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BOSON FERMION SYMMETRIES AND DYNAMICAL SUPERSYMMETRIES FOR ODD-ODD NUCLEI

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Abstract

The concept of boson-fermion symmetries and supersymmetries is applied to odd-odd nuclei.

The approach to even-even and odd-even nuclei based on nuclear symmetries in IBM/IBFM has received much attention in recent years [ARI76, ARI78, ARI79,BAL81,BAL83,IAC79,IAC80].

In this report we discuss the extension of this concept to odd-odd nuclei. Odd-odd nuclei provide richer and more complex structure, and the residual proton-neutron interaction appears explicitly in the boson-fermion interaction.

Odd-odd nuclei are described as mixed system of bosons and fermions (proton and neutron) by the Hamiltonian

$$H = H_B + H_F + V_{BF} \ . \qquad (1)$$

Here, H_B is identical to the IBM Hamiltonian, H_F includes one-fermion and fermion-fermion interaction terms and V_{BF} is the boson-fermion interaction. The computer code for diagonalizing Hamiltonian (1) for odd-odd nuclei has been recently written [VRE84]. As a residual proton-neutron force the surface delta-, spin- and tensor-interaction were included. Computations have been performed for some particular cases [BRA84,VRE84,PAA84, PAA85,MEY85].

Particularly, the Hamiltonian (1) was diagonalized in the case of a proton particle j_p and a neutron particle j_n coupled to the SU(3) core. The computed energy pattern exhibits two regular low-lying bands based on the states of angular momenta $J = j_p + j_n$ and $J = |j_p - j_n|$ [PAA84,PAA85, PAA85a]. In comparison to the rotational model, these two bands are the truncated analogs of the Gallagher-Moszkowski bands based on the Nilsson

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states $\mu_p=j_p$, $\mu_n=j_n$ [PAA84] . It was shown [PAA85a] that the wave functions of the states of $J=j_p+j_n$ band can be brought to the form of the approximate SUSY wave function of the analogs of Nilsson states in odd-A nuclei [SUN84].The energies of this band follow the J(J+1) energy rule, with the same moment of inertia as for the boson core.

The band structure in odd-odd nuclei has been also explored in the case of O(6) boson core [BRA84].

Here we discuss the boson-fermion system for odd-odd nuclei using the concept of dynamical symmetry and supersymmetry. The dimension of the fermionic subspace is $n=n_\pi+n_\nu$, where $n_\pi(n_\nu)$ is the total number of components of the angular momenta of the unpaired protons (neutrons). One can construct the $(n_\pi+n_\nu)^2$ generators of the group $U_F(n_\pi+n_\nu)$, where the subscript F reminds that a fermionic realization is employed. In general, the group structure of the Hamiltonian is

$$U_B(6) \times U_F(n_\pi+n_\nu) \quad . \tag{2}$$

There is a second, alternative approach. One could assume that the unpaired neutron and the unpaired proton form a quasibound state. The total number of components of the angular momenta of this quasi-bound state is given by $n_\pi n_\nu$. Then we introduce a pair of new bosonic creation and annihilation operators associated with each level of this subsystem, c_I^+ ,c_J, I,J = $1,2,\ldots,n_\pi n_\nu$. The $(n_\pi n_\nu)^2$ operators $\bar{G}_{IJ} = c_I^+ c_J$ constitute a bosonic realization of the $U(n_\pi n_\nu)$ algebra. The group structure of the corresponding Hamiltonian is

$$U_B(6) \times U_F(n_\pi n_\nu), \tag{3}$$

where the appropriate representation of $U(n_\pi n_\nu)$ is the fundamental representation (1,0,0,...,0).

To find analytical solutions to the eigenvalue problem of the either Hamiltonian, associated with (2) or (3), the key idea is the use of the isomorphisms between groups in the two chains, one starting with $U_B(6)$ and the other one starting with either $U_F(n_\pi+n_\nu)$ or $U_F(n_\pi n_\nu)$. (This idea is along the line of approach to boson-fermion symmetries for odd-even nuclei, which was introduced in ref. [IAC80].) Groups obtained by joining two chains transform simultaneously bosons into bosons and fermions into fermions.

Some special solutions associated with the group structure (3) have been studied by two of us [HÜB84, PAA85,HÜB85a].

