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Effects of Non-Zero Dispersion at Crab Cavities on the Beam Dynamics*

Yong Ho Chin

Exploratory Studies Group
Accelerator & Fusion Research Division
Lawrence Berkeley Laboratory, Berkeley, CA 94720

Introduction

The idea of crab crossing, which has been proposed to allow a non-zero crossing angle without a loss of luminosity, is based on avoiding the excitation of synchro-betatron resonances in a storage ring collider.[1] The validity of the crab crossing scheme relies on the cancellation of kick effects at a crab cavity by those at another crab cavity located on the other side of the interaction point (IP). For the effects to be cancelled exactly, the energy change due to the beam-beam interaction must also be cancelled by these two cavities. We can show, however, that the effect of the energy change is not cancelled exactly if the dispersion, η, and its derivative with respect to s, η', are non-zero at the crab cavities. Consequently, the crab crossing scheme may induce synchro-betatron resonances with, and even without, the beam-beam effect. We show an example of stopbands due to synchro-betatron resonances when only crab kicks are taken into account. We also present a stability criterion that can be used to determine tolerable values of η and η', or the crossing angle, when beam-beam effects are included.

Matrix Analysis

The first step is to calculate the transformation matrix of a crab kick for non-zero dispersion at the crab cavity. Here we assume that the crossing is done in the horizontal plane. We define the longitudinal coordinate, τ, by the longitudinal position relative to the center of the bunch, and the horizontal displacement of a particle, x, measured

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from the beam axis. Their canonical conjugates are the horizontal angle, x', and the relative energy deviation, δ, respectively. We restrict ourselves to the case where the kick is linear in these coordinates. The transformation matrix for zero dispersion is already known: it is given by

$$\begin{pmatrix} x \\ x' \\ \tau \\ \delta \end{pmatrix}_{\text{after a kick}} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & a & 0 \\ 0 & 0 & 1 & 0 \\ a & 0 & 0 & 1 \end{pmatrix}}_{\equiv \mathbf{K}} \begin{pmatrix} x \\ x' \\ \tau \\ \delta \end{pmatrix}_{\text{before a kick}} \tag{1}$$

where

$$a = \frac{\tan\varphi}{\beta_r} \tag{2}$$

with $\beta_r \equiv \sqrt{\beta_x^* \beta_c}$, φ is half the crab crossing angle, β_x^* and β_c are the beta functions at the IP and at the crab cavity, respectively. The non-zero (4,1)element of the matrix $\mathbf{K}$ is essential to ensure symplecticity.[2] The emergence of this term may be understood as follows: symplecticity means that the equations of motion can be derived from a Hamiltonian(say, $\mathcal{H}$).

$$\frac{dx'}{ds} = \frac{\partial \mathcal{H}}{\partial x} = a\tau \tag{3}$$

Therefore,

$$\mathcal{H} = ax\tau + \cdots. \tag{4}$$

Then, the equation of motion of energy is

$$\frac{d\delta}{ds} = \frac{\partial \mathcal{H}}{\partial \tau} = ax + \cdots. \tag{5}$$

When there is non-zero dispersion, the betatron motion and the synchrotron motion are coupled. The transformation matrix from the pure transverse $(\tilde{x}, \tilde{x}')$ and the pure longitudinal $(\tilde{\tau}, \tilde{\delta})$ coordinates to the mixed coordinate (x, x', τ, δ) is[3]

$$\begin{pmatrix} x \\ x' \\ \tau \\ \delta \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & \eta \\ 0 & 1 & 0 & \eta' \\ -\eta' & \eta & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\equiv \mathbf{E}} \begin{pmatrix} \tilde{x} \\ \tilde{x}' \\ \tilde{\tau} \\ \tilde{\delta} \end{pmatrix} \tag{6}$$

The transformation matrix of a crab kick for non-zero dispersion, $\mathbf{T}_1$, is then given by

$$\mathbf{T}_1 = \mathbf{E}\mathbf{K}\mathbf{E}^{-1}. \tag{7}$$

The explicit form is

$$\mathbf{T}_1 = \begin{pmatrix} 1-a\eta & 0 & 0 & -a\eta^2 \\ -2a\eta' & 1+a\eta & a & -a\eta\eta' \\ a\eta\eta' & -a\eta^2 & 1-a\eta & 0 \\ a & 0 & 0 & 1+a\eta \end{pmatrix} \tag{8}$$

