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REPORT ON SOME BEAM-BEAM FUNCTIONAL DEPENDENCIES IN SPEAR

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1. INTRODUCTION

A considerable amount of experimental results on beam-beam effects in SPEAR is available. We have analyzed the results which give the functional dependences of some important machine parameters. The data have been taken from machine physics experiments carried out during the period December 1977 to October 1978, and from records of the operation runs.

2. DEFINITIONS

The experimental data cover the energy range 1.5 - 3.9 GeV. There are two circulating bunches (one electron, one positron) and consequently, two interaction regions. Unless otherwise specified, the symbols, definitions and values of the relevant parameters referred to in this report are the following:

ν_{xo} = horizontal unperturbed betatron tune = 5.28

ν_{yo} = vertical unperturbed betatron tune = 5.18

β_x^* = horizontal beta function at the interaction points = 1.2 m

β_y^* = vertical beta function at the interaction points = 10 cm

η_x^* = horizontal dispersion at the interaction points = 0.00186 m

The luminosity is defined as [1]

$$\mathcal{L} = \frac{1}{4e^2 f} \frac{i^+ i^-}{A_{int}} \qquad (1)$$

where

$$A_{int} = \pi \sigma_x \sigma_y$$

f = revolution frequency

e = electron charge

A_{int} = effective interaction area

i^+, i^- = beam currents

and where σ_x and σ_y are the standard deviations of the Gaussian distributions of the transverse density of the beam at the interaction points.

The 'space charge parameter', ξ, is the vertical linear tune shift due to the beam-beam forces if the vertical tune change between interaction points is not too close to an integer. It is a measure of the strength of the beam-beam interaction. For a Gaussian beam with head-on collision, it is given by[1]

$$\xi = \frac{r_e}{2\pi} \frac{N_B \beta_y^*}{\gamma \sigma_y (\sigma_x+\sigma_y)} \qquad (2)$$

where

r_e = classical electron radius

N_B = number of particles per bunch

γ = (energy/rest energy)

The linear tune shift, $\Delta\nu_y$, is the tune shift due to the linear component of the space charge forces; it is related to the space charge parameter introduced earlier by the relationship:

$$\cos 2\pi \ (\nu_{yo}+\Delta\nu_y) = \cos 2\pi \ \nu_{yo} - 2\pi \xi \sin 2\pi \ \nu_{yo}. \qquad (3)$$

3. LUMINOSITY AS A FUNCTION OF BEAM CURRENT

Eq.(1) predicts the dependence, if $i^+ = i^- = i$,

$$\mathcal{L} \sim i^2 .$$

Experiments performed at different times [(2)] indicate that, above a certain intensity, the luminosity at ~2 GeV does not scale like i^2 but, rather, like $i^{1.3}$. This is shown in Fig.(1); the luminosity starts departing from the quadratic law when the beam current reaches about 2 mA, at 1.95 GeV; this corresponds to a space charge parameter

$$\xi = 0.016$$

Whereas the luminosity depends on many machine parameters in a way which is not completely understood, the functional dependence of the luminosity on the beam current is rather reproducible. It is, for instance, not very sensitive to the amplitude and shape of the residual closed orbit distortions and to the coupling between horizontal and vertical planes; it is also insensitive to the 'bunch lengthening cavity', which drastically modifies the beam density function in the longitudinal phase space. The same experiment[(3)] of luminosity versus current with a vertical beta function at the interaction points of 20 cm (instead of 10 cm) gave a dependence $\mathcal{L} \sim i^{1.45}$ at 2 GeV.

Since the other factor intervening in the luminosity is the beam cross section, we postulate, in order to fit the experimental data, that the transverse beam size increases with current. In order to test the above hypothesis, the horizontal and vertical beam sizes, as given by the synchrotron light monitors, have been measured.[(4)] The results are given in Tables 1 and 2 below. In these tables we have recorded the vertical and horizontal beam sizes in mm as a function of the colliding beams currents. The beam size in Table 1 is full width measured at 60.6% of the peak of the density distribution; if the current density is truly Gaussian, this beam dimension corresponds to two standard deviations.

Similarly, Table 2 gives the full beam sizes at 14.5% of the peak; for a Gaussian distribution, they should be 2 times the values listed in Table 1 (4 standard deviations). Any departure from this factor 2 is a qualitative indication on how much the distribution differs from a Gaussian.

The ratios between the beam sizes in Tables 1 and 2 are listed in Table 3.

The positron beam blows up much more than the electron beam, in the vertical plane. This is a phenomenon which is known to occur close to the beam-beam limit: depending on some machine parameters (phasing of the rf cavities, horizontal dispersion), in a not understood way, one or the other of the two colliding beams blows up.[6]

A least square fit of the logarithm of the vertical beam size against the logarithm of the current gave the following slopes (values of K for a dependence $\sigma \sim i^{K}$):

vertical; 2 standard deviations, electrons, $K = 0.28 \pm 0.08$
vertical; 2 standard deviations, positrons, $K = 0.59 \pm 0.10$
vertical; 4 standard deviations, electrons, $K = 0.40 \pm 0.08$
vertical: 4 standard deviations, positrons, $K = 0.65 \pm 0.12$

The least square fit of the logarithm of the luminosity versus the logrithm of the beam current gave the dependence $\mathcal{L} \sim i^{1.41 \pm 0.09}$.

