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ABSTRACT

A kinetic ballooning mode equation retaining full finite ion Larmor radius and ion magnetic drift resonance effects is derived by employing the high n ballooning mode formalism. We find that the critical β is smaller than the ideal MHD critical β, except when $\eta_i = 0$ ($\eta_i \equiv d\ln T_i/d\ln N$) they are identical. The finite Larmor radius effects reduce the growth rate but do not stabilize the mode. The ion magnetic drift resonance effects are destabilizing.

I. INTRODUCTION

It is believed that the plasma confinement in high temperature axisymmetric tokamaks may be limited by the instability of magnetohydrodynamic (MHD) ballooning modes. According to ideal MHD theory the ballooning modes can be driven unstable by the combined effects of the magnetic curvature and the pressure gradient. When β (the ratio of the kinetic pressure to the magnetic pressure) increases to a critical value, β_c, an unstable mode can develop and balloons in the bad curvature region. Many calculations[1-5] have been done to determine β_c from the ideal MHD equations. One of the basic assumptions of ideal MHD theory is that the parallel electric field perturbation $E_\parallel$ vanishes. However, $E_\parallel$ may become finite when kinetic effects such as finite Larmor radius, magnetic drift and Landau resonances, trapped particles and collisional effects are included. Especially, in the high n limit these kinetic effects can be significant. It is, therefore, of interest to examine whether these kinetic effects could modify the stability of the ballooning modes and hence β_c.

Previous investigations[6,7] of the kinetic effects on the MHD ballooning modes have made use of the assumption that the ion magnetic drift frequency, ω_{Di}, is small compared with the mode frequency, ω, and is treated perturbatively without including resonances. However, this assumption is not always valid, especially when the modes are near marginal stability. In this paper we will remove this limitation and investigate the stability of the ballooning modes by retaining full finite ion Larmor radius and ion magnetic drift resonance effects. Effects due to trapped particles, Landau resonances, and collisons will be ignored.

Employing the high n ballooning mode and WKB formalism,[2-5] we have derived, in Section II, a set of eigenmode equations governing both drift and shear Alfvén waves. If terms of order $(\omega_{ti}/\omega)^2$ are neglected (ω_{ti} is the ion transit frequency), we can obtain a single second order differential equation describing the kinetic ballooning modes. Then, we establish its relationship to the ideal MHD ballooning mode equation. In Section III, we first observe that for $\eta_i = 0$ ($\eta_i \equiv d\ell nT_i/d\ell nN$) the kinetic ballooning mode equation reduces to the ideal MHD ballooning mode equation at marginal stability with $\omega = \omega_{*i}$ (ω_{*i} is the ion diamagnetic drift frequency). This means that β_c is identical to the ideal MHD β_c and is independent of the ion Larmor radius and magnetic drift resonance effects. For $\eta_i \neq 0$, β_c is found to be smaller than the ideal MHD β_c, and numerical solutions with parametric variations of β, etc. are presented in detail. Finally, a summary is given in Section IV.

II. FORMULATION

Let us consider low β plasma in an axisymmetric, large aspect ratio torus with concentric, circular magnetic surfaces. We use a (r, θ, ζ) coordinate system, where r is the minor radius, θ is the poloidal angle, and ζ is the toroidal angle. The equilibrium magnetic field is given by $\vec{B} = B_o (1 - r\cos\theta/R)[\hat{\zeta} + (r/qR)\hat{\theta}]$. The perturbed quantities can be expressed in the form

$$\Phi = \phi(r, \theta)\exp[i(m\theta - n\zeta - \omega t)] .$$

The linearized ion gyrokinetic equation is given by

$$[\omega - \omega_{ti}\hat{v}_\parallel(s - i\frac{\partial}{\partial\theta}) + \hat{\omega}_{di}]\, H_i$$

$$= (\frac{eF_M}{T})_i\ \{\omega - \omega_{*i}[1 + \eta_i(\hat{v}^2 - \frac{3}{2})]\}\ J_o(\frac{k_\perp v_\perp}{\Omega_i})\ (\phi - v_\parallel A_\parallel/c) \quad (1)$$

where $\omega_{ti} = v_i/qR$ is the ion transit frequency, $s = (r - r_o)/\Delta r_s$, r_o is the minor radius of the reference mode rational surface with $m = nq(r_o)$, $\Delta r_s = 1/k_\theta\hat{s}$, $k_\theta = m/r_o$, $\hat{s} = (rq'/q)$ at $r = r_o$, $\vec{k}_\perp = -i[(\hat{\theta}/r)\partial/\partial\theta + \hat{r}\partial/\partial r]$, $\Omega_i = eB/m_ic$, $\eta_i = (d\ln T_i/d\ln N)$, the magnetic drift frequency is $\omega_{di} \cong (2\epsilon_n/\tau)\,\omega_{*e}\,(\hat{v}_\parallel^{\,2} + \hat{v}_\perp^{\,2}/2)(\cos\theta - i\hat{s}\sin\theta\ \partial/\partial s)$ $\tau = T_e/T_i$, $\epsilon_n = r_n/R$, r_n is the density scale length, R is the major radius of the torus, q is the safety factor, $\omega_{*e} = (cT_e/eB)\ (k_\theta/r_n)$, $\omega_{*i} = -\omega_{*e}/\tau$, $\hat{v} = v/v_i$, $v_i^{\,2} = 2T_i/m_i$, F_M is the local Maxwellian distribution function, Φ is the perturbed electrostatic potential, $A_\parallel$ is the parallel component of the perturbed vector potential, and H_i is related to the perturbed ion distribution f_i by $f_i = -(e\Phi/T_i)F_M + H_i$. Since we will work in the limit $\omega_{ti} < \omega$, the θ dependence of $v_\parallel$ and $v_\perp$ will be ignored.

