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## CONTROLLER DESIGN FOR A TELEOPERATOR SYSTEM WITH DISSIMILAR KINEMATICS AND FORCE FEEDBACK\*

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# CONTROLLER DESIGN FOR A TELEOPERATOR SYSTEM WITH DISSIMILAR KINEMATICS AND FORCE FEEDBACK\*

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## ABSTRACT

The purpose of this paper is to develop a controller for dissimilar kinematic teleoperator systems, which include a force/torque sensor mounted on the slave. Due to improved modern microprocessor computing capability and the trend toward redundant slaves, the next generation of teleoperator systems will likely incorporate dissimilar kinematics in their design; consequently, a need exists for a workable control scheme for these systems. The control scheme presented in this paper incorporates the work and ideas of numerous researchers over the past 40 years. The master controller and the orientation representation using Euler parameters for both the master and slave will be the main focus of this paper. The implementation of the master controller on a 6-degrees-of-freedom (DOF) master is also discussed. Only a brief summary of the overall control strategy will be presented.

## INTRODUCTION

In the late 1940s Goertz [6] and his colleagues at Argonne National Laboratory developed one of the earliest recognizable mechanical master/slave manipulators without force-reflection and later with force-reflecting capabilities. Later, in the early 1950s, Goertz and his colleagues developed electric master/slave manipulators where each slave joint servo was tied directly to the master joint servo since both the master and slave were kinematically similar. The control structure for these manipulators was the classical position-position controller. A positional difference between the slave and an object in its environment is reflected back as a drive signal to the master to push the human operator away from the object. The positional-positional control scheme has been the basic controller for almost all master/slave manipulators used by industry up to the present. There is another scheme that originated with Goertz which was never implemented. This scheme was the position/force controller [5]. The basic structure is simple: the slave's controller is a positional controller and the master's controller is a torque or force controller; however, the complexity of the stability problem plus poor torque sensors probably convinced Goertz that the position-force controller was not a

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viable substitute for the positional-positional controller. In the late 1960s, Flatau [3] built a master/slave manipulator using a position-force controller. While aware of the technical difficulties associated with such a scheme, a master/slave manipulator was finally built in 1969 on an extremely limited budget. Some of the basic benefits of such a control scheme were shown, even though the overall performance reliability was poor, simply because this system was based on analog technology and it would be at least 10 years before suitable microprocessor technology would be available to improve reliability. The next generation of teleoperator systems will incorporate both dissimilar kinematics and force feedback loops.

When the master and slave are not kinematically similar, the design of the controller is particularly difficult. There are three major problems associated with this design: (1) orientation representation, (2) accurate and stable force-reflection, and (3) redundancy resolution. Representing the orientation between the master and slave and using this information for force reflection is one of the more difficult problems in the control of any teleoperated system that is not kinematically similar. The objective of this paper is to show how to incorporate Euler parameters (which are related to quaternions) into the controller design. To achieve accurate and stable force-reflection, a type of stiffness controller will be designed for both the master and the slave, plus a state-dependent force-feedback term to ensure stability. Based on the use of both the master and slave Jacobians [9], accurate force reflection for kinematically dissimilar manipulators is possible. When the slave has more than 6 DOF, redundancy resolution needs to be addressed. With dissimilar kinematic designs, simple joint positional differences are no longer adequate for a force-reflecting manipulator. Because there are space limitations and since this research is ongoing, the master manipulator will be the focus of this paper. A brief discussion will be given concerning the slave controller; however, details will be deferred until a later paper. The results are applied to a specific 6-DOF master manipulator and a 7-DOF slave manipulator at Oak Ridge National Laboratory (ORNL).

## DEFINITION OF EULER PARAMETERS

### Difficulties with Present Schemes

Only three variables are needed to represent orientation, implying that there is considerable redundancy in a rotational matrix composed of nine terms. Euler angles (differing from Euler parameters) such as roll, pitch, and yaw have difficulties when applied to teleoperated systems having a master dissimilar from the slave. These difficulties can be summarized as follows: (1) Euler angles introduce artificial singularities, and (2) they are not a natural representation for force reflection. Clearly, another method of representing orientations that does not produce these types of singularities is desirable. Such a scheme is possible with Euler parameters.