When two beams with Gaussian density distribution of different vertical standard deviations collide, the effective interaction dimension σ_y in the expression of the luminosity (Eq. 1) must be replaced by

$$\sqrt{\sigma_y^{+2} + \sigma_y^{-2}}$$

where σ_y^{+} and σ_y^{-} are the r.m.s. vertical beam sizes of the two beams. Thus, in Table 1, the luminosity at the higher current levels is largely determined by the beam size of the blown-up beam (positrons), which scales with the beam current like $\sigma_y^{+} \sim i^{0.59 \pm 0.10}$.

We find, therefore, good agreement between the dependence of luminosity and beam size with current: the empirical dependence for the luminosity was found to be $\mathcal{L} \sim i^{1.41 \pm 0.09}$ which implies a vertical beam size dependence as $\sigma_y \sim i^{0.59}$, which agrees with the measured

$\sigma_y \sim i^{0.59 \pm 0.10}$. We also observe in Table 3 that, at higher beam currents, the ratios of the beam sizes measured at 60.5% and 14.5% of the peak differ from the value 2; this implies that the density distribution departs from a Gaussian, and that the tails spread out more than the core. The horizontal beam sizes experience very little blow-up; note that the value of the horizontal dispersion function (η_x^*) at the light monitor positions is 0.98 m.

Another interesting result comes from the direct measurements of the tune shift with colliding beams[7]; the fit of the experimental data shows a dependence $\Delta\nu_y \sim i^{1/3}$ at 2.4 GeV. Since (Eq.2), $\xi \sim \frac{i}{\sigma_x \sigma_y}$ (if $\sigma_y << \sigma_x$), there seems to be qualitative agreement with the proposed empirical law $\sigma_y \sim i^{0.60}$.

We don't have recent data from machine physics experiments on the dependence of luminosity on current at different energies. We do have, however, operation data, where the luminosity and the current are recorded periodically. The range of recorded values covers a rather restricted interval, the upper limit being just below the beam-beam limit, for stable operation. We have selected most of the runs at 1.5, 2.5 and 3.7 GeV in the period July 1976 to June 1978. Each set of measurements consists of about 30 points (luminosity-current) at a given energy. A linear least square fit of the logarithm of the luminosity (in units 10^{30}cm^{-2} sec^{-1}) versus the logarithm of the current (in mA) was computed; the slope of the straight line gives the value of the exponent α, if the dependence $\mathscr{L} \sim i^{\alpha}$ is admitted.

The results of the analysis are summarized in Fig. 2. In it, the exponent α versus the energy is plotted. Each point in the graph corresponds to one set of measurements. Figs. 3, 4, and 5 show some examples of the original data (logarithm of the luminosity versus the logarithm of the beam current) at different energies.

From the data in Fig. 2 we can draw the following conclusions:

a) At 3.7 GeV the dependence $\mathscr{L} \sim i^{1.6}$ is quite clear. Since we can imagine that the operation runs were done under different conditions of closed orbit, coupling, rf, we can say that the above empirical law is independent of these parameters. It is to be noted that at 3.7 GeV the beam current is farther away from the beam-beam limit than at lower energies, due to a current limitation imposed by another reason (heating caused by interaction with cavity-like objects).

b) At lower energies, the dependence of the exponent α is less clear, due to a large scatter. At 1.5 GeV, the scatter of the original data is so large as to make the analysis meaningless. There seems to be a trend for the exponent α to decrease with the energy. In our interpretation, the beam blow-up is greater at lower energies.

c) In no case does the exponent α approach the theoretical value 2, except at very low current.

d) The data from machine physics experiments fit in the trend of the operation data.

e) The way in which the electron or the positron beams blow-up, as mentioned earlier on, can be affected, in a not understood way, by various machine parameters such as horizontal dispersion and phasing of rf cavities.(6) Clearly, this is not, operationally, a very easily reproducible phenomenon, as the above parameters are often changed, either purposely or due to orbit errors. On the opposite, the data from the operation runs analyzed in Fig 2 show that the functional depencence of luminosity on beam current is reproducible within the limits of the fluctuations. If, as we have postulated, the beam size and luminosity dependences with beam current are related, we must conclude that the two beams blow-up, erratic as it might appear, is such as to give a reproducible luminosity dependence on current.

We propose then the following empirical law:

$$\mathscr{L} \sim I^{\alpha(E)} . \tag{4}$$

Note that this law is only valid above a current threshold which depends on the energy (2 mA at 2 GeV).

4. DEPENDENCE OF LUMINOSITY ON ENERGY

The theoretical dependence is, from 1) $\mathscr{L} \sim \frac{1}{E^2}$, since the beam cross section goes like $\frac{1}{E^2}$. Fig. 6 shows the luminosity versus energy at different intensities.(8) The luminosity increases up to ~2-2.5 GeV (depending on the beam current), then decays approximately with the expected dependence. On the grounds of observations made in Section 3, we may assume:

$$\mathscr{L} = \frac{k\, i^{\alpha(E)}}{E^2} \qquad (5)$$

where k is a constant.*

Eq. 5, solved for $\alpha(E)$, gives:

$$\alpha(E) = \frac{\ln \quad + 2 \ln E - \ln k}{\ln i} \qquad (6)$$

The factor k depends on machine parameters, in particular, coupling. We have taken the data from Fig 6, 1[illegible] mA case, and, knowing $\mathscr{L}$ and E, have solved Eq. 6 for $\alpha(E)$. For k we have taken an average value from the operation data, namely k = 0.37 if $\mathscr{L}$ is in 10^{30} cm^{-2} sec^{-1}, i in mA, E in GeV. The results for $\alpha(E)$ are shown as squares in Fig. 2, and they confirm the trend observed from the measurements of luminosity as a function of the beam current.

