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A CONVERGENT SPECTRAL REPRESENTATION FOR THREE-DIMENSIONAL INVERSE MHD EQUILIBRIA

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An essential aspect of the inverse moment method for computing MHD equilibria[1] is the appropriate parameterization of magnetic flux surfaces. Let (R,ϕ,Z) denote a cylindrical coordinate system, where ϕ is the toroidal angle. Then, a closed flux surface ρ = const may be represented in the form

$$
\begin{aligned}
R &= R(\rho,\theta,\phi) \ , \\
Z &= Z(\rho,\theta,\phi) \ ,
\end{aligned}
\tag{1}
$$

where θ is a poloidal angle, $0 \le \theta \le 2\pi$; $R(\theta + 2\pi,\ \phi) = R(\theta,\phi)$; and $Z(\theta + 2\pi,\ \phi) = Z(\theta,\phi)$. Both R and Z may be expanded in double Fourier series in θ and ϕ:

$$
\begin{aligned}
R &= \sum \left[R^c_{mn}\cos(m\theta - n\phi) + R^s_{mn}\sin(m\theta - n\phi)\right] \\
Z &= \sum \left[Z^c_{mn}\cos(m\theta - n\phi) + Z^s_{mn}\sin(m\theta - n\phi)\right] \ .
\end{aligned}
\tag{2}
$$

The MHD force balance equation[1,2] is subsequently used to determine the Fourier coefficients $R_{mn}(\rho)$ and $Z_{mn}(\rho)$ appearing in Eq. (2).

A difficulty with the representation given by Eq. (2) arises from the nonuniqueness of the poloidal angle θ. Note that the form of Eq. (2) is invariant under transformations given by

$$
\theta = \theta' + \lambda(\rho,\theta',\phi) \ , \tag{3}
$$

where λ is a periodic function of θ' and ϕ satisfying $|d\lambda/d\theta'| < 1$. Thus, the Fourier series for flux surfaces described in (θ,ϕ)

coordinates have the same form as those in (θ',ϕ). Hence, Eq. (2) does not provide a unique representation. [Of course, the values of the expansion coefficients will be different in the (θ,ϕ) and (θ',ϕ) systems.] Although any finite truncation of the moment equations used to determine (R_{mn}, Z_{mn}) may have a unique solution, the nonuniqueness of Eq. (2) implies that such truncations will be numerically ill-conditioned as the limit of infinitely many equations is approached.

In practice, the threshold for ill-conditioned behavior and the consequent lack of numerical convergence may be as few as three poloidal harmonics [$m \geq 2$ in Eq. (2)], the exact number depending weakly on the complexity of the flux surface topology. Figure 1 illustrates the numerical consequences for MHD equilibria when a unique poloidal angle is not prescribed. Three toroidal cross sections of a 12-period, helical-axis torsatron are shown. The top row of graphs show equilibrium magnetic flux surfaces (solid contours), computed using the method described in Ref. 2, for which no constraint was applied to the R and Z spectra. The bottom row is composed of similar results for the case when θ is unique. The dashed lines are contours of constant θ. In all of the graphs, the harmonics $m = 0, 1, 2$ and $n = -12, 0, 12$ were retained in the Fourier series for R and Z. In the top row of Fig. 1, the physically relevant flux contours were nearly in equilibrium when the computation ended. It was not possible, however, to converge the MHD force residuals any further. This was due to the random motion of the θ contours, which reflected the nonuniqueness of Eq. (2). In contrast, the bottom row of Fig. 1 illustrates an