In the following, we will employ the ballooning mode formalism which uses $\epsilon \equiv 1/n$ as an expansion parameter to develop an asymptotic solution of Eq. (1). The perturbed quantities have short perpendicular and long parallel wave lengths and can be expressed by the eikonal representation

$$\phi = \hat{\phi}(\theta,s,\epsilon)\ \exp[-\ i\ S(\theta,s)/\epsilon] \quad (2)$$

where S describes the rapid cross field variations and $\vec{B} \cdot \nabla S = 0$. In the axisymmetric case, S can be expressed as

$$S(\theta,s) = (m/n - q)\ \theta + \int k_q(s)dq$$

$$= -\Delta r_\varepsilon q'[s\theta - \int k_q ds] \tag{3}$$

where $k_q(s)/\varepsilon$ is the radial wavenumber in (θ,q) coordinates. Insert Eq. (2) into Eq. (1) and expanding in powers of ε, we find at lowest order

$$\{\omega - i\ \omega_{ti}\ \hat{v}_\parallel\ \partial/\partial\theta + \omega_{Di}\}\ \hat{H}_i$$

$$= (eF_M/T)_i\ \{\omega - \omega_{*i}[1 + \eta_i\ (\hat{v}^2 - 3/2)]\}\ J_o\ (\hat{\phi} - v_\parallel \hat{A}_\parallel/c) \tag{4}$$

$$\equiv G_i$$

where

$$\omega_{Di} = 2\ \omega_{*e}\ (\varepsilon_n/\tau)\ (\hat{v}_\parallel^{\ 2} + \hat{v}_\perp^{\ 2}/2)\ (\cos\theta + \hat{s}\ (\theta - k_q)\ \sin\theta),$$

$$J_o = J_o(b_i^{\ 1/2}),\ b_i = 2b_\theta(1 + \hat{s}^2\theta^2)\hat{v}^2/\tau,\ b_\theta = \tau k_\theta^{\ 2}\rho_i^{\ 2}/2,$$

and

$$\rho_i = v_i/\Omega_i.$$

In Eq. (4) we will choose $k_q = 0$ so that the perturbations are centered at the outside of the torus. This choice of k_q is made because we have learned from numerical experience that the maxima of $\mathrm{Im}(\omega)$ occur at $k_q \approx 2\pi N$ ($N = 0, 1\ldots$) for up-down symmetric equilibrium. Equation (4) is defined over an infinite range in θ without periodicity constraint. The boundary condition for $\hat{H}_i$ is that $\hat{H}_i$ decays sufficiently fast as $|\theta| \to \infty$.

In the limit $\omega_{ti} \ll \omega$, Eq. (4) can be solved to yield

$$\hat{H}_i = \frac{G_i}{\omega + \omega_{Di}} + \frac{i\omega_{ti}\hat{v}_\parallel}{(\omega + \omega_{Di})^2}\frac{\partial G_i}{\partial\theta} - \frac{\omega_{ti}^2\hat{v}_\parallel^2}{(\omega + \omega_{Di})^3}\frac{\partial^2 G_i}{\partial\theta^2} + 0\left(\frac{\omega_{ti}}{\omega}\right)^3 . \tag{5}$$

The ion density and current perturbations, δn_i and $j_{\parallel i}$, can be calculated from Eq. (5) and are given by

$$\begin{aligned}\frac{\delta n_i}{N} &= -e\hat{\phi}/T_i + \int \hat{H}_i J_o d^3 v \\ &= -\tau\phi + \int \frac{J_o^2 F_o \Omega_{Si} d^3\hat{v}}{(\Omega + \Omega_{Di})}\phi + \frac{\Omega_{ti}^2}{\Omega}\int \frac{J_o^2 F_o \hat{v}_\parallel^2 \Omega_{si} d^3\hat{v}}{(\Omega + \Omega_{Di})^2}\frac{\partial^2 A}{\partial\theta^2} \\ &\quad -\Omega_{ti}^2 \int \frac{J_o^2 F_o \hat{v}_\parallel^2 \Omega_{si} d^3\hat{v}}{(\Omega + \Omega_{Di})^3}\frac{\partial^2\phi}{\partial\theta^2} + 0\left(\frac{\Omega_{ti}}{\Omega}\right)^3 ,\end{aligned} \tag{6}$$

and

$$\begin{aligned}\frac{\partial}{\partial\theta} j_{\parallel i} &= \frac{\partial}{\partial\theta}\int e v_\parallel \hat{H}_i J_o d^3 v \\ &= \frac{N e v_i \Omega_{ti}}{i\Omega}\int J_o^2 F_o \hat{v}_\parallel^2 \Omega_{si}\left(\frac{\partial^2 A/\partial\theta^2}{\Omega + \Omega_{Di}} - \frac{\Omega\partial^2\phi/\partial\theta^2}{(\Omega + \Omega_{Di})^2}\right) d^3\hat{v} \\ &\quad + 0\left(\frac{\Omega_{ti}}{\Omega}\right)^2 ,\end{aligned} \tag{7}$$

where

$$\phi = e\hat{\Phi}/T_e,\ A = e\hat{A}/T_e,\ \hat{A} = \int^\theta d\theta \hat{A}_\parallel\ \omega v_i/i\omega_{ti}c,\ \Omega = \omega/\omega_{*e},\ \Omega_{Di} = \omega_{Di}/\omega_{*e},$$

$$\Omega_{ti} = \omega_{ti}/\omega_{*e}\ ,\ \Omega_{si} = \tau\Omega + 1 + \eta_i(\hat{v}^2 - 3/2) \text{ and } F_o = (1/\pi^{3/2})\exp(-\hat{v}^2).$$

We will consider all the electrons to be highly circulating and collisions and trapped electron effects are not retained. The linearized gyrokinetic equation for electrons is given by

$$[\Omega - i\Omega_{te}\hat{v}_\parallel \frac{\partial}{\partial\theta} + \Omega_{De}]\, \hat{H}_e = (eF_M/T)_e\, \Omega_{se}\, (\hat{\Phi} - v_\parallel \hat{A}_\parallel/c) \equiv G_e \tag{8}$$

where

$$\Omega_{De} = -\tau\Omega_{Di},\ \Omega_{te} = v_e/(qR\omega_{*e}),\ v_e^{\,2} = 2T_e/m_e,$$

$$\Omega_{se} = \Omega - [1 + \eta_e(\hat{v}^2 - 3/2)],\ \hat{v} = v/v_e,\ \text{and}\ \eta_e = d\ell nT_e/d\ell nN.$$

Since we are interested in the limit $\Omega \ll \Omega_{te}$, the θ dependence of $\hat{v}_\parallel$ will be ignored and Eq. (8) can be solved with the asymptotic decaying boundary condition to give

$$\hat{H}_e = \int^{\theta} d\theta\, \frac{iG_e}{\Omega_{te}\hat{v}_\parallel} + \int^{\theta} d\theta'\, \frac{\Omega + \Omega_{De}}{\Omega_{te}\hat{v}_\parallel} \int^{\theta'} d\theta''\, \frac{G_e}{\Omega_{te}\hat{v}_\parallel} + O\left(\frac{\Omega}{\Omega_{te}}\right)^3 . \tag{9}$$

