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THE BILINEAR NODAL TRANSPORT METHOD IN WEIGHTED DIAMOND DIFFERENCE FORM*

by

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Nodal methods have been developed and implemented for the numerical solution of the discrete-ordinates neutron transport equation.[1,2] Numerical testing of these methods and comparison of their results to those obtained by conventional methods have established the high accuracy of nodal methods.[1,2] Furthermore, it has been suggested that the linear-linear approximation is the most computationally efficient, practical nodal approximation.[2] Indeed, this claim has been substantiated by comparing the accuracy in the solution, and the CPU time required to achieve convergence to that solution by several nodal approximations, as well as the diamond difference scheme.[2,3]

Two types of linear-linear nodal methods have been developed in the literature: analytic linear-linear (NLL) methods in which the "transverse-leakage" terms are devied analytically,[1,2] and approximate linear-linear (PLL) methods in which these terms are approximated.[2-4] In spite of their higher accuracy, NLL methods result in very complicated discrete-variable equations that exhibit a high degree of coupling, thus requiring special solution algorithms. On the other hand, the sacrificed accuracy in PLL methods is compensated for by the simple discrete-variable equations and diamond difference-like solution algorithm.[3-5] In this Summary we outline the development of an NLL nodal method, the Bilinear method,[6] which can be written in a weighted diamond difference (WDD) form with one spatial weight per dimension which is analytically derived, rather than preassigned in an *ad hoc* fashion.

First, the domain of the problem is divided into computational cells, or nodes, which in two-dimensional Cartesian geometry are closed rectangles whose edges are parallel to the x- and y-axes. Zeroth and first order (i.e., up to bilinear) spatial moments of the discrete-ordinates transport equation are taken yielding expressions for the local balance of the neutron flux moments over each node. These expressions involve zeroth, first and bilinear nodal moments, as well as zeroth and first order transverse moments (evaluated at node surfaces) of the angular flux. [Nodal and transverse moments of the flux have been defined previously; see Refs. 1 or 3.] Additional equations relating these quantities are needed in order to close the set of discrete-variable equations. In conventional WDD schemes, equations of the form

$$\psi_n = \frac{(1+\alpha_n)}{2}\,\psi_n(+a) + \frac{(1-\alpha_n)}{2}\,\psi_n(-a) \;, \tag{1}$$

are assumed where ψ_n and ψ_n (±a) are the nodal moment and the y-moment evaluated at the node x-surfaces, of the angular flux in the nth angular direction, respectively, and α_n is a preassigned x-direction spatial weight associated with direction n. An equation similar to Eq. (1), but with y-dependent quantities on the RHS is also used. The flux moments appearing in Eq. (1) represent only the zeroth moment of the flux, since higher order moments are not commonly incorporated in WDD methods.

In order to derive the WDD form, we take the zeroth x- and the zeroth y-moments (0,0) of the transport equation, then separately take its first x- and zeroth y-moments (1,0) to obtain

$$\frac{\mu_n}{2a}\left[\psi_{n,x,0}(+a)-\psi_{n,x,0}(-a)\right] + Y_{n,0,0}+\sigma\psi_{n,0,0} = S_{n,0,0} , \tag{2}$$

$$\frac{\mu_n}{2a}\left[\psi_{n,x,0}(+a)-2\psi_{n,0,0}+\psi_{n,x,0}(-a)\right] + Y_{n,1,0}+\sigma\psi_{n,1,0} = S_{n,1,0} , \tag{3}$$

respectively, where Y_n is the y-leakage of the nth angular flux, and $\psi_{n,x,0}$ ($\pm a$) are the transverse-averaged nth angular flux evaluated at x = $\pm a$. Next, we transverse-average the transport equation with respect to y and integrate it exactly using an integrating factor of the form: exp $(\sigma x/\mu_n)$. We expand the integrating factor in a Legendre polynomial of order one cnly in the integral involving the leakage and source moments, to obtain

$$\frac{\mu_n}{2a}\left[e^{\sigma a/\mu_n}\ \psi_{n,x,0}(+a)-e^{-\sigma a/\mu_n}\ \psi_{n,x,0}(-a)\right] =$$

$$E_{n,0}\left[S_{n,0,0}-Y_{n,0,0}\right] + E_{n,1}\left[S_{n,1,0}-Y_{n,1,0}\right] . \tag{4}$$

