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Fixed Points: Fermion Mass Predictions

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ABSTRACT

The renormalization group equations for heavy fermion Higgs-Yukawa coupling constants possess low energy fixed points. We predict the masses of fourth generation quarks and leptons, or an ultra-heavy top quark. These also correspond to upper bounds on fermion masses in SU(5)-like theories.

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FIXED POINTS: FERMION MASS PREDICTIONS

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Familiar theoretical relationships exist amongst the masses of the light fermions in SU(5):¹

$$\frac{m_d}{m_e} \approx \frac{m_s}{m_\mu} \approx 4 \pm \frac{1}{2} \qquad \frac{m_b}{m_\tau} \approx 2.75 \pm .25 \qquad (1)$$

These result from essentially three ingredients: (a) the Higgs mechanism which relates the fermion mass to the Higgs-Yukawa (HY) coupling constant by $m_f = g_f \langle \phi^0 \rangle / \sqrt{2} \approx g_f \cdot (175 \text{ GeV})$; (b) the renormalization group (RG) equations which describe the evolution of the (HY) couplings down from M_X to low energies and (c) the boundary conditions from SU(5), assuming a 5 and 24 of Higgs, which relate $g_{-1/3}^{\text{quark}}(M_X) = g_{-1}^{\text{lepton}}(M_X)$ within a given generation. Thus one obtains:

$$\frac{m_{-1/3}}{m_{-1}} = \frac{g_{-1/3}(\mu)}{g_{-1}(\mu)} \approx \left(\frac{\alpha_3(\mu)}{\alpha_3(M_X)} \right)^{8/2b_0} + (\text{electroweak}) \Big|_{\mu = m_{\text{quark}}} \qquad (2)$$

and the numerical results of eq. (1) follow to one loop accuracy with $\Lambda_g \sim .4 \text{ GeV}$. We note that ingredients (b) and (c) do not apply to models with composite Higgs bosons (e.g. ETC) or composite fermions on a scale of $M' < M_X$ since the RG equations demand pointlike particles over the entire range of the desert. Though the results for m_d/m_e and m_s/m_μ are questionable we note that they are qualitatively correct. We take the "prediction" for m_b/m_τ to be a successful result of SU(5) and we seek to extend the above analysis to much heavier objects; either a very heavy top-quark or a heavy fourth generation of quarks and leptons.

For very heavy quarks and leptons the RG equations of the HY coupling constants, which lead to eq. (2), must be modified to include the effects of the (now large) HY couplings themselves. The equations thus become nonlinear in g_f when $g_f \sim (g_2 \text{ or } g_3)$ and will thus fix the absolute scale of g_f . Hence, the relevant fermion mass scale for which these effects become important is expected to be, assuming a single 5 of Higgs, $m_{\text{quark}} \sim g_3(200) \cdot 175 \sim 240 \text{ GeV}$ (with several isodoublet Higgs having VEVs this scale decreases).

Furthermore, as first emphasized by Froggatt and Nielsen² and more recently by Pendleton and Ross,³ there will be "fixed points" which determine $g_i(\mu)$ independent of $g_i(M_X)$ (though in general dependent upon μ , M_X , g_3 , g_2 , g_1 , etc.). Thus for sufficiently large $g_i(M_X)$ (typically $g_i(M_X) \geq 1$) we can hope to make predictions for heavy fermion masses from the fixed point structure of the RG equations without knowing any details of the initial conditions at M_X ! However, the nature of these fixed points is slightly subtle,⁴ as we now show.

Pendleton and Ross considered the possibility of a heavy t-quark (we will focus upon a fourth generation below which we expect to be a more realistic possibility; the t-quark will serve as a paradigm for the mathematics) for which the RG equations of g_t and g_3 ($\equiv g_{QCD}$) become:

$$\left. \begin{aligned} 16\pi^2 \frac{d}{dt} \ln g_t &= \left(\frac{9}{2} g_t^2 - 8g_3^2 \right) \\ 16\pi^2 \frac{d}{dt} \ln g_3 &= -b_0 g_3^2 ; \quad t = \ln \mu, \quad b_0 = 7 \end{aligned} \right\} \quad (3)$$

Combining:

$$16\pi^2 \frac{d}{dt} \ln (g_t/g_3) = \frac{9}{2} g_t^2 - (8 - b_0) g_3^2 = \frac{9}{2} g_t^2 - g_3^2 \quad (4)$$

and hence, the Pendleton-Ross "quasi-stable fixed point" is the vanishing of the rhs of eq. (4):

$$g_t(\mu) = \sqrt{\frac{2}{9}} g_3(\mu) \quad (5)$$

If $g_t(\mu)$ is ever near the value given by eq. (5) in terms of $g_3(\mu)$, it will remain "locked in" to this relationship for all subsequent, decreasing μ . Taking $\mu \sim 100$ GeV one has $m_{top} = g_t \cdot 175 = 110$ GeV, which becomes 135 GeV upon including electroweak corrections.

