of the model's degrees of freedom, equation (2) models the Frequency Response Functions (FRF) of the problem.

To represent the different formulations for FEM updating, we now introduce the identified responses in equations (1) and (2). Measured (or identified) quantities are referred to using the subscript $(\ )_{\mathrm{id}}$: for example, $\omega_{\mathrm{id}}$ denotes a particular identified frequency of interest, not to be confused with its numerical counterpart $\omega_j$. However, since the computational model derives from a mathematical idealization of the structure, it usually presents discrepancies compared to the real test article: material and geometrical properties are known to specified manufacturing tolerances, joints are assumed linear, dissipative effects are neglected, etc. Clearly, the equilibrium can only be achieved by introducing either out–of–balance forces or secondary admissible displacement fields. Each one of these two options is summarized and the residues they naturally lead to are presented below.

## (a) Force Residuals

Substituting $\omega_{\mathrm{id}}$ and $\{\mathbf{\Phi}_{\mathrm{id}}\}$ in equation (1) introduces force residuals since the equilibrium can be satisfied only if:

$$\left([\mathbf{K}] - \omega_{\mathrm{id}}^2[\mathbf{M}]\right)\{\mathbf{\Phi}_{\mathrm{id}}\} = \{\mathbf{R}_{\mathrm{F}}\} \qquad (3)$$

Residual forces $\{\mathbf{R}_{\mathrm{F}}\}$ appearing in the right–hand side of equation (3) are nonzero as long as the FEM is not a *perfect* representation of the identified dynamics $\left(\omega_{\mathrm{id}}; \{\mathbf{\Phi}_{\mathrm{id}}\}\right)$. Physically, they represent out–of–balance forces that must be applied in order to enforce the equilibrium. Note that a similar definition may be obtained from equation (2), in which case force residuals $\{\mathbf{R}_{\mathrm{F}}\}$ are defined as:

$$\left([\mathbf{K}] - \omega_s^2[\mathbf{M}]\right)\{\mathbf{u}_{\mathrm{id}}\} - \{\mathbf{f}\} = \{\mathbf{R}_{\mathrm{F}}\} \qquad (4)$$

As mentioned previously, it is assumed in this work that the experimental and numerical models have matching dimensions. Otherwise, products such as $[\mathbf{K}]\{\mathbf{\Phi}_{\mathrm{id}}\}$ in equation (3) could not be carried out. This unrealistic assumption is necessary for establishing the mathematical results presented below but the reader should not forget that, in reality, incomplete measurements can only deteriorate these results since the problem becomes highly nonlinear.

## (b) Displacement Residuals

Due to conditioning issues, it might be better to define residues that are consistent with displacement units instead of force units: various authors have shown that displacement–based residues are less sensitive to ill–conditioning of the stiffness matrix as well as more robust with respect to measurement errors [2]. If so chosen, the displacement field associated to out–of–balance forces $\{\mathbf{R}_{\mathrm{F}}\}$ may be obtained by solving the following static problem:

$$[\mathbf{K}]\{\mathbf{R}_{\mathrm{D}}\} = \{\mathbf{R}_{\mathrm{F}}\} \qquad (5)$$

It can be easily verified that this definition is equivalent to allowing a secondary admissible displacement field that compensates for the previous out–of–balance forces. Then, the equilibrium is described by the equation below:

$$[\mathbf{K}]\left(\{\mathbf{\Phi}_{\mathrm{id}}\} - \{\mathbf{R}_{\mathrm{D}}\}\right) = \omega_{\mathrm{id}}^2[\mathbf{M}]\{\mathbf{\Phi}_{\mathrm{id}}\} \qquad (6)$$

## (c) Error in Constitutive Law

Another popular formulation of the updating problem involves a displacement–based residue defined as:

$$\{\mathbf{R}_{\mathrm{ECL}}\} = \{\mathbf{\Phi}_{\mathrm{id}}\} - \{\mathbf{\Psi}\} \qquad (7)$$

This approach is known as Error in Constitutive Law (ECL) and is well documented in the literature [3]. Modeling errors in the FEM are indicated by *large* components of $\{\mathbf{R}_{\text{ECL}}\}$. This allows in theory a precise localization of adjustments brought to the model. Hence the residue for ECL is completely determined by the knowledge of the admissible displacement field $\{\Psi\}$.