The transformation matrix from the crab cavity(say, #1) to the IP, $\mathbf{R}_1$, is given by

$$\mathbf{R}_1 = \begin{pmatrix} 0 & \beta_r & 0 & 0 \\ -\frac{1}{\beta_r} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{9}$$

where the phase advance between crab cavity #1 and the IP is $\pi/2$. Here, we have assumed that the Twiss parameter α is zero at both the crab cavity and the IP for simplicity. (This assumption is satisfied in Ritson's design of a crab crossing scheme that will be described later.) Similarly, the transformation matrix from the IP to the second crab cavity on the other side of the IP, where Twiss parameters are mirror symmetrical to those at crab cavity #1 with respect to the IP, is given by

$$\mathbf{R}_2 = \begin{pmatrix} 0 & \beta_r & 0 & 0 \\ -\frac{1}{\beta_r} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{10}$$

The transformation matrix of a kick at crab cavity #2 is

$$\mathbf{T}_2 = \begin{pmatrix} 1-a\eta & 0 & 0 & -a\eta^2 \\ 2a\eta' & 1+a\eta & a & a\eta\eta' \\ -a\eta\eta' & -a\eta^2 & 1-a\eta & 0 \\ a & 0 & 0 & 1+a\eta \end{pmatrix} \tag{11}$$

where we have assumed(as is usually the case) that the dispersion at crab cavity #2 is equal to that at crab cavity #1, while its derivative changes its sign.

The transformation matrix from crab cavity #2 to crab cavity #1 through the rest of the ring is

$$\mathbf{A} = \begin{pmatrix} \cos\mu_{21} & \beta_c \sin\mu_{21} & 0 & 0 \\ -\frac{\sin\mu_{21}}{\beta_c} & \cos\mu_{21} & 0 & 0 \\ 0 & 0 & \cos\phi_{21} & -\frac{\alpha_p}{\Omega}\sin\phi_{21} \\ 0 & 0 & \frac{\Omega}{\alpha_p}\sin\phi_{21} & \cos\phi_{21} \end{pmatrix} \tag{12}$$

where

$$\mu_{21} = 2\pi(Q_x - \frac{1}{2}) \tag{13}$$

with the horizontal tune Q_x, $\phi_{21} = 2\pi Q_s$ with the synchrotron tune Q_s, α_p is the momentum compaction factor, and $\Omega = \frac{2\pi Q_s}{C}$ where C is the circumference. In the above formulation, we have neglected the synchrotron phase advance between crab cavities, which is smaller than that of the rest of the ring by a factor of $\sim Q_s/(2Q_x)$.

Now, we have all the transformation matrixes for the whole ring excluding the effect of the beam-beam interaction(denoted BBI). The effect of the BBI cannot be written as a simple transformation matrix since it includes higher-order nonlinear forces. However, it can be expressed by an inhomogeneous equation as

$$\begin{pmatrix} x \\ x' \\ \tau \\ \delta \end{pmatrix}_{\text{after BBI}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \\ \tau \\ \delta \end{pmatrix}_{\text{before BBI}} + \begin{pmatrix} 0 \\ -F\cos^2\varphi \\ 0 \\ F\sin\varphi\cos\varphi \end{pmatrix} \tag{14}$$

where F is the beam-beam kick of a particle integrated over the incoming bunch.

Let us conceptually divide the ring into two parts: the first part goes from crab cavity #1 to #2 including their kicks, and the second part includes the rest of the ring. This is schematically shown in Fig. 1. The first part is expressed by

$$\mathbf{T}_2 \times \mathbf{R}_2 \times \text{BBI} \times \mathbf{R}_1 \times \mathbf{T}_1 \tag{15}$$

Namely,

$$\begin{pmatrix} x \\ x' \\ \tau \\ \delta \end{pmatrix}_{\#2} = \underbrace{\begin{pmatrix} -1+2a\eta-2a^2\eta^2 & 0 & 0 & -2a^2\eta^3 \\ 6a^2\eta\eta' & -1-2a\eta-2a^2\eta^2 & -2a^2\eta & 2a\eta\eta'+4a^2\eta^2\eta' \\ 2a\eta\eta'-4a^2\eta^2\eta' & 2a^2\eta^3 & 1-2a\eta+2a^2\eta^2 & -2a^2\eta^3\eta' \\ 2a^2\eta & 0 & 0 & 1+2a\eta+2a^2\eta^2 \end{pmatrix}}_{\equiv \mathbf{C}}$$