The electron density perturbation, δn_e, and current perturbation, $j_{\parallel e}$, can be obtained from Eq. (9) and are given by

$$\delta n_e/N = e\hat{\Phi}/T_e + \int \hat{H}_e\, d^3v = \phi - \Omega - 1/\Omega\ A + O\,(\Omega/\Omega_{te})^2, \tag{10}$$

and

$$\frac{\partial}{\partial\theta} j_{\parallel e} = -\frac{\partial}{\partial\theta} \int ev_{\parallel} \hat{H}_e d^3v$$

$$= -\frac{Nev_e}{i\Omega_{te}\Omega} [\Omega(\Omega - 1)(\phi - A) + 2\epsilon_n(\Omega - 1 - \eta_e)f_\theta A] + O\left(\frac{\Omega}{\Omega_{te}}\right)^2 , \qquad (11)$$

where $f_\theta = \cos\theta + \hat{s}\theta \sin\theta$. Comparing Eq. (7) with Eq. (11) we note that $\partial j_{\parallel i}/\partial\theta$ is of order $(\Omega_{ti}/\Omega)^2$ smaller than $\partial j_{\parallel e}/\partial\theta$.

The basic set of coupled one dimensional equations governing the eigenmodes of the system can now be written in terms of the solutions of the perturbed quantities given by Eqs. (7), (8), (10), and (11). These are the quasi-neutrality condition and the parallel component of Ampere's law along $\vec{B}$. The quasi-neutrality condition can be expressed as

$$\frac{\Omega - 1}{\Omega}(\phi - A) = -\left(\frac{\tau\Omega + 1}{\Omega}\right)\phi + \int \frac{J_0^2 F_0 \Omega_{si} d^3v}{(\Omega + \Omega_{Di})}\phi - \Omega_{ti}^2 \int \frac{J_0^2 F_0 \hat{v}_\parallel^2 \Omega_{si} d^3v}{(\Omega + \Omega_{Di})^2}\left[\frac{\partial^2\phi/\partial\theta^2}{(\Omega + \Omega_{Di})} - \frac{\partial^2 A/\partial\theta^2}{\Omega}\right] , \qquad (12)$$

and the parallel Ampere's law is given by

$$\frac{\partial}{\partial\theta}(1 + \hat{s}^2\theta^2)\frac{\partial A}{\partial\theta} = \Omega_A^{-2} \Big[\Omega(\Omega - 1)(\phi - A) + 2\epsilon_n(\Omega - 1 - \eta_e)f_\theta A - \Omega_{ti}^2 \int \frac{J_0^2 F_0 \hat{v}_\parallel^2 \Omega_{si}}{(\Omega + \Omega_{Di})}\left[\frac{\partial^2 A}{\partial\theta^2} - \frac{\Omega}{(\Omega + \Omega_{Di})}\frac{\partial^2\phi}{\partial\theta^2}\right] d^3v\Big], \qquad (13)$$

where $\Omega_A^{-2} = [\beta/2(1 + 1/\tau)](q/\epsilon_n)^2$ and $\beta = 8\pi N(T_e + T_i)/B^2$. Since the parallel electric field perturbation $E_\parallel$ is proportional to $\partial/\partial\theta\ (\phi - A)$, the right hand side of Eq. (12) represents the kinetic ion contributions from finite Larmor radius and magnetic drift resonance effects. If we further neglect terms of order $(\Omega_{ti}/\Omega)^2$, Eqs. (12) and (13) can be reduced to a single second order differential equation, i.e.,

$$\frac{\partial}{\partial\theta}\,(1 + \hat{s}^2\theta^2)\,\frac{\partial A}{\partial\theta} = \Omega_A^{-2}\,\{2\varepsilon_n(\Omega - 1 - \eta_e)f_\theta - \Omega(\Omega - 1)$$

$$+ (\Omega - 1)^2/[1 + \tau - \int \frac{J_o^{\,2}F_o\Omega_{si}d^3v}{(\Omega + \Omega_{Di})}]\}\ A. \qquad (14)$$

Equation (14) describes the kinetic ballooning mode with even A in θ and can be solved as an eigenvalue equation for Ω under the boundary condition that A decays asymptotically as $|\theta| \to \infty$. We can now compare Eq. (14) with the analogous MHD equations.[2-5] If Ω_{Di}/Ω and b_θ are taken as small parameters and terms of order $(\Omega_{Di}/\Omega)^2$ and $[b_\theta\ (1 + \hat{s}^2\theta^2)]^2$ are neglected, then Eq. (14) reduces to[6]

$$\frac{\partial}{\partial\theta}\,(1 + \hat{s}^2\theta^2)\,\frac{\partial}{\partial\theta}\,A = -\Omega_A^{-2}\{2\varepsilon_n[1 + \eta_e + (1 + \eta_i)/\tau]f_\theta$$

$$+ \Omega\ [\Omega + (1 + \eta_i)/\tau]b_\theta(1 + \hat{s}^2\theta^2)\}A. \qquad (15)$$

In the limit $b_\theta \to 0$, Eq. (15) corresponds exactly to the ideal, single-fluid MHD ballooning mode equation. From Eq. (15) we see that the eigenfrequency at marginal stability is given by $\Omega = -(1 + \eta_i)/2\tau$, and the critical β_c versus b_θ is shown in Fig. (1) by solving Eq. (15) numerically for the parameters: $\hat{s} = 0.5$, $\varepsilon_n = 0.2$, $q = 2$, $\tau = 1$, and $\eta_e = \eta_i = 0$. Figure 1 clearly shows the finite Larmor radius stabilization of the MHD ballooning modes. In the limit $b_\theta \to 0$, β_c approaches to the ideal MHD critical β. However, the assumptions, $\Omega_{Di}/\Omega \ll 1$ and $b_\theta(1 + \hat{s}^2\theta^2)/\tau \ll 1$, that lead to Eq. (15) become invalid at marginal stability because the mode structure becomes very broad and extends to $\theta \sim O(10^2)$. Therefore, the result in Fig. 1 is incorrect and we must employ Eq. (14) to study the ballooning modes near marginal stability.