The ECL defines the admissible displacement field $\{\Psi\}$ as the solution of an inverse iteration step, that is:

$$[\mathbf{K}]\{\Psi\} = \omega_{\text{id}}^2[\mathbf{M}]\{\Phi_{\text{id}}\} \qquad (8)$$

Actually, it is trivial to verify that equation (6) may be obtained by substituting equation (7) into equation (8). Therefore, residues (5) and (7) are equal as long as the stiffness matrix is nonsingular, leading to:

$$[\mathbf{K}]\{\mathbf{R}_{\text{ECL}}\} = \{\mathbf{R}_{\text{F}}\} \qquad (9)$$

In the case of a singular matrix $[\mathbf{K}]$ (which accounts for a free–floating structure), the only difference between these two residuals is a rigid body motion, which should have no impact on the updating since rigid body modes carry no strain energy.

## (d) Error in Inertia Law

Finally, a fourth category of residue is considered. Proceeding as before, a residual vector that characterizes the Error in Inertia Law (EIL) may be defined as:

$$\{\mathbf{R}_{\text{EIL}}\} = \{\Phi_{\text{id}}\} - \{\Psi\} \qquad (10)$$

where the admissible displacement field $\{\Psi\}$ is, this time, obtained by:

$$\omega_{\text{id}}^2[\mathbf{M}]\{\Psi\} = [\mathbf{K}]\{\Phi_{\text{id}}\} \qquad (11)$$

Numerically, the only difference between equations (8) and (11) is that the mass matrix is inverted in equation (11) instead of the stiffness matrix previously. Publications have presented the EIL as a generalization of the ECL and it has been observed that the EIL may be better at localizing modeling errors which affect the mass matrix [3]. Using the same procedure as before, residues associated with the EIL can be shown to be obtained from the force residues as:

$$-\omega_{\text{id}}^2[\mathbf{M}]\{\mathbf{R}_{\text{EIL}}\} = \{\mathbf{R}_{\text{F}}\} \qquad (12)$$

Of course, a practical implementation of these residues for model updating would have to address the spatial incompleteness issue: identified vectors would require modal expansion, or the FEM matrices would have to be reduced to the size of the experimental model. Then, differences may appear between them depending on the numerical implementation and equations (5), (9) and (12) may not hold anymore. However, these results are useful in the *ideal* case (i.e., full measurement set) because they illustrate relationships between the residues. In the following, only the force residue defined in equations (3-4) is considered since others may be obtained from $\{\mathbf{R}_{\text{F}}\}$ as shown in equations (5), (9) and (12).

## (e) Formulation of the Updating

After one of the above residue has been chosen, the formulation of model updating is a constrained optimization problem where a given norm of the residue is minimized given a set of constraints:

$$\min_{\{\mathbf{p}\}} \|\mathbf{R}\| \quad \text{with } \mathcal{C}(\mathbf{p}) \geq \mathbf{0} \qquad (13)$$

Constraints can be chosen for enforcing the physical nature of the solution, for example, by requiring that the updated physical variables stay within acceptable range. In equation (13), vector $\{\mathbf{p}\}$ represents the optimization variables, that is, most usually, geometrical and material parameters. When modal expansion is considered, identified vectors are expanded to match the size of the FEM and expansions $\{\Phi_2\}$ constitute additional optimization variables. The problem is then formulated as:

$$\min_{\{\mathbf{p};\Phi_2\}} \|\mathbf{R}(\mathbf{p};\Phi_2)\| \qquad (14)$$

where modal residual vectors are defined as:

$$\{\mathbf{R}\} = \left([\mathbf{K}(\mathbf{p})] - \omega_{\text{id}}^2[\mathbf{M}(\mathbf{p})]\right)\left\{\begin{array}{c}\Phi_{\text{id}}\\ \Phi_2\end{array}\right\} \qquad (15)$$

The same formulation would apply to FRF data or even static deflection data, starting with equation (4). An example is the Sensitivity–based Element By Element (SB–EBE) updating method that may also account for the coupling between modal expansion and parameter correction [4], [5] and may be generalized to identify damping models [6].