5. MAXIMUM ACHIEVABLE LUMINOSITY

The operation record data of the maximum achievable luminosity in the range 1.6 ÷ 2.5 GeV are plotted, in logarithmic scales, in Fig. 7; the fitting gives the dependence

$$\mathscr{L} \sim E^4$$

This agrees with the assumption of a beam-beam limit independent of energy. The assumption made earlier that $\mathscr{L} \sim \frac{i^{\alpha(E)}}{E^2}$ should give, however, a faster dependence of luminosity on energy. If, for instance, $\mathscr{L} \sim \frac{i^{1.5}}{E^2}$

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*The luminosity in Eq. (5) must not be confused with the maximum achievable luminosity, which scales like E^4

and $\xi_{max} \sim \frac{I_{max}^{0.5}}{E^3}$, $\mathcal{L}_{max} \sim E^7$ (as experienced in Adone). We don't know the reason for this discrepancy. The assumption of a maximum allowable tune shift independent of energy can be subject to discussion. Also, the empirical law $\mathcal{L} \sim \frac{I^{\alpha(E)}}{E^2}$ may break down when working very close to the beam-beam limit.

6. LUMINOSITY AND SPACE CHARGE PARAMETERS AS A FUNCTION OF β_y^*

From beam size and linear optics considerations, one should find, at low current

$$\mathcal{L} \sim \frac{1}{\sqrt{\beta_y^*}} \tag{7}$$

At higher current, increasing β_y^* increases the beam-beam tune shift and the luminosity should fall more rapidly. The results of two experiments[9] are summarized in Figs. 8, 9 and 10. The two sets of points refer to two different conditions: in one case the beam current was 7 mA and the vertical tune ν_{y_0} = 5.123; the second case had a beam current of 10 mA and the vertical tune was ν_{y_0} = 5.174. Fig. 8 shows the rather surprising result that, apart from the first point of the 10 mA case, the luminosity does not critically depend on the value of the vertical beta function at the interaction points, up to β_y^* = 20 cms. Beyond this value, the luminosity falls, as one expects.

In Fig. 9 we plot the value of the space charge parameter calculated from the luminosity for the two experiments described above; in Fig. 10 the the real vertical tune shift is plotted. These results indicate that the luminosity, for a fixed current, does not depend on the value of the beam-beam tune shift, if this is less than 0.045. Clearly, these surprising results require further investigation.

7. DEPENDENCE OF LUMINOSITY ON THE UNPERTURBED VERTICAL TUNE

We have analyzed the results of three experiments in which the luminosity was measured as a function of the unperturbed vertical tune.[10]

Fig. 11 shows that the luminosity decreases as the tune increases. At first sight this looks like a reasonable result, as one could argue that the luminosity decreases because the real tune shift increases with the vertical tune. In fact, the luminosity decreases with increasing tune faster than it would just to keep the real tune shift constant. This is shown in Fig. 12, where the total vertical tune shift (i.e., tune shift per interaction region times the number of interaction regions) is plotted as a function of the unperturbed vertical tune. Fig. 12 shows that the total tune shift is not constant, but decreases with increasing tune. In Fig. 13 we plot the luminosity versus the 'perturbed tune' (nominal vertical tune + beam-beam linear tune shift): this plot indicates that the luminosity decreases as the tune approaches a certain value ($6\ \nu_y = 31$?). These results justify the question: is it the tune shift alone which is the critical parameter of the beam-beam interaction or a combination of the tune shift and the value of the working point?

One should add here that in spite of the low tune experiment results, the operation of SPEAR is more stable, and maximum luminosity is obtained, with a nominal vertical tune of 5.18, which is higher than any in the range we have just considered. Perhaps other effects, like synchro-betatron resonances, impose other conditions on the choice of the tunes. Nevertheless, in the region we have considered, the conclusions drawn in this section stand.

8. CONCLUSIONS

We have analyzed some machine physics results and operation data in SPEAR. There is reproducible evidence of a vertical beam blow-up, whose extent is a function of the beam current. The rate of the blow-up with current shows little dependence on machine parameters like closed orbit, coupling, rf conditions.

The dependence of the luminosity on the vertical beta function at the interaction points gave the unexpected result that, up to $\beta_y^* = 20$ cms, the luminosity depends little on this parameter. The maximum tolerable beam-beam tune shift seems to depend on the value of the nominal vertical tune, although this dependence is not, as yet, clear. The maximum achievable luminosity during operation runs scales like the 4-th power of the energy.

9. ACKNOWLEDGEMENTS

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   p.82, 4/18/78, experimentalists V. Brown, T. Taylor, K. Underwood, H. Wiedemann.

