

F-1888

8-1

PROCEEDINGS OF THE
THIRD TOPICAL CONFERENCE ON
RADIO FREQUENCY PLASMA HEATING

FG03-78ET 53039

MASTER

HELD AT THE

CALIFORNIA INSTITUTE OF TECHNOLOGY

PASADENA, CALIFORNIA

JANUARY 11 - 13, 1978

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

PROCEEDINGS OF THE
THIRD TOPICAL CONFERENCE ON
RADIO FREQUENCY PLASMA HEATING

held at the
CALIFORNIA INSTITUTE OF TECHNOLOGY
Pasadena, California

January 11-13, 1978

Additional copies may be ordered at a
cost of \$10.00 from:

Plasma Laboratory, M.S. 116-81
California Institute of Technology
Pasadena, California 91125

DISCLAIMER

This book was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

Handwritten signature/initials

TABLE OF CONTENTS

THIRD TOPICAL CONFERENCE ON RADIO FREQUENCY PLASMA HEATING

Session A: Lower Hybrid Heating - I

January 11/78 - 8:30 A.M.

M. KRISTIANSEN presiding

- A1 - Theory of Plasma Heating in the Lower-Hybrid Range of Frequencies (LHRF)
ABRAHAM BERS, Massachusetts Institute of Technology (30 min)
- A2 - A Survey of the Status of Lower Hybrid Heating Experiments
W. M. HOOKE, Princeton University (30 min)
- A3 - Heating of Ions by Lower Hybrid in the WEGA Tokamak
WEGA GROUP (30 min)
- A4 - Lower Hybrid Wave Heating Experiments at General Atomic
CHARLES MOELLER and DOUBLET IIA GROUP, General Atomic Co.
- A5 - Effect of Lower Hybrid Waves on the Evolution of the Doublet IIA Plasma
D. F. VASLOW, N. H. BROOKS, R. L. FREEMAN, E. J. HECKMAN, T. H. JENSEN,
J. LOHR, J. L. LUXON, A. MAHDAVI, C. P. MOELLER, J. C. RIORDAN, R. D.
STAV and J. F. TOOKER, General Atomic Co.
- A6 - Demonstration of Accessibility and Inaccessibility of Slow Plasma Waves
R. W. MOTLEY, S. BERNABEI, W. M. HOOKE, R. McWILLIAMS and L. OLSON,
Princeton University
- A7 - Limitations on Lower Hybrid Heating of Dense, Hot Plasmas
R. W. HARVEY, J. M. RAWLS, M. P. HACKER and V. C. CHAN, General Atomic Co.
- A8 - Simulation of Lower-Hybrid Heating in a Bounded Plasma
V. K. DECYK, J. M. DAWSON and G. J. MORALES, U.C.L.A.

Session B: Lower Hybrid Heating - II

January 11/78 - 2:00 P.M.

G. MORALES presiding

- B1 - Electric Field Distribution Associated with Lower Hybrid Heating of
Tokamak Plasmas
M. BRAMBILLA, CEA Grenoble
- B2 - Coupling of Lower Hybrid Waves in the Doublet IIA High Power RF Heating
Experiments
V. S. CHAN, C. MOELLER and R. W. GOULD, General Atomic Co.
- B3 - Lower Hybrid Experiments in the Petula Tokamak
C. M. SINGH, P. BRIAND and L. DUPAS, CEN Grenoble
- B4 - Lower Hybrid Wave Heating in the DC Octopole
P. I. PETERSEN, V. S. CHAN, S. EJIMA, R. J. LA HAYE, C. P. MOELLER and
J. C. WESLEY, General Atomic Co.
- B5 - Parametric Decay of Lower Hybrid and Ion Cyclotron Waves of Finite
Wavenumber in a Plasma
V. K. TRIPATHI, C. S. LIU and C. GREBOGI, University of Maryland

- B6 - Ergodic Behavior of Lower Hybrid Decay Wave Trajectories
E. OTT, J.-M. WERSINGER and J. FINN, Cornell University
- B7 - Harmonic Generation in Lower-Hybrid Heating
J. L. SPERLING and R. W. HARVEY, General Atomic Co.
- B8 - Effect of Convective Loss on Resonant Decay of Cold Lower Hybrid Waves
K. L. WONG, J. R. WILSON, Princeton U. and M. PORKOLAB, M.I.T.
- B9 - RF Coupling near the Lower Hybrid Resonance
R. B. MICHIE, M. C. CLARK and M. E. OAKES, U. of Texas, Austin
- B10 - Lower Hybrid RF Heating of the MIT Versator II and Alcator-C Tokamaks
M. PORKOLAB, G. BEKEFI and R. PARKER, M.I.T. and S. BERNABEI, R. MOTLEY and W. M. HOOKE, Princeton U.
- B11 - Proposal of Lower Hybrid Heating Experiment on the Frascati Tokamak
F. DE MARCO and F. SANTINI, CNEN Frascati.
- B12 - Ion Heating by Ion Cyclotron Parametric Instability in a Multispecies Plasma
K. YATSUI, M. SHIMADA, S. OKAMOTO and M. YOKOYAMA, Osaka U.

Session C: Ion Cyclotron Heating - I

January 12/78 - 8:30 A.M.

L. SHOHET presiding

- C1 - Plasma Heating in the Ion-Cyclotron Range of Frequencies
F. W. PERKINS, Princeton U. (30 min)
- C2 - Experimental Aspects of RF Heating of Tokamak Plasmas in the Ion Cyclotron Range of Frequencies (ICRF)
J. C. HOSEA, Princeton U. (30 min)
- C3 - Ion Cyclotron Heating by Circularly Polarized RF Fields
Y. YASAKA, S. KOMORI and R. ITATANI, Kyoto U.
- C4 - Preliminary RF Experiments on the ERASMUS Tokamak
F. P. BHATNAGAR, G. BOSIA, M. BURES, A. M. MESSIAEN, G. TELESKA, P. E. VANDENPLAS, R. R. WEYNANTS, Euratom (Ecole Royale Militaire)
- C5 - Efficiency of Antenna Coupling to the Fast Alfvén Modes in Macrotron
R. J. TAYLOR and G. J. MORALES, U.C.L.A.
- C6 - Fast Magnetosonic Modes in the Caltech Tokamak
D. Q. HWANG and R. W. GOULD, Caltech
- C7 - Fast Wave Generation in the PLT Tokamak
P. L. COLESTOCK, J. C. HOSEA, F. J. PAOLONI, H. R. THOMPSON and H. TAKAHASHI, Princeton U.
- C8 - Energy Loss and Edge Density Measurements during ICRF Experiments on ATC
H. HSUAN, R. J. HAWRYLUK, S. SUCKEWER and H. TAKAHASHI, Princeton U.
- C9 - Modification of Plasma Edge Temperature by RF Heating
S. SUCKEWER and R. J. HAWRYLUK, Princeton U.

Session D: Ion Cyclotron Heating - II

January 12/78 - 2:00 P.M.

A. BERS presiding

- D1 - Ion Cyclotron Heating in the Wisconsin Supported Toroidal Octupole and Quadrupole
A. P. BIDDLE, K. J. MILLER, J. C. SPROTT, U. of Wisconsin
- D2 - The Parametric Instabilities near the Ion Cyclotron Range of Frequency in a Single and Multi-Ion Species Plasma
M. ONO, Princeton U., M. PORKOLAB, M.I.T., R. P. H. CHANG, Bell Labs
- D3 - Alfvén Wave Heating with High-Q Eigenmodes
C.F.F. KARNEY and F. W. PERKINS, Princeton U.
- D4 - Heating of the Various Plasma Species of a Tokamak via the Ion-Ion Hybrid Resonance
J. JACQUINOT, EUR-CEA, Fontenay
- D5 - Mode Conversion and Absorption in a Deuterium Plasma with a Hydrogen Impurity
D. G. SWANSON, U. So. California
- D6 - Fast Wave Heating via Mode Conversion in the Ion Cyclotron Range of Frequencies (ICRF)
B. D. McVEY and J. E. SCHARER, U. of Wisconsin
- D7 - Scaling of Ion Cyclotron Frequency Range Heating to Reactor Size
J. SCHARER, J. BEYER and D. T. BLACKFIELD, U. of Wisconsin

Session E: Lower Hybrid Heating - III

January 12/78 - 3:45 P.M.

W. HOOKE presiding

- E1 - Reevaluation of the Effect of Damping on Linear Mode Conversion
P. M. BELLAN, Caltech
- E2 - Collisional Absorption of Lower Hybrid Waves near the Mode Conversion Layer
J. J. SCHUSS, M. PORKOLAB and R. R. PARKER, M.I.T.
- E3 - Ion-Cyclotron-Harmonic Heating Using the Lower-Hybrid Wave
S. PURI, Max-Planck, Garching
- E4 - A Lower Hybrid Heating System for a Tokamak Reactor
J. BROOKS, S. HARKNESS, J. JUNG, B. MISRA, A. MORETTI, J. NOREN, H. STEVENS, Argonne Natl. Lab.
- E5 - A Steady State Toroidal Reactor Driven by Microwave Power in the Lower-Hybrid Range of Frequencies
A. BERS and N. J. FISCH, M.I.T.
- E6 - Current Generation by High Power RF Fields
N. J. FISCH and A. BERS, M.I.T.

Session F: Alfvén Wave, TTMP, and Other Heating

January 13/78 - 8:30 A.M.

T. JENSEN presiding

- F1 - Transit Time Magnetic Pumping in Hydrogen and Deuterium Discharges
PETULA GROUP, CEN, Grenoble (30 min)
- F2 - Advances in Plasma Heating with Alfvén Waves
J. L. SHOHET, U. of Wisconsin (30 min)
- F3 - Technical Aspects of Alfvén Wave Heating
A. HASEGAWA, Bell Labs
- F4 - Non-Ideal Effects on Alfvén Wave Heating
G. CONN, Michigan State and J. A. TATARONIS, N.Y.U.
- F5 - The Continuous MHD Spectrum for Alfvén Wave Heating in a Straight Stellarator
J. N. TALMADGE and J. L. SHOHET, U. of Wisconsin
- F6 - Resonant Magnetic Pumping at Very Low Frequency
N. CANOBBIO, CEA Grenoble
- F7 - ECR Upper-Hybrid Heating in the Culham Levitron
A. C. RIVIERE, M. W. ALCOCK and T. N. TODD, Culham Lab
- F8 - Magnetoacoustic Oscillations of a Cylindrical Plasma Column
B. BLACKWELL, G. COLLINS, R. CROSS and J. LEHANE, U. of Sydney
- F9 - Microwave Absorption Studies near the Plasma Frequency
B. L. WRIGHT, M. E. BANTON, H. DREICER, J. C. INGRAHAM, Los Alamos

Session G: General Theory Relating to Heating

January 13/78 - 2:00 P.M.

P. BELLAN presiding

- G1 - Stochastic Heating of an Ion Beam by Lower Hybrid Waves
E. LAZZARO, Lab. Fisica del Plasma, Milan
- G2 - Stochastic Plasma Heating by Localized RF Fields
H. SUGAI and S. TAKEDA, Nagoya U.
- G3 - Whistler Cavity Eigenmodes in Tokamaks
C. CHU and J. L. SPERLING, General Atomic Co.
- G4 - Universal Formula for Quasistatic Second-Order Density Perturbation by a Cold Magnetoplasma Wave
A. N. KAUFMAN, J. R. CARY and N. R. PEREIRA, Lawrence-Berkeley Lab
- G5 - Ponderomotive Effects in a Magnetized Plasma
V. KRAPCHEV and A. BERS, M.I.T.
- G6 - Three-Dimensional Effects on Nonlinear Mode-Converted Lower-Hybrid Waves
H. H. KUEHL, U. So. California
- G7 - Complex Modified K-DV Equation and Nonlinear Propagation of Lower Hybrid Waves
A. SEN, C.F.F. KARNEY and A. BERS, M.I.T. and N. R. PEREIRA, Lawrence-Berkeley Lab

- G8 - Energy Flow and Tunnelling of Resonance Cones near the Lower Hybrid
C. L. GRABBE, Caltech
- G9 - The Effect of Field Gradients on Wave Propagation in Tokamaks
K. A. CONNOR, Rensselaer, and P. L. COLESTOCK, Princeton U.

LIST OF PARTICIPANTS

THEORY OF PLASMA HEATING IN THE LOWER HYBRID RANGE OF FREQUENCIES (LHRF)

ABRAHAM BERS

Plasma Fusion Center
Massachusetts Institute of Technology
Cambridge, Mass. 02139

We review the status of theoretical understanding of various problems relevant to the supplementary heating of tokamak-type plasmas with rf power in the LHRF ($\Omega_i \ll \omega \sim \omega_{pi} \ll \Omega_e$).

I. Linear Coupling and Propagation

The linear excitation of fields in the plasma can be described in terms of the waves that can propagate from the vacuum boundary into the plasma. The cold-plasma dispersion relation is¹

$$K_{\perp} n_{\perp}^4 + [(K_{\parallel} + K_{\perp})(n_{\parallel}^2 - K_{\perp}) + K_x^2] n_{\perp}^2 + K_{\parallel} [(n_{\parallel}^2 - K_{\perp})^2 - K_x^2] = 0 \quad (1)$$

where $n_{\parallel} = (ck_{\parallel}/\omega)$ and $n_{\perp} = (ck_{\perp}/\omega)$ are the indices of refraction parallel and perpendicular, respectively, to $\bar{B}_0$, and for $\Omega_i \ll \omega \ll \Omega_e$ we have

$$K_{\parallel} \approx 1 - \frac{\omega_{pe}^2}{\omega^2} \quad (2); \quad K_{\perp} \approx 1 + \frac{\omega_{pe}^2}{\Omega_e^2} - \frac{\omega_{pi}^2}{\omega^2} \quad (3); \quad K_x \approx \frac{\omega_{pe}^2}{\omega \Omega_e} \quad (4)$$

For plasma parameters (2-4) that vary slowly with position compared to the wavelength, (1) can be used to describe the local propagation characteristics. In a two-dimensional model we let $\bar{B}_0$ be along the homogeneous $\hat{z}$ -direction and solve (1) for propagation in the $\hat{x}$ -direction in which the plasma is weakly inhomogeneous. Thus, at a frequency ω , any excited n_z -spectrum remains unchanged and (1) can be solved for n_x as a function of x with n_z a parameter. An examination of the Allis-(CMA) diagrams¹ for the frequency regime of interest reveals that inside the plasma ($\omega_p \gg \omega$) there are only waves with phase velocities smaller than c . Since at the plasma edge--vacuum-- $n_x^2 = 1 - n_z^2$, wave propagation to the interior of the plasma requires that the excited spectrum have $n_z^2 > 1$. A detailed analysis gives the lowest n_z for which the n_x^2 roots of (1) are positive-real to the interior of the plasma, the so-called accessibility n_z^2 :

$$n_{z0}^2 = \left(\sqrt{K_{\perp}} + \frac{K_x}{\sqrt{K_{\parallel}}} \right)^2 \quad (5)$$

Assuming $\bar{B}_0$ is constant and only the plasma density varies with x , one can distinguish two cases:^{3,4,5}

(a) $\omega^2 < \omega_h^2 = \Omega_e \Omega_i$, and $\left(\frac{\omega_{pim}}{\omega} \right)^2 > \frac{(\omega/\omega_h)^2}{1 - (\omega/\omega_h)^2}$, for which

$$n_{z0}^2 = \frac{1}{1 - (\omega/\omega_h)^2} \quad (6)$$

(b) $\omega^2 < \omega_h^2$ and $\left(\frac{\omega_{pim}}{\omega} \right)^2 < \frac{(\omega/\omega_h)^2}{1 - (\omega/\omega_h)^2}$, or $\omega^2 > \omega_h^2$, for which

$$n_{z0}^2 = 1 + 2\left(\frac{\omega_{pem}}{\Omega_e}\right)^2 - \left(\frac{\omega_{pim}}{\omega}\right)^2 + 2\left(\frac{\omega_{pem}}{\Omega_e}\right) \sqrt{1 + \left(\frac{\omega_{pem}}{\Omega_e}\right)^2 - \left(\frac{\omega_{pim}}{\omega}\right)^2} \quad (7)$$

where the subscript m indicates evaluation at the maximum density. For $n_z > n_{z0}$, two types of waves can propagate to the interior of the plasma. These correspond to the two distinct n_x^2 solutions of (1), the larger of which we call "slow" branch and the smaller "fast" branch.

The waves on the slow-branch of n_x^2 are essentially electrostatic plasma waves (EPW). For $n_z \gg n_{z0}$ their cold-plasma dispersion relation is approximately

$$n_x^2 \approx \frac{-K_{\parallel}}{K_{\perp}} n_z^2 \quad (8)$$

and the relative E -field components are $\bar{E} \sim E_x (1; 0; (K_{\perp}/K_{\parallel})^{1/2})$. Their group velocity ray-trajectories, obtainable from $v_{gz}/v_{gx} = -(\partial n_x^2 / \partial n_z) = (-K_{\parallel}/K_{\perp})^{1/2}$, are independent of n_z and can be used to readily locate and evaluate the fields in the plasma from a finite extent excitation.^{6,7} The power-flow density in these waves is given by

$$S_x \approx \frac{\epsilon_0 |E_x|^2}{2} \frac{\omega K_{\perp}}{k_x} \quad (9)$$

The cold-plasma dispersion relation (8) predicts a resonance in propagation ($n_x^2 \rightarrow \infty$) when $K_{\perp} \rightarrow 0$, i.e. at $\omega = \omega_{pi}/(1 + \omega_{pe}^2/\Omega_e^2)^{1/2} \equiv \omega_{LH}$, the lower-hybrid frequency. The leading-order corrections to (8), from a Vlasov description, due to finite electron and ion temperature effects modify the dispersion and introduce Landau-damping of the waves.⁸ The real part of the dispersion relation is then

$$-ak_x^4 + K_{\perp} k_x^2 + K_{\parallel} k_z^2 = 0 \quad (10)$$

where

$$a = 3\left(\frac{v_{Ti}}{\omega}\right)^2 \left(\frac{\omega_{pi}}{\omega}\right)^2 + \frac{3}{4} \left(\frac{v_{Te}}{\Omega_e}\right)^2 \left(\frac{\omega_{pe}}{\Omega_e}\right)^2 \quad (11)$$

and $v_T^2 = (\kappa T/m)$. Equation (10) shows that in addition to the cold plasma branch there is an ion-thermal branch to k_x^2 . For a given ω and k_z these merge at a plasma density given by

$$K_{\perp}^2 = 4ak_z^2(-K_{\parallel}) \quad (12)$$

where a reflection or wave-conversion will occur. Energy propagating into the plasma on the cold plasma branch can turn around and propagate back out on the thermal branch. Thus, in a hot plasma, the lower-hybrid resonance layer is never accessible. The position x_{wc} at which wave conversion occurs is from (12)

$$\omega_{LH}^2(x_{wc}) = \omega^2 \left[1 + 2\sqrt{3} \frac{k_z v_{Te}}{\omega} \left(\frac{T_i}{T_e}\right)^{1/2} \left(1 + \frac{1}{4} \frac{T_e}{T_i} \frac{\omega^4}{\omega_h^4}\right)^{1/2} \right]^{-1} \quad (13)$$

The wave conversion layer may exist in a plasma even if the resonance layer does not (i.e. $\omega > \omega_h$), thus x_{wc} can be made to occur near the maximum density, which as we shall see is desirable for heating. The linear damping of slow-branch waves is mainly by electron-Landau damping. Ion-cyclotron-harmonic damping is by comparison negligible for most cases of interest, except very near an ion-cyclotron harmonic;⁹ however, the problem of the two wave conversions (ion-plasma and Bernstein) in the presence of damping and $B_0(x)$ has not been solved fully. The local, spatial electron-Landau damping is

given by the imaginary part of k_x , obtained by perturbation analysis applied to the Vlasov description of waves,^{10,11}

$$k_{xi} = \sqrt{\pi} k \zeta^3 \exp(-\zeta^2) \quad (14)$$

where $\zeta = (\omega/\sqrt{2} k_z v_{Te})$ and $k = (k_x^2 + k_z^2)^{1/2}$ is obtained from (10). The exponential dependence in (14) shows that the temperature profile has a profound effect on where a particular k_z (i.e. n_z) of the excited spectrum becomes strongly damped; large k_z will damp on the outside while too small a k_z may not damp at all. Taken together--accessibility wave conversion, and electron Landau damping--these considerations make it very difficult to penetrate the slow-branch waves to the center of plasmas with temperatures exceeding $T_e \approx 5 \text{ keV}$.^{3,5}

The waves on the fast branch of n_x^2 have very favorable penetration characteristics for hot plasmas. Choosing $\omega^2 < \omega_h^2$ (6) predicts that n_{z0}^2 can be close to unity. If in addition $(n_z^2 - 1) < (\omega_{pi}/\omega)^2$ the approximate dispersion relation of the fast branch is that of the high-frequency Alfvén wave (HFAW)

$$n_x^2 \approx \left(\frac{c}{c_A}\right)^2 = \left(\frac{\omega_{pi}}{\Omega_i}\right)^2 \quad (15)$$

where $c_A = B_0/(\mu_0 n_0 m_i)^{1/2}$ is the Alfvén speed, and $\bar{E} \sim E_x(1; -i\Omega_i/\omega; n_z \omega^2/\omega_{pi} \Omega_e)$. For this wave $v_{gz}/v_{gx} \approx n_z \omega^2/\omega_{pi} \Omega_i$, and the power-flow density and spatial electron-Landau damping rate are

$$S_x \approx \frac{\epsilon_0 |E_x|^2}{2} c_A \left(\frac{\omega_{pi}}{\omega}\right)^2 \quad (16); \quad k_{xi} = k_x \left(\frac{\omega^2}{\omega_{pi} \omega_h}\right)^2 n_z^2 \sqrt{\pi} \zeta^3 e^{-\zeta^2} \quad (17)$$

For higher-frequencies ($\omega^2 \gtrsim \omega_h^2$) and larger n_z , specifically $n_z^2 \gg K_\perp + (\omega_{pe}/\Omega_e)^2$, the approximate dispersion relation of the fast-branch is that of the whistler wave (WW) at an angle ($n_x^2 \gg n_z^2$) to $\bar{B}_0$,

$$n_x^2 \approx \frac{K_x^2}{n_z^2} \quad (18)$$

For this wave $\bar{E} \sim E_x(i; -i n_z^2/K_x; \omega/\Omega_e)$, $v_{gz}/v_{gx} \approx K_x/n_z^2$, and

$$S_x \approx \frac{\epsilon_0 |E_x|^2}{2} \frac{\omega}{k_x} n_z^2 \quad (19); \quad k_{xi} \approx k_x \left(\frac{\omega_{pe}}{\Omega_e n_z}\right)^2 \sqrt{\pi} \zeta^3 e^{-\zeta^2} \quad (20)$$

are the power-flow density and electron Landau damping rates, respectively. Comparing the propagation of these three types of waves inside an inhomogeneous plasma, we note from the above equations that: (a) The EPW E -fields remain spatially confined, and penetrate slowly; $|E_z| \sim n_0^{-1/4}$ while $|E_x| \sim n_0^{1/4}$; they can obtain the largest k_x and k_{xi}/k_x . (b) The HFAW E -fields penetrate rapidly but disperse spatially; $|E_x|$ and $|E_y| \sim n_0^{-1/4}$ while $|E_z| \sim n_0^{-3/4}$. (c) The WW E -field structure is in between the above two; $|E_x|$ and $|E_z| \sim n_0^{1/2}$ while $|E_y| \sim n_0^{-1/2}$. In each of the three types of waves we have $k_x \gg k_z$ and $|E_x|$ is the largest field component. In addition, in both of the fast waves $|E_y| \gg |E_z|$.

For $1 < n_z < n_{z0}$ the fast and slow branches couple.^{11,12} For the conditions of (6) this coupling occurs inside the density profile; the fields do not penetrate beyond a certain density (ω_{pi}^2/ω^2 equal to the rhs of the second inequality associated with (6)) and they thus form "surface wave" fields.

At low densities, near the plasma edge, both the slow and the fast branches have cutoffs ($n_x^2 = 0$), at densities corresponding to respectively,

$$\omega_{pe}^2(x_{co}^s) = \omega^2 \quad (21); \quad \omega_{pe}^2(x_{co}^f) = \omega \Omega_e (n_z^2 - 1) \quad (22)$$

From the plasma edge (or wall) to these cutoffs the waves are evanescent ($k_x^2 < 0$) and the externally applied fields must be able to "tunnel through" these regions. For parameters corresponding to the EPW and the HFAW these tunnelling regions, respectively given by (21) and (22), are of negligible extent. However, for the WW (22) places an upper bound on the n_z that penetrate.^{4,5}

At the plasma wall we encounter the region where the fields couple to the plasma from some structure to which the external RF power is applied. For the frequency regime of interest a natural structure at the wall is that of a waveguide opening. This was indeed suggested in the original ideas of tokamak heating in the LHRF, both at MIT¹³ and Princeton.¹⁴ However, as the above considerations of accessibility, electron-Landau damping, and cutoff indicate the structure at the wall should impose a slow-wave spectrum in the z -direction (i.e. along $\bar{B}_0$) and of limited extent in n_z . This was later recognized, independently at MIT¹⁵ and Grenoble,¹⁶ to require a structure consisting of phased arrays of waveguide openings. More recently slow-wave structures have also been used.¹⁷ The most detailed linear coupling theory has been developed for a two-dimensional array of phased waveguides exciting the slow-branch, i.e. with the incident waveguide field parallel to $\bar{B}_0$.¹⁸ This theory predicts the amount of power reflected into the waveguides as a function of the relative phasing of the fields in the waveguides, and plasma parameters. The crucial parameters involve the position of the waveguide opening relative to the cutoff density (21) and the scale of the density gradient in that vicinity, L_{nc} , through the parameter $\alpha^{1/3}$, where $\alpha = (\omega_{pm}^2 / \omega^2 k_o L_{nc})$. Minimum reflection is obtained when the free-space waveguide opening is as close as possible to the cutoff density and $\alpha^{1/3} \sim 10 - 50$, for typical tokamak plasma characteristics and a four-waveguide array; in this case it is also important to have the outside two waveguides carry only about 1/2 to 1/4 of the power that the inside two waveguides carry, so that all waveguides have about the same reflection coefficient and the transmitted power spectrum below n_{za} is a minimum.¹⁹ If coupling to the fast wave is ignored,²⁰ the transmitted field is described by Airy functions of argument $[(1 - n_z^2)^{1/3} \alpha^{1/3} x]$ which match onto the electrostatic WKB solution for x just above the cutoff density. From a calculation of the effective reflection coefficient in the waveguide, R , and the excited power spectrum,^{18,19} one can estimate the power P_s lost to surface modes ($n_z \leq n_{za}$), and the effective spectrum which will propagate on the slow branch as electrostatic waves ($n_z > n_{za}$). From the latter we find effective values for k_{zc} , where the spectrum is peaked, and the L_z and H_y extent of the fields that have penetrated. From power conservation, and assuming that the excited spectra for $z > 0$ and $z < 0$ are identical, we can now write $S_x(x) = S_i \eta / 2$ where S_x is given by (9), $S_i = c(\lambda/\lambda g) \epsilon_0 |E_{zg}|^2 / 4$ is the power flow density in a waveguide, and $\eta = [1 - R^2 - (P_s/P_i)] (LH/L_z H_y) (a/r)$; here, LH is the waveguide array area, (a/r) is a focusing factor for the power cylindrical coordinates ($a - x \approx r$ where a is the minor radius of the plasma), and P_s can also include any

power that goes into the fast waves and/or dissipation near the edge. We thus find the electrostatic fields in the plasma

$$\frac{|E_x(x)|^2}{|E_{zR}|^2} = \frac{\eta}{4} \left(\frac{\lambda}{\lambda_g} \right) n_{zc} \left(\frac{-K_{||}(x)}{K_{\perp}^3(x)} \right)^{1/2} \quad (24); \quad \frac{|E_z(x)|^2}{|E_{zR}|^2} = \frac{\eta}{4} \left(\frac{\lambda}{\lambda_g} \right) \frac{n_{zc}}{[-K_{||}(x)K_{\perp}(x)]^{1/2}} \quad (25)$$

which are valid for $x \lesssim x_{wc}$ (viz. (13)). We note that in addition to the cylindrical geometry enhancement in η there is also a toroidal geometry enhancement in $n_{zc} (\sim R_0/R)$, where R_0 is the outside major radius of the tokamak. In applying the above calculation of the fields to a tokamak plasma one must also account for shear due to the plasma current.²¹ In addition, the above ignores the enhancement of the fields in the vicinity of the group velocity ray trajectories that emanate from the waveguide wall edges.^{19,22} Much less is known for coupling to the fast-branch waves from waveguide arrays.^{23,24} In this case the incident waveguide field must be oriented perpendicularly to $\bar{B}_0$, and hence the wave in the waveguide must be slowed down.^{4,5} As an example, arrays of ridged-waveguides can be used.

II. Nonlinear Coupling and Propagation

The high-power required for supplementary heating together with the restricted access areas for coupling lead to large amplitude electric fields in both the coupling and propagation regions of the plasma. Important nonlinear effects can arise because of the ponderomotive force density on the plasma. In a fluid plasma model this nonlinear force density can be derived from the low-frequency component of $-n_0 m (\bar{v} \cdot \nabla) \bar{v}$ where $\bar{v}$ is the induced velocity by the high-frequency field. The z-component (i.e. along $\bar{B}_0$) of this force density on electrons, for the fields of interest, can be written as²⁵

$$F_{ez} = -\frac{\epsilon_0}{4} \frac{\partial}{\partial z} \left[\frac{\omega_{pe}^2}{\omega^2} |E_z|^2 - \frac{\omega_{pe}^2}{\Omega_e^2} (|E_x|^2 + |E_y|^2) + \frac{i\omega_{pe}^2}{\omega\Omega_e} (E_x^* E_y - E_x E_y^*) \right]. \quad (26)$$

This has also been derived by other means,^{26,27} and can be written as $F_{ez} = -(\partial/\partial z)\mathcal{E}_p$, where $\mathcal{E}_p$ is a (ponderomotive) potential energy density. Under the assumptions of a Maxwellian distribution and charge neutrality, this leads to a modified plasma density²⁷

$$n = n_0 \exp \left(\frac{-\mathcal{E}_p}{n_0 (T_e + T_i)} \right). \quad (27)$$

Near the plasma edge, where the temperature is small, the high field regions near the waveguide fins, and along the rays of these fins, can give rise to important density modifications which can effect both the coupling and propagation of the power. A satisfactory description of this density modification, including the question of its evolution to a steady-state,²⁸ has not yet been produced. Further inside the plasma, where the temperature is higher, this effect is weaker and its consequences on the propagating fields can be described in terms of selfmodulation and parametric excitations. Such perturbation analyses have been carried out mostly for the EPW excitations, which we now review.

Selfmodulation for EPW in a homogeneous plasma and in two dimensions is described by²⁹

$$\left(K_{\perp} \frac{\partial^2}{\partial x^2} + K_{\parallel} \frac{\partial^2}{\partial z^2}\right)\phi + \left(a \frac{\partial^4}{\partial x^4} + b \frac{\partial^4}{\partial x^2 \partial z^2} + c \frac{\partial^4}{\partial z^4}\right)\phi + \frac{\epsilon_0}{4} \left[\chi_{\perp} \frac{\partial}{\partial x} \frac{|\nabla\phi|^2}{n_0 T} \frac{\partial\phi}{\partial x} + \chi_{\parallel} \frac{\partial}{\partial z} \frac{|\nabla\phi|^2}{n_0 T} \frac{\partial\phi}{\partial z}\right] = 0. \quad (28)$$

Here the first linear operator on the potential ϕ describes the cold plasma propagation (viz. (8)). The second linear operator describes the dispersive correction due to thermal effects, where a is given by (11) $b = 6(v_{Ti} \omega_{pi}/\omega^2)^2 - (v_{Te} \omega_{pe}/\omega \Omega_e)^2$, and $c = 3(v_{Ti} \omega_{pi}/\omega^2)^2 + 3(v_{Te} \omega_{pe}/\omega^2)^2$, are all obtained from a Vlasov description. The last term describes the correction due to nonlinear effects, $\chi_{\perp} = 1 - K_{\perp}$, $\chi_{\parallel} = 1 - K_{\parallel}$; it is obtained by using the first-order term of the expansion of (27) in $K_{\perp}$ and $K_{\parallel}$ of the linear, cold-plasma operator and setting $E_y = 0$ in (26) for the two-dimensional problem. When thermal and nonlinear corrections are ignored the solution of (28) for $z > 0$ is just $\phi(x - z/g) \equiv \phi(\xi)$ where $g = v_{gz}/v_{gx} = (-K_{\parallel}/K_{\perp})^{1/2}$. With both corrections we let ϕ acquire a slow variation with x , $\phi(\xi, x)$. Introducing the normalized variables, $v = [\epsilon_0/4n_0(T_e + T_i)]^{1/2} \partial\phi/\partial\xi$, $\xi = \xi g[(\chi_{\perp} g^2 + \chi_{\parallel})/(ag^4 + bg^2 + c)]^{1/2}$, and $\tau = (x/2gK_{\perp})[(\chi_{\perp} g^2 + \chi_{\parallel})^3/(ag^4 + bg^2 + c)]^{1/2}$ we obtain the nonlinear equation

$$\frac{\partial v}{\partial \tau} + \frac{\partial}{\partial \xi} (|v|^2 v) + \frac{\partial^3 v}{\partial \xi^3} = 0 \quad (29)$$

which is the complex modified Kortweg-de Vries equation (CMKDV).³⁰ Since this is to represent power flow into the plasma ($x > 0$), the linear potential in the absence of the corrections, $\phi(\xi)$, must be complex--it contains only the positive k_z spectrum--and hence v must also be complex. If v is taken as real (29) reduces to the real MKDV equation;³¹ this equation has known soliton solutions, obtainable by the inverse scattering transform (IST) method,³² but cannot represent power flow into the plasma.²⁹ Unfortunately, for v complex (29) is not solvable by the IST method. Numerical solutions of this equation indicate that the nonlinearity may generate reflections,^{33,34} but solutions that satisfy proper boundary conditions in x have yet to be generated. A further simplification of (28) or (29) can be obtained if one assumes that the k_z -spectrum of ϕ is quasi-monochromatic around some k_{zc} . This is not unreasonable for the EPW fields that penetrate when excited by a properly phased array of four, or more, waveguides.³⁴ We then let $\phi(x, z) = \psi(\xi, x) \exp(ik_{zc}z - ik_{xc}x)$, with $\xi = z - gx$ and the slow x -variation in the complex envelope function ψ to be due to the weak effects of both (thermal) dispersion and nonlinearity. One can then show that ψ satisfies a nonlinear Schrödinger equation (NLSE) which has well-known solutions.^{35,36} Using a new normalization for the variables $q = \psi k_{zc} [\epsilon_0/4n_0(T_e + T_i)]^{1/2} / [6(a + bg^2 + cg^{-4})]^{1/2}$, and $\tau = x k_{xc} g^2 3(a + bg^{-2} + cg^{-4}) / 2K_{\perp}$, the NLSE is

$$i \frac{\partial q}{\partial \tau} + \frac{\partial^2 q}{\partial \xi^2} + 2|q|^2 q = 0 \quad (30)$$

Detailed solutions of this equation can be obtained by IST.³⁷ If $q(\tau = 0)$ is real and non-negative its τ -asymptotic solution will have N solitons provided $(N - 1/2)\pi < \int q(\tau = 0) d\xi < (N + 1/2)\pi$. We can therefore expect that self-modulation is significant when the field amplitude is such that the threshold for one soliton is exceeded. In un-normalized form this becomes, approximately, for $\omega_{pi}(x_0) < \omega < \omega_h$,

$$q_0 \equiv q(\tau = 0) L_z \approx \left[\frac{\epsilon_0 |E_z|^2}{4n(T_e + T_i)} \right]^{1/2} \frac{L_z}{3\sqrt{2} \lambda_{De}} \approx \left(\frac{|E_z L_z|}{12 T_e} \right)_{x_0} \geq \frac{\pi}{2} \quad (31)$$

where in the last equality we assumed $T_e = T_i$ and the temperature is in volts. It should be noted that $r = 0$ is to be taken as any position x_0 where the electrostatic approximation is valid. For typical parameters of the central, quasi-homogeneous, region of an ohmically heated tokamak plasma, with applied RF powers equaling the ohmic heating power, and fields extending typically of a four waveguide array excitation structure, the inequality (31) is not satisfied by one to two orders of magnitude. Even if the inequality (31) were satisfied, it has been recently shown^{38,39} that these two-dimensional solitons are structurally unstable due to perturbations in the third-dimension (y) which was ignored in (28). In contrast, it has also been argued that cylindrical effects (also ignored in (28)) near mode-conversion can lead to a collapse (as in the Langmuir problem) and possibly strong heating.⁴⁰ Similar effects are predicted for the mode-converted wave,⁴¹ but both of these do not account for thresholds due to wave damping and inhomogeneity. As we move from the central region of the plasma toward the edge E_z increases (by about an order-of-magnitude) and T_e drops (by about one-to-two orders-of-magnitude) so that inequality (31) is now easily satisfied. Here, however, the plasma is inhomogeneous and the condition (31) is necessary but not sufficient for soliton formation. There is in addition a threshold that depends upon the inhomogeneity and the finite extent of the fields, namely⁴²

$$q_0 \tau_0 \gtrsim 10^{-1} ; \quad \tau_0 \approx \frac{9}{(r+4)} \left(\frac{\omega_{pe}}{\omega} \right)_{x_0}^3 \left(\frac{\lambda_{De}}{L_z} \right)_{x_0}^2 k_{zc} x_0 \gtrsim \frac{1}{16\pi} \quad (32)$$

where the local density and temperature profiles near x_0 have been modeled by $n = n_0(x/x_N)^r$ and $T = T_c(x/x_T)$. For tokamak plasmas, the inequalities (32) are almost never satisfied. Thus the plasma inhomogeneity strongly inhibits soliton formation even in two-dimensions.

Parametric interactions of waves can be generated by ponderomotive forces or other nonlinearities. A vast number of papers have treated such interactions and we cannot do justice to them in this short review. We only describe one or two of the important ones in each of the following categories: resonant 3-wave, quasimode (or induced scattering) coupling to a lower sideband, and quasimode coupling to both an upper and lower sideband (also known as OTSI). These interactions become important only when the thresholds due to damping, inhomogeneity, and especially finite pump extent are overcome. In the two and three-dimensional structure of the externally excited fields, appropriate threshold conditions have been formulated only recently.⁴³ Resonant 3-wave interactions that may be significant are: the decay of $(EPW)_1 \rightarrow (EPW)_2 + (EPW)_3$, especially near the plasma edge;^{44,45} the decay of $(EPW)_1 \rightarrow (EPW)_2 + (ICW)_3$ (ICW = ion cyclotron wave) where $T_e > T_i$;⁴⁶ and the decay of a lower-hybrid wave to two ion-Bernstein waves $(LHW)_1 \rightarrow (IBW)_2 + (IBW)_3$, which has the largest growth rate in an infinite, homogeneous plasma for $k_{\perp} a_i \gg 1$.⁴⁷ The nonlinear evolution of resonant 3-wave interactions can lead to pump-depletion, which is now well understood,^{48,49} or it may saturate due to the onset of trapping or nonlinear damping, which have not been explored in detail for these interactions. Quasimode coupling between the pump and a sideband can take place either through electron Landau damping^{50,51} or ion-cyclotron (harmonic) damping.⁵² The nonlinear evolution of the interaction⁵³ has also been modeled via coherent pump depletion,^{54,55} or by cascading of the sideband,

in the random phase approximation.^{56,57} Finally, the quasimode coupling between the pump and upper and lower sidebands, has only been treated linearly and, even there, incompletely^{51,58} since dephasing effects due to inhomogeneity have been ignored so far. The dominant nonlinearity in all of these interactions arises from the $(\bar{E} \times \bar{B}_0)$ motion of the electrons (last term in (26)), and realistic thresholds require, among other things, $(E_x/B_0)/v_{Ti} \sim 1$ to 10.

III. Heating Mechanisms

Linear damping of the excited waves inside the plasma would provide the simplest heating mechanism, and this was indeed the original suggestion for the EPW excitation:⁵⁹ ion-heating by ion-cyclotron harmonic damping, after the second mode-conversion, of quasi ion-Bernstein waves. Amazingly enough, this linear problem in inhomogeneous n_0 and $\bar{B}_0$ has still not been solved. Ion-cyclotron harmonic damping on the ion-thermal wave, after the first mode conversion, is also possible,⁹ but this has also not been resolved for a spatially varying $\bar{B}_0$ field. In contrast, electron heating by electron-Landau damping on the incoming warm plasma wave is well understood.^{10,11} Collisional damping in the hot plasmas of interest is too weak to be alone a dominant heating mechanism, although enhancements in such electron damping at high densities, and in ion-ion damping for $k_{\perp} a_i \gg 1$, must be reckoned with. All of the above heating mechanisms are modified by nonlinear effects. In fact, nonlinear effects and heating mechanisms are likely to control the scaling of this type of supplementary heating. These nonlinear mechanisms are: stochastic ion heating by perturbation of their cyclotron orbit; plateau formation, and possibly trapping, in stochastic electron-Landau heating; and heating by parametric excitation of damped waves. The first two can be effective directly on the externally excited fields or the parametrically generated fields.

Stochastic ion-heating for the fields of interest, and inside the plasma, can set in when the following conditions are satisfied:⁶⁰

$$E_x/B_0 \geq (\Omega_i/\omega)^{1/3} (\omega/4k_x) \quad (33)$$

where the ions that are heated ($v_{\perp i} \sim \omega/k_x$) remain in the finite extent of the field for a time (L_z/v_{Ti}) of 10 to 20 cyclotron periods, but are maxwellized in a time $(\sim 1/\nu_{ii})$ comparable to their "collision" time with the field $(\sim 2\pi R/v_{Ti})$. For the lower-hybrid wave near wave conversion in the center of the plasma, (33) is much easier satisfied than the required field strength threshold for parametric instabilities. Under the above conditions, the power delivered to the ions per unit area of the field extent perpendicular to $\bar{B}_0$ is approximately

$$S_{Di} \approx v_{Ti} m_i (\omega/k_x) v_{tr} n_0 \exp - [(w/k_x - v_{tr})^2 / 2v_{ti}^2] \quad (34)$$

where $v_{tr} = \sqrt{qE_x/m_i k_x}$, and (33) can be interpreted as the short-time energy transfer condition via trapping.⁶¹ This type of ion heating will be most effective near wave-conversion, in the central region

of the plasma, where (for typical fields $v_{tr} \ll v_{Ti}$) $(\omega/k_x v_{Ti}) \sim 1$ to 3 and (34) can be significant. The ratio $L_z S_x / S_{Di}$, where S_x is given by e.g. (9), gives the absorption length which should be smaller than one-half or one-quarter, of the minor radius a of the plasma.

The conditions for effective heating by electron-Landau damping depend critically upon: the ratio of the effective quasilinear diffusion coefficient to the collisional one, $D \equiv D_{QL}/D_c$; the position and width of the parallel phase velocity spectrum of the fields, $w_0 \equiv (\omega/k_z v_{Te})$ and $\Delta \equiv (\delta(\omega/k_z)/v_{Te})$; and the spatial damping length relative to the plasma radius, $\alpha = k_{xi} a$. For most tokamak heating situations, the E_z -field amplitude near the plasma center and its extent are such that $(\tau_{ec}/\tau_{tr}) \ll 1$, so that trapping effects can be dealt with as corrections,¹⁰ but D ranges between 10 to 100 so that plateau formation is the dominant nonlinear effect.¹¹ Under these conditions, a one-dimensional, steady-state solution for the electron velocity distribution function in the resonant region has been obtained.⁶²

$$f_{er} = C \exp \int^w - [w/(1+w^2 D)] dw. \quad (35)$$

From this we can calculate: $k_{xi} (\sim \partial f_{er} / \partial w)$; the power density dissipated in the plasma

$$P_{De} = \frac{nm v_{Te}^2}{\tau_o} \int w D \frac{\partial f_{er}}{\partial w} dw, \quad (36)$$

where $\tau_o = \tau_e / w^3$; and the heating rate. For heating in the central region of the plasma, one can simply require $\alpha_x \approx 1/2$, but a profile calculation is usually necessary. When the applied field spectrum in (ω/k_z) is unidirectional, broad $\Delta > 1$, and intense $D > \Delta/w_0^2$, the fields can generate a substantial current carried by the electrons in the raised plateau (35). This can be used, e.g., to control directly the current profile for stability at the highest β . Furthermore, choosing an appropriate combination of Δ , w_0 , and slow and fast waves within Δ , one can efficiently generate a steady-state current to confine reactor-grade plasmas.⁶³

Heating analyses for parametric excitations in the realistic geometries of the fields have yet to be done. Even for infinite extent pump fields the different weak turbulence analyses of heating by quasimode coupling that have been carried out^{53,56,57} have conflicting models and conclusions, so that further work is still required.

Acknowledgement

This work was supported in part by the U. S. Department of Energy (Contract EG-77-G-01-4107) and in part by the National Science Foundation (Grant ENG 77-00340).

References

1. W. P. Allis, S. J. Buchsbaum, A. Bers, Waves in Anisotropic Plasmas M.I.T. Press, Cambridge, 1963;
- T. H. Stix, Theory of Plasma Waves, McGraw Hill, New York 1962
2. V. E. Golant, Sov. Phys., Tech. Phys. 16,1980(1972)
3. F. Troyon, and F. W. Perkins, Proc. 2nd Top. Conf. RF Plasma Heating, Lubbock, Texas(1974)
4. A. Bers, C. F. F. Karney, K. Theilhaber, MIT-RLE QPR. 114, 123(1974)
5. A. Bers, Proc. US-Australia Plasma Seminar, R. C. Cross ed., Univ. of Sydney, Sydney(1975)

6. R. J. Briggs, R. R. Parker, Phys. Rev. Lett. 29, 852(1972)
7. P. M. Bellan, M. Porkolab, Phys. Fluids 17, 1592(1974)
8. V. M. Glagolev, Plasma Phys. 14, 301 and 315(1972)
9. M. D. Simonuti, Phys. Fluids 18, 1524(1975); S. S. Pesic, Proc. VII ECCFPF Ausanne(1975) p.176
10. A. Bers, F. W. Chambers, N. J. Fisch, MIT-RLE, PRR 76/23(1976)
11. A. Bers, Proc. 3rd Varenna Symp. Plasma Heating, Ed. Compositori, Bologna, 1976, p.99
12. K. Theilhaber, A. Bers, MIT-RLE, PRR 76/25(1976)
13. R. R. Parker, MIT-RLE, QPR 102, 97(1971)
14. W. Hooke, Internat. Plasma School; Varenna 1971; EUR-5064, p.310 Luxembourg 1974
15. A. Bers, C. F. F. Karney, MIT-RLE, QPR 114, 123(1974)
16. P. Lallia, Proc. 2nd Top. Conf. RF Plasma Heating, Lubbock, Texas 1974, C3
17. C. P. Moeller et. al., Bull. Amer. Phys. Soc. 22, 1162(1977)
18. M. Brambilla, Nucl. Fusion 16, 47 1976; Proc. 3rd Varenna Symp. Plasma Heating, Ed. Compositori, Bologna 1976, p. 84
19. V. Krapchev, A. Bers, MIT-RLE, PRR 77/18-2(1977); Nucl. Fusion (1978)
20. K. C. Wong, T. W. Tang, Plasma Phys. 18, 922(1976)
21. J. L. Kulp, G. L. Johnston, A. Bers, MIT-RLE, PRR 76/9(1976)
22. M. Brambilla - this conference
23. K. Theilhaber, A. Bers, MIT-RLE, QPR 116, 117(1975); PRR 76/26(1976)
24. R. L. Berger, F. W. Perkins, F. Troyon, PPPL-1366(1977)
25. G. L. Johnston et. al., MIT-RLE, PRR 76/8(1976)
26. J. R. Cary, A. N. Kaufman, Phys. Rev. Lett. 39, 403(1977)
27. V. Krapchev, A. Bers - this conference
28. G. J. Morales, Phys. Fluids 20, 1164(1977)
29. C. F. F. Karney et. al., MIT-RLE, PRR 76/11(1976)
30. G. L. Johnston et. al., MIT-RLE, PRR 76/18(1976)
31. G. J. Morales, Y. C. Lee, Phys. Rev. Lett. 35, 930(1975)
32. M. J. Ablowitz et. al., Stud. Appl. Math. 53, 249(1974)
33. H. H. Kuehl, private communication
34. A. Sen, C. F. F. Karney, A. Bers - this conference
35. A. C. Newell, D. J. Kaup, Bull. Amer. Phys. Soc. 21, 1095(1976)
36. C. F. F. Karney, MIT-RLE, PRR 77/10 (1977); MACSYMA Conf. Berkeley 1977
37. J. Satsuma, N. Yajima, Suppl. Progr. Theo. Phys. 55, 284(1974)
38. A. Sen, et. al., Nucl. Fusion 18, 171(1978)
39. N. R. Pereira, A. Sen, A. Bers, Phys. Fluids (Jan. 1978)
40. S. L. Musher, B. I. Sturman, JETP. Lett. 22, 265(1975)
41. H. H. Kuehl, Phys. Fluids 19, 1972(1976); also this conference
42. G. Leclert et. al., MIT-RLE, PRR 77/25(1977)
43. A. H. Reiman, A. Bers, MIT-RLE, PRR 77/20(1977); Phys. Fluids (1978)
44. E. Ott, Phys. Fluids 18, 566(1975)
45. A. H. Reiman, A. Bers, Bull. Amer. Phys. Soc. 22, 1164(1977)
46. C. F. F. Karney, A. Bers, MIT-RLE, QPR 113, 105(1974); 116, 94(1975)
47. D. C. Watson, A. Bers, Phys. Fluids 20, 1704(1977)
48. A. Bers, D. J. Kaup, A. H. Reiman, Phys. Rev. Lett. 37, 182(1976)
49. A. H. Reiman, A. Bers, D. J. Kaup, Phys. Rev. Lett. 39, 245 and 850(1977)
50. M. Porkolab, Phys. Fluids 17, 1432(1974)
51. R. L. Berger et. al., Phys. Fluids 20 1864(1977)
52. M. Porkolab, Phys. Fluids (Dec 1977); J. L. Sperling, C. Chu GA-A14537(1977)
53. A. Rogister, Phys. Rev. Lett. 34, 80(1975); A. R., G. Hasselberg, Phys. Fl. 19, 108 (1976)
54. L. Chen, R. L. Berger, Nucl. Fusion 17, 779 (1977)
55. A. Sen, C. F. F. Karney, A. Bers, MIT-RLE, PRR 77/19(1977)
56. L. Chen, R. L. Berger, Phys. Fluids 20, 808(1977)
57. A. Hasegawa, L. Chen, Phys. Fluids 18 1321(1975)
58. G. L. Johnston, A. Bers, MIT-RLE, PRR 766/22(1976)
59. T. H. Stix, Phys. Rev. Lett. 15, 878(1965)
60. C. F. F. Karney, A. Bers, Phys. Rev. Lett. 39, 550(1977)
61. C. F. F. Karney, A. Bers, MIT-RLE, PRR 76/7(1976); A. Bers, Proc. 3rd Varenna Symp. Plasma Heating, Ed. Compositori, Bologna, 1976, p.110
62. N. J. Fisch, A. Bers, MIT-RLE, PRR 77/31(1977); also this conference
63. A. Bers, N. J. Fisch, -OSP Inv. Rec. (1977); also this Conference

A Survey of the Status of Lower Hybrid Heating Experiments in Toroidal Devices*

W. M. Hooke
Plasma Physics Laboratory, Princeton University
Princeton, New Jersey 08540

I. Introduction

From a purely engineering viewpoint, lower hybrid heating is the most attractive technique for heating toroidal plasmas. Taking finite plasma temperature and coupler design restraints into account, the range 1 - 5 GHz encompasses today's research devices and tomorrow's reactors, be they of TFTR or Alcator genre. It has long been demonstrated that 1/2 MW, continuous wave, sources can be constructed in this frequency interval. Like all other RF sources the power generator can be placed hundreds of feet from the plasma, an advantage ever more acutely appreciated by those associated with neutral beam heating. Furthermore the waveguide grill or phased array conceived at Grenoble and experimentally demonstrated as an effective coupler by the Princeton group, offers an extraordinarily clean coupling structure. But the major shortfall for lower hybrid heating is now the lack of knowledge of the physics of the penetration and heating processes. Therefore, because of the favorable technological aspects, the situation is highly leveraged so as to make progress in the physical understanding of lower hybrid heating of extreme interest and importance.

In Table 1 we show a summary of recent, existing, and proposed lower hybrid heating experiments in Europe and the United States. It is clear from this table that this field is growing rapidly and that experimentalists in this area are striving to provide the physics base necessary for multimegawatt experiments in the future. A corresponding growth in theoretical studies has taken place in recent years and this activity will be summarized in a review paper by A. Bers.

II. Summary of Results

A. Coupling

1. The Phased Waveguide Array

Before 1975 coupling to the desired mode was a subject of uncertainty and controversy. Since that time, effective and efficient coupling has been accomplished with waveguides, magnetic loops, and electrostatic slow wave structures. But because of its relatively simple configuration within the vacuum chamber, the phased waveguide array appears to be the most promising coupler for the future. Also, the waveguide array is the only configuration for which a quantitative theory exists, i.e., the theory of M. Brambilla.

The first experimental evidence for the effectiveness of the phased array was obtained on a linear test device, H-1, at Princeton. These experiments quantitatively confirmed the theory of Brambilla. In 1976 a phased array of four waveguides was attached to the ATC Tokamak during the last days of its lifetime. In Fig. 1 we show the experimental and theoretical reflected wave intensity in each waveguide for two different phase configurations. On ATC we found that the overall transmission coefficient was typically 80% or

more and could be increased to greater than 90% by proper positioning of the plasma column within the vacuum chamber. It should be noted that these high transmission coefficients were achieved without matching circuitry. Therefore the usual high standing wave ratio between matching elements and coupler are absent in these experiments. This "natural" match of plasma load to transmission line means that the problems associated with large reactive fields at transmission windows and plasma surface are minimized. The absence of tuned circuit elements also means that the coupling is rather insensitive to plasma conditions.

2. Magnetic Loops

On Wega two magnetic loops are inserted near the plasma surface. The loops are oriented so that the RF magnetic field is in the θ or poloidal direction. The extent of the loop in the z or toroidal field direction and the relative phasing of the loops determine the all important k_z power-spectrum. The polarization of the magnetic and electric fields make this system the magnetic loop analogue of a two waveguide phased array. Without tuners the match for the loops is not quite as good as for the waveguides, but with tuning, the fractional transmission of RF energy to the plasma can easily exceed 90%.

3. An Electrostatic Slow Wave Coupler

At General Atomic an electrostatic coupler is being used on the Doublet II-A Tokamak. With this coupler energy is fed into an extended electrostatic slow wave structure with an imposed periodicity which determines the k_z power-spectrum. In terms of reflection coefficient this system is extremely effective, with negligible reflection occurring even without tuning. By adjusting the plasma limiter position the coupling can be made strong enough to insure virtually negligible reflection yet weak enough so that the antenna as a whole is excited. In this way one should achieve a very well defined k_z spectrum. The values of n_z used in the GA experiment are 11, 14, and 16, and the antennas contain about 20 elements.

4. A Controversy, A Potential Problem, and A Promising Diagnostic

a. A Controversy

It is a well known result that for

$$n_z^2 > n_{\text{CRIT}}^2 = 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} \quad , \quad \omega = \omega_{\text{LH}}$$

the lower hybrid resonance layer is accessible. Waves which are generated with $1 < n_z < n_{\text{CRIT}}$ will not reach the resonance layer but will couple to the fast mode, and waves with smaller values of n_z will penetrate only slightly and become surface waves. (An experimental demonstration of the accessibility condition is given in the paper A6 by Motley et al. in the Proceedings of this conference.) However, at least partly because the ray trajectories in this region depend on k_z there is some controversy concerning

the relation between the self-consistent power spectrum and the energy transmitted uselessly (or perhaps harmfully) along the plasma surface. It appears that this is merely an academic controversy for couplers with four or more elements and a Fourier peak comfortably above n_{CRIT} . However, for some devices, the port space available does not allow room for more than two elements, and this uncertainty becomes very important. This is a linear problem and the uncertainties should dissolve before even a modest attack by theorists and experimentalists in the coming year.

b. A Potential Problem

The rosy status of lower hybrid wave coupling could be disturbed if at high power levels the plasma is pathologically modified in the vicinity of the couplers. An increase in recycling might lead to a density increase or pondermotive forces might decrease the plasma density. Either of these effects, if intense enough, could lead to an increase in the reflectivity of the coupler. Some evidence for the pondermotive effect has been recently reported by the General Atomic group working on the Dc Octopole. The effect of high power levels on coupling to tokamak plasmas is an area that should be studied carefully in the near future. These experiments will have to be performed with care to avoid confusion with trivial effects such as arcing at a feed-through or impurity generation in a dirty device.

One of the few major differences between the ATC and Wega results is the large density increase (sometimes as much as a factor of two) associated with the RF on the Wega Tokamak. On ATC the density increase was around 10-15 %. It is possible that the density increase on Wega may be higher because of increased recycling of filling gas and impurities associated with higher electric fields resulting from the poorer match with the loops and the relatively small cross sectional area of the loops compared with the waveguide openings. On the other hand the ATC vacuum vessel walls were relatively far away from much of the plasma surface, and this separation may have reduced the wall-surface interaction effects. But very recent evidence from the JFT-2 tokamak in Tokai points to the coupler being the difference. A private communication from T. Nagashima reports that the average density increase is less than 10% for power and pulse lengths similar to those used at Grenoble. The coupler used on JFT-2 is a quadruple phased array, and the vacuum vessel cross section is of conventional design as is Wega.

c. A Promising Diagnostic

External coupling measurements are extremely useful, but they do not provide direct information for wave intensity and penetration within the plasma. On ATC an unsuccessful attempt was made to scatter CO_2 infra-red laser radiation off the density perturbations associated with the waves. There is an urgent need for a wave diagnostic which measures wave amplitude and k vector in the tokamak experiments. Recently on the H-1 device at PPL, microwaves (8 mm) have been successfully scattered off lower hybrid waves, and absolute power measurements of lower hybrid waves generated by a waveguide phased array have been made for the first time. This technique should be fairly easily extrapolated to tokamak conditions and will be attempted on the Versator II experiment which starts sometime in 1978.

B. Ion Heating

Ion heating was first observed in a tokamak plasma in the ATC plasma. A plot of the ion temperature versus time based on measurements of perpendicularly emitted charge exchange neutrals is shown in Fig. 2. The basic weakness of this diagnostic was its failure to give information concerning the radial distribution of the fast neutrals. Measurement of a rather rapid decay of ion temperatures $t \leq 2-3$ ms and some rather skimpy measurements of $T_{||}$ showing relatively little or no increase in temperature indicated that much of the observed heating might be taking place on the plasma surface. However, spectroscopic measurements of Doppler broadened impurity radiation indicated that ion heating was taking place well within the plasma. In Fig. 3 we show the O^{VII} temperature increase. Light from this impurity comes from within the inner third of the plasma cross section. Analysis of light from the surface, C^{IV} , showed no temperature increase. The ATC device was dismantled before the discrepancies between these ion temperature diagnostics could be resolved. In addition to the heating of the body of the ion distribution, an extended high energy tail was induced by the RF, Fig. 4. On ATC and on Wega this tail has been observed to decay in 80-100 μ seconds and it appears that these very energetic particles are generated near the ion surface. Evidence of heating of the body of the ion energy distribution plus an RF induced tail of 1-2 keV effective temperature appears to be common to all tokamak lower hybrid heating experiments.

In recent months a systematic study of ion heating on the Wega tokamak at power levels of nearly 200 kW has confirmed the existence of ion heating. A plot of the $T_{||}$ increase from fast neutral analysis on Wega is shown in Fig. 5. The time response of the charge exchange diagnostic on Wega is faster than that on ATC, and the uncertainties concerning the exact decay rate of $T_{||}$ were greatly reduced. The important point in Fig. 5. is that the decay time is several milliseconds indicating that the hot ions are on well contained orbits deep within the plasma volume. The D_2 gas injection curve represents a control experiment whereby the observed RF induced density increase is duplicated with a puff of deuterium gas so that the effect of density increase can be subtracted. Spectroscopic measurements of ion heating in Wega can be seen in Fig. 6.

Further detailed studies of $T_{||}$ ion from fast neutral analysis and Doppler broadening measurements of $T_{||}$ and $T_{\perp}$ indicate without ambiguity that ions are heated well within the plasma volume. A detailed report of the Wega measurements is included in the Proceedings of this conference.

Of course the question of the physical mechanism for the ion heating process is still unanswered. On both ATC and Wega ion heating is seen only if plasma conditions are adjusted so that $\omega \approx \omega_{LH}$ somewhere in the plasma column. On ATC ion heating was observed if $\omega \leq 1.4 \omega_{LH}$, and a definite power threshold was observed (Fig. 7.). In Wega ion heating appears only if the linear turning point exists within the plasma column. Thus, one experiment (ATC) indicates nonlinear ion heating, while the other (Wega) is consistent with a linear interpretation. However, it is too early to say that these data are inconsistent, since operationally the ion heating is observed at $\omega \approx \omega_{LH}$ and detailed analysis of the Wega results will be necessary to answer this question.

C. Electron Heating

According to theory electron heating can occur in a variety of ways at these frequencies since: nonlinear effects tend to dump more energy into electrons than ions; collisional electron heating may conceivably be important on high density devices in the resonance layer; Landau damping by electrons should be very strong under conditions easily achieved experimentally.

1. ATC

In the last days of the existence of ATC in 1976, a Landau heating experiment was attempted using a quadrupole phased array. Because of a phasing error for which there was too little time to detect and correct, and because the electron temperature was much lower than expected, the phase velocity of the waves was 2-3 times faster than the optimum for Landau damping. No body heating of electrons was observed, but a strong interaction with the electron tail was indicated by soft and hard x-ray detectors. Two soft x-ray detectors designed to detect electron heating in the 2-10 keV region showed a strong response to the RF power. In addition to an enhanced signal in each detector, the high energy channel showed the larger increase, so that the effective temperature of the tail rose. These detectors were scanned radially and indicated that the increase in the x-rays came from a region 3-4 cm in diameter near the plasma center (Fig. 8).

2. Doublet IIA

The most ambitious existing heating experiment in the LH frequency range is the Landau heating experiment which is underway at General Atomic on Doublet IIA. This is not lower hybrid heating since $\omega^2 > \omega_{ce}^2$, and, therefore, no hybrid resonance can exist in this plasma for any value of density. However, the G.A. group has very recently assembled ≈ 450 kW of RF power, and they have constructed a radically different electrostatic coupler which, in terms of reflection coefficient, is the most effective system constructed to date. On the characteristic time scale for RF experiments this work must be considered as barely having begun. Even so some very encouraging results have begun to appear. In Fig. 9 we show the first evidence for electron heating seen in Doublet IIA. The change in plasma resistivity in Fig. 9 is to my knowledge the first RF-induced decrease in plasma resistance at frequencies below the electron cyclotron frequency. Table II summarizes the conclusions of the G.A. group. Details of this work can be found in papers A4, A5, and B2 in the Proceedings.

Landau heating is a linear mechanism which should be comparable in effectiveness and predictability to cyclotron heating. Now that lower hybrid experimentalists can control k_z with the new brand of coupling structures, we should be able to achieve $\omega/k_z \approx V_{th}$ just as easily as we have been able to satisfy the $\omega = \omega_{ce}$ condition for several decades.

One of the persistent fears in lower hybrid heating experiments is that nonlinear absorption of plasma waves at the surface might prevent effective wave penetration. The G.A. experiment is characterized by low B field (~ 8 kG) low temperatures $T_{e0} \lesssim 300$ eV and couplers which are relatively extended in the Z-direction. All of these factors tend to lower thresholds for most of the dominant nonlinear decay processes, so that the Doublet IIA experiment may have problems with nonlinear surface effects. However, if such is not the case for the G.A. experiment, then we can say that nonlinear

effects at the surface are unlikely to bedevil any of the toroidal experiments planned on other devices.

3. Wega

When the Wega power level was doubled in the Spring of 1977 several effects appeared which are different from those reported in the other experiments. These effects are:

- (1) a density increasing at the rate of $2 \times 10^{15} \text{ cm}^{-3} \text{ s}^{-1}$ at 180 kW of power, (Fig. 10),
- (2) a loop voltage increase from ~ 2 to 3.3 volts in 6.3 msec,
- (3) a peak electron temperature increase from ~ 500 eV without RF to ~ 750 eV with 135 kW of RF power. The temperature at the surface appears to be slightly lower in the presence of RF.

This is a collection of startling results and needs to be studied in detail. However, it appears at the present time that the study of these phenomena on Wega will be delayed by many months at least since that device will be converted from a tokamak to a stellarator during the coming year.

At first glance these changes might be ascribed to RF-associated impurity production. However, preliminary analysis indicates that impurities come in a time scale shorter than, for example, the temperature increase. In any case it should be borne in mind that waveguide coupling experiments at similar powers in ATC and JFT-2 have shown much smaller effects, i.e., $\Delta n/n \leq 1.5\%$, $\Delta n/\Delta t \leq 3 \times 10^{14} \text{ cm}^{-3} \text{ sec}^{-1}$, $\Delta V_{\text{loop}}/V \leq 10\%$ and $\Delta T_e/T_i < 30\%$.

4. Dc Octopole

Electron heating has been achieved on the Dc Octopole using electrostatic couplers. In Fig. 11 we show the observed electron heating versus frequency in the Octopole. Heating is attributed to Landau damping and collisions. The observed electron heating appears to be consistent with a 100% efficiency.

A variety of interesting observations have been made in this experiment. One of these is the measurement of resonance cones when the Landau damping conditions are satisfied, and the disappearance of these cones when conditions are such that damping should be very weak. In the latter situation the entire volume of the plasma becomes filled with waves, the wave amplitude profile coinciding with the radial density profile. This result is interpreted in terms of waves traveling around the torus many times before leaving the plasma or damping. This interpretation is bolstered by the observation of a spatial incoherence for these waves. If weakly damped waves can fill the volume of a tokamak, like light trapped in a light pipe, then our physical concept of the physics of lower hybrid heating, in certain circumstances, would undergo interesting revisions.

D. Heating Efficiency

Definitions of heating efficiency vary from one experimental group to another. Some define efficiency in terms of simple heat capacity arguments, while others use detailed computer models. Most of the electron temperature measurements have been made with Thomson scattering which is, in general, a sufficiently noisy method to allow one to assume a large flow of undetected energy to the electrons. Finally the efficiency is a simple quotable number and one becomes quickly involved with the politics of funding. For these reasons experimentalists are loathe to quote heating efficiency measurements. Energy flow channels in both ICRH and lower hybrid heating have been inadequately diagnosed. In general I think it is fair to say, however, that for both heating techniques at least

half of the RF energy is unaccounted for. (This would not be true if the electron heating in Wega is directly due to the absorption of RF power). Documenting the energy flow is probably the highest priority for experimentalists in the coming years.

E. Lower Hybrid Wave Interaction with an Injected Ion Beam

Perhaps the biggest surprise in the ATC experiment was the observation of a strong interaction between the RF and a tangentially-injected 26 keV neutral beam. Charge exchange analysis at several angles, with respect to the beam direction, indicated that the perpendicular energy of the circulating beam was significantly increased with no observed decrease in the parallel energy. The neutron yield associated with the deuterium beam in a deuterium plasma increased by $\sim 50\%$. The angle of observation of the fast neutrals determines the ratio $V_{\perp}/V_{\parallel}$. The increase in flux of particles vs. energy is shown for four angles of observation in Fig. 12. The solid circles are measured values without RF, and the open circles denote values of flux with the RF on. Two theories have been developed to explain the absorption; one involves cyclotron harmonic damping¹ the other, paper G-1 in the Proceedings, concludes that stochastic heating can explain the observed effect. In either case the theories predict that the RF absorption should become even stronger in larger devices in which beams will have greater perpendicular energy. The potential importance of this technique as an energy clumper or accelerator is obvious.

In addition the beam-wave interaction could provide an important wave diagnostic. It is clear from these results that we should continue experimental and theoretical studies involving the interaction between beams and waves.

III. Future Directions

The basic challenge in lower hybrid heating appears to be in the mastering of the physics. The following areas need sorely to be studied:

1. The surface plasma in the vicinity of the coupler.

What is the effect of high power on the tenuous plasma at the surface which determines the coupling effectiveness? Are ponderomotive effects at the surface an important limitation on applied power densities? Is recycling increased by the RF power and how serious is the problem of impurity injection at high power levels and long pulses? Are surface density fluctuations in tokamak plasmas sufficient to prevent effective penetration? Are decay waves at the surface soaking up significant amounts of energy? To what extent is the energy flow in high energy ions on poorly contained orbits near the surface? Up until now experiments have operated in the $\sim 1 \text{ kW cm}^{-2}$ range. However, some experiments, Alcator A and Petula, will operate in the 5 kW cm^{-2} , which is the power density needed in future heating experiments on large devices.

2. The physics of the absorption process.

Near the resonance region do linear or nonlinear processes dominate? What is the net result of quasilinear effects on Landau heating? (These are very much non-academic questions since they determine the frequency to be used in the experiments.) Are collisional effects important in high density ($\sim 10^{15} \text{ cm}^{-3}$) plasmas? How does the absorbed wave energy divide between ions and electrons, and to what extent is the energy flow into the bulk and the tail of the energy distributions? Can one "clamp" the energy of an injected neutral beam with RF energy at these frequencies? Can the Landau process be used to drive significant dc currents? And so it goes.

At the beginning of this paper, reference was made to the expanding research in lower hybrid heating. However, Table 1 may be somewhat misleading. Most, if not all of the experiments listed, are threatened by competition from higher priority projects. For example, the ATC device was terminated after a few days of running with the quadruple Landau heating array, the Wega tokamak is to be converted into a Stellarator in which there is inadequate space for waveguide couplers, and Doublet IIA will probably be terminated in the midst of its RF experimentation. With the possible exception of the FT tokamak, the other experiments are likely to be seriously limited because of shared running time. Given the wide range of physics problems, a few of which have been posed here, it appears unlikely that by the early 1980's that we will have sufficient input to answer the following question: Should we commence construction for a lower hybrid heating experiment to be ready in the mid or late 1980's at a power level of tens of megawatts and a cost of several tens of millions of 1978 dollars? We won't be able to answer that question, but nonetheless it will arise.

Lower hybrid experiments, in particular, and RF experiments, in general, need well diagnosed, state of the art, confinement systems which are dedicated primarily to RF heating studies.

REFERENCES

* Work supported by U.S.D.O.E., Contract EY-76-C-02-3073.

¹ A. H. Kritz, S. M. Mahajan, R. L. Berger, R. J. Goldston, R. W. Motley, W. M. Hooke, and S. Bernabei, PPPL-1390; also accepted for publication in Nuc. Fusion.

FIGURE CAPTIONS

Fig. 1. The transmission coefficient for the quadruple waveguide array used on the ATC Tokamak. The bars in the histogram represent the experimental results, and the small black rectangles are calculated from Brambilla Theory. The theoretical and measured values for each transmission for the phased array are shown at the right. Relative phases for each waveguide are shown below the corresponding waveguide number.

Fig. 2. Temperature of the bulk of the ion energy distribution on ATC as measured from fast neutrals emitted perpendicular to the magnetic field. The RF power was 125 kW.

Fig. 3. Ion temperature based on Doppler broadening measurements of O^{VII} with 5 ms of RF power.

Fig. 4. The RF-induced tail of the ion energy distribution measured from perpendicularly emitted fast neutrals.

Fig. 5. Perpendicular ion temperature measured on the Wega Tokamak (Berchtesgaden Conference).

Fig. 6. O^{VII} Doppler temperature measured on the Wega Tokamak (1977 Berchtesgaden Conference).

Fig. 7. Ion temperature and decay wave amplitude vs. power on ATC. At lower densities the heating threshold was increased to 60 kW.

Fig. 8. The electron temperature vs. radius as measured with Thomson scattering and soft x-rays on ATC. The RF induced peak is an effective temperature well out on the tail. The tail temperature exceeds the range of the x-ray diagnostic, and these measurements should be considered as qualitative in nature.

Fig. 9. Effect of RF power on the Doublet IIA Tokamak. (From the GA report at the Atlanta, APS, meeting, 1977.)

Fig. 10. Density changes induced in Wega by RF power of varying pulse length (Berchtesgaden Conference 1977).

Fig. 11. RF-generated plasma temperature increase in the Dc Octopole experiment. The radiated power is the total RF energy flow to the plasma.

Fig. 12. Relative flux as function of the energy of detected particles. The solid circles correspond to measurements without incident RF power while the open circles denote the measurements with the incident RF on.

TABLE 1.

LOWER HYBRID HEATING EXPERIMENTS ON TOROIDAL DEVICES

	Laboratory	Device	Coupler	Power (kW)	Frequency (GHz)	
RECENT	PPL	ATC	waveguide	200	0.8	3 configurations single, double, quadruple waveguide
	MIT	Alcator A	waveguide	80	2.5	single waveguide
EXISTING	Grenoble	WEGA	magnetic loops	200	0.5	to be terminated for tokamak to stellarator conversion
	GA	Doublet II	electrostatic	~500	.8 & .915	
	Tokai	JFT-2	waveguide	150	0.65	quadruple waveguide
	Grenoble	Petula	waveguide	1000 (short pulse 100 μ s)	1.25	double waveguide
	MIT	Alcator A	waveguide	100	2.5	double waveguide
	GA	Octopole	electrostatic	watts	10-150 MHz	
FUTURE	MIT-PPL	Versator	waveguide	180	0.8	to start in spring 1978
	Frascati	FT	waveguide	500	2.5	approved
	MIT-PPL	Alcator C	waveguide	1000 - 2000	4.0 - 4.5	virtually approved
	PPL	PLT	waveguide	800	0.8	late 1979
	Grenoble	WEGA	waveguide	400 200	0.8 0.5	possibly after reconversion to tokamak
	GA	Doublet II	electrostatic	1,500	2.5	
	Argonne		waveguide	5,000 12,000	1.3, 2.85	100 ps pulse lengths
	UCLA	Micro, Macrotron	electrostatic	50-500	0.3	

MACROSCOPIC CHANGES INTERPRETED AS A CHANGE
IN CONDUCTANCE AND CONDUCTIVITY TEMPERATURE

No RF	RF (85 kW)
$I_p^{max} = 33 \text{ kA}$ ($I_p = 0$)	38 kA
$V(I_p^{max}) = 2.0 \text{ V}$	1.22 V
$R^{-1} = 17 \text{ kmho}$	31 kmho
$P_{OH} = 66 \text{ kW}$	46 kW
$P_{RF} = 0 \text{ kW}$	85 kW
$P_{TOT} = 66 \text{ kW}$	131 kW
$T_e = 140 \text{ eV}$	$T_e = 213 \text{ eV}$

Table 2.

THE CONDUCTANCE HAS INCREASED 82%
THE CONDUCTIVITY TEMPERATURE 50%

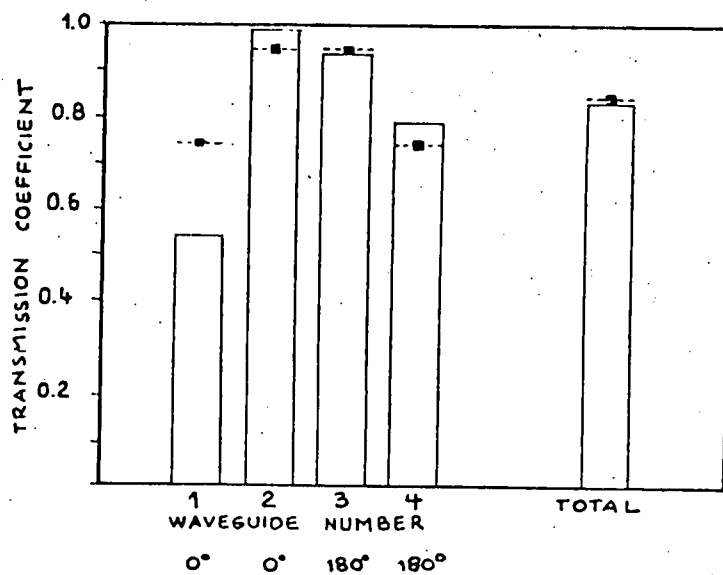


Fig. 1(a).

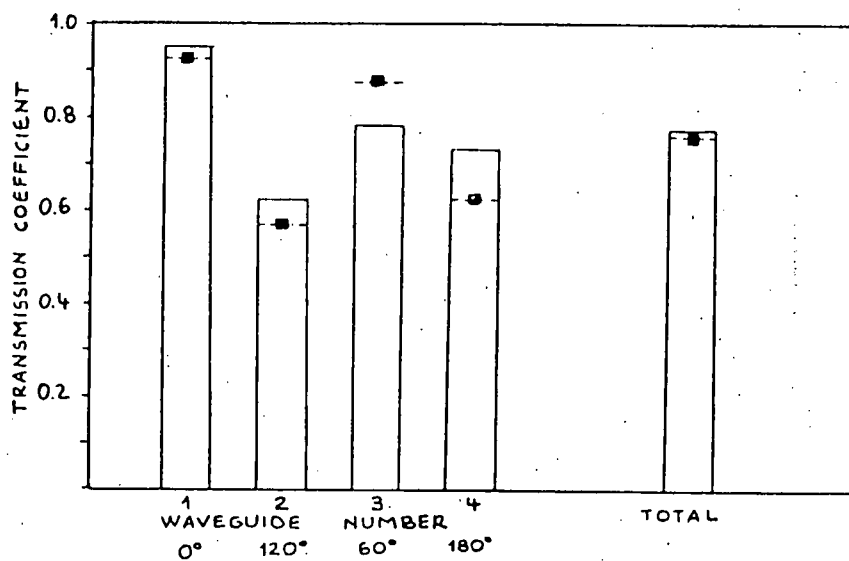


Fig. 1(b).

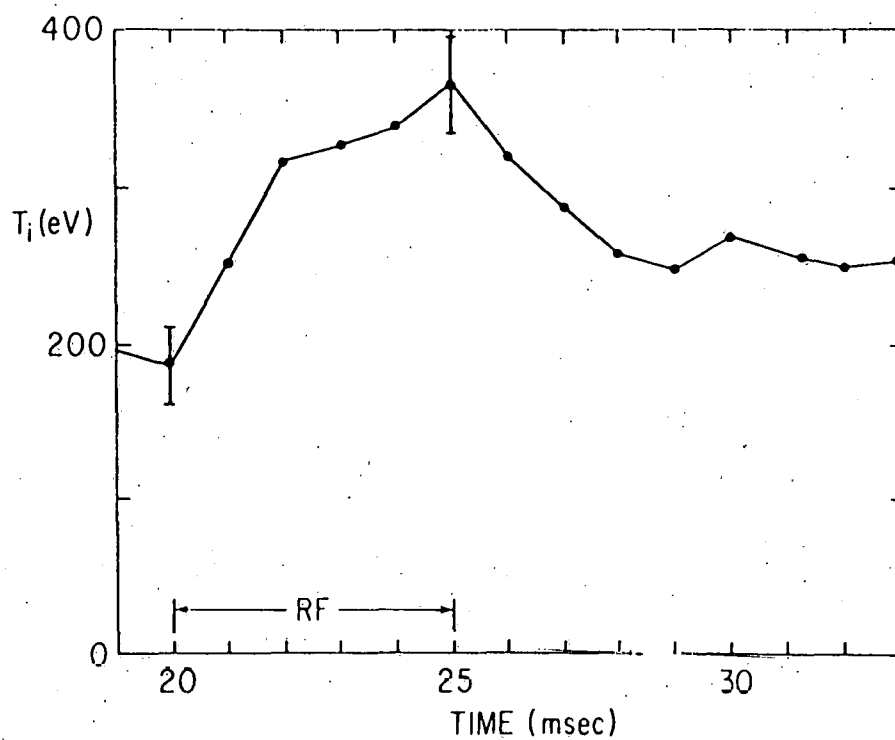


Fig. 2.

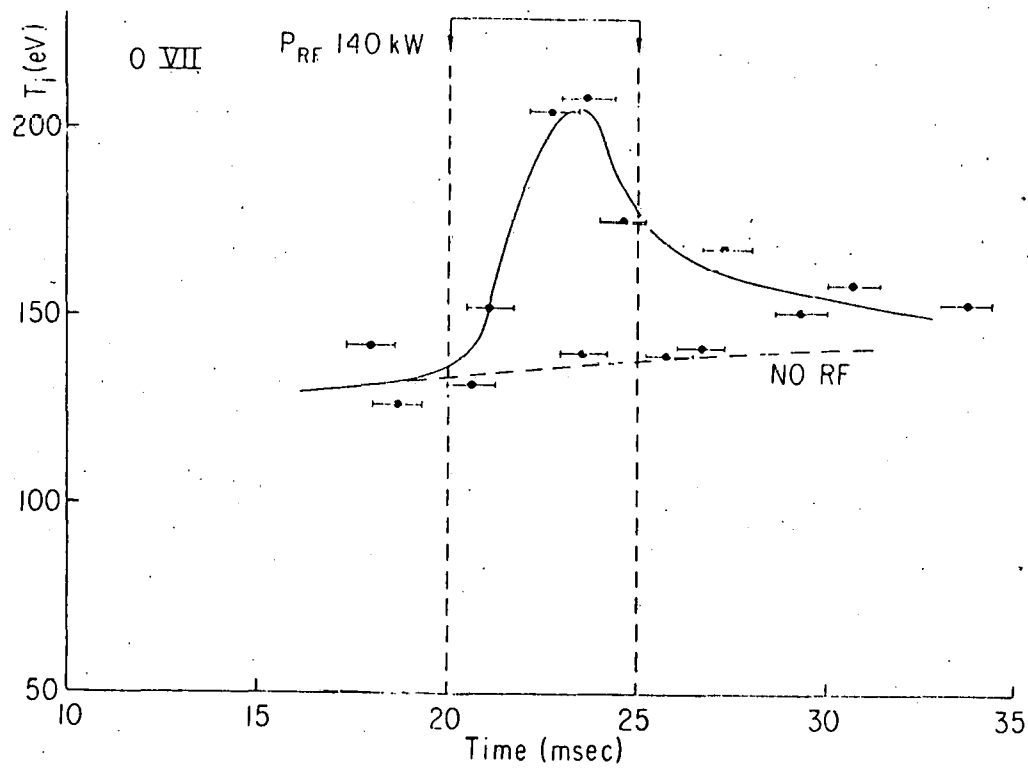


Fig. 3.

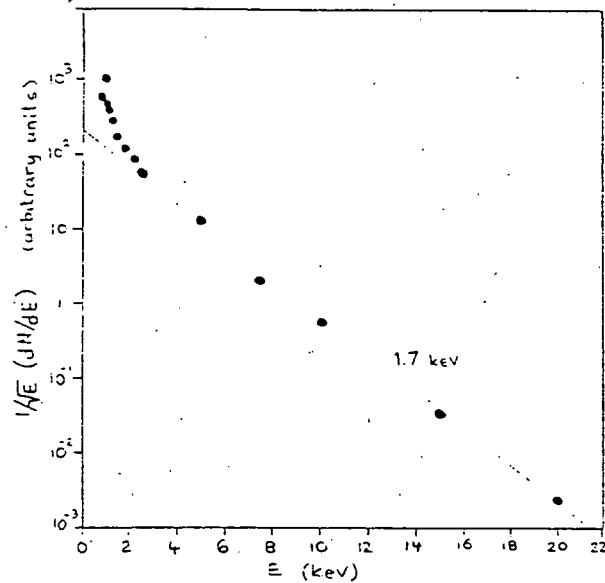


Fig. 4.

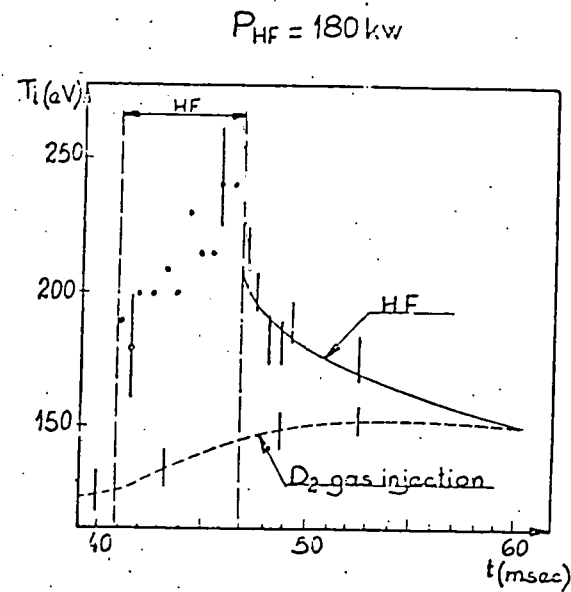


Fig. 5.

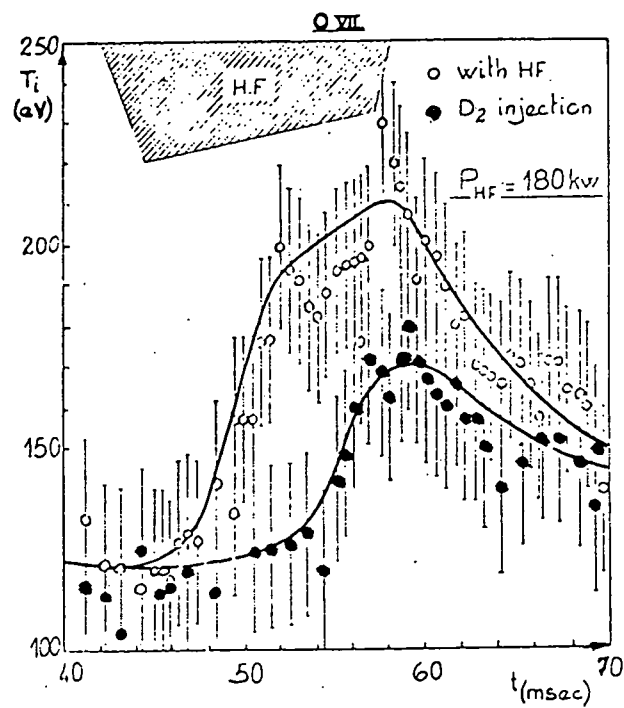


Fig. 6.

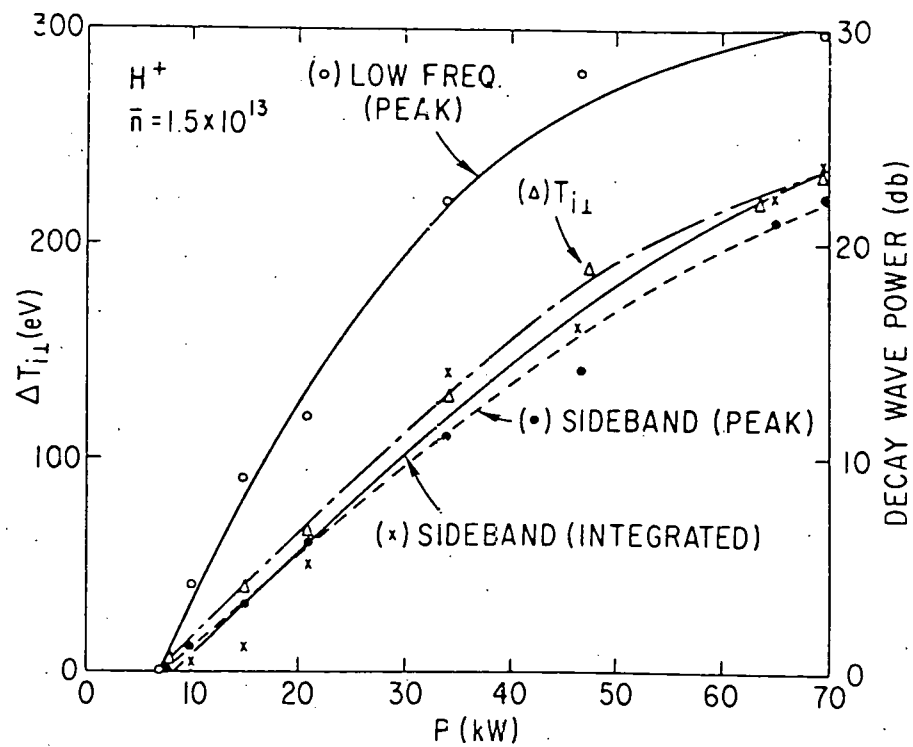


Fig. 7.

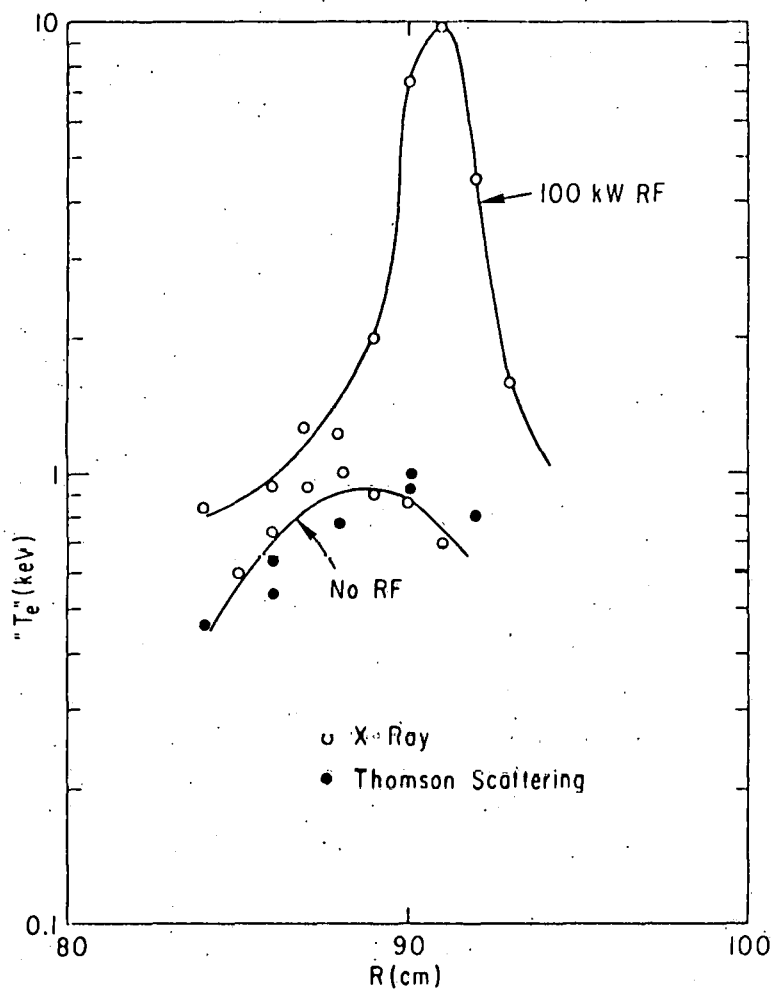
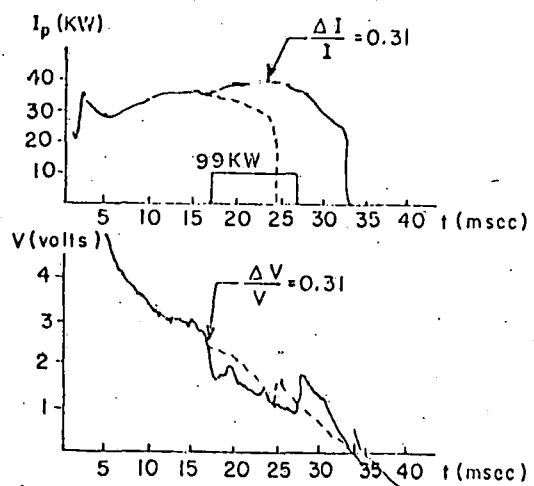


Fig. 8.



NO RF: $P_{OH} = 96 \text{ kW}$

99 kW RF: $P_{OH} = 52 \text{ kW}$

$\Delta(PWR) = 55 \text{ kW}$

Fig. 9.

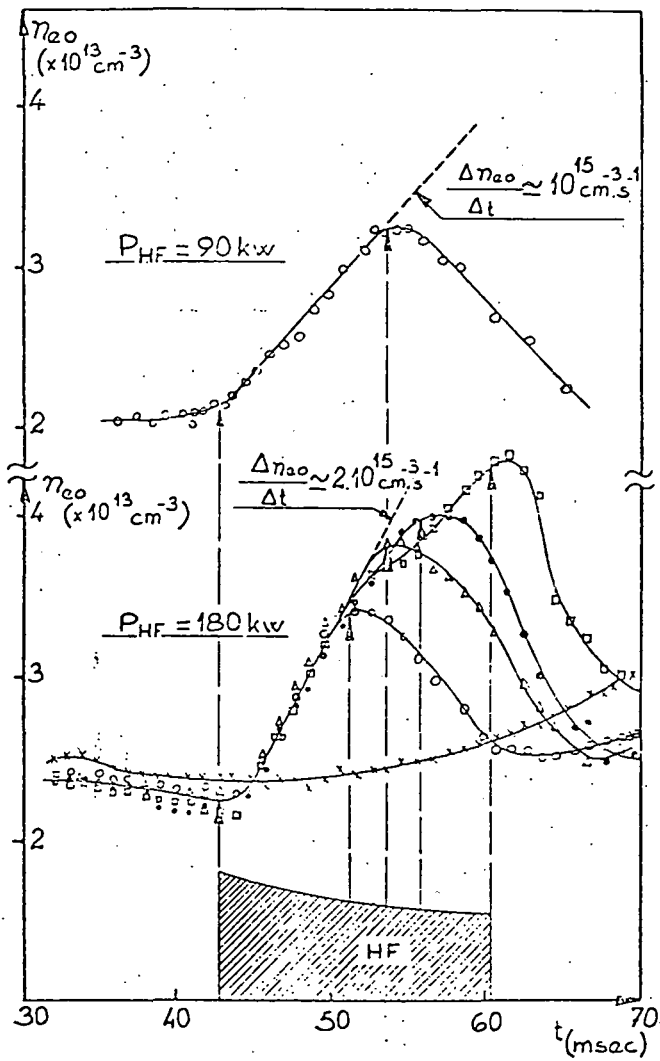


Fig. 10.

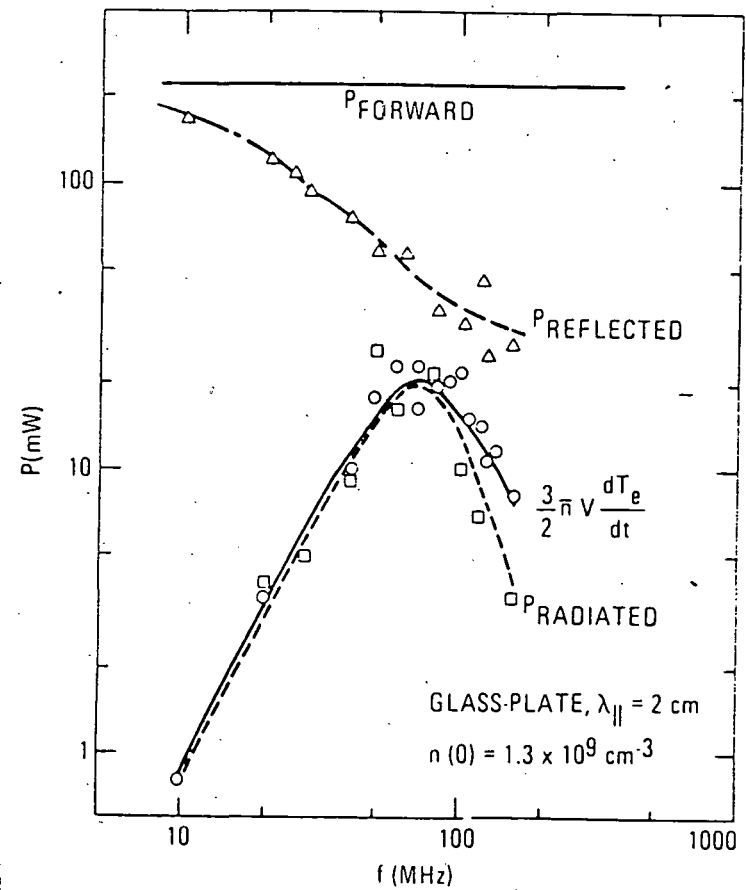


Fig. 11.

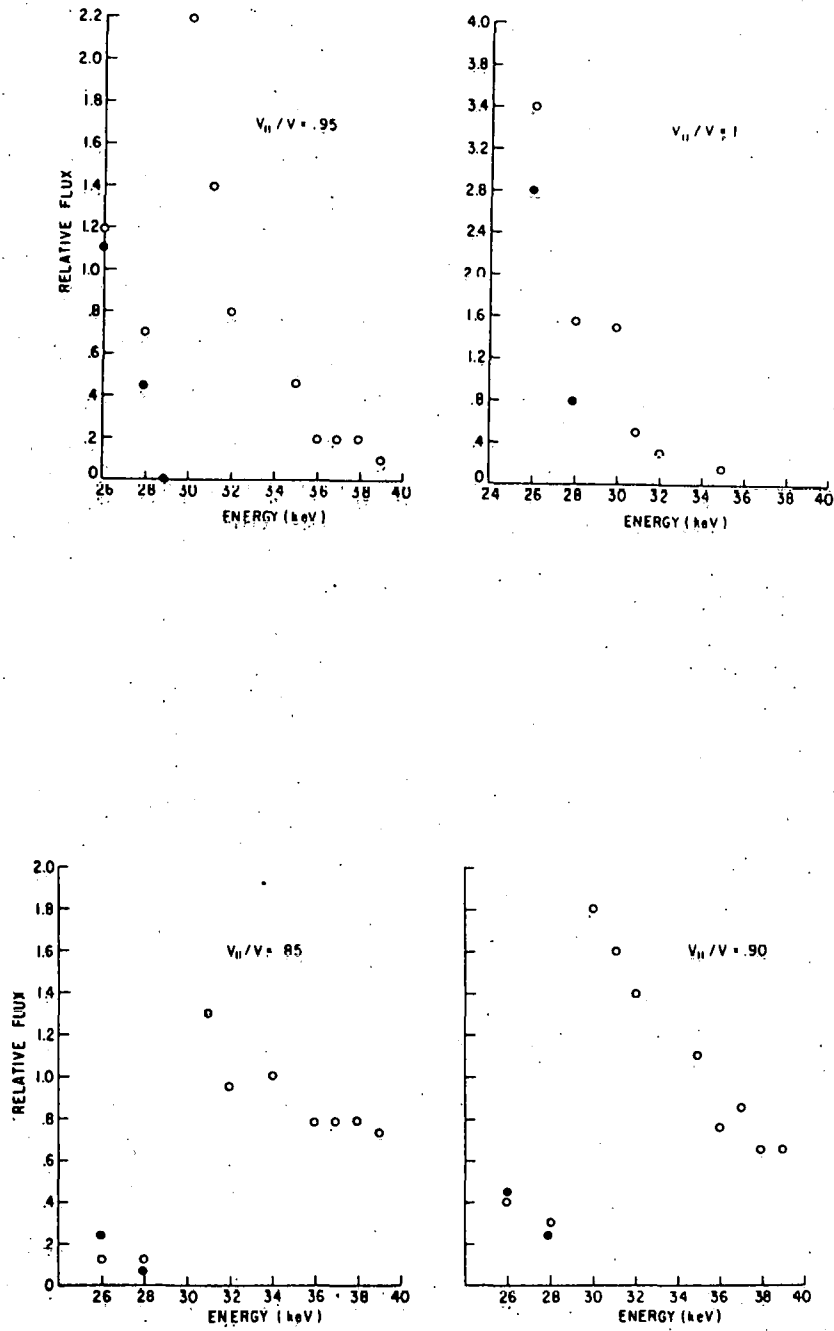


Fig. 12.

HEATING OF IONS BY LOWER HYBRID IN THE WEGA TOKAMAK

C. GORMEZANO°, P. BLANC°, M. DURVAUX°,°° W. HESS°, G. ICHTCHENKO°;
 P. LALLIA°, R. MAGNE°, T.K. NGUYEN°, H.D. PACHER°,° G.W. PACHER°,°
 F. SÖLDNER°,° G. TONON°, J.G. WEGROWE°°

°ASSOCIATION EURATOM-CEA

Département de Physique du Plasma et de la Fusion Contrôlée
 Service IGn - Centre d'Etudes Nucléaires
 85 X - 38041 GRENOBLE CEDEX (France)

°°MAX-PLANCK-INSTITUT FUER PLASMAPHYSIK, EURATOM-ASSOC.
 GARCHING, Fed. Rep. of Germany

°°°E.R.M. BRUXELLES (Belgium)

Experimental results presented here have been obtained in the WEGA facility operated as a Tokamak. The characteristics of this experiment have been presented elsewhere (1). The lay-out of WEGA is presented in Fig. 1. The main parameters are the following : $a = 15$ cm, $R = 72$ cm, $B = 14.4$ kGauss, $I_p = 50$ kA, deuterium gas, mean electron density : $1.5 - 3.10^{13} \text{cm}^{-3}$. H.F. generator characteristics are : $f = 500$ MHz, $P_{\text{peak}} = 180$ kW, maximum pulse duration $\Delta t = 12$ ms.

In the present experiments an average H.F. power of 135 kW has been applied for 10 ms. With these parameters the lower hybrid frequency (cold plasma approximation) is approximately at the center. In WEGA, the H.F. energy has to be coupled by means of two small loops due to the small size of the available ports. As sketched in Fig. 1, they are fed in phase but produce by their geometry electric fields which are 180° out of phase. Between 60 % and 70 % of the power is coupled without matching and more than 95 % with. The spectrum which can be obtained by Fourier analysis of this two element structure is rather wide and not optimized. Furthermore, it is not certain that the loops excite mainly the slow H.F. wave. However, an idealized estimate of that excitation spectrum shows that only about 40 % of the H.F. power ($1.6 < N_{\parallel} < 5$) should penetrate to the center of the plasma. Measurements with Langmuir probes (2) have given a density of a few times 10^{10}cm^{-3} on the magnetic field line of the H.F. loops, well above the cut-off density of 3.10^9cm^{-3} at 500 MHz.

The application of lower hybrid in the WEGA plasma entails drastic changes in the ion population, i.e. ion heating, and in the electron population, i.e. density increase and changes of profiles together with a loop voltage increase. This will be presented in the first

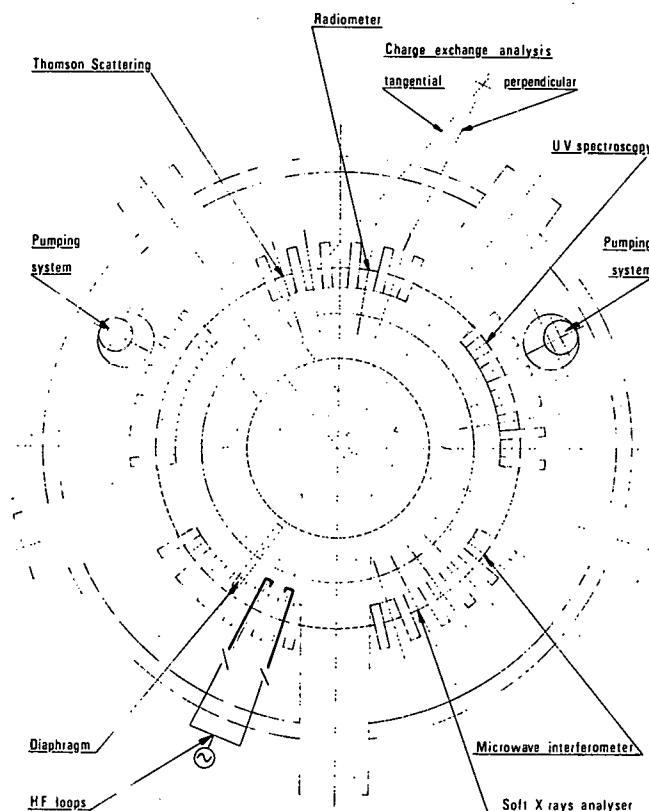


Fig. 1 : Lay-out of the WEGA facility

two paragraphs. Preliminary measurements of the H.F. emitted spectra will be presented in the third paragraph.

1 - HEATING OF IONS.

H.F. heating near the resonance is expected to give the target particles a kick in energy perpendicular to the magnetic field. Therefore, it is not surprising to observe a fast ion tail in the analysis of the neutrals escaping from the plasma in a direction perpendicular to the magnetic axis. This tail appears to be maxwellian with a mean energy ranging between 1 and 2 keV (Fig. 2a). The only measurements available are in a limited region of velocity and physical space of the toroidal plasma, far from the loops. If this region were characteristic of the entire plasma, the power lost by this population through charge exchange would be in the range of 1 kW. The lifetime of these fast ions has been observed (3) to be 100 μ s after the H.F. is turned off. This small lifetime together with some other experimental indications leads to the assumption that these ions are localized in the periphery of the plasma. With these assumptions a crude estimate of the power required to sustain this fast ion population is in the range of few kilowatts.

When the H.F. is turned off, this fast ion tail disappears in a few hundred microseconds and a thermalization of the ion population was deduced from the fact that a decay time of the bulk ion temperature of several milliseconds was observed. Of course, the real test of thermalization is in the measurement of the ion temperature by charge exchange analysis parallel to the toroidal magnetic field. This has been made possible by drilling a 1 cm hole in a toroidal coil. The result of this measurement appears in Fig. 2b. The flux of a fast ion tail, if any, is lower by at least one order of magnitude than the corresponding flux in a perpendicular direction. Therefore, the direct measurement of the central ion temperature is now reliable during the H.F.

