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TITLE: THREE DIMENSIONAL NUMERICAL STUDY OF INHOMOGENEOUS CHAOTIC INFLATION

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THREE-DIMENSIONAL NUMERICAL STUDY OF INHOMOGENEOUS CHAOTIC INFLATION

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ABSTRACT

The inflationary scenario was motivated to explain the large scale homogeneity of the universe. However, most of the work in this area has been carried out in homogeneous spacetimes! We present a 3D study solving Einstein's equations for inhomogeneous cosmologies to analyze the effects of matter and spacetime inhomogeneities on inflation. We use York's conformal approach to solve the initial value problem. Preliminary results of the evolution of inhomogeneous initial data are presented.

The inflationary universe scenario[1] has generated great interest because it offers the possibility of explaining the homogeneity, isotropy and flatness of the universe, and the origin of density perturbations. Among inflationary models, chaotic inflation[2] has emerged with great appeal because of its lack of an initial thermal state and the simplicity of its potential. In chaotic inflation the inflaton field ϕ is chaotically initially displaced in some regions from the minimum of its potential, and it evolves slowly towards the minimum. The rapid expansion (inflation) is due to its potential energy density. We use $V(\phi) = \lambda\phi^4/4$ for the effective potential. For potentials with sufficiently small curvature ($\lambda \approx 10^{-14}$) to obtain density perturbations of the proper amplitude, ϕ initially knows nothing about the position of the minimum of the effective potential. Thus the inflaton field can take initially any value, but in order to obtain sufficient inflation to solve the standard astrophysical conundra, $\phi_o > 4.5M_p$. Our numerical study of the influence of inhomogeneities on inflation is based on nonlinear deviations from the homogeneous solutions. We are interested in addressing the question of which truly chaotic initial conditions will lead to inflation; that is, if there exists a large enough region of space where inflation is a generic description of early universe evolution.

We use York's[3] conformal approach to solve the initial value problem of general relativity. The Hamiltonian constraint becomes

$$8\Delta\psi - R\psi + A_{ij}A^{ij}\psi^{-7} - \frac{2}{3}K^2\psi^5 + 2\kappa\rho\psi^{-3} = 0, \tag{1}$$

where R is the Ricci scalar of the initial conformal 3-slice, K the trace of the extrinsic curvature, A_{ij} its traceless part, and $\rho = \frac{1}{2}(\eta^2+\nabla_i\phi\nabla^i\phi)+V$ is the energy density, with η the inflaton momentum. Solutions of this elliptic equation yield the conformal factor ψ. The physical metric is recovered by the conformal transformation $h_{ij} \rightarrow \psi^4 h_{ij}$. From the momentum constraint we get the elliptic equation

$$(\Delta_l W)^i - \frac{2}{3}\psi^6\nabla^i K - \kappa J^i = 0, \tag{2}$$

where $J^i = -\eta\nabla^i\phi$ is the momentum density. This gives the vector potential W^i which determines the longitudinal part of the extrinsic curvature. We choose[4] inflaton field

$$\phi(r^i,t) = \phi_h(t)[1+\alpha\sum_{lmn=1}^{2}\frac{1}{lmn}\sin x_l \sin y_m \sin z_n], \tag{3}$$

where $x_l = 2\pi x l/L + \theta_l$ with θ_l random phases. We assume trace of the extrinsic curvature given by $K = K_o(1+\beta\psi)$, where $K_o = -3H$ with H the Hubble constant. With this choice, Eq. 1 and Eq. 2 remain coupled. We use $\phi_o = 2.5M_p, \lambda = 10^{-3}, \alpha = 0.04, \beta = 1.0$ and the size of the grid L is approximately six horizon lengths. Then the total energy density is $3.7\times10^{-2}M_p^4$ and distributed as follows: 27% potential, 71% kinetic and 2% spatial gradients. Once the initial data is obtained, the evolution problem consists of evolving the inflaton field ϕ, its momentum η, the 3D metric h_{ij}, and the extrinsic curvature K_{ij}. We choose zero shift and unit lapse as gauge conditions, and the constraints are only used to check deviations of our data from Einstein data. The figures show a model which evolves through inflation to reheating. This computation models a volume expansion of over 10^{24}.

