A COMPREHENSIVE AND CRITICAL REVIEW OF THE PREDICTIVE PROPERTIES OF THE VARIOUS MASS MODELS*

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Abstract: Since the publication of the 1975 Mass Predictions approximately 300 new atomic masses have been reported. These data come from a variety of experimental studies using diverse techniques and they span a mass range from the lightest isotopes to the very heaviest. It is instructive to compare these data with the 1975 predictions and several others (Möller and Nix, Monahan and Serduke, Uno and Yamada) which appeared later. Extensive numerical and graphical analyses have been performed to examine the quality of the mass predictions from the various models and to identify features in these models that require correction. In general, there is only rough correlation between the ability of a particular model to reproduce the measured mass surface which had been used to refine its adjustable parameters and that model's ability to predict correctly the new masses. For some models distinct systematic features appear when the new mass data are plotted as functions of relevant physical variables. Global intercomparisons of all the models are made first, followed by several examples of types of analysis performed with individual mass models.

I. INTRODUCTION

Shortly after the publication of the 1975 Mass Predictions,[1] a program of spectroscopic studies at BNL of new neutron-rich nuclei resulted in the measurement of $Q_{\beta-}$ for several of these nuclides.[2-4] Comparison of the measured $Q_{\beta-}$ values with predictions from the various mass models revealed rather large differences among the models. The new nuclei, while small in number, were the most neutron-rich of their particular element and were located in regions where little new mass information had been reported. As such, these data seemed to signal a useful way to examine the predictive properties of the various mass models. Definitive and critical tests of the models clearly would have to await more broadly based mass information. The tables of experimental masses as provided by Wapstra in 1977,[5] 1981,[6] and 1983[7] have been used subsequently for extensive comparisons of newly reported masses with predictions from the models as published in 1975 and from those models which appeared later (Möller and Nix,[8] Monahan and Serduke,[9] Uno and Yamada[10] In most cases the 1975 Mass Predictions were based on fits to the 1975 Wapstra-Bos mass table. The models of Myers and of Bauer used the 1971 masses while later predictions of Möller and Nix and of Uno and Yamada used the 1977 Wapstra mass table. With now nearly 300 new masses reported since 1975, systematic features in some of the mass models have become evident through the application of some specialized numerical and graphical analysis techniques. A theoretical understanding of these features will serve as a basis for refinement of the models.

II. NEW MASSES AND GLOBAL COMPARISONS

The 1983 Wapstra-Audi Atomic Mass Adjustment[7] contains 278 new masses beyond those found in the 1975 table. In general the new data come from measurements of very neutron-rich or proton-rich isotopes throughout the periodic system. The long isotopic sequences of Na, Rb, and Cs nuclei extend well beyond the usual region of known masses. Mass measurements of alpha-decaying isotopes originating from ^{176}Hg, ^{178}Hg and neutron-deficient trans-Pb isotopes also extend the region of known masses well beyond that determined in 1975. These new data in particular are very sensitive tests of the mass models. But before individual models are discussed it is instructive to examine the global characteristics of how well the 1975 masses and the new masses are reproduced by the models. Table 1 lists root-mean-square deviations of 12 models for the 1975 (or 1977) masses and those new masses which were not present in the data base used to refine model parameters. The models are listed in approximately the order of the increasingly larger number of adjustable parameters used, i.e., the more "ab-initio" approaches (liquid drop or droplet) precede the shell models which precede those models based on mass relationships. The rms-deviations of the 1975('77) masses exhibit the trend noted by Tondeur[11]--more adjustable parameters result in better fits to the measured masses. It is noteworthy that this trend is not repeated when the new masses are considered. All models give larger rms-deviations for the new masses. The final column of Table 1 lists the ratio of the rms-deviations of the new masses to those of the data base used to construct the mass model. It measures, therefore, the extent to which predictions of new masses by a particular model reflects the goodness-of-fit that the model achieved on the data base used to refine its adjustable parameters. One notes that models using mass relationships (and large numbers of parameters) exhibit much larger rms-deviations for the new masses as compared to 1975('77) masses. The more fundamental approaches show a slightly enlarged (about 1.1-1.5 times) rms-deviation for the new masses. The model of Liran and Zeldes, while not having the best fit to the 1975 masses, shows the smallest rms-deviations for the new masses.