The main output of this measurement is the demonstration that the energy input to the ion population is thermalized. The time evolution of the central ion temperature is indicated in Fig. 3 for a H.F. power of 120 kW. This shows almost a doubling of the ion temperature together with a decay time of 15 ms while the decay time of the temperature increment is 6 ms. If one assumes that when the H.F. is turned off, the energy lifetime as well as the time derivative of the ion density do not change, the comparison of the characteristic time before and after the H.F. is turned off indicates a local deposition of 100 mW/cm³ by the H.F.

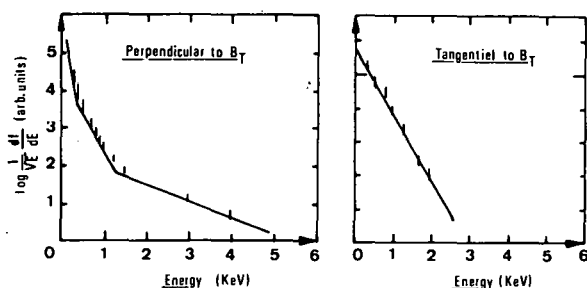


Fig. 2 : Charge exchange analysis :
a) perpendicular to B_T
b) tangential to B_T

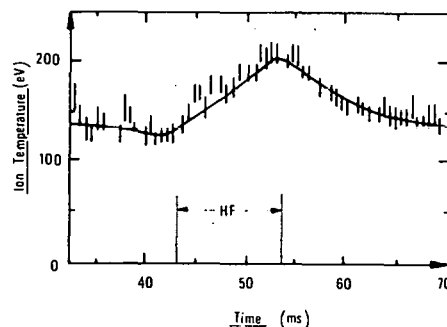


Fig. 3 : Time variation of ion temperature deduced from tangential charge exchange analysis.

Observation of Doppler line broadening of impurity lines can provide information on the ion temperature at different plasma radii. A theoretical estimate of the emitted intensity of various impurity lines has been performed. The charge state distribution of impurity ions as a function of radius has been computed using the Fontenay model (4), which assumes a constant radial diffusion velocity, and a constant impurity concentration. Experimental profiles for electron temperature and electron density are injected into a numerical code (5). These profiles will be presented later in the paper. The output of the code is shown on Fig. 4 for two set of experimental data : i) before application of the H.F. ii) 6 ms after application of the H.F. and for the OVII, CV and CIV impurity lines where the diffusion velocity, the magnitude of which is uncritical, has been taken to be $1.5 \cdot 10^3 \text{ cm s}^{-1}$. The point of maximum emission for both OVII and CV lines does not change, while the maximum emission for the CIV line is slightly displaced towards the outskirts of the plasma.

Roughly, the OVII line is emitted at a radius of $a/3$, the CV line at $r = 2a/3$ and CIV at $r = a$. The time evolution of the ion temperature which can be inferred from the Doppler broadening of these lines is shown in Fig. 5 together with the time variation of their total intensity (the integration time being 0.5 ms). An average power of 135 kW was applied in that case. The edge temperature shows no increase while the temperature increases by 60 % at the location of the CV line emission and more doubles at the location of the OVII line emission.

The time variation of the ion temperature from the latter impurity line is shown in Fig. 6 on a different time scale. It is worthwhile to note that the characteristic times agree well with the observed decay time from charge exchange analysis, and that the ion temperature after the H.F. falls back to its initial level before

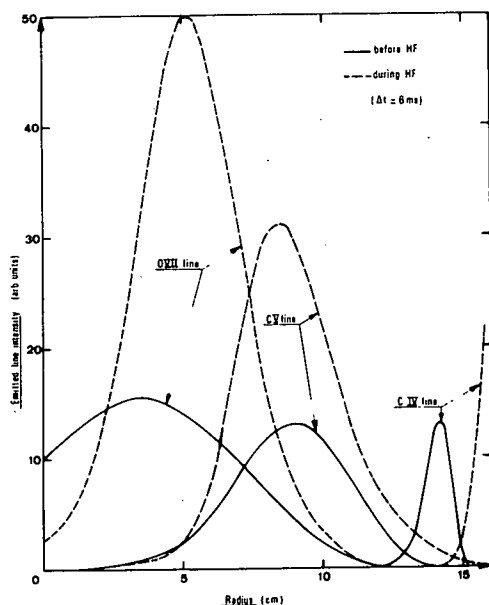


Fig. 4 : Computed impurity line emission assuming :
 $v_d = 1.5 \cdot 10^3 \text{ cm s}^{-1}$.

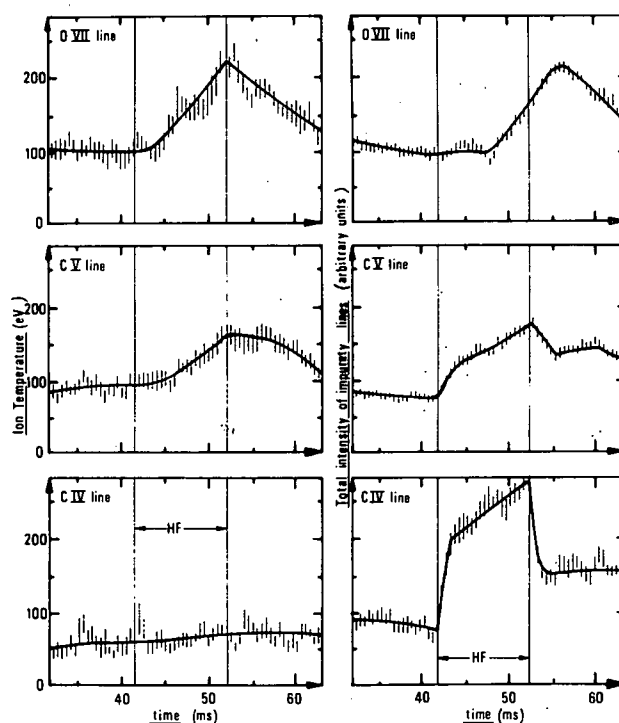


Fig. 5 : Ion temperature and total intensity from various impurity lines versus time : OVII : $r = 4-5 \text{ cm}$, CV : $r = 9 \text{ cm}$, CIV : $r = 14-16 \text{ cm}$.

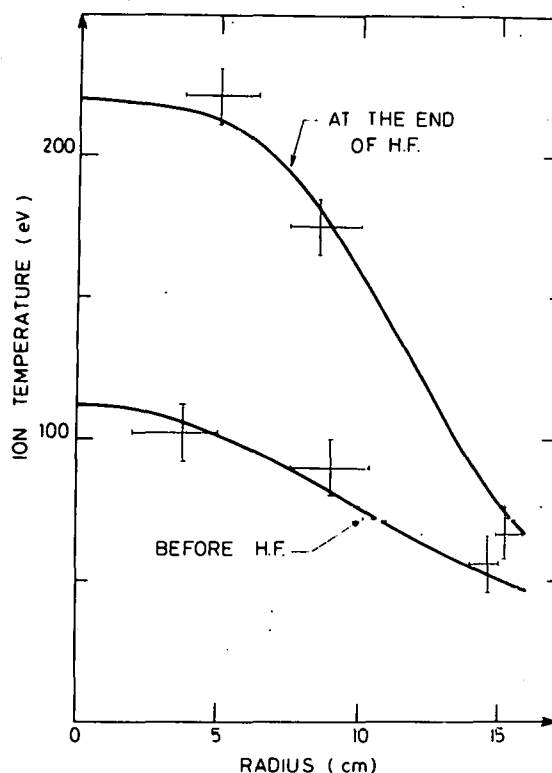
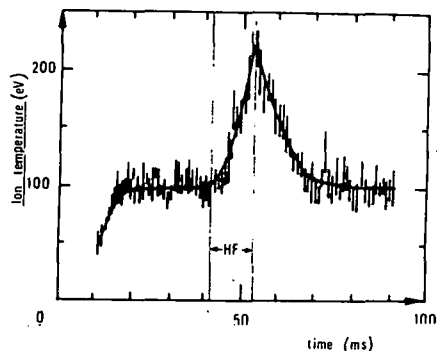


Fig. 6 : Time variation of ion temperature from Doppler broadening of OVII.

Fig. 7 : Ion temperature profile before and at end of H.F. from impurity line broadening.

the H.F.. The radial variation of the ion temperature before and at the end of the application of the H.F. is shown in Fig. 7. The corresponding substantial increase of the ion temperature is not accompanied by any drastic change in the temperature profile.

From the right side of Fig. 5, it can be noted that while the CIV intensity increases immediately when the H.F. is applied, the OVII intensity at one third of the radius does not change at all for 6 ms after the H.F. is applied. Although, owing to the complexity of the phenomena involved in the creation of OVII line emission, this is not a proof that impurities are not coming into the central part of the plasma for 6 ms., it can be tentatively concluded that a certain flux of impurities impinges the plasma edge immediately after the application of the H.F. but then needs 5 to 10 ms. to reach the center. This fact, together with return of the ion temperature back to the original temperature after the H.F. is turned off while the line intensity is still increasing are strong indications that impurities are not responsible for the observed ion heating.

Since, at a given magnetic field, the electron density is an important parameter for the location of the resonance layer, it has been attempted to optimize the heating effect by carrying out experiments at different starting densities. Such an effect is illustrated on Fig. 8, which represents the wave conversion zone according to linear theory (6) for typical plasma conditions before and at the end of the H.F.. The experimental results obtained with an averaged H.F. power of 135 kW are indicated in table 1 where the ion temperatures are deduced from OVII line broadening. The best increase in ion temperature $\Delta T/T$ is obtained for the lowest starting density for which the turning point is located at the center of the plasma. The highest final ion temperature is obtained for the lowest final density.

The location of the turning point for approximately the corresponding plasma conditions is also indicated in Table 1. A code simulation using the Fontenay code (7) has shown that the transfer of the deposited power to the bulk plasma is less and less efficient when the location of the coupling zone is displaced towards the periphery and almost nil when $r/a > 0.5$. Therefore an optimum of the heating can be expected when the resonance layer stays in the inner part of the plasma for the entire H.F. duration. The observed heating agrees well with this hypothesis. From table 1, it can also be seen that the increase in ion temperature which can be attributed to the density increase is small compared to the heating observed for the best plasma condition. This has been further confirmed by simulating the electron density increase by means of deuterium gas puff experiment as reported elsewhere (8).

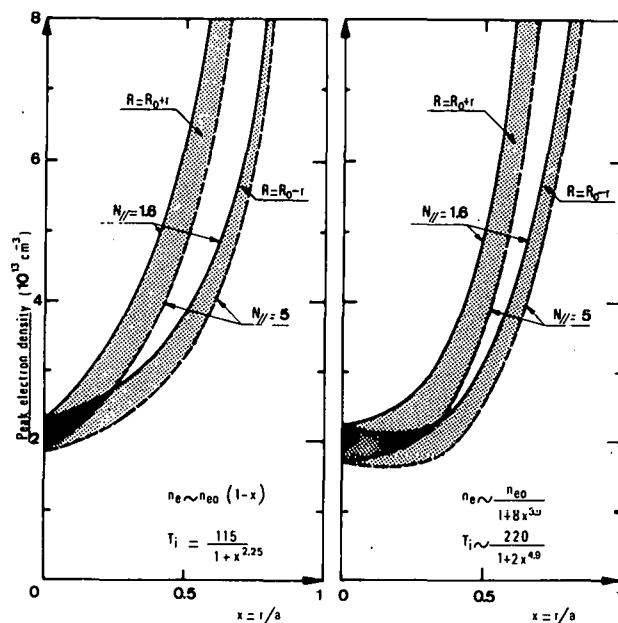


Fig. 8 : Computed turning point location as a function of peak density for conditions a) at beginning of H.F. and b) at end of H.F.

TABLE I

starting peak density (10^{13} cm^{-3})	1.8	2.4	2.6	3.6
turning point location (r/a)	~ 0	~ 0.1	~ 0.2	~ 0.3
final central density (10^{13} cm^{-3})	2.8	3.4	3.8	4.6
turning point location (r/a)	~ 0.35	~ 0.45	~ 0.50	~ 0.55
starting ion temperature (eV)	100	105	115	138
final ion temperature (eV)	192	170	180	174

2 - BEHAVIOUR OF THE ELECTRON POPULATION.

The main effects engendered on the electrons by the application of the H.F. are a large density increase together with an increase in the loop voltage. These effects are illustrated in Fig. 9 where the time variation of the mean density and the product of the plasma current and loop voltage : $I \times V_p$ are plotted. Since I is almost a constant, the variation of this product $I \times V_p$ is due to the variation of V_p the loop voltage. This product would represent also the ohmic power if the plasma were at equilibrium and therefore inductive effects could be excluded.

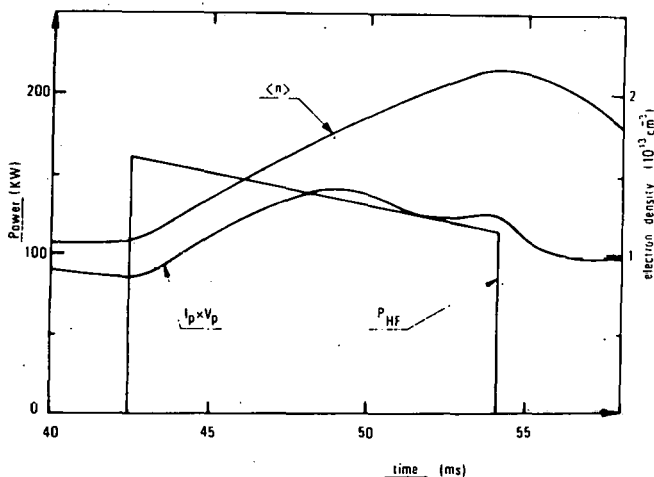


Fig. 9 : Time variation of line density, H.F. power, and the product of plasma current and loop voltage.

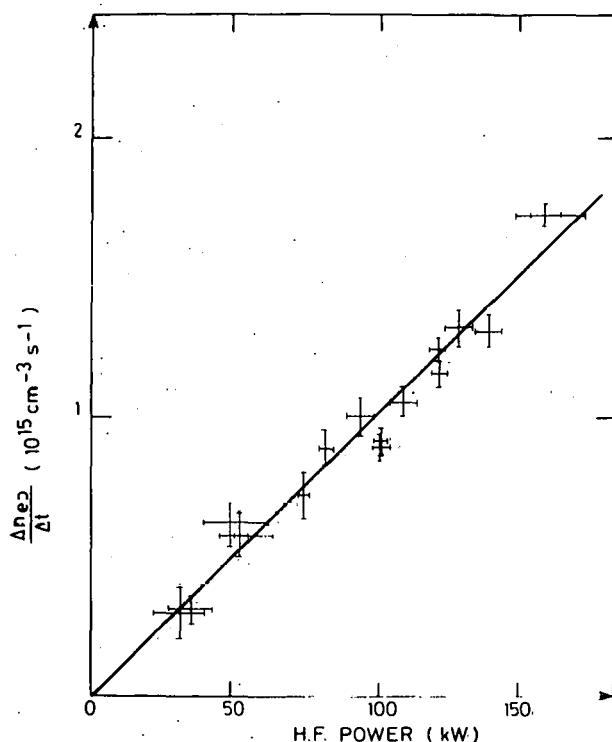


Fig. 10 : Time rate of change of peak density versus H.F. input power, from interferometry assuming parabolic profile.

The rate of increase of the line density as plotted in Fig. 10 is proportional to the applied H.F. power with a rate of $d \langle n \rangle / dt = 1.1 \cdot 10^{13} \text{ cm}^{-3} / \text{s/kW}$. In Fig. 10, the peak density is computed assuming a parabolic profile. This rate has not changed very much for all the various experimental conditions except when the working gas was changed. For the case of hydrogen, the density increase was half as large as for the case of deuterium. It can be noted that the density begins to increase right after the H.F. is applied and decreases immediately after the H.F. is turned off. This density increase presently limits the ion heating.

Although a clear picture of the behaviour of the electron population is not yet available, Thomson scattering data giving n_e and T_e profiles have been obtained during the first 6 ms after H.F. application for the following set of conditions :

$$n_{e0} (\text{start}) = 2 \cdot 10^{13} \text{ cm}^{-3}, \quad \langle P_{\text{HF}} \rangle = 135 \text{ kW}, \quad \Delta t = 10 \text{ ms}.$$

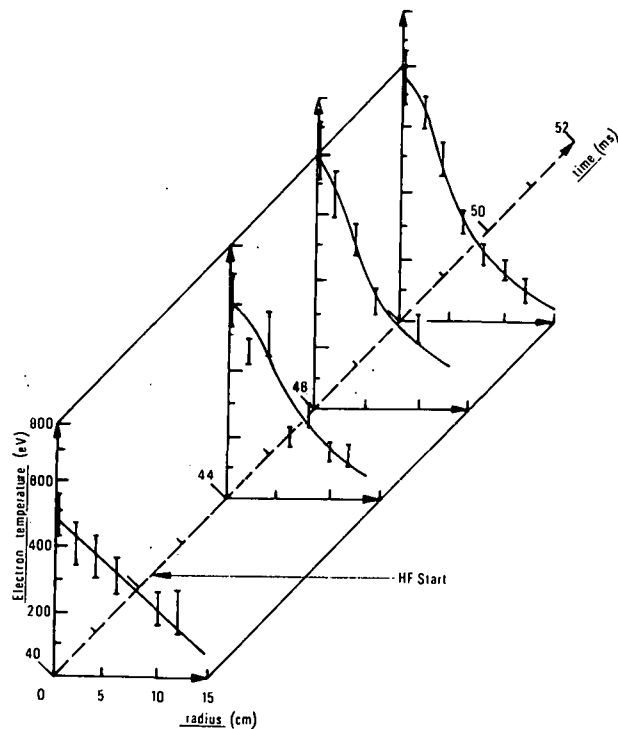


Fig. 11 : Time evolution of electron temperature profiles.

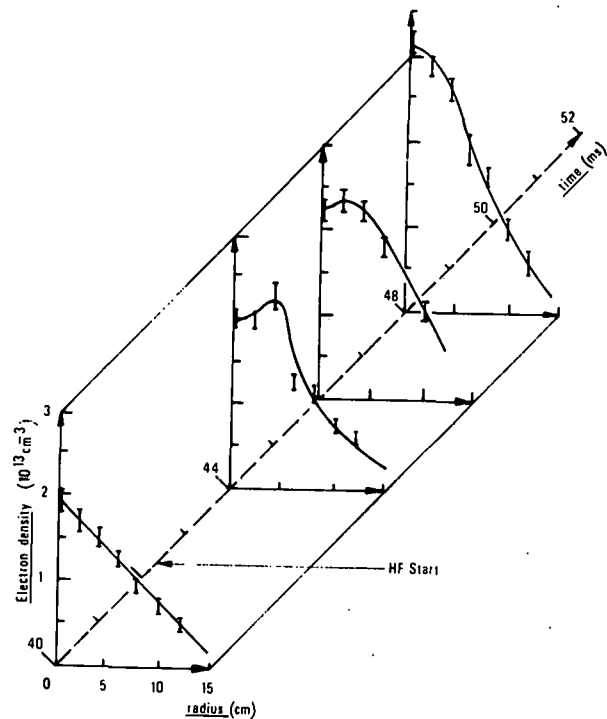


Fig. 12 : Time evolution of electron density profiles.

The obtained profiles are shown in Fig. 11 and 12. Before the H.F., the T_e and n_e profiles are roughly triangular. During the H.F., the central values increase and the profiles become considerably more peaked. The corresponding time behaviour of the electron energy content is plotted in Fig. 13. In 6 ms, doubling of the energy content is obtained. After that time, sawteeth in the soft X ray data appear, indicating that q values lower than 1 occur in the plasma and therefore some peaking of the current profile has occurred.

The corresponding peaking factor has been tentatively estimated by assuming that Z_{eff} is constant along the plasma radius and inductive effects may be neglected, that is that the current density varies as $T_e^{3/2}$. This is shown in Fig. 14 where the time variation of the ratio of q at the edge ($q = 4.5$) to q at the center which is equivalent to the ratio of the central current density to the average current density is plotted. From this peaking factor, the electron temperature and the loop voltage, one can calculate an average $Z_{effective}$ by assuming that : i) the peaking factor is attributed to changes in profiles, ii) inductive effects are negligible. The average Z_{eff} calculated in this way increases from 2 to 5 during the H.F.. However, a local measurement of Z_{eff} at the center, deduced from soft X-ray emission, by assuming only oxygen impurities, shows a smaller increase of Z_{eff} , from 2.5 to 4. The dashed line in Fig. 14 indicates this Z_{eff} (the electron profiles, which are not known after 6 ms., are then assumed to be constant). While this would indicate, as did the OVII measurements, a lower than average Z_{eff} at the center, the fact that the sawteeth start only after 7 ms, would demand a higher than average resistivity or a lower electric field at the center. This discrepancy may be due to inductive effects, which have not yet been completely evaluated, but may also

Fig. 13 : Time evolution of electron energy content and of "electron energy replacement time" defined as the ratio between the energy content and the product $I_p \times V_p$.

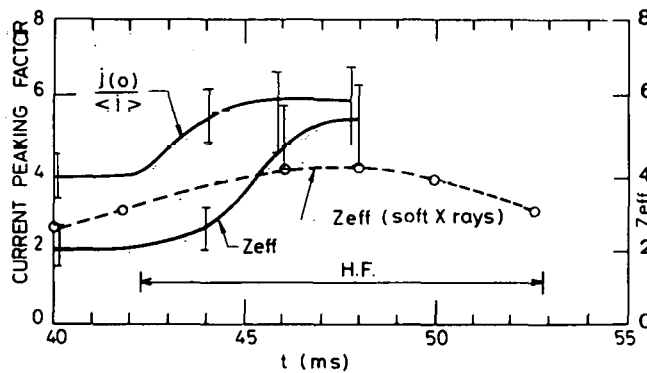
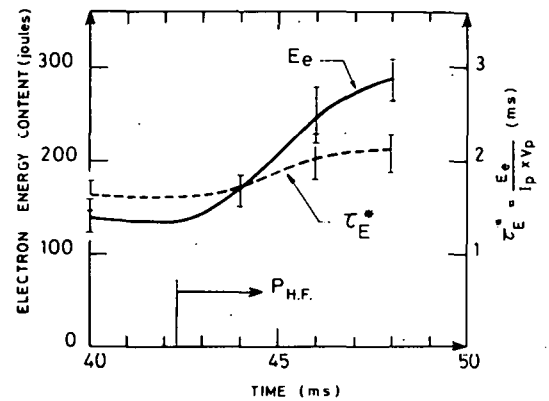


Fig. 14 : Time evolution of current peaking factor, average Z_{eff} inferred from loop voltage, and Z_{eff} at center from soft X-rays. Z_{eff} and electric field are assumed constant radially.

indicate an anomalous resistivity at the center. More accurate measurements are needed to clear up this point.

If one can assume that the product $I_p \times V_p$ represents the ohmic power and that only this power is applied to the electrons, the ratio between the electron energy content and this product is a measure of the electron energy lifetime. The time variation of this pseudo lifetime : τ_E shown in Fig. 13 indicates an increase from 1.7 to 2.1 ms. By assuming that no H.F. power is transmitted to the electrons, one could estimate by what amount the lifetime is upgraded or degraded by the H.F. provided that the appropriate scaling law is known. The scaling law where τ_E is roughly proportional to the density would give a larger increase in the lifetime than above but this scaling law (9) is not necessary applicable in a non equilibrium state and in the presence of additional heating.

3 - H.F. EMITTED SPECTRA.

During the H.F., decay spectra obtained with a magnetic pick-up loop oriented in the horizontal plane i.e. observing the poloidal component of the H.F. magnetic field and with an electric probe observing the radial component of the electric field, are shown in Fig. 15. The spectrum of Fig. 15c presents a linear shape on a logarithmic scale indicating that the induced waves decay exponentially away from the pump on the low-frequency side. In addition on the expanded scale of the spectrum of Fig. 15a a broadening of the pump of about 4 MHz is seen, the bandwidth of the detector being 0.3 MHz. Although the spectrum obtained with the electric probe is very similar to the others (Fig. 15b), here the low frequency component can be observed whereas this could not be observed on the magnetic probe because of its poor response at low frequencies. Even though the harmonics of

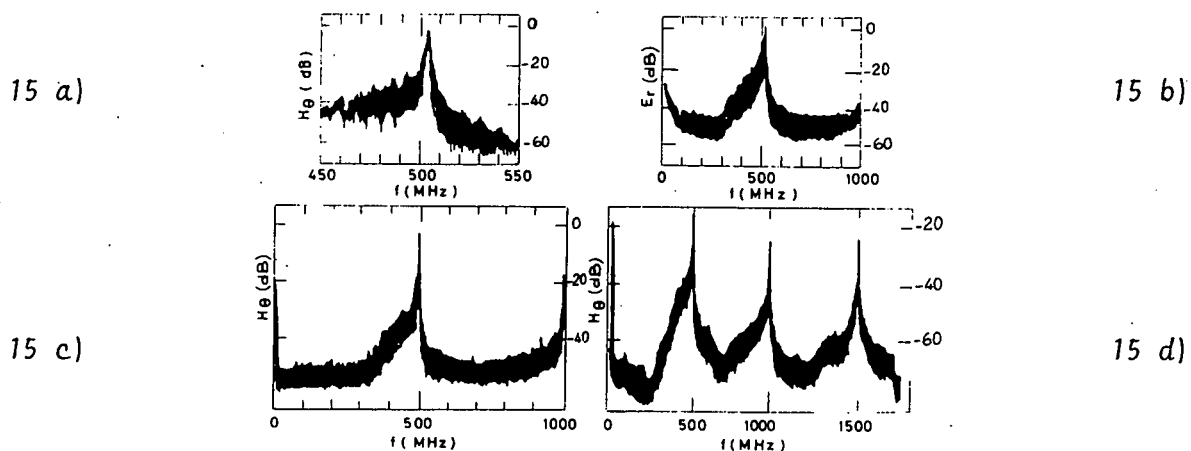


Fig. 15 : H.F. decay spectra as function of frequency : a), c), and d) H_0 from pickup loops and b) E_r from electric probe.

500 MHz are produced by the generator, it is worthwhile to note that each harmonic has its own sideband (Fig. 15d).

These measurements are performed at the edge of the plasma. Improved diagnostics are required in order to obtain information from the inside of the plasma. The power in the decay spectrum is about proportional to the input power for $P_{HF} > 20$ kW, and its shape is independent of both starting density and input power.

CONCLUSIONS.

Several conclusions about the behaviour of the electron population may be stated. While a scaling for the density increase including geometrical effects as well as launching structure parameters is not available, the density increase for the present parameters is proportional to the H.F. energy. A peaking of the electron density and temperature profiles occurs. The electron energy content doubles. This heating is consistent with ohmic heating alone, but this implies that the electron energy confinement time increases less than the density increases. Anomalous resistivity, if any, cannot yet be separated from inductive effects.

Although a comprehensive study of the effects of lower hybrid heating on the ion population is not presented here, some definite conclusions can be stated. Significant bulk ion heating with a thermalization of the heat deposition is observed, as indicated by the parallel temperature increase and the long decay time after the H.F. The central ion temperature is more than doubled, and the ion energy content is tripled. This heating does not seem to be due to impurity heating or increased electron-ion heat transfer due to the density increase.

The best heating efficiency is found when the resonance layer stays close to the center during the entire H.F. pulse. Although data such as the neutral gas density profile are not available and therefore the correct power balance equation cannot be written, the amount of power transferred to the ions by the H.F. can be estimated to be between 15 and 50 kW.

REFERENCES.

- (1) - G. TONON et al., Third International Congress on Waves and Instabilities in Plasmas, Palaiseau, France (June 1977)
See also EUR-CEA-FC 914, Sept. 1977.
- (2) - G. ICHTCHENKO, Bull. Am. Phys. Soc. 22 (Oct. 1977), p. 1169.
- (3) - P. LALLIA, G.W. PACHER, H.D. PACHER, Third Symposium on Plasma Heating in Toroidal Devices, Varenna, (Sept. 6-17 1976) p. 79.
- (4) - C. BRETON et al., EUR-CEA-FC 822 (1976).
- (5) - R. BARDET, private communication.
- (6) - M. BRAMBILLA, P. LALLIA, T.K. NGUYEN, Third International Meeting Theoretical and Experimental Aspects of Heating of Toroidal Plasmas, Grenoble (1976), Vol. II, p. 283-304.
- (7) - Fontenay code MAKOKOT.
- (8) - J.G. WEGROWE et al., 8th European Conference on Controlled Fusion and Plasma Physics, Prague, (Sept. 19-23 1977), Vol. I, p. 17.
- (9) - J. HUGILL, J. SHEFFIELD, The JET project EUR-5791 e.
Submitted to Nuclear Fusion.

Sup

LOWER HYBRID WAVE HEATING EXPERIMENTS AT GENERAL ATOMIC*

by

Charles Moeller and Doublet IIA Group
General Atomic Company, San Diego, California 92138

ABSTRACT

Plasma heating experiments utilizing Landau damping of the lower hybrid wave at frequencies above ω_{LH} are currently in progress in Doublet IIA and the dc Octopole tokamak. Both the Octopole and Doublet IIA have demonstrated excellent coupling using slow wave antennas. The dc Octopole operating in its normal mode has demonstrated penetration and highly efficient electron heating. Increased electrical conductivity has been observed in the Doublet IIA experiments. In the current experiments the auxiliary power can significantly exceed the ohmic power input, the ratio being as high as 5.

We have been conducting rf heating experiments for several years at General Atomic, first in the dc Octopole, and more recently in Doublet IIA. Most of the work has concerned electron heating by Landau damping of the lower hybrid wave at frequencies above ω_{LH} .

In these experiments, because of the $n_{||} = k_{||}c/\omega$ required for Landau damping, we have used slow wave structures. Even in Doublet IIA, which requires an $n_{||} \gtrsim 10$, it was not practical to use the multiwaveguide grill scheme, both because of access difficulties and because the reflected power for that large an $n_{||}$ would have been unacceptable without external tuning. The physics of the wave launching by these slow wave structures is identical to that of the waveguide grill, however, so that results obtained with them are equivalent to electron heating experiments using waveguide grills.

In all of our slow wave structures, the radiating elements are fed from a common transmission line, with the expected plasma loading such that only a small fraction of the input power is radiated by each element. In order to control the plasma loading, we use two methods to produce a small evanescent layer between the plasma and radiating element.

In the Octopole, it is possible to use a glass plate to define the plasma boundary with considerable confidence. A drawing of this structure, which is a strip-line version of a bifilar helix sandwiched between two glass plates, is shown in Fig. 1. This structure, in the absence of plasma, is a balanced 100 Ω transmission line, which is terminated outside the vacuum chamber so that the plasma loading can be measured. With this inherently broadband structure, it is possible to vary the $n_{||} = k_{||}c/\omega$ over a wide range by varying ω . The details of these measurements will be given in Paper B4, the salient points being that as long as $\omega/k_{||} > v_{eth}$ near the antenna, the loading is consistent with theoretical predictions based on the

*Work supported by Department of Energy Contract EY-76-C-03-0167, Project Agreement 38.

cold plasma model; we saw only electron heating as long as $\omega \gg \omega_{LH}$; the measured plasma loading is entirely accounted for by the measured increase in T_e in the body of the plasma. This last point is important in showing that there were no anomalous processes whereby the power was lost on the plasma edge rather than penetrating to the plasma interior.

Although a dielectric sheet provides a certain means of limiting the plasma, it is unacceptable with high temperature plasmas, so that an alternative scheme using metallic limiters is also used, the Octopole version being shown in Fig. 2. In this case, the radiating elements capacitively load the feedline, but the overall structure is still approximately a 100Ω balanced line at our frequencies of interest (< 100 MHz). The behavior of the plasma loading in this case was observed to be no different than with the glass plate, giving us some confidence in the effectiveness of these limiters.

The antennas being used in Doublet IIA, which use metallic limiters, are all of the design shown in Fig. 3. The principal difference from the Octopole antenna is that the radiating elements are $\lambda/2$ resonators so that they are self-supporting and do not load the feedline. The electric field presented to the plasma is very similar to that of the waveguide grill structure, the principal difference being the absence of limiters to control the plasma density in front of the grill.

The plasma loading and self-consistent radiated spectrum calculations will be given in Paper B2. The main results are that the plasma loading has a broad maximum as a function of plasma density gradient, so that the voltage along the antenna and the resulting radiated spectrum do not vary greatly with the density gradient at the antenna. We have three pairs of antennas located symmetrically on the top and bottom of the machine with $n_{||}$'s of 11, 14, and 16. The experimental data we have thus far on the antenna loading as a function of plasma density is rather qualitative and is typified by Fig. 4. At 20 msec, the plasma began to shift radially, reducing its effective diameter and the density

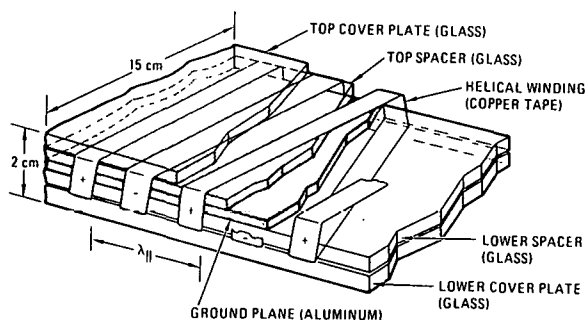


Fig. 1. Octopole slow wave antenna using a glass plate to limit the plasma

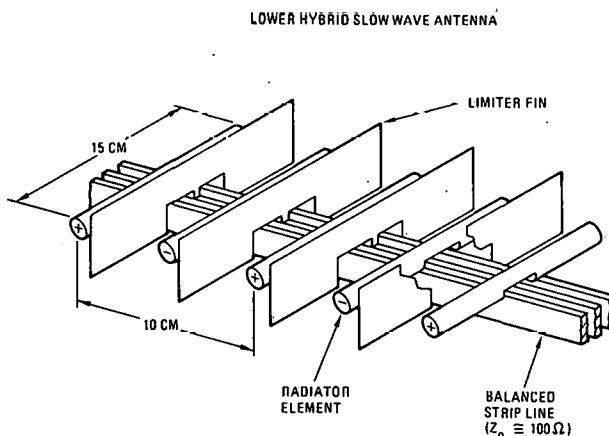


Fig. 2. Octopole slow wave antenna using grounded metal plates to limit the plasma

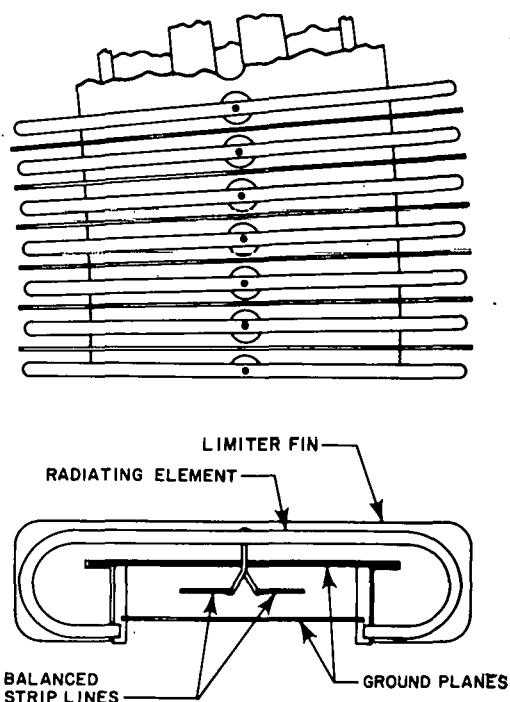


Fig. 3. Doublet IIA slow wave antenna

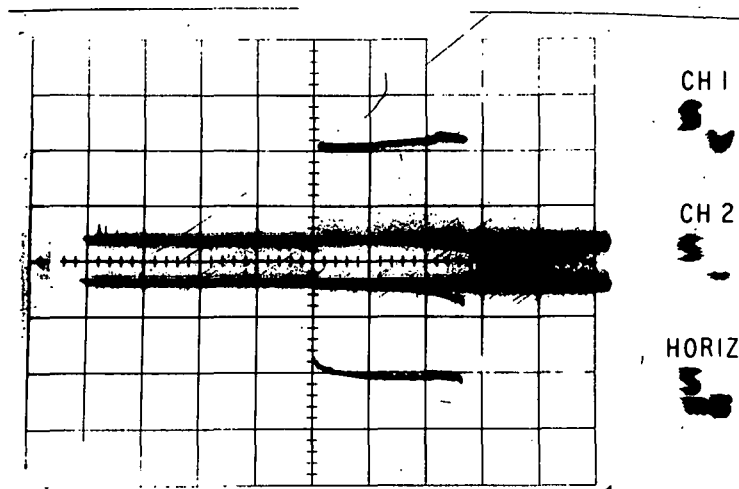


Fig. 4. Forward and reflected power, 100 kW/div

Upper pair of traces $n_{||} = 11$
Lower pair of traces $n_{||} = 14$

at the antenna. This indicates that the antennas are not highly over-coupled at the normal density. Note that the $n_{||} = 14$ is more strongly affected than the $n_{||} = 11$. We have routinely operated the $n_{||} = 11$ and 14 antennas at 80 kW each, and the $n_{||} = 16$ antennas at 60 kW each. Although we have not yet obtained a consistent picture from all of our diagnostics, the Thomson scattering dissenting, we have several indications that the wave is penetrating and adding energy to the electrons. The details will be given in Paper B2, but the main points are as follows: (1) we observe a substantial (as much as a factor of 2) increase in the average plasma conductivity upon application of the rf, which has rise and fall times of the order of the electron energy confinement time, (2) the radiometer at the second harmonic of the electron cyclotron frequency shows a substantial increase upon application of the rf, and (3) the soft X-ray ratios show a substantial increase when the rf is applied.

CONCLUSION

We have observed efficient electron heating in the Octopole, a machine in which the energy confinement is understood and is long compared to the time required to significantly raise T_e . In Doublet IIA, as in all tokamaks, the electron energy loss mechanisms are less well understood. We at present have evidence that power is reaching the interior and adding energy to the electrons, but the effects of the rf may be dominated by the more numerous energy loss mechanisms in a high temperature plasma. This is suggested by the great variation of the rf effects with machine and plasma conditions.

Further investigations could be significant not only to electron heating experiments using Landau damping of the lower hybrid wave but possibly to other electron heating experiments.

Dup

EFFECT OF LOWER HYBRID WAVES ON THE EVOLUTION OF THE DOUBLET IIA PLASMA*

by

D. F. Vaslow, Neil H. Brooks, R. L. Freeman, E. J. Heckman,
T. H. Jensen, John Lohr, J. L. Luxon, Ali Mahdavi,
Charles P. Moeller, John C. Riordan, R. D. Stav, and J. F. Tooker
General Atomic Company, San Diego, California 92138

ABSTRACT

RF power at 800 MHz and 915 MHz has been launched into Doublet IIA plasmas using slow wave antennas. The maximum rf power coupled is more than a factor of four larger than the ohmic power for circular discharges. Changes in the plasma current and loop voltage have been observed with the onset of rf power for low impurity level discharges.

1. INTRODUCTION

The basic approach of the Doublet IIA rf heating experiments was addressed in the paper by C. Moeller, *et al.*, in these proceedings.¹ Landau damping of lower hybrid waves by electrons represents an efficient means of transferring energy from the waves to the plasma, and for tokamak reactor plasmas the close coupling of electrons and ions insures effective ion heating even though the energy is deposited mainly into the electrons. Waves above the lower hybrid frequency can be efficiently coupled by waveguides,² loops,³ or slow wave structures.¹ The relatively modest electron temperatures in Doublet IIA (100-400 eV) require a large $n_{||}$ (11, 14, and 16 are used) for optimum electron Landau damping.

RF power levels of up to 400 kW have been coupled to circular discharges with ohmic inputs of approximately 100 kW. The impurity level of the discharges appears to be the main parameter which determines the macroscopic behavior of the plasma during rf heating. When only a short period of discharge cleaning preparation is used, the dominant change in the discharge characteristics is an increase by up to a factor of two in the rate of density increase and increased impurity radiation from all ionization states of oxygen. After considerable discharge cleaning preparation, the rf power absorption appears as an increase in the electrical conductivity of the plasma. Under these conditions wave penetration to the interior is indicated by increases in line radiation from the higher ionization states of oxygen while only small increases are observed in the radiation from the lower ionization states.

The experiments have been conducted in two phases. The first phase was a test of the antenna coupling and heating with 100 kW of rf power at 800 MHz with clean machine conditions. The second phase to investigate rf heating with power levels larger than the ohmic power input has started with 500 kW of rf power available and improved diagnostic capability. Both experimental phases are described separately in the following sections and the operating parameters for each phase are shown in Table 1.

*Work supported by Department of Energy Contract EY-76-C-03-0167, Project Agreement 38.

TABLE 1
TYPICAL DOUBLET IIA OPERATING PARAMETERS

	100 kW	500 kW
Operating gas	Hydrogen	Hydrogen
Toroidal magnetic field (kG)	6.4	7.6
Plasma current (kA)	30	35
Ohmic power (kW)	70	100
Central electron temperature	150	300
Average electron density (cm^{-3})	0.7×10^{13}	1.2×10^{13}
Z_{eff}	1.5	3

2. 100 kW EXPERIMENTS

This first phase of the experiments was carried out with 100 kW of power at 800 MHz and demonstrated excellent coupling with the General Atomic slow wave structures. Good power absorption in the plasma was indicated by a significant increase in the electrical conductivity of the plasma during the application of the rf power to low Z_{eff} circular discharges. The increase in conductivity resulting from the rf power application as seen in Fig. 1 corresponds to an increase of the electron temperature from 140 to 210 eV assuming $Z_{\text{eff}} = 1$. The hard and soft X-ray signals indicate that this increased conductivity is not due to runaway or slideaway electrons. The Thomson scattering data was not reliable during these experiments, so the increased conductivity temperature could not be confirmed by Thomson scattering measurements.

Spectroscopic observations of these discharges were made through a radial viewing port displaced 10 cm from the plasma center. No increase in radiation from the low ionization states of oxygen or iron ions was observed (Fig. 1) indicating no global increase in impurity content. Power absorption at the spatial location of the OIV shell is demonstrated by an increase in the radiation from the OIV line. A localization of heating towards the outside is not unexpected due to the low electron temperatures resulting from the reduced impurity level of these discharges. Consequently, the Landau damping is expected to be weak so the waves may be collisionally damped in the outer regions of the plasma.

3. 500 kW EXPERIMENTS

The second phase of the experiments is in progress at higher power levels and with improved diagnostic capabilities. A total rf power in excess of 500 kW is presently available with 350 kW at 800 MHz (upgraded P.P.P.L. supplies) and 200 kW at 915 MHz. An additional pair of slow wave structures for 915 MHz with $n_{\parallel} = 16$ has been installed and all six antennas can now be placed at the surface of a circular discharge in the midplane of the machine. This allows use of the microwave interferometer and the Thomson scattering system is operative. A radiometer system for monitoring electron cyclotron second harmonic radiation has been installed. Central electron temperatures around 300 eV have been obtained. This temperature

range is optimum for linear Landau damping for our antennas if the temperature profile is not too peaked. The dominant physical mechanism determining the influence of the rf heating on the discharge behavior still appears to be the impurity level of the discharge although the machine has not yet operated with the "clean" conditions utilized for the 100 kW experiments. Thomson scattering measurement of these discharges have shown a small density increase in the central region and no increase in the electron temperature as a result of application of rf power. In contrast, the radiometer shows a significant increase in the signal at the second harmonic of the electron cyclotron frequency indicating a temperature increase. Figure 2 shows an example of a short 350 kW rf pulse applied to a circular discharge. The increase in the electron cyclotron second harmonic radiation from the region of the major radius decays about 1 msec after the rf pulse ends. The radiometer is tuned to the frequency corresponding to the major radius but does not view the plasma directly so radiation from a vertical cord is received after multiple reflections from the vacuum vessel walls. Since the radiometer measurement is sensitive to nonthermal radiation, the interpretation of these signals is not straightforward. However, no increase is detected in the hard X-ray radiation. Longer rf pulses lead to even larger increases in the radiometer signal, but are often accompanied by minor disruptions.

4. CONCLUSION

Good coupling using slow wave structures has been demonstrated, and evidence exists for wave penetration and power absorption. The central electron temperature as measured by Thomson scattering does not show any significant increase as a result of application of rf power. The increase in electron energy as indicated by the radiometer measurement may be at locations not accessible to the Thomson scattering system.

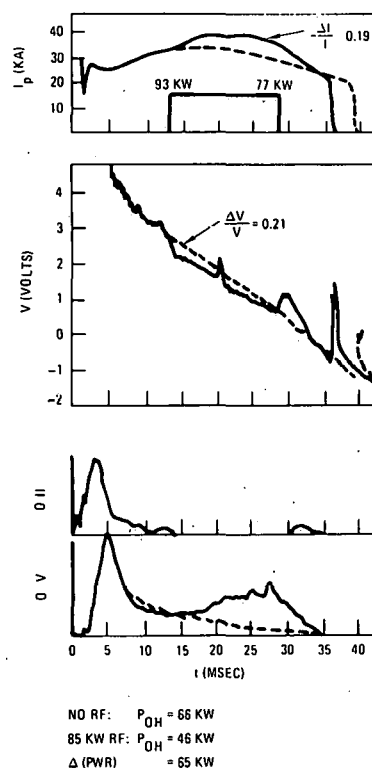


Fig. 1. An increase in the electrical conductivity of a circular plasma due to the application of rf power is illustrated by the solid curves showing the temporal dependence of the plasma current and voltage. The behavior of the oxygen line radiation is shown in the last two traces.

References

1. C. Moeller, et al. (these proceedings).
2. S. Bernabei, et al., Third Symposium on Plasma Heating in Toroidal Devices, Vienna, Sept. 6-16 (1976) p. 68.
3. G. Tonon, et al., "Lower Hybrid Frequency Heating in Toroidal Devices With Emphasis on Wega Results," IPP Report: 2/233, September 1977.

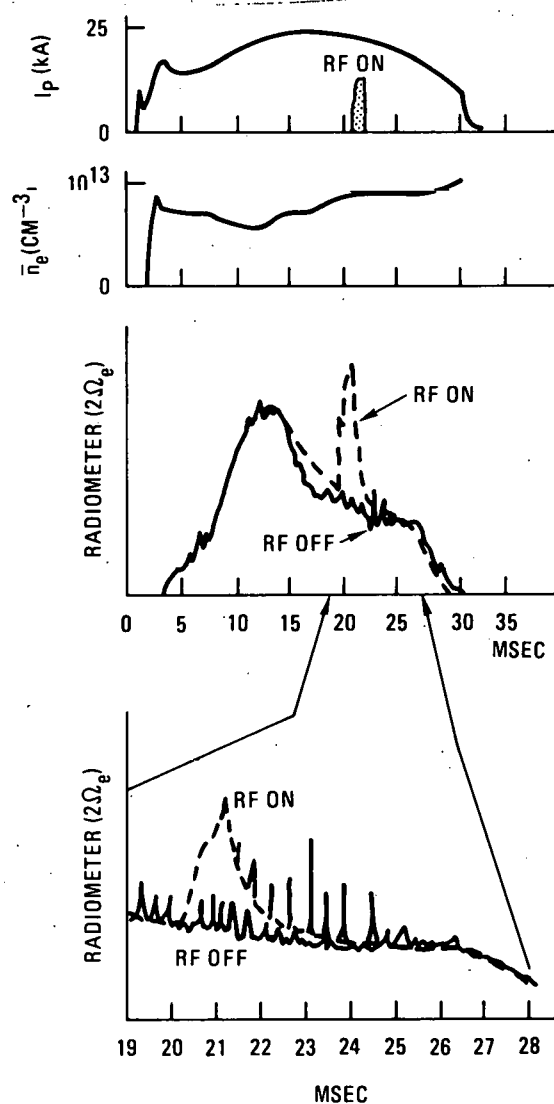


Fig. 2. The application of a 1 msec pulse of rf power at the 150 kW level to a circular discharge results in a large increase in the radiation at the second harmonic of the electron cyclotron frequency. The last trace shows the radiometer signal with a faster time sweep at the time of the heating pulse.

Demonstration of Accessibility and Inaccessibility for Slow Plasma Waves*

R. W. Motley, S. Bernabei, W. M. Hooke, R. McWilliams, and L. Olson,
Plasma Physics Laboratory, Princeton University

In the frequency interval $\omega_{LH} < \omega < \omega_{pe}$ and
for $\omega_{ci}\omega_{ce} < \omega^2$ probe measures of plasma waves
launched from phased waveguides show penetration only
if the accessibility criterion is satisfied.

- - - - -

The accessibility criterion^{1,2} for the propagation of plasma waves in the frequency interval between ω_{LH} and ω_{pe} is important because it imposes severe limitations on the design of slow wave structures for lower hybrid heating of plasmas; yet there exists no direct experimental evidence for its relevance.

Dispersion characteristics of lower hybrid waves for the particular case $\omega^2 > \omega_{ci}\omega_{ce}$ are illustrated in Fig. 1. Waves propagating along the upper sectors of the loops are called "slow waves"; along the lower sectors, "fast waves". A slow wave launched from outside the plasma, after tunnelling through a non-propagating low density region (where $n_1^2 < 0$), may propagate inward to a certain maximum density n_1 , at which point it may either reflect or convert to a fast wave. If $\omega^2 \gg \omega_{ci}\omega_{ce}$

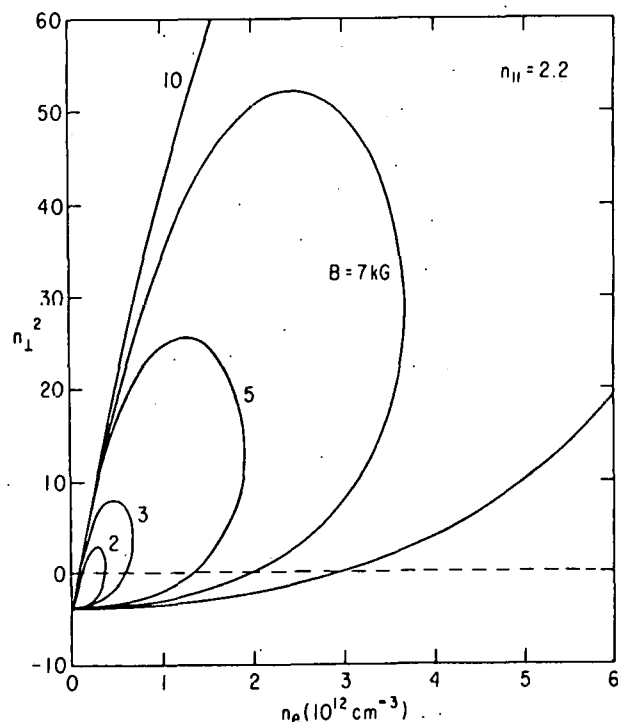


Fig. 1. Dispersion curves for $\omega_{LH} < \omega < \omega_{pe}$; $\omega^2 > \omega_{ci}\omega_{ce}$.

$$\frac{n_1}{n_c} = 1 + \left[\frac{(n_H^2 - 1) \omega_{ce}}{2n_H \omega} \right]^2, \quad (1)$$

where n_c is the cutoff density for $n_{\omega} = 0$.³ For a given geometry Eq. (1) predicts a critical field B_c that varies as $n_e^{1/2}$.

We have investigated the properties of waves launched by a phased twin waveguide into an overdense plasma. The target plasma was an rf-excited, 2 m long, 10 cm dia. argon plasma confined by magnetic field < 16 kG.⁴ Two adjacent waveguides were positioned at the outer radius of the plasma column, and excited out of phase ($E \parallel B_0$) by a 20 - 2000 W, 2.45 GHz magnetron. Typical transmission coefficients of 80-90% were achieved in the overdense plasma⁵ ($n \sim 2 \times 10^{12} \text{ cm}^{-3}$, $n/n_c \sim 25$). Wave propagation was studied by means of a triaxial probe moveable axially and radially.

Radial scans of the RF signal are shown in Fig. 2. If the magnetic field was high, two radial peaks were observed, one located just outside the waveguide throat and the other well inside the waveguide plasma column. Radial scans at increasing axial distances showed that the surface waves did not penetrate; the body waves moved across the plasma at an angle of $\sim 15^\circ$ with the field, consistent with the expected resonance cone angle $\theta = \omega/\omega_{pe} \sim 0.2 - 0.25$. Body wave excitation was enhanced if the waveguide phasing was $\sim 180^\circ$ (short wave excitation); 0° phasing (long wave excitation) reduced the signal by about a factor of 3, as expected.

We have also measured the radial and axial wavenumbers by means of an interferometer. The surface waves, in the high field

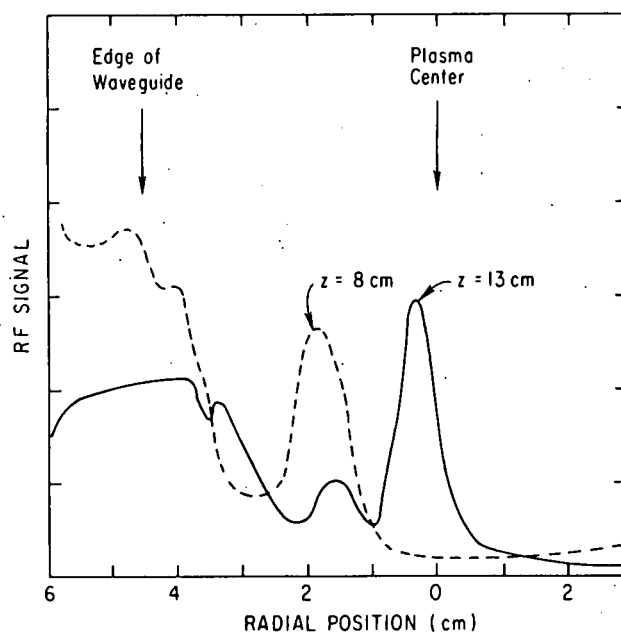


Fig. 2. Radial wave profiles measured 8 and 13 cm from the waveguide; $B = 6.5$ kG, $n_{\max} = 1.6 \times 10^{12} \text{ cm}^{-3}$.

cases, are of long wavelength, $n_{||} = 1 - 1.2$, as shown in Fig. 3, with no measurable phase change radially. The body waves are of short wavelength; $n_{||} = 2 - 2.6$ and $n_{\perp} = 9 - 12$, the ratio $n_{\perp}/n_{||} = 4 - 5$ being consistent with the approximate electrostatic dispersion relation $n_{\perp}/n_{||} = \omega_{pe}/\omega = (n/n_c)^{1/2}$.

Quite a different pattern was observed at lower fields, if $B < B_c$. As shown in Fig. 4, the surface wave intensity increased strikingly below

$B = 3$ kG, relative to the interior wave. In this low field regime

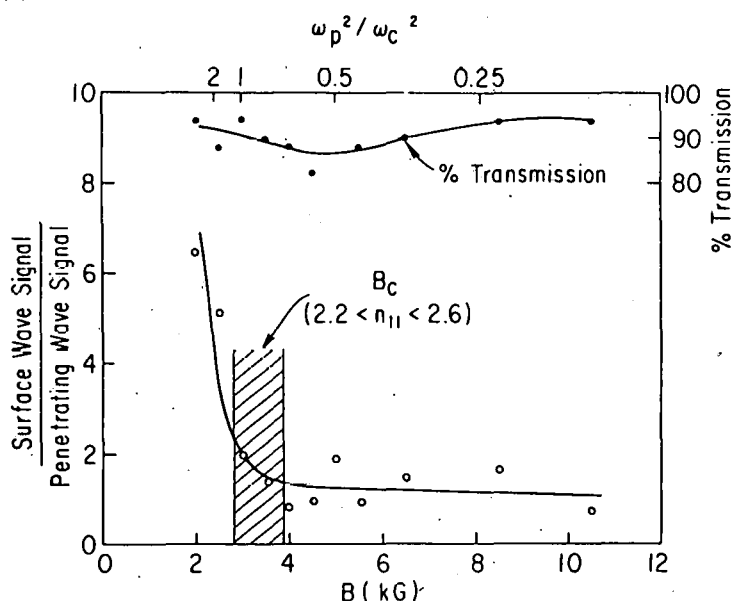


Fig. 4. Waveguide transmission coefficient and ratio of surface wave/interior wave as a function of magnetic field.

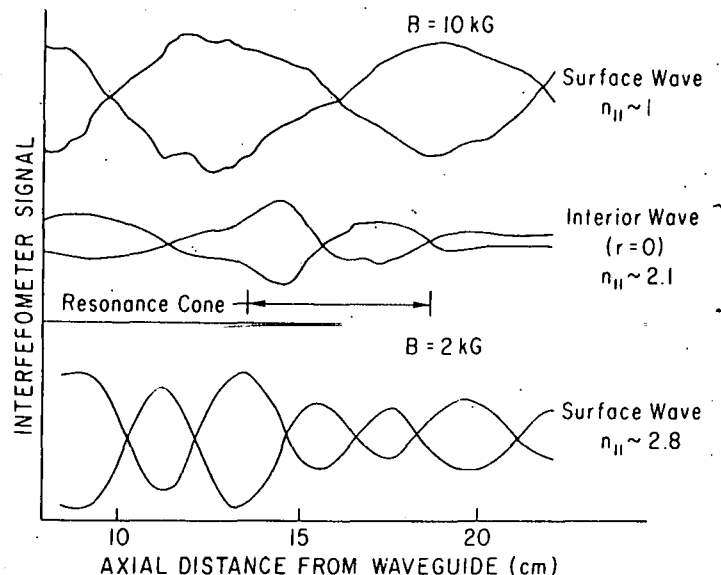


Fig. 3. Axial phase measurements of surface ($r = 4$ cm) and interior ($r = 0$) waves above and below B_c .

where $\omega_p^2/\omega_c^2 \gtrsim 1$, the interferometer measurements show $n_{||} \sim 2 - 2.8$ for the surface wave, which was observed not to penetrate the plasma column. The existence of a critical field of ~ 3 kG is consistent with Eq. (2) as shown in the figure, since the twin waveguide excites a band of wavelengths extending to $n_{||} \leq 3$, according to Brambilla.⁶

It is interesting to note that the excitation of an almost pure surface wave does not lead to increased reflection from the waveguides.

In conclusion we have shown for $\omega^2 \gg \omega_{ci} \omega_{pe}$ that lower hybrid waves can penetrate on overdense plasma only if the magnetic field exceeds a critical value consistent with the (modified) accessibility criterion. This criterion forms the basis for the design of lower hybrid slow wave structures designed to heat toroidal plasmas.

REFERENCES

* This work was supported by the U.S. Department of Energy, Contract EY-76-C-02-3073.

¹ T. H. Stix, The Theory of Plasma Waves (McGraw-Hill, New York, 1962).

² V. E. Golant, Zh. Tekh. Fiz. 41, 2492 (1971) [Sov. Phys. Tech. Phys. - JETP 16, 1980 (1972)].

³ F. Troyon and F. W. Perkins, in Proc. of Second Topical Conf. of RF Plasma Heating (Texas Tech. Univ., Lubbock, Texas, 1974) paper B4.

⁴ R. W. Motley, S. Bernabei, W. M. Hooke, and D. L. Jassby, J. Appl. Phys. 46, 3286 (1975).

⁵ S. Bernabei, M. A. Heald, W. M. Hooke, F. J. Paoloni, Phys. Rev. Lett. 34, 866 (1975).

⁶ M. Brambilla, Nucl. Fusion 16, 47 (1976).

Dug
A7-1

LIMITATIONS ON LOWER HYBRID HEATING OF DENSE, HOT PLASMAS*

by

R. W. Harvey, J. M. Rawls, M. P. Hacker, and V. S. Chan
General Atomic Company, San Diego, California 92138

ABSTRACT

Accessibility and quasilinear Landau damping considerations limit the density and temperature of plasma which may be effectively heated by lower hybrid (LH) waves. In conjunction with a global thermonuclear ignition condition, it is found that LH heating to ignition is possible if the plasma size and magnetic field satisfy $aB^2 \geq 70$ (meters-tesla²).

1. INTRODUCTION

Heating of experimental plasmas by lower hybrid (LH) waves is currently the subject of considerable effort. It is hoped that this heating can be readily extended to the regime of thermonuclear plasma ignition. There is a difficulty though, in that the wave refractive index parallel to the magnetic field, $n_{\parallel}$, must be large enough for the wave to propagate to a given density, yet small enough to avoid excessive Landau damping. Using linear Landau damping, Troyon and Perkins¹ found quite severe limits on LH heating of hot, dense plasma. Fortunately, consideration of quasilinear plateau formation in the electron distribution results in substantially reduced Landau absorption, and hence greater penetration. Nevertheless, Coulomb collisions tend to re-thermalize the quasilinear plateau, hence the damping does not become entirely negligible. This paper re-examines limits on LH heating of thermonuclear plasmas, using a quasilinear-collisional model^{2,3} to determine the Landau damping.

2. ACCESSIBILITY TO HIGH DENSITY REGION AND CHOICE OF OPERATING FREQUENCY

Physics considerations entering into the choice of the two wave parameters fixed by the heating system — operating frequency, ω , and parallel refractive index, $n_{\parallel}$ — are shown in Fig. 1. Frequency is normalized to $\omega_{LH/\max}$, the maximum LH frequency attained in the plasma. We assume that the $n_{\parallel}$ of a wave remains approximately constant along a ray trajectory. If $n_{\parallel} < 1$, the wave reflects from the $\omega = \omega_{pe}$ layer near the plasma edge. If $\omega/\omega_{LH/\max} < 1$, the LH resonance layer occurs within the plasma and the wave energy will be strongly damped before penetrating this layer. Linear mode conversion to thermal modes occurs for $\omega/\omega_{LH/\max}$ typically up to 1.5, even for $n_{\parallel} = 2$. Experimentally⁴, parametric instabilities give rise to considerable damping in the frequency range up to $\omega/\omega_{LH/\max} = 2$. This suggests operating at $\omega/\omega_{LH/\max} \geq 2$. In order to avoid excessive Landau damping $n_{\parallel}$ must be bounded. To maximize the accessible density n_1 at given $n_{\parallel}$, yet avoid parametric instabilities, we choose $\omega = 2\omega_{LH}$. Assuming a 50-50 D-T mixture, this gives

$$n_1 \Big|_{\omega = 2\omega_{LH}} = f_1(n_{\parallel}) B^2, \quad (1)$$

where (MKS units)

$$f_1(n_{\parallel}) \equiv 9.7 \times 10^{18} \left\{ 28 n_{\parallel}^2 + 3 - 8 n_{\parallel} \left[3(4 n_{\parallel}^2 + 1) \right]^{1/2} \right\}.$$

*Work supported by Department of Energy Contract EY-76-C-03-0167, Project Agreement 38.

3. QUASILINEAR-COLLISIONAL LIMIT

ON $n_{||}$

The upper bound on $n_{||}$ such LH energy penetrates the plasma at a given temperature, is found using a quasilinear-collisional theory.^{2,3} We assume a slab model of the plasma, in which all spatial variations occur in the x-direction, and the $\underline{B}$ -field is aligned in the z-direction. The electron distribution function is averaged over velocity perpendicular to $\underline{B}$ to obtain $f(v_{||})$; also, it represents a spatial average over a poloidal flux surface. The electric field power spectrum $\xi(k_{||}, x)$ is assumed to be equal to $\epsilon_0 E_{rms}^2(x)/(2\Delta k_{||})$ if $|k_{||} - k_{||0}| < \Delta k_{||}/2$, and zero otherwise. E_{rms} is the rms wave electric field strength averaged over a poloidal flux surface, and $k_{||0}$ and $\Delta k_{||}$ are, respectively, the central value and width of the wave number spectrum excited by the antenna.

The basic steady state equations for f and ξ are

$$\frac{\partial}{\partial v_{||}} D_w(v_{||}) \frac{\partial f}{\partial v_{||}} + \frac{\partial}{\partial v_{||}} \left(D_{coll} \frac{\partial f}{\partial v_{||}} + \frac{3}{2} \frac{v_{||}}{\tau_D} f \right) = 0, \quad (2)$$

and

$$\frac{\partial}{\partial x} \left[\xi \frac{\partial(\omega \epsilon)}{\partial \omega} v_{gx} \right] = P_{ABS}(x), \quad (3)$$

where D_w is the quasilinear diffusion coefficient,⁵ $P_{ABS} = 2 \gamma \partial(\omega \epsilon)/\partial \omega \xi$, γ is the Landau damping decrement,

$$D_{coll} = 3 v_{Te}^2 / 2 \tau_D, \quad \tau_D = 2 \pi \epsilon_0^2 m_e^2 v_{ph}^3 / (e^4 n_e \ln \Lambda),$$

ϵ is the LH dielectric coefficient, v_{gx} is the perpendicular group velocity, $v_{ph} \equiv \omega/k_{||} = c/n_{||}$ is the parallel wave phase velocity and $v_{Te} = (T_e/m_e)^{1/2}$ is the electron thermal velocity. The collision term in Eq. (2) involving D_{coll} and τ_D has been derived by Zakharov and Karpman.⁶

Assuming that f is approximately Maxwellian except for the flattening caused by quasilinear diffusion, then Eqs. (2) and (3) give γ reduced by quasilinear effects,

$$\gamma = \frac{1}{2} \left(\frac{\pi}{2} \right)^{1/2} v_{gx} k_{||} v_{ph}^3 v_{Te}^{-3} \exp \left(- v_{ph}^2 / 2 v_{Te}^2 \right) / \left(1 + D_w / D_{coll} \right). \quad (4)$$

It is useful to evaluate the length, $L_{1/2}$, over which the wave intensity decreases by one half. Noting that $D_w/D_{coll} \gg 1$ for rf heating, and approximating the average rf flux at the plasma center by the applied flux S_{APP} , Eqs. (2) and (3) give

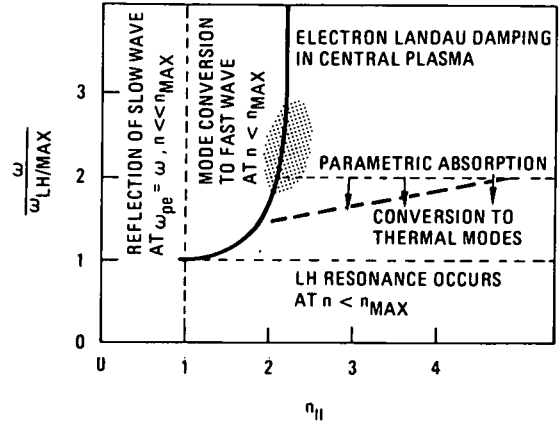


Fig. 1. The key physical mechanisms operating for various choices of $n_{||}$ and ω . The shaded region optimizes heating of the plasma center.

$$L_{1/2} = \frac{1}{2} \frac{S_{APP}}{P_{ABS} \Delta k_{\parallel}} = A_T n_e^{-2} T_e^{1/2} S_{APP} \exp\left(\frac{v_{ph}^2}{2v_{Te}^2}\right), \quad (5)$$

$$\text{where } A_T = (2\pi)^{3/2} \epsilon_0^2 m_e^{1/2} / \left[3 e^4 \ln \Lambda (\Delta k_{\parallel} / k_{\parallel}) \right].$$

Stipulating that $L_{1/2}$ must be greater than a given length $L_{1/2 o}$ for effective heating of the plasma center, we obtain the heating criterion

$$\frac{v_{ph}}{v_{Te}} > \left[2 \ln \left(\frac{L_{1/2 o} n_e^2}{A_T T_e^{1/2} S_{APP}} \right) \right]^{1/2} \equiv C_p. \quad (6)$$

C_p is a weak function of its arguments. At $n_e = 3 \times 10^{20} \text{ m}^{-3}$, $T_e = 7 \text{ keV}$, $S_{APP} = 400 \text{ kW/m}^2$, $\Delta k_{\parallel} / k_{\parallel} = 0.1$, and $L_{1/2 o} = 0.2 \text{ m}$, one obtains $C_p = 4.1$, giving $n_{\parallel} < c / (v_{Te} C_p) = 2.1$. The condition for the maximum central density and temperature that can be heated by LH radiation is obtained by substituting this $n_{\parallel}$ limit into Eq. (1), giving

$$\frac{n_e}{B^2} \leq f_1 \left(n_{\parallel} = \frac{c}{v_{Te} C_p} \right), \quad (7)$$

4. HEATING TO IGNITION WITH LH RADIATION

In order to LH heat to ignition, the region of n - T space defined by Eq. (7) must have a non-null intersection with that region in which the fusion power exceeds the sum of transport and radiative losses. For the case of a 50-50 D-T mixture with no impurities, the latter condition reads

$$\frac{1}{4} n_e^2 \langle \sigma v \rangle_{D-T} * 3.5 \text{ MeV} > \frac{3n_e T}{\tau_E} + C_B n_e^2 T^{1/2}. \quad (8)$$

For Alcator energy confinement time scaling, i.e., $\tau_E = 5.0 \times 10^{-21} n_e a^2$, the global ignition condition amounts to a lower bound on the line-average density, namely,

$$n_e a > f_2(T) \equiv 8.1 \times 10^{30} T^{1/2} / \left[T^{-2/3} \exp(-1.1 \times 10^{-4} T^{-1/3}) - 4.0 \times 10^{12} Z_{eff} T^{1/2} \right]^{1/2}. \quad (9)$$

In this representation, n_e and T are the central values, and the ignition condition has been evaluated by averaging over parabolic density and temperature profiles.

A criterion that lower hybrid heating be capable of heating to ignition is simply,

$$aB^2 > f_2(T) / f_1 \left(\frac{c m_e^{1/2}}{T_e^{1/2} C_p} \right), \quad (10)$$

obtained by taking the ratio of the above rf and global ignition conditions. The density dependence of aB^2 through C_p in Eq. (10) is significant. Figure 2 shows $(aB^2)_{\min}$ obtained by minimizing Eq. (10) as a function of T_e , and $(a)_{\min}$ satisfying Eq. (9), as functions of n_e . The minimizing temperature in Eq. (10) is $8.0 \pm 0.5 \text{ keV}$ over the whole range. The largest

conceivable value of a for a reactor is ~ 3 meters. Hence aB^2 must be greater than 70. This condition is generally much stronger than that for ignition alone.

However, this criterion is not as prohibitive as it might appear at first sight. For one thing, the plasma size enters the derivation only through the $\tau \propto a^2$ relation and it is believed that the correct treatment for noncircular cross sections involves the replacement

$$a \rightarrow \left(\frac{2\kappa^2}{1 + \kappa^2} \right)^{1/2} a, \quad (11)$$

where κ is the height-to-width ratio. This reduces aB^2 for ignition heating by a factor of 1.34 for $\kappa = 3$ — typical of doublet designs. Secondly, one has the freedom to position and phase the antenna structure such that the wave enters the plasma on the high magnetic field side, thereby enhancing the effective B . Further, it may be possible to optimize the penetration by choosing a frequency somewhat different from $2\omega_{LH}/\max$. There is also the distinct possibility that the very powerful quasilinear diffusion (much greater than collisional) may result in a different picture of the physics than the 1D quasilinear diffusion, possibly giving greater penetration. Damping may also be reduced by propagating only waves antiparallel to the net electron motion associated with the plasma current.⁷

As a final remark, note that the quantity aB^2 increases as one progresses in the scenario TNS-DEMO-power plant so that electron Landau damping of lower hybrid waves remains promising as a reactor heating scheme. In addition, if transport is in fact reduced at higher temperatures in conformity with a scaling law of the form $\tau \propto na^2 T^{1/2}$, a form which is not inconsistent with existing data, then the condition on aB^2 is eased by nearly a factor of 2.

References

1. F. Troyon and F. W. Perkins, *Proc. of the Second Topical Conference on RF Plasma Heating*, B-4, Texas Tech. University, Lubbock, Texas (1974).
2. J. M. Antonsen and E. Ott, Sherwood Theory Meeting, Madison, Wisconsin (1976).
3. A. Bers, F. W. Chambers, N. J. Fisch, and G. L. Johnston, Sherwood Theory Meeting, Madison, Wisconsin (1976).
4. M. Porkolab, S. Barnabei, W. M. Hooke, R. W. Motley, and T. Nagashima, *Phys. Rev. Lett.* **38**, 230 (1977).
5. A. Bers, F. W. Chambers, and N. J. Fisch, RLE Report, PR 118, MIT, Cambridge, Massachusetts (1976).
6. V. E. Zakharov, and V. I. Karpman, *Sov. Phys. JETP* **16**, 351 (1963).
7. S. Yoshikawa, personal communication (1977).

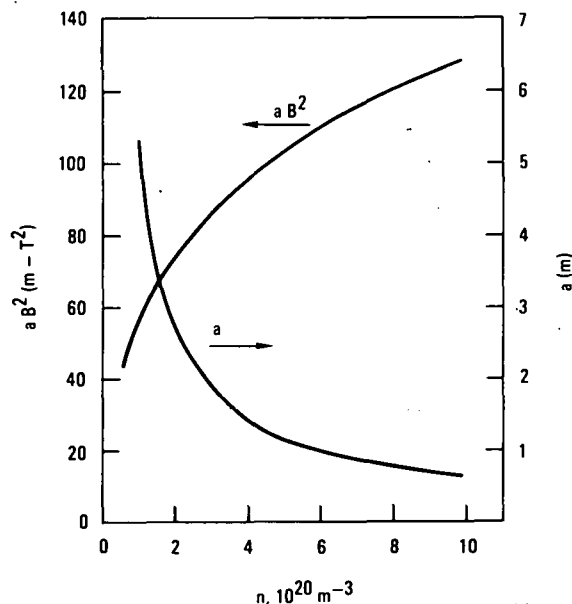


Fig. 2. Minimum aB^2 and a as a function of central density n_e , in order to heat a plasma to ignition

SIMULATION OF LOWER-HYBRID HEATING IN BOUNDED PLASMA

V. K. Decyk, J. M. Dawson, and G. J. Morales*
Department of Physics, University of California
Los Angeles, California 90024

A computer simulation study of plasma heating by lower hybrid waves has been undertaken. An electrostatic particle simulation code, described elsewhere,¹ is used to model a two-dimensional, magnetized plasma slab, periodic in the direction parallel to the magnetic field and bounded and subject to various boundary conditions in the direction perpendicular to it. As a first step in studying this problem, we have considered the launching of lower hybrid waves by external sources and their propagation in a finite but homogeneous slab bounded on both sides by vacuum. The ratio of electron cyclotron to plasma frequency has been taken as $\Omega_e/\omega_{pe} = 1$ and the ion-electron mass ratio $M_i/m_e = 64$. The source frequency varied between the ion plasma frequency ω_{pi} and $3\omega_{pi}$, and particles were reflected elastically from the boundaries. Since the normal modes of oscillation of a bounded plasma form a discrete set, the system responds differently depending on whether or not the frequency and wavelength of the source match simultaneously those of a normal mode of the plasma. For perfect matching, the resonant mode grows secularly with time, while if there is a mismatch, the plasma responds with non-resonant driven oscillations.

For a point source exciter, resonance cones have been observed to be created if the frequency is chosen so that the cones from each of the periodic sources interfere constructively. The resonance cones are observed to be broader than expected from cold plasma theory because the shorter wavelength components of the potential are quickly absorbed by electron Landau damping at the surface. Secondly, a large potential sheath develops around the source itself, which further reduces the penetration of the source. Superimposed on the driven oscillations, there is a modulation of

the electric field energy at the ion cyclotron harmonics. This was observed to occur whenever the physical size of the exciter was comparable or smaller than the ion Larmor radius. The plasma heating observed in this case occurs at the surface, arising mainly from an increase of electron thermal velocity parallel to the magnetic field and without the creation of energetic tails in the velocity distribution function.

When two short capacitor plates oscillating out of phase are used as an exciter, the "filtering" effect of the plasma surface plays a dominant role. The frequency and length of the source is chosen so that the wavenumber spectrum is broadly peaked around those modes whose phase velocity is near the electron thermal velocity v_{th} . The simulation results show that most of the modes are absorbed at the plasma surface, and only the longest wavelength mode is able to penetrate to the plasma interior. In other respects, such as the presence of the plasma sheath surrounding the exciter, the electron heating at the surface, and the ion cyclotron modulation, the results obtained with the capacitor plates are quite similar to those of the point source.

A phased array of capacitor plates has also been used as the exciting structure. The frequency and length of each element has been chosen so that the wavenumber spectrum is sharply peaked around a single mode whose phase velocity equals the electron thermal velocity. The simulation results show that there exist no driven oscillations in the plasma interior. The major observations in this case are the familiar electron surface heating and the presence of significant ion heating, which is also largest at the plasma surface. This ion heating comes from strong ion acceleration away from the exciting structure (across the magnetic field), and results in energetic tails in the ion velocity distribution in the direction perpendicular to the magnetic field.

The excitation of a bounded plasma resonance has also been considered. From a knowledge of the dispersion relation of the bounded plasma slab, the frequency and length of a capacitor plate exciter was chosen so that the wavenumber spectrum peaks at a normal mode whose parallel phase velocity is about $6 v_{th}$. The simulation results show that the plasma heating is dominated by electrons. The resonant mode grows in time until the amplitude is large enough to begin to trap electrons in the bulk of the velocity distribution. When this happens, many electrons are accelerated to velocities as large as $10 v_{th}$, and the wave loses most of its energy. Electron heating is localized in space and follows the pattern of the parallel electric field energy for the resonant mode. This offers a possibility for controlling the electron temperature profile in space. Nonlinear density changes associated with the ponderomotive force are also significant. Ions gain energy by being expelled from the density cavities by the ambi-polar potential associated with the ponderomotive force. When the density cavities formed relax, the resonant mode is excited again, and the process repeats.

* Work Supported by USDOE and ONR.

¹Viktor K. Decyk and John M. Dawson, UCLA Center for Plasma Physics and Fusion Engineering Report PPG-331 (1977). To be published.

ELECTRIC FIELD DISTRIBUTION ASSOCIATED WITH LOWER HYBRID
HEATING OF A TOKAMAK PLASMA

M. BRAMBILLA

ASSOCIATION EURATOM-CEA
Département de Physique du Plasma et de la Fusion Contrôlée
Service IGn - Centre d'Etudes Nucléaires
85 X - 38041 GRENOBLE CEDEX (France)

Most auxiliary heating methods for Tokamak plasmas are bound to transmit a considerable power through relatively small access ports. In the case of h.f heating this implies large electric fields near the coils or antennas used for the excitation of the waves.

In the Lower Hybrid frequency range best use of the available access is made by the Grill /1/-/3/. The nominal electric field required to satisfy the power needs of a Tokamak is then of the order of 1 kV/cm, rather independently of the size of the device.

The actual electric field near the Grill aperture will always be larger than nominal, due to the inhomogeneities which can be expected near metal edges. Fig. 1 shows the field E_z in the left guide of a two guides Grill, excited with $\Delta\phi = 180^\circ$. Ten evanescent eigenmodes are included, using the Grill numerical code /2/. The strong peak of the electric field near the central conductor helps to reduce the jump of B_y across this plane.

Inside the plasma these already large electric fields will be amplified not only by the usual WKB factor, but also by the peculiar structure of the so called "resonance cones" /4/. Indeed, in the cold-plasma electrostatic approximation the electric field diverges logarithmically along the "rays" (group velocity trajectories) starting from each discontinuity of the electric field at the antenna.

Of course an upper limit to the actual electric field can be deduced from the fact that parallel wavelengths shorter than a Debye length λ_D cannot propagate. Thus

$$E_{\text{peak}} \lesssim E_{\text{antenna}} \cdot \text{Log} (D/\lambda_D)$$

where D is a typical dimension of the antenna. We note that this quite large upper limit can indeed be approached in the vicinity of metallic antennas located without screening in the shadow of the limiter, where the local plasma frequency ω_{pe} exceeds the applied frequency ω by a large factor.

For a Grill the situation is more favorable, as long as one can expect $\omega_{pe}^2 \leq \omega^2$ near the wall. Then it is well known /2/ that the surface admittance of the plasma decreases with increasing $n_{//}$, so that the "short wavelength catastrophe" of the electric field no longer occurs. Let us write

$$\vec{E} = \int dk_{//} \vec{E}(k_{//}) e^{i(k_{//}z - k_0 \int q_s dx)} \quad (1)$$

where $q_s = (n_{//}^2 - \epsilon_{xx})^{1/2} (-\epsilon_{zz}/\epsilon_{xx})^{1/2}$ is the perpendicular index of the slow wave. The integral extends over values of $k_{//}$ which satisfy the accessibility condition ; z is the direction of the static magnetic field, and x the direction of the density gradient. Then for the largest component, E_x , we have

$$E_x(k_{//}) = \frac{n_{//}}{(n_{//}^2 - \epsilon_{xx}(x))^{1/2}} \left(-\frac{\epsilon_{zz}(x)}{\epsilon_{xx}(x)} \right)^{1/2} \left(\frac{n_{//}^2 - \epsilon_{xx}(0)}{n_{//}^2 - \epsilon_{xx}(x)} \right)^{1/2} \frac{\epsilon_a(k_{//})}{Z(k_{//})}$$

where $\epsilon_a(k_{//})$ is the Fourier transform of the antenna electric field along z , and /2/

$$Z(k_{//}) = Cte * (1 - \alpha^2)^{1/2} I_{-1/3}(\eta) + e^{\pi i/3} I_{1/3}(\eta)$$

with $\eta = (2/3) \kappa_L L (1 - \alpha^2)^{3/2}$, $\alpha^2 = \omega_{pe}^2 / \omega^2$ and $L^{-1} = n_{\text{cut-off}}^{-1} (dn/dx)$ evaluated at the Grill mouth.

It has recently been shown /5/ that by taking $\alpha^2 = 1$ and neglecting all but the fundamental waveguide modes in $\epsilon_a(k_{//})$ the integral (1) can be done in closed form, and is finite everywhere. If $\alpha^2 < 1$, Z^{-1} decreases exponentially for large $k_{//}$ and convergence is even more rapid, but for reasonable values of dn/dx the results for $\alpha^2 = 0$ are very similar to those for $\alpha^2 = 1$.

Evaluations along the lines of Ref. /4/, /5/ result however in an unphysical discontinuity of the electric field E_z across the plane of the antenna, in spite of the fact that the boundary conditions are formally

satisfied. This is of course due to the fact that the antenna electric field is not evaluated selfconsistently. Including a sufficient number of evanescent modes to describe the field in the Grill, the discontinuity is eliminated : The field in the antenna plane already shows peaks near the metal edges, while the peaks inside the plasma are somewhat less pronounced than predicted by /4/, /5/. We stress that evanescent eigenmodes are very important for the evaluation of the electric field, but influence little the reflection coefficient of the Grill and the power spectrum transmitted. This is because waves with large $k_{//}$ are almost electrostatic inside the plasma, and thus contribute little to the Poynting vector.

Further inside the plasma two new effects limit the amplitude of the electric field. First Electron Landau Damping rapidly eliminates the largest values of $k_{//}$. For the reasons explained above, this has a dramatic effect on the peak electric field, even if the power absorbed by the electrons is relatively small. Secondly, as easily deduced from the expression of q_s , the group velocity along the static magnetic field increases slightly with decreasing $k_{//}$ when electromagnetic corrections are taken into account. Thus while the internal edges of the resonance cones remain relatively well defined, the region of constructive interference rapidly broadens towards the outside. This depresses further the peak value of the field.

Fig. 2 shows the maximum value of E_x , normalized to the nominal field at the antenna, as a function of x , taking into account ELD and depletion of the spectrum by linear wave transformation. The Grill is the same as in Fig. 1, with $f = 0.85$ GHz, $b = 5$ cm ; density and temperature profiles simulate those measured on the WEGA Tokamak, with central values $n_0 = 5 \cdot 10^{13}$, $T_{i0} = 240$ eV, $T_{e0} = 800$ eV. The most striking result is that the peak electric field decreases rapidly (curve a) towards the inside, in spite of the large increase of the WKB factor (curve b), even before the region of efficient power absorption (curve c).

These results have a bearing on the kind of nonlinear effects which can be expected to accompany LHR heating. Clearly the most dangerous region is the very edge of the plasma, where the electric field is largest and the sound speed smallest, thus maximizing the coupling coefficient for parametric instabilities. It could be hard to avoid unwanted nonlinear effects there, as repeatedly observed in experiments. On the other hand these effects

could involve the absorption of a minor fraction of the total transmitted power, just enough to eliminate the largest values of $k_{//}$ and to reduce the electric field to a tolerable level.

REFERENCES :

- /1/ - P. LALLIA, Proc. 2^d Topical Conf. on R.F. Plasma Heating, Lubbock 1974, paper C3.
- /2/ - M. BRAMBILLA, Nuclear Fusion 16, 47, 1976 and VII Europ. Conf. on Controlled Fusion and Plasma Physics, Lausanne 1975, Vol.2, p. 222.
- /3/ - S. BERNABEI et al., Nuclear Fusion, 17, 929, (1975).
- /4/ - P.M. BELLAN, M. PORCOLAB, Phys. Fluids, 17, 1952 (1974).
- /5/ - V. KRAPCHEV, A. BERS, PRR 77/18-2 (MIT), August (1977) to be published.

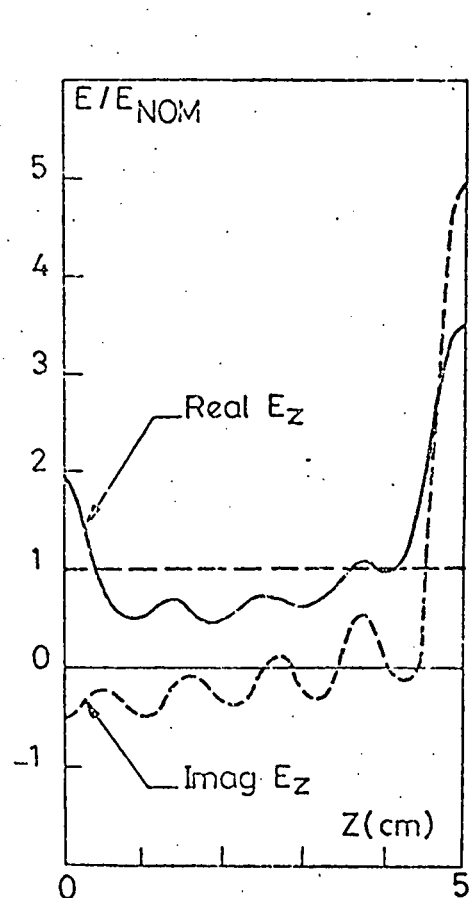


FIG. 1

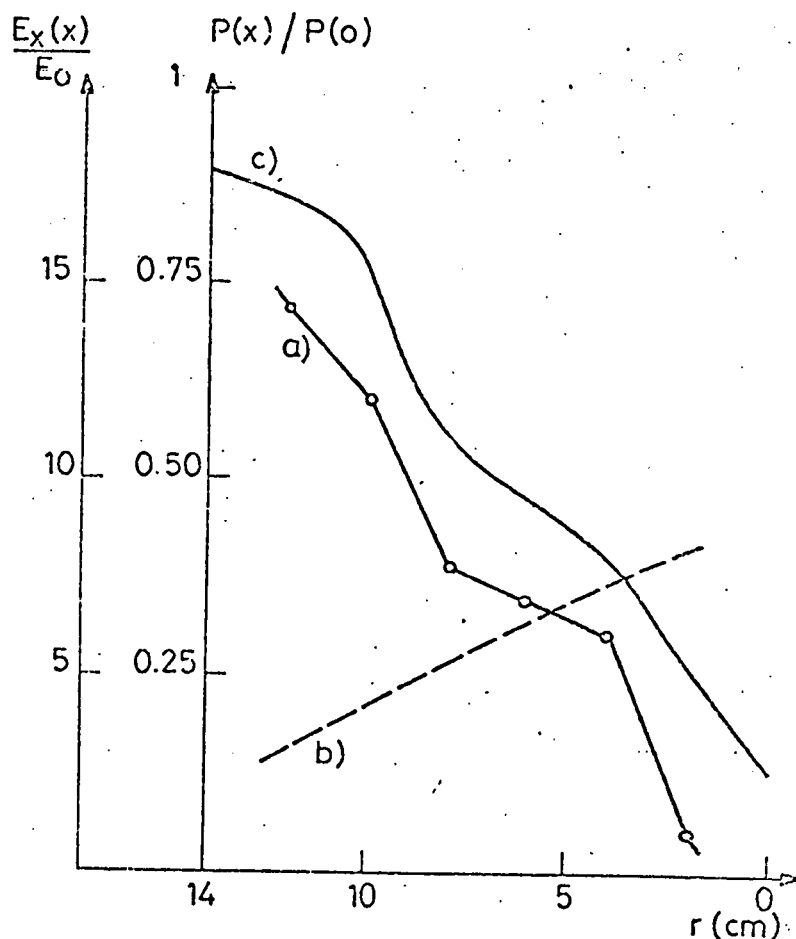


FIG. 2

Sup

COUPLING OF LOWER HYBRID WAVES IN THE DOUBLET IIA HIGH POWER RF HEATING EXPERIMENTS*

by

V. S. Chan, Charles Moeller, and R. W. Gould**
General Atomic Company, San Diego, California 92138

ABSTRACT

In the Doublet IIA high power rf heating experiments, a slow wave is launched by an electrostatic antenna above the lower hybrid frequency. Such an antenna has been proposed previously for generating a well-defined large parallel wavenumber spectrum. In the present experiments, a total of six structures with $n_{||}$ ranging from 11 to 16 at 800 and 915 MHz at power up to 100 kW/structure are being used. The coupling and radiated spectrum are analyzed using the theory developed for these antennas. They are found to be insensitive to changes in plasma parameters.

1. INTRODUCTION

Hitherto, the majority of lower hybrid wave heating experiments have been designed to heat ions through parametric processes or linear mode conversion near the lower hybrid resonance layer.¹ However, it is also possible theoretically that the slow wave branch above the lower hybrid frequency is capable of direct electron heating by Landau damping. Indeed, in a reactor size plasma where the equilibration time is less than the energy confinement time, this offers a viable alternative for bulk heating of tokamak plasmas. In addition, the possibility of current profile control by affecting the local electron temperature presents an attractive approach to stabilizing macroscopic instabilities. The General Atomic lower hybrid heating experiments are designed to explore the feasibility of these ideas.

Unlike experiments that are designed to heat ions, where the only requirement on the parallel wavelength is that accessibility condition be satisfied ($ck_{||}/\omega > 2$), electron heating experiments also require that $\omega/k_{||}$ be sufficiently close to the electron thermal velocity. For existing tokamaks $ck_{||}/\omega$ as large as 10 or bigger may be necessary. As a result, the evanescent layer which may not be a factor in ion heating experiments becomes significant. In the waveguide "grill" scheme, once ω and $k_{||}$ are chosen, there exist no other variables to compensate for this evanescence. In our experiments, we chose to use a slow wave structure of the more conventional type first analyzed by Golant.²

By placing grounded plates at the point of zero potential between alternately biased radiating elements (Fig. 1), the antenna coupling becomes much less sensitive to changes in plasma conditions. These plates provide an upper limit on the plasma loading of each radiating element to prevent a condition in which the input power would all be radiated by the first few elements. This controlled plasma loading results in a power spectrum which

* Work supported by Department of Energy Contract EY-76-C-03-0167, Project Agreement 38.

** Permanent address: California Institute of Technology

is relatively independent of plasma density gradient. In order to make the most efficient use of the slow wave structure, the impedance of the feedline can be made to increase away from the input to compensate for the plasma loading and thereby maintain the voltage constant along the feedline. A brief description of a quantitative analysis of the structure³ and the numerical results for our parameters are given in the following sections.

2. THEORY

The coupled plasma-antenna system is subdivided into two regions. Inside the antenna, we expand the field bounded by each pair of adjacent plates in a Fourier cosine series in z for $0 \leq x \leq h$:

$$\phi(x, z) = \sum_{p=1}^M \sum_{n=1}^{\infty} \left(a_{np} e^{-k_n x} + b_{np} e^{k_n x} \right) \cos k_n (z - z_p) \Theta_p(z) \quad (1)$$

Here, M is the number of elements in the structure, z_p , the position of the p^{th} element, $z_p = 2(p-1)d$, $k_n = \pi(2n-1)/2d$, and Θ_p is a step function:

$$\Theta_p(z) = \begin{cases} 1 & \text{if } z_p - d \leq z \leq z_p + d \\ 0 & \text{elsewhere} \end{cases}$$

Inside the plasma, the field is represented as a Fourier integral

$$\phi(z, x) = \int_{-\infty}^{\infty} \tilde{\phi}(k_{||}, x) e^{+ik_{||}z} dk_{||}$$

with each Fourier component governed by the electrostatic dispersion relation

$$\nabla \cdot \vec{\kappa} \cdot \nabla \tilde{\phi} = 0 \quad , \quad (2)$$

where $\vec{\kappa}$ is the plasma dielectric tensor. Here, we have neglected coupling to the fast mode and the radiation approximation is taken.

By matching boundary conditions at the interface, we can relate the a_{np} 's and b_{np} 's in terms of plasma and wave parameters. Both these quantities change with plasma loading. However, the limiter plates permit only loose coupling between the antenna and plasma, hence the potential at $x = 0$ normalized by the voltage of the radiating elements (V_p) is essentially unaffected by the plasma loading. As a result, we have

$$a_{np} + b_{np} = A_{np} \quad , \quad (3)$$

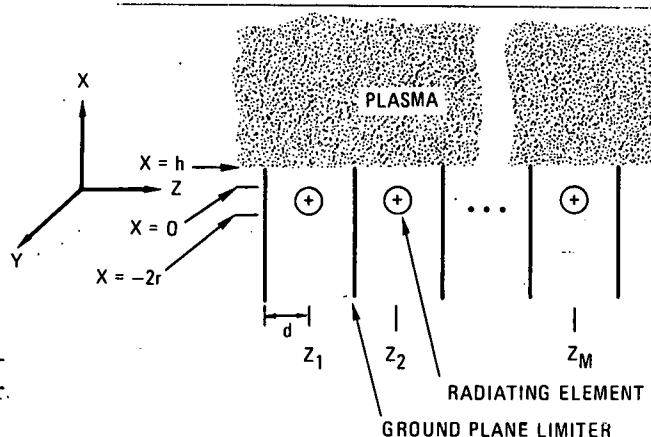


Fig. 1. Idealized version of the slow wave antenna - side view

where the A_{np} 's are the Fourier coefficients of $\phi(0,z)/V_p$ with no plasma loading. They can be evaluated by conformal mapping techniques. The spectrum (in $k_{||}$) is given by

$$P(k_{||}) = \frac{1}{2} \tilde{E}_z(k_{||}, h) \cdot \tilde{E}_z^*(k_{||}, h) \operatorname{Re} Y(k_{||}) \quad (4)$$

$$\text{with } Y(k_{||}) = \frac{-i\omega}{\mu_0 k_{||} c} \frac{\partial \tilde{E}_z}{\partial x}(k_{||}, h) / \tilde{E}_z(k_{||}, h)$$

determined from Eq. (2) and $\tilde{E}_z$ determined from Eq. (1). The total power radiated at the p th element can also be calculated using the Poynting theorem. These quantities are determined in terms of the V_p 's, the voltages at the elements. V_p has to be determined iteratively. Starting with the infinite line approximation (VSWR = 1), the plasma loading is estimated through the power radiated/section. The new voltage along the feedline is then calculated which in turn yields a new plasma loading. The procedure converges in a few iterations. In the next section, this procedure will be applied to analyze the antennas in the experiments.

3. APPLICATION TO EXPERIMENTAL CASE

For our examples, the frequency is fixed at 800 MHz, and the fundamental wavelength $\lambda_{||} = 2.68$ cm ($d = 0.67$ cm), making $n_{||} \approx 14$. The elements are 0.15 cm in radius and the total structure length is 30 cm. The impedance of the balanced line increases linearly from 100 ohms to 200 ohms along the structure length.

In Fig. 2 we show the plasma loading expressed in terms of the conductance/unit length, G , as a function of the plate height. We note that the loading becomes progressively weaker as h increases, due to the increase in evanescence. Thus the presence of the grounded plates justifies the loose coupling approximation. In subsequent examples as well as in the experiment, h is set equal to 0.2 cm.

Next, we examine the sensitivity of coupling to changes in plasma conditions. In Fig. 3, the loading (in terms of G) is shown as a function of density gradient. Clearly, this case is much less sensitive to the density gradient length than the case when the plates are not present (Fig. 4). That the evanescence imposed by the plates dominates the evanescence due to the plasma is clear.

Finally, the radiated power spectrum, as shown in Fig. 5, is also quite insensitive to changes in density gradient length. Instead of a peak at $n_{||} = \pi c / 2\omega d = 14$, there are two peaks at $n_{||} = 14 \pm 1$, which are the consequence of the finite phase velocity along the feedline.

In summary, we have demonstrated by detailed calculations that as a result of the presence of the grounded plates, the antenna coupling is only a weak function of changes in the plasma. The radiated spectrum is well-defined and peaks approximately at the desired wavenumber.

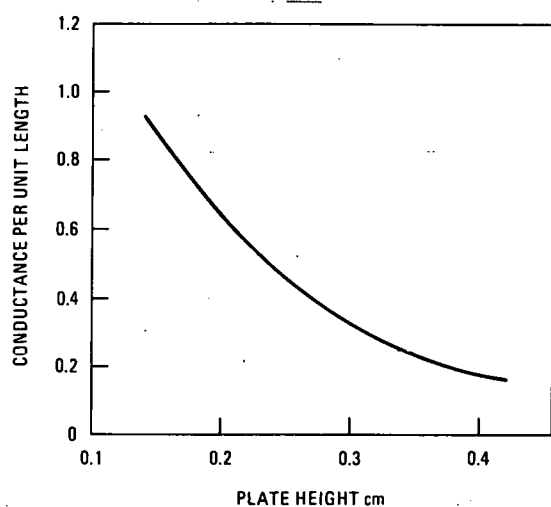


Fig. 2. Calculated plasma loading versus plate height

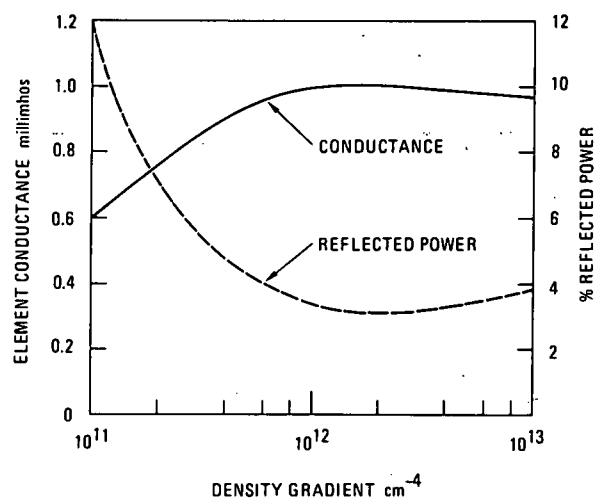
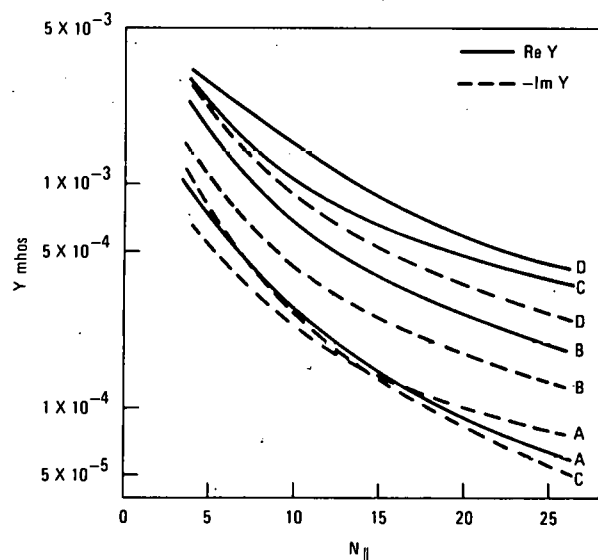


Fig. 3. Calculated plasma loading versus density gradient



A = 10^{11} cm^{-4} ; B = 10^{12} cm^{-4} ;
 C = 10^{12} cm^{-4} with initial density
 of 10^{11} cm^{-3} ;
 D = 10^{13} cm^{-4}

Fig. 4. Calculated surface admittance of the plasma at the antenna

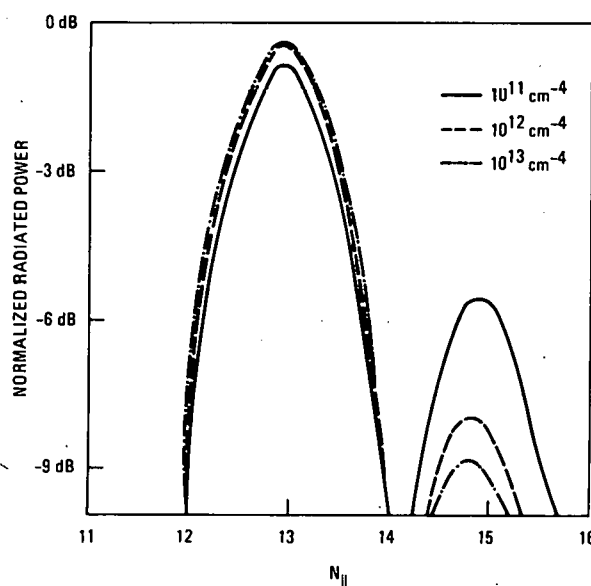


Fig. 5. The calculated radiated spectrum

References

1. M. Porkolab, et al., Phys. Rev. Lett. **38**, 230 (1977).
 G. W. Pacher, et al., Bull. Am. Phys. Soc. **22**, 1169 (1977), p8F1.
2. V. E. Golant, Sov. Phys.-Tech. Phys. **16**, 1980 (1972).
3. V. S. Chan, et al., Sherwood Theory Meeting, San Diego, CA 1977, p2.

C.M. SINGH, P. BRIAND, L. DUPAS

ASSOCIATION EURATOM-CEA

Département de Physique du Plasma et de la Fusion Contrôlée

Service IGn - Centre d'Etudes Nucléaires

85 X - 38041 GRENOBLE CEDEX (France)

The study of lower hybrid waves on Petula Tokamak /1/ addresses itself to the following questions (i) wave excitation by phased waveguide arrays, (ii) technological problems encountered with high power microwave circuitry, and (iii) non linear processes induced by large amplitude pump waves. An r.f. generator was used to deliver a maximum power of 1.0 MW, 100 μ s at 1.25 GHz to a two waveguide grill ($E_{\max} = 4$ kV/cm). The data reported here were taken in a deuterium plasma, $n_e \sim 3.5 \times 10^{13} \text{ cm}^{-3}$, $T_e \sim 500$ eV, $T_i \sim 200$ eV and $B_\phi = 15$ kG. The choice of the plasma parameters and the frequency of the pump was made so as to avoid the linear mode conversion layer in the plasma.

REFLECTION COEFFICIENT.

The microwave circuit was conceived to measure the "natural" (without matching network) reflection coefficient in each waveguide as a function of the phase ϕ between the incident electric fields emitted by the GRILL /2/. The reflection coefficients were measured for 3 radial positions of the GRILL and in a large R.F. power range.

Fig. 1 shows the dependence of the measured reflection coefficient in each guide and the total reflection coefficient $R(\phi)$ on the radial position at a low power level (2 kW). For the "4 cm from limiter" and "2 cm from limiter" positions, the behaviour of the curves is in good agreement with the theory /3/. The curves for guide 1 and guide 2 cross for $\phi = 0$ and for $\phi = \pi$. But the variations with ϕ are more pronounced for the farthest radial position. The minimum values of R are respectively .40 and .20 for these two positions. For the "1.6 cm from limiter" position, the curves do not cross. However, at other time in the plasma discharge, they can have the same feature as in the previous positions. But the minimum $R(\phi = \pi)$ takes its lowest value which is .15. This "1.6 cm from limiter" radial position was adopted for all of the following experiments.

Other $R(\phi)$ measurements were made at higher power levels (Fig. 2). At 10 kW, the maximum R occurs for $\phi = \pi/2$. At 950 kW, the lower curves correspond to a transient regime starting with the r.f. pulse. The upper curve corresponds to a steady but somewhat noisy state after the transient regime. The R minimum value is about .05 for $\phi = \pi$.

The $R(\phi)$ curves for 2 kW, 10 kW, 950 kW indicate the influence of the power level. R was measured for $\phi = \pi$ and $\phi = 0$ in the 1 kW - 1 MW range and plotted in the Fig. 3. For a given power, the lower bar represents a minimum value generally occurring in a transient regime, and the upper bar represents a steady regime. In the 1 kW-300 kW range, the upper values are found between .1 and .2 for $\phi = \pi$ and between .3 and .75 for $\phi = 0$. In the 300 kW-600 kW range, the $R(\phi = \pi)$ values are less than .05 and the $R(\phi = 0)$ values are about .1. In the range 600 kW-1 MW, the $R(\phi = \pi)$ values are increasing with power up to .2 and the $R(\phi = 0)$ values are increasing also up to .5. The 3 power dependent regimes which can be deduced from the $R(P)$ curves, are also visible on the detected probe signal evolution with the R.F. power.

PROBE SIGNALS.

Fig. 4 and 5 show the square law detected signal with r.f. probes. The probes were toroidally located at 24 cm and 218 cm from the Grill, in the equatorial plane. One observes that for $P < 300$ kW, close to the Grill, the signal increases linearly with power. However far from the Grill, it increases as $P^{1.5}$ indicating increased scattering of the pump wave by the plasma. The signal tends to saturate between 300 and 600 kW. On further increasing the power, in Figs. 4 and 5, it seems to fall off. This signal behavior, together with the observations made on the reflection coefficient, suggests the presence of three regimes of nonlinear phenomena in our experiment. The possible mechanisms can be :

- 1 - The parametric regime ($P < 300$ kW). ($E_{\perp} \times B_{\phi}$) electron drift driven parametric instabilities.
- 2 - The transitional regime ($300 \text{ kW} < P < 600 \text{ kW}$) characterized by $1/2 \epsilon_0 E_{\text{grill}}^2 \sim 5\% (nkT)$ and ponderomotive force effects, assuming a plasma of $2.10^{11} \text{ cm}^{-3}$ density and temperature of 50 eV at the location of the Grill.
- 3 - The turbulent regime ($P > 600$ kW) - ($E_{\text{grill}} \times B_{\theta}$) electron drift, electron oscillation drift driven instabilities, ponderomotive force effects.