TABLE 1

Beam size; 2 standard deviations for a gaussian distribution of current density.

Estimated measurement error on beam size : 5% absolute (systematic), 2% random (r.m.s.)

Error in reading plotted output : ±0.03 mm. The instrumental resolution, 0.12 mm has been subtracted in r-m-s fashion from the row data.

i^+ (mA)	i^- (mA)	$2\sigma_y^-$ (mm)	$2\sigma_y^+$ (mm)	$2\sigma_x^-$ (mm)	$2\sigma_x^+$ (mm)	$\mathcal{L}$ (10^{30} cm^{-2} x sec^{-1})
7.85	7.85	0.64	1.43	1.76	2.08	0.87
5.70	5.64	0.63	0.93	1.65	1.91	0.67
3.58	3.51	0.64	0.76	1.58	1.78	0.34
2.15	2.13	0.47	0.53	1.58	1.73	0.17
1.02	1.00	0.37	0.40	1.58	1.73	0.05
Theoretical (for 10% coupling)		0.28	0.28	1.81	1.81	

TABLE 2

Beam size; 4 standard deviations for a gaussian distribution of current density.

i^+ (mA)	i^- (mA)	$4\sigma_y^-$ (mm)	$4\sigma_y^+$ (mm)	$4\sigma_x^-$ (mm)	$4\sigma_x^+$ (mm)	$\mathscr{L}$ (10^{30} cm^{-2} x sec^{-1})
7.85	7.85	1.57	3.10	3.53	4.05	0.87
5.70	5.64	1.48	2.05	3.19	3.69	0.67
3.58	3.51	1.38	1.47	3.06	3.51	0.34
2.15	2.13	.91	1.01	3.16	3.36	0.17
1.02	1.00	0.72	0.79	3.16	3.42	0.05
Theoretical (10% coupling)		0.56	0.56	3.62	3.62	

TABLE 3

Ratios between the values in Table 2 and the corresponding ones in Table 1 (the ratios should have the values 2 for gaussian distributions).

i^+ (mA)	i^- (mA)	$\frac{4\sigma_y^-}{2\sigma_y^-}$	$\frac{4\sigma_y^+}{2\sigma_y^+}$	$\frac{4\sigma_x^-}{2\sigma_x^-}$	$\frac{4\sigma_x^+}{2\sigma_x^+}$
7.85	7.85	2.45 ±0.09	2.17 ±0.04	2.00 ±0.03	1.95 ±0.03
5.70	5.64	2.35 ±0.09	2.20 ±0.06	1.93 ±0.03	1.93 ±0.03
3.58	3.51	2.16 ±0.09	1.93 ±0.08	1.94 ±0.03	1.97 ±0.03
2.15	2.13	1.94 ±0.13	1.90 ±0.11	2.00 ±0.03	1.94 ±0.03
1.02	1.00	1.94 ±0.16	1.97 ±0.15	2.00 ±0.03	1.98 ±0.03

