SAND98-0607C
SAND--98-0607C

Efficient Monte Carlo Probability Estimation with Finite Element Response Surfaces built from Progressive Lattice Sampling[1]

CONF-980419--

Vicente J. Romero[2] and Susan D. Bankston[3]
Thermal Sciences Dept. 9113
Sandia National Laboratories
Albuquerque, New Mexico, 87185

RECEIVED
MAR 17 1998
OSTI

ABSTRACT

The concept of "progressive Lattice Sampling" as a basis for generating successive finite element response surfaces that are increasingly effective in matching actual response functions is investigated here. The goal is optimal response-surface generation, which achieves an adequate representation of system behavior over the relevant parameter space of a problem with a minimum of computational and user effort. Such is important in global optimization and in estimation of system probabilistic response, which are both made much more viable by replacing large complex computer models of system behavior by fast-running accurate approximations. This paper outlines the methodology for Finite-Element / Lattice-Sampling (FE/LS) response surface generation and examines the effectiveness of progressively refined FE/LS response surfaces in "decoupled" Monte Carlo analysis of several model problems. The proposed method is in all cases more efficient (generally orders of magnitude more efficient) than direct Monte Carlo evaluation, with no appreciable loss of accuracy. Thus, when arriving at probabilities or distributions by Monte Carlo, it appears to be more efficient to expend computer-model function evaluations on building a FE/LS response surface than to expend them in direct Monte Carlo sampling. Furthermore, the marginal efficiency of the FE/LS decoupled Monte Carlo approach increases as the size of the computer model increases, which is a very favorable property.

1. Finite Element Response Surfaces based on Global Lattice Sampling

The simple formulation underlying Finite-Element / Lattice-Sampling (FE/LS) response surfaces is biefly sketched here. Reference [1] contains more detailed information. Figure 1 shows unit squares representing an appropriately mapped 2-D parameter space where the mapped parameters both vary between 0 and 1, inclusive. Various discretizations of the parameter space into finite elements are shown. Each discretization Level is associated with an increasing number of node or sample points where a test is to be run or a function evaluation (FEV) of a model is to be performed. The number and location of sample points determine the number and character of finite elements covering the parameter space.

- **Level 1** ◆ a bilinear global approximation based on 4 sample points (evaluations of the analytic function) at the 4 corners of the domain
- **Level 2** ● one sample point is added to the center of the parameter space, which is then subdivided into 4 triangles as shown, the associated global response-surface representation being comprised of 4 linear triangular finite elements supported by the 5 sample points
- **Level 3** ▼ 4 sample points are added and the global response surface is reconstituted as one Lagrangian 9-node biquadratic quadrilateral finite element
- **Level 4** ■ 4 more sample points are added for a total of 13, and the global response surface is rediscretized into one Lagrangian 9-node biquadratic quadrilateral finite element and 4 linear-to-quadratic transition triangles at the four corners of the domain
- **Level 5** ▲ 12 sample points are added and the global response surface is subdivided into four Lagrangian biquadratic elements supported by a rectangular grid of 25 points
- **Level 6** ○ 16 sample points are added for a total of 41, and the global response surface is subdivided into four Lagrangian biquadratic quads., 4 quadratic triangles, and 4 linear-to-quadratic transition triangles at the four corners of the domain

---

[1]This work was supported by the United States Department of Energy under Contract DE-AL04-94AL8500. This paper is declared a work of the U.S. Government and is not subject to copyright protection in the United States.

[2]Senior Member of Technical Staff

[3]Summer Undergraduate Intern, presently at Texas A&M University

The Levels are built by adding new sample points to the sample points of the previous Levels so that progressive global refinement of the parameter space occurs with a minimum of new samples. Of course, other "Lattice" type schemes are possible, including classical experimental design, which is also presently being investigated. One of the nice features of Lattice-type schemes is that they are conceptually simple and seem to be straightforwardly extendable to arbitrary dimensions. Other advantages of the approach are elaborated in [2].

2. Multimodal Test Function and Successive FE/LS Approximations

In the context of reliability or failure probability problems, a multimodal function of the random or uncertain variables might arise, for example, in an application involving relative times of failure of system components that either catalize or retard potentially catastrophic events in a nuclear power plant. Figure 2 shows various representations of an analytic multimodal surface defined by the equation

$$\text{response(p1,p2)}=\left[0.8r + 0.35\sin\left(2.4\pi\frac{r}{\sqrt{2}}\right)\right][1.5\sin(1.3\theta)]$$

on the domain $0 \le p1, p2 \le 1$

where $r = \sqrt{(p1)^2 + (p2)^2}$, $\theta = \operatorname{atan}\left(\frac{p2}{p1}\right)$.

Figure 2 shows a plot of the analytic function along with representations corresponding to the 6 levels of parameter space resolution depicted in Figure 1. The plots are drawn from point-to-point linear interpolation off a 21x21 grid of samples evaluated with the various finite element approximations. After about Level 5 (25 samples) the finite element approximations appear to match the topology of the exact surface very well. Thus, the "convergence rate" vs. number of analytic function evaluations would appear to be fairly high even for this highly varying surface. A quantitative assessment of convergence rate is made in the next section. In using the response surface in nondeterministic analysis or global optimization it would seem to make sense to switch from further global refinement to more localized refinement after about Level 6 so that probabilities or local optima could be most efficiently converged upon with the fewest number of added samples.

3. Performance of FE/LS Response Surfaces on Model Probability Quantification Problems

For Monte Carlo (MC) probabilistic analysis, the prediction of physical response of deterministic (nonstochastic) systems can be decoupled from probabilistic Monte Carlo sampling via intermediate "surrogate" models, such as the FE/LS global response-surface models in Figure 1, that may run orders of magnitude faster than full computational physics models. Such an approach is here coined a "decoupled" Monte Carlo approach because the Monte Carlo analysis is decoupled from the running of the full computer model through use of the surrogate model. Because of the numerical noise associated with complex physics simulations (see *e.g.* [3] and [4]), surrogate models can also be much more effective in reliability-based approaches[5] to nondeterministic analysis if they meet certain differentiability (smoothness) criteria that can make the optimization process in these approaches more affordable (see [3]). The following is an assessment of direct and decoupled Monte Carlo analysis applied to several probability quantification test problems.

Exact and Approximate "Failure Regions" Corresponding to Various Response Thresholds

Figures 3 - 6 show the exact and approximate parameter regions (shaded) where parameter combinations result in responses that exceed the respective threshold levels 0.2, 0.5, 1.0, and 1.5. These thresholds could be viewed as failure thresholds, above which the system response (say that the response quantity is shear stress) indicates potential failure of the system. The shaded regions are therefore referred to as "failure regions" in the parameter space.

The test function and response thresholds prescribed here are particularly "good" because they test a large and diverse set of attributes of both Monte Carlo sampling and Lattice sampling. In the upcoming quantitative analysis it is found that the data spans a large range of probabilities, on the orders 10^{-4}, 10^{-2}, 10^{-1}, and 1. Thus, a large portion of the parameter space is being investigated in this regard.

As Figure 3 shows, the 0.2 threshold generates a failure region with very high-order geometry that standard reliability-type methods[3] (which can be applied to ascertain probability at this threshold level) cannot approximate very effectively. The Finite-Element / Lattice-Sampling (FE/LS) methodology requires at least 13 function evaluations (Level 4) before the approximation begins to adequately resemble the actual failure region.

The 0.5 threshold creates two separate failure regions (see Figure 4), one of which is a semi-circular-like failure "island" of high-order geometry. Because of the disjoint regions and complexity of the geometry, reliability-type methods are not practical for assessing probability at the 0.5 threshold level. The FE/LS methodology requires about 25 FEVs (Level 5) before the approximate failure region appears to adequately resemble the exact one.

The 1.0 and 1.5 thresholds in Figures 5 and 6 yield failure regions that are much more accommodating to reliability methods. The boundaries between 'failure' and 'no failure' are seemingly adequately approximated as linear (FORM[5]) or quadratic (SORM[6]) curves. As established quantitatively below, the FE/LS approach requires a Level 4 approximation (13 FEVs) before convergence for the 1.0 threshold, though the 9 FEV Level 3 approximation does an adequate job. Though the 1.5 threshold creates a failure region that would seem from Figure 6 to be approximated adequately with 13 FEVs in Level 4, analysis reveals that for the low probability of failure associated with this region even the Level 6 representation with 41 FEVs does not appear to be sufficient. Of course, problems like this will require a transition from global refinement (like the FE/LS procedure described here) to a more local method like might be found in adaptive finite element meshing. Algorithms are presently being devised and tested at Sandia for global and local refinement and their integration into efficient and versatile hybrid algorithms.

