A Range-to-Target Algorithm for a Continuous-Wave Ground Penetrating Radar

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ABSTRACT

Many geologic situations of interest to oil and gas exploration, and to enhanced recover methods, occur in media whose conductivity is too large to permit the use of pulsed GPRs because of severe dispersion. A continuous-wave radar is not affected by dispersion, and can use the round-trip phase, rather than time, to give an estimate of range. In this paper a range-to-target algorithms is developed for targets which exhibit a crude hyperbolic phase response. This new algorithm minimizes a difference function over both a $2n\pi$-phase interval and a wavelength interval to provide the range. Only crude initial estimates of the electrical parameters of the host media are required to initiate the algorithm. The furnished range may be the distance to some point within the target rather than to a point upon the illuminated surface because the target is three-dimensional and its electrical parameters can take on any value. This error can be reduced by a sufficiently high operating frequency. Examples are given for a variety of targets, media, range and operating frequency using simulated data.

Key words: CW Radar, GPR, range, range-to-target

THE ALGORITHM

Several assumptions are required for the development of the algorithm, namely: (1) The media between the radar and the target is homogeneous and isotropic; (2) The transmitting and receiving antennas of the GPR are oriented and sufficiently separated to reduce crosstalk to a negligible value; (3) Measurements are made at traverse intervals that do not exceed one-tenth of the separation distance between antennas; (4) The maximum radius of curvature of the target is much less than the minimum range.

Traverse Geometry

Suppose that the radar is translated in uniform increments along the line separating its antennas, and approaches, illuminates, and passes-by a discrete target as shown in Figure 1. The slant range is given by:

$$R_{p,i} = \lambda \phi_i / 4\pi \qquad \text{meters. (1)}$$

$R_{p,i}$ is the *propagation* form of the slant range measured from the midpoint of the antenna separation at the ith traverse position z_i ; λ is the wavelength in the host media, and ϕ_i is the measured roundtrip phase. The minimum phase is denoted by ϕ_0 , and the minimum radial range is denoted by ρ_0 given by:

$$\rho_0 = \lambda \phi_0 / 4\pi \qquad \text{meters. (2)}$$

There are several difficulties with using Equations 1 and 2. In practice the phase is measured modulo 2π, and the number of 2π-increments is unknown. Also, there is a phase shift upon reflection which is included in the phase measurement. Additionally, the wavelength is unknown, and a final problem is that the two unknowns, λ and ϕ_i , multiply each other!

The geometric form of the slant range from the midpoint of the antenna separation can also be written from Figure 1:

$$R_{g,i} = \sqrt{\rho_0^2 + z_i^2} \qquad \text{meters. (3)}$$

The geometric slant range is an approximation to one-half the sum of R1 and R2 shown in Figure 1. An examination of the error in a recent report (Caffey, 1997) shows that the maximum error is less than 1% if the antenna separation divided by ρ_0 is less than 1/4.

The propagation and geometric forms of the slant range are the basis for the algorithm. The relation between phase and traverse distance is shown to be hyperbolic by equating the two forms of slant range and using the definition of ρ_0 :

$$\frac{\phi_i^2}{\phi_0^2} - \frac{z_i^2}{\rho_0^2} = 1 . \qquad (4)$$

This is the standard from of an hyperbola which is centered at $\phi_0 = 0$ with foci at $\rho_0 \sqrt{1 + 16\pi^2 / \lambda^2}$ perpendicular to the traverse line.

Difference Function

The key idea in the algorithm consists of minimizing the magnitude of the difference between the squares of $R_{g,i}$ and $R_{p,i}$, namely:

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$$\text{Dif Ftn} = \left| \left(z_i^2 + \left[\lambda_j \phi_0 / 4\pi \right]^2 \right) - \left(\lambda_j \phi_i / 4\pi \right)^2 \right|, \quad (5)$$

where $i = 0, 1, 2 \ldots;$ and the λ_j are trial values of wavelength, $j = 1, 2, \ldots j_{max}$. At $i = 0$, $z_i = 0$, $\phi_i = \phi_0$, and the Dif Ftn is zero.

Both ϕ_0 and ϕ_i are now augmented with trial values so that adjustments can be introduced:

$$\Phi_i = \phi_i + \psi_k + 2N\pi \quad ; \quad i = 0, 1, 2 \ldots \quad (6)$$

where:

N = the phase adjustment index which allows trial restorations of the 2π-increments which may have been removed; $N = 0, 1, \ldots N_{max}$;

and

Ψ_k = trial values of reflection phase, $k = 1, 2, \ldots k_{max}$.