In the following, test–analysis correlation measures are summarized and relationships between these measures and residues $\{\mathbf{R}_\mathrm{F}\}$ are sought after in an attempt to establish useful *a priori* error indicators.

# Test Analysis Correlation

For the purpose of test–analysis correlation, we will consider the following three indicators:

- Frequency difference;
- Modal Assurance Criterion (MAC); and
- Mode Shape Difference (MSD).

This choice is motivated by the simplicity of these measures and by their popularity in the structural dynamics community. Frequency difference consists of simply correlating an identified frequency with a numerical frequency: $(\omega_{\mathrm{id}} - \omega_j)$ and the MAC measures the spatial correlation between two vectors relative to the mass matrix:

$$\mathbf{MAC}_j = \frac{\left(\mathbf{\Phi}_{\mathrm{id}};\mathbf{\Phi}_j\right)_{\mathrm{m}}^2}{\left(\mathbf{\Phi}_{\mathrm{id}};\mathbf{\Phi}_{\mathrm{id}}\right)_{\mathrm{m}}\left(\mathbf{\Phi}_j;\mathbf{\Phi}_j\right)_{\mathrm{m}}} \qquad (16)$$

where $(\mathbf{u};\mathbf{v})_{\mathrm{m}} = \{\mathbf{u}\}^T[\mathbf{M}]\{\mathbf{v}\}$ for any two vectors $\{\mathbf{u}\}$ and $\{\mathbf{v}\}$.

Since the MAC is known to be a quite forgiving correlation indicator, it should in practice only be used for *modal pairing*. Thus, whenever frequency and mode shape differences are considered in the following, it is assumed that they involve modes that have already been paired (with a MAC value greater than 80%, typically).

Instead of using the MAC for correlating mode shapes, a better practice is to rely on a mode shape difference, here defined as:

$$\mathbf{MSD}_j = \frac{\left\|\mathbf{\Phi}_{\mathrm{id}} - \mathbf{\Phi}_j\right\|}{\left\|\mathbf{\Phi}_{\mathrm{id}}\right\|} \qquad (17)$$

Mode shape differences are to be used with caution since they require test and analysis vectors to receive the same normalization but they are otherwise more meaningful of the correlation between two modes compared to the MAC. The question addressed in the remainder is to know whether frequency and mode shape differences may be bounded by residues used to measure the degree of correlation during the updating. The basic mathematical tools used for establishing these results are exposed in the following Section.

# Math Tools Used

Results presented below have been obtained by starting from the definition (3) of force residues and by combining the following three mathematical tools:

- Spectral Decomposition Theorem;
- Cauchy–Schwartz inequality; and
- Triangular inequality.

For completeness, these tools are enounced briefly below. In addition, the norm considered here is the Frobenius (or Euclidean) norm $\|\ \|_2$ defined as:

$$\|\mathbf{u}\|_2^2 = \{\mathbf{u}\}^T\{\mathbf{u}\} = \sum_{i=1\cdots N} \mathbf{u}(i)^2 \qquad (18)$$

for a vector quantity of length $N$ and:

$$\begin{aligned}\|\mathbf{A}\|_2^2 &= \mathbf{Trace}\left([\mathbf{A}]^T[\mathbf{A}]\right) \\ &= \sum_{j=1\cdots M}\sum_{i=1\cdots N} \mathbf{A}(i,j)^2\end{aligned} \qquad (19)$$

for a $N \times M$ matrix. Note that, in a finite dimensional space, all norms are equivalent. Therefore, results presented below can be generalized to any choice of norm. The Euclidean norm is used here for the only reason that it simplifies greatly derivations.