Model Joint Probability Density Function (JPDF)

The joint probability density function (JPDF) used in the following is depicted in Figure 7. It is a truncated 2-D standard-normal JPDF of independent normally distributed uncertain parameters *p1* and *p2* with means 0.5 and truncation limits 0 and 1. The standard deviations σ are set such that truncation of the individual distributions occurs at $\pm 3\sigma$, *i.e.* σ=0.5/3, so that the effect of truncation is relatively small.

Figure 8 shows convergence behavior for means and standard deviations for *p1* and *p2* populations ranging from 10^2 to 10^6 samples as generated by the LHS[7] code. The means of both parameters appear to converge to their terminal values within about 100 samples, while the standard deviations stabilize within 1000 samples. (We found that double precision has to be used in the SUN SPARCstation10 LHS computations in order to establish convergence, as the single precision results showed a suspicious divergent character for population sizes 10^5 and greater.) The population sizes used here extend to 10^6 because it was found that the probabilities computed for some of the following test problems converge much more slowly than the mean and standard deviations do. Presently at Sandia the convergence rates of moments and probabilities of such distributions are being explored in connection with automated convergence assessment, incremental sampling, and adaptive termination of Monte Carlo sampling when convergence within user-prescribed tolerances occurs.

Failure Probability Calculation Method

In the following, failure probabilities are estimated by evaluating a particular exact or approximate response function shown in Figure 2 at all *(p1,p2)* parameter sets in a given population of Latin Hypercube[7] samples. The resulting response values are then examined to determine the number of responses at or above the threshold value. This number is then divided by the total number of samples in the population to arrive at a failure probability for the given threshold level and population size.

Assessment of Convergence Behavior for Direct and Decoupled Monte Carlo Sampling

In the following we will talk about both "level-wise" convergence and Monte Carlo convergence. 'Level-wise convergence' refers to the convergence of a probability result as the number of FEVs increase through the various FE/LS approximation Levels (while the number of Monte Carlo samples of each response surface is the same). 'Monte Carlo convergence' refers to the convergence of a probability result as the number of MC samples of a given exact or approximate response surface increases.

For the four thresholds in Figures 3 - 6, failure probabilities from decoupled (*i.e.* using the various approximation Levels 1 - 6) and direct (*i.e.* using the exact function) Monte Carlo sampling are listed in Tables 1- 4 and plotted in Figures 9 - 12. All results were triple-checked for accuracy. The abscissa values 1 - 7 in the data represent the various levels of FE/LS response surface approximations, with Level 7 corresponding to the exact function itself.

Figure 9 shows the convergence behavior for the 0.2 threshold. The very prominent level-wise "adjustment" that occurs in going from level 3 to 4 at all population sizes corresponds to the markedly lower shaded areas in Levels 2 and 3 of Figure 3. Level 1, though also having a grossly different configuration from the exact region, is fairly close in overall area, and the distribution of the area benefits by coincidence from the circular symmetry of the JPDF. A general (non-axisymmetric) distribution combined with the Level 1 approximation would certainly not yield such close values to the exact. The large undulation in the Lattice Sampling convergence plot is representative of the oscillatory convergence behavior normally exhibited by sampling methods. One thousand LHS samples appear to be enough for MC convergence of the exact result (Level 7), though results at some of the lower levels of approximation take an

Table 1 Probability of failure for various numbers of Latin Hypercube Monte Carlo samples of exact function (Level 7) and successive FE/LS Levels. (Threshold = 0.2)

Level of Approximation	Number of Latin Hypercube Monte Carlo Samples				
	10^2	10^3	10^4	10^5	10^6
1	0.97000	0.97600	0.98000	0.97936	0.97899
2	0.96000	0.95500	0.95580	0.95602	0.95608
3	0.95000	0.94600	0.94190	0.94093	0.94171
4	0.99000	0.98300	0.98110	0.98091	0.98112
5	0.98000	0.98600	0.98440	0.98461	0.98445
6	0.98000	0.98600	0.98380	0.98383	0.98388
7 (exact)	0.98000	0.98600	0.98460	0.98448	0.98439

Table 2 Probability of failure for various numbers of Latin Hypercube Monte Carlo samples of exact function (Level 7) and successive FE/LS Levels. (Threshold = 0.5)

Level of Approximation	Number of Latin Hypercube Monte Carlo Samples				
	10^2	10^3	10^4	10^5	10^6
1	0.70000	0.71700	0.71070	0.71179	0.71374
2	0.45000	0.45000	0.44290	0.44439	0.44574
3	0.46000	0.47200	0.47910	0.47671	0.47708
4	0.53000	0.47600	0.49570	0.49278	0.49371
5	0.46000	0.42400	0.46790	0.46721	0.46592
6	0.45000	0.42400	0.44850	0.44715	0.44587
7 (exact)	0.44000	0.42900	0.45090	0.45058	0.44921

order of magnitude more samples for convergence. The level-wise convergence for the structured Lattice Sampling scheme appears to occur by Level 5 (25 samples) for all LHS populations.

Figure 10 shows the convergence behavior for the 0.5 threshold. A trend of oscillatory level-wise convergence is observed, with the Level 6 results based on 41 FEVs being essentially converged to the probabilities from the exact function. The FE/LS method takes more FEVs to satisfactorily capture the complexity and multiplicity of the failure regions for this case (see Figure 4) vs. the simpler single region for the 0.2 threshold (see Figure 3). For each of the global response surfaces (Levels 1 - 6) the LHS sampling converges in about 10,000 samples, demonstrating slight oscillatory convergence to that point. It may be remarked that the number of LHS samples required before convergence is established is generally an order or magnitude greater than for the 0.2 threshold. This is probably partially a result of the complexity and multiplicity of the failure regions and of the fact that the total probability has decreased from an order of 1.0 for the 0.2 threshold to the order of 0.1 for this threshold.

Figure 11 shows the convergence behavior for the 1.0 threshold. For the relatively simple failure region shown in Figure 5 it takes only about 16 FEVs (Level 4) for

Table 3 Probability of failure for various numbers of Latin Hypercube Monte Carlo samples of exact function (Level 7) and successive FE/LS Levels. (Threshold = 1.0)

Level of Approximation	Number of Latin Hypercube Monte Carlo Samples				
	10^2	10^3	10^4	10^5	10^6
1	9.0000e-02	6.1000e-02	6.9300e-02	6.9950e-02	7.0624e-02
2	4.0000e-02	4.9000e-02	4.7900e-02	4.9560e-02	4.9359e-02
3	3.0000e-02	1.6000e-02	1.4000e-03	1.4210e-02	1.3946e-02
4	2.0000e-02	1.2000e-02	9.3000e-03	8.2900e-03	8.1160e-03
5	2.0000e-02	1.2000e-02	8.2000e-03	7.5800e-03	7.3930e-03
6	2.0000e-02	1.2000e-02	8.1000e-03	7.3900e-03	7.2140e-03
7 (exact)	2.0000e-02	1.2000e-02	8.7000e-03	7.7600e-03	7.5600e-03

Table 4 Probability of failure for various numbers of Latin Hypercube Monte Carlo samples of exact function (Level 7) and successive FE/LS Levels. (Threshold = 1.5)

Level of Approximation	Number of Latin Hypercube Monte Carlo Samples				
	10^2	10^3	10^4	10^5	10^6
1	0.0000	0.0000	1.0000e-03	5.1000e-04	5.3700e-04
2	0.0000	0.0000	1.0000e-03	5.5000e-04	5.4400e-04
3	0.0000	0.0000	1.0000e-04	1.0000e-04	1.0000e-04
4	0.0000	0.0000	1.0000e-04	1.6000e-04	1.5600e-04
5	0.0000	0.0000	1.0000e-04	2.2000e-04	1.9400e-04
6	0.0000	0.0000	1.0000e-04	2.1000e-04	2.2800e-04
7 (exact)	0.0000	0.0000	2.0000e-04	2.3000e-04	2.5200e-04

probabilities of the various populations to converge level-wise. Though the failure region is much simpler to resolve than for the 0.5 threshold above, 10,000 LHS samples are still required before MC convergence occurs. This is presumably because, though the failure region is easier to resolve in this case, the probability level drops to the order of 0.01.