$$\times \begin{pmatrix} x \\ x' \\ \tau \\ \delta \end{pmatrix}_{\#1} + \underbrace{\begin{pmatrix} -\beta_r F\cos^2\varphi + \eta F\cos\varphi\sin\varphi - \frac{\eta^2}{\beta_r}F\sin^2\varphi \\ -2\eta' F\sin\varphi\cos\varphi + \frac{\eta\eta'}{\beta_r}F\sin^2\varphi \\ \eta\eta' F\cos\varphi\sin\varphi \\ \frac{\eta}{\beta_r}F\sin^2\varphi \end{pmatrix}}_{\equiv \mathbf{B}} \tag{16}$$

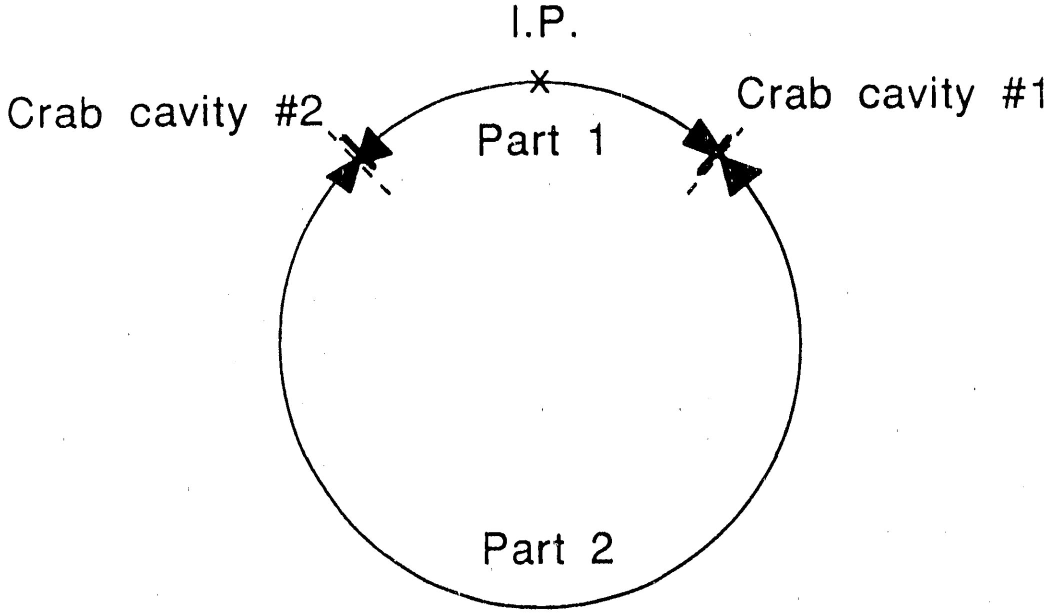

Figure 1: Schematic configuration of the ring with crab cavities.

It looks complicated, but reduces to a simple form if we insert $\eta = \eta' = 0$ into the above equation:

$$\begin{pmatrix} x \\ x' \\ \tau \\ \delta \end{pmatrix}_{\#2} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \\ \tau \\ \delta \end{pmatrix}_{\#1} + \begin{pmatrix} -\beta_\tau F \cos^2 \varphi \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{17}$$

The effect of the crab kicks is cancelled, and the energy change due to the BBI disappears. As a matter of fact, if both η and η' at crab cavity #2 are reversed in sign and equal in magnitude to those at crab cavity #1, we obtain the same result as above. This explains the physical origin of the remnant of crab kick effects. Namely, in order for the kick effects generated by the two cavities to cancel each other, it is essential that the physical coordinates of a particle, x and τ, are transformed to $-x$ and τ (when the BBI is neglected) as the particle advances from crab cavity #1 to #2. The decoupled pure coordinates, $\tilde{x}$ and $\tilde{\tau}$, are transformed to $-\tilde{x}$ and $\tilde{\tau}$. Thus, the physical coordinates, x

and τ, can transform as required only when η and η' both change sign at the two crab cavities (see Eq.(6)).