Physically we must ask, however, given an arbitrary initial $g_t(M_X)$ what is the most probable final result for $g_t(\mu)$? In fact, eq. (4) is just the "Bernoulli equation" and can be solved analytically:

$$g_t^2(\mu) = \frac{g_t^2(M_X) (g_3^2(\mu)/g_3^2(M_X))^{8/b_0}}{\left\{ 1 + \frac{9g_t^2(M_X)}{2g_3^2(M_X)} \left(\left(\frac{g_3^2(\mu)}{g_3^2(M_X)} \right)^{1/b_0} - 1 \right) \right\}} \quad (6)$$

In the limit $\left(g_3^2(\mu)/g_3^2(M_X)\right)^{1/b_0} \gg 1$ we reach the PR fixed point:

$$g_t^2(\mu) \rightarrow \frac{2}{9} \left(\frac{g_3^2(\mu)}{g_3^2(M_X)} \right)^{7/b_0} g_3^2(M_X) = \frac{2}{9} g_3^2(\mu) \Big|_{b_0=7} \quad (7)$$

However, to be at a fixed point in the sense that $g_t(M_X)$ no longer influences $g_t(\mu)$ it is sufficient that

$$\frac{9}{2} \frac{g_t^2(M_X)}{g_3^2(M_X)} \left(\left(\frac{g_3^2(\mu)}{g_3^2(M_X)} \right)^{1/b_0} - 1 \right) \gg 1 \quad (8)$$

which can easily occur for $\left(g_3^2(\mu)/g_3^2(M_X)\right)^{1/b_0} \sim 1$, long before the limit leading to the PR fixed point. Defining $R = g_3^2(\mu)/g_3^2(M_X)$ we now assume $1/b_0 \ln R \sim 0$. Expanding eq. (6) one finds:

$$g_t^{*2}(\mu) \equiv g_t^2(\mu) = \frac{2b_0}{9} \frac{g_3^2(\mu)}{\ln R} \left\{ 1 + \frac{1}{2b_0} \ln R + \frac{7}{12b_0^2} (\ln R)^2 + \dots \right\} \quad (9)$$

Equation (9) defines a moving fixed point in μ and M_X which we refer to as the "intermediate fixed point." It is the physically interesting asymptotic behavior for $g_t(\mu)$ as $\mu/M_X \rightarrow 0$, but sets in before the decoupling limit, $\mu \gtrsim m_t$.

In Figure 1 we illustrate how an arbitrary initial $g_t(M_X)$ tends to be swept toward $g_t^*(\mu)$ (we also plot the decoupling limit, $\mu = 175 g_t(\mu)$) provided $g_t(M_X) \gtrsim 1$. This is a valid perturbative estimate provided $g_t^2(\mu)/16\pi^2 \lesssim 1$ or $g_t \lesssim 4\pi$. Hence, there is a large perturbative domain of attraction corresponding to $g_t^*(\mu) \sim 1.3$ for $\mu = 200$ GeV. The resulting "prediction" for our hypothetical single heavy t-quark is 240 GeV ($\pm 10\%$) including full electroweak corrections.⁴ This is also equivalent to the absolute upper bound of Cabibbo et al.⁵ Is this a reasonable value for the mass of the physical t-quark? Recently Buras⁶ has obtained a limit of $m_t \lesssim 33$ GeV and we remark that a t-quark heavier than ~ 200 GeV will destroy the quantitatively successful m_b/m_t relationship.⁷ Otherwise, this is consistent with all bounds.