### Euler Parameters

Many of the matrix and vector relationships that are stated but not proven can be found in Yuan's work [13] or at least in his references. Let frame A,  $\{A\}$ , and frame B,  $\{B\}$ , be two arbitrary frames that are initially coincident. If  $\{A\}$  is fixed and  $\{B\}$  is rotated about a normalized vector  $\hat{K}$  by an angle  $\theta$  according to

the right-hand rule, then the rotational matrix,  ${}^B R$ , relating a vector in  $\{B\}$  to  $\{A\}$  can be written in terms of  ${}^A \hat{K}$  and  $\theta$ . Defining the following Euler parameters:

$$\epsilon_1 = k_1 \sin(\theta/2) , \quad (1)$$

$$\epsilon_2 = k_2 \sin(\theta/2) , \quad (2)$$

$$\epsilon_3 = k_3 \sin(\theta/2) , \quad (3)$$

$$\epsilon_4 = \cos(\theta/2) , \quad (4)$$

where  ${}^A \hat{K} = [k_1, k_2, k_3]^T$ . Let the first three Euler parameter terms be combined into a vector

$$\hat{\epsilon} = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{bmatrix} , \quad (5)$$

which is given with respect to  $\{A\}$  since  ${}^A \hat{K}$  is given with respect to  $\{A\}$ . In this paper, the Euler parameters will be represented by the set  $\{ \epsilon_4, \hat{\epsilon} \}$ .

Time derivatives of the Euler parameters will be used in the design of the stiffness controller. The Euler parameter rates can be written as:

$$\dot{\epsilon}_4 = -\frac{1}{2} \hat{\epsilon}^T \hat{\omega} , \quad (6)$$

$$\dot{\hat{\epsilon}} = \frac{1}{2} (\epsilon_4 I_3 - \hat{\epsilon}^x) \hat{\omega} , \quad (7)$$

where  $\hat{\omega}$  is the angular velocity vector with respect to  $\{A\}$ , and  $\hat{\epsilon}^x$  in Eq. (7) is a matrix that is a function of the first three Euler parameters and is defined in Yuan [13].

The relative orientation between two rotational matrices can be easily defined in terms of the Euler parameters. Let  ${}^M R$  and  ${}^S R$  be two arbitrary matrices relating frames  $\{M\}$  and  $\{S\}$  to frame  $\{0\}$ , respectively. The rotational matrix  ${}^M S R$  describing the orientational differences between these two frames is

$${}^M S R = ({}^M R)^T {}^S R . \quad (8)$$

The Euler parameters of  ${}^M S R$ ,  $\{ \delta\epsilon_4, \delta\hat{\epsilon} \}$ , can be written in terms of the Euler parameters of  ${}^M R$ ,  $\{ \epsilon_M, \hat{\epsilon}_M \}$ , and  ${}^S R$ ,  $\{ \epsilon_S, \hat{\epsilon}_S \}$  [13] as:

$$\delta\hat{\epsilon} = \epsilon_M \hat{\epsilon}_S - \epsilon_S \hat{\epsilon}_M - \hat{\epsilon}_M^x \hat{\epsilon}_S , \quad (9)$$

and

$$\delta \hat{\epsilon}_4 = \epsilon_M \hat{\epsilon}_S + \hat{\epsilon}_M^T \hat{\epsilon}_S , \quad (10)$$

where  $\delta \hat{\epsilon}$  is with respect to  $\{M\}$ .

## STIFFNESS CONTROLLER USING EULER PARAMETERS

### Master

The master manipulator will incorporate a stiffness controller [11]. The torque signal is

$$\tau_m = J_m^T \{ [K_{pm} (x_s - x_m) + K_{vm} (\dot{x}_s - \dot{x}_m)] \} + \tau_{m \text{ grav}} + v J_{mf}^T F_s . \quad (11)$$

where the  $m$  subscript indicates master terms and

- $J_m$  = master Jacobian,
- $J_{mf}$  = master force-reflecting Jacobian,
- $K_{pm}$  and  $K_{vm}$  = positional and velocity gain matrices (typically diagonal matrices), respectively,
- $\tau_{m \text{ grav}}$  = torque signal to compensate for gravity effects,
- $x_s$  and  $\dot{x}_s$  = slave position and velocity, respectively,
- $v$  = force stability term (to be discussed next),
- $F_s$  = force/torque signal measured at the slave end-effector,
- $x_m$  and  $\dot{x}_m$  = master position and velocity, respectively.

The force stability term,  $v$ , can be either a negative constant or a function of the slave force vector and the master velocity vector, such as  $v = -k_{\text{for}} \dot{x}_m^T F_s$ . In the past,  $v$  has been set to a negative constant, which is physically appealing but has the potential for pumping energy into the system, thereby creating an unstable condition. The advantage of using an expression such as  $v = -k_{\text{for}} \dot{x}_m^T F_s$  is that global stability can be shown [9]; however, the physical interpretation and meaning of this term is lost. Only the case where  $v$  is set to a negative constant will be discussed in this paper.