The expansion coefficients of the integrating factor, $E_{n,0}$ and $E_{n,1}$, are given by

$$E_{n,0} \equiv \frac{1}{\epsilon_n}\sinh(\epsilon_n) ,$$

$$E_{n,1} \equiv \frac{3}{\epsilon_n}\left[\cosh(\epsilon_n) - \frac{1}{\epsilon_n}\sinh(\epsilon_n)\right] , \quad \epsilon_n \equiv \sigma a/\mu_n . \tag{5}$$

Finally, we use Eqs. (2) and (3) to eliminate the RHS of Eq. (4), to obtain

$$\psi_{n,(0,m)} + 3\,\alpha_n \psi_{n,(1,m)} = \frac{(1+\alpha_n)}{2}\,\psi_{n,m}(+a) + \frac{(1-\alpha_n)}{2}\,\psi_{n,m}(-a), \quad m = 0,1 , \tag{6}$$

for m = 0, where

$$\alpha_n \equiv \frac{1-(1/\epsilon_n)\tanh(\epsilon_n)}{\tanh(\epsilon_n) - (3/\epsilon_n)[1-(1/\epsilon_n)\tanh(\epsilon_n)]}, \quad \epsilon_n = \sigma_T a/\mu_n . \tag{7}$$

The m = 1 case can be derived similarly by taking the (1,0) and (1,1) moments of the transport equation in the first step, and by taking its first y-transverse moment then applying the integrating factor and proceeding in a similar fashion as above.

Equation (6) is a generalization of the WDD equation, Eq. (1), to include a linear spatial component of the flux in addition to the constant component present in conventional WDD schemes. Moreover, it is an analytic LL nodal method, even though it has a simpler form than existing PLL schemes,[3,4] which permits using a WDD algorithm to solve the discrete-variable equations. Also, it requires the storage of only one parameter, α_n, per node, per dimension, per distinct discrete-ordinate, compared to a larger storage requirement even in current PLL methods.[3] It is worth noting that the simplification of the final form of NLL methods presented here is possible only because we have retained the bilinear flux moment which has been ignored by previous authors.[2-4]

In order to verify the derivation of our method we implemented our equations in a computer program, and used it to solve a one-group sion of the well-logging problem[3] on various meshes, with an S-6 EQN type angular quadrature, and a 10^{-5} pointwise relative convergence criterion. The quantity of interest in this problem is the response of the detector located in the steel region[3] 8 cm above the source. The average flux in the detector as calculated by the Bilinear method is compared in Table 1 to that calculated by other nodal methods,[7] as well as to the h^2-extrapolated value.[7] The comparison indicates the correctness and very high accuracy of our Bilinear nodal method.

We have shown that a highly accurate NLL nodal scheme, the Bilinear Nodal method, for the two-dimensional neutron transport equation can be written in a simple, single spatial-weight, WDD form, so that the final equations can be solved via a WDD algorithm. This result can be generalized to three-dimensional geometries, and higher order spatial approximations (e.g., quadratic-quadratic). Users familiar only with WDD methods now have easy access to the high accuracy and computational efficiency of NLL nodal methods through the method described here.

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Table 1. Comparison of the detector response in the well-logging problem[3] as calculated by several nodal methods. The h^2- extrapolated value is 1.7170.[7] Except for the Bilinear Method results, all entries were obtained from Ref. 6. The % error [≡ 100×(ϕ-1.717)/1.717] for each value is given in parentheses.

Node Size	CL Method [Ref. 1]	CQ Method [Ref. 1]	LN Method [Ref. 4]	BL Method
8 cm	1.0826 (-36.9)	1.3434 (-21.8)	1.6475 (-4.0)	1.4517 (-15.5)
4 cm	1.5709 (-8.51)	1.6030 (-6.6)	1.7326 (.9)	1.6840 (-1.9)
2 cm	1.6795 (-2.2)	1.6831 (-2.0)	1.7188 (.1)	1.7137 (-.2)