III. ANALYSIS METHODS FOR INDIVIDUAL MODELS

For all of the new data the differences between the predictions of the 12 models and the new experimental masses as reported by Wapstra-Audi have been computed. The convention used was Δ = Calculated Mass-Experimental Mass, so $\Delta > 0$ indicates that the model has predicted a nucleus to be not bound enough and $\Delta < 0$ corresponds to nuclei being predicted to be too well bound. Histograms of the frequency of Δ with energy are plotted. Δ-values have also been plotted as functions of relevant physical quantities, e.g., Z, A, N, isospin, distance from shell closure (Z or N), and distance from stability. This lattermost quantitiy is expressed as "neutrons-from-stability" (NFS) and is computed from the following relationship:

$$NFS = N - Z - (0.4\ A^2)/(200 + A)$$

Plots of Δ-values as a function of NFS are particularly useful since each nuclide is then placed into one of four quadrants. Its position in the quadrant is determined by whether it is neutron rich (NFS > 0) or proton rich (NFS < 0) and whether it is not bound enough (Δ > 0) or too well bound (Δ < 0).

IV. ANALYSIS RESULTS AND COMMENTS ON INDIVIDUAL MODELS

A. Liran and Zeldes: The Δ frequency histogram for this model which gave the smallest rms-deviation for the new masses in shown in Fig. 1. The few cases with large delta values are labeled. It is significant that all of these are at the very edges of the measured mass surface and some of them (^{34}Na and ^{32}Mg) have experimental mass uncertainties assigned to them that are comparable to their Δ-values. Three of these nuclei also lie near N = 20. For this model a plot of Δ-values versus NFS is shown in Fig. 2. The few large Δ-values, |Δ| > 1.5 MeV, scatter into three of the four quadrants; the cases in the "proton-rich, too well bound" quadrant all cluster with |Δ| < 0.3 MeV.

