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New Anomaly in Axial-Vector Ward Identity*

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Abstract

It is shown how a new type of anomaly, in addition to the Adler-Bell-Jackiw anomaly, can occur in a gauge theory with γ_5 couplings. Such an anomaly renders standard theories of quantum flavor dynamics non-renormalizable.

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One peculiar feature encountered in renormalization of a gauge theory involving γ_5 couplings is the ABJ anomaly[1] which destroys renormalizability unless it is cancelled.[2,3] Here we exhibit a new anomaly which further undermines renormalizability and which is not, in general, cancelled by the usual restrictions.

The crucial point is that the renormalized lagrangian must itself be gauge invariant so that gauge invariance is preserved order by order in the renormalized perturbation series and unitarity is satisfied.

Consider an abnormal-parity triangle embedded in a general Feynman diagram as indicated in Fig.1. The usual procedure[4] is first to regularize dimensionally all meson loops and define counter-terms by their $(4-n)^{-1}$ poles [here n = space-time dimensionality], then perform the Dirac trace for n=4 and finally regularize separately the femion loop momentum integral by any converient method. Such a procedure appears dangerous, especially when the abnormal-parity fermion loop is itself involved in an overlapping divergence; as a possible example, see Fig.2. That this danger is real will now be shown by a computation of the triangular vertex. Only a sufficient outline is presented; further details will be given elsewhere.[5]

Consider the triangle vertex with kinematics defined as shown in Fig.3. The full vertex ($\Gamma^{abc}_{\mu\nu\lambda}$) involves first adding the crossed diagram by

$$I^{abc}_{\mu\nu\lambda}(p_1,p_2;m_\alpha,m_\beta,m_\gamma;n) = T^{abc}_{\mu\nu\lambda}(p_1,p_2;m_\alpha,m_\beta,m_\gamma;n)$$

$$+ T^{bac}_{\nu\gamma\lambda}(p_2,p_1;m_\alpha,m_\beta,m_\gamma;n) \qquad (1)$$

and then summing over flavors $\{\alpha,\beta,\gamma\}$ which belong to a generation g of fermions where g contains sufficient flavors to cancel the ABJ anomaly

$$\Gamma^{abc}_{\mu\nu\lambda} = \sum_{\{\alpha,\beta,\gamma\}cg} I^{abc}_{\mu\nu\lambda}(p_1,p_2;m_\alpha,m_\beta,m_\gamma;n) \quad . \tag{2}$$

Let the flavor-group matrices $T^a_{\alpha\beta}$ be understood to include any relevant coupling constant as well as sign (±1) corresponding to right-or left-handed helicity. Then

$$T^{abc}_{\mu\nu\lambda} = T^a_{\alpha\beta}\, T^c_{\beta\gamma}\, T^b_{\gamma\alpha}\, t_{\mu\nu\lambda}(p_1,p_2;m_\alpha,m_\beta,m_\gamma;n) \tag{3}$$

Now, $t_{\mu\nu\lambda}$ is given by the usual Feynman rules as

$$t_{\mu\nu\lambda} = \int \frac{d^4k}{(2\pi)^4} \frac{1}{[(k+p_2)^2 - m_\gamma^2][k^2 - m_\alpha^2][(k-p_1)^2 - m_\beta^2]}$$

$$\mathrm{Tr}[\gamma_5(\not{k} + \not{p}_2 + m_\gamma)\gamma_\nu(\not{k} + m_\alpha)\gamma_\mu(\not{k} - \not{p}_1 + m_\beta)\gamma_\lambda] \tag{4}$$

Performing the Dirac trace (in 4 dimensions) and introducing Feynman parameters now gives

$$t_{\mu\nu\lambda} = \frac{8i}{(2\pi)^4} \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4k}{(k^2 + 2k\cdot Q - M^2)^3}$$

$$[T_{\delta\epsilon\zeta;\mu\nu\lambda}(k+p)_\delta k_\epsilon (k-p_1)_\zeta$$

$$+ \epsilon_{\mu\nu\lambda\alpha}(m_\alpha m_\beta (k+p)_\alpha - m_\beta m_\gamma k_\alpha + m_\gamma m_\alpha (k-p_1)_\alpha)] \tag{5}$$

where

$$Q_\mu = xp_{2\mu} - yp_{1\mu} \tag{6}$$

$$M^2 = m_\alpha^2(1-x-y) + \left(m_\gamma^2 - p_2^2\right)x + \left(m_\beta^2 - p_1^2\right)y \tag{7}$$

$$T_{\delta\epsilon\zeta,\mu\nu\lambda} = \epsilon_{\rho\delta\nu\epsilon}(g_{\rho\mu}g_{\zeta\lambda} - g_{\rho\zeta}g_{\lambda\mu} + g_{\rho\lambda}g_{\mu\zeta})$$

$$- \epsilon_{\rho\mu\zeta\lambda}(g_{\rho\delta}g_{\nu\epsilon} - g_{\rho\nu}g_{\delta\epsilon} + g_{\rho\epsilon}g_{\delta\nu}) \tag{8}$$