Figure 12 shows the convergence behavior for the 1.5 threshold. For the very simple triangular failure region shown in Figure 6 the results are not yet quite converged even by Level 6 for most MC populations. Though the triangular failure region is seemingly very easy to resolve, even very small changes in the approximation can affect the number of LHS samples falling inside the region, and since the total number of samples falling in the region is relatively small (probability is on the order of 0.0001 for this case), a small difference in the number of samples falling inside it can have a large relative effect. The effect is particularly accute at the lower levels of approximation, where small inaccuracies in the size of the region (see Figure 6) contribute to relatively large percentage changes in its size and therefore in the number of samples falling within its boundaries. The convergence of LHS sampling is also not as fast for this case as for the other cases. The results using the exact function (Level 7) do not seem to have converged even with 10^6 samples, though some of the other Levels look converged at this number of samples. The reason for the slow Monte Carlo convergence is of course the very low probabilities involved.

Efficiency Comparison of Direct and Decoupled Monte Carlo Sampling

Toward a valid comparison of the convergence rates of direct and decoupled LHS Monte Carlo approaches, the following reasoning was used to establish a basis for comparison. Over the convergence plots Figures 9 - 12, a population size of 10,000 samples in all cases gives nearly the same probabilities as those obtained with 1,000,000 samples. Therefore, for convenience, 10^4 is taken as the convergence limit of population size for these problems. Assuming that each sample takes 1 second of CPU time (which is actually about 2 orders of magnitude longer than we experienced running the FE/LS INTERP research code on these 2-random-variable problems), we arrive at a total execution time of 2.78 hours. Now, let us also assume that we have a large finite element model that takes an average of 2.78 CPU hours for each function evaluation in our problem. Therefore, to achieve a Level 1 decoupled Monte Carlo (10,000 sample) estimation of failure probability, we add the time required for the 4 FEVs in Level 1 to an equivalent 1 FEV of CPU time required to sample the Level 1 response surface 10,000 times via the INTERP code. Thus, the total equivalent CPU time for the analysis is 5 FEVs. We can then compare the results of 5 FEVs used in this manner to the results of using 5 FEVs in a direct Monte Carlo evaluation of probability. Likewise, a Level 6 analysis requires 41 + 1 = 42 FEV, which we can compare against direct Monte Carlo evaluation with 42 samples. Now, if the finite element model only takes an average of 1 CPU minute to run, a similar accounting shows that a Level 1 evaluation requires 4 + 167 = 171 FEVs, while a Level 6 evaluation requires 41 + 167 = 208 FEVs.

We see that for decoupled Monte Carlo analysis at a given FE/LS approximation Level, the equivalent number of direct MC samples decreases as the CPU time of the involved model increases, *e.g.* from 171 to 5 and from 208 to 42 when model FEV increases from 1 CPU minute to 2.78 CPU hr. Therefore, *everything else being equal, relative to direct Monte Carlo analysis the marginal efficiency of the Finite-Element / Lattice-Sampling decoupled Monte Carlo (FELSDMC) approach increases as the size of the finite element model increases, which is a very favorable indicator for the FELSDMC method.*

The *absolute* efficiencies of the direct and decoupled LHS Monte Carlo approaches can be compared for the parameters of the present problems by examining Figures 13 - 16. The abscissa in the plots are mapped in Table 5 to equivalent numbers of FEVs. The circles on the plots mark data generated by the FELSDMC method. The triangles on the plots mark data generated by direct Monte Carlo sampling where it is assumed that the model takes 60 times as long to run as the response surface approximation does. The X's on the plots mark data generated by direct Monte Carlo sampling with a model assumed to take even 60 times longer still. In all cases (for all functions and threshold or probability levels), the FELSDMC method converges fastest toward the "exact" result obtained from 10,000 samples of the

Table 5 Mapping of abscissa in Figures 13 - 16 to equivalent numbers of function evaluations

		Equivalent Number of FEVs			
		DMCFELS w/ 10^4 1-sec. MC samples	direct LHS (2.78 hr. CPU model)	DMCFELS w/ 10^4 1-sec. MC samples	direct LHS (1min. CPU model)
stage	1	5	5	5	171
	2	6	6	6	172
	3	10	10	10	176
	4	14	14	14	180
	5	26	26	26	192
	6	42	42	42	208

analytic function. The next fastest convergence rate occurs for direct MC sampling where FEVs are assumed to take 1 CPU minute (relative to 1 CPU second assumed per run of the INTERP code). Despite the slower convergence, the equivalent computational costs shown in Table 5 are from 5 to 43 times the cost of the decoupled Monte Carlo approach. The slowest convergence occurs under the assumption that FEVs take 2.78 CPU hours each, in which case the direct Monte Carlo equivalent costs shown in the table are from 2% to 25% more than decoupled MC even though convergence is much slower. The data also suggests that the convergence advantages of the FELSDMC method become more pronounced as the magnitude of the probability being resolved decreases. This is to be expected for the smooth, non-stochastic, low-order functions tested here.

4. Conclusion

Data presented here and elsewhere for other model problems[1] indicates that geometry and topology convergence in the Finite-Element / Lattice-Sampling scheme (i.e. level-wise convergence) occurs with orders of magnitude less samples than it takes for LHS Monte Carlo sampling to converge to a probability estimate. In all cases the Level 6 results based on 41 samples approximate the "exact" results of Level 7 extremely closely. That is, running any nondeterministic analysis method off of the Level 6 response surfaces would yield essentially the same results as running the method off the exact function itself. In fact, when a complex finite element model is used as the "exact" function, running derivative-based optimization and reliability methods off the actual model might actually be worse than running off an appropriate FE/LS representation because of stochastic numerical noise associated with large mechanics simulations (see e.g. [3] and [4]).

Certainly, the indications here are that, assuming it takes orders of magnitude more CPU time to run a simulation with the phenomenological model than to run the FE/LS surrogate model (which will usually be the case), the decoupled Monte Carlo approach with Finite-Element Lattice-Sampling response surfaces can require orders of magnitude less computer time overall than direct Monte Carlo analysis with the phenomenological model itself - with no appreciable loss of accuracy. Furthermore, the marginal efficiency of the FELSDMC approach increases as the size of the computational model increases.

Therefore, the cumulative experience to date suggests that when estimating probability with a Monte Carlo approach, it is much more efficient to expend computer model function evaluations on building FE/LS response surfaces that to expend them in a direct Monte Carlo simulation.

REFERENCES

[1] Romero, V.J., and Bankston, S.D., "Finite-Element / Progressive-Lattice-Sampling Response-Surface Methodology and Application to Testbed Probability Quantification Problems," Sandia National Laboratories report SANDxx-yyyy, to be printed Feb 1998

[2] Romero, V. J., "Efficient Propagation of Uncertainty and Probabilistic Behavior through Engineering Models via Decoupled Monte Carlo with Finite Element Response Surfaces Built from Structured Sampling," in preparation for submission to International Journal for Numerical Methods in Engineering.

[3] Romero, V. J., "Noise and Bias vs. Model Resolution in Complex Physics Simulations and a Simple Response Surface Approach for Making Numerical Optimization More Affordable," to be submitted to Computer Modeling and Simulation in Engineering

[4] Eldred, M.S., Outka, Bohnhoff, W.J., Witkowski, W.P., Romero, V.J., Ponslet, E.J., and Chen, K.S., "Optimization of Complex Mechanics Simulations with Object-Oriented Software Design," in Computer Modeling and Simulation in Engineering, Vol. 1 No. 3, August, 1996, pp. 323-352.

[5] Wu, Y.-T., Millwater, H.R., and Cruse, T.A., "An Advanced Probabilistic Structural Analysis Method for Implicit Performance Functions", AIAA Journal J 17624, pp. 1-34.

[6] Wu, Y.-T., and Wirsching, P. H., "New Algorithm for Structural Reliability Estimation", ASCE Journal of Engineering Mechanics (Sept. 1987), vol. 113, no. 9, pp. 1319-1336.