Detailed results on reflection coefficient and wave penetration measurements are presented elsewhere /4/. The parametric decay spectrum is being studied in the three regimes.

FAST ION POPULATION.

If the parametrically excited waves are electrostatic in nature, one expects that a fraction of the ions resonating ($v_i \sim v_{ph} \sim v_{T,A}$) with the wave may gain energy. Fig. 6 shows the charge exchange neutral signal versus time. One observes a large burst of signal on the 1.1 keV channel, with a decay time of 100 μs , indicating the existence of a fast ion population.

CONCLUSIONS.

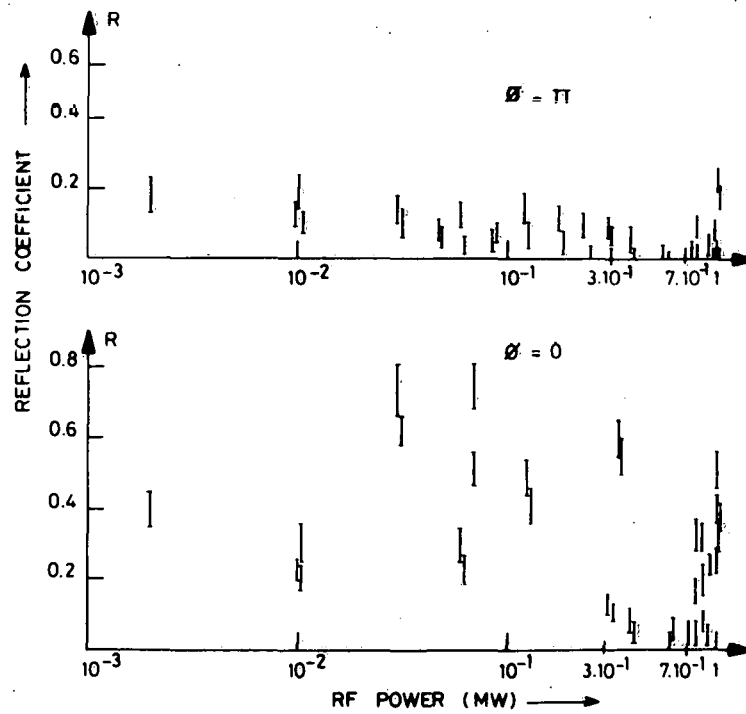
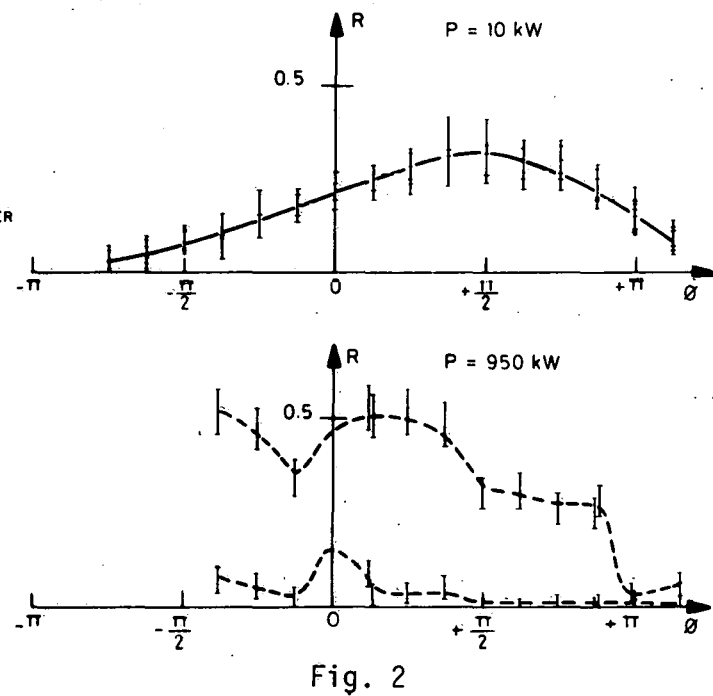
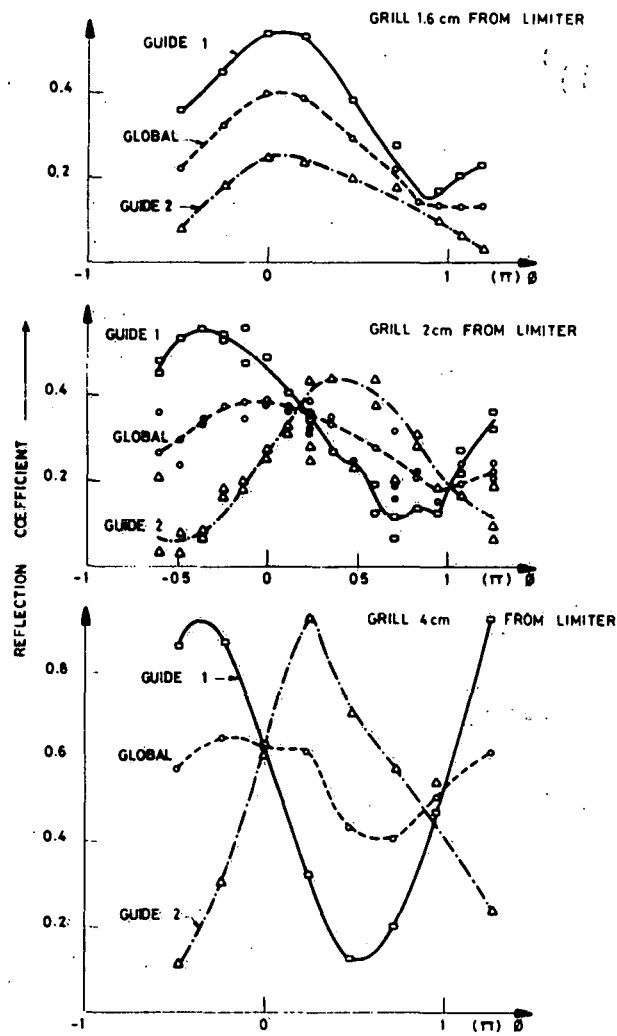
This first demonstration of the operation of phased waveguide arrays at 1 MW power level ($E_{\text{max}} = 4 \text{ kV/cm}$, $P = 8.5 \text{ kW/cm}^2$) has shown the capability of the microwave hardware in the L H R H. These results indicate that there are effects due to non linear processes on Grill coupling and wave penetration.

ACKNOWLEDGEMENTS.

The authors would like to thank Drs. G. MELIN and S. GOLOVATO for their help in various stages of this project.

REFERENCES.

- /1/ - R. BARDET et al., Nuclear Fusion, 16, 4, p. 579 (1976).
- /2/ - L. DUPAS, G. MELIN, C.M. SINGH, Dph-PFC-Note Interne n° 1170 (1977).
- /3/ - M. BRAMBILLA, Nuclear Fusion, 16, 4, p. 47 (1976).
- /4/ - EUR-CEA-FC N° 934 (1978).



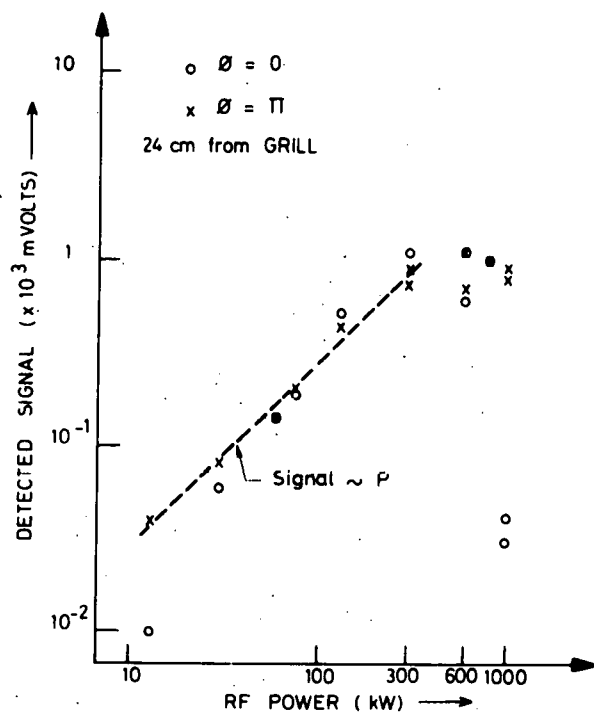


Fig. 4

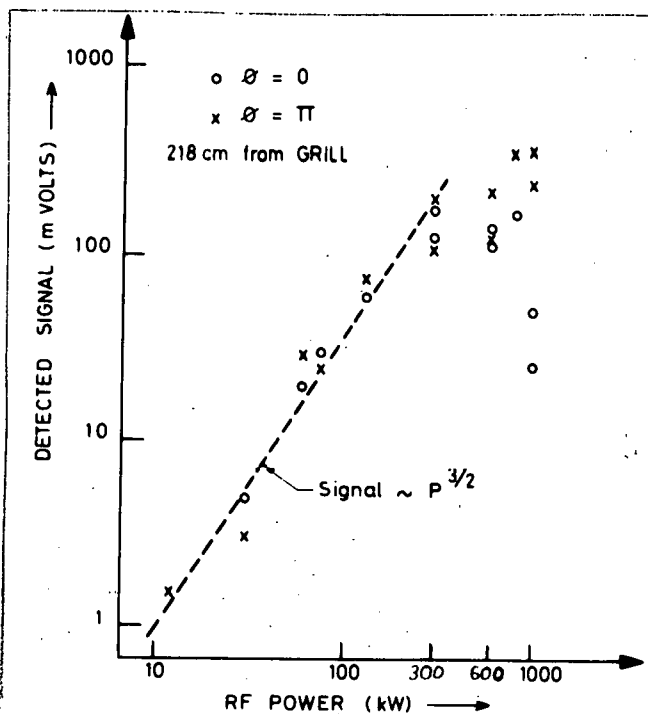


Fig. 5

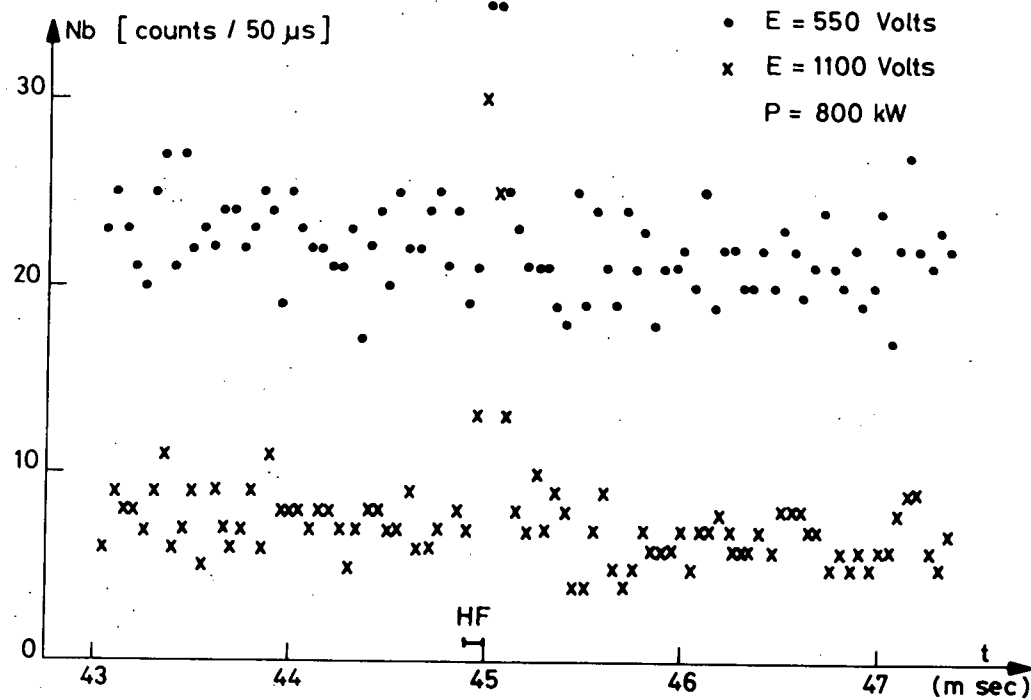


Fig. 6

LOWER HYBRID HEATING IN THE DC OCTOPOLE*

by

Peter I. Petersen, V. S. Chan, S. Ejima, Robert J. La Haye
Charles P. Moeller, and John C. Wesley
General Atomic Company, San Diego, California 92138

ABSTRACT

Efficient electron rf heating is observed in a low density Li^+ plasma in the dc Octopole with toroidal field. The rf is launched from a slow wave ($n_{\parallel} \approx 200$) electrostatic antenna and the wave energy is absorbed by collisional and Landau damping. At low frequencies ($\omega \approx 0.5-3 \omega_{\text{LH}}$) both ions and electrons are heated. Because of the low density and the low power level it has been possible to use Langmuir probes and an electrostatic particle energy analyzer to measure plasma parameters. The density and applied frequency were varied over a large range ($1 \times 10^8 \leq n \leq 5 \times 10^9$ and $1 \text{ MHz} \leq f \leq 150 \text{ MHz}$) to deduce scaling relations. Extrapolation of the experimental results to tokamak temperatures and densities suggests that lower hybrid electron heating holds considerable promise as an efficient auxiliary heating method for large tokamaks.

1. INTRODUCTION

The dc Octopole is an internal ring device, with four current carrying rings submerged in the plasma and two rings external to the plasma that carry the return current. The typical poloidal field is 135 gauss. The toroidal magnetic field (~ 520 gauss) is produced by a 36 turn toroidal solenoid outside the vacuum tank. The Li^+ plasma used for the heating studies is created by irradiating a lithium target with a 10 joule laser pulse, producing a plasma with an initial central density $n(0) \approx 10^{10} \text{ cm}^{-3}$ that cools to about 1 eV within 10 msec. Without rf heating, temperature and density continue to decay slowly, typically to 0.2 eV and $2 \times 10^8 \text{ cm}^{-3}$ at 600 msec, with approximately equal electron and ion temperatures and a nearly uniform temperature profile. Temperatures are measured with a gridded electrostatic analyzer. The resulting electron temperatures agree within $\pm 30\%$ with those inferred from a $2 \omega_{\text{ce}}$ radiometer. Langmuir probe density measurements are calibrated against a 10 cm microwave interferometer.

2. RF SOURCES AND ANTENNAS

A Wavetek signal generator was used as an rf source. The signal was amplified by a 100 W rf power amplifier before being transmitted to the antenna. Directional couplers allowed the forward and reflected power of the open-circuited antenna to be measured with sufficient precision to obtain the power absorbed by the plasma. Two different types of antennas were used to launch the slow waves. One consisted of a periodic array of short nonresonant lateral elements oriented normal to the toroidal field,

*Work supported in part by the Electric Power Research Institute, EPRI Contract No. RP 323-2.

driven from alternate sides of a balanced transmission feedline with a characteristic impedance of 100Ω . The distance between the lateral elements fixes the parallel wavelength. Limiters were placed between the dipoles to keep the plasma away from the radiating elements in order to have well-defined conditions around the antenna. Since the plasma is blocked from the area between the radiating elements and since the antenna has many dipoles, the $k_{\parallel}$ spectrum is expected to be fairly narrowly centered around $k_{\parallel} = \pi/L$, where L is the distance between the dipoles. The k_p spectrum is not known because of the complex field geometry. Calculations using the ray tracing code indicate that the ray path is slightly dependent on k_p . In the experiment we did find, however, that there was no dependence of the heating rate on $k_{\parallel}$ and k_p , which is in agreement with the interpretation of the experiment that all the power that was radiated from the antenna was absorbed by the plasma. The other antenna type consists of a balanced stripline wound in helical fashion with a constant toroidal pitch around a supporting ground plane sandwiched between two glass plates. The whole antenna was also sandwiched between two glass plates to insure well-defined boundary conditions.

3. EXPERIMENTAL RESULTS

Typical heating results are shown in Fig. 1 with three different antenna wavelengths for a forward power of 5.5 W into each antenna. The differences in the heating observed between these antennas are due primarily to the differences in the absorption of coupled power by the plasma. The short wavelength antennas heat only the electrons, with the highest heating rates at higher frequencies. The long wavelength (80 cm) antenna heats the electrons at higher frequencies, but also heats the ions at low frequencies, $f \leq 3 f_{LH} \approx 9$ MHz.

Propagation and absorption of the lower hybrid wave has been studied using an rf probe located near the antenna. The probe is located ~ 15 cm beyond the end of the antenna, with a radial position that can be adjusted over the plasma cross section. Measurements of the rf wave amplitude reveal the existence of resonance cones. At high densities the wave propagates nearly parallel to the magnetic field and the resonance cone is confined to the outer edge of the plasma. As the density is decreased, the wave is able to propagate more nearly normal to the magnetic field before it reaches the probe, so that the wave penetrates further into the plasma. The radial positions of the

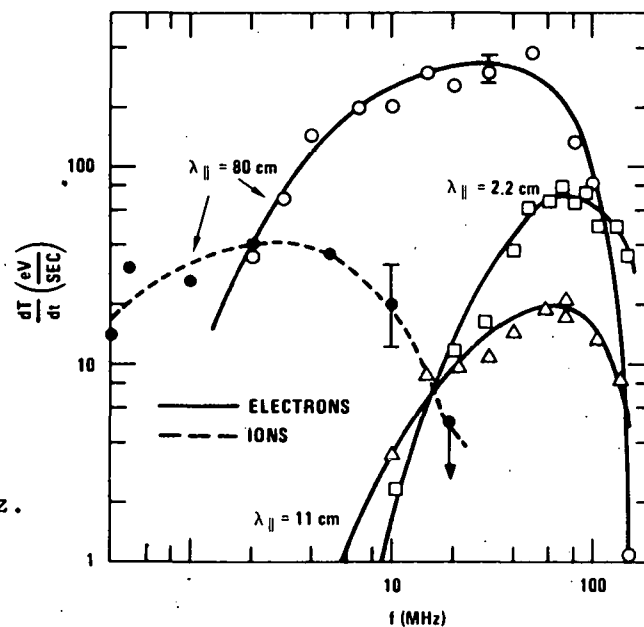


Fig. 1. Electron and ion heating

resonance cone maxima are found to be in reasonable agreement with theoretical calculations.

The principal damping mechanisms expected at frequencies sufficiently above the LH resonance are collisional and Landau damping. Appreciable Landau damping is expected for wave phase velocities comparable to or less than the electron thermal velocity, or for the 0.4 eV electron temperature typical of the Octopole experiments, $n_{\parallel} \geq 200$. For faster waves ($n \leq 200$), collisional damping of the wave by electron-ion collisions provides a weak, but still effective, damping mechanism. Even under the lowest collisionality conditions encountered in the Octopole, the collisional damping time is of the order 100 μ sec, more than two orders of magnitude shorter than the electron energy confinement time.

Measurements of the electron temperature after heating show that, as for the unheated plasma, the electron temperature is nearly uniform over the plasma region. Accordingly, it is possible to estimate the total increase in the plasma thermal energy content using $\Delta U = 3n\Delta T\bar{V}/2$. The rf power radiated by the antenna is measured directly by comparing the difference in the reflected power from the open-circuited antenna with and without plasma. When correction is made for the resistive losses, the calculated radiated power equals ΔU within $\pm 45\%$. Thus, the observed heating rate measures coupling, not absorption.

At high frequencies the coupling depends on ω/ω_{pe} . Maximum coupling is obtained for $\omega/\omega_{pe} \approx 0.2$. At higher frequencies, the coupling begins to fall owing to an increase in width of the evanescent layer between the antenna and the plasma. The coupling falls by a factor of 10 at $\omega/\omega_{pe} = 0.5$, and is negligible for $\omega/\omega_{pe} \approx 0.6$. At low frequencies, warm plasma effects become significant, and the coupling depends on $\omega/k_{\parallel}v_e$. A detailed analysis¹ shows that the antenna coupling drops dramatically for $\omega/k_{\parallel}v_e < 1$.

Coupling is also affected by the location of the antenna relative to the density profile, the magnitude of the peak density, and by the rf input power level. As the antenna is withdrawn from a position well inside the plasma, coupling is independent of the position, z , up to some critical position z_0 , beyond which it falls as $\exp -(z - z_0)/\delta$. The position of z_0 as a function of n and ω corresponds to $\omega/\omega_{pe}(z_0) = 1$. The measured coupling decrease is in agreement with evanescent decay of the wave in the region with $\omega/\omega_{pe} < 1$. Coupling at a fixed radial position with $z < z_0$ is independent of the input power P up to some critical value P_0 . Above this limit, the total coupled power increases only as $P^{0.5}$. The P_0 limit scales approximately as $nT\lambda_{\parallel}^2Z_0^{-1}$, where Z_0 is the antenna impedance, suggesting that this decrease in relative coupling arises due to the ponderomotive force pushing the plasma away from the antenna. Typical values are $P_0 \approx 2$ W for $Z_0 = 100 \Omega$, $\lambda_{\parallel} = 2.2$ cm and $n \approx 10^9$ cm⁻³. At input powers less than P_0 , coupling at the optimal value of ω/ω_{pe} scales as n up to 5×10^9 cm⁻³.

The absorption mechanism responsible for the ion heating has not been identified. The experimental results are consistent with absorption near the lower hybrid resonance layer owing to linear and parametric mode conversion, but the data is not sufficient to be able to say much more. No

observable power threshold for either the electron or ion heating has been found.

An interesting, but as yet unexplained dependence of coupling on the ratio of plasma density and rf frequency, has been observed in the Octopole rf studies. The rf is applied continuously at a low power that is detectable with the probe, but does not produce any appreciable plasma heating. The rf level in the plasma exhibits a series of resonances or increases in amplitude as the density decays, with the densities where the maxima occur being approximately in the ratio 1, 4, 9, and 16. If the rf frequency is changed, the peaks shift to a density n that scales as ω^2 . The phenomenon is caused by a global increase in rf level throughout the plasma, rather than a local resonance at the rf probe. No definite cause for this improved coupling has been found. The frequency and density scaling suggests that the effect is some sort of an eigenmode property.

Although the absolute powers used in the Octopole are modest, both the power input per plasma particle and the heating rates demonstrated (10^{-15} W/electron and 3 keV/s, respectively) approach those required for tokamak heating. At the same specific power input, our results extrapolate to a steady-state temperature rise of about 1 eV/kW in a small tokamak ($n = 2 \times 10^{13}$, $\tau_E = 2$ msec), identical to present generation neutral beam heating results. While the validity of such extrapolations remains to be verified in an actual experiment, our present results suggest that lower hybrid heating of electrons shows considerable promise for efficient heating of tokamak devices.

The one noticeable nonlinear process observed in the Octopole, the reduction of coupling efficiency at high power inputs, is not expected to pose any significant limitation on tokamak heating. The power limit found in the Octopole is about 20 W/m². Extrapolating this to the higher edge temperatures and densities characteristic of tokamaks (typically $\sim 10^{13}$ cm⁻³ and 10 eV) gives an equivalent power limit of about 5 MW/m². This ponderomotive coupling limit is comparable to or in excess of the limitation on power density imposed by electrical breakdown in the slow wave structure.

Another significant qualitative aspect of the Octopole rf heating experiments is the demonstration of the effectiveness of collisional damping as an electron heating mechanism.

References

1. V. Chan, "Lower Hybrid Wave Penetration in the Octopole," General Atomic Co. Report No. GA-A14440, June 1977.

PARAMETRIC DECAY OF LOWER HYBRID AND ION CYCLOTRON WAVES OF FINITE WAVENUMBER IN A PLASMA

V. K. Tripathi, C. S. Liu, and C. Grebogi
Department of Physics and Astronomy
University of Maryland
College Park, Maryland 20742

ABSTRACT

A method is developed to solve the Vlasov equation for the nonlinear response of magnetized ions in the presence of a finite wavelength pump wave. The contribution of ion nonlinearity to the coupling coefficients for various channels of parametric decay, quasimode decay and OTSI of lower hybrid and ion cyclotron waves is obtained and a detailed comparison of the growth rates of these channels has been made. In the case of lower hybrid pump, the ion nonlinearity is important only for the decay into ion Bernstein waves ($\omega \gg k_z v_e$, $k_\perp \rho_i > 1$). For an ion cyclotron pump at ω_{ci} and $2\omega_{ci}$ the contribution of ions predominates for all channels of decay; OTSI and three wave decay into ion acoustic and ion cyclotron waves are found to have large growth rates.

1. General Approach

In order to obtain the nonlinear response of ions, we write the Vlasov equation in terms of the guiding center coordinates $\underline{x}_g$, magnetic moment μ , polar angle of perpendicular velocity θ and parallel momentum p_z as

$$\frac{\partial F_1}{\partial t} + \dot{\mu} \frac{\partial F_1}{\partial \mu} + \dot{\theta} \frac{\partial F_1}{\partial \theta} - e \frac{\partial \Phi'}{\partial z} \frac{\partial F_1}{\partial p_z} + \dot{\underline{x}}_g \cdot \frac{\partial F_1}{\partial \underline{x}_g} = 0 \quad (1)$$

where $\underline{x}_g = \underline{x} + \rho \sin \theta$, $y_g = y - \rho \cos \theta$, $z_g = z$,

$$\rho = v_\perp / \omega_{ci}, \quad \omega_{ci} = eB / m_i c, \quad \mu = m_i v_\perp^2 / 2\omega_{ci}.$$

Using the equation of motion one could easily deduce

$$\begin{aligned} \dot{\mu} &= - \frac{e}{\omega_{ci}} \nabla_\perp \Phi' \cdot \underline{v}_\perp = \frac{\partial H}{\partial \theta} \\ \dot{\theta} &= - (\omega_{ci} + \frac{\partial}{\partial \mu} e\Phi') = - \frac{\partial H}{\partial \mu} \end{aligned} \quad (2)$$

$$\dot{\underline{x}}_g = - \frac{e}{m_i \omega_{ci}} \frac{\partial \Phi'}{\partial y_g}, \quad \dot{y}_g = \frac{e}{m_i \omega_{ci}} \frac{\partial \Phi'}{\partial x_g}, \quad \dot{z}_g = \frac{p_z}{m_i}$$

where

$$H = \mu \omega_{ci} + e\Phi' + p_z^2 / 2m_i, \quad (3)$$

$$\Phi' = \Phi_0 + \Phi + \Phi_\pm,$$

Φ_0 , Φ , and $\Phi_\pm$ are the pump ($\omega_0, \underline{k}_0$), low frequency perturbation ($\omega, \underline{k}$) and the high frequency side bands ($\omega \pm \omega_0$, $\underline{k} \pm \underline{k}_0$), respectively;

$$\Phi_0 = \phi_0 \exp[-i(\omega_0 t - \underline{k}_0 \cdot \underline{x})] = \phi_0 \exp[-i(\omega_0 t - \underline{k}_0 \cdot \underline{x}_g)] \sum_n \exp(-in\theta) I_n(k_{0\perp} \rho). \quad (4)$$

* Work supported by CTP (University of Maryland), DOE, NSF, and CNPq.

Since (μ, θ) , (x_g, y_g) and (p_z, z) form the canonical set of variables, Eq. (1) follows directly from the equation of continuity of ion density in the six dimensional space of the new variables. Eq. (1) offers straightforward solutions for linear response. Using the linear solutions one could easily obtain the higher order solutions. The expressions for nonlinear ion densities turn out to be very compact.

For the response of electrons, one could easily use the drift kinetic equation¹ because $k_{\perp} \rho_i < 1$ for all channels of interest.

2. Lower Hybrid Decay

In the case of a lower hybrid pump

$$\omega_0 = \omega_{LH} (1 + k_{0z}^2 m_i / k_0^2 m)^{1/2} \quad (5)$$

we assume the high frequency sidebands to be lower hybrid waves. The low frequency perturbations could be a mode or a quasimode. The growth rates and the conditions of existence of various channels of decay are summarized in Table I.

The contribution of ion nonlinearity to the coupling coefficients is important only when the low frequency mode has long parallel wavelength ($k_{\parallel} v_e / \omega < 1$) and short perpendicular wavelength ($k_{\perp} \rho_i > 1$), viz., for ion Bernstein modes $\omega \approx n\omega_{ci}$, $n \gg 10$; Watson and Bers² have obtained a similar result for the case of lower hybrid decay into two Bernstein modes. For all other channels $\mathbf{E} \times \mathbf{B}$ electron nonlinearity predominates; the coupling coefficients reduce to those obtained by Porkolab.³ The quasimode decay (i.e., nonlinear Landau damping) and OTSI are equally important and most predominant channels of decay. The modulational instability has extremely small growth rate and hence the envelope solitons of lower hybrid waves are ruled out.

3. Ion Cyclotron Decay

An ion cyclotron pump around ω_{ci} decays into a low frequency ion acoustic wave ($\omega = k_z c_s$) and a scattered ion cyclotron wave. The growth rate, in the limit of $k_{\perp} \rho_i < 1$ turns out to be

$$\gamma^2 \approx \frac{e^2 \phi_0^2 \phi_0^* k_0^4 k_1^2 c_s^2}{16 m_i^2 \omega_{ci}^4} \frac{\omega}{\omega_{ci}}, \quad k_1 = k - k_0.$$

The pump around ω_{ci} is also unstable to oscillating two stream instability. The solution of Eq. (1) for this case is, however, unmanageable. Liu and Tripathi⁴ have employed fluid approach to study OTSI and obtained the following expression for growth rate

$$\gamma^2 = -\Delta^2 - \Delta \frac{|\mathbf{k} \cdot \mathbf{v}_0|^2}{2(\omega_0^2 - \omega_{ci}^2 - k^2 v_i^2)} \frac{k^2 c_s^2}{\omega_0}; \quad k_{\perp} \rho_i < 1, \quad k_0 \ll k$$

where $\mathbf{v}_0$ is the ion drift in the presence of the pump and Δ is the frequency mismatch

$$\Delta \equiv (\omega_0^2 - \omega_{ci}^2 - k^2 v_i^2 + c_s^2) / 2\omega_0.$$

For a pump around $2\omega_{ci}$, an additional channel of decay is open, viz., decay into two ion cyclotron waves of ω_{ci} each. We consider a deuterium plasma with a Δ fraction of hydrogen and study the decay of a long wavelength pump wave around

$2\omega_{ci}$ ($k_{0i}\rho_i < 1$). When the decay wave also possess long wavelengths ($k_{1i}\rho_i < 1$), the resonance conditions can be satisfied only for $\Delta > 0.33$. The growth rate turns out to be

$$\gamma^2 = \frac{e^2 k_{0i}^4 \phi_0 \phi_0^*}{16 m_i^2 \omega_{ci}^2 (1-\Delta)^2}.$$

In the opposite case of $k_{1i}\rho_i > 1$, the resonance conditions can be satisfied for arbitrary values of Δ , however the growth rate is an order of magnitude smaller.

References

- ¹V. K. Tripathi, C. Grebogi, and C. S. Liu, Phys. Fluids 20, 1525 (1977).
- ²D. C. Watson and A. Bers, Phys. Fluids 20, 1704 (1977).
- ³M. Porkolab, Phys. Fluids 17, 1432 (1974).
- ⁴C. S. Liu and V. K. Tripathi, to be published (1978).

Table I

Growth Rates for Various Channels of Lower Hybrid Decay*

Low Frequency Mode	Growth Rate
(AI) Lower hybrid wave $\omega \gg k_z v_e$ $\omega_0 > 2\omega_{LH}$, $k_0 \neq 0$	$\gamma_0 = \frac{\omega_{LH}^2 \omega_0 - 2\omega k c_s}{4(\omega_0 - \omega)^{3/2} \omega^{3/2}} \left(\frac{\omega^2}{\omega_{LH}^2} - 1 \right) \sin \delta_1 \frac{u}{c_s}$
(AII) Ion Bernstein wave $\omega \gg k_z v_e$, $\omega - \omega_{ci} \gg k_z v_i$ (i) $k_1 \rho_i \ll 1$, $\omega < \omega_{ci}$ (ii) ** $k_1 \rho_i > 1$, $\omega = \eta \omega_{ci}$	$\gamma_0 = \frac{\omega_{LH}}{\omega_0} \left(\frac{\omega_0^2}{\omega_{LH}^2} - 1 \right)^{1/4} (\omega_0 \omega_{ci})^{1/2} \left(\frac{T_e}{2T_i} \right)^{1/2} k_1 \rho_i \sin \delta_1 \frac{u}{4c_s}$ $\gamma_0 \approx \frac{\omega_{LH}}{\omega_0} \frac{k_1^{1/2} \rho_i^{1/2}}{2\pi^{1/4}} \left(\frac{T_e}{2T_i} \right)^{1/2} \left(\frac{\omega_1^2}{\omega_{LH}^2} - 1 \right)^{1/2} \cdot \frac{ 2\omega - \omega_0 ^{1/2}}{\omega^{1/2} (\omega_0 - \omega)^{1/2}} \cdot \omega_{ci}^{3/2} u/c_s$
(AIII) Ion Bernstein reactive quasimode (four wave decay) $\omega \gg k_z v_e$, $k_1 \rho_i < 1$	$\gamma_0 = \left\{ \frac{\omega_{LH}^2 \sin \delta_1}{16\omega_0^3} (\omega_0^2 - \omega_{LH}^2) k^2 c_s^2 \right\}^{1/3} \left(\frac{u}{c_s} \right)^{2/3}$
(AIV) Modulational instability $k_z \gg k_1$ $\omega \gg k_z v_e$ $k_1 \rho_i \ll 10^{-4}$	$\gamma_0 = \frac{k_z v_e}{\omega_0} \omega \left(\frac{m}{m_i} \right)^{3/2} \left(1 - \frac{\omega_{LH}^2}{\omega_0^2} \right) \left(4 - \frac{3\omega_{LH}^2}{\omega_0^2} \right)^{1/2} \cdot \sin \delta_1 \cdot \frac{u}{c_s}$
(BI) Lower hybrid quasimode $\omega \approx k_z v_e$ $\omega \gg \omega_{ci}$	$\gamma_0 = 0.6 \frac{\omega_{LH}^2}{8\omega_0} \sin \delta_1 \frac{u^2}{c_s^2}$
(BII) Ion cyclotron quasimode $\omega \approx k_z v_e$ $\omega = \omega_{ci}$	$\gamma_0 = 0.6 \frac{\omega_{LH}^2}{8\omega_0} \cdot \sin \delta_1 \frac{u^2}{c_s^2}$
(CI) Ion acoustic wave $\omega_0 > 4\omega_{LH}$, $T_e > 4T_i$, $\omega \gg \omega_{ci}$	$\gamma_0 = \frac{\omega_{LH}}{4\omega_0} (\omega \omega_0)^{1/2} \sin \delta_1 \frac{u}{c_s}$
(CII) Ion cyclotron wave $k_z v_e \gg \omega \omega_{ci}$, $k_1 \rho_i > 1$ (i) $\omega - \omega_{ci} \gg \gamma_0$ (ii) $\omega \gg \gamma_0 \gg \omega_{ci}$	$\gamma_0 = \frac{\omega_{LH}}{\omega_0} 0.15 \frac{(\omega_0 \omega_{ci})^{1/2}}{1 + T_i/T_e} \sin \delta_1 \frac{u}{c_s}$ $\gamma_0 = \frac{\omega_{LH}^2}{2\omega_0 (1 + T_i/T_e)} \cdot \frac{x}{1+x^2} \frac{u^2}{4c_s^2};$ $x = \sqrt{\pi} \omega / k_1 v_i$
(CIII) Oscillating two stream instability $\omega \ll k_z v_e$, $k_z v_e$	$\gamma_0 = \frac{\omega_{LH}^2}{8\omega_0 (1 + T_i/T_e)} \frac{u^2}{c_s^2}$

* The high frequency sideband is a lower hybrid wave.

** Ion nonlinearity is important only for this channel; here we have assumed $\sin \delta_1 \approx \epsilon_0 \delta_1$. For all other channels $\underline{E} \times \underline{B}$ electron nonlinearity predominates.

Ergodic Behavior of Lower Hybrid Decay Wave Trajectories*

E. Ott, J.-M. Wersinger and J. Finn

Laboratory of Plasma Studies
Cornell University, Ithaca, NY 14853

Recently, it has been demonstrated for slab or straight cylindrical geometry that the combined effects of magnetic shear and density gradient can lead to the trapping of lower hybrid waves in a radially localized region¹ provided that these waves have significant wavenumber components in the direction $\mathbf{B}_0 \times \nabla n$ (the poloidal direction in a torus). If these features persist in toroidal geometry they could have important implications for lower hybrid wave heating of tokamaks.^{1,2} In lower hybrid heating the radial localization of the decay waves could allow them to grow as absolute instabilities. The mechanism would be the following: the decay wave product increases as it passes through the finite pump region (e.g. due to nonresonant coupling³). It decays via Landau damping as it propagates around the torus in regions not occupied by the pump, but can then experience further growth when it re-enters the pump region. Absolute instability would occur (even if the exponentiation on one pass through the pump region is small) provided that on average there is net growth on one transit around the torus. In the cylindrical case the perturbed fields may be taken to have the dependence $E_{\perp} \sim \exp(im\theta)$ where θ is the angle of cylindrical symmetry and m is a constant. The constancy of m is used in obtaining the radial localization of the ray. On the other hand, actual situations involve toroidal geometry, and m is no longer a "conserved quantity" of the system. In this case, recent work on the theory of Hamiltonian systems suggests the possibility that some or all of the ray trajectories undergo ergodic wandering. The following question then arises: are appropriate lower hybrid decay waves radially confined in toroidal plasmas due to magnetic shear and density gradient, or does the ergodic property of ray trajectories lead to their escape to the resonance? It is the purpose of the present paper to investigate these questions.

1. The Model

The trajectory of a wave packet which satisfies a local dispersion relation $\omega = F(\mathbf{k}, \mathbf{r})$ is a solution of the ray equations,

*A more complete report of this work will appear in Physics of Fluids.

$$\frac{dk}{dt} = - \frac{\partial F}{\partial \tilde{\kappa}} ; \quad \frac{d\tilde{\kappa}}{dt} = \frac{\partial F}{\partial k} . \quad (1)$$

The ray equations are identical in form to Hamilton's equations of motion for a particle with momentum k and Hamiltonian F . We use the cold fluid dispersion relation for lower hybrid waves $F = \omega_{pi} [1 + (m_i/m_e)(k_{||}/k)^2]^{1/2} [1 + (\omega_{pe}/\omega_{ce})^2(k_{\perp}/k)^2]^{-1/2}$, where $k_{||}^2 = (k \cdot B)^2/B^2$, $k_{\perp}^2 = k^2 - k_{||}^2$, $\omega_{pe,i}$ and $m_{e,i}$ are the electron and ion plasma frequencies and masses, and ω_{ce} is the electron gyrofrequency. In the usual toroidal coordinate system (r, θ, ϕ) the major radius is $R = R_0[1 + (r/R_0)\cos\theta]$ and $\tilde{\kappa} = k_r r_0 + \frac{m}{r} \theta_0 + \frac{n}{R} \phi_0$. We assume an axisymmetric toroidal equilibrium so that $\partial F/\partial \phi = 0$ and from (1) n is a constant. To obtain a self-consistent toroidal equilibrium we use Shafranov's aspect ratio expansion.⁴ The lowest order terms are the following cylindrical equilibrium quantities: $B_{\phi} = B_0(\nabla |B_0| = 0)$, the density profile is parabolic $N(r) = N(0)[1 - (r/a)^2]$ and the q -profile is $q(r) = \alpha[1 + \lambda(r/a)^2] \equiv rB_{\phi}(r)/(R_0 B_{\theta}(r))$, where α and λ are constants. The toroidal corrections to these quantities introduce a functional dependence in θ and a radial component of magnetic field $B_r(r, \theta)$.

2. The Results

The dispersion relation may be written $\omega = F(r, \theta, k_r, m, n)$. In cylindrical geometry ($\epsilon \rightarrow 0$) $\partial F/\partial \theta = 0$, hence m is a constant and the dispersion relation may be solved for $k_r(r)$. We present such solutions (level curves for m) in Fig. 1, where we observe the existence of a separatrix enclosing trapped orbits. Clearly, for finite inverse aspect ratio $\epsilon = a/R_0$, m is no longer an invariant of the ray motion. However, even in the presence of toroidal effects there may still be some other (more complicated) invariant of the motion, $\tilde{m} = \tilde{m}(r, \theta, k_r, m)$, which takes the place of m . In order to test for the existence of such an invariant we employ the surface of section method: If there exists an invariant $\tilde{m}$, then the equations $\omega = F(r, \theta, k_r, m)$ and $\tilde{m} = \tilde{m}(r, \theta, k_r, m)$ confine the ray path to a surface in (r, θ, k_r) space. If we cut this surface with a plane (the "surface of section") then the ray path will lie on a curve in this plane. On the other hand, if the ray path is observed not to lie on a curve, then there is no invariant $\tilde{m}$ (in such a case the ray wanders ergodically).

The Hamiltonian F is periodic in θ with period 2π so that we take the

k_r - r plane with $\theta = 2\ell\pi$, $\ell = 0, 1, 2, \dots$ to be our surface of section. We obtain a complete set of initial values by solving the dispersion for m as a function of r , with fixed ω, n, k_r and $\theta = 2\pi\ell$.

The ray equations, (1), were solved numerically. In Fig. 2 we present a typical result in the k_r - r surface of section plane, exhibiting ergodic orbits coexisting with smooth curves and evidence of two islands. This result was obtained for $\epsilon = 0.1$, $\omega = 0.92 \omega_{\text{LH}}(0)$, $n = -100$ and $q(r) = 0.5(1 + 3r^2/a^2)$. This figure is generated from four different initial positions along r , with $k_r = 0$, labeled (a), (b), (c), (d) in Fig. 2, and one initial condition labeled (e) with $k_r \neq 0$. The distribution of points is symmetric with respect to the reflection $k_r \rightarrow -k_r$. The two closed curves labeled I and II, generated by initial position (e), are islands. The rays having initial positions r at (a) and (b) exhibit ergodic behavior. These rays make many trips around in the poloidal direction (typically fifty) remaining in a relatively confined region in the k_r - r plane (cf. Fig. 2) before escaping to the resonance. They typically make two transits around the torus in the ϕ -direction for one transit in the θ -direction.

The preceeding has pointed out typical features of ergodic behavior for lower hybrid ray trajectories in the case of a low inverse aspect ratio ($\epsilon = 0.1$). We now investigate the behavior for larger ϵ . We integrated the ray equations for $\epsilon = 0.2$, for several wave frequencies, namely $\omega/\omega_{\text{LH}}(0) = 0.99, 0.96, 0.92, 0.86$ and for q -profiles with $q(a) = 3, 4, 6$, $q(0) = 1$. In all of these cases no trapped orbits were found. Typically the rays made several trips around in the toroidal direction before obtaining large values of k (i.e. approaching the resonance). The number of such transits ranged from 5 to 20, and it appears that more transits could be obtained by proper choice of initial conditions.

3. Conclusion

For moderate aspect ratio tokamaks, trapped lower hybrid ray trajectories of the type illustrated in Fig. 1 were not found. Rather, we find that for $\epsilon > 0.2$ toroidal effects destroy the constant of the motion necessary for ray trapping. However, even when $\epsilon = 0.2$, we find that the ray can make several transits around the torus, experiencing several interactions with the pump, before escaping to the resonance. This aspect of the problem requires further study.

This work was supported by ERDA.

References

1. E. Ott and C. S. Liu, Cornell University Laboratory of Plasma Studies Report - LPS 224, May 1976.
2. R. L. Berger, L. Chen and F. W. Perkins, Princeton Plasma Physics Laboratory Report - PPPL 1307, November 1976.
3. M. Porkolab, Phys. Fluids 17, 1432 (1975).
4. V. D. Shafranov, in Reviews of Plasma Physics, Vol. 2, edited by M. A. Leontovich (Consultants Bureau, New York, 1966) pp. 125-131.

Figure Captions

Figure 1. Level curves of m in the $k_r - r$ plane for the straight cylindrical model. The values of the parameters are $\omega = 0.92 \omega_{LH}(0)$, $n = -100$, $q(r) = [1 + 3(r/a)^2]/2$, $\omega_{pe}(0) = \omega_{ce}(0)$, $120 \leq m \leq 160$. The dashed curve is the separatrix ($m = 137$).

Figure 2. Behavior of rays in the $k_r - r$ surface of section for $\varepsilon = 0.1$, $\omega = 0.92 \omega_{LH}(0)$, $n = -100$ and $q(r) = [1 + 3(r/a)^2]k$. The circles and the triangles are generated by initial values at (a) and (b), respectively. The initial values (c) and (d) generate smooth curves as in cylindrical geometry. The initial value (e) generates the two islands I and II. This figure fits in the rectangle appearing in Fig. 1.

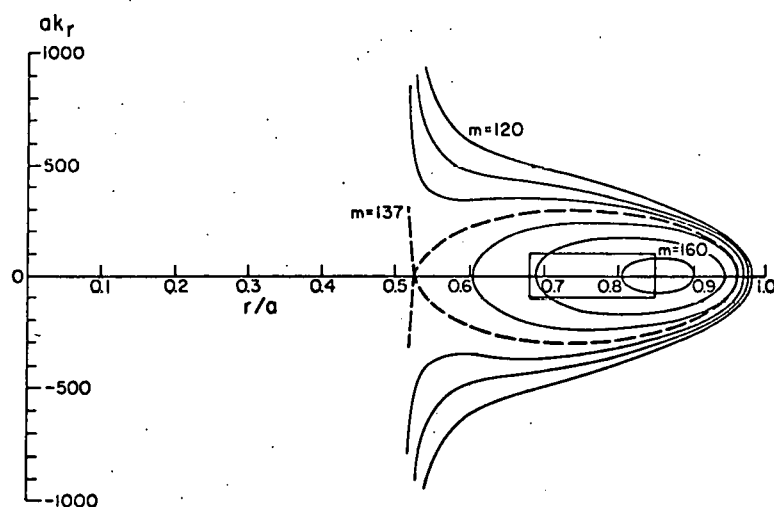


Figure 1

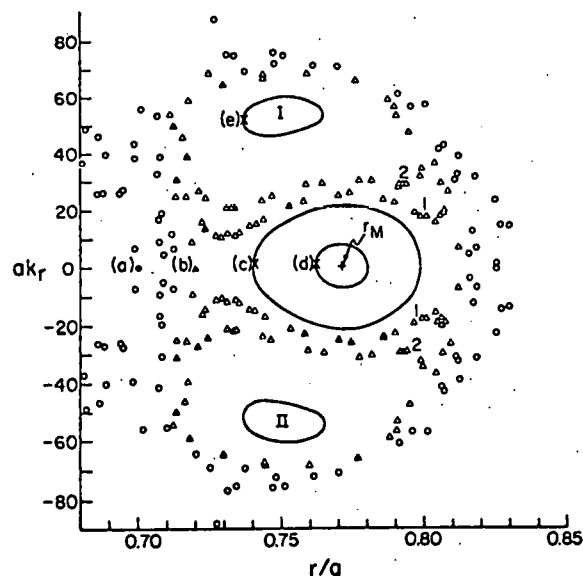


Figure 2

Harmonic Generation in Lower-Hybrid Heating.

J. L. SPERLING and R. W. HARVEY, General Atomic Co.*--
Using kinetic theory we show that a lower-hybrid or whistler wave with frequency, ω_0 , can destabilize lower-hybrid waves with frequencies which are harmonics of ω_0 . The energy required to destabilize a harmonic mode with frequency, $n\omega_0$, is provided by the driving RF source and the internal plasma energy in the ratio of ω_0 to $(n-1)\omega_0$, respectively. The harmonic generation of lower-hybrid waves could not only alleviate the condensation problem which has arisen in recent cascade theories, but could also enhance wave penetration by providing a means whereby lower-hybrid wave energy could be transformed into higher frequencies.

*Supported by US DOE Contract EY-76-C-03-0167, Project Agreement No. 38.

Effect of Convective Loss on Resonant Decay of Cold Lower Hybrid Waves*

K. L. Wong and J. R. Wilson
Plasma Physics Laboratory, Princeton University,
Princeton, New Jersey 08540

and

M. Porkolab
M.I.T., Cambridge, Mass. 02139

The resonant decay of cold lower hybrid waves was investigated in the Princeton L-3 device in an argon plasma with $n \sim 1-5 \times 10^{10}/\text{c.c.}$, $B \sim 400 \text{ G} - 2 \text{ kG}$, $f_0 \sim 50-80 \text{ MHz}$ ($\omega_0 \geq 10 \omega_{\text{LH}}$). The decay waves were identified to be ion-acoustic waves and lower hybrid waves. Because of convective loss in the finite extent pump field, coupling due to $E_{\parallel}$ dominates that due to $\vec{E} \times \vec{B}$ and the sideband follows the resonance cone trajectory of the pump. The acoustic wave has a frequency much lower than the pump ($\sim 10^{-2} \omega_0$). It increases with magnetic field and pump frequency. The energy in the sideband can exceed 10% of the pump, and the parallel wavelength is several times shorter. Therefore, this decay can lead to enhanced Landau heating of the plasma electrons.

INTRODUCTION

It is well known that lower hybrid waves propagate along the resonance cone trajectory which is highly localized in space. The finite extent of the wave has a significant effect on the parametric decay processes because convective loss can be an important stabilizing mechanism. In a uniform pump field, $\vec{E} \times \vec{B}$ coupled decay should dominate and the threshold depends only on the damping rate of the decay waves (γ_1, γ_2), namely, $\gamma_0^2 > \gamma_1 \gamma_2$ where γ_0 is the growth rate. In a pump field of finite extent L , the threshold for convective instability is loosely defined by $\kappa L > \pi$ where κ is the spatial amplification factor. For a one dimensional slab,¹

$$\kappa = \frac{-V_2\gamma_1 - V_1\gamma_2 + \sqrt{(V_2\gamma_1 + V_1\gamma_2)^2 - 4V_1V_2(\gamma_1\gamma_2 - \gamma_0^2)}}{2V_1V_2}$$

where V_1, V_2 are the group velocities of the decay waves. The frequency of the decay wave should be at the maximum of κ . In this paper we present experimental data showing that in a finite extent lower hybrid wave, the parallel coupled decay (due to $E_{\parallel}$) can dominate the $\vec{E} \times \vec{B}$ coupled decay.

EXPERIMENT

The experiment was performed in the Princeton L-3 device which is a 4-meter long linear plasma column 10 cm in diameter, with argon plasma density $n \sim 1-5 \times 10^{10}/\text{c.c.}$, magnetic field $B \sim 400 \text{ G} - 2 \text{ kG}$, electron temperature $T_e \sim 1-3 \text{ eV}$, ion temperature $T_i < 0.1 \text{ eV}$. The lower hybrid wave was launched by a slow wave structure² consists of four to eight alternately phased rings driven by an rf oscillator. When the rf power was raised above a threshold value, parametric decay occurred with $\omega_1 \sim 10^{-2} \omega_0$. It was found that ω_1 decreases with pump frequency ω_0 , pump wavelength λ_0 and magnetic field β . In order to identify the daughter waves, the wavelengths were measured by interferometry as shown in Fig. 1. Different probes capable of scanning in the axial, radial and azimuthal directions were used to measure wavelengths in all directions. There was no measurable wavelength in the azimuthal direction and we set a lower bound of $\lambda_\theta > 2 \text{ cm}$. Therefore, the parametric decay was mainly driven by the parallel coupling. From the measured frequencies and wavelengths, the low frequency decay wave was identified to be ion acoustic wave propagating nearly perpendicularly to the pump resonance cone and the sideband was a lower hybrid wave propagating closely along the pump wave packet since it obeyed the same dispersion as the pump. This is apparent in the k -vector diagram as well as the data shown in Fig. 2. A calibrated double tip probe was used to measure the decay threshold. The decay can be observed when the pump field is above the collisional threshold ($\sim 3.5 \text{ V/cm}$). At $r > 2 \text{ cm}$, $E_\parallel \leq 5 \text{ V/cm}$ and the decay waves have very small amplitudes. The sideband electric field is about 25 db below the pump. Near the center of the plasma column, the pump electric field can go up to 10 V/cm and the amplitude of the decay waves are much higher. The energy in the sideband can exceed 10% of the pump. Fig. 3 shows the relative magnitude of the pump and the sideband waves as a function of rf input power near the center of the plasma column. It is apparent that a significant fraction of energy goes to the sideband. Since the sideband has wavelength several times shorter than the pump, it can be easily dissipated by electron Landau damping.³ Figure 4 shows some raw data for the measurement of the electron parallel distribution function measured by a 5-grid electrostatic electron energy analyzer. Coincident with the parametric decay, an enhanced electron tail was observed with energy between 40 - 100 eV, in and above the energy range where Landau damping should occur.

REFERENCES

- * This work was supported by U.S.D.O.E. Contract EX-76-C-3073.
- ¹ M. Porkolab and R. P. H. Chang, Phys. Fluids 13, 2054 (1970); also D. Pesme et al., Phys. Rev. Lett. 31, 203 (1973).
- ² P. Bellan and M. Porkolab, Phys. Rev. Lett. 34, 124 (1975).
- ³ F. J. Paoloni, R. W. Motley, W. M. Hooke, and S. Bernabei, Phys. Rev. Lett. 39, 1081 (1977).

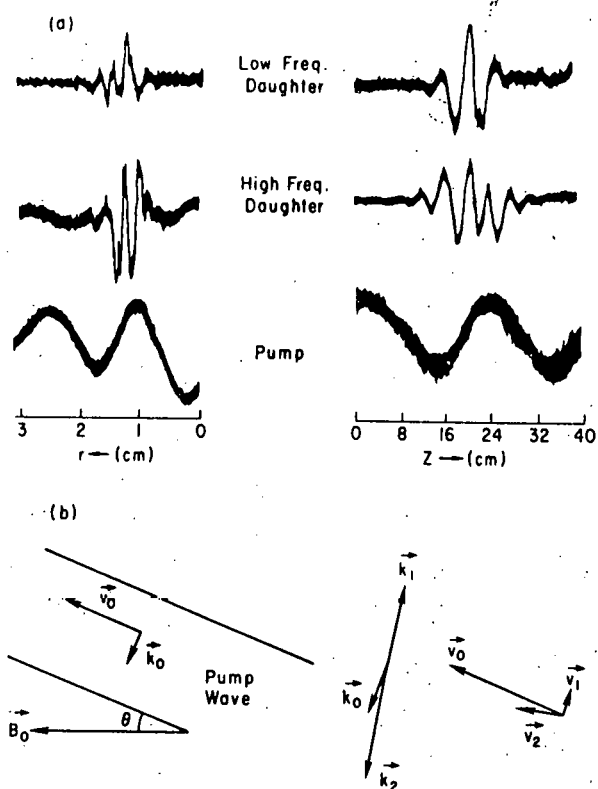


Fig. 1. Measurement of wavelengths by interferometry.

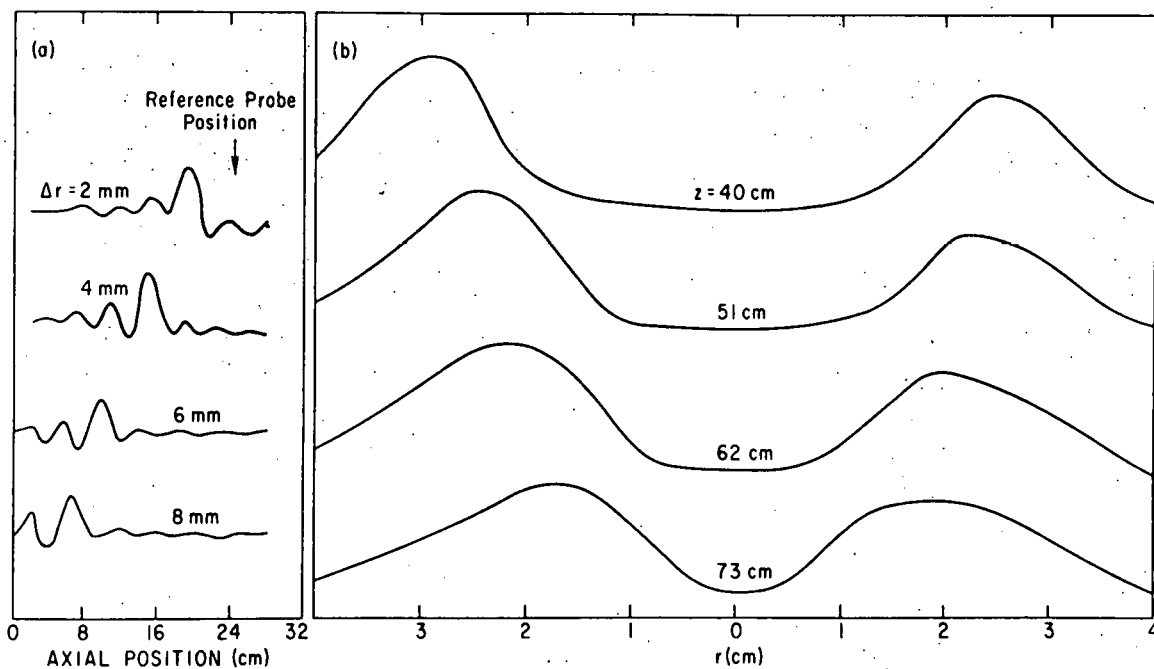


Fig. 2. Conical trajectory of the sideband wave.

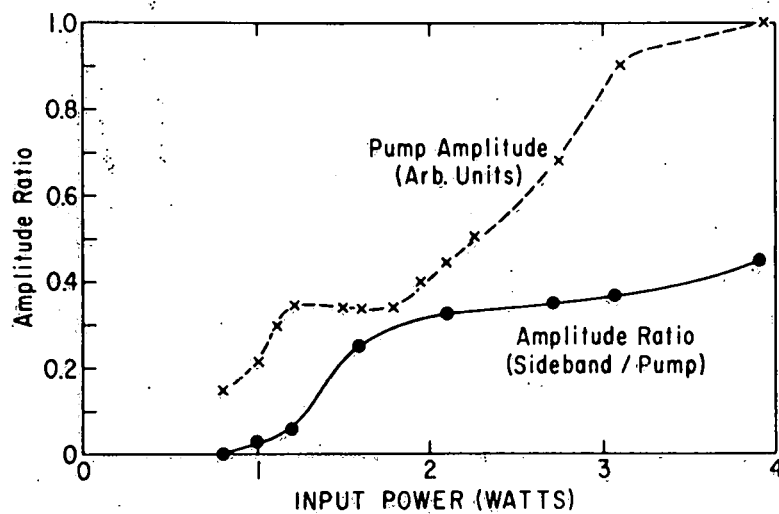


Fig. 3. Pump and sideband amplitudes at various power levels.

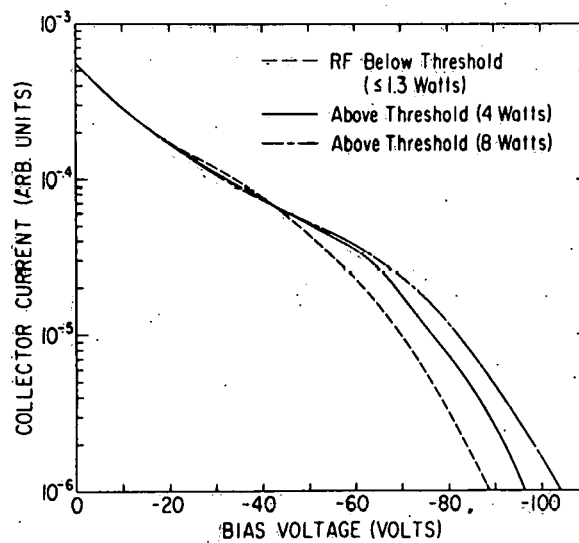


Fig. 4. Enhanced electron tail produced by the sideband lower hybrid wave.

RF Coupling Near the Lower Hybrid Resonance*

R.B. Michie, M.C. Clark** and M.E. Oakes
 University of Texas at Austin
 Austin, Texas 78712

Abstract

A pulsed turbulently heated washer gun produced plasma was used to study propagation and absorption around the lower hybrid resonance. Efficient coupling of the rf power was observed under collisionless conditions. The dependence of power absorbed and rf amplitude on magnetic field and density was measured. Additional power absorption measurements were made on a colder plasma produced in the same magnetic mirror by discharging the capacitor into a neutral gas backfill. Variations with density, magnetic field and ion mass were studied. A comparison with the results on the hot plasma is presented.

Introduction

A variety of experiments on lower hybrid heating have been conducted on plasmas which were collisional; such studies have shown rather strong heating under this condition. Less is known about the behavior of waves under collisionless conditions. Theoretical evidence^{1,2} indicates mode conversion is unlikely unless collisions are reduced drastically. It has also been found that a number of nonlinear effects have rather low thresholds³ and thus are likely to be important under conditions of low collisional damping. In addition there appear to be few good estimates of the steady state impedance presented to an antenna by a collisionless plasma. For primarily these reasons we set about producing a hot plasma.

Experiment

In Fig. 1 we show the schematic diagram of the experiment. The plasma is first produced by a hydrogenated titanium stacked washer gun and injected into the mirror field shown in Fig. 2. A capacitor discharge produced a turbulent hot plasma with a 600 eV electron temperature determined from diamagnetic loop and x-ray measurements. The critically damped plasma current and strong microwave emission are evidence of the turbulence present. The rf power is provided by a 2 kw-148 MHz amplifier driving the antenna shown in Fig. 3. In Fig. 4 we show the experimental parameters and lower hybrid resonance for the driving frequency.

Fig. 5 (upper section) shows the dependence of percent power radiated upon magnetic field strengths for various times after the peak in the diamagnetic signal. Each point represents the average of several shots. It is apparent that

strong coupling to the plasma is present for the hot plasma (1 μ s) and for slightly cooler conditions (5 μ s). The level of turbulence as indicated by the microwave emission is stronger at the early time (1 μ s). The lower hybrid resonance will enter the plasma (at the mirror points) for a center field of 1.3 kG. The (5 μ s) results is consistent with absorption at a lower hybrid critical layer. The (1 μ s) turbulent results appear rather flat which may suggest a large anomalous collision frequency. These are necessarily conjectures since measurement inside the hot plasma is not possible.

A second group of measurements were done on a plasma created by firing the machine with a 1 mTorr backfill. Fig. 5 (center and lower sections) shows the results of these measurements. Again we find the radiated power increasing as the lower hybrid enters the machine, the coupling however saturates at a level which is $\sim 1/n(\text{cm}^{-3})$. As the rf power is increased we see that the saturated level is independent of density. The saturated value is near the low density value. Similar measurements were made on the warm plasma with a number of other gases. The results are shown in Fig. 6. Again the characteristics rise and saturation $\sim 1/n$ is seen. However the lower hybrid resonance can not account for the onset of coupling since in argon the necessary onset field is never in the machine. The similarities of these warm plasma results with the 5 μ s hot plasma results discussed earlier would seem to indicate this is not a collisional effect ($\nu/\omega \sim .16$ for the warm plasma). While the detailed boundary value problem has not been completed a much idealized model indicates that Landau damping on the slow wave may account for these results. It should also be noted that in the warm plasma rf probes indicate field penetration to the plasma center. Since the cone angle is very shallow for these conditions it is thought that multiple reflection of the cone is occurring from the ends of the machine or a significant amount of the fast wave is excited; phase measurements are required in order to decide. The phase studies are in progress.

*This work was supported by the Texas Atomic Energy Research Foundation.

**Kirtland Air Force Base, New Mexico

References

1. Moore, B.N. and Oakes, M.E., Phys. Fluids 15, 144 (1972).
2. Bellan, P.M. and Porkolab, M., Phys. Fluids 17, 1592 (1974).
3. Porkolab, M., Bull. Am. Phys. Soc. 22, 1123 (1977).

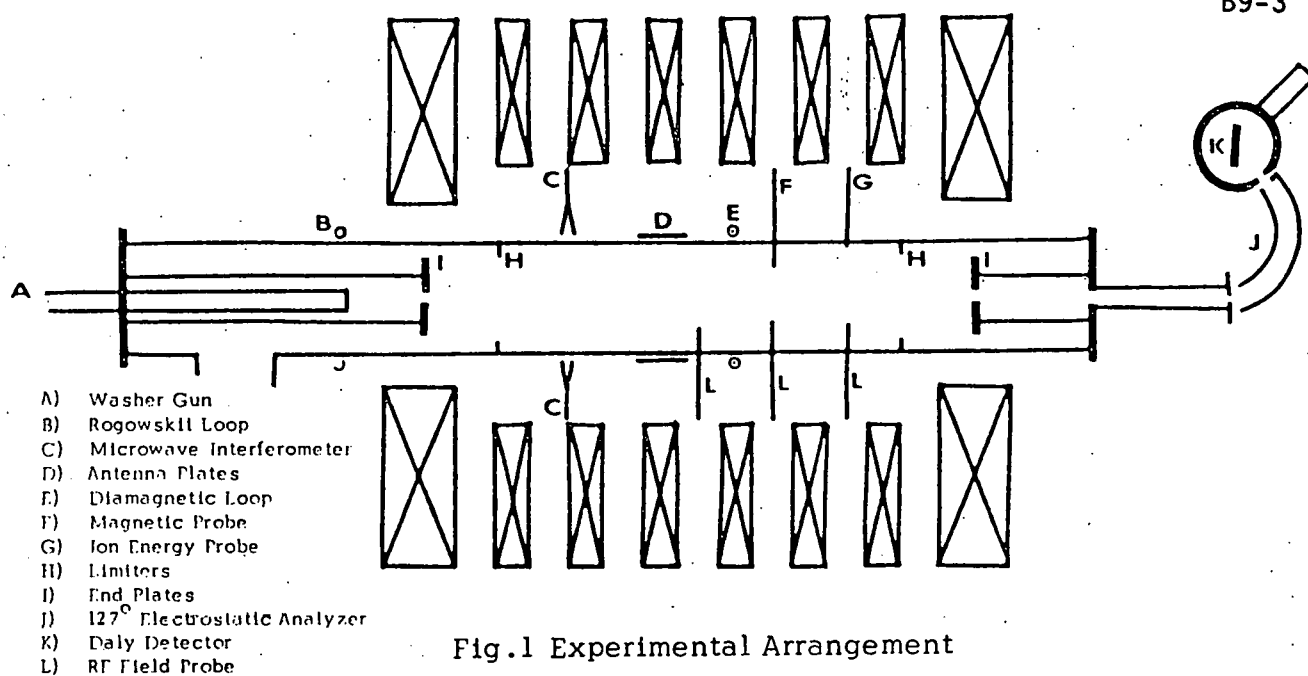


Fig.1 Experimental Arrangement

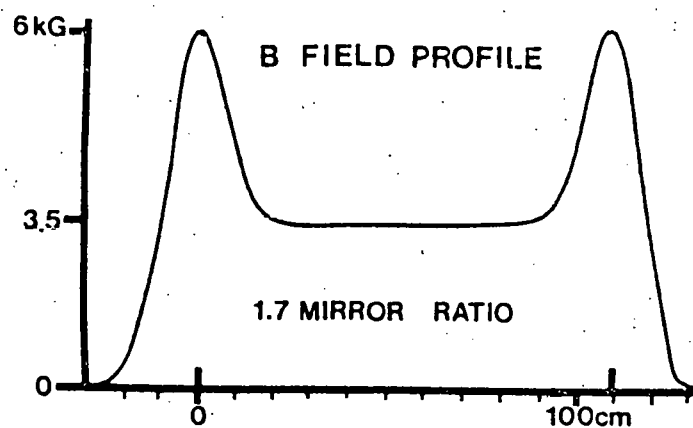


Fig.2 Mirror Field Profile

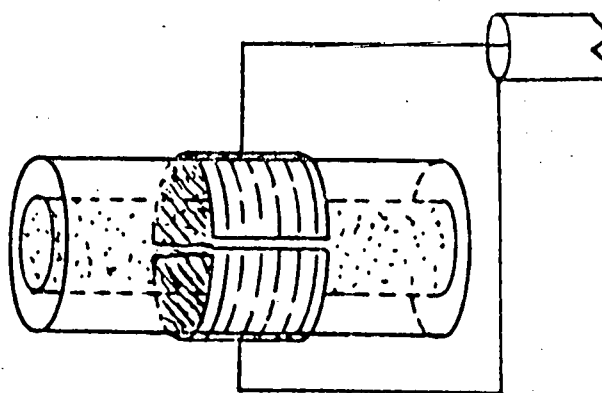


Fig.3 Antenna

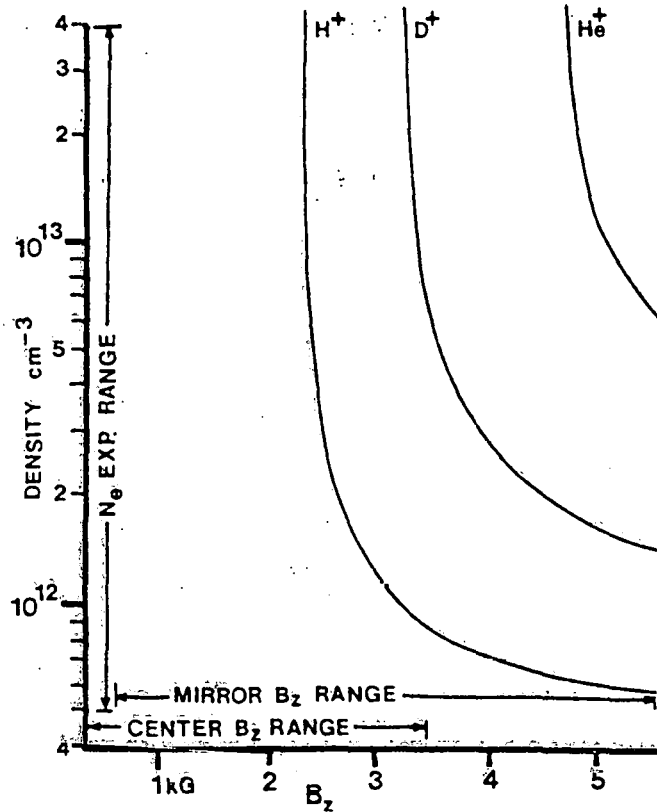


Fig. 4 Lower Hybrid Density vs. Magnetic Field

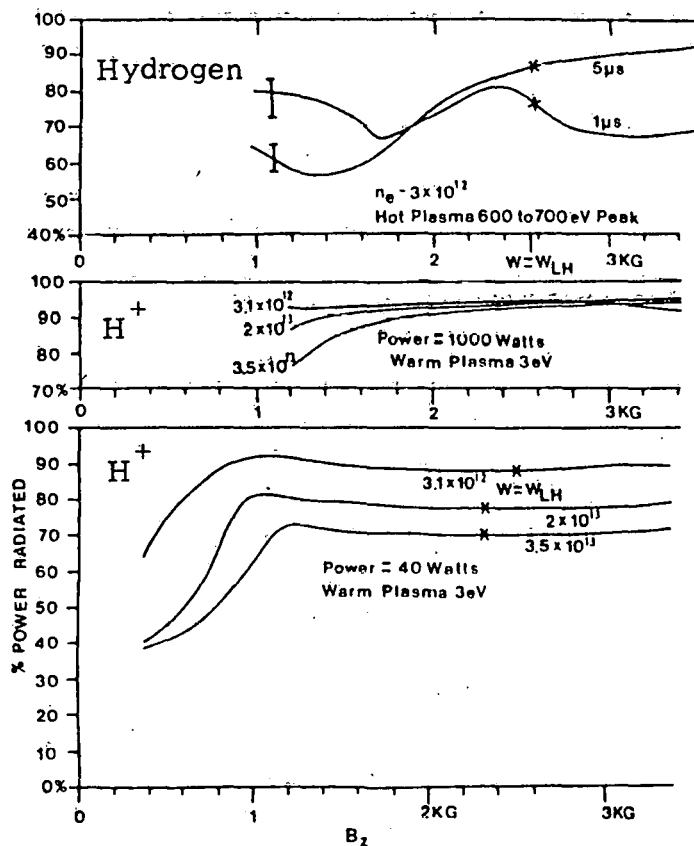


Fig. 5 Percent of Incident Power Radiated Into Hot plasma (Upper Section) and Warm Plasma (Center and Lower Section)

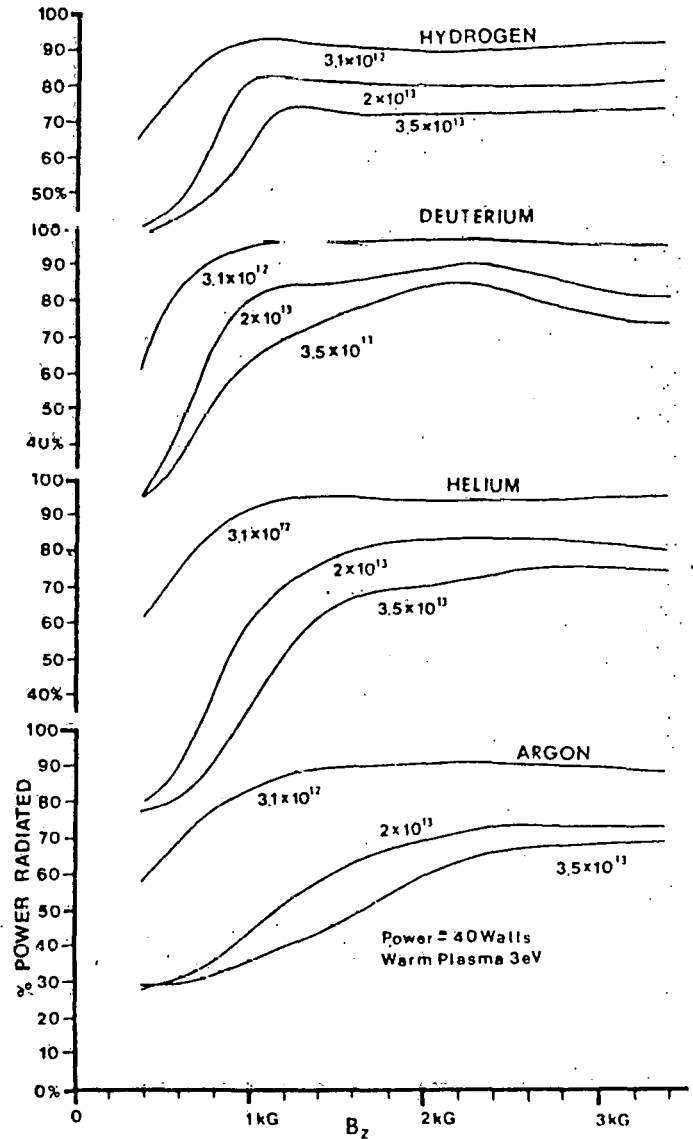


Fig. 6 Percent of Incident Power Radiated Into Warm Plasma of Hydrogen, Deuterium, Helium, and Argon

Lower-Hybrid Heating of the MIT Versator II and Alcator Tokamaks

M. Porkolab, G. Bekefi and R. Parker,
 Massachusetts Institute of Technology, Cambridge, Ma. 01239, and
 S. Bernabei, W.M. Hooke and R. Motley,
 Plasma Physics Laboratory, Princeton University, Princeton, N.J. 08540

We describe here the expected power spectrum of several lower-hybrid heating experiments planned at MIT, namely that of the Versator II research tokamak, and that of the Alcator A and C tokamaks. The power spectrum of the phase-arrayed waveguide grill planned for each device is calculated by using the Brambilla theory,¹ which was suitably normalized to calculate the power-spectrum according to the prescription of Bernabei and Fidone.² The relevant machine parameters are shown in Table I.

While the Versator II device will have sufficiently large ports (14 cm wide, 30 cm high) to test a number of different sized grills, the Alcator devices have previously designed ports with restricted dimensions. In Table I we also show the expected plasma parameters, as well as the planned microwave frequency and maximum power available.

In Figs. 1-3 we show computer calculations of relative power spectra for Versator II. Figure 1(a) corresponds to a grill of four units, phased $0\pi 0\pi$, with equal amplitudes in each of the four waveguides. We see that in this particular case the reflection coefficient is $R = 0.11$, and the surface component of the power spectrum P_s (i.e. $N_z \lesssim N_{zc} = 1.8$ in a hydrogen plasma) is $P_s = 0.14 P_0$, (where P_0 is the incident power), yielding a total efficiency of $\eta = 0.77$. In Fig. 1(b) we show the spectrum for unequal amplitudes. We note that by reducing the relative power levels in the outer two waveguides to approximately 1/4 of that in the inner elements, the surface wave component is almost completely eliminated (i.e., $P_s = 0.02 P_0$).³ Note that the reflection coefficient is not greatly altered ($R = 0.09$). Although in principle in some cases one may eliminate surface heating by this technique, we note that the total power transmitted through a given port area has been reduced by 38%, and the power transmitted to the center of the plasma column has been reduced by more than 20%. In Figs. 2 and 3 we show results for grills of 6 and 8 elements. In these cases the phases of subsequent waveguides are increased in steps of $\pi/2$ (Fig. 2) and $2\pi/3$ (Fig. 3), so that in both cases a nearly unidirectional power flow is produced (albeit a 10% power-flow in the negative N_z components). Note that unless the density gradient is uncomfortably large, as shown in Fig. 3 (i.e. $\nabla n \approx n_c$, where $\omega = \omega_{lh}$ at $n = n_c$) an undesirably large reflection results (i.e., $R \approx 0.30$). Thus, Landau heating using a unidirectional power spectrum may be difficult to achieve under realistic experimental conditions. We note that in Versator II for an expected peak electron temperature of $T_e \approx 400$ eV, we need $N_z \approx 11$ for Landau heating.

In Fig. 4 we show the results for Alcator A which will have a split waveguide (2 elements) of a total width of 2.6 cm. In this case we find that for reasonable density gradients ($\nabla n \approx 5 \times 10^{12} - 5 \times 10^{13} \text{ cm}^{-4}$) the reflection coefficient varies in the range $R \approx 0.05 - 0.10$. Using $f = 2.45$ GHz, $B = 7$ Tesla, $f \approx f_{lh}$ for $n \approx 1.9 \times 10^{14} \text{ cm}^{-3}$ in an H+ plasma and $N_{zc} \approx 1.20$. The surface component is approximately $P_s \approx 0.15 P_0$, so that a maximum efficiency of $\eta \approx 0.75$ appears feasible (in reality one should use somewhat lower values since $N \approx N_{zc}$ may penetrate too slowly). However, in a D₂ gas $N_{zc} \approx 1.54$ and $P_s \approx 0.3 P_0$, so that a large fraction of the power will not penetrate.

In Fig. 5 we show a 4 waveguide grill design for Alcator C. In this case the port size is 4.4x25 cm, so that we plan to stack vertically four sets of 6 cm high waveguides. The relevant data are given in the figure captions. We see that regardless of the relative wave amplitudes, in a D⁺ plasma $N_{zc} = 2$ and the surface power is in the range $P_s = (0.09-0.11)P_0$ for cases (a)-(c). Thus, we see that in deuterium varying the relative power levels in adjacent waveguides offers little improvement in reducing the surface wave component.

In Fig. 6 we show a 6 waveguide grill design for Alcator C. Case (a)

corresponds to a phasing of $(2\pi/3)$ for adjacent waveguides, and case (b) corresponds to a phasing of π . Assuming a temperature of $T_e \approx 2$ KeV near the center, case (b) would be appropriate for electron Landau damping of the incident wave ($v_{ph}/v_{te} = 3$ for $N_z \approx 5$). The surface wave component in D^+ for both cases (a) and (b) corresponds to about $P_s \approx 0.10 P_0$, and the reflection coefficient is $R = 0.12$ for $\nabla n = 2 \times 10^{14} \text{ cm}^{-4}$. For $B = 10$ Tesla and $f = 4.0$ GHz the critical density is $n = 2.9 \times 10^{15} \text{ cm}^{-3}$ in D^+ ; however, for a temperature of $T \approx (1-2) \text{ keV}$ the mode conversion layer occurs at considerably lower densities, i.e. $n \approx 10^{15} \text{ cm}^{-3}$.

In summary, we discussed here several waveguide grill designs for the purposes of lower-hybrid heating of three MIT tokamaks. In addition, several other cases have also been considered in some detail. Using a grill of four or more elements, under favorable conditions as much as 80% of the incident power could penetrate to the center of the plasma column in both the Versator II and Alcator C tokamaks. At $B > 70 \text{ kG}$ a 2 waveguide grill in Alcator A gives an acceptable penetration only in an H^+ plasma. We also find that (a) by varying the relative power level of adjacent waveguides, in certain cases one can reduce the surface wave level; (b) phasing waveguides to produce a nearly unidirectional flow of power and to provide electron Landau damping at the same time requires large values of ∇n for an acceptably low value of R , the reflection coefficient.

Acknowledgements

We wish to thank Dr. M. Brambilla for making his computer code for calculating the Fourier spectra available to us.

This research has been supported by the U.S. DOE.

References

1. M. Brambilla, Nucl. Fusion 16, 47 (1976).
2. S. Bernabei and I. Fidone, Third Symp. on Plasma Heating in Tor. Dev., Varenna, 1976 (Editrice Compositori, Bologna, Italy, 1976).
3. Similar results have also been obtained by V. Krapchev and A. Bers (private communication and to be published).

Table I

Machine	B (Tesla)	a (cm)	Port Size w x h (cm)	n_0 (cm^{-3})	f (GHz)	P (MW)	Δt (msec)
Versator II	1.5	14	14 x 30	5×10^{13}	0.8	0.2	10
Alcator A	7	11	2.6 x 8	1×10^{15}	2.4	0.1	20
Alcator C	10(14)	17	4.4 x 25	$(1-2) \times 10^{15}$	4.0	1.6	200

Figure Captions

1. 4 wgd grill for Versator II; $\Delta\phi=\pi$; dimensions: $w=2.35\text{cm}$, $d=0.64\text{cm}$ (w =wgd width, d =wall thickness); $\bar{V}_n=5\times 10^{12}\text{cm}^{-4}$, $f=0.8\text{GHz}$, $B=1.5\text{ Tesla}$, $N_{ZC}(H^+)=1.8$.
 - a. Equal power in each wgd; $R=0.11$, $P_S=0.14P_0$
 - b. Power in outer 2 wgds = 1/4 of inner 2 wgds; $R=0.09$, $P_S=0.02P_0$.
2. Grill with $\Delta\phi=\pi/2$; dimensions: $w=0.8\text{cm}$, $d=0.25\text{cm}$; $f=0.8\text{GHz}$. $B=1.5\text{ Tesla}$, $\bar{V}_n=5\times 10^{12}\text{cm}^{-4}$.
 - a. 6 wgd grill; $R=0.29$, $P_S=0.08P_0$ in H^+ .
 - b. 8 wgd grill; $R=0.29$, $P_S=0.10P_0$ in H^+ .
3. 6 wgd grill, $\Delta\phi=2\pi/3$; dimensions: $w=0.9\text{cm}$, $d=0.25\text{cm}$; $f=0.8\text{GHz}$, $B=1.5\text{ Tesla}$, $\bar{V}_n=5\times 10^{13}\text{cm}^{-4}$;
 - a. Power spectrum in $+N_Z$ components; $P_S=0.10P_0$.
 - b. Power spectrum in $-N_Z$ components; $P(-N_Z)=0.12P_0$.
(not shown; $\bar{V}_n=1\times 10^{13}\text{cm}^{-4}$, $R=0.28$; $\bar{V}_n=2\times 10^{12}\text{cm}^{-4}$, $R=0.44$.)
4. Split wgd grill (2 wgd) for Alcator A; $\Delta\phi=\pi$; dimensions: $w=1.20\text{cm}$, $d=0.20\text{cm}$; $f=2.45\text{GHz}$, $B=7.0\text{ Tesla}$, $N_{ZC}(H^+)=1.20$; (a) $\bar{V}_n=1\times 10^{13}\text{cm}^{-4}$, $R=0.06$; (b) $\bar{V}_n=5\times 10^{13}\text{cm}^{-4}$, $R=0.10$; $P_S=0.15P_0$.
5. 4 wgd grill for Alcator C; $\Delta\phi=\pi$; dimensions: $w=0.95\text{cm}$, $d=0.20\text{cm}$; $f=4.0\text{GHz}$, $B=10\text{ Tesla}$, $N_{ZC}(D^+)=2.0$; $\bar{V}_n=5\times 10^{13}\text{cm}^{-4}$.
 - a. Equal power in each wgd; $R=0.12$, $P_S(D^+)=0.11P_0$.
 - b. Power in outer 2 wgds=1/2 of inner 2 wgds; $R=0.10$, $P_S=0.11P_0$.
 - c. Power in outer 2 wgds=1/4 of inner 2 wgds; $R=0.09$, $P_S=0.09P_0$ (not shown).
6. 6 wgd grill for Alcator C; dimensions: $w=0.60\text{cm}$, $d=0.16\text{cm}$; $f=4.0\text{GHz}$, $B=10\text{ Tesla}$, $\bar{V}_n=2\times 10^{14}\text{cm}^{-4}$. (a) $\Delta\phi=2\pi/3$; (b) $\Delta\phi=\pi$; For both (a) and (b), $R=0.12$, $P_S(D^+)=0.10P_0$, $P_S(H^+)\approx 0.05P_0$.

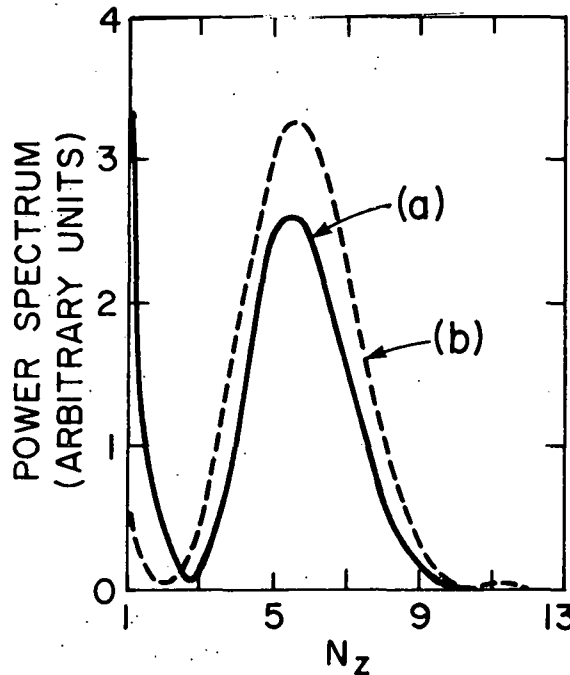


Fig. 1

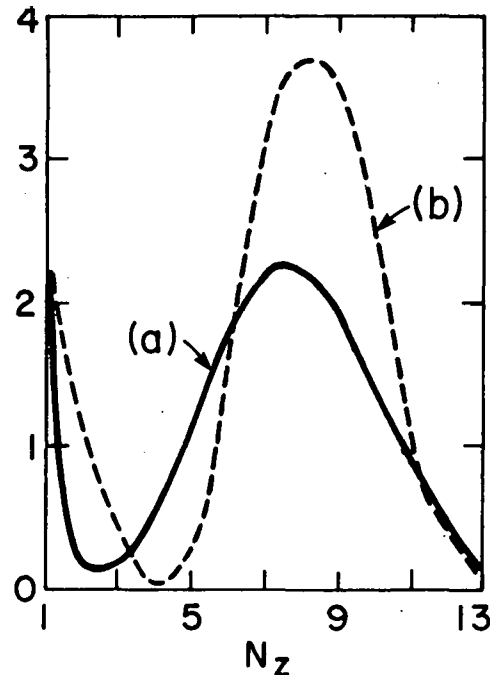


Fig. 2

Fig. 3

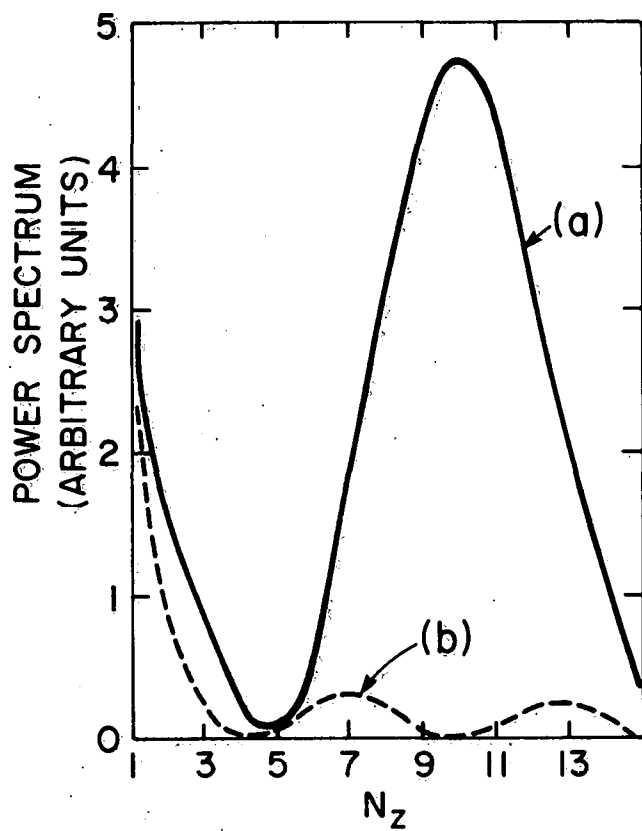


Fig. 4

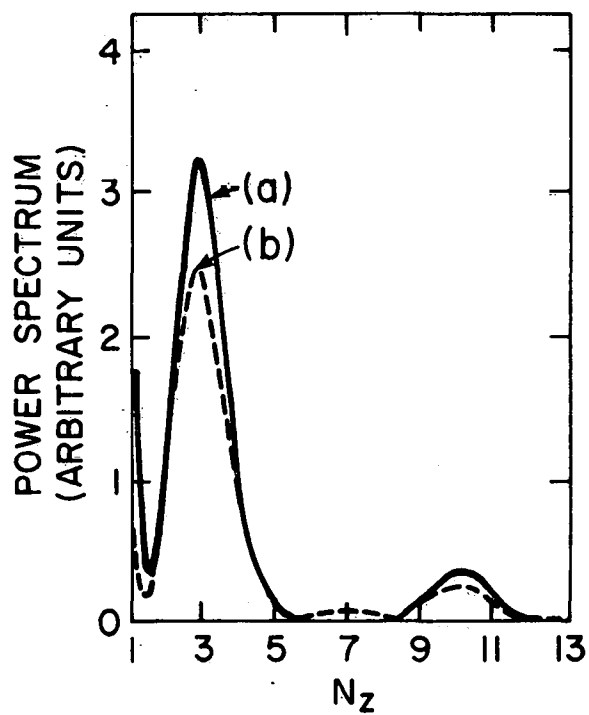
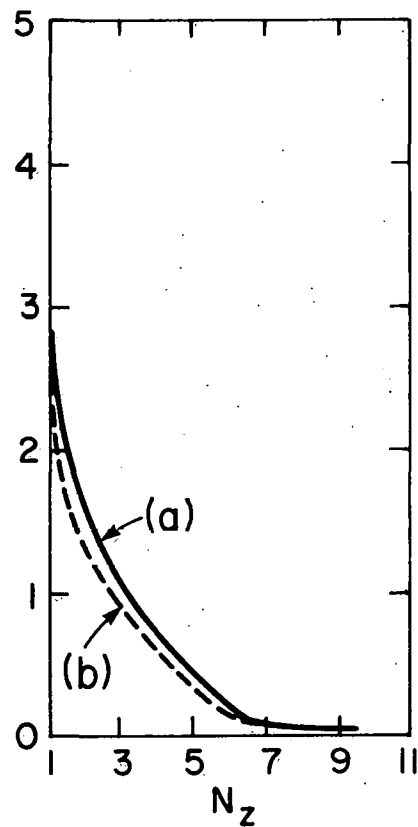


Fig. 5

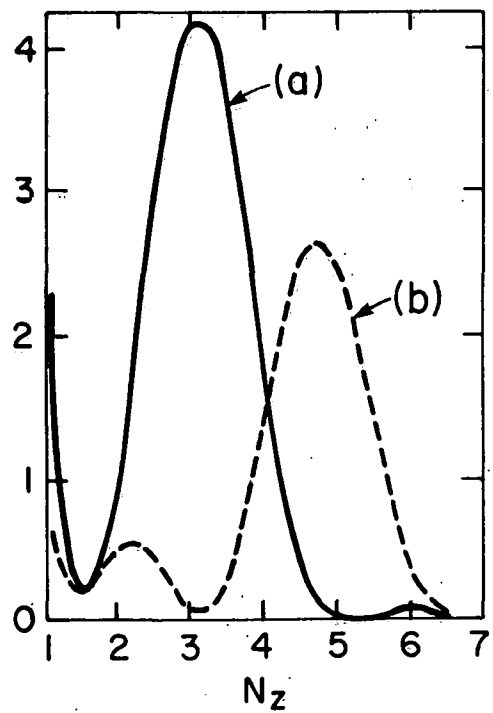


Fig. 6

PROPOSAL OF A LH HEATING EXPERIMENT
ON THE FRASCATI TOKAMAK (FT)

F. De Marco, P. Santini

Associazione EURATOM-CNEN sulla Fusione, Centro di Frascati,
Frascati, Rome, Italy.

ABSTRACT: The main features of the LH heating experiment proposed for the FT tokamak are discussed with regard to coupling and penetration. Heating simulations from a transport code are also reported. The actual size of the machine and its ports, and the values of the typical magnetic field and the expected plasma density, are shown to indicate which values of the main parameters of the RF system are to be chosen.

INTRODUCTION

A high field Tokamak is generally a device in which the characteristics of the toroidal magnet and the not large dimensions do not allow a wide access to the plasma. In particular the port holes of FT are so narrow to present serious difficulties for an experiment of neutral injection. On the other hand, if a relatively high performance of the FT device is considered, heating at ICRF presents some problems of contamination due to the necessity of insulated antennas in a small volume close to the plasma.

The high plasma density at which FT can work makes possible the use of waveguides for launching RF power at lower hybrid frequency. As a matter of fact central densities in the range $1 \div 4 \times 10^{14} \text{ cm}^{-3}$ should be surely reached so that a frequency of several gigahertz can be used together with suitably small waveguides. The high magnetic field ($\sim 80 \text{ kG}$) has several beneficial effects. If the density is not pushed to the extreme, the ratio $\omega_{pe}^2/\omega_{ce}^2$ can be quite low allowing a good accessibility even with the simplest coupling structure, a two waveguides array. Moreover parametric decay instabilities seem to show higher thresholds ($\propto B^2$) so RF power depletion at the border should be more unlikely to occur.

For the relevant poloidal field and low ripple of the FT, particles even with large values of perpendicular energy are fairly confined. Fig. 1 shows the loss cones for particles hitting the limiter for a current of 500 kA. (The axes are labeled in keV). This is a most distinctive feature between the experiment planned on FT and the previous or present ones on other tokamaks. It will be interesting to see how much the loss of high energetic ions observed up to now is affected. In the following table we give the FT nominal parameters and the actual ones we have chosen for the LH experiment.

TABLE I

FT Nominal Parameters

Torus Major radius R_0	: 83 cm
Limiter inner radius a_L	: 20.5 cm
Maximum toroidal field B_0	: 100 kG
Plasma current (at $q = 3$)	: 845 kA
Dimensions of a port:	
- vertical	: 16 cm
- horizontal	: 4.2 cm

Parameters for the LH Experiment

Toroidal field	: 80 kG
Plasma current ($q = 4$)	: 500 kA

In the FT there is some shortage of accesses to the plasma caused by the tight construction of the magnet. Since in LH heating many problems are still to be investigated, we intend to use no more than one window to inject the RF power and to use the others to have as much diagnostics as possible. The philosophical aim of the experiment is to use enough power to detect and study sizeable effects rather than to increase the temperature several times; for this reason we took also plasma and machine parameters that should be easy to achieve in a routine operation.

In the first section we shall describe the reasons of the choice of the power and the frequency. In the second section we shall discuss the accessibility and penetration characteristics. In the third section we shall discuss the power absorption processes and show some preliminary numerical results. In the fourth section we describe briefly the diagnostics.

1. FREQUENCY AND POWER

It is commonly accepted that the extra heating power should be at least of the same order of the ohmic power to detect any effect. It is difficult to guess the value of the ohmic heating power because, as it is well known, it depends on the plasma cleanliness, temperature profile and then on the not yet well understood processes of recycling at the wall and transport.

But for densities equal or larger than 10^{14} cm^{-3} , it is reasonable to assume a Z_{eff} close to 1 since experiments have shown that the absolute level of impurities does not increase as the density. Moreover the Alcator empirical relation

$$\tau_E(s) = 7 \times 10^{-19} n(\text{cm}^{-3}) a^2(\text{cm}) \quad (1)$$

with the Spitzer formula gives the value of the mean temperature depending weakly on the actual profile. If we assume the scaling (1), the temperature T_e and then the ohmic power will not depend on density. In our case $\langle T_e \rangle = 500 \text{ eV}$ and $P_0 \approx 450 \text{ kW}$ ($\tau_E = 40 \text{ msec}$ for $n = 1.3 \cdot 10^{14} \text{ cm}^{-3}$ in hydrogen and $\tau_E = 82 \text{ msec}$ for $n = 3 \cdot 10^{14} \text{ cm}^{-3}$ in deuterium). Consistently with the density range $1 \div 4$ in 10^{14} cm^{-3} , we can use hydrogen and/or deuterium plasma with LH frequency $\approx 2.5 \text{ GHz}$. At this frequency components and high power tubes are easily available on the market. This frequency fits well with the height of the ports of FT (vacuum wavelength $\lambda_0 = 12 \text{ cm}$). Then we have chosen the following values for the parameters of the experiment:

TABLE II

$I = 500 \text{ kA}$	$B = 80 \text{ kG}$
$P_{RF} = 500 \text{ kW}$	RF frequency = 2.5 GHz
Cold resonant density (H)	= $1.7 \times 10^{14} \text{ cm}^{-3}$
Cold resonant density (D)	= $4.9 \times 10^{14} \text{ cm}^{-3}$
Resonant density with temperature corrections for $n_{ } = 2$ and $T_i = 1 \text{ keV}$	
(H)	= 1.3×10^{14}
(D)	= 3.6×10^{14}

The above parameters have the characteristics we are looking for: power and frequency suitable for the FT windows. Indeed, electric fields $\approx 4 \text{ kV/cm}$ in the waveguide can be reasonably tolerated. As we shall see in next section, a two waveguides array shows an adequate accessibility since the critical value of $n_{||}$ is close to one due to the high magnetic field. The machine should operate without trouble in the indicated range of densities. The power is consistent with the expected ohmic heating power and suitable RF components are already available on the market.

2. ACCESSIBILITY AND PENETRATION

As it is well known, only slow waves with $n_{\parallel} > n_{\parallel c} \equiv (1 - \omega^2/\omega_{ci}\omega_{ce})^{-1/2}$ can propagate toward the linear conversion point (LCP); all the waves with $n_{\parallel} < 1$ are evanescent while for $1 < n_{\parallel} < n_{\parallel c}$ the slow wave travels up to a point where it is converted in the fast mode and travels no further. To excite the proper waves a phased waveguides array has been proposed by Lallia⁽¹⁾; the theory of this array has been done by Brambilla⁽²⁾ and the theory has been verified experimentally by Hooke, Bernabei et al.^(3,4). We have used this theory to study the transmission and penetration of power with a two and four waveguides array in our case. The theory gives the spectra as a function of $n_{\parallel}$, the transmission coefficient (TC) in the waveguides having as a parameter the plasma density gradient in front of the waveguides before the cut off layer. We have also calculated the accessible fraction (AF), i.e. the product of TC with the fraction of the spectrum for $n_{\parallel} > n_{\parallel c}$ (namely: $AF = TC \cdot \int_{n_{\parallel c}}^{\infty} P_x dn_{\parallel} / \int_{-\infty}^{\infty} P_x dn_{\parallel}$ where P_x is the perpendicular component of the Poynting vector). Actually for high $n_{\parallel}$ power can be depleted through electron Landau damping (ELD) absorption at the edge. But this fact is unlikely to happen in practice because the temperature profile should be too flat (cfr. next section).

In Fig. 2 we show the TC and AF curves at 2.5 GHz with a two waveguide array both for H and D as a function of the density gradient. We can see that the AF is good especially for H. In Fig. 3 we show a spectrum for a value of the density gradient. The relevant $n_{\parallel}$ are about 2 and then ion heating should be dominant. We note that the maximum of AF is broad over density gradients $\approx 10^{12} \div 10^{13} \text{ cm}^{-4}$ which correspond to the expected values at the plasma edge in FT. Fig. 4 shows the same calculations of Fig. 2 but for 3 GHz. The TC is about the same but the AF decreases since $n_{\parallel c}$ is greater than in the previous case. Figure 5 shows the case of 4 waveguides. The AF is good but at very high density gradients. For lower density gradients the TC of the internal waveguides is very bad. This can give considerable problems in the high power tubes and lines. For this case, Fig. 6 shows the spectrum. The relevant $n_{\parallel}$ are quite high so that only if T_e is very peaked, ELD can occur at the center. Figure 7 shows the TC and AF curves for 3.5 GHz and 4 waveguides. There is not a large improvement with regard to the features shown in Fig. 5.

From what has been seen, the accessibility of the two waveguides array at 2.5 GHz is satisfactory for density gradients as expected at the edge of the FT discharge. If much higher density gradients will be found, then we consider the possibility to use a 4 waveguides array in a second phase of the experiment.

3. ABSORPTION AND HEATING

The accessible part of the RF power penetrating beyond the cut off layer, should be absorbed mainly in the core of the plasma column. In fact, absorption at the plasma periphery would in general cause a rise of the energy of the tails of the particle distribution function and a fast loss of this energy out of the discharge.

Absorption in plasma periphery (where we suppose $n = 10^{13} \text{ cm}^{-3}$ and $T_e = 100 \text{ eV}$) can take place by linear parallel electron Landau damping. As a criterium, we can assume that ELD is efficient when $\omega/k_{\parallel} v_e \leq 3.5$ (with $v_e^2 = 2T_e/m_e$) which yields

$$n_{\parallel} \sqrt{T_e} \geq 150 \quad (2)$$

where T_e is in eV. For $T_e \sim 100 \text{ eV}$, only the fraction of the power spectrum for $n_{\parallel} \geq 15$ will be absorbed, that is negligible in our case. Also decay instabilities can cause an energy depletion of the RF flux. For an incident power $\approx 400 \text{ kW}$ penetrating into the plasma, the electric field component E_{0z} in one of the re-

sonance cones i. of the order of 3 kV/cm. At the plasma periphery the net electric field of the pump wave is $|E_0| = E_{0x} \approx E_{0z}(\omega_{pe}/\omega)^{1/2} = 11 \text{ kV/cm}$ for the conditions here considered, i.e., $\omega/\omega_{LH} \sim 3$, and for a hydrogen plasma (here z is along the B_0 field and x is in the radial direction). The ratio of the electron drift in the pump field $V = cE_{0x}/B_0$ to the sound speed $c_s = (T_e/m_i)^{1/2}$ is then found using the expression

$$\frac{V}{c_s} = 102 \frac{E_{0x}}{B_0} \frac{\sqrt{\mu}}{\sqrt{T_e}} \quad (3)$$

where E_{0x} in V/cm, B_0 in Gauss, $\mu = m_i/m_H$ and T_e in eV. Then, for $T_e \sim 100 \text{ eV}$, $B_0 = 70 \text{ kG}$ (for external plasma periphery), $E_{0x} = 11 \text{ kV/cm}$, $\mu = 1$, we obtain from (3) that $V/c_s \sim 1.7$.

The most severe conditions for the onset of decay instabilities are determined by the finite extent of the pump field along and across the magnetic field⁽⁵⁾. According to Ref. (6) resonant decay instabilities into cold lower hybrid waves and ion cyclotron or acoustic waves are likely to show the lowest threshold in the plasma periphery for $\omega_0 \geq 3\omega_{LH}$. Indeed, non resonant decay into ion cyclotron quasi-modes and LH waves show here a threshold $(V/c_s)_t > 3$. Among the resonant instabilities, those into i.c. waves for $k_{\perp} \rho_i \ll 1$ and $T_e \sim T_i$ also show high thresholds in V/c_s . Lower thresholds are found for $k_{\perp} c_s \geq \omega_{ci}$, i.e. for decay into acoustic waves which on the other hand require $T_e > 3T_i$. Typically, these show a minimum threshold $(V/c_s)_t \sim 0.3$ due to the finite x-extension of the pump and $(V/c_s) \sim 2$ for the z-extension of it. Indeed the latter high value of $(V/c_s)_t$ is consistent with the predictions of Ref. (7) concerning the finite extension across the resonant cone, which give in our case power thresholds much above 500 kW. The reason is mainly because these thresholds scale as $R^{2/3}n$ and then high magnetic fields and not too high peripheral densities are favoured. Furthermore, since the condition $T_e > 3T_i$ is not likely to be satisfied at the plasma periphery, even the excitation of acoustic waves can be prevented there.

These features are reversed as the RF power propagates into the plasma core. As $\omega_0 < 2\omega_{LH}$ decay instabilities into quasi-modes can take place in the warm plasma region since their thresholds decrease significantly. How much the pump energy is depleted by these non linear mechanisms is an open question, since it depends on the non linear evolution and saturation of these instabilities. We can also investigate in the proposed experiment these problems by observing the emitted spectra and the amount of heating for different values of the central density. Nevertheless, for standard experimental situations, the density will be properly chosen so that the linear conversion point (LCP) be present as close as possible to the center of the plasma column for typical values of $n_{\parallel} (\leq 3)$. The reason is that decay instabilities, stochastic absorption processes⁽⁸⁾, ion cyclotron damping and ELD may not be sufficient for significant power depletion before the wave reaches the LCP. If this is the case, absorption and heating will take place mainly after linear conversion to hot plasma modes.

The excited spectra of the two w.g. array, show most of the power for $n_{\parallel} \leq 3$ and this make us confident to expect ion heating overcoming the electron heating (unless T_e reaches high values as shown hereafter).