Replacing the ϕ_i with Φ_i in Equation (5), the Dif Ftn becomes:

$$\left| 16\pi^2 z_i^2 + \lambda_j^2 (\phi_0 - \phi_i)(\phi_0 + \phi_i + 2\psi_k + 4\pi N) \right| \quad (7)$$

The Φ_i, like the ϕ_i, must always be ≥ 0 to prohibit the computation of a negative range. The sum $(\phi_i + \psi_k)$ may span the range 0-to-2π each for N. Accordingly, $-\phi_i \leq \psi_k \leq (2\pi - \phi_i)$; that is, negative values of ψ_k are permitted. The range of ψ_k is searched with a uniform increment $\Delta\psi$ which is a free choice.

If the mean difference function over the (ϕ_i, z_i) is minimized over the indicial set $\{j, k\}$ for each N, including the point $\phi_i = \phi_0$, the global minimum should provide the range-to-target which is most consistent with the data. This process is called the *difference method*.

Another method is based on the fact that Equation (7) can be set to zero and solved directly for ψ_k:

$$2\psi_k = \frac{(4\pi z)^2}{(\phi_i - \phi_o)\lambda_j^2} - (\phi_i + \phi_k + 4\pi N), \quad i \neq 0. \quad (8)$$

The ψ_k obtained for each (i, j, N) are averaged over the $(i - 1)$ data points to obtain an average trial value. This average ψ_k is used with λ_j to obtain a minimum of the mean difference function as before. This process is called the *direct method* because trial values of ψ_k are 'directly' obtained, and it is preferred because the tiresome difficulty of making guesses for $\Delta\psi$ is avoided.

Wavelength Estimates

Trial values of wavelength, the λ_j, must be supplied based on estimates of the lower and upper limits of relative dielectric constant and conductivity of the host media. It is assumed that the relative magnetic permeability is unity. The wavelength limits are computed via the real part of the propagation factor of the host media. The lower limit, λ_{min}, is computed from $(\varepsilon_r, \sigma)_{max}$, and λ_{max} is computed from $(\varepsilon_r, \sigma)_{min}$. The wavelength interval is searched with a uniform increment $\Delta\lambda = (\lambda_{max} - \lambda_{min}) / j_{max-1}$ where j_{max} is a free choice.

Range Constraints

A minimum and maximum range exists for each choice of N. The radial distance, in terms of the search variables, is written as:

$$\rho_{i,j,k,N} = \frac{\lambda_j \Phi_i}{4\pi} = \frac{\lambda_j (\phi_i + \psi_k + 2\pi N)}{4\pi}, \quad (9)$$

and

$$\rho_{min,N} = N\lambda_{min} / 2, \quad \rho_{max,N} = (N + 1)\lambda_{max} / 2.$$

These extremes provide overlapping range-intervals for successive choices of N as shown below.

N	ρ_{min}	ρ_{max}
0	0	$0.5\lambda_{max}$
1	$0.5\lambda_{min}$	λ_{max}
2	λ_{min}	$1.5\lambda_{max}$
3	$1.5\lambda_{min}$	$2.0\lambda_{max}$

An Effect of Limited Precision

A problem arises from the limited precision of the phase measurement itself. The several values of ϕ_i near the apex of the hyperbola often have the same value. If the number of such values is odd, the middle value is chosen as ϕ_0 ; if the number is even, the mean value of the two z_i in the center is inserted as z_0 and a ϕ_0 is likewise inserted. In either case the z_i are recalculated so that only $z_0 = 0$.

Computer Code

The code for the algorithm, over 600 non-commented source statements, is a double-precision Fortran program. Input data are checked, and intermediate results are examined to ensure that they are within the constraints imposed by wavelength, ψ-limits, and range.

Psi-Break

The code proceeds by finding a minimum value of Equation (8) for increasing values of N. There is both a range and a minimum Dif Ftn for each N, and the problem is to know which range to accept because the global minimum Dif Ftn may not occur at the correct range as shown in this partial output listing:

N	Wavelength, meters	Minimum Mean Dif Ftn	Range, meters	ψ_k, degrees
0	10.2	.220	3.16	-45.3
1	10.1	.143E-02	9.38	40.3
2	8.1	.151E-03	12.15	91.3
3	7.1	.116E-03	14.01	72.3
4	6.6	.101E-03	15.11	-60.3

The minimum Dif Ftn just decreases with increasing N in this listing. The correct solution becomes evident as an effect of the range constraint. In Equation (9), as N increases, the λ_j decrease with possible increases in ψ_k permitted by ρ_{max}. Eventually, the range constraint breaks the increase of ψ_k –values as shown above beginning with $N = 3$. The value of N at which the decrease begins is termed the "Psi-break", and a correct range has not been observed at a Psi-break or greater N. Hence the solutions for N = 3,4 are discarded, and the Dif Ftn for $N = 2$ is about one-tenth the value for $N = 1$. This identifies the range as 12.2 meters.