III. RESULTS

In this section we present the results of an eigenmode analysis of kinetic ballooning modes (with the even A solution) by solving Eq. (14) numerically. Let us first consider the $\eta_i = 0$ case. We observe that for $\Omega = -1/\tau$ (i.e., $\omega = \omega_{*i}$), Eq. (14) reduces exactly to the ideal MHD ballooning mode equation at marginal stability[2-5] (with $\Omega = 0$), i.e.,

$$\frac{\partial}{\partial\theta}\,(1 + \hat{s}^2\theta^2)\,\frac{\partial}{\partial\theta}\,A + \beta_c\,(q^2/\epsilon_n)\,[1 + \eta_e/(1 + 1/\tau)](\cos\theta + \acute{s}\theta\,\sin\theta)\,A = 0. \tag{16}$$

Therefore, the critical β obtained from Eq. (16) is exactly the same as the ideal MHD β_c and is independent of the kinetic ion contributions due to finite Larmor radius and magnetic drift resonance effects. Note that this conclusion of β is unchanged when we employ the self-consistent equilibrium, and the second critical β also occurs at $\omega = \omega_{*i}$. It is not difficult to understand this result physically because at marginal stability with $\Omega = -1/\tau$, the ions behave adiabatically and the parallel electric field perturbation $E_\parallel$ vanishes identically which corresponds to the ideal MHD assumption. This phenomenon can also be seen in Fig. 2 where the numerical eigenfrequencies from solving Eq. (14) are shown versus β for the parameters: $\hat{s} = 0.5$, $\epsilon_n = 0.2$, $q = 2$,

$\tau = 1$, $b_\theta = 0.1$ and $\eta_i = \eta_e = 0$. The broken curve shows the growth rates of the ideal MHD ballooning modes ($b_\theta \to 0$ limit). We note that the growth rates of the kinetic ballooning modes are reduced due to finite Larmor radius and magnetic drift effects, but reach marginal stability at the same β as the ideal MHD modes.

Note that in solving Eq. (14) we have made the approximation[8]

$$\Omega_{Di} = 2(\epsilon_n/\tau)\ (2\hat{v}_\parallel^2) f_\theta \tag{17}$$

so that the velocity space integral in Eq. (14) can be carried out analytically to yield

$$\int J_o^{\,2} F_o \Omega_{si} d^3v/(\Omega + \Omega_{Di} = [(\tau\Omega + 1 - \eta_i/2)\ \Gamma_o + \eta_i b(\Gamma_1 - \Gamma_o)]\ D_o + \eta_i \Gamma_o D_1 \tag{18}$$

where

$$D_o = (\tau/4\epsilon_n f_\theta) Z(\xi)/\xi,\ D_1 = (\tau/4\epsilon_n f_\theta)[1 + \xi Z(\xi)],\ \xi^2 = -(\Omega\tau/4\epsilon_n f_\theta),$$

Z is the plasma dispersion function, $\Gamma_{0,1} = I_{0,1}$ (b) $\exp(-b)$ and $b = b_\theta$ $(1 + \hat{s}^2\theta^2)/\tau$. In obtaining D_0 and D_1, we must perform the proper analytic continuation of the integration contour and ξ in the complex $\hat{v}_\parallel$ space. This approximation deforms the resonant surface in velocity space into a plane without affecting the basic magnetic drift resonance effects.

In addition to the eigenmode (branch I) described in Fig. 2, there exists another eigenmode branch (branch II). For $\eta_i = 0$, branch II remains stable for all β. Figure 3 shows the dependence of the eigenfrequency Ω on η_i for these two branches of eigenmodes with parameters: $\eta_e = 0$, $\tau = 1.0$, $\epsilon_n = 0.2$, $b_\theta = 0.1$, $q = 2.0$, $\hat{s} = 0.5$, and $\beta = 0.017$. Branch I is unstable for small η_i and becomes stable for large η_i. On the other hand, branch II is stable for small η_i, but becomes highly destabilized as η_i increases. For both branches the real frequencies decrease as η_i increases. Figure 4 shows the eigenfrequency Ω versus η_e for $\eta_i = 0$, $\tau = 1.0$, $\epsilon_n = 0.2$, $b_\theta = 0.1$, $q = 2.0$, $\hat{s} = 0.5$, and $\beta = 0.017$. Here, contrary to the η_i dependence in Fig. 4, branch I is destabilized by η_e and branch II remains stable for all η_e shown. For both branches the real frequencies increase with η_e.

Figures 5(a) and 5(b) show the eigenfunctions $A(\theta)$ of branches I and II for $\eta_e = \eta_i = 0$, $\beta = 0.017$, $\tau = 1.0$, $\epsilon_n = 0.2$, $b_\theta = 0.1$, $\hat{s} = 0.5$, and $q = 2.0$. The eigenfrequencies are $\Omega = -0.896 + 0.11i$ for branch I and $\Omega = -1.59 - 0.073i$ for branch II. The modes are near marginal stability and have very broad structure. Since the ion magnetic drift frequency ω_{Di} is secular in θ, the perturbative treatment of ω_{Di} term breaks down for $\theta \gtrsim (\tau/2\epsilon_n\hat{s}) \approx 5$. Also the assumption $b_\theta(1 + \hat{s}^2\theta^2)/\tau \ll 1$ is not valid for $\theta \gtrsim (\tau/b_\theta\hat{s}^2)^{1/2} \approx 6$.

We have studied the behavior of these two branches of eigenmodes in the (η_e, η_i) plane. We find that for the set of parameters described in Figs. 3 and 4 there is a branch point at $(\eta_i, \eta_e) \approx (0.71, 0.35)$ where these two branches coincide and become a double root with $\Omega \approx (-1.8, 0.185)$. If we vary η_i and η_e along a closed path in (η_i, η_e) plane around the branch point once, we find that branch I interchanges with branch II.

For $\eta_i \neq 0$ the critical β is different from the ideal MHD β_c and is usually smaller. The dependence of the eigenfrequency of the most unstable mode on β is displayed in Fig. 6 for $\eta_i = \eta_e = 1.0$, $\hat{s} = 0.5$, $\epsilon_n = 0.2$, $q = 2.0$, $b_\theta = 0.1$, and $\tau = 1.0$. The mode becomes marginally stable at $\beta_c = 0.006$ with eigenfrequency $\Omega \approx -4.0$. The growth rate of the ideal MHD mode is also shown in Fig. 6 for comparison. Figure 7 shows the dependence of Ω on b_θ for $\hat{s}$ = 0.017 and the other parameters are the same as in Fig. 6. The finite Larmor radius effect can reduce the growth rate, but does not stabilize the mode even when b_θ is larger than unity. The real frequency decreases as b_θ increases. For $b_\theta \lesssim 10^{-3}$, the condition $(\omega_{ti}/\omega) \ll 1$ is not well satisfied and the solutions are questionable.