It is easiest to employ standard dimensional regularization formulae to perform the k-integration giving

$$t_{\mu\nu\lambda} = -\frac{1}{4\pi^2}\int \frac{dx\,dy}{(-Q^2-M^2)^{3-n/2}}\left[\Gamma\left(3-\frac{n}{2}\right)A_{\mu\nu\lambda} + (-Q^2-M^2)\,\Gamma\left(2-\frac{n}{2}\right)B_{\mu\nu\lambda}\right] \tag{9}$$

with the tensors $A_{\mu\nu\lambda}, B_{\mu\nu\lambda}$ given by

$$A_{\mu\nu\lambda} = \epsilon_{\mu\nu\lambda\alpha}\left(A^1 p_{1\alpha} + A^2 p_{2\alpha}\right) + \epsilon_{\mu\nu\alpha\beta}p_{1\alpha}p_{2\beta}\left(A^3 p_{1\lambda} + A^4 p_{2\lambda}\right)$$

$$+ \epsilon_{\mu\lambda\alpha\beta}p_{1\alpha}p_{2\beta}\left(A^5 p_{1\nu} + A^6 p_{2\nu}\right) + e_{\nu\lambda\alpha\beta}p_{1\alpha}p_{2\beta}\left(A^7 p_{1\mu} + A^8 p_{2\mu}\right) \tag{10}$$

$$B_{\mu\nu\lambda} = \epsilon_{\mu\nu\lambda\alpha}\left(B^1 p_{1\alpha} + B^2 p_{2\alpha}\right) \tag{11}$$

where

$$A_1 = -p_1\cdot p_2 y(1-y)(1-2x) + x(1-x)(1-y)p_2^2 - y^2(1-y)p_1^2$$

$$+ m_\alpha m_\beta y - m_\beta m_\gamma y + m_\gamma m_\alpha(y-1) \tag{12}$$

$$A_2 = -p_1 \cdot p_2 xy(1-2x) + x^2(1-x)p_2^2 - xy^2 p_1^2$$
$$+ m_\alpha m_\beta (1-x) + m_\beta m_\gamma x - m_\gamma m_\alpha x \tag{13}$$

$$A_3 = -A_7 = -y(1-y) \tag{14}$$

$$A_4 = -A_8 = xy \tag{15}$$

$$A_5 = y(1-2x-y) \tag{16}$$

$$A_6 = -x(2-2x-y) \tag{17}$$

$$B_1 = (3y-1) + \tfrac{1}{2}(4-n)(1-y) \tag{18}$$

$$B_2 = (1-3x) + \tfrac{1}{2}(4-n)x \tag{19}$$

The vector and axial-vector Ward identities require contraction of $t_{\mu\nu\lambda}$ with $p_{1\mu}$, $p_{2\nu}$, q_λ respectively. Then we use

$$p_{1\mu}A_{\mu\nu\lambda} = \epsilon_{\nu\lambda\alpha\beta}p_{1\alpha}p_{2\beta}\left[x(-Q^2-M^2) + p_1^2 y(1-x-y) + (\delta m^2)_1\right] \tag{20}$$

$$p_{2\nu}A_{\mu\nu\lambda} = \epsilon_{\mu\lambda\alpha\beta}p_{1\alpha}p_{2\beta}\left[-y(-Q^2-M^2) - p_2^2 x(1-x-y) + (\delta m^2)_2\right] \tag{21}$$

$$q_\lambda A_{\mu\nu\lambda} = \epsilon_{\mu\nu\alpha\beta}p_{1\alpha}p_{2\beta}\left[(x+y-2)(-Q^2-M^2) + \left(p_1^2 y + p_2^2 x\right)(1-x-y) + (\delta m^2)_3\right] \tag{22}$$

$$(\delta m^2)_1 = x\left[m_\alpha^2(1-x-y) + m_\gamma^2 x + m_\beta^2 y\right]$$
$$+ m_\alpha m_\beta (1-x) + m_\beta m_\gamma x - m_\gamma m_\alpha x \tag{23}$$

$$(\delta m^2)_2 = -y\left[m_\alpha^2(1-x-y) + m_\gamma^2 x + m_\beta^2 y\right] + m_\alpha m_\beta y - m_\beta m_\gamma y + m_\gamma m_\alpha (y-1) \tag{24}$$

$$(\delta m^2)_3 = (x+y-2)\left[m_\alpha^2(1-x-y) + m_\gamma^2 x + m_\beta^2 y\right] + m_\alpha m_\beta (1-x-y) - m_\beta m_\gamma y + m_\gamma m_\alpha (y-1) \tag{25}$$

The corresponding results for $B_{\mu\nu\lambda}$ can be read off immediately from Eqs.11,18 and 19.