EXAMPLES

Three examples are given below which demonstrate the algorithm in two increasingly conductive soils and also in a crosswell mode. The transmitter is a magnetic dipole whose moment is along the positive X-axis of a Cartesian coordinate system. The receiver is an electric dipole, centered on the positive Z-axis, and 1.5m above the transmitter in the first two examples. A spherical target is centered on the Y-axis, and the antenna-pair is translated upwards along the Z-axis from beneath the XY-plane. The back-scattered fields are computed from the Debye potentials given by March(1953).

Moderate Soil Example

The Boulder Creek granite formation near Raymond, Colorado, has been extensively measured *in situ* at frequencies up to 25MHz (Grubb *et al.*, 1976). At 7.6MHz the parameters are $\varepsilon_r = 9$ and $\sigma = 1.6$mS/m which provide a wavelength of 12.88m. A 1m OD spherical target is centered 2 wavelengths, or 25.76m, away. The target ε_r is the same as the host media, but two conductivities are used: 4-times the host value and 1/4th of the host value. The traverses extend ±7.65m in 0.15m increments, and provide maximum phases of 328° and 127° centered on $\phi_0 = 266.4°$ and 64.8°

respectively. The two range estimates, using just ϕ_0 and the three points as ϕ_0 is approached, are identical: 25.8m ± 0.05m which is 4cm beyond the target center.

Conductive Soil Example

This is a set of 3 examples to show the effect of increasing frequency upon the determination of range with a 2m OD target centered at 13m. The host conductivity is 13mS/m, and the target conductivity is 6.5mS/m. An $\varepsilon_r = 6$ is used for both host and target. Frequencies of 0.65, 2.6, and 10.4MHz are used so that the illuminated surface is at minimum distance, in host wavelengths, of 0.35, 0.72, and 1.6 respectively. Figure 2 is a plot of the data from the 0.65MHz traverse. The traverse extends ±10m on each side of ϕ_0, and the phase values for all frequencies are within 64.4° of ϕ_0. The results are shown below.

Frequency, MHz	Range, meters	N
0.65	13.95± 0.10	0
2.60	13.23 ± 0.05	1
10.40	12.13 ± 0.03	2

All three range estimates are within the target, but move from the rear toward the front with increasing frequency. However, operating at 2.5MHz and 10.4MHz requires a power increase of 11dB and 47dB respectively compared to 0.65MHz.

Crosswell Example

If the antennas are horizontally separated, and one is vertically traversed past the other, the hyperbolic phase response is still obtained. In this example the antennas are placed 31.4m apart in the Boulder Creek media at 10.2MHz for which the conductivity is 1.75mS/m and ε_r is about 8.2. Because the path is now one-way, the algorithm is modified by replacing the factor of 4π by 2π in Equation (1) *et seq.* There is no reflection phase, so the ψ_k are set to zero. The computed range is 31.4m ± 0.2m which is in excellent agreement with the separation distance.

References
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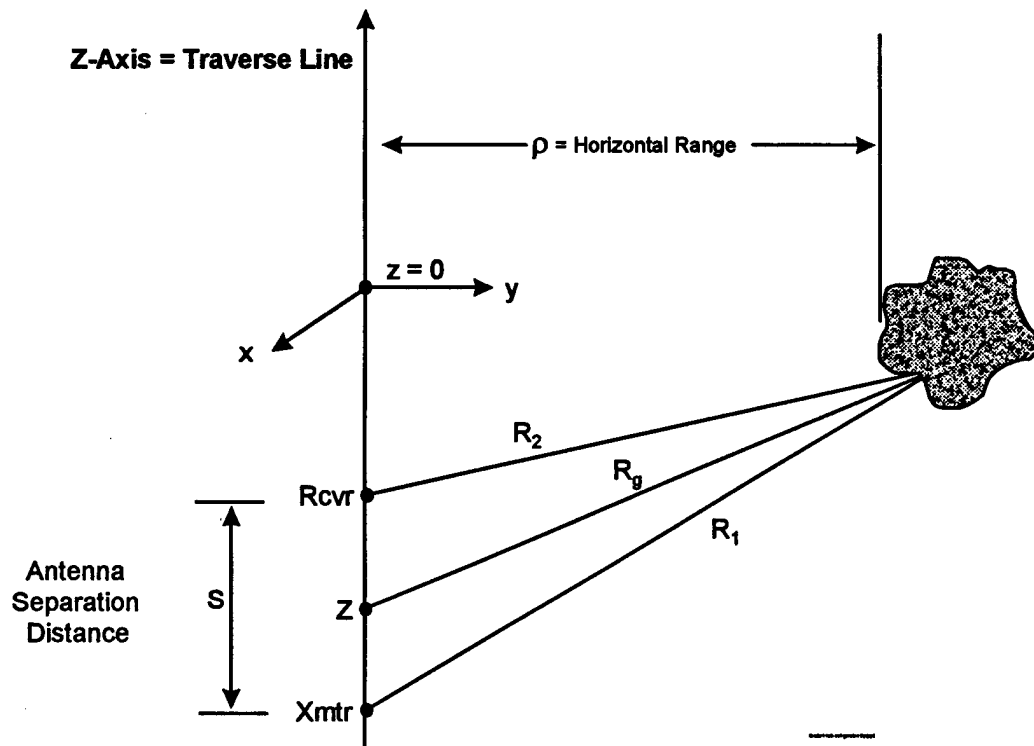
Z-Axis = Traverse Line