For the Kraft manipulator, counterbalance weights have been incorporated in its design, making  $\tau_{m \text{ grav}} \equiv 0$ ; consequently, for the rest of the discussion in this paper, it will be assumed to be zero. The external force,  $F_s$ , is the force that is fed back from a force torque sensor located on the wrist of the slave.

### Slave Controller and Dynamics

The slave manipulator considered in this paper has 7 DOF. The slave manipulator will incorporate a stiffness controller [11, 9]. The torque signal is

$$\tau_s = J^T [K_{ps} (x_m - x_s) + K_{vs} (\dot{x}_m - \dot{x}_s)] + \tau_{s \text{ grav}} + \tau_{red} , \quad (12)$$

where the  $s$  subscript indicates slave terms and

- $J^T$  = transpose of the slave Jacobian,
- $K_{ps}$  and  $K_{vs}$  = positional and velocity gain matrices (typically diagonal matrices), respectively,
- $\tau_{s\text{grav}}$  = torque signal to compensate for gravity,
- $x_s$  and  $\dot{x}_s$  = slave position and velocity, respectively,
- $x_m$  and  $\dot{x}_m$  = master position and velocity, respectively,
- $\tau_{red}$  = redundancy torque.

The redundancy torque,  $\tau_{red}$ , will be defined, based on extended task-space techniques [10, 2]. The rationale is to add additional constraints to the system such that the end-effector Jacobian can be extended to full rank. For a detail discussion, see Jansen's work [8].

Assume that feedforward compensation has been incorporated to make  $\tau_{s\text{grav}} \equiv 0$ ; consequently, for the rest of the discussion in this paper it will be set to zero. The redundancy torque,  $\tau_{red}$ , is the signal used to exploit the redundancy of the extra DOF without resorting to pseudoinverse techniques.

If, at steady state, the manipulator is stationary, then it can be shown [8] that the governing equation describing the system dynamic response reduces to

$$K_{ps} (x_s - x_m) + F_{s\text{ext}} - \bar{J}^T \tau_{red} = 0 \quad (13)$$

where  $\bar{J} = M^{-1} J^T (J M^{-1} J^T)^{-1}$  is the generalized inverse that minimizes kinetic energy [12].

Equation (13) indicates that the stiffness seen by the end-effector depends only on the difference between the slave and master positions and the redundancy torque. If  $\bar{J}^T \tau_{red}$  can either be made small or zero, then the slave external force is proportional to the positional differences in Cartesian coordinates in steady state. Again, see the references [8] for details pertaining to why this term will always be small.

The main difficulty with Eq. (12) is in representing the slave position,  $x_s$ , and the master position,  $x_m$ . Both  $x_s$  and  $x_m$  are vectors and have to be at least the dimensions of  $6 \times 1$ , since six pieces of information are required to specify the spatial location and orientation in three-dimensional space. The first three terms of these vectors should be the linear Cartesian position (i.e., the  $x$ ,  $y$ ,  $z$  coordinates).

In Eq. (12), replace the first three terms in  $x_s - x_m$  with  $\Delta x$ . The first three terms in  $\Delta x$  will be the linear Cartesian position difference between the slave and master with respect to the base frame. The next three variables in  $x_s - x_m$ , as proposed in this paper, should be the  $\hat{\delta\epsilon}$  vector. The stiffness controller of the master and slave will be modified to include Euler parameters

$$\tau_m = J_m^T \left\{ K_{pm} \begin{bmatrix} \Delta \hat{x} \\ \Delta \hat{\dot{x}} \\ \Delta \hat{\ddot{x}} \end{bmatrix} + K_{vm} \begin{bmatrix} \Delta \hat{x} \\ \Delta \hat{\dot{x}} \\ \Delta \hat{\ddot{x}} \end{bmatrix} \right\} + \tau_{m \text{grav}} + v J_m^T F_s , \quad (14a)$$

$$\tau_s = - J_s^T \left\{ K_{ps} \begin{bmatrix} \Delta \hat{x} \\ \Delta \hat{\dot{x}} \\ \Delta \hat{\ddot{x}} \end{bmatrix} + K_{vs} \begin{bmatrix} \Delta \hat{x} \\ \Delta \hat{\dot{x}} \\ \Delta \hat{\ddot{x}} \end{bmatrix} \right\} + \tau_{s \text{grav}} + \tau_{red} . \quad (14b)$$