[7] Iman, R.L., and Shortencarier, M.J., "A FORTRAN77 Program and User's Guide for the Generation of Latin Hypercube and Random Samples to Use with Computer Models," Sandia National Laboratories report SAND83-2365 (RG), printed March 1984.

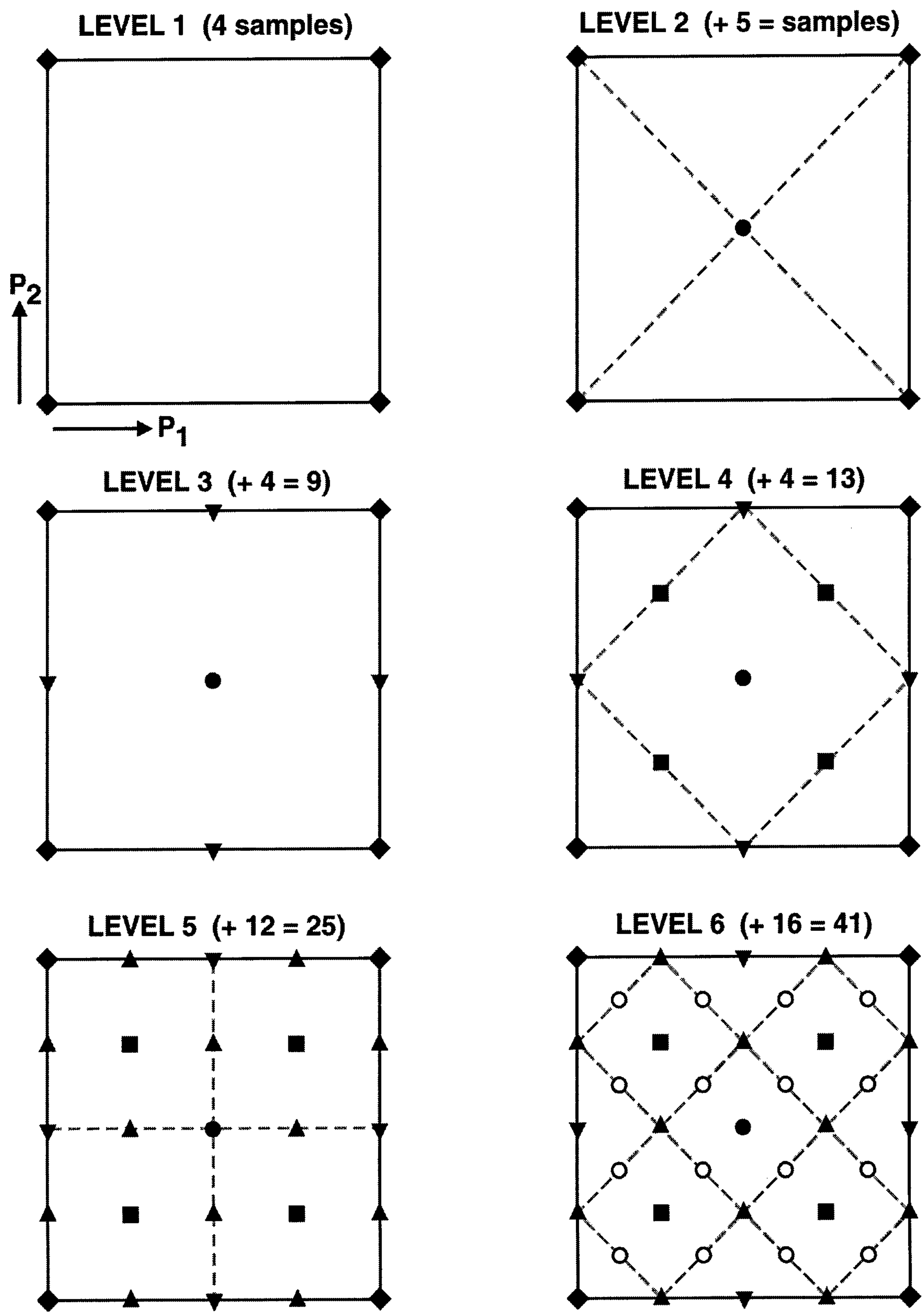

Figure 1: 2-D Lattice Sampling Levels and associated discretization of the parameter space.