Probably more interesting and relevant to the real world are the consequences for a fourth SU(5) generation. Here we must numerically integrate the equations for g_E, g_T, g_B (where T, B, E refer to +2/3,

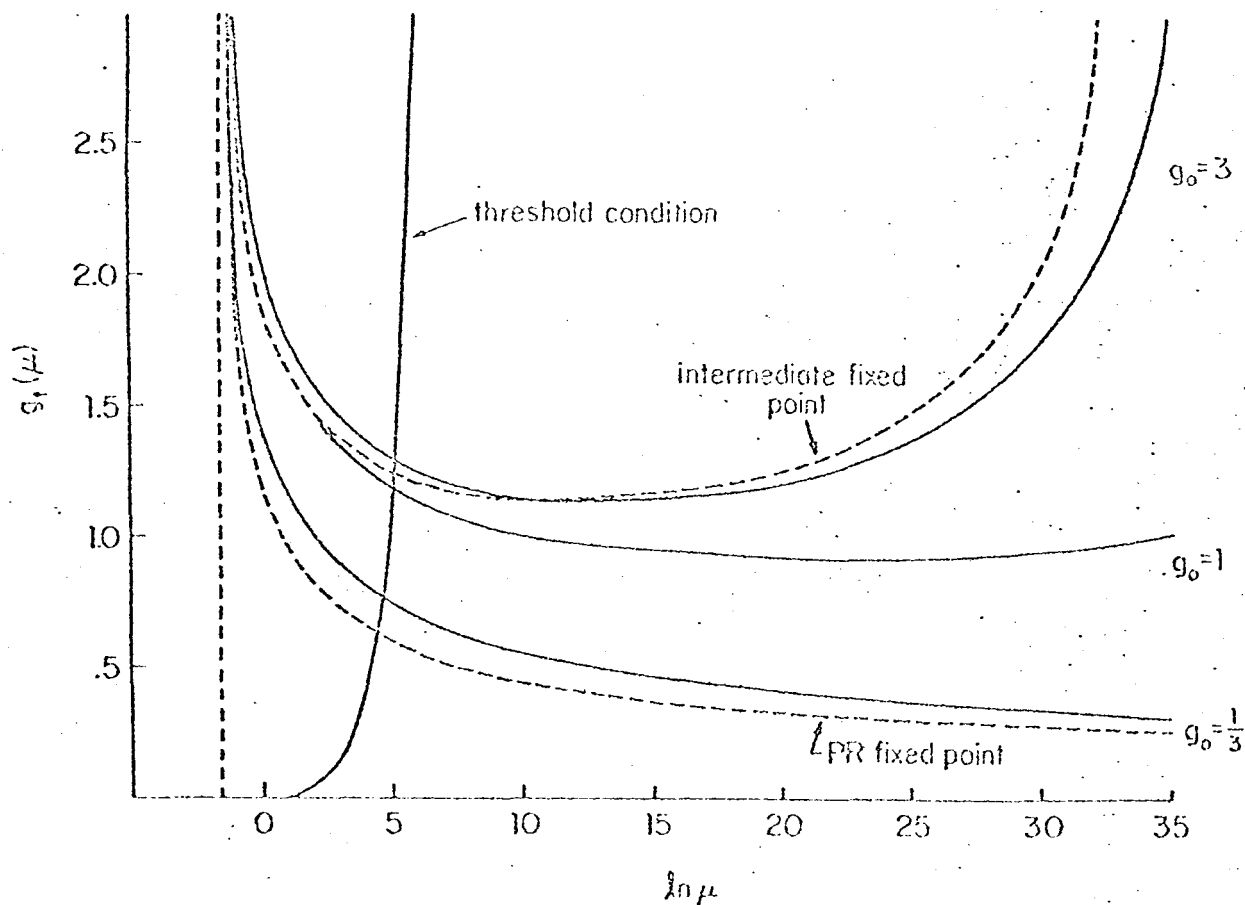


Figure 1. The evolution of $g_t(\mu)$ for three initial values, $g_t(M_X)$. Only the portion to the right of the "threshold condition" is physical. The intermediate fixed point of eq. (9) is the dotted line.

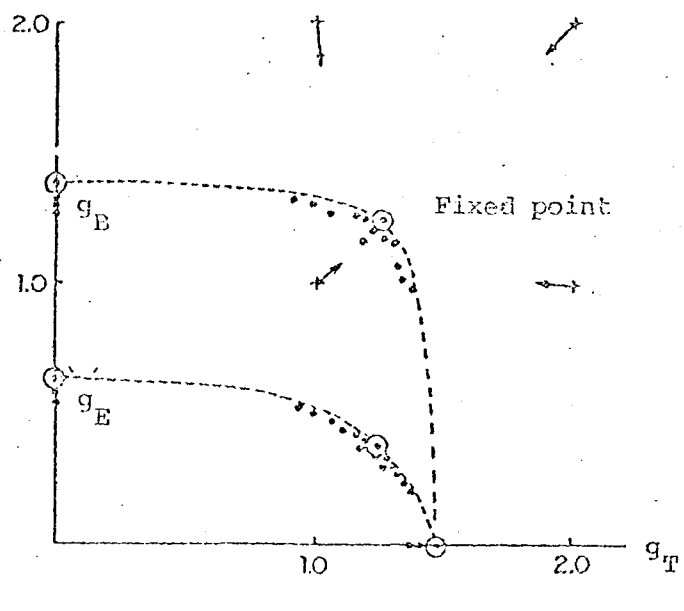


Figure 2. The results of the numerical integration of eq. (10) for a 5×5 integer array of $(g_T(M_X), g_E(M_X) = g_E(M_X))$. Points cluster about the fixed points and boundary curves.