Synchro-betatron Resonances Induced by Crab Kicks

The transformation (16) suggests that synchro-betatron resonances may be excited even without the beam-beam interaction($F = 0$). The stability of motion can be examined by calculating eigenvalues of the total transformation matrix in the ring, $\mathbf{A} \times \mathbf{C}$. If the absolute value of an eigenvalue is greater than one, the particle motion is unstable. Figure 2 shows an example of the stopband with a crossing half-angle of 20 mrad as a function of η at the crab cavities and the fractional part of the horizontal tune, δQ_x. The parameters used are summarized in Table 1. They are partially taken from Ritson's design of an IP with crab crossing.[4] Arbitrarily, η' has been set to zero. Actually, the stopband has only a weak dependence on η', since η' changes its sign at the two cavities and therefore its effect is nearly cancelled.

Table 1. Parameters of the sample crab crossing design after Ritson.

Circumference, C (m)	2200
Bunch length, σ_s (cm)	1.0
Beta function at IP β_x^* (m)	0.4
Beta function at the crab cavities, β_c (m)	19
Emittance, ϵ_x (nm-rad)	100
Crab crossing half-angle, φ (mrad)	20
Synchrotron tune, Q_s	0.040
Momentum compaction factor, α_p	0.00115

We can see the stopbands at tunes

$$\delta Q_x \approx 0, \tag{18}$$

$$\delta Q_x \approx 0.5, \tag{19}$$

$$\delta Q_x - Q_s \approx 0. \tag{20}$$

Since we assume that a crab kick is linear in x and τ, only the linear resonance $\delta Q_x - Q_s \approx 0$ appears. In reality, a crab kick may include nonlinear terms in x and