In order to simulate the LH heating for a typical FT plasma discharge we have used a simplified Duchs-type transport code supplemented with extra ion and electron power input, which is described by the average over a magnetic surface of the power absorbed by linear parallel electron and perpendicular ion Landau damping evaluated at each time step⁽⁹⁾. The latter mechanism can be really appropriate if a nonlinear broadening of the order $\geq \omega_{ci}$ affects the linear resonances $\omega \approx m\omega_{ci}$ (where here $m \gg 1$, $k_{\perp} \rho_i > 1$, $k_{\parallel} \rho_i < 1$). Anyhow, it can simulate also the other ion absorption mechanisms mentioned above since the corresponding ion heating takes place in general very close to the LCP. The RF power is injected for $\Delta t = 100 \text{ msec}$ in a discharge with

$I = 500$ kA. The energy losses are mainly described by pseudoclassical heat conductivity enhanced by a factor 45 for electrons and neoclassical heat conductivity enhanced by a factor 15 for ions, so that the ion energy confinement time is ≈ 80 msec.

In Figs 8a) and b), two waves are launched with $n_{||} = 2.5$ and 3 respectively for a density $n_{max} = 1.2 \cdot 10^{14}$ cm $^{-3}$. For $n_{||} = 2.5$ ion heating is dominant and electron temperature rises only via e-i collisional interactions. The case with $n_{||} = 3$ shows that initially ions heat up but later, as the electron temperature increases, electron Landau damping becomes dominant. For other typical cases see Ref. 9.

4. DIAGNOSTICS

Given the spirit of the experiment, diagnostics play a fundamental role. Some restrictions are imposed by the window shortage of FT.

The first and absolutely necessary aim is to measure the evolution of T_i and T_e and overthermal tails as a function of space and time.

For this purpose we intend to have:

For the ions:

- a) Active neutral beam. A beam of good optical quality (divergence about 0.2 deg.) will be fired vertically and the ion temperature will be measured by a 7 channel neutral analyser looking horizontally. Space resolution will be obtained by tilting the analyser. The characteristics of the beam are $I = 2$ A, $E = 60$ keV.
- b) Doppler broadening. The broadening of impurity lines in the region 1000-2000 Å will be measured. Also the broadening of the hydrogen lines of the charge exchange products by the active beam will be observed if possible.

For the electrons:

- a) Thomson scattering
- b) Cyclotron emission
- c) Soft and hard x-Rays

The second aim is to measure the amplitude of the wave inside the plasma directly. This is a very difficult task and several methods have been suggested as Stark broadening, use of non-linear refractive index, and collective scattering. We are studying which method is most reliable and can be fitted to the characteristics of the FT and other diagnostics.

We intend to thank Drs S. Bernabei for useful discussions and M. Brambilla also for the use of his code.

REFERENCES

- 1) LALLIA, P., Proceedings 2nd Topical Conference, Lubbock 1974, Paper C3
- 2) BRAMBILLA, M., Nucl. Fusion **16**, 47 (1976)
- 3) BERNABEI, S., HEALD, M.A., HOOKE, W.M., PAOLONI, F.J., Phys. Rev. Letts **34**, 866 (1975)
- 4) BERNABEI, S., HEALD, M.A., HOOKE, W.M., MOTLEY, R.W., PAOLONI, F.J., BRAMBILLA, M., GETTY, W.D., Nucl. Fusion **17**, 929 (1977)
- 5) PESME, D., LAVAL, G., PELLAT, R., Phys. Rev. Letts **31**, 203 (1973)
- 6) PORKOLAB, M., 3rd Symposium on Plasma Heating in Toroidal Devices, Varenna, 1976, p. 118
- 7) BERGER, R.L., CHEN, L., KAW, P.K., PERKINS, F.W., Princeton PPL Report-1308
- 8) BERS, A., 3rd Symposium on Plasma Heating in Toroidal Devices, Varenna, 1976, p. 110; and STIX, T.M., idem p. 159
- 9) BORNATICI, M., SANTIHI, F., TARONI, A., 3rd Symposium on Plasma Heating in Toroidal Devices, Varenna, 1976, p. 128
- 10) BERRY, L.A., 2nd Symposium on Plasma Heating in Toroidal Devices, Varenna, 1974, p. 151

FIGURE CAPTIONS

- Fig. 1 Loss cones for different positions of r due to ion orbits which hit the limiter. Axes are labeled in keV. These results are scaled from those of Ref. (10).
- Fig. 2 Transmission coefficient and accessible fraction for a two waveguide array and a frequency of 2.5 GHz as a function of the density gradient for a H and D plasma.
- Fig. 3 Spectra of the wave energy $|E_k|^2$ in arbitrary units and the power flux P_x , normalized with $\int n_{||} P_x$, excited by the two waveguides array for 2.5 GHz.
- Fig. 4 The same as in Fig. 2 for a H plasma compared with 3 GHz.
- Fig. 5 The same as in Fig. 2 for a four waveguides array and 2.5 GHz.
- Fig. 6 The same as in Fig. 3 for a four waveguides and array 2.5 GHz.
- Fig. 7 The same as in Fig. 2 for a 4 waveguides array at 3.5 GHz for a D-plasma
- Fig. 8 Heating simulation of ions and electrons for an absorbed power of 450 kW at 2.5 GHz concentrated at two values of $n_{||} = q_0$ is the value of q on the magnetic axis

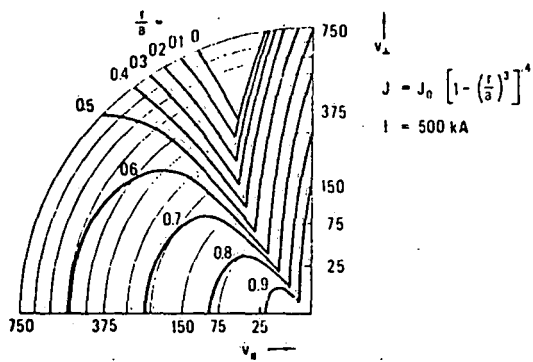


Fig. 1

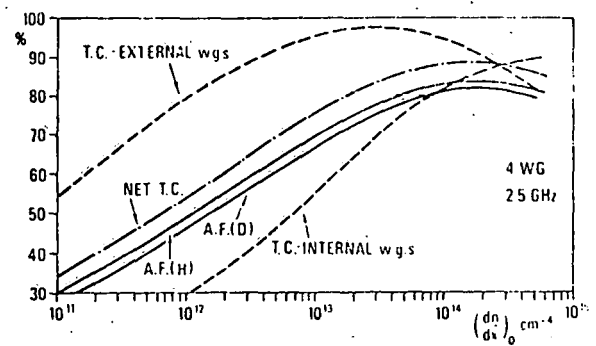


Fig. 5

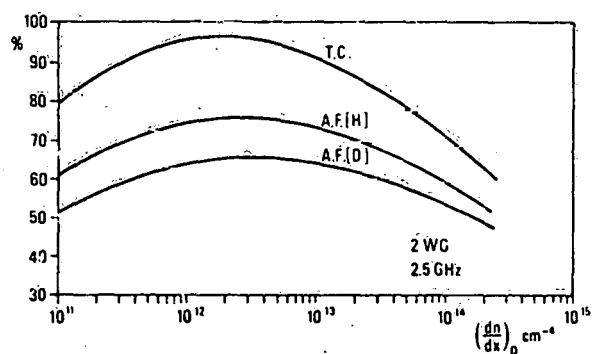


Fig. 2

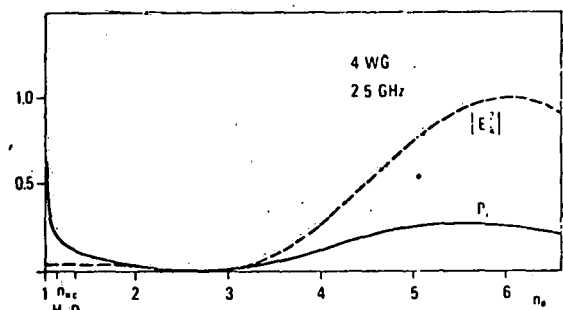


Fig. 6

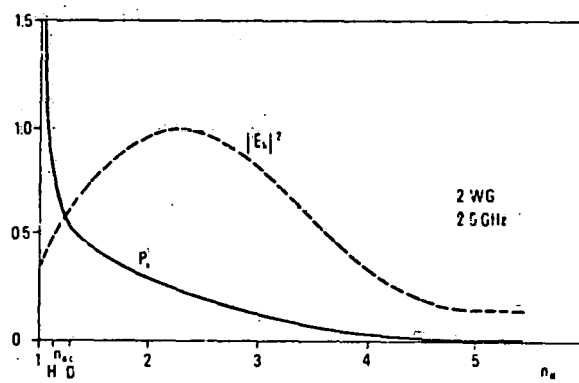


Fig. 3

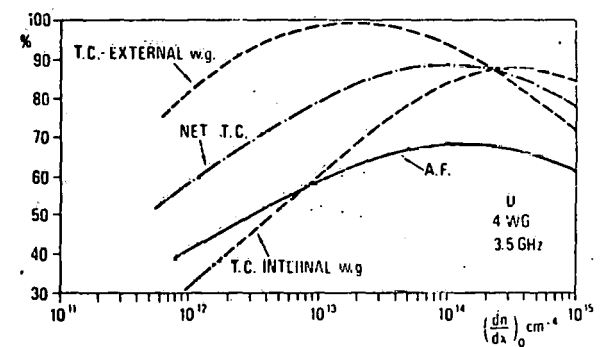


Fig. 7

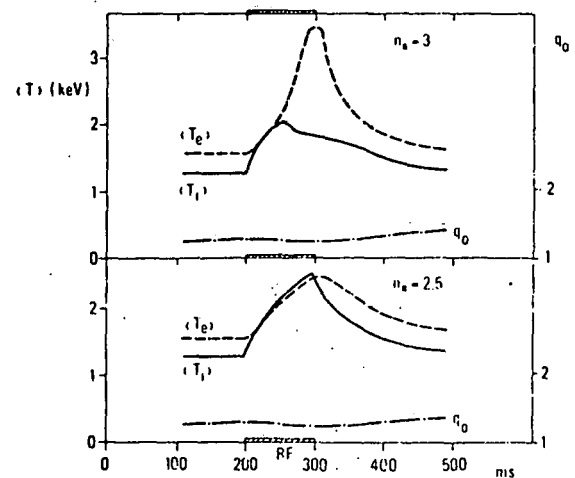
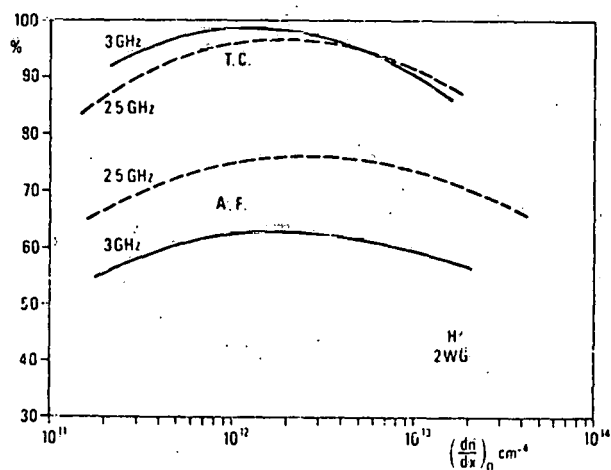


Fig. 8

Ion Heating by Ion Cyclotron Parametric Instability in a Multispecies Plasma

K. Yatsui, M. Shimada, S. Okamoto, and M. Yokoyama⁺

Department of Electrical Engineering, Faculty of Engineering Science, Osaka University, Toyonaka, Osaka 560, Japan

Abstracts Experimental studies are presented of an ion cyclotron parametric instability (ICPI) into two electrostatic ion cyclotron waves (ESICW) in a two-ion-species (He-Ne) plasma that is driven by a relative drift of ions. At high levels of beam modulation above a certain threshold, the beam pumps the decay above, and correspondingly ion heating takes place rapidly. The instability threshold is determined in a parameter space. The results are compared with the corresponding theory, and reasonable agreement is obtained.

Introduction In this paper, we present experimental and theoretical studies of ICPI into two ESICW and associated ion heating in a two-ion-species plasma by modulating an electron beam near the sum of the ion cyclotron frequencies (Ω_i).¹⁻³ If a density modulation operates under a condition such that $T_t \ll 1/f_0$ (T_t : transit time of electrons, f_0 : modulation frequency), there arises a rf excess charge, hence producing a radial electric field.⁴ For this case, we may regard the beam as an electrode (or exciter). Since the ICPI results from a relative drift of different ions, associated ion heating should be important for additional heating schemes in tokamaks that are usually composed of several types of ions.

Experiment The experiments were carried out in the Osaka University B-2 linear device.⁵ The ~~vacuum~~ chamber was 13 cm in i.d. and 145 cm long. An electron beam obtained by a magnetron injection gun of perveance $G \approx 10^{-5}$ was modulated by a 1-kw cw transmitter, where the transmitter output was connected in series to the dc power supply of the electron gun. Typical experimental parameters were as follows: beam voltage $V_b = 200$ V, beam current $I_b \approx 30$ mA, longitudinal magnetic field strength $B_0 \approx 700$ G, plasma density $n \approx 6 \times 10^8$ cm⁻³, almost equally mixed-gas pressure $p(\text{He-Ne}) \approx 8 \times 10^{-5}$ Torr, electron temperature $T_{e\parallel}$ (parallel to $\vec{B}_0$) ≈ 10 eV $\gg T_{i\perp}$ (ion temperature perpendicular to $\vec{B}_0$). The temperatures above are those in the absence of beam modulation. Such a fairly high temperatures of ions appear to be due to the presence of a crossfield current.⁵

Figure 1 shows the (a) frequency f and (b) amplitude A of oscillations excited vs. modulation voltage V_0 of the transmitter. We modulate the beam at $f_0 \approx 450$ kHz, which is ~ 1.4 times as large as the sum of the ion cyclotron frequencies (Ω_i) of He and Ne ions. If we increase V_0 above a certain critical voltage V_c , i.e., $V_0 > V_c \approx 70$ V, two types of new modes (f_1, f_2) appear

suddenly. The amplitudes of the f_1 and f_2 modes strongly grow at $V_0 > V_c$. The frequency of the f_1 mode is always higher than $f_{ci}(\text{Ne})$, and that of the f_2 mode is larger than $f_{ci}(\text{He})$. The frequency matching condition is satisfied by $f_0 = f_1 + f_2$.

Next, we have measured the instability threshold V_c vs. f_0 , and plotted it in Fig. 2 (• - experimental). Here, the value of V_c was converted into v_{12} , a relative drift velocity of two ions.^{1,2} As seen from Fig. 2, there appears resonantly a minimum threshold at $f_0 \approx 400$ kHz, which corresponds to ~ 1.3 times as large as the sum of $f_{ci}(\text{He})$ and $f_{ci}(\text{Ne})$. These characteristics will be discussed later.

Ion-temperature measurements have also been carried out by use of a gridded probe, the result of which is shown in Fig. 3. From Fig. 3, ion heating resonantly takes place at $f_0 \approx 400$ kHz, where a minimum threshold have appeared (cf. Fig. 2). For the resonance condition, ions are rapidly heated at $V_0 > V_c$, where there appeared two types of new modes. For the non-resonance condition ($f_0 = 485$ kHz), effective heating does

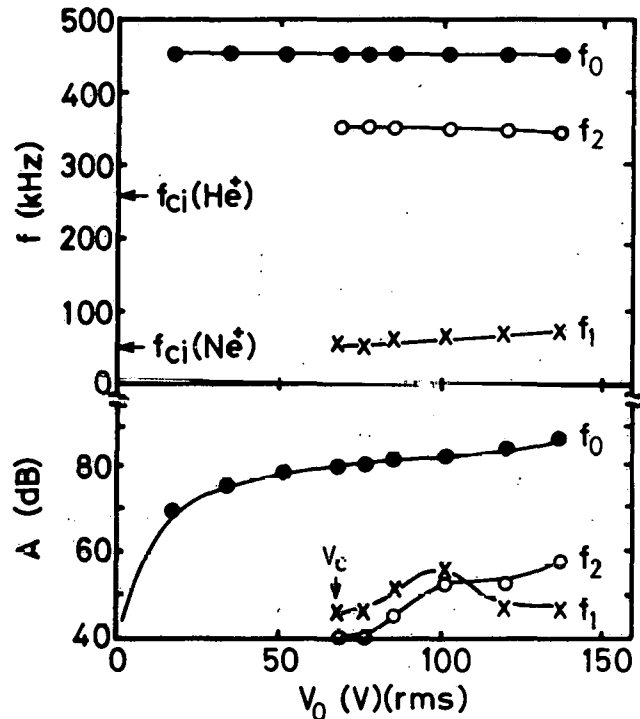


Fig. 1 Frequency f and amplitude A vs. modulation voltage V_0 of the transmitter.

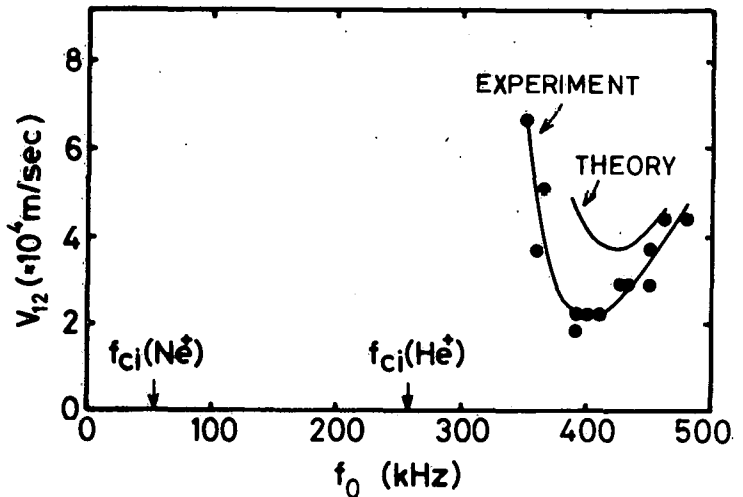


Fig. 2 Threshold relative-drift velocity of two ions v_{12} vs. modulation frequency f_0 .

not occur.

Furthermore, we have also made phase-shift measurements. From these studies, we have found that the f_1 and f_2 modes propagate almost perpendicular to $\vec{B}_0$, i.e., $k_{\parallel}/k_{\perp} = 1/(18-23)^{1-3}$. The wavenumber matching condition is satisfied by $m_1 + m_2 = m_0 = 0$, where $|m_{1,2}| = 1$. Here, m_0 , m_1 , and m_2 are the azimuthal mode numbers of the f_0 , f_1 , and f_2 modes, respectively.

From the experimental studies presented above, we consider that new modes (f_1 , f_2) are ESICW in a two-ion-species plasma.³ Next, we theoretically consider this type of ICPI.

Theory Using a hot-electron, cold-ion plasma model, we may write the dispersion relation in the form,¹⁻³

$$\begin{aligned} \omega^2 &= \Omega_i^2 + r_i (k_{\perp} c_{si})^2 & (\lambda_i < 1), \\ \omega^2 &= \Omega_i^2 \left(1 + \frac{2}{\sqrt{\pi}} \frac{r_i T}{k_{\perp} g_i}\right) & (\lambda_i > 1). \end{aligned} \quad (1)$$

where $i = 1, 2$, $\lambda_i = (k_{\perp} g_i)^2/2$, $r_i = n_i/n_e$, $c_{si}^2 = T_e/m_i$, $T = T_e/T_i$ (where we assumed $T_1 = T_2$). Using eq. (1), we have estimated the phase velocity of the f_1 and f_2 modes. The comparison between theory and experiment has been found to be reasonable.¹⁻³

Next, we obtain a theoretical estimate of the threshold drift velocity and the growth rates of ICPI in a multispecies plasma. Using a standard technique for the parametric decay with a dipole approximation as well as the quasi-neutral condition, we have derived the coupled equation,¹⁻³

$$\epsilon(\omega)\epsilon(\omega+\omega_0)\epsilon(\omega-\omega_0) - \frac{1}{4} (k\lambda_e)^{-4} \mu_{12}^2 \left\{ \epsilon(\omega+\omega_0) + \epsilon(\omega-\omega_0) \right\} = 0. \quad (2)$$

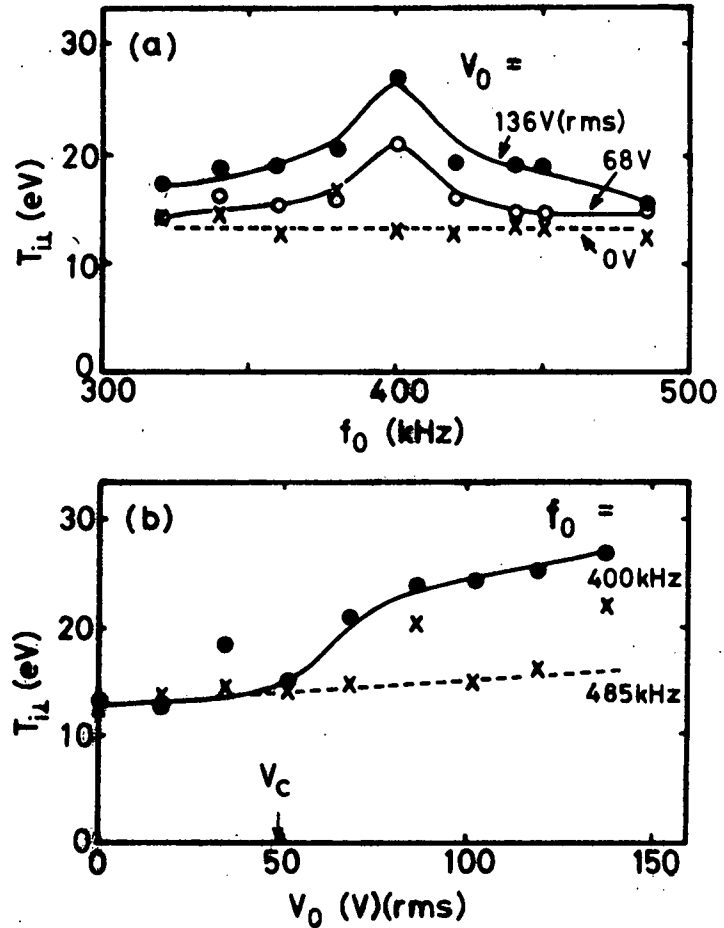


Fig. 3 (a) T_{11} vs. f_0 , (b) T_{11} vs. V_0 .

where $\mu_{12}(= \mu_1 - \mu_2)$ is the coupling constant. From eq. (2), we approximately obtain the threshold of ICPI in the form:

$$v_{12} = \frac{4\omega_0}{kT(k_{1vT1})(k_{1vT2})} \sqrt{\frac{\gamma_1 \gamma_2 \omega_1 \omega_2}{r_1 r_2}} \quad (\lambda_i < 1),$$

$$v_{12} = \frac{2\omega_0}{kT\omega_{c1}\omega_{c2}} \sqrt{\frac{\pi(k_{1\rho_1})(k_{1\rho_2})\gamma_1 \gamma_2 \omega_1 \omega_2}{r_1 r_2}} \quad (\lambda_i > 1). \quad (3)$$

Finally, the growth rates well above threshold can be written by³

$$\gamma_0 = \frac{1}{4} \mu_{12} T(k_{1vT1})(k_{1vT2}) \sqrt{\frac{r_1 r_2}{\omega_1 \omega_2}} \quad (\lambda_i < 1),$$

$$\gamma_0 = \frac{1}{2} \mu_{12} T\omega_{c1}\omega_{c2} \sqrt{\frac{r_1 r_2}{\pi(k_{1\rho_1})(k_{1\rho_2})\omega_1 \omega_2}} \quad (\lambda_i > 1). \quad (4)$$

Discussions and Concluding Remarks Using eq. (2), we have carried out the numerical calculation of the instability threshold for the experimental parameters, the result of which is plotted in Fig. 2 (solid line). As seen from Fig. 2, reasonable agreement is obtained. Thus, we may conclude that rapid heating of ions³ observed is ascribed to ICPI into two ESICW in a two-ion-species plasma. The extension of the studies above might be useful to RF heating schemes in tokamaks that are usually composed of several ions.

References

⁺Permanent address: Institute of Laser Engineering, Osaka University, Suita, Osaka 565, Japan.

¹K. Yatsui, M. Shimada, T. Imai, and M. Yokoyama, Proc. 3rd Symp. on Plasma Heating in Toroidal Devices, Varenna, Italy, (Editrice Compositori, Bologna, 1976) p. 146.

²K. Yatsui, M. Shimada, and M. Yokoyama, Phys. Lett. A61, 459(1977).

³K. Yatsui, M. Shimada, S. Okamoto, and M. Yokoyama, in preparation.

⁴K. Yatsui and T. Imai, Phys. Rev. Lett. 35, 1279(1975); Also, K. Yatsui, M. Shimada, and M. Yokoyama, J. Phys. Soc. Japan, 41, 1085 (1976).

⁵K. Yatsui et al., J. Phys. Soc. Japan, 40, 540(1976); Also, ibid. 42, 652 (1977); Also, Appl. Phys. Lett. 25, 637 (1974).

A Review of Plasma Heating by Waves
in the Ion - Cyclotron Range of Frequencies

Francis W. Perkins

Plasma Physics Laboratory, P. O. Box 451
Princeton, N. J. 08540

ABSTRACT

This brief review attempts to cover three aspects of wave heating in the Ion-Cyclotron Range of Frequencies. First, we outline the many interesting physics phenomena which occur in this frequency regime and indicate how they can be profitably utilized in tokamak and other heating experiments. Second, we enumerate the key current theoretical problems. Thirdly, we show how the ICRF program for the PLT tokamak can address most of the physics questions.

INTRODUCTION AND OVERVIEW

Heating plasma waves in the ion-cyclotron range of frequencies has a long history. Indeed, one of the most impressive achievements of the C-Stellarator research program was to reach, via ion-cyclotron resonance heating, an ion temperature of roughly 500 eV in a device whose confinement followed the Bohm diffusion law, based on the much colder electron temperature [1]. The C-Stellarator research concentrated on heating via a true ion-cyclotron absorption process. The dominant attention today, however, focusses on tokamaks where, by dint of different geometrical symmetry properties, the ion-cyclotron absorption process of the C-Stellarator is not available. Instead, both theory and experiment have shown the existence of high-Q toroidal eigenmodes [2, 3] which, evidently, suffer relatively weak damping. Far from being a disadvantage, the high-Q toroidal eigenmodes offer in fact an excellent way to couple

energy from external circuits such as antennas. Happily, the absorption of energy from the toroidal eigenmodes is dominated by plasma processes rather than the resistive wall. Since the damping processes are weak, they can be calculated with some degree of precision, although mode conversion processes often play a role here.

The large variety of damping processes is one of the real advantages of the ICRF approach to plasma heating. Starting at the low frequency end, even if $\omega < \Omega_i$ for all ion species, there is still wave absorption by what has become known as the Alfvén wave absorption process [4], and a paper at this meeting [5] shows that this absorption process, which heats electrons via mode conversion [6], can be used in conjunction with high-Q toroidal eigenmodes. When the frequency of the impressed wave is equal to the gyrofrequency of a minority ion species, with a concentration below roughly 1%, then this ion species is strongly heated and a two-component plasma can result [7]. On the other hand, if the minority species concentration exceeds 1%, then very strong damping attributable to the ion-ion hybrid resonance sets in and toroidal eigenmodes disappear [3, 8, 9]. A number of recent calculations show that the complicated mode conversion processes lead to principally electron heating [9, 10, 11, 12] although some ion heating can result. Operationally, the ion-ion hybrid resonance can provide for tokamaks what the ion-cyclotron resonance did for tokamaks: strong absorption of a wave impinging on the resonant layer. But in the tokamak ion-ion hybrid resonance case, it is electron heating which results. When there is no competition from minority absorption and the ion-ion hybrid resonance, finite gyroradius effects give rise to weak second harmonic absorption which permits high-Q toroidal eigenmodes [13]. In tokamak geometry, second harmonic heating is particularly important because it represents the only mechanism for absorbing wave energy into a majority ion species. Happily, second harmonic heating of He^3 (or T) in a proton (or D) plasma can proceed without the degeneracies that mask second harmonic deuterium heating with fundamental proton (or ion-ion hybrid) absorption. However, when the plasma β is large for a tokamak (e.g. $\beta > 0.01$),

mode conversion again comes into play at the second harmonic, and electron heating may occur [14, 15]. At the highest frequency of the ICRF regime ($\omega > \Omega_i$ for all ion species) electron transit time damping attenuates the toroidal eigenmodes [13]. Indeed, electron transit-time damping represents a promising way to utilize high-Q eigenmodes to heat electrons.

What has happened to fundamental resonance absorption by a majority ion species? A basic difference between tokamak and stellarator geometry is that, in tokamaks, the wave polarization is almost purely right-circular at the point where $\omega = \Omega_i$ leading to very weak fundamental absorption [13]. A recent calculation [9] shows that this fundamental absorption is weaker than the absorption associated with mode conversion at the "perpendicular" ion-cyclotron resonance, which is located near the plasma periphery. Mode conversion calculations show attempts to heat via the fundamental resonance should lead to principally surface electron heating.

The richness of these phenomena constitutes one of the most attractive features of plasma heating in the Ion Cyclotron Range of Frequencies. A possible heating scheme for almost any mission exists. For example, electron heating at a specified radius (to control conductivity or pressure profiles) could be brought about by high-Q Alfvén wave absorption or the ion-ion hybrid resonance. Bulk electron heating can be effected by strong fundamental heating of a minority ion species, which in turn heats electrons by collisional decay, or by transit-time electron heating. Transit-time electron heating by travelling waves could maintain a toroidal circuit, and provide much higher heating for a given current than Ohmic heating. Many of the present problems of tokamaks stem from the fact that Ohmic heating does not provide enough power.

The advent of the Tandem Mirror concept [16] may return ion-cyclotron heating physics to the same considerations that prevailed for the C-Stellarator. In both these devices, ion-cyclotron waves launched in straight uniform magnetic fields propagate toward a weaker magnetic field region where they are absorbed by the ion-cyclotron resonance method. On the other hand, the evanescent-wave/single-particle picture which provided a good model for ion heating in the

Wisconsin octupole may be entirely adequate for the tandem mirror. Research is barely starting in this area.

Outstanding Theoretical Problems

As experiments both in the United States and France have firm plans to undertake major high power heating experiments in large tokamaks, the theoretical program must deal with the issues presented by large tokamak experiments. Of these, the first is how to understand the consequences of having many modes in a tokamak. To achieve heating in the central region of tokamaks, modes of low azimuthal mode number (e. g., $m=0$, $m=1$) must be used because higher- m modes, which will be allowed for the first time, are localized toward the outside of the plasma. Since their surface electric fields are large, high- m modes will couple well to antennas with appreciable high- m components. Hence, eigenmode-tracking schemes will have to be designed to recognize the spatial structure of the modes they are tracking. This can be accomplished by a suitable array of magnetic probes.

If minority absorption continues to play a major role as seems probable for hydrogen-deuterium experiments, then Fokker-Planck studies of the thermal runaway which will be experienced by the minority species is important to predict the type of velocity distribution which will be observed, and to check on ion orbit effects. Minority heating experiments will lead to velocity distributions at least as extreme as neutral injection experiments due to the small concentration of minority ions involved.

A more complete understanding of energy absorption at the second harmonic resonance is required to assess the feasibility of an ICRF-driven high- T_i , moderate T_e Tokamak Fusion Test Reactor.

The PLT Program

Table I shows how the planned PLT program can investigate most of the physics processes described above.

TABLE I - ICRF Physics for PLT

Physics	Frequency	Plasma	B _o	Remarks
High-Q/Alfvén Electron Heating	25 MHz	protons	45 kG	$\omega/\Omega_1 = 0.4$
Fundamental Minority Heating	55 MHz	p, D	35 kG	$\eta_p < 10^{-2}$
	25 MHz			$\eta_D < 10^{-2}$
$\omega = 2\Omega_1$ Heating	55 MHz	H _e ³ , P	27 kG	no degeneracies
		pure D	35 kG	$\eta_p < 10^{-3}$
Electron Transit Time	55 MHz	D	25 kG	$2\Omega_p > \omega > \Omega_p$
Ion-Ion Hybrid	55 MHz	p-D	40 kG	no eigenmodes expected
Fundamental Majority	55 MHz	p	35 kG	surface electron heating

ACKNOWLEDGMENT

This research was supported by the Department of Energy Contract
no. EY-76-C-02-3073.

BIBLIOGRAPHY

- [1] M. A. Rothman, R. M. Sinclair, I. G. Brown, and J. C. Hosea, Ion Cyclotron Heating in the Model C Stellarator, *Phys. Fluids* 12, 2211 (1969).
- [2] J. Adam, M. Chance, H. Eubank, W. Getty, E. Hinnov, W. M. Hooke, J. Hosea, F. Jobes, F. W. Perkins, R. Sinclair, J. Sperling H. Takahashi, Wave Generation and Heating in the ST-Tokamak at the Fundamental and Harmonic Ion-Cyclotron Frequencies, 5th International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Tokyo (1974) Vol. I, p. 65.
- [3] T.F.R. Group, Magnetosonic Wave Generation and Damping in the TFR Tokamak Near the Ion-Cyclotron Frequencies, 6th Conference on Plasma Physics and Controlled Nuclear Fusion Research, Berchtesgaden, FRG, paper IAEA CM 35/G8.
- [4] J. A. Tataronis, Energy Absorption in the Continuous Spectrum of Ideal MHD, *J. Plasma Physics* 13, 87 (1975).
- [5] C. F. F. Karney and F. W. Perkins, Alfvén Wave Heating With High-Q Eigenmodes, paper D3, Third Topical Conference on Radio Frequency Plasma Heating (1978).
- [6] A. Hasegawa and Liu Chen, Kinetic Process of Plasma Heating due to Alfvén Wave Absorption, *Phys. Rev. Letters* 35, 370 (1975).
- [7] T. H. Stix, Fast-Wave Heating of a Two Component Plasma, *Nuclear Fusion* 15, 737 (1975).
- [8] H. Takahashi, C. C. Daughney, R. A. Ellis, Jr., R. J. Goldston, H. Hsuan, T. Nagashima, F. J. Paoloni, A. J. Sivo, S. Suckewer, Ion Heating in the ATC Tokamak in the Ion-Cyclotron Range of Frequencies, *Phys. Rev. Letters* 39, 31 (1977).
- [9] F. W. Perkins, Heating Tokamaks Via the Ion-Cyclotron and Ion-Ion Hybrid Resonances, *Nucl. Fusion* 17, 1197 (1977).
- [10] D. G. Swanson, Mode Conversion and Tunneling at the Two-Ion Hybrid Resonance, *Phys. Rev. Letters* 36, 316 (1976).
- [11] J. Jacquinet, B. D. McVey, and J. E. Scharer, Mode Conversion of the Fast Magnetosonic Wave in a Deuterium-Hydrogen Tokamak Plasma, *Phys. Rev. Letters* 39, 88 (1977).

- [12] J. Jacquinet, Heating of the Various Plasma Species of a Tokamak Via the Ion-Ion Hybrid Resonance, paper D4, Third Topical Conference on Radio Frequency Plasma Heating (1978).
- [13] F. W. Perkins, Hydromagnetic Wave Heating of Tokamak Plasmas, Symposium on Plasma Heating and Injection, Varenna, Italy (1972).
- [14] D. G. Swanson and Y. C. Ngan, Warm-Plasma Effects on Fast-Alfvén-Wave Cavity Resonance, Phys. Rev. Letters 35, 517 (1975).
- [15] R. R. Weynants, Ion Heating at Twice the Ion-Cyclotron Frequency in Reactor-Oriented Machines, Phys. Rev. Letters 33, 78 (1974).
- [16] T. K. Fowler and B. G. Logan, The Tandem Mirror Reactor, UCRL-78740, Lawrence Livermore Laboratory (1976)
- [17] J. D. Barter and J. C. Sprott, Ion-Cyclotron Resonance Heating in the Wisconsin Supported Toroidal Octupole, Plasma Physics 19, 945 (1977).

EXPERIMENTAL ASPECTS OF RF HEATING OF TOKAMAK PLASMAS IN THE ION CYCLOTRON
RANGE OF FREQUENCIES (ICRF) *

J. C. Hosea
Plasma Physics Laboratory
Princeton University

Abstract

Several aspects of ICRF heating of tokamak plasmas have been investigated over the past few years and promising outlook for its use to heat large hot tokamak plasmas is emerging. For fixed frequency operation, wave generating efficiencies ($P_{\text{waves}}/P_{\text{coil}}$) exceeding 90% have been easily obtained and bulk ion heating efficiencies ($\Delta W_i/\Delta W_{\text{rf}}$ for $\Delta t \ll \tau_i$) have progressively improved with improvements in ion tail confinement--e.g., from ~20% in ST to ~40% in ATC. In larger machines, (TFR, PLT, TFTR) ion tail confinement should improve to such an extent that ion heating should no longer be encumbered by tail losses. The wave coupling, wave power deposition pattern, and consequently, the induced impurity influx for tokamak plasmas strongly depend on the modes excited and their damping mechanisms. Only the desirable modes with good power deposition patterns should be selected with antenna design and/or mode tracking in the case of sufficiently high Q modes. Strict control of minority species is necessary for selecting the desired damping mechanism and decrement to optimize overall heating efficiency.

I. Introduction

Considerable progress has been made over the past few years in applying rf heating in the ion cyclotron range of frequencies (ICRF) to tokamaks. Wave excitation at efficiencies in excess of 90% with the use of suitably designed antennae has been demonstrated¹⁻⁴ and ion heating efficiencies in the range of ~20% to ~40%¹⁻⁵ have been achieved. These relatively high heating efficiencies (considering the tokamak regimes under investigation) have been obtained for fast wave excitation in the vicinity of $\Omega = \omega/\omega_{ci} = 2$ and $\Omega = 1^4$ whereas a much reduced efficiency has been observed for slow wave excitation at $\Omega = 1$.^{1,2} Also, to date, the rf energy transfer to the plasma has been accompanied by electron density enhancement and current channel constriction. In fact, a maximum limit has been placed on the rf energy transfer by the onset of the disruptive instability. Hence, the ongoing ICRF heating efforts are being directed toward optimizing fast wave heating efficiency while avoiding deleterious effects on the plasma equilibrium state.

In order to substantially improve the fast wave performance in heating tokamaks, the mechanisms which influence heating and plasma response (e.g., density buildup and stability) must be understood and controlled. Possible determining factors include:

1. unconfined energetic ions
2. surface heating through a) undesirable mode excitation and
b) toroidal and minority species effects on the damping
properties of desirable modes

3. electron heating.

The experimental approach which is being adopted to deal with these factors over the next several years incorporates:

1. large machine operation
2. mode selection and tracking
3. choice of damping mechanism (s) and the damping strength profile through $\Omega = 2/1$ resonance layer positioning and minority concentration control
4. extensive diagnosis of the ion and electron energy balance and of the total plasma response.

The elements of this approach are addressed in this paper and are thought to be sufficient to assure the successful application of ICRF heating on existing large tokamaks. This occurrence would both benefit ongoing confinement studies by extending the temperature range and set the stage for extrapolation of ICRF heating to the reactor regime.

II. Large Machine Operation

a) Ion tail confinement

ICRF heating is found to produce energetic perpendicular ion tails¹⁻⁸ (deuteron and/or proton) for both $\Omega = 1$ and 2 resonant conditions in deuterium. (Also, perpendicular proton tails have been observed in the vicinity of the antenna in the absence of resonant conditions and toroidal eigenmodes.^{6,7}) This is as expected for $\Omega = 1$ and 2 resonance deuterium heating¹, for $\Omega = 1/2$ resonance heating in the presence of a small concentration (of order of a few percent) of protons for the $\Omega = 2$ resonance condition in deuterium^{3,9-11}, and for heating via mode conversion at the ion-ion hybrid layer,¹²⁻¹⁴ especially for proton concentrations ($.01 \lesssim n_d \lesssim .3$) amenable to thermal runaway. In the ST tokamak,^{1,2,15} the ion tail could have been composed primarily of deuterons since the extended operation in deuterium should have reduced the proton concentration to a small fraction of one percent. In TM-1-VCh^{4,8}, mass discriminating neutral charge exchange detection revealed the formation of deuteron tails at higher rf powers and proton tails at even modest powers for proton concentrations of a few percent. This latter result was also observed on T-4.^{6,7} For ATC^{3,5}, the proton tail was probably dominant since ICRF operation was interspersed with hydrogen experimentation. Regardless of the specie involved, the important fact remains that a significant ion tail has been observed in all ICRF heating experiments to date.

Since much of the rf energy transferred to the ions goes to the tail ions, ICRF heating can be compared to perpendicular neutral beam injection heating in that confinement of the ion tail is essential for optimizing the bulk ion heating efficiency.^{2,15,16} Ion tail confinement was very poor in ST over the plasma cross-section. It was significantly improved in ATC in the core of the plasma, although it remained poor near the plasma surface. This improved confinement probably contributed to the observed enhancement of the ion heating efficiency. (Some improvement of the mode deposition

pattern probably contributed as well as discussed later.) Ion tail losses should virtually disappear over the entire plasma cross-section in the large, high-current machines (e.g., TFR, PLT, PDX, T-10).

b. Mode selection and tracking

The modes which may be excited in a tokamak are classified by m , v , and possibly n wave numbers for a $f_v(r) \exp(i\omega t + im\theta - ik_\phi R\phi)$ rf field description ($k_\phi R \rightarrow n$ for toroidal eigenmodes). As the machine size (minor radius) increases the number of modes m, v which may be excited increases accordingly.¹⁶ What is more important, the power deposition profile can be strongly dependent on the mode excited and care must be taken to select the mode (s) which assures optimal heating over the plasma volume.^{16,17}

Such selection was not possible in ST since only the $m = -1$, $v = 1$ mode could be excited for heating conditions.^{1,2} For resonant $\Omega = 2$ heating, a two-step density profile analysis¹⁶ reveals that $|\nabla E_\ell|^2$ (E_ℓ is the left-hand field component), the dominant ion energy source term, was very large in the external low density region relative to the value over the bulk of the plasma. A refined many-step density profile analysis¹⁷ places the peak of $|\nabla E_\ell|^2$ in the vicinity of the limiter for ST. In addition, the Poynting flux in the surface of the plasma is found to be large, ^{2,16,17} indicative of large fields which could have resulted in surface heating through other damping mechanisms. Thus, the $m = -1$, $v = 1$ mode tended to heat the plasma surface in ST where ion confinement was minimal.

These analysis^{16,17} further reveal that although the Poynting flux (fields) becomes well confined to the bulk plasma in ATC, TFR, and PLT, the $m = -1$, $v = 1$ mode remains detrimental with regard to $|\nabla E_\ell|^2$ resonant surface heating at $\Omega = 2$. Also, they show that high $m, v = 1$ modes must also be avoided if surface heating is to be minimized. The $m = -1$, $v = 1$ mode was not excluded in ATC so that it is probable that the heating efficiency was reduced somewhat due to surface heating. Furthermore, the persistence of the density enhancement and the rf energy transfer limit might have resulted at least in part from this surface heating. In TFR, the $m = -1$, $v = 1$ mode must also be selectively avoided to minimize $\Omega = 2$ resonance surface heating. For PLT, this mode is so far from onset that coupling to it should be unimportant, but care must be taken to discriminate against high m modes.

Mode selection can be accomplished through antenna design, plasma parameter control, and mode locking.^{16,17} Antenna selection is provided by limiting the excitation spectrum (m, k_ϕ) with appropriate arrays. For example, in TFR two half-turn coils encompassing the plasma can be phased to favor $m = 0$ and/or $m = \pm 1$ excitation.¹⁸ For the preliminary testing phase on PLT, a half-turn coil, located on the larger major radius-side of the plasma (Fig. 1) is being used to favor low $|m|$ excitation.

Plasma parameter control refers to programming the discharge conditions such as density, toroidal magnetic field, and minority concentration (see next section) to favor excitation of specific modes. In the case of toroidal eigenmodes, the density and/or magnetic field can be programmed during the discharge to approximately track relatively low Q modes over reasonable time spans. Such a case is demonstrated in Fig. 2 for excitation in PLT. Here the density is maintained approximately constant over a few tenths of a second and it is possible to stay on a mode for ~50 msec.

Obviously mode tracking through feedback control of the rf source frequency will be required to stay on modes (especially with high Q) for long time spans; rf heating pulse durations of up to 300 msec are planned for PLT. Development of feedback systems is being pursued and some success with a constant phase feedback loop has been obtained on TFR where mode locking for relatively high Q modes has been achieved for ~15 msec.¹⁸ For long rf pulse duration, a combination of plasma parameter control, frequency programming, and one of the alternative forms of feedback will probably be required for successful mode selection over the variety of discharge conditions which should be explored.

III. Damping Mechanisms

In addition to the dependence on the mode selected, the power deposition over the plasma volume depends strongly on the nature of the damping mechanism(s) involved. Many damping mechanisms are possible over the ion cyclotron range of frequencies and these can compete in a complicated fashion over the cross-section of a toroidal plasma to heat ions and/or electrons. Hence, it is important to quantitatively understand these mechanisms individually and, when applicable, together if control is to be exercised over the damping profile and, when toroidal eigenmodes are permitted, the mode Q . In this section some of the wave damping aspects important for near term deuterium heating experiments are considered briefly.

a) $\Omega_D = 2$

For a pure deuterium discharge, fast wave $\Omega = 2$ resonance heating occurs along an approximately vertical chord through the plasma cross-section [$2\Omega(R) = \omega$]. Since heating of the more energetic ions is favored^{1,9}, this is a very attractive regime for heating the hotter core of the plasma. However, since the damping decrements can be relatively small^{9,10}, high Q toroidal eigenmodes can be obtained (depending on the $\beta_i = 8\pi n_i k T_i / B^2$ of the plasma¹²). Sophisticated mode tracking capability could be required to stay on the modes and, as well, the large stored fields could result in some surface heating by as yet undetermined processes.

When the concentration of protons $n_H/n_D \geq \beta_i$, $\Omega_H = 1$ resonance absorption becomes dominant over $\Omega_D = 2$ resonance absorption¹². This minority proton resonance heating⁹⁻¹³ will reduce the mode Q , and at only one or two percent proton concentration could completely damp toroidal eigenmodes. As long as the damping length is $\geq d$, the plasma diameter, the coupling will be set by the usual antenna radiation resistance and efficient wave excitation can be maintained¹⁶ (as for slow waves at $\Omega = 1^{1,2,4}$). However, the surface fields can be large in this case and again could result in surface heating. (Antenna design and plasma parameter control can still be used to minimize surface fields.¹⁶) Also, as discussed earlier, proton runaway can occur and this would set a limit on the maximum rate of energy transfer if the protons are to be confined.

When protons are present, the ion-ion hybrid resonance is introduced into the plasma along a resonance layer which approximately parallels (on the smaller major radius side) the $\Omega_D = 2$ resonance layer.^{9,12,13,14,20} The presence of this layer can enhance both the proton and deuteron

resonance damping³, and as well, introduces damping through mode conversion to electrostatic waves which can heat protons, deuterons, and electrons.^{12,14} Mode conversion damping dominates when $n_H/n_d \gtrsim nT_i^{1/2}(\text{keV})/200$

(n is $k_\phi R$) and appears to account for the order of magnitude enhancement of damping observed with $\Omega_D = 2$ at the center of the plasma relative to that for $\Omega_D = 2$ at the edge in ATC³, TM-1-VCh^{4,8}, TFR¹⁸, and PLT²¹ when the proton concentration exceeds a few percent. (Note however, that the damping was not so markedly enhanced in T-4⁶.) The comments made above with regard to surface heating and proton runaway for minority proton resonance heating then apply also to mode conversion heating. These two processes are in competition and must be treated simultaneously to determine the proton concentration above which the mode conversion damping dominates.

It is clear that control of the minority proton concentration is extremely important for optimizing ICRF heating at $\Omega_D = 2$. Extended operation in deuterium will be required to reach the low level which permits resonant $\Omega = 2$ heating. However, it may prove desirable to operate in the mode conversion regime by selecting a given proton concentration which assures that damping occurs in the core of the plasma, prevents proton runaway, and permits reasonably low Q eigenmode excitation.¹⁴ For a given concentration the mode Q can be further controlled by shifting the $\Omega_D = 2$ resonance layer an acceptable distance from the center of the plasma (see Fig. 2).

A final consideration for $\Omega_D = 2$ heating is the influence of mode conversion to ion Bernstein modes at the elevated temperatures which should be obtained ($T_i \gtrsim 4\text{keV}$) at high power levels ($\sim 5\text{ MW}$ in PLT).²²⁻²⁴ Waves propagating from the larger major radius side of the plasma traverse the $\Omega_D = 2$ resonance but are then reflected from the fast wave cutoff and/or absorbed in the mode conversion region. In principal, such propagation may be used to heat the core of the plasma, although the role of wave propagation around the surface of the plasma into the heavily damped mode conversion region is difficult to predict.

b) $\Omega_D = 1$

Wave damping in deuterium at $\Omega = 1$ is predicted to be very weak due to the small left-hand component of the electric field at high densities.⁹ Also, mode conversion at the slow wave resonance at the border of the plasma can produce a damping decrement comparable to or greater than that along the $\Omega = 1$ resonance.¹² This slow wave mode conversion produces surface heating of the electrons.¹⁹ However, at sufficiently elevated temperatures and with the regime selected to minimize the mode conversion effect (large $k_\phi R$, placement of the mode conversion region in the low density cold region of the plasma), fundamental heating may still prove to be feasible and should be explored. Note that high Q eigenmodes are obtained in PLT with the resonant $\Omega = 1$ layer located inside the major radius of the plasma,²¹ suggesting that mode conversion is weak there.

Considerable progress has been made in developing the theory for the individual damping processes and in predicting their relative importance for certain conditions. However, a comprehensive, quantitative theory incorporating all the known processes simultaneously over the entire cross-section of the toroidal plasma, which is ultimately required for predicting

the power deposition profile, is still forthcoming. Experimental investigation of the total energy balance for ions and electrons over the cross-section of the plasma at large rf power levels will be required to document the role of the various mechanisms in heating the plasma and to optimize the deposition profile.

IV. Diagnosis of ICRF Tokamak Heating

Both ion and electron heating, impurity effects, and instability behavior must be carefully diagnosed over the cross-section to determine the performance of ICRF heating as in the case for ohmic heating, neutral beam injection heating, and other auxiliary heating schemes. Arriving at such a global energy balance for the ions and electrons is the primary goal of much of tokamak research and, consequently, a variety of diagnostics have been developed and are now available for this purpose. For example, $T_i(r,t)$ ²⁵ and $T_e(r,t)$ ^{26,27} can be measured in several ways:

- 1) $T_i(r,t)$ - active charge exchange measurements with mass discrimination
 - fast scanning Doppler broadening measurements
 - neutron flux analysis
- 2) $T_e(r,t)$ - electron cyclotron emission
 - Thomson scattering
 - x-ray PHA.

For ions, mass discrimination charge exchange measurements are particularly valuable for investigating minority heating.^{4,6-8} However, the neutron flux analysis will be complicated by the presence of deuteron tails. For electrons, the electron cyclotron measurements permit detailed investigation of the time evolution of T_e over the cross-section of the plasma as has already been demonstrated for neutral beam injection heating,²⁸ and as well, permit millimeter resolution along R of the spatial perturbation of T_e which might result for mode conversion heating.

Electron energy loss through both light and heavy impurity radiation is now being measured with adequate accuracy²⁹ to permit (along with measurements of T_e and n_e) evaluation of the power input to the electrons from ICRF heating. Furthermore, the role of impurities with regard to surface heating can be monitored.

Instability effects on energy confinement are difficult to determine unambiguously but instability changes induced by ICRF heating can be monitored over the discharge.^{27,30} Hopefully, these can be minimized to give an adequate measurement of the energy balance and possibly controlled with selection of the ICRF heating regime.

V. Discussion

With proper wave excitation, selection of the damping profile, and extensive diagnosis of the ion and electron energy balance and the plasma response, a definitive test of ICRF heating of existing large tokamaks should be forthcoming in the next few years. In this paper, considerations

for deuterium discharges, with minority concentrations of protons, have been outlined. Many of these considerations apply as well to the D-T regime envisioned for reactors except that the damping characteristics can change markedly.^{10,12,31,32} Even this regime can and will be modeled in the existing devices using a He^3 -p plasma.¹²

The emphasis of this paper has been on demonstrating efficient ICRF heating in the near term. Once this goal has been achieved the stage will be set for designing systems for reactor devices.

It is a pleasure to acknowledge the participation of the U.S., French, and Soviet ICRF enthusiasts in the specification of the ICRF critical areas and the steps to be taken in the ICRF research program.

* Supported by U.S. DOE Contract EY-76-C-02-3073.

References

1. J. Adam, M. Chance, H. Eubank, W. Getty, E. Hinnov, W. Hooke, J. Hosea, F. Jobes, F. Perkins, R. Sinclair, J. Sperling, and H. Takahashi, Proceedings 5th International Conference on Plasma Physics and Controlled Nuclear Fusion Research (IAEA, Vienna, 1975) 65.
2. J. Hosea, Proceedings 2nd Symposium on Plasma Heating in Toroidal Devices (Varennia, 1974) 61; Princeton University Report MATT - 1129 (1975).
3. H. Takahashi, Third International Congress on Waves and Instabilities in Plasma (Ecole Polytechniques, Palaiseau, June, 1977); Princeton University Report PPPL-1374 (1977).
4. V. L. Vdovin, N. V. Shapotkovskii, and V. D. Rusanov, Proceedings 3rd International Meeting on Theoretical and Experimental Aspects of Toroidal Plasmas (C.E.A., Grenoble, 1976) Vol. 2, p. 349.
5. H. Takahashi, C. C. Daughney, R. A. Ellis, R. J. Goldston, H. Hsuan, T. Nagashima, F. J. Paoloni, A. J. Sivo, and S. Suckewer, Phys. Rev. Letters 39 (1977) 31.
6. V. V. Buzankin, V. A. Vershkov, N. V. Ivanov, I. A. Kavan, V. A. Krupin, I. A. Popov, I. B. Semenov, and Yu. A. Sokolov, Proceedings of the 6th International Conference on Plasma Physics and Controlled Nuclear Fusion Research (IAEA, Vienna, 1977) 61.
7. N. V. Ivanov, I. A. Kovan, and Yu. A. Sokolov, JETP Lett. 24 (1976) 316.
8. V. L. Vdovin, V. D. Rusanov, and N. V. Shapotkovskii, JETP Lett. 24 (1976) 374.
9. J. Adam and A. Samain, Association Euratom - C.E.A. Report EUR-CEA-FC-579 (Fontenay-aux-Roses, 1971); J. Adam, Report EUR-CEA-FC-711 (Fontenay-aux-Roses, 1973).
10. T. H. Stix, Nuclear Fusion 15 (1975) 737.

11. R. Klima, A. V. Longinov, and K. N. Stepanov, Sov. Phys. Tech. Phys. 21 (1976) 409.
12. F. W. Perkins, Nuclear Fusion 17 (1977) 1197.
13. R. Klima, A. V. Longinov, and K. N. Stepanov, Nuclear Fusion 15 (1975) 1157.
14. J. Jacquinot, this conference, paper D4.
15. H. Takahashi, Bull. Amer. Phys. Soc. 19 (1974) 959; Princeton University Report MATT-1140 (1975).
16. J. Hosea, Proceedings 3rd Symposium and Workshop on Plasma Heating in Toroidal Devices (Varenna, 1976); Princeton University Report PPPL-1309 (1976).
17. J. Adam and J. Jacquinot, Association Euratom - C.E.A. Report EUR-CEA-FC-886 (1977).
18. Equipe TFR, Proceedings 6th International Conference on Plasma Physics and Controlled Nuclear Fusion (IAEA, Vienna, 1977) 39; Proceedings 3rd International Meeting on Theoretical and Experimental Aspects of Heating of Toroidal Plasmas (C.E.A., Grenoble, 1976) Vol. I, p. 87; Proceedings 3rd Symposium and Workshop on Plasma Heating in Toroidal Devices (Varenna, 1976).
19. F. W. Perkins, M. Chance, and J. M. Kindel, Proceedings 3rd International Symposium on Toroidal Plasma Confinement (Garching, 1973) paper B8.
20. J. Jacquinot, B. D. McVey, and J. E. Scharer, Phys. Rev. Letters 39 (1977) 88.
21. P. L. Colestock, J. C. Hosea, F. J. Paoloni, H. R. Thompson, and H. Takahashi, this conference, paper C7.
22. R. R. Weynants, Phys. Rev. Letters 33 (1974) 78.
23. D. G. Swanson and Y. C. Ngan, Phys. Rev. Letters 35 (1975) 517; Y. C. Ngan and D. G. Swanson, Phys. Fluids 20 (1977) 1920.
24. B. D. McVey and J. E. Scharer, this conference, paper D6.
25. M. Brusati, S. L. Davies, J. C. Hosea, J. D. Strachan, and S. Suckewer, submitted for publication; Princeton University Report PPPL-1403 (1977).
26. D. J. Grove, et al., Proceedings 6th International Conference on Plasma Physics and Controlled Nuclear Fusion (IAEA, Vienna, 1977).
27. J. C. Hosea, V. Arunasalam, and R. Cano, Phys. Rev. Letters 39 (1977) 408.
28. V. Arunasalam, P. C. Efthimion, and J. Hosea, to be presented at the 2nd International Conference on High Temperature Plasma Diagnostics (Santa Fe, March, 1978); D. A. Boyd, *ibid.*

29. S. von Goeler, V. Arunasalam, et. al., Eighth Conference on Controlled Fusion and Plasma Physics (Prague, Czechoslovakia, September, 1977).
30. N. R. Sauthoff, S. von Goeler, and W. Stodiek, submitted for publication; Princeton University Report PPPL-1379 (1978).
31. D. G. Swanson, Phys. Rev. Letters 36 (1976) 316.
32. J. E. Scharer, B. D. McVey, and T. K. Mau, Proceedings 3rd International Meeting on Theoretical and Experimental Aspects of Heating of Toroidal Plasmas (C.E.A., Grenoble, 1976) Vol. 1, p. 79.

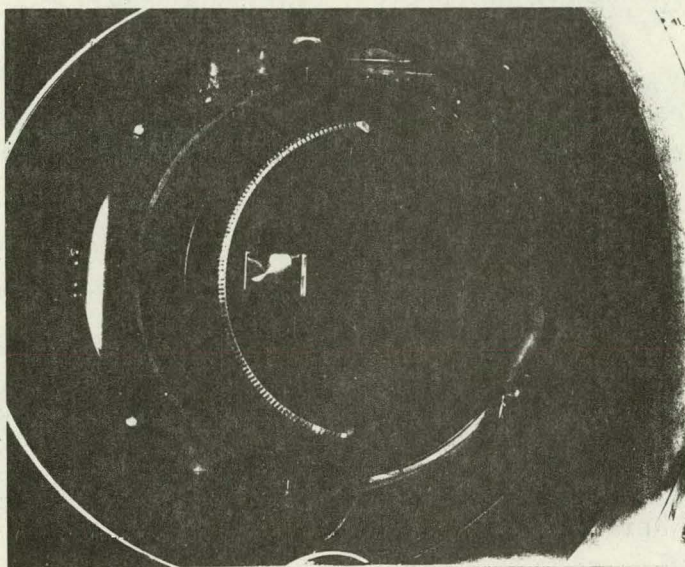


Fig. 1. PLT ICRF
Test Coil

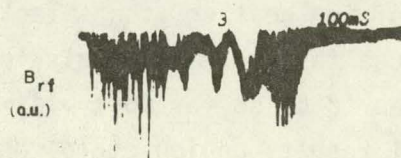
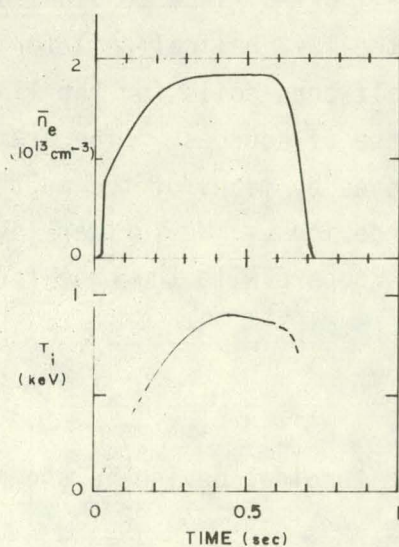


Fig. 2. Modes Excited
for $\Omega_D = 2$ Resonance
at $r/a \approx -0.25$



ION CYCLOTRON HEATING BY CIRCULARLY POLARIZED RF FIELDS

Y.Yasaka, S.Komori, and R.Itatani

Dept. of Electronics, Kyoto University, Kyoto 606 Japan

Introduction

Ion cyclotron heating has now been recognized as one of the most promising methods for heating a fusion plasma. Many experiments were made using slow waves or fast waves in toroidal or linear machines, in which several types of rf coils were used to launch waves. In the Uragan Stellarator [1], azimuthally asymmetric mode (rf fields which vary as $\exp[j(m\theta - k_{\parallel}z + \omega t)]$ with $m = \pm 1$) generated by a Kharkov coil propagated freely along the torus and brought an uniform heating of the entire plasma. It should be noted that the transverse fields of waves are maximum on axis for the $m = \pm 1$ modes. Recently, a detailed investigation concerning the mode effect on coil loading was performed by Knox et al.[2] using a helical antenna with four-phase rf source. They indicated that a proper mode selection of excited waves may increase the efficiency of wave generation. In the viewpoint of ion heating, the selective excitation of the $m = \pm 1$ mode is attractive because this mode is purely left-hand circularly polarized on axis, and continues to be so for most plasma radii.

Here, we describe the experimental results on ion cyclotron wave generation by circularly polarized rf fields ($m = \pm 1$ or -1) in a toroidal and a linear plasma. The rf coils we employ are the $l = 2$ helical coil for the toroidal device and the composition of three half-turn coils for the linear device. These coils are energized by multi-phase rf sources. The preferential coupling of rf power to the $m = \pm 1$ slow waves by means of the multi-phase rf systems is to be revealed in both the devices. We further investigate the associated ion heating in detail, and compare with the case for linearly polarized rf fields ($m = \pm 1$).

Experimental results

1. Toroidal device

The experimental arrangement of the toroidal device is shown in Fig.1.

The rf coil is the $l = 2$ helical coil of one-field-period ($\lambda_{||} = 30$ cm), which is wound around the glass section of the torus with an electrostatic shield inside. Feeding the pulsed four-phase current ($f = 3.5$ MHz) to the coil, we can establish the rotating rf fields. A hydrogen plasma of 4 cm in minor radius and 25 cm in major radius is produced by Joule heating in a toroidal field up to 6 kG. The plasma density as measured by a microwave interferometer ranged from 1×10^{12} to 1×10^{13} cm $^{-3}$. Figure 2(a) shows the increment of the loading resistance versus the toroidal field for the cases of the $m = +1$ and $m = -1$ mode. For the $m = +1$ (field rotation in the left-hand sense with respect to the toroidal magnetic field), the resonant peak of the loading was observed at $\omega / \omega_{ci} \sim 0.8$, while for the $m = -1$, no resonant coupling was observed. The wave field measured with a small loop is plotted in Fig. 2(b) versus the toroidal field. The resonant excitation of the $m = +1$ slow wave is clearly seen within the predicted parameter range for the $m = +1$ operation. The field of the $m = -1$ slow wave is little, if any, and shows no dependence on the toroidal field. Figure 3 gives the increase in ion energy density as a function of the net rf power into the plasma and rf coil systems for three cases of operating modes. The ion temperature was measured with an electrostatic energy analyzer directed perpendicularly. The ion temperature rise from 14 eV to 42 eV was attained for a power of ~ 45 kW in the $m = +1$ operation. Figure 3 shows that the perpendicular ion energy density increases linearly with the incident rf power and, furthermore, the overall efficiency of ion heating for the $m = +1$ mode is better than for the $m = \pm 1$ mode. When the rf coil is operated in $m = \pm 1$ mode, the plasma density decreases notably during heating.

2. Linear device

A potassium plasma column 3 cm in diameter and 1.3 m long is formed in the single-ended Q machine 'HIEI' shown in Fig. 4. Typical plasma parameters are as follows; the density range $5 \times 10^9 < n_0 < 1 \times 10^{11}$ cm $^{-3}$, initial temperatures $T_i, T_e \sim 0.25$ eV, and the axial uniform magnetic field $B \leq 10$ kG. The rf coil which consists of three half-turn coils as shown in Fig. 4, is fed by a three=

phase rf source ($f = 0.35$ MHz). By operating the rf coil in the $m = +1$ mode, efficient coupling of the rf power to the $m = +1$ slow wave was confirmed through the measurements of the loading and the wave propagation, and the ion heating up to 46 eV (200 times that of the initial plasma) was achieved. The heating efficiency is given in Fig.5 versus the initial plasma density for the $m = +1$ and $m = \pm 1$ operation. Typical values of the efficiency are $\sim 10\%$ for the $m = +1$, $\sim 5\%$ for the $m = \pm 1$, and few % for the $m = -1$ mode. Rather small values of the efficiency come from lack of confinement. Thus, the pure $m = +1$ operation of the rf coil indeed improves the efficiency of ion cyclotron heating over the $m = \pm 1$ operation by almost a factor of 2. Figure 6 shows the radial density profiles for the $m = \pm 1$ and $m = +1$ mode as a function of time. The radial enhanced diffusion of the plasma is notable for the $m = \pm 1$ case. Especially, near $r = 2$ cm, the density reaches more than 6 times that of initial one at the peak of the rf current. For the $m = +1$, no enhanced radial diffusion is seen, hence, the density decay is mainly due to axial loss resulting from lack of confinement.

Summary

It is shown experimentally that the pure $m = +1$ operation of the rf coil improves the efficiency of ion cyclotron heating over the $m = \pm 1$ operation in both the toroidal and linear plasmas. The enhanced radial loss of the plasma is observed for the case of the $m = \pm 1$. The $m = \pm 1$ operation of the rf coil launches the $m = +1$ and $m = -1$ fields travelling oppositely in azimuthal direction. In such a case, radial convection of ions is created due to nonlinear interaction of two travelling waves. [3] This convection may enhance the radial particle loss and be a cause of inferior efficiency for the $m = \pm 1$ operation.

1. A.G.Dicky et al. 3rd Int. Symposium on Toroidal Plasma Confinement (Garching, 1973) E17
2. S.O.Knox et al. J. App. Phys. 46 (1975) 2516
3. F.F.Chen et al. Phys. Fluids 13 (1970) 687

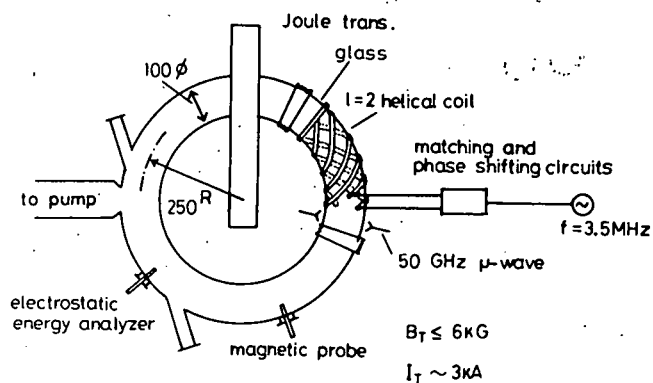


Fig.1

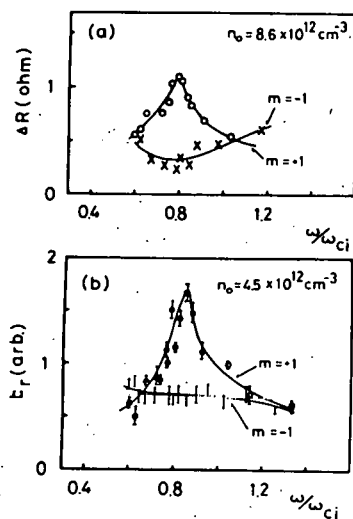


Fig.2

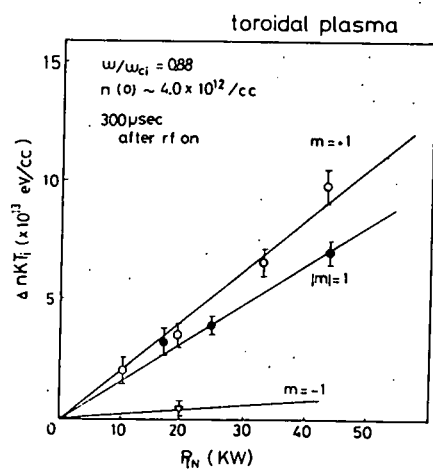


Fig.3

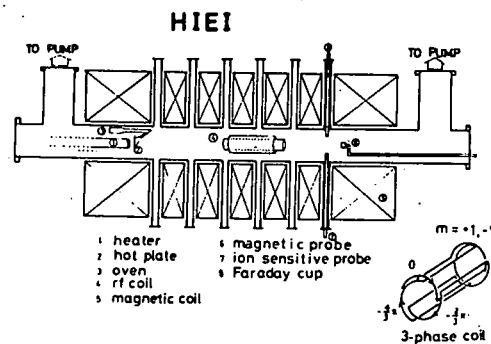


Fig.4

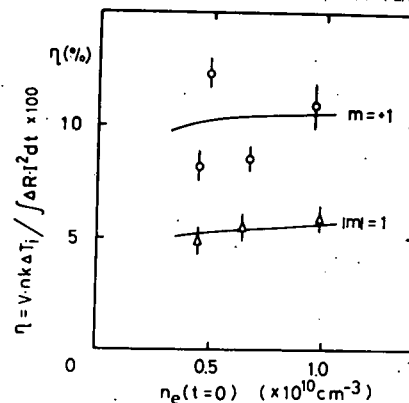


Fig.5

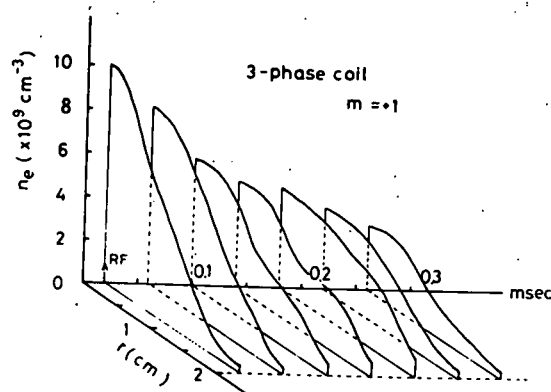
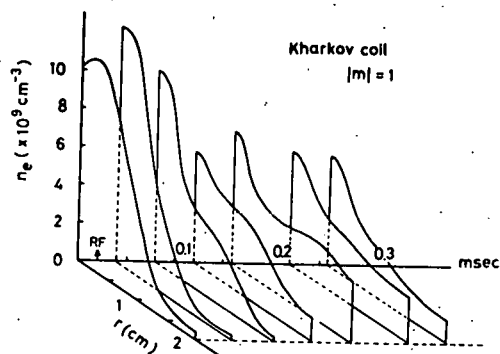


Fig.6

PRELIMINARY RF EXPERIMENTS ON THE ERASMUS TOKAMAK.

V.P. Bhatnagar, G. Bosia⁺, M. Bures⁺⁺, J. Jacquinot^{*}, A.M. Messiaen^{**}, G. Telesca,
P.E. Vandenplas, R.R. Weynants^x.

Laboratoire de Physique des Plasmas - Laboratorium voor Plasmafysica
Association "Euratom-Etat belge" - Associatie "Euratom-Belgische Staat"
Ecole Royale Militaire - 1040 Brussels - Belgium - Koninklijke Militaire School

I. Tracking scheme on ERASMUS.

The ERASMUS tokamak is a low aspect ratio (~ 2.5), low toroidal field (~ 4 kG) device whose main characteristics have been given in refs. [1] and [2]. The study of the coupling of rf energy to ERASMUS by magnetosonic resonances (see [3] and [4] and the references cited therein) has now started. The resonances correspond to toroidal cavity eigenmodes characterized by the radial r , azimuthal n and toroidal k eigenvalues. The exciting structure of ERASMUS consists of a single loop of inductance L_0 shown on fig. 1. At a given magnetosonic resonance of frequency ω_r (given r , n , k and plasma parameters), the coil impedance becomes $Z_{\text{coil}} = -i\omega L_0 \{1 - K^2/(y + i/Q)\}$ [4,6] where $y = 2(\omega - \omega_r)/\omega_r$ is the detuning, Q the quality factor of the resonance and K the coupling coefficient depending on geometrical factors, L_0 , r , n and k . The resonance frequency ω_r shifts during the shot as a result of the time dependence of the plasma density N . With constant rf coil voltage and fixed frequency ω the energy absorption is $E = C_1 K^2/(4L_0)$ as the mode considered is being swept through (C_1 depends on dN/dt). This value is Q -independent due to lack of resonance tracking. A tentative solution for this tracking is proposed here: the loop is used as the tank coil of a power oscillator (fig. 1). Such a system oscillates at a frequency such that $\text{Im}(Z_{\text{coil}}) = -(\omega C_T)^{-1}$, $C_T = C_1 C_2/(C_1 + C_2)$ is the capacitance of the tank circuit. As ω_r changes with plasma parameters the frequency ω of the oscillator will track ω_r from $(L_0 C_T)^{-1/2}$ in a relative frequency domain $\leq K^2 Q/4$ [4],[6]. Due to this tracking the energy absorption $E \approx C_1 (KQ)^2/(48\pi L_0)$. This indicates that KQ must be $\gtrsim 6$ to improve the amount of energy absorbed and this explains why a full loop is used to maximize K (for a side loop with aperture angle 2α , $K_\alpha = K_\pi (\alpha/\pi)$). For ERASMUS $K \approx 3 - 6 \times 10^{-2}$ and $L_0 \approx 1.4 \mu\text{H}$.

II. Low power experiments on ERASMUS.

The setup is shown on fig. 2. Receiving probes are placed along the wall at locations A, B, C and display of B_{tor} (called EM signal) or E_r (called ES signal) is possible. The launching antenna can be fed by a tuned generator (with quality factor Q_T) or a current generator. We use a current generator to measure Q on the probe response curve in order to avoid the loading by the Q_T of the tuned generator ($1/Q_{\text{probe}} = 1/Q + K^2 Q_T$).

+ On leave from Lab. di Cosmogeofisica del CNR (Torino, Italy). ++ Euratom Visiting Scientist from Ass. Euratom-Swedish Board ESD (Stockholm, Sweden).

* Visiting Scientist from Ass. Euratom-CEA, Fontenay-aux-Roses (France).

** Maître de Recherches au FNRS. x Bevoegdverklaard Navorser bij het NFWO.

(i) Resonance spectrum. Fig. 3 shows typical response curves in D (the time average central density $\bar{N}_0$ decreases continuously during the second part of the shot). Fig. 4 displays the resonances on a $\bar{N}_0 - \omega$ diagram. Lines are drawn through the representative points which can be followed from shot to shot ; for large ω or $\bar{N}_0$ the density of resonances becomes high and the continuity is difficult to establish due to the spread in position. The ratios between the positions of the first three resonances correspond to the theoretical ones (plasma in cylindrical coordinates [4] corrected for nonuniformity) for $n = 0$, $r = 1$, $k = 1, 2, 3$ but they occur at frequencies ~ 1.3 lower than the theoretical ones. (ii) Q value. The resonance curve shown on fig. 3 is the output of a receiver through a filter of 5 kHz bandwidth. If the output is displayed with a bandwidth of ~ 30 kHz a fine structure consisting of "spikes" appears around each resonance. See fig. 5 where a filtered and unfiltered display are compared. This splitting-up of each resonances in many spikes is consistent with the level of density fluctuation seen on the 4 mm interferometer. Fig. 6 shows qualitatively the effect of a regular fluctuation with frequency $\omega_f/2\pi$ and amplitude N_f . $\bar{N}$ is the mean density which decreases with time. The 3dB density width $\Delta N_{1/2}$ of the resonance considered (with maximum at density N_r) is indicated on the upper curve. It is linked to the Q of the resonance by the relation $\Delta N_{1/2} \approx 2N_r/Q$. A spike appears each time that the density becomes close to N_r (this is also observed on TFR but with a lower level of fluctuations [7]). Each mode is split in spikes whose envelope is approximately a trapeze for regular fluctuations. One distinguishes the characteristic times a) $\Delta t_1 = 2N_f(d\bar{N}/dt)^{-1}$ b) $\Delta t_2 = \Delta N_{1/2} (d\bar{N}/dt)^{-1} = 2\bar{N}(Qd\bar{N}/dt)^{-1}$ c) $\Delta t_3 = \Delta N_{1/2} (\omega_f N_f)^{-1} = 2\bar{N}(\omega_f N_f Q)^{-1}$. From data at $\omega > 2\omega_{ci}$ in D we can presently derive the following rough estimates : $0.1 < \Delta t_1 < 1$ ms, $0.1 < \Delta t_2 < 0.2$ ms, $\Delta t_3 < 50 \mu s$ which is consistent with $50 < Q < 150$ and the observed $1.5 \% \lesssim N_f/\bar{N} \lesssim 15 \%$ and $\omega_f/2\pi = 0(10 \text{ kHz})$. With filtering we measure, of course, an apparent Q_{app} such that $1/Q_{app} = 1/Q + N_f/(2\bar{N})$. (iii) EM and ES response. As seen on fig. 7 a non-shielded probe gives simultaneously an EM ($\propto B_T$) and an ES ($\propto E_r$) response. The shape of the ES response is different from the EM one and its magnitude signal is significantly larger. Furthermore large peaks are seen on the probe signals up to 10 ms after the end of the tokamak current. Such peaks appear mostly on the ES signal and correspond probably to transmission in the afterglow low density plasma.

III. High power experiments.

The main observations made during some pulsed high power experiments are the following : (i) Up to 10 kV rf voltage on the excitation loop no visible effect is seen on plasma current loop voltage and position of the plasma. Only a 3 to 10 % increase of density is observed. (ii) The same type of spiky resonance structure and same ratio between EM and ES response of the probes is observed. (iii) During the rf pulse, jumps in oscillator frequency (~ 100 kHz) correlated with dips in rf coil current appear but are not correlated with the maxima of receiving probe signal. These frequency jumps appear more related to negative spikes of the loop voltage and thus to a capacitive effect due to column displacement. (iv) Preliminary measurements of the absorbed power by means of $V_{coil} I_{coil} \cos \phi$ do not show a large increase of absorption due to magneto-acoustic resonances. This is in disagreement with resonance loading seen on TFR and T4 [8]. An experiment studying the effect of an electrostatic shield on the

rf coil is planned to see, in comparable plasma conditions, whether this will influence the resonance loading. (v) Presence of strong harmonic 2 generation has also been noticed in the presence of magnetosonic resonances. The level can exceed e.g. 30 % of harmonic 1 level for $I_{\text{coil}} \approx 100$ A at 6 MHz. Such a quadratic non-linear effect can also produce a d.c. helical deformation of the plasma and affect its stability. This harmonic generation will be studied in detail.

References.

- [1] V.P. Bhatnagar et al., Proc. Vth Int. Conf. Plasma Phys. and Contr. Fusion Research (Berchtesgaden 1976) Vol. I, p. 359.
- [2] A.M. Messiaen et al., Proc. Workshop on Transport and MHD plasma theory, Varenna 1977, Italy (to be edited by Euratom).
- [3] Vdovin et al., Proc. Vth Eur. Conf. Contr. Fusion and Plasma Phys., (Moscow 1973) Vol. I, p. 553 ; A.M. Messiaen et al., ibid., Vol. I, p. 545 ; J. Adam et al., Proc. Vth Int. Conf. Plasma Phys. and Nucl. Fusion Research (Tokyo 1974) Vol. I, p.65.
- [4] R.R. Weynants et al., Proc. Workshop on Transport and MHD plasma theory, Varenna 1977, Italy (to be edited by Euratom).
- [5] J. Adam, J. Jacquinet, report EUR-CEA-FC-886, 1977 Fontenay-aux-Roses, France.
- [6] A.M. Messiaen et al., L.P.P. report 64, 1977 Brussels.
- [7] The TFR Group, Plasma Physics, 19 (1977), 349.
- [8] The TFR Group, Proc. Vth Int. Conf. Plasma Phys. and Contr. Fusion Research (Berchtesgaden 1976) Vol. III, p. 39 ; V.V. Buzankin, ibid, Vol. III, p. 61.

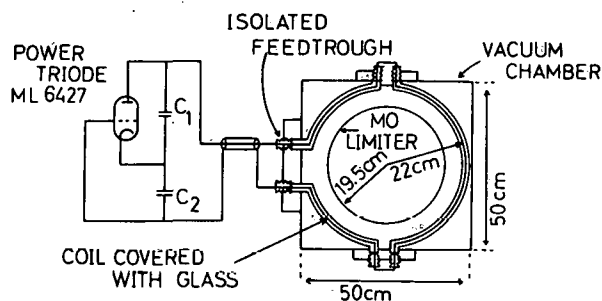


Fig. 1. ERASMUS coil system and power oscillator schematic.

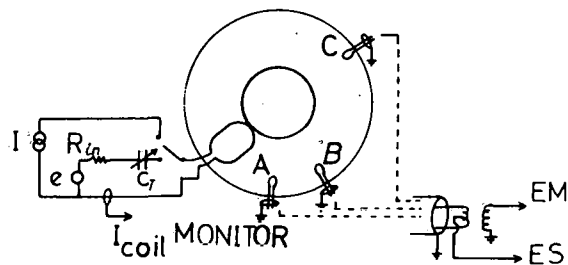


Fig. 2. Low power setup.

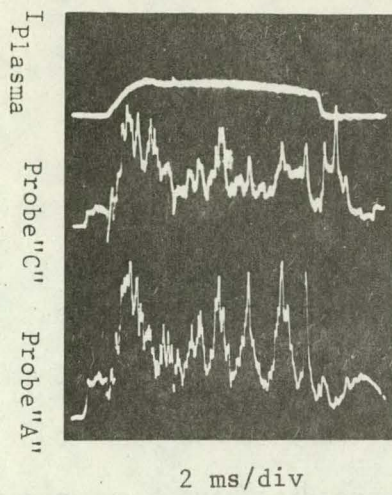


Fig. 3. Plasma current, response curves of probes "C" and "A" in deuterium at 10 MHz.

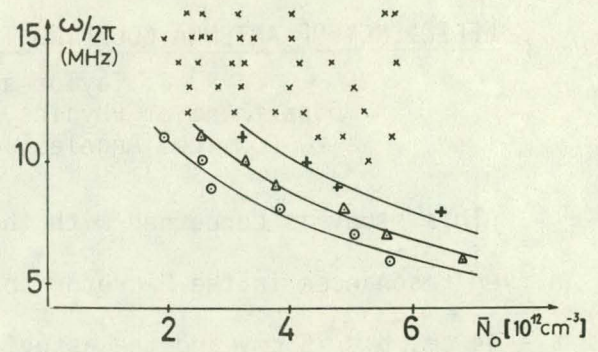


Fig. 4. Resonance loci on a $\bar{N}_0 - \omega$ diagram.

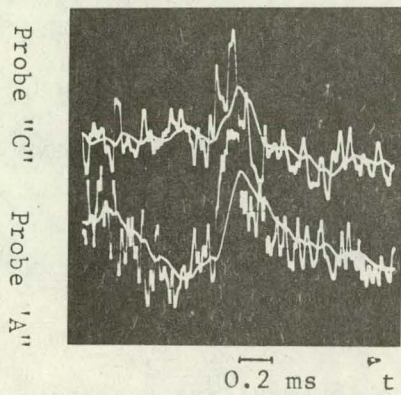


Fig. 5. Comparison between a filtered and an unfiltered resonance display.

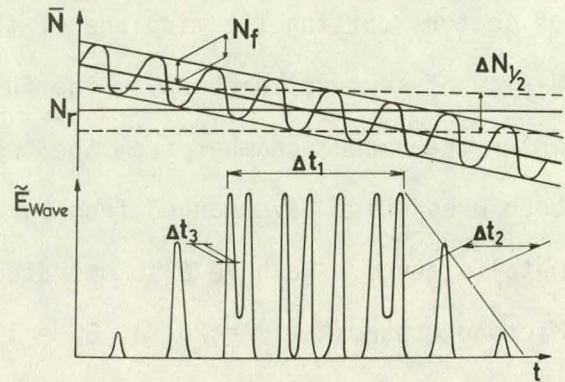


Fig. 6. Effect of a regular density fluctuation on the resonance display.

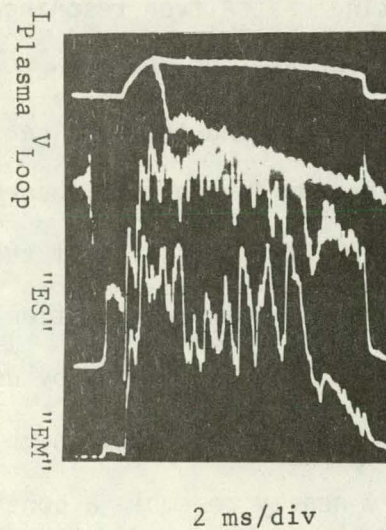


Fig. 7. Plasma current, loop voltage, "ES" and "EM" probe display ("ES" display is 20dB less sensitive than the "EM" one). (9MHz).

EFFICIENCY OF ANTENNA COUPLING TO THE FAST ALFVEN MODES IN MACROTOR*

R. J. Taylor and G. J. Morales
Department of Physics, University of California
Los Angeles, California 90024

This study is concerned with the low power ($<100\text{W}$) excitation of Fast Alfven resonances in the Macrotor tokamak ($I_p = 70 \text{ Ka}$, $B_t = 2 \text{ KG}$, $R = 95 \text{ cm}$, $a = 44 \text{ cm}$, $b = 75 \text{ cm}$) and the associated loading of the antennae used to excite these resonances. The following antennae geometries have been used: (a) long straight rods parallel to the inner and outer walls of the vacuum chamber, covering the full vertical extent of the plasma, (b) short **U**-type of sectors cutting the midplane of the plasma on the outer side, (c) long **U**-type of sectors which cover the full horizontal extent of the plasma and enter the vacuum chamber from the top. Various shielding arrangements have been used which have ranged from bare to heavy combinations of glass and metal sleeves. We have also operated antennae structures in the smaller Microtor tokamak ($I_p = 20 \text{ Ka}$, $B_t = 10 \text{ KG}$, $R = 30 \text{ cm}$, $a = 10 \text{ cm}$, $B = 12 \text{ cm}$) which have fully enclosed the plasma. Both of these tokamaks are operated **without** a limiter and no special antenna limiters have been introduced.

Most of the antennae structures used (perhaps with the exception of the bare antennae) are capable of exciting Fast Alfven resonances. In Microtor we have made a systematic study of the density dependence of these modes by means of controlled gas puffing. It is found that the real part of the dispersion relation is well described by cold plasma theory. For the studies in Macrotor we have observed the slow time evolution of the cavity resonances due to the natural density drop during a shot. We routinely observe the evolution of a full spectrum (0-20 MHz) of resonances by using frequency sweeping of a broad-band antenna. We observe that at high density the resonances are so crowded that they nearly resemble a continuum, and as the density drops, the continuum evolves into well resolved resonances. The

typical Q values of these resonances are of the order of 40-50. Fine splitting of the peaks can be observed as well as the rapid changes in phase across each of the peaks.

The major effect found in this study has been that antennae structures can be heavily loaded by the presence of plasma in their neighborhood, even when subjected to heavy shielding. This parasitic loading has been observed to occur in all the antennae studied and exists over the wide range of densities investigated, i.e., $n_e = 10^{10}\text{cm}^{-3}$ (in discharge cleaning) to $n_e = 5 \times 10^{13}\text{cm}^{-3}$ (in Microtor). From the frequency response function of each antenna we have verified that the dominant plasma effect consists of a resistive drop in antenna Q accompanied by a small shift in the resonant frequency. The parasitic loading increases with plasma density, antenna dimensions, and frequency. The frequency dependence has been investigated to values up to and including the lower-hybrid frequency. Typically, for a one meter long antenna the signal at the end of the antenna drops by 0.2 db/MHZ, thus having important consequences for antenna structures designed for lower-hybrid heating.

The experiments in Microtor showed no evidence for a correlation between the excitation of Alfvén resonances and antenna loading. Due to the small dimensions of Microtor all the ICRF experiments were performed at high density ($n_e \sim 2-5 \times 10^{13}\text{cm}^{-3}$) and accordingly experienced a strong parasitic loading which overwhelmed the resonant loading. Upper bound estimates on these experiments indicate that 70% of the applied power went into the plasma but less than 5% appeared as resonances.

Due to the large dimensions of Macrotor one is able to perform Fast Alfvén experiments at lower densities ($n_e < 10^{13}\text{cm}^{-3}$), thus reducing the parasitic loading. In this machine one is able to observe a definite correlation between antenna loading and resonances for well shielded antennae, and

during the low density portion of the shot. The resonances can produce loading of the order of 0.5 to 1Ω . However, this wave loading is masked by the parasitic loading during the early part of the shot when the density is high.

*Work supported by USDOE.

FAST MAGNETOSONIC MODES IN THE CALTECH TOKAMAK*

D. Q. Hwang and R. W. Gould

California Institute of Technology, Pasadena, California 91125

Fast magnetosonic cavity eigenmodes are being studied in the Caltech tokamak. The toroidal field axis is 4 kG ($f_{ci} = 6$ MHz) and varies by a factor of 2 from the inner to outer wall. The resonant modes have been observed both in transmission and in input resistance measurements. The loop antenna can be matched to the source, either on or off resonance. The resonant input resistance can be up to 1.3Ω , 3 to 4 times the off-resonant resistance. This off-resonance resistance is not much higher than the "vacuum" resistance. The complex input impedance and transmission characteristics are being determined.

A. Introduction

The use of the fast magnetosonic cavity modes as a method to heat plasma has been proposed theoretically for a long time.¹⁻³ Some low and high power experiments have been performed in some of the large tokamaks around the world.^{4,5} As a result of these experiments, some new questions have arisen. We are trying to investigate these eigenmodes in detail at low power, as preparation for future high power heating experiments. Power of 10-300 watts has been used, and frequency range is between 1.2 and 3 times the ion cyclotron frequency.

B. Experimental Setup

The arrangement of the transmitting and receiving antennas is shown in Fig. 1. The transmitting antenna consists of a 2-turn copper loop enclosed in glass. The dimensions of the loop are 6.5 x 2.5 cm. Because of the small antenna size, its coupling coefficient is very low. Therefore, in order to measure the plasma loading impedance we find it necessary to minimize the antenna resistance. A capacitive matching network, also shown in Fig. 1, is used in order to tune out the antenna inductance and transform the antenna resistance to 50Ω . The transmitting antenna is driven by a 300-watt broad band amplifier. The receiving probe is a 6-turn loop, located 180° toroidally opposite the transmitter.

For plasma loading measurements, a directional coupler is used to obtain the incident and reflected voltages into the antenna system. A r.f. current probe is placed around the antenna to measure the current in the antenna; thus, we can determine the plasma loading resistance using the following equation where R_{ant} is the antenna resistance in vacuum:

$$R = ((P_{inc} - P_{ref})/I^2) - R_{ant} \quad (1)$$

The phase between the incident and reflected voltages has been determined so that the complex reflection coefficient and complex impedance can be calculated.

C. Hydrogen Plasma Parameters

The toroidal magnetic field on axis of the Caltech tokamak is 4 kG (corresponding to $f_{ci} = 6$ MHz); and varies by a factor of two from the inner to outer wall. The electron temperature of the plasma, obtained from Langmuir probe data, is between 50 and 100 eV. Due to pulse discharge cleaning (as proposed by Taylor) the tokamak wall condition is such that the line-averaged

*This work supported by the U.S. Department of Energy.

electron density decays rapidly from $7 \times 10^{12} \text{ cm}^{-3}$ to $1.5 \times 10^{12} \text{ cm}^{-3}$ in the first two milliseconds (Fig. 2). Therefore, most of the eigenmodes appear in the early phase of the tokamak discharge.

D. Transmission Measurements

The first part of the low power experiment is to study the toroidal eigenmodes using the receiving probe. The transmitted signal is very easily observed. Since our transmitting antenna is small compared to the size of the tokamak, various toroidal eigenmodes can be excited. The particular mode excited depends on the input frequency and the plasma density. As the input frequency increases, more eigenmodes can be excited. This effect is shown in Fig. 3, where the peaks in amplitude correspond to the resonances. A given resonance is excited both during the buildup and during the decay of the plasma density.

In Fig. 4 we have plotted the locations of the resonance peaks in frequency vs. density. The figure shows both the theoretical uniform cold cylindrical plasma cutoffs and the experimental data. Assumptions of m_e (electron mass) = 0, and conducting wall boundary conditions are made. One finds the data points fall in the region around the cutoffs, indicating cold plasma theory seems to be valid here.

To further confirm that the peaks in transmission are associated with toroidal modes, we have measured the phase of the received signal with respect to the reference signal. As expected, the peaks in amplitude correspond to a rapid change in phase (Fig. 5).

E. Plasma Loading

In order to efficiently couple r.f. energy into the plasma, it is essential to know the plasma loading resistance. From the measurement of the incident power, the reflected power, and the antenna current, the plasma loading resistance can be determined. Fig. 6 shows a comparison of the transmitted signal with the reflected voltage and antenna current. In this case the antenna is tuned so that during the plasma discharge when there is no toroidal mode present, the antenna is matched to the 50Ω source. When a toroidal resonance occurs the antenna becomes mismatched; therefore, the reflected voltage increases and the r.f. current decreases. We use Eq. 1 to calculate the plasma loading resistance, which is the first trace in Fig. 6. Plasma loading at resonance peaks can be as high as 1.3Ω , which is 3 to 4 times the resistance of the antenna. This is an encouraging result, for it tells us that as much as 75% of the power can be delivered into the plasma, and that only 25% of the power is lost in the antenna.

The phase between the incident and reflected voltage has been measured (Fig. 7). Corresponding to a peak in amplitude, there is a shift in phase, indicating a change in complex reflection coefficient.

F. Resonance Matching

In order to deliver the maximum power into the resonance peaks, it is necessary to match the impedance at a toroidal mode resonance to 50Ω . This process is difficult because both the real and the imaginary parts of the impedance are changing very rapidly. Therefore, it requires some care to transform the resonance loading into 50Ω at the precise point where the real part of the impedance is a maximum and the imaginary part is zero. Fig. 8 shows an example where this has been done.

G. Conclusions

We have observed the toroidal fast-wave eigenmodes in both transmission and antenna loading. The density dependence of the modes agrees satisfactorily with cold plasma theory. Phase variations of the transmitted signal are consistent with the amplitude resonance. Antenna loading resistance at resonance is as high as 1.3Ω . This makes heating using these eigenmodes very encouraging.

References

1. T. Stix, Nucl. Fus. 15, 737 (1975).
2. D. G. Swanson, Phys. Fluids 17, 2241 (1974).
3. F. J. Paoloni, Matt. Report 1119, Princeton (1975).
4. J. Adam, M. Chance, H. Eubank, W. Getty, E. Hinov, W. Hooke, J. Hosea, J. Jobes, F. Perkins, R. Sinclair, J. Sperling and H. Takahashi, Proc. Vth Int. Conf. on Plasma Physics & Controlled Nuc. Fusion Res., Tokyo 1974, Vol. I, p. 65.
5. TFR Group, 3d Conf. on Theoretical & Exp. Heating of Toroidal Plasma, Grenoble, 1976, Vol. 1, p. 87.

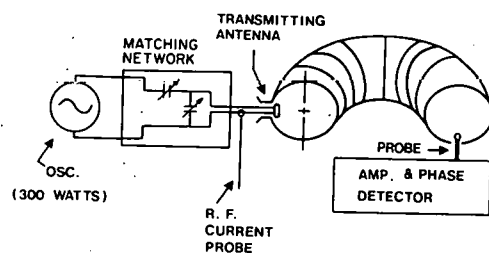


Fig. 1. Schematic of the experimental setup

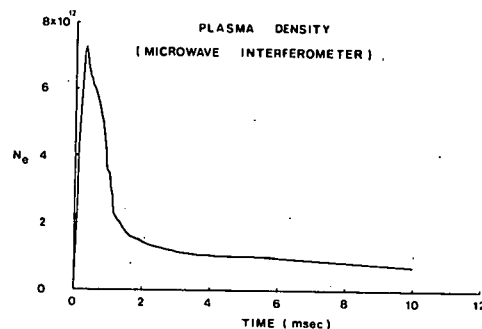


Fig. 2. Electron Density (4 mm microwave interferometer)

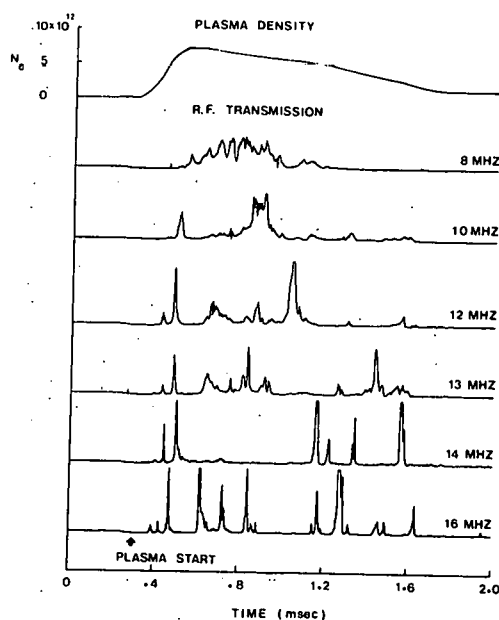


Fig. 3. Transmission Measurements vs. Typical Density Decay

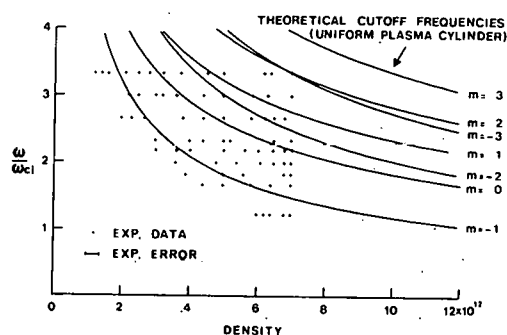


Fig. 4. Theoretical cold plasma cutoffs vs. experimental data

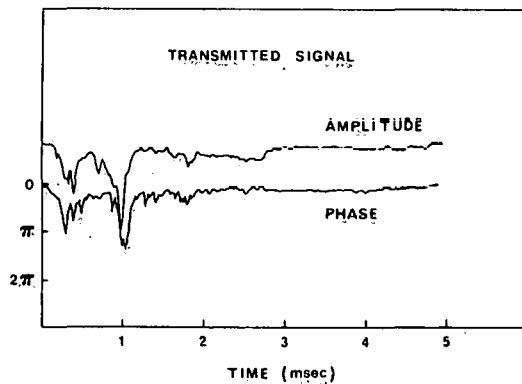


Fig. 5. Phase change of the received signal

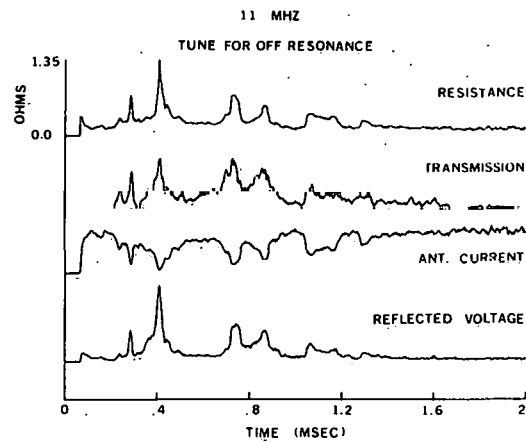


Fig. 6. Calculated plasma loading resistance. Antenna is tuned so that it is matched to 50Ω when no resonance is present.

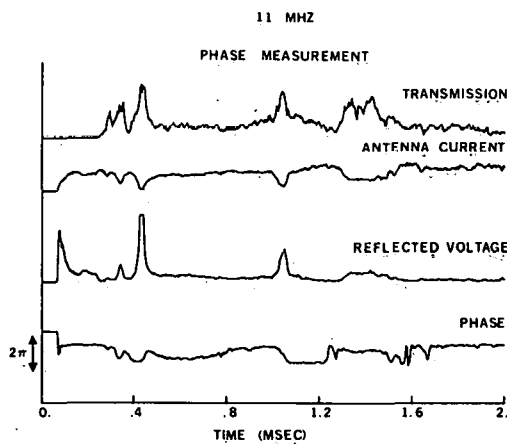


Fig. 7. Phase between incident and reflected voltage

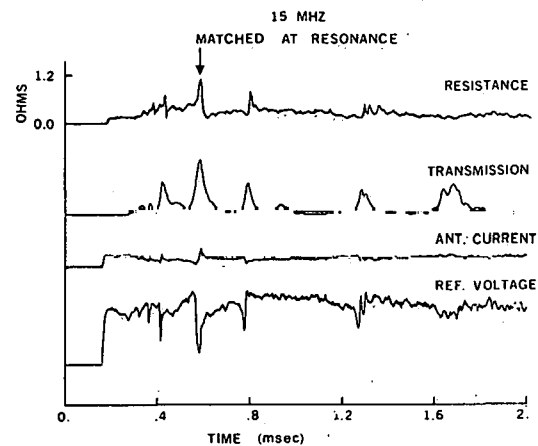


Fig. 8. Antenna is matched to 50Ω at one of the resonance peaks

FAST WAVE GENERATION IN THE PLT TOKAMAK

by

P. L. Colestock, J. C. Hosea, F. J. Paoloni,
H. R. Thompson, and H. Takahashi
Plasma Physics Laboratory
Princeton University
Princeton, New Jersey 08540

ABSTRACT

Large tokamaks are capable of sustaining a great number of eigenmodes of the fast hydromagnetic wave in the ion cyclotron range of frequencies. The coupling and cavity Q of these eigenmodes depend on the damping decrement and dispersion characteristics which in turn are functions of the magnetic field, density and concentration ratio in a multispecies plasma. We examine fast wave generation in PLT in terms of a cylindrical plasma model and in terms of a WKB plane wave model modified to include toroidal and rotational transform effects. Preliminary measurements of toroidal eigenmode properties of waves excited in PLT by a one-half turn coil will also be presented.

I. Introduction

The new generation of larger cross-section tokamaks offers the opportunity for exciting many more eigenmodes of the fast magnetosonic wave than were possible in previous experiments. Such a situation allows considerable flexibility in optimizing RF energy deposition under optimum plasma conditions.^{1,2} The coupling to the eigenmodes depends first on the dispersion characteristics and the damping decrements which are complicated by toroidal geometry effects and the presence of impurities.^{3,4} Second the coupling depends on the antenna geometry. In this paper we examine aspects of fast wave propagation in a deuterium plasma in the PLT device using the cold plasma dispersion relation evaluated locally including the effects of toroidality and a small concentration of proton impurity. Preliminary measurements of the toroidal eigenmode properties of waves excited in PLT by a one-half turn coil are then presented and discussed in terms of the cold plasma theory and compared to other measurements.

II. Cold Plasma Theory

Considerable progress has been made in understanding fast wave propagation in a torus by including the variation of the toroidal field with major radius.⁴ Since both the parallel and perpendicular

wavelengths are functions of position it is useful to plot k_x^2 as determined from the complete cold plasma dispersion relation evaluated locally across the plasma cross-section. Although the eigenmodes themselves cannot be determined without considering the boundaries of the system, it is nevertheless instructive to determine regions of evanescence and propagation and the accessibility of the latter from the boundary. An equivalent but alternative view of the wave propagation may be found by plotting those values of k_z which separate regions of real, imaginary or complex k_x as a function of position. Such a plot assuming a parabolic density profile and a $1/R$ toroidal field variation is given in Fig. 1(a) for the case where the deuterium fundamental occurs in the plasma. Above a critical value of k_z , much of the plasma is cutoff to both slow and fast waves with the exception of a narrow region on the inside of the torus. Below this value, however, fast waves are free to propagate being damped mainly by the deuterium fundamental cyclotron layer. The effect of toroidality on the eigenmodes is taken into account by allowing the local parallel wavenumber to vary inversely with major radius and the results for a typical imposed boundary value are shown in Fig. 1(b). It is noted that the slow wave can exist on the inside of the torus and Perkins (ref. 4) finds that mode conversion to this slow wave branch as a loss mechanism can be comparable to damping at the fundamental cyclotron layer. From Fig. 1(a) it is observed that the slow wave exists only on the inside of the cyclotron layer at all wavelengths and coupling from the inside must necessarily pass through this region as was the case in the ST device. In PLT, however, coupling from the outside allows direct coupling to the fast waves which may circumvent significant slow wave coupling and make possible fast wave heating at the fundamental.

A case of interest also occurs near the deuterium second harmonic as shown in Fig. 2(a). Due to a small amount of hydrogen present the ion-ion hybrid occurs close to the second harmonic layer and introduces a small evanescent zone into the fast wave region. The corresponding dispersion relation roots for typical conditions are plotted in Fig. 2(b). Damping in this region is complicated by the fact that energy may be lost into the ion-ion hybrid branch by mode conversion or either by deuterium second harmonic or hydrogen fundamental damping which also may be enhanced by the presence of the ion-ion hybrid layer. Analysis of this layer has revealed that well below impurity concentrations of about 1% the effect of the ion-ion hybrid should be small so that the damping effects may be separated.^{3,4} The test of this theory must be made in a particularly clean device and this is planned to be done in PLT by running for extended periods exclusively with deuterium.