LUMINØSITY VERSUS CURRENT

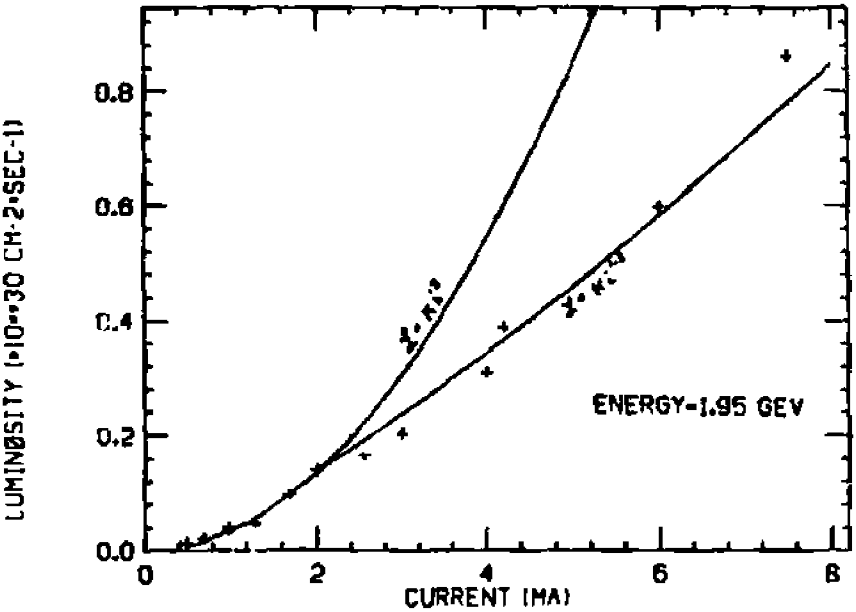


FIG.1

ALFA(E) VERSUS ENERGY

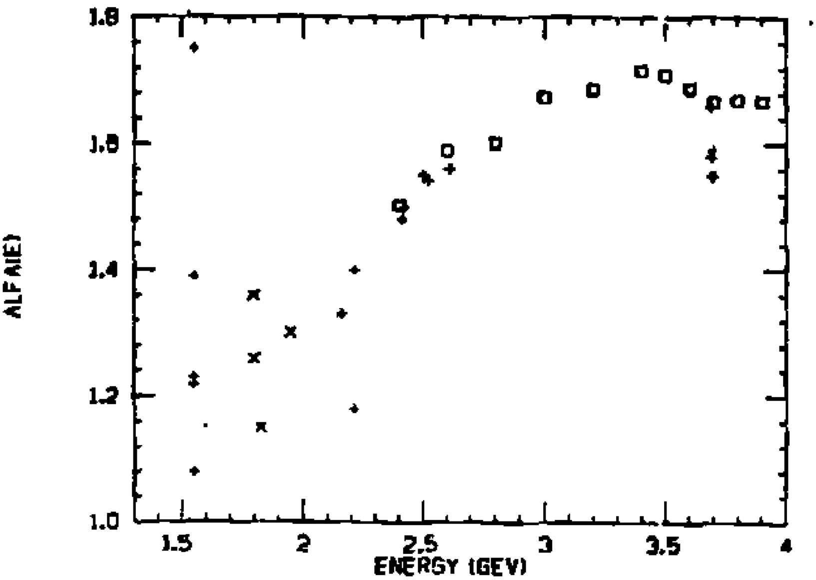


FIG.2

+ - ØPERATIØN DATA

x - MACHINE PHYSICS EXPERIMENTS DATA