ρ = Horizontal Range

z = 0

y

x

R_2

R_g

R_1

Rcvr

z

Xmtr

S

Antenna
Separation
Distance

Figure 1 - Measurement Geometry

Magnitude

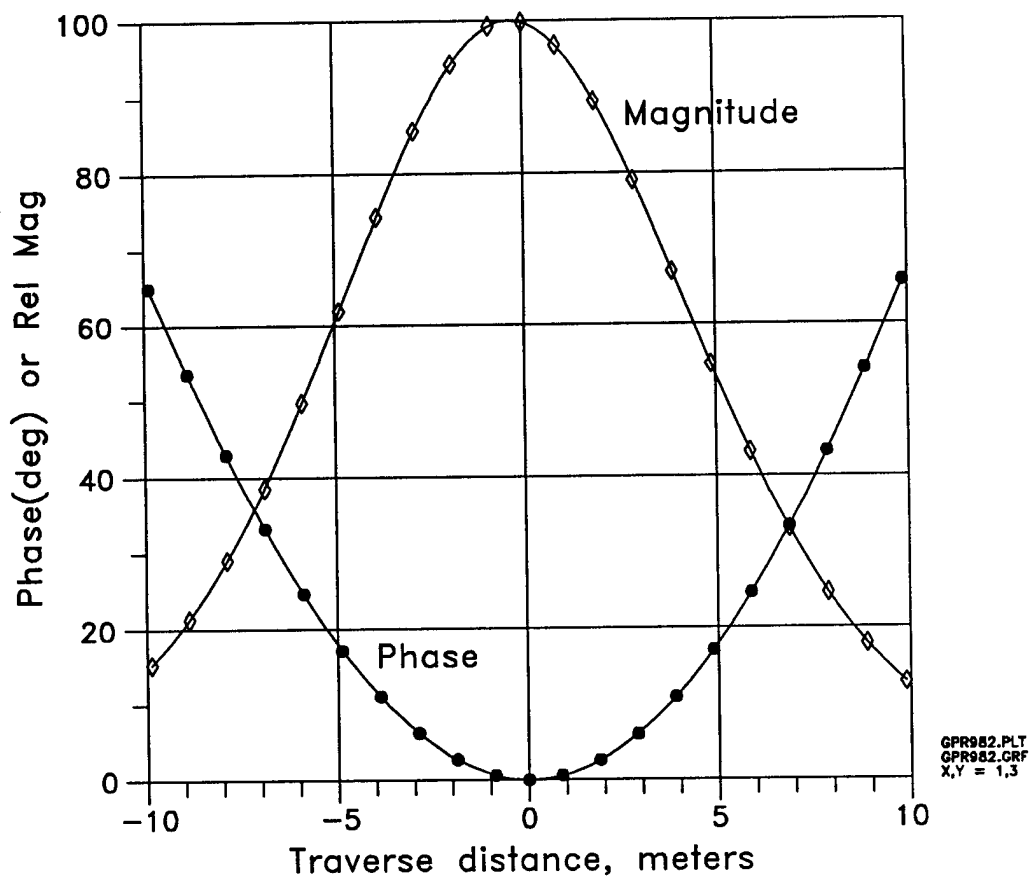Phase

GPR982.PLT
GPR982.GRF
X,Y = 1,3

Traverse distance, meters

Figure 10 — Traverse past a sphere in soil
Phase at center = 109 degrees

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