III. Experimental Results

An experimental program is currently underway on the PLT device to investigate eigenmode properties and antenna loading at modest powers. Waves are excited by a one-half turn coil on the outside edge of the plasma chosen so that low order $m=0, \pm 1$ modes are excited preferentially. Preliminary measurements of wave

amplitude and phase have been made using an array of magnetic probes spaced both toroidally and poloidally around the torus. The results of a frequency sweep over both fundamental and second harmonic ranges at constant magnetic field and density are shown in Fig. 3 for a probe located on the top of the torus. The response is characterized by a densely packed spectrum of high Q modes which are observed as peaks in the wave amplitude (and for the sweep about $\Omega_D=1$ in the antenna loading as well). The abrupt increase in wave amplitude at 20 MHz could be attributable to the onset of a mode group or to the position of the $\Omega_D=1$ layer. Further study is required to determine whether mode onset, slow wave coupling at the coil ($\Omega=1$ at the edges of the coil at 20 MHz) or a change in fundamental damping, as observed for $\Omega_H=1$ in TFR⁵, is responsible for this effect. Nonetheless it is clear that mode damping is small for the case when the cyclotron layer is on the inner half of the plasma which suggests that fast wave $\Omega_D=1$ resonance heating at the deuterium fundamental may be feasible.

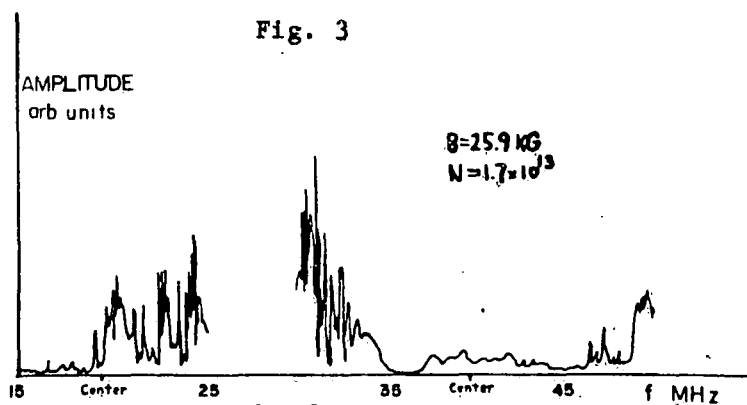
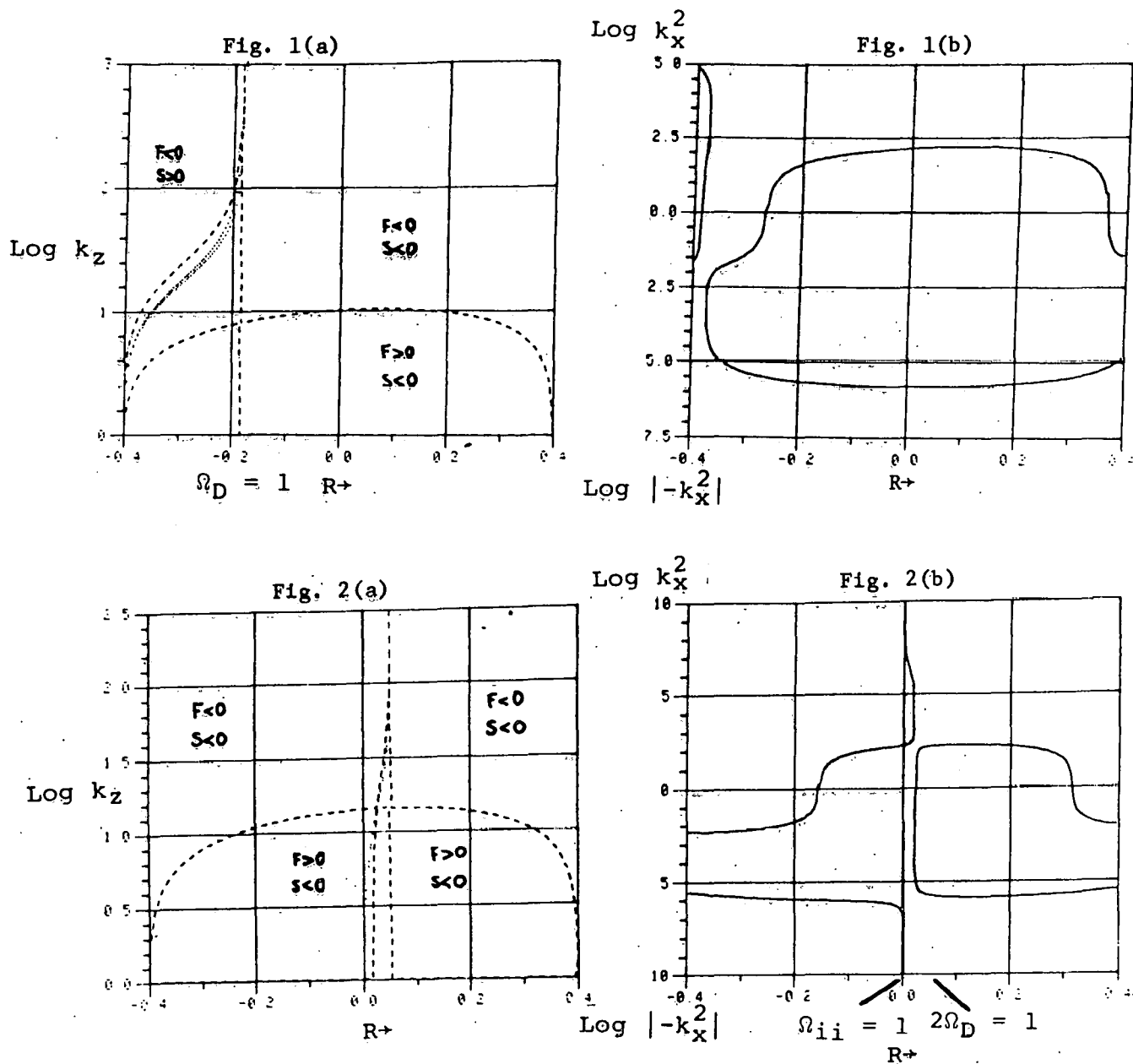
A region of strong damping occurs from 33 to 48 MHz where the peak eigenmode amplitude is reduced by about a factor of 10 from that observed when the ion-ion hybrid -- second harmonic layer pair is at the two edges of the plasma column. This result is similar to previously reported results for this case.⁵⁻⁷ Similar behavior is observed in a constant frequency scan made by varying the magnetic field and these measurements show an approximately linear decrease in amplitude as the hybrid layer approaches the plasma center from the inside major radius side of the plasma. This result continues the trend at the upper end of the frequency scan shown in Fig. 3. The damping observed here is stronger than that attributable to second harmonic damping alone in the absence of the ion-ion hybrid layer. At present there is insufficient experimental evidence to determine whether the main channel of energy flow is into the ion-ion hybrid branch or whether the hybrid layer enhances deuterium second harmonic or hydrogen fundamental damping by increasing the field gradients in the vicinity of the cyclotron layer.⁶ It is also important to note that the modes can be of sufficiently high Q so as to require mode tracking in order to optimize coupling over long discharge times. Finally we note that the poloidal phasing indicates the presence of $m=0, \pm 1$ modes as expected.

In summary, we report the existence of a densely packed spectrum of fast wave eigenmodes in both the deuterium fundamental and second harmonic ranges. High Q eigenmodes exist when the fundamental is well within the plasma column which suggests fast wave heating at $\Omega=1$ may be feasible. Strong damping occurs when the ion-ion hybrid layer is present and considerably more work will be necessary to determine the precise nature of the damping mechanism.

References

1. Hosea, J. C., Third Symposium and Workshop on Plasma Heating in Toroidal Devices, Varenna, 1976; PPPL-1309.
2. Adam, J. and Jacquinet, J., Assoc. EURATOM, EUR-CEA-FC 866, 1977.
3. Jacquinet, J., McVey, B. D. and Scharer, J. E., Phys. Rev. Lett. 39, 88 (1977).
4. Perkins, F. W., Nucl. Fusion 17, 1197 (1977); PPPL-1336.
5. Equipe TFR, 6th Conf. on Plasma Physics and Controlled Nuclear Fusion Research Berchtesgaden (IAEA, Vienna, vol II, p. 39).

6. Takahashi, H., Proc. 3d International Congress on Waves and Instabilities in Plasmas, Palaiseau, 1977; PPPL-1374.
7. Vdovin, V. L., Shapotkovskii, N. V., Rusanov, V. D., 3d International Conf. on Theoretical and Experimental Aspects of Heating Toroidal Plasmas, vol. 2, Grenoble, 1976.



Energy Loss and Edge Density Measurements During ICRF Experiments on ATC*

H. Hsuan, R. J. Hawryluk, S. Suckewer and H. Takahashi
Princeton University, Plasma Physics Laboratory
Princeton, New Jersey, USA 08540

Ion heating of tokamak plasmas by RF heating¹ in the ion cyclotron range of frequencies (ICRF) has been demonstrated in ST¹ and ATC². As reported previously³, bulk ion heating efficiency of 10 to 40% was obtained in the ATC experiments. In addition to ion heating, significant profile modification in the plasma periphery was observed as determined from Langmuir probe, bolometer and spectroscopic measurements. This paper will discuss principally the Langmuir probe in the plasma periphery and the bolometer measurements of energy loss. Another paper by Suckewer and Hawryluk⁴ will discuss the spectroscopic measurements and the importance of plasma periphery modification by RF power in reducing high Z impurity radiation.

In the ATC experiments, a single coil was used to couple the RF power near the second harmonic deuterium cyclotron frequency. The coil of 135° arc encased in a ceramic sheath was located in the minor azimuthal plane on the large major radius side of the plasma. The maximum average RF power was 150 kW for a ~ 10 msec pulse length.

In order to measure the edge density profile, a tungsten Langmuir probe, 2 mm long and 1 mm in diameter, was biased to collect the ion saturation current. A low pass filter (DC to 1 kHz) was used to obtain the average ion density. The probe was radially scannable from the vacuum vessel wall to the limiter radius. In a normal discharge without RF, large density fluctuations of 50% or more of the average value (up to a few hundred kHz) were observed in the plasma periphery. Typical probe data is shown in Fig. 1. With the application of the ICRF probe, the amplitude of these fluctuations did not change but the fluctuations became peaked at some frequencies. Furthermore, the ion density profile became much flatter during the RF pulse as indicated in Fig. 2. For a typical ATC plasma of $\langle n \rangle \approx 1. \times 10^{13} \text{ cm}^{-3}$, the measured limiter edge density was about $1. \text{ to } 2. \times 10^{12} \text{ cm}^{-3}$ without the ICRF pulse and decreased to $\leq 1. \times 10^{12} \text{ cm}^{-3}$ with the pulse; and the density at the vacuum vessel wall was about $3. \times 10^{10} \text{ cm}^{-3}$ without the pulse and $3. \text{ to } 8. \times 10^{10} \text{ cm}^{-3}$ with the pulse. The density profile was approximately linear in both cases.

The flattening of the density profile in the plasma periphery indicates a local enhancement of the particle diffusion coefficient. An increase in the diffusion coefficient is also suggested by the CO₂ laser measurements of the low frequency turbulence by Surko and Slusher⁵. In these measurements, the low frequency turbulence increased during ICRF heating. Furthermore, the increase was localized in the edge region ($0.7 \leq r/a \leq 1$), and the location of the maximum amplitude was a function of wavelength of the density fluctuation.

Nonetheless, the intensity of the D _{β} line and hence the gross particle replacement time changed during the ICRF pulse by typically < 50%. One dimensional simulations of the density profile for ATC with a Monte Carlo treatment of the

neutral simulations⁶ indicates that for ATC discharges the neutral source function is not strongly peaked near the plasma periphery but is relatively uniform over the plasma volume. Thus, even large increases in the diffusion coefficient in the plasma periphery would result in relatively small changes in the intensity of the D_β line.

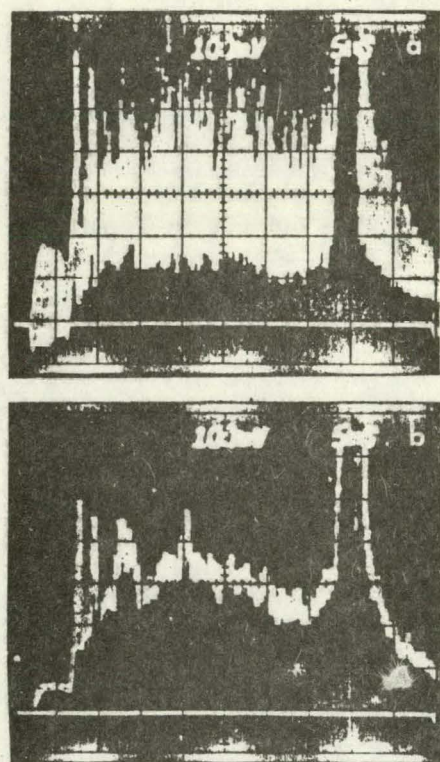


Fig. 1. Ion saturation current for a normal discharge without RF pulse at 7 cm from the plasma limiter radius, and 1 cm from the wall.

a) with a (DC to 30 kHz) filter
b) with a (DC to 1 kHz) filter
Vertical scale $\sim 2 \times 10^{10} \text{ cm}^{-3}/\text{Division}$ (for 10 eV electrons) horizontal scale 5 msec/Division.

The detectors at the equatorial plane were radially movable to within 5mm of the plasma-limiter edge. They were particularly sensitive to ions with a large banana orbit. In the direction parallel to the plasma current, a factor of three higher loss was observed than in the anti-parallel direction during the RF pulse. However, the total loss due to direct banana orbits was estimated to be less than 10% of the total RF input energy.

Bolometers in the radial direction in a normal discharge without RF heating only detected about 30% of the ohmic heating power during the high current phase of the discharge incident on the walls.⁷ Although the energy loss to the limiters was the dominant part in a normal ATC discharge, this loss was not

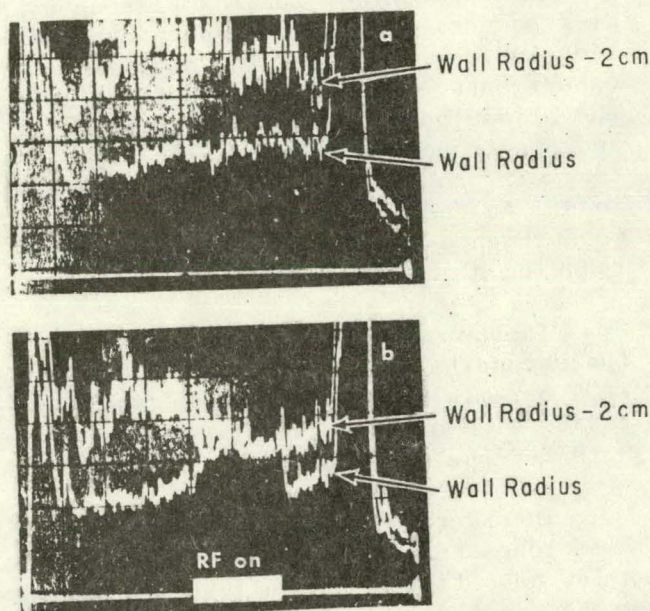


Fig. 2. Comparison of ion saturation current for

a) without,
b) with ICRF power.

The wall was about 8 cm from the plasma limiter radius. Vertical scale $\sim 2 \times 10^{10} \text{ cm}^{-3}/\text{Division}$ (for 10 eV electrons), horizontal scale 5 msec/Division.

The energy loss was monitored by bolometers and an absolutely calibrated monochromator. Bolometers were constructed to measure losses in the direction parallel and anti-parallel to the plasma current.

measured during the ICRF experiments. Thus, the energy balance during the ICRF experiments was only partially obtained. During the ICRF pulse the amount of energy lost, through increased radiation and charge exchange, determined by radial bolometer measurements would correspond to 40-100% of the RF energy input (See Fig. 3). Comparison with spectroscopic measurements indicate that an important part of the energy intercepted by the bolometer was due to radiation from the plasma periphery. Detailed spectroscopic measurements of carbon and oxygen lines indicated that the increased radiation was located in the plasma periphery and not in the plasma core for RF powers < 100 kW. Under some plasma conditions large $m = 2$ MHD oscillations were observed. In this case, the impurity radiation increased throughout the plasma and a higher loss was observed on the bolometer⁸.

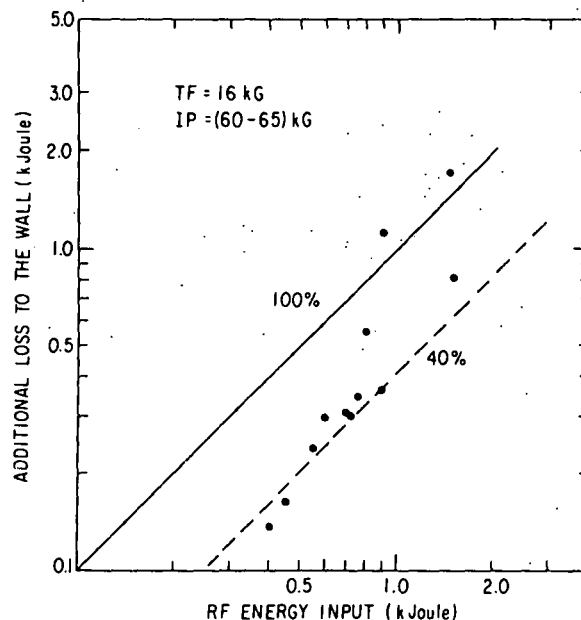


Fig. 3. The increase loss detected by the bolometer vs. the ICRF energy input for the same toroidal field strength and the same plasma current.

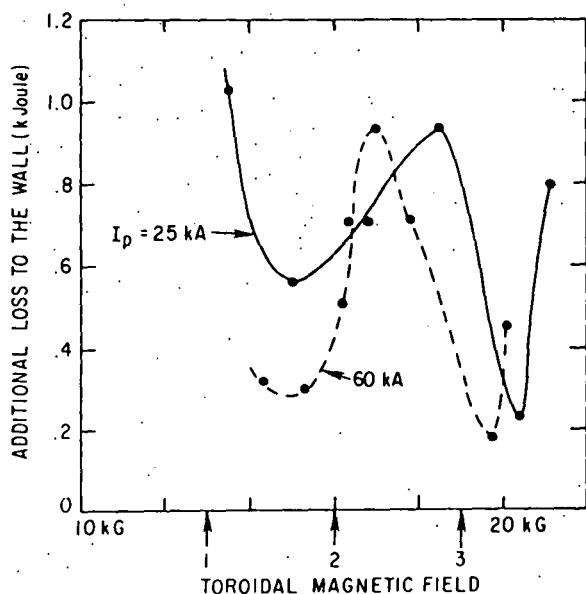


Fig. 4. The increased loss detected by the bolometer for about 80 Kjoule ICRF input vs. the toroidal magnetic field. Point 2 corresponds to the second harmonic resonance frequency occurring at the center, and points 1 and 3 correspond to that at the edge.

The radial bolometric loss was also observed to strongly depend upon the toroidal field strength as shown in Fig. 4. The bolometer signal indicated a broad maximum when the deuterium second harmonic cyclotron frequency was near the plasma center. On one hand, the broad maximum occurred when the wave damping was also strong. On the other hand, the radiated power measured spectroscopically was not strongly dependent on the toroidal field and was generally less than the loss measured by the bolometer. These observations seem to suggest that the wave energy is strongly absorbed by ions which are subsequently lost through charge exchange and intercepted by the bolometer.

In summary, during the ICRF pulse in ATC, significant modification of the density in the plasma periphery was observed indicating a local enhancement of the diffusion coefficient. In the overall energy balance, $\leq 10\%$ of the RF energy was lost through direct banana orbits,

while the increased loss through radiation and charge exchange corresponds to 40 to 100% of the RF energy input. Spectroscopic measurements of the power radiated indicated that a large fraction of the loss to the walls was from the plasma periphery and not the core.

ACKNOWLEDGEMENT

We wish to acknowledge the helpful discussions with Drs. R. A. Ellis, F. W. Perkins as well as the helpful assistance of A. J. Sivo, R. Shoemaker, D. B. Bollenbacher, H. H. Higgins, and W. T. Ringle.

*Work supported by U.S.D.O.E. Contract EY-76-C-02-3073.

REFERENCES

- ¹J. Adam, J. Hosea, et al., "Wave Generation and Heating in the ST Tokamak at the Fundamental and Harmonic Ion Cyclotron Frequencies", in Proc. of the 5th Conference on Plasma Physics and Cont. Nucl. Fusion Res., Tokyo, 1974, IAEA, Vienna, 1975, Vol. II, p. 65.
- ²H. Takahashi, et al., Phys. Rev. Lett., 39, p. 31 (1977).
- ³H. Takahashi, PPPL-1374, also to be published in Journal de Physique, Special issue (1977).
- ⁴S. Suckewer, R. J. Hawryluk in this conference.
- ⁵C. Surko and R. Slusher, Phys. Rev. Lett. 37, 1747 (1976).
- ⁶M. H. Hughes and D. E. Post, PPPL-1335 (1977).
- ⁷H. Hsuan, K. Bol, and R. A. Ellis, Nucl. Fusion 15, p. 657 (1975).
- ⁸P. Stüdiér, H. Hsuan, R. R. Smith and H. Takahashi, Bull. Am. Phys. Soc., 21, 1157 (1976).

Modification of Plasma Edge Temperature by RF Heating*

S. Suckewer and R. J. Hawryluk
Princeton University, Plasma Physics Laboratory
Princeton, New Jersey, USA 08540

Recent studies of ohmically heated discharges on the Princeton Large Torus (PLT) have shown a strong correlation between the plasma edge temperature and high Z impurity radiation¹ (in particular tungsten) as shown on Fig. 1. When the plasma periphery edge temperature was decreased in PLT by either cooling the plasma periphery by low Z (oxygen or carbon) radiation or operating at high densities (either helium or recently deuterium discharges), the high Z radiation was found to decrease substantially². As will be discussed below, spectroscopic measurements conducted during RF experiments on the Adiabatic Toroidal Compressor³ (ATC) and recent computational modeling of impurity transport⁴ demonstrated the feasibility of modifying the edge plasma temperature by applying RF power without increasing the low Z impurity content in the core of the discharge. Thus, it may be possible to control high Z impurity radiation, which can significantly degrade tokamak operation, by actively modifying the plasma edge temperature.

In ion cyclotron range of frequency (ICRF) experiments on ATC⁵, a 25 MHz generator was used corresponding to the second harmonic deuterium frequency at $B_T = 16.4$ kGauss.

The maximum average RF power was ≈ 150 kW and the pulse length was typically ~ 10 msec. In lower hybrid experiments⁶, a waveguide array coupled the RF power (at 800 MHz) to the plasma. The maximum RF power was 160-180 kW for ≈ 5 msec. In both experiments, the impurity concentration and Doppler broadening of various impurity lines was measured. An absolutely calibrated 1m Ebert-Fastie monochromator with a vibrating LiF plate was used to obtain the time dependence of the line emission and of the ion temperature during a single discharge.

During RF heating, the edge ion temperature as determined from the broadening of the CIV 1548Å line ($r/a \sim .8-.9$) showed a significant decrease as shown in Fig. 2. For higher RF powers, the edge ion temperature decreased further. At the same time, the ion temperature near the center as determined from the broadening of CV and OVII lines ($r/a \sim .3-.5$) and charge exchange measurements

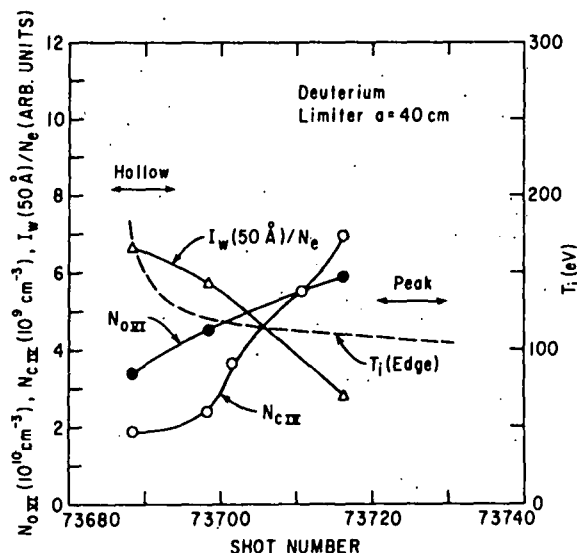


Fig. 1 Correlation between the ion edge temperature (from broadening of the CIV 1548Å line) oxygen and carbon density and the tungsten band intensity at 50Å. Hollow and peak refer to T_e profile.

increased substantially,^{3,5,6} Thomson scattering measurements⁷ of the electron temperature profile which extended to within 3 cm of the plasma-limiter interface also showed a significant decrease in the edge electron temperature during RF heating and relatively small changes in the plasma core. Thus, during RF heating especially ICRF, plasma edge cooling was quite evident.

Also as shown in Fig. 2 during RF heating the power radiated as determined from measurements of carbon and oxygen lines increased substantially. For RF heating powers of 30-60 kW, the intensity of CIV and OVI lines increased by a factor of two to three and the enhancement increased with increasing RF power. Emission of CIII, CII and OII lines increased by a similar factor (1.5-2.5). The background plasma lines (D or H) changed by a much smaller factor (< 50%),

and often showed even smaller changes. The intensity of the CV 2271Å and OVII 1623Å lines did not change as shown in Fig. 3. These lines because of their higher ionization and excitation energies radiated nearer the plasma center ($r/a \approx .3-.5$) and indicated that the concentration of low Z impurities in the plasma core did not increase. On the other hand, the emission of the CIV and OVI lines were located not far from the plasma edge ($r/a \sim .8-.9$). The change in the line average electron density during RF heating by only 20-30% cannot explain the enhanced radiation of the lower ionization states. Also, Thomson scattering density measurements near the plasma edge did not change while Langmuir probe measurements in the shadow of the limiter showed a significant decrease near the plasma limiter interface⁸.

In addition to modifying the plasma edge temperature profile and enhancing the power radiated, a RF power appears to change the transport coefficients in the plasma periphery as described in Ref. 8. At modest power levels (< 120-150 kW ATC) neither the gross particle replacement times nor the energy replacement times changed significantly. Thus, the modifications in plasma transport were localized in the plasma periphery. Nonetheless, at high power levels when large MHD activity ($m = 2$ modes) were typically observed (possibly as a result of excessive plasma edge cooling), the intensity of all the impurity lines increased substantially.

The spectroscopic measurements of impurity line intensity were compared with the results of a one dimensional time dependent computer simulation of the transport of carbon or oxygen impurities by solving

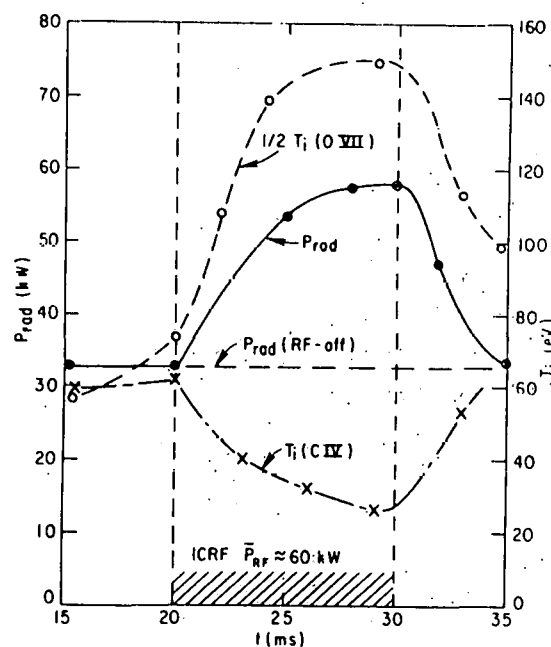


Fig. 2. Spectroscopic measurements of power radiated and ion temperature (from CIV lines in the edge and OVII lines in the core) during ICRF heating.

$$\frac{\partial n_j}{\partial t} = + \frac{\partial}{\partial r} r \left[D_A(r, t) \frac{\partial n_j}{\partial r} - \Gamma_j^{P.S.} \right] + S_j, \quad (1)$$

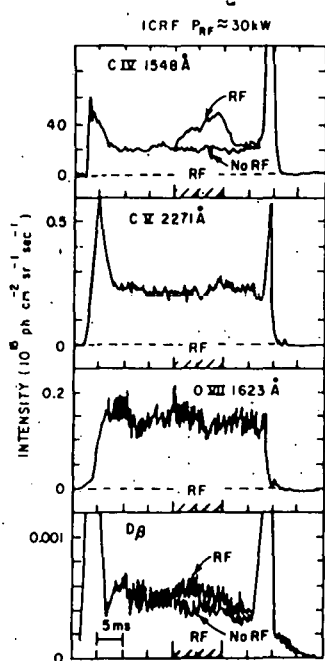


Fig. 3. Oscilloscope traces of the absolute intensity of the CIV, CV and OVII lines during ICRF heating. For CV and OVII lines, the trace without RF heating was indistinguishable and hence not shown.

presented in Fig. 4 showing a significant increase in the CIV and OVI lines emission and only a small change in CV and OVII. In this calculation, the reflection coefficient was assumed to be one. The power radiated by the lower ionization states tends to increase with decreasing edge electron temperature, until the temperature becomes too low whereupon the power radiated becomes constant and can even decrease. This functional dependence of the power radiated on electron temperature can be beneficially used in altering the power balance in the plasma periphery. In this way, the effect of relatively small changes in the edge diffusion coefficient can be enhanced. Similarly, a small increase in the electron heat conductivity in the plasma periphery during the RF pulse can result in a large decrease in the edge electron temperature because of the increased power radiated. In the ATC experiments, the decrease in plasma edge temperature may be due to an increase in the power radiated as well as a possible increase in the electron heat conductivity.

for each ionization state using the measured Thomson scattering profiles of density and temperature. $D_A(r, t)$ is the anomalous diffusion coefficient, $\Gamma_j^{P.S.}$ is the neoclassical flux due to Pfirsch-Schlüter diffusion⁹, S_j is the source term due to ionization and recombination. The inclusion of the neoclassical flux is motivated by the results of Cohen et al. on ATC¹⁰ while the need for anomalous transport by the experiments of Schmidt et al. on FM-11¹¹. Radiation losses were calculated from

$$P_{rad} = \int n_j n_e \xi_j^k E_j^k dV, \quad (2)$$

where ξ_j^k and E_j^k are the excitation rate coefficients and transition energy for ionization state j and line k . In Eq. (2), the population in the excited state is assumed to be in equilibrium with the ground state density. The results of rather extensive calculations indicated that for constant density and temperature profiles if the flux of neutral low Z impurities increased during RF heating then the power radiated by all of the ionization state would have increased by approximately the same factor. This was true for various forms of the anomalous diffusion coefficient. In order to obtain even qualitative agreement with experiments, it was necessary to include both the measured decrease in plasma edge temperature and assume a small increase ($\sim 50\%$) in the anomalous diffusion coefficient in the plasma periphery ($r/a > .9$). The results of the computer code are

In conclusion by minor modification of the transport coefficient in the edge region, it is possible to considerably increase the low-Z radiation and hence affect the edge plasma temperature. By carefully controlling both the RF power and the mode, through the design of the coupling structure, it appears to be possible to radiate a significant fraction of the input power without contaminating the core of the discharge with low Z impurities. Furthermore by cooling the plasma periphery, it should be possible, as in PLT, to decrease the high Z radiation.

ACKNOWLEDGEMENT

We are deeply indebted to S. Bernabei, K. Bol, C. C. Daughney, R. A. Ellis, Jr., P. C. Efthimion, H. P. Furth, E. Hinnov, W. M. Hooke, J. C. Hosea, H. Hsuan, D. M. Meade, R. W. Motley, J. A. Schmidt and H. Takahashi for their cooperation, interesting discussions and valuable comments.

*Work supported by U.S.D.O.E. Contract EY-76-C-02-3073.

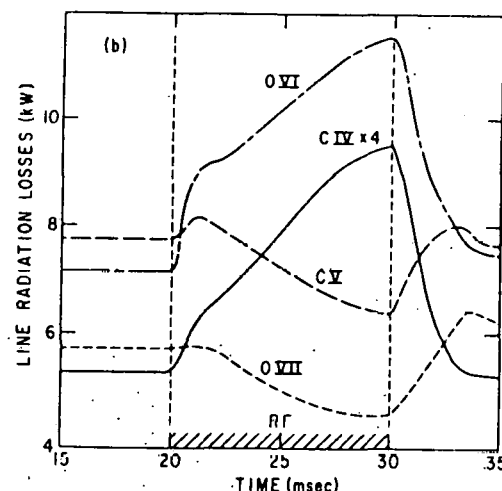


Fig. 4. Calculation of CIV, CV, OVI and OVII line radiation losses during RF heating.

REFERENCES

- ¹ S. Suckewer, et al., Bull Amer. Phys. Soc. 21, 1082 (1977).
- ² H. Hsuan, private communication, to be published.
- ³ S. Suckewer and E. Hinnov, Nucl. Fusion 17, 945 (1977).
- ⁴ R. J. Hawryluk, S. Suckewer and D. E. Post, in Conference on Atomic Processes in High Temperature Plasmas (Knoxville, Tennessee, 16-18 February 1977).
- ⁵ H. Takahashi, et al., Phy. Rev. Lett. 39, 31 (1977).
- ⁶ S. Bernabei, C. C. Daughney, W. M. Hooke, R. W. Motley, T. Nagashima, M. Porkolab and S. Suckewer, "Proc. 3rd Symposium on Plasma Heating in Toroidal Devices" (Editrice Compositio-Bologna, 1976), Varenna, Italy, p. 68 (1976).
- ⁷ C. C. Daughney, private communication.
- ⁸ H. Hsuan, R. J. Hawryluk, S. Suckewer and H. Takahashi, Third Topical Conference on Radio Frequency Plasma Heating, California Institute of Technology, 11-13 January 1978, paper C8 "Energy Loss and Edge Density Measurements During ICRF Experiments on ATC".
- ⁹ S. P. Hirshman, Phys. of Fluids 20, 589 (1976).
- ¹⁰ S. A. Cohen, J. L. Cecchi, E. S. Marmor, Phys. Rev. Lett. 35, 1507 (1975).
- ¹¹ J. A. Schmidt, N. R. Sauthoff and R. J. Hawryluk, Phys. Rev. Lett. 33, 1272 (1974).

ION CYCLOTRON HEATING IN THE WISCONSIN SUPPORTED TOROIDAL OCTUPOLE AND QUADRUPOLE

by

A. P. Biddle, K. J. Miller, and J. C. Sprott
Department of Physics, University of Wisconsin
Madison, Wisconsin 53706

ABSTRACT

Ion cyclotron heating at the fundamental frequency in the supported octupole has produced ~ 600 eV ions at densities 10^{10} cm^{-3} . Conversion to a quadrupole configuration has demonstrated comparable temperatures, but at reduced densities and with higher wall reflux. The launching structure consists of a single turn, insulated, unshielded loop, coaxial to the four/two confining field producing hoops. ICRF techniques for the Tokapole II device are also considered.

INTRODUCTION

Recent work¹ in the supported octupole has provided substantial increases in ion temperatures, ~ 600 eV, for low density $< 10^{10} \text{ cm}^{-3}$ hydrogen plasmas. Conversion to a quadrupole configuration has shown unfavorable trends in confinement and coupling efficiencies, suggesting the advantages of higher order multipoles.

EXPERIMENTAL APPARATUS

The basic launching structure throughout has been a flat, copper hoop 5 cm wide, supported 1.7 cm above the vacuum tank floor by a teflon insulator. The hoop is parallel to the main hoops, extending 327° around the machine, and is fed through the floor via teflon feedthroughs. The antenna has been insulated from contact with the plasma by both a glass and later, in an attempt to reduce reflux, a MACOR top cover. No matching device is needed between the antenna and transmitter, as the hoop is the tank circuit inductor, paralleled by external capacitors. This insures tracking of the plasma reactance, preventing detuning. Frequency shifts were $\sim 0.1\%$ of the unloaded frequency.

The transmitter, Fig. 1, itself is currently operable over a frequency range from 1.9 MHz to 2.6 MHz. It consists of a two tube push-pull water-cooled

oscillator, and a delay line type power supply, storing 4 MJ. The output power pulse from the delay line is a square wave, selectable in 100 μ s steps up to 1 ms. RF power levels of 1.8 MW out have been achieved, though power supply voltage limitations have prevented regular use of this level. The two tube version will also drive lower impedances, and hence higher plasma densities than the single tube oscillator that we previously used.

The principle diagnostic consisted of a 127° curved plate electrostatic analyser², sampling the velocity distribution from the field null on the machine minor axis through a high permeability extractor tube. Repeated shots at high reproducibility allow point by point plotting of the distribution. Plasma loading of the tank circuit could also be measured. Density, both neutral and plasma, could not be measured directly during the pulse because of RF interference, but could be inferred from after-glow measurements.

Fig. 2a and 2b show the locations of the hoops, launching device, and ion extractor pipe in the octupole and quadrupole, respectively. The unusual diagonal configuration of the quadrupole was necessitated by the requirement for the field null to be accessible to the ion extractor pipe while simultaneously placing the highest density plasma near the launching structure.

EXPERIMENTAL RESULTS

Fig. 3 gives the actual achieved ion temperatures for the heated component versus the electric field at the antenna surface. The two modes of operation are comparable in terms of temperatures. However, the density of heated ions was reduced for several reasons in the quadrupole case. Particle confinement times in the unoptimized quadrupole were $\sim 500 \mu$ s, about $\frac{1}{2}$ that of the octupole.³ Most important was the large spatial separation of the launching structure from the densest plasma. The ECRH pre-ionization plasma peaks much nearer the hoops than in the octupole case. It has been shown³ that even the vacuum RF field is attenuated sharply by separation from the antenna. Because the plasma density peaks further off the separatrix than in the octupole case, it is spatially further from the antenna, and hence samples a weaker rf field, with reduced heating efficiency. Also contributing to the lower density is the dependence of the $|B|$ of the quadrupole on the distance from the minor axis. The enhanced losses at high temperatures, primarily to charge exchange and hoop supports, requires supplemental ECRH heating to maintain the plasma density. This was supplied by a 5 KW 2.45 GHz and a 10 KW 9.0 GHz source. But $|B_1| / |B_2| \sim .67$ and 2.45 for the respective microwave resonance zones, thus causing a spatial separation of the plasma generating and heating zones. The density of ions inferred from the electrostatic analyser shows a larger component of cold ions than is found in the octupole case.

Fig. 4 is a typical analyser plot, showing the usual three component Maxwellian of the quadrupole. An octupole case is qualitatively similar, except as noted above. The lowest energy component consists of the weakly heated plasma and the reflux component. The intermediate is the strongly

heated component, and the decrease above ~ 600 eV represents the point where the particles' gyrodiameter becomes comparable to the size of the machine.

Fig. 5 shows ion temperature as a function of time. The full 1 ms RF pulse is used here. One can see that the highest temperature is achieved almost immediately, and then falls steadily as wall reflux raises the neutral pressure. The wall cleanliness has been shown to be the critical factor in achievable temperature.⁴

FUTURE EXPERIMENTS

This machine has recently been converted to a tokamak configuration for the purpose of gaining experience with a pure tokamak prior to replacing it with a device called TOKAPOLE II. This device, while similar to an octupole in that it contains four current-carrying rings, will also have a toroidal field ~ 4.4 kG on axis, and is expected to produce toroidal currents ~ 50 kA. It has been optimized to produce a tokamak topology embedded in that of an octupole, and will be the subject of eigenmode studies. Initial plans call for low power, 1 kW, loading measurements at 14 MHz. Initial calculations indicated that at least low order mode numbers should be attainable with moderate densities, $\sim 5 \times 10^{12} \text{ cm}^{-3}$. Preliminary work will be with a single turn, insulated unshielded loop, as well as with electrostatically driven antennas. After loading parameters are established, high power experiments will be done.

CONCLUSION

Past experience has shown the ICRH heating in multipole geometry can be effective if proper design optimization is used. Fast wave ICRH should offer advantages in both loading efficiency and energy deposition controls though wall reflux is expected to continue as a major obstacle.

ACKNOWLEDGEMENT

This work was supported by the U. S. Department of Energy.

REFERENCES

1. J. D. Barter and J. C. Sprott, Plasma Physics 19, 945 (1977).
2. C. W. Erickson, Ph.D. Thesis, University of Wisconsin (1970).
3. D. Holly, private communication.
4. J. D. Barter, Ph.D. Thesis, University of Wisconsin (1976).

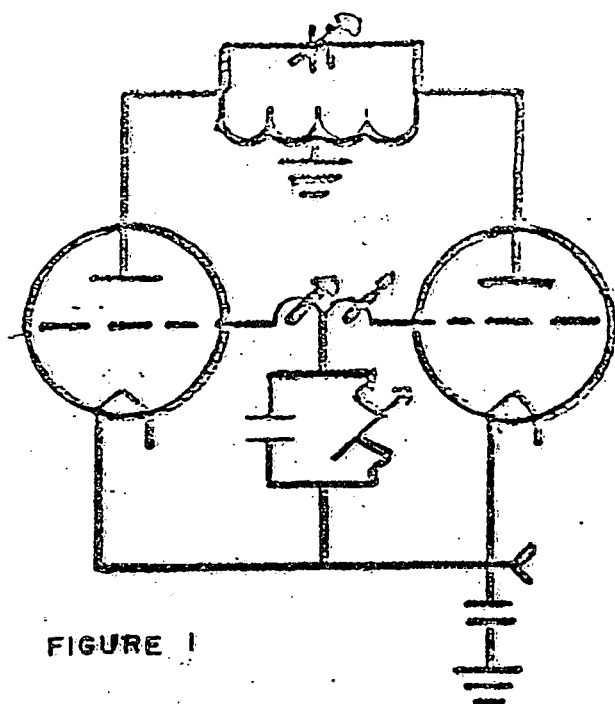


FIGURE 1

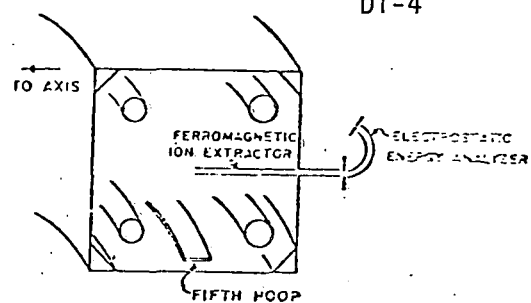


FIGURE 2a

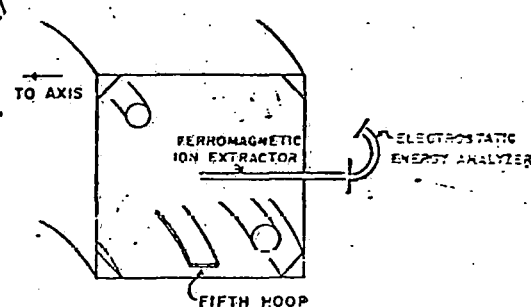
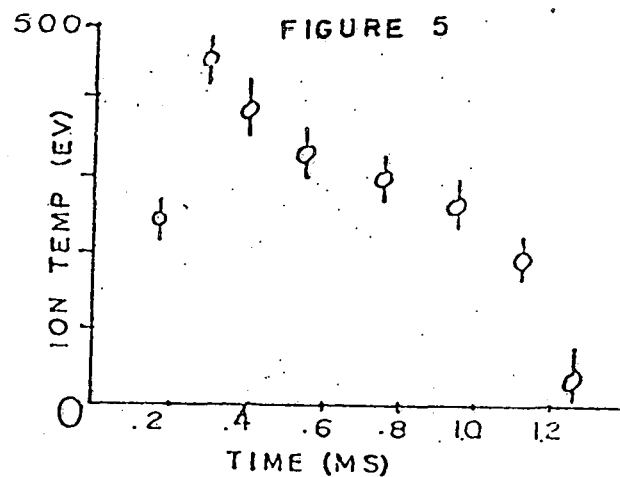
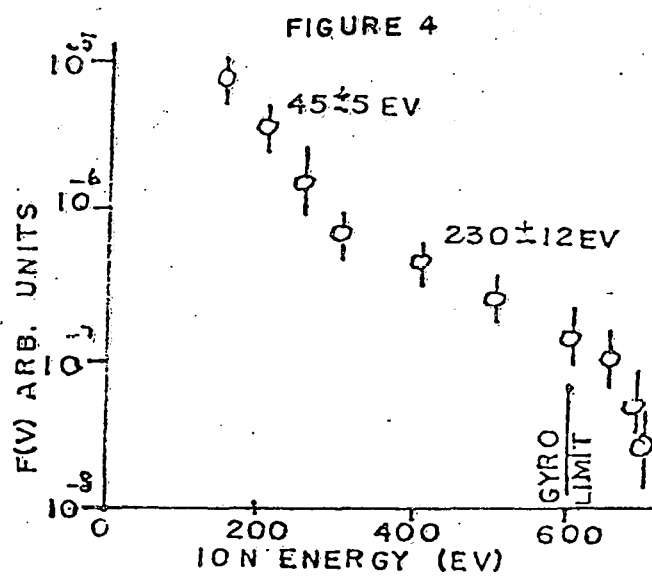
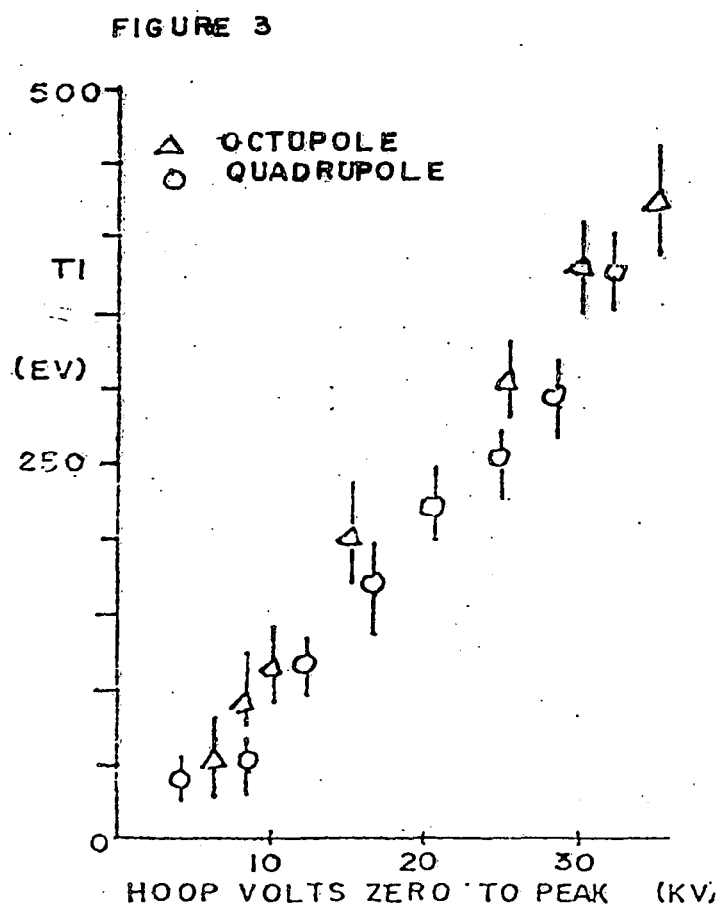


FIGURE 2b



Parametric Instabilities Near the Ion Cyclotron Frequency in Single and Multi-ion Species Plasma

M. Ono

Plasma Physics Laboratory, Princeton University
Princeton, New Jersey 08540

M. Porkolab

Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

and

R. P. H. Chang

Bell Telephone Laboratories
Murray Hill, New Jersey 07974

We present experimental and theoretical investigations of parametric instabilities near the ion cyclotron frequency (i.e., $\omega = 1.3 \omega_{ci} - 10 \omega_{ci}$). In a multi-ion species plasma, the kinetic ion-ion hybrid mode is parametrically excited. The dispersion relation and the threshold for the excitation are measured. In a low density plasma (i.e., $\omega_{pi} \geq \omega_{ci}$), and non-resonant ion cyclotron quasi-mode ($\omega_1 \approx \omega_{ci}$) can be parametrically excited. Associated with this decay, we have observed a strong ion heating.

One of the attractive means to supplementally heat the tokamak plasma is to apply an intense RF electric field in the ion cyclotron range of frequency via an RF induction coil.¹ Because of the multi-ion species nature of the reactor plasma, it is of importance to understand the possible nonlinear phenomena associated with such RF electric fields in both a single and a multi-ion species plasma. In this paper, we present a result related to parametric excitation of electrostatic waves in the vicinity of the ion cyclotron frequency.^{2,3}

One can write a dispersion relation of such parametric coupling equation in the following form:

$$\epsilon \epsilon^- + \epsilon \sum_{\sigma} \frac{|\mu_{\sigma}|^2}{4} (\chi_{\sigma} - 2 \chi_{\sigma}^-) - \sum_{\sigma} \frac{\mu_{\sigma}}{2} (\chi_{\sigma} - \chi_{\sigma}^-) \sum_{\sigma} \frac{\mu_{\sigma}^*}{2} (\chi_{\sigma} - \chi_{\sigma}^-) = 0, \quad (1)$$

where $\epsilon = 1 + \sum_{\sigma} \chi_{\sigma}$ is the plasma dielectric constant for the low frequency mode and $\epsilon^- = 1 + \sum_{\sigma} \chi_{\sigma}^-$ is the plasma dielectric constant for the lower sideband which is usually resonant and accordingly the upper sideband contribution is neglected; $\mu_{\sigma} \ll 1$ is the ratio of drift excursion of σ species to the decay wavelength. The summation σ is over both electron and all the ion species.

We have experimentally investigated the parametric phenomena in the ion cyclotron range of frequency in Princeton's L-4 device.^{2,3} The range of the experimental parameter was as follows: the magnetic field, $B_0 \leq 4.2$ kG; the helium ion cyclotron frequency, $f_{ci}(\text{He}) \leq 1.6$ MHz; the plasma density, $n_0 \approx 3 \times 10^8 - 10^{10} \text{ cm}^{-3}$; the temperature, $T_e = 3-5$ eV and $T_i < 0.1$ eV; the neutral gas filling pressure in the experimental region, $P \approx 4 \times 10^{-5} - 1.5 \times 10^{-4}$ torr; the pump frequency, $f_0 \approx 2 - 4$ MHz. A Faraday shielded RF induction coil was used to impress a radially uniform RF electric field of up to 15 volt/cm.

In a helium-neon plasma, we have observed parametric excitation of the kinetic ion-ion hybrid mode (two ion species versions of the electrostatic ion cyclotron waves) whose typical decay frequency spectrum is shown in Fig. 1(a). The decay frequencies were observed to occur above each ion cyclotron frequency and varied with the magnetic field and relative ion concentration ratio in accord with the theory. They decay disappeared when either one of the ion species was removed. In Fig. 1(b), the dots show the experimentally measured values of $\omega(k)$, as determined by interferometric measurements of the wave numbers of the decay waves. In the same figure, we plotted the theoretical dispersion curves of the kinetic ion-ion hybrid modes for various ion temperatures. We see that the best agreement between theory and experiment is obtained by assuming $T_i \approx 1/40$ eV (or T_i is approximately equal to room temperature). We have also propagated the kinetic hybrid mode linearly and the resulting dispersion relation agrees well with the parametrically obtained dispersion relation. The coupling mechanism and the threshold for this decay can be obtained from Eq. (1) by letting the low frequency mode to be also resonant, $\epsilon_R = 0$, and noting $\chi_1 - \chi_1^- = (\chi_2 - \chi_2^-)$, and $\chi_e = \chi_e^-$, Eq. (1) can be written in the following form:⁴

$$\frac{|\mu_1 - \mu_2|^2}{4} = \frac{\text{Im } \epsilon \text{ Im } \epsilon^-}{[\text{Re}(\chi_1 - \chi_1^-)]^2}, \quad (2)$$

where $\mu_1 - \mu_2$ is the relative ion drift velocity. The measured threshold agrees well with the threshold predicted by Eq. (2). We note that the parametric coupling goes to zero when one of the ion species is being removed.

When the plasma density was reduced such that $\omega_0 \approx \omega_{ci} + \omega_{pi}$ and $\omega_{pi} \gtrsim \omega_{ci}$, a nonresonant ion cyclotron quasi-mode ($\omega_1 \approx \omega_{ci}$) was parametrically excited even in a single ion species plasma. A typical decay frequency spectrum obtained in a helium plasma is shown in the insert of Fig. 2(a). (We note that similar decay spectra were observed in a neon plasma and also in a helium-neon plasma. In Fig. 2(a), we have plotted the observed decay frequencies (where the decay amplitude is maximum for a given RF power) versus the magnetic field strength, with the pump frequency fixed at 4 MHz. The measured parallel wavelength of the low frequency mode is of the order of 15 ~ 16 cm and thus $\omega/k_{||} \ll V_{Te}$. The parallel wavelength of the lower sideband is of the order of $\lambda_{||2} \gtrsim 150$ cm (which is the order of the machine length) and hence this is a fluid mode (i.e., $\omega/k_{||} \gg V_{Te}$). The measured dispersion relation of the low frequency mode is shown in Fig. 2(b) by the circles, and this

shows a clear deviation from the theoretical dispersion relationship of the electrostatic ion cyclotron wave which is shown by a solid curve (except when $\omega/\Omega_{He} \approx 1.1$). The parametric coupling equation for this decay can be written from Eq. (1) in the following form:⁵

$$\frac{|\mu|^2}{4} = - \frac{\epsilon \epsilon^-}{\chi_e \chi_i}, \quad (3)$$

where $|\mu|^2 = |\mu_e - \mu_i|^2$. We have used the following ordering in electron and ion susceptibilities $\chi_i, \chi_e \gg \chi_i^-, \chi_e^-, 1$, which is satisfied in this experiment. Using Eq. (3), we have calculated the threshold for the experimental parameters and found them to agree well with the experimentally observed threshold. Since this decay is induced by the relative drift of electrons and ions, it is relatively insensitive to the presence of a second ion species which is in accord with the experimental observation. Using a perpendicular ion energy analyzer which could detect energetic ions with energy $E_i > 0.5$ eV, efficient ion heating associated with this decay was observed. In Fig. 3, we plotted the collected ion current versus the bias voltage on a semi-log paper. We see that there is a considerable amount of hot ion tail produced as a result of the ion cyclotron quasi-mode excitation. Using typical tokamak plasma parameters, we computed that this type of decay may be excited at the edge of the tokamak plasma near the RF induction coil with a relatively low RF electric field of 60 volt/cm.

We thank P. Bellan, L. Johnson, E. Hinnov, K.-L. Wong and M. Yamada for their helpful discussions, and J. Johnson, J. Taylor, R. McWilliams and W. L. Hsu for their technical assistance.

Work supported by U.S.D.O.E. Contract EY=76-C-02-3073.

REFERENCES

- ¹J. Adam et al., in Proceedings of the Fifth Conference on Plasma Physics and Controlled Nuclear Fusion Research, (Tokyo, Japan, 1974), Vol. II, p. 65; H. Takahashi et al., Phys. Rev. Lett. **39**, 31 (1977).
- ²M. Ono, R. P. H. Chang, M. Porkolab, Phys. Rev. Lett. **38**, 962 (1977).
- ³M. Ono, M. Porkolab, R. P. H. Chang, Princeton PPPL-1395 (1977).
- ⁴J. L. Sperling and F. W. Perkins, Phys. Fluids **17**, 1857 (1974).
- ⁵M. Porkolab, Phys. Fluids **17**, 1432 (1974).

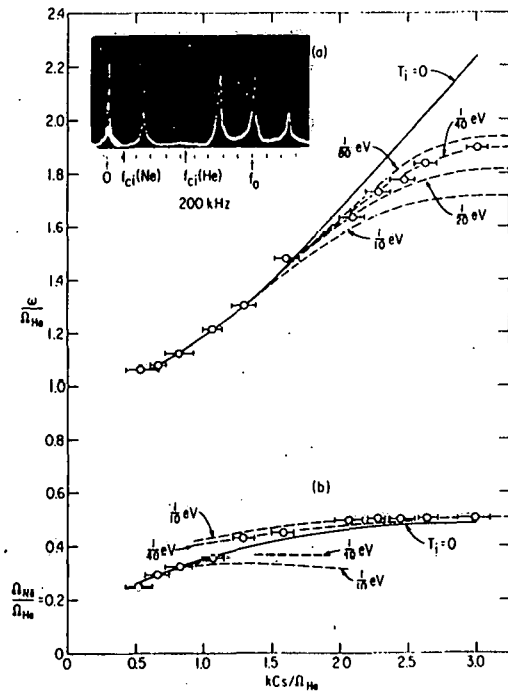


Fig. 1. (a) Parametric Decay Spectrum. $f_{ci}(He)$, $f_{ci}(Ne)$ are the helium, neon ion cyclotron frequencies. ($B = 2.9$ kG, $f_0 = 2$ MHz, He:Ne = 4:6). (b) Dispersion curve of kinetic ion-ion hybrid mode. Solid and dashed curves, theory. (He:Ne = 4:6, C_s is the helium acoustic speed). Circles are the experimentally measured values.

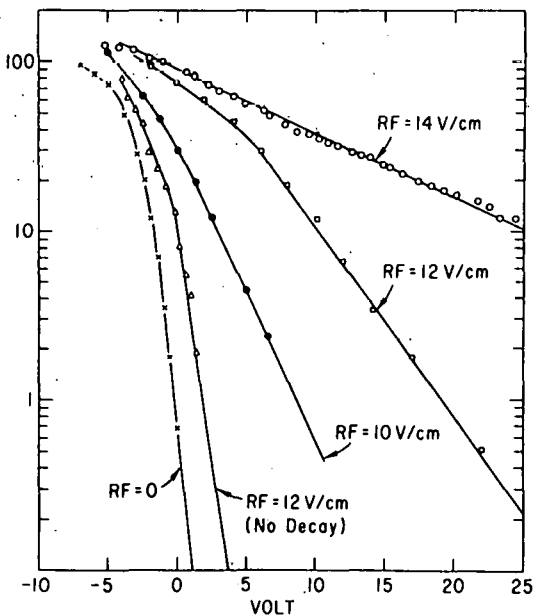


Fig. 3. The perpendicular ion energy analyzer ion collector current versus the bias voltage. The parameters are pump RF electric field.

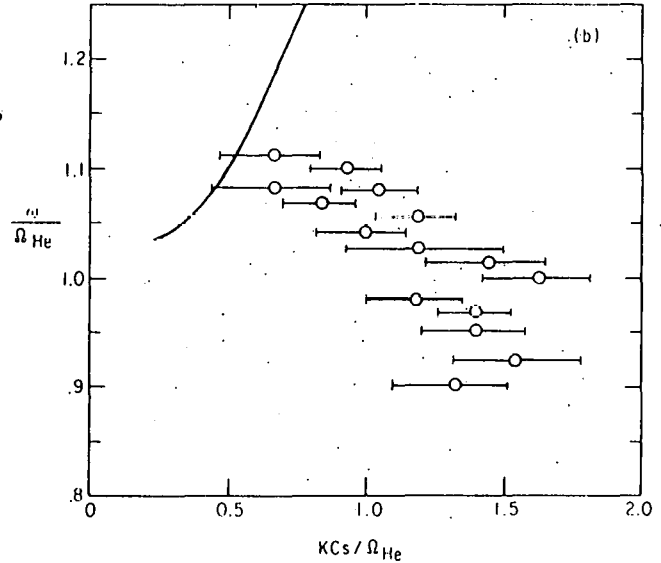
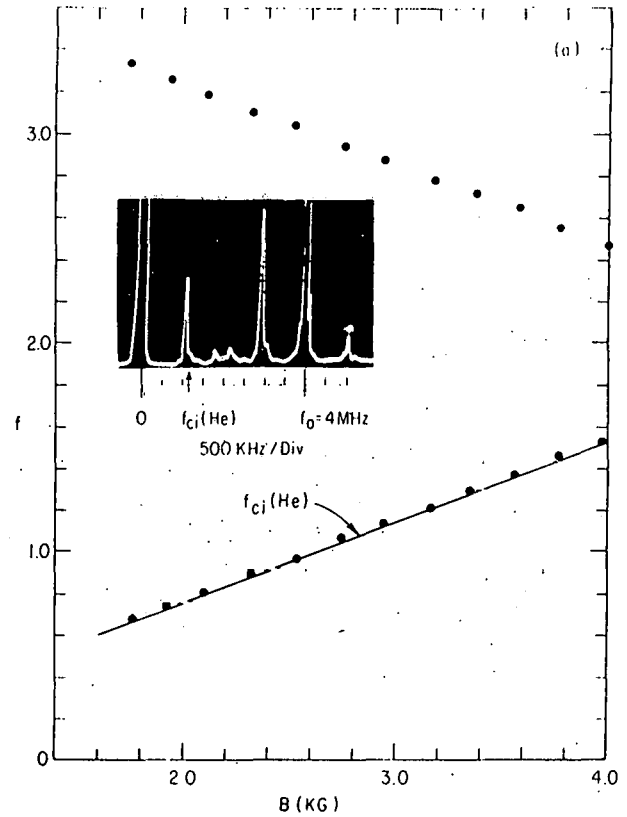


Fig. 2. (a) The observed decay frequency versus magnetic field strength. $f_0 = 4$ MHz. The insert shows a typical decay spectrum in a helium plasma. $B_0 = 3.15$ kG. (b) Measured dispersion relation of the ion cyclotron quasi-mode (circles) and the theoretical dispersion relation of the electrostatic ion cyclotron wave (solid curve). $f_0 = 4$ MHz. $B_0 = 3.15$ kG.

Alfvén Wave Heating with High-Q Eigenmodes*

by

C. F. F. Karney and F. W. Perkins

Princeton Plasma Physics Laboratory, Princeton University, Princeton, NJ 08540

We consider the equation governing the mode structure of Alfvén waves. Since the Alfvén speed is a function of density, the compressional Alfvén wave changes from evanescent to propagating at some point in the plasma. However the presence of the shear Alfvén resonance changes this cutoff into a cutoff-resonance-cutoff triplet. By analytically and numerically studying the properties of the governing equation, we determine the absorption due to the resonance and Q for the eigenmode. We show that heating is possible using relatively high- Q eigenmodes.

The dispersion relation for Alfvén waves may be calculated using the cold two-fluid equations, assuming quasineutrality. The result is

$$D(\omega, k) = (A - k^2)(A - k_{\parallel}^2) - D^2 \quad (1)$$

where $A = k_A^2 \Omega_i^2 / (\Omega_i^2 - \omega^2)$, $D = (\omega / \Omega_i) A$, $k_A = \omega / v_A$, $v_A = c \Omega_i / \omega_{pi} = B / (\mu_0 n m_i)^{1/2}$. Consider a slab model with B constant and parallel to $\hat{z}$ and the density gradient parallel to $\hat{x}$. Then if we impose k_y and k_z the solution of (1) for k_x is

$$k_x^2 = \frac{(A + D - k_z^2)(A - D - k_z^2)}{A - k_z^2} - k_y^2. \quad (2)$$

It is clear from (2) that as the wave penetrates the plasma (note $A, D \propto n$) it passes a cutoff followed by a resonance and then a second cutoff. The wave is propagating on the high density side and evanescent on the low density side of this cutoff-resonance-cutoff triplet.

In order to determine the effect of the resonance on the propagation we convert (2) back into a differential equation,

$$(d^2/dx^2)\phi + k_x^2\phi = 0 \quad (3)$$

where k_x is given by (2) and ϕ is an appropriate mixture of the various field amplitudes. We will assume a linear density gradient, $n(x) = n_0 x/L$, where n_0 is the maximum density. Taking $k_A(x_R) = k_z$ and introducing a rescaled independent coordinate, $X = (x - x_R) k_{A0}^{2/3} / L^{1/3}$, where $k_{A0} = k_A(L)$, we obtain

$$\frac{d^2\phi}{dX^2} + \left[\frac{(X - N^2 s)(X + N^2 s)}{X + N^2 s^2} - M^2 \right] \phi = 0 \quad (4)$$

where $N = k_z L^{1/3} / k_{A0}^{2/3}$, $M = k_y L^{1/3} / k_{A0}^{2/3}$ and $s = \omega / \Omega_i$. We consider the case $s < 1$. If the shear Alfvén resonance is not too close to the center of the plasma then $N < 1$. Therefore for simplicity we replace the denominator in (4) by X . We may write (4) in a generic form,

*Work supported by U.S. Department of Energy (Contract EY-76-C-02-3073)

$$\frac{d^2\phi}{dX^2} + \frac{(X + \delta_1)(X - \delta_2)}{X} \phi = 0 \quad (5)$$

where $\delta_2 = \frac{1}{2}M^2 + (\frac{1}{4}M^4 + N^4s^2)^{1/2}$ and $\delta_1 = N^4s^2/\delta_2$.

The solution to (5) is singular at $X = 0$. As $|X| \rightarrow \infty$, $\phi \rightarrow \text{Ai}[-(X + \delta_1 - \delta_2)]$ or $\text{Bi}[-(X + \delta_1 - \delta_2)]$. We will start at $X = -\infty$ with the Ai solution, which decays as $X \rightarrow -\infty$. Then by integrating above the singularity at $X = 0$ we obtain $\phi = A \text{Ai}[-(X + \delta_1 - \delta_2)] + B \text{Bi}[-(X + \delta_1 - \delta_2)]$ at $X = \infty$. (We determine that we must go above the singularity by letting ω have a small positive imaginary part, corresponding to a growing wave.) By regarding this mixture of solutions as the sum of an incident wave (from the right) and a reflected wave we may determine the fraction of the incident energy absorbed as

$$q = 4 \text{Im}(A^*B)/|A - iB|^2 \approx 4 \text{Im}(B/A) \quad (6)$$

for $|B| \ll |A|$. Note that $q > 0$ corresponds to dissipation.

In solving (5) we treat N^2s as small. We can then distinguish two cases of interest:

- (1) $M \ll 1, \delta_2 \ll 1, \delta_1 \ll 1$
- (2) $M \gg 1, \delta_2 = M^2 \gg 1, \delta_1\delta_2 = N^4s^2 \ll 1$

Case 1. $\delta_1, \delta_2 \ll 1$

For $|X| \gg 1$ we have $\phi = \text{Ai}(-X)$ or $\text{Bi}(-X)$. We solve (5) in the connection region by developing series solutions about the singular point $X = 0$,

$$\phi = \phi_1(X) = f(-X) - \delta_1\delta_2 \log(X)g(-X) + O(\delta^2)$$

$$\phi = \phi_2(X) = -g(-X) + O(\delta^2)$$

where $c_1 f(-X) = \text{Bi}(-X) + \sqrt{3} \text{Ai}(-X)$, $c_2 g(-X) = \text{Bi}(-X) - \sqrt{3} \text{Ai}(-X)$, $c_1 = \text{Ai}(0)$, $c_2 = -\text{Ai}'(0)$. For large X we may treat $\log(X)$ as a constant, in which case ϕ_1 and ϕ_2 are linear combinations of the Airy functions. If we do the connection of the solutions at $X = \pm X_0$, we find that the solution which becomes $\text{Ai}(-X)$ as $X \rightarrow -\infty$ is

$$\phi = \text{Ai}(-X) + i 1.24 \delta_1 \delta_2 \text{Bi}(-X) + O(\delta^2) \quad (7)$$

for $X \rightarrow \infty$, where $1.24 = \pi c_1 / (2\sqrt{3}c_2)$. (Here we have only retained the $O(\delta^2)$ term which contributes to q .) Thus from (6) the dissipation is

$$q = 5 \delta_1 \delta_2 \quad (8)$$

Case 2. $\delta_2 \gg 1, \delta_1\delta_2 \ll 1$

In this case the problem simplifies since, far from the cutoff at $X = -\delta_1$ and the resonance at $X = 0$, the solution is $\text{Ai}(\delta_2 - X)$ or $\text{Bi}(\delta_2 - X)$, while close to these points the factor $X - \delta_2$ in (5) is approximately $-\delta_2$. In the latter case the resulting equation is one for which the solution may be written as an integral,

$$\phi = \int_C \frac{1}{t^2 - \kappa^2} \left[\frac{t + \kappa}{t - \kappa} \right]^\epsilon e^{Xt} dt \quad (9)$$

where $\epsilon = \frac{1}{2}\delta_1\sqrt{\delta_2} \ll 1$ and $\kappa = \sqrt{\delta_2}$. This solution enables us to make the connection between the Airy solutions. The end points of C may be chosen where the integrand vanishes at $|t| \rightarrow \infty$. For $X < 0$ we pick $C = C_1$ shown in Fig. 1a, since the integral then is of the order of $e^{\kappa X}$ and so matches onto $\text{Ai}(\delta_2 - X)$. For $X > 0$, C_1 becomes the contour shown in Fig. 1b. We split C_1 into two pieces C_a and C_b (see Fig. 1c). With appropriate choice of cuts the integral over C_a is purely imaginary, since the integrand takes on complex conjugate values either side of the real t axis. Letting $\epsilon \rightarrow 0$ in (9) we may evaluate the integral as the residue of the pole at $t = \kappa$ and obtain $\int_{C_a} \approx (i\pi/\kappa) e^{\kappa X}$. For the integral over C_b we let $(t - \kappa)^\epsilon = e^{-i\pi\epsilon}(\kappa - t)^\epsilon$. Taking the factor $e^{i\pi\epsilon}$ out of the integral the remaining terms again give a purely imaginary result for the integral. Thus $\int_{C_b} \approx (i\pi/\kappa) e^{i\pi\epsilon} e^{-\kappa X}$, where as before we have evaluated the integral by letting $\epsilon \rightarrow 0$. The solution for $X > 0$ is (dividing out $i\pi/\kappa$)

$$\phi = e^{\kappa X} + i\pi\epsilon e^{-\kappa X}.$$

(As before the terms we have neglected do not affect q .) Matching this onto the Airy functions gives

$$\phi = \text{Ai}(\delta_2 - X)/\text{Ai}(\delta_2) + i\pi\epsilon \text{Bi}(\delta_2 - X)/\text{Bi}(\delta_2) \quad (10)$$

so we obtain

$$q = 4\pi\epsilon \text{Ai}(\delta_2)/\text{Bi}(\delta_2) \approx \pi\delta_1\sqrt{\delta_2} \exp(-\frac{4}{3}\delta_2^{3/2}). \quad (11)$$

In Fig. 2 we compare the results (8) and (11) with the results obtained by numerically integrating (5). We note that if we use (8) for $\delta_2 < 0.3$ and (11) for $\delta_2 > 0.3$ we are in error by at most a factor of 2 in q .

Finally we note that q is the fractional energy lost per pass of the wave through the machine, whereas $1/Q$ is defined as the fractional energy lost per wave period. Thus Q is approximately ℓ/q where ℓ is the mode number in the x (or radial) direction. As an example consider a plasma with $B = 35$ kG, $n_0 = 3 \times 10^{13} \text{ cm}^{-3}$, $R = 2$ m, and $a = 50$ cm. If we choose a toroidal mode number of $n = 5$, and a wave frequency of 25 MHz ($= \frac{1}{2}f_{cl}$), then $\delta_1\delta_2 = 5 \times 10^{-3}$. From Fig. 2, we then see that values of Q greater than 100 are possible.

Acknowledgment.

The series solutions (Case 1) were obtained using MACSYMA, a symbolic computation system at the Laboratory for Computer Science, Massachusetts Institute of Technology.

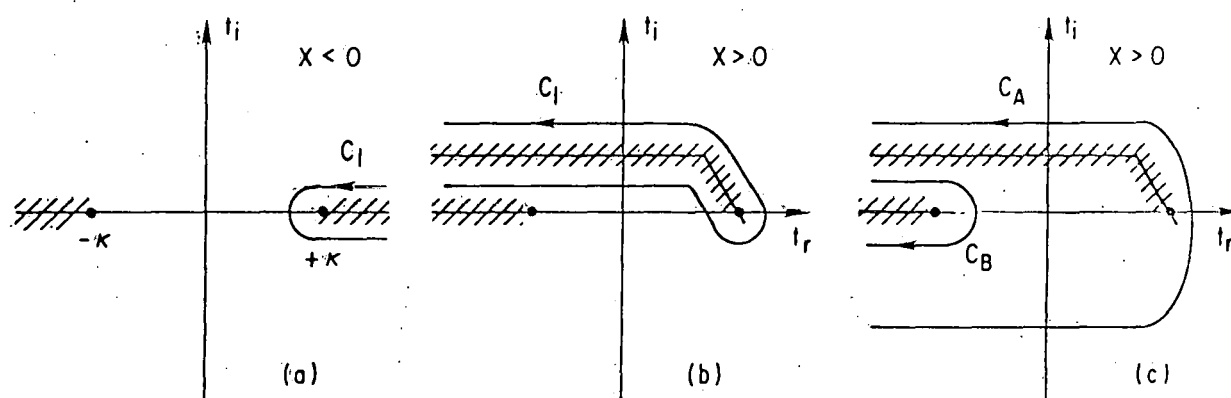


Fig. 1. The contours used in evaluating (9).

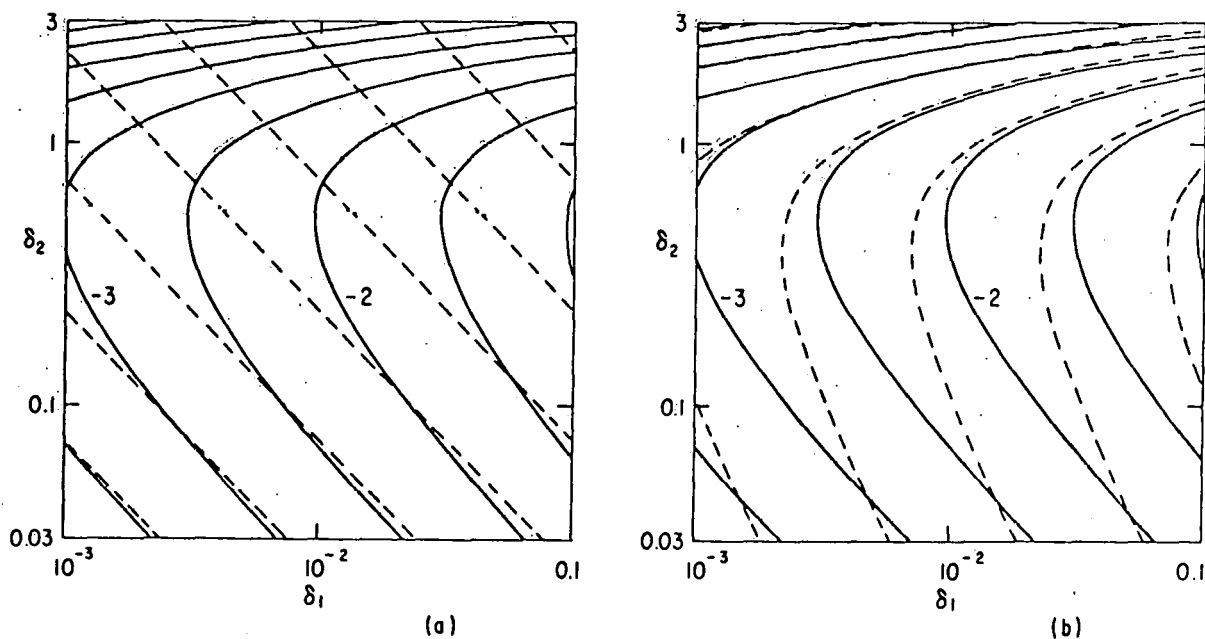


Fig. 2. Contours of $\log_{10}(q)$. The contours are equally spaced at 5 dB intervals. Solid lines give q as determined by integrating (5) numerically. Dashed lines show q as given by (a) (8) and (b) (11).

HEATING THE VARIOUS PLASMA SPECIES VIA THE PROTON-DEUTERON HYBRID RESONANCE

J. JACQUINOT

ASSOCIATION EURATOM-CEA SUR LA FUSION

Département de Physique du Plasma et de la Fusion Contrôlée

Centre d'Etudes Nucléaires

Boîte Postale n° 6... 92260 FONTENAY-AUX-ROSES (FRANCE)

A series of recent papers /1, 4/ has established the idea that even a small amount of hydrogen impurity at fundamental resonance in a deuterium plasma modifies entirely the damping mechanism of the fast magnetosonic wave (FMW). The FMW energy is linearly mode-converted into a slow electrostatic mode (EW) in the ion-ion hybrid resonance layer. The overall damping of the FMW eigen modes can be calculated without knowing the fate of the EW /2 - 4/. Obviously, the evaluation of the heating performance requires to know the trajectory and damping of the EW which contain the information on the localisation and heating rate of the various plasma species. Perkins /3/ has presented preliminary solutions to this problem. In a Tokamak geometry, he showed the paramount importance of the effects of the finite temperature and the poloidal field component normal to the mode conversion layer. In this paper we consider again the same problem but we include the damping effects of the minority species and the integration of the deposited power in each species along the wave trajectory. Moreover we find necessary to solve the wave equation for an arbitrary magnitude of the damping decrement.

We consider a one dimensional slab model with constant poloidal field and density but including the major radius variation (along axis ox) of the magnetic field intensity. Let oz be the direction of the magnetic axis and oy the direction of the mode conversion layer. Snell's law demands that waves propagate with fixed real values of ω , k_z , k_y imposed by the FMW but k_x , assumed complex, is permitted to vary according to the dispersion relation calculated from the dielectric tensor :

$$\begin{aligned}
 K_{11} &= \sum_{\sigma} \Omega_p^2 \left[\left(1 - \frac{\lambda^2}{2}\right) \left\{ \frac{1}{\Omega_c^2 - 1} + \frac{w_1}{2(1 - \Omega_c)} \right\} + \frac{\lambda^2}{2} \left\{ \frac{1}{4\Omega_c^2 - 1} + \frac{w_2}{2(1 - 2\Omega_c)} \right\} \right] \\
 K_{12} &= -K_{21} = \frac{-i \Omega_p^2 p_e}{\Omega_{ce}} + \sum_{\sigma} i \Omega_p^2 \Omega_c \left[\frac{(1 - \lambda^2)}{\Omega_c^2 - 1} \left(1 - \frac{w_1}{2\Omega_c} (1 + \Omega_c)\right) \right. \\
 &\quad \left. + \frac{\lambda^2}{4\Omega_c^2 - 1} \left(1 - \frac{w_2}{4\Omega_c} (1 + 2\Omega_c)\right) \right] \\
 K_{22} &= \sum_{\sigma} \Omega_p^2 \left\{ \frac{1 + 3\lambda^2/2}{\Omega_{\sigma}^2 - 1} \left[1 - w_1 \frac{(1 + \Omega_{\sigma})}{2} \right] + \frac{\lambda^2/2}{4\Omega_{\sigma}^2 - 1} \left[1 - w_{2\sigma} \frac{1 + 2\Omega_{\sigma}}{2} \right] \right. \\
 &\quad \left. - \lambda^2 \right\} - \frac{w_p^2}{\omega^2} \lambda_c^2 (1 - w_e)
 \end{aligned}$$

$$\begin{aligned}
K_{33} &= \frac{2\Omega_p^2}{k_{\parallel}^2 c^2} \frac{w_e \omega^2}{\beta_e \Omega_{ce}^2} \\
K_{13} &= K_{31} = - \Sigma \Omega_p^2 \frac{k_u}{2k_w \Omega_c^2} \left[w_{1\sigma} \left(1 - \frac{\lambda^2}{2}\right) + \frac{\lambda^2}{4} w_{2\sigma} \right] \\
K_{32} &= -K_{23} = \frac{i}{4} \frac{k_u}{k_w} \left\{ \Sigma \frac{\Omega_p^2}{\Omega_c^2} \left[\frac{1 - \lambda^2}{2} w_{1\sigma} + \frac{\lambda^2}{2} w_{2\sigma} \right] - \frac{\Omega_{pe}^2 w_e}{\Omega_{ce}} \right\}
\end{aligned}$$

where $\Omega_p = \omega_p/\omega$, $\Omega_c = \omega_c/\omega$, $w = (1 + \zeta Z(\zeta))$, Z is the plasma dispersion function, $\zeta_{1,2} = (\omega - (2) \omega_c)/k_{\parallel} V$. The index w denotes the direction of the total magnetic field in the plane xoy . It is assumed that $k_y = k_z = 0$. Using the definitions of ref. 3, the angle (zow) is defined as $\epsilon = y/r \cos \theta/Rq$. Then $k_w \approx k_z + \epsilon k_x$; $k_u \approx k_x$. The wave power going to each plasma species is :

$$\frac{\partial W_{\sigma}}{\partial t} = \frac{1}{16\pi} \omega \bar{E}^* \cdot (-i\bar{K}_{\sigma} + i\bar{K}_{\sigma}^+) \cdot \bar{E}$$

and the corresponding damping decrement along the axis ox :

$$k_{i\sigma} = - \frac{1}{2 S_x} \frac{\partial W_{\sigma}}{\partial t}, \quad k_i = k_{ie} + k_{id} + k_{ih}$$

where the Poynting vector and its component are given by :

$$\bar{S} = \frac{c}{16\pi} (\bar{E}^* \times \bar{B} + c.c.); \quad S_x = S_u + \epsilon S_w$$

Finally we integrate numerically the set of 4 differential equations governing the energy transfer between the wave and the plasma species :

$$dP_w = -2 P_w k_i dx; \quad dP_{\sigma} = 2 P_w k_{i\sigma} dx.$$

Results :

The physics of the problem is best illustrated with an example using the parameters obtained in the TFR 600 plasma at mid-radius :

$$n_e = 5.10^{13} \text{ cm}^{-3}; \quad T_e = T_{id} = T_{ih} = 500 \text{ ev}; \quad n_p = n_h/n_d = 0.2; \quad B = 40 \text{ kG};$$

$\nu = 60.8 \text{ MHz}$; $\epsilon = 6.7 \cdot 10^{-2}$ ($q = 1.5$). Fig. 1 shows the solution of the dispersion relation in the region where the electromagnetic FMW couples to the electrostatic branch. The striking feature of the hot plasma dispersion relation is that the energy converted in the electrostatic branch can travel only to the low field side of the Tokamak. The slope of the curve is such that the wk_B approximation, $k_{\parallel} \gg k_{\parallel}^{-1} dk_{\parallel}/dx$, will be valid only when the mode conversion region is far from the cyclotron resonance, i.e. for $n_p \gtrsim 0.1$. Fig. 2 represents the damping decrements related to each species. Electron Landau damping dominates on the high field side but further along the trajectory the fundamental and harmonic damping of respectively the protons and deuterons take over. The resulting energy

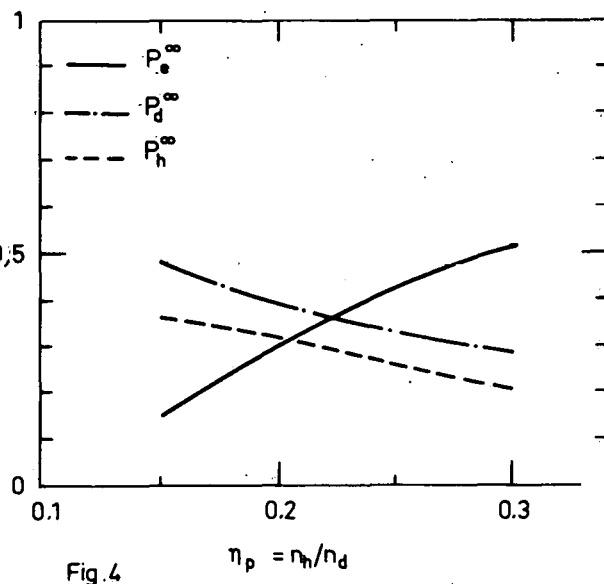
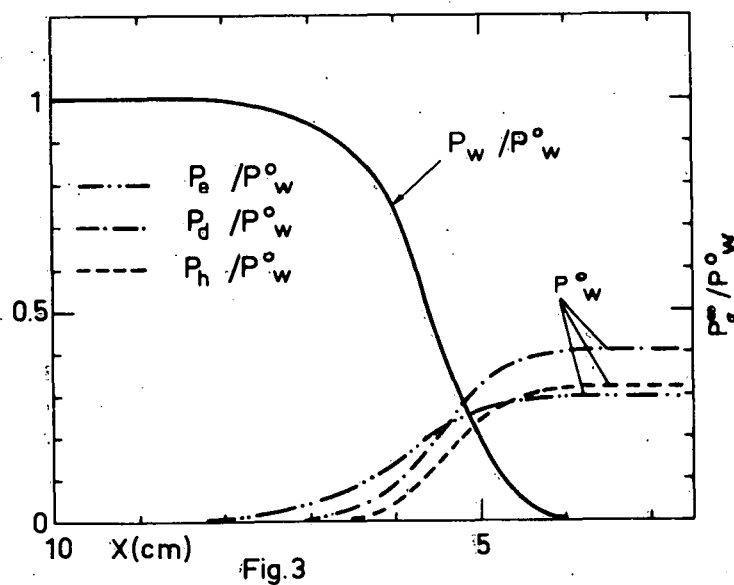
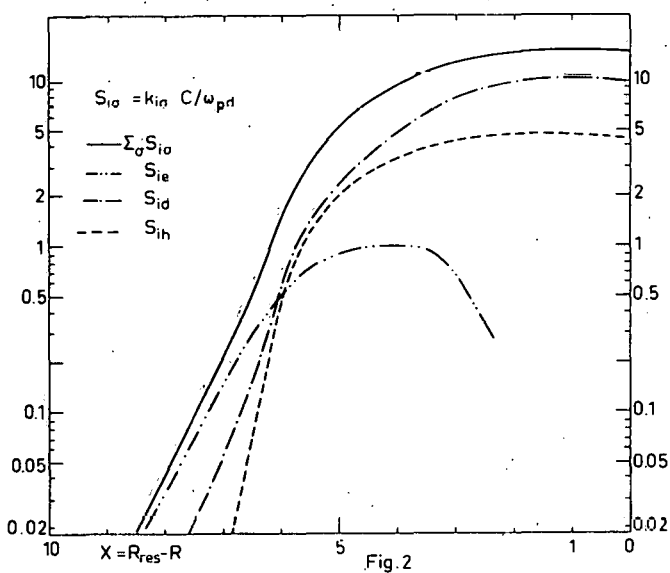
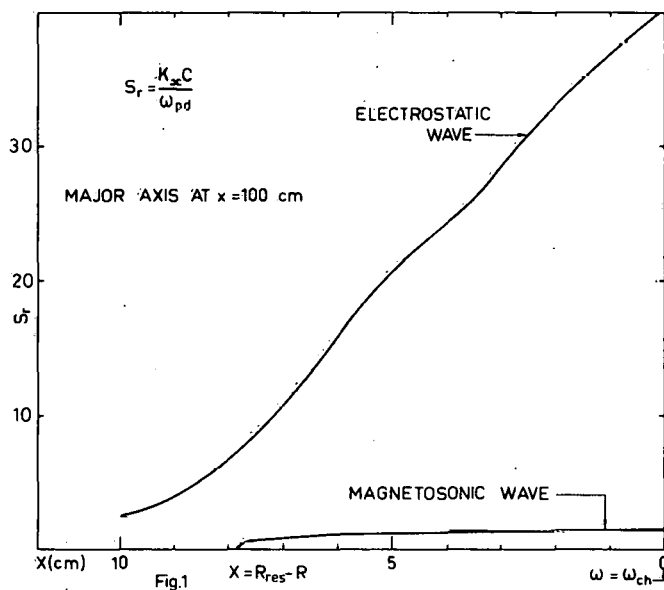
exchange between wave and particles is seen on fig. 3. Clearly the wave is entirely absorbed before it has a chance to travel far from the mode conversion layer. The exact share of energy going to each species depends quite sensitively on the parameters ϵ , η , temperatures. For the present generation of tokamaks and for moderate values of η , the 2 ion components receive most of the energy but the protons and deuterons receive a comparable amount. This means that the energy received per proton is about 5 to 10 times larger than for the other species. If the energy equipartition time is not small compared to the heating time of the protons then T_{ih} becomes larger than T_{id} and this new situation results in a further increase of the energy received per proton (table 1) suggesting a runaway process. Such a proton high energy tail formation has been observed in the Russian experiments /5, 6/. Exaggerated perpendicular heating of the protons may also increase the number of poorly confined localized particles. This effect is a possible candidate for the deleterious effects observed in ICRF experiments. Our model suggests to avoid the situations of exaggerated proton heating i.e. when $10^{-2} \lesssim \eta \lesssim 5 \cdot 10^{-2}$ and to work at a high density regime where the equipartition time is τ_{small} .

It is a pleasure to acknowledge enlightening discussions with Drs. J. ADAM and R. AYMAR.

References

- /1/ Equipe TFR, Proc. 6th Int. Conf. Berchtesgaden, (1976) 3, IAEA, Vienna (1977) 39.
- /2/ J. JACQUINOT, B.D. Mc VEY, and J.E. SCHARER, Phys. Rev. Letters 39 (1977) 88.
- /3/ F.W. PERKINS, Nuclear Fusion 176 (1977) 1197.
- /4/ I. FIDONE, L. GOMBEROFF, EUR-CEA-FC-905 (1977).
- /5/ U.L. VDOVIN et al, Proc. 3rd Int. Meeting, Grenoble 1976, 2 Grenoble (1976).
- /6/ V.U. BUZANKIN et al Proc. 6th Int. Conf. Berchtesgaden (1976) 3, IAEA, Vienna (1977) 61.

$T_{ih}(\text{ev})$	ϵ	P_e^∞/P_w°	P_d^∞/P_w°	P_h^∞/P_w°	$T_e = T_{id} = 500 \text{ ev}$ $n_e = 5.10^{13} \text{ cm}^{-3}$ $B = 40 \text{ kG}$ $v = 60.8 \text{ kG}$ $k_z = 10 \text{ m}^{-1}$
500	$6.7 \cdot 10^{-2}$	29.4	38.8	31.8	
1000	$6.7 \cdot 10^{-2}$	14.8	34.1	51.1	
500	5.10^{-2}	37.2	54.6	8.2	



MODE CONVERSION AND ABSORPTION IN A DEUTERIUM PLASMA WITH A HYDROGEN IMPURITY

D. G. Swanson

University of Southern California

Introduction

Since experiments attempting to use harmonic ion cyclotron heating in tokamaks with deuterium normally have ordinary hydrogen in the machine as an impurity, it has been noted that the two-ion hybrid resonance may completely mask the harmonic heating.^{1, 2} It is the purpose of this paper to derive an approximate dispersion relation relevant for $\omega \approx 2\omega_{cD} = \omega_{cH}$ with a toroidally varying magnetic field. Electron Landau damping shall be neglected and the K_{zz} component of the dielectric tensor shall be taken to be infinite. The exact dispersion relation with $K_{zz} \rightarrow \infty$ may be written as³

$$(k_{\parallel}^2 - K'_1)(k_{\parallel}^2 + k_{\perp}^2 - K'_1 - 2K'_0) + K'_2{}^2 = 0 \quad (1)$$

where $\vec{K}' = (\omega^2/c^2)\vec{K}$ and K_0 , K_1 and K_2 are defined in ref. 3. The K'_0 term will be neglected since its resonant terms are higher order in $\lambda_D = \frac{1}{k_{\perp}} \frac{k_{\perp}^2 \rho_{LD}}{\rho_{LD}}$ or λ_H than corresponding terms in K'_1 and K'_2 and λ_D is taken to be small. Writing out K'_1 and K'_2 to first order in λ_D and zero order in λ_H , they are

$$K'_1 = \frac{\omega^2}{c^2} \left\{ 1 + \frac{\omega_{pD}^2}{2\omega k_{\parallel} v_D} \left[Z(\zeta_{1D}) + Z(\zeta_{-1D}) + \lambda_D (Z(\zeta_{2D}) + Z(\zeta_{-2D})) \right] + \frac{\omega_{pH}^2}{2\omega k_{\parallel} v_H} \left[Z(\zeta_{1H}) + Z(\zeta_{-1H}) \right] \right\}$$

$$K'_2 = \frac{i\omega^2}{c^2} \left\{ \frac{\omega_{pD}^2}{2\omega k_{\parallel} v_D} \left[Z(\zeta_{-1D}) - Z(\zeta_{1D}) + \lambda_D (Z(\zeta_{-2D}) - Z(\zeta_{2D})) \right] + \frac{\omega_{pH}^2}{2\omega k_{\parallel} v_H} \left[Z(\zeta_{-1H}) - Z(\zeta_{1H}) \right] - \frac{\omega_{pD}^2}{\omega \omega_{cD}} - \frac{\omega_{pH}^2}{\omega \omega_{cH}} \right\}$$

where

$$\zeta_{\pm nD} = \frac{\omega \pm n\omega_{cD}}{k_{\parallel} v_D}, \quad \zeta_{\pm 1H} = \frac{\omega \pm \omega_{cH}}{k_{\parallel} v_H}$$

and ω_{cD} and ω_{cH} are the deuterium and hydrogen cyclotron frequencies, respectively, and v_D and v_H are the corresponding thermal speeds. Using the large argument expansion for $Z(\zeta)$ for all nonresonant terms, introducing the variable $\chi = 8(R_0 - R)/R$ where the magnetic field varies as $B(R) = B_0 R_0/R$, the dielectric tensor components may be approximated by

$$K'_1 = \frac{\omega^2}{V_A^2} \left(-\frac{1}{3} + \frac{2\eta_p F_1}{\chi} + \frac{\lambda_D F_2}{\chi} \right) \quad (2)$$

$$K'_2 = i \frac{\omega^2}{V_A^2} \left(-\frac{2}{3} + \frac{2\eta_p F_1}{\chi} + \frac{\lambda_D F_2}{\chi} \right) \quad (3)$$

where $F_1 = -\zeta_{-1H} Z(\zeta_{-1H})$ and $F_2 = -\zeta_{-2H} Z(\zeta_{-2H})$ so that far from resonance $F_1 \rightarrow 1$ and $F_2 \rightarrow 1$. It has been assumed that $n_D \approx n_e$ and the definition $\eta_p = n_H/n_e$ has been introduced.

To develop the dispersion relation, it is convenient to introduce the definitions $p = k_{\perp} V_A / \omega$ and $D^2 = -k_{\perp}^2 V_A^2 / \omega^2$ so that $\lambda_D \rightarrow -2\beta_H D^2$ and using these in Eq. (1) with Eqs. (2) and (3) one may write

$$D^4 + \left[2 \left(\frac{1}{3} - p^2 \right) - \frac{\eta_p F_1}{\beta_D F_2} + \frac{\left(\frac{1}{3} + p^2 \right)}{2\beta_D F_2} \chi \right] D^2 + \frac{\left(\frac{1}{3} - p^2 \right) (1 + p^2)}{2\beta_D F_2} \chi - \frac{2\eta_p F_1}{\beta_D F_2} \left(\frac{1}{3} - p^2 \right) = 0. \quad (4)$$

This is now transformed into a differential equation by letting $u = \omega(R_0 - R)/V_A = \omega_{pD} R_0 \chi / 4c$ and $D^2 \rightarrow \frac{d^2}{du^2}$. The variable u is then stretched by the factor α so $z' = \alpha u$. The final step is to replace the functions F_1 and F_2 by their asymptotic values on the left hand side and the difference between the exact and asymptotic values are placed on the right hand side. With

these changes, Eq. (4) may be written

$$y^{iv} + (\lambda^2 z^2 + C_0)y'' + (\lambda^2 z' + C_1)y = \lambda^2 z' \left(1 - \frac{1}{F_2}\right)(y'' + y) + \frac{\eta_p}{2\beta_D} \left(\frac{F_1}{F_2} - 1\right) \left(y'' + \frac{2(\frac{1}{3} - p^2)}{\alpha^2} y\right)$$

where

$$\lambda^2 = 2c(1+3p^2)/3\omega_{pD}R_0\beta_D\alpha^3$$

$$C_1 = -2\eta_p(1-3p^2)/3\beta_D\alpha^4$$

$$\alpha^2 = (1+p^2)(1-3p^2)/(1+3p^2)$$

$$\zeta_1 = \frac{-z'}{\alpha n \sqrt{2\beta_D}}$$

$$C_0 = [2(1-3p^2)/3 - \eta_p/\beta_D]/\alpha^2$$

$$\zeta_2 = \frac{-z'}{\alpha n \sqrt{\beta_D}} = \sqrt{2}\zeta_1.$$

The final step is to shift the origin so that the left hand side is in the standard form for the mode conversion-tunneling equation. This is accomplished by defining $\lambda^2 z' + C_0 = \lambda^2 z$ with the result

$$y^{iv} + \lambda^2 zy'' + (\lambda^2 z + \gamma)y = (\lambda^2 z + \gamma - C_1) \left(1 - \frac{1}{F_2}\right)(y'' + y) + \frac{\eta_p}{\beta_D \alpha^2} \left(\frac{F_1}{F_2} - 1\right) \left[y'' + \frac{2(1-3p^2)}{3\alpha^2} y\right], \quad (5)$$

where $\gamma = C_1 - C_0$. The tunneling factor is given by

$$\eta = \frac{\pi}{2} \left(\frac{1+\gamma}{\lambda^2}\right) = \frac{\pi\omega_{pD}R_0}{4c} \left(\frac{1-3p^2}{1+3p^2}\right)^{\frac{3}{2}} \frac{(\alpha^2\beta_D + \eta_p)}{(1+p^2)^{\frac{1}{2}}}, \quad \gamma = \left(\frac{1+3p^2}{1+p^2}\right) \left[-\frac{2}{3} + \frac{\eta_p}{3\beta_D(1+p^2)}\right]$$

and if the inhomogeneous terms in Eq. (5) are neglected the transmission coefficient is simply $\exp(-\eta)$.

From the above analysis, several special cases may be easily examined. The first limit is when $p^2 \ll \frac{1}{3}$, in which case $\alpha^2 \approx 1$ and one may approximate

$$\eta \approx \frac{\pi\omega_{pD}R_0}{4c} (\beta_D + \eta_p).$$

From this result it is immediately apparent that mode conversion will be dominated by the two-ion hybrid resonance when $\eta_p \gg \beta_D$ and harmonic

effects will dominate only when $\beta_D \gg \eta_p$, which is a severe restriction.

For this special case Eq. (5) can be simplified somewhat to read

$$y^{iv} + \lambda^2 z y'' + (\lambda^2 z + \gamma) \dot{y} = (\lambda^2 z + \gamma + \frac{2\eta_p}{3\beta_D}) (1 - \frac{1}{F_2}) (y'' + \dot{y}) + \frac{\eta_p}{\beta_D} (\frac{F_1}{F_2} - 1) (y'' + \frac{2}{3} \dot{y}) \quad (6)$$

with

$$\lambda^2 = 2c / 3\omega_p R_0 \beta_D$$

$$\zeta_1 = \frac{z_0 - z}{n \sqrt{2\beta_D}}$$

$$\gamma = -\frac{2}{3} + \frac{\eta_p}{3\beta_D}$$

$$\zeta_2 = \frac{z_0 - z}{n \sqrt{\beta_D}}$$

$$F_K = -\zeta_K Z(\zeta_K)$$

$$z_0 = \left(\frac{2}{3} - \frac{\eta_p}{\beta_D} \right) / \lambda^2.$$

It may be noted from the inhomogeneous term of Eq. (6) that the fundamental absorption source term vanishes both at resonance ($z = z_0$) and asymptotically, so that in spite of the potentially large coefficient, η_p / β_D , fundamental absorption may not be very strong. Harmonic absorption does not vanish at resonance, however, so it may dominate cyclotron absorption processes.

Another special case is when $k_{\parallel} \rightarrow \omega / \sqrt{3} V_A$ ($p^2 \rightarrow \frac{1}{3}$) in which case mode coupling disappears ($\eta \rightarrow 0$). In this limit, one needs to go back to the very beginning because neglected terms may now dominate, but the basic conclusion appears to be that mode conversion virtually disappears in this limit, and for $p^2 = \frac{1}{6}$, η is only 7% of the value it would be with $p^2 \ll \frac{1}{3}$.

References

1. J. E. Scharer, B. D. McVey, & T. K. Mau, *Nuc. Fusion* 17, 2(1977); J. Jacquinet, B. D. McVey, & J. E. Scharer, *Phys. Rev. Lett.* 39, 88(1977).
2. F. W. Perkins, Princeton Plasma Phys. Lab Report PPPL-1336, 1977 (to be published in *Nuc. Fusion*).
3. D. G. Swanson, *Phys. Fluids* 10, 428(1967).

Fast Wave Heating Via Mode Conversion
in the Ion Cyclotron Range of Frequencies (ICRF)*

B.D. McVey and J.E. Scharer

Electrical and Computer Eng. Department
University of Wisconsin
Madison, Wisconsin 53706

We investigate fast wave heating of a large tokamak plasma via linear mode conversion of the fast wave to an Ion Bernstein wave. For the plasma considered, typically half of the fast wave energy is coupled to the Ion Bernstein for one pass through the second harmonic ion cyclotron resonance zone. Using a ray tracing analysis, the spatial absorption of the energy in the Ion Bernstein wave is calculated.

We have numerically solved the complete 3×3 linear dispersion relation as derived from kinetic theory assuming a Maxwellian plasma. From this analysis, we obtain an algebraically tractable dispersion relation suitable for defining the ray trajectories. A Hamiltonian treatment is used to calculate the ray trajectories ($\frac{d\vec{r}}{dt} = \frac{\partial \omega}{\partial \vec{k}}$, $\frac{d\vec{k}}{dt} = \frac{\partial \omega}{\partial \vec{r}}$). We expand these equations in component form in cylindrical coordinates which are suitable for a tokamak geometry. The resulting six first-order differential equations along with two others calculating the local damping decrement are solved numerically. Typical results are shown in the accompanying figures.

We consider second harmonic heating of a pure deuterium plasma of the following parameters ($R_0 = 250$ cm, $a = 85$ cm, $n_{pk} = 3 \times 10^{13}$ cm⁻³, $T_{pk} = 4$ keV, $q_0 = 1.2$, $q_s = 3.0$, and $B_{tor} = 50$ kG). The usual $1/R$ variation in toroidal magnetic field, parabolic density and temperature profiles, and rotational transform are assumed. In the figures, the three hatch marks indicate positions where 1%, 50% and 90% of the wave energy has been absorbed.

*Work supported by NSF Grant 75-19259

The first figure is a projection in the minor cross section of the ray trajectories, and the second indicates the projection in the toroidal direction. The fast wave propagates inward perpendicular to the magnetic field with a slight convergence of the rays due to the density gradient. Near the second harmonic resonance layer, it mode converts to an Ion Bernstein wave. The Ion Bernstein wave propagates radially outward and tends to follow the total magnetic field. Rays 1 through 3 are absorbed via electron Landau damping of the Ion Bernstein mode. Ray 4 has some ion cyclotron damping near the mode conversion layer. Ray 5 is completely absorbed near the mode conversion layer. This heating is due to enhancement of ion cyclotron damping by the rotational transform.