Spectral decomposition expresses the factorization of self–adjoint operators over a $N$–dimensional basis. Its application to symmetric, positive definite stiffness and mass matrices provides the following decomposition:

$$[\mathbf{K}] = [\mathbf{\Phi}]^{-T}[\mathbf{\Omega}]^2[\mathbf{\Phi}]^{-1}, \quad [\mathbf{M}] = [\mathbf{\Phi}]^{-T}[\mathbf{\Phi}]^{-1} \qquad (20)$$

where $[\mathbf{\Phi}]$ represents the mode shape matrix where vector $\{\mathbf{\Phi}_j\}$ is stored in the $j$–th column, and $[\mathbf{\Omega}]$

is a diagonal matrix storing cyclic frequencies $\omega_j$. Furthermore, if all modes are available, the mode shape matrix is orthogonal which may be written as:

$$[\Phi]^T [\mathbf{M}] [\Phi] = [\mathbf{Id}] \qquad (21)$$

As a result, any power of the dynamic stiffness matrix may be decomposed on the same mode shape basis according to:

$$\begin{array}{c} \left([\mathbf{K}] - \omega_{\mathrm{id}}^2 [\mathbf{M}]\right)^p = \\ [\mathbf{M}]^q [\Phi] \left([\Omega]^2 - \omega_{\mathrm{id}}^2 [\mathbf{Id}]\right)^p [\Phi]^T [\mathbf{M}]^q \end{array} \qquad (22)$$

where $p$ and $q$ are integer numbers such that $|p| = 1$ and $q = 1$ if $p = 1$ and $q = 0$ if $p = -1$. Equation (22) is used for proving the estimations below with either $p = 1$ or $p = -1$.

The other math equations used in the demonstrations are the Cauchy–Schwartz inequality:

$$\|\mathbf{AB}\|_2 \leq \|\mathbf{A}\|_2 \|\mathbf{B}\|_2 \qquad (23)$$

and the triangular inequality:

$$\|\mathbf{A} + \mathbf{B}\|_2 \leq \|\mathbf{A}\|_2 + \|\mathbf{B}\|_2 \qquad (24)$$

which can easily be established for any $N \times N$ matrices $[\mathbf{A}]$ and $[\mathbf{B}]$.

# Spectral Decomposition of Residues

First of all, we want to investigate the relationship between the objective function being minimized and the metrics used for correlation. Starting from equation (3) and using equation (22) with $p = 1$, it is easy to verify that the inverse mass–weighted norm of modal residues is expressed as a combination of the MACs and frequency differences such as:

$$\begin{array}{c} \dfrac{\{\mathbf{R_F}\}^T [\mathbf{M}]^{-1} \{\mathbf{R_F}\}}{(\Phi_{\mathrm{id}}; \Phi_{\mathrm{id}})_{\mathrm{m}}} = \\ \sum\limits_{j=1\cdots N} (\Phi_j; \Phi_j)_{\mathrm{m}} \left(\omega_{\mathrm{id}}^2 - \omega_j^2\right)^2 \mathbf{MAC}_j \end{array} \qquad (25)$$

Clearly, equation (25) shows that minimizing modal residues leads to a FEM that reproduces the identified data since having $\omega_j = \omega_{\mathrm{id}}$ or $\mathbf{MAC}_j = \mathbf{0}$ is a necessary condition for having $\left\|\mathbf{R_F}\right\| = \mathbf{0}$. However, its practical use is limited by the fact that the entire modal spectrum must be available ($j = 1 \cdots N$) for equation (25) to be implemented. Extracting all the frequency content of a FEM is most impractical and computationally expensive, especially when dealing with large models. It is of far greater interest to isolate one specific frequency or mode shape difference and to relate it to the norm of modal residues, as seen below.

