APPLICATIONS OF DIGITAL SIGNAL PROCESSING FOR NOISE REMOVAL FROM PLASMA DIAGNOSTICS

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Abstract

The use of digital signal processing techniques for removal of noise components present in plasma diagnostic signals is discussed, particularly with reference to diamagnetic loop signals. These signals contain noise due to power supply ripple in addition to plasma characteristics. The application of noise canceling techniques, such as adaptive noise canceling and model-based estimation, will be discussed. The use of computer codes such as **SIG** is described.

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Background

The Tandem Mirror Experiment-Upgrade (TMX-U) at Lawrence Livermore National Laboratory uses magnetic confinement techniques to contain high temperature deuterium plasmas for fusion research. Twenty-four large electromagnets are used to shape and contain the plasma while heating systems are used to raise its temperature to millions of degrees. The heating systems include neutral beam injectors, low frequency RF for ion heating and microwave RF for electron heating. Specialized diagnostic instruments are used to measure plasma parameters during these experiments and must do so without disturbing the plasma characteristics.

A parameter of significant importance to magnetically confined plasmas is the diamagnetism of the plasma. The diamagnetism is a measure of energy density stored in hot particles and is used to determine the plasma beta (β) and the heating rates in the plasma. This paper is concerned with the techniques for improving the signal available for estimating plasma diamagnetism.

In the TMX-U, a single-turn loop transformer—the so-called diamagnetic loop (DML) is used as the sensor for the plasma diamagnetism. The DML sensor is subjected to various noise sources which make the plasma estimation problem difficult. Variations of the magnetic field used to contain the plasma are present because of feedback circuits and ripple currents in the main power system. In many cases the signal that is used to determine the plasma diamagnetism is so badly corrupted with coherent frequency noise (ripple) that the plasma perturbation due to diamagnetism is not even visible. When the signals are approaching the noise level, or when the feedback control system has introduced a trend to the data, this approach is no longer satisfactory. A more sophisticated technique must be used for the processing of the measured signals. It must incorporate trend removal with the capability of removing the coherent noise without affecting the frequency content of the plasma perturbation itself. In this paper we will show how the problem can be viewed as a noise cancelling problem which can be solved using an identification approach.

In the next section we develop the noise canceller using a system identification approach. Next we summarize the algorithm implementation using a solution to the generalized Levinson problem, then we discuss the design of the processor for the plasma estimation problem and summarize the results in the final section.

Noise Cancelling Via System Identification

In this section we develop the algorithm for the noise canceller using an identification approach. The concept of noise cancelling evolves naturally from applications in the biomedical (EKGs, patient monitoring, speech, etc.) and seismological areas [11-13]. Ideally, for noise cancelling to be effective the measured data contains little or no signal information for a period of time so that the only information recorded is the noise, therefore, when the signal occurs it is uncorrelated with the reference noise (e.g. pulses in radar, etc.). The initial algorithms developed were adaptive requiring long data records in order for the algorithm to converge, new approaches eliminate this requirement [13,14]. Variations from the ideal case still met with success. For example, even if signal information is present in the reference record, a reasonable signal estimate can still be obtained. Also, independent measurements can be used rather than the same data record partitioned into reference and signal plus reference. The removal of 60 Hz disturbances can be accomplished by measuring the line voltage as the reference, for instance. In any case, the plasma diagnostics required for monitoring fusion reaction is an ideal candidate for cancelling, since the reference noise can be obtained directly from the measured signal plus noise record, the signal is uncorrelated with the noise, and the onset of the plasma is known.

The fundamental noise cancelling problem evolves by assuming that the noise, n', corrupting the signal, s, and reference, r', is passed through linear systems, h_1 and h_2, that is,

$$y(t) = s(t) + h_1(t) * n'(t) + v_1(t), \tag{1}$$
$$r'(t) = h_2(t) * n'(t) + v_2(t) \tag{2}$$

where

- y is the measured data
- s is the signal
- n' is the disturbance or noise
- v is the random disturbance or noise
- r' is the measured reference noise, and
- h is the sensor or measurement system dynamics.

The convolution operation $*$ is defined by

$$h(t) * n(t) = \sum_{i=1}^{N} h(i)n(t-i) = H(q^{-1})n(t)$$

for

$$H(q^{-1}) = h(0) + h(1)q^{-1} + \cdots + h(N)q^{-N},$$

and q is a shift or delay operator (i.e., $q^{-i}n(t) = n(t-i)$).