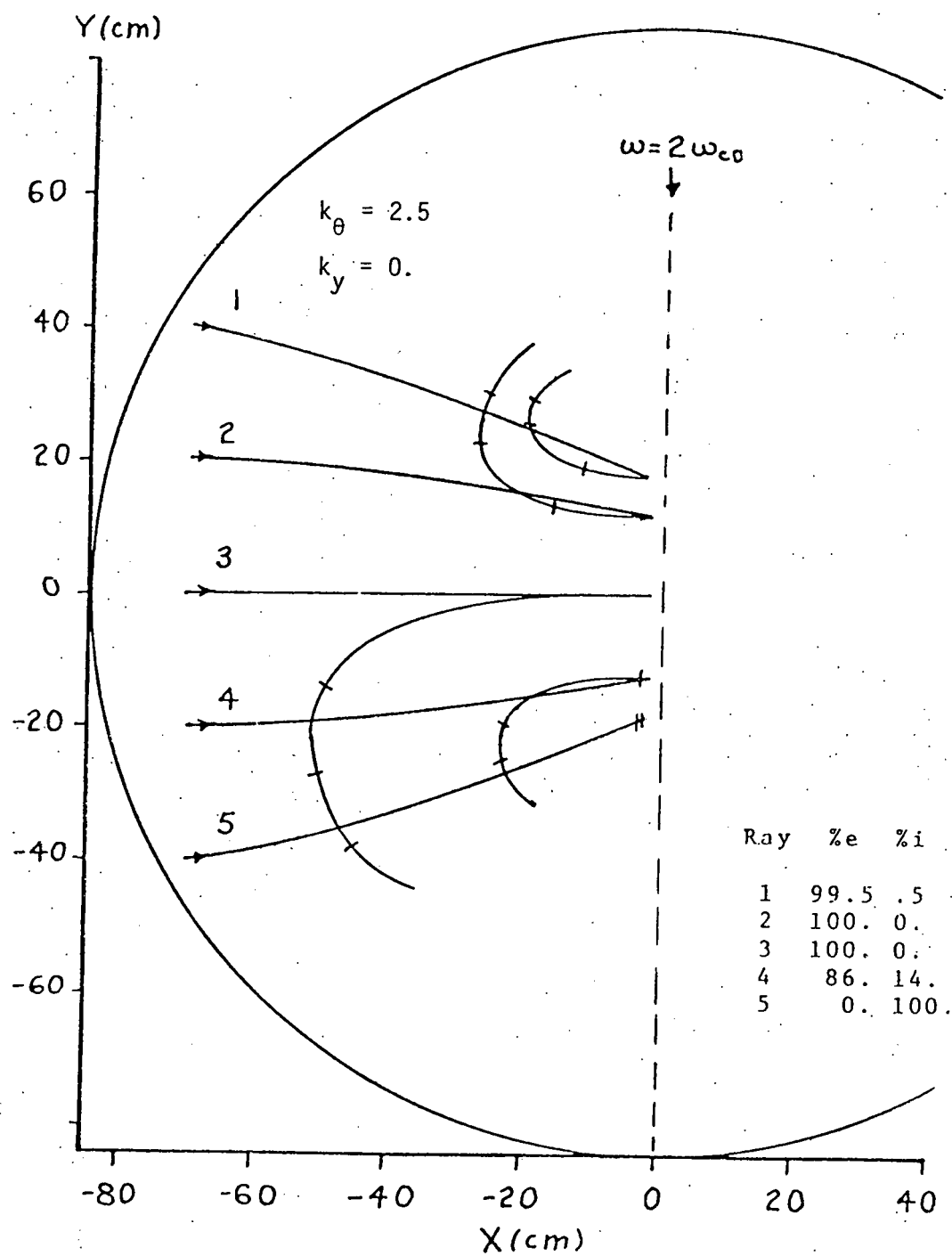


Figure 1

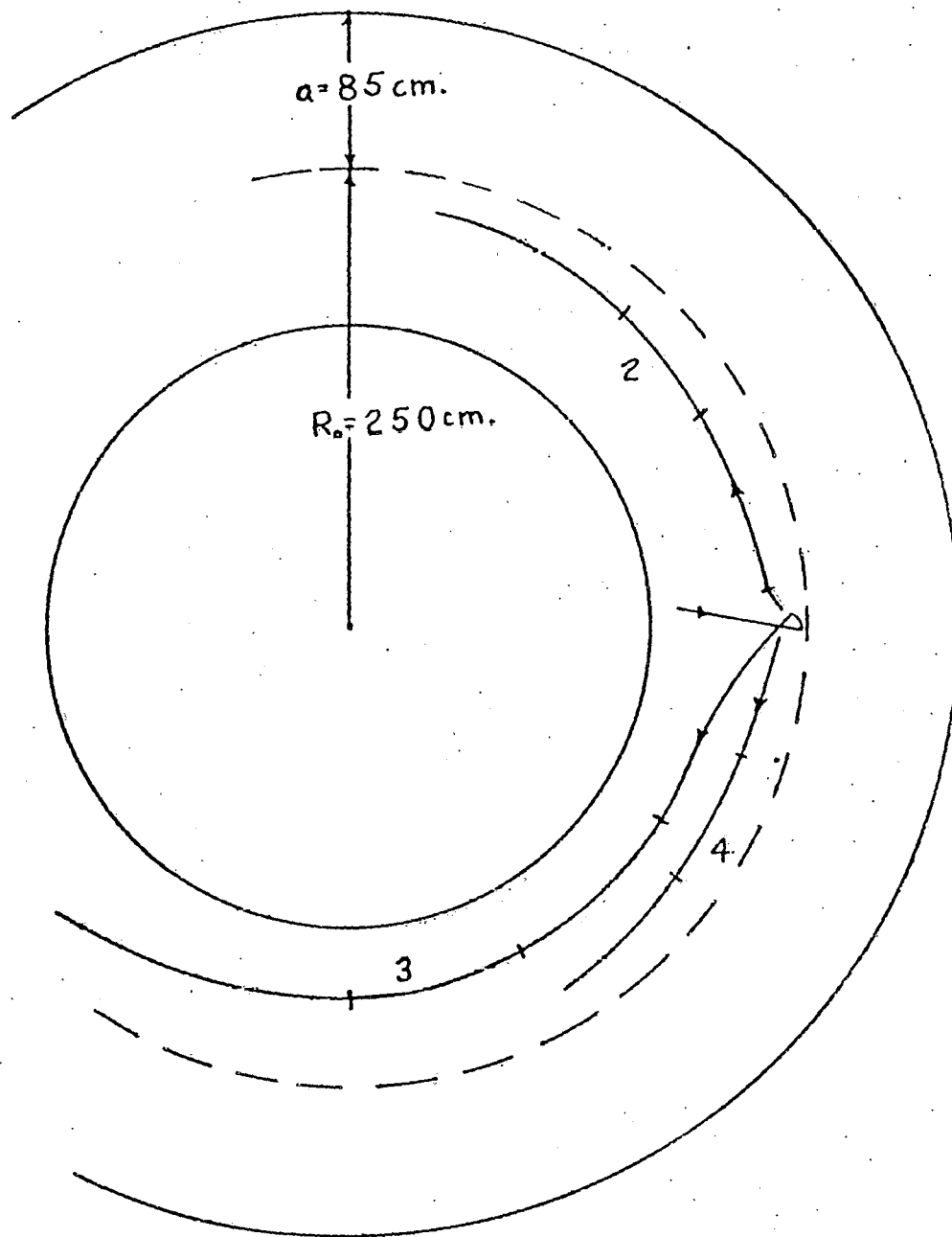


Figure 2

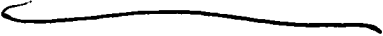
Scaling of Ion Cyclotron Frequency Range Heating to Reactor Size*

J. Scharer, J. Beyer, and D. T. Blackfield

University of Wisconsin
Madison, Wisconsin 53706Abstract

We present studies of the coupling of wave energy and heating in the ion cyclotron frequency range for the startup phase of tokamak reactors. The effect of the dense mode spectrum and fast wave mode conversion processes are considered in the coupler design. Both coil and cavity aperture coupling systems are considered. Another criterion which is considered is the design of an array coupling system which optimizes mode selectivity to achieve plasma core heating with efficient coupling. It must also be compatible with the high neutron flux emanating from the plasma and the access constraints imposed by a high field, high density, minimum size tokamak reactor.

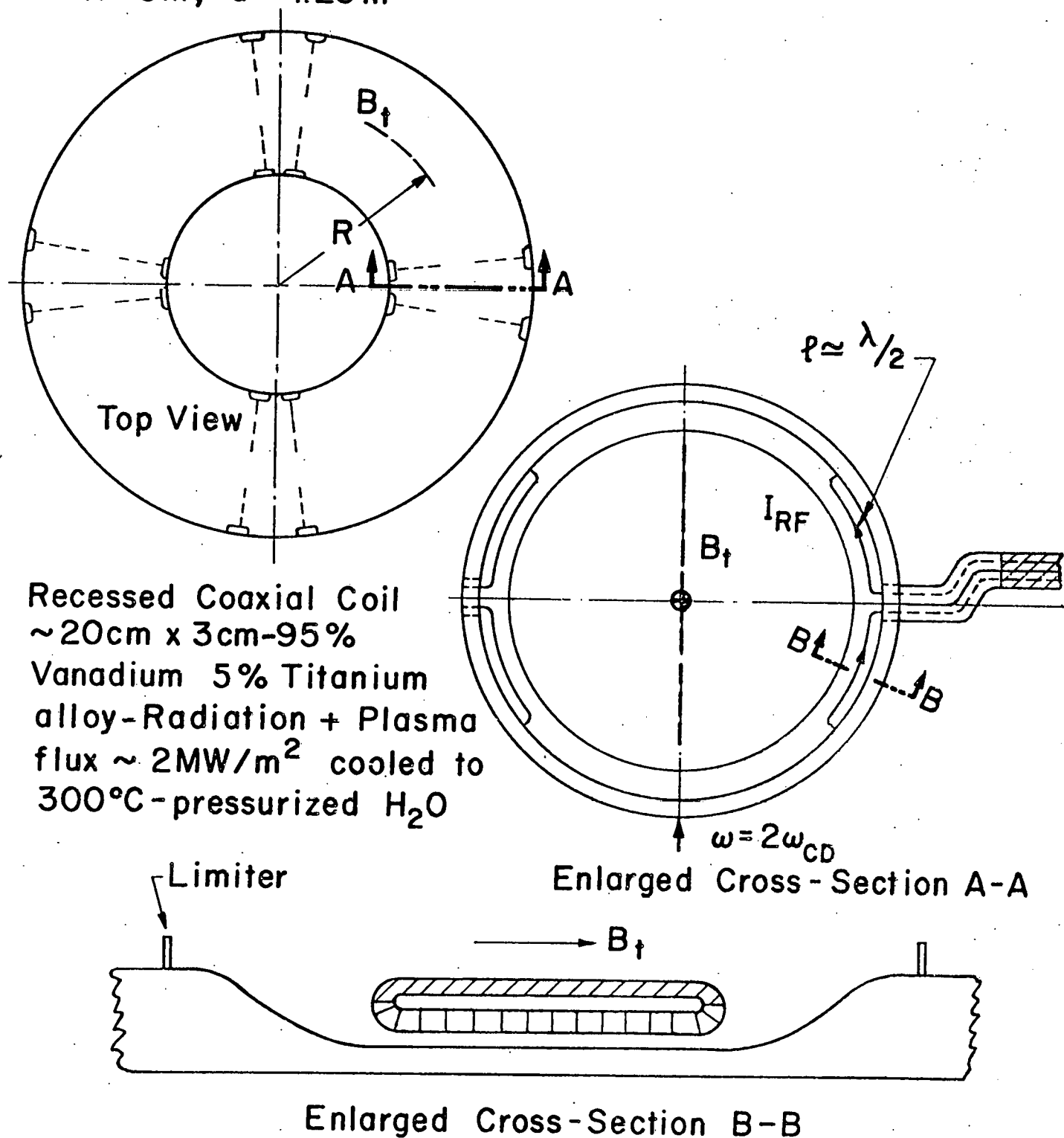
*Work supported by ERDA Contract AT(11-1)-S-02-2272

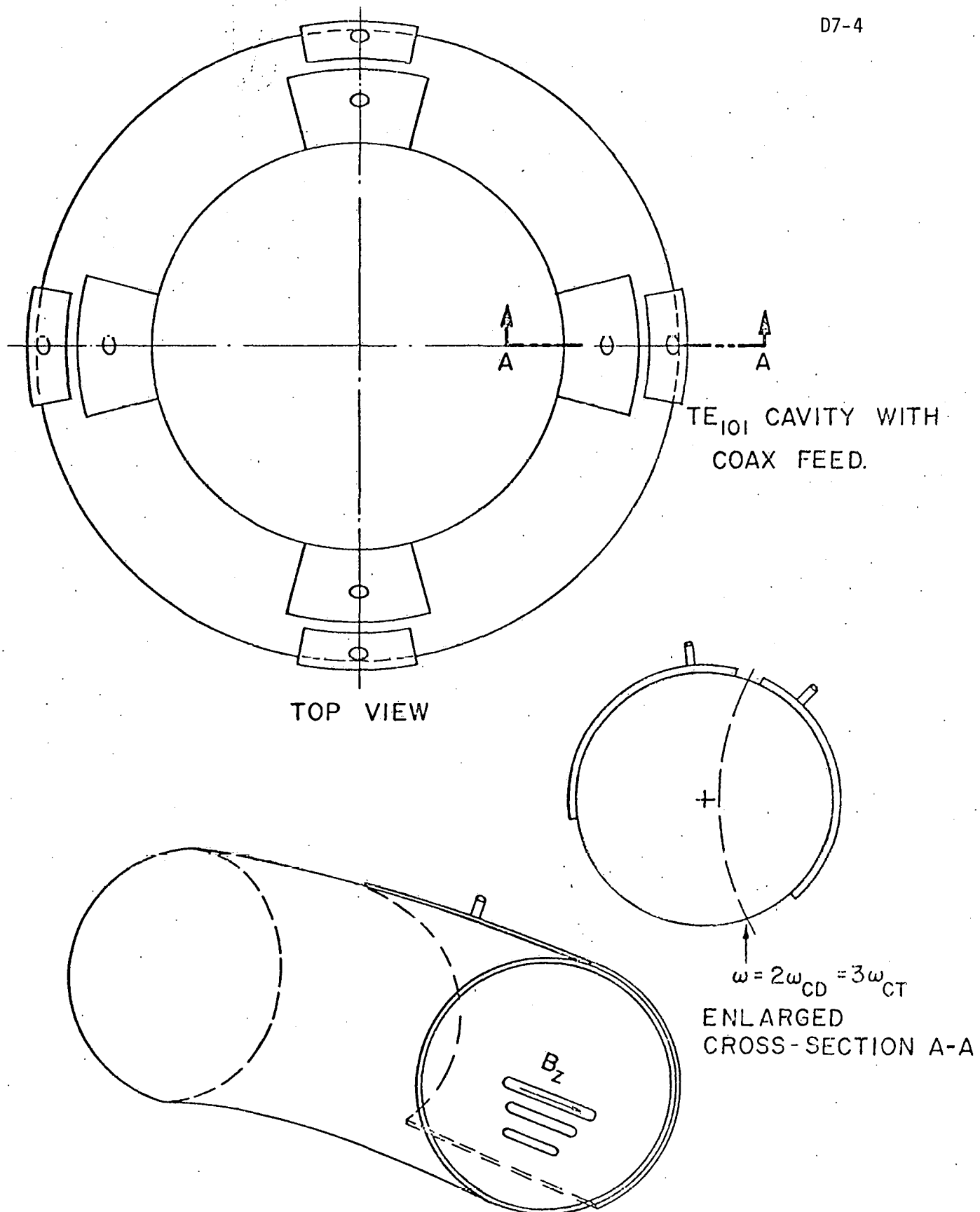


SCALING OF $2\omega_{ci}$ HEATING TO A TOKAMAK REACTOR

1. RF POWER LEVELS REQUIRED ~ 100 MW @ ~ 100 MHz FOR ~ 1 SEC FOR A 800 MW_e D-T REACTOR. $n \approx 10^{14}/\text{cm}^3$, $B_0 \approx 66$ kG, AND $T_i \approx 6\text{-}10$ keV.
2. WAVE PROPAGATION AND HEATING MECHANISMS.
 - A. FINITE ION GYRORADIUS ($k_{\perp} \rho_i > 0$). E_+ , $\frac{\omega - 2\omega_{ci}(R)}{k_{\parallel} v_i} \lesssim 1$
 - B. ELECTRON LANDAU (E_z) AND TRANSIT-TIME (B_z) DAMPING
 $1 < \omega/k_{\parallel} v_e < 10$.
 - C. FAST ALFVEN WAVE EIGENMODES AND MODE CONVERSION PROCESSES DUE TO THERMAL EFFECTS NEAR $2\omega_{CD}$, ω_{ii} .
3. WAVE COUPLING (COIL OR APERTURE).
 - A. WAVE MODE SPECTRUM (r-RADIAL, m-POLOIDAL, n-TOROIDAL).
 - B. LAUNCHING STRUCTURE SPECTRUM (m,n).
 - C. IMPEDANCE SEEN BY LAUNCHING STRUCTURE, DEPENDENCE ON EIGENMODE AND MODE CONVERSION ABSORPTION PROCESSES.
4. REACTOR ENVIRONMENTAL COMPATIBILITY OF LAUNCHING STRUCTURE.
 - A. TECHNOLOGY-ARCING, MATERIALS, COOLING, RADIATION DAMAGE.
 - B. LAYOUT, MINIMIZE WALL SURFACE AREA OF STRUCTURE.

NUWMAK Base design of RF Coil Coupling Array
 80MW for 3 sec. start up to ignition.
 8RF Units, 10MW/Unit. $f = 90\text{MHz} = 2f_{\text{CD}}$
 $R = 5\text{m}$, $a = 1.25\text{m}$





SURFACE APERTURE COUPLING - 90MHz, 10MW/UNIT

RE-EVALUATION OF THE EFFECT OF DAMPING ON LINEAR MODE CONVERSION

by

P. M. Bellan

California Institute of Technology, Pasadena, CA

ABSTRACT

A re-evaluation of the phase integrals in the "modified-for-damping" linear mode conversion shows that in contrast to a previous calculation there is no special enhancement of damping right at the mode conversion layer. Thus the amplitude of the cold (incoming) wave may be adequately described by the ordinary WKB attenuation incurred in propagating from the antenna to the mode conversion layer. Similarly, the amplitude of the hot (converted) wave is determined by the product of (i) the attenuation incurred while propagating away from the mode conversion layer, and (ii) the total cold plasma wave attenuation.

A re-evaluation of the phase integrals involved in the "modified-for-damping" linear mode conversion calculation shows that, in contrast to the prediction of Sec. V of Ref. 1, there is no special enhancement of damping right at the mode conversion layer.²

For simplicity let us consider the effects of perpendicular damping only. As in Ref. 1 we then have

$$K_{xx} \rightarrow K_{xx_r} + iK_{xx_i}$$

in general. Near the resonant layer

$$xK'_{xx} \rightarrow xK'_{xx_r} + iK_{xx_i}$$

since K_{xx_i} is approximately constant near the resonant layer. Equation (37)³ becomes

$$\frac{d^4 E_z}{d\xi^4} + b \frac{d^3 E_z}{d\xi^3} + (\xi + i\bar{\epsilon}) \frac{d^2 E_z}{d\xi^2} + \frac{dE_z}{d\xi} + \mu E_z = 0$$

where $\bar{\epsilon} = \alpha^{-1/3} K_{xx_r}^{1/3} K_{xx_i} / K'_{xx_r}$. All the analysis up to Eq. (40) holds for the damping case if we simply replace ξ by $\xi + i\bar{\epsilon}$. Replacing the non-dimensional variables (ξ, μ etc.) by the dimensional variables (x, k_z etc.) in Eq. (41) gives formally the same result as for the non-damping case except now x is replaced by $x + i\epsilon$ where $\epsilon = K_{xx_i} / K'_{xx_r}$. Thus Eq. (41) becomes

$$E_z(x, z) = i|k_z|^{1/2} \left\{ \frac{\alpha}{[(x+i\epsilon)K'_{xx_r}]^3} \right\}^{1/4} \exp \left[\frac{2}{3} i \left(\frac{K'_{xx_r}}{\alpha} \right)^{1/2} (x+i\epsilon)^{3/2} \right] \\ + [-(x+i\epsilon)K'_{xx_r} K_{zz}]^{-1/4} \exp \left\{ 2i \left[\frac{-k_z^2 (x+i\epsilon) K_{zz}}{K'_{xx_r}} \right]^{1/2} \right\}$$

The quantities $(x+i\epsilon)^{1/2}$, $(x+i\epsilon)^{3/2}$ in the phases of the above expression can be written as integrals,

$$\begin{aligned}\frac{2}{3} (x+i\epsilon)^{3/2} &= \int_{-i\epsilon}^x (x+i\epsilon)^{1/2} dx \\ 2(x+i\epsilon)^{1/2} &= \int_{-i\epsilon}^x (x+i\epsilon)^{-1/2} dx\end{aligned}$$

Note the lower limit at $x = -i\epsilon$. This was missed in Ref. 1 because damping was introduced after matching the boundary layer solutions to the WKB solutions, rather than before. Using these integrals Eq. (42) becomes

$$\begin{aligned}E_z(x, z) &= i|k_z|^{1/2} \left(\frac{\alpha}{K_{xx}}\right)^{1/4} \exp\left[i \int_{-i\epsilon}^x \left(-\frac{K_{xx}}{\alpha}\right)^{1/2} dx\right] \\ &+ (-K_{xx} K_{zz})^{-1/4} \exp\left[i|k_z| \int_{-i\epsilon}^x \left(-\frac{K_{zz}}{K_{xx}}\right)^{1/2} dx\right]\end{aligned}$$

where $K_{xx} = xK'_{xx} + iK_{xx}$. The above expression is undetermined with respect to an overall constant, but this may be determined by requiring that the second (i.e., cold) term correspond to the cold plasma wave launched from the antenna located at x_0 . This correspondence is achieved by multiplying the entire expression by the constant

$$\exp\left[i|k_z| \int_{x_0}^{-i\epsilon} \left(-\frac{K_{zz}}{K_{xx}}\right)^{1/2} dx\right],$$

giving:

$$\begin{aligned}E_z(x, k_z) &= i|k_z|^{1/2} \left(\frac{\alpha}{K_{xx}}\right)^{1/4} \exp\left[i \int_{-i\epsilon}^x \left(-\frac{K_{xx}}{\alpha}\right)^{1/2} dx + i|k_z| \int_{x_0}^{-i\epsilon} \left(-\frac{K_{zz}}{K_{xx}}\right)^{1/2} dx\right] \\ &+ (-K_{xx} K_{zz})^{-1/4} \exp\left[i|k_z| \int_{x_0}^x \left(-\frac{K_{zz}}{K_{xx}}\right)^{1/2} dx\right]\end{aligned}$$

This expression is the same as the non-damping case, Eq. (43), except that here K_{xx} is complex and the integration limits at $x = 0$ are replaced by limits at $x = -i\epsilon$.

The cold plasma wave experiences the usual damping coming from $\text{Im} \int_{x_0}^x |k_z| (-K_{zz}/K_{xx})^{1/2} dx$. The hot plasma phase has two terms, one being the explicit hot plasma wave phase, the other being the phase shift incurred by the cold plasma wave in going from the antenna to the resonant layer. A consideration of the real part of the hot plasma phase in the region just beyond the mode conversion layer will tell us whether or not there is any strong attenuation at the mode conversion layer.

Let us now consider the real part, γ , of the total hot plasma wave phase, where

$$\gamma = \text{Re} \left[i \int_{-i\epsilon}^x \left(\frac{K_{xx}}{\alpha} \right)^{1/2} dx + i |k_z| \int_{x_0}^{-i\epsilon} \left(-\frac{K_{zz}}{K_{xx}} \right)^{1/2} dx \right]$$

Let us split the region $0 < x < x_0$ into two subregions, namely $0 \leq x \leq L$ and $L \leq x < x_0$ where L is the scale length near the mode conversion layer (i.e., $K_{xxr} \approx xK'_{xxr}$ for $0 \leq x \leq L$). Since we are interested in the value of γ in the region just after the mode conversion layer, we calculate γ for the subregion $0 \leq x \leq L$. Taking advantage of the Taylor expansion for K_{xx} in this region we obtain

$$\begin{aligned} \gamma &= \text{Re} \left[i \int_{-i\epsilon}^x \left(\frac{xK'_{xxr} + iK_{xx1}}{\alpha} \right)^{1/2} dx + i |k_z| \int_L^{-i\epsilon} \left(-\frac{K_{zz}}{xK'_{xxr} + iK_{xx1}} \right)^{1/2} dx \right. \\ &\quad \left. + i |k_z| \int_{x_0}^L \left(-\frac{K_{zz}}{K_{xx}} \right)^{1/2} dx \right] \\ &= \text{Re} \left[i \frac{2}{3} \left(\frac{K'_{xxr}}{\alpha} \right)^{1/2} (x+i\epsilon)^{3/2} - 2i |k_z| \left(-\frac{K_{zz}}{K'_{xxr}} \right)^{1/2} (L+i\epsilon)^{1/2} \right. \\ &\quad \left. + i |k_z| \int_{x_0}^L \left(-\frac{K_{zz}}{K_{xx}} \right)^{1/2} dx \right] \end{aligned}$$

The last term is just the damping of the incoming cold plasma wave from x_0 to L ; this gives the attenuation of the cold plasma wave outside (but not including) the mode conversion layer. Let us now consider the first two terms. Note that $K'_{xxr} \approx L^{-1}$ so $\epsilon \approx LK_{xx1} \ll L$, since $K_{xx1} \ll 1$. Then there exists a region where $\epsilon \ll x \ll L$. Expanding $(x+i\epsilon)^{3/2}$, $(L+i\epsilon)^{1/2}$ in this region, we find

$$\gamma = -\left(\frac{K'_{xxr}}{\alpha} \right)^{1/2} \epsilon x^{1/2} + |k_z| \left(-\frac{K_{zz}}{K'_{xxr}} \right)^{1/2} \frac{\epsilon}{L^{1/2}} + \text{Re} \left[i |k_z| \int_{x_0}^L \left(-\frac{K_{zz}}{K_{xx}} \right)^{1/2} dx \right]$$

Surprisingly, the sign of the second term is positive; however, a comparison of the magnitudes of the first and second terms shows that the first term is always larger than the second, providing the criterion, Eq. (40) for mode conversion holds, i.e.,

$$\left| \frac{K'_{xx} x^3}{\alpha} \right| \gg \left| \frac{k_z^2 K_{zz} x}{K'_{xx}} \right| \gg 1$$

γ includes, in principle, any special damping occurring at the mode conversion layer. Clearly, there is no special enhancement of damping at the mode conversion layer, since the dominant terms in γ (i.e., first and third) are just the ordinary WKB damping experienced by waves outside the conversion layer.

Acknowledgments: The author would like to thank Dr. M. Porkolab for suggesting that this calculation be re-evaluated, and Dr. V. Krapchev for criticism of the previous calculation.

References

1. P. Bellan and M. Porkolab, Phys. Fluids 17, 1592 (1974).
2. A similar result has been obtained by V. Krapchev (private communication) and J. Schuss, M. Porkolab, and R. R. Parker (see following paper, this conference).
3. The equation numbers used here are those of Ref. 1.

COLLISIONAL ABSORPTION OF LOWER HYBRID WAVES NEAR THE MODE CONVERSION LAYER

by

J. J. Schuss, M. Porkolab, and R. R. Parker
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

ABSTRACT

Mutually consistent numerical and analytical solutions are obtained that describe the mode conversion and collisional damping of lower hybrid waves near the hot plasma mode conversion layer in a linearly increasing density profile. From these solutions simple analytical expressions are derived for the total wave absorption within a given distance of the mode conversion layer. These results are in agreement with a WKB analysis of wave damping and show that the collisional power absorption is not strongly localized to the mode conversion point. Collisional absorption appears to be unimportant in tokamaks with $T_e \geq 1$ keV and $n_e < 10^{14} \text{ cm}^{-3}$; however for $n_e \geq 10^{15} \text{ cm}^{-3}$ and $T_e \approx 1$ keV this absorption could be significant.

The mode conversion of a lower hybrid electrostatic wave into an ion wave in a nonuniform plasma has been studied theoretically^{1,2}; it has been suggested that collisions could cause significant power absorption near the lower hybrid layer.² In this paper the relevant equation describing this process is solved both numerically and analytically in the presence of collisions. The resulting solutions are valid closer to the mode conversion point in a hot plasma than those obtained previously and present new simple expressions for the wave energy absorbed near this layer.

We assume a linear density profile in the x direction in a magnetized plasma with wave frequency $\omega_{ce} \gg \omega \gg \omega_{ci}$. The electrons are given a full Vlasov treatment while the ions are assumed unmagnetized. For $k_x^2 T_e / m_e \omega_{ce}^2 \ll 1$ and $k_x^2 T_i / M_i \omega^2 \ll 1$ we can expand the resulting $\bar{\epsilon}(k, \omega) = \bar{\epsilon}_0(\omega) + \bar{\epsilon}_1(\omega) k_x + \bar{\epsilon}_2(\omega) k_x^2$. Letting $ik_x = \partial/\partial x$ the electrostatic wave equation $\nabla \cdot \bar{\epsilon} \cdot \nabla \phi = 0$ becomes

$$\frac{\partial}{\partial x} \left[\epsilon_{xx0} - \epsilon_{xx2} \frac{\partial^2}{\partial x^2} \right] \frac{\partial \phi}{\partial x} - k_z^2 \epsilon_{zz0} \phi = 0 \quad (1)$$

Near the mode conversion layer Eq. (1) can be expanded as

$$\frac{\partial}{\partial X} \left[-X + i\epsilon + \Delta \frac{\partial^2}{\partial X^2} \right] \frac{\partial \phi}{\partial X} + \phi = 0 \quad (2)$$

where $X = \frac{x}{L} - \frac{n_z^2 (\alpha_0 - \epsilon_0)}{m_e} \frac{\omega^2 L^2}{c^2}$

$$\epsilon = \frac{\nu_{\text{eff}}}{\omega} (\alpha_0 - 1) n_z^2 (\alpha_0 - \epsilon_0) \frac{M_i}{m_e} \frac{\omega^2 L^2}{c^2}$$

$$\Delta = \frac{1}{2} n_z^2 (\alpha_0 - \epsilon_0)^3 \left(\frac{M_i}{m_e} \right)^3 \frac{\omega^4 L^4}{c^4}$$

$$a = \frac{3}{4} \frac{(\alpha_0 - 1)^2}{\alpha_0 - \epsilon_0} \frac{T_e}{M_i c^2} + 3 (\alpha_0 - \epsilon_0) \frac{T_i}{M_i c^2}$$

$$L = L_0 / (1 - \epsilon_0) \quad L_0 \equiv \left| \frac{1}{n} \frac{dn}{dx} \right|^{-1} \text{ at the mode conversion layer.}$$

$$\epsilon_o = \epsilon_{xxo} \text{ at the mode conversion layer}$$

$$\alpha_o = 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} \text{ at the mode conversion layer}$$

$$\nu_{eff} = 3.34 \frac{ne^4 \log \Lambda}{T_e^{\frac{3}{2}} m_e^{\frac{1}{2}}}$$

Typically for tokamak conditions $\epsilon \sim 1-10$ and $\Delta \sim 10^{10}$. The dissipative term comes from adding to ϵ_{xxo} the collisional value $\delta\epsilon_{xxo} = i\nu_{eff}/\omega (\omega_{pe}^2/\omega_{ce}^2)$.³ A WKB analysis of Eq. (2) reveals that the mode conversion point is at $X = -2\Delta^{\frac{1}{2}}$. Equation (2) is solved by numerically integrating through this point, selecting only the solution with a purely outgoing ion mode. Figure 1 shows a series of such solutions for $\varphi(X)$ with successively larger values of ϵ . These solutions reveal that the log of ion mode power decreases linearly with increasing ϵ .

Equation (2) can also be solved by Fourier transform methods.^{1,2} Letting $\xi = X - i\epsilon$ we can obtain the solution

$$\varphi = \int_C dk \exp [ik\xi - i/k - \log(k) + i\Delta k^3/3] \quad (3)$$

which is correct when contour C is chosen so that at the end points $k^2 \exp [ik\xi - i/k - \log(k) + i\Delta k^3/3] \rightarrow 0$. An asymptotic saddle point solution of Eq. (3) can be formed for $\xi \ll 0$ and $\xi \gg 0$ by using the contours of Figs. 2a and 2b respectively. The contour of Fig. 2b for $\xi \gg 0$ guarantees that φ is exponentially decaying away from the mode conversion point. The contour of Fig. 2a for $\xi \ll 0$ results in the solution

$$\begin{aligned} \varphi(X) = & \beta_1^{-1} \exp \left[-\frac{3\pi i}{4} + ik_1 \xi - 1/k_1 - \log(k_1) + i\Delta k_1^3/3 \right] \\ & + \beta_2^{-1} \exp \left[\frac{3\pi i}{4} + ik_2 \xi - 1/k_2 - \log(k_2) + i\Delta k_2^3/3 \right] \end{aligned} \quad (4)$$

where

$$\begin{aligned} k_{1,2}^2 = & -\xi/2\Delta \pm (\xi^2/4\Delta^2 - 1/\Delta)^{\frac{1}{2}} \\ \beta_{1,2}^2 = & \left| \frac{2}{k_{1,2}} \right| (\xi k_{1,2}^2 + 2) \end{aligned}$$

and the 1,2 subscripts refer to the lower hybrid and ion modes respectively. k_1 and k_2 are chosen with real parts < 0 . For $|\xi| \gg \Delta^{\frac{1}{2}}$ these solutions are of the same form as those of Bellan and Porkolab.² For $|\xi| \ll \Delta^{\frac{1}{2}}$ but $X < -2\Delta^{\frac{1}{2}}$

$$k_{1,2}^2 \approx \frac{1}{\sqrt{\Delta}} \pm \sqrt{\frac{|\delta X|}{\Delta^{\frac{3}{2}}}} \quad (5)$$

where $\xi = -2\Delta^{\frac{1}{2}} + \delta X$. These solutions are valid so long as the two saddle points are distinct, or that $\beta^2(k_1 - k_2)^2 \gg 1$. This condition becomes

$$|\delta X| > \Delta^{\frac{1}{3}} \quad (6)$$

Thus Eq. (4) is valid closer to the mode conversion layer than previous solutions.²

Using Eq. (4) we can express the power flux of the two waves as

$$S_{1,2} = \exp [I_{1,2} + I_{1,2}^*] \quad (7)$$

where $I_{1,2} = ik_{1,2}\xi - 1/k_{1,2} + i\Delta k_{1,2}^3/3$. In Fig. 3 the value of $\log(S_2/S_1)$ of Eq. (7) is compared with that found in the numerical solutions to Eq. (2) for $|\delta X| > \Delta^{1/3}$;

the solutions are seen to be mutually consistent. The reduction in the log of the amplitude of the power flux is proportional to the collisional ϵ .

Equation (7) can be simplified for $\Delta^{\frac{1}{2}} > |\delta X| > \Delta^{1/3}$:

$$\begin{aligned} S_1 &= \exp[-2\epsilon/\Delta^{\frac{1}{4}} + \epsilon |\delta X|^{\frac{1}{2}}/\Delta^{\frac{1}{2}}] \\ S_2 &= \exp[-2\epsilon/\Delta^{\frac{1}{4}} - \epsilon |\delta X|^{\frac{1}{2}}/\Delta^{\frac{1}{2}}] \end{aligned} \quad (8a)$$

and for $|\delta X| \gg \Delta^{\frac{1}{2}}$:

$$\begin{aligned} S_1 &= \exp[-2\epsilon/|X|^{\frac{1}{2}}] \\ S_2 &= \exp[-2\epsilon |X|^{\frac{1}{2}}/\Delta^{\frac{1}{2}}] \end{aligned} \quad (8b)$$

S_1/S_2 in Eq. (8) is also consistent with the amplitudes obtained from a simple WKB analysis which expresses the wave power as $\exp(2R_e \int ik_x dx)$.

From Eq. (8) we see that substantial absorption will occur when $\log(S_1/S_2) \sim 1$ or in a distance $|\delta X| \sim \Delta/4\epsilon^2$. This becomes in real space

$$\delta x_0 = ac^2/[4v_{\text{eff}}^2(\alpha_0 - 1)^2 L] \quad (9)$$

If δx_0 is small compared to the plasma dimensions, collisional absorption at the mode conversion layer may absorb a significant fraction of the wave power. For a deuterium plasma with $f = 2.45$ GHz, $n_z = 3$, and $B_0 = 60$ kG, $T_e \approx 1$ keV $\approx T_i$, $n_e \approx 4 \times 10^{14} \text{ cm}^{-3}$ and $L = 50$ cm we find that $\delta x_0 \sim 15$ cm and is too large to indicate strong collisional absorption. However, for the same conditions with $f = 4.0$ GHz, $B_0 = 100$ kG, and $n_e \approx 10^{15} \text{ cm}^{-3}$ we find that $\delta x_0 \sim 2.5$ cm; thus at high densities $n_e > 10^{15} \text{ cm}^{-3}$ and $T_e \approx 1$ keV collisional absorption could be an important source of electron heating.

ACKNOWLEDGEMENT

One author (M.P.) would like to acknowledge his valuable discussions with P. Bellan and V. Krapchev concerning this work. This work is supported by D.O.E. Grant No. EG-77-G-01-4108.

REFERENCES

1. T.H. Stix, Phys. Rev. Lett. 15, 878 (1965).
2. P.M. Bellan and M. Porkolab, Phys. Fluids 17, 1592 (1974).
3. K.D. Harms, Nucl. Fusion 16, 753 (1976).

FIGURES

- Fig. 1. ϕ vs $\bar{X}$ for increasing values of ϵ ; $\Delta = 1 \times 10^8$ and $\bar{X} = X/(10 \Delta^{\frac{1}{3}})$. In (a) $\epsilon = 5$, in (b) $\epsilon = 50$, in (c) $\epsilon = 500$, and in (d) $\epsilon = 5000$. At $\bar{X} = -1$, $\log(S_2/S_1) = -0.075$ for (a), -0.69 for (b), and -7.0 for (c), which within the numerical accuracy of the program is consistent with $\log(S_2/S_1) \propto \epsilon$.
- Fig. 2. Saddle points and contours for asymptotic evaluation of Eq. (3). (a) is for $X < 0$, and (b) is for $X > 0$.
- Fig. 3. Graph of $\log(S_2/S_1)$ vs $\bar{X}$ determined from both the saddle point solution and the numerical solution of Eq. (2). $\bar{X} = X/(10 \Delta^{\frac{1}{3}})$ here. The region $|\delta X| < \Delta^{\frac{1}{3}}$ is not graphed, as it is outside the range of validity of the saddle point solution.

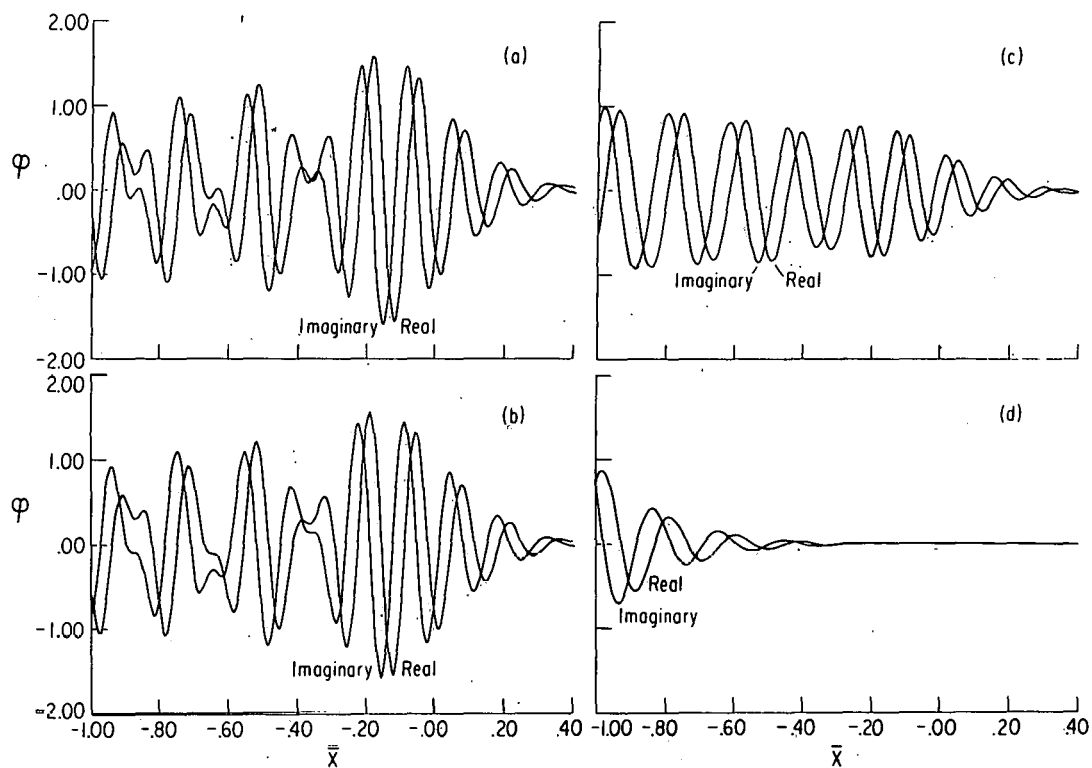


Fig. 1

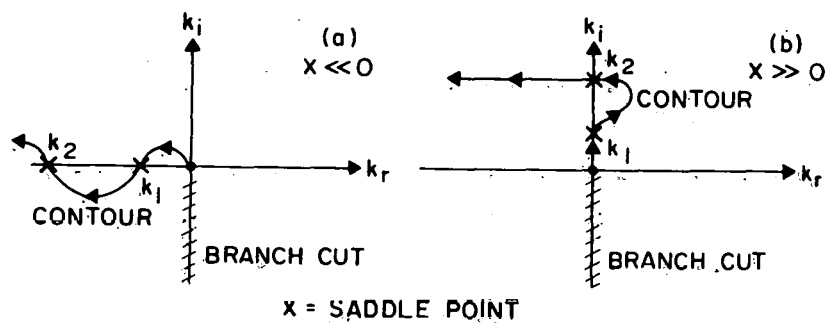


Fig. 2

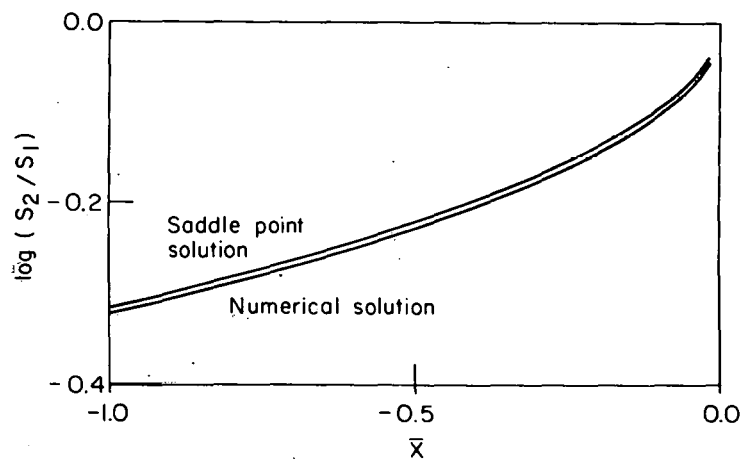


Fig. 3

ION-CYCLOTRON-HARMONIC HEATING USING THE LOWER-HYBRID WAVE

Satish Puri

Max-Planck-Institut für Plasmaphysik, 8046 Garching, FRG and EURATOM

1. EFFECT OF MAGNETIC FIELD GRADIENTS

Figure 1 shows the lower hybrid wave dispersion characteristics in the presence of simultaneous density and magnetic field gradients for an isotropic, maxwellian ($T_i = T_e$) and "locally" homogeneous plasma. These curves have very much the appearance of the electrostatic Bernstein¹ modes with the addition of the lower-hybrid wave approaching from the upper left. After conversion into the plasma² and the electrostatic waves^{3,4} respectively the lower-hybrid wave is fully absorbed at the cyclotron harmonic resonance acting as a "singular turning point".

Cyclotron Harmonics as Singular Turning Points

Near the harmonics $|k_x| \rightarrow \infty$ and the electrostatic wave assumes the approximate form

$$\epsilon_{xx} = 0 = 1 + \frac{\omega_p^2}{\omega \omega_c} \epsilon_0 e^{-\lambda} \sum_{n=-\infty}^{\infty} \frac{n}{\lambda} I_n(\lambda) Z(\epsilon_n) \quad (1)$$

$$\text{where } \epsilon_n = \frac{\omega - n\omega_c}{k_z v_{th}}, \quad \lambda = \frac{1}{2} k_x^2 r_c^2,$$

$I_n(\lambda)$ is the modified Bessel function and $Z(\epsilon_n)$ is the Fried's function defined as

$$Z(\epsilon_n) = i\pi^{1/2} e^{-\epsilon_n^2} + \pi^{1/2} \int_{-\infty}^{\infty} \frac{e^{-x^2}}{x - \epsilon_n} dx, \quad \text{Im}(\epsilon_n) > 0 \quad (2)$$

But for the existence of finite k_z , Eq. (1) is the familiar dispersion relation for the electrostatic ion-cyclotron-harmonic waves. The finite k_z has the effect of introducing dissipation near the harmonics. In Ref. (5) it was shown that for the case of purely perpendicular propagation ($k_z = 0$), the cyclotron harmonics are "true singularities" of the dispersion relation namely $\lambda \rightarrow \infty$ and $i \infty$ respectively immediately above and below the harmonic frequencies. Thus in the neighbourhood of the cyclotron harmonics the dispersion relation (1) possesses the singular form

$$k_x^2 - q(x - x_0)^{-1/2} = 0 \quad (3)$$

Absorption at the Singular Layer

The differential equation for the electric field near the singular turning point $s = x - x_0$ may be expressed as,

$$\frac{d^2 E}{ds^2} + q s^\mu E = 0 \quad (4)$$

with the Hankel function solution,

$$E = s^{1/2} H_{2\nu}(\nu^{-1} q^{1/2} s^\nu) \quad (5)$$

having the asymptotic forms (for $-1 \leq \mu < 0$, and with the proper choice for the sign of $q^{1/2}$),

$$E \sim A e^{-i q^{1/2} X} + B e^{i q^{1/2} X}, \quad x \rightarrow \infty \quad (6)$$

and

$$E \sim A e^{-\alpha q X} e^{i \beta q X} + B e^{\alpha q X} e^{-i \beta q X}, \quad x \rightarrow -\infty \quad (7)$$

where $\alpha = \nu^{-1} \sin(\pi|\mu|/2)$, $\beta = \nu^{-1} \cos(\pi|\mu|/2)$, $X = \nu^{-1} |x|^\nu$,

and $\nu = (\mu + 2)/2$.

In order for E to remain finite at $x \rightarrow -\infty$, one obtains from (7) that $B \equiv 0$, i.e., the absence of a reflected wave thereby implying complete absorption of the incident wave at the singular layer⁶.

2. EFFECT OF IMPURITIES

A trivial effect of impurities would be to change the effective plasma frequency and thereby shift the location of the lower-hybrid layer and the first wave conversion region.

More importantly, though, the presence of impurities introduces a series of cyclotron harmonics (one set for each Z/m) in the path of the wave. The amount of energy intercepted at one of these harmonics at $x = x_0$ is reflected in the damping decrement Δ as,

$$\Delta = \int_{x_0 - \epsilon}^{x_0 + \epsilon} k_{xi} dx. \quad (8)$$

From the dispersion relation

$$a n_x^4 + b n_x^2 + c = 0$$

one obtains on differentiation and retaining the dominant terms only,

$$\delta n_x \approx (n_x / 2 n_z^2) \delta \epsilon_{xx}, \quad (9)$$

where we have omitted terms containing $\delta \epsilon_{xy}$ and $\delta \epsilon_{zz}$ which are odd in x and make no net contribution to the integral in (8). Since $n_z^2 \approx 1$, we obtain from (8) and (9)

$$\Delta \approx \int_{x_0 - \epsilon}^{x_0 + \epsilon} k_x \delta \epsilon_{xxi}^* dx, \quad (10)$$

where the asterisk denotes impurity contribution. On substituting $\delta \epsilon_{xxi}$ from (1) and (2) in (10),

$$\Delta \approx \pi^{1/2} \frac{\omega_{pi}^{*2}}{\omega \omega_{ci}^*} \eta_0^* e^{-\lambda^*} \frac{n^*}{\lambda^*} I_{n^*}(\lambda^*) k_x \int_{-\infty}^{\infty} e^{-\eta_{n^*}^{*2}} dx, \quad (11)$$

where all quantities varying slowly with x have been taken out of the integral. Also, since $\exp(-\eta_{n^*}^{*2})$ is a rapidly decaying function of position, the limits of integration have been extended to $\pm \infty$. Assuming a linearly varying magnetic field with a characteristic gradient length equal to plasma radius,

$$\omega_{ci}^*(x) = (d\omega_{ci}^*/dx)(x - x_0) = (\omega_{ci}^*/R)(x - x_0),$$

and one obtains from (11),

$$\Delta = \frac{\omega_{pi}^{*2}}{\omega_{ci}^{*2}} \frac{\sqrt{2} \pi R}{\tau_{ci}^*} \frac{e^{-\lambda^*} I_{n^*}(\lambda^*)}{\lambda^{*1/2}}. \quad (12)$$

For typical parameters ($\lambda^* \sim 1$), Δ may amount to less than one part in a million.

Thus there is no significant diversion of the wave energy into impurity heating near the plasma edge as the wave approaches the lower-hybrid layer. Following wave conversions, however, the wave may encounter an impurity ion-cyclotron harmonic and the possibility of impurity heating assumes importance in the plasma interior ($\lambda^* \gg 1$).

3. HEATING AT LOW-CYCLOTRON HARMONICS

For a more uniform deposition of the wave energy in the plasma it may be advantageous to heat the plasma at a low-cyclotron harmonic (e.g., $\omega = \omega_{ci}$).⁷

The wave conversions then occur close to the plasma edge (Fig. 2). However, for the small n^* , (12) shows that the presence of even a small amount of impurities could result in considerable impurity heating close to the plasma edge. This heating can cause an unstable runaway condition because the hotter impurity ions so produced become ever more efficient in removing energy from the wave. Such an adverse condition, in principle, could also occur if during the fast-wave heating the antenna inadvertently launches a component of the TM electromagnetic wave. For heating tritium ions at ω_{CT} it may yet be possible to avoid impurities with $Z^*/m^* < 1/3$ in a thermonuclear plasma.

REFERENCES

1. I.B. Bernstein, Phys. Rev. 109, 10 (1958).
2. T.H. Stix, Phys. Rev. Lett. 15, 878 (1965).
3. M. Simonutti (private communication).
4. K.G. Wong and T. Tang, Second International Conference on Waves and Instabilities in Plasmas, Innsbruck (1975), paper H7.
5. S. Puri, Phys. Letters 55A, 99 (1975).
6. K.G. Budden, Proc. Roy. Soc. A227, 517 (1955).
7. S. Puri, Proceedings of the Second International Conference on Waves and Instabilities in Plasmas, Innsbruck, P1-P64 (1975).

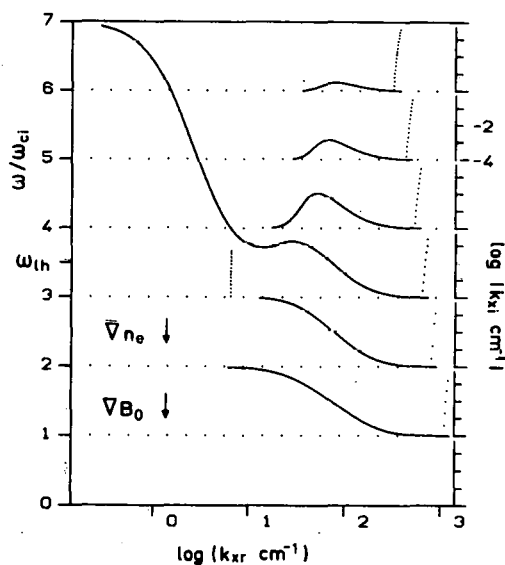


Fig. 1
Dispersion characteristics of the lower-hybrid wave for $T = 100$ eV, $n_z = 1.5$, $B_0 = 60$ kG and $\omega/\omega_{ci} = 3.5$ for deuterium. Density n_i increases linearly from 0 to $2 n_{ih}$ as ω/ω_{ci} varies from 7 to 0.

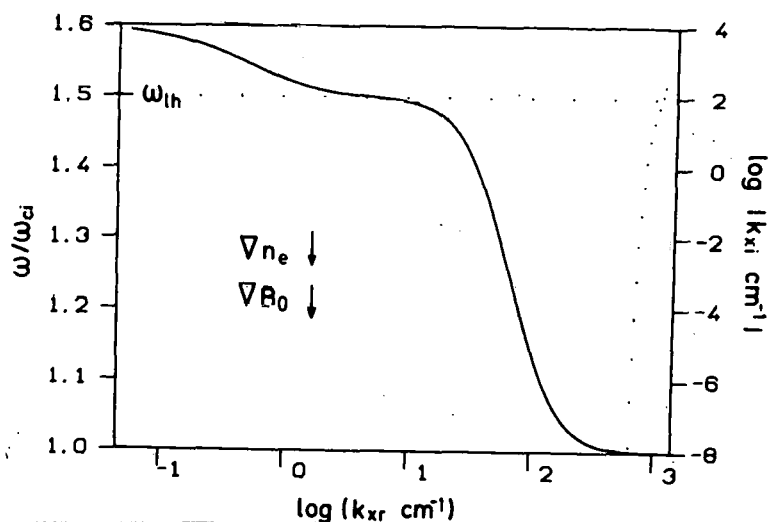


Fig. 2
Same as Fig. 1 for $\omega/\omega_{ci} = 1.5$.

A LOWER HYBRID HEATING SYSTEM FOR AN IGNITION TOKAMAK*

J. Brooks, S. Harkness, J. Jung,
A. Moretti, J. Norem, and H. Stevens
Argonne National Laboratory
Argonne, Illinois 60439

Summary We have attempted to design a complete Lower Hybrid Resonance Heating System (LHRH) that could be used for TFTR, TNS, EPR, or a reactor. In addition to plasma physics constraints, we have considered those imposed by neutron radiation, surface heating of waveguides, sputtering, multipactoring, vacuum systems, materials, window design, engineering, maintenance and assembly. The system uses a Lallia-Brambilla grill which is fed by a number of waveguides entering the reactor by means of a labyrinth.

Plasma Physics Considerations It has been shown^{1,2} that plasma heating to ignition should be possible with power levels of the order of 50 MW, with heating times from 4 to 10 s. Additional constraints are imposed by the requirements that this power must penetrate to the center of a 1 to 2 m minor radius plasma. Heating by means of radio frequency (rf) power in the lower hybrid frequency range was studied because 1) the availability of high power sources in this frequency range (1 to 5 GHz). 2) the experimental success of the grill launching structure proposed and studied by Lallia and Brambilla³. 3) the overall simplicity and flexibility of the resulting system, 4) the general compatibility of this system with the high-radiation environment, and 5) bulk heating of plasma has been demonstrated⁴.

The recent ATC data⁵ have shown that parametric instabilities in the plasma itself are excited when the driving frequency f is near the lower hybrid frequency, f_{LH} . These parametric instabilities⁰ cause power deposition near the edges of the plasma. Both theoretical and experimental evidence indicates that the effects of parametric instabilities should be negligible for $f > 2 f_{LH}$. At these frequencies the primary mode of power absorption will be Electron Landau Damping (ELD). The range of parallel refractive indexes ($n_{||}$) appropriate to ELD depends on the plasma temperature. For efficient coupling, the phase velocity of the wave should be comparable to three or four times the electron thermal speed. An additional constraint is imposed by the requirement that $n_{||}^2 > 1 + (\omega_{pe}^2 / \omega_{ce}^2)$.

A possible scenario for heating to ignition has been studied. The density is quickly raised to nearly its final value and the temperature is raised smoothly over about 7 s. The power required to do this need not be constant. The low temperature plasma will require $n_{||}$ values which will be rather inefficiently radiated by the grill structure. Fortunately, the conduction losses of the low-temperature plasma will also be small.

RF Engineering The launcher for supplying 50MW of supplemental heating consists of 10 grills composed of ~ 40 narrow-height WR-430 waveguides forming a rectangular array, 4 waveguides high and 10 waveguides wide. Each vertical column of the grill is fed by a 500 kW CW klystron amplifier, the X3070. The X3070 is a high-power modulating anode klystron tunable from 2.320 to 2.456 GHz.

The windows are similar in design to the klystron window but only rated 125 kW CW. The only component in the waveguide system that is not standard is the dc break. This could be readily designed around a contact-less resonant choke flange arrangement in which the space between the choke and flange, as well as the groove, is filled with ceramic. The gap and groove are sized to hold off the expected dc fault voltage and present a short circuit at the waveguide junction at rf.

The overall dc to rf efficiency is 41%, without thermal recovery, assuming a transmission efficiency for the grill of 90% (95% has been achieved in a four-element grill on ATC), 53% as the conversion efficiency of the klystron, 0.4 dB as the insertion loss of the waveguide transmission line, and 95% as the ac to dc conversion efficiency of the dc power supply.

System Engineering The overall plan of the rf system (Fig. 1) is based on the following arguments: 1) the major part of the waveguide run from the klystrons to the reactor will be pressurized SF_6 , and 2) the windows separating the pressurized and evacuated sections should be removed from as much as possible for the neutron flux. Pump down of the evacuated section will be most efficient if it is short, and the conflicting constraints of low neutron flux and short waveguide sections have been solved by means of a two-bend labyrinth. This will minimize the time between breakdown, when the neutral pressure may be $\sim 10^{-4}$ torr, and the time when the rf can be turned on, which seems to be dependent only on when a pressure of $\sim 10^{-6}$ torr can be achieved. A dc break is required in the feed waveguide either before or after the window to prevent possible ground loop interference with sensitive instrumentation and protection of personnel and equipment from ground faults. Toroidal vacuum systems on rf-heated tokamaks may require penetrations in shielding comparable to that required by neutral beams.

Since the electron cyclotron resonance (ECR) equals 2.4 GHz for a magnetic field of 1 kG special care will be necessary to insure that breakdown will not occur where $B_{OH} + B_T + B_{EF}$ equal ~ 1 kG. This effect can be minimized 1) by insuring that ECR occurs in the pressurized guide, 2) by placing compensation coils to perturb the field from the resonance value elsewhere.

Materials The choice of waveguide materials is constrained by the requirements for 1) low resistivity, 2) dimensional stability, and 3) good mechanical and thermal properties. On the basis of resistivity, copper and aluminum are the most obvious choices but copper has roughly 60% lower resistivity values, making it somewhat more preferable. Other considerations that lead to the choice of Cu are its higher melting point (1083° vs. 660° C for Al) leading to more tolerance for temperature transients, higher strength, and compatibility with Be, which should allow the use of a Be coating.

The surface material of the waveguide is critical for two reasons: 1) impurity sputtering effects would be minimized if a low Z coating was applied to all surfaces exposed to the plasma, and 2) multipactoring would be minimized if the surfaces chosen had secondary electron yields for electrons less than one. Both these constraints can be met if the waveguides are coated everywhere with a thin layer of Be. This treatment has also been advocated for the whole reactor first wall¹. Neutron-induced surface damage, which would continuously alter

the surface roughness at the $10\ \mu$ in level is not a serious problem for rf in the 1 to 5 GHz band. However, higher-frequency heating systems (electron cyclotron heating at 70-120 GHz) may be quite sensitive to this effect.

The constraints that limit the window material choices are 1) long life-time under high neutron and gamma fluxes at temperatures of $\sim 100^\circ\text{C}$, 2) high dielectric strength, and 3) low loss tangent (< 0.0002). A tentative selection of window material would be Y_2O_3 if radiation resistance was the most important consideration, and BeO or Al_2O_3 otherwise.

Shielding A labyrinth with two bends was calculated using a detailed Monte Carlo model⁶, because 1) it is doubtful that more than two bends could be required, and 2) all the required data for zero and one bend systems can be obtained from the two bend results. The results show that small ducts are highly desirable and lifetimes of ~ 10 years can be expected from Al_2O_3 windows if removed from direct neutron flux.

Cooling of Waveguides Neutron heating amounts to about $1\text{W}/\text{cm}^3$ of material, and steady state surface heating loads are about $20\text{W}/\text{cm}^2$. The waveguides and grill structure should operate at low temperatures ($< 100^\circ\text{C}$) to avoid increased radiation-induced swelling, which takes place at a higher rate at high temperatures. The grill itself, composed of 0.5 cm thick walls, would be cooled using channels running within the walls. The inlet and exhaust manifolds would occupy the space between the horizontal rows of waveguides.

Conclusions This system⁷ has a number of advantages over neutral beams⁸, which are the primary alternative. The grill is simpler than a neutral beam injector. Cost estimates that have been made for comparable heating systems show the rf system to be about 30% cheaper, (\$18 M for rf vs. \$24 M for neutral beams, exclusive of power supplies). A large part of the cost advantage for rf systems is due to the fact that a significantly smaller building is required for the reactor, however, additional torus pumping is required. A parallel study⁹ of LHRH, based on somewhat different assumptions, has confirmed the engineering feasibility of rf heating.

The major uncertainties in this design are in the physics of the launcher, and the coupling and penetration of the power into the plasma. These points require more experimental results.

Acknowledgements We have had useful discussions with B. Reed, S. Bernabei, and W. M. Hooke of Princeton Plasma Physics Laboratory and with R. W. Harvey of the General Atomic Company.

References

1. W. M. Stacey, Jr. et al., "Tokamak Experimental Power Reactor Conceptual Design," ANL/CTR/76-3, Argonne National Laboratory (August 1976).
2. "TNS Scoping Studies, Interim Status Report," General Atomic Company and Argonne National Laboratory, GA-A14412, ANL/FPP/77-2 (May 1977).

3. M. Brambilla, "The Theory of the Grill," Third Symposium on Plasma Heating in Toroidal Devices, E. Sindoni, ed., Varenna, Italy (1976).
4. P. R. Lallia, et al., "Ion Measurements during Lower Hybrid Experiments in Wega," Third Symposium on Plasma Heating in Toroidal Devices, E. Sindoni, ed., Varenna, Italy (1976).
5. M. Porkolab, et al., Phys. Rev. Lett. 38, 230 (1977).
6. E. M. Gelbard et al., "Monte Carlo Work at Argonne National Laboratory," in Proc. NEACRP mtg. of a Monte Carlo Study Group, ANL-75-2, Argonne National Laboratory (1974).
7. J. Brooks, et al., "A Lower Hybrid Heating System for a Ignition Tokamak," Seventh Symposium on Engineering Problems in Fusion Research, Knoxville, TN (1977).
8. J. Fasolo, et al., "A Neutral Beam System for an Ignition Tokamak," Seventh Symposium on Engineering Problems in Fusion Research, Knoxville, TN (1977).
9. B. Reed, "Preliminary Report on the Development of Radio Frequency Auxiliary Heating for TEPR-1," Princeton Plasma Physics Laboratory Report (to be published).

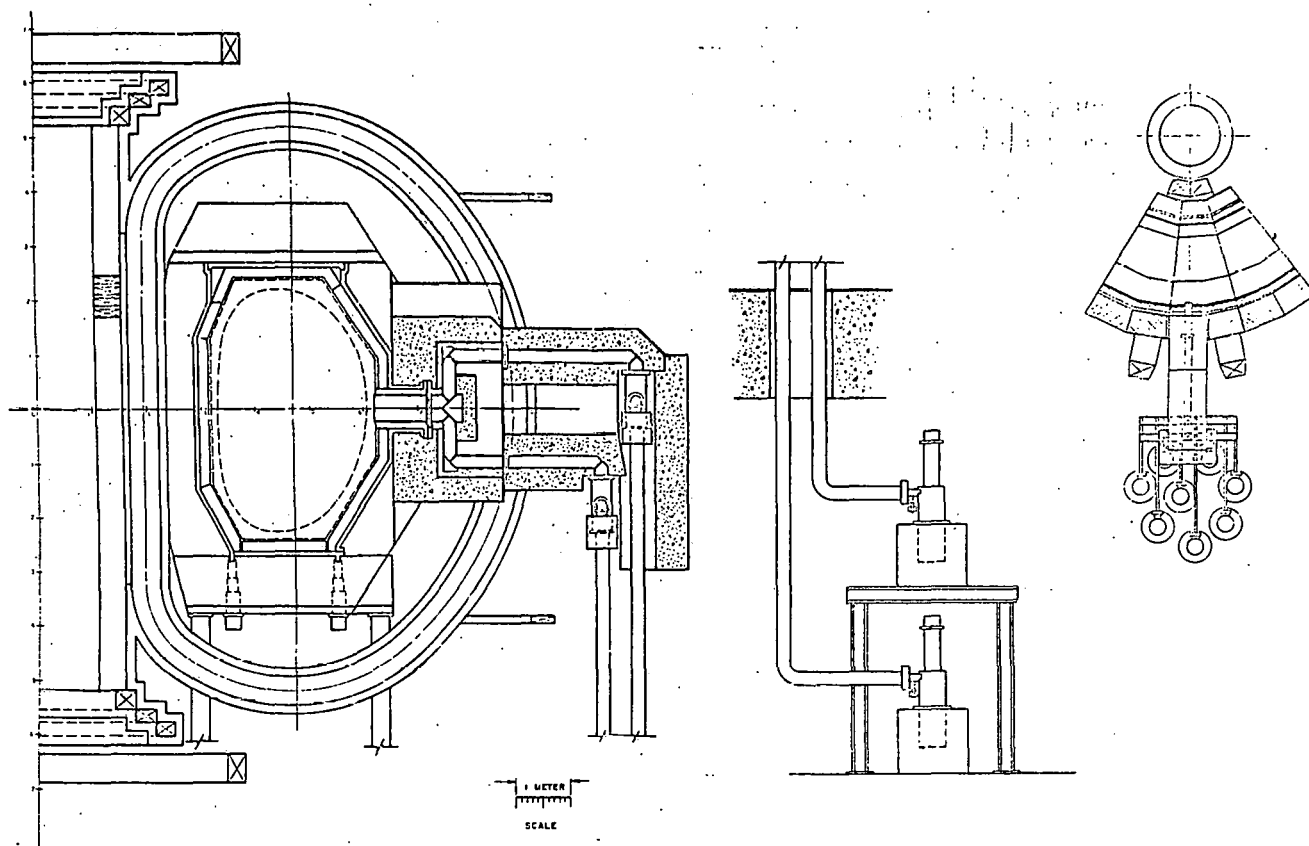


Fig. 1 Overall Plan of the rf System, on the Argonne Experimental Power Reactor

A Steady State Toroidal Reactor Driven by Microwave Power in the Lower-Hybrid Range of Frequencies*

by

A. Bers and N. J. Fisch

Plasma Fusion Center, M. I. T., Cambridge, MA 02139

At high cw-powers an end-fire array of phased waveguides at the plasma wall can be made to deliver net momentum to plasma electrons in the tail of its velocity distribution function thus generating an appreciable steady-state current which confines and maintains the temperature of a compact reactor-type plasma.¹ Proof-of-principle experiments on current tokamaks are also suggested.

The evolution of the electron distribution function is due to collisions (e-e and e-i) and quasi-linear diffusion by the applied electric fields. For the present we consider only the distribution function parallel to the magnetic field (i.e. the distribution function integrated over perpendicular velocities)

$$\frac{\partial f}{\partial \tau} = \frac{\partial}{\partial w} \left[\frac{1}{w^3} \left(\frac{\partial f}{\partial w} + w f \right) \right] + \frac{\partial}{\partial w} D \frac{\partial f}{\partial w} \quad (1)$$

where the following normalizations have been introduced: $w = v_z/v_T$ ($v_T^2 = T_e/m_e$), $\tau = t/\tau_0$ ($\tau_0 = \tau_e/w^3 \approx 10^{-6} T_{\text{keV}}^{3/2}/n_{14}$), $D = D_{QL}\tau_0/v_T^2$. The steady state solution of this equation, described in the adjoining paper, is

$$f = c \exp \left[- \int^w \frac{w dw}{1 + w^3 D} \right] \quad (2)$$

For large-amplitude fields, such that $Dw^3 = w\Delta \gg 1$, where $\Delta = w_2 - w_1$ is the extent of phase velocities of the field spectrum, the distribution function in the resonant region is approximately given by

$$f \approx A \exp \left[\frac{1}{Dw} - \frac{1}{4D^2 w^4} \dots \right] \quad (3)$$

and takes on the shape of an essentially flat and raised plateau, as shown in Fig. 1. The electrons in this raised plateau constitute a current in the plasma. In addition, through collisions with electrons in the bulk of the distribution function they also heat the plasma. From the above equations we can calculate the power dissipated and the current generated by the fields:

*Work supported by U.S. Department of Energy (Contract EG-77-G-01-4107)

$$P_D = \frac{nmv_T^2}{\tau_0} Q_{RF} F_1 \approx 3 \times 10^9 \frac{n_{14}^2}{T_{kev}^{1/2}} a_{10}^2 R_m F_1 \text{ Watts}, F_1 = \frac{\exp(-w_1^2/2)}{\sqrt{2\pi}} \ln \frac{w_2}{w_1} \quad (4)$$

$$I = env_T A_{RF} F_2 \approx 7 \times 10^6 n_{14} T_{kev}^{1/2} a_{10}^2 F_2 \text{ Amps}, F_2 = \frac{\exp(-w_1^2/2)}{\sqrt{2\pi}} \left(\frac{w_2^2 - w_1^2}{2} \right) \quad (5)$$

where Q_{RF} and A_{RF} are respectively the volume and area occupied by the fields, a_{10} is the minor radius extent of the fields in units of 10 cm, R_m is the major radius in meters, n_{14} is the plasma density in units of $10^{14}/\text{cm}^3$, and T_{kev} is the electron temperature in units of 1 keV.

For steady-state operation we now require that the power dissipated by the fields equal the power loss by the plasma, and that the current generated by the fields be sufficient for maintaining equilibrium and stability so that the ohmic-current, which was used to prepare the plasma, can be turned off. For the power loss we shall assume that the energy confinement time τ_E has Alcator-type scaling:² $\tau_E \approx 3.5 \times 10^{-3} \bar{n}_{14} a_{10}^2 \text{ sec}$. We then have,

$$P_L = \frac{3neT_{ev}}{\tau_E} Q \approx 10^6 T_{kev} R_m \text{ Watts} \quad (6)$$

For the required equilibrium and stability current we start with

$$I_0 = \frac{2\pi a}{\mu_0} B_p \approx 1.4 \times 10^5 \left(\frac{n_{14} T_{kev}}{\beta_p} \right)^{1/2} a_{10} \text{ Amps} \quad (7)$$

where B_p and β_p are respectively the poloidal magnetic field and the poloidal plasma beta. We shall require that the current be sufficient to confine the α -particles ($I_\alpha \approx 12.5 \text{ kA}/(R/a)^3$) and we shall allow ourselves to adjust the toroidal magnetic field B_T and the aspect ratio $R/a (\sim \beta_p)$ to remain within technologically reasonable bounds and consistent with MHD stability (safety factor $q_s \gtrsim 2$, and total plasma beta $\beta \sim 1\text{-}10\%$).

The required phase velocity range (w_1, w_2) for the applied fields can now be determined from $P_D = P_L$ and $I = I_0 = I_\alpha$, giving F_1 and F_2 in terms of plasma parameters. In determining w_1 and w_2 it is convenient to consider the ratio of these functions

$$\frac{F_1}{F_2} \equiv F_3 = \frac{2 \ln(w_2/w_1)}{w_2^2 - w_1^2} \approx 5 \times 10^{-2} \left(\frac{T_{kev}}{n_{14}} \right)^{3/2} \frac{\beta_p^{1/2}}{a_{10}} \approx 1.7 \times 10^{-4} \frac{T_{kev}^2}{n_{14}} \left(\frac{R}{a} \right) \quad (8)$$

which describes the effective voltage (P_D/I) and resistivity. Since F_3 does not contain the rapid exponential variation with w_1 it allows for a first-cut determination of w_1 with reasonable choices of (w_2/w_1) . Figures 2-4 display the functions F_1 - F_3 .

Consider the following reactor-type example: For $\tau_E \sim 10^{14} \text{ sec/cm}^3$ let $n_{14} \sim 4$, $a_{10} \sim 5$

and let $\beta_p \sim (R/a) \sim 5$ and $T_{kev} \sim 10$. We then find approximately the operating point designated R on Figures 2-4 with $w_1 \sim 4$ and $(w_2/w_1) \sim 1.5$ giving a current $I \sim 2 \times 10^6$ Amps and requiring $P_D \sim 20 \times 10^6$ Watts. Assuming $q_a \sim 2$ and letting $B_T \sim 100$ kG we find a plasma $\beta \sim 5\%$. For this "compact reactor" the fusion power out is only about 300×10^6 Watts with a wall loading of 2.5×10^6 Watts/m². In addition the current may not be sufficient to confine enough of the α -particles, and the aspect ratio too small for the stresses associated with the large magnetic field. However, using noncircular cross sections and different spectrum locations gives enough flexibility for achieving workable reactor designs.

The location of the spectrum and the desired plasma temperature dictates the required design of the waveguide array at the wall of the plasma. For our example we find $1.2 < n_z < 1.8$. For this range to be accessible we must choose the frequency below the lower-hybrid frequency.⁴ For the high-densities of the reactor this can be chosen near $f \sim 1$ GHz which will also permit large powers to be carried by the waveguides. In addition, in this frequency regime it is advantageous to excite not only the slow waves which will heat efficiently but also the fast waves (high-frequency Alfvén waves) which can be used mainly to produce the current. A waveguide array of interleaved waveguides at an angle to B_T or ridged-waveguides, as shown in Fig. 5, should have sufficient flexibility to accomplish this. Simple calculations with profiles show that the slow-waves, which would be phased to be near w_1 , have a damping length comparable to the chosen plasma radius, and damp near the center. The fast-wave spectrum phased to produce mainly current would be placed near w_2 . For the required power one obtains reasonable, non-breakdown fields by allowing 1% of half the wall area to be for the phased-array waveguides.

Finally we point out the feasibility of proof-of-principle experiments (omit last equality in (8)) on current tokamak machines. These are indicated on Figs 2-4 for parameters of Alcator A ($n_{14} \sim 1$, $T_{kev} \sim 2$) and Versator II ($n_{14} \sim 4$, $T_{kev} \sim 8$). In addition to maintaining a temperature about twice what is obtained with ohmic heating alone, the RF would produce appreciable currents (150 kA in Alcator A and 30 kA in Versator II) with modest powers (Alcator A 500 kW, Versator II 100 kW).

References

1. A. Bers and N. J. Fisch, M.I.T. - O.S.P. Invention Record (1977).
2. M. Gaudreau *et. al.* Phys. Rev. Lett. 39, 1266 (1977).
3. D.G. McAlees, ORNL-TM-4661 (1974)
4. A. Bers, in Proc. US - Australian Workshop on Plasma Waves, R. C. Cross ed., Sydney, 1975.

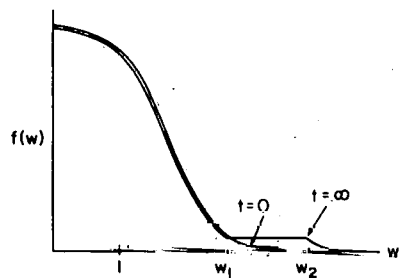


Fig. 1

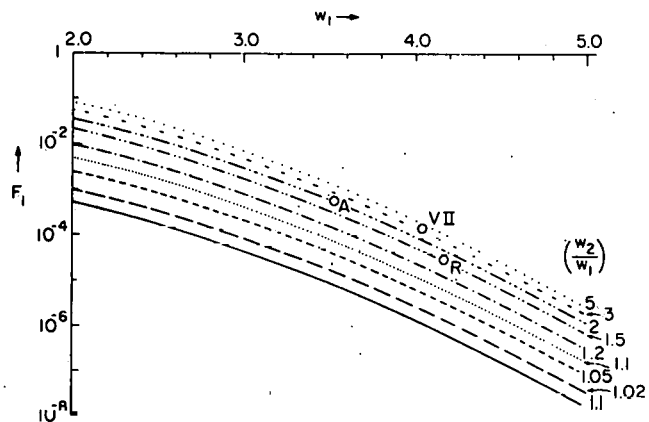


Fig. 2

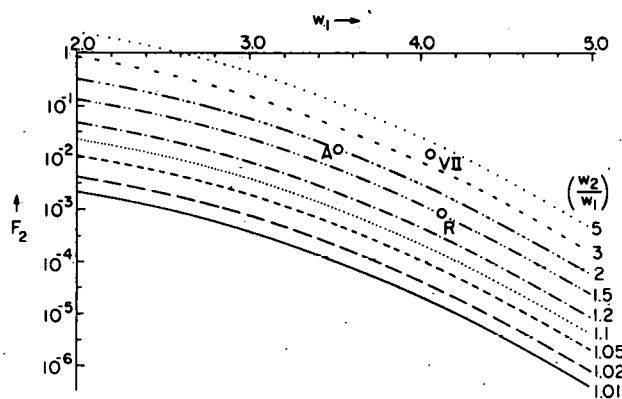


Fig. 3

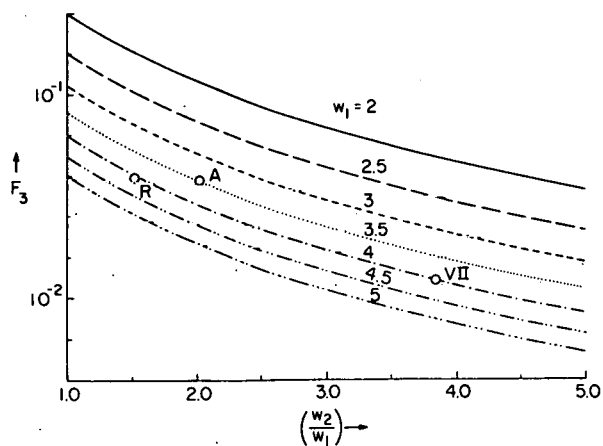


Fig. 4

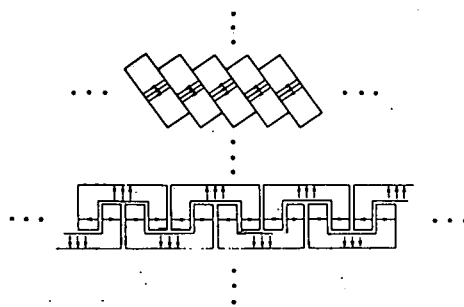


Fig. 5

Current Generation by High Power RF Fields*

by

N. J. Fisch and A. Bers

Plasma Fusion Center, Massachusetts Institute of Technology, Cambridge, MA 02139

Continuous toroidal electron currents, which sustain the poloidal magnetic field in tokamaks, may be generated by injecting net momentum waves into the plasma via phased waveguide arrays. We show that a significant current can be turned on in a relatively short time with modest power dissipation.

Plasma confinement in Tokamak fusion devices is maintained, in part, by a poloidal magnetic field sustained by a toroidal current. The current is usually driven by an inductively produced DC electric field, so that the tokamak operates only in a pulsed mode. For steady-state tokamak operation, a method of continuously driving the toroidal current is essential. One scheme of producing continuous current relies upon the Landau damping of RF waves with a net parallel momentum, such as might be absorbed from an endfire array of phased waveguides. The wave momentum, largely absorbed by the electrons, results in an electric current. Unfortunately, the amount of RF power required to produce such a current, if it sees the full plasma resistivity, is too large to be practical.⁽¹⁾ We show, however, that if the RF power is intense enough and the spectrum is broad enough, it is possible to establish a current largely carried by high-velocity electrons. These electrons experience much less dynamical friction than thermal electrons, resulting in less power dissipated. The generation of toroidal currents by RF power then becomes of interest.⁽²⁾

The presence of the RF power results in parallel velocity diffusion which competes with the collisional relaxation of the plasma, so that the evolution of the space-averaged electron velocity distribution is governed by

$$\partial f(\mathbf{v}, t) / \partial t = \partial / \partial v_z D_{QL}(v_z) \partial f(\mathbf{v}, t) / \partial v_z + (\partial f / \partial t)_c, \quad (1)$$

where we have neglected any DC electric fields and where D_{QL} is the quasilinear diffusion coefficient and $(\partial f / \partial t)_c$ is the Fokker-Planck collision operator,⁽³⁾ which describes both parallel and perpendicular velocity scattering. Our main interest, however, is in the dynamics in the parallel direction, where the quasilinear diffusion tends to flatten the distribution and the collisions tend to restore it to a Maxwellian. These dynamics are retained when eq. (1) is integrated over the perpendicular velocity direction, where f is assumed to be Maxwellian, obtaining for high-velocity electrons in a singly ionized plasma⁽⁴⁾

$$\frac{\partial f}{\partial \tau} = \frac{\partial}{\partial w} D(w) \frac{\partial f}{\partial w} + \frac{\partial}{\partial w} \left(\frac{1}{w^3} \frac{\partial f}{\partial w} + \frac{1}{w^2} f \right), \quad (2)$$

where we have normalized $w = v_z / v_{th}$, $\tau = \nu_0 t$, $\nu_0 = \nu w^3$, $\nu = \omega_p^4 \ln \Lambda / 2 \pi n v_z^3$, and $D(w) = D_{QL} / \nu_0 v_{th}^2$.

*Work supported by U.S. Department of Energy (Grant EG 77-G-01-4107)

In one-dimensionalizing eq. (1), we have neglected the perpendicular velocity space structure; however, if this structure is independent of v_z , it does not affect the current generated by the RF power, and if it is nearly Maxwellian, the power dissipated is, similarly, negligibly affected. Although the perpendicular structure, which manifests itself mainly in a flattening in the resonant region of velocity space, may be important in other plasma dynamics, e.g. interaction with a DC electric field, it is not expected to qualitatively affect the calculations that we perform. The exception to this intuition occurs for a high Z_{eff} plasma, where the perpendicular dynamics may dominate to the extent that RF runaway results.⁽⁵⁾

The power dissipated and current generated by the RF may be calculated from the steady-state solution of eq. (2), in which f evolves to a state where it is slowly being heated but otherwise not evolving, and is given by

$$f = c \exp\left(-\int^w \frac{wdw}{1 + w^3 D(w)}\right), \quad (3)$$

where c is a constant determined by conservation of particles. The problem of interest for RF heating schemes and especially for current generation is when $D(w)$ is effectively infinite in a finite region, say $w_1 < w < w_2$, and vanishes elsewhere. The steady-state solution is then Maxwellian where there is no RF and flat in the resonant region. The plateau in the resonant region acts as a beam of fast electrons with current density

$$J = 2.1 \times 10^8 n T^{1/2} f(w_1)(w_2^2 - w_1^2)/2 \text{ amps/m}^2, \quad (4)$$

where n and T are in "practical units," i.e. n is normalized to 10^{14} cm^{-3} and T is normalized to 1 KeV. The power dissipated is easily calculated, among other ways, by demanding that eq. (2), in the absence of the RF, but instantaneously in the steady-state, conserve energy, thus obtaining

$$P_D = 1.5 \times 10^{10} n^2 T^{-1/2} f(w_1) \ln(w_2/w_1) \text{ watts/m}^3. \quad (5)$$

In order to appreciate the power cost for generating the current it is helpful to define an effective RF resistivity

$$\eta_{RF} = P_D/J^2 = \frac{3 \times 10^{-7}}{T^{3/2}} \frac{\ln(w_2/w_1)}{f(w_1)(w_2^2 - w_1^2)^{3/4}} \text{ Ohm-m} \approx \frac{3 \times 10^{-7}}{T^{3/2}} \frac{n/n_p}{w_1^3} \text{ Ohm-m}, \quad (6)$$

where the last approximate equality was written for $\Delta/w \ll 1$, and n_p/n is the fraction of electrons in the plateau.

Note that for $\Delta/w \ll 1$, we can write

$$\eta_{RF}/\eta_{||} \approx 10(n/n_p)/w_1^3, \quad (7)$$

where $\eta_{||}$ is the Spitzer resistivity. Eq. (7) implies that whereas the RF current heats more effectively than the ohmic current at low current levels (n_p/n small), as the current level increases, or as the spectrum is shifted to higher phase velocities, the RF current begins to heat less effectively than the ohmic current.

The current generated by RF power differs in several important ways, other than simply in

the amount of power dissipated, from an inductively generated current. It must be appreciated that when net momentum waves are damped, the plasma itself acquires a net momentum, and begins to rotate. A similar situation occurs in unidirectional neutral beam injection. A calculation determining the extent of the plasma rotation is beyond the scope of the present study; we note, however, that momentum may be lost to particles leaving the plasma or to particles trapped in ripple fields⁽⁶⁾ (and their supporting coils). In any case, one may write momentum balance equations of the form

$$(\partial/\partial t + \gamma_e) m_e n_0 v_{De} = \nu_{ee}^{P/B} m_e n_p v_p + \nu_{ei}^{B/B} n_0 m_e (v_{Di} - v_{De}) , \quad (8)$$

$$(\partial/\partial t + \gamma_i) m_i n_0 v_{Di} = \nu_{ei}^{P/B} m_e n_p v_p + \nu_{ie}^{B/B} n_0 m_i (v_{De} - v_{Di}) , \quad (9)$$

where $\nu_{ee}^{P/B}$, for example, indicates the collision frequency for slowing down or momentum transfer of plateau with bulk electrons and γ_e and γ_i model any momentum sinks for electrons and ions respectively. Equations (8) and (9) describe the transfer of momentum from the fast plateau electrons, at velocity v_p and density n_p to the bulk ion and electron distributions, drifting respectively velocities v_{Di} and v_{De} , which, in turn, experience a mutual friction to the extent that these drift velocities differ. Solving eqs. (8) and (9), not for the overall toroidal rotation, which necessitates an evaluation of γ_e and γ_i , but for the relative drift between the ions and electrons, and assuming $\gamma_e, \gamma_i \ll \nu_{ei}^{B/B}$, we find a steady-state bulk current, which is seen to be negligible compared to the plateau current, i.e.

$$\frac{J_B}{J_p} = \frac{qn_0(v_{De} - v_{Di})}{qn_p v_p} \approx \frac{\nu_{ee}^{P/B}}{\nu_{ei}^{B/B}} \sim \frac{1}{w_1^3} \ll 1. \quad (10)$$

An RF driven current also differs from an inductively driven current in that there is no DC electric field in steady-state operation. This is because the current is forced volumetrically rather than, as through a resistor, from a boundary. However, as the RF current is turned on in a tokamak, it does generate a time-varying magnetic field, which, in turn, induces a toroidal DC electric field that opposes the motion of the plateau electrons. The DC field instantaneously produces a counter-current of electrons, primarily in the bulk of the velocity distribution (since most electrons are situated there), so as to oppose any abrupt change in the flux linkage to the plasma. The counter-current decays in an L/R time of the tokamak, where L and R are the plasma inductance and resistance, after which the RF current flows in the absence of the DC field.

When even very intense RF power is turned on, there is, initially, very little RF current, since the number of electrons initially in the resonant region is quite small. We can define an RF current turn-on time, τ_{t-o} , occurring on a collisional time scale, during which bulk electrons are collisionally scattered into the resonant region to form the "raised" plateau that is characteristic of the time-asymptotic distribution. If τ_{t-o} is less than L/R, then both the RF plateau current and bulk counter-current are turned on in a time τ_{t-o} , the latter decaying in a time L/R. In the event that the RF power is turned on in an inductively driven tokamak, the current is transferred from bulk carriers to plateau carriers in a time τ_{t-o} , but the total current does not change, although the DC electric field is effectively shut off. In the event that the RF current exceeds the original

Ohmic current in less than an L/R time, the RF current will begin to drive the primary transformer coils in reverse. In steady-state tokamak operation, it is desired to switch from the Ohmic current to the RF current, so that after a turn-on time the Ohmic coils should be disconnected. It should be noticed that the absence of a DC electric field in the steady-state operation implies that no runaways are produced.

It remains to calculate the turn-on time, τ_{t-o} . We assume in the calculation that during the turn-on, the bulk electron temperature is essentially constant; in other words, that $\tau_{t-o} \ll \tau_h$, where τ_h is a heating time defined, in normalized units, by $\tau_h = 4 \exp(w_1^2/2)/\ln(w_2/w_1)$. It may be seen, upon use of eq. (5), that in the steady-state the bulk electron temperature is roughly doubled in a time τ_h .

A rigorous derivation, given elsewhere,^(5,7) corroborates the following rough calculation of the turn-on time. The flux of electrons from the region $w < w_1$ into the resonant region is found by successive approximations of eq. (2), assuming in the lowest order that $\partial f/\partial \tau = 0$, but with slowly varying boundary conditions on f at $w = 0$ and $w = w_1$, obtaining

$$S(w_1) \approx [f(0, \tau) \exp(-w_1^2/2) - f(w_1, \tau)]/w_1^2. \quad (11)$$

A comparison of the time-asymptotic state with the initial distribution indicates that $f(0, \tau)$ is slowly varying compared to $f(w_1, \tau)$. Furthermore, for problems of interest, i.e. $\Delta w_1 > 1$, where $\Delta = w_2 - w_1$, very few electrons are scattered into the region $w > w_2$. Therefore, we have, by conservation of electrons,

$$S(w_1) = \Delta \partial f(w_1, \tau) / \partial \tau, \quad (12)$$

where we used the fact that under intense RF excitation, the resonant region is always nearly flat. Treating $f(0, \tau)$ as constant in solving eqs. (11) and (12), we find $\tau_{t-o} \approx \Delta w_1^2$.

References

1. T. Ohkawa, Nucl. Fusion, 10, 185 (1970).
2. A. Bers and N. J. Fisch, these proceedings.
3. B. A. Trubnikov, Particle Interaction in a Fully Ionized Plasma, in Reviews of Plasma Physics, Vol. 1 (Consultants Bureau, New York, 1965).
4. A. A. Vedenov, Theory of a Weakly Turbulent Plasma, in Reviews of Plasma Physics, Vol. 3 (Consultants Bureau, New York, 1967).
5. N. J. Fisch, Heating and Confining a Toroidal Plasma with RF Power, Ph. D. Thesis, Dept. of Electrical Engineering and Computer Science, M. I. T. (to be submitted Jan. 1978).
6. J. W. Connor and J. W. Cordey, Nucl. Fusion, 14, 185 (1975).
7. N. J. Fisch and A. Bers, M.I.T. Plasma Research Report 77-31 (1977).

TRANSIT TIME MAGNETIC PUMPING IN HYDROGEN AND DEUTERIUM DISCHARGES

R. Bardet, M. Bernard, G. Briffod, M. Clement, A. Gauthier,
M. Gregoire, P. Grelott, M. Hesse, F. Parlange, D. Pinet, E. Porrot,
G. Rey, B. Taquet, J. Weissee, S. Zymanskii

PETULA GROUP
Association Euratom CEN
Grenoble, France

In the Petula tokamak the toroidal magnetic field can be modulated up to 1.5% by RF coils equally spaced around the torus. These coils provide two oppositely travelling waves, the phase velocity of which is in the range of ion thermal velocity in order to get transit time magnetic pumping. The heating experiments have been performed both with hydrogen and deuterium under the following conditions: $T_e = 600-900$ eV, $T_i = 180-250$ eV, $n_e = 1.8-2.2 \times 10^{13}$. The TTMP operating frequency is 150 kHz with a 10 ms pulse duration. Maximum heating effect is obtained with three wavelengths along the torus. Neutral charge exchange measurement and Doppler broadening of oxygen line show a 30 to 45% temperature increase. Loop voltage, impurity content, and electron density remain unaffected during the heating process. For the case of one wavelength, the wave velocity no longer matches the ion velocity, so that the temperature shows but a small increase which is due to higher spatial harmonics of the wave. All these experimental situations, as well as extrapolation to larger machines, can be simulated by a one-dimensional numerical code which solves the energy balance equation for ions. A good agreement exists between calculated and experimental temperatures in the case of Petula.

ADVANCES IN PLASMA HEATING WITH ALFVÉN WAVES*

J. L. Shohet
The University of Wisconsin

ABSTRACT

The choice of Alfvén waves as a mechanism for transferring energy to hot plasmas is attractive from two aspects: (1) low frequency and (2) strong coupling to the plasma. Recent experiments, both in linear and toroidal geometry, have yielded encouraging results. In linear geometry, absorption of the Alfvén wave in a theta-pinch plasma has been seen, along with loading of the launching structure corresponding to that predicted by theory over a wide range of frequencies. In toroidal geometry, good absorption of Alfvén waves has been seen in several stellarator and stellarator-type devices, although these experiments have also exhibited enhanced transport in addition to the heating. Furthermore, Alfvén waves have been used to achieve dynamic stabilization and increased heating efficiency when operated in a non-linear regime. Launching structures for Alfvén wave heating are often large, since they usually need to generate waves whose lengths are comparable to the dimensions of the experiment. Theoretical work has predicted, in a one-dimensional screw-pinch geometry, for both ideal MHD and guiding center models, the location and magnitude of the absorption, as well as the impedance seen by the launching structure. Ion heating has been measured experimentally and predicted theoretically, although, in general, it is not a direct result of the Alfvén wave itself. Rather, a mode conversion, which is then subsequently followed by ion heating appears to be necessary. There are now, in fact, enough processes which have been predicted theoretically to account for the thermalization of the energy, which, together with the experimental measurements, show that the estimates made for the heating can be believed.

It is the purpose of this paper to provide a review of recent developments in theory and experiment regarding the use of Alfvén waves for heating plasmas. The first section of this paper provides a brief theoretical introduction and subsequent sections discuss several experiments, which have been carried out in both linear and toroidal geometry. The final section summarizes the results and provides a listing of problems needing further attention.

I. Theoretical modeling

Alfvén wave heating (AWH) can be considered most simply from the point of view of ideal MHD. For linearized MHD waves, in an infinite homogeneous plasma, the following dispersion relation can be written

$$(\omega^2 - k^2 V_a^2 \cos^2 \theta) [\omega^4 - (V_a^2 + V_s^2) k^2 \omega^2 + V_a^2 V_s^2 k^4 \cos^2 \theta] = 0 \quad (1)$$

In equation (1) k is the magnitude of the propagation vector, θ is the angle between k and the d.c. magnetic field B . V_a is the Alfvén speed $\frac{B}{\sqrt{\rho_m \mu_0}}$ and V_s is the sound speed $\sqrt{\frac{\gamma P}{\rho_m}}$.

Equation (1) shows that 3 modes of propagation can be obtained. The first term on the left hand side is the Alfvén wave, while the other two, obtained from the second term, are the fast and slow magnetosonic waves. In the cold plasma limit ($V_s = 0$), equation (1) reduces to only two modes: the left hand side of equation (1) as before, and the right hand side to a single "compressional" Alfvén wave. The left hand side is thus the "shear" Alfvén wave. Since $V_s \neq 0$, the "shear" wave will be called simply the Alfvén wave. Phase velocity envelopes of these waves show that the fast wave is relatively isotropic with respect to variation in θ , but both the slow and Alfvén waves are anisotropic. For propagation entirely parallel to the magnetic field ($k_{\perp} = 0$, $\theta = 0$), both the slow and the Alfvén waves in the case of a homogeneous plasma, exhibit a cutoff at the same frequency.

$$\omega^2 - k_{\parallel}^2 V_a^2 = 0 \quad (2)$$

In inhomogeneous plasmas, local energy absorption of the slow and Alfvén wave can be achieved, at the location in the plasma of resonances. For ideal MHD, resonances appear in the case of the Alfvén and slow waves ($k_{\perp} \rightarrow \infty$). Equation (2) defines both the resonance and cutoff frequencies, for a given $k_{\parallel}$ in the case of the Alfvén wave. The slow wave resonance frequency is defined by the expression

$$\omega^2 - k_{\parallel}^2 V_c^2 = 0 \quad (3)$$

where V_c , the cusp velocity is: $V_c = \sqrt{\frac{V_a^2 V_s^2}{V_a^2 + V_s^2}}$.

At any point in an inhomogeneous plasma, the Alfvén and cusp speeds become functions of position. At any point where the above resonance conditions are satisfied, localized absorption is possible.

Absorption of energy is possible for these waves in the case of an inhomogeneous plasma. However, the dispersion relation in equation (1) is then replaced by a set of differential equations. The solutions of these differential equations are characterized by singularities which occur in space at the point where the applied frequency equals the local value of the now spatially dependent resonance frequency. It has been shown,¹ however, that absorption of the slow wave is much smaller than for the Alfvén wave. Accordingly, most experiments have concentrated on the Alfvén wave as the major source of heating.

In the case of an axisymmetric screw pinch, a picture of the propagation and absorption of Alfvén waves may be obtained. First, it should be noted that the screw pinch (basically a straight tokamak) has magnetic surfaces that coincide with surfaces of

constant magnetic field. If it is assumed that all quantities vary only in the radial direction, then the resonant condition(s) described by equation (2) will, by definition, exist on the magnetic surfaces themselves. Thus, the Alfvén waves can be considered to be oscillations of the magnetic field lines which are located on the magnetic surfaces. The resonant layer exactly corresponds to a particular magnetic oscillator. In fact, each magnetic surface may be considered as a separate oscillator of differing frequency from its neighboring surfaces, making in effect, a continuum of oscillators. The existence of such a continuum provides a mathematical basis for absorption, such as is the case for Landau damping.

Theoretical work using these concepts has now been developed by several authors.^{2,3} It must also be pointed out that the mechanism for thermalization of the absorbed energy using ideal MHD is not specified--also analogously to the case of Landau damping. Application of the concepts of kinetic theory has resulted in the development of several mechanisms for the absorption,⁴ such as linear mode conversion, etc.

In systems where the magnetic surfaces do not correspond to the surfaces described by equation (2) such as in the case of a tokamak, stellarator, etc., the picture becomes a bit more obscure. Alfvén waves must still be characterized by oscillations of the magnetic field lines, and these lines must still describe magnetic surfaces. Accordingly, any resonance condition must show that these resonant surfaces exist on the magnetic surfaces. Thus the simple condition described in equation (2) can no longer apply. Instead of an algebraic expression for the resonance, a differential eigenvalue equation describing the resonance must be solved on each magnetic surface.

The loading of a particular launch coil by the plasma may then be computed for the ideal MHD case, which should determine the effectiveness of that launching structure in coupling energy to the plasma. Figure 1 shows a typical calculated loading of a helical launch coil⁵ by the plasma as a function of normalized frequency. Note this quantity, Ω_N , is proportional to plasma density. The peak in the real part of the impedance occurs at a frequency related to the resonance of a surface wave. In case of a sharp boundary, the frequency of the surface wave is purely real. Introducing a finite gradient into the density profile leads to a damping of this surface wave due to the excitation of localized Alfvén waves. In order to determine the optimal condition for absorption, a comparison of the effects of various tokamak plasma profiles (e.g., PLT, ORMAK, ST, etc.), on the helical coil impedance have recently been computed.⁶ Thermalization of the energy then requires application of kinetic theory for an explanation of its effects, as well as possible effects on plasma transport and containment.

The following sections cover the experimental attempts to examine the nature of Alfvén wave heating in linear and toroidal geometries.

II. Linear Geometry

The first experimental evidence⁷ of Alfvén Wave Heating (AWH) was seen in the ISAR-I 5-meter θ -pinch at Garching with $m=1$ windings. A standing $m=1$ helical oscillation was observed in the plasma column. It decayed after 2-3 oscillation periods. This damping was several orders of magnitude larger than that predicted by classical effects such as resistivity and viscosity. The damping could be accounted for by considering the excitation of localized Alfvén waves in the plasma.⁶ Subsequently, a linear theta-screw pinch experiment at Lausanne has been used to examine Alfvén wave heating.⁸ The launch coil was helical, driven by an LC oscillator, and wound on the surface of a quartz tube. The basic parameters of the experiment are as follows:

Length of tube	142 cm	Average beta	0.24
Tube radius	2.6 cm	Location of resonance	$r=0.57$ cm
Launch coil radius	3 cm	Pitch of coil	$m=1$
Maximum axial field	16 kG	Frequency	900 Khz
Plasma density	$1.3 \times 10^{16}/\text{cm}^3$	Wavelength	57 cm, wound in opposite series to the rotational transform for the screw pinch
Average temperature	48 eV		

When the launch coil was excited, $m=1$ kink-like modes appeared. Two different operating conditions were used. (A) c.w. operation. In this case, as long as the driving frequency was less than the Alfvén resonant frequency at the center of the tube, the kink-motion appeared and was not strongly damped. When the frequency was raised above the resonant point, the kink motion was strongly damped. (B) Two cycles of r.f. When the excitation was removed, the kind was nearly critically damped and disappeared quickly.

In order to determine the amount of energy coupled to the plasma two calculations were made. First, the actual power transported to the plasma by the external circuit was determined from a measurement of the damping of the current in the external circuit, after subtracting the ohmic losses in the external circuit. It was then compared to the same signal with and without plasma and is plotted as function of frequency as the upper curve in Figure 2. The thermal power is found by introducing a source term corresponding to the heating into the differential equation governing the plasma beta. This is plotted as W in Figure 2. It can be seen that both curves show a peak. The frequency which this peak occurs corresponds very well to the theoretically predicted peak shown in Figure 1 for the plasma and coil parameters of the experiment. Figure 3 shows the time history of the temperature obtained from the pressure balance during the discharge with and without r.f. Note that the temperature with heating remains continuously above that without heating. No evidence of enhanced loss was seen.

III. Toroidal Geometry

In this case of toroidal geometry, 4 experiments have been performed, all basically on stellarator and stellarator-type devices. It must be emphasized at the outset that all of these devices are

truly non-axisymmetric and thus the 1-dimensional theory previously developed may not directly apply.

A. Proto-Cleo. For this experiment at Wisconsin, Proto-Cleo operated as an $\ell=3$, 7 field period stellarator, with a major radius of 0.4 meters and an average plasma minor radius of 5 cm.⁹ Plasma density is approximately $10^{12}/\text{cm}^3$ and ion and electron temperatures are of the order of 10-20 eV. The toroidal field is 3 KG. τ_E was 1 msec. No ohmic current was used, the plasma being formed by injection of a hydrogen-impregnated titanium electrode gun. Plasma densities are low enough so that the resonance condition (2) cannot be directly applied if the wave is considered to propagate only toroidally. Thus a launching structure that excites both toroidal and poloidal modes is required. A cross section of such a launching structure in Proto-Cleo is shown in Figure 4. The r.f. coil is the helical winding that goes three times around the major axis of the torus in going once around the minor axis. It is inside the higher-pitched confining field windings. RF was launched with an up-to-200 Kw generator operating at pulse lengths of 1 millisecond. Figure 5 shows a plot of the radial electron temperature profile immediately after the rf was cut off, normalized to the temperature profile at the same time without the presence of r.f. Several peaks are observed. These correspond roughly to the locations in the plasma where the resonance occurs, assuming 1-dimensional theory. Both the electron and ion temperatures appeared to be doubled. Figure 6 shows the electron temperature ratio at a fixed position as a function of density. The heating efficiency appears to improve with lower density, corresponding to moving toward lower values of normalized frequency on Figure 1, but a peaking in efficiency was not found. The temperature increase was approximately linear with rf amplitude. In addition, enhanced transport was observed, also varying approximately linearly with respect to the amplitude of the rf voltage applied to the launch coil. No significant difference in amplitude of the rf voltage profile appeared with and without plasma. Figure 7 shows the values of rf field amplitude, density, and temperature profiles when plasma was produced by filling the vacuum tank with gas and letting the rf coil itself break down the gas. Note that the rf field amplitude now peaks near the location of the resonance.

B. Uragon II. Alfvén waves were launched in the Uragon II stellarator¹⁰ at Kharkov with a much shorter coil. Figure 8 shows a diagram of the discharge chamber of the $\ell=2$ race-track stellarator. The toroidal field was up to 20 kG, and the minor radius is approximately 10 cm. The launch coil was a set of 4 bars excited by a rf generator whose pulse length varied between 1 and 3 ms. Densities were up to $5 \times 10^{12}/\text{cm}^3$. Figure 9 shows the dependence of the input rf current as a function of frequency with and without plasma. The main experimental mode of operation was under the conditions of having the launch coil break down the gas, in a similar manner to that used in Proto-Cleo without plasma filling. No ohmic heating was used. Figure 10 shows the wave amplitude parallel and perpendicular to the magnetic field in a plane parallel to the magnetic axis, at an angle of 45 degrees with respect to the vertical (a longitudinal cross section). It can be seen that the wave amplitude parallel to B is much smaller than the amplitude perpendicular to B, at least on the average. The radial profile of the

perpendicular component of the wave amplitude is shown in Figure 11. No local peaking is seen. Figure 12 shows the plasma temperature, density and rf field amplitude as a function of toroidal magnetic field. Note that peaks in these quantities occur. Estimates show that the positions of the rf field maxima correspond to the regime of excitation of longitudinal Alfvén modes, according to the formula

$$\lambda_{11} = \frac{V_a}{nf} = \frac{L}{n}$$

where L is the length of the torus (1036 cm) V_a is the Alfvén speed and n is the longitudinal mode number. One can observe that heating also occurs at these maximum values of rf magnetic field, but that the plasma density also falls, as is the case in Proto-Cleo. These measurements were all made 180 degrees around the torus from the rf coil.

C. Heliotron-D. The 3rd Alfvén heating experiment was performed at Kyoto in Heliotron-D,¹¹ an $\ell=2$ ohmically heated torsatron with an external toroidal field. The toroidal field was up to 3 KG. Figure 13 shows a cross section of the device. Its launch coil goes 1/4 of the way around the torus and is very similar to the launching system in UragonII. Heliotron-D has a sufficiently high density so as to excite Alfvén waves by use of pure toroidal mode, whose number is in this case 2. The poloidal mode number is also 2. Up to 1 MW of rf was supplied. Plasma densities were as high as $2 \times 10^{13}/\text{cm}^3$. Figure 14 shows launch coil impedance as a function of density. It is very similar to that shown for Proto-Cleo with the additional fact that the peak in the loading with density as predicted in Figure 1 is shown. Figure 15 shows the behavior of the electron temperature, and ion temperature as a function of time for various values of ohmic heating currents and densities.

It can be seen that after an initial increase in temperature, there appears to be a decrease in most of these quantities. It should be noted that a doubling of both T_e and T_i does occur, from approximately 100-200 eV, and although no pump-out is observed, there does appear to be some ionization due to the rf which may mask enhanced transport.

D. R-02. The R-02 stellarator at Sukuhmi is a small, very high aspect ratio, ohmically heated $\ell=2$ stellarator of toroidal field 15 kG and ohmic heating currents up to 2 kA. An Alfvén wave is launched with a separate helical quadrupole winding which is capable of approximately 1% modulation of the toroidal field.¹² Plasma densities are of the order of $5 \times 10^{13}/\text{cm}^3$ and electron temperatures between 10-20 eV. Figure 16 shows a diagram of the device. Figure 17 shows the plasma energy density as a function of magnetic field for various plasma densities. The peaks correspond to Alfvén resonances to within a factor of approximately 2. The most interesting part of this experiment however, was that the dependence of the heating on the RF field strength was found to be non-linear. Above a certain threshold of rf amplitude, intense heating of the plasma occurred as shown in Figure 18. The threshold value is virtually independent of the plasma density and toroidal magnetic field values. The rf field amplitudes in the plasma appear to be much larger than those present with no magnetic field.

Diamagnetic temperatures up to 500 eV were measured, and were mainly due to an increase in measured ion temperature. Several theories for the threshold and absorption mechanism are under study. Among them are a parametric decay instability into two ion acoustic branches or one ion acoustic and one Alfvén branch, followed by ion heating and subsequent damping of the ion acoustic branch. It is important to note, however, that the energy containment time for this experiment is of the order to 30 microseconds.

IV. Summary and Conclusions

From the experimental work reported here, it can be seen that several important points can be made regarding Alfvén wave heating. They are:

- (1) Energy appears to penetrate into the body of the plasma
- (2) Heating occurs under the appropriate resonant conditions
- (3) Enhanced transport appears to be an important problem
- (4) No convincing evidence for specific mechanisms of thermalization has been seen experimentally
- (5) Launching structures are relatively large.

Future experiments in Alfvén wave heating might be to:

- (1) Examine Alfvén wave heating in axisymmetric toroidal devices such as tokamaks
- (2) Provide an understanding for anomalous transport and variation of plasma temperatures with time during the rf heating phase
- (3) Extrapolate to higher field and larger devices.

Future theoretical work might be to:

- (1) Develop a non-axisymmetric theory for Alfvén wave heating
- (2) Examine the mechanisms for anomalous transport
- (3) Develop smaller launching structures
- (4) Scale towards fusion reactor parameters.

* This work was supported by the National Science Foundation under Grant ENG 77-14820.