o - CALCULATED FRØM THE LUMINØSITY VERSUS ENERGY DATA

1.5 GEV

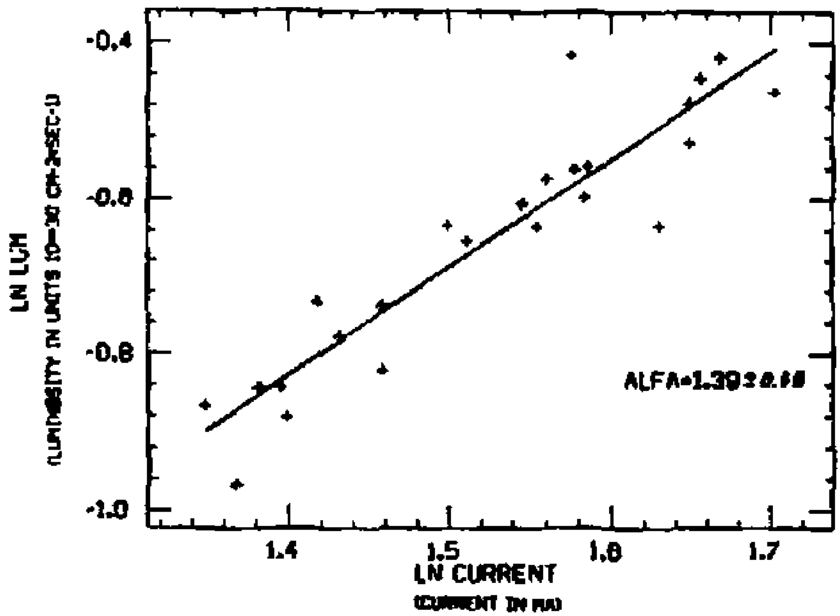


FIG.3

2.5 GEV

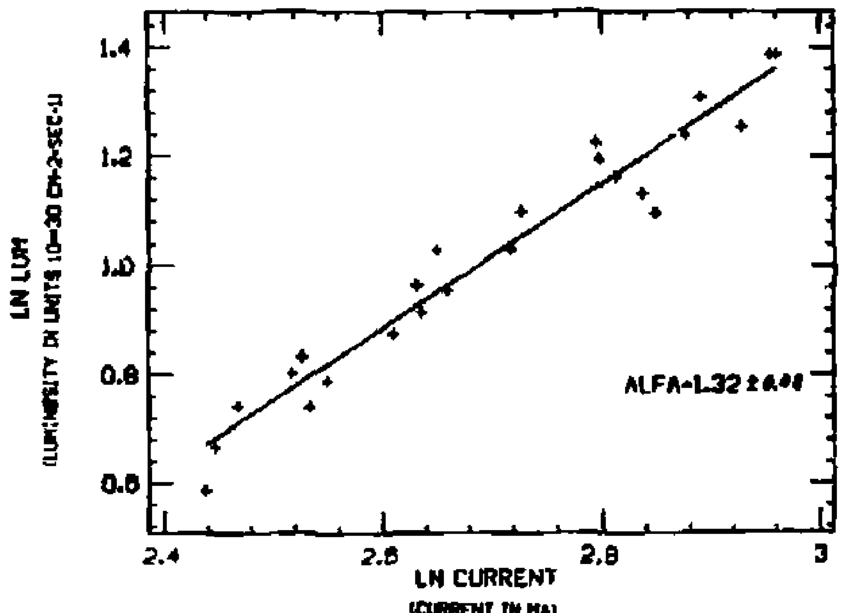


FIG.4

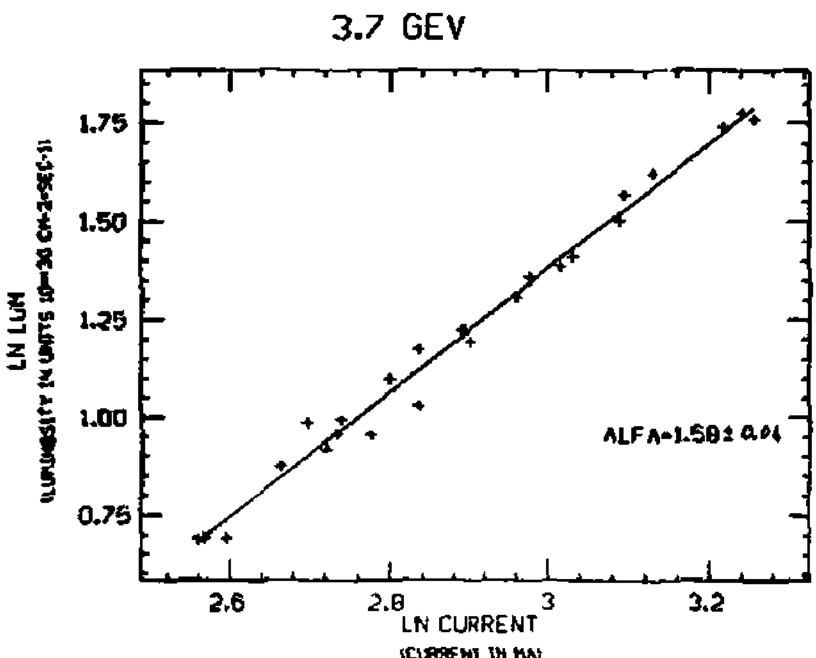
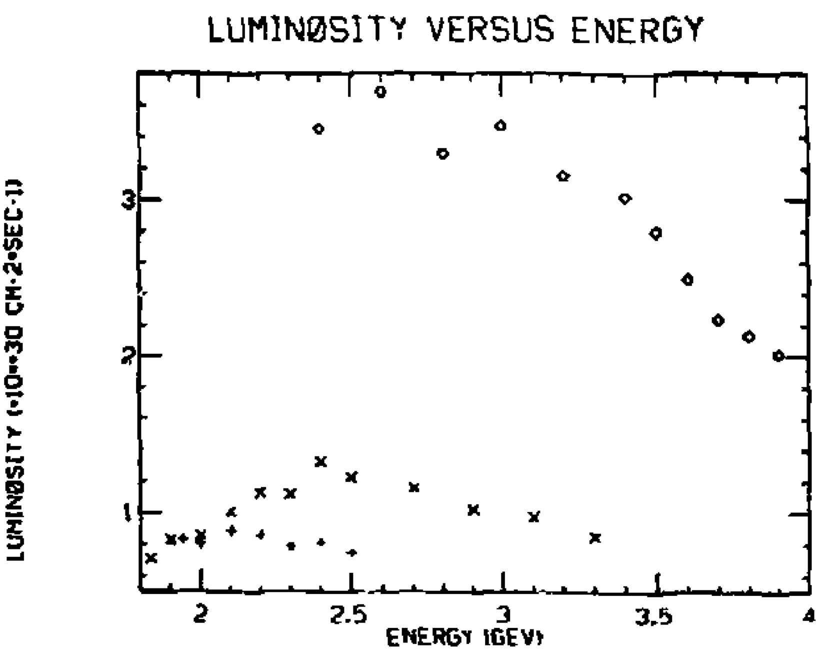