Thus, using these polynomial relations the convolution equations can be expressed as

$$y(t) = s(t) + H_1(q^{-1})n'(t) + v_1(t), \tag{3}$$
$$r'(t) = H_2(q^{-1})n'(t) + v_2(t). \tag{4}$$

If we assume that H_2 is invertible, then we obtain

$$n'(t) = H_2^{-1}(q^{-1})r'(t) - H_2^{-1}(q^{-1})v_2(t).$$

Substituting for n' in the measurement equation, we obtain

$$y(t) = s(t) + H_1(q^{-1})H_2^{-1}(q^{-1})\Big[r'(t) - v_2(t)\Big] + v_1(t),$$

or more simply,

$$y(t) = s(t) + n(t) + v(t) = s(t) + H(q^{-1})r(t) + v(t) \qquad (5)$$

where $n(t) = H(q^{-1})r(t)$, with $H(q^{-1}) = H_1(q^{-1})H_2^{-1}(q^{-1})$, $r(t) = r'(t) - v_2(t)$, and $v(t) = v_1(t)$.

Equation (5) *defines an input/output model* for the noise cancelling problem with the input sequence given by $\{r(t)\}$ and the output by $\{y(t)\}$. Using this formulation, we see that the noise cancelling problem is aimed at removing the coherent noise, $n(t)$, from the contaminated measurement, $y(t)$. This problem differs from that of classical signal estimation† because more information about *the characteristics of the noise is available in the reference data*. The solution to this problem is achieved by first developing an estimate of the coherent noise, $\hat{n}(t)$, and then subtracting it, that is,

$$\epsilon(t) = y(t) - \hat{n}(t). \qquad (6)$$

The estimate, $\hat{n}(t)$, removes or cancels the reference noise, as is easily seen by substituting the estimator,

$$\hat{n}(t) = \hat{H}(q^{-1})r(t) \qquad (7)$$

above to obtain

$$\epsilon(t) = s(t) + \Big[H(q^{-1}) - \hat{H}(q^{-1})\Big]r(t) + v(t). \qquad (8)$$

Clearly, as $\hat{H} \rightarrow H$ then $\epsilon \rightarrow s + v$. If the random measurement noise, $v(t)$ were minimal (small variance), then $\epsilon \rightarrow s$, i.e., the estimator would provide an estimate of s as well, however, for v significant, further processing must be used to obtain a reasonable estimate of s.

Thus, we see that noise cancelling can be viewed as a two step process:

1. Obtain the minimum variance estimate of the noise, $n(t)$, and
2. Subtract the estimated noise, $\hat{n}(t)$, from the measured data, $y(t)$.

The minimum variance estimator is obtained by finding the $H(q^{-1})$ that minimizes the criterion,

$$J(t) = E\{\epsilon^2(t)\}$$

where the error is given by $\epsilon(t) = y(t) - \hat{n}(t)$.

The solution to this problem is obtained by differentiating J with respect to each of the $h(i)$, setting the result to zero and solving the resulting set of equations, i.e.,

$$\frac{\partial J(t)}{\partial h(k)} = \frac{\partial}{\partial h(k)}E\{\epsilon^2(t)\}$$
$$= 2E\{\epsilon(t)\frac{\partial \epsilon(t)}{\partial h(k)}\}$$

The error gradient is found by substituting Eqn. 7 for $\hat{n}(t)$ to obtain

$$\frac{\partial \epsilon(t)}{\partial h(k)} = -r(t-k),$$

and therefore,

$$\frac{\partial J(t)}{\partial h(k)} = -2E\{(y(t) - \sum_{i=1}^{N} h(i)r(t-i))r(t-k)\},$$

$$= -2\Big(E\{y(t)r(t-k)\} - \sum_{i=1}^{N} h(i)E\{r(t-i)r(t-k)\}\Big),$$

Setting this expression to zero and solving, we obtain the so-called *normal equations*

$$R_{yr}(k) = \sum_{i=1}^{N} h(i)R_r(k-i), \;\; k = 1,\cdots,N. \qquad (9)$$

Carrying out the summations, we obtain the set of linear vector-matrix equations,