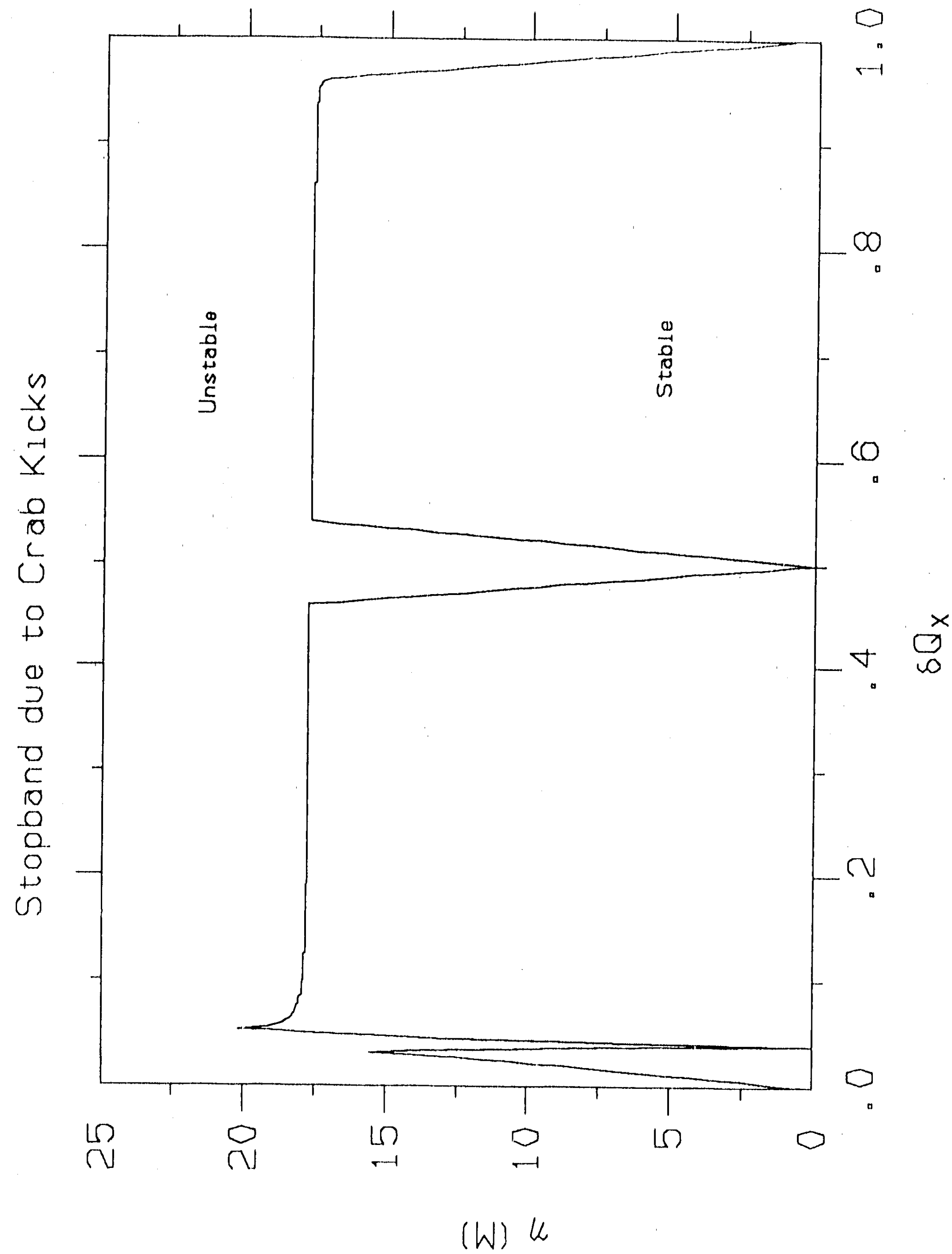

Figure 2: Stopband due to synchro-betatron resonances excited by crab kicks only.

τ, so nonlinear resonances may also be excited. With these parameters, the unstable region spreads out over the entire tune range and has a weak tune dependence when η becomes larger than ~ 17 m. A similar stopband pattern is observed in the analysis of the mode-coupling beam instability due to a localized impedance,[5,6] where the change of stopband pattern is explained by the cause of instability changing from a resonance to a mode-coupling instability.

Beam-beam Effects and Tolerances

Next, let us consider the effects of the beam-beam interaction. The changes in τ and δ arising from the BBI (see Eq.(16)) are a source of synchro-betatron resonance. Let us compare these longitudinal coordinate changes with that due to a non-zero crossing angle without the crab scheme.[7] The energy change due to the crossing angle 2φ is

$$\Delta\delta = F \sin\varphi. \tag{21}$$

In the present case with the crab scheme, the energy change is much smaller than $F\sin\varphi$, since $\frac{\eta}{\beta_r}\sin\varphi \ll 1$. Therefore, the τ change is likely to give the main contribution to synchro-betatron resonances. Otherwise, the effects will be negligible. The τ change, $\eta\eta' F\cos\varphi\sin\varphi$, is equivalent to an energy change of $\frac{\eta\eta'}{\alpha C}F\cos\varphi\sin\varphi$. Thus, the effective crossing angle that gives the same strength of synchro-betatron resonance as produced by the crossing angle φ without a crab scheme, is

$$\sin\varphi_{eff} \approx \frac{\eta\eta'}{\alpha C}\cos\varphi\sin\varphi, \tag{22}$$

which may be approximated by

$$\varphi_{eff} \approx \frac{\eta\eta'}{\alpha C}\varphi. \tag{23}$$

The maximum tolerable crossing angle that will excite weak enough synchro-betatron resonances is given by

$$(\varphi_{eff})_{max} \approx \frac{\sigma_x^*}{2\sigma_s}, \tag{24}$$

where σ_x^* is the rms horizontal beam size at the IP and σ_s is the rms bunch length. The condition (24) expresses the situation where two beams are still well overlapping, so that the crossing angle may not be well defined for particles whose orbits have angular divergence from the betatron oscillation. If we insert Eq. (23) into Eq. (24), we obtain a tolerance for the dispersion times its derivative at a crab cavity or alternatively, for the crossing angle when the dispersion and its slope are given:

$$\eta\eta' \leq \frac{\sigma_x^*}{2\sigma_s}\frac{\alpha C}{\varphi}. \tag{25}$$

With the parameters in Table 1, this criterion becomes numerically

$$\eta\eta' \leq 1.27 \text{ m}. \tag{26}$$

Conclusions

The present analysis in terms of a matrix formulation can predict only linear synchro-betatron resonances, as shown in Fig. 2, when the effects of crab kicks are considered without the beam-beam interaction. In reality, however, the intrinsic nonlinearity of the electromagnetic field in crab cavities will induce nonlinear resonances as well. This nonlinearity may be enhanced by various effects: not only by errors in the crab cavity performance, but also by dynamic effects such as a beam entering a crab cavity off axis. The latter may be another source of synchro-betatron resonances by itself, just as in the case of non-zero dispersion. So far, most tolerance studies for a crab crossing scheme have been done in terms of errors in crab cavity performance. Further work on the beam dynamics effects may also be needed.

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