Now we systematically expand the right-hand-side of the Ward identities in (4-n) using

$$\Gamma(3 - \tfrac{n}{2}) = 1 + \tfrac{1}{2}(4-n)\Gamma'(1) + \cdots \tag{26}$$

$$\Gamma(2 - \tfrac{n}{2}) = 2(4-n)^{-1} + \Gamma'(1) + \cdots \tag{27}$$

$$\frac{1}{(-Q^2-M^2)^{2-n/2}} = 1 - \tfrac{1}{2}(4-n)\,\ln(-Q^2-M^2) + \tfrac{1}{8}(4-n)^2 \ln^2(-Q^2-M^2) + \cdots \tag{28}$$

We insist that the vector Ward identities be maintained since they involve the electromagnetic current conservation. Thus we write

$$t'_{\mu\nu\lambda} = t_{\mu\nu\lambda} + c_{\mu\nu\lambda} \tag{29}$$

and <u>define</u> the contact term $c_{\mu\nu\lambda}$ such that

$$p_{1\mu} t'_{\mu\nu\lambda} = 0 \tag{30}$$

$$p_{2\nu} t'_{\mu\nu\lambda} = 0 \tag{31}$$

A sufficiently general decomposition is

$$c_{\mu\nu\lambda} = \epsilon_{\mu\nu\lambda\alpha}\left(\gamma^1 p_{1\alpha} + \gamma^2 p_{2\alpha}\right) \tag{32}$$

and the functions γ^1, γ^2 are provided as power series in (4-n) by the requirements of Eqs.30,31.

We may then compute the (unique) result for the axial-vector Ward identity. After straightforward algebra, the result is

$$q_\lambda t'_{\mu\nu\lambda} = \frac{1}{4\pi^2}\, \epsilon_{\mu\nu\alpha\beta} p_{1\alpha} p_{2\beta} [\alpha_0 + \alpha_1 (4-n) + ---] \tag{33}$$

with

$$\alpha_o = 1 \tag{34}$$

This is the ABJ anomaly, and is cancelled in Eq.2 above provided that

$$\mathrm{Tr}[T^c\{T^a,T^b\}_+] = 0 \tag{35}$$

which is the usual cancellation condition.[2]

More interesting is

$$\alpha_1 = \tfrac{1}{2}\Gamma'(1)\alpha_o - \int_0^1 dx \int_0^{1-x} dy\, \ell n(-Q^2-M^2) \tag{36}$$

The first term in Eq.36 is harmless, but the second term makes the flavor sum in Eq.2 intractable, in general, because M^2 depends on $\{\alpha,\beta,\gamma\}$ through the fermion masses in Eq.7. This term depends on the energies p_1^2, p_2^2, q^2 and hence a necessary and sufficient condition for its cancellation (which must hold for <u>all</u> energies) is degeneracy of $m_\alpha, m_\beta, m_\gamma$ as would be the case in an abelian theory without non-diagonal matrix elements.

Note that although we have chosen to employ dimensional regularization, the result is expect to be independent of this choice.

Also, the new anomaly occurs in an individual Feynman diagram (or two diagrams if we count the crossed diagram) and inter-diagrammatic cancellations are possible. Because 3-loop diagrams are involved, checking this explicitly is technically difficult.

To conclude and summarise, let us note the two possibilities for the standard SU(2) $\times$ U(1) model of leptons[6] extended to quarks with the GIM mechanism[7] and further extended to six flavors to accommodate CP violation[8] and experiment.[9,10]

The first possibility is that the new anomaly is cancelled between different Feynman diagrams at fixed order in perturbation theory. That such miraculous cancellation occur might be suggested by the fact that in a temperature bath at sufficiently high T the symmetry is restored, the fermions are mass-degenerate (massless) and the new anomaly is absent. But this requires both that all quark and lepton masses arise from the Higgs mechanism and that the cancellation be demonstrated.

The alternative possibility is that the cancellation simply does not occur. Then either the quark-lepton generations such as (e,u,d), (μ,c,s), (τ,t,b) are mass-degenerate--such degeneracy is certainly ruled out physically--or the usual quantum flavor dynamics is shown to be non-renormalizable.