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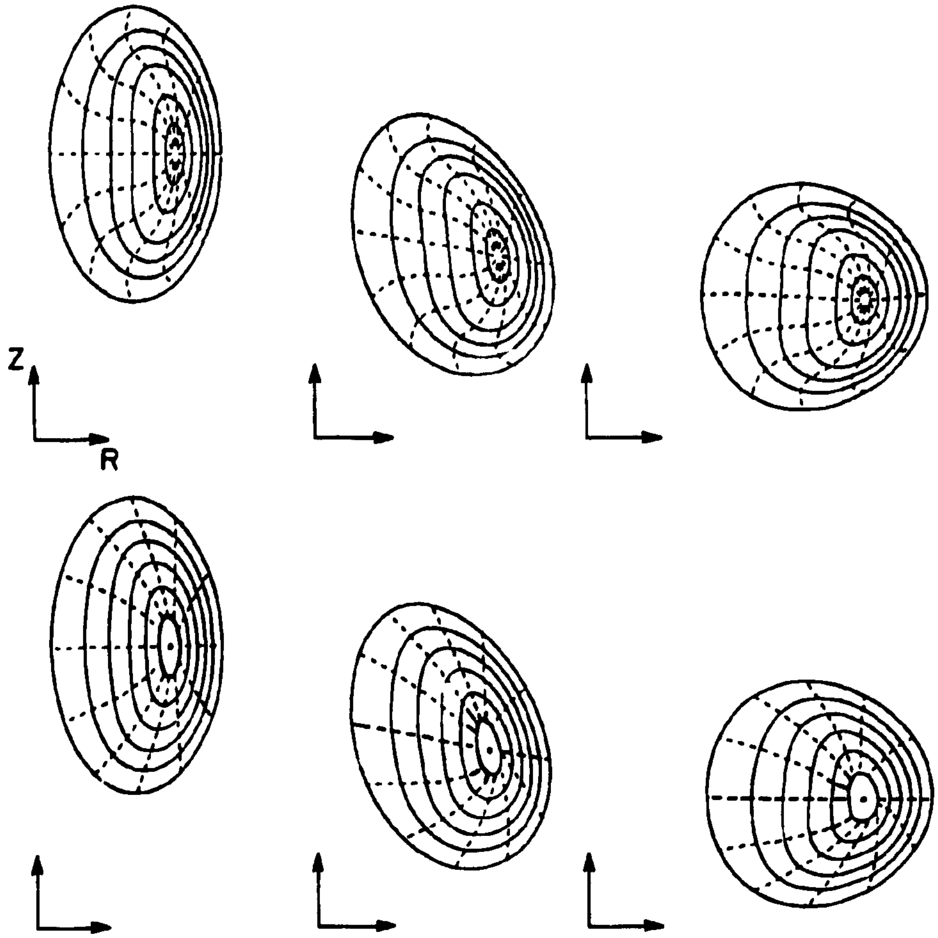

Fig. 1. Equilibrium magnetic flux surfaces for a helical-axis, 12-field-period torsatron. The solid lines are flux contours, and the dashed lines are constant poloidal angle contours. The top row shows the consequences of solving an ill-conditioned set of equations, namely, the wavy motion of θ contours. The bottom row is an equilibrium obtained when θ is uniquely prescribed.

equilibrium solution for which the angle coordinate was prescribed by the representation in Eq. (9) and a numerical stationary state was achieved. Thus, the apparently random waviness of the θ contours is a graphical signature of poor numerical convergence due to an underdetermined system of equations.

It is, therefore, desirable to employ Fourier series expansions of Eq. (1) that are both unique and guaranteed to exist for a certain class of flux surface topologies. The poloidal angle θ can be determined in several ways to yield a unique representation. For example, a prescription for the angular variation of the transformation Jacobian between flux and cylindrical coordinates defines θ uniquely. This is the "straight" magnetic field line coordinate system.[3] Such an angle choice, however, is generally incompatible[2] with rapid convergence of the series in Eq. (2).

The convergence properties of Eq. (2) may be improved by using a geometric description of the flux surfaces. A polar representation of the form

$$R = R_0(\phi) + r(\rho,\theta,\phi)\cos\theta \,, \qquad Z = Z_0(\phi) + r(\rho,\theta,\phi)\sin\theta \tag{4}$$

provides such a description, where $R_0(\phi)$ and $Z_0(\phi)$ are periodic functions of ϕ that describe the axis of the polar system and r is the polar radius, which has a Fourier expansion of the form given in Eq. (2). The angle θ is also uniquely defined by Eq. (4):

$$\theta = \tan^{-1}\left[\frac{Z - Z_0(\phi)}{R - R_0(\phi)}\right] . \tag{5}$$

[Equation (4) is limited to starlike domains, since r is assumed to be a single-valued function of θ.] Once the polar axis is determined, the angle θ is independent of r, and hence θ is an Eulerian angle variable.