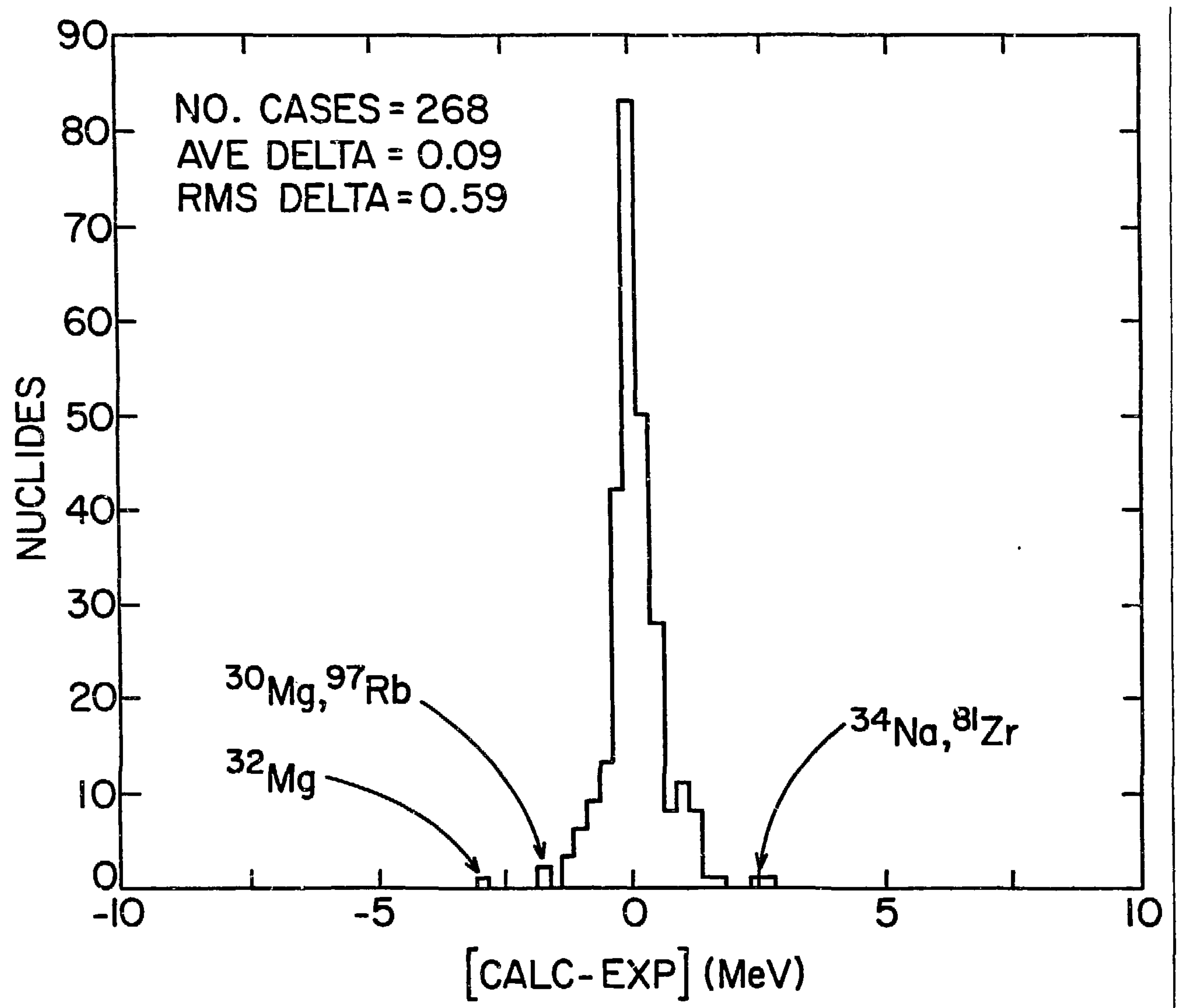

Fig. 1. A frequency histogram for the model of Liran and Zeldes. Isotopic labels identify cases with large Δ-values.

Table 1. Root-Mean-Square Deviations.

Model	Data Base Used	RMS-Δ for 1975 (or 1977 Masses)	RMS-Δ for New Masses	Ratio
Myers	1971	1.327 MeV	1.380	1.04
Groote et al.	1975	0.718	1.096	1.53
Seeger and Howard	1971	0.718	0.954	1.33
Möller and Nix	1977	(0.835)	0.970	1.16
Bauer	1971	1.506	1.772	1.18
Beiner et al.	1975	2.747	3.125	1.14
Liran and Zeldes	1975	0.276	0.589	2.13
Uno and Yamada	1977	(0.393)	1.100	2.80
Comay and Kelson	1975	0.312	1.314	4.21
Jänecke, Garvey-Kelson	1975	0.212	1.361	6.42
Monahan and Serduke	1975	0.159	0.695	4.37
Jänecke and Eynon	1975	0.363	0.952	2.62