In the recent work [BAL85] we give a detailed study of various level schemes for odd-odd nuclei obtained by analytical solutions of Hamiltonian associated with either (2) or (3). The isomorphisms between the two group chains are elaborated for the case where the unpaired nucleons occupy some or all of levels with j = 1/2,3/2,5/2 and the analytical expressions for the corresponding energy eigenvalues are given.

In a further step, such solutions are embedded into a supergroup and new chains arising from such embedding are given.

Now we look for correlations in the spectra of the four neighboring nuclei: the even-even nucleus with (Z,A),the odd-even nucleus with (Z,A+1), the even-odd nucleus with (Z+1,A+1), and the odd-odd nucleus with (Z+1,A+2). Such correlations arise if the Hamiltonian associated with (2) has a supergroup structure $U(6/n_\pi+n_\nu)$. Consequently, the properties of these four nuclei could be related by the symmetry operations of this supergroup. In particular, we have obtained analytical expressions for the eigenvalues of this Hamiltonian in terms of the eigenvalues of the Casimir operators of chains of supergroups starting with $U(6/n_\pi+n_\nu)$ and terminating with $Spin^{B+F}(3)$ [BAL85] . Such a situation was termed a dynamical supersymmetry in the previous investigations of odd-even nuclei [BAL81].

The concept of dynamical supersymmetries was successfully used to connect the properties of even-even nuclei with the neighboring odd-even nuclei. If we want to study, say, the correlation between an even-even nucleus and the next, odd-proton nucleus, the first decomposition in the supergroup chain is

$$U(6/n_\pi + n_\nu) \supset U(6/n_\pi) \times U(n_\nu) \ . \tag{4}$$

One can then continue the chain with the decompositions of the supergroup $U(6/n_\pi)$ as was done in ref. [BAL81].

The appropriate representation of the group $U(n_\nu)$ in the case of an odd-proton nucleus is a singlet, hence the existence of the group $U(n_\nu)$ in the chain does not affect any of the quantum numbers. A similar situation arises in the case of correlations between an even-even nucleus and the neighboring odd-neutron nucleus.

In this case we start with, what we call, the canonical decomposition

$$U(6/n_\pi + n_\nu) \supset U_B(6) \times U_F(n_\pi + n_\nu) \supset U_B(6) \times U_F(n_\pi) \times U_F(n_\nu) , \tag{5}$$

and continue the chains in various possible forms [BAL85]. The difference to the case of boson-fermion dynamical symmetries is that several nuclei are now placed in the same supermultiplet. Consequently, parameters appearing in the energy formulae take the same values for all nuclei in the same supermultiplet.

As an illustration of dynamical supersymmetry with canonical decomposition we construct a supermultiplet starting from the even-even nucleus ^{194}Pt. In this region the proton shell is dominated by $j = 3/2$ and the neutron shell by $j = 1/2, 3/2, 5/2$. Hence $n_\pi = 4$ and $n_\nu = 12$. The relevant representation of the appropriate supergroup $U(6/16)$ is the one with $\mathcal{N} = 7$. Various nuclei are placed in the tensor product representations as follows:

$$U(6/16) \supset U_B(6) \times U_F(16) \supset U_B(6) \times U_F^{(\pi)}(4) \times U_F^{(\nu)}(12)$$

$$\begin{aligned}
|7\} &= ([7],\{0\}) &&= \underset{\underline{^{194}\mathrm{Pt}}}{([7],\{0\},\{0\})} \\
&+ ([6],\{1\}) &&= \underset{^{195}\mathrm{Au}}{([6],\{1\},\{0\})} + \underset{^{195}\mathrm{Pt}}{([6],\{0\},\{1\})} \\
&+ ([5],\{1^2\}) &&= \underset{^{196}\mathrm{Hg}^*}{([5],\{1^2\},\{0\})} + \underset{^{196}\mathrm{Au}}{([5],\{1\},\{1\})} + \underset{^{196}\mathrm{Pt}^*}{([5],\{0\},\{1^2\})} \\
&+ \ldots
\end{aligned} \tag{6}$$

In the above expression asterisk over a given symbol denotes the two-quasiparticle states in that nucleus. Eq. (6) is illustrated below

Number of unpaired neutrons \ Number of unpaired protons	0	1	2
0	$^{194}_{78}$Pt	$^{195}_{79}$Au	$^{196}_{80}$Hg*
1	$^{195}_{78}$Pt	$^{196}_{79}$Au	$^{197}_{80}$Hg**
2	$^{196}_{78}$Pt*	$^{197}_{79}$Au**	$^{198}_{80}$Hg***

Asterisk * denotes two-quasiparticle states in even-even and ** denotes three-quasiparticle states in odd-even nuclei. Dashed line separates nuclei belonging to the same representation of the direct product group $U_B(6) \times U_F(n_\pi + n_\nu)$.