# Error Estimation for Frequencies

To propose error bounds which could be used for estimating the *a priori* quality of a model compared to test data, we start, once more, from definition (3) and decompose the dynamic stiffness matrix using equation (22) and $p = 1$. Then, keeping in mind that $\|\Phi\|_2 = 1$ since $[\Phi]$ is an orthogonal matrix, the following result is straightforward:

**Theorem 1:**

$$\boxed{\frac{\left\|\mathbf{R_F}\right\|_2}{\left\|\Phi_{\mathrm{id}}\right\|_2} \leq \max_{j=1\cdots N} \left|\omega_{\mathrm{id}}^2 - \omega_j^2\right|} \qquad (26)$$

Equation (26) states that the worse possible frequency error is an upper bound of the norm of the residual vector. Furthermore, since only the Cauchy–Schwartz inequality is used for establishing this result, the upper bound is generally the smallest possible.

A similar, yet more useful, result may be obtained when expressing the mode shape vector as a function of $\{\mathbf{R_F}\}$ in equation (3). Then, the inverse of the dynamic stiffness matrix is decomposed using equation (22) and $p = -1$ to lead:

**Theorem 2:**

$$\boxed{\min_{j=1\cdots N} \left|\omega_{\mathrm{id}}^2 - \omega_j^2\right| \leq \frac{\left\|\mathbf{R_F}\right\|_2}{\left\|\Phi_{\mathrm{id}}\right\|_2}} \qquad (27)$$

This result shows that the degree of correlation of a model (measured here with the frequency difference) is controlled by the norm of the residual vector, divided by the norm of the identified mode shape. In other words, *a priori* measures of the model's quality relative to test data may be obtained without having to extract the model's eigenpairs.

## Error Estimation for Mode Shapes

This Section generalizes previous results to mode shape differences. Again, the first result is obtained using the spectral decomposition of the dynamic stiffness matrix (equation (22) with $p = 1$) while the second one is obtained by factoring the inverse dynamic stiffness matrix (equation (22) and $p = -1$).

**Theorem 3:**

$$\frac{\left\|\mathbf{R}_{\mathrm{F}}\right\|_2}{\left\|\mathbf{\Phi}_{\mathrm{id}}\right\|_2} \leq \left|\omega_{\mathrm{id}}^2 - \omega_k^2\right| + \mathbf{MSD}_k \max_{j=1\cdots N} \left|\omega_k^2 - \omega_j^2\right| \quad (28)$$

**Theorem 4:**

$$\mathbf{MSD}_k \leq \frac{\left|\omega_{\mathrm{id}}^2 - \omega_k^2\right| + \dfrac{\left\|\mathbf{R}_{\mathrm{F}}\right\|_2}{\left\|\mathbf{\Phi}_{\mathrm{id}}\right\|_2}}{\min_{j=1\cdots N} \left|\omega_k^2 - \omega_j^2\right|} \quad (29)$$

This last result shows how to control the correlation of mode shapes without having to extract them explicitly from the FEM matrices. However, this estimator is useful only to the extent where the distribution of analytical eigenvalues is known in the frequency range of interest.

## Impact of Incomplete Instrumentation

Since correlation estimators can not be obtained easily when some of the model's degrees of freedom are not measured, an example is presented here that illustrates how results deteriorate with incomplete measurements. The test article is a steel plate used in the automotive industry. The discretization counts 1,845 nodes and 5,535 degrees of freedom. A total of 1,356 volume elements with different geometries (hexahedra, pentahedra and tetrahedra) are used for modeling the structure. The test structure is instrumented at 15 nodal points evenly distributed on the upper surface of the plate, providing a total of 30 acceleration measurements. The first four mode shapes are paired and results of the correlation are presented in Table 1.

Clearly, the correlation is excellent for the lowest frequency modes, with MAC values (not included in Table 1) over 90%. Hence, this example provides a perfect illustration since our correlation estimator has been obtained as an upper bound that can only increase as the test–analysis correlation becomes worse.