References

1. J.A. Tataronis & W. Grossmann, Nucl. Fusion 16, 667 (1976).
2. W. Grossmann & J.A. Tataronis, Z. Physik 261 203, 217 (1973).
3. L. Chen & A. Hasegawa, Phys. Fluids 17, 1399 (1974).
4. A. Hasegawa & L. Chen, Phys. Rev. Let. 35, 370 (1975).
5. S.N. Golovato, J.L. Shohet & J.A. Tataronis, IEEE Trans. on Plasma Sci. PS-5, 952 (1977).
6. J.A. Tataronis & W. Grossmann, Courant Inst. Report MF-84 (1977).
7. W. Grossmann, M. Kaufmann & J. Neuhauser, Nucl. Fusion 13, 462 (1973).
8. A. Pochelon & R. Keller, Bericht der Herbsttagung der Schweiz. Physikal. 50, 172 (1977).
9. S.N. Golovato, J.L. Shohet & J.A. Tataronis, Phys. Rev. Let. 37, 1272 (1976) and S.N. Golovato & J.L. Shohet, Phys. Fluids, to be published.
10. A.G. Dikij, et al., Plasma Physics & Controlled Fusion Research (IAEA, Vienna, 1977), v. II, p. 129.
11. K. Uo, et al., Plasma Physics & Controlled Fusion Research (IAEA, Vienna, 1977), v. II, p. 103 and T. Obiki, et al., Phys. 39, 812 (1977).
12. R. A. Demirkhanov, et al., Plasma Physics & Controlled Fusion Research, (IAEA, Vienna, 1977), v. III, p. 31.

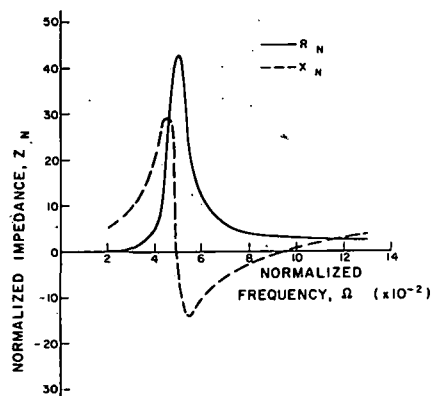


Fig. 1. Normalized impedance of a helical loading coil.

$\Omega = \Omega_N = \frac{\omega r_p}{V_{ao}}$ r_p is the plasma radius. V_{ao} is the Alfvén speed on the magnetic axis.

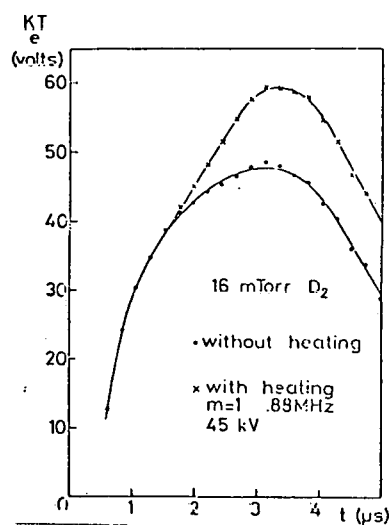


Fig. 3. Time evolution of temperature with and without rf heating.

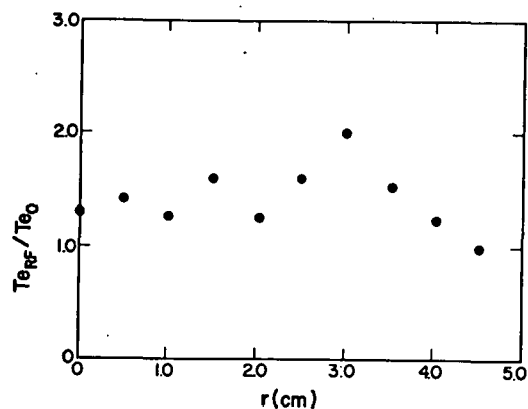


Fig. 5 T_e with rf/ T_e without rf

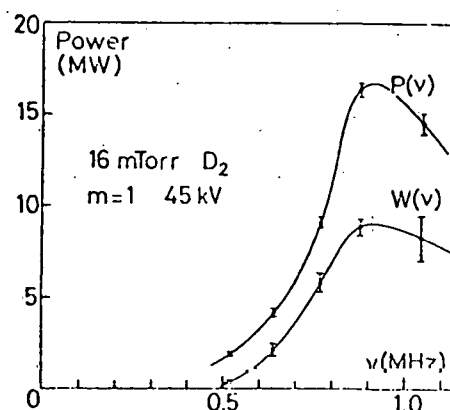


Fig. 2. Thermal power W and rf power P versus excitation frequency for the theta pinch.

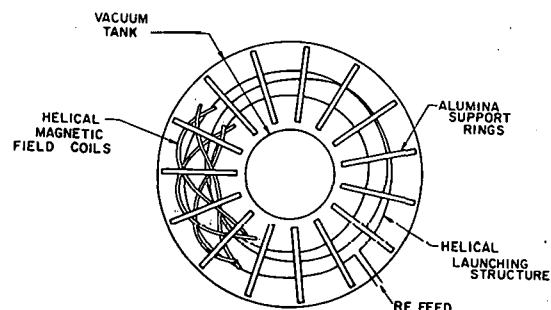


Fig. 4. Cross section of Proto-Cleo Stellarator with Alfvén wave coil installed.

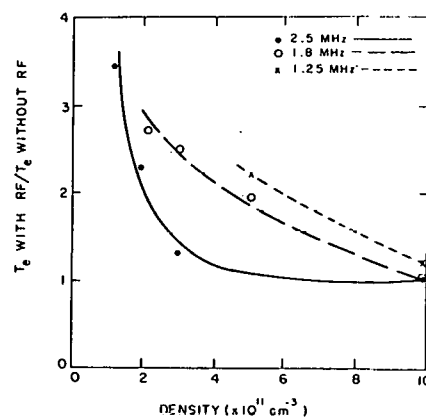


Fig. 6. T_e with rf/ T_e without rf as a function of density.

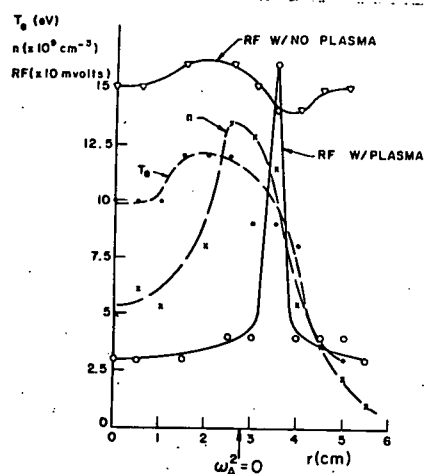


Fig. 7. Profiles of T_e , rf field, and n_e for coil breakdown of the filling gas.

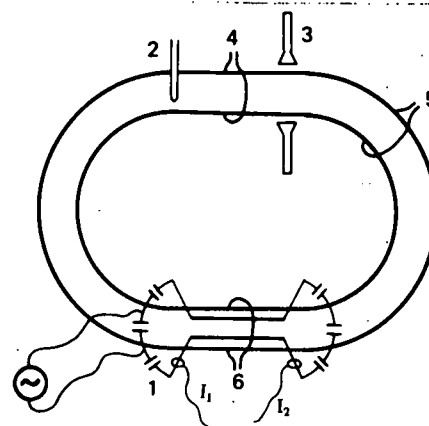


Fig. 8. Schematic of Uragon II stellarator.

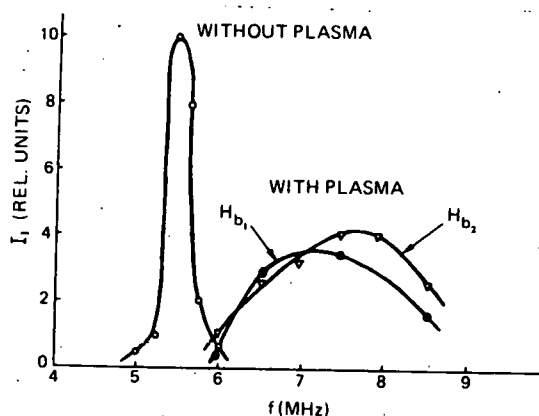


Fig. 9. Rf current versus frequency without plasma and with plasma.

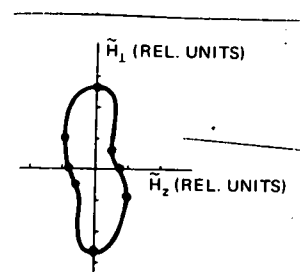


Fig. 10. Longitudinal scan of rf field amplitudes.

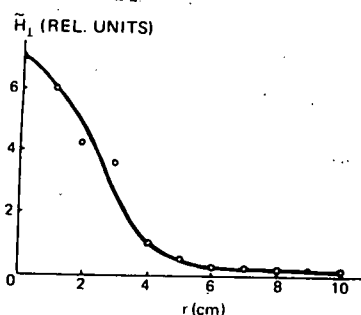


Fig. 11. RF field profile.

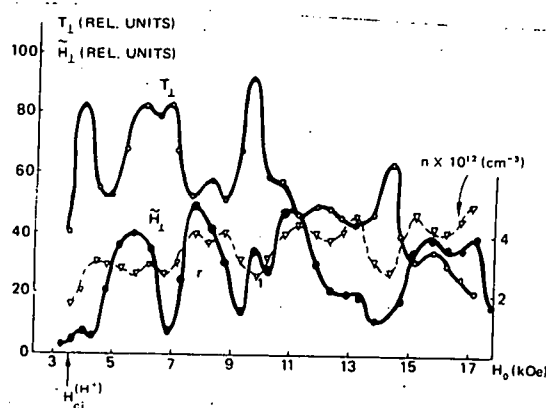


Fig. 12. Temperature, density and rf field amplitudes versus toroidal field.

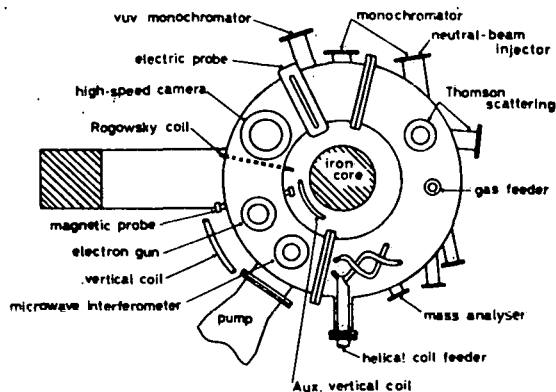


Fig. 13. Schematic of Heliotron-D. The rf field coil is not shown, but extends approximately 90 degrees around the torus.

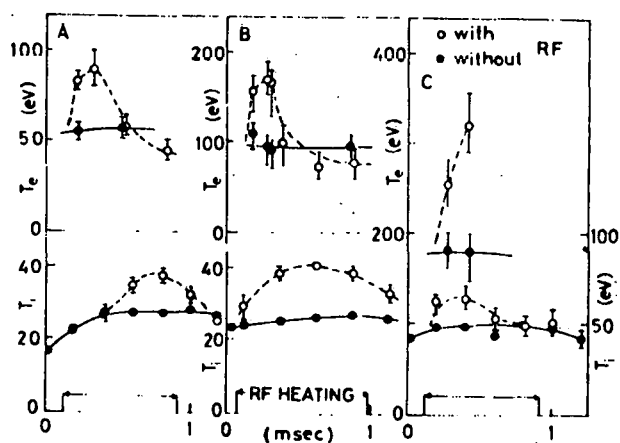


Fig. 15. Dependences of the electron and ion temperatures on time. $B_0 = 2.9 \text{ kG}$. (a), $n_e = 1 \times 10^{13} \text{ cm}^{-3}$, $I_{OH} = 12 \text{ kA}$; (b), $n_e = 6 \times 10^{12} \text{ cm}^{-3}$, $I_{OH} = 12.4 \text{ kA}$; (c), $n_e = 6 \times 10^{12} \text{ cm}^{-3}$, $I_{OH} = 14.5 \text{ kA}$.

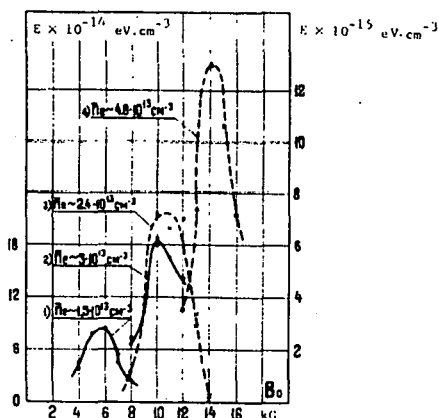


Fig. 17. Diamagnetism versus magnetic field for various plasma densities in the presence of rf versus toroidal magnetic field.

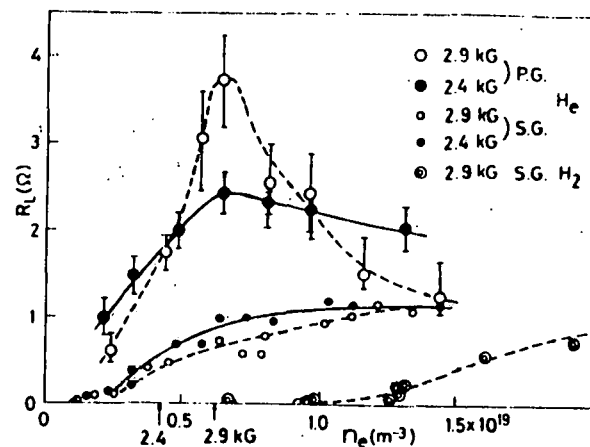


Fig. 14. Real part of the load coil impedance versus density.

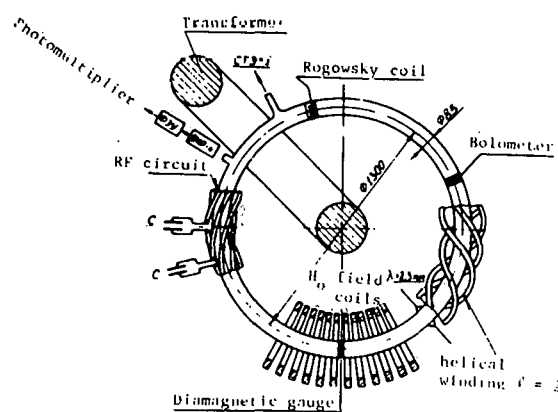


Fig. 16. Schematic of RO-2 stellarator.

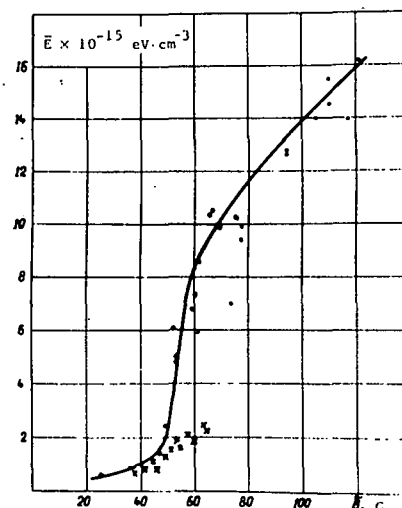


Fig. 18. Diamagnetism versus rf field amplitude.

TECHNICAL ASPECTS OF ALFVÉN WAVE HEATING

Akira Hasegawa
Bell Laboratories
Murray Hill, New Jersey 07974

ABSTRACT

Elementary processes of energy coupling to the plasma in Alfvén wave heating will be discussed in its relation to the antenna design for the optimum heating efficiency. The talk will also include other technical aspects of the problem, such as the calculation of antenna impedance, heating rate and efficiency, and RF power input to produce required plasma temperature within a given energy confinement time.

The Alfvén wave heating in a collisionless plasma occurs by the dissipation of the kinetic Alfvén wave, $\omega \simeq k_{\parallel} v_A (1 + k_{\perp}^2 \rho_i^2)$, which is excited by the resonant mode conversion of the externally applied RF field. Here v_A is the Alfvén speed and ρ_i is the ion gyroradius. The resonant mode conversion occurs at the surface $r = r_0$ at which the frequency ω_0 of the externally applied RF field satisfies the local Alfvén resonant condition $\omega_0 = k_{\parallel}(r_0) v_A(r_0)$ for the prescribed value of the parallel wave number $k_{\parallel}$.

In a low β plasma, $\beta \simeq m_e/m_i$, the electron Landau damping dominates the linear absorption processes and electrons are quickly heated.¹ The heating rate is given by $\gamma_e \simeq \omega_0 v_A / v_{Te}$. However because $v_A \lesssim v_{Te}$, the electron heating due to the linear Landau damping will saturate by the plateau formation in the electron distribution function. The time needed for the saturation τ_s is given by

$$\omega_0 \tau_s \simeq \frac{v_A}{v_{Te}} \left(\frac{n_0 T_e}{B^2 / 2\mu_0} \right)^{3/4}, \quad (1)$$

where $\tilde{B}$ is the magnetic field amplitude of the kinetic Alfvén wave. For a nominal value of the heating design, τ_s becomes approximately 100 μsec .

After the saturation of the electron heating, ions are heated more slowly by the nonlinear Landau damping.¹ The nonlinear heating rate of ions is given by $\gamma_i \simeq 0.1 \omega_{ci}^2 / (\omega_0 \beta) (\tilde{B}/B_0)^2$.

The amplitude of the kinetic Alfvén wave $\tilde{B}$ is related to the externally applied oscillating magnetic field $\tilde{B}_{ext}$ through the usual Airy function asymptotic relation by

$$\tilde{B} = \tilde{B}_{ext} (a/\rho_i)^{1/2}, \quad (2)$$

where a is the minor radius. Typically $\tilde{B} \simeq 20 \sim 30 \tilde{B}_{ext}$.

If the kinetic Alfvén wave is completely dissipated in the plasma, the optimum energy absorption rate is obtained by choosing $k_0 \simeq 1/a$ and is given by

$$\frac{dW}{dt} \simeq \omega_0 \frac{\tilde{B}_{ext}^2}{2\mu_0} V, \quad (3)$$

where V is the plasma volume facing the coupling coil.

The oscillating RF field with a typical frequency of 1 MHz is launched by a coil placed at the wall helically wound with respect to the toroidal magnetic field. The Ohmic (skin) resistance R of the coil is given by

$$R = R_s \ell / d,$$

where d and ℓ are the width and the length of the coil, and R_s is the skin resistance $\sim 10^{-4}$ Ohms for a copper and $\sim 10^{-3}$ Ohms for a tungsten.

On the other hand, the radiation resistance R_r is given by

$$R_r \sim \frac{\omega_0}{2\pi} \mu_0 L \sim \omega_0 L,$$

where L is the coil inductance. Hence the heating efficiency of the coil η is given by

$$\eta = \frac{1}{1 + R/R_r}, \quad (4)$$

where

$$\frac{R}{R_r} \sim \frac{1}{2\pi \times 10^2 d(m)}, \quad (5)$$

even for a tungsten coil.

If $d \sim 0.1$ m is used $R/R_r \ll 1$ and $\eta \sim 1$. The Q value of the coil itself is given by $\omega_0 L/R \sim 2\pi \times 10^2 d$. The Q value of the coil including the radiation impedance is order unity because of the high efficiency of the resonant absorption.

The induced terminal voltage V of the coil is given by

$$V = \omega_0 LI. \quad (6)$$

To produce 10 G amplitude of $\tilde{B}_{ext}$ at 1 MHz with a coil of 1 m length produces $V \sim 100$ Volt.

REFERENCES

1. A. Hasegawa and L. Chen, Phys. Fluids 14, 1924 (1976).

NON-IDEAL EFFECTS ON ALFVÉN WAVE HEATING

G. Conn

Michigan State University
East Lansing, Michigan 48824

and

J.A. Tataronis

New York University
Courant Institute of Mathematical Sciences
New York, New York 10012

Abstract: We consider effects of dispersion and resistive dissipation on r f heating of plasmas through localized Alfvén waves. We employ a two fluid warm plasma model with the electron fluid coupled to the ion fluid through collisions, and impose a scaling which yields the Ohm's law $\underline{E} + \underline{v} \times \underline{B} = \eta \underline{J} + (\epsilon/\rho) \underline{J} \times \underline{B}$ where ρ and ϵ are respectively the mass density and the mass to charge ratio of the ions. We extend a previous analysis with $\epsilon \equiv 0^1$ by introducing dispersion through the Hall effect $(\epsilon/\rho) \underline{J} \times \underline{B}$. For small η , we show that a boundary layer of order $\epsilon^{1/2}$ exists about localized Alfvén waves, and by the method of matched asymptotic expansions we describe the resolution of the MHD Alfvén wave singularities through discrete eigenmodes.

Alfvén wave excitation has been proposed in the past as a means of providing supplementary heating of fusion plasmas.¹⁻⁴ The basis of this heating method is found in the singular nature of the Alfvén wave. In linearized ideal magnetohydrodynamics (MHD), Alfvén waves are localized on magnetic flux surfaces of the equilibrium and are characterized by spatial singularities which render the corresponding eigenfunctions of the linearized MHD operator non-square integrable and the energy content of the mode unbounded. These singularities are a consequence of the absence of dissipative and dispersive processes in ideal MHD. The effects of dissipation through the inclusion of resistivity in Ohm's law have been examined by Kappraff and Tataronis¹ who show that resistivity gives rise to

a boundary layer about the Alfvén wave singularities. Within this boundary layer, the amplitude of the excited wave is finite but large, and the absorbed energy is dissipated. This paper concerns itself with the effects of a further modification of Ohm's law, specifically with the effects of dispersion resulting from the Hall current. The Hall current introduces the ion cyclotron frequency which allows an analysis of Alfvén wave heating at frequencies higher than those contained in ideal MHD.

The plasma equilibrium we assume is the cylindrically symmetric theta pinch. With respect to an (r, θ, z) coordinate system, the equilibrium magnetic field is directed parallel to the longitudinal z -axis, and all equilibrium quantities depend only on the radial coordinate r . The linearized fluid equations considered here are essentially those of ideal MHD¹ but with Ohm's law written in the form $\underline{E} + \underline{v} \times \underline{B} = \eta \underline{J} + (\epsilon/\rho) \underline{J} \times \underline{B}$, where $\underline{E}$ is the electric field, $\underline{v}$ is the plasma velocity, $\underline{J}$ is the current density, η is the resistivity, ρ is the mass density, and ϵ is the mass to charge ratio of the ions. The Hall current gives rise to the term $(\epsilon/\rho) \underline{J} \times \underline{B}$. We point out here that in many cases the resistive term can be neglected with respect to the Hall term in Ohm's law. Specifically, with ω_{ce} the electron cyclotron frequency and τ the electron-ion collision time, the condition $\omega_{ce} \tau \gg 1$ implies $|\eta \underline{J}| \ll |(\epsilon/\rho) \underline{J} \times \underline{B}|$.⁵ If we assume Spitzer resistivity and typical Tokamak parameters (electron density $\sim 10^{14} \text{ cm}^{-3}$, electron temperature $\sim 2000 \text{ e.v.}$), one obtains that $\omega_{ce} \tau \gg 1$ implies $|\underline{B}| \gg 10^{-3} \text{ Gauss}$. The toroidal magnetic field of a Tokamak is typically of order 30kG. Therefore, in the analysis below, we write Ohm's law as $\underline{E} + \underline{v} \times \underline{B} = (\epsilon/\rho) \underline{J} \times \underline{B}$.

We assume that the linearized variables depend only on time and the coordinate z in the form $\exp i(\omega t - kz)$. With incompressibility $\nabla \cdot \underline{v} = 0$ as an equation of state, one obtains the following fourth order system of differential equations for v_r , the radial component of $\underline{v}$, and P , $i\omega$ times the total linearized pressure (kinetic plus magnetic):

$$\frac{dP}{dr} + \frac{A}{r} r v_r = - \frac{1}{rA} \left(\frac{\epsilon \omega}{\mu_0} f \right)^2 \left[r \frac{d}{dr} \left(\frac{1}{r} \frac{dr v_r}{dr} \right) - k^2 r v_r \right], \quad (1)$$

$$\frac{A}{r} \frac{dr v_r}{dr} + k^2 P = \left\{ \frac{d}{dr} - \frac{r}{f} \left[\frac{d}{dr} \left(\frac{f}{r} \right)' + \frac{1}{r} \frac{df}{dr} \right] \right\} \left(\frac{dP}{dr} + \frac{A}{r} r v_r \right), \quad (2)$$

where μ_0 is the vacuum permeability, $f = kB(r)$, $A = \rho(r) [-\omega^2 + \omega_a^2(r)]$, and $\omega_a^2(r) (\equiv f^2/\rho\mu_0)$ is the square of the local Alfvén wave frequency. Neglecting the Hall effect in equations (1) and (2) by setting $\epsilon \equiv 0$, one recovers the second order system of ideal MHD and the singular solution $v_r \sim \ln(r - r_0)$ in the vicinity of r_0 where $\omega_a^2(r_0) = \omega^2$. With $\epsilon \neq 0$, this singularity is not present. Assuming ϵ is small, we introduce a boundary layer about r_0 and scale equations (1) and (2) appropriately. The solutions of the scaled equations in the layer are matched to the corresponding quantities in the region outside the layer, which to lowest order in ϵ is described by ideal MHD. The relevant boundary layer scaling can be shown to be $r - r_0 \sim O(\epsilon^{1/2})$. Therefore, introducing the normalized radial variable $s \equiv (r - r_0)/\epsilon^{1/2}$ with $s \sim O(1)$, we obtain that in the layer, to lowest order in ϵ , $P = \text{const.}$ while v_r satisfies,

$$\frac{d}{ds} \left[\frac{1}{\alpha s} \left(\frac{\omega f}{\mu_0} \right)^2 \frac{dU}{ds} \right] + \alpha s U = -k^2 P \quad (3)$$

where we have written $A \approx \epsilon^{1/2} \alpha s$ and have set $U \equiv (1/r) dr v_r / ds$. Equation (3) can be solved explicitly for U in terms of the two linearly independent Fresnel integrals $C(s)$ and $S(s)$:

$$U = qP [C(s) \sin(ps^2) - S(s) \cos(ps^2)] + D_1 \sin(ps^2) + D_2 \cos(ps^2), \quad (4)$$

where p and q are constants depending on α , and D_1 and D_2 are constants of integration. From the definition of U , we obtain $rv_r = \int ds rU + \text{const.}$, and upon application of the asymptotic expansions of the Fresnel integrals we find that as $s \rightarrow \pm \infty$, $rv_r \sim \ln|s| + O(1) + O(1/s)G$ where G is function of order one and linear in $\cos(ps^2)$ and $\sin(ps^2)$. The logarithm appearing in the inner expression of v_r can thus be matched to the logarithm in the outer expression of v_r . We point out that U in equation (4) contains terms which oscillate in s . These spatial oscillations suggest the existence of radial waves which would give rise to a discrete spectrum in place of the continuous spectrum of ideal MHD.

These discrete modes would absorb the energy transferred to the plasma by an external source and dissipation of this energy would occur for example through resistivity in Ohm's law.

References

1. J. Kappraff and J. A. Tataronis, J. Plasma Phys. 18, 209 (1977).
2. J. A. Tataronis and W. Grossmann, Z. Phys. 261, 203 and 217 (1973).
3. A. Hasegawa and L. Chen, Phys. Rev. Lett. 32, 454 (1974).
4. J. A. Tataronis and W. Grossmann, New York Univ., Courant Institute of Mathematical Sciences Report COO-3077-102, MF-84, September 1977.
5. M. J. Lighthill, Proc. Roy. Soc. A 252, 1014 (1960).

The Continuous MHD Spectrum for Alfvén Wave Heating in Non-Axisymmetric Systems

J. A. Tataronis
Courant Institute of Mathematical Sciences
New York University
New York, New York 10012

and

J. L. Shohet and J. N. Talmadge
The University of Wisconsin
Madison, Wisconsin 53706

ABSTRACT

Recent experimental results for Alfvén wave heating in the Proto-Cleo stellarator,¹ have demonstrated electron and ion heating as well as evidence of resonant layers. It is thus of interest to extend present one-dimensional screw pinch theory² to arbitrary geometries, possessing no definite axis of symmetry. The MHD equations are transformed to flux surface coordinates and the continuous spectrum is then determined by a set of eigenvalue equations which are evaluated on each flux surface.

Alfvén wave heating (AWH) is based on the existence of spatial singularities which are predicted by the equations of ideal MHD. The surfaces over which these singularities exist are characteristics of the linearized MHD equations. These equations are a system of hyperbolic partial differential equations.

As an example of the determination of the characteristics, we consider the case of a linear theta-pinch with $m=0$ symmetry. In this situation, the well-known equation for the radial component of the linearized velocity v_r is:

$$\frac{d}{dr} \left[\frac{A}{r} \frac{d}{dr} (rv_r) \right] - k^2 \frac{A}{r} rv_r = 0 \quad (1)$$

where

$$A = \rho(r) \left[-\omega^2 + \omega_A^2(r) \right]$$

ω_A is the Alfvén resonance frequency defined as:

$$\omega_A^2 = k^2 B_z^2(r) / \rho(r) .$$

The singularities of equation (1) occur at surfaces where

$$\omega^2 = \omega_A^2(r)$$

For this case, the singular surfaces are determined by an algebraic relation. However, this equation is actually obtained as a result of a simplification of a Fourier transformed differential equation defined in the surface, which in the case of the theta pinch is given by:

$$\left(\rho \omega^2 + B_z^2 \frac{\partial^2}{\partial z^2} \right) \phi = 0 \quad (2)$$

Equation (2) is an ordinary differential equation defined on the flux surfaces of the theta-pinch, which are simply, $r=\text{const.}$

The question to be asked at this time is what the equivalent equation might be in other more complicated geometries wherever a Fourier transform with respect to the variables which span the flux surface cannot be carried out, due to the lack of symmetry, such as in the case of a tokamak or a stellarator.

For the particular case of an axisymmetric toroidal configuration, e.g., a tokamak, this question has been answered by Pao.³ He has shown that corresponding to each flux surface there exists an ordinary differential equation which must be solved for its eigenvalues ω . The eigenvalues are function of the flux surfaces ψ . In the case of a straight stellarator, after transforming to helical coordinates, a similar procedure can be followed resulting in an equivalent eigenvalue equation.

In cases where the equilibria possess no axis of symmetry, such as toroidal stellarators, the corresponding equation will be a partial differential equation in the 2 flux surface variables. It is the purpose of this paper to discuss the derivation of this partial differential equation and its relationship to Alfvén Wave Heating.

Our basic equations are the ideal linearized MHD equations, which can be written as follows assuming a time dependence of $\exp(i\omega t)$:

$$i\omega \rho \tilde{v} = (B \cdot \nabla) \tilde{b} + (\tilde{b} \cdot \nabla) B - \nabla(p_1 + B \cdot \tilde{b}) \quad (3)$$

$$i\omega \tilde{b} = (B \cdot \nabla) \tilde{v} - (\tilde{v} \cdot \nabla) B - B \nabla \cdot \tilde{v} \quad (4)$$

$$i\omega p_1 = -\tilde{v} \cdot \nabla P - \gamma P \nabla \cdot \tilde{v} \quad (5)$$

Equation (5) is the adiabatic equation of state. $\tilde{b}$ is the linearized magnetic field, B is the equilibrium magnetic field, $\tilde{v}$ is the plasma velocity, P is the equilibrium pressure and γ is the ratio of the specific heats.

What we wish to now consider is a development of the general method used to obtain the differential equation defined on each flux surface. The form that this system takes depends upon the specific application.

Equations (3)-(5) form a system of hyperbolic partial differential equations in the 3 spatial variables. The singular surfaces associated with the continuum are the characteristic surfaces of the system. To derive the eigenvalue equations for such a system, we introduce 3 new variables in place of the spatial coordinates: ψ , χ , ξ . The characteristic surfaces are given by the expression $\psi = \text{const}$. In addition, $\chi = \text{const}$ and $\xi = \text{const}$ are surface variables which are defined on and span the surface $\psi = \text{const}$.

It can be shown in ideal MHD that ψ satisfies the equation

$$B \cdot \nabla \psi = 0 \quad (6)$$

That is, $\psi = \text{const}$ is the equilibrium flux surface. In terms of these new variables, the gradient operator takes the form

$$\nabla = \nabla \psi \frac{\partial}{\partial \psi} + \nabla \chi \frac{\partial}{\partial \chi} + \nabla \xi \frac{\partial}{\partial \xi} \quad (7)$$

Using equation (7), and from the definition of ψ , the operator $B \cdot \nabla$ is defined in equation (8)

$$B \cdot \nabla = (B \cdot \nabla \chi) \frac{\partial}{\partial \chi} + (B \cdot \nabla \xi) \frac{\partial}{\partial \xi} \quad (8)$$

That is, $B \cdot \nabla$ contains only derivatives with respect to the surface variables. Using equations (7) and (8), we obtain, from equations (3)-(5), expressions for the ψ derivative of $p_1 + B \cdot \tilde{b}$ and of v_ψ where $v_\psi = \tilde{v} \cdot \nabla \psi$. Specifically, we have

$$\frac{\partial}{\partial \psi} (p_1 + B \cdot \tilde{b}) = g \quad (9)$$

and

$$\frac{\partial v_\psi}{\partial \psi} = h \quad (10)$$

where g and h are functions of the linearized variables, containing no derivatives with respect to ψ . The remaining equations obtained from equations (3)-(5) form a system of partial differential equations in the variables χ and ξ . That is, no ψ derivatives appear. This system can be written in the form

$$L\tilde{X} = \tilde{f}(p^*, v_\psi) \quad (11)$$

where $p^* = p_1 + B \cdot b$.

In equation (11), L is a differential operator depending on ω , and containing derivatives only with respect to the surface variables χ & ξ . $\tilde{X}$ is a vector whose components are those of b & v except v_ψ , while $\tilde{f}$ is a vector that is a function only of p^* and v_ψ .

The procedure we now follow is to solve equation (11) for $\tilde{X}$, substitute this solution into equations (9) and (10), which can then be solved for v_ψ and p^* . We therefore see that v_ψ and p^* may be advanced in ψ only if the operator L has an inverse. That is only if the equation

$$L\tilde{X} = 0 \quad (12)$$

possesses the trivial solution $\tilde{X} = 0$.

If, for a given ω , equation (12) possesses a non-trivial solution for $\tilde{X}$, equation (11) is singular. The condition given by equation (12) is the differential equation defining the continuous spectrum⁴ which reduces to equation (2) for the case of the theta-pinch, and to Pao's system of differential equations in the case of a tokamak.

In stellarator geometry, the form of the operator L will be determined. We point out that in coupling to the plasma, the frequency of the driving source must be identical to one of the eigenvalues of equation (12), which also defines the surface to which energy is transferred.

* This work supported by NSF under Grant ENG 77-14820 and U.S. DOE under contract No. EY-76-C-02-3077.

References

1. S. N. Golovato, J. L. Shohet & J. A. Tataronis, Phys. Rev. Let. 37, 1272 (1976).
2. J. A. Tataronis and W. Grossmann, Nucl. Fusion 16, 667 (1976).
3. Y. P. Pao, Nucl. Fusion 15, 631 (1975).
4. R. W. Gould, Proc. US-Aust. Plasma Waves Workshop, Texas Tech, U. (1975).

RESONANT MAGNETIC PUMPING AT VERY LOW FREQUENCY

Ernesto CANOBBIO

ASSOCIATION EURATOM-CEA

Département de Physique du Plasma et de la Fusion Contrôlée
Service IGn - Centre d'Etudes Nucléaires
 85 X - 38041 GRENOBLE CEDEX (France)

ABSTRACT : We propose to exploit for plasma heating purposes the very low frequency limit of the Alfvén wave resonance condition, which reduces essentially to safety factor $q = m/n$, a rational number. It is shown that a substantial fraction of the total RF-energy can be absorbed by the plasma. The lowest possible frequency value is determined by the maximum tolerable width of the RF-magnetic islands which develop near the singular surface. The obvious interest of the proposed scheme is the low frequency value ($f \lesssim 10$ KHz) which allows the RF-coils to be protected by stainless steel or even to be put outside the liner.

The spectrum of possible auxiliary plasma heating methods reduces drastically if one considers a truly thermonuclear environment. The potentially most interesting remaining approaches are :

- 1 - High frequency (≥ 1 GHz) heating methods using wave guide systems which are compatible with reasonably small wall apertures (e.g. those foreseen for neutral injection). Being extensively studied by many authors, they will not be considered in this paper.
- 2 - Low frequency heating schemes using RF-coils which can be completely protected by stainless-steel (or titanium) or can even be put outside a metallic first wall. Indeed, insulating free surfaces (e.g. ceramics) should be avoided as far as possible within the first wall, if only for the degradation they would suffer from the thermonuclear neutron flux. The useful frequencies, therefore, are those which correspond to skin depths (in stainless-steel and titanium) around and above 1 cm, that is to say $f \lesssim 10$ KHz.

Besides their interest for RF-power deposition, the very low frequency schemes are attractive for dynamic stabilisation of the MHD modes which control the value of the toroidal current of a Tokamak, and hence the ohmic heating level.

The use of horizontal coils carrying properly dephased axisymmetric RF-currents in the KHz range, has been proposed recently /1/. RF-power is dissipated into a toroidal plasma by perpendicular Landau damping of an essentially MHD pump wave. The vertical phase velocity of the pump matches the value of the toroidal drift velocity of a suprathermal ion population. The heating efficiency turns out to be strongly dependent on the plasma parameters.

A systematically much higher heating efficiency can in principle be achieved /2/ at equally-low frequencies if one gives up axisymmetry and exploits the very low frequency limit of the shear Alfvén-wave resonance condition :

$$\omega^2 = |\vec{k} \cdot \vec{v}_A|^2 \equiv \left| \frac{m}{r} B_\theta - \frac{n}{R} B_\phi \right|^2 / 4\pi\rho \quad (1)$$

where m/r is the poloidal-and n/R the toroidal wave number, and ρ is the plasma mass density. This limit reduces essentially to

$$q = m/n \quad (2)$$

where $q = r B_\phi / R B_\theta$ is the usual safety factor. We will consider, in particular, the case of frequencies $\omega < \nu$, where ν is the electron collision frequency. If the singular surface $q(r) = 1$ is present - at least marginally - in the flat central region of the plasma, then the most appropriate poloidal and toroidal mode combination is $m = n$ (but not necessarily $m = n = 1$). In the absence of the $q = 1$ surface, other singular surfaces can obviously be considered. What should in any case be avoided is the presence of mode numbers matching condition (2) on the very edge of the plasma, because of the adverse effects they would have on thermal insulation and plasma confinement. The appropriate wave numbers are produced either by helical RF-coils (all the harmonics they can create have essentially the same helicity) or by the two annular plates located above and below the plasma ring as proposed for torsional TTMP heating /3/. The latter solution is most probably the best way of producing shear modes with the required helicity, because it does not imply putting RF-coils in the high- B_ϕ inner region of the toroidal device.

Wave propagation and power deposition can be studied in the usual collisional tearing mode stability scheme /4/, that is to say by assuming that each singular surface is embedded in a thin resistive layer which separates two perfectly conducting plasma regions. Another important approximation is the use of the Tokamak ordering $B_\phi \gg B_\theta$ to expand the magneto-hydrodynamic equations to lowest order in the inverse aspect ratio. Thus we replace the torus with a cylinder of length $L = 2\pi R$. Power deposition can conveniently be derived by means of a heuristic procedure (as in the tearing mode case /4/) which gives the correct parametric dependence of the relevant quantities while producing the numerical factors with sufficient accuracy in view of the various uncertainties inherent to the model. In this model RF-power can only be deposited in the resistive layer. Assuming that within this layer (whose radius is r_s and thickness Δ) both the electrical resistivity and the RF-current density j are essentially uniform, we write for the total RF-power absorbed :

$$P_{RF} \approx 4\pi^2 R r_s \Delta \cdot \eta j^2 \quad (3)$$

Notice, incidentally, that in order to have $P_{RF} > P_{Ohmic}$ the RF-current amplitude in the resistive layer must exceed the ohmic j_{dc} -current. Introducing the surface current $j^* \approx \Delta \cdot j$ which is determined by the jump of the tangent components of the RF $\vec{B}$ -field across the layer, $\langle \vec{B}_t \rangle$, the mean power density, absorbed is

$$\bar{P} \approx \frac{r_s}{2\pi a^2} |\langle \vec{B}_t \rangle|^2 \cdot \frac{\eta c^2}{4\pi \Delta} \quad (4)$$

where a is the minor radius of the plasma. Within the resistive layer the radial components of Faraday's law and of momentum balance law are found to involve only B_r and v_r , the radial components of the $\vec{B}$ and $\vec{v}$ disturbances :

$$\partial B_r / \partial t = \vec{B}_\theta \cdot \text{grad } v_r + \frac{c^2 \eta}{4\pi} \cdot \nabla^2 B_r \quad (5)$$

$$4\pi \rho \partial v_r / \partial t \approx \vec{B}_\theta \cdot \text{grad } B_r \quad (6)$$

(here only the tension of the $\vec{B}$ -lines and not the pressure gradient is retained as appropriate to shear disturbances). Since power dissipation requires some relative motion of fluid and $\vec{B}$ -field lines, we see that in the resistive layer ω must exceed the Alfvén frequency $k \cdot \vec{v}_A$, which characterizes plasma motions with frozen-in $\vec{B}$ -lines. Thus Δ is defined by

$$\omega \approx |\vec{k} \cdot \vec{v}_A| \approx \Delta |\vec{k} \cdot \vec{v}_A|' \quad (7)$$

where the prime denotes radial derivative. Notice that in view of the large sheath currents required for powerful heating, the linearization of the problem is questionable. Thus, the $\vec{B}$ -field and the derivatives contained in Eq. (7) should rather be interpreted as actual quantities including the RF-perturbations. B_r is assumed continuous across the layer so that B_r' is not especially large. This derivative, however, suffers an apparent discontinuity at r_s as measured by

$$\Delta' \equiv \{B_r'(r_s + \frac{\Delta}{2}) - B_r'(r_s - \frac{\Delta}{2})\} / B_r(r_s)$$

Then, within the layer, $\nabla^2 B_r \approx \frac{i\Delta'}{\Delta} B_r$, and Eq. (5) gives

$$\frac{c^2 \eta}{4\pi \Delta} \approx \frac{-\omega}{\Delta'} \quad (8)$$

With Eq. (8), Eq. (4) becomes

$$\bar{P} \approx \left(\frac{r_s}{a}\right)^2 \cdot \frac{\omega |\langle \vec{B}_t \rangle|^2}{2\pi r_s |\Delta'|} \quad (9)$$

The problem is now reduced to calculate $\langle \vec{B}_t \rangle$ and Δ' from the ideal MHD equations outside the tearing layer. As ω is supposed to be small compared with the natural frequencies of the confined plasma, $\bar{P}$ is accurately given to first order in ω and we only need to consider the zero-frequency MHD solutions. If we assume helical symmetry, all quantities have to be functions of only r and $\tau \equiv m\theta + kz$ (with $k = n/R$). This, together with the Tokamak ordering and the assumption that the axial $\vec{B}$ -component is uniform, permits to give $\vec{B}$ in term of only one flux function, ϕ , which can be calculated with a Stokes equation once the functional dependence of $dp/d\phi$ is specified by physical arguments. In the assumed ordering the appropriate equation of state is $\text{div } \vec{v} \approx 0$. The solution to the linearized Stokes equation can then be given in terms of known transcendental functions /5/. A particularly simple case is the force-free situation /6/. The arbitrary coefficients in front of the solutions within the plasma can then be found in terms of the vacuum field by considering a sharp free-boundary plasma and the usual jump conditions across the perturbed boundary (which is a magnetic surface).

It remains to discuss the width of the RF-magnetic islands which develop near the singular surfaces as a result of the applied helical RF-fields. This width is an important quantity, as no pressure gradient can be sustained across a magnetic island, even if only neoclassical diffusion processes operate /7/. The usual estimate /8/ of the full width of the islands, w , gives in our case :

$$w \approx 4 \left\{ -B_r \frac{d}{dr} \left(\frac{mB_\theta}{r} \right) \right\}^{1/2} \approx 4 a (B_r/B_\theta)^{1/2} \quad (10)$$

(here B_r is the radial component of the $\underline{\text{RF}}\text{-}\vec{B}$ field) which inserted into Eq. (9) gives in order of magnitude

$$\bar{P} \approx \omega \frac{B_\theta^2}{8\pi} \left(\frac{w}{4a} \right)^4 \quad (11)$$

If a given amount of power has to be dissipated into a given plasma, the factor $\omega (w/a)^4$ must be kept constant. As a result, the lowest possible frequency value is determined by the maximum tolerable w .

Comparing Eq. (11) with the corresponding estimate for Transit Time Magnetic Pumping with $B_0/B \approx 3 \cdot 10^{-3}$ -which should represent the reactor requirement /9/- one finds for $f \geq$ a few kHz, $w \leq 0.2$ a. However a more precise derivation of $\bar{P} = \bar{P}(w)$ is required to assess this crucial point. Notice that the island problem is avoided if the proposed scheme is used to heat multiple configurations (Doublets, octupoles, etc.) near the hyperbolic axes : this requires properly dephased axisymmetric horizontal RF-coils, similar to those of Ref./1/.

REFERENCES :

- /1/ - E. CANOBBIO, "Plasma Physics and Controlled Nuclear Fusion Research", I.A.E.A, Vienna (1977), III, 19 ; see also E. CANOBBIO, VIII European Conference on Controlled Fusion and Plasma Physics, Prague (1977) I, 160.
- /2/ - E. CANOBBIO, VIII European Conference on controlled Fusion and Plasma Physics, Prague (1977) I, 161.
- /3/ - F. KOEHLIN and A. SAMAIN, Phys. Rev. Letters, 26, (1971) 490.
- /4/ - H.P. FURTH, J. KILLEEN, and M.N. ROSENBLUTH, Phys. Fluids, 6 (1963) 459 ; R.D. HAZELTINE, D. DOBROT, and T.S. WANG, Phys. Fluids 18 (1975) 1778.
- /5/ - D. CORREA and D. LORTZ, Nuclear Fusion, 12 (1972) 127.
- /6/ - S. CHANDRASEKHAR and P.C. KENDALL, Astrophys. J. 126 (1957) 457 ; D. VOSLAMBER and D.K. CALLEBAUT, Phys. Rev. 128 (1962) 2016.
- /7/ - T. OHKAWA, Phys. Letters 38 A (1972) 21.
- /8/ - A.B. RECHESER and T.H. STIX, Phys. Rev. Letters 36 (1976) 587.
- /9/ - E. CANOBBIO, Second Topical Conference on Radio Frequency Plasma Heating, Lubbock (Texas) 1974, paper C.1 ; see also E. CANOBBIO, VI European Conference on Controlled Fusion and Plasma Physics (1973) II, 81.

ECR AND UPPER HYBRID HEATING IN THE CULHAM LEVITRON

A C Riviere, M W Alcock and T N Todd
 Culham Laboratory, Abingdon, Oxon. OX14 3DB, U.K.
 (Euratom/UKAEA Fusion Association)

ABSTRACT: Electron temperature and density profiles have been measured for microwave plasmas in the Culham Levitron. At a power level of 16.5 W the heating is located where $\omega = \omega_c$, whereas at higher power levels heating occurs between ω_c and the upper hybrid resonance. At the highest powers (2.4 kW, 10 GHz) electron densities have been reached ($> 4 \cdot 10^{12} \text{ cm}^{-3}$) such that $\omega_p^2 \geq 3 \omega^2$. The heating process is thought to be due to linear conversion of electromagnetic waves at the upper hybrid resonance into electron plasma waves which then propagate into the high density region and heat the electrons by Landau damping.

INTRODUCTION: The linear conversion of electromagnetic waves into slow electrostatic waves at plasma resonance is well known^(1,2). The slow wave energy can be transferred to the plasma by Landau damping when $k \cdot B$ becomes finite. Here we present results on the plasmas produced by microwave power in the Culham Levitron⁽³⁾. The data is consistent with linear conversion at the upper hybrid resonance and has similar features to the results obtained by Golant et al.⁽⁴⁾ and Anisimov et al.⁽⁵⁾.

EXPERIMENTAL CONDITIONS: The Culham Levitron is a toroidal containment system with the main field provided by a superconducting ring winding (major diameter = 60 cm, minor diameter = 9.3 cm). Magnetic shear is provided by a toroidal field and the flux surface shapes are determined by the combination of ring and vertical field components. All the fields are d.c. The ring is freely floated within the plasma and operated for these experiments at a current (I) between 110 and 140 kA for heating at 10 GHz and at $I = 180$ kA for heating at 16.25 GHz. The field distribution for $I = 180$ kA and toroidal field current of 140 kA is shown in Figure 1 with the ECR surface ($\omega_c = \omega$) at 1.3 cm from the ring ($\omega_c/2\pi$ is the electron cyclotron frequency and $\omega/2\pi$ the applied frequency). The field strength decreases with distance from the ring so that outside the ECR surface $\omega_c < \omega$. Steady state plasma was produced for 3.5 s by applying microwave power through a horn 10 cm from the ring on the equatorial plane. The average electron density was measured with a 4 mm interferometer and an average value for electron temperature was obtained from an absolute measurement of the intensity of the 5876 Å HeI line. Profiles of electron density (n_e) and temperature (T_e) were measured with a swept double probe working at 10 kHz⁽⁶⁾. The probe was moved through the plasma in 2.5 s to obtain a

complete profile in each discharge. The agreement between the density derived from the probe and the average density was $\pm 10\%$ when allowance was made for ion mass etc. The average temperature obtained from the HeI light intensity was within 20% of that calculated from the temperature profile.

RESULTS: Profiles of n_e and T_e were measured for microwave power levels from 10 W to 2.4 kW, ring currents from 110 to 180 kA and gas pressures from 10^{-6} to 10^{-4} torr. Those for power levels of 16.5, 30, 60 and 165 W (10^{-5} torr He, $I = 110$ kA) are shown in Figure 2. The small dip in the n_e profile was always present but is not understood. The size of the dip was insensitive to changes in power level, gas pressure and operating frequency and does not affect the main conclusions. The interesting feature is that at low power (16.5 W) the T_e profile shows that the heating is located at the ECR surface but at higher powers the profiles broaden towards the weaker magnetic field region where $\omega_c < \omega$. The upper hybrid resonance (UHR), where $\omega_{UH}^2 = \omega_p^2 + \omega_c^2$, is marked for each profile (ω_p is the plasma frequency). The power lost by ionization and excitation calculated from the known neutral density, n_e and T_e is shown in Figure 3. If we neglect thermal conduction then the power appears to be absorbed over a broad band between ECR and UHR. The absence of heating where $\omega_c > \omega$ is further illustrated in Figure 4 where profiles are shown for three locations of the ECR surface. At the highest powers the 'cut-off' density corresponding to $\omega_p^2/\omega^2 = 1$ is exceeded by a factor of three as shown in Figure 5.

Plasmas were also produced with two input horns on opposite sides of the plasma, with the horn directed to the small major radius side of the ring and with the horn at different distances from the ring but all showed the same characteristics. In addition no change was observed when the plane of polarization was rotated.

DISCUSSION: The broad absorption profile we observe could be explained by the plasma resonance which exists for all n_e above the upper hybrid resonance and $\omega_p^2 \leq \omega^2$, $\omega_c^2 < \omega^2$, given the appropriate angle of propagation with respect to the magnetic field. However these resonances are inaccessible if Snell's law ($n \cos \theta = \text{constant}$) is obeyed and they do not exist for $\omega_p^2 > \omega^2$. The preferred explanation is that conversion occurs mainly at the upper hybrid resonance and energy is then transported into the plasma by slow electrostatic waves which propagate freely across the field⁽⁷⁾. In this case there is no restriction on ω_p^2/ω^2 and the high density region can be reached. Propagation is not possible for the electrostatic modes when $\omega_c > \omega$, so again an inner boundary to the heating process is formed where $\omega_c = \omega$.

ACKNOWLEDGEMENTS: The authors gratefully acknowledge the contribution to this work made by members of the Levitron team led by D R Sweetman and in particular, M F Payne, T Edlington, P R Collins and N R Ainsworth. They would also like to acknowledge useful discussions with D E T F Ashby and C N Lashmore-Davies.

REFERENCES

1. T H Stix, Physical Review Letters 15, 878 (1965).
2. V E Golant and A D Piliya, Soviet Physics Uspekhi 14, 413 (1972).
3. N R Ainsworth et al., Proc. 7th European Conf. on Contr. Fusion and Plasma Phys. Vol. III (Lausanne, 1975) p.127.
4. V E Golant et al., Proc. 6th European Conf. on Contr. Fusion and Plasma Phys. Vol. I (Moscow, 1973) p.587.
5. A I Anisimov et al. Sov. Phys. Tech. Phys., Vol.18, p.459 (1973).
6. D E T F Ashby, W H W Fletcher and T N Todd - to be published.
7. F W Crawford, Radio Science 69D, 789 (1965).

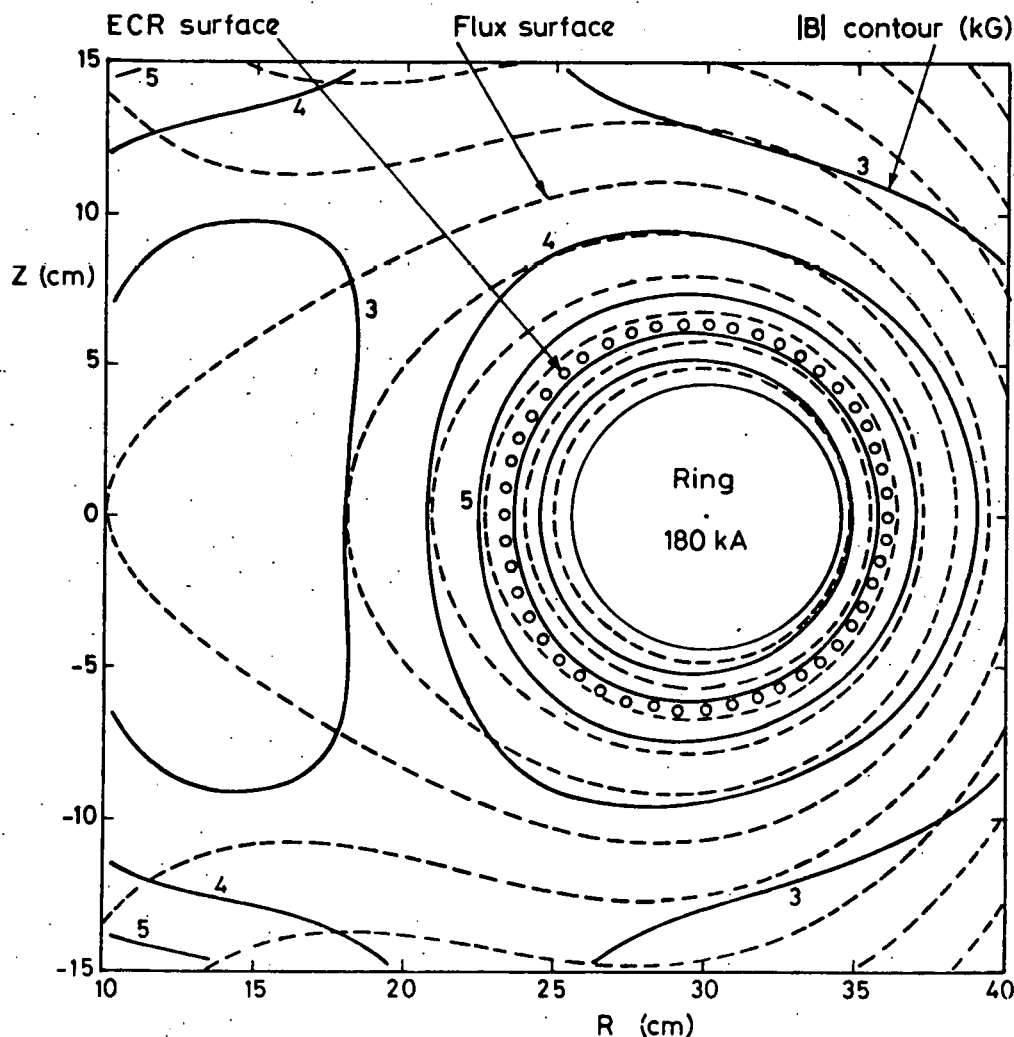


Fig.1 Magnetic field plot for a ring current of 180 kA and toroidal field current of 140 kA. The ECR surface corresponds to $\omega/2\pi = 16.25$ GHz. Profiles were measured on the plane $z = 0$ between $R \approx 35$ and $R = 40$ cm.

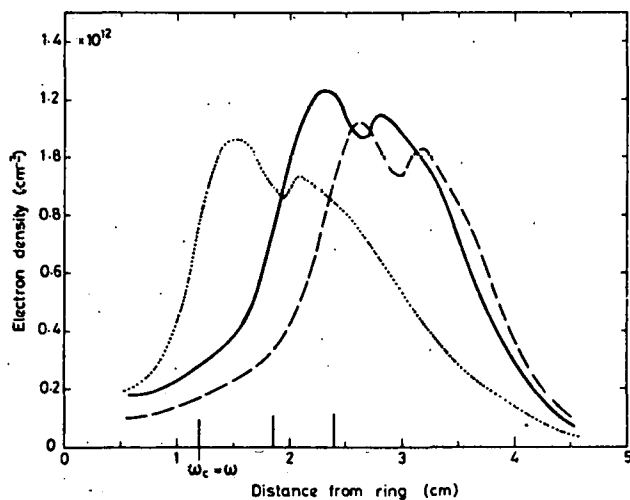
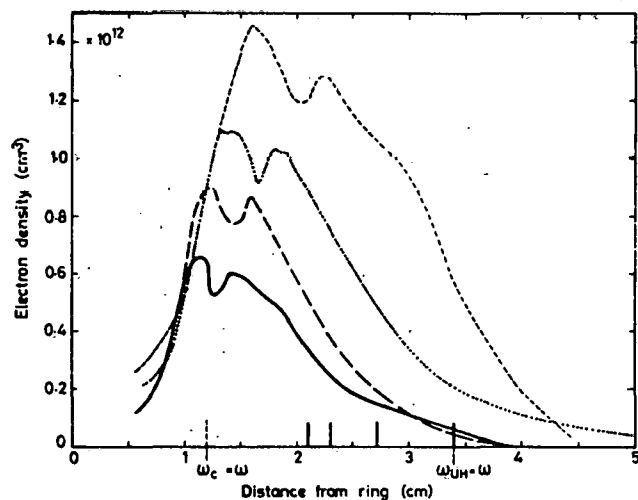
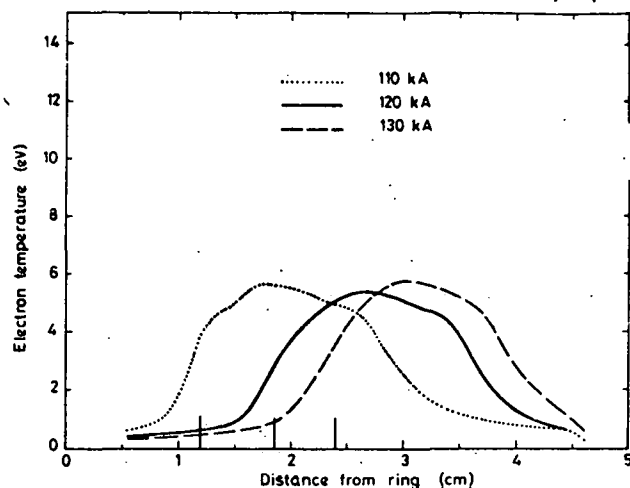
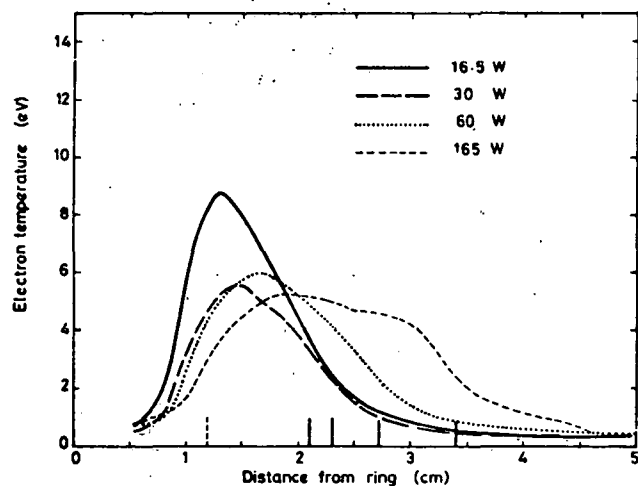


Fig.2 Profiles of n_e and T_e measured with the swept double probe with $I = 110$ kA, $\omega/2\pi = 10$ GHz and 10^{-5} torr of He. The parameter is microwave power.

Fig.4 Profiles of n_e and T_e measured with the swept double probe with $\omega/2\pi = 10$ GHz, 3.5×10^{-5} torr of He and 75 W of power. The parameter is ring current.

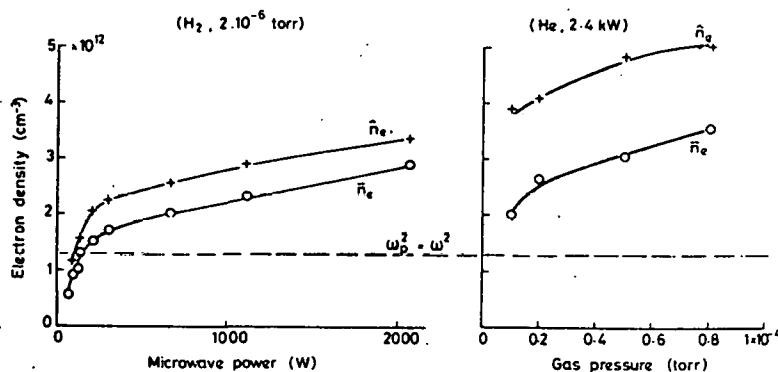
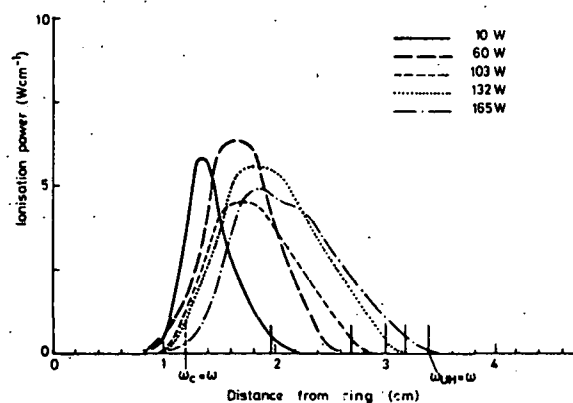


Fig.3 Profiles of power lost by ionization and excitation calculated from measured n_e and T_e (conditions as for Figure 2)

Fig.5 Electron densities achieved at high power levels with $I = 100$ kA and $\omega/2\pi = 10$ GHz. $\bar{n}_e$ = average density from interferometer, $\hat{n}_e$ = peak density from probe measurements

MAGNETOACOUSTIC OSCILLATIONS OF A CYLINDRICAL PLASMA COLUMN

B. Blackwell, G. Collins, R. Cross and J. Lehané

Wills Plasma Physics Department,
University of Sydney, N.S.W. 2006, Australia.

INTRODUCTION

The magneto-acoustic wave travelling perpendicular to a magnetic field exhibits resonance behaviour when the transit time of the wave across the plasma column is a multiple of the wave period.¹ If the resonances are generated at low power their detection provides a powerful method for diagnosing the properties of the plasma. This is particularly so if a detection system is used which is external to the plasma. Measurements of this nature using a 100 Watt oscillator are described. The generation of the magneto-acoustic resonances at high power offers a method of heating a plasma,² a method which seems attractive in that large currents can be induced in the interior of the plasma³ rather than at the surface. Preliminary measurements of this type using a 10 megawatt pulsed oscillator are also described.

2. EXPERIMENTAL METHOD

2.1 Plasma preparation and diagnosis

The experiment was conducted using a cylindrical glass vessel of length 2.65 m, inner diameter 15.2 cm and wall thickness 0.72 cm, surrounded by a set of field coils to provide an axial magnetic field uniform in time and space to within 3% during the formation and decay of the plasma. For the low power measurements this field was 0.8 tesla and for the high power measurements 0.1-1 tesla. The ends of the vessel were sealed with pyrex glass end plates each holding a 13 cm diameter stainless steel electrode. Argon was pumped continuously through the vessel at a filling pressure of 10 m torr. The plasma was formed by discharging a lumped LC transmission line between the end electrodes to give a constant current pulse of 13.3 kA for 360 μ sec. After 360 μ sec the current reversed, giving a second pulse of 11 kA for 360 μ sec.

The properties of the plasma were determined as follows. The average electron temperature, T during current flow was determined from plasma resistivity measurements using the ratio of the voltage across the electrodes to the plasma current. Values of T of 1.3, 1.75, 2.1, 1.6 and 2.1×10^4 °K at times, t , after gas breakdown of 50, 100, 150, 250 and 300 μ sec respectively were hence obtained. Magnetic probe measurements of the radial distribution of the current density implied that the radial variation of temperature could be approximated by a parabolic distribution.

The electron density was measured at radii of 0, 5.7 and 7.0 cm using a He-Ne laser interferometer operating at 633 nm and designed to resolve fringe shifts as small as 0.1 fringe. The electron density measurements are given in Fig. 1. Comparing these with the initial filling density of $3.3 \times 10^{20} \text{ m}^{-3}$ it would seem that most of the argon is doubly ionised during the discharge. This result is consistent with Saha equilibrium for the observed values of T at the end of each half cycle of current, is consistent with spectroscopic observations of the plasma and is also consistent with a simple energy input calculation.

2.2 Wave generation and detection at low power

Magneto-acoustic waves were launched by the antenna shown in Fig. 2. It consisted of 20 separate low resistance single turn loops connected as five 4 turn

solenoids. A current equalising resistor in series with each solenoid allowed the solenoids to be connected in parallel. The antenna was connected in series with a 25Ω resistor and driven by a 100 watt amplifier electronically controlled to deliver a constant R.F. current of amplitude 400 mA regardless of impedance changes in the antenna during plasma formation and decay. The R.F. frequency was varied from shot to shot over the range 0.2 to 5.0 MHz.

External detection of the resonance, which loads the solenoid and hence changes its impedance, was made by measuring the impedance of the centre 4-turn solenoid as a function of frequency. The impedance was monitored continuously in a four-terminal configuration by feeding signals from differential "signal" and "reference" probes (shown in Fig. 2) to an electronic network analyser. The analyser employs phase sensitive detectors to convert the amplitude ratio and phase difference between the signal and reference probes into two D.C. output signals, one being directly proportional to the reactance between the signal probes, the other being directly proportional to the series resistance between the signal probes. The resonance was also detected by measuring the amplitude of the axial component of the wave magnetic field, $|b_z|$, at zero radius.

2.3 Wave generation and detection at high power

Waves were generated at high power using a 10 MW lumped line oscillator of the Lietti type⁴ which was designed to produce a sine wave burst of 10 equal amplitude cycles at a frequency of 0.28 MHz. To match the impedance of the oscillator an antenna consisting of two 6 turn solenoids connected in parallel was used. Current equalisation was achieved by winding the solenoids with interleaved turns rather than by using series resistors.

As for the measurements at low power the resonances were detected by measuring $|b_z|$ on axis with a magnetic probe. External detection was achieved by measuring the voltage and current in the solenoid.

3. RESULTS AND DISCUSSION

3.1 Low power measurements

Results which illustrate the detection of the resonance at a particular time in the life of the plasma are shown in Fig. 3. The resonance is indicated by the changes which occur in the inductance, L and effective series resistance, R of the solenoid and also by the change in the magnetic field $|b_z|$ on axis.

Results which show the position of the resonance at various times throughout the life of the plasma are shown in Fig. 4. As can be seen a strong fundamental resonance was observed well into the plasma afterglow. There is however no strong evidence for harmonic resonances.

The curves fitted to the points in Figs. 3 and 4 are best fit theoretical curves based on a 3-fluid description of the plasma similar to that of Frommelt and Jones.⁵ The detailed shapes of the theoretical resonance curves depend on the assumed form of the radial distribution of temperature and ion density. These were assumed to be either uniform or parabolic of the form

$$n_i = 2n_0(1 - r^2/r_0^2) \quad \text{or} \quad n_i = n_0 \quad \text{for } r < r_0$$

$$\text{and} \quad n_i = 0 \quad \text{for } r > r_0 \quad (r = r_0 \text{ is the plasma boundary})$$

$$T = T_0 \quad \text{or} \quad T = T_0 - (T_0 - 300) \times (r/0.076)^2$$

By fitting simultaneously to the results for L , R and $|b_z|$ as a function of frequency

the appropriate distributions and values of n_o , r_o and T_o were obtained. Up to 650 μ sec the best fits were obtained with a parabolic temperature distribution and a uniform density distribution. The parameters obtained are given in Table 1.

Table I Best Fit Parameters

t (μ sec)	150	250	350	450	650
r_o (cm)	6.1	6.1	6.6	6.6	6.6
$n_o \times 10^{20} \text{ m}^{-3}$	4.2	3.5	3.0	2.8	2.5
$T_o \times 10^4 \text{ }^\circ\text{K}$	1.4	1.1	1.2	1.05	0.85

The ion density results in this table cannot be compared directly with the electron density results given in Fig. 1 due to the uncertainty in the ion charge number. The results indicate however that little, if any, of the original filling gas is lost during the discharge and that a slight compression of the gas occurs during the discharge.

The electron temperature results given in Table I are appreciably lower than those given in §2.1. The reason for this discrepancy is not known but it does appear that the temperatures in Table I are unrealistically low in that they imply that the plasma volume between $r = 5$ cm and $r = 6.1$ cm which is known to contain doubly ionised argon is at a temperature below $5 \times 10^3 \text{ }^\circ\text{K}$.

3.2 High power measurements

Measurements with the high power oscillator were made at a fixed frequency of 0.28 MHz and a constant oscillator voltage, scanning through the resonance by varying the magnetic field. The measurements again show the existence of magneto-acoustic resonances but as well show the existence of a non-linear effect not present at low powers. As is to be expected this effect was most marked at low fields and at the resonant field where the ratios of the perturbation field to the steady field are highest. The effect which has previously been observed by Fässler et al⁶ is illustrated in Fig. 5 which shows b_z on axis for the resonant field of 0.23 tesla and for other fields. There is no sign of harmonic distortion at the highest field but at the other fields it is quite evident.

No direct measurement of temperature has yet been made and so there is no direct evidence of plasma heating at these high powers. The total light emitted from the plasma has however been observed. The pronounced increase in this which occurs for resonance conditions is apparent from Fig. 6 where the total light is given for the resonant field and for fields away from resonance.

4. CONCLUSION

Magneto-acoustic resonances have been excited at both low and high powers. At low powers an external method of detection of the resonances has been demonstrated which provides a most useful method of plasma diagnosis. Preliminary measurements at high power show the existence of non-linear effects and give some indication of possible plasma heating.

REFERENCES

1. CANTIENI, E. and SCHNEIDER, H. 1963 *Helv. Phys. Acta.*, **36**, 993.
2. IVANOV, N.V. and KOVAN, I.A. 1975 *Nucl. Fus. Supplement*, p.41.
3. CROSS, R.C. 1970 *Can. Jnl. Phys.*, **48**, 2888.
4. LIETTI, A. 1969 *Rev. Sci. Insts.*, **40**, 473.
5. FROMMELT, J. and JONES, I. 1975 *J. Plas. Phys.*, **14**, 373.
6. FÄSSLER, K., VACLAVIK, J., SCHNEIDER, H. 1969 *Helv. Phys. Acta*, **42**, 23.

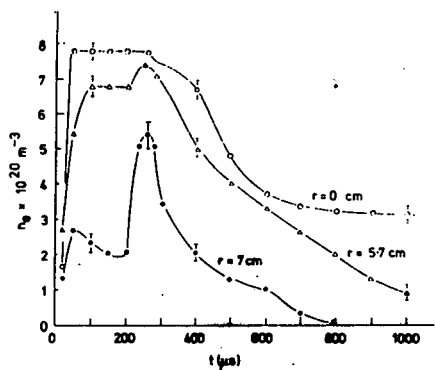


Fig. 1 Electron density as a function of time at different radial positions (filling pressure 10 m torr).

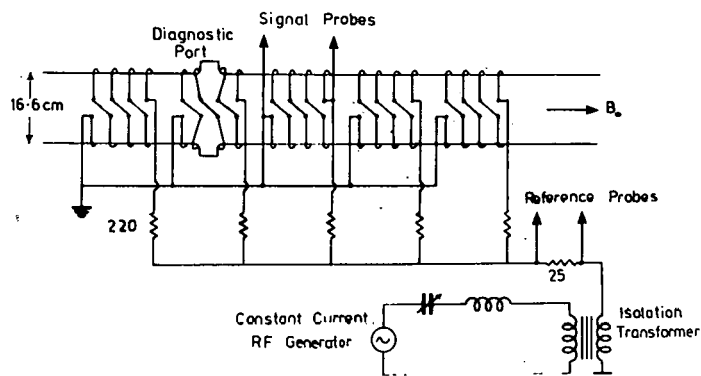


Fig. 2 Guarded solenoid configuration used to launch and detect magneto-acoustic waves.

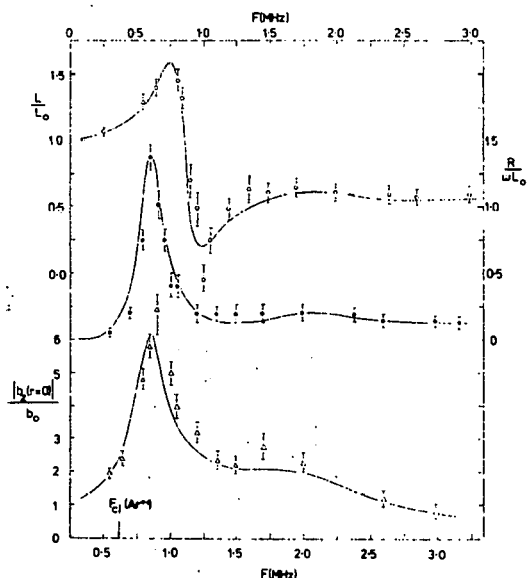


Fig. 3 Experimental values of L/L_0 , $R/\omega L_0$, and $|b_z(r=0)|/b_0$ as a function of wave frequency at a time $t=150 \mu\text{sec}$ after gas breakdown. The upper frequency scale refers to L/L_0 only. L_0 is the vacuum inductance and b_0 the vacuum value of $|b_z(r=0)|$. The theoretical curves correspond to the parameters given in Table 1.

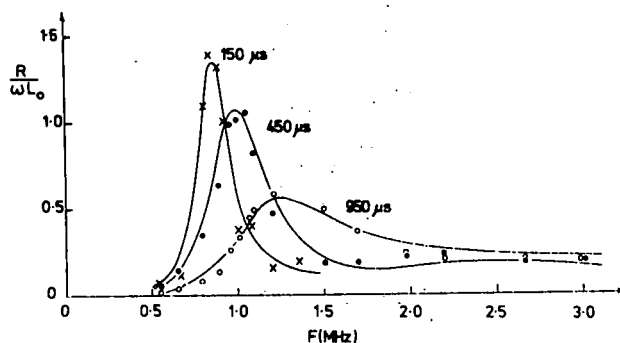


Fig. 4 Experimental values of $R/\omega L_0$ as a function of wave frequency for three different times after gas breakdown. The solid curves are theoretical ones corresponding to the parameters given in Table 1.

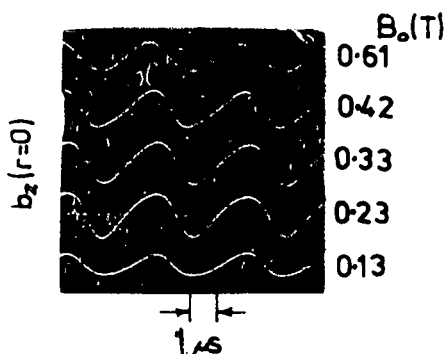


Fig. 5 b_z on axis 250 μsec after gas breakdown for the resonant field of 0.23 tesla and other fields. The vertical scale is 0.1 tesla/large division.

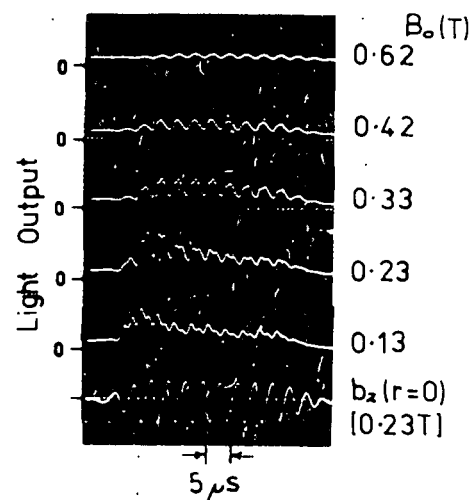


Fig. 6 Total light emitted from the plasma for the resonant field of 0.23 tesla and for other fields.

MICROWAVE ABSORPTION STUDIES NEAR THE PLASMA FREQUENCY*

B. L. Wright, Martin E. Banton, H. Dreicer, and J. C. Ingraham

Los Alamos Scientific Laboratory
of the University of California
Los Alamos, New Mexico

To study the rate of electromagnetic wave absorption by a plasma, we use two high-Q microwave resonators that surround an alkali plasma column (Fig. 1). The resonators are operated in the TM_{010} mode (rf electric field parallel to static magnetic field) at resonant frequencies, $\omega/2\pi$, near 2.0 GHz. The fully ionized plasma is in thermal equilibrium, $T_e = T_i = 0.2$ eV, with a density that can be varied to obtain values of ω_p/ω near unity. The rf absorption rate is measured by first resonantly exciting a mode with a short pulse of microwave power and then monitoring the decay of its rf field energy after the external source is switched off. After correction for loss in the copper resonator walls, the observed decay rate can be converted to an absolute rate of microwave absorption by the plasma alone or to an equivalent value of its ac electrical resistivity. Our earlier work concentrated on the relation between the steady-state resistivity and the rf field strength, E_0 . At low amplitudes the resistivity is found to be precisely that expected from electron-ion collisional effects, but for more intense fields, enhanced, nonlinear absorption results from the onset of parametric instabilities.¹ Detailed analysis of the threshold for instability, detection of hot tails on the electron velocity distribution, and observation of the spectrum of plasma fluctuations near the pump frequency suggest that both the oscillating-two-stream and the parametric decay instability are operative in generating large-amplitude electron and ion waves with associated particle trapping.

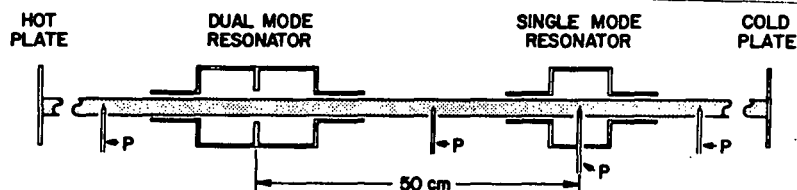


Fig. 1.

Transient Growth of Parametric Instabilities

More recent studies have treated the temporal development of pump-field absorption during the initial period when the plasma fluctuations are still unstably growing. It is found that a pump field raised quickly above threshold will at first exhibit only classical, collisional absorption until the fluctuations reach sufficient amplitude to cause a sharp decrease (pump depletion) in the rf field. As the field strength, E_0 , is raised, the time delay required for the onset of enhanced absorption is decreased. Field-dependent growth rates measured in this way agree with theoretical models for the instabilities involved. Langmuir probes outside the resonator indicate also that pump depletion is accompanied by the generation of a burst of fast electrons sufficient to produce a peak on the tail of the longitudinal velocity distribution at about 50 times the mean thermal energy. For the case $\omega_p^2/\omega^2 \approx 0.90$ we find that the density of fast electrons is a small fraction (less than one percent) of the total electron density but that (at the higher rf powers used) the total energy in fast electrons accounts for all the microwave energy absorbed through the parametric process. This transient effect

*Work performed under the auspices of the U.S.D.o.E.

is similar to that previously reported¹ in the saturated steady state, but now we can analyze its temporal development. By applying time-of-flight corrections to the probe data, the rate of nonclassical absorption of electromagnetic energy and the rate of production of fast-electron energy in the resonator can be plotted and compared (Fig. 2). It is clear from such a plot that throughout most of the period of pump depletion, electromagnetic energy is first transferred to plasma fluctuations and that only subsequently is the fluctuation energy efficiently converted to that of fast electrons.

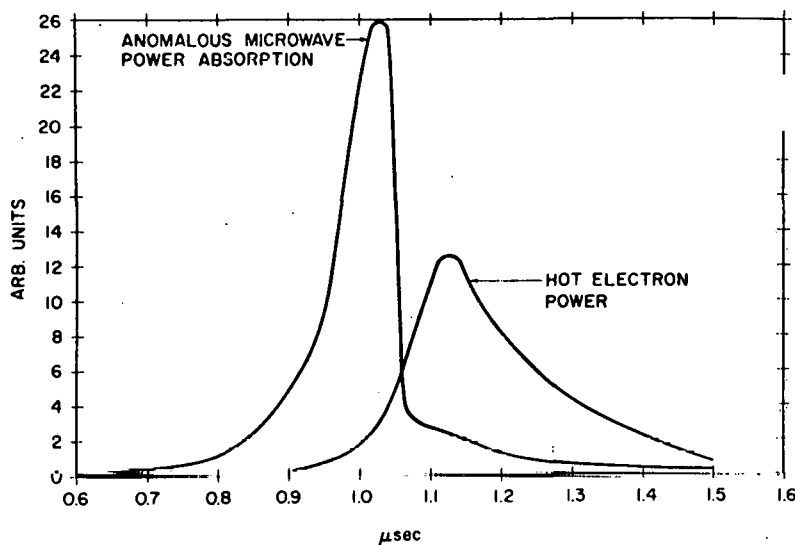


Fig. 2.

Absorption of a Weak Field by Controlled Levels of Turbulence

The electron and ion fluctuations created by parametric instabilities have the nature of a turbulent spectrum of plasma waves whose effect on the weak-field ac resistivity--at frequencies other than that of the pump that generated them--can itself be an object of study. To examine this problem we use a dual-mode resonator (see Fig. 1) that is capable of supporting two nearly degenerate modes (even and odd symmetry) with overlapping fields of the TM_{010} character. We use one of these modes as a strong driver field to generate and maintain plasma fluctuations via parametric processes. The other mode is then used as a weak subthreshold test field to monitor the ac resistivity at a somewhat different frequency. For $\omega_p^2/\omega^2 \approx 0.85$ we have measured noticeable enhancement of the weak-field resistivity at test-driver frequency separations of ± 11 , ± 15 , ± 28 and ± 86 MHz. (For comparison, the ion plasma frequency, $\omega_{pi}/2\pi$, is around 7.2 MHz.) Figure 3 shows data taken during the saturated, steady state with the test-field frequency 28 MHz above that of the driver field. The quantity plotted is the effective collision rate, ν_{eff} , resistivity, normalized to the weak-field classical value, ν_0 . It is shown for both modes as a function of the driver power, $P \propto E_0^2$, normalized to the threshold level, P_T . As the driver field passes above threshold it experiences a jump in its absorption rate (crosses) due to the onset of parametric instabilities. The test field (spots) also shows enhanced absorption but only at the higher driver powers. In these experiments the test-field absorption is found to be linear in the weak test-field amplitude; i.e., its measured enhanced resistivity is independent of the (subthreshold) test power. Figure 4 shows a transient analysis of the period of unstable growth similar to that of Fig. 2. In this case the test-field frequency is 15 MHz above that of the driver field. Again strongly enhanced nonlinear absorption of the driver field is accompanied by enhanced linear absorption of the test field. We attribute the test field behavior to stimulated absorption processes similar to those of parametric decay but in which the requisite plasma fluctuations are maintained by the separate driver field rather than being generated solely by the absorption process. This difference in mechanism and the fact that different portions of the fluctuation spectrum are involved could account for the differences in behavior exhibited by the test and driver fields.

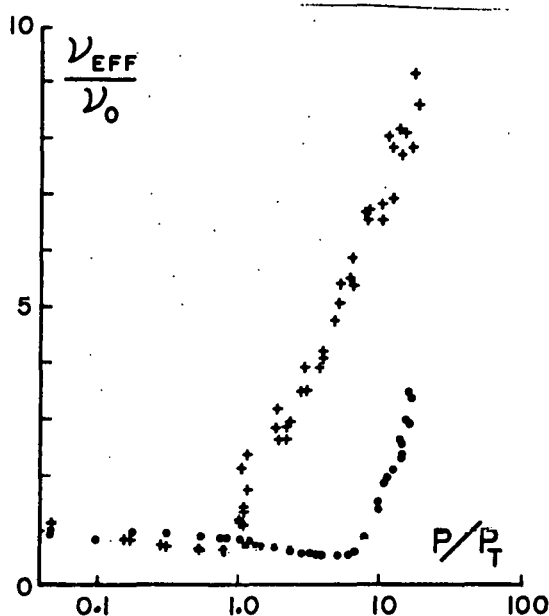


Fig. 3.

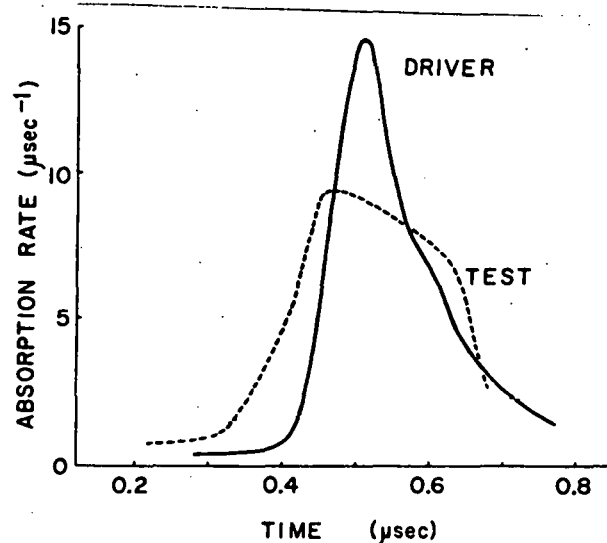


Fig. 4.

Modification of Electron-Ion Collision Rate by Intense RF Fields

The dual-mode resonator has also been used to confirm earlier work on high-field effects that do not involve parametric instability. We have already reported² measurements taken under conditions for which the plasma was parametrically stable but for which the oscillatory velocity of an electron in the rf field, $V_E = eE_0/m\omega$, was comparable to the random thermal velocity, $V_T = [2kT/m]^{1/2}$. A pronounced reduction of ac resistivity was observed in this case, a reduction attributed to the strong modification of electron orbits as they interact with the screened potentials of nearby ions. Through use of the dual-mode resonator, we have now applied simultaneous weak and strong fields at a frequency separation of 28 MHz. Figure 5 shows a plot of the normalized collision rates of both the strong field (crosses) and the weak field (spots) as functions of the ac amplitude parameter, V_E/V_T , associated with the strong field. The modification of electron orbits by the strong field is seen to affect the absorption of the weak field so that indeed both fields show identical reductions in collisional resistivity. This observation supports the idea that an intense ac field can produce a genuine, frequency-independent reduction of the electron-ion collision rate of the sort that might affect other collision-dependent transport properties as well.

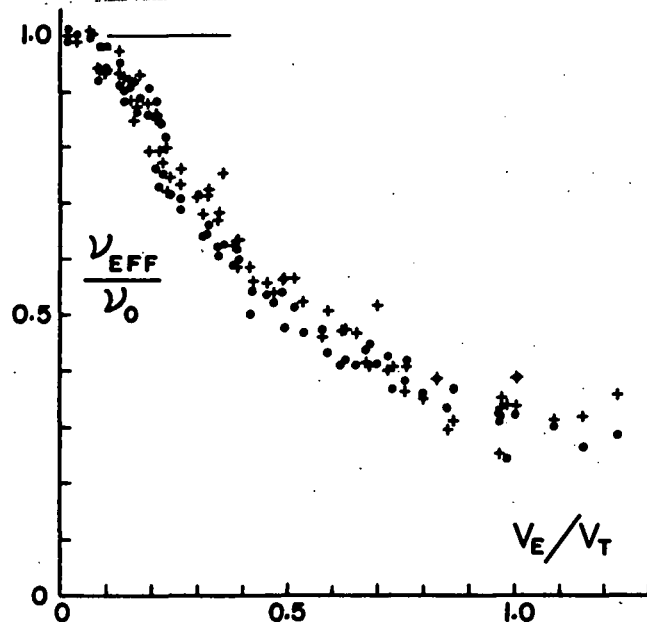


Fig. 5.

Observation of Negative Inverse Bremsstrahlung Absorption

We have used the single-mode resonator in the range $0.35 < \omega_p^2/\omega^2 < 0.50$ to look for a net negative collisional absorption of an rf field that has been predicted³ for the case of a fully-ionized plasma with an electron drift speed, V_D , comparable to the thermal speed, V_T . The measurement was accomplished by applying a dc current pulse along the plasma column while simultaneously observing the decay of a microwave field in the resonator. The current pulse width was kept short enough ($\approx 0.4 \mu s$) that instabilities of the Buneman type could not grow sufficiently to influence the electron drift speed and the microwave absorption measurement. Electron heating by the current pulse, though significant, was likewise kept to a minimum. Careful analysis of our results shows that indeed there is a range of V_D/V_T for which the plasma gives a negative contribution to the absorption of microwaves by the system; i.e. for which the observed decay of rf energy in the resonator proceeds somewhat more slowly than is measured for the copper walls alone. Figure 6 shows a plot of our results with the drift speed, V_D , expressed in units of the thermal speed, V_{T0} , that existed before application of the current pulse. The solid curve in the figure represents our own calculation based upon the Dawson-Oberman formalism.⁴ It uses the model of a drifting Maxwellian electron velocity distribution and includes the effect of a strong magnetic field.

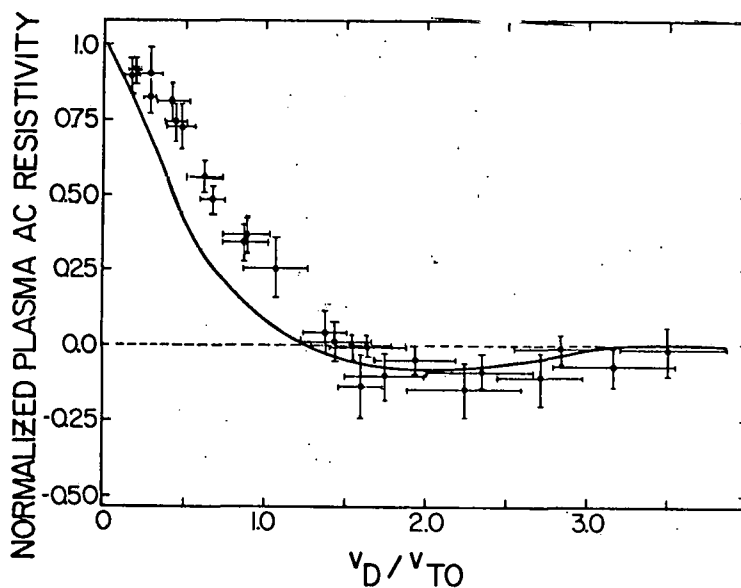


Fig. 6.

REFERENCES

1. H. Dreicer, D. B. Henderson, and J. C. Ingraham, Phys. Rev. Lett. 26, 1616 (1971); H. Dreicer, R. F. Ellis, and J. C. Ingraham, Phys. Rev. Lett. 31, 426 (1973).
2. J. H. Brownell, H. Dreicer, R. F. Ellis, and J. C. Ingraham, Phys. Rev. Lett. 33, 1210 (1974).
3. T. Musha and F. Yoshida, Phys. Rev. Lett. 1, 8 (1958).
4. J. M. Dawson and C. R. Oberman, Phys. Fluids 5, 517 (1961).

STOCHASTIC INTERACTION OF AN ION BEAM WITH LOCALIZED WAVES IN A CLOSED CONFINEMENT SYSTEM.

E.Lazzaro

Laboratorio di Fisica del Plasma C.N.R-Ass.EURATOM, MILAN-ITALY

Recent experiments on the ATC plasma¹ have demonstrated that a tangentially injected 26 Kev ion beam, result of neutral beam charge exchange reactions, interacts strongly with the lower hybrid waves, which, as it is well known², are localized in the "resonance cones". The observation show that the beam perpendicular energy grows significantly when the R.F. power is turned on, while the background plasma ions are not appreciably heated. The model of interaction considered here takes into account that cyclotron harmonic absorption should be inefficient because the resonance cone width in the region of interest is smaller than the ions gyroradius, but, on the other hand, the closed toric configuration should allow a possibility, for a test ion, of crossing repeatedly the wave field region, and gain (or lose) some energy at each traversal. It can be shown that this type of interaction develops stochastic properties, which lead to perpendicular heating growing linearly in time. In the present model the wave field is taken in the form³, valid in the cartesian geometry shown in Fig. 1, with symmetry in the y direction, in a cold uniform plasma region

$$E_z(x, z, t) = -E_0 \sum_{n=1}^{N_0} H(\xi_n - \xi) H(\xi_{n+1} - \xi) \sin(k_0 \xi - \omega t), \quad E_x = \sqrt{\epsilon_{33}/\epsilon_{11}} E_z.$$

with ξ_n the cold plasma dielectric tensor $\xi = z + \sqrt{\epsilon_{33}/\epsilon_{11}} x$ one of the resonance cone variable, $\xi_n = 2\pi nR$, $k_0 = n_{||} \omega/c$, and $\omega_B^2 = eE_0 k_0 / m$, $\gamma^2 = (1 + \frac{\epsilon_{33}}{\epsilon_{11}}) \omega_B^2$, $\eta = L/2\pi R$, where ω is the wave frequency, L the parallel width of the cone, R the tori major radius; $n_{||}$ is the parallel index of refraction, v_z the parallel ion speed and $T = 2\pi R/v_z$, $t_0 = L/v_z$, the times of revolution around the torus, and of traversal of the force region. The scaling $t_0^{1/2} \ll 1/\Omega <$

$\omega/k_0 < T$ does not allow a guiding center description in the interaction region. With the change of variables $\varphi = k_0 \xi - \omega t$ the equations of motion can be cast in the form

$$\ddot{\varphi} + \Omega^2 \varphi + \gamma_L^2 \sum_{n=1}^{N_0} H(\xi - \xi_n) H(\xi_n + L - \xi) \sin \varphi = -\Omega^2 k_0 \bar{z} \quad (1)$$

$$\ddot{\bar{z}} = -\frac{\omega_B^2}{K_0} \sum_{n=1}^{N_0} H(\xi - \xi_n) H(\xi_n + L - \xi) \sin \varphi \quad (2)$$

$$\bar{z} = z - \omega t / k_0$$

$$\dot{W}_\perp = -\sqrt{\epsilon_{33}/\epsilon_{11}} (\omega_B^2 / k_0) \dot{x}_L \sum_{n=1}^{N_0} H(\xi - \xi_n) H(\xi_n + L - \xi) \sin \varphi \quad (3)$$

where $W_\perp$ is the perpendicular energy and φ is the wave phase perceived by the particle. The increase of the perpendicular energy is $\frac{W_\perp}{W_{||}} = O(m_i/m_e)$ as can be checked directly from equation (2), and is due to the polarization typical of the L.H. waves². Taking advantage: now of $\eta < 1$ $K\bar{z} = \psi$

the force model is rather drastically reduced to impulses³ occurring at intervals of T . Putting $\dot{\varphi} = I$, $\dot{\psi} = J$, the equations of motion can be reduced to the following finite difference scheme³

$$\begin{aligned} \bar{\varphi} &= \varphi + T \bar{I} \\ \bar{\psi} &= \psi + T \bar{J} \\ \bar{I} &= I - \Omega^2 T (\bar{\varphi} + \varphi) - \eta v^2 T \sin \varphi \\ \bar{J} &= J - \eta \omega_B^2 T \sin \varphi \end{aligned} \quad (4)$$

which is an area preserving mapping of the old variables φ, ψ, I, J into the "new" variables. Since in our problem $\Omega/v \ll 1$ we consider now the reduced form of (4)

$$\begin{aligned} \bar{\varphi} &= \varphi + T \bar{I} \\ \bar{I} &= I - \eta v^2 T \sin \varphi \end{aligned}$$

which is well known to represent a fundamental model of stochasticity⁴. It is known⁴ that initially neighboring phases evolve as $\Delta \varphi_n = K^n \Delta \varphi_0$, with $K = \eta v^2 T^2$ if $K > 1$ while if $K < 1$ the motion is stable. The condition $K > 1$ is easily met in the present case, since it requires that the "trapping time" $1/v$ related to the field amplitude, be less

Than the geometric mean of the time of revolution T around the torus, and the time t_0 of traversal of the field region. Since φ behaves as a random variable, because of the properties of (4) the φ averaged single particle transverse energy increment can be evaluated as $\langle \Delta W_{\perp} \rangle_{\varphi} = \frac{1}{2} \eta^2 T^2 \left(\frac{e E_{\perp}}{m} \right)^2 N_0$ which is proportional to the number N_0 of transits through the force layer. In fig. 2-5 the evolution of the phase space (φ, I) is shown as the perturbation strength increases, clearly showing the onset of stochastic behavior. For a distribution function $f(\varphi, I, t)$, a diffusion equation can be written with a diffusion tensor built from the mean square velocity steps, and the time of "collision" with the field.

$$D_{\parallel} = \frac{\langle (\Delta v_{\parallel})^2 \rangle}{\Delta t} = \frac{k T}{2 \Delta t} \frac{\omega_p^4}{k^2} ; \quad D_{\perp} \approx \frac{\epsilon_{33}}{\epsilon_{11}} D_{\parallel} \quad \Delta t = 2\pi R / v_z$$

The perpendicular heating rate turns out to be $\dot{W}_{\perp} = \frac{\omega_p^4 t_0^2 \epsilon_{33} \bar{v}_{\parallel}}{2 \epsilon_{11} k^2 R}$, proportional to $\bar{v}_{\parallel}$ thereby accounting for the observed interaction between beam and R.F., and not between R.F. and background ions. The evolution of the average parallel energy turns out to be $\bar{v}_{\parallel}^2 = (t_0^2 \omega_p^4 / k^2 R)^2 t^2$ as in ref. 5. The flux of particles collected by a system of energy analyzers monitoring the beam characteristics can be evaluated from $n(t) = n_0 \exp(-v_{\text{analyzer}}^2 / 2 D_{\perp} t)$, and this is in good qualitative agreement with the experimental signal. The numerical results of the integration of the full equations of motion are shown in fig. 6; for various values of $E_{\perp}$ the perpendicular energy in KeV is plotted versus number of traversals, occurring at intervals $T \sim 3.45 \times 10^{-6}$ sec, with ATC data. The author is grateful to W. Hooke, S. Bernabei, R. Motley for useful discussions: References.

¹ W. Hooke, S. Bernabei, R. Motley, private communication (1977)

² P. Bellan, M. Porkolab, Phys. Fluids 17, 1592 (1974)

³ E. Lazzaro Proc. Workshop on Theory of magnetically confined plasmas- Varenna 1977 edit. R. Pozzoli, T. E. Stringer, G. Leotta, Commiss. Europ. Community-Luxembourg.

⁴ G. M. Zaslavskii, B. V. Filonenko, Sov. Phys. Usp. 14 549 (1972)

⁵ A. I. Akhiezer, A. S. Bakai, Fiz. Plasmii 2 654 (1976)

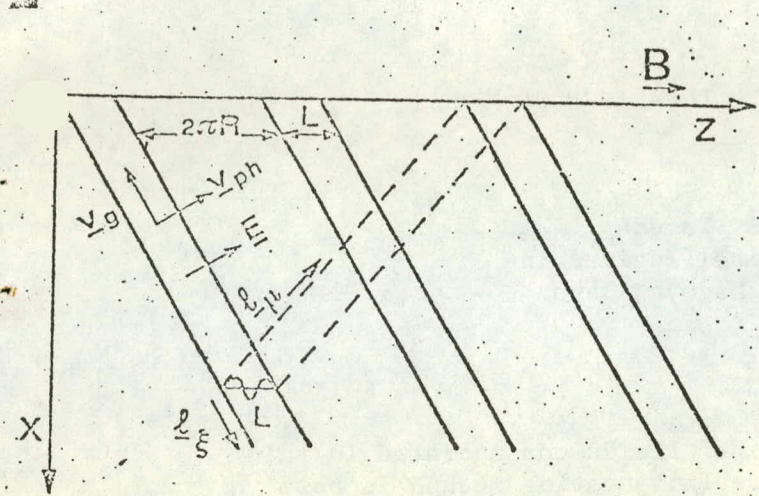


FIG. 1

$\alpha = 10^{-4}$
 $\beta = 10$

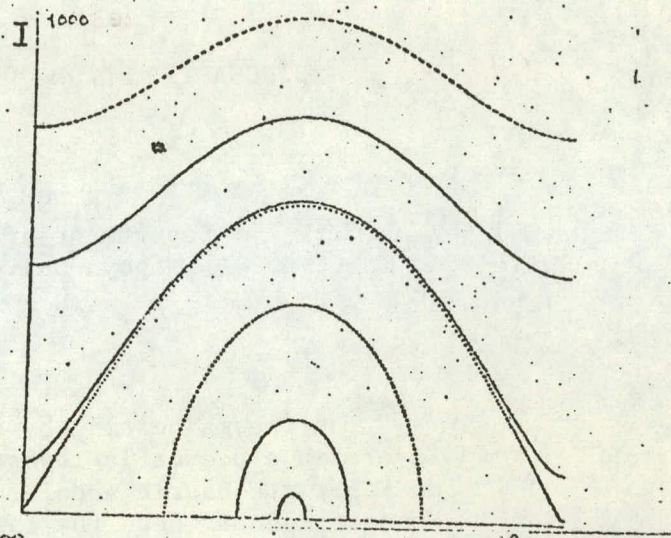


FIG. 2

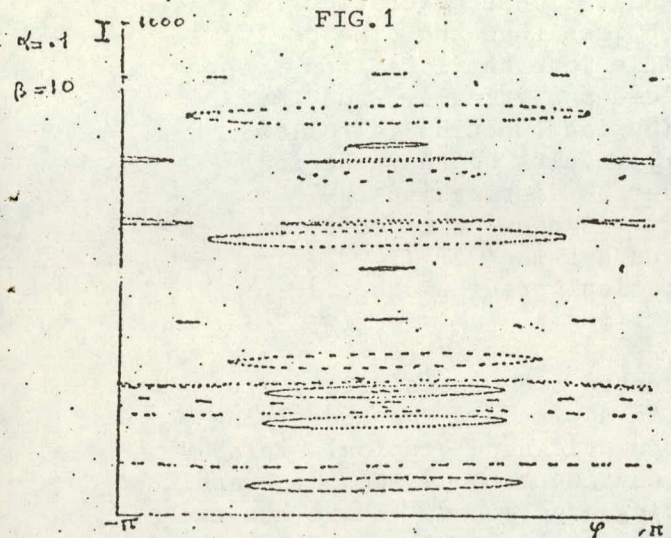


FIG. 3

$\alpha = 0.2$
 $\beta = 10$

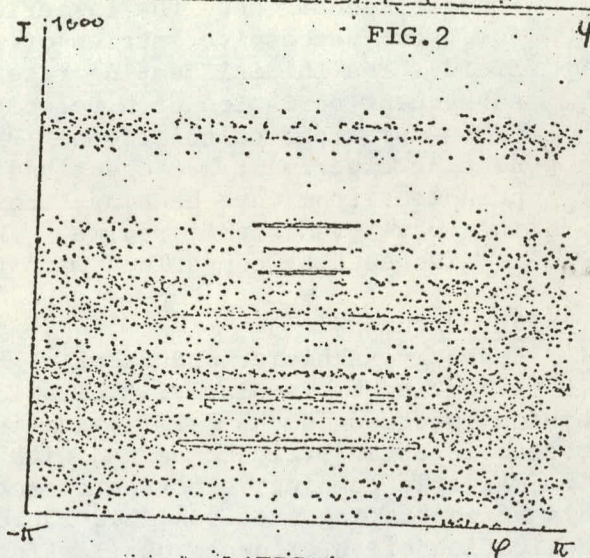


FIG. 4

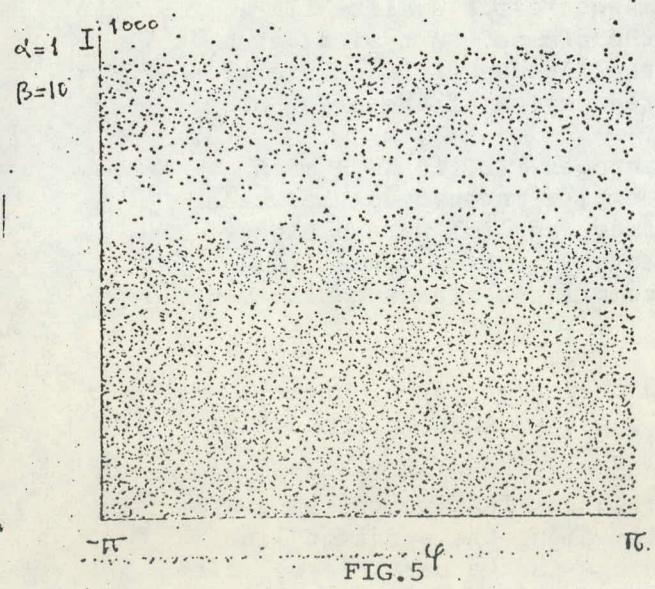


FIG. 5

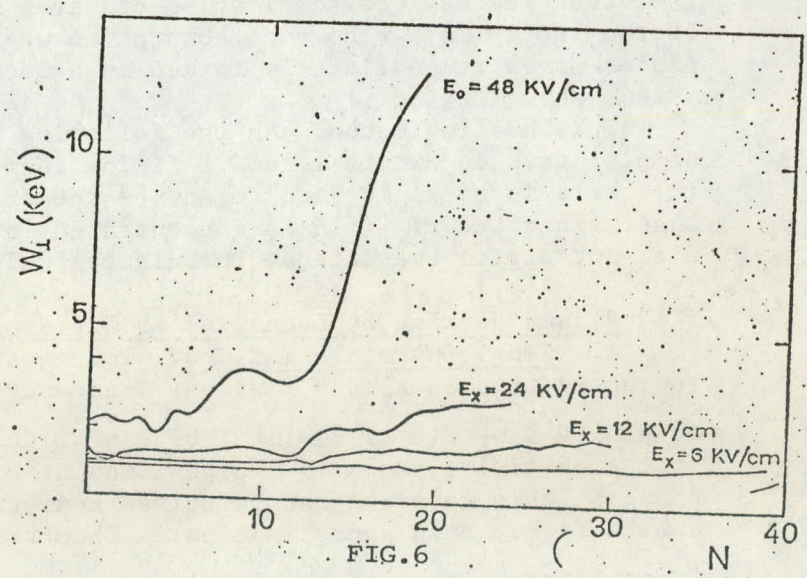


FIG. 6

STOCHASTIC PLASMA HEATING BY LOCALIZED RF FIELDS

by

H. Sugai and S. Takeda
 Department of Electrical Engineering
 Nagoya University, Nagoya, Japan

ABSTRACT

The plasma heating by coherent rf fields concentrated in thin layers in a plasma is investigated. This heating method is based on Fermi stochastic acceleration; the transit-time acceleration occurs randomly when the rf period is much less than the time between two successive entries of the particle into the localized field. The initial heating rate is provided analytically, and the subsequent variation of a velocity distribution function is followed by numerical calculations. Preliminary results of the electron heating experiment are described. In order to distinguish the phenomena from wave heating, the frequency has been selected to be in a wave evanescence regime. The observed heating ($\Delta T_e/T_e = 2-7$) is discussed by comparison with the theoretical predictions.