A new possibility of supersymmetry arises when $n_\pi = n_\nu = n$. In this case, using fermionic creation and annihilation operators it is possible to construct the generators of the symplectic group Usp (2n). Consequently a supergroup chain starting with decomposition into the orthosymplectic group

$$U(6/2n) \supset Osp\ (6/2n) \qquad (7)$$

might be relevant in such cases. The properties of orthosymplectic supergroups are studied in refs. [JAR79,BAL82]. The appealing aspect of this case is that it emphasizes the residual force between the unpaired neutron and the unpaired proton, while retaining the supersymmetry scheme. The main problem, however, is that this group decomposition does not conserve the boson number N.

However, there is an intermediate situation with the decomposition

$$U(6/2n) \supset SU_B(6) \times SU_F(2n) \supset SU_B(6) \times Sp_F(2n) \qquad (8)$$

The representation of Sp(2n) contains the n-dimensional fundamental representation (1,0,0,...,0,0) of SU(n), which we denote $\square$ and its conjugate representation (1,1,1,...,1,0) which we denote $\boxdot$.

In general, the proper bases to describe nuclei with one unpaired nucleon, would be given by linear combinations of the bases of SU(n):

$$|\text{isotope}\rangle = \cos\Theta\,|\square\rangle + \sin\Theta\,|\boxdot\rangle \qquad (9)$$

$$|\widetilde{\text{isotope}}\rangle = -\sin\Theta\,|\square\rangle + \cos\Theta\,|\boxdot\rangle \qquad (10)$$

where $|\square\rangle$ denotes the basis of the representation $\square$ and $|\boxdot\rangle$ denotes that of $\boxdot$. Here we consider the case $\Theta = 0$, but the results can easily be generalized for the finite Θ case. (The choice $\Theta = 0$ would be physically transparent if, say, the unpaired neutrons are particles and the unpaired protons are holes, or vice versa, since it is reasonable to consider conjugate representations for holes.)

A good place to look for such a supersymmetry is again the Pt-Au region. In this region the odd neutron and proton occupy mostly levels with $j = 1/2, 3/2, 5/2$. In this case we have $n_\pi = n_\nu = n = 12$. The resulting supersymmetry has then a U(6/24) structure. Again if we start from the even-even nucleus ^{194}Pt with $\mathcal{N} = 7$ and O(6) core, we employ the tensor product representation

$$U(6/24) \supset U_B(6) \times U_F(24) \supset U_B(6) \times Sp_F(24) \supset U_B(6) \times SU_F(12) \qquad (11)$$

which gives the corresponding energy formula [BAL85]. A typical spectrum of the low-lying states in Sp(24) scheme is shown in fig.1 and compared to the experimental states of ^{196,198}Au.

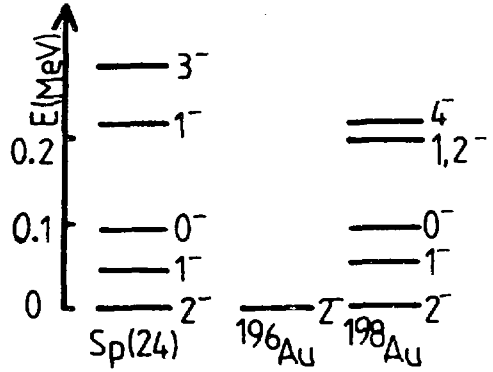

Fig.1 Sp(24) scheme for low-lying states in comparison to the experimental data in ^{196,198}Au.

A comparison of the Sp(24) spectrum with the experimental spectra of ^{196}Au and ^{198}Au seems encouraging. This is particularly interesting since the previous attempts to describe such nuclei using the canonical chain could not account for the observed ground state spin [BAL,BAR84]. Experimental studies of the odd-odd nuclei is very desirable to decide whether or not the Sp (24) chain is applicable in this region.