In the control algorithm, there are two Jacobians: the *force-reflecting* Jacobian,  $J_{mf}$ , and the *master* Jacobian,  $J_m$ . The *force-reflecting* Jacobian is the standard manipulator Jacobian [11]. The *force-reflecting* Jacobian is a transformation relating the joint rates to the Cartesian rates. The *force-reflecting* Jacobian of the master can be written as

$$J_{mf} \dot{q} = \begin{bmatrix} \dot{x}_m \\ \hat{\omega}_m \end{bmatrix} , \quad (15)$$

where  $\dot{q}$  (6×1 vector) is the joint actuator rate and  $\hat{\omega}$  is the angular velocity vector (3×1 vector) with respect to the base frame. The master Jacobian differs from the *force-reflecting* Jacobian since the angular rates will be based on Euler parameters. Define the (3×3) matrix,  $W$ , which relates the angular velocities  $\hat{\omega}$  to  $\hat{\dot{\omega}}$ , (i.e.,  $\hat{\dot{\omega}} = W \hat{\omega}$ ) [8]. The master Jacobian,  $J_m$ , is simply

$$J_m = \begin{bmatrix} I_3 & 0 \\ 0 & W \end{bmatrix} J_{mf} . \quad (16)$$

Similarly, the slave Jacobian,  $J_s$ , and slave *force-reflecting* Jacobian,  $J_{sf}$ , can be defined:

$$J_{sf} \dot{q} = \begin{bmatrix} \dot{x}_s \\ \hat{\omega}_s \end{bmatrix} , \quad (17)$$

$$J_s = \begin{bmatrix} I_3 & 0 \\ 0 & W \end{bmatrix} J_{sf} , \quad (18)$$

where

$$W = 0.5 \left[ \hat{\epsilon}_s \hat{\epsilon}_M^T + (\epsilon_s I_3 - \hat{\epsilon}_s^x)(\epsilon_M I_3 - \hat{\epsilon}_M^x) \right] . \quad (19)$$

It can be shown that  $W$  has no artificial singularities [8]. Stability of the proposed controller can be shown, based on Liapunov stability methods [8].

## DISCUSSION AND APPLICATION TO THE KRAFT MASTER

### Hardware

The controller was implemented on the Kraft master controller, shown schematically in Fig. 1. Since the master is far from an ideal master, suitable compensation was required to achieve the desired performance. The Kraft KMC 9100-MC is a lightweight 6-DOF master arm designed, manufactured, and sold by Kraft Telerobotics, Inc., of Overland Park, Kansas. Position is measured at each joint by potentiometers. The first five joints are actuated by ac servomotors for force feedback and the wrist roll is not actuated.

### Implementation

The control algorithm was programmed in the C language on a Motorola 68020 with a 68881 floating-point coprocessor. The control algorithm was optimized by factoring the Jacobians so that common terms were not recalculated and by a special assembly language routine that determines the sine and cosine of each joint angle, simultaneously. When implemented, the master code ran at ~60 Hz, including the communication overhead.

Torque vs applied signal was measured in the laboratory for each of the five actuated joints on the Kraft master. Experimental results show that all of the joints required ~10 to 20% of the full-scale signal to move. To optimize between good backdrivability and force sensitivity when using a master with this amount of large "dead" region, some compensation is required. A simple form of compensation is an offset function, called the preload function (PLF), which is the inverse of a deadband function [4].

An analysis was performed using descriptive function methods to determine if limit cycles (a limit cycle defined as an initial condition-independent periodic oscillation occurring in dissipative systems [4]) would be present when applying the preload function. The results indicated that, as long as there were not significant amounts of backlash, then no limit cycle would occur. This is true for the Kraft master arm.

The PLF was implemented on the master controller. For most of the joints, the preload was set to ~10% except for wrist pitch and yaw. For these joints, the preload was reduced to ~5% to avoid a chattering at the switching line between plus and minus preload. The PLF did not introduce any instabilities and it significantly improved the force reflection.

## GAIN SELECTION FOR THE STIFFNESS CONTROLLERS

While both the master and the slave incorporate a type of stiffness controller, the purpose differs from robotic operation. For robotic stiffness control, the interaction between the environment and the robot arm is specified [1]. For teleoperation, the purpose is to reflect the environmental forces and stiffness accurately to the human operator. The human operator will vary impedance according to changes in the slave impedance [7]. To achieve this, the positional gain matrices for both the master and slave,  $K_{pm}$ , and,  $K_{ps}$ , will be tuned so that they are made large but not so large as to produce limit cycles [4].