The evolution of the ballooning modes is also examined as the parameters, $\hat{s}$ and ε_n, are varied. The fixed parameters are $\eta_e = \eta_i = 1.0$, $\beta = 0.02$, $b_\theta = 0.1$, $q = 2.0$, and $\tau = 1.0$. In Fig. 8(a) (with $\varepsilon_n = 0.2$) the shear stabilization of the ballooning modes occurs at $\hat{s} = 1.15$. Both the real frequency, Ω_r, and the growth rate, Ω_i, decrease as $\hat{s}$ increases. Figure 8(b) displays the dependence of Ω on ε_n with $\hat{s} = 0.5$. There are two stability boundaries with $\varepsilon_{n1} = 0.0135$ and $\varepsilon_{n2} = 0.306$. For small ε_n $(|\omega_{Di}/\omega| \ll 1)$ the magnetic drift resonance effect is exponentially small and the modes evolve from a pair of almost complex conjugates into two real Ω solutions as ε_n decreases below ε_{n1}. This differs from the ideal MHD results in that there is only one critical ε_n for a low β equilibrium with concentric, circular magnetic surfaces.

IV. CONCLUSIONS

In this paper we have employed the high n ballooning mode formalism to derive the eigenmode equation for kinetic ballooning modes of arbitrary wavelengths in the limit $\omega_{ti} < \omega < \omega_{te}$. The ion magnetic drift frequency, ω_{Di}, is not taken to be smaller than the mode frequency, ω, so that the ion magnetic drift resonances are retained. In the limits $\omega_{Di}/\omega \ll 1$ and $b_\theta \to 0$, the eigenmode equation reduces to the ideal MHD ballooning mode equation. The eigenmode equation is then analyzed numerically by using a shooting code. Two branches of eigenmodes are found to exist. For $\eta_i = 0$ we have observed that the critical β of branch I which occurs at $\omega = \omega_{*i}$ is identical to the ideal MHD β_c and is independent of the finite Larmor radius and magnetic drift resonance effects. Branch II remains stable for all β at $\eta_i = 0$, but can be more unstable than branch I as η_i is varied. For $\eta_i \neq 0$ the numerical results have shown that the critical β is smaller than the ideal MHD β_c and the finite Larmor radius effect can reduce the growth rate, but does not stabilize the ballooning modes. The ion magnetic drift resonance effect is destabilizing. The dependence of Ω on ε_n shows that there are two stability boundaries in ε_n. This is different from the ideal MHD prediction of only one critical ε_n for a low β equilibrium with concentric, circular magnetic surfaces.

Finally, this work is incomplete since kinetic effects, such as trapped particles, Landau resonances, and collisions[9,10], are suppressed here in order to concentrate on the finite ion Larmor radius and ion magnetic drift resonance effects. The self-consistent MHD equilibrium must also be included before we can determine the critical β of tokamaks.

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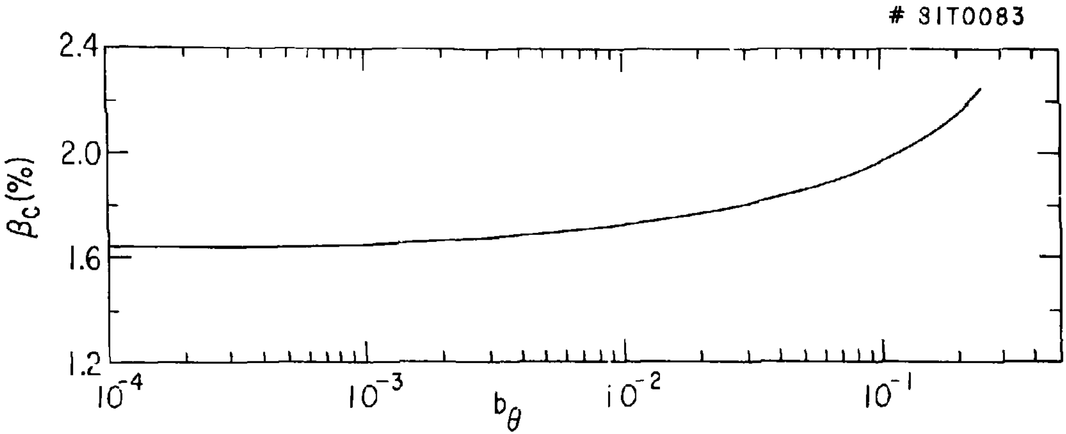

Fig. 1 Critical β versus b_θ for $\hat{s} = 0.5$, $\epsilon_n = 0.2$, $q = 2.0$, $\tau = 1.0$, and $\eta_i = \eta_e = 0$ by solving Eq. (15).

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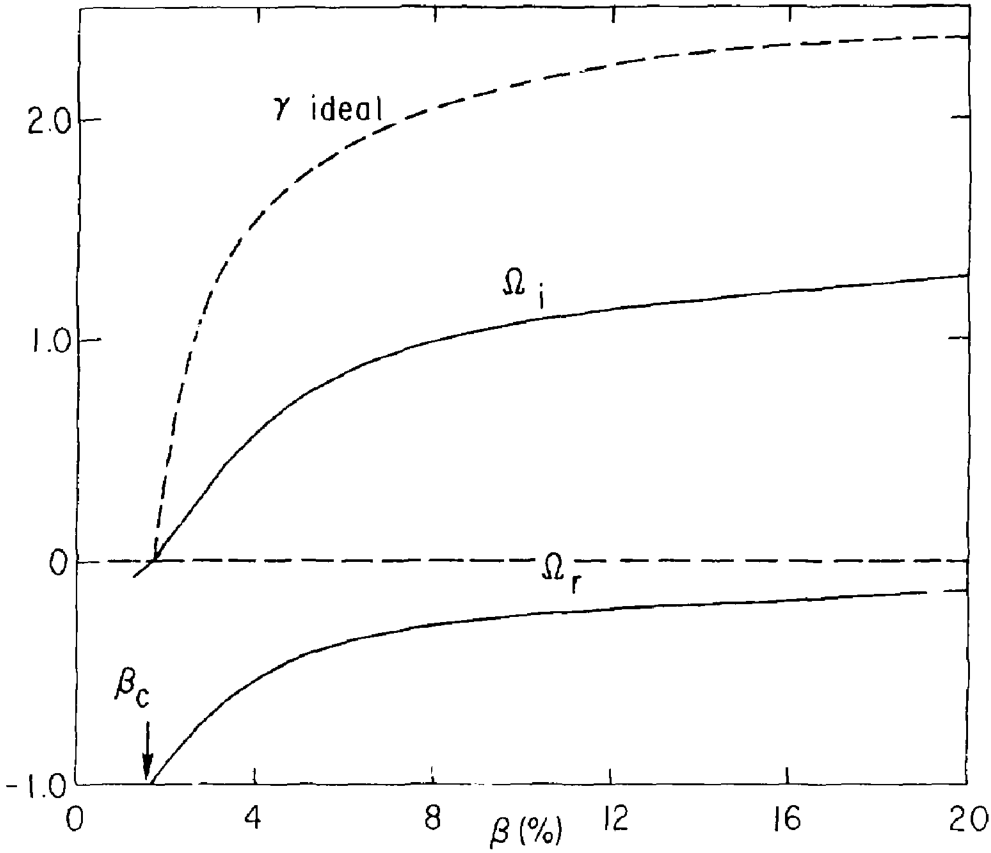


Fig. 2 The dependence of Ω on β for $b_\theta = 0.1$, $\hat{s} = 0.5$, $\varepsilon_n = 0.2$, $q = 2.0$, $\tau = 1.0$, and $\eta_e = \eta_i = 0$. The ideal MHD growth rate is also shown.