There are several difficulties with Eq. (4) that can degrade the convergence rate for the series expansion of r. These are related to the location of the mean flux surface position [$m = 0$ components of Eq. (4)] and mean radius [$m = 1$ components of Eq. (4)] as functions of ρ. Suppose that at the surface $\rho = \rho_0$, Eq. (4) represents the elliptical flux surface $\rho_0^2 = (R - R_0')^2 + Z^2/\kappa^2$. Define $\Delta = R_0 - R_0' < \rho_0/2$. First, consider the case $\kappa = 1$ (circular surface) but $\Delta^2 \neq 0$. Then

$$r = a + \left(\rho_0^2 - \Delta^2 + a^2\right)^{1/2} , \tag{6a}$$

where $a = \Delta \cos \theta$. For $\Delta \ll 1$, the $m = 1$ component of r approximates a shift Δ relative to the magnetic axis. Next, consider the case $\kappa \neq 1$, $\Delta = 0$. Then

$$r = b\rho_0(1 + \epsilon \cos 2\theta)^{-1/2} , \tag{6b}$$

where $b = \left[2/(1 + \kappa^2)\right]^{1/2}$ and $\epsilon = (\kappa^2 - 1)/(\kappa^2 + 1)$. For $\epsilon \ll 1$, the $m = 2$ component of r approximates the effect of elongation. However, when either $2\Delta/\rho_0$ or ϵ approaches unity, corresponding to a significant shift or elongation, respectively, the spectrum of r given in Eq. (6)

will begin to broaden substantially, and the single mode approximations for shift and elongation cease to be valid.

This spectral broadening can be eliminated by allowing the center and ellipticity of each flux surface to be represented exactly. Introducing the complex quantity $\varsigma = R + iZ = \varsigma_0 + r\exp(i\theta)$, where $\varsigma_0 = R_0 + iZ_0$, Eq. (4) can be recast as follows:

$$\varsigma = \xi_0(\rho,\phi) + \xi_1(\rho,\phi)e^{-i\theta} + \bar{r}(\rho,\theta,\phi)e^{i\theta} , \tag{7}$$

where r and $\bar{r}$ are real. Note that ξ_0 and ξ_1 are functions of the local surface ρ $\left[\xi_0(0,\phi) = \varsigma_0(\phi)\right]$ but are independent of θ. The quantity ξ_0 is defined to be the m = 0 component of ς,

$$\xi_0 \equiv \oint \varsigma \frac{d\theta}{2\pi} , \tag{8a}$$

where the loop integral is over one complete period in θ. Equation (8a) and Eq. (7) together represent a constraint on $\bar{r}$:

$$\int \bar{r}\, e^{i\theta}\, d\theta = 0 . \tag{8b}$$

Since $\bar{r}$ is real, this implies that there is no m = 1 component of $\bar{r}$. If N denotes the number of toroidal modes with nonzero mode numbers, then Eq. (8b) is composed of 2(N + 1) mode amplitudes of $\bar{r}$, which are replaced in Eq. (7) by the same number of ξ_0 modes. In a similar way,

$$\xi_1 \equiv \oint \varsigma\, e^{i\theta} \frac{d\theta}{2\pi} . \tag{8c}$$

Together with Eq. (7), this implies the additional constraint on $\bar{r}$:

$$\int \bar{r}\, e^{2i\theta}\, d\theta = 0 \ . \tag{8d}$$

Thus, there is no m = 2 component of $\bar{r}$. Once again, the number of amplitudes in Eqs. (8c) and (8d) is the same, so that no information is lost or introduced in going from the polar form in Eq. (4) to Eq. (7). In this way, the angle θ remains unique.

It is convenient now to display the real form of Eq. (7) for the original cylindrical coordinates:

$$R = \sum_{n=0}^{N} \left(R^c_{0n}\cos n\phi + R^s_{0n}\sin n\phi\right)$$
$$+ \sum_{n=-N}^{N} \left[R^c_{1n}\cos(\theta - n\phi) + R^s_{1n}\sin(\theta - n\phi)\right] + \bar{r}\cos\theta \ ,$$
$$Z = \sum_{n=0}^{N} \left(Z^c_{0n}\cos n\phi + Z^s_{0n}\sin n\phi\right)$$
$$+ \sum_{n=-N}^{N} \left[R^s_{1n}\cos(\theta - n\phi) - R^c_{1n}\sin(\theta - n\phi)\right] + \bar{r}\sin\theta \ ,$$
$$\bar{r} = \sum_{n=0}^{N} \left(r^c_{0n}\cos n\phi + r^s_{0n}\sin n\phi\right)$$
$$+ \sum_{m=3;\,n}^{N} \left[r^c_{mn}\cos(m\theta - n\phi) + r^s_{mn}\sin(m\theta - n\phi)\right] \ . \tag{9}$$