$$\begin{pmatrix} R_{yr}(1) \\ \vdots \\ R_{yr}(N) \end{pmatrix} =$$

$$\begin{pmatrix} R_r(0) & R_r(-1) & \cdots & R_r(1-N) \\ \vdots & \vdots & \ddots & \vdots \\ R_r(N-1) & R_r(N-2) & \cdots & R_r(0) \end{pmatrix} \begin{pmatrix} h(1) \\ \vdots \\ h(N) \end{pmatrix},$$

or solving for $\mathbf{h}$ we obtain

$$\underline{\hat{h}}(N) = \mathbf{R}_r^{-1}\underline{R}_{yr}(N) \qquad (10)$$

It is straightforward to show that the corresponding error variance, $\tilde{R}$, is given by

$$\tilde{R} = R_y(0) - \underline{R}'_{ry}(N)\mathbf{R}_r^{-1}\underline{R}_{yr}(N) \qquad (11)$$

This set of linear equations can be solved using standard techniques in linear algebra or since the covariance matrix to be inverted has a Toeplitz structure, a more efficient technique employing the generalized Levinson approach can be used. An example of how this process can be applied is described in the next section. The computer code SIG was used for the signal analysis work.

Plasma Estimation Using the Noise Canceller

In this section, we analyze the acquired diamagnetic loop (DML) sensor measurements and show how the data can be processed to retain the essential information required for post experimental analysis. The measured DML data is analog (anti-alias) filtered and digitized at a 25 KHz sampling rate (40 μs sample interval). A typical experiment generates a transient signal (plasma) which is recorded for approximately 650 ms. Pre-processed data (decimated etc.) and the frequency spectrum are shown in Fig. 1 along with an expanded section of the transient pulse and noise. We note that the raw data is contaminated with a sinusoidal drift, linear trend, and random noise as well as sinusoidal disturbances at harmonics of 60 Hz, the largest at 360 Hz caused by the feedback circuits and ripple currents in the main power system. The pulse is also contaminated by these disturbances. We also note that some of the plasma information appears as high energy spikes (pulses) *riding* on the slower plasma build-up pulse.

A processor must be developed to eliminate these disturbances, yet preserve all of the essential features of the transient plasma pulse and associated energy spikes. This application is ideally suited for *noise cancelling*. The basic requirements of the data are that a reference file of noise and of the signal and noise are available. For best results, the signal and noise should not be correlated. These conditions are satisfied by the DML measure

† Actually this estimation problem is a system identification problem as noted by (Ljung and Soderstrom, 1983)

ment data, since the onset of the measurement consists only of the disturbances (trend and sinusoids), and the signal is available at the time of the transient plasma pulse.

During the operation of the TMX experiment a "shot" (generation of a plasma in the experiment vessel) terminates after a few seconds, during this time data are collected and displayed so that the experimenter can adjust process parameters and criteria and perform another shot within a five (5) minute time period. So we see that even though the processor need not be on-line, it still must function in a real-time enviroment. Clearly, post-experimental analysis creates no restrictions on the processor design and alloted computational time. So we analyzed the performance of the processor to function for both real-time and post-experimental modes of operation. We studied the performance of the processor by varying its length **N** . The real-time processor must perform reasonably well enough to enable the experimenter to make the necessary decisions regarding the selection of process parameters for the next shot.

After some preliminary runs of the processor over various data sets we decided to use **N = 512** weights for the post-experimental design since it produced excellent results. Using the post-experimental design as a standard we then evaluated various designs for weights in the range of $8 < N < 512$.

Before we discuss the comparisions, let us consider the heuristic operation of the processor. The crucial step in the design of the canceller is the estimation of the optimal noise filter $\hat{h}$ which is required to produce the minimum variance estimate of the noise, $\hat{n}$. In essence we expect the filter to match the corresponding noise spectrum in magnitude and phase. This means that we expect the optimal filter to pass the spectral peaks of the noise and attenuate any signal information not contained in the reference. These results are confirmed as shown by the performance of the 512-weight filter shown in Fig. 2a. Here we see that the filter passbands enable most of the noise resonances to pass while signal energy is attenuated. The real-time design is shown below in Fig. 2b. We see that the 64-weight filter still passes much of the noise energy but does not spectrally match the noise as well as the 512-weight filter, since there are fewer weights. These results are again confirmed in Fig. 3 where the estimated and actual noise spectra are shown. Again we see that the 512-weight produces a much better spectral match, than the 64-weight design due to its increased resolution. Note that the highest energy noise spectral peaks were matched by both processors reasonably well thereby eliminating these disturbances in the cancelling operation. Intermediate designs for the real-time processor fall in between these results where selecting higher number of weights resulted in better processor performance.