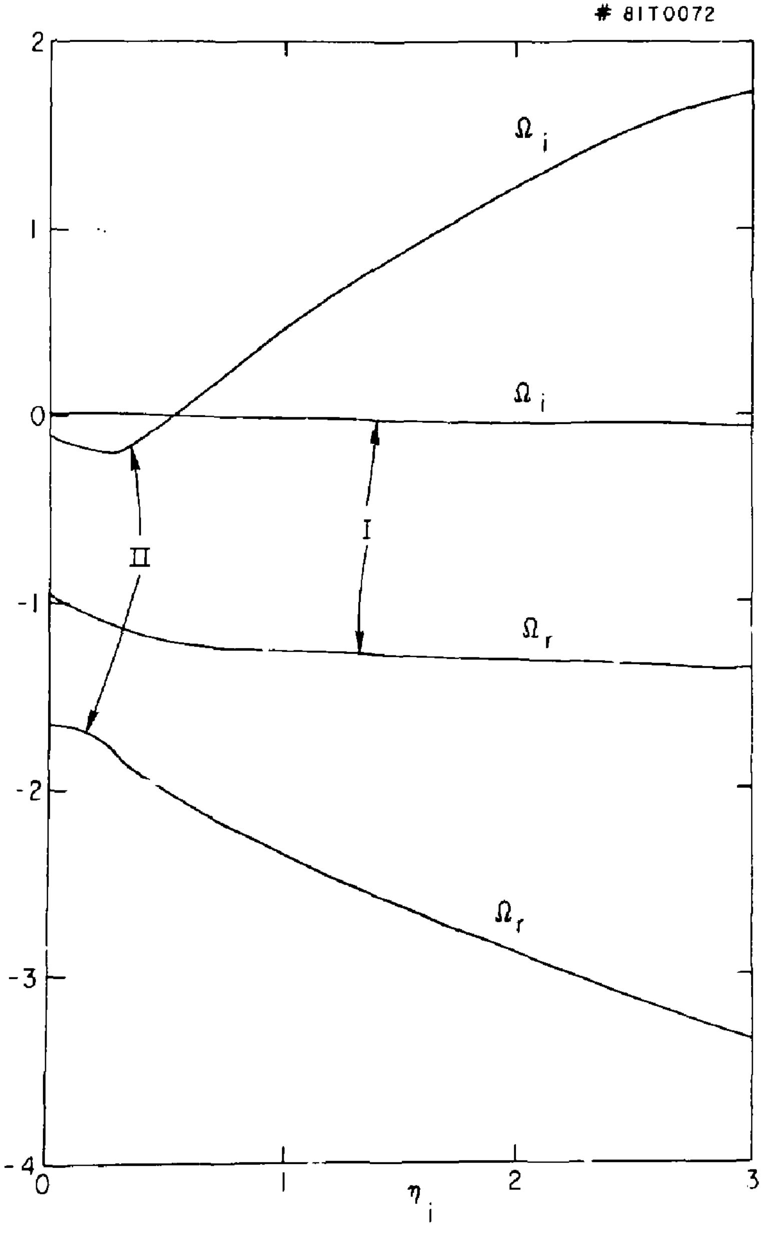


Fig. 3 The dependence of Ω on η_i for two branches of eigenmodes. The fixed parameters are $\beta = 0.017$, $b_\theta = 0.1$, $\hat{s} = 0.5$, $\epsilon_n = 0.2$, $q = 2.0$, $\tau = 1.0$, and $\eta_e = 0$.