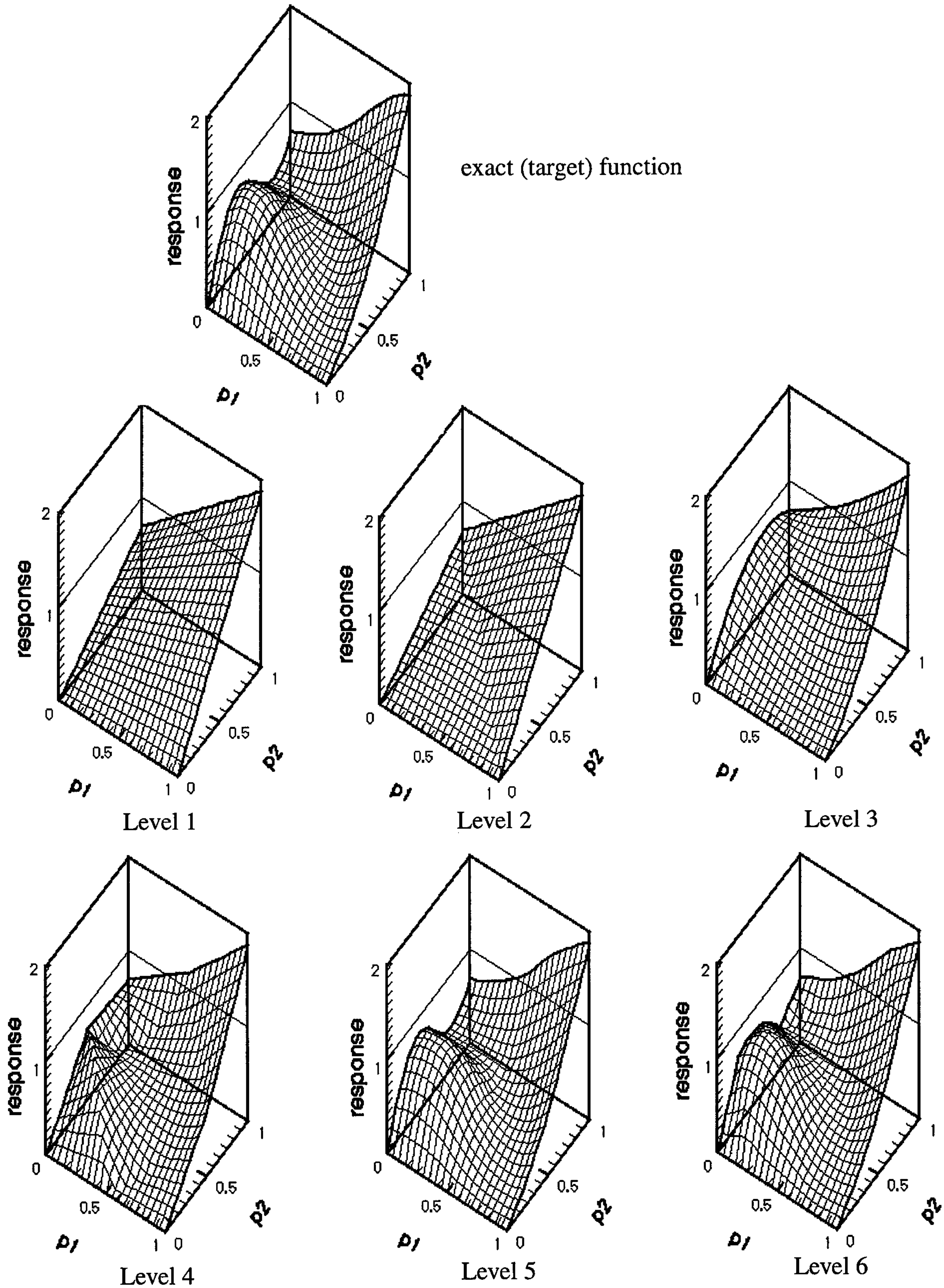

Figure 2: Various Lattice Sampling / Finite Element approximation levels

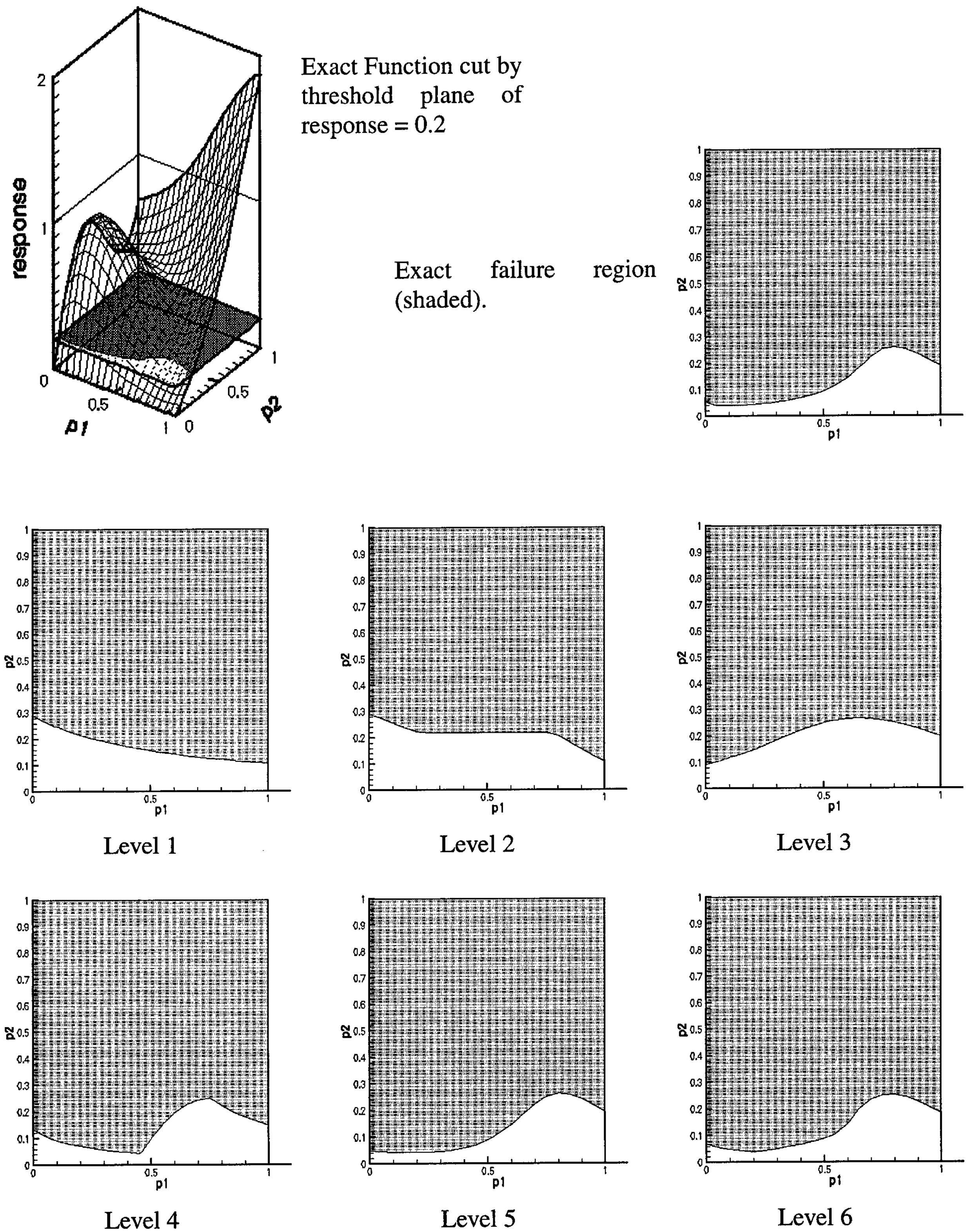

Figure 3: Parameter space "failure regions" (shaded) for exact and approximate response surfaces and a response exceedence threshold of 0.2

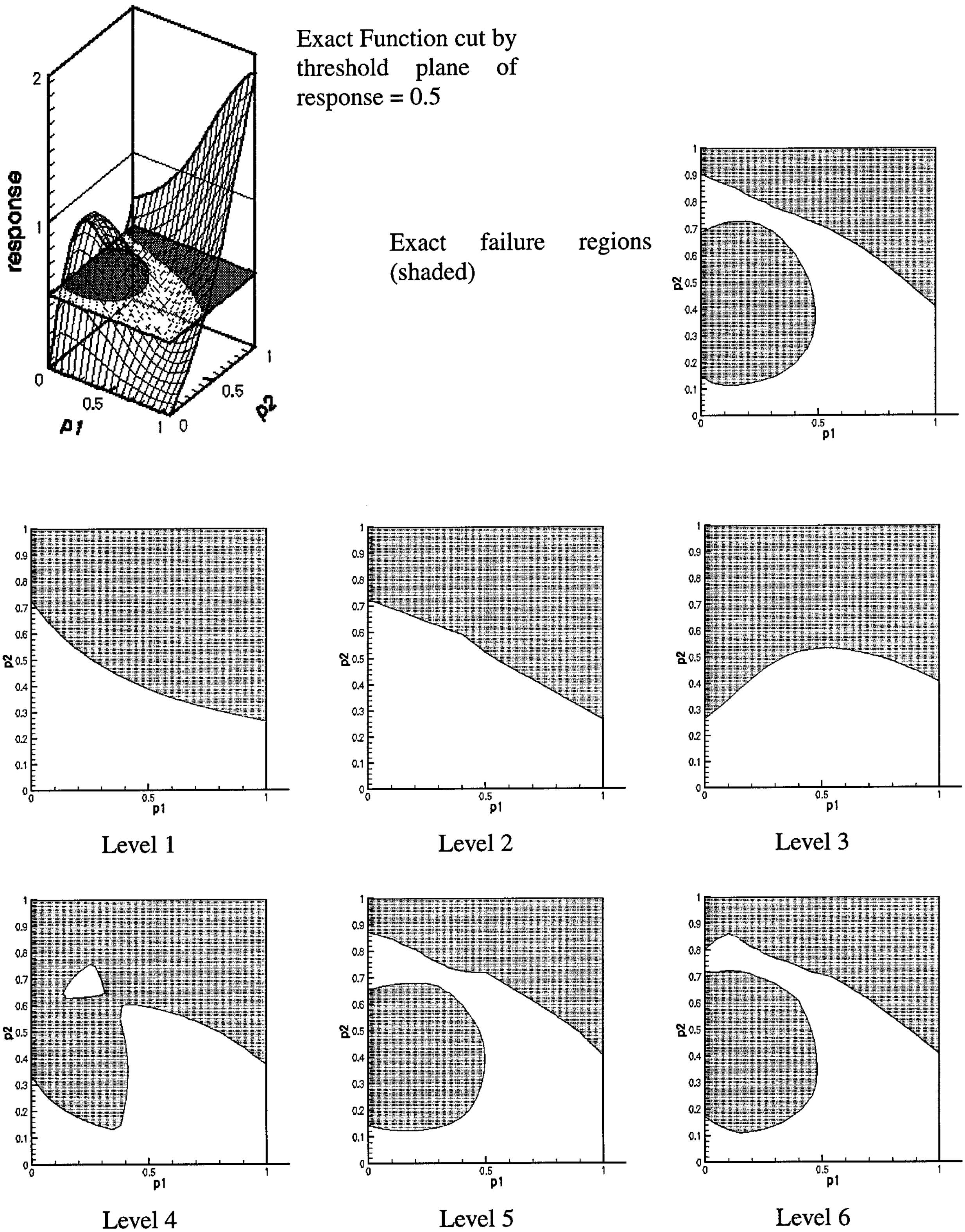

Figure 4: Parameter space "failure regions" (shaded) for exact and approximate response surfaces and a response exceedence threshold of 0.5.