It should be mentioned that we chose to use the FIR (all-zero) solution to this problem, rather than the IIR (pole-zero) as suggested in [11] or [14] because initial attempts at identifying the optimal noise filter were unsatisfactory primarily because of the high resonances (sinusoids) in the data. The IIR identifiers could identify the frequencies but usually overestimated the damping which proved detrimental when the estimated noise was cancelled (subtracted) from the signal plus noise measurements.

The noise canceller algorithm was constructed using various commands in **SIG**. Both the post-experimental and real-time designs were run on the data set described in Fig. 1 and the results of the 512-weight design is shown in Fig. 4 and the 64-weight design in Fig. 5. Here we see the raw and processed data and corresponding spectra. A closer examination of the estimated transient pulse shows that not only have the disturbances been removed, but that the integrity of the pulse has been maintained and all of the high frequency energy spikes have been preserved. We see that the 512-weight processor has clearly eliminated the trends and sinusoidal disturbances and retained the transient plasma information quite well while the real-time (64-weight) processor has not performed as well as evidenced by some remaining (though small) sinusoidal disturbances. However, for the real-time requirements it is satisfactory.

Once these disturbances have been removed, the processed signal can be integrated to remove or deconvolve the effects of the differentiating DML probe and provide an estimate of the stored energy build-up in the machine.

Summary

In this paper we have developed a noise cancelling algorithm using the system identification approach and applied it to the problem of estimating a transient plasma pulse for the magnetic fusion experiment (TMX-U). We have developed solutions for both post-experimental analysis and real-time processing and analyzed the performance of the corresponding processors. More effort will continue in developing processors for the experiments and they will utilize model-based signal processing ideas [19].

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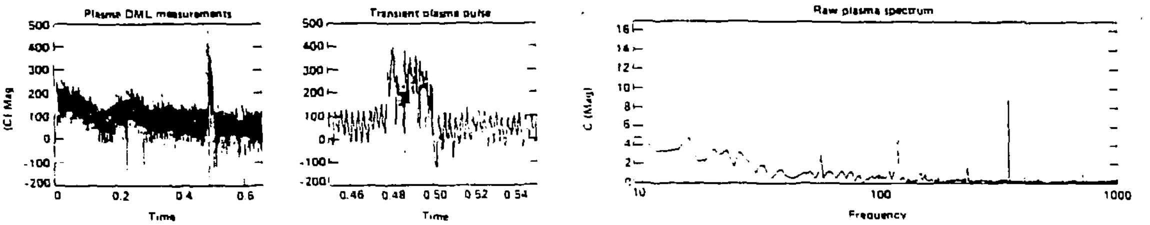

Figure 1. Preprocessed diamagnetic loop measurement data and spectrum.

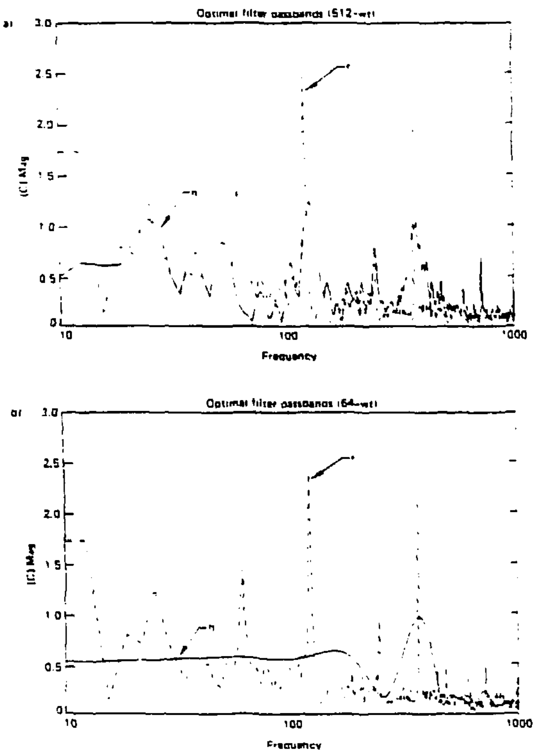

Figure 2. Optimal noise filter and noise spectra (a) post filter and (b) real-time filter.