FIG.5



O = 14 MA
X = 8 MA
+ = 6.5 MA

FIG.6

MAXIMUM LUMINØSITY VERSUS ENERGY

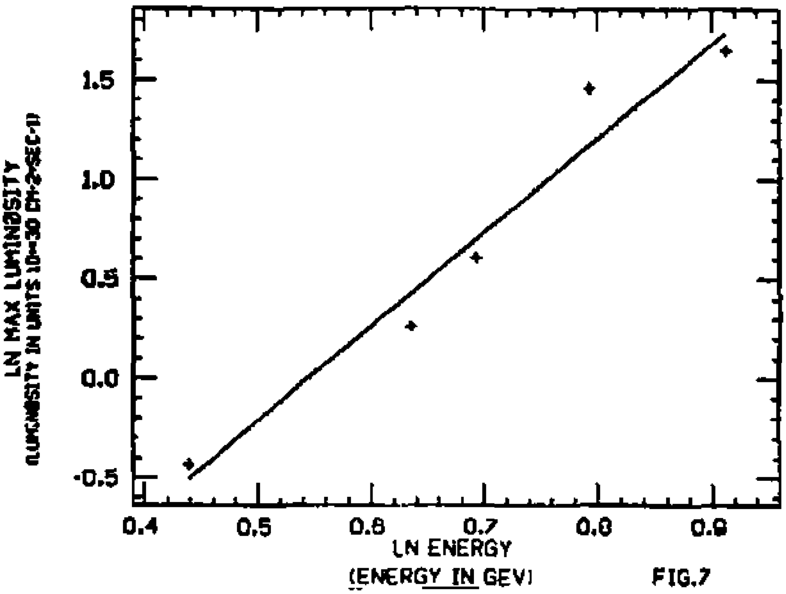


FIG.7

LUMINØSITY VERSUS BETAY

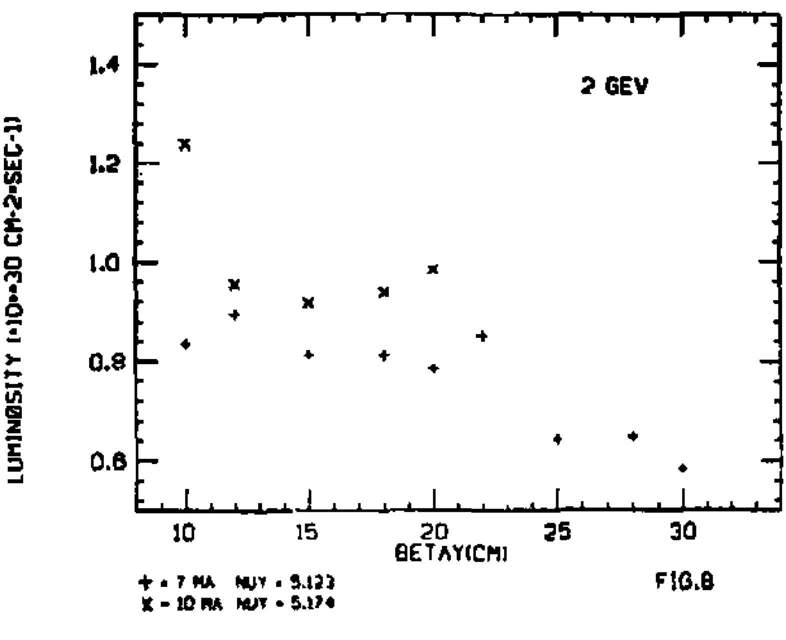


FIG.8

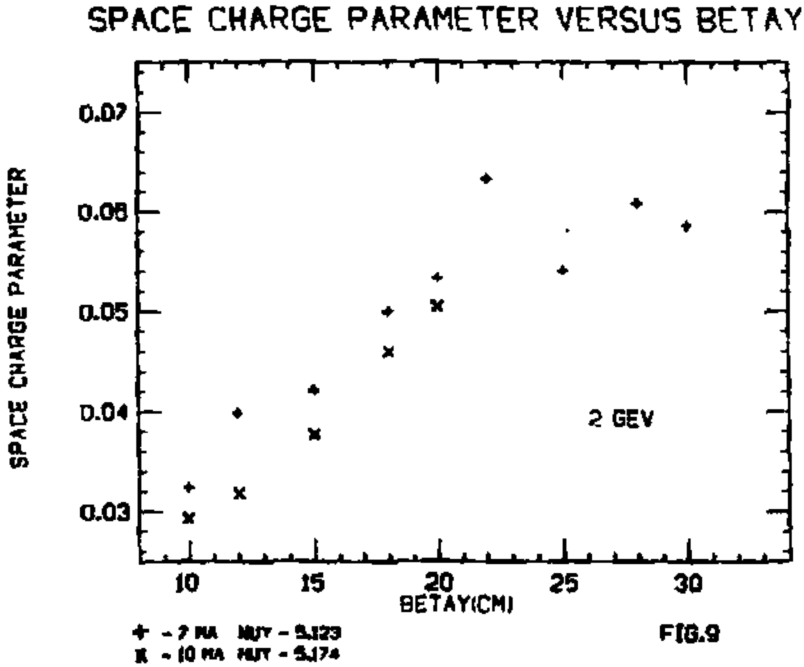

SPACE CHARGE PARAMETER VERSUS BETAY
SPACE CHARGE PARAMETER
0.07
0.06
0.05
0.04
0.03
10
15
20
25
30
BETAY(CM)
2 GEV
+ - 7 MA NUY - 5.123
x - 10 MA NUY - 5.174

FIG.9

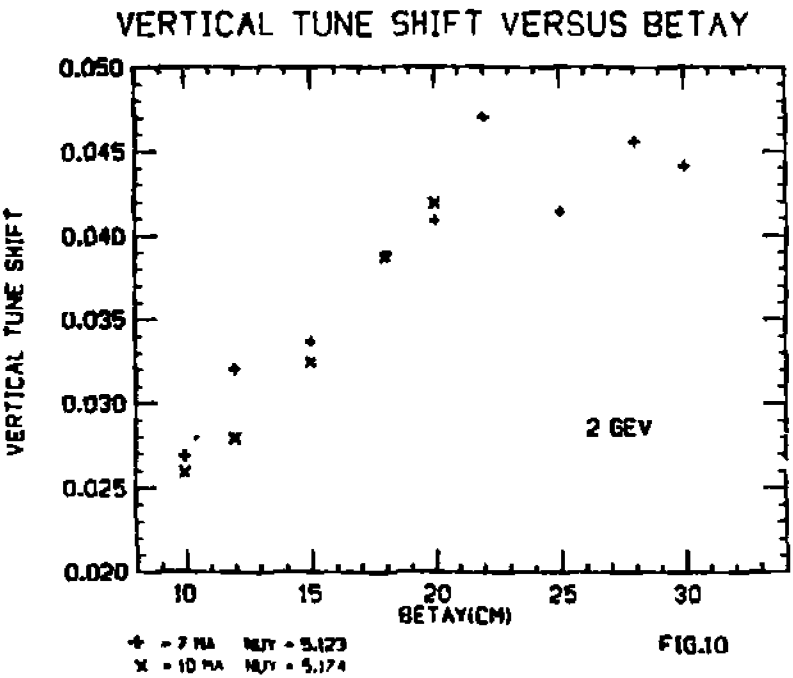

VERTICAL TUNE SHIFT VERSUS BETAY
VERTICAL TUNE SHIFT
0.050
0.045
0.040
0.035
0.030
0.025
0.020
10
15
20
25
30
BETAY(CM)
2 GEV
+ - 7 MA NUY - 5.123
x - 10 MA NUY - 5.174