Denavit-Hartenberg Table

$\theta_i$ (deg)	$d_i$ (m)	$\alpha_{i-1}$ (deg)	$a_{i-1}$ (m)	$\theta_i$ (deg)
$\theta_1$	0	0	0	shown 0
$\theta_2$	$d_2$	90°	0	+90
$\theta_3$	0	0	$a_2$	-90
$\theta_4$	$d_4$	-90°	$a_3$	0
$\theta_5$	0	90°	0	+90
$\theta_6$	$d_6$	90°	0	0

Note:  $d_2 = 0.073$  m,  $d_4 = 0.084$  m,  
 $d_6 = 0.114$  m  
 $a_2 = 0.178$  m,  $a_3 = 0.203$  m

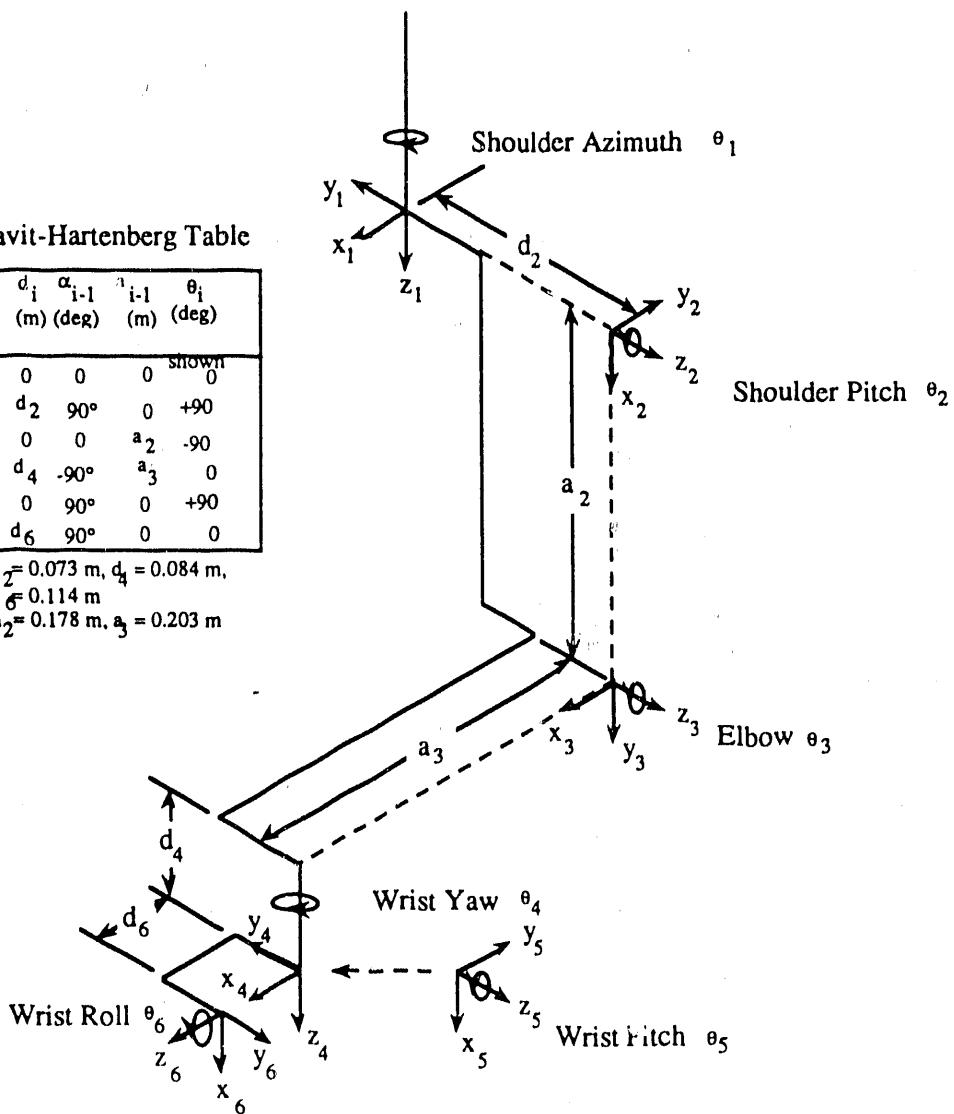


Fig. 1. Kinematic diagram and Denavit-Hartenberg table for the Kraft master controller.