81T0071

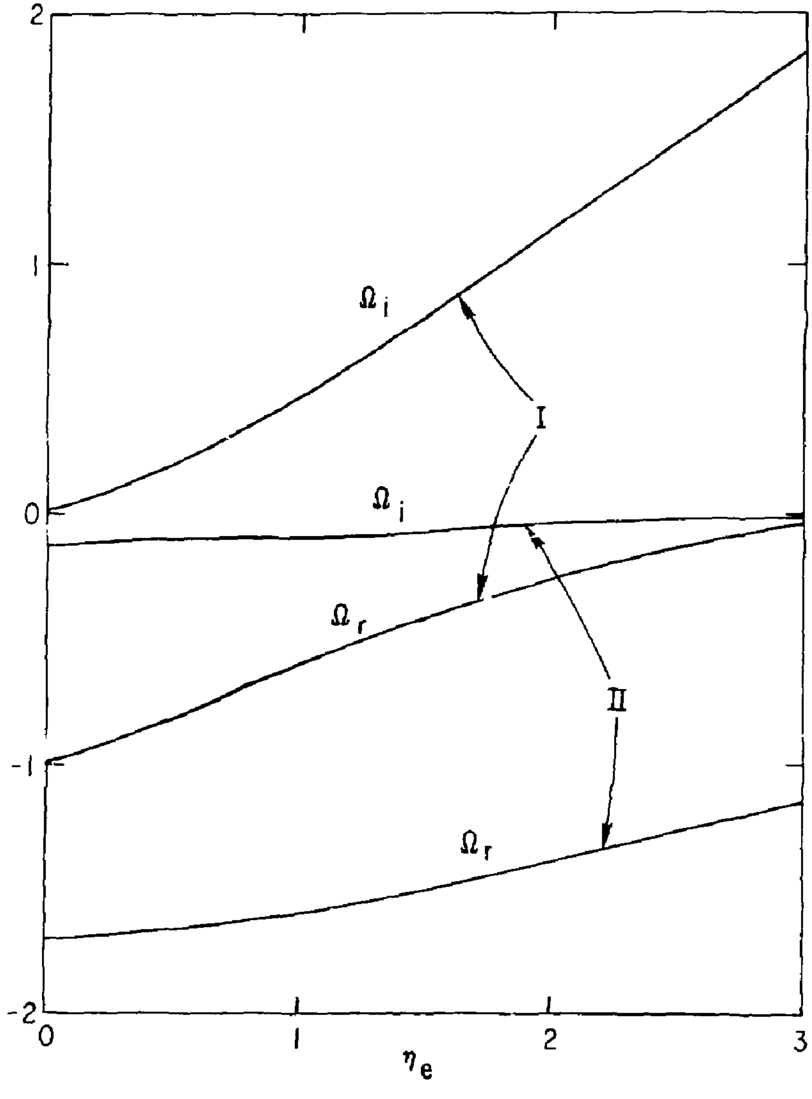
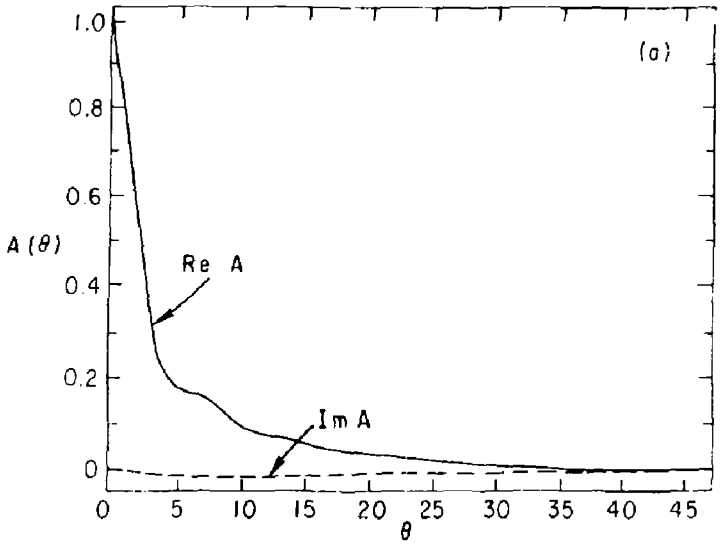


Fig. 4 The dependence of Ω on η_e for two branches of eigenmodes with η_i = 0. The other parameters are the same as in Fig. 3.

81T0078



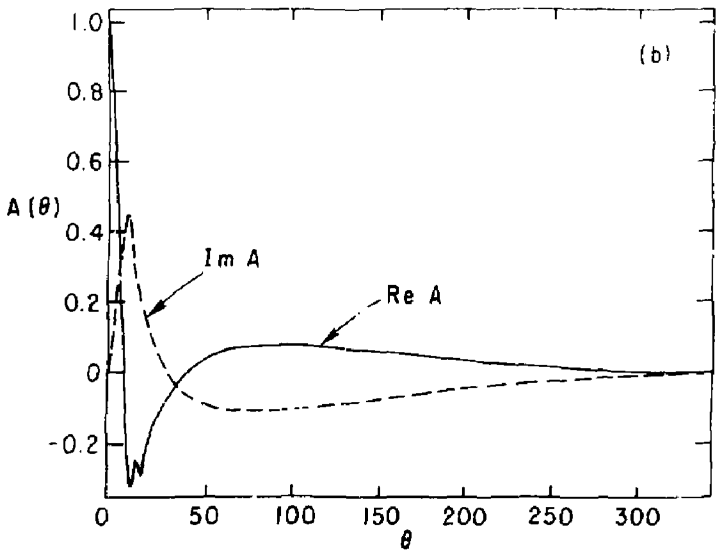


Fig. 5 Plot of the eigenfunction of two branches of eigenmodes (for β = 0.02, b_θ = 0.1, $\hat{s}$ = 0.5, ε_n = 0.2, q = 2.0, τ = 1.0, and $\eta_e = \eta_i$ = 0): (a) branch I with Ω = (- 0.896, 0.11), (b) branch II with Ω = (- 1.588, - 0.073).