-1/3 quarks and the -1 lepton respectively; we assume a light neutrino here):

$$\begin{aligned}
16\pi^2 \frac{d}{dt} \ln g_T &= \frac{9}{2} g_T^2 + \frac{3}{2} g_B^2 + g_E^2 - 8g_3^2 - \frac{9}{4} g_2^2 - \frac{17}{12} (g')^2 \\
16\pi^2 \frac{d}{dt} \ln g_B &= \frac{9}{2} g_B^2 + \frac{3}{2} g_T^2 + g_E^2 - 8g_3^2 - \frac{9}{4} g_2^2 - \frac{5}{12} (g')^2 \\
16\pi^2 \frac{d}{dt} \ln g_E &= \frac{5}{2} g_E^2 + 3g_T^2 + 3g_B^2 - \frac{9}{4} g_2^2 - \frac{15}{4} (g')^2 \quad . \quad (10)
\end{aligned}$$

The results of this analysis have been detailed in ref. (4). For lack of space we quote only the essential results.

(I) There is a nontrivial fixed point for these equations with all g_i nonvanishing analogous to the intermediate fixed point discussed above. It corresponds to:

$$m_T = 220 \text{ GeV} \quad m_B = 215 \text{ GeV} \quad m_E = 60 \text{ GeV} \quad (11)$$

to within 10% uncertainty at one loop. We've also assumed no Cabibbo mixing to lighter generations (the results are quite insensitive to this; rotating say the B quark maximally (90°) out of the weak current for this generation displaces the fixed point 27%).

(II) The bound of Cabibbo et al.⁵ generalizes to an allowed region in the space of g_T , g_B and g_E . Only points within this region are physical (have finite values of g_T , g_B , g_E over the entire desert). Moreover, the boundary of this region acts somewhat like a "generalized fixed point" since arbitrary, large initial points $g_T(M_X)$, $g_B(M_X)$ and $g_E(M_X)$ are mapped preferentially to the fixed point or the boundary, the fixed point lying on the boundary. Figure 2 illustrates the distribution of values of $g_B(200 \text{ GeV})$ and $g_E(200 \text{ GeV})$ vs. $g_T(200 \text{ GeV})$ resulting from the numerical integration of eq. (10) for an initial 5×5 array of points $(g_B(M_X), g_T(M_X))$ and $(g_E(M_X) \equiv g_B(M_X), g_T(M_X))$. We see that the points cluster near the fixed point along the boundary curve (the fixed point corresponds to the mass values of eq. (11)). Hence, in addition to a relationship between m_B and m_E , we also obtain a further relationship between all three masses m_B , m_E and m_T for a sufficiently heavy fourth generation! These results can be generalized to many succeeding heavy generations.

(III) These heavy masses are fully consistent with known bounds on fermion masses, e.g. the ρ -parameter,⁸ unitarity⁹ and the stability of

the Higgs potential.¹⁰

Recently we have considered the effects upon the standard evolution of g_1 , g_2 and g_3 in SU(5) from large g_F , at the two loop level, and thus the effects upon M_X and $\sin^2 \theta_W$.¹¹ These are found to be miniscule with:

$$\frac{\Delta M_X}{M_X} \lesssim 2.5\% \text{ per heavy fermion} \quad (12a)$$

$$0 > \frac{\Delta \sin^2 \theta_W}{\sin^2 \theta_W} \gtrsim -.15\% \text{ per heavy fermion} \quad (12b)$$

where eq. (12a) is the change in M_X from the large HY coupling effects but does not include the change from a new flavor threshold. Surprisingly, the change in M_X coming from the addition of a fourth generation neglecting HY effects, but with $m_T, m_B \sim 200$ GeV is found to be a factor of ~ 1.25 , much less than the 1.8 quoted earlier¹² but consistent with recent estimates of Marciano.¹³ The effects of a fourth generation on m_b/m_t are not known at present.

Searches for heavy quarks in the 200 to 240 GeV region and leptons in the vicinity of 60 GeV may make interesting grist for the Tevatron and collider mills of the future.

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