The m = 0 components of $\bar{r}$ combine with the R_{0n} and R_{1n} terms in Eq. (9) to yield exact shifted ellipses, while the remaining m ≥ 3 terms of $\bar{r}$ represent perturbations of elliptical topology. Near the magnetic [illegible]s

($\rho = 0$) where the flux surfaces degenerate to ellipses,[1] $R_{1n} \sim \rho$, $r_{0n} \sim \rho$, and $r_{mn} \le O(\rho^2)$ for $m \ge 3$. In contrast to the original polar representation, Eq. (4), the poloidal angle θ in Eq. (9) is a Lagrangian coordinate, depending on $\bar{r}$ as well as on the surface geometry.

An obvious extension of the resummation process given by Eq. (7) is to project out terms of the form $\zeta_k(\rho,\phi)\exp(-ik\theta)$, thus annihilating the $(k + 1)$th mode amplitudes of $\bar{r}$. As $k \to \infty$, only the $m = 0$ terms of $\bar{r}$ survive, and the (R_{mn}, Z_{mn}) coefficients for $m \ge 2$ are then related as follows:

$$R^c_{mn} = -Z^s_{mn} \ , \tag{10a}$$

$$R^s_{mn} = Z^c_{mn}. \tag{10b}$$

The Riemann mapping theorem can be used to prove that the representation given by Eq. (10) is unique and describes any sufficiently smooth, closed flux surface. Consider the closed curve obtained by the intersection of a flux surface with the plane $\phi = \phi_0$. Let $\nu = R + iZ$ denote this curve in the complex ν plane, and consider also the plane of the complex variable $\omega = \rho\exp(i\theta)$. By the mapping theorem, there is a unique function $\nu = F(\omega)$ that maps the interior of the unit circle $|\omega| < 1$ onto the exterior of the curve in the ν plane such that $\omega = 0$ maps to infinity in the ν plane and the positive real axis near $\omega = 0$ maps to the positive real axis near infinity in the ν plane. Then, $F(\omega)$ has a Laurent expansion,

$$F(\omega) = \frac{a_{-1}}{\omega} + \sum_{n=0} (a_n + ib_n)\omega^n \ , \tag{11}$$

where a_n and b_n are real. (Previously,[1] because of the assumed vertical symmetry of the flux surfaces, it could be argued that $b_n = 0$.) Setting $\omega = \exp(-i\theta)$ in Eq. (11) and separating real and imaginary parts yields a representation for the curve in the ν plane that is the map of the unit circle:

$$R = a_0 + (a_{-1} + a_1)\cos\theta + b_1 \sin\theta + \sum_{n=2} (a_n \cos n\theta + b_n \sin n\theta) \ ,$$

$$Z = b_0 + (a_{-1} - a_1)\sin\theta + b_1 \cos\theta + \sum_{n=2} (-a_n \sin n\theta + b_n \cos n\theta) \ . \tag{12}$$

Allowing a_n and b_n to be periodic functions of ϕ and introducing Fourier series in ϕ, it follows that Eq. (12) is equivalent to the representation given in Eq. (10).

In practice, it is sometimes desirable to represent the boundary flux surface in a different form from Eq. (9). In this case, it is still possible to use the representation in Eq. (9) for the perturbations $\tilde{R}$ and $\tilde{Z}$ from the scaled boundary shape, where $\tilde{R} = R - R_b(\rho,\theta,\phi)$, $\tilde{Z} = Z - Z_b(\rho,\theta,\phi)$, and the ρ dependence of (R_b,Z_b) is chosen so that $\tilde{R}(\rho = 1) = \tilde{Z}(\rho = 1) = 0$ and $R_b(\rho = 0) = Z_b(\rho = 0) = 0$. For example, scaling the boundary Fourier coefficients by ρ^m, for $m \geq 1$, would achieve this behavior.

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ABSTRACT

By rearranging terms in a polar representation for the cylindrical spatial coordinates (R,ϕ,Z), a renormalized Fourier series moment expansion is obtained that possesses superior convergence properties in mode number space. This convergent spectral representation also determines a unique poloidal angle and thus resolves the underdetermined structure of previous moment expansions. A conformal mapping technique is used to demonstrate the existence and uniqueness of the new representation.