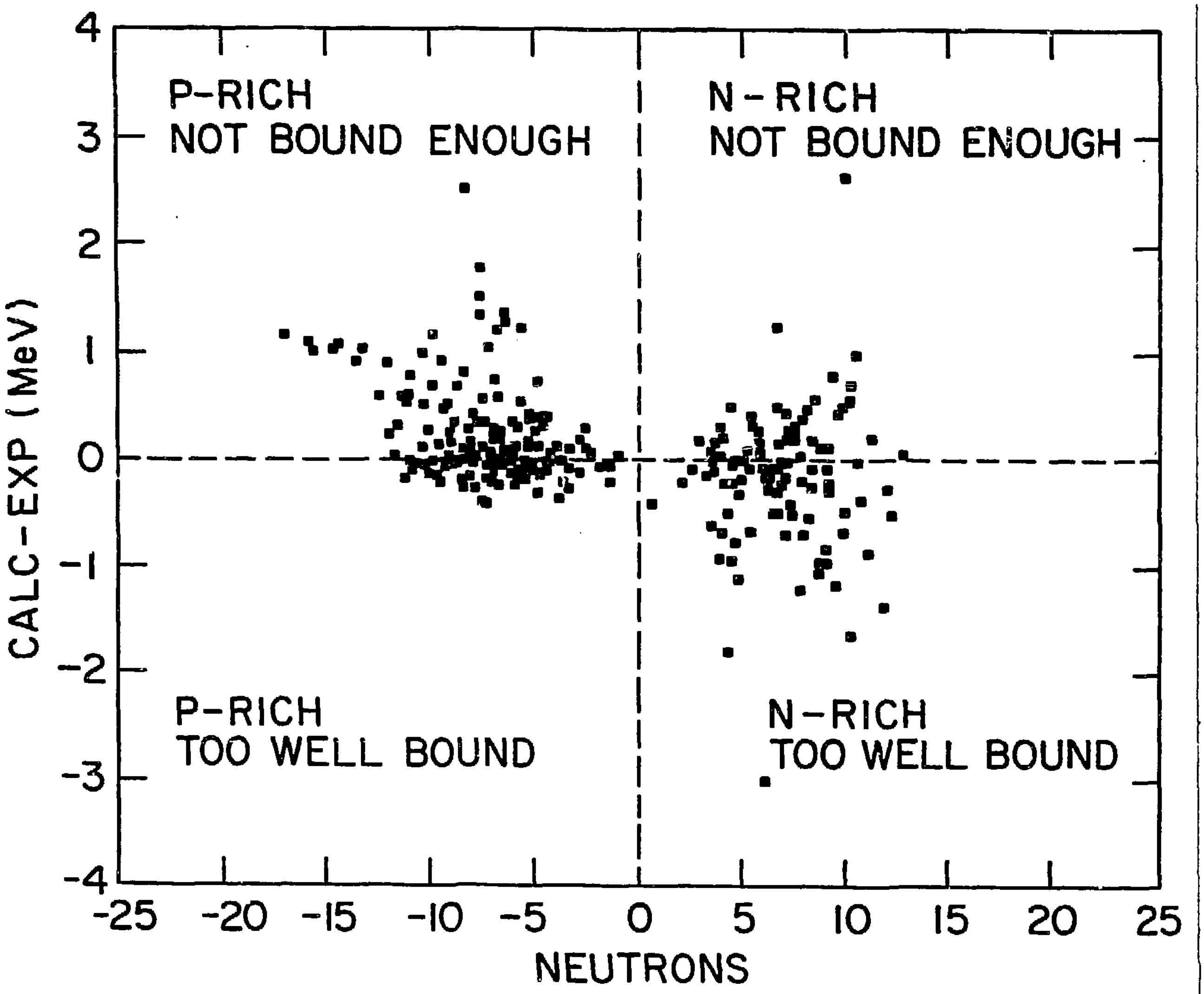

Fig. 2. Δ-values plotted as a function of neutrons from stability for the model of Liran and Zeldes.

B. Comay and Kelson (also Jänecke, Garvey-Kelson): Fig. 3 displays Δ-values as a function of NFS. Many nuclei cluster along the Δ = 0 line. Two light nuclei, $^{33,34}Na$, appear with large Δ-values in the upper right hand quadrant. Other large delta values occur in either the upper left hand quadrant or the lower right hand quadrant. The former come from α-emitting nuclei originating from either ^{176}Hg or ^{178}Hg, while the latter come mostly from the most neutron-rich Rb or Cs nuclei. In both instances deviations increase with distance from stability and appear, at least in first order, to be proportional to T_z^3. As a consequence of these trends, predictions of the location of the neutron and proton drip-lines by this model (or any other showing similar trends) should be examined closely. If the trends are not reversed the proton drip-line is predicted too close to stability, while the neutron drip-line will be too far from stability. This will clearly be of concern when the model is used as input for r-process nucleosynthesis calculations. More detailed analysis of models of this type and their connection to other types of mass models can be found in the contribution of Jänecke to this conference.

C. Myers: Δ-values versus NFS for the Semiempirical Droplet Model of Myers are shown in Fig. 4. Despite a large amount of scatter, a trend is evident of nuclei