As another illustrative application of supersymmetry extension to odd-odd nuclei we consider the spectrum of odd-odd $^{62}_{29}Cu_{33}$ if one assumes that the even-even nucleus $^{64}_{30}Zn_{34}$, odd-even nucleus $^{63}_{29}Cu_{34}$ and odd-odd nucleus $^{62}_{29}Cu_{33}$ correspond to the members of the same super-multiplet. In a simplified presentation with odd proton and odd neutron restricted only to $j = 3/2$ configurations and with $U_B(5)$ boson core, the resulting energy formula [BAL85] gives the spectrum presented in fig.2.

We note that this spectrum is obtained using the supersymmetry relation to the neighboring nuclei ^{63}Cu and ^{64}Zn, without adjusting any parameter to ^{62}Cu.

We note that the idea of symmetry and supersymmetry was further extended to hypernuclei by two of us [HÜB85,HÜB85a, PAA85].

Concluding, we point out that the application of the idea of dynamical symmetry and supersymmetry to odd-odd nuclei leads to a series of energy formulas, and gives a new insight into the nuclear structure. Therefore, more detailed studies of odd-odd nuclei are highly desirable.

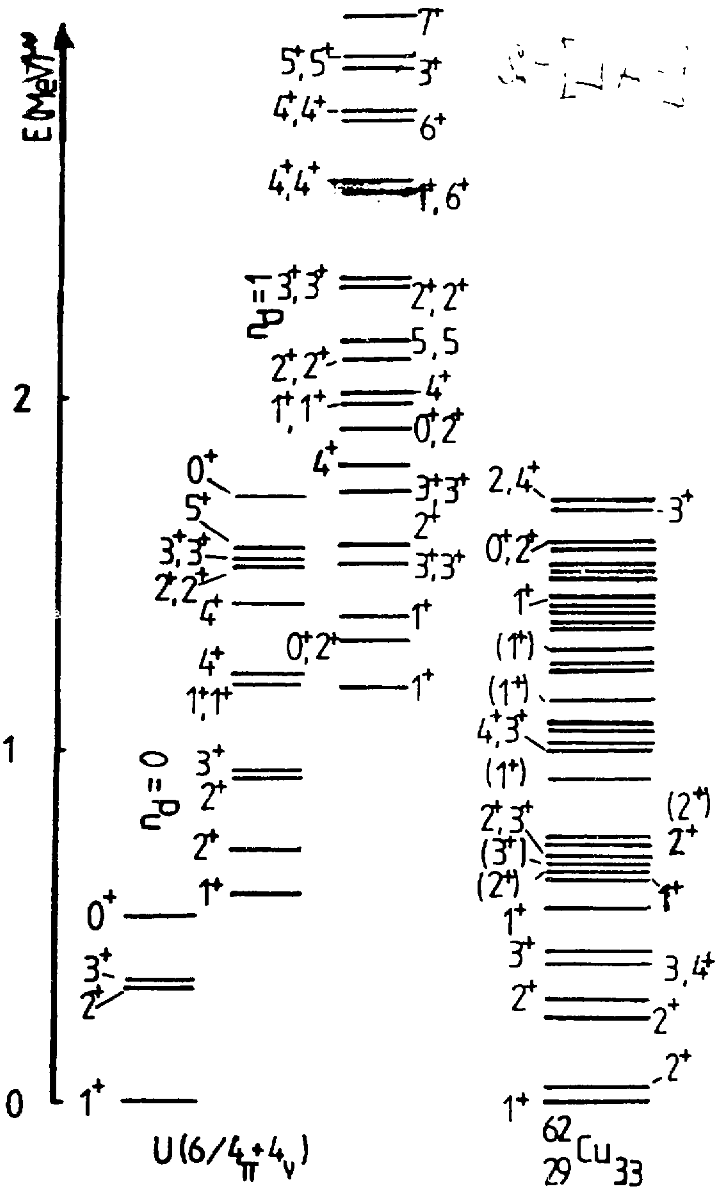

Fig.2 $U(6/4_\pi + 4_\nu)$ spectrum of ^{62}Cu in comparison to the experimental states.

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