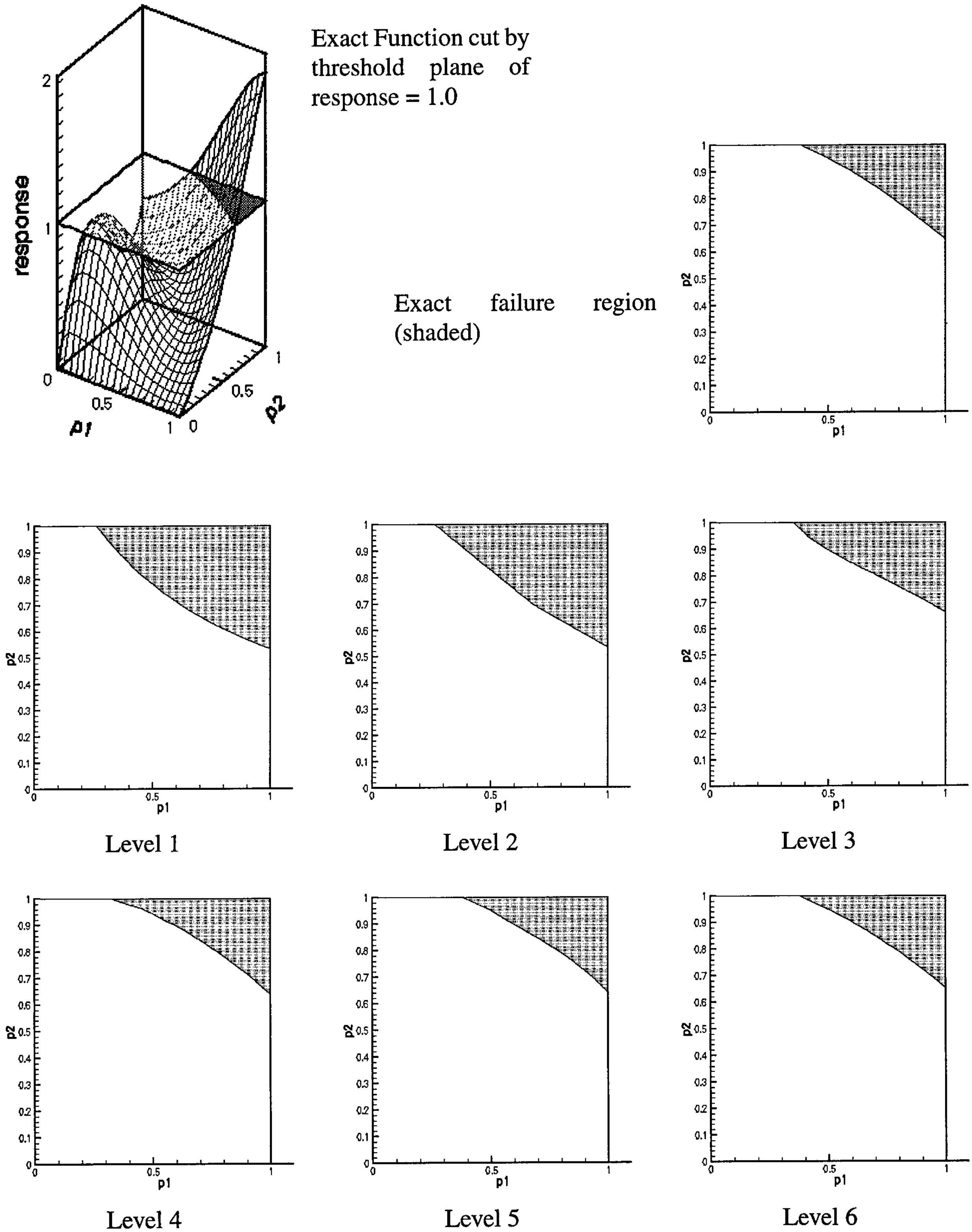

Figure 5: Parameter space "failure regions" (shaded) for exact and approximate response surfaces and a response exceedence threshold of 1.0.

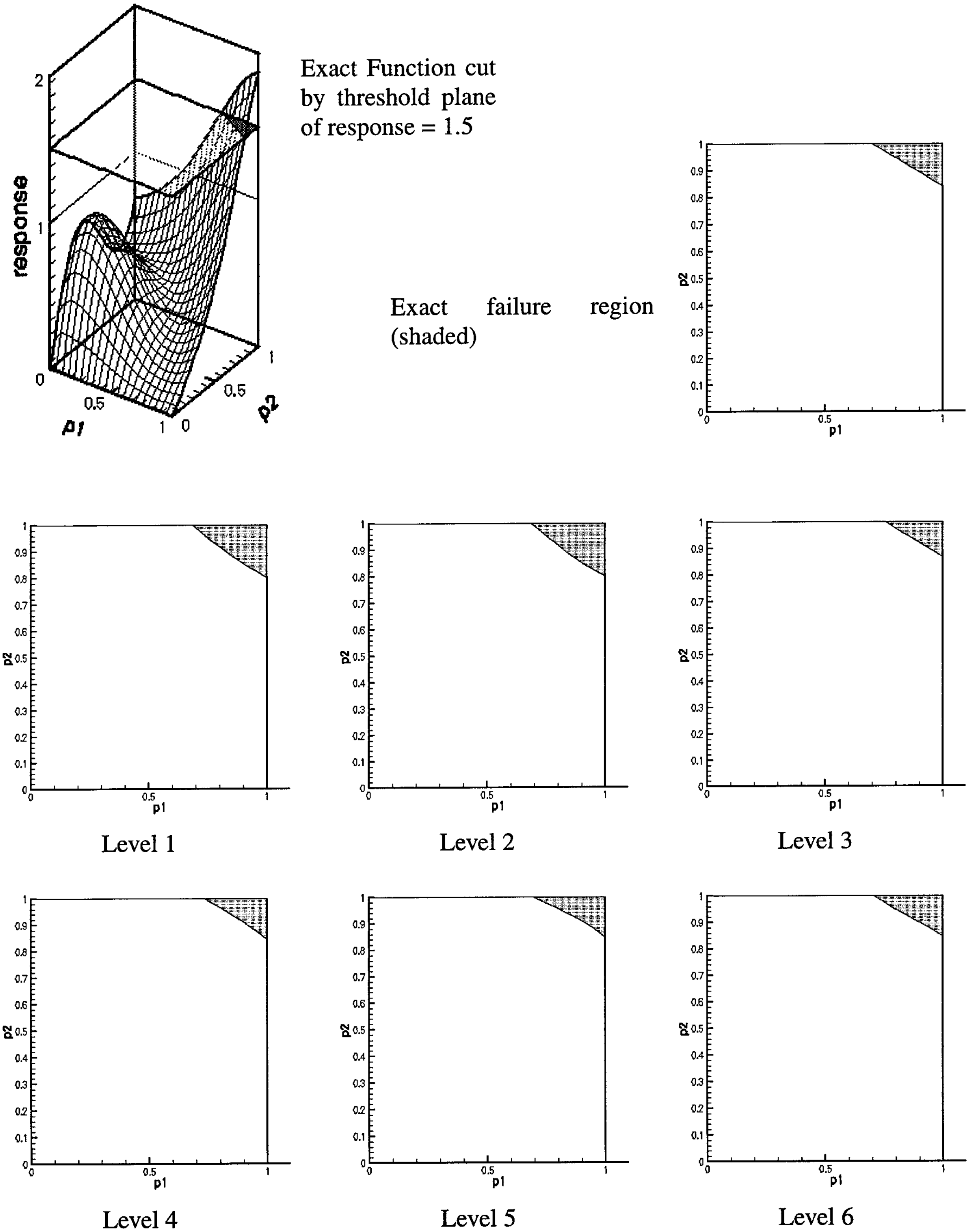

Figure 6: Parameter space "failure regions" (shaded) for exact and approximate response surfaces and a response exceedence threshold of 1.5.

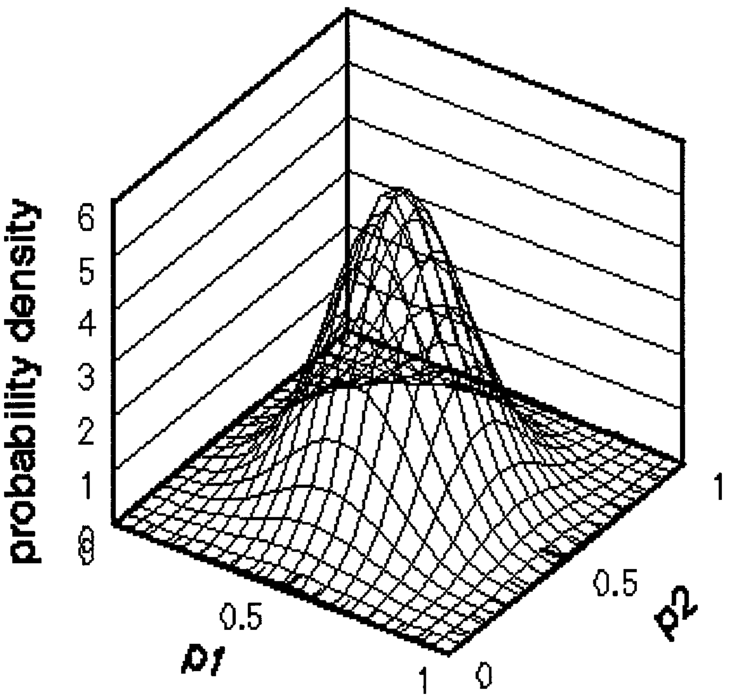

Figure 7: Joint Probability Density Function describing the random variables in the problem: normally distributed parameters *p1* and *p2* with means 0.5, std. deviations 0.167, and trunctation of the unit square parameter space at 3σ above and below the means.