81T0084

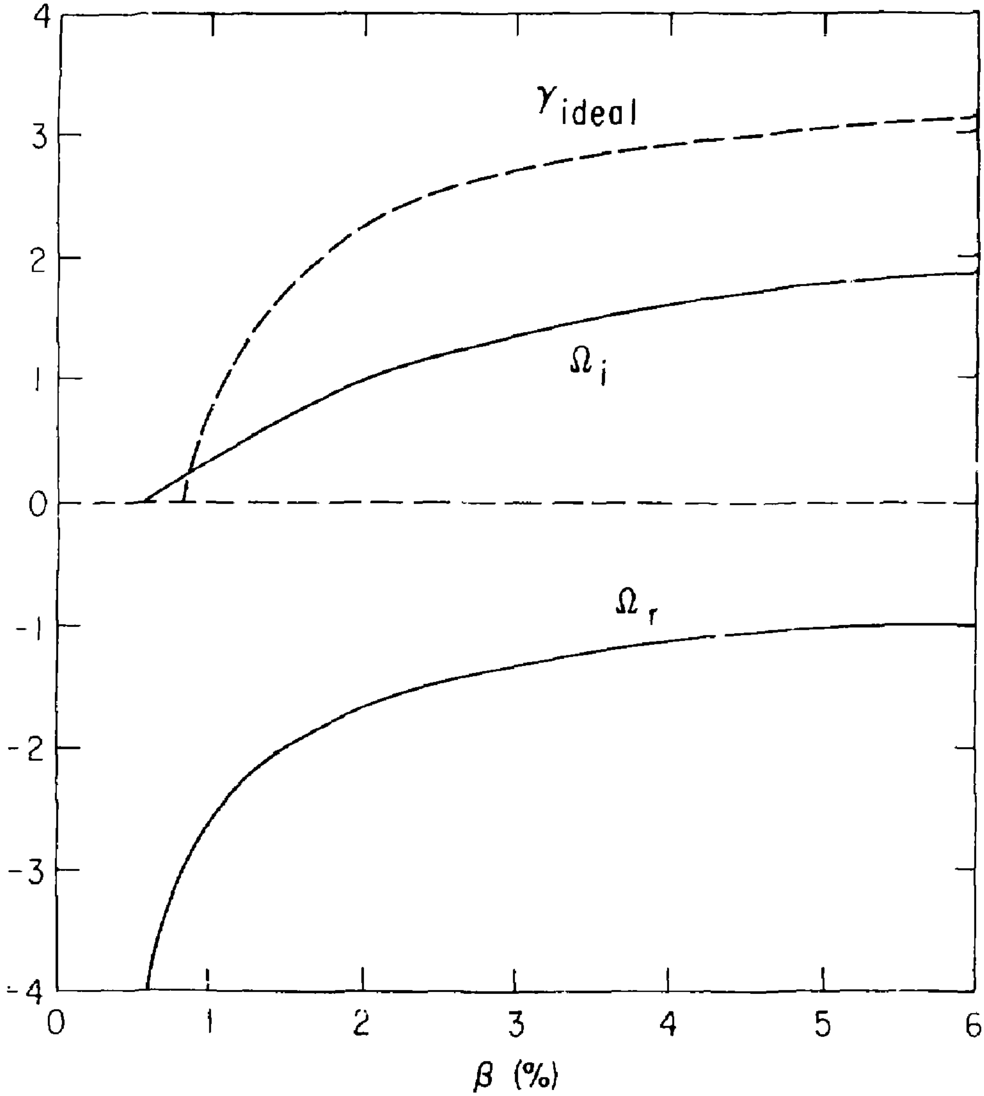


Fig. 6 The dependence of Ω on β for $b_\theta = 0.1$, $\hat{s} = 0.5$, $\epsilon_n = 0.2$, $q = 2.0$, $\tau = 1.0$ and $\eta_e = \eta_i = 1.0$. The corresponding ideal MHD growth rate is also shown.

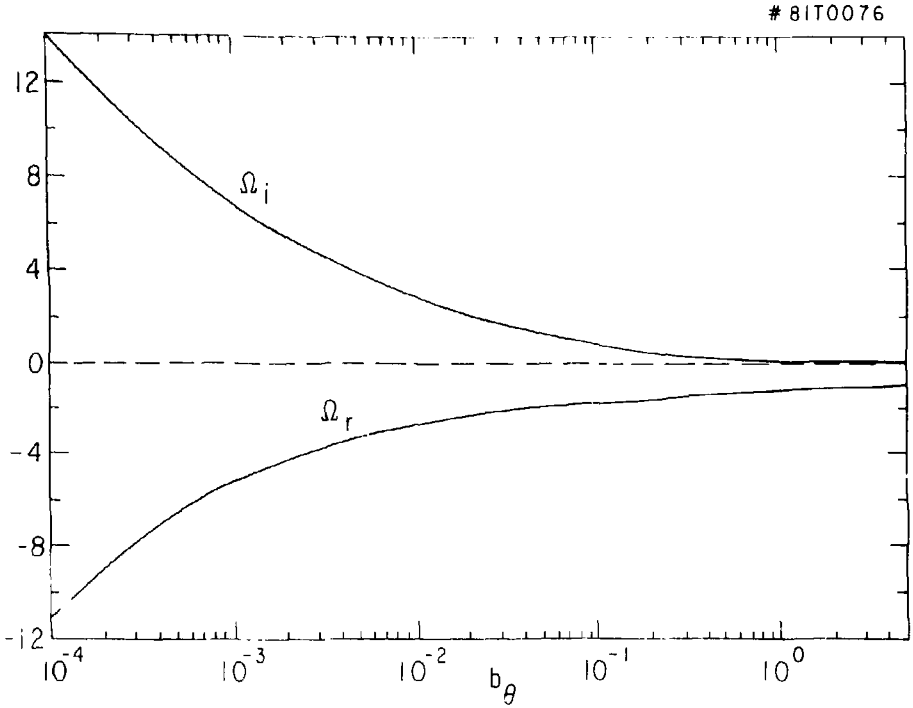


Fig. 7 The dependence of [illegible] on [illegible] for [illegible]

= 1.0, and [illegible] = 1.0.

\# 81T0075

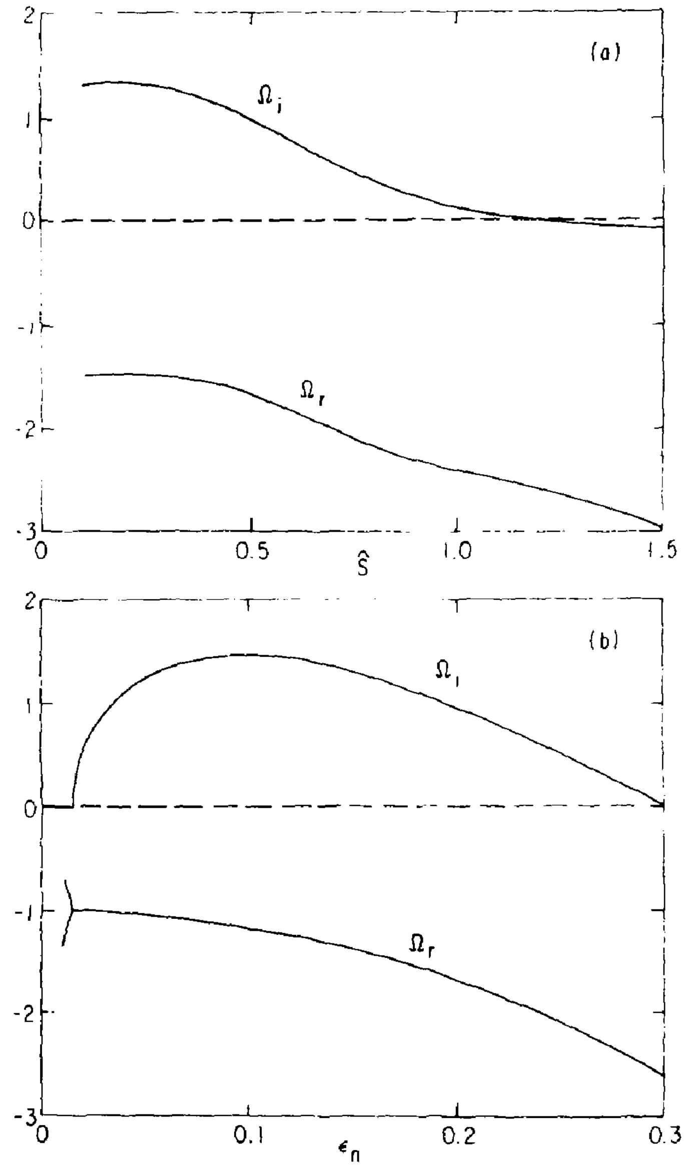


Fig. 8 The dependence of Ω on (a) $\hat{s}$ for ϵ_n = 0.2, and (b) ϵ_n for $\hat{s}$ = 0.5. The fixed parameters are β = 0.02, b_θ = 0.1, τ = 1.0, q = 2.0, and η_e = η_i = 1.0.