FIG.10

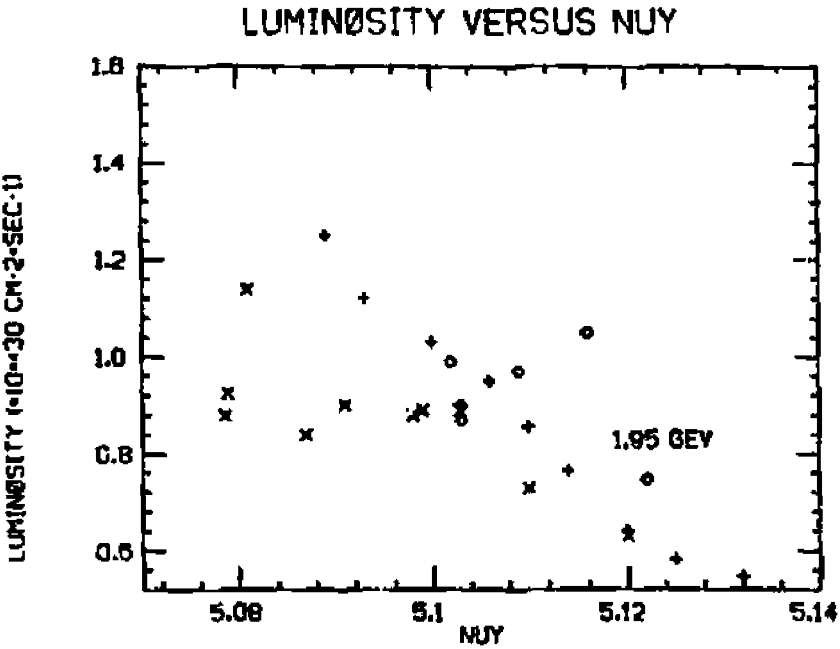

LUMINØSITY VERSUS NUY
LUMINØSITY (*10**30 CM-2*SEC-1)
1.6
1.4
1.2
1.0
0.8
0.6
5.08
5.1
5.12
5.14
NUY
1.95 GEV
+ = 7.5 MA
X = 7.0 MA
◇ = 8.2 MA

FIG.11

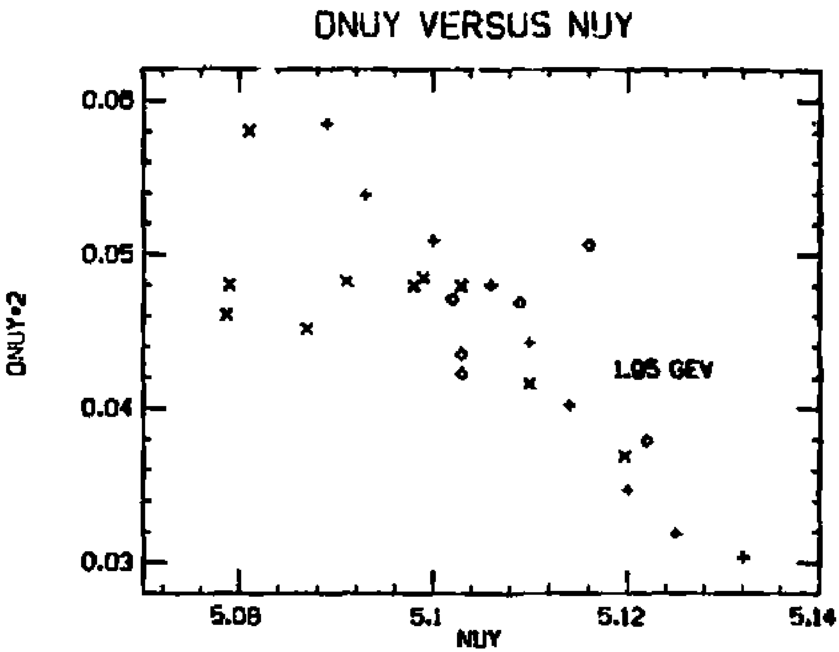

DNUY VERSUS NUY
DNUY*2
0.06
0.05
0.04
0.03
5.08
5.1
5.12
5.14
NUY
1.95 GEV
+ = 7.5 MA
X = 7.0 MA
◇ = 8.2 MA

FIG.12

LUMINOSITY VERSUS PERTURBED NUY

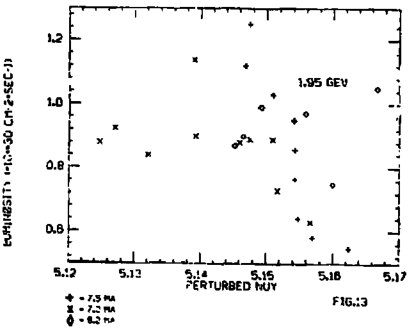


FIG.13

DISCLAIMER