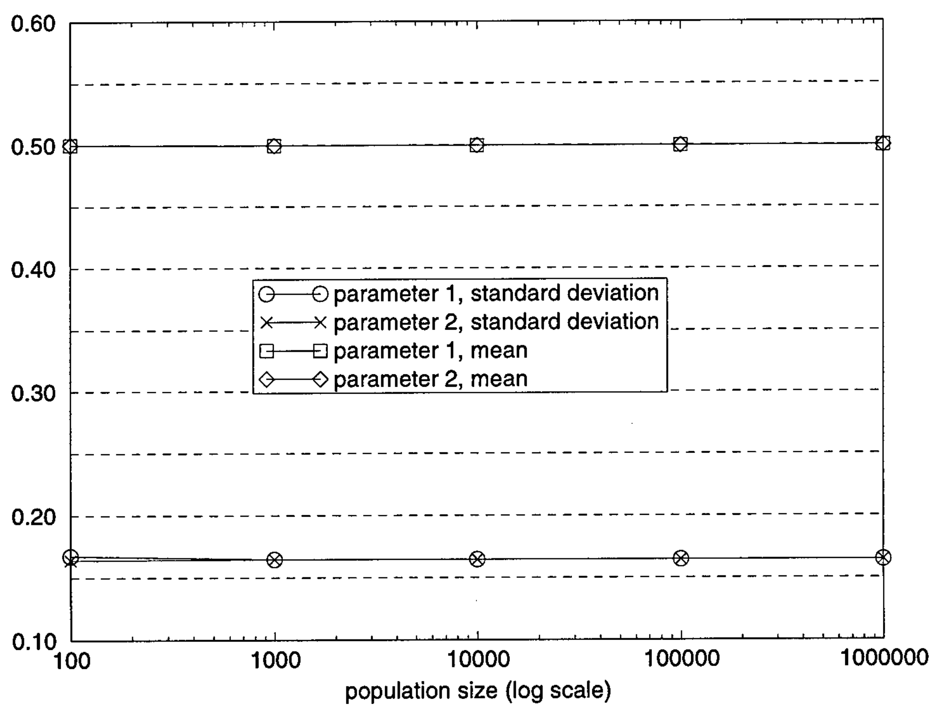

Figure 8: Convergence behavior of means and standard deviations of LHS-generated populations from the above JPDF for 100 to 1,000,000 samples in decades

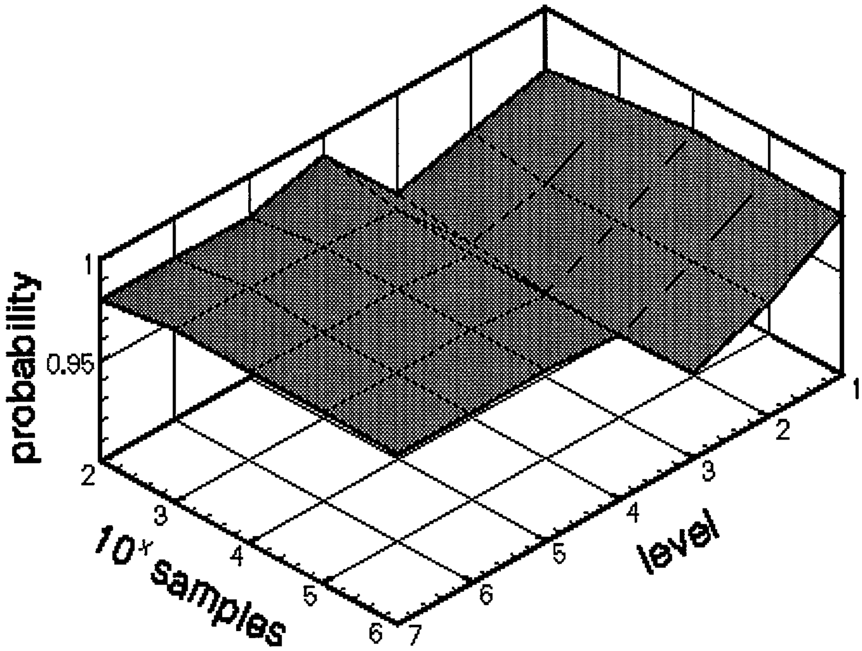

Figure 9: Convergence behavior for LHS Monte Carlo sampling and Progressive Lattice sampling for a threshold of 0.2.

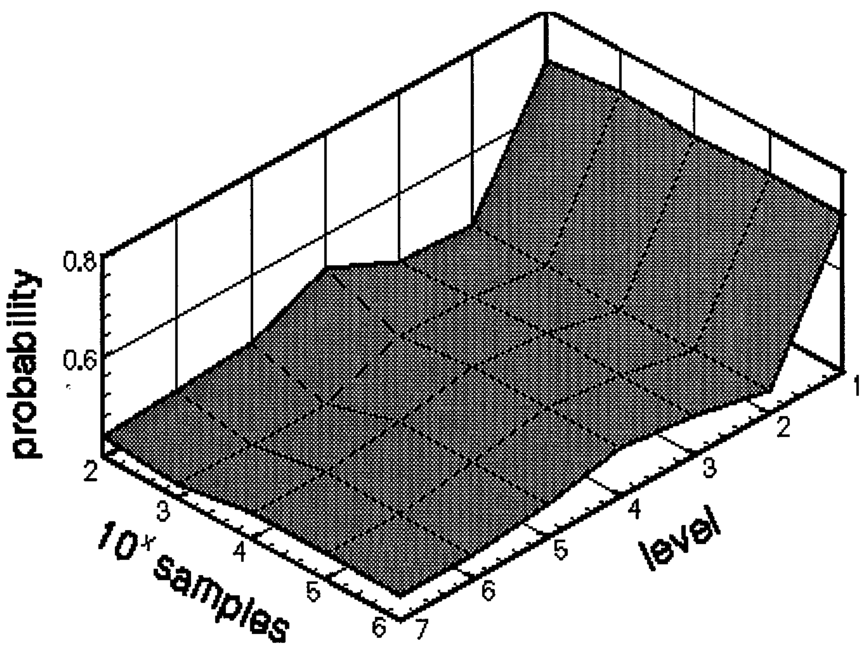

Figure 10: Convergence behavior for LHS Monte Carlo sampling and Progressive Lattice sampling for a threshold of 0.5.

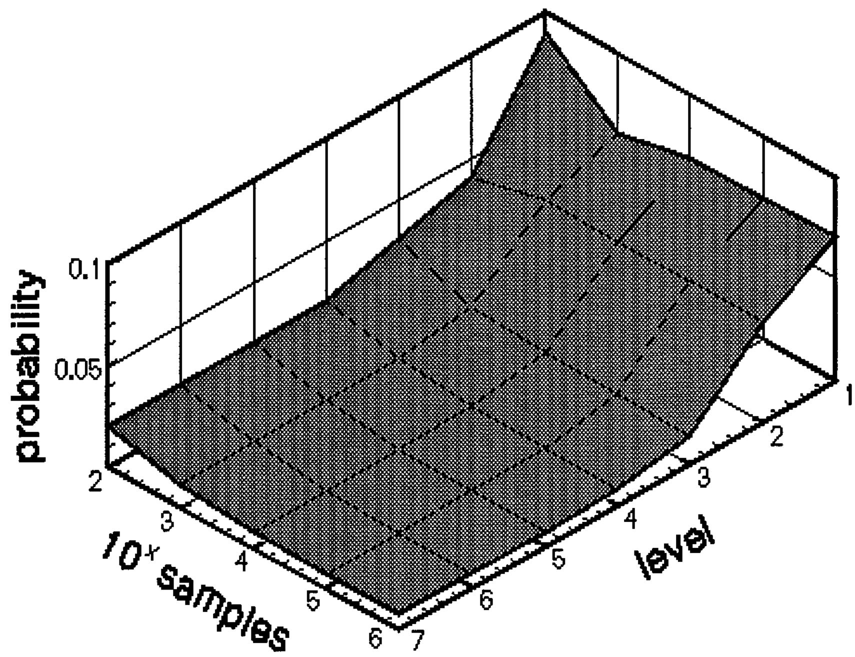

Figure 11: Convergence behavior for LHS Monte Carlo sampling and Progressive Lattice sampling for a threshold of 1.0.