**Table 1. Test Analysis Correlation.**

| Test Frequency | Model Frequency | Relative Error |
|---|---|---|
| 1,311.2 Hz | 1,334.8 Hz | 1.8 % |
| 2,421.9 Hz | 2,421.2 Hz | 0.0 % |
| 2,968.9 Hz | 2,972.6 Hz | 0.1 % |
| 3,414.9 Hz | 3,471.1 Hz | 1.6 % |

To illustrate the influence of incomplete instrumentation, finite element mode shapes have been substituted to test mode shapes (since the correlation between the two sets is excellent) to estimate the error estimator of eigenvalues. Starting with a full measurement set, nodes have been eliminated one by one to generate an increasing number on non–measured degrees of freedom. Hence, modal expansion is used for re–constructing the full vectors based on measurements available [4]. Table 2 shows the initial quality of the model, when all degrees of freedom are assumed measured (no modal expansion required).

**Table 2. Initial Model Quality.**

| Difference $\left\|\omega_{\mathrm{id}}^2 - \omega_j^2\right\|$ | Estimator (Upper Bound) |
|---|---|
| $24.65 \times 10^5$ | $4.29 \times 10^5$ |
| $1.34 \times 10^5$ | $6.39 \times 10^5$ |
| $8.68 \times 10^5$ | $5.39 \times 10^5$ |
| $152.78 \times 10^5$ | $4.88 \times 10^5$ |

Units in Table 2 are sec.$^{-2}$.

It can be seen in Table 2 that eigenvalue differences for modes 1, 3 and 4 are larger than the error estimators. Also, the model is much better at predicting mode number 2 where the eigenvalue difference is smaller than the upper bound, which is consistent with the excellent figure for mode 2 in Table 1. This means that model updating may improve the representation of modes 1, 3 and 4 but it is unlikely to improve that of the second mode.

Finally, Figure 1 depicts the influence of incomplete instrumentation. As nodal points are removed from the measurement set, modal expansion is implemented to interpolate the missing information, which tends to increase the value of the upper bound. This is shown in Figure 1 where error estimators increase for all four modes as the number of measurements is being reduced.

# Conclusion

Error estimators are presented that, combined with test data modal parameters, evaluate the quality of a model relative to measurements available. Results reported here apply to the full measurement case and are obtained using the decomposition of mass and stiffness matrices on their basis of eigenvectors. Hence, these estimators provide upper bounds for the error committed when the model is used for predicting a particular mode. As such, they are useful for estimating if model updating may improve the representation of a given measured dynamics. Moreover, they are computationally efficient (less expensive than an eigenpair extraction). This theory is however restricted to the full measurement case and taking into account non–measured degrees of freedom is currently being further investigated.

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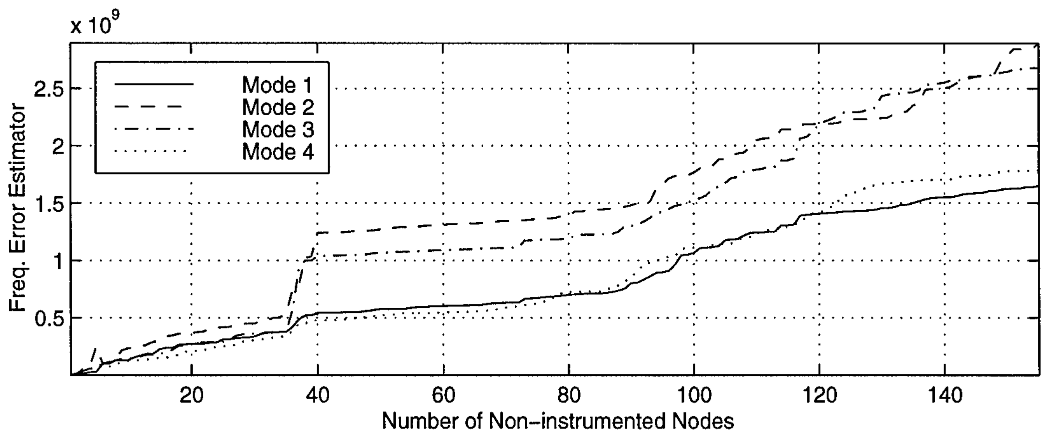


**Figure 1. Effect on the Estimator of Incomplete Measurements.**

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