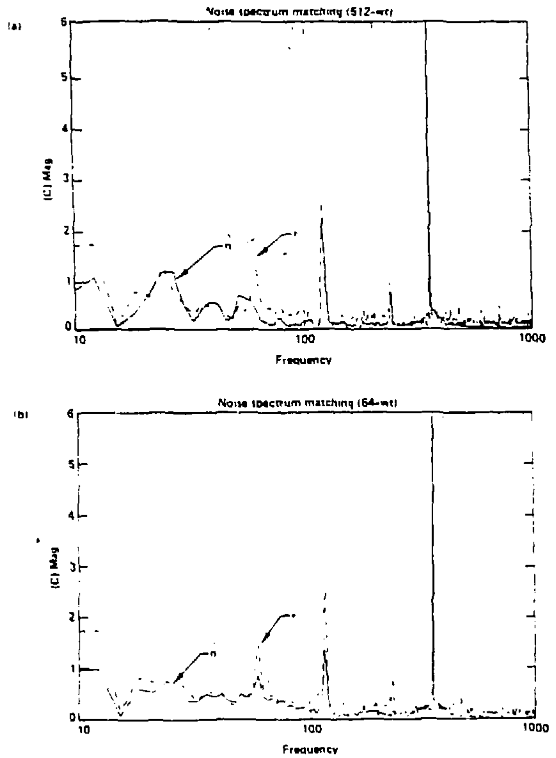

Figure 3. Optimal noise spectral matching (a) post filter and (b) real-time filter.

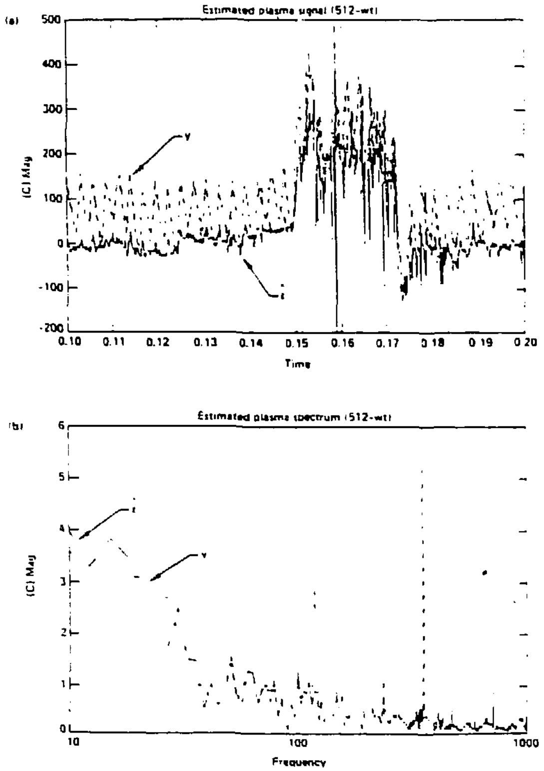

Figure 4. Post-experiment nosie canceller design (a) plasma pulse and (b) spectra.

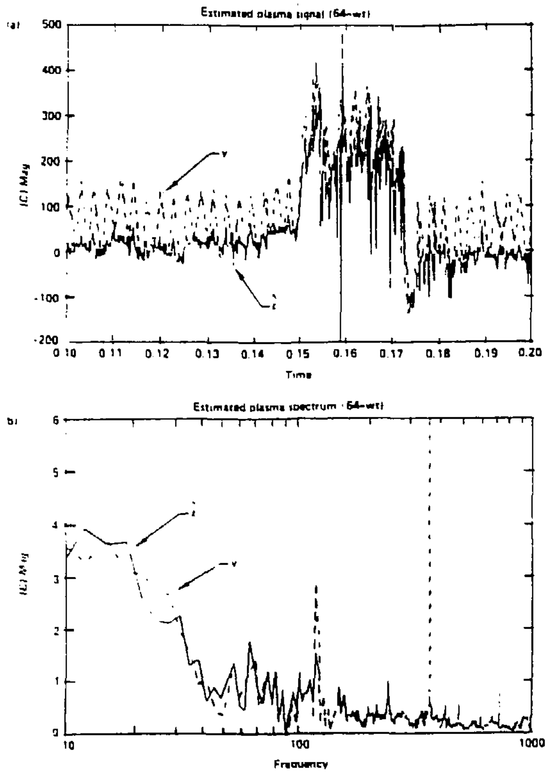

Figure 5. Real-time noise canceller design (a) plama pulse and (b) spectra.