I. Introduction

Recently, Akhiezer and Bakai^{1,2} have discussed a new possibility of plasma heating by regular rf fields localized in a plasma. The underlying physical process is the Fermi mechanism of stochastic acceleration. This process is extensively concerned with various heating methods such as transit time magnetic pumping, electron cycrotron heating, and so on. There are two ways of generating the localized fields in the plasma. One is the local application of an external oscillating electromagnetic field, like TTMP. The other is the local excitation of waves in the plasma. A typical example of the latter is a resonance absorption where the intense field of electron plasma waves is nonlinearly formed in a localized region of the order of a few Debye lengths.

This heating method has the following advantages: (1) it is a much simpler task to excite local rf fields than to excite intense volume waves, (2) there is no rigid requirement on the frequency, and (3) we can expect that, since the rf field is concentrated in a certain region, this field will not excite the intense turbulences which complicate plasma confinement.

II. Plasma Heating by Localized RF Fields

A. Single-particle motions. The equation of motion for a particle subjected to a localized field of frequency ω , phase θ , and amplitude E is $m d^2x(t)/dt^2 = qE(x(t))\cos(\omega t + \theta)$, in which q and m are the charge and mass of the particle, and x is its position at time t . For fast particles and/or small E , this equation can be solved analytically along the straight line orbits (i.e., Born approximation)³. Then the change in the particle velocity in an interaction with the localized field is given by $\Delta v = (q/m) \int_{-\infty}^{\infty} dt E(vt)$

$\cos(\omega t + \theta)$. The scaled phase-averaged energy change, $\langle m(\Delta v)^2 \rangle / (mv_E^2/2)$, is found to be $(\gamma^2/2)\exp(-\gamma^2)$ for a Gaussian field of width d ; and to be $\gamma^2 / 2(1 + \gamma^2)^2$ for a skin field $E(x) = (E_0/2) \exp(-|x|/d)$, where $v_E = qE_0/m\omega$ and $\gamma = \omega d/v$. It should be noted that, for both types of the localized field, the energy change becomes maximum at $\gamma=1$, i.e., at the rf period equal to the transit time.

B. Stochastic heating. We assume that the charged particles, moving in a confinement system of length L , from time to time intersect the localized field of width d . When the time interval between two successive transit through the field, $\Delta t = L/v$, is much less than $2\pi/\omega$, the particles are accelerated at random phase; the Fermi stochastic acceleration acts.^{1,2} In this case, the evolution of the velocity distribution function f can be described by a Fokker-Planck equation of the form

$$\frac{\partial f}{\partial t} = \frac{1}{2} \frac{\partial^2}{\partial v^2} \left(\left[\frac{\langle (\Delta v)^2 \rangle}{\Delta t} \right] f \right), \quad (1)$$

where Δv is the velocity change over the time interval Δt , and the angle brackets denote a phase average. Multiplying Eq. (1) by v^2 and integrating over v with a Maxwellian distribution as the initial condition, we find the heating rate:

$$(\partial T / \partial t) / T \Big|_{t=0} = (v_E / \bar{v})^2 (\bar{v} / L) h(\bar{\gamma}), \quad (2)$$

where $\bar{v} = (kT/m)^{1/2}$, $\bar{\gamma} = \omega d / \bar{v}$, and $h(\bar{\gamma}) = (4\bar{\gamma}^4 / \sqrt{2\pi}) \int_0^\infty [u^3 / (u^2 + \bar{\gamma}^2)^3] \exp(-u^2/2) du$. As seen in Fig. 1, the heating factor $h(\bar{\gamma})$ increases by $\bar{\gamma}^4$ for $\bar{\gamma} \ll 1$ and reaches its maximum for $\bar{\gamma} \approx 1$, and it decreases by $1/\bar{\gamma}^2$ for $\bar{\gamma} \gg 1$.

C. Time evolution of velocity distribution function.

We proceed to solve Eq. (1) numerically with a Maxwell distribution as the initial condition. Figure 2 displays the distribution function for $\bar{\gamma} = 1$, with a scaled time $\tau = (1/8)(v_E / \bar{v})^2 (\bar{v} / L) t$ as a parameter. It is evidently seen from Fig. 2 that the localized fields efficiently accelerate some particles having $v/\bar{v} \approx \bar{\gamma}$. The corresponding increase of the temperature defined by $kT = \int_{-\infty}^{\infty} mv^2 f dv$ is plotted as a function of the time. The initial heating rate well agrees with the value given by Eq. (2).

Akhiezer and Bakai² showed in the rough estimation that the temperature increases in proportion to the square of the time. In our exact numerical calculation, however, the temperature exhibits an essentially linear time dependence, probably due to the rapid deformation of the distribution function.

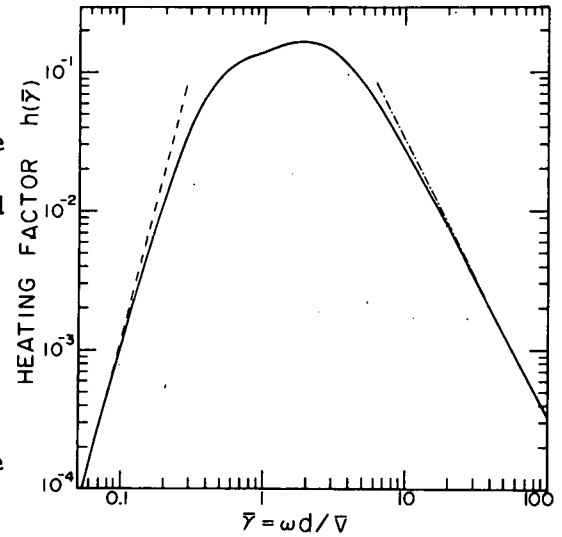


Fig. 1

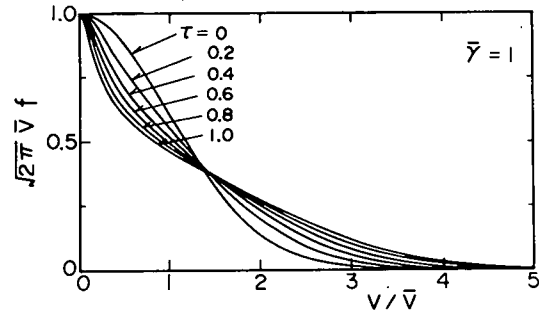


Fig. 2

The next section describes a basic model experiment in which local rf field gives rise to the electron heating.

III. Experiment on Electron Heating

Figure 3 shows a diagram of the experimental apparatus and method. A plasma is produced in argon at a pressure of 3×10^{-4} Torr by a pulsed discharge between an oxide coated cathode and a mesh anode. The experiment is performed in the initial phase of the afterglow plasma with the following parameters: plasma density $N = 3 \times 10^{11} \text{ cm}^{-3}$, electron temperature $kT_e = 1.5 \text{ eV}$, ion temperature $kT_i = 0.2 \text{ eV}$, plasma length $L = 40 \text{ cm}$, plasma diameter = 10 cm, axial magnetic field = 400 G. The rf power ($P \leq 50 \text{ W}$) at $\omega/2\pi = 800 \text{ MHz}$ is applied, in the pulsed mode, to a linear electric dipole (3.5 cm in length) located in the center of the plasma column.

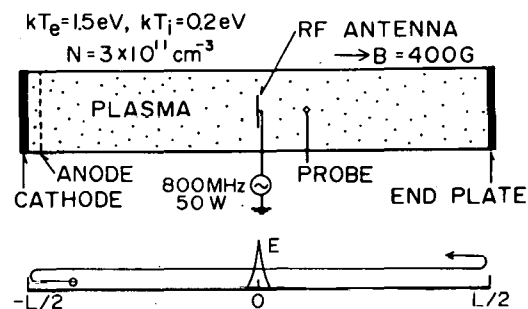


Fig. 3

In order to distinguish the phenomena from wave heating, the frequency has been selected to be in a wave evanescence regime, $\omega^2 \gg \omega_{pe}^2 > \omega_{ce}^2$, where ω_{pe} and ω_{ce} are the electron plasma and the electron cyclotron frequencies, respectively. The actual rf field in the plasma has been found to localize in the region of a few mm around the antenna. The electron temperature is measured with a Langmuir probe method; the measurement is performed at the position far from the antenna so as to prevent it from errors due to the intense localized field at the antenna.

Since $\omega \gg \nu_{ei}$ (collision frequency) and $L \lesssim \lambda_{ei}$ (mean free path), the electron can be regarded collisionless. The electrons accelerated by the localized field are reflected back at the ends with ambipolar potential sheath. The period $2\pi/\omega$ is much less than the time interval $L/\bar{v}_e$ of "collisions" with the localized field, so that the Fermi stochastic acceleration is undertaken.

The efficient electron heating is observed as shown in Fig. 4, where the normalized temperature increment $\Delta T_e/T_e$ is plotted as a function of the power P for different times t after turn-on of the rf signal. It is found in the figure that the temperature rise is almost proportional to the power for $\Delta T_e/T_e < 1$. The deviation from its linear dependence is attributed to the decrease of the plasma density accompanied with the heating.

The electron temperature increases linearly with time until $t \approx 10 \mu\text{sec}$, and then it begins to saturate in time.

We measured the initial heating rate for $t < 10 \mu\text{sec}$. Figure 5 shows the heating time, $\tau_H \equiv [(\partial T_e / \partial t) / T_e]^{-1}$ as a function of the power P .

It is clearly seen that the heating time is inversely proportional to P , except for the high power

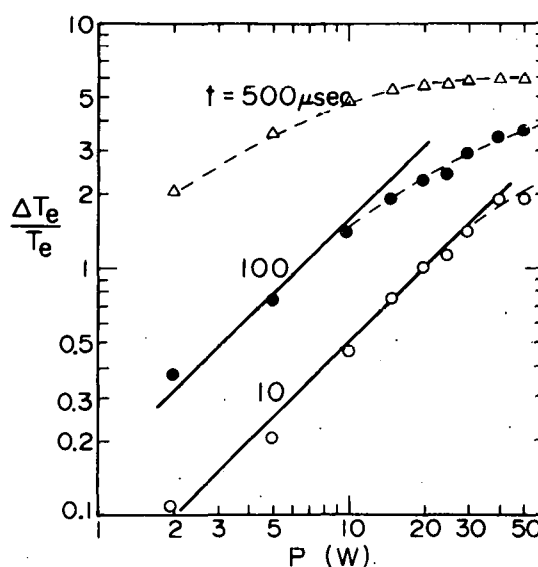


Fig. 4

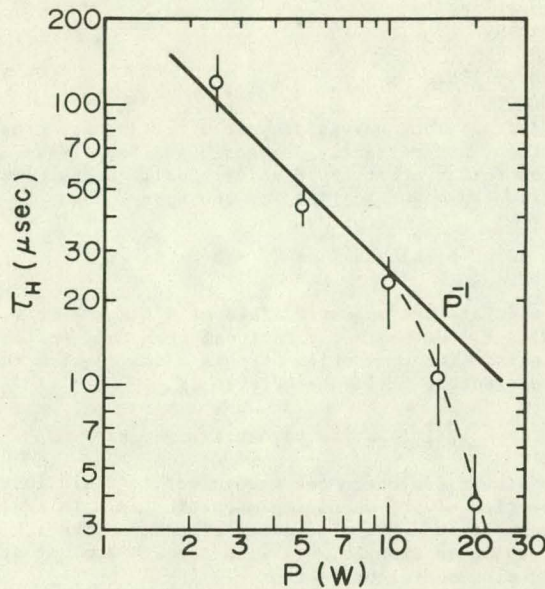


Fig. 5

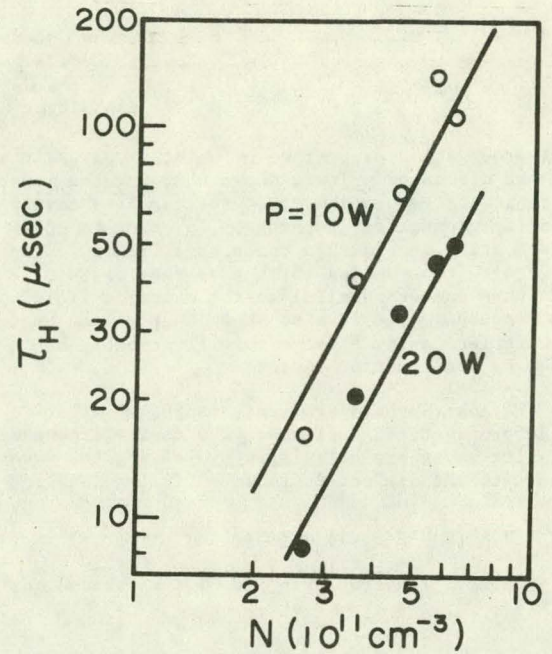


Fig. 6

region where the plasma density modification occurs.

Figure 6 shows the heating time τ_H as a function of the plasma density N at different powers. The solid lines in the figure indicate the slope for $\tau_H \propto N^2$.

IV. Comparison to Theories

The antenna near-zone field consists of both the radiation and the induction fields. In the wave evanescence regime, the radiation field is shielded in skin depth c/ω , while the static potential of the induction field is screened in Debye^{pe} length λ_D . In the experiments^{4,5} almost the same magnitude of the electron heating has been observed in the region of whistler wave propagation ($\omega^2 < \omega_{ce}^2 < \omega_{pe}^2$). This means that the origin of the heating is not the wave radiation field but the induction field. Therefore, the scale length of the localized field is determined by $d \approx \lambda_D$, and then $\bar{\gamma} \approx \omega \lambda_D / v_e < 1$. From Eq. (2), we find the corresponding heating time:

$$\tau_H \approx 0.081 (L/v_e) (\bar{\gamma}_e/v_e)^2 (\omega_{pe}/\omega)^4 \propto P^{-1} N^2 T_e^{1/2} L \omega^{-4}.$$

The predicted dependence of τ_H on P and N is clearly demonstrated in the experiment, as shown in Figs. 5 and 6. Also, the dependence on L , T_e , and ω is recognized to agree qualitatively with the theories. It is difficult to measure the absolute amplitude of the rf field. According to Eq. (2), however, we can estimate $v_e \approx \bar{\gamma}_e$ at $\tau_H \approx 20 \mu\text{sec}$ for the parameters in our experiment.

References

1. A. I. Akhiezer and A. S. Bakai, Sov. Phys.-Doklady **16**, 1065(1972).
2. A. I. Akhiezer and A. S. Bakai, Sov. J. Plasma Phys. **2**, 359(1976).
3. G. J. Morales and Y. C. Lee, Phys. Rev. Letters **33**, 1534(1974).
4. R. L. Stenzel, Phys. Fluids **19**, 865(1976).
5. H. Sugai, M. Maruyama, M. Sato, and S. Takeda, to be published.

WHISTLER CAVITY EIGENMODES IN TOKAMAKS*

Cheng Chu and J. L. Sperling
General Atomic Company
P. O. Box 81608
San Diego, California 92138

The propagation properties of whistler waves in a tokamak are discussed. It is shown that for the lower-hybrid range of frequencies, the wave can be a cavity eigenmode in present day tokamaks. In order to optimize the heating deep within the plasma, it is desirable to excite those modes which have the largest possible wave number parallel to the magnetic field for a given frequency. It is also shown that plasma heating by whistler cavity eigenmodes will probably be dominated by nonlinear processes.

In the lower hybrid frequency range, whistler waves can propagate in a plasma. The cavity eigenmodes for whistler waves are calculated by solving the wave equation with the dielectric tensor $\vec{\kappa}(\vec{r})$

$$\nabla \times [\nabla \times \vec{E}(\vec{r}, t)] = (\omega^2/c^2) \vec{\kappa} \cdot \vec{E}(\vec{r}, t) \quad (1)$$

in the geometry illustrated in Fig. 1. We assume an



infinitely long waveguide with a rectangular cross section, and perfectly conducting walls. Inside, we allow for an arbitrary density profile and a nonuniform z-directed magnetic field of the form

$$B_z(x) = B_0 [1 + (x - a/2)/R]^{-1} \quad (2)$$

to simulate the magnetic field of a tokamak of major radius R. We neglect rotational transform and shear effects. The symmetries of this model require that the wave electric fields be written as

$$\vec{E}(\vec{r}, t) = \vec{E}(x, y) \exp[i(kz - \omega t)] \quad (3)$$

For situations where the wave electric field in the direction of the confining magnetic field is small, the eigenmodes and the associated eigenwavenumbers k, can be evaluated from Eq. (1) with the 2 x 2 minor of the cold plasma dielectric tensor [1]

$$\vec{\kappa} = \begin{pmatrix} S & -iD \\ iD & S \end{pmatrix} \quad (4)$$

$$\text{where } S = 1 - \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\omega^2 - \Omega_{\alpha}^2} \quad (5a)$$

$$\text{and } D = \sum_{\alpha} \frac{\Omega_{\alpha}}{\omega} \frac{\omega_{p\alpha}^2}{\omega^2 - \Omega_{\alpha}^2}, \quad (5b)$$

and $\omega_{p\alpha}$ and $\Omega_{\alpha}(\vec{r})$ are the local plasma frequency and cyclotron frequency for species α , respectively.

For a given frequency, the numerical solution of the wave equation leads to a number of eigenmodes with different values of the parallel wavenumber k. A general trend of the eigenmode structures is for the wave fields to be less spread out over the plasma profile and to become more concentrated in the high density regions of the plasma as k increases. As a typical example, the eigenmode with the largest value of k, the "cut-off mode," at a frequency of 800 MHz is shown in Figs. (2a-b) for a doublet profile with plasma parameters similar to those of the Doublet IIA tokamak at General Atomic[2]. Upon neglecting the ellipticity of of the mode structure shown in Fig. (2a), it is apparent that the dominant right-hand circularly polarized component of the wave electric field $E_R \equiv E_x - iE_y$ for the "cut-off mode" has little poloidal variation about its peak value. Correspondingly, the wave magnetic field in the z-direction has an m = 1 variation [e.g. $B_z \sim \exp(im\theta)$, m = 1]. These features of the mode structure agree well with the analytic solution of the differential equation

$$\nabla_{\perp}^2 E_R + 2 [(\omega^2/c^2)(S + D) - k^2] E_R = 0 \quad (6)$$

which is valid for modes near the cutoff. Furthermore, examination of Figs. (2a-b) suggests that the maximum value of E_R is shifted away from the peak density towards the low magnetic field strength side of the

*Work supported in part by Department of Energy, Contract No. EY-76-C-03-0167, Project Agreement 38.

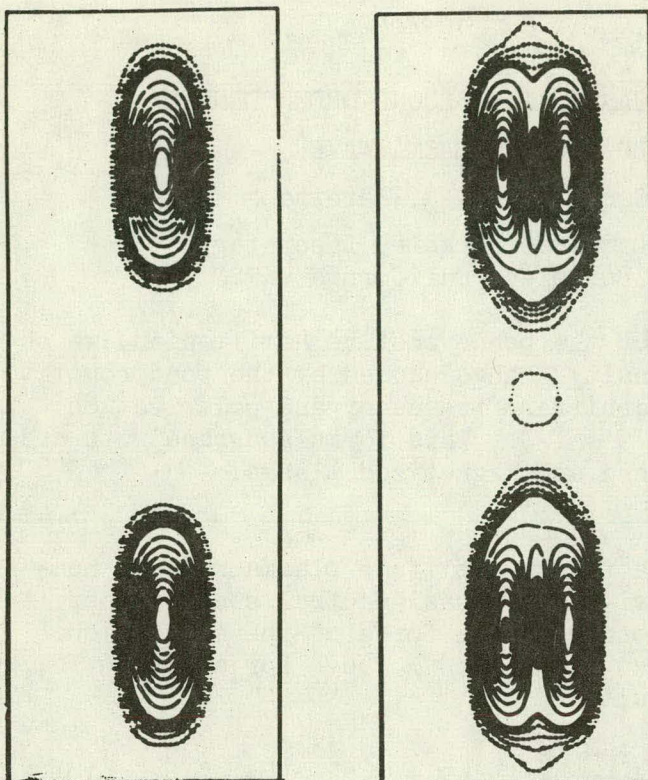


Fig. 2. Contours for $|E_R|^2$ and $|B_z|^2$ are plotted in Fig. 2(a) and (b) respectively. The solid and dotted lines represent contours of constant $|E_R|^2$ and $|B_z|^2$. The separation of solid and dotted lines denote 10% and 2% variations in the square of the field amplitudes, respectively. The parameters are $\omega = 2\pi \times 800$ MHz, $kc/\omega = 11.56$, $B_0 = 10^4$ gauss, $\bar{n} \approx 3.3 \times 10^{13} \text{ cm}^{-3}$, $R = 66$ cm, $a = 35$ cm, and $b = 104$ cm.

plasma. This point is also verified by the analytic solution of Eq. (6) which indicates that the maximum of E_R is located at the place where (n/B) peaks.

The phase speed of the cavity whistler modes in the direction of the confining field has a minimum value for the cut-off mode. The value for this minimum phase speed is given approximately by the expression

$$\frac{\omega}{k} = v_{A_0} \left(1 + \frac{\omega}{\Omega_1} \right)^{1/2} \left(1 - \frac{\omega}{|\Omega_e|} \right)^{1/2} \quad (7)$$

where v_{A_0} is the Alfvén speed evaluated at the point where (n/B) is a maximum. In general, this speed is larger than the electron thermal speed, and so the linear damping of the whistler wave is expected to be weak. Our computer calculations show that this is indeed true. Hence, in view of the weak linear absorption and the large electric field strength, nonlinear

processes can be expected to play a crucial role in the heating of plasma by whistlers. Since the electric fields are strongest in the interior of the plasma, heating via nonlinear processes should occur deep within the plasma.

In addition to being able to have relatively low field amplitudes in the outer regions of the plasma, the whistler cavity modes have electric field components which primarily lie orthogonal to the magnetic field. Therefore, as compared to heating by lower-hybrid waves, for example, it is less likely that whistlers will excite parametric instabilities in the low density portions of plasma. A final significant practical advantage for whistler cavity mode heating is that the excitation of cavity eigenmodes requires lower voltages and currents in the launching structures than does the launching of strongly damped modes like lower hybrid waves[3]. Optimal launching of short parallel wavelength whistler cavity modes may require a slow wave structure in which the rf electric field is oriented in the poloidal direction while the components of the rf magnetic field are in the density gradient and the confining magnetic field (see Fig. 3) directions.

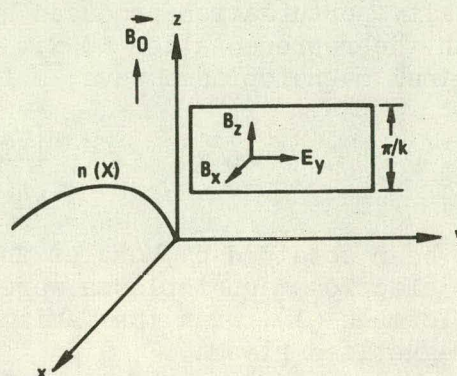


Fig. 3. Diagram of the wave field orientation at the launching structure for short parallel wavelength whistler cavity modes. The length of the structure in the z -direction is π/k . The wave magnetic field has components in the confining magnetic field (z -) and the density gradient (x -) directions. There is a component of the wave electric field in the poloidal (y -) direction.

References

- [1] Stix, T. H., *The Theory of Plasma Waves*, McGraw-Hill, New York (1962) Ch. 1.
- [2] Fisher, R. K., et al., General Atomic Report GA-A14146.
- [3] Stix, T. H., *Nucl. Fusion* **15** (1975) 737.

A UNIVERSAL FORMULA FOR THE QUASISTATIC SECOND-ORDER DENSITY PERTURBATION BY A COLD MAGNETOPLASMA WAVE*

A. N. Kaufman, J. R. Cary, and N. R. Pereira

Department of Physics and Lawrence Berkeley Laboratory
University of California, Berkeley, California 94720

Using the general expression for the ponderomotive Hamiltonian, we obtain the quasi-static quasi-neutral density change caused by the ponderomotive force of a cold magnetoplasma wave of arbitrary frequency and polarization: $\delta n(\underline{x}) = -(4\pi)^{-1} [|\underline{E}(\underline{x})|^2 - |\underline{B}(\underline{x})|^2] / (T_e + T_i)$. This formula agrees with and extends previous results for unmagnetized and magnetized plasma.

In studying the modulation of a finite-amplitude plasma wave, a number of authors have calculated the quasi-static quasi-neutral second-order density perturbation produced by the ponderomotive force of the modulation. With the representation $\phi(\underline{x}, t) = \phi(\underline{x}) \exp(-i\omega t) + \text{c.c.}$ for a longitudinal magnetoplasma wave, the result¹

$$\delta n(\underline{x}) = - \frac{|\nabla \phi(\underline{x})|^2}{4\pi(T_e + T_i)} \quad (1)$$

has been obtained by Morales and Lee² for lower-hybrid waves, and by Shukla³ for electron magnetoplasma waves. The former authors remarked on the identity of formula (1) with the familiar expression for Langmuir wave modulation in unmagnetized plasma.

It is natural to inquire into the universality of formula (1). In this paper, we show that it does indeed apply to any longitudinal cold-plasma wave (for a single ion species⁴); i. e., the three solutions⁵ $\omega(\theta)$ of $\epsilon_L(\omega, \theta) = 0$, where $\epsilon_L = \underline{k} \cdot \underline{\epsilon}(\omega) \cdot \underline{k}$.

More importantly, we show that formula (1) can be simply generalized to apply to a cold plasma wave of any polarization, i. e., to a wave with non-zero $\nabla \times \underline{E}$. Here we use a local plane-wave⁶ representation $\underline{E}(\underline{x}, t) = \underline{E}(\underline{x}) \exp(i\underline{k} \cdot \underline{x} - i\omega t) + \text{c. c.}$, with $\underline{B}(\underline{x}) = (c/\omega) \underline{k} \times \underline{E}$. The generalization, derived below, is

$$\delta n(\underline{x}) = - \frac{|\underline{E}(\underline{x})|^2 - |\underline{B}(\underline{x})|^2}{4\pi(T_e + T_i)} \quad (2)$$

We note first that it reduces to (1) when $\underline{B} = 0$. Secondly, for the transverse unmagnetized case, where $|\underline{B}|^2 = (kc/\omega)^2 |\underline{E}|^2 = (1 - \omega_p^2/\omega^2) |\underline{E}|^2$, formula (2) becomes $\delta n/n = - (e^2/m\omega^2) |\underline{E}|^2 / (T_e + T_i)$, the familiar result⁷.

Formula (2) can be used for any cold-magnetoplasma wave, e. g., lower hybrid in the electromagnetic region⁸, fast-magnetosonic-whistler⁹, Alfvén¹⁰, ordinary and extraordinary, etc., so long as $\underline{B} = (c/\omega) \underline{k} \times \underline{E}$ is a valid approximation (When it is not, use formula (7) below.)

Our derivation begins with the standard expression¹¹ for the quasi-static density perturbation, of species s , caused by the ponderomotive potential energy $\Psi_s(\underline{x})$ of an oscillation center¹² and by the self-consistent electric potential $\Phi(\underline{x})$:

$$\frac{\delta n_s(\underline{x})}{n_s^0} = - \frac{\Psi_s(\underline{x}) + e_s \Phi(\underline{x})}{T_s} \quad (3)$$

For two species (electrons and singly-charged ions), we impose quasi-neutrality ($\delta n_e = \delta n_i$, $n_e^0 = n_i^0$) to eliminate Φ , and obtain the relation

$$\frac{\delta n(\underline{x})}{n^0} = \frac{\Psi_e(\underline{x}) + \Psi_i(\underline{x})}{T_e + T_i} \quad (4)$$

Our expression for $\Psi_s(\underline{x})$ is based on a useful relation¹³ for the ponderomotive Hamiltonian¹⁴ of an oscillation center. In the cold-species limit, Eq. (3) of Ref. (13) reduces to

$$n_s(\underline{x}) \Psi_s(\underline{x}) = -(4\pi)^{-1} \underline{E}^*(\underline{x}) \cdot \underline{\chi}_\omega^s(\underline{x}) \cdot \underline{E}(\underline{x}), \quad (5)$$

with the representation $\underline{E}(\underline{x}, t) \equiv \underline{E}(\underline{x}) \exp(-i\omega t) + \text{c. c.}$, where $\underline{\chi}_\omega$ is the well-known¹⁵ cold-species susceptibility. (We note that $\underline{\chi}$ is proportional to density, so that Ψ is density-independent; but the dependence of $\underline{\chi}$ on possibly nonuniform magnetic field $\underline{B}_0(\underline{x})$ appears in Ψ .)

Inserting (5) into (4), we have

$$\delta n(\underline{x}) = \frac{\underline{E}^*(\underline{x}) \cdot (\underline{\chi}_\omega^e + \underline{\chi}_\omega^i) \cdot \underline{E}(\underline{x})}{4\pi(T_e + T_i)} \quad (6)$$

Now we use the field equation $(\underline{\chi}_\omega^e + \underline{\chi}_\omega^i) \cdot \underline{E}(\underline{x}) = -\underline{E}(\underline{x}) + (ic/\omega) \nabla \times \underline{B}(\underline{x})$, where $\underline{B}(\underline{x}) = (c/i\omega) \nabla \times \underline{E}(\underline{x})$, to obtain

$$\delta n(\underline{x}) = - \frac{|\underline{E}(\underline{x})|^2 - |\underline{B}(\underline{x})|^2 - (c/\omega) \text{Im } \nabla \cdot \underline{E}^*(\underline{x}) \times \underline{B}(\underline{x})}{4\pi(T_e + T_i)} \quad (7)$$

Finally, for a local plane wave, with $\underline{E}(\underline{x}) \equiv \underline{\tilde{E}}(\underline{x}) \exp ik \cdot \underline{x}$ and $\underline{B} = (c/\omega) k \times \underline{\tilde{E}}$, one may drop the complex Poynting term in (7), as higher order in $k \nabla \ln \tilde{E}$; the result is then Eq. (2).

Two points should be kept in mind in applying (4): second-order magnetic perturbations may be of significance¹⁶; and the quasi-static assumption may be invalid.¹⁷

Footnotes and References

- * Supported by the U. S. Department of Energy.
- 1. The numerical factor in the denominator is sometimes given incorrectly as 8π . If one uses $\phi(\underline{x}, t) = \text{Re } \phi(\underline{x}) \exp(-i\omega t)$, the factor should be 16π .
- 2. G. J. Morales and Y. C. Lee, Phys. Rev. Lett. 35, 930 (1975).
- 3. P. K. Shukla, J. Plasma Phys. 18, 249 (1977).
- 4. For more than one ion species, formulas (1) and (2) generalize to less beautiful forms.
- 5. We note that for the lowest-frequency solution (ion-cyclotron wave), the cold plasma model may be invalid.
- 6. More correctly, $\exp i\mathbf{k} \cdot \mathbf{x} \rightarrow \exp i\theta(\mathbf{x})$, with $\mathbf{k}(\mathbf{x}) = \nabla\theta$.
- 7. P. Kaw, G. Schmidt, and T. Wilcox, Phys. Fluids 16, 1522 (1973).
- 8. G. J. Morales, Phys. Fluids 20, 1164 (1977).
- 9. R. L. Berger and F. W. Perkins, Bull. APS Oct. 1977; K. H. Spatschek, M. Y. Yu, and P. K. Shukla, J. Geophys. Res. 81, 1413 (1976).
- 10. J. A. Tataronis and W. Grossmann, Nuc. Fusion 16, 667 (1976); D. J. Kaup and A. C. Newell, Bull. APS Oct. 1977.
- 11. In a Vlasov treatment, T_s represents the effective temperature of a velocity distribution parallel to the magnetic field, if the wave is localized. For a cavity mode, the temperature is not an adiabatic invariant.
- 12. G. Schmidt, Physics of High Temperature Plasmas, Sec. 2-9.
- 13. J. R. Cary and A. N. Kaufman, Phys. Rev. Lett. 39, 402 (1977).
- 14. R. Dewar, Phys. Fluids 16, 1102 (1973); S. Johnston, Phys. Fluids 19, 93 (1976).
- 15. T. Stix, The Theory of Plasma Waves, Sec. 1-2.
- 16. A. N. Kaufman and L. Stenflo, Phys. Scripta 11, 269 (1975); M. Porkolab and M. V. Goldman, Phys. Fluids 19, 879 (1976).
- 17. N. R. Pereira, R. N. Sudan and J. Denavit, Phys. Fluids 20, 271 (1977).

Ponderomotive Effects in a Magnetized Plasma*

by

V. Krapchev and A. Bers

Plasma Fusion Center

Massachusetts Institute of Technology, Cambridge, MA 02139

We derive the selfconsistent nonlinear electromagnetic equations for the electric fields in a cold plasma, immersed in a constant magnetic field and driven by an RF source. In the approximation of nonrelativistic particles and by neglecting the nonresonant diffusion term in the equation for the slowly varying part of the distribution function, one can write a closed set of equations for the electric fields.

The selfconsistent nonlinear Maxwell's equations are:

$$\bar{\nabla} \times (\bar{\nabla} \times \bar{E}) + \frac{1}{c^2} \frac{\partial^2 \bar{E}}{\partial t^2} = -\frac{4\pi}{c^2} \frac{\partial \bar{j}(E)}{\partial t} \quad (1)$$

where the electric field in the plasma $\bar{E} = \frac{1}{2} \bar{\mathcal{E}}(\bar{r}, t) e^{-i\omega t} + \text{c.c.}$ is driven by an external source of frequency $\Omega_i < \omega < \Omega_e$ and arbitrary polarization. $\bar{\mathcal{E}}(\bar{r}, t)$ has a slow time dependence. From the fluid picture we can write the current in the form:

$$\bar{j}(E) = q(n_i^S \bar{v}_i^F - n_e^F \bar{v}_e^S - n_e^S \bar{v}_e^F) \quad (2)$$

where i(e) denotes ions(electrons) and S(F) denotes slow(fast) time components. $\bar{v}_{i,e}^F = \frac{1}{2} \bar{v}_{i,e} e^{-i\omega t} + \text{c.c.}$ The fast components of the velocity are proportional to the electric field, and the slow components of the velocity are quadratic in the electric field. Ponderomotive effects on the ions are neglected and the only nonlinearity is due to charge separation. The electrons are subject both to ponderomotive effects and fields arising from charge separation. We shall show that for nonrelativistic electrons when the effect of the induced magnetic field is neglected $\bar{v}_e^S = 0$.

With these approximations we can write equation (1) in the form:

$$\bar{\nabla} \times (\bar{\nabla} \times \bar{\mathcal{E}}) - \frac{\omega^2}{c^2} \bar{\mathcal{E}} - \frac{2i\omega}{c^2} \frac{\partial \bar{\mathcal{E}}}{\partial t} = \frac{4\pi q}{c^2} i\omega (n_i^S \bar{v}_i - n_e^S \bar{v}_e) \quad (3)$$

The amplitudes of the fast components of the velocity are given by the well-known

*Work supported by U.S. Department of Energy (Contract EG-77-G-01-4107) and the National Science Foundation (Grant ENG77-00340)

cold plasma relation:

$$\bar{v}_{e,i} = \bar{\alpha}_{e,i} \bar{\mathcal{E}} \quad (4)$$

where $\bar{\alpha}_{e,i}$ is the usual mobility tensor.

The slow time densities are found from the steady state distribution function, satisfying the equation:

$$\frac{1}{m} (\bar{F}_{e,i} \cdot \bar{\nabla}_v) f_{e,i} + (\bar{v} \cdot \bar{\nabla}) f_{e,i} = 0 \quad (5)$$

Here we have neglected the nonresonant diffusion term, which leads to an apparent temperature due to nonlinear interaction with the waves.⁽¹⁾ For ions the force in (5) comes from the charge separation:

$$\bar{F}_i = -q \bar{\nabla} \phi \quad (6)$$

where ϕ satisfies the equation:

$$\nabla^2 \phi = -4\pi q (n_i^S - n_e^S) \quad (7)$$

The electrons are subjected to ponderomotive and charge separation forces.

$$\bar{F}_e = \bar{F}_p + q \bar{\nabla} \phi \quad (8)$$

The ponderomotive force $\bar{F}_p$ is found from the time average of the Lorentz force equation for a single electron over the cyclotron and driving frequencies.

$$\frac{d^2 \bar{r}(t)}{dt^2} = \frac{q}{m_e} \left(\frac{1}{2} \bar{\mathcal{E}} e^{-i\omega t} + \text{c.c.} + \frac{1}{c} \frac{d\bar{r}(t)}{dt} \times \bar{B}_0 + \frac{1}{2i\omega} \frac{d\bar{r}(t)}{dt} \times (\bar{\nabla} \times \bar{\mathcal{E}}) e^{-i\omega t} + \text{c.c.} \right) \quad (9)$$

The trajectory of the particle can be written as:

$$\bar{r}(t) = \bar{r}_0(t) + \bar{r}_\omega(t) + \bar{r}_c(t) \quad (10)$$

where $\bar{r}_G(t) \equiv \bar{r}_0(t) + \bar{r}_\omega(t)$ is the guiding centre of the particle, $\bar{r}_0(t)$ is the slow time motion, $\bar{r}_\omega(t)$ is the oscillation and $\bar{r}_c(t)$ is the cyclotron motion. We write the equation for the guiding centre and separate the fast time and slow time terms.⁽²⁾ With B_0 in the z direction and $\bar{r}_\omega(t) = \frac{1}{2} \bar{r}_\omega e^{-i\omega t} + \text{c.c.}$ we find:

$$r_{\omega x,y} = - \frac{q}{m_e (\Omega_e^2 - \omega^2)} \left(\bar{\mathcal{E}} + \frac{iq}{m_e c \omega} \bar{\mathcal{E}} \times \bar{B}_0 \right)_{x,y} \quad (11a)$$

$$r_{\omega z} = \frac{q}{m_e \omega^2} \mathcal{E}_z \quad (11b)$$

Notice that here we have neglected the terms arising from the cross-product of the slow-time velocity and the induced magnetic field:

$$\frac{1}{\omega} \frac{d\bar{r}_0}{dt} \times (\bar{\nabla} \times \bar{\mathcal{E}}) \ll \bar{\mathcal{E}}$$

The equation for the motion of the particle on the slow time scale is:

$$\frac{d^2 \bar{r}_0(t)}{dt^2} = -\frac{q}{m_e} \left\{ \frac{1}{4} (\bar{r}_0^* \bar{\nabla}) \bar{\mathcal{E}} + \text{c.c.} + \frac{1}{4} \bar{r}_0^* \times (\bar{\nabla} \times \bar{\mathcal{E}}) + \text{c.c.} + \frac{1}{c} \frac{d\bar{r}_0(t)}{dt} \times (\bar{B}_0 + O(\mathcal{E}^2)) \right\} \quad (12)$$

We substitute in (12) the results from (11a-b) and find:

$$\frac{d^2 \bar{r}_0(t)}{dt^2} \equiv \frac{\bar{F}_p}{m_e} = \frac{1}{m_e} (-\bar{\nabla} \phi_p - \frac{q}{c} \frac{d\bar{r}_0}{dt} \times \bar{B}_0) \quad (13)$$

where we have neglected: $O(\mathcal{E}^2) \ll B_0$. The ponderomotive potential ϕ_p is given by the well-known formula:^(3,4)

$$\phi_p = \frac{q^2}{4m_e} \left\{ \frac{1}{\omega^2} |\mathcal{E}|^2 - \frac{1}{\Omega_e^2 - \omega^2} (|\mathcal{E}_x|^2 + |\mathcal{E}_y|^2) - \frac{i\Omega_e}{\omega(\Omega_e^2 - \omega^2)} (\mathcal{E}_x^* \mathcal{E}_y - \mathcal{E}_y^* \mathcal{E}_x) \right\} \quad (14)$$

If the effect of the induced magnetic field is included in the fast components r_ω , the force on the r.h.s. of equation (12) can not be written as a gradient of a scalar potential.

We require that in the limit $\bar{\mathcal{E}} \rightarrow 0$ the slow-time distribution function is a Maxwellian. From equations (5,8,13) we find for the electrons:

$$f_e = A_e \exp\left(-\frac{1}{\kappa T_e} \left(\frac{m_e v^2}{2} + \phi_p - q\phi\right)\right) \quad (15)$$

where A_e is a normalization factor. Similarly from (5,6) we get for the ions:

$$f_i = A_i \exp\left(-\frac{1}{\kappa T_i} \left(\frac{m_i v^2}{2} + q\phi\right)\right) \quad (16)$$

Now we can easily find the slow time densities.

$$n_e^S = n_0 \exp\left(-\frac{1}{\kappa T_e} (\phi_p - q\phi)\right) \quad (17a)$$

$$n_i^S = n_0 \exp\left(-\frac{q\phi}{\kappa T_i}\right) \quad (17b)$$

n_0 is the unperturbed density distribution. With the results (17a-b) substituted in (7) we obtain one equation for the unknown quantities $\bar{\mathcal{E}}$, ϕ . The other three equations which complete the selfconsistent set of equations can be written from (3,5,17) in the form:

$$\bar{\nabla} \times (\bar{\nabla} \times \bar{\mathcal{E}}) - \frac{2i\omega}{c^2} \frac{\partial \bar{\mathcal{E}}}{\partial t} = \frac{\omega^2}{c^2} \bar{K} \bar{\mathcal{E}} \quad (18)$$

The nonlinear dielectric tensor has the following elements:

$$K_\perp = 1 - \frac{\tilde{\omega}_i^2}{\omega^2} + \frac{\tilde{\omega}_e^2}{\Omega_e^2}, \quad K_x = \frac{\tilde{\omega}_e^2}{\omega \Omega_e}, \quad K_\parallel = 1 - \frac{\tilde{\omega}_e^2}{\omega^2}$$

where $\tilde{\omega}_{e,i}^2 = \frac{4\pi q^2 n_{e,i}}{m_{e,i}}$ and the selfmodulated $n_{e,i}$ are given by (17a,b).

As an example we apply the nonlinear equations to the problem of coupling of RF

power to the slow wave at the edge of the plasma. We treat a steady state problem for a two dimensional model. At low densities $E_y, K_x \rightarrow 0, K_{\perp} \rightarrow 1$ and $\phi_p = \frac{q^2}{4m_e \omega^2} |\mathcal{E}_z|^2$.

Furthermore we assume charge neutrality:

$$n_e = n_i = n_0 \exp\left(-\frac{\phi_p}{\kappa(T_e + T_i)}\right) \quad (19)$$

In this approximation the problem is reduced to the equation:

$$\frac{\partial^2}{\partial x^2} \mathcal{E}_z + \left(\frac{\partial^2}{\partial z^2} + 1\right) K_{\parallel} \mathcal{E}_z = 0 \quad (20)$$

where we have introduced dimensionless variables.

This is a nonlinear Klein-Gordon equation which describes the electromagnetic coupling problem at low densities. For a weakly inhomogeneous plasma and small nonlinear effects the problem can be simplified to a well-known equation. We seek a solution for the electric field of the form:

$$\mathcal{E}_z = \mathcal{E}\left(x, \xi = z - \left(\frac{n_z^2}{n_z^2 - 1} \left(\frac{\omega_e^2}{\omega^2} - 1\right)\right)^{1/2} x\right) e^{-in_x x + in_z z} \quad (21)$$

where n_x, n_z satisfy the dispersion relation at low densities:

$$n_x^2 + (n_z^2 - 1)\left(1 - \frac{\omega_e^2}{\omega^2}\right) = 0 \quad (22)$$

From (20) we find the following nonlinear Schrödinger equation:

$$2in_x \frac{\partial \mathcal{E}}{\partial x} + \left(\frac{\omega_e^2}{\omega^2} - 1\right) \frac{\partial^2 \mathcal{E}}{\partial \xi^2} + \frac{q^2}{4m_e \omega^2 \kappa(T_e + T_i)} (n_z^2 - 1) \frac{\omega_e^2}{\omega^2} |\mathcal{E}|^2 \mathcal{E} = 0 \quad (23)$$

which describes the evolution of solitary waves along the resonance cone.

References

- [1] R. E. Aamodt, M. C. Vella, Phys. Rev. Lett. 39, 1273 (1977).
- [2] A. I. Morozov, L. S. Solov'ev, Reviews of Plasma Phys. V. 2, Consultants Bureau, New York (1966).
- [3] G. L. Johnston, C. F. F. Karney, F. Y. F. Chu, A. Bers, M.I.T., RLE Progress Report #117, p. 197 (1976).
- [4] J. R. Cary, A. N. Kaufman, Phys. Rev. Lett. 39, 403 (1977).

THREE-DIMENSIONAL EFFECTS ON NONLINEAR MODE-CONVERTED LOWER-HYBRID WAVES

H. H. Kuehl

University of Southern California, Los Angeles, California 90007

Electrostatic plasma waves with frequencies near the lower-hybrid resonance frequency are of current interest because of possible application to the heating of tokamak plasmas. According to linear theory, cold-plasma waves excited by an rf source near the surface of a magnetically confined plasma propagate in resonance cones¹ into the plasma until they reach the vicinity of the lower-hybrid resonant layer. There the incoming cold-plasma wave may convert² to a slow hot-plasma wave. Nonlinear effects in two-dimensional geometry due to the ponderomotive force acting on the cold-plasma wave³ as well as the mode-converted wave⁴ have recently been considered. More recently, the effect of the third dimension on the cold-plasma wave with a wave packet type of excitation has been studied.^{5,6} In particular, it has been shown that the third dimension introduces an additional dispersive term in the equation for the cold-plasma wave amplitude which renders a two-dimensional soliton unstable to perturbations in the third dimension. Numerical investigation shows that the soliton breaks up into bunches which move apart and spread the energy throughout the plasma.⁶ Here, we consider the effect of the third dimension on the nonlinear mode-converted wave. In particular, we derive the appropriate three-dimensional differential equation from which it follows that a soliton is unstable to a perturbation in the third dimension which leads to collapse.

In order to include the third dimension, we generalize the analysis of Ref. 4. The approximate dispersion relation for the mode-converted lower-hybrid wave is given by⁴

$$Ak_{\perp}^4 - K_{\perp}k_{\perp}^2 - K_{\parallel}k_z^2 = 0, \quad (1)$$

where $k_{\perp}^2 = k_x^2 + k_y^2$, $K_{\perp}$ and $K_{\parallel}$ are the dielectric tensor elements, and A is proportional to the square of the thermal velocity. Equation (1) is equivalent to the differential equation for the potential ϕ

$$\nabla_{\perp}^2 (A \nabla_{\perp}^2 \Phi) + \nabla_{\perp} \cdot (K_{\perp} \nabla_{\perp} \Phi) + \partial_z (K_{\parallel} \partial_z \Phi) = 0, \quad (2)$$

where $\nabla_{\perp} = \hat{x} \partial_x + \hat{y} \partial_y$. In the two-dimensional analysis of Ref. 4, the dependence of Φ on y was not included. In the present analysis, we retain a weak dependence on the third dimension y . The pondermotive force causes a density change δn , whereby we may write

$$K_{\perp} = K_{\perp 0} + (K_{\perp 0} - 1) \delta n / n_0, \quad K_{\parallel} = K_{\parallel 0} + (K_{\parallel 0} - 1) \delta n / n_0, \quad (3)$$

$$A = A_0 (1 + \delta n / n_0), \quad (4)$$

where the zero subscripts denote unperturbed values. Treating the third term in Eq. (2) and $\delta n / n_0$ as small first order corrections,⁴ and inserting Eqs. (3) and (4) into Eq. (2), we obtain

$$A_0 \nabla_{\perp}^4 \Phi + K_{\perp 0} \nabla_{\perp}^2 \Phi + (K_{\perp 0} - 1) \nabla_{\perp} \cdot \left(\frac{\delta n}{n_0} \nabla_{\perp} \Phi \right) + A_0 \nabla_{\perp}^2 \left(\frac{\delta n}{n_0} \nabla_{\perp}^2 \Phi \right) + K_{\parallel 0} \partial_z^2 \Phi = 0, \quad (5)$$

where the last three terms on the left-hand side are first-order quantities. If these terms are neglected as well as the y dependence of Φ , Eq. (5) has the solution

$$\Phi = \varphi(z) \exp[i(K_{\perp 0} / A_0)^{\frac{1}{2}} x], \quad (6)$$

where $\varphi(z)$ is an arbitrary function. In order to include the effects of the last three terms and a weak y dependence in Eq. (5), we let

$$\Phi = \varphi(x, y, z) \exp[i(K_{\perp 0} / A_0)^{\frac{1}{2}} x]. \quad (7)$$

Since the dependence of φ on x and y is assumed weak, we henceforth retain only the lowest derivatives of φ with respect to x and y . Furthermore, because the nonlinear terms in Eq. (5) are of first order, we need evaluate $\delta n / n_0$ only to lowest order, i. e., we neglect derivatives of φ with respect to x and y when computing $\delta n / n_0$. This leads to the expression for $\delta n / n_0$ identical to that given in Ref. 4. Finally, inserting Eqs. (7) and the expression for $\delta n / n_0$ into Eq. (5), retaining only the lowest derivatives of φ with respect to x and y , and normalizing φ according to

$$u = \left(\frac{\epsilon_0 K_{\perp 0}}{4n_0 \kappa (T_e + T_i) A_0} \right)^{\frac{1}{2}} \varphi, \quad (8)$$

we obtain

$$i\partial_x u + \alpha |u|^2 u + \beta \partial_z^2 u + \gamma \partial_y^2 u = 0, \quad (9)$$

where

$$\alpha = \left[\frac{(1-K_{\perp})}{2K_{\perp}} \left(\frac{K_{\perp}}{A} \right)^{\frac{1}{2}} \right]_0, \quad \beta = \left[\frac{-K_{\parallel}}{2K_{\perp}} \left(\frac{A}{K_{\perp}} \right)^{\frac{1}{2}} \right]_0, \quad \gamma = \frac{1}{2} \left(\frac{A}{K_{\perp}} \right)_0^{\frac{1}{2}}. \quad (10)$$

Equation (9) is the nonlinear differential equation governing the amplitude of three-dimensional mode-converted lower-hybrid waves, and is valid for sufficiently weak dependence on the y coordinate. It is the nonlinear Schrödinger equation with an additional term $\gamma \partial_y^2 u$. Because $\alpha, \beta, \gamma > 0$, this equation (with appropriate normalization of x, y, z) is identical to the equation governing nonlinear Langmuir waves when ion inertia is neglected.^{7,8} It is interesting to note that the equation governing cold-plasma lower-hybrid waves is similar in form to Eq. (9) except that the sign of the last term on the left-hand side is reversed.^{5,6} This causes the two types of lower-hybrid waves to evolve quite differently. In particular, for certain weak perturbations in the y direction, a two dimensional cold-wave soliton is unstable which produces a breakup into smaller pieces which cannot keep themselves together but spread out.⁶ However, the instability in the Langmuir soliton to such a perturbation causes the soliton to eventually collapse, producing large local values for the amplitude.⁸ Thus, since Eq. (9) governing the mode-converted lower-hybrid wave is identical in form to that governing Langmuir waves, we conclude that a soliton in the mode-converted wave will also experience such a collapse.

This work was supported by the National Science Foundation under grant ENG-7680670.

1. H.H. Kuehl, Phys. Fluids 5, 1095(1962); R.J. Briggs and R.R. Parker, Phys. Rev. Lett. 29, 852(1972).
2. T.H. Stix, Phys. Rev. Lett. 15, 878(1965); M.D. Simonutti, Phys. Fluids 18, 1524(1975).
3. G.J. Morales and Y.C. Lee, Phys. Rev. Lett. 35, 930(1975).
4. H.H. Kuehl, Phys. Fluids 19, 1972(1976).
5. A. Sen, C.F.F. Karney, G.L. Johnston and A. Bers, RLE-PRR 77/16, M.I. T. (June 1977).
6. N.R. Pereira, A. Sen, and A. Bers, Phys. Fluids, to be published.
7. V.E. Zakharov, Zh. Eksp. Teor. Fiz. 62, 1745(1972) [Sov. Phys.-HETP 35, 908(1972)].
8. N.R. Pereira, R.N. Sudan, and J. Denavit, Phys. Fluids 20, 936(1977).

Complex Modified K-DV Equation and Nonlinear Propagation of Lower Hybrid Waves*

by

A. Sen, C. F. F. Karney,[‡] A. Bers, and N. R. Pereira[§]

Plasma Fusion Center, M. I. T., Cambridge, MA 02139

The nonlinear steady state propagation of lower hybrid waves in a uniform plasma can be described by a "Complex" Modified Korteweg De Vries equation, $v_\tau + (|v|^2 v)_\xi + v \xi \xi \xi = 0$, where v the amplitude of the electric field is complex. This equation is not amenable to analytic solution by the Inverse Scattering Transform method and we solve it numerically. In the limit of a narrow spectrum excitation at the boundary it approximately reduces to a nonlinear Schrödinger equation and we obtain envelope soliton solutions. For broader spectrums we obtain "MKDV type solitons" with constant phases. We discuss these solutions in terms of satisfying the radiation condition inside the plasma and the limitations posed on the choice of "initial" conditions by nonlinear reflections.

In a cold homogeneous plasma the linear steady state propagation of lower hybrid waves is along resonance cones. Treating nonlinear and dispersive (thermal) effects as small perturbations along any single resonance cone the propagation equation can be derived as¹:

$$v_\tau + (|v|^2 v)_\xi + v \xi \xi \xi = 0 \quad (1)$$

where v is proportional to the electric field amplitude ($\approx \partial \phi / \partial x$), $\xi \sim (x - cz)$ is the stretched characteristic coordinate (c is the ratio of cold group velocities) and $\tau \sim x$ characterises the perturbative effects of nonlinearity and dispersion. If v is assumed real, (1) reduces to the Modified Korteweg-de Vries (MKDV) equation² which has soliton solutions. However in general v is complex and then (1) is the correct nonlinear equation to solve. Unlike the MKDV equation, this equation, the "Complex Modified K-dV" (CMKDV) equation, does not appear to have an infinite set of conservation laws and is not amenable to solution by the Inverse Scattering Transform method. In this paper we therefore present the results of numerically integrating (1).

Equation (1) has a conservation law

$$\int_{-\infty}^{\infty} |v(\tau, k)|^2 / k \, dk = \text{const.} \quad (2)$$

*Work supported by U.S. Department of Energy (Contract EG-77-G-01-4107) and the National Science Foundation (Grant ENG77-00340)

[‡] Princeton Plasma Physics Laboratory, Princeton University, Princeton, NJ 08540

[§] Lawrence Berkeley Laboratory, University of California, Berkeley, CA 94720

where k is the transform variable for ξ . This is just the same as the power flow conservation law $\int_{-\infty}^{\infty} S_x dx$. (Note that $S_x(k) \propto |v_k|^2/k$.) Thus when imposing "initial" conditions to (1) at $\tau = 0$ (i.e. $x = 0$), we require that $v(0, k < 0) = 0$ so that all spectral components of the initial pulse carry power into the plasma. This may be achieved by ensuring that $v(0, \xi)$ has the form

$$v(0, \xi) = \frac{1}{2} w(\xi) + \frac{1}{2\pi i} \oint_{-\infty}^{\infty} \frac{w(\xi')}{\xi' - \xi} d\xi'. \quad (3)$$

For the numerical integration of (1) we find it more convenient to choose initial conditions of the form

$$v(0, \xi) = A \operatorname{sech}(\xi) \exp(ik_0 \xi). \quad (4)$$

For $k_0 > 1$ this only has a small tail of negative k components which we do not expect to materially effect the results. We note that if $k_0 = 0$, we may obtain the MKDV soliton by letting $A = \sqrt{2}$. The soliton threshold is $A = 1/\sqrt{2}$.

We begin first by looking at $k_0 \gg 1$. In this case (1) approximately reduces to the Nonlinear Schrödinger equation (NLSE)³. If we make no approximations (1) becomes

$$u_\eta + iu_{\xi\xi} + 2i|u|^2u + u_{\xi\xi\xi}/(3k_0) + 2(|u|^2u)_{\xi}/k_0 = 0 \quad (5)$$

where $v(\tau, \xi) = \sqrt{6}u(\eta, \xi) \exp[i(k_0\xi + k_0^3\tau)]$, $\xi = \xi + 3k_0^2\tau$, $\eta = 3k_0\tau$. Note that in the limit of $k_0 \rightarrow \infty$ (when (5) becomes the NLSE) the single soliton solution is obtained when $A = \sqrt{6}$ in (4), i.e. $\sqrt{3}$ times the area of the soliton for the case $k_0 = 0$. We integrate (5) with an initial condition of $u = \operatorname{sech}(\xi)$. We find that for $k_0 > 2\frac{1}{2}$ the pulse is long-lived and remains sech-like but it has a finite velocity in the η, ξ frame (unlike the case $k_0 \rightarrow \infty$). For $k_0 < 2\frac{1}{2}$ the pulse rapidly loses its identity, and the NLSE approximation is no longer valid.

This brings us to the other limit to consider, namely $k \sim 1$. (This is important in 2 and 4 waveguide excitations.) We show in Fig. 1 the results of integrating (1) with $A = 4.0$ and $k_0 = 1.2$. Note that the pulse breaks up into "solitons" confirming the results of Kuehl⁴. These solitons have the form

$$v(\tau, \xi) = \exp(i\theta) \sqrt{2}a \operatorname{sech}[a(\xi - \xi_0 - a^2\tau)], \quad (6)$$

where θ is a constant. This is just the solution of the MKDV equation multiplied by a complex constant. These solutions have the property that quite general initial conditions lead to the formation of solitons at large τ . (Note the characteristic ordering of the solitons in Fig. 1, with the tallest and fastest leading the others.) These solitons also have the property that they are unaffected by collisions with each other (except for a translation of the soliton), when they all have the same phase, θ . However the collision

of two solitons with different values of θ , is inelastic, in that some "radiation" is produced. In Fig. 2, we show the collision of two solitons with $a_1 = \sqrt{2}$, $a_2 = 1/\sqrt{2}$, $\theta_1 = 0$, and $\theta_2 = \frac{1}{2}\pi$. We see that after the collision the tallest soliton remains unaffected, while the shorter one has lost some of its amplitude. The phases of these solitons are nearly interchanged, with $\theta_1 = 80^\circ$ and $\theta_2 = 10^\circ$.

The problem with the solution shown in Fig. 1 is that the solitons carry no net power. So that Fig. 1 shows a situation where all the power injected at the boundary ends up being carried by the radiation. The solitons correspond to field structures going off to $\tau = \infty$ in which there are equal amounts of positive and negative power flow. This "steady state" is obviously not accessible in a finite time; so we must ask ourselves what assumptions we have made that causes us this trouble. The answer is that the nonlinear term in (1) can cause internal reflection of the power (by changing the sign of k). In systems where there is internal reflection we must specify a radiation condition at the far end of the system. Taking this point to be $\tau = \tau_1$, we should require that $v(\tau_1, k < 0) = 0$, i.e. power only flows outwards at the far boundary. Note that this condition is violated in Fig. 1. At $\tau = 0$ we should only impose the incident power; thus we should specify $v(0, k > 0)$, but we may not specify $v(0, k < 0)$ which must emerge as part of the solution such that the radiation condition is satisfied at the far end. This is rather difficult to achieve numerically and we do not yet have a "steady state" solution satisfying these boundary conditions.

Summarizing, we understand the nonlinear behaviour for the narrow spectrum excitations ($k \gg \Delta k$) where selfmodulation effects can lead to envelope soliton structures. For $k \sim \Delta k$ the problem is not completely solved but it appears that nonlinear internal reflections can occur. In either case, for 2 or 4 waveguide excitations for tokamak plasmas, practical electric field amplitudes and their spatial extent are likely to be below the threshold condition for soliton formation⁵ and the nonlinear reflections would then also be relatively unimportant.

Acknowledgement

We would like to acknowledge Bruce Edwards for much help with the numerical work and Flora Chu for valuable comments.

References

1. G. L. Johnston *et al.*, PRR 76/18-2, M.I.T. (July 1976).
2. G. J. Morales and Y. C. Lee, Phys. Rev. Letts. 35, 930 (1975).
3. A. Bers *et al.*, Bull. Am. Phys. Soc. 21, 1156 (1976) and A. C. Newell and D. J. Kaup *idem* 21, 1095 (1976).
4. H. H. Kuehl, Private Communication.
5. A. Bers, Third Symposium on Plasma Heating in Toroidal Devices, Varenna, Sept. 6-17, 1976, pp. 99-103.

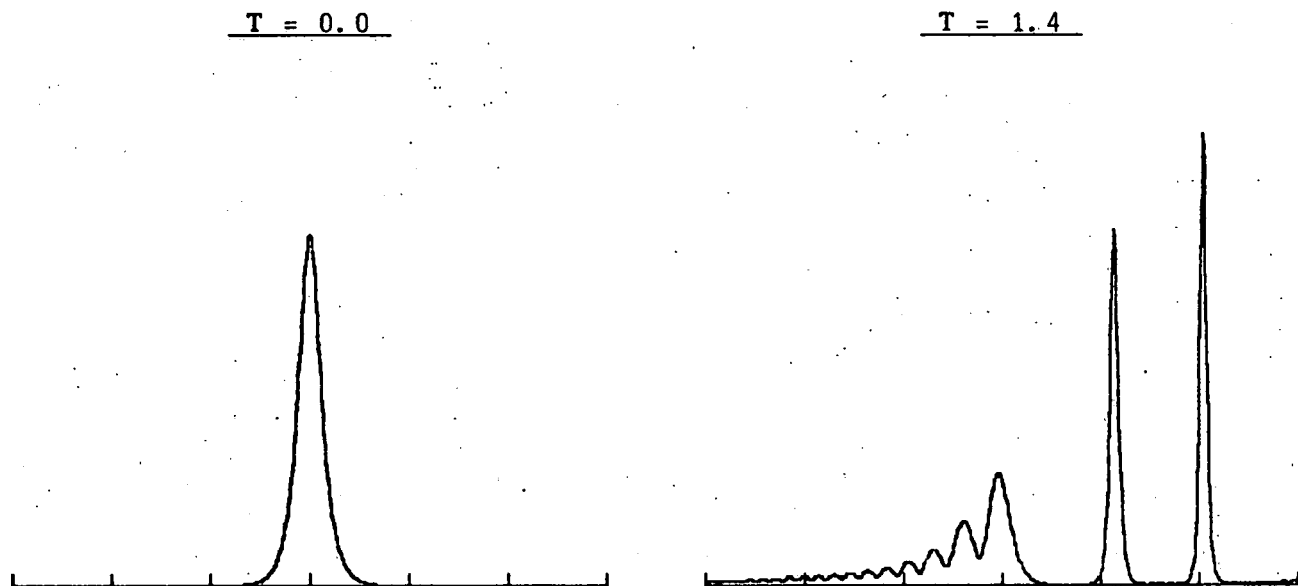


Fig. 1. Evolution of an initial sech pulse with $A=4$, $k_0=1.2$, into "MKDV" type solitons and "radiation".

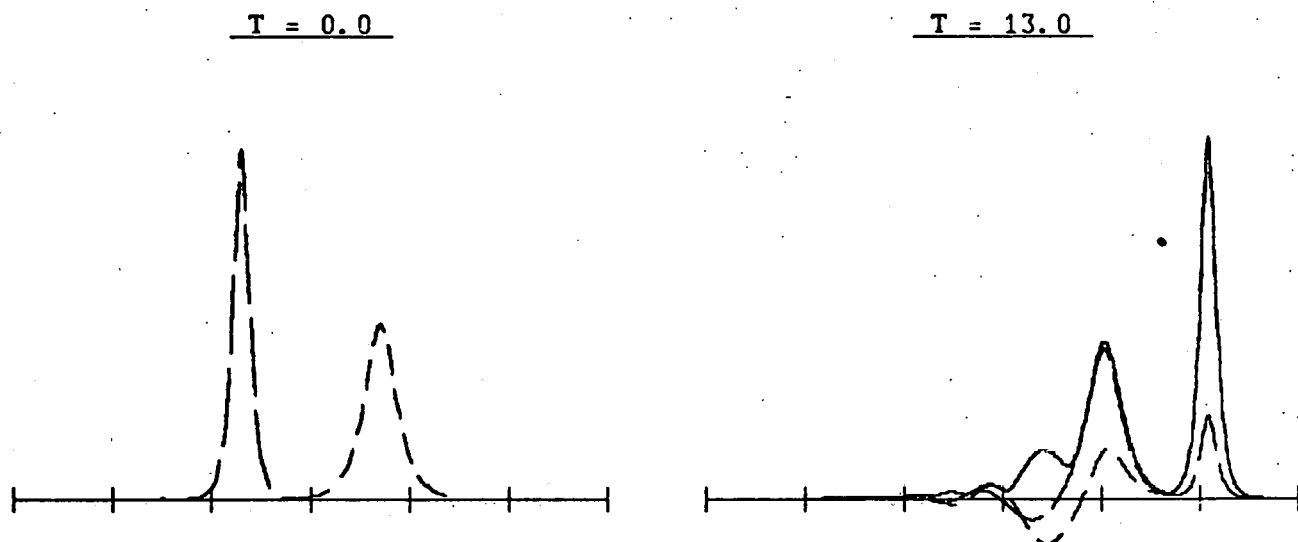


Fig. 2. Collision of two solitons initially placed apart with $a_1 = \sqrt{2}$, $a_2 = 1/\sqrt{2}$, $\theta_1 = 0$, and $\theta_2 = \frac{1}{2}\pi$. At $\tau=13$, the tall soliton has passed through the shorter one but there is some change in the phases and generation of some radiation. Large dashed lines denote $\text{Re}(v)$, short dashed lines $\text{Im}(v)$ and solid lines $\text{abs}(v)$.

ENERGY FLOW AND TUNNELLING OF RESONANCE CONES NEAR THE LOWER HYBRID

by

C. L. Grabbe

California Institute of Technology

ABSTRACT

The potential and power flow along resonance cones which tunnel across a thin high density evanescent layer separating two lower hybrid layers is considered and compared with the potential and power flow along the cones incoming to the lower hybrid layer and the mode converted cone going out on the same side, by use of energy theorems for quasistatic fields discussed previously. The relative importance of damping mechanisms along each of these cones is discussed.

There has been recent interest in the possible mode conversion of resonance cones near the lower hybrid resonance and its significance for lower hybrid heating. Recently, the tunnelling of resonance cones through the high density evanescent layer separating two hybrid layers has been studied under certain conditions as well.^{1,2} These predicted processes are summarized in Fig. 1. It is the purpose of this paper to compare the relative power flow and absorption along these cones.

To investigate theoretically the energy flow associated with quasistatic propagating resonance cones, energy conservation theorems for quasistatic fields must be used. Two forms of the theorem will be stated here, which are somewhat complementary and can be combined to get useful expressions to apply to the resonance cones. From the fluid equations one may obtain^{2,3}

$$\frac{\partial U(\vec{r}, t)}{\partial t} + \vec{\nabla} \cdot \vec{S}(\vec{r}, t) = P_R(\vec{r}, t) \quad (1)$$

where the energy density U is the sum of components associated with potential energy of the fields, particle kinetic energy, and energy of thermal oscillations

$$U(\vec{r}, t) = \frac{\epsilon_0}{2} |\nabla \phi|^2 + \sum_{\alpha} \frac{n_{\alpha} m_{\alpha} \vec{v}_{\alpha}^2}{2} + \sum_{\alpha} \frac{\gamma k T_{\alpha} n_{\alpha}^2}{2 n_0} \quad (2)$$

the power flux $\vec{S}(\vec{r}, t)$ is the sum of regular current, displacement current, and pressure oscillation components

$$\vec{S}(\vec{r}, t) = \phi * (\vec{J} - \epsilon_0 \frac{\partial(\nabla \phi)}{\partial t}) + \sum_{\alpha} \gamma k T_{\alpha} n_{\alpha} \vec{v}_{\alpha} \quad (3)$$

and the power absorption (due to collisions) is

$$P_R = -n_0 \sum_{\alpha} m_{\alpha} \vec{v}_{\alpha}^2 \nu_{\alpha} \quad (4)$$

Similarly, from the kinetic equations we obtain the $(\vec{k}, \omega)$ -space form

$$\tilde{U}(\vec{k}, \omega) \delta \omega + \tilde{\vec{S}}(\vec{k}, \omega) \cdot \delta \vec{k} = \tilde{P}_R(\vec{k}, \omega) \quad (5)$$

where

$$\tilde{U}(\vec{k}, \omega) = \frac{\epsilon_0}{2} |\phi(k, \omega)|^2 \vec{k} \cdot \left[\frac{\partial}{\partial \omega} (\omega \vec{k}_h) \right] \quad (6)$$

$$\tilde{S}(\vec{k}, \omega) = \frac{\epsilon_0}{2} |\phi(\vec{k}, \omega)|^2 \vec{k} \cdot [2\overline{K}_h + \frac{\partial \overline{K}_h}{\partial \vec{k}} \cdot \vec{k}] \quad (7)$$

$$\tilde{P}_R(\vec{k}, \omega) = \frac{\epsilon_0 \omega}{4} |\phi(\vec{k}, \omega)|^2 \vec{k} \cdot \overline{K}_a \cdot \vec{k} \quad (8)$$

and $\overline{K}_h$ and $\overline{K}_a$ are the Hermitian and anti-Hermitian parts of the dielectric tensor. The form of $\overline{K}_a$ gives collisionless as well as collisional damping contributions to $\tilde{P}_R$.

If we write $\tilde{S}(\vec{k}, \omega) = \tilde{\alpha}(\vec{k}) |\phi(\vec{k}, \omega)|^2$ and $P_R = \beta(\vec{k}) |\phi(\vec{k}, \omega)|^2$, then with the aid of the fluid energy conservation theorem, we may derive a form for $\tilde{S}(\vec{r})$ and $P_R(\vec{r})$ which is valid whenever $\phi(\vec{r})$ can be evaluated by asymptotic saddle point methods, i.e., when most of the value of $\phi(\vec{r})$ comes in the vicinity of the saddle point $k_z = k_0$ (z is the direction of B_0)

$$\tilde{S}(\vec{r}, t) \approx \tilde{\alpha}(k_z = k_0) |\phi(\vec{r}, t)|^2 \quad (9)$$

$$P_R(\vec{r}, t) \approx \beta(k_z = k_0) |\phi(\vec{r}, t)|^2 \quad (10)$$

(Note k_x is a fixed function of k_z through the local dispersion relation.) The forms given in Eqs. (9)-(10) assumes $\tilde{\alpha}$ and β are slowly varying near $k_z = k_0$, which is valid if the collisionless contributions to β are small.

From the WKB form of the resonance cone fields for a point gap source and their saddle point values of k_z ,^{2,4} we obtain expressions for $\tilde{S}(\vec{r})$ for each of the resonance cones. For the incoming cone we have

$$\begin{aligned} \tilde{S}(\vec{r}) \approx \omega \epsilon_0 \left| F\left(\frac{z - g(x)}{[3q(x)]^{1/3}}\right) \right|^2 \left[\frac{z - \text{Re } g(x)}{3q(x)^{7/3}} \right]^{1/2} \\ \times \{ [-K_{\parallel} K_{\perp}]^{1/2} \hat{x} + K_{\parallel} \hat{z} \} \end{aligned} \quad (11)$$

where $F(\zeta) = \text{Ai}(\zeta) - i\text{Gi}(\zeta)$, $g(x)$ is the WKB phase (in cold plasma theory) from $x=0$ to x divided by k_z for a given k_z component and $q(x)$ is the thermal correction to $g(x)$ divided by k_z^2 . [See Refs. (2) and (4) for a formal definition of g and q .] The form of the cone field and the relative power flow along this cone structure are shown in Fig. 2. Similarly, for the mode-converted cone we have

$$\begin{aligned} \tilde{S}(\vec{r}) \approx \frac{\omega \epsilon_0}{2 [p(x_{h1}) - p(x)]} \left| D_{-1/2} \left[\frac{z - g(x_{h1})}{2 [p(x_{h1}) - p(x)]} \right] \right|^2 \\ \times \left\{ - \left[\frac{K_{\perp}^3(x)}{\alpha(x)} \right]^{1/2} \hat{x} + \frac{[z - \text{Re } g(x_{h1})]}{2 [p(x_{h1}) - p(x)]} \hat{z} \right\} \end{aligned} \quad (12)$$

where $D_{-1/2}$ is a Whittaker's function, $p(x)$ is the thermal correction to the phase of this thermal wave, $x = x_h$ is the hybrid layer, and $\alpha(x)$ is the thermal part of $K_{\perp}$. The structure of this cone and the relative power flow along it is shown in Fig. 3. Finally, the tunnelled "cold plasma" (X-mode) cone takes the form

$$\tilde{S}(\vec{r}) \approx \omega \epsilon_0 \left| F \left(\frac{z - 2g(x_{h1}) + g(x) + i\eta(x_{h2})}{\{3[2q(x_{h1}) - q(x) + iQ(x_{h2})]\}^{1/3}} \right) \right|^2 \times$$

$$\times \frac{[z - 2g(x_{h1}) + g(x) + iD(x_{h2})]^{1/2}}{\sqrt{3}[2q(x_{h2}) - q(x) + iQ(x_{h2})]^{7/6}} \{ [-K_{\parallel} K_{\perp}]^{1/2} \hat{x} + K_{\parallel} \hat{z} \} \quad (13)$$

where $|k_z| D(x_{h2})$ is the exponent of decay of a given k_z component between x_{h1} and x_{h2} . This cone field and associated power flow is shown in Fig. 4 for a special case of a thin evanescent layer.

We see from the figures that for the incoming cone, the energy flow along the higher order peaks is not much smaller than along the main peak; the power flux concentration falls off rather slowly with peak order. (This means the power flow along resonance cones is not nearly so localized for a point gap source as naive cold plasma theory would suggest.) For the converted cone the power density varies more rapidly across the cone structure, i.e., there is a greater contrast between the power flow along the maximum and the minimum of the cone fields than for the incoming cone. Also, the power density falls off much faster as one goes from the main peak to the secondary peaks, i.e., there is a greater concentration of power along the main peak relative to the secondary peaks than for the incoming cone. Thus mode conversion causes a focusing of the power flow along the cone more toward the main peak.

On the tunnelled cone, the higher order peaks have decayed through the evanescent layer at a faster rate than the main peak, while the main peak has broadened somewhat. We see that there is a greater concentration along the main peak relative to the secondary peaks for this cone also. At first this result may seem somewhat paradoxical; it is known that a resonance cone in cold plasma theory will broaden upon passing through an evanescent layer. However, in the warm plasma case, the cone is made up of both a cold plasma component and a thermal wave. The latter component is dispersive and causes the whole cone to spread out as it propagates away from the source, with the higher k_z components being the most dispersive. When the cone passes through the evanescent region, the higher k_z 's decay the most rapidly. Then dispersion of the thermal component is reduced, while the cold plasma component is broadened. The former effect dominates, and the resonance cone field and power flux shown in Fig. 4 results when the evanescent layer is sufficiently thin.

Expressions for P_R may similarly be directly obtained for each cone to identify the points of concentration of the various absorption mechanisms. This may be related to a damping exponent of the potential of the form (valid in the asymptotic region of the cone) $\phi(x,z) = \phi_0(x,z)e^{-\Gamma(x,z)}$, where ϕ_0 is the undamped potential, by the energy conservation theorem:

$$P_R = -2 \left\{ S_x \frac{d\Gamma}{dx} + S_z \frac{d\Gamma}{dz} \right\} \quad (14)$$

This may be solved directly to give Γ as a function of the dielectric tensor independently of the value of ϕ_0 . The nature of the potential damping of the resonance cones is discussed elsewhere^{2,5}. One important conclusion is that the collisional damping rate on the incoming cone is significantly smaller than predicted by Bellan and Porkolab⁶, so that observation of mode conversion may still be possible in linear devices.²

REFERENCES

1. C. Grabbe, Bull. Am. Phys. Soc. 21, 1075 (1976).
2. C. Grabbe, Ph.D. Thesis, Caltech, August 1977.
3. C. Grabbe, Bull. Am. Phys. Soc. 22, 1073 (1977).
4. K. Ko and H. Kuehl, Phys. Fluids 18, 1816 (1975).
5. C. Grabbe, Caltech Report, December 1977.
6. P. Bellan and M. Porkolab, Phys. Fluids 17, 1592 (1974).

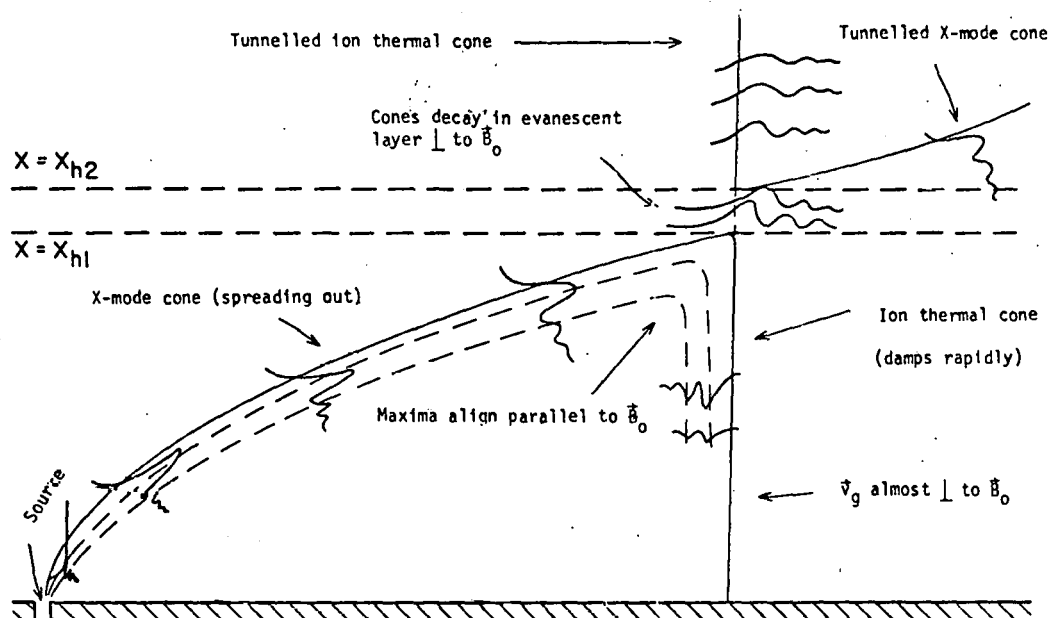


Fig. 1 Diagram showing the way the resonance cones propagate into the hybrid layer and through the evanescent layer. Solid lines indicate cold plasma cone lines, and dotted lines the trajectories of the peaks.

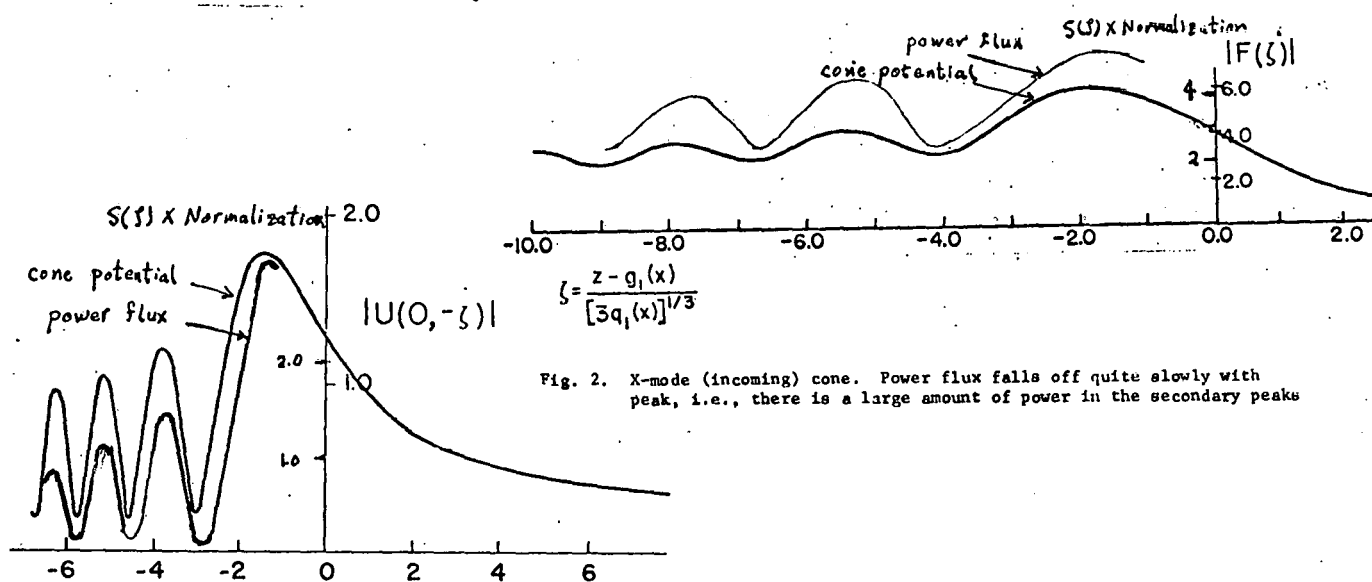


Fig. 2. X-mode (incoming) cone. Power flux falls off quite slowly with peak, i.e., there is a large amount of power in the secondary peaks

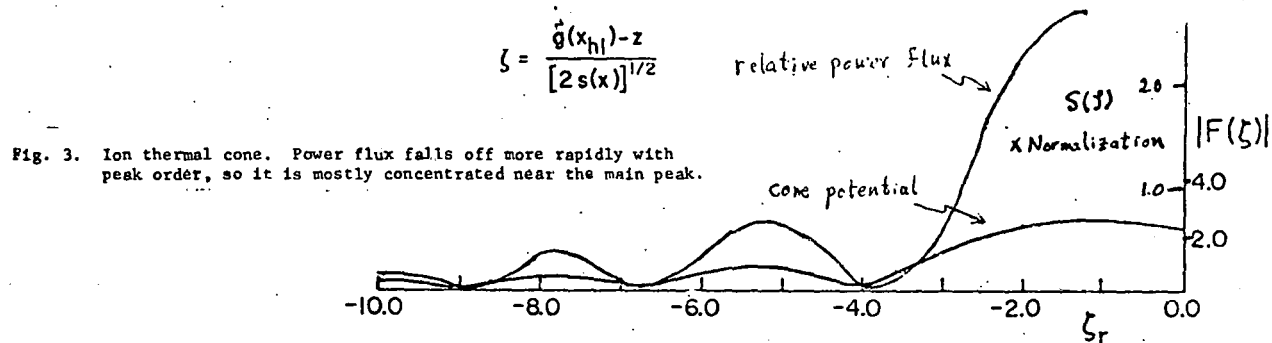


Fig. 3. Ion thermal cone. Power flux falls off more rapidly with peak order, so it is mostly concentrated near the main peak.

Cone structure after tunnelling through evanescent layer ($\zeta_i = 1.0$)

Fig. 4. Tunnelled X-mode cone. The "thermal component" of the wave has decayed quite rapidly.

THE EFFECT OF FIELD GRADIENTS ON WAVE PROPAGATION IN TOKAMAKS

K. A. Connor

Rensselaer Polytechnic Institute, Troy, New York

and

P. L. Colestock

Princeton University, Princeton, New Jersey

ABSTRACT

Wave propagation in a torus differs from results predicted by plane wave theory due to both the toroidal curvature and rotational transform of the field lines. In particular field gradients in tokamaks cause the wavelength to vary along a field line which gives rise to focusing and cusping not present in the uniform field, planar case. We examine the effects of field gradients in a simplified tokamak model using a ray tracing approach. A corresponding planar model is developed which includes similar field gradients and the associated wave phenomena. Results are shown for electrostatic waves valid in the geometrical optics approximation including finite temperature effects.

I. INTRODUCTION

Wave propagation in tokamaks has been explored in great detail, using planar or cylindrical one dimensionally inhomogeneous plasma models; relatively little attention has been given, however, to the problem of determining the effects on the wave propagation of the complicated two dimensional inhomogeneities which actually occur. One of the first attempts was made by Briggs and Parker [1], who analyzed the case of lower hybrid wave propagation in a plasma whose density gradient was not quite perpendicular to the magnetic field line. This implies, of course, a small density gradient along the field line which causes the parallel wavelength along the direction of propagation to vary. In ideal MHD tokamak equilibria, such a density gradient does not occur; magnetic field gradients occur in the combined toroidal and poloidal field. The result can be a reflection or a cusp point as seen in the Briggs and Parker analysis. For this reason, in the second section of this paper we re-examine the Briggs and Parker problem from a ray tracing point of view, including the effects of the gradient on mode conversion. In the third section we develop an approximate planar plasma model which includes field gradients similar to that seen traveling along a field line in a tokamak and reconsider the lower hybrid wave propagation and mode conversion problem. The results of this analysis are then compared to the ray tracing solution found using the more accurate, but tedious, cylindrical tokamak model.

II. AXIAL DENSITY GRADIENTS

Briggs and Parker found that the lower hybrid resonance cones from a source of finite size showed an asymmetry in the axial direction in a system whose x axis was aligned along the density gradient, but whose z axis was slightly skewed

from the direction of the magnetic field. In particular, a cusp point was seen to occur in one of the resonance cone branches as shown in Fig. 1A. Further investigation shows that the cusp occurs at the lower hybrid resonance layer for perpendicular propagation, whereas the asymptote of the resonance cone is the true wave resonance, or so-called "oblique resonance". This resonance occurs at a lower density than the lower hybrid layer as determined by the expression

$$\omega_{\text{res}}^2 \sim \omega_{\text{pi}}^2 \left(1 + \frac{m_i}{m_e} \cot^2 \theta\right) \quad (1)$$

where θ is the angle between the field and the density gradient. Reflection from the lower hybrid layer is to be expected since it forms a boundary between elliptic and hyperbolic regions in the plasma, but the index of refraction remains finite since equation (1) is not satisfied.

Identical results may be obtained by solving for the electrostatic group velocity rays in a system whose axis is aligned along the magnetic field lines. It is relatively easy to add thermal effects to this approach using the conventional warm plasma electrostatic dispersion relation producing a mode conversion of the lower hybrid waves to thermal waves. The spatial dispersion of these waves may be significantly altered by the axial density gradient, as shown in Fig. 1B. The gradient tends to increase the axial wave number in the direction of increasing density and causes negative k_z values to pass through zero giving rise to a turning point. As the temperature is increased, the fraction of waves reaching the turning point diminishes. It is also worth noting that propagation out the plane of the figure may be easily studied as well. However, since the plasma is assumed to be uniform in the $\hat{y}$ direction, no new wave phenomena occur out of the plane.

3. AXIAL FIELD GRADIENTS

In simple tokamak equilibria, density gradients do not normally occur along the field lines. A picture of the gradients in a tokamak may be found by orienting a coordinate system along the field lines in a magnetic surface as suggested by Hamada [2]. The field lines become straight lines normal to the direction of the density gradient; however, the three dimensional system is non-orthogonal due to the magnetic shear which leads to a more complex wave equation. Nonetheless it is possible to expand the wave equation in a simple form that provides a good approximation when shear is small. To simplify the expressions we adopt the cylindrical tokamak model in which the magnetic surfaces are circles centered on the axis. We choose coordinates in which the ξ coordinate measures distance along the field line on a constant r surface.

$$\begin{aligned} \rho &= r \\ \eta &= \theta - \gamma \frac{z}{r} \\ \xi &= \sqrt{1 + \gamma^2} z \end{aligned} \quad (2)$$

where $\gamma = B_\theta/B_z$ is small. When this transformation is carried out, the resulting wave equation may be expanded in powers of γ retaining only the lowest order terms.

The second order terms of the wave equation from which the ray equations are derived are identical to the cylindrical case, but the dielectric tensor components vary periodically along the field lines due to the toroidal field variation across the plasma,

$$B_t = B_o \left(1 + \frac{r}{R_o} \cos \theta\right)^{-1} \quad (3)$$

The associated mod B surfaces are shown in Fig. 2. The solution to the ray equations in this approximate ρ, η, ξ system for the lower hybrid case with thermal corrections are shown in Fig. 3. Since lower hybrid waves depend only weakly on the magnetic field in the low density limit, there is only a slight departure from the planar solution as seen in the small oscillation of the rays. Near the resonance layer, however, a fraction of the rays may become trapped between successive mod B maxima in a process entirely analogous to that observed in the Briggs and Parker problem. The spatial period of the oscillation is the distance along the field line required to make one minor circuit of the torus.

The problem may also be approached in a direct manner for formulating the dielectric tensor in cylindrical coordinates including both a poloidal and toroidal field. All components of the cold plasma dielectric tensor are, in general, non-zero and depend on the angle between the axis and the local direction of the field line. This straightforward, but tedious, procedure yields a simple physical picture. The projections of the rays excited by a point source at the plasma edge for the lower hybrid case excluding thermal effects, are shown in Fig. 4. It is of interest to note that wave trapping occurs between successive points on the resonance layer as was seen in the planar model. The planar solution indicates, however, that mode conversion causes the wave energy to escape the trapping region or avoid it all together. It may be concluded that where significant mode conversion is likely to occur that the occurrence of cusps or reflections from the lower hybrid layer is minimal.

IV. REFERENCES

- [1]. R. J. Briggs and R. R. Parker, Phys. Rev. Lett., 29, 852 (1972).
- [2]. S. Hamada, Nucl. Fusion, 1, 23 (1962).

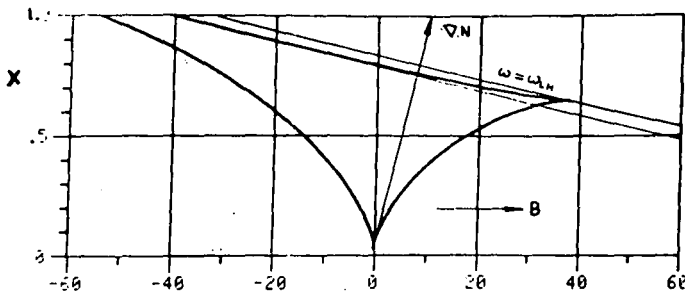
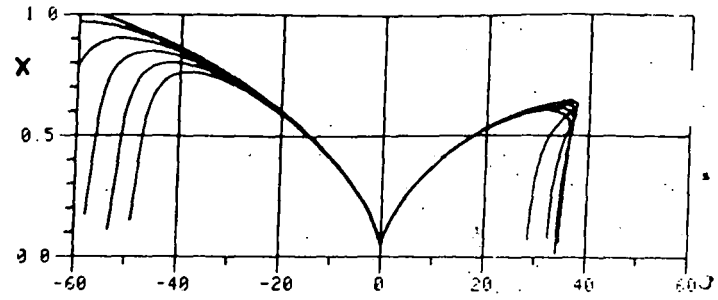


Fig. 1(a) $T_e = T_i = 0$



$T_e \neq 0$
 $T_i \neq 0$ Fig. 1(b)

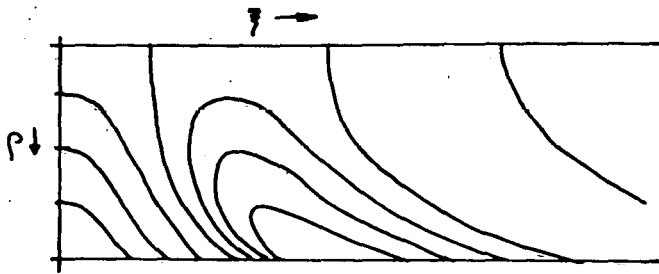


Fig. 2 Mod B $\eta = 0$

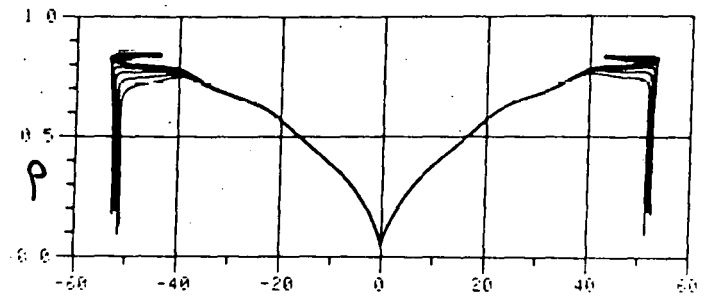
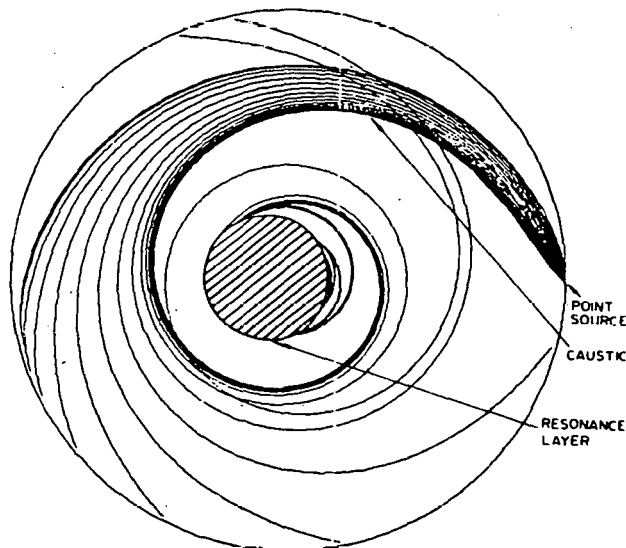


Fig. 3 $T_e \neq 0$
 $T_i \neq 0$



$T_e = T_i = 0$

Fig. 4

LIST OF PARTICIPANTS
THIRD TOPICAL CONFERENCE ON RADIO FREQUENCY PLASMA HEATING
California Institute of Technology
January 11-13, 1978

Charles Armentrout
General Atomic
P.O. Box 81608
San Diego, CA 92138

William P. Ballard
Electronics Research Lab
Stanford University
Stanford, CA 94305

James D. Barter
T.R.W., Inc.
R1/2020
Redondo Beach, CA 90278

Donald B. Batchelor
Oak Ridge National Lab.
Oak Ridge, Tenn. 37830

D. E. Bowen
Litton Industries
960 Industrial Road
San Carlos, CA 94070

Paul Bellan
M.S. 116-81
California Inst. of Technology
Pasadena, CA 91125

Stefano Bernabei
Plasma Physics Laboratory
Princeton University
Princeton, N.J. 08540

A. Bers
Massachusetts Inst. of Tech.
Room 38-260
Cambridge, Mass. 02139

Alan P. Biddle
Dept. of Physics
U. of Wisconsin
Madison, WI 53706

Marco Brambilla
CEN Grenoble--Fusion
85X 38041
Grenoble CEDEX, France

Ernesto Canobbio
CLN--Fusion
85X 38041
Grenoble CEDEX, France

Vincent Chan
General Atomic
P.O. Box 81608
San Diego, CA 92138

Francis F. Chen
Boelter 7731
U.C.L.A.
Los Angeles, CA 90024

Cheng Chu
General Atomic, E-130
P.O. Box 81608
San Diego, CA 92138

Patrick Colestock
Plasma Physics Lab.
Princeton University
Princeton, N.J. 08540

Gerard Conn
Michigan State University
East Lansing, MI.

Kenneth A. Connor
Rensselaer Polytechnic Inst.
Troy, New York 12181

John R. Cory
Lawrence Berkeley Lab.
Berkeley, CA 94720

George Craig
U.C. Lawrence Livermore Lab.
P.O. Box 808
Livermore, CA 94550

Viktor K. Decyk
Physics Department
U.C.L.A.
Los Angeles, CA 90024

Nat Fisch
Massachusetts Inst. of Tech.
Cambridge, MA 02139

Eric Fredrickson
1115 E. Cordova St #115
Pasadena, CA 91106

Richard L. Freeman
General Atomic
P.O. Box 81608
San Diego, CA 92138

Burton D. Fried
Department of Physics
U.C.L.A.
Los Angeles, CA 90024

Edward Garelis
U.C. Lawrence Livermore Lab.
P.O. Box 808
Livermore, CA 94550

Claude Gormezano
CENG--Fusion
85X 38041
Grenoble, CEDEX, France

Roy W. Gould
M.S. 116-81
California Institute of Tech.
Pasadena, CA 91125

Crockett L. Grabbe
M.S. 116-81
California Inst. of Technology
Pasadena, CA 91125

Madhu Gupta
Physics Department
U.C.L.A.
Los Angeles, CA 90024

Glenn Greene
M.S. 116-81
California Inst. of Technology
Pasadena, CA 91125

Michel Grenoire
CENG--Fusion
85X 38041
Grenoble, CEDEX, France

Mark P. Hacker
General Atomic
P.O. Box 81608
San Diego, CA 92138

A. Hasegawa
Bell Laboratories
Murray Hill, N.J. 07974

Robert W. Harvey (E-040)
General Atomic
P.O. Box 81608
San Diego, CA 92138

Richard Hawrluk
Plasma Physics Lab.
Princeton University
Princeton, N.J. 08540

Rob La Haye
General Atomic
P.O. Box 81608
San Diego, CA 92138

Mark Hedemann
M.S. 116-81
California Institute of Tech.
Pasadena, CA 91125

W. Hooke
Plasma Physics Laboratory
Princeton University
Princeton, N.J. 08540

Joel C. Hosea
Plasma Physics Laboratory
Princeton University
Princeton, N.J. 08540

Hulbert Hsuan
Plasma Physics Laboratory
Princeton University
Princeton, N.J. 08540

David Hwang
M.S. 116-81
California Inst. of Technology
Pasadena, CA 91125

J. Hyman
Hughes Research Labs.
3011 Malibu Canyon Rd
Malibu, CA 90265

Abram R. Jacobson
Los Alamos Scientific Lab.
P.O. Box 1663
Los Alamos, N.M. 87544

J. Jacquinot
CFA
Fontenay-aux-Roses (Seine)
FRANCE

B. W. James
University of Sydney
Physics Dept.
Sydney, AUSTRALIA

Torkil H. Jensen
General Atomic
P.O. Box 81608
San Diego, CA 92138

Shayne Johnston, Physics Dept.
Columbia University
New York, N.Y. 10027

C. Karney
Plasma Physics Lab.
Princeton University
Princeton, N.J. 08540

D. J. Kaup
Dept. of Physics
Clarkson Coll. Tech.
Potsdam, N.Y. 13676

Allan N. Kaufman, 50A-3139
U.C. Lawrence Livermore Lab.
P.O. Box 808
Berkeley, CA 94720

Gary D. Kerbel
564 Charlotte Way #31
Livermore, CA 94550

Vladimir Krapchev
80 Park St
Brookline, MA 02146

N. Krishansen
Texas Tech University
Physics Department
Lubbock, Texas

Randy Kubena
M.S. 116-81 Steele Lab
California Inst. of Technology
Pasadena, CA 91125

H. Kuehl
E.E. Department
U. Southern California
Los Angeles, CA 90007

J. A. Lehané
School of Physics
University of Sydney
Sydney, Australia

Fritz Leuterer
Inst. für Plasmaphysik
D-8046 Garching
GERMANY

Bruce Levine
M.S. 116-81
California Inst. of Technology
Pasadena, CA 91125

Robert G. Littlejohn
5317 Lawton Ave
Oakland, CA 94618

James Luxon
General Atomic
P.O. Box 81608
San Diego, CA 92138

Ken Mackenzie
Dept. of Physics
U.C.L.A.
Los Angeles, CA 90024

John J. McClure
Department of Energy
Div. Magnetic Fusion Energy
Washington, D. C. 20545

Roger McWilliams
Plasma Physics Lab.
Princeton University
Princeton, N.J. 08540

Gerard Melin
Lawrence Livermore Lab.
P.O. Box 808
Berkeley, CA 94720

Andre Messiaen
Plasma Physics Lab.
Ecole Royale Militaire
1040 Brussels, BELGIUM

Robert Michie
Physics Dept.
University of Texas
Austin, TEXAS 78712

Charles Moeller
General Atomic
P.O. Box 81608
San Diego, CA 92138

George J. Morales
Physics Dept.
University of California
Los Angeles, CA 90024

Alfred Moretti
7012 Blackburn
Downers Grove, Ill. 60515

Robert W. Motley
Plasma Physics Lab.
Princeton University
Princeton, N.J. 08540

John H. Mullin
McDonnell Douglas Research
RR1 Box 253
Chesterfield, MO 63017

Masayuki Ono
Plasma Physics Lab
Princeton University
Princeton, N.J. 08540

Lena Oren
University of California
Physics Department
Los Angeles, CA 90024

E. Ott
Dept. of Elect. Eng.
Cornell University
Ithaca, N.Y. 14853

Francois J. Parlange
CENG SIG
85X 38041
Grenoble, CEDEX, France

N. R. Pereira
Lawrence Livermore Lab
B50A-3140
Livermore, CA 94550

Francis Perkins
Plasma Physics Lab
Princeton University
Princeton, N.J. 08540

Pete Petersen
General Atomic
P.O. Box 81608
San Diego, CA 92138

Joseph S. Pettibone
Lawrence Livermore Lab
P.O. Box 808
Livermore, CA 94550

M. L. Pool
Ohio State University
Dept. of Physics
Columbus, Ohio 43210

M. Porkolab
Massachusetts Inst. of Tech.
Room 36-213
Cambridge, MA 02139

Satish Puri
Max-Planck Institute
8046 Garching bei München
GERMANY

Steven Ratliff (UCLA)
1827 E. Michelle St
West Covina, CA 91791

Michael E. Read
Code 7740
Naval Research Lab
Washington, D.C. 20375

Mark Rhodes
15421 Harvard St
Santa Monica, CA 90404

John Riordan
General Atomic
P.O. Box 81608
San Diego, CA 92138

A. C. Riviere
Culham Laboratory
UKAEA Abingdon
OX14 3DB ENGLAND

Stephen Rosenthal
Electronics Research Lab
Stanford, CA 94305

N. Rynn
Physics Department
University of California
Irvine, CA

Francis Santini
Frascati
Euratom-CNEN Gas Ionizzati
C.P. 65, Frascati, ITALY

Motoyasu Sato
Plasma Physics Lab
Kyoto University
Gokasho, Uji, JAPAN

John Scharer
Electr. & Computer Eng. Dept.
University of Wisconsin
Madison, WIS. 53706

Abhijit Sen
Physics Department
M.I.T.
Cambridge, Mass. 02139

J. L. Shohet
Dept. of Elec. & Computer Eng.
University of Wisconsin
Madison, Wis. 53706

Mario Simonutti
TRW Systems R1-2020
Redondo Beach, CA 90278

J. C. Sprott
Dept. of Elec. & Computer Eng.
University of Wisconsin
Madison, Wis. 53706

Ron Stambaugh
General Atomic
P.O. Box 81608
San Diego, CA 92138

Phil Stone
U.S. Department of Energy
Washington, D.C. 20545

Hideo Sugai
Dept. of E.E.
Nagoya University
Nagoya, JAPAN

Gary Swanson
E.E. Department
U. Southern California
Los Angeles, CA 90007

Haruo Takeyama
Hiroshima University
Hiroshima 730, JAPAN

T. Tajima
Physics Department
U.C.L.A.
Los Angeles, CA 90024

Samuel Talmadge
2567 Boelter Hall
U.C.L.A.
Los Angeles, CA 90024

John Tataronis
NYU Courant Inst. of Math. Sci.
251 Mercer Street
New York, N.Y. 10012

R. J. Taylor
2567 Boelter Hall
U.C.L.A.
Los Angeles, Ca. 90024

C. B. Thorington
Dept. of Physics
U.C.L.A.
Los Angeles, CA 90024

Dr. James Turner
Div. Magnetic Fusion Energy
Department of Energy
Washington, D.C. 20545

Jean-Georges Wegrowe
CEN-G STG
85X 38041
Grenoble, CEDEX, France

W. S. Williamson
Hughes Research Lab
3011 Malibu Canyon Rd
Malibu, CA 90265

Bradford L. Wright
Los Alamos Scientific Lab
P.O. Box 1663
Los Alamos, N.M. 87544

Stewart Zweben
Physics Dept.
U.C.L.A.
Los Angeles, CA 90024

V. K. Tripathi
Physics & Astronomy Dept.
University of Maryland
College Park, MD 20742

N. A. Uckan
Union Carbide Corp
P.O. Box Y
Oak Ridge National Lab
Oak Ridge, Tenn 37831

Ki Whang
Dept. of Physics
U.C.L.A.
Los Angeles, CA 90024

John Willis
Rt. 13, Box 151
Frederick, MD 21701

S. V. Yadavalli
868 Thornwood Dr
Palo Alto, CA 94303

Y. Yasaka
Dept. of Electronics
Kyoto University
Kyoto 606, JAPAN

Alvin W. Trivelpiece
Maxwell Laboratories
8835 Balboa Ave
San Diego, CA 92123

Dale Vaslow
General Atomic
P.O. Box 81608
San Diego, CA 92138

David A. Whelan
2567 Boelter Hall
U.C.L.A.
Los Angeles, CA 90024

King-Lap Mong
Plasma Physics Lab
Princeton University
Princeton, N.J. 08540

Y. K. Yau
T.R.W.
Redondo Beach, CA 90278

Kiyoshi Yatsui
E.E. Department
Osaka University
Toyonaka, Osaka 560, JAPAN