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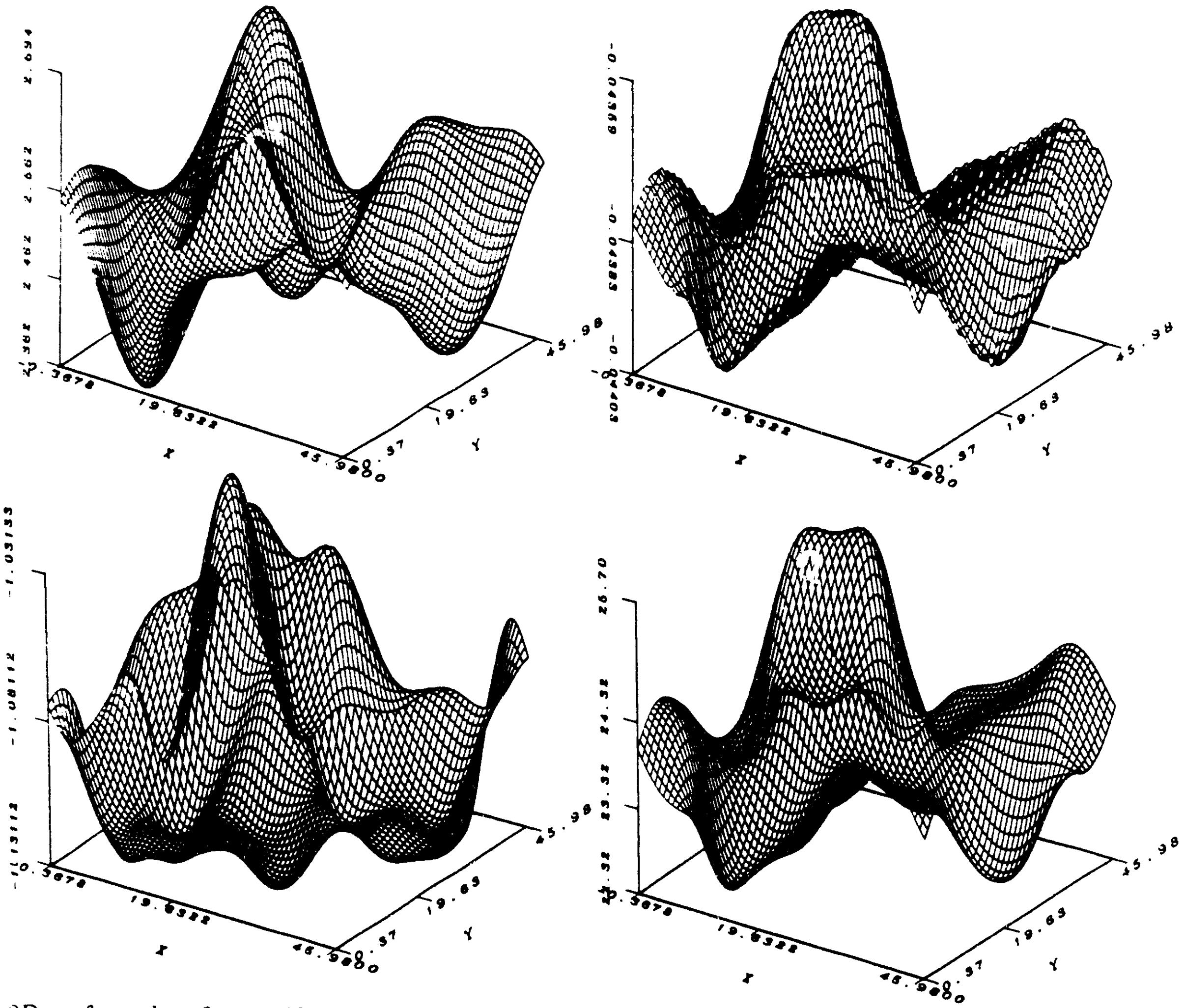
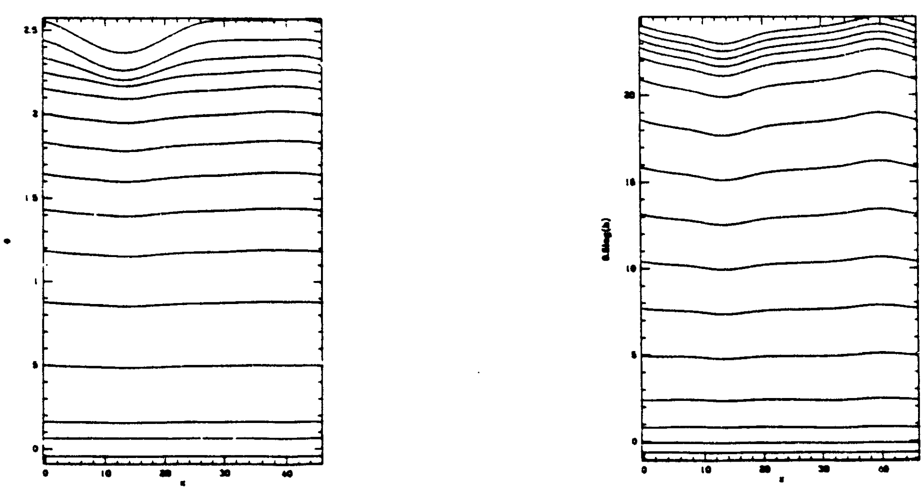
References

1. K. Olive, *Phys. Rep.* **190** (1990) 307.
2. A. D. Linde, *Phys. Lett.* **162B** (1985) 281.
3. J. W. York, in *Sources of Gravitational Radiation*, ed. L. L. Smarr (Cambridge University Press, Cambridge, 1979).
4. P. Laguna, H. Kurki-Suonio and R. A. Matzner, to be published *Phys. Rev. D* (1991).



1. 2D surface plots from a (64 × 64 × 5) run at cycles 0, 1024 for (a): the inflaton field ϕ top, and (b): the volume expansion $\log_{10}\sqrt{\det h_{ij}}$ bottom. Initially (left) there are 3.8 horizon lengths in our computational grid. The simulation ends after 20 e-folds with 157.8 horizon lengths in our grid. The grid cell size first exceeds the horizon size at about timestep 80.



2. 1D plots of the left front edge of the surface plots every 64 cycles for (a): the inflaton field ϕ left and (b): the volume expansion $\log_{10}\sqrt{\det h_{ij}}$ right. Time evolution runs down for the inflaton field and up for the determinant of the metric. ϕ decreases throughout the evolution, achieving a negative value as reheating begins. The volume factor $\sqrt{\det h}$ increases by about 10^{24} during the evolution.

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