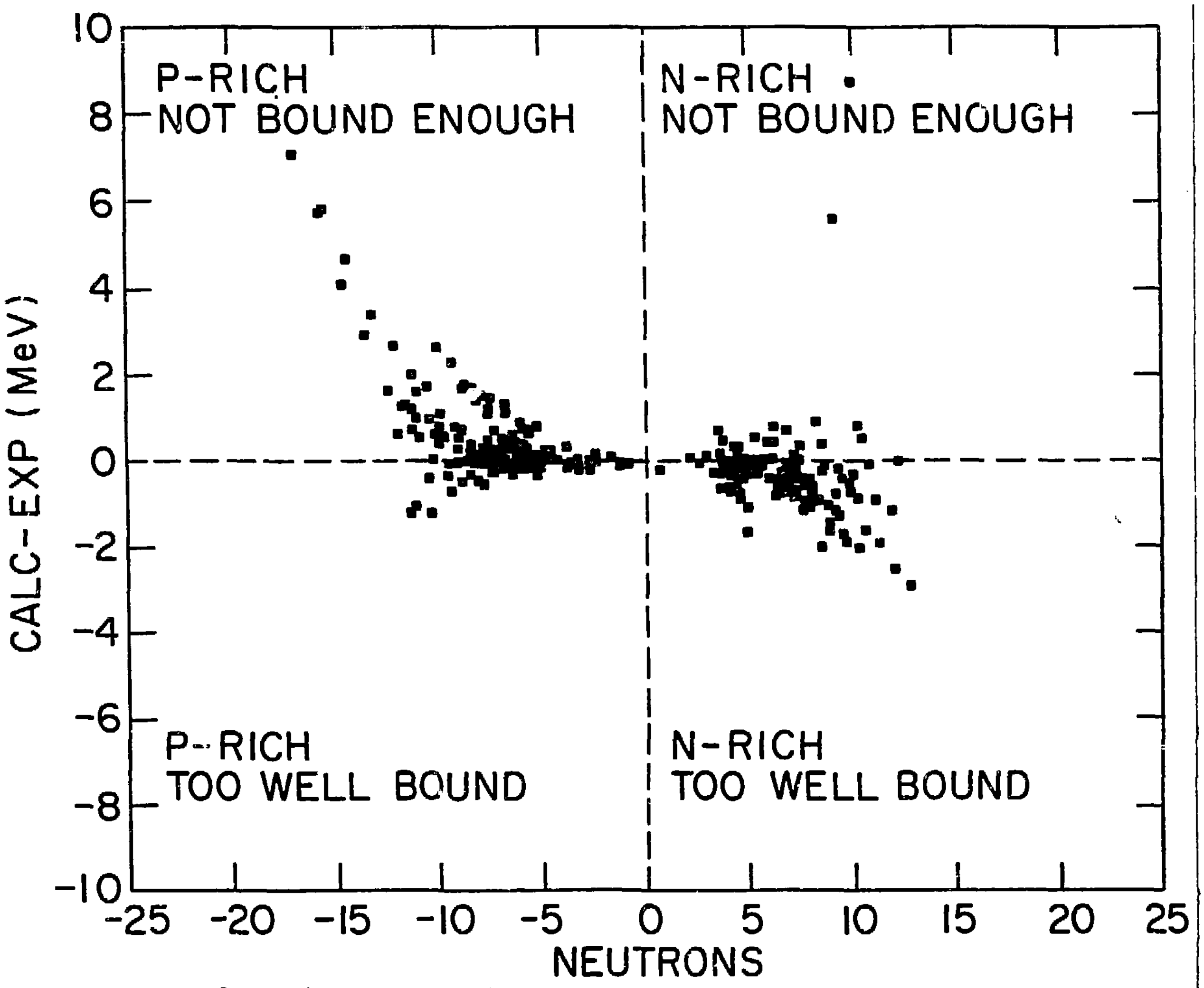

Fig. 3. Δ-values plotted as a function of neutrons from stability for the model of Comay and Kelson.

near stability being not bound enough and nuclei far from stability being too well bound. this occurs on both sides of stability for the new masses and it points to features, recognized before in the droplet model, which were thought to compensate each other. The droplet model generally places the predicted bottom of the valley of β-stability slightly above the measured masses and the curvature of the mass surface is generally greater than that predicted. The new mass data indicate that while these two aspects do compensate approximately near stability, the overly gentle curvature of the predicted surface eventually predominates and this results in nuclei far from stability being too well bound. The Myers model has demonstrated remarkable ability to predict correctly nuclear charge radii. The trends in mass predictions of the type shown in Fig. 4 appear to be an unavoidable trade-off which has to be accepted when predictions of both masses and charge radii are made by the droplet model. The combined droplet and finite range model (contribution of P. Möller to this conference) may represent a solution that might result in both good mass predictions and correct predictions of charge radii.