## CONCLUSION AND SUMMARY

This paper presents a formulation of a controller for a teleoperator system with dissimilar kinematics and force feedback. The controller is a stiffness controller for both the master and the slave. A mathematical problem associated with representing orientations using Euler angles has been described and Euler parameters are proposed as an alternative. Euler parameters are superior to Euler angles, not only because they do not introduce artificial singularities, but also because they are a natural representation for force reflection. Basic properties of Euler parameters are presented, specifically those that pertain to stiffness control. The stiffness controller for both the master and the slave is formulated using the Euler parameters to represent orientation. Stability can be shown as well for the proposed controller.

The master controller is presently implemented on a 6-DOF, force-reflecting Kraft master manipulator and runs at a loop rate of ~60Hz. The stiffness controller has worked well on the Kraft master manipulator. With the Euler parameter formulation, artificial singularities were not present, unlike the Euler angle formulation. Further, the magnitude (Euclidean norm) of  $\hat{\delta\epsilon}$  is proportional to the sine of the half angular difference between frames, as can be seen in Eqs. (1) through (3). For small angular displacements,  $\hat{\delta\epsilon}$  is proportional to the angular difference between frames, which is the feel an operator desires. As the angular displacements increase, the sine function will act as a saturation function, thus preventing feedback of excessive forces.

The controller was capable of being tuned to produce the "feel" of Cartesian springs attached at a "home" location. The controller could also create artificial walls and surfaces defined in Cartesian space to restrict operation in "forbidden zones." With proper gain settings, these surfaces could be given a "repelling" feel such that the operator would have to exert a strong force to pass through the surface. This ability makes this control algorithm highly useful for obstacle avoidance when operating in a cluttered environment. Dangerous obstacles can be defined in the master's Cartesian space such that the operator cannot move the master to a location that would drive the slave into the obstacle. A future paper will discuss the overall performance of the controller with particular emphasis on slave performance.

## REFERENCES

1. C. H. An, C. G. Atkeson, and J. M. Hollerbach, Model-Based Control of a Robotic Manipulator (MIT Press, Cambridge, Massachusetts 1988).
2. R. Colbaugh, H. Seraji, and K. L. Glass, "Obstacle Avoidance for Redundant Robots Using Configuration Control", *Journal of Robotic Systems* 6 (6), 721-744 (1989).
3. C. R. Flatau, "Compact Servo Master-Slave Manipulator with Optimized Communication Links", *Proceedings of the 17<sup>th</sup> Conference on Remote Systems Technology*, 154-164 (1969).
4. A. Gelb and W. E. V. Velde, Multiple-Input Describing Functions and Nonlinear System Design (McGraw-Hill, Inc., New York, N.Y., 1968).
5. R. C. Goertz and F. Bevilacqua, "A Force-Reflecting Positional Servomechanism", *Nucleonics*, 43-44 (November 1952).
6. R. C. Goertz and W. M. Thompson, "Electronically Controlled Manipulator", *Nucleonics*, 46-47 (November 1954).
7. N. Hogan, "Impedance Control: An Approach to Manipulation: Part 1-3 Theory", *Journal of Dynamic Systems, Measurement, and Control*, 107, 1-24 (March 1985).
8. J. F. Jansen, R. L. Kress, and S. M. Babcock, "Controller Design for a Force-Reflecting Teleoperator System with Kinematically Dissimilar Master and Slave", *Proceedings of Dynamic Systems, Measurement, and Control*, (in publication, 1990).
9. F. Miyazaki, S. Matsubayashi, T. Yoshimi, and S. Arimoto, "A New Control Methodology Toward Advanced Teleoperation of Master-Slave Robot Systems", *1986 IEEE International Conference on Robotics and Automation*, 997-1002.

10. S. Y. Oh, D. Orin, and M. Bach, "An inverse Kinematic solution for kinematically redundant Robot Manipulators", *Journal of Robotic Systems*, 1, 235-249 (1984).
11. J. K. Salisbury, "Active Stiffness Control of a Manipulator in Cartesian Coordinates", *IEEE Conference Decision and Control*, Albuquerque, New Mexico, 95-100 (November 1980).
12. D. E. Whitney, "The Mathematics of Coordinated Control of Prosthetic Arms and Manipulators", *ASME Journal of Dynamic Systems Measurement and Control* (303-309).
13. Joseph S.-C. Yuan, "Closed-Loop Manipulator Control Using Quaternion", *IEEE Transactions of Robotics and Automation* 4, (4) 434-440 (August 1988).

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