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Figure Captions

Fig. 1 General Feynman diagram containing triangle anomaly.

Fig. 2 Specific example of overlapping divergence.

Fig. 3 Kinematics for triangle diagram.

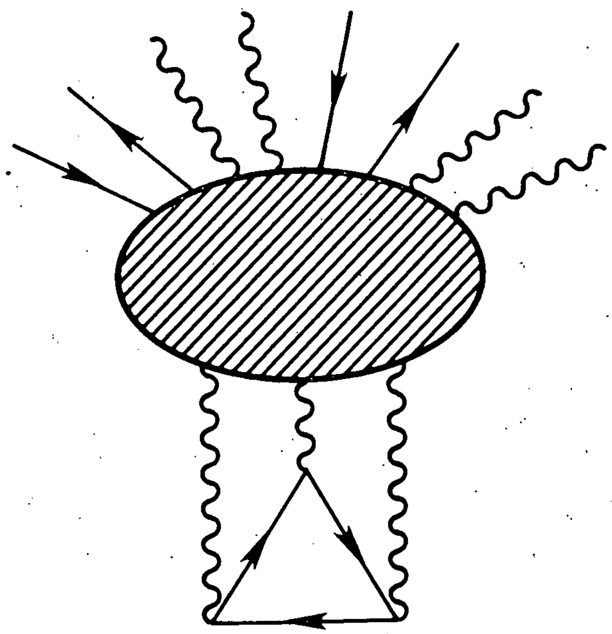


Fig. 1

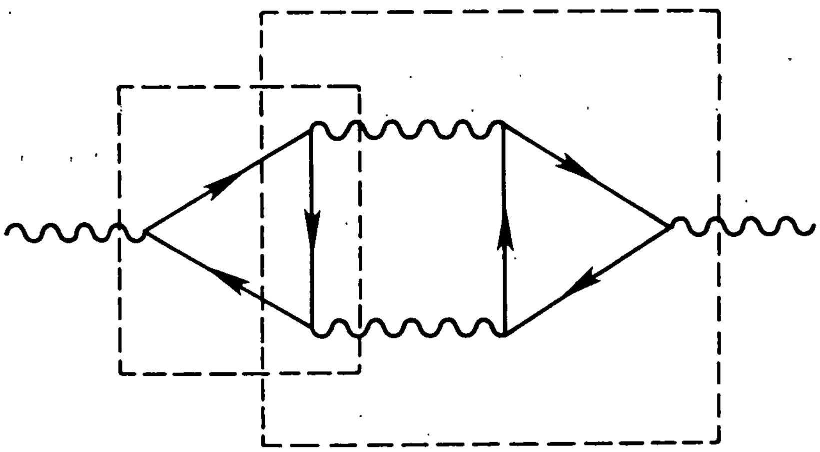
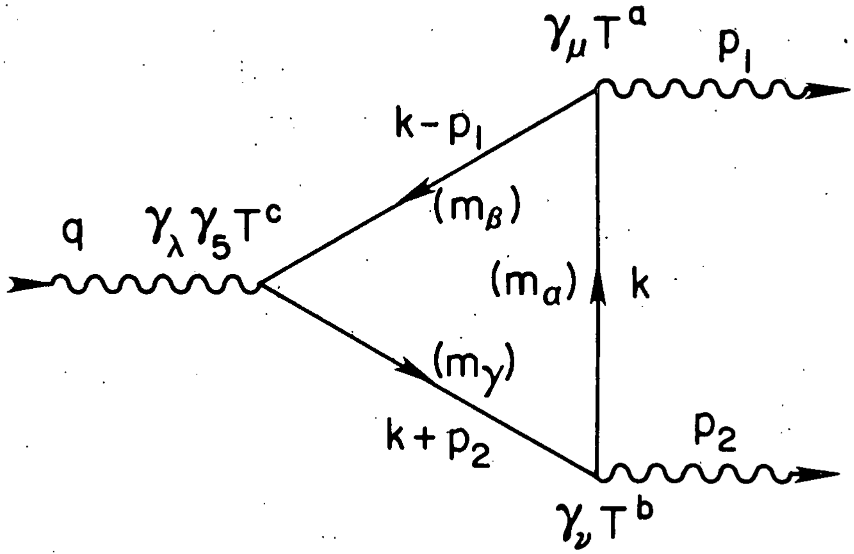


Fig. 2



$\gamma_\mu T^a$
p_1
$k-p_1$
(m_β)
q
$\gamma_\lambda \gamma_5 T^c$
(m_α)
k
(m_γ)
$k+p_2$
p_2
$\gamma_\nu T^b$

Fig. 3