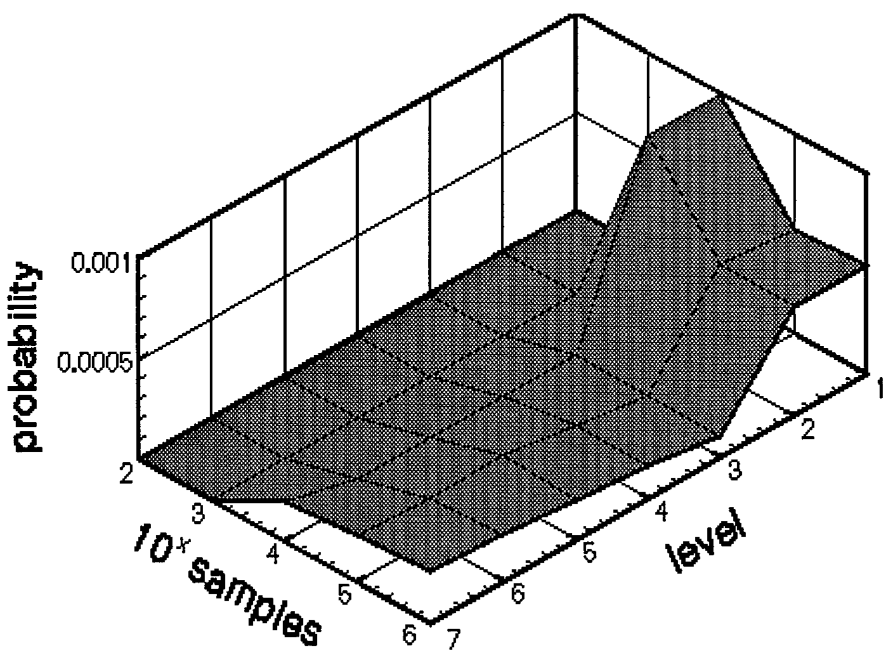

Figure 12: Convergence behavior for LHS Monte Carlo sampling and Progressive Lattice sampling for a threshold of 1.5.

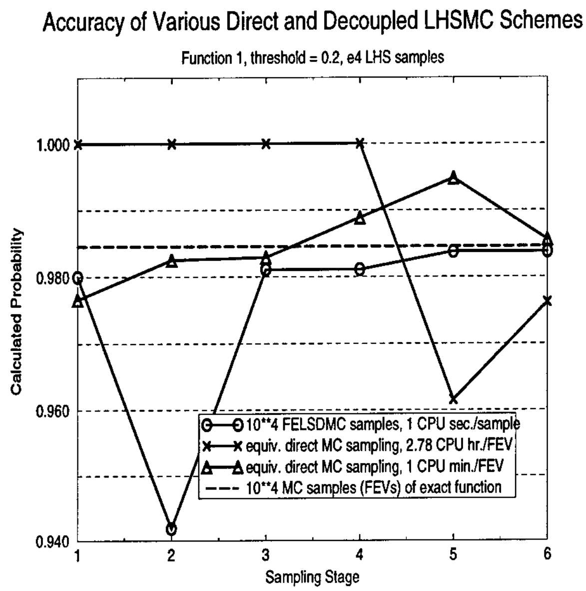

Figure 13: Comparison of various instances of direct and decoupled Monte Carlo Latin Hypercube Sampling for a threshold of 0.2.

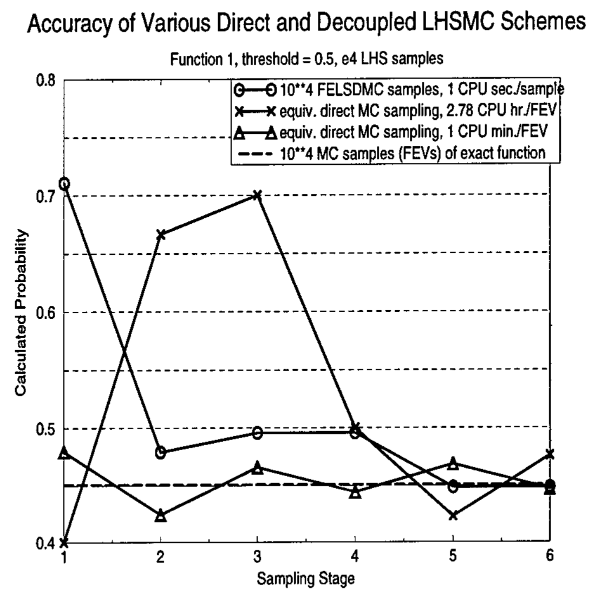

Figure 14: Comparison of various instances of direct and decoupled Monte Carlo Latin Hypercube Sampling for a threshold of 0.5.

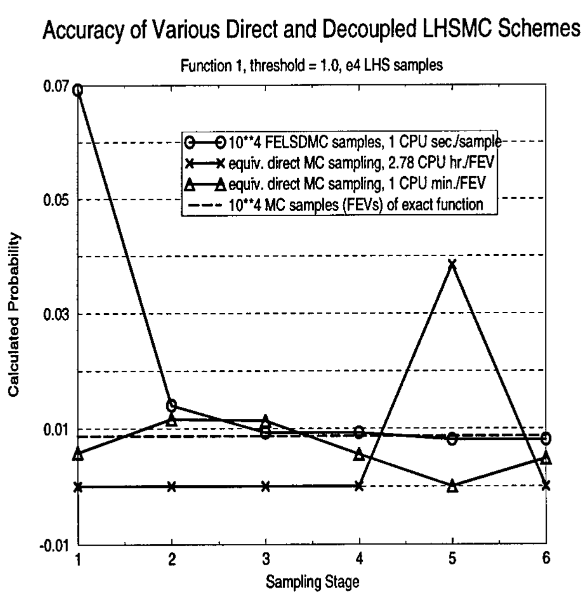

Figure 15: Comparison of various instances of direct and decoupled Monte Carlo Latin Hypercube Sampling for a threshold of 1.0.

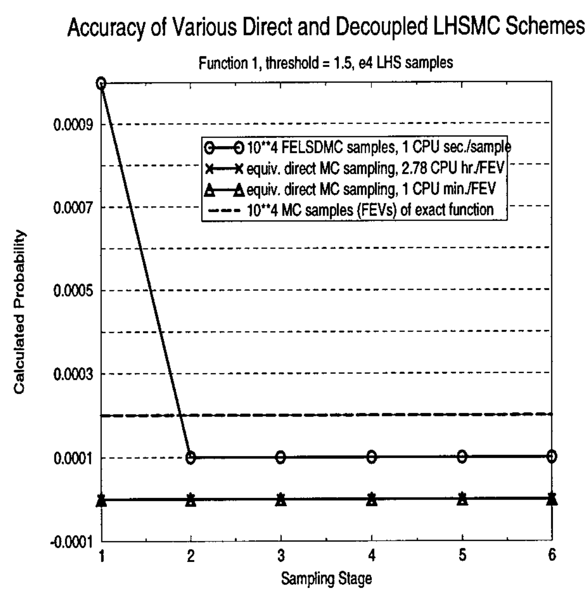

Figure 16: Comparison of various instances of direct and decoupled Monte Carlo Latin Hypercube Sampling for a threshold of 1.5.

M98002809

Report Number (14) SAND--98-0607C

CONF-980419--

Publ. Date (11) 19980310

Sponsor Code (18) DOE/CR, XF

UC Category (19) UC-900, DOE/ER

DOE