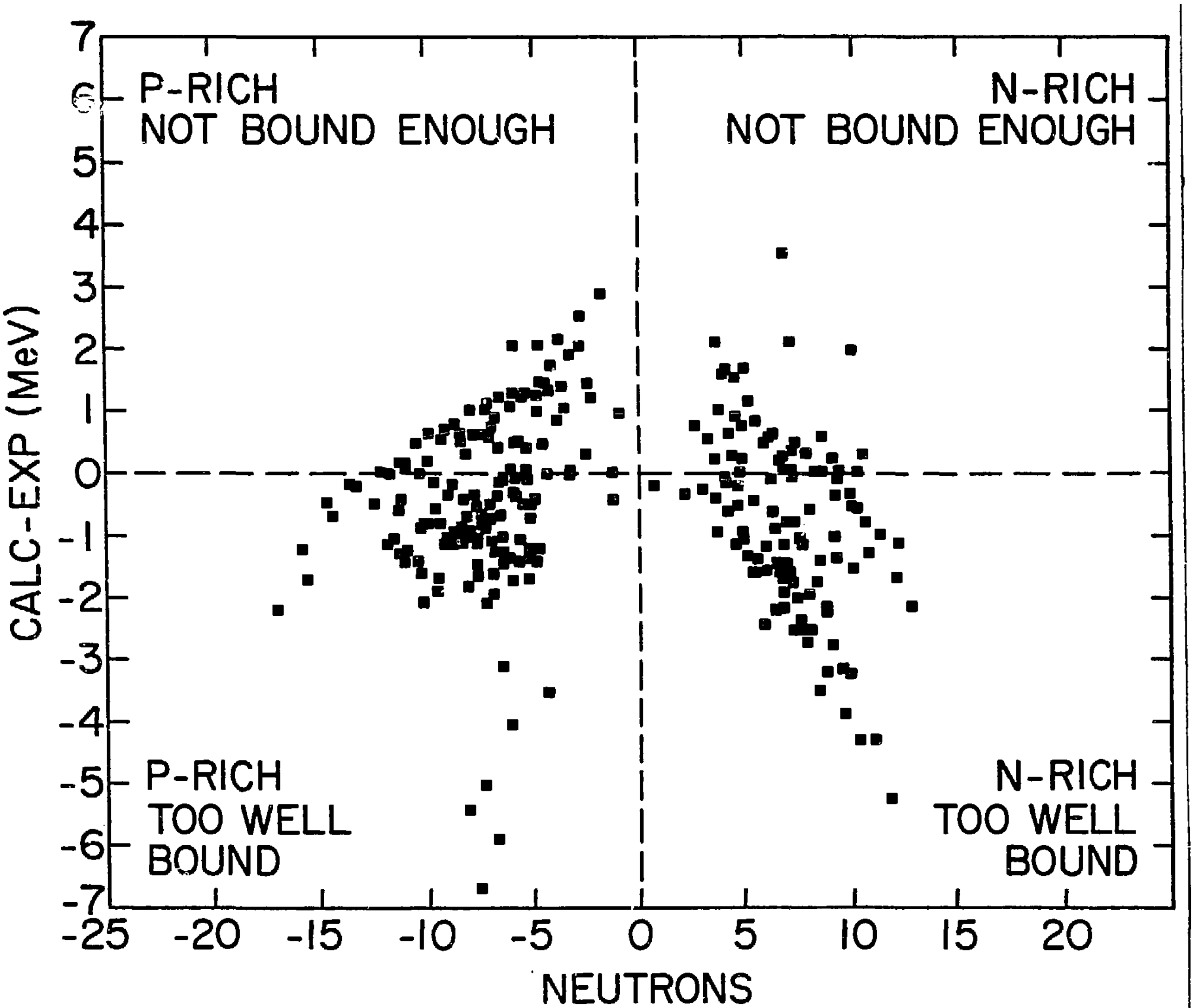

Fig. 4. Δ-values plotted as a function of neutrons from stability for the model of Myers.

V. CONCLUSIONS

The predictive properties of various mass models have been examined by using a variety of numerical and graphical techniques and the nearly 300 new atomic masses reported since the last comprehensive set of mass predictions appeared. Systematic features in some of the models have been identified. These may serve as a basis for improvement of the models in two ways: (1) correction of those features in the models that resulted in poor predictions for whatever reason; and (2) the use of a new larger data base (with significant numbers of nuclei in regions quite far from stability) to refine model parameters so that extrapolations into unmeasured mass regions will be more accurate. It is clearly time to undertake an effort to prepare a comprehensive update of mass predictions which will replace that done in 